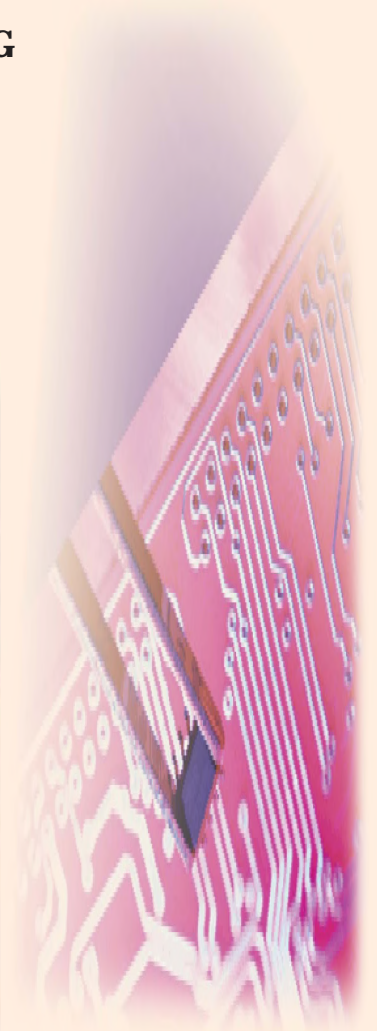
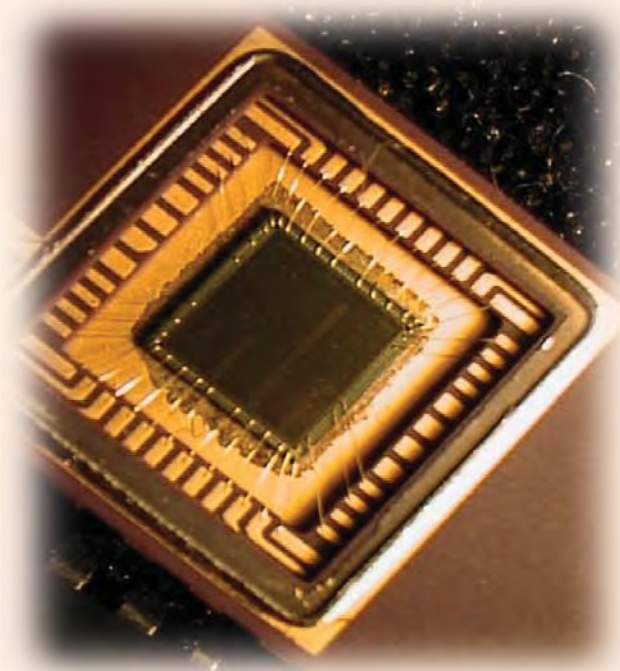
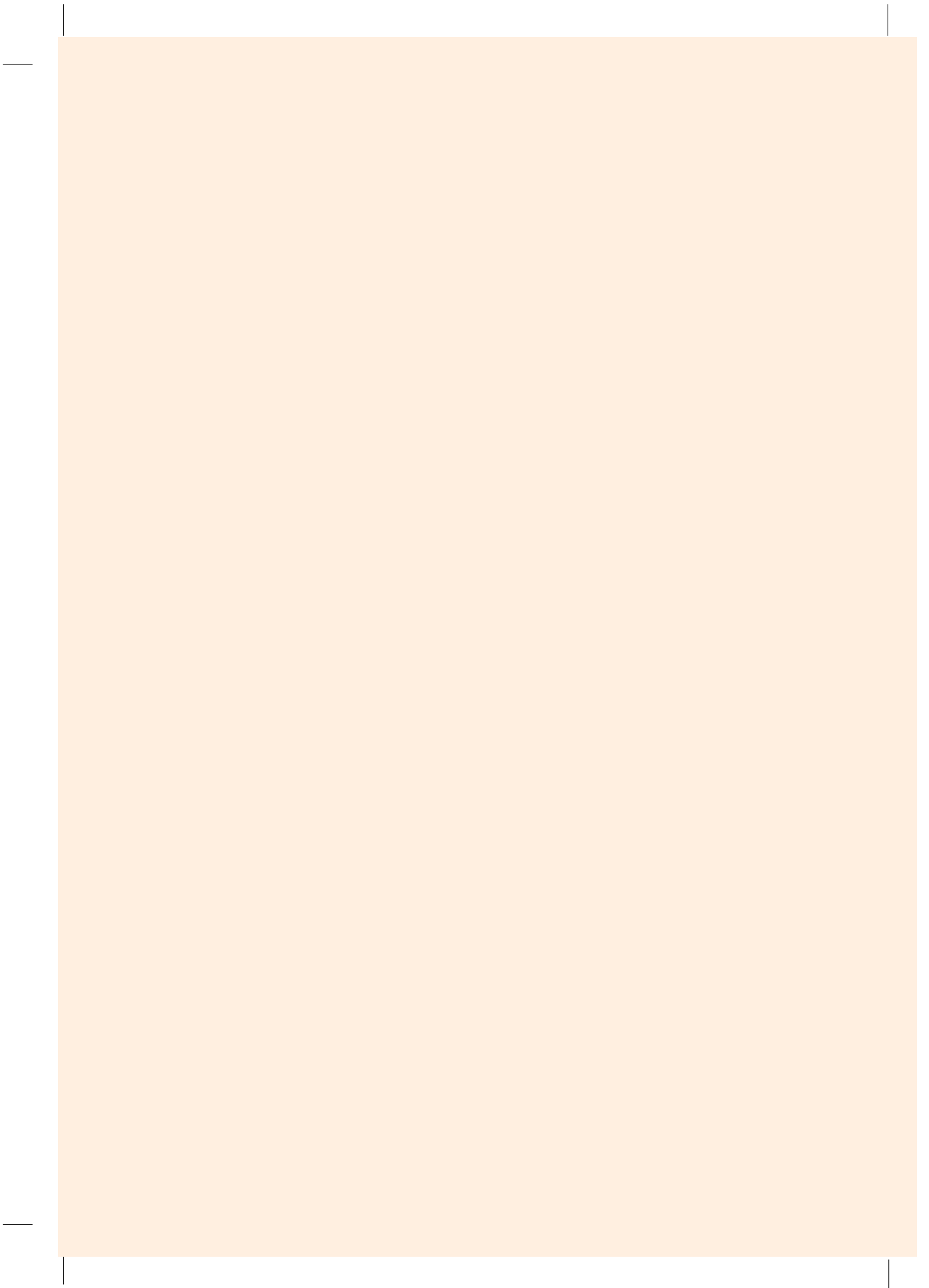


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**A TEXTBOOK OF**  
**ELECTRICAL**  
**TECHNOLOGY**

**VOLUME I**  
**BASIC ELECTRICAL ENGINEERING**  
**IN**  
**S.I. SYSTEM OF UNITS**





# A TEXTBOOK OF ELECTRICAL TECHNOLOGY

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## Preface to the Twenty-Third Revised Multicoloured Edition

**A**uthors feel happy to present to their esteemed readers this revised first **multicoloured** edition of Vol. I of “A Textbook of Electrical Technology”. To provide a comprehensive treatment of topics in “Basic Electrical Engineering” both for electrical as well as non-electrical students pursuing their studies in civil, mechanical, mining, textile, chemical, industrial, environmental, aerospace, electronic and computer engineering, information technology both at the Degree and Diploma level. Based on the suggestions received from our esteemed readers, both from India and abroad, the scope of the book has been enlarged according to their requirements.

Establishment of Technological Universities have taken place in recent past. This resulted into a pool of expert manpower within a large area. Unification of syllabi has taken place and the question papers set during the last 4-5 years have a wider variety and are of more inquisitive nature. Solutions to these with brief logical reasonings have been added for the benefit of our student community.

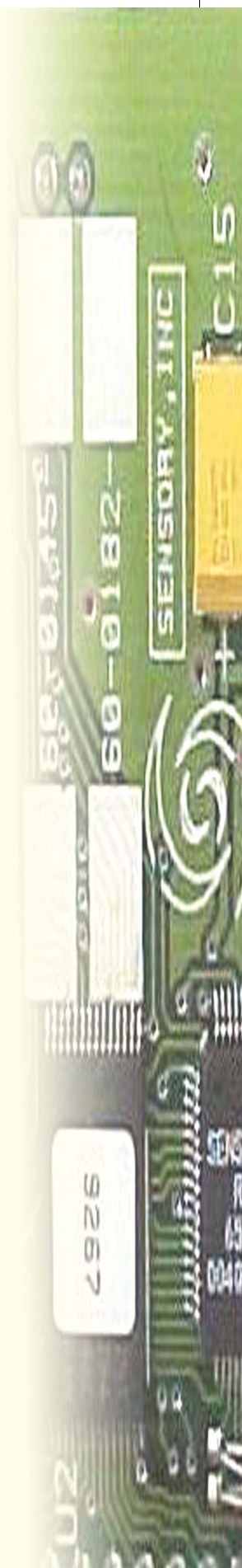
Many universities include a brief coverage on methods of “Electrical Power Generation”, in their first and basic paper on this subject. Hence, this revision includes an introductory chapter on this topic.

It is earnestly hoped that with these extensive additions and revisions, this revised edition will prove even more useful to our numerous readers in developing more confidence while appearing at national competitive examinations.

I would like to thank my Publishers particularly Mr. Ravindra Kumar Gupta, M.D. and Mr. Bhagirath Kaushik, Regional Manager (Western India) of S. Chand & Company Ltd., for the personal interest they look in the publishing of this revised and enlarged edition.

Our student-friends, teacher-colleagues, Booksellers and University authorities have been showing immense faith and affection in our book, which is acknowledged with modesty and regards. We are sure that this revised edition will satisfy their needs to a still greater extent and serve its cause more effectively.

**S.G. TARNEKAR**



## Preface to the Twenty-second Edition

The primary objective of Vol. I of **A Textbook of Electrical Technology** is to provide a comprehensive treatment of topics in “Basic Electrical Engineering” both for electrical as well as non-electrical students pursuing their studies in civil, mechanical, mining, textile, chemical, industrial, environmental, aerospace, electronic and computer engineering both at the Degree and Diploma level. Based on the suggestions received from our esteemed readers, both from India and abroad, the scope of the book has been enlarged according to their requirements. Almost half the solved examples have been deleted and replaced by latest examination papers set upto 1994 in different engineering colleges and technical institutions in India and abroad.

Following major additions/changes have been made in the present edition :

1. Three new chapters entitled (a) *A.C. Network Analysis* (b) *A.C. Filter Networks* and (c) *Fourier Series* have been added thereby widening the scope of the book.

2. The chapter on *Network Theorems* has been updated with the addition of Millman’s Theorem (as applicable to voltage and current sources or both) and an article on Power Transfer Efficiency relating to Maximum Power Transfer Theorem.

3. The additions to the chapter on *Capacitors* include detailed articles on Transient Relations during Capacitor Charging and Discharging Cycles and also the Charging and Discharging of a Capacitor with Initial Charge.

4. Chapter on *Chemical Effects of Current* has been thoroughly revised with the inclusion of Electronic Battery Chargers, Static Uninterruptable Power Supply (UPS) Systems, High Temperature Batteries, Secondary Hybrid Cells, Fuel Cells and Aircraft and Submarine Batteries.

5. A detailed description of Thermocouple Ammeter has been added to the chapter on Electrical Instruments.

6. The chapter on *Series A.C. Circuits* has been enriched with many articles such as Determination of Upper and Lower Half-power Frequencies, Value of Edge Frequencies and Relation between Resonant Power and Off-resonance Power.

It is earnestly hoped that with these extensive additions and revisions, the present edition will prove even more useful to our numerous readers than the earlier ones.

As ever before, we are thankful to our publishers particularly Sh. Ravindra Kumar Gupta for the personal interest he took in the expeditious printing of this book and for the highly attractive cover design suggested by him. Our sincere thanks go to their hyperactive and result-oriented overseas manager for his globe-trotting efforts to popularise the book from one corner of the globe to the other. Lastly we would love to record our sincere thanks to two brilliant ladies; Mrs. Janaki Krishnan from ever-green fairy land of Kerala and Ms. Shweta Bhardwaj from the fast-paced city of Delhi for the secretarial support they provided us during the preparation of this book.

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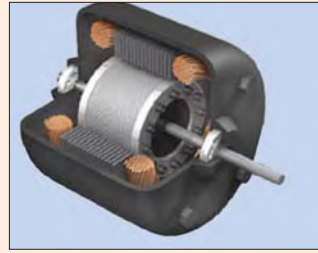


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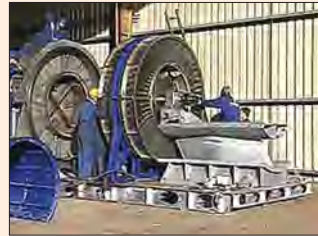
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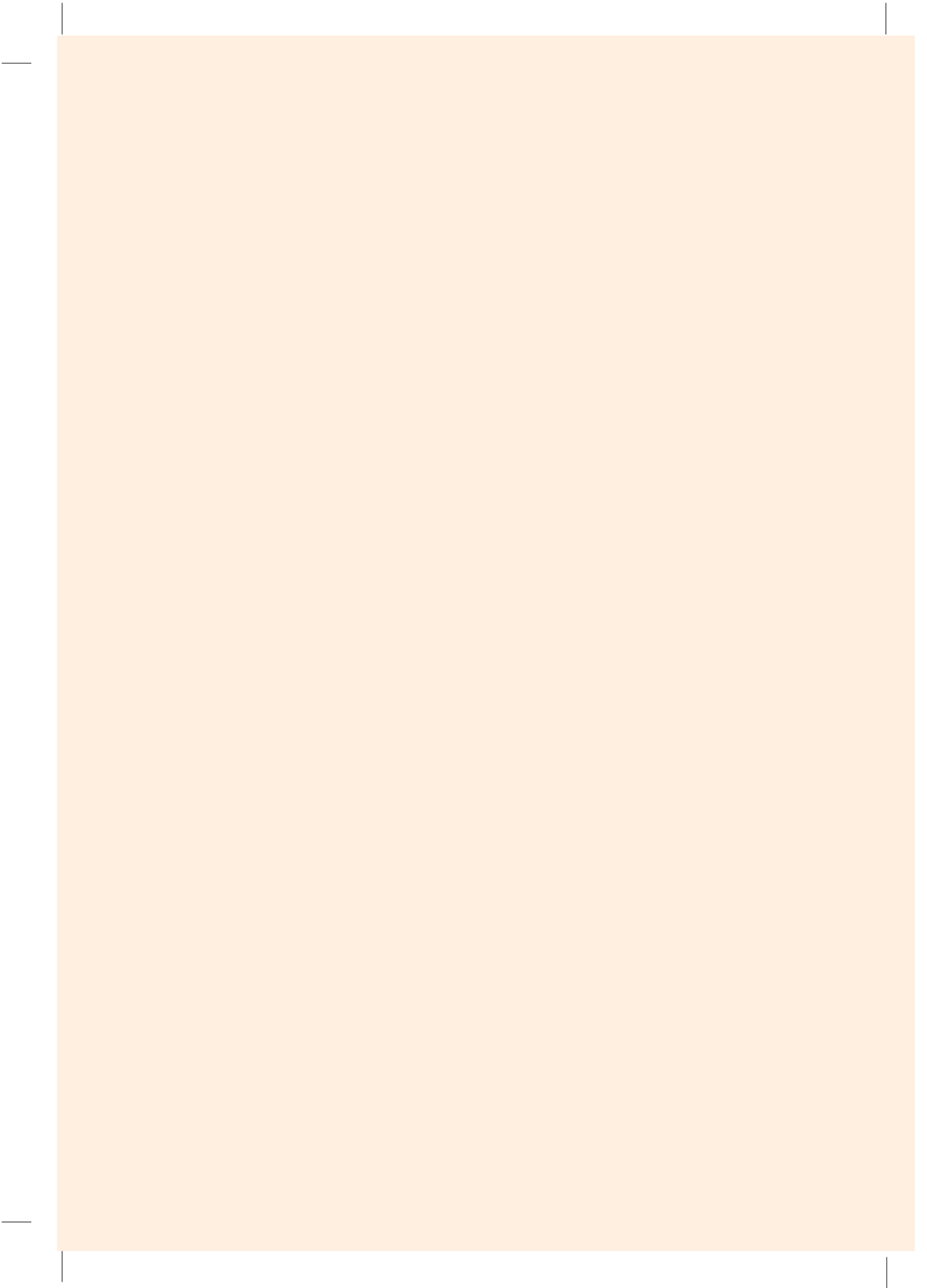
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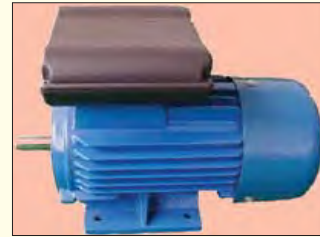
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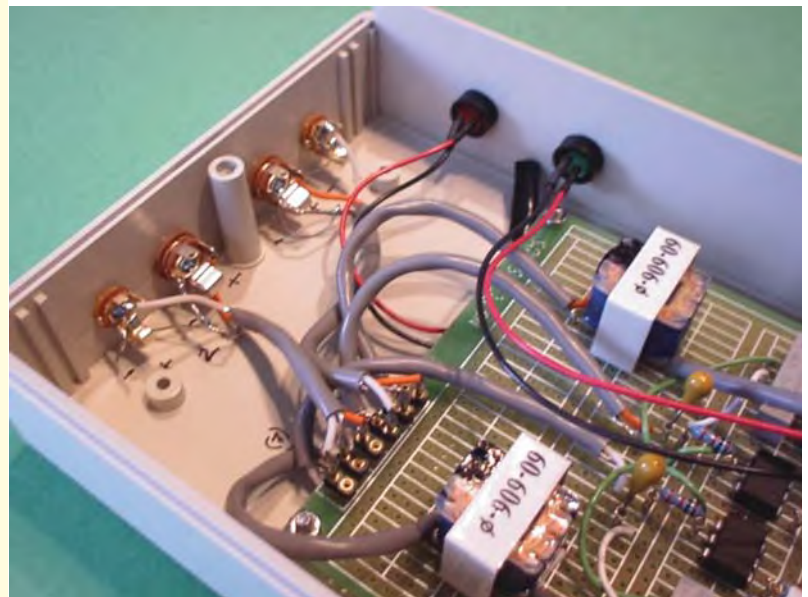
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# CHAPTER 1

## Learning Objectives

- Electron Drift Velocity
- Charge Velocity and Velocity of Field Propagation
- The Idea of Electric Potential Resistance
- Unit of Resistance
- Law of Resistance
- Units of Resistivity Conductance and Conductivity
- Temperature Coefficient of Resistance
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- Ohm's Law
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# ELECTRIC CURRENT AND OHM'S LAW



Ohm's law defines the relationship between voltage, resistance and current. This law is widely employed while designing electronic circuits

### 1.1. Electron Drift Velocity

Suppose that in a conductor, the number of free electrons available per  $\text{m}^3$  of the conductor material is  $n$  and let their axial drift velocity be  $v$  metres/second. In time  $dt$ , distance travelled would be  $v \times dt$ . If  $A$  is area of cross-section of the conductor, then the volume is  $vAdt$  and the number of electrons contained in this volume is  $vAdt$ . Obviously, all these electrons will cross the conductor cross-section in time  $dt$ . If  $e$  is the charge of each electron, then total charge which crosses the section in time  $dt$  is  $dq = nAev dt$ .

Since current is the rate of flow of charge, it is given as

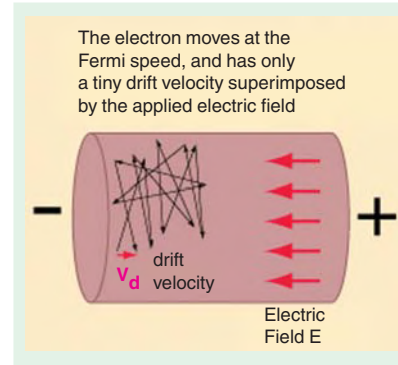
$$i = \frac{dq}{dt} = \frac{nAev dt}{dt} \quad \therefore i = nAev$$

Current density,  $J = i/A = nev$  ampere/metre<sup>2</sup>

Assuming a normal current density  $J = 1.55 \times 10^6 \text{ A/m}^2$ ,  $n = 10^{29}$  for a copper conductor and  $e = 1.6 \times 10^{-19}$  coulomb, we get

$$1.55 \times 10^6 = 10^{29} \times 1.6 \times 10^{-19} \times v \quad \therefore v = 9.7 \times 10^{-5} \text{ m/s} = 0.58 \text{ cm/min}$$

It is seen that contrary to the common but mistaken view, the electron drift velocity is rather very slow and is independent of the current flowing and the area of the conductor.



**N.B.** Current density *i.e.*, the current per unit area, is a vector quantity. It is denoted by the symbol  $\vec{J}$ .

Therefore, in vector notation, the relationship between current  $I$  and  $\vec{J}$  is :

$$I = \vec{J} \cdot \vec{a} \quad [\text{where } \vec{a} \text{ is the vector notation for area 'a'}]$$

For extending the scope of the above relationship, so that it becomes applicable for area of any shape, we write :

$$I = \int \vec{J} \cdot d\vec{a}$$

The magnitude of the current density can, therefore, be written as  $J \cos \alpha$ .

**Example 1.1.** A conductor material has a free-electron density of  $10^{24}$  electrons per metre<sup>3</sup>. When a voltage is applied, a constant drift velocity of  $1.5 \times 10^{-2}$  metre/second is attained by the electrons. If the cross-sectional area of the material is  $1 \text{ cm}^2$ , calculate the magnitude of the current. Electronic charge is  $1.6 \times 10^{-19}$  coulomb. **(Electrical Engg. Aligarh Muslim University)**

**Solution.** The magnitude of the current is

$$i = nAev \text{ amperes}$$

Here,

$$n = 10^{24}; A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

$$e = 1.6 \times 10^{-19} \text{ C}; v = 1.5 \times 10^{-2} \text{ m/s}$$

$\therefore$

$$i = 10^{24} \times 10^{-4} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-2} = \mathbf{0.24 \text{ A}}$$

### 1.2. Charge Velocity and Velocity of Field Propagation

The speed with which charge drifts in a conductor is called the **velocity of charge**. As seen from above, its value is quite low, typically fraction of a metre per second.

However, the *speed* with which the effect of e.m.f. is experienced at all parts of the conductor resulting in the flow of current is called the **velocity of propagation of electrical field**. It is independent of current and voltage and has high but constant value of nearly  $3 \times 10^8 \text{ m/s}$ .

**Example 1.2.** Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section  $1 \text{ cm}^2$  and length 10 km. Free electron density of copper =  $8.5 \times 10^{28}$  per  $\text{m}^3$ . How long will it take the electric charge to travel from one end of the conductor to the other?

**Solution.**  $i = neAv$  or  $v = i/neA$

$$\therefore v = 1/(8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}) = 7.35 \times 10^{-7} \text{ m/s} = \mathbf{0.735 \mu\text{m/s}}$$

Time taken by the charge to travel conductor length of 10 km is

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{10 \times 10^3}{7.35 \times 10^{-7}} = 1.36 \times 10^{10} \text{ s}$$

Now, 1 year =  $365 \times 24 \times 3600 = 31,536,000$  s

$$t = 1.36 \times 10^{10}/31,536,000 = \mathbf{431 \text{ years}}$$

### 1.3. The Idea of Electric Potential

In Fig. 1.1, a simple voltaic cell is shown. It consists of copper plate (known as anode) and a zinc rod (*i.e.* cathode) immersed in dilute sulphuric acid ( $\text{H}_2\text{SO}_4$ ) contained in a suitable vessel. The chemical action taking place within the cell causes the electrons to be removed from copper plate and to be deposited on the zinc rod at the same time. This transfer of electrons is accomplished through the agency of the diluted  $\text{H}_2\text{SO}_4$  which is known as the electrolyte. The result is that zinc rod becomes negative due to the deposition of electrons on it and the copper plate becomes positive due to the removal of electrons from it. The large number of electrons collected on the zinc rod is being attracted by anode but is prevented from returning to it by the force set up by the chemical action within the cell.

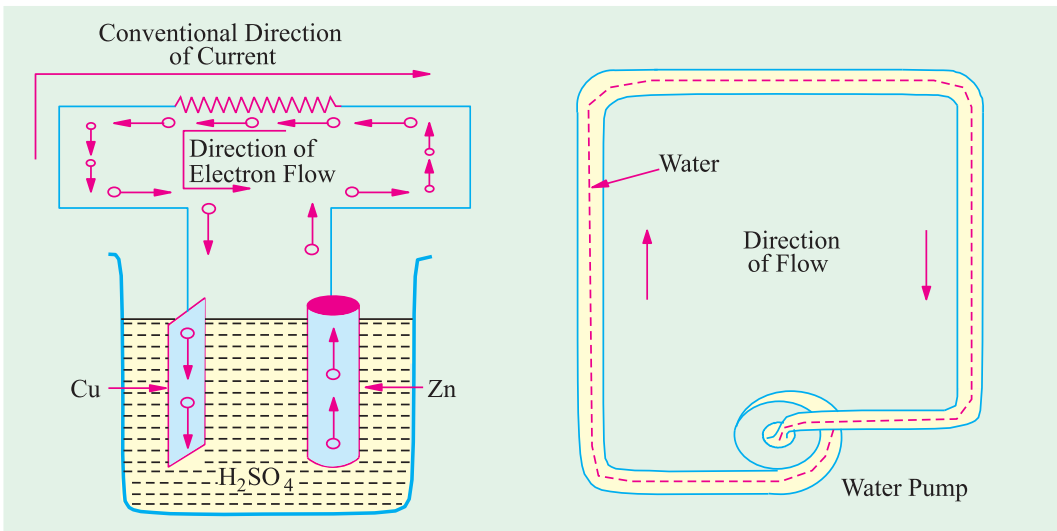


Fig. 1.1.

Fig. 1.2

But if the two electrodes are joined by a wire **externally**, then electrons rush to the anode thereby equalizing the charges of the two electrodes. However, due to the continuity of chemical action, a continuous difference in the number of electrons on the two electrodes is maintained which keeps up a continuous flow of current through the external circuit. The action of an electric cell is similar to that of a water pump which, while working, maintains a continuous flow of water *i.e.*, water current through the pipe (Fig. 1.2).

It should be particularly noted that the direction of **electronic** current is from zinc to copper in the external circuit. However, the direction of **conventional** current (which is given by the direction

of flow of positive charge) is from copper to zinc. In the present case, there is no flow of positive charge as such from one electrode to another. But we can look upon the arrival of electrons on copper plate (with subsequent decrease in its positive charge) as equivalent to an actual departure of positive charge from it.

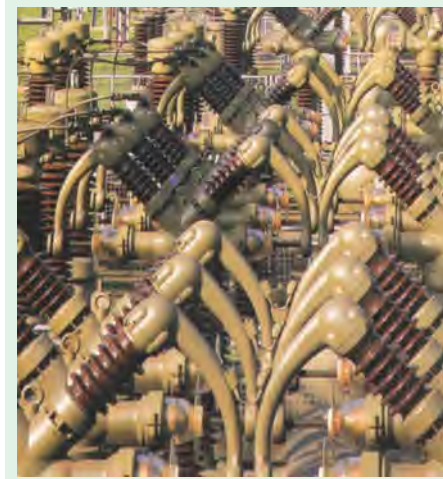
When zinc is negatively charged, it is said to be at negative potential with respect to the electrolyte, whereas anode is said to be at positive potential relative to the electrolyte. Between themselves, copper plate is assumed to be at a higher potential than the zinc rod. The difference in potential is continuously maintained by the chemical action going on in the cell which supplies energy to establish this potential difference.

### 1.4. Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (*i.e.*, electrons) through it.

Metals (as a class), acids and salts solutions are good conductors of electricity. Amongst pure metals, silver, copper and aluminium are very good conductors in the given order.\* This, as discussed earlier, is due to the presence of a large number of free or loosely-attached electrons in their atoms. These vagrant electrons assume a directed motion on the application of an electric potential difference. These electrons while flowing pass *through* the molecules or the atoms of the conductor, collide and other atoms and electrons, thereby producing heat.

Those substances which offer relatively greater difficulty or hindrance to the passage of these electrons are said to be relatively poor conductors of electricity like bakelite, mica, glass, rubber, p.v.c. (polyvinyl chloride) and dry wood etc. Amongst good insulators can be included fibrous substances such as paper and cotton when dry, mineral oils free from acids and water, ceramics like hard porcelain and asbestos and many other plastics besides p.v.c. It is helpful to remember that electric friction is similar to friction in Mechanics.



Cables are often covered with materials that do not carry electric current easily

### 1.5. The Unit of Resistance

The practical unit of resistance is ohm.\*\* A conductor is said to have a resistance of one ohm if it permits one ampere current to flow through it when one volt is impressed across its terminals.

For insulators whose resistances are very high, a much bigger unit is used *i.e.*, mega-ohm =  $10^6$  ohm (the prefix 'mega' or mego meaning a million) or kilo-ohm =  $10^3$  ohm (kilo means thousand). In the case of very small resistances, smaller units like milli-ohm =  $10^{-3}$  ohm or micro-ohm =  $10^{-6}$  ohm are used. The symbol for ohm is  $\Omega$



George Simon Ohm

\* However, for the same resistance per unit length, cross-sectional area of aluminium conductor has to be 1.6 times that of the copper conductor but it weighs only half as much. Hence, it is used where economy of weight is more important than economy of space.

\*\* After George Simon Ohm (1787-1854), a German mathematician who in about 1827 formulated the law known after his name as Ohm's Law.



Table 1.1. Multiples and Sub-multiples of Ohm

Prefix	Its meaning	Abbreviation	Equal to
Mega-	One million	M $\Omega$	$10^6 \Omega$
Kilo-	One thousand	k $\Omega$	$10^3 \Omega$
Centi-	One hundredth	–	–
Milli-	One thousandth	m $\Omega$	$10^{-3} \Omega$
Micro-	One millionth	$\mu \Omega$	$10^{-6} \Omega$

### 1.6. Laws of Resistance

The resistance  $R$  offered by a conductor depends on the following factors :

- (i) It varies directly as its length,  $l$ .
- (ii) It varies inversely as the cross-section  $A$  of the conductor.
- (iii) It depends on the nature of the material.
- (iv) It also depends on the temperature of the conductor.

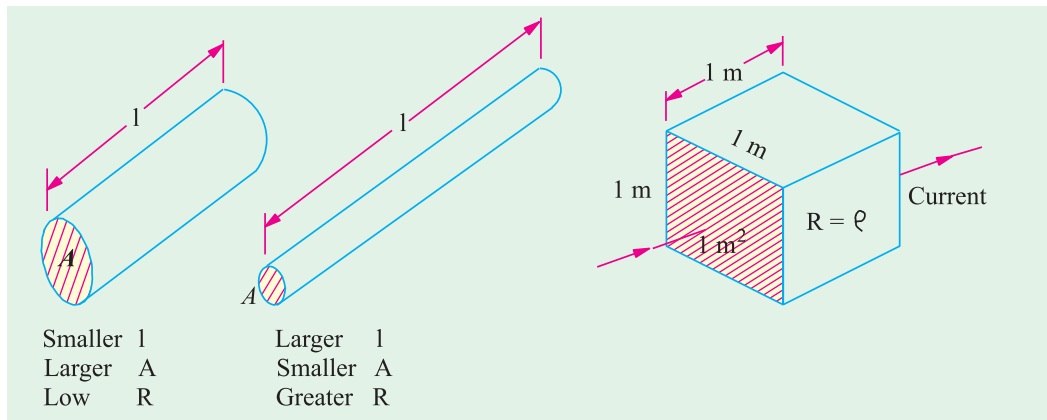


Fig. 1.3.

Fig. 1.4

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A} \quad \dots(i)$$

where  $\rho$  is a constant depending on the nature of the material of the conductor and is known as its **specific resistance or resistivity**.

If in Eq. (i), we put

$$l = 1 \text{ metre} \quad \text{and} \quad A = 1 \text{ metre}^2, \text{ then } R = \rho \quad (\text{Fig. 1.4})$$

Hence, specific resistance of a material may be defined as **the resistance between the opposite faces of a metre cube of that material**.

### 1.7. Units of Resistivity

From Eq. (i), we have

$$\rho = \frac{AR}{l}$$

In the S.I. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

Hence, the unit of resistivity is ohm-metre ( $\Omega\text{-m}$ ).

It may, however, be noted that resistivity is sometimes expressed as so many ohm per  $\text{m}^3$ . Although, it is incorrect to say so but it means the same thing as ohm-metre.

If  $l$  is in centimetres and  $A$  in  $\text{cm}^2$ , then  $\rho$  is in ohm-centimetre ( $\Omega\text{-cm}$ ).

Values of resistivity and temperature coefficients for various materials are given in Table 1.2. The resistivities of commercial materials may differ by several per cent due to impurities etc.

**Table 1.2. Resistivities and Temperature Coefficients**

<i>Material</i>	<i>Resistivity in ohm-metre at 20°C (<math>\times 10^8</math>)</i>	<i>Temperature coefficient at 20°C (<math>\times 10^4</math>)</i>
Aluminium, commercial	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–5
Constantan or Eureka	49	+0.1 to –0.4
Copper (annealed)	1.72	39.3
German Silver (84% Cu; 12% Ni; 4% Zn)	20.2	2.7
Gold	2.44	36.5
Iron	9.8	65
Manganin (84% Cu ; 12% Mn ; 4% Ni)	44 – 48	0.15
Mercury	95.8	8.9
Nichrome (60% Cu ; 25% Fe ; 15% Cr)	108.5	1.5
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	$5 \times 10^{14}$	
Bakelite	$10^{10}$	
Glass	$10^{10} - 10^{12}$	
Mica	$10^{15}$	
Rubber	$10^{16}$	
Shellac	$10^{14}$	
Sulphur	$10^{15}$	

**Example 1.3.** A coil consists of 2000 turns of copper wire having a cross-sectional area of  $0.8 \text{ mm}^2$ . The mean length per turn is 80 cm and the resistivity of copper is  $0.02 \mu\Omega\text{-m}$ . Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.

(F.Y. Engg. Pune Univ. May 1990)

**Solution.** Length of the coil,  $l = 0.8 \times 2000 = 1600 \text{ m}$  ;  
 $A = 0.8 \text{ mm}^2 = 0.8 \times 10^{-6} \text{ m}^2$ .

$$R = \rho \frac{l}{A} = 0.02 \times 10^{-6} \times 1600 / 0.8 \times 10^{-6} = \mathbf{40 \Omega}$$

$$\text{Power absorbed} = V^2 / R = 110^2 / 40 = \mathbf{302.5 \text{ W}}$$



**Example 1.4.** An aluminium wire 7.5 m long is connected in a parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is  $0.017 \mu\Omega\text{-m}$  ; that of the aluminium is  $0.028 \mu\Omega\text{-m}$ .

(F.Y. Engg. Pune Univ. May 1991)

**Solution.** Let the subscript 1 represent aluminium and subscript 2 represent copper.

$$R_1 = \rho \frac{l_1}{a_1} \text{ and } R_2 = \rho_2 \frac{l_2}{a_2} \quad \therefore \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{a_1}{a_2}$$

$$\therefore a_2 = a_1 \cdot \frac{R_1}{R_2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \quad \dots (i)$$

Now

$$I_1 = 3 \text{ A} ; I_2 = 5 - 3 = 2 \text{ A.}$$

If  $V$  is the common voltage across the parallel combination of aluminium and copper wires, then

$$V = I_1 R_1 = I_2 R_2 \quad \therefore R_1 / R_2 = I_2 / I_1 = 2/3$$

$$a_1 = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

Substituting the given values in Eq. (i), we get

$$a_2 = \frac{\pi}{4} \times \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} = 0.2544 \text{ m}^2$$

$$\therefore \pi \times d_2^2 / 4 = 0.2544 \quad \text{or} \quad d_2 = \mathbf{0.569 \text{ mm}}$$

**Example 1.5. (a)** A rectangular carbon block has dimensions  $1.0 \text{ cm} \times 1.0 \text{ cm} \times 50 \text{ cm}$ .  
**(i)** What is the resistance measured between the two square ends ? **(ii)** between two opposing rectangular faces / Resistivity of carbon at  $20^\circ\text{C}$  is  $3.5 \times 10^{-5} \Omega\text{-m}$ .

**(b)** A current of 5 A exists in a  $10\text{-}\Omega$  resistance for 4 minutes **(i)** how many coulombs and **(ii)** how many electrons pass through any section of the resistor in this time ? Charge of the electron =  $1.6 \times 10^{-19} \text{ C}$ .

(M.S. Univ. Baroda)

**Solution.**

**(a) (i)**

$$R = \rho l / A$$

Here,

$$A = 1 \times 1 = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2 ; l = 0.5 \text{ m}$$

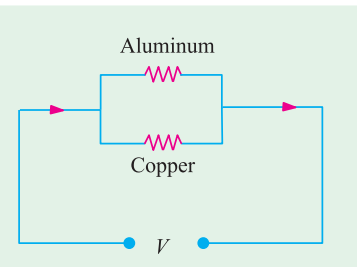
$\therefore$

$$R = 3.5 \times 10^{-5} \times 0.5 / 10^{-4} = \mathbf{0.175 \Omega}$$

**(ii)** Here,

$$l = 1 \text{ cm} ; A = 1 \times 50 = 50 \text{ cm}^2 = 5 \times 10^{-3} \text{ m}^2$$

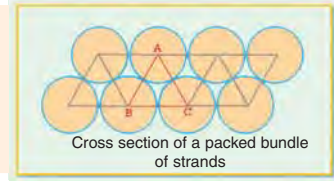
$$R = 3.5 \times 10^{-5} \times 10^2 / 5 \times 10^{-3} = \mathbf{7 \times 10^{-5} \Omega}$$



$$(b) (i) \quad Q = It = 5 \times (4 \times 60) = 1200 \text{ C}$$

$$(ii) \quad n = \frac{Q}{e} = \frac{1200}{1.6 \times 10^{-19}} = 75 \times 10^{20}$$

**Example 1.6.** Calculate the resistance of 1 km long cable composed of 19 strands of similar copper conductors, each strand being 1.32 mm in diameter. Allow 5% increase in length for the 'lay' (twist) of each strand in completed cable. Resistivity of copper may be taken as  $1.72 \times 10^{-8} \Omega\text{-m}$ .

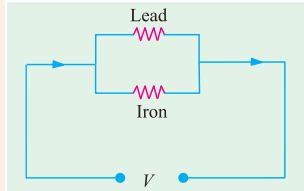


**Solution.** Allowing for twist, the length of the strands.  
 $= 1000 \text{ m} + 5\% \text{ of } 1000 \text{ m} = 1050 \text{ m}$

Area of cross-section of 19 strands of copper conductors is  
 $19 \times \pi \times d^2/4 = 19 \pi \times (1.32 \times 10^{-3})^2/4 \text{ m}^2$

$$\text{Now,} \quad R = \rho \frac{l}{A} = \frac{1.72 \times 10^{-8} \times 1050 \times 4}{19 \pi \times 1.32^2 \times 10^{-6}} = 0.694 \Omega$$

**Example 1.7.** A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 percent more current than the latter and the latter is 47 percent longer than the former. Determine the ratio of their cross sectional areas.



(Elect. Engg. Nagpur Univ. 1993)

**Solution.** Let suffix 1 represent lead and suffix 2 represent iron. We are given that

$$\rho_1/\rho_2 = 49/24; \text{ if } i_2 = 1, i_1 = 1.8; \text{ if } l_1 = 1, l_2 = 1.47$$

$$\text{Now,} \quad R_1 = \rho_1 \frac{l_1}{A_1} \quad \text{and} \quad R_2 = \rho_2 \frac{l_2}{A_2}$$

Since the two wires are in parallel,  $i_1 = V/R_1$  and  $i_2 = V/R_2$

$$\therefore \quad \frac{i_2}{i_1} = \frac{R_1}{R_2} = \frac{\rho_1 l_1}{\rho_2 l_2} \times \frac{A_2}{A_1}$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{i_2}{i_1} \times \frac{\rho_2 l_2}{\rho_1 l_1} = \frac{1}{1.8} \times \frac{24}{49} \times 1.47 = 0.4$$

**Example 1.8.** A piece of silver wire has a resistance of 1  $\Omega$ . What will be the resistance of manganin wire of one-third the length and one-third the diameter, if the specific resistance of manganin is 30 times that of silver.

(Electrical Engineering-I, Delhi Univ.)

**Solution.** For silver wire,  $R_1 = \frac{l_1}{A_1}$ ; For manganin wire,  $R = \rho_2 \frac{l_2}{A_2}$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2 \times l_2 \times A_1}{\rho_1 \times l_1 \times A_2}$$

$$\text{Now} \quad A_1 = \pi d_1^2/4 \quad \text{and} \quad A_2 = \pi d_2^2/4 \quad \therefore A_1/A_2 = d_1^2/d_2^2$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2 \times l_2 \times \left(\frac{d_1}{d_2}\right)^2}{\rho_1 \times l_1}$$

$$R_1 = 1 \Omega; l_2/l_1 = 1/3, (d_1/d_2)^2 = (3/1)^2 = 9; \rho_2/\rho_1 = 30$$

$$\therefore \quad R_2 = 1 \times 30 \times (1/3) \times 9 = 90 \Omega$$

**Example 1.9.** The resistivity of a ferric-chromium-aluminium alloy is  $51 \times 10^{-8} \Omega\text{-m}$ . A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between (a) opposite ends and (b) opposite sides. (Electric Circuits, Allahabad Univ.)

**Solution.** (a) As seen from Fig. 1.5 (a) in this case,

$$l = 15 \text{ cm} = 0.15 \text{ m}$$

$$A = 6 \times 0.014 = 0.084 \text{ cm}^2 \\ = 0.084 \times 10^{-4} \text{ m}^2$$

$$R = \rho \frac{l}{A} = \frac{51 \times 10^{-8} \times 0.15}{0.084 \times 10^{-4}} \\ = 9.1 \times 10^{-3} \Omega$$

(b) As seen from Fig. 1.5 (b) here

$$l = 0.014 \text{ cm} = 14 \times 10^{-5} \text{ m}$$

$$A = 15 \times 6 = 90 \text{ cm}^2 = 9 \times 10^{-3} \text{ m}^2$$

$$\therefore R = 51 \times 10^{-8} \times 14 \times 10^{-5} / 9 \times 10^{-3} = 79.3 \times 10^{-10} \Omega$$

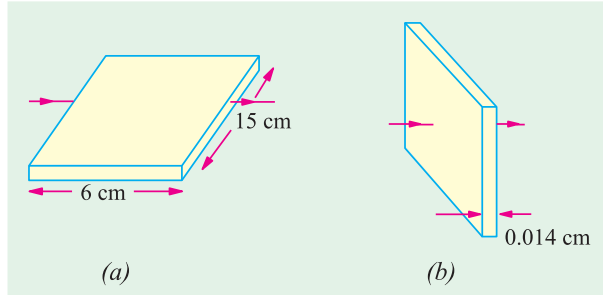


Fig. 1.5

**Example 1.10.** The resistance of the wire used for telephone is 35  $\Omega$  per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is  $1.95 \times 10^{-8} \Omega\text{-m}$ , what is the cross-sectional area of the wire? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used?

**Solution.** Here  $R = 35 \Omega$ ,  $l = 1 \text{ km} = 1000 \text{ m}$ ;  $\rho = 1.95 \times 10^{-8} \Omega\text{-m}$

$$\text{Now, } R = \rho \frac{l}{A} \text{ or } A = \frac{\rho l}{R} \therefore A = \frac{1.95 \times 10^{-8} \times 1000}{35} = 55.7 \times 10^{-8} \text{ m}^2$$

If the second case, if the wire is of the material but weighs 20 kg/km, then its cross-section must be greater than that in the first case.

$$\text{Cross-section in the second case} = \frac{20}{5} \times 55.7 \times 10^{-8} = 222.8 \times 10^{-8} \text{ m}^2$$

$$\text{Length of wire} = 2 \times 8 = 16 \text{ km} = 16000 \text{ m} \therefore R = \rho \frac{l}{A} = \frac{1.95 \times 10^{-8} \times 16000}{222.8 \times 10^{-8}} = 140.1 \Omega$$

### Tutorial Problems No. 1.1

- Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of  $0.1 \text{ mm}^2$  if the wire is made of manganin having a resistivity of  $50 \times 10^{-8} \Omega\text{-m}$ .  
If the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased? [500  $\Omega$ ; 9 times]
- A cube of a material of side 1 cm has a resistance of  $0.001 \Omega$  between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length? [0.064  $\Omega$ ]
- A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 per cent more current than the latter and the latter is 47 per cent longer than the former. Determine the ratio of their cross-sectional area. [2.5 : 1]
- A rectangular metal strip has the following dimensions :  
 $x = 10 \text{ cm}$ ,  $y = 0.5 \text{ cm}$ ,  $z = 0.2 \text{ cm}$   
Determine the ratio of resistances  $R_x$ ,  $R_y$ , and  $R_z$  between the respective pairs of opposite faces.  
[ $R_x : R_y : R_z : 10,000 : 25 : 4$ ] (Elect. Engg. A.M.Ae. S.I.)
- The resistance of a conductor  $1 \text{ mm}^2$  in cross-section and 20 m long is  $0.346 \Omega$ . Determine the specific resistance of the conducting material. [ $1.73 \times 10^{-8} \Omega\text{-m}$ ] (Elect. Circuits-1, Bangalore Univ. 1991)
- When a current of 2 A flows for 3 micro-seconds in a copper wire, estimate the number of electrons crossing the cross-section of the wire. (Bombay University, 2000)

**Hint :** With 2 A for 3  $\mu$  Sec, charge transferred = 6  $\mu$ -coulombs

$$\text{Number of electrons crossed} = 6 \times 10^{-6} / (1.6 \times 10^{-19}) = 3.75 \times 10^{13}$$

### 1.8. Conductance and Conductivity

Conductance ( $G$ ) is reciprocal of resistance\*. Whereas resistance of a conductor measures the **opposition** which it offers to the flow of current, the conductance measures the **inducement** which it offers to its flow.

$$\text{From Eq. (i) of Art. 1.6, } R = \rho \frac{l}{A} \text{ or } G = \frac{1}{\rho} \cdot \frac{A}{l} = \frac{\sigma A}{l}$$

where  $\sigma$  is called the **conductivity or specific conductance** of a conductor. The unit of conductance is siemens (S). Earlier, this unit was called mho.

It is seen from the above equation that the conductivity of a material is given by

$$\sigma = G \frac{l}{A} = \frac{G \text{ siemens} \times l \text{ metre}}{A \text{ metre}^2} = G \frac{l}{A} \text{ siemens/metre}$$

Hence, the unit of conductivity is siemens/metre (S/m).

### 1.9. Effect of Temperature on Resistance

The effect of rise in temperature is :

- (i) to **increase** the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line (Fig. 1.6). As would be presently clarified, metals have a positive temperature co-efficient of resistance.
- (ii) to **increase** the resistance of alloys, though in their case, the increase is relatively small and irregular. For some high-resistance alloys like Eureka (60% Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.
- (iii) to **decrease** the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a **negative** temperature-coefficient of resistance.

### 1.10. Temperature Coefficient of Resistance

Let a metallic conductor having a resistance of  $R_0$  at  $0^\circ\text{C}$  be heated of  $t^\circ\text{C}$  and let its resistance at this temperature be  $R_t$ . Then, considering normal ranges of temperature, it is found that the increase in resistance  $\Delta R = R_t - R_0$  depends

- (i) directly on its initial resistance
- (ii) directly on the rise in temperature
- (iii) on the nature of the material of the conductor.

$$\text{or } R_t - R_0 \propto R_0 \times t \text{ or } R_t - R_0 = \alpha R_0 t \quad \dots(i)$$

where  $\alpha$  (alpha) is a constant and is known as the *temperature coefficient of resistance* of the conductor.

$$\text{Rearranging Eq. (i), we get } \alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\Delta R}{R_0 \times t}$$

$$\text{If } R_0 = 1 \Omega, t = 1^\circ\text{C, then } \alpha = \Delta R = R_t - R_0$$

Hence, the temperature-coefficient of a material may be defined as :

**the increase in resistance per ohm original resistance per  $^\circ\text{C}$  rise in temperature.**

$$\text{From Eq. (i), we find that } R_t = R_0 (1 + \alpha t) \quad \dots(ii)$$

\* In a.c. circuits, it has a slightly different meaning.

It should be remembered that the above equation holds good for both rise as well as fall in temperature. As temperature of a conductor is decreased, its resistance is also decreased. In Fig. 1.6 is shown the temperature/resistance graph for copper and is practically a straight line. If this line is extended backwards, it would cut the temperature axis at a point where temperature is  $-234.5^{\circ}\text{C}$  (a number quite easy to remember). It means that theoretically, the resistance of copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 1.6 it is seen that :

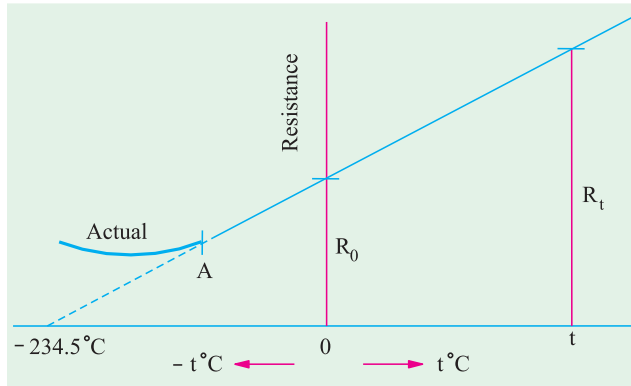


Fig. 1.6

$$\frac{R_t}{R_0} = \frac{t + 234.5}{234.5} = \left(1 + \frac{t}{234.5}\right)$$

$$\therefore R_t = R_0 \left(1 + \frac{t}{234.5}\right) \text{ or } R_t = R_0 (1 + \alpha t) \text{ where } \alpha = 1/234.5 \text{ for copper.}$$

### 1.1.1. Value of $\alpha$ at Different Temperatures

So far we did not make any distinction between values of  $\alpha$  at different temperatures. But it is found that value of  $\alpha$  itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at  $0^{\circ}\text{C}$ , then  $\alpha$  has the value of  $\alpha_0$ . At any other initial temperature  $t^{\circ}\text{C}$ , value of  $\alpha$  is  $\alpha_t$  and so on. It should be remembered that, for any conductor,  $\alpha_0$  has the maximum value.

Suppose a conductor of resistance  $R_0$  at  $0^{\circ}\text{C}$  (point A in Fig. 1.7) is heated to  $t^{\circ}\text{C}$  (point B). Its resistance  $R_t$  after heating is given by

$$R_t = R_0 (1 + \alpha_0 t) \quad \dots(i)$$

where  $\alpha_0$  is the temperature-coefficient at  $0^{\circ}\text{C}$ .

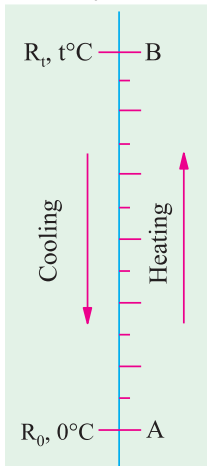


Fig. 1.7

Now, suppose that we have a conductor of resistance  $R_t$  at temperature  $t^{\circ}\text{C}$ . Let this conductor be **cooled** from  $t^{\circ}\text{C}$  to  $0^{\circ}\text{C}$ . Obviously, now the initial point is B and the final point is A. The final resistance  $R_0$  is given in terms of the initial resistance by the following equation

$$R_0 = R_t [1 + \alpha_t (-t)] = R_t (1 - \alpha_t \cdot t) \quad \dots(ii)$$

From Eq. (ii) above, we have  $\alpha_t = \frac{R_t - R_0}{R_t \times t}$

Substituting the value of  $R_t$  from Eq. (i), we get

$$\alpha_t = \frac{R_0 (1 + \alpha_0 t) - R_0}{R_0 (1 + \alpha_0 t) \times t} = \frac{\alpha_0}{1 + \alpha_0 t} \therefore \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t} \quad \dots(iii)$$

In general, let  $\alpha_1 =$  tempt. coeff. at  $t_1^{\circ}\text{C}$  ;  $\alpha_2 =$  tempt. coeff. at  $t_2^{\circ}\text{C}$ . Then from Eq. (iii) above, we get

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \quad \text{or} \quad \frac{1}{\alpha_1} = \frac{1 + \alpha_0 t_1}{\alpha_0}$$

Similarly,

$$\frac{1}{\alpha_2} = \frac{1 + \alpha_0 t_2}{\alpha_0}$$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) \quad \text{or} \quad \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) \quad \text{or} \quad \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

Values of  $\alpha$  for copper at different temperatures are given in Table No. 1.3.

**Table 1.3. Different values of  $\alpha$  for copper**

Tempt. in °C	0	5	10	20	30	40	50
$\alpha$	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

In view of the dependence of  $\alpha$  on the initial temperature, we may define **the temperature coefficient of resistance at a given temperature as the change in resistance per ohm per degree centigrade change in temperature from the given temperature.**

In case  $R_0$  is not given, the relation between the known resistance  $R_1$  at  $t_1^\circ\text{C}$  and the unknown resistance  $R_2$  at  $t_2^\circ\text{C}$  can be found as follows :

$$R_2 = R_0 (1 + \alpha_0 t_2) \quad \text{and} \quad R_1 = R_0 (1 + \alpha_0 t_1)$$

$$\therefore \frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1} \quad \dots (iv)$$

The above expression can be simplified by a little approximation as follows :

$$\frac{R_2}{R_1} = (1 + \alpha_0 t_2) (1 + \alpha_0 t_1)^{-1}$$

$$= (1 + \alpha_0 t_2) (1 - \alpha_0 t_1)$$

$$= 1 + \alpha_0 (t_2 - t_1)$$

[Using Binomial Theorem for expansion and neglecting squares and higher powers of  $(\alpha_0 t_1)$ ]

$$\therefore R_2 = R_1 [1 + \alpha_0 (t_2 - t_1)] \quad \text{[Neglecting product } (\alpha_0^2 t_1 t_2)]$$

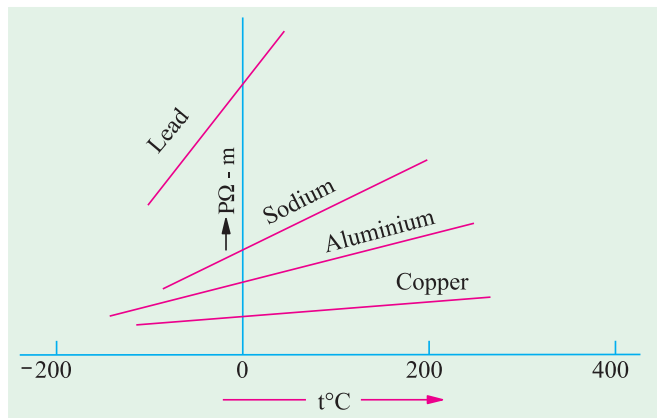
For more accurate calculations, Eq. (iv) should, however, be used.

## 1.12. Variations of Resistivity with Temperature

Not only resistance but specific resistance or resistivity of metallic conductors also increases with rise in temperature and **vice-versa**.

As seen from Fig. 1.8 the resistivities of metals vary linearly with temperature over a significant range of temperature—the variation becoming non-linear both at very high and at very low temperatures. Let, for any metallic conductor,

$$\rho_1 = \text{resistivity at } t_1^\circ\text{C}$$



**Fig. 1.8**



$\rho_2$  = resistivity at  $t_2^\circ\text{C}$

$m$  = Slope of the linear part of the curve

Then, it is seen that

$$m = \frac{\rho_2 - \rho_1}{t_2 - t_1}$$

$$\text{or } \rho_2 = \rho_1 + m(t_2 - t_1) \quad \text{or} \quad \rho_2 = \rho_1 \left[ 1 + \frac{m}{\rho_1}(t_2 - t_1) \right]$$

The ratio of  $m/\rho_1$  is called the **temperature coefficient of resistivity** at temperature  $t_1^\circ\text{C}$ . It may be defined as numerically equal to the fractional change in  $\rho_1$  per  $^\circ\text{C}$  change in the temperature from  $t_1^\circ\text{C}$ . It is almost equal to the temperature-coefficient of resistance  $\alpha_1$ . Hence, putting  $\alpha_1 = m/\rho_1$ , we get

$$\rho_2 = \rho_1 [1 + \alpha_1(t_2 - t_1)] \quad \text{or} \quad \text{simply as } \rho_t = \rho_0(1 + \alpha_0 t)$$

**Note.** It has been found that although temperature is the most significant factor influencing the resistivity of metals, other factors like pressure and tension also affect resistivity to some extent. For most metals except lithium and calcium, increase in pressure leads to decrease in resistivity. However, resistivity increases with increase in tension.

**Example 1.11.** A copper conductor has its specific resistance of  $1.6 \times 10^{-6}$  ohm-cm at  $0^\circ\text{C}$  and a resistance temperature coefficient of  $1/254.5$  per  $^\circ\text{C}$  at  $20^\circ\text{C}$ . Find (i) the specific resistance and (ii) the resistance - temperature coefficient at  $60^\circ\text{C}$ . (F.Y. Engg. Pune Univ. Nov.)

**Solution.**  $\alpha_{20} = \frac{\alpha_0}{1 + \alpha_0 \times 20}$  or  $\frac{1}{254.5} = \frac{\alpha_0}{1 + \alpha_0 \times 20} \quad \therefore \alpha_0 = \frac{1}{234.5}$  per  $^\circ\text{C}$

(i)  $\rho_{60} = \rho_0(1 + \alpha_0 \times 60) = 1.6 \times 10^{-6} (1 + 60/234.5) = 2.01 \times 10^{-6} \Omega\text{-cm}$

(ii)  $\alpha_{60} = \frac{\alpha_0}{1 + \alpha_0 \times 60} = \frac{1/234.5}{1 + (60/234.5)} = \frac{1}{294.5}$  per  $^\circ\text{C}$

**Example 1.12.** A platinum coil has a resistance of  $3.146 \Omega$  at  $40^\circ\text{C}$  and  $3.767 \Omega$  at  $100^\circ\text{C}$ . Find the resistance at  $0^\circ\text{C}$  and the temperature-coefficient of resistance at  $40^\circ\text{C}$ .

(Electrical Science-II, Allahabad Univ.)

**Solution.**  $R_{100} = R_0(1 + 100 \alpha_0)$  ... (i)

$R_{40} = R_0(1 + 40 \alpha_0)$  ... (ii)

$\therefore \frac{3.767}{3.146} = \frac{1 + 100 \alpha_0}{1 + 40 \alpha_0}$  or  $\alpha_0 = 0.00379$  or  $1/264$  per  $^\circ\text{C}$

From (i), we have  $3.767 = R_0(1 + 100 \times 0.00379) \quad \therefore R_0 = 2.732 \Omega$

Now,  $\alpha_{40} = \frac{\alpha_0}{1 + 40 \alpha_0} = \frac{0.00379}{1 + 40 \times 0.00379} = \frac{1}{304}$  per  $^\circ\text{C}$

**Example 1.13.** A potential difference of  $250 \text{ V}$  is applied to a field winding at  $15^\circ\text{C}$  and the current is  $5 \text{ A}$ . What will be the mean temperature of the winding when current has fallen to  $3.91 \text{ A}$ , applied voltage being constant. Assume  $\alpha_{15} = 1/254.5$ . (Elect. Engg. Pune Univ.)

**Solution.** Let  $R_1$  = winding resistance at  $15^\circ\text{C}$ ;  $R_2$  = winding resistance at unknown mean temperature  $t_2^\circ\text{C}$ .

$\therefore R_1 = 250/5 = 50 \Omega$ ,  $R_2 = 250/3.91 = 63.94 \Omega$

Now  $R_2 = R_1 [1 + \alpha_{15}(t_2 - t_1)] \quad \therefore 63.94 = 50 \left[ 1 + \frac{1}{254.5}(t_2 - 15) \right]$

$\therefore t_2 = 86^\circ\text{C}$

**Example 1.14.** Two coils connected in series have resistances of  $600\ \Omega$  and  $300\ \Omega$  with temp. coeff. of  $0.1\%$  and  $0.4\%$  respectively at  $20^\circ\text{C}$ . Find the resistance of the combination at a temp. of  $50^\circ\text{C}$ . What is the effective temp. coeff. of combination ?

**Solution.** Resistance of  $600\ \Omega$  resistor at  $50^\circ\text{C}$  is  $= 600 [1 + 0.001 (50 - 20)] = 618\ \Omega$

Similarly, resistance of  $300\ \Omega$  resistor at  $50^\circ\text{C}$  is  $= 300 [1 + 0.004 (50 - 20)] = 336\ \Omega$

Hence, total resistance of combination at  $50^\circ\text{C}$  is  $= 618 + 336 = 954\ \Omega$

Let  $\beta$  = resistance-temperature coefficient at  $20^\circ\text{C}$

Now, combination resistance at  $20^\circ\text{C} = 900\ \Omega$

Combination resistance at  $50^\circ\text{C} = 954\ \Omega$

$\therefore 954 = 900 [1 + \beta (50 - 20)] \therefore \beta = 0.002$

**Example 1.15.** Two wires  $A$  and  $B$  are connected in series at  $0^\circ\text{C}$  and resistance of  $B$  is 3.5 times that of  $A$ . The resistance temperature coefficient of  $A$  is  $0.4\%$  and that of the combination is  $0.1\%$ . Find the resistance temperature coefficient of  $B$ . (Elect. Technology, Hyderabad Univ.)

**Solution.** A simple technique which gives quick results in such questions is illustrated by the

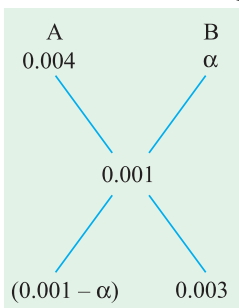


Fig. 1.9

diagram of Fig. 1.9. It is seen that  $R_B/R_A = 0.003/(0.001 - \alpha)$

or  $3.5 = 0.003/(0.001 - \alpha)$

or  $\alpha = 0.000143^\circ\text{C}^{-1}$  or  $0.0143\%$

**Example 1.16.** Two materials  $A$  and  $B$  have resistance temperature coefficients of  $0.004$  and  $0.0004$  respectively at a given temperature. In what proportion must  $A$  and  $B$  be joined in series to produce a circuit having a temperature coefficient of  $0.001$  ? (Elect. Technology, Indore Univ.)

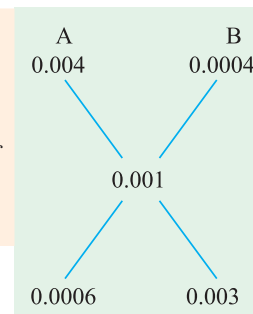


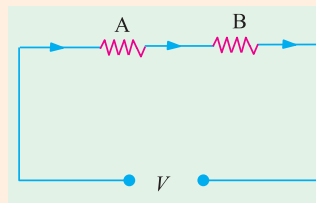
Fig. 1.10

**Solution.** Let  $R_A$  and  $R_B$  be the resistances of the two wires of materials  $A$  and  $B$  which are to be connected in series. Their ratio may be found by the simple technique shown in Fig. 1.10.

$$\frac{R_B}{R_A} = \frac{0.003}{0.0006} = 5$$

Hence,  $R_B$  must be 5 times  $R_A$ .

**Example 1.17.** A resistor of  $80\ \Omega$  resistance, having a temperature coefficient of  $0.0021$  per degree  $C$  is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material  $A$ , the resistance is  $80\ \text{ohm per } 100\ \text{metres}$  and the temperature coefficient is  $0.003$  per degree  $C$ . For material  $B$ , the corresponding figures are  $60\ \text{ohm per metre}$  and  $0.0015$  per degree  $C$ . Calculate suitable lengths of wires of materials  $A$  and  $B$  to be connected in series to construct the required resistor. All data are referred to the same temperature.



**Solution.** Let  $R_a$  and  $R_b$  be the resistances of suitable lengths of materials  $A$  and  $B$  respectively which when joined in series will have a combined temperature coeff. of  $0.0021$ . Hence, combination resistance at any given temperature is  $(R_a + R_b)$ . Suppose we heat these materials through  $t^\circ\text{C}$ .

When heated, resistance of  $A$  increases from  $R_a$  to  $R_a (1 + 0.003 t)$ . Similarly, resistance of  $B$  increases from  $R_b$  to  $R_b (1 + 0.0015 t)$ .

$$\begin{aligned} \therefore \text{combination resistance after being heated through } t^\circ\text{C} \\ = R_a (1 + 0.003 t) + R_b (1 + 0.0015 t) \end{aligned}$$

The combination  $\alpha$  being given, value of combination resistance can be also found directly as

$$\begin{aligned} &= (R_a + R_b) (1 + 0.0021 t) \\ \therefore (R_a + R_b) (1 + 0.0021 t) &= R_a (1 + 0.003 t) + R_b (1 + 0.0015 t) \end{aligned}$$

$$\text{Simplifying the above, we get } \frac{R_b}{R_a} = \frac{3}{2} \quad \dots(i)$$

$$\text{Now } R_a + R_b = 80 \, \Omega \quad \dots(ii)$$

Substituting the value of  $R_b$  from (i) into (ii) we get

$$R_a + \frac{3}{2} R_a = 80 \quad \text{or } R_a = 32 \, \Omega \quad \text{and } R_b = 48 \, \Omega$$

If  $L_a$  and  $L_b$  are the required lengths in metres, then

$$L_a = (100/80) \times 32 = 40 \, \text{m} \quad \text{and } L_b = (100/60) \times 48 = 80 \, \text{m}$$

**Example 1.18.** A coil has a resistance of  $18 \, \Omega$  when its mean temperature is  $20^\circ\text{C}$  and of  $20 \, \Omega$  when its mean temperature is  $50^\circ\text{C}$ . Find its mean temperature rise when its resistance is  $21 \, \Omega$  and the surrounding temperature is  $15^\circ\text{C}$ . (Elect. Technology, Allahabad Univ.)

**Solution.** Let  $R_0$  be the resistance of the coil and  $\alpha_0$  its temp. coefficient at  $0^\circ\text{C}$ .

$$\text{Then, } 18 = R_0 (1 + \alpha_0 \times 20) \quad \text{and } 20 = R_0 (1 + 50 \alpha_0)$$

Dividing one by the other, we get

$$\frac{20}{18} = \frac{1 + 50 \alpha_0}{1 + 20 \alpha_0} \quad \therefore \alpha_0 = \frac{1}{250} \text{ per}^\circ\text{C}$$

If  $t^\circ\text{C}$  is the temperature of the coil when its resistance is  $21 \, \Omega$ , then,

$$21 = R_0 (1 + t/250)$$

Dividing this equation by the above equation, we have

$$\frac{21}{18} = \frac{R_0 (1 + t/250)}{R_0 (1 + 20 \alpha_0)}; \quad t = 65^\circ\text{C}; \text{ temp. rise} = 65 - 15 = 50^\circ\text{C}$$

**Example 1.19.** The coil of a relay takes a current of  $0.12 \, \text{A}$  when it is at the room temperature of  $15^\circ\text{C}$  and connected across a  $60\text{-V}$  supply. If the minimum operating current of the relay is  $0.1 \, \text{A}$ , calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance-temperature coefficient of the coil material is  $0.0043 \text{ per}^\circ\text{C}$  at  $6^\circ\text{C}$ .

**Solution.** Resistance of the relay coil at  $15^\circ\text{C}$  is  $R_{15} = 60/0.12 = 500 \, \Omega$

Let  $t^\circ\text{C}$  be the temperature at which the minimum operating current of  $0.1 \, \text{A}$  flows in the relay coil. Then,  $R_t = 60/0.1 = 600 \, \Omega$

$$\text{Now } R_{15} = R_0 (1 + 15 \alpha_0) = R_0 (1 + 15 \times 0.0043) \quad \text{and } R_t = R_0 (1 + 0.0043 t)$$

$$\therefore \frac{R_t}{R_{15}} = \frac{1 + 0.0043 t}{1.0654} \quad \text{or } \frac{600}{500} = \frac{1 + 0.0043 t}{1.0645} \quad \therefore t = 65.4^\circ\text{C}$$

If the temperature rises above this value, then due to increase in resistance, the relay coil will draw a current less than  $0.1 \, \text{A}$  and, therefore, will fail to operate.

**Example 1.20.** Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at  $25^\circ\text{C}$ . What proportion of current will pass through each if the temperature is raised to  $100^\circ\text{C}$ ? The temperature coefficients of resistance at  $0^\circ\text{C}$  are  $0.0043/^\circ\text{C}$  and  $0.0063/^\circ\text{C}$  for copper and iron respectively. (Principles of Elect. Engg. Delhi Univ.)

**Solution.** Since the copper and iron conductors carry equal currents at  $25^\circ\text{C}$ , their resistances are the same at that temperature. Let each be  $R \, \text{ohm}$ .

$$\text{For copper, } R_{100} = R_1 = R [1 + 0.0043 (100 - 25)] = 1.3225 R$$

$$\text{For iron, } R_{100} = R_2 = R [1 + 0.0063 (100 - 25)] = 1.4725 R$$

If  $I$  is the current at  $100^\circ\text{C}$ , then as per current divider rule, current in the copper conductor is

$$I_1 = I \frac{R_2}{R_1 + R_2} = I \frac{1.4725 R}{1.3225 R + 1.4725 R} = 0.5268 I$$

$$I_2 = I \frac{R_1}{R_1 + R_2} = I \frac{1.3225 R}{2.795 R} = 0.4732 I$$

Hence, copper conductor will carry **52.68%** of the total current and iron conductor will carry the balance i.e. **47.32%**.

**Example 1.21.** The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20°C of 4.3 μΩ-cm. If α = 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 watts at a filament temp. of 2420°C?

**Solution.** Electric power generated =  $I^2 R$  watts =  $V^2/R$  watts

$$\therefore V^2/R = 60 \quad \text{or} \quad 240^2/R = 60$$

$$\text{Resistance at } 2420^\circ\text{C} \quad R_{2420} = \frac{240 \times 240}{60} = 960 \Omega$$

$$\text{Now} \quad R_{2420} = R_{20} [1 + (2420 - 20) \times 0.005]$$

$$\text{or} \quad 960 = R_{20} (1 + 12)$$

$$\therefore R_{20} = 960/13 \Omega$$

$$\text{Now} \quad \rho_{20} = 4.3 \times 10^{-6} \Omega\text{-cm} \quad \text{and} \quad A = \frac{\pi(0.002)^2}{4} \text{ cm}^2$$

$$\therefore l = \frac{A \times R_{20}}{\rho_{20}} = \frac{\pi (0.002)^2 \times 960}{4 \times 13 \times 4.3 \times 10^{-6}} = 54 \text{ cm}$$

**Example 1.22.** A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of Cu at 20°C = 1.724 × 10<sup>-6</sup> ohm-cm and resistance temp. coeff. of Cu at 0°C = 0.0043/°C.

**Solution.** The semi-circular ring is shown in Fig. 1.11.

$$\text{Mean radius of ring} = (6 + 9)/2 = 7.5 \text{ cm}$$

$$\text{Mean length between end faces} = 7.5 \pi \text{ cm} = 23.56 \text{ cm}$$

$$\text{Cross-section of the ring} = 3 \times 4 = 12 \text{ cm}^2$$

$$\text{Now } \alpha_0 = 0.0043/^\circ\text{C}; \alpha_{20} = \frac{0.0043}{1 + 20 \times 0.0043} = 0.00396$$

$$\rho_{50} = \rho_{20} [1 + \alpha_0 (50 - 20)] \\ = 1.724 \times 10^{-6} (1 + 30 \times 0.00396) = 1.93 \times 10^{-6} \Omega\text{-cm}$$

$$R_{50} = \frac{\rho_{50} \times l}{A} = \frac{1.93 \times 10^{-6} \times 23.56}{12} = 3.79 \times 10^{-6} \Omega$$

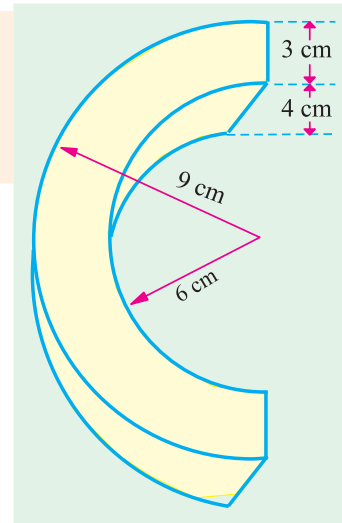


Fig.1.11

### Tutorial Problems No. 1.2

1. It is found that the resistance of a coil of wire increases from 40 ohm at 15°C to 50 ohm at 60°C. Calculate the resistance temperature coefficient at 0°C of the conductor material.

[1/165 per °C] (Elect. Technology, Indore Univ.)

2. A tungsten lamp filament has a temperature of 2,050°C and a resistance of 500 Ω when taking normal working current. Calculate the resistance of the filament when it has a temperature of 25°C. Temperature coefficient at 0°C is 0.005/°C.

[50 Ω] (Elect. Technology, Indore Univ.)

3. An armature has a resistance of  $0.2 \Omega$  at  $150^\circ\text{C}$  and the armature Cu loss is to be limited to 600 watts with a temperature rise of  $55^\circ\text{C}$ . If  $\alpha_0$  for Cu is  $0.0043/^\circ\text{C}$ , what is the maximum current that can be passed through the armature? [50.8 A]
4. A d.c. shunt motor after running for several hours on constant voltage mains of 400 V takes a field current of 1.6 A. If the temperature rise is known to be  $40^\circ\text{C}$ , what value of extra circuit resistance is required to adjust the field current to 1.6 A when starting from cold at  $20^\circ\text{C}$ ? Temperature coefficient =  $0.0043/^\circ\text{C}$  at  $20^\circ\text{C}$ . [36.69  $\Omega$ ]
5. In a test to determine the resistance of a single-core cable, an applied voltage of 2.5 V was necessary to produce a current of 2 A in it at  $15^\circ\text{C}$ .
  - (a) Calculate the cable resistance at  $55^\circ\text{C}$  if the temperature coefficient of resistance of copper at  $0^\circ\text{C}$  is  $1/235$  per  $^\circ\text{C}$ .
  - (b) If the cable under working conditions carries a current of 10 A at this temperature, calculate the power dissipated in the cable. [(a) 1.45  $\Omega$  (b) 145 W]
6. An electric radiator is required to dissipate 1 kW when connected to a 230 V supply. If the coils of the radiator are of wire 0.5 mm in diameter having resistivity of  $60 \mu\Omega\text{-cm}$ , calculate the necessary length of the wire. [1732 cm]
7. An electric heating element to dissipate 450 watts on 250 V mains is to be made from nichrome ribbon of width 1 mm and thickness 0.05 mm. Calculate the length of the ribbon required (the resistivity of nichrome is  $110 \times 10^{-8} \Omega\text{-m}$ ). [631 m]
8. When burning normally, the temperature of the filament in a 230 V, 150 W gas-filled tungsten lamp is  $2,750^\circ\text{C}$ . Assuming a room temperature of  $16^\circ\text{C}$ , calculate (a) the normal current taken by the lamp (b) the current taken at the moment of switching on. Temperature coefficient of tungsten is  $0.0047 \Omega/\Omega^\circ\text{C}$  at  $0^\circ\text{C}$ . [(a) 0.652 A (b) 8.45 A] (Elect. Engg. Madras Univ.)
9. An aluminium wire 5 m long and 2 mm diameter is connected in parallel with a wire 3 m long. The total current is 4 A and that in the aluminium wire is 2.5 A. Find the diameter of the copper wire. The respective resistivities of copper and aluminium are 1.7 and  $2.6 \mu\Omega\text{-m}$ . [0.97 mm]
10. The field winding of d.c. motor connected across 230 V supply takes 1.15 A at room temp. of  $20^\circ\text{C}$ . After working for some hours the current falls to 0.26 A, the supply voltage remaining constant. Calculate the final working temperature of field winding. Resistance temperature coefficient of copper at  $20^\circ\text{C}$  is  $1/254.5$ . [70.4 $^\circ\text{C}$ ] (Elect. Engg. Pune Univ.)
11. It is required to construct a resistance of  $100 \Omega$  having a temperature coefficient of  $0.001$  per  $^\circ\text{C}$ . Wires of two materials of suitable cross-sectional area are available. For material A, the resistance is  $97 \Omega$  per 100 metres and for material B, the resistance is  $40 \Omega$  per 100 metres. The temperature coefficient of resistance for material A is  $0.003$  per  $^\circ\text{C}$  and for material B is  $0.0005$  per  $^\circ\text{C}$ . Determine suitable lengths of wires of materials A and B. [A : 19.4 m, B : 200 m]
12. The resistance of the shunt winding of a d.c. machine is measured before and after a run of several hours. The average values are 55 ohms and 63 ohms. Calculate the rise in temperature of the winding. (Temperature coefficient of resistance of copper is  $0.00428$  ohm per ohm per  $^\circ\text{C}$ ). [36 $^\circ\text{C}$ ] (London Univ.)
13. A piece of resistance wire, 15.6 m long and of cross-sectional area  $12 \text{ mm}^2$  at a temperature of  $0^\circ\text{C}$ , passes a current of 7.9 A when connected to d.c. supply at 240 V. Calculate (a) resistivity of the wire (b) the current which will flow when the temperature rises to  $55^\circ\text{C}$ . The temperature coefficient of the resistance wire is  $0.00029 \Omega/\Omega/^\circ\text{C}$ . [(a) 23.37  $\mu\Omega\text{-m}$  (b) 7.78 A] (London Univ.)
14. A coil is connected to a constant d.c. supply of 100 V. At start, when it was at the room temperature of  $25^\circ\text{C}$ , it drew a current of 13 A. After sometime, its temperature was  $70^\circ\text{C}$  and the current reduced to 8.5 A. Find the current it will draw when its temperature increases further to  $80^\circ\text{C}$ . Also, find the temperature coefficient of resistance of the coil material at  $25^\circ\text{C}$ . [7.9 A;  $0.01176^\circ\text{C}^{-1}$ ] (F.Y. Engg. Univ.)
15. The resistance of the filed coils with copper conductors of a dynamo is  $120 \Omega$  at  $25^\circ\text{C}$ . After working for 6 hours on full load, the resistance of the coil increases to  $140 \Omega$ . Calculate the mean temperature rise of the field coil. Take the temperature coefficient of the conductor material as  $0.0042$  at  $0^\circ\text{C}$ . [43.8 $^\circ\text{C}$ ] (Elements of Elec. Engg. Bangalore Univ.)

### 1.13. Ohm's Law

This law applies to electric to electric conduction through good conductors and may be stated as follows :

**The ratio of potential difference ( $V$ ) between any two points on a conductor to the current ( $I$ ) flowing between them, is constant, provided the temperature of the conductor does not change.**

In other words,  $\frac{V}{I} = \text{constant}$  or  $\frac{V}{I} = R$

where  $R$  is the resistance of the conductor between the two points considered.

Put in another way, it simply means that provided  $R$  is kept constant, current is directly proportional to the potential difference across the ends of a conductor. However, this linear relationship between  $V$  and  $I$  does not apply to all non-metallic conductors. For example, for silicon carbide, the relationship is given by  $V = KI^m$  where  $K$  and  $m$  are constants and  $m$  is less than unity. It also does not apply to non-linear devices such as Zener diodes and voltage-regulator (VR) tubes.

**Example 1.23.** A coil of copper wire has resistance of  $90\ \Omega$  at  $20^\circ\text{C}$  and is connected to a  $230\text{-V}$  supply. By how much must the voltage be increased in order to maintain the current constant if the temperature of the coil rises to  $60^\circ\text{C}$ ? Take the temperature coefficient of resistance of copper as  $0.00428$  from  $0^\circ\text{C}$ .

**Solution.** As seen from Art. 1.10

$$\frac{R_{60}}{R_{20}} = \frac{1 + 60 \times 0.00428}{1 + 20 \times 0.00428} \quad \therefore R_{60} = 90 \times \frac{1.2568}{1.0856} = 104.2\ \Omega$$

Now, current at  $20^\circ\text{C} = 230/90 = 23/9\ \text{A}$

Since the wire resistance has become  $104.2\ \Omega$  at  $60^\circ\text{C}$ , the new voltage required for keeping the current constant at its previous value =  $104.2 \times 23/9 = 266.3\ \text{V}$

$\therefore$  increase in voltage required =  $266.3 - 230 = 36.3\ \text{V}$

**Example 1.24.** Three resistors are connected in series across a  $12\text{-V}$  battery. The first resistor has a value of  $1\ \Omega$ , second has a voltage drop of  $4\ \text{V}$  and the third has a power dissipation of  $12\ \text{W}$ . Calculate the value of the circuit current.

**Solution.** Let the two unknown resistors be  $R_2$  and  $R_3$  and  $I$  the circuit current

$$\therefore I^2 R_3 = 12 \quad \text{and} \quad IR_3 = 4 \quad \therefore R_3 = \frac{3}{4} R_2^2. \quad \text{Also, } I = \frac{4}{R_2}$$

Now,  $I(1 + R_2 + R_3) = 12$

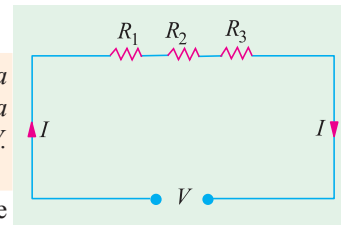
Substituting the values of  $I$  and  $R_3$ , we get

$$\frac{4}{R_2} \left( 1 + R_2 + \frac{3}{4} R_2^2 \right) = 12 \quad \text{or} \quad 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore R_2 = \frac{8 \pm \sqrt{64 - 48}}{6} \quad \therefore R_2 = 2\ \Omega \quad \text{or} \quad \frac{2}{3}\ \Omega$$

$$\therefore R_3 = \frac{3}{4} R_2^2 = \frac{3}{4} \times 2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4} \left( \frac{2}{3} \right)^2 = \frac{1}{3}\ \Omega$$

$$\therefore I = \frac{12}{1 + 2 + 3} = 2\ \text{A} \quad \text{or} \quad I = \frac{12}{1 + (2/3) + (1/3)} = 6\ \text{A}$$



### 1.14. Resistance in Series

When some conductors having resistances  $R_1$ ,  $R_2$  and  $R_3$  etc. are joined end-on-end as in Fig. 1.12, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points  $A$  and  $D$  is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that ( $i$ ) current is the same through all the three conductors

(ii) but voltage drop across each is different due to its different resistance and is given by Ohm's Law and (iii) sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point A to D as shown in Fig. 1.13.

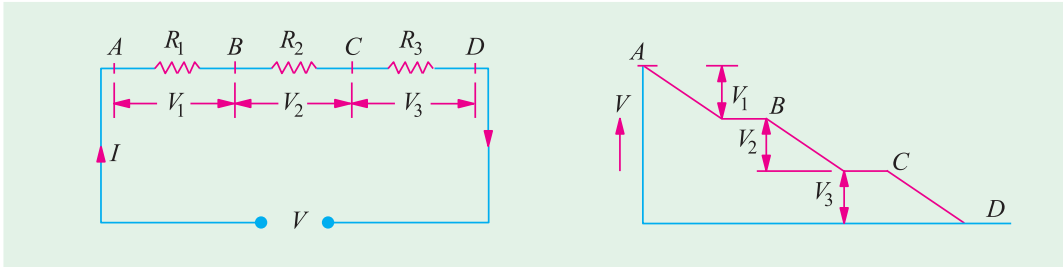


Fig. 1.12

Fig. 1.13

$$\therefore V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad \text{---Ohm's Law}$$

$$\text{But } V = IR$$

where R is the equivalent resistance of the series combination.

$$\therefore IR = IR_1 + IR_2 + IR_3 \quad \text{or } R = R_1 + R_2 + R_3$$

Also 
$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

### 1.15. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 1.14 is shown a 24-V battery connected across a series combination of three resistors.

Total resistance  $R = R_1 + R_2 + R_3 = 12 \Omega$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V \cdot \frac{R_1}{R} = 24 \times \frac{2}{12} = 4 \text{ V}$$

$$V_2 = V \cdot \frac{R_2}{R} = 24 \times \frac{4}{12} = 8 \text{ V}$$

$$V_3 = V \cdot \frac{R_3}{R} = 24 \times \frac{6}{12} = 12 \text{ V}$$

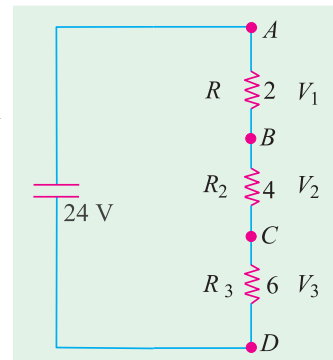


Fig.1.14

### 1.16. Resistances in Parallel

Three resistances, as joined in Fig. 1.15 are said to be connected in parallel. In this case (i) p.d. across all resistances is the same (ii) current in each resistor is different and is given by Ohm's Law and (iii) the total current is the sum of the three separate currents.

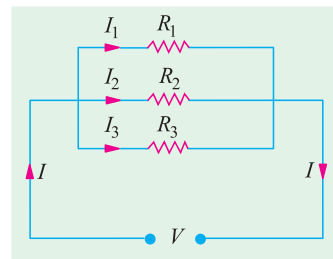


Fig.1.15

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now,

$$I = \frac{V}{R} \text{ where } V \text{ is the applied voltage.}$$

$R$  = equivalent resistance of the parallel combination.

∴

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also

$$G = G_1 + G_2 + G_3$$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

**Example 1.25.** What is the value of the unknown resistor  $R$  in Fig. 1.16 if the voltage drop across the  $500 \Omega$  resistor is 2.5 volts? All resistances are in ohm. (Elect. Technology, Indore Univ.)

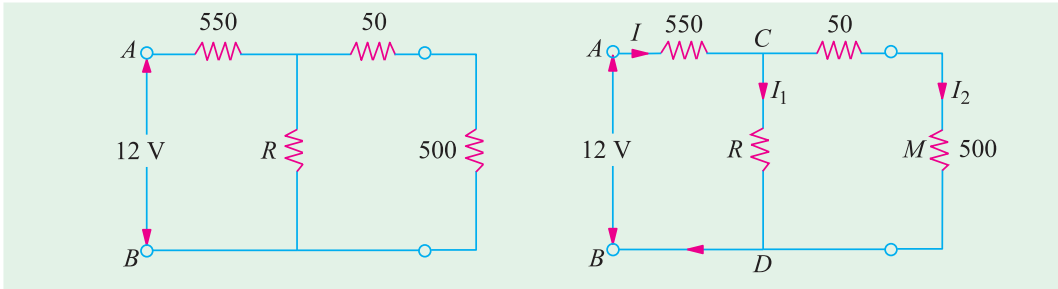


Fig. 1.16

**Solution.** By direct proportion, drop on  $50 \Omega$  resistance =  $2.5 \times 50/500 = 0.25 \text{ V}$

Drop across CMD or CD =  $2.5 + 0.25 = 2.75 \text{ V}$

Drop across  $550 \Omega$  resistance =  $12 - 2.75 = 9.25 \text{ V}$

$$I = 9.25/550 = 0.0168 \text{ A}, \quad I_2 = 2.5/500 = 0.005 \text{ A}$$

$$I_1 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

∴

$$0.0118 = 2.75/R; \quad R = \mathbf{233 \Omega}$$

**Example 1.26.** Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.

**Solution.** Resistance between A and C (Fig. 1.17).

$$= 6 \parallel 3 = 2 \Omega$$

Resistance of branch ACD =  $18 + 2 = 20 \Omega$

Now, there are two parallel paths between points A and D of resistances  $20 \Omega$  and  $5 \Omega$

Hence, resistance between A and D =  $20 \parallel 5 = 4 \Omega$

∴ Resistance between A and B =  $4 + 8 = 12 \Omega$

Total circuit current =  $60/12 = 5 \text{ A}$

Current through  $5 \Omega$  resistance =  $5 \times \frac{20}{25} = 4 \text{ A}$

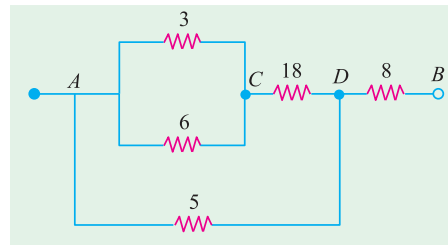


Fig. 1.17

—Art. 1.25



$$\begin{aligned} \text{Current in branch } ACD &= 5 \times \frac{5}{25} = 1 \text{ A} \\ \therefore \text{ P.D. across } 3 \Omega \text{ and } 6 \Omega \text{ resistors} &= 1 \times 2 = 2 \text{ V} \\ \text{P.D. across } 18 \Omega \text{ resistors} &= 1 \times 18 = 18 \text{ V} \\ \text{P.D. across } 5 \Omega \text{ resistors} &= 4 \times 5 = 20 \text{ V} \\ \text{P.D. across } 8 \Omega \text{ resistors} &= 5 \times 8 = 40 \text{ V} \end{aligned}$$

**Example 1.27.** A circuit consists of four 100-W lamps connected in parallel across a 230-V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter?

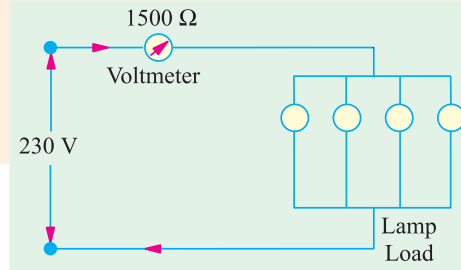


Fig.1.18

**Solution.** The circuit is shown in Fig. 1.18. The wattage of a lamp is given by :

$$W = I^2 R = V^2/R$$

$$\therefore 100 = 230^2/R \quad \therefore R = 529 \Omega$$

Resistance of each lamp under stated condition is  $= 6 \times 529 = 3174 \Omega$

Equivalent resistance of these four lamps connected in parallel  $= 3174/4 = 793.5 \Omega$

This resistance is connected in series with the voltmeter of 1500 Ω resistance.

$$\therefore \text{total circuit resistance} = 1500 + 793.5 = 2293.5 \Omega$$

$$\therefore \text{circuit current} = 230/2293.5 \text{ A}$$

According to Ohm's law, voltage drop across the voltmeter  $= 1500 \times 230/2293.5 = 150 \text{ V (approx)}$

**Example 1.28.** Determine the value of R and current through it in Fig. 1.19, if current through branch AO is zero. (Elect. Engg. & Electronics, Bangalore Univ.)

**Solution.** The given circuit can be redrawn as shown in Fig. 1.19 (b). As seen, it is nothing else but Wheatstone bridge circuit. As is well-known, when current through branch AO becomes zero, the bridge is said to be balanced. In that case, products of the resistances of opposite arms of the bridge become equal.

$$\therefore 4 \times 1.5 = R \times 1; R = 6 \Omega$$

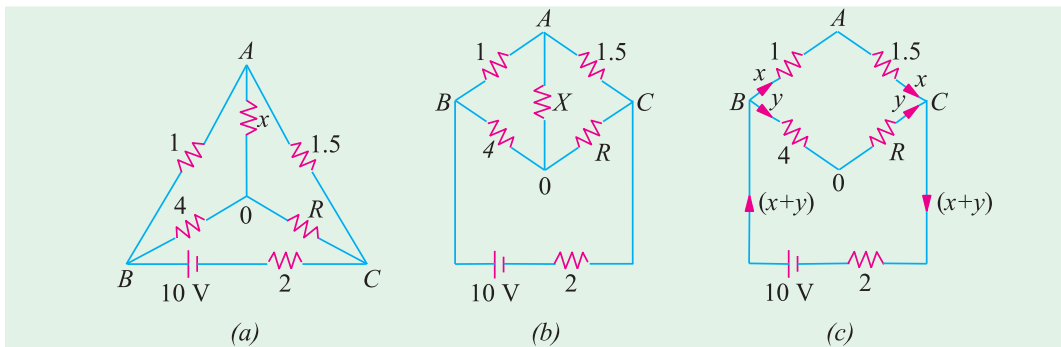


Fig.1.19

Under condition of balance, it makes no difference if resistance X is removed thereby giving us the circuit of Fig. 1.19 (c). Now, there are two parallel paths between points B and C of resistances  $(1 + 1.5) = 2.5 \Omega$  and  $(4 + 6) = 10 \Omega$ .  $R_{BC} = 10 \parallel 2.5 = 2 \Omega$

Total circuit resistance  $= 2 + 2 = 4 \Omega$ . Total circuit current  $= 10/4 = 2.5 \text{ A}$

This current gets divided into two parts at point B. Current through R is

$$y = 2.5 \times 2.5/12.5 = 0.5 \text{ A}$$

**Example 1.29.** In the unbalanced bridge circuit of Fig. 1.20 (a), find the potential difference that exists across the open switch  $S$ . Also, find the current which will flow through the switch when it is closed.

**Solution.** With switch open, there are two parallel branches across the 15-V supply. Branch  $ABC$  has a resistance of  $(3 + 12) = 15 \Omega$  and branch  $ADC$  has a resistance of  $(6 + 4) = 10 \Omega$ . Obviously, each branch has 15 V applied across it.

$$V_B = 12 \times 15/15 = 12 \text{ V}; V_D = 4 \times 15/(6 + 4) = 6 \text{ V}$$

$$\therefore \text{p.d. across points } B \text{ and } D = V_B - V_D = 12 - 6 = 6 \text{ V}$$

When  $S$  is closed, the circuit becomes as shown in Fig. 1.20 (b) where points  $B$  and  $D$  become electrically connected together.

$$R_{AB} = 3 \parallel 6 = 2 \Omega \quad \text{and} \quad R_{BC} = 4 \parallel 12 = 3 \Omega$$

$$R_{AC} = 2 + 3 = 5 \Omega \quad ; \quad I = 15/5 = 3 \text{ A}$$

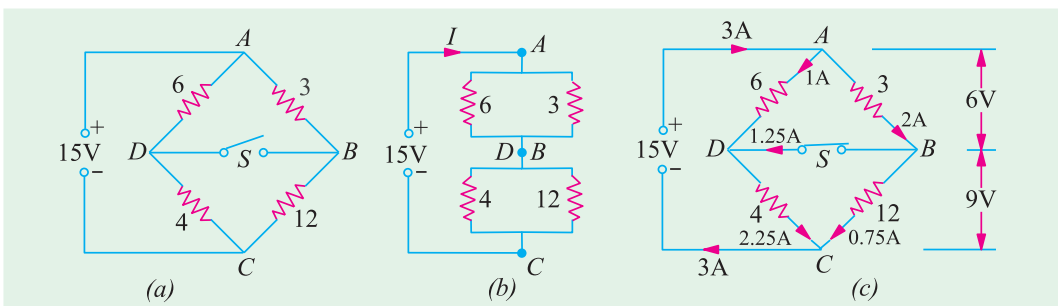


Fig. 1.20

Current through arm  $AB = 3 \times 6/9 = 2 \text{ A}$ . The voltage drop over arm  $AB = 3 \times 2 = 6 \text{ V}$ . Hence, drop over arm  $BC = 15 - 6 = 9 \text{ V}$ . Current through  $BC = 9/12 = 0.75 \text{ A}$ . It is obvious that at point  $B$ , the incoming current is 2 A, out of which 0.75 A flows along  $BC$ , whereas remaining  $2 - 0.75 = 1.25 \text{ A}$  passes through the switch.

As a check, it may be noted that current through  $AD = 6/6 = 1 \text{ A}$ . At point  $D$ , this current is joined by 1.25 A coming through the switch. Hence, current through  $DC = 1.25 + 1 = 2.25 \text{ A}$ . This fact can be further verified by the fact that there is a voltage drop of 9 V across 4 Ω resistor thereby giving a current of  $9/4 = 2.25 \text{ A}$ .

**Example 1.30.** A 50-ohm resistor is in parallel with 100-ohm resistor. Current in 50-ohm resistor is 7.2 A. How will you add a third resistor and what will be its value if the line-current is to be 12.1 amp?

[Nagpur Univ., Nov. 1997]

**Solution.** Source voltage =  $50 \times 7.2 = 360 \text{ V}$ , Current through 100-ohm resistor = 3.6 A

Total current through these two resistors in parallel = 10.8 A

For the total line current to be 12.1 A, third resistor must be connected in parallel, as the third branch, for carrying  $(12.1 - 10.8) = 1.3 \text{ A}$ . If  $R$  is this resistor  $R = 360/1.3 = 277 \text{ ohms}$

**Example 1.31.** In the circuit shown in Fig. 1.21, calculate the value of the unknown resistance  $R$  and the current flowing through it when the current in branch  $OC$  is zero.

[Nagpur Univ., April 1996]

**Solution.** If current through  $R$ -ohm resistor is  $I$  amp,  $AO$  branch carries the same current, since, current through the branch  $CO$  is zero. This also means that the nodes  $C$  and  $O$  are at the equal potential. Then, equating voltage-drops, we have  $V_{AO} = V_{AC}$ .

This means branch  $AC$  carries a current of  $4I$ .

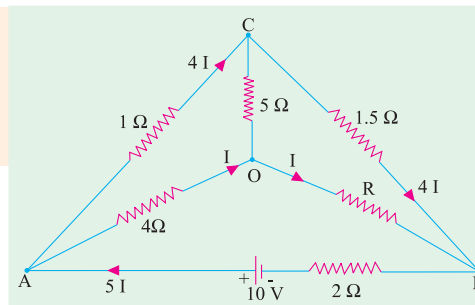


Fig. 1.21

This is current of  $4I$  also flows through the branch  $CB$ . Equating the voltage-drops in branches  $OB$  and  $CB$ ,

$$1.5 \times 4I = RI, \text{ giving } R = 6 \Omega$$

At node  $A$ , applying  $KCL$ , a current of  $5I$  flows through the branch  $BA$  from  $B$  to  $A$ . Applying  $KVL$  around the loop  $BAOB$ ,  $I = 0.5$  Amp.

**Example 1.32.** Find the values of  $R$  and  $V_s$  in Fig. 1.22. Also find the power supplied by the source.  
[Nagpur University, April 1998]

**Solution.** Name the nodes as marked on Fig. 1.22. Treat node  $A$  as the reference node, so that  $V_A = 0$ . Since path  $ADC$  carries  $1$  A with a total of  $4$  ohms resistance,  $V_C = +4$  V.

Since  $V_{CA} + 4$ ,  $I_{CA} = 4/8 = 0.5$  amp from  $C$  to  $A$ . Applying  $KCL$  at node  $C$ ,  $I_{BC} = 1.5$  A from  $B$  to  $C$ . Along the path  $BA$ ,  $1$  A flows through  $7$ -ohm resistor.

$$V_B = +7 \text{ Volts. } V_{BC} = 7 - 4 = +3.$$

This drives a current of  $1.5$  amp, through  $R$  ohms. Thus  $R = 3/1.5 = 2$  ohms.

Applying  $KCL$  at node  $B$ ,  $I_{FB} = 2.5$  A from  $F$  to  $B$ .

$V_{FB} = 2 \times 2.5 = 5$  volts,  $F$  being higher than  $B$  from the view-point of Potential. Since  $V_B$  has already been evaluated as  $+7$  volts,  $V = 12$  volts (w.r. to  $A$ ). Thus, the source voltage  $V_s = 12$  volts.

**Example 1.33.** In Fig. 1.23 (a), if all the resistances are of  $6$  ohms, calculate the equivalent resistance between any two diagonal points.  
[Nagpur Univ. April 1998]

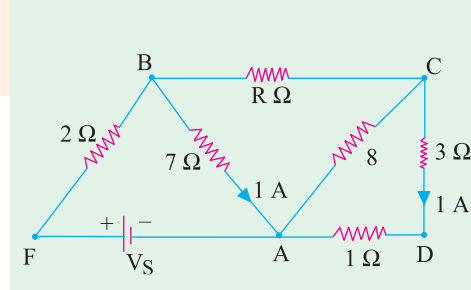


Fig. 1.22

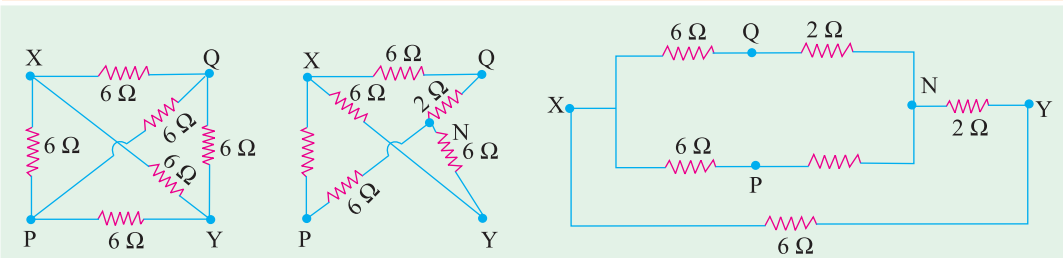


Fig. 1.23 (a)

Fig. 1.23 (b)

Fig. 1.23 (c)

**Solution.** If  $X$ - $Y$  are treated as the concerned diagonal points, for evaluating equivalent resistance offered by the circuit, there are two ways of transforming this circuit, as discussed below :

**Method 1 :** Delta to Star conversion applicable to the delta of  $PQY$  introducing an additional node  $N$  as the star-point. Delta with  $6$  ohms at each side is converted as  $2$  ohms as each leg of the star-equivalent. This is shown in Fig. 1.23 (b), which is further simplified in Fig. 1.23 (c). After handling series-parallel combinations of resistances,

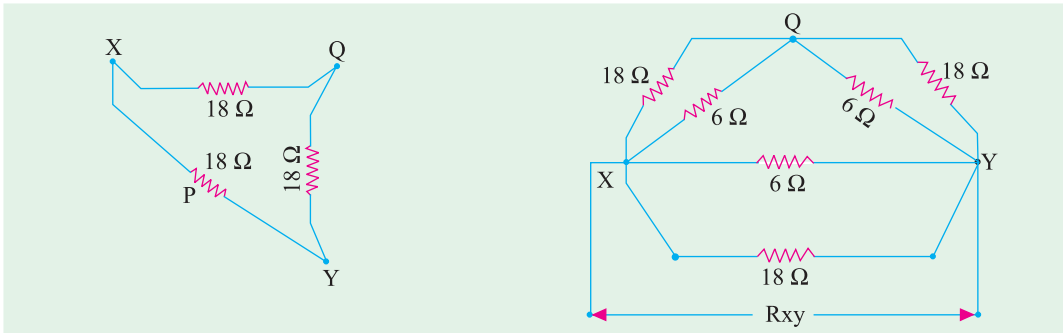


Fig. 1.23 (d)

Fig. 1.23 (e)

Total resistance between  $X$  and  $Y$  terminals in Fig. 1.23 (c) comes out to be  $3$  ohms.

**Method 2 :** Star to Delta conversion with  $P$  as the star-point and  $XYQ$  to be the three points of concerned converted delta. With star-elements of 6 ohms each, equivalent delta-elements will be 18 ohms, as Fig. 1.23 (d). This is included while redrawing the circuit as in Fig. 1.23 (e).

After simplifying, the series-parallel combination results into the final answer as  $R_{XY} = 3$  ohms.

**Example 1.34.** For the given circuit find the current  $I_A$  and  $I_B$ . [Bombay Univ. 1991]

**Solution.** Nodes  $A, B, C, D$  and reference node 0 are marked on the same diagram.

$I_A$  and  $I_B$  are to be found.

Apply KCL at node  $A$ . From  $C$  to  $A$ , current =  $7 + I_B$

At node 0, KCL is applied, which gives a current of  $7 + I_A$  through the 7 volt voltage source. Applying KCL at node  $B$  gives a current  $I_A - I_B$  through 2-ohm resistor in branch  $CB$ . Finally, at node  $A$ , KCL is applied. This gives a current of  $7 + I_B$  through 1-ohm resistor in branch  $CA$ .

Around the Loop  $OCBO$ ,  $2(I_A - I_B) + 1 \cdot I_A = 7$

Around the Loop  $CABC$ ,  $1(7 + I_B) + 3I_B - 2(I_A - I_B) = 0$

After rearranging the terms,  $3I_A - 2I_B = 7$ ,  $2I_A + 6I_B = -7$

This gives  $I_A = 2$  amp,  $I_B = -0.5$  amp.

This means that  $I_B$  is 0.5 amp from  $B$  to  $A$ .

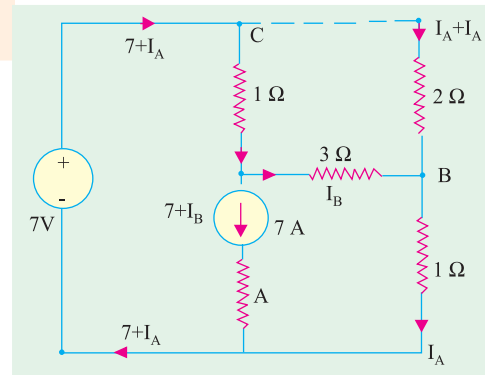


Fig. 1.24

**Example 1.35.** Find  $R_{AB}$  in the circuit, given in Fig. 1.25. [Bombay Univ. 2001]

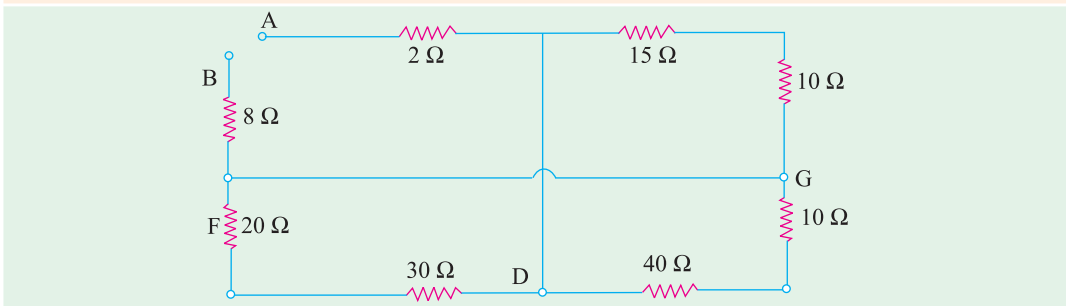


Fig. 1.25 (a)

**Solution.** Mark additional nodes on the diagram,  $C, D, E, F, G$ , as shown. Redraw the figure as in 1.25 (b), and simplify the circuit, to evaluate  $R_{AB}$ , which comes out to be 22.5 ohms.

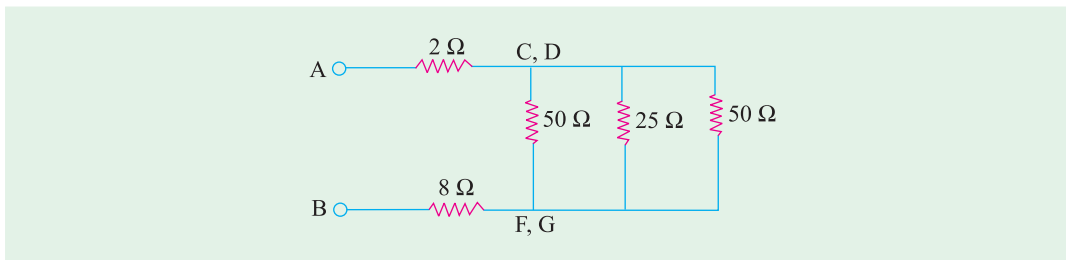


Fig. 1.25 (b)

**Example 1.36.** Find current through 4 resistance.

[Bombay Univ. 2001]

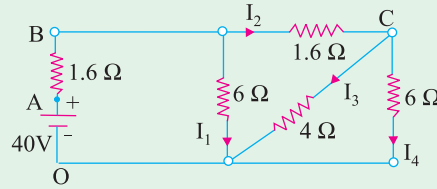


Fig. 1.26

**Solution.** Simplifying the series-parallel combinations, and solving the circuit, the source current is 10 amp. With respect to 0,  $V_A = 40$ ,  $V_B = 40 - 16 = 24$  volts.

$$I_1 = 4 \text{ amp, hence } I_2 = 6 \text{ amp}$$

$$V_C = V_B - I_2 \times 1.6 = 24 - 9.6 = 14.4 \text{ volts}$$

$$I_3 = 14.4/4 = 3.6 \text{ amp, which is the required answer. Further } I_4 = 24 \text{ amp.}$$

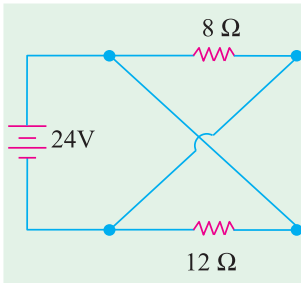


Fig. 1.27

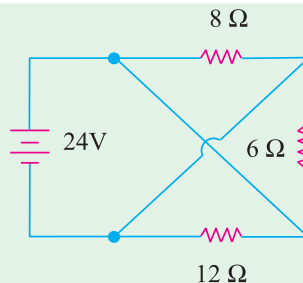


Fig. 1.28

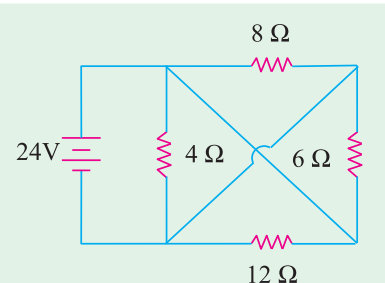


Fig. 1.29

**Tutorial Problems No. 1.3**

- Find the current supplied by the battery in the circuit of Fig. 1.27. [5 A]
- Compute total circuit resistance and battery current in Fig. 1.28. [8/3 ohm; 9 A]
- Calculate battery current and equivalent resistance of the network shown in Fig. 1.29. [15 A; 8/5 ohm]
- Find the equivalent resistance of the network of Fig. 1.30 between terminals A and B. All resistance values are in ohms. [6 ohm]
- What is the equivalent resistance of the circuit of Fig. 1.31 between terminals A and B? All resistances are in ohms. [4 ohm]
- Compute the value of battery current I in Fig. 1.32. All resistances are in ohm. [6 A]

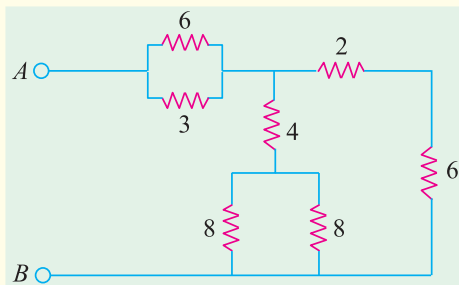


Fig. 1.30

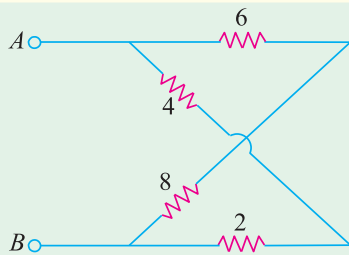


Fig. 1.31

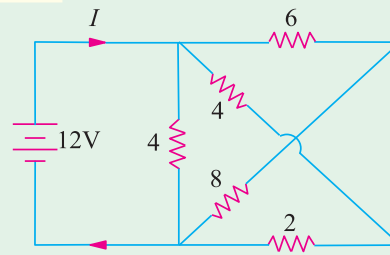


Fig. 1.32

7. Calculate the value of current  $I$  supplied by the voltage source in Fig. 1.33. All resistance values are in ohms. **(Hint : Voltage across each resistor is 6 V) [1 A]**
8. Compute the equivalent resistance of the circuit of Fig. 1.34 between points (i)  $ab$  (ii)  $ac$  and (iii)  $bc$ . All resistances values are in ohm. **[(i) 6  $\Omega$  (ii) 4.5  $\Omega$  (iii) 4.5  $\Omega$ ]**

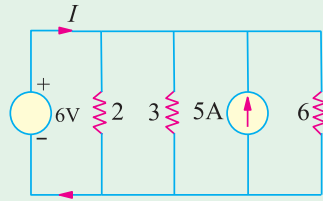


Fig. 1.33

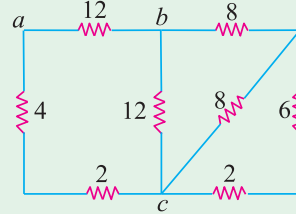


Fig. 1.34

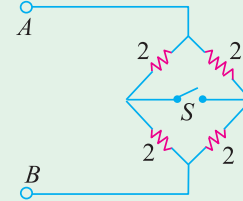


Fig. 1.35

9. In the circuit of Fig. 1.35, find the resistance between terminals  $A$  and  $B$  when switch is (a) open and (b) closed. Why are the two values equal? **[(a) 2  $\Omega$  (b) 2  $\Omega$ ]**
10. The total current drawn by a circuit consisting of three resistors connected in parallel is 12 A. The voltage drop across the first resistor is 12 V, the value of second resistor is 3  $\Omega$  and the power dissipation of the third resistor is 24 W. What are the resistances of the first and third resistors? **[2  $\Omega$ ; 6  $\Omega$ ]**
11. Three parallel connected resistors when connected across a d.c. voltage source dissipate a total power of 72 W. The total current drawn is 6 A, the current flowing through the first resistor is 3 A and the second and third resistors have equal value. What are the resistances of the three resistors? **[4  $\Omega$ ; 8  $\Omega$ ; 8  $\Omega$ ]**
12. A bulb rated 110 V, 60 watts is connected with another bulb rated 110-V, 100 W across a 220 V mains. Calculate the resistance which should be joined in parallel with the first bulb so that both the bulbs may take their rated power. **[302.5  $\Omega$ ]**
13. Two coils connected in parallel across 100 V supply mains take 10 A from the line. The power dissipated in one coil is 600 W. What is the resistance of the other coil? **[25  $\Omega$ ]**
14. An electric lamp whose resistance, when in use, is 2  $\Omega$  is connected to the terminals of a dry cell whose e.m.f. is 1.5 V. If the current through the lamp is 0.5 A, calculate the internal resistance of the cell and the potential difference between the terminals of the lamp. If two such cells are connected in parallel, find the resistance which must be connected in series with the arrangement to keep the current the same as before. **[1  $\Omega$  ; 1 V ; 0.5  $\Omega$ ] (Elect. Technology, Indore Univ.)**
15. Determine the current by the source in the circuit shown below. **(Bombay Univ. 2001)**

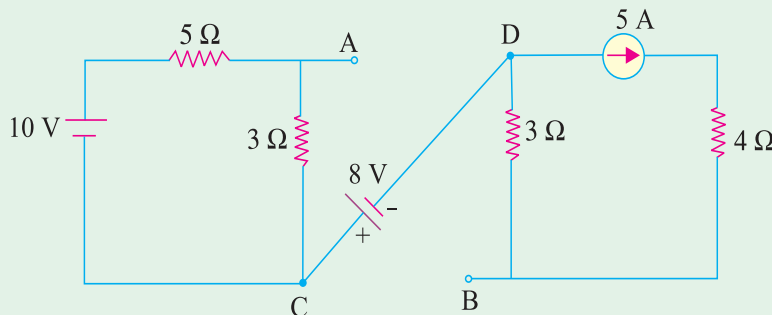


Fig. 1.36. (a)

**Hint.** Series-parallel combinations of resistors have to be dealt with. This leads to the source current of 28.463 amp.

16. Find the voltage of point  $A$  with respect to point  $B$  in the Fig. 1.36 (b). Is it positive with respect to  $B$ ?  
(Bombay University, 2000)

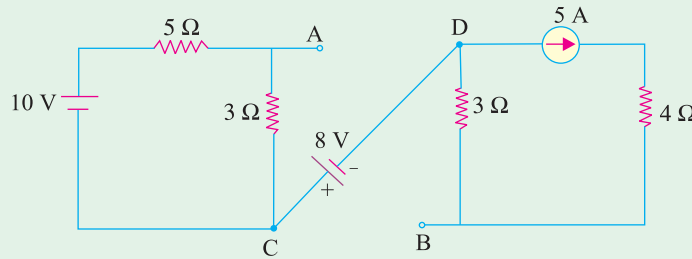


Fig. 1.36 (b)

Hint. If  $V_A = 0$ ,  $V_C = -1.25 \times 3 = -3.75$  V  
 $V_D = -3.75 - 8 = -11.75$  V  
 $V_B = V_D + 15 = +3.25$  volts  
 Thus, the potential of point  $A$  with respect to  $B$  is  $-3.25$  V.

### 1.17. Types of Resistors

#### (a) Carbon Composition

It is a combination of carbon particles and a binding resin with different proportions for providing desired resistance. Attached to the ends of the resistive element are metal caps which have axial leads of tinned copper wire for soldering the resistor into a circuit. The resistor is enclosed in a plastic case to prevent the entry of moisture and other harmful elements from outside. Billions of carbon composition resistors are used in the electronic industry every year. They are available in power ratings of 1/8, 1/4, 1/2, 1 and 2 W, in voltage ratings of 250, 350 and 500 V. They have low failure rates when properly used.

Such resistors have a tendency to produce electric noise due to the current passing from one carbon particle to another. This noise appears in the form of a hiss in a loudspeaker connected to a hi-fi system and can overcome very weak signals. That is why carbon composition resistors are used where performance requirements are not demanding and where low cost is the main consideration. Hence, they are extensively used in entertainment electronics although better resistors are used in critical circuits.

#### (b) Deposited Carbon

Deposited carbon resistors consist of ceramic rods which have a carbon film deposited on them. They are made by placing a ceramic rod in a methane-filled flask and heating it until, by a gas-cracking process, a carbon film is deposited on them. A helix-grinding process forms the resistive path. As compared to carbon composition resistors, these resistors offer a major improvement in lower current noise and in closer tolerance. These resistors are being replaced by metal film and metal glaze resistors.

#### (c) High-Voltage Ink Film

These resistors consist of a ceramic base on which a special resistive ink is laid down in a helical band. These resistors are capable of withstanding high voltages and find extensive use in cathode-ray circuits, in radar and in medical electronics. Their resistances range from 1 k $\Omega$  to 100,000 M $\Omega$  with voltage range upto 1000 kV.

#### (d) Metal Film

Metal film resistors are made by depositing vaporized metal in vacuum on a ceramic-core rod. The resistive path is helix-ground as in the case of deposited carbon resistors. Metal film resistors have excellent tolerance and temperature coefficient and are extremely reliable. Hence, they are very suitable for numerous high grade applications as in low-level stages of certain instruments although they are much more costlier.

**(e) Metal Glaze**

A metal glaze resistor consists of a metal glass mixture which is applied as a thick film to a ceramic substrate and then fired to form a film. The value of resistance depends on the amount of metal in the mixture. With helix-grinding, the resistance can be made to vary from  $1\ \Omega$  to many mega-ohms.

Another category of metal glaze resistors consists of a tinned oxide film on a glass substrate.

**(f) Wire-wound**

Wire-wound resistors are different from all other types in the sense that no film or resistive coating is used in their construction. They consist of a ceramic-core wound with a drawn wire having accurately-controlled characteristics. Different wire alloys are used for providing different resistance ranges. These resistors have highest stability and highest power rating.

Because of their bulk, high-power ratings and high cost, they are not suitable for low-cost or high-density, limited-space applications. The completed wire-wound resistor is coated with an insulating material such as baked enamel.

**(g) Cermet (Ceramic Metal)**

The cermet resistors are made by firing certain metals blended with ceramics on a ceramic substrate. The value of resistance depends on the type of mix and its thickness. These resistors have very accurate resistance values and show high stability even under extreme temperatures. Usually, they are produced as small rectangles having leads for being attached to printed circuit boards (PCB).

**1.18. Nonlinear Resistors**

Those elements whose  $V-I$  curves are not straight lines are called nonlinear elements because their resistances are nonlinear resistances. Their  $V-I$  characteristics can be represented by a suitable equation.

Examples of nonlinear elements are filaments of incandescent lamps, diodes, thermistors and varistors. A varistor is a special resistor made of carborundum crystals held together by a binder. Fig. 1.37 (a) shows how current through a varistor increase rapidly when the applied voltage increases beyond a certain amount (nearly 100 V in the present case).

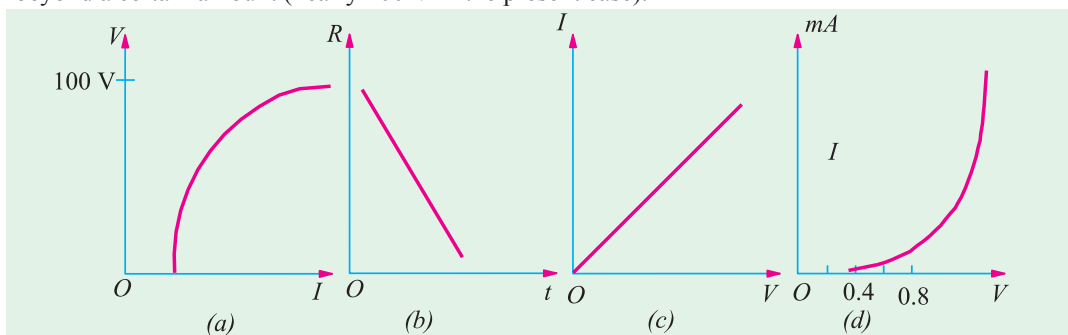


Fig. 1.37

There is a corresponding rapid decrease in resistance when the current increases. Hence, varistors are generally used to provide over-voltage protection in certain circuits.

A thermistor is made of metallic oxides in a suitable binder and has a large negative coefficient of resistance *i.e.*, its resistance decreases with increase in temperature as shown in Fig. 1.30 (b). Fig. 1.30 (c) shows how the resistance of an incandescent lamp increases with voltage whereas Fig. 1.30 (d) shows the  $V-I$  characteristics of a typical silicon diode. For a germanium diode, current is related to its voltage by the relation.

$$I = I_o (e^{V/0.026} - 1)$$



### 1.19. Varistor (Nonlinear Resistor)

It is a voltage-dependent metal-oxide material whose resistance decreases sharply with increasing voltage. The relationship between the current flowing through a varistor and the voltage applied across it is given by the relation  $i = ke^n$  where  $i$  = instantaneous current,  $e$  is the instantaneous voltage and  $\eta$  is a constant whose value depends on the metal oxides used. The value of  $\eta$  for silicon-carbide-based varistors lies between 2 and 6 whereas zinc-oxide-based varistors have a value ranging from 25 to 50.

The zinc-oxide-based varistors are primarily used for protecting solid-state power supplies from low and medium surge voltage in the supply line. Silicon-carbide varistors provide protection against high-voltage surges caused by lightning and by the discharge of electromagnetic energy stored in the magnetic fields of large coils.

### 1.20. Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire (Fig. 1.38), they are said to be *short-circuited*. Since 'short' has practically zero resistance, it gives rise to two important facts :

- (i) no voltage can exist across it because  $V = IR = I \times 0 = 0$
- (ii) current through it (called short-circuit current) is very large (theoretically, infinity)

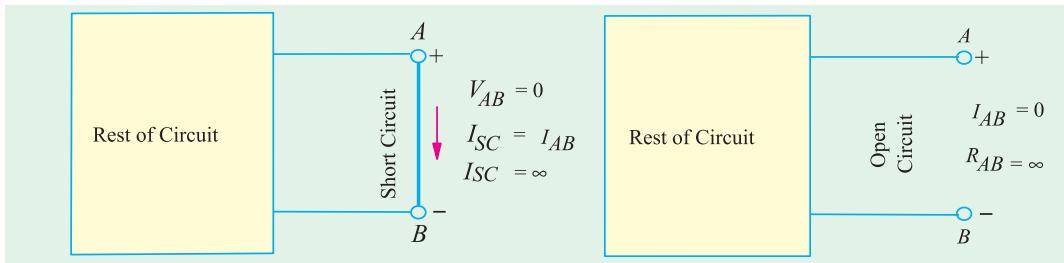


Fig. 1.38

Fig. 1.39

Two points are said to be open-circuited when there is no direct connection between them (Fig. 1.39). Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

- (i) resistance between the two points is infinite.
- (ii) there is no flow of current between the two points.

### 1.21. 'Shorts' in a Series Circuit

Since a dead (or solid) short has almost zero resistance, it causes the problem of excessive current which, in turn, causes power dissipation to increase many times and circuit components to burn out.

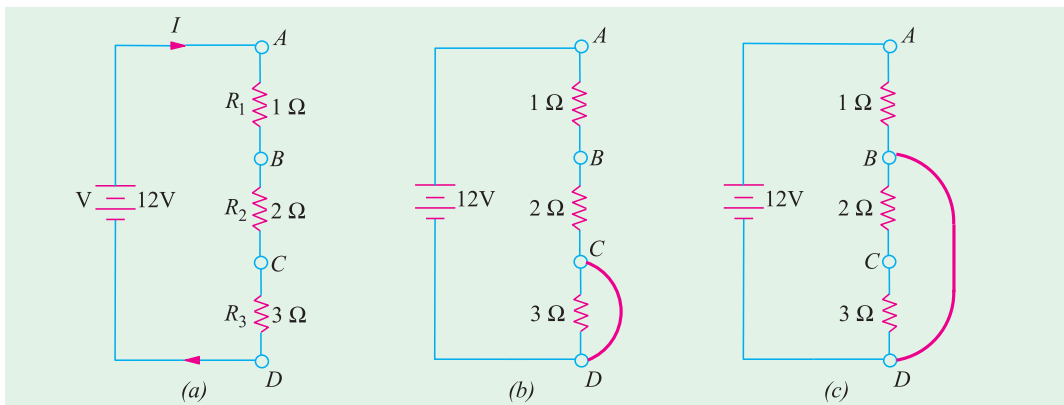


Fig. 1.40

In Fig. 1.40 (a) is shown a normal series circuit where

$$V = 12 \text{ V}, R = R_1 + R_2 + R_3 = 6 \Omega$$

$$I = V/R = 12/6 = 2 \text{ A}, P = I^2 R = 2^2 \times 6 = 24 \text{ W}$$

In Fig. 1.40 (b), 3- $\Omega$  resistor has been shorted out by a resistanceless copper wire so that  $R_{CD} = 0$ . Now, total circuit resistance  $R = 1 + 2 + 0 = 3 \Omega$ . Hence,  $I = 12/3 = 4 \text{ A}$  and  $P = 4^2 \times 3 = 48 \text{ W}$ .

Fig. 1.40 (c) shows the situation where both 2  $\Omega$  and 3  $\Omega$  resistors have been shorted out of the circuit. In this case,

$$R = 1 \Omega, I = 12/1 = 12 \text{ A} \quad \text{and} \quad P = 12^2 \times 1 = 144 \text{ W}$$

Because of this excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

### 1.22. 'Opens' in a Series Circuit

In a normal series circuit like the one shown in Fig. 1.41 (a), there exists a current flow and the voltage drops across different resistors are proportional to their resistances. If the circuit becomes 'open' anywhere, following two effects are produced :

(i) since 'open' offers infinite resistance, circuit current becomes zero. Consequently, there is no voltage drop across  $R_1$  and  $R_2$ .

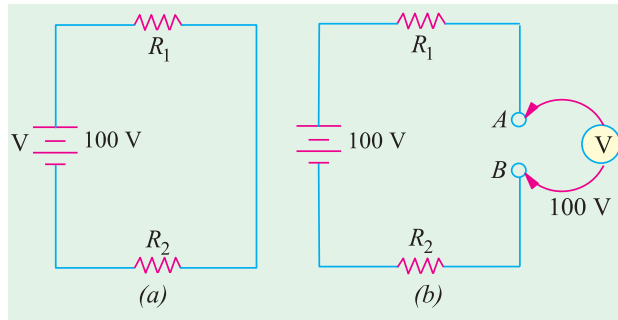


Fig. 1.41

(ii) **whole of the applied voltage (i.e. 100 V in this case) is felt across the 'open' i.e. across terminals A and B [Fig. 1.41 (b)].**

The reason for this is that  $R_1$  and  $R_2$  become negligible as compared to the infinite resistance of the 'open' which has practically whole of the applied voltage dropped across it (as per Voltage Divider Rule of art. 1.15). Hence, voltmeter in Fig. 1.41 (b) will read nearly 100 V i.e. the supply voltage.

### 1.23. 'Opens' in a Parallel Circuit

Since an 'open' offers infinite resistance, there would be no current in that part of the circuit where it occurs. In a parallel circuit, an 'open' can occur either in the main line or in any parallel branch.

As shown in Fig. 1.42 (a), an open in the main line prevents flow of current to **all branches**. Hence, neither of the two bulbs glows. However, full applied voltage (i.e. 220 V in this case) **is available across the open**.

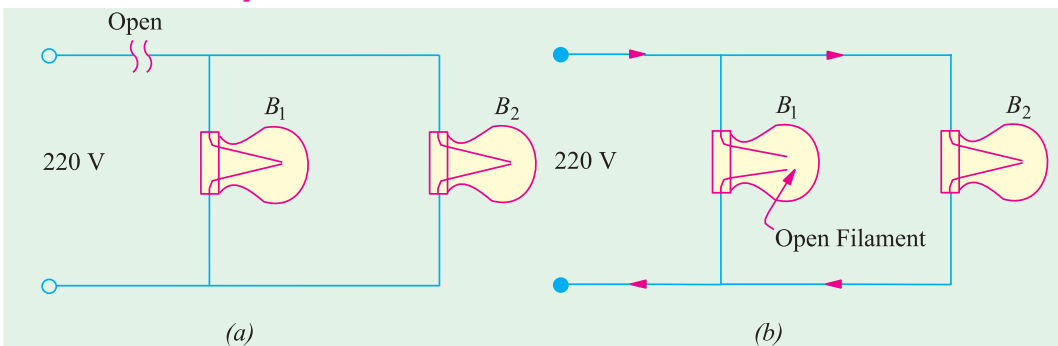


Fig. 1.42

In this Fig. 1.42 (b), 'open' has occurred in branch circuits of  $B_1$ . Since there is no current in this branch,  $B_1$  will not glow. However, as the other bulb remains connected across the voltage supply, it would keep operating normally.

It may be noted that if a voltmeter is connected across the open bulb, it will read full supply voltage of 220 V.

### 1.24. 'Shorts' in Parallel Circuits

Suppose a 'short' is placed across  $R_3$  (Fig. 1.43). It becomes directly connected across the battery and draws almost infinite current because not only its own resistance but that of the connecting wires  $AC$  and  $BD$  is negligible. Due to this excessive current, the wires may get hot enough to burn out unless the circuit is protected by a fuse.

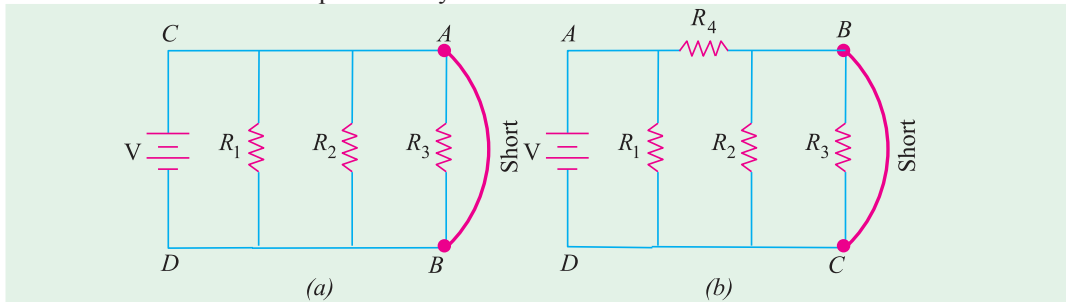


Fig. 1.43

Following points about the circuit of Fig. 1.43 (a) are worth noting.

1. not only is  $R_3$  short-circuited but both  $R_1$  and  $R_2$  are also shorted out *i.e.* **short across one branch means short across all branches**.
2. there is no current in shorted resistors. If there were three bulbs, they will not glow.
3. the shorted components are not damaged, For example, if we had three bulbs in Fig. 1.43 (a), they would glow again when circuit is restored to normal conditions by removing the short-circuited.

It may, however, be noted from Fig. 1.43 (b) that a short-circuit across  $R_3$  may short out  $R_2$  but not  $R_1$  since it is protected by  $R_4$ .

### 1.25. Division of Current in Parallel Circuits

In Fig. 1.44, two resistances are joined in parallel across a voltage  $V$ . The current in each branch, as given in Ohm's law, is

$$I_1 = V/R_1 \text{ and } I_2 = V/R_2$$

$$\therefore \frac{I_1}{I_2} = \frac{R_1}{R_2}$$

$$\text{As } \frac{I}{R_1} = G_1 \text{ and } \frac{I}{R_2} = G_2$$

$$\therefore \frac{I_1}{I_2} = \frac{G_1}{G_2}$$

Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus :

$$\text{Now } I_1 + I_2 = I; \therefore I_2 = I - I_1 \quad \therefore \frac{I_1}{I - I_1} = \frac{R_2}{R_1} \text{ or } I_1 R_1 = R_2 (I - I_1)$$

$$\therefore I_1 = I \frac{R_1}{R_1 - R_2} = I \frac{G_1}{G_1 + G_2} \text{ and } I_2 = I \frac{R_1}{R_1 - R_2} = I \cdot \frac{G_1}{G_1 + G_2}$$

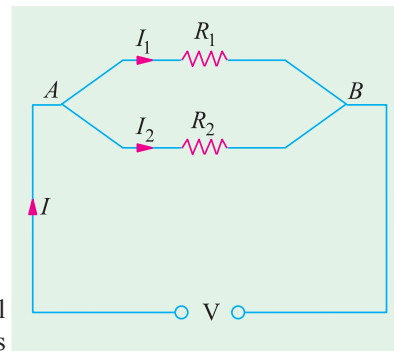


Fig. 1.44

This Current Divider Rule has direct application in solving electric circuits by Norton's theorem (Art. 2.25).

Take the case of three resistors in parallel connected across a voltage  $V$  (Fig. 1.45). Total current is  $I = I_1 + I_2 + I_3$ . Let the equivalent resistance be  $R$ . Then

$$V = IR$$

Also  $V = I_1 R_1 \therefore IR = I_1 R_1$

or  $\frac{I}{I_1} = \frac{R_1}{R}$  or  $I_1 = IR/R_1 \dots (i)$

Now  $\frac{I}{R} = \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3}$

$$R = \frac{R_1 R_2 R_3}{R_2 R_3 + R_2 R_1 + R_1 R_3}$$

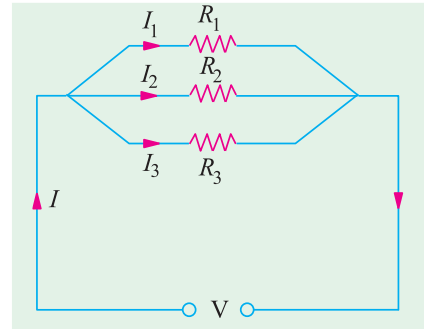


Fig. 1.45

From (i) above,  $I_1 = I \left( \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right) = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$

Similarly,  $I_2 = I \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} = I \cdot \frac{G_2}{G_1 + G_2 + G_3}$

$$I_3 = I \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} = I \cdot \frac{G_3}{G_1 + G_2 + G_3}$$

**Example 1.37.** A resistance of  $10 \Omega$  is connected in series with two resistances each of  $15 \Omega$  arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be  $1.5 \text{ A}$  with  $20 \text{ V}$  applied?

(Elements of Elect. Engg.-1; Bangalore Univ.)

**Solution.** The circuit connections are shown in Fig. 1.46.

Drop across  $10\text{-}\Omega$  resistor =  $1.5 \times 10 = 15 \text{ V}$

Drop across parallel combination,  $V_{AB} = 20 - 15 = 5 \text{ V}$

Hence, voltage across each parallel resistance is  $5 \text{ V}$ .

$$I_1 = 5/15 = 1/3 \text{ A}, I_2 = 5/15 = 1/3 \text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6 \text{ A}$$

$$\therefore I_3 R = 5 \text{ or } (5/6) R = 5 \text{ or } R = 6 \Omega$$

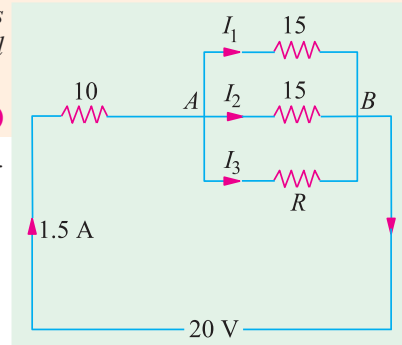


Fig. 1.46

**Example 1.38.** If  $20 \text{ V}$  be applied across  $AB$  shown in Fig. 1.40, calculate the total current, the power dissipated in each resistor and the value of the series resistance to have the total current.

(Elect. Science-II, Allahabad Univ. 1992)

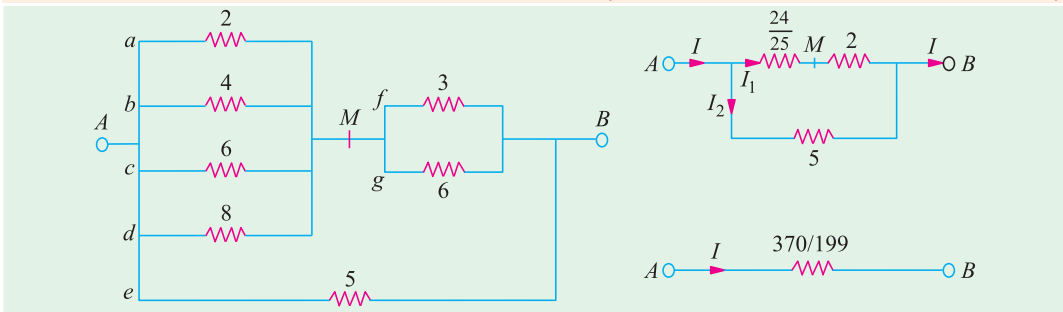


Fig. 1.47

**Solution.** As seen from Fig. 1.47.  $R_{AB} = 370/199 \Omega$

Hence, total current =  $20 \div 370/199 = 10.76 \text{ A}$

$$I_1 = 10.76 \times 5(5 + 74.25) = 6.76 \text{ A}; I_2 = 10.76 - 6.76 = 4 \text{ A}$$

$$I_f = 6.76 \times 6/9 = 4.51 \text{ A}; I_g = 6.76 - 4.51 = 2.25 \text{ A}$$

Voltage drop across  $A$  and  $M$ ,  $V_{AM} = 6.76 \times 24/25 = 6.48 \text{ V}$

$$I_a = V_{AM}/2 = 6.48/2 = 3.24 \text{ A}; I_b = 6.48/4 = 1.62 \text{ A}; I_c = 6.48/6 = 1.08 \text{ A}$$

$$I_d = 6.48/8 = 0.81 \text{ A}, I_e = 20/5 = 4 \text{ A}$$

**Power Dissipation**

$$P_a = I_a^2 R_a = 3.24^2 \times 2 = 21 \text{ W}, P_b = 1.62^2 \times 4 = 10.4 \text{ W}, P_c = 1.08^2 \times 6 = 7 \text{ W}$$

$$P_d = 0.81^2 \times 8 = 5.25 \text{ W}, P_e = 4^2 \times 5 = 80 \text{ W}, P_f = 4.51^2 \times 3 = 61 \text{ W}$$

$$P_g = 2.25^2 \times 6 = 30.4 \text{ W}$$

The series resistance required is  $370/199 \Omega$

Incidentally, total power dissipated =  $I^2 R_{AB} = 10.76^2 \times 370/199 = 215.3 \text{ W}$  (as a check).

**Example 1.39.** Calculate the values of different currents for the circuit shown in Fig. 1.48. What is the total circuit conductance? and resistance?

**Solution.** As seen,  $I = I_1 + I_2 + I_3$ . The current division takes place at point  $B$ .

As seen from Art. 1.25.

$$I = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$$

$$= 12 \times \frac{0.1}{0.6} = 2 \text{ A}$$

$$I_2 = 12 \times 0.2/0.6 = 4 \text{ A}$$

$$I_3 = 12 \times 0.3/0.6 = 6 \text{ A}$$

$$G_{BC} = 0.1 + 0.2 + 0.3 = 0.6 \text{ S}$$

$$\frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6} \text{ S}^{-1} \therefore R_{AC} = 1/G_{AC} = 25/6 \Omega$$

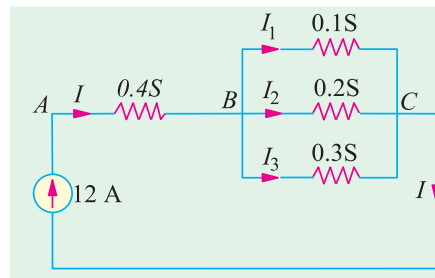


Fig. 1.48

**Example 1.40.** Compute the values of three branch currents for the circuits of Fig. 1.49 (a). What is the potential difference between points  $A$  and  $B$ ?

**Solution.** The two given current sources may be combined together as shown in Fig. 1.49 (b). Net current =  $25 - 6 = 19 \text{ A}$  because the two currents flow in opposite directions.

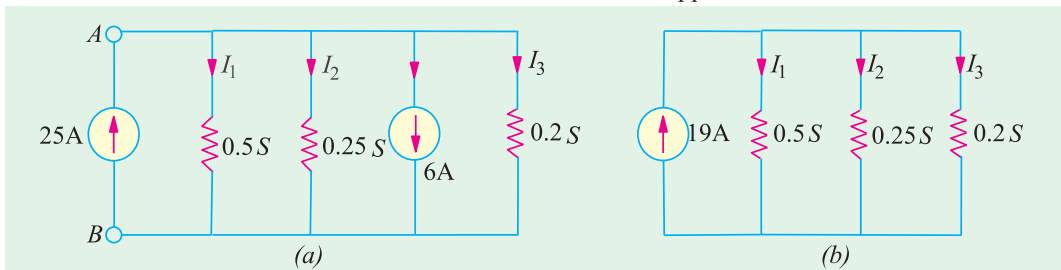


Fig. 1.49

Now,  $G = 0.5 + 0.25 + 0.2 = 0.95 \text{ S}; I_1 = I \frac{G_1}{G} = 19 \times \frac{0.5}{0.95} = 10 \text{ A}$

$$I_2 = I \frac{G_2}{G} = 19 \times \frac{0.25}{0.95} = 5 \text{ A}; I_3 = I \frac{G_3}{G} = 19 \times \frac{0.2}{0.95} = 4 \text{ A}$$

$$V_{AB} = I_1 R_1 = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} \therefore V_{AB} = \frac{10}{0.5} = 20 \text{ A}$$

The same voltage acts across the three conductances.

**Example 1.41.** Two conductors, one of copper and the other of iron, are connected in parallel and at 20°C carry equal currents. What proportion of current will pass through each if the temperature is raised to 100°C? Assume  $\alpha$  for copper as 0.0042 and for iron as 0.006 per °C at 20°C. Find also the values of temperature coefficients at 100°C.

(Electrical Engg. Madras Univ.)

**Solution.** Since they carry equal current at 20°C, the two conductors have the same resistance at 20°C i.e.  $R_{20}$ . As temperature is raised, their resistances increase through unequally.

$$\text{For Cu, } R_{100} = R_{20} (1 + 80 \times 0.0042) = 1.336 R_{20}$$

$$\text{For iron } R'_{100} = R_{20} (1 + 80 \times 0.006) = 1.48 R_{20}$$

As seen from Art. 1.25, current through Cu conductor is

$$I_1 = I \times \frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.48 R_{20}}{2.816 R_{20}} = 0.5256 I \text{ or } 52.56\% \text{ of } I$$

Hence, current through Cu conductor is 52.56 per cent of the total current. Obviously, the remaining current i.e. 47.44 per cent passes through iron.

Or current through iron conductor is

$$I_2 = I \times \frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.336 R_{20}}{2.816 R_{20}} = 0.4744 I \text{ or } 47.44\% \text{ of } I$$

$$\text{For Cu, } \alpha_{100} = \frac{1}{(1/0.0042) + 80} = 0.00314^\circ\text{C}^{-1}$$

$$\text{For iron, } \alpha_{100} = \frac{1}{(1/0.006) + 80} = 0.0040^\circ\text{C}^{-1}$$

**Example 1.42.** A battery of unknown e.m.f. is connected across resistances as shown in Fig. 1.50. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter? What is the e.m.f. of the battery? (Basic Elect. Engg.; Bangladesh Univ., 1990)

**Solution.** Current through 8 Ω resistance =  $20/8 = 2.5 \text{ A}$

This current is divided into two parts at point A; one part going along path AC and the other along path ABC which has a resistance of 28 Ω

$$I_2 = 2.5 \times \frac{11}{(11 + 28)} = 0.7$$

Hence, ammeter reads **0.7 A**.

Resistance between A and C =  $(28 \times 11/39) \text{ ohm}$ .

Total circuit resistance =  $8 + 11 + (308/39) = 1049/39 \Omega$

$$\therefore E = 2.5 \times 1049/39 = 67.3 \text{ V}$$

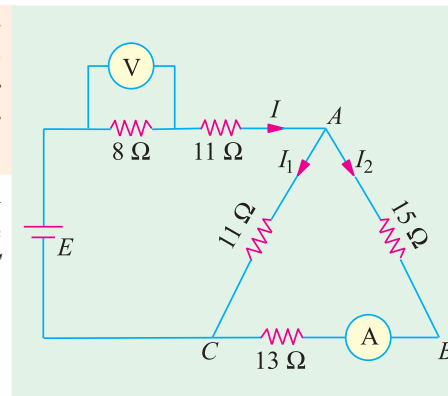
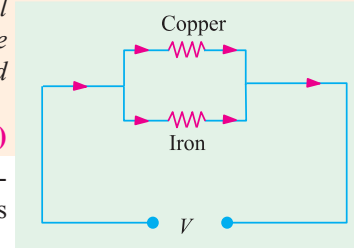


Fig. 1.50

## 1.26. Equivalent Resistance

The equivalent resistance of a circuit (or network) between its any two points (or terminals) is given by that **single** resistance which can replace the entire given circuit between **these two points**. It should be noted that resistance is always between two **given** points of a circuit and can have different

values for different point-pairs as illustrated by Example 1.42. It can usually be found by using series and parallel laws of resistances. Concept of equivalent resistance is essential for understanding network theorems like Thevenin's theorem and Norton's theorem etc. discussed in Chapter 2.

**Example 1.43.** Find the equivalent resistance of the circuit given in Fig. 1.51 (a) between the following points (i) A and B (ii) C and D (iii) E and F (iv) A and F and (v) A and C. Numbers represent resistances in ohm.

**Solution. (i) Resistance Between A and B**

In this case, the entire circuit to the right side of AB is in parallel with 1 Ω resistance connected directly across points A and B.

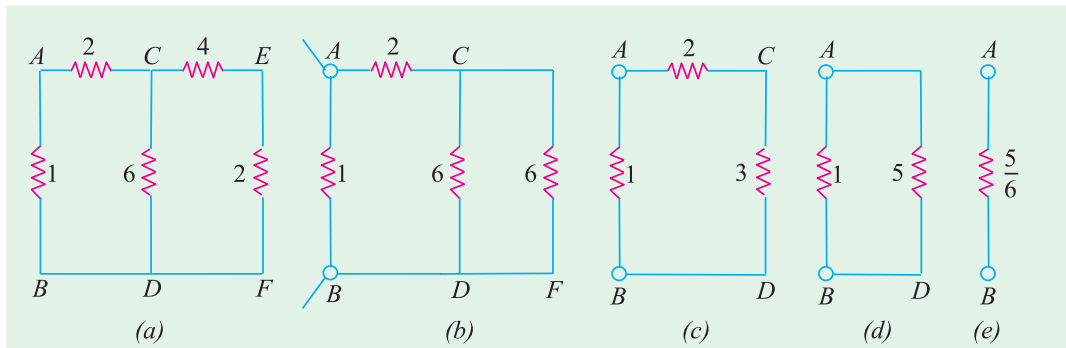


Fig. 1.51

As seen, there are two parallel paths across points C and D; one having a resistance of 6 Ω and the other of  $(4 + 2) = 6 \Omega$ . As shown in Fig. 1.51 (c), the combined resistance between C and D is  $= 6 \parallel 6 = 3 \Omega$ . Further simplifications are shown in Fig. 1.51 (d) and (e). As seen,  $R_{AD} = 5/6 \Omega$

**(ii) Resistance between C and D**

As seen from Fig. 1.51 (a), there are three parallel paths between C and D (i) CD itself of 6 Ω (ii) CEFD of  $(4 + 2) = 6 \Omega$  and (iii) CABD of  $(2 + 1) = 3 \Omega$ . It has been shown separately in Fig. 1.52 (a). The equivalent resistance  $R_{CD} = 3 \parallel 6 \parallel 6 = 1.5 \Omega$  as shown in Fig. 1.52 (b).

**(iii) Resistance between E and F**

In this case, the circuit to the left side of EF is in parallel with the 2 Ω resistance connected directly across E and F. This circuit consists of a 4 Ω resistance connected in series with a parallel

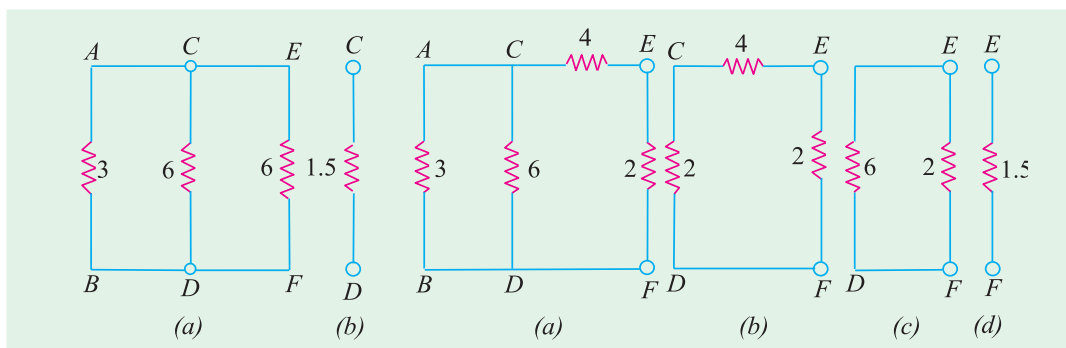


Fig. 1.52

Fig. 1.53

circuit of  $6 \parallel (2 + 1) = 2 \Omega$  resistance. After various simplifications as shown in Fig. 1.53,  $R_{EF} = 2 \parallel 6 = 1.5 \Omega$

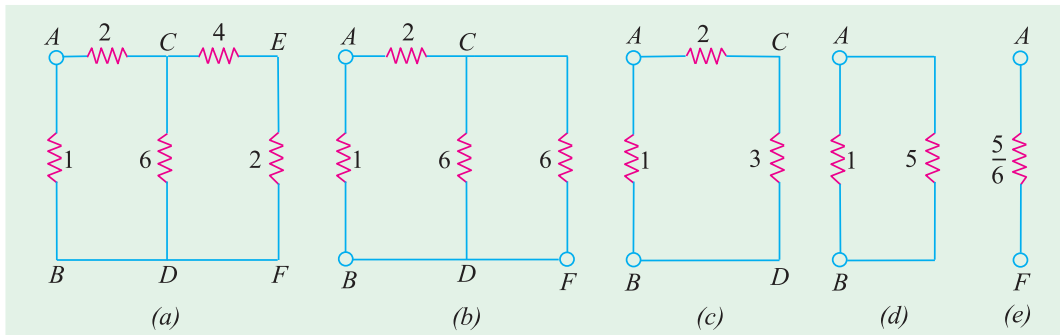


Fig. 1.54

**(iv) Resistance Between A and F**

As we go from  $A$  and  $F$ , there are two possible routes to begin with: one along  $ABDF$  and the other along  $AC$ . At point  $C$ , there are again two alternatives, one along  $CDF$  and the other along  $CEF$ .

As seen from Fig. 1.54 (b),  $R_{CD} = 6 \parallel 6 = 3 \Omega$ . Further simplification of the original circuit as shown in Fig. 1.54 (c), (d) and (e) gives  $R_{AF} = 5/6 \Omega$

**(v) Resistance Between A and C**

In this case, there are two parallel paths between  $A$  and  $C$ ; one is directly from  $A$  to  $C$  and the other is along  $ABD$ . At  $D$ , there are again two parallel paths to  $C$ ; one is directly along  $DC$  and the other is along  $DFEC$ .

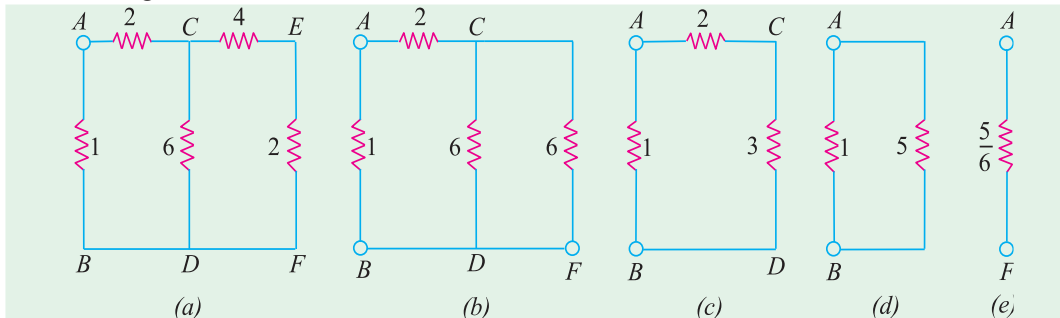


Fig. 1.55

As seen from Fig. 1.55 (b),  $R_{CD} = 6 \parallel 6 = 3 \Omega$ . Again, from Fig. 1.55 (d),  $R_{AC} = 2 \parallel 4 = 4/3 \Omega$

**Example 1.44.** Two resistors of values  $1 \text{ k}\Omega$  and  $4 \text{ k}\Omega$  are connected in series across a constant voltage supply of  $100 \text{ V}$ . A voltmeter having an internal resistance of  $12 \text{ k}\Omega$  is connected across the  $4 \text{ k}\Omega$  resistor. Draw the circuit and calculate

- true voltage across  $4 \text{ k}\Omega$  resistor before the voltmeter was connected.
- actual voltage across  $4 \text{ k}\Omega$  resistor after the voltmeter is connected and the voltage recorded by the voltmeter.
- change in supply current when voltmeter is connected.
- percentage error in voltage across  $4 \text{ k}\Omega$  resistor.

**Solution.** (a) True voltage drop across  $4 \text{ k}\Omega$  resistor as found by voltage-divider rule is  $100 \times 4/5 = 80 \text{ V}$

Current from the supply =  $100/(4 + 1) = 20 \text{ mA}$

(b) In Fig. 1.56, voltmeter has been joined across the  $4 \text{ k}\Omega$  resistor. The equivalent resistance between  $B$  and  $C = 4 \times 12/16 = 3 \text{ k}\Omega$

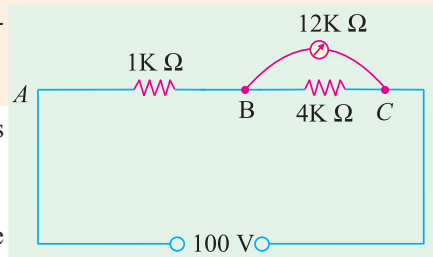


Fig. 1.56



Drop across  $B$  and  $C = 100 \times 3/(3 + 1) = 75$  V.

(c) Resistance between  $A$  and  $C = 3 + 1 = 4 \text{ k}\Omega$

New supply current =  $100/4 = 25 \text{ mA}$

$\therefore$  increase in current =  $25 - 20 = 5 \text{ mA}$

(d) Percentage error in voltage =  $\frac{\text{actual voltage} - \text{true voltage}}{\text{true voltage}} = \frac{(75 - 80)}{80} \times 100 = -6.25\%$

The reduction in the value of voltage being measured is called voltmeter loading effect because voltmeter loads down the circuit element across which it is connected. Smaller the voltmeter resistance as compared to the resistance across which it is connected, greater the loading effect and, hence, greater the error in the voltage reading. Loading effect cannot be avoided but can be minimized by selecting a voltmeter of resistance much greater than that of the network across which it is connected.

**Example 1.45.** In the circuit of Fig. 1.57, find the value of supply voltage  $V$  so that  $20\text{-}\Omega$  resistor can dissipate  $180 \text{ W}$ .

**Solution.**  $I_4^2 \times 20 = 180 \text{ W}; I_4 = 3 \text{ A}$

Since  $15 \text{ }\Omega$  and  $20 \text{ }\Omega$  are in parallel,

$$I_3 \times 15 = 3 \times 20 \quad \therefore I_3 = 4 \text{ A}$$

$$I_2 = I_3 + I_4 = 4 + 3 = 7 \text{ A}$$

Now, resistance of the circuit to the right of point  $A$  is

$$= 10 + 15 \times 20/35 = 130/7 \text{ }\Omega$$

$$\therefore I_1 \times 25 = 7 \times 130/7$$

$$\therefore I_1 = 26/5 \text{ A} = 5.2 \text{ A}$$

$$\therefore I = I_1 + I_2 = 5.2 + 7 = 12.2 \text{ A}$$

Total circuit resistance

$$R_{AE} = 5 + 25 \parallel 130/7 = 955/61 \text{ }\Omega$$

$$\therefore V = I \cdot R_{AE} = 12.2 \times 955/61 = 191 \text{ V}$$

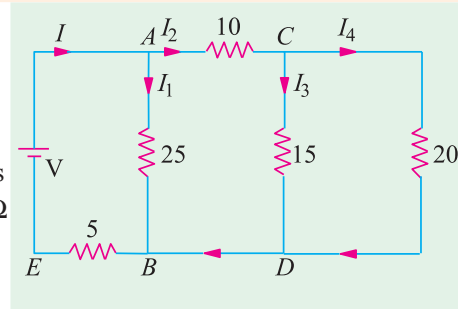


Fig. 1.57

**Example 1.46.** For the simple ladder network shown in Fig. 1.58, find the input voltage  $V_i$  which produces a current of  $0.25 \text{ A}$  in the  $3 \text{ }\Omega$  resistor. All resistances are in ohm.

**Solution.** We will assume a current of  $1 \text{ A}$  in the  $3 \text{ }\Omega$  resistor. The voltage necessary to produce  $1 \text{ A}$  bears the same ratio to  $1 \text{ A}$  as  $V_i$  does to  $0.25 \text{ A}$  because of the linearity of the network. It is known as Current Assumption technique.

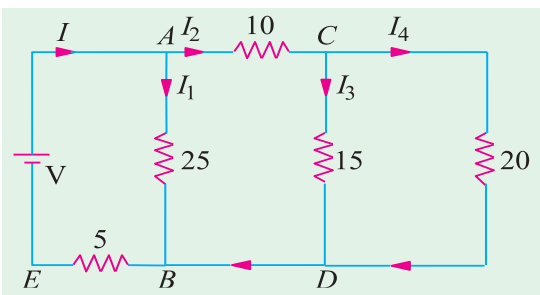


Fig. 1.58

Taking the proportion, we get

$$\frac{80}{1} = \frac{V_i}{0.25} \quad \therefore V_i = 80 \times 0.25 = 20 \text{ V}$$

Since  $R_{cdef} = R_{cf} = 6 \text{ }\Omega$

Hence,  $I_{cf} = 1 \text{ A}$

and  $V_{cf} = V_{cdef} = 1 \times 6 = 6 \text{ V}$ .

Also,  $I_{bc} = 1 + 1 = 2 \text{ A}$

$$V_{bg} = V_{bb} + V_{ef} = 2 \times 5 + 6 = 16 \text{ V}$$

$$I_{bg} = 16/8 = 2 \text{ A}$$

$$I_{ab} = I_{bc} + I_{bg} = 2 + 2 = 4 \text{ A}$$

$$V_i = V_{ab} + V_{bg} + V_{gh} = 4 \times 7 + 16 + 4 \times 9 = 80 \text{ V}$$

**Example 1.47.** In this circuit of Fig. 1.59, find the value  $R_1$  and  $R_2$  so that  $I_2 = I_1/n$  and the input resistance as seen from points  $A$  and  $B$  is  $R$  ohm.

**Solution.** As seen, the current through  $R_2$  is  $(I_1 - I_2)$ . Hence, p.d. across points  $C$  and  $D$  is  $R_2(I_1 - I_2) = (R_1 + R)I_2$  or  $R_2 I_1 = (R_1 + R_2 + R)I_2$

$$\therefore \frac{I_1}{I_2} = \frac{R_1 + R_2 + R}{R_2} = n \quad \dots(i)$$

The input resistance of the circuit as viewed from terminals  $A$  and  $B$  is required to be  $R$ .

$$\begin{aligned} \therefore R &= R_1 + R_2 \parallel (R_1 + R) \\ &= R_1 + \frac{R_1 + R}{n} \quad \dots \text{using Eq. (i)} \end{aligned}$$

$$R(n-1) = R_1(n+1)$$

$$\therefore R_1 = \frac{n-1}{n+1}R \quad \text{and} \quad R_2 = \frac{R_1 + R}{(n-1)} = \frac{2n}{n^2 - 1}R$$

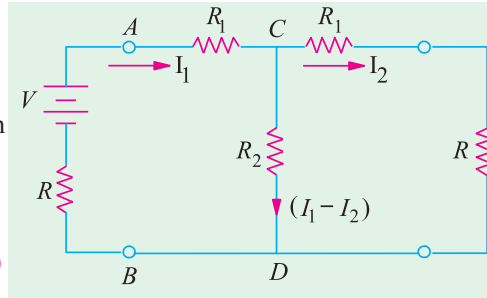


Fig. 1.59

### 1.27. Duality Between Series and Parallel Circuits

There is a certain peculiar pattern of relationship between series and parallel circuits. For example, in a series circuit, current is the same whereas in a parallel circuit, voltage is the same. Also, in a series circuit, individual voltages are added and in a parallel circuit, individual currents are added. It is seen that while comparing series and parallel circuits, voltage takes the place of current and current takes the place of voltage. Such a pattern is known as “duality” and the two circuits are said to be duals of each other.

As arranged in Table 1.4 the equations involving voltage, current and resistance in a series circuit have a corresponding dual counterparts in terms of current, voltage and conductance for a parallel circuit.

Table 1.4

Series Circuit	Parallel Circuit
$I_1 = I_2 = I_3 = \dots\dots\dots$	$V_1 = V_2 = V_3 = \dots\dots\dots$
$V_T = V_1 + V_2 + V_3 + \dots\dots\dots$	$I_1 = I_1 + I_2 + I_3 + \dots\dots\dots$
$R_T = R_1 + R_2 + R_3 + \dots\dots\dots$	$G_T = G_1 + G_2 + G_3 + \dots\dots\dots$
$I = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} = \dots\dots\dots$	$V = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} = \dots\dots\dots$
Voltage Divider Rule $V_1 = V_T \frac{R_1}{R_T}, V_2 = V_T \frac{R_2}{R_T}$	Current Divider Rule $I_1 = I_T \frac{G_1}{G_T}, I_2 = I_T \frac{G_2}{G_T}$

#### Tutorial Problems No. 1.4

- Using the current-divider rule, find the ratio  $I_L/I_S$  in the circuit shown in Fig. 1.60. [0.25]
- Find the values of variables indicated in the circuit of Fig. 1.61. All resistances are in ohms.

(a) 40 V (b) 21 V; 15 V (c) -5 A; 3 A

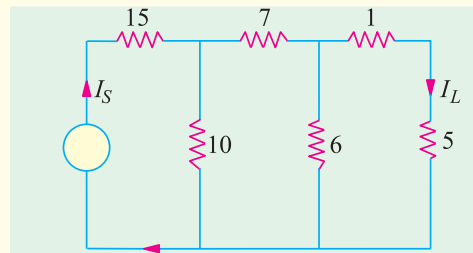


Fig. 1.60

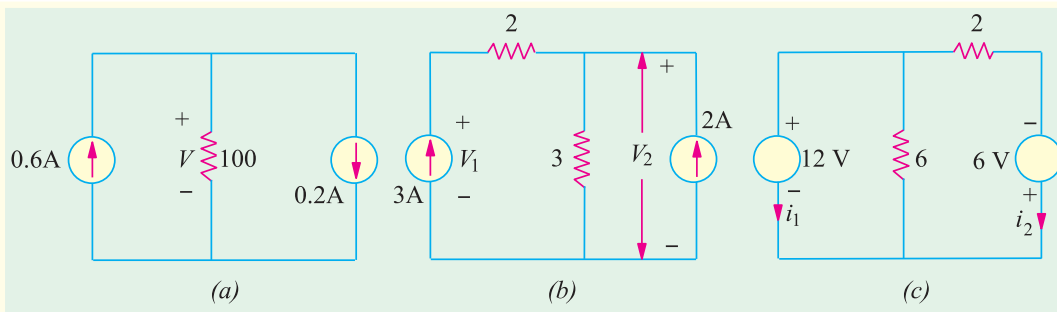


Fig. 1.61

3. An ohmmeter is used for measuring the resistance of a circuit between its two terminals. What would be the reading of such an instrument used for the circuit of Fig. 1.62 at point (a)  $AB$  (b)  $AC$  and (c)  $BC$ ? All resistances are in ohm.

**[(a) 25  $\Omega$  (b) 24  $\Omega$  (c) 9  $\Omega$ ]**

4. Find the current and power supplied by the battery to the circuit of Fig. 1.63 (i) under normal conditions and (ii) when a 'short' occurs across terminals  $A$  and  $B$ . All resistances are in kilo-ohm.

**[(i) 2 mA; 24 mW; (ii) 34 mA; 36 mW]**

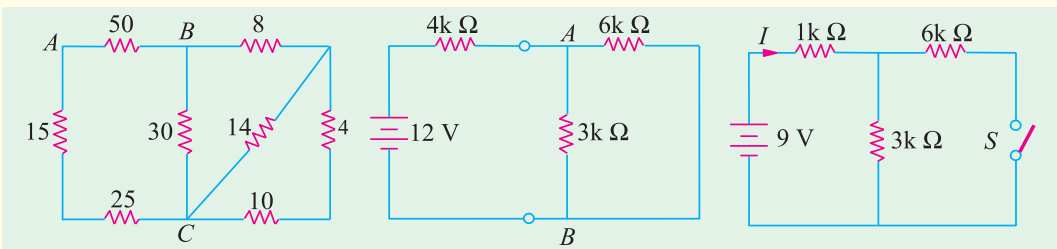


Fig. 1.62

Fig. 1.63

Fig. 1.64

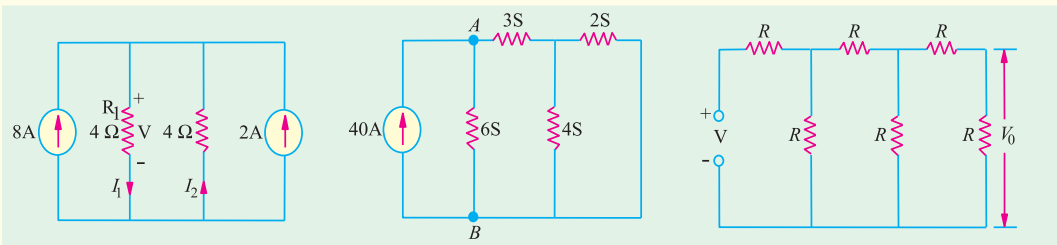


Fig. 1.65

Fig. 1.66

Fig. 1.67

5. Compute the values of battery current  $I$  and voltage drop across 6 k $\Omega$  resistor of Fig. 1.64 when switch  $S$  is (a) closed and (b) open. All resistance values are in kilo-ohm.

**[(a) 3mA; 6 V; (b) 2.25 mA; 0V]**

6. For the parallel circuit of Fig. 1.65 calculate (i)  $V$  (ii)  $I_1$  (iii)  $I_2$ .

**[(i) 20 V; (ii) 5 A; (iii) -5 A]**

7. Find the voltage across terminals  $A$  and  $B$  of the circuit shown in Fig. 1.66. All conductances are in siemens (S).

**[5 V]**

8. Prove that the output voltage  $V_0$  in the circuit of Fig. 1.67 is  $V/13$ .

9. A fault has occurred in the circuit of Fig. 1.68. One resistor has burnt out and has become an open. Which is the resistor if current supplied by the battery is 6 A? All resistances are in ohm.

**[4  $\Omega$ ]**

10. In Fig. 1.69 if resistance between terminals  $A$  and  $B$  measures 1000  $\Omega$ , which resistor is open-circuited. All conductance values are in milli-siemens (mS).

**[0.8 mS]**

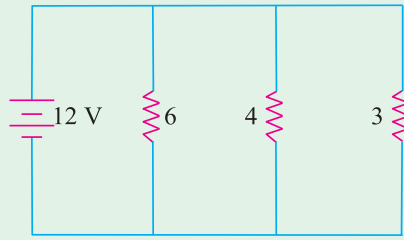


Fig. 1.68

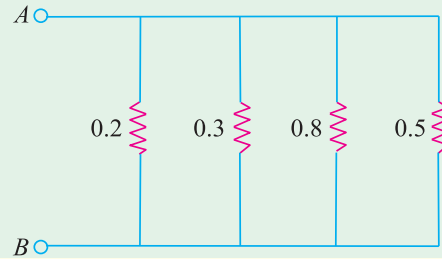


Fig. 1.69

11. In the circuit of Fig. 1.70, find current (a)  $I$  and (b)  $I_1$ . [(a) 2 A; (b) 0.5 A]

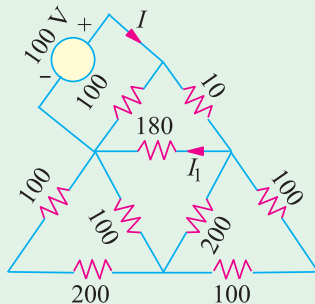


Fig. 1.70

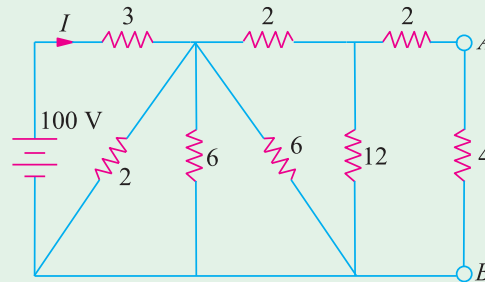


Fig. 1.71

12. Deduce the current  $I$  in the circuit of Fig. 1.71. All resistances are in ohms. [25 A]
13. Two resistors of  $100\ \Omega$  and  $200\ \Omega$  are connected in series across a 4-V cell of negligible internal resistance. A voltmeter of  $200\ \Omega$  resistance is used to measure P.D. across each. What will the voltage be in each case? [1 V across  $100\ \Omega$ ; 2 V across  $200\ \Omega$ ]
14. Using series-parallel combination laws, find the resistance between terminals  $A$  and  $B$  of the network shown in Fig. 1.72. [4 R]

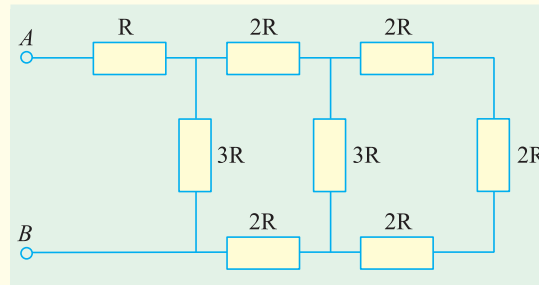


Fig. 1.72

15. A resistance coil  $AB$  of  $100\ \Omega$  resistance is to be used as a potentiometer and is connected to a supply at 230 V. Find, by calculation, the position of a tapping point  $C$  between  $A$  and  $B$  such that a current of 2 A will flow in a resistance of  $50\ \Omega$  connected across  $A$  and  $C$ . [43.4  $\Omega$  from  $A$  to  $C$ ] (London Univ.)
16. In the circuit shown in Fig. 1.73, calculate (a) current  $I$  (b) current  $I_1$  and (c)  $V_{AB}$ . All resistances are in ohms. [(a) 4 A (b) 0.25 A (c) 4 V]

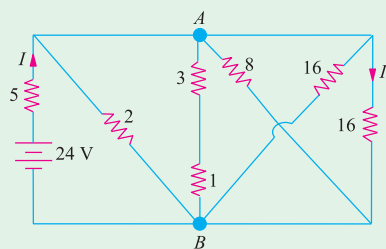


Fig. 1.73

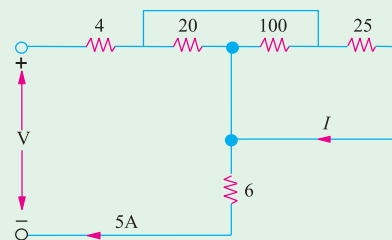


Fig. 1.74

17. In the circuit given in Fig. 1.74, calculate (a) current through the  $25\ \Omega$  resistor (b) supply voltage  $V$ . All resistances are in ohms. **[(a) 2 A (b) 100 V]**
18. Using series and parallel combinations for the electrical network of Fig. 1.75, calculate (a) current flowing in branch  $AF$  (b) p.d. across branch  $CD$ . All resistances are in ohms. **[(a) 2 A (b) 1.25 V]**

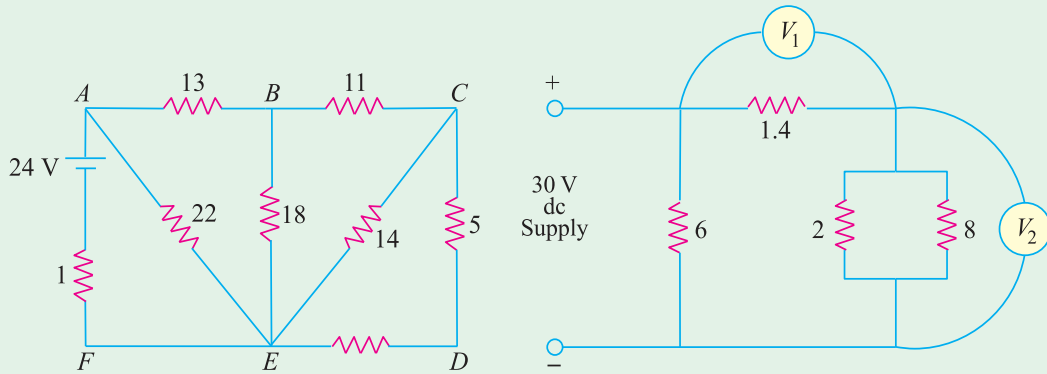


Fig. 1.75

Fig. 1.76

19. Neglecting the current taken by voltmeters  $V_1$  and  $V_2$  in Fig. 1.76, calculate (a) total current taken from the supply (b) reading on voltmeter  $V_1$  and (c) reading on voltmeter  $V_2$ .

**[(a) 15 A (b) 14 V (c) 16 V]**

20. Find the equivalent resistance between terminals  $A$  and  $B$  of the circuit shown in Fig. 1.77. Also, find the value of currents  $I_1$ ,  $I_2$ , and  $I_3$ . All resistances are in ohm.

**[8  $\Omega$ ;  $I_1 = 2$  A;  $I_2 = 0.6$  A;  $I_3 = 0.4$  A]**

21. In Fig. 1.78, the  $10\ \Omega$  resistor dissipates 360 W. What is the voltage drop across the  $5\ \Omega$  resistor? **[30 V]**

22. In Fig. 1.79, the power dissipated in the  $10\ \Omega$  resistor is 250 W. What is the total power dissipated in the circuit? **[850 W]**

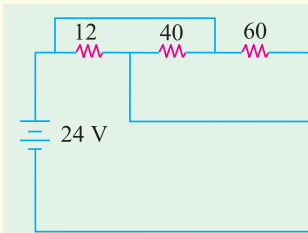


Fig. 1.77

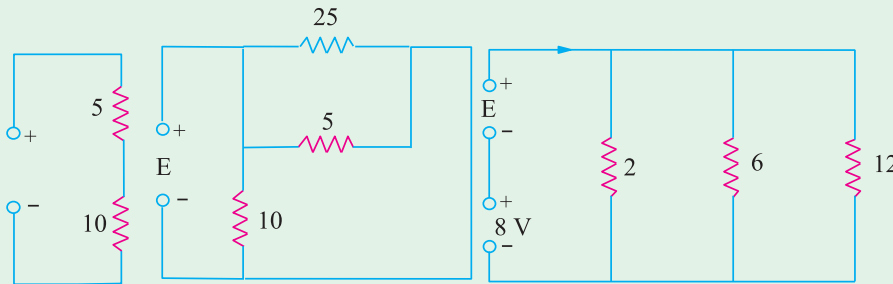


Fig. 1.78

Fig. 1.79

Fig. 1.80

23. What is the value of  $E$  in the circuit of Fig. 1.80? All resistances are in ohms. **[4 V]**

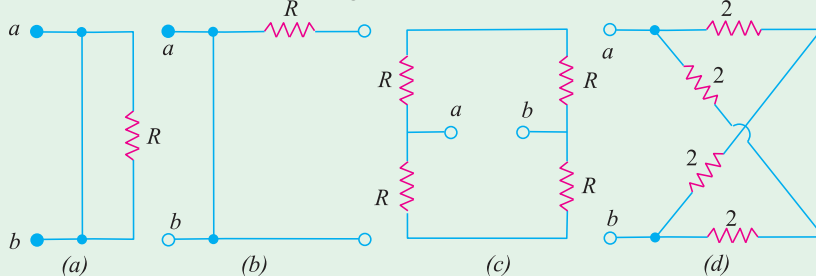


Fig. 1.81

24. Find the equivalent resistance  $R_{a-b}$  at the terminals  $a-b$  of the networks shown in Fig. 1.81.

[(a) 0 (b) 0 (c)  $R$  (d)  $2\ \Omega$ ]

25. Find the equivalent resistance between terminals  $a$  and  $b$  of the circuit shown in Fig. 1.82 (a). Each resistor has a value of  $1\ \Omega$ .

[5/11  $\Omega$ ]

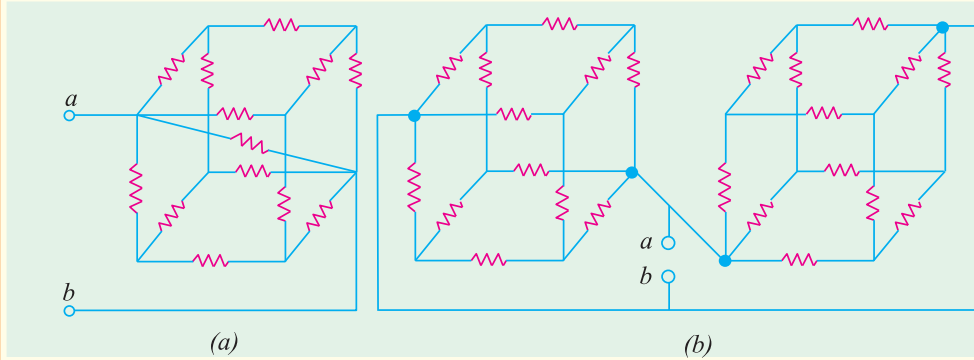


Fig. 1.82

26. Find the equivalent resistance between terminals  $a$  and  $b$  of the circuit shown in Fig. 1.82 (b). Each resistor has a value of  $1\ \Omega$ .

[5/12  $\Omega$ ]

27. Two resistors of value  $1000\ \Omega$  and  $4000\ \Omega$  are connected in series across a constant voltage supply of  $150\ \text{V}$ . Find (a) p.d. across  $4000\ \Omega$  resistor (b) calculate the change in supply current and the reading on a voltmeter of  $12,000\ \Omega$  resistance when it is connected across the larger resistor.

[(a)  $120\ \text{V}$  (b)  $7.5\ \text{mA}$ ;  $112.5\ \text{V}$ ]

## 1.28. Relative Potential

It is the voltage of one point in a circuit with respect to that of another point (usually called the reference or common point).

Consider the circuit of Fig. 1.83 (a) where the most negative end-point  $C$  has been taken as the reference. With respect to point  $C$ , both points  $A$  and  $B$  are positive though  $A$  is more positive than  $B$ . The voltage of point  $B$  with respect to that of  $C$  i.e.  $V_{BC} = +30\ \text{V}$ .

Similarly,  $V_{AC} = +(20 + 30) = +50\ \text{V}$ .

In Fig. 1.83 (b), the most positive end point  $A$  has been taken as the reference point. With respect to  $A$ , both  $B$  and  $C$  are negative though  $C$  is more negative than  $B$ .

$V_{BA} = -20\ \text{V}$ ,  $V_{CA} = -(20 + 30) = -50\ \text{V}$

In Fig. 1.83 (c), mid-point  $B$  has been taken as the reference point. With respect to  $B$ ,  $A$  is at positive potential whereas  $C$  is at a negative potential.

Hence,  $V_{AB} = +20\ \text{V}$  and  $V_{CB} = -30\ \text{V}$  (of course,  $V_{BC} = +30\ \text{V}$ )

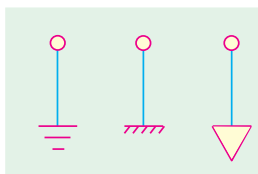


Fig. 1.84

It may be noted that **any point** in the circuit can be chosen as the reference point to suit our requirements. This point is often called **ground** or **earth** because originally it meant a point in a circuit which was **actually** connected to earth either for safety in power systems or for efficient radio reception and transmission. Although, this meaning still exists, yet it has become usual today for **'ground'** to mean any point in the circuit which is connected to a large metallic object such as the metal chassis of a transmit-

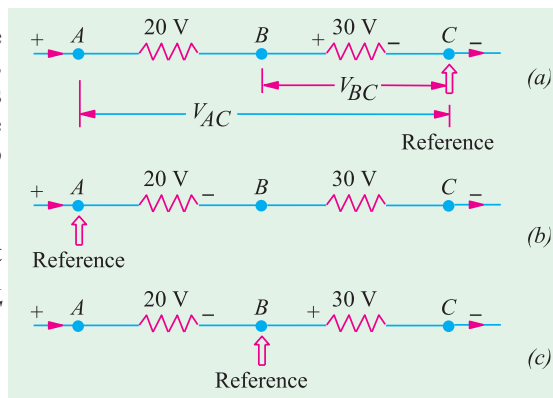


Fig. 1.83

ter, the aluminium chassis of a receiver, a wide strip of copper plating on a printed circuit board, frame or cabinet which supports the whole equipment. Sometimes, reference point is also called **common** point. The main advantage of using a ground system is to simplify our circuitry by saving on the amount of wiring because ground is used as the return path for many circuits. The three commonly-used symbols for **ground** are shown in Fig. 1.84.

**Example 1.48.** In Fig. 1.85, calculate the values of (i)  $V_{AF}$  (ii)  $V_{EA}$  and (iii)  $V_{FB}$ .

**Solution.** It should be noted that  $V_{AF}$  stands for the potential of point  $A$  with respect to point  $F$ . The easiest way of finding it is to start from the reference point  $F$  and go to point  $A$  along any available path and calculate the algebraic sum of the voltages met on the way. Starting from point  $F$  as we go to point  $A$ , we come across different battery voltages. Taking the sign convention given in Art. 1.28, we get

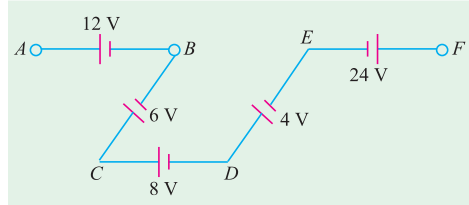


Fig. 1.85

$$(i) \quad V_{AF} = -24 + 4 + 8 - 6 + 12 = -6 \text{ V}$$

The negative sign shows that point  $A$  is negative with respect to point  $F$  by 6 V.

$$(ii) \quad \text{Similarly,} \quad V_{EA} = -12 + 6 - 8 - 4 = -18 \text{ V}$$

$$(iii) \quad \text{Starting from point } B, \text{ we get } V_{FB} = 6 - 8 - 4 + 24 = 18 \text{ V.}$$

Since the result is positive it means that point  $F$  is at a higher potential than point  $B$  by 18 V.

**Example 1.49.** In Fig. 1.86 compute the relative potentials of points  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  which (i) point  $A$  is grounded and (ii) point  $D$  is grounded. Does it affect the circuit operation or potential difference between any pair of points?

**Solution.** As seen, the two batteries have been connected in *series opposition*. Hence, net circuit voltage =  $34 - 10 = 24 \text{ V}$   
 Total circuit resistance =  $6 + 4 + 2 = 12 \Omega$   
 Hence, the circuit current =  $24/12 = 2 \text{ A}$   
 Drop across  $2 \Omega$  resistor =  $2 \times 2 = 4 \text{ V}$ , Drop across  $4 \Omega$  resistor =  $2 \times 4 = 8 \text{ V}$   
 Drop across  $6 \Omega$  resistor =  $2 \times 6 = 12 \text{ V}$

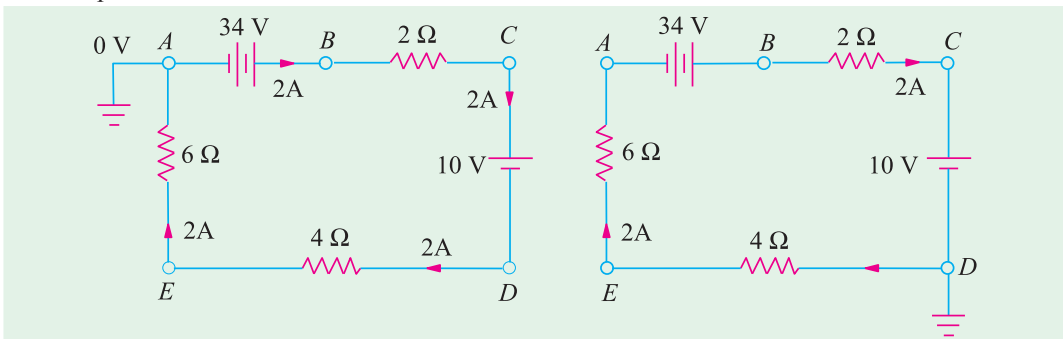


Fig. 1.86

Fig. 1.87

(i) Since point  $B$  is directly connected to the positive terminal of the battery whose negative terminal is earthed, hence  $V_B = +34 \text{ V}$ .

Since there is a fall of 4 V across  $2 \Omega$  resistor,  $V_C = 34 - 4 = 30 \text{ V}$

As we go from point  $C$  to  $D$  i.e. from positive terminal of 10-V battery to its negative terminal, there is a *decrease* in potential of 10 V. Hence,  $V_D = 30 - 10 = 20$  i.e. point  $D$  is 20 V above the ground  $A$ .

Similarly,  $V_E = V_D - \text{voltage fall across } 4 \Omega \text{ resistor} = 20 - 8 = +12 \text{ V}$

Also  $V_A = V_E$  -fall across  $6\ \Omega$  resistor =  $12(2 \times 6) = 0\ \text{V}$

(ii) In Fig. 1.87, point  $D$  has been taken as the ground. Starting from point  $D$ , as we go to  $E$  there is a fall of  $8\ \text{V}$ . Hence,  $V_E = -8\ \text{V}$ . Similarly,  $V_A = -(8 + 12) = -20\ \text{V}$ .

As we go from  $A$  to  $B$ , there is a sudden increase of  $34\ \text{V}$  because we are going from negative terminal of the battery to its positive terminal.

$$\therefore \begin{aligned} V_B &= -20 + 34 = +14\ \text{V} \\ V_C &= V_B - \text{voltage fall across } 2\ \Omega \text{ resistor} = 14 - 4 = +10\ \text{V}. \end{aligned}$$

It should be so because  $C$  is connected directly to the positive terminal of the  $10\ \text{V}$  battery.

Choice of a reference point does not in any way affect the operation of a circuit. Moreover, it also does not change the voltage across any resistor or between any pair of points (as shown below) because the ground current  $i_g = 0$ .

#### Reference Point A

$$\begin{aligned} V_{CA} &= V_C - V_A = 30 - 0 = +30\ \text{V}; V_{CE} = V_C - V_E = 30 - 12 = +18\ \text{V} \\ V_{BD} &= V_B - V_D = 34 - 20 = +14\ \text{V} \end{aligned}$$

#### Reference Point D

$$\begin{aligned} V_{CA} &= V_C - V_A = 10 - (-20) = +30\ \text{V}; V_{CE} = V_C - V_E = 10 - (-8) = +18\ \text{V} \\ V_{BD} &= V_B - V_D = 14 - 0 = +14\ \text{V} \end{aligned}$$

**Example 1.50.** Find the voltage  $V$  in Fig. 1.88 (a). All resistances are in ohms.

**Solution.** The given circuit can be simplified to the final form shown in Fig. 1.88 (d). As seen, current supplied by the battery is  $1\ \text{A}$ . At point  $A$  in Fig. 1.88 (b), this current is divided into two equal parts of  $0.5\ \text{A}$  each.

Obviously, voltage  $V$  represents the potential of point  $B$  with respect to the negative terminal of the battery. Point  $B$  is above the ground by an amount equal to the voltage drop across the series combination of  $(40 + 50) = 90\ \Omega$

$$V = 0.5 \times 90 = 45\ \text{V}.$$

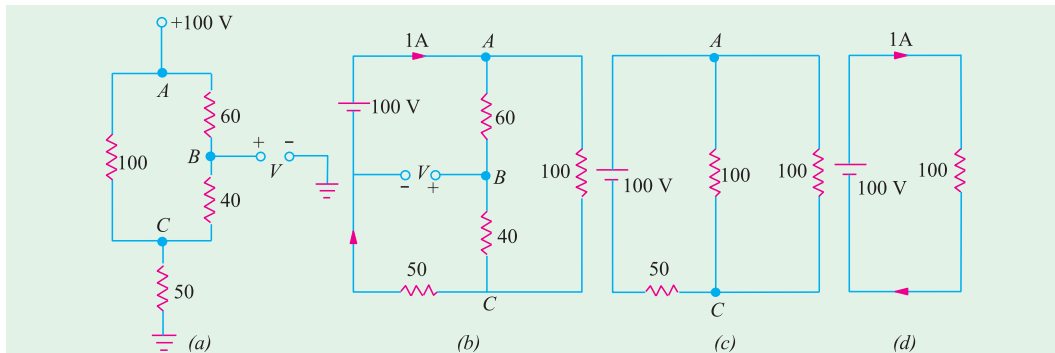


Fig. 1.88

### 1.29. Voltage Divider Circuit

A voltage divider circuit (also called potential divider) is a series network which is used to feed other networks with a number of different voltages and derived from a single input voltage source.

Fig. 1.89 (a) shows a simple voltage divider circuit which provides two output voltages  $V_1$  and  $V_2$ . Since no load is connected across the output terminals, it is called an **unloaded** voltage divider.

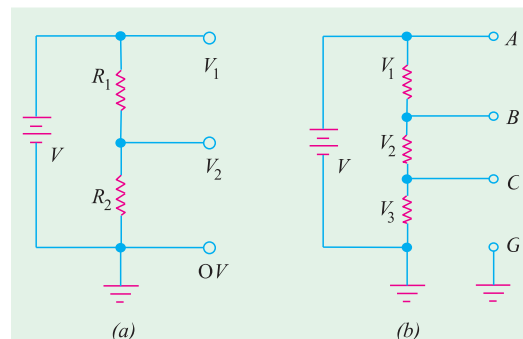


Fig. 1.89



As seen from Art. 1.15.

$$V_1 = V \frac{R_1}{R_1 + R_2} \text{ and } V_2 = V \cdot \frac{R_2}{R_1 + R_2}$$

The ratio  $V_2/V$  is also known as voltage-ratio **transfer function**.

$$\text{As seen, } \frac{V_2}{V} = \frac{R_2}{R_1 + R_2} = \frac{1}{1 + R_1/R_2}$$

The voltage divider of Fig. 1.89 (b) can be used to get six different voltages :

$$V_{CG} = V_3, V_{BC} = V_2, V_{AB} = V_1, V_{BG} = (V_2 + V_3), V_{AC} = (V_1 + V_2) \text{ and } V_{AG} = V$$

**Example 1.51.** Find the values of different voltages that can be obtained from a 12-V battery with the help of voltage divider circuit of Fig. 1.90.

**Solution.**  $R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8 \Omega$   
 Drop across  $R_1 = 12 \times 4/8 = 6 \text{ V}$   
 $\therefore V_B = 12 - 6 = 6 \text{ V}$  above ground  
 Drop across  $R_2 = 12 \times 3/8 = 4.5 \text{ V}$   
 $\therefore V_C = V_B - 4.5 = 6 - 4.5 = 1.5$   
 Drop across  $R_3 = 12 \times 1/8 = 1.5 \text{ V}$

Different available load voltages are :

- (i)  $V_{AB} = V_A - V_B = 12 - 6 = 6 \text{ V}$
- (ii)  $V_{AC} = 12 - 1.5 = 10.5 \text{ V}$
- (iii)  $V_{AD} = 12 \text{ V}$
- (iv)  $V_{BC} = 6 - 1.5 = 4.5 \text{ V}$
- (v)  $V_{CD} = 1.5 \text{ V}$

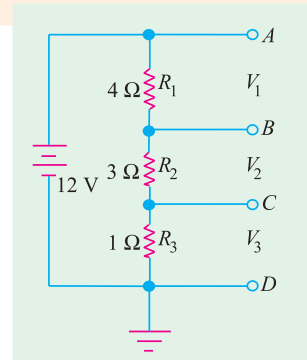


Fig. 1.90

**Example 1.52.** What are the output voltages of the unloaded voltage divider shown in Fig. 1.91 ? What is the direction of current through AB ?

**Solution.** It may be remembered that both  $V_1$  and  $V_2$  are with respect to the ground.

$$R = 6 + 4 + 2 = 12 \Omega$$

$$\therefore V_1 = \text{drop across } R_2 = 24 \times 4/12 = +8 \text{ V}$$

$$V_2 = \text{drop across } R_3 = -24 \times 2/12 = -4 \text{ V}$$

It should be noted that point B is at negative potential with respect to the ground.

Current flows from A to B i.e. from a point at a higher potential to a point at a lower potential.

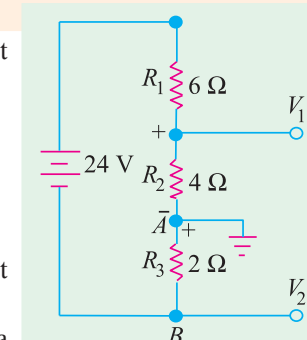


Fig. 1.91

**Example 1.53.** Calculate the potentials of point A, B, C and D in Fig. 1.92. What would be the new potential values if connections of 6-V battery are reversed ? All resistances are in ohm.

**Solution.** Since the two batteries are connected in additive series, total voltage around the circuit is  $= 12 + 6 = 18 \text{ V}$ . The drops across the three resistors as found by the voltage divider rule as shown in Fig. 1.92 (a) which also indicates their proper polarities. The potential of any point in the circuit can be found by starting from the ground point G (assumed to be at 0V) and going to the point either in clockwise direction or counter-clockwise direction. While going around the circuit, the rise in potential would be taken as positive and the fall in potential as negative. (Art. 2.3). Suppose we start from point G and proceed in the clockwise direction to point A. The only potential met on the way is the battery voltage which is taken as positive because there is a rise of potential since we are going from its negative to positive terminal. Hence,  $V_A = +12 \text{ V}$ .

$$V_B = 12 - 3 = 9 \text{ V}; V_C = 12 - 3 - 6 = 3 \text{ V}$$

Similarly,  $V_D = 12 - 3 - 6 - 9 = -6 \text{ V}$ .

It is also obvious that point  $D$  must be at  $-6 \text{ V}$  because it is directly connected to the negative terminal of the 6-V battery.

We would also find the potentials of various points by starting from point  $G$  and going in the counter-clockwise direction. For example,  $V_B = -6 + 9 + 6 = 9 \text{ V}$  as before.

The connections of the 6-V battery have been reversed in Fig. 1.92 (b). Now, the net voltage around the circuit is  $12 - 6 = 6 \text{ V}$ . The drop over the  $1 \Omega$  resistor is  $= 6 \times 1/(1 + 2 + 3) = 1 \text{ V}$ ; Drop over  $2 \Omega$  resistor is  $= 6 \times 2/6 = 2 \text{ V}$ . Obviously,  $V_A = +12 \text{ V}$ ,  $V_B = 12 - 1 = 11 \text{ V}$ ,  $V_C = 12 - 1 - 2 = 9 \text{ V}$ . Similarly,  $V_D = 12 - 1 - 2 - 3 = +6 \text{ V}$ .

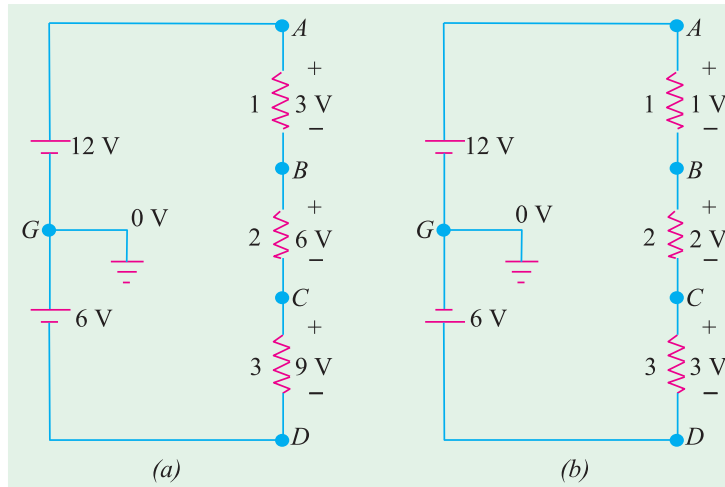


Fig. 1.92

**Example 1.54.** Using minimum number of components, design a voltage divider which can deliver  $1 \text{ W}$  at  $100 \text{ V}$ ,  $2 \text{ W}$  at  $-50 \text{ V}$  and  $1.6 \text{ W}$  at  $-80 \text{ V}$ . The voltage source has an internal resistance of  $200 \Omega$  and supplies a current of  $100 \text{ mA}$ . What is the open-circuit voltage of the voltage source? All resistances are in ohm.

**Solution.** From the given load conditions, the load currents are as follows:

$$I_{L1} = 1/100 = 10 \text{ mA},$$

$$I_{L2} = 2/50 = 40 \text{ mA},$$

$$I_{L3} = 1.6/80 = 20 \text{ mA}$$

For economising the number of components, the internal resistance of  $200 \Omega$  can be used as the series dropping resistance. The suitable circuit and the ground connection are shown in Fig. 1.93.

Applying Kirchoff's laws to the closed circuit  $ABCD$ , we have

$$V - 200 \times 100 \times 10^{-3} - 100 - 80 = 0 \quad \text{or} \quad V = 200 \text{ V}$$

$$I_1 = 100 - 10 = 90 \text{ mA} \quad \therefore R_1 = 100 \text{ V}/90 \text{ mA} = 1.11 \text{ k}\Omega$$

$$I_3 = 100 - 20 = 80 \text{ mA}; \text{ voltage drop across } R_3 = -50 - (-80) = 30 \text{ V}$$

$$\therefore R_3 = 30 \text{ V}/80 \text{ mA} = 375 \Omega$$

$$I_2 + 40 = 80 \quad \therefore I_2 = 40 \text{ mA}; R_2 = 50 \text{ V}/40 \text{ mA} = 1.25 \text{ k}\Omega$$

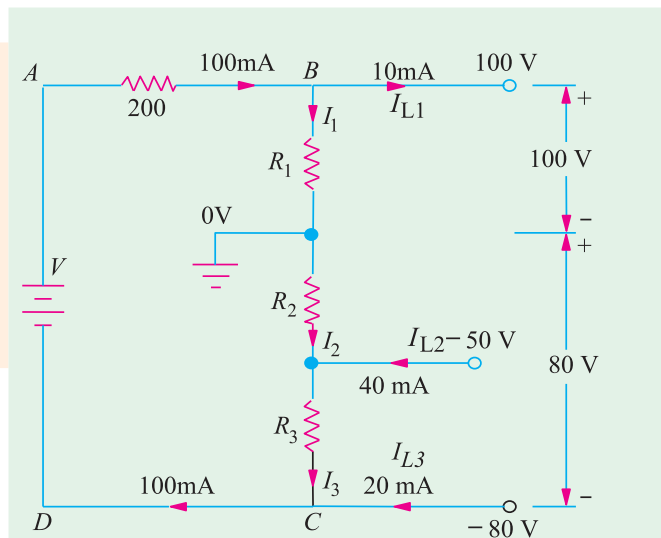


Fig. 1.93

**Example 1.55.** Fig. 1.94 shows a transistor with proper voltages established across its base, collector and emitter for proper function. Assume that there is a voltage drop  $V_{BE}$  across the base-emitter junction of  $0.6\text{ V}$  and collector current  $I_C$  is equal to collector current  $I_E$ . Calculate (a)  $V_1$  (b)  $V_2$  and  $V_B$  (c)  $V_4$  and  $V_E$  (d)  $I_E$  and  $I_C$  (e)  $V_3$  (f)  $V_C$  (g)  $V_{CE}$ . All resistances are given in kilo-ohm.

**Solution.** (a) The  $250\text{ k}$  and  $50\text{ k}$  resistors form a voltage-divider bias network across  $20\text{ V}$  supply.

$$\therefore V_1 = 20 \times \frac{250}{300} = 16.7\text{ V}$$

$$(b) V_2 = 20 - 16.7 = 3.3\text{ V}$$

The voltage of point B with respect to ground is  $V_2 = 3.3\text{ V}$

$$(c) V_E = V_2 - V_{BE} = 3.3 - 0.6 = 2.7\text{ V. Also } V_4 = 2.7\text{ V}$$

$$(d) I_E = V_4 / 2 = 2.7\text{ V} / 2\text{ k} = 1.35\text{ mA. It also equals } I_C.$$

$$(e) V_3 = \text{drop across collector resistor} = 1.35\text{ mA} \times 8\text{ k} = 10.8\text{ V}$$

$$(f) \text{Potential of point C is } V_C = 20 - 10.8 = 9.2\text{ V}$$

$$(g) V_{CE} = V_C - V_E = 9.2 - 2.7 = 6.5\text{ V}$$

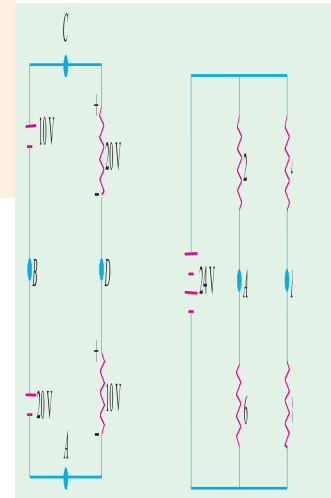


Fig. 1.94

### Tutorial Problems No. 1.5

- A direct-current circuit comprises two resistors, 'A' of value  $25\text{ ohms}$ , and 'B' of unknown value, connected in parallel, together with a third resistor 'C' of value  $5\text{ ohms}$  connected in series with the parallel group. The potential difference across C is found to be  $90\text{ V}$ . If the total power in the circuit is  $4320\text{ W}$ , calculate :
  - the value of resistor B,
  - the voltage applied to the ends of the whole circuit,
  - the current in each resistor.

*(Mumbai University 2002) (Nagpur University, Summer 2002)*
- A current of  $5\text{ A}$  flows through a non inductive resistance connected in series with a choke coil when supplied at  $250\text{ V}$ ,  $50\text{ Hz}$ . If voltage across resistance is  $125\text{ V}$  and across coil is  $200\text{ V}$  calculate :
  - impedance, resistance and reactance of coil
  - power in coil
  - total power consumed in the circuit
  - draw phasor diagram.

*(Pune University 2002) (Nagpur University, Winter 2003)*
- Define temp. coefficient of resistance.

$$\text{Prove } \alpha_{t_1} = \frac{\alpha_0}{(1 + \alpha_0 t_1)}$$

where  $\alpha_0$  = temp. coeff. of resistance at  $0^\circ\text{C}$ .

*(Gujrat University, Summer 2003)*

- A resistance wire  $10\text{ m}$  long and cross section area  $10\text{ mm}^2$  at  $0^\circ\text{C}$  passes a current of  $10\text{ A}$ , when connected to a d.c. supply of  $200\text{ volts}$ . Calculate :
  - resistivity of the material
  - current which will flow through the wire when the temp. rises to  $50^\circ\text{C}$ . Given  $\alpha_0 = 0.0003\text{ per }^\circ\text{C}$ .

*(Mumbai University, 2003) (Gujrat University, Summer 2003)*
- Why domestic appliances are connected in parallel ? Give comparison with series ckt.
 

*(B.P.T.U., Orissa 2003) (Gujrat University, Summer 2003)*
- Two wires A and B made up of same material, wire B has twice the length of wire A and having half the diameter to that of A. Calculate the ratio  $R_B/R_A$ .
 

*(Gujrat University, Summer 2003)*
- A resistor of  $12\ \Omega$  is connected in series with a combination of  $15\ \Omega$  and  $20\ \Omega$  resistor in parallel. When a voltage of  $120\text{ V}$  is applied across the whole circuit find the current taken from the supply.
 

*(V.T.U., Belgaum, Karnataka University, Summer 2002)*

8. A network is arranged as shown in Fig. 1.95. Determine the value of currents in each resistor.  
(V.T.U., Belgaum, Karnataka University, Summer 2002)

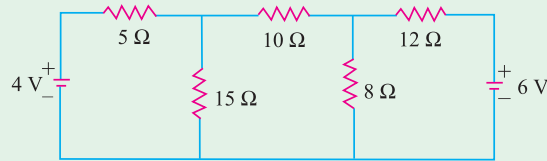


Fig. 1.95

9. A resistance of  $100\Omega$  is connected in series with  $100\mu\text{F}$  capacitor across  $200\text{V}$ ,  $60\text{Hz}$  supply. Find the impedance, current and power factor.  
(V.T.U., Belgaum, Karnataka University, Summer 2002)
10. An EMF whose instantaneous value is  $100\sin(314t - \pi/4)$  volts is applied to a circuit and the current flowing through it is  $20\sin(314t - 1.5708)$  Amperes. Find the frequency and the values of circuit elements, assuming a series combination of circuit elements.  
(V.T.U., Belgaum, Karnataka University, Winter 2003)
11. An inductive coil draws a current of  $2\text{A}$ , when connected to a  $230\text{V}$ ,  $50\text{Hz}$  supply. The power taken by the coil is  $100$  watts. Calculate the resistance and inductance of the coil.  
(Pune University, 2003) (V.T.U., Belgaum University, Winter 2003)
12. Find the resistance between the terminals A and B for the network shown in Fig.1.96.  
(Pune University, 2003) (V.T.U., Belgaum University, Winter 2003)

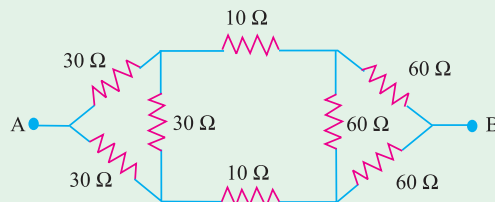


Fig. 1.96

13. A network is arranged as shown in Fig 1.97 Determine the current in each resistance using loop current method.  
(V.T.U., Belgaum, Karnataka University, Winter 2003)

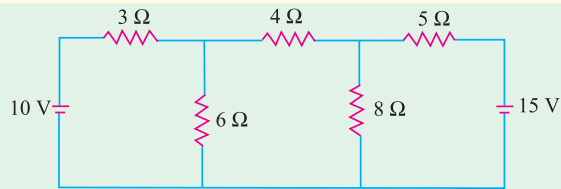


Fig. 1.97

14. A resistor of  $12\Omega$  is connected in series with a combination of  $15\Omega$  and  $20\Omega$  resistor in parallel. When a voltage of  $120\text{V}$  is applied across the whole circuit. Find the current taken from the supply.  
(V.T.U., Belgaum, Karnataka University, Winter 2004)
15. Four wires a, b, c and d are connected at a common point. The currents flowing in a, b and c towards the common point are  $i_a = 6\sin\left(\omega t + \frac{\pi}{3}\right)$ ,  $i_b = 5\cos\left(\omega t + \frac{\pi}{3}\right)$  and  $i_c = 3\cos\left(\omega t + \frac{2\pi}{3}\right)$ . Determine the current in the fourth wire.  
(V.T.U., Belgaum, Karnataka University, Winter 2004)
16. Two resistors  $R_1 = 2500\Omega$  and  $R_2 = 4000\Omega$  are in series across a  $100\text{V}$  supply. The voltage drop across  $R_1$  and  $R_2$  are successively measured by a voltmeter having a resistance of  $50,000\Omega$  Find the sum of the two readings.  
(V.T.U., Belgaum, Karnataka University, Winter 2004)

17. Explain 'resistance', 'reactance' and 'impedance'. (RGPV, Bhopal December 2002)
18. A 4 ohm resistor is connected to a 10 mH inductor across a 100 V, 50 Hz voltage source. Find input current, voltage drops across resistor and inductor, power factor of the circuit and the real power consumed in the circuit. (Mumbai University 2002) (RGPV, Bhopal December 2003)
19. Define and explain the terms MMF, Reluctance, Permeance, flux density and fringing. (RGPV, Bhopal December 2003)
20. Find the value of resistance (R), if source current is 6 amp and source voltage is 66 V is shown in Fig.1.98 (Pune University 2003) (Nagpur University, Winter 2002)

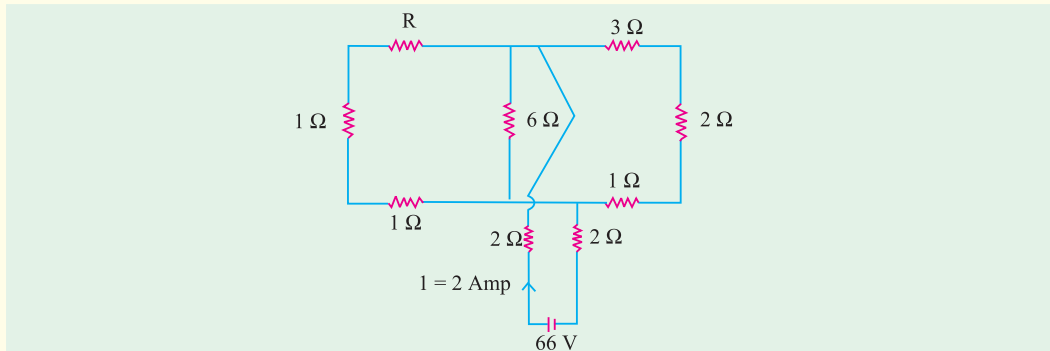


Fig. 1.98

21. Determine a non-negative value of R such that the power consumed by the 2-Ω resistor in the Fig.1.99 is shown maximum. (Pune University 2003)(Engineering Services Examination 2003)

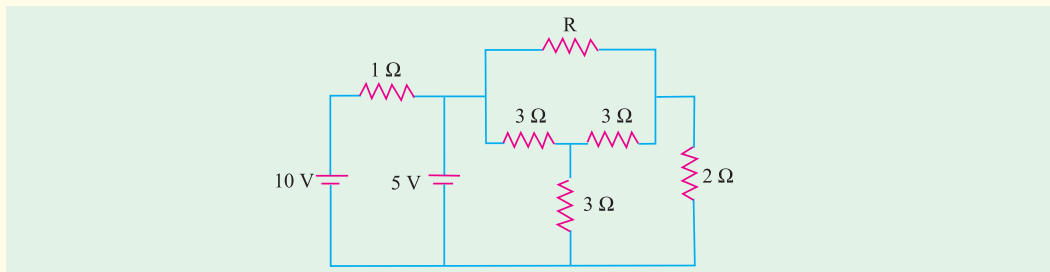


Fig. 1.99

## OBJECTIVE TESTS -1

- A 100  $\mu\text{A}$  ammeter has an internal resistance of 100  $\Omega$ . For extending its range to measure 500  $\mu\text{A}$ , the shunt required is of resistance (in  $\Omega$ )  
 (a) 20.0 (b) 22.22  
 (c) 25.0 (d) 50.0  
 (GATE 2001)
- Resistances  $R_1$  and  $R_2$  have, respectively, nominal values of 10  $\Omega$  and 5  $\Omega$ , and tolerances of  $\pm 5\%$  and  $\pm 10\%$ . The range of values for the parallel combination of  $R_1$  and  $R_2$  is  
 (a) 3.077  $\Omega$  to 3.636  $\Omega$   
 (b) 2.805  $\Omega$  to 3.371  $\Omega$   
 (c) 3.237  $\Omega$  to 3.678  $\Omega$   
 (d) 3.192  $\Omega$  to 3.435  $\Omega$   
 (GATE 2001)
- The open circuit impedance of a certain length of a loss-less line is 100  $\Omega$ . The short circuit impedance of the same line is also 100  $\Omega$ . The characteristic impedance of the line is  
 (a) 100  $\sqrt{2}$   $\Omega$  (b) 50  $\Omega$   
 (c)  $\frac{100}{\sqrt{2}}$   $\Omega$  (d) 100  $\Omega$   
 (ESE 2001)
- The current in the given circuit with a dependent voltage source is  
 (a) 10A (b) 12 A  
 (c) 14 A (d) 16 A  
 (ESE 2001)

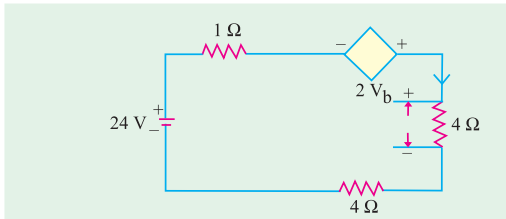


Fig. 1.100

5. The value of resistance 'R' shown in the given Fig. 1.101 is

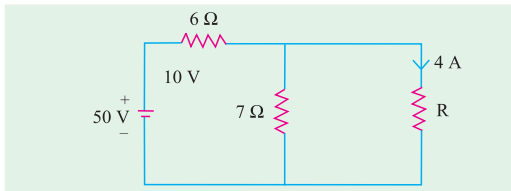


Fig. 1.101

- (a) 3.5 Ω                      (b) 2.5 Ω  
(c) 1 Ω                         (d) 4.5 Ω

(ESE 2001)

6. For the circuit shown in the given Fig. 1.102 the current I is given by

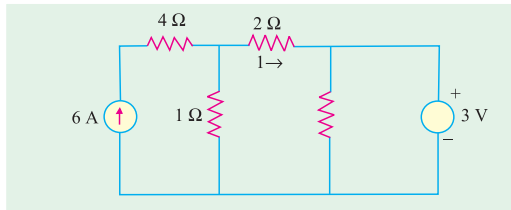


Fig. 1.102

- (a) 3 A                         (b) 2 A  
(c) 1 A                         (d) zero

(Pune University 2003) (ESE 2001)

7. The value of V in the circuit shown in the given Fig. 1.103 is

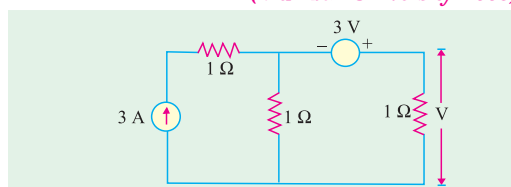


Fig. 1.103

- (a) 1 V                         (b) 2 V  
(c) 3 V                         (d) 4 V

(GATE 2003) (ESE 2001)

8. In the circuit shown in Fig. 1.104, the value of  $V_s$  is 0, when  $I = 4A$ . The value of I when  $V_s = 16V$ , is

- (a) 6 A                         (b) 8 A  
(c) 10 A                        (d) 12 A

(GATE 2003) (ESE 2003)

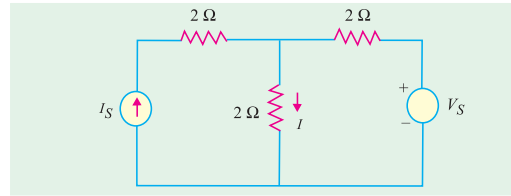


Fig. 1.104

9. The linear network as in Fig. 1.105 has only resistors. If  $I_1 = 8A$  and  $I_2 = 12A$ ; V is found to be 80 V.  $V = 0$  when  $I_1 = -8A$  and  $I_2 = 4A$ . Then the value of V when  $I_1 = I_2 = 10A$ , is

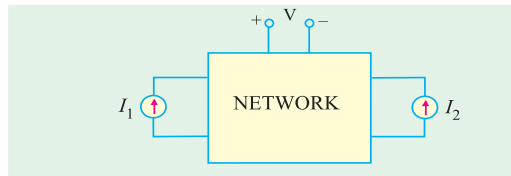


Fig. 1.105

- (a) 25 V                        (b) 50  
(c) 75 V                        (d) 100 V

(GATE 2003) (ESE 2003)

10. In Fig. 1.106, the value of R is

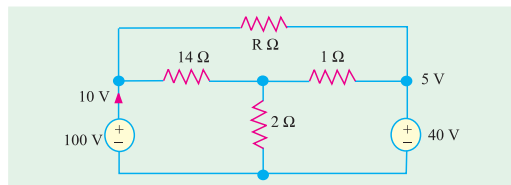


Fig. 1.106

- (a) 10 Ω                        (b) 18 Ω  
(c) 24 Ω                        (d) 12 Ω

(GATE 2003)

11. In the circuit shown in Fig. 1.107, the switch S is closed at time  $t = 0$ . The voltage across the inductance at  $t = 0^+$ , is

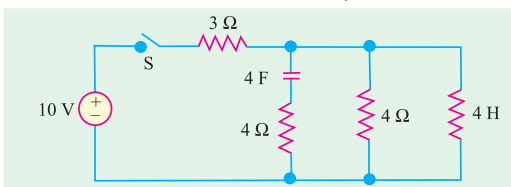


Fig. 1.107

- (a) 2 V                         (b) 4 V  
(c) -6 V                        (d) 8 V

(GATE 2003)

12. The rms value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 A is

- (a) 14.1 A                      (b) 17.3 A  
(c) 22.4 A                      (d) 30.0 A

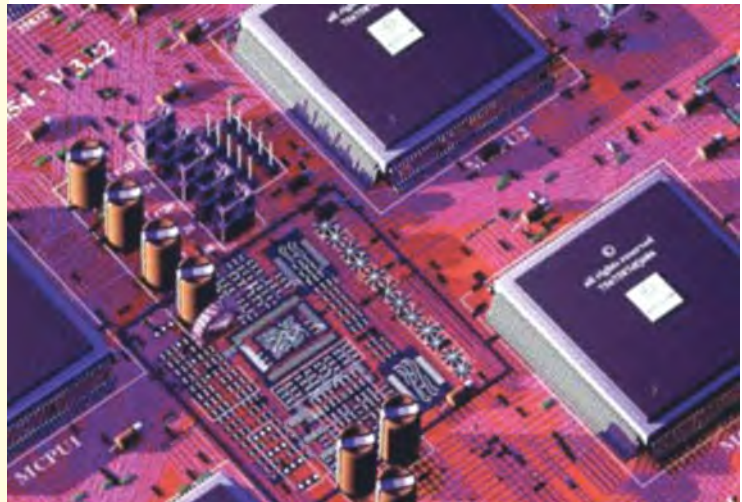
(GATE 2004)

# C H A P T E R 2

## Learning Objectives

- Electric Circuits and Network Theorems
- Kirchhoff's Laws
- Determination of Voltage Sign
- Assumed Direction of Current
- Solving Simultaneous Equations
- Determinants
- Solving Equations with Two Unknowns
- Solving Equations With Three Unknowns
- Independent and Dependent Sources
- Maxwell's Loop Current Method
- Mesh Analysis Using Matrix Form
- Nodal Analysis with Voltage Sources
- Nodal Analysis with Current Sources
- Source Conversion
- Ideal Constant-Voltage Source
- Ideal Constant-Current Source
- Superposition Theorem
- Thevenin Theorem
- How to Thevenize a Given Circuit ?
- General Instructions for Finding Thevenin Equivalent Circuit
- Reciprocity Theorem
- Delta/Star Transformation
- Star/Delta Transformation
- Compensation Theorem
- Norton's Theorem
- How to Nortanize a Given Circuit ?
- General Instructions for Finding Norton Equivalent Circuit
- Millman's Theorem
- Generalised Form of Millman's Theorem
- Maximum Power Transfer Theorem
- Power Transfer Efficiency

## DC NETWORK THEOREMS



↑ Network theorems help to determine the unknown values of current, resistance and voltage etc, in electric networks

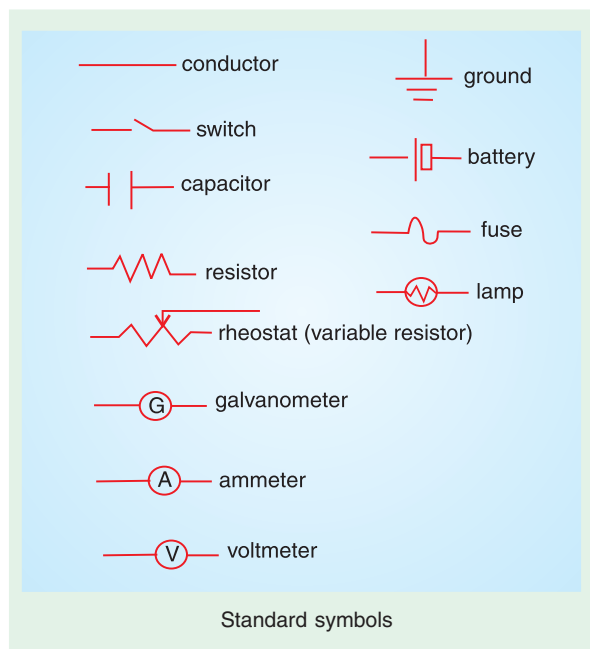
## 2.1. Electric Circuits and Network Theorems

There are certain theorems, which when applied to the solutions of electric networks, wither simplify the network itself or render their analytical solution very easy. These theorems can also be applied to an a.c. system, with the only difference that impedances replace the ohmic resistance of d.c. system. Different electric circuits (according to their properties) are defined below :

1. **Circuit.** A circuit is a closed conducting path through which an electric current either flows or is intended flow.
2. **Parameters.** The various elements of an electric circuit are called its parameters like resistance, inductance and capacitance. These parameters may be *lumped or distributed*.
3. **Liner Circuit.** A linear circuit is one whose parameters are constant *i.e.* they do not change with voltage or current.
4. **Non-linear Circuit.** It is that circuit whose parameters change with voltage or current.
5. **Bilateral Circuit.** A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.
6. **Unilateral Circuit.** It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.
7. **Electric Network.** A combination of various electric elements, connected in any manner whatsoever, is called an electric network.
8. **Passive Network** is one which contains no source of e.m.f. in it.
9. **Active Network** is one which contains one or more than one source of e.m.f.
10. **Node** is a junction in a circuit where two or more circuit elements are connected together.
11. **Branch** is that part of a network which lies between two junctions.
12. **Loop.** It is a close path in a circuit in which no element or node is encountered more than once.
13. **Mesh.** It is a loop that contains no other loop within it. For example, the circuit of Fig. 2.1 (a) has even branches, six nodes, three loops and two meshes whereas the circuit of Fig. 2.1 (b) has four branches, two nodes, six loops and three meshes.

It should be noted that, unless stated otherwise, an electric network would be assumed passive in the following treatment.

We will now discuss the various network theorems which are of great help in solving complicated networks. Incidentally, a network is said to be completely





solved or analyzed when all voltages and all currents in its different elements are determined.

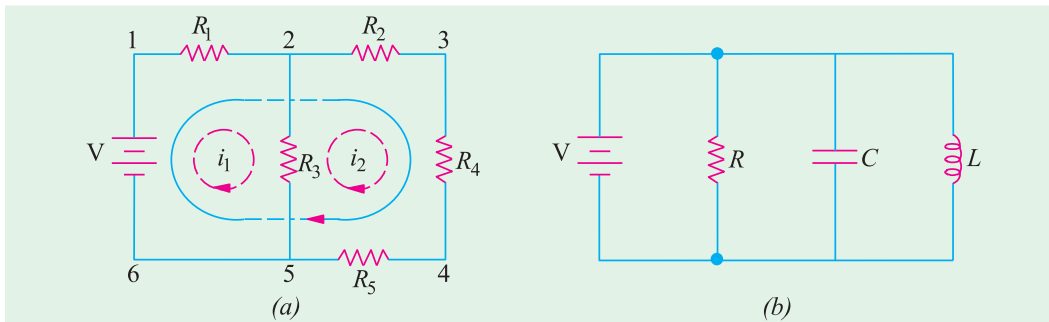


Fig. 2.1

There are two general approaches to network analysis :

**(i) Direct Method**

Here, the network is left in its original form while determining its different voltages and currents. Such methods are usually restricted to fairly simple circuits and include Kirchhoff's laws, Loop analysis, Nodal analysis, superposition theorem, Compensation theorem and Reciprocity theorem etc.

**(ii) Network Reduction Method**

Here, the original network is converted into a much simpler equivalent circuit for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are : Delta/Star and Star/Delta conversions. Thevenin's theorem and Norton's Theorem etc.

## 2.2. Kirchhoff's Laws \*

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter. Kirchhoff's laws, two in number, are particularly useful **(a)** in determining the equivalent resistance of a complicated network of conductors and **(b)** for calculating the currents flowing in the various conductors. The two-laws are :

**1. Kirchhoff's Point Law or Current Law (KCL)**

It states as follows :

***in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.***

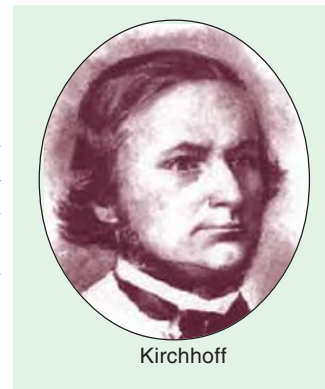
Put in another way, it simply means that the total current *leaving* a junction is equal to the total current *entering* that junction. It is obviously true because there is no accumulation of charge at the junction of the network.

Consider the case of a few conductors meeting at a point A as in Fig. 2.2 (a). Some conductors have currents leading to point A, whereas some have currents leading away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$$

or 
$$I_1 + I_4 - I_2 - I_3 - I_5 = 0 \quad \text{or} \quad I_1 + I_4 = I_2 + I_3 + I_5$$

or **incoming currents = outgoing currents**



Kirchhoff

\* After Gustave Robert Kirchhoff (1824-1887), an outstanding German Physicist.

Similarly, in Fig. 2.2 (b) for node A

$$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \quad \text{or} \quad I = I_1 + I_2 + I_3 + I_4$$

We can express the above conclusion thus :  $\Sigma I = 0$

....at a junction

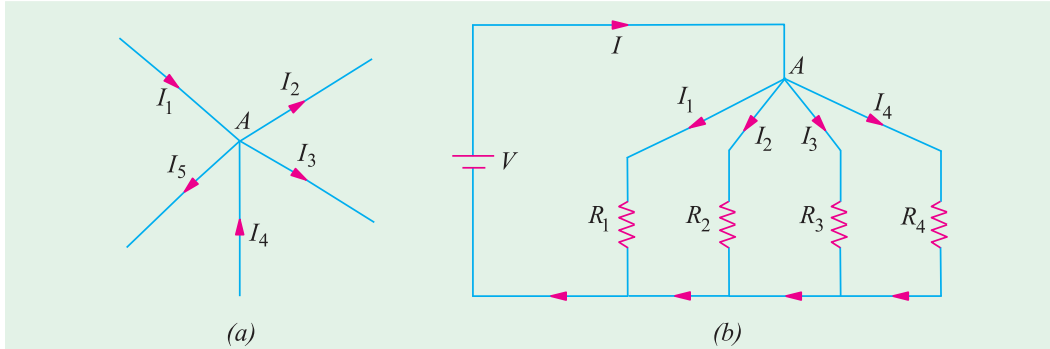


Fig. 2.2

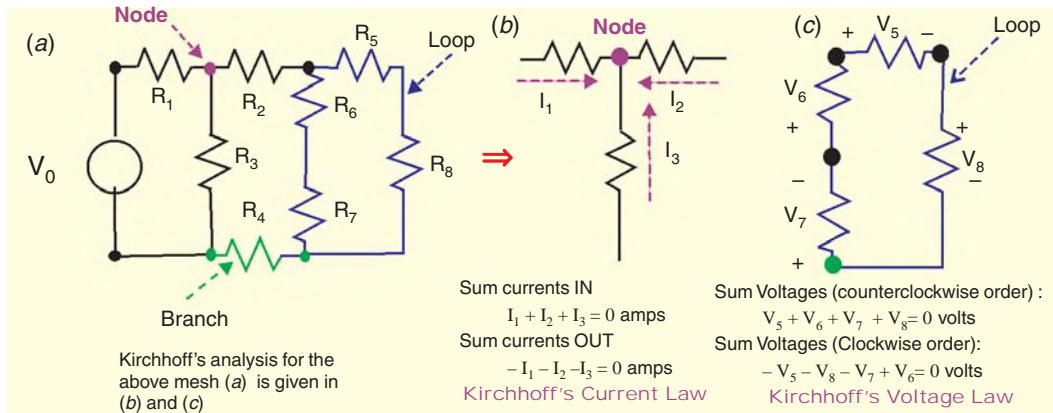
## 2. Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows :

**The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.f.s. in that path is zero.**

In other words,  $\Sigma IR + \Sigma e.m.f. = 0$  ....round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.



The basis of this law is this : If we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence, it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

## 2.3. Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.f.s., otherwise results will come out to be wrong. Following sign conventions is suggested :

### (a) Sign of Battery E.M.F.

A rise in voltage should be given a + ve sign and a fall in voltage a -ve sign. Keeping this in

mind, it is clear that as we go from the  $-ve$  terminal of a battery to its  $+ve$  terminal (Fig. 2.3), there is a *rise* in potential, hence this voltage should be given a  $+ve$  sign. If, on the other hand, we go from  $+ve$  terminal to  $-ve$  terminal, then there is a *fall* in potential, hence this voltage should be preceded

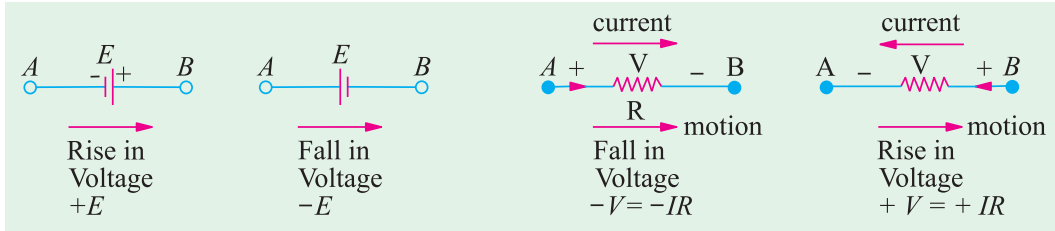


Fig. 2.3

Fig. 2.4

by a  $-ve$  sign. ***It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.***

### (b) Sign of IR Drop

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the current, then there is a *fall* in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken  $-ve$ . However, if we go in a direction opposite to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

***It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.***

Consider the closed path ABCDA in Fig. 2.5. As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

$I_1R_1$	is $-ve$	(fall in potential)
$I_2R_2$	is $-ve$	(fall in potential)
$I_3R_3$	is $+ve$	(rise in potential)
$I_4R_4$	is $-ve$	(fall in potential)
$E_2$	is $-ve$	(fall in potential)
$E_1$	is $+ve$	(rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

$$\text{or } I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

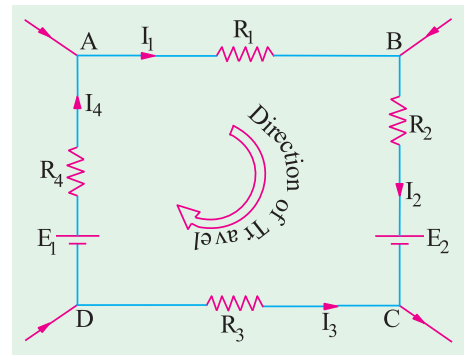


Fig. 2.5

## 2.4. Assumed Direction of Current

In applying Kirchhoff's laws to electrical networks, the question of assuming proper direction of current usually arises. The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the same as actual direction (Example 2.10). ***However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.***

**Note.** It should be noted that Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account (See Example 2.14).

## 2.5. Solving Simultaneous Equations

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy.

## 2.6. Determinants

The symbol  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$  is called a determinant of the second order (or  $2 \times 2$  determinant) because it contains two rows ( $ab$  and  $cd$ ) and two columns ( $ac$  and  $bd$ ). The numbers  $a$ ,  $b$ ,  $c$  and  $d$  are called the elements or constituents of the determinant. Their number in the present case is  $2^2 = 4$ .

The evaluation of such a determinant is accomplished by cross-multiplication is illustrated below :

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The above result for a second order determinant can be remembered as

**upper left times lower right minus upper right times lower left**

The symbol  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  represents a third-order determinant having  $3^2 = 9$  elements. It may be evaluated (or expanded) as under :

1. Multiply each element of the first row (or alternatively, first column) by a determinant obtained by omitting the row and column in which it occurs. (It is called minor determinant or just minor as shown in Fig. 2.6).

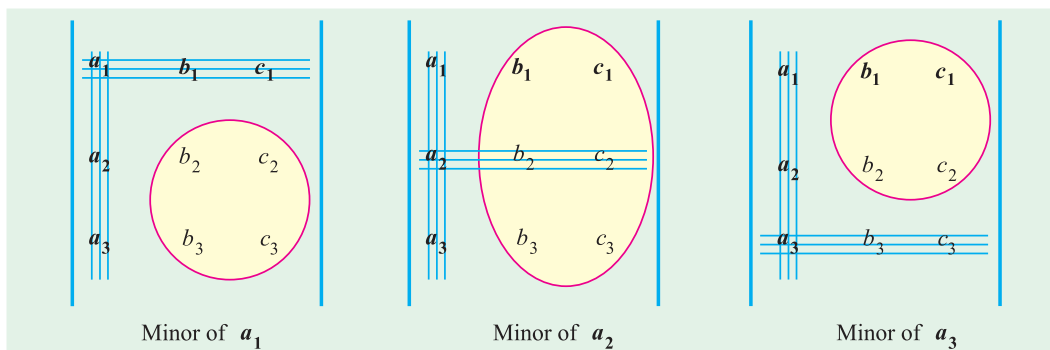


Fig. 2.6

2. Prefix + and -sing alternately to the terms so obtained.
3. Add up all these terms together to get the value of the given determinant.

Considering the first column, minors of various elements are as shown in Fig. 2.6.

Expanding in terms of first column, we get

$$\begin{aligned} \Delta &= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \\ &= a_1 (b_2c_3 - b_3c_2) - a_2 (b_1c_3 - b_3c_1) + a_3 (b_1c_2 - b_2c_1) \quad \dots(i) \end{aligned}$$

Expanding in terms of the first row, we get

$$\begin{aligned}\Delta &= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \\ &= a_1 (b_2 c_3 - b_3 c_2) - b_1 (a_2 c_3 - a_3 c_2) + c_1 (a_2 b_3 - a_3 b_2)\end{aligned}$$

which will be found to be the same as above.

**Example 2.1.** Evaluate the determinant  $\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}$

**Solution.** We will expand with the help of 1st column.

$$\begin{aligned}D &= 7 \begin{vmatrix} -2 & -2 \\ -4 & 11 \end{vmatrix} - (-3) \begin{vmatrix} -3 & -4 \\ -4 & 11 \end{vmatrix} + (-4) \begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix} \\ &= 7 [(6 \times 11) - (-2 \times -2)] + 3 [(-3 \times 11) - (-4 \times -2)] - 4 [(-3 \times -2) - (-4 \times 6)] \\ &= 7 (66 - 4) + 3 (-33 - 8) - 4 (6 + 24) = \mathbf{191}\end{aligned}$$

## 2.7. Solving Equations with Two Unknowns

Suppose the two given simultaneous equations are

$$\begin{aligned}ax + by &= c \\ dx + ey &= f\end{aligned}$$

Here, the two unknown are  $x$  and  $y$ ,  $a, b, d$  and  $e$  are coefficients of these unknowns whereas  $c$  and  $f$  are constants. The procedure for solving these equations by the method of determinants is as follows :

- Write the two equations in the matrix form as  $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$
- The **common** determinant is given as  $\Delta = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = ae - bd$
- For finding the determinant for  $x$ , replace the coefficients of  $x$  in the original matrix by the constants so that we get determinant  $\Delta_1$  given by  $\Delta_1 = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = (ce - bf)$
- For finding the determinant for  $y$ , replace coefficients of  $y$  by the constants so that we get  $\Delta_2 = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = (af - cd)$
- Apply Cramer's rule to get the value of  $x$  and  $y$   
 $x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd}$  and  $y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$

**Example 2.2.** Solve the following two simultaneous equations by the method of determinants :

$$\begin{aligned}4i_1 - 3i_2 &= 1 \\ 3i_1 - 5i_2 &= 2\end{aligned}$$

**Solution.** The matrix form of the equations is  $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = (4 \times -5) - (-3 \times 3) = -11$$

$$\Delta_1 = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix} = (1 \times -5) - (-3 \times 2) = 1$$

$$\Delta_2 = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (4 \times 2) - (1 \times 3) = 5$$

$$\therefore i_1 = \frac{\Delta_1}{\Delta} = \frac{1}{-11} = -\frac{1}{11}; \quad i_2 = \frac{\Delta_2}{\Delta} = -\frac{5}{11}$$

## 2.8. Solving Equations With Three Unknowns

Let the three simultaneous equations be as under :

$$\begin{aligned} ax + by + cz &= d \\ ex + fy + gz &= h \\ jx + ky + lz &= m \end{aligned}$$

The above equations can be put in the matrix form as under :

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

The value of common determinant is given by

$$\Delta = \begin{vmatrix} a & b & c \\ e & f & g \\ j & k & l \end{vmatrix} = a(fl - gk) - e(bl - ck) + j(bg - cf)$$

The determinant for  $x$  can be found by replacing coefficients of  $x$  in the original matrix by the constants.

$$\therefore \Delta_1 = \begin{vmatrix} d & b & c \\ h & f & g \\ m & k & l \end{vmatrix} = d(fl - gk) - h(bl - ck) + m(bg - cf)$$

Similarly, determinant for  $y$  is given by replacing coefficients of  $y$  with the three constants.

$$\Delta_2 = \begin{vmatrix} a & d & c \\ e & h & g \\ j & m & l \end{vmatrix} = a(hl - mg) - e(dl - mc) + j(dg - hc)$$

In the same way, determinant for  $z$  is given by

$$\Delta_3 = \begin{vmatrix} a & b & d \\ e & f & h \\ j & k & m \end{vmatrix} = a(fm - hk) - e(bm - dk) + j(bh - df)$$

As per Cramer's rule  $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$

**Example 2.3.** Solve the following three simultaneous equations by the use of determinants and Cramer's rule

$$\begin{aligned} i_1 + 3i_2 + 4i_3 &= 14 \\ i_1 + 2i_2 + i_3 &= 7 \\ 2i_1 + i_2 + 2i_3 &= 2 \end{aligned}$$

**Solution.** As explained earlier, the above equations can be written in the form

$$\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 1(4 - 1) - 1(6 - 4) + (3 - 8) = -9$$

$$\Delta_1 = \begin{vmatrix} 14 & 3 & 4 \\ 7 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 14(4 - 1) - 7(6 - 4) + 2(3 - 8) = 18$$

$$\Delta_2 = \begin{bmatrix} 1 & 14 & 4 \\ 1 & 7 & 1 \\ 2 & 2 & 2 \end{bmatrix} = 1(14 - 2) - 1(28 - 8) + 2(14 - 28) = -36$$

$$\Delta_3 = \begin{bmatrix} 1 & 3 & 14 \\ 1 & 2 & 7 \\ 2 & 1 & 2 \end{bmatrix} = 1(4 - 7) - 1(6 - 14) + 2(21 - 28) = -9$$

According to Cramer's rule,

$$i_1 = \frac{-1}{-9} = \frac{1}{9} \text{ A}; \quad i_2 = \frac{18}{-9} = -2 \text{ A}; \quad i_3 = \frac{-36}{-9} = 4 \text{ A}$$

**Example 2.4.** What is the voltage  $V_s$  across the open switch in the circuit of Fig. 2.7 ?

**Solution.** We will apply KVL to find  $V_s$ . Starting from point A in the clockwise direction and using the sign convention given in Art. 2.3, we have

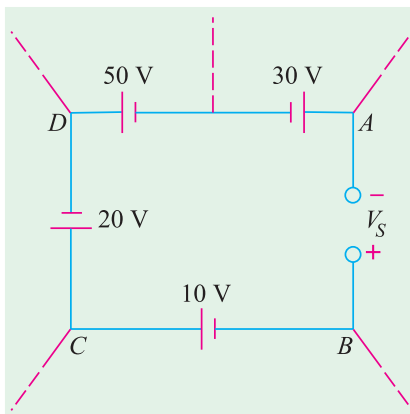


Fig. 2.7

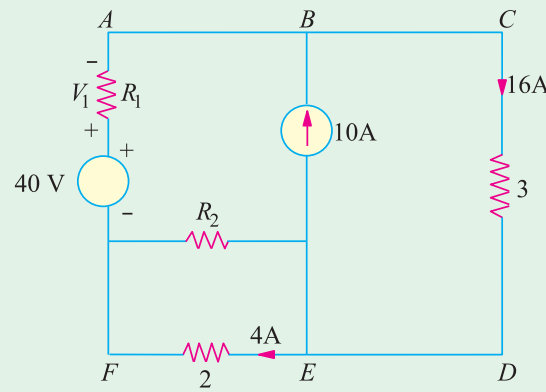


Fig. 2.8

$$+V_s + 10 - 20 - 50 + 30 = 0 \quad \therefore V_s = 30 \text{ V}$$

**Example 2.5.** Find the unknown voltage  $V_1$  in the circuit of Fig. 2.8.

**Solution.** Initially, one may not be clear regarding the solution of this question. One may think of Kirchhoff's laws or mesh analysis etc. But a little thought will show that the question can be solved by the simple application of Kirchhoff's voltage law. Taking the outer closed loop **ABCDEF** and applying KVL to it, we get

$$-16 \times 3 - 4 \times 2 + 40 - V_1 = 0; \quad \therefore V_1 = -16 \text{ V}$$

The negative sign shows there is a fall in potential.

**Example 2.6.** Using Kirchhoff's Current Law and Ohm's Law, find the magnitude and polarity of voltage  $V$  in Fig. 2.9 (a). Directions of the two current sources are as shown.

**Solution.** Let us arbitrarily choose the directions of  $I_1, I_2$  and  $I_3$  and polarity of  $V$  as shown in Fig. 2.9.(b). We will use the sign convention for currents as given in Art. 2.3. Applying KCL to node A, we have

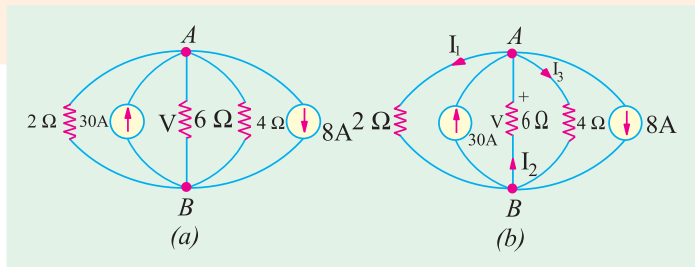


Fig. 2.9

$$-I_1 + 30 + I_2 - I_3 - 8 = 0$$

$$\text{or } I_1 - I_2 + I_3 = 22 \quad \dots(i)$$

Applying Ohm's law to the three resistive branches in Fig. 2.9 (b), we have

$$I_1 = \frac{V}{2}, I_3 = \frac{V}{4}, I_2 = -\frac{V}{6} \quad (\text{Please note the -ve sign.})$$

Substituting these values in (i) above, we get

$$\frac{V}{2} - \left(-\frac{V}{6}\right) + \frac{V}{4} = 22 \quad \text{or } V = 24 \text{ V}$$

$$\therefore I_1 = V/2 = 24/2 = 12 \text{ A}, I_2 = -24/6 = -4 \text{ A}, I_3 = 24/4 = 6 \text{ A}$$

The negative sign of  $I_2$  indicates that actual direction of its flow is opposite to that shown in Fig. 2.9 (b). Actually,  $I_2$  flows from A to B and not from B to A as shown.

Incidentally, it may be noted that all currents are outgoing except 30A which is an incoming current.

**Example 2.7.** For the circuit shown in Fig. 2.10, find  $V_{CE}$  and  $V_{AG}$ .

(F.Y. Engg. Pune Univ.)

**Solution.** Consider the two battery circuits of Fig. 2.10 separately. Current in the 20 V battery circuit ABCD is  $20/(6+5+9) = 1\text{A}$ . Similarly, current in the 40 V battery circuit EFGH is  $40/(5+8+7) = 2\text{A}$ . Voltage drops over different resistors can be found by using Ohm's law.

For finding  $V_{CE}$  i.e. voltage of point C with respect to point E, we will start from point E and go to C via points H and B. We will find the algebraic sum of the voltage drops met on the way from point E to C. Sign convention of the voltage drops and battery e.m.fs. would be the same as discussed in Art. 2.3.

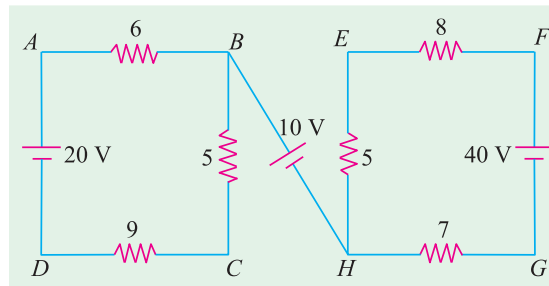


Fig. 2.10

$$\therefore V_{CE} = (-5 \times 2) + (10) - (5 \times 1) = -5\text{V}$$

The negative sign shows that point C is negative with respect to point E.

$$V_{AG} = (7 \times 2) + (10) + (6 \times 1) = 30 \text{ V.}$$

The positive sign shows that point A is at a positive potential of 30 V with respect to point G.

**Example 2.8.** Determine the currents in the unbalanced bridge circuit of Fig. 2.11 below. Also, determine the p.d. across BD and the resistance from B to D.

**Solution.** Assumed current directions are as shown in Fig. 2.11.

Applying Kirchhoff's Second Law to circuit DACD, we get

$$-x - 4z + 2y = 0 \quad \text{or } x - 2y + 4z = 0 \quad \dots(1)$$

Circuit ABCA gives

$$-2(x - z) + 3(y + z) + 4z = 0 \quad \text{or } 2x - 3y - 9z = 0 \quad \dots(2)$$

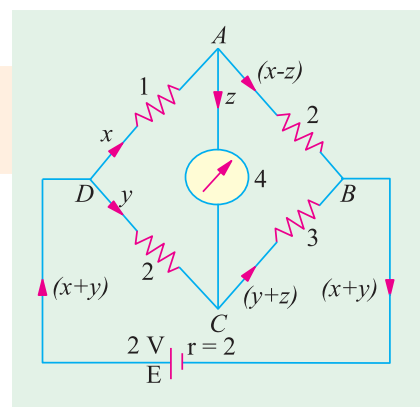


Fig. 2.11



Circuit *DABED* gives

$$-x - 2(x - z) - 2(x + y) + 2 = 0 \text{ or } 5x + 2y - 2z = 2 \quad \dots(3)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$-y + 17z = 0 \quad \dots(4)$$

Similarly, multiplying (1) by 5 and subtracting (3) from it, we have

$$-12y + 22z = -2 \text{ or } -6y + 11z = -1 \quad \dots(5)$$

Eliminating  $y$  from (4) and (5), we have  $91z = 1$  or  $z = 1/91$  A

From (4);  $y = 17/91$  A. Putting these values of  $y$  and  $z$  in (1), we get  $x = 30/91$  A

Current in *DA* =  $x = 30/91$  A Current in *DC* =  $y = 17/91$  A

$$\text{Current in } AB = x - z = \frac{30}{91} - \frac{1}{91} = \frac{29}{91} \text{ A}$$

$$\text{Current in } CB = y - z = \frac{17}{91} - \frac{1}{91} = \frac{16}{91} \text{ A}$$

$$\text{Current in external circuit} = x + y = \frac{30}{91} + \frac{17}{91} = \frac{47}{91} \text{ A}$$

$$\text{Current in } AC = z = \frac{1}{91} \text{ A}$$

Internal voltage drop in the cell =  $2(x + y) = 2 \times 47/91 = 94/91$  V

$$\therefore \text{P.D. across points } D \text{ and } B = 2 \times \frac{94}{91} = \frac{188}{91} \text{ V}^*$$

Equivalent resistance of the bridge between points *D* and *B*

$$= \frac{\text{p.d. between points } B \text{ and } D}{\text{current between points } B \text{ and } D} = \frac{188/91}{47/91} = \frac{188}{47} = 1.87 \Omega \text{ (approx)}$$

### Solution By Determinants

The matrix from the three simultaneous equations (1), (2) and (3) is

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{vmatrix} = 1(6 + 18) - 2(4 - 8) + 5(18 + 12) = 182$$

$$\Delta_1 = \begin{vmatrix} 0 & -2 & 4 \\ 0 & -3 & -9 \\ 2 & 2 & -2 \end{vmatrix} = 0(6 + 18) - 0(4 - 8) + 2(18 + 12) = 60$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 4 \\ 2 & 0 & -9 \\ 5 & 2 & -2 \end{vmatrix} = 34, \Delta_3 = \begin{vmatrix} 1 & -2 & 0 \\ 2 & -3 & 0 \\ 5 & 2 & 2 \end{vmatrix} = 2$$

$$\therefore x = \frac{-1}{182} = \frac{30}{91} \text{ A}, y = \frac{34}{182} = \frac{17}{91} \text{ A}, z = \frac{2}{182} = \frac{1}{91} \text{ A}$$

**Example 2.9.** Determine the branch currents in the network of Fig. 2.12 when the value of each branch resistance is one ohm. (Elect. Technology, Allahabad Univ. 1992)

**Solution.** Let the current directions be as shown in Fig. 2.12.

Apply Kirchoff's Second law to the closed circuit *ABDA*, we get

$$5 - x - z + y = 0 \text{ or } x - y + z = 5 \quad \dots(i)$$

\* P.D. between *D* and *B* = drop across *DC* + drop across *CB* =  $2 \times 17/91 + 3 \times 18/91 = 88/91$  V.

Similarly, circuit  $BCDB$  gives

$$-(x-z) + 5 + (y+z) + z = 0$$

$$\text{or } x - y - 3z = 5 \quad \dots(ii)$$

Lastly, from circuit  $ADCEA$ , we get

$$-y - (y+z) + 10 - (x+y) = 0$$

$$\text{or } x + 3y + z = 10 \quad \dots(iii)$$

From Eq. (i) and (ii), we get,  $z = 0$

Substituting  $z = 0$  either in Eq. (i) or (ii) and in Eq. (iii), we get

$$x - y = 5 \quad \dots(iv)$$

$$x + 3y = 10 \quad \dots(v)$$

Subtracting Eq. (v) from (iv), we get

$$-4y = -5 \quad \text{or } y = 5/4 = 1.24 \text{ A}$$

Eq. (iv) gives  $x = 25/4 \text{ A} = 6.25 \text{ A}$

Current in branch  $AB$  = current in branch  $BC$  = **6.25 A**

Current in branch  $BD$  = 0; current in branch

$AD$  = current in branch  $DC$  = **1.25 A**; current in branch  $CEA$  =  $6.25 + 1.25 = 7.5 \text{ A}$ .

**Example 2.10.** State and explain Kirchhoff's laws. Determine the current supplied by the battery in the circuit shown in Fig. 2.12 A. **(Elect. Engg. I, Bombay Univ.)**

**Solution.** Let the current distribution be as shown in the figure. Considering the close circuit  $ABCA$  and applying Kirchhoff's Second Law, we have

$$-100x - 300z + 500y = 0$$

$$\text{or } x - 5y + 3z = 0 \quad \dots(i)$$

Similarly, considering the closed loop  $BCDB$ , we have

$$-300z - 100(y+z) + 500(x-z) = 0$$

$$\text{or } 5x - y - 9z = 0 \quad \dots(ii)$$

Taking the circuit  $ABDEA$ , we get

$$-100x - 500(x-z) + 100 - 100(x+y) = 0$$

$$\text{or } 7x + y - 5z = 1 \quad \dots(iii)$$

The value of  $x$ ,  $y$  and  $z$  may be found by solving the above three simultaneous equations or by the method of determinants as given below :

Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 240, \Delta_1 = \begin{bmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{bmatrix} = 48$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 24, \Delta_3 = \begin{bmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{bmatrix} = 24$$

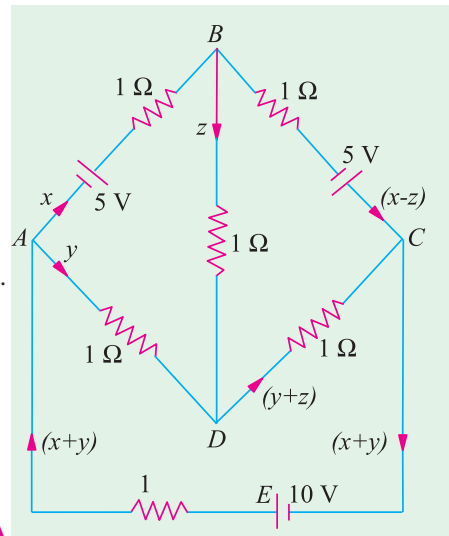


Fig. 2.12

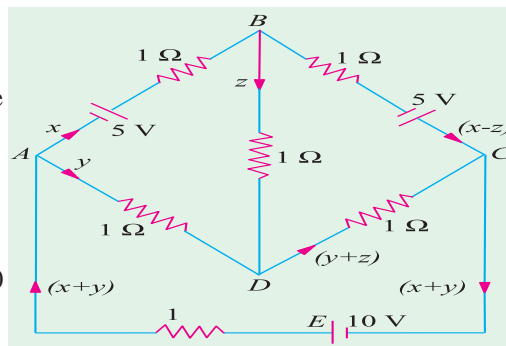


Fig. 2.12 A

$$\therefore x = \frac{48}{240} \frac{1}{5} \text{ A}; y = \frac{24}{240} \frac{1}{10} \text{ A}; z = \frac{24}{240} \frac{1}{10} \text{ A}$$

Current supplied by the battery is  $x + y = 1/5 + 1/10 = 3/10 \text{ A}$ .

**Example 2.11.** Two batteries A and B are connected in parallel and load of  $10 \Omega$  is connected across their terminals. A has an e.m.f. of  $12 \text{ V}$  and an internal resistance of  $2 \Omega$ ; B has an e.m.f. of  $8 \text{ V}$  and an internal resistance of  $1 \Omega$ . Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.

(F.Y. Engg. Pune Univ.)

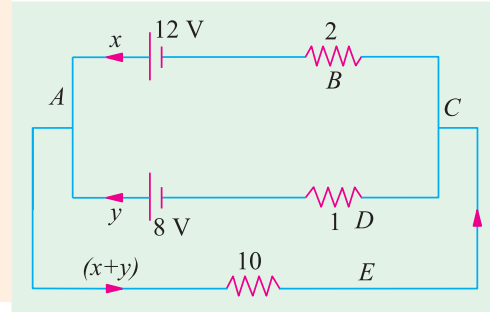


Fig. 2.13

**Solution.** Applying KVL to the closed circuit ABCDA of Fig. 2.13, we get

$$-12 + 2x - 1y + 8 = 0 \quad \text{or} \quad 2x - y = 4 \quad \dots(i)$$

Similarly, from the closed circuit ADCEA, we get

$$-8 + 1y + 10(x + y) = 0 \quad \text{or} \quad 10x + 11y = 8 \quad \dots(ii)$$

From Eq. (i) and (ii), we get

$$x = 1.625 \text{ A} \quad \text{and} \quad y = -0.75 \text{ A}$$

The negative sign of  $y$  shows that the current is flowing into the  $8\text{-V}$  battery and not out of it. In other words, it is a charging current and not a discharging current.

Current flowing in the external resistance =  $x + y = 1.625 - 0.75 = 0.875 \text{ A}$

P.D. across the external resistance =  $10 \times 0.875 = 8.75 \text{ V}$

**Note.** To confirm the correctness of the answer, the simple check is to find the value of the external voltage available across point A and C with the help of the two parallel branches. If the value of the voltage comes out to be the same, then the answer is correct, otherwise it is wrong. For example,  $V_{CBA} = -2 \times 1.625 + 12 = 8.75 \text{ V}$ . From the second branch  $V_{CDA} = 1 \times 0.75 + 8 = 8.75 \text{ V}$ . Hence, the answer found above is correct.

**Example 2.12.** Determine the current  $x$  in the  $4\text{-}\Omega$  resistance of the circuit shown in Fig. 2.13 (A).

**Solution.** The given circuit is redrawn with assumed distribution of currents in Fig. 2.13 (B). Applying KVL to different closed loops, we get

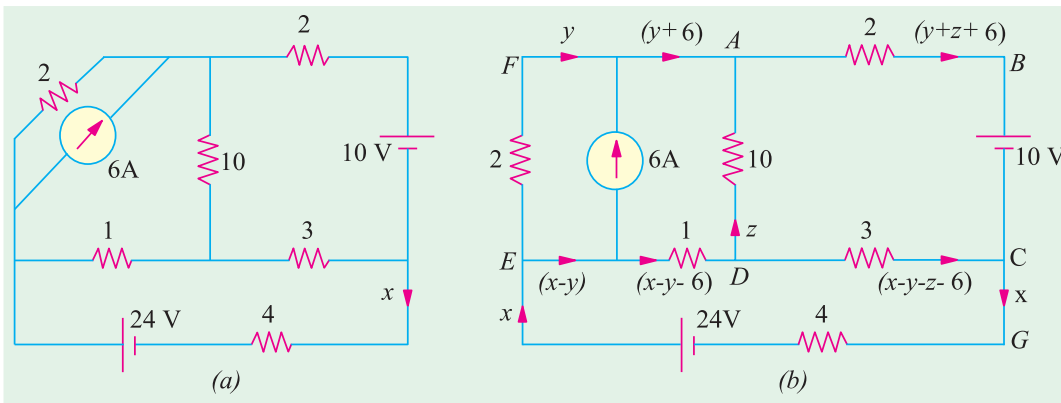


Fig. 2.13 A

**Circuit EFADE**

$$-2y + 10z + (x - y - 6) = 0 \quad \text{or} \quad x - 3y + 10z = 6 \quad \dots(i)$$

**Circuit ABCDA**

$$2(y + z + 6) - 10 + 3(x - y - z - 6) - 10z = 0 \quad \text{or} \quad 3x - 5y - 14z = 40 \quad \dots(ii)$$

**Circuit EDCGE**

$$-(x - y - 6) - 3(x - y - z - 6) - 4x + 24 = 0 \quad \text{or} \quad 8x - 4y - 3z = 48 \quad \dots(iii)$$

From above equations we get  $x = 4.1 \text{ A}$

**Example 2.13.** Applying Kirchhoff's laws to different loops in Fig. 2.14, find the values of  $V_1$  and  $V_2$ .

**Solution.** Starting from point A and applying Kirchhoff's voltage law to loop No.3, we get

$$-V_3 + 5 = 0 \quad \text{or} \quad V_3 = 5 \text{ V}$$

Starting from point A and applying Kirchhoff's voltage law to loop No. 1, we get

$$10 - 30 - V_1 + 5 = 0 \quad \text{or} \quad V_1 = -15 \text{ V}$$

The negative sign of  $V_1$  denotes that its polarity is opposite to that shown in the figure.

Starting from point B in loop No. 3, we get

$$-(-15) - V_2 + (-15) = 0 \quad \text{or} \quad V_2 = 0$$

**Example 2.14.** In the network of Fig. 2.15, the different currents and voltages are as under :

$$i_2 = 5e^{-2t}, \quad i_4 = 3 \sin t \quad \text{and} \quad v_3 = 4e^{-2t}$$

Using KCL, find voltage  $v_1$ .

**Solution.** According to KCL, the algebraic sum of the currents meeting at junction A is zero i.e.

$$\begin{aligned} i_1 + i_2 + i_3 + (-i_4) &= 0 \\ i_1 + i_2 + i_3 - i_4 &= 0 \end{aligned} \quad \dots(i)$$

Now, current through a capacitor is given by  $i = C dv/dt$

$$\therefore i_3 = C \frac{dv_3}{dt} = 2 \frac{d(4e^{-2t})}{dt} = 16e^{-2t}$$

Substituting this value in Eq (i) above, we get

$$i_1 + 5e^{-2t} - 16e^{-2t} - 3 \sin t = 0$$

$$\text{or} \quad i_1 = 3 \sin t + 11e^{-2t}$$

The voltage  $v_1$  developed across the coil is

$$\begin{aligned} v_1 &= L \frac{di_1}{dt} = 4 \cdot \frac{d}{dt} (3 \sin t + 11e^{-2t}) \\ &= 4 (3 \cos t - 22e^{-2t}) = 12 \cos t - 88e^{-2t} \end{aligned}$$

**Example 2.15.** In the network shown in Fig. 2.16,  $v_1 = 4\text{V}$ ,  $v_4 = 4 \cos 2t$  and  $i_3 = 2e^{-t/3}$ . Determine  $i_2$ .

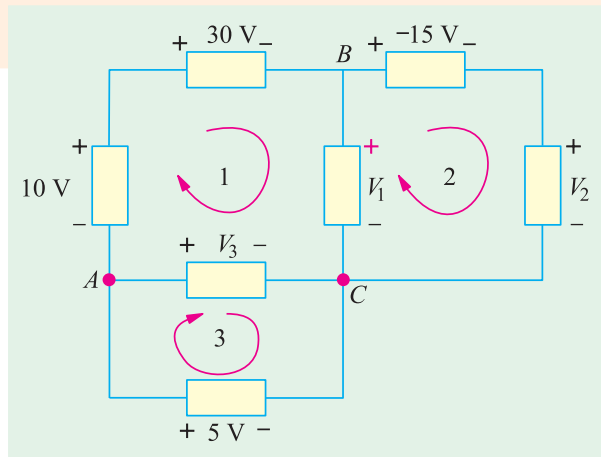


Fig. 2.14

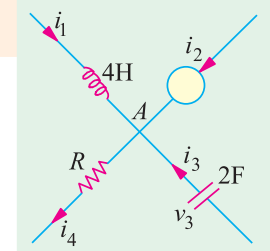


Fig. 2.15

**Solution.** Applying KVL to closed mesh ABCDA, we get

$$-v_1 - v_2 + v_3 + v_4 = 0$$

Now 
$$v_3 = L \frac{di_3}{dt} = 6 \cdot \frac{d}{dt} (2e^{-t/3}) = -4e^{-t/3}$$

$$\therefore -4 - v_2 - 4e^{-t/3} + 4 \cos 2t = 0$$
  
 or 
$$v_2 = 4 \cos 2t - 4e^{-t/3} - 4$$

Now, 
$$i_2 = C \frac{dv_2}{dt} = 8 \frac{d}{dt} (4 \cos 2t - 4e^{-t/3} - 4)$$

$$\therefore i_2 = 8 \left( -8 \sin 2t + \frac{4}{3} e^{-t/3} \right) = -64 \sin 2t + \frac{32}{3} e^{-t/3}$$

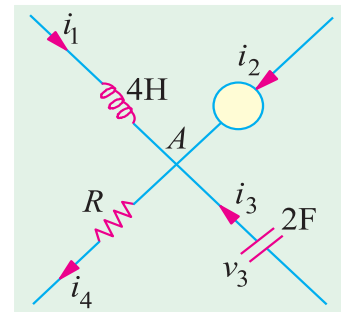


Fig. 2.16

**Example 2.16.** Use nodal analysis to determine the voltage across 5 Ω resistance and the current in the 12 V source. [Bombay University 2001]

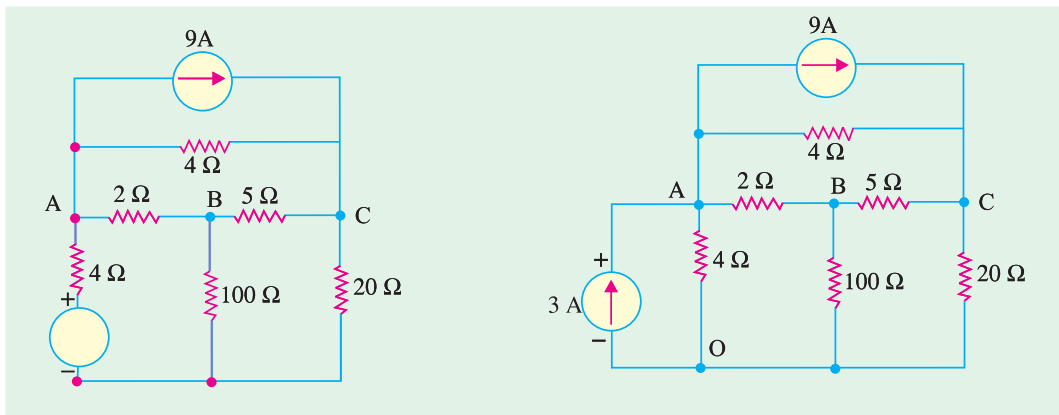


Fig. 2.17 (a)

Fig. 2.17 (b)

**Solution.** Transform the 12-volt and 4-ohm resistor into current-source and parallel resistor.

Mark the nodes O, A, B and C on the diagram. Self- and mutual conductance terms are to be written down next.

At A,  $G_{aa} = 1/4 + 1/2 + 1/4 = 1$  mho

At B,  $G_{bb} = 1/2 + 1/5 + 1/100 = 0.71$  mho

At C,  $G_{cc} = 1/4 + 1/5 + 1/20 = 0.50$  mho

Between A and B,  $G_{ab} = 0.5$  mho,

Between B and C,  $G_{bc} = 0.2$  mho,

Between A and C,  $G_{ac} = 0.25$  mho.

Current Source matrix : At node A, 3 amp incoming and 9 amp outgoing currents give a net outgoing current of 6 amp. At node C, incoming current = 9 amp. At node B, no current source is

connected. Hence, the current-source matrix is :

$$\begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

The potentials of three nodes to be found are :  $V_A, V_B, V_C$

$$\begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

For evaluating  $V_A$ ,  $V_B$ ,  $V_C$ , following steps are required.

$$\Delta = \begin{vmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{vmatrix} = 1 \times (0.7105 - 0.04) + 0.5(-0.25 - 0.05) - 0.25(0.1 + 0.71 \times 0.25)$$

$$= 0.315 - 0.15 - 0.069375 = 0.095625$$

$$\Delta_a = \begin{vmatrix} -6 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ 9 & -0.20 & +0.5 \end{vmatrix} = +0.6075$$

$$\Delta_b = \begin{vmatrix} 1 & -6 & -0.25 \\ -0.5 & 0 & -0.20 \\ -0.25 & 9 & 0.50 \end{vmatrix} = 1.125$$

$$\Delta_c = \begin{vmatrix} 1 & -0.5 & -6 \\ -0.5 & 0.71 & 0 \\ -0.25 & -0.20 & 9 \end{vmatrix} = 2.2475$$

$$V_A = \Delta_a / \Delta = +0.6075 / 0.095625 = 6.353 \text{ volts}$$

$$V_B = \Delta_b / \Delta = 1.125 / 0.095625 = 11.765 \text{ volts}$$

$$V_C = \Delta_c / \Delta = 2.2475 / 0.095625 = 23.503 \text{ volts}$$

Hence, voltage across 5-ohm resistor =  $V_C - V_B = 14.18$  volts. Obviously,  $B$  is positive w.r. to  $A$ . From these node potentials, current through 100-ohm resistor is 0.118 amp; (i) current through 20 ohm resistor is 1.294 amp.

(ii) Current through 5-ohm resistor =  $14.18/5 = 2.836$  amp.

(iii) Current through 4-ohm resistor between  $C$  and  $A = 19.53/4 = 4.883$  amp

**Check :** Apply *KCL* at node  $C$

Incoming current = 9 amp, from the source.

Outgoing currents as calculated in (i), (ii) and (iii) above =  $1.294 + 2.836 + 4.883 \cong 9$  amp

(iv) Current through 2-ohm resistor =  $(V_B - V_A)/2 = 2.706$  amp, from  $B$  to  $A$ .

(v) Current in  $A-O$  branch =  $6.353/4 = 1.588$  amp

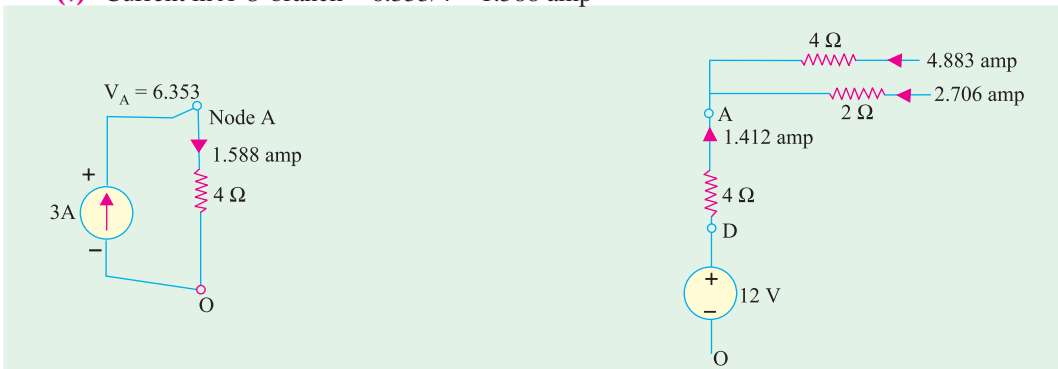


Fig. 2.17 (c) Equivalent

Fig. 2.17 (d) Actual elements

In Fig. 2.17 (c), the transformed equivalent circuit is shown. The 3-amp current source ( $O$  to  $A$ ) and the current of 1.588 amp in  $A-O$  branch have to be interpreted with reference to the actual circuit, shown in Fig. 2.17 (d), where in a node  $D$  exists at a potential of 12 volts w.r. to the reference node. The 4-ohm resistor between  $D$  and  $A$  carries an upward current of  $\{(12 - 6.353)/4 =\}$  1.412 amp, which is nothing but 3 amp into the node and 1.588 amp away from the node, as in Fig. 2.17 (c), at node  $A$ . The current in the 12-V source is thus 1.412 amp.

**Check.** *KCL* at node  $A$  should give a check that incoming currents should add-up to 9 amp.

$$1.412 + 2.706 + 4.883 \cong 9 \text{ amp}$$

**Example 2.17.** Determine current in 5-Ω resistor by any one method.

(Bombay University 2001)

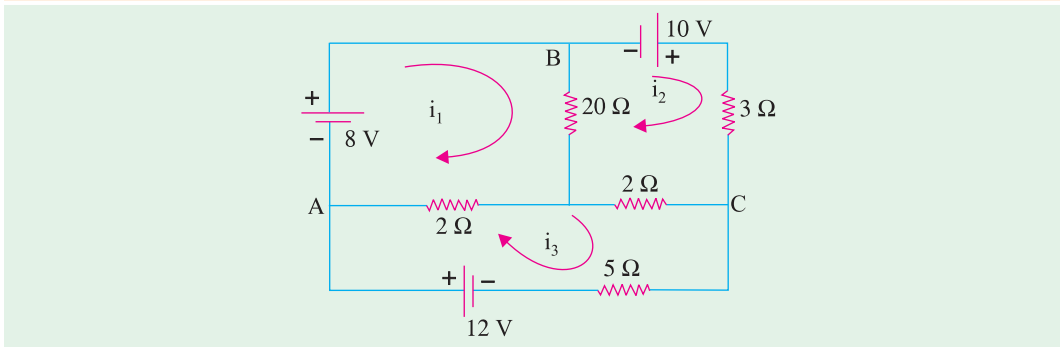


Fig. 2.18 (a)

**Solution (A).** Matrix-method for Mesh analysis can be used. Mark three loops as shown, in Fig. 2.18 (a). Resistance-matrix should be evaluated for current in 5-ohm resistor. Only,  $i_3$  is to be found.

$$R_{11} = 3, R_{22} = 6, R_{33} = 9 \quad R_{12} = 1, R_{23} = 2, R_{13} = 2$$

Voltage-source will be a column matrix with entries serially as : + 8 Volts, + 10 Volts, + 12 Volts.

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -2 \\ -2 & -2 & 9 \end{vmatrix} = 3 \times (54 - 4) + 1(-9 - 4) - 2(2 + 12) = 109$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 8 \\ -1 & 6 & 10 \\ -2 & -2 & 12 \end{vmatrix} = 396$$

$$i_3 = \Delta_3 / \Delta = 396 / 109 = 3.633 \text{ amp.}$$

**Solution (B).** Alternatively, Thevenin’s theorem can be applied.

For this, detach the 5-ohm resistor from its position, Evaluate  $V_{TH}$  at the terminals X-Y in Fig. 2.18 (b) and de-activating the source, calculate the value of  $R_{TH}$  as shown in Fig. 2.18 (c).

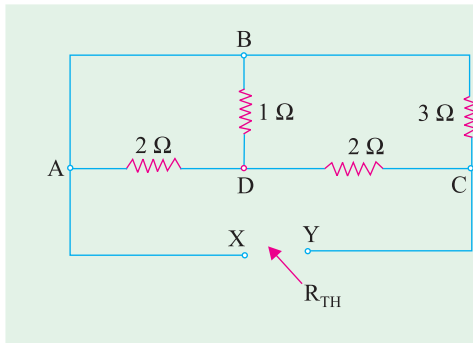


Fig. 2.18 (b)

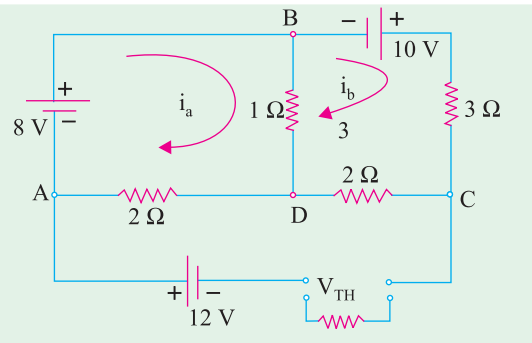


Fig. 2.18 (c)

By observation, Resistance-elements of  $2 \times 2$  matrix have to be noted.

$$R_{aa} = 3, R_{bb} = 5, R_{ab} = 1$$

$$\begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} \begin{vmatrix} i_a \\ i_b \end{vmatrix} = \begin{vmatrix} +8 \\ +10 \end{vmatrix}$$

$$i_a = \left| \begin{array}{cc} 8 & -1 \\ 10 & 6 \end{array} \right| \div \left| \begin{array}{cc} 3 & -1 \\ -1 & 6 \end{array} \right| = 58/17 = 3.412 \text{ amp}$$

$$i_b = \left| \begin{array}{cc} 3 & 8 \\ -1 & 10 \end{array} \right| \div (17) = 38/17 = 2.2353 \text{ amp}$$

$$V_{XY} = V_{TH} = 12 + 2i_a + 2i_b = 23.3 \text{ Volts,}$$

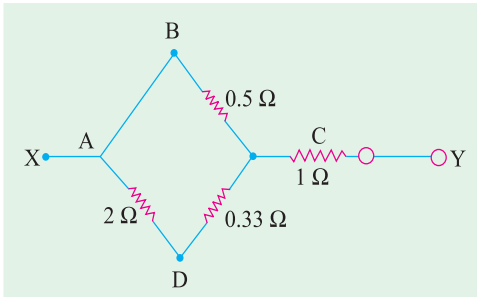


Fig. 2.18 (d)

with y positive w.r. to X.  $R_{TH}$  can be evaluated from Fig. 2.18 (c), after transforming delta configuration at nodes B-D-C to its equivalent star, as shown in Fig. 2.18 (d)

Further simplification results into :

$$R_{XY} = R_{TH} = 1.412 \text{ ohms}$$

$$\text{Hence, Load Current} = V_{TH}/(R_L + R_{TH}) = 23.3/6.412 = 3.643 \text{ amp.}$$

This agrees with result obtained earlier.

**Example 2.18 (a).** Determine the voltages 1 and 2 of the network in Fig. 2.19 (a) by nodal analysis. (Bombay University, 2001)

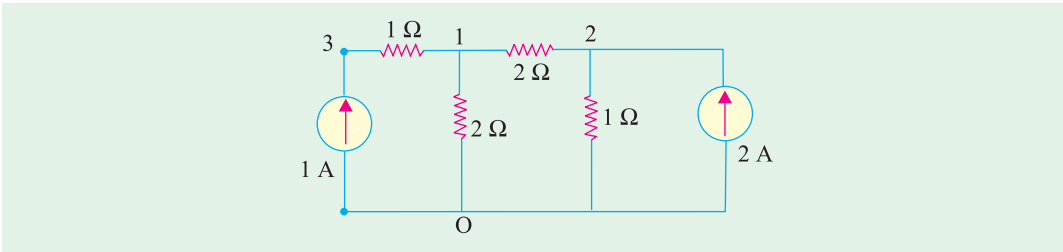


Fig. 2.19 (a)

**Solution.** Write the conductance matrix for the network, with nodes numbered as 1, 2, 4 as shown.

$$g_{11} = 1 + 0.5 + 0.5 = 2 \text{ mho, } g_{22} = 1 + 0.5 = 1.5 \text{ mho,}$$

$$g_{33} = 1 \text{ mho, } g_{12} = 0.5 \text{ mho, } g_{23} = 0, g_{13} = 1 \text{ mho}$$

$$\Delta = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25, \quad \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 & -1 \\ -0.5 & 2 & 0 \\ -1 & 1 & 1.0 \end{vmatrix} = 2.5$$

This gives  $V_1 = \Delta_1/\Delta = 2.50/1.25 = 2 \text{ Volts}$

And  $V_2 = \Delta_2/\Delta = 2.50/1.25 = 2 \text{ Volts}$

It means that the 2-ohm resistor between nodes 1 and 2 does not carry current.

**Example 2.18 (b).** In the circuit of Fig. 2.19 (b), find current through 1-Ω resistor using both THEVENIN's theorem and SUPERPOSITION theorem.



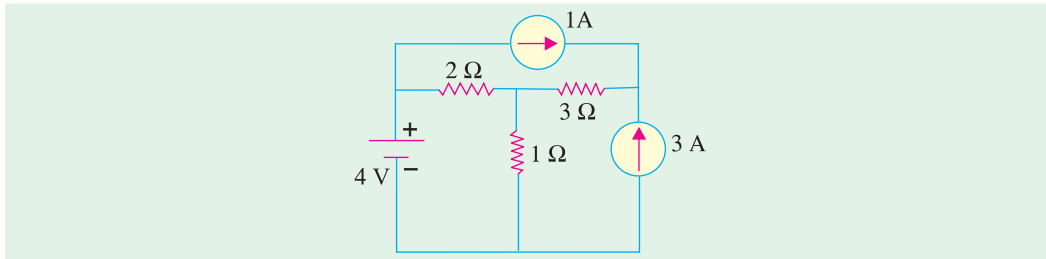


Fig. 2.19 (b)

**Solution. (i)** By Thevenin's Theorem :

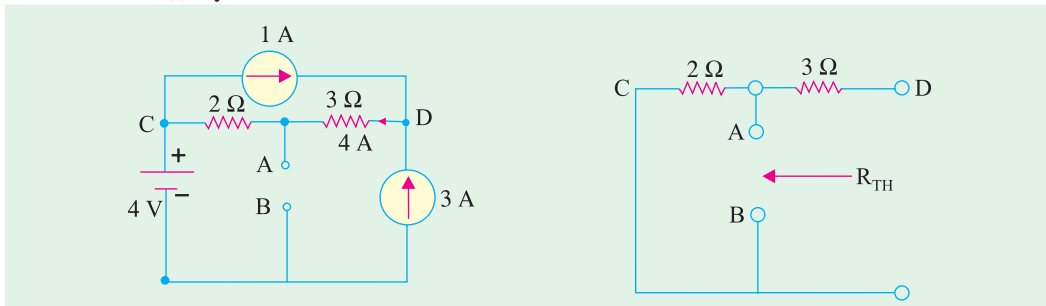


Fig. 2.19 (c)

Fig. 2.19 (d)

Take  $V_B = 0$ . Then  $V_A = 4 + 8 = 12$ , since from  $D$  to  $C$ , a current of  $4\text{ A}$  must flow, as shown in Fig. 2.19 (b), applying KCL of Node  $D$ .

$$V_{TH} = V_{AB} = 12 \text{ volts}$$

From Fig. 2.19 (d),  $R_{TH} = 2 \text{ ohms}$

$$I_L = 12 / (2 + 1) = 4 \text{ amp}$$

**(ii)** By Superposition Theorem : One source acts at a time. Current through  $A-B$  ( $1\text{ ohm}$ ) is to be calculated due to each source and finally all these contributions added.

Due to  $4\text{-V}$  source :

$1\text{-ohm}$  resistor carries a current of  $4/3$  amp from  $A$  to  $B$ , as shown in Fig. 2.19 (e).

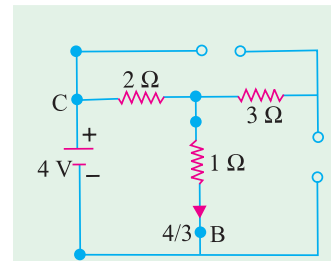


Fig. 2.19 (e).  $4\text{-V}$  Source acts

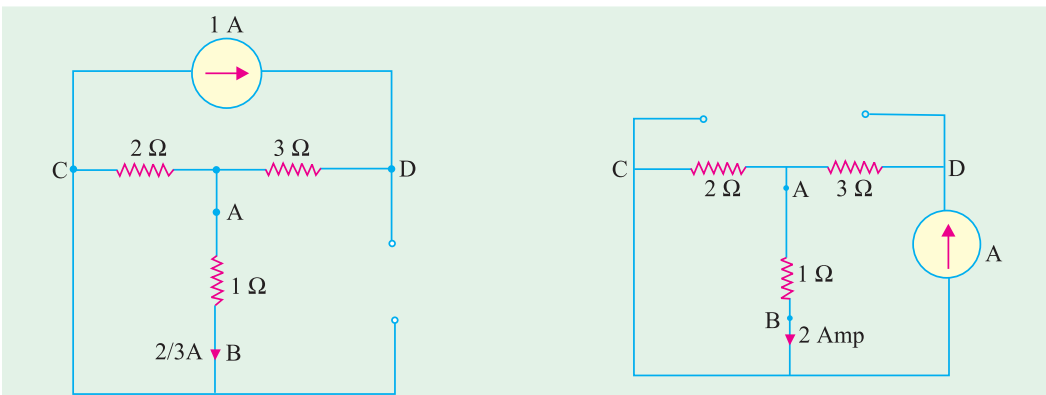


Fig. 2.19 (f).  $1\text{-A}$  Source acts

Fig. 2.19 (g).  $3\text{-A}$  Source acts

Due to  $1\text{-A}$  source :  $2/3$  Amp from  $A$  to  $B$ , as shown in Fig. 2.19 (f)

Due to  $3\text{-A}$  source :  $2$  Amp from  $A$  to  $B$  as shown in Fig. 2.19 (g)

Total current =  $4$  amp from  $A$  to  $B$ .

## 2.9. Independent and Dependent Sources

Those voltage or current sources, which do not depend on any other quantity in the circuit, are called independent sources. An independent d.c. voltage source is shown in Fig. 2.20 (a) whereas a time-varying voltage source is shown in Fig. 2.20 (b). The positive sign shows that terminal A is positive with respect to terminal B. In other words, potential of terminal A is  $v$  volts higher than that of terminal B.

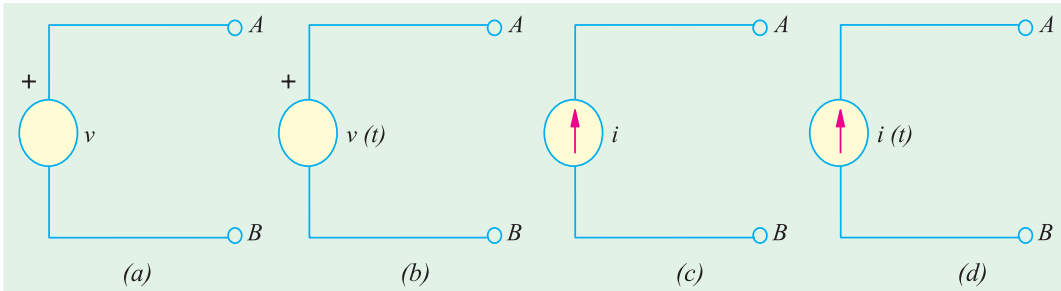


Fig. 2.20

$$\Delta_2 = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25, \quad \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

Similarly, Fig. 2.20 (c) shows an ideal constant current source whereas Fig. 2.20 (d) depicts a time-varying current source. The arrow shows the direction of flow of the current at any moment under consideration.

A dependent voltage or current source is one which depends on some other quantity in the circuit which may be either a voltage or a current. Such a source is represented by a diamond-shaped symbol as shown in Fig. 2.21 so as not to confuse it with an independent source. There are four possible dependent sources :

1. Voltage-dependent voltage source [Fig. 2.21 (a)]
2. Current-dependent voltage source [Fig. 2.21 (b)]
3. Voltage-dependent current source [Fig. 2.21 (c)]
4. Current-dependent current source [Fig. 2.21 (d)]

Such sources can also be either constant sources or time-varying sources. Such sources are often met in electronic circuits. As seen above, the voltage or current source is dependent on the and is

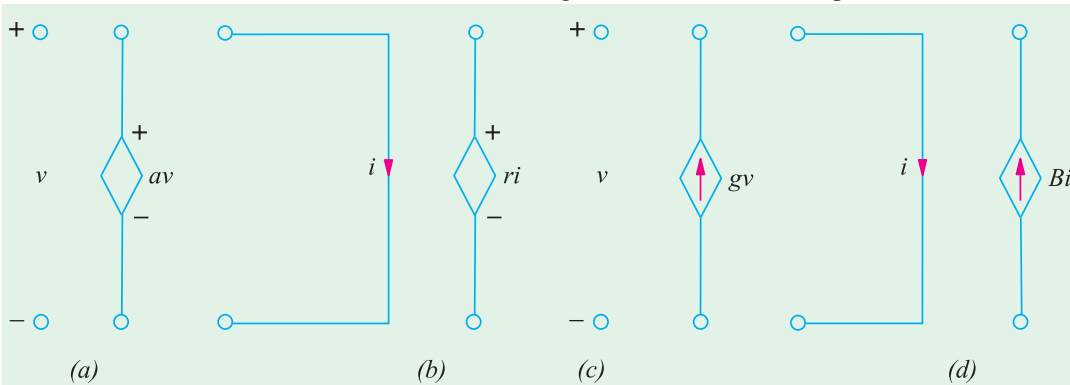


Fig. 2.21

proportional to another current or voltage. The constants of proportionality are written as  $a$ ,  $r$ ,  $g$  and  $\beta$ . The constants  $a$  and  $\beta$  have no units,  $r$  has the unit of ohms and  $g$  has the unit of siemens.

Independent sources actually exist as physical entities such as a battery, a *d.c.* generator and an alternator etc. But dependent sources are parts of *models* that are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.

**Example 2.19.** Using Kirchhoff's current law, find the values of the currents  $i_1$  and  $i_2$  in the circuit of Fig. 2.22 (a) which contains a current-dependent current source. All resistances are in ohms.

**Solution.** Applying KCL to node A, we get

$$2 - i_1 + 4i_1 - i_2 = 0 \quad \text{or} \quad -3i_1 + i_2 = 2$$

By Ohm's law,  $i_1 = v/3$  and  $i_2 = v/2$

Substituting these values above, we get

$$-3(v/3) + v/2 = 2 \quad \text{or} \quad v = -4 \text{ V}$$

$$\therefore i_1 = -4/3 \text{ A and } i_2 = -4/2 = -2 \text{ A}$$

The value of the dependent current source is  $= 4i_1 = 4 \times (-4/3) = -16/3 \text{ A}$ .

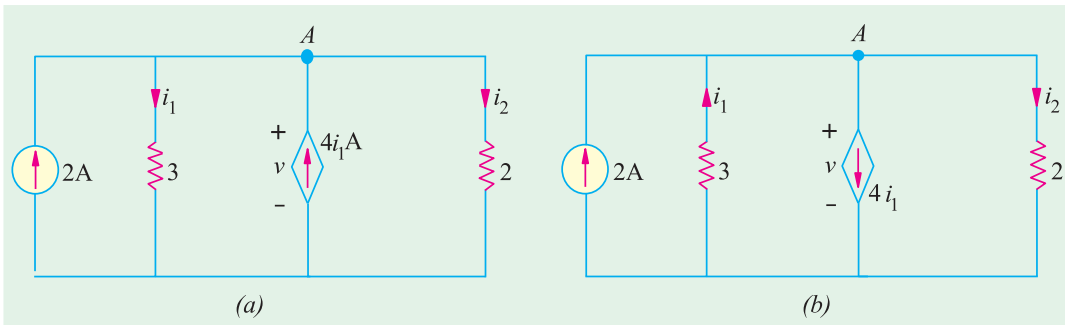


Fig. 2.22

Since  $i_1$  and  $i_2$  come out to be negative, it means that they flow upwards as shown in Fig. 2.22(b) and not downwards as presumed. Similarly, the current of the dependent source flows downwards as shown in Fig. 2.22 (b). It may also be noted that the sum of the upwards currents equals that of the downward currents.

**Example 2.20.** By applying Kirchhoff's current law, obtain the values of  $v$ ,  $i_1$  and  $i_2$  in the circuit of Fig. 2.23 (a) which contains a voltage-dependent current source. Resistance values are in ohms.

**Solution.** Applying KCL to node A of the circuit, we get

$$2 - i_1 + 4v - i_2 = 0 \quad \text{or} \quad i_1 + i_2 - 4v = 2$$

Now,  $i_1 = v/3$  and  $i_2 = v/6$

$$\therefore \frac{v}{3} + \frac{v}{6} - 4v = 2 \quad \text{or} \quad v = \frac{-4}{7} \text{ V}$$

$$\therefore i_1 = \frac{4}{21} \text{ A and } i_2 = \frac{2}{21} \text{ A and } 4v = 4 \times \frac{-4}{7} = \frac{-16}{7} \text{ V}$$

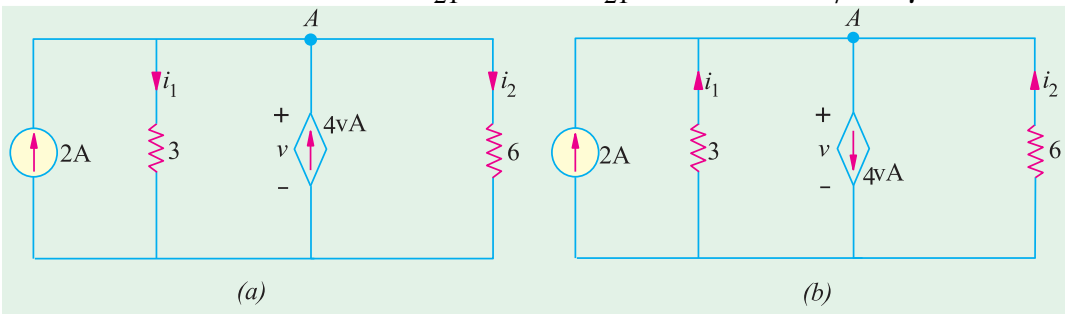


Fig. 2.23

Since  $i_1$  and  $i_2$  come out to be negative and value of current source is also negative, their directions of flow are opposite to those presumed in Fig. 2.23 (a). Actual current directions are shown in Fig. 2.23 (b).

**Example 2.21.** Apply Kirchhoff's voltage law, to find the values of current  $i$  and the voltage drops  $v_1$  and  $v_2$  in the circuit of Fig. 2.24 which contains a current-dependent voltage source. What is the voltage of the dependent source? All resistance values are in ohms.

**Solution.** Applying KVL to the circuit of Fig. 2.24 and starting from point A, we get

$$-v_1 + 4i - v_2 + 6 = 0 \quad \text{or} \quad v_1 - 4i + v_2 = 6$$

Now,  $v_1 = 2i$  and  $v_2 = 4i$

$$\therefore 2i - 4i + 4i = 6 \quad \text{or} \quad i = 3\text{A}$$

$$\therefore v_1 = 2 \times 3 = 6\text{V} \quad \text{and} \quad v_2 = 4 \times 3 = 12\text{V}$$

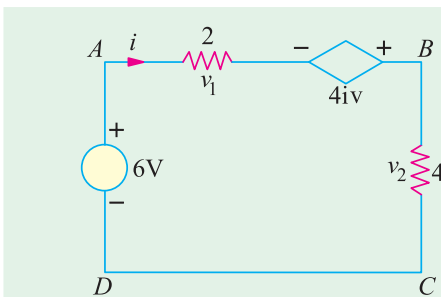


Fig. 2.24

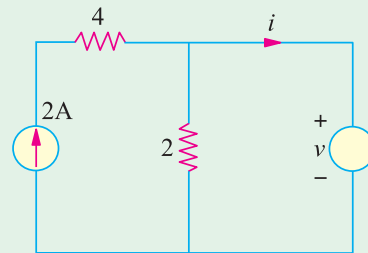


Fig. 2.25

$$\text{Voltage of the dependent source} = 4i = 4 \times 3 = 12\text{V}$$

**Example 2.22.** In the circuit shown in Fig. 2.25, apply KCL to find the value of  $i$  for the case when (a)  $v = 2\text{V}$ , (b)  $v = 4\text{V}$  (c)  $v = 6\text{V}$ . The resistor values are in ohms.

**Solution.** (a) When  $v = 2\text{V}$ , current through  $2\ \Omega$  resistor which is connected in parallel with the  $2\text{V}$  source  $= 2/2 = 1\text{A}$ . Since the source current is  $2\text{A}$ ,  $i = 2 - 1 = 1\text{A}$ .

(b) When  $v = 4\text{V}$ , current through the  $2\ \Omega$  resistor  $= 4/2 = 2\text{A}$ . Hence  $i = 2 - 2 = 0\text{A}$ .

(c) When  $v = 6\text{V}$ , current through the  $2\ \Omega$  resistor  $= 6/2 = 3\text{A}$ . Since current source can supply only  $2\text{A}$ , the balance of  $1\text{A}$  is supplied by the voltage source. Hence,  $i = -1\text{A}$  i.e. it flows in a direction opposite to that shown in Fig. 2.25.

**Example 2.23.** In the circuit of Fig. 2.26, apply KCL to find the value of current  $i$  when (a)  $K = 2$  (b)  $K = 3$  and (c)  $K = 4$ . Both resistances are in ohms.

**Solution.** Since  $6\ \Omega$  and  $3\ \Omega$  resistors are connected in parallel across the  $24\text{-V}$  battery,  $i_1 = 24/6 = 4\text{A}$ .

Applying KCL to node A, we get  $i - 4 + 4K - 8 = 0$  or  $i = 12 - 4K$ .

(a) When  $K = 2$ ,  $i = 12 - 4 \times 2 = 4\text{A}$

(b) When  $K = 3$ ,  $i = 12 - 4 \times 3 = 0\text{A}$

(c) When  $K = 4$ ,  $i = 12 - 4 \times 4 = -4\text{A}$

It means that current  $i$  flows in the opposite direction.

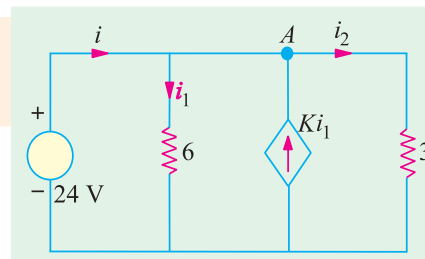


Fig. 2.26

**Example 2.24.** Find the current  $i$  and also the power and voltage of the dependent source in Fig. 2.72 (a). All resistances are in ohms.

**Solution.** The two current sources can be combined into a single source of  $8 - 6 = 2$  A. The two parallel  $4 \Omega$  resistances when combined have a value of  $2 \Omega$  which, being in series with the  $10 \Omega$  resistance, gives the branch resistance of  $10 + 2 = 12 \Omega$ . This  $12 \Omega$  resistance when combined with the other  $12 \Omega$  resistance gives a combination resistance of  $6 \Omega$ . The simplified circuit is shown in Fig. 2.27 (b.)

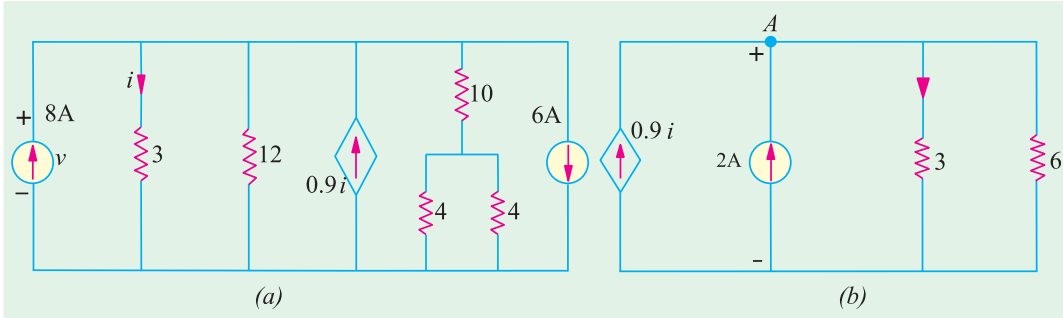


Fig. 2.27

Applying KCL to node A, we get

$$0.9i + 2 - i - V/6 = 0 \quad \text{or} \quad 0.6i = 12 - v$$

Also  $v = 3i \therefore i = 10/3$  A. Hence,  $v = 10$  V.

The power furnished by the current source =  $v \times 0.9 i = 10 \times 0.9 (10/3) = 30$  W.

**Example 2.25.** By using voltage-divider rule, calculate the voltages  $v_x$  and  $v_y$  in the network shown in Fig. 2.28.

**Solution.** As seen, 12 V drop in over the series combination of 1, 2 and 3  $\Omega$  resistors. As per voltage-divider rule  $v_x =$  drop over  $3 \Omega = 12 \times 3/6 = 6$  V.

The voltage of the dependent source =  $12 \times 6 = 72$  V.

The voltage  $v_y$  equals the drop across  $8 \Omega$  resistor connected across the voltage source of 72 V.

Again using voltage-divider rule, drop over  $8 \Omega$  resistor =  $72 \times 8/12 = 48$  V.

Hence,  $v_y = -48$  V. The negative sign has been given because positive and negative signs of  $v_y$  are actually opposite to those shown in Fig. 2.28.

**Example 2.26.** Use KCL to find the value of  $v$  in the circuit of Fig. 2.29.

**Solution.** Let us start from ground and go to point  $a$  and find the value of voltage  $v_a$ . Obviously,  $5 + v = v_a$  or  $v = v_a - 5$ . Applying KCL to point, we get

$$6 - 2 - v + (5 - v_a)/1 = 0 \quad \text{or} \quad 6 - 2 - (v_a - 5) + (5 - v_a) = 0 \quad \text{or} \quad v_a = 7 \text{ V}$$

Hence,  $v = v_a - 5 = 7 - 5 = 2$  V. Since it turns out to be positive, its sign as indicated in the figure is correct.

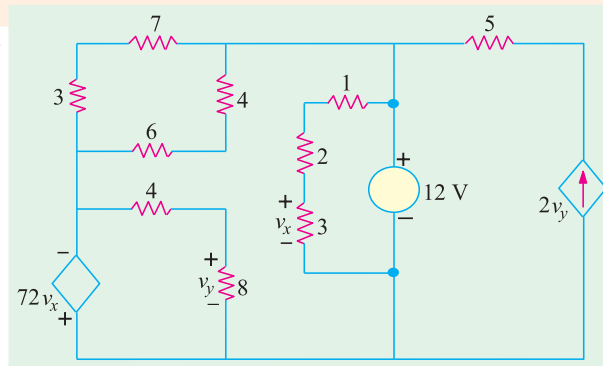


Fig. 2.28

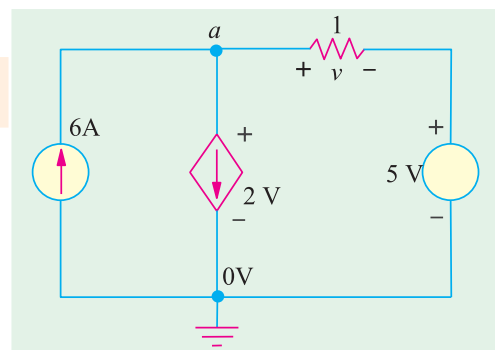


Fig. 2.29

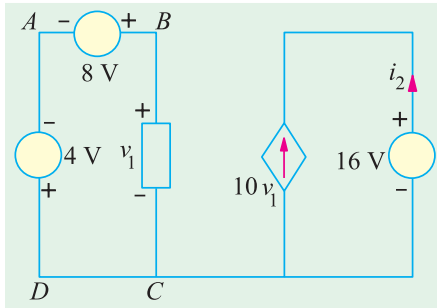


Fig. 2.30

**Example 2.27. (a) Basic Electric Circuits by Cunningham.**

Find the value of current  $i_2$  supplied by the voltage-controlled current source (VCCS) shown in Fig. 2.30.

**Solution.** Applying KVL to the closed circuit ABCD, we have  $-4 + 8 - v_1 = 0 \therefore v_1 = 4 \text{ V}$

The current supplied by VCCS is  $10 v_1 = 10 \times 4 = 40 \text{ A}$ . Since  $i_2$  flows in an opposite direction to this current, hence  $i_2 = -40 \text{ A}$ .

**Example 2.27. (b).** Find the voltage drop  $v_2$  across the current-controlled voltage source (CCVS) shown in Fig. 2.28.

**Solution.** Applying KCL to point A, we have  $2 + 6 - i_1 = 0$  or  $i_1 = 8 \text{ A}$ .

Application of KVL to the closed circuit on the right hand side gives  $5 i_1 - v_2 = 0$  or  $v_2 = 5 i_1 = 5 \times 8 = 40 \text{ V}$ .

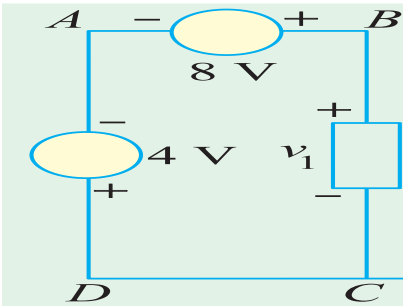


Fig. 2.31

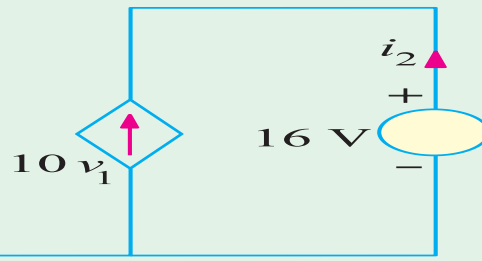


Fig. 2.32

**Example 2.28.** Find the values of  $i_1$ ,  $v_1$ ,  $v_x$  and  $v_{ab}$  in the network of Fig. 2.32 with its terminals  $a$  and  $b$  open.

**Solution.** It is obvious that  $i_1 = 4 \text{ A}$ . Applying KVL to the left-hand closed circuit, we get  $-40 + 20 - v_1 = 0$  or  $v_1 = -20 \text{ V}$ .

Similarly, applying KVL to the second closed loop, we get

$$v_1 - v_x + 4v_1 - 50 = 0 \text{ or } v_x = 5 v_1 - 50 = -5 \times 20 - 50 = -150 \text{ V}$$

Again applying KVL to the right-hand side circuit containing  $v_{ab}$ , we get

$$50 - 4v_1 - 10 v_{ab} = 0 \text{ or } v_{ab} = 50 - 4(-20) - 10 = 120 \text{ V}$$

**Example 2.29 (a).** Find the current  $i$  in the circuit of Fig. 2.33. All resistances are in ohms.

**Solution.** The equivalent resistance of the two parallel paths across point  $a$  is  $3 \parallel (4 + 2) = 2 \Omega$ . Now, applying KVL to the closed loop, we get  $24 - v - 2v - 2i = 0$ . Since  $v = 2i$ , we get  $24 - 2i - 2(2i) - 2i = 0$  or  $i = 3 \text{ A}$ .

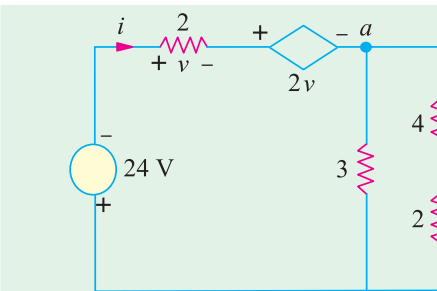


Fig. 2.33

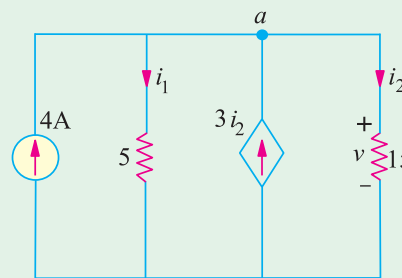


Fig. 2.34

**Example 2.29. (b)** Determine the value of current  $i_2$  and voltage drop  $v$  across  $15 \Omega$  resistor in Fig. 2.34.

**Solution.** It will be seen that the dependent current source is related to  $i_2$ . Applying KCL to node  $a$ , we get  $4 - i_1 + 3i_2 - i_2 = 0$  or  $4 - i_1 + 3 i_2 = 0$ .

Applying ohm's law, we get  $i_1 = v/5$  and  $i_2 = v/15$ .

Substituting these values in the above equation, we get  $4 - (v/5) + 3(v/15) = 0$  or  $v = 60 \text{ V}$  and  $i_2 = 4 \text{ A}$ .

**Example 2.29 (c).** In the circuit of Fig. 2.35, find the values of  $i$  and  $v$ . All resistances are in ohms.

**Solution.** It may be noted that  $12 + v = v_a$  or  $v = v_a - 12$ . Applying KCL to node  $a$ , we get

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \quad \text{or} \quad v_a = 4 \text{ V}$$

Hence,  $v = 4 - 12 = -8 \text{ V}$ . The negative sign shows that its polarity is opposite to that shown in Fig. 2.35. The current flowing from the point  $a$  to ground is  $4/2 = 2 \text{ A}$ . Hence,  $i = -2 \text{ A}$ .

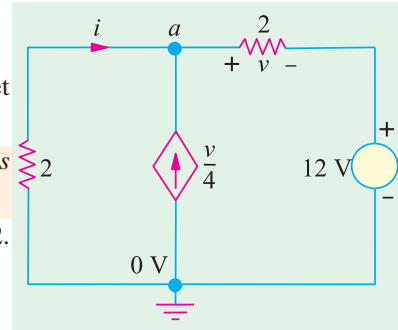


Fig. 2.35

### Tutorial Problems No. 2.1

1. Apply KCL to find the value of  $I$  in Fig. 2.36.

[8 A]

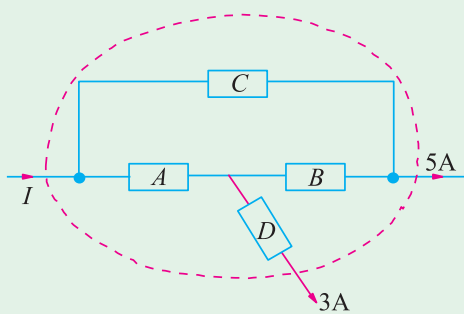


Fig. 2.36

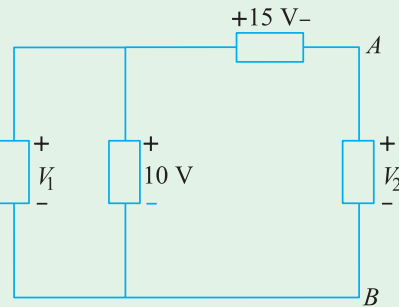


Fig. 2.37

2. Applying Kirchhoff's voltage law, find  $V_1$  and  $V_2$  in Fig. 2.37.

[ $V_1 = 10 \text{ V}$ ;  $V_2 = 5 \text{ V}$ ]

3. Find the values of currents  $I_2$  and  $I_4$  in the network of Fig. 2.38.

[ $I_2 = 4 \text{ A}$ ;  $I_4 = 5 \text{ A}$ ]

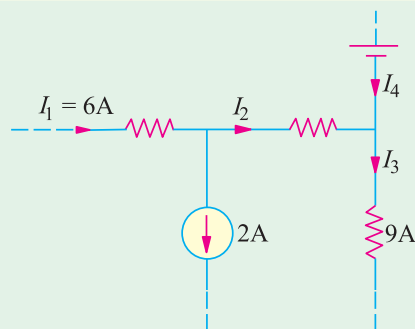


Fig. 2.38

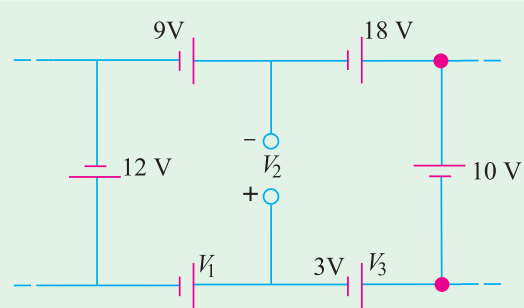


Fig. 2.39

4. Use Kirchhoff's law, to find the values of voltages  $V_1$  and  $V_2$  in the network shown in Fig. 2.39.

[ $V_1 = 2 \text{ V}$ ;  $V_2 = 5 \text{ V}$ ]

5. Find the unknown currents in the circuits shown in Fig. 2.40 (a).

$[I_1 = 2 \text{ A} ; I_2 = 7 \text{ A}]$

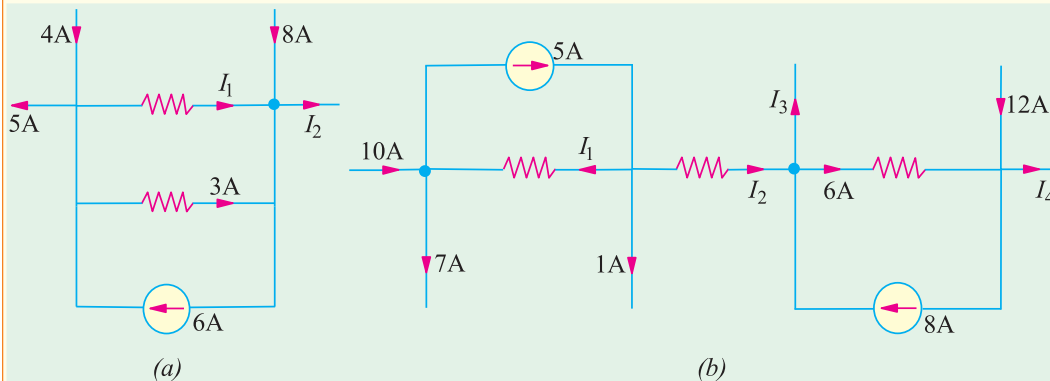


Fig. 2.40

6. Using Kirchhoff's current law, find the values of the unknown currents in Fig. 2.40 (b).

$[I_1 = 2 \text{ A} ; I_2 = 2 \text{ A} ; I_3 = 4 \text{ A} ; I_4 = 10 \text{ A}]$

7. In Fig. 2.41, the potential of point A is  $-30 \text{ V}$ . Using Kirchhoff's voltage law, find (a) value of  $V$  and (b) power dissipated by  $5 \Omega$  resistance. All resistances are in ohms.

$[100 \text{ V} ; 500 \text{ W}]$

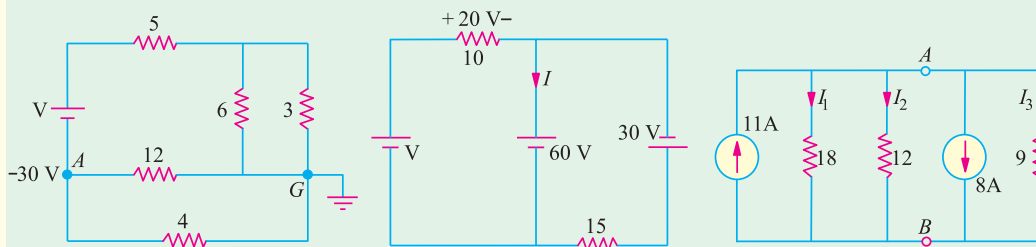


Fig. 2.41

Fig. 2.42

Fig. 2.43

8. Using KVL and KCL, find the values of  $V$  and  $I$  in Fig. 2.42. All resistances are in ohms.

$[80 \text{ V} ; -4 \text{ A}]$

9. Using KCL, find the values  $V_{AB}$ ,  $I_1$ ,  $I_2$  and  $I_3$  in the circuit of Fig. 2.43. All resistances are in ohms.

$[V_{AB} = 12 \text{ V} ; I_1 = 2/3 \text{ A} ; I_2 = 1 \text{ A} ; I_3 = 4/3 \text{ A}]$

10. A bridge network  $ABCD$  is arranged as follows :

Resistances between terminals  $A-B$ ,  $B-C$ ,  $C-D$ ,  $D-A$ , and  $B-D$  are 10, 20, 15, 5 and 40 ohms respectively. A 20 V battery of negligible internal resistance is connected between terminals  $A$  and  $C$ . Determine the current in each resistor.

$[AB = 0.645 \text{ A} ; BC = 0.678 \text{ A} ; AD = 1.025 \text{ A} ; DB = 0.033 \text{ A} ; DC = 0.992 \text{ A}]$

11. Two batteries  $A$  and  $B$  are connected in parallel and a load of  $10 \Omega$  is connected across their terminals.  $A$  has an e.m.f. of 12 V and an internal resistance of  $2 \Omega$  ;  $B$  has an e.m.f. of 8 V and an internal resistance of  $1 \Omega$ . Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the p.d. across the external resistance.

$[I_A = 1.625 \text{ A (discharge)}, I_B = 0.75 \text{ A (charge)} ; 0.875 \text{ A} ; 8.75 \text{ V}]$

12. The four arms of a Wheatstone bridge have the following resistances ;  $AB = 100$ ,  $BC = 10$ ,  $CD = 4$ ,  $DA = 50$  ohms.

A galvanometer of 20 ohms resistance is connected across  $BD$ . Calculate the current through the galvanometer when a potential difference of 10 volts is maintained across  $AC$ .

$[0.00513 \text{ A}]$  [Elect. Tech. Lond. Univ.]

13. Find the voltage  $V_{da}$  in the network shown in Fig. 2.44 (a) if  $R$  is  $10 \Omega$  and (b)  $20 \Omega$

$[(a) 5 \text{ V} (b) 5 \text{ V}]$

14. In the network of Fig. 2.44 (b), calculate the voltage between points  $a$  and  $b$  i.e.  $V_{ab}$ .

$[30 \text{ V}]$  (Elect. Engg. I, Bombay Univ.)



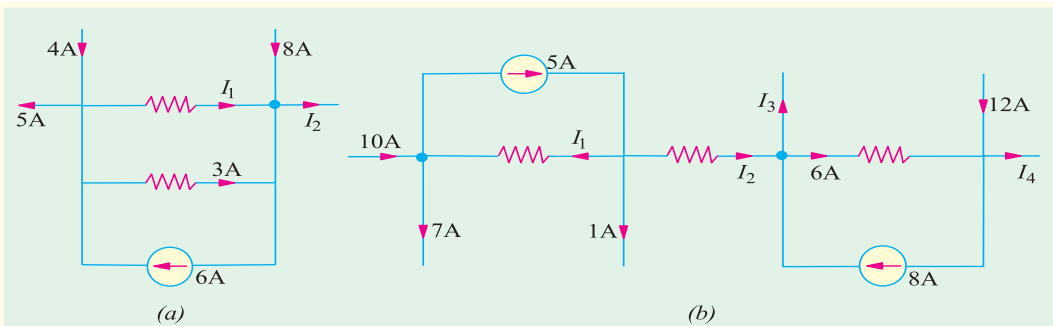


Fig. 2.44

[Hint : In the above two cases, the two closed loops are independent and no current passes between them].

15. A battery having an *E.M.F.* of 110 V and an internal resistance of  $0.2 \Omega$  is connected in parallel with another battery having an *E.M.F.* of 100 V and internal resistance  $0.25 \Omega$ . The two batteries in parallel are placed in series with a regulating resistance of  $5 \Omega$  and connected across 200 V mains. Calculate the magnitude and direction of the current in each battery and the total current taken from the supply mains.

[ $I_A = 11.96$  (discharge);  $I_B = 30.43$  A (charge) : 18.47 A] (Elect Technology, Sumbhal Univ.)

16. Three batteries *P*, *Q* and *R* consisting of 50, 55 and 60 cells in series respectively supply in parallel a common load of 100 A. Each cell has a e.m.f. of 2 V and an internal resistance of  $0.005 \Omega$ . Determine the current supplied by each battery and the load voltage.

[1.2 A ; 35.4 A : 65.8 A : 100.3 V] (Basic Electricity, Bombay Univ.)

17. Two storage batteries are connected in parallel to supply a load having a resistance of  $0.1 \Omega$ . The open-circuit e.m.f. of one battery (*A*) is 12.1 V and that of the other battery (*B*) is 11.8 V. The internal resistances are  $0.03 \Omega$  and  $0.04 \Omega$  respectively. Calculate (i) the current supplied at the lead (ii) the current in each battery (iii) the terminal voltage of each battery.

[(i) 102.2 A (ii) 62.7 A (A), 39.5 A (B) (iii) 10.22 V] (London Univ.)

18. Two storage batteries, *A* and *B*, are connected in parallel to supply a load the resistance of which is  $1.2 \Omega$ . Calculate (i) the current in this load and (ii) the current supplied by each battery if the open-circuit e.m.f. of *A* is 12.5 V and that of *B* is 12.8 V, the internal resistance of *A* being  $0.05 \Omega$  and that of *B*  $0.08 \Omega$ .

[(i) 10.25 A (ii) 4 (A), 6.25 A (B)] (London Univ.)

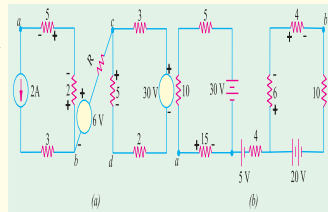


Fig. 2.45

19. The circuit of Fig. 2.45 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. Both resistances are in ohms. [8 A ; 1920 W]

20. Find the equivalent resistance between terminals *a* and *b* of the network shown in Fig. 2.46. [2  $\Omega$ ]

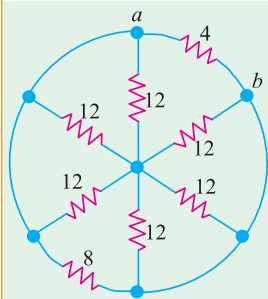


Fig. 2.46

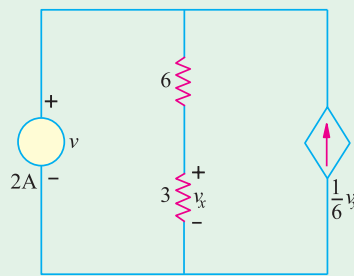


Fig. 2.47

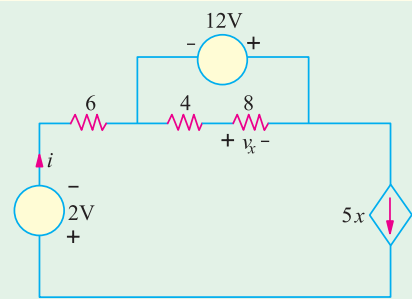


Fig. 2.48

21. Find the value of the voltage  $v$  in the network of Fig. 2.47. [36 V]
22. Determine the current  $i$  for the network shown in Fig. 2.48. [-40 A]
23. State and explain Kirchhoff's current law. Determine the value of  $R_S$  and  $R_p$  in the network of Fig. 2.49 if  $V_2 = V_1/2$  and the equivalent resistance of the network between the terminals  $A$  and  $B$  is  $100 \Omega$ .  
[ $R_S = 100/3 \Omega$   $R_p = 400/3 \Omega$ ] (Elect. Engg. I, Bombay Univ.)
24. Four resistance each of  $R$  ohms and two resistances each of  $S$  ohms are connected (as shown in Fig. 2.50) to four terminals  $AB$  and  $CD$ . A p.d. of  $V$  volts is applied across the terminals  $AB$  and a resistance of  $Z$  ohm is connected across the terminals  $CD$ . Find the value of  $Z$  in terms of  $S$  and  $R$  in order that the current at  $AB$  may be  $V/Z$ .

Find also the relationship that must hold between  $R$  and  $S$  in order that the p.d. at the points  $EF$  be  $V/2$ .  
[ $Z = \sqrt{R(R + 2S)}$ ;  $S = 4R$ ]

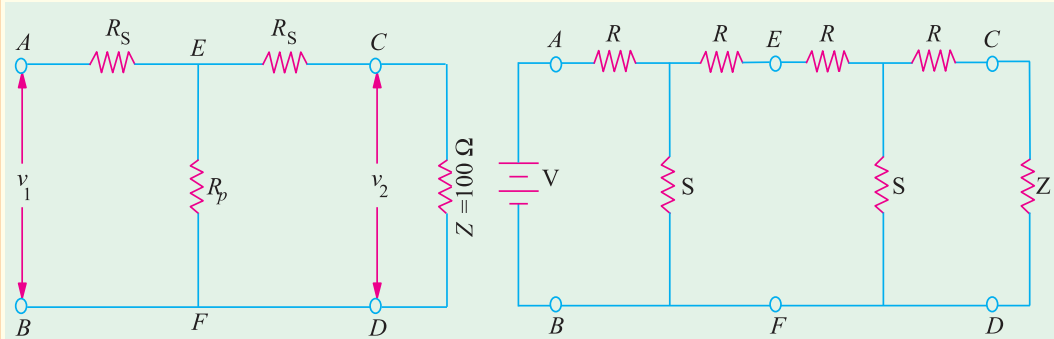


Fig. 2.49

Fig. 2.50

## 2.10. Maxwell's Loop Current Method

This method which is particularly well-suited to coupled circuit solutions employs a system of **loop** or **mesh** currents instead of **branch** currents (as in Kirchhoff's laws). Here, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. This method eliminates a great deal of tedious work involved in the branch-current method and is best suited when energy sources are voltage sources rather than current sources. Basically, this method consists of writing loop voltage equations by Kirchhoff's voltage law in terms of unknown loop currents. As will be seen later, the number of independent equations to be solved reduces from  $b$  by Kirchhoff's laws to  $b - (j - 1)$  for the loop current method where  $b$  is the number of branches and  $j$  is the number of junctions in a given network.

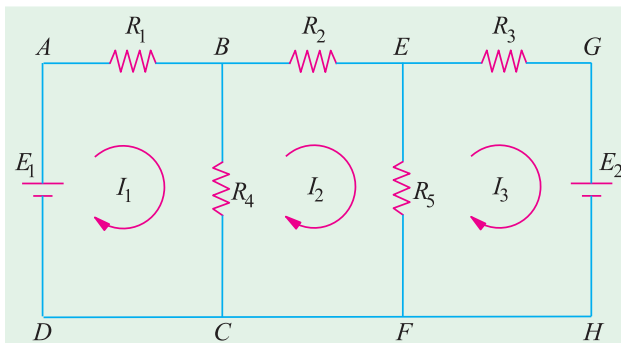


Fig. 2.51

Fig. 2.51 shows two batteries  $E_1$  and  $E_2$  connected in a network consisting of five resistors. Let the loop currents for the three meshes be  $I_1, I_2$  and  $I_3$ . It is obvious that current through  $R_4$  (when considered as a part of the first loop) is  $(I_1 - I_2)$  and that through  $R_5$  is  $(I_2 - I_3)$ . However, when  $R_4$  is considered part of the second loop, current through it is  $(I_2 - I_1)$ . Similarly, when  $R_5$  is considered part of the third loop, current through it is  $(I_3 - I_2)$ . Applying Kirchhoff's voltage law to the three loops, we get,

$$E_1 - I_1 R_1 - R_4 (I_1 - I_2) = 0 \quad \text{or} \quad I_1 (R_1 + R_4) - I_2 R_4 - E_1 = 0 \quad \dots \text{loop 1}$$

Similarly,  $-I_2R_2 -R_5 (I_2 -I_3) -R_4 (I_2 -I_1) = 0$

or  $I_2 R_4 -I_2 (R_2 + R_4 + R_5) + I_3R_5 = 0$

...loop 2

Also  $-I_3R_3 -E_2 -R_5 (I_3 -I_2) = 0$  or  $I_2R_5 -I_3 (R_3 + R_5) -E_2 = 0$

...loop 3

The above three equations can be solved not only to find loop currents but branch currents as well.

## 2.11. Mesh Analysis Using Matrix Form

Consider the network of Fig. 2.52, which contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be designated as  $I_1, I_2$  and  $I_3$  and all the three may be assumed to flow in the clockwise direction for obtaining symmetry in mesh equations.

Applying KVL to mesh (i), we have

$$E_1 -I_1R_1 -R_3 (I_1 -I_3) -R_2 (I_1 -I_2) = 0$$

or  $(R_1 + R_2 + R_3) I_1 -R_2I_2 -R_3I_3 = E_1$  ... (i)

Similarly, from mesh (ii), we have

$$E_2 -R_2 (I_2 -I_1) -R_5 (I_2 -I_3) -I_2R_4 = 0$$

or  $-R_2I_1 + (R_2 + R_4 + R_5) I_2 -R_5I_3 = E_2$  ... (ii)

Applying KVL to mesh (iii), we have

$$E_3 -I_3R_7 -R_5 (I_3 -I_2) -R_3 (I_3 -I_1) -I_3 R_6 = 0$$

or  $-R_3I_1 -R_5I_2 + (R_3 + R_5 + R_6 + R_7) I_3 = E_3$  ... (iii)

It should be noted that signs of different items in the above three equations have been so changed as to make the items containing self resistances positive (please see further).

The matrix equivalent of the above three equations is

$$\begin{bmatrix} + (R_1 + R_2 + R_3) & -R_2 & -R_3 \\ -R_2 & + (R_2 + R_4 + R_5) & -R_5 \\ -R_3 & -R_5 & + (R_3 + R_5 + R_6 + R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It would be seen that the first item is the first row *i.e.*  $(R_1 + R_2 + R_3)$  represents the self resistance of mesh (i) which equals the sum of all resistance in mesh (i). Similarly, the second item in the first row represents the mutual resistance between meshes (i) and (ii) *i.e.* the sum of the resistances common to mesh (i) and (ii). Similarly, the third item in the first row represents the mutual-resistance of the mesh (i) and mesh (iii).

The item  $E_1$ , in general, represents the algebraic sum of the voltages of all the voltage sources acting around mesh (i). Similar is the case with  $E_2$  and  $E_3$ . The sign of the *e.m.f.*s is the same as discussed in Art. 2.3 *i.e.* while going along the current, if we pass from negative to the positive terminal of a battery, then its *e.m.f.* is taken positive. If it is the other way around, then battery *e.m.f.* is taken negative.

In general, let

$R_{11}$  = self-resistance of mesh (i)

$R_{22}$  = self-resistance of mesh (ii) *i.e.* sum of all resistances in mesh (ii)

$R_{33}$  = Self-resistance of mesh (iii) *i.e.* sum of all resistances in mesh (iii)

$R_{12} = R_{21} = -[\text{Sum of all the resistances common to meshes (i) and (ii)}]^*$

$R_{23} = R_{32} = -[\text{Sum of all the resistances common to meshes (ii) and (iii)}]^*$

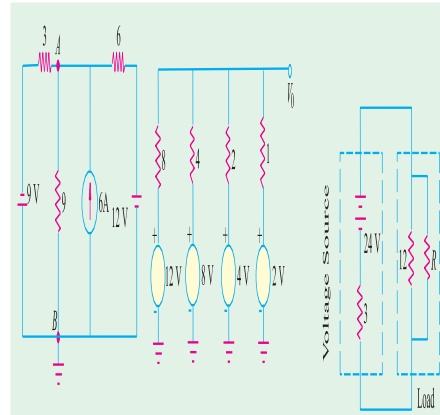


Fig. 2.52

\* Although, it is easier to take all loop currents in one direction (Usually clockwise), the choice of direction for any loop current is arbitrary and may be chosen independently of the direction of the other loop currents.

$$R_{31} = R_{13} = -[\text{Sum of all the resistances common to meshes (i) and (iii)}] *$$

Using these symbols, the generalized form of the above matrix equivalent can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

If there are  $m$  independent meshes in any linear network, then the mesh equations can be written in the matrix form as under :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & \dots & R_{1m} \\ R_{21} & R_{22} & R_{23} & \dots & R_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ R_{31} & R_{32} & R_{33} & \dots & R_{3m} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ \dots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \dots \\ \dots \\ E_m \end{bmatrix}$$

The above equations can be written in a more compact form as  $[R_m][I_m] = [E_m]$ . It is known as Ohm's law in matrix form.

In the end, it may be pointed out that the directions of mesh currents can be selected arbitrarily. If we assume each mesh current to flow in the clockwise direction, then

(i) All self-resistances will always be positive and (ii) all mutual resistances will always be negative. We will adapt this sign convention in the solved examples to follow.

The above main advantage of the generalized form of all mesh equations is that they can be easily remembered because of their symmetry. Moreover, for any given network, these can be written by inspection and then solved by the use of determinants. It eliminates the tedium of deriving simultaneous equations.

**Example. 2.30.** Write the impedance matrix of the network shown in Fig. 2.53 and find the value of current  $I_3$ . **(Network Analysis A.M.I.E. Sec. B.W. 1980)**

**Solution.** Different items of the mesh-resistance matrix  $[R_m]$  are as under :

$$R_{11} = 1 + 3 + 2 = 6 \Omega; R_{22} = 2 + 1 + 4 = 7 \Omega; R_{33} = 3 + 2 + 1 = 6 \Omega;$$

$$R_{12} = R_{21} = -2 \Omega; R_{23} = R_{32} = -1 \Omega; R_{13} = R_{31} = -3 \Omega;$$

$$E_1 = +5 \text{ V}; E_2 = 0; E_3 = 0.$$

The mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{vmatrix} = 6(42 - 1) + 2(-12 - 3) - 3(2 + 21) = 147$$

$$\Delta_3 = \begin{vmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{vmatrix} = 6 + 2(5) - 3(-35) = 121$$

$$I_3 = \Delta_3 / \Delta = \frac{121}{147} = 0.823 \text{ A}$$

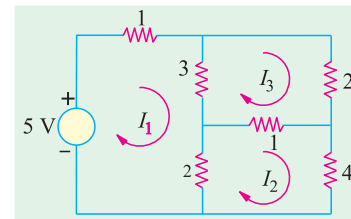


Fig. 2.53

\* In general, if the two currents through the common resistance flow in the same direction, then the mutual resistance is taken as negative. One the other hand, if the two currents flow in the same direction, mutual resistance is taken as positive.

**Example 2.31.** Determine the current supplied by each battery in the circuit shown in Fig. 2.54. (Electrical Engg. Aligarh Univ.)

**Solution.** Since there are three meshes, let the three loop currents be shown in Fig. 2.51.

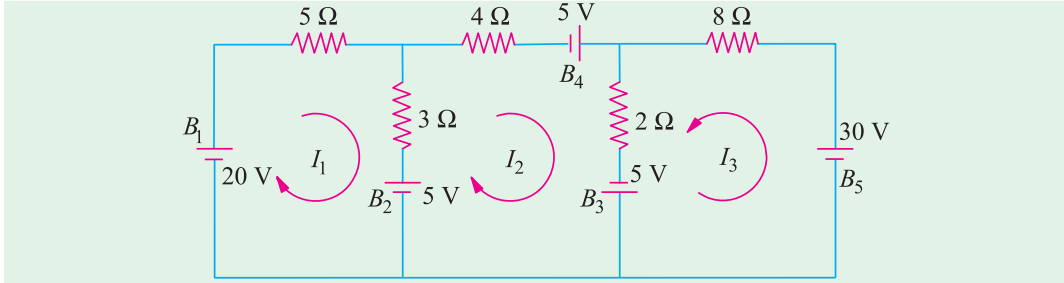


Fig. 2.54

For loop 1 we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0 \quad \text{or} \quad 8I_1 - 3I_2 = 15 \quad \dots(i)$$

For loop 2 we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0 \quad \text{or} \quad 3I_1 - 9I_2 + 2I_3 = -15 \quad \dots(ii)$$

Similarly, for loop 3, we get

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0 \quad \text{or} \quad 2I_2 - 10I_3 = 35 \quad \dots(iii)$$

Eliminating  $I_1$  from (i) and (ii), we get  $63I_2 - 16I_3 = 165 \quad \dots(iv)$

Similarly, for  $I_2$  from (iii) and (iv), we have  $I_2 = 542/299 \text{ A}$

From (iv),  $I_3 = -1875/598 \text{ A}$

Substituting the value of  $I_2$  in (i), we get  $I_1 = 765/299 \text{ A}$

Since  $I_3$  turns out to be negative, actual directions of flow of loop currents are as shown in Fig. 2.55.

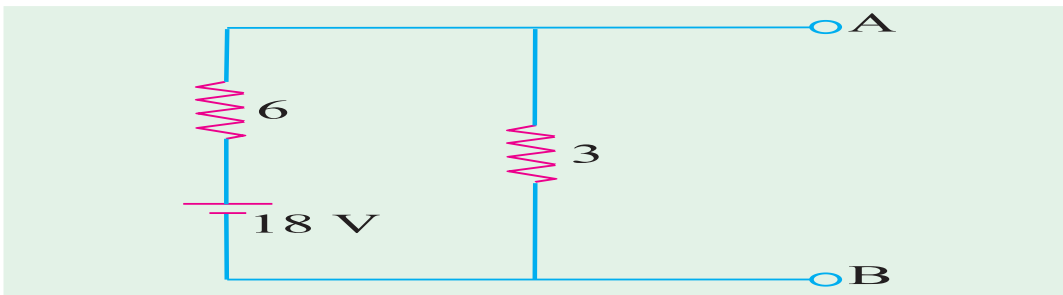


Fig. 2.55

Discharge current of  $B_1 = 765/299 \text{ A}$   
 Charging current of  $B_2 = I_1 - I_2 = 220/299 \text{ A}$   
 Discharge current of  $B_3 = I_2 + I_3 = 2965/598 \text{ A}$   
 Discharge current of  $B_4 = I_2 = 545/299 \text{ A}$ ; Discharge current of  $B_5 = 1875/598 \text{ A}$

**Solution by Using Mesh Resistance Matrix.**

The different items of the mesh-resistance matrix  $[R_m]$  are as under :

$$R_{11} = 5 + 3 = 8 \Omega; R_{22} = 4 + 2 + 3 = 9 \Omega; R_{33} = 8 + 2 = 10 \Omega$$

$$R_{12} = R_{21} = -3 \Omega; R_{13} = R_{31} = 0; R_{23} = R_{32} = -2 \Omega$$

$$E_1 = \text{algebraic sum of the voltages around mesh (i)} = 20 - 5 = 15 \text{ V}$$

$$E_2 = 5 + 5 + 5 = 15 \text{ V}; E_3 = -30 - 5 = -35 \text{ V}$$

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90 - 4) + 3(-30) = 598$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(90 - 4) - 15(-30) - 35(6) = 1530$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150 - 70) + 3(150 + 0) = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(-315 + 30) + 3(105 + 30) = -1875$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \frac{765}{299} \text{ A}; I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \frac{545}{299} \text{ A}; I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598} \text{ A}$$

**Example 2.32.** Determine the current in the 4-Ω branch in the circuit shown in Fig. 2.56.

(Elect. Technology, Nagpur Univ.)

**Solution.** The three loop currents are as shown in Fig. 2.53 (b).

For loop 1, we have

$$-1(I_1 - I_2) - 3(I_1 - I_3) - 4I_1 + 24 = 0 \quad \text{or} \quad 8I_1 - I_2 - 3I_3 = 24 \quad \dots(i)$$

For loop 2, we have

$$12 - 2I_2 - 12(I_2 - I_3) - 1(I_2 - I_1) = 0 \quad \text{or} \quad I_1 - 15I_2 + 12I_3 = -12 \quad \dots(ii)$$

Similarly, for loop 3, we get

$$-12(I_3 - I_2) - 2I_3 - 10 - 3(I_3 - I_1) = 0 \quad \text{or} \quad 3I_1 + 12I_2 - 17I_3 = 10 \quad \dots(iii)$$

Eliminating  $I_2$  from Eq. (i) and (ii) above, we get,  $119I_1 - 57I_3 = 372$  ... (iv)

Similarly, eliminating  $I_2$  from Eq. (ii) and (iii), we get,  $57I_1 - 111I_3 = 6$  ... (v)

From (iv) and (v) we have,

$$I_1 = 40,950/9,960 = 4.1 \text{ A}$$

**Solution by Determinants**

The three equations as found above are

$$8I_1 - I_2 - 3I_3 = 24$$

$$I_1 - 15I_2 + 12I_3 = -12$$

$$3I_1 + 12I_2 - 17I_3 = 10$$

Their matrix form is  $\begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{vmatrix} = 664, \quad \Delta_1 = \begin{vmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{vmatrix} = 2730$$

$$\therefore I_1 = \Delta_1/\Delta = 2730/664 = 4.1 \text{ A}$$

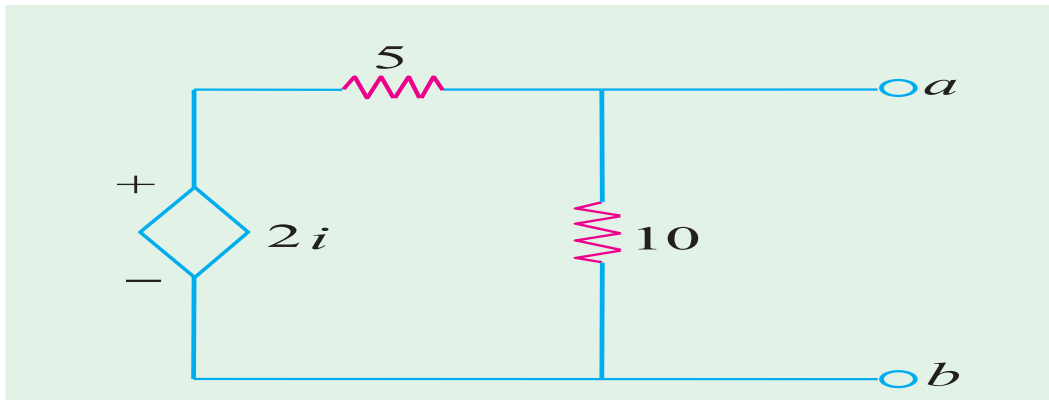


Fig. 2.56

**Solution by Using Mesh Resistance Matrix**

For the network of Fig. 2.53 (b), values of self resistances, mutual resistances and e.m.f.'s can be written by more inspection of Fig. 2.53.

$$R_{11} = 3 + 1 + 4 = 8 \Omega; R_{22} = 2 + 12 + 1 = 15 \Omega; R_{33} = 2 + 3 + 12 = 17 \Omega$$

$$R_{12} = R_{21} = -1; R_{23} = R_{32} = -12; R_{13} = R_{31} = -3$$

$$E_1 = 24 \text{ V}; E_2 = 12 \text{ V}; E_3 = -10 \text{ V}$$

The matrix form of the above three equations can be written by inspection of the given network as under :-

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

$$\Delta = 8(255 - 144) + 1(-17 - 36) - 3(12 + 45) = 664$$

$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{bmatrix} = 24(255 - 144) - 12(-17 - 36) - 10(12 + 45) = 2730$$

$$\therefore I_1 = \frac{-1}{664} \frac{2730}{664} \quad \mathbf{4.1 \text{ A}}$$

It is the same answer as found above.

**Tutorial Problems No. 2.2**

1. Find the ammeter current in Fig. 2.57 by using loop analysis.

[1/7 A] (Network Theory Indore Univ. 1981)

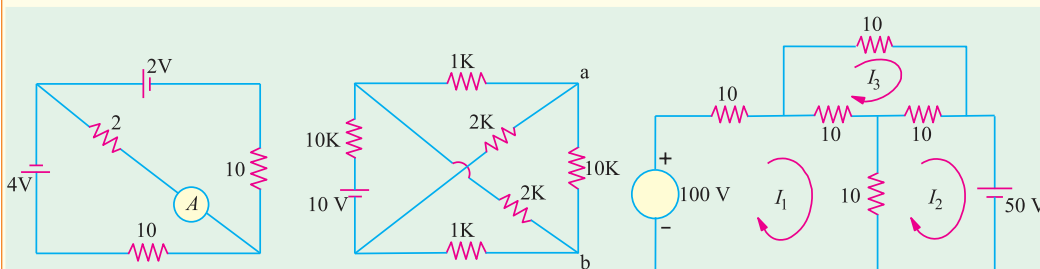


Fig. 2.57

Fig. 2.58

Fig. 2.59

2. Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals *a-b* of the circuit shown in Fig. 2.58. **[2.65 V] (Elect. Technology, Indore Univ.)**

3. Apply loop current method to find loop currents  $I_1$ ,  $I_2$  and  $I_3$  in the circuit of Fig. 2.59.

**[ $I_1 = 3.75$  A,  $I_2 = 0$ ,  $I_3 = 1.25$  A]**

## 2.12. Nodal Analysis With Sources

The node-equation method is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law. However, like loop current method, nodal method also

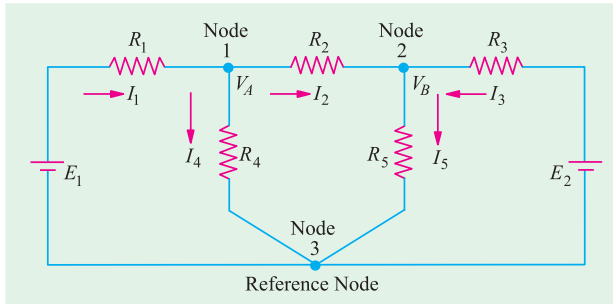


Fig. 2.60

has the advantage that a minimum number of equations need be written to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

For the application of this method, every junction in the network where three or more branches meet is regarded a node. One of these is regarded as the

reference node or datum node or zero-potential node. Hence the number of simultaneous equations to be solved becomes  $(n - 1)$  where  $n$  is the number of independent nodes. These node equations often become simplified if all voltage sources are converted into current sources (Art. 2.12).

### (i) First Case

Consider the circuit of Fig. 2.60 which has three nodes. One of these *i.e.* node 3 has been taken in as the reference node.  $V_A$  represents the potential of node 1 with reference to the datum node 3. Similarly,  $V_B$  is the potential difference between node 2 and node 3. Let the current directions which have been chosen arbitrary be as shown.

For node 1, the following current equation can be written with the help of KCL.

$$I_1 = I_4 + I_2$$

$$\text{Now } I_1 R_1 = E_1 - V_A \quad \therefore I_1 = (E_1 - V_A)/R_1 \quad \dots(i)$$

$$\text{Obviously, } I_4 = V_A/R_4 \quad \text{Also, } I_2 R_2 = V_A - V_B \quad (\because V_A > V_B)$$

$$\therefore I_2 = (V_A - V_B)/R_2$$

Substituting these values in Eq. (i) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have.

$$V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \quad \dots(ii)$$

The current equation for node 2 is  $I_5 = I_2 + I_3$

$$\text{or } \frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \quad \dots(iii)$$

$$\text{or } V_B \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \quad \dots(iv)$$

Though the above nodal equations (ii) and (iii) seem to be complicated, they employ a very simple and systematic arrangement of terms which can be written simply by inspection. Eq. (ii) at node 1 is represented by



1. The product of node potential  $V_A$  and  $(1/R_1 + 1/R_2 + 1/R_4)$  i.e. the sum of the reciprocals of the branch resistance connected to this node.
2. **Minus** the ratio of adjacent potential  $V_B$  and the interconnecting resistance  $R_2$ .
3. **Minus** ratio of adjacent battery (or generator) voltage  $E_1$  and interconnecting resistance  $R_1$ .
4. All the above set to zero.

Same is the case with Eq. (iii) which applies to node 2.

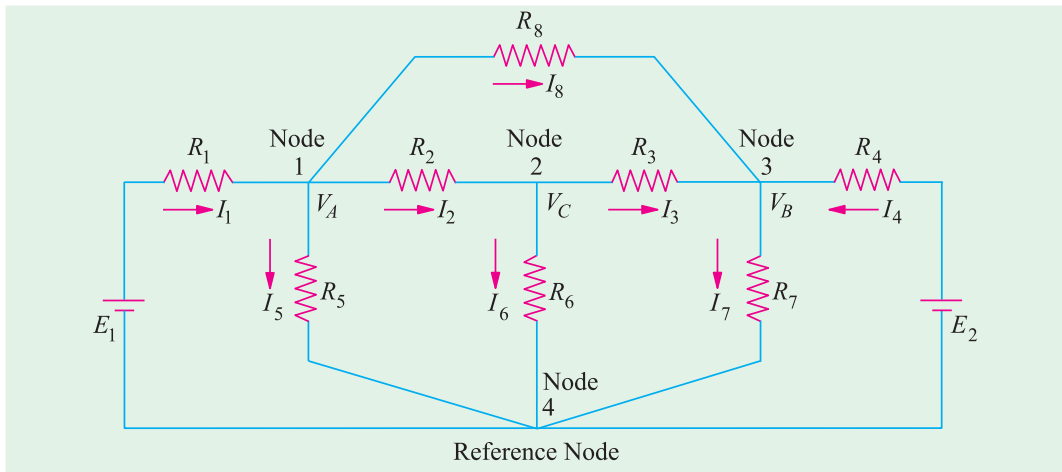


Fig. 2.61

Using conductances instead of resistances, the above two equations may be written as

$$V_A (G_1 + G_2 + G_4) - V_B G_2 - E_1 G_1 = 0 \quad \dots (iv)$$

$$V_B (G_2 + G_3 + G_5) - V_A G_2 - E_2 G_3 = 0 \quad \dots (v)$$

To emphasize the procedure given above, consider the circuit of Fig. 2.61.

The three node equations are 
$$V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_8} \right) - \frac{V_C}{R_2} - \frac{V_B}{R_8} - \frac{E_1}{R_1} = 0 \quad (\text{node 1})$$

$$V_C \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6} \right) - \frac{V_A}{R_2} - \frac{V_B}{R_3} = 0 \quad (\text{node 2})$$

$$V_B \left( \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_7} + \frac{1}{R_8} \right) - \frac{V_C}{R_3} - \frac{V_A}{R_8} - \frac{E_2}{R_4} = 0 \quad (\text{node 3})$$

After finding different node voltages, various currents can be calculated by using Ohm's law.

**(ii) Second Case**

Now, consider the case when a third battery of e.m.f.  $E_3$  is connected between nodes 1 and 2 as shown in Fig. 2.62.

It must be noted that as we travel from node 1 to node 2, we go from the  $-ve$  terminal of  $E_3$  to its  $+ve$  terminal. Hence, according to the sign convention given in Art. 2.3,  $E_3$  must be taken as *positive*. However, if we travel from node 2 to node 1, we go from the  $+ve$  to the  $-ve$  terminal

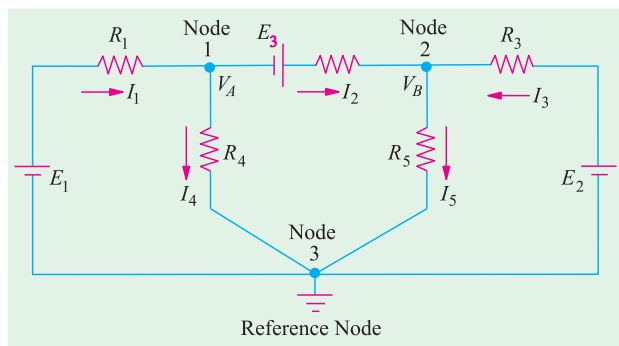


Fig. 2.62

of  $E_3$ . Hence, **when viewed from node 2,  $E_3$  is taken negative.**

#### For node 1

$$\begin{aligned}
 & I_1 - I_4 - I_2 = 0 \text{ or } I_1 = I_4 + I_2 \text{ -as per KCL} \\
 \text{Now,} & I_1 = \frac{E_1 - V_A}{R_1}; I_2 = \frac{V_A + E_3 - V_B}{R_2}; I_4 = \frac{V_A}{R_4} \\
 \therefore & \frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A + E_3 - V_B}{R_2} \\
 \text{or} & V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_2}{R_2} = 0 \quad \dots(i)
 \end{aligned}$$

It is exactly the same expression as given under the First Case discussed above except for the additional term involving  $E_3$ . This additional term is taken as  $+E_3/R_2$  (and not as  $-E_3/R_2$ ) because this third battery is so connected that when viewed from node 1, it represents a *rise* in voltage. Had it been connected the other way around, the additional term would have been taken as  $-E_3/R_2$ .

#### For node 2

$$\begin{aligned}
 & I_2 + I_3 - I_5 = 0 \text{ or } I_2 + I_3 = I_5 \text{ -as per KCL} \\
 \text{Now, as before,} & I_2 = \frac{V_A + E_3 - V_B}{R_2}, I_3 = \frac{E_2 - V_B}{R_3}, I_5 = \frac{V_B}{R_5} \\
 \therefore & \frac{V_A + E_3 - V_B}{R_2} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_5} \\
 \text{On simplifying, we get} & V_B \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0 \quad \dots(ii)
 \end{aligned}$$

As seen, the additional term is  $-E_3/R_2$  (and not  $+E_3/R_2$ ) because as viewed from this node,  $E_3$  represents a **fall** in potential.

It is worth repeating that the additional term in the above Eq. (i) and (ii) can be either  $+E_3/R_2$  or  $-E_3/R_2$  depending on whether it represents a rise or fall of potential when viewed from **the node under consideration.**

**Example 2.33.** Using Node voltage method, find the current in the  $3\Omega$  resistance for the network shown in Fig. 2.63. **(Elect. Tech. Osmania Univ.)**

**Solution.** As shown in the figure node 2 has been taken as the reference node. We will now find the value of node voltage  $V_1$ . Using the technique developed in Art. 2.10, we get

$$V_1 \left( \frac{1}{5} + \frac{1}{2} + \frac{1}{2} \right) - \frac{4}{2} - \left( \frac{4+2}{5} \right) = 0$$

The reason for adding the two battery voltages of 2 V and 4 V is because they are connected in additive series. Simplifying above, we get  $V_1 = 8/3$  V. The current flowing through the  $3\Omega$  resistance towards node 1 is  $= \frac{6 - (8/3)}{(3+2)} = \frac{2}{3}$  A

#### Alternatively

$$\begin{aligned}
 \frac{6 - V_1}{5} + \frac{4}{2} - \frac{V_1}{2} &= 0 \\
 12 - 2V_1 + 20 - 5V_1 &= 0
 \end{aligned}$$

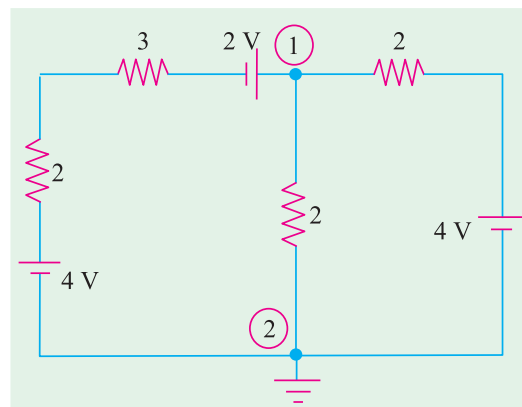


Fig. 2.63

$$7V_1 = 32$$

$$\text{Also } \frac{6 - V_1}{5} + \frac{4 - V_1}{2} = \frac{V_1}{2}$$

$$12 - 2V_1 + 20 - 5V_1 = 5V_1$$

$$12V_1 = 32; V_1 = 8/3$$

**Example 2.34.** Frame and solve the node equations of the network of Fig. 2.64. Hence, find the total power consumed by the passive elements of the network. (Elect. Circuits Nagpur Univ.)

**Solution.** The node equation for node 1 is

$$V_1 \left( 1 + 1 + \frac{1}{0.5} \right) - \frac{V_2}{0.5} - \frac{15}{1} = 0$$

or  $4V_1 - 2V_2 = 15$  ... (i)

Similarly, for node 2, we have

$$V_1 \left( 1 + \frac{1}{2} + \frac{1}{0.5} \right) - \frac{V_2}{0.5} - \frac{20}{1} = 0$$

or  $4V_1 - 7V_2 = -40$  ... (ii)

∴  $V_2 = 11$  volt and  $V_1 = 37/4$  volt  
Now,

$$I_1 = \frac{15 - 37/4}{1} = \frac{23}{4} \text{ A} = 5.75 \text{ A}; I_2 = \frac{11 - 37/4}{0.5} = 3.5 \text{ A}$$

$$I_4 = 5.75 + 3.5 = 9.25 \text{ A}; I_3 = \frac{20 - 11}{1} = 9 \text{ A}; I_5 = 9 - 3.5 = 5.5 \text{ A}$$

The passive elements of the network are its five resistances. Total power consumed by them is  $= 5.75^2 \times 1 + 3.5^2 \times 0.5 + 9^2 \times 1 + 9.25^2 \times 1 + 5.5^2 \times 2 = 266.25$

**Example 2.35.** Find the branch currents in the circuit of Fig. 2.65 by using (i) nodal analysis and (ii) loop analysis.

**Solution. (i) Nodal Method**

The equation for node A can be written by inspection as explained in Art. 2-12.

$$V_A \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_3}{R_2} = 0$$

Substituting the given data, we get,

$$V_A \left( \frac{1}{6} + \frac{1}{2} + \frac{1}{3} \right) - \frac{6}{6} - \frac{V_B}{2} + \frac{5}{2} = 0 \quad \text{or } 2V_A - V_B = -3 \quad \dots (i)$$

For node B, the equation becomes

$$V_B \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0$$

$$\therefore V_B \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \right) - \frac{10}{4} - \frac{V_A}{2} - \frac{5}{2} = 0 \quad \therefore V_B - \frac{V_A}{2} = 5 \quad \dots (ii)$$

From Eq. (i) and (ii), we get,

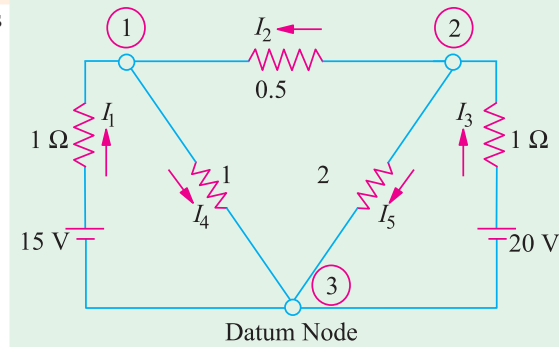


Fig. 2.64

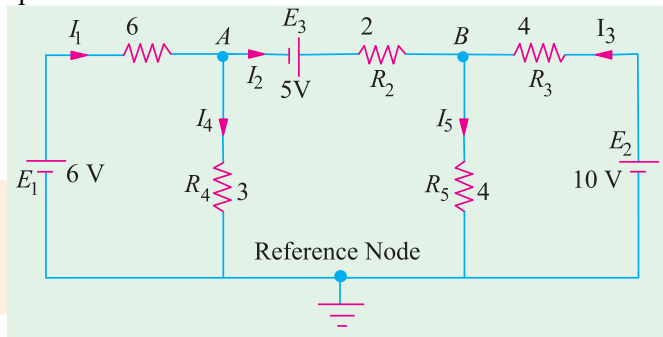


Fig. 2.65

$$V_A = \frac{4}{3} V, V_B = \frac{17}{3} V$$

$$I_1 = \frac{E_1 V_A}{R_1} = \frac{6 \cdot 4/3}{6} = \frac{7}{9} \text{ A}$$

$$I_2 = \frac{V_A E_3 V_B}{R_2} = \frac{(4/3) \cdot 5 \cdot (17/3)}{2} = \frac{1}{3} \text{ A}$$

$$I_3 = \frac{E_2 V_B}{R_3} = \frac{10 \cdot 17/3}{4} = \frac{17}{12} \text{ A}$$

$$I_4 = \frac{V_A}{R_4} = \frac{4/3}{3} = \frac{4}{9} \text{ A}, I_5 = \frac{V_B}{R_5} = \frac{17/3}{4} = \frac{17}{12} \text{ A}$$

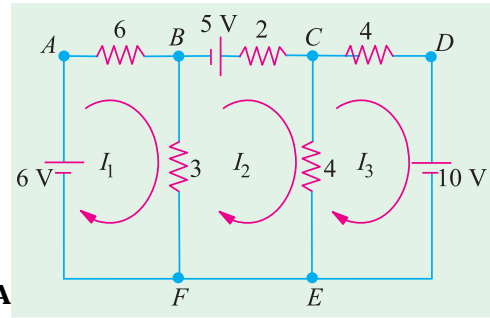


Fig. 2.66

**(ii) Loop Current Method**

Let the direction of flow of the three loop currents be as shown in Fig. 2.66.

**Loop ABFA :**

$$-6I_1 - 3(I_1 - I_2) + 6 = 0$$

or

$$3I_1 - I_2 = 2$$

...**(i)****Loop BCEFB :**

$$+5 - 2I_2 - 4(I_2 - I_3) - 3(I_2 - I_1) = 0$$

or

$$3I_1 - 9I_2 + 4I_3 = -5$$

...**(ii)****Loop CDEC :**

$$-4I_3 - 10 - 4(I_3 - I_2) = 0 \text{ or } 2I_2 - 4I_3 = 5$$

...**(iii)**

The matrix form of the above three simultaneous equations is

$$\begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}; \Delta = \begin{vmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{vmatrix} = 84 - 12 - 0 = 72$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 0 \\ -5 & -9 & 4 \\ 5 & 2 & -4 \end{vmatrix} = 56; \Delta_2 = \begin{vmatrix} 3 & 2 & 0 \\ 3 & -5 & 4 \\ 0 & 5 & -4 \end{vmatrix} = 24; \Delta_3 = \begin{vmatrix} 3 & -1 & 2 \\ 3 & -9 & -5 \\ 0 & 2 & 5 \end{vmatrix} = -78$$

$$\therefore I_1 = \Delta_1/\Delta = 56/72 = 7/9 \text{ A}; I_2 = \Delta_2/\Delta = 24/72 = 1/3 \text{ A}$$

$$I_3 = \Delta_3/\Delta = -78/72 = -13/12 \text{ A}$$

The negative sign of  $I_3$  shows that it is flowing in a direction opposite to that shown in Fig. 2.64 *i.e.* it flows in the CCW direction. The actual directions are as shown in Fig. 2.67.

The various branch currents are as under :

$$I_{AB} = I_1 = \frac{7}{9} \text{ A}; I_{BF} = I_1 - I_2 = \frac{7}{9} - \frac{1}{3} = \frac{4}{9} \text{ A}$$

$$I_{BC} = I_2 = \frac{1}{3} \text{ A}; I_{CE} = I_2 - I_3 = \frac{1}{3} - \left(-\frac{13}{12}\right) = \frac{17}{12} \text{ A}$$

$$I_{DC} = I_3 = \frac{13}{12} \text{ A}$$

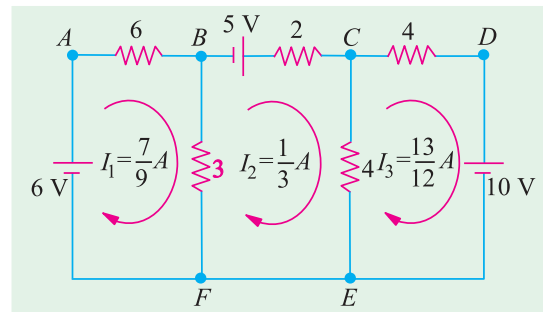


Fig. 2.67

**Solution by Using Mesh Resistance Matrix**

From inspection of Fig. 2.67, we have

$$R_{11} = 9; R_{22} = 9; R_{33} = 8$$

$$R_{12} = R_{21} = -3 \Omega; R_{23} = R_{32} = -4 \Omega; R_{13} = R_{31} = 0 \Omega$$

$$E_1 = 6 \text{ V}; E_2 = 5 \text{ V}; E_3 = -10 \text{ V}$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} = 9(72 - 16) + 3(-24) = 432$$

$$\Delta_1 = \begin{vmatrix} 6 & -3 & 0 \\ 5 & 9 & -4 \\ -10 & -4 & 8 \end{vmatrix} = 6(72 - 16) - 5(-24) - 10(12) = 336$$

$$\Delta_2 = \begin{vmatrix} 9 & 6 & 0 \\ -3 & 5 & -4 \\ 0 & -10 & 8 \end{vmatrix} = 9(40 - 40) + 3(48) = 144$$

$$\Delta_3 = \begin{vmatrix} 9 & -3 & 6 \\ -3 & 9 & 5 \\ 0 & -4 & -10 \end{vmatrix} = 9(-90 + 90) - 3(30 + 24) = -468$$

$$I_1 = \Delta_1/\Delta = 336/432 = 7/9 \text{ A}$$

$$I_2 = \Delta_2/\Delta = 144/432 = 1/3 \text{ A}$$

$$I_3 = \Delta_3/\Delta = -468/432 = -13/12 \text{ A}$$

These are the same values as found above.

### 2.13. Nodal Analysis with Current Sources

Consider the network of Fig. 2.68 (a) which has two current sources and three nodes out of which 1 and 2 are independent ones whereas No. 3 is the reference node.

The given circuit has been redrawn for ease of understanding and is shown in Fig. 2.68 (b). The current directions have been taken on the assumption that

1. both  $V_1$  and  $V_2$  are positive with respect to the reference node. That is why their respective currents flow from nodes 1 and 2 to node 3.
2.  $V_1$  is positive with respect to  $V_2$  because current has been shown flowing from node 1 to node 2.

A positive result will confirm our assumption whereas a negative one will indicate that actual direction is opposite to that assumed.

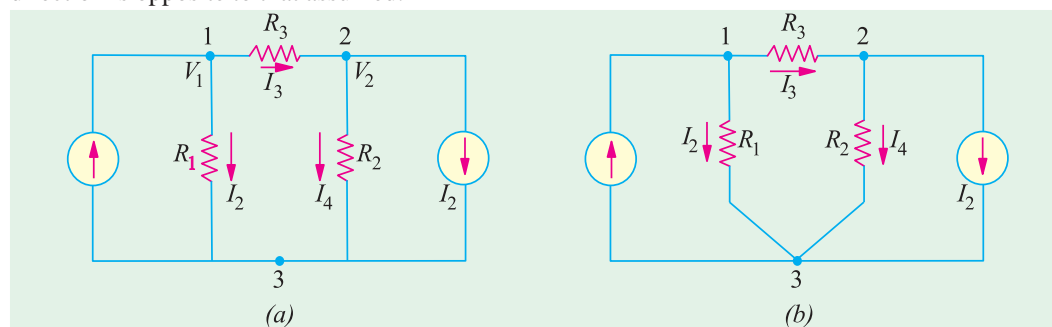


Fig. 2.68

We will now apply *KCL* to each node and use Ohm's law to express branch currents in terms of node voltages and resistances.

#### Node 1

$$I_1 - I_2 - I_3 = 0 \quad \text{or} \quad I_1 = I_2 + I_3$$

$$\begin{aligned} \text{Now} \quad I_2 &= \frac{V_1}{R_1} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{R_3} \\ \therefore \quad I_1 &= \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} \quad \text{or} \quad V_1 \left( \frac{1}{R_1} + \frac{1}{R_3} \right) - \frac{V_2}{R_3} = I_1 \quad \dots(i) \end{aligned}$$

**Node 2**

$$\begin{aligned} I_3 - I_2 - I_4 &= 0 \quad \text{or} \quad I_3 = I_2 + I_4 \\ \text{Now,} \quad I_4 &= \frac{V_2}{R_2} \quad \text{and} \quad I_3 = \frac{V_1 - V_2}{-R_3} \quad \text{--as before} \\ \therefore \quad \frac{V_1 - V_2}{R_3} &= I_2 + \frac{V_2}{R_2} \quad \text{or} \quad V_2 \left( \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_1}{R_3} = -I_1 \quad \dots(ii) \end{aligned}$$

The above two equations can also be written by simple inspection. For example, Eq. (i) is represented by

**1. product** of potential  $V_1$  and  $(1/R_1 + 1/R_3)$  i.e. sum of the reciprocals of the branch resistances connected to this node.

**2. minus** the ratio of adjoining potential  $V_2$  and the interconnecting resistance  $R_3$ .

**3.** all the above equated to the current supplied by the current source connected to this node.

This current is taken *positive* if flowing *into* the node and *negative* if flowing *out* of it (as per sign convention of Art. 2.3). Same remarks apply to Eq. (ii) where  $I_2$  has been taken negative because it flows *away* from node 2.

In terms of branch conductances, the above two equations can be put as

$$V_1 (G_1 + G_3) - V_2 G_3 = I_1 \quad \text{and} \quad V_2 (G_2 + G_3) - V_1 G_3 = -I_2$$

**Example 2.36.** Use nodal analysis method to find currents in the various resistors of the circuit shown in Fig. 2.69 (a).

**Solution.** The given circuit is redrawn in Fig. 2.66 (b) with its different nodes marked 1, 2, 3 and 4, the last one being taken as the reference or datum node. The different node-voltage equations are as under :

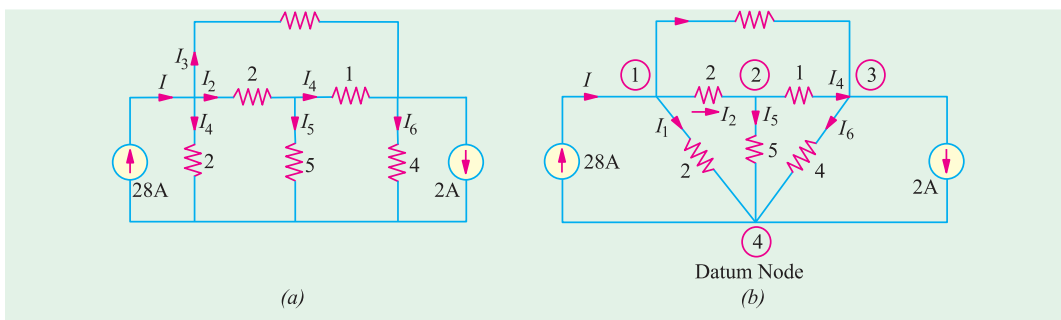


Fig. 2.69

$$\begin{aligned} \text{Node 1} \quad V_1 \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{10} \right) - \frac{V_2}{2} - \frac{V_3}{10} &= 8 \\ \text{or} \quad 11V_1 - 5V_2 - V_3 - 280 &= 0 \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{Node 2} \quad V_2 \left( \frac{1}{2} + \frac{1}{5} + 1 \right) - \frac{V_1}{2} - \frac{V_3}{1} &= 0 \\ \text{or} \quad 5V_1 - 17V_2 + 10V_3 &= 0 \quad \dots(ii) \end{aligned}$$

$$\text{Node 3} \quad V_3 \left( \frac{1}{4} + 1 + \frac{1}{10} \right) - \frac{V_2}{1} - \frac{V_1}{10} = -2$$

or  $V_1 + 10 V_2 - 13.5 V_3 - 20 = 0$

...(iii)

The matrix form of the above three equations is

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 1424.5 - 387.5 - 67 = 970$$

$$\Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34,920, \quad \Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19,400$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{vmatrix} = 15,520$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34,920}{970} = 36 \text{ V}, \quad V_2 = \frac{\Delta_2}{\Delta} = \frac{19,400}{970} = 20 \text{ V}, \quad V_3 = \frac{\Delta_3}{\Delta} = \frac{15,520}{970} = 16 \text{ V}$$

It is obvious that all nodes are at a higher potential with respect to the datum node. The various currents shown in Fig. 2.69 (b) can now be found easily.

$$I_1 = V_1/2 = 36/2 = \mathbf{18 \text{ A}}$$

$$I_2 = (V_1 - V_2)/2 = (36 - 20)/2 = \mathbf{8 \text{ A}}$$

$$I_3 = (V_1 - V_3)/10 = (36 - 16)/10 = \mathbf{2 \text{ A}}$$

It is seen that total current, as expected, is  $18 + 8 + 2 = \mathbf{28 \text{ A}}$

$$I_4 = (V_2 - V_3)/1 = (20 - 16)/1 = \mathbf{4 \text{ A}}$$

$$I_5 = V_2/5 = 20/5 = \mathbf{4 \text{ A}}, \quad I_6 = V_3/4 = 16/4 = \mathbf{4 \text{ A}}$$

**Example 2.37.** Using nodal analysis, find the different branch currents in the circuit of Fig. 2.70 (a). All branch conductances are in siemens (i.e. mho).

**Solution.** Let the various branch currents be as shown in Fig. 2.70 (b). Using the procedure detailed in Art. 2.11, we have

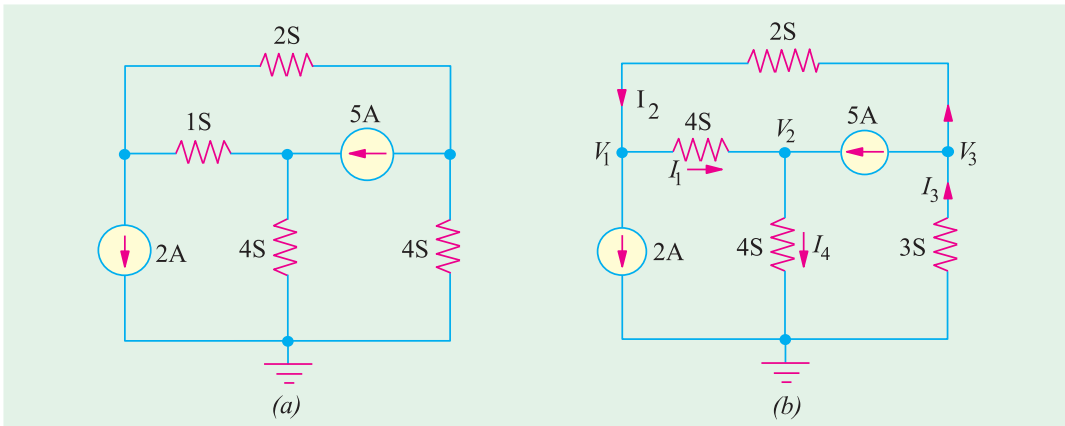


Fig. 2.70

**First Node**

$$V_1 (1 + 2) - V_2 \times 1 - V_3 \times 2 = -2 \quad \text{or} \quad 3V_1 - V_2 - 2V_3 = -2 \quad \dots(i)$$

**Second Node**

$$V_2 (1 + 4) - V_1 \times 1 = 5 \quad \text{or} \quad V_1 - 5V_2 = -5 \quad \dots(ii)$$

**Third Node**

$$V_3 (2 + 3) - V_1 \times 2 = -5 \quad \text{or} \quad 2V_1 - 5V_3 = 5 \quad \dots(iii)$$

Solving for the different voltages, we have

$$V_1 = -\frac{3}{2} \text{ V}, V_2 = \frac{7}{10} \text{ V and } V_3 = -\frac{8}{5} \text{ V}$$

$$I_1 = (V_1 - V_2) \times 1 = (-1.5 - 0.7) \times 1 = \mathbf{-2.2 \text{ A}}$$

$$I_2 = (V_3 - V_1) \times 2 = [-1.6 - (-1.5)] \times 2 = \mathbf{-0.2 \text{ A}}$$

$$I_4 = V_2 \times 4 = 4 \times (7/10) = \mathbf{2.8 \text{ A}}$$

$$I_3 = 2 + 2.8 = \mathbf{4.8 \text{ A}}$$

As seen,  $I_1$  and  $I_2$  flow in directions opposite to those originally assumed (Fig. 2.71).

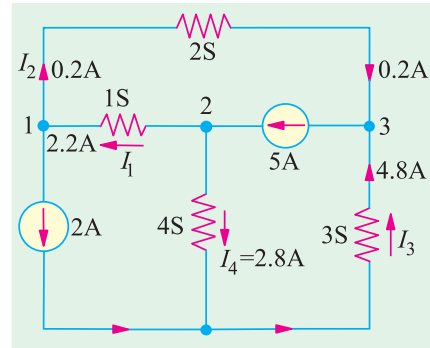


Fig. 2.71

**Example 2.38.** Find the current  $I$  in Fig. 2.72 (a) by changing the two voltage sources into their equivalent current sources and then using Nodal method. All resistances are in ohms.

**Solution.** The two voltage sources have been converted into their equivalent current sources in Fig. 2.72 (b). The circuit has been redrawn as shown in Fig. 2.72 (c) where node No. 4 has been

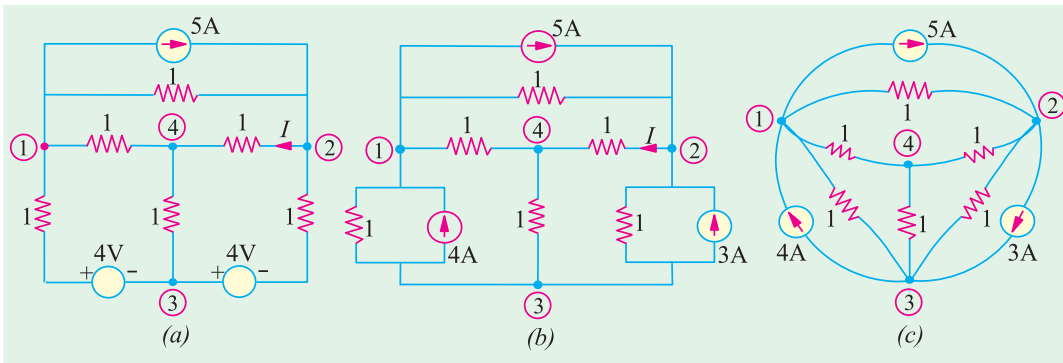


Fig. 2.72

taken as the reference node or common ground for all other nodes. We will apply *KCL* to the three nodes and taken currents coming towards the nodes as positive and those going away from them as negative. For example, current going away from node No. 1 is  $(V_1 - V_2)/1$  and hence would be taken as negative. Since 4 A current is coming towards node No. 1, it would be taken as positive but 5 A current would be taken as negative.

$$\text{Node 1 : } -\frac{(V_1 - 0)}{1} - \frac{(V_1 - V_2)}{1} - \frac{(V_1 - V_3)}{1} - 5 + 4 = 0$$

$$\text{or } 3V_1 - V_2 - V_3 = -1 \quad \dots(i)$$

$$\text{Node 2 : } -\frac{(V_2 - 0)}{1} - \frac{(V_2 - V_3)}{1} - \frac{(V_2 - V_1)}{1} + 5 - 3 = 0$$

$$\text{or } V_1 - 3V_2 + V_3 = -2 \quad \dots(ii)$$

$$\text{Node 3 : } -\frac{(V_3 - 0)}{1} - \frac{(V_3 - V_1)}{1} - \frac{(V_3 - V_2)}{1} - 4 + 3 = 0$$

$$\text{or } V_1 + V_2 - 3V_3 = 1 \quad \dots(iii)$$

The matrix form of the above three equations is

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$



$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(9-1) - 1(3+1) + 1(-1-3) = 16$$

$$\Delta_2 = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(6-1) - 1(3+1) + 1(-1-2) = 8$$

$$\therefore V_2 = \Delta_2/\Delta = 8/16 = 0.5 \text{ V}$$

$$\therefore I = V_2/1 = 0.5 \text{ A}$$

**Example 2.39.** Use Nodal analysis to determine the value of current  $i$  in the network of Fig. 2.73.

**Solution.** We will apply KCL to the two nodes 1 and 2. Equating the incoming currents at node 1 to the outgoing currents, we have

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3i$$

As seen,  $i = V_1/8$ . Hence, the above equation becomes

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3 \frac{V_1}{8}$$

$$\text{or } 3V_1 - V_2 = 24$$

Similarly, applying KCL to node No. 2, we get

$$\frac{V_1 - V_2}{4} + 3i = \frac{V_2}{6} \text{ or } \frac{V_1 - V_2}{4} + 3 \frac{V_1}{8} = \frac{V_2}{6} \text{ or } 3V_1 = 2V_2$$

From the above two equations, we get

$$V_1 = 16 \text{ V} \quad \therefore i = 16/8 = 2 \text{ A.}$$

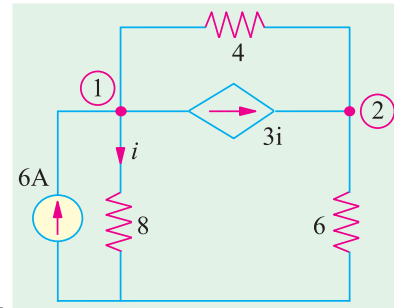


Fig. 2.73

**Example 2.40.** Using Nodal analysis, find the node voltages  $V_1$  and  $V_2$  in Fig. 2.74.

**Solution.** Applying KCL to node 1, we get

$$8 - 1 - \frac{V_1}{3} - \frac{(V_1 - V_2)}{6} = 0$$

$$\text{or } 3V_1 - V_2 = 42 \quad \dots(i)$$

Similarly, applying KCL to node 2, we get

$$1 + \frac{(V_1 - V_2)}{6} - \frac{V_2}{15} - \frac{V_2}{10} = 0$$

$$\text{or } V_1 - 2V_2 = -6 \quad \dots(ii)$$

Solving for  $V_1$  and  $V_2$  from Eqn. (i) and (ii), we get

$$V_1 = 18 \text{ V and } V_2 = 12 \text{ V.}$$

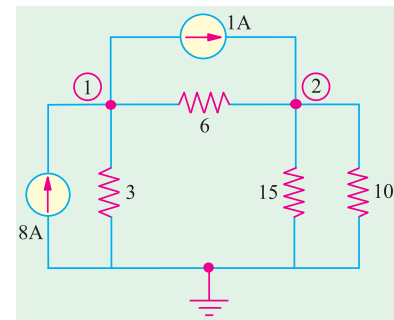


Fig. 2.74

## 2.14. Source Conversion

A given voltage source with a series resistance can be converted into (or replaced by) and equivalent current source with a parallel resistance. Conversely, a current source with a parallel resistance can be converted into a voltage source with a series resistance. Suppose, we want to convert the voltage source of Fig. 2.75 (a) into an equivalent current source. First, we will find the value of current supplied by the source when a 'short' is put across in terminals A and B as shown in Fig. 2.75 (b). This current is  $I = V/R$ .

A current source supplying this current  $I$  and having the same resistance  $R$  connected in **parallel** with it represents the equivalent source. It is shown in Fig. 2.75 (c). Similarly, a current source of  $I$  and a parallel resistance  $R$  can be converted into a voltage source of voltage  $V = IR$  and a resistance

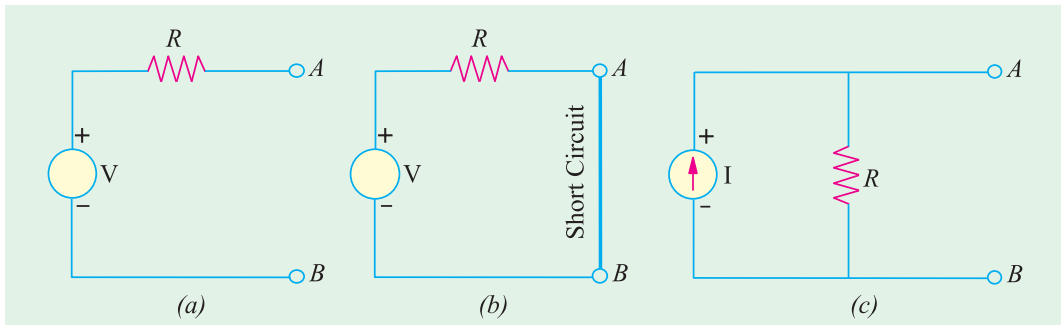


Fig. 2.75

$R$  in series with it. It should be kept in mind that a voltage source-series resistance combination is equivalent to (or replaceable by) a current source-parallel resistance combination if, and only if their

1. respective open-circuit voltages are equal, and
2. respective short-circuit currents are equal.

For example, in Fig. 2.75 (a), voltage across terminals  $A$  and  $B$  when they are open (*i.e.* open-circuit voltage  $V_{OC}$ ) is  $V$  itself because there is no drop across  $R$ . Short-circuit current across  $AB = I = V/R$ .

Now, take the circuit of Fig. 2.75 (c). The open-circuit voltage across  $AB =$  drop across  $R = IR = V$ . If a short is placed across  $AB$ , whole of  $I$  passes through it because  $R$  is completely shorted out.

**Example 2.41.** Convert the voltage source of Fig. 2.73 (a) into an equivalent current source.

**Solution.** As shown in Fig 2.76 (b), current obtained by putting a short across terminals  $A$  and  $B$  is  $10/5 = 2$  A.

Hence, the equivalent current source is as shown in Fig. 2.76 (c).

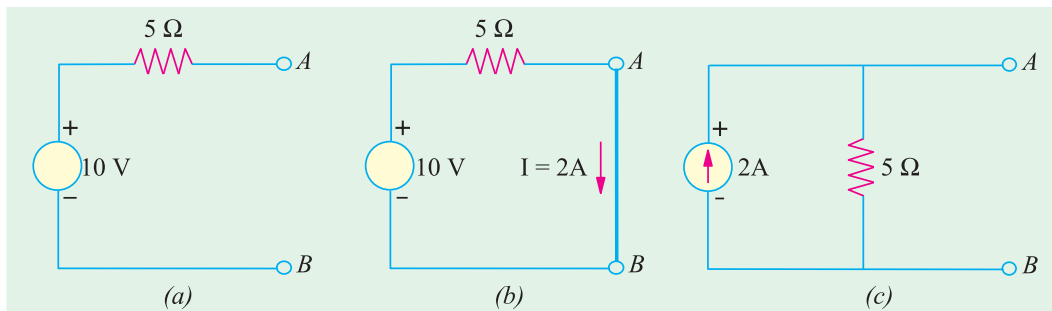


Fig. 2.76

**Example 2.42.** Find the equivalent voltage source for the current source in Fig. 2.77 (a).

**Solution.** The open-circuit voltage across terminals  $A$  and  $B$  in Fig. 2.77 (a) is

$$\begin{aligned} V_{OC} &= \text{drop across } R \\ &= 5 \times 2 = 10 \text{ V} \end{aligned}$$

Hence, voltage source has a voltage of **10 V** and the same resistance of  $2 \Omega$  through connected in series [Fig. 2.77 (b)].

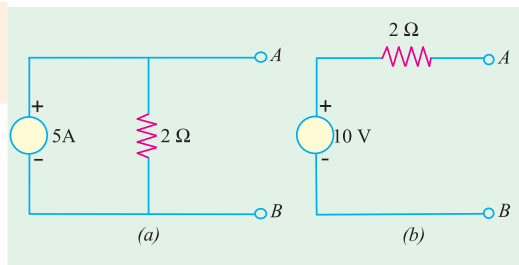


Fig. 2.77

**Example 2.43.** Use Source Conversion technique to find the load current  $I$  in the circuit of Fig. 2.78 (a).

**Solution.** As shown in Fig. 2.78 (b), 6-V voltage source with a series resistance of  $3\ \Omega$  has been converted into an equivalent 2 A current source with  $3\ \Omega$  resistance in parallel.

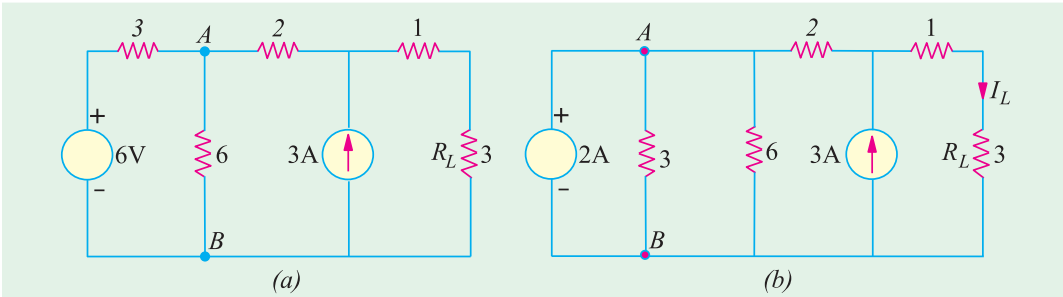


Fig. 2.78

The two parallel resistances of  $3\ \Omega$  and  $6\ \Omega$  can be combined into a single resistance of  $2\ \Omega$  as shown in Fig. 2.79 (a).

The two current sources cannot be combined together because of the  $2\ \Omega$  resistance present between points A and C. To remove this hurdle, we convert the 2 A current source into the equivalent 4 V voltage source as shown in Fig. 2.79 (b). Now, this 4 V voltage source with a series resistance of  $(2 + 2) = 4\ \Omega$  can again be converted into the equivalent current source as shown in Fig. 2.80 (a). Now, the two current sources can be combined into a single 4-A source as shown in Fig. 2.80 (b).

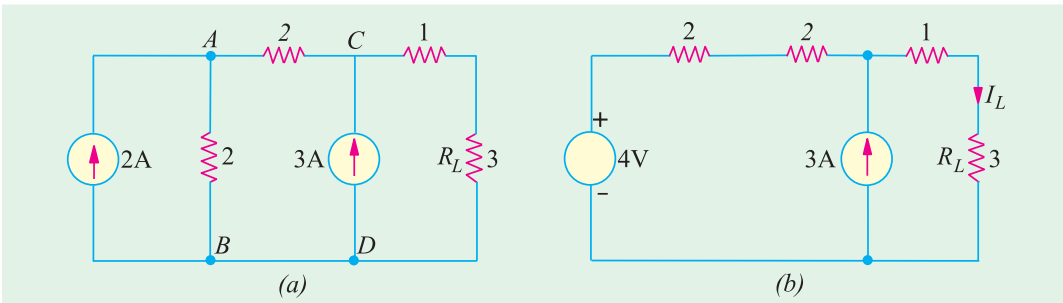


Fig. 2.79

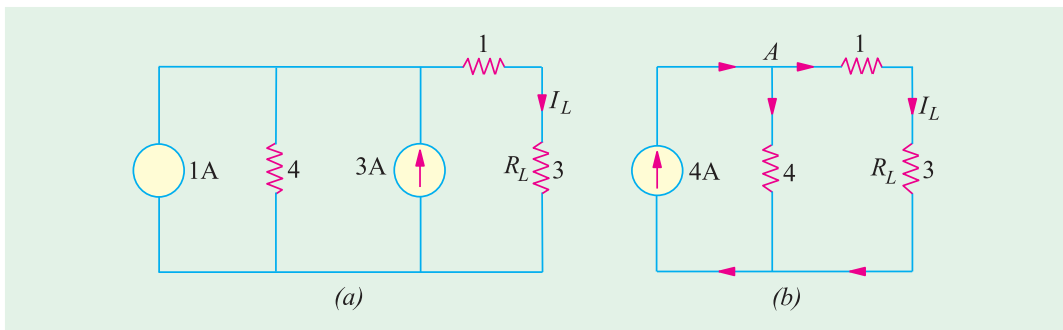


Fig. 2.80

The 4-A current is divided into two equal parts at point A because each of the two parallel paths has a resistance of  $4\ \Omega$ . Hence  $I_1 = 2\ \text{A}$ .

**Example 2.44.** Calculate the direction and magnitude of the current through the  $5\ \Omega$  resistor between points A and B of Fig. 2.81 (a) by using nodal voltage method.

**Solution.** The first thing is to convert the voltage source into the current sources as shown in Fig. 2.81 (b). Next, the two parallel resistances of  $4\ \Omega$  each can be combined to give a single resistance of  $2\ \Omega$  [Fig. 2.82 (a)]. Let the current directions be as indicated.

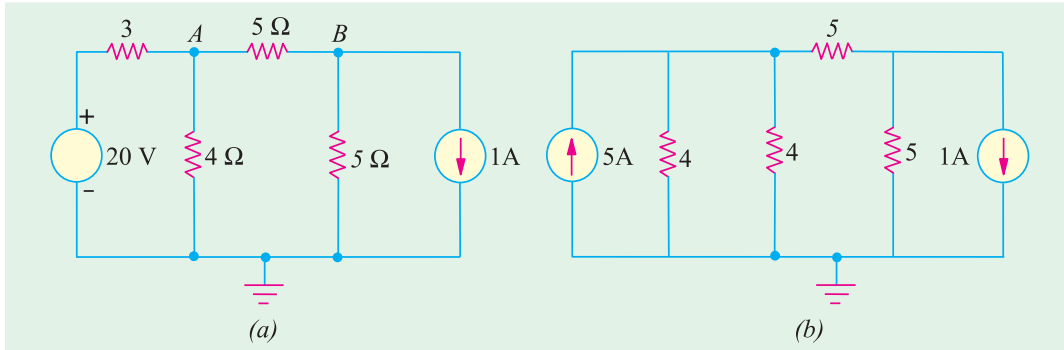


Fig. 2.81

Applying the nodal rule to nodes 1 and 2, we get

**Node 1**

$$V_1 \left( \frac{1}{2} + \frac{1}{5} \right) - \frac{V_2}{5} = 5 \quad \text{or} \quad 7V_1 - 2V_2 = 50 \quad \dots(i)$$

**Node 2**

$$V_2 \left( \frac{1}{5} + \frac{1}{5} \right) - \frac{V_1}{5} = -1 \quad \text{or} \quad V_1 - 2V_2 = 5 \quad \dots(ii)$$

Solving for  $V_1$  and  $V_2$ , we get  $V_1 = \frac{15}{2}\ \text{V}$  and  $V_2 = \frac{5}{4}\ \text{V}$ .

$$I_2 = \frac{V_1 - V_2}{5} = \frac{15/2 - 5/4}{5} = \frac{15/2 - 5/4}{5} = \frac{15/4 - 5/4}{5} = \frac{10/4}{5} = \frac{2.5}{5} = 0.5\ \text{A}$$

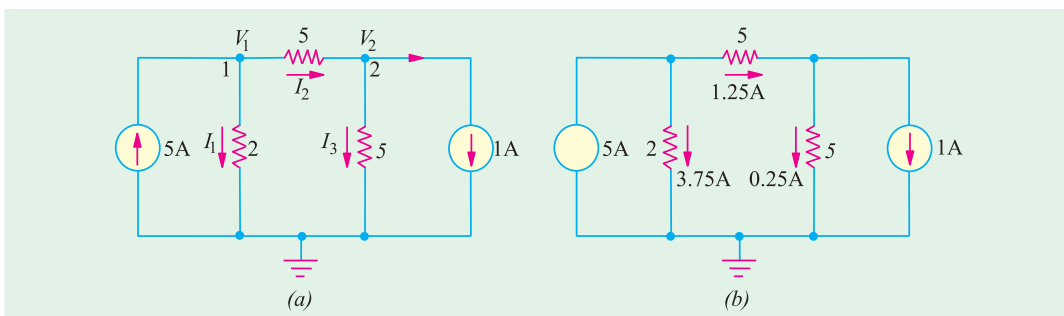


Fig. 2.82

Similarly,  $I_1 = V_1/2 = 15/4 = 3.75\ \text{A}$ ;  $I_3 = V_2/5 = 5/20 = 0.25\ \text{A}$ .

The actual current distribution becomes as shown in Fig. 2.79 (b).

**Example 2.45.** Replace the given network by a single current source in parallel with a resistance.

[Bombay University 2001]

**Solution.** The equivalence is expected for a load connected to the right-side of terminals A and B. In this case, the voltage-source has no resistive element in series. While handling such cases, the 3-ohm resistor has to be kept aside, treating it as an independent and separate loop. This voltage source will circulate a current of  $20/3$  amp in the resistor, and will not appear in the calculations.

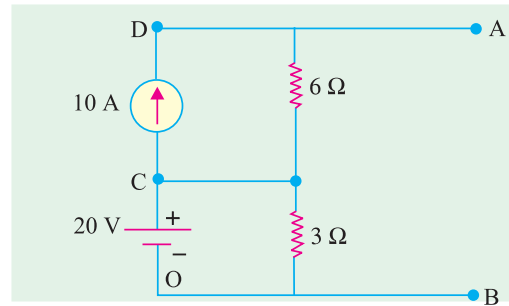


Fig. 2.83 (a)

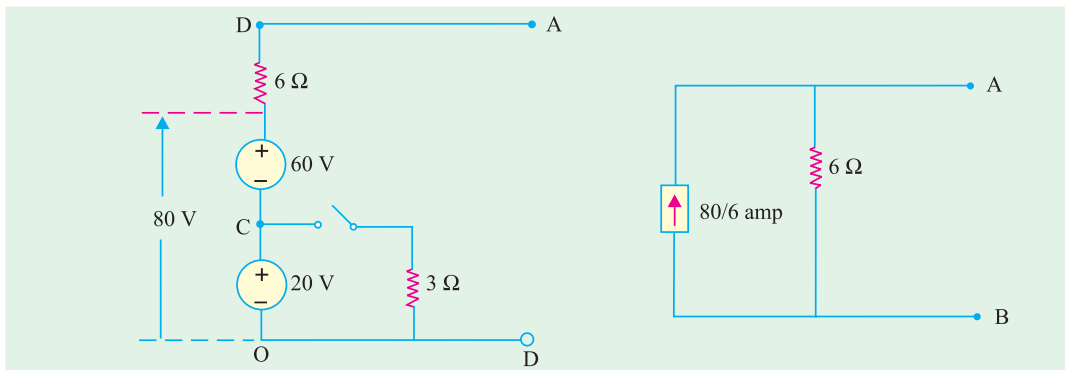


Fig. 2.83 (b)

Fig. 2.83 (c)

This step does not affect the circuit connected to A-B.  
Further steps are shown in Fig. 2.83 (b) and (c)

### Tutorial Problems No. 2.3

1. Using Maxwell's loop current method, calculate the output voltage  $V_o$  for the circuits shown in Fig. 2.84. [(a) 4 V (b)  $-150/7$  V (c)  $V_o = 0$  (d)  $V_o = 0$ ]

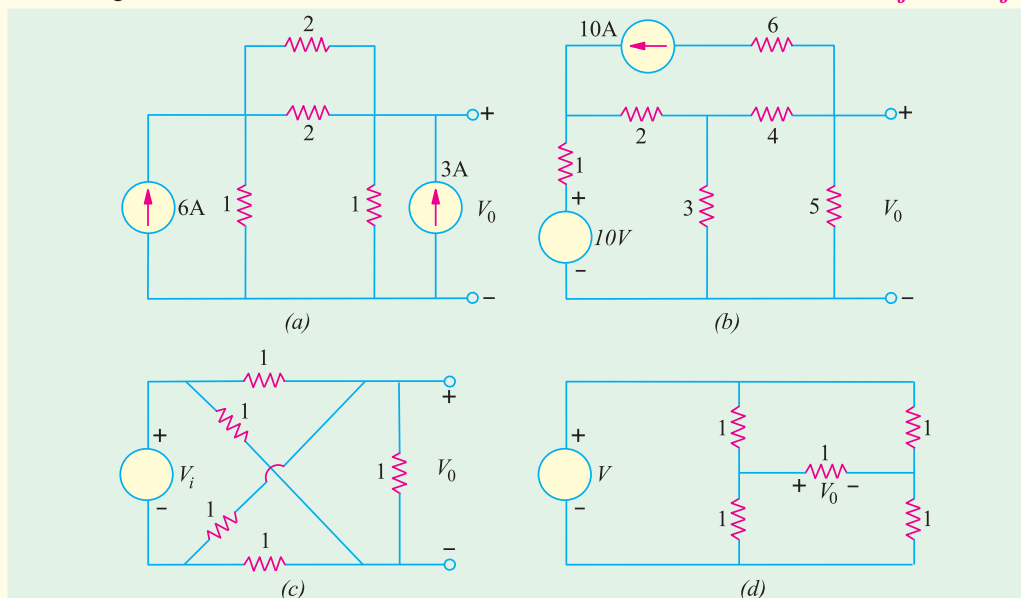


Fig. 2.84

2. Using nodal voltage method, find the magnitude and direction of current  $I$  in the network of Fig. 2.85.

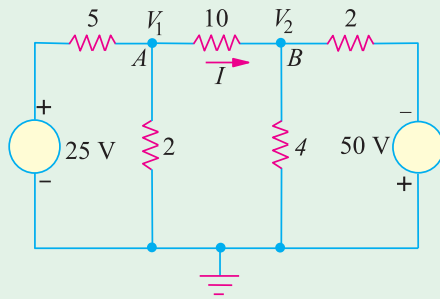


Fig. 2.85

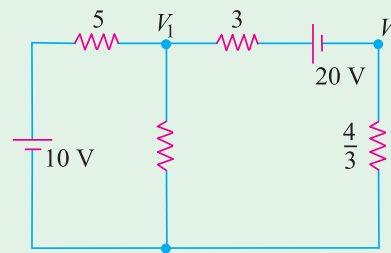
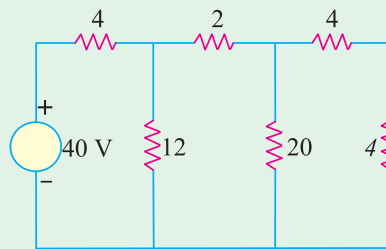
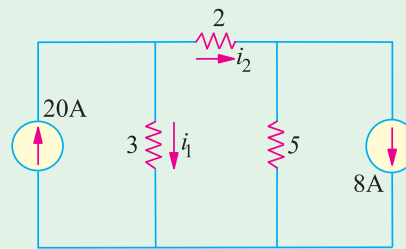


Fig. 2.86

3. By using repeated source transformations, find the value of voltage  $v$  in Fig. 2.87 (a). [8 V]



(a)



(b)

Fig. 2.87

4. Use source transformation technique to find the current flowing through the  $2\ \Omega$  resistor in Fig. 2.87 (b). [10 A]

5. With the help of nodal analysis, calculate the values of nodal voltages  $V_1$  and  $V_2$  in the circuit of Fig. 2.86. [7.1 V; -3.96 V]

6. Use nodal analysis to find various branch currents in the circuit of Fig. 2.88.

[Hint : Check by source conversion.]

[ $I_{ac} = 2\ \text{A}$ ;  $I_{ab} = 5\ \text{A}$ ,  $I_{bc} = 0$ ]

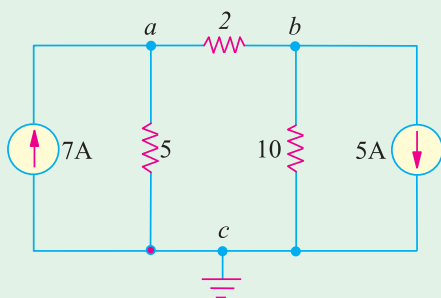


Fig. 2.88

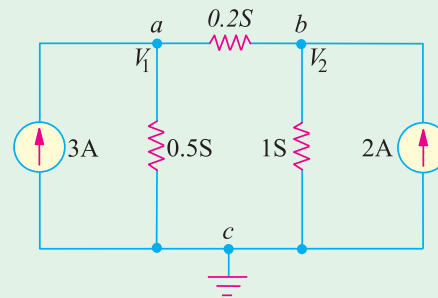


Fig. 2.89

7. With the help of nodal analysis, find  $V_1$  and  $V_2$  and various branch currents in the network of Fig. 2.85. [5 V, 2.5 V;  $I_{ac} = 2.5\ \text{A}$ ;  $I_{ab} = 0.5\ \text{A}$ ;  $I_{bc} = 2.5\ \text{A}$ ]

8. By applying nodal analysis to the circuit of Fig. 2.90, find  $I_{ab}$ ,  $I_{bd}$  and  $I_{bc}$ . All resistance values are in ohms. [ $I_{ab} = \frac{22}{21}\ \text{A}$ ,  $I_{bd} = \frac{10}{7}\ \text{A}$ ,  $I_{bc} = \frac{-8}{21}\ \text{A}$ ]

[Hint : It would be helpful to convert resistance into conductances.]

9. Using nodal voltage method, compute the power dissipated in the 9-Ω resistor of Fig. 2.91. [81 W]

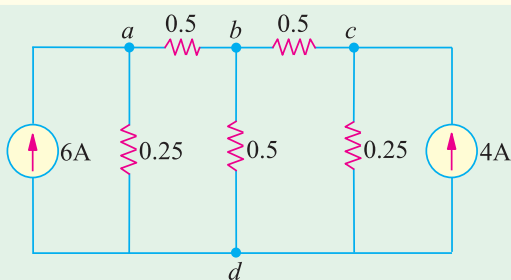


Fig. 2.90

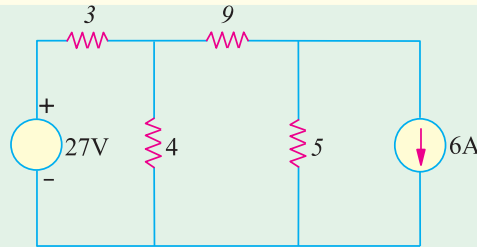


Fig. 2.91

10. Write equilibrium equations for the network in Fig. 2.92 on nodal basis and obtain the voltage  $V_1$ ,  $V_2$  and  $V_3$ . All resistors in the network are of 1 Ω [Network Theory and Fields, Madras Univ.]

11. By applying nodal method of network analysis, find current in the 15 Ω resistor of the network shown in Fig. 2.93. [3.5 A] [Elect. Technology-1, Gwalior Univ.]

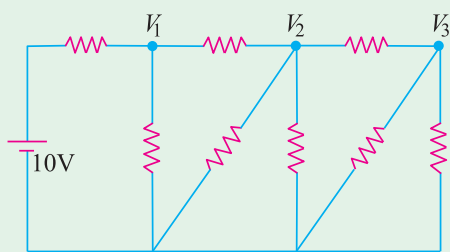


Fig. 2.92

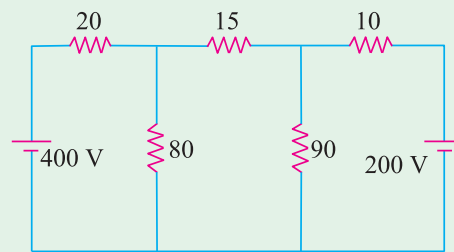


Fig. 2.93

### 2.15. Ideal Constant-Voltage Source

It is that voltage source (or generator) whose output voltage remains absolutely constant whatever the change in load current. Such a voltage source must possess **zero internal resistance so that internal voltage drop in the source is zero**. In that case, output voltage provided by the source would remain constant **irrespective of the amount of current drawn from it**. In practice, none such ideal constant-voltage source can be obtained. However, smaller the internal resistance  $r$  of a voltage source, closer it comes to the ideal sources described above.

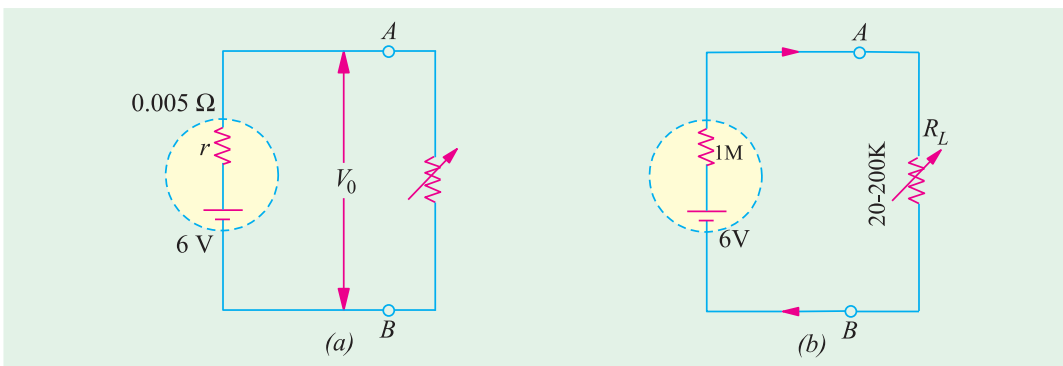


Fig. 2.94

Suppose, a 6-V battery has an internal resistance of 0.005 Ω [Fig. 2.94 (a)]. When it supplies no current *i.e.* it is on no-load,  $V_o = 6\text{ V}$  *i.e.* output voltage provided by it at its output terminals A and B

is 6 V. If load current increases to 100 A, internal drop =  $100 \times 0.005 = 0.5$  V. Hence,  $V_o = 6 - 0.5 = 5.5$  V.

Obviously an output voltage of 5.5 – 6 V can be considered constant as compared to wide variations in load current from 0 A to 100 A.

## 2.16. Ideal Constant-Current Source

It is that voltage source whose internal resistance is infinity. In practice, it is approached by a source which possesses very high resistance as compared to that of the external load resistance. As shown in Fig. 2.94 (b), let the 6-V battery or voltage source have an internal resistance of  $1 \text{ M}\Omega$  and let the load resistance vary from 20 K to 200 K. The current supplied by the source varies from  $6/1.02 = 5.9 \mu\text{A}$  to  $6/1.2 = 5 \mu\text{A}$ . As seen, even when load resistance increases 10 times, current decreases by  $0.9 \mu\text{A}$ . Hence, the source can be considered, for all practical purposes, to be a constant-current source.

## 2.17. Superposition Theorem

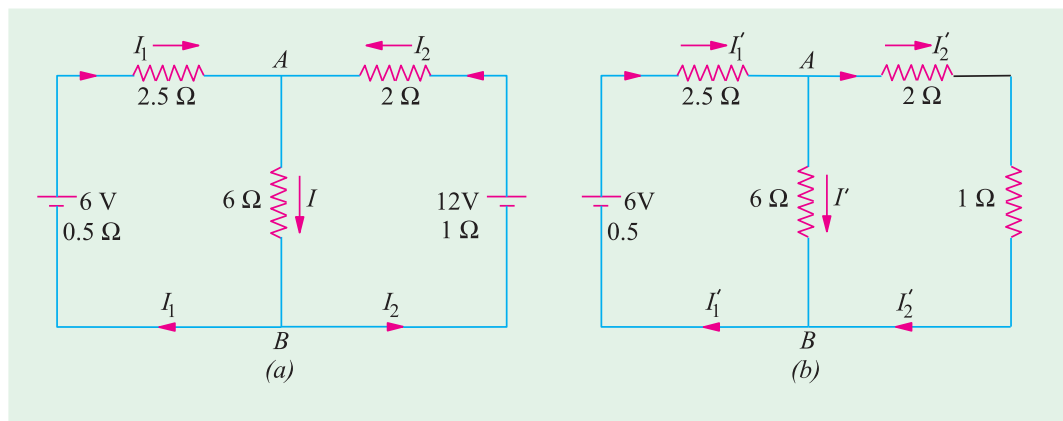


Fig. 2.95

According to this theorem, if there are a number of e.m.f.s. acting simultaneously in any linear bilateral network, then each e.m.f. acts independently of the others *i.e.* as if the other e.m.f.s. did not exist. The value of current in any conductor is the algebraic sum of the currents due to each e.m.f. Similarly, voltage across any conductor is the algebraic sum of the voltages which each e.m.f. would have produced while acting singly. In other words, current in or voltage across, any conductor of the network is obtained by superimposing the currents and voltages due to each e.m.f. in the network. It is important to keep in mind that this theorem is applicable only to **linear** networks where current is

**linearly** related to voltage as per Ohm's law.

Hence, this theorem may be stated as follows :

**In a network of linear resistances containing more than one generator (or source of e.m.f.), the current which flows at any point is the sum of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistances.**

### Explanation

In Fig. 2.95 (a)  $I_1$ ,  $I_2$  and  $I$  represent the values of

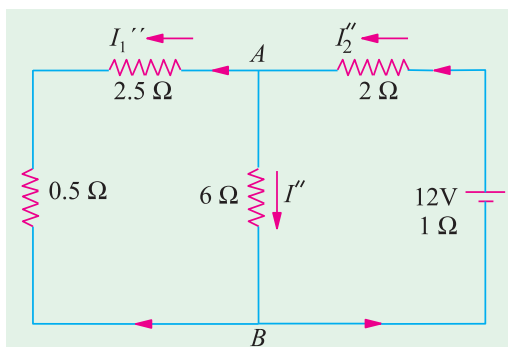


Fig. 2.96



currents which are due to the simultaneous action of the two sources of e.m.f. in the network. In Fig. 2.95 (b) are shown the current values which would have been obtained if left-hand side battery had acted alone. Similarly, Fig. 2.96 represents conditions obtained when right-hand side battery acts alone. By combining the current values of Fig. 2.95 (b) and 2.96 the actual values of Fig. 2.95 (a) can be obtained.

$$\text{Obviously, } I_1 = I_1' - I_1''', I_2 = I_2' - I_2''', I = I' + I''.$$

**Example 2.46.** In Fig. 2.95 (a) let battery e.m.fs. be 6 V and 12 V, their internal resistances 0.5  $\Omega$  and 1  $\Omega$ . The values of other resistances are as indicated. Find the different currents flowing in the branches and voltage across 60-ohm resistor.

**Solution.** In Fig. 2.95 (b), 12-volt battery has been removed though its internal resistance of 1  $\Omega$  remains. The various currents can be found by applying Ohm's Law.

It is seen that there are two parallel paths between points A and B, having resistances of 6  $\Omega$  and (2 + 1) = 3  $\Omega$

$$\therefore \text{ equivalent resistance} = 3 \parallel 6 = 2 \Omega$$

$$\text{Total resistance} = 0.5 + 2.5 + 2 = 5 \Omega \quad \therefore I_1' = 6/5 = \mathbf{1.2 \text{ A.}}$$

This current divides at point A inversely in the ratio of the resistances of the two parallel paths.

$$\therefore I = 1.2 \times (3/9) = 0.4 \text{ A.} \quad \text{Similarly, } I_2' = 1.2 \times (6/9) = 0.8 \text{ A}$$

In Fig. 2.96, 6 volt battery has been removed but not its internal resistance. The various currents and their directions are as shown.

The equivalent resistance to the left to points A and B is = 3  $\parallel$  6 = 2  $\Omega$

$$\therefore \text{ total resistance} = 1 + 2 + 2 = 5 \Omega \quad \therefore I_2''' = 12/5 = 2.4 \text{ A}$$

At point A, this current is divided into two parts,

$$I' = 2.4 \times 3/9 = 0.8 \text{ A, } I_1''' = 2.4 \times 6/9 = 1.6 \text{ A}$$

The actual current values of Fig. 2.95 (a) can be obtained by superposition of these two sets of current values.

$$\therefore I_1 = I_1' - I_1''' = 1.2 - 1.6 = \mathbf{-0.4 \text{ A}} \text{ (it is a charging current)}$$

$$I_2 = I_2' - I_2''' = 0.8 - 2.4 = \mathbf{-1.6 \text{ A}}$$

$$I = I' + I'' = 0.4 + 0.8 = \mathbf{1.2 \text{ A}}$$

$$\text{Voltage drop across 6-ohm resistor} = 6 \times 1.2 = \mathbf{7.2 \text{ V}}$$

**Example 2.47.** By using Superposition Theorem, find the current in resistance R shown in Fig. 2.97 (a)

$$R_1 = 0.005 \Omega, R_2 = 0.004 \Omega, R = 1 \Omega, E_1 = 2.05 \text{ V}, E_2 = 2.15 \text{ V}$$

Internal resistances of cells are negligible. **(Electronic Circuits, Allahabad Univ. 1992)**

**Solution.** In Fig. 2.97 (b),  $E_2$  has been removed. Resistances of 1  $\Omega$  and 0.04  $\Omega$  are in parallel across points A and C.  $R_{AC} = 1 \parallel 0.04 = 1 \times 0.04/1.04 = 0.038 \Omega$ . This resistance is in series with 0.05  $\Omega$ . Hence, total resistance offered to battery  $E_1 = 0.05 + 0.038 = 0.088 \Omega$ .  $I = 2.05/0.088 = 23.3 \text{ A}$ . Current through 1- $\Omega$  resistance,  $I_1 = 23.3 \times 0.04/1.04 = 0.896 \text{ A}$  from C to A.

When  $E_1$  is removed, circuit becomes as shown in Fig. 2.97 (c). Combined resistance of paths CBA and CDA is = 1  $\parallel$  0.05 = 1  $\times$  0.05/1.05 = 0.048  $\Omega$ . Total resistance offered to  $E_2$  is = 0.04 + 0.048 = 0.088  $\Omega$ . Current  $I = 2.15/0.088 = 24.4 \text{ A}$ . Again,  $I_2 = 24.4 \times 0.05/1.05 = 1.16 \text{ A}$ .

To current through 1- $\Omega$  resistance when both batteries are present

$$= I_1 + I_2 = 0.896 + 1.16 = \mathbf{2.056 \text{ A.}}$$

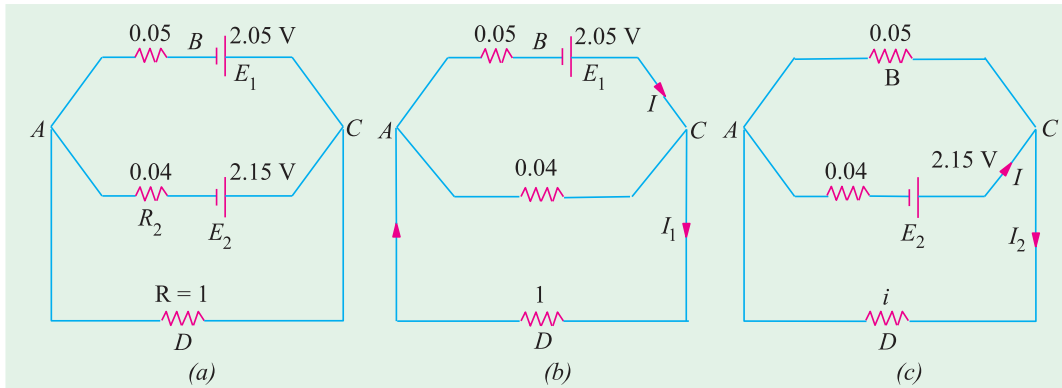


Fig. 2.97

**Example 2.48.** Use Superposition theorem to find current  $I$  in the circuit shown in Fig. 2.98 (a). All resistances are in ohms. **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

**Solution.** In Fig. 2.98 (b), the voltage source has been replaced by a short and the 40 A current sources by an open. Using the current-divider rule, we get  $I_1 = 120 \times 50/200 = 30$  A.

In Fig. 2.98 (c), only 40 A current source has been considered. Again, using current-divider rule  $I_2 = 40 \times 150/200 = 30$  A.

In Fig. 2.98 (d), only voltage source has been considered. Using Ohm's law,

$$I_3 = 10/200 = 0.05 \text{ A.}$$

Since  $I_1$  and  $I_2$  cancel out,  $I = I_3 = 0.005$  A.

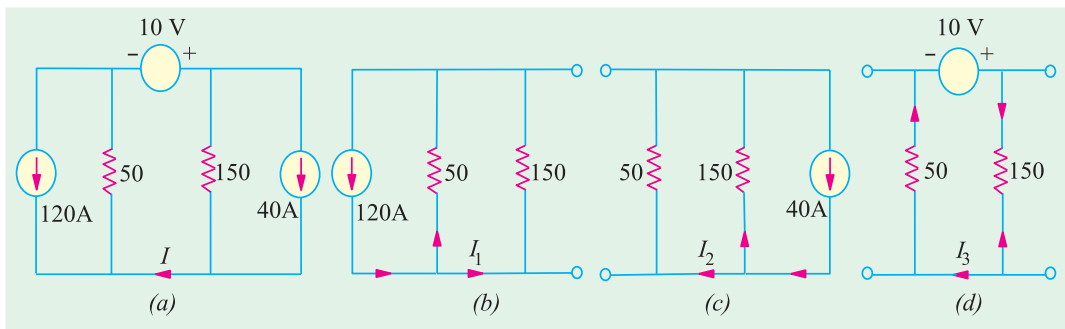


Fig. 2.98

**Example 2.49.** Use superposition theorem to determine the voltage  $v$  in the network of Fig. 2.99(a).

**Solution.** As seen, there are three independent sources and one dependent source. We will find the value of  $v$  produced by each of the three independent sources when acting alone and add the three values to find  $v$ . It should be noted that unlike independent source, a dependent source cannot be set to zero *i.e.* it cannot be 'killed' or deactivated.

Let us find the value of  $v_1$  due to 30 V source only. For this purpose we will replace current source by an open circuit and the 20 V source by a short circuit as shown in Fig. 2.99 (b). Applying KCL to node 1, we get

$$\frac{(30 - v_1)}{6} - \frac{v_1}{3} + \frac{(v_1/3 - v_1)}{2} = 0 \quad \text{or} \quad v_1 = 6 \text{ V}$$

Let us now keep 5 A source alive and 'kill' the other two independent sources. Again applying KCL to node 1, we get, from Fig. 2.99 (c).

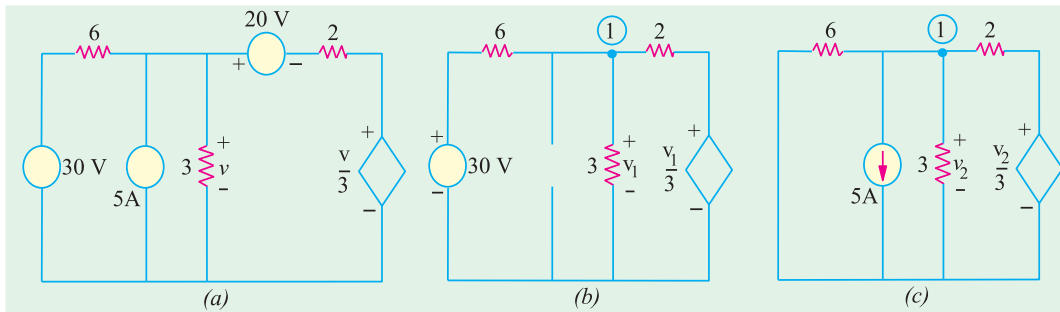


Fig. 2.99

$$\frac{v_2}{6} - 5 - \frac{v_2}{3} + \frac{(v_2/3 - v_2)}{2} = 0 \quad \text{or} \quad v_2 = -6 \text{ V}$$

Let us now ‘kill’ 30 V source and 5 A source and find  $v_3$  due to 20 V source only. The two parallel resistances of 6 Ω and 3 Ω can be combined into a single resistance of 2 Ω. Assuming a circulating current of  $i$  and applying KVL to the indicated circuit, we get, from Fig. 2.100.

$$-2i - 20 - 2i - \frac{1}{3}(-2i) = 0 \quad \text{or} \quad i = 6 \text{ A}$$

Hence, according to Ohm’s law, the component of  $v$  that corresponds to 20 V source is  $v_3 = 2 \times 6 = 12 \text{ V}$ .  $\therefore v = v_1 + v_2 + v_3 = 6 - 6 + 12 = 12 \text{ V}$ .

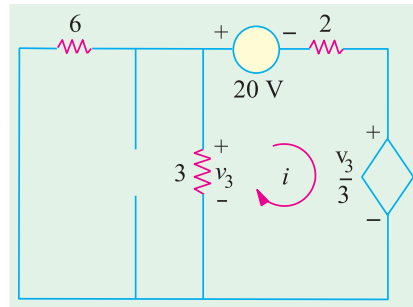


Fig. 2.100

**Example 2.50.** Using Superposition theorem, find the current through the 40 W resistor of the circuit shown in Fig. 2.101 (a). (F.Y. Engg. Pune Univ. May 1990)

**Solution.** We will first consider when 50 V battery acts alone and afterwards when 10-V battery is alone in the circuit. When 10-V battery is replaced by short-circuit, the circuit becomes as shown in Fig. 2.101 (b). It will be seen that the right-hand side 5 Ω resistor becomes connected in parallel with 40 Ω resistor giving a combined resistance of  $5 \parallel 40 = 4.44 \Omega$  as shown in Fig. 101 (c). This 4.44 Ω resistance is in series with the left-hand side resistor of 5 Ω giving a total resistance of  $(5 + 4.44) = 9.44 \Omega$ . As seen there are two resistances of 20 Ω and 9.44 Ω connected in parallel. In Fig. 2.101 (c) current  $I = 50/9.44 = 5.296 \text{ A}$ .

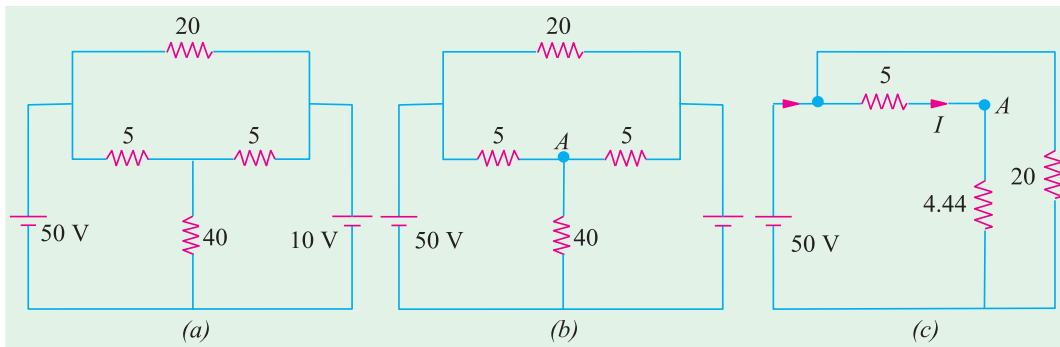


Fig. 2.101

At point A in Fig. 2.101 (b) there are two resistances of 5 Ω and 40 Ω connected in parallel, hence, current  $I$  divides between them as per the current-divider rule. If  $I_1$  is the current flowing through the 40 Ω resistor, then

$$I_1 = I \frac{5}{5 + 40} = 5.296 \frac{5}{45} = \mathbf{0.589 \text{ A}}$$

In Fig. 2.102 (a), 10 V battery acts alone because 50-V battery has been removed and replaced by a short-circuit.

As in the previous case, there are two parallel branches of resistances 20  $\Omega$  and 9.44  $\Omega$  across the 10-V battery. Current  $I$  through 9.44  $\Omega$  branch is  $I = 10/9.44 = 1.059 \text{ A}$ . This current divides at point  $B$  between 5  $\Omega$  resistor and 40  $\Omega$  resistor. Current through 40  $\Omega$  resistor  $I_2 = 1.059 \times 5/45 = 0.118 \text{ A}$ .

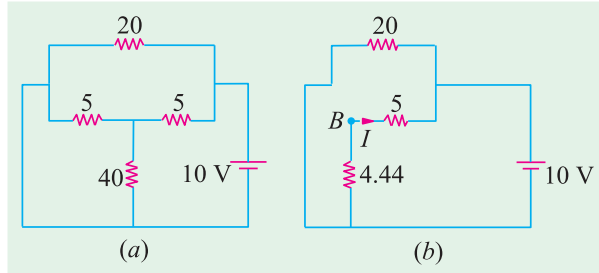


Fig. 2.102

According to the Superposition theorem, total current through 40  $\Omega$  resistance is  $= I_1 + I_2 = 0.589 + 0.118 = \mathbf{0.707 \text{ A}}$ .

**Example 2.51.** Solve for the power delivered to the 10  $\Omega$  resistor in the circuit shown in Fig. 2.103 (a). All resistances are in ohms. (Elect. Science - I, Allahabad Univ. 1991)

**Solution.** The 4-A source and its parallel resistance of 15  $\Omega$  can be converted into a voltage source of  $(15 \times 4) = 60 \text{ V}$  in series with a 15  $\Omega$  resistance as shown in Fig. 2.103 (b).

Now, we will use Superposition theorem to find current through the 10  $\Omega$  resistances.

#### When 60-V Source is Removed

When 60-V battery is removed the total resistance as seen by 2 V battery is  $= 1 + 10 \parallel (15 + 5) = 7.67 \Omega$

The battery current  $= 2/7.67 \text{ A} = 0.26 \text{ A}$ . At point  $A$ , this current is divided into two parts. The current passing through the 10  $\Omega$  resistor from  $A$  to  $B$  is

$$I_1 = 0.26 \times (20/30) = 0.17 \text{ A}$$

#### When 2-V Battery is Removed

Then resistance seen by 60 V battery is  $= 20 + 10 \parallel 1 = 20.9 \Omega$ . Hence, battery current  $= 60/20.9 = 2.87 \text{ A}$ . This current divides at point  $A$ . The current flowing through 10  $\Omega$  resistor from  $A$  to  $B$  is

$$I_2 = 2.87 \times 1/(1 + 10) = 0.26 \text{ A}$$

Total current through 10  $\Omega$  resistor due to two batteries acting together is  $= I_1 + I_2 = 0.43 \text{ A}$ .

Power delivered to the 10  $\Omega$  resistor  $= 0.43^2 \times 10 = 1.85 \text{ W}$ .

**Example 2.52.** Compute the power dissipated in the 9-W resistor of Fig. 2.104 by applying the Superposition principle. The voltage and current sources should be treated as ideal sources. All resistances are in ohms.

**Solution.** As explained earlier, an ideal constant-voltage source has zero internal resistances whereas a constant-current source has an infinite internal resistance.

#### (i) When Voltage Source Acts Alone

This case is shown in Fig. 2.104 (b) where constant-current source has been replaced by an open-circuit *i.e.* infinite resistance (Art. 2.16). Further circuit simplification leads to the fact that total resistances offered to voltage source is  $= 4 + (12 \parallel 15) = 32/3 \Omega$  as shown in Fig. 2.104 (c).

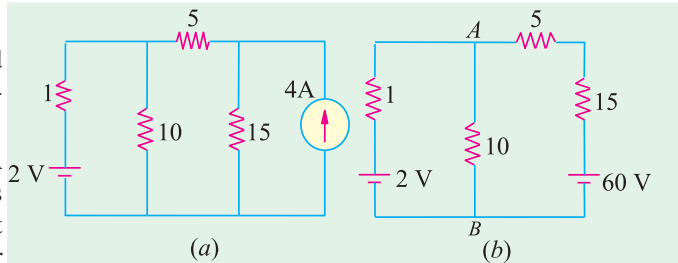


Fig. 2.103

Hence current =  $32 \div 32/3 = 3$  A. At point A in Fig. 2.104 (d), this current divides into two parts. The part going along AB is the one that also passes through 9 Ω resistor.

$$I = 3 \times 12 / (15 + 12) = 4/3 \text{ A}$$

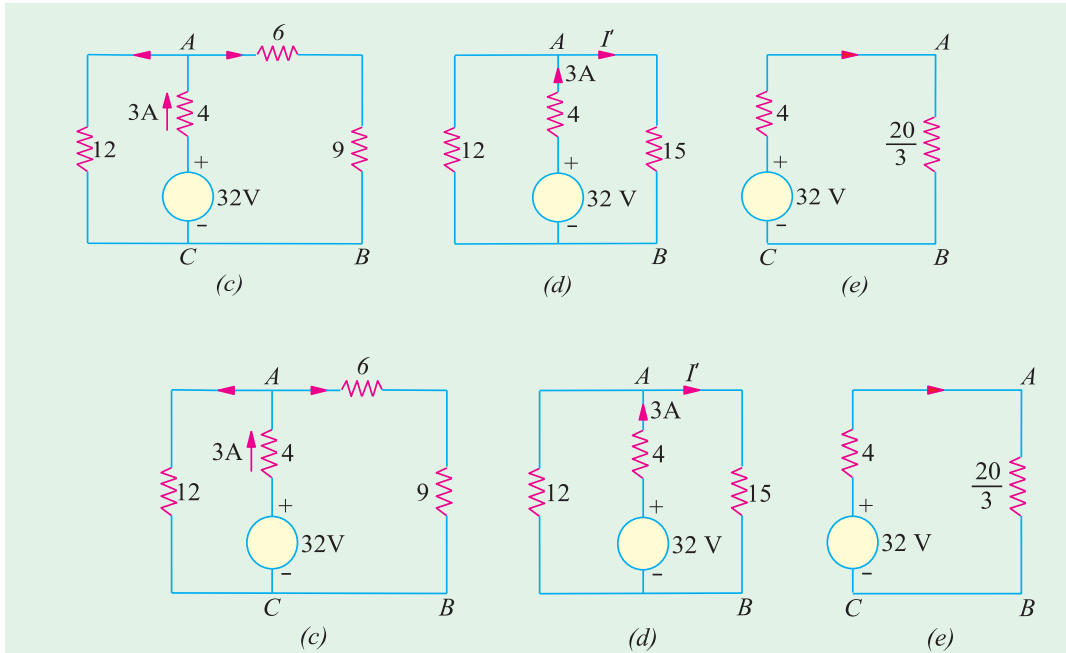


Fig. 2.104

**(ii) When Current Source Acts Alone**

As shown in Fig. 2.105 (a), the voltage source has been replaced by a short-circuit (Art 2.13). Further simplification gives the circuit of Fig. 2.105 (b).

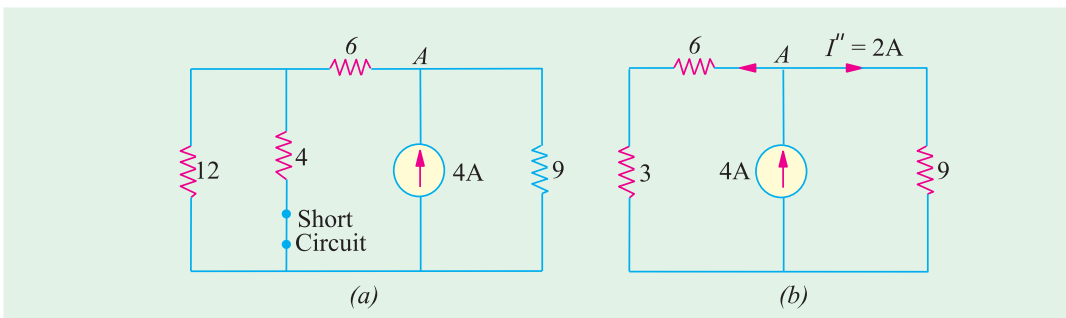


Fig. 2.105

The 4 - A current divides into two equal parts at point A in Fig. 2.105 (b). Hence  $I = 4/2 = 2$  A. Since both  $I'$  and  $I''$  flow in the same direction, total current through 9-Ω resistor is

$$I = I' + I'' = (4/3) + 2 = (10/3) \text{ A}$$

Power dissipated in 9 Ω resistor =  $I^2 R = (10/3)^2 \times 9 = 100 \text{ W}$

**Example 2.53(a).** With the help of superposition theorem, obtain the value of current I and voltage  $V_0$  in the circuit of Fig. 2.106 (a).

**Solution.** We will solve this question in three steps. First, we will find the value of I and  $V_0$  when current source is removed and secondly, when voltage source is removed. Thirdly, we would combine the two values of I and  $V_0$  in order to get their values when both sources are present.

**First Step**

As shown in Fig. 2.106 (b), current source has been replaced by an open-circuit. Let the values of current and voltage due to 10 V source be  $I_1$  and  $V_{01}$ . As seen  $I_1 = 0$  and  $V_{01} = 10$  V.

**Second Step**

As shown in Fig. 2.106 (c), the voltage source has been replaced by a short circuit. Here  $I_2 = -5$  A and  $V_{02} = 5 \times 10 = 50$  V.

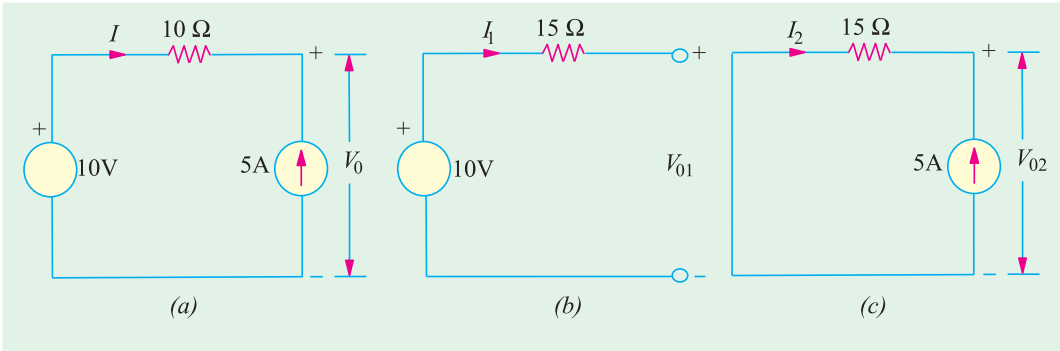


Fig. 2.106

**Third Step**

By applying superposition theorem, we have

$$I = I_1 + I_2 = 0 + (-5) = -5 \text{ A}$$

$$V_0 = V_{01} + V_{02} = 10 + 50 = 60 \text{ V}$$

**Example 2.53(b).** Using Superposition theorem, find the value of the output voltage  $V_0$  in the circuit of Fig. 2.107.

**Solution.** As usual, we will break down the problem into three parts involving one source each.

**(a) When 4 A and 6 V sources are killed\***

As shown in Fig. 2.108 (a), 4 A source has been replaced by an open circuit and 6 V source by a short-circuit. Using the current-divider rule, we find current  $i_1$  through the  $2 \Omega$  resistor =  $6 \times 1/(1 + 2 + 3) = 1$  A  $\therefore V_{01} = 1 \times 2 = 2$  V.

**(b) When 6 A and 6 V sources are killed**

As shown in Fig. 2.108 (b), 6 A source has been replaced by an open-circuit and 6 V source by a short-circuit. The current  $i_2$  can again be found with the help of current-divider rule because there are two parallel paths across the current source. One has a resistance of  $3 \Omega$  and the other of  $(2 + 1) = 3 \Omega$ . It means that current divides equally at point A.

$$\text{Hence, } i_2 = 4/2 = 2 \text{ A} \quad \therefore V_{02} = 2 \times 2 = 4 \text{ V}$$

**(c) When 6 A and 4 A sources are killed**

As shown in Fig. 2.108 (c), drop over  $2 \Omega$  resistor =  $6 \times 2/6 = 2$  V. The potential of point B with respect to point A is =  $6 - 2 = +4$  V. Hence,  $V_{03} = -4$  V.

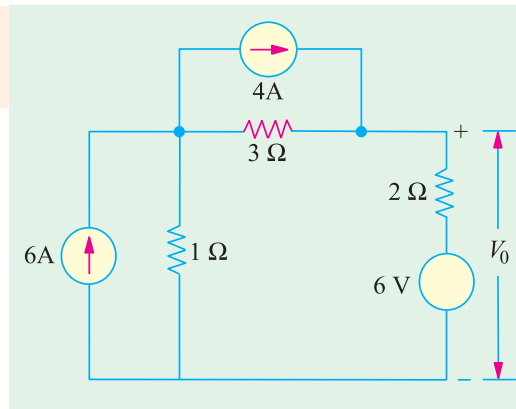


Fig. 2.107

\* The process of setting of voltage source of zero is called *killing* the sources.

According to Superposition theorem, we have

$$V_0 = V_{01} + V_{02} + V_{03} = 2 + 4 - 4 = 2 \text{ V}$$

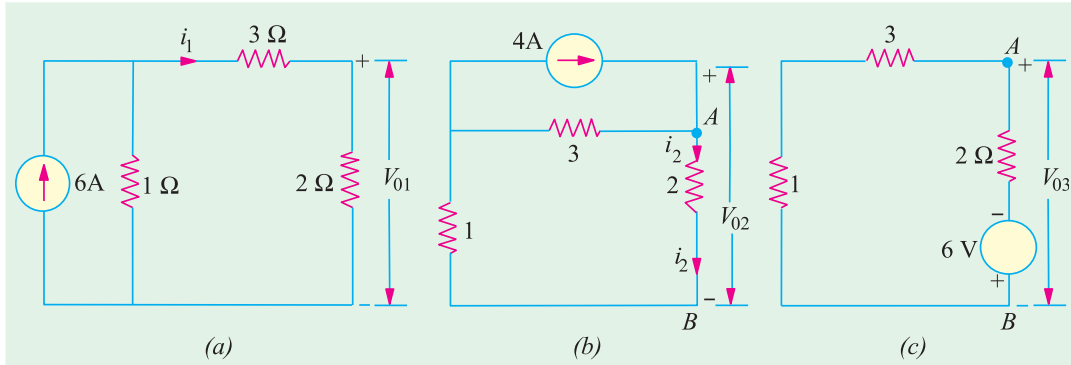


Fig. 2.108

**Example 2.54.** Use Superposition theorem, to find the voltage  $V$  in Fig. 2.109 (a).

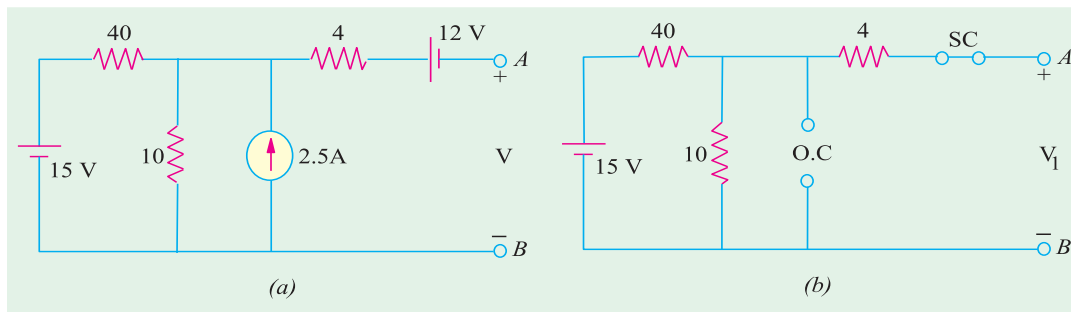


Fig. 2.109

**Solution.** The given circuit has been redrawn in Fig. 2.109 (b) with 15 - V battery acting alone while the other two sources have been killed. The 12 - V battery has been replaced by a short-circuit and the current source has been replaced by an open-circuit (O.C) (Art. 2.19). Since the output terminals are open, no current flows through the 4 Ω resistor and hence, there is no voltage drop across it. Obviously  $V_1$  equals the voltage drop over 10 Ω resistor which can be found by using the voltage-divider rule.

$$V_1 = 15 \times 10 / (40 + 10) = 3 \text{ V}$$

Fig. 2.110 (a) shows the circuit when current source acts alone, while two batteries have been killed. Again, there is no current through 4 Ω resistor. The two resistors of values 10 Ω and 40 Ω are

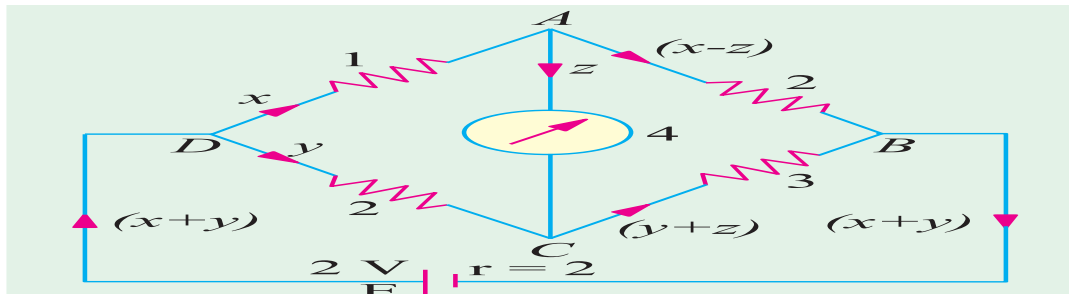


Fig. 2.110

in parallel across the current source. Their combined resistances is  $10 \parallel 40 = 8 \Omega$

$$\therefore V_2 = 8 \times 2.5 = 20 \text{ V with point A positive.}$$

Fig. 2.110 (b) shows the case when 12 V battery acts alone. Here,  $V_3 = -12 \text{ V}^*$ . Minus sign has been taken because negative terminal of the battery is connected to point A and the positive terminal to point B. As per the Superposition theorem,

$$V = V_1 + V_2 + V_3 = 3 + 20 - 12 = 11 \text{ V}$$

**Example 2.55.** Apply Superposition theorem to the circuit of Fig. 2.107 (a) for finding the voltage drop  $V$  across the  $5 \Omega$  resistor.

**Solution.** Fig. 2.111 (b) shows the redrawn circuit with the voltage source acting alone while the two current sources have been ‘killed’ i.e. have been replaced by open circuits. Using voltage-divider principle, we get

$V_1 = 60 \times 5 / (5 + 2 + 3) = 30 \text{ V}$ . It would be taken as positive, because current through the  $5 \Omega$  resistances flows from A to B, thereby making the upper end of the resistor positive and the lower end negative.

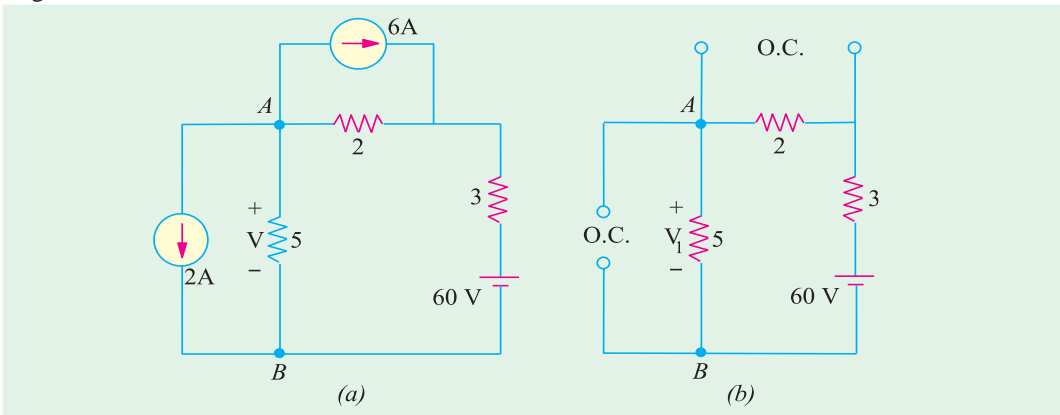


Fig. 2.111

Fig. 2.112 (a) shows the same circuit with the 6 A source acting alone while the two other sources have been ‘killed’. It will be seen that 6 A source has to parallel circuits across it, one having a resistance of  $2 \Omega$  and the other  $(3 + 5) = 8 \Omega$ . Using the current-divider rule, the current through the  $5 \Omega$  resistor =  $6 \times 2 / (2 + 3 + 5) = 1.2 \text{ A}$ .

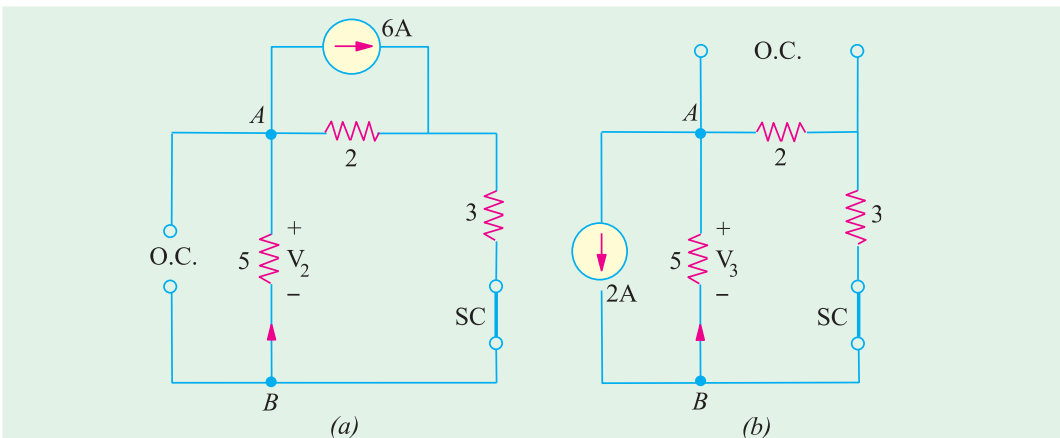


Fig. 2.112

\* Because Fig. 2.110 (b) resembles a voltage source with an internal resistance =  $4 + 10 \parallel 40 = 12 \Omega$  and which is an open-circuit.



$\therefore V_2 = 1.2 \times 5 = 6 \text{ V}$ . It would be taken **negative** because current is flowing from  $B$  to  $A$ . *i.e.* point  $B$  is at a higher potential as compared to point  $A$ . Hence,  $V_2 = -6 \text{ V}$ .

Fig. 2.112 (b) shows the case when 2-A source acts alone, while the other two sources are dead. As seen, this current divides equally at point  $B$ , because the two parallel paths have equal resistances of  $5 \Omega$  each. Hence,  $V_3 = 5 \times 1 = 5 \text{ V}$ . It would also be taken as negative because current flows from  $B$  to  $A$ . Hence,  $V_3 = -5 \text{ V}$ .

Using Superposition principle, we get

$$V = V_1 + V_2 + V_3 = 30 - 6 - 5 = \mathbf{19 \text{ V}}$$

**Example 2.56.** (b) Determine using superposition theorem, the voltage across the 4 ohm resistor shown in Fig. 2.113 (a) [Nagpur University, Summer 2000]

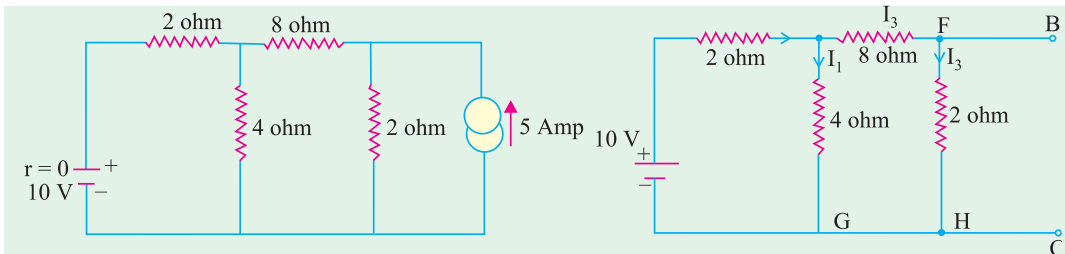


Fig. 2.113 (a)

Fig. 2.113 (b)

**Solution.** Superposition theorem needs one source acting at a time.

**Step I :** De-acting current source.

The circuit is redrawn after this change in Fig. 2.113 (b)

$$I_1 = \frac{10}{2 + \frac{4 \times (8 + 2)}{4 + (8 + 2)}} = \frac{10}{2 + \frac{40}{14}} = 2.059 \text{ amp}$$

$$I_2 = \frac{2.059}{14} \times 10 = 1.471 \text{ amp, in downward direction}$$

**Step II :** De-activate the voltage source.

The circuit is redrawn after the change, in Fig. 2.113 (c)

With the currents marked as shown.

$I_d = 2I_c$  relating the voltage drops in Loop ADC.

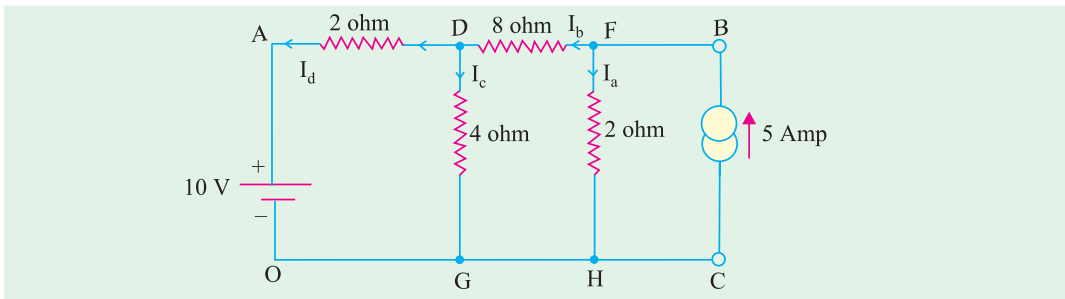


Fig. 2.113 (c)

Thus  $I_b = 3 I_c$ .

Resistance of parallel combination of

$$2 \text{ and } 4 \text{ ohms} = \frac{2 \times 4}{2 + 4} = 1.333 \Omega$$

$$\text{Resistance for flow of } I_b = 8 + 1.333 = 9.333 \Omega$$

The 5-amp current from the sources gets divided into  $I_b (= 3 I_c)$  and  $I_a$ , at the node F.

$$I_b = 3 I_c = \frac{2.0}{2.0 + 9.333} \times 5 = 0.8824$$

$\therefore I_c = 0.294$  amp, in downward direction.

**Step III.** Apply superposition theorem, for finding the total current into the 4-ohm resistor  
 = Current due to Current source + Current due to Voltage source  
 =  $0.294 + 1.471 = 1.765$  amp in downward direction.

**Check.** In the branch  $AD$ ,

The voltage source drives a current from  $A$  to  $D$  of 2.059 amp, and the current source drives a current of  $I_d (= 2I_c)$  which is 0.588 amp, from  $D$  to  $A$ .

The net current in branch  $AD$

$$= 2.059 - 0.588 = 1.471 \text{ amp} \quad \dots \text{eqn. (a)}$$

With respect to  $O$ ,  $A$  is at a potential of + 10 volts.

Potential of  $D$  with respect to  $O$

$$\begin{aligned} &= (\text{net current in resistor}) \times 4 \\ &= 1.765 \times 4 = + 7.06 \text{ volts} \end{aligned}$$

Between  $A$  and  $D$ , the potential difference is  $(10 - 7.06)$  volts

Hence, the current through this branch

$$= \frac{10 - 7.06}{2} = 1.47 \text{ amp from } A \text{ to } D \quad \dots \text{eqn (b)}$$

This is the same as eqn. (a) and hence checks the result, obtained previously.

**Example 2.57.** Find the current flowing in the branch  $XY$  of the circuit shown in Fig. 2.114 (a) by superposition theorem. [Nagpur University, April 1996]

**Solution.** As shown in Fig. 2.114 (b), one source is de-activated. Through series-parallel combinations of resistances, the currents due to this source are calculated. They are marked as Fig. 2.114 (b).

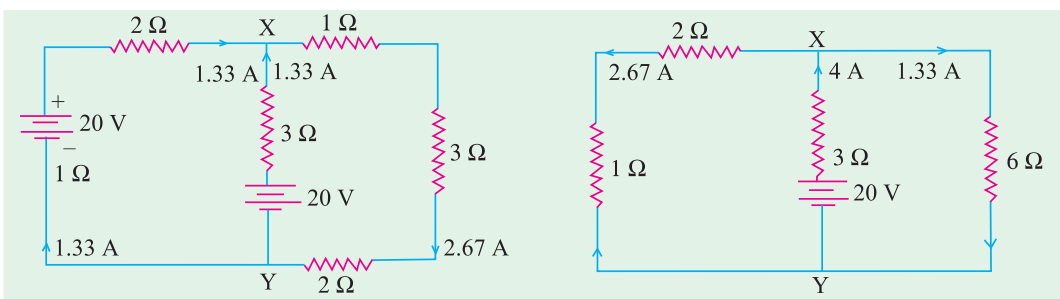


Fig. 2.114 (a)

Fig. 2.114 (b)

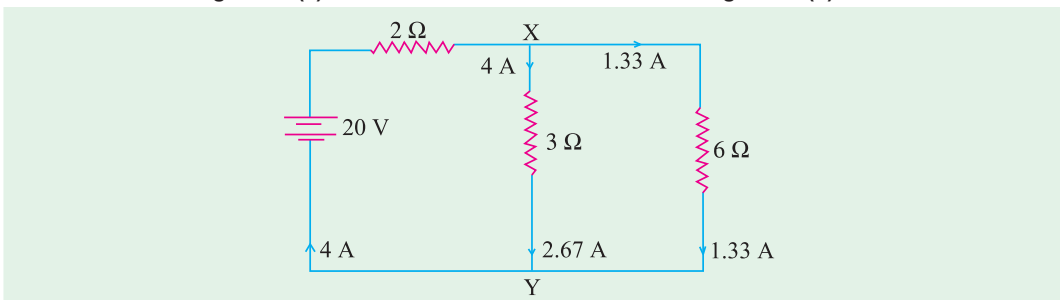


Fig. 2.114 (c)

In the next step, second source is de-activated as in Fig. 2.114 (c). Through simple series parallel resistances combinations, the currents due to this source are marked on the same figure.

According to the superposition theorem, the currents due to both the sources are obtained after adding the individual contributions due to the two sources, with the final results marked on Fig. 2.114 (a). Thus, the current through the branch XY is 1.33 A from Y to X.

**Example 2.58.** Find the currents in all the resistors by Superposition theorem in the circuit shown in Fig. 2.115 (a). Calculate the power consumed. [Nagpur University, Nov. 1996]

**Solution.** According to Superposition theorem, one source should be retained at a time, deactivating remaining sources. Contributions due to individual sources are finally algebraically added to get the answers required. Fig. 2.115 (b) shows only one source retained and the resultant currents in all branches/elements. In Fig. 2.115 (c), other source is shown to be in action, with concerned currents in all the elements marked.

To get the total current in any element, two component-currents in Fig. 2.115 (b) and Fig. 2.115 (c) for the element are to be algebraically added. The total currents are marked on Fig. 2.115 (a).

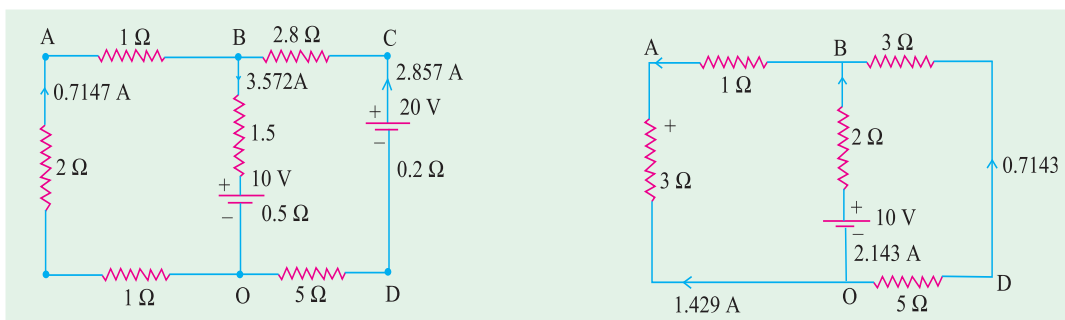


Fig. 2.115 (a)

Fig. 2.115 (b)

All resistors are in ohms

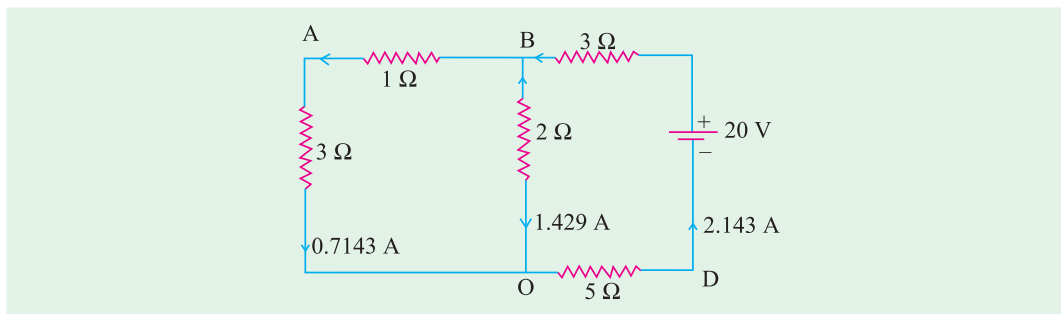


Fig. 2.115 (c)

Power loss calculations. (i) from power consumed by resistors :

$$\text{Power} = (0.7147^2 \times 4) + (3.572^2 \times 2) + (2.875^2 \times 8) = 92.86 \text{ watts}$$

(ii) From Source-power.

$$\text{Power} = 10 \times 3.572 + 20 \times 2.857 = 92.86 \text{ watts}$$

### Tutorial Problems No. 2.4.

1. Apply the principle of Superposition to the network shown in Fig. 2.116 to find out the current in the 10 Ω resistance. [0.464 A] (F.Y. Engg. Pune Univ.)
2. Find the current through the 3 Ω resistance connected between C and D Fig. 2.117.

[1 A from C to D] (F.Y. Engg. Pune Univ.)

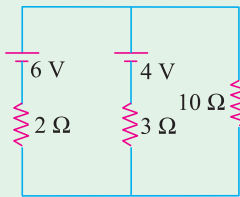


Fig. 2.116

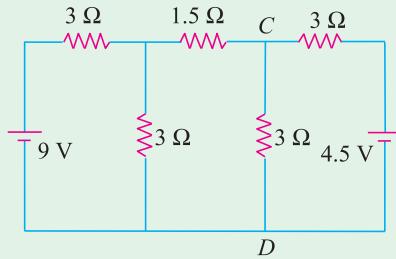


Fig. 2.117

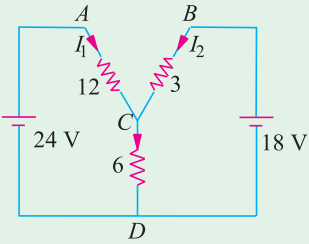


Fig. 2.118

3. Using the Superposition theorem, calculate the magnitude and direction of the current through each resistor in the circuit of Fig. 2.118.  **$I_1 = 6/7 \text{ A}; I_2 = 10/7 \text{ A}; I_3 = 16/7 \text{ A}$**

4. For the circuit shown in Fig. 2.119 find the current in  $R = 8 \Omega$  resistance in the branch AB using superposition theorem.  **$[0.875 \text{ A}]$  (F.Y. Engg. Pune Univ.)**

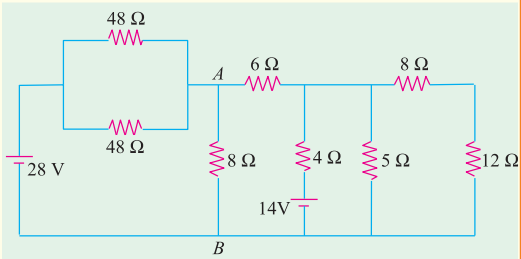


Fig. 2.119

5. Apply superposition principle to compute current in the 2-Ω resistor of Fig. 2.120. All resistors are in ohms.  **$[I_{ab} = 5 \text{ A}]$**

6. Use Superposition theorem to calculate the voltage drop across the 3 Ω resistor of Fig. 2.121. All resistance values are in ohms.  **$[18 \text{ V}]$**

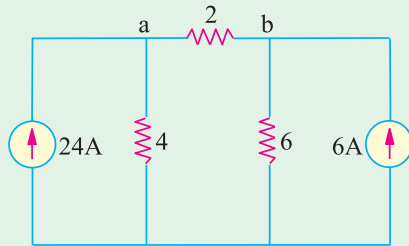


Fig. 2.120

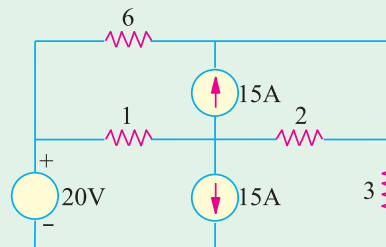


Fig. 2.121

7. With the help of Superposition theorem, compute the current  $I_{ab}$  in the circuit of Fig. 2.122. All resistances are in ohms.  **$[I_{ab} = -3 \text{ A}]$**

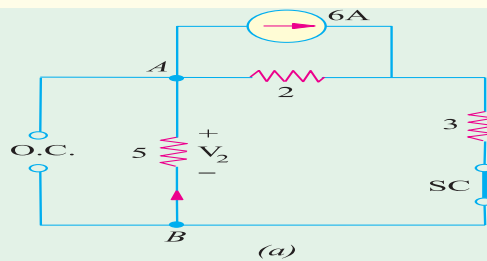


Fig. 2.122

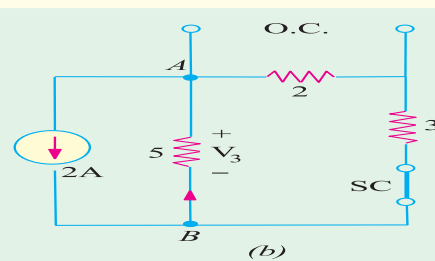


Fig. 2.123

8. Use Superposition theorem to find current  $I_{ab}$  in the circuit of Fig. 2.123. All resistances are in ohms.  **$[100 \text{ A}]$**

9. Find the current in the 15 Ω resistor of Fig. 2.124 by using Superposition principle. Numbers represent resistances in ohms.  **$[2.8 \text{ A}]$**

10. Use Superposition principle to find current in the 10-Ω resistor of Fig. 2.125. All resistances are in ohms. [1 A]
11. State and explain Superposition theorem. For the circuit of Fig. 2.126.
- (a) determine currents  $I_1$ ,  $I_2$  and  $I_3$  when switch  $S$  is in position  $b$ .
- (b) using the results of part (a) and the principle of superposition, determine the same currents with switch  $S$  in position  $a$ .

[(a) 15 A, 10 A, 25 A (b) 11 A, 16 A, 27 A] (Elect. Technology Vikram Univ.)

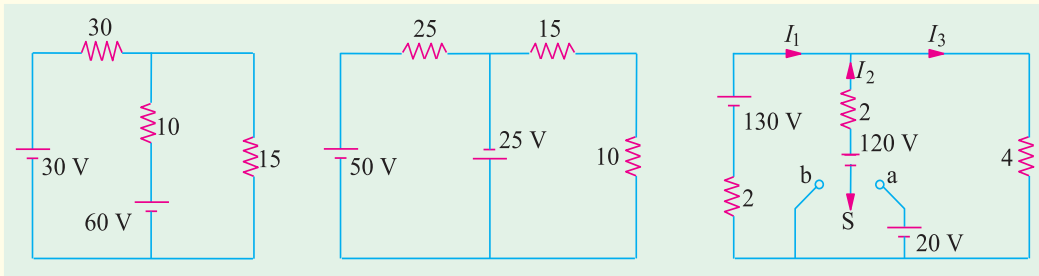
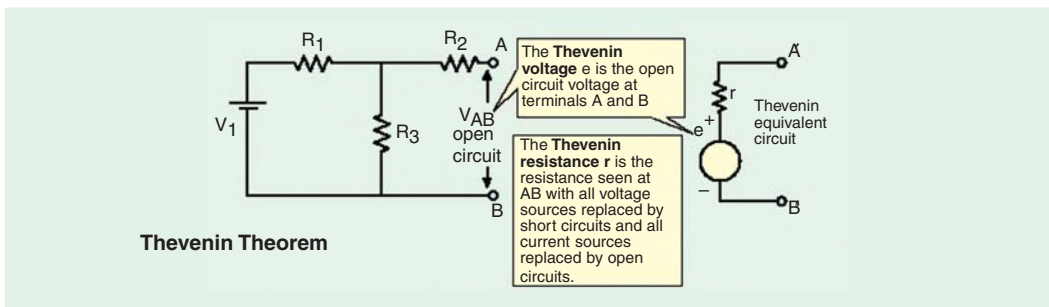


Fig. 2.124

Fig. 2.125

Fig. 2.126

### 2.18. Thevenin Theorem



It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by **a single voltage source with a series resistance**. It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.

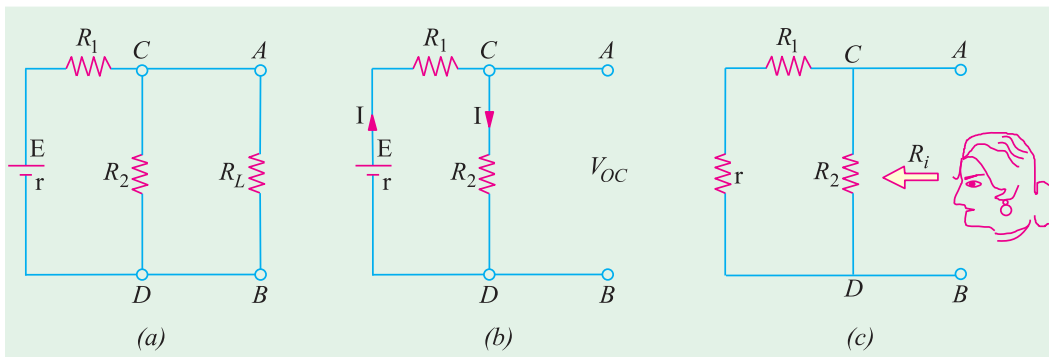


Fig. 2.127

Suppose, it is required to find current flowing through load resistance  $R_L$ , as shown in Fig. 2.127 (a). We will proceed as under :

1. Remove  $R_L$  from the circuit terminals  $A$  and  $B$  and redraw the circuit as shown in Fig. 2.127 (b). Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage  $V_{oc}$  which appears across terminals  $A$  and  $B$  when they are open *i.e.* when  $R_L$  is removed.

As seen,  $V_{oc} = \text{drop across } R_2 = IR_2$  where  $I$  is the circuit current when  $A$  and  $B$  are open.

$$I = \frac{E}{R_1 + R_2 + r} \quad \therefore V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r} \quad [r \text{ is the internal resistance of battery}]$$

It is also called 'Thevenin voltage'  $V_{th}$ .

3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance  $r$  behind and redraw the circuit, as shown in Fig. 2.127 (c). When viewed *inwards* from terminals  $A$  and  $B$ , the circuit consists of two parallel paths : one containing  $R_2$  and the other containing  $(R_1 + r)$ . The equivalent resistance of the network, as viewed from these terminals is given as

$$R = R_2 \parallel (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called, \* Thevenin resistance  $R_{sh}$  (though, it is also sometimes written as  $R_i$  or  $R_0$ ).

Consequently, as viewed from terminals  $A$  and  $B$ , the whole network (excluding  $R_L$ ) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals  $V_{oc}$  (or  $V_{sh}$ ) and whose internal resistance equals  $R_{sh}$  (or  $R_i$ ) as shown in Fig. 2.128.

4.  $R_L$  is now connected back across terminals  $A$  and  $B$  from where it was temporarily removed earlier. Current flowing through  $R_L$  is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$

It is clear from above that any network of resistors and voltage sources (and current sources as well) when viewed from any points  $A$  and  $B$  in the network, can be replaced by a single voltage source and a single resistance\*\* in series with the voltage source.

After this replacement of the network by a single voltage source with a series resistance has been accomplished, it is easy to find current in any load resistance joined across terminals  $A$  and  $B$ . This theorem is valid even for those linear networks which have a nonlinear load.

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

**The current flowing through a load resistance  $R_L$  connected across any two terminals  $A$  and  $B$  of a linear, active bilateral network is given by  $V_{oc} \parallel (R_i + R_L)$  where  $V_{oc}$  is the open-circuit voltage (*i.e.* voltage across the two terminals when  $R_L$  is removed) and  $R_i$  is the internal resistance of the network as viewed back into the open-circuited network from terminals  $A$  and  $B$  with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.**

\* After the French engineer M.L. Thevenin (1857-1926) who while working in Telegraphic Department published a statement of the theorem in 1893.

\*\* Or impedance in the case of a.c. circuits.

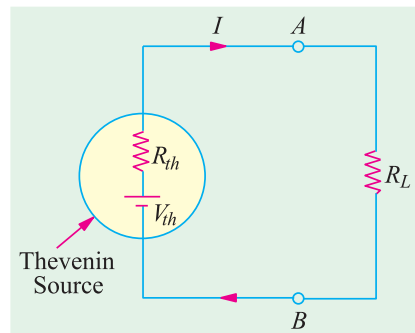


Fig. 2.128

### 2.19. How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance  $R_L$ ) whose current is required.
2. Find the open-circuit voltage  $V_{oc}$  which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage  $V_{th}$ .
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance  $R_{th}$  or  $T_i$ .
4. Replace the entire network by a single Thevenin source, whose voltage is  $V_{th}$  or  $V_{oc}$  and whose internal resistance is  $R_{th}$  or  $R_i$ .
5. Connect  $R_L$  back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through  $R_L$  by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

**Example 2.59.** Convert the circuit shown in Fig. 2.129 (a), to a single voltage source in series with a single resistor.

(AMIE Sec. B, Network Analysis Summer 1992)

**Solution.** Obviously, we have to find equivalent Thevenin circuit. For this purpose, we have to calculate (i)  $V_{th}$  or  $V_{AB}$  and (ii)  $R_{th}$  or  $R_{AB}$ .

With terminals A and B open, the two voltage sources are connected in subtractive series because they oppose each other. Net voltage around the circuit is  $(15 - 10) = 5$  V and total resistance is  $(8 + 4) = 12 \Omega$ . Hence circuit current is  $= 5/12$  A. Drop across  $4 \Omega$  resistor  $= 4 \times 5/12 = 5/3$  V with the polarity as shown in Fig. 2.129 (a).

$$\therefore V_{AB} = V_{th} = +10 + 5/3 = 35/3 \text{ V.}$$

Incidentally, we could also find  $V_{AB}$  while going along the parallel route  $BFEA$ .

Drop across  $8 \Omega$  resistor  $= 8 \times 5/12 = 10/3$  V.  $V_{AB}$  equal the algebraic sum of voltages met on the way from B to A. Hence,  $V_{AB} = (-10/3) + 15 = 35/3$  V.

As shown in Fig. 2.129 (b), the single voltage source has a voltage of  $35/3$  V.

For finding  $R_{th}$ , we will replace the two voltage sources by short-circuits. In that case,  $R_{th} = R_{AB} = 4 \parallel 8 = 8/3 \Omega$

**Example 2.60.** State Thevenin's theorem and give a proof. Apply this theorem to calculate the current through the  $4 \Omega$  resistor of the circuit of Fig. 2.130 (a).

(A.M.I.E. Sec. B Network Analysis W.)

**Solution.** As shown in Fig. 2.130 (b),  $4 \Omega$  resistance has been removed thereby open-circuiting the terminals A and B. We will now find  $V_{AB}$  and  $R_{AB}$  which will give us  $V_{th}$  and  $R_{th}$  respectively. The potential drop across  $5 \Omega$  resistor can be found with the help of voltage-divider rule. Its value is  $= 15 \times 5/(5 + 10) = 5$  V.

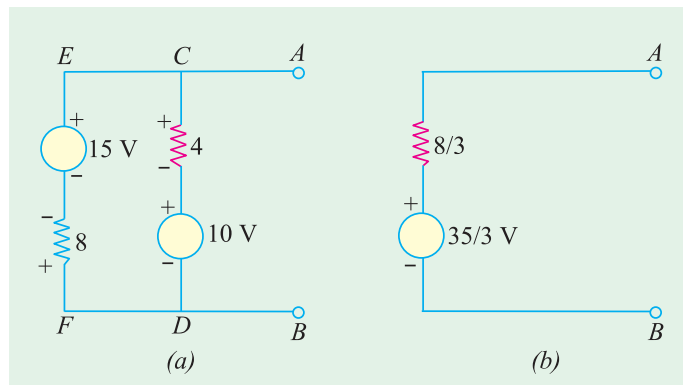


Fig. 2.129

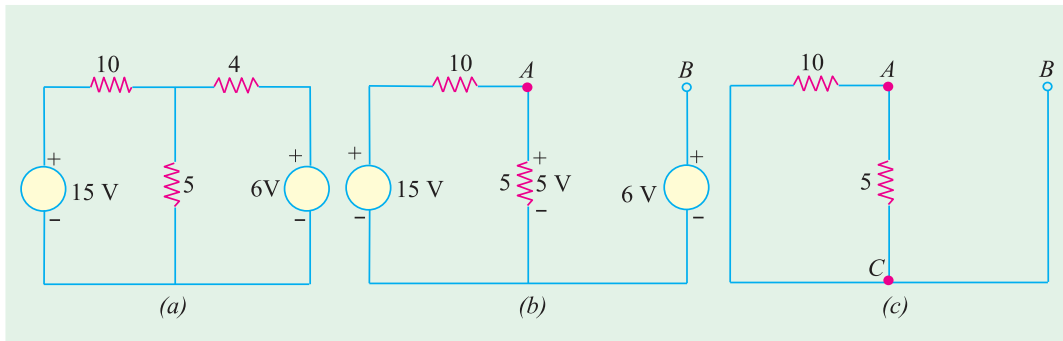


Fig. 2.130

For finding  $V_{AB}$ , we will go from point  $B$  to point  $A$  in the clockwise direction and find the algebraic sum of the voltages met on the way.

$$\therefore V_{AB} = -6 + 5 = -1 \text{ V.}$$

It means that point  $A$  is negative with respect to point  $E$ , or point  $B$  is at a higher potential than point  $A$  by one volt.

In Fig. 2.130 (c), the two voltage source have been short-circuited. The resistance of the network as viewed from points  $A$  and  $B$  is the same as viewed from points  $A$  and  $C$ .

$$\therefore R_{AB} = R_{AC} = 5 \parallel 10 = 10/3 \Omega$$

Thevenin's equivalent source is shown in Fig. 2.131 in which  $4 \Omega$  resistor has been joined back across terminals  $A$  and  $B$ . Polarity of the voltage source is worth nothing.

$$\therefore I = \frac{1}{(10/3) + 4} = \frac{3}{22} = 0.136 \text{ A} \quad \text{From } E \text{ to } A$$

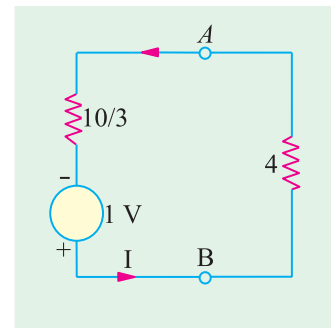


Fig. 2.131

**Example 2.61.** With reference to the network of Fig. 2.132 (a), by applying Thevenin's theorem find the following :

- (i) the equivalent e.m.f. of the network when viewed from terminals  $A$  and  $B$ .
- (ii) the equivalent resistance of the network when looked into from terminals  $A$  and  $B$ .
- (iii) current in the load resistance  $R_L$  of  $15 \Omega$ . (Basic Circuit Analysis, Nagpur Univ. 1993)

**Solution.** (i) Current in the network before load resistance is connected [Fig. 2.132 (a)]  
 $= 24/(12 + 3 + 1) = 1.5 \text{ A}$

$$\therefore \text{voltage across terminals } AB = V_{oc} = V_{th} = 12 \times 1.5 = 18 \text{ V}$$

Hence, so far as terminals  $A$  and  $B$  are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(ii) There are two parallel paths between points  $A$  and  $B$ . Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point  $A$  and  $B$  is [Fig. 2.132 (c)]

$$R_i = R_{th} = 12 \times 4/(12 + 4) = 3 \Omega$$

(iii) When load resistance of  $15 \Omega$  is connected across the terminals, the network is reduced to the structure shown in Fig. 2.132 (d).



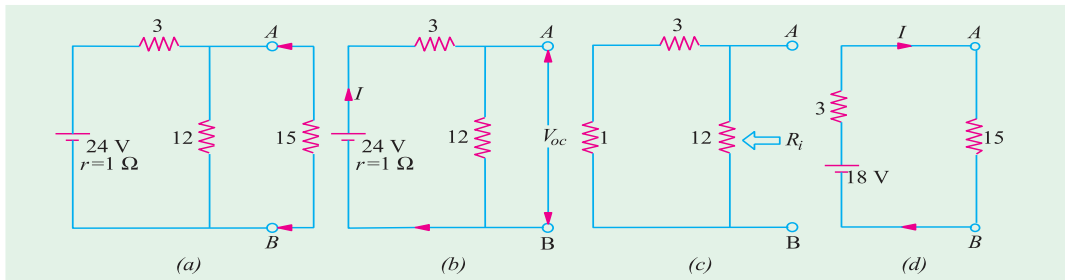


Fig. 2.132

$$I = V_{th}/(R_{th} + R_L) = 18/(15 + 3) = 1 \text{ A}$$

**Example 2.62.** Using Thevenin theorem, calculate the current flowing through the 4-Ω resistor of Fig. 2.133 (a).

**Solution.** (i) Finding  $V_{th}$

If we remove the 4-Ω resistor, the circuit becomes as shown in Fig. 2.133 (b). Since full 10 A current passes through 2 Ω resistor, drop across it is  $10 \times 2 = 20 \text{ V}$ . Hence,  $V_B = 20 \text{ V}$  with respect to the common ground. The two resistors of 3 Ω and 6 Ω are connected in series across the 12 V battery. Hence, drop across 6 Ω resistor =  $12 \times 6/(3 + 6) = 8 \text{ V}$ .

$$\therefore V_A = 8 \text{ V with respect to the common ground}^*$$

$$\therefore V_{th} = V_{BA} = V_B - V_A = 20 - 8 = 12 \text{ V—with } B \text{ at a higher potential}$$

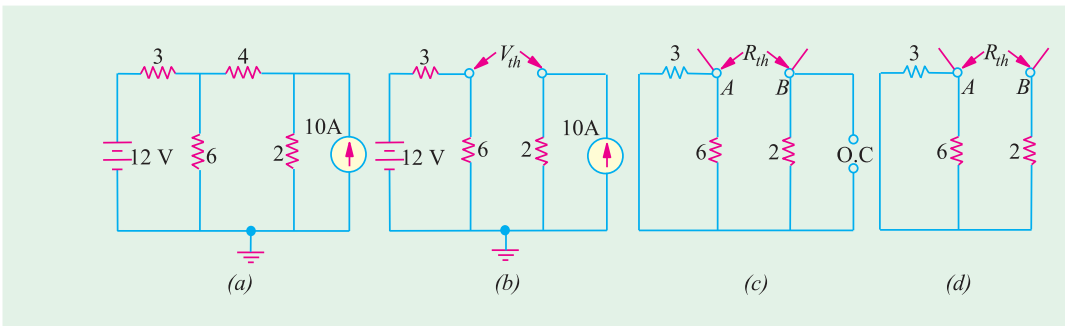


Fig. 2.133

(ii) Finding  $R_{th}$

Now, we will find  $R_{th}$  i.e. equivalent resistance of the network as looked back into the open-circuited terminals A and B. For this purpose, we will replace both the voltage and current sources. Since voltage source has no internal resistance, it would be replaced by a short circuit i.e. zero resistance. However, current source would be removed and replaced by an 'open' i.e. infinite resistance (Art. 1.18). In that case, the circuit becomes as shown in Fig. 2.133 (c). As seen from Fig. 2.133 (d),  $R_{th} = 6 \parallel 3 + 2 = 4 \Omega$ . Hence, Thevenin's equivalent circuit consists of a voltage source of 12 V and a series resistance of 4 Ω as shown in Fig. 2.134 (a). When 4 Ω resistor is connected across terminals A and B, as shown in Fig. 2.134 (b).

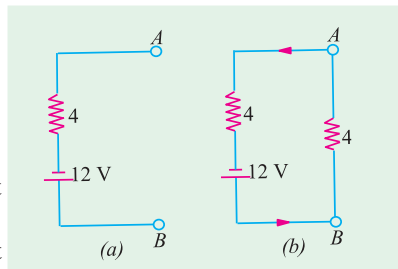


Fig. 2.134

$$I = 12/(4 + 4) = 1.5 \text{ A—from } B \text{ to } A$$

\* Also,  $V_A = 12 - \text{drop across } 3\text{-}\Omega \text{ resistor} = 12 - 12 \times 3/(6 + 3) = 12 - 4 = 8 \text{ V}$

**Example 2.63.** For the circuit shown in Fig. 2.135 (a), calculate the current in the 10 ohm resistance. Use Thevenin's theorem only.

(Elect. Science-I Allahabad Univ. 1992)

**Solution.** When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.135 (b).

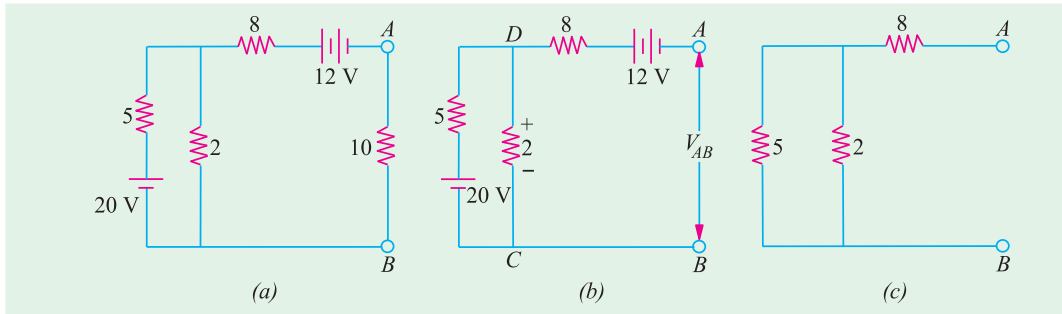


Fig. 2.135

Now, we will find the open-circuit voltage  $V_{AB} = V_{th}$ . For this purpose, we will go from point  $B$  to point  $A$  and find the algebraic sum of the voltages met on the way. It should be noted that with terminals  $A$  and  $B$  open, there is no voltage drop on the 8 Ω resistance. However the two resistances of 5 Ω and 2 Ω are connected in series across the 20-V battery. As per voltage-divider rule, drop on 2 Ω resistance =  $20 \times 2/(2 + 5) = 5.71$  V with the polarity as shown in figure. As per the sign convention of Art.

$$V_{AB} = V_{th} = + 5.71 - 12 = -6.29 \text{ V}$$

The negative sign shows that point  $A$  is negative with respect to point  $B$  or which is the same thing, point  $B$  is positive with respect to point  $A$ .

For finding  $R_{AB} = R_{th}$ , we replace the batteries by short-circuits as shown in Fig. 2.128 (c).

$$\therefore R_{AB} = R_{th} = 8 + 2 \parallel 5 = 9.43 \text{ } \Omega$$

Hence, the equivalent Thevenin's source with respect to terminals  $A$  and  $B$  is as shown in Fig. 2.136. When 10 Ω resistance is reconnected across  $A$  and  $B$ , current through it is  $I = 6.24/(9.43 + 10) = 0.32$  A.

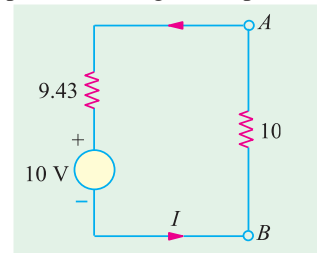


Fig. 2.136 (a)

**Example 2.64.** Using Thevenin's theorem, calculate the p.d. across terminals  $A$  and  $B$  in Fig. 2.137 (a).

**Solution. (i) Finding  $V_{oc}$**

First step is to remove 7 Ω resistor thereby open-circuiting terminals  $A$  and  $B$  as shown in Fig. 2.137 (b). Obviously, there is no current through the 1 Ω resistor and hence no drop across it. Therefore  $V_{AB} = V_{oc} = V_{CD}$ . As seen, current  $I$  flows due to the combined action of the two batteries. Net voltage in the  $CDFE$  circuit =  $18 - 6 = 12$  V. Total resistance =  $6 + 3 = 9$  Ω. Hence,  $I = 12/9 = 4/3$  A

$$V_{CD} = 6 \text{ V} + \text{drop across } 3 \text{ } \Omega \text{ resistor} = 6 + (4/3) \times 3 = 10 \text{ V}^*$$

$$\therefore V_{oc} = V_{th} = 10 \text{ V.}$$

**(ii) Finding  $R_i$  or  $R_{th}$**

As shown in Fig. 2.137 (c), the two batteries have been replaced by short-circuits (SC) since their internal resistances are zero. As seen,  $R_i = R_{th} = 1 + 3 \parallel 6 = 3$  Ω. The Thevenin's equivalent circuit is as shown in Fig. 2.137 (d) where the 7 Ω resistance has been reconnected across terminals  $A$  and  $B$ .

\* Also,  $V_{CD} = 18 - \text{drop across } 6 \text{ } \Omega \text{ resistor} = 18 - (4/3) \times 6 = 10 \text{ V}$

The p.d. across this resistor can be found with the help of Voltage Divider Rule (Art. 1.15).

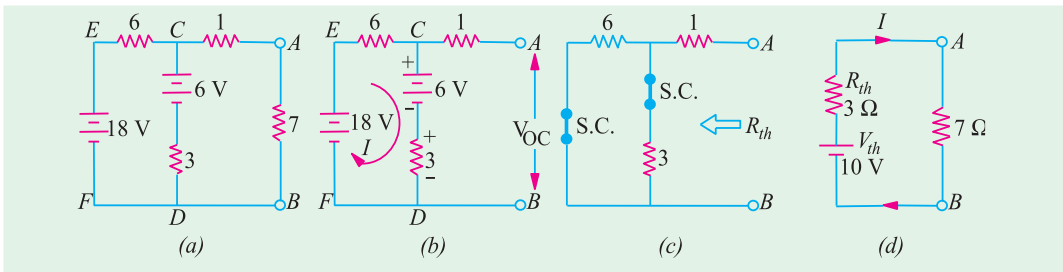


Fig. 2.137

**Example 2.65.** Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of the network shown in Fig. 2.138 (a) if the load is (a)  $2\ \Omega$  (b)  $1\ \Omega$ .

(Elect. Technology, Gwalior Univ.)

**Solution.** For finding open-circuit voltage  $V_{oc}$  or  $V_{th}$  across terminals A and B, we must first find current  $I_2$  flowing through branch CD. Using Maxwell's loop current method (Art. 2.11), we have from Fig. 2.131 (a).

$$-2 I_1 - 4 (I_1 - I_2) + 8 = 0 \quad \text{or} \quad 3 I_1 - 2 I_2 = 4$$

$$\text{Also} \quad -2 I_2 - 2 I_2 - 4 - 4 (I_2 - I_1) = 0 \quad \text{or} \quad I_1 - 2 I_2 = 1$$

From these two equations, we get  $I_2 = 0.25\ \text{A}$

As we go from point D to C, voltage rise =  $4 + 2 \times 0.25 = 4.5\ \text{V}$

Hence,  $V_{CD} = 4.5$  or  $V_{AB} = V_{th} = 4.5\ \text{V}$ . Also, it may be noted that point A is positive with respect to point B.

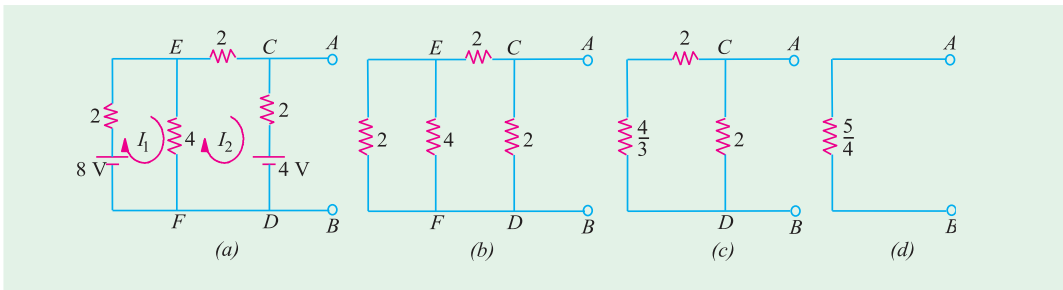


Fig. 2.138

In Fig. 2.138 (b), both batteries have been removed. By applying laws of series and parallel combination of resistances, we get  $R_i = R_{th} = 5/4\ \Omega = 1.25\ \Omega$

(i) When  $R_L = 2\ \Omega$ ;  $I = 4.5 / (2 + 1.25) = \mathbf{1.38\ \text{A}}$

(ii) When  $R_L = 1\ \Omega$ ;  $I = 4.5 / (1 + 1.25) = \mathbf{2.0\ \text{A}}$

**Note.** We could also find  $V_{oc}$  and  $R_i$  by first Thevenining part of the circuit across terminals E and F and then across A and B (Ex. 2.62).

**Example 2.66.** The four arms of a Wheatstone bridge have the following resistances :

$AB = 100$ ,  $BC = 10$ ,  $CD = 4$ ,  $DA = 50\ \Omega$ . A galvanometer of  $20\ \Omega$  resistance is connected across BD. Use Thevenin's theorem to compute the current through the galvanometer when a p.d. of  $10\ \text{V}$  is maintained across AC.

(Elect. Technology, Vikram Univ. of Ujjain)

**Solution.** (i) When galvanometer is removed from Fig. 2.139 (a), we get the circuit of Fig. 2.139 (b).

(ii) Let us next find the open-circuit voltage  $V_{oc}$  (also called Thevenin voltage  $V_{th}$ ) between points  $B$  and  $D$ . Remembering that  $ABC$  (as well as  $ADC$ ) is a potential divider on which a voltage drop of 10 V takes place, we get

$$\text{Potential of } B \text{ w.r.t. } C = 10 \times 10/110 = 10/11 = 0.909 \text{ V}$$

$$\text{Potential of } D \text{ w.r.t. } C = 10 \times 4/54 = 20/27 = 0.741 \text{ V}$$

$$\therefore \text{ p.d. between } B \text{ and } D \text{ is } V_{oc} \text{ or } V_{th} = 0.909 - 0.741 = 0.168 \text{ V}$$

(iii) Now, remove the 10-V battery retaining its internal resistance which, in this case, happens to be zero. Hence, it amounts to short-circuiting points  $A$  and  $C$  as shown in Fig. 2.139 (d).

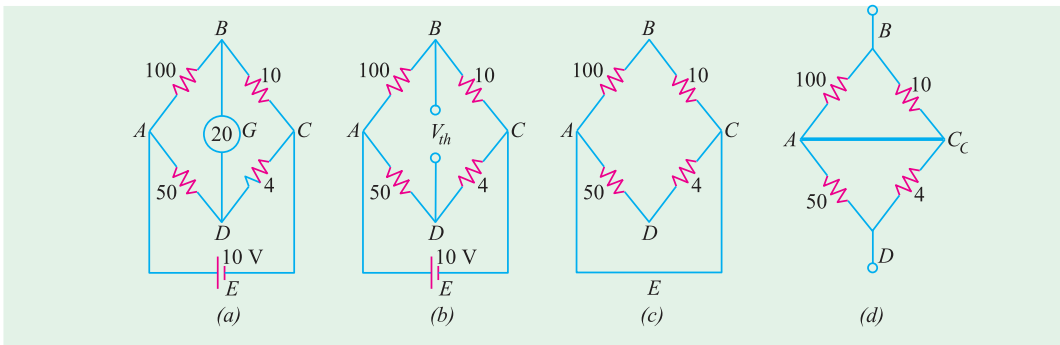


Fig. 2.139

(iv) Next, let us find the resistance of the whole network as viewed from points  $B$  and  $D$ . It may be easily found by noting that electrically speaking, points  $A$  and  $C$  have become one as shown in Fig. 2.140 (a). It is also seen that  $BA$  is in parallel with  $BC$  and  $AD$  is in parallel with  $CD$ . Hence,  $R_{BD} = 10 \parallel 100 + 50 \parallel 4 = 12.79 \Omega$

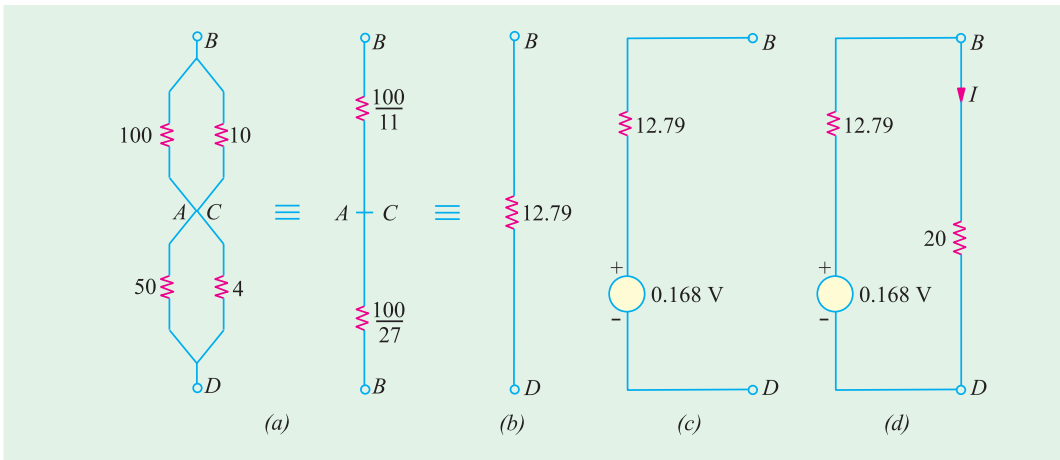


Fig. 2.140

(v) Now, so far as points  $B$  and  $D$  are connected, the network has a voltage source of 0.168 V and internal resistance  $R_i = 12.79 \Omega$ . This Thevenin's source is shown in Fig. 2.140 (c).

(vi) Finally, let us connect the galvanometer (initially removed) to this Thevenin source and calculate the current  $I$  flowing through it. As seen from Fig. 2.140 (d).

$$I = 0.168 / (12.79 + 20) = 0.005 \text{ A} = \mathbf{5 \text{ mA}}$$

**Example 2.67.** Determine the current in the  $1\ \Omega$  resistor across  $AB$  of network shown in Fig. 2.141 (a) using Thevenin's theorem. (Network Analysis, Nagpur Univ. 1993)

**Solution.** The given circuit can be redrawn, as shown in Fig. 2.141 (b) with the  $1\ \Omega$  resistor removed from terminals  $A$  and  $B$ . The current source has been converted into its equivalent voltage source as shown in Fig. 2.141 (c). For finding  $V_{th}$  we will find the currents  $x$  and  $y$  in Fig. 2.141 (c). Applying KVL to the first loop, we get

$$3 - (3 + 2)x - 1 = 0 \quad \text{or} \quad x = 0.4\ \text{A}$$

$$\therefore V_{th} = V_{AB} = 3 - 3 \times 0.4 = 1.8\ \text{V}$$

The value of  $R_{th}$  can be found from Fig. 2.141 (c) by replacing the two voltage sources by short-circuits. In this case  $R_{th} = 2 \parallel 3 = 1.2\ \Omega$

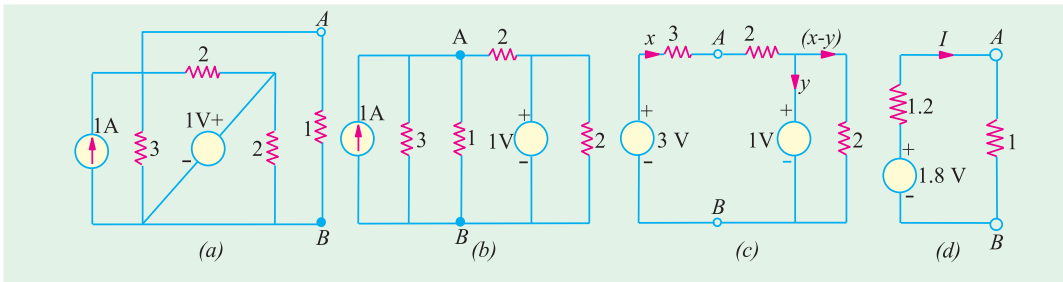


Fig. 2.141

Thevenin's equivalent circuit is shown in Fig. 2.141 (d). The current through the reconnected  $1\ \Omega$  resistor is  $= 1.8 / (1.2 + 1) = 0.82\ \text{A}$ .

**Example 2.68.** Find the current flowing through the  $4\ \Omega$  resistor in Fig. 2.142 (a) when (i)  $E = 2\ \text{V}$  and (ii)  $E = 12\ \text{V}$ . All resistances are in series.

**Solution.** When we remove  $E$  and  $4\ \Omega$  resistor, the circuit becomes as shown in Fig. 2.142 (b). For finding  $R_{th}$  i.e. the circuit resistance as viewed from terminals  $A$  and  $B$ , the battery has been short-circuited, as shown. It is seen from Fig. 2.142 (c) that  $R_{th} = R_{AB} = 15 \parallel 30 + 18 \parallel 9 = 16\ \Omega$

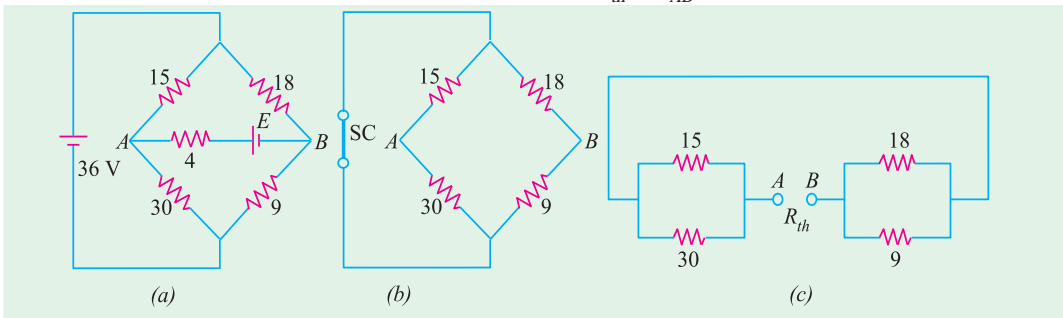


Fig. 2.142

We will find  $V_{th} = V_{AB}$  with the help of Fig. 2.143 (a) which represents the original circuit, except with  $E$  and  $4\ \Omega$  resistor removed. Here, the two circuits are connected in parallel across the  $36\ \text{V}$  battery. The potential of point  $A$  equals the drop on  $30\ \Omega$  resistance, whereas potential of point  $B$  equals the drop across  $9\ \Omega$  resistance. Using the voltage,

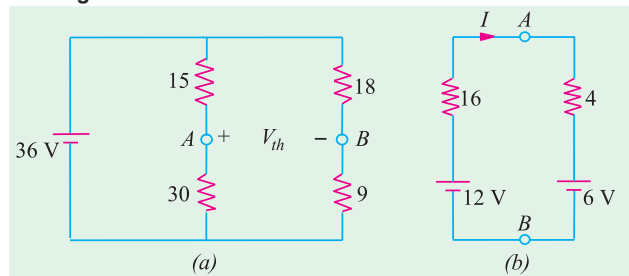


Fig. 2.143

divider rule, we have

$$V_A = 30 \times 30/45 = 24 \text{ V}$$

$$V_B = 36 \times 9/27 = 12 \text{ V}$$

$$\therefore V_{AB} = V_A - V_B = 24 - 12 = 12 \text{ V}$$

In Fig. 2.143 (b), the series combination of  $E$  and  $4 \Omega$  resistors has been reconnected across terminals  $A$  and  $B$  of the Thevenin's equivalent circuit.

$$(i) I = (12 - E)/20 = (12 - 2)/20 = 0.5 \text{ A} \quad (ii) I = (12 - 12)/20 = 0$$

**Example 2.69.** Calculate the value of  $V_{th}$  and  $R_{th}$  between terminals  $A$  and  $B$  of the circuit shown in Fig. 2.144 (a). All resistance values are in ohms.

**Solution.** Forgetting about the terminal  $B$  for the time being, there are two parallel paths between  $E$  and  $F$ : one consisting of  $12 \Omega$  and the other of  $(4 + 8) = 12 \Omega$ . Hence,  $R_{EF} = 12 \parallel 12 = 6 \Omega$ . The source voltage of  $48 \text{ V}$  drops across two  $6 \Omega$  resistances connected in series. Hence,  $V_{EF} = 24 \text{ V}$ . The same  $24 \text{ V}$  acts across  $12 \Omega$  resistor connected directly between  $E$  and  $F$  and across two series-connected resistance of  $4 \Omega$  and  $6 \Omega$  connected across  $E$  and  $F$ . Drop across  $4 \Omega$  resistor  $= 24 \times 4/(4 + 8) = 8 \text{ V}$  as shown in Fig. 2.144 (c).

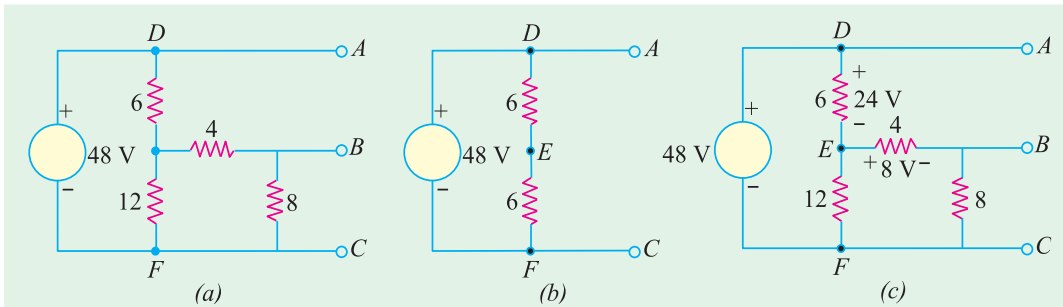


Fig. 2.144

Now, as we go from  $B$  to  $A$  via point  $E$ , there is a rise in voltage of  $8 \text{ V}$  followed by another rise in voltage of  $24 \text{ V}$  thereby giving a total voltage drop of  $32 \text{ V}$ . Hence  $V_{th} = 32 \text{ V}$  with point  $A$  positive.

For finding  $R_{th}$ , we short-circuit the  $48 \text{ V}$  source. This short circuiting, in effect, combines the points  $A, D$  and  $F$  electrically as shown in Fig. 2.145 (a). As seen from Fig. 2.145 (b),

$$R_{th} = V_{AB} = 8 \parallel (4 + 4) = 4 \Omega$$

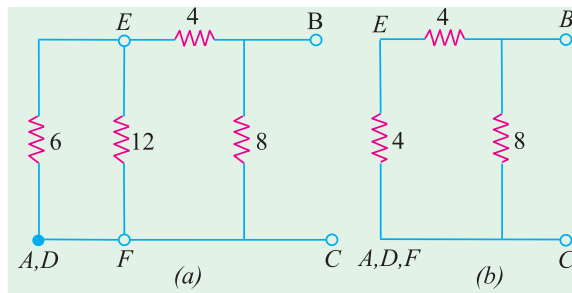


Fig. 2.145

**Example 2.70.** Determine Thevenin's equivalent circuit which may be used to represent the given network (Fig. 2.146) at the terminals  $AB$ .

(Electrical Eng.; Calcutta Univ.)

**Solution.** The given circuit of Fig. 2.146 (a) would be solved by applying Thevenin's theorem twice, first to the circuit to the left of point  $C$  and  $D$  and then to the left of points  $A$  and  $B$ . Using this technique, the network to the left of  $CD$  [Fig. 2.146 (a)] can be replaced by a source of voltage  $V_1$  and series resistance  $R_{t1}$  as shown in Fig. 2.146 (b).

$$V_1 = \frac{12 \times 6}{(6 + 1 + 1)} = 9 \text{ volts and } R_{t1} = \frac{6 \times 2}{(6 + 2)} = 1.5 \Omega$$

Similarly, the circuit of Fig. 2.146 (b) reduced to that shown in Fig. 2.146 (c)

$$V_2 = \frac{9}{(6 + 2)} \times \frac{6}{(1.5 + 6)} = 5.68 \text{ volts and } R_{t2} = \frac{6 \times 3.5}{9.5} = 2.21$$

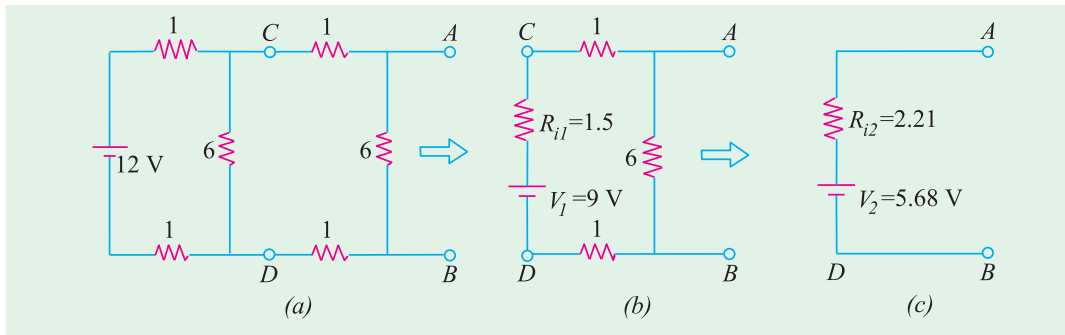


Fig. 2.146

**Example 2.71.** Use Thevenin's theorem, to find the value of load resistance  $R_L$  in the circuit of Fig. 2.147 (a) which results in the production of maximum power in  $R_L$ . Also, find the value of this maximum power. All resistances are in ohms.

**Solution.** We will remove the voltage and current sources as well as  $R_L$  from terminals A and B in order to find  $R_{th}$  as shown in Fig. 2.147 (b).

$$R_{th} = 4 + 6 \parallel 3 = 6 \Omega$$

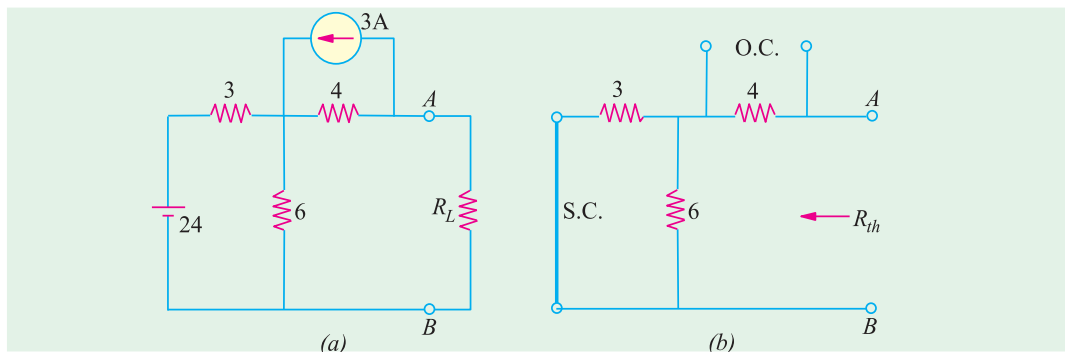


Fig. 2.147

In Fig. 2.147 (a), the current source has been converted into the equivalent voltage source for convenience. Since there is no current through the 4 Ω resistance (and hence no voltage drop across it),  $V_{th}$  equals the algebraic sum of battery voltage and drop across 6 Ω resistor. As we go along the path  $BDCA$ , we get,

$$V_{th} = 24 \times \frac{6}{6+3} - 12 = 4 \text{ V}$$

The load resistance has been reconnected to the Thevenin's equivalent circuit as shown in Fig. 2.148 (b). For maximum power transfer,  $R_L = R_{th} = 6 \Omega$

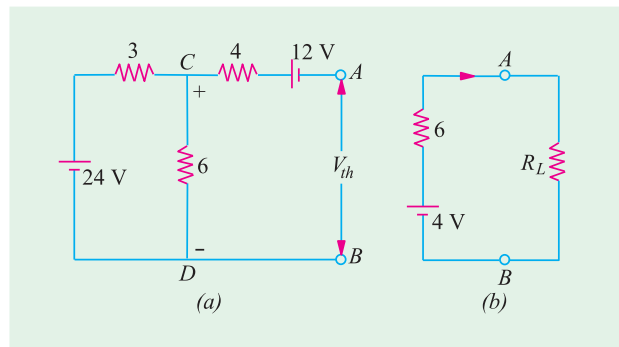


Fig. 2.148

Now, 
$$V_L = \frac{1}{2} V_{th} = \frac{1}{2} \times 4 = 2 \text{ V}; P_{L \max} = \frac{V_L^2}{R_L} = \frac{2^2}{6} = 0.67 \text{ W}$$

**Example 2.72.** Use Thevenin's theorem to find the current flowing through the  $6\ \Omega$  resistor of the network shown in Fig. 2.149 (a). All resistances are in ohms.

(Network Theory, Nagpur Univ. 1992)

**Solution.** When  $6\ \Omega$  resistor is removed [Fig. 2.149 (b)], whole of  $2\ \text{A}$  current flows along  $DC$  producing a drop of  $(2 \times 2) = 4\ \text{V}$  with the polarity as shown. As we go along  $BDCA$ , the total voltage is

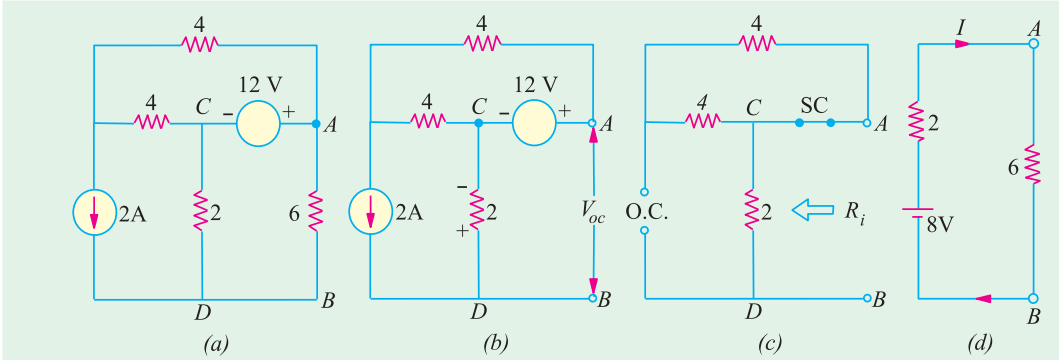


Fig. 2.149

$$= -4 + 12 = 8\ \text{V}$$

—with  $A$  positive w.r.t.  $B$ .

Hence,

$$V_{oc} = V_{th} = 8\ \text{V}$$

For finding  $R_i$  or  $R_{th}$   $18\ \text{V}$  voltage source is replaced by a short-circuit (Art- 2.15) and the current source by an open-circuit, as shown in Fig. 2.149 (c). The two  $4\ \Omega$  resistors are in series and are thus equivalent to an  $8\ \Omega$  resistance. However, this  $8\ \Omega$  resistor is in parallel with a short of  $0\ \Omega$ . Hence, their equivalent value is  $0\ \Omega$ . Now this  $0\ \Omega$  resistance is in series with the  $2\ \Omega$  resistor. Hence,  $R_i = 2 + 0 = 2\ \Omega$ . The Thevenin's equivalent circuit is shown in Fig. 2.149 (d).

$\therefore$

$$I = 8/(2 + 6) = 1\ \text{Amp}$$

—from  $A$  to  $B$

**Example 2.73.** Find Thevenin's equivalent circuit for the network shown in Fig. 2.150 (a) for the terminal pair  $AB$ .

**Solution.** It should be carefully noted that after coming to point  $D$ , the  $6\ \text{A}$  current has only one path to reach its other end  $C$  i.e., through  $4\ \Omega$  resistor thereby creating and  $IR$  drop of  $6 \times 4 = 24\ \text{V}$  with polarity as shown in Fig. 2.150 (b). No part of it can go along  $DE$  or  $DF$  because it would not find any path back to point  $C$ . Similarly, current due to  $18\text{-V}$  battery is restricted to loop  $EDFE$ . Drop across  $6\ \Omega$  resistor =  $18 \times 6/(6 + 3) = 12\ \text{V}$ . For finding  $V_{AB}$ , let us start from  $A$  and go to  $B$  via the *shortest* route  $ADFB$ . As seen from Fig. 2.150 (b), there is a *rise* of  $24\ \text{V}$  from  $A$  to  $D$  but a *fall* of  $12\ \text{V}$ .

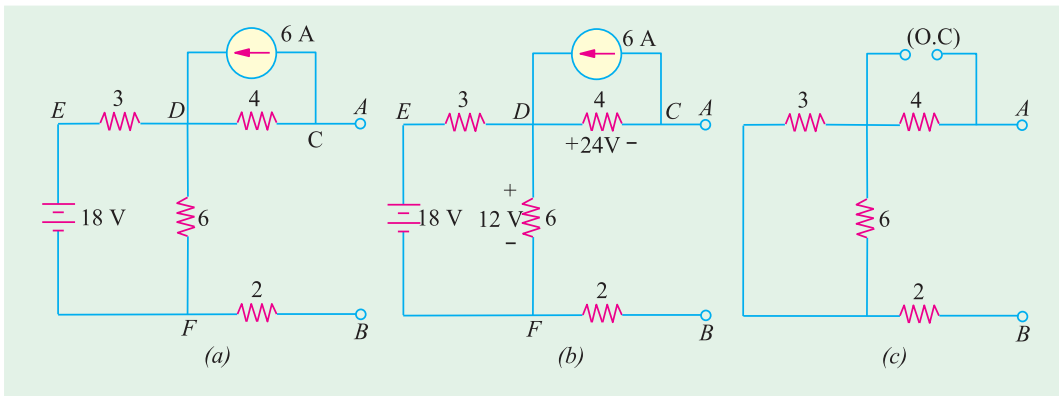


Fig. 2.150



from  $D$  to  $F$ . Hence,  $V_{AB} = 24 - 12 = 12$  V with point  $A$  negative w.r.t. point  $B$ \*. Hence,  $V_{th} = V_{AB} = -12$  V (or  $V_{BA} = 12$  V).

For finding  $R_{th}$ , 18 V battery has been replaced by a short-circuit and 6 A current source by an open-circuit, as shown in Fig. 2.150 (c).

$$\begin{aligned} \text{As seen,} \quad R_{th} &= 4 + 6 \parallel 3 + 2 \\ &= 4 + 2 + 2 = 8 \Omega \end{aligned}$$

Hence, Thevenin's equivalent circuit for terminals  $A$  and  $B$  is as shown in Fig. 2.151. It should be noted that if a load resistor is connected across  $AB$ , current through it will flow from  $B$  to  $A$ .

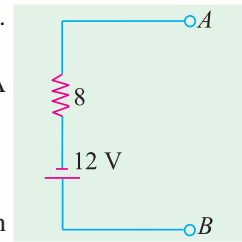


Fig. 2.151

**Example 2.74.** The circuit shown in Fig. 2.152 (a) contains two voltage sources and two current sources. Calculate (a)  $V_{th}$  and (b)  $R_{th}$  between the open terminals  $A$  and  $B$  of the circuit. All resistance values are in ohms.

**Solution.** It should be understood that since terminals  $A$  and  $B$  are open, 2 A current can flow only through 4  $\Omega$  and 10  $\Omega$  resistors, thus producing a drop of 20 V across the 10  $\Omega$  resistor, as shown in Fig. 2.152 (b). Similarly, 3 A current can flow through its own closed circuit between  $A$  and  $C$  thereby producing a drop of 24 V across 8  $\Omega$  resistor as shown in Fig. 2.152 (b). Also, there is no drop across 2  $\Omega$  resistor because no current flows through it.

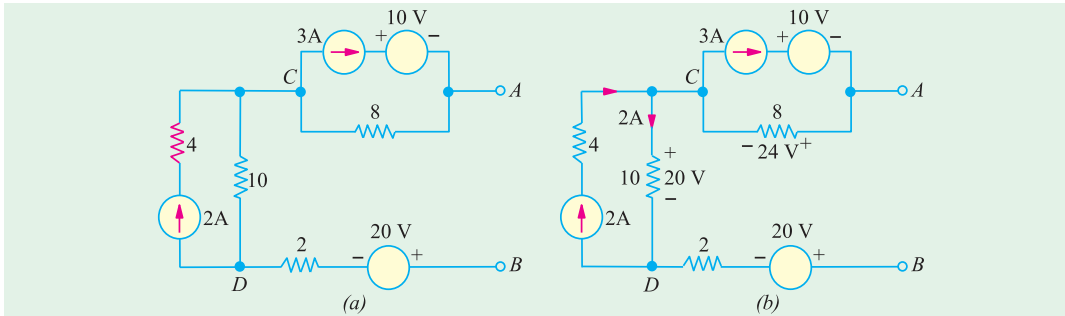


Fig. 2.152

Starting from point  $B$  and going to point  $A$  via points  $D$  and  $C$ , we get

$$V_{th} = -20 + 20 + 24 = 24$$
 V

—with point  $A$  positive.

For finding  $R_{th}$ , we will short-circuit the voltage sources and open-circuit the current sources, as shown in Fig. 2.153. As seen,  $R_{th} = R_{AB} = 8 + 10 + 2 = 20 \Omega$ .

**Example 2.75.** Calculate  $V_{th}$  and  $R_{th}$  between the open terminals  $A$  and  $B$  of the circuit shown in Fig. 2.154 (a). All resistance values are in ohms.

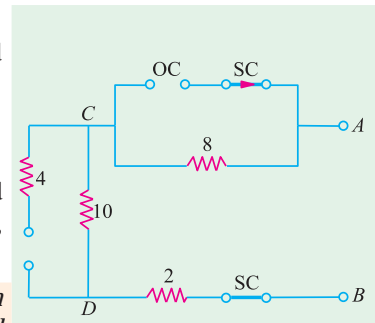


Fig. 2.153

**Solution.** We will convert the 48 V voltage source with its series resistance of 12  $\Omega$  into a current source of 4 A, with a parallel resistance of 12  $\Omega$ , as shown in Fig. 2.154 (b).

In Fig. 2.154 (c), the two parallel resistance of 12  $\Omega$  each have been combined into a single resistance of 6  $\Omega$ . It is obvious that 4 A current flows through the 6  $\Omega$  resistor, thereby producing a drop of  $6 \times 4 = 24$  V. Hence,  $V_{th} = V_{AB} = 24$  V with terminal  $A$  negative. In other words  $V_{th} = -24$  V.

If we open-circuit the 8 A source and short-circuit the 48-V source in Fig. 2.154 (a),  $R_{th} = R_{AB} = 12 \parallel 12 = 6 \Omega$ .

\* Incidentally, had 6 A current been flowing in the opposite direction, polarity of 24 V drop would have been reversed so that  $V_{AB}$  would have equalled  $(24 + 12) = 36$  V with  $A$  positive w.r.t. point  $B$ .

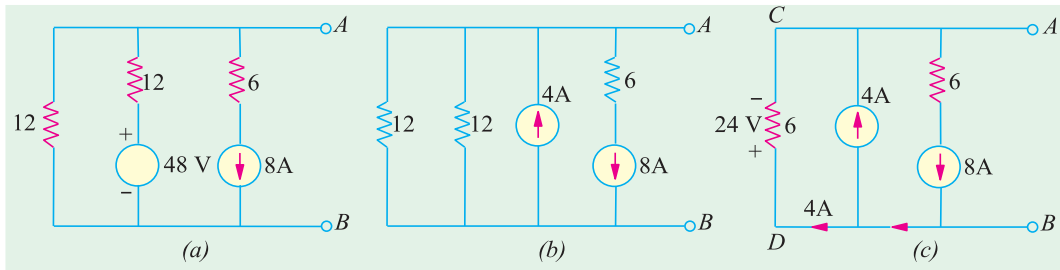


Fig. 2.154

**Example 2.76.** Calculate the value of  $V_{th}$  of  $R_{th}$  between the open terminals  $A$  and  $B$  of the circuit shown in Fig. 2.155 (a). All resistance values are in ohms.

**Solution.** It is seen from Fig. 2.155 (a) that positive end of the 24 V source has been shown connected to point  $A$ . It is understood that the negative terminal is connected to the ground terminal  $G$ . Just to make this point clear, the given circuit has been redrawn in Fig. 2.155 (b) as well as in Fig. 2.155 (c).

Let us start from the positive terminal of the battery and go to its negative terminal  $G$  via point  $C$ . We find that between points  $C$  and  $G$ , there are two parallel paths : one of  $6\ \Omega$  resistance and the

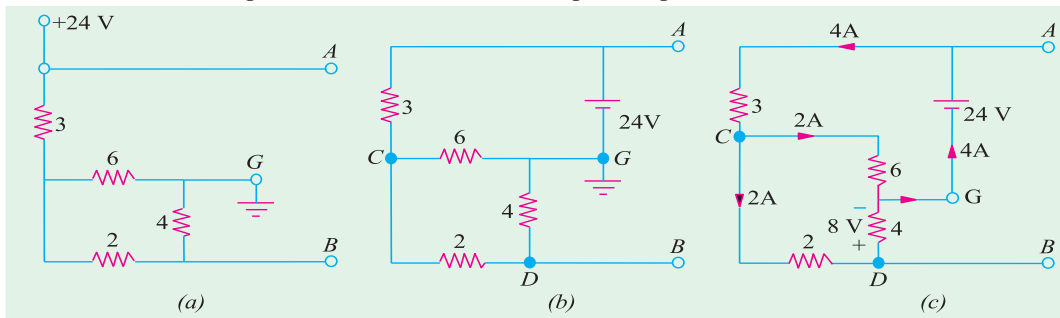


Fig. 2.155

other of  $(2 + 4) = 6\ \Omega$  resistance, giving a combined resistance of  $6 \parallel 6 = 3\ \Omega$ . Hence, total resistance between positive and negative terminals of the battery  $= 3 + 3 = 6\ \Omega$ . Hence, battery current  $= 24/6 = 4\ \text{A}$ . As shown in Fig. 2.155 (c), this current divides equally at point  $C$ . Let us go from  $B$  to  $A$  points  $D$  and  $G$  and total up the potential difference between the two,  $V_{th} = V_{AB} = -8\ \text{V} + 24\ \text{V} = 16\ \text{V}$  with point  $A$  positive.

For finding  $R_{th}$ , let us replace the voltage source by a short-circuit, as shown in Fig. 2.156 (a). It connects one end each of  $6\ \Omega$  resistor and  $4\ \Omega$  resistor directly to point  $A$ , as shown in Fig. 2.156 (b). The resistance of branch  $DCG = 2 + 6 \parallel 3 = 4\ \Omega$ . Hence  $R_{th} = R_{AB} = 4 \parallel 4 = 2\ \Omega$

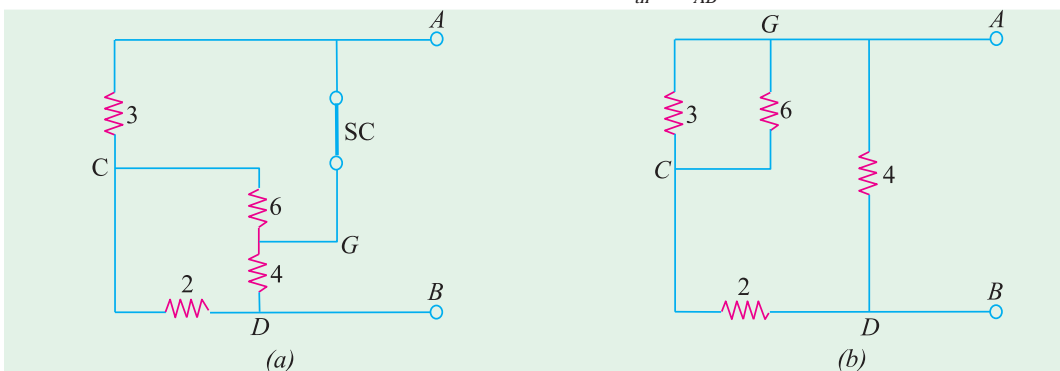


Fig. 2.156

**Example 2.77.** Calculate the power which would be dissipated in the 8-Ω resistor connected across terminals A and B of Fig. 2.157 (a). All resistance values are in ohms.

**Solution.** The open-circuit voltage  $V_{oc}$  (also called Thevenin's voltage  $V_{th}$ ) is that which appears across terminals A and B. This equals the voltage drop across 10 Ω resistor between points C and D. Let us find this voltage. With AB an open-circuit, 120-V battery voltage acts on the two parallel paths EF and ECDF. Hence, current through 10 Ω resistor is

$$I = 120/(20 + 10 + 20) = 2.4 \text{ A}$$

Drop across 10-Ω resistor,  $V_{th} = 10 \times 2.4 = 24 \text{ V}$

Now, let us find Thevenin's resistance  $R_{th}$  i.e. equivalent resistance of the given circuit when looked into from terminals A and B. For this purpose, 120 V battery is removed. The results in shorting the 40-Ω resistance since internal resistance of the battery is zero as shown in Fig. 2.157 (b).

$$\therefore R_i \text{ or } R_{th} = 16 + \frac{10 \times (20 + 20)}{10 + (20 + 20)} + 16 = 40 \text{ } \Omega$$

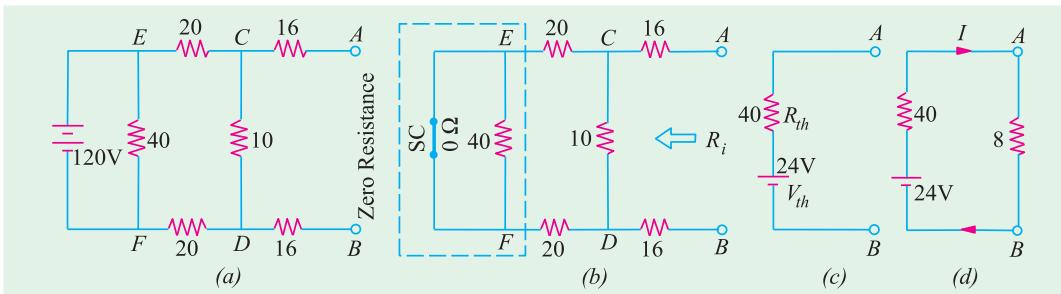


Fig. 2.157

Thevenin's equivalent circuit is shown in Fig. 2.157 (c). As shown in Fig. 2.157 (d), current through 8-Ω resistor is

$$I = 24/(40 + 8) = \frac{1}{2} \text{ A} \quad P = I^2 R = \left(\frac{1}{2}\right)^2 \times 8 = 2 \text{ W}$$

**Example 2.78.** With the help of Thevenin's theorem, calculate the current flowing through the 3-Ω resistor in the network of Fig. 2.158 (a). All resistances are in ohms.

**Solution.** The current source has been converted into an equivalent voltage source in Fig. 158 (b).

(i) **Finding  $V_{oc}$**  As seen from Fig. 2.158 (c),  $V_{oc} = V_{CD}$ . In closed circuit CDFEC, net voltage =  $24 - 8 = 16 \text{ V}$  and total resistance =  $8 + 4 + 4 = 16 \text{ } \Omega$ . Hence, current =  $16/16 = 1 \text{ A}$ .

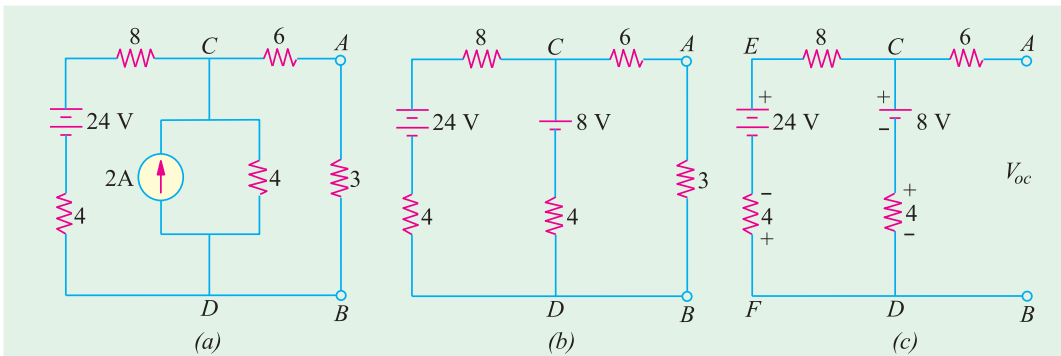


Fig. 2.158

Drop over the 4-Ω resistor in branch CD = 4 × 1 = 4 V with a polarity which is in series addition with 8-V battery.

Hence,  $V_{oc} = V_{th} = V_{CD} = 8 + 4 = 12 \text{ V}$

**(ii) Finding  $R_i$  or  $R_{th}$ .** In Fig. 2.159 (a), the two batteries have been replaced by short-circuits because they do not have any internal resistance.

As seen,  $R_i = 6 + 4 \parallel (8 + 4) = 9 \text{ } \Omega$

The Thevenin's equivalent circuit is as shown in Fig. 2.159 (b).

$I = 12 / (9 + 3) = 1 \text{ A}$

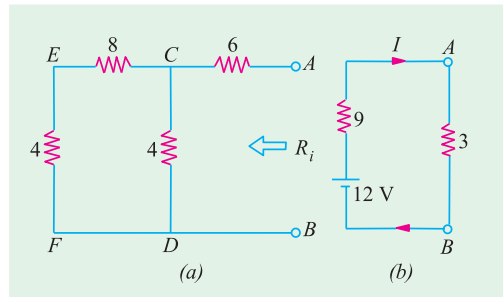


Fig. 2.159

**Example 2.79.** Using Thevenin and Superposition theorems find complete solution for the network shown in Fig. 2.160 (a).

**Solution.** First, we will find  $R_{th}$  across open terminals A and B and then find  $V_{th}$  due to the voltage sources only and then due to current source only and then using Superposition theorem, combine the two voltages to get the single  $V_{th}$ . After that, we will find the Thevenin equivalent.

In Fig. 2.160 (b), the terminals A and E have been open-circuited by removing the 10 V source and the 1 Ω resistance. Similarly, 24 V source has been replaced by a short and current source has been replaced by an infinite resistance i.e. by open-circuit. As seen,  $R_{AB} = R_{th} = 4 \parallel 4 = 2 \text{ } \Omega$

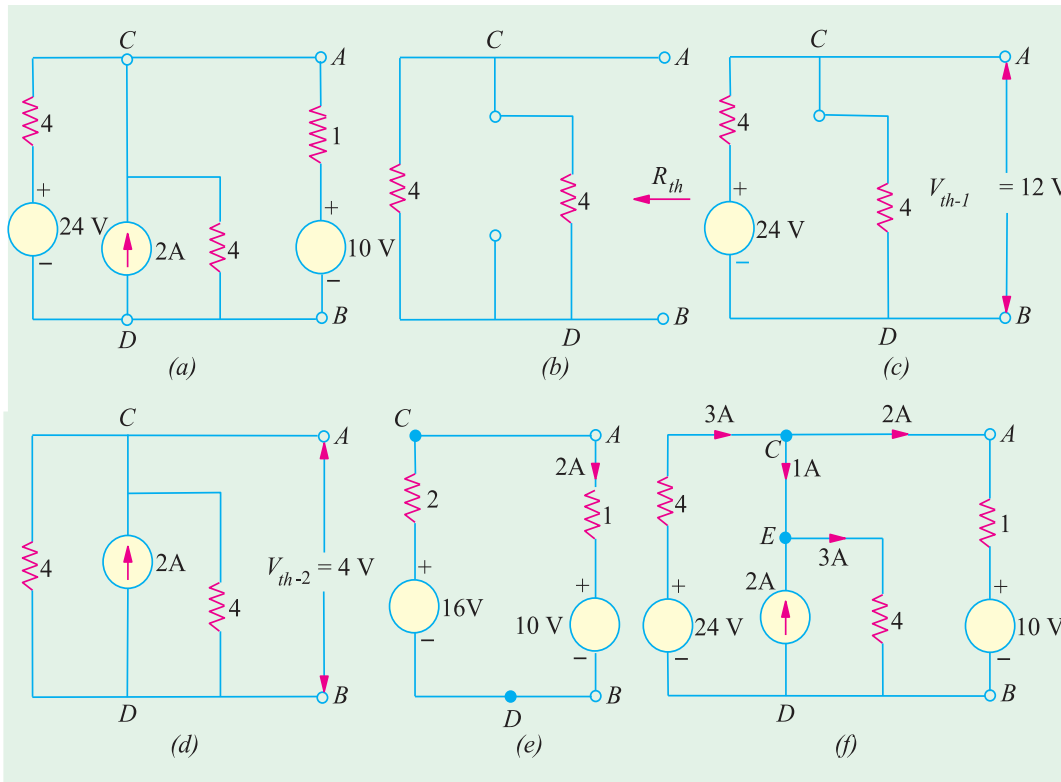


Fig. 2.160

We will now find  $V_{th-1}$  across  $AB$  due to 24 V source only by open-circuiting the current source. Using the voltage-divider rule in Fig. 2.160 (c), we get  $V_{AB} = V_{CD} = V_{th-1} = 24/2 = 12$  V.

Taking only the current source and short-circuiting the 24 V source in Fig. 2.160 (d), we find that there is equal division of current at point  $C$  between the two  $4 \Omega$  parallel resistors. Therefore,  $V_{th-2} = V_{AB} = V_{CD} = 1 \times 4 = 4$  V.

Using Superposition theorem,  $V_{th} = V_{th-1} + V_{th-2} = 12 + 4 = 16$  V. Hence, the Thevenin's equivalent consists of a 16 V source in series with a  $2 \Omega$  resistance as shown in Fig. 2.160 (e) where the branch removed earlier has been connected back across the terminals  $A$  and  $B$ . The net voltage around the circuit is  $16 - 10 = 6$  V and total resistance is  $2 + 1 = 3 \Omega$ . Hence, current in the circuit is  $6/3 = 2$  A. Also,  $V_{AB} = V_{AD} = 16 - (2 \times 2) = 12$  V. Alternatively,  $V_{AB}$  equals  $(2 \times 1) + 10 = 12$  V.

Since we know that  $V_{AB} = V_{CD} = 12$  V, we can find other voltage drops and various circuit currents as shown in Fig. 2.160 (f). Current delivered by the 24-V source to the node  $C$  is  $(24 - V_{CD})/4 = (24 - 12)/4 = 3$  A. Since current flowing through branch  $AB$  is 2 A, the balance of 1 A flows along  $CE$ . As seen, current flowing through the  $4 \Omega$  resistor connected across the current source is  $(1 + 2) = 3$  A.

**Example 2.80.** Use Superposition Theorem to find  $I$  in the circuit of Fig. 2.161.

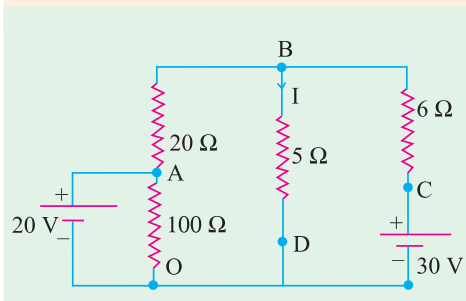


Fig. 2.161. Given Circuit

[Nagpur Univ. Summer 2001]

**Solution.** At a time, one source acts and the other is de-activated, for applying Superposition theorem. If  $I_1$  represents the current in 5-ohm resistor due to 20-V source, and  $I_2$  due to 30-V source,

$$I = I_1 + I_2$$

Due to 20-V source, current into node  $B$

$$= 20/(20 + 5/6) = 0.88 \text{ amp}$$

Out of this,  $I_1 = 0.88 \times 6/11 = 0.48$  amp

Due to 30-V source, current into node  $B$

$$= 30/(6 + 5/20) = 3 \text{ amp}$$

Out of this,  $I_2 = 3 \times 20/25 = 2.4$  amp

Hence,  $I = 2.88$  amp

Alternatively, Thevenin's theorem can be applied at nodes  $BD$  after removing 5-ohms resistor from its position. Following the procedure to evaluate  $V_{TH}$  and  $R_{TH}$ ,

Thevenin-voltage,  $V_{TH} = 27.7$  Volts

and  $R_{TH} = 4.62$  Ohms

Current,  $I = 27.7/(4.62 + 5) = 2.88$  amp

## 2.20. General Instructions for Finding Thevenin Equivalent Circuit

So far, we have considered circuits which consisted of resistors and independent current or voltage sources only. However, we often come across circuits which contain both independent and dependent sources or circuits which contain only dependent sources. Procedure for finding the value of  $V_{th}$  and  $R_{th}$  in such cases is detailed below :

### (a) When Circuit Contains Both Dependent and Independent Sources

(i) The open-circuit voltage  $V_{oc}$  is determined as usual with the sources activated or 'alive'.

(ii) A short-circuit is applied across the terminals  $a$  and  $b$  and the value of short-circuit current  $I_{th}$  is found as usual.

(iii) Thevenin resistance  $R_{th} = v_{oc}/I_{sh}$ . It is the same procedure as adopted for Norton's theorem. Solved examples 2.81 to 2.85 illustrate this procedure.

**(b) When Circuit Contains Dependent Sources Only****(i)** In this case,  $v_{oc} = 0$ **(ii)** We connect 1 A source to the terminals  $a$  and  $b$  and calculate the value of  $v_{ab}$ .**(iii)**  $R_{th} = v_{ab} / 1 \Omega$ 

The above procedure is illustrated by solved examples.

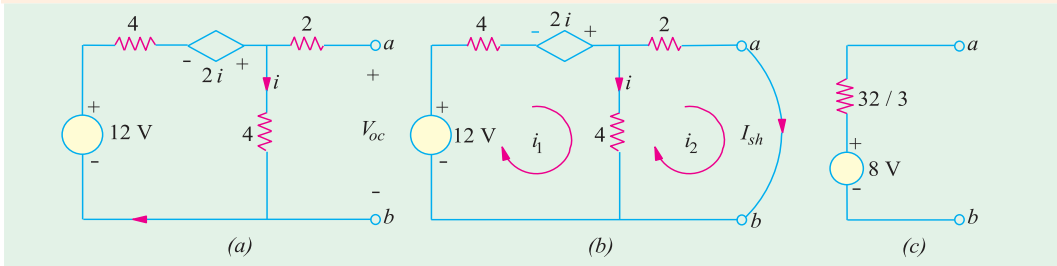
**Example 2.81.** Find Thevenin equivalent circuit for the network shown in Fig. 2.162 (a) which contains a current controlled voltage source (CCVS).

Fig. 2.162

**Solution.** For finding  $V_{oc}$  available across open-circuit terminals  $a$  and  $b$ , we will apply KVL to the closed loop.

$$\therefore 12 - 4i - 2i - 4i = 0 \quad \therefore i = 2 \text{ A}$$

Hence,  $V_{oc} =$  drop across  $4 \Omega$  resistor  $= 4 \times 2 = 8 \text{ V}$ . It is so because there is no current through the  $2 \Omega$  resistor.For finding  $R_{th}$ , we will put a short-circuit across terminals  $a$  and  $b$  and calculate  $I_{sh}$  as shown in Fig. 2.162 (b). Using the two mesh currents, we have
$$12 - 4i_1 + 2i - 4(i_1 - i_2) = 0 \text{ and } -8i_2 - 4(i_2 - i_1) = 0. \text{ Substituting } i = (i_1 - i_2) \text{ and Simplifying the above equations, we have}$$

$$12 - 4i_1 + 2(i_1 - i_2) - 4(i_1 - i_2) = 0 \quad \text{or} \quad 3i_1 - i_2 = 6 \quad \dots(i)$$

Similarly, from the second equation, we get  $i_1 = 3i_2$ . Hence,  $i_2 = 3/4$  and  $R_{th} = V_{oc}/I_{sh} = 8/(3/4) = 32/3 \Omega$ . The Thevenin equivalent circuit is as shown in Fig. 2.162 (c).**Example 2.82.** Find the Thevenin equivalent circuit with respect to terminals  $a$  and  $b$  of the network shown in Fig. 2.163 (a).**Solution.** It will be seen that with terminals  $a$  and  $b$  open, current through the  $8 \Omega$  resistor is  $v_{ab}/4$  and potential of point  $A$  is the same that of point  $a$  (because there is no current through  $4 \Omega$  resistor). Applying KVL to the closed loop of Fig. 2.163 (a), we get

$$6 + (8 \times v_{ab}/4) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = 12 \text{ V}$$

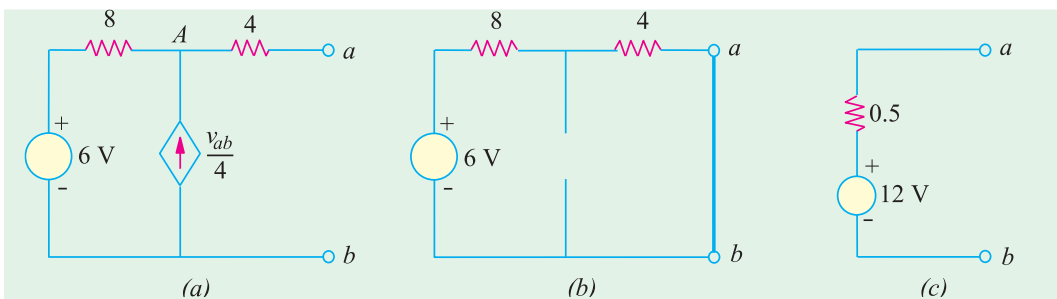


Fig. 2.163

It is also the value of the open-circuit voltage  $v_{oc}$ .

For finding short-circuit current  $i_{sh}$ , we short-circuit the terminals  $a$  and  $b$  as shown in Fig. 2.163 (b). Since with  $a$  and  $b$  short-circuited,  $v_{ab} = 0$ , the dependent current source also becomes zero. Hence, it is replaced by an open-circuit as shown. Going around the closed loop, we get

$$12 - i_{sh}(8 + 4) = 0 \quad \text{or} \quad i_{sh} = 6/12 = 0.5 \text{ A}$$

Hence, the Thevenin equivalent is as shown in Fig. 2.163 (c).

**Example 2.83.** Find the Thevenin equivalent circuit for the network shown in Fig. 2.164 (a) which contains only a dependent source.

**Solution.** Since circuit contains no independent source,  $i = 0$  when terminals  $a$  and  $b$  are open. Hence,  $v_{oc} = 0$ . Moreover,  $i_{sh}$  is zero since  $v_{oc} = 0$ .

Consequently,  $R_{sh}$  cannot be found from the relation  $R_{th} = v_{oc}/i_{sh}$ . Hence, as per Art. 2.20, we will connect a 1 A current source to terminals  $a$  and  $b$  as shown in Fig. 2.164 (b). Then by finding the value of  $v_{ab}$ , we will be able to calculate  $R_{th} = v_{ab}/1$ .

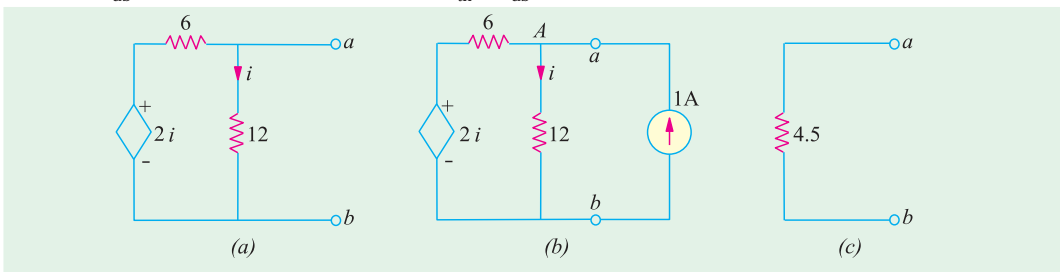


Fig. 2.164

It should be noted that potential of point  $A$  is the same as that of point  $a$  i.e. voltages across  $12 \Omega$  resistor is  $v_{ab}$ . Applying KCL to point  $A$ , we get

$$\frac{2i - v_{ab}}{6} - \frac{v_{ab}}{12} + 1 = 0 \quad \text{or} \quad 4i - 3v_{ab} = -12$$

Since  $i = v_{ab}/12$ , we have  $4(v_{ab}/12) - 3v_{ab} = -12$  or  $v_{ab} = 4.5 \text{ V} \therefore R_{th} = v_{ab}/1 = 4.5/1 = 4.5 \Omega$

The Thevenin equivalent circuit is shown in Fig. 2.164 (c).

**Example 2.84.** Determine the Thevenin equivalent circuit as viewed from the open-circuit terminals  $a$  and  $b$  of the network shown in Fig. 2.165 (a). All resistances are in ohms.

**Solution.** It would be seen from Fig. 2.165(a) that potential of node  $A$  equals the open-circuit terminal voltage  $v_{oc}$ . Also,  $i = (v_s - v_{oc})/(80 + 20) = (6 - v_{oc})/100$ .

Applying KCL to node,  $A$  we get

$$\frac{6 - V_{oc}}{100} + \frac{9 \times (6 - v_{oc})}{100} - \frac{V_{oc}}{10} = 0 \quad \text{or} \quad V_{oc} = 3 \text{ V}$$

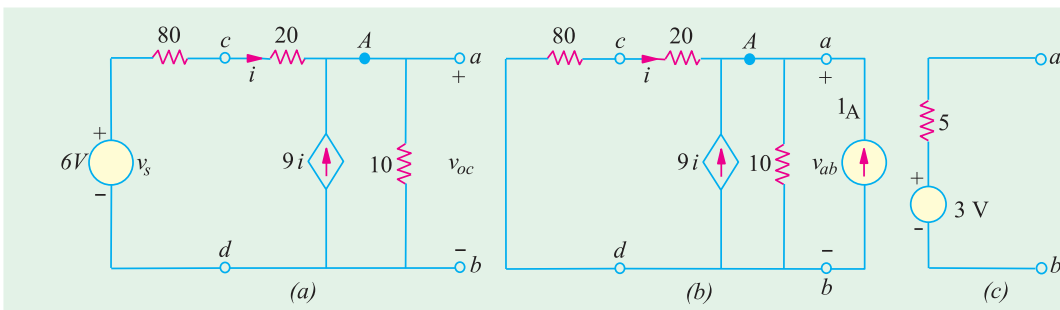


Fig. 2.165

For finding the Thevenin's resistance with respect to terminals  $a$  and  $b$ , we would first 'kill' the independent voltage source as shown in Fig. 2.165 (b). However, the dependent current source cannot be 'killed'. Next, we will connect a current source of 1 A at terminals  $a$  and  $b$  and find the value of  $v_{ab}$ . Then, Thevenin's resistance  $R_{th} = v_{ab}/1$ . It will be seen that current flowing away from node  $A$  i.e. from point  $c$  to  $d$  is  $v_{ab}/100$ . Hence,  $i = -v_{oc}/100$ . Applying KCL to node  $A$ , we get

$$-\frac{v_{ab}}{100} + 9 \left( -\frac{v_{ab}}{100} \right) - \frac{v_{ab}}{10} + 1 = 0 \quad \text{or} \quad v_{ab} = 5 \text{ V}$$

$\therefore R_{th} = 5/1 = 5 \Omega$ . Hence, Thevenin's equivalent source is as shown in Fig. 2.165 (c).

**Example 2.85.** Find the Thevenin's equivalent circuit with respect to terminals  $a$  and  $b$  of the network shown in Fig. 2.166 (a). All resistances are in ohms.

**Solution.** It should be noted that with terminals  $a$  and  $b$  open, potential of node  $A$  equals  $v_{ab}$ . Moreover,  $v = v_{ab}$ . Applying KCL to node  $A$ , we get

$$-5 - \frac{v_{ab}}{15} + \frac{1}{10} \left[ \left( \frac{v_{ab}}{3} + 150 \right) - v_{ab} \right] = 0 \quad \text{or} \quad v_{ab} = 75 \text{ V}$$

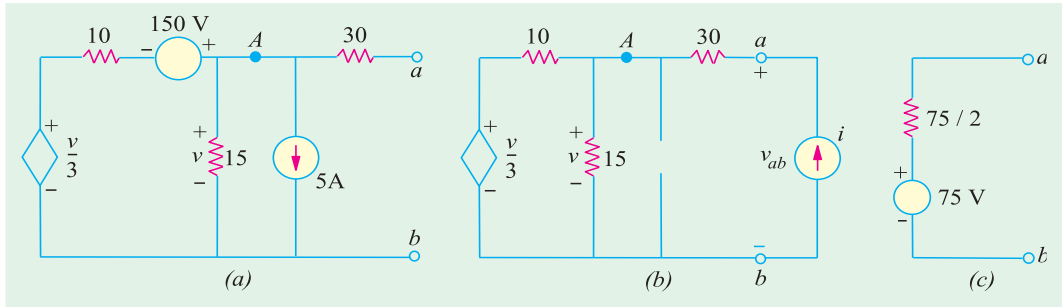


Fig. 2.166

For finding  $R_{th}$ , we will connect a current source of  $iA$  \* across terminals  $a$  and  $b$ . It should be particularly noted that in this case the potential of node  $A$  equals  $(v_{ab} - 30i)$ . Also,  $v = (v_{ab} - 30i) =$  potential of node  $A$ , Applying KCL to node  $A$ , we get from Fig. 2.166 (b).

$$i = \frac{(v_{ab} - 30i)}{15} + \frac{1}{10} \left[ \left( \frac{v_{ab} - 30i}{3} \right) - (v_{ab} - 30i) \right] = 0$$

$\therefore 4v_{ab} = 150i$  or  $v_{ab}/i = 75/2 \Omega$ . Hence,  $R_{th} = v_{ab}/i = 75/2 \Omega$ . The Thevenin's equivalent circuit is shown in Fig. 2.166 (c).

## 2.21. Reciprocity Theorem

It can be stated in the following manner :

**In any linear bilateral network, if a source of e.m.f.  $E$  in any branch produces a current  $I$  in any other branch, then the same e.m.f.  $E$  acting in the second branch would produce the same current  $I$  in the first branch.**

In other words, it simply means that  $E$  and  $I$  are mutually transferrable. The ratio  $E/I$  is known as the **transfer** resistance (or impedance in a.c. systems). Another way of stating the above is that the receiving point and the sending point in a network are interchangeable. It also means that interchange of an **ideal** voltage sources and an **ideal** ammeter in any network will not change the ammeter reading. Same is the case with the interchange of an ideal current source and an **ideal** voltmeter.

\* We could also connect a source of 1 A as done in Ex. 2.83.



**Example 2.86.** In the network of Fig. 2.167 (a), find (a) ammeter current when battery is at A and ammeter at B and (b) when battery is at B and ammeter at point A. Values of various resistances are as shown in diagram. Also, calculate the transfer resistance.

**Solution.** (a) Equivalent resistance between points C and B in Fig. 2.167 (a) is

$$= 12 \times 4/16 = 3 \Omega$$

∴ Total circuit resistance

$$= 2 + 3 + 4 = 9 \Omega$$

∴ Battery current =  $36/9 = 4 \text{ A}$

∴ Ammeter current

$$= 4 \times 12/16 = 3 \text{ A.}$$

(b) Equivalent resistance between points C and D in Fig. 2.167 (b) is

$$= 12 \times 6/18 = 4 \Omega$$

Total circuit resistance =  $4 + 3 + 1 = 8 \Omega$

Battery current =  $36/8 = 4.5 \text{ A}$

∴ Ammeter current =  $4.5 \times 12/18 = 3 \text{ A}$

Hence, ammeter current in both cases is the same.

Transfer resistance =  $36/3 = 12 \Omega$

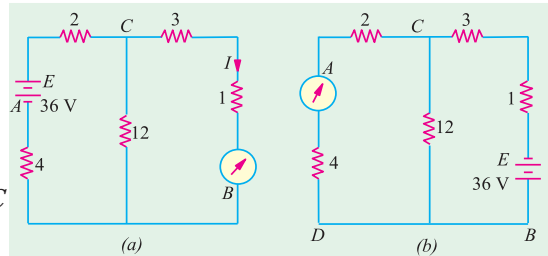


Fig. 2.167

**Example 2.87.** Calculate the currents in the various branches of the network shown in Fig. 2.168 and then utilize the principle of Superposition and Reciprocity theorem together to find the value of the current in the 1-volt battery circuit when an e.m.f. of 2 volts is added in branch BD opposing the flow of original current in that branch.

**Solution.** Let the currents in the various branches be as shown in the figure. Applying Kirchoff's second law, we have

For loop ABDA ;  $-2I_1 - 8I_3 + 6I_2 = 0$  or  $I_1 - 3I_2 + 4I_3 = 0$  ... (i)

For loop BCDB,  $-4(I_1 - I_3) + 5(I_2 + I_3) + 8I_3 = 0$  or  $4I_1 - 5I_2 - 17I_3 = 0$  ... (ii)

For loop ABCEA,  $-2I_1 - 4(I_1 - I_3) - 10(I_1 + I_2) + 1 = 0$  or  $16I_1 + 10I_2 - 4I_3 = 1$  ... (iii)

Solving for  $I_1, I_2$  and  $I_3$ , we get  $I_1 = 0.494 \text{ A}$ ;  $I_2 = 0.0229 \text{ A}$ ;  $I_3 = 0.0049 \text{ A}$

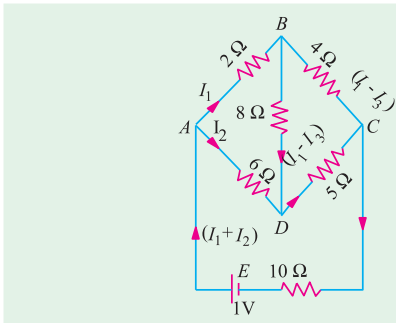


Fig. 2.168

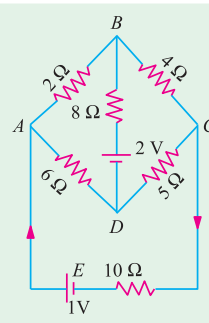


Fig. 2.169

∴ Current in the 1 volt battery circuit is  $I_1 + I_2 = 0.0723 \text{ A}$ .

The new circuit having 2 - V battery connected in the branch BD is shown in Fig. 2.169. According to the Principle of Superposition, the new current in the 1 - volt battery circuit is due to the superposition of two currents; one due to 1 - volt battery and the other due to the 2 - volt battery when each acts independently.

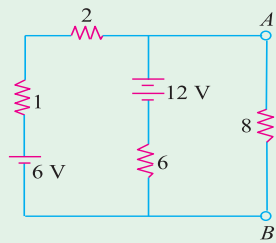
The current in the external circuit due to 1 - volt battery when 2 - volt battery is not there, as found above, is 0.0723 A.

Now, according to Reciprocity theorem; if 1 - volt battery were transferred to the branch *BD* (where it produced a current of 0.0049 A), then it would produce a current of 0.0049 A in the branch *CEA* (where it was before). Hence, a battery of 2 - V would produce a current of  $(-2 \times 0.0049) = -0.0098$  A (by proportion). The negative sign is used because the 2 - volt battery has been so connected as to oppose the current in branch *BD*,

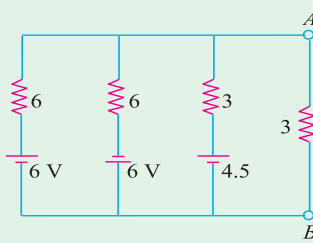
$\therefore$  new current in branch *CEA* = 0.0723 - 0.0098 = **0.0625 A**

**Tutorial Problems No. 2.5**

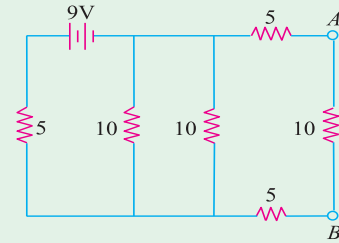
1. Calculate the current in the 8- $\Omega$  resistor of Fig. 2.170 by using Thevenin's theorem. What will be its value of connections of 6-V battery are reversed? **[0.8 A ; 0 A]**
2. Use Thevenin's theorem to calculate the p.d. across terminals *A* and *B* in Fig. 2.171. **[1.5 V]**



**Fig. 2.170**

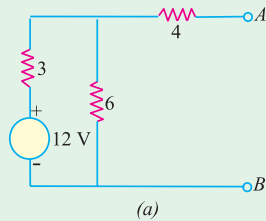


**Fig. 2.171**

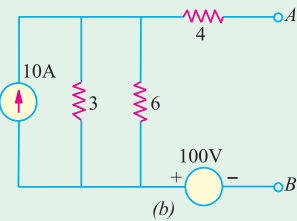


**Fig. 2.172**

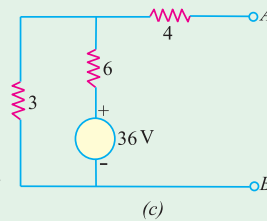
3. Compute the current flowing through the load resistance of 10  $\Omega$  connected across terminals *A* and *B* in Fig. 2.172 by using Thevenin's theorem.
4. Find the equivalent Thevenin voltage and equivalent Thevenin resistance respectively as seen from open-circuited terminals *A* and *B* to the circuits shown in Fig. 2.173. All resistances are in ohms.



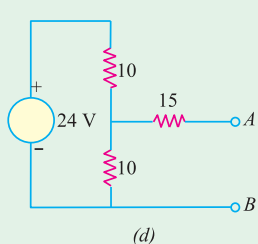
(a)



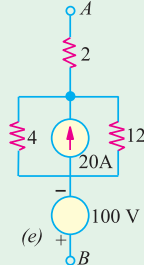
(b)



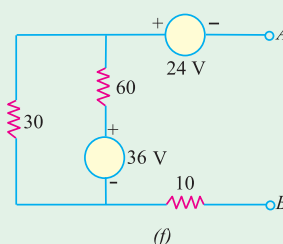
(c)



(d)



(e)



(f)

**Fig. 2.173**

**[(a) 8 V, 6  $\Omega$  ; (b) 120 V, 6  $\Omega$  ; (c) 12 V, 6  $\Omega$  ; (d) 12 V, 20  $\Omega$  ; (e) -40 V, 5  $\Omega$  ; (f) -12 V, 30  $\Omega$ ]**

5. Find Thevenin's equivalent of the circuits shown in Fig. 2.174 between terminals *A* and *B*.

[(a)  $V_{th} = I \frac{R_1 R_2}{R_1 + R_2} + V \frac{R_2}{R_1 + R_2}$ ;  $R_{th} = \frac{R_1 R_2}{R_1 + R_2}$  (b)  $V_{th} = \frac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$ ;  $R_{th} = \frac{R_1 R_2}{R_1 + R_2}$

(c)  $V_{th} = -IR$ ;  $R_{th} = R_1$  (d)  $V_{th} = -V_1 - IR$ ,  $R_{th} = R$  (e) Not possible]

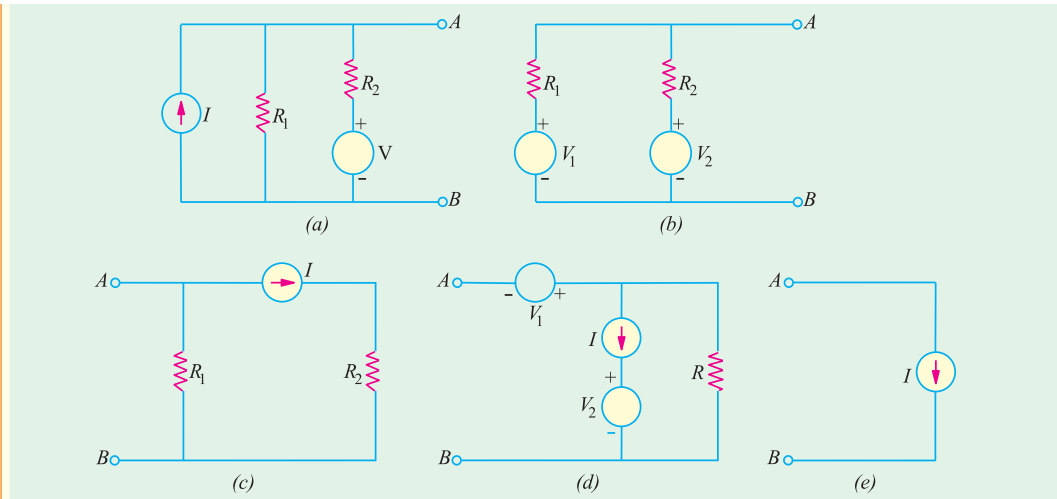


Fig. 2.174

6. The four arms of a Wheatstone bridge have the following resistances in ohms.

$$AB = 100, BC = 10, CD = 5, DA = 60$$

A galvanometer of 15 ohm resistance is connected across  $BD$ . Calculate the current through the galvanometer when a potential difference of 10 V is maintained across  $AC$ .

[Elect. Engg. A.M.Ae. S.I.Dec. 1991] [4.88 mA]

7. Find the Thevenin equivalent circuit for the network shown in Fig. 2.175.

[(a) 4 V; 8 Ω (b) 6 V; 3 Ω (c) 0V; 2/5 Ω]

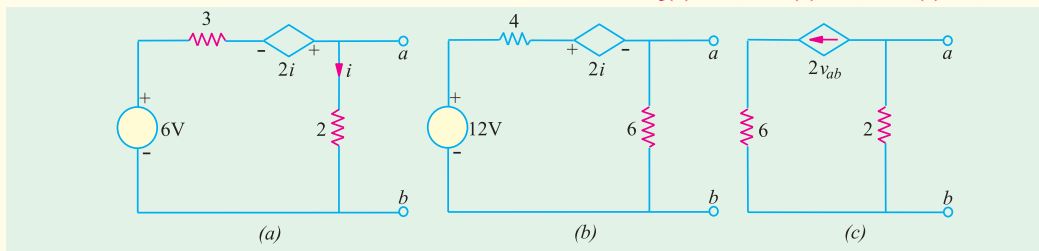


Fig. 2.175

8. Use Thevenin's theorem to find current in the branch  $AB$  of the network shown in Fig. 2.176.

[1.84 A]

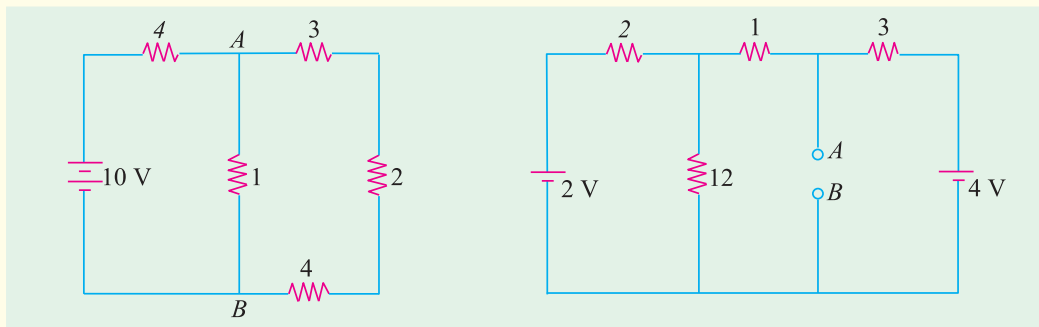


Fig. 2.176

Fig. 2.177

9. In the network shown in Fig. 2.177 find the current that would flow if a  $2\text{-}\Omega$  resistor was connected between points  $A$  and  $B$  by using.

(a) Thevenin's theorem and (b) Superposition theorem. The two batteries have negligible resistance.

[0.82 A]

10. State and explain Thevenin's theorem. By applying Thevenin's theorem or otherwise, find the current through the resistance  $R$  and the voltage across it when connected as shown in Fig. 2.178.

[60.49 A, 600.49 V] (*Elect. and Mech. Technology, Osmania Univ.*)

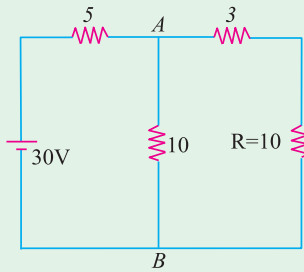


Fig. 2.178

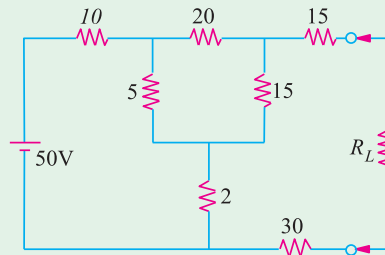


Fig. 2.179

11. State and explain Thevenin's theorem.

For the circuit shown in Fig. 2.179, determine the current through  $R_L$  when its value is  $50\ \Omega$ . Find the value of  $R_L$  for which the power drawn from the source is maximum.

(*Elect. Technology I, Gwalior Univ.*)

12. Find the Thevenin's equivalent circuit for terminal pair  $AB$  for the network shown in Fig. 2.180.

[ $V_{th} = -16\text{ V}$  and  $R_{th} = 16\ \Omega$ ]

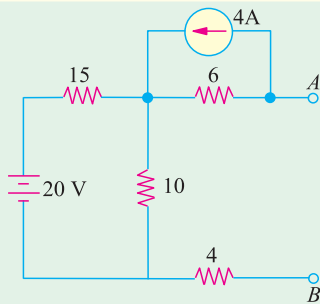


Fig. 2.180

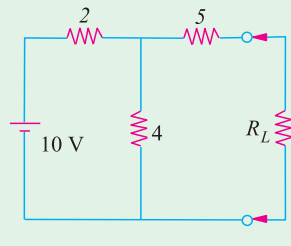


Fig. 2.181

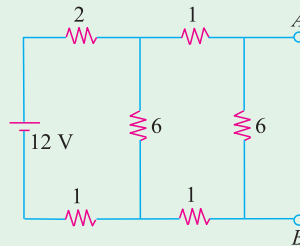


Fig. 2.182

13. For the circuit shown in Fig. 2.181, determine current through  $R_L$  when it takes values of  $5$  and  $10\ \Omega$ .

[0.588 A, 0.408 A] (*Network Theorem and Fields, Madras Univ.*)

14. Determine Thevenin's equivalent circuit which may be used to represent the network of Fig. 2.182 at the terminals  $AB$ .

[ $V_{th} = 4.8\text{ V}$ ,  $R_{th} = 2.4\ \Omega$ ]

15. For the circuit shown in Fig. 2.183 find Thevenin's equivalent circuit for terminal pair  $AB$ .

[6 V, 6  $\Omega$ ]

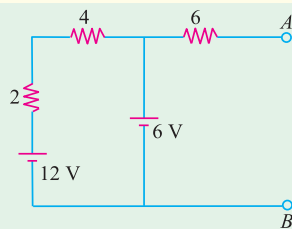


Fig. 2.183

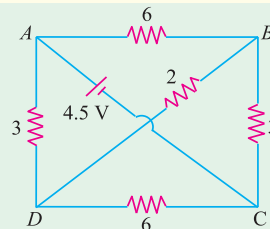


Fig. 2.184

16.  $ABCD$  is a rectangle whose opposite side  $AB$  and  $DC$  represent resistances of  $6\ \Omega$  each, while  $AD$  and  $BC$  represent  $3\ \Omega$  each. A battery of e.m.f.  $4.5\ \text{V}$  and negligible resistances is connected between diagonal points  $A$  and  $C$  and a  $2\text{-}\Omega$  resistance between  $B$  and  $D$ . Find the magnitude and direction of the current in the  $2\text{-}\Omega$  resistor by using Thevenin's theorem. The positive terminal is connected to  $A$ . (Fig. 2.184) [0.25 A from  $D$  to  $B$ ] (*Basic Electricity Bombay Univ.*)

## 2.22. Delta/Star\* Transformation

In solving networks (having considerable number of branches) by the application of Kirchhoff's Laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated network can be simplified by successively replacing delta meshes by equivalent star system and *vice versa*.

Suppose we are given three resistances  $R_{12}$ ,  $R_{23}$  and  $R_{31}$  connected in delta fashion between terminals 1, 2 and 3 as in Fig. 2.185 (a). So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances  $R_1$ ,  $R_2$  and  $R_3$  connected in star as shown in Fig. 2.185 (b).

These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements. Let us find this condition.

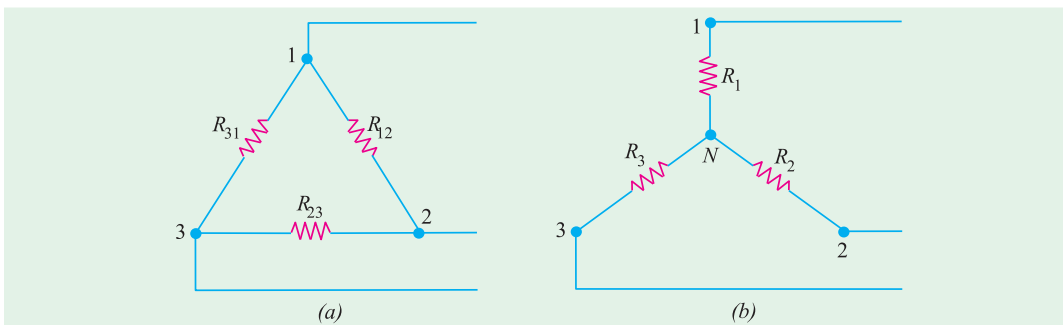


Fig. 2.185

First, take delta connection : Between terminals 1 and 2, there are two parallel paths; one having a resistance of  $R_{12}$  and the other having a resistance of  $(R_{12} + R_{31})$ .

$$\therefore \text{Resistance between terminals 1 and 2 is} = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$$

Now, take star connection : The resistance between the same terminals 1 and 2 is  $(R_1 + R_2)$ .

As terminal resistances have to be the same

$$\therefore R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \dots (i)$$

Similarly, for terminals 2 and 3 and terminals 3 and 1, we get

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \dots (ii)$$

$$\text{and} \quad R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad \dots (iii)$$

Now, subtracting (ii) from (i) and adding the result to (iii), we get

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}; R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \text{ and } R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

\* In Electronics, star and delta circuits are generally referred to as  $T$  and  $\pi$  circuits respectively.

### How to Remember ?

It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star. Hence, it should be remembered that : **resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.**

### 2.23. Star/Delta Transformation

This transformation can be easily done by using equations (i), (ii) and (iii) given above. Multiplying (i) and (ii), (ii) and (iii), (iii) and (i) and adding them together and then simplifying them, we get

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_1 + R_3 + \frac{R_3 R_1}{R_2}$$

### How to Remember ?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

**Example 2.88.** Find the input resistance of the circuit between the points A and B of Fig 2.186 (a).  
(AMIE Sec. B Network Analysis Summer 1992)

**Solution.** For finding  $R_{AB}$ , we will convert the delta CDE of Fig. 2.186 (a) into its equivalent star as shown in Fig. 2.186 (b).

$$R_{CS} = 8 \times 4/18 = 16/9 \Omega; \quad R_{ES} = 8 \times 6/18 = 24/9 \Omega; \quad R_{DS} = 6 \times 4/18 = 12/9 \Omega$$

The two parallel resistances between S and B can be reduced to a single resistance of  $35/9 \Omega$

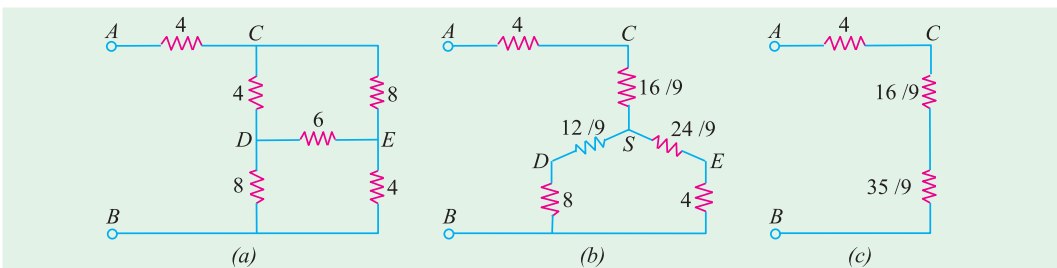


Fig 2.186

As seen from Fig. 2.186 (c),  $R_{AB} = 4 + (16/9) + (35/9) = 87/9 \Omega$

**Example 2.89.** Calculate the equivalent resistance between the terminals A and B in the network shown in Fig. 2.187 (a).  
(F.Y. Engg. Pune Univ.)

**Solution.** The given circuit can be redrawn as shown in Fig. 2.187 (b). When the delta BCD is converted to its equivalent star, the circuit becomes as shown in Fig. 2.187 (c).

Each arm of the delta has a resistance of  $10 \Omega$ . Hence, each arm of the equivalent star has a resistance  $= 10 \times 10/30 = 10/3 \Omega$ . As seen, there are two parallel paths between points A and N, each having a resistance of  $(10 + 10/3) = 40/3 \Omega$ . Their combined resistance is  $20/3 \Omega$ . Hence,  $R_{AB} = (20/3) + 10/3 = 10 \Omega$

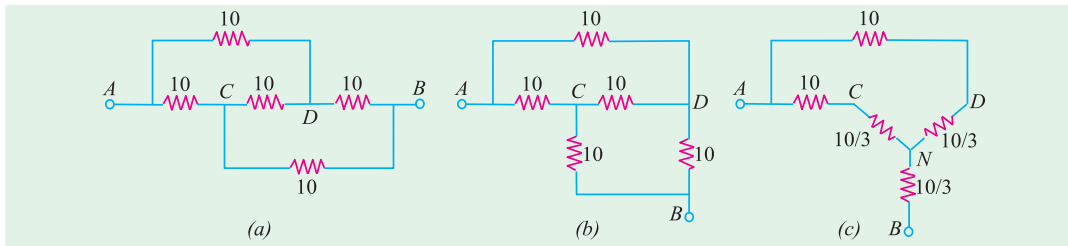


Fig. 2.187

**Example 2.90.** Calculate the current flowing through the 10 Ω resistor of Fig. 2.188 (a) by using any method. (Network Theory, Nagpur Univ. 1993)

**Solution.** It will be seen that there are two deltas in the circuit i.e. ABC and DEF. They have been converted into their equivalent stars as shown in Fig. 2.188 (b). Each arm of the delta ABC has a resistance of 12 Ω and each arm of the equivalent star has a resistance of 4 Ω. Similarly, each arm of the delta DEF has a resistance of 30 Ω and the equivalent star has a resistance of 10 Ω per arm.

The total circuit resistance between A and F =  $4 + 48 \parallel 24 + 10 = 30 \Omega$ . Hence  $I = 180/30 = 6 \text{ A}$ .  
 Current through 10 Ω resistor as given by current-divider rule =  $6 \times 48/(48 + 24) = 4 \text{ A}$ .

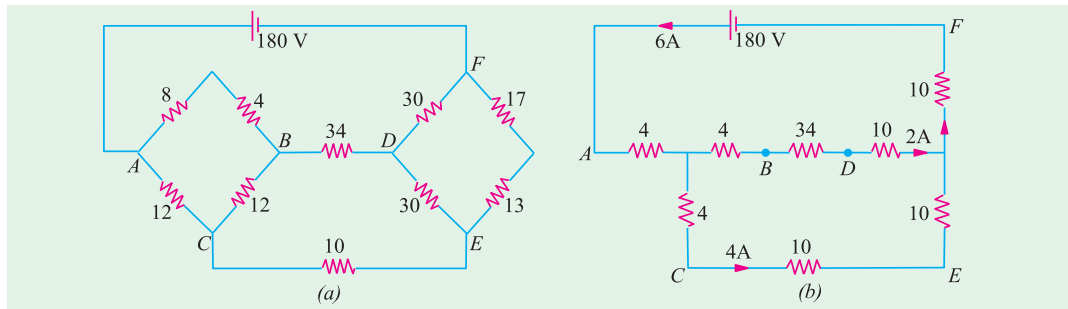


Fig. 2.188

**Example 2.91.** A bridge network ABCD has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by star/delta transformation, the network resistance as viewed from the battery terminals. (Basic Electricity, Bombay Univ.)

(Basic Electricity, Bombay Univ.)

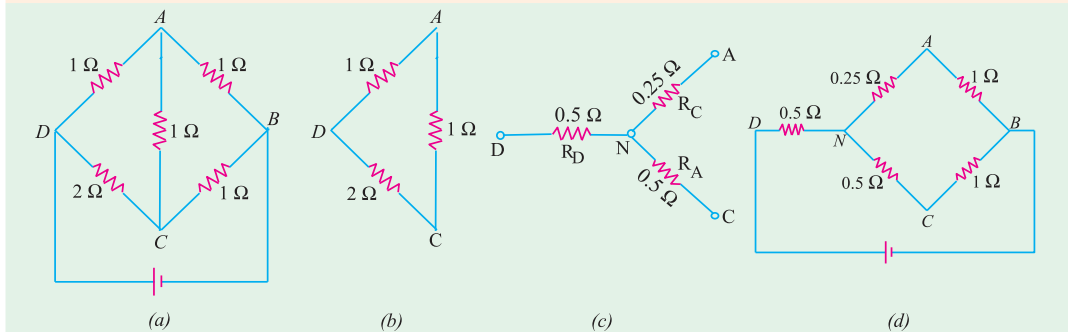


Fig. 2.189

**Solution.** As shown in Fig. 2.189 (b), delta DAC has been reduced to its equivalent star.

$$R_D = \frac{2 \times 1}{2 + 1 + 1} = 0.5 \Omega, \quad R_A = \frac{1}{4} = 0.25 \Omega, \quad R_C = \frac{2}{4} = 0.5 \Omega$$

Hence, the original network of Fig. 2.189 (a) is reduced to the one shown in Fig. 2.189 (d). As seen, there are two parallel paths between points  $N$  and  $B$ , one of resistance  $1.25 \Omega$  and the other of resistance  $1.5 \Omega$ . Their combined resistance is

$$= \frac{1.25 \times 1.5}{1.25 + 1.5} = \frac{15}{22} \Omega$$

Total resistance of the network between points  $D$  and  $B$  is

$$= 0.5 \frac{15}{22} = \frac{13}{11} \Omega$$

**Example 2.92.** A network of resistances is formed as follows as in Fig. 2.190 (a)

$AB = 9 \Omega$ ;  $BC = 1 \Omega$ ;  $CA = 1.5 \Omega$  forming a delta and  $AD = 6 \Omega$ ;  $BD = 4 \Omega$  and  $CD = 3 \Omega$  forming a star. Compute the network resistance measured between (i)  $A$  and  $B$  (ii)  $B$  and  $C$  and (iii)  $C$  and  $A$ . (Basic Electricity, Bombay Univ. 1980)

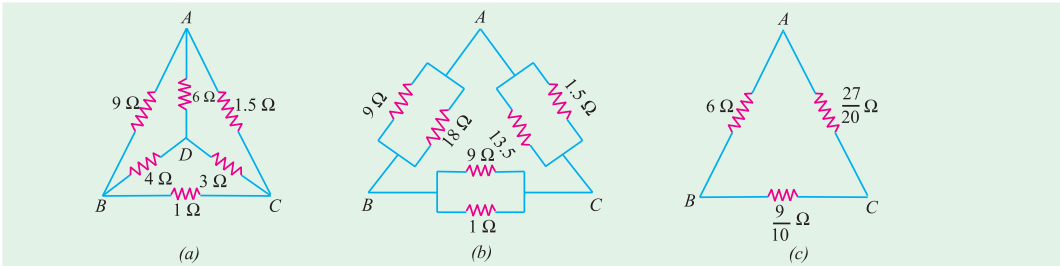


Fig. 2.190

**Solution.** The star of Fig. 2.190 (a) may be converted into the equivalent delta and combined in parallel with the given delta  $ABC$ . Using the rule given in Art. 2.22, the three equivalent delta resistance of the given star become as shown in Fig. 2.190 (b).

When combined together, the final circuit is as shown in Fig. 2.190 (c).

(i) As seen, there are two parallel paths across points  $A$  and  $B$ .

(a) one directly from  $A$  to  $B$  having a resistance of  $6 \Omega$  and

(b) the other via  $C$  having a total resistance

$$= \frac{27}{20} \frac{9}{10} = 2.25 \quad R_{AB} = \frac{6 \times 2.25}{(6 + 2.25)} = \frac{18}{11} \Omega$$

$$(ii) \quad R_{BC} = \frac{\frac{9}{10} \times 6}{\frac{9}{10} + 6} = \frac{27}{20} \frac{6}{\frac{69}{10}} = \frac{441}{550} \Omega \quad (iii) \quad R_{CA} = \frac{\frac{27}{20} \times 6}{\frac{27}{20} + 6} = \frac{621}{550} \Omega$$

**Example 2.93.** State Norton's theorem and find current using Norton's theorem through a load of  $8 \Omega$  in the circuit shown in Fig. 2.191 (a). (Circuit and Field Theory, A.M.I.E. Sec. B, 1993)

**Solution.** In Fig. 2.191 (b), load impedance has replaced by a short-circuit.

$$I_{SC} = I_N = 200/2 = 100 \text{ A.}$$

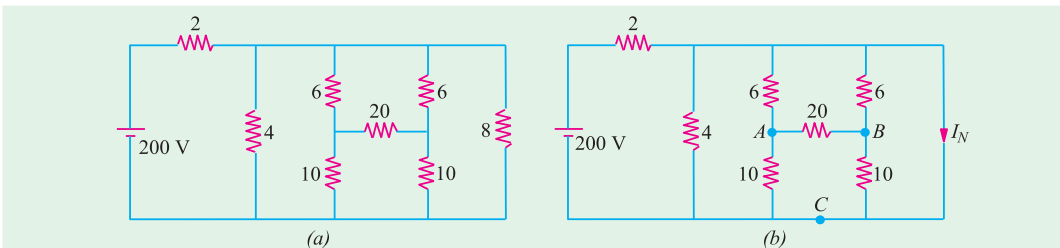


Fig. 2.191



Norton's resistance  $R_N$  can be found by looking into the open terminals of Fig. 2.191 (a). For this purpose  $\Delta ABC$  has been replaced by its equivalent Star. As seen,  $R_N$  is equal to  $8/7 \Omega$

Hence, Norton's equivalent circuit consists of a 100 A source having a parallel resistance of  $8/7 \Omega$  as shown in Fig. 2.192 (c). The load current  $I_L$  can be found by using the Current Divider rule.

$$I_L = 100 \times \frac{(8/7)}{8 + (8/7)} = 12.5 \text{ A}$$

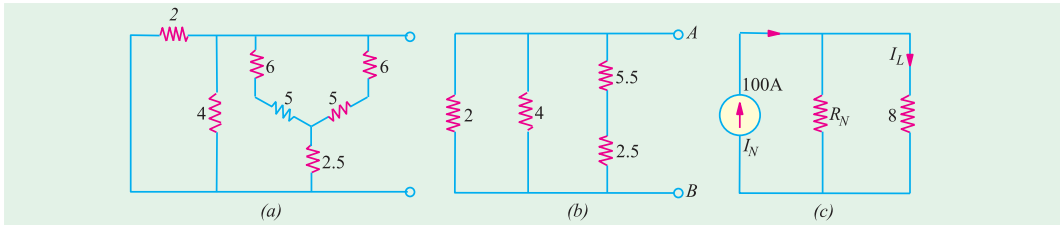


Fig. 2.192

**Example 2.94.** Use delta-star conversion to find resistance between terminals 'AB' of the circuit shown in Fig. 2.193 (a). All resistances are in ohms. [Nagpur University April 1999]

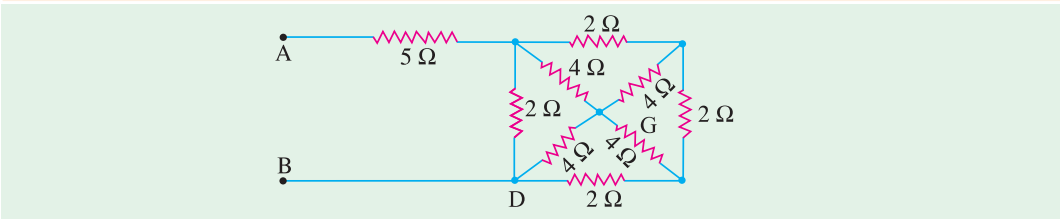


Fig. 2.193 (a)

**Solution.** First apply delta-star conversion to CGD and EDF, so as to redraw the part of the circuit with new configuration, as in Fig. 2.193 (b).

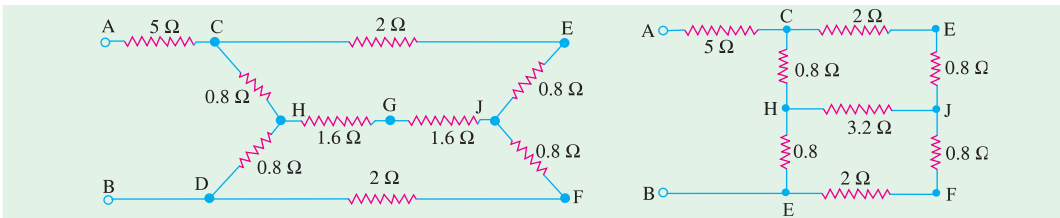


Fig. 2.193 (b)

Fig. 2.193 (c)

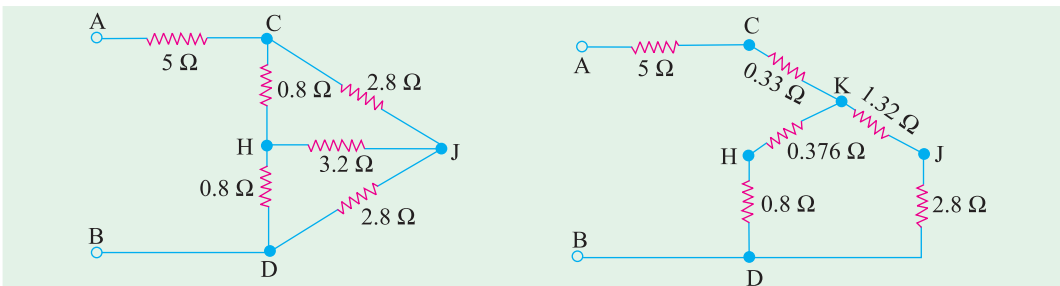


Fig. 2.193 (d)

Fig. 2.193 (e)

Simplify to reduce the circuit to its equivalents as in Fig. 2.193 (c) and later as in Fig. 2.193 (d). Convert CHJ to its equivalent star as in Fig. 2.193 (e). With the help of series-parallel combinations, calculate  $R_{AD}$  as

$$R_{AB} = 5.33 + (1.176 \times 4.12/5.296) = 6.245 \text{ ohms}$$

**Note :** Alternatively, after simplification as in Fig. (d). “CDJ – H” star-configuration can be transformed into delta. Node H then will not exist. The circuit has the parameters as shown in Fig. 2.193 (f). Now the resistance between C and J (and also between D and J) is a parallel combination of 7.2 and 2.8 ohms, which 2.016 ohms. Along CJD, the resistance between terminals AB then obtained as :

$$\begin{aligned} R_{AB} &= 5.0 + (1.8 \times 4.032/5.832) \\ &= 5.0 + 1.244 = 6.244 \text{ ohms} \end{aligned}$$

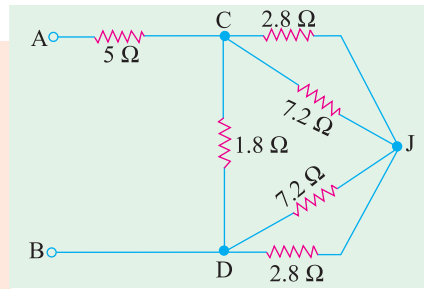


Fig. 2.193 (f)

**Example 2.94 (a).** Find the resistance at the A-B terminals in the electric circuit of Fig. 2.193 (g) using  $\Delta$ -Y transformation. [U.P. Technical University, 2001]

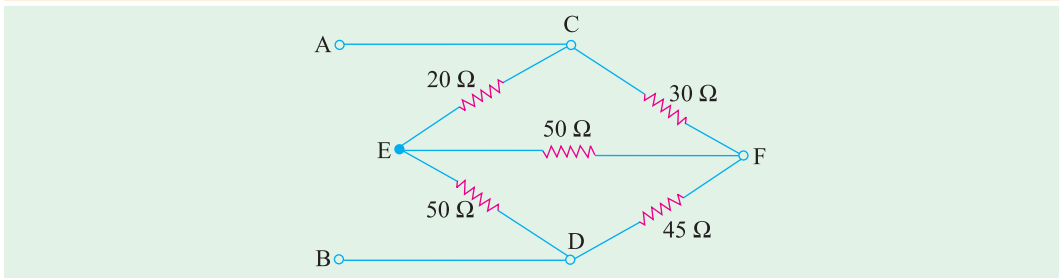


Fig. 2.193 (g)

**Solution.** Convert delta to star for nodes C, E, F. New node N is created. Using the formulae for this conversion, the resistances are evaluated as marked in Fig. 2.193 (h). After handling series parallel combinations for further simplifications.

$$R_{AB} = 36 \text{ ohms.}$$

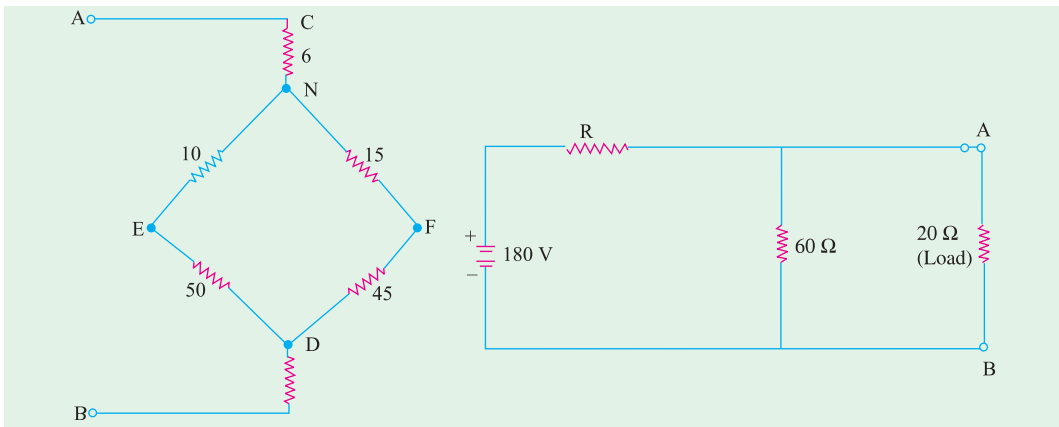


Fig. 2.193 (h)

Fig. 2.193 (i)

**Example 2.94 (b).** Consider the electric circuit shown in Fig. 2.193 (i)

**Determine :** (i) the value of R so that load of 20 ohm should draw the maximum power; (ii) the value of the maximum power drawn by the load. [U.P. Technical University, 2001]

**Solution.** Maximum power transfer takes place when load resistance = Thevenin's Resistance = 20 ohms, here

$$\begin{aligned} R/60 &= 20 \text{ ohms, giving } R = 30 \text{ ohms} \\ V_{TH} &= 180 \times (60/90) = 120 \text{ volts} \\ \text{Current through load} &= 120/40 = 3 \text{ amps} \\ \text{Maximum Power Load} &= 180 \text{ watts} \end{aligned}$$

Tutorial Problems No. 2.6

Delta/Star Conversion

- Find the current in the  $17\ \Omega$  resistor in the network shown in Fig. 2.194 (a) by using (a) star/delta conversion and (b) Thevenin's theorem. The numbers indicate the resistance of each member in ohms. [10/3A]
- Convert the star circuit of Fig. 2.194 (b) into its equivalent delta circuit. Values shown are in ohms. Derive the formula used. (Elect. Technology, Indor Univ.)

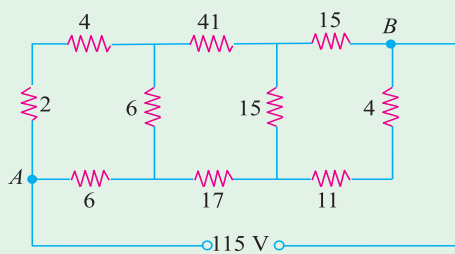


Fig. 2.194 (a)

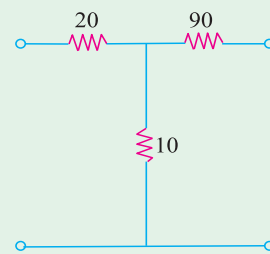


Fig. 2.194 (b)

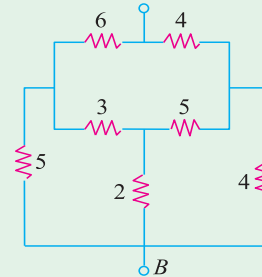


Fig. 2.195

- Determine the resistance between points  $A$  and  $B$  in the network of Fig. 2.195. [4.23  $\Omega$ ] (Elect. Technology, Indor Univ.)
- Three resistances of  $20\ \Omega$  each are connected in star. Find the equivalent delta resistance. If the source of e.m.f. of  $120\ \text{V}$  is connected across any two terminals of the equivalent delta-connected resistances, find the current supplied by the source. [60  $\Omega$ , 3A] (Elect. Engg. Calcutta Univ.)

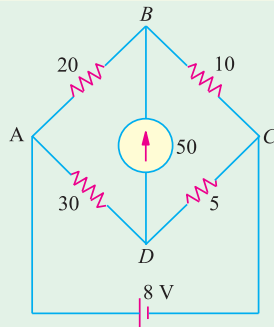


Fig. 2.196

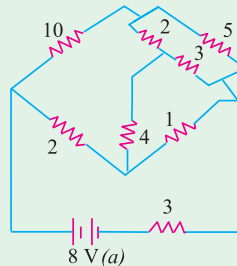
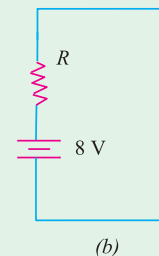


Fig. 2.197



- Using delta/star transformation determine the current through the galvanometer in the Wheatstone bridge of Fig. 2.196. [0.025 A]
- With the aid of the delta star transformation reduce the network given in Fig. 2.197 (a) to the equivalent circuit shown at (b) [R = 5.38  $\Omega$ ]
- Find the equivalent resistance between points  $A$  and  $B$  of the circuit shown in Fig. 2.198. [1.4 R]
- By first using a delta-star transformation on the mesh  $ABCD$  of the circuit shown in Fig. 2.199, prove that the current supplied by the battery is  $90/83\ \text{A}$ .

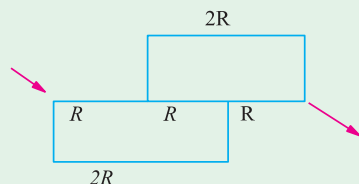


Fig. 2.198

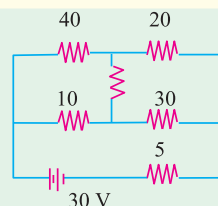


Fig. 2.199

## 2.24. Compensation Theorem

This theorem is particularly useful for the following two purposes :

(a) For analysing those networks where the values of the branch elements are varied and for studying the effect of tolerance on such values.

(b) For calculating the sensitivity of bridge network.

As applied to d.c. circuits, it may be stated in the following for ways :

(i) In its simplest form, this theorem asserts that **any resistance  $R$  in a branch of a network in which a current  $I$  is flowing can be replaced, for the purposes of calculations, by a voltage equal to  $- IR$ .**

OR

(ii) **If the resistance of any branch of network is changed from  $R$  to  $(R + \Delta R)$  where the current flowing originally is  $I$ , the change of current at any other place in the network may be calculated by assuming that an e.m.f.  $- I \cdot \Delta R$  has been injected into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.**

**Example 2.95.** Calculate the values of new currents in the network illustrated in Fig. 2.200 when the resistor  $R_3$  is increased (in place of  $s$ ) by 30 %.

**Solution.** In the given circuit, the values of various branch currents are

$$I_1 = 75/(5 + 10) = 5 \text{ A}$$

$$I_2 = I_3 = 2.5 \text{ A}$$

Now, value of

$$R_3 = 20 + (0.3 \times 20) = 26 \Omega$$

$$\therefore \Delta R = 6 \Omega$$

$$V = -I_3 \Delta R$$

$$= -2.5 \times 6 = -15 \text{ V}$$

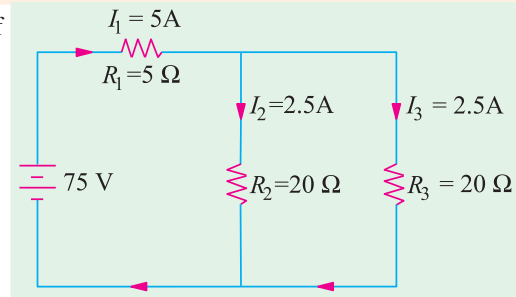


Fig. 2.200

The compensating currents produced by this voltage are as shown in Fig. 2.201 (a).

When these currents are added to the original currents in their respective branches the new current distribution becomes as shown in Fig. 2.201 (b)

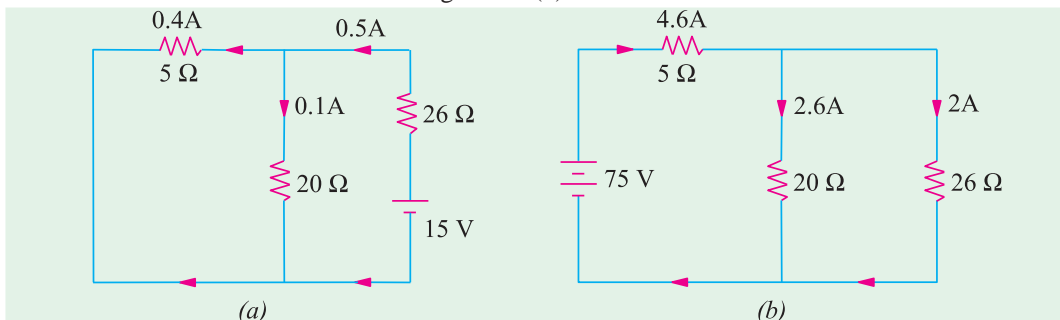


Fig. 2.201

## 2.25. Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. In fact, it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

This theorem may be stated as follows :

(i) **Any two-terminal active network containing voltage sources and resistance when viewed from its output terminals, is equivalent to a constant-current source and a parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.**

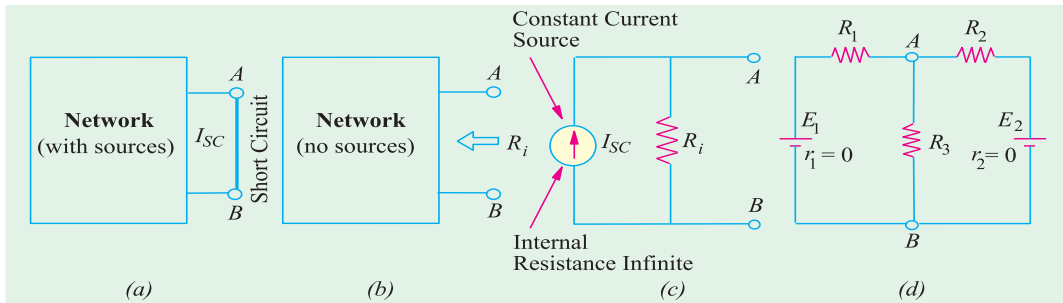


Fig. 2.202

### Explanation

As seen from Fig. 2.202 (a), a short is placed across the terminals  $A$  and  $B$  of the network **with all its energy sources present**. The short-circuit current  $I_{SC}$  gives the value of constant-current source.

For finding  $R_p$ , all sources have been removed as shown in Fig. 2.202 (b). The resistance of the network when looked into from terminals  $A$  and  $B$  gives  $R_p$ .

The Norton's\* equivalent circuit is shown in Fig. 2.202 (c). It consists of an ideal constant-current source of infinite internal resistance (Art. 2.16) having a resistance of  $R_i$  connected in parallel with it. Solved Examples 2.96, 2.97 and 2.98 etc. illustrate this procedure.

(ii) Another useful generalized form of this theorem is as follows :

**The voltage between any two points in a network is equal to  $I_{SC} \cdot R_i$  where  $I_{SC}$  is the short-circuit current between the two points and  $R_i$  is the resistance of the network as viewed from these points with all voltage sources being replaced by their internal resistances (if any) and current sources replaced by open-circuits.**

Suppose, it is required to find the voltage across resistance  $R_3$  and hence current through it [Fig. 2.202 (d)]. If short-circuit is placed between  $A$  and  $B$ , then current in it due to battery of e.m.f.  $E_1$  is  $E_1/R_1$  and due to the other battery is  $E_2/R_2$ .

$$\therefore I_{SC} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1 G_1 + E_2 G_2$$

where  $G_1$  and  $G_2$  are branch conductances.

Now, the internal resistance of the network as viewed from  $A$  and  $B$  simply consists of three resistances  $R_1$ ,  $R_2$  and  $R_3$  connected in parallel between  $A$  and  $B$ . Please note that here load resistance  **$R_3$  has not been removed**. In the first method given above, it has to be removed.

$$\begin{aligned} \therefore \frac{1}{R_i} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3 \\ \therefore R_i &= \frac{1}{G_1 + G_2 + G_3} \end{aligned} \quad \therefore V_{AB} = I_{SC} \cdot R_i = \frac{E_1 G_1 + E_2 G_2}{G_1 + G_2 + G_3}$$

Current through  $R_3$  is  $I_3 = V_{AB}/R_3$ .

Solved example No. 2.96 illustrates this approach.

\* After E.L. Norton, formerly an engineer at Bell Telephone Laboratory, U.S.A.

## 2.26. How To Nortonize a Given Circuit ?

This procedure is based on the first statement of the theorem given above.

1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.
2. Compute the short-circuit current  $I_{SC}$ .
3. Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits *i.e.* by infinite resistance.
4. Next, find the resistance  $R_1$  (also called  $R_N$ ) of the network as looked into from the given terminals. It is exactly the same as  $R_{th}$  (Art. 2.16).
5. The current source ( $I_{SC}$ ) joined in parallel across  $R_1$  between the two terminals gives Norton's equivalent circuit.

As an example of the above procedure, please refer to Solved Example No. 2.87, 88, 90 and 91 given below.

**Example 2.96.** Determine the Thevenin and Norton equivalent circuits between terminals A and B for the voltage divider circuit of Fig. 2.203 (a).

**Solution. (a) Thevenin Equivalent Circuit**

$$\text{Obviously, } V_{th} = \text{drop across } R_2 = E \frac{R_2}{R_1 + R_2}$$

When battery is replaced by a short-circuit.

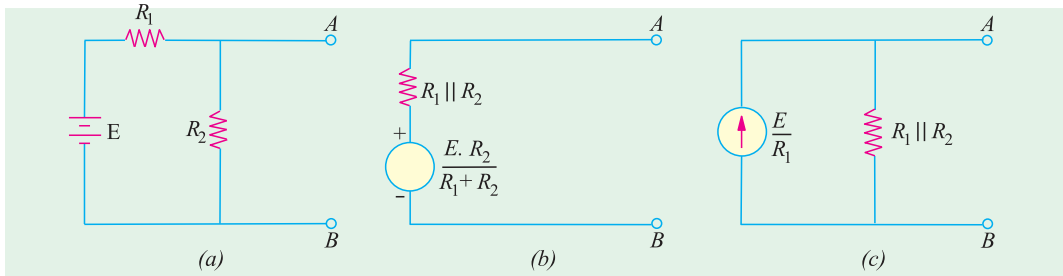


Fig. 2.203

$$R_i = R_1 \parallel R_2 = R_1 R_2 / (R_1 + R_2)$$

Hence, Thevenin equivalent circuit is as shown in Fig. 2.203 (b).

**(b) Norton Equivalent Circuit**

A short placed across terminals A and B will short out  $R_2$  as well. Hence,  $I_{SC} = E/R_1$ . The Norton equivalent resistance is exactly the same as Thevenin resistance except that it is connected in parallel with the current source as shown in Fig. 2.203 (c)

**Example 2.97.** Using Norton's theorem, find the constant-current equivalent of the circuit shown in Fig. 2.204 (a).

**Solution.** When terminals A and B are short-circuited as shown in Fig. 2.204 (b), total resistance of the circuit, as seen by the battery, consists of a  $10 \Omega$  resistance in series with a parallel combination of  $10 \Omega$  and  $15 \Omega$  resistances.

$$\therefore \text{ total resistance} = 10 + \frac{15 \times 10}{15 + 10} = 16 \Omega$$

$$\therefore \text{ battery current } I = 100/16 = 6.25 \text{ A}$$

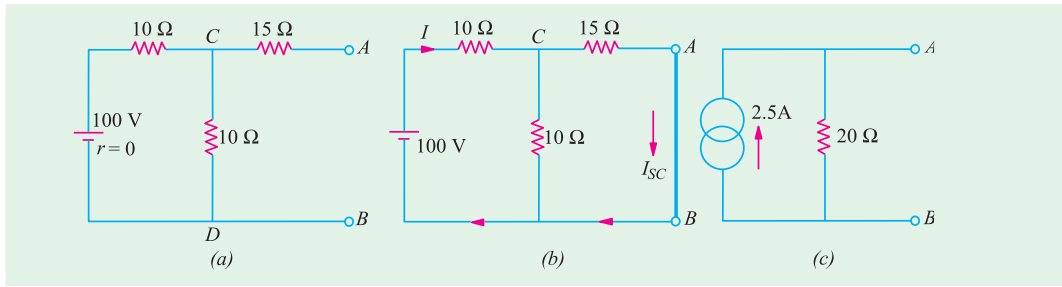


Fig. 2.204

This current is divided into two parts at point C of Fig. 2.204 (b).

Current through A B is  $I_{SC} = 6.25 \times 10/25 = 2.5 \text{ A}$

Since the battery has no internal resistance, the input resistance of the network when viewed from A and B consists of a 15 Ω resistance in series with the parallel combination of 10 Ω and 10 Ω  
Hence,  $R_1 = 15 + (10/2) = 20 \text{ Ω}$

Hence, the equivalent constant-current source is as shown in Fig. 2.204 (c).

**Example 2.98.** Apply Norton's theorem to calculate current flowing through 5 – Ω resistor of Fig. 2.05 (a).

**Solution. (i)** Remove 5 – Ω resistor and put a short across terminals A and B as shown in Fig. 2.205 (b). As seen, 10 –Ω resistor also becomes short-circuited.

**(ii)** Let us now find  $I_{SC}$ . The battery sees a parallel combination of 4 Ω and 8 Ω in series with a 4 Ω resistance. Total resistance seen by the battery =  $4 + 4 \parallel 8 = 20/3 \text{ Ω}$  Hence,  $I = 20 + 20/3 = 3 \text{ A}$ . This current divides at point C of Fig. 2.205 (b). Current going along path CAB gives  $I_{SC}$ . Its value =  $3 \times 4/12 = 1 \text{ A}$ .

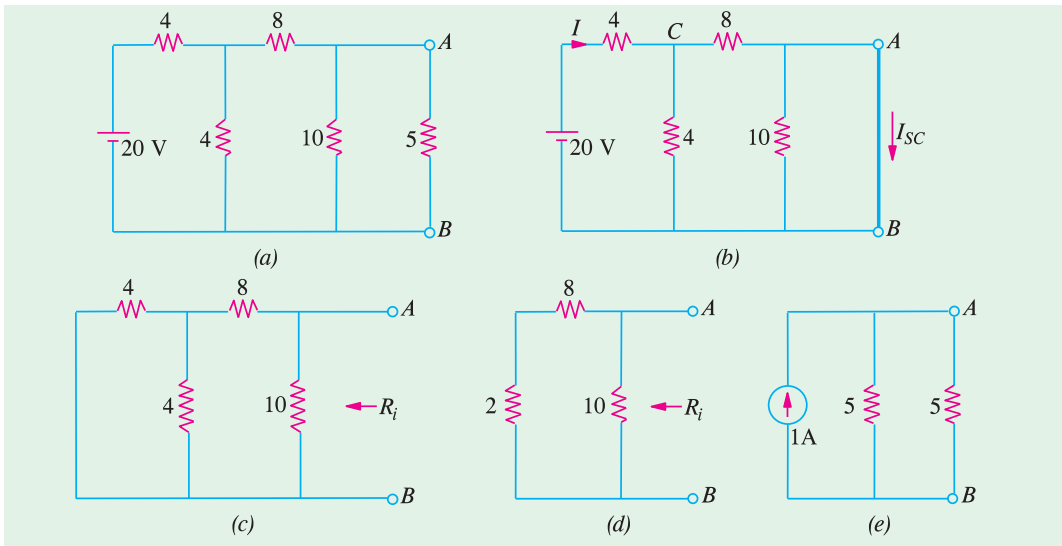


Fig. 2.205

**(iii)** In Fig. 2.205 (c), battery has been removed leaving behind its internal resistance which, in this case, is zero.

Resistance of the network looking into the terminals A and B in Fig. 2.205 (d) is

$$R_i = 10 \parallel 10 = 5 \text{ Ω}$$

**(iv)** Hence, Fig. 2.205 (e), gives the Norton's equivalent circuit.

(v) Now, join the  $5\text{-}\Omega$  resistance back across terminals  $A$  and  $B$ . The current flowing through it, obviously, is  $I_{AB} = 1 \times 5/10 = 0.5\text{ A}$ .

**Example 2.99.** Find the voltage across points  $A$  and  $B$  in the network shown in Fig. 2.206 (a) by using Norton's theorem.

**Solution.** The voltage between points  $A$  and  $B$  is  $V_{AB} = I_{SC} R_i$   
 where  $I_{SC}$  = short-circuit current between  $A$  and  $B$   
 $R_i$  = Internal resistance of the network as viewed from points  $A$  and  $B$ .

When short-circuit is placed between  $A$  and  $B$ , the current flowing in it due to  $50\text{-V}$  battery is  
 $= 50/50 = 1\text{ A}$  – from  $A$  to  $B$   
 Current due to  $100\text{ V}$  battery is  $= 100/20 = 5\text{ A}$  – from  $B$  to  $A$   
 $I_{SC} = 1 - 5 = -4\text{ A}$  – from  $B$  to  $A$

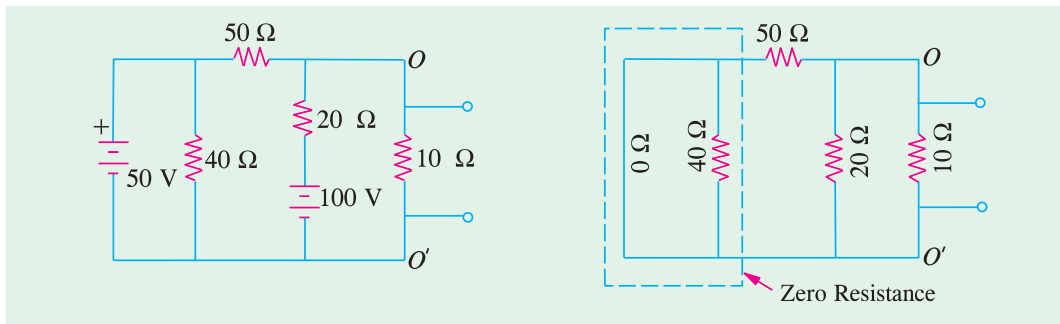


Fig. 2.206 (a)

Fig. 2.206 (b)

Now, suppose that the two batteries are removed so that the circuit becomes as shown in Fig. 2.206 (b). The resistance of the network as viewed from points  $A$  and  $B$  consists of three resistances of  $10\text{ }\Omega$ ,  $20\text{ }\Omega$  and  $50\text{ }\Omega$  ohm connected in parallel (as per second statement of Norton's theorem).

$$\therefore \frac{1}{R_i} = \frac{1}{10} + \frac{1}{20} + \frac{1}{50}; \quad \text{hence } R_i = \frac{100}{17}\text{ }\Omega$$

$$\therefore V_{AB} = -4 \times 100/17 = -23.5\text{ V}$$

The negative sign merely indicates that point  $B$  is at a higher potential with respect to the point  $A$ .

**Example 2.100.** Using Norton's theorem, calculate the current flowing through the  $15\text{ }\Omega$  load resistor in the circuit of Fig. 2.207 (a). All resistance values are in ohm.

**Solution. (a) Short-Circuit Current  $I_{SC}$**

As shown in Fig. 2.207 (b), terminals  $A$  and  $B$  have been shorted after removing  $15\text{ }\Omega$  resistor. We will use Superposition theorem to find  $I_{SC}$ .

**(i) When Only Current Source is Present**

In this case,  $30\text{-V}$  battery is replaced by a short-circuit. The  $4\text{ A}$  current divides at point  $D$  between parallel combination of  $4\text{ }\Omega$  and  $6\text{ }\Omega$ . Current through  $6\text{ }\Omega$  resistor is

$$I_{sc}' = 4 \times 4/(4 + 6) = 1.6\text{ A} \quad \text{– from } B \text{ to } A$$

**(ii) When Only Battery is Present**

In this case, current source is replaced by an open-circuit so that no current flows in the branch  $CD$ . The current supplied by the battery constitutes the short-circuit current

$$\therefore I_{sc}'' = 30/(4 + 6) = 3\text{ A} \quad \text{– from } A \text{ to } B$$

$$\therefore I_{sc} = I_{sc}'' - I_{sc}' = 3 - 1.6 = 1.4\text{ A} \quad \text{– from } A \text{ to } B$$



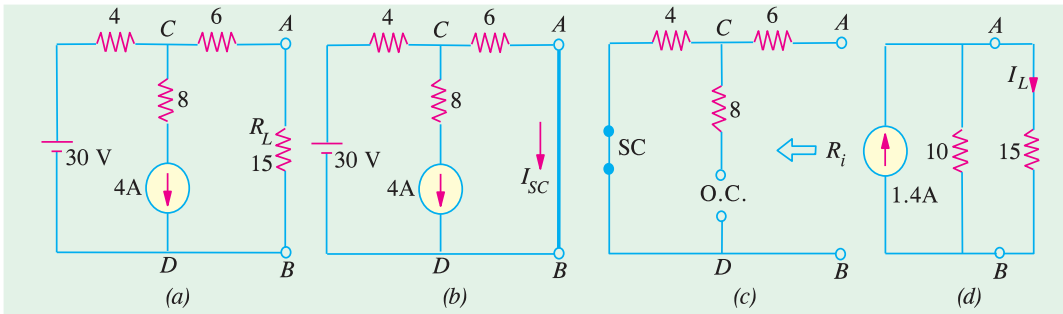


Fig. 2.207

**(b) Norton's Parallel Resistance**

As seen from Fig. 2.207 (c)  $R_1 = 4 + 6 = 10 \Omega$ . The  $8 \Omega$  resistance does not come into the picture because of an open in the branch  $CD$ .

Fig. 2.207 (d) shows the Norton's equivalent circuit along with the load resistor.

$$I_L = 1.4 \times 10 / (10 + 15) = 0.56 \text{ A}$$

**Example 2.101.** Using Norton's current-source equivalent circuit of the network shown in Fig. 2.208 (a), find the current that would flow through the resistor  $R_2$  when it takes the values of 12, 24 and 36  $\Omega$  respectively. [Elect. Circuits, South Gujarat Univ.]

**Solution.** In Fig. 2.208 (b), terminals  $A$  and  $B$  have been short-circuited. Current in the shorted path due to  $E_1$  is  $= 120/40 = 3 \text{ A}$  from  $A$  to  $B$ . Current due to  $E_2$  is  $180/60 = 3 \text{ A}$  from  $A$  to  $B$ . Hence  $I_{SC} = 6 \text{ A}$ . With batteries removed, the resistance of the network when viewed from open-circuited terminals is  $40 \parallel 60 = 24 \Omega$

- (i) When  $R_L = 12 \Omega$   $I_L = 6 \times 24 / (24 + 12) = 4 \text{ A}$
- (ii) When  $R_L = 24 \Omega$   $I_L = 6/2 = 3 \text{ A}$ .
- (iii) When  $R_L = 36 \Omega$   $I_L = 6 \times 24 / (24 + 36) = 2.4 \text{ A}$ .

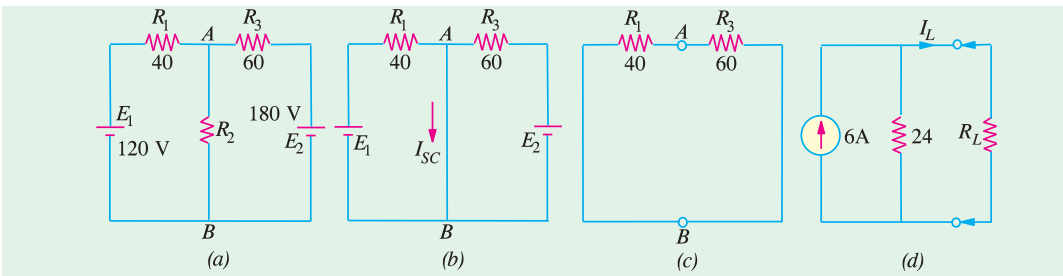
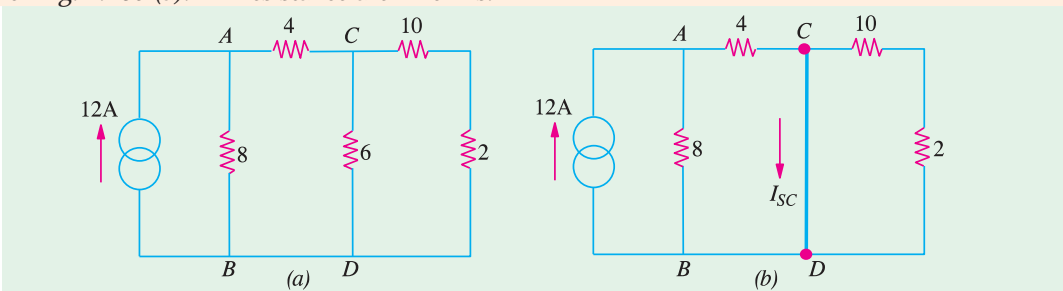


Fig. 2.208

**Example 2.102.** Using Norton's theorem, calculate the current in the  $6\text{-}\Omega$  resistor in the network of Fig. 2.209 (a). All resistance are in ohms.



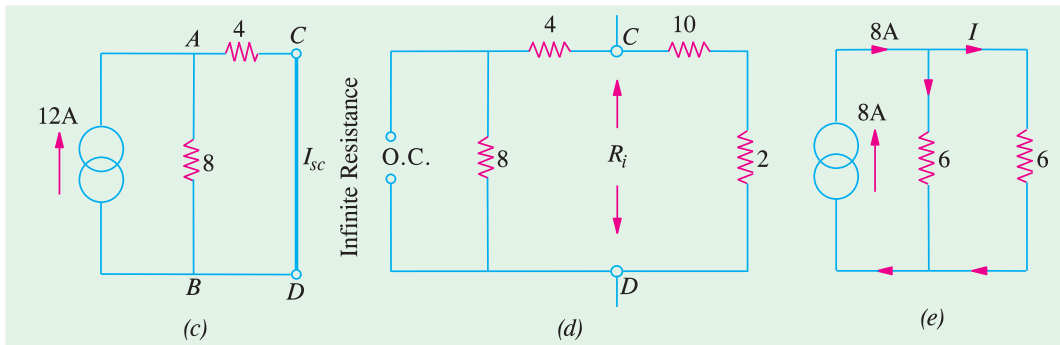


Fig. 2.209

**Solution.** When the branch containing  $6\text{ }\Omega$  resistance is short-circuited, the given circuit is reduced to that shown in Fig. 2.209 (b) and finally to Fig. 2.209 (c). As seen, the  $12\text{ A}$  current divides into two unequal parts at point A. The current passing through  $4\text{ }\Omega$  resistor forms the short-circuit current  $I_{SC}$ .

Resistance  $R_i$  between points C and D when they are open-circuited is

$$R_i = \frac{(4\text{ }\Omega \parallel 8\text{ }\Omega) + (10\text{ }\Omega \parallel 2\text{ }\Omega)}{6} = 6\text{ }\Omega$$

It is so because the constant-current source has *infinite* resistance *i.e.*, it behaves like an open circuit as shown in Fig. 2.209 (d).

Hence, Norton's equivalent circuit is as shown in Fig. 2.209 (e). As seen current of  $8\text{ A}$  is divided equally between the two equal resistances of  $6\text{ }\Omega$  each. Hence, current through the required  $6\text{ }\Omega$  resistor is  $4\text{ A}$ .

$$I_{SC} = 12 \times \frac{8}{8+4} = 8\text{ A}$$

**Example 2.103.** Using Norton's theorem, find the current which would flow in a  $25\text{ }\Omega$  resistor connected between points N and O in Fig. 2.210 (a). All resistance values are in ohms.

**Solution.** For case of understanding, the given circuit may be redrawn as shown in Fig. 2.210 (b). Total current in short-circuit across ON is equal to the sum of currents driven by different batteries through their respective resistances.

$$I_{SC} = \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5\text{ A}$$

The resistance  $R_i$  of the circuit when looked into from point N and O is

$$\frac{1}{R_i} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{7}{20}\text{ }\Omega; \quad R_i = \frac{20}{7}\text{ }\Omega = 2.86\text{ }\Omega$$

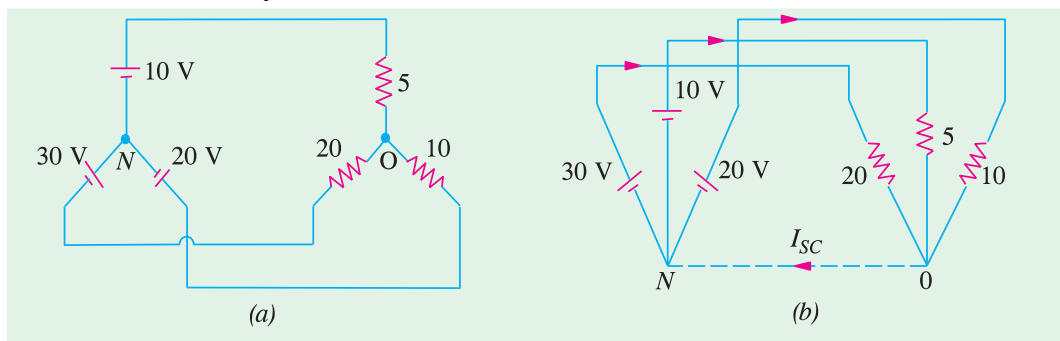


Fig. 2.210

Hence, given circuit reduces to that shown in Fig. 2.211 (a).

Open-circuit voltage across  $NO$  is  $= I_{SC}R_i$   
 $= 5.5 \times 2.86 = 15.73 \text{ V}$

Hence, current through  $25\text{-}\Omega$  resistor connected across  $NO$  is [Fig. 2.211 (b)]

$$I = 15.73/25 = 0.63 \text{ A}$$

$$\text{or } I = 5.5 \frac{2.86}{2.86 + 25} = 0.56 \text{ A.}$$

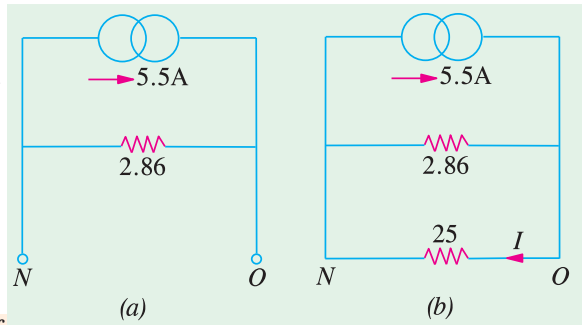


Fig. 2.211

**Example 2.104.** With the help of Norton's theorem, find  $V_o$  in the circuit shown in Fig. 2.212 (a). All resistances are in ohms.

**Solution.** For solving this circuit, we will Nortonise the circuit to the left to the terminals  $1-1'$  and to the right of terminals  $2-2'$ , as shown in Fig. 2.212 (b) and (c) respectively.

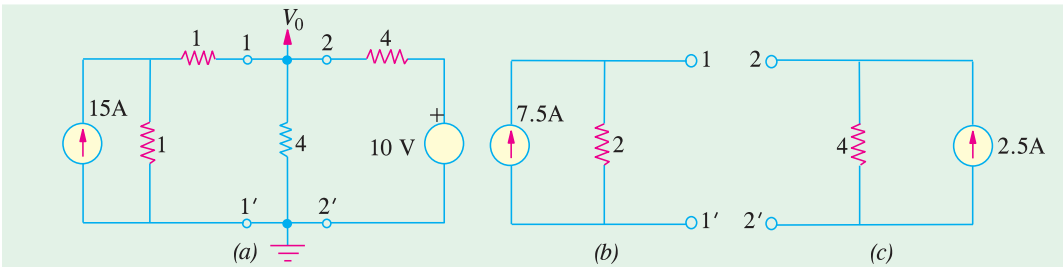


Fig. 2.212

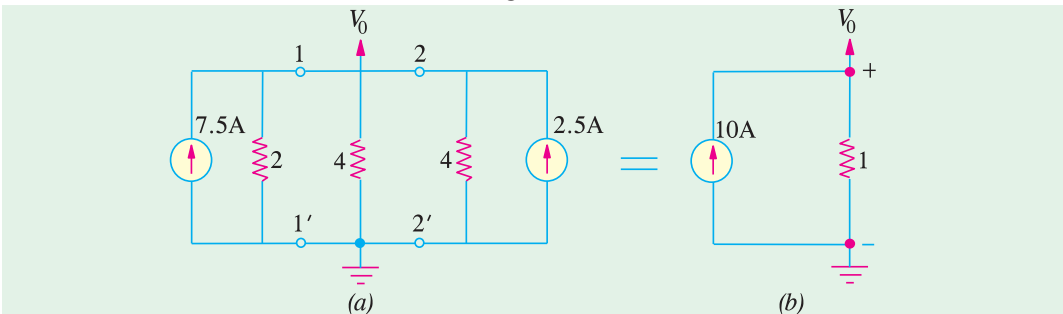


Fig. 2.213

The two equivalent Norton circuits can now be put back across terminals  $1-1'$  and  $2-2'$ , as shown in Fig. 2.213 (a).

The two current sources, being in parallel, can be combined into a single source of  $7.5 + 2.5 = 10 \text{ A}$ . The three resistors are in parallel and their equivalent resistance is  $2 \parallel 4 \parallel 4 = 1 \text{ }\Omega$ . The value of  $V_o$  as seen from Fig. 2.213 (b) is  $V_o = 10 \times 1 = 10 \text{ V}$ .

**Example 2.105.** For the circuit shown in Fig. 2.214 (a), calculate the current in the  $6 \text{ }\Omega$  resistance by using Norton's theorem. (Elect. Tech. Osmania Univ. Feb. 1992)

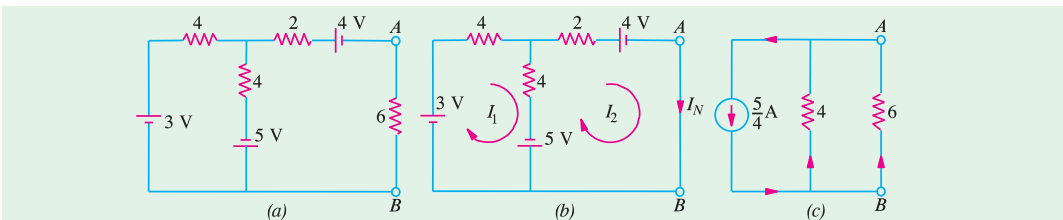


Fig. 2.214

**Solution.** As explained in Art. 2.19, we will replace the  $6\ \Omega$  resistance by a short-circuit as shown in Fig. 2.214 (b). Now, we have to find the current passing through the short-circuited terminals  $A$  and  $B$ . For this purpose we will use the mesh analysis by assuming mesh currents  $I_1$  and  $I_2$ .

From mesh (i), we get

$$3 - 4 I_1 - 4 (I_1 - I_2) + 5 = 0 \quad \text{or} \quad 2 I_1 - I_2 = 2 \quad \dots(i)$$

From mesh (ii), we get

$$-2 I_2 - 4 - 5 - 4 (I_2 - I_1) = 0 \quad \text{or} \quad 4 I_1 - 6 I_2 = 9 \quad \dots(ii)$$

From (i) and (ii) above, we get  $I_2 = -5/4$

The negative sign shows that the actual direction of flow of  $I_2$  is opposite to that shown in Fig. 2.214 (b). Hence,  $I_{sh} = I_N = I_2 = -5/4\ \text{A}$  i.e. current flows from point  $B$  to  $A$ .

After the terminals  $A$  and  $B$  are open-circuited and the three batteries are replaced by short-circuits (since their internal resistances are zero), the internal resistance of the circuit, as viewed from these terminals' is

$$R_i = R_N = 2 + 4 \parallel 4 = 4\ \Omega$$

The Norton's equivalent circuit consists of a constant current source of  $5/4\ \text{A}$  in parallel with a resistance of  $4\ \Omega$  as shown in Fig. 2.214 (c). When  $6\ \Omega$  resistance is connected across the equivalent circuit, current through it can be found by the current-divider rule (Art).

$$\text{Current through } 6\ \Omega \text{ resistor} = \frac{5}{4} \times \frac{4}{10} = 0.5 \text{ from } B \text{ to } A$$

## 2.27. General instructions For Finding Norton Equivalent Circuit

Procedure for finding Norton equivalent circuit of a given network has already been given in Art. That procedure applies to circuits which contain resistors and independent voltage or current sources. Similar procedures for circuits which contain both dependent and independent sources or only dependent sources are given below :

### (a) Circuits Containing Both Dependent and Independent Sources

- (i) Find the open-circuit voltage  $v_{oc}$  with all the sources activated or 'alive'.
- (ii) Find short-circuit current  $i_{sh}$  by short-circuiting the terminals  $a$  and  $b$  but with all sources activated.
- (iii)  $R_N = V_{oc} / i_{sh}$

### (b) Circuits Containing Dependent Sources Only

- (i)  $i_{sh} = 0$ .
- (ii) Connect  $1\ \text{A}$  source to the terminals  $a$  and  $b$  calculate  $v_{ab}$ .
- (iii)  $R_N = v_{ab} / 1$ .

**Example 2.106.** Find the Norton equivalent for the transistor amplifier circuit shown in Fig. 2.215 (a). All resistances are in ohms.

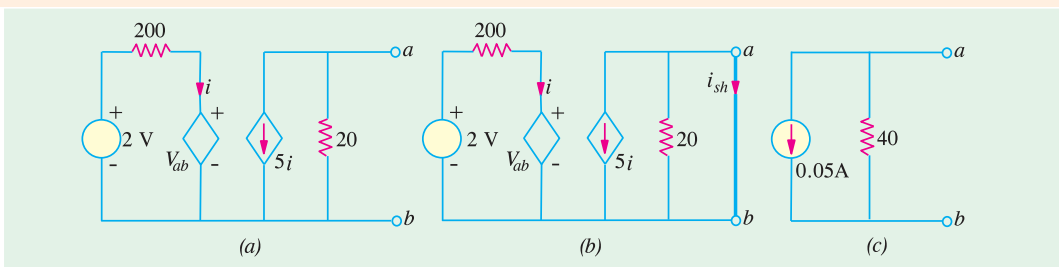


Fig. 2.215

**Solution.** We have to find the values of  $i_{sh}$  and  $R_N$ . It should be noted that when terminals  $a$  and  $b$  are short-circuited,  $v_{ab} = 0$ . Hence, in that case, we find from the left-hand portion of the circuit that  $i = 2/200 = 1/100\text{A} = 0.01\text{ A}$ . As seen from Fig. 2.215 (b), the short-circuit across terminals  $a$  and  $b$ , short circuits  $20\ \Omega$  resistance also. Hence,  $i_{sh} = -5 i = -5 \times 0.01 = -0.05\text{ A}$ .

Now, for finding  $R_N$ , we need  $v_{oc} = v_{ab}$  from the left-hand portion of the Fig. 2.215 (a). Applying KVL to the closed circuit, we have

$$2 - 200 i - v_{ab} = 0 \quad \dots(i)$$

Now, from the right-hand portion of the circuit, we find  $v_{ab} = \text{drop over } 20\ \Omega \text{ resistance} = -20 \times 5i = -100 i$ . The negative sign is explained by the fact that current flows from point  $b$  towards point  $a$ . Hence,  $i = -v_{ab}/100$ . Substituting this value in Eqn. (i) above, we get

$$2 - 200 (-v_{ab}/100) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = -2\text{ V}$$

$$\therefore R_N = v_{ab}/i_{sh} = -2/-0.05 = 40\ \Omega$$

Hence, the Norton equivalent circuit is as shown in Fig. 2.215 (c).

**Example 2.107.** Using Norton's theorem, compute current through the  $1\text{-}\Omega$  resistor of Fig. 2.216.

**Solution.** We will employ source conversion technique to simplify the given circuit. To begin with, we will convert the three voltage sources into their equivalent current sources as shown in Fig. 2.216 (b) and (c). We can combine together the two current sources on the left of  $EF$  but cannot combine the  $2\text{-A}$  source across  $CD$  because of the  $3\text{-}\Omega$  resistance between  $C$  and  $E$ .

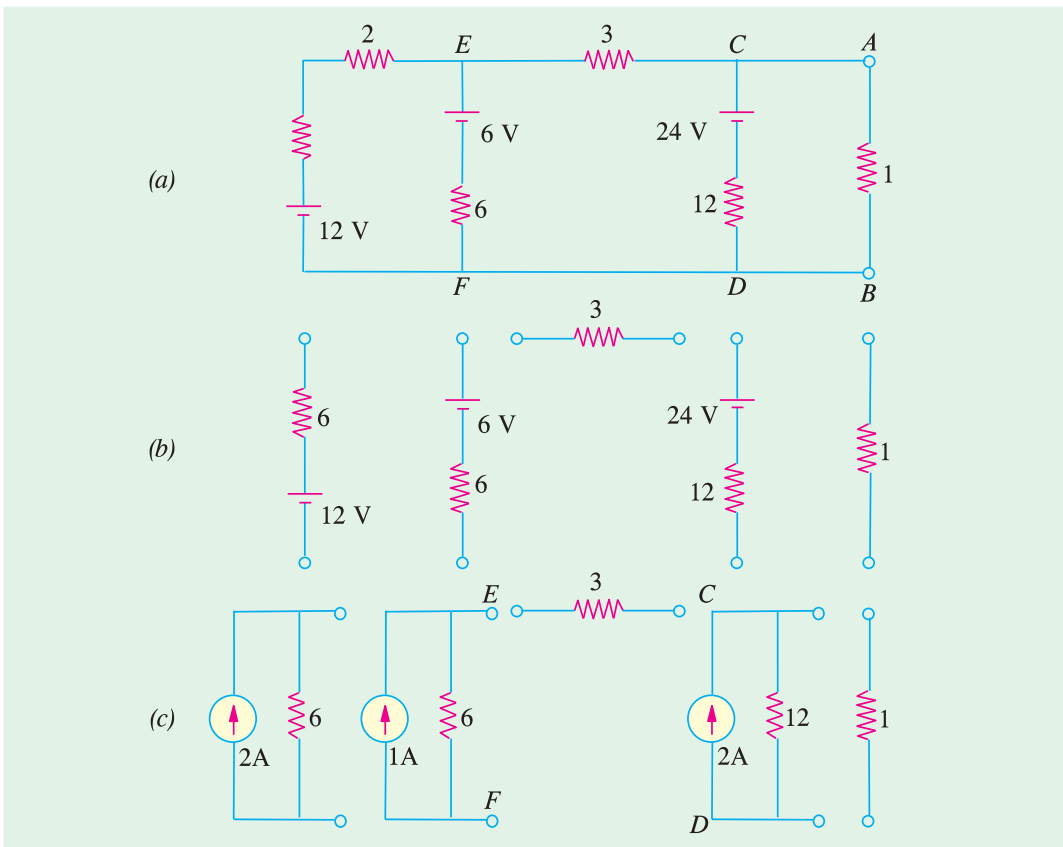


Fig. 2.216

In Fig. 2.217 (b), the two current sources at the left-hand side of  $3\ \Omega$  resistor have been replaced by a single  $(2\text{ A} + 1\text{ A}) = 3\text{ A}$  current source having a single parallel resistance  $6\ \parallel\ 6 = 3\ \Omega$

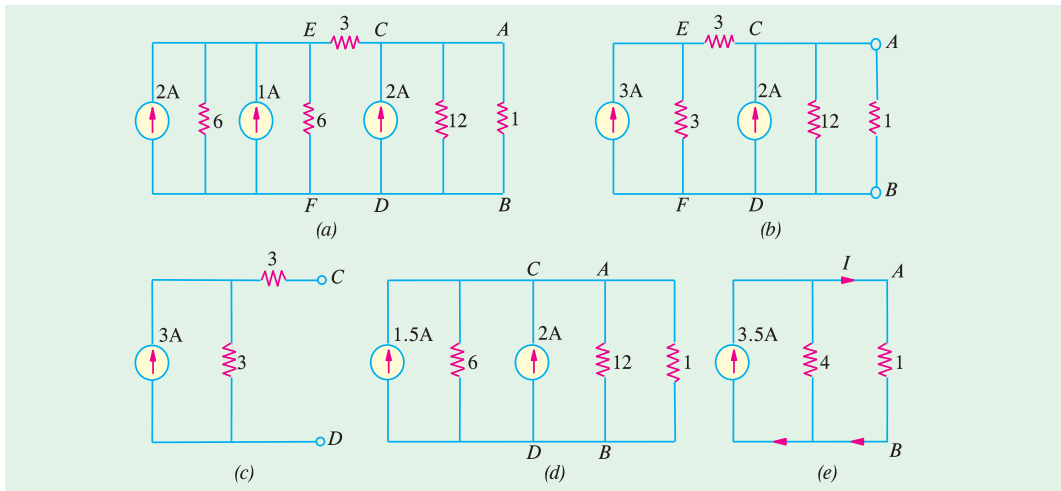


Fig. 2.217

We will now apply Norton's theorem to the circuit on the left-hand side of CD [Fig. 2.217 (c)] to convert it into a single current source with a single parallel resistor to replace the two  $3\ \Omega$  resistors. As shown in Fig. 2.217 (d), it yields a  $1.5\ \text{A}$  current source in parallel with a  $6\ \Omega$  resistor. This current source can now be combined with the one across  $CD$  as shown in Fig. 2.217 (e). The current through the  $1\text{-}\Omega$  resistor is

$$I = 3.5 \times 4 / (4 + 1) = 2.8\ \text{A}$$

**Example 2.108.** Obtain Thevenin's and Norton's equivalent circuits at  $AB$  shown in Fig. 2.218 (a). [Elect. Network, Analysis Nagpur Univ. 1993]

**Solution.** *Thevenin's Equivalent Circuit*

We will find the value of  $V_{th}$  by using two methods (i) KVL and (ii) mesh analysis.

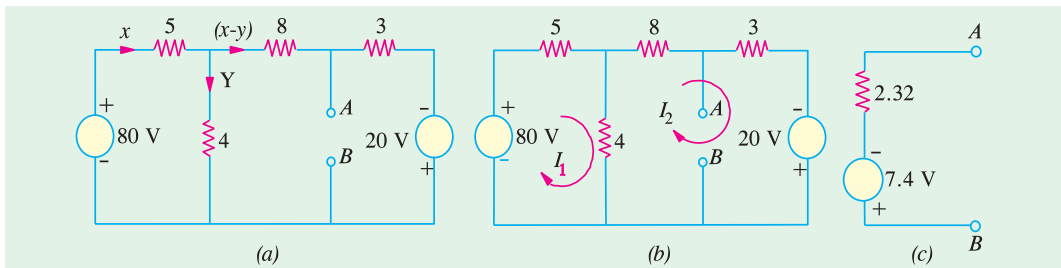


Fig. 2.218

(a) Using KVL

If we apply KVL to the first loop of Fig. 2.218 (a), we get

$$80 - 5x - 4y = 0 \quad \text{or} \quad 5x + 4y = 80 \quad \dots(i)$$

From the second @ loop, we have

$$-11(x-y) + 20 + 4y = 0 \quad \text{or} \quad 11x - 15y = 20 \quad \dots(ii)$$

From (i) and (ii), we get  $x = 10.75\ \text{A}$ ;  $y = 6.56\ \text{A}$  and  $(x-y) = 4.2\ \text{A}$ .

Now,  $V_{th} = V_{AB}$  i.e. voltage of point  $A$  with respect to point  $B$ . For finding its value, we start from point  $B$  and go to point  $A$  either via  $3\ \Omega$  resistance or  $4\ \Omega$  resistance or  $(5 + 8) = 13\ \Omega$  resistance and take the algebraic sum of the voltage met on the way. Taking the first route, we get

$$V_{AB} = -20 + 3(x-y) = -20 + 3 \times 4.2 = -7.4\ \text{V}$$

It shows that point  $A$  is negative with respect to point  $B$  or, which is the same thing, point  $B$  is positive with respect to point  $A$ .

**(b) Mesh Analysis [Fig. 2.218 (b)]**

Here,  $R_{11} = 9$ ;  $R_{22} = 15$ ;  $R_{21} = -4$

$$\therefore \begin{vmatrix} 9 & -4 \\ -4 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 80 \\ 20 \end{vmatrix}; \Delta = 135 - 16 = 119$$

$$\Delta_1 = \begin{vmatrix} 80 & -4 \\ 20 & 15 \end{vmatrix} = 1280; \Delta_2 = \begin{vmatrix} 9 & 80 \\ -4 & 20 \end{vmatrix} = 500$$

$$I_1 = 1280/119 = 10.75 \text{ A}; I_2 = 500/119 = 4.2 \text{ A}$$

Again  $V_{AB} = -20 + 12.6 = -7.4 \text{ V}$

**Value of  $R_{th}$** 

For finding  $R_{th}$ , we replace the two voltage sources by short-circuits.

$$\therefore R_{th} = R_{AB} = 3 \parallel (8 + 4 \parallel 5) = 2.32 \Omega$$

The Thevenin's equivalent circuit becomes as shown in Fig. 2.219 (c). It should be noted that point  $B$  has been kept positive with respect to point  $A$  in the Fig.

**Example 2.109.** Find current in the 4 ohm resistor by any three methods.

[Bombay University 2000]

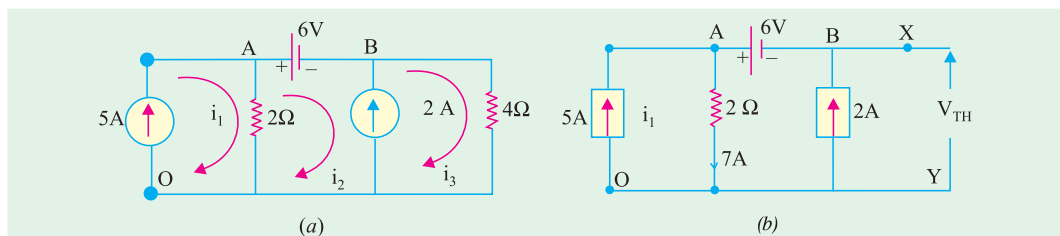


Fig. 2.219

**Solution. Method 1 :** Writing down circuit equations, with given conditions, and marking three clockwise loop-currents as  $i_1$ ,  $i_2$  and  $i_3$ .

$$\begin{aligned} \mathbf{i}_1 &= 5 \text{ A, due to the current source of 5 Amp} \\ V_A - V_B &= 6 \text{ V, due to the voltage source of 6 Volts} \\ \mathbf{i}_3 - \mathbf{i}_2 &= 2 \text{ A, due to the current source of 2 Amp.} \\ V_A &= (\mathbf{i}_1 - \mathbf{i}_2) 2, \quad V_B = \mathbf{i}_3 \times 4 \end{aligned}$$

With these equations, the unknowns can be evaluated.

$$2(i_1 - i_2) - 4i_3 = 6, \quad 2(5 - i_2) - 4(2 + i_2) = 6$$

This gives the following values :  $i_2 = -2/3 \text{ Amp.}$ ,  $i_3 = 4/3 \text{ Amp.}$

$$V_A = 34/3 \text{ volts, } V_B = 16/3 \text{ volts}$$

**Method 2 :** Thevenin's theorem : Redraw the circuit with modifications as in Fig. 2.219 (b)

$$R_{TH} = +14 - 6 = 8 \text{ V}$$

$R_{TH} = 2 \text{ ohms}$ , looking into the circuit form  $X$ - $Y$  terminals after de-activating the sources

$$I_L = 8/(2 + 4) = 4/3 \text{ Amp.}$$

**Method 3 :** Norton's Theorem : Redraw modifying as in Fig. 2.219 (c)

$$I_N = 2 + 2 = 4 \text{ Amp.}$$

This is because,  $X$  and  $Y$  are at ground potential, 2-ohm resistor has to carry 3 A and hence from 5-Amp. source, 2-Amp current is driven into  $X$ - $Y$  nodes.

$$R_N = 2 \text{ ohms}$$

Then the required current is calculated as shown in Fig. 2.219 (d)

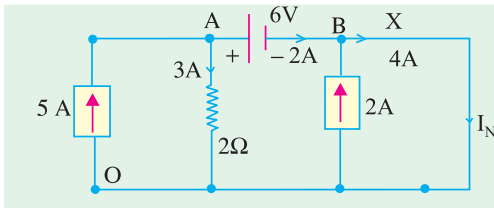


Fig. 2.219 (c) Evaluation of  $I_N$

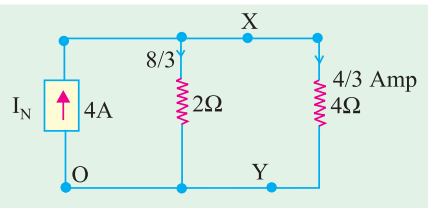


Fig. 2.219 (d)

**Note :** One more method is described. This transforms the sources such that the current through 4-ohm resistor is evaluated, as in final stage shown in Fig. 2.219 (j) or in Fig. 2.219 (k).

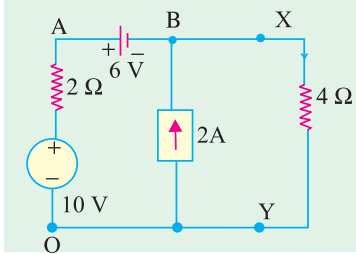


Fig. 2.219 (e)

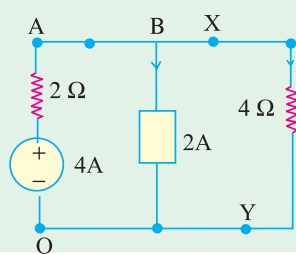


Fig. 2.219 (f)

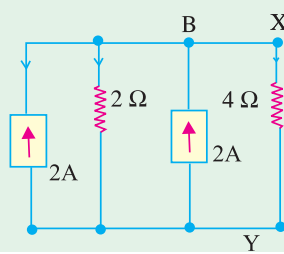


Fig. 2.219 (h)

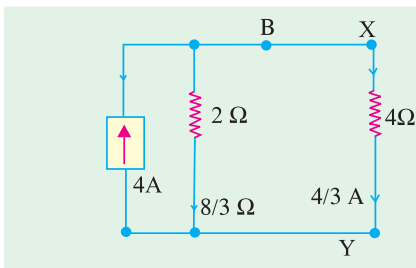


Fig. 2.219 (j)

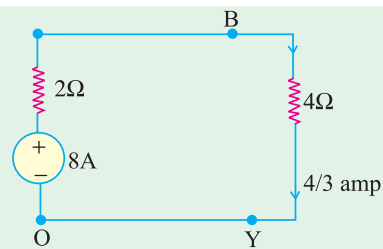


Fig. 2.219 (k)

**Example 2.109. (a).** Find Mesh currents  $i_1$  and  $i_2$  in the electric circuit of Fig. 2.219 (m)

[U.P. Tech. University, 2001]

**Solution.** Mark the nodes as shown in Fig. 2.219 (m).

Treat  $O$  as the reference node.

From the dependent current source of  $3i_1$  amp between  $B$  and  $O$ ,

$$i_2 - i_1 = 3i_1 \quad \text{or} \quad 4i_1 = i_2 \quad \dots(a)$$

$V_B$  is related to  $V_A$ ,  $V_C$  and the voltage across resistors concerned

$$V_B = V_A - i_1 \times 1 = 4 - i_1$$

$$V_B = V_C + i_2 \times 2 = 3 + 2i_2$$

Hence  $4 - i_1 = 3 + 2i_2 \quad \dots(b)$

From equations (a) and (b) above,  $i_1 = 1/9$  amp and  $i_2 = 4/9$  amp

Substituting these,  $V_B = 35/9$  volts

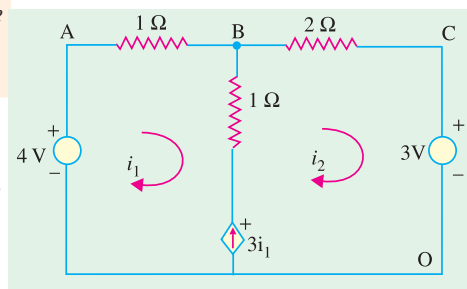


Fig. 2.219 (m)

**Example 2.109 (b).** Determine current through 6 ohm resistance connected across A-B terminals in the electric circuit of - 2.219 (n), using Thevenin's Theorem. [U.P. Tech. Univ. 2001]



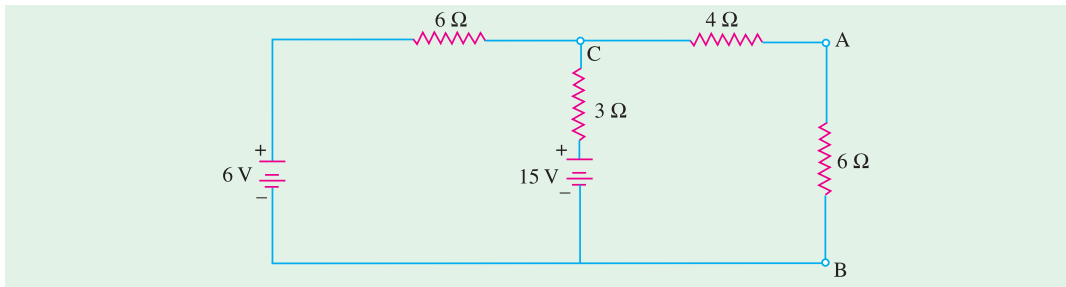


Fig. 2.219 (n)

**Solution.** Applying Thevenin’s theorem, after detaching the 6-ohm resistor from terminals  $A - B$ ,

$$V_{TH} = V_C = 15 - 1 \times 3 = 12 \text{ volts}$$

$$R_{TH} = 4 + 3/6 = 6 \text{ ohms}$$

$$I_L = 12/(6 + 6) = \text{lamp}$$

**Example 2.109 (c).** Applying Kirchoff’s Current Law, determine current  $I_s$  in the electric circuit of Fig. 2.219 (p). Take  $V_o = 16 \text{ V}$ . [U.P. Tech. Univ. 2001]

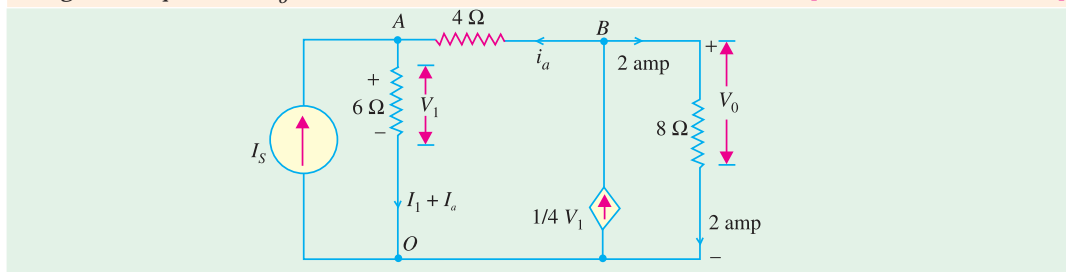


Fig. 2.219 (p)

**Solution.** Mark the nodes  $A, B$ , and  $O$  and the currents associated with different branches, as in Fig. 2.219 (p).

Since  $V_o = 16 \text{ V}$ , the current through 8-ohm resistor is 2 amp.

KCL at node B :  $1/4 V_1 = 2 + i_a$  ... (a)

KCL at node A :  $I_s + i_a = V_1/6$  ... (b)

Further,  $V_A = V_1, V_B = 16, V_B - V_1 = 4i_a$  ... (c)

From (a) and (c),  $i_a = 1 \text{ amp}$ . This gives  $V_1 - V_A = 12 \text{ volts}$ , and  $I_s = 1 \text{ amp}$

The magnitude of the dependent current source = 3 amp

**Check :** Power from 1 amp current source =  $1 \times 12 = 12 \text{ W}$

Power from dependent C.S. of 3 A =  $3 \times 16 = 48 \text{ W}$

Sum of source-output-power = 60 watts

Sum of power consumed by resistors =  $2^2 \times 6 + 1^2 \times 4 + 2^2 \times 8 = 60 \text{ watts}$

The power from sources equal the consumed by resistors. This confirms that the answers obtained are correct.

### Norton’s Equivalent Circuit

For this purpose, we will short-circuit the terminals  $A$  and  $B$  find the short-circuit currents produced by the two voltage sources. When viewed from the side of the 80-V source, a short across  $AB$  short-circuits everything on the right side of  $AB$ . Hence, the circuit becomes as shown in Fig. 2.230 (a). The short-circuit current  $I_1$  can be found with the help of series-parallel circuit technique. The total resistance offered to the 80 -V source is  $5 + 4 \parallel 8 = 23/3 \Omega$

$$\therefore I = 80 \times 3/23 = 10.43 \text{ A}; \quad \therefore I_1 = 10.43 \times 4/12 = 3.48 \text{ A}.$$

When viewed from the side of the 20-V source, a short across  $AB$  short-circuits everything beyond  $AB$ . In the case, the circuit becomes as shown in Fig. 2.230 (b). The short circuit current flowing from  $B$  to  $A = 20/3 = 6.67 \text{ A}$ .

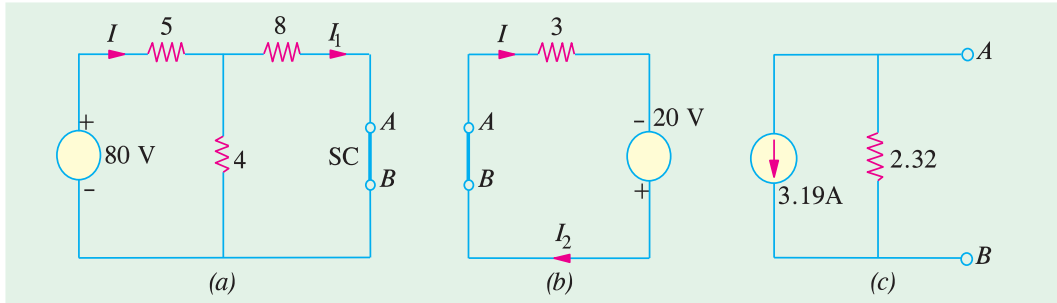


Fig. 2.220

$$\text{Total short-circuit current} = 6.67 - 3.48 = 3.19 \text{ A} \quad \dots \text{ from } B \text{ to } A.$$

$$R_N = R_{th} = 3 \parallel (8 + 4 \parallel 5) = 2.32 \Omega$$

Hence, the Norton's equivalent circuit becomes as shown in Fig. 2.220 (c).

## 2.28. Millman's Theorem

This theorem can be stated either in terms of voltage sources or current sources or both.

### (a) As Applicable to Voltage Sources

This Theorem is a combination of Thevenin's and Norton's theorems. It is used for finding the common voltage across any network which contains a number of parallel voltage sources as shown in Fig. 2.221 (a). Then common voltage  $V_{AB}$  which appears across the output terminals  $A$  and  $B$  is affected by the voltage sources  $E_1$ ,  $E_2$  and  $E_3$ . The value of the voltage is given by

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

This voltage represents the Thevenin's voltage  $V_{th}$ . The resistance  $R_{th}$  can be found, as usual, by replacing each voltage source by a short circuit. If there is a load resistance  $R_L$  across the terminals  $A$  and  $B$ , then load current  $I_L$  is given by

$$I_L = V_{th}/(R_{th} + R_L)$$

If as shown in Fig. 2.222 (b), a branch does not contain any voltage source, the same procedure is used except that the value of the voltage for that branch is equated to zero as illustrated in Example 2.210.

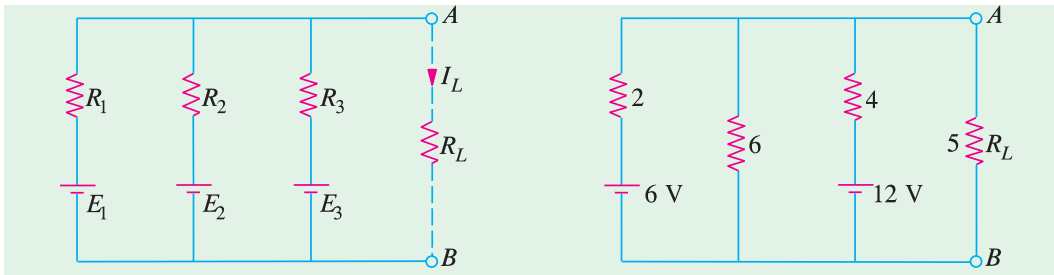


Fig. 2.221

Fig. 2.222

**Example 2.110.** Use Millman's theorem, to find the common voltage across terminals  $A$  and  $B$  and the load current in the circuit of Fig. 2.222.

**Solution.** As per Millman's Theorem,

$$V_{AB} = \frac{6/2 + 0/6 + 12/4}{1/2 + 1/6 + 1/4} = \frac{6}{11/12} = 6.55 \text{ V}$$

$\therefore$

$$V_{th} = 6.55 \text{ V}$$

$$R_{th} = 2 \parallel 6 \parallel 4 = 12/11 \Omega$$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{6.55}{(12/11) + 5} = 1.05 \text{ A}$$

**(b) As Applicable to Current Sources**

This theorem is applicable to a mixture of parallel voltage and current sources that are reduced to a single final equivalent source which is either a constant current or a constant voltage source. This theorem can be stated as follows :

Any number of constant current sources which are directly connected in parallel can be converted into a single current source whose current is the algebraic sum of the individual source currents and whose total internal resistances equals the combined individual source resistances in parallel.

**Example 2.111.** Use Millman's theorem, to find the voltage across and current through the load resistor  $R_L$  in the circuit of Fig. 2.223 (a).

**Solution.** First thing to do is to convert the given voltage sources into equivalent current sources. It should be kept in mind that the two batteries are connected in opposite direction. Using source conversion technique given in Art. 1.14 we get the circuit of Fig. 2.223 (b).

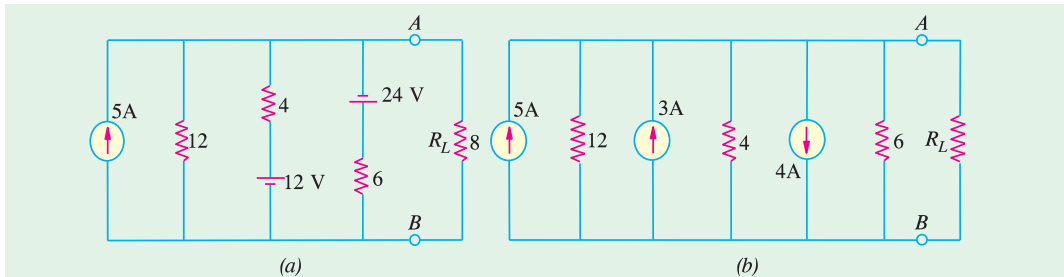


Fig. 2.223

The algebraic sum of the currents =  $5 + 3 - 4 = 4 \text{ A}$ . The combined resistance is  $= 12 \parallel 4 \parallel 6 = 2 \Omega$ . The simplified circuit is shown in the current-source form in Fig. 2.224 (a) or voltage source form in Fig. 2.224 (b).

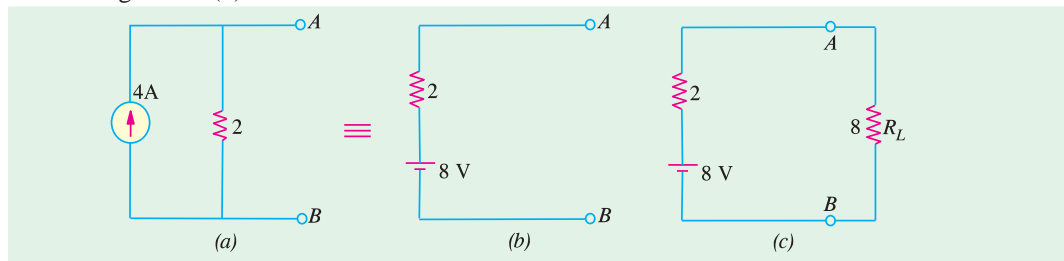


Fig. 2.224

As seen from Fig. 2.224 (c).

$$I_L = 8/(2 + 8) = 0.8 \text{ A} ; V_L = 8 \times 0.8 = 6.4 \text{ V}$$

Alternatively,  $V_L = 8 \times 8/(2 + 8) = 6.4 \text{ V}$

Following steps are necessary when using Millman's Theorem :

1. convert all voltage sources into their equivalent current sources.
2. calculate the algebraic sum of the individual dual source currents.

- if found necessary, convert the final current source into its equivalent voltage source.

As pointed out earlier, this theorem can also be applied to voltage sources which must be initially converted into their constant current equivalents.

### 2.29. Generalised Form of Millman's Theorem

This theorem is particularly useful for solving many circuits which are frequently encountered in both electronics and power applications.

Consider a number of admittances  $G_1, G_2, G_3 \dots G_n$  which terminate at common point  $0'$  (Fig. 2.225). The other ends of the admittances are numbered as 1, 2, 3... $n$ . Let  $O$  be any other point in the network. It should be clearly understood that it is not necessary to know anything about the inter-connection between point  $O$  and the end points 1, 2, 3... $n$ . However, what is essential to know is the voltage drops from  $O$  to 1,  $O$  to 2, ...  $O$  to  $n$  etc.

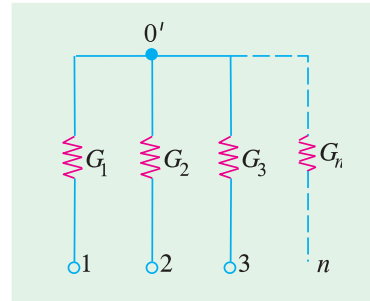


Fig. 2.225

According to this theorem, the voltage drop from  $O$  to  $0'$  ( $V_{00'}$ ) is given by

$$V_{00'} = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + \dots + V_{0n}G_n}{G_1 + G_2 + G_3 + \dots + G_n}$$

#### Proof

Voltage drop across	$G_1 = V_{10}' = (V_{00}' - V_{01})$
Current through	$G_1 = I_{10}' = V_{10}' G_1 = (V_{00}' - V_{01}) G_1$
Similarly,	$I_{20}' = (V_{00}' - V_{02}) G_2$
	$I_{30}' = (V_{00}' - V_{03}) G_3$
	.....
	.....
	$I_{n0}' = (V_{00}' - V_{0n}) G_n$

and

By applying *KCL* to point  $0'$ , we get

$$I_{10}' + I_{20}' + \dots + I_{n0}' = 0$$

Substituting the values of these currents, we get

$$V_{00'} = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + \dots + V_{0n}G_n}{G_1 + G_2 + G_3 + \dots + G_n}$$

#### Precaution

It is worth repeating that only those resistances or admittances are taken into consideration which terminate at the common point. All those admittances are ignored which do not terminate at the common point even though they are connected in the circuit.

**Example 2.112.** Use Millman's theorem to calculate the voltage developed across the  $40 \Omega$  resistor in the network of Fig. 2.226.

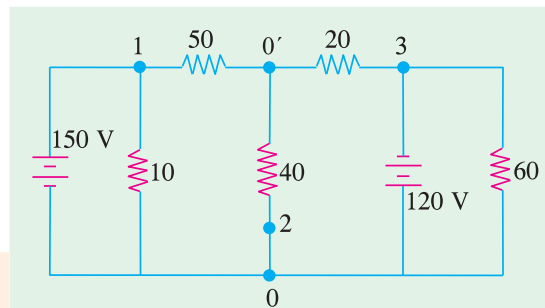


Fig. 2.226

**Solution.** Let the two ends of the  $40\ \Omega$  resistor be marked as 0 and  $0'$ . The end points of the three resistors terminating at the common point  $0'$  have been marked 1, 2 and 3. As already explained in Art. 2.29, the two resistors of values  $10\ \Omega$  and  $60\ \Omega$  will not come into the picture because they are not directly connected to the common point  $0'$ .

Here,

$$V_{01} = -150\ \text{V}; \quad V_{02} = 0; \quad V_{03} = 120\ \text{V}$$

$$G_1 = 1/50; \quad G_2 = 1/40; \quad G_3 = 1/20$$

$$\therefore V_{00}' = \frac{(-150/50) + (0/40) + (120/20)}{(1/50) + (1/40) + (1/20)} = 31.6\ \text{V}$$

It shows that point 0 is at a higher potential as compared to point  $0'$ .

**Example 2.113.** Calculate the voltage across the  $10\ \Omega$  resistor in the network of Fig. 2.227 by using (a) Millman's theorem (b) any other method.

**Solution. (a)** As shown in the Fig. 2.227 we are required to calculate voltage  $V_{00}'$ . The four resistances are connected to the common terminal  $0'$ .

Let their other ends be marked as 1, 2, 3 and 4 as shown in Fig. 2.227. Now potential of point 0 with respect to point 1 is (Art. 1.25)  $-100\ \text{V}$  because (see Art. 1.25)

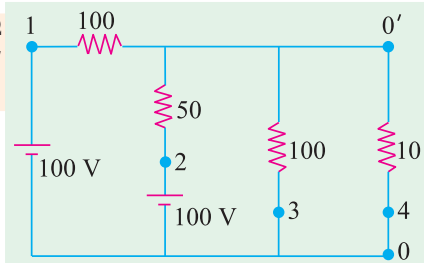


Fig. 2.227

$$\therefore V_{01} = -100\ \text{V}; \quad V_{02} = -100\ \text{V}; \quad V_{03} = 0\ \text{V}; \quad V_{04} = 0\ \text{V}.$$

$$G_1 = 1/100 = 0.01\ \text{Siemens}; \quad G_2 = 1/50 = 0.02\ \text{Siemens};$$

$$G_3 = 1/100 = 0.01\ \text{Siemens}; \quad G_4 = 1/10 = 0.1\ \text{Siemens}$$

$$\begin{aligned} \therefore V_{00}' &= \frac{V_{01} G_1 + V_{02} G_2 + V_{03} G_3 + V_{04} G_4}{G_1 + G_2 + G_3 + G_4} \\ &= \frac{100 \quad 0.01 \quad (-100) \quad 0.02 \quad 0 \quad 0.01 \quad 0 \quad 0.1}{0.01 \quad 0.02 \quad 0.01 \quad 0.1} \quad \frac{3}{0.14} \quad \mathbf{21.4\ \text{V}} \end{aligned}$$

Also,  $V_{00}' = -V_{00}' = \mathbf{21.4\ \text{V}}$

**(b)** We could use the source conversion technique (Art. 2.14) to solve this question. As shown in Fig. 2.228 (a), the two voltage sources and their series resistances have been converted into current sources with their parallel resistances. The two current sources have been combined into a single resistance current source of  $3\ \text{A}$  and the three parallel resistances have been combined into a single resistance of  $25\ \Omega$ . This current source has been reconverted into a voltage source of  $75\ \text{V}$  having a series resistance of  $25\ \Omega$  as shown in Fig. 2.228 (c).

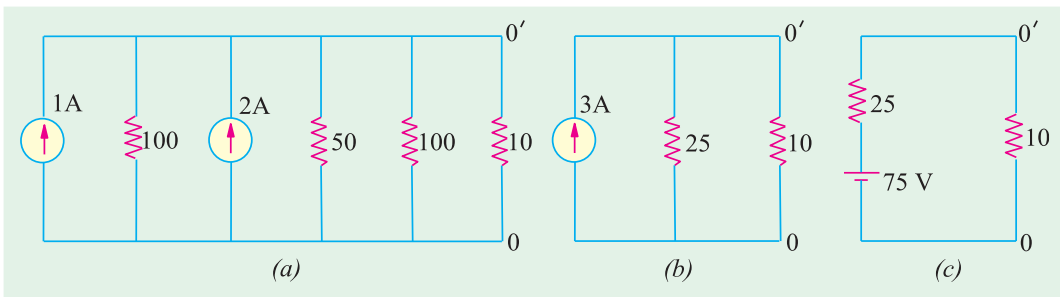


Fig. 2.228

Using the voltage divider formula (Art. 1.15), the voltage drop across  $10\ \Omega$  resistance is  $V_{0'0} = 75 \times 10/(10 + 25) = 21.4\ \text{V}$ .

**Example 2.114.** In the network shown in Fig. 2.229, using Millman's theorem, or otherwise find the voltage between  $A$  and  $B$ . (Elect. Engg. Paper-I Indian Engg. Services 1990)

**Solution.** The end points of the different admittances which are connected directly to the common point  $B$  have been marked as 1, 2 and 3 as shown in the Fig. 2.229. Incidentally,  $40\ \Omega$  resistance will not be taken into consideration because it is not directly connected to the common point  $B$ . Here  $V_{01} = V_{A1} = -50\ \text{V}$ ;  $V_{02} = V_{A2} = 100\ \text{V}$ ;  $V_{03} = V_{A3} = 0\ \text{V}$ .

$$\therefore V_{00}' = V_{AB} = \frac{(-50/50) + (100/20) + (0/10)}{(1/50) + (1/20) + (1/10)} = 23.5\ \text{V}$$

Since the answer comes out to be positive, it means that point  $A$  is at a higher potential as compared to point  $B$ .

The detailed reason for not taking any notice of  $40\ \Omega$  resistance are given in Art. 2.29.

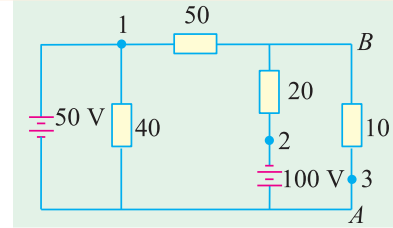


Fig. 2.229

### 2.30. Maximum Power Transfer Theorem

Although applicable to all branches of electrical engineering, this theorem is particularly useful for analysing communication networks. The overall efficiency of a network supplying maximum power to any branch is 50 per cent. For this reason, the application of this theorem to power transmission and distribution networks is limited because, in their case, the goal is high efficiency and not maximum power transfer.

However, in the case of electronic and communication networks, very often, the goal is either to receive or transmit maximum power (through at reduced efficiency) specially when power involved is only a few milliwatts or microwatts. Frequently, the problem of maximum power transfer is of crucial significance in the operation of transmission lines and antennas.

As applied to d.c. networks, this theorem may be stated as follows :

**A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.**

In Fig. 2.230 (a), a load resistance of  $R_L$  is connected across the terminals  $A$  and  $B$  of a network which consists of a generator of e.m.f.  $E$  and internal resistance  $R_g$  and a series resistance  $R$  which, in fact, represents the lumped resistance of the connecting wires. Let  $R_i = R_g + R =$  internal resistance of the network as viewed from  $A$  and  $B$ .

According to this theorem,  $R_L$  will abstract maximum power from the network when  $R_L = R_i$ .

**Proof.** Circuit current  $I = \frac{E}{R_L + R_i}$

Power consumed by the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2} \quad \dots(i)$$

For  $P_L$  to be maximum,  $\frac{dP_L}{dR_L} = 0$ .

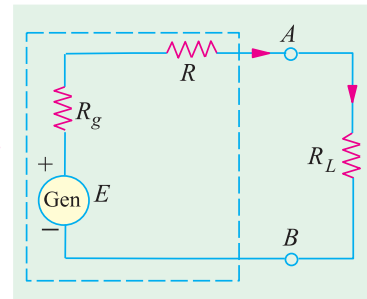


Fig. 2.230

Differentiating Eq. (i) above, we have

$$\frac{dP_L}{dR_L} = E^2 \left[ \frac{1}{(R_L + R_i)^2} + R_L \left( \frac{-2}{(R_L + R_i)^3} \right) \right] = E^2 \left[ \frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right]$$

$$\therefore 0 = E^2 \left[ \frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right] \quad \text{or} \quad 2R_L = R_L + R_i \quad \text{or} \quad R_L = R_i$$

It is worth noting that under these conditions, the voltage across the load is hold the open-circuit voltage at the terminals  $A$  and  $B$ .

$$\therefore \text{Max. power is } P_{L \max.} = \frac{E^2 R_L}{4 R_L^2} = \frac{E^2}{4 R_L} = \frac{E^2}{4 R_i}$$

Let us consider an a.c. source of internal impedance  $(R_1 + jX_1)$  supplying power to a load impedance  $(R_L + jX_L)$ . It can be proved that maximum power transfer will take place when the modulus of the load impedance is equal to the modulus of the source impedance *i.e.*  $|Z_L| = |Z_1|$

Where there is a completely free choice about the load, the maximum power transfer is obtained when load impedance is the complex conjugate of the source impedance. For example, if source impedance is  $(R_1 + jX_1)$ , then maximum transfer power occurs, when load impedance is  $(R_1 - jX_1)$ . It can be shown that under this condition, the load power is  $= E^2/4R_1$ .

**Example 2.115.** In the network shown in Fig. 2.231 (a), find the value of  $R_L$  such that maximum possible power will be transferred to  $R_L$ . Find also the value of the maximum power and the power supplied by source under these conditions. **(Elect. Engg. Paper I Indian Engg. Services)**

**Solution.** We will remove  $R_L$  and find the equivalent Thevenin's source for the circuit to the left of terminals  $A$  and  $B$ . As seen from Fig. 2.231 (b)  $V_{th}$  equals the drop across the vertical resistor of  $3\Omega$  because no current flows through  $2\Omega$  and  $1\Omega$  resistors. Since  $15\text{ V}$  drops across two series resistors of  $3\Omega$  each,  $V_{th} = 15/2 = 7.5\text{ V}$ . Thevenin's resistance can be found by replacing  $15\text{ V}$  source with a short-circuit. As seen from Fig. 2.231 (b),  $R_{th} = 2 + (3 \parallel 3) + 1 = 4.5\Omega$ . Maximum power transfer to the load will take place when  $R_L = R_{th} = 4.5\Omega$

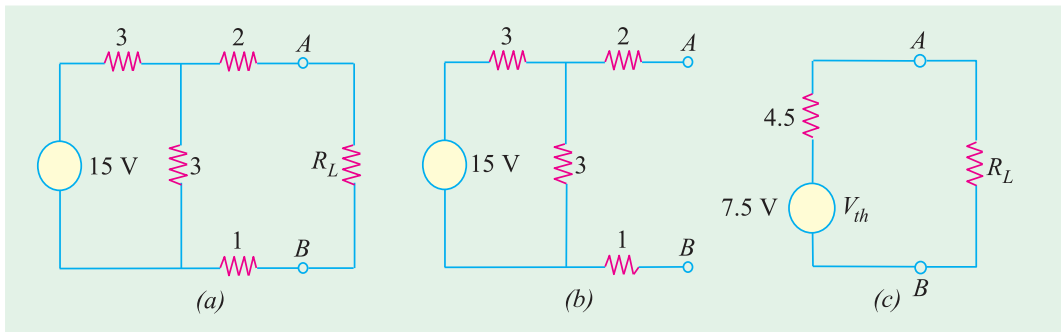


Fig. 2.231

Maximum power drawn by  $R_L = V_{th}^2/4 \times R_L = 7.5^2/4 \times 4.5 = 3.125\text{ W}$ .

Since same power is developed in  $R_{th}$ , power supplied by the source  $= 2 \times 3.125 = 6.250\text{ W}$ .

**Example 2.116.** In the circuit shown in Fig. 2.232 (a) obtain the condition from maximum power transfer to the load  $R_L$ . Hence determine the maximum power transferred.

**(Elect. Science-I Allahabad Univ. 1992)**

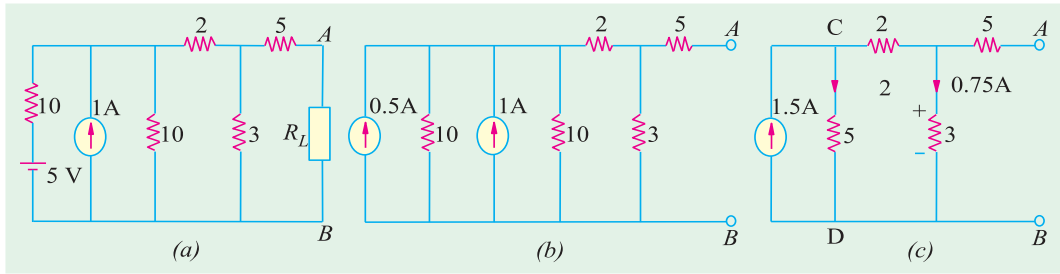


Fig. 2.232

**Solution.** We will find Thevenin's equivalent circuit to the left of terminals  $A$  and  $B$  for which purpose we will convert the battery source into a current source as shown in Fig. 2.232 (b). By combining the two current sources, we get the circuit of Fig. 2.232 (c). It would be seen that open circuit voltage  $V_{AB}$  equals the drop over  $3\Omega$  resistance because there is no drop on the  $5\Omega$  resistance connected to terminal  $A$ . Now, there are two parallel path across the current source each of resistance  $5\Omega$ . Hence, current through  $3\Omega$  resistance equals  $1.5/2 = 0.75\text{ A}$ . Therefore,  $V_{th} = V_{th} = 3 \times 0.75 = 2.25\text{ V}$  with point  $A$  positive with respect to point  $B$ .

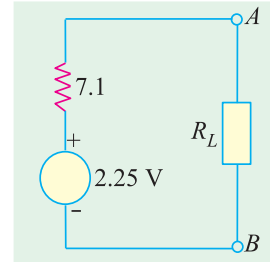


Fig. 2.233

For finding  $R_{AB}$ , current source is replaced by an infinite resistance.

$$\therefore R_{AB} = R_{th} = 5 + 3 \parallel (2 + 5) = 7.1\ \Omega$$

The Thevenin's equivalent circuit alongwith  $R_L$  is shown in Fig. 2.233. As per Art. 2.30, the condition for MPT is that  $R_L = 7.1\ \Omega$

$$\text{Maximum power transferred} = V_{th}^2 / 4R_L = 2.25^2 / 4 \times 7.1 = 0.178\text{ W} = 178\text{ mW}.$$

**Example 2.117.** Calculate the value of  $R$  which will absorb maximum power from the circuit of Fig. 2.234 (a). Also, compute the value of maximum power.

**Solution.** For finding power, it is essential to know both  $I$  and  $R$ . Hence, it is essential to find an equation relating  $I$  to  $R$ .

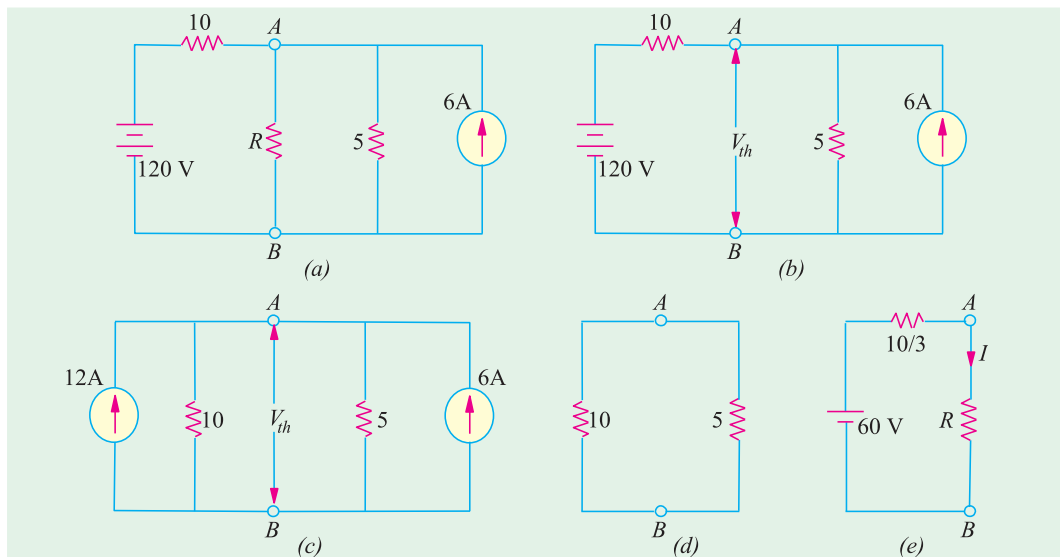


Fig. 2.234



Let us remove  $R$  and find Thevenin's voltage  $V_{th}$  across  $A$  and  $B$  as shown in Fig. 2.234 (b). It would be helpful to convert 120 V, 10- $\Omega$  source into a constant-current source as shown in Fig. 2.234 (c). Applying  $KCL$  to the circuit, we get

$$\frac{V_{th}}{10} + \frac{V_{th}}{5} = 12 + 6 \quad \text{or} \quad V_{th} = 60 \text{ V}$$

Now, for finding  $R_i$  and  $R_{th}$ , the two sources are reduced to zero. Voltage of the voltage-source is reduced to zero by short-circuiting it whereas current of the current source is reduced to zero by open-circuiting it. The circuit which results from such source suppression is shown in Fig. 2.234 (d). Hence,  $R_i = R_{th} = 10 \parallel 5 = 10/3 \Omega$ . The Thevenin's equivalent circuit of the network is shown in Fig. 2.234 (e).

According to Maximum Power Transfer Theorem,  $R$  will absorb maximum power when it equals  $10/3 \Omega$ . In that case,  $I = 60 \div 20/3 = 9 \text{ A}$

$$P_{max} = I^2 R = 9^2 \times 10/3 = 270 \text{ W}$$

### 2.31. Power Transfer Efficiency

If  $P_L$  is the power supplied to the load and  $P_T$  is the total power supplied by the voltage source, then power transfer efficiency is given by  $\eta = P_L/P_T$ .

Now, the generator or voltage source  $E$  supplies power to both the load resistance  $R_L$  and to the internal resistance  $R_i = (R_g + R)$ .

$$P_T = P_L + P_i \quad \text{or} \quad E \times I = I^2 R_L + I^2 R_i$$

$$\therefore \eta = \frac{P_L}{P_T} = \frac{I^2 R_L}{I^2 R_L + I^2 R_i} = \frac{R_L}{R_L + R_i} = \frac{1}{1 + (R_i/R_L)}$$

The variation of  $\eta$  with  $R_L$  is shown in Fig. 2.235 (a). The maximum value of  $\eta$  is unity when  $R_L = \infty$  and has a value of 0.5 when  $R_L = R_i$ . It means that under maximum power transfer conditions, the power transfer efficiency is only 50%. As mentioned above, maximum power transfer condition is important in communication applications but in most power systems applications, a 50% efficiency is undesirable because of the wasted energy. Often, a compromise has to be made between the load power and the power transfer efficiency. For example, if we make  $R_L = 2 R_i$ , then

$$P_L = 0.222 E^2/R_i \quad \text{and} \quad \eta = 0.667.$$

It is seen that the load power is only 11% less than its maximum possible value, whereas the power transfer efficiency has improved from 0.5 to 0.667 *i.e.* by 33%.

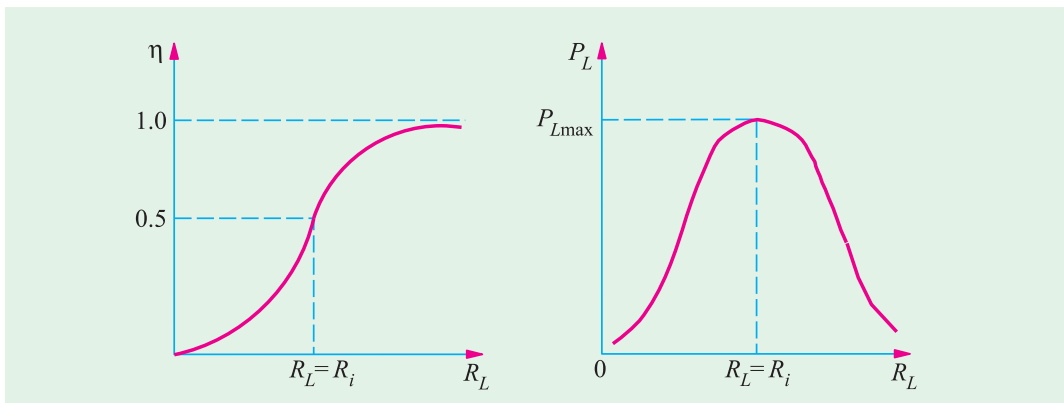


Fig. 2.235

**Example 2.118.** A voltage source delivers 4 A when the load connected to it is 5 Ω and 2 A when the load becomes 20 Ω. Calculate

(a) maximum power which the source can supply (b) power transfer efficiency of the source with  $R_L$  of 20 Ω (c) the power transfer efficiency when the source delivers 60 W.

**Solution.** We can find the values of  $E$  and  $R_i$  from the two given load conditions.

(a) When  $R_L = 5 \Omega$ ,  $I = 4 \text{ A}$  and  $V = IR_L = 4 \times 5 = 20 \text{ V}$ , then  $20 = E - 4 R_i$  ... (i)

When  $R_L = 20 \Omega$ ,  $I = 2 \text{ A}$  and  $V = IR_L = 2 \times 20 = 40 \text{ V}$   $\therefore 40 = E - 2 R_i$  ... (ii)

From (i) and (ii), we get,  $R_i = 10 \Omega$  and  $E = 60 \text{ V}$

When  $R_L = R_i = 10 \Omega$

$$P_{L \max} = \frac{E^2}{4R_i} = \frac{60 \times 60}{4 \times 10} = 90 \text{ W}$$

(b) When  $R_L = 20 \Omega$ , the power transfer efficiency is given by

$$\eta = \frac{R_L}{R_L + R_i} = \frac{20}{30} = 0.667 \text{ or } 66.7\%$$

(c) For finding the efficiency corresponding to a load power of 60 W, we must first find the value of  $R_L$ .

$$\text{Now, } P_L = \left( \frac{E}{R_i + R_L} \right)^2 R_L$$

$$\therefore 60 = \frac{60^2 \times R_L}{(R_L + 10)^2} \text{ or } R_L^2 - 40 R_L + 100 = 0$$

Hence  $R_L = 37.32 \Omega$  or  $2.68 \Omega$

Since there are two values of  $R_L$ , there are two efficiencies corresponding to these values.

$$\eta_1 = \frac{37.32}{37.32 + 10} = 0.789 \text{ or } 78.9\%, \quad \eta_2 = \frac{2.68}{12.68} = 0.211 \text{ or } 21.1\%$$

It will be seen from above, the  $\eta_1 + \eta_2 = 1$ .

**Example 2.119.** Two load resistance  $R_1$  and  $R_2$  dissipate the same power when connected to a voltage source having an internal resistance of  $R_i$ . Prove that (a)  $R_i^2 = R_1 R_2$  and (b)  $\eta_1 + \eta_2 = 1$ .

**Solution.** (a) Since both resistances dissipate the same amount of power, hence

$$P_L = \frac{E^2 R_1}{(R_1 + R_i)^2} = \frac{E^2 R_2}{(R_2 + R_i)^2}$$

Cancelling  $E^2$  and cross-multiplying, we get

$$R_1 R_2^2 + 2R_1 R_2 R_i + R_1 R_i^2 = R_2 R_1^2 + 2R_1 R_2 R_i + R_2 R_i^2$$

Simplifying the above, we get,  $R_i^2 = R_1 R_2$

(b) If  $\eta_1$  and  $\eta_2$  are the two efficiencies corresponding to the load resistances  $R_1$  and  $R_2$ , then

$$\eta_1 + \eta_2 = \frac{R_1}{R_1 + R_i} + \frac{R_2}{R_2 + R_i} = \frac{2 R_1 R_2 + R_i(R_1 + R_2)}{R_1 R_2 + R_i^2 + R_i(R_1 + R_2)}$$

Substituting  $R_i^2 = R_1 R_2$ , we get

$$\eta_1 + \eta_2 = \frac{2 R_1 R_2 + R_i(R_1 + R_2)}{2 R_1 R_2 + R_i(R_1 + R_2)} = 1$$

**Example 2.120.** Determine the value of  $R_L$  for maximum power at the load. Determine maximum power also. The network is given in the Fig. 2.236 (a). [Bombay University 2001]

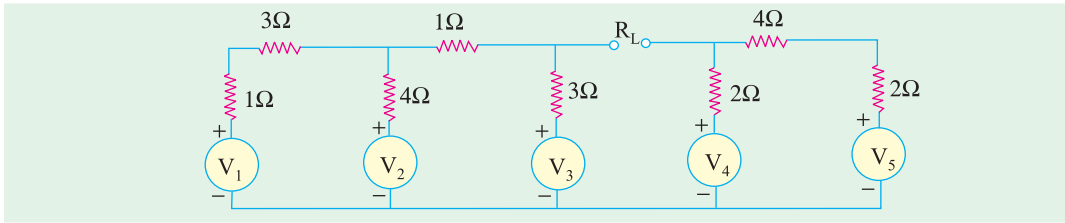


Fig. 2.236 (a)

**Solution.** This can be attempted by Thevenin's Theorem. As in the circuit, with terminals  $A$  and  $B$  kept open, from the right hand side,  $V_B$  (w.r. to reference node 0) can be calculated.  $V_4$  and  $V_5$  will have a net voltage of 2 volts circulating a current of  $(2/8) = 0.25$  amp in clockwise direction.

$$V_B = 10 - 0.25 \times 2 = 9.5 \text{ volts.}$$

On the Left-hand part of the circuit, two loops are there.  $V_A$  (w.r. to 0) has to be evaluated. Let the first loop (with  $V_1$  and  $V_2$  as the sources) carry a clockwise current of  $i_1$  and the second loop (with  $V_2$  and  $V_3$  as the sources), a clockwise current of  $i_2$ . Writing the circuit equations.

$$\begin{aligned} 8i_1 - 4i_2 &= +4 \\ -4i_1 + 8i_2 &= +4 \end{aligned}$$

This gives  $i_1 = 1$  amp,  $i_2 = 1$  amp

Therefore,  $V_A = 12 + 3 \times 1 = 15$  volts.

Thevenin -voltage,  $V_{TH} = V_A - V_B = 15 - 9.5 = 5.5$  volts

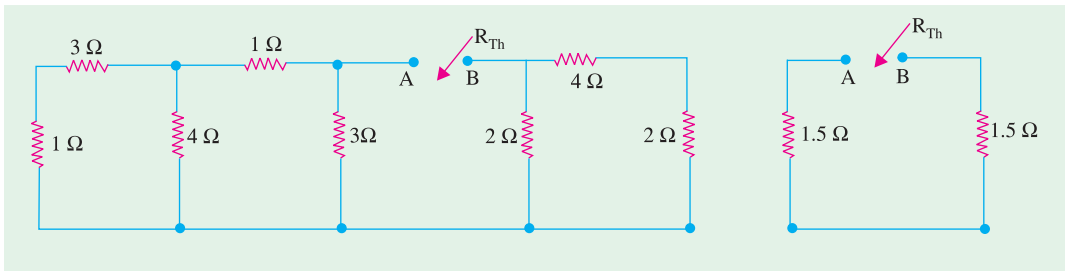


Fig. 2.236 (b)

Fig. 2.236 (c)

Solving as shown in Fig. 2.236 (b) and (c).

$$R_{TH} = 3 \text{ ohms}$$

For maximum power transfer,  $R_L = 3$  ohms

$$\text{Current} = 5.5/6 = 0.9167 \text{ amp}$$

$$\text{Power transferred to load } 0.9167^2 \times 3 = 2.52 \text{ watts.}$$

**Example 2.121.** For the circuit shown below, what will be the value of  $R_L$  to get the maximum power? What is the maximum power delivered to the load? [Bombay University 2001]

**Solution.** Detach  $R_L$  and apply Thevenin's Theorem.

$$V_{TH} = 5.696 \text{ volts, } R_{TH} = 11.39 \Omega$$

$R_L$  must be 11.39 ohms for maximum power transfer.

$$P_{\max} = 0.712 \text{ watt.}$$

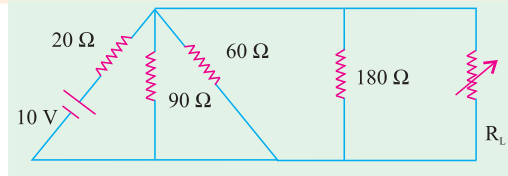


Fig. 2.237

**Example 2.122.** Find the maximum power in ' $R_L$ ' which is variable in the circuit shown below in Fig. 2.238. [Bombay University, 2001]

**Solution.** Apply Thevenin's theorem. For this  $R_L$  has to be detached from nodes A and B. Treat O as the reference node.

$$V_A = 60 \text{ V}, V_B = V_C + 2 = 50 + 2 = 52 \text{ V}$$

Thus,  $V_{TH} = V_{AB} = 8$  volts, with A positive w.r. to B,  $R_{TH} = (60//40) + (50//50) = 49$  ohms

Hence, for maximum power,  $R_L = 49$  ohms

With this  $R_L$ , Current =  $8/98$  amp = 0.08163 amp

Power to Load =  $I^2 R_L = 0.3265$  watt

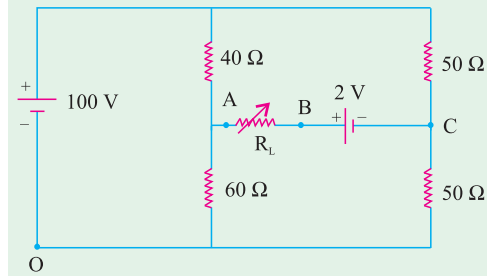


Fig. 2.238

**Example 2.123.** Find  $V_A$  and  $V_B$  by "nodal analysis" for the circuit shown in Fig. 2.239 (a).

[Bombay University]

**Solution.** Let the conductance be represented by  $g$ . Let all the sources be current sources. For this, a voltage-source in series with a resistor is transformed into its equivalent current source. This is done in Fig. 2.239 (b).

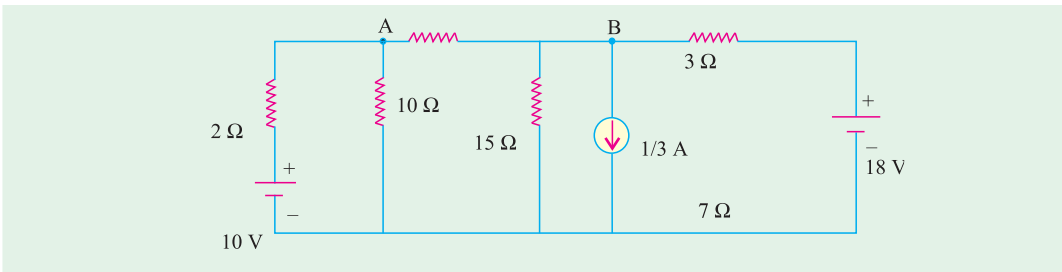


Fig. 2.239 (a)

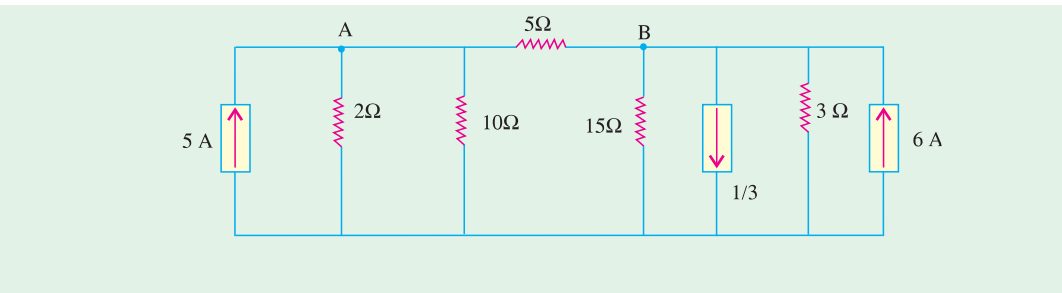


Fig. 2.239 (b). All Current Sources

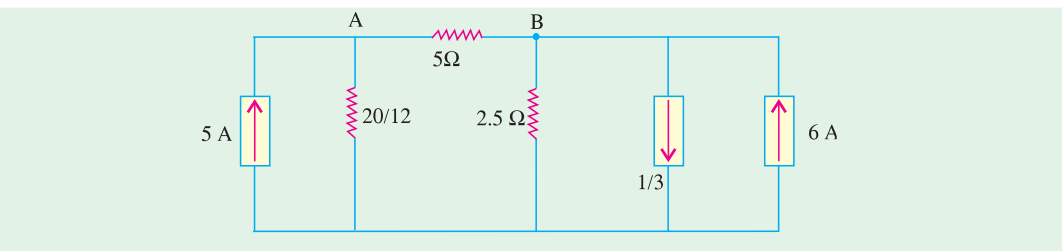


Fig. 2.239 (c)

Observing the circuit,  $g_{11} = (1/5) + 0.6 = 0.8$ ,  $g_{22} = 0.40 + 0.2 = 0.6$   
 $g_{12} = 0.2$ , Current sources : + 5 amp into 'A' + 5.67 amp into 'B'

$$\Delta = \begin{bmatrix} 0.8 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} = 0.44$$

$$\Delta_1 = \begin{bmatrix} 5 & -0.2 \\ 5.67 & 0.6 \end{bmatrix} = 4.134$$

$$\Delta_2 = \begin{bmatrix} 0.8 & 5 \\ -0.2 & 5.67 \end{bmatrix} = 5.526$$

$$V_A = 4.134/0.44 = 9.4 \text{ volts,}$$

$$V_B = 5.536/0.44 = 12.6 \text{ volts.}$$

Current in 5-ohm resistor

$$= (V_B - V_A)/5 = 0.64 \text{ amp}$$

**Check :** Apply Thevenin's Theorem :

$$V_A = 10 \times (10/12) = 8.333 \text{ V}$$

$$V_B = (17/3) \times 2.5 = 14.167 \text{ V}$$

$$V_{TH} = 14.167 - 8.333 = 5.834 \text{ V}$$

$$R_{TH} = 4.167$$

$$I_5 = 5.834/(4.167 + 5) = 0.64 \text{ A}$$

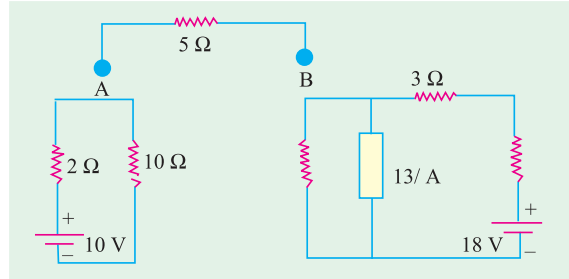


Fig. 2.239 (d) Thevenized Circuit

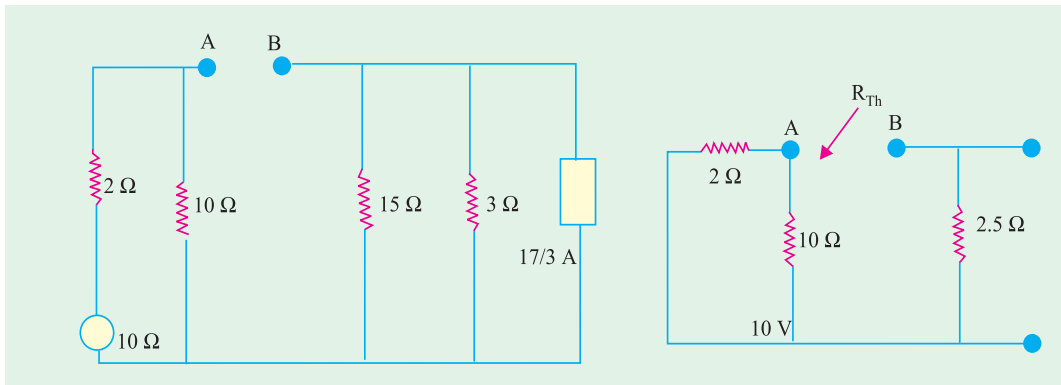


Fig. 2.239 (e) Right side simplified

Fig. 2.239 (f) Evaluating  $R_{TH}$

**Example. 2.124.** Find the magnitude  $R_L$  for the maximum power transfer in the circuit shown in Fig. 2.240 (a). Also find out the maximum power.

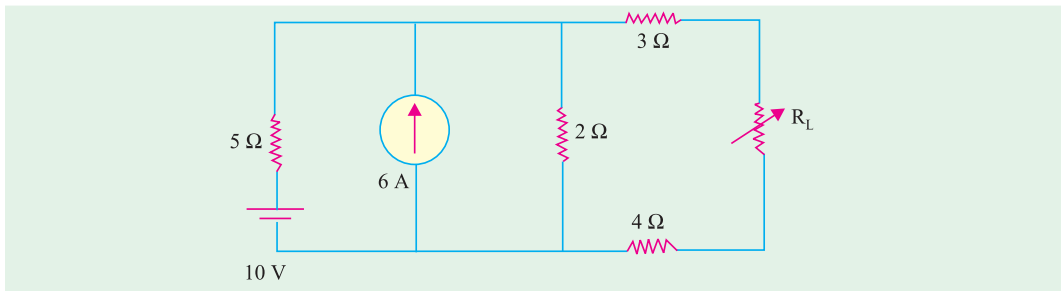


Fig. 2.240 (a)

**Solution.** Simplify by source transformations, as done in Fig. 2.240 (b), (c), (d)

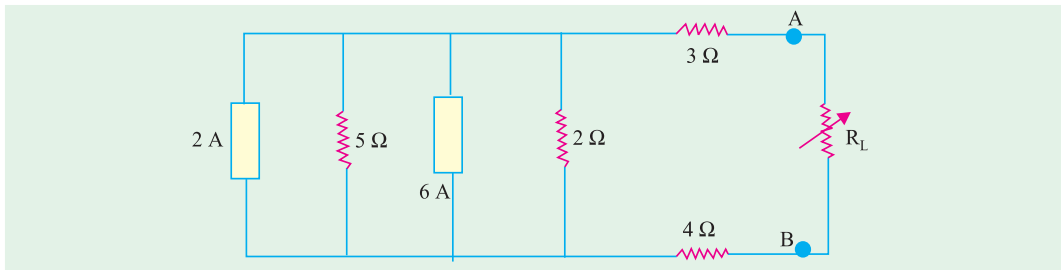


Fig. 2.240 (b)

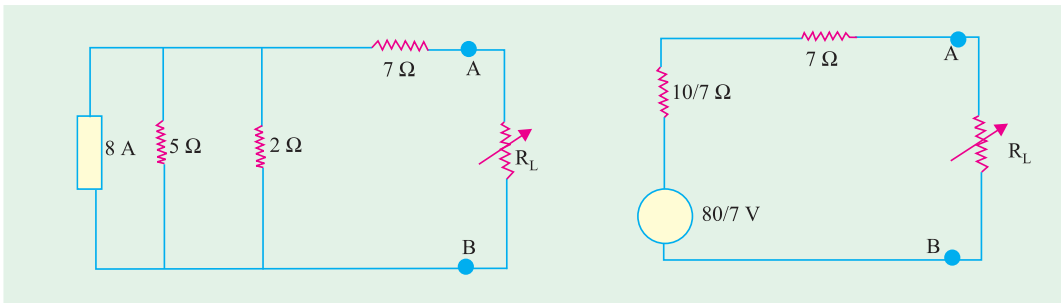


Fig. 2.240 (c)

Fig. 2.240 (d)

For maximum power,  $R_L = 7 + (10/7) = 8.43 \Omega$   
 Maximum power  $= [(80/7)/16.68]^2 \times 8.43 = 3.87 \text{ watts.}$

### Tutorial Problems No. 2.6

#### (a) Norton Theorem

- Find the Thevenin and Norton equivalent circuits for the active network shown in Fig. 2.241 (a). All resistance are in ohms. **[Hint : Use Superposition principle to find contribution of each source]**  
**[10 V source, series resistor = 5 Ω ; 2 A source, parallel resistance = 5 Ω]**
- Obtain the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 2.241 (b). All resistance values are in ohms.  
**[15 V source, series resistance = 5 Ω ; 3 A source, parallel resistance = 5 Ω]**

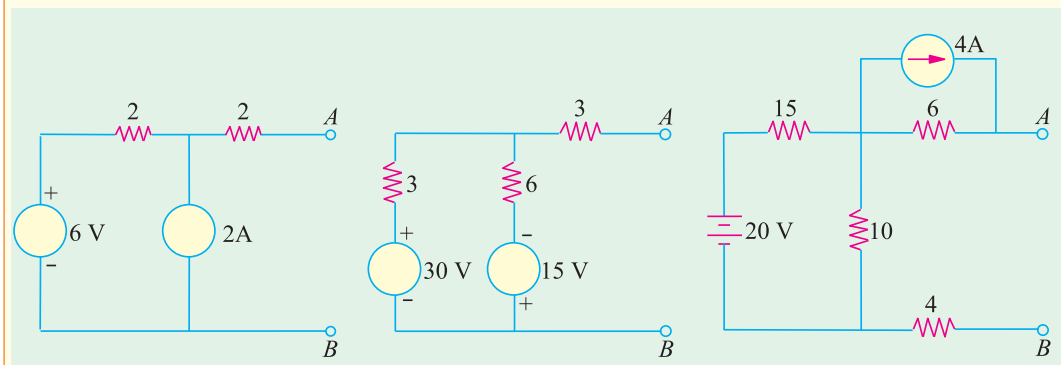


Fig. 2.241 (a)

Fig. 2.241 (b)

Fig. 2.241 (c)

- Find the Norton equivalent circuit for the active linear network shown in Fig. 2.241 (c). All resistances are in ohms. **Hint :** It would be easier to first find Thevenin's equivalent circuit].  
**[2 A source; parallel resistance = 16 Ω]**

4. Find Norton's equivalent circuit for the network shown in Fig. 2.249. Verify it through its Thevenin's equivalent circuit. [1 A, Parallel resistance = 6 Ω]
5. State the Tellegen's theorem and verify it by an illustration. Comment on the applicability of Tellegen's theorem on the types of networks. (Circuit and Field Theory, A.M.I.E. Sec. B, 1993)

**Solution.** Tellegen's Theorem can be stated as under :

For a network consisting of  $n$  elements if  $i_1, i_2, \dots, i_n$  are the currents flowing through the elements satisfying Kirchhoff's current law and  $v_1, v_2, \dots, v_n$  are the voltages across these elements satisfying Kirchhoff's law, then

$$\sum_{k=1}^n v_k i_k = 0$$

where  $v_k$  is the voltage across and  $i_k$  is the current through the  $k_{th}$  element. In other words, according to Tellegen's Theorem, the sum of instantaneous powers for the  $n$  branches in a network is always zero.

This theorem has wide applications. It is valid for any lumped network that contains any elements linear or non-linear, passive or active, time-variant or time-invariant.

**Explanation :** This theorem will be explained with the help of the simple circuit shown in Fig. 2.242. The total resistance seen by the battery is  $= 8 + 4 \parallel 4 = 10 \Omega$

Battery current  $I = 100/10 = 10$  A. This current divides equally at point B,

- Drop over 8 Ω resistor =  $8 \times 10 = 80$  V
- Drop over 4 Ω resistor =  $4 \times 5 = 20$  V
- Drop over 1 Ω resistor =  $1 \times 5 = 5$  V
- Drop over 3 Ω resistor =  $3 \times 5 = 15$  V

According to Tellegen's Theorem,  
 $= 100 \times 10 - 80 \times 10 - 20 \times 5 - 5 \times 5 - 15 \times 5 = 0$

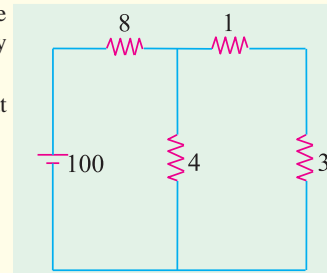


Fig. 2.242

**(b) Millman's Theorem**

6. Use Millman's theorem, to find the potential of point A with respect to the ground in Fig. 2.243. [ $V_A = 8.18$  V]
7. Using Millman's theorem, find the value of output voltage  $V_0$  in the circuit of Fig. 2.244. All resistances are in ohms. [4 V]

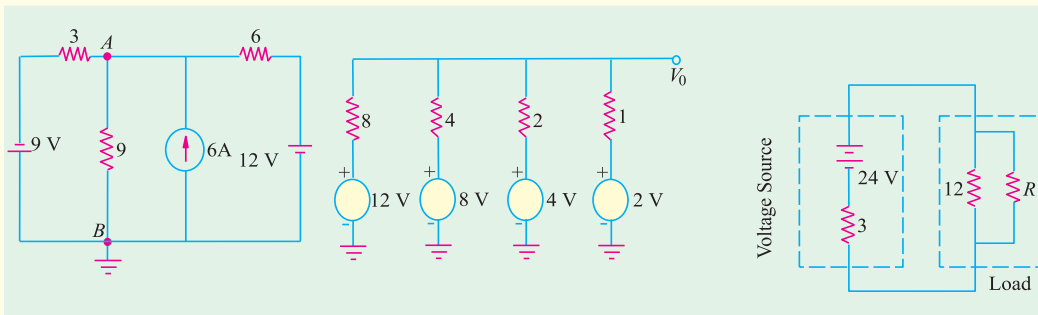


Fig. 2.243

Fig. 2.244

Fig. 2.245

**(b) MPT Theorem**

8. In Fig. 2.245 what value of  $R$  will allow maximum power transfer to the load ? Also calculate the maximum total load power. All resistances are in ohms. [4 Ω ; 48 W]
9. Use superposition theorem to find currents in various branches of the ckt in Fig. 2.246. (B.P.T.U., Orissa 2003) (Nagpur University, Summer 2002)

10. Find the resistance between point A and B for the circuit shown in Fig. 2.247.

(Nagpur University, Winter 2002)

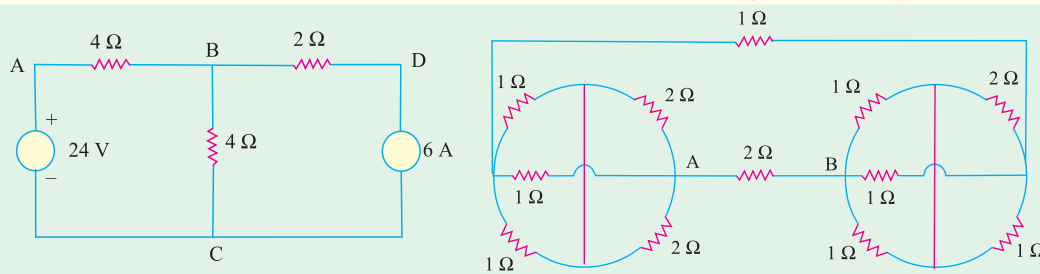


Fig. 2.246

Fig. 2.247

11. Apply the superposition theorem and find the current through 25 ohm resistance of the circuit shown in Fig. 2.248.

(Mumbai University 2002) (Nagpur University, Summer 2003)

12. Find the total current flowing through the circuit shown in Fig. 2.249 using star-delta transformation if the circuit is excited by 39 volts and the value of each resistor connected in circuit is 4 ohms.

(Ravishankar University, Raipur 2003) (Nagpur University, Summer 2003)

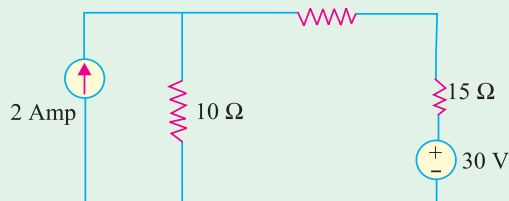


Fig. 2.248

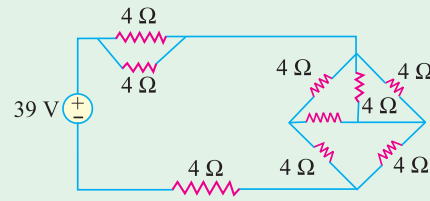


Fig. 2.249

13. Compute the power dissipated in the 9 ohm resistor in the Fig. 2.250 by applying Superposition Theorem. The voltage and current sources should be treated as ideal. All resistances are in ohm.

(Mumbai University 2003) (Nagpur University, Winter 2003)

14. Find the current in 11 ohm resistor in the Fig. 2.251 using star/delta conversion. All resistances are in ohm.

(Nagpur University, Winter 2003)

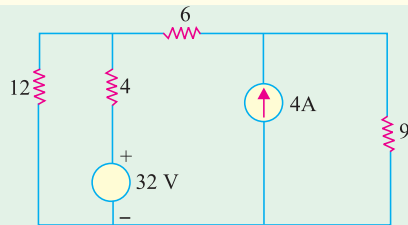


Fig. 2.250

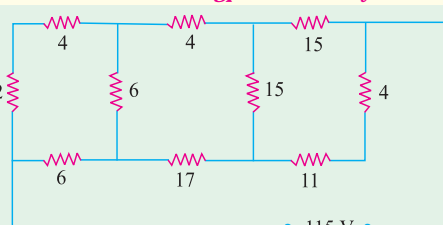


Fig. 2.251

15. Calculate current-flowing through “2 ohms” resistor in Fig. 2.252 by using Superposition theorem.

(Mumbai University 2003) (Nagpur University, Summer 2004)

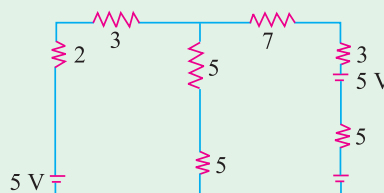


Fig. 2.252. All resistance are in ohms.



16. State and explain Superposition Theorem. *(Pune University 2003) (Nagpur University, Summer 2004)*
17. A cast iron ring of 40 cm diameter is wound with a coil. The coil carries a current of 3 amp and produces a flux of 3 mwb in the air gap. The length of air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate no. of turns of the coil. *(Nagpur University, Summer 2004)*
18. Using superposition theorem, calculate the current  $I_{AB}$  in the given circuit of Fig. 2.253. *(Gujrat University, Summer 2003)*
19. Using delta-star transformation, determine the current drawn from the source in the given circuit Fig.2.254. *(Gujrat University, Summer 2003)*

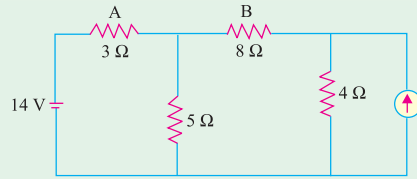


Fig. 2.253

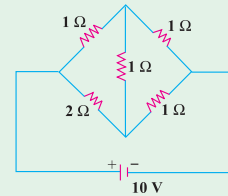


Fig. 2.254

20. State and explain Kirchoff's laws applied to electric circuit. *(Gujrat University, Summer2003)*
21. State Kirchoff's laws. *(Madras University, April 2002)*
22. Three resistances  $R_{ab}$ ,  $R_{bc}$  and  $R_{ca}$  are connected in delta. Obtain expressions for their equivalent star resistances. *(V.T.U., Belgaum Karnataka University, February 2002)*
23. In the circuit, shown in Fig. 2.255 determine the value of E so that the current I = 0. Use mesh method of analysis. *(V.T.U., Belgaum Karnataka University, January/February 2004)*
24. In Fig. 2.256 derive the expressions to replace a delta connected resistances by an equivalent star connected resistances. Determine the resistance between a and b. All the resistance and 1Ω each. *(V.T.U., Belgaum Karnataka University, January/February 2004)*

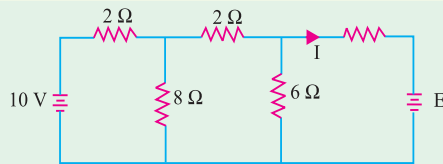


Fig. 2.255

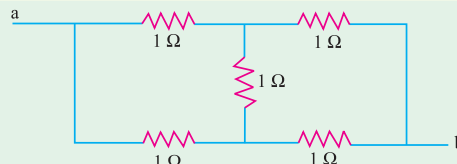


Fig. 2.256

25. Determine the values of I and R in the circuit shown in the Fig. 2.257. *(ESE 2003)*
26. In the circuit shown in the Fig. 2.258, S is closed at time  $t=0$ . Determine  $i_C(t)$  and the time constant. *(Pune University 2003) (ESE 2003)*
27. In the circuit shown in the Fig. 2.259, S is closed at  $t=0$ . Find the current  $i_C(t)$  through the capacitor at  $t=0$ . *(Pune University 2003) (ESE 2003)*

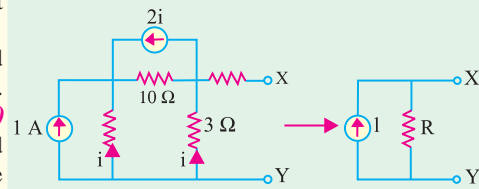


Fig. 2.257

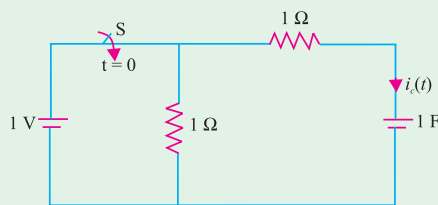


Fig. 2.258

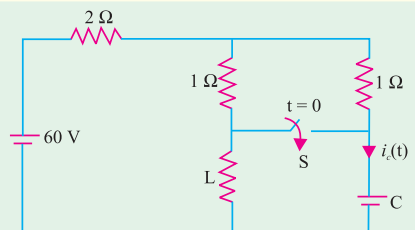


Fig. 2.259

## OBJECTIVE TESTS – 2

- Kirchhoff's current law is applicable to only
  - closed loops in a network
  - electronic circuits
  - junctions in a network
  - electric circuits.
- Kirchhoff's voltage law is concerned with
  - $IR$  drops
  - battery e.m.fs.
  - junction voltages
  - both (a) and (b)
- According to *KVL*, the *algebraic* sum of all  $IR$  drops and e.m.f.s in any closed loop of a network is always
  - zero
  - positive
  - negative
  - determined by battery e.m.fs.
- The *algebraic* sign of an  $IR$  drop is primarily dependent upon the
  - amount of current flowing through it
  - value of  $R$
  - direction of current flow
  - battery connection.
- Maxwell's loop current method of solving electrical networks
  - uses branch currents
  - utilizes Kirchhoff's voltage law
  - is confined to single-loop circuits
  - is a network reduction method.
- Point out of the **WRONG** statement. In the node-voltage technique of solving networks, choice of a reference node does not
  - affect the operation of the circuit
  - change the voltage across any element
  - alter the p.d. between any pair of nodes
  - affect the voltages of various nodes.
- For the circuit shown in the given Fig. 2.260, when the voltage  $E$  is 10 V, the current  $i$  is 1 A. If the applied voltage across terminal C-D is 100 V, the short circuit current

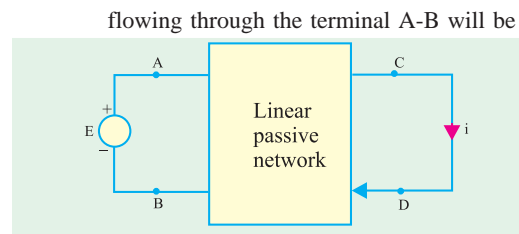


Fig. 2.260

- 0.1 A
  - 1 A
  - 10 A
  - 100 A
- (ESE 2001)

- The component inductance due to the internal flux-linkage of a non-magnetic straight solid circular conductor per metre length, has a constant value, and is independent of the conductor-diameter, because
  - All the internal flux due to a current remains concentrated on the peripheral region of the conductor.
  - The internal magnetic flux-density along the radial distance from the centre of the conductor increases proportionately to the current enclosed
  - The entire current is assumed to flow along the conductor-axis and the internal flux is distributed uniformly and concentrically
  - The current in the conductor is assumed to be uniformly distributed throughout the conductor cross-section

(ESE 2003)

- Two ac sources feed a common variable resistive load as shown in Fig. 2.261. Under the maximum power transfer condition, the power absorbed by the load resistance  $R_L$  is

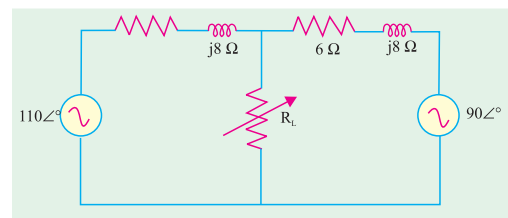


Fig. 2.261

- 2200 W
  - 1250 W
  - 1000 W
  - 625 W
- (GATE 2003)

## ANSWERS

1. c    2. d    3. a    4. c    5. b    6. d

## CHAPTER 3

**Learning Objectives**

- Effect of Electric Current
- Joule's Law of Electric Heating
- Thermal Efficiency
- S.I. Units
- Calculation of Kilo-watt Power of a Hydroelectric Station

**WORK, POWER  
AND  
ENERGY**

Today, life without electricity is highly unimaginable. Electric locomotives, heaters, and fans are some of the appliances and machines which convert electricity into work and energy

### 3.1. Effect of Electric Current

It is a matter of common experience that a conductor, when carrying current, becomes hot after some time. As explained earlier, an electric current is just a directed flow or drift of electrons through a substance. The moving electrons as they pass through molecules of atoms of that substance, collide with other electrons. This electronic collision results in the production of heat. This explains why passage of current is always accompanied by generation of heat.

### 3.2. Joule's Law of Electric Heating

The amount of work required to maintain a current of  $I$  amperes through a resistance of  $R$  ohm for  $t$  second is

$$\begin{aligned} \text{W.D.} &= I^2 R t \text{ joules} \\ &= VI t \text{ joules} && (\because R = V/I) \\ &= Wt \text{ joules} && (\because W = VI) \\ &= V^2 t/R \text{ joules} && (\because I = V/R) \end{aligned}$$

This work is converted into heat and is dissipated away. The amount of heat produced is

$$H = \frac{\text{work done}}{\text{mechanical equivalent of heat}} = \frac{W.D.}{J}$$

$$\begin{aligned} \text{where } J &= 4,186 \text{ joules/kcal} = 4,200 \text{ joules / kcal (approx)} \\ \therefore H &= I^2 R t / 4,200 \text{ kcal} = VI t / 4,200 \text{ kcal} \\ &= Wt / 4,200 \text{ kcal} = V^2 t / 4,200 R \text{ kcal} \end{aligned}$$

### 3.3. Thermal Efficiency

It is defined as the ratio of the heat actually utilized to the total heat produced electrically. Consider the case of the electric kettle used for boiling water. Out of the total heat produced (i) some goes to heat the apparatus itself i.e. kettle (ii) some is lost by radiation and convection etc. and (iii) the rest is utilized for heating the water. Out of these, the heat utilized for useful purpose is that in (iii). Hence, thermal efficiency of this electric apparatus is the ratio of the heat utilized for heating the water to the total heat produced.

Hence, the relation between heat produced electrically and heat absorbed usefully becomes

$$\frac{VIt}{J} \times \eta = ms (\theta_2 - \theta_1)$$

**Example 3.1.** The heater element of an electric kettle has a constant resistance of  $100 \Omega$  and the applied voltage is  $250 \text{ V}$ . Calculate the time taken to raise the temperature of one litre of water from  $15^\circ\text{C}$  to  $90^\circ\text{C}$  assuming that  $85\%$  of the power input to the kettle is usefully employed. If the water equivalent of the kettle is  $100 \text{ g}$ , find how long will it take to raise a second litre of water through the same temperature range immediately after the first.

(Electrical Engineering, Calcutta Univ.)



James Joule\*



In an electric kettle, electric energy is converted into heat energy.

\* James Joule was born in Salford, England, in 1818. He was a physicist who is credited with discovering the law of conservation of energy. Joule's name is used to describe the international unit of energy known as the joule.

**Solution.** Mass of water = 1000 g = 1 kg ( $\because 1 \text{ cm}^3$  weight 1 gram)  
 Heat taken by water =  $1 \times (90 - 15) = 75 \text{ kcal}$   
 Heat taken by the kettle =  $0.1 \times (90 - 15) = 7.5 \text{ kcal}$   
 Total heat taken =  $75 + 7.5 = 82.5 \text{ kcal}$   
 Heat produced electrically  $H = I^2 R t / J \text{ kcal}$   
 Now,  $I = 250/100 = 2/5 \text{ A}$ ,  $J = 4,200 \text{ J/kcal}$ ;  $H = 2.5^2 \times 100 \times t / 4200 \text{ kcal}$   
 Heat actually utilized for heating one litre of water and kettle  
 =  $0.85 \times 2.5^2 \times 100 \times t / 4,200 \text{ kcal}$   
 $\therefore \frac{0.85 \times 6.25 \times 100 \times t}{4,200} = 82.5 \quad \therefore t = \mathbf{10 \text{ min } 52 \text{ second}}$

In the second case, heat would be required only for heating the water because kettle would be already hot.

$$\therefore 75 = \frac{0.85 \times 6.25 \times 100 \times t}{4,200} \quad \therefore t = \mathbf{9 \text{ min } 53 \text{ second}}$$

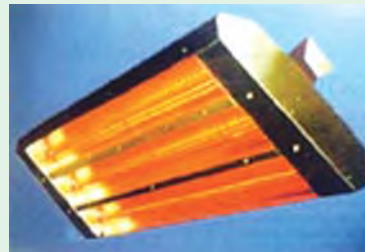
**Example 3.2.** Two heater A and B are in parallel across supply voltage  $V$ . Heater A produces 500 kcal in 200 min. and B produces 1000 kcal in 10 min. The resistance of A is 10 ohm. What is the resistance of B? If the same heaters are connected in series across the voltage  $V$ , how much heat will be produced in kcal in 5 min? **(Elect. Science - II, Allahabad Univ. 1992)**

**Solution.** Heat produced =  $\frac{V^2 t}{JR} \text{ kcal}$   
 For heater A,  $500 = \frac{V^2 \times (20 \times 60)}{10 \times J} \quad \dots(i)$   
 For heater B,  $1000 = \frac{V^2 \times (10 \times 60)}{R \times J} \quad \dots(ii)$

From Eq. (i) and (ii), we get,  $R = \mathbf{2.5 \Omega}$



(a)



(b)



(c)



(d)

In this a, b, and c are heaters which convert electric energy into heat; and d is the electric bulb which converts electric energy into light and heat

When the two heaters are connected in series, let  $H$  be the amount of heat produced in kcal. Since combined resistance is  $(10 + 2.5) = 12.5 \Omega$ , hence

$$H = \frac{V^2 \times (5 \times 60)}{12.5 \times J} \quad \dots (iii)$$

Dividing Eq. (iii) by Eq. (i), we have  $H = 100 \text{ kcal}$ .

**Example 3.3.** An electric kettle needs six minutes to boil 2 kg of water from the initial temperature of  $20^\circ\text{C}$ . The cost of electrical energy required for this operation is 12 paise, the rate being 40 paise per kWh. Find the kW-rating and the overall efficiency of the kettle.

(F.Y. Engg. Pune Univ.)

**Solution.** Input energy to the kettle =  $\frac{12 \text{ paise}}{40 \text{ paise/kWh}} = 0.3 \text{ kWh}$

$$\text{Input power} = \frac{\text{energy in kWh}}{\text{Time in hours}} = \frac{0.3}{(6/60)} = 3 \text{ kW}$$

Hence, the power rating of the electric kettle is 3 kW

Energy utilised in heating the water

$$= mst = 2 \times 1 \times (100 - 20) = 160 \text{ kcal} = 160 / 860 \text{ kWh} = 0.186 \text{ kWh.}$$

Efficiency = output/input =  $0.186/0.3 = 0.62 = 62\%$ .

### 3.4. S.I. Units

**1. Mass.** It is quantity of matter contained in a body.

Unit of mass is kilogram (kg). Other multiples commonly used are :

$$1 \text{ quintal} = 100 \text{ kg, } 1 \text{ tonne} = 10 \text{ quintals} = 1000 \text{ kg}$$

**2. Force.** Unit of force is newton (N). Its definition may be obtained from Newton's Second Law of Motion i.e.  $F = ma$ .

If  $m = 1 \text{ kg}$  ;  $a = 1 \text{ m/s}^2$ , then  $F = 1 \text{ newton}$ .

Hence, one newton is that force which can give an acceleration of  $1 \text{ m/s}^2$  to a mass of 1 kg.

Gravitational unit of force is kilogram-weight (kg-wt). It may be defined as follows :

or

It is the force which can impart an acceleration of  $9.8 \text{ m/s}^2$  to a mass of 1 kg.

It is the force which can impart an acceleration of  $1 \text{ m/s}^2$  to a mass of 9.8 kg.

Obviously,  $1 \text{ kg-wt.} = 9.8 \text{ N}$

**3. Weight.** It is the force with which earth pulls a body downwards. Obviously, its units are the same as for force.

(a) Unit of weight is newton (N)

(b) Gravitational unit of weight is kg-wt.\*

**Note.** If a body has a mass of  $m \text{ kg}$ , then its weight,  $W = mg \text{ newtons} = 9.8 \text{ newtons}$ .

**4. Work,** If a force  $F$  moves a body through a distance  $S$  in its direction of application, then

$$\text{Work done } W = F \times S$$

(a) Unit of work is joule (J).

If, in the above equation,  $F = 1 \text{ N}$  ;  $S = 1 \text{ m}$  ; then work done = 1 m.N or joule.

Hence, one joule is the work done when a force of 1 N moves a body through a distance of 1 m in the direction of its application.

(b) Gravitational unit of work is m-kg. wt or m-kg\*\*.

\* Often it is referred to as a force of 1 kg, the word 'wt' being omitted. To avoid confusion with mass of 1 kg, the force of 1 kg is written in engineering literature as kgf instead of kg. wt.

\*\* Generally the work 'wt' is omitted and the unit is simply written as m-kg.

If  $F = 1 \text{ kg-wt}$ ;  $S = 1 \text{ m}$ ; then W.D. =  $1 \text{ m-kg}$ .  $Wt = 1 \text{ m-kg}$ .  
Hence, one m-kg is the work done by a force of one kg-wt when applied over a distance of one metre.

Obviously,  $1 \text{ m-kg} = 9.8 \text{ m-N}$  or J.

**5. Power.** It is the rate of doing work. Its units is watt (W) which represents 1 joule per second.

$$1 \text{ W} = 1 \text{ J/s}$$

If a force of  $F$  newton moves a body with a velocity of  $v \text{ m./s}$  then

$$\text{power} = F \times v \text{ watt}$$

If the velocity  $v$  is in km/s, then

$$\text{power} = F \times v \text{ kilowatt}$$

**6. Kilowatt-hour (kWh) and kilocalorie (kcal)**

$$1 \text{ kWh} = 1000 \times 1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J}$$

$$1 \text{ kcal} = 4,186 \text{ J} \quad \therefore 1 \text{ kWh} = 36 \times 10^5 / 4,186 = 860 \text{ kcal}$$

**7. Miscellaneous Units**

(i) 1 watt hour (Wh) =  $1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600 \text{ J}$

(ii) 1 horse power (metric) =  $75 \text{ m-kg/s} = 75 \times 9.8 = 735.5 \text{ J/s}$  or watt

(iii) 1 kilowatt (kW) =  $1000 \text{ W}$  and 1 megawatt (MW) =  $10^6 \text{ W}$

### 3.5. Calculation of Kilo-watt Power of a Hydroelectric Station

Let  $Q$  = water discharge rate in cubic metres/second ( $\text{m}^3/\text{s}$ ),  $H$  = net water head in metre (m).  
 $g = 9.81$ ,  $\eta$ ; overall efficiency of the hydroelectric station expressed as a fraction.

Since  $1 \text{ m}^3$  of water weighs  $1000 \text{ kg}$ ., discharge rate is  $1000 Q \text{ kg/s}$ .

When this amount of water falls through a height of  $H$  metre, then energy or work available per second or available power is

$$= 1000 QgH \text{ J/s} \quad \text{or} \quad W = QgH \text{ kW}$$

Since the overall station efficiency is  $\eta$  power actually available is  $= 9.81 \eta QH \text{ kW}$ .

**Example 3.4.** A de-icing equipment fitted to a radio aerial consists of a length of a resistance wire so arranged that when a current is passed through it, parts of the aerial become warm. The resistance wire dissipates  $1250 \text{ W}$  when  $50 \text{ V}$  is maintained across its ends. It is connected to a d.c. supply by  $100 \text{ metres}$  of this copper wire, each conductor of which has resistance of  $0.006 \Omega/\text{m}$ .

Calculate

- (a) the current in the resistance wire  
(b) the power lost in the copper connecting wire  
(c) the supply voltage required to maintain  $50 \text{ V}$  across the heater itself.

**Solution.** (a) Current = wattage/voltage =  $1250/50 = 25 \text{ A}$   
(b) Resistance of one copper conductor =  $0.006 \times 100 = 0.6 \Omega$   
Resistance of both copper conductors =  $0.6 \times 2 = 1.2 \Omega$   
Power loss =  $I^2 R \text{ watts} = 25^2 \times 1.2 = 750 \text{ W}$   
(c) Voltage drop over connecting copper wire =  $IR \text{ volt} = 25 \times 1.2 = 30 \text{ V}$   
 $\therefore$  Supply voltage required =  $50 + 30 = 80 \text{ V}$

**Example 3.5.** A factory has a  $240\text{-V}$  supply from which the following loads are taken :  
Lighting : Three hundred  $150\text{-W}$ , four hundred  $100 \text{ W}$  and five hundred  $60\text{-W}$  lamps  
Heating :  $100 \text{ kW}$   
Motors : A total of  $44.76 \text{ kW}$  ( $60 \text{ b.h.p.}$ ) with an average efficiency of  $75 \text{ percent}$   
Misc. : Various load taking a current of  $40 \text{ A}$ .

Assuming that the lighting load is on for a period of 4 hours/day, the heating for 10 hours per day and the remainder for 2 hours/day, calculate the weekly consumption of the factory in kWh when working on a 5-day week.

What current is taken when the lighting load only is switched on ?

**Solution.** The power consumed by each load can be tabulated as given below :

	Power consumed
Lighting	$300 \times 150 = 45,000 = 45 \text{ kW}$
	$400 \times 100 = 40,000 = 40 \text{ kW}$
	$500 \times 60 = 30,000 = 30 \text{ kW}$
	Total = 115 kW
Heating	= 100 kW
Motors	= $44.76/0.75 = 59.7 \text{ kW}$
Misc.	= $240 \times 40/1000 = 9.6 \text{ kW}$

Similarly, the energy consumed/day can be tabulated as follows :

	Energy consumed / day
Lighting	= $115 \text{ kW} \times 4 \text{ hr} = 460 \text{ kWh}$
Heating	= $100 \text{ kW} \times 10 \text{ hr} = 1,000 \text{ kWh}$
Motors	= $59.7 \text{ kW} \times 2 \text{ hr} = 119.4 \text{ kWh}$
Misc.	= $9.6 \text{ kW} \times 2 \text{ hr} = 19.2 \text{ kWh}$
Total daily consumption	= 1,598.6 kWh
Weekly consumption	= $1,598.6 \times 5 = 7,993 \text{ kWh}$
Current taken by the lighting load alone	= $115 \times 1000/240 = 479 \text{ A}$



A factory needs electric power for lighting and running motors

**Example 3.6.** A Diesel-electric generating set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35% (a) calculate the mass of oil required per hour (b) the electric energy generated per tonne of the fuel.

**Solution.** Output = 25 kW, Overall  $\eta = 0.35$ ,

$$\text{Input} = 25/0.35 = 71.4 \text{ kW}$$

$$\therefore \text{input per hour} = 71.4 \text{ kWh} = 71.4 \times 860 = 61,400 \text{ kcal}$$

Since 1 kg of fuel-oil produces 12,500 kcal

(a)  $\therefore$  mass of oil required =  $61,400/12,500 = 4.91 \text{ kg}$

(b) 1 tonne of fuel = 1000 kg

Heat content =  $1000 \times 12,500 = 12.5 \times 10^6 \text{ kcal}$

$$= 12.5 \times 10^6/860 = 14,530 \text{ kWh}$$

Overall  $\eta = 0.35\%$   $\therefore$  energy output =  $14,530 \times 0.35 = 5,088 \text{ kWh}$

**Example 3.7.** The effective water head for a 100 MW station is 220 metres. The station supplies full load for 12 hours a day. If the overall efficiency of the station is 86.4%, find the volume of water used.

**Solution.** Energy supplied in 12 hours =  $100 \times 12 = 1200 \text{ MWh}$

$$= 12 \times 10^5 \text{ kWh} = 12 \times 10^5 \times 3^5 \times 10^5 \text{ J} = 43.2 \times 10^{11} \text{ J}$$

Overall  $\eta = 86.4\% = 0.864$   $\therefore$  Energy input =  $43.2 \times 10^{11}/0.864 = 5 \times 10^{12} \text{ J}$

Suppose  $m$  kg is the mass of water used in 12 hours, then  $m \times 9.81 \times 220 = 5 \times 10^{12}$

$$\therefore m = 5 \times 10^{12}/9.81 \times 220 = 23.17 \times 10^8 \text{ kg}$$

Volume of water =  $23.17 \times 10^8/10^3 = 23.17 \times 10^5 \text{ m}^3$

( $\therefore 1 \text{ m}^3$  of water weighs  $10^3 \text{ kg}$ )



Diesel electric generator set



**Example 3.8.** Calculate the current required by a 1,500 volts d.c. locomotive when drawing 100 tonne load at 45 km.p.h. with a tractive resistance of 5 kg/tonne along (a) level track (b) a gradient of 1 in 50. Assume a motor efficiency of 90 percent.

**Solution.** As shown in Fig. 3.1 (a), in this case, force required is equal to the tractive resistance only.

$$\begin{aligned} \text{(a) Force required at the rate of 5 kg-wt/tonne} &= 100 \times 5 \text{ kg-wt.} = 500 \times 9.81 = 4905 \text{ N} \\ \text{Distance travelled/second} &= 45 \times 1000/3600 = 12.5 \text{ m/s} \\ \text{Power output of the locomotive} &= 4905 \times 12.5 \text{ J/s or watt} = 61,312 \text{ W} \\ \eta = 0.9 \quad \therefore \text{Power input} &= 61,312/0.9 = 68,125 \text{ W} \\ \therefore \text{Current drawn} &= 68,125/1500 = \mathbf{45.41 \text{ A}} \end{aligned}$$

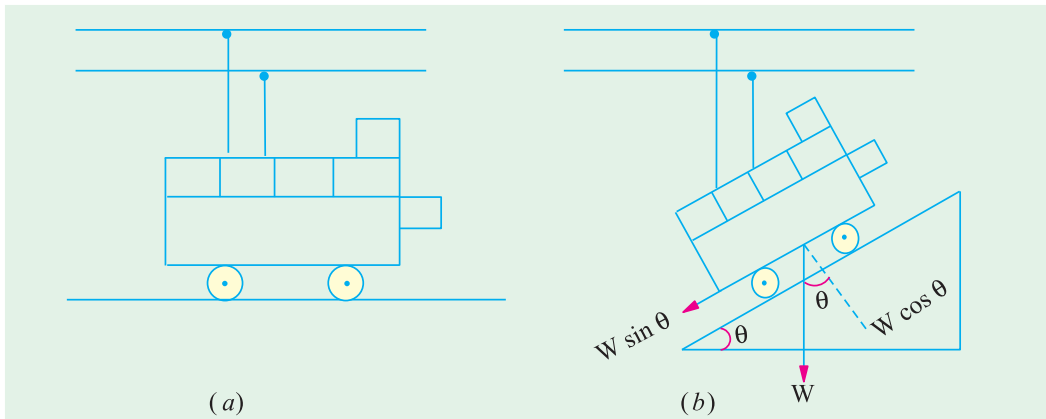


Fig. 3.1

(b) When the load is drawn along the gradient [Fig. 3.1 (b)], component of the weight acting downwards =  $100 \times 1/50 = 2$  tonne-wt = 2000 kg-wt =  $2000 \times 9.81 = 19,620 \text{ N}$

$$\text{Total force required} = 19,620 + 4,905 = 24,525 \text{ N}$$

$$\text{Power output} = \text{force} \times \text{velocity} = 24,525 \times 12.5 \text{ watt}$$

$$\text{Power input} = 24,525 \times 12.5/0.9 \text{ W}; \text{ Current drawn} = \frac{24,525 \times 12.5}{0.9 \times 1500} = \mathbf{227 \text{ A}}$$

**Example 3.9.** A room measures 4 m × 7 m × 5 m and the air in it has to be always kept 15°C higher than that of the incoming air. The air inside has to be renewed every 35 minutes. Neglecting radiation loss, calculate the rating of the heater suitable for this purpose. Take specific heat of air as 0.24 and density as 1.27 kg/m<sup>3</sup>.

$$\text{Solution. Volume of air to be changed per second} = 4 \times 7 \times 5/35 = 60 = 1/15 \text{ m}^3$$

$$\text{Mass of air to be changed/second} = (1/15) \times 1.27 \text{ kg}$$

$$\text{Heat required/second} = \text{mass/second} \times \text{sp. heat} \times \text{rise in temp.}$$

$$= (1.27/15) \times 0.24 \times 15 \text{ kcal/s} = 0.305 \text{ kcal/s}$$

$$= 0.305 \times 4186 \text{ J/s} = \mathbf{1277 \text{ watt.}}$$

**Example 3.10.** A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.

(Principles of Elect. Engg.-I, Jadavpur Univ.)

$$\text{Solution. Angular speed } \omega = 2\pi \text{ N}/60 \text{ rad/s} = 2\pi \times 75/60 = 7.85 \text{ rad/s}$$

Power required for rotating the engine at this angular speed is

$$P = \text{torque} \times \text{angular speed} = \omega T \text{ watt} = 60 \times 7.85 = \mathbf{471 \text{ W}}$$

$$\begin{aligned}\text{Energy required per start is} &= \text{power} \times \text{time per start} = 471 \times 8 = 3,768 \text{ watt-s} = 3,768 \text{ J} \\ &= 3,768/3600 = 1.047 \text{ Wh}\end{aligned}$$

$$\begin{aligned}\text{Energy drawn from the battery taking into consideration the efficiency of the motor and gearing} \\ &= 1.047/0.25 = 4.188 \text{ Wh}\end{aligned}$$

$$\text{No. of start possible with a fully-charged battery} = 100/4.188 = \mathbf{24 \text{ (approx.)}}$$

**Example 3.11.** Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75°C. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy? Assume efficiencies of the heating equipment and lifting equipment to be 90% and 70% respectively. (Elect. Engg. A.M. Ae. S.I.)

**Solution.** Mass of water heated = 45 kg. Heat required =  $45 \times 75 = 3,375 \text{ kcal}$

Heat produced electrically =  $3,375/0.9 = 3,750 \text{ kcal}$ . Now, 1 kcal = 4,186 J

∴ electrical energy expended =  $3,750 \times 4,186 \text{ J}$

Energy available for lifting the load is =  $0.7 \times 3,750 \times 4,186 \text{ J}$

If  $h$  metre is the height through which the load of 5 tonnes can be lifted, then potential energy of the load =  $mgh$  joules =  $5 \times 1000 \times 9.81 h$  joules

$$\therefore 5000 \times 9.81 \times h = 0.7 \times 3,750 \times 4,186 \quad \therefore h = \mathbf{224 \text{ metres}}$$

**Example 3.12.** An hydro-electric station has a turbine of efficiency 86% and a generator of efficiency 92%. The effective head of water is 150 m. Calculate the volume of water used when delivering a load of 40 MW for 6 hours. Water weighs 1000 kg/m<sup>3</sup>.

**Solution.** Energy output =  $40 \times 6 = 240 \text{ MWh}$   
 $= 240 \times 10^3 \times 36 \times 10^5 = 864 \times 10^9 \text{ J}$

Overall  $\eta = 0.86 \times 0.92$  ∴ Energy input =  $\frac{864 \times 10^9}{0.86 \times 0.92} = 10.92 \times 10^{11} \text{ J}$

Since the head is 150 m and 1 m<sup>3</sup> of water weighs 1000 kg, energy contributed by each m<sup>3</sup> of water =  $150 \times 1000 \text{ m-kg (wt)} = 150 \times 1000 \times 9.81 \text{ J} = 147.2 \times 10^4 \text{ J}$

$$\therefore \text{Volume of water for the required energy} = \frac{10.92 \times 10^{11}}{147.2 \times 10^4} = \mathbf{74.18 \times 10^4 \text{ m}^3}$$

**Example 3.13.** An hydroelectric generating station is supplied from a reservoir of capacity 6 million m<sup>3</sup> at a head of 170 m.

(i) What is the available energy in kWh if the hydraulic efficiency be 0.8 and the electrical efficiency 0.9?

(ii) Find the fall in reservoir level after a load of 12,000 kW has been supplied for 3 hours, the area of the reservoir is 2.5 km<sup>2</sup>.

(iii) If the reservoir is supplied by a river at the rate of 1.2 m<sup>3</sup>/s, what does this flow represent in kW and kWh/day? Assume constant head and efficiency.

Water weighs 1 tonne/m<sup>3</sup>.

(Elect. Engineering-I, Osmania Univ.)

**Solution. (i)** Wt. of water  $W = 6 \times 10^6 \times 1000 \text{ kg wt} = 6 \times 10^9 \times 9.81 \text{ N}$

Water head = 170 m

Potential energy stored in this much water

$$= Wh = 6 \times 10^9 \times 9.81 \times 170 \text{ J} = 10^{12} \text{ J}$$

Overall efficiency of the station =  $0.8 \times 0.9 = 0.71$

$$\begin{aligned}\therefore \text{energy available} &= 0.71 \times 10^{12} \text{ J} = 71 \times 10^{11}/36 \times 10^5 \\ &= \mathbf{2 \times 10^6 \text{ kWh}}\end{aligned}$$

(ii) Energy supplied =  $12,000 \times 3 = 36,000 \text{ kWh}$

Energy drawn from the reservoir after taking into consideration the overall efficiency of the station

$$= 36,000/0.71 = 5 \times 10^4 \text{ kWh}$$

$$= 5 \times 10^4 \times 36 \times 10^5 = 18 \times 10^{10} \text{ J}$$

If  $m$  kg is the mass of water used in two hours, then, since water head is 170 m

$$mgh = 18 \times 10^{10}$$

or  $m \times 9.81 \times 170 = 18 \times 10^{10} \therefore m = 1.08 \times 10^8 \text{ kg}$

If  $h$  metre is the fall in water level, then

$$h \times \text{area} \times \text{density} = \text{mass of water}$$

$$\therefore h \times (2.5 \times 10^6) \times 1000 = 1.08 \times 10^8 \therefore h = 0.0432 \text{ m} = \mathbf{4.32 \text{ cm}}$$

(iii) Mass of water stored per second =  $1.2 \times 1000 = 1200 \text{ kg}$

$$\text{Wt. of water stored per second} = 1200 \times 9.81 \text{ N}$$

$$\text{Power stored} = 1200 \times 9.81 \times 170 \text{ J/s} = \mathbf{2,000 \text{ kW}}$$

$$\text{Power actually available} = 2,000 \times 0.72 = 1440 \text{ kW}$$

$$\text{Energy delivered /day} = 1440 \times 24 = \mathbf{34,560 \text{ kWh}}$$

**Example 3.14.** The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is  $45 \times 10^{10} \text{ kg}$ . What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85%. Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.

(Power System, Allahabad Univ.)

**Solution.** Actual head available =  $230 - 30 = 200 \text{ m}$

Energy available at the turbine house =  $mgh$

$$= 45 \times 10^{10} \times 9.81 \times 200 = 88.29 \times 10^{13} \text{ J}$$

$$= \frac{88.29 \times 10^{13}}{36 \times 10^5} = \mathbf{24.52 \times 10^7 \text{ kWh}}$$

Overall  $\eta = 0.85$

$$\therefore \text{Energy output} = 24.52 \times 10^7 \times 0.85 = \mathbf{20.84 \times 10^7 \text{ kWh}}$$

The kinetic energy of water is just equal to its loss of potential energy.

$$\frac{1}{2} mv^2 = mgh \therefore v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 200} = 62.65 \text{ m/s}$$

Power available from a mass of  $m$  kg when it flows with a velocity of  $v$  m/s is

$$P = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times 62.65^2 \text{ J/s or W}$$

Equating this to the maximum demand on the station, we get

$$\frac{1}{2} m 62.65^2 = 45 \times 10^6 \therefore m = 22,930 \text{ kg/s}$$

If  $A$  is the total area of the pipes in  $\text{m}^2$ , then the flow of water is  $Av \text{ m}^3/\text{s}$ . Mass of water flowing/second =  $Av \times 10^3 \text{ kg}$  ( $\therefore 1 \text{ m}^3$  of water = 1000 kg)

$$\therefore A \times v \times 10^3 = 22,930 \text{ or } A = \frac{22,930}{62.65 \times 10^3} = 0.366 \text{ m}^2$$

If ' $d$ ' is the diameter of each pipe, then  $\pi d^2/4 = 0.183 \therefore d = \mathbf{0.4826 \text{ m}}$

**Example 3.15.** A large hydel power station has a head of 324 m and an average flow of 1370 cubic metres/sec. The reservoir is a lake covering an area of 6400 sq. km, Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate

(i) the available electric power ;

(ii) the number of days this power could be supplied for a drop in water level by 1 metre.

(AMIE Sec. B Power System I (E-6) Winter)



Hydroelectric generators

**Solution. (i)** Available power =  $9.81 \eta QH \text{ kW} = (0.9 \times 0.95) \times 1370 \times 324 = 379,524 \text{ kW} = 379.52 \text{ MW}$ .

**(ii)** If  $A$  is the lake area in  $\text{m}^2$  and  $h$  metre is the fall in water level, the volume of water used is  $= A \times h = \text{m}^3$ . The time required to discharge this water is  $Ah / Q$  second.

Now,  $A = 6400 \times 10^6 \text{ m}^2$ ;  $h = 1 \text{ m}$ ;  $Q = 1370 \text{ m}^3 / \text{s}$ .

$\therefore t = 6400 \times 10^6 \times 1 / 1370 = 4.67 \times 10^6 \text{ second} = 540686 \text{ days}$

**Example 3.16.** The reservoir area of a hydro-electric generating plant is spread over an area of 4 sq km with a storage capacity of 8 million cubic-metres. The net head of water available to the turbine is 70 metres. Assuming an efficiency of 0.87 and 0.93 for water turbine and generator respectively, calculate the electrical energy generated by the plant.

Estimate the difference in water level if a load of 30 MW is continuously supplied by the generator for 6 hours.

(Power System I-AMIE Sec. B)

**Solution.** Since 1 cubic metre of water weighs 1000 kg., the reservoir capacity =  $8 \times 10^6 \text{ m}^3 = 8 \times 10^6 \times 1000 \text{ kg.} = 8 \times 10^9 \text{ kg.}$

Wt. of water,  $W = 8 \times 10^9 \text{ kg.}$  Wt.  $8 \times 10^9 \times 9.81 = 78.48 \times 10^9 \text{ N.}$  Net water head = 70 m.

Potential energy stored in this much water =  $Wh = 78.48 \times 10^9 \times 70 = 549.36 \times 10^{10} \text{ J}$

Overall efficiency of the generating plant =  $0.87 \times 0.93 = 0.809$

Energy available =  $0.809 \times 549.36 \times 10^{10} \text{ J} = 444.4 \times 10^{10} \text{ J}$   
 $= 444.4 \times 10^{10} / 36 \times 10^5 = 12.34 \times 10^5 \text{ kWh}$

Energy supplied in 6 hours =  $30 \text{ MW} \times 6 \text{ h} = 180 \text{ MWh}$   
 $= 180,000 \text{ kWh}$

Energy drawn from the reservoir after taking into consideration, the overall efficiency of the station =  $180,000 / 0.809 = 224,500 \text{ kWh} = 224,500 \times 36 \times 10^5$   
 $= 80.8 \times 10^{10} \text{ J}$

If  $m \text{ kg.}$  is the mass of water used in 6 hours, then since water head is 70 m,  
 $mgh = 80.8 \times 10^{10}$  or  $m \times 9.81 \times 70 = 80.8 \times 10^{10} \therefore m = 1.176 \times 10^9 \text{ kg.}$

If  $h$  is the fall in water level, then  $h \times \text{area} \times \text{density} = \text{mass of water}$

$\therefore h \times (4 \times 10^6) \times 1000 = 1.176 \times 10^9 \therefore h = 0.294 \text{ m} = 29.4 \text{ cm.}$

**Example 3.17.** A proposed hydro-electric station has an available head of 30 m, catchment area of  $50 \times 10^6 \text{ sq.m.}$  the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies : Penstock 95%, Turbine 80% and Generator 85.  
 (Elect. Engg. AMIETE Sec. A Part II)

**Solution.** Volume of water available =  $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7 \text{ m}^3$

Mass of water available =  $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10} \text{ kg}$

This quantity of water is available for a period of one year. Hence, quantity available per second =  $4.2 \times 10^{10} / 365 \times 24 \times 3600 = 1.33 \times 10^3$ .

Available head = 30 m

Potential energy available =  $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3 \text{ J}$

Since this energy is available per second, hence power available is =  $391 \times 10^3 \text{ J/s} = 391 \times 10^3 \text{ W} = 391 \text{ kW}$

Overall efficiency =  $0.95 \times 0.80 \times 0.85 = 0.646$

The power that could be generated =  $391 \times 0.646 = 253 \text{ kW.}$

**Example 3.18.** In a hydro-electric generating station, the mean head (i.e. the difference of height between the mean level of the water in the lake and the generating station) is 400 metres. If the overall efficiency of the generating stations is 70%, how many litres of water are required to generate 1 kWh of electrical energy? Take one litre of water to have a mass of 1 kg.



In a hydel plant, potential energy of water is converted into kinetic energy and then into electricity.

(F.Y. Engg. Pune Univ.)

**Solution.** Output energy = 1 kWh =  $36 \times 10^5$  J

Input energy =  $36 \times 10^5 / 0.7 = 5.14 \times 10^6$  J

If  $m$  kg. water is required, then

$mgh = 5.14 \times 10^6$  or  $m \times 9.81 \times 400 = 5.14 \times 10^6$ ,  $\therefore = 1310$  kg.

**Example 3.19.** A 3-tonne electric-motor-operated vehicle is being driven at a speed of 24 km/hr upon an incline of 1 in 20. The tractive resistance may be taken as 20 kg per tonne. Assuming a motor efficiency of 85% and the mechanical efficiency between the motor and road wheels of 80%, calculate

(a) the output of the motor

(b) the current taken by motor if it gets power from a 220-V source.

Calculate also the cost of energy for a run of 48 km, taking energy charge as 40 paise/kWh.

**Solution.** Different forces acting on the vehicle are shown in Fig. 3.2.

Wt. of the vehicle =  $3 \times 10^3 = 3000$  kg-wt

Component of the weight of the vehicle acting downwards along the slope =  $3000 \times 1/20 = 150$  kg-wt

Tractive resistance =  $3 \times 20 = 60$  kg-wt  
Total downward force =  $150 + 60 = 210$  kg-wt

=  $210 \times 9.81 = 2,060$  N

Distance travelled/second =  $24,000/3600 = 20/3$  m/s

Output at road wheels =  $2,060 \times 20/3$  watt

Mechanical efficiency = 80% or 0.8

(a) Motor output =  $\frac{2,060 \times 20}{3 \times 0.8} = 17,167$  W

(b) Motor input =  $17,167 / 0.85 = 20,200$  W

Current drawn =  $20,200 / 220 = 91.7$  A

Motor power input = 20,200 W = 20.2 kW

Time for 48 km run = 2 hr.

$\therefore$  Motor energy input =  $20.2 \times 2 = 40.4$  kWh

Cost = Rs.  $40.4 \times 0.4 = \text{Rs. } 16$  paise 16

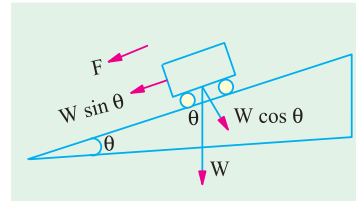


Fig. 3.2

**Example 3.20.** Estimate the rating of an induction furnace to melt two tonnes of zinc in one hour if it operates at an efficiency of 70%. Specific heat of zinc is 0.1. Latent heat of fusion of zinc is 26.67 kcal per kg. Melting point is 455°C. Assume the initial temperature to be 25°C.

(Electric Drives and Utilization Punjab Univ.)

**Solution.** Heat required to bring 2000 kg of zinc from 25°C to the melting temperature of 455°C =  $2000 \times 0.1 \times (455 - 25) = 86,000$  kcal.

Heat of fusion or melting =  $mL = 2000 \times 26.67 = 53,340$  kcal

Total heat reqd. =  $86,000 + 53,340 = 139,340$  kcal

Furnace input =  $139,340 / 0.7 = 199,057$  kcal

Now, 860 kcal = 1 kWh  $\therefore$  furnace input =  $199,057 / 860 = 231.5$  kWh.

Power rating of furnace = energy input/time =  $231.5$  kWh/1 h = **231.5 kW.**

**Example 3.21.** A pump driven by an electric motor lifts  $1.5 \text{ m}^3$  of water per minute to a height of 40 m. The pump has an efficiency of 90% and motor has an efficiency of 85%. Determine : (a) the power input to the motor. (b) The current taken from 480 V supply. (c) The electric energy consumed when motor runs at this load for 4 hours. Assume mass of  $1 \text{ m}^3$  of water to be 1000 kg.

(Elect. Engg. Pune Univ.)

**Solution.** (a) Weight of the water lifted =  $1.5 \text{ m}^3 = 1.5 \times 1000 = 1500$  kg. Wt =  $1500 \times 9.8 = 14700$  N.

Height = 40 m; time taken = 1 min. = 60 s

$\therefore$  Motor output power =  $14700 \times 40 / 60 = 9800$  W

Combined pump and motor efficiency =  $0.9 \times 0.85$

$\therefore$  Motor power input =  $9800 / 0.9 \times 0.85 = 12810$  W = **12.81 kW.**

(b) Current drawn by the motor =  $12810/480 = 26.7 \text{ A}$

Electrical energy consumed by the motor =  $12.81 \text{ kW} \times 4 \text{ h} = 51.2 \text{ kWh}$ .

**Example 3.22.** An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWh supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230 V d.c. Assume the efficiency of the motor at 90%.  
(Elect. Engg. A.M. Ae. S.I. June)

**Solution.** Work done by the lift =  $Wh = mgh = (5 \times 1000) \times 9.8 \times 30 = 1.47 \times 10^6 \text{ J}$

Since 25% of the electric current input is wasted, the energy supplied to the lift is 75% of the input.

$\therefore$  input energy to the lift =  $1.47 \times 10^6 / 0.75 = 1.96 \times 10^6 \text{ J}$

Now,  $1 \text{ kWh} = 26 \times 10^5 \text{ J}$

$\therefore$  energy input to the lift =  $1.96 \times 10^6 / 36 \times 10^5 = 0.544 \text{ kWh}$

Motor energy output =  $1.96 \times 10^6 \text{ J}$ ;  $\eta = 0.9$

Motor energy input =  $1.96 \times 10^6 / 0.9 = 2.18 \times 10^6 \text{ J}$ ; time taken =  $27 \times 60 = 1620 \text{ second}$

Power rating of the electric motor = work done/time taken

=  $2.18 \times 10^6 / 1620 = 1.345 \times 10^3 \text{ J/s} = 1345 \text{ W}$

Current taken by the motor =  $1345/230 = 5.85 \text{ A}$

**Example 3.23.** An electrical lift make 12 double journey per hour. A load of 5 tonnes is raised by it through a height 50 m and it returns empty. The lift takes 65 seconds to go up and 48 seconds to return. The weight of the cage is 1/2 tonne and that of the counterweight is 2.5 tonne. The efficiency of the hoist is 80 per cent that of the motor is 85 %. Calculate the hourly consumption in kWh.  
(Elect. Engg. Pune Univ.)

**Solution.** The lift is shown in Fig. 3.3.

Weight raised during upward journey  
=  $5 + 1/2 - 2.5 = 3 \text{ tonne} = 3000 \text{ kg-wt}$

Distance travelled = 50 m

Work done during upward journey  
=  $3000 \times 50 = 15 \times 10^4 \text{ m-kg}$

Weight raised during downward journey  
=  $2.5 - 0.5 = 2 \text{ tonne} = 2000 \text{ kg}$

Similarly, work done during downward journey  
=  $2000 \times 50 = 10 \times 10^4 \text{ m-kg}$ .

Total work done per double journey  
=  $15 \times 10^4 + 10 \times 10^4 = 25 \times 10^4 \text{ m-kg}$

Now,  $1 \text{ m-kg} = 9.8 \text{ joules}$

$\therefore$  Work done per double journey =  $9.8 \times 25 \times 10^4 \text{ J} = 245 \times 10^4 \text{ J}$

No. of double journey made per hour = 12

$\therefore$  work done per hour =  $12 \times 245 \times 10^4 = 294 \times 10^5 \text{ J}$

Energy drawn from supply =  $294 \times 10^5 / 0.8 \times 0.85 = 432.3 \times 10^5 \text{ J}$

Now,  $1 \text{ kWh} = 36 \times 10^5 \text{ J}$

$\therefore$  Energy consumption per hour =  $432.3 \times 10^5 / 36 \times 10^5 = 12 \text{ kWh}$

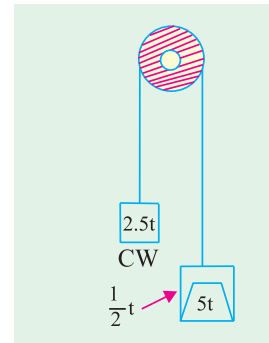


Fig. 3.3

**Example 3.24.** An electric hoist makes 10 double journey per hour. In each journey, a load of 6 tonnes is raised to a height of 60 meters in 90 seconds. The hoist cage weighs 1/2 tonne and has a balance load of 3 tonnes. The efficiency of the hoist is 80 % and of the driving motor 88 %. Calculate (a) electric energy absorbed per double journey (b) hourly energy consumption in kWh (c) hp (British) rating of the motor required (d) cost of electric energy if hoist works for 4 hours/day for 30 days. Cost per kWh is 50 paise.  
(Elect. Power - 1, Bangalore Univ.)

- Solution.**
- Wt. of cage when fully loaded =  $6\frac{1}{2}$  tonne-wt.
- Force exerted on upward journey =  $6\frac{1}{2} - 3 = 3\frac{1}{2}$  tonne-wt.  
 $= 3\frac{1}{2} \times 1000 = 3,500$  kg-wt.
- Force exerted on downward journey =  $3 - \frac{1}{2} = 2\frac{1}{2}$  tonnes-wt. = 2500 kg-wt
- Distance moved = 60 m
- Work done during upward journey =  $3,500 \times 60$  m-kg
- Work done during downward journey =  $2,500 \times 60$  m-kg
- Work done during each double journey =  $(3,500 + 2,500) \times 60 = 36 \times 10^4$  m-kg  
 $= 36 \times 10^4 \times 9.81 = 534 \times 10^4$  J
- Overall  $\eta = 0.80 \times 0.88$
- $\therefore$  Energy input per double journey =  $534 \times 10^4 / 0.8 \times 0.88 = 505 \times 10^4$  J
- (a) Electric energy absorbed per double journey =  $505 \times 10^4 / 36 \times 10^5 = 1.402$  kWh
- (b) Hourly consumption =  $1.402 \times 10 = 14.02$  kWh
- (c) Before calculating the rating of the motor, maximum rate of working should be found. It is seen that maximum rate of working is required in the upward journey.
- Work done =  $3,500 \times 60 \times 9.81 = 206 \times 10^4$  J
- Time taken = 90 second
- B.H.P of motor =  $\frac{206 \times 10^4}{90 \times 0.8 \times 746} = 38.6$  (British h.p.)
- (d) Cost =  $14.02 \times (30 \times 4) \times 50/100 = \text{Rs. } 841.2$

**Example 3.25.** A current of 80 A flows for 1 hr, in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 tonne must move in order that its kinetic energy shall be equal to the energy dissipated in the resistance.

(Elect. Engg. A.M.A.e. S.I.)

- Solution.** Energy dissipated in the resistance =  $VIt = 2 \times 80 \times 3600 = 576,000$  J
- A weight of one tonne represents a mass of one tonne i.e., 1000 kg. Its kinetic energy is =  $(1/2) \times 1000 \times v^2 = 500 v^2$
- $\therefore 500 v^2 = 576,000 \quad \therefore v = 1152$  m/s.

### Tutorial Problems No. 3.1

- A heater is required to give 900 cal/min on a 100 V. d.c. circuit. What length of wire is required for this heater if its resistance is 3  $\Omega$  per metre? **[53 metres]**
- A coil of resistance 100  $\Omega$  is immersed in a vessel containing 500 gram of water of 16° C and is connected to a 220-V electric supply. Calculate the time required to boil away all the water (1kcal = 4200 joules, latent heat of steam = 536 kcal/kg). **[44 min 50 second]**
- A resistor, immersed in oil, has 62.5  $\Omega$  resistance and is connected to a 500-V d.c. supply. Calculate
  - the current taken
  - the power in watts which expresses the rate of transfer of energy to the oil.
  - the kilowatt-hours of energy taken into the oil in 48 minutes. **[8A ; 4000 W ; 3.2 kWh]**
- An electric kettle is marked 500-W, 230 V and is found to take 15 minutes to raise 1 kg of water from 15° C to boiling point. Calculate the percentage of energy which is employed in heating the water. **[79 per cent]**
- An aluminium kettle weighing 2 kg holds 2 litres of water and its heater element consumes a power of 2 kW. If 40 percent of the heat supplied is wasted, find the time taken to bring the kettle of water to boiling point from an initial temperature of 20° C. (Specific heat of aluminium = 0.2 and Joule's equivalent = 4200 J/kcal.) **[11.2 min]**

6. A small electrically heated drying oven has two independent heating elements each of  $1000\ \Omega$  in its heating unit. Switching is provided so that the oven temperature can be altered by rearranging the resistor connections. How many different heating positions can be obtained and what is the electrical power drawn in each arrangement from a 200 V battery of negligible resistance ?  
**[Three, 40, 20 and 80 W]**
7. Ten electric heaters, each taking 200 W were used to dry out on site an electric machine which had been exposed to a water spray. They were used for 60 hours on a 240 V supply at a cost of twenty paise/kWh. Calculate the values of following quantities involved :  
 (a) current (b) power in kW (c) energy in kWh (d) cost of energy.  
**[(a) 8.33 A (b) 2 kW (c) 120 kWh (d) Rs. 24]**
8. An electric furnace smelts 1000 kg of tin per hour. If the furnace takes 50 kW of power from the electric supply, calculate its efficiency, given : the smelting temp. of tin =  $235^\circ\text{C}$  ; latent heat of fusion = 13.31 kcal/kg; initial temperature =  $15^\circ\text{C}$  ; specific heat = 0.056. Take  $J = 4200\ \text{J/kcal}$ .  
**[59.8%] (Electrical Engg.-I, Delhi Univ.)**
9. Find the useful rating of a tin-smelting furnace in order to smelt 50 kg of tin per hour. Given : Smelting temperature of tin =  $235^\circ\text{C}$ , Specific heat of tin = 0.055 kcal/kg-K. Latent heat of liquefaction = 13.31 kcal per kg. Take initial temperature of metal as  $15^\circ\text{C}$ . **[1.5 kW]**  
**(F.Y. Engg. Pune Univ.)**
10. State the relation between  
 (i) Kcal and kWh (ii) Horse power and watts (iii) kWh and joule (watt sec) (iv) K.E and joules.  
**(Gujrat University, Summer 2003)**
11. The electrical load in a small workshop consists of 14 lamps, each rated at 240 V, 60 W and 3 fans each rated at 240 V, 1 kW. What is the effective resistance of the total load, total current and energy utilised if run for 8 hrs.  
**(Pune University 2003) (Gujrat University, Summer 2003)**

### OBJECTIVE TESTS – 3

1. If a 220 V heater is used on 110 V supply, heat produced by it will be — as much.  
 (a) one-half (b) twice  
 (c) one-fourth (d) four times
2. For a given line voltage, four heating coils will produce maximum heat when connected  
 (a) all in parallel (b) all in series  
 (c) with two parallel pairs in series  
 (d) one pair in parallel with the other two in series
3. The electric energy required to raise the temperature of a given amount of water is 1000 kWh. If heat losses are 25%, the total heating energy required is — kWh.  
 (a) 1500 (b) 1250  
 (c) 1333 (d) 1000
4. One kWh of energy equals nearly  
 (a) 1000 W (b) 860 kcal  
 (c) 4186 J (d) 735.5 W
5. One kWh of electric energy equals  
 (a) 3600 J (b) 860 kcal  
 (c) 3600 W (d) 4186 J
6. A force of 10,000 N accelerates a body to a velocity 0.1 km/s. This power developed is — kW  
 (a) 1,00,000 (b) 36,000  
 (c) 3600 (d) 1000
7. A 100 W light bulb burns on an average of 10 hours a day for one week. The weekly consumption of energy will be — unit/s  
 (a) 7 (b) 70  
 (c) 0.7 (d) 0.07  
**(Principles of Elect. Engg. Delhi Univ.)**
8. Two heaters, rated at 1000 W, 250 volts each, are connected in series across a 250 Volts 50 Hz A.C. mains. The total power drawn from the supply would be — watt.,  
 (a) 1000 (b) 500  
 (c) 250 (d) 2000  
**(Principles of Elect. Engg. Delhi Univ.)**

### ANSWERS

1. c 2. a 3. c 4. b 5. b 6. d 7. a 8. b



### Learning Objectives

- Static Electricity
- Absolute and Relative Permittivity of a Medium
- Laws of Electrostatics
- Electric Field
- Electrostatic Induction
- Electric Flux and Faraday Tubes
- Field Strength or Field Intensity or Electric Intensity ( $E$ )
- Electric Flux Density or Electric Displacement  $D$
- Gauss Law
- The Equations of Poisson and Laplace
- Electric Potential and Energy
- Potential and Potential Difference
- Potential at a Point
- Potential of a Charged Sphere
- Equipotential Surfaces
- Potential and Electric Intensity Inside a Conducting Sphere
- Potential Gradient
- Breakdown Voltage and Dielectric Strength
- Safety Factor of Dielectric
- Boundary Conditions

# ELECTRO- STATICS



Lightning and thunder are created when the static electricity concentrated on the clouds suddenly discharges

### 4.1. Static Electricity

In the preceding chapters, we concerned ourselves exclusively with electric current *i.e.* electricity in motion. Now, we will discuss the behaviour of static electricity and the laws governing it. In fact, electrostatics is that branch of science which deals with the phenomena associated with electricity at rest.

It has been already discussed that generally an atom is electrically neutral *i.e.* in a normal atom the aggregate of positive charge of protons is exactly equal to the aggregate of negative charge of the electrons.

If, somehow, some electrons are removed from the atoms of a body, then it is left with a preponderance of positive charge. It is then said to be positively-charged. If, on the other hand, some electrons are added to it, negative charge out-balances the positive charge and the body is said to be negatively charged.

In brief, we can say that positive electrification of a body results from a deficiency of the electrons whereas negative electrification results from an excess of electrons.

The total deficiency or excess of electrons in a body is known as its charge.

### 4.2. Absolute and Relative Permittivity of a Medium

While discussing electrostatic phenomenon, a certain property of the medium called its **permittivity** plays an important role. Every medium is supposed to possess two permittivities :

(i) absolute permittivity ( $\epsilon$ ) and (ii) relative permittivity ( $\epsilon_r$ ).

For measuring relative permittivity, vacuum or free space is chosen as the reference medium. It has an absolute permittivity of  $8.854 \times 10^{-12}$  F/m

Absolute permittivity  $\epsilon_0 = 8.854 \times 10^{-12}$  F/m

Relative permittivity,  $\epsilon_r = 1$

Being a ratio of two similar quantities,  $\epsilon_r$  has no units.

Now, take any other medium. If its relative permittivity, as compared to vacuum is  $\epsilon_r$ , then its absolute permittivity is  $\epsilon = \epsilon_0 \epsilon_r$  F/m

If, for example, relative permittivity of mica is 5, then, its absolute permittivity is

$$\epsilon = \epsilon_0 \epsilon_r = 8.854 \times 10^{-12} \times 5 = 44.27 \times 10^{-12} \text{ F/m}$$



Charles Augustin de Coulomb\*

### 4.3. Laws of Electrostatics

**First Law.** Like charges of electricity repel each other, whereas unlike charges attract each other.

**Second Law.** According to this law, the force exerted between two *point* charges (i) is directly proportional to the product of their strengths (ii) is inversely proportional to the square of the distance between them.

This law is known as Coulomb's Law and can be expressed mathematically as :

$$F \propto \frac{Q_1 Q_2}{d^2} \quad \text{or} \quad F = k \frac{Q_1 Q_2}{d^2}$$

In vector form, the Coulomb's law can be written as 
$$\vec{F} = \frac{Q_1 Q_2}{d^2} \hat{d} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} = \frac{Q_1 Q_2}{d^2} \vec{d}$$

\* Coulomb is better known for his law which states that the force between two point charges is proportional to each charge and inversely proportional to the square of the distance between them.

where  $\hat{d}$  is the unit vector *i.e.* a vector of unit length in the direction of distance  $d$ , *i.e.*  $d = \frac{d}{\hat{d}}$  (where  $\vec{d}$  is the vector notation for  $d$ , which is a scalar notation).

Therefore, explicit forms of this law are :

$$\vec{F}_{21} = k \frac{Q_1 Q_2}{d_{12}^2} \hat{d}_{12} = k \frac{Q_1 Q_2}{d_{12}^3} \vec{d}_{12}$$

where  $\vec{F}_{21}$  is the force on  $Q_2$  due to  $Q_1$  and  $\hat{d}_{12}$  is the unit vector in direction from  $Q_1$  to  $Q_2$

and  $\vec{F}_{12} = k \frac{Q_1 Q_2}{d_{21}^2} \hat{d}_{21} = k \frac{Q_1 Q_2}{d_{21}^3} \vec{d}_{21}$  where  $F_{12}$  is the force on  $Q_1$  due to  $Q_2$  and  $\hat{d}_{21}$  is the unit vector in the direction from  $Q_2$  to  $Q_1$ .

where  $k$  is the constant of proportionality, whose value depends on the system of units employed. In S.I. system, as well as M.K.S.A. system  $k = 1/4\pi\epsilon$ . Hence, the above equation becomes.

$$F = \frac{Q_1 Q_2}{4 \pi \epsilon d^2} = \frac{Q_1 Q_2}{4 \pi \epsilon_0 \epsilon_r d^2}$$

If  $Q_1$  and  $Q_2$  are in colomb,  $d$  in metre and  $\epsilon$  in fard/metre, then  $F$  is in newtons

Now  $\frac{1}{4 \pi \epsilon_0} = \frac{1}{8.854 \times 10^{-12}} = 8.9878 \times 10^9 \text{ (approx.)}$

Hence, Coulomb's Law can be written as

$$F = 9 \times 10^9 \frac{Q_1 Q_2}{\epsilon_r d^2} \quad \text{---in a medium}$$

$$= 9 \times 10^9 \frac{Q_1 Q_2}{d^2} \quad \text{---in air or vacuum ... (i)}$$

If in Eq. (i) above

$$Q_1 = Q_2 = Q \text{ (say), } d = 1 \text{ metre; } F = 9 \times 10^9 \text{ N}$$

then

$$Q^2 = 1 \text{ or } Q = \pm 1 \text{ coulomb}$$

Hence, one coulomb of charge may be defined as **that charge (or quantity of electricity) which when placed in air (strictly vacuum) from an equal and similar charge repels it with a force of  $9 \times 10^9 \text{ N}$ .**

Although coulomb is found to be a unit of convenient size in dealing with electric current, yet, from the standpoint of electrostatics, it is an enormous unit. Hence, its submultiples like micro-coulomb ( $\mu \text{ C}$ ) and micro-microcoulomb ( $\mu \mu \text{ C}$ ) are generally used.

$$1 \mu \text{ C} = 10^{-6} \text{ C; } 1 \mu \mu \text{ C} = 10^{-12} \text{ C}$$

It may be noted here that relative permittivity of air is one, of water 81, of paper between 2 and 3, of glass between 5 and 10 and of mica between 2.5 and 6.

**Example 4.1.** Calculate the electrostatic force of repulsion between two  $\alpha$ -particles when at a distance of  $10^{-13} \text{ m}$  from each other. Charge of an  $\alpha$ -particle is  $3.2 \times 10^{-19} \text{ C}$ . If mass of each particle is  $6.68 \times 10^{-27} \text{ N-m}^2/\text{kg}^2$ .

**Solution.** Here  $Q_1 = Q_2 = 3.2 \times 10^{-19} \text{ C, } d = 10^{-13} \text{ m}$

$$F = 9 \times 10^9 \times \frac{3.2 \times 10^{-19} \times 3.2 \times 10^{-19}}{(10^{-13})^2} = 9.2 \times 10^{-2} \text{ N}$$

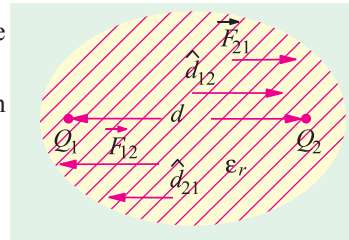
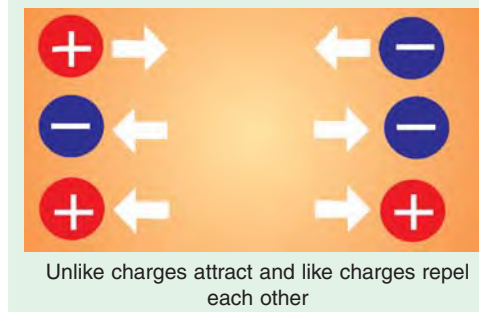


Fig. 4.1



The force of gravitational attraction between the two particles is given by

$$F = G \frac{m_1 m_2}{d^2} = \frac{6.67 \times 10^{-11} \times (6.68 \times 10^{-27})^2}{(10^{-13})^2} = 2.97 \times 10^{-37} \text{ N}$$

Obviously, this force is negligible as compared to the electrostatic force between the two particles.

**Example 4.2.** Calculate the distance of separation between two electrons (in vacuum) for which the electric force between them is equal to the gravitation force on one of them at the earth surface.

$$\text{mass of electron} = 9.1 \times 11^{-31} \text{ kg, charge of electron} = 1.6 \times 10^{-19} \text{ C.}$$

**Solution.** Gravitational force on one electron.

$$= mg \text{ newton} = 9.1 \times 10^{-31} \times 9.81 \text{ N}$$

Electrostatic force between the electrons

$$= 9 \times 10^9 \frac{Q^2}{d^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{d^2} \text{ N}$$

Equating the two forces, we have

$$\frac{9 \times 10^9 \times 2.56 \times 10^{-38}}{d^2} = 9.1 \times 10^{-31} \times 9.81 \quad \therefore d = 5.08 \text{ m}$$

**Example 4.3. (a)** Three identical point charges, each  $Q \Omega$  coulombs, are placed at the vertices of an equilateral triangle 10 cm apart. Calculate the force on each charge.

**(b)** Two charges  $Q$  coulomb each are placed at two opposite corners of a square. What additional charge “ $q$ ” placed at each of the other two corners will reduce the resultant electric force on each of the charges  $Q$  to zero?

**Solution. (a)** The equilateral triangle with its three charges is shown in Fig. 4.2 (a). Consider the charge  $Q$  respectively. These forces are equal to each other and each is

$$F = 9 \times 10^9 \frac{Q^2}{0.1^2} = 9 \times 10^{11} Q^2 \text{ newton}$$

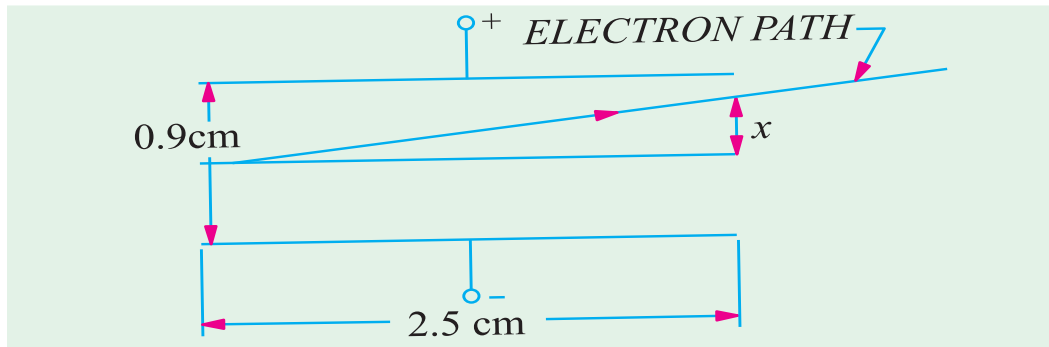


Fig. 4.2

Since the angle between these two equal forces is  $60^\circ$ , their resultant is

$$= 2 \times F \times \cos 60^\circ/2 = \sqrt{3} F = 9 \times 10^{11} \times \sqrt{3} Q^2 \text{ Newton}$$

The force experienced by other charges is also the same.

**(b)** The various charges are shown in Fig. 4.2 (b). The force experienced by the charge  $Q$  at point  $C$  due to the charge  $Q$  at point  $A$  acts along  $ACM$  and is

$$= 9 \times 10^9 \frac{Q^2}{(\sqrt{2}d)^2} = 4.5 \times 10^9 Q^2/d^2 \text{ newton} \quad \dots(i)$$

where  $d$  is the side of the square in metres.

If the charges  $q$  are negative, they will exert attractive forces on the charge  $Q$  at point  $C$  along  $CB$  and  $CD$  respectively. Each force is

$$= -9 \times 10^9 \frac{Qq}{d^2} \text{ newton}$$

Since these two forces are at right angles to each other, their resultant is

$$= -\sqrt{2} \times 9 \times 10^9 \frac{qQ}{d^2}$$

If net force on charge  $Q$  at point  $C$  is to be zero, then (i) must equal (ii),

$$\therefore 4.5 \times 10^9 \frac{Q^2}{d^2} = -9 \times 10^9 \sqrt{2} \frac{qQ}{d^2} \quad \therefore q = -\frac{Q}{2\sqrt{2}} \text{ coulomb}$$

**Example 4.4.** The small identical conducting spheres have charges of  $2.0 \times 10^{-9} \text{ C}$  and  $-0.5 \times 10^{-9}$  respectively. When they are placed 4 cm apart, what is the force between them? If they are brought into contact and then separated by 4 cm, what is the force between them?

(Electromagnetic Theory, A.M.I.E. Sec B.)

**Solution.**  $F = 9 \times 10^9 \frac{Q_1 Q_2}{d^2} = 9 \times 10^9 \times (-0.5 \times 10^{-9}) / 0.04^2 = -56.25 \times 10^{-7} \text{ N}$ . When two identical spheres are brought into contact with each other and then separated, each gets half of the total charge. Hence,

$$Q_1 = Q_2 = [2 \times 10^{-9} + (-0.5 \times 10^{-9}) / 2] = 0.75 \times 10^{-9} \text{ C}$$

When they are separated by 4 cm,

$$F = 9 \times 10^9 \times (0.75 \times 10^{-9})^2 / 0.04^2 = 0.316 \times 10^{-5} \text{ N}$$

**Example 4.5.** Determine resultant force on  $3 \mu\text{C}$  charge due to  $-4 \mu\text{C}$  and  $10 \text{ nC}$  charges. All these three point charges are placed on the vertices of equilateral triangle  $ABC$  of side 50 cm.

[Bombay University, 2001]

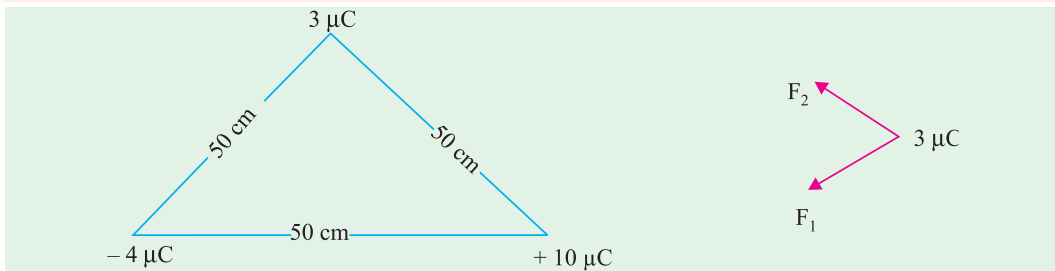


Fig. 4.3 (a)

Fig. 4.3 (b)

**Solution.**

$$F_2 = \frac{Q_1 Q_2}{4 \pi \epsilon_0 d^2} = \frac{3 \times 10^{-6} \times 10 \times 10^{-9}}{4 \times 8.854 \times 10^{-12} \times 0.50^2} = 1.08 \times 10^{-3} \text{ Newton, in the direction shown}$$

Similarly,

$$F_1 = 0.432 \text{ Newton, in the direction shown.}$$

Resultant of  $F_1$  and  $F_2$  has to be found out.

**Example 4.6.** A capacitor is composed of 2 plates separated by a sheet of insulating material 3 mm thick and of relative permittivity 4. The distance between the plates is increased to allow the insertion of a second sheet of 5 mm thick and of relative permittivity  $\epsilon_r$ . If the equivalent capacitance is one third of the original capacitance. Find the value of  $\epsilon_r$ . [Bombay University, 2001]

**Solution.**  $C_1 \frac{\epsilon_0 \epsilon_r A}{d} = k(4/3)$ , where  $k = \epsilon_0 A \times 10^{+3}$

The composite capacitor [with one dielectric of  $\epsilon_{r1} = 4$  and other dielectric of  $\epsilon_{r2}$  as relative

permittivity has a capacitance of  $C/3$ . Two capacitors are effectively in series. Let the second dielectric contribute a capacitor of  $C_2$ .

$$K \cdot (4/9) = \frac{C_1 C_2}{C_1 + C_2} = \frac{K \cdot (4/3) \cdot C_2}{K \cdot (4/3) + C_2}$$

This gives

$$C_2 = 2/3 K$$

$$(2/3) K = \frac{\epsilon_0 \epsilon_{r2} A}{5 \times 10^{-3}}$$

$$E_{r2} = 10/3 \cdot K / \epsilon_0 A \times 10^{-3}$$

$$= 10/3 \epsilon_0 A \times 103 / \epsilon_0 A \times 10^{-3}$$

$$= 10/3 = \mathbf{3.33}$$

#### 4.4. Electric Field

It is found that in the medium around a charge a force acts on a positive or negative charge when placed in that medium. If the charge is sufficiently large, then it may create such a huge stress as to cause the electrical rupture of the medium, followed by the passage of an arc discharge.

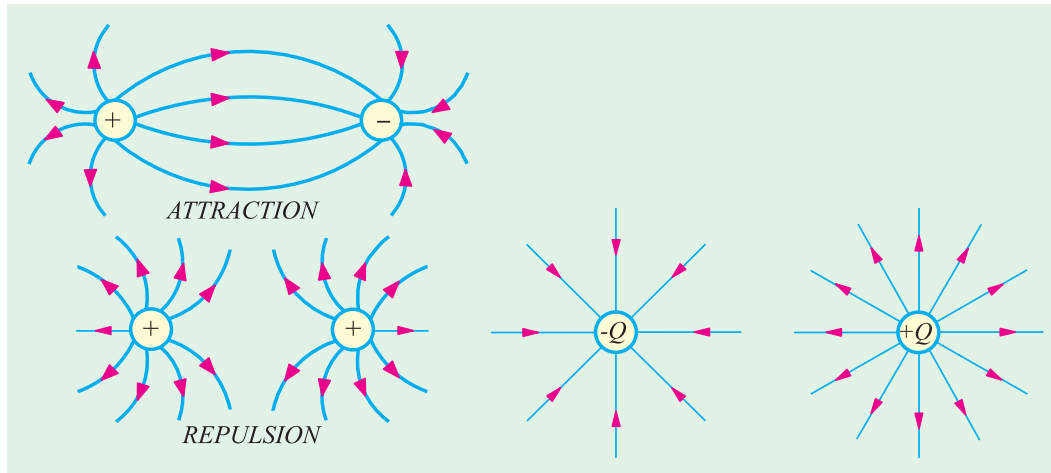


Fig. 4.4 (a)

Fig. 4.4 (b)

The region in which the stress exists or in which electric forces act, is called an electric field or electrostatic field.

The stress is represented by imaginary lines of forces. The direction of the lines of force at any point is the direction along which a unit positive charge placed at that point would move if free to do so. It was suggested by Faraday that the electric field should be imagined to be divided into **tubes of force** containing a fixed number of **lines** of force. He assumed these tubes to be elastic and having the property of contracting longitudinally and repelling laterally. With the help of these properties, it becomes easy to explain (i) why unlike charges attract each other and try to come nearer to each other and (ii) why like charges repel each other [Fig. 4.4 (a)].

However, it is more common to use the term lines of force. These lines are supposed to emanate from a positive charge and end on a negative charge [Fig. 4.4 (b)]. These lines always leave or enter a conducting surface normally.

#### 4.5. Electrostatic Induction

It is found that when an uncharged body is brought near a charged body, it acquires some charge. This phenomenon of an uncharged body getting charged merely by the nearness of a charged body is known as **induction**. In Fig. 4.5, a positively-charged body  $A$  is brought close to a perfectly-insulated

uncharged body  $B$ . It is found that the end of  $B$  nearer to  $A$  gets negatively charged whereas further end becomes positively charged. The negative and positive charges of  $B$  are known as **induced charges**. The negative charge of  $B$  is called 'bound' charge because it must remain on  $B$  so long as positive charge of  $A$  remains there. However, the positive charge on the farther end of  $B$  is called free charge. In Fig. 4.6, the body  $B$  has been earthed by a wire. The positive charge flows to earth leaving negative charge behind. If next  $A$  is removed, then this negative charge will also go to earth, leaving  $B$  uncharged. It is found that :

- (i) a positive charge induces a negative charge and **vice-versa**.
- (ii) each of the induced charges is equal to the inducing charge.

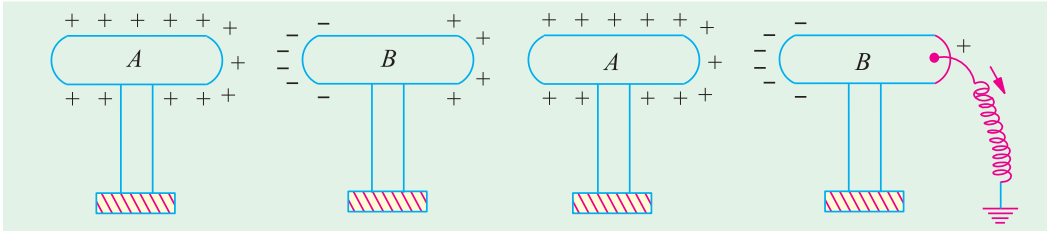


Fig. 4.5

Fig. 4.6

#### 4.6. Electric Flux and Faraday Tubes

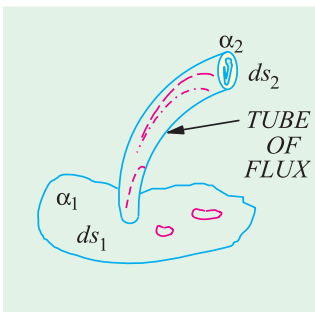


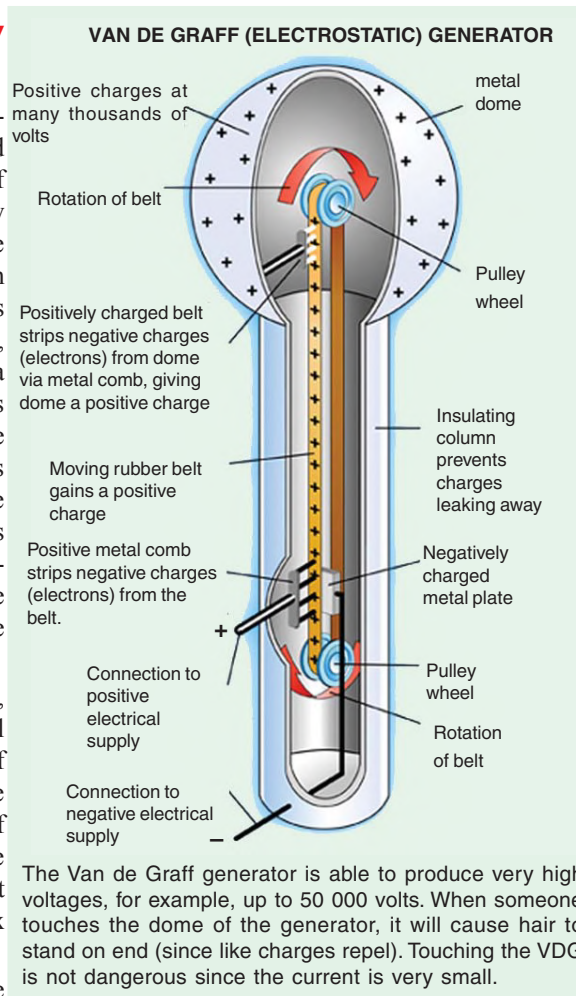
Fig. 4.7

Consider a small closed curve in an electric field (Fig. 4.7). If we draw lines of force through each point of this closed curve, then we get a tube as shown in the figure. It is called the

**tube of the electric flux**. It may be defined as the region of space enclosed within the tubular surface formed by drawing lines of force through every point of a small closed curve in the electric field.

Since lines of force end on conductors, the two ends of a flux tube will consist of small area  $ds_1$  and  $ds_2$  on the conductor surfaces. If surface charge densities over these areas are  $\sigma_1$  and  $-\sigma_2$ , then charges at the two ends of the flux tube will be  $\sigma_1 ds_1$  and  $-\sigma_2 ds_2$ . These charges are assumed to be always equal but opposite to each other. The strength of a flux tube is represented by the charge at its ends.

A **unit tube of flux** is one in which the



end charge is one unit of charge.

In the S.I. system of units, one such tube of flux is supposed to start from a positive charge of one coulomb and terminate on a negative charge of the same amount.

A **unit tube of flux** is known as Faraday tube. If the charge on a conductor is  $\pm Q$  coulombs, then the **number of Faraday tubes starting or terminating on it also  $Q$** .

The number of Faraday tubes of flux passing through a surface in an electric field is called the *electric flux* (or dielectric flux) through that surface. Electric flux is represented by the symbol  $\psi$ . Since electric flux is numerically equal to the charge, it is measured in coulombs.

Hence, 
$$\psi = Q \text{ coulombs}$$

**Note.** It may also be noted that ‘tubes of flux’ passing per unit area through a medium are also supposed to measure the ‘electric displacement’ of that dielectric medium. In that case, they are referred to as **lines of displacement** and are equal to  $\epsilon$  times the lines of force (Art. 4.8). It is important to differentiate between the ‘tubes of flux’ and ‘lines of force’ and to remember that if  $Q$  is the charge, then

$$\text{tubes of flux} = Q \text{ and lines of force} = Q/\epsilon$$

#### 4.7. Field Strength or Field Intensity or Electric Intensity ( $E$ )

Electric intensity at any point within an electric field may be defined in either of the following three ways :

(a) It is given by the force experienced by a unit positive charge placed at that point. Its direction is the direction along which the force acts.

Obviously, the unit of  $E$  is newton/coulomb ( $N/C$ ).

For example, if a charge of  $Q$  coulombs placed at a particular point  $P$  within an electric field instances a force of  $F$  newton, then electric field at that point is given by

$$E = F/Q \text{ N/C}$$

The value of  $E$  within the field due to a point charge can be found with help of Coulomb’s laws. Suppose it is required to find the electric field at a point  $A$  situated at a distance of  $d$  metres from a charge of  $Q$  coulombs. Imagine a positive charge of one coulomb placed at that point (Fig. 4.8). The force experienced by this charge is

$$F = \frac{Q \times 1}{4 \pi \epsilon_0 \epsilon_r d^2} \text{ N} \quad \text{or} \quad \vec{F} = \frac{Q \times 1}{4 \pi \epsilon_0 \epsilon_r d_{PA}^2} \hat{d}_{PA}$$

$$\therefore E = \frac{Q \times 1}{4 \pi \epsilon_0 \epsilon_r d_{PA}^2} \text{ N/C}$$

$$= 9 \times 10^9 \frac{Q}{\epsilon_r d_{PA}^2} \text{ N/C} \quad \left. \vphantom{\frac{Q}{\epsilon_r d_{PA}^2}} \right\} \text{ in a medium}$$

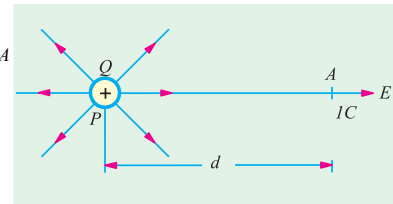


Fig. 4.8

or in vector notation,

$$\vec{E}(d) = 9 \times 10^9 \frac{Q}{\epsilon_r d^2} \hat{d} \quad \text{where } \vec{E}(d) \text{ denotes } \vec{E} \text{ as a function of } d$$

$$\left. \begin{aligned} &= \frac{Q}{4 \pi \epsilon_0 d^2} \text{ N/C} \\ &= 9 \times 10^9 \frac{Q}{d^2} \text{ N/C} \end{aligned} \right\} \text{ in air}$$

(b) Electric intensity at a point may be defined as equal to the **lines of force** passing normally through a unit cross-section at that point. Suppose, there is a charge of  $Q$  coulombs. The number of



lines of force produced by it is  $Q/\epsilon$ . If these lines fall normally on an area of  $A \text{ m}^2$  surrounding the point, then electric intensity at that point is

$$E = \frac{Q/\epsilon}{A} = \frac{Q}{\epsilon A}$$

Now  $Q/A = D$  —the flux density over the area

$$\therefore E = \frac{D}{\epsilon} = \frac{D}{\epsilon_0 \epsilon_r}$$

—in a medium

$$= \frac{D}{\epsilon_0}$$

—in air

The unit of  $E$  is volt/metre.

(c) Electric intensity at any point in an electric field is **equal to the potential gradient at that point.**

In other words,  $E$  is equal to the rate of fall of potential in the direction of the lines of force.

$$\therefore E = \frac{-dV}{dx}$$

Obviously, the unit of  $E$  is volt/metre.

It may be noted that  $E$  and  $D$  are vector quantities having magnitude and direction.

$$\therefore \text{In vector notation, } \vec{D} = \epsilon_0 \vec{E}$$

**Example 4.7.** Point charges in air are located as follows :

$+5 \times 10^{-8} \text{ C}$  at  $(0, 0)$  metres,  $+4 \times 10^{-8} \text{ C}$  at  $(3, 0)$  metres and  $-6 \times 10^{-8} \text{ C}$  at  $(0, 4)$  metres. Find electric field intensity at  $(3, 4)$  metres.

**Solution.** Electric intensity at point  $D(3, 4)$  due to positive charge at point  $A$  is

$$E_1 = 9 \times 10^9 Q/d^2 = 9 \times 10^9 \times 5 \times 10^{-8}/5^2 = 18 \text{ V/m}$$

As shown in Fig. 4.9, it acts along  $AD$ .

Similarly, electric intensity at point  $D$  due to positive charge at point  $B$  is  $E_2 = 9 \times 10^9 \times 4 \times 10^{-8}/4^2 = 22.5 \text{ V/m}$ . It acts along  $BD$ .

$$E_3 = 9 \times 10^9 \times 6 \times 10^{-8}/3^2 = 60 \text{ V/m. It acts along } DC.$$

The resultant intensity may be found by resolving  $E_1, E_2$  and  $E_3$  into their  $X$ -and  $Y$ -components. Now,  $\tan \theta = 4/3$ ;  $\theta = 53^\circ 8'$ .

$$\begin{aligned} X\text{-component} &= E_1 \cos \theta - E_2 = 18 \cos 53^\circ 8' - 60 \\ &= -49.2 \end{aligned}$$

$$Y\text{-component} = E_1 \sin \theta + E_2 = 18 \sin 53^\circ 8' + 22.5 = 36.9$$

$$\therefore E = \sqrt{(-49.2)^2 + 36.9^2} = 61.5 \text{ V/m.}$$

It acts along  $DE$  such that  $\tan \phi = 36.9/49.2 = 0.75$ .

Hence  $\phi = 36.9^\circ$ .

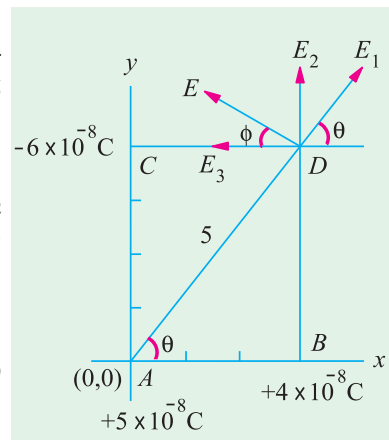


Fig. 4.9

**Example 4.8.** An electron has a velocity of  $1.5 \times 10^7 \text{ m/s}$  at right angles to the uniform electric field between two parallel deflecting plates of a cathode-ray tube. If the plates are  $2.5 \text{ cm}$  long and spaced  $0.9 \text{ cm}$  apart and p.d. between the plates is  $75 \text{ V}$ , calculate how far the electron is deflected sideways during its movement through the electric field. Assume electronic charge to be  $1.6 \times 10^{-19} \text{ coulomb}$  and electronic mass to be  $9.1 \times 10^{-31} \text{ kg}$ .

**Solution.** The movement of the electron through the electric field is shown in Fig. 4.10. Electric intensity between the plates is  $E = dV/dx = 75/0.009 = 8,333 \text{ V/m}$ .

$$\begin{aligned} \text{Force on the electron is } F &= QE \\ &= 8,333 \times 1.6 \times 10^{-19} = 1.33 \times 10^{-15} \text{ N.} \end{aligned}$$

Since the deflection  $x$  is small as compared to the length of the plates, time taken by the electron to travel through the electric field is  $= 0.025/1.5 \times 10^7 = 1.667 \times 10^{-9} \text{ s}$

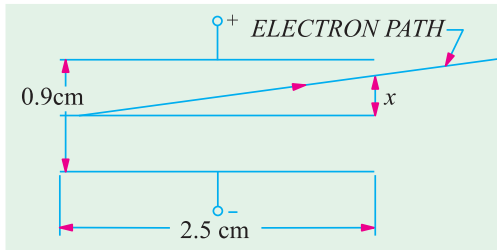
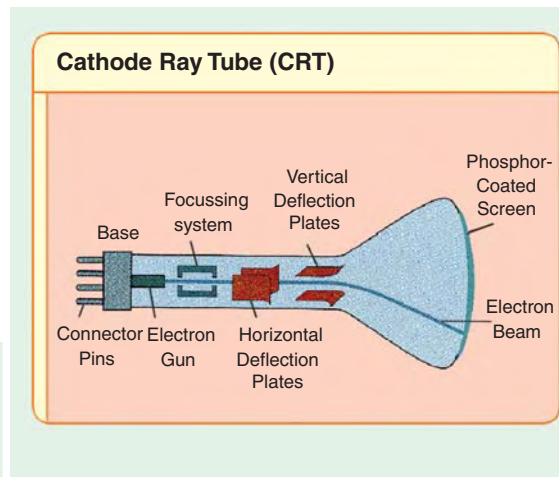


Fig. 4.10



Now, force = mass  $\times$  acceleration

$$\begin{aligned} \therefore \text{Transverse acceleration is} \\ &= \frac{1.33 \times 10^{-15}}{9.1 \times 10^{-31}} = 1.44 \times 10^{15} \text{ m/s}^2 \end{aligned}$$

Final transverse velocity of the electron = acceleration  $\times$  time  
 $= 1.44 \times 10^{15} \times 1.667 \times 10^{-9} = 2.4 \times 10^6 \text{ m/s}$

$\therefore$  sideways or transverse movement of the electron is

$$\begin{aligned} x &= (\text{average velocity}) \times \text{time} \\ &= \frac{1}{2} \times 2.4 \times 10^6 \times 1.667 \times 10^{-9} = \mathbf{2 \text{ mm (approx.)}^*} \end{aligned}$$

#### 4.8. Electric Flux Density or Electric Displacement

It is given by the normal flux per unit area.

If a flux of  $\Psi$  coulombs passes normally through an area of  $A \text{ m}^2$ , then flux density is

$$D = \frac{\Psi}{A} \text{ C/m}^2$$

It is related to electric field intensity by the relation

$$\begin{aligned} D &= \epsilon_0 \epsilon_r E && \dots \text{in a medium} \\ &= \epsilon_0 E && \dots \text{in free space} \end{aligned}$$

In other words, the product of electric intensity  $E$  at any point within a dielectric medium and the absolute permittivity  $\epsilon (= \epsilon_0, \epsilon_r)$  at the same point is called the **displacement** at that point.

Like electric intensity  $E$ , electric displacement  $D^{**}$  is also a vector quantity (see 4.7) whose direction at every point is the same as that of  $E$  but whose magnitude is  $\epsilon_0 \epsilon_r$  times  $E$ . As  $E$  is represented by lines of force, similarly  $D$  may also be represented by lines called lines of electric

\* The above result could be found by using the general formula

$$x = \frac{1}{2} \left( \frac{e}{m} \right) \left( \frac{V}{d} \right) \left( \frac{l}{v} \right)^2 \text{ metres}$$

where  $e/m$  = ratio of the charge and mass of the electron

$V$  = p.d. between plates in volts;  $d$  = separation of the plates in metres

$l$  = length of the plates in metres;  $v$  = axial velocity of the electron in m/s.

\*\* A more general definition of displacement  $D$  is that  $D = \epsilon_0 \epsilon_r E + P$  where  $P$  is the polarisation of the dielectric and is equal to the dipole moment per unit volume.

**displacement.** The tangent to these lines at any point gives the direction of  $D$  at that point and the number of lines per unit area perpendicular to their direction is numerically equal to the electric displacement at that point. Hence, the number of lines of electric displacement per unit area ( $D$ ) is  $\epsilon_0 \epsilon_r$  times the number of lines of force per unit area at that point.

It should be noted that whereas the value of  $E$  depends on the permittivity of the surrounding medium, that of  $D$  is independent of it.

One useful property of  $D$  is that its surface integral over any closed surface equals the enclosed charge (Art. 4.9).

Let us find the value of  $D$  at a point distant  $r$  metres from a point charge of  $Q$  coulombs. Imagine a sphere of radius  $r$  metres surrounding the charge. Total flux =  $Q$  coulombs and it falls normally on a surface area of  $4\pi r^2$  metres. Hence, electric flux density.

$$D = \frac{\Psi}{4\pi r^2} = \frac{Q}{4\pi r^2} \text{ coulomb/metre}^2 \text{ or } D = \frac{Q}{4\pi r^2} r = r \text{ (in vector notation)}$$

#### 4.9. Gauss\* Law

Consider a point charge  $Q$  lying at the centre of a sphere of radius  $r$  which surrounds it completely [Fig. 4.11 (a)]. The total number of **tubes of flux** originating from the charge is  $Q$  (but number of lines of force is  $Q/\epsilon_0$ ) and are normal to the surface of the sphere. The electric field  $E$  which equals  $Q/4\pi\epsilon_0 r^2$  is also normal to the surface. As said earlier, total number of lines of force passing perpendicularly through the whole surface of the sphere is

$$= E \times \text{Area} = \frac{Q}{4\pi\epsilon_0 r^2} \times 4\pi r^2 = \frac{Q}{\epsilon_0}$$

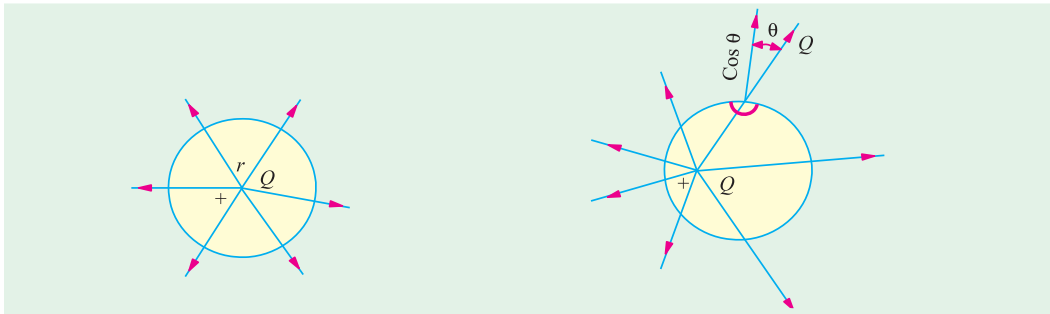


Fig. 4.11

Now, suppose we draw another sphere surrounding the charge [Fig. 4.11 (b)] but whose centre does not lie at the charge but elsewhere. In this case also, the number of tubes of flux emanating from the charge is  $Q$  and lines of force is  $Q/\epsilon_0$  though they are not normal to the surface. These can, however, be split up into  $\cos \theta$  components and  $\sin \theta$  components. If we add up  $\sin \theta$  components all over the surface, they will be equal to zero. But if we add up  $\cos \theta$  components over the whole surface of the sphere, the normal flux will again come out to be  $Q$  (or lines of force will come out to be  $Q/\epsilon_0$ ). Hence, it shows that irrespective of where the charge  $Q$  is placed within a closed surface completely surrounding it, the total normal flux is  $Q$  and the total number of lines of force passing out normally is  $Q/\epsilon_0$ .

In fact, as shown in Fig. 4.12, if there are placed charges of value  $Q_1, Q_2, -Q_3$  inside a closed surface, the total *i.e.* net charge enclosed by the surface is  $(Q_1 + Q_2 - Q_3)/\epsilon_0$  through the closed surface.

\* After the German mathematician and astronomer Karel Freidrich Gauss (1777-1855).

This is the meaning of Gauss's law which may be stated thus : the surface integral of the normal component of the electric intensity  $E$  over a closed surface is equal to  $1/\epsilon_0$  times the total charge inside it.

Mathematically,  $\oint E_n ds = Q/\epsilon_0$  (where the circle on the integral sign indicates that the surface of integration is a closed surface).

$$\text{or} \quad \int \epsilon_n \epsilon_0 ds = Q, \text{ i.e.} \quad \int D_n ds = Q \quad [\because D_n = \epsilon_0 E_n]$$

$$\text{or} \quad \int \epsilon_0 E \cos \theta ds = Q, \text{ i.e.} \quad \int D \cos \theta ds = Q$$

$$\text{or} \quad \int \epsilon_0 E_{ds} \cos \theta = Q, \text{ i.e.,} \quad \int D ds \cos \theta = Q$$

when  $E$  and  $D$  are not normal to the surface but make an angle  $\theta$  with the normal (perpendicular) to the surface as shown in Fig. 4.13.

**Proof.** In Fig. 4.13, let a surface  $S$  completely surround a quantity of electricity or charge  $Q$ . Consider a small surface area  $ds$  subtending a small solid angle  $d\omega$  at point charge  $Q$ . The field intensity at  $ds$  is  $E = \frac{Q}{4\pi\epsilon_0 d^2}$  where  $d$  is the distance between  $Q$  and  $ds$ .

In vector notation,  $\int \epsilon_0 \vec{E} \cdot \vec{ds} = Q$  i.e.  $\int \vec{D} \cdot \vec{ds} = Q = \int_V \rho dv$  (where  $\rho$  is the volume density of charge in the volume enclosed by closed surface  $S$ ).

Thus  $\int_S \vec{D} \cdot \vec{ds} = \int_V \rho dv$  is the vector statement of Gauss Law\* and its alternative statement is  $\nabla D = \rho$

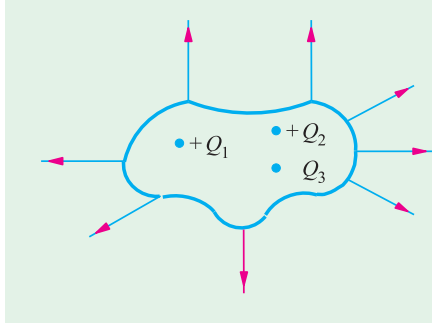


Fig. 4.12

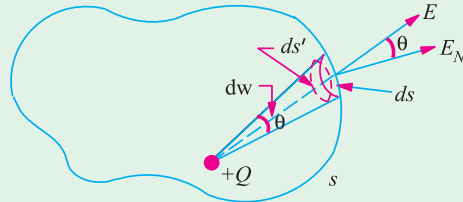


Fig. 4.13

The normal component of the intensity  $E_n = E \cos \theta$

$\therefore$  No. of lines of force passing normally through the area  $ds$  is

$$= E_n ds = E ds \cos \theta = E \cdot ds \text{ in vector notation}$$

$$\text{Now} \quad ds \cos \theta = ds' \quad \therefore E \cdot ds' = \frac{Q}{4\pi\epsilon_0 d^2} \cdot ds'$$

$$\text{Now} \quad ds' / d^2 = d\omega$$

$$\text{Hence, the number of lines of force passing normally is} = \frac{Q}{4\pi\epsilon_0} d\omega$$

\* This results from the application of the Divergence theorem, also called the Gauss' Theorem, viz.,

$\int_S \nabla D \cdot d\mathbf{v} = \int D \cdot ds$  where vector operator called 'del' is defined as

$$\nabla = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z$$

Total number of lines of force over the whole surface

$$= \frac{Q}{4\pi\epsilon_0} \int_s d\omega = \frac{Q}{4\pi\epsilon_0} \times 4\pi = \frac{Q}{\epsilon_0}$$

where sign  $\oint$  denotes integration around the whole of the closed surface *i.e.* surface integral.

If the surface passes through a material medium, then the above law can be generalized to include the following :

**the surface integral of the normal component of  $D$  over a closed surface equals the free charge enclosed by the surface.**

$$\text{As before } D = \frac{Q}{4\pi\epsilon d^2}. \text{ The normal component } D_n = D \cos \theta = \frac{Q}{4\pi d^2} \times \cos \theta$$

Hence, the normal electric flux from area  $ds$  is

$$d\psi = D_n \times ds = \frac{Q}{4\pi d^2} \cdot \cos \theta \cdot ds = \frac{Q}{4\pi d^2} \cdot ds'$$

$$\therefore d\psi = \frac{Q}{4\pi} \left( \frac{ds'}{d^2} \right) = \frac{Q}{4\pi} d\omega$$

$$\text{or } \psi = \int \frac{Q}{4\pi} \cdot d\omega = \frac{Q}{4\pi} \int d\omega = \frac{Q}{4\pi} \times 4\pi = Q \quad \therefore \Psi = Q$$

which proves the statement made above.

Hence, we may state Gauss's law in two slightly different ways.

$$\int_3 E_n \cdot ds = \int_3 E \cdot \cos \theta \cdot ds = Q/\epsilon_0 \quad \text{or} \quad \epsilon_0 \int_3 E_n \cdot ds = Q$$

$$\text{and } \int_3 D_n \cdot ds = \int_3 D_n \cdot ds = Q$$

(vector statement is given above)

#### 4.10. The Equations of Poisson and Laplace

These equations are useful in the solution of many problems concerning electrostatics especially the problem of space charge\* present in an electronic valve. The two equations can be derived by applying Gauss's theorem. Consider the electric field set up between two charged plates  $P$  and  $Q$  [Fig. 4.14 (a)]. Suppose there is some electric charge present in the space between the two plates. It is, generally, known as the space charge. Let the space charge density be  $\rho$  coulomb/metre<sup>3</sup>. It will be assumed that the space charge density varies from one point of space to the another but is uniform throughout any thin layer taken parallel to the plates  $P$  and  $Q$ . If  $X$ -axis is taken perpendicular to the plates, then  $\rho$  is assumed to depend on the value of  $x$ . It will be seen from Fig. 4.14 (a) that the value of electric intensity  $E$  increases with  $x$  because of the space charge.

Now, consider a thin volume element of cross-section  $A$  and thickness  $\Delta x$  as shown in Fig. 4.14 (b). The values of electric intensity at the two opposite faces of this element are  $E$  and  $(E + \Delta E)$ . If  $dE/dx$  represents the rate of increases of electric intensity with distance, then

$$\Delta E = \frac{\partial E}{\partial x} \times \Delta x \quad \therefore E + \Delta E = E + \frac{\partial E}{\partial x} \times \Delta x$$

The surface integral of electric intensity over the right-hand face of this element is

\* Such a space charge exists in the space between the cathode and anode of a vacuum tube.

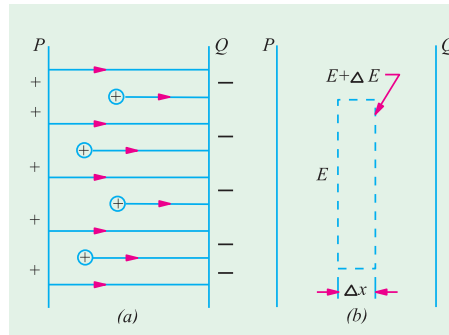


Fig. 4.14

$$= \left( E + \frac{\partial E}{\partial x} \cdot \Delta x \right) A$$

The surface integral over the left-hand face of the element is  $= -E \times A$

The negative sign represents the fact that  $E$  is directed inwards over this face.

The surface integral over the entire surface, *i.e.*, the closed surface of the element is

$$= \left( E + \frac{\partial E}{\partial x} \cdot \Delta x \right) A - E \times A = A \cdot \Delta x \cdot \frac{\partial E}{\partial x}$$

From symmetry it is evident that along with  $y$  and  $z$  there is no field.

Now, according to Gauss's theorem (Art. 4.9), the surface integral of electric intensity over a closed surface is equal to  $1/\epsilon_0$  time the charge within that surface.

Volume of the element,  $dV = A \times \Delta x$ ; charge  $= \rho A \cdot \Delta x$

$$\therefore A \cdot \Delta x \cdot \frac{\partial E}{\partial x} = \rho A \cdot \Delta x \cdot \frac{1}{\epsilon_0} \quad \text{or} \quad \frac{\partial E}{\partial x} = \frac{\rho}{\epsilon_0}$$

$$\text{Now} \quad E = -\frac{\partial V}{\partial x} \quad \therefore \frac{\partial E}{\partial x} = \frac{\partial}{\partial x} \left( -\frac{dV}{dx} \right) = -\frac{\partial^2 V}{\partial x^2} \quad \therefore \frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\epsilon_0}$$

It is known as Poisson's equation in one dimension where potential varies with  $x$ .

When  $V$  varies with  $x$ ,  $y$  and  $z$ , then  $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0} = \nabla^2 V$  in vector notation.

If, as a special case, where space charge density is zero, then obviously,

$$\frac{\partial^2 V}{\partial x^2} = 0$$

In general, we have  $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$  or  $\nabla^2 V = 0$  in vector notation where  $\nabla^2$  is defined (in cartesian co-ordinates) as the operation

$$\nabla^2 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

It is known as Laplace's equation.

#### 4.11. Electric Potential and Energy

We know that a body raised above the ground level has a certain amount of mechanical potential energy which, by definition, is given by the amount of work done in raising it to that height. If, for example, a body of 5 kg is raised against gravity through 10 m, then the potential energy of the body is  $5 \times 10 = 50$  m-kg. wt.  $= 50 \times 9.8 = 490$  joules. The body falls because there is attraction due to gravity and always proceeds from a place of higher potential energy to one of lower potential energy. So, we speak of gravitational potential energy or briefly 'potential' at different points in the earth's gravitational field.

Now, consider an electric field. Imagine an isolated positive charge  $Q$  placed in air (Fig. 4.15). Like earth's gravitational field, it has its own electrostatic field which theoretically extends upto infinity. If the charge  $X$  is very far away from  $Q$ , say, at infinity, then force on it is practically zero. As  $X$  is brought nearer to  $Q$ , a force of repulsion acts on it (as similar charges repel each other), hence work or energy is required to bring it to a point like  $A$  in the electric field. Hence, when at point  $A$ , charge  $X$  has

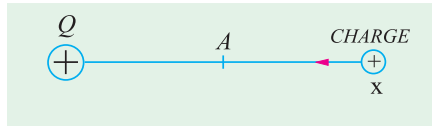


Fig. 4.15



Using Van De Graff Generator, artificial lightning can be created in the laboratory, in a miniature scale.

some amount of electric potential energy. Similar other points in the field will also have some potential energy. In the gravitational field, usually 'sea level' is chosen as the place of 'zero' potential. In electric field infinity is chosen as the theoretical place of 'zero' potential although, in practice, earth is chosen as 'zero' potential, because earth is such a large conductor that its potential remains practically constant although it keeps on losing and gaining electric charge every day.

#### 4.12. Potential and Potential Difference

As explained above, the force acting on a charge at infinity is zero, hence 'infinity' is chosen as the theoretical place of zero electric potential. Therefore, potential at any point in an electric field may be defined as

**numerically equal to the work done in bringing a positive charge of one coulomb from infinity to that point against the electric field.**

The unit of this potential will depend on the unit of charge taken and the work done.

If, in shifting one coulomb from infinity to a certain point in the electric field, the work done is one joule, then potential of that point is one volt.

Obviously, potential is work per unit charge,

$$\therefore 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Similarly, potential difference (p.d.) of one volt exists between two points if one joule of work is done in shifting a charge of one coulomb from one point to the other.

#### 4.13. Potential at a Point

Consider a positive point charge of  $Q$  coulombs placed in air. At a point  $x$  metres from it, the force on one coulomb positive charge is  $Q/4\pi\epsilon_0 x^2$  (Fig. 4.16). Suppose, this one coulomb charge is moved towards  $Q$  through a small distance  $dx$ . Then, work done is

$$dW = \frac{Q}{4\pi\epsilon_0 x^2} \times (-dx)$$

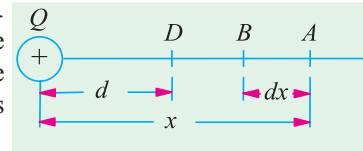


Fig. 4.16

The negative sign is taken because  $dx$  is considered along the negative direction of  $x$ .

The total work done in bringing this coulomb of positive charge from infinity to any point  $D$  which is  $d$  metres from  $Q$  is given by

$$\begin{aligned} W &= - \int_{x=\infty}^{x=d} Q \cdot \frac{dx}{4\pi\epsilon_0 x^2} = - \frac{Q}{4\pi\epsilon_0} \int_{\infty}^d \frac{dx}{x^2} \\ &= - \frac{Q}{4\pi\epsilon_0} \left[ -\frac{1}{x} \right]_{\infty}^d = - \frac{Q}{4\pi\epsilon_0} \left[ -\frac{1}{d} - \left( -\frac{1}{\infty} \right) \right] = \frac{Q}{4\pi\epsilon_0 d} \text{ joules} \end{aligned}$$

By definition, this work in joules is numerically equal to the potential of that point in volts.

$$\therefore V = \frac{Q}{4\pi\epsilon_0 d} = 9 \times 10^9 \frac{Q}{d} \text{ volt} \quad \text{---in air}$$

$$\text{and } V = \frac{Q}{4\pi\epsilon_0 \epsilon_r d} = 9 \times 10^9 \frac{Q}{\epsilon_r d} \text{ volt} \quad \text{---in medium}$$

We find that as  $d$  increases,  $V$  decreases till it becomes zero at infinity.

#### 4.14. Potential of a Charged Conducting Sphere

The above formula  $V = Q/4\pi\epsilon_0 \epsilon_r d$  applies only to a charge concentrated at a point. The problem

of finding potential at a point outside a charged sphere sounds difficult, because the charge on the sphere is distributed over its entire surface and so, is not concentrated at a point. But the problem is easily solved by noting that the lines of force of a charged sphere, like  $A$  in Fig. 4.17 spread out normally from its surface. If produced backwards, they meet at the centre of  $A$ . Hence for finding the potentials at points outside the sphere, we can imagine the charge on the sphere as concentrated at its centre  $O$ . If  $r$  is the radius of sphere in metres and  $Q$  its charge in coulomb then, potential of its surface is  $Q/4\pi\epsilon_0 r$  volt and electric intensity is  $Q/4\pi\epsilon_0 r^2$ . At any other point ' $d$ ' metres from the centre of the sphere, the corresponding values are  $Q/4\pi\epsilon_0 d$  and  $Q/4\pi\epsilon_0 d^2$  respectively with  $d > r$  as shown in Fig. 4.18 though its starting point is coincident with that of  $r$ . The variations of the potential and electric intensity with distance for a charged sphere are shown in Fig. 4.18.

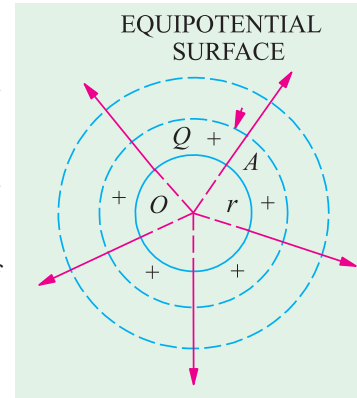


Fig. 4.17

#### 4.15. Equipotential Surfaces

An equipotential surface is a surface in an electric field such that all points on it are at the same potential. For example, different spherical surfaces around a charged sphere are equipotential surfaces. One important property of an equipotential surface is that the direction of the electric field strength and flux density is always at right angles to the surface. Also, electric flux emerges out normal to such a surface. If, it is not so, then there would be some component of  $E$  along the surface resulting in potential difference between various points lying on it which is contrary to the definition of an equipotential surface.

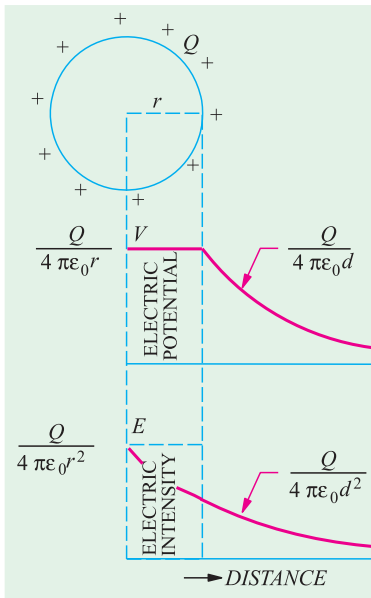


Fig. 4.18

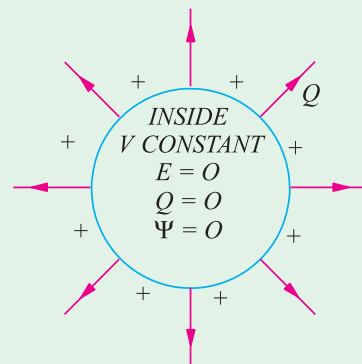


Fig. 4.19

#### 4.16. Potential and Electric Intensity Inside a Conducting Sphere

It has been experimentally found that when charge is given to a conducting body say, a sphere then it resides entirely on its outer surface *i.e.*, within a conducting body whether hollow or solid, the charge is zero. Hence, (i) flux is zero (ii) field intensity is zero (iii) all points within the conductor are at the same potential as at its surface (Fig. 4.19).

**Example 4.9.** Three concentric spheres of radii 4, 6 and 8 cm have charges of +8, -6 and +4  $\mu\text{C}$  respectively. What are the potentials and field strengths at points, 2, 5, 7 and 10 cm from the centre.



**Solution.** As shown in Fig. 4.20, let the three spheres be marked  $A, B$  and  $C$ . It should be remembered that (i) the field intensity outside a sphere is the same as that obtained by considering the charge at its centre (ii) inside the sphere, the field strength is zero (iii) potential anywhere inside a sphere is the same as at its surface.

(i) Consider point 'a' at a distance of 2 cm from the centre  $O$ . Since it is inside all the spheres, field strength at this point is zero.

Potential at 'a'

$$\begin{aligned} &= \sum \frac{Q}{4\pi\epsilon_0 d} = 9 \times 10^9 \sum \frac{Q}{d} \\ &= 9 \times 10^9 \left( \frac{8 \times 10^{-12}}{0.04} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{1.35 \text{ V}} \end{aligned}$$

(ii) Since point 'b' is outside sphere  $A$  but inside  $B$  and  $C$ .

$$\begin{aligned} \therefore \text{Electrical field} &= \frac{Q}{4\pi\epsilon_0 d^2} = 9 \times 10^9 \frac{Q}{d^2} \text{ N/C} \\ &= 9 \times 10^9 \times \frac{8 \times 10^{-12}}{0.05^2} = \mathbf{28.8 \text{ N/C}} \end{aligned}$$

$$\text{Potential at 'b'} = 9 \times 10^9 \times \left( \frac{8 \times 10^{-12}}{0.05} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{0.99 \text{ V}}$$

(iii) The field strength at point 'c' distant 7 cm from centre  $O$

$$= 9 \times 10^9 \times \left[ \frac{8 \times 10^{-12}}{0.07^2} - \frac{6 \times 10^{-12}}{0.07^2} \right] = \mathbf{3.67 \text{ N/C}}$$

$$\text{Potential at 'c'} = 9 \times 10^9 \times \left[ \frac{8 \times 10^{-12}}{0.07} - \frac{6 \times 10^{-12}}{0.07} + \frac{4 \times 10^{-12}}{0.08} \right] = \mathbf{0.71 \text{ V}}$$

(iv) Field strength at 'd' distant 10 cm from point  $O$  is

$$= 9 \times 10^9 \times \left[ \frac{8 \times 10^{-12}}{0.1^2} - \frac{6 \times 10^{-12}}{0.1^2} + \frac{4 \times 10^{-12}}{0.1^2} \right] = \mathbf{5.4 \text{ N/C}}$$

$$\text{Potential at 'd'} = 9 \times 10^9 \times \left[ \frac{8 \times 10^{-12}}{0.1} - \frac{6 \times 10^{-12}}{0.1} + \frac{4 \times 10^{-12}}{0.1} \right] = \mathbf{0.54 \text{ V}}$$

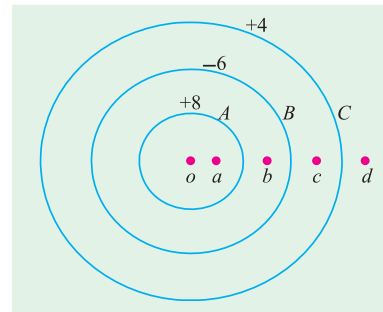


Fig. 4.20

**Example 4.10.** Two positive point charges of  $12 \times 10^{-10} \text{ C}$  and  $8 \times 10^{-10} \text{ C}$  are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.

**Solution.** Suppose the  $12 \times 10^{-10} \text{ C}$  charge to be fixed. Now, the potential of a point 10 cm from this charge

$$= 9 \times 10^9 \frac{12 \times 10^{-10}}{0.1} = 108 \text{ V}$$

The potential of a point distant 6 cm from it

$$= 9 \times 10^9 \times \frac{12 \times 10^{-10}}{0.06} = 180 \text{ V}$$

$\therefore$  potential difference =  $180 - 108 = 72 \text{ V}$

Work done = charge  $\times$  p.d. =  $8 \times 10^{-10} \times 72 = \mathbf{5.76 \times 10^{-8} \text{ joule}}$

**Example 4.11.** A point charge of  $10^{-9} \text{ C}$  is placed at a point  $A$  in free space. Calculate :

(i) the intensity of electrostatic field on the surface of sphere of radius 5 cm and centre  $A$ .

(ii) the difference of potential between two points 20 cm and 10 cm away from the charge at A.  
(Elements of Elect.-I, Banglore Univ. 1987)

**Solution. (j)**  $E = Q/4\pi\epsilon_0 r^2 = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times (5 \times 10^{-2})^2 = 3,595 \text{ V/m}$

(ii) Potential of first point  $= Q/4\pi\epsilon_0 d = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.2 = 45 \text{ V}$

Potential of second point  $= 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.1 = 90 \text{ V}$

$\therefore$  p.d. between two points  $= 90 - 45 = 45 \text{ V}$

#### 4.17. Potential Gradient

It is defined as the rate of change of potential with distance in the direction of electric force

i.e.  $\frac{dV}{dx}$

Its unit is volt/metre although volt/cm is generally used in practice. Suppose in an electric field of strength  $E$ , there are two points  $dx$  metre apart. The p.d. between them is

$$dV = E \cdot (-dx) = -E \cdot dx \quad \therefore E = -\frac{dV}{dx} \quad \dots(i)$$

*The -ve sign indicates that the electric field is directed outward, while the potential increases inward.*

*Hence, it means that electric intensity at a point is equal to the negative potential gradient at that point.*

#### 4.18. Breakdown Voltage and Dielectric Strength

An insulator or dielectric is a substance within which there are no mobile electrons necessary for electric conduction. However, when the voltage applied to such an insulator exceeds a certain value, then it breaks down and allows a heavy electric current (much larger than the usual leakage current) to flow through it. If the insulator is a solid medium, it gets punctured or cracked.

The disruptive or breakdown voltage of an insulator is the minimum voltage required to break it down.\*

Dielectric strength of an insulator or dielectric medium is given by the *maximum potential difference which a unit thickness of the medium can withstand without breaking down.*

In other words, the dielectric strength is given by the potential gradient necessary to cause breakdown of an insulator. Its unit is volt/metre (V/m) although it is usually expressed in kV/mm.

For example, when we say that the dielectric strength of air is 3 kV/mm, then it means that the maximum p.d. which one mm thickness of air can withstand across it without breaking down is 3 kV or 3000 volts. If the p.d. exceeds this value, then air insulation breaks down allowing large electric current to pass through.

Dielectric strength of various insulating materials is very important factor in the design of high-voltage generators, motors and transformers. Its value depends on the thickness of the insulator, temperature, moisture, content, shape and several other factors.

For example doubling the thickness of insulation does not double the safe working voltage in a machine.\*\*

\* Flashover is the disruptive discharge which takes place over the surface of an insulator and occurs when the air surrounding it breaks down. Disruptive conduction is luminous.

\*\* The relation between the breakdown voltage  $V$  and the thickness of the dielectric is given approximately by the relation  $V = At^{2/3}$  where  $A$  is a constant depending on the nature of the medium and also on the thickness  $t$ . The above statement is known as Baur's law.

**Note.** It is obvious that the electric intensity  $E$ , potential gradient and dielectric strength are dimensionally equal.

#### 4.19. Safety Factor of a Dielectric

It is given by the ratio of the dielectric strength of the insulator and the electric field intensity established in it. If we represent the dielectric strength by  $E_{bd}$  and the actual field intensity by  $E$ , then safety factor

$$k = E_{bd}/E$$

For example, for air  $E_{bd} = 3 \times 10^6$  V/m. If we establish a field intensity of  $3 \times 10^5$  V/m in it, then,  $k = 3 \times 10^6 / 3 \times 10^5 = 10$ .

#### 4.20. Boundary Conditions

There are discontinuities in electric fields at the boundaries between conductors and dielectrics of different permittivities. The relationships existing between the electric field strengths and flux densities at the boundary are called the boundary conditions.

With reference to Fig. 4.21, first boundary conditions is that the normal component of flux density is continuous across a surface.

As shown, the electric flux approaches the boundary  $BB$  at an angle  $\theta_1$  and leaves it at  $\theta_2$ .  $D_{1n}$  and  $D_{2n}$  are the normal components of  $D_1$  and  $D_2$ . According to first boundary condition,

$$D_{1n} = D_{2n} \quad \dots (i)$$

The second boundary condition is that the tangential field strength is continuous across the boundary

$$\therefore E_{1t} = E_{2t} \quad \dots (ii)$$

In Fig. 4.21, we see that

$$D_{1n} = D_1 \cos \theta_1 \quad \text{and} \quad D_{2n} = D_2 \cos \theta_2$$

$$\text{Also} \quad E_1 = D_1/\epsilon_1 \quad \text{and} \quad E_{1t} = D_1 \sin \theta_1/\epsilon_1$$

$$\text{Similarly,} \quad E_2 = D_2/\epsilon_2 \quad \text{and} \quad E_{2t} = D_2 \sin \theta_2/\epsilon_2$$

$$\therefore \frac{D_{1n}}{E_{1t}} = \frac{\epsilon_1}{\tan \theta_1} \quad \text{and} \quad \frac{D_{2n}}{E_{2t}} = \frac{\epsilon_2}{\tan \theta_2}$$

$$\text{Since} \quad D_{1n} = D_{2n} \quad \text{and} \quad E_{1t} = E_{2t} \quad \therefore \frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}$$

This gives the law of electric flux refraction at a boundary.

It is seen that if  $\epsilon_1 > \epsilon_2$ ,  $\theta_1 > \theta_2$ .

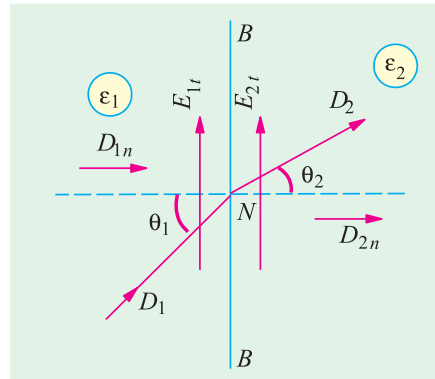


Fig. 4.21

**Table No. 4.1**  
**Dielectric Constant and Strength**  
(\*indicates average value)

Insulating material	Dielectric constant or relative permittivity ( $\epsilon_r$ )	Dielectric Strength in kV/mm
Air	1.0006	3.2
Asbestos*	2	2
Bakelite	5	15
Epoxy	3.3	20

Glass	5-12	12-100
Marble*	7	2
Mica	4-8	20-200
Micanite	4-5-6	25-35
Mineral Oil	2.2	10
Mylar	3	400
Nylon	4.1	16
Paper	1.8-2.6	18
Paraffin wax	1.7-2.3	30
Polyethylene	2.3	40
Polyurethane	3.6	35
Porcelain	5-6.7	15
PVC	3.7	50
Quartz	4.5-4.7	8
Rubber	2.5-4	12-20
Teflon	2	20
Vacuum	1	infinity
Wood	2.5-7	---

**Example 4.12.** Find the radius of an isolated sphere capable of being charged to 1 million volt potential before sparking into the air, given that breakdown voltage of air is 30,000 V/cm.

**Solution.** Let  $r$  metres be the radius of the spheres, then

$$V = \frac{Q}{4\pi\epsilon_0 r} = 10^6 \text{ V} \quad \dots(i)$$

Breakdown voltage = 30,000 V/cm =  $3 \times 10^6$  V/m  
 Since electric intensity equals breakdown voltage

$$\therefore E = \frac{Q}{4\pi\epsilon_0 r^2} = 3 \times 10^6 \text{ V/m} \quad \dots(ii)$$

Dividing (i) by (ii), we get  $r = 1/3 = 0.33$  metre

**Example 4.13.** A parallel plate capacitor having waxed paper as the insulator has a capacitance of 3800 pF, operating voltage of 600 V and safety factor of 2.5. The waxed paper has a relative permittivity of 4.3 and breakdown voltage of  $15 \cdot 10^6$  V/m. Find the spacing  $d$  between the two plates of the capacitor and the plate area.

**Solution.** Breakdown voltage  $V_{bd} = \text{operating voltage} \times \text{safety factor} = 600 \cdot 2.5 = 1500$  V

$$V_{bd} = d \times E_{bd} \quad \text{or} \quad d = 1500/15 \times 10^6 = 10^{-4} \text{ m} = 0.1 \text{ mm}$$

$$C = \epsilon_0 \epsilon_r A/d \quad \text{or} \quad A = Cd/\epsilon_0 \epsilon_r = 3800 \times 10^{-9} \times 10^{-4}/8.854 \times 10^{-12} \cdot 4.3 = 0.01 \text{ m}^2$$

**Example 4.14.** Two brass plates are arranged horizontally, one 2 cm above the other and the lower plate is earthed. The plates are charged to a difference of potential of 6,000 volts. A drop of oil with an electric charge of  $1.6 \times 10^{-19}$  C is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop ?

**Solution.** The electric intensity is equal to the potential gradient between the plates.

$$g = 6,000/2 = 3,000 \text{ volt/cm} = 3 \times 10^5 \text{ V/m}$$

$$\therefore E = 3 \times 10^5 \text{ V/m} \quad \text{or} \quad \text{N/C}$$

$$\therefore \text{force on drop} = E \times Q = 3 \times 10^5 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14} \text{ N}$$

$$\text{Wt. of drop} = mg \text{ newton}$$

$$\therefore m \times 9.81 = 4.8 \times 10^{-14} \quad \therefore m = 4.89 \times 10^{-15} \text{ kg}$$

**Example 4.15.** A parallel-plate capacitor has plates 0.15 mm apart and dielectric with relative permittivity of 3. Find the electric field intensity and the voltage between plates if the surface charge is  $5 \times 10^{-4} \mu\text{C}/\text{cm}^2$ .

(Electrical Engineering, Calcutta Univ.)

**Solution.** The electric intensity between the plates is

$$E = \frac{D}{\epsilon_0 \epsilon_r} \text{ volt/metre;}$$

$$\text{Now, } \sigma = 5 \times 10^{-4} \mu\text{C}/\text{cm}^2 = 5 \times 10^{-6} \text{ C}/\text{m}^2$$

Since, charge density equals flux density

$$\therefore E = \frac{D}{\epsilon_0 \epsilon_r} = \frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3} = 188,000 \text{ V}/\text{m} = \mathbf{188 \text{ kV}/\text{m}}$$

$$\text{Now potential difference } V = E \times dx = 188,000 \times (0.15 \times 10^{-3}) = \mathbf{2.82 \text{ V}}$$

**Example 4.16.** A parallel-plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon ( $\epsilon_r = 2.0$ ) 6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100 V is applied across the capacitor, find the electric field ( $E_0$ ) in the air; electric field  $E_t$  in Teflon, flux density  $D_a$  in air; flux density  $D_t$  in Teflon and potential difference  $V_t$  across Teflon slab. (Circuit and Field Theory, A.M.I.E. Sec. B)

**Solution.**

$$C = \frac{\epsilon_0 A}{(d_1/\epsilon_{r1} + d_2/\epsilon_{r2})} = \frac{8.854 \times 10^{-12} \times (0.5)^2}{(6 \times 10^{-3}/2) + (4 \times 10^{-3}/1)} = 3.16 \times 10^{-10} \text{ F}$$

$$Q = CV = 3.16 \times 10^{-10} \times 100 = 31.6 \times 10^{-9} \text{ C}$$

$$D = Q/A = 31.6 \times 10^{-9}/(0.5)^2 = 1.265 \times 10^{-7} \text{ C}/\text{m}^2$$

The charge or flux density will be the same in both media i.e.  $D_a = D_t = D$

$$\text{In air, } E_0 = D/\epsilon_0 = 1.265 \times 10^{-7}/8.854 \times 10^{-12} = 14,280 \text{ V}/\text{m}$$

$$\text{In Teflon, } E_t = D/\epsilon_0 \epsilon_r = 14,280/2 = 7,140 \text{ V}/\text{m}$$

$$V_t = E_t \times d_t = 7,140 \times 6 \times 10^{-3} = \mathbf{42.8 \text{ V}}$$

**Example 4.17.** Calculate the dielectric flux in micro-coulombs between two parallel plates each 35 cm square with an air gap of 1.5 mm between them, the p.d. being 3,000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7,500 V.

(Elect. Engg.-I, Nagpur Univ.)

**Solution.** The capacitance of the two parallel plates is

$$C = \epsilon_0 \epsilon_r A/d \quad \text{Now, } \epsilon_r = 1 \quad \text{—for air}$$

$$A = 35 \times 35 \times 10^{-4} = 1,225 \times 10^{-4} \text{ m}^2; \quad d = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore C = \frac{8.854 \times 10^{-12} \times 1,225 \times 10^{-4}}{1.5 \times 10^{-3}} \text{ F} = 7.22 \times 10^{-16} \text{ F}$$

$$\text{Charge } Q = CV = 7.22 \times 10^{-16} \times 3,000 \text{ coulomb}$$

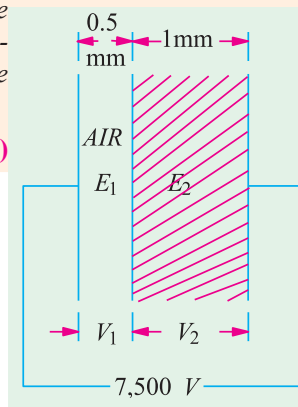
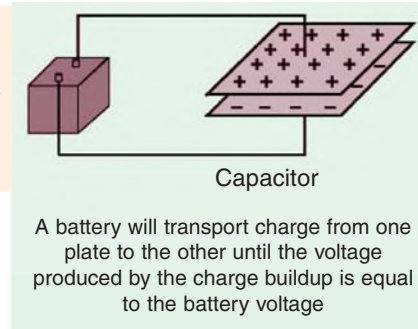


Fig. 4.22

$$\begin{aligned} \text{Dielectric flux} &= 7.22 \times 3,000 \times 10^{-10} \text{ C} \\ &= 2.166 \times 10^{-6} \text{ C} = \mathbf{2.166 \mu\text{C}} \end{aligned}$$

With reference to Fig. 4.23, we have

$$V_1 = E_1 x_1 = 0.5 \times 10^{-3} E_1; V_2 = 10^{-3} E_2$$

$$\text{Now } V = V_1 + V_2$$

$$\therefore 7,500 = 0.5 \times 10^{-3} E_1 + 10^{-3} E_2$$

$$\text{or } E_1 + 2 E_2 = 15 \times 10^6 \quad \dots(i)$$

$$\text{Also } D = \epsilon_0 \epsilon_{r1} E_1 = \epsilon_0 \epsilon_{r2} E_2 \quad \therefore E_1 = 6 E_2 \quad \dots(ii)$$

From (i) and (ii), we obtain  $E_1 = 11.25 \times 10^6 \text{ V/m}; E_2 = 1.875 \times 10^6 \text{ V/m}$

**Example 4.18.** An electric field in a medium with relative permittivity of 7 passes into a medium of relative permittivity 2. If  $E$  makes an angle of  $60^\circ$  with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric?

(Elect. Engg. Nagpur Univ.)

**Solution.** As seen from Art. 4.19.

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2} \quad \text{or} \quad \frac{\tan 60^\circ}{\tan \theta_2} = \frac{7}{2} \quad \therefore \tan \theta_2 = \sqrt{3} \times 2/7 = 4.95 \quad \text{or} \quad \theta_2 = \mathbf{26^\circ 20'}$$

**Example 4.19.** Two parallel sheets of glass having a uniform air gap between their inner surfaces are sealed around their edges (Fig. 4.23). They are immersed in oil having a relative permittivity of 6 and are mounted vertically. The glass has a relative permittivity of 3. Calculate the values of electric field strength in the glass and the air when that in the oil is  $1.2 \text{ kV/m}$ . The field enters the glass at  $60^\circ$  to the horizontal.

**Solution.** Using the law of electric flux refraction, we get (Fig. 4.23).

$$\tan \theta_2 / \tan \theta_1 = \epsilon_2 / \epsilon_1 = \epsilon_0 \epsilon_{r2} / \epsilon_0 \epsilon_{r1} = (\epsilon_{r2} / \epsilon_{r1})$$

$$\begin{aligned} \therefore \tan \theta_2 &= (6/3) \tan 60^\circ \\ &= 2 \times 1.732 = 3.464; \\ \theta_2 &= 73.9^\circ \end{aligned}$$

Similarly

$$\begin{aligned} \tan \theta_3 &= (\epsilon_{r3} / \epsilon_{r2}) \tan \theta_2 = (1/6) \tan 73.9^\circ \\ &= 0.577; \therefore \theta_3 = 30^\circ \end{aligned}$$

As shown in Art. 4.20.

$$\begin{aligned} D_{1n} &= D_{2n} \quad \text{or} \quad D_1 \cos \theta_1 = D_2 \cos \theta_2 \\ \therefore D_2 &= D_1 \times \cos \theta_1 / \cos \theta_2 \quad \text{or} \quad \epsilon_0 \epsilon_{r2} E_2 \\ &= \epsilon_0 \epsilon_{r1} E_1 \times \cos \theta_1 / \cos \theta_2 \\ \therefore 6 E_2 &= 3 \times 1.2 \times 10^3 \times \cos 60^\circ / \cos 73.9^\circ \\ \therefore E_2 &= 1082 \text{ V/m} \end{aligned}$$

$$\begin{aligned} \text{Now, } \epsilon_0 \epsilon_{r3} E_3 \cos \theta_3 &= \epsilon_0 \epsilon_{r2} E_2 \cos \theta_2 \\ \therefore E_3 &= E_2 (\epsilon_{r2} / \epsilon_{r3}) \times (\cos \theta_2 / \cos \theta_3) \\ &= 1082 (6/1) (\cos 73.9^\circ / \cos 30^\circ) = \mathbf{2079 \text{ V/m}} \end{aligned}$$

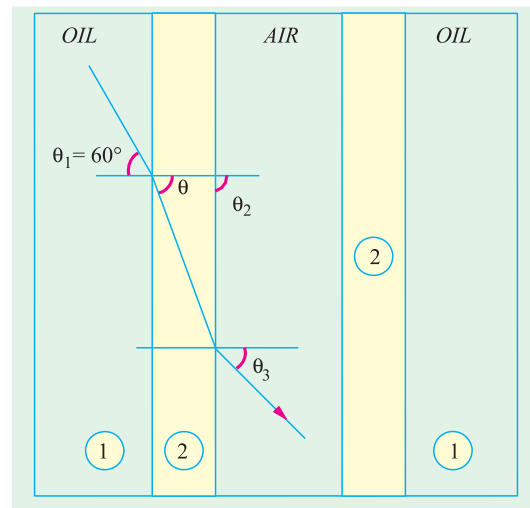


Fig. 4.23

## Tutorial Problems No. 4.1

- Two parallel metal plates of large area are spaced at a distance of 1 cm from each other in air and a p.d. of 5,000 V is maintained between them. If a sheet of glass 0.5 cm thick and having a relative permittivity of 6 is introduced between the plates, what will be the maximum electric stress and where will it occur ?  
[8.57 kV/cm; in air]
- A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 1.5 cm and relative permittivity 7 is introduced between the plates.  
[ $1.4 \times 10^8$  V/m]
- A capacitor is made up of two parallel circular metal discs separated by three layers of dielectric of equal thickness but having relative permittivities of 3, 4 and 5 respectively. The diameter of each disc is 25.4 cm and the distance between them is 6 cm. Calculate the potential gradient in each dielectric when a p.d. of 1,500 V is applied between the discs.  
[319.2; 239.4; 191.5 kV/m]
- A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 0.5 cm and relative permittivity 5 is introduced between the plates.  
[ $0.625 \times 10^4$  V/m]
- The capacitance of a capacitor formed by two parallel metal plates, each having an effective surface area of  $50 \text{ cm}^2$  and separated by a dielectric 1 mm thick, is  $0.0001 \mu\text{F}$ . The plates are charged to a p.d. of 200 V. Calculate (a) the charge stored (b) the electric flux density (c) the relative permittivity of the dielectric.  
[(a)  $0.02 \mu\text{C}$  (b)  $4 \mu\text{C}/\text{m}^2$  (c) 2.26]
- A capacitor is constructed from two parallel metallic circular plates separated by three layers of dielectric each 0.5 cm thick and having relative permittivity of 4, 6 and 8 respectively. If the metal discs are 15.25 cm in diameter, calculate the potential gradient in each dielectric when the applied voltage is 1,000 volts.  
(Elect. Engg.-I Delhi Univ.)
- A point electric charge of  $8 \mu\text{C}$  is kept at a distance of 1 metre from another point charge of  $-4 \mu\text{C}$  in free space. Determine the location of a point along the line joining two charges where in the electric field intensity is zero.  
(Elect. Engineering, Kerala Univ.)
- In a given R-L circuit,  $R = 35\Omega$  and  $L = 0.1\text{H}$ . Find (i) current through the circuit (ii) power factor if a 50 Hz frequency, voltage  $V = 220\angle 30^\circ$  is applied across the circuit.  
(RGPV, Bhopal 2001)
- Three voltage represented by  $e_1 = 20 \sin \omega t$ ,  $e_2 = 30 \sin (\omega t + 45^\circ)$  and  $e_3 = \sin (\omega t + 30^\circ)$  are connected in series and then connected to a load of impedance  $(2 + j 3) \Omega$ . Find the resultant current and power factor of the circuit. Draw the phasor diagram.  
(B.P.T.U. Orissa 2003) (RGPV Bhopal 2001)

## OBJECTIVE TESTS – 4

- The unit of absolute permittivity of a medium is  
(a) joule/coulomb  
(b) newton-metre  
(c) farad/metere  
(d) farad/coulomb
- If relative permittivity of mica is 5, its absolute permittivity is  
(a)  $5 \epsilon_0$   
(b)  $5/\epsilon_0$   
(c)  $\epsilon_0/5$   
(d)  $8.854 \times 10^{-12}$
- Two similar electric charges of 1 C each are placed 1 m apart in air. Force of repulsion between them would be nearly..... newton  
(a) 1  
(b)  $9 \times 10^9$   
(c)  $4 \pi$   
(d)  $8.854 \times 10^{-12}$
- Electric flux emanating from an electric charge of  $+Q$  coulomb is  
(a)  $Q/\epsilon_0$   
(b)  $Q/\epsilon_r$   
(c)  $Q/\epsilon_0\epsilon_r$   
(d)  $Q$
- The unit of electric intensity is  
(a) joule/coulomb  
(b) newton/coulomb

- (c) volt/metre  
(d) both (b) and (c)
6. If  $D$  is the electric flux density, then value of electric intensity in air is  
(a)  $D/\epsilon_0$  (b)  $D/\epsilon_0\epsilon_r$   
(c)  $dV/dt$  (d)  $Q/\epsilon A$
7. For any medium, electric flux density  $D$  is related to electric intensity  $E$  by the equation  
(a)  $D = \epsilon_0 E$  (b)  $D = \epsilon_0\epsilon_r E$   
(c)  $D = E/\epsilon_0\epsilon_r$  (d)  $D = \epsilon_0 E/\epsilon_r$
8. Inside a conducting sphere, ...remains constant  
(a) electric flux  
(b) electric intensity  
(c) charge  
(d) potential
9. The SI unit of electric intensity is  
(a) N/m  
(b) V/m  
(c) N/C  
(d) either (b) or (c)
10. According to Gauss's theorem, the surface integral of the normal component of electric flux density  $D$  over a closed surface containing charge  $Q$  is  
(a)  $Q$  (b)  $Q/\epsilon_0$   
(c)  $\epsilon_0 Q$  (d)  $Q^2/\epsilon_0$
11. Which of the following is zero inside a charged conducting sphere?  
(a) potential  
(b) electric intensity  
(c) both (a) and (b)  
(d) both (b) and (c)
12. In practice, earth is chosen as a place of zero electric potential because it  
(a) is non-conducting  
(b) is easily available  
(c) keeps losing and gaining electric charge every day  
(d) has almost constant potential.

## ANSWERS

- |      |      |      |       |       |       |
|------|------|------|-------|-------|-------|
| 1. c | 2. a | 3. b | 4. d  | 5. d  | 6. a  |
| 7. b | 8. d | 9. d | 10. a | 11. c | 12. d |

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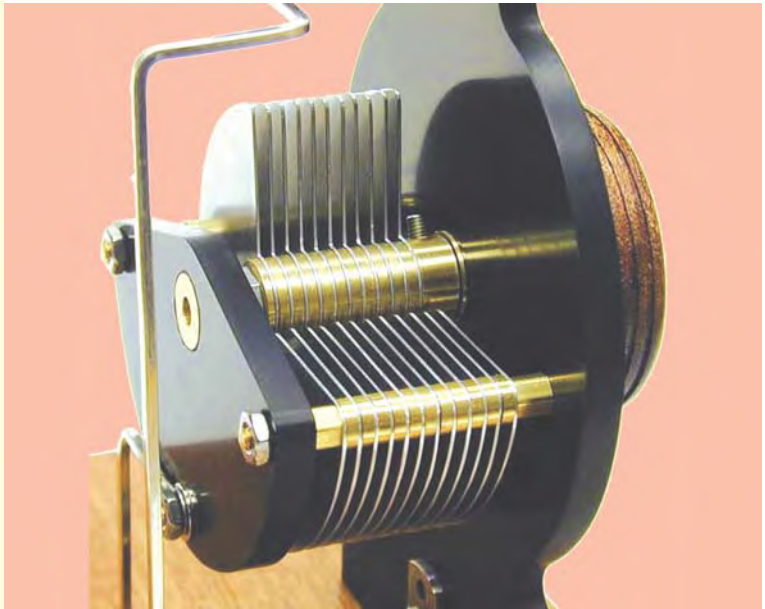


# C H A P T E R 5

## Learning Objectives

- Capacitor
- Capacitance
- Capacitance of an Isolated Sphere
- Spherical Capacitor
- Parallel-plate Capacitor
- Special Cases of Parallel-plate Capacitor
- Multiple and Variable Capacitors
- Cylindrical Capacitor
- Potential Gradient in Cylindrical Capacitor
- Capacitance Between two Parallel Wires
- Capacitors in Series
- Capacitors in Parallel
- Cylindrical Capacitor with Compound Dielectric
- Insulation Resistance of a Cable Capacitor
- Energy Stored in a Capacitor
- Force of Attraction Between Oppositely-charged Plates
- Current-Voltage Relationships in a Capacitor
- Charging of a Capacitor
- Time Constant
- Discharging of a Capacitor
- Transient Relations during Capacitor Charging Cycle
- Transient Relations during Capacitor Discharging Cycle
- Charging and Discharging of a Capacitor with Initial Charge

## CAPACITANCE



The above figure shows a variable capacitor. A capacitor stores electric charge and acts a small reservoir of energy

### 5.1. Capacitor

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called **dielectric**. The conducting surfaces may be in the form of either circular (or rectangular) plates or be of spherical or cylindrical shape. The purpose of a capacitor is to store electrical energy by electrostatic stress in the dielectric (the word ‘condenser’ is a misnomer since a capacitor does not ‘condense’ electricity as such, it merely stores it).

A parallel-plate capacitor is shown in Fig. 5.1. One plate is joined to the positive end of the supply and the other to the negative end or is earthed. It is experimentally found that in the presence of an earthed plate *B*, plate *A* is capable of withholding more charge than when *B* is not there. When such a capacitor is put across a battery, there is a momentary flow of electrons from *A* to *B*. As negatively-charged electrons are withdrawn from *A*, it becomes positive and as these electrons collect on *B*, it becomes negative. Hence, a p.d. is established between plates *A* and *B*. The transient flow of electrons gives rise to charging current. The strength of the charging current is maximum when the two plates are uncharged but it then decreases and finally ceases when p.d. across the plates becomes slowly and slowly equal and opposite to the battery e.m.f.



Fig. 5.1

### 5.2. Capacitance

The property of a capacitor to ‘store electricity’ may be called its capacitance.

As we may measure the capacity of a tank, not by the total mass or volume of water it can hold, but by the mass in kg of water required to raise its level by one metre, similarly, the capacitance of a capacitor is defined as **“the amount of charge required to create a unit p.d. between its plates.”**

Suppose we give  $Q$  coulomb of charge to one of the two plate of capacitor and if a p.d. of  $V$  volts is established between the two, then its capacitance is

$$C = \frac{Q}{V} = \frac{\text{charge}}{\text{potential difference}}$$

Hence, capacitance is the **charge required per unit potential difference**.

By definition, the unit of capacitance is coulomb/volt which is also called **farad** (in honour of Michael Faraday)

∴

$$1 \text{ farad} = 1 \text{ coulomb/volt}$$

One farad is defined as **the capacitance of a capacitor which requires a charge of one coulomb to establish a p.d. of one volt between its plates**.

One farad is actually too large for practical purposes. Hence, much smaller units like microfarad ( $\mu\text{F}$ ), nanofarad (nF) and micro-microfarad ( $\mu\mu\text{F}$ ) or picofarad (pF) are generally employed.

$$1 \mu\text{F} = 10^{-6} \text{ F}; 1 \text{ nF} = 10^{-9} \text{ F}; 1 \mu\mu\text{F} \text{ or } \text{pF} = 10^{-12} \text{ F}$$

Incidentally, capacitance is that property of a capacitor which delays and change of voltage across it.

### 5.3. Capacitance of an Isolated Sphere

Consider a charged sphere of radius  $r$  metres having a charge of  $Q$  coulomb placed in a medium



A capacitor stores electricity

of relative permittivity  $\epsilon_r$ , as shown in Fig. 5.2.

It has been proved in Art 4.13 that the free surface potential  $V$  of such a sphere with respect to infinity (in practice, earth) is given by

$$V = \frac{Q}{4\pi\epsilon_0\epsilon_r r} \quad \therefore \frac{Q}{V} = 4\pi\epsilon_0\epsilon_r r$$

By definition,  $Q/V =$  capacitance  $C$

$$\therefore \begin{aligned} &= 4\pi\epsilon_0\epsilon_r r \text{ F} && \text{– in a medium} \\ &= 4\pi\epsilon_0 r \text{ F} && \text{– in air} \end{aligned}$$

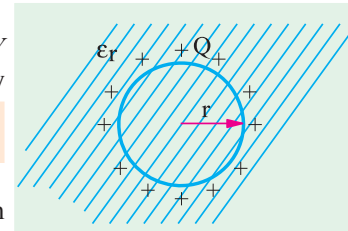


Fig. 5.2

**Note :** It is sometimes felt surprising that an isolated sphere can act as a capacitor because, at first sight, it appears to have one plate only. The question arises as to which is the second surface. But if we remember that the surface potential  $V$  is with reference to infinity (actually earth) then it is obvious that the other surface is earth. The capacitance  $4\pi\epsilon_0 r$  exists between the surface of the sphere and earth.

### 5.4. Spherical Capacitor

#### (a) When outer sphere is earthed

Consider a spherical capacitor consisting of two concentric spheres of radii ‘ $a$ ’ and ‘ $b$ ’ metres as shown in Fig. 5.3. Suppose, the inner sphere is given a charge of  $+Q$  coulombs. It will induce a charge of  $-Q$  coulombs on the inner surfaces which will go to earth. If the dielectric medium between the two spheres has a relative permittivity of  $\epsilon_r$ , then the free surface potential of the inner sphere due to its own charge  $Q/4\pi\epsilon_0\epsilon_r a$  volts. The potential of the inner sphere due to  $-Q$  charge on the outer sphere is  $-Q/4\pi\epsilon_0\epsilon_r b$  (remembering that potential anywhere inside a sphere is the same as that its surface).

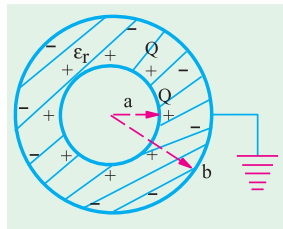


Fig. 5.3

$\therefore$  Total potential difference between two surfaces is

$$\begin{aligned} V &= \frac{Q}{4\pi\epsilon_0\epsilon_r a} - \frac{Q}{4\pi\epsilon_0\epsilon_r b} \\ &= \frac{Q}{4\pi\epsilon_0\epsilon_r} \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{4\pi\epsilon_0\epsilon_r} \left( \frac{b-a}{ab} \right) \\ \frac{Q}{V} &= \frac{4\pi\epsilon_0\epsilon_r ab}{b-a} \quad \therefore C = 4\pi\epsilon_0\epsilon_r \frac{ab}{b-a} \text{ F} \end{aligned}$$

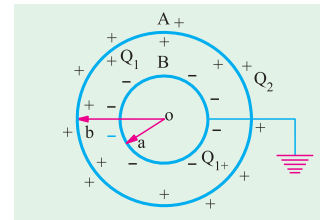


Fig. 5.4

#### (b) When inner sphere is earthed

Such a capacitor is shown in Fig. 5.4. If a charge of  $+Q$  coulombs is given to the outer sphere  $A$ , it will distribute itself over both its inner and outer surfaces. Some charge  $Q_2$  coulomb will remain on the outer surface of  $A$  because it is surrounded by earth all around. Also, some charge  $+Q_1$  coulombs will shift to its inner side because there is an earthed sphere  $B$  inside  $A$ .

Obviously,  $Q = Q_1 + Q_2$

The inner charge  $+Q_1$  coulomb on  $A$  induces  $-Q_1$  coulomb on  $B$  but the other induced charge of  $+Q_1$  coulomb goes to earth.

Now, there are two capacitors connected in parallel :

(i) One capacitor consists of the inner surface of  $A$  and the outer surface of  $B$ . Its capacitance, as found earlier, is

$$C_1 = 4\pi\epsilon_0\epsilon_r \frac{ab}{b-a}$$

(ii) The second capacitor consists of outer surfaces of  $B$  and earth. Its capacitance is  $C_2 = 4\pi\epsilon_0 b$  –if surrounding medium is air. Total capacitance  $C = C_1 + C_2$ .

### 5.5. Parallel-plate Capacitor

#### (i) Uniform Dielectric-Medium

A parallel-plate capacitor consisting of two plates  $M$  and  $N$  each of area  $A$  m<sup>2</sup> separated by a thickness  $d$  metres of a medium of relative permittivity  $\epsilon_r$  is shown in Fig. 5.5. If a charge of  $+Q$  coulomb is given to plate  $M$ , then flux passing through the medium is  $\psi = Q$  coulomb. Flux density in the medium is

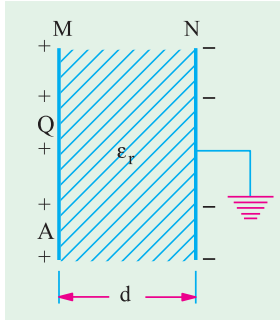


Fig. 5.5

$$D = \frac{\Psi}{A} = \frac{Q}{A}$$

Electric intensity  $E = V/d$  and  
 $D = \epsilon E$

$$\text{or} \quad \frac{Q}{A} = \epsilon \frac{V}{d} \quad \therefore \frac{Q}{V} = \frac{\epsilon A}{d}$$

$$\therefore C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ farad} \quad \text{— in a medium} \quad \dots (i)$$

$$= \frac{\epsilon_0 A}{d} \text{ farad} \quad \text{— with air as medium}$$

#### (ii) Medium Partly Air

As shown in Fig. 5.6, the medium consists partly of air and partly of parallel-sided dielectric slab of thickness  $t$  and relative permittivity  $\epsilon_r$ . The electric flux density  $D = Q/A$  is the same in both media. But electric intensities are different.

$$E_1 = \frac{D}{\epsilon_0 \epsilon_r} \quad \dots \text{ in the medium}$$

$$E_2 = \frac{D}{\epsilon_0} \quad \dots \text{ in air}$$

$$\begin{aligned} \text{p.d. between plates, } V &= E_1 \cdot t + E_2 (d-t) \\ &= \frac{D}{\epsilon_0 \epsilon_r} t + \frac{D}{\epsilon_0} (d-t) = \frac{D}{\epsilon_0} \left( \frac{t}{\epsilon_r} + d-t \right) \\ &= \frac{Q}{\epsilon_0 A} [d - (t - t/\epsilon_r)] \end{aligned}$$

$$\text{or} \quad \frac{Q}{V} = \frac{\epsilon_0 A}{[d - (t - t/\epsilon_r)]} \quad \text{or} \quad C = \frac{\epsilon_0 A}{[d - (t - t/\epsilon_r)]} \quad \dots (ii)$$

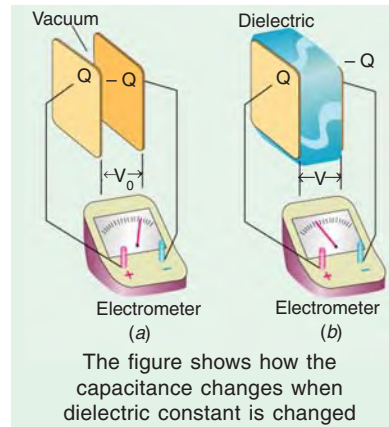
If the medium were totally air, then capacitance would have been

$$C = \epsilon_0 A/d$$

From (ii) and (iii), it is obvious that when a dielectric slab of thickness  $t$  and relative permittivity  $\epsilon_r$  is introduced between the plates of an air capacitor, then its capacitance increases because as seen from (ii), the denominator decreases. The distance between the plates is effectively reduces by  $(t - t/\epsilon_r)$ . To bring the capacitance back to its original value, the capacitor plates will have to be further separated by that much distance in air. Hence, the new separation between the two plates would be

$$= [d + (t - t/\epsilon_r)]$$

The expression given in (i) above can be written as  $C = \frac{\epsilon_0 A}{d/\epsilon_r}$



If the space between the plates is filled with slabs of different thickness and relative permittivities, then the above expression can be generalized into  $C = \frac{\epsilon_0 A}{\sum d/\epsilon_r}$

The capacitance of the capacitor shown in Fig. 5.7 can be written as

$$C = \frac{\epsilon_0 A}{\left(\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}} + \frac{d_3}{\epsilon_{r3}}\right)}$$

**(iii) Composite Medium**

The above expression may be derived independently as given under :

If  $V$  is the total potential difference across the capacitor plates and  $V_1, V_2, V_3$ , the potential differences across the three dielectric slabs, then

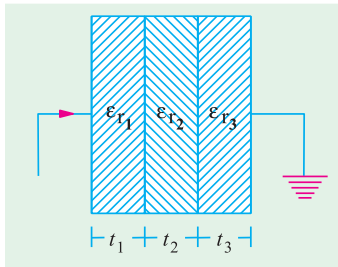


Fig. 5.7

$$\begin{aligned} V &= V_1 + V_2 + V_3 = E_1 t_1 + E_2 t_2 + E_3 t_3 \\ &= \frac{D}{\epsilon_0 \epsilon_{r1}} \cdot t_1 + \frac{D}{\epsilon_0 \epsilon_{r2}} \cdot t_2 + \frac{D}{\epsilon_0 \epsilon_{r3}} \cdot t_3 \\ &= \frac{D}{\epsilon_0} \left( \frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} + \frac{t_3}{\epsilon_{r3}} \right) = \frac{Q}{\epsilon_0 A} \left( \frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} + \frac{t_3}{\epsilon_{r3}} \right) \end{aligned}$$

$$\therefore C = \frac{Q}{V} = \frac{\epsilon_0 A}{\left(\frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} + \frac{t_3}{\epsilon_{r3}}\right)}$$

**5.6. Special Cases of Parallel-plate Capacitor**

Consider the cases illustrated in Fig. 5.8.

(i) As shown in Fig. 5.8 (a), the dielectric is of thickness  $d$  but occupies only a part of the area. This arrangement is equal to two capacitors in parallel. Their capacitances are

$$C_1 = \frac{\epsilon_0 A_1}{d} \quad \text{and} \quad C_2 = \frac{\epsilon_0 \epsilon_r A_2}{d}$$

Total capacitance of the parallel-plate capacitor is

$$C = C_1 + C_2 = \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 \epsilon_r A_2}{d}$$

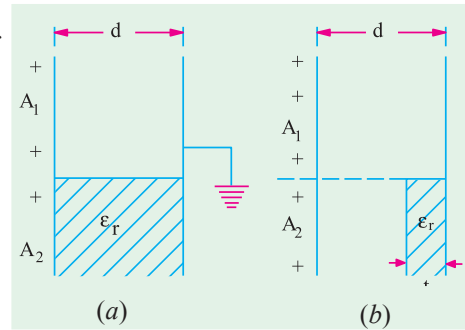


Fig. 5.8

(ii) The arrangement shown in Fig. 5.8 (b) consists of two capacitors connected in parallel.

(a) one capacitor having plate area  $A_1$  and air as dielectric. Its capacitance is  $C_1 = \frac{\epsilon_0 A_1}{d}$

(b) the other capacitor has dielectric partly air and partly some other medium. Its capacitance is [Art 5.5 (ii)].  $C_2 = \frac{\epsilon_0 A_2}{[d - (t - t/\epsilon_r)]}$ . Total capacitance is  $C = C_1 + C_2$

**5.7. Multiple and Variable Capacitors**

Multiple capacitors are shown in Fig. 5.9 and Fig. 5.10.

The arrangement of Fig. 5.9. is equivalent to two capacitors joined in parallel. Hence, its capacitance is double that of a single capacitor. Similarly, the arrangement of Fig. 5.10 has four times the capacitance of single capacitor.

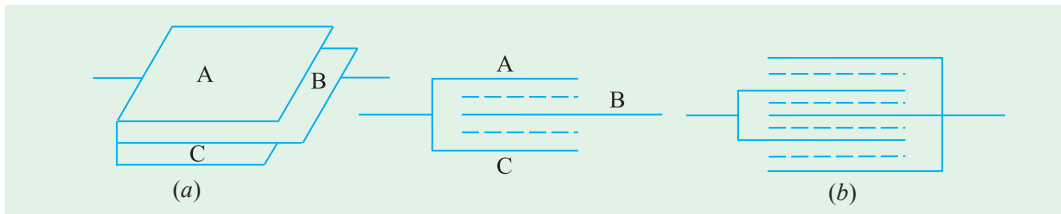


Fig. 5.9

Fig. 5.10

If one set of plates is fixed and the other is capable of rotation, then capacitance of such a multiplate capacitor can be varied. Such variable-capacitance air capacitors are widely used in radio work (Fig. 5.11). The set of fixed plates  $F$  is insulated from the other set  $R$  which can be rotated by turning the knob  $K$ . The common area between the two sets is varied by rotating  $K$ , hence the capacitance between the two is altered. Minimum capacitance is obtained when  $R$  is completely rotated out of  $F$  and maximum when  $R$  is completely rotated in *i.e.* when the two sets of plates completely overlap each other.

The capacitance of such a capacitor is

$$= \frac{(n-1) \cdot \epsilon_0 \epsilon_r A}{d}$$

where  $n$  is the number of plates which means that  $(n-1)$  is the number of capacitors.

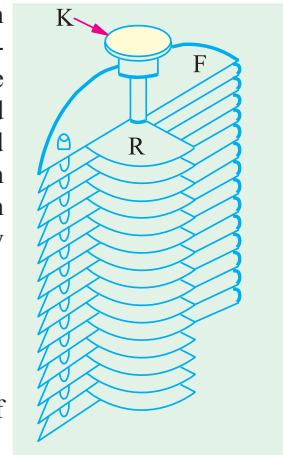


Fig. 5.11

**Example 5.1.** The voltage applied across a capacitor having a capacitance of  $10 \mu F$  is varied thus :

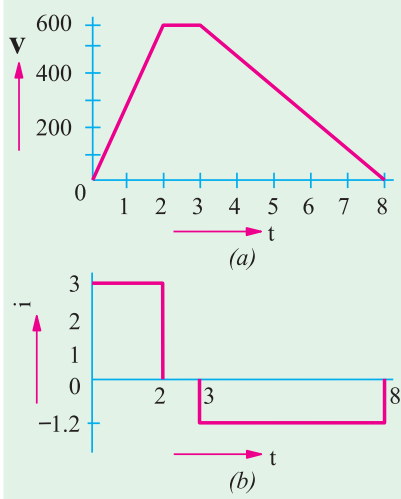


Fig. 5.12

The p.d. is increased uniformly from 0 to 600 V in seconds. It is then maintained constant at 600 V for 1 second and subsequently decreased uniformly to zero in five seconds. Plot a graph showing the variation of current during these 8 seconds. Calculate (a) the charge (b) the energy stored in the capacitor when the terminal voltage is 600.

(Principles of Elect. Engg.-I, Jadavpur Univ.)

**Solution.** The variation of voltage across the capacitor is as shown in Fig. 5.12 (a).

The charging current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt} (Cv) = C \cdot \frac{dv}{dt}$$

Charging current during the first stage

$$= 10 \times 10^{-6} \times (600/2) = 3 \times 10^{-3} \text{ A} = 3 \text{ mA}$$

Charging current during the second stage is zero because  $dv/dt = 0$  as the voltage remains constant.

Charging current through the third stage

$$= 10 \times 10^{-6} \times \left( \frac{0-600}{5} \right) = -1.2 \times 10^{-3} \text{ A} = -1.2 \text{ mA}$$

The waveform of the charging current or capacitor current is shown in Fig. 5.12 (b).

(a) Charge when a steady voltage of 600 V is applied is  $= 600 \times 10 \times 10^{-6} = 6 \times 10^{-3} \text{ C}$

(b) Energy stored  $= \frac{1}{2} C V^2 = \frac{1}{2} \times 10^{-5} \times 600^2 = 1.8 \text{ J}$

**Example 5.2.** A voltage of  $V$  is applied to the inner sphere of a spherical capacitor, whereas the outer sphere is earthed. The inner sphere has a radius of  $a$  and the outer one of  $b$ . If  $b$  is fixed and  $a$  may be varied, prove that the maximum stress in the dielectric cannot be reduced below a value of  $4 V/b$ .

**Solution.** As seen from Art. 5.4,

$$V = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \dots(i)$$

As per Art. 4.15, the value of electric intensity at any radius  $x$  between the two spheres is given by  $E = \frac{Q}{4\pi\epsilon_0\epsilon_r x^2}$  or  $Q = 4\pi\epsilon_0\epsilon_r x^2 E$

Substituting this value in (i) above, we get

$$V = \frac{4\pi\epsilon_0\epsilon_r x^2 E}{4\pi\epsilon_0\epsilon_r} \left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{or} \quad E = \frac{V}{(1/a - 1/b)x^2}$$

As per Art. 5.9, the maximum value of  $E$  occurs as the surface of inner sphere *i.e.* when  $x = a$ . For  $E$  to be maximum or minimum,  $dE/da = 0$ .

$$\therefore \frac{d}{da} \left(\frac{1}{a} - \frac{1}{b}\right) a^2 = 0 \quad \text{or} \quad \frac{d}{da} (a - a^2/b) = 0$$

$$\text{or} \quad 1 - 2a/b = 0 \quad \text{or} \quad a = b/2$$

$$\text{Now,} \quad E = \frac{V}{(1/a - 1/b)x^2} \quad \therefore E_{\max} = \frac{V}{(1/a - 1/b)a^2} = \frac{V}{(a - a^2/b)}$$

$$\text{Since, } a = b/2 \quad \therefore E_{\max} = \frac{V}{(b/2 - b^2/4b)} = \frac{4bV}{2b^2 - b^2} = \frac{4bV}{b^2} = \frac{4V}{b}$$

**Example 5.3.** A capacitor consists of two similar square aluminium plates, each  $10 \text{ cm} \times 10 \text{ cm}$  mounted parallel and opposite to each other. What is their capacitance in  $\mu\text{F}$  when distance between them is  $1 \text{ cm}$  and the dielectric is air? If the capacitor is given a charge of  $500 \mu\text{C}$ , what will be the difference of potential between plates? How will this be affected if the space between the plates is filled with wax which has a relative permittivity of 4?

**Solution.**

$$C = \epsilon_0 A/d \text{ farad}$$

Here

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}; \quad A = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$$

$$d = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$\therefore C = \frac{8.854 \times 10^{-12} \times 10^{-2}}{10^{-2}} = 8.854 \times 10^{-12} \text{ F} = \mathbf{8.854 \mu\text{F}}$$

$$\text{Now} \quad C = \frac{Q}{V} \quad \therefore V = \frac{Q}{C} \quad \text{or} \quad V = \frac{500 \times 10^{-12} \text{ C}}{8.854 \times 10^{-12} \text{ F}} = \mathbf{56.5 \text{ volts.}}$$

When wax is introduced, their capacitance is increased four times because

$$C = \epsilon_0 \epsilon_r A/d \quad \text{F} = 4 \times 8.854 = 35.4 \mu\text{F}$$

The p.d. will obviously decrease to one fourth value because charge remains constant.

$$\therefore V = 56.5/4 = \mathbf{14.1 \text{ volts.}}$$

**Example 5.4.** The capacitance of a capacitor formed by two parallel metal plates each  $200 \text{ cm}^2$  in area separated by a dielectric  $4 \text{ mm}$  thick is  $0.0004 \text{ microfarads}$ . A p.d. of  $20,000 \text{ V}$  is applied. Calculate (a) the total charge on the plates (b) the potential gradient in  $\text{V/m}$  (c) relative permittivity of the dielectric (d) the electric flux density. **(Elect. Engg. I Osmania Univ.)**

**Solution.**

$$C = 4 \times 10^4 \mu\text{F}; \quad V = 2 \times 10^4 \text{ V}$$

$$(a) \therefore \text{Total charge} \quad Q = CV = 4 \times 10^4 \times 2 \times 10^4 \mu\text{C} = 8 \mu\text{C} = \mathbf{8 \times 10^6 \text{ C}}$$

$$(b) \text{ Potential gradient} \quad = \frac{dV}{dx} = \frac{2 \times 10^4}{4 \times 10^{-3}} = \mathbf{5 \times 10^6 \text{ V/m}}$$

$$(c) \quad D = Q/A = 8 \times 10^6 / 200 \times 10^{-4} = \mathbf{4 \times 10^4 \text{ C/m}^2}$$

$$(d) \quad E = 5 \times 10^6 \text{ V/m}$$

$$\text{Since } D = \epsilon_0 \epsilon_r E \quad \therefore \epsilon_r = \frac{D}{\epsilon_0 \times E} = \frac{4 \times 10^4}{8.854 \times 10^{-12} \times 5 \times 10^6} = \mathbf{9}$$

**Example 5.5.** A parallel plate capacitor has 3 dielectrics with relative permittivities of 5.5, 2.2 and 1.5 respectively. The area of each plate is  $100 \text{ cm}^2$  and thickness of each dielectric 1 mm. Calculate the stored charge in the capacitor when a potential difference of 5,000 V is applied across the composite capacitor so formed. Calculate the potential gradient developed in each dielectric of the capacitor. (Elect. Engg. A.M.Ae.S.I.)

**Solution.** As seen from Art. 5.5,

$$C = \frac{\epsilon_0 A}{\left(\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}} + \frac{d_3}{\epsilon_{r3}}\right)} = \frac{8.854 \times 10^{-12} \times (100 \times 10^{-4})}{\left(\frac{10^{-3}}{5.5} + \frac{10^{-3}}{2.2} + \frac{10^{-3}}{1.5}\right)} = \frac{8.854 \times 10^{-14}}{10^{-3} \times 0.303} = 292 \text{ pF}$$

$$Q = CV = 292 \times 10^{-12} \times 5000 = 146 \times 10^{-8} \text{ coulomb}$$

$$D = Q/A = 146 \times 10^{-8} / (100 \times 10^{-4}) = 146 \times 10^{-6} \text{ C/m}^2$$

$$g_1 = E_1 = D/\epsilon_0 \epsilon_{r1} = 146 \times 10^{-6} / 8.854 \times 10^{-12} \times 5.5 = 3 \times 10^6 \text{ V/m}$$

$$g_2 = E_2 = D/\epsilon_0 \epsilon_{r2} = 7.5 \times 10^6 \text{ V/m}; g_3 = D/\epsilon_0 \epsilon_{r3} = 11 \times 10^6 \text{ V/m}$$

**Example 5.6.** An air capacitor has two parallel plates  $10 \text{ cm}^2$  in area and 0.5 cm apart. When a dielectric slab of area  $10 \text{ cm}^2$  and thickness 0.4 cm was inserted between the plates, one of the plates has to be moved by 0.4 cm to restore the capacitance. What is the dielectric constant of the slab? (Elect. Technology, Hyderabad Univ. 1992)

**Solution.** The capacitance in the first case is

$$C_a = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \times 10 \times 10^{-4}}{0.5 \times 10^{-2}} = \frac{\epsilon_0}{5}$$

The capacitor, as it becomes in the second case, is shown in Fig. 5.13. The capacitance is

$$C_m = \frac{\epsilon_0 A}{\Sigma d / \epsilon_r} = \frac{\epsilon_0 \times 10^{-3}}{\left(\frac{0.5 \times 10^{-3}}{\epsilon_r}\right) + \left(\frac{5}{\epsilon_r}\right)}$$

$$\text{Since, } C_a = C_m \therefore \frac{\epsilon_0}{5} = \frac{\epsilon_0}{(5/\epsilon_r + 4)} \therefore \epsilon_r = 5$$

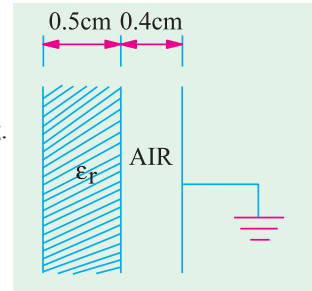


Fig. 5.13

**Note.** We may use the relation derived in Art. 5.5 (ii)

$$\text{Separation} = (t - t/\epsilon_1) \therefore 0.4 = (0.5 - 0.5/\epsilon_r) \text{ or } \epsilon_r = 5$$

**Example 5.7.** A parallel plate capacitor of area,  $A$ , and plate separation,  $d$ , has a voltage,  $V_0$ , applied by a battery. The battery is then disconnected and a dielectric slab of permittivity  $\epsilon_1$  and thickness,  $d_1$ , ( $d_1 < d$ ) is inserted. (a) Find the new voltage  $V_1$  across the capacitor; (b) Find the capacitance  $C_0$  before and its value  $C_1$  after the slab is introduced. (c) Find the ratio  $V_1/V_0$  and the ratio  $C_1/C_0$  when  $d_1 = d/2$  and  $\epsilon_1 = 4 \epsilon_0$ .

(Electromagnetic Fields and Waves AMIETE (New Scheme) June 1990)

**Solution. (b)**

$$C_0 = \frac{\epsilon_0 A}{d}; C_1 = \frac{A}{\left(\frac{(d-d_1)}{\epsilon_0} + \frac{d_1}{\epsilon_1}\right)}$$

$$\text{Since } d_1 = d/2 \text{ and } \epsilon_1 = 4 \epsilon_0 \therefore C_1 = \frac{A}{\left(\frac{d}{2\epsilon_0} + \frac{d}{2 \times 4 \epsilon_0}\right)} = \frac{8 \epsilon_0 A}{5d}$$

(a) Since the capacitor charge remains the same

$$Q = C_0 V_0 = C_1 V_1 \therefore V_1 = V_0 \frac{C_0}{C_1} = V_0 \times \frac{\epsilon_0 A}{d} \times \frac{5d}{8 \epsilon_0 A} = \frac{5V_0}{8}$$



(c) As seen from above,  $V_1 = V_0/8$ ;  $C_1/C_0 = \frac{8\epsilon_0 A}{5d} \times \frac{d}{\epsilon_0 A} = \frac{5}{8}$

### Tutorial Problems No. 5.1

1. Two parallel plate capacitors have plates of an equal area, dielectrics of relative permittivities  $\epsilon_{r1}$  and  $\epsilon_{r2}$  and plate spacing of  $d_1$  and  $d_2$ . Find the ratio of their capacitances if  $\epsilon_{r1}/\epsilon_{r2} = 2$  and  $d_1/d_2 = 0.25$ .

[ $C_1/C_2 = 8$ ]

2. A capacitor is made of two plates with an area of  $11 \text{ cm}^2$  which are separated by a mica sheet 2 mm thick. If for mica  $\epsilon_r = 6$ , find its capacitance. If, now, one plate of the capacitor is moved further to give an air gap 0.5 mm wide between the plates and mica, find the change in capacitance.

[29.19 pF, 11.6 pF]

3. A parallel-plate capacitor is made of two plane circular plates separated by  $d$  cm of air. When a parallel-faced plane sheet of glass 2 mm thick is placed between the plates, the capacitance of the system is increased by 50% of its initial value. What is the distance between the plates if the dielectric constant of the glass is 6?

[ $0.5 \times 10^{-3} \text{ m}$ ]

4. A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two circular plates, each having an area of  $100 \text{ cm}^2$  separated by a dielectric 1 mm thick. If the capacitance is  $3 \times 10^{-4} \mu\text{F}$ , calculate  
(a) the total electric flux in coulomb  
(b) the electric flux density and  
(c) the relative permittivity of the dielectric.

[(a)  $3 \times 10^{-6} \text{ C}$  (b)  $3 \times 10^{-4} \mu\text{C/m}^2$  (c) 3.39]

5. Two slabs of material of dielectric strength 4 and 6 and of thickness 2 mm and 5 mm respectively are inserted between the plates of a parallel-plate capacitor. Find by how much the distance between the plates should be changed so as to restore the potential of the capacitor to its original value.

[5.67 mm]

6. The oil dielectric to be used in a parallel-plate capacitor has a relative permittivity of 2.3 and the maximum working potential gradient in the oil is not to exceed  $10^6 \text{ V/m}$ . Calculate the approximate plate area required for a capacitance of  $0.0003 \mu\text{F}$ , the maximum working voltage being 10,000 V.

[ $147 \times 10^3 \text{ m}^2$ ]

7. A capacitor consist of two metal plates, each 10 cm square placed parallel and 3 mm apart. The space between the plates is occupied by a plate of insulating material 3 mm thick. The capacitor is charged to 300 V.

(a) the metal plates are isolated from the 300 V supply and the insulating plate is removed. What is expected to happen to the voltage between the plates?

(b) if the metal plates are moved to a distance of 6 mm apart, what is the further effect on the voltage between them. Assume throughout that the insulation is perfect.

[300  $\epsilon_r$ ; 600  $\epsilon_r$ ; where  $\epsilon_r$  is the relative permittivity of the insulating material]

8. A parallel-plate capacitor has an effecting plate area of  $100 \text{ cm}^2$  (each plate) separated by a dielectric 0.5 mm thick. Its capacitance is  $442 \mu\text{F}$  and it is raised to a potential differences of 10 kV. Calculate from first principles

- (a) potential gradient in the dielectric (b) electric flux density in the dielectric  
(c) the relative permittivity of the dielectric material.

[(a) 20 kV/mm (b)  $442 \mu\text{C/m}^2$  (c) 2.5]

9. A parallel-plate capacitor with fixed dimensions has air as dielectric. It is connected to supply of p.d. V volts and then isolated. The air is then replaced by a dielectric medium of relative permittivity 6. Calculate the change in magnitude of each of the following quantities.

- (a) the capacitance (b) the charge (c) the p.d. between the plates  
(d) the displacement in the dielectric (e) the potential gradient in the dielectric.

[(a) 6 : 1 increase (b) no change (c) 6 : 1 decrease (d) no change (e) 6 : 1 decrease]

### 5.8. Cylindrical Capacitor

A single-core cable or cylindrical capacitor consisting two co-axial cylinders of radii  $a$  and  $b$  metres, is shown in Fig. 5.14. Let the charge per metre length of the cable on the outer surface of the inner cylinder be  $+Q$  coulomb and on the inner surface of the outer cylinder be  $-Q$  coulomb. For all practical purposes, the charge  $+Q$  coulomb/metre on the surface of the inner cylinder can be supposed to be located along its axis. Let  $\epsilon_r$  be the relative permittivity of the medium between the two cylinders. The outer cylinder is earthed.

Now, let us find the value of electric intensity at any point distant  $x$  metres from the axis of the inner cylinder. As shown in Fig. 5.15, consider an imaginary co-axial cylinder of radius  $x$  metres and length one metre between the two given cylinders. The electric field between the two cylinders is radial as shown. Total flux coming out radially from the curved surface of this imaginary cylinder is  $Q$  coulomb. Area of the curved surface  $= 2\pi x \times 1 = 2\pi x$  m<sup>2</sup>.

Hence, the value of electric flux density on the surface of the imaginary cylinder is

$$D = \frac{\text{flux in coulomb}}{\text{area in metre}^2} = \frac{\Psi}{A} = \frac{Q}{A} \text{ C/m}^2 \therefore D = \frac{Q}{2\pi x} \text{ C/m}^2$$

The value of electric intensity is

$$E = \frac{D}{\epsilon_0 \epsilon_r} \quad \text{or} \quad E = \frac{Q}{2\pi \epsilon_0 \epsilon_r x} \text{ V/m}$$

Now,  $dV = -E dx$

$$\begin{aligned} \text{or} \quad V &= \int_b^a -E \cdot dx = \int_b^a -\frac{Q dx}{2\pi \epsilon_0 \epsilon_r x} \\ &= \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \int_b^a \frac{dx}{x} = \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \left| \log_e x \right|_b^a \\ &= \frac{-Q}{2\pi \epsilon_0 \epsilon_r} (\log_e a - \log_e b) = \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \log_e \left( \frac{a}{b} \right) = \frac{Q}{2\pi \epsilon_0 \epsilon_r} \log_e \left( \frac{b}{a} \right) \\ \frac{Q}{V} &= \frac{2\pi \epsilon_0 \epsilon_r}{\log_e \left( \frac{b}{a} \right)} \therefore C = \frac{2\pi \epsilon_0 \epsilon_r}{2.3 \log_{10} \left( \frac{b}{a} \right)} \text{ F/m} \left( \log_e \left( \frac{b}{a} \right) = 2.3 \log_{10} \left( \frac{b}{a} \right) \right) \end{aligned}$$

The capacitance of  $l$  metre length of this cable is  $C = \frac{2\pi \epsilon_0 \epsilon_r l}{2.3 \log_{10} \left( \frac{b}{a} \right)} \text{ F}$

In case the capacitor has compound dielectric, the relation becomes

$$C = \frac{2\pi \epsilon_0 l}{\sum \log_e \left( \frac{b}{a} \right) / \epsilon_r} \text{ F}$$

The capacitance of 1 km length of the cable in  $\mu\text{F}$  can be found by putting  $l = 1$  km in the above expression.

$$C = \frac{2\pi \times 8.854 \times 10^{-12} \times \epsilon_r \times 1000}{2.3 \log_{10} \left( \frac{b}{a} \right)} \text{ F/km} = \frac{0.024 \epsilon_r}{\log_{10} \left( \frac{b}{a} \right)} \mu\text{F/km}$$

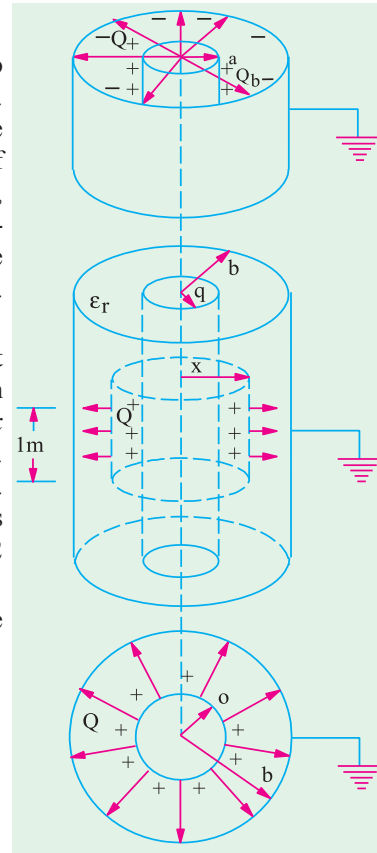


Fig. 5.14

### 5.9. Potential Gradient in a Cylindrical Capacitor

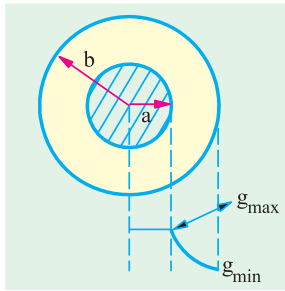


Fig. 5.15

It is seen from Art. 5.8 that in a cable capacitor

$$E = \frac{Q}{2\pi\epsilon_0\epsilon_r x} \text{ V/m}$$

where  $x$  is the distance from cylinder axis to the point under consideration.

Now  $E = g \therefore g = \frac{Q}{2\pi\epsilon_0\epsilon_r x} \text{ V/m} \dots (i)$

From Art. 5.8, we find that  $V = \frac{Q}{2\pi\epsilon_0\epsilon_r} \log_e \left(\frac{b}{a}\right)$  or  $Q = \frac{2\pi\epsilon_0\epsilon_r V}{\log_e \left(\frac{b}{a}\right)}$

Substituting this value of  $Q$  in (i) above, we get

$$g = \frac{2\pi\epsilon_0\epsilon_r V}{\log_e \left(\frac{b}{a}\right) \times 2\pi\epsilon_0\epsilon_r x} \text{ V/m} \text{ or } g = \frac{V}{x \log_e \left(\frac{b}{a}\right)} \text{ V/m} \text{ or } g = \frac{V}{2.3 x \log_{10} \left(\frac{b}{a}\right)} \text{ volt/metre}$$

Obviously, potential gradient varies inversely as  $x$ .

Minimum value of  $x = a$ , hence maximum value of potential gradient is

$$g_{max} = \frac{V}{2.3 a \log_{10} \left(\frac{b}{a}\right)} \text{ V/m} \dots (ii)$$

Similarly,

$$g_{max} = \frac{V}{2.3 b \log_{10} \left(\frac{b}{a}\right)} \text{ V/m}$$

**Note.** The above relation may be used to obtain most economical dimension while designing a cable. As seen, greater the value of permissible maximum stress  $E_{max}$ , smaller the cable may be for given value of  $V$ . However,  $E_{max}$  is dependent on the dielectric strength of the insulating material used.

If  $V$  and  $E_{max}$  are fixed, then Eq. (ii) above may be written as

$$E_{max} = \frac{V}{a \log_e \left(\frac{b}{a}\right)} \text{ or } a \log_e \left(\frac{b}{a}\right) = \frac{V}{E_{max}} \therefore \frac{b}{a} = e^{k/a} \text{ or } b = a e^{k/a}$$

For most economical cable  $db/da = 0$

$$\therefore \frac{db}{da} = 0 = e^{k/a} + a(-k/a^2)e^{k/a} \text{ or } a = k = V/E_{max} \text{ and } b = ae = 2.718 a$$

**Example 5.8.** A cable is 300 km long and has a conductor of 0.5 cm in diameter with an insulation covering of 0.4 cm thickness. Calculate the capacitance of the cable if relative permittivity of insulation is 4.5. (Elect. Engg. A.M.Ae. S.I.)

**Solution.** Capacitance of a cable is  $C = \frac{0.024 \epsilon_r}{\log_{10} \left(\frac{b}{a}\right)} \mu \text{ F/km}$

Here,  $a = 0.5/2 = 0.25 \text{ cm}$  ;  $b = 0.25 + 0.4 = 0.65 \text{ cm}$  ;  $b/a = 0.65/0.25 = 2.6$  ;  $\log_{10}^{2.6} = 0.415$

$$\therefore C = \frac{0.024 \times 4.5}{0.415} = 0.26$$

Total capacitance for 300 km is  $= 300 \times 0.26 = 78 \mu \text{ F}$ .

**Example 5.9.** In a concentric cable capacitor, the diameters of the inner and outer cylinders are 3 and 10 mm respectively. If  $\epsilon_r$  for insulation is 3, find its capacitance per metre.

A p.d. of 600 volts is applied between the two conductors. Calculate the values of the electric force and electric flux density : (a) at the surface of inner conductor (b) at the inner surface of outer conductor.

**Solution.**  $a = 1.5 \text{ mm}$  ;  $b = 5 \text{ mm}$  ;  $\therefore b/a = 5/1.5 = 10/3$  ;  $\log_{10}\left(\frac{10}{3}\right) = 0.523$

$$C = \frac{2\pi\epsilon_0\epsilon_r l}{2.3 \log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 3 \times 1}{2.3 \times 0.523} = 138.8 \times 10^{-12} \text{ F} = \mathbf{138.8 \text{ pF}}$$

(a)  $D = Q/2\pi a$

Now  $Q = CV = 138.8 \times 10^{-12} \times 600 = 8.33 \times 10^{-9} \text{ C}$

$$D = 8.33 \times 10^{-9} / 2\pi \times 1.5 \times 10^{-3} = \mathbf{8.835 \mu \text{ C/m}^2}$$

$$E = D/\epsilon_0\epsilon_r = \mathbf{332.6 \text{ V/m}}$$

(b)  $D = \frac{8.33 \times 10^{-9}}{2\pi \times 5 \times 10^{-3}} \text{ C/m}^2 = \mathbf{2.65 \mu \text{ C/m}^2}$  ;  $E = D/\epsilon_0\epsilon_r = \mathbf{99.82 \text{ V/m}}$ .

**Example 5.10.** The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. Calculate the radius of the lead sheath which covers the rubber insulation and the cable capacitance per metre. A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3. The rubber insulation has a relative permittivity of 4 and breakdown field strength of  $18 \times 10^6 \text{ V/m}$ .

**Solution.** As shown in Art. 5.9,  $g_{max} = \frac{V}{2.3 a \log_{10}\left(\frac{b}{a}\right)}$

Now,  $g_{max} = E_{max} = 18 \times 10^6 \text{ V/m}$  ;  $V = \text{breakdown voltage } x$

Safety factor =  $10^4 \times 3 = 30,000 \text{ V}$

$$\therefore 18 \times 10^6 = \frac{30,000}{2.3 \times 2.25 \times 10^{-3} \times \log_{10}\left(\frac{b}{a}\right)} \therefore \frac{b}{a} = 2.1 \text{ or } b = 2.1 \times 2.25 = 4.72 \text{ mm}$$

$$C = \frac{2\pi\epsilon_0\epsilon_r l}{2.3 \log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 4 \times 1}{2.3 \log_{10}(2.1)} = \mathbf{3 \times 10^{-9} \text{ F}}$$

## 5.10. Capacitance Between Two Parallel Wires

This case is of practical importance in overhead transmission lines. The simplest system is 2-wire system (either *d.c.* or *a.c.*). In the case of *a.c.* system, if the transmission line is long and voltage high, the charging current drawn by the line due to the capacitance between conductors is appreciable and affects its performance considerably.

With reference to Fig. 5.16, let

$d$  = distance between centres of the wires  $A$  and  $B$

$r$  = radius of each wire ( $\leq d$ )

$Q$  = charge in coulomb/metre of each wire\*

Now, let us consider electric intensity at any point  $P$  between conductors  $A$  and  $B$ .

Electric intensity at  $P^*$  due to charge  $+Q$  coulomb/metre on  $A$  is

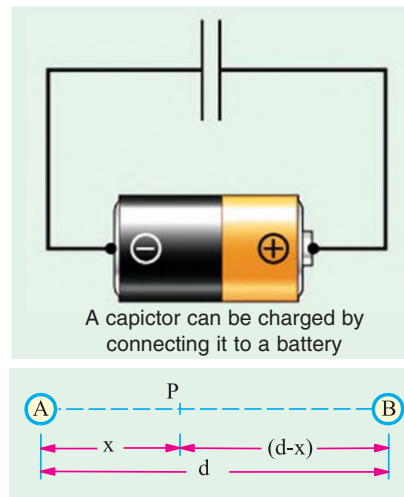


Fig. 5.16

\* If charge on  $A$  is  $+Q$ , then on  $B$  will be  $-Q$ .

$$= \frac{Q}{2\pi\epsilon_0\epsilon_r x} \text{ V/m}$$

... towards B.

Electric intensity at  $P$  due to charge  $-Q$  coulomb/metre on  $B$  is

$$= \frac{Q}{2\pi\epsilon_0\epsilon_r (d-x)} \text{ V/m}$$

... towards B.

$$\text{Total electric intensity at } P, E = \frac{Q}{2\pi\epsilon_0\epsilon_r} \left( \frac{1}{x} + \frac{1}{d-x} \right)$$

Hence, potential difference between the two wires is

$$V = \int_r^{d-r} E dx = \frac{Q}{2\pi\epsilon_0\epsilon_r} \int_r^{d-r} \left( \frac{1}{x} + \frac{1}{d-x} \right) dx$$

$$V = \frac{Q}{2\pi\epsilon_0\epsilon_r} \left[ \log_e x - \log_e (d-x) \right]_r^{d-r} = \frac{Q}{\pi\epsilon_0\epsilon_r} \log_e \frac{d-r}{r}$$

$$\text{Now } C = Q/V \therefore C = \frac{\pi\epsilon_0\epsilon_r}{\log_e \frac{(d-r)}{r}} = \frac{\pi\epsilon_0\epsilon_r}{2.3 \log_{10} \frac{(d-r)}{r}} = \frac{\pi\epsilon_0\epsilon_r}{2.3 \log_{10} \left( \frac{d}{r} \right)} \text{ F/m (approx.)}$$

$$\text{The capacitance for a length of } l \text{ metres } C = \frac{\pi\epsilon_0\epsilon_r}{2.3 \log_{10} \left( \frac{d}{r} \right)} \text{ F}$$

The capacitance per kilometre is

$$C = \frac{\pi \times 8.854 \times 10^{-12} \times \epsilon_r \times 100 \times 10^6}{2.3 \log_{10} \left( \frac{d}{r} \right)} = \frac{0.0121 \epsilon_r}{\log_{10} \left( \frac{d}{r} \right)} \mu \text{ F/km}$$

**Example 5.11.** The conductors of a two-wire transmission line (4 km long) are spaced 45 cm between centre. If each conductor has a diameter of 1.5 cm, calculate the capacitance of the line.

**Solution.** Formula used  $C = \frac{\pi\epsilon_0\epsilon_r}{2.3 \log_{10} \left( \frac{d}{r} \right)} \text{ F}$

Here  $l = 4000$  metres ;  $r = 1.5/2$  cm ;  $d = 45$  cm ;  $\epsilon_r = 1$ —for air  $\therefore \frac{d}{r} = \frac{45 \times 2}{1.5} = 60$

$$C = \frac{\pi \times 8.854 \times 10^{-12} \times 4000}{2.3 \log_{10} 60} = 0.0272 \times 10^{-6} \text{ F}$$

[or  $C = 4 \frac{0.0121}{\log_{10} 60} \text{ } 0.0272 \mu \text{F}$ ]

### 5.11. Capacitors in Series

With reference of Fig. 5.17, let

$$C_1, C_2, C_3 = \text{Capacitances of three capacitors}$$

$$V_1, V_2, V_3 = \text{p.ds. across three capacitors.}$$

$$V = \text{applied voltage across combination}$$

$$C = \text{combined or equivalent or joining capacitance.}$$

In series combination, charge on all capacitors is the same but p.d. across each is different.

$$\begin{aligned} \therefore V &= V_1 + V_2 + V_3 \\ \text{or } \frac{Q}{C} &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\ \text{or } \frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \end{aligned}$$

For a changing applied voltage,

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{dV_3}{dt}$$

We can also find values of  $V_1$ ,  $V_2$  and  $V_3$  in terms of  $V$ . Now,  $Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = CV$

$$\text{where } C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1} = \frac{C_1 C_2 C_3}{\Sigma C_1 C_2}$$

$$\therefore C_1 V_1 = CV \text{ or } V_1 = V \frac{C}{C_1} = V \cdot \frac{C_2 C_3}{\Sigma C_1 C_2}$$

$$\text{Similarly, } V_2 = V \cdot \frac{C_1 C_3}{\Sigma C_1 C_2} \text{ and } V_3 = V \cdot \frac{C_1 C_2}{\Sigma C_1 C_2}$$

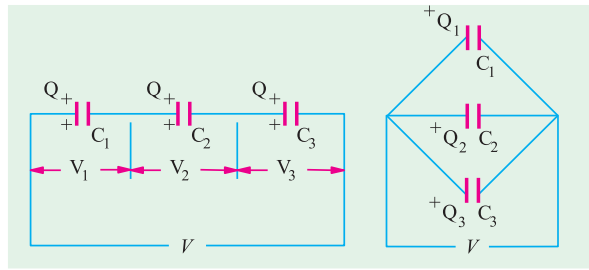


Fig. 5.17

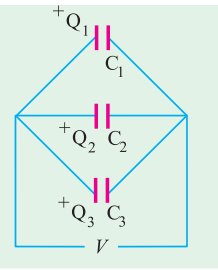


Fig. 5.18

## 5.12. Capacitors in Parallel

In this case, p.d. across each is the same but charge on each is different (Fig. 5.18).

$$\therefore Q = Q_1 + Q_2 + Q_3 \text{ or } CV = C_1 V + C_2 V + C_3 V \text{ or } C = C_1 + C_2 + C_3$$

For such a combination,  $dV/dt$  is the same for all capacitors.

**Example 5.12.** Find the  $C_{eq}$  of the circuit shown in Fig. 5.19. All capacitances are in  $\mu F$ .

(Basic Circuit Analysis Osmania Univ. Jan./Feb. 1992)

**Solution.** Capacitance between C and D =  $4 + 1 \parallel 2 = 14/3 \mu F$ .

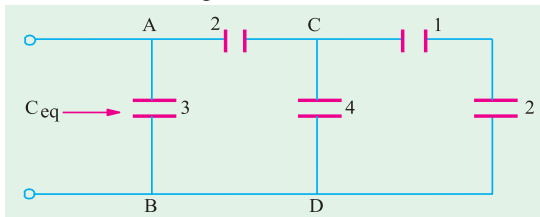


Fig. 5.19

Capacitance between A and B i.e.  $C_{eq} = 3 + 2 \parallel 14/3 = 4.4 \mu F$

**Example 5.13.** Two capacitors of a capacitance  $4 \mu F$  and  $2 \mu F$  respectively, are joined in series with a battery of e.m.f.  $100 V$ . The connections are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor.

**Solution.** When joined in series, let  $V_1$  and  $V_2$  be the voltages across the capacitors. Then as charge across each is the same.

$$\therefore 4 \times V_1 = 2V_2 \quad \therefore V_2 = 2V_1 \quad \text{Also } V_1 + V_2 = 100$$

$$\therefore V_1 + 2V_1 = 100 \quad \therefore V_1 = 100/3 \text{ V} \quad \text{and } V_2 = 200/3 \text{ V}$$

$$\therefore Q_1 = Q_2 = (200/3) \times 2 = (400/3) \mu C$$

$$\therefore \text{Total charge on both capacitors} = 800/3 \mu C$$

When joined in parallel, a redistribution of charge takes place because both capacitors are reduced to a common potential  $V$ .

Total charge =  $800/3 \mu C$ ; total capacitance =  $4 + 2 = 6 \mu F$

$$\therefore V = \frac{800}{3 \times 6} = \frac{400}{9} \text{ volts}$$

Hence

$$Q_1 = (400/9) \times 4 = 1600/9 = 178 \mu\text{C}$$

$$Q_2 = (400/9) \times 2 = 800/9 = 89 \mu\text{C (approx.)}$$

**Example 5.14.** Three capacitors A, B, C have capacitances 10, 50 and 25  $\mu\text{F}$  respectively.

Calculate (i) charge on each when connected in parallel to a 250 V supply (ii) total capacitance and (iii) p.d. across each when connected in series. (Elect. Technology, Gwalior Univ.)

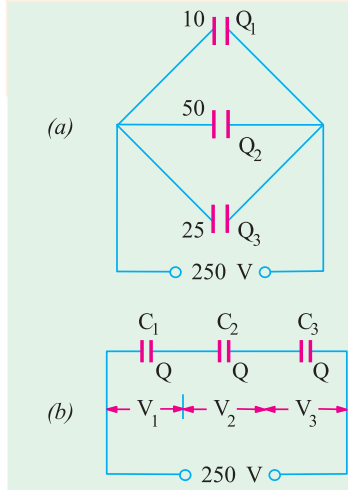


Fig. 5.20

**Solution.** (i) Parallel connection is shown in Fig. 5.20 (a). Each capacitor has a p.d. of 250 V across it.

$$Q_1 = C_1 V = 10 \times 250 = 2500 \mu\text{C}; Q_2 = 50 \times 250 = 12,500 \mu\text{C}$$

$$Q_3 = 25 \times 250 = 6,750 \mu\text{C}.$$

$$(ii) C = C_1 + C_2 + C_3 = 10 + 50 + 25 = 85 \mu\text{F}$$

(iii) Series connection is shown in Fig. 5.20 (b). Here charge on each capacitor is the same and is equal to that on the equivalent single capacitor.

$$1/C = 1/C_1 + 1/C_2 + 1/C_3; C = 25/4 \mu\text{F}$$

$$Q = CV = 25 \times 250/4 = 1562.5 \mu\text{C}$$

$$Q = C_1 V_1; V_1 = 1562.5/10 = 156.25 \text{ V}$$

$$V_2 = 1562.5/25 = 62.5 \text{ V}; V_3 = 1562.5/50 = 31.25 \text{ V}.$$

**Example 5.15.** Find the charges on capacitors in Fig. 5.21 and the p.d. across them.

**Solution.** Equivalent capacitance between points A and B is

$$C_2 + C_3 = 5 + 3 = 8 \mu\text{F}$$

Capacitance of the whole combination (Fig. 5.21)

$$C = \frac{8 \times 2}{8 + 2} = 1.6 \mu\text{F}$$

Charge on the combination is

$$Q_1 = CV = 100 \times 1.6 = 160 \mu\text{C}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{160}{2} = 80 \text{ V}; V_2 = 100 - 80 = 20 \text{ V}$$

$$Q_2 = C_2 V_2 = 3 \times 10^{-6} \times 20 = 60 \mu\text{C}$$

$$Q_3 = C_3 V_2 = 5 \times 10^{-6} \times 20 = 100 \mu\text{C}$$

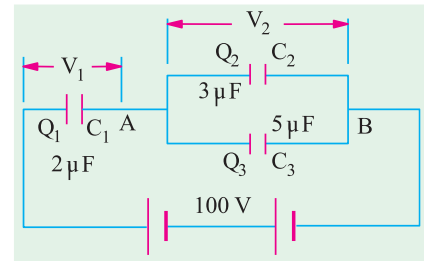


Fig. 5.21

**Example 5.16.** Two capacitors A and B are connected in series across a 100 V supply and it is observed that the p.d.s. across them are 60 V and 40 V respectively. A capacitor of 2  $\mu\text{F}$  capacitance is now connected in parallel with A and the p.d. across B rises to 90 volts. Calculate the capacitance of A and B in microfarads.

**Solution.** Let  $C_1$  and  $C_2 \mu\text{F}$  be the capacitances of the two capacitors. Since they are connected in series [Fig. 5.22 (a)], the charge across each is the same.

$$\therefore 60 C_1 = 40 C_2 \text{ or } C_1/C_2 = 2/3 \quad \dots(i)$$

In Fig. 5.22 (b) is shown a capacitor of 2  $\mu\text{F}$  connected across capacitor A. Their combined capacitance =  $(C_1 + 2) \mu\text{F}$

$$\therefore (C_1 + 2) 10 = 90 C_2 \text{ or } C_1/C_2 = 2/3 \quad \dots(ii)$$

Putting the value of  $C_2 = 3C_1/2$  from (i) in (ii) we get

$$\frac{C_1 + 2}{3C_1/2} = 9 \quad \therefore C_1 + 2 = 13.5 C_1$$

or

$$C_1 = 2/12.4 = 0.16 \mu F \text{ and}$$

$$C_2 = (3/2) \times 0.16 = 0.24 \mu F$$

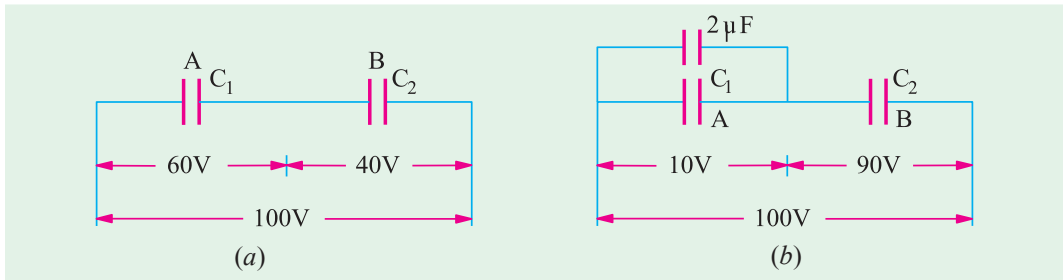


Fig. 5.22

**Example 5.17.** Three capacitors of  $2 \mu F$ ,  $5 \mu F$  and  $10 \mu F$  have breakdown voltage of  $200 V$ ,  $500 V$  and  $100 V$  respectively. The capacitors are connected in series and the applied direct voltage to the circuit is gradually increased. Which capacitor will breakdown first? Determine the total applied voltage and total energy stored at the point of breakdown. [Bombay University 2001]

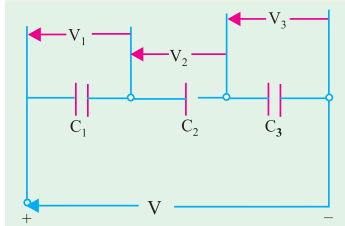


Fig. 5.23

**Solution.**  $C_1$  of  $2 \mu F$ ,  $C_2$  of  $5 \mu F$ , and  $C_3$  of  $10 \mu F$  are connected in series. If the equivalent single capacitor is  $C$ ,

$$1/C = 1/C_1 + 1/C_2 + 1/C_3, \text{ which gives } C = 1.25 \mu F$$

If  $V$  is the applied voltage,

$$V_1 = V \times C/C_1 = V \times (1.25/2)$$

$$= 62.5 \% \text{ of } V$$

$$V_2 = V \times (C/C_2) = C \times (1.25/5) = 25 \% \text{ of } V$$

$$V_3 = V \times (C/C_3) = V \times (1.25/10) = 12.5 \% \text{ of } V$$

If  $V_1 = 200$  volts,  $V = 320$  volts and  $V_2 = 80$  volts,  $V_3 = 40$  volts.

It means that, first capacitor  $C_1$  will breakdown first.

$$\text{Energy stored} = 1/2 CV^2 = 1/2 \times 1.25 \times 10^{-6} \times 320 \times 320 = 0.064 \text{ Joule}$$

**Example 5.18.** A multiple plate capacitor has 10 plates, each of area 10 square cm and separation between 2 plates is 1 mm with air as dielectric. Determine the energy stored when voltage of 100 volts is applied across the capacitor. [Bombay University 2001]

**Solution.** Number of plates,  $n = 10$

$$C = \frac{(n-1)\epsilon_0}{d} = \frac{9 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 79.7 \text{ pF}$$

$$\text{Energy stored} = 1/2 \times 79.7 \times 10^{-12} \times 100 \times 100 = 0.3985 \mu J$$

**Example 5.19.** Determine the capacitance between the points A and B in figure 5.24 (a). All capacitor values are in  $\mu F$ .

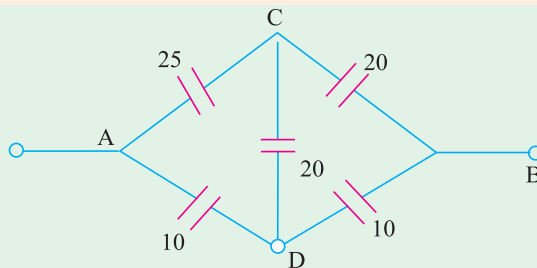


Fig. 5.24 (a)



**Solution.** Capacitances are being dealt with in this case. For simplifying this, Delta to star transformation is necessary. Formulae for this transformation are known if we are dealing with resistors or impedances. Same formulae are applicable to capacitors provided we are aware that capacitive reactance is dependent on reciprocal of capacitance.

Further steps are given below :

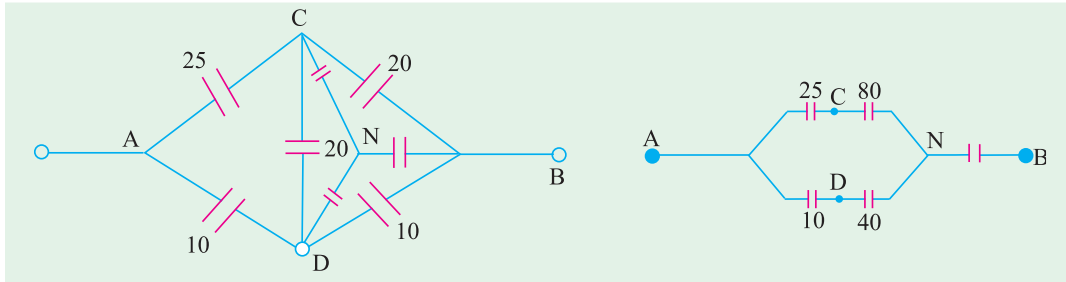


Fig. 5.24 (b)

Fig. 5.24 (c)

Reciprocals of capacitances taken first :

Between B-C — 0.05, Between B-D — 0.10

Between C-D — 0.05, Sum of these three = 0.20

For this delta, star-transformation is done :

Between N-C :  $0.05 \times 0.05 / 0.20 = 0.0125$ , its reciprocal =  $80 \mu F$

Between N-B :  $0.05 \times 0.10 / 0.20 = 0.025$ , its reciprocal =  $40 \mu F$

Between N-D :  $0.05 \times 0.10 / 0.20 = 0.025$ , its reciprocal =  $40 \mu F$

This is marked on Fig. 5.24 (c).

With series-parallel combination of capacitances, further simplification gives the final result.

$$C_{AB} = 16.13 \mu F$$

**Note :** Alternatively, with ADB as the vertices and C treated as the star point, star to delta transformation can be done. The results so obtained agree with previous effective capacitance of  $16.14 \mu F$ .

**Example 5.20. (a)** A capacitor of  $10 \text{ pF}$  is connected to a voltage source of  $100 \text{ V}$ . If the distance between the capacitor plates is reduced to  $50 \%$  while it remains, connected to the  $100 \text{ V}$  supply. Find the new values of charge, energy stored and potential as well as potential gradient. Which of these quantities increased by reducing the distance and why ?

[Bombay University 2000]

**Solution.**

(i)  $C = 10 \text{ pF}$

(ii)  $C = 20 \text{ pF}$ , distance halved

Charge =  $1000 \text{ p Coul}$

Charge =  $2000 \text{ p-coul}$

Energy =  $1/2 CV^2 = 0.05 \mu \text{ J}$

Energy =  $0.10 \mu \text{ J}$

Potential gradient in the second case will be twice of earlier value.

**Example 5.20 (b).** A capacitor  $5 \mu F$  charged to  $10 \text{ V}$  is connected with another capacitor of  $10 \mu F$  charged to  $50 \text{ V}$ , so that the capacitors have one and the same voltage after connection. What are the possible values of this common voltage ?

[Bombay University 2000]

**Solution.** The clearer procedure is discussed here.

Initial charges held by the capacitors are represented by equivalent voltage sources in Fig. 5.25 (b). The circuit is simplified to that in Fig. 5.25 (c). This is the case of  $C_1$  and  $C_2$  connected in series and excited by a  $40\text{-V}$  source. If  $C$  is the equivalent capacitance of this series-combination,

$$1/C = 1/C_1 + 1/C_2$$

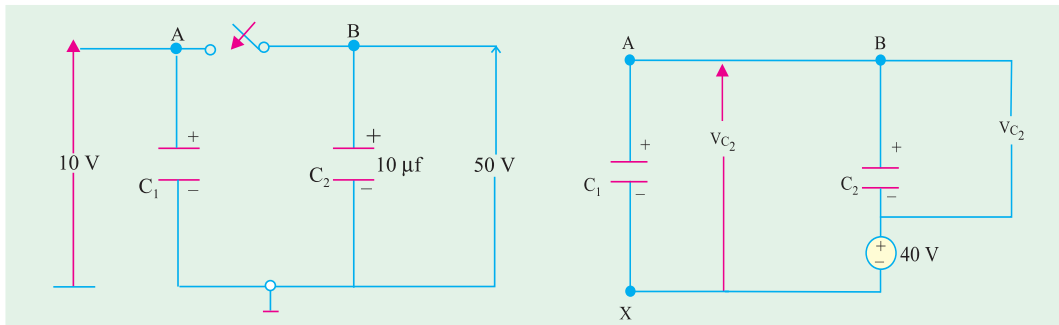


Fig. 5.25 (a)

Fig. 5.25 (c) Simplification

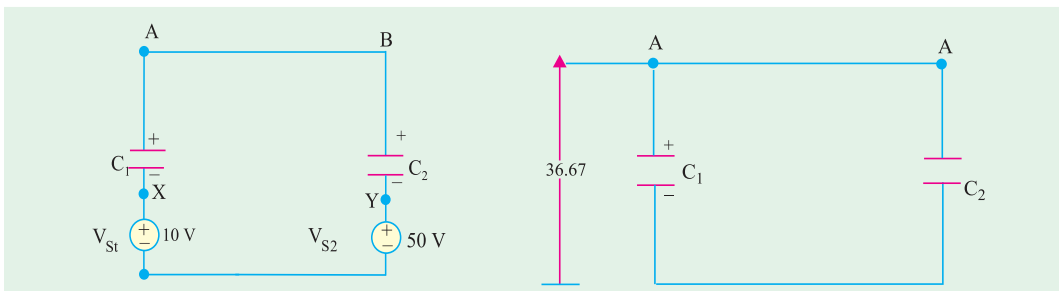


Fig. 5.25 (b). Initial charge represented by equiv-source

Fig. 5.25 (d). Final condition

This gives  $C = 3.33 \mu\text{F}$

In Fig. (c),  $V_{C1} = 40 \times C/C_1 = 40 \times 3.33/5 = 26.67$  volts

$V_{S1}$  and  $V_{S2}$  are integral parts of  $C_1$  and  $C_2$  in Fig. 5.25 (c),

Voltage across  $C_1 = 10 + 26.67 = 36.67$  (A w.r. to 0)

Voltage across  $C_2 = 50 - 13.33 = 36.67$ , (B w.r. to 0)

Thus, the final voltage across the capacitor is 36.67 volts.

**Note :** If one of the initial voltages on the capacitors happens to be the opposite to the single equivalent source voltage in Fig. 5.25 (c) will be 60 volts. Proceeding similarly, with proper care about signs, the final situation will be the common voltage will be 30 volts.

### 5.13. Cylindrical Capacitor with Compound Dielectric

Such a capacitor is shown in Fig. 5.26

Let  $r_1$  = radius of the core

$r_2$  = radius of inner dielectric  $\epsilon_{r1}$

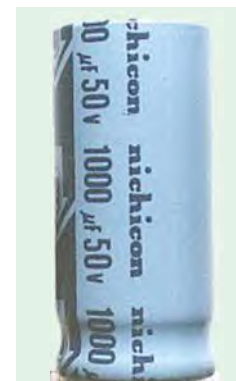
$r_3$  = radius of outer dielectric  $\epsilon_{r2}$

Obviously, there are two capacitors joined in series.

Now

$$C_1 = \frac{0.024 \epsilon_{r1}}{\log_{10} (r_2/r_1)} \mu\text{F/km} \text{ and } C_2 = \frac{0.024 \epsilon_{r2}}{\log_{10} (r_3/r_2)} \mu\text{F/M}$$

$$\text{Total capacitance of the cable is } C = \frac{C_1 C_2}{C_1 + C_2}$$



A cylindrical Capacitor

Now for capacitors joined in series, charge is the same.

$$\therefore Q = C_1 V_1 = C_2 V_2$$

$$\text{or } \frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{\epsilon_{r1} \log_{10} (r_3/r_2)}{\epsilon_{r1} \log_{10} (r_2/r_1)}$$

From this relation,  $V_2$  and  $V_1$  can be found,

$$g_{max} \text{ in inner capacitor } \frac{V_1}{2.3 r_1 \log_{10}(r_2/r_1)}$$

(Art. 5.9)

$$\text{Similarly, } g_{max} \text{ for outer capacitor } = \frac{V_2}{2.3 r_2 \log_{10}(r_3/r_2)}$$

$$\begin{aligned} \therefore \frac{g_{max}}{g_{max}} &= \frac{V_1}{2.3 r_1 \log_{10}(r_2/r_1)} \div \frac{V_2}{2.3 r_2 \log_{10}(r_3/r_2)} \\ &= \frac{V_1 r_2}{V_2 r_1} \times \frac{\log_{10} (r_3/r_2)}{\log_{10} (r_2/r_1)} = \frac{C_2 r_2}{C_1 r_1} \times \frac{\log_{10} (r_3/r_2)}{\log_{10} (r_2/r_1)} \left( \because \frac{V_1}{V_2} = \frac{C_2}{C_1} \right) \end{aligned}$$

Putting the values of  $C_1$  and  $C_2$ , we get

$$\frac{g_{max 1}}{g_{max 2}} = \frac{0.024 \epsilon_{r2}}{\log_{10}(r_3/r_2)} \times \frac{\log_{10} (r_3/r_2)}{0.024 \epsilon_{r1}} = \frac{r_2}{r_1} \times \frac{\log_{10} (r_2/r_1)}{\log_{10} (r_2/r_1)} \therefore \frac{g_{max 1}}{g_{max 2}} = \frac{\epsilon_{r2} \cdot r_2}{\epsilon_{r1} \cdot r_1}$$

Hence, voltage gradient is inversely proportional to the permittivity and the inner radius of the insulating material.

**Example 5.21.** A single-core lead-sheathed cable, with a conductor diameter of 2 cm is designed to withstand 66 kV. The dielectric consists of two layers A and B having relative permittivities of 3.5 and 3 respectively. The corresponding maximum permissible electrostatic stresses are 72 and 60 kV/cm. Find the thicknesses of the two layers. **(Power Systems-I, M.S. Univ. Baroda)**

**Solution.** As seen from Art. 5.13.

$$\frac{g_{max 1}}{g_{max 2}} = \frac{\epsilon_{r2} \cdot r_2}{\epsilon_{r1} \cdot r_1} \text{ or } \frac{72}{60} = \frac{3 \times r_2}{3.5 \times 1} \text{ or } r_2 = 1.4 \text{ cm}$$

$$\text{Now, } g_{max} = \frac{V_1 \times \sqrt{2}}{2.3 r_1 \log_{10} r_2/r_1} \quad \dots \text{Art. 5.9}$$

where  $V_1$  is the r.m.s. values of the voltage across the first dielectric.

$$\therefore 72 = \frac{V_1 \times \sqrt{2}}{2.3 \times 1 \times \log_{10} 1.4} \text{ or } V_1 = 17.1 \text{ kV}$$

$$\text{Obviously, } V_2 = 66 - 17.1 = 48.9 \text{ kV}$$

$$\text{Now, } g_{max 2} = \frac{V_2 \times \sqrt{2}}{2.3 r_2 \log_{10} (r_3/r_2)} \therefore 60 = \frac{48.9}{2.3 \times 1.4 \log_{10} (r_3/r_2)}$$

$$\therefore \log_{10} (r_3/r_2) = 0.2531 = \log_{10} (1.79) \therefore \frac{r_3}{r_2} = 1.79 \text{ or } r_3 = 2.5 \text{ cm}$$

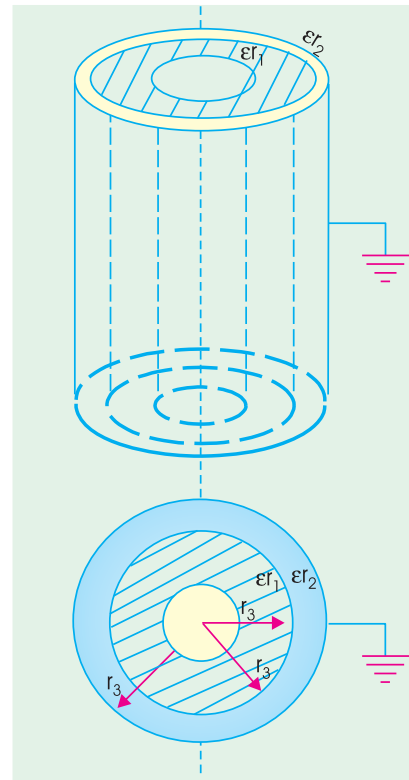


Fig. 5.26

Thickness of first dielectric layer =  $1.4 - 1.0 = 0.4 \text{ cm}$ .

Thickness of second layer =  $2.5 - 1.4 = 1.1 \text{ cm}$ .

### 5.14. Insulation Resistance of a Cable Capacitor

In a cable capacitor, useful current flows along the axis of the core but there is always present some leakage of current. This leakage is radial *i.e.* at right angles to the flow of useful current. The

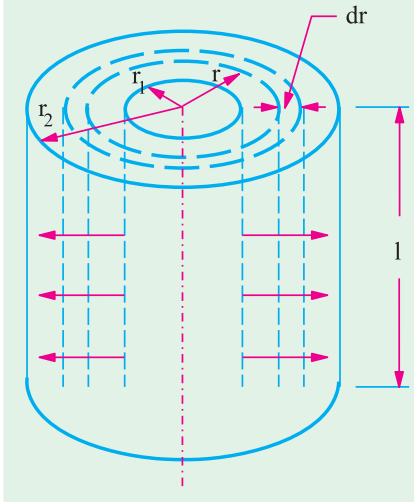


Fig. 5.27

resistance offered to this radial leakage of current is called **insulation resistance** of the cable. If cable length is greater, then leakage is also greater. It means that more current will leak. In other words, insulation resistance is decreased. Hence, we find that insulation resistance is inversely proportional to the cable length. This insulation resistance is not to be confused with conductor resistance which is directly proportional to the cable length.

Consider  $l$  metre of a single-core cable of inner-radius  $r_1$  and outer radius  $r_2$  (Fig. 5.27). Imagine an annular ring of radius ' $r$ ' and radial thickness ' $dr$ '.

If resistivity of insulating material is  $\rho$ , then resistance of the this narrow ring is  $dR = \frac{\rho dr}{2\pi r \times l} = \frac{\rho dr}{2\pi r l}$ . ∴ Insulation resistance of  $l$  metre length of cable is

$$\int dR = \int_{r_1}^{r_2} \frac{\rho dr}{2\pi r l} \text{ or } R = \frac{\rho}{2\pi r l} \left| \log_e (r) \right|_{r_1}^{r_2}$$

$$R = \frac{\rho}{2\pi l} \log_e (r_2/r_1) = \frac{2.3 \rho}{2\pi l} \log_{10} (r_2/r_1) \Omega$$

It should be noted

(i) that  $R$  is inversely proportional to the cable length

(ii) that  $R$  depends upon the ratio  $r_2/r_1$  and NOT on the thickness of insulator itself.

**Example 5.22.** A liquid resistor consists of two concentric metal cylinders of diameters  $D = 35 \text{ cm}$  and  $d = 20 \text{ cm}$  respectively with water of specific resistance  $\rho = 8000 \Omega \text{ cm}$  between them. The length of both cylinders is  $60 \text{ cm}$ . Calculate the resistance of the liquid resistor.

(Elect. Engg. Aligarh Univ.,)

**Solution.**  $r_1 = 10 \text{ cm}$ ;  $r_2 = 17.5 \text{ cm}$ ;  $\log_{10} (1.75) = 0.243$   
 $\rho = 8 \times 10^3 \Omega\text{-cm}$ ;  $l = 60 \text{ cm}$ .

Resistance of the liquid resistor  $R = \frac{2.3 \times 8 \times 10^3}{2\pi \times 60} \times 0.243 = 11.85 \Omega$ .

**Example 5.23.** Two underground cables having conductor resistances of  $0.7 \Omega$  and  $0.5 \Omega$  and insulation resistance of  $300 M\Omega$  respectively are joined (i) in series (ii) in parallel. Find the resultant conductor and insulation resistance.

(Elect. Engineering, Calcutta Univ.)

**Solution.** (i) The conductor resistance will add like resistances in series. However, the leakage resistances will decrease and would be given by the reciprocal relation.

Total conductor resistance =  $0.7 + 0.5 = 1.2 \Omega$

If  $R$  is the combined leakage resistance, then

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{600} \quad \therefore R = 200 M\Omega$$

(ii) In this case, conductor resistance is  $= 0.7 \times 0.5 / (0.7 + 0.5) = 0.3 \Omega$  (approx)

Insulation resistance  $= 300 + 600 = 900 \text{ M } \Omega$

**Example 5.24.** The insulation resistance of a kilometre of the cable having a conductor diameter of 1.5 cm and an insulation thickness of 1.5 cm is 500 M  $\Omega$ . What would be the insulation resistance if the thickness of the insulation were increased to 2.5 cm ?

(Communication Systems, Hyderabad Univ. 1992)

**Solution.** The insulation resistance of a cable is

$$\text{First Case} \quad R = \frac{2.3 \rho}{2\pi l} \log_{10} (r_2/r_1)$$

$$r_1 = 1.5/2 = 0.75 \text{ cm} ; r_2 = 0.75 + 1.5 = 2.25 \text{ cm}$$

$$\therefore r_2/r_1 = 2.25/0.75 = 3 ; \log_{10} (3) = 0.4771 \quad \therefore 500 = \frac{2.3 \rho}{2\pi l} \times 0.4771 \quad \dots(i)$$

**Second Case**

$$r_1 = 0.75 \text{ cm} \text{ --as before } r_2 = 0.75 + 2.5 = 3.25 \text{ cm}$$

$$r_2/r_1 = 3.25/0.75 = 4.333 ; \log_{10} (4.333) = 0.6368 \quad \therefore R = \frac{2.4 \rho}{2\pi l} \times 0.6368 \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{R}{500} = \frac{0.6368}{0.4771} ; R = 500 \times 0.6368 / 0.4771 = 667.4 \text{ M } \Omega$$

## 5.15. Energy Stored in a Capacitor

Charging of a capacitor always involves some expenditure of energy by the charging agency. This energy is stored up in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.

To begin with, when the capacitor is uncharged, little work is done in transferring charge from one plate to another. But further instalments of charge have to be carried against the repulsive force due to the charge already collected on the capacitor plates. Let us find the energy spent in charging a capacitor of capacitance  $C$  to a voltage  $V$ .

Suppose at any stage of charging, the p.d. across the plates is  $v$ . By definition, it is equal to the work done in shifting one coulomb from one plate to another. If 'dq' is charge next transferred, the work done is

$$dW = v.dq$$

$$\text{Now} \quad q = Cv \quad \therefore dq = C.dv \quad \therefore dW = Cv.dv$$

Total work done in giving  $V$  units of potential is

$$W = \int_0^V Cv.dv = C \left[ \frac{v^2}{2} \right]_0^V \quad \therefore W = \frac{1}{2} CV^2$$

If  $C$  is in farads and  $V$  is in volts, then  $W = \frac{1}{2} CV^2$  joules  $= \frac{1}{2} QV$  joules  $= \frac{Q^2}{2C}$  joules

If  $Q$  is in coulombs and  $C$  is in farads, the energy stored is given in joules.



Capacitors on a motherboard

**Note :** As seen from above, energy stored in a capacitor is  $E = \frac{1}{2} CV^2$

Now, for a capacitor of plate area  $A \text{ m}^2$  and dielectric of thickness  $d$  metre, energy per unit volume of dielectric medium.

$$\frac{1}{2} \frac{CV^2}{Ad} = \frac{1}{2} \frac{A}{d} \cdot \frac{V^2}{Ad} = \frac{1}{2} \frac{V^2}{d} = \frac{1}{2} E^2 = \frac{1}{2} DE = D^2/2 \text{ joules/m}^3 *$$

It will be noted that the formula  $\frac{1}{2} DE$  is similar to the expression  $\frac{1}{2}$  stress  $\times$  strain which is used for calculating the mechanical energy stored per unit volume of a body subjected to elastic stress.

**Example 5.25.** Since a capacitor can store charge just like a lead-acid battery, it can be used at least theoretically as an electrostatic battery. Calculate the capacitance of 12-V electrostatic battery which the same capacity as a 40 Ah, 12 V lead-acid battery.

**Solution.** Capacity of the lead-acid battery = 40 Ah = 40  $\times$  36 As = 144000 Coulomb

Energy stored in the battery =  $QV = 144000 \times 12 = 1.728 \times 10^6 \text{ J}$

Energy stored in an electrostatic battery =  $\frac{1}{2} CV^2$

$$\therefore \frac{1}{2} \times C \times 12^2 = 1.728 \times 10^6 \therefore C = 2.4 \times 10^4 \text{ F} = 24 \text{ kF}$$

**Example 5.26.** A capacitor-type stored-energy welder is to deliver the same heat to a single weld as a conventional welder that draws 20 kVA at 0.8 pf for 0.0625 second/weld. If  $C = 2000 \mu\text{F}$ , find the voltage to which it is charged. **(Power Electronics, A.M.I.E. Sec B, 1993)**

**Solution.** The energy supplied per weld in a conventional welder is

$$W = VA \times \cos \phi \times \text{time} = 20,000 \times 0.8 \times 0.0625 = 1000 \text{ J}$$

Now, energy stored in a capacitor is  $(1/2) CV^2$

$$\therefore W = \frac{1}{2} CV^2 \text{ or } V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 1000}{2000 \times 10^{-6}}} = 1000 \text{ V}$$

**Example 5.27.** A parallel-plate capacitor is charged to 50  $\mu\text{C}$  at 150 V. It is then connected to another capacitor of capacitance 4 times the capacitance of the first capacitor. Find the loss of energy. **(Elect. Engg. Aligarh Univ.)**

**Solution.**  $C_1 = 50/150 = 1/3 \mu\text{F}$ ;  $C_2 = 4 \times 1/3 = 4/3 \mu\text{F}$

**Before Joining**

$$E_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times \left(\frac{1}{3}\right) 10^{-6} \times 150^2 = 37.5 \times 10^{-4} \text{ J}; E_2 = 0$$

$$\text{Total energy} = 37.5 \times 10^{-4} \text{ J}$$

**After Joining**

When the two capacitors are connected in parallel, the charge of 50  $\mu\text{C}$  gets redistributed and the two capacitors come to a common potential  $V$ .

$$V = \frac{\text{total charge}}{\text{total capacitance}} = \frac{50 \mu\text{C}}{[(1/3) + (4/3)] \mu\text{F}} = 30 \text{ V}$$

$$E_1 = \frac{1}{2} \times (1/3) \times 10^{-6} \times 30^2 = 1.5 \times 10^{-4} \text{ J}$$

$$E_2 = \frac{1}{2} \times (4/3) \times 10^{-6} \times 30^2 = 6.0 \times 10^{-4} \text{ J}$$

$$\text{Total energy} = 7.5 \times 10^{-4} \text{ J}; \text{ Loss of energy} = (37.5 - 7.5) \times 10^{-4} = 3 \times 10^{-2} \text{ J}$$

The energy is wasted away as heat in the conductor connecting the two capacitors.

\* It is similar to the expression for the energy stored per unit volume of a magnetic field.

**Example 5.28.** An air-capacitor of capacitance  $0.005 \mu F$  is connected to a direct voltage of  $500 V$ , is disconnected and then immersed in oil with a relative permittivity of  $2.5$ . Find the energy stored in the capacitor before and after immersion. (Elect. Technology : London Univ.)

**Solution.** Energy before immersion is

$$E_1 = \frac{1}{2} CV^2 = \frac{1}{2} \times 0.005 \times 10^{-6} \times 500^2 = 625 \times 10^{-6} \text{ J}$$

When immersed in oil, its capacitance is increased  $2.5$  times. Since charge is constant, voltage must become  $2.5$  times. Hence, new capacitance is  $2.5 \times 0.005 = 0.0125 \mu F$  and new voltage is  $500/2.5 = 200 V$ .

$$E_2 = \frac{1}{2} \times 0.0125 \times 10^{-6} \times (200)^2 = 250 \times 10^{-6} \text{ J}$$

**Example 5.29.** A parallel-plate air capacitor is charged to  $100 V$ . Its plate separation is  $2 \text{ mm}$  and the area of each of its plate is  $120 \text{ cm}^2$ .

Calculate and account for the increase or decrease of stored energy when plate separation is reduced to  $1 \text{ mm}$

(a) at constant voltage (b) at constant charge.

**Solution.** Capacitance in the first case

$$C_1 = \frac{\epsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 120 \times 10^{-4}}{2 \times 10^{-3}} = 53.1 \times 10^{-12} \text{ F}$$

Capacitance in the second case i.e. with reduced spacing

$$C_2 = \frac{8.854 \times 10^{-12} \times 120 \times 10^{-4}}{1 \times 10^{-3}} = 106.2 \times 10^{-12} \text{ F}$$

(a) When Voltage is Constant

$$\begin{aligned} \text{Change in stored energy } dE &= \frac{1}{2} C_2 V^2 - \frac{1}{2} C_1 V^2 \\ &= \frac{1}{2} \times 100^2 \times (106.2 - 53.1) \times 10^{-12} = 26.55 \times 10^{-8} \text{ J} \end{aligned}$$

This represents an increase in the energy of the capacitor. This extra work has been done by the external supply source because charge has to be given to the capacitor when its capacitance increases, voltage remaining constant.

(b) When Charge Remains Constant

$$\text{Energy in the first case } E_1 = \frac{1}{2} \frac{Q^2}{C_1}; \text{ Energy in the second case, } E_2 = \frac{1}{2} \frac{Q^2}{C_2}$$

$$\begin{aligned} \therefore \text{ change in energy is } dE &= \frac{1}{2} Q^2 \left( \frac{1}{53.1} - \frac{1}{106.2} \right) \times 10^{12} \text{ J} \\ &= \frac{1}{2} (C_1 V_1)^2 \left( \frac{1}{53.1} - \frac{1}{106.2} \right) \times 10^{12} \text{ J} \\ &= \frac{1}{2} (53.1 \times 10^{-12})^2 \times 10^4 \times 0.0094 \times 10^{12} \\ &= 13.3 \times 10^{-8} \text{ joules} \end{aligned}$$

Hence, there is a decrease in the stored energy. The reason is that charge remaining constant, when the capacitance is increased, then voltage must fall with a consequent decrease in stored energy

$$(E = \frac{1}{2} QV)$$

**Example 5.30.** A point charge of  $100 \mu\text{C}$  is embedded in an extensive mass of bakelite which has a relative permittivity of 5. Calculate the total energy contained in the electric field outside a radial distance of (i) 100 m (ii) 10 m (iii) 1 m and (iv) 1 cm.

**Solution.** As per the Coulomb's law, the electric field intensity at any distance  $x$  from the point charge is given by  $E = Q/4\pi\epsilon x^2$ . Let us draw a spherical shell of radius  $x$  as shown in Fig. Another spherical shell of radius  $(x + dx)$  has also been drawn. A differential volume of the space enclosed between the two shells is  $dv = 4\pi x^2 dx$ . As per Art. 5.15, the energy stored per unit volume of the electric field is  $(1/2)DE$ . Hence, differential energy contained in the small volume is

$$dW = \frac{1}{2} DE dv = \frac{1}{2} \epsilon E^2 dv = \frac{1}{2} \epsilon \left( \frac{Q}{4\pi\epsilon x^2} \right)^2 4\pi x^2 dx = \frac{Q^2}{8\pi\epsilon} \cdot \frac{dx}{x^2}$$

Total energy of the electric field extending from  $x = R$  to  $x = \infty$  is

$$W = \frac{Q^2}{8\pi\epsilon} \int_R^\infty x^{-2} dx = \frac{Q^2}{8\pi\epsilon R} = \frac{Q^2}{8\pi\epsilon_0 \epsilon_r R}$$

(i) The energy contained in the electric field lying outside a radius of  $R = 100$  m is

$$W = \frac{(100 \times 10^{-6})^2}{8\pi \times 8.854 \times 10^{-12} \times 5 \times 100} = \mathbf{0.90 \text{ J}}$$

(ii) For  $R = 10$  m,  $W = 10 \times 0.09 = \mathbf{0.90 \text{ J}}$

(iii) For  $R = 1$  m,  $W = 100 \times 0.09 = \mathbf{9 \text{ J}}$

(iv) For  $R = 1$  cm,  $W = 10,000 \times 0.09 = \mathbf{900 \text{ J}}$

**Example 5.31.** Calculate the change in the stored energy of a parallel-plate capacitor if a dielectric slab of relative permittivity 5 is introduced between its two plates.

**Solution.** Let  $A$  be the plate area,  $d$  the plate separation,  $E$  the electric field intensity and  $D$  the electric flux density of the capacitor. As per Art. 5.15, energy stored per unit volume of the field is  $(1/2)DE$ . Since the space volume is  $d \times A$ , hence,

$$W_1 = \frac{1}{2} D_1 E_1 \times dA = \frac{1}{2} \epsilon_0 E_1^2 \times dA = \frac{1}{2} \epsilon_0 dA \left( \frac{V_1}{d} \right)^2$$

When the dielectric slab is introduced,

$$\begin{aligned} W_2 &= \frac{1}{2} D_2 E_2 \times dA = \frac{1}{2} \epsilon E_2^2 \times dA = \frac{1}{2} \epsilon_0 \epsilon_r dA \left( \frac{V_2}{d} \right)^2 \\ &= \frac{1}{2} \epsilon_0 \epsilon_r dA \left( \frac{V_2}{\epsilon_r d} \right)^2 = \frac{1}{2} \epsilon_0 dA \left( \frac{V_1}{d} \right)^2 \frac{1}{\epsilon_r} \therefore W_2 = \frac{W_1}{\epsilon_r} \end{aligned}$$

It is seen that the stored energy is reduced by a factor of  $\epsilon_r$ . Hence, change in energy is

$$dW = W_1 - W_2 = W_1 \left( 1 - \frac{1}{\epsilon_r} \right) = W_1 \left( 1 - \frac{1}{5} \right) = W_1 \times \frac{4}{5} \therefore \frac{dW}{W_1} = \mathbf{0.8}$$

**Example 5.32.** When a capacitor  $C$  charges through a resistor  $R$  from a d.c. source voltage  $E$ , determine the energy appearing as heat. **[Bombay University, 2000]**

**Solution.** R-C series circuit switched on to a d.c. source of voltage  $E$ , at  $t = 0$ , results into a current  $i(t)$ , given by

$$i(t) = (E/R) e^{-t/\tau}$$

where

$$t = RC$$

$$\Delta W_R = \text{Energy appearing as heat in time } \Delta t$$



$$\begin{aligned}
 &= i^2 R \Delta t \\
 \Delta W_R &= \text{Energy appearing as heat in time } \Delta t \\
 &= i^2 R \Delta t \\
 W_R &= R \int_0^\infty i^2 dt \\
 &= R (E/R)^2 \int_0^\infty (\epsilon^{-t/\tau})^2 dt = \frac{1}{2} CE^2
 \end{aligned}$$

**Note :** Energy stored by the capacitor at the end of charging process =  $1/2 CE^2$   
Hence, energy received from the source =  $CF$ .

### 5.16. Force of Attraction Between Oppositely-charged Plates

In Fig. 5.28 are shown two parallel conducting plates *A* and *B* carrying constant charges of  $+Q$  and  $-Q$  coulombs respectively. Let the force of attraction between the two be  $F$  newtons. If one of the plates is pulled apart by distance  $dx$ , then work done is

$$= F \times dx \text{ joules} \quad \dots(i)$$

Since the plate charges remain constant, no electrical energy comes into the arrangement during the movement  $dx$ .

$\therefore$  Work done = change in stored energy

$$\text{Initial stored energy} = \frac{1}{2} \frac{Q^2}{C} \text{ joules}$$

If capacitance becomes  $(C - dC)$  due to the movement  $dx$ , then

$$\text{Final stored energy} = \frac{1}{2} \frac{Q^2}{(C - dC)} = \frac{1}{2} \cdot \frac{Q^2}{C} \cdot \frac{1}{1 - \frac{dC}{C}} = \frac{1}{2} \frac{Q^2}{C} \left( 1 + \frac{dC}{C} \right) \text{ if } dC \ll C$$

$$\therefore \text{Change in stored energy} = \frac{1}{2} \frac{Q^2}{C} \left( 1 + \frac{dC}{C} \right) - \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{C^2} \cdot dC \quad \dots(ii)$$

$$\text{Equating Eq. (i) and (ii), we have } F \cdot dx = \frac{1}{2} \frac{Q^2}{C^2} \cdot dC$$

$$F = \frac{1}{2} \frac{Q^2}{C^2} \cdot \frac{dC}{dx} = \frac{1}{2} V^2 \cdot \frac{dC}{dx} \quad (\because V = Q/C)$$

$$\text{Now } C = \frac{\epsilon A}{x} \therefore \frac{dC}{dx} = -\frac{\epsilon A}{x^2}$$

$$\therefore F = -\frac{1}{2} V^2 \cdot \frac{\epsilon A}{x^2} = -\frac{1}{2} \epsilon A \left( \frac{V}{x} \right)^2 \text{ newtons} = -\frac{1}{2} \epsilon A E^2 \text{ newtons}$$

This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of  $V$  volts. The negative sign shows that it is a force of attraction.

**Example 5.33.** A parallel-plate capacitor is made of plates 1 m square and has a separation of 1 mm. The space between the plates is filled with dielectric of  $\epsilon_r = 25.0$ . If 1 k V potential difference is applied to the plates, find the force squeezing the plates together.

(Electromagnetic Theory, A.M.I.E. Sec B, 1993)

**Solution.** As seen from Art. 5.16,  $F = -(1/2) \epsilon_0 \epsilon_r A E^2$  newton  
Now  $E = V/d = 1000/1 \times 10^{-3} = 10^6$  V/m

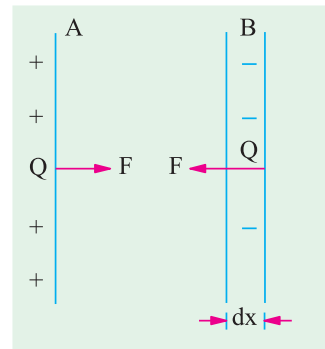


Fig. 5.28

$$\therefore F = -\frac{1}{2} \epsilon_0 \epsilon_r A E^2 = -\frac{1}{2} \times 8.854 \times 10^{-12} \times 25 \times 1 \times (10^6)^2 = -1.1 \times 10^{-4} \text{ N}$$

### Tutorial Problems No. 5.2

- Find the capacitance per unit length of a cylindrical capacitor of which the two conductors have radii 2.5 and 4.5 cm and dielectric consists of two layers whose cylinder of contact is 3.5 cm in radius, the inner layer having a dielectric constant of 4 and the outer one of 6.  
[440 pF/m]
- A parallel-plate capacitor, having plates 100 cm<sup>2</sup> area, has three dielectrics 1 mm each and of permittivities 3, 4 and 6. If a peak voltage of 2,000 V is applied to the plates, calculate :  
(a) potential gradient across each dielectric  
(b) energy stored in each dielectric.  
[8.89 kV/cm; 6.67 kV/cm ; 4.44 kV/cm ; 1047, 786, 524 × 10<sup>-7</sup> joule]
- The core and lead-sheath of a single-core cable are separated by a rubber covering. The cross-sectional area of the core is 16 mm<sup>2</sup>. A voltage of 10 kV is applied to the cable. What must be the thickness of the rubber insulation if the electric field strength in it is not to exceed 6 × 10<sup>6</sup> V/m ?  
[2.5 mm (approx)]
- A circular conductor of 1 cm diameter is surrounded by a concentric conducting cylinder having an inner diameter of 2.5 cm. If the maximum electric stress in the dielectric is 40 kV/cm, calculate the potential difference between the conductors and also the minimum value of the electric stress.  
[18.4 kV ; 16 kV/cm]
- A multiple capacitor has parallel plates each of area 12 cm<sup>2</sup> and each separated by a mica sheet 0.2 mm thick. If dielectric constant for mica is 5, calculate the capacitance.  
[265.6 μF]
- A p.d. of 10 kV is applied to the terminals of a capacitor of two circular plates each having an area of 100 sq. cm. separated by a dielectric 1 mm thick. If the capacitance is 3 × 10<sup>-4</sup> microfarad, calculate the electric flux density and the relative permittivity of the dielectric.  
[D = 3 × 10<sup>-4</sup> C/m<sup>2</sup>, ε<sub>r</sub> = 3.39] (City & Guilds, London)
- Each electrode of a capacitor of the electrolytic type has an area of 0.02 sq. metre. The relative permittivity of the dielectric film is 2.8. If the capacitor has a capacitance of 10 μF, estimate the thickness of the dielectric film.  
[4.95 × 10<sup>-8</sup> m] (I.E.E. London)

### 5.17. Current-Voltage Relationships in a Capacitor

The charge on a capacitor is given by the expression  $Q = CV$ . By differentiating this relation, we get

$$i = \frac{dQ}{dt} = \frac{d}{dt} (CV) = C \frac{dV}{dt}$$

Following important facts can be deduced from the above relations :

- since  $Q = CV$ , it means that the voltage across a capacitor is proportional to *charge*, not the *current*.
- a capacitor has the ability to store charge and hence to provide a sort of memory.
- a capacitor can have a voltage across it even when there is *no current flowing*.
- from  $i = c dV/dt$ , it is clear that current in the capacitor is present only when voltage on it changes with time. If  $dV/dt = 0$  *i.e.* when its voltage is constant or for d.c. voltage,  $i = 0$ . Hence, the capacitor behaves like an *open circuit*.

(v) from  $i = C \, dV/dt$ , we have  $dV/dt = i/C$ . It shows that for a given value of (charge or discharge) current  $i$ , rate of change in voltage is inversely proportional to capacitance. Larger the value of  $C$ , slower the rate of change in capacitive voltage. Also, capacitor voltage **cannot change instantaneously**.

(vi) the above equation can be put as  $dv = \frac{i}{C} \cdot dt$

Integrating the above, we get  $\int dv = \frac{1}{C} \int i \cdot dt$  or  $dv = \frac{1}{C} \int_0^t i \, dt$

**Example 5.34.** The voltage across a  $5 \mu\text{F}$  capacitor changes uniformly from 10 to 70 V in 5 ms. Calculate (i) change in capacitor charge (ii) charging current.

**Solution.**  $Q = CV \quad \therefore dQ = C \cdot dV$  and  $i = C \, dV/dt$

(i)  $dV = 70 - 10 = 60 \text{ V}$ ,  $\therefore dQ = 5 \times 60 = 300 \mu\text{C}$ .

(ii)  $i = C \cdot dV/dt = 5 \times 60/5 = 60 \text{ mA}$

**Example 5.35.** An uncharged capacitor of  $0.01 \text{ F}$  is charged first by a current of  $2 \text{ mA}$  for 30 seconds and then by a current of  $4 \text{ mA}$  for 30 seconds. Find the final voltage in it.

**Solution.** Since the capacitor is initially uncharged, we will use the principle of Superposition.

$$V_1 = \frac{1}{0.01} \int_0^{30} 2 \times 10^{-3} \cdot dt = 100 \times 2 \times 10^{-3} \times 30 = 6 \text{ V}$$

$$V_2 = \frac{1}{0.01} \int_0^{30} 4 \times 10^{-3} \cdot dt = 100 \times 4 \times 10^{-3} \times 30 = 12 \text{ V}; \quad \therefore V = V_1 + V_2 = 6 + 12 = 18 \text{ V}$$

**Example 5.36.** The voltage across two series-connected  $10 \mu\text{F}$  capacitors changes uniformly from 30 to 150 V in 1 ms. Calculate the rate of change of voltage for (i) each capacitor and (ii) combination.

**Solution.** For series combination

$$V_1 = V \frac{C_2}{C_1 + C_2} = \frac{V}{3} \text{ and } V_2 = V \cdot \frac{C_1}{C_1 + C_2} = \frac{2V}{3}$$

When  $V = 30 \text{ V}$   $V_1 = V/3 = 30/3 = 10 \text{ V}$ ;  $V_2 = 2V/3 = 2 \times 30/3 = 20 \text{ V}$

When  $V = 150 \text{ V}$   $V_1 = 150/3 = 50 \text{ V}$  and  $V_2 = 2 \times 150/3 = 100 \text{ V}$

(i)  $\therefore \frac{dV_1}{dt} = \frac{(50 - 10)}{1 \text{ ms}} = 40 \text{ kV/s}$ ;  $\frac{dV_2}{dt} = \frac{(100 - 20) \text{ V}}{1 \text{ ms}} = 80 \text{ kV/s}$

(ii)  $\frac{dV}{dt} = \frac{(150 - 30)}{1 \text{ ms}} = 120 \text{ kV/s}$

It is seen that  $dV/dt = dV_1/dt + dV_2/dt$ .

## 5.18. Charging of a Capacitor

In Fig. 5.29. (a) is shown an arrangement by which a capacitor  $C$  may be charged through a high resistance  $R$  from a battery of  $V$  volts. The voltage across  $C$  can be measured by a suitable voltmeter. When switch  $S$  is connected to terminal (a),  $C$  is charged but when it is connected to  $b$ ,  $C$  is short circuited through  $R$  and is thus discharged. As shown in Fig. 5.29. (b), switch  $S$  is shifted to  $a$  for charging the capacitor for the battery. The voltage across  $C$  does not rise to  $V$  instantaneously but builds up slowly *i.e.* exponentially and not linearly. Charging current  $i_c$  is maximum at the start *i.e.* when  $C$  is uncharged, then it decreases exponentially and finally ceases when p.d. across capacitor plates becomes equal and opposite to the battery voltage  $V$ . At any instant during charging, let

$v_c$  = p.d. across  $C$ ;  $i_c$  = charging current

$q$  = charge on capacitor plates

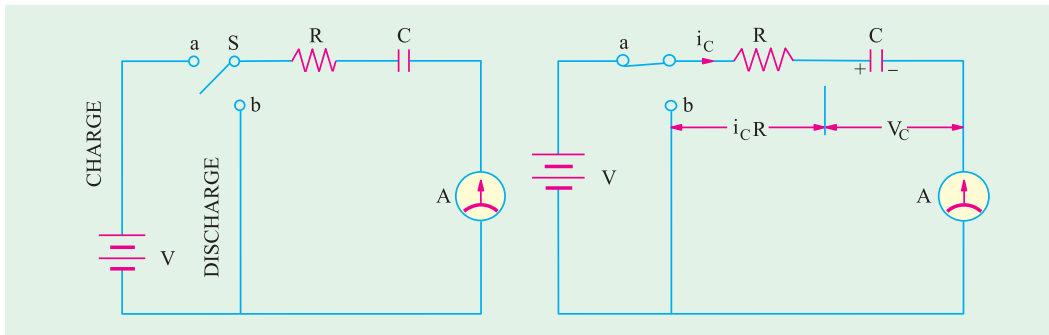


Fig. 5.29

The applied voltage  $V$  is always equal to the sum of :

(i) resistive drop ( $i_c R$ ) and (ii) voltage across capacitor ( $v_c$ )

$$\therefore V = i_c R + v_c \quad \dots(i)$$

$$\text{Now } i_c = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C \frac{dv_c}{dt} \therefore V = v_c + CR \frac{dv_c}{dt} \quad \dots(ii)$$

$$\text{or } -\frac{dv_c}{V - v_c} = -\frac{dt}{CR}$$

$$\text{Integrating both sides, we get } \int \frac{-dV_c}{V - v_c} = -\frac{1}{CR} \int dt; \therefore \log_e (V - v_c) = -\frac{t}{CR} + K \quad \dots(iii)$$

where  $K$  is the constant of integration whose value can be found from initial known conditions. We know that at the start of charging when  $t = 0$ ,  $v_c = 0$ .

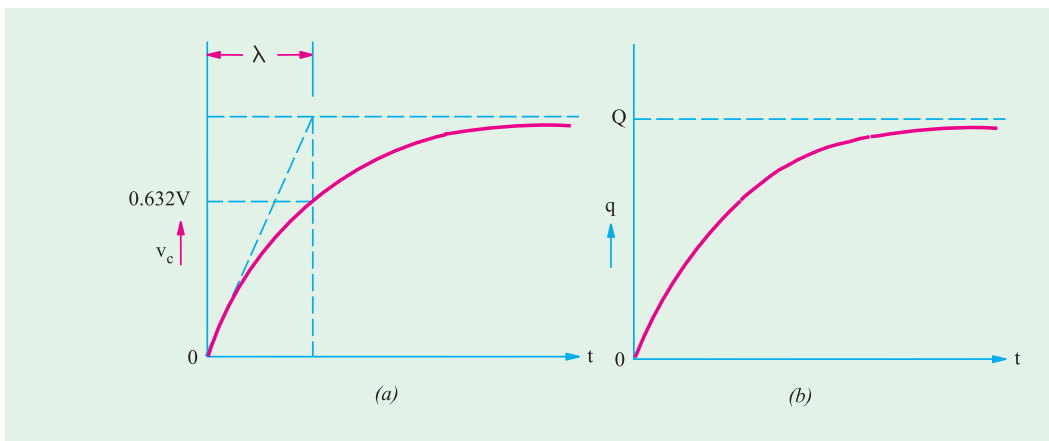
Substituting these values in (iii), we get  $\log_e V = K$

$$\text{Hence, Eq. (iii) becomes } \log_e (V - v_c) = \frac{-t}{CR} + \log_e V$$

$$\text{or } \log_e \frac{V - v_c}{V} = \frac{-t}{CR} = -\frac{t}{\lambda} \text{ where } \lambda = CR = \text{time constant}$$

$$\therefore \frac{V - v_c}{V} = e^{-t/\lambda} \text{ or } v_c = V(1 - e^{-t/\lambda}) \quad \dots(iv)$$

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 5.27.(a)



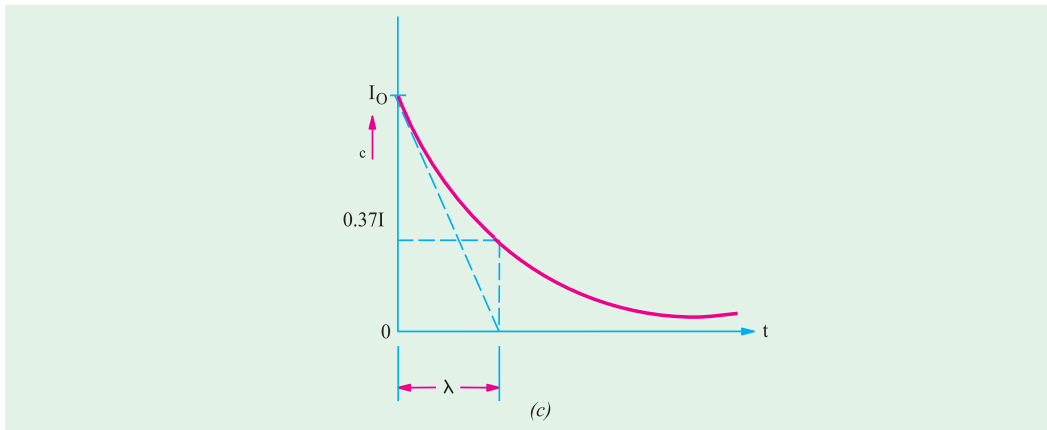


Fig. 5.30

Now  $v_c = q/C$  and  $V = Q/C$

Equation (iv) becomes  $\frac{q}{c} = \frac{Q}{c} (1 - e^{-t/\lambda}) \therefore q = Q (1 - e^{-t/\lambda})$  ... (v)

We find that increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time (Fig. 5.30 b). Now,  $i_c = dq/dt$ .

Differentiating both sides of Eq. (v), we get

$$\begin{aligned} \frac{dq}{dt} &= i_c = Q \frac{d}{dt} (1 - e^{-t/\lambda}) = Q \left( + \frac{1}{\lambda} e^{-t/\lambda} \right) \\ &= \frac{Q}{\lambda} e^{-t/\lambda} = \frac{CV}{CR} e^{-t/\lambda} \quad (\because Q = CV \text{ and } \lambda = CR) \end{aligned}$$

$$\therefore i_c = \frac{V}{R} \cdot e^{-t/\lambda} \text{ or } i_c = I_0 e^{-t/\lambda} \quad \dots (vi)$$

where  $I_0 = \text{maximum current} = V/R$

Exponentially rising curves for  $v_c$  and  $q$  are shown in Fig. 5.30 (a) and (b) respectively. Fig. 5.30 (c) shows the curve for exponentially decreasing charging current. It should be particularly noted that  $i_c$  decreases in magnitude only but its direction of flow remains the same *i.e.* positive.

As charging continues, charging current decreases according to equation (vi) as shown in Fig. 5.30 (c). It becomes zero when  $t = \infty$  (though it is almost zero in about 5 time constants). Under steady-state conditions, the circuit appears only as a capacitor which means it acts as an open-circuit. Similarly, it can be proved that  $v_R$  decreases from its initial maximum value of  $V$  to zero exponentially as given by the relation  $v_R = V e^{-t/\lambda}$ .

### 5.19. Time Constant

(a) Just at the start of charging, p.d. across capacitor is zero, hence from (ii) putting  $v_c = 0$ , we get

$$V = CR \frac{dv_c}{dt}$$

$$\therefore \text{initial rate of rise of voltage across the capacitor is}^* = \left( \frac{dv_c}{dt} \right)_{t=0} = \frac{V}{CR} = \frac{V}{\lambda} \text{ volt/second}$$

If this rate of rise were maintained, then time taken to reach voltage  $V$  would have been  $V + V/CR = CR$ . This time is known as **time constant** ( $\lambda$ ) of the circuit.

\* It can also be found by differentiating Eq. (iv) with respect to time and then putting  $t = 0$ .

Hence, time constant of an  $R$ - $C$  circuit is defined as **the time during which voltage across capacitor would have reached its maximum value  $V$  had it maintained its initial rate of rise.**

(b) In equation (iv) if  $t = \lambda$ , then

$$v_c = V(1 - e^{-t/\lambda}) = V(1 - e^{-1}) = V(1 - \frac{1}{e}) = V\left(1 - \frac{1}{2.718}\right) = 0.632 V$$

Hence, time constant may be defined as **the time during which capacitor voltage actually rises to 0.632 of its final steady value.**

(c) From equation (vi), by putting  $t = \lambda$ , we get

$$i_c = I_0 e^{-\lambda/\lambda} = I_0 e^{-1} = I_0/2.718 \cong 0.37 I_0$$

Hence, the constant of a circuit is also the **time during which the charging current falls to 0.37 of its initial maximum value (or falls by 0.632 of its initial value).**

## 5.20. Discharging of a Capacitor

As shown in Fig. 5.31 (a), when  $S$  is shifted to  $b$ ,  $C$  is discharged through  $R$ . It will be seen that the discharging current flows in a direction opposite to that the charging current as shown in Fig. 5.31(b). Hence, if the direction of the charging current is taken positive, then that of the discharging current will be taken as negative. To begin with, the discharge current is maximum but then decreases exponentially till it ceases when capacitor is fully discharged.

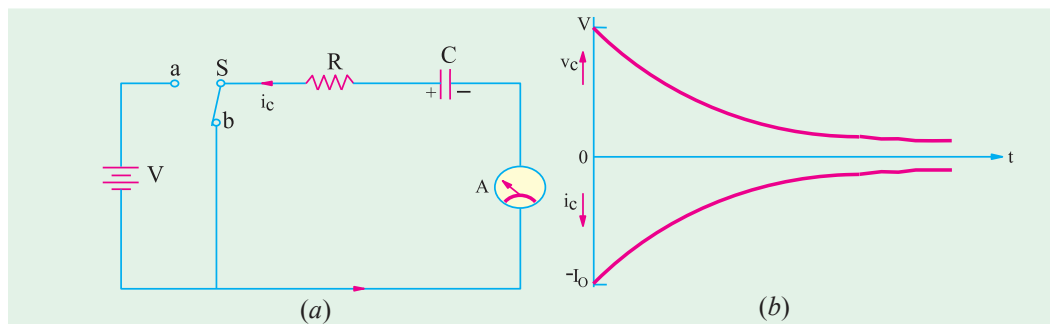


Fig. 5.31

Since battery is cut of the circuit, therefore, by putting  $V = 0$  in equation (ii) of Art. 5.18, we get

$$0 = CR \frac{dv_c}{dt} - v_c \quad \text{or} \quad v_c = CR \frac{dv_c}{dt} \quad \left( i_c = C \frac{dv_c}{dt} \right)$$

$$\therefore \frac{dv_c}{v_c} = \frac{dt}{CR} \quad \text{or} \quad \frac{dv_c}{v_c} = \frac{1}{CR} dt \quad \log_e v_c = \frac{t}{CR} + k$$

At the start of discharge, when  $t = 0$ ,  $v_c = V$   $\therefore \log_e V = 0 + K$ ; or  $\log_e V = K$

Putting this value above, we get

$$\log_e v_c = -\frac{t}{\lambda} + \log_e V \quad \text{or} \quad \log_e v_c/V = -t/\lambda$$

$$\text{or} \quad \frac{v_c}{V} = e^{-t/\lambda} \quad \text{or} \quad v_c = V e^{-t/\lambda}$$

$$\text{Similarly,} \quad q = Q e^{-t/\lambda} \quad \text{and} \quad i_c = -I_0 e^{-t/\lambda}$$

It can be proved that

$$v_R = -V e^{-t/\lambda}$$

The fall of capacitor potential and its discharging current are shown in Fig. 5.32 (b).

One practical application of the above charging and discharging of a capacitor is found in digital

control circuits where a square-wave input is applied across an  $R$ - $C$  circuit as shown in Fig. 5.32 (a). The different waveforms of the current and voltages are shown in Fig. 5.32 (b), (c), (d), (e). The sharp voltage pulses of  $V_R$  are used for control circuits.

**Example 5.37.** Calculate the current in and voltage drop across each element of the circuit shown in Fig. 5.33 (a) after switch  $S$  has been closed long enough for steady-state conditions to prevail.

Also, calculate voltage drop across the capacitor and the discharge current at the instant when  $S$  is opened.

**Solution.** Under steady-state conditions, the capacitor becomes fully charged and draws no current. In fact, it acts like an open circuit with the result that no current flows through the  $1\text{-}\Omega$  resistor. The steady state current  $I_{SS}$  flows through loop  $ABCD$  only.

Hence,

$$I_{SS} = 100/(6 + 4) = 10 \text{ A}$$

Drop

$$V_6 = 100 \times 6/(6 + 4) = 60 \text{ V}$$

$$V_4 = 100 \times 4/10 = 40 \text{ V}$$

$$V_1 = 0 \times 2 = 0 \text{ V}$$

Voltage across the capacitor = drop across  $B-C = 40 \text{ V}$

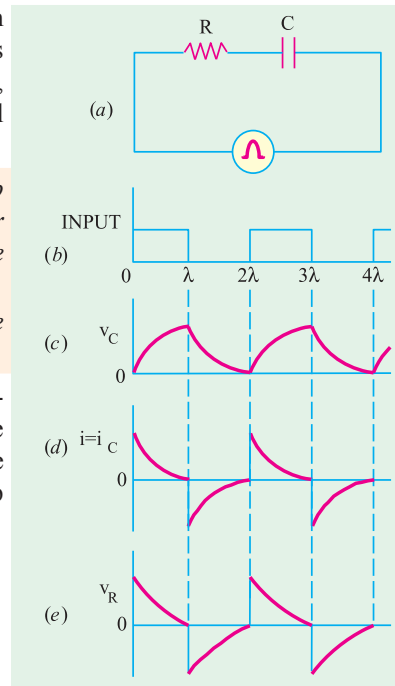


Fig. 5.32

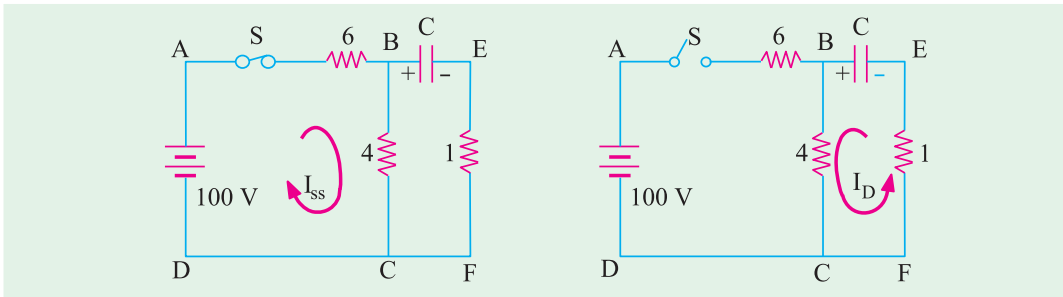


Fig. 5.33

**Switch Open**

When  $S$  is opened, the charged capacitor discharges through the loop  $BCFE$  as shown in Fig. 5.33 (b). The discharge current is given by

$$I_D = 40/(4 + 1) = 8 \text{ A}$$

As seen, it flows in a direction opposite to that of  $I_{SS}$ .

**Example 5.38. (a)** A capacitor is charged through a large non-reactive resistance by a battery of constant voltage  $V$ . Derive an expression for the instantaneous charge on the capacitor.

**(b)** For the above arrangement, if the capacitor has a capacitance of  $10 \mu\text{F}$  and the resistance is  $1 \text{ M}\Omega$  calculate the time taken for the capacitor to receive 90% of its final charge. Also, draw the charge/time curve.

**Solution. (a)** For this part, please refer to Art. 5.18.

**(b)**  $\lambda = CR = 10 \times 10^{-6} \times 1 \times 10^6 = 10 \text{ s}$  ;  $q = 0.9 Q$

Now,  $q = Q (1 - e^{-t/\lambda}) \therefore 0.9 Q = Q (1 - e^{-t/10})$  or  $e^{t/10} = 10$

$\therefore 0.1 t \log_e e = \log_e 10$  or  $0.1 t = 2.3 \log_{10} 10 = 2.3$  or  $t = 23 \text{ s}$

The charge/time curve is similar to that shown in Fig. 5.27 (b).

**Example 5.39.** A resistance  $R$  and a  $4 \mu\text{F}$  capacitor are connected in series across a  $200 \text{ V}$  d.c. supply. Across the capacitor is a neon lamp that strikes (glows) at  $120 \text{ V}$ . Calculate the value of  $R$  to make the lamp strike (glow) 5 seconds after the switch has been closed.

(Electrotechnics-I.M.S. Univ. Baroda)

**Solution.** Obviously, the capacitor voltage has to rise  $120 \text{ V}$  in 5 seconds.

$$\therefore 120 = 200 (1 - e^{-5/\lambda}) \quad \text{or} \quad e^{5/\lambda} = 2.5 \quad \text{or} \quad \lambda = 5.464 \text{ second.}$$

$$\text{Now, } \lambda = CR \quad \therefore R = 5.464/4 \times 10^{-6} = \mathbf{1.366 \text{ M}\Omega}$$

**Example 5.40.** A capacitor of  $0.1 \mu\text{F}$  is charged from a  $100\text{-V}$  battery through a series resistance of  $1,000 \text{ ohms}$ . Find

- (a) the time for the capacitor to receive 63.2 % of its final charge.  
 (b) the charge received in this time (c) the final rate of charging.  
 (d) the rate of charging when the charge is 63.2% of the final charge.

(Elect. Engineering, Bombay Univ.)

**Solution.** (a) As seen from Art. 5.18 (b), 63.2% of charge is received in a time equal to the time constant of the circuit.

$$\text{Time required} = \lambda = CR = 0.1 \times 10^{-6} \times 1000 = 0.1 \times 10^{-3} = \mathbf{10^{-4} \text{ second}}$$

$$\text{(b) Final charge, } Q = CV = 0.1 \times 100 = 10 \mu\text{C}$$

$$\text{Charge received during this time is} = 0.632 \times 10 = \mathbf{6.32 \mu\text{C}}$$

(c) The rate of charging at any time is given by Eq. (ii) of Art. 5.18.

$$\frac{dv}{dt} = \frac{V - v}{CR}$$

$$\text{Initially } v = 0, \text{ Hence } \frac{dv}{dt} = \frac{V}{CR} = \frac{100}{0.1 \times 10^{-6} \times 10^3} = \mathbf{10^6 \text{ V/s}}$$

$$\text{(d) Here } v = 0.632 \text{ V} = 0.632 \times 100 = 63.2 \text{ volts}$$

$$\therefore \frac{dv}{dt} = \frac{100 - 63.2}{10^{-4}} = \mathbf{368 \text{ kV/s}}$$

**Example 5.41.** A series combination having  $R = 2 \text{ M}\Omega$  and  $C = 0.01 \mu\text{F}$  is connected across a d.c. voltage source of  $50 \text{ V}$ . Determine

- (a) capacitor voltage after  $0.02 \text{ s}$ ,  $0.04 \text{ s}$ ,  $0.06 \text{ s}$  and  $1 \text{ hour}$   
 (b) charging current after  $0.02 \text{ s}$ ,  $0.04 \text{ s}$ ,  $0.06 \text{ s}$  and  $0.1 \text{ s}$ .

$$\text{Solution. } \lambda = CR = 2 \times 10^6 \times 0.01 \times 10^{-6} = 0.02 \text{ second}$$

$$I_m = V/R = 50/2 \times 10^6 = 25 \mu\text{A.}$$

While solving this question, it should be remembered that (i) in each time constant,  $v_c$  increases further by 63.2% of its balance value and (ii) in each constant,  $i_c$  decreases to 37% its previous value.

(a) (i)  $t = 0.02 \text{ s}$

Since, initially at  $t = 0$ ,  $v_c = 0 \text{ V}$  and  $V_e = 50 \text{ V}$ , hence, in one time constant

$$v_c = 0.632 (50 - 0) = \mathbf{31.6 \text{ V}}$$

(ii)  $t = 0.04 \text{ s}$

This time equals two time-constants.

$$\therefore v_c = 31.6 + 0.632 (50 - 31.6) = \mathbf{43.2 \text{ V}}$$

(iii)  $t = 0.06 \text{ s}$

This time equals three time-constants.

$$\therefore v_c = 43.2 + 0.632 (50 - 43.2) = \mathbf{47.5 \text{ V}}$$



Since in one hour, steady-state conditions would be established,  $v_c$  would have achieved its maximum possible value of **50 V**.

(b) (i)  $t = 0.02$  s,  $i_c = 0.37 \times 25 = 9.25 \mu\text{A}$

(ii)  $t = 0.4$  s,  $i_c = 0.37 \times 9.25 = 3.4 \mu\text{A}$

(iii)  $t = 0.06$  s,  $i_c = 0.37 \times 3.4 = 1.26 \mu\text{A}$

(iv)  $t = 0.1$  s, This time equals 5 time constants. In this time, current falls almost to **zero** value.

**Example 5.42.** A voltage as shown in Fig. 5.43 (a) is applied to a series circuit consisting of a resistance of  $2 \Omega$  in series with a pure capacitor of  $100 \mu\text{F}$ . Determine the voltage across the capacitor at  $t = 0.5$  millisecond. [Bombay University, 2000]

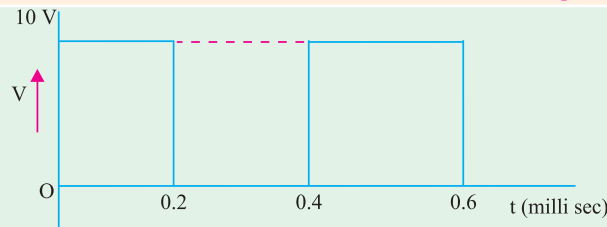


Fig. 5.34 (a)

**Solution.**

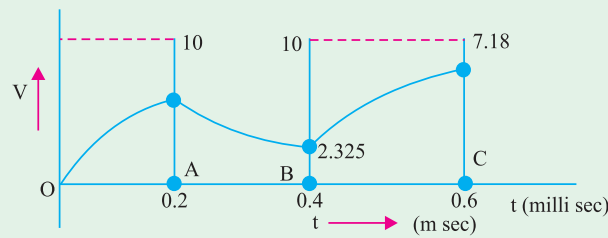


Fig. 5.34 (b)

$$\tau = RC = 0.2 \text{ milli-second}$$

Between 0 and 0.2 m sec;

$$v(t) = 10 [1 - \exp(-t/\tau)]$$

At  $t = 0.2$ ,  $v(t) = 6.32$  volts

Between 0.2 and 0.4 m Sec, counting time from A indicating it as  $t_1$

$$v(t_1) = 6.32 \exp(t_1/\tau)$$

At point B,  $t_1 = 0.2$ ,  $V = 2.325$

Between 0.4 and 0.6 m Sec, time is counted from B with variable as  $t_2$ ,

$$v(t_2) = 2.325 + (10 - 2.325) [1 - \exp(-t_2/\tau)]$$

At C,  $t_2 = 0.2$ ,  $V = 7.716$  volts.

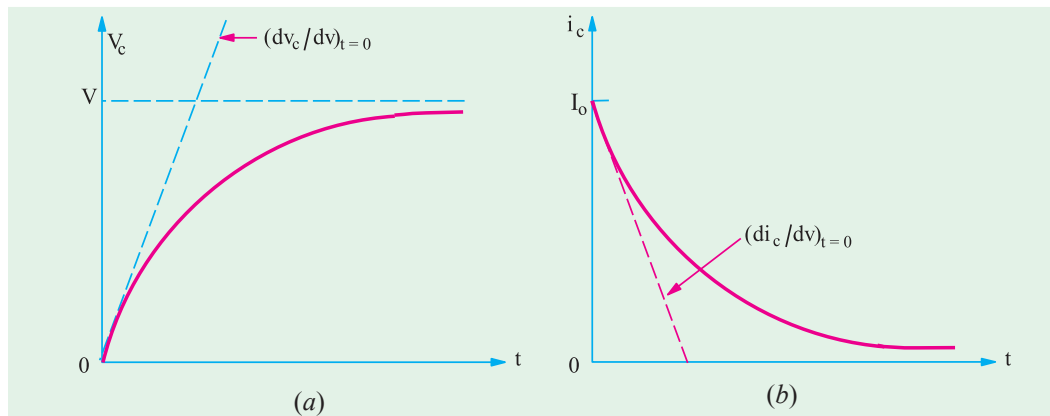
## 5.21. Transient Relations During Capacitor Charging Cycle

Whenever a circuit goes from one steady-state condition to another steady-state condition, it passes through a transient state which is of short duration. The first steady-state condition is called the **initial condition** and the second steady-state condition is called the **final condition**. In fact, transient condition lies in between the initial and final conditions. For example, when switch  $S$  in Fig. 5.35 (a) is not connected either to  $a$  or  $b$ , the  $RC$  circuit is in its initial steady state with no current and

hence no voltage drops. When  $S$  is shifted to point  $a$ , current starts flowing through  $R$  and hence, transient voltages are developed across  $R$  and  $C$  till they achieve their final steady values. The period during which current and voltage changes take place is called **transient condition**.

The moment switch  $S$  is shifted to point 'a' as shown in Fig. 5.35 (b), a charging current  $i_c$  is set up which starts charging  $C$  that is initially uncharged. At the beginning of the transient state,  $i_c$  is maximum because there is no potential across  $C$  to oppose the applied voltage  $V$ . It has maximum value  $= V/R = I_0$ . It produces maximum voltage drop across  $R = i_c R = I_0 R$ . Also, initially,  $v_c = 0$ , but as time passes,  $i_c$  decreases gradually so does  $v_R$  but  $v_c$  increases exponentially till it reaches the final steady value of  $V$ . Although  $V$  is constant,  $v_R$  and  $v_c$  are variable. However, at any time  $V = v_R + v_c = i_c R + v_c$ .

At the beginning of the transient state,  $i_c = I_0$ ,  $v_c = 0$  but  $v_R = V$ . At the end of the transient state,  $i_c = 0$  hence,  $v_R = 0$  but  $v_c = V$ .



The initial rates of change of  $v_c$ ,  $v_R$  and  $i_c$  are given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{\lambda} \text{ volt/second,}$$

$$\left(\frac{dv_R}{dt}\right)_{t=0} = \frac{I_0 R}{\lambda} = -\frac{V}{\lambda} \text{ volt/second}$$

$$\left(\frac{di_c}{dt}\right)_{t=0} = \frac{I_0}{\lambda} \text{ where } I_0 = \frac{V}{R}$$

These are the initial rates of change. However, their rate of change at any time during the charging transient are given as under :

$$\frac{dv_c}{dt} = \frac{V}{\lambda} e^{-t/\lambda}, \quad \frac{di_c}{dt} = -\frac{dv_R}{dt} = -\frac{V}{\lambda} e^{-t/\lambda}$$

It is shown in Fig. 5.35 (c).

It should be clearly understood that a negative rate of change means a decreasing rate of change. It does not mean that the concerned quantity has reversed its direction.

## 5.22. Transient Relations During Capacitor Discharging Cycle

As shown in Fig. 5.36 (b), switch  $S$  has been shifted to  $b$ . Hence, the capacitor undergoes the discharge cycle. Just before the transient state starts,  $i_c = 0$ ,  $v_R = 0$  and  $v_c = V$ . The moment transient

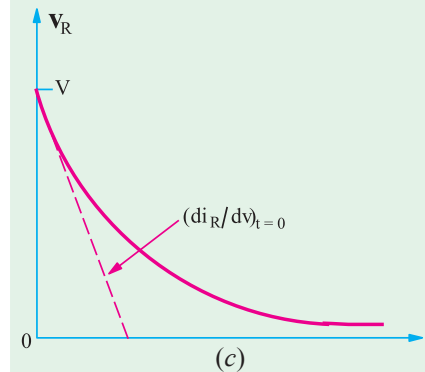


Fig. 5.35

state begins,  $i_c$  has maximum value and decreases exponentially to zero at the end of the transient state. So does  $v_c$ . However, during discharge, all rates of change have polarity opposite to that during charge. For example,  $dv_c/dt$  has a positive rate of change during charging and negative rate of change during discharging.

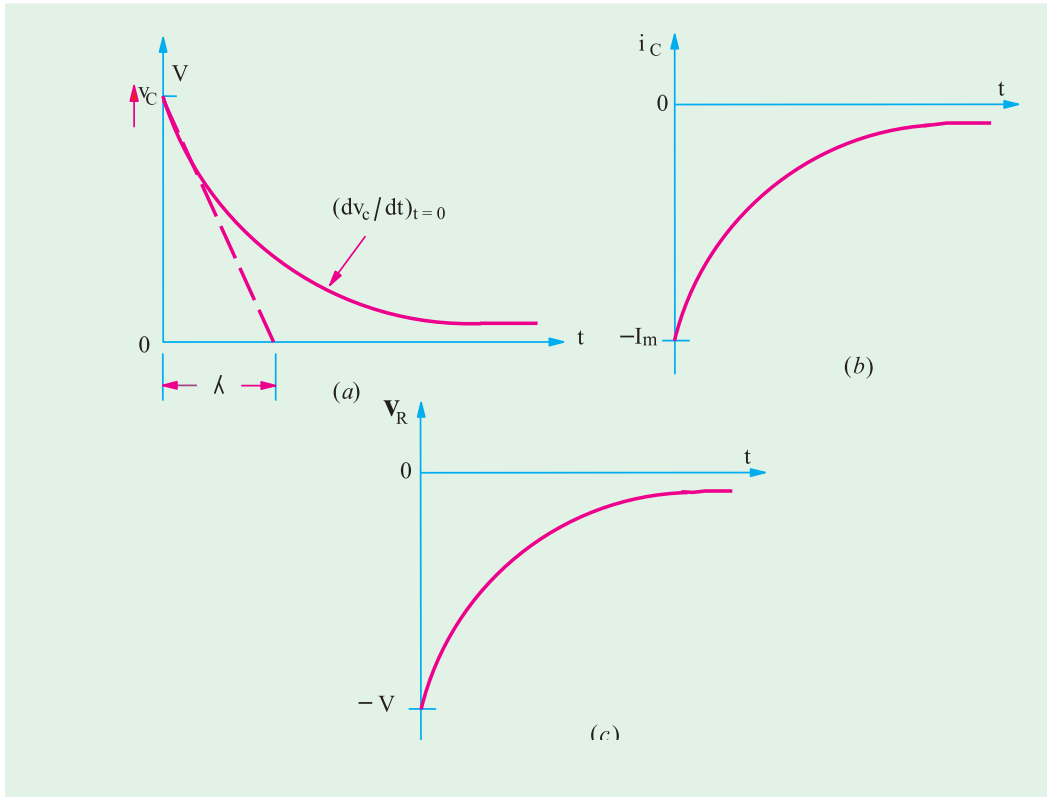


Fig. 5.36

Also, it should be noted that during discharge,  $v_c$  maintains its original polarity whereas  $i_c$  reverses its direction of flow. Consequently, during capacitor discharge,  $v_R$  also reverses its direction.

The various rates of change at *any time* during the discharge transients are as given in Art.

$$\frac{dv_c}{dt} = -\frac{V}{\lambda} e^{-t/\lambda}; \quad \frac{di_c}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda}; \quad \frac{dv_R}{dt} = \frac{V}{\lambda} e^{-t/\lambda}$$

These are represented by the curves of Fig. 5.32.

### 5.23. Charging and Discharging of a capacitor with Initial Charge

In Art. 5.18, we considered the case when the capacitor was initially uncharged and hence, had no voltage across it. Let us now consider the case, when the capacitor has an initial potential of  $V_0$  (less than  $V$ ) which opposes the applied battery voltage  $V$  as shown in Fig. 5.37 (a).

As seen from Fig. 5.37 (b), the initial rate of rise of  $v_c$  is now somewhat less than when the capacitor is initially uncharged. Since the capacitor voltage rises from an initial value of  $v_0$  to the final value of  $V$  in one time constant, its initial rate of rise is given by

$$\left( \frac{dv_c}{dt} \right)_{t=0} = \frac{V - V_0}{\lambda} = \frac{V - V_0}{RC}$$

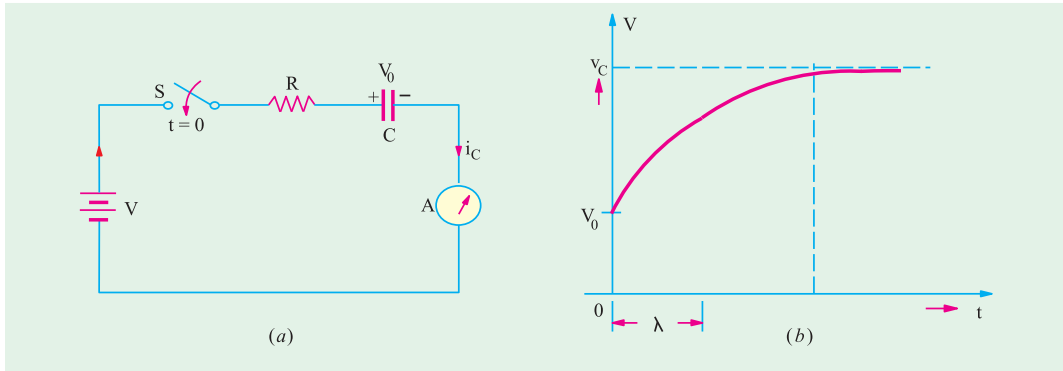


Fig. 5.37

The value of the capacitor voltage at *any time* during the charging cycle is given by

$$v_c = (V - V_0)(1 - e^{-t/\lambda}) + V_0$$

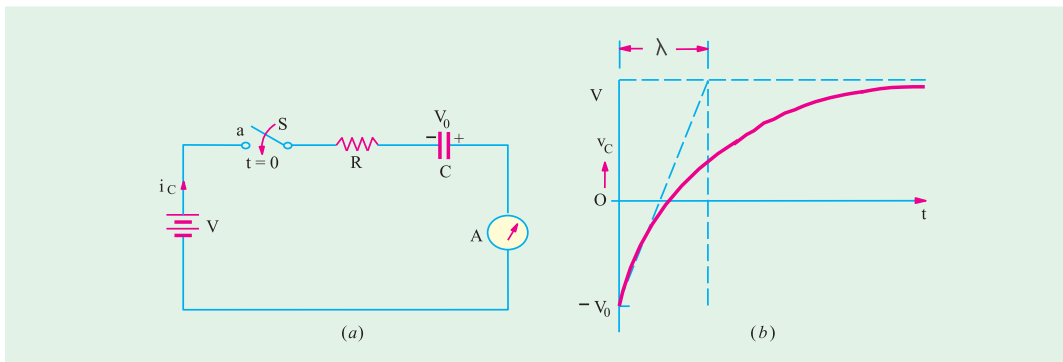


Fig. 5.38

However, as shown in Fig. 5.38 (a), if the initial capacitor voltage is negative with respect to the battery voltage *i.e.* the capacitor voltage is series aiding the battery voltage, rate of change of  $v_c$  is steeper than in the previous case. It is so because as shown in Fig. 5.38 (b), in one time period, the voltage change =  $V - (-V_0) = (V + V_0)$ . Hence, the initial rate of change of voltage is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V + V_0}{\lambda} = \frac{V + V_0}{RC}$$

The value of capacitor voltage at *any time* during the charging cycle is given by

$$v_c = (V + V_0)(1 - e^{-t/\lambda}) - V_0$$

The time required for the capacitor voltage to attain any value of  $v_c$  during the charging cycle is given by

$$t = \lambda \ln \left( \frac{V - V_0}{V - v_c} \right) = RC \ln \left( \frac{V - V_0}{V - v_c} \right) \quad \dots \text{when } V_0 \text{ is positive}$$

$$t = \lambda \ln \left( \frac{V + V_0}{V - v_c} \right) = RC \ln \left( \frac{V + V_0}{V - v_c} \right) \quad \dots \text{when } V_0 \text{ is negative}$$

**Example 5.43.** In Fig. 5.39, the capacitor is initially uncharged and the switch  $S$  is then closed. Find the values of  $I$ ,  $I_1$ ,  $I_2$  and the voltage at the point  $A$  at the start and finish of the transient state.

**Solution.** At the moment of closing the switch *i.e.* at the start of the transient state, the capacitor acts as a short-circuit. Hence, there is only a resistance of  $2\ \Omega$  in the circuit because  $1\ \Omega$  resistance is shorted out thereby grounding point *A*. Hence,  $I_1 = 0$ ;  $I = I_2 = 12/2 = 6\text{ A}$ . Obviously,  $V_A = 0\text{ V}$ .

At the end of the transient state, the capacitor acts as an open-circuit. Hence,

$$I_2 = 0 \text{ and } I = I_1 = 12/(2 + 1) = 4\text{ A. } V_A = 6\text{ V.}$$

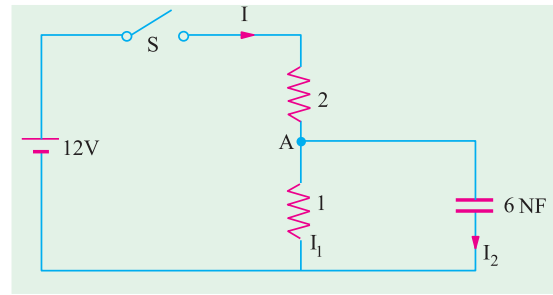


Fig. 5.39

**Example 5.44.** Calculate the values of  $i_2, i_3, v_2, v_3, v_a, v_c$  and  $v_L$  of the network shown in Fig. 5.40 at the following times :

- (i) At time,  $t = 0 +$  immediately after the switch *S* is closed ;
- (ii) At time,  $t \rightarrow \infty$  *i.e.* in the steady state. (Network Analysis AMIE Sec. B Winter 1990)

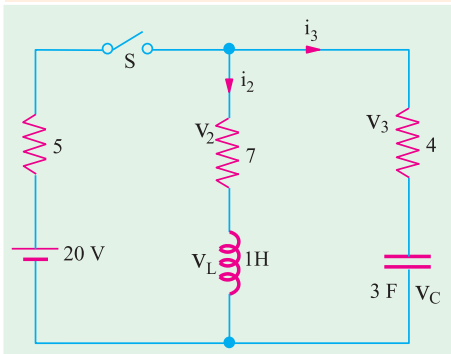


Fig. 5.40

**Solution.** (i) In this case the coil acts as an open circuit, hence  $i_2 = 0$ ;  $v_2 = 0$  and  $v_L = 20\text{ V}$ .

Since a capacitor acts as a short circuit  $i_3 = 20/(5 + 4) = 9 = 20/9\text{ A}$ . Hence,  $v_3 = (20/9) \times 4 = 80/9\text{ V}$  and  $v_c = 0$ .

(ii) Under steady state conditions, capacitor acts as an open circuit and coil as a short circuit. Hence,  $i_2 = 20/(5 + 7) = 20/12 = 5/3\text{ A}$ ;  $v_2 = 7 \times 5/3 = 35/3\text{ V}$ ;  $v_L = 0$ . Also  $i_3 = 0, v_3 = 0$  but  $v_c = 20\text{ V}$ .

**Example 5.45.** If in the RC circuit of Fig. 5.36;  $R = 2\text{ M}\Omega, C = 5\text{ mF}$  and  $V = 100\text{ V}$ , calculate

- (a) initial rate of change of capacitor voltage
- (b) initial rate of change of capacitor current

- (c) initial rate of change of voltage across the  $2\text{ M}\Omega$  resistor
- (d) all of the above at  $t = 80\text{ s}$ .

**Solution.** (a)  $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{2 \times 10^6} \frac{100}{5 \times 10^{-6}} \frac{100}{10} = 10\text{ V/s}$

(b)  $\left(\frac{di_c}{dt}\right)_{t=0} = \frac{I_0}{10} \frac{V/R}{10} = \frac{100/2}{10} \frac{10^6}{10} = -5\mu\text{A/s}$

(c)  $\left(\frac{dv_R}{dt}\right)_{t=0} = \frac{V}{10} \frac{100}{10} = -10\text{ V/s}$

(d) All the above rates of change would be zero because the transient disappears after about  $5\lambda = 5 \times 10 = 50\text{ s}$ .

**Example 5.46.** In Fig. 5.41 (a), the capacitor *C* is fully discharged, since the switch is in position 2. At time  $t = 0$ , the switch is shifted to position 1 for 2 seconds. It is then returned to position 2 where it remains indefinitely. Calculate

- (a) the maximum voltage to which the capacitor is charged when in position 1.
- (b) charging time constant  $\lambda_1$  in position 1.
- (c) discharging time constant  $\lambda_2$  in position 2.
- (d)  $v_c$  and  $i_c$  at the end of 1 second in position 1.

- (e)  $v_c$  and  $i_c$  at the instant the switch is shifted to position 2 at  $t = 1$  second.  
 (f)  $v_c$  and  $i_c$  after a lapse of 1 second when in position 2.  
 (g) sketch the waveforms for  $v_c$  and  $i_c$  for the first 2 seconds of the above switching sequence.

**Solution.** (a) We will first find the voltage available at terminal 1. As seen the net battery voltage around the circuit =  $40 - 10 = 30$  V. Drop across  $30$  K resistor =  $30 \times 30 / (30 + 60) = 10$  V. Hence, potential of terminal 1 with respect to ground  $G = 40 - 10 = 30$  V. Hence, capacitor will charge to a maximum voltage of  $30$  V when in position 1.

(b) Total resistance,  $R = [(30 \text{ K} \parallel 60 \text{ K}) + 10 \text{ K}] = 30 \text{ K}$

$\therefore \lambda_1 = RC = 30 \text{ K} \times 10 \mu\text{F} = 0.3 \text{ s}$

(c)  $\lambda_2 = 10 \text{ K} \times 10 \mu\text{F} = 0.1 \text{ s}$

(d)  $v_c = V(1 - e^{-t/\lambda_1}) = 30(1 - e^{-1/0.3}) = 28.9 \text{ V}$

$$i_c = \frac{V}{R} e^{-t/\lambda_1} = \frac{30 \text{ V}}{30 \text{ K}} e^{-1/0.3} = 1 \times 0.0361 = 0.036 \text{ mA}$$

(e)  $v_c = 28.9 \text{ V}$  at  $t = 1^+$  s at position 2 but  $i_c = -28.9 \text{ V} / 10 \text{ K} = -2.89 \text{ mA}$  at  $t = 1^+$  s in position 2.

(f)  $v_c = 28.9 e^{-t/\lambda_2} = 28.9 e^{-1/0.1} = 0.0013 \text{ V} \approx 0 \text{ V}$ .

$$i_c = 28.9 e^{-t/\lambda_2} = -2.89 e^{-1/0.1} = 0.00013 \text{ mA} \approx 0.$$

The waveform of the capacitor voltage and charging current are sketched in Fig. 5.41 (b).

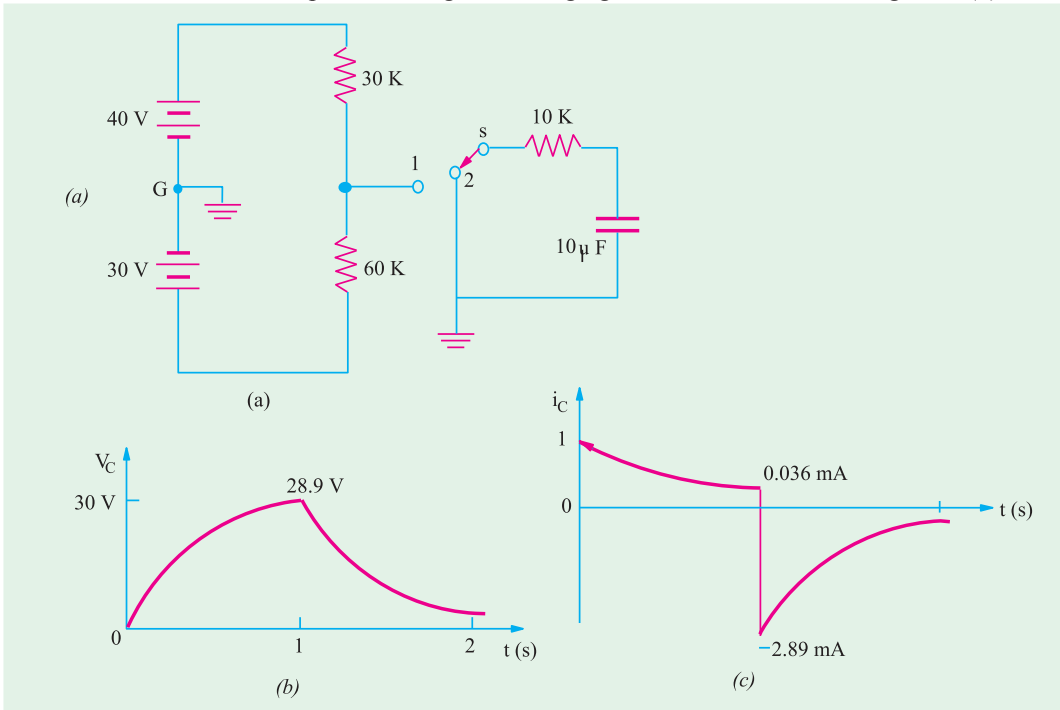


Fig. 5.41

**Example 5.47.** In the  $RC$  circuit of Fig. 5.42,  $R = 2 \text{ M}\Omega$  and  $C = 5 \mu\text{F}$ , the capacitor is charged to an initial potential of  $50 \text{ V}$ . When the switch is closed at  $t = 0^+$ , calculate

- (a) initial rate of change of capacitor voltage and  
 (b) capacitor voltage after a lapse of 5 times the time constant i.e.  $5\lambda$ .

If the polarity of capacitor voltage is reversed, calculate

- (c) the values of the above quantities and
- (d) time for  $v_c$  to reach  $-10$  V,  $0$  V and  $95$  V.

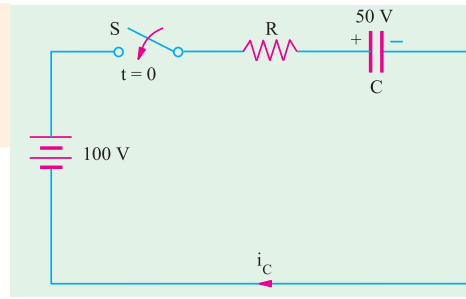


Fig. 5.42

**Solution. (a)**  $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - V_0}{\lambda}$   
 $= \frac{V - V_0}{RC} = \frac{100 - 50}{10} = 5 \text{ V/s}$

(b)  $v_c = (V - V_0)(1 - e^{-t/\lambda}) + V_0$   
 $= (100 - 50)(1 - e^{-5 \times 10/10}) + 50 = 49.7 + 50 = 99.7 \text{ V}$

(c) When  $V_0 = -50 \text{ V}$ ,  $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - (-V_0)}{\lambda} = \frac{V + V_0}{\lambda} = \frac{150}{10} = 15 \text{ V/s}$

$v_c = (V - V_0)(1 - e^{-t/\lambda}) + V_0 = [100 - (-50)](1 - e^{-5}) + (-50)$   
 $= 150(1 - e^{-5}) - 50 = 99 \text{ V}$

(d)  $t = \lambda \ln\left(\frac{V - V_0}{V - v_c}\right) = 10 \ln\left[\frac{100 - (-50)}{100 - (-10)}\right] = 10 \ln\left(\frac{150}{110}\right) = 3.1 \text{ s}$

$t = 10 \ln\left[\frac{100 - (-50)}{100 - (0)}\right] = 10 \ln\left(\frac{150}{100}\right) = 4.055 \text{ s}$

$t = 10 \ln\left[\frac{100 - (-50)}{100 - 95}\right] = 10 \ln\left(\frac{150}{5}\right) = 34 \text{ s}$

**Example 5.48.** The uncharged capacitor, if it is initially switched to position 1 of the switch for 2 sec and then switched to position 2 for the next two seconds. What will be the voltage on the capacitor at the end of this period? Sketch the variation of voltage across the capacitor. [Bombay University 2001]

**Solution.** Uncharged capacitor is switched to position 1 for 2 seconds. It will be charged to 100 volts instantaneously since resistance is not present in the charging circuit. After 2 seconds, the capacitor charged to 100 volts will get discharged through R-C circuit with a time constant of

$\tau = RC = 1500 \times 10^{-3} = 1.5 \text{ sec}$ .

Counting time from instant of switching over to position 2, the expression for voltage across the capacitor is  $V(t) = 100 \exp(-t/\tau)$

After 2 seconds in this position,

$v(t) = 100 \exp(-2/1.5) = 26/36 \text{ Volts}$ .

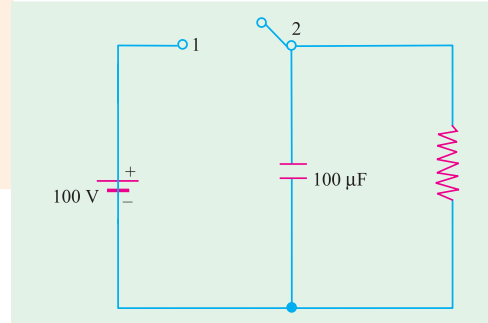


Fig. 5.43

**Example 5.49.** There are three passive elements in the circuit below and a voltage and a current are defined for each. Find the values of these six qualities at both  $t = 0^-$  and  $t = 0^+$ .

[Bombay University, 2001]

**Solution.** Current source  $4u(t)$  means a step function of 4 amp applied at  $t = 0$ . Other current source of 5 amp is operative throughout.

At  $t = 0^-$ , 5 amp source is operative. This unidirectional constant current establishes a steady current of 5 amp through 30-ohm resistor and 3-H inductor. Note that positive  $V_R$  means a rise from right to left.

At  $t = 0$   
 $V_R = -150$  Volts (Since right-terminal of Resistor is + ve)  
 $i_L = 5$  amp  
 $V_L = 0$ , it represents the voltage between  $B$  and  $O$ .  
 $i_C = 0$   
 $V_C = 150$  volts  $= V_{BO} +$  (Voltage between  $A$  and  $B$  with due regards to sign).  
 $= 0 - (-150) = + 150$  volts

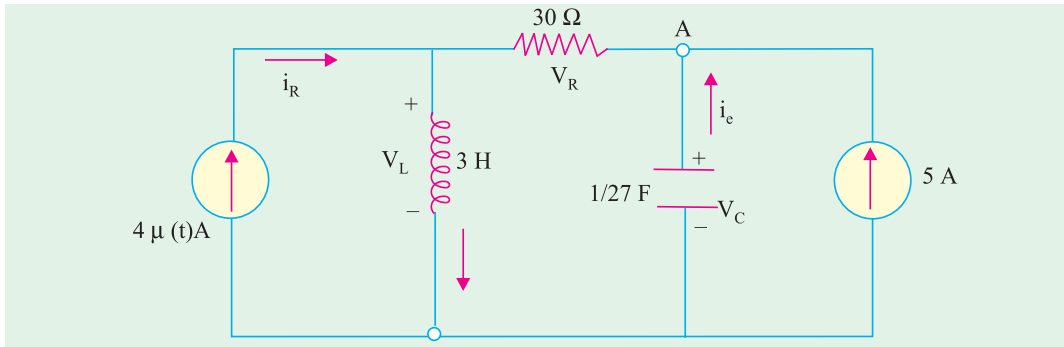


Fig. 5.44 (a)

At  $t = 0_+$ , 4 amp step function becomes operative. Capacitive-voltage and Inductance-current cannot change abruptly.

Hence  $i_L(0_+) = 5$  amp

$V_C(0_+) = 150$  amp

$V_C(0_+) = 150$  volts, with node  $A$  positive with respect to  $O$ .

With these two values known, the waveforms for current sources are drawn in Fig. 5.44 (b).

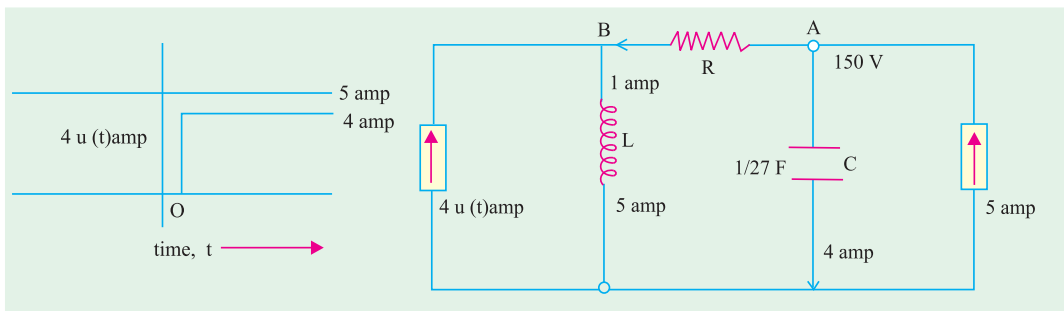


Fig. 5.44 (b)

Fig. 5.44 (c)

Remaining four parameters are evaluated from Fig. 5.44 (c).

$V_L = V_B = V_A - (30 \times 1) = 120$  Volts

$i_R = 1$  amp,  $V_R = -30$  Volts

$i_C = 4$  amp in downward direction.

**Additional Observation.** After 4 amp source is operative, final conditions (at  $t$  tending to infinity) are as follows.

Inductance carries a total direct current of 9 amp, with  $V_L = 0$ .

Hence,  $V_B = 0$ .

$i_R = 5$  amp,  $V_R = -150$  volts

$V_C = 150$  volts,  $i_C = 0$

**Example 5.50.** The voltage as shown in Fig. 5.45 (a) is applied across **(i)** A resistor of 2 ohms **(ii)** A capacitor of 2 F. Find and sketch the current in each case up to 6 seconds.



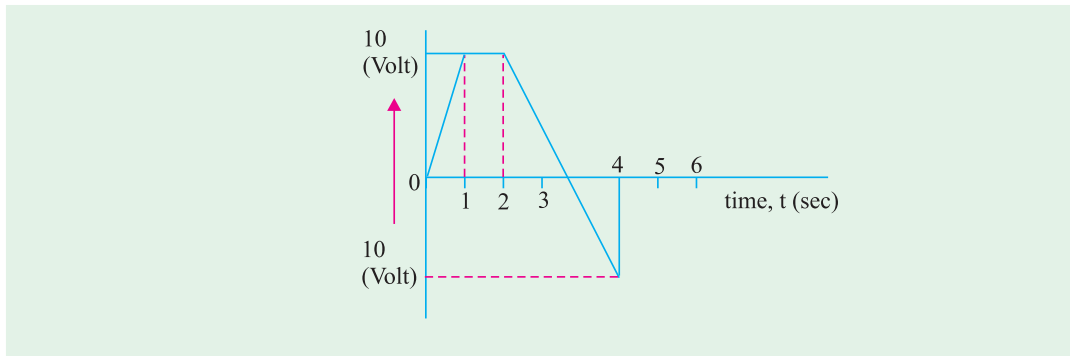


Fig. 5.45 (a)

[Bombay University 1998]

Solution.

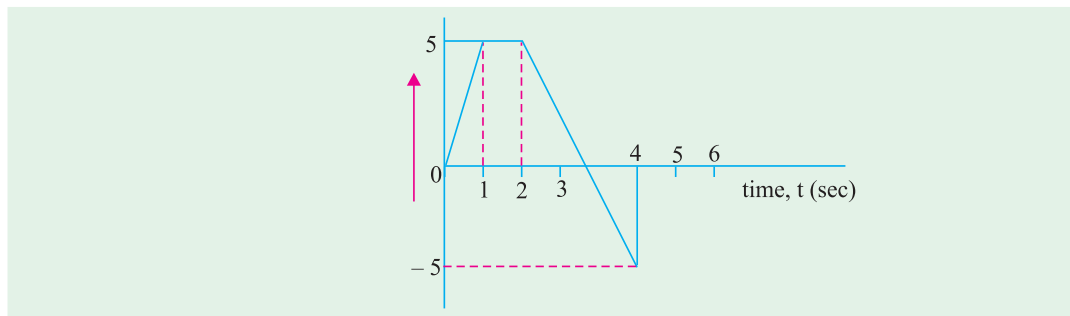


Fig. 5.45 (b) Current in a Resistor of 2 ohms  $i_R = V(t)/2$  amp

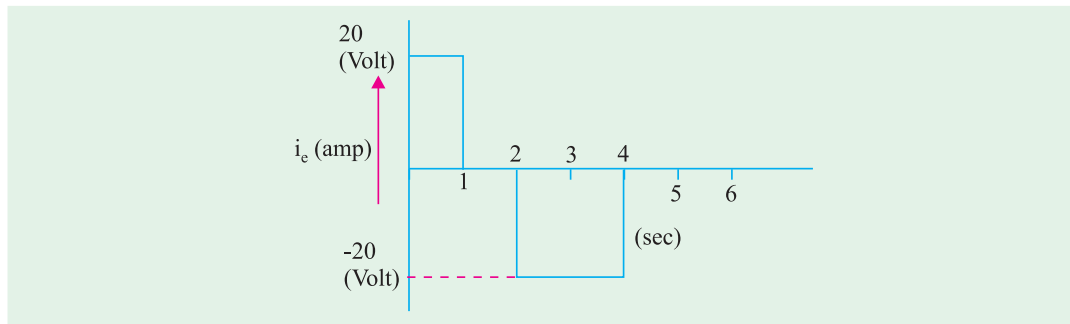


Fig. 5.45 (c) Current thro 2-F capacitor,  $i_C = C (dv/dt)$

**Example 5.51.** Three capacitors  $2 \mu F$ ,  $3 \mu F$ , and  $5 \mu F$  are connected in series and charged from a  $900 V$  d.c. supply. Find the voltage across condensers. They are then disconnected from the supply and reconnected with all the +ve plates connected together and all the -ve plates connected together. Find the voltages across the combinations and the charge on each capacitor after reconnections. Assume perfect insulation. [Bombay University, 1998]

**Solution.** The capacitors are connected in series. If  $C$  is the resultant capacitance.

$$1/C = 1/C_1 + 1/C_2 + 1/C_3, \text{ which gives } C = (30/31) \mu F$$

$$V_1 = 900 \times (30/31)/2 = 435.5 \text{ volts}$$

$$V_2 = 900 \times (30/31)/3 = 290.3 \text{ volts}$$

$$V_3 = 900 \times (30/31)/5 = 174.2 \text{ volts}$$

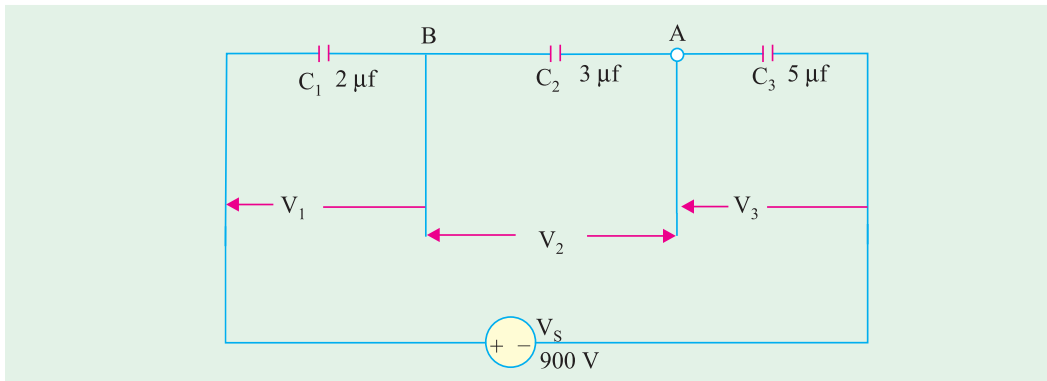


Fig. 5.46

In series connection, charge held by each capacitor is same. If it is denoted by  $Q$ .

$$Q = 435 \times 2 \times 10^{-6} = 871 \mu \text{ coulombs}$$

Three capacitors hold a total charge of  $(3 \times 871) = 2613 \mu \text{ coulombs}$

With parallel connection of these three capacitors, equivalent capacitance,  $C' = C_1 + C_2 + C_3 = 10 \mu\text{F}$

Since,  $Q' = C', 2613 \times 10^{-6} = 10 \times 10^{-6} \times V'$

or  $V' = 261 \text{ volts.}$

Charge on each capacitor after reconnection is as follows :

$$Q_1' = C_1 V_1 = 2 \times 10^{-6} \times 261 = \mathbf{522 \mu\text{-coulombs}}$$

$$Q_2' = C_2 V_1 = 3 \times 10^{-6} \times 261 = \mathbf{783 \mu\text{-coulombs}}$$

$$Q_3' = C_3 V_2 = 5 \times 10^{-6} \times 261 = \mathbf{1305 \mu\text{-coulombs}}$$

### Tutorial Problems No. 5.3

1. For the circuit shown in Fig. 5.47 calculate (i) equivalent capacitance and (ii) voltage drop across each capacitor. All capacitance values are in  $\mu\text{F}$ .

**[(i)  $6 \mu\text{F}$  (ii)  $V_{AB} = 50 \text{ V}, V_{BC} = 40 \text{ V}$ ]**

2. In the circuit of Fig. 5.48 find (i) equivalent capacitance (ii) drop across each capacitor and (iii) charge on each capacitor. All capacitance values are in  $\mu\text{F}$ .

**[(i)  $1.82 \mu\text{F}$  (ii)  $V_1 = 50 \text{ V}; V_2 = V_3 = 20 \text{ V}; V_4 = 40 \text{ V}$   
(iii)  $Q_1 = 200 \mu\text{C}; Q_2 = 160 \mu\text{C}; Q_3 = 40 \mu\text{C}; Q_4 = 200 \mu\text{C}$ ]**

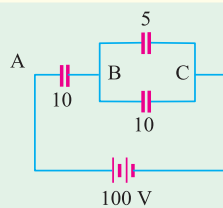


Fig. 5.47

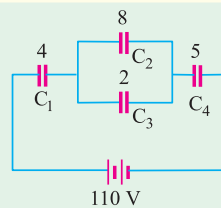


Fig. 5.48

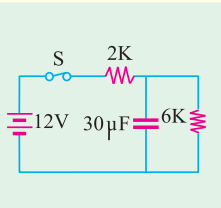


Fig. 5.49

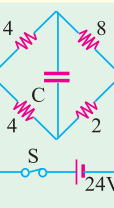


Fig. 5.50

3. With switch in Fig. 5.49 closed and steady-state conditions established, calculate (i) steady-state current (ii) voltage and charge across capacitor (iii) what would be the discharge current at the instant of opening the switch ?

**[(i)  $1.5 \text{ mA}$  (ii)  $9 \text{ V}; 270 \mu\text{C}$  (iii)  $1.5 \text{ mA}$ ]**

4. When the circuit of Fig. 5.50 is in steady state, what would be the p.d. across the capacitor ? Also, find the discharge current at the instant  $S$  is opened.

**[ $8 \text{ V}; 1.8 \text{ A}$ ]**

5. Find the time constant of the circuit shown in Fig. 5.51.

[200  $\mu$ S]

6. A capacitor of capacitance  $0.01 \mu\text{F}$  is being charged by  $1000 \text{ V d.c.}$  supply through a resistor of  $0.01 \text{ megaohm}$ . Determine the voltage to which the capacitor has been charged when the charging current has decreased to  $90\%$  of its initial value. Find also the time taken for the current to decrease to  $90\%$  of its initial value.

[100 V, 0.1056 ms]

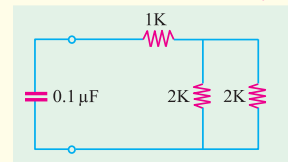


Fig. 5.51

7. An  $8 \mu\text{F}$  capacitor is being charged by a  $400 \text{ V}$  supply through  $0.1 \text{ mega-ohm}$  resistor. How long will it take the capacitor to develop a p.d. of  $300 \text{ V}$ ? Also what fraction of the final energy is stored in the capacitor? [1.11 Second, 56.3% of full energy]

8. An  $10 \mu\text{F}$  capacitor is charged from a  $200 \text{ V}$  battery  $250 \text{ times/second}$  and completely discharged through a  $5 \Omega$  resistor during the interval between charges. Determine

(a) the power taken from the battery.

(b) the average value of the current in  $5 \Omega$  resistor.

[(a) 50 W (b) 0.5 A]

9. When a capacitor, charged to a p.d. of  $400 \text{ V}$ , is connected to a voltmeter having a resistance of  $25 \text{ M}\Omega$  the voltmeter reading is observed to have fallen to  $50 \text{ V}$  at the end of an interval of  $2 \text{ minutes}$ . Find the capacitance of the capacitor. [2.31  $\mu\text{F}$ ] (App. Elect. London Univ.)

10. A capacitor and a resistor are connected in series with a d.c. source of  $V$  volts. Derive an expression for the voltage across the capacitor after ' $t$ ' seconds during discharging.

(Gujrat University, Summer 2003)

11. Derive an expression for the equivalent capacitance of a group of capacitors when they are connected (i) in parallel (ii) in series. (Gujrat University, Summer 2003)

12. The total capacitance of two capacitors is  $0.03 \mu\text{F}$  when joined in series and  $0.16 \mu\text{F}$  when connected in parallel. Calculate the capacitance of each capacitor. (Gujrat University, Summer 2003)

13. In a capacitor with two plates separated by an insulator  $3 \text{ mm}$  thick and of relative permittivity of  $4$ , the distance between the plates is increased to allow the insertion of a second insulator  $5 \text{ mm}$  thick and relative permittivity  $E$ . If the capacitance so formed is one third of the original capacitance, find  $E$ . (VTU., Belgaum Karnataka University, February 2002)

14. Derive an expression for the capacitance of a parallel plate capacitor.

(VTU., Belgaum Karnataka University, Summer 2002)

15. Three capacitors A, B and C are charged as follows

A =  $10 \mu\text{F}$ ,  $100 \text{ V}$     B =  $15 \mu\text{F}$ ,  $150 \text{ V}$     C =  $25 \mu\text{F}$ ,  $200 \text{ V}$

They are connected in parallel with terminals of like polarities together. Find the voltage across the combination. (VTU., Belgaum Karnataka University, Summer 2002)

16. Prove that average power consumed by a pure capacitance is zero.

(VTU., Belgaum Karnataka University, Summer 2002)

17. Current drawn by a pure capacitor of  $20 \mu\text{F}$  is  $1.382 \text{ A}$  from  $220 \text{ V AC}$  supply. What is the supply frequency? (VTU., Belgaum Karnataka University, Summer 2003)

18. Find the equivalent capacitance between the points A and B of the network shown in fig. 1.

(VTU., Belgaum Karnataka University, Summer 2003)

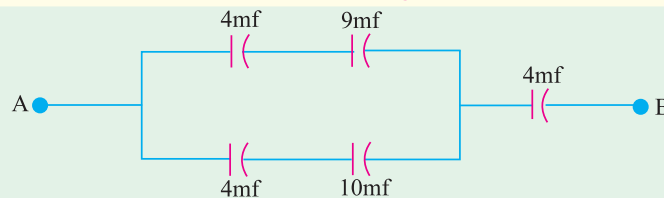


Fig. 5.52

19. Three capacitors of  $1, 2$  and  $3 \text{ micro farads}$  are connected in series across a supply voltage of  $100 \text{ V}$ . Find the equivalent capacitance of the combination and energy stored in each capacitor. (Mumbai University 2003) (VTU. Belgaum Karnataka University, Winter 2003)

20. Consider a parallel plate capacitor, the space between which is filled by two dielectric of thickness  $d_1$  and  $d_2$  with relative permittivities  $\epsilon_1$  and  $\epsilon_2$  respectively. Derive an expression for the capacitance between the plates. (VTU. Belgaum Karnataka University, Winter 2004)

21. A capacitor consists of two plates of area  $0.16\text{m}^2$  spaced 6mm apart. This space is filled with a layer of 1mm thick paper of relative permittivity 2, and remaining space with glass of relative permittivity 5. A dc voltage of 10kV is applied between the plates. Determine the electric field strength in each dielectric. *(V.T.U. Belgaum Karnataka University, Winter2004)*
22. In a give R-L circuit,  $R = 35\Omega$  and  $L = 0.1\text{H}$ . Find (i) current through the circuit (ii) power factor if a 50 Hz frequency, voltage  $V = 220\angle 30^\circ$  is applied across the circuit. *(RGPV Bhopal 2001)*
23. Three voltage represented by  $e_1 = 20 \sin \omega t$ ,  $e_2 = 30 \sin (\omega t + 45^\circ)$  and  $e_3 = \sin (\omega t + 30^\circ)$  are connected in series and then connected to a load of impedance  $(2 + j 3) \Omega$  Find the resultant current and power factor of the circuit. Draw the phasor diagram. *(Mumbai University, 2002) (RGPV Bhopal 2001)*

### OBJECTIVE TESTS – 5

1. A capacitor consists of two  
 (a) insulation separated by a dielectric  
 (b) conductors separated by an insulator  
 (c) ceramic plates and one mica disc  
 (d) silver-coated insulators
2. The capacitance of a capacitor is NOT influenced by  
 (a) plate thickness  
 (b) plate area  
 (c) plate separation  
 (d) nature of the dielectric
3. A capacitor that stores a charge of 0.5 C at 10 volts has a capacitance of .....farad.  
 (a) 5 (b) 20  
 (c) 10 (d) 0.05
4. If dielectric slab of thickness 5 mm and  $\epsilon_r = 6$  is inserted between the plates of an air capacitor with plate separation of 8 mm, its capacitance is  
 (a) decreased (b) almost doubled  
 (c) almost halved (d) unaffected
5. For the circuit shown in the given figure, the current through  $L$  and the voltage across  $C_2$  are respectively

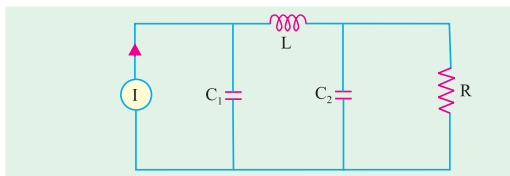


Fig. 5.53

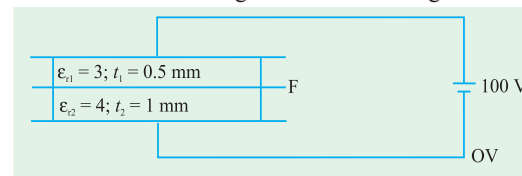


Fig. 5.54

- (a) 52 V (b) 60 V  
 (c) 67 V (d) 33 V  
*(GATE 2003)*

### ANSWERS

1. b 2. a 3. d 4. b

# CHAPTER 6

## Learning Objectives

- Laws of Magnetic Force
- Magnetic Field Strength ( $H$ )
- Magnetic Potential
- Flux per Unit Pole
- Flux Density ( $B$ )
- Absolute Permeability ( $\mu$ ) and Relative Permeability ( $\mu_r$ )
- Intensity of Magnetisation ( $I$ )
- Susceptibility ( $K$ )
- Relation Between  $B$ ,  $H$ ,  $I$  and  $K$
- Boundary Conditions
- Weber and Ewing's Molecular Theory
- Curie Point. Force on a Current-carrying Conductor Lying in a Magnetic Field
- Ampere's Work Law or Ampere's Circuital Law
- Biot-Savart Law
- Savart Law
- Force Between two Parallel Conductors
- Magnitude of Mutual Force
- Definition of Ampere
- Magnetic Circuit
- Definitions
- Composite Series Magnetic Circuit
- How to Find Ampere-turns ?
- Comparison Between Magnetic and Electric Circuits
- Parallel Magnetic Circuits
- Series-Parallel Magnetic Circuits
- Leakage Flux and Hopkinson's Leakage Coefficient
- Magnetisation Curves
- Magnetisation curves by Ballistic Galvanometer
- Magnetisation Curves by Fluxmeter

## MAGNETISM AND ELECTRO-MAGNETISM



Designing high speed magnetic levitation trains is one of the many applications of electromagnetism. Electromagnetism defines the relationship between magnetism and electricity

### 6.1. Absolute and Relative Permeabilities of a Medium

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities :

(i) absolute permeability ( $\mu$ ) and (ii) relative permeability ( $\mu_r$ ).

For measuring relative permeability, vacuum or free space is chosen as the reference medium. It is allotted an absolute permeability of  $\mu_0 = 4\pi \times 10^{-7}$  henry/metre. Obviously, relative permeability of vacuum with reference to itself is unity. Hence, for free space,

$$\text{absolute permeability} \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\text{relative permeability} \quad \mu_r = 1.$$

Now, take any medium other than vacuum. If its relative permeability, as compared to vacuum is  $\mu_r$ , then its absolute permeability is  $\mu = \mu_0 \mu_r$  H/m.

### 6.2. Laws of Magnetic Force

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two *isolated* point poles. It may be noted here that, in view of the fact that magnetic poles always exist in pairs, it is impossible, in practice, to get an isolated pole. The concept of an isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be point poles for all practical purposes (Fig. 6.1). By using a torsion balance, he found that the force between two magnetic poles placed in a medium is

- (i) directly proportional to their pole strengths
- (ii) inversely proportional to the square of the distance between them and
- (iii) inversely proportional to the absolute permeability of the surrounding medium.



Fig. 6.1

Fig. 6.2

For example, if  $m_1$  and  $m_2$  represent the magnetic strength of the two poles (its unit as yet being undefined),  $r$  the distance between them (Fig. 6.2) and  $\mu$  the absolute permeability of the surrounding medium, then the force  $F$  is given by

$$F \propto \frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad F = k \frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad \vec{F} = \frac{k m_1 m_2}{\mu r^2} \hat{r} \quad \text{in vector form}$$

where  $\hat{r}$  is a unit vector to indicate direction of  $r$ .

$$\text{or} \quad \vec{F} = k \frac{m_1 m_2}{r^3} \vec{r} \quad \text{where } \vec{F} \text{ and } \vec{r} \text{ are vectors}$$

In the S.I. system of units, the value of the constant  $k$  is  $= 1/4\pi$ .

$$F = \frac{m_1 m_2}{4\pi \mu r^2} \text{ N} \quad \text{or} \quad F = \frac{m_1 m_2}{4\pi \mu_0 \mu_r r^2} \text{ N} \quad \text{— in a medium}$$

$$\text{In vector form,} \quad \vec{F} = \frac{m_1 m_2}{4\pi \mu r^3} \vec{r} = \frac{m_1 m_2}{4\pi \mu_0 r^2} \text{ N}$$

If, in the above equation,

$$m_1 = m_2 = m \text{ (say)} ; r = 1 \text{ metre} ; F = \frac{1}{4\pi \mu_0} \text{ N}$$

Then  $m^2 = 1$  or  $m = \pm 1$  weber\*

Hence, a unit magnetic pole may be defined as *that pole which when placed in vacuum at a distance of one metre from a similar and equal pole repels it with a force of  $1/4\pi \mu_0$  newtons.*\*\*

### 6.3. Magnetic Field Strength (H)

Magnetic field strength at any point within a magnetic field is numerically equally to the force experienced by a  $N$ -pole of one weber placed at that point. Hence, unit of  $H$  is N/Wb.

Suppose, it is required to find the field intensity at a point  $A$  distant  $r$  metres from a pole of  $m$  webers. Imagine a similar pole of one weber placed at point  $A$ . The force experienced by this pole is

$$F = \frac{m \times 1}{4\pi\mu_0 r^2} \text{ N} \quad \therefore H = \frac{m}{4\pi\mu_0 r^3} \text{ N/Wb (or A/m)}^{***} \text{ or oersted.}$$

Also, if a pole of  $m$  Wb is placed in a uniform field of strength  $H$  N/Wb, then force experienced by the pole is  $= mH$  newtons.

It should be noted that field strength is a vector quantity having both magnitude and direction

$$\therefore \vec{H} = \frac{m}{4\pi\mu_0 r^2} \hat{r} = \frac{m}{4\pi\mu_0 r^3} r$$

It would be helpful to remember that following terms are sometimes interchangeably used with field intensity : Magnetising force, strength of field, magnetic intensity and intensity of magnetic field.

### 6.4. Magnetic Potential

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a  $N$ -pole of one weber from infinity to that point against the force of the magnetic field. It is given by

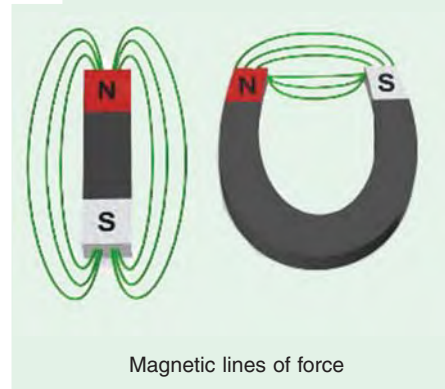
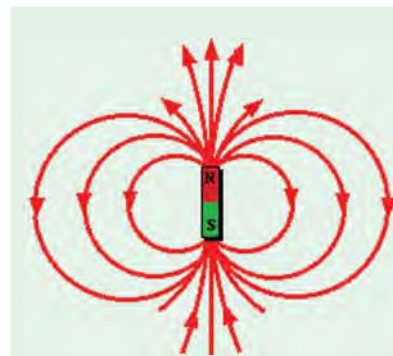
$$M = \frac{m}{4\pi\mu_0 r} \text{ J/Wb} \quad \dots(\text{Art. 4.13})$$

It is a scalar quantity.

### 6.5. Flux per Unit Pole

A unit  $N$ -pole is supposed to radiate out a flux of one weber. Its symbol is  $\Phi$ . Therefore, the flux coming out of a  $N$ -pole of  $m$  weber is given by

$$\Phi = m \text{ Wb}$$



\* To commemorate the memory of German physicist Wilhelm Edward Weber (1804-1891).

\*\* A unit magnetic pole is also defined as that magnetic pole which when placed at a distance of one metre from a very long straight conductor carrying a current of one ampere experiences a force of  $1/2\pi$  newtons (Art. 6.18).

\*\*\* It should be noted that N/Wb is the same thing as ampere/metre (A/m) or just A/m cause 'turn' has no units

### 6.6. Flux Density (B)

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter  $B$  and is measured in weber/meter<sup>2</sup>. It is a Vector Quantity.

If  $\Phi$  Wb is the total magnetic flux passing normally through an area of  $A$  m<sup>2</sup>, then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

**Note.** Let us find an expression for the flux density at a point distant  $r$  metres from a unit  $N$ -pole (*i.e.* a pole of strength 1 Wb.) Imagine a sphere of radius  $r$  metres drawn round the unit pole. The flux of 1 Wb radiated out by the unit pole falls normally on a surface of  $4\pi r^2$  m<sup>2</sup>. Hence

$$B = \frac{\Phi}{A} = \frac{1}{4\pi r^2} \text{ Wb/m}^2$$

### 6.7. Absolute Permeability ( $\mu$ ) and Relative Permeability ( $\mu_r$ )

In Fig. 6.3 is shown a bar of a magnetic material, say, iron placed in a uniform field of strength  $H$  N/Wb. Suppose, a flux density of  $B$  Wb/m<sup>2</sup> is developed in the rod.

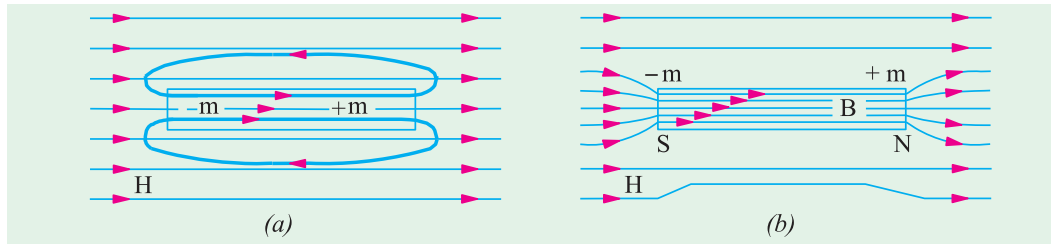


Fig. 6.3

Then, the absolute permeability of the material of the rod is defined as

$$\mu = B/H \text{ henry/metre or } B = \mu H = \mu_0 \mu_r H \text{ Wb/m}^2 \quad \dots (i)$$

When  $H$  is established in air (or vacuum), then corresponding flux density developed in air is

$$B_0 = \mu_0 H$$

Now, when iron rod is placed in the field, it gets magnetised by induction. If induced pole strength in the rod is  $m$  Wb, then a flux of  $m$  Wb emanates from its  $N$ -pole, re-enters its  $S$ -pole and continues from  $S$  to  $N$ -pole within the magnet. If  $A$  is the face or pole area of the magnetised iron bar, the *induction* flux density in the rod is

$$B_i = m/A \text{ Wb/m}^2$$

Hence, total flux density in the iron rod consists of two parts [Fig. 6.3 (b)].

(i)  $B_0$  –flux density in air even when rod is not present

(ii)  $B_i$  –induction flux density in the rod

$$B = B_0 + B_i = \mu_0 H + m/A$$

Eq. (i) above may be written as  $B = \mu_r \cdot \mu_0 H = \mu_r B_0$

$$\therefore \mu_r = \frac{B}{B_0} = \frac{B(\text{material})}{B_0(\text{vacuum})} \quad \dots \text{for same } H$$

Hence, relative permeability of a material is equal to **the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetising force.**

### 6.8. Intensity of Magnetisation (I)

It may be defined as the induced pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let  $m$  = pole strength induced in the bar in Wb



$A$  = face or pole area of the bar in  $\text{m}^2$

Then

$$I = m/A \text{ Wb/m}^2$$

Hence, it is seen that intensity of magnetisation of a substance may be defined as **the flux density produced in it due to its own induced magnetism.**

If  $l$  is the magnetic length of the bar, then the product ( $m \times l$ ) is known as its magnetic moment  $M$ .

$$\therefore I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M}{V} = \text{magnetic moment/volume}$$

### 6.9. Susceptibility (K)

Susceptibility is defined as **the ratio of intensity of magnetisation  $I$  to the magnetising force  $H$ .**

$$\therefore K = I/H \text{ henry/metre.}$$

### 6.10. Relation Between B, H, I and K

It is obvious from the above discussion in Art. 6.7 that flux density  $B$  in a material is given by

$$B = B_0 + m/A = B_0 + I \quad \therefore B = \mu_0 H + I$$

$$\text{Now absolute permeability is } \mu = \frac{B}{H} = \frac{\mu_0 H + I}{H} = \mu_0 + \frac{I}{H} \quad \therefore \mu = \mu_0 + K$$

$$\text{Also } \mu = \mu_0 \mu_r \quad \therefore \mu_0 \mu_r = \mu_0 + K \text{ or } \mu_r = 1 + K/\mu_0$$

For ferro-magnetic and para-magnetic substances,  $K$  is positive and for diamagnetic substances, it is negative. For ferro-magnetic substance (like iron, nickel, cobalt and alloys like nickel-iron and cobalt-iron)  $\mu_r$  is much greater than unity whereas for para-magnetic substances (like aluminium),  $\mu_r$  is slightly greater than unity. For diamagnetic materials (bismuth)  $\mu_r < 1$ .

**Example 6.1.** The magnetic susceptibility of oxygen gas at  $20^\circ\text{C}$  is  $167 \times 10^{-11} \text{ H/m}$ . Calculate its absolute and relative permeabilities.

**Solution.**

$$\mu_r = 1 + \frac{K}{\mu_0} = 1 + \frac{167 \times 10^{-11}}{4\pi \times 10^{-7}} = \mathbf{1.00133}$$

$$\text{Now, absolute permeability } \mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1.00133 = \mathbf{12.59 \times 10^{-7} \text{ H/m}}$$

### 6.11. Boundary Conditions

The case of boundary conditions between two materials of different permeabilities is similar to that discussed in Art. 4.19.

As before, the two boundary conditions are

(i) the normal component of flux density is continuous across boundary.

$$B_{1n} = B_{2n} \quad \dots(i)$$

(ii) the tangential component of  $H$  is continuous across boundary

$$H_{1t} = H_{2t}$$

As proved in Art. 4.19, in a similar way, it can be shown

$$\text{that } \frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This is called the law of magnetic flux refraction.

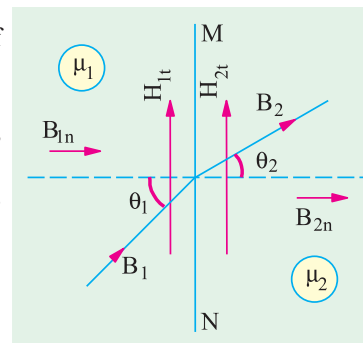


Fig. 6.4

### 6.12. Weber and Ewing's Molecular Theory



Fig. 6.5

This theory was first advanced by Weber in 1852 and was, later on, further developed by Ewing in 1890. The basic assumption of this theory is **that molecules of all substances are inherently magnets in themselves, each having a N and S pole.** In an unmagnetised state, it is supposed that these small molecular

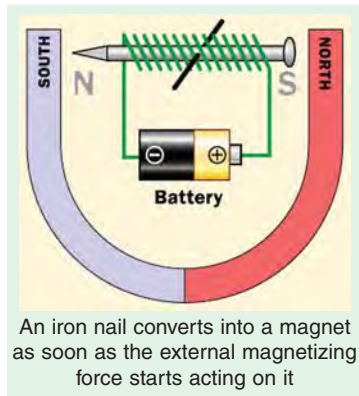
magnets lie in all sorts of haphazard manner forming more or less closed loops (Fig. 6.5). According to the laws of attraction and repulsion, these closed magnetic circuits are satisfied internally, hence there is no resultant external magnetism exhibited by the iron bar. But when such an iron bar is placed in a magnetic field or under the influence of a magnetising force, then these molecular magnets start turning round their axes and orientate themselves more or less along straight lines parallel to the direction of the magnetising force. This linear arrangement of the molecular magnets results in *N* polarity at one end of the bar and *S* polarity at the other (Fig. 6.6). As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment, thus accounting for the magnetic saturation. On this theory, the hysteresis loss is supposed to be due to molecular friction of these turning magnets.



Fig. 6.6



Molecular magnets which are randomly arranged in the unmagnetised state, get oriented under the influence of an external magnetizing force



An iron nail converts into a magnet as soon as the external magnetizing force starts acting on it

Because of the limited knowledge of molecular structure available at the time of Weber, it was not possible to explain firstly, as to why the molecules themselves are magnets and secondly, why it is impossible to magnetise certain substances like wood etc.

The first objection was explained by Ampere who maintained that orbital movement of the electrons round the atom of a molecule constituted a flow of current which, due to its associated magnetic effect, made the molecule a magnet.

Later on, it became difficult to explain the phenomenon of diamagnetism (shown by materials like water, quartz, silver and copper etc.) erratic behaviour of ferromagnetic (intensely magnetisable) substances like iron, steel, cobalt, nickel and some of their alloys etc. and the paramagnetic (weakly magnetisable)

substances like oxygen and aluminium etc. Moreover, it was asked : if molecules of all substances are magnets, then why does not wood or air etc. become magnetised ?

All this has been explained satisfactorily by the atom-domain theory which has superseded the molecular theory. It is beyond the scope of this book to go into the details of this theory. The interested reader is advised to refer to some standard book on magnetism. However, it may just be mentioned that this theory takes into account not only the planetary motion of an electron but its rotation about its own axis as well. This latter rotation is called 'electron spin'. The gyroscopic behaviour of an electron gives rise to a magnetic moment which may be either positive or negative. A substance is ferromagnetic or diamagnetic accordingly as there is an excess of unbalanced positive spins or negative spins. Substances like wood or air are non-magnetisable because in their case, the positive and negative electron spins are equal, hence they cancel each other out.

### 6.13. Curie Point

As a magnetic material is heated, its molecules vibrate more violently. As a consequence, individual molecular magnets get out of alignment as the temperature is increased, thereby reducing the magnetic strength of the magnetised substance. Fig. 6.7 shows the approximate decrease of magnetic strength with rise in temperature. Obviously, it is possible to partially or even completely destroy the magnetic properties of a material by heating. The temperature at which the vibrations of the molecular magnets become so random

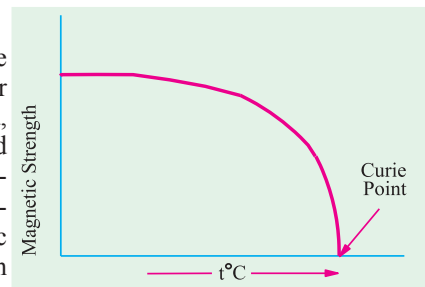


Fig. 6.7

and out of alignment as to reduce the magnetic strength to zero is called Curie point. More accurately, it is that critical temperature above which ferromagnetic material becomes paramagnetic.

## ELECTROMAGNETISM

### 6.14 . Force on a Current-carrying Conductor Lying in a Magnetic Field

It is found that whenever a current-carrying conductor is placed in magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. In Fig. 6.8 is shown a conductor  $XY$  lying at right angles to the uniform horizontal field of flux density  $B$   $\text{Wb/m}^2$  produced by two solenoids  $A$  and  $B$ . If  $l$  is the length of the conductor lying within this field and  $I$  ampere the current carried by it, then the magnitude of the force experienced by it is

$$\vec{F} = BIl = \mu_0 \mu_r HIl \text{ newton}$$

Using vector notation  $\vec{F} = I \vec{l} \times \vec{B}$  and  $F = IlB \sin \theta$  where  $\theta$  is the angle between  $\vec{l}$  and  $\vec{B}$  which is  $90^\circ$  in the present case

or

$$F = IlB \sin 90^\circ = IlB \text{ newtons} \quad (\because \sin 90^\circ = 1)$$

The direction of this force may be easily found by Fleming's left-hand rule.

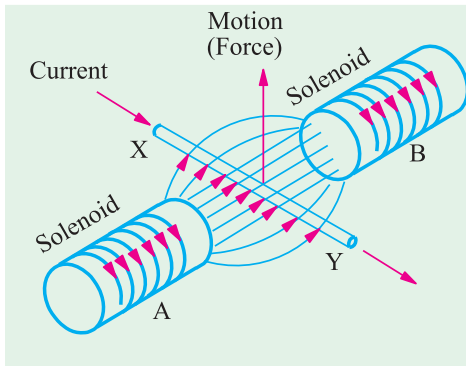


Fig. 6.8

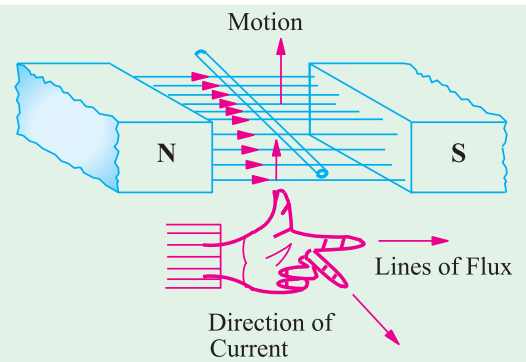


Fig. 6.9

Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field and the second finger that of the current, then thumb gives the direction of the motion. It is illustrated in Fig. 6.9.

Fig. 6.10 shows another method of finding the direction of force acting on a current carrying conductor. It is known as Flat Left Hand rule. The force acts in the direction of the thumb obviously, the direction of motor of the conductor is the same as that of the force.

It should be noted that no force is exerted on a conductor when it lies parallel to the magnetic field. In general, if the conductor lies at an angle  $\theta$  with the direction of the field, then  $B$  can be resolved into two components,  $B \cos \theta$  parallel to and  $B \sin \theta$  perpendicular to the conductor. The former produces no effect whereas the latter is responsible for the motion observed. In that case,

$F = BIl \sin \theta$  newton, which has been expressed as cross product of vector above.\*

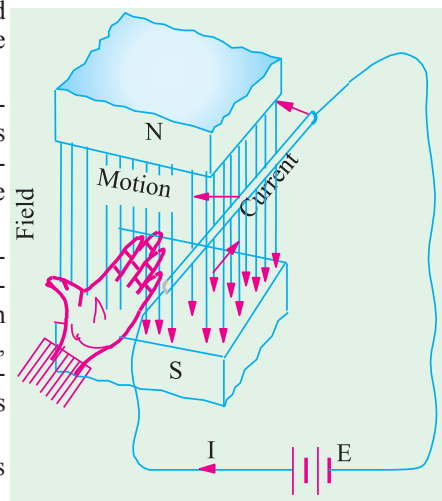


Fig. 6.10

\* It is simpler to find direction of Force (Motion) through cross product of given vectors  $I \vec{l}$  and  $\vec{B}$ .

### 6.15. Ampere's Work Law or Ampere's Circuital Law

The law states that m.m.f.\* (magnetomotive force corresponding to e.m.f. *i.e.* electromotive force of electric field) around a closed path is equal to the current enclosed by the path. Mathematically,  $\int \vec{H} \cdot d\vec{s} = I$  amperes where  $\vec{H}$  is the vector representing magnetic field strength in dot product with vector  $d\vec{s}$  of the enclosing path  $S$  around current  $I$  ampere and that is why line integral ( $\oint$ ) of dot product  $\vec{H} \cdot d\vec{s}$  is taken.

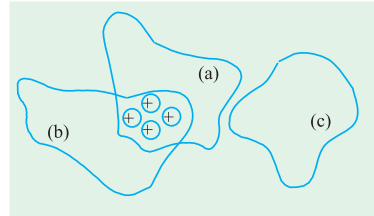


Fig. 6.11

Work law is very comprehensive and is applicable to all magnetic fields whatever the shape of enclosing path *e.g.* (a) and (b) in Fig. 6.11. Since path c does not enclose the conductor, the m.m.f. around it is zero.

The above work Law is used for obtaining the value of the magnetomotive force around simple idealized circuits like (i) a long straight current-carrying conductor and (ii) a long solenoid.

#### (i) Magnetomotive Force around a Long Straight Conductor

In Fig. 6.12 is shown a straight conductor which is assumed to extend to infinity in either direction. Let it carry a current of  $I$  amperes upwards. The magnetic field consists of circular lines of force having their plane perpendicular to the conductor and their centres at the centre of the conductor.

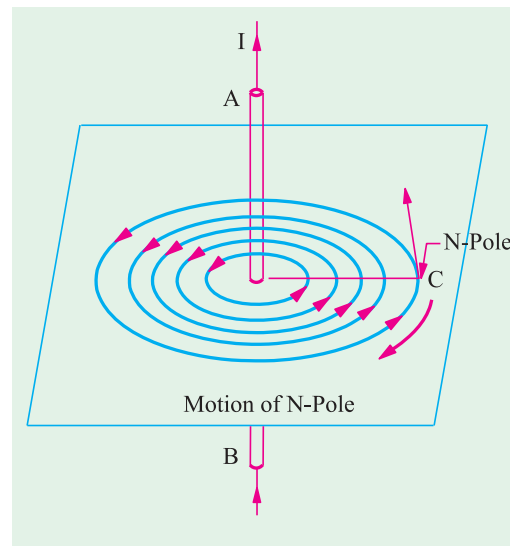


Fig. 6.12

Suppose that the field strength at point  $C$  distant  $r$  metres from the centre of the conductor is  $H$ . Then, it means that if a unit  $N$ -pole is placed at  $C$ , it will experience a force of  $H$  newtons. The direction of this force would be tangential to the circular line of force passing through  $C$ . If this unit  $N$ -pole is moved once round the conductor *against* this force, then work done *i.e.*

$$\text{m.m.f.} = \text{force} \times \text{distance} = I$$

$$\text{i.e. } I = H \times 2\pi r \text{ joules} = \text{Amperes}$$

$$\text{or } H = \frac{I}{2\pi r}$$

$$= \int \vec{H} \cdot d\vec{s} \text{ Joules} = \text{Amperes} = I$$

Obviously, if there are  $N$  conductors (as shown in Fig. 6.13), then

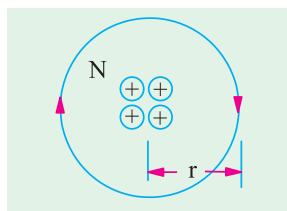


Fig. 6.13

and

$$H = \frac{NI}{2\pi r} \text{ A/m or Oersted}$$

$$B = \mu_0 \frac{NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \quad \dots \text{in air}$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \quad \dots \text{in a medium}$$

\*\* M.M.F. is not a force, but is the work done.

**(ii) Magnetic Field Strength of a Long Solenoid**

Let the Magnetic Field Strength along the axis of the solenoid be  $H$ . Let us assume that

(i) the value of  $H$  remains constant throughout the length  $l$  of the solenoid and

(ii) the volume of  $H$  outside the solenoid is negligible.

Suppose, a unit  $N$ -pole is placed at point  $A$  outside the solenoid and is taken once round the completed path (shown dotted in Fig. 6.14) in a direction opposite to that of  $H$ . Remembering that the force of  $H$  newtons acts on the  $N$ -pole only over the length  $l$  (it being negligible elsewhere), the work done in one round is

$$= H \times l \text{ joules} = \text{Amperes}$$

The 'ampere-turns' linked with this path are  $NI$  where  $N$  is the number of turns of the solenoid and  $I$  the current in amperes passing through it. According to Work Law

$$H \times l = NI \quad \text{or} \quad H = \frac{NI}{l} \text{ A/m or Oersted}$$

$$\text{Also } B = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \quad \text{or tesla ...in air}$$

$$= \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \quad \text{or tesla ...in a medium}$$

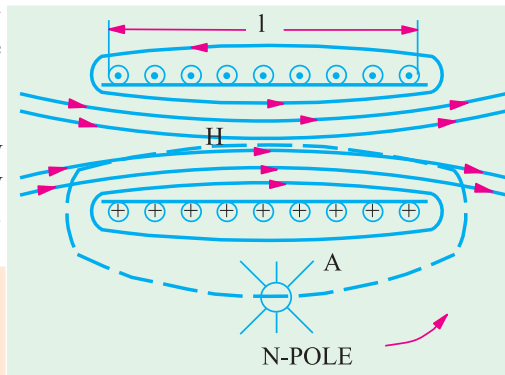
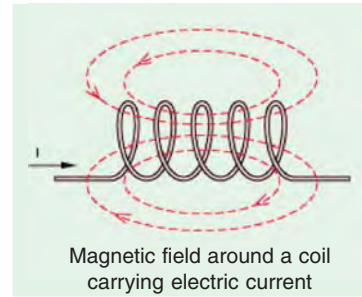


Fig. 6.14

**6.16. Biot-Savart Law\***

The expression for the magnetic field strength  $dH$  produced at point  $P$  by a vanishingly small length  $dl$  of a conductor carrying a current of  $I$  amperes (Fig. 6.15) is given by

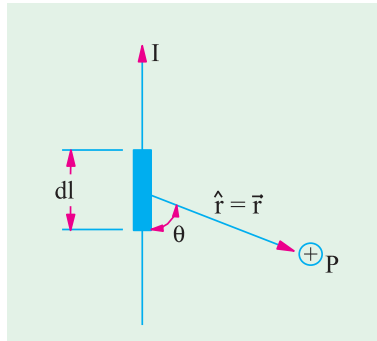


Fig. 6.15

$$dH = \frac{Idl \sin \theta}{4\pi r^2} \text{ A/m}$$

$$\text{or } d\vec{H} = (Id\vec{l} \times \hat{r}) / 4\pi r^2 \text{ in vector form}$$

The direction of  $d\vec{H}$  is perpendicular to the plane containing both ' $d\vec{l}$ ' and ' $\vec{r}$ ' i.e. entering.

$$\text{or } dB_0 = \frac{\mu_0 Idl}{4\pi r^2} \sin \theta \text{ Wb/m}^2$$

$$\text{and } d\vec{B}_0 = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2} \text{ in vector form}$$

**6.17. Applications of Biot-Savart Law****(i) Magnetic Field Strength Due to a Finite Length of Wire Carrying Current**

Consider a straight wire of length  $l$  carrying a steady current  $I$ . We wish to find magnetic field strength ( $H$ ) at a point  $P$  which is at a distance  $r$  from the wire as shown in Fig. 6.16.

\* After the French mathematician and physicist Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841) a well-known French physicist.

The magnetic field strength  $d\vec{H}$  due to a small element  $dl$  of the wire shown is

$$d\vec{H} = \frac{I d\vec{l} \times \hat{s}}{4\pi s^2} \text{ (By Biot-Savart Law)}$$

or  $d\vec{H} = \frac{Idl \sin \theta}{4\pi \times s^2} \hat{u}$  (where  $\hat{u}$  is unit vector perpendicular to plane containing  $d\vec{l}$  and  $\hat{s}$  and into the plane.)

or  $d\vec{H} = \frac{Idl \cos \phi}{4\pi s^2} \hat{u}$  ...[ $\because \theta$  and  $\phi$  are complementary angles]

The magnetic field strength due to entire length  $l$  :

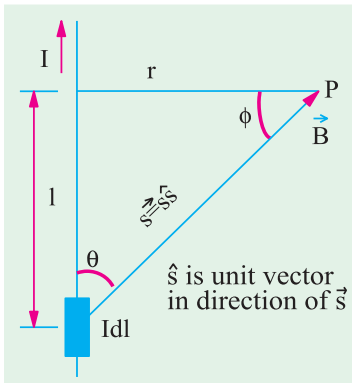


Fig. 6.16

$$\begin{aligned} \vec{H} &= \frac{I}{4\pi} \left[ \int_0^l \frac{\cos \phi dl}{s^2} \right] \hat{u} \\ &= \frac{I}{4\pi} \left[ \int_0^l \frac{r/s}{s^2} dl \right] \hat{u} \quad \left( \cos \phi = \frac{r}{s} \text{ in Fig. 6.16} \right) \\ &= \frac{Ir}{4\pi} \left[ \int_0^l \frac{dl}{s^3} \right] \hat{u} = \frac{Ir}{4\pi} \left[ \int_0^l \frac{dl}{(r^2 + l^2)^{3/2}} \right] \hat{u} \end{aligned}$$

( $\because r$  is constant) ;  $s = \sqrt{r^2 + l^2}$  in Fig. 6.16

$$= \frac{Ir}{4\pi r^3} \left[ \int_0^l \frac{dl}{[1 + (r/l)^2]^{3/2}} \right] \hat{u} \text{ (Taking } r^3 \text{ out from denominator)}$$

To evaluate the integral most simply, make the following substitution

$$\frac{l}{r} = \tan \phi \text{ in Fig. 6.16}$$

$\therefore l = r \tan \phi \therefore dl = r \sec^2 \phi d\phi$  and  $1 + (r/l)^2 = 1 + \tan^2 \phi = \sec^2 \phi$  and limits get transformed i.e. become 0 to  $\phi$ .

$$\begin{aligned} \vec{H} &= \frac{Ir}{4r^3} \int_0^\phi \frac{r \sec^2 d}{\sec^2} \hat{u} = \frac{Ir^2}{4r^3} \int_0^\phi \cos d \hat{u} = \frac{1}{4r} \sin \phi \hat{u} \\ &= \frac{I}{4\pi r} \sin \phi \hat{u} \end{aligned}$$

**N.B.** For wire of infinite length extending it at both ends i.e.  $-\infty$  to  $+\infty$  the limits of integration would be  $-\frac{\pi}{2}$  to  $+\frac{\pi}{2}$ , giving  $\vec{H} = \frac{I}{4\pi r} \times 2\hat{u}$  or  $\vec{H} = \frac{I}{2\pi r} \hat{u}$ .

**(ii) Magnetic Field Strength along the Axis of a Square Coil**

This is similar to (i) above except that there are four conductors each of length say,  $2a$  metres and carrying a current of  $I$  amperes as shown in Fig. 6.17. The Magnetic Field Strengths at the axial point  $P$  due to the opposite sides  $ab$  and  $cd$  are  $H_{ab}$  and  $H_{cd}$  directed at right angles to the planes containing  $P$  and  $ab$  and  $P$  and  $cd$  respectively. Now,  $H_{ab}$  and  $H_{cd}$  are numerically equal,

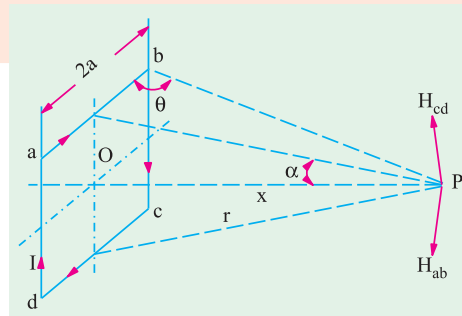


Fig. 6.17

hence their components at right angles to the axis of the coil will cancel out, but the axial components will add together. Similarly, the other two sides  $da$  and  $bc$  will also give a resultant axial component only.

As seen from Eq. (ii) above,

$$H_{ab} = \frac{I}{4\pi r} [\cos \theta - \cos (180^\circ - \theta)] = \frac{I \cdot 2 \cos \theta}{4\pi r} = \frac{I \cos \theta}{2\pi r}$$

Now 
$$r = \sqrt{a^2 + x^2} \quad \therefore H_{ab} = \frac{I \cdot \cos \theta}{2\pi \sqrt{a^2 + x^2}}$$

Its axial components is 
$$H_{ab'} = H_{ab} \cdot \sin \alpha = \frac{I \cos \theta}{2\pi \sqrt{a^2 + x^2}} \cdot \sin \alpha$$

All the four sides of the rectangular coil will contribute an equal amount to the resultant magnetic field at  $P$ . Hence, resultant magnetising force at  $P$  is

$$H = 4 \times \frac{I \cos \theta}{2\pi \sqrt{a^2 + x^2}} \cdot \sin \alpha,$$

Now 
$$\cos \theta = \frac{a}{\sqrt{(2a^2 + x^2)}} \text{ and } \sin \alpha = \frac{a}{\sqrt{a^2 + x^2}}$$

$$\therefore H = \frac{2a^2 \cdot I}{\pi (a^2 + x^2) \cdot \sqrt{x^2 + 2a^2}} \text{ AT/m.}$$

In case, value of  $H$  is required at the centre  $O$  of the coil, then putting  $x = 0$  in the above expression,

we get

$$H = \frac{2a^2 \cdot I}{\pi a^2 \cdot \sqrt{2} \cdot a} = \frac{\sqrt{2} \cdot I}{\pi a} \text{ AT/m}$$

**Note.** The last result can be found directly as under. As seen from Fig. 6.18, the field at point  $O$  due to any side is, as given by Eq. (i)

$$= \frac{I}{4\pi a} \int_{\pi/4}^{-\pi/4} \sin \theta \cdot d\theta = \frac{I}{4\pi a} [-\cos \theta]_{45^\circ}^{-45^\circ} = \frac{I}{4\pi a} \cdot 2 \cos 45^\circ = \frac{I}{4\pi a} \cdot \frac{2}{\sqrt{2}}$$

Resultant magnetising force due to all sides is

$$H = 4 \times \frac{1}{4\pi a} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{2} I}{\pi a} \text{ AT/m} \quad \dots \text{as found above}$$

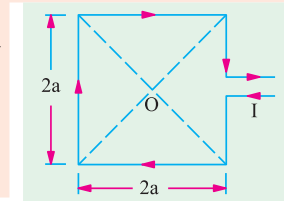


Fig. 6.18

**(iii) Magnetising Force on the Axis of a Circular Coil**

In Fig. 6.19 is shown a circular one-turn coil carrying a current of  $I$  amperes. The magnetising force at the axial point  $P$  due to a small element ' $dl$ ' as given by Laplace's Law is

$$\left| \vec{dH} \right| = \frac{I dl}{4\pi (r^2 + x^2)}$$

The direction of  $dH$  is at right angles to the line  $AP$  joining point  $P$  to the element ' $dl$ '. Now,  $dH$  can be resolved into two components :

- (a) the axial component  $dH' = dH \sin \theta$
- (b) the vertical component  $dH'' = dH \cos \theta$

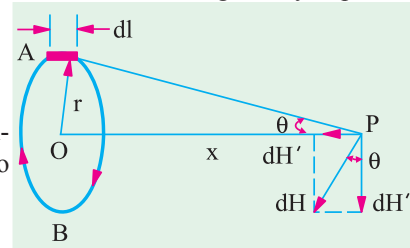


Fig. 6.19

Now, the vertical component  $dH \cos \theta$  will be cancelled by an equal and opposite vertical component of  $dH$  due to element ' $dl$ ' at point  $B$ . The same applies to all other diametrically opposite pairs of  $dl$ 's taken around the coil. Hence, the resultant magnetising force at  $P$  will be equal to the sum of all the axial components.

$$\begin{aligned} \therefore H &= \sum dH' = \sum dH \sin \theta \int dl = \sum \frac{I \cdot dl \cdot r}{4\pi (r^2 + x^2)^{3/2}} \int dl \quad \left( \sin \theta = \frac{r}{\sqrt{r^2 + x^2}} \right) \\ &= \frac{I \cdot r}{4\pi (r^2 + x^2)^{3/2}} \int_0^{2\pi r} dl = \frac{I \cdot r \cdot 2\pi r}{4\pi (r^2 + x^2)^{3/2}} = \frac{I r^2}{2 (r^2 + x^2)^{3/2}} \\ &= \frac{I}{2r} \cdot \frac{r^3}{(r^2 + x^2)^{3/2}} \quad \therefore H = \frac{I \sin^3 \theta}{2r} \text{ AT/m} \end{aligned}$$

$$\text{or } H = \frac{NI}{2r} \sin^3 \theta \text{ AT/m} \quad \text{--for an } N\text{-turn coil} \quad \dots (iii)$$

In case the value of  $H$  is required at the centre  $O$  of the coil, then putting  $\theta = 90^\circ$  and  $\sin \theta = 1$  in the above expression, we get

$$H = \frac{I}{2r} \text{ --for single-turn coil} \quad \text{or} \quad H = \frac{NI}{2r} \text{ --for } N\text{-turn coil}$$

**Note.** The magnetising force  $H$  at the centre of a circular coil can be directly found as follows :

With reference to the coil shown in the Fig. 6.20, the magnetising force  $dH$  produced at  $O$  due to the small element  $dl$  (as given by Laplace's law) is

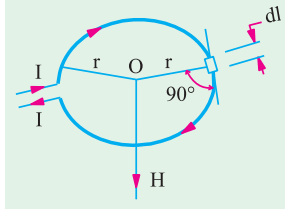


Fig. 6.20

$$dH = \frac{I \cdot dl \sin \theta}{4\pi r^2} = \frac{I \cdot dl}{4\pi r^2} \quad (\because \sin \theta = \sin 90^\circ = 1)$$

$$\therefore \sum dH = \sum \frac{I \cdot dl}{4\pi r^2} = \frac{I}{4\pi r^2} \sum dl \quad \text{or} \quad H = \frac{I \cdot 2\pi r}{4\pi r^2} = \frac{I}{2r}$$

$$\therefore H = \frac{I}{2r} \text{ AT/m --for 1-turn coil ; } \frac{NI}{2r} \text{ AT/m --for } N\text{-turn coil.}$$

#### (iv) Magnetising Force on the Axis of a Short Solenoid

Let a short solenoid having a length of  $l$  and radius of turns  $r$  be uniformly wound with  $N$  turns each carrying a current of  $I$  as shown in Fig. 6.21. The winding density *i.e.* number of turns per unit length of the solenoid is  $N/l$ . Hence, in a small element of length  $dx$ , there will be  $N \cdot dx/l$  turns. Obviously, a very short element of length of the solenoid can be regarded as a concentrated coil of very short axial length and having  $N \cdot dx/l$  turns. Let  $dH$  be the magnetising force contributed by

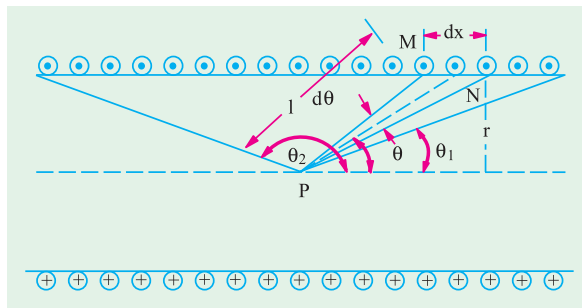


Fig. 6.21

the element  $dx$  at any axial point  $P$ . Then, substituting  $dH$  for  $H$  and  $N \cdot dx/l$  for  $N$  in Eq. (iii), we get

$$dH = \frac{N \cdot dx}{l} \cdot \frac{I}{2r} \cdot \sin^3 \theta$$

$$\text{Now} \quad dx \cdot \sin \theta = r \cdot d\theta / \sin \theta^* \quad \therefore dx = r \cdot d\theta / \sin^2 \theta$$

Substituting this value of  $dx$  in the above equation, we get

$$dH = \frac{NI}{2l} \sin \theta \cdot d\theta$$

Total value of the magnetising force at  $P$  due to the whole length of the solenoid may be found by integrating the above expression between proper limits.

\* Because  $l \sin \theta = r$  :  $l = r / \sin \theta$ . Now,  $M/N = l \cdot d\theta = r \cdot d\theta / \sin \theta$ . Also,  $MN = dx \cdot \sin \theta$ , hence  $dx = r \cdot d\theta / \sin^2 \theta$ .



$$\begin{aligned} \therefore H &= \frac{NI}{2l} \int_{\theta_1}^{\theta_2} \sin \theta \cdot d\theta = \frac{NI}{2l} \left[ -\cos \theta \right]_{\theta_1}^{\theta_2} \\ &= \frac{NI}{2l} (\cos \theta_1 - \cos \theta_2) \quad \dots (iv) \end{aligned}$$

The above expression may be used to find the value of  $H$  at any point of the axis, either inside or outside the solenoid.

(i) At mid-point,  $\theta_2 = (\pi - \theta_1)$ , hence  $\cos \theta_2 = -\cos \theta_1$

$$\therefore H = \frac{2NI}{2l} \cos \theta_1 = \frac{NI}{l} \cos \theta_1$$

Obviously, when the solenoid is very long,  $\cos \theta_1$  becomes nearly unity. In that case,

$$H = \frac{NI}{l} \text{ AT/m} \quad \text{---Art. 6.15 (ii)}$$

(ii) At any point on the axis inside a very long solenoid but not too close to either end,  $\theta_1 \cong 0$  and  $\theta_2 \cong \pi$  so that  $\cos \theta_1 \cong 1$  and  $\cos \theta_2 \cong -1$ . Then, putting these values in Eq. (iv) above, we have

$$H \cong \frac{NI}{2l} \times 2 = \frac{NI}{l}$$

It proves that inside a very long solenoid,  $H$  is practically constant at all axial points excepts those lying too close to either end of the solenoid.

(iii) Towards either end of the solenoid,  $H$  decreases and exactly at the ends,  $\theta_1 = \pi/2$  and  $\theta_2 \cong \pi$ , so that  $\cos \theta_1 = 0$  and  $\cos \theta_2 = -1$ . Hence, from Eq. (iv) above, we get

$$H = \frac{NI}{2l}$$

In other words, value of  $H$  is decreased to half the normal value well inside the solenoid.

**Example 6.2.** Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air. Draw a curve showing the variation of  $B$  from the conductor surface outwards if its diameter is 2 mm.

**Solution.** As seen from Art. 6.15 (i),

$$H = \frac{I}{2\pi r} = \frac{250}{2\pi \times 0.05} = 795.6 \text{ AT/m}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 795.6 = 10^{-3} \text{ Wb/m}^2$$

$$\text{In general, } B = \frac{\mu_0 I}{2\pi r}$$

Now, at the conductor surface,  $r = 1 \text{ mm} = 10^{-3} \text{ m}$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 250}{2\pi \times 10^{-3}} = 0.05 \text{ Wb/m}^2$$

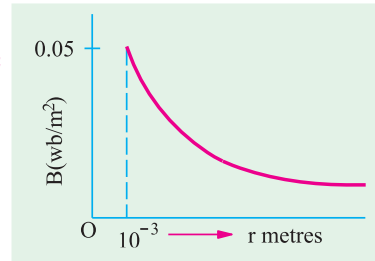


Fig. 6.22

The variation of  $B$  outside the conductor is shown in Fig. 6.22.

**Example 6.3.** A wire 2.5 m long is bent (i) into a square and (ii) into a circle. If the current flowing through the wire is 100 A, find the magnetising force at the centre of the square and the centre of the circle. (Elec. Measurements; Nagpur Univ. 1992)

**Solution.** (i) Each side of the square is  $2a = 2.5/4 = 0.625 \text{ m}$

Value of  $H$  at the centre of the square is [Art 6.17 (ii)]

$$= \frac{\sqrt{2} I}{\pi a} = \frac{\sqrt{2} \times 100}{\pi \times 0.3125} = 144 \text{ AT/m} \quad (ii) \quad 2\pi r = 2.5 ; r = 0.398 \text{ m}$$

Value of  $H$  at the centre is  $= I/2r = 100/2 \times 0.398 = 125.6 \text{ AT/m}$

**Example 6.4.** A current of 15 A is passing along a straight wire. Calculate the force on a unit magnetic pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit magnetic pole when carrying a current of 15 A. (Elect. Engg. Calcutta Univ.)

**Solution.** By the force on a unit magnetic pole is meant the magnetising force  $H$ .  
 For a straight conductor [Art 6.15 (i)]  $H = I/2\pi r = 15/2\pi \times 0.15 = 50/\pi$  AT/m  
 Now, the magnetising force at the centre of a loop of wire is [Art. 6.17 (iii)]  
 $= I/2r = I/D = 15/D$  AT/m

Since the two magnetising forces are equal

$$\therefore 50/\pi = 15/D; D = 15\pi/50 = 0.9426 \text{ m} = \mathbf{94.26 \text{ cm.}}$$

**Example 6.5.** A single-turn circular coil of 50 m. diameter carries a direct current of  $28 \times 10^4$  A. Assuming Laplace's expression for the magnetising force due to a current element, determine the magnetising force at a point on the axis of the coil and 100 m. from the coil. The relative permeability of the space surrounding the coil is unity.

**Solution.** As seen from Art 6.17 (iii),  $H = \frac{I}{2r} \cdot \sin^3 \theta$  AT/m

$$\text{Here } \sin \theta = \frac{r}{\sqrt{r^2 + x^2}} = \frac{25}{\sqrt{25^2 + 100^2}} = 0.2425$$

$$\sin^3 \theta = (0.2425)^3 = 0.01426 \quad \therefore H = \frac{28 \cdot 10^4}{2 \cdot 25} \cdot 0.01426 = \mathbf{76.8 \text{ AT/m}}$$

## 6.18. Force Between Two Parallel Conductors

(i) **Currents in the same direction.** In Fig. 6.23 are shown two parallel conductors  $P$  and  $Q$  carrying currents  $I_1$  and  $I_2$  amperes in the same direction *i.e.* upwards. The field strength in the space between the two conductors is decreased due to the two fields there being in opposition to each other. Hence, the resultant field is as shown in the figure. Obviously, the two conductors are attracted towards each other.

(ii) **Currents in opposite directions.** If, as shown in Fig. 6.24, the parallel conductors carry currents in opposite directions, then field strength is increased in the space between the two conductors due to the two fields being in the same direction there. Because of the lateral repulsion of the lines of the force, the two conductors experience a mutual force of repulsion as shown separately in Fig. 6.24 (b).

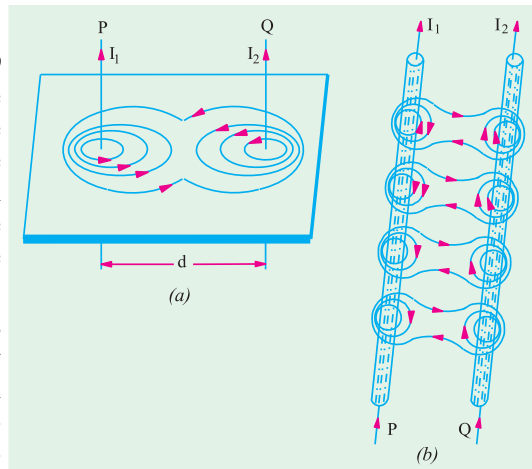


Fig. 6.23

## 6.19. Magnitude of Mutual Force

It is obvious that each of the two parallel conductors lies in the magnetic field of the other conductor. For example, conductor  $P$  lies in the magnetic field of  $Q$  and  $Q$  lies in the field of  $P$ . If ' $d$ ' metres is the distance between them, then flux density at  $Q$  due to  $P$  is [Art. 6.15 (i)]

$$B = \frac{\mu_0 I_1}{2\pi d} \text{ Wb/m}^2$$

If  $l$  is the length of conductor  $Q$  lying in this flux density, then force (either of attraction or repulsion) as given in Art. 6.14 is

$$F = BI_2 l \text{ newton} \quad \text{or} \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

Obviously, conductor  $P$  will experience an equal force in the opposite direction.

The above facts are known as Laws of Parallel Currents and may be stated as follows :

- (i) Two parallel conductors attract each other if currents through them flow in the *same* direction and repel each other if the currents through them flow in the *opposite* directions.
- (ii) The force between two such parallel conductors is proportional to the product of current strengths and to the length of the conductors considered and varies inversely as the distance between them.

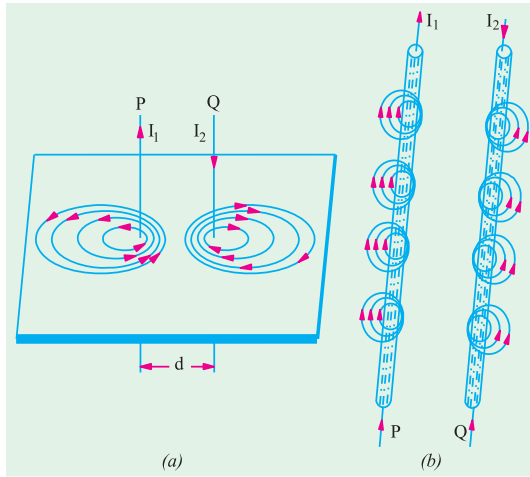


Fig. 6.24

## 6.20. Definition of Ampere

It has been proved in Art. 6.19 above that the force between two infinitely long parallel current-carrying conductors is given by the expression

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N} \quad \text{or} \quad F = \frac{4\pi \times 10^{-7} I_1 I_2 l}{2\pi d} = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N}$$

The force per metre run of the conductors is

$$F = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N/m}$$

If  $I_1 = I_2 = 1$  ampere (say) and  $d = 1$  metre, then  $F = 2 \times 10^{-7}$  N

Hence, we can define one ampere current as **that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of  $2 \times 10^{-7}$  N per metre length.**

**Example 6.6.** Two infinite parallel conductors carry parallel currents of 10 amp. each. Find the magnitude and direction of the force between the conductors per metre length if the distance between them is 20 cm. **(Elect. Engg. Material - II Punjab Univ. May 1990)**

**Solution.** 
$$F = 2 \times 10^{-7} \frac{10 \times 10 \times 1}{0.2} \text{ N} = 10^{-4} \text{ N}$$

The direction of force will depend on whether the two currents are flowing in the same direction or in the opposite direction. As per Art. 6.19, it would be a force of attraction in the first case and that of repulsion in the second case.

**Example 6.7.** Two long straight parallel wires, standing in air 2 m apart, carry currents  $I_1$  and  $I_2$  in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is  $2.4 \times 10^{-4}$  N, evaluate  $I_1$  and  $I_2$ .

**Solution.** As seen from Art. 6.17, the magnetic intensity of a long straight current-carrying conductor is

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

Also, it is seen from Fig. 6.23 that when the two currents flow in the same direction, net field strength midway between the two conductors is the difference of the two field strengths.

Now,  $H_1 = I_1/2\pi$  and  $H_2 = I_2/2\pi$  because  $r = 2/1 = 2$  metre

$$\therefore \frac{I_1}{2\pi} - \frac{I_2}{2\pi} = 7.95 \quad \therefore I_1 - I_2 = 50 \quad \dots(i)$$

Force per unit length of the conductors is  $F = 2 \times 10^{-7} I_1 I_2 / d$  newton

$$\therefore 2.4 \times 10^{-4} = 2 \times 10^{-7} I_1 I_2 / 2 \quad \therefore I_1 I_2 = 2400 \quad \dots(ii)$$

Substituting the value of  $I_1$  from (i) in (ii), we get

$$(50 + I_2)I_2 = 2400 \quad \text{or} \quad I_2^2 + 50I_2 - 2400 = 0$$

$$\text{or} \quad (I_2 + 80)(I_2 - 30) = 0 \quad \therefore I_2 = 30 \text{ A} \quad \text{and} \quad I_1 = 50 + 30 = 80 \text{ A}$$

### Tutorial Problems No. 6.1

- The force between two long parallel conductors is 15 kg/metre. The conductor spacing is 10 cm. If one conductor carries twice the current of the other, calculate the current in each conductor.

[6,060 A; 12,120 A]

- A wire is bent into a plane to form a square of 30 cm side and a current of 100 A is passed through it. Calculate the field strength set up at the centre of the square.

[300 AT/m]

(Electrotechnics - I, M.S. Univ. Baroda)

## MAGNETIC CIRCUIT

### 6.21. Magnetic Circuit

It may be defined as the route or path which is followed by magnetic flux. The law of magnetic circuit are quite similar to (but not the same as) those of the electric circuit.

Consider a solenoid or a toroidal iron ring having a magnetic path of  $l$  metre, area of cross section  $A$  m<sup>2</sup> and a coil of  $N$  turns carrying  $I$  amperes wound anywhere on it as in Fig. 6.25.

Then, as seen from Art. 6.15, field strength inside the solenoid is

$$H = \frac{NI}{l} \text{ AT/m}$$

$$\text{Now} \quad B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2$$

$$\text{Total flux produce } \Phi = B \times A = \frac{\mu_0 \mu_r ANI}{l} \text{ Wb}$$

$$\therefore \Phi = \frac{NI}{l / \mu_0 \mu_r A} \text{ Wb}$$

The numerator ' $NI$ ' which produces magnetization in the magnetic circuit is known as magnetomotive force (m.m.f.). Obviously, its unit is ampere-turn (AT)\*. It is analogous to e.m.f. in an electric circuit.

The denominator  $\frac{l}{\mu_0 \mu_r A}$  is called the **reluctance** of the circuit and is analogous to resistance in electric circuits.

$$\therefore \text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}} \quad \text{or} \quad \Phi = \frac{F}{S}$$

Sometimes, the above equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuits *i.e.*

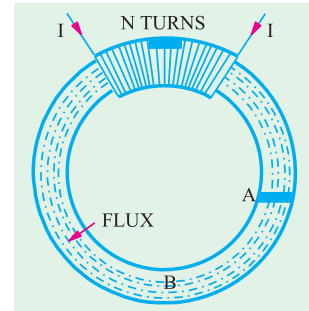


Fig. 6.25

\* Strictly speaking, it should be only 'ampere' because turns have no unit.

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}} \quad \text{or} \quad I = \frac{V}{R}$$

## 6.22. Definitions Concerning Magnetic Circuit

**1. Magnetomotive force (m.m.f.).** It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one point to another, similarly, m.m.f. between two points is measured by the work done in joules in carrying a unit magnetic pole from one point to another.

**2. Ampere-turns (AT).** It is the unit of magnetomotive force (m.m.f.) and is given by the product of number of turns of a magnetic circuit and the current in amperes in those turns.

**3. Reluctance.** It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even in form. Its units is **AT/Wb.\***

$$\text{reluctance} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}; \quad \text{resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words, the reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since  $1 \text{ AT/Wb} = 1/\text{henry}$ , the unit of reluctance is “reciprocal henry.”

**4. Permeance.** It is reciprocal of reluctance and implies the ease or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry.

**5. Reluctivity.** It is specific reluctance and corresponds to resistivity which is ‘specific resistance’.

## 6.23. Composite Series Magnetic Circuit

In Fig. 6.26 is shown a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ( $\mu_r = 1$ ). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\begin{aligned} \therefore \quad \text{total reluctance} &= \sum \frac{l}{\mu_0 \mu_r A} \\ &= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g} \\ \therefore \quad \text{flux } \Phi &= \frac{\text{m.m.f.}}{\frac{l}{\mu_0 \mu_r A}} \end{aligned}$$

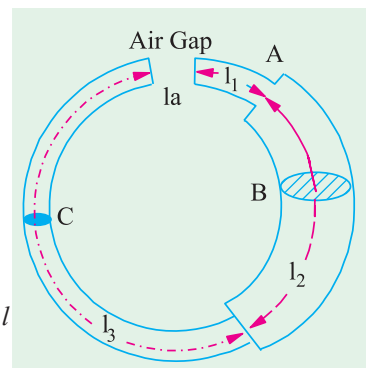


Fig. 6.26

## 6.24. How to Find Ampere-turns ?

It has been shown in Art. 6.15 that  $H = NI/l$  AT/m or  $NI = H \times l$

$$\therefore \quad \text{ampere-turns AT} = H \times l$$

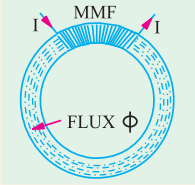
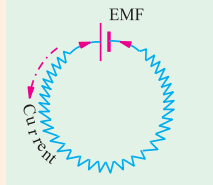
Hence, following procedure should be adopted for calculating the total ampere turns of a composite magnetic path.

\* From the ratio  $\Phi = \frac{\text{m.m.f.}}{\text{reluctance}}$ , it is obvious that reluctance = m.m.f./ $\Phi$ . Since m.m.f. is in ampere-turns and flux in webers, unit of reluctance is ampere-turn/weber (AT/Wb) or A/Wb.

- (i) Find  $H$  for each portion of the composite circuit. For air,  $H = B/\mu_0$ , otherwise  $H = B/\mu_0\mu_r$ .
- (ii) Find ampere-turns for each path separately by using the relation  $AT = H \times l$ .
- (iii) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

## 6.25. Comparison Between Magnetic and Electric Circuits.

### SIMILARITIES

Magnetic Circuit	Electric Circuit
 <p style="text-align: center;">Fig. 6.27</p>	 <p style="text-align: center;">Fig. 6.28</p>
<ol style="list-style-type: none"> <li>1. Flux = <math>\frac{\text{m.m.f.}}{\text{reluctance}}</math></li> <li>2. M.M.F. (ampere-turns)</li> <li>3. Flux <math>\Phi</math> (webers)</li> <li>4. Flux density <math>B</math> (<math>\text{Wb}/\text{m}^2</math>)</li> <li>5. Reluctance <math>S = \frac{l}{\mu A} \left( = \frac{l}{\mu_0 \mu_r A} \right)</math></li> <li>6. Permeance (= 1/reluctance)</li> <li>7. Reluctivity</li> <li>8. Permeability (= 1/reductivity)</li> <li>9. Total m.m.f. = <math>\Phi S_1 + \Phi S_2 + \Phi S_3 + \dots</math></li> </ol>	<ol style="list-style-type: none"> <li>Current = <math>\frac{\text{e.m.f.}}{\text{resistance}}</math></li> <li>E.M.F. (volts)</li> <li>Current <math>I</math> (amperes)</li> <li>Current density (<math>\text{A}/\text{m}^2</math>)</li> <li>resistance <math>R = \rho \frac{l}{A} = \frac{l}{\rho A}</math></li> <li>Conductance (= 1/resistance)</li> <li>Resistivity</li> <li>Conductivity (= 1/resistivity)</li> <li>9. Total e.m.f. = <math>IR_1 + IR_2 + IR_3 + \dots</math></li> </ol>

### DIFFERENCES

1. Strictly speaking, flux does not actually 'flow' in the sense in which an electric current flows.
2. If temperature is kept constant, then resistance of an electric circuit is constant and is independent of the current strength (or current density). On the other hand, the reluctance of a magnetic circuit does depend on flux (and hence flux density) established in it. It is so because  $\mu$  (which equals the slope of  $B/H$  curve) is not constant even for a given material as it depends on the flux density  $B$ . Value of  $\mu$  is large for low value of  $B$  and **vice-versa**. Hence, reluctance is small ( $S = l/\mu A$ ) for small values of  $B$  and large for large values of  $B$ .
3. Flow of current in an electric circuit involves continuous expenditure of energy but in a magnetic circuit, energy is needed only creating the flux initially but not for maintaining it.

## 6.26. Parallel Magnetic Circuits

Fig. 6.29 (a) shown a parallel magnetic circuit consisting of two parallel magnetic paths  $ACB$  and  $ADB$  acted upon by the same  $m.m.f.$  Each magnetic path has an average length of  $2(l_1 + l_2)$ . The flux produced by the coil wound on the central core is divided equally at point  $A$  between the two outer parallel paths. The reluctance offered by the two parallel paths is = half the reluctance of each path.

Fig. 6.29 (b) shows the equivalent electrical circuit where resistance offered to the voltage source is  $R \parallel R = R/2$

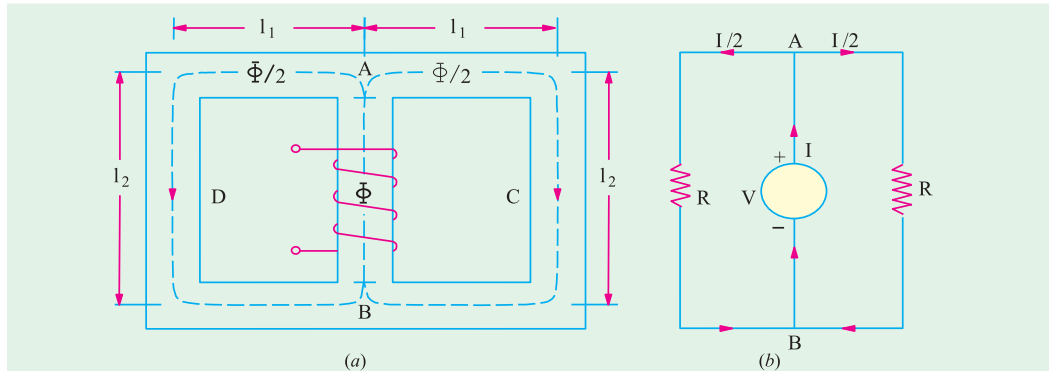


Fig. 6.29

It should be noted that reluctance offered by the central core AB has been neglected in the above treatment.

### 6.27. Series-Parallel Magnetic Circuits

Such a circuit is shown in Fig. 6.30 (a). It shows two parallel magnetic circuits  $ACB$  and  $ACD$  connected across the common magnetic path  $AB$  which contains an air-gap of length  $l_g$ . As usual, the flux  $\Phi$  in the common core is divided equally at point  $A$  between the two parallel paths which have equal reluctance. The reluctance of the path  $AB$  consists of (i) air gap reluctance and (ii)

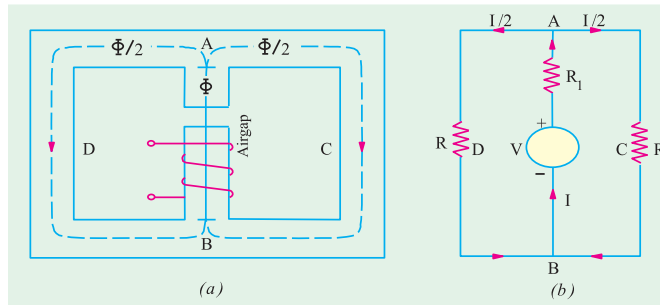


Fig. 6.30

the reluctance of the central core which comparatively negligible. Hence, the reluctance of the central core  $AB$  equals only the air-gap reluctance across which are connected two equal parallel reluctances. Hence, the  $m.m.f.$  required for this circuit would be the sum of (i) that required for the air-gap and (ii) that required for either of two paths (not both) as illustrated in Ex. 6.19, 6.20 and 6.21.

The equivalent electrical circuit is shown in Fig. 6.30 (b) where the total resistance offered to the voltage source is  $= R_1 + R \parallel R = R_1 + R/2$ .

### 6.28. Leakage Flux and Hopkinson's Leakage Coefficient

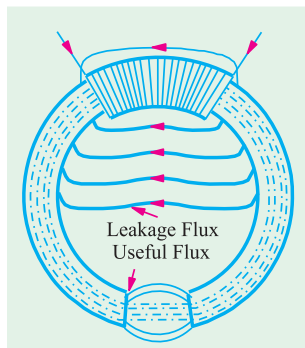


Fig. 6.31

Leakage flux is the flux which follows a path not intended for it. In Fig. 6.31 is shown an iron ring wound with a coil and having an air-gap. The flux in the air-gap is known as the useful flux because it is only this flux which can be utilized for various useful purposes.

It is found that it is impossible to confine all the flux to the iron path only, although it is usually possible to confine most of the electric current to a definite path, say a wire, by surrounding it with insulation. Unfortunately, there is no known insulator for magnetic flux. Air, which is a splendid insulator of electricity, is unluckily a fairly good magnetic conductor. Hence, as shown, some of the flux leaks through air surrounding the iron ring. The presence of leakage flux can be detected by a compass. Even in the best designed dynamos, it is found

that 15 to 20% of the total flux produced leaks away without being utilised usefully.

If,  $\Phi_t$  = total flux produced ;  $\Phi$  = useful flux available in the air-gap, then

$$\text{leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} \quad \text{or} \quad \lambda = \frac{\Phi_t}{\Phi}$$

In electric machines like motors and generators, magnetic leakage is undesirable, because, although it does not lower their power efficiency, yet it leads to their increased weight and cost of manufacture. Magnetic leakage can be minimised by placing the exciting coils or windings as close as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilized for useful purposes.

It is also seen from Fig. 6.31 that there is fringing or spreading of lines of flux at the edges of the air-gap. This fringing increases the effective area of the air-gap.

The value of  $\lambda$  for modern electric machines varies between 1.1 and 1.25.

### 6.29. Magnetisation Curves

The approximate magnetisation curves of a few magnetic materials are shown in Fig. 6.32.

These curves can be determined by the following methods provided the materials are in the form of a ring :

(a) By means of a ballistic galvanometer and (b) By means of a fluxmeter.

### 6.30. Magnetisation Curves by Ballistic Galvanometer

In Fig. 6.33 shown the specimen ring of uniform cross-section wound uniformly with a coil  $P$  which is connected to a battery  $B$  through a reversing switch  $RS$ , a variable resistance  $R_1$  and an ammeter. Another secondary coil  $S$  also wound over a small portion of the ring and is connected through a resistance  $R$  to a ballistic galvanometer  $BG$ .

The current through the primary  $P$  can be adjusted with the help of  $R_1$ . Suppose the primary current is  $I$ . When the primary current is reversed by means of  $RS$ , then flux is reversed through  $S$ , hence an induced e.m.f. is produced in it which sends a current through  $BG$ . This current is of very short duration. The first deflection or 'throw' of the  $BG$  is proportional to the quantity of electricity or charge passing through it so long as the time taken for this charge to flow is short as compared with the time of one oscillation.

If  $\theta$  = first deflection or 'throw' of the galvanometer when primary current  $I$  is reversed.

$k$  = ballistic constant of the galvanometer *i.e.* charge per unit deflection.

then, charge passing through  $BG$  is  $k\theta$  coulombs

...(i)

Let  $\Phi$  = flux in Wb produced by primary current of  $I$  amperes ;  $t$  = time of reversal of flux ; then

rate of change of flux =  $\frac{2\Phi}{t}$  Wb/s

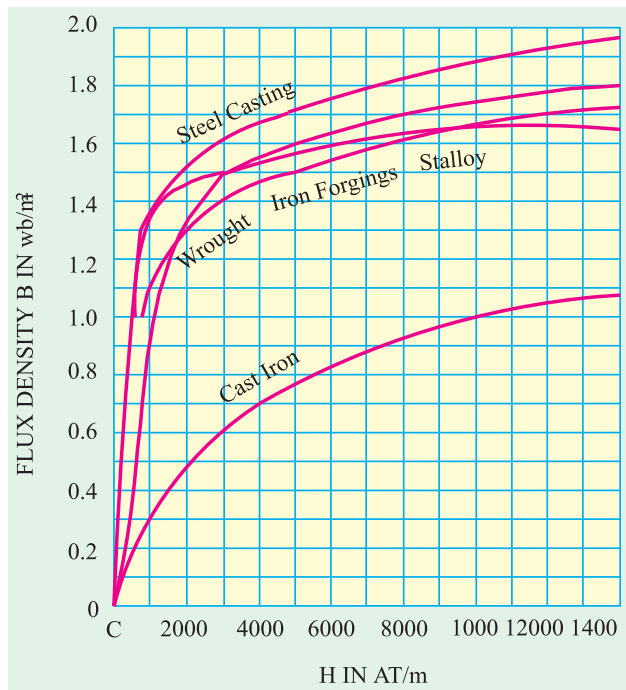


Fig. 6.32



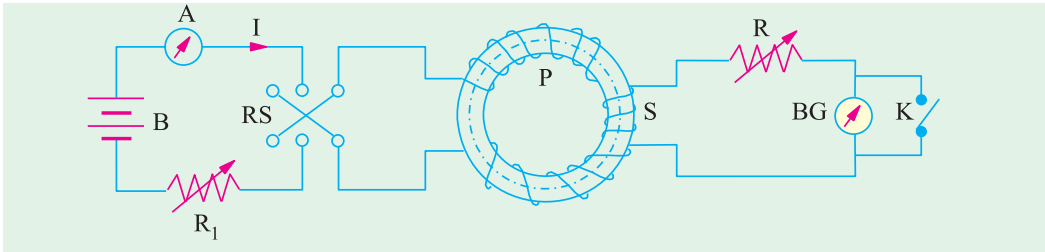


Fig. 6.33

If  $N_2$  is the number of turns in secondary coil  $S$ , then average e.m.f. induces in it is

$$= N_2 \cdot \frac{2\Phi}{t} \text{ volt.}$$

Secondary current or current through  $BG = \frac{2N_2 \Phi}{R_s t}$  amperes

where  $R_s$  is the total resistance of the secondary circuit.

Charge flowing through  $BG = \text{average current} \times \text{time} = \frac{2N_2 \Phi}{R_s t} \times t = \frac{2N_2 \Phi}{R_s}$  coulomb ... (ii)

Equation (i) and (ii), we get  $k\theta = \frac{2N_2 \Phi}{R_s} \therefore \Phi = \frac{k\theta R_s}{2N_2}$  Wb

If  $A \text{ m}^2$  is the cross-sectional area of the ring, then flux density is

$$B = \frac{\Phi}{A} = \frac{k\theta R_s}{2N_2 A} \text{ Wb/m}^2$$

If  $N_1$  is the number of primary turns and  $l$  metres the mean circumference of the ring, then, magnetising force  $H = N_1 I / l$  AT/m.

The above experiment is repeated with different values of primary current and from the data so obtained, the  $B/H$  curves or magnetisation curves can be drawn.

### 6.31. Magnetisation Curves by Fluxmeter

In this method, the  $BG$  of Fig. 6.31 is replaced by a fluxmeter which is just a special type of ballistic galvanometer. When current through  $P$  is reversed, the flux is also reversed. The deflection of the fluxmeter is proportional to the change in flux-linkages of the secondary coil. If the flux is reversed from  $+\Phi$  to  $-\Phi$ , the change in flux-linkages in secondary  $S$  in  $= 2\Phi N_2$ .

If  $\theta =$  corresponding deflection of the fluxmeter

$C =$  fluxmeter constant *i.e.* weber-turns per unit deflection.

then, change of flux-linkages in  $S = C\theta$

$$\therefore 2\Phi N_2 = C\theta \quad \text{or} \quad \Phi = \frac{C\theta}{N_2} \text{ Wb}; \quad B = \frac{\Phi}{A} = \frac{C\theta}{2N_2 A} \text{ Wb/m}^2$$

**Example 6.8.** A fluxmeter is connected to a search-coil having 600 turns and mean area of  $4 \text{ cm}^2$ . The search coil is placed at the centre of an air-cored solenoid 1 metre long and wound with 1000 turns. When a current of 4 A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.

(Measurements-I, Nagpur Univ. 1991)

**Solution.** Magnetising force of the solenoid is  $H = NI/l$  AT/m

$$B = \mu_0 H = \mu_0 NI/l = 4\pi \times 10^{-7} \times 1000 \times 4/1 = 16\pi \times 10^{-4} \text{ Wb/m}^2$$

Flux linked with the search coil is  $\Phi = BA = 64\pi \times 10^{-8}$  Wb

Total change of flux-linkages on reversal

$$= 2 \times 64\pi \times 10^{-8} \times 600 \text{ Wb-turns} \quad \text{---Art. 6.29}$$

$$= 7.68\pi \times 10^{-4} \text{ Wb-turns}$$

Fluxmeter constant  $C$  is given by =  $\frac{\text{Change in flux-linkages}}{\text{deflection produced}}$

$$= 7.68\pi \times 10^{-4}/20 = \mathbf{1.206 \times 10^{-4} \text{ Wb-turns/division}}$$

**Example 6.9.** A ballistic galvanometer, connected to a search coil for measuring flux density in a core, gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180 ohm. The galvanometer constant is 100  $\mu\text{C}$  per scale division. The search coil has an area of 50  $\text{cm}^2$ , wound with 1000 turns having a resistance of 20 ohm. Calculate the flux density in the core. **(Elect. Instru & Measu. Nagpur Univ. 1992)**

**Solution.** As seen from Art. 6.28.

$$k\theta = 2N_2\Phi/R_s \quad \text{or} \quad \Phi = k\theta R_s/2N_2 \text{ Wb}$$

$$\therefore BA = k\theta R_s/2N_2 \quad \text{or} \quad B = k\theta R_s/2N_2 A$$

Here  $k = 100 \mu\text{C/division} = 100 \times 10^{-6} = 10^{-4} \text{ C/division}$

$$\theta = 100; A = 50 \text{ cm}^2 = 5 \times 10^{-3} \text{ m}^2$$

$$R_s = 180 + 20 = 200 \Omega$$

$$\therefore B = 10^{-4} \times 100 \times 200/2 \times 1000 \times 5 \times 10^{-3} = \mathbf{0.2 \text{ Wb/m}^2}$$

**Example 6.10.** A ring sample of iron, fitted with a primary and a secondary winding is to be tested by the method of reversals to obtain its  $B/H$  curve. Give a diagram of connections explain briefly how the test could be carried out.

In such a test, the primary winding of 400 turns carries a current of 1.8 A. On reversal, a change of  $8 \times 10^{-3}$  Wb-turns is recorded in the secondary winding of 10 turns. The ring is made up of 50 laminations, each 0.5 mm thick with outer and inner diameters of 25 and 23 cm respectively. Assuming uniform flux distribution, determine the values of  $B$ ,  $H$  and the permeability.

**Solution.** Here, change of flux-linkages =  $2\Phi N_2 = 8 \times 10^{-3}$  Wb-turns

$$\therefore 2\Phi \times 10 = 8 \times 10^{-3} \quad \text{or} \quad \Phi = 4 \times 10^{-4} \text{ Wb and } A = 2.5 \times 10^{-4} \text{ m}^2$$

$$\therefore B = \frac{4 \times 10^{-4}}{2.5 \times 10^{-4}} = 1.6 \text{ Wb/m}^2; H = \frac{NI}{l} = \frac{400 \times 1.8}{0.24\pi} = 955 \text{ AT/m}$$

Now  $\mu_0 \mu_r = \frac{B}{H}; \mu_r = \frac{B}{\mu_0 H} = \frac{1.6}{4\pi \times 10^{-7} \times 955} = \mathbf{1333}$

**Example 6.11.** An iron ring of 3.5  $\text{cm}^2$  cross-sectional area with a mean length of 100 cm is wound with a magnetising winding of 100 turns. A secondary coil of 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 micro-coulomb per scale division, the total resistance of the secondary circuit being 2000  $\Omega$ . On reversing a current of 10 A in the magnetising coil, the galvanometer gave a throw of 200 scale divisions. Calculate the flux density in the specimen and the value of the permeability at this flux density. **(Elect. Measure, A.M.I.E Sec.B. 1992)**

**Solution.** Reference may please be made to Art. 6.28.

Here  $N_1 = 100; N_2 = 200; A = 3.5 \times 10^{-4} \text{ m}^2; l = 100 \text{ cm} = 1 \text{ m}$

$$k = 10^{-6} \text{ C/division, } \theta = 200 \text{ divisions; } R_s = 2000 \Omega; I = 10 \text{ A}$$

$$B = \frac{k\theta R_s}{2N_2 A} = \frac{10^{-6} \times 200 \times 2000}{2 \times 200 \times 3.5 \times 10^{-4}} = 1.43 \text{ Wb/m}^2$$

Magnetising force  $H = N_1 I/l = 100 \times 10/1 = 1000 \text{ AT/m}$

$$\mu = \frac{B}{H} = \frac{1.43}{1000} = \mathbf{1.43 \times 10^{-3} \text{ H/m}}$$

**Note.** The relative permeability is given by  $\mu_r = \mu / \mu_0 = 1.43 \times 10^3 / 4\pi \times 10^{-7} = 1137$ .

**Example 6.12.** An iron ring has a mean diameter of 0.1 m and a cross-section of  $33.5 \times 10^{-6} \text{ m}^2$ . It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert Magnetic standard with 10 turns and a flux of  $2.5 \times 10^{-4}$  gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen. **(Elect. Measu. A.M.I.E. Sec B, 1991)**

**Solution.** Length of the magnetic path  $l = \pi D = 0.1 \pi \text{ m}$

Magnetising Force,  $H = NI/l = 320 \times 10 / 0.1 \pi = 10,186 \text{ AT/m}$

Flux density  $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times \mu_r \times 10,186 = 0.0128 \mu_r$  ... (i)

Now, from Hilbert's Magnetic standard, we have

$$2.5 \times 10^{-4} \times 10 = K \times 102, K = 2.45 \times 10^{-5}$$

On reversing a current of 10 A in the magnetising winding, total change in Weber-turns is  $2\Phi N_s = 2.45 \times 10^{-5} \times 272$  or  $2 \times 220 \times \Phi = 2.45 \times 10^{-5} \times 272$  or  $\Phi = 1.51 \times 10^{-5} \text{ Wb}$

$$\therefore B = \Phi/A = 1.51 \times 10^{-5} / 33.5 \times 10^{-6} = 0.45 \text{ Wb/m}^2$$

Substituting this value in Eq. (i), we have  $0.0128 \mu_r = 0.45$ ,  $\therefore \mu_r = 35.1$

**Example 6.13.** A laminated soft iron ring of relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area  $500 \text{ mm}^2$ . A radial air-gap of 1 mm width is cut in the ring which is wound with 1000 turns. Calculate the current required to produce an air-gap flux of 0.5 mWb if leakage factor is 1.2 and stacking factor 0.9. Neglect fringing.

**Solution.** Total AT reqd.  $= \Phi_g S_g + \Phi_i S_i = \frac{\Phi_g l_g}{\mu_0 A_g} + \frac{\Phi_i l_i}{\mu_0 \mu_r A_i B}$

Now, air-gap flux  $\Phi_g = 0.5 \text{ mWb} = 0.5 \times 10^{-3} \text{ Wb}$ ,  $l_g = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$ ;  $A_g = 500 \text{ mm}^2 = 500 \times 10^{-6} \text{ m}^2$

Flux in the iron ring,  $\Phi_i = 1.2 \times 0.5 \times 10^{-3} \text{ Wb}$

Net cross-sectional area  $= A_i \times \text{stacking factor} = 500 \times 10^{-6} \times 0.9 \text{ m}^2$

$$\therefore \text{total AT reqd.} = \frac{0.5 \times 10^{-3} \times 1 \times 10^{-3}}{4\pi \times 10^{-7} \times 500 \times 10^{-6}} + \frac{1.2 \times 0.5 \times 10^{-3} \times 800 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000 \times (0.9 \times 500 \times 10^{-6})} = 1644$$

$$\therefore I = 1644 / 1000 = 1.64 \text{ A}$$

**Example 6.14.** A ring has a diameter of 21 cm and a cross-sectional area of  $10 \text{ cm}^2$ . The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of  $8 \times 10^{-4} \text{ Wb}$ . The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.

**(Elect. Circuits, South Gujarat Univ.)**

**Solution.**  $\Phi = 8 \times 10^{-4} \text{ Wb}$ ;  $A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2$ ;  
 $B = 8 \times 10^{-4} / 10^{-3} = 0.8 \text{ Wb/m}^2$

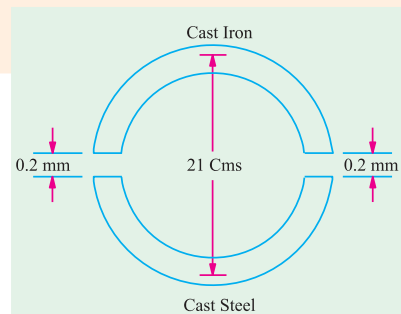
**Air gap**

$$H = B/\mu_0 = 0.8 / 4\pi \times 10^{-7} = 6.366 \times 10^5 \text{ AT/m}$$

$$\begin{aligned} \text{Total air-gap length} &= 2 \times 0.2 = 0.4 \text{ mm} \\ &= 4 \times 10^{-4} \text{ m} \end{aligned}$$

$$\therefore \text{AT required} = H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$$

**Cast Steel Path** (Fig. 6.34)



**Fig. 6.34**

$$H = B/\mu_0 \mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796 \text{ AT/m}$$

$$\text{path} = \pi D/2 = 21 \pi/2 = 33 \text{ cm} = 0.33 \text{ m}$$

$$\text{AT required} = H \times l = 796 \times 0.33 = 263$$

### Cast Iron Path

$$H = 0.8/\pi \times 10^{-7} \times 166 = 3,835 \text{ AT/m ; path} = 0.33 \text{ m}$$

$$\text{AT required} = 3,835 \times 0.33 = 1265$$

$$\text{Total AT required} = 255 + 263 + 1265 = \mathbf{1783.}$$

**Example 6.15.** A mild steel ring of 30 cm mean circumference has a cross-sectional area of  $6 \text{ cm}^2$  and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air-gap of 1 mm in the magnetic circuit. It is found that a current of 4 A in the winding, produces a flux density of 1 T in the air-gap. Find (i) the relative permeability of the mild steel and (ii) inductance of the winding. (F.E. Engg. Pune Univ.)

### Solution. (a) Steel ring

$$H = B/\mu_0 \mu_r = 1/4\pi \times 10^{-7} \times \mu_r \text{ AT/m} = 0.7957 \times 10^7/\mu_r \text{ AT/m}$$

$$\text{m.m.f.} = H \times l = (0.7957 \times 10^7/\mu_r) \times 29.9 \times 10^{-2} = 0.2379 \times 10^6/\mu_r \text{ AT}$$

### (b) Air-gap

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 0.7957 \times 10^6 \text{ AT/m}$$

$$\text{m.m.f. reqd.} = H \times l = 0.7957 \times 10^6 \times (1 \times 10^{-3}) = 795.7 \text{ AT}$$

$$\text{Total m.m.f.} = (0.2379 \times 10^6/\mu_r) + 795.7$$

$$\text{Total m.m.f. available} = NI = 500 \times 4 = 2000 \text{ AT}$$

$$(i) \therefore 2000 = (0.2379 \times 10^6/\mu_r) + 795.7 \quad \therefore \mu_r = \mathbf{197.5}$$

$$(ii) \text{ Inductance of the winding} = \frac{N\Phi}{I} = \frac{NBA}{I} = \frac{500 \times 1 \times 6 \times 10^{-4}}{4} = \mathbf{0.075 \text{ H}}$$

**Example 6.16.** An iron ring has a X-section of  $3 \text{ cm}^2$  and a mean diameter of 25 cm. An air-gap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 200 turns through which a current of 2 A is passed. If the total magnetic flux is 0.24 mWb, find the relative permeability of iron, assuming no magnetic leakage. (Elect. Engg. A.M.Ae.S.I., June 1992)

$$\text{Solution. } \Phi = 0.24 \text{ mWb; } A = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2;$$

$$B = \Phi/A = 0.24 \times 10^{-3}/3 \times 10^{-4} = 0.8 \text{ Wb/m}^2$$

$$\text{AT for iron ring} = H \times l = (B/\mu_0 \mu_r) \times l = (0.8/4\pi \times 10^{-7} \times \mu_r) \times 0.25 = 1.59 \times 10^5/\mu_r$$

$$\text{AT for air-gap} = H \times l = (B/\mu_0) \times l = (0.8/4\pi \times 10^{-7}) \times 0.4 \times 10^{-3} = 255$$

$$\text{Total AT reqd.} = (1.59 \times 10^5/\mu_r) + 255 ; \text{ total AT provided} = 200 \times 2 = 400$$

$$\therefore (1.59 \times 10^5/\mu_r) + 255 = 400 \quad \text{or } \mu_r = \mathbf{1096.}$$

**Example 6.17.** A rectangular iron core is shown in Fig. 6.35. It has a mean length of magnetic path of 100 cm, cross-section of  $(2 \text{ cm} \times 2 \text{ cm})$ , relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns  $N_a = 335$ ,  $N_b = 600$  and  $N_c = 600$ ; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap. (F.Y. Engg. Pune Univ.)

**Solution.** By applying the Right-Hand Thumb rule, it is found that fluxes produced by the current  $I_a$  and  $I_b$  are directed in the clockwise direction through the iron core whereas that produced by current  $I_c$  is directed in the anticlockwise direction through the core.

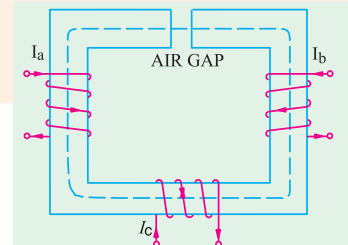


Fig. 6.35

$$\therefore \text{total m.m.f.} = N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136 \text{ AT}$$

$$\text{Reluctance of the air-gap} = \frac{l}{\mu_0 A} = \frac{5 \times 10^3}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.946 \times 10^6 \text{ AT/Wb}$$

$$\text{Reluctance of the iron path} = \frac{l}{\mu_0 \mu_r A} = \frac{100 - (0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^6 \text{ AT/Wb}$$

$$\text{Total reluctance} = (9.946 + 1.414) \times 10^6 = 11.36 \times 10^6 \text{ AT/Wb}$$

The flux in the air-gap is the same as in the iron core.

$$\text{Air-gap flux} = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1136}{11.36 \times 10^6} = 100 \times 10^{-6} \text{ Wb} = \mathbf{100 \mu\text{Wb}}$$

**Example 6.18.** A series magnetic circuit comprises of three sections (i) length of 80 mm with cross-sectional area 60 mm<sup>2</sup>, (ii) length of 70 mm with cross-sectional area 80 mm<sup>2</sup> and (iii) air-gap of length 0.5 mm with cross-sectional area of 60 mm<sup>2</sup>. Sections (i) and (ii) are of a material having magnetic characteristics given by the following table.

$H$ (AT/m)	100	210	340	500	800	1500
$B$ (Tesla)	0.2	0.4	0.6	0.8	1.0	1.2

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 Tesla in the air-gap. Neglect magnetic leakage. **(F.E. Pune Univ. May 1990)**

**Solution. Section (i)** It has the same cross-sectional area as the air-gap. Hence, it has the same flux density i.e. 0.7 Tesla as in the air-gap. The value of the magnetising force  $H$  corresponding to this flux density of 0.7 T as read from the  $B$ - $H$  plot is 415 AT/m.

$$\text{m.m.f. reqd} = H \times l = 415 \times (80 \times 10^{-3}) = 33.2 \text{ AT}$$

**Section (ii)** Since its cross-sectional area is different from that of the air-gap, its flux density would also be different even though, being a series circuit, its flux would be the same.

$$\text{Air-gap flux} = B \times L = 0.7 \times (60 \times 10^{-6}) = 42 \times 10^{-6} \text{ Wb}$$

$$\text{Flux density in this section} = 42 \times 10^{-6} / 80 \times 10^{-6} = 0.525 \text{ T}$$

The corresponding value of the  $H$  from the given graph is 285 AT/m

$$\text{m.m.f. reqd, for this section} = 285 \times (70 \times 10^{-3}) = 19.95 \text{ AT.}$$

**Air-gap**

$$H = B/\mu_0 = 0.7/4\pi \times 10^{-7} = 0.557 \times 10^6 \text{ AT/m}$$

$$\therefore \text{m.m.f. reqd.} = 0.557 \times 10^6 \times (0.5 \times 10^{-3}) = 278.5 \text{ AT}$$

$$\text{Total m.m.f. reqd.} = 33.2 + 19.95 + 278.5 = 331.6$$

$$\therefore NI = 331.6 \text{ or } I = 331.6/4000 = \mathbf{0.083 \text{ A}}$$

**Example 6.19.** A magnetic circuit made of mild steel is arranged as shown in Fig. 6.36. The central limb is wound with 500 turns and has a cross-sectional area of 800 mm<sup>2</sup>. Each of the outer limbs has a cross-sectional area of 500 mm<sup>2</sup>. The air-gap has a length of 1 mm. Calculate the current required to set up a flux of 1.3 mWb in the central limb assuming no magnetic leakage and fringing. Mild steel required 3800 AT/m to produce flux density of 1.625 T and 850 AT/m to produce flux density of 1.3 T.

**(F.Y. Engg. Pune Univ.)**

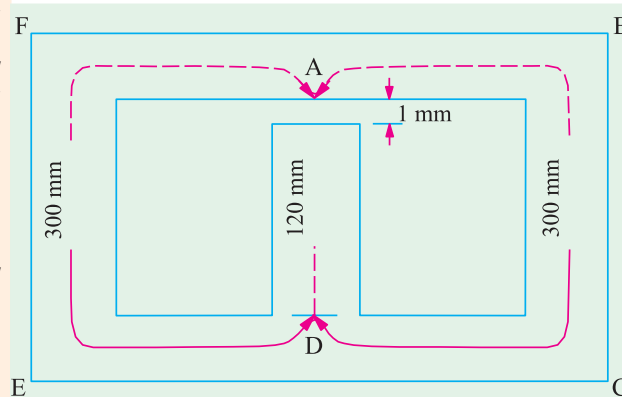


Fig. 6.36

**Solution.** Flux in the central limb is  $= 1.3 \text{ mWb} = 1.3 \times 10^{-3} \text{ Wb}$   
 Cross section  $A = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$   
 $\therefore B = \Phi/A = 1.3 \times 10^{-3} / 800 \times 10^{-6}$   
 $= 1.625 \text{ T}$

Corresponding value of  $H$  for this flux density is given as  $3800 \text{ AT/m}$ .

Since the length of the central limb is  $120 \text{ mm}$ , m.m.f. required is  $= H \times l = 3800 \times (120 \times 10^{-3})$   
 $= 456 \text{ AT/m}$ .

### Air-gap

Flux density in the air-gap is the same as that in the central limb.

$H = B/\mu_0 = 1.625/4\pi \times 10^{-7} = 0.1293 \times 10^7 \text{ AT/m}$

Length of the air-gap  $= 1 \text{ mm} = 10^{-3} \text{ m}$

m.m.f. reqd. for the air-gap  $= H \times l = 0.1293 \times 10^7 \times 10^{-3} = 1293 \text{ AT}$ .

The flux of the central limb divides equally at point A in figure along the two parallel path  $ABCD$  and  $AFED$ . We may consider either path, say  $ABCD$  and calculate the m.m.f. required for it. The same m.m.f. will also send the flux through the other parallel path  $AFED$ .

Flux through  $ABCD = 1.3 \times 10^{-3}/2 = 0.65 \times 10^{-3} \text{ Wb}$

Flux density  $B = 0.65 \times 10^{-3} / 500 \times 10^{-6} = 1.3 \text{ T}$

The corresponding value of  $H$  for this value of  $B$  is given at  $850 \text{ AT/m}$ .

$\therefore$  m.m.f. reqd. for path  $ABCD = H \times l = 850 \times (300 \times 10^{-3}) = 255 \text{ AT}$

As said above, this, m.m.f. will also send the flux in the parallel path  $AFED$ .

Total m.m.f. reqd.  $= 456 + 1293 + 255 = 2004 \text{ AT}$

Since the number of turns is  $500$ ,  $I = 2004/500 = 4 \text{ A}$ .

**Example 6.20.** A cast steel d.c. electromagnet shown in Fig. 6.37 has a coil of  $1000$  turns on its central limb. Determine the current that the coil should carry to produce a flux of  $2.5 \text{ mWb}$  in the air-gap. Neglect leakage. Dimensions are given in cm. The magnetisation curve for cast steel is as under :

Flux density ( $\text{Wb/m}^2$ ) :	0.2	0.5	0.7	1.0	1.2
Amp-turns/metre :	300	540	650	900	1150

(Electrotechnics-I ; M.S. Univ. Baroda)

**Solution.** Two points should be noted

(i) there are two (equal) parallel paths  $ACDE$  and  $AGE$  across the central path  $AE$ .

(ii) flux density in either parallel path is half of that in the central path because flux divides into two equal parts at point A.

Total m.m.f. required for the whole electromagnet is equal to the sum of the following three m.m.fs.

- (i) that required for path  $EF$
- (ii) that required for air-gap
- (iii) that required for either of the two parallel paths ; say, path  $ACDE$

Flux density in the central limb and air gap is  
 $= 2.5 \times 10^{-3} / (5 \times 5) \times 10^{-4} = 1 \text{ Wb/m}^2$

Corresponding value of  $H$  as found from the given data is  $900 \text{ AT/m}$ .

$\therefore$  AT for central limb  $= 900 \times 0.3 = 270$

$H$  in air-gap  $= B/\mu_0 = 1/4\pi \times 10^{-7} = 79.56 \times 10^4 \text{ AT/m}$

AT required  $= 79.56 \times 10^4 \times 10^{-3} = 795.6$

Flux density in path  $ACDE$  is  $0.5 \text{ Wb/m}^2$  for which corresponding value of  $H$  is  $540 \text{ AT/m}$ .

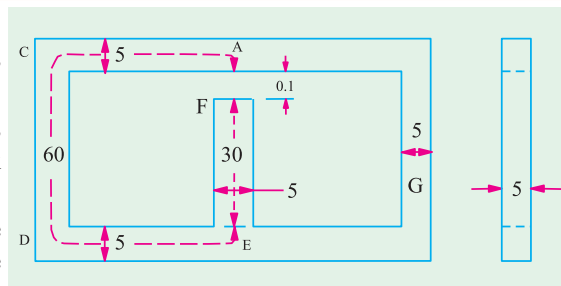


Fig. 6.37

$\therefore$  AT required for path  $ACDE = 540 \times 0.6 = 324$

Total AT required =  $270 + 795.6 + 324 = 1390$ ; Current required =  $1390/1000 = 1.39 \text{ A}$

**Example 6.21.** A cast steel magnetic structure made for a bar of section  $8 \text{ cm} \times 2 \text{ cm}$  is shown in Fig. 6.35. Determine the current that the 500 turn-magnetising coil on the left limb should carry so that a flux of  $2 \text{ mWb}$  is produced in the right limb. Take  $\mu_r = 600$  and neglect leakage.

(Elect. Technology Allahabad Univ. 1993)

**Solution.** Since path  $C$  and  $D$  are in parallel with each other w.r.t. path  $E$  (Fig. 6.38), the m.m.f. across the two is the same.

$$\begin{aligned} \Phi_1 S_1 &= \Phi_2 S_2 \\ \therefore \Phi_1 \times \frac{15}{\mu A} &= 2 \times \frac{25}{\mu A} \\ \therefore \Phi_1 &= 10/3 \text{ mWb} \\ \therefore \Phi &= \Phi_1 + \Phi_2 = 16/3 \text{ mWb} \end{aligned}$$

Total AT required for the whole circuit is equal to the sum of

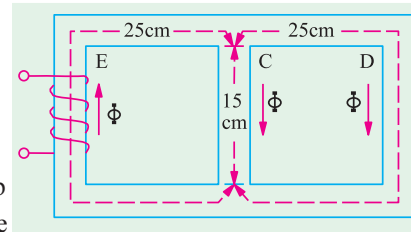


Fig. 6.38

(i) that required for path  $E$  and (ii) that required for either of the two paths  $C$  or  $D$ .

$$\text{Flux density in path } E = \frac{16 \times 10^{-3}}{3 \times 4 \times 10^{-4}} = \frac{40}{3} \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{40 \times 0.25}{3 \times 4\pi \times 10^{-7} \times 600} = 4,420$$

$$\text{Flux density in path } D = \frac{2 \times 10^{-3}}{4 \times 10^{-4}} = 5 \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{5}{4\pi \times 10^{-7} \times 600} \times 0.25 = 1658$$

$$\text{Total AT} = 4,420 + 1,658 = 6,078 ;$$

$$\text{Current needed} = 6078/500 = 12.16 \text{ A}$$

**Example 6.22.** A ring of cast steel has an external diameter of  $24 \text{ cm}$  and a square cross-section of  $3 \text{ cm}$  side. Inside and cross the ring, an ordinary steel bar  $18 \text{ cm} \times 3 \text{ cm} \times 0.4 \text{ cm}$  is fitted with negligible gap. Calculating the number of ampere-turns required to be applied to one half of the ring to produce a flux density of  $1.0 \text{ weber per metre}^2$  in the other half. Neglect leakage. The  $B$ - $H$  characteristics are as below :

	For Cast Steel			For Ordinary Plate			
$B$ in $\text{Wb/m}^2$	1.0	1.1	1.2	$B$ in $\text{Wb/m}^2$	1.2	1.4	1.45
Amp-turn/m	900	1020	1220	Amp-turn/m	590	1200	1650

(Elect. Technology, Indore Univ.)

**Solution.** The magnetic circuit is shown in Fig. 6.39.

The m.m.f. (or AT) produced on the half  $A$  acts across the parallel magnetic circuit  $C$  and  $D$ . First, total AT across  $C$  is calculated and since these amp-turns are also applied across  $D$ , the flux density  $B$  in  $D$  can be estimated. Next, flux density in  $A$  is calculated and therefore, the AT required for this flux density. In fact, the total AT (or m.m.f.) required is the sum of that required for  $A$  and that of either for the two parallel paths  $C$  or  $D$ .

Value of flux density in  $C = 1.0 \text{ Wb/m}^2$

Mean diameter of the ring =  $(24 + 18)/2 = 21 \text{ cm}$

Mean circumference =  $\pi \times 21 = 66$  cm

Length of path  $A$  or  $C = 66/2 = 33$  cm = 0.33 m

Value of AT/m for a flux density of 1.0

Wb/m<sup>2</sup> as seen from the given  $B.H$  characteristics = 900 AT/m

$\therefore$  Total AT for path  $C = 900 \times 0.33 = 297$ . The same ATs. are applied across path  $D$ .

Length of path  $D = 18$  cm = 0.18 m  $\therefore$  AT/m for path  $D = 297/0.18 = 1650$

Value of  $B$  corresponding to this AT/m from given table is = 1.45 Wb/m<sup>2</sup>

Flux through  $C = B \times A = 1.0 \times 9 \times 10^{-4} = 9 \times 10^{-4}$  Wb

Flux through  $D = 1.45 \times (3 \times 0.4 \times 10^{-4}) = 1.74 \times 10^{-4}$  Wb

$\therefore$  Total flux through  $A = 9 \times 10^{-4} + 1.74 \times 10^{-4} = 10.74 \times 10^{-4}$  Wb.

Flux density through  $A = 10.74 \times 10^{-4} / 9 \times 10^{-4} = 1.193$  Wb/m<sup>2</sup>

No. of AT/m reqd. to produce this flux density as read from the given table = 1200 (approx.)

$\therefore$  Amp-turns required for limb  $A = 1200 \times 0.33 = 396$

Total AT required = 396 + 297 = **693**

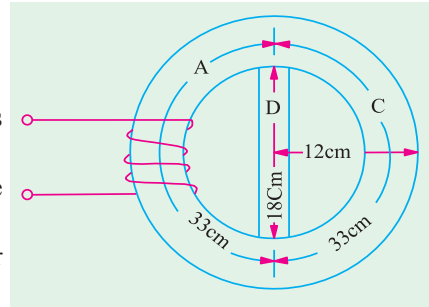


Fig. 6.39

**Example 6.23.** Show how the ampere-turns per pole required to produce a given flux in d.c. generator are calculated.

Find the ampere-turns per pole required to produce a flux of 40 mWb per pole in a machine with a smooth core armature and having the following dimensions :

Length of air gap = 5 mm      Area of air-gap = 500 cm<sup>2</sup>

Length of pole = 12 cm      Sectional area of pole core = 325 cm<sup>2</sup>

Relative permeability of pole core = 1,500

Length of magnetic path in yoke between pole = 65 cm

Cross-sectional area of yoke = 450 cm<sup>2</sup> ; Relative permeability of yoke = 1,200

Leakage coefficient = 1.2

The ampere-turns for the armature core may be neglected.

**Solution.** Air-gap  $\Phi = 40$  mWb =  $4 \times 10^{-2}$  Wb ;  $A = 500 \times 10^{-4} = 5 \times 10^{-3}$  m<sup>2</sup>

$\therefore B = 4 \times 10^{-2} / 5 \times 10^{-3} = 0.8$  Wb/m<sup>2</sup> ;  $H = B / \mu_0 = 0.8 / 4\pi \times 10^{-7} = 63.63 \times 10^4$  AT/m

Air-gap length =  $5 \times 10^{-3}$  m ; AT reqd. =  $63.63 \times 10^4 \times 5 \times 10^{-3} = 3181.5$

#### Pole Core

$\Phi = 1.2 \times 4 \times 10^{-2} = 4.8 \times 10^{-2}$  Wb ;  $A = 325 \times 10^{-4}$  m<sup>2</sup>

$B = 4.8 \times 10^{-2} / 325 \times 10^{-4} = 1.477$  Wb/m<sup>2</sup>

$H = B / \mu_0 \mu_r = 1.477 / 4\pi \times 10^{-7} \times 1,500 = 783$  AT/m

Pole length = 0.12 m ; AT reqd. =  $783 \times 0.12 = 94$

#### Yoke Path

flux = half the pole flux =  $0.5 \times 4 \times 10^{-2} = 2 \times 10^{-2}$  Wb

$A = 450$  cm<sup>2</sup> =  $45 \times 10^{-3}$  m<sup>2</sup> ;  $B = 2 \times 10^{-2} / 45 \times 10^{-3} = 4/9$  Wb/m<sup>2</sup>

$H = \frac{4/9}{4\pi \times 10^{-7} \times 1,200} = 294.5$  AT/m Yoke length = 0.65 m

At reqd =  $294.5 \times 0.65$ , Total AT/pole =  $3181.5 + 94 + 191.4 = \mathbf{3,467}$



**Example 6.24.** A shunt field coil is required to develop 1,500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of turn of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be  $2 \times 10^{-6} \mu\Omega\text{-cm}$  at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of  $3 \text{ A/mm}^2$ .

(Basis Elect. Machines Nagpur Univ. 1992)

**Solution.**

$$NI = 1,500 \text{ (given) or } N \cdot \frac{V}{R} = N \cdot \frac{60}{R} = 1,500$$

$$\therefore R = \frac{N}{25} \text{ ohm}$$

$$\text{Also } R = \rho \cdot \frac{l}{A} = \frac{2 \times 10^{-6} \times 50 N}{A}$$

$$\therefore \frac{N}{25} = \frac{10^{-4} n}{A}$$

$$\text{or } A = 25 \times 10^{-4} \text{ cm}^2 \text{ or } A = 0.25 \text{ mm}^2$$

$$\therefore \frac{\pi D^2}{4} = 0.25$$

$$\text{or } D = \mathbf{0.568 \text{ mm}}$$

$$\text{Current in the coil} = 3 \times 0.25 = 0.75 \text{ A}$$

Now,

$$NI = 1,500;$$

$$\therefore N = 1,500/0.75 = \mathbf{2,000}$$

**Example 6.25.** A wooden ring has a circular cross-section of 300 sq. mm and a mean diameter of the ring is 200 mm. It is uniformly wound with 800 turns.

Calculate :

(i) the field strength produced in the coil by a current of 2 amperes : (assume = 1)

(ii) the magnetic flux density produced by this current and

(iii) the current required to produce a flux density of  $0.02 \text{ wb/m}^2$ .

[Nagpur University (Summer 2000)]

**Solution.** The question assumes that the flux-path is through the ring, as shown by the dashed line, in figure, at the mean diameter, in Fig. 6.40.

With a current of 2 amp,

$$\text{Coil m.m.f.} = 800 \times 2 = 1600 \text{ AT}$$

$$\begin{aligned} \text{Mean length of path} &= \pi \times 0.2 \\ &= 0.628 \text{ m} \end{aligned}$$

$$(i) \quad H = \frac{1600}{0.628} = 2548 \text{ amp-turns/meter}$$

$$(ii) \quad B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 2548 \\ = 3.20 \times 10^{-3} \text{ Wb/m}^2$$

This Flux density is produced by a coil current of 2-amp

(iii) For producing a flux of  $0.02 \text{ Wb/m}^2$ , the coil current required is

$$2 \times \frac{0.02}{0.0032} = \mathbf{12.5 \text{ amp}}$$

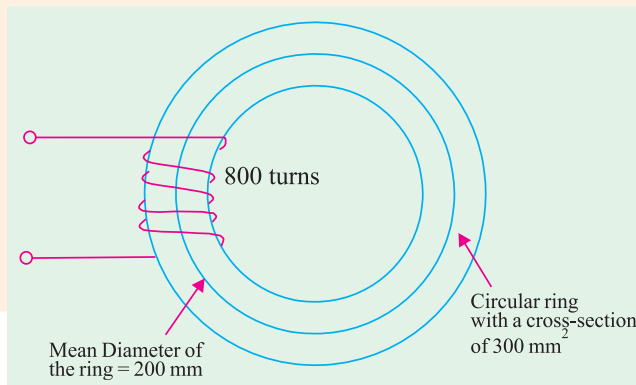


Fig. 6.40

**Example 6.26.** A magnetic core in the form of a closed circular ring has a mean length of 30 cm and a cross-sectional area of  $1 \text{ cm}^2$ . The relative permeability of iron is 2400. What direct-current will be needed in the coil of 2000 turns uniformly wound around the ring to create a flux of  $0.20 \text{ mWb}$  in iron? If an air-gap of 1 mm is cut through the core perpendicular to the direction of this flux, what current will now be needed to maintain the same flux in the air gap?

[Nagpur University Nov. 1999]

**Solution.**

$$\begin{aligned} \text{Reluctance of core} &= \frac{1}{\mu_0 \mu_r} \frac{L}{a} = \frac{1}{10\pi \times 10^{-7} \times 2400} \times \frac{30 \times 10^2}{1 \times 10^{-4}} \\ &= \frac{30 \times 10^{-9}}{4\pi \times 2400 \times 1} = 995223 \text{ MKS units} \end{aligned}$$

$$\phi = 0.2 \times 10^{-3} \text{ Wb}$$

$$\begin{aligned} \text{MMF required} &= \phi \times \text{Rel} \\ &= 0.2 \times 10^{-3} \times 995223 = 199 \text{ amp-turns} \end{aligned}$$

Direct current required through the 2000 turn coil

$$= \frac{199}{2000} = 0.0995 \text{ amp}$$

Reluctance of 1 mm air gap

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{1 \times 10^{-3}}{1 \times 10^{-4}} = \frac{10^8}{4\pi} = 7961783 \text{ MKS units}$$

Addition of two reluctances

$$= 995223 + 7961783 = 8957006 \text{ MKS units}$$

MMF required to establish the given flux

$$= 0.2 \times 10^{-3} \times 8957006 = \mathbf{1791 \text{ amp turns}}$$

Current required through the coil

$$= \frac{1791}{2000} \quad \mathbf{0.8955 \text{ amp}}$$

**Note :** Due to the high permeability of iron, which is given here as 2400, 1 mm of air-gap length is equivalent magnetically to 2400 mm of length through the core, for comparison of mmf required.

**Example 6.27.** An iron-ring of mean length 30 cm is made up of 3 pieces of cast-iron. Each piece has the same length, but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 – cm. Piece. If a coil of 1000 turns is wound on the ring, find the value of the current has to carry to produce a flux density of 0.5 Wb/m<sup>2</sup> in the air gap. B-H curve data of cast-iron is as follows :

$B$ (Wb/m <sup>2</sup> ) :	0.10	0.20	0.30	0.40	0.50	0.60
$H$ (AT/m) :	280	680	990	1400	2000	2800

Permeability of free space =  $4\pi \times 10^{-7}$

Neglect Leakage and fringing effects.

[Nagpur University, November 1998]

**Solution.** From the data given, plot the B-H curve for cast-iron

The magnetic circuit has four parts connected in series

**Part 1.** Air-gap 0.5 mm length,  $B = 0.5 \text{ wb/m}^2$ , and

Permeability of free sapce is known

$$H_1 = B/\mu_0 = 0.5 \times 10^7/(4\pi) = 398100$$

$$\text{AT for gap} = (0.5 \times 10^{-3}) \times H_1 = 199$$

**Part 2.** 2.5 cm diameter, 10 – cm long, cast-iron ring portion  $B$  and  $H$  are to be related with the help of given data. In this, out of 10 cms. 0.5 mm air-gap is cut, and this portion of ring will have cast-iron length of 99.5 mm.

For

$$B = 0.5 \text{ wb/m}^2, \quad H_2 = 2000 \text{ AT/m}$$

$$\text{AT}_2 = 2000 \times 9.95 \times 10^{-2} = 199$$

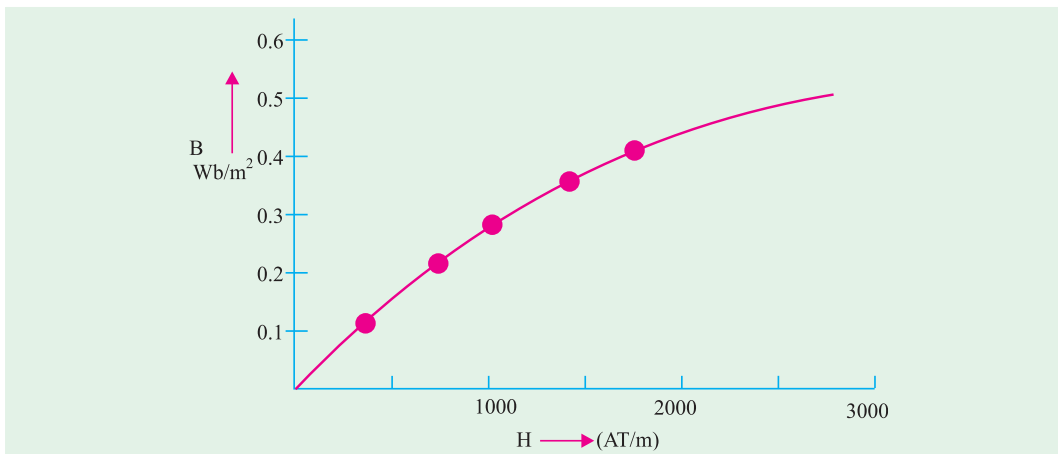


Fig. 6.41

**Part 3.** 3-cm diameter, 10-cm long, cast-iron ring-portion.

$$\text{Here } B = 0.50 \times (2.5/3)^2 = 0.347 \text{ Wb/m}^2$$

For this  $B$ ,  $H$  is read from  $B$ - $H$  curve.

$$H_3 = 1183 \text{ AT/m}$$

$$AT_3 = 1183 \times 10 \times 10^{-2} = 118.3$$

**Part 4.** 4 cm. Diameter, 10 cm long, cast-iron ring portion.

$$\text{Here, } B = 0.50 \times (2.5 \times 4)^2 \times 0.195 \text{ Wb/m}^2$$

From,  $B$ - $H$  curve, corresponding  $H$  is 661

$$AT_4 = 661 \times 10 \times 10^{-2} = 66 \text{ AT}$$

Since all these four parts in series, the total m.m.f. required is obtained by adding the above terms.

$$AT = 199 + 199 + 118 + 66 = 582$$

$$\text{Coil Current} = 582/1000 = 0.582 \text{ amp}$$

**Additional observations.**

(a) The 2.5-cm diameter portion of the ring has  $H = 2000$  for  $B = 0.5 \text{ Wb/m}^2$ . From this, the relative permeability of cast-iron can be found out.

$$\mu_0 \mu_r = 0.5/2000, \text{ giving } \mu_r = 199$$

An air-gap of 0.5 mm is equivalent of 99.5 mm of cast-iron length. Hence, the two m.m.fs. are equal to 199 each.

(b) The common flux for this circuit is obtained from flux-density and the concerned area.

$$\begin{aligned} \text{Hence } \phi &= 0.5 \times (\pi/4) \times (2.5 \times 10^{-2})^2 = 0.02453 \times 10^{-2} \\ &= 0.2453 \text{ mWb} \end{aligned}$$

Reluctance of total magnetic circuit

$$\begin{aligned} &= \text{m.m.f./flux} = 582/(2.453 \times 10^{-4}) \\ &= 2372650 \text{ MKS units} \end{aligned}$$

**Example 6.28.** A steel-ring of 25 cm mean diameter and of circular section 3 cm in diameter has an air-gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 amp. Calculate : (i) Magneto motive force (ii) Flux density (iii) Magnetic flux (iv) Relative permeability. Neglect magnetic leakage and assume that iron path takes 35 % of total magneto motive force. [Nagpur University, April 1996]

**Solution.** From the given data, length of mean path in the ring ( $= L_m$ ) is to be calculated. For a mean diameter of 25 cm, with 1.5 mm of air-gap length.

$$L_m = (\pi \times 0.25) - (1.5 \times 10^{-3}) = 0.7835 \text{ m}$$

$$\text{Cross-sectional area of a 3 cm diameter ring} = 7.065 \times 10^{-4} \text{ sq.m.}$$

$$\text{Total m.m.f. due to coil} = 700 \times 2 = 1400 \text{ amp-turns}$$

Since iron-path takes 355 of the total mmf, it is 490.

Remaining mmf of 910 is consumed by the air-gap.

$$\text{Corresponding } H \text{ for air-gap} = 910 / (1.5 \times 10^{-3}) = 606666 \text{ amp-turns/m.}$$

If Flux density is  $B_g$ , we have

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 606666 = 0.762 \text{ Wb/m}^2$$

Iron-ring and air-gap are in series hence their flux is same. Since the two have same cross-sectional area, the flux density is also same. The ring has a mean length of 0.7835 m and needs an mmf of 490. For the ring.

$$H = 490 / 0.7845 = 625.4 \text{ amp-turns / m}$$

$$\mu_0 \mu_r = B/H = 0.752 / 625.4 = 1.218 \times 10^{-3}$$

$$\mu_r = (1.218 \times 10^{-3}) / (4\pi \times 10^{-7}) = 970$$

$$\text{Flux} = \text{Flux density} \times \text{Cross-sectional area} = 0.762 \times 7.065 \times 10^{-4} = 0.538 \text{ milli-webers}$$

**Check.**  $\mu_r$  of 970 means that 1.5 mm of air-gap length is equivalent to  $(1.5 \times 10^{-3} \times 970) = 1.455$  m of length through iron as a medium. With this equivalent.

$$\frac{\text{mmf of ring}}{\text{mmf for (ring + air-gap)}} = \frac{0.785}{0.785 + 1.455} = 0.35$$

which means that 35 % of total mmf is required for the ring

**Example 6.29. (a)** Determine the amp-turns required to produce a flux of 0.38 mWb in an iron-ring of mean diameter 58 cm and cross-sectional area of 3 sq. cm. Use the following data for the ring :

$B \text{ Wb/m}^2$	0.5	1.0	1.2	1.4
$\mu_r$	2500	2000	1500	1000

**(b)** If a saw-cut of 1mm width is made in the ring, calculate the flux density in the ring, with the mmf remaining same as in (a) above. **[Nagpur University, Nov. 1996]**

**Solution.** Plot the  $B$ - $\mu_r$  curve as in Fig. 6.42

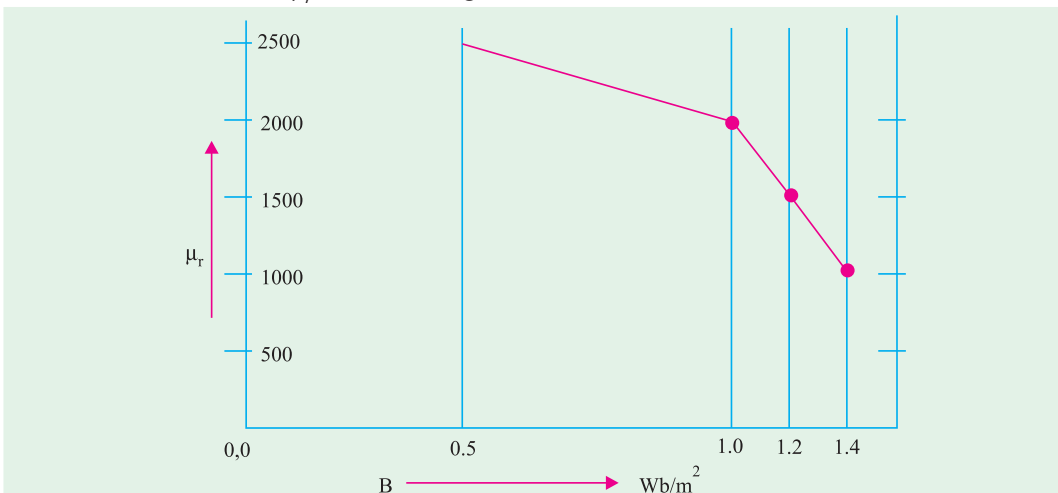


Fig. 6.42

(a) Cross-sectional area = 3 sq. cm =  $3 \times 10^{-4}$  sq. m.

$$\text{Flux} = 38 \text{ mWb} = 0.38 \times 10^{-3} \text{ Wb}$$

$$\text{Flux density, } B = \text{flux/area} = (0.38 \times 10^{-3}) / (3 \times 10^{-4}) = 1.267 \text{ Wb/m}^2$$

Looking into the table relating  $B$  and  $\mu_r$ , interpolation is required for evaluating  $\mu_r$  for  $B = 1.267$  Wb/m<sup>2</sup>. After 1.2 Wb/m<sup>2</sup>,  $\mu_r$  decreases by 500 for a rise of 0.2 in  $B$ . Interpolation results into :

$$\mu_r = 1500 - \frac{0.067}{0.20} \times 500 = 1332$$

For mean diameter of path in the ring as 0.58 m, the length of the magnetic path in the ring is

$$l_m = p \times 0.58 = 1.8212 \text{ m}$$

Since

$$B = \mu_0 \mu_r H,$$

$$H = 1.267 / (4\pi \times 10^{-7} \times 1332) = 757$$

Hence, the required m.m.f. is

$$757 \times 1.8212 = 1378 \text{ amp-turns}$$

(b) If a saw-cut of 1 mm is cut in the ring,  $B$  is to be calculated with a m.m.f. of 1378. Now the magnetic circuit has two components in series : the ring with its  $B$ - $\mu_r$  curve in Fig. 6.42 and the air-gap. Since  $B$  is not known,  $\mu_r$  cannot be accurately known right in the initial steps. The procedure to solve the case should be as follows :

Let  $B$  the flux density as a result of 1378 amp-turns due to the coil.

**For air-gap.**

$$H_g = B_g / (4\pi \times 10^{-7}) = 0.796 \times 10^6 \text{ AT/m}$$

$$AT_g = H_g \times I_g = 0.796 \times 10^6 \times 1 \times 10^{-3} \times B_g = 796 B_g$$

Due to the air-gap, the flux-density is expected to be between 0.5 and 1 Wb/m<sup>2</sup>, because, in (a) above,  $\mu_r$  (for  $B = 1.267$  Wb/m<sup>2</sup>) is 1332. One mm air-gap is equivalent to 1332 mm of path added in iron-medium. This proportional increase in the reluctance of the magnetic circuit indicates that flux density should fall to a value in between 0.5 and 1 Wb/m<sup>2</sup>.

**For Iron-ring.** With flux density expected to be as mentioned above, interpolation formula for  $\mu_r$  can be written as :

$$\mu_r = 2500 - 500 [(B_g - 0.50) / 0.50] = 3000 - 1000 B_g$$

$$H_i = B_g / (\mu_0 \mu_r) = B_g / [\mu_0 (3000 - 1000 B_g)]$$

$$\text{Total m.m.f.} = AT_g + AT_i = 1378, \text{ as previously calculated}$$

$$\text{Hence, } 1378 = \frac{1.8212 \times B_g}{\mu_0 (3000 - 1000 B_g)} + 796 B_g$$

This is a quadratic equation in  $B_g$  and the solution, which gives  $B_g$  in between 0.5 & 1.0 Wb/m<sup>2</sup> is acceptable.

This results into

$$B_g = 0.925 \text{ Wb/m}^2$$

Corresponding

$$\mu_r = 3000 - 1000 \times 0.925 = 2075$$

**Example 6.30.** An iron-ring of mean diameter 19.1 cm and having a cross-sectional area of 4 sq. cm is required to produce a flux of 0.44 mWb. Find the coil-mmf required.

If a saw-cut 1 mm wide is made in the ring, how many extra amp-turns are required to maintain the same flux ?

$B$  -  $\mu_r$  curve is as follows :

$B$ (Wb/m <sup>2</sup> )	0.8	1.0	1.2	1.4
$\mu_r$	2300	2000	1600	1100

[Nagpur University, April 1998]

**Solution.** For a mean-diameter of 19.1 cm, Length of mean path,  $l_m = \pi \times 0.191 = 0.6$  m

Cross-sectional area = 4 sq.cm =  $4 \times 10^{-4} \text{ m}^2$

Flux,  $\phi = 0.44 \text{ mWb} = 0.44 \times 10^{-3} \text{ Wb}$

Flux density,  $B = 0.44 \times 10^{-3} / (4 \times 10^{-4}) = 1.1 \text{ Wb/m}^2$

For this flux density,  $\mu_r = 1800$ , by simple interpolation.

$H = B / (\mu_0 \mu_r) = 1.1 \times 10^7 / (4\pi \times 1800) = 486.5 \text{ amp-turns/m}$ .

m.m.f required =  $H \times l_m = 486.5 \times 0.60 = 292$

A saw-cut of 1 mm, will need extra mmf.

$H_g = B_g / \mu_0 = 1.1 \times 10^7 / (4\pi) = 875796$

$AT_g = H_g \times l_g = 875796 \times 1.0 \times 10^{-3} = 876$

Thus, additional mmf required due to air-gap = 876 amp-turns

**Example 6.31.** A 680-turn coil is wound on the central limb of a cast steel frame as shown in Fig. 6.43 (a) with all dimensions in cms. A total flux of 1.6 mWb is required in the air-gap. Find the current in the magnetizing coil. Assume uniform flux distribution and no leakage. Data for B-H curve for cast steel is given. [Nagpur University, November 1997]

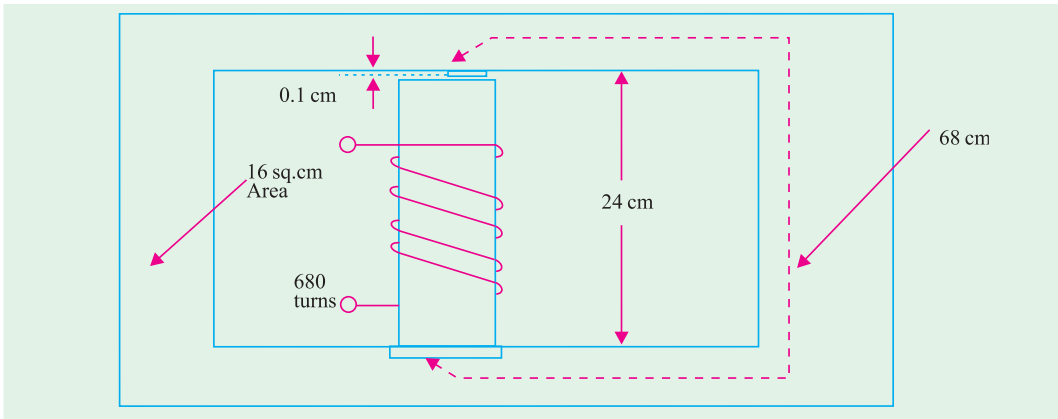


Fig. 6.43 (a)

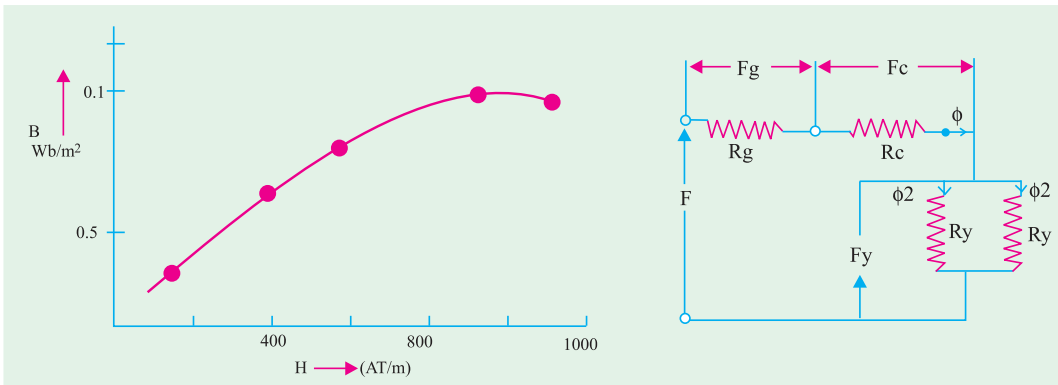


Fig. 6.43 (b)

Fig. 6.43 (c)

**Solution.**

$\phi = 1.6 \text{ mWb}$  through air-gap and central limb

$\phi/2 = 0.8 \text{ mWb}$  through yokes

Corresponding flux densities are :

$$B_g = B_c = 1.6 \text{ mWb} / (16 \times 10^{-4}) = 1.0 \text{ Wb/m}^2$$

$$B_y = 0.8 \text{ m Wb}/(16 \times 10^{-4}) = 0.50 \text{ Wb}/\text{m}^2$$

MMF-Calculations :

(a) For Air gap : For  $B_g$  of  $1 \text{ Wb}/\text{m}^2$ ,  $H_g = 1.0/\mu_o$

$$\begin{aligned} AT_g = H_g \times l_g &= [1/(4\pi \times 10^{-7})] \times (0.1 \times 10^{-2}) \\ &= 796 \text{ amp turns} \end{aligned}$$

(b) For Central limb :  $AT_c = H_c \times l_c = 900 \times 0.24 = 216$

$\therefore$  For  $B_c = 1.00$ ,  $H_c$  from data =  $900 \text{ AT}/\text{m}$

The yokes are working at a flux-density of  $0.50 \text{ Wb}/\text{m}^2$ . From the given data and the corresponding plot, interpolation can be done for accuracy.

$$\begin{aligned} H_y &= 500 + [(0.5 - 0.45)/(0.775 - 0.45)] \times 200 \\ &= 530 \text{ AT}/\text{m} \end{aligned}$$

$$F_y = 530 \times 0.68 = 360$$

$$\text{Total mmf required} = 796 + 216 + 360 = 1372$$

$$\text{Hence, coil-current} = 1372/680 = 2.018 \text{ A}$$

**Example 6.32.** For the magnetic circuit shown in fig. 6.44 the flux in the right limb is  $0.24 \text{ mWb}$  and the number of turns wound on the central-limb is 1000. Calculate (i) flux in the central limb (ii) the current required.

The magnetization curve for the core is given as below :

$H \text{ (AT/m)}$ :	200	400	500	600	800	1060	1400
$B \text{ (Nb}/\text{m}_2)$ :	0.4	0.8	1.0	1.1	1.2	1.3	1.4

Neglect Leakage and fringing. [Rajiv Gandhi Technical University, Bhopal, Summer 2001]

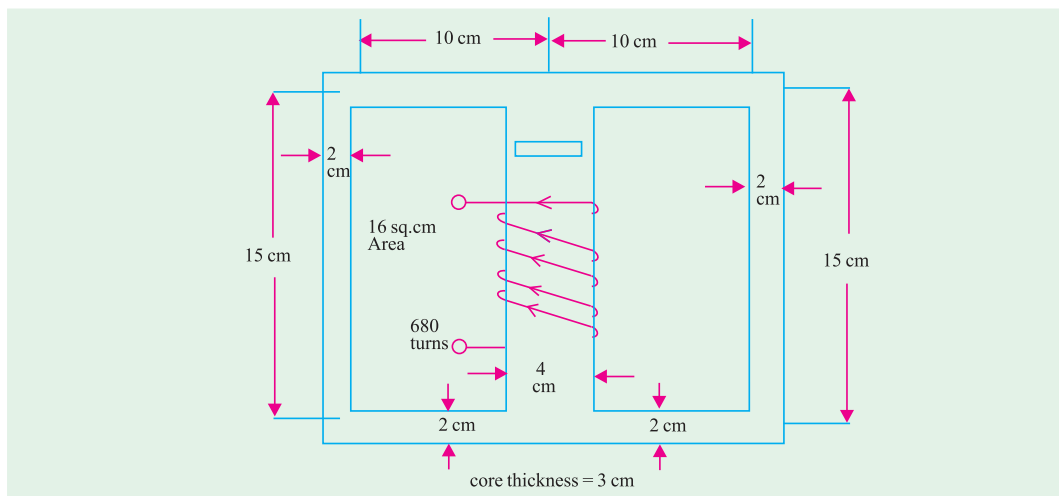


Fig. 6.44

**Solution.** Area of cross-section of side-limbs =  $2 \times 3 = 6 \text{ sq.cm}$

Area of cross-section of core =  $3 \times 4 = 12 \text{ sq.cm}$

Flux in side Limbs =  $0.24 \text{ mWb}$

Flux density in side Limbs =  $(0.24 \times 10^{-3})/(6 \times 10^{-4}) = 0.4 \text{ Wb}/\text{m}^2$

Since the coil is wound on the central limb and the magnetic circuit is symmetrical, the flux in the central limb =  $0.48 \text{ mWb}$ . Flux density in the central limb =  $(0.48 \times 10^{-3})/(12 \times 10^{-4}) = 0.4 \text{ Wb}/\text{m}^2$

For the flux density of  $0.40 \text{ Wb}/\text{m}^2$ ,  $H = 200 \text{ AT}/\text{m}$

Central Limb has a path length of 15 cm.

Other part carrying 0.24 mWb has a total path length of 35 cm.

Total mmf required =  $(200 \times 0.15) + (200 \times 0.35) = 100 \text{ AT}$

Hence, coil current =  $100/1000 = 0.1 \text{ Amp}$ .

**Example 6.33.** A ring composed of three sections. The cross-sectional area is  $0.001 \text{ m}^2$  for each section. The mean arc length are  $l_a = 0.3 \text{ m}$ ,  $l_b = 0.2 \text{ m}$ ,  $l_c = 0.1 \text{ m}$ . An air-gap length of  $0.1 \text{ mm}$  is cut in the ring.  $\mu_r$  for sections a, b, c are 5000, 1000, and 10,000 respectively. Flux in the air gap is  $7.5 \times 10^{-4} \text{ Wb}$ . Find (i) mmf (ii) exciting current if the coil has 100 turns, (iii) reluctances of the sections. [Nagpur University April 1999]

**Solution.**

Area =  $0.001 \text{ sq.m}$

$$l_a = 0.3 \text{ m}, l_b = 0.2 \text{ m}, l_c = 0.1 \text{ m}, l_g = 0.1 \times 10^{-3} \text{ m}$$

$$\mu_{ra} = 5000, \mu_{rb} = 1000, \mu_{rc} = 10,000 \quad \mu_o = 4\pi \times 10^{-7}$$

$$\phi = 7.5 \times 10^{-4} \text{ Wb}$$

(iii) Calculations of Reluctances of four parts of the magnetic circuit :

(a) Reluctance of air gap,  $R_{eg} = \frac{1}{\mu_o} \times \frac{0.1 \times 10^{-3}}{0.001} = \frac{1000}{4\pi \times 0.001} = 79618$

(b) Reluctance of section 'a' of ring

$$= R_{ea} = \frac{1}{\mu_o \mu_{ra}} \times \frac{0.3}{0.001} = \frac{10^7 \times 0.3}{4\pi \times 47770 \times 5000 \times 0.001} = 47770$$

(c) Reluctance of section 'b' of the ring

$$= R_{eb} = \frac{1}{\mu_o \mu_{rb}} \times \frac{0.20}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 15923.6$$

(d) Reluctance of section 'c' of the ring

$$= R_{ec} = \frac{1}{\mu_o \mu_{rc}} \times \frac{0.10}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 7961$$

$$\text{Total Reluctance} = R_{eg} + R_{ea} + R_{eb} + R_{ec} = 294585$$

(i) Total mmf required = Flux  $\times$  Reluctance  
 $= 7.5 \times 10^{-4} \times 294585 = 221 \text{ amp-turns}$

(ii) Current required =  $221/100 = 2.21 \text{ amp}$

### Tutorial Problems No. 62

- An iron specimen in the form of a closed ring has a 350-turn magnetizing winding through which is passed a current of 4A. The mean length of the magnetic path is 75 cm and its cross-sectional area is  $1.5 \text{ cm}^2$ . Wound closely over the specimen is a secondary winding of 50 turns. This is connected to a ballistic galvanometer in series with the secondary coil of 9-mH mutual inductance and a limiting resistor. When the magnetising current is suddenly reversed, the galvanometer deflection is equal to that produced by the reversal of a current of 1.2 A in the primary coil of the mutual inductance. Calculate the  $B$  and  $H$  values for the iron under these conditions, deriving any formula used.

[1.44 Wb/m<sup>2</sup> ; 1865 AT/m] (London Univ.)

- A moving-coil ballistic galvanometer of  $150 \Omega$  gives a throw of 75 divisions when the flux through a search coil, to which it is connected, is reversed.

Find the flux density in which the reversal of the coil takes place, given that the galvanometer constant is  $110 \mu\text{C}$  per scale division and the search coil has 1400 turns, a mean area of  $50 \text{ cm}^2$  and a resistance of  $20 \Omega$  [0.1 Wb/m<sup>2</sup>] (Elect. Meas. & Measuring Inst. Gujarat Univ.)



3. A fluxmeter is connected to a search coil having 500 turns and mean area of  $5 \text{ cm}^2$ . The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5 A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the flux-meter constant.  
**[ $10^{-4} \text{ Wb-turn/division}$ ] (Elect. Meas. & Measuring Inst., M.S. Univ. Baroda)**
4. An iron ring of mean length 50 cms has an air gap of 1 mm and a winding of 200 turns. If the permeability of iron is 300 when a current of 1 A flows through the coil, find the flux density.  
**[ $94.2 \text{ mWb/m}^3$ ] (Elect. Engg. A.M.Ae.S.I.)**
5. An iron ring of mean length 100 cm with an air gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. When a current of 3 A flows in the winding, determine the flux density. Neglect fringing.  
**[ $0.523 \text{ Wb/m}^2$ ] (Elect. Engg. & Electronic Bangalore Univ. 1990)**
6. A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having a mean circumference of 40 mm and cross-sectional area of  $50 \text{ mm}^2$ . If a current of 25 amps is passed through the coil, find (i) m.m.f. (ii) reluctance of the ring and (iii) flux.  
**[(i)  $7500 \text{ AT}$  (ii)  $0.7 \times 10^6 \text{ AT/Wb}$  (iii)  $10.7 \text{ mWb}$ ]  
 (Elect. Engg. & Electronics Bangalore Univ.)**
7. A specimen ring of transformer stampings has a mean circumference of 40 cm and is wound with a coil of 1,000 turns. When the currents through the coil are 0.25 A, 1 A and 4 A the flux densities in the stampings are 1.08, 1.36 and  $1.64 \text{ Wb/m}^2$  respectively. Calculate the relative permeability for each current and explain the differences in the values obtained.  
**[1,375,434,131]**
8. A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area  $12 \text{ cm}^2$  throughout. A current of 2A in the magnetising coil of 200 turns produces a total flux of 1.2 mWb in the iron. Calculate :  
 (a) the flux density in the iron  
 (b) the absolute and relative permeabilities of iron  
 (c) the reluctance of the circuit  
**[ $1 \text{ Wb/m}^2$  ; 0.002, 1,590 ;  $3.33 \times 10^5 \text{ AT/Wb}$ ]**
9. A coil of 500 turns and resistance  $20 \Omega$  is wound uniformly on an iron ring of mean circumference 50 cm and cross-sectional area  $4 \text{ cm}^2$ . It is connected to a 24-V d.c. supply. Under these conditions, the relative permeability of iron is 800. Calculate the values of :  
 (a) the magnetomotive force of the coil  
 (b) the magnetizing force  
 (c) the total flux in the iron  
 (d) the reluctance of the ring  
**[(a)  $600 \text{ AT}$  (b)  $1,200 \text{ AT/m}$  (c)  $0.483 \text{ mWb}$  (d)  $1.24 \times 10^6 \text{ AT/Wb}$ ]**
10. A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is  $6 \text{ cm}^2$  and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :
- |                                    |     |       |       |       |                 |
|------------------------------------|-----|-------|-------|-------|-----------------|
| Flux density ( $\text{Wb/m}^2$ ) : | 1.2 | 1.35  | 1.45  | 1.55  |                 |
| Magnetizing force (AT/m) :         | 500 | 1,000 | 2,000 | 4,500 | <b>[6.35 A]</b> |
11. An iron-ring of mean length 30 cm is made of three pieces of cast iron, each has the same length but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 cm piece. If a coil of 1,000 turns is wound on the ring, find the value of the current it has to carry to produce a flux density of  $0.5 \text{ Wb/m}^2$  in the air gap.  $B/H$  characteristic of cast-iron may be drawn from the following :
- |                       |     |     |     |       |       |        |                 |
|-----------------------|-----|-----|-----|-------|-------|--------|-----------------|
| $B (\text{Wb/m}^2)$ : | 0.1 | 0.2 | 0.3 | 0.4   | 0.5   | 0.6    |                 |
| (AT/m) :              | 280 | 620 | 990 | 1,400 | 2,000 | 2,8000 | <b>[0.58 A]</b> |
- Permeability of free space =  $4\pi \times 10^7 \text{ H/m}$ . Neglect leakage and fringing.
12. The length of the magnetic circuit of a relay is 25 cm and the cross-sectional area is  $6.25 \text{ cm}^2$ . The length of the air-gap in the operated position of the relay is 0.2 mm. Calculate the magnetomotive force required to produce a flux of 1.25 mWb in the air gap. The relative permeability of magnetic material at this flux density is 200. Calculate also the reluctance of the magnetic circuit when the

relay is in the unoperated position, the air-gap then being 8 mm long (assume  $\mu_r$  remains constant).

[2307 AT,  $1.18 \times 10^7$  AT/Wb]

13. For the magnetic circuit shown in Fig. 6.45, all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the centre limb as shown.

Calculate the m.m.f. required to produce a flux of 1.7 mWb in the centre limb. Neglect the leakage and fringing. The magnetisation data for the material is as follows :

$H$ (AT/m) :	400	440	500	600	800	
$B$ (Wb/m <sup>2</sup> ) :	0.8	0.9	1.0	1.1	1.2	[1,052 AT]

14. In the magnetic circuit shown in Fig. 6.46 a coil of 500 turns is wound on the centre limb. The magnetic paths  $A$  to  $B$  by way of the outer limbs have a mean length of 100 cm each and an effective cross-sectional area of 2.5 cm<sup>2</sup>. The centre limb is 25 cm long and 5 cm<sup>2</sup> cross-sectional area. The air-gap is 0.8 cm long. A current of 9.2 A through the coil is found to produce a flux of 0.3 mWb.

15. The magnetic circuit of a choke is shown in Fig. 6.47. It is designed so that the flux in the central core is 0.003 Wb. The cross-section is square and a coil of 500 turns is wound on the central core. Calculate the exciting current. Neglect leakage and assume the flux to be uniformly distributed along the mean path shown dotted. Dimensions are in cm.

The characteristics of magnetic circuit are as given below :

$B$ (Wb/m <sup>2</sup> ) :	0.38	0.67	1.07	1.2	1.26
$H$ (AT/m) :	100	200	600	1000	1400

(Elect. Technology I. Gwalior Univ.)

16. A 680-turn coil is wound on the central limb of the cast steel sheet frame as shown in Fig. 6.48 where dimensions are in cm. A total flux of 1.6 mWb is required to be in the gap. Find the current required in the magnetising coil. Assume gap density is uniform and all lines pass straight across the gap. Following data is given :

$H$ (AT/m) :	300	500	700	900	1100
$B$ (Wb/m <sup>2</sup> ) :	0.2	0.45	0.775	1.0	1.13

(Elect. Technology ; Indore Univ.)

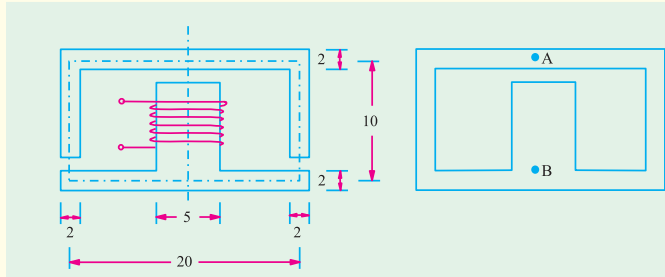


Fig. 6.45

Fig. 6.46

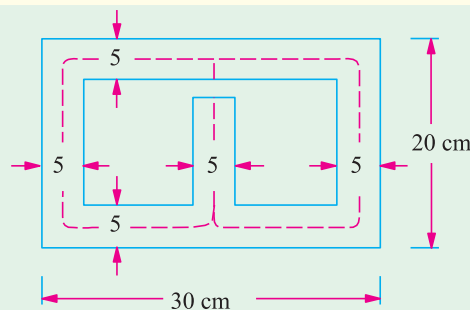


Fig. 6.47

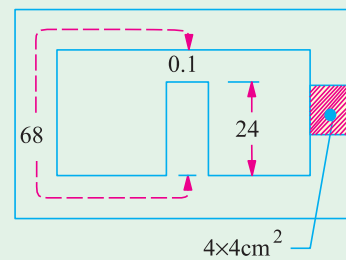


Fig. 6.48

17. In the magnetic circuit of Fig. 6.49, the core is composed of annealed sheet steel for which a stacking factor of 0.9 should be assumed. The core is 5 cm thick. When  $\Phi_A = 0.002$  Wb,  $\Phi_B = 0.0008$  Wb and  $\Phi_C = 0.0012$  Wb. How many amperes much each coil carry and in what direction ? Use of the

following magnetisation curves can be made for solving the problem.

$B$ ( $\text{Wb/m}^2$ ):	0.2	0.4	0.6	0.8	1.0	1.4	1.6	1.8
$H$ ( $\text{AT/m}^2$ ):	50	100	130	200	320	1200	3800	10,000

(Elect. Technology, Vikram Univ.)

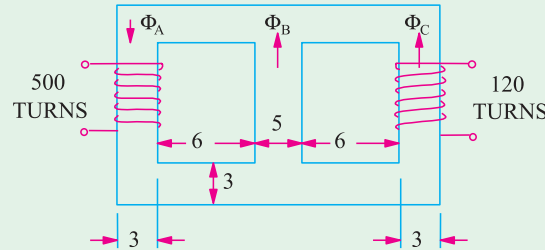


Fig. 6.49

18. A magnetic circuit with a uniform cross-sectional area of  $6 \text{ cm}^2$  consists of a steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetization curve of cast steel are :

$B$ ( $\text{Wb/m}^2$ ):	0.12	0.14	0.16	0.18	0.20
$H$ ( $\text{AT/m}$ ):	200	230	260	290	320

[0.1128 m Wb] (City & Guilds, London)

19. Explain the terms related to magnetic circuits :

(i) reluctance (ii) flux density (iii) magnetomotive force (Nagpur University, Summer 2002)

20. A metal ring of mean diameter of 80 cm is made out of two semi-circular pieces of cast iron and cast steel separated at junctions by pieces of copper each of 1 mm thickness. If the ring is uniformly wound with 1000 turns, calculate the value of current required to produce a flux density of 0.85 wb/nV in the ring.

Given that relative permeability of cast iron as 200, that of cast steel is 1200 and for copper,  $\mu_r = 1$ .

(Nagpur University, Summer 2002)

21. A 1154 turns coil is wound on the central limb of the case steel frame shown in Fig. 6.50. A total flux of 1.6 mwb is required in the air gap. Find the current required through the gap. Assume that the gap density is uniform and there is no leakage. Frame dimensions are given in cm. Take permeability of cast steel as 1,200.

(Nagpur University, Winter 2002)

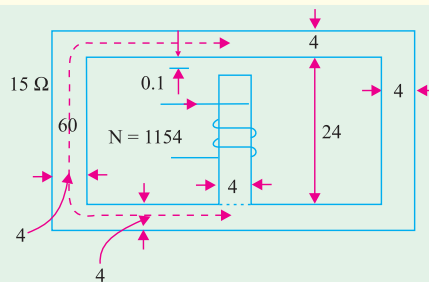


Fig. 6.50

22. Explain the terms related to magnetic circuits :

(i) Reluctance (ii) Flux density (iii) Coercive force (iv) Magnetomotive force (v) Residual flux.

(Nagpur University, Summer 2003)

23. Compare electric and magnetic circuit by their similarities and dissimilarities.

(Nagpur University, Winter 2003)

24. Compare electric and magnetic circuits with respect to their similarities and dissimilarities.

(Nagpur University, Summer 2004)

- 25.** A steel wire of 25 cm mean diameter and circular cross section 3 cm in diameter has an airgap of 1 mm wide. It is wound with a coil of 700 turns carrying a current of 2 A.  
Calculate : (i) m.m.f. (ii) Flux density (iii) Reluctance (iv) Relative permeability.  
Assume that iron path take 30% of total m.m.f. *(Gujrat University, Summer 2003)*
- 26.** What is a search coil in magnetic measurements? *(Anna University, April 2002)*
- 27.** Name the magnetic squares used to find iron loss. *(Anna University, April 2002)*
- 28.** What is a magnetic circuit? A magnetic circuit is made up of 3 limbs A, B and C in parallel. The reluctances of the magnetic paths of A, B and C in AT/mWb are 312, 632.6 and 520 respectively. An exciting coil of 680 turns is wound on limb B. Find the exciting current to produce of flux of lmwb in the limb A. *(V.T.U., Belgaum Karnataka University, February 2002)*
- 29.** An iron ring of 300cm mean circumference with a cross section of 5cm<sup>2</sup> is wound uniformly with 350 turns of wire. Find the current required to produce a flux of 0.5 Mw in iron. Take relative permeability of iron as 400. *(V.T.U. Belgaum Karnataka University, July/August 2002)*
- 30.** What is Biot-Savart law? Explain briefly. Find the magnetic field due to a small circular loop carrying current I at distances from loop that are large compared with its dimensions.  
*(Agra Univ. 1978 Supp.)*
- 31.** Magnetic potential *(Mumbai University, 2002) (RGPV, Bhopal 2001)*
- 32.** Flux density *(Pune University, 2002) (RGPV, Bhopal 2001)*
- 33.** Susceptibility *(Mumbai University, 2002) (RGPV, Bhopal 2001)*
- 34.** Define mmf, flux, reluctance, absolute and relative permeabilities with reference to magnetic circuits.  
*(U.P. Technical University 2003) (RGPV, Bhopal 2002)*
- 35.** Discuss B-H curve of a ferro-magnetic material and explain the following.  
(i) Magnetic saturation (ii) Hysteresis (iii) Residual magnetism (iv) Coercive force  
*(RGPV, Bhopal 2002)*
- 36.** What is meant by leakage and fringing? Define leakage coefficient.  
*(RGPV, Bhopal 2002)*
- 37.** Define the following terms (any five) :  
(i) MMF (ii) Reluctance (iii) Permeance (iv) Magnetisation curve (v) flux density  
(vi) Magnetizing force (vii) Susceptibility (viii) Relative permeability (ix) Magnetic potential  
*(RGPV, Bhopal 2002)*
- 38.** Distinguish between leakage and fringing of flux. *(RGPV, Bhopal 2002)*
- 39.** Explain fringing of magnetic flux, magnetic leakage, saturation of ferowegnetic materials, B-H Curve, hysteresis and eddy current losses. *(RGPV, Bhopal 2003)*

### OBJECTIVE TESTS – 6

- 1.** Relative permeability of vacuum is  
(a)  $4\pi \times 10^{-7}$  H/m (b) 1 H/m  
(c) 1 (d)  $1/4\pi$
- 2.** Unit of magnetic flux is  
(a) weber (b) ampere-turn  
(c) tesla (d) coulomb
- 3.** Point out the WRONG statement.  
The magnetising force at the centre of a circular coil varies.  
(a) directly as the number of its turns  
(b) directly as the current  
(c) directly as its radius  
(d) inversely as its radius
- 4.** A pole of driving point admittance function implies  
(a) zero current for a finite value of driving voltage  
(b) zero voltage for a finite value of driving current  
(c) an open circuit condition  
(d) None of (a), (b) and (c) mentioned in the question *(ESE 2001)*

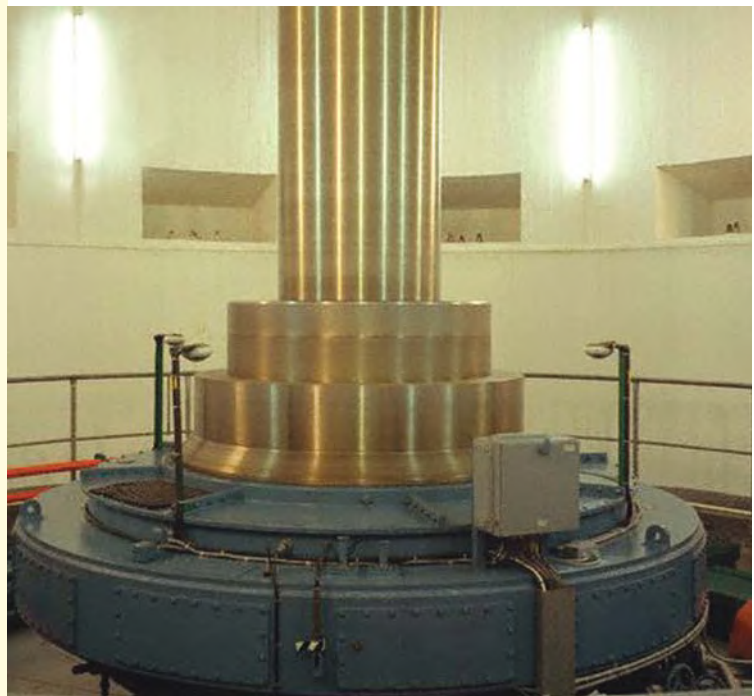
### ANSWERS

- 1.** c      **2.** a      **3.** a

## CHAPTER 7

**Learning Objectives**

- Relation Between Magnetism and Electricity
- Production of Induced E.M.F. and Current
- Faraday's Laws of Electromagnetic Induction
- Direction of Induced E.M.F. and Current
- Lenz's Law
- Induced E.M.F.
- Dynamically-induced E.M.F.
- Statically-induced E.M.F.
- Self-Inductance
- Coefficient of Self-Inductance ( $L$ )
- Mutual Inductance
- Coefficient of Mutual Inductance ( $M$ )
- Coefficient of Coupling
- Inductances in Series
- Inductances in Parallel

**ELECTRO-MAGNETIC INDUCTION**

The above figure shows the picture of a hydro-electric generator. Electric generators, motors, transformers, etc., work based on the principle of electromagnetic induction

## 7.1. Relation Between Magnetism and Electricity

It is well known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. It can be said that when electrons are in motion, they produce a magnetic field. The converse of this is also true *i.e.* when a magnetic field embracing a conductor moves *relative* to the conductor, it produces a flow of electrons in the conductor. This phenomenon whereby an e.m.f. and hence current (*i.e.* flow of electrons) is induced in any conductor which is cut across or is cut by a magnetic flux is known as **electromagnetic induction**. The historical background of this phenomenon is this :

After the discovery (by Oersted) that electric current produces a magnetic field, scientists began to search for the converse phenomenon from about 1821 onwards. The problem they put to themselves was how to ‘convert’ magnetism into electricity. It is recorded that Michael Faraday\* was in the habit of walking about with magnets in his pockets so as to constantly remind him of the problem. After nine years of continuous research and experimentation, he succeeded in producing electricity by ‘converting magnetism’. In 1831, he formulated basic laws underlying the phenomenon of electromagnetic induction (known after his name), upon which is based the operation of most of the commercial apparatus like motors, generators and transformers etc.

## 7.2. Production of Induced E.M.F. and Current

In Fig. 7.1 is shown an insulated coil whose terminals are connected to a sensitive galvanometer *G*. It is placed close to a stationary bar magnet initially at position *AB* (shown dotted). As seen, some flux from the *N*-pole of the magnet is linked with or threads through the coil but, as yet, there is no deflection of the galvanometer. Now, suppose that the magnet is **suddenly** brought closer to the coil in position *CD* (see figure). Then, it is found that there is a jerk or a sudden but a momentary deflection

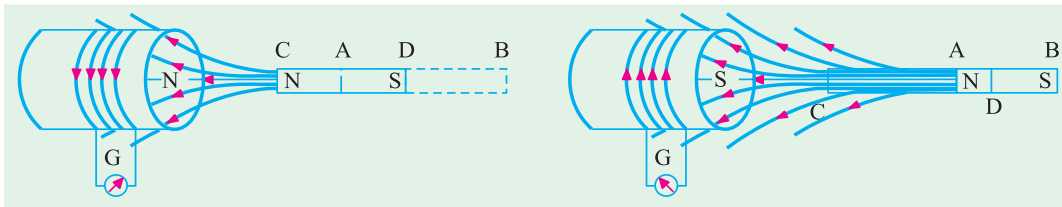


Fig. 7.1.

Fig. 7.2.

in the galvanometer and that this **lasts so long as the magnet is in motion relative to the coil, not otherwise**. The deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*. It should be noted that due to the approach of the magnet, flux linked with the coil is increased.

Next, the magnet is **suddenly** withdrawn away from the coil as in Fig. 7.2. It is found that again there is a **momentary** deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary. It is important to note that this deflection is in a direction opposite to that of Fig. 7.1. Obviously, due to the withdrawal of the magnet, flux linked with the coil is decreased.

The deflection of the galvanometer indicates the production of e.m.f. in the coil. The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of flux linking with the coil. This e.m.f. exists so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f. in a stationary conductor. In fact, the same results can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

\* Michael Faraday (1791-1867), an English physicist and chemist.

The direction of this electromagnetically-induced e.m.f. is as shown in the two figures given on back page.

The production of this electromagnetically-induced e.m.f. is further illustrated by considering a conductor  $AB$  lying within a magnetic field and connected to a galvanometer as shown in Fig. 7.3. It is found that whenever this conductor is moved up or down, a **momentary** deflection is produced in the galvanometer. It means that some transient e.m.f. is induced in  $AB$ . The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) **depends on the quickness of the movement of  $AB$ .**

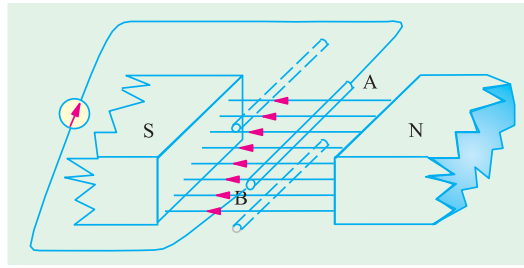


Fig. 7.3

From this experiment we conclude that whenever a conductor cuts or *shears* the magnetic flux, an e.m.f. is always induced in it.

It is also found that if the conductor is moved parallel to the direction of the flux so that it does not cut it, then no e.m.f. is induced in it.

### 7.3. Faraday's Laws of Electromagnetic Induction

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction.

**First Law.** It states :

Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

*or*

Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

**Second Law.** It states :

The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

**Explanation.** Suppose a coil has  $N$  turns and flux through it changes from an initial value of  $\Phi_1$  webers to the final value of  $\Phi_2$  webers in time  $t$  seconds. Then, remembering that by flux-linkages mean the product of number of turns and the flux linked with the coil, we have

Initial flux linkages =  $N\Phi_1$ , add Final flux linkages =  $N\Phi_2$

$$\therefore \text{induced e.m.f. } e = \frac{N\Phi_2 - N\Phi_1}{t} \text{ Wb/s or volt or } e = N \frac{\Phi_2 - \Phi_1}{t} \text{ volt}$$

Putting the above expression in its differential form, we get

$$e = \frac{d}{dt} (N\Phi) = N \frac{d\Phi}{dt} \text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it (Art. 7.5).

$$e = -N \frac{d\Phi}{dt} \text{ volt}$$

**Example 7.1.** The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and residual magnetism is 0.002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.

**Solution.** Total number of turns,  $N = 6 \times 500 = 3000$

Total initial flux =  $6 \times 0.02 = 0.12$  Wb

Total residual flux =  $6 \times 0.002 = 0.012$  Wb

Change in flux,  $d\Phi = 0.12 - 0.012 = 0.108$  Wb

Time of opening the circuit,  $dt = 0.02$  second

$$\therefore \text{Induced e.m.f.} = N \frac{d\Phi}{dt} \text{ volt} = 3000 \times \frac{0.108}{0.02} = \mathbf{16,200 \text{ V}}$$

The direction of this induced e.m.f. is the same as the initial direction of the exciting current.

**Example 7.2.** A coil of resistance  $100 \Omega$  is placed in a magnetic field of  $1 \text{ mWb}$ . The coil has  $100$  turns and a galvanometer of  $400 \Omega$  resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in  $1/10$ th second from the given field to a field of  $0.2 \text{ mWb}$ .

**Solution.** Induced e.m.f. =  $N \cdot \frac{d\Phi}{dt}$  volt

Here  $d\Phi = 1 - 0.2 = 0.8 \text{ mWb} = 0.8 \times 10^{-3} \text{ Wb}$

$$dt = 1/10 = 0.1 \text{ second}; N = 100$$

$$e = 100 \times 0.8 \times 10^{-3} / 0.1 = \mathbf{0.8 \text{ V}}$$

$$\text{Total circuit resistance} = 100 + 400 = 500 \Omega$$

$$\therefore \text{Current induced} = 0.8/500 = 1.6 \times 10^{-3} \text{ A} = \mathbf{1.6 \text{ mA}}$$

**Example 7.3.** The time variation of the flux linked with a coil of  $500$  turns during a complete cycle is as follows :

$$\Phi = 0.04 (1 - 4t/T) \text{ Weber} \quad 0 < t < T/2$$

$$\Phi = 0.04 (4t/T - 3) \text{ Weber} \quad T/2 < t < T$$

where  $T$  represents time period and equals  $0.04$  second. Sketch the waveforms of the flux and induced e.m.f. and also determine the maximum value of the induced e.m.f..

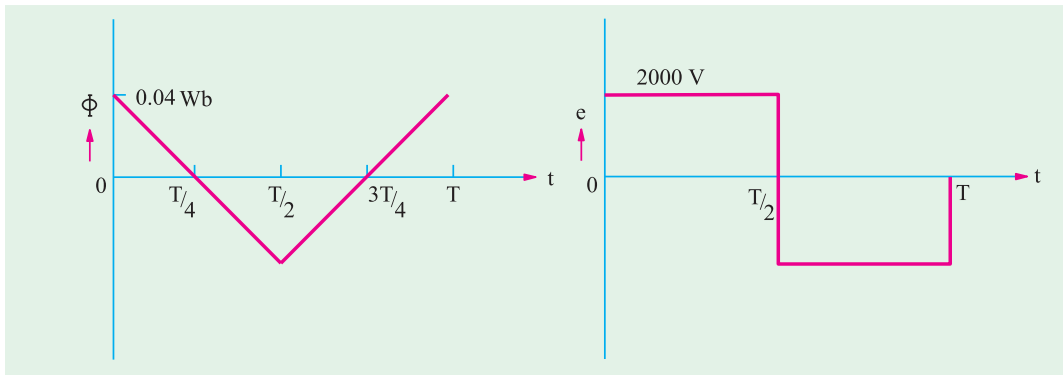


Fig. 7.4.

**Solution.** The variation of flux is linear as seen from the following table.

$t$ (second) :	0	$T/4$	$T/2$	$3T/4$	$T$
$\Phi$ (Weber) :	0.04	0	-0.04	0	0.04

The induced e.m.f. is given by  $e = -N \frac{d\Phi}{dt}$

From  $t = 0$  to  $t = T/2$ ,  $\frac{d\Phi}{dt} = -0.04 \times 4/T = -4 \text{ Wb/s} \therefore e = -500 (-4) = 2000 \text{ V}$

From  $t = T/2$  to  $t = T$ ,  $\frac{d\Phi}{dt} = 0.04 \times 4/T = 4 \text{ Wb/s} \therefore e = -500 \times 4 = -2000 \text{ V}$ .

The waveforms are selected in Fig. 7.4.

#### 7.4. Direction of induced e.m.f. and currents

There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or Flat-hand rule or Lenz's Law. Fleming's rule



(Fig. 7.5) is used where induced e.m.f. is due to flux-cutting (*i.e.*, dynamically induced e.m.f.) and Lenz's when it is used to change by flux-linkages (*i.e.*, statically induced e.m.f.).

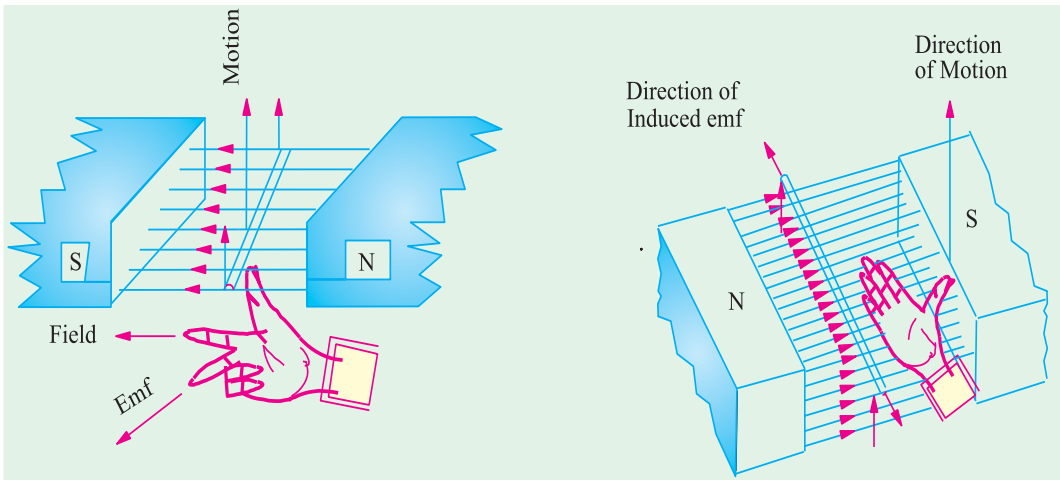


Fig. 7.5.

Fig. 7.6.

Fig. 7.6 shows another way of finding the direction of the induced e.m.f. It is known as Right Flat-hand rule. Here, the front side of the hand is held perpendicular to the incident flux with the thumb pointing in the direction of the motion of the conductor. The direction of the fingers give the direction of the induced e.m.f. and current.

### 7.5. Lenz's Law

The direction of the induced current may also be found by this law which was formulated by Lenz\* in 1835. This law states, in effect, that electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

This statement will be clarified with reference to Fig. 7.1 and 7.2. It is found that when *N*-pole of the bar magnet approaches the coil, the induced current set up by induced e.m.f. flows in the **anti-clockwise** direction in the coil as seen from the magnet side. The result is that face of the coil becomes a *N*-pole and so tends to oppose the onward approach of the *N*-Pole of the magnet (like poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

When the magnet is withdrawn as in Fig. 7.2, the induced current flows in the clockwise direction thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to withdraw against this attractive force of the *S*-pole of coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy. Imagine for a moment that when *N*-pole of the magnet (Fig. 7.1) approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then, due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause which produces it. In the present case, it is relative motion of the magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direction to oppose this relative motion *i.e.*, the approach or withdrawal of the magnet.

\* After the Russian born geologist and physicist Heinrich Friedrich Emil Lenz (1808 - 1865).

### 7.6. Induced e.m.f.

Induced e.m.f. can be either **(i) dynamically induced** or **(ii) statically induced**. In the first case, usually the field is stationary and conductors cut across it (as in d.c. generators). But in the second case, usually the conductors or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).

### 7.7. Dynamically induced e.m.f.

In Fig. 7.7, a conductor  $A$  is shown in cross-section, lying  $m^2$  within a uniform magnetic field of flux density  $B \text{ Wb/m}^2$ . The arrow attached to  $A$  shows its direction of motion. Consider the conditions shown in Fig. 7.7 (a) when  $A$  cuts across at right angles to the flux. Suppose  $l$  is its length lying within the field and let it move a distance  $dx$  in time  $dt$ . Then area swept by it is  $l dx$ . Hence, flux cut =  $l dx \times B$  webers.

$$\begin{aligned} \text{Change in flux} &= B l dx \text{ weber} \\ \text{Time taken} &= dt \text{ second} \end{aligned}$$

Hence, according to Faraday's Laws (Art. 7.3.) the e.m.f. induced in it (known as dynamically induced e.m.f.) is

$$\text{rate of change of flux linkages} = \frac{B l dx}{dt} = B l \frac{dx}{dt} = B l v \text{ volt where } \frac{dx}{dt} = \text{velocity}$$

If the conductor  $A$  moves at an angle  $\theta$  with the direction of flux [Fig. 7.7 (b)] then the induced e.m.f. is  $e = B l v \sin \theta$  volts =  $l v \times B$  (i.e. as cross product vector  $v$  and  $B$ ).

The direction of the induced e.m.f. is given by Fleming's Right-hand rule (Art. 7.5) or Flat-hand rule and most easily by vector cross product given above.

It should be noted that generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

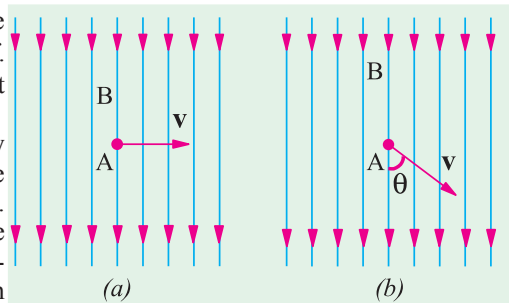
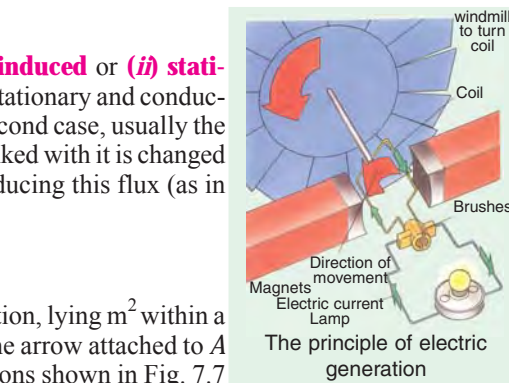


Fig. 7.7

**Example 7.4.** A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density  $1.5 \text{ Wb/m}^2$  with a velocity of 50 metre/second. Calculate the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of  $30^\circ$  to the direction of the field.

**Solution.** Here  $B = 1.5 \text{ Wb/m}^2$   $l = 1 \text{ m}$   $v = 50 \text{ m/s}$ ;  $e = ?$   
 Now  $e = B l v = 1.5 \times 1 \times 50 = 75 \text{ V}$ .  
 In the second case  $\theta = 30^\circ$   $\therefore \sin 30^\circ = 0.5$   $\therefore e = 75 \times 0.5 = 37.5 \text{ V}$

**Example 7.5.** A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 500 rpm about an axis at right angle to a uniform field of  $0.5 \text{ Wb/m}^2$ . Calculate the instantaneous value of induced e.m.f. when the plane of the coil is (i) at right angle to the plane of the field. (ii) in the plane of the field. (iii) at  $45^\circ$  with the field direction. (Elect. Engg. A.M.Ae. S.I. Dec. 1991)

**Solution.** As seen from Art. 12.2, e.m.f. induced in the coil would be zero when its plane is at right angles to the plane of the field, even though it will have maximum flux linked with it. However, the coil will have maximum e.m.f. induced in it when its plane lies parallel to the plane of the field even though it will have minimum flux linked with it. In general, the value of the induced e.m.f. is given by  $e = \omega N \Phi_m \sin \theta = E_m \sin \theta$  where  $\theta$  is the angle between the axis of zero e.m.f. and the plane of the coil.

Here,  $f = 500/60 = 25/3 \text{ r.p.s}$ ;  $N = 100$ ;  $B = 0.5 \text{ Wb/m}^2$ ;  $A = (10 \times 10) \times 10^{-4} = 10^{-2} \text{ m}^2$ .

$\therefore E_m = 2 \pi f N B A = 2 \pi (25/3) \times 100 \times 0.5 \times 10^{-2} = 26.2 \text{ V}$  (i) since  $\theta = 0$ ;  $\sin \theta = 0$ ; therefore,

$e = 0$ . (ii) Here,  $\theta = 90^\circ$ ;  $e = E_m \sin 90^\circ = 26.2 \times 1 = 26.2 \text{ V}$  (iii)  $\sin 45^\circ = 1/\sqrt{2}$ ;  $e = 26.2 \times 1/\sqrt{2} = 18.5 \text{ V}$

**Example 7.6.** A conducting rod  $AB$  (Fig. 7.8) makes contact with metal rails  $AD$  and  $BC$  which are 50 cm apart in a uniform magnetic field of  $B = 1.0 \text{ Wb/m}^2$  perpendicular to the plane  $ABCD$ . The total resistance (assumed constant) of the circuit  $ABCD$  is  $0.4 \Omega$ .

(a) What is the direction and magnitude of the e.m.f. induced in the rod when it is moved to the left with a velocity of 8 m/s?

(b) What force is required to keep the rod in motion?

(c) Compare the rate at which mechanical work is done by the force  $F$  with the rate of development of electric power in the circuit.

**Solution.** (a) Since  $AB$  moves to the left, direction of the induced current, as found by applying Fleming's Right-hand rule is from  $A$  to  $B$ . Magnitude of the induced e.m.f. is given by

$$e = \beta lv \text{ volt} = 1 \times 0.5 \times 8 = \mathbf{4 \text{ volt}}$$

(b) Current through  $AB = 4/0.4 = 10 \text{ A}$

Force on  $AB$  i.e.  $F = BIl = 1 \times 10 \times 0.5 = \mathbf{5 \text{ N}}$

The direction of this force, as found by applying Fleming's left-hand rule, is to the right.

(c) Rate of doing mechanical work =  $F \times v = 5 \times 8 = \mathbf{40 \text{ J/s or W}}$

Electric power produced =  $e i = 4 \times 10 = 40 \text{ W}$

From the above, it is obvious that the mechanical work done in moving the conductor against force  $F$  is converted into electric energy.

**Example 7.7** In a 4-pole dynamo, the flux/pole is 15 mWb. Calculate the average e.m.f. induced in one of the armature conductors, if armature is driven at 600 r.p.m.

**Solution.** It should be noted that each time the conductor passes under a pole (whether  $N$  or  $S$ ) it cuts a flux of 15 mWb. Hence, the flux cut in one revolution is  $15 \times 4 = 60 \text{ mWb}$ . Since conductor is rotating at  $600/60 = 10 \text{ r.p.s.}$  time taken for one revolution is  $1/10 = 0.1 \text{ second}$ .

$$\therefore \text{average e.m.f. generated} = N \frac{d\Phi}{dt} \text{ volt}$$

$$N = 1; \quad d\Phi = 60 \text{ mWb} = 6 \times 10^{-2} \text{ Wb}; \quad dt = 0.1 \text{ second}$$

$$\therefore e = 1 \times 6 \times 10^{-2} / 0.1 = \mathbf{0.6 \text{ V}}$$

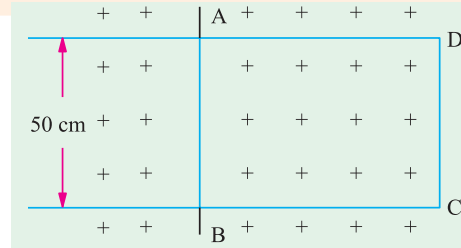


Fig. 7.8

### Tutorial Problems No. 7.1

1. A conductor of active length 30 cm carries a current of 100 A and lies at right angles to a magnetic field of strength  $0.4 \text{ Wb/m}^2$ . Calculate the force in newtons exerted on it. If the force causes the conductor to move at a velocity of 10 m/s, calculate (a) the e.m.f. induced in it and (b) the power in watts developed by it. **[12 N; 1.2 V, 120 W]**
2. A straight horizontal wire carries a steady current of 150 A and is situated in a uniform magnetic field of  $0.6 \text{ Wb/m}^2$  acting vertically downwards. Determine the magnitude of the force in kg/metre length of conductor and the direction in which it works. **[9.175 kg/m horizontally]**
3. A conductor, 10 cm in length, moves with a uniform velocity of 2 m/s at right angles to itself and to a uniform magnetic field having a flux density of  $1 \text{ Wb/m}^2$ . Calculate the induced e.m.f. between the ends of the conductor. **[0.2 V]**

### 7.8. Statically Induced E.M.F.

It can be further sub-divided into (a) *mutually induced e.m.f.* and (b) *self-induced e.m.f.*

(a) **Mutually-induced e.m.f.** Consider two coils  $A$  and  $B$  lying close to each other (Fig. 7.9).

Coil  $A$  is joined to a battery, a switch and a variable resistance  $R$  whereas coil  $B$  is connected

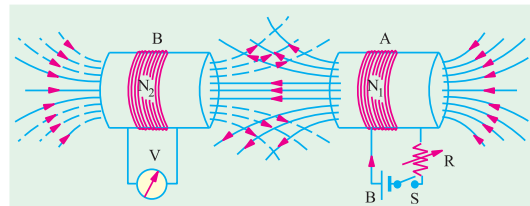


Fig. 7.9

to a sensitive voltmeter  $V$ . When current through  $A$  is established by closing the switch, its magnetic field is set up which partly links with or threads through the coil  $B$ . As current through  $A$  is changed, the flux linked with  $B$  is also changed. Hence, mutually induced e.m.f. is produced in  $B$  whose magnitude is given by Faraday's Laws (Art. 7.3) and direction by Lenz's Law (Art. 7.5).

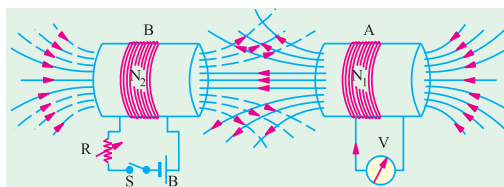


Fig. 7.10

If, now, battery is connected to  $B$  and the voltmeter across  $A$  (Fig. 7.10), then the situation is reversed and now a change of current in  $B$  will produce mutually-induced e.m.f. in  $A$ .

It is obvious that in the examples considered above, there is no movement of any conductor, the flux variations being brought about by variations in current strength only. Such an **e.m.f.** induced in one coil by the influence of the other coil is called (statically but) mutually induced e.m.f.

**(b) Self-induced e.m.f.** This is the e.m.f. induced in a coil due to **the change of its own flux linked with it**. If current through the coil (Fig. 7.11) is changed, then the flux linked with its own turns will also change, which will produce in it what is called **self-induced e.m.f.** The direction of this induced e.m.f. (as given by Lenz's law) would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter e.m.f. of self-induction.

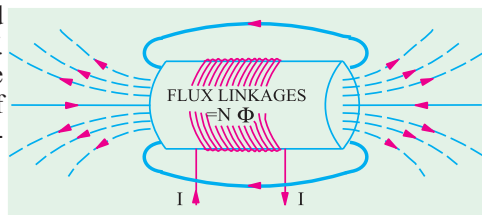


Fig. 7.11

## 7.9. Self-inductance

Imagine a coil of wire similar to the one shown in Fig. 7.11 connected to a battery through a rheostat. It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery. As will be fully explained later on, this energy is stored in the additional flux produced.

If, now an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction. This property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as **self-inductance**. It is quantitatively measured in terms of coefficient of self induction  $L$ . This property is analogous to inertia in a material body. We know by experience that initially it is difficult to set a heavy body into motion, but once in motion, it is equally difficult to stop it. Similarly, in a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it. Hence, self-induction is sometimes analogously called **electrical inertia or electromagnetic inertia**.

## 7.10. Coefficient of Self-induction ( $L$ )

It may be defined in any one of the three ways given below :

### (i) First Method for $L$

The coefficient of self-induction of a coil is defined as

**the weber-turns per ampere in the coil**

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil.

Consider a solenoid having  $N$  turns and carrying a current of  $I$  amperes. If the flux produced is  $\Phi$  webers, the weber-turns are  $N\Phi$ . Hence, weber-turns per ampere are  $N\Phi/I$ .

By definition, 
$$L = \frac{N\Phi}{I}$$
. The unit of self-induction is henry\*.

\* After the American scientist Joseph Henry (1797 - 1878), a company of Faraday.

If in the above relation,  $N\Phi = 1$  Wb-turn,  $I = 1$  ampere, then  $L = 1$  henry ( $H$ )

**Hence a coil is said to have a self-inductance of one henry if a current of 1 ampere when flowing through it produced flux-linkages of 1 Wb-turn in it.**

Therefore, the above relation becomes  $L = \frac{N\Phi}{I}$  henry

**Example 7.8.** The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of  $50 \Omega$  when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.

**Solution.** Formula used :  $L = \frac{N\Phi}{I} H$

Current through coil =  $230/50 = 4.6$  A  $\Phi = 0.005$  Wb ;  $N = 960$

$$L = \frac{960 \times 0.005}{4.6} = \mathbf{1.0435 H} \quad \text{Energy stored} = \frac{1}{2} L I^2 = \frac{1}{2} \times 1.0435 \times 4.6^2 = \mathbf{11.04 J}$$

### Second Method for L

We have seen in Art. 6.20 that flux produced in a solenoid is

$$\Phi = \frac{NI}{l/\mu_0\mu_r A} \quad \therefore \frac{\Phi}{I} = \frac{N}{l/\mu_0\mu_r A} \quad \text{Now } L = N \frac{\Phi}{I} = N \cdot \frac{N}{l/\mu_0\mu_r A} H$$

$$\therefore L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S} H \quad \text{or} \quad L = \frac{\mu_0\mu_r AN^2}{l} H$$

It gives the value of self-induction in terms of the dimensions of the solenoid\*.

**Example 7.9.** An iron ring 30 cm mean diameter is made of square of iron of  $2 \text{ cm} \times 2 \text{ cm}$  cross-section and is uniformly wound with 400 turns of wire of  $2 \text{ mm}^2$  cross-section. Calculate the value of the self-inductance of the coil. Assume  $\mu_r = 800$ . **(Elect. Technology, I, Gwalior Univ.)**

**Solution.**  $L = \mu_0\mu_r AN^2/l$ . Here  $N = 400$  ;  $A = 2 \times 2 = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$  ;  $l = 0.3 \pi \text{ m}$  ;  $\mu_r = 800$

$$\therefore L = 4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4} (400)^2 / 0.3\pi = \mathbf{68.3 \text{ mH}}$$

**Note.** The cross-section of the wire is not relevant to the given question.

### Third Method for L

It will be seen from Art. 7.10 (i) above that  $L = \frac{N\Phi}{I}$   $\therefore N\Phi = LI$  or  $-N\Phi = -LI$

Differentiating both sides, we get  $-\frac{d}{dt} (N\Phi) = -L \cdot \frac{dI}{dt}$  (assuming  $L$  to be constant) ;

$$-N \cdot \frac{d\Phi}{dt} = -L \cdot \frac{dI}{dt}$$

As seen from Art. 7.3,  $-N \cdot \frac{d\Phi}{dt} = \text{self-induced e.m.f.}$   $\therefore e_L = -L \frac{dI}{dt}$

If  $\frac{dI}{dt} = 1$  ampere/second and  $e_L = 1$  volt, then  $L = 1 H$

**Hence, a coil has a self-inductance of one henry if one volt is induced in it when current through it changes at the rate of one ampere/second.**

**Example 7.10.** If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying current of 10 A, calculate the inductance of the coil. If this current is uniformly reversed in 0.01 second, calculating the induced electromotive force.

**Solution.**  $L = N\Phi/I = 150 \times 0.01/10 = \mathbf{0.15 H}$

Now,  $e_L = L dI/dt$  ;  $dI = -10 - (-10) = 20$  A

$$\therefore e_L = 0.15 \times 20/0.01 = \mathbf{300 V}$$

\* In practice, the inductance of a short solenoid is given by  $L = K\mu_0\mu_r AN^2/l$  where  $K$  is Nagaoka's constant.

**Example 7.11.** An iron rod, 2 cm in diameter and 20 cm long is bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0.5 A is passed through this coil, the flux density in the coil is 0.5 Wb/m<sup>2</sup>. Assuming that all the flux is linked with every turn of the coil, what is (a) the B/H ratio for the iron (b) the inductance of the coil? What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10% of its former value in 0.001 second? **(Principle of Elect. Engg. Jadavpur Univ.)**

**Solution.**  $H = NI/l = 3000 \times 0.2 = 7500 \text{ AT/m}$   $B = 0.5 \text{ Wb/m}^2$

(a) Now,  $\frac{B}{H} = \frac{0.5}{7500} = 6.67 \times 10^{-5} \text{ H/m}$ . Also  $\mu_r = B/\mu_a H = 6.67 \times 10^{-5}/4\pi \times 10^{-7} = 53$

(b)  $L = \frac{N\Phi}{I} = \frac{300 \times \pi \times (0.02)^2 \times 0.5}{4 \times 0.5} = 0.94 \text{ H}$

$e_L = N \frac{d\Phi}{dt}$  volt;  $d\Phi = 90\%$  of original flux =  $\frac{0.9 \times \pi \times (0.02)^2 \times 0.5}{4} = 0.45 \pi \times 10^{-4} \text{ Wb}$

$dt = 0.001$  second  $\therefore e_L = 3000 \times 0.45\pi \times 10^{-4}/0.001 = 424 \text{ V}$

**Example 7.12.** A circuit has 1000 turns enclosing a magnetic circuit 20 cm<sup>2</sup> in section. With 4 A, the flux density is 1.0 Wb/m<sup>2</sup> and with 9 A, it is 1.4 Wb/m<sup>2</sup>. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 seconds. **(Elect. Engineering-1, Delhi Univ.)**

**Solution.**  $L = N \frac{d\Phi}{dI} = N \frac{d(BA)}{dI} = NA \frac{dB}{dI}$  henry =  $1000 \times 20 \times 10^{-4} (1.4 - 1)/(9 - 4) = 0.16 \text{ H}$

Now,  $e_L = L \cdot dI/dt$ ;  $dI = (9 - 4) = 5 \text{ A}$ ,  $dt = 0.05 \text{ s}$   $\therefore e_L = 0.16 \times 5/0.05 = 16 \text{ V}$

**Example 7.13.** A direct current of one ampere is passed through a coil of 5000 turns and produces a flux of 0.1 mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil? What would be the voltage developed across the coil if the current were interrupted in 10<sup>-3</sup> second? What would be the maximum voltage developed across the coil if a capacitor of 10 μF were connected across the switch breaking the d.c. supply?

**Solution.**  $L = N\Phi/I = 5000 \times 10^{-4} = 0.5 \text{ H}$ ; Induced e.m.f. =  $L \cdot \frac{dI}{dt} = \frac{0.5 \times 1}{10^{-3}} = 500 \text{ V}$

The energy stored in the coil is =  $\frac{1}{2} L I^2 = \frac{1}{2} \times 0.5 \times 1^2 = 0.25 \text{ J}$

When the capacitor is connected, then the voltage developed would be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of the voltage,

then  $\frac{1}{2} CV^2 = \frac{1}{2} L I^2$ ;  $\frac{1}{2} \times 10 \times 10^{-6} V^2 = 0.25$  or  $V = 224 \text{ volt}$

**Example 7.14. (a)** A coil of 1000 turns is wound on a toroidal magnetic core having a reluctance of 10<sup>4</sup> AT/Wb. When the coil current is 5 A and is increasing at the rate of 200 A/s, determine.

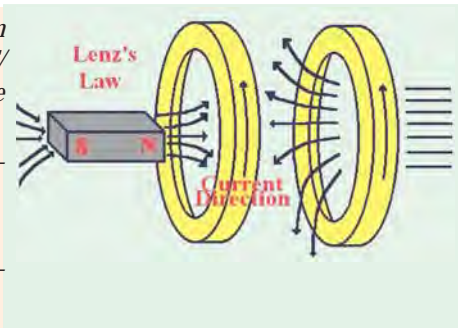
(i) energy stored in the magnetic circuit (ii) voltage applied across the coil

Assume coil resistance as zero.

(b) How are your answers affected if the coil resistance is 2 Ω

**(Elect. Technology, Hyderabad Univ. 1991)**

**Solution. (a)**  $L = N^2/S = 1000^2/10^6 = 1 \text{ H}$



(i) Energy stored =  $\frac{1}{2} LI^2 = \frac{1}{2} \times 1 \times 5^2 = 12.5 \text{ J}$

(ii) Voltage applied across coil = self-induced e.m.f. in the coil =  $L \cdot di/dt = 1 \times 200 = 200 \text{ V}$

(b) Though there would be additional energy loss of  $5^2 \times 2 = 50 \text{ W}$  over the coil resistance, energy stored in the coil would remain the same. However, voltage across the coil would increase by an amount =  $5 \times 2 = 10 \text{ V}$  i.e., now its value would be **210 V**.

### 7.11. Mutual Inductance

In Art. 7.8 (Fig. 7.9) we have that any change of current in coil *A* is always accompanied by the production of mutually-induced e.m.f. in coil *B*. Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the coefficient of mutual induction *M*.

**Example 7.15.** A single element has the current and voltage functions graphed in figure 7.12. (a) and (b). Determine the element. [Bombay University 2001]

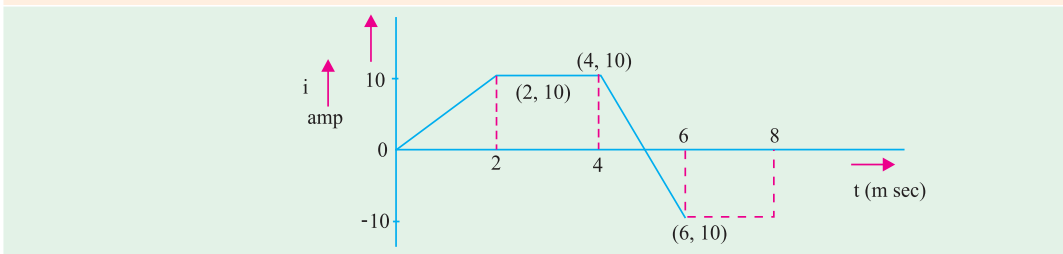


Fig. 7.12 (a)

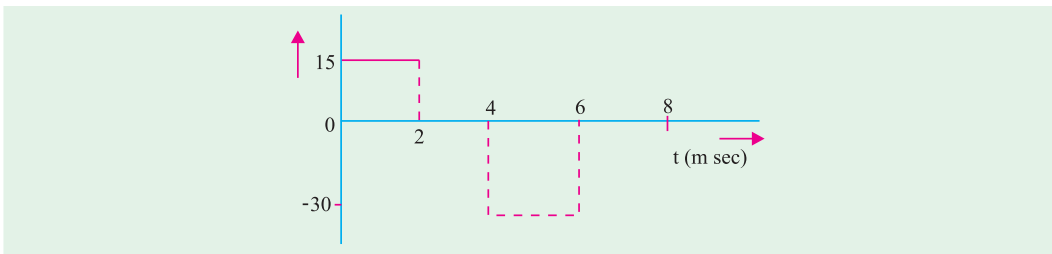


Fig. 7.12 (b)

**Solution.** Observations from the graph are tabulated below.

Sr. No.	Between time	$di/dt$ amp/sec	<i>V</i>	<i>L</i>	
1	0 - 2 m Sec	5000	15	$15/5000$	= 3mH
2	2 - 4 m Sec	0	0	—	
3	4 - 6 m Sec	- 10,000	- 30	$- 30 / (- 10,000)$	= 3 mH
4	6 - 8 m Sec	0	0	—	

The element is a 3-mH inductor.

### 7.12. Coefficient of Mutual Inductance (*M*)

It can also be defined in three ways as given below :

**(i) First Method for *M***

Let there be two magnetically-coupled coils having  $N_1$  and  $N_2$  turns respectively (Fig. 7.9). Coefficient of mutual inductance between the two coils is defined as

*the weber-turns in one coil due to one ampere current in the other.*

Let a current  $I_1$  ampere when flowing in the first coil produce a flux  $\Phi_1$  webers in it. **It is supposed that whole of this flux links with the turns of the second coil\***. Then, flux-linkages i.e., webers-turns in the *second* coil for unit current in the *first* coil are  $N_2 \Phi_1 / I_1$ . Hence, by definition

$$M = \frac{N_2 \Phi_1}{I_1}$$

If weber-turns in *second* coil due to one ampere current in the *first* coil i.e.  $N_2 \Phi_1 / I_1 = 1$  then, as seen from above,  $M = 1\text{H}$ .

**Hence, two coils are said to have a mutual inductance of 1 henry is one ampere current when flowing in one coil produces flux-linkages of one Wb-turn in the other.**

**Example 7.16.** Two identical coils  $X$  and  $Y$  of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in  $X$  produces a flux of 0.5 mWb in it, find the mutual inductance between  $X$  and  $Y$ . (Elect. Engg. A.M.Ae.S.I.)

**Solution.** Formula used  $M = \frac{N_2 \Phi_1}{I_1}$  H; Flux produced in  $X = 0.5$  mWb  $= 0.5 \times 10^{-3}$  Wb

Flux linked with  $Y = 0.5 \times 10^{-3} \times 0.8 = 0.4 \times 10^{-3}$  Wb;  $M = \frac{1000 \times 0.4 \times 10^{-3}}{5} = 0.08$  H

**Example 7.17.** A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 AT/metre. There is a small concentrated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils in each case if the concentrated coil has 120 turns on an effective diameter of (a) 8 cm and (b) 12 cm.

(Elect. Science - II Allahabad Univ. 1992)

**Solution.** The two cases (a) and (b) are shown in Fig. 7.13 (a) and (b) respectively.

(a) Let  $I_1$  be the current flowing through the solenoid. Then

$$B = \mu_0 H \times \mu_0 N I_1 / l = 2500 \mu_0 I_1 \text{ Wb/m}^2 \quad \dots l = 1 \text{ m}$$

$$\text{Area of search coil} \quad A_1 = \frac{\pi}{4} \times 8^2 \times 10^{-4} = 16\pi \times 10^{-4} \text{ m}^2$$

Flux linked with search coil is

$$\Phi = B A_1 = 2500 \mu_0 I_1 \times 16\pi \times 10^{-4} = 15.79 I_1 \times 10^{-6} \text{ Wb}$$

$$\therefore M = \frac{N_2 \Phi_1}{I_1} = \frac{120 \times 15.79 I_1 \times 10^{-6}}{I_1} = 1.895 \times 10^{-3} \text{ H}$$

(b) Since the field strength outside the solenoid is negligible, the effective area of the search coil, in this case, equals the area of the long solenoid.

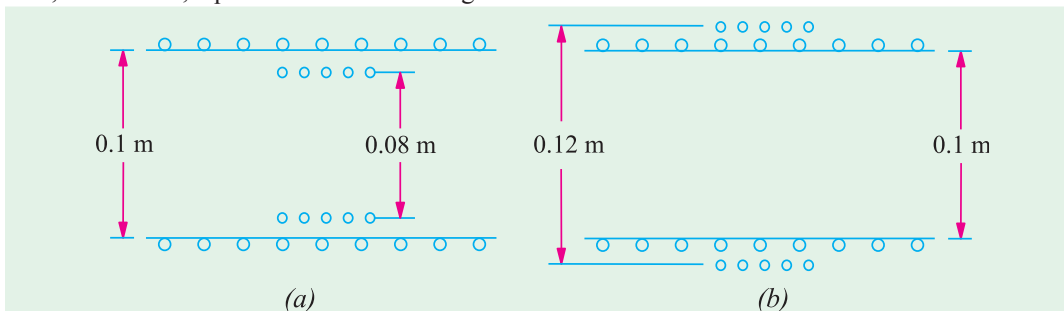


Fig. 7.13

$$A_2 = \frac{\pi}{4} \times 10^2 \times 10^{-4} = \frac{\pi}{4} 10^{-2} \text{ m}^2;$$

\* If whole of this flux does not link with turns of the second coil, then only that part of the flux which is actually linked is taken instead. (Ex. 7.13 and 7.17). In general,  $M = N_2 \Phi_2 / I_1$ .



$$\Phi = BA_2 = 2500 \mu_0 I_1 \times \frac{\pi}{4} \times 10^{-2} = 24.68 I_1 \times 10^{-6} \text{ Wb}$$

$$M = \frac{120 \times 24.68 I_1 \times 10^{-6}}{I_1} = 2.962 \times 10^{-3} \text{ H}$$

**Example 7.18.** A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate (i) the inductance of the coil (ii) the e.m.f. induced in the coil when a current of 5 A is switched off, assuming the current to fall to zero in 1 milli second and (iii) the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.

(F. E. Pune Univ.)

**Solution.** (i) Inductance of the first coil =  $\frac{N\Phi}{I} = \frac{900 \times 0.5 \times 10^{-3}}{3} = 0.15 \text{ H}$

(ii) e.m.f. induced  $e_1 = L \frac{di}{dt} = 0.15 \times \frac{(5-0)}{1 \times 10^{-3}} = 750 \text{ V}$

(iii)  $M \frac{N_2 \Phi_1}{I_1} = \frac{600 \times 0.5 \times 10^{-3}}{3} = 0.1 \text{ H}$

(ii) **Second Method for M**

We will now deduce an expression for coefficient of mutual inductance in terms of the dimensions of the two coils.

$$\text{Flux in the first coil } \Phi_1 = \frac{N_2 I_1}{l / \mu_0 \mu_r A} \text{ Wb ; Flux/ampere} = \frac{\Phi_1}{I_1} = \frac{N_1}{l / \mu_0 \mu_r A}$$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having  $N_2$  turns, the weber-turns in it due to the flux/ampere in the first coil is

$$M = \frac{N_2 \Phi_1}{I_1} = \frac{N_2 N_1}{l / \mu_0 \mu_r A} \quad \therefore M = \frac{\mu_0 \mu_r A N_1 N_2}{l} \text{ H}$$

Also  $M = \frac{N_1 N_2}{l / \mu_0 \mu_r A} = \frac{N_1 N_2}{\text{reluctance}} = \frac{N_1 N_2}{S} \text{ H}$

**Example 7.19.** If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A ; calculate the inductance of the coil. If this current is uniformly reversed in 0.1 second, calculate the induced e.m.f. If a second coil of 100 turns is uniformly wound over the first coil, find the mutual inductance between the coils.

(F. E. Pune Univ.)

**Solution.**

$$L_1 = N_1 \Phi_1 / I_1 = 150 \times 0.01 / 10 = 0.15 \text{ H}$$

$$e = L \times di/dt = 0.15 \times [10 - (-10)] / 0.1 = 1 = 30 \text{ V}$$

$$M = N_2 \Phi_1 / I_1 = 100 \times 0.01 / 10 = 0.1 \text{ H}$$

(iii) **Third Method for M**

As seen from Art. 7.12 (i)  $M = \frac{N_2 \Phi_1}{I_1} \quad \therefore N_2 \Phi_1 = MI_1 \quad \text{or} \quad -N_2 \Phi_1 = -MI_1$

Differentiating both sides, we get :  $-\frac{d}{dt}(N_2 \Phi_1) = -M \frac{dI_1}{dt}$  (assuming  $M$  to be constant)

Now,  $-\frac{d}{dt}(N_2 \Phi_1) =$  mutually-induced e.m.f. in the second coil =  $e_M \quad \therefore e_M = -M \frac{dI_1}{dt}$

If  $dI_1/dt = 1 \text{ A/s}$  ;  $e_M = 1 \text{ volt}$ , then  $M = 1 \text{ H}$ .

**Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other.**

**Example 7.20.** Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of area of cross-section 100 sq.cm. and mean length 200 cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil. **(Elect. Engg. I, JNT Univ., Warangal)**

**Solution.** Formula used :  $M = \frac{N_1 N_2}{l/\mu_0 \mu_r A} H$ ,  $N_1 = 30$  ;  $N_2 = 600$  ;  $A = 100 \times 10^{-4} = 10^{-2} \text{m}^2$ ,  $l = 2\text{m}$

$$\therefore M = \mu_0 \mu_r A \frac{N_1 N_2}{l} = 4\pi \times 10^{-7} \times 2000 \times 10^{-2} \times 30 \times 600/2 = \mathbf{0.226 H}$$

$$dI_1 = 20 - 0 = 20 \text{ A} ; dt = 0.02 \text{ s} ; e_M = M dI_1/dt = 0.226 \times 20/0.2 = \mathbf{226 V}$$

**Example 7.21.** Two coils A and B each having 1200 turns are placed near each other. When coil B is open-circuited and coil A carries a current of 5 A, the flux produced by coil A is 0.2 Wb and 30% of this flux links with all the turns of coil B. Determine the voltage induced in coil B on open-circuit when the current in the coil A is changing at the rate of 2 A/s.

**Solution.** Coefficient of mutual induction between the two coils is  $M = N_2 \Phi_2 / I_1$

Flux linked with coil B is 30 per cent of 0.2 Wb i.e. 0.06 Wb

$$\therefore M = 1200 \times 0.06/5 = 14.4 \text{ H}$$

Mutually-induced e.m.f. in coil B is  $e_M = M dI_1/dt = 14.4 \times 2 = \mathbf{28.8 V}$

**Example 7.22.** Two coils are wound side by side on a paper-tube former. An e.m.f. of 0.25 V is induced in coil A when the flux linking it changes at the rate of  $10^3$  Wb/s. A current of 2 A in coil B causes a flux of  $10^5$  Wb to link coil A. What is the mutual inductance between the coils ?

**(Elect. Engg-I, Bombay Univ.)**

**Solution.** Induced e.m.f. in coil A is  $e = N_1 \frac{d\Phi}{dt}$  where  $N_1$  is the number of turns of coil A.

$$\therefore 0.25 = N_1 \times 10^{-3} \quad \therefore N_1 = 250$$

Now, flux linkages in coil A due to 2 A current in coil B =  $250 \times 10^5$

$$\therefore M = \frac{\text{flux linkages in coil A}}{\text{current in coil B}} = 250 \times 10^5/2 = \mathbf{1.25 \text{ mH}}$$

### 7.13. Coefficient of Coupling

Consider two magnetically-coupled coils A and B having  $N_1$  and  $N_2$  turns respectively. Their individual coefficients of self-induction are,

$$L_1 = \frac{N_1^2}{l/\mu_0 \mu_r A} \quad \text{and} \quad L_2 = \frac{N_2^2}{l/\mu_0 \mu_r A}$$

The flux  $\Phi_1$  produced in A due to a current  $I_1$  ampere is  $\Phi_1 = \frac{N_1 I_1}{l/\mu_0 \mu_r A}$

Suppose a fraction  $k_1$  of this flux i.e.  $k_1 \Phi_1$  is linked with coil B.

Then  $M = \frac{k_1 \Phi_1 \times N_2}{I_1}$  where  $k_1 \leq 1$ .

Substituting the value of  $\Phi_1$ , we have,  $M = k_1 \times \frac{N_1 N_2}{l/\mu_0 \mu_r A}$  ... (i)

Similarly, the flux  $\Phi_2$  produced in B due to  $I_2$  ampere in it is  $\Phi_2 = \frac{N_2 I_2}{l/\mu_0 \mu_r A}$

Suppose a fraction  $k_2$  of this flux i.e.  $k_2 \Phi_2$  is linked with A.

Then  $M = \frac{k_2 \Phi_2 \times N_1}{I_2} = k_2 \frac{N_1 N_2}{l/\mu_0 \mu_r A}$  ... (ii)

Multiplying Eq. (i) and (ii), we get

$$M^2 = k_1 k_2 \frac{N_1^2}{l/\mu_0 \mu_r A} \times \frac{N_2^2}{l/\mu_0 \mu_r A} \quad \text{or} \quad M^2 = k_1 k_2 L_1 L_2$$

Putting  $\sqrt{k_1 k_2} = k$ , we have  $M = k\sqrt{L_1 L_2}$  or  $k = \frac{M}{\sqrt{L_1 L_2}}$

The constant  $k$  is called the **coefficient of coupling** and may be defined as the ratio of **mutual inductance actually present between the two coils to the maximum possible value**. If the flux due to one coil completely links with the other, then value of  $k$  is unity. If the flux of one coil does not at all link with the other, then  $k = 0$ . In the first case, when  $k = 1$ , coils are said to be tightly coupled and when  $k = 0$ , the coils are magnetically isolated from each other.

**Example 7.23.** Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

**Solution.** Now,  $e_M = M dI_1/dt$  ...Art. 7.12

$$M = \frac{e_M}{dI_1/dt} = \frac{11.25}{1500} = 7.5 \times 10^{-3} \text{ H} = \mathbf{7.5 \text{ mH}}$$

Now,  $L_1 = \frac{N_1 \Phi_1}{I_1} \quad \therefore \quad \frac{\Phi_1}{I_1} = \frac{L_1}{N_1} = \frac{15 \times 10^{-3}}{750} = \mathbf{2 \times 10^{-5} \text{ Wb/A}}$  ...Art. 7.10

Now,  $k = \frac{M}{\sqrt{L_1 L_2}} = \frac{7.5 \times 10^{-3}}{\sqrt{15 \times 15 \times 10^{-3}}} = \frac{7.5 \times 10^{-3}}{15 \times 10^{-3}} = 0.5 = \mathbf{50\%}$  ( $\because L_1 = L_2 = L$ ) ...Art. 7.13

**Example 7.24.** Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60 % of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0.6 mWb while the same current in B produces 0.8 mWb. Determine (i) mutual inductance and (ii) coupling coefficient.

**Solution.** (i) Flux/ampere in A =  $0.6/5 = 0.12 \text{ mWb}$

$$\text{Flux linked with B} = 0.12 \times 0.6 = 0.072 \text{ mWb}$$

$$\therefore M = 0.072 \times 10^{-3} \times 16,000 = \mathbf{1.15 \text{ H}}$$

Now,  $L_1 = \frac{12,500 \times 0.6}{5} = 150 \times 10^{-3} \text{ H}$  ;  $L_2 = \frac{16,000 \times 0.8}{5} = 256 \times 10^{-3} \text{ H}$

(ii)  $k = \frac{M}{\sqrt{L_1 L_2}} = 1.15 / \sqrt{1.5 \times 2.56} = \mathbf{0.586}$

**Note.** We could find  $k$  in another way also. Value of  $k_1 = 0.6$ , that of  $k_2$  could also be found, then  $k = \sqrt{k_1 k_2}$ .

**Example 7.25.** Two magnetically-coupled coils have a mutual inductance of 32 mH. What is the average e.m.f. induced in one, if the current through the other changes from 3 to 15 mA in 0.004 second? Given that one coil has twice the number of turns in the other, calculate the inductance of each coil. Neglect leakage.

**Solution.**  $M = 32 \times 10^{-3} \text{ H}$  ;  $dI_1 = 15 - 3 = 12 \text{ mA} = 12 \times 10^{-3} \text{ A}$  ;  $dt = 0.004 \text{ second}$

$$\text{Average e.m.f. induced} = M \frac{dI_1}{dt} = \frac{32 \times 10^{-3} \times 12 \times 10^{-3}}{0.004} = \mathbf{96 \times 10^{-3} \text{ V}}$$

Now  $L_1 = \mu_0 N^2 A/l = k N^2$  where  $k = \mu_0 A/l$  (taking  $\mu_r = 1$ )

$$L_2 = \frac{(2N)^2 \mu_0 A}{2l} = 2kN^2 ; \frac{L_2}{L_1} = \frac{2kN^2}{kN^2} = 2 \quad \therefore L_2 = 2L_1$$

Now  $M = \sqrt{L_1 L_2} = \sqrt{2L_1 \times L_1} = 32$ ,  $L_1 = 32/\sqrt{2} = \mathbf{16/\sqrt{2} \text{ mH}}$  ;  $L_2 = 2 \times 16/\sqrt{2} = \mathbf{32\sqrt{2} \text{ mH}}$

**Example 7.26.** Two coils, A and B, have self inductances of  $120\ \mu\text{H}$  and  $300\ \mu\text{H}$  respectively. A current of  $1\ \text{A}$  through coil A produces flux linkages of  $100\ \mu\text{Wb}$  turns in coil B. Calculate (i) the mutual inductance between the coils (ii) the coupling coefficient and (iii) the average e.m.f. induced in coil B if a current of  $1\ \text{A}$  in coil A is reversed at a uniform rate in  $0.1\ \text{sec}$ .

(F. E. Pune Univ.)

**Solution. (i)** 
$$M = \frac{\text{flux-linkages of coil B}}{\text{current in coil A}} = \frac{100 \times 10^{-6}}{1} = 100\ \mu\text{H}$$

**(ii)** 
$$M = k\sqrt{L_1 L_2} \quad \therefore k = \frac{M}{\sqrt{L_1 L_2}} = \frac{100 \times 10^{-6}}{\sqrt{120 \times 10^{-6} \times 300 \times 10^{-6}}} = 0.527$$

**(iii)** 
$$e_2 = M \times di/dt = (100 \times 10^{-6}) \times 2/0.1 = 0.002\ \text{V} \text{ or } 2\ \text{mV}.$$

### 7.14. Inductances in Series

(i) Let the two coils be so joined in series that their fluxes (or m.m.fs) are additive i.e., in the same direction (Fig. 7.14).

Let  $M$  = coefficient of mutual inductance  
 $L_1$  = coefficient of self-inductance of 1st coil  
 $L_2$  = coefficient of self-inductance of 2nd coil.

Then, self induced e.m.f. in A is  $e_1 = -L_1 \frac{di}{dt}$

Mutually-induced e.m.f. in A due to change of current in B

is  $e' = -M \frac{di}{dt}$

Self-induced e.m.f. in B is  $e_2 = -L_2 \frac{di}{dt}$

Mutually-induced e.m.f. in B due to change of current in A is  $e_2' = -M \frac{di}{dt}$

(All have -ve sign, because both self and mutually induced e.m.fs. are in opposition to the applied e.m.f.). Total induced e.m.f. in the combination =  $-\frac{di}{dt} (L_1 + L_2 + 2M)$  ... (i)

If  $L$  is the equivalent inductance then total induced e.m.f. in that single coil would have been

$$= -L \frac{di}{dt} \quad \dots (ii)$$

Equating (i) and (ii) above, we have  $L = L_1 + L_2 + 2M$

(ii) When the coils are so joined that their fluxes are in opposite directions (Fig. 7.15).

As before  $e_1 = -L_1 \frac{di}{dt}$

$$e_1' = +M \frac{di}{dt} \quad (\text{mark this direction})$$

$$e_2 = -L_2 \frac{di}{dt} \quad \text{and} \quad e_2' = +M \frac{di}{dt}$$

$$\text{Total induced e.m.f.} = -\frac{di}{dt} (L_1 + L_2 - 2M)$$

$\therefore$  Equivalent inductance

$$L = L_1 + L_2 - 2M$$

In general, we have :

$$L = L_1 + L_2 + 2M$$

and

$$L = L_1 + L_2 - 2M$$

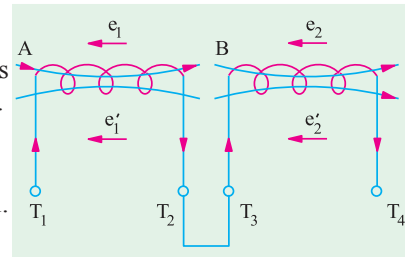


Fig. 7.14

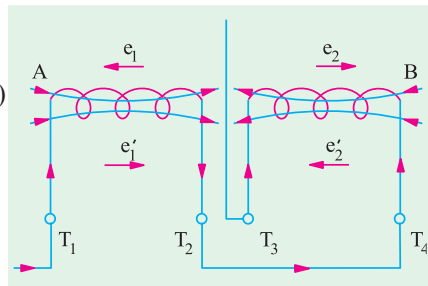


Fig. 7.15

... if m.m.fs are additive

... if m.m.fs. are subtractive

**Example 7.27.** Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.

**Solution. (a)**  $L = L_1 + L_2 + 2M$  or  $1.9 = L_1 + L_2 + 2M$  ... (i)

**(b)** Here  $L = L_1 + L_2 - 2M$  or  $0.7 = L_1 + L_2 - 2M$  ... (ii)

Subtracting (ii) from (i), we get

$$1.2 = 4M \quad \therefore M = 0.3 \text{ H}$$

Putting this value in (i) above, we get  $L_1 + L_2 = 1.3 \text{ H}$  ... (iii)

We know that, in general,  $M = k\sqrt{L_1L_2}$

$$\therefore \sqrt{L_1L_2} = \frac{M}{k} = \frac{0.3}{0.5} = 0.6 \quad \therefore L_1L_2 = \mathbf{0.36}$$

From (iii), we get  $(L_1 + L_2)^2 - 4L_1L_2 = (L_1 - L_2)^2$

$$\therefore (L_1 - L_2)^2 = 0.25 \quad \text{or} \quad L_1 - L_2 = 0.5 \quad \dots \text{(iv)}$$

From (iii) and (iv), we get  $L_1 = \mathbf{0.9 \text{ H}}$  and  $L_2 = \mathbf{0.4 \text{ H}}$

**Example 7.28.** The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate (a) mutual inductance and (b) coupling coefficient.

(Elect. Technology, Univ. of Indore)

**Solution. (j)**  $L = L_1 + L_2 + 2M$  or  $0.6 = L_1 + L_2 + 2M$  ... (i)

and  $0.1 = L_1 + L_2 - 2M$  ... (ii)

**(a)** From (i) and (ii) we get,  $M = \mathbf{0.125 \text{ H}}$

Let  $L_1 = 0.2 \text{ H}$ , then substituting this value in (i) above, we get  $L_2 = 0.15 \text{ H}$

**(b)** Coupling coefficient  $k = M\sqrt{L_1L_2} = 0.125 / \sqrt{0.2 \times 0.15} = \mathbf{0.72}$

**Example 7.29.** Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80 mH. Calculate the self inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.

(F. E. Pune Univ.)

**Solution.** If each coil has an inductance of  $L$  henry, then  $L_1 = L_2 = L$ ;  $M = k\sqrt{L_1L_2} = k\sqrt{L \times L} = kL$

When connected in series cumulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L(1 + 0.25) = 2.5L$$

$$\therefore 2.5L = 80 \quad \text{or} \quad L = \mathbf{32 \text{ mH}}$$

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L(1 - k) = 2L(1 - 0.25)$$

$$\therefore 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48 \text{ mH}}$$

**Example 7.30.** Two coils with terminals  $T_1, T_2$  and  $T_3, T_4$  respectively are placed side by side. When measured separately, the inductance of the first coil is 1200 mH and that of the second is 800 mH. With  $T_2$  joined to  $T_3$ , the inductance between  $T_1$  and  $T_4$  is 2500 mH. What is the mutual inductance between the two coils? Also, determine the inductance between  $T_1$  and  $T_3$  when  $T_2$  is joined to  $T_4$ .

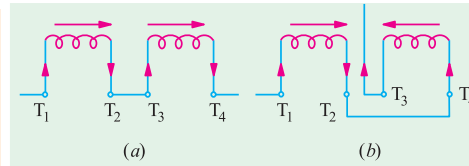


Fig. 7.16

(Electrical Circuit, Nagpur Univ. 1991)

**Solution.**  $L_1 = 1200 \text{ mH}, L_2 = 800 \text{ mH}$

Fig. 7.16 (a) shows additive series.

$$\therefore L = L_1 + L_2 + 2M$$

or  $2500 = 1200 + 800 + 2M$ ;  $M = \mathbf{250 \text{ mH}}$

Fig. 7.16 (b) shows the case of subtractive or opposing series.

Here,  $L = L_1 + L_2 - 2M = 1200 + 800 - 2 \times 250 = 1500 \text{ mH}$

**Example 7.31.** The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2 H, depending on the relative directions of the current in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate

(a) the mutual inductance between the two coils

(b) the self-inductance of coil B

(c) the coupling factor between the coils.

(d) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A per second in the series circuit. **(Elect. Technology, Hyderabad Univ. 1992)**

**Solution.** (a) Combined inductance is given by  $L = L_1 + L_2 \pm 2M$

$$\therefore 0.5 = L_1 + L_2 + 2M \quad \dots(i), \quad 0.2 = L_1 + L_2 - 2M \quad \dots(ii)$$

Subtracting (ii) from (i), we have  $4M = 0.3$  or  $M = 0.075 \text{ H}$

(b) Adding (i) and (ii) we have  $0.7 = 2 \times 0.2 + 2L_2 = 0.15 \text{ H}$

(c) Coupling factor or coefficient is  $k = M / \sqrt{L_1 L_2} = 0.075 / \sqrt{0.2 \times 0.15} = 0.433$  or **43.4%**

$$(d) \quad e_1 = L_1 \frac{di}{dt} \pm M \frac{di}{dt}$$

$$\therefore e_1 = (0.2 + 0.075) \times 1000 = 275 \text{ V} \quad \dots \text{'cumulative connection'}$$

$$= (0.2 - 0.075) \times 1000 = 125 \text{ V} \quad \dots \text{'differential connection'}$$

**Example 7.32.** Find the equivalent inductance  $L_{AB}$  in Fig. 7.17

**(Bombay University, 2001)**

**Solution.** Series Parallel combination of Inductors has to be dealt with. Note that there is no mutual coupling between coils.  
 $L_{AB} = 0.5 + [0.6 \times 0.3 / (0.3 + 0.3)] = 0.7 \text{ H}$

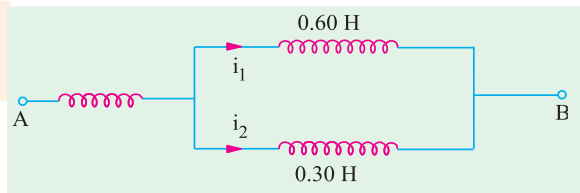


Fig. 7.17

## 7.15. Inductance in Parallel

In Fig. 7.18, two inductances of values  $L_1$  and  $L_2$  henry are connected in parallel. Let the coefficient of mutual inductance between the two be  $M$ . Let  $i$  be the main supply current and  $i_1$  and  $i_2$  be the branch currents

$$\text{Obviously, } i = i_1 + i_2$$

$$\therefore \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \dots(i)$$

In each coil, both self and mutually induced e.m.fs. are produced. Since the coils are in parallel, these e.m.fs. are equal. For a case when self-induced e.m.f.,

we get

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \therefore L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\text{or } \frac{di_1}{dt} (L_1 - M) = \frac{di_2}{dt} (L_2 - M) \quad \therefore \frac{di_1}{dt} = \left( \frac{L_2 - M}{L_1 - M} \right) \frac{di_2}{dt} \quad \dots(ii)$$

$$\text{Hence, (i) above becomes } \frac{di}{dt} = \left[ \left( \frac{L_2 - M}{L_1 - M} \right) + 1 \right] \frac{di_2}{dt} \quad \dots(iii)$$

If  $L$  is the equivalent inductance, then  $e = L \frac{di}{dt}$  = induced e.m.f. in the parallel combination

$$= \text{induced e.m.f. in any one coil} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

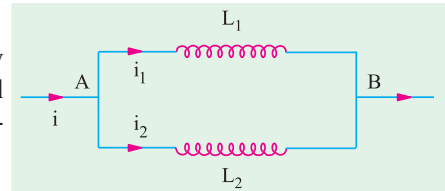


Fig. 7.18

$$\therefore \frac{di}{dt} = \frac{1}{L} \left( L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) \quad \dots (iv)$$

Substituting the value of  $di_1/dt$  from (ii) in (iv), we get  $\frac{di}{dt} = \frac{1}{L} \left[ L_1 \left( \frac{L_2 - M}{L_1 - M} \right) + M \right] \frac{di_2}{dt} \quad \dots (v)$

Hence, equating (iii) to (iv), we have  $\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L} \left[ L_1 \left( \frac{L_2 - M}{L_1 - M} \right) + M \right]$

or  $\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L} \left( \frac{L_1 L_2 - M^2}{L_1 - M} \right)$

$\therefore L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$  when mutual field assists the separate fields.

Similarly,  $L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$  when the two fields oppose each other.

**Example 7.33.** Two coils of inductances 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if (i) mutual inductance assists the self-inductance (ii) mutual inductance opposes the self-inductance.

**Solution.** (i)  $L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 6 - 3^2}{4 + 6 - 2 \times 3} = \frac{15}{4} = 3.75 \text{ H}$

(ii)  $L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{24 - 9}{16} = \frac{15}{16} = 0.94 \text{ H (approx.)}$

### Tutorial Problems No. 7.2

- Two coils are wound close together on the same paxolin tube. Current is passed through the first coil and is varied at a uniform rate of 500 mA per second, inducing an e.m.f. of 0.1 V in the second coil. The second coil has 100 turns. Calculate the number of turns in the first coil if its inductance is 0.4 H. **[200 turns]**
- Two coils have 50 and 500 turns respectively are wound side by side on a closed iron circuit of section  $50 \text{ cm}^2$  and mean length 120 cm. Estimate the mutual inductance between the coils if the permeability of iron is 1000. Also, find the self-inductance of each coil. If the current in one coil grows steadily from zero to 5A in 0.01 second, find the e.m.f. induced in the other coil. **[M = 0.131 H, L<sub>1</sub> = 0.0131 H, L<sub>2</sub> = 1.21 H, E = 65.4 V]**
- An iron-cored choke is designed to have an inductance of 20 H when operating at a flux density of  $1 \text{ Wb/m}^2$ , the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding, given that the magnetic flux path has a mean length of 22 cm in the iron core and of 1 mm in air-gap that its cross-section is  $10 \text{ cm}^2$ . Neglect leakage and fringing. **[4100]**
- A non-magnetic ring having a mean diameter of 30 cm and a cross-sectional area of  $4 \text{ cm}^2$  is uniformly wound with two coils A and B, one over the other. A has 90 turns and B has 240 turns. Calculate from first principles the mutual inductance between the coils. Also, calculate the e.m.f. induced in B when a current of 6 A in A is reversed in 0.02 second. **[11.52 μH, 6.912 mV]**
- Two coils A and B, of 600 and 100 turns respectively are wound uniformly around a wooden ring having a mean circumference of 30 cm. The cross-sectional area of the ring is  $4 \text{ cm}^2$ . Calculate (a) the mutual inductance of the coils and (b) the e.m.f. induced in coil B when a current of 2 A in coil A is reversed in 0.01 second. **[(a) 100.5 μH (b) 40.2 mV]**
- A coil consists of 1,000 turns of wire uniformly wound on a non-magnetic ring of mean diameter 40 cm and cross-sectional area  $20 \text{ cm}^2$ . Calculate (a) the inductance of the coil (b) the energy stored in the magnetic field when the coil is carrying a current of 15 A (c) the e.m.f. induced in the coil if this current is completely interrupted in 0.01 second. **[(a) 2mH (b) 0.225 joule (c) 3V]**
- A coil of 50 turns having a mean diameter of 3 cm is placed co-axially at the centre of a solenoid 60 cm long, wound with 2,500 turns and carrying a current of 2 A. Determine mutual inductance of the arrangement. **[0.185 mH]**
- A coil having a resistance of  $2 \Omega$  and an inductance of 0.5 H has a current passed through it which

varies in the following manner ; (a) a uniform change from zero to 50 A in 1 second (b) constant at 50 A for 1 second (c) a uniform change from 50 A to zero in 2 seconds. Plot the current graph to a time base. Tabulate the p.d. applied to the coil during each of the above periods and plot the graph of p.d. to a time base. **[(a) 25 to 125 V (b) 100 V (c) 87.5 V to -12.5 V]**

9. A primary coil having an inductance of  $100 \mu\text{H}$  is connected in series with a secondary coil of  $240 \mu\text{H}$  and the total inductance of the combination is measured as  $146 \mu\text{H}$ . Determine the coefficient of coupling.

10. Find the total inductance measured from A-B terminals, in Fig. 7.19. **[62.6%] (Circuit Theory, Jadavpur Univ.)**

**[Hint :  $L = 100 + 50 - (2 \times 60) = 30 \mu\text{H}$ , due to opposite senses of currents with respect to dot-markings.]**

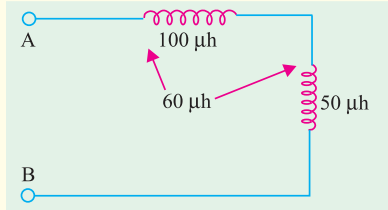


Fig. 7.19

11. Given that relative permeability of cast iron as 200, that of cast steel is 1200 and for Copper  $\mu_0 = 1$ . **(Nagpur University, Summer 2003)**
12. State Faraday's laws of electromagnetic induction. Distinguish between statically induced emf and dynamically induced emf with examples. **(V.T.U., Belgaum Karnataka University, February 2002)**

13. State : (i) Fleming's right hand rule, and (ii) Fleming's left hand rule.

Mention their applications.

**(V.T.U., Belgaum Karnataka University, Winter 2003)**

14. Define : (i) Self inductance, and (ii) Mutual inductance.

Mention their units and formula to calculate each of them. Derive an expression for the energy stored in an inductor of self inductance 'L' henry carrying the current of 'I' amperes.

**(V.T.U., Belgaum Karnataka University, Winter 2003)**

15. State and explain Faraday's laws of electro magnetic induction, Lenz's Law. Fleming's right hand rule and Fleming's left hand rule. **(V.T.U., Belgaum Karnataka University, Summer 2003)**

16. A coil of 300 turns wound on a core of non magnetic material has an inductance of  $10\text{mH}$ . Calculate (i) the flux produced by a current of  $5\text{A}$  (ii) the average value of the emf induced when a current of  $5\text{A}$  is reversed in 8 mills seconds. **(V.T.U., Belgaum Karnataka University, Summer 2003)**

## OBJECTIVE TESTS – 7

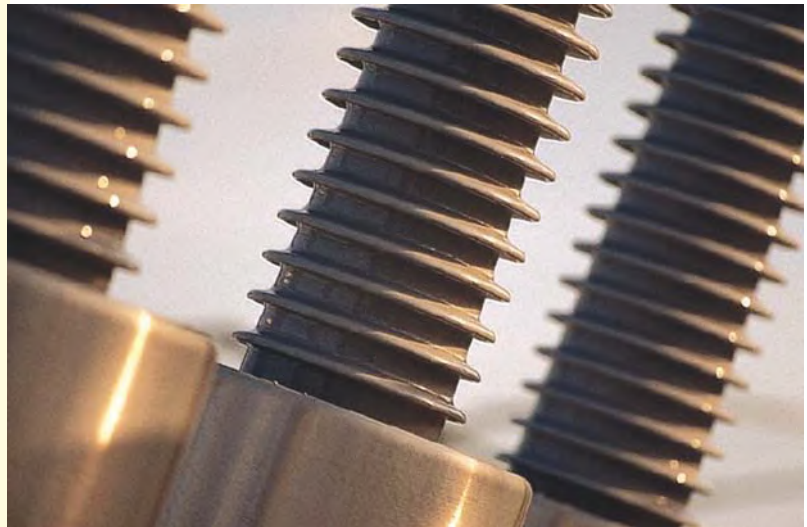
- According to Faraday's Laws of Electromagnetic Induction, an e.m.f. is induced in a conductor whenever it
  - lies in a magnetic field
  - cuts magnetic flux
  - moves parallel to the direction of the magnetic field
  - lies perpendicular to the magnetic flux.
- A pole of driving point admittance function implies
  - zero current for a finite value of driving voltage
  - zero voltage for a finite value of driving current
  - an open circuit condition
  - None of (a), (b) and (c) mentioned in the question **(ESE 2001)**
- The inductance of a long solenoid of length  $1000 \text{ mm}$  wound uniformly with  $3000$  turns on a cylindrical paper tube of  $60 \text{ mm}$  diameter is
  - $3.2 \mu\text{H}$
  - $3.2 \text{ mH}$
  - $32.0 \text{ mH}$
  - $3.2 \text{ H}$**(GATE 2004)**
- A moving iron ammeter produced a full scale torque of  $240 \mu\text{Nm}$  with a deflection of  $120^\circ$  at a current of  $10 \text{ A}$ . The rate of change of self inductance ( $\mu\text{H}/\text{radian}$ ) of the instrument at full scale is
  - $2.0 \mu\text{H}/\text{radian}$
  - $4.8 \mu\text{H}/\text{radian}$
  - $12.0 \mu\text{H}/\text{radian}$
  - $114.6 \mu\text{H}/\text{radian}$**(GATE 2004)**
- The self-inductance of a long cylindrical conductor due to its internal flux linkages is  $k \text{ H/m}$ . If the diameter of the conductor is doubled, then the selfinductance of the conductor due its internal flux linkages would be
  - $0.5 \text{ K H/m}$
  - $\text{K H/m}$
  - $1.414 \text{ K H/m}$
  - $4 \text{ K H/m}$**(GATE)**



## CHAPTER 8

**Learning Objectives**

- Magnetic Hysteresis
- Area of Hysteresis Loop
- Properties and Application of Ferromagnetic Materials
- Permanent Magnet Materials
- Steinmetz Hysteresis Law
- Energy Stored in Magnetic Field
- Rate of Change of Stored Energy
- Energy Stored per Unit Volume
- Lifting Power of Magnet
- Rise of Current in Inductive Circuit
- Decay of Current in Inductive Circuit
- Details of Transient Current Rise in  $R$ - $L$  Circuit
- Details of Transient Current Decay in  $R$ - $L$  Circuit
- Automobile Ignition System

**MAGNETIC  
HYSTERESIS**

Magnetic hysteresis is one of the important considerations in choosing and designing the cores of transformers and other electric machines

### 8.1. Magnetic Hysteresis

It may be defined as the lagging of magnetisation or induction flux density ( $B$ ) behind the magnetising force ( $H$ ). Alternatively, it may be defined as that quality of a magnetic substance, due to which energy is dissipated in it, on the reversal of its magnetism.

Let us take an unmagnetised bar of iron  $AB$  and magnetise it by placing it within the field of a solenoid (Fig. 8.1). The field  $H (= NI/l)$  produced by the solenoid is called the magnetising force. The value of  $H$  can be increased or decreased by increasing or decreasing current through the coil. Let  $H$  be increased in steps from zero up to a certain maximum value and the corresponding values of flux density ( $B$ ) be noted. If we plot the relation between  $H$  and  $B$ , a curve like  $OA$ , as shown in Fig. 8.2, is obtained. The material becomes magnetically saturated for  $H = OM$  and has at that time a maximum flux density of  $B_{max}$  established through it.

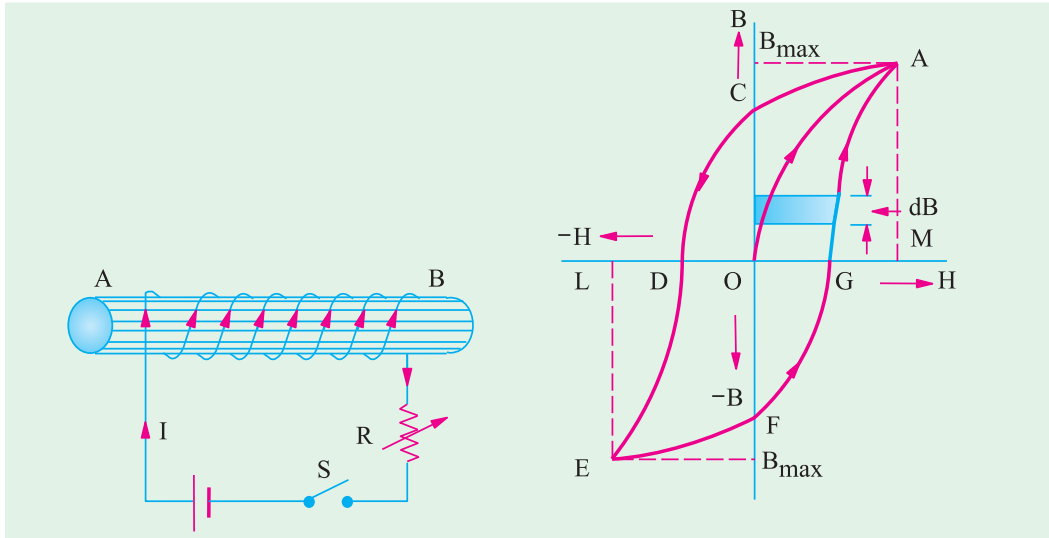


Fig. 8.1

Fig. 8.2

If  $H$  is now decreased gradually (by decreasing solenoid current), flux density  $B$  will not decrease along  $AO$ , as might be expected, but will decrease less rapidly along  $AC$ . When  $H$  is zero,  $B$  is not but has a definite value  $B_r = OC$ . It means that on removing the magnetising force  $H$ , the iron bar is not completely demagnetised. This value of  $B (= OC)$  measures the **retentivity or remanence** of the material and is called the remanent or residual flux density  $B_r$ .

To demagnetise the iron bar, we have to apply the magnetising force in the reverse direction. When  $H$  is reversed (by reversing current through the solenoid), then  $B$  is reduced to zero at point  $D$  where  $H = OD$ . This value of  $H$  required to wipe off residual magnetism is known as **coercive force** ( $H_c$ ) and is a measure of the **coercivity** of the material *i.e.* its 'tenacity' with which it holds on to its magnetism.

If, after the magnetisation has been reduced to zero, value of  $H$  is further increased in the 'negative' *i.e.* reversed direction, the iron bar again reaches a state of magnetic saturation, represented by point  $L$ . By taking  $H$  back from its value corresponding to negative saturation, ( $= OL$ ) to its value for positive saturation ( $= OM$ ), a similar curve  $EFGA$  is obtained. If we again start from  $G$ , the same curve  $GACDEFG$  is obtained once again.\*

\* In fact, when  $H$  is varied a number of times between fixed positive and negative maxima, the size of the loop becomes smaller and smaller till the material is cyclically magnetised. A material is said to be cyclically magnetised when for each increasing (or decreasing) value of  $H$ ,  $B$  has the same value in successive cycles.

It is seen that  $B$  always lag behind  $H$ . The two never attain zero value simultaneously. This lagging of  $B$  behind  $H$  is given the name 'hysteresis' which literally means '*to lag behind*'. The closed loop  $ACDEFGA$  which is obtained when iron bar is taken through one complete cycle of magnetisation is known as 'hysteresis loop'.

By one cycle of magnetisation of a magnetic material is meant its being carried through one reversal of magnetisation, as shown in Fig. 8.3.

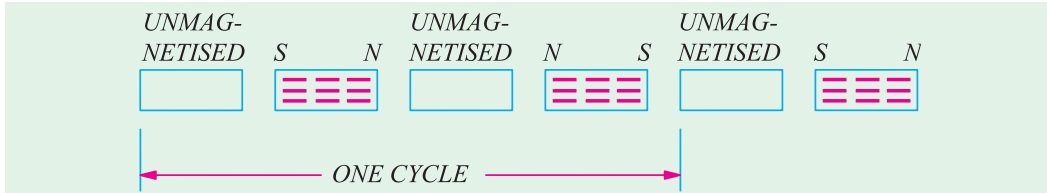


Fig. 8.3

## 8.2. Area of Hysteresis Loop

Just as the area of an indicator diagram measures the energy made available in a machine, when taken through one cycle of operation, so also the area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

According to *Weber's Molecular Theory* of magnetism, when a magnetic material is magnetised, its molecules are forced along a straight line. So, energy is spent in this process. Now, if iron has no retentivity, then energy spent in straightening the molecules could be recovered by reducing  $H$  to zero in the same way as the energy stored up in a spring can be recovered by allowing the spring to release its energy by driving some kind of load. Hence, in the case of magnetisation of a material of *high retentivity*, all the energy put into it originally for straightening the molecules is not recovered when  $H$  is reduced to zero. We will now proceed to find this loss of energy per cycle of magnetisation.

Let  $l$  = mean length of the iron bar ;  $A$  = its area of cross-section;  $N$  = No. of turns of wire of the solenoid.

If  $B$  is the flux density at any instant, then  $\Phi = BA$ .

When current through the solenoid changes, then flux also changes and so produces an induced e.m.f. whose value is

$$e = N \frac{d\Phi}{dt} \text{ volt} = N \frac{d}{dt} (BA) = NA \frac{dB}{dt} \text{ volt} \quad (\text{neglecting -ve sign})$$

$$\text{Now} \quad H = \frac{NI}{l} \quad \text{or} \quad I = \frac{Hl}{N}$$

The power or rate of expenditure of energy in maintaining the current ' $I$ ' against induced e.m.f. ' $e$ ' is

$$= e I \text{ watt} = \frac{Hl}{N} \times NA \frac{dB}{dt} = AlH \frac{dB}{dt} \text{ watt}$$

$$\text{Energy spent in time 'dt'} = AlH \frac{dB}{dt} \times dt = Al.H.dB \text{ joule}$$

Total net work done for one cycle of magnetisation is  $W = Al \oint H dB$  joule

where  $\oint$  stands for integration over the whole cycle. Now, ' $H dB$ ' represents the shaded area in Fig. 8.2. Hence,  $\oint H dB$  = area of the loop *i.e.* the area between the  $B/H$  curve and the  $B$ -axis

$$\therefore \text{work done/cycle} = A_l \times (\text{area of the loop}) \text{ joule. Now } A_l = \text{volume of the material}$$

$$\therefore \text{net work done/cycle/m}^3 = (\text{loop area}) \text{ joule, or } W_h = (\text{Area of } B/H \text{ loop}) \text{ joule m}^3/\text{cycle}$$

### Precaution

*Scale of  $B$  and  $H$  should be taken into consideration while calculating the actual loop area.*

For example, if the scales are,  $1 \text{ cm} = x \text{ AT/m}$  –for  $H$  and  $1 \text{ cm} = y \text{ Wb/m}^2$  –for  $B$   
then

$$W_h = xy (\text{area of } B/H \text{ loop}) \text{ joule/m}^3/\text{cycle}$$

In the above expression, loop area has to be in  $\text{cm}^2$ .

As seen from above, hysteresis loop measures the energy dissipated due to hysteresis which appears in the form of heat and so raises the temperature of that portion of the magnetic circuit which is subjected to magnetic reversal. The shape of the hysteresis loop depends on the nature of the magnetic material (Fig. 8.4).

Loop 1 is for hard steel. Due to its high retentivity and collectivity, it is well suited for making permanent magnets. But due to large hysteresis loss (as shown by large loop area) it is not suitable for rapid reversals of magnetisation. Certain alloys of aluminium, nickel and steel called Alnico alloys have been found extremely suitable for making permanent magnets.

Loop 2 is for wrought iron and cast steel. It shows that these materials have high permeability and fairly good coercivity, hence making them suitable for cores of electromagnets.

Loop 3 is for alloyed sheet steel and it shows high permeability and low hysteresis loss. Hence, such materials are most suited for making armature and transformer cores which are subjected to rapid reversals of magnetisation.

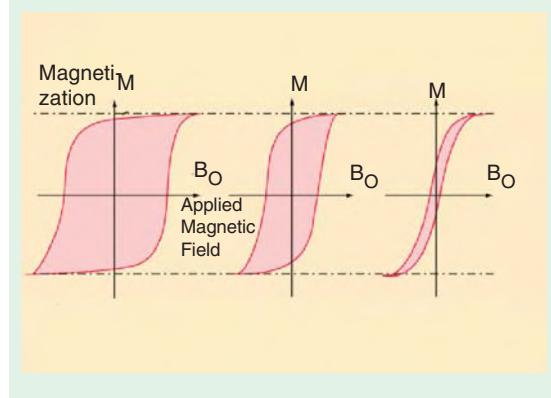
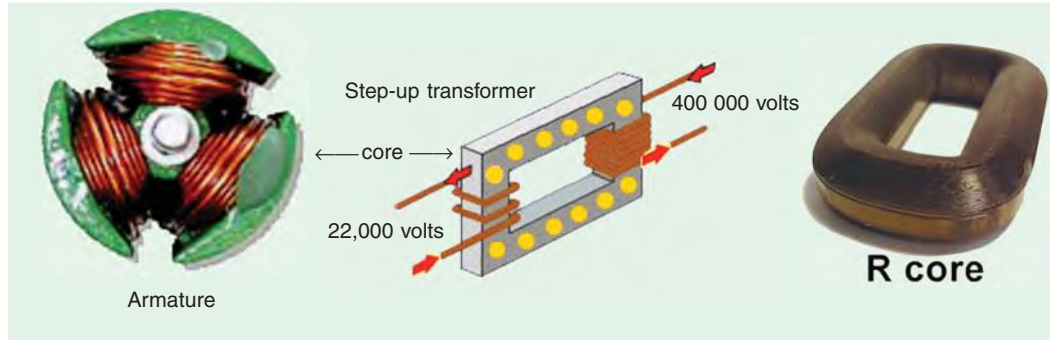


Fig. 8.4

### 8.3. Properties and Applications of Ferromagnetic Materials

Ferromagnetic materials having low retentivities are widely used in power and communication apparatus. Since silicon iron has high permeability and saturation flux density, it is extensively used



in the magnetic circuits of electrical machines and heavy current apparatus where a high flux density is desirable in order to limit the cross-sectional area and, therefore, the weight and cost. Thin silicon-iron laminations (clamped together but insulated from each other by varnish, paper or their own surface scale) are used in the construction of transformer and armature cores where it is essential to minimize hysteresis and eddy-current losses.

In field systems (where flux remains constant), a little residual magnetism is desirable. For such systems, high permeability and high saturation flux density are the only important requirements which are adequately met by fabricated rolled steel or cast or forged steel.

Frequencies used in line communication extend up to 10 MHz whereas those used in radio vary from about 100 kHz to 10 GHz. Hence, such material which have high permeability and low losses are very desirable. For these applications, nickel-iron alloys containing up to 80 per cent of nickel and a small percentage of molybdenum or copper, cold rolled and annealed are very suitable.

### 8.4. Permanent Magnet Materials

Permanent magnets find wide application in electrical measuring instruments, magnetos, mag-

netic chucks and moving-coil loudspeakers etc. In permanent magnets, high retentivity as well as high coercivity are most desirable in order to resist demagnetisation. In fact, the product  $B_r H_c$  is the best criterion for the merit of a permanent magnet. The material commonly used for such purposes are carbon-free iron-nickel-aluminium copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses  $B_r H_c$  value of about  $40,000 \text{ J/m}^3$  as compared with  $2,500 \text{ J/m}^3$  for chromium-steel.

**Example 8.1.** The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of  $1.3 \text{ Wb/m}^2$  has an area of  $93 \text{ cm}^2$ , the scales being  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$  and  $1 \text{ cm} = 50 \text{ AT/m}$ . Calculate the hysteresis loss in watts when  $1500 \text{ cm}^3$  of the same material is subjected to an alternating flux density of  $1.3 \text{ Wb/m}^2$  peak value of a frequency of  $65 \text{ Hz}$ .

(Electromechanics, Allahabad Univ, 1992)

**Solution.**

$$\begin{aligned} \text{Loss} &= xy \text{ (area of } B/H \text{ loop) J/m}^3/\text{cycle} \\ &= 0.1 \times 50 \times 93 = 465 \text{ J/m}^3/\text{cycle} \\ \text{Volume} &= 1500 \text{ cm}^3 = 15 \times 10^{-4} \text{ m}^3; \text{ No. of reversals/second} = 65 \\ \therefore W_h &= 465 \times 15 \times 10^{-4} \times 65 \text{ J/s} = \mathbf{45.3 \text{ W}} \end{aligned}$$

**Note.** The given value of  $B_{max} = 1.3 \text{ Wb/m}^2$  is not required for solution.

**Example 8.2.** Calculate the hourly loss of energy in kWh in a specimen of iron, the hysteresis loop of which is equivalent in area to  $250 \text{ J/m}^3$ . Frequency  $50 \text{ Hz}$ ; specific gravity of iron  $7.5$ ; weight of specimen  $10 \text{ kg}$ .

(Electrical Engg. Materials, Nagpur Univ. 1991)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= 250 \text{ J/m}^3/\text{cycle}, \text{ Mass of iron} = 10 \text{ kg} \\ \text{Volume of iron specimen} &= 10/7.5 \times 10^3 \text{ m}^3 = 10^2/7.5 \text{ m}^3 \\ \text{No. of cycles of reversals/hr} &= 60 \times 50 = 3000 \\ \therefore \text{loss/hour} &= 250 \times (10^2/7.5) \times 3000 = 1000 \text{ J} = 1000/36 \times 10^5 = \mathbf{27.8 \times 10^5 \text{ kWh}} \end{aligned}$$

**Example 8.3.** The hysteresis loop for a certain magnetic material is drawn to the following scales:  $1 \text{ cm} = 200 \text{ AT/m}$  and  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$ . The area of the loop is  $48 \text{ cm}^2$ . Assuming the density of the material to be  $7.8 \times 10^3 \text{ kg/m}^3$ , calculate the hysteresis loss in watt/kg at  $50 \text{ Hz}$ .

(Elect. Circuits & Fields, Gujarat Univ.)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= xy \text{ (area of } B/H \text{ loop) J/m}^3/\text{cycle} \\ \text{Now, } 1 \text{ cm} &= 200 \text{ AT/m}; 1 \text{ cm} = 0.1 \text{ Wb/m}^2 \\ \therefore x &= 200, y = 0.1, \text{ area of loop} = 48 \text{ cm}^2 \\ \therefore \text{loss} &= 200 \times 0.1 \times 48 = 960 \text{ J/m}^3/\text{cycles}, \text{ Density} = 7.8 \times 10^3 \text{ kg/m}^3 \\ \text{Volume of 1 kg of material} &= \text{mass/density} = 1/7.8 \times 10^3 \text{ m}^3 \\ \therefore \text{loss} &= 960 \times 1/7.8 \times 10^3 \text{ J/cycle No. of reversals/second} = 50 \\ \therefore \text{loss} &= 960 \times 50 \times 10^{-3}/7.8 = 6.15 \text{ J/s or watt} \\ \therefore \text{hysteresis loss} &= \mathbf{6.15 \text{ watt/kg.}} \end{aligned}$$

**Example 8.4.** Determine the hysteresis loss in an iron core weighing  $50 \text{ kg}$  having a density of  $7.8 \times 10^3 \text{ kg/m}^3$  when the area of the hysteresis loop is  $150 \text{ cm}^2$ , frequency is  $50 \text{ Hz}$  and scales on X and Y axes are:  $1 \text{ cm} = 30 \text{ AT/cm}$  and  $1 \text{ cm} = 0.2 \text{ Wb/m}^2$  respectively.

(Elements of Elect. Engg-1, Bangalore Univ.)

**Solution.**

$$\begin{aligned} \text{Hysteresis loss} &= xy \text{ (area of } B/H \text{ loop) J/m}^3/\text{cycle} \\ 1 \text{ cm} &= 30 \text{ AT/cm} = 3000 \text{ AT/m}; 1 \text{ cm} = 0.2 \text{ Wb/m}^2 \\ x &= 3000, y = 0.2, A = 150 \text{ cm}^2 \\ \therefore \text{loss} &= 3000 \times 0.2 \times 150 = 90,000 \text{ J/m}^3/\text{cycle} \\ \text{Volume of 50 kg of iron} &= m/\rho = 50/7.8 \times 10^3 = 6.4 \times 10^{-3} \text{ m}^3 \\ \therefore \text{loss} &= 90,000 \times 6.4 \times 10^{-3} \times 50 = 28,800 \text{ J/s or watts} = \mathbf{28.8 \text{ kW}} \end{aligned}$$

**Example 8.5.** In a transformer core of volume  $0.16 \text{ m}^3$ , the total iron loss was found to be  $2,170 \text{ W}$  at  $50 \text{ Hz}$ . The hysteresis loop of the core material, taken to the same maximum flux density, had an area of  $9.0 \text{ cm}^2$  when drawn to scales of  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$  and  $1 \text{ cm} = 250 \text{ AT/m}$ . Calculate the total iron loss in the transformer core if it is energised to the same maximum flux density but at a frequency of  $60 \text{ Hz}$ .

**Solution.**  $W_h = xy \times (\text{area of hysteresis loop})$  where  $x$  and  $y$  are the scale factors.  
 $W_h = 9 \times 0.1 \times 250 = 225 \text{ J/m}^3/\text{cycle}$

**At 50 Hz**

Hysteresis loss =  $225 \times 0.16 \times 50 = 1,800 \text{ W}$  ; Eddy-current loss =  $2,170 - 1800 = 370 \text{ W}$

**At 60 Hz**

Hysteresis loss =  $1800 \times 60/50 = 2,160 \text{ W}$  ; Eddy-current loss =  $370 \times (60/50)^2 = 533 \text{ W}$

Total iron loss =  $2,160 + 533 = 2,693 \text{ W}$

### Tutorial Problems No. 8.1

- The area of a hysteresis loop of a material is  $30 \text{ cm}^2$ . The scales of the co-ordinates are :  $1 \text{ cm} = 0.4 \text{ Wb/m}^2$  and  $1 \text{ cm} = 400 \text{ AT/m}$ . Determine the hysteresis power loss if  $1.2 \times 10^{-3} \text{ m}^3$  of the material is subjected to alternating flux density at 50 Hz. **[288 W] (Elect. Engg., Aligarh Univ)**
- Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of the hysteresis loop represents 240 joules/ $\text{m}^3$  and the density of iron is  $7800 \text{ kg/m}^3$ . **[138,240] (Principles of Elect. Engg. I, Jadavpur Univ.)**
- The hysteresis loop of a specimen weighing 12 kg is equivalent to 300 joules/ $\text{m}^3$ . Find the loss of energy per hour at 50 Hz. Density of iron is  $7500 \text{ kg/m}^3$ . **[86,400] (Electrotechnics - I, Gawahati Univ.)**
- The area of the hysteresis loop for a steel specimen is  $3.84 \text{ cm}^2$ . If the ordinates are to the scales :  $1 \text{ cm} = 400 \text{ AT/m}$  and  $1 \text{ cm} = 0.5 \text{ Wb/m}^2$ , determine the power loss due to hysteresis in  $1,200 \text{ cm}^3$  of the steel if it is magnetised from a supply having a frequency of 50 Hz. **[46.08 W]**
- The armature of a 4-pole d.c. motor has a volume of  $0.012 \text{ m}^3$ . In a test on the steel iron used in the armature carried out to the same value of maximum flux density as exists in the armature, the area of the hysteresis loop obtained represented a loss of  $200 \text{ J/m}^3$ . Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m. **[72 W]**
- In a magnetisation test on a sample of iron, the following values were obtained.

$H$ (AT/m)	1,900	2,000	3,000	4,000	4,500	3,000	1,000	0	-1,000	-1,900
$B$ (Wb/ $\text{m}^2$ )	0	0.2	0.58	0.7	0.73	0.72	0.63	0.54	0.38	0

Draw the hysteresis loop and find the loss in watts if the volume of iron is  $0.1 \text{ m}^3$  and frequency is 50 Hz. **[22 kW]**

### 8.5. Steinmetz Hysteresis Law

It was experimentally found by Steinmetz that hysteresis loss per  $\text{m}^3$  per cycle of magnetisation of a magnetic material depends on (i) the maximum flux density established in it i.e.  $B_{\max}$  and (ii) the magnetic quality of the material.

$$\therefore \text{Hysteresis loss } W_h \propto B_{\max}^{1.6} \text{ joule/m}^3/\text{cycle} = \eta B_{\max}^{1.6} \text{ joule/m}^3 \text{ cycle}$$

where  $\eta$  is a constant depending on the nature of the magnetic material and is known as **Steinmetz hysteresis coefficient**. The index 1.6 is empirical and holds good if the value of  $B_{\max}$  lies between 0.1 and  $1.2 \text{ Wb/m}^2$ . If  $B_{\max}$  is either lesser than  $0.1 \text{ Wb/m}^2$  or greater than  $1.2 \text{ Wb/m}^2$ , the index is greater than 1.6.

$$\therefore W_h = \eta B_{\max}^{1.6} fV \text{ J/s or watt}$$

where  $f$  is frequency of reversals of magnetisation and  $V$  is the volume of the magnetic material.

The armatures of electric motors and generators and transformer cores etc. which are subjected to rapid reversals of magnetisation should, obviously, be made of substances having low hysteresis coefficient in order to reduce the hysteresis loss.

**Example 8.6.** A cylinder of iron of volume  $8 \times 10^{-3} \text{ m}^3$  revolves for 20 minutes at a speed of 3,000 r.p.m in a two-pole field of flux density  $0.8 \text{ Wb/m}^2$ . If the hysteresis coefficient of iron is 753.6 joule/ $\text{m}^3$ , specific heat of iron is 0.11, the loss due to eddy current is equal to that due to hysteresis and 25% of the heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as  $7.8 \times 10^3 \text{ kg/m}^3$ . **(Elect. Engineering-I, Osmania Univ.)**

**Solution.** An armature revolving in a multipolar field undergoes one magnetic reversal after passing under a pair of poles. In other words, number of magnetic reversals in the same as the number of pair of poles. If  $P$  is the number of poles, the magnetic reversals in one revolution are  $P/2$ . If speed of armature rotation is  $N$  r.p.m, then number of revolutions/second =  $N/60$ .

$$\begin{aligned}\text{No. of reversals/second} &= \text{reversals in one revolutions} \times \text{No. of revolutions/second} \\ &= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ reversals/second}\end{aligned}$$

$$\text{Here } N = 3,000 \text{ r.p.m ; } P = 2 \therefore f = \frac{3,000 \times 2}{120} = 50 \text{ reversals/second}$$

According to Steinmetz's hysteresis law,  $W_h = \eta B_{\max}^{1.6} f V$  watt

Note that  $f$  here stands for magnetic reversals/second and not for mechanical frequency armature rotation.

$$W_h = 753.6 \times (0.8)^{1.6} \times 50 \times 8 \times 10^{-3} = 211 \text{ J/s}$$

$$\text{Loss in 20 minutes} = 211 \times 1,200 = 253.2 \times 10^3 \text{ J}$$

$$\text{Eddy current loss} = 253.2 \times 10^3 \text{ J; Total loss} = 506.4 \times 10^3 \text{ J}$$

$$\text{Heat produced} = 506.4 \times 10^3 / 4200 = 120.57 \text{ kcal ; Heat utilized} = 120.57 \times 0.75 = 90.43 \text{ kcal}$$

$$\text{Heat absorbed by iron} = (8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 t \text{ kcal}$$

$$\therefore (8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 \times t = 90.43 \quad \therefore t = 13.17^\circ\text{C}$$

**Example 8.7.** The area of the hysteresis loop obtained with a certain specimen of iron was  $9.3 \text{ cm}^2$ . The coordinates were such that  $1 \text{ cm} = 1,000 \text{ AT/m}$  and  $1 \text{ cm} = 0.2 \text{ Wb/m}^2$ . Calculate (a) the hysteresis loss per  $\text{m}^3$  per cycle and (b) the hysteresis loss per  $\text{m}^3$  at a frequency of 50 Hz if the maximum flux density were  $1.5 \text{ Wb/m}^2$  (c) calculate the hysteresis loss per  $\text{m}^3$  for a maximum flux density of  $1.2 \text{ Wb/m}^2$  and a frequency of 30 Hz, assuming the loss to be proportional to  $B_{\max}^{1.6}$ .

(Elect. Technology, Allahabad Univ. 1991)

$$\text{Solution. (a) } W_h = xy \times (\text{area of } B/H \text{ loop})$$

$$= 1,000 \times 0.2 \times 9.3 = 1860 \text{ J/m}^2/\text{cycle}$$

$$\text{(b) } W_h = 1,860 \times 50 \text{ J/s/m}^3 = 93,000 \text{ W/m}^3$$

$$\text{(c) } W_h = B_{\max}^{1.8} f V W \text{ For a given specimen, } W_h \propto B_{\max}^{1.8} f$$

$$\text{In (b) above, } 93,000 \propto 1.5^{1.8} \times 50 \text{ and } W_h \propto 1.2^{1.8} \times 30$$

$$\therefore \frac{W_h}{93,000} = \left( \frac{1.2}{1.5} \right)^{1.8} \times \frac{30}{50}; W_h = 93,000 \times 0.669 \times 0.6 = 37,360$$

**Example 8.8.** Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak density reached is  $1.3 \text{ Wb/m}^2$  and the frequency is 25 Hz. Assume Steinmetz coefficient as  $628 \text{ J/m}^3$  and density of iron as  $7.8 \times 10^3 \text{ kg/m}^3$ .

What will be the area of  $B/H$  curve of this specimen if  $1 \text{ cm} = 12.4 \text{ AT/m}$  and  $1 \text{ cm} = 0.1 \text{ Wb/m}^2$ .

(Elect. Engg. ; Madras Univ.)

$$\text{Solution. } W_h = \eta B_{\max}^{1.6} f V \text{ watt ; volume } V = \frac{50}{7.8 \times 10^3} = 6.41 \times 10^{-3} \text{ m}^3$$

$$\therefore W_h = 628 \times 1.3^{1.6} \times 25 \times 6.41 \times 10^{-3} = 152 \text{ J/s}$$

$$\text{Loss in one hour} = 152 \times 3,600 = 551,300 \text{ J}$$

$$\text{As per Steinmetz law, hysteresis loss} = \eta B_{\max}^{1.6} \text{ J/m}^3/\text{cycle}$$

Also, hysteresis loss =  $xy$  (area of  $B/H$  loop)

Equating the two, we get

$$628 \times 1.3^{1.6} = 12.5 \times 0.1 \times \text{loop area}$$

$$\therefore \text{loop area} = 628 \times 1.3^{1.6} / 1.25 = 764.3 \text{ cm}^2$$

## Tutorial Problems No. 8.2

1. In a certain transformer, the hysteresis loss is 300 W when the maximum flux density is 0.9 Wb/m<sup>2</sup> and the frequency 50 Hz. What would be the hysteresis loss if the maximum flux density were increased to 1.1 Wb/m<sup>2</sup> and the frequency reduced to 40 Hz. Assume the hysteresis loss over this range to be proportional to  $B_{\max}^{1.7}$  [337 W]
2. In a transformer, the hysteresis loss is 160 W when the value of  $B_{\max} = 1.1$  Wb/m<sup>2</sup> and when supply frequency is 60 Hz. What would be the loss when the value of  $B_{\max}$  is reduced to 0.9 Wb/m<sup>2</sup> and the supply frequency is reduced to 50 Hz. [97 W] (*Elect. Engg. II, Bangalore Univ.*)

## 8.6. Energy Stored in a Magnetic Field

For establishing a magnetic field, energy must be spent, though no energy is required to *maintain* it. Take the example of the exciting coils of an electromagnet. The energy supplied to it is spent in two ways (i) part of it goes to meet  $I^2R$  loss and is lost once for all (ii) part of it goes to create flux and is stored in the magnetic field as potential energy and is similar to the potential energy of a raised weight. When a weight  $W$  is raised through a height of  $h$ , the potential energy stored in it is  $W_h$ . Work is done in *raising* this weight but once raised to a certain height, no further expenditure of energy is required to *maintain* it at that position. This mechanical potential energy can be recovered, so can be the electrical energy stored in the magnetic field.

When current through an inductive coil is gradually changed from zero to maximum value  $I$ , then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this stored energy may be found in the following two ways :

(i) **First Method.** Let, at any instant,

$i$  = instantaneous value of current ;  $e$  = induced e.m.f. at that instant =  $L.di/dt$

Then, work done in time  $dt$  in overcoming this opposition is

$$dW = ei dt = L \frac{di}{dt} \times i \times dt = Li di$$

Total work done in establishing the maximum steady current of  $I$  is

$$\int_0^W dW = \int_0^I Li.di = LI^2 \text{ or } W = \frac{1}{2} LI^2$$

This work is stored as the energy of the magnetic field  $\therefore E = \frac{1}{2} LI^2$  joules

(ii) **Second Method**

If current grows uniformly from zero value to its maximum steady value  $I$ , then average current is  $I/2$ . If  $L$  is the inductance of the circuit, then self-induced e.m.f. is  $e = LI/t$  where ' $t$ ' is the time for the current change from zero to  $I$ .

$$\begin{aligned} \therefore \text{Average power absorbed} &= \text{induced e.m.f.} \times \text{average current} \\ &= L \frac{1}{t} \times \frac{1}{2} I = \frac{1}{2} \frac{LI^2}{t} \end{aligned}$$

$$\text{Total energy absorbed} = \text{power} \times \text{time} = \frac{1}{2} \frac{LI^2}{t} \times t = \frac{1}{2} LI^2$$

$$\therefore \text{energy stored } E = \frac{1}{2} LI^2 \text{ joule}$$

It may be noted that in the case of series-aiding coils, energy stored is

$$E = \frac{1}{2} (L_1 + L_2 + 2M) I^2 = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 + M I^2$$

Similarly, for series-opposing coils,  $E = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 - M I^2$



**Example 8.9.** Reluctance of a magnetic circuit is known to be  $10^5$  AT/Wb and excitation coil has 200 turns. Current in the coil is changing uniformly at 200 A/s. Calculate (a) inductance of the coil (b) voltage induced across the coil and (c) energy stored in the coil when instantaneous current at  $t = 1$  second is 1 A. Neglect resistance of the coil. (Elect. Technology, Univ. of Indore, 1987)

**Solution.** (a)  $L = N^2/S = 200^2/10^5 = 0.4 \text{ H}$   
 (b)  $e_L = L \, dI/dt = 0.4 \times 200 = 80 \text{ V}$   
 (c)  $E = \frac{1}{2} L I^2 = 0.5 \times 0.4 \times 1^2 = 0.2 \text{ J}$

**Example 8.10.** An iron ring of 20 cm mean diameter having a cross-section of  $100 \text{ cm}^2$  is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of  $1 \text{ Wb/m}^2$  if the relative permeability of iron is 1000. What is the value of energy stored?

(Elect. Engg-I, Nagpur Univ. 1992)

**Solution.**  $B = \mu_0 \mu_r NI/l \text{ Wb/m}^2$   
 $\therefore 1 = 4\pi \times 10^{-7} \times 1000 \times 400 I/0.2\pi$  or  $I = 1.25 \text{ A}$   
 Now,  $L = \mu_0 \mu_r AN^2/l = 4\pi \times 10^{-7} \times 10^3 \times (100 \times 10^{-4}) \times (400)^2/0.2\pi = 32 \text{ H}$   
 $E = \frac{1}{2} LI^2 = \frac{1}{2} \times 32 \times 1.25^2 = 2.5 \text{ J}$

## 8.7. Rate of Change of Stored Energy

As seen from Art. 8.6,  $E = \frac{1}{2} L I^2$ . The rate of change of energy can be found by differentiating the above equation

$$\frac{dE}{dt} = \frac{1}{2} \left[ L \cdot 2I \cdot \frac{dI}{dt} + I^2 \frac{dL}{dt} \right] = LI \cdot \frac{dI}{dt} + \frac{1}{2} I^2 \frac{dL}{dt}$$

**Example 8.11.** A relay (Fig. 8.5) has a coil of 1000 turns and an air-gap of area  $10 \text{ cm}^2$  and length 1.0 mm. Calculate the rate of change of stored energy in the air-gap of the relay when

(i) armature is stationary at 1.0 mm from the core and current is 10 mA but is increasing at the rate of 25 A/s.

(ii) current is constant at 20 mA but inductance is changing at the rate of 100 H/s.

**Solution.**  $L = \frac{\mu_0 N^2 A}{l_g}$   
 $= \frac{4\pi \times 10^{-7} \times (10^3)^2 \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 1.26 \text{ H}$

(i) Here,  $dI/dt = 25 \text{ A/s}$ ,  $dL/dt = 0$  because armature is stationary.

$$\therefore \frac{dE}{dt} = LI \frac{dI}{dt} = 1.26 \times 10 \times 10^{-3} \times 25 = 0.315 \text{ W}$$

(ii) Here,  $dL/dt = 100 \text{ H/s}$ ;  $dI/dt = 0$  because current is constant.

$$\therefore \frac{dE}{dt} = \frac{1}{2} I^2 \frac{dL}{dt} = \frac{1}{2} (20 \times 10^{-3})^2 \times 100 = 0.02 \text{ W}$$

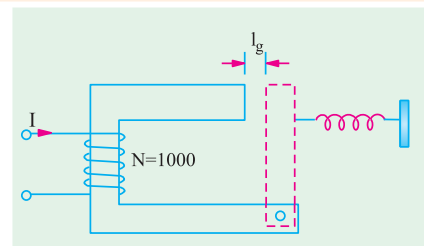


Fig. 8.5

## 8.8. Energy Stored Per Unit Volume

It has already been shown that the energy stored in a magnetic field of length  $l$  metre and of cross-section  $A \text{ m}^2$  is  $E = \frac{1}{2} L I^2$  joule or  $E = \frac{1}{2} \times \frac{\mu_0 \mu_r AN^2}{l} \cdot I^2$  joule

$$\text{Now } H = \frac{NI}{l} \therefore E = \left(\frac{NI}{l}\right)^2 \times \frac{1}{2} \mu_0 \mu_r Al = \frac{1}{2} \mu_0 \mu_r H^2 \times Al \text{ joule}$$

Now,  $Al$  = volume of the magnetic field in  $\text{m}^3$

$$\therefore \text{energy stored/m}^3 = \frac{1}{2} \mu_0 \mu_r H^2 = \frac{1}{2} BH \text{ joule} \quad (\because \mu_0 \mu_r H = B)$$

$$= \frac{B^2}{2 \mu_0 \mu_r} \text{ joule} \quad \text{—in a medium}$$

$$\text{or} \quad = \frac{B^2}{2 \mu_0} \text{ joule} \quad \text{—in air}$$

### 8.9. Lifting Power of a Magnet

In Fig. 8.6 let,  $P$  = pulling force in newtons between two poles and  $A$  = pole area in  $\text{m}^2$

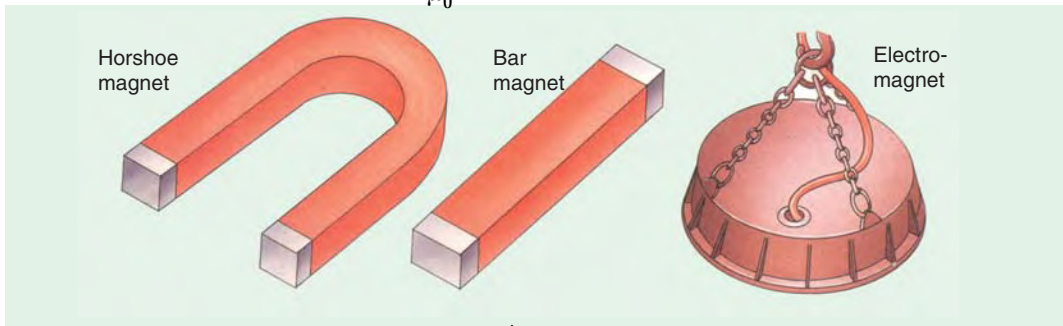
If one of the poles (say, upper one) is pulled apart against this attractive force through a distance of  $dx$  metres, then work done =  $P \times dx$  joule ... (i)

This work goes to provide energy for the additional volume of the magnetic field so created.

Additional volume of the magnetic field created is

$$= A \times dx \text{ m}^3$$

$$\text{Rate of energy requirement is} = \frac{B^2}{2 \mu_0} \text{ joule/m}^3$$



$$\therefore \text{energy required for the new volume} = \frac{B^2}{2 \mu_0} \times A dx \quad \text{... (ii)}$$

Equating (i) and (ii), we get,

$$P \cdot dx = \frac{B^2 \times A \cdot dx}{2 \mu_0}$$

$$\therefore P = \frac{B^2 A}{2 \mu_0} \text{ N} = 4,00,000 \frac{B^2}{A} \text{ N}$$

$$\text{or} \quad P = \frac{B^2 A}{2 \mu_0} \text{ N/m}^2 = 4,00,000 B^2 \text{ N/m}^2$$

$$\text{Also} \quad P = \frac{B^2 A}{9.81 \times 2 \mu_0} = \frac{B^2 A}{19.62 \mu_0} \text{ kg-wt}$$

**Example 8.12.** A horse-shoe magnet is formed out of a bar of wrought iron 45.7 cm long, having a cross-section of  $6.45 \text{ cm}^2$ . Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.

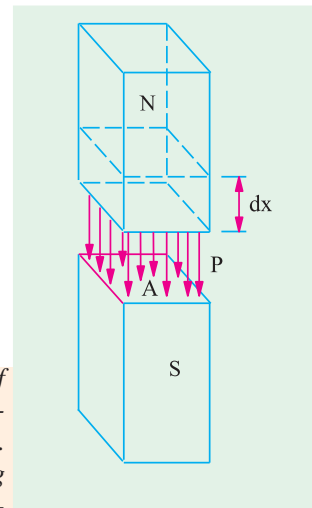


Fig. 8.6

(Elect. Engg. A.M.Ae. S.I., June, 1992)

**Solution.** Horse-shoe magnet is shown in Fig. 8.7.

Force of attraction of each pole =  $68/2 = 34 \text{ kg} = 34 \times 9.81 = 333.5 \text{ N}$

$$A = 6.45 \text{ cm}^2 = 6.45 \times 10^{-4} \text{ m}^2$$

$$\text{Since } F = \frac{B^2 A}{2\mu_0} \text{ N}$$

$$\therefore 333.5 = \frac{B^2 \cdot 6.45 \cdot 10^{-4}}{2 \cdot 4 \cdot 10^{-7}}$$

$$B = \sqrt{1.3} = 1.14 \text{ Wb/m}^2$$

$$\text{and } H = B/\mu_0 \mu_r = 1.14/4\pi \times 10^{-7} \times 700 = 1296 \text{ AT/m}$$

Length of the plate =  $45.7 \text{ cm} = 0.457 \text{ m}$

$$\therefore \text{AT required} = 1296 \times 0.457 = 592.6$$

$$\text{No. of turns} = 500 \times 2 = 1000 \quad \therefore \text{current required} = 592.6/1000 = \mathbf{0.593 \text{ A}}$$

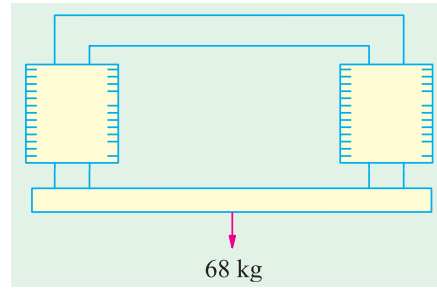


Fig. 8.7

**Example 8.13.** The pole face area of an electromagnet is  $0.5 \text{ m}^2/\text{pole}$ . It has to lift an iron ingot weighing  $1000 \text{ kg}$ . If the pole faces are parallel to the surface of the ingot at a distance of  $1 \text{ millimetre}$ , determine the coil m.m.f. required. Assume permeability of iron to be infinity at the permeability of free space is  $4\pi \times 10^{-7} \text{ H/m}$ .  
(Elect. Technology, Univ. of Indore)

**Solution.** Since iron has a permeability of infinity, it offers zero reluctance to the magnetic flux.

$$\text{Force at two poles} = 2 \times B^2 A / 2\mu_0 = B^2 A / \mu_0$$

$$\therefore B^2 \times 0.5/4\pi \times 10^{-7} = 1000 \times 9.8, B = 0.157 \text{ Wb/m}^2$$

$$\therefore H = 0.157/4\pi \times 10^{-7} = 125 \times 10^3 \text{ AT/m}, l = 2 \times 1 = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\therefore \text{AT required} = 125 \times 10^3 \times 2 \times 10^{-3} = \mathbf{250}.$$

**Example 8.14.** A soft iron ring having a mean circumference of  $40 \text{ cm}$  and cross-sectional area of  $3 \text{ cm}^2$  has two radial saw cuts made at diametrically opposite points. A brass plate  $0.5 \text{ mm}$  thick is inserted in each gap. The ring is wound with  $800$  turns. Calculate the magnetic leakage and fringing. Assume the following data for soft iron :

$B \text{ (Wb/m}^2\text{)} :$	0.76	1.13	1.31	1.41	1.5
$H \text{ (AT/m)} :$	50	100	150	200	250

(Elect. Engineering-I, Delhi Univ.)

**Solution.** It should be noted that brass is a non-magnetic material.

$$\text{Force at one separation} = B^2 A / 2\mu_0 \text{ newton.}$$

$$\text{Force at both separations} = B^2 A / \mu_0 \text{ newton.}$$

$$\text{Now } F = 12 \text{ kg wt} = 12 \times 9.81 = 117.7 \text{ N}$$

$$\therefore 117.7 = B^2 \times 3 \times 10^{-4} / 4\pi \times 10^{-7}; B = 0.7 \text{ Wb/m}^2$$

If  $B/H$  curve is drawn, it will be found that for  $B = 0.7 \text{ Wb/m}^2$ , value of  $H = 45 \text{ AT/m}$ .

Now, length of iron path =  $40 \text{ cm} = 0.4 \text{ m}$ . AT required for iron path =  $45 \times 0.4 = 18$

Value of  $H$  in the non-magnetic brass plates =  $B/\mu_0 = 0.7/4\pi \times 10^{-7} = 557,042 \text{ AT/m}$

Total thickness of brass plates =  $0.5 \times 2 = 1 \text{ mm}$

$$\text{AT required} = 557,042 \times 1 \times 10^{-3} = 557, \text{ Total AT needed} = 18 + 557 = 575$$

$$\therefore \text{magnetising current required} = 575/800 = \mathbf{0.72 \text{ A}}$$

**Example 8.15.** The arm of a d.c. shunt motor starter is held in the 'ON' position by an electro-magnet having a pole face area of  $4 \text{ cm}^2$  and air gap of  $0.6 \text{ mm}$ . The torque exerted by the spring is  $12 \text{ N-m}$  and effective radius at which the force is exerted is  $15 \text{ cm}$ . What is the minimum number of AT required to keep the arm in the 'ON' position ?

**Solution.** The arm is shown in Fig. 8.8

Let  $F$  be the force in newtons exerted by the two poles of the electromagnet.

$$\text{Torque} = \text{Force} \times \text{radius}$$

$$\therefore 12 = F \times 0.15; \quad F = 80 \text{ N}$$

$$\text{Force per pole} = 80/2 = 40 \text{ N}$$

$$\text{Now} \quad F = \frac{B^2 A}{2\mu_0} \text{ N} \quad \therefore 40 = \frac{B^2 \times 4 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}}$$

$$\therefore B = 0.5 \text{ Wb/m}^2 \quad \therefore H = 0.5/\pi \times 10^{-7} \text{ AT/m}$$

$$\text{Total air-gap} = 2 \times 0.6 \times 10^{-3} = 1.2 \times 10^{-3} \text{ m}$$

$$\therefore \text{AT reqd.} = HI = \frac{0.5 \times 1.2 \times 10^{-3}}{4\pi \times 10^{-7}} = 477$$

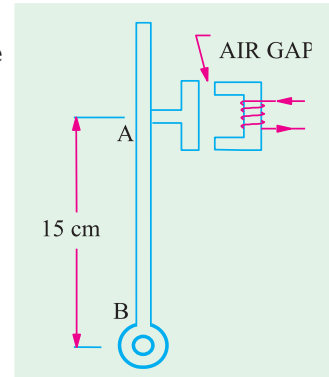


Fig. 8.8

**Example 8.16.** The following particulars are taken from the magnetic circuit of a relay; Mean length of iron circuit = 20 cm; length of air gap = 2 mm, number of turns on core = 8000, current through coil = 50 mA, relative permeability of iron = 500. Neglecting leakage, what is the flux density in the air-gap? If the area of the core is  $0.5 \text{ cm}^2$ , what is the pull exerted on the armature?

$$\text{Solution. Flux} \quad \Phi = \frac{NI}{\sum l/\mu_0 \mu_r A}$$

$$\text{Now, m.m.f} = NI = 8000 \times 50 \times 10^{-3} = 400 \text{ AT}$$

$$\text{Total circuit reluctance} = \frac{l}{\sum \mu_0 \mu_r A} \text{ AT/Wb or } H^{-1}$$

$$= \frac{0.2}{500 \times 4\pi \times 10^{-7} \times 0.5 \times 10^{-4}} + \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.5 \times 10^{-4}}$$

$$\Phi = \frac{400}{12 \times 10^7 / \phi} \text{ Wb}; \quad \text{Flux density } B = \frac{\Phi}{A} = \frac{400\pi}{12 \times 10^7 \times 0.5 \times 10^{-4}} = 0.21 \text{ Wb/m}^2$$

$$\text{The pull on the armature} = \frac{B^2 A}{2\mu_0} \text{ N} = \frac{0.21^2 \times 0.5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 0.87 \text{ N}$$

### Tutorial Problems No. 8.2

- An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1,000 turns and also find the energy stored in it if the current rises from zero to 5 A.  
[0.7 mH; 8.7 mJ] (Elect. Engg. and Electronic Bangalore Univ. 1998)
- An air-cored solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calculate (i) the self inductance (ii) the energy stored in the magnetic field when a current of 2 A flows in the solenoid.  
[(i) 0.2468 H (ii) 0.4936 J] (F.E. Pune Univ.)
- Determine the force required to separate two magnetic surfaces with contact area of  $100 \text{ cm}^2$  if the magnetic flux density across the surface is  $0.1 \text{ Wb/m}^2$ . Derive formula used, if any.  
[39.8 N] (Elect. Engg. A.M.Ae.S.I.)
- In a telephone receiver, the size of each of the two poles is  $1.2 \text{ cm} \times 0.2 \text{ cm}$  and the flux between each pole and the diaphragm is  $3 \times 10^{-6} \text{ Wb}$ ; with what force is the diaphragm attracted to the poles?  
[0.125 N] (Elect. Engg. A.M.Ae.S.I. June 1991)
- A lifting magnet is required to raise a load of 1,000 kg with a factor of safety of 1.5. If the flux density across the pole faces is  $0.8 \text{ Wb/m}^2$ , calculate the area of each pole.  
[577  $\text{cm}^2$ ]
- Magnetic material having a surface of  $100 \text{ cm}^2$  are in contact with each other. They are in a magnetic circuit of flux. 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces.  
[3,978 N] (Elect. Engg. Kerala Univ.)

7. A steel ring having a mean diameter of 35 cm and a cross-sectional area of  $2.4 \text{ cm}^2$  is broken by a parallel-sided air-gap of length 1.2 cm. Short pole pieces of negligible reluctance extend the effective cross-sectional area of the air-gap to  $12 \text{ cm}^2$ . Taking the relative permeability of steel as 700 and neglecting leakage, determine (a) the current necessary in 300 turns of wire wound on the ring to produce a flux density in the air-gap of  $0.25 \text{ Wb/m}^2$  (b) the tractive force between the poles.

[(a) 13.16 A (b) 29.9 N]

8. A cast iron ring having a mean circumference of 40 cm and a cross-sectional area of  $3 \text{ cm}^2$  has two radial saw-cuts at diametrically opposite points. A brass plate is inserted in each gap (thickness 0.5 mm). If the ring is wound with 800 turns, calculate the magnetising current to exert a total pull of 3 kg between the two halves. Neglect any magnetic leakage and fringing and assume the magnetic data for the cast iron to be :

$B \text{ (Wb/m}^2\text{)}$ :	0.2	0.3	0.4	0.5	
$H \text{ (AT)}$ :	850	1150	1500	2000	[1.04 A]

9. A magnetic circuit in the form of an inverted U has an air-gap between each pole and the armature of 0.05 cm. The cross-section of the magnetic circuit is  $5 \text{ cm}^2$ . Neglecting magnetic leakage and fringing, calculate the necessary exciting ampere-turns in order that the armature may exert a pull of 15 kg. The ampere-turns for the iron portion of the magnetic circuit may be taken as 20 percent of those required for the double air-gap.

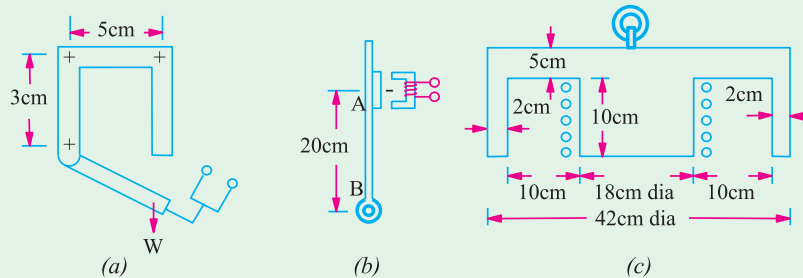


Fig. 8.9

10. In Fig. 8.9 (a) is shown the overload trip for a shunt motor starter. The force required to lift the armature is equivalent to a weight of  $W = 0.8165 \text{ kg}$  positioned as shown. The air-gaps in the magnetic circuit are equivalent to a single gap of 0.5 cm. The cross-sectional area of the circuit is  $1.5 \text{ cm}^2$  throughout and the magnetisation curve is as follows :

$H \text{ (AT/m)}$ :	1000	2000	3000	4000
$B \text{ (Wb/m}^2\text{)}$ :	0.3	0.5	0.62	0.68

Calculate the number of turns required if the trip is to operate when 80 A passes through the coil.

[21 turns]

11. The armature of a d.c. motor starter is held in the 'ON' position by means of an electromagnet [Fig 8.9. (b)]. A spiral spring exerts a mean counter torque of 8 N-m on the armature in this position after making allowance for the weight of the starter arm. The length between the centre of the armature and the pivot on the starter arms is 20 cm and cross sectional area of each pole face of the electromagnet  $3.5 \text{ cm}^2$ .

Find the minimum number of AT required on the electromagnet to keep the arm in the 'ON' position when the air-gap between the armature and the electromagnet is 0.5 mm. (Neglect the AT needed for the iron of the electromagnet).

[301 AT]

12. A cylindrical lifting magnet of the form shown in Fig. 8.9 (c) has a winding of 200 turns which carries a current of 5 A. Calculate the maximum lifting force which could be exerted by the magnet on a flat iron sheet 5 cm thick. Why would this value not be realized in practice ? The relative permeability of the iron can be taken as 500.

[698 N]

### 8.10. Rise of current in an Inductive Circuit

In Fig. 8.10 is shown a resistance of  $R$  in series with a coil of self-inductance  $L$  henry, the two being put across a battery of  $V$  volt. The  $R$ - $L$  combination becomes connected to battery when switch

$S$  is connected to terminal 'a' and is short-circuited when  $S$  is connected to 'b'. The inductive coil is assumed to be resistanceless, its actual small resistance being included in  $R$ .

When  $S$  is connected to 'a' the  $R$ - $L$  combination is suddenly put across the voltage of  $V$  volt. Let us take the instant of closing  $S$  as the starting zero time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

We will now investigate the growth of current  $i$  through such an inductive circuit.

The applied voltage  $V$  must, at any instant, supply not only the ohmic drop  $iR$  over the resistance  $R$  but must also overcome the e.m.f. of self inductance *i.e.*  $L di/dt$ .

$$\begin{aligned} \therefore V &= v_R + v_L = iR + \frac{di}{dt} \\ \text{or } (V - iR) &= L \frac{di}{dt} \therefore \frac{di}{V - iR} = \frac{dt}{L} \end{aligned} \quad \dots(i)$$

Multiplying both sides by  $(-R)$ , we get  $(-R) \frac{di}{(V - iR)} = -\frac{R}{L} dt$

$$\text{Integrating both sides, we get } \int \frac{(-R) di}{(V - iR)} = \int dt \therefore \log_e^{V - iR} = -\frac{R}{L} t + K \quad \dots(ii)$$

where  $e$  is the Napierian logarithmic base = 2.718 and  $K$  is constant of integration whose value can be found from the initial known conditions.

To begin with, when  $t = 0$ ,  $i = 0$ , hence putting these values in (ii) above, we get

$$\log_e^V = K$$

Substituting this value of  $K$  in the above given equation, we have

$$\log_e^{V - iR} = \frac{R}{L} t + \log_e^V \text{ or } \log_e^{V - iR} - \log_e^V = -\frac{R}{L} t$$

$$\text{or } \log_e \frac{V - iR}{V} = -\frac{R}{L} t = -\frac{1}{\lambda} \text{ where } L/R = \lambda \text{ 'time constant'}$$

$$\therefore \frac{V - iR}{V} = e^{-t/\lambda} \text{ or } i = \frac{V}{R} (1 - e^{-t/\lambda})$$

Now,  $V/R$  represents the maximum steady value of current  $I_m$  that would eventually be established through the  $R$ - $L$  circuit.

$$\therefore i = I_m (1 - e^{-t/\lambda}) \quad \dots(iii)$$

This is an exponential equation whose graph is shown in Fig. 8.11. It is seen from it that current rise is rapid at first and then decreases until at  $t = \infty$  it becomes zero. Theoretically, current does not reach its maximum steady value  $I_m$  until infinite time. However, in practice, it reaches this value in a relatively short time of about  $5\lambda$ .

The rate of rise of current  $di/dt$  at any stage can be found by differentiating Eq. (iii) above w.r.t. time. However, the *initial* rate of rise of

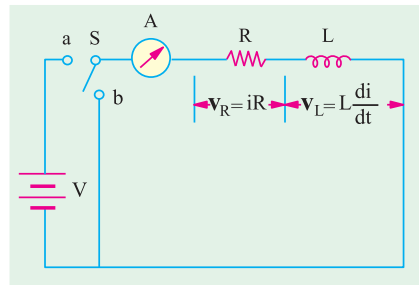


Fig. 8.10

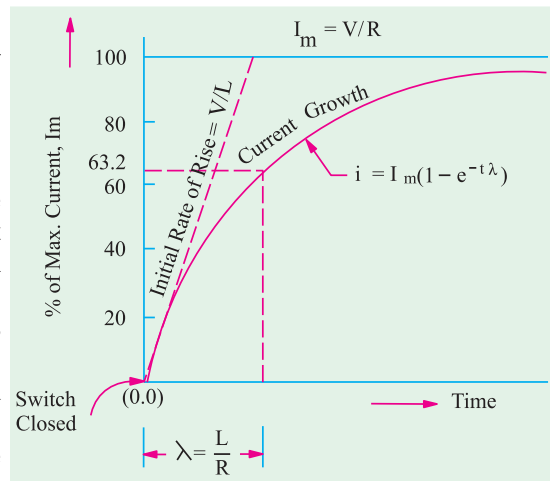


Fig. 8.11

current can be obtained by putting  $t = 0$  and  $i = 0$  in (i)\* above.

$$\therefore V = 0 \times R + L \frac{di}{dt} \text{ or } \frac{di}{dt} = \frac{V}{L}$$

The constant  $\lambda = L/R$  is known as the **time-constant** of the circuit. It can be variously defined as :

(i) It is the **time** during which current would have reached its maximum value of  $I_m (= V/R)$  had it maintained its initial rate of rise.

$$\text{Time taken} = \frac{I_m}{\text{initial rate of rise}} = \frac{V/R}{V/L} = \frac{L}{R}$$

But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0.632 of its maximum values as shown below :

Putting  $t = L/R = \lambda$  in Eq. (iii) above, we get

$$i = I_m (1 - e^{-\lambda/\lambda}) = I_m \left(1 - \frac{1}{e}\right) = I_m \left(1 - \frac{1}{2.718}\right) = 0.632 I_m$$

(ii) Hence, the time-constant  $\lambda$  of an  $R$ - $L$  circuit may also be defined as the time during which the current **actually** rises to 0.632 of its maximum steady value (Fig. 8.11).

This delayed rise of current in an inductive circuit is utilized in providing time lag in the operation of electric relays and trip coils etc.

### 8.11. Decay of Current in an Inductive Circuit

When the switch  $S$  (Fig. 8.10) is connected to point 'b', the  $R$ - $L$  circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit.

The equation for decay of current with time is found by putting  $V = 0$  in Eq. (i) of Art. 8.10

$$0 = iR + L \frac{di}{dt} \text{ or } \frac{di}{i} = -\frac{R}{L} dt$$

Integrating both sides, we have

$$\int \frac{di}{i} = -\frac{R}{L} \int dt \therefore \log i = \frac{R}{L} t + K \quad \dots (i)$$

Now, at the instant of switching off current,  $i = I_m$  and if time is counted from this instant, then  $t = 0$

$$\therefore \log_e I_m = 0 + K$$

Putting the value of  $K$  in Eq (i) above, we get,

$$\log_e i = -\frac{t}{\lambda} = \log_e I_m$$

$$\therefore \log_e i/I_m = -\frac{t}{\lambda}$$

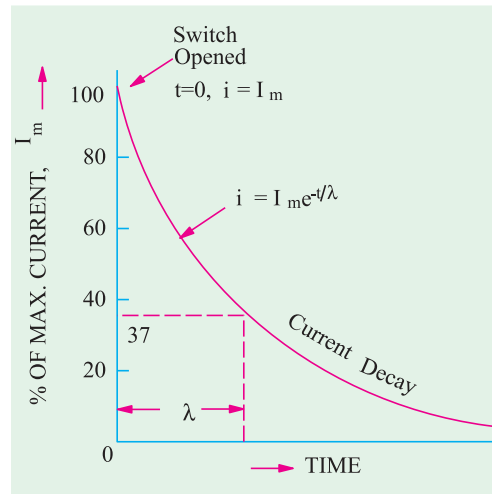


Fig. 8.12

\* Initial value of  $di/dt$  can also be found by differentiating Eq. (iii) and putting  $t = 0$  in it. In fact, the three quantities  $V, L, R$  give the following various combinations :

$V/R = I_m$  - the maximum final steady current.

$V/L =$  initial rate of rise of current.

$L/R =$  time constant of the circuit.

The first rule of switching is that the current flowing through an inductance cannot change instantaneously.

The second rule of switching is that the voltage across a capacitor cannot change instantaneously.

$$\therefore \frac{i}{I_m} = e^{-t/\lambda}$$

$$\text{or } i = I_m e^{-t/\lambda} \quad \dots(ii)$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about  $5\lambda$ .

Again, putting  $t = \lambda$  in Eq. (ii) above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37 I_m.$$

Hence, time constant ( $\lambda$ ) of an  $R$ - $L$  circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).

**Example 8.17.** A coil having an effective resistance of  $20 \Omega$  and an inductance of  $5 \text{ H}$ . is suddenly connected across a  $50\text{-V}$  dc supply. What is the rate at which energy is stored in the field of the coil when current is (a)  $0.5 \text{ A}$  (b)  $1.0 \text{ A}$  and (c) steady? Also find the induced EMF in the coil under the above conditions.

**Solution.** (a) Power input =  $50 \times 0.5 = 25 \text{ W}$

Power wasted as heat =  $i^2 R = 0.5^2 \times 25 = 6.25 \text{ W}$ . Hence, rate of energy storage in the coil field is  $25 - 6.25 = 18.75 \text{ W}$  or  $\text{J/s}$ . (b) Power input =  $50 \times 1 = 50 \text{ W}$

Power lost as heat =  $1^2 \times 25 = 25 \text{ W}$ .  $\therefore$  Rate of energy storage in field =  $50 - 25 = 25 \text{ W}$  or  $\text{J/s}$ .

(c) Steady value of current =  $50/25 = 2 \text{ A}$ . Power input =  $50 \times 2 = 100 \text{ W}$

Power lost as heat =  $2^2 \times 25 = 100 \text{ W}$

Rate of energy storage in field =  $100 - 100 = 0$ ; Now,  $V = iR + e_L = V - iR$

(a)  $e_L = 50 - 0.5 \times 25 = 37.5 \text{ V}$  (b)  $e_L = 50 - 1 \times 25 = 25 \text{ V}$  (c)  $e_L = 50 - 2 \times 25 = 0 \text{ V}$ .

**Example 8.18.** A coil having a resistance of  $10 \Omega$  and an inductance of  $4 \text{ H}$  is switched across a  $20\text{-V}$  dc source. Calculate (a) time required by the current to reach 50% of its final steady value and (b) value of the current after  $0.5$  second.

**Solution.** The rise of current through an inductive circuit is given by the equation  $i = I(1 - e^{-t/\lambda})$ . It may be written as

$$e^{-t/\lambda} = \frac{I - i}{I} \quad \text{or} \quad \frac{I}{e^{t/\lambda}} = \frac{I - i}{I} \quad \text{or} \quad e^{t/\lambda} = \frac{I}{I - i}$$

Taking logs of both sides, we have

$$\frac{t}{\lambda} \cdot \log_e = \log_e \frac{I}{I - i} = I_n \frac{I}{I - i}$$

$$\therefore \frac{Rt}{L} = R_n \frac{I}{I - i} \quad \text{or} \quad t = \frac{L}{R} I_n \frac{I}{I - i}$$

(a) Now,  $I = V/R = 20/10 = 2 \text{ A}$

$$\therefore t = \frac{4}{10} I_n \frac{2}{(2 - 1)} = \frac{4}{10} \times 0.693 = 0.2777 \text{ s}$$

(b)  $\lambda = L/R = 4/10 = 0.4 \text{ s}$  and  $t = 0.5 \text{ s}$

$$\therefore i = 2(1 - e^{-t/0.4})$$

**Example 8.19.** With reference to the circuit shown in Fig. 8.13, calculate :

- (i) the current taken from the d.c. supply at the instant of closing the switch
- (ii) the rate of increase of current in the coil at the instant of switch
- (iii) the supply and coil currents after the switch has been closed for a long time
- (iv) the maximum energy stored in the coil
- (v) the e.m.f. induced in the coil when the switch is opened.

**Solution.** (i) When switch  $S$  is closed (Fig. 8.13), the supply d.c. voltage of  $120 \text{ V}$  is applied



across both arms. The current in  $R_2$  will immediately become  $120/30 = 4$  A. However, due to high inductance of the second arm, there would be no instantaneous flow of current in it. Hence current taken from the supply at the instant of switching on will be **4 A**.

(ii) Since at the *instant* of switching on, there is no current through the inductor arm, no potential drop will develop across  $R_1$ . The whole of the supply voltage will be applied across the inductor. If  $di/dt$  is the rate of increase of current through the inductor at the instant of switching on, the back e.m.f. produced in it is  $L \cdot di/dt$ . This e.m.f. is equal and opposite to the applied voltage.

$$120 = L \, di/dt \quad \text{or} \quad di/dt = 120/2 = \mathbf{60 \text{ A/s}}$$

(iii) When switch has been closed for a sufficiently long time, current through the inductor arm reaches a steady value =  $120/R_1 = 120/15 = \mathbf{8 \text{ A}}$

$$\text{Current through } R_2 = 120/30 = 4 \text{ A}; \text{ Supply current} = 8 + 4 = \mathbf{12 \text{ A}}$$

(iv) Maximum energy stored in the inductor arm

$$= \frac{1}{2} LI^2 = \frac{1}{2} \times 2 \times 8^2 = \mathbf{64 \text{ J}}$$

(v) When switch is opened, current through the inductor arm cannot change immediately because of high self-inductance of the inductor. Hence, inductance current remains at 8 A. But the current through  $R_2$  can change immediately. After the switch is opened, the inductor current path lies through  $R_1$  and  $R_2$ . Hence, e.m.f. induced in the inductor at the instant of switching off is  $= 8 \times (30 + 15) = \mathbf{360 \text{ V}}$ .

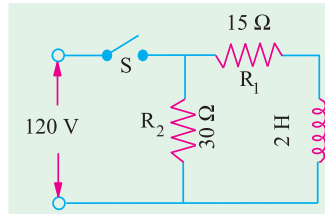


Fig. 8.13

**Example 8.20.** A coil has a time constant of 1 second and an inductance of 8 H. If the coil is connected to a 100 V d.c. source, determine :

- (i) the rate of rise of current at the instant of switching (ii) the steady value of the current and (iii) the time taken by the current to reach 60% of the steady value of the current.

(Electrotechnics-I, M.S. Univ. Baroda)

**Solution.**  $\lambda = L/R$ ;  $R = L/\lambda = 8/1 = 8 \text{ ohm}$

(i) Initial  $di/dt = V/L = 100/8 = \mathbf{12.5 \text{ A/s}}$  (ii)  $I_M = V/R = 100/8 = \mathbf{12.5 \text{ A}}$

(iii) Here,  $i = 60\%$  of  $12.5 = 7.5 \text{ A}$

Now,  $i = I_m (1 - e^{-t/\lambda})$   $\therefore 7.5 = 12.5 (1 - e^{-t/1})$ ;  $t = \mathbf{0.915 \text{ second}}$

**Example 8.21.** A d.c. voltage of 80 V is applied to a circuit containing a resistance of  $80 \Omega$  in series with an inductance of 20 H. Calculate the growth of current at the instant (i) of completing the circuit (ii) when the current is 0.5 A and (iii) when the current is 1 A.

(Circuit Theory, Jadavpur Univ.)

**Solution.** The voltage equation for an  $R$ - $L$  circuit is

$$V = iR + L \frac{di}{dt} \quad \text{or} \quad L \frac{di}{dt} = V - iR \quad \text{or} \quad \frac{di}{dt} = \frac{1}{L} (V - iR)$$

(i) when  $i = 0$ ;  $\frac{di}{dt} = \frac{1}{L} (V - 0 \times R) = \frac{V}{L} = \frac{80}{20} = \mathbf{4 \text{ A/s}}$

(ii) when  $i = 0.5 \text{ A}$ ;  $\frac{di}{dt} = \frac{80 - 0.5 \times 80}{20} = \mathbf{2 \text{ A/s}}$

(iii) when  $i = 1 \text{ A}$ ;  $\frac{di}{dt} = \frac{80 - 80 \times 1}{20} = \mathbf{0}$ .

In other words, the current has become steady at 1 ampere.

**Example 8.22.** The two circuits of Fig. 8.14 have the same time constant of 0.005 second. With the same d.c. voltage applied to the two circuits, it is found that the steady state current of circuit (a) is 2000 times the initial current of circuit (b). Find  $R_1$ ,  $L_1$  and  $C$ .

(Elect. Engg.-I, Bombay Univ.)

**Solution.** The time constant of circuit 8.14 (a) is  $\lambda = L_1/R_1$  second, and that of circuit 8.14 (b) is  $\lambda = CR_2$  second.

$$\begin{aligned} \therefore L_1/R_1 &= 0.005 \\ C \times 2 \times 10^6 &= 0.005, C = 0.0025 \times 10^{-6} = \mathbf{0.0025 \mu F} \end{aligned}$$

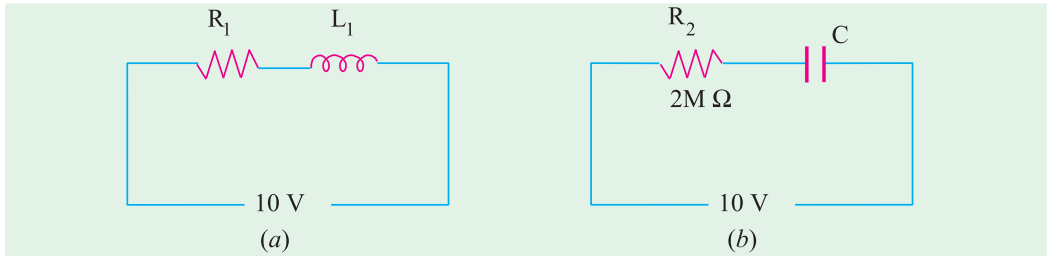


Fig. 8.14

Steady-state current of circuit 8.14 (a) is  $= V/R_1 = 10/R_1$  amperes.

Initial current of circuit 8.14 (b)  $= V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6} A^*$

Now  $10/R_1 = 2000 \times 5 \times 10^{-6} \therefore R_1 = 1000 \Omega$

Also  $L_1/R_1 = 0.005 \therefore L_1 = 1000 \times 0.005 = \mathbf{5 H}$

**Example 8.23.** A constant voltage is applied to a series R-L circuit at  $t = 0$  by closing a switch. The voltage across L is 25 V at  $t = 0$  and drops to 5 V at  $t = 0.025$  second. If  $L = 2 H$ , what must be the value of R ? (Elect. Engg.-I Bombay Univ.)

**Solution.** At  $t = 0, i = 0$ , hence there is no  $iR$  drop and the applied voltage must equal the back e.m.f. in the coil. Hence, the voltage across L at  $t = 0$  represents the applied voltage.

At  $t = 0.025$  second, voltage across L is 5 V, hence voltage across

$R = 25 - 5 = 20 V \therefore iR = 20 V$  -at  $t = 0.025$  second. Now  $i = I_m (1 - e^{-t/\lambda})$

Here  $I_m = 25/R$  ampere,  $t = 0.025$  second  $\therefore i = \frac{25}{R} (1 - e^{-0.025/\lambda})$

$$R \times \frac{25}{R} (1 - e^{-0.025/\lambda}) = 20 \quad \text{or} \quad e^{-0.025/\lambda} = 5 \quad \therefore 0.025/\lambda = 2.3 \log_{10} 5 = 1.6077$$

$\therefore \lambda = 0.025/1.6077$  Now  $\lambda = L/R = 2/R \therefore 2/R = 0.025/1.6077 \therefore \mathbf{R = 128.56 \Omega}$

**Example 8.24.** A circuit of resistance R ohms and inductance L henries has a direct voltage of 230 V applied to it. 0.3 second second after switching on, the current in the circuit was found to be 5 A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5 A at 0.3 second second after short-circuiting the coil. Find the value of R and L. (Basic Electricity, Bombay Univ.)

**Solution.** For growth ;  $5 = I_m (1 - e^{-0.3/\lambda})$  ...(i)

For decay;  $5 = I_m e^{-0.3/\lambda}$

Equating the two, we get,  $I_m e^{-0.3/\lambda} = (1 - e^{-0.3/\lambda}) I_m$

or  $2 e^{-0.3/\lambda} = 1 \therefore e^{-0.3/\lambda} = 0.5$  or  $\lambda = 0.4328$

Putting this value in (i), we get,

$$5 = I_m = e^{0.3/0.4328} \quad \text{or} \quad I_m = 5 e^{0.3/0.4328} = 5 \times 2 = 10 A.$$

Now,  $I_m = V/R \therefore 10 = 230/R$  or  $R = 230/10 = 23 \Omega$  (approx.)

As  $\lambda = L/R = 0.4328 ; L = 0.4328 \times 23 = \mathbf{9.95 H}$

\* Because just at the time of starting the current, there is no potential drop across C so that the applied voltage is dropped across  $R_2$ . Hence, the initial charging current  $= V/R_2$ .

**Example 8.25.** A relay has a coil resistance of  $20\ \Omega$  and an inductance of  $0.5\ \text{H}$ . It is energized by a direct voltage pulse which rises from  $0$ – $10\ \text{V}$  instantaneously, remains constant for  $0.25$  second and then falls instantaneously to zero. If the relay contacts close when the current is  $200\ \text{mA}$  (increasing) and open when it is  $100\ \text{mA}$  (decreasing), find the total time during which the contacts are closed.

**Solution.** The time constant of the relay coil is

$$\lambda = L/R = 0.5/20 = 0.025\ \text{second}$$

Now, the voltage pulse remains constant at  $10\ \text{V}$  for  $0.25$  second which is long enough for the relay coil current to reach its steady value of  $V/R = 10/20 = 0.5\ \text{A}$

Let us now find the value of time required by the relay coil current to reach a value of  $200\ \text{mA} = 0.2\ \text{A}$ . Now  $i = I_m (1 - e^{-t/\lambda}) \quad \therefore 0.2 = 0.5 (1 - e^{-t/0.025}) \quad \therefore e^{-40t} = 5/3$

$$\therefore t = 0.01276\ \text{second}$$

Hence, relay contacts close at  $t = 0.01276$  second and will remain closed till current falls to  $100\ \text{mA}$ . Let us find the time required by the current to fall from  $0.5\ \text{A}$  to  $0.1\ \text{A}$ .

At the end of the voltage pulse, the relay current decays according to the relation

$$i = I_m e^{-t/\lambda} \quad \therefore 0.1 = 0.5 e^{-t/0.025} \quad \therefore e^{-40t} = 5$$

$$\therefore t = 0.04025\ \text{second after the end of the voltage pulse.}$$

Hence, the time for which contacts remain closed is

$$= (0.25 - 0.01276) + 0.04025\ \text{second} = \mathbf{277.5\ \text{milli-second (approx)}}$$

## 8.12. Details of Transient Current Rise in an R-L Circuit

As shown in Fig. 8.15 (a), when switch  $S$  is shifted to position  $a$ , the  $R$ - $L$  circuit is suddenly energized by  $V$ . Since a coil opposes any change in current, the initial value of current is zero at  $t = 0$  and but then it rises exponentially, although its rate of rise keeps decreasing. After some time, it reaches a maximum value of  $I_m$  when it becomes constant *i.e.* its rate of rise becomes zero. Hence, just at the start of the transient

state,  $i = 0$ ,  $V_R = 0$  and  $V_L = V$  with its polarity opposite to that of battery voltage as shown in Fig. 8.15 (a). Both  $i$  and  $V_R$  rise exponentially during the transient state, as shown in Fig. 8.15 (b) and (c) respectively. However  $V_L$  decreases exponentially to zero from its initial maximum value of  $V = I_m R$ . It does not become negative during the transient rise of current through the circuit.

Hence, during the transient rise of current, the following equations hold good :

$$i = \frac{V}{R} (1 - e^{-t/\lambda}) = I_m (1 - e^{-t/\lambda}); \quad V_R = iR = V (1 - e^{-t/\lambda}) = I_m R (1 - e^{-t/\lambda}); \quad v_L = Ve^{-t/\lambda}$$

If  $S$  remains at 'a' long enough,  $i$  reaches a steady value of  $I_m$  and  $V_R$  equals  $I_m R$  but since  $di/dt = 0$ ,  $v_L = 0$ .

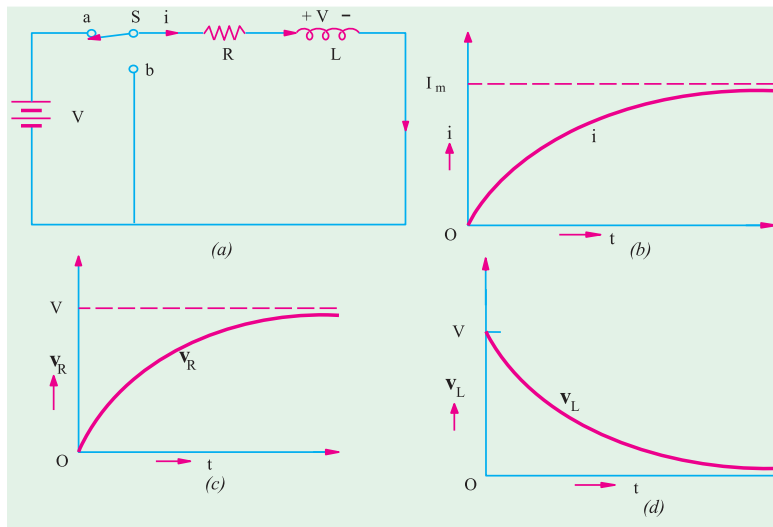


Fig. 8.15

**Example 8.26.** A voltage as shown in Fig. 8.16 is applied to an inductor of 0.2 H, find the current in the inductor at  $t = 2$  sec.

**Solution.**  $L(di/dt) = 2$  volts, for  $t$  between 0 and 1 sec, and corresponding value of  $di/dt = 2/0.2 = 10$  amp/sec, uniform during this period.

After  $t > 1$  voltage is zero, hence  $di/dt = 0$

Current variation is marked on the same diagram.

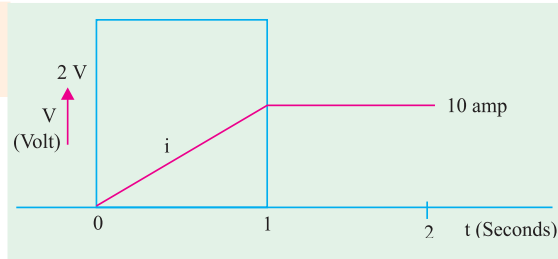


Fig. 8.16

### 8.13. Details of Transient Current Decay in an R-L Circuit

Now, let us consider the conditions during the transient decay of current when  $S$  is shifted to point 'b'. Just at the start of the decay condition, the following values exist in the circuit.

$i = I_m = V/R$ ,  $v_R = I_m R = V$  and since initial  $di/dt$  is maximum,  $v_L = -V = -I_m R$ .

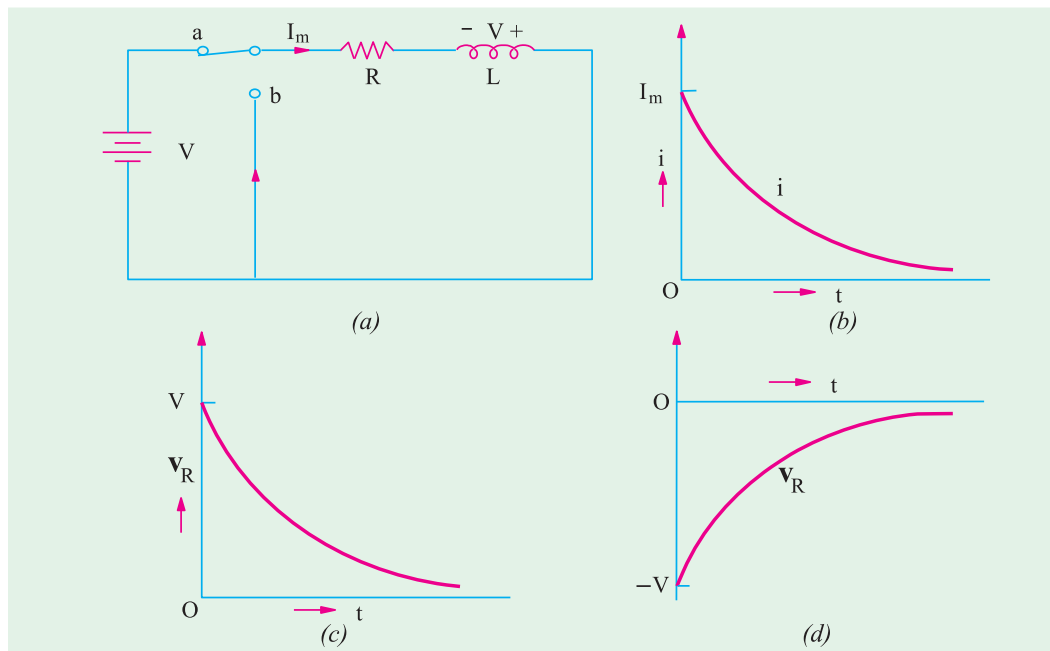


Fig. 8.17

The change in the polarity of voltage across the coil in Fig. 8.17 (a) is worth noting. Due to its property of self-induction, the coil will not allow the circuit current to die immediately, but only gradually. In fact, by reversing the sign of its voltage, the coil tends to maintain the flow of current in the original direction. Hence, as the decay continues  $i$  decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (b). Similarly,  $v_R$  decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (c). However,  $v_L$  is reversed in polarity and decreases exponentially from its initial value of  $-V$  to zero as shown in Fig. 8.17 (d).

During the transient decay of current and voltage, the following relations hold good :

$$i = i_L = I_m e^{-t/\lambda} = \frac{V}{R} e^{-t/\lambda}$$

$$v_R = V e^{-t/\lambda} = I_m R e^{-t/\lambda}$$

$$v_L = -V e^{-t/\lambda} = -I_m R e^{-t/\lambda}$$

### 8.14. Automobile Ignition System

Practical application of mutual induction is found in the single-spark petrol-engine ignition system extensively employed in automobiles and air-engines. Fig. 8.18 shows the circuit diagram of such a system as applied to a 4-cylinder automobile engine.

It has a spark coil (or induction coil) which consists of a primary winding (of a few turns) and a secondary winding (of a large number of turns)

wound on a common iron core (for increasing mutual induction). The primary circuit (containing battery  $B$ ) includes a 'make and break contact' actuated by a timer cam. The secondary circuit includes the rotating blade of the distributor and the spark gap in the spark plug as shown in Fig. 8.17. The timer cam and the distributor are mounted on the same shaft and are geared to rotate at exactly half the speed of the engine shaft. It means that in the case of automobile engines (which are four-cycle engines) each cylinder is fired only once for every two revolutions of the engine shaft.

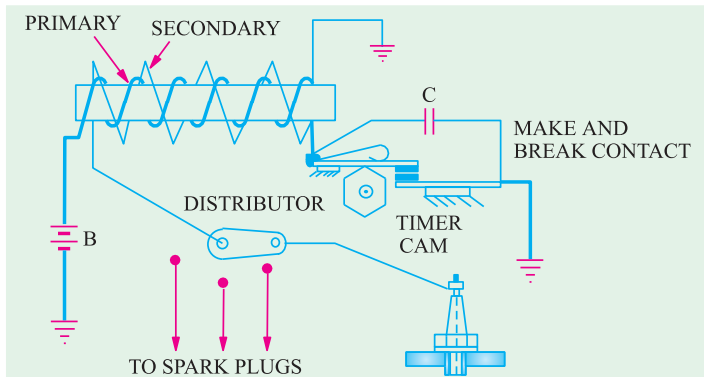


Fig. 8.18

#### Working

When timer cam rotates, it alternately closes and opens the primary circuit. During the time primary circuit is closed, current through it rises exponentially after the manner shown in Fig. 8.11 and so does the magnetic field of the primary winding. When the cam suddenly opens the primary circuit, the magnetic field collapses rapidly thereby producing a very large e.m.f. in secondary by mutual induction. During the time this large e.m.f. exists, the distributor blade rotates and connects the secondary winding across the proper plug and so the secondary circuit is completed except for the spark gap in the spark plug. However, the induced e.m.f. is large enough to make the current jump across the gap thus producing a spark which ignites the explosive mixture in the engine cylinder.

The function of capacitor  $C$  connected across the 'make and break' contact is two-fold :

- (i) to make the break rapid so that large e.m.f. is induced in secondary and
- (ii) to reduce sparking and burning at the 'make-and-break' contact thereby prolonging their life.

#### Tutorial Problems No. 8.3

1. A relay has a resistance of  $300\ \Omega$  and is switched on to a  $110\ \text{V d.c.}$  supply. If the current reaches 63.2 per cent of its final steady value in 0.002 second, determine
  - (a) the time-constant of the circuit
  - (b) the inductance of the circuit
  - (c) the final steady value of the current
  - (d) the initial rate of rise of current.

[(a) 0.002 second (b) 0.6 H (c) 0.366 A (d) 183 A/second]
2. A coil with a self-inductance of  $2.4\ \text{H}$  and resistance  $12\ \Omega$  is suddenly switched across a  $120\text{-V d.c.}$  supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.1 second, the final steady value of the current and the time taken for the current to reach 5 A.
 

[(a) 0.2 second; 3.94 A; 10 A; 0.139 second]
3. A circuit whose resistance is  $20\ \Omega$  and inductance  $10\ \text{H}$  has a steady voltage of  $100\ \text{V}$  suddenly applied to it. For the instant 0.5 second after the voltage is applied, determine (a) the total power input to the circuit (b) the power dissipated in the resistance. Explain the reason for the difference between (a) and (b).
 

[(a) 316 W (b) 200 W]
4. A lighting circuit is operated by a relay of which the coil has a resistance of  $5\ \Omega$  and an inductance of  $0.5\ \text{H}$ . The relay coil is supplied from a  $6\text{-V d.c.}$  source through a push-button switch. The relay operates when the current in the relay coil attains a value of  $500\ \text{mA}$ . Find the time interval between the pressing of the push-button and the closing of lighting circuit.
 

[53.8 ms]

5. The field winding of a separately-excited d.c. generator has an inductance of 60 H and a resistance of 30  $\Omega$ . A discharge resistance of 50  $\Omega$  is permanently connected in parallel with the winding which is excited from a 200-V supply. Find the value of decay current 0.6 second after the supply has been switched off. [3.0 A]
6. The field winding of a dynamo may be taken to have a constant inductance of 120 H and an effective resistance of 30  $\Omega$ . When it is carrying a current of 5 A, the supply is interrupted and a resistance of 50  $\Omega$  is connected across the winding. How long will it take for the current to fall to 1.0? [2.415 s]
7. A 200-V d.c. supply is suddenly switched to a relay coil which has a time constant of 3 milli-second. If the current in the coil reaches 0.2 A after 3 milli-second, determine the final steady value of the current and the resistance of the coil. [0.316 A; 632  $\Omega$ ; 1.896 H]
8. Explain the terms related to magnetic circuits :  
(i) coercive force (ii) residual flux. (Nagpur University, Summer 2002)

9. The B-H characteristic of cast iron may be drawn from the following :-

B(Wb/m <sup>2</sup> )	H(AT/m)
0.1	280
0.2	620
0.3	990
0.4	1400
0.5	2000
0.6	2800

(Nagpur University, Winter 2003)

10. Derive an expression for the energy stored in the magnetic field of a coil of an inductance L henry. (Gujrat University, June/July 2003)
11. A cast steel ring has a circular cross-section of 3 cm in diameter and a mean circumference of 80 cm. A 1 mm air gap is cut out in the ring which is wound with a coil of 600 turns. Estimate the current required to establish a flux of 0.75 mWb in the air gap. Neglect fringing and leakage. The magnetization data of the material is as under :

(R.G.P.V. Bhopal University, June 2004)

H (At/m)	B (T)
200	0.10
400	0.32
600	0.60
800	0.90
1000	1.08
1200	1.18
1400	1.27
1600	1.32
1800	1.36

12. What is the difference between B.H. curve and hysteresis loop? (Anna University, April 2002)
13. Explain with neat diagram how can you obtain B.H. curve and hysteresis loop of ring specimens. (Anna University, April 2002)
14. Derive an expression for energy stored in an inductance. (V.T.U., Belgaum Karnataka University, Summer 2002)
15. Derive an expression for the energy stored in a magnetic circuit. (V.T.U., Belgaum Karnataka University, January/February 2003)
16. What is hysteresis and eddy current losses? What are the undesirable effects of eddy currents and hysteresis loss? How can they be minimised? Mention some applications of eddy currents. (U.P. Technical University, 2002) (RGPV Bhopal June 2002)

### OBJECTIVE TESTS – 8

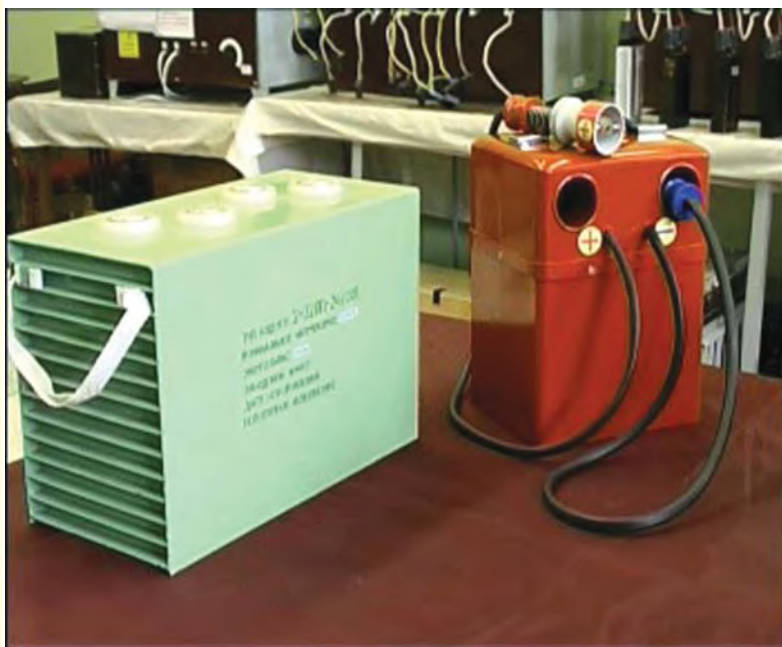
1. Permanent magnets are normally made of  
(a) aluminium (b) wrought iron  
(c) cast iron (d) alnico alloys
2. A coil of 1000 turns is wound on a core. A current of 1 A flowing through the coil creates a core flux of 1 mWb. The energy stored in the magnetic field is  
(a) 0.25 J (b) 0.5 J  
(c) 1 J (d) 2 J (ESE 2003)

# C H A P T E R 9

## Learning Objectives

- Faraday's Laws of electrolysis
- Value of Back e.m.f.
- Primary and Secondary Batteries
- Classification of Lead Storage Batteries
- Formation of Plates of Lead-acid Cells
- Plante Process
- Faure Process
- Positive Pasted Plates
- Negative Pasted Plates
- Comparison : Plante and Faure Plates
- Internal Resistance and Capacity of a Cell
- Electrical Characteristics of the Lead-acid Cell
- Battery Ratings
- Indications of a Fully-Charged Cell
- Voltage Regulators
- End-cell Control System
- Charging Systems
- Constant-current System-Constant-voltage System
- Trickle Charging
- Sulphation-Causes and Cure
- Mains operated Battery Chargers
- Car Battery Charger
- Automobile Battery Charger
- Static Uninterruptable Power Systems
- Alkaline Batteries
- Nickel-iron or Edison Batteries
- Nickel-Cadmium Batteries
- Silver-zinc Batteries
- High Temperature Batteries
- Secondary Hybrid Cells
- Fuel Cells

## ELECTRO-CHEMICAL POWER SOURCES



Electrochemical cells convert chemical energy into electrical energy

### 9.1. Faraday's Laws of Electrolysis

From his experiments, Faraday deduced two fundamental laws which govern the phenomenon of electrolysis. These are :

(i) **First Law.** The mass of ions liberated at an electrode is directly proportional to the quantity of electricity *i.e.* charge which passes through the electrolyte.

(ii) **Second Law.** The masses of ions of different substances liberated by the same quantity of electricity are proportional to their chemical equivalent weights.



Michael Faraday

#### Explanation of the First Law

If  $m$  = mass of the ions liberated,  $Q$  = quantity of electricity =  $I \times t$  where  $I$  is the current and  $t$  is the time, then according to the first law  $m \propto Q$  or  $m = Z Q$  or  $m = Z I t$

where  $Z$  is a constant and is known as the electrochemical equivalent (*E.C.E.*) of the substance.

If  $Q = 1$  coulomb *i.e.*  $I = 1$  ampere and  $t = 1$  second, then  $m = Z$ .

Hence, ***E.C.E. of a substance is equal to the mass of its ions liberated by the passage of one ampere current for one second through its electrolytic solution or by the passage of a charge of one coulomb.***

In fact, the constant  $Z$  is composite and it depends on the valency and atomic weight of the substance concerned. Its value is given by  $Z = \left( \frac{1}{F} \cdot \frac{a}{v} \right)$  where  $a$  is the atomic weight,  $v$  the valency and  $F$  is Faraday's constant. It is so because  $m$  is proportional to atomic weight, since each ion carries a definite charge. Obviously, the charge carried by an ion is proportional to its valency. Now, consider the molecules of sulphuric acid and copper sulphate. The sulphion  $\text{SO}_4^-$  in the acid molecule is combined with two positive hydrogen ions, whereas in  $\text{CuSO}_4$  molecule, it is combined only with one positive (bivalent)  $\text{Cu}^{++}$  ion. It is seen that a copper ion being bivalent carries twice the charge of a hydrogen ion which is univalent (monovalent). It means that in order to transfer a given quantity of electricity, only one-half as many bivalent copper ions as univalent hydrogen ions will be required. In other words, greater is the valency of an ion, smaller is the number of ions needed to carry a given quantity of electricity or charge which means that the mass of an ion liberated is inversely proportional to its valency.

$$\therefore m = \left( \frac{1}{F} \cdot \frac{a}{v} \right) I t = \left( \frac{1}{F} \cdot \frac{a}{v} \right) Q = \frac{E}{F} \cdot Q$$

where  $E$  is the chemical equivalent weight ( $= a/v$ ).

The constant  $F$  is known as Faraday's constant. The value of Faraday's constant can be found thus. It is found that one coulomb liberates 0.001118 gram of silver. Moreover, silver is univalent and its atomic weight is 107.88. Hence, substituting these values above, we find that

$$0.001118 = \frac{1}{F} \cdot 107.88 \times 1$$

$$\therefore F = 107.88 / 0.001118 = 96,500 \text{ coulomb, which corresponds to } 96,500/3600 = 26.8 \text{ Ah}$$

Faraday's constant is defined as ***the charge required to liberate one gram-equivalent of any substance.***

For all substances,  $\frac{\text{chemical equivalent (E)}}{\text{electrochemical equivalent (Z)}} = \text{Faraday's constant (F)} = 96,500 \text{ coulomb}$   
or  $F = E/Z$



### Explanation of the Second Law

Suppose an electric current is passed for the same time through acidulated water, solution of  $\text{CuSO}_4$  and  $\text{AgNO}_3$ , then for every 1.0078 (or 1.008) gram of hydrogen evolved, 107.88 gram of silver and 31.54 gram of Cu are liberated. The values 107.88 and 31.54 represent the equivalent weights\* of silver and copper respectively *i.e.* their atomic weights (as referred to hydrogen) divided by their respective valencies.

**Example 9.1.** Calculate the time taken to deposit a coating of nickel 0.05 cm thick on a metal surface by means of a current of 8 A per  $\text{cm}^2$  of surface. Nickel is a divalent metal of atomic weight 59 and of density 9  $\text{gram/cm}^3$ . Silver has an atomic weight of 108 and an E.C.E. of 1.118 mg/C.

**Solution.** Wt. of nickel to be deposited per  $\text{cm}^2$  of surface =  $1 \times 0.05 \times 9 = 0.45$  g

$$\text{Now } \frac{\text{E.C.E. of Ni}}{\text{E.C.E. of Ag}} = \frac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}}$$

$$\therefore \text{E.C.E. of Ni} = 1.118 \times 10^{-3} \times \frac{(59/2)}{108} = 0.0003053 \text{ g/C}$$

(chemical equivalent = atomic wt./valency)

$$\text{Now } m = Zit \quad \therefore 0.45 = 0.0003053 \times 8 \times t; t = 184 \text{ second} = \mathbf{3 \text{ min. } 4 \text{ second}}$$

**Exmample 9.2.** If 18.258 gm of nickel are deposited by 100 amp flowing for 10 minutes, how much copper would be deposited by 50 amp for 6 minutes : Atomic weight of nickel = 58.6 and that of copper 63.18. Valency of both is 2. (Electric Power AMIE Summer 1991)

**Solution.** From Faraday's first law, we get  $m = Zit = m \left( \frac{1}{F} \cdot \frac{a}{v} \right) It$ .

If  $m_1$  is the mass of nickel deposited and  $m_2$  that of copper, then

$$m_1 = 18.258 = \left( \frac{1}{F} \cdot \frac{58.6}{2} \right) \times 100 (10 \times 60), \left( \frac{1}{F} \cdot \frac{63.18}{2} \right) \times 50 \times (6 \times 60)$$

$$\therefore \frac{m_2}{18.258} = \frac{31.59}{29.3} \times \frac{18,000}{60,000} \quad \therefore m_2 = \mathbf{5.905 \text{ gm}}$$

**Example 9.3.** The cylindrical surface of a shaft of diameter 12 cm and length 24 cm is to be repaired by electrodeposition of 0.1 cm thick nickel on it. Calculate the time taken if the current used is 100 A. The following data may be used :

Specific gravity of nickel = 8.9 ; Atomic weight of nickel = 58.7 (divalent) ; E.C.E. of silver = 1.2 mg/C; Atomic weight of silver = 107.9. (Elect. Engg. A.M.Ae. S.I. June, 1991)

$$\text{Solution. Curved surface of the shaft} = \pi D \times l = \pi \times 12 \times 24 \text{ cm}^2$$

$$\text{Thickness of nickel layer} = 0.1 \text{ cm}$$

$$\text{Volume of nickel to be deposited} = 12\pi \times 24 \times 0.1 = 90.5 \text{ cm}^3$$

$$\text{Mass of nickel deposited} = 90.5 \times 8.9 = 805.4 \text{ g}$$

$$\text{Chemical equivalent of Ni} = \frac{\text{atomic weight}}{\text{valency}} = \frac{58.7}{2} = 29.35$$

\* The electro-chemical equivalents and chemical equivalents of different substance are inter-related thus :

$$\frac{\text{E.C.E. of A}}{\text{E.C.E. of B}} = \frac{\text{chemical equivalent of A}}{\text{chemical equivalent of B}}$$

Further, if  $m_1$  and  $m_2$  are masses of ions deposited at or liberated from an electrode,  $E_1$  and  $E_2$  their chemical equivalents and  $Z_1$  and  $Z_2$  their electrochemical equivalent weights, then

$$m_1/m_2 = E_1/E_2 = Z_1/Z_2$$

$$\begin{aligned} \text{Now} \quad \frac{E.C.E. \text{ of Ni}}{E.C.E. \text{ of Ag}} &= \frac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}} \\ \therefore \quad \frac{E.C.E. \text{ of Ni}}{1.12} &= \frac{29.35}{107.9} \\ \therefore \quad E.C.E. \text{ of Ni} &= 1.12 \times 29.35/107.9 = 0.305 \text{ mg/C} \\ \text{Now} \quad m &= ZIt \\ \therefore 805.4 &= 0.305 \times 10^{-3} \times 100 \times t \quad \therefore t = 26,406 \text{ second or } \mathbf{7 \text{ hr, } 20 \text{ min } 7 \text{ s}} \end{aligned}$$

**Example 9.4.** The worn-out part of a circular shaft 15 cm in diameter and 30 cm long is to be repaired by depositing on it 0.15 cm of Nickel by an electro-depositing process. Estimate the quantity of electricity required and the time if the current density is to be 25 mA/cm<sup>2</sup>. The current efficiency of the process may be taken as 95 per cent. Take E.C.E. for nickel as 0.3043 mg/coulomb and the density of nickel as 8.9 g/cm<sup>3</sup>. (Elect. Power-I, Bangalore Univ.)

**Solution.** Curved surface area of shaft =  $\pi D \times l = \pi \times 15 \times 30 = 1414 \text{ cm}^2$ .  
 Thickness of nickel layer = 0.15 cm  
 Volume of nickel to be deposited =  $1414 \times 0.15 = 212 \text{ cm}^3$   
 Mass of nickel to be deposited =  $212 \times 8.9 = 1887 \text{ gram}$   
 Now,  $m = ZQ$ ;  $Q = m/Z = 1887/0.343 \times 10^{-3} = \mathbf{62 \times 10^5 \text{ C}}$   
 Now, current density =  $15 \times 10^{-3} \text{ A/cm}^2$ ;  $A = 1414 \text{ cm}^2$   
 $I = 25 \times 10^{-3} \times 1414 = 35.35 \text{ A}$   
 Since  $Q = It \quad \therefore t = 62 \times 10^5/35.35 = 1.7 \times 10^5 \text{ s} = \mathbf{47.2 \text{ hr.}}$

**Example 9.5.** A refining plant employs 1000 cells for copper refining. A current of 5000 A is used and the voltage per cell is 0.25 volt. If the plant works for 100 hours/week, determine the annual output of refined copper and the energy consumption in kWh per tonne. The electrochemical equivalent of copper is 1.1844 kg/1000 Ah. (Electric Drives and Utilization, Punjab Univ. Jan. 1991)

**Solution.** Total cell voltage =  $0.25 \times 1000 = 250 \text{ V}$ ;  $I = 5000 \text{ A}$ ; plant working time = 100 hour/week =  $100 \times 52 = 5200 \text{ hour/year}$ ;  $Z = 1.1844 \text{ kg/1000 Ah}$ ;  $1 \text{ Ah} = 1 \times 60 \times 60 = 3600 \text{ C}$ ;  
 $\therefore Z = 1.1844 \text{ kg/1000} \times 3600 = 0.329 \times 10^{-6} \text{ kg/C}$ .  
 According to Faraday's law of Electrolysis, the amount of refined copper produced per year is  
 $m = ZIt = 0.329 \times 10^{-6} \times 5000 \times (5200 \times 3600) = 3079 \text{ kg} = \mathbf{3.079 \text{ tonne.}}$   
 Hence, annual output of refined copper = 3.079 tonne  
 Energy consumed per year =  $250 \times 5000 \times 5200/1000 = 6500 \text{ kWh}$   
 This is the energy consumed for refining 3.079 tonne of copper  
 $\therefore$  Energy consumed per tonne =  $6500/3.079 = \mathbf{2110 \text{ kWh/tonne.}}$

**Example 9.6.** A sheet of iron having a total surface area of 0.36 m<sup>2</sup> is to be electroplated with copper to a thickness of 0.0254 mm. What quantity of electricity will be required? The iron will be made the cathode and immersed, together with an anode of pure copper, in a solution of copper sulphate.

(Assume the mass density of copper =  $8.96 \times 10^3 \text{ kg m}^{-3}$ ; E.C.E. of copper =  $32.9 \times 10^{-8} \text{ kg C}^{-1}$   
 Current density =  $300 \text{ Am}^{-2}$ ) (AMIE Sec. B Utilisation of Electric Power Summer 1992)

**Solution.** Area over which copper is to be deposited =  $0.36 \text{ m}^2$   
 Thickness of the deposited copper =  $0.0254 \times 10^{-3} \text{ m}$   
 Volume of deposited copper =  $0.36 \times 0.0254 \times 10^{-3} = 9.144 \times 10^{-6} \text{ m}^3$   
 Mass of copper deposited = volume  $\times$  density  
 $= 9.144 \times 10^{-6} \times 8.96 \times 10^3 = 0.0819 \text{ kg}$   
 Now,  $m = ZQ \quad \therefore Q = m/Z = 0.0819/32.9 \times 10^{-8} = \mathbf{248936 \text{ C}}$

### Tutorial Problems No. 9.1

1. A steady current was passed for 10 minutes through an ammeter in series with a silver voltameter and 3.489 grams of silver were deposited. The reading of the ammeter was 5A. Calculate the percentage error. Electrochemical equivalent of silver = 1.1138 mg/C.  
[3.85 %] (City and Guilds, London)
2. Calculate the ampere-hours required to deposit a coating of silver 0.05 mm thick on a sphere of 5 cm radius. Assume electrochemical equivalent of silver = 0.001118 and density of silver to be 10.5 g cm<sup>3</sup>.  
[Utilization of Elect. Power, A.M.I.E. Summer]

### 9.2. Polarisation of Back E.M.F.

Let us consider the case of two platinum electrodes dipped in dilute sulphuric acid solution. When a small potential difference is applied across the electrodes, no current is found to flow. When, however, the applied voltage is increased, a time comes when a temporary flow of current takes place. The  $H^+$  ions move towards the cathodes and  $O^-$  ions move towards the anode and are absorbed there. These absorbed ions have a tendency to go back into the electrolytic solution, thereby leaving them as oppositely-charged electrodes. This tendency produces an e.m.f. that is in opposition to the applied voltage which is consequently reduced.

*This opposing e.m.f. which is produced in an electrolyte due to the absorption of gaseous ions by the electrolyte from the two electrodes is known as the back e.m.f. of electrolysis or polarisation.*

The value of this back e.m.f. is different from different electrolytes. The minimum voltage required to decompose an electrolyte is called the *decomposition* voltage for that electrolyte.

### 9.3. Value of Back E.M.F.

For producing electrolysis, it is necessary that the applied voltage must be greater than the back e.m.f. of electrolysis for that electrolyte. The value of this back e.m.f. of electrolysis can be found thus :

Let us, for example, find the decomposition voltage of water. We will assume that the energy required to separate water into its constituents (*i.e.* oxygen and hydrogen) is equal to the energy liberated when hydrogen and oxygen combine to form water. Let  $H$  kcal be the amount of heat energy absorbed when 9 kg of water are decomposed into 1 kg of hydrogen and 8 kg of oxygen. If the electrochemical equivalent of hydrogen is  $Z$  kg/coulomb, then the passage of  $q$  coulomb liberates  $Zq$  kg of hydrogen. Now,  $H$  is the heat energy required to release 1 kg of hydrogen, hence for releasing  $Zq$  kg of hydrogen, heat energy required is  $HZq$  kcal to  $JHZq$  joules. If  $E$  is the decomposition voltage, then energy spent in circulating  $q$  coulomb of charge is  $Eq$  joule. Equating the two amounts of energies, we have

$$Eq = JHZq \text{ or } E = JHZ$$

where  $J$  is 4200 joule/kcal.

The e.m.f. of a cell can be calculated by determining the two electrode potentials. The electrode potential is calculated on the assumption that the electrical energy comes entirely from the heat of the reactions of the constituents. Let us take a zinc electrode. Suppose it is given that 1 kg of zinc when dissolved liberates 540 kcal of heat and that the electrochemical equivalent of zinc is  $0.338 \times 10^{-6}$  kg/coulomb. As calculated above,

$$E = JHZ = 4200 \times 540 \times 0.338 \times 10^{-6} = 0.76 \text{ volt}$$

The electrode potentials are usually referred to in terms of the potential of a standard hydrogen electrode *i.e.* an electrode of hydrogen gas at normal atmospheric pressure and in contact with a normal acid solution. In table No. 9.1 are given the electrode potentials of various elements as referred to the standard hydrogen electrode. The elements are assumed to be in normal solution and at atmospheric pressure.

In the case of Daniel cell having copper and zinc electrodes, the copper electrode potential with respect to hydrogen ion is +0.345 V and that of the zinc electrode is -0.758 V. Hence, the cell e.m.f. is  $0.345 - (-0.758) = 1.103$  volt. The e.m.f. of other primary cells can be found in a similar way.

Table No. 9.1

Electrode	Potential (volt)	Electrode	Potential (volt)
Cadmium	-0.398	Mercury	+0.799
Copper	+0.345	Nickel	-0.231
Hydrogen	0	Potassium	-2.922
Iron	-0.441	Silver	+0.80
Lead	-0.122	Zinc	-0.758

**Example 9.7.** Calculate the weight of zinc and  $MnO_2$  required to produce 1 ampere-hour in a leclanche cell.

Atomic weights : Mn, 55 ; O, 16 ; Zn, 65. E.C.E. of hydrogen =  $1.04 \times 10^{-8}$  kg/C.

**Solution.** 1 ampere-hour = 3600 A-s = 3600 C

Wt. of hydrogen liberated =  $Zq = 1.04 \times 10^{-8} \times 3600 = 37.44 \times 10^{-6}$  kg

Now, the chemical reactions in the cell are

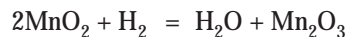


It is seen that 1 *atom* of zinc is used up in liberating *two atoms* of hydrogen. In other words, to produce 2 kg of hydrogen, 65 kg of zinc will have to go into chemical combination.

$\therefore$  Zinc required to produce  $37.44 \times 10^{-6}$  kg of hydrogen =  $37.44 \times 10^{-6} \times 65/2$

$$= 1.217 \times 10^{-3} \text{ kg}$$

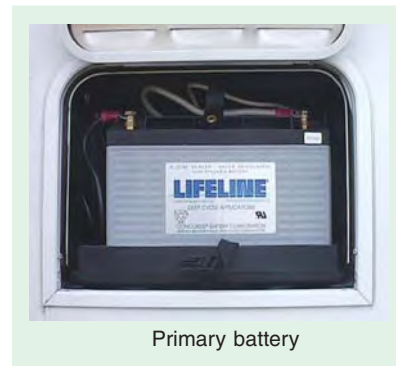
The hydrogen liberated combines with manganese dioxide as under :



Atomic weight of  $2MnO_2 = 2(55 + 16 \times 2) = 174$

It is seen that 174 kg of  $MnO_2$  combine with 2 kg of hydrogen, hence Wt. of  $MnO_2$  needed to combine with  $37.44 \times 10^{-6}$  kg of hydrogen =  $37.44 \times 10^{-6} \times 174/2 = 3.258 \times 10^{-3}$  kg.

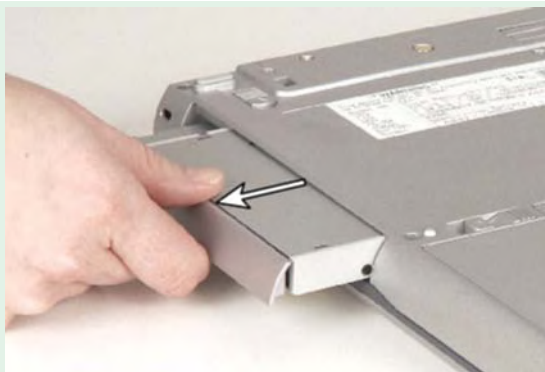
Hence, for 1 ampere-hour,  $1.217 \times 10^{-3}$  kg of zinc and  $3.258 \times 10^{-3}$  kg of  $MnO_2$  are needed.



Primary battery

#### 9.4. Primary and Secondary Batteries

An electric battery consists of a number of electrochemical cells, connected either in series or



Secondary battery

parallel. A cell, which is the basic unit of a battery, may be defined as a power generating device, which is capable of converting stored chemical energy into electrical energy. If the stored energy is inherently present in the chemical substances, it is called a primary cell or a non-rechargeable cell. Accordingly, the battery made of these cells is called primary battery. The examples of primary cells are Leclanche cell, zinc-chlorine cell, alkaline-manganese cell and metal air cells etc.

If, on the other hand, energy is induced in the chemical substances by applying an external source, it is called a secondary cell or rechargeable cell. A battery made out of these cells is called a secondary battery or storage battery or rechargeable battery. Examples of secondary cells are lead-acid cell, nickel-cadmium cell, nickel-iron cell, nickel-zinc cell, nickel-hydrogen cell, silver-zinc cell and high temperature cells like lithium-chlorine cell, lithium-sulphur cell, sodium-sulphur cell etc.

### 9.5. Classification of Secondary Batteries Based on Their Use

Various types of secondary batteries can be grouped in to the following categories as per their use :

#### 1. Automotive Batteries or SLI Batteries or Portable Batteries

These are used for starting, lighting and ignition (SLI) in internal-combustion-engined vehicles. Examples are; lead-acid batteries, nickel-cadmium batteries etc.

#### 2. Vehicle Traction Batteries or Motive Power Batteries or Industrial Batteries

These are used as a motive power source for a wide variety of vehicles. Lead-acid batteries, nickel-iron batteries, silver-zinc batteries have been used for this purpose. A number of advance batteries including high-temperature batteries are under development for electric vehicle (EV) use. These high-temperature batteries like sodium-sulphur and lithium-iron sulphide have energy densities in the range of 100-120 Wh/kg.

#### 3. Stationary Batteries.

These fall into two groups (a) standby power system which is used intermittently and (b) load-levelling system which stores energy when demand is low and, later on, uses it to meet peak demand.

### 9.6. Classification of Lead Storage Batteries

Lead storage batteries may be classified according to the service which they provide.

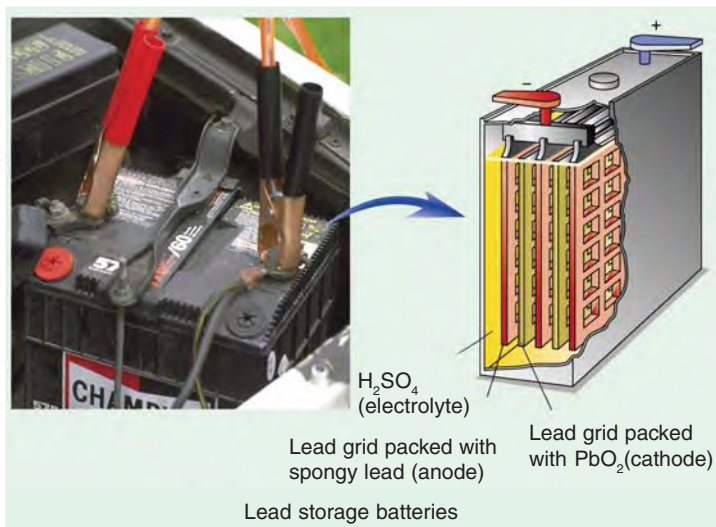
#### 1. SLI Batteries

The primary purpose of these batteries is to supply power for engine starting, lighting and ignition (SLI) of vehicles propelled by IC engines such as automobiles, buses, lorries and other heavy road vehicles and motor cycles etc. Usually, these batteries provide 12 V and consist of six series-connected lead-acid cells with capacity of the order of 100 Ah. Their present-day energy density is about 45 Wh/kg and 75 Wh/dm<sup>3</sup>.

These days 'maintenance-free' (MF) SLI batteries have been designed, which do not require the addition of water

throughout their normal service life of 2-5 years. MF versions of the SLI batteries are constructed of such material that no gassing occurs during charging. In MF batteries, the electrolyte is either absorbed within the microporous separators and the plates or is immobilized with suitable gelling agents.

These days the SLI batteries are charged from an alternator (AC generator) and not from dynamo (DC generator). The alternating current produced by the alternator is converted into direct current by



a full-wave bridge rectifier, which uses semi-conductor diodes. In this arrangement, no cutout is needed and the transistorised voltage controller regulates the alternator output to suit the electrical load and the state of charge of the battery. The battery is charged under constant-voltage conditions.

### 2. Vehicle Traction Batteries

The recent universal concern over the levels of toxic gases (particularly in urban areas) emitted by the IC engines has revived interest in electric traction. There has been great development in the use of battery-powered vehicles, primarily industrial trucks and commercial road vehicles of various types like 'milk floats' (*i.e.* bottled-milk delivery trucks), fork lift trucks, mining, airport tractors, aircraft service vehicles, electric cars and, more recently, in robotics and guided vehicles.

Traction batteries are of higher quality than *SLI* batteries. They provide constant output voltage, high volumetric capacity, good resistance to vibration and a long service life. They can withstand prolonged and deep discharges followed by deep recharges usually on a daily basis. The voltage of traction batteries varies from 12 V to 240 V and they have a cycle life of 1000-1500 cycles.

A number of advanced batteries and under development for *EV* use (*i*) room temperature batteries like zinc-nickel oxide battery (75 Wh/kg) and zinc-chlorine hydrate battery (80 Wh/kg) and (*ii*) high-temperature batteries like sodium-sulphur battery (120 Wh/kg) and lithium-iron sulphide battery (100 Wh/kg).

### 3. Stationary Batteries

Their use falls into two groups :

(*a*) as standby power system and (*b*) as load-levelling system.

In the standby applications, the battery is used to power essential equipment or to provide alarms or emergency lighting, in case of break-down in the main power supply. Standby applications have increased in recent years with increasing demand for uninterruptable power systems (*UPS*) and a tremendous growth in new telecommunication networks. The *UPS* provides 'clean' a.c supply free of sags or surges in the line voltage, frequency variations, spikes and transients to modern computer and electronic equipment. Banks of sealed lead-acid (*SLA*) standby batteries have been recently used in telecommunication systems and for *UPS* applications.

Recently, advanced lead-acid batteries have been used for load-levelling purpose in the electric generating plants. A 100 M Wh lead-acid battery load-levelling system could occupy a building two and a half storey high and an area of about 250,000 m<sup>2</sup>.

## 9.7. Parts of a Lead-acid Battery

A battery consists of a number of cells and each cell of the battery-consists of (*a*) positive and negative plants (*b*) separators and (*c*) electrolyte, all contained in one of the many compartments of the battery container.\* Different parts of a lead-acid battery are as under :

(*i*) **Plates.** A plate consists of a lattice type of grid of cast antimonial lead alloy which is covered with *active* material (Art. 9.8). The grid not only serves as a support for the fragile active material but also conducts electric current. Grid for the positive and negative plates are often of the same design although negative plate grids are made somewhat lighter. As discussed in Art. 9.10, positive plates are usually Plante plates whereas negative plates are generally of Faure or pasted type.

(*ii*) **Separators.** These are thin sheets of a porous material placed between the positive and negative plates for preventing contact between them and thus avoiding internal short-circuiting of the battery. A separator must, however, be sufficiently porous to allow diffusion or circulation of electrolyte between the plates. These are made of especially-treated cedar wood, glass wool mat, microporous rubber (mipor), microporous plastics (plastipore, miplast) and perforated p.v.c. as shown in Fig. 9.1. In addition to good porosity, a separator must possess high electrical resistance and mechanical strength.

\* The most common form of lead-acid cell used for marine applications is the tubular cell which consists of 'armoured' tubular positive plate of standard flat negative plate.

(iii) **Electrolyte.** It is dilute sulphuric acid which fills the cell compartment to immerse the plates completely.

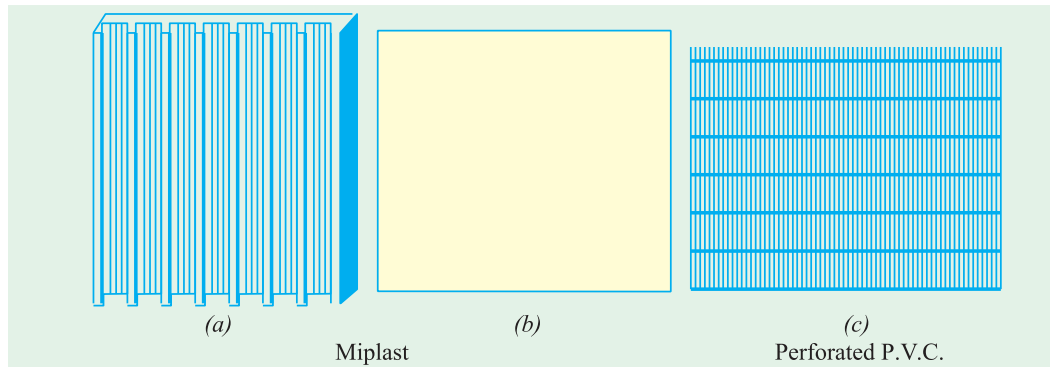


Fig. 9.1

(iv) **Container.** It may be made of vulcanised rubber or moulded hard rubber (ebonite), moulded plastic, ceramics, glass or celluloid. The vulcanised rubber containers are used for car service, while glass containers are superior for lighting plants and wireless sets. Celluloid containers are mostly used for portable wireless set batteries. A single monoblock type container with 6 compartments generally used for starting batteries is shown in Fig. 9.2. Full details of a Russian 12-CAM-28 lead-acid battery parts are shown in Fig. 9.3. Details of some of these parts are as follows:

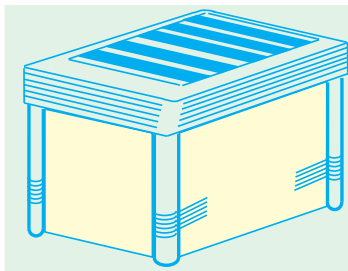


Fig. 9.2

(a) **Bottom Grooved Support Blocks.** These are raised ribs, either fitted in the bottom of the container or made with the container itself. Their function is to

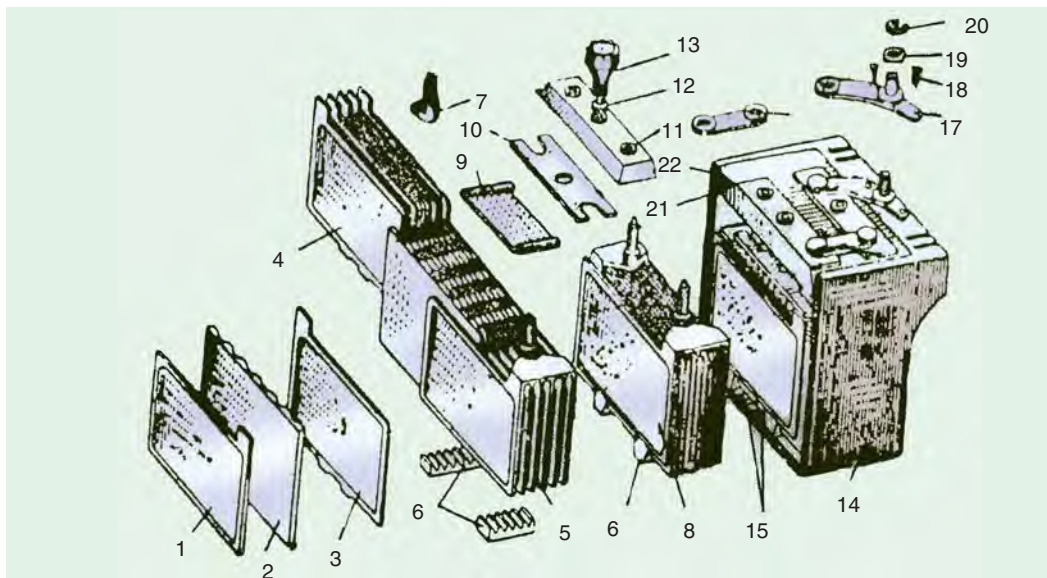


Fig. 9.3. (Courtesy MIR Publishers, Moscow)

1. -ve plate 2. separator 3. + ve plate. 4. + ve group 5. -ve group 6. -ve group grooved support block 7. lug 8. plate group 9. guard screen 10. guard plate 11. cell cover 12. plug washer 13. vent plug 14. monoblock jar 15. supporting prisms of + ve group 16. inter-cell connector 17. terminal lug 18. screw 19. washer 20. nut 21. rubber packing 22. sealing compound.

support the plates and hold them in position and at the same time protect them from short-circuits that would otherwise occur as a result of fall of the active material from the plates onto the bottom of the container.

(b) **Connecting Bar.** It is the lead alloy link which joins the cells together in series connecting the positive pillar of one cell to the negative pillar of the next one.

(c) **Terminal Post or Pillar.** It is the upward extension from each connecting bar which passes through the cell cover for cable connections to the outside circuits. For easy identification, the negative terminal post is smaller in diameter than the positive terminal post.

(d) **Vent Plugs or Filler Caps.** These are made of polystyrene or rubber and are usually screwed in the cover. Their function is to prevent escape of electrolyte but allow the free exit of the gas. These can be easily removed for topping up or taking hydrometer readings.

(e) **External Connecting Straps.** These are the antimonial lead alloy flat bars which connect the positive terminal post of one cell to the negative of the next across the top of the cover. These are of very solid construction especially in starting batteries because they have to carry very heavy currents.

## 9.8. Active materials of a Lead-acid Cell

Those substances of the cell which take active part in chemical combination and hence absorb or produce electricity during charging or discharging, are known as **active materials** of the cell.

The active materials of a lead-acid cell are :

1. **Lead peroxide** ( $\text{PbO}_2$ ) for +ve plate 2. **Sponge Lead** (Pb) for -ve plate 3. **Dilute Sulphuric Acid** ( $\text{H}_2\text{SO}_4$ ) as electrolyte.

### 1. Lead Peroxide

It is a combination of lead and oxygen, is dark chocolate brown in colour and is quite hard but brittle substance. It is made up of one atom of lead (Pb) and two atoms of oxygen ( $\text{O}_2$ ) and its chemical formula is  $\text{PbO}_2$ . As said earlier, it forms the positive active material.

### 2. Sponge Lead

It is pure lead in soft sponge or porous condition. Its chemical formula is Pb and forms the negative active material.

### 3. Dilute Sulphuric Acid

It is approximately 3 parts water and one part sulphuric acid. The chemical formula of the acid is  $\text{H}_2\text{SO}_4$ . The positive and negative plates are immersed in this solution which is known as electrolyte. It is this medium through which the current produces chemical changes.

**Hence, the lead-acid cell depends for its action on the presence of two plates covered with  $\text{PbO}_2$  and Pb in a solution of dilute  $\text{H}_2\text{SO}_4$  of specific gravity 1.21 or nearabout.**

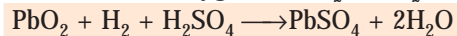
Lead in the form of  $\text{PbO}_2$  or sponge Pb has very little mechanical strength, hence it is supported by plates of pure lead. Those plates covered with or otherwise supporting  $\text{PbO}_2$  are known as +ve plates and those supporting sponge lead are called -ve plates. The +ve and -ve plates are arranged alternately and are connected to two common +ve and -ve terminals. These plates are assembled in a suitable jar or container to make a complete cell as discussed in Art. 9.4 above.

## 9.9. Chemical changes

### (i) DISCHARGING (Fig. 9.4)

When the cell is fully charge, its positive plate or anode is  $\text{PbO}_2$  (dark chocolate brown) and the negative plate or cathode is Pb (slate grey). When the cell discharges *i.e.* it sends current through the external load, then  $\text{H}_2\text{SO}_4$  is dissociated into positive  $\text{H}^+$  and negative  $\text{SO}_4^-$  ions. As the current within the cell is flowing from cathode to anode,  $\text{H}_2$  ions move to anode and  $\text{SO}_4^-$  ions move to the cathode.

At anode ( $\text{PbO}_2$ ),  $\text{H}_2$  combines with the oxygen of  $\text{PbO}_2$  and  $\text{H}_2\text{SO}_4$  attacks lead to form  $\text{PbSO}_4$ .





At the cathode (Pb),  $\text{SO}_4$  combines with it to form  $\text{PbSO}_4$

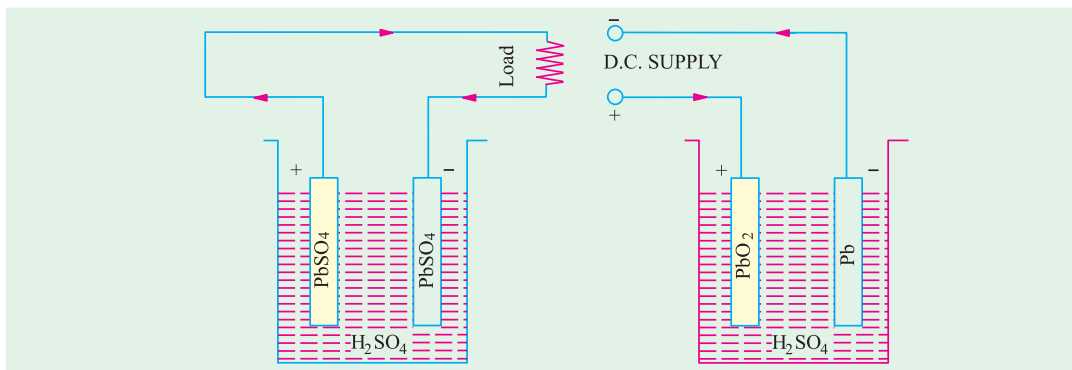
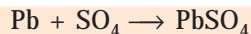


Fig. 9.4

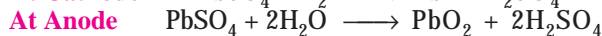
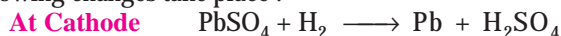
Fig. 9.5

It will be noted that during discharging :

- (i) Both anode and cathode become  $\text{PbSO}_4$  which is somewhat whitish in colour.
- (ii) Due to formation of water, specific gravity of the acid decreases.
- (iii) Voltage of the cell decreases. (iv) The cell gives out energy.

**(ii) CHARGING (Fig. 9.5)**

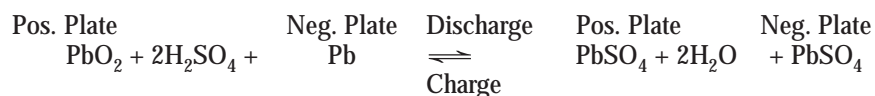
When the cell is recharged, the  $\text{H}_2$  ions move to cathode and  $\text{SO}_4$  ions go to anode and the following changes take place :



Hence, the anode and cathode again become  $\text{PbO}_2$  and Pb respectively.

- (i) The anode becomes dark chocolate brown in colour ( $\text{PbO}_2$ ) and cathode becomes grey metallic lead (Pb).
- (ii) Due to consumption of water, specific gravity of  $\text{H}_2\text{SO}_4$  is increased.
- (iii) There is rise in voltage. (iv) Energy is absorbed by the cell.

The charging and discharging of the cell can be represented by a single reversible equation given below :



For discharge, the equation should be read from left to right and for charge from right to left.

**Example 9.8.** Estimate the necessary weight of active material in the positive and negative plates of a lead-acid secondary cell per ampere-hour output (atomic weight of lead 207, valency 2, E.C.E. of hydrogen  $0.0104 \times 10^{-6}$  kg/C).

**Solution.** Wt. of hydrogen evolved per ampere-hour =  $0.0104 \times 10^{-6} \times 3,600$   
 =  $37.44 \times 10^{-6}$  kg

During discharge, reaction at cathode is  $\text{Pb} + \text{H}_2\text{SO}_4 = \text{PbSO}_4 + \text{H}_2$

As seen, 207 kg of lead react chemically to liberate 2 kg of hydrogen.

Hence, weight of Pb needed per ampere-hour =  $37.44 \times 10^{-6} \times 207/2 = 3.876 \times 10^{-3}$  kg

At anode the reaction is :  $\text{PbO}_2 + \text{H}_2 \longrightarrow \text{PbO} + \text{H}_2\text{O}$

Atomic weight of  $\text{PbO}_2 = 207 + 32 = 239$

∴ Wt. of  $\text{PbO}_2$  going into combination per ampere-hour =  $37.44 \times 10^{-6} \times 239/2 = 4.474 \times 10^{-3}$  kg

Therefore, quantity of active material required per ampere-hour is : lead  $3.876 \times 10^{-3}$  kg and lead peroxide  $4.474 \times 10^{-3}$  kg.

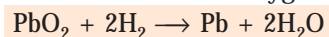
### 9.10. Formation of Plates of Lead-acid Cells

There are, in general, two methods of producing the active materials of the cells and attaching them to lead plates. These are known after the names of their inventors. (1) **Plante** plates or formed plates (2) **Faure** plates or pasted plates.

### 9.11. Plante Process

In this process, two sheets of lead are taken and immersed in dilute  $H_2SO_4$ . When a current is passed into this lead-acid cell from a dynamo or some other external source of supply, then due to electrolysis, hydrogen and oxygen are evolved. At anode, oxygen attacks lead converting it into  $PbO_2$  whereas cathode is unaffected because hydrogen can form on compound with Pb.

If the cell is now discharged (or current is reversed through it), the peroxide-coated plate becomes cathode, so hydrogen forms on it and combines with the oxygen of  $PbO_2$  to form water thus :



At the same time, oxygen goes to anode (the plate previously unattached) which is lead and reacts to form  $PbO_2$ . Hence, the anode becomes covered with a thin film of  $PbO_2$ .

By continuous reversal of the current or by charging and discharging the above electrolytic cell, the thin film of  $PbO_2$  will become thicker and thicker and the polarity of the cell will take increasingly longer time to reverse. Two lead plates after being subjected to hundreds of reversals will acquire a skin of  $PbO_2$  thick enough to possess sufficiently high capacity. This process of making positive plates is known as **formation**. The negative plates are also made by the same process. They are turned from positive to negative plates by reversing the current through them until whole  $PbO_2$  is converted into sponge lead. Although Plante positives are very commonly used for stationary work, Plante negatives have been completely replaced by the Faure or pasted type plates as discussed in Art. 9.13. However, owing to the length of time required and enormous expenditure of electrical energy, this process is commercially impracticable. The process of formation can be accelerated by forming agents such as acetic, nitric or hydrochloric acid or their salts but still this method is expensive and slow and plates are heavy.

### 9.12. Structure of Plante Plates

It is seen that since active material on a Plante plate consists of a thin layer of  $PbO_2$  formed on and from the surface of the lead plate, it must be made of large superficial area in order to get an appreciable volume of it. An ordinary lead plate subjected to the forming process as discussed above will have very small capacity. Its superficial area and hence its capacity, can be increased by grooving or laminating. Fig. 9.6 shows a Plante positive plate which consists of a pure lead grid with finely laminated surfaces. The construction of these plates consists of a large number of thin vertical laminations which are strengthened at intervals by horizontal binding ribs. This results in an increase of the superficial area 10 to 12 times that possessed by a plain lead sheet of the same overall dimensions. The above design makes possible the expansion of the plate structure to accommodate the increase in mass and the value of the active material ( $PbO_2$ ) which takes place when the cell goes through a series of chemical changes during each cycle of charge or discharge. The expansions of the plate structure takes place downwards where there is room left for such purpose. Usually, a Plante positive plate expands by about 10% or so of its length during the course of its useful life.

Another type of Plante positive plate is the 'rosette' plate which consists of a perforated cast grid or framework of lead alloy with 5 to 12 per cent of antimony holding rosettes or spirals of corrugated pure lead tape. The rosettes (Fig. 9.7) provide the active material of the positive plate and, during formation, they expand in the holes of the grid which are countersunk on both sides of the grid. The advantages of such plates are that the lead-antimony grid is itself unaffected by the chemical action and the complete plate is exceptionally strong.

Other things being equal, the life of a Plante plate is in direct proportion to the weight of lead metal in it, because as the original layer of  $PbO_2$  slowly crumbles away during the routine charging

and discharging of the cell, fresh active material is formed out of the underlying lead metal. Hence, the capacity of such a plate lasts as long as the plate itself. In this respect, Plante plate is superior to the Faure or pasted plate.



Fig. 9.6

Fig. 9.7

Fig 9.8

### 9.13. Faure Process

In the making of Faure plates, the active material is mechanically applied instead of being electrochemically produced out of lead plate itself as in Plante process. The active material which is in the form of red lead ( $Pb_3O_4$ ) or litharge  $PbO$  or the mixture of the two in various proportions, is pressed into the interstices of a thin lead grid or lattice work of intersecting ribs which also serves as conductor of current. The plates after being thus pasted are allowed to dry and harden, are then assembled in weak solution of  $H_2SO_4$  of specific gravity 1.1 to 1.2 and are formed by passing an electric current between them. If plates are meant to be positive, they are connected up as anodes, if negative, then as cathodes. The oxygen evolved at the anode converts the lead oxide ( $Pb_3O_4$ ) into peroxide ( $PbO_2$ ) and at cathode the hydrogen reduces  $PbO$  to sponge lead by abstracting the oxygen.

### 9.14. Positive Pasted Plates

Formation of positive plate involves converting lead oxide into  $PbO_2$ . A high lead oxide like  $Pb_3O_4$  is used for economy both in current and time, although in practice, a mixture of  $Pb_3O_4$  and  $PbO$  is taken—the latter being added to assist in the setting or cementation of the plate.

### 9.15. Negative Pasted Plates

Faure process is much better adopted for making a negative rather than a positive plate. The negative material *i.e.* sponge lead is quite tough instead of being hard and brittle like  $PbO_2$  and, moreover, it undergoes a comparatively negligible change in volume during the charging and discharging of the cell. Hence, it has no tendency to disintegrate or shed out of the grid although it does tend to lose its porosity and become dense and so lose capacity. Hence, in the manufacture of the pasted negatives, a small percentage of certain substances like powdered pumic or graphite or magnesium sulphate or barium sulphate is added to increase the porosity of the material. If properly handled, a paste made with  $H_2SO_4$ , glycerine and  $PbO$  (or mixture of  $PbO$  and  $Pb_3O_4$ ) results in a very good negative, because glycerine is carbonised during formation and so helps in keeping the paste porous.

Faure plates are in more general use because they are cheaper and have a high (capacity/weight) ratio than Plante plates. Because of the lightness and high capacity/weight ratio, such plates are used

practically for all kinds of portable service like electric vehicles, train lighting, car-lighting and strating etc. But their life is shorted as compared to Plante plates.

### 9.16. Structure of Faure Plates

Usually, the problem of Faure type grid is relatively simple as compared to the Plante type. In the case of Faure plates, the grid serves simply as a support for the active material and a conductor for the current and as a means for distributing the current evenly over the active material. Unlike Plante plates, it is not called upon to serve as a kind of reservoir from which fresh active material is continuously being formed for replacing that which is lost in the wear and tear of service. Hence, this makes possible the use of an alloy of lead and antimony which, as pointed out earlier, resists the attack of acid and 'forming' effect of current more effectively than pure lead and is additionally much harder and stiffer.

Because of the hardening effect of antimony, it is possible to construct very thin light plates which possess sufficient rigidity to withstand the expensive action of the positive active material. Simplest type of grid consists of a meshwork of vertical and horizontal ribs intersecting each other thereby forming a number of rectangular spaces in which the paste can be pressed and allowed to set. Such a thin grid has the disadvantage that there is not much to 'key' in the paste and due to a great shock or vibration the pellets are easily 'started' and so fall out.

A much better support to the active material can be given by the construction illustrated in Fig. 9.9 which is known as 'basket' type or screened grid. The paste instead of being in isolated pellets forms a continuous sheet contained and supported by the horizontal ribs of the grid. With this arrangement the material can be very effectively keyed in.

Another type of grid structure used in pasted plates is shown in Fig. 9.10.

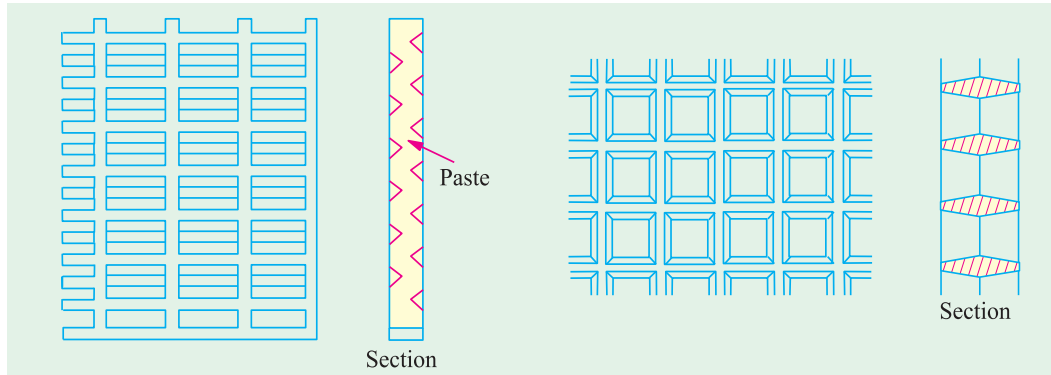


Fig. 9.9

Fig. 9.10

### 9.17. Comparison : Plante and Faure Plates

1. Plante plates have a longer life and can withstand rapid discharging (as in traction work) better than Faure's.
2. They are less liable to disintegration when in use than Faure's plates.
3. They are heavier and more expensive than Faure plates.
4. Plante plates have less capacity-to-weight ratio, values being 12 to 21 Ah per kg of plate, the corresponding values for Faure plate being 65 to 90 Ah/kg.

### 9.18. Internal Resistance and Capacity of a Cell

The secondary cell possesses internal resistance due to which some voltage is lost in the form of potential drop across it when current is flowing. Hence, the internal resistance of the cell has to be kept to the minimum.

One obvious way to lessen internal resistance is to increase the size of the plates. However, there

is a limit to this because the cell will become too big to handle. Hence, in practice, it is usual to multiply the number of plate inside the cell and to join all the negative plates together and all the positives ones together as shown in Fig. 9.11.

The effect is equivalent to joining many cells in parallel. At the same time, the length of the electrolyte between the electrodes is decreased with a consequent reduction in the internal resistance.

The 'capacity' of a cell is given by the product of current in amperes and the time in hours during which the cell can supply current until its e.m.f. falls to 1.8 volt. It is expressed in ampere-hour (Ah).

The interlacing of plates not only decreases the internal resistance but additionally increases the capacity of the cell also. There is always one more negative plate than the positive plates *i.e.* there is a negative plate at both ends. This gives not only more mechanical strength but also assures that both sides of a positive plate are used.

Since in this arrangement, the plates are quite close to each other, something must be done to make sure that a positive plate does not touch the negative plate otherwise an internal short-circuit will take place. The separation between the two plates is achieved by using separators which, in the case of small cells, are made of treated cedar wood, glass, wool mat, microporous rubber and microporous plastic and in the case of large stationary cells, they are in the form of glass rods.

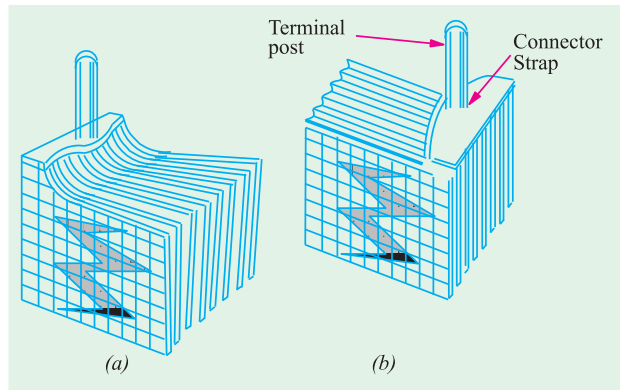


Fig. 9.11

### 9.19. Two Efficiencies of the Cell

The efficiency of a cell can be considered in two ways :

1. **The quantity or ampere-hour (Ah) efficiency**
2. **The energy or watt-hour (Wh) efficiency**

The Ah efficiency does not take into account the varying voltages of charge and discharge. The Wh efficiency does so and is always less than Ah efficiency because average p.d. during discharging is less than that during charging. Usually, during discharge the e.m.f. falls from about 2.1 V to 1.8 V whereas during charge it rises from 1.8 volt to about 2.5 V.

$$\text{Ah efficiency} = \frac{\text{amp-hour discharge}}{\text{amp-hour charge}}$$

The Ah efficiency of a lead-acid cell is normally between 90 to 95%, meaning that about 100 Ah must be put back into the cell for every 90-95 Ah taken out of it. Because of gassing which takes place during the charge, the Ah available for delivery from the battery decreases. It also decreases (i) due to self-discharge of the plates caused due to local reactions and (ii) due to leakage of current because of faulty insulation between the cells of the battery.

The Wh efficiency varies between 72-80%.

If Ah efficiency is given, Wh efficiency can be found from the following relation :

$$\text{Wh efficiency} = \text{Ah efficiency} \times \frac{\text{average volts on discharge}}{\text{average volts on charge}}$$

From the above, it is clear that anything that increases the charge volts or reduces the discharge volts will decrease Wh efficiency. Because high charge and discharge rates will do this, hence it is advisable to avoid these.

### 9.20. Electrical Characteristics of the Lead-acid Cell

The three important features of an accumulator, of interests to an engineer, are (1) voltage (2) capacity and (3) efficiency.

### 1. Voltage

The open-circuit voltage of a fully-charged cell is approximately 2.2 volt. This value is not fixed but depends on (a) length of time since it was last charged (b) specific gravity-voltage increasing with increase in sp. gravity and *vice versa*. If sp. gravity comes near to density of water *i.e.* 1.00 then voltage of the cell will disappear altogether (c) temperature-voltage increases, though not much, with increase in temperature.

The variations in the terminal p.d. of a cell on charge and discharge are shown in Fig. 9.12. The voltage-fall depends on the rate of discharge. Rates of discharge are generally specified by the number of hours during which the cell will sustain the rate in question before falling to 1.8 V. The voltage falls rapidly in the beginning (rate of fall depending on the rate of discharge), then vary slowly up to 1.85 and again suddenly to 1.8 V.

The voltage should not be allowed to fall to lower than 1.8 V, otherwise hard insoluble lead sulphate is formed on the plate which increases the internal resistance of the cell.

The general form of the voltage-time curves corresponding to 1-, 3- 50 and 10- hour rates of corresponding to the steady currents which would discharge the cell in the above mentioned times (in hour). It will be seen that both the terminal voltage and the rate at which the voltage and the rate at which the voltage falls, depend on the rate of discharge. The more rapid fall in voltage at higher rates of discharge is due to the rapid increase in the internal resistance of the cell.

During charging, the p.d. increases (Fig. 9.12). The curve is similar to the discharge curve reversed but is everywhere higher due to the increased density of  $H_2SO_4$  in the pores of the positive plate.

### 2. Capacity

It is measured in amp-hours (Ah). The capacity is always given at a specified rate of discharge (10-hour discharge in U.K., 8-hour discharge in U.S.A.). However, motor-cycle battery capacity is based on a 20-hour rate (at 30° C). The capacity depends upon the following :

(a) **Rate of discharge.** The capacity of a cell, as measured in Ah, depends on the discharge rate. It decreases with increased rate of discharge. Rapid rate of discharge means greater fall in p.d. of the cell due to internal resistance of the cell. Moreover, with rapid discharge the weakening of the acid in the pores of the plates is also greater. Hence, the chemical change produced at the plates by 1 ampere for 10 hours is not the same as produced by 2 A for 5 hours or 4 A for 2.5 hours. It is found that a cell having a 100 Ah capacity at 10 hour discharge rate, has its capacity reduced to 82.5 Ah at 5-hour rate and 50 Ah at 1-hour rate. The variation of capacity with discharge rate is shown in Fig. 9.14.

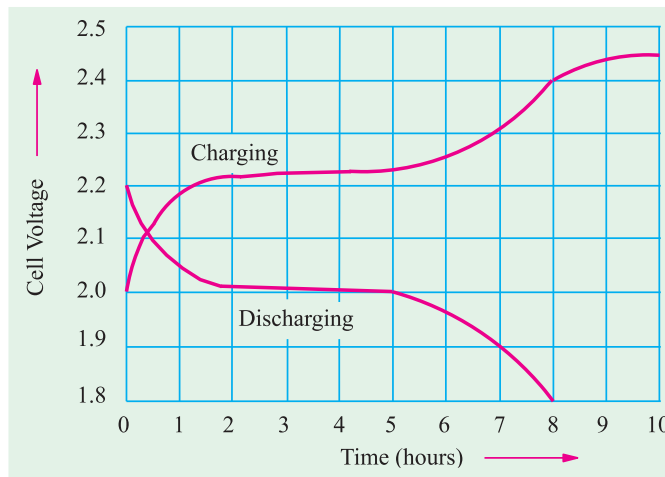


Fig. 9.12

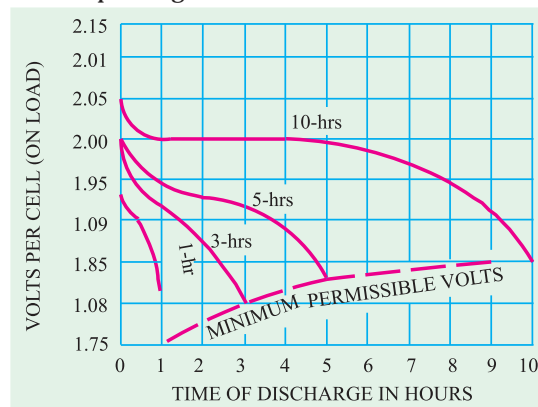


Fig. 9.13

(b) **Temperature.** At high temperature, (i) chemical reactions within the cell take place more vigorously.

(ii) the resistance of the acid is decreased and

(iii) there is a battery diffusion of the electrolyte.

Hence, high temperature increases the capacity of the lead-acid cell. Apparently, it is better to operate the battery at a high temperature. However, at high temperatures :

(a) the acid attacks the antimony-lead alloy grid, terminal posts and wooden separators.

(b) the paste is rapidly changed into lead sulphate. Sulphation is always accompanied by expansion of paste particularly at the positive plates and results in buckling and cracking of the grid.

Hence, it is not advisable to work batteries above 40° C.

As temperature is lowered, the speed of chemical reactions is decreased. Moreover, cell resistance also increases. Consequently, the capacity of the cell decreases with decrease in temperature till at freezing point the capacity is reduced to zero even though the battery otherwise be fully charged.

(c) **Density of electrolyte.** As the density of electrolyte affects the internal resistance and the vigour of chemical reaction, it has an important effect on the capacity. Capacity increases with the density.

(d) **Quantity of active material.** Since production of electricity depends on chemical action taking place within the cells, it is obvious that the capacity of the battery must depend directly upon the kind and amount of the active material employed. Consider the following calculations:

The gram-equivalent of lead is 103.6 gram and Faraday's constant is 96,500 coulombs which is =  $96,500/3600 = 26.8$  Ah. Hence, during the delivery of one Ah by the cell, the quantity of lead expended to form lead sulphate at the negative plate is  $103.6/26.8 = 3.86$  gram.

Similarly, it can be calculated that, at the same time, 4.46 gram of  $PbO_2$  would be converted into lead sulphate at the positive plate while 3.66 gram of acid would be expended to form 0.672 gram of water. It is obvious that for obtaining a cell of a greater capacity, it is necessary to provide the plates with larger amounts of active material.

### 3. Efficiency

It has already been discussed in Art. 9.19

## 9.21. Battery Ratings

Following standards have been adopted, both by industry and government organisations to get a fair picture of battery quantity :

### 1. Ampere-hour Capacity

It is a function of the total plate area *i.e.* size of the individual plate multiplied by the number of plates. For measuring this capacity, the battery is discharged continuously for 20 hours and its current output supplied to a standard load is measured. Suppose that a battery delivers 4A current for 20 hours. Hence, its rating is 80 Ah which is stamped on the battery case.

### 2. Reserve Capacity

It is one of the newly-developed rating standards and is more realistic because it provides a double-check on the Ah figures. The capacity is given by the number of minutes a battery will tolerate a 25 A drain without dropping below 10.5 V. Higher this rating, better the battery.

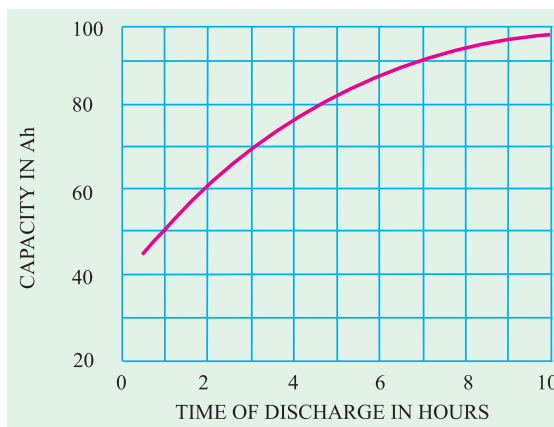


Fig. 9.14

### 3. Zero Cranking Power

It was the first cold weather rating and is applicable in relation to crafts which ply in freezing weather. This zero-degree performance test gives a valuable insight into battery quality. Large batteries are tested at a 300 A drain with battery chilled to  $-18^{\circ}\text{C}$  ( $0^{\circ}\text{F}$ ) whereas smaller sizes are tested at half this value.

This test consists of two part (a) the battery is first chilled to  $-18^{\circ}\text{C}$  ( $0^{\circ}\text{F}$ ) and the load applied for 5 second. Meanwhile, the voltage output is recorded. It is the first part of the zero-cranking-power rating. (b) The test is continued further till voltage drops to 5 V. The number of minutes it takes to reduce the voltage to 5 V forms the second half of the rating. Higher both the digits, better the battery quality.

### 4. Cold Cranking Power

This simple rating is applied to all 12-V storage batteries regardless of their size. The battery is loaded at  $-18^{\circ}\text{C}$  ( $0^{\circ}\text{F}$ ) till the total voltage drops to 7.2 V. The output current in amperes is measured for 30 seconds. Higher the output, better the battery.

**Example 9.9.** An alkaline cell is discharged at a steady current of 4 A for 12 hours, the average terminals voltage being 1.2 V. To restore it to its original state of charge, a steady current of 3 A for 20 hours is required, the average terminal voltage being 1.44 V. Calculate the ampere-hour (Ah) efficiency and Wh efficiency in this particular case.

(Principles of Elect. Engg.-I, Jadavpur Univ.)

**Solution.** As discussed in Art. 9.19

$$\text{Ah efficiency} = \frac{\text{Ah of discharge}}{\text{Ah of charge}} = \frac{12 \times 4}{20 \times 3} = 0.8 \text{ or } 80\%$$

$$\text{Wh efficiency} = \text{Ah effi.} \times \frac{\text{Av. volts on discharge}}{\text{Av. volts on charge}} = \frac{0.8 \times 1.2}{1.44} = 0.667 \text{ or } 66.7\%$$

**Example 9.10.** A discharged battery is charged at 8 A for 2 hours after which it is discharged through a resistor of  $R \Omega$ . If discharge period is 6 hours and the terminal voltage remains fixed at 12 V, find the value of  $R$  assuming the Ah efficiency of the battery as 80%.

**Solution.** Input amp-hours =  $8 \times 2 = 16$

Efficiency = 0.8  $\therefore$  Ah output  $16 \times 0.8 = 12.8$

Discharge current =  $12.8/6 \text{ A}$   $\therefore R = \frac{12}{12.8/6} = \frac{6 \times 12}{12.8} = 5.6 \Omega$

## 9.22. Indications of a Full-charged Cell

The indications of a fully-charged cell are :

(i) gassing (ii) voltage (iii) specific gravity and (iv) colour of the plates.

(i) Gassing

When the cell is fully charged, it freely gives off hydrogen at cathode and oxygen at the anode, the process being known as "Gassing". Gassing at *both* plates indicates that the current is no longer doing any useful work and hence should be stopped. Moreover, when the cell is fully charged, the electrolyte assumes a milky appearance.

(ii) Voltage

The voltage ceases to rise when the cell becomes fully-charged. The value of the voltage of a fully-charged cell is a variable quantity being affected by the rate of charging, the temperature and specific gravity of the electrolyte etc. The approximate value of the e.m.f. is 2.1 V or so.

(iii) Specific Gravity of the Electrolyte

A third indication of the state of charge of a battery is given by the specific gravity of the electrolyte. We have seen from the chemical equations of Art. 9.9, that during discharging, the density of electrolyte decreases due to other production of water, whereas it increases during charging due to the



absorption of water. The value of density when the cell is fully charged is 1.21 and 1.18 when discharged up to 1.8 V. Specific gravity can be measured with a suitable hydrometer which consists of a float, a chamber for the electrolyte and a squeeze bulb.

**(iv) Colour**

The colour plates, on full charge is deep chocolate brown for positive plate and clear slate gray for negative plate and the cell looks quite brisk and alive.

### 9.23. Applications of Lead-acid Batteries

Storage batteries are these days used for a great variety and range of purposes, some of which are summarised below :

1. In Central Stations for supplying the whole load during light load periods, also to assist the generating plant during peak load periods, for providing reserve emergency supply during periods of plant breakdown and finally, to store energy at times when load is light for use at time when load is at its peak value.
2. In private generating plants both for industrial and domestic use, for much the same purpose as in Central Stations.
3. In sub-stations, they assist in maintaining the declared voltage by meeting a part of the demand and so reducing the load on and the voltage drop in, the feeder during peak-load periods.
4. As a power source for industrial and mining battery locomotives and for road vehicles like cars and trucks.
5. As a power source for submarines when submerged.
6. Marine applications include emergency or stand-by duties in case of failure of ship's electric supply, normal operations where batteries are subjected to regular cycles of charge and discharge and for supplying low-voltage current to bells, telephones, indicators and warning systems etc.
7. For petrol motor-car starting and ignition etc.
8. As a low voltage supply for operating purposes in many different ways such as high-tension switchgear, automatic telephone exchange and repeater stations, broadcasting stations and for wireless receiving sets.

### 9.24. Voltage Regulators

As explained in Art. 9.20, the voltage of a battery varies over a considerable range while under discharge. Hence, it is necessary to find some means to control the battery voltage upto the end so as to confine variations within reasonable limits – these limits being supplied by the battery.

The voltage control systems may be hand-operated or automatic. The simplest form of hand-operated control consists of a rheostat having a sufficient number of steps so that assistance can be inserted in the circuit when battery is fully charged and gradually cut out as the discharge continues, as shown in Fig. 9.15.

The above system can be designed for automatic operation as shown in Fig. 9.16. A rise in voltage results in the release of pressure on the carbon block rheostat, thereby increasing its resistance whereas a fall in voltage results in increasing the pressure on the block thereby decreasing its resistance. By this automatic variation of control resistance, variations in battery voltage are automatically controlled.

### 9.25. End-cell Control System

The use of rheostat for controlling the battery

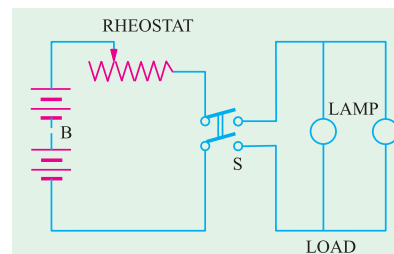


Fig. 9.15

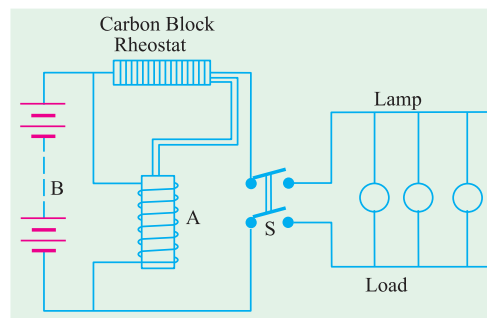


Fig. 9.16

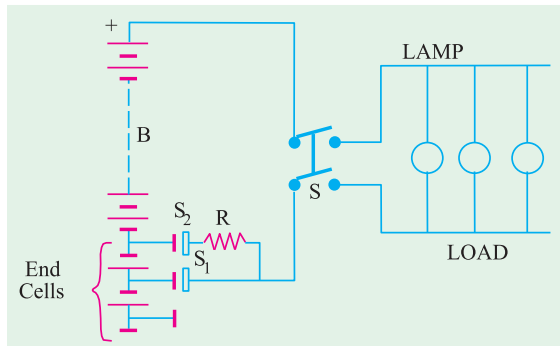


Fig. 9.17

where from it derives the name **end cell**. By moving the contact arm of the switch to the left or right, cells are cut in or out of the discharge circuit and so the voltage is varied accordingly.

For making the end-cell switch operate without opening the circuit or shortcircuiting the cells during its passage from one cell to another, an auxiliary constraint  $S_2$  is employed.  $S_2$  prevents the circuit from being open entirely but has sufficient resistance  $R$  between it and the main contact arm  $S_1$  to prevent any objectionably large current to flow on short-circuit. The above mechanism usually incorporates devices for preventing the stoppage of the switch in the short-circuit position.

### 9.26. Number of End-cells

For maintaining a supply voltage of  $V$  volts from a battery of lead-acid cells when the latter are approaching their discharge voltage of 1.83 (depending on the discharge rate), the number of cells required is  $V/1.83$ . When the battery is fully charged with each cell having an e.m.f. of 2.1 V, then the number of cells required is  $V/2.1$ . Hence, the number of end-cells required is  $(V/1.83 - V/2.1)$ . These are connected to a regulating switch which adds them in series with the battery one or two at a time, as the discharge proceeds.

### 9.27. Charging Systems

In various installations, batteries are kept floating on the line and are so connected that they are being charged when load demands are light and automatically discharged during peak periods when load demands are heavy or when the usual power supply fails or is disconnected. In some other installations, the battery is connected to the feeder circuit as and when desired, allowed to discharge to a certain point, then removed and re-charged for further requirements.

For batteries other than the 'floating' and 'system-governed' type, following two general methods (though there are some variations of these) are employed.

(i) *The Constant-current System* and (ii) *The Constant-voltage System*.

### 9.28. Constant-current System

In this method, the charging current is kept constant by varying the supply voltage to overcome the increased back e.m.f. of cells. If a charging booster (which is just a shunt dynamo directly driven by a motor) is used, the current supplied by it can be kept constant by adjusting its excitations. If charged on a d.c. supply, the current is controlled by varying the rheostat connected in the circuit. The value of charging current should be so chosen that there would be no

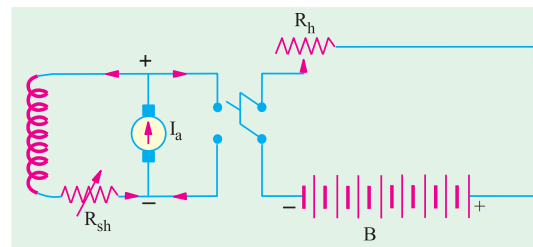


Fig. 9.18

voltage is objectionable especially in large-capacity installations where the  $I^2R$  loss would be considerable. Hence other more economical systems have been developed and put into use. One such system is the end-cell control system. It consists of suitable regulator switches which cut one or more of a selected number of cells out of the circuit when the battery is fully charged and into the circuit again as the discharge continues. To make the process of cutting cells in and out at the battery circuit simple, the group of cells selected for this control is situated at one end of the battery

excessive gassing during final stages of charging and, also, the cell temperature does not exceed 45°C. This method takes a comparatively longer time.

### 9.29. Constant-voltage System

In this method, the voltage is kept constant but it results in very large charging current in the beginning when the back e.m.f. of the cells is low and a small current when their back e.m.f. increases on being charged.

*With this method, time of charging is almost reduced to half. It increases the capacity by approximately 20% but reduces the efficiency by 10% or so.*

#### Calculations

When a secondary cell or a battery of such cells is being charged, then the emf of the cells acts in opposition to the applied voltage. If  $V$  is the supply voltage which sends a charging current of  $I$  against the back e.m.f.  $E_b$ , then input is  $VI$  but the power spent in overcoming the opposition (Fig. 9.19) is  $E_b I$ . This power  $E_b I$  is converted into the chemical energy which is stored in the cell. The charging current can be found from the following equation :

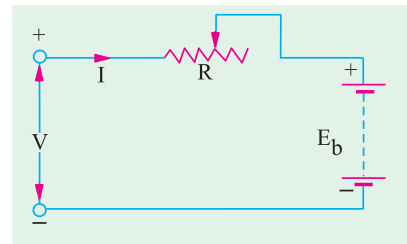


Fig. 9.19

$$I = \frac{V - E_b}{R}$$

where

$R$  = total circuit resistance including internal resistance of the battery

$I$  = charging current

By varying  $R$ , the charging current can be kept constant throughout.

**Example 9.11.** A battery of accumulators of e.m.f. 50 volt and internal resistance  $2 \Omega$  is charged on 100 volt direct means. What series resistance will be required to give a charging current of 2 A?

If the price of energy is 50 paise per kWh, what will it cost to charge the battery for 8 hours and what percentage of energy supplied will be used in the form of heat ?

**Solution.** Applied voltage = 100 V; Back e.m.f. of the battery = 50 V  
 Net charging voltage =  $100 - 50 = 50$  V  
 Let  $R$  be the required resistance, then  $2 = 50/(R + 2)$ ;  $R = 46/2 = 23 \Omega$   
 Input for eight hours =  $100 \times 2 \times 8 = 1600$  Wh = 1.6 kWh  
 Cost =  $50 \times 1.5 = 80$  paise; Power wasted on total resistance =  $25 \times 2^2 = 100$  W  
 Total input =  $100 \times 2 = 200$  W : Percentage waste =  $100 \times 100/200 = 50 \%$

**Example 9.12.** A 6-cell, 12-V battery is to be charged at a constant rate of 10 A from a 24-V d.c. supply. If the e.m.f. of each cell at the beginning and end of the charge is 1.9 V and 2.4 V, what should be the value of maximum resistance to be connected in series with the battery. Resistance of the battery is negligible.

#### Solution. Beginning of Charging

Total back e.m.f. of battery =  $6 \times 1.9 = 11.4$  volt

Net driving voltage =  $24 - 11.4 = 12.6$  V ;  $R_{\max} = 12.6/10 = 1.26 \Omega$

#### End of Charging

Back e.m.f. of battery =  $6 \times 2.4 = 14.4$  volt

Net driving voltage =  $24 - 14.4 = 9.6$  V ;  $R_{\min} = 9.6/10 = 0.96 \Omega$

**Example 9.13.** Thirty accumulators have to be charged from their initial voltage of 1.8 V using a direct current supply of 36 volt. Each cell has an internal resistance of  $0.02 \Omega$  and can be charged at 5 amperes. Sketch a circuit by which this can be done, calculating the value of any resistance or resistances used. What will be the current taken from the mains towards the end of the charging period when the voltage has risen to 2.1 volt per cell ?

**Solution.** Since the supply voltage (36 V) is less than the back e.m.f. of the 30 cell battery (54 V), hence the cells are divided into two equal groups and placed in parallel across the supply for charging as shown in Fig. 9.20. It would be economical to use a separate resistance  $R$  in series with each group.

$$\begin{aligned} \text{Here } V &= 36 \text{ V}, E_b = 15 \times 1.8 = 27 \text{ V} \\ \text{Internal resistance of each parallel group} \\ &= 15 \times 0.02 = 0.3 \Omega \\ \text{Charging current} = 5 \text{ A } \therefore 5 &= \frac{36 - 27}{R + 0.3} \end{aligned}$$

$$R = 1.5 \Omega$$

Now, when the voltage per cell becomes 2.1 V, then back e.m.f. of each parallel group =  $15 \times 2.1 = 31.5 \text{ V}$

$$\therefore \text{Charging current} = \frac{36 - 31.5}{1.5 + 0.3} = 2.5 \text{ A}$$

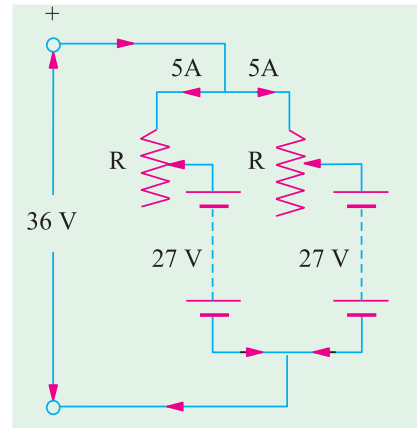


Fig. 9.20

**Example 9.14.** A battery of 60 cells is charged from a supply of 250 V. Each cell has an e.m.f. of 2 volts at the start of charge and 2.5 V at the end. If internal resistance of each cell is  $0.1 \Omega$  and if there is an external resistance of  $19 \Omega$  in the circuit, calculate (a) the initial charging current (b) the final charging current and (c) the additional resistance which must be added to give a finishing charge of 2 A rate.

**Solution.** (a) Supply voltage  $V = 250 \text{ V}$

Back e.m.f. of the battery  $E_b$  at start =  $60 \times 2 = 120 \text{ V}$  and at the end =  $60 \times 2.5 = 150 \text{ V}$

$$\text{Internal resistance of the battery} = 60 \times 0.1 = 6 \Omega$$

$$\text{Total circuit resistance} = 19 + 6 = 25 \Omega$$

$$\text{(a) Net charging voltage at start} = 250 - 120 = 130 \text{ V}$$

$$\therefore \text{Initial charging current} = 130/25 = 5.2 \text{ A}$$

$$\text{(b) Final charging current} = 100/25 = 4 \text{ A}$$

(c) Let  $R$  be the external resistance, then

$$2 = \frac{100}{R + 6} \quad \therefore R = 88/2 = 44 \Omega$$

$$\therefore \text{Additional resistance required} = 44 - 19 = 25 \Omega$$

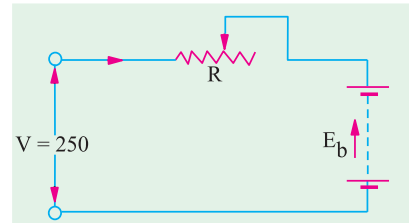


Fig. 9.21

**Example 9.15.** Two hundred and twenty lamps of 100 W each are to be run on a battery supply at 110 V. The cells of the battery when fully charged have an e.m.f. of 2.1 V each and when discharged 1.83 V each. If the internal resistance per cell is  $0.00015 \Omega$  (i) find the number of cells in the battery and (ii) the number of end cells. Take the resistance of the connecting wires as  $0.005 \Omega$

**Solution.** Current drawn by lamps =  $220 \times 100/110 = 200 \text{ A}$

$$\text{Voltage drop on the resistance of the connecting wires} = 0.005 \times 200 = 1.0 \text{ V}$$

$$\text{Battery supply voltage} = 110 + 1 = 111 \text{ V}$$

Terminal voltage/cell when fully charged and supplying the load

$$= 2.1 - (200 \times 0.00015) = 2.08 \text{ V}$$

Terminal voltage/cell when discharged

$$= 1.83 - (200 \times 0.00015) = 1.8 \text{ V}$$

(i) No. of cells in the battery =  $111/2.08$   
 = 53.4 say, **54**

(ii) No. of cells required when discharged  
 =  $111/1.8 = 62$

Hence, number of end cells =  $62 - 54 = 8$

The connections are shown in Fig. 9.22.

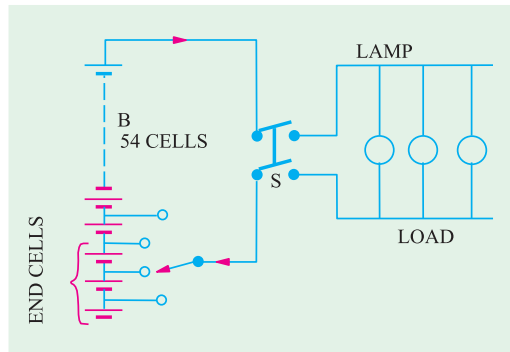


Fig. 9.22

**Example 9.16.** A storage battery consists of 55 series-connected cells each of internal resistance  $0.001 \Omega$  and e.m.f.  $2.1 \text{ V}$ . Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring  $20 \times 25 \text{ cm}$ . If full-load current per cell is  $0.01 \text{ A per cm}^2$  of positive plate surface, find (i) full-load terminal voltage of the battery and (ii) power wasted in the battery if the connectors have a total resistance of  $0.025 \Omega$

**Solution.** Since both sides of a positive plate are utilized, the area of both sides will be taken into consideration.

Total area (both sides) of ten positive plates =  $2 \times 20 \times 25 \times 10 = 10,000 \text{ cm}^2$

Full load current =  $10,000 \times 0.01 = 100 \text{ A}$

Voltage drop in battery and across connectors =  $100 [(55 \times 0.001) + 0.025] = 8 \text{ V}$

Battery e.m.f. =  $55 \times 1.2 = 115.5 \text{ V}$

(i) Battery terminal voltage on full-load =  $115.5 - 8 = 107.5 \text{ V}$

(ii) Total resistance =  $(55 \times 0.001) + 0.025 = 0.08 \Omega$ ; Power loss =  $100^2 \times 0.08 = 800 \text{ W}$ .

**Example 9.17.** A charging booster (shunt generator) is to charge a storage battery of 100 cells each of internal resistance  $0.001 \Omega$ . Terminal p.d. of each cell at completion of charge is  $2.55 \text{ V}$ . Calculate the e.m.f. which the booster must generate to give a charging current of  $20 \text{ A}$  at the end of charge. The armature and shunt field resistances of the generator are  $0.2$  and  $258 \Omega$  respectively and the resistance of the cable connectors is  $0.05 \Omega$

**Solution.** Terminal p.d. per cell =  $2.55 \text{ volt}$

The charging voltage across the battery must be capable of overcoming the back e.m.f. and also to supply the voltage drop across the internal resistance of the battery.

Back e.m.f. =  $100 \times 2.55 = 255 \text{ V}$

Voltage drop on internal resistance

$$= 100 \times 0.001 \times 20 = 2 \text{ V}$$

$\therefore$  P.D. across points A and B =  $255 + 2 = 257 \text{ V}$

P.D. across terminals C and D of the generator

$$= 257 + (20 \times 0.05) = 258 \text{ V}$$

$\therefore$   $I_{sh} = 258/258 = 1 \text{ A}; I_a = 20 + 1 = 21 \text{ A}$

$\therefore$   $I_a R_a = 21 \times 0.2 = 4.2 \text{ V}$

$\therefore$  Generated e.m.f. =  $258 + 4.2 = 262.2 \text{ V}$

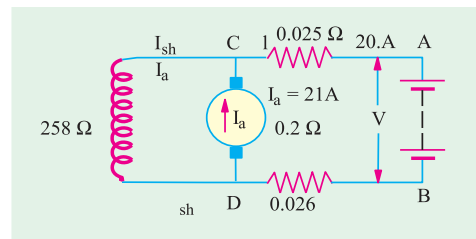


Fig. 9.23

### Tutorial Problems No. 9.2

1. A 60-cell storage battery having a capacity of 360 Ah takes 8 hours when charged by a dc generator at a voltage of 220 V. Calculate the charging current and the range of the rheostat required to ensure a constant charging current. The emf of each cell is 1.8 V at the beginning of charging and 2.7 V at the end of the charging. Ignore the internal resistance of the cell.

[45 A; 2.45 to 1.29  $\Omega$ ]

2. A storage battery consists of 55 series connected cells each of internal resistance 0.001  $\Omega$  and e.m.f. 2.1 V. Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring 20  $\times$  25 cm. If full-load current per cell is 0.01 A per cm<sup>2</sup> of positive plate surface, find (i) full-load terminal voltage of the battery and (ii) power wasted in the battery if the connectors have a total resistance of 0.025  $\Omega$ .

[(i) 107.5 V (ii) 800 W]

### 9.30. Trickle Charging

When a storage battery is kept entirely as an emergency reserve, it is very essential that it should be found fully charged and ready for service when an emergency arises. Due to leakage action and other open-circuit losses, the battery deteriorates even when idle or on open-circuit. Hence, to keep it fresh, the battery is kept on a trickle charge. The rate of trickle charge is small and is just sufficient to balance the open-circuit losses. For example, a standby battery for station bus-bars capable of giving 2000 A for 1 hour or 400 Ah at the 10-hr rate, will be having a normal charging rate of 555 A, but a continuous 'trickle' charge of 1 A or so will keep the cells fully charged (without any gassing) and in perfect condition. When during an emergency, the battery gets discharged, it is recharged at its normal charging rate and then is kept on a continuous trickle charge.

### 9.31. Sulphation-Causes and Cure

If a cell is left incompletely charged or is not fully charged periodically, then the lead sulphate formed during discharge, is not converted back into PbO<sub>2</sub> and Pb. Some of the unreduced PbSO<sub>4</sub> which is left, gets deposited on the plates which are then said to be sulphated. PbSO<sub>4</sub> is in the form of minute crystals which gradually increase in size if not reduced by thoroughly charging the cells. It increases the internal resistance of the cell thereby reducing its efficiency and capacity. Sulphation also sets in if the battery is overcharged or left discharged for a long time.

Sulphated cells can be cured by giving them successive overcharges, for which purpose they are cut out of the battery during discharge, so that they can get two charges with no intervening discharge. The other method, in which sulphated cells need not be cut out of the battery, is to continue charging them with a '*milking booster*' even after the battery as a whole has been charged. A milking booster is a motor-driven low-voltage dynamo which can be connected directly across the terminals of the sulphated cells.

### 9.32. Maintenance of Lead-acid Cells

The following important points should be kept in mind for keeping the battery in good condition:

1. Discharging should not be prolonged after the minimum value of the voltage for the particular rate of discharge is reached.
2. It should not be left in discharged condition for long.
3. The level of the electrolyte should always be 10 to 15 mm above the top of the plates which must not be left exposed to air. Evaporation of electrolyte should be made up by adding distilled water occasionally.
4. Since acid does not vaporise, none should be added.

5. Vent openings in the filling plug should be kept open to prevent gases formed within from building a high pressure.
6. The acid and corrosion on the battery top should be washed off with a cloth moistened with baking soda or ammonia and water.
7. The battery terminals and metal supports should be cleaned down to bare metal and covered with vaseline or petroleum jelly.

### 9.33. Main Operated Battery Chargers

A battery charger is an electrical device that is used for putting energy into a battery. The battery charger changes the a.c. from the power line into d.c. suitable for charger. However, d.c. generator and alternators are also used as charging sources for secondary batteries.

In general, a mains-operated battery charger consists of the following elements :

1. A step-down transformer for reducing the high a.c. mains voltage to a low a.c. voltage.
2. A half-wave or full-wave rectifier for converting alternating current into direct current.
3. A charger-current limiting element for preventing the flow of excessive charging current into the battery under charge.
4. A device for preventing the reversal of current *i.e.* discharging of the battery through the charging source when the source voltage happens to fall below the battery voltage.

In addition to the above, a battery charger may also have circuitry to monitor the battery voltage and automatically adjust the charging current. It may also terminate the charging process when the battery becomes fully charged. However, in many cases, the charging process is not totally terminated but only the charging rate is reduced so as to keep the battery on trickle charging. These requirements have been illustrated in Fig. 9.24.

Most of the modern battery chargers are fully protected against the following eventualities :

- (a) They are able to operate into a short-circuit.
- (b) They are not damaged by a reverse-connected battery.
- (c) They are operated into a totally flat battery.
- (d) They can be regulated both for current and voltage.

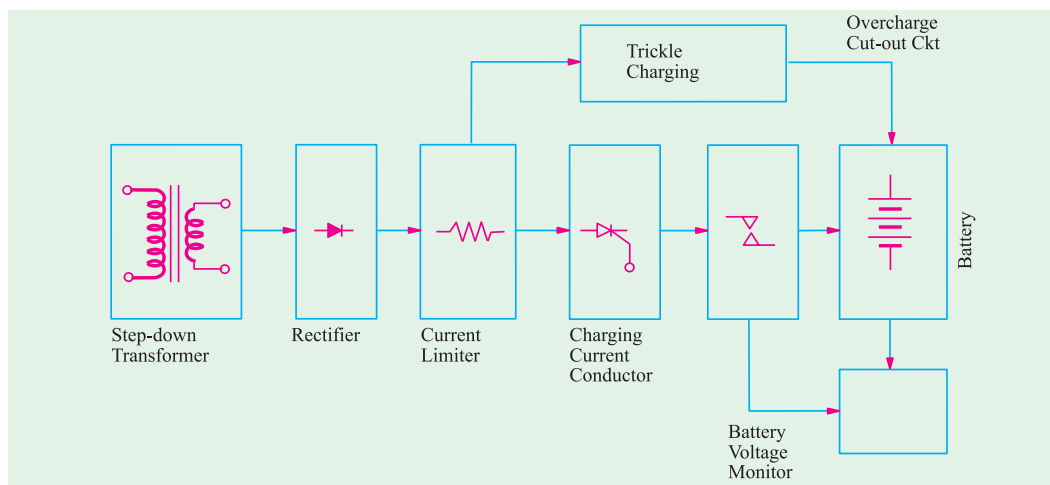


Fig. 9.24

### 9.34. Car Battery Charger Using SCR

Fig. 9.25 shows the circuitry of a very simple lead-acid battery charger which has been provided

protection from load short-circuit and from reverse battery polarity. The *SCR* is used as a half-wave rectifier as well as switching element to terminate the high-current charging process when battery gets fully-charged.

### Working

The *SCR* acts as a half-wave rectifier during only the positive half-cycles of the secondary voltage when point *M* in Fig. 9.25 is at a positive potential. The *SCR* does not conduct during the negative half-cycle of the secondary voltage when point *M* achieves negative potential. When *M* is at positive potential, the *SCR* is triggered into conduction because of the small gate current  $I_g$  passing via  $R_1$  and diode  $D_1$ . In this way, the charging current  $I$  after passing through  $R_5$  enters the battery which is being charged.

In the initial state, when the battery voltage is low, the potential of point *A* is also low (remember that  $R_3$ ,  $R_4$  and preset resistor  $R_6$  are connected across the battery via  $R_5$ ) which means that the forward bias on the base of transistor *T* is not sufficient to make it conduct and thereby stop the conduction of *SCR*. Hence, *SCR* keeps conducting, consequently, keeps charging the battery through the current limiting resistor  $R_5$ .

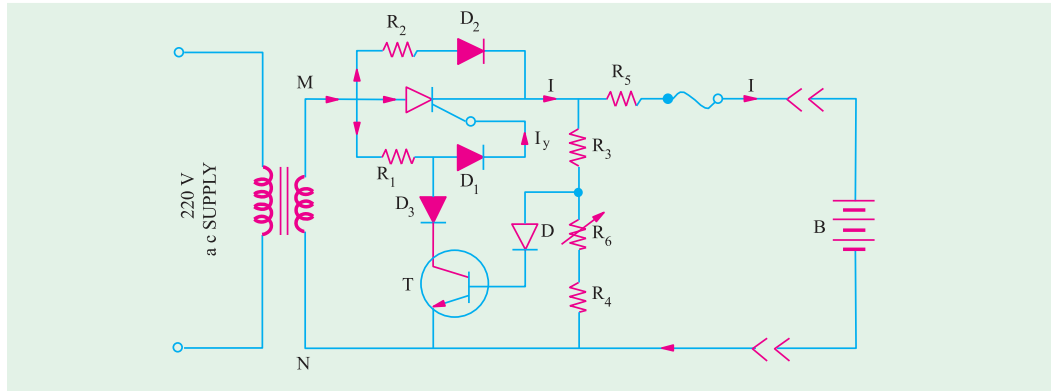


Fig. 9.25

As the battery gets progressively charged, its voltage rises and when it becomes fully charged, the potential of point *A* increases thereby increasing the forward bias of *T* which starts conducting. In that case, *T* bypasses the triggering gate current of the *SCR* via  $R_1$  and  $D_3$ . Since the *SCR* can no longer be triggered, the charging process stops. However, a small trickle charging current keeps flowing via  $R_2$  and  $D_2$ . The function of diode  $D_2$  is to prevent reverse flow of the current through the battery when point *M* has negative potential during the negative cycle of the secondary voltage. The value of trickle charging current is determined by  $R_2$  because  $R_5$  has a fixed but small value. The resistor  $R_5$  also limits the flow of excessive charging current when the charger is connected to a completely dead battery.

The charger described above is not suitable for fast charging because it utilizes half-wave rectification. Most of the mains-operated chargers working on a single-phase supply use a full-wave rectifier consisting of a center-tapped transformer and two diodes or a bridge circuit using four diodes.

### 9.35. Automobile Battery Charger Using Full-wave Rectifier

The battery charger shown in Fig. 9.26, is used to recharge run-down lead-acid batteries in automobiles without removing them from their original mountings and without any need for constant attention. When the battery is fully charged, the circuit automatically switches from charging current to trickle charging and an indicator lamp lights up to provide a visual indication of this condition.



As shown in Fig. 9.26, diodes  $D_1$  and  $D_2$  form a full-wave rectifier to provide pulsating direct current for charging the battery. The battery is charged through the SCR which is also used as switch to terminate the charging process when the battery becomes fully charged. The two transistors  $T_1$  and  $T_2$  together form an electronic switch that has two stable states *i.e.* the ON state in which  $T_1$  and  $T_2$  conduct and the OFF state in which  $T_1$  and  $T_2$  do not conduct. The ON-OFF state of this switch is decided by the battery voltage and setting of the “current adjust” potentiometer  $R_6$ .

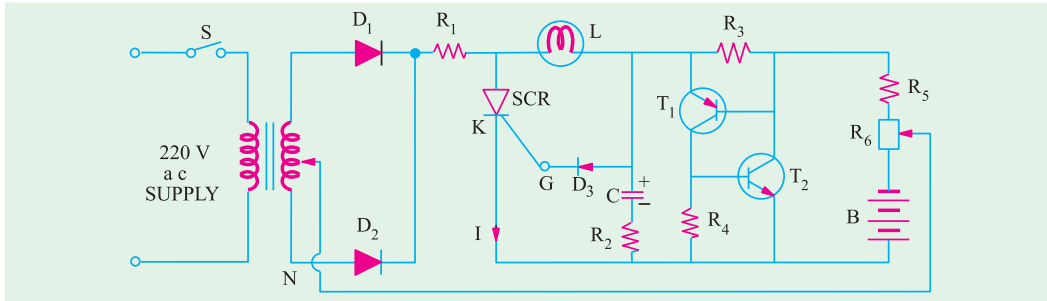


Fig. 9.26

### Working

When switch  $S$  is turned on, the full-wave rectified output of  $D_1$  and  $D_2$  charges capacitor  $C$  through  $R_1$ , lamp  $L$  and  $R_2$ . In a very short time, capacitor voltage rises high enough to make diode  $D_3$  conduct the gate current thereby triggering SCR into conduction during each half-cycle of the output voltage. Hence, full charging current is passed through the cathode  $K$  of the SCR to the positive terminal of the battery whose negative terminal is connected directly to the center tap of the step-down transformer. Resistor  $R_1$  limits the charging current to a safe value in order to protect the rectifier diodes  $D_1$  and  $D_2$  in case the load happens to be a “dead” battery.

When the battery is being charged and has low voltage, the two transistors  $T_1$  and  $T_2$  remain in the non-conducting state. However, when the battery voltage rises and finally the battery becomes fully-charged, the two transistors  $T_1$  and  $T_2$  (which form a regenerative switch) are triggered into conduction at a point set by  $R_6$ . In this way,  $T_1$  and  $T_2$  provide a low-impedance discharge path for  $C$ . Hence,  $C$  discharges through  $R_2$  and the  $T_1$ - $T_2$  switch, thereby cutting off the gate current of the SCR which stops conducting thereby terminating the battery charge. Thereafter, small trickle charge current keeps on flowing into the battery via  $L$  and the regenerative switch formed by  $T_1$  and  $T_2$ . A glowing lamp  $L$  indicates that the battery is under trickle charging.

Fig. 9.27 shows the same circuit as shown in Fig. 9.26 except that the two-diode full-wave rectifier has been replaced by a full-wave bridge rectifier using four diodes.

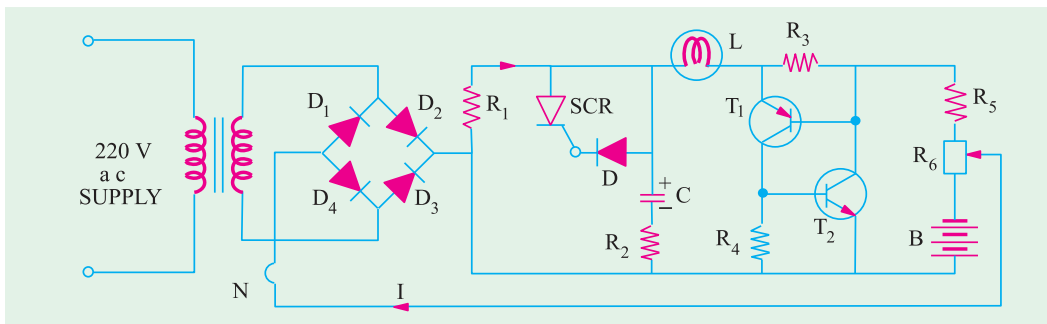


Fig. 9.27

### 9.36. Static Uninterruptable Power Systems (UPS)

The function of a *UPS* is to ensure absolute continuity of power to the computerised control systems thereby protecting critical equipment from electrical supply failure. A *UPS* makes it possible to provide a 'clean' reliable supply of alternating current free of sags or surges in the line voltage, frequency variation, spikes and transients. *UPS* systems achieve this by rectifying the standard mains supply, using the direct current to charge the standby battery and to produce 'clean' alternating current by passing through an inverter and filter system.

#### Components of a *UPS* System

The essential components of a *UPS* system as shown in Fig. 9.28 are as under :

1. A rectifier and thyristor-controlled battery charger which converts the *AC* input into regulated *DC* output and keeps the standby battery fully charged.
2. A standby battery which provides *DC* input power to inverter during voltage drops or on failure of the normal mains *AC* supply.
3. An inverter which converts *DC* to clean *AC* thus providing precisely regulated output voltage and frequency to the load as shown.

#### Working

As shown in Fig. 9.28 the main flow of energy is from the rectifier to the inverter with the standby battery kept on 'float'. If the supply voltage falls below a certain level or fails completely, the battery output to the inverter maintains a clean a.c. supply. When the mains power supply is resorted, the main energy flow again starts from the rectifier to the inverter but, in addition, the rectifier recharges the battery. When the standby battery gets fully charged, the charging current is automatically throttled back due to steep rise in the back e.m.f. of the battery. An automatic/manual bypass switch is used to connect the load either directly to the mains a.c. supply or to the inverter a.c. supply.

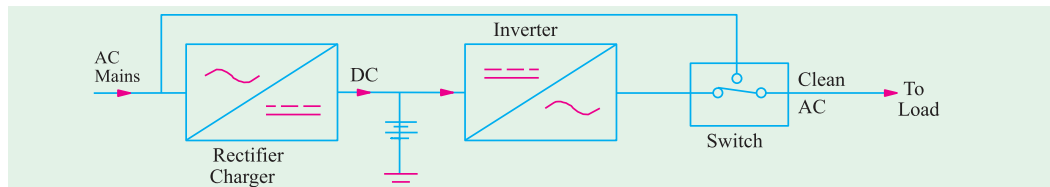


Fig. 9.28

Depending on the application, the voltage of the *UPS* standby batteries may be anywhere between 12 V and 400 V. Typical values are 24 V, 48 V, 110 V and 220 V with currents ranging from a few amperes to 2000 A. Fig. 9.28 shows Everon 4-kVA on-line *UPS* system which works on 170 V-270 V a.c. input and provides an a.c. output voltage of 230 V at 50 Hz frequency with a voltage stability of  $\pm 2\%$  and frequency stability of  $\pm 1\%$ . It has zero change over time and has audio beeper which indicates mains fail and battery discharge. It provides 100% protection against line noise, spikes, surges and radio frequency interference. It is manufactured by Everon Electro Systems Pvt. Ltd. New Delhi.

### 9.37. Alkaline Batteries

Such batteries are ideally suited for portable work. Like lead-acid cells, the alkaline cells also consist of positive and negative plates immersed in an electrolyte. The plates and the electrolyte are placed in a suitable container.

The two types of alkaline batteries which are in general use are :

- (i) nickel-iron type of Edison type.
- (ii) nickel-cadmium type of Jungnor type which is commercially known as NIFE battery.

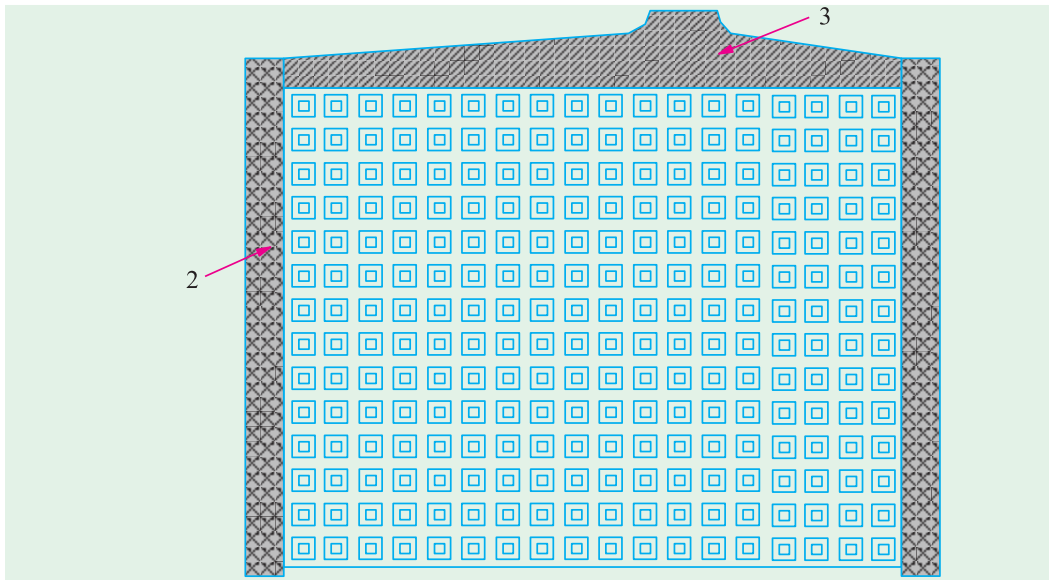


Fig. 9.29

Another alkaline battery which differs from the above only in the mechanical details of its plates is known as Alkum battery which uses nickel hydroxide and graphite in the positive plates and a powdered alloy of iron and chromium in the negative plates.

Silver-zinc type of alkaline batteries are also made whose active material for the positive plate is silver oxide ( $\text{Ag}_2\text{O}$ ) and for negative plate is zinc oxide and zinc powder. The electrodes or plates of the alkaline cells are designed to be either of the enclosed-pocket type or open-pocket type. In the case of enclosed-pocket type plates, the active material is inside perforated metal envelopes whereas in the other type, the active material is outside directly in contact with the electrolyte. As shown in Fig. 9.29, the active material of the enclosed-pocket type plates is enclosed in nickel-plated perforated steel pockets or packs which are pressed into the steel frames of the plates.

The open-pocket type plates are made of the following three materials :

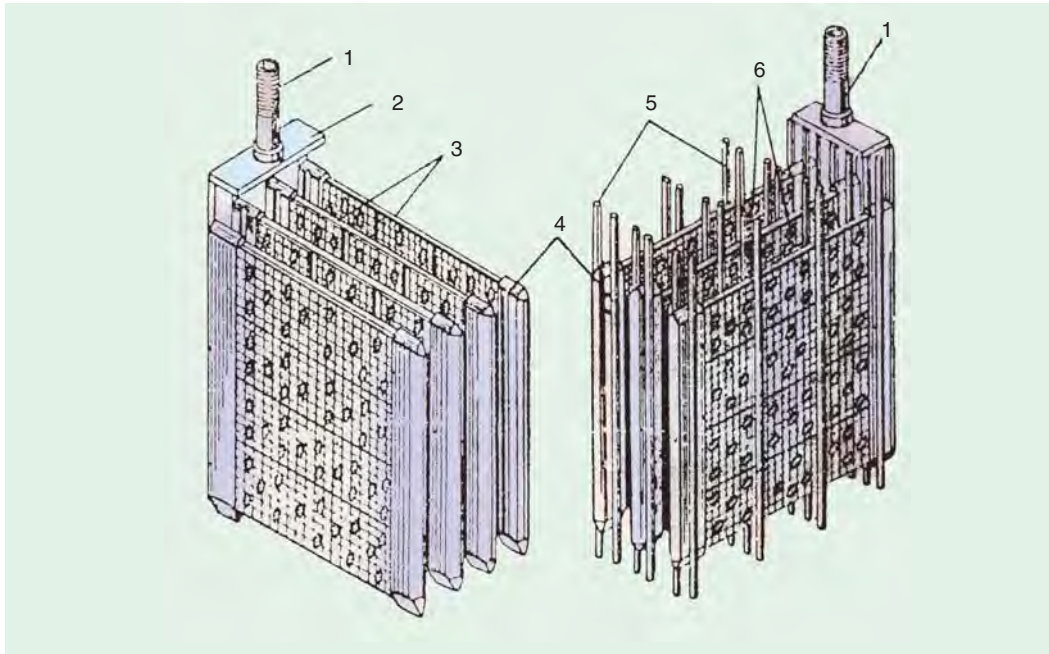
- (i) metal-ceramic plate—the frame of the plate is a nickel-plated steel grid with the active material pressed in under a pressure of 800 to 1900  $\text{kg}/\text{cm}^2$ .
- (ii) foil plate—the base of such a plate is a thin nickel foil coated with a layer of nickel suspension deposited by a spray technique.
- (iii) pressed plates—the base member of these plates is a nickel-plated pressed steel grid. The active material is pressed into them at a pressure of about 400  $\text{kg}/\text{cm}^2$ .

### 9.38. Nickel-iron and Edison Batteries

There is revived interest in the nickel-iron battery because it seems to be one of the few systems which may be developed into a high-energy density battery for electric vehicles. Since long the two main designs for this battery have been the tubular positive type and the flat pocket plate type although cells with sintered type negative are also being manufactured.

The active materials in a nickel-iron cell are :

- (i) Nickel hydroxide  $\text{Ni}(\text{OH})_2$  or apple green nickel peroxide  $\text{NiO}_2$  for the positive plate. About 17 per cent of graphite is added to increase conductivity. It also contains an activating additive barium hydroxide which is about 2 per cent of the active material. This additive increases the service life of the plates.



**Fig. 9.30** Plates groups of an alkaline cell

(a) + ve group (b) – ve group, 1–terminal post 2–connecting strap, 3–plates.  
4–plates side members. 5–ebonite spacer sticks. 6–pockets.

(ii) powdered iron and its oxides for the negative plate. Small quantities of nickel sulphate and ferrous sulphide are added to improve the performance of the coil.

(iii) the electrolyte is 21 per cent solution of caustic potash KOH (potassium hydrate) to which is added a small quantity of lithium hydrate LiOH for increasing the capacity of the cell.

As shown in Fig. 9.30, plates of the same polarity with their pockets filled, are assembled into cells groups for which purpose they are welded to a common strap having a threaded post.

The number of negative plates is one more than the positive plates. The extreme negative plates are electrically connected to the container. Ebonite separating sticks are placed between the positive and negative plates to prevent any short-circuiting.

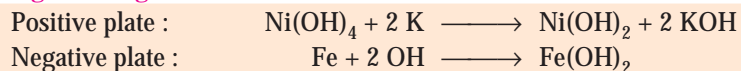
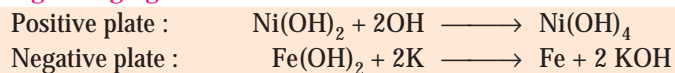
The steel containers of the batteries are press-formed from steel and the joints are welded. The body and the cover are nickel-plated and have a dull finish. However, it should be kept in mind that since these containers are electrically alive, no loose wires should touch them owing to the danger of severe sparking from short-circuits.

### 9.39. Chemical Changes

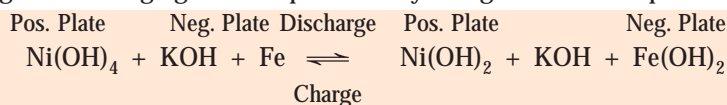
The exact nature of the chemical changes taken place in such a cell is not clearly understood because the exact formula for the nickel oxide is not yet well established but the action of the cell can be understood by assuming the peroxide  $\text{NiO}_2$  or its hydrated form  $\text{Ni(OH)}_4$ .

First, let us assume that at positive plate, nickel oxide is in its hydrate form  $\text{Ni(OH)}_4$ . During discharge, electrolyte KOH splits up into positive K ions and negative OH ions. The K ions go to anode and reduce  $\text{Ni(OH)}_4$  to  $\text{Ni(OH)}_2$ . The OH ions travel towards the cathode and oxidise iron. During charging, just the opposite reactions take place *i.e.* K ions go to cathode and OH ions go to anode. The chemical reactions can be written thus :



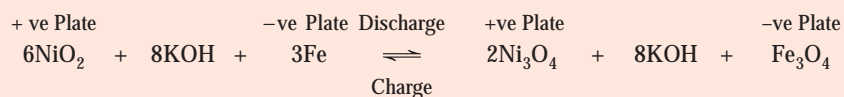
**During discharge****During Charging**

The charging and discharging can be represented by a single reversible equation thus :



It will be observed from the above equation that as no water is formed, there is no overall change in the strength of the electrolyte. Its function is merely to serve as a conductor or as a vehicle for the transfer of OH ions from one plate to another. Hence, the specific gravity of the electrolyte remains practically constant, both during charging and discharging. That is why only a small amount of electrolyte is required which fact enables the cells to be small in bulk.

**Note.** If, however, we assume the nickel oxide to be in the form  $\text{NiO}_2$ , then the above reactions can be represented by the following reversible equation :

**9.40. Electrical Characteristics**

The e.m.f. of an Edison cell, when fully charged, is nearly 1.4 V which decreases rapidly to 1.3 V and then try slowly on 1.1 or 1.0 V on discharge. The average discharge voltage for a 5-hour discharge rate is 1.2 V. Hence, for the same average value of the voltage, an alkali accumulator will consist of 1.6 to 1.7 times as many cells as in a lead-acid battery. Internal resistance of an alkali cell is nearly five times that of the lead-acid cell, hence there is a relatively greater difference between its terminal voltage when charging and discharging.

The average charging voltage for an alkali cell is about 1.7 V. The general shapes of the charge and discharge curves for such cells are, however, similar to those for lead-acid cells. The rated capacity of nickel accumulators usually refers to either 5-hour or 8-hour discharge rate unless stated otherwise.

The plates of such cells have greater mechanical strength because of all-steel construction. They are comparatively lighter because (i) their plates are lighter and (ii) they require less quantity of electrolyte. They can withstand heavy charge and discharge currents and do not deteriorate even if left discharged for long periods.

Due to its relatively higher internal resistance, the efficiencies of an Edison cell are power than those of the lead acid cell. On the average, its Ah efficiency is about 80% and Wh efficiency 60 or 50%. It has an average density of 50 Wh/kg.

With increase in temperature, e.m.f. is increased slightly but capacity increases by an appreciable amount. With decrease in temperature, the capacity decreases becoming practically zero at  $4^\circ\text{C}$  even through the cell is fully charged. This is serious drawback in the back in the case of electrically driven vehicles in cold weather and previous have to be taken to heat up the battery before starting, though, in practice, the  $I^2 R$  loss in the internal resistance of the battery is sufficient to keep the battery cells warm when running.

The principal disadvantage of the Edison battery on nickel-iron battery is its high initial cost (which will probably be sufficiently reduced when patents expire). At present, an Edison battery costs approximately twice as much as a lead-acid battery designed for similar service. But since the alkaline battery outlasts an indeterminate number of lead-acid batteries, it is cheaper in the end.

Because of their lightness, compact construction, increased mechanical strength, ability to withstand rapid charging and discharging without injury and freedom from corrosive liquids and fumes, alkaline batteries are ideally suited for traction work such as propulsion of electric factory trucks, mine locomotives, miner's lamps, lighting and starting of public service vehicles and other services involving rough usage etc.

### 9.41. Nickel-Cadmium Batteries

The reactive materials in a nickel-cadmium cell (Fig. 9.31) are :

(i)  $\text{Ni(OH)}_4$  for the positive plate exactly as in the nickel-iron cell.

(ii) a mixture of cadmium or cadmium oxide and iron mass to which is added about 3 per cent of solar oil for stabilizing the electrode capacity. The use of cadmium results in reduced internal resistance of the cell.

(iii) the electrolyte is the same as in the nickel-iron cell.

The cell grouping and plate arrangement is identical with nickel-iron batteries except that the number of positive plates is more than the negative plates. Such batteries are more suitable than nickel-iron batteries for floating duties in conjunction with a charging dynamo because, in their case, the difference between charging and discharging e.m.f.s is not as great as in nickel-iron batteries.

Nickel-cadmium sintered plate batteries were first manufactured by Germans for military aircrafts and rockets. Presently, they are available in a variety of designs and sizes and have energy density going upto 55 Wh/kg. Their capacity is less affected by high discharge rates and low operating temperature than any other rechargeable batteries. Since such batteries have very low open-circuit losses, they are well-suited for pleasure yatches and launches which may be laid up for long periods. They are also used in commercial airliners, military aeroplanes and helicopters for starting main engines or auxiliary turbines and for emergency power supply.

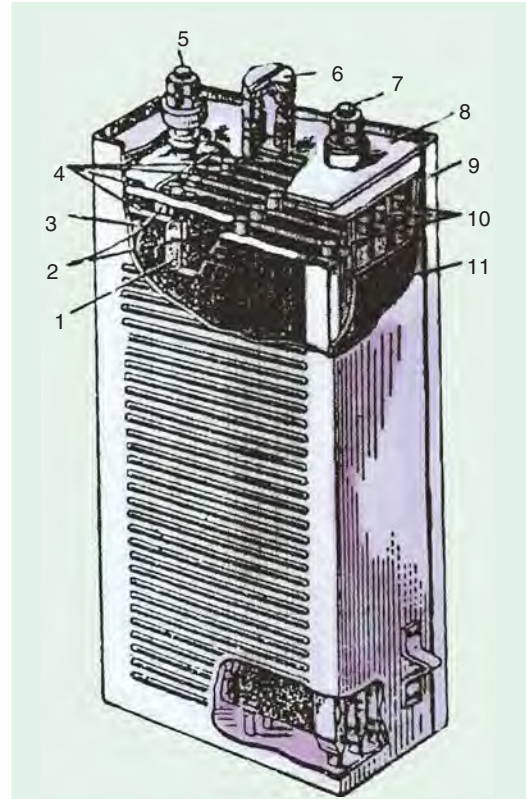


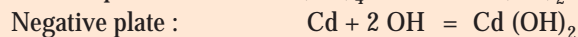
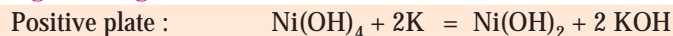
Fig. 9.31. Parts of Nickel-cadium alkaline cell.

- 1-active material 2-ebonite spacer sticks 3-pocket element 4-positive plates 5-positive terminal post 6-vent plug 7-negative terminal post 8-cover 9-container 10-negative plates 11-ebonite plate.

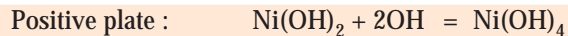
### 9.42. Chemical Changes

The chemical changes are more or less similar to those taking place in nickel-iron cell. As before, the electrolyte is split up into positive K ions and negative OH ions. The chemical reactions at the two plates are as under :

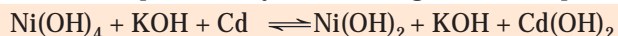
#### During discharge



#### During Charging



The above reaction can be represented by the following reversible equation :



### 9.43. Comparison : Lead-acid and Edison Cells

The relative strong and weak points of the cells have been summarised below :

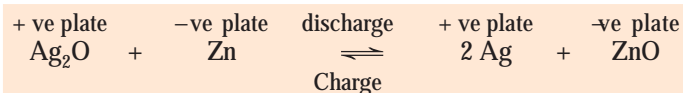
Particulars	Lead-acid cell	Edison cell
1. Positive Plate	PbO <sub>2</sub> lead peroxide	Nickel hydroxide Ni(OH) <sub>2</sub> or NiO <sub>2</sub>
2. Negative Plate	Sponge lead	Iron
3. Electrolyte	diluted H <sub>2</sub> SO <sub>4</sub>	KOH
4. Average e.m.f.	2.0 V/cell	1.2 V/cell
5. Internal resistance	Comparatively low	Comparatively higher
6. Efficiency :	90–95%	nearly 80%
amp-hour watt-hour	72–80%	about 60%
7. Cost	Comparatively less than alkaline line cell	almost twice that of Pb-acid cell
8. Life	gives nearly 1250 charges and discharges	Easy maintenance five years at least
9. Strength	Needs much care and maintenance. Sulphation occurs often due to incomplete charge of discharge.	robust, mechanically strong, can withstand vibration, is light, unlimited rates of charge and discharge. Can be left discharged, free from corrosive liquids and fumes.

Moreover, as compared to lead-acid, the alkaline cells operate much better at low temperature, do not emit obnoxious fumes, have very small self-discharge and their plates do not buckle or smell.

### 9.44. Silver-Zinc Batteries

The active material of the positive plates is silver oxide which is pressed into the plate and then subjected to a heat treatment. The active material of the negative plates is a mixture of zinc powder and zinc oxide.

The chemical changes taking place within the cell can be represented by the following single equation :



A silver-zinc cell has a specific capacity (*i.e.* capacity per unit weight) 4 to 5 times greater than that of other type of cells. Their ground applications are mainly military *i.e.* communications equipment, portable radar sets and night-vision equipment. Moreover, comparatively speaking, their efficiency is high and self-discharge is small. Silver-zinc batteries can withstand much heavier discharge currents than are permissible for other types and can operate over a temperature range of  $-20^\circ\text{C}$  to  $+60^\circ\text{C}$ . Hence, they are used in heavy – weight torpedoes and for submarine propulsion. It has energy density of 150 Wh/kg. Its life time in wet condition is 1-2 years and the dry storage life is upto 5 years. However, the only disadvantage of silver-zinc battery or cell is its higher cost.

### 9.45. High Temperature Batteries

It is a new group of source which requires operating temperatures above the ambient. They possess the advantages of high specific energy and power coupled with low cost. They are particularly suitable for vehicle traction and load levelling purpose in the electric supply industry. We will briefly describe the following cell from which high-temperature batteries are made.

### 1. Lithium/Chlorine Cell

It has an emf of 3.5 V, a theoretical specific energy of 2200 Wh/kg at 614°C and operating temperature of 650°C.

### 2. Lithium/Sulphur Cell

It has an emf of 2.25 V, specific energy of 2625 Wh/kg and an operating temperature of 365°C.

### 3. Lithium-Aluminium/Iron-Sulphide Cells

The emf of these cells is 1.3 V and a theoretical specific energy of 450 Wh/kg.

### 4. Sodium/Sulphur Cells

It utilises liquid sodium as negative electrode and sulphur as positive electrode and employs polycrystalline beta alumina as solid electrolyte. It was conceived in the 1960s by J.T. Kummer and N. Weber. The cell reaction can be written as  $2 \text{Na} + 3 \text{S} = \text{Na}_2\text{S}_3$ . The announcement of sodium/sulphur battery based on beta alumina was made by Ford Motor Company of USA in 1966. The open-circuit voltage of the cell is 2.1 V and it has a specific energy of 750 Wh/kg with an operating temperature of 350°C. The two unique features of this cell are (1) a Faradaic efficiency of 100% and an ampere-hour capacity which is invariant with discharge rate and (2) high self-life (which is critical for certain space applications).

## 9.46. Secondary Hybrid Cells

A hybrid cell may be defined as a galvanic electrochemical generator in which one of the active reagents is in the gaseous state *i.e.* the oxygen of the air. Such cells take advantage of both battery and fuel cell technology. Examples of such cells are :

### 1. Metal-air cells such as iron oxygen and zinc oxygen cells.

The Zn/O<sub>2</sub> cell has an open-circuit voltage of 1.65 V and a theoretical energy density of 1090 Wh/kg. The Fe/O<sub>2</sub> cell has an OCV of 1.27 V and energy density of 970 Wh/kg.

### 2. Metal-halogen cells such as zinc-chlorine and zinc-bromine cells.

The zinc-chlorine cell has an OCV of 2.12 V at 25°C and a theoretical energy density of 100 Wh/kg. Such batteries are being developed for EV and load levelling applications. The zinc-bromine cell has an OCV of 1.83 V at 25°C and energy density of 400 Wh/kg.

### 3. Metal-hydrogen cells such as nickel-hydrogen cell.

Such cells have an OCV of 1.4 V and a specific energy of about 65 Wh/kg. Nickel-hydrogen batteries have captured large share of the space battery market in recent years and are rapidly replacing Nickel/cadmium batteries as the energy storage system of choice. They are acceptable for geosynchronous orbit applications where not many cycles are required over the life of the system (1000 cycles, 10 years).

The impetus for research and development of metal-air cells has arisen from possible EV applications where energy density is a critical parameter. An interesting application suggested for a secondary zinc-oxygen battery is for energy storage on-board space craft where the cell could be installed inside one of the oxygen tanks thereby eliminating need for gas supply pipes and valves etc. These cells could be recharged using solar converters.

Some of the likely future developments for nickel-hydrogen batteries are (1) increase in cycle life for low earth orbit applications upto 40,000 cycles (7 years) (2) increase in the specific energy upto 100 W/kg for geosynchronous orbit applications and (3) development of a bipolar nickel-hydrogen battery for high pulse power applications.

## 9.47. Fuel Cells

As discussed earlier, a secondary battery produces electric current by oxidation-reduction chemical reaction. Similar chemical reactions take place in fuel cells but there is a basic difference between the two. Whereas in secondary batteries the



Fuel Cells

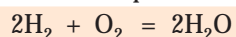


chemical energy is stored in the positive and negative electrodes, in fuel cells the oxidant and the fuel are stored outside the cells and must be fed to the electrodes continuously during the time the fuel cell supplies electric current. This gives an advantage to the fuel cells over the storage battery because fuels can be quickly replenished which is similar to filling up to the petrol tank of a car. Moreover, storage batteries when fully discharged take several hours to be recharged.

### 9.48. Hydrogen-Oxygen Fuel Cells

The first fuel battery was designed by F.T. Bacon in 1959. The construction of a simple fuel cell is shown in Fig. 9.32. The electrodes are made from sintered nickel plates having a coarse pore surface and a fine pore surface, the two surfaces being for gas and electrolyte respectively. The electrolyte used in KOH is of about 85 per cent concentration. The water vapour formed as a byproduct of the reaction is removed by condensation from the stream of hydrogen passing over the back of the fuel.

The two electrodes of the fuel cell are fed with a continuous stream of hydrogen and oxygen (or air) as shown. The oxygen and hydrogen ions react with the potassium hydroxide electrolyte at the surface of the electrodes and produce water. The overall cell reaction is



The basic reaction taking place in the cells are shown in more details in the Fig. 9.32.

Fuel cell batteries have been used in the manned Apollo space mission for on-board power supply and also for power supply in unmanned satellites and space probes. These batteries have also been used for tractors, fork-lift trucks and golf carts etc. Research is being carried out to run these batteries with natural gas and alcohol. Fuel cell systems are particularly useful where electrical energy is required for long periods. Such applications include (1) road and rail traction (2) industrial trucks (3) naval craft and submarine (4) navigational aids and radio repeater stations etc.

### 9.49. Batteries for Aircraft

The on-board power requirements in aircraft have undergone many changes during the last three or four decades. The jet engines of the aircraft which require starting currents of about 1000 A, impose a heavy burden on the batteries. However, these days this load is provided by small turbogenerator sets and since batteries are needed only to start them, the power required is much less. These batteries possess good high-rate capabilities in order to supply emergency power for upto 1 h in the event of the generator failure. However, their main service is as a standby power for miscellaneous on-board equipment. Usually, batteries having 12 cells (of a nominal voltage of 24 V) with capacities of 18 and 34 Ah at the 10 h rate are used. In order to reduce weight, only light-weight high-impact polystyrene containers and covers are used and the cells are fitted with non-spill ventplugs to ensure complete unspillability in any aircraft position during aerobatics. Similarly, special plastic manifolds are moulded into the covers to provide outlet for gases evolved during cycling.

### 9.50. Batteries for Submarines

These batteries are the largest units in the traction service. In older types of submarines, the lead storage battery was the sole means of propulsion when the submarine was fully submerged and, additionally supplied the 'hotel load' power for lights, instruments and other electric equipment. When the introduction of the snorkel breathing tube made it possible to use diesel engines for propulsion, battery was kept in reserve for emergency use only. Even modern nuclear-powered submarines use storage batteries for this purpose. These lead-acid batteries may be flat, pasted plate or tubular

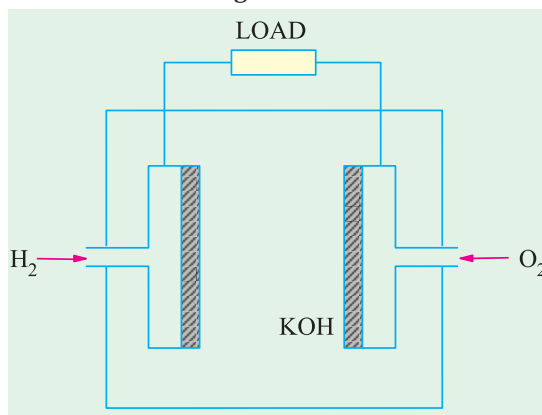


Fig. 9.32

positive plate type with 5 h capacities ranging from 10,000 to 12,000 Ah. One critical requirement for this service is that the rate of evolution of hydrogen gas on open-circuit should not exceed the specified low limit.

Double plate separation with the help of felted glass fibre mats and microscoporous separators is used in order to ensure durability, high performance and low standing losses.

### OBJECTIVE TESTS – 9

1. Active materials of a lead-acid cell are :  
 (a) lead peroxide (b) sponge lead  
 (c) dilute sulphuric acid (d) all the above
2. During the charging of a lead-acid cell :  
 (a) its cathode becomes dark chocolate brown in colour  
 (b) its voltage increases  
 (c) it gives out energy  
 (d) specific gravity of  $H_2SO_4$  is decreased
3. The ratio of Ah efficiency to Wh efficiency of a lead-acid cell is :  
 (a) always less than one (b) just one  
 (c) always greater than one  
 (d) either (a) or (b)
4. The capacity of a cell is measured in :  
 (a) watt-hours (b) watts  
 (c) amperes (d) ampere-hours
5. The capacity of a lead-acid cell does NOT depend on its :  
 (a) rate of charge  
 (b) rate of discharge  
 (c) temperature  
 (d) quantity of active material
6. As compared to constant-current system, the constant-voltage system of charging a lead-acid cell has the advantage of :  
 (a) avoiding excessive gassing  
 (b) reducing time of charging  
 (c) increasing cell capacity  
 (d) both (b) and (c).
7. Sulphation in a lead-acid battery occurs due to:  
 (a) trickle charging  
 (b) incomplete charging  
 (c) heavy discharging (d) fast charging
8. The active materials of a nickel-iron battery are.  
 (a) nickel hydroxide  
 (b) powdered iron and its oxides  
 (c) 21% solution of caustic potash  
 (d) all of the above.
9. During the charging and discharging of a nickel iron cell :  
 (a) its e.m.f. remains constant  
 (b) water is neither formed nor absorbed  
 (c) corrosive fumes are produced  
 (d) nickel hydroxide remains unsplit
10. As compared to a lead-acid cell, the efficiency of a nickel-iron cell is less due to its :  
 (a) lower e.m.f.  
 (b) smaller quantity of electrolyte used  
 (c) higher internal resistance  
 (d) compactness.
11. Trickle charging of a storage battery helps to :  
 (a) prevent sulphation  
 (b) keep it fresh and fully charged  
 (c) maintain proper electrolyte level  
 (d) increase its reserve capacity
12. A dead storage battery can be revived by :  
 (a) a dose of  $H_2SO_4$   
 (b) adding so-called battery restorer  
 (c) adding distilled water  
 (d) none of the above
13. The sediment which accumulates at the bottom of a lead-acid battery consists largely of :  
 (a) lead-peroxide (b) lead-sulphate  
 (c) antimony-lead alloy (d) graphite
14. The reduction of battery capacity at high rates of discharge is primarily due to :  
 (a) increase in its internal resistance  
 (b) decrease in its terminal voltage  
 (c) rapid formation of  $PbSO_4$  on the plates  
 (d) non-diffusion of acid to the inside active materials.
15. Floating battery systems are widely used for :  
 (a) power stations  
 (b) emergency lighting  
 (c) telephone exchange installation  
 (d) all of the above
16. Any charge given to the battery when taken off the vehicle is called :  
 (a) bench charge (b) step charge  
 (c) float charge (d) trickle charge

### ANSWERS

1. d 2. b 3. c 4. d 5. a 6. d 7. b 8. d 9. b 10. c 11. b 12. d 13. c 14. c 15. d 16. a

# CHAPTER 10

## Learning Objectives

- Absolute and Secondary Instruments
- Deflecting Torque
- Controlling Torque
- Damping Torque
- Moving-iron Ammeters and Voltmeters
- Moving-coil Instruments
- Permanent Magnet Type Instruments
- Voltmeter Sensitivity
- Electrodynamic or Dynamometer Type Instruments
- Hot-wire Instruments
- Megger
- Induction Voltmeter
- Wattmeters
- Energy Meters
- Electrolytic Meter
- Ampere-hour Mercury Motor Meter
- Friction Compensation
- Commutator Motor Meters
- Ballistic Galvanometer
- Vibration Galvanometer
- Vibrating-reed Frequency Meter
- Electrodynamic Frequency Meter
- Moving-iron Frequency Meter
- Electrodynamic Power Factor Meter
- Moving-iron Power Factor Meter
- Nalder-Lipman Moving-iron Power Factor Meter
- D.C. Potentiometer
- A.C. Potentiometers
- Instrument Transformers
- Potential Transformers.

## ELECTRICAL INSTRUMENTS AND MEASUREMENTS



Electrical instruments help us to measure the changes in variables such as voltage, current and resistance

### 10.1. Absolute and Secondary Instruments

The various electrical instruments may, in a very broad sense, be divided into *(i) absolute* instruments and *(ii) secondary instruments*. **Absolute instruments are those which give the value of the quantity to be measured, in terms of the constants of the instrument and their deflection only.** No previous calibration or comparison is necessary in their case. The example of such an instrument is tangent galvanometer, which gives the value of current, in terms of the tangent of deflection produced by the current, the radius and number of turns of wire used and the horizontal component of earth's field.

Secondary instruments are those, in which the value of electrical quantity to be measured can be determined from the deflection of the instruments, only when they have been pre-calibrated by comparison with an absolute instrument. Without calibration, the deflection of such instruments is meaningless.

It is the secondary instruments, which are most generally used in everyday work; the use of the absolute instruments being merely confined within laboratories, as standardizing instruments.



An absolute instrument

### 10.2. Electrical Principles of Operation

All electrical measuring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are :

1. Magnetic effect - for ammeters and voltmeters usually.
2. Electrodynamical effect - for ammeters and voltmeters usually.
3. Electromagnetic effect - for ammeters, voltmeters, wattmeters and watt-hour meters.
4. Thermal effect - for ammeters and voltmeters.
5. Chemical effect - for d.c. ampere-hour meters.
6. Electrostatic effect - for voltmeters only.

Another way to classify secondary instruments is to divide them into *(i) indicating instruments*, *(ii) recording instruments* and *(iii) integrating instruments*.

Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured *at the time* at which it is being measured. Their indications are given by pointers moving over calibrated dials. Ordinary ammeters, voltmeters and wattmeters belong to this class.

Recording instruments are those, which, instead of indicating by means of a pointer and a scale the instantaneous value of an electrical quantity, give a **continuous record** or the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked pen which rests lightly on a chart or graph, that is moved at a uniform and low speed, in a direction perpendicular to that of the deflection of the pen. The path traced out by the pen presents a continuous record of the variations in the deflection of the instrument.

Integrating instruments are those which measure and register by a set of dials and pointers either the **total** quantity of electricity (in amp-hours) or the **total** amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time. This summation gives the product of time and the electrical quantity but gives no direct indication as to the **rate** at which the quantity or energy is being supplied because their registrations are independent of this rate provided the current flowing through the instrument is sufficient to operate it.

Ampere-hour and watt-hour meters fall in this class.

### 10.3. Essentials of Indicating Instruments

As defined above, indicating instruments are those which indicate the value of the quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale and which is attached to a moving system pivoted in jewelled bearings. The moving system is subjected to the following three torques :

1. A deflecting (or operating) torque
2. A controlling (or restoring) torque
3. A damping torque.

### 10.4. Deflecting Torque

The deflecting or operating torque ( $T_d$ ) is produced by utilizing one or other effects mentioned in Art. 10.2 *i.e.* magnetic, electrostatic, electrodynamic, thermal or inductive etc. The actual method of torque production depends on the type of instrument and will be discussed in the succeeding paragraphs. This deflecting torque causes the moving system (and hence the pointer attached to it) to move from its 'zero' position *i.e.* its position when the instrument is disconnected from the supply.

### 10.5. Controlling Torque

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque opposes the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques are equal. The deflecting torque ensures that currents of different magnitudes shall produce deflections of the moving system in proportion to their size. Without such a torque, the pointer would swing over to the maximum deflected position irrespective of the magnitude of the current to be measured. Moreover, in the absence of a restoring torque, the pointer once deflected, would not return to its zero position on removing the current. The controlling or restoring or balancing torque in indicating instruments is obtained either by a spring or by gravity as described below :

#### (a) Spring Control

A hair-spring, usually of phosphor-bronze, is attached to the moving system of the instrument as shown in Fig. 10.1 (a).

With the deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring produces restoring torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to a position of rest (or equilibrium) when the deflecting torque ( $T_d$ ) and controlling torque ( $T_c$ ) are equal. For example, in permanent-magnet, moving-coil type of instruments, the deflecting torque is proportional to the current passing through them.

$$\therefore T_d \propto I$$

and for spring control  $T_c \propto \theta$

$$\text{As } T_c = T_d$$

$$\therefore \theta \propto I$$

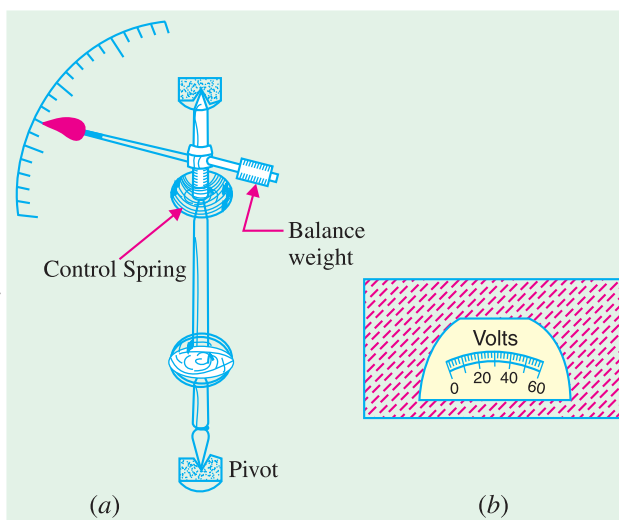


Fig. 10.1

Since deflection  $\theta$  is directly proportional to current  $I$ , the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range as shown in Fig. 10.1 (b).

To ensure that controlling torque is proportional to the angle of deflection, the spring should have a fairly large number of turns so that angular deformation per unit length, on full-scale deflection, is small. Moreover, the stress in the spring should be restricted to such a value that it does not produce a permanent set in it.

Springs are made of such materials which

- (i) are non-magnetic
- (ii) are not subject to much fatigue
- (iii) have low specific resistance-especially in cases where they are used for leading current in or out of the instrument
- (iv) have low temperature-resistance coefficient.

The exact expression for controlling torque is  $T_c = C\theta$  where  $C$  is *spring constant*. Its value is given by  $C = \frac{Ebt^3}{L}$  N-m/rad. The angle  $\theta$  is in radians.

### (b) Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system such that the two exert torques in the opposite directions. The usual arrangements is shown in Fig. 10.2(a).

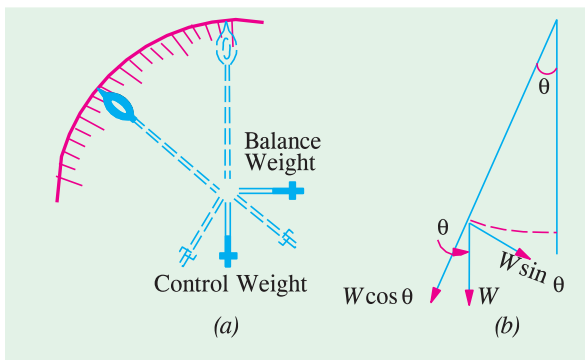


Fig. 10.2

Fig. 10.2(a).

It is seen from Fig. 10.2 (b) that the controlling or restoring torque is proportional to the sine of the angle of deflection *i.e.*

$$T_c \propto \sin \theta$$

The degree of control is adjusted by screwing the weight up or down the carrying system

It  $T_d \propto I$

then for position of rest

$$T_d = T_c$$

or  $I \propto \sin \theta$  (not  $\theta$ )

It will be seen from Fig. 10.2 (b) that as  $\theta$  approaches  $90^\circ$ , the distance  $AB$  increases by a relatively small amount for a given change in the angle than when  $\theta$  is just increasing from its zero value. Hence, gravity-controlled instruments have scales which are not uniform but are cramped or crowded at their lower ends as shown in Fig. 10.3.

As compared to spring control, the disadvantages of gravity control are :

1. it gives cramped scale
2. the instrument has to be kept vertical.

However, gravity control has the following advantages :

1. it is cheap
2. it is unaffected by temperature
3. it is not subjected to fatigue or deterioration with time.

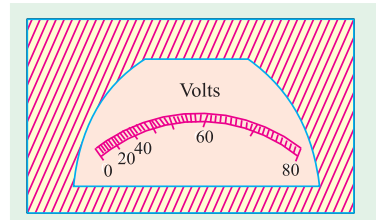


Fig. 10.3

**Example 10.1** The torque of an ammeter varies as the square of the current through it. If a current of 5 A produces a deflection of  $90^\circ$ , what deflection will occur for a current of 3 A when the instrument is (i) spring-controlled and (ii) gravity-controlled.

(Elect. Meas. Inst and Meas. Jadavpur Univ.)

**Solution.** Since deflecting torque varies as (current)<sup>2</sup>, we have  $T_d \propto I^2$

For spring control,  $T_c \propto \theta \therefore \theta \propto I^2$

For gravity control,  $T_c \propto \sin \theta \therefore \sin \theta \propto I^2$

(i) For spring control  $90^\circ \propto 5^2$  and  $\theta \propto 3^2$ ;  $\theta = 90^\circ \times 3^2/5^2 = 32.4^\circ$

(ii) For gravity control  $\sin 90^\circ \propto 5^2$  and  $\sin \theta \propto 3^2$

$\sin \theta = 9/25 = 0.36$ ;  $\theta = \sin^{-1}(0.36) = 21.1^\circ$ .

## 10.6. Damping Torque

A damping force is one which acts on the moving system of the instrument **only when it is moving** and always opposes its motion. Such stabilizing or damping force is necessary to bring the pointer to rest **quickly**, otherwise due to inertia of the moving system, the pointer will oscillate about its final deflected position for quite some time before coming to rest in the steady position. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be **dead-beat**. Any increase of damping above this limit *i.e.* overdamping will make the instruments slow and lethargic. In Fig. 10.4 is shown the effect of damping on the variation of position with time of the moving system of an instrument.

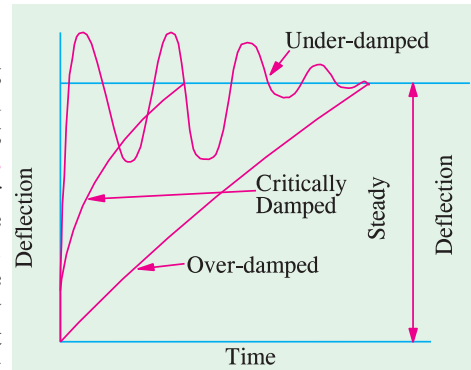


Fig. 10.4

In Fig. 10.4 is shown the effect of damping on the variation of position with time of the moving system of an instrument.

The damping force can be produced by (i) **air frictions** (ii) **eddy currents** and (iii) **fluid friction** (used occasionally).

Two methods of air-friction damping are shown in Fig. 10.5 (a) and 10.5 (b). In Fig. 10.5 (a), the light aluminium piston attached to the moving system of the instrument is arranged to travel with

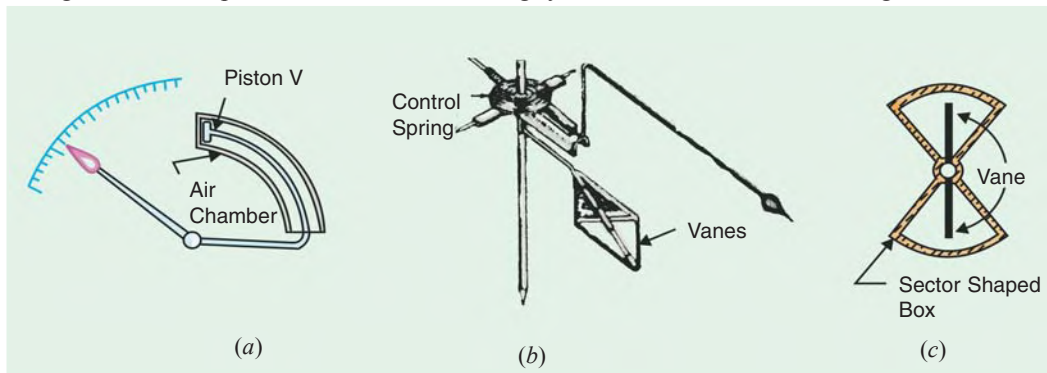


Fig. 10.5

a very small clearance in a fixed air chamber closed at one end. The cross-section of the chamber is either circular or rectangular. Damping of the oscillation is affected by the compression and suction actions of the piston on the air enclosed in the chamber. Such a system of damping is not much favoured these days, those shown in Fig. 10.5 (b) and (c) being preferred. In the latter method, one or two light aluminium vanes are mounted on the spindle of the moving system which move in a closed sector-shaped box as shown.

Fluid-friction is similar in action to the air friction. Due to greater viscosity of oil, the damping is more effective. However, oil damping is not much used because of several disadvantages such as objectionable creeping of oil, the necessity of using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The eddy-current form of damping is the most efficient of the three. The two forms of such a damping are shown in Fig. 10.6 and 10.7. In Fig. 10.6 (a) is shown a thin disc of a conducting but **non-magnetic** material like copper or aluminium mounted on the spindle which carries the moving system and the pointer of the instrument. The disc is so positioned that its edges, when in rotation, cut the magnetic flux between the poles of a permanent magnet. Hence, eddy currents are produced in the disc which flow and so produce a damping force in such a direction as to oppose the very cause producing them (Lenz's Law Art. 7.5). Since the cause producing them is the rotation of the disc, these eddy current retard the motion of the disc and the moving system as a whole.

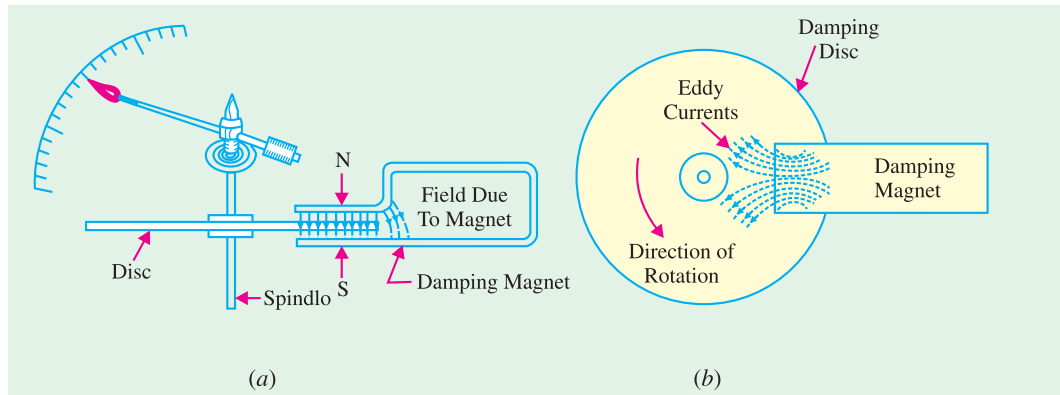


Fig. 10.6

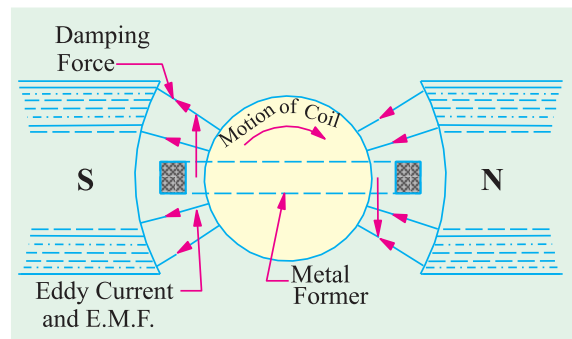


Fig. 10.7

In Fig. 10.7 is shown the second type of eddy-current damping generally employed in permanent-magnet moving coil instruments. The coil is wound on a thin light aluminium former in which eddy currents are produced when the coil moves in the field of the permanent magnet. The directions of the induced currents and of the damping force produced by them are shown in the figure.

The various types of instruments and the order in which they would be discussed in this chapter are given below.

### Ammeters and voltmeters

1. Moving-iron type (both for D.C./A.C.)
  - (a) **the attraction type**
  - (b) **the repulsion type**
2. Moving-coil type
  - (a) **permanent-magnet type** (for D.C. only)
  - (b) **electrodynamical or dynamometer type** (for D.C./A.C.)
3. Hot-wire type (both for D.C./A.C.)
4. Induction type (for A.C. only)
  - (a) **Split-phase type**
  - (b) **Shaded-pole type**
5. Electrostatic type-for voltmeters only (for D.C./A.C.)



## Wattmeter

6. **Dynamometer type** (both for D.C./A.C.),
7. **Induction type** (for A.C. only)
8. **Electrostatic type** (for D.C. only)

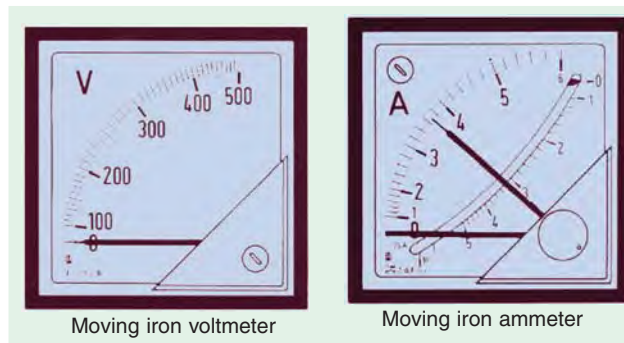
## Energy Meters

9. **Electrolytic type** (for D.C. only)
10. **Motor Meters**
  - (i) **Mercury Motor Meter:** For d.c. work only. Can be used as amp-hour or watt-hour meter.
  - (ii) **Commutator Motor Meter:** Used on D.C./A.C. Can be used as Ah or Wh meter.
  - (iii) **Induction type.** For A.C. only.
11. Clock meters (as Wh-meters).

## 10.7. Moving-iron Ammeters and Voltmeters

There are two basic forms of these instruments *i.e.* the **attraction** type and the **repulsion** type. The operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field and that of repulsion type depends on the repulsion of two adjacent pieces of iron magnetised by the same magnetic field. For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil.

In case the instrument is to be used as an ammeter, the coil has comparatively fewer turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit. In case it is to be used as a voltmeter, the coil has high impedance so as to draw as small a current as possible since it is connected in parallel with the circuit. As the current through the coil is small, it has large number of turns in order to produce sufficient ampere-turns.



Moving iron voltmeter

Moving iron ammeter

## 10.8. Attraction Type M.I. Instruments

The basic working principle of an attraction-type moving-iron instrument is illustrated in Fig. 10.8. It is well-known that if a piece of an unmagnetised soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet. Hence, if we pivot an oval-shaped disc of soft iron on a spindle between bearings near the coil (Fig. 10.8), the iron disc will swing into the coil when the latter has an electric current passing through it. As the field strength would be strongest at the centre of

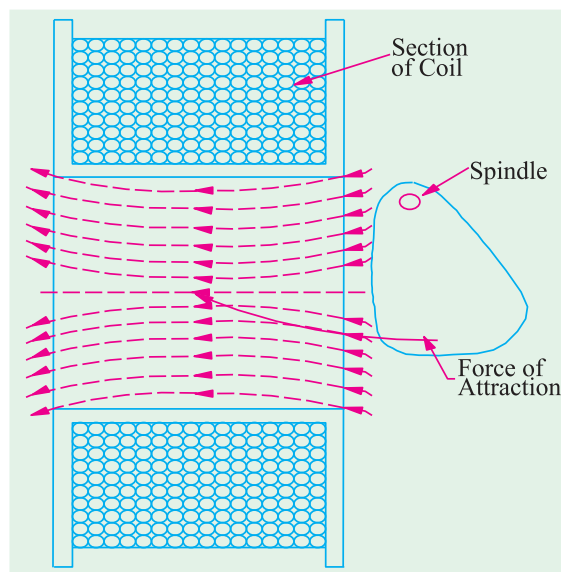


Fig. 10.8

the coil, the ovalshaped iron disc is pivoted in such a way that the greatest bulk of iron moves into the centre of the coil. If a pointer is fixed to the spindle carrying the disc, then the passage of current through the coil will cause the pointer to deflect. The amount of deflection produced would be greater when the current producing the magnetic field is greater. Another point worth noting is that **whatever the direction of current through the coil, the iron disc would always be magnetised in such a way that it is pulled inwards**. Hence, such instruments can be used both for direct as well as alternating currents.

A sectional view of the actual instrument is shown in Fig. 10.9. When the current to be measured is passed through the coil or solenoid, a magnetic field is produced, which attracts the eccentrically-mounted disc inwards, thereby deflecting the pointer, which moves over a calibrated scale.

### Deflecting Torque

Let the axis of the iron disc, when in zero position, subtend an angle of  $\phi$  with a direction perpendicular to the direction of the field  $H$  produced by the coil. Let the deflection produced be  $\theta$  corresponding to a current  $I$  through the coil. The magnetisation of iron disc is proportional to the component of  $H$  acting along the axis of the disc *i.e.* proportional to  $H \cos [90 - (\phi + \theta)]$  or  $H \sin (\theta + \phi)$ . The force  $F$  pulling the disc inwards is proportional to  $MH$  or  $H^2 \sin (\theta + \phi)$ . If the permeability of iron is assumed constant, then,  $H \propto I$ . Hence,  $F \propto I^2 \sin (\theta + \phi)$ . If this force acted at a distance of  $l$  from the pivot of the rotating disc, then deflecting torque  $T_d = Fl \cos (\theta + \phi)$ . Putting the value of  $F$ , we get

$$T_d \propto I^2 \sin (\theta + \phi) \times l \cos (\theta + \phi) \propto I^2 \sin 2 (\theta + \phi) = KI^2 \sin 2 (\theta + \phi) \quad \dots \sin l \text{ is constant}$$

If spring-control is used, then controlling torque  $T_c = K' \theta$

In the steady position of deflection,  $T_d = T_c$

$$\therefore KI^2 \sin 2 (\theta + \phi) = K' \theta ; \text{ Hence } \theta \propto I^2$$

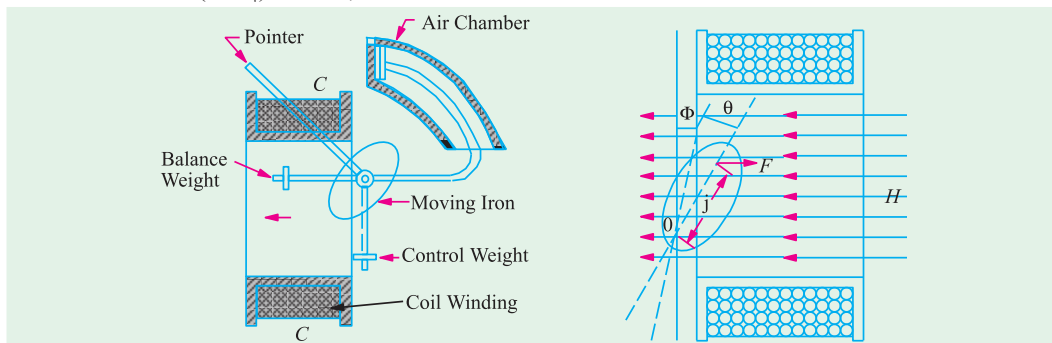


Fig. 10.9

Fig. 10.10

If A.C. is used, then  $\theta \propto I_{\text{r.m.s.}}^2$

However, if gravity-control is used, then  $T_c = K_1 \sin \theta$

$$\therefore KI^2 \sin 2 (\theta + \phi) = K_1 \sin \theta \quad \therefore \sin \theta \propto I^2 \sin 2 (\theta + \phi)$$

In both cases, the scales would be uneven.

### Damping

As shown, air-friction damping is provided, the actual arrangement being a light piston moving in an air-chamber.

### 10.9. Repulsion Type M.I. Instruments

The sectional view and cut-away view of such an instrument are shown in Fig. 10.11 and 10.12. It consists of a fixed coil inside which are placed two soft-iron rods (or bars)  $A$  and  $B$  parallel to one another and along the axis of the coil. One of them *i.e.*  $A$  is fixed and the other  $B$  which is movable carries a pointer that moves over a calibrated scale. When the current to be measured is passed

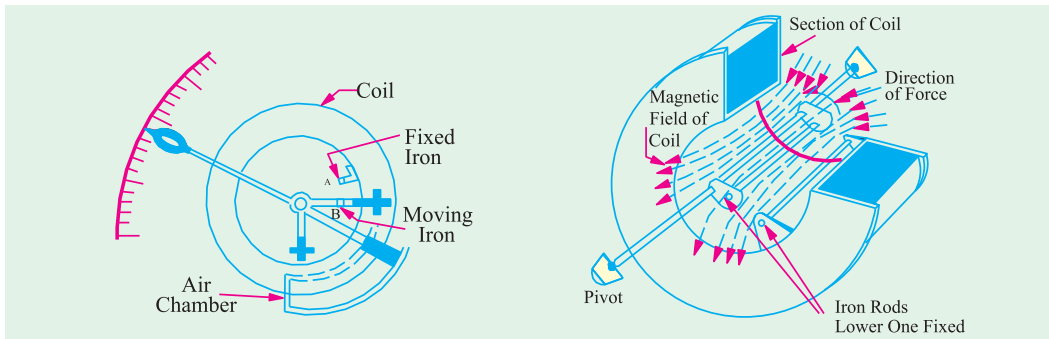


Fig. 10.11

Fig. 10.12

through the fixed coil, it sets up its own magnetic field which magnetises the two rods similarly *i.e.* the adjacent points on the lengths of the rods will have the same magnetic polarity. Hence, they repel each other with the result that the pointer is deflected against the controlling torque of a spring or gravity. The force of repulsion is approximately proportional to the square of the current passing through the coil. Moreover, whatever may be the direction of the current through the coil, the two rods will be magnetised similarly and hence will repel each other.

In order to achieve uniformity of scale, two tongue-shaped strips of iron are used instead of two rods. As shown in Fig. 10.13 (a), the fixed iron consists of a tongue-shaped sheet iron bent into a cylindrical form, the moving iron also consists of another sheet of iron and is so mounted as to move parallel to the fixed iron and towards its narrower end [Fig. 10.13 (b)].

**Deflecting Torque**

The deflecting torque is due to the repulsive force between the two similarly magnetised iron rods or sheets.

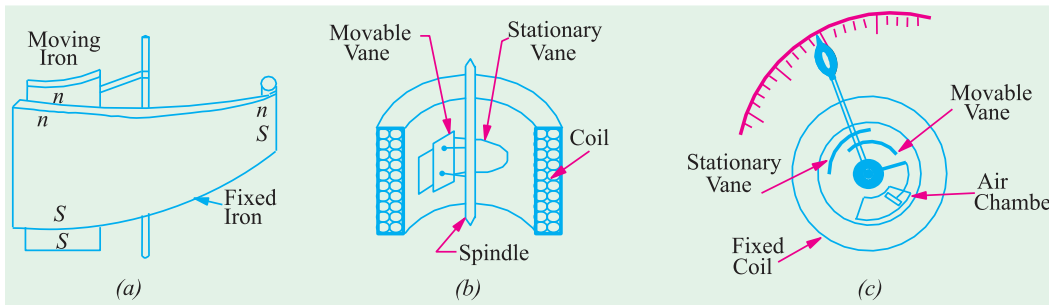


Fig. 10.13

Instantaneous torque  $\propto$  repulsive force  $\propto m_1 m_2$  ...product of pole strengths

Since pole strength are proportional to the magnetising force  $H$  of the coil,

$\therefore$  instantaneous torque  $\propto H^2$

Since  $H$  itself is proportional to current (assuming **constant** permeability) passing through the coil,  $\therefore$  instantaneous torque  $\propto I^2$

Hence, the deflecting torque, which is proportional to the mean torque is, in effect, proportional to the mean value of  $I^2$ . Therefore, when used on a.c. circuits, the instrument reads the r.m.s. value of current.

Scales of such instruments are uneven if rods are used and uniform if suitable-shaped pieces of iron sheet are used.

The instrument is either gravity-controlled or as in modern makes, is spring-controlled.

Damping is pneumatic, eddy current damping cannot be employed because the presence of a permanent magnet required for such a purpose would affect the deflection and hence, the reading of the instrument.

Since the polarity of both iron rods reverses simultaneously, the instrument can be used both for a.c. and d.c. circuits *i.e.* instrument belongs to the unpolarised class.

### 10.10. Sources of Error

There are two types of possible errors in such instruments, firstly, those which occur both in a.c. and d.c. work and secondly, those which occur in a.c. work alone.

#### (a) Errors with both d.c. and a.c. work

(i) **Error due to hysteresis.** Because of hysteresis in the iron parts of the moving system, readings are higher for descending values but lower for ascending values.

The hysteresis error is almost completely eliminated by using Mumetal or Perm-alloy, which have negligible hysteresis loss.

(ii) **Error due to stray fields.** Unless shielded effectively from the effects of stray external fields, it will give wrong readings. Magnetic shielding of the working parts is obtained by using a covering case of cast-iron.

#### (b) Errors with a.c. work only

Changes of frequency produce (i) change in the impedance of the coil and (ii) change in the magnitude of the eddy currents. The increase in impedance of the coil with increase in the frequency of the alternating current is of importance in voltmeters (Ex. 10.2). For frequencies higher than the one used for calibration, the instrument gives lower values. However, this error can be removed by connecting a capacitor of suitable value in parallel with the swamp resistance  $R$  of the instrument. It can be shown that the impedance of the whole circuit of the instrument becomes independent of frequency if  $C = L/R^2$  where  $C$  is the capacitance of the capacitor.

### 10.11. Advantages and Disadvantages

Such instruments are cheap and robust, give a reliable service and can be used both on a.c. and d.c. circuits, although they cannot be calibrated with a high degree of precision with d.c. on account of the effect of hysteresis in the iron rods or vanes. Hence, they are usually calibrated by comparison with an alternating current standard.

### 10.12. Deflecting Torque in terms of Change in Self-induction

The value of the deflecting torque of a moving-iron instrument can be found in terms of the variation of the self-inductance of its coil with deflection  $\theta$ .

Suppose that when a direct current of  $I$  passes through the instrument, its deflection is  $\theta$  and inductance  $L$ . Further suppose that when current changes from  $I$  to  $(I + dI)$ , deflection changes from  $\theta$  to  $(\theta + d\theta)$  and  $L$  changes to  $(L + dL)$ . Then, the increase in the energy stored in the magnetic field is

$$dE = d\left(\frac{1}{2}LI^2\right) = \frac{1}{2}L2I.dI + \frac{1}{2}I^2dL = LI.dI + \frac{1}{2}I^2.dL \text{ joule.}$$

If  $T\frac{1}{2}$  N-m is the controlling torque for deflection  $\theta$ , then extra energy stored in the control system is  $T \times d\theta$  joules. Hence, the total increase in the stored energy of the system is

$$LI.dI + \frac{1}{2}I^2.dL + T \times d\theta \quad \dots(i)$$

The e.m.f. induced in the coil of the instrument is  $e = N \cdot \frac{d\Phi}{dt}$  volt

where

$d\phi$  = change in flux linked with the coil due to change in the position of the disc or the bars

$dt$  = time taken for the above change ;  $N = N_0$ . of turns in the coil

Now  $L = NF/I \quad \therefore \Phi = LI/N \quad \therefore \frac{d\Phi}{dt} = \frac{1}{N} \cdot \frac{d}{dt}(LI)$

Induced e.m.f.  $e = N \cdot \frac{1}{N} \cdot \frac{d}{dt}(LI) = \frac{d}{dt}(LI)$

The energy drawn from the supply to overcome this back e.m.f is

$$= e.Idt = \frac{d}{dt}(LI).Idt = I.d(LI) = I(L.dI + I.dL) = LI.dI + I^2.dL \quad \dots(ii)$$

Equating (i) and (ii) above, we get  $LI.dI + \frac{1}{2}I^2.dL + T.d\theta = LI.dI + I^2.dL \therefore T = \frac{1}{2}I^2 \frac{dL}{d\theta}$  N-m

where  $dL/d\theta$  is henry/radian and I in amperes.

### 10.13. Extension of Range by Shunts and Multipliers

**(i) As Ammeter.** The range of the moving-iron instrument, when used as an ammeter, can be extended by using a suitable shunt across its terminals. So far as the operation with direct current is concerned, there is no trouble, but with alternating current, the division of current between the instrument and shunt changes with the change in the applied frequency. For a.c. work, both the inductance and resistance of the instrument and shunt have to be taken into account.

Obviously, 
$$\frac{\text{current through instruments, } i}{\text{current through shunt, } I_s} = \frac{R_s + j\omega L_s}{R + j\omega L} = \frac{Z_s}{Z}$$

where  $R, L$  = resistance and inductance of the instrument  
 $R_s, L_s$  = resistance and inductance of the shunt

It can be shown that above ratio *i.e.* the division of current between the instrument and shunt would be independent of frequency if the time-constants of the instrument coil and shunt are the same *i.e.* if  $L/R = L_s/R_s$ . The multiplying power ( $N$ ) of the shunt is given by

$$N = \frac{I}{i} = 1 + \frac{R}{R_s}$$

where  $I$  = line current ;  $i$  = full-scale deflection current of the instrument.

**(ii) As Voltmeter.** The range of this instrument, when used as a voltmeter, can be extended or multiplied by using a high non-inductive resistance  $R$  connected in series with it, as shown in Fig. 10.14. This series resistance is known as ‘multiplier’ when used on d.c. circuits. Suppose, the range of the instrument is to be extended from  $v$  to  $V$ . Then obviously, the excess voltage of  $(V-v)$  is to be dropped across  $R$ . If  $i$  is the full-scale deflection current of the instrument, then

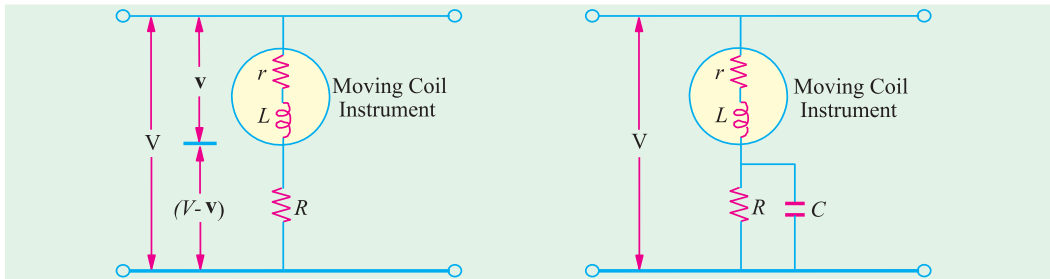


Fig. 10.14

Fig. 10.15

$$iR = V - v; R = \frac{V - v}{i} = \frac{V - ir}{i} = \frac{V}{i} - r$$

Voltage magnification =  $V/v$ . Since  $iR = V - v; \therefore \frac{iR}{v} = \frac{V}{v} - 1$

or 
$$\frac{iR}{ir} = \frac{V}{v} - 1 \therefore \frac{V}{v} = \left(1 + \frac{R}{r}\right)$$

Hence, greater the value of  $R$ , greater is the extension in the voltage range of the instrument.

For d.c. work, the principal requirement of  $R$  is that its value should remain constant *i.e.* it should have low temperature-coefficient. But for a.c. work it is essential that total impedance of the voltmeter and the series resistance  $R$  should remain as nearly constant as possible at different frequencies. That is why  $R$  is made as non-inductive as possible in order to keep the inductance of the whole circuit to the minimum. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting  $R$  by a capacitor  $C$  as shown in Fig. 10.15. In case  $r \ll R$ , the impedance of the voltmeter circuit will remain practically constant (for frequencies upto 1000 Hz) provided.

$$C = \frac{L}{(1 + \sqrt{2})R^2} = 0.41 \frac{L}{R^2}$$

**Example 10.2.** A 250-volt moving-iron voltmeter takes a current of 0.05 A when connected to a 250-volt d.c. supply. The coil has an inductance of 1 henry. Determine the reading on the meter when connected to a 250-volt, 100-Hz a.c. supply. (Elect. Engg., Kerala Univ.)

**Solution.** When used on d.c. supply, the instrument offers ohmic resistance only. Hence, resistance of the instrument =  $250/0.05 = 5000 \Omega$

When used on a.c. supply, the instrument offers **impedance** instead of ohmic resistance.

$$\text{impedance at 100 Hz} = \sqrt{5000^2 + (2\pi \times 100 \times 1)^2} = 5039.3 \Omega$$

$$\therefore \text{voltage of the instrument} = 250 \times 5000/5039.3 = \mathbf{248 \text{ V}}$$

**Example 10.3.** A spring-controlled moving-iron voltmeter reads correctly on 250-V d.c. Calculate the scale reading when 250-V a.c. is applied at 50 Hz. The instrument coil has a resistance of 500  $\Omega$  and an inductance of 1 H and the series (non-reactive) resistance is 2000  $\Omega$

(Elect. Instru. & Measure. Nagpur Univ. 1992)

**Solution.** Total circuit resistance of the voltmeter is

$$= (r + R) = 500 + 2,000 = 2,500 \Omega$$

Since the voltmeter reads correctly on direct current supply, its full-scale deflection current is =  $250/2500 = 0.1 \text{ A}$ .

When used on a.c. supply, instrument offers an impedance

$$Z = \sqrt{2500^2 + (2\pi \times 50 \times 1)^2} = 2.520 \Omega \quad \therefore I = 0.099 \text{ A}$$

$$\therefore \text{Voltmeter reading on a.c. supply} = 250 \times 0.099/0.1 = \mathbf{248 \text{ V}^*}$$

**Note.** Since swamp resistance  $R = 2,000 \Omega$ , capacitor required for compensating the frequency error is

$$C = 0.41 L/R^2 = 0.41 \times 1/2000^2 = 0.1 \mu\text{F}.$$

**Example 10.4.** A 150-V moving-iron voltmeter intended for 50 Hz has an inductance of 0.7 H and a resistance of 3 k $\Omega$ . Find the series resistance required to extend the range of the instrument to 300 V. If this 300-V, 50-Hz instrument is used to measure a d.c. voltage, find the d.c. voltage when the scale reading is 200 V. (Elect. Measur, A.M.I.E. Sec B, 1991)

**Solution.** Voltmeter reactance =  $2\pi \times 50 \times 0.7 = 220 \Omega$

Impedance of voltmeter =  $(3000 + j 220) = 3008 \Omega$

When the voltmeter range is doubled, its impedance has also to be doubled in order to have the same current for full-scale deflection. If  $R$  is the required series resistance, then  $(3000 + R)^2 + 220^2 = (2 \times 3008)^2 \quad \therefore R = 3012 \Omega$

When used on d.c. supply, if the voltmeter reads 200 V, the actual applied d.c. voltage would be =  $200 \times (\text{Total A.C. Impedance})/\text{total d.c. resistance} = 200 \times (2 \times 3008)/(3000 + 3012) = 200 \times (6016 \times 6012) = 200.134 \text{ V}$ .

**Example 10.5.** The coil of a moving-iron voltmeter has a resistance of 5,000  $\Omega$  at 15°C at which temperature it reads correctly when connected to a supply of 200 V. If the coil is wound with wire whose temperature coefficient at 15°C is 0.004, find the percentage error in the reading when the temperature is 50°C.

In the above instrument, the coil is replaced by one of 2,000  $\Omega$  but having the same number of turns and the full 5,000  $\Omega$  resistance is obtained by connecting in series a 3,000  $\Omega$  resistor of negligible temperature-coefficient. If this instrument reads correctly at 15°C, what will be its percentage error at 50°C.

**Solution.** Current at 15°C =  $200/5,000 = 0.04 \text{ A}$

Resistance at 50°C is  $R_{50} = R_{15} (1 + \alpha_{15} \times 35)$

$$\therefore R_{50} = 5,000 (1 + 35 \times 0.004) = 5,700 \Omega$$

\* or reading =  $250 \times 2500/2520 = \mathbf{248 \text{ V}}$ .

$$\begin{aligned} \therefore \text{current at } 50^{\circ}\text{C} &= 200/5,700 \\ \therefore \text{reading at } 50^{\circ}\text{C} &= \frac{200 \times (200/5,700)}{0.04} = 175.4 \text{ V or } = 200 \times 5000/5700 = 175.4 \text{ V} \\ \therefore \text{\% error} &= \frac{175.4 - 200}{200} \times 100 = \mathbf{-12.3\%} \end{aligned}$$

In the second case, swamp resistance is  $3,000 \Omega$  whereas the resistance of the instrument is only  $2,000 \Omega$

$$\begin{aligned} \text{Instrument resistance at } 50^{\circ}\text{C} &= 2,000 (1 + 35 \times 0.004) = 2,280 \Omega \\ \therefore \text{total resistance at } 50^{\circ}\text{C} &= 3,000 + 2,280 = 5,280 \Omega \\ \therefore \text{current at } 50^{\circ}\text{C} &= 200/5,280 \text{ A} \\ \therefore \text{instrument reading} &= 200 \times \frac{200/5,280}{0.04} = 189.3 \text{ V} \\ \therefore \text{percentage error} &= \frac{189.3 - 200}{200} \times 100 = \mathbf{-5.4 \%} \end{aligned}$$

**Example 10.6.** The change of inductance for a moving-iron ammeter is  $2 \mu\text{H}/\text{degree}$ . The control spring constant is  $5 \times 10^{-7} \text{ N-m}/\text{degree}$ .

The maximum deflection of the pointer is  $100^{\circ}$ , what is the current corresponding to maximum deflection ?  
(Measurement & Instrumentation Nagpur Univ. 1993)

**Solution.** As seen from Art. 10.12 the deflecting torque is given by

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m}$$

Control spring constant =  $5 \times 10^{-7} \text{ N-m}/\text{degree}$

Deflection torque for  $100^{\circ}$  deflection =  $5 \times 10^{-7} \times 100 = 5 \times 10^{-5} \text{ N-m}$ ;  $dL/d\theta = 2 \mu\text{H}/\text{degree} = 2 \times 10^{-6} \text{ H}/\text{degree}$ .

$$\therefore 5 \times 10^{-5} = \frac{1}{2} I^2 \times 2 \times 10^{-6} \quad \therefore I^2 = \mathbf{50} \quad \text{and} \quad I = 7.07 \text{ A}$$

**Example 10.7.** The inductance of attraction type instrument is given by  $L = (10 + 5\theta - \theta^2) \mu\text{H}$  where  $\theta$  is the deflection in radian from zero position. The spring constant is  $12 \times 10^{-6} \text{ N-m}/\text{rad}$ . Find out the deflection for a current of  $5 \text{ A}$ .

(Elect. and Electronics Measurements and Measuring Instruments Nagpur Univ. 1993)

**Solution.**  $L = (10 + 5\theta - \theta^2) \times 10^{-6} \text{ H}$

$$\therefore \frac{dL}{d\theta} = (0 + 5 - 2 \times \theta) \times 10^{-6} = (5 - 2\theta) \times 10^{-6} \text{ H}/\text{rad}$$

Let the deflection be  $\theta$  radians for a current of  $5 \text{ A}$ , then deflecting torque,

$$T_d = 12 \times 10^{-6} \times \theta \text{ N-m}$$

Also,

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad \dots \text{Art.}$$

Equating the two torques, we get

$$12 \times 10^{-6} \times \theta = \frac{1}{2} \times 5^2 \times (5 - 2\theta) \times 10^{-6} \quad \therefore \theta = \mathbf{1.689 \text{ radian}}$$

### Tutorial Problems No. 10.1

1. Derive an expression for the torque of a moving-iron ammeter. The inductance of a certain moving-iron ammeter is  $(8 + 4\theta - \frac{1}{2} \theta^2) \mu\text{H}$  where  $\theta$  is the deflection in radians from the zero position. The control-spring torque is  $12 \times 10^{-6} \text{ N-m}/\text{rad}$ . Calculate the scale position in radians for a current of  $3 \text{ A}$ .  
[1.09 rad] (I.E.E. London)

2. An a.c. voltmeter with a maximum scale reading of  $50\text{-V}$  has a resistance of  $500 \Omega$  and an inductance of  $0.09 \text{ henry}$ . The magnetising coil is wound with  $50$  turns of copper wire and the remainder of the circuit is a non-inductive resistance in series with it. What additional apparatus is needed to make this instrument read correctly on both d.c. circuits or frequency  $60$  ?  
[0.44  $\mu\text{F}$  in parallel with series resistance]

3. A 10-V moving-iron ammeter has a full-scale deflection of 40 mA on d.c. circuit. It reads 0.8% low on 50 Hz a.c. Hence, calculate the inductance of the ammeter. [115.5 mH]

4. It is proposed to use a non-inductive shunt to increase the range of a 10-A moving iron ammeter to 100 A. The resistance of the instrument, including the leads to the shunt, is  $0.06 \Omega$  and the inductance is  $15 \mu H$  at full scale. If the combination is correct on a.d.c circuit, find the error at full scale on a 50 Hz a.c. circuit.

[3.5 %](London Univ.)

### 10.14. Moving-coil Instruments

There are two types of such instruments (i) *permanent-magnet type* which can be used for d.c. work only and (ii) *the dynamometer type* which can be used both for a.c. and d.c. work.

### 10.15. Permanent Magnet Type Instruments

The operation of a permanent-magnet moving-coil type instrument is based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

#### Construction

As its name indicates, the instrument consists of a permanent magnet and a rectangular coil of many turns wound on a light aluminium or copper former inside which is an iron core as shown in

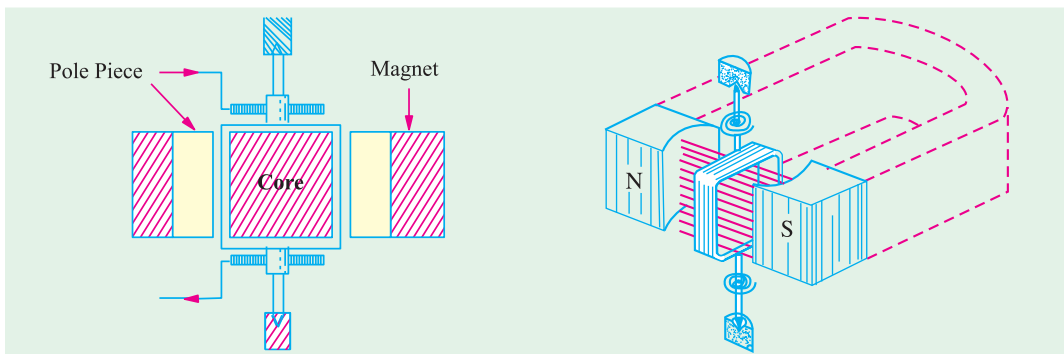


Fig. 10.16.

Fig. 10.17

Fig. 10.16. The powerful U-shaped permanent magnet is made of Alnico and has soft-iron end-pole pieces which are bored out cylindrically. Between the magnetic poles is fixed a soft iron cylinder whose function is (i) to make the field radial and uniform and (ii) to decrease the reluctance of the air path between the poles and hence increase the magnetic flux. Surrounding the core is a rectangular coil of many turns wound on a light aluminium frame which is supported by delicate bearings and to which is attached a light pointer. The aluminium frame not only provides support for the coil but also provides damping by eddy currents induced in it. The sides of the coil are free to move in the two air-gaps between the poles and core as shown in Fig. 10.16 and Fig. 10.17. Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of lending the current in and out of the coil. The two springs are spiralled in opposite directions in order to neutralize the effects of temperature changes.

#### Deflecting Torque

When current is passed through the coil, force acts upon its both sides which produce a deflecting torque as shown in Fig. 10.18. Let

$B$  = flux density in  $Wb/m^2$

$l$  = length or depth of the coil in metre

$b$  = breadth of coil in metre

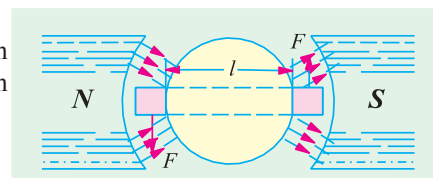


Fig. 10.18



$N$  = number of turns in the coil

If  $I$  ampere is the current passing through the coil, then the magnitude of the force experienced by each of its sides is =  $BIL$  newton

For  $N$  turns, the force on each side of the coil is =  $NBIL$  newton

$$\begin{aligned} \therefore \text{deflecting torque } T_d &= \text{force} \times \text{perpendicular distance} \\ &= NBIL \times b = NBI(L \times b) = NBIA \text{ N-m} \end{aligned}$$

where  $A$  is the face area of the coil.

It is seen that if  $B$  is constant, then  $T_d$  is proportional to the current passing through the coil *i.e.*  $T_d \propto I$ .

Such instruments are invariable spring-controlled so that  $T_c \propto \text{deflection } \theta$ .

Since at the final deflected position,  $T_d = T_c \therefore \theta \propto I$

Hence, such instruments have uniform scales. Damping is electromagnetic *i.e.* by eddy currents induced in the metal frame over which the coil is wound. Since the frame moves in an intense magnetic field, the induced eddy currents are large and damping is very effective.

### 10.16. Advantage and Disadvantages

The permanent-magnet moving-coil (PMMC) type instruments have the following advantages and disadvantages :

#### Advantages

1. they have low power consumption.
2. their scales are uniform and can be designed to extend over an arc of  $170^\circ$  or so.
3. they possess high (torque/weight) ratio.
4. they can be modified with the help of shunts and resistances to cover a wide range of currents and voltages.
5. they have no hysteresis loss.
6. they have very effective and efficient eddy-current damping.
7. since the operating fields of such instruments are very strong, they are not much affected by stray magnetic fields.

#### Disadvantages

1. due to delicate construction and the necessary accurate machining and assembly of various parts, such instruments are somewhat costlier as compared to moving-iron instruments.
2. some errors are set in due to the aging of control springs and the permanent magnets.

Such instruments are mainly used for d.c. work only, but they have been sometimes used in conjunction with rectifiers or thermo-junctions for a.c. measurements over a wide range or frequencies.

Permanent-magnet moving-coil instruments can be used as ammeters (with the help of a low resistance shunt) or as voltmeters (with the help of a high series resistance).

The principle of permanent-magnet moving-coil type instruments has been utilized in the construction of the following :

1. For a.c. galvanometer which can be used for detecting extremely small d.c. currents. A galvanometer may be used either as an ammeter (with the help of a low resistance) or as a voltmeter (with the help of a high series resistance). Such a galvanometer (of pivoted type) is shown in Fig. 10.19.
2. By eliminating the control springs, the instrument can be used for measuring the quantity of electricity passing through the coil. This method is used for *fluxmeters*.



Fig. 10.19

3. If the control springs of such an instrument are purposely made of large moment of inertia, then it can be used as ballistic galvanometer.

### 10.17. Extension of Range

#### (i) As Ammeter

When such an instrument is used as an ammeter, its range can be extended with the help of a low-resistance shunt as shown in Fig. 10.12 (a). This shunt provides a bypath for extra current because it is connected across (*i.e.* in parallel with) the instrument. These shunted instruments can be made to record currents many times greater than their normal full-scale deflection currents. The ratio of maximum current (with shunt) to the full-scale deflection current (without shunt) is known as the 'multiplying power' or 'multiplying factor' of the shunt.

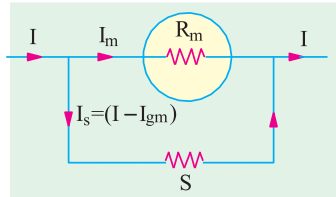


Fig. 10.20 (a)

Let  $R_m$  = instrument resistance  
 $S$  = shunt resistance  
 $I_m$  = full-scale deflection current of the instrument  
 $I$  = line current to be measured

As seen from Fig. 10.20 (a), the voltage across the instrument coil and the shunt is the same since both are joined in parallel.

$$\therefore I_m \times R_m = S I_s = S (I - I_m) \quad \therefore S = \frac{I_m R_m}{(I - I_m)}; \text{ Also } \frac{I}{I_m} = \left(1 + \frac{R_m}{S}\right)$$

$$\therefore \text{ multiplying power} = \left(1 + \frac{R_m}{S}\right)$$

Obviously, lower the value of shunt resistance, greater its multiplying power.

#### (ii) As voltmeter

The range of this instrument when used as a voltmeter can be increased by using a high resistance in series with it [Fig. 10.20 (b)].

Let  $I_m$  = full-scale deflection current  
 $R_m$  = galvanometer resistance  
 $v$  =  $R_m I_m$  = full-scale p.d. across it  
 $V$  = voltage to be measured  
 $R$  = series resistance required

Then it is seen that the voltage drop across  $R$  is  $V - v$

$$\therefore R = \frac{V - v}{I_m} \text{ or } R \cdot I_m = V - v$$

Dividing both sides by  $v$ , we get

$$\frac{R I_m}{v} = \frac{V}{v} - 1 \quad \text{or} \quad \frac{R \cdot I_m}{I_m R_m} = \frac{V}{v} - 1 \quad \therefore \frac{V}{v} = \left(1 + \frac{R}{R_m}\right)$$

$$\therefore \text{ voltage multiplication} = \left(1 + \frac{R}{R_m}\right)$$

Obviously, larger the value of  $R$ , greater the voltage multiplication or range. Fig. 10.20 (b) shown a voltmeter with a single multiplier resistor for one range. A multi-range voltmeter requires on multiplier resistor for each additional range.

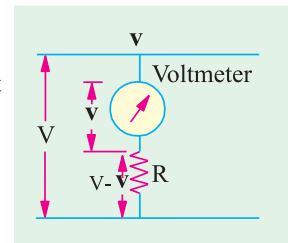


Fig. 10.20 (b)

**Example 10.8.** A moving coil ammeter has a fixed shunt of  $0.02 \Omega$  with a coil circuit resistance of  $R = 1 \text{ k}\Omega$  and need potential difference of  $0.5 \text{ V}$  across it for full-scale deflection.

(1) To what total current does this correspond ?

(2) Calculate the value of shunt to give full scale deflection when the total current is  $10 \text{ A}$  and  $75 \text{ A}$ .

(Measurement & Instrumentation Nagpur Univ. 1993)

**Solution.** It should be noted that the shunt and the meter coil are in parallel and have a common p.d. of 0.5 V applied across them.

$$(1) \therefore I_m = 0.5/1000 = 0.0005 \text{ A}; I_s = 0.5./0.02 = 25 \text{ A}$$

$$\therefore \text{line current} = \mathbf{25.0005 \text{ A}}$$

$$(2) \text{ When total current is } 10 \text{ A, } I_s = (10 - 0.0005) = 9.9995 \text{ A}$$

$$\therefore S = \frac{I_m R_m}{I_s} = \frac{0.0005 \times 1000}{9.9995} = 0.05 \Omega$$

$$\text{When total current is } 75 \text{ A, } I_s = (75 - 0.0005) = 74.9995 \text{ A}$$

$$\therefore S = 0.0005 \times 1000/74.9995 = \mathbf{0.00667 \Omega}$$

**Example 10.9.** A moving-coil instrument has a resistance of  $10 \Omega$  and gives full-scale deflection when carrying a current of  $50 \text{ mA}$ . Show how it can be adopted to measure voltage up to  $750 \text{ V}$  and currents upto  $1000 \text{ A}$ .  
(Elements of Elect. Engg.I, Bangalore Univ.)

**Solution. (a) As Ammeter.**

As discussed above, current range of the meter can be extended by using a shunt across it [Fig. 10.21 (a)].

Obviously,

$$10 \times 0.05 = S \times 99.95$$

$$\therefore S = \mathbf{0.005 \Omega}$$

**(b) As Voltmeter.** In this

case, the range can be extended by using a high resistance  $R$  in series with it.

[Fig. 10.21 (b)]. Obviously,  $R$  must drop a voltage of  $(750 - 0.5) = 749.5 \text{ V}$  while carrying  $0.05 \text{ A}$ .

$$\therefore 0.05 R = 749.5 \text{ or } R = \mathbf{14.990 \Omega}$$

**Example 10.10.** How will you use a P.M.M.C. instrument which gives full scale deflection at  $50 \text{ mV}$  p.d. and  $10 \text{ mA}$  current as

(1) Ammeter 0 - 10 A range

(2) Voltmeter 0-250 V range (Elect. Instruments & Measurements Nagpur Univ. 1993)

**Solution.** Resistance of the instrument  $R_m = 50 \text{ mV}/10 \text{ mA} = 5$

**(i) As Ammeter**

full-scale meter current,  $I_m = 10 \text{ mA} = 0.01 \text{ A}$

shunt current  $I_s = I - I_m = 10 - 0.01 = 9.99 \text{ A}$

$$\text{Reqd. shunt resistance, } S = \frac{I_m R_m}{(I - I_m)} = \frac{0.01 \times 5}{9.99} = 0.0005 \Omega$$

**(ii) As Voltmeter**

Full-scale deflection voltage,  $v = 50 \text{ mV} = 0.05 \text{ V}; V = 250 \text{ V}$

$$\text{Reqd. series resistance, } R = \frac{V - v}{I_m} = \frac{250 - 0.05}{0.01} = 24,995 \Omega$$

**Example 10.11.** A current galvanometer has the following parameters :

$B = 10 \times 10^{-3} \text{ Wb/m}^2$ ;  $N = 200$  turns,  $l = 16 \text{ mm}$ ;

$d = 16 \text{ mm}$ ;  $k = 12 \times 10^{-9} \text{ Nm/radian}$ .

Calculate the deflection of the galvanometer when a current of  $1 \mu\text{A}$  flows through it.

(Elect. Measurement Nagpur Univ. 1993)

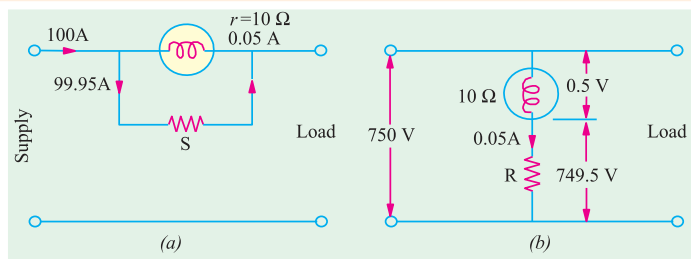


Fig. 10.21

**Solution.** Deflecting torque  $T_d = NBIA \text{ N-m} = 200 \times (10 \times 10^{-3}) \times (1 \times 10^{-6}) \times (1 \times 10^{-3}) \times (16 \times 10^{-3}) \text{ N-m} = 512 \times 10^{-12} \text{ N-m}$

Controlling torque  $T_c = \text{controlling spring constant} \times \text{deflection} = 12 \times 10^9 \times \theta \text{ N-m}$

Equating the deflecting and controlling torques, we have  $12 \times 10^9 \times \theta = 512 \times 10^{-12}$

$$\therefore \theta = 0.0427 \text{ radian} = 2.45^\circ$$

**Example 10.12.** The coil of a moving coil permanent magnet voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of  $120 \times 10^{-6} \text{ N-m}$  when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is  $0.5 \text{ Wb/m}^2$ , estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected. (Elect. Mesur. AMIE Sec. B Summer 1991)

**Solution.** Let  $I$  be the current for full-scale deflection. Deflection torque  $T_d = NBIA$   
 $= 100 \times 0.5 \times I \times (1200 \times 10^{-6}) = 0.06 I \text{ N-m}$

Controlling torque  $T_c = 120 \times 10^{-6} \text{ N-m}$

In the equilibrium position, the two torques are equal i.e.  $T_d = T_c$ .

$$\therefore 0.06 I = 120 \times 10^{-6} \quad \therefore I = 2 \times 10^{-3} \text{ A.}$$

Since the instrument is meant to read 1 volt per division, its full-scale reading is 100 V.

$$\text{Total resistance} = 100/2 \times 10^{-3} = 50,000 \Omega$$

Since voltmeter coil resistance is negligible, it represents the additional required resistance.

**Example 10.13.** Show that the torque produced in a permanent-magnet moving-coil instrument is proportional to the area of the moving coil.

A moving-coil voltmeter gives full-scale deflection with a current of 5 mA. The coil has 100 turns, effective depth of 3 cm and width of 2.5 cm. The controlling torque of the spring is 0.5 cm for full-scale deflection. Estimate the flux density in the gap. (Elect. Meas, Marathwads Univ.)

**Solution.** The full-scale deflecting torque is  $T_d = NBIA \text{ N-m}$

where  $I$  is the full-scale deflection current ;  $I = 5 \text{ mA} = 0.005 \text{ A}$

$$T_d = 100 \times B \times 0.005 \times (3 \times 2.5 \times 10^{-4}) = 3.75 \times 10^{-4} B \text{ N-m}$$

The controlling torque is

$$\begin{aligned} T_c &= 0.5 \text{ g-cm} = 0.5 \text{ g. wt.cm} = 0.5 \times 10^{-3} \times 10^{-2} \text{ kg wt-m} \\ &= 0.5 \times 10^{-5} \times 9.8 = 4.9 \times 10^{-5} \text{ N-m} \end{aligned}$$

For equilibrium, the two torques are equal and opposite.

$$\therefore 4.9 \times 10^{-5} = 3.75 \times 10^{-4} B \quad \therefore B = 0.13 \text{ Wb/m}^2$$

**Example 10.14.** A moving-coil milliammeter has a resistance of  $5 \Omega$  and a full-scale deflection of 20 mA. Determine the resistance of a shunt to be used so that the instrument could measure currents upto 500 mA at  $20^\circ \text{C}$ . What is the percentage error in the instrument operating at a temperature of  $40^\circ \text{C}$ ? Temperature co-efficient of copper = 0.0039 per  $^\circ \text{C}$ .

(Measu. & Instrumentation, Allahabad Univ. 1991)

**Solution.** Let  $R_{20}$  be the shunt resistance at  $20^\circ \text{C}$ . When the temperature is  $20^\circ \text{C}$ , line current is 500 mA and shunt current is  $(500-20) = 480 \text{ mA}$ .

$$\therefore 5 \times 20 = R_{20} \times 480, \quad R_{20} = 1/4.8 \Omega$$

If  $R_{40}$  is the shunt resistance at  $40^\circ \text{C}$ , then

$$R_{40} R_{20} (1 + 20 \alpha) = \frac{1}{4.8} (1 + 0.0039 \times 20) = \frac{1.078}{4.8} \Omega$$

$$\text{Shunt current at } 40^\circ \text{C is } = \frac{5 \times 20}{1.078/4.8} = 445 \text{ mA}$$

Line current =  $445 + 20 = 465 \text{ mA}$

Although, line current would be only 465 mA, the instrument will indicate 500 mA.

$$\therefore \text{error} = 35/500 = 0.07 \text{ or } 7\%$$

**Example 10.15.** A moving-coil millivoltmeter has a resistance of  $20\Omega$  and full-scale deflection of  $120^\circ$  is reached when a potential difference of  $100\text{ mV}$  is applied across its terminals. The moving coil has the effective dimensions of  $3.1\text{ cm} \times 2.6\text{ cm}$  and is wound with 120 turns. The flux density in the gap is  $0.15\text{ Wb/m}^2$ . Determine the control constant of the spring and suitable diameter of copper wire for coil winding if 55% of total instrument resistance is due to coil winding.  $\rho$  for copper =  $1.73 \times 10^{-6}\ \Omega\text{ cm}$ .  
(Elect. Inst. and Meas. M.S. Univ. Baroda)

**Solution.** Full-scale deflection current is  $= 100/20 = 5\text{ mA}$

Deflecting torque for full-scale deflection of  $120^\circ$  is

$$T_d = NBIA = 120 \times 0.15 \times (5 \times 10^{-3}) \times (3.1 \times 2.6 \times 10^{-4}) = 72.5 \times 10^{-6}\text{ N-m}$$

Control constant is defined as the deflecting torque per radian (or degree) or deflection of moving coil. Since this deflecting torque is for  $120^\circ$  deflection.

$$\text{Control constant} = 72.5 \times 10^{-6}/120 = \mathbf{6.04 \times 10^{-7}\text{ N-m/degree}}$$

Now, resistance of copper wire = 55% of  $20\ \Omega = 11\ \Omega$

Total length of copper wire =  $120 \times 2(3.1 + 2.6) = 1368\text{ cm}$

$$\text{Now } R = \rho l/A \quad \therefore A = 1.73 \times 10^{-6} \times 1368/11 = 215.2 \times 10^{-6}\text{ cm}^2$$

$$\therefore \pi d^2/4 = 215.2 \times 10^{-6}$$

$$\therefore d = \sqrt{215.2 \times 4 \times 10^{-6}/\pi} = 16.55 \times 10^{-3}\text{ cm} = \mathbf{0.1655\text{ mm}}$$

### 10.18. Voltmeter Sensitivity

It is defined in terms of resistance per volt ( $\Omega/\text{V}$ ). Suppose a meter movement of  $1\text{ k}\Omega$  internal resistance has a full-scale deflection current of  $50\ \mu\text{A}$ . Obviously, full-scale voltage drop of the meter movement is  $= 50\ \mu\text{A} \times 1000\ \Omega = 50\text{ mV}$ . When used as a voltmeter, its sensitivity would be  $1000/50 \times 10^{-3} = 20\text{ k}\Omega/\text{V}$ . It should be clearly understood that a sensitivity of  $20\text{ k}\Omega/\text{V}$  means that the total resistance of the circuit in which the above movement is used should be  $20\text{ k}\Omega$  for a full-scale deflection of  $1\text{ V}$ .

### 10.19. Multi-range Voltmeter

It is a voltmeter which measures a number of voltage ranges with the help of different series resistances. The resistance required for each range can be easily calculated provided we remember one basic fact that the sensitivity of a meter movement is always the same regardless of the range selected. Moreover, the full-scale deflection current is the same in every range. For any range, the total circuit resistance is found by multiplying the sensitivity by the full-scale voltage for that range.

For example, in the case of the above-mentioned  $50\ \mu\text{A}$ ,  $1\text{ k}\Omega$  meter movement, total resistance required for  $1\text{ V}$  full-scale deflection is  $20\text{ k}\Omega$ . It means that an additional series resistance of  $19\text{ k}\Omega$  is required for the purpose as shown in Fig. 10.22 (a).

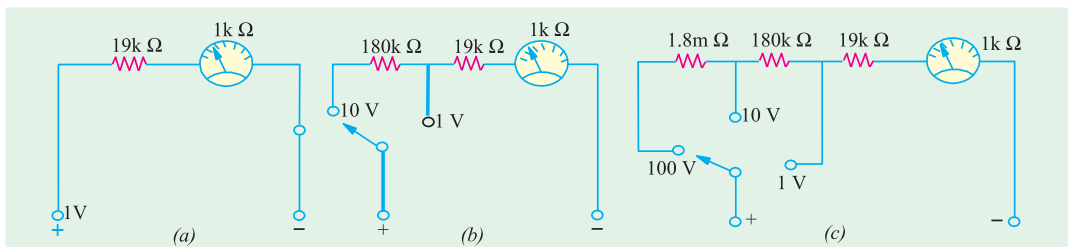


Fig. 10.22

For  $10\text{-V}$  range, total circuit resistance must be  $(20\text{ k}\Omega/\text{V})(10\text{ V}) = 200\text{ k}\Omega$ . Since total resistance for  $1\text{ V}$  range is  $20\text{ k}\Omega$ , the series resistance  $R$  for  $10\text{-V}$  range  $= 200 - 20 = 180\text{ k}\Omega$  as shown in Fig. 10.22 (b).

For the range of  $100\text{ V}$ , total resistance required is  $(20\text{ k}\Omega/\text{V})(100\text{ V}) = 2\text{ M}\Omega$ . The additional resistance required can be found by subtracting the existing two-range resistance from the total

resistance of  $2\text{ M}\Omega$ . Its value is

$$= 2\text{ M}\Omega - 180\text{ k}\Omega - 19\text{ k}\Omega - 1\text{ k}\Omega = 1.8\text{ M}\Omega$$

It is shown in Fig. 10.22 (c).

**Example 10.16.** A basic  $I'$  Arsonval movement with internal resistance  $R_m = 100\ \Omega$  and full scale deflection current  $I_f = 1\text{ mA}$  is to be converted into a multirange d.c. voltmeter with voltage ranges of 0–10 V, 0–50 V, 0–250 V and 0–500 V. Draw the necessary circuit arrangement and find the values of suitable multipliers. **(Instrumentation AMIE Sec. B Winter 1991)**

**Solution.** Full-scale voltage drop =  $(1\text{ mA})(100\ \Omega) = 100\text{ mV}$ . Hence, sensitivity of this movement is  $100/100 \times 10^{-3} = 1\text{ k}\Omega/\text{V}$ .

**(i) 0–10 V range**

Total resistance required =  $(1\text{ k}\Omega/\text{V})(10\text{ V}) = 10\text{ k}\Omega$ . Since meter resistance is  $1\text{ k}\Omega$  additional series resistance required for this range  $R_1 = 10 - 1 = 9\text{ k}\Omega$

**(ii) 0–50 V range**

$$R_T = (1\text{ k}\Omega/\text{V})(50\text{ V}) = 50\text{ k}\Omega; R_2 = 50 - 1 = 49\text{ k}\Omega$$

**(iii) 0–250 V range**

$$R_T = (1\text{ k}\Omega/\text{V})(250\text{ V}) = 250\text{ k}\Omega; R_3 = 250 - 50 = 200\text{ k}\Omega$$

**(iv) 0–500 V range**

$$R_T = (1\text{ k}\Omega/\text{V})(500\text{ V}) = 500\text{ k}\Omega; R_4 = 500 - 250 = 250\text{ k}\Omega$$

The circuit arrangement is similar to the one shown in Fig. 10.22

### Tutorial problem No. 10.2

1. The flux density in the gap of a 1-mA (full scale) moving ‘-coil’ ammeter is  $0.1\text{ Wb/m}^2$ . The rectangular moving-coil is 8 mm wide by 1 cm deep and is wound with 50 turns. Calculate the full-scale torque which must be provided by the springs.  **$[4 \times 10^{-7}\text{ N-m}]$  (App. Elec. London Univ.)**

2. A moving-coil instrument has 100 turns of wire with a resistance of  $10\ \Omega$ , an active length in the gap of 3 cm and width of 2 cm. A p.d. of 45 mV produces full-scale deflection. The control spring exerts a torque of  $490.5 \times 10^{-7}\text{ N-m}$  at full-scale deflection. Calculate the flux density in the gap.  **$[0.1817\text{ Wb/m}^2]$  (I.E.E. London)**

3. A moving-coil instrument, which gives full-scale deflection with 0.015 A has a copper coil having a resistance of  $1.5\ \Omega$  at  $15^\circ\text{C}$  and a temperature coefficient of  $1/234.5$  at  $0^\circ\text{C}$  in series with a swamp resistance of  $3.5\ \Omega$  having a negligible temperature coefficient.

Determine (a) the resistance of shunt required for a full-scale deflection of 20 A and (b) the resistance required for a full-scale deflection of 250 V.

If the instrument reads correctly at  $15^\circ\text{C}$ , determine the percentage error in each case when the temperature is  $25^\circ\text{C}$ .  **$[(a) 0.00376\ \Omega; 1.3\% (b) 16,662\ \Omega, \text{negligible}]$  (App. Elect. London Univ.)**

4. A direct current ammeter and leads have a total resistance of  $1.5\ \Omega$ . The instrument gives a full-scale deflection for a current of 50 mA. Calculate the resistance of the shunts necessary to give full-scale ranges of 2.5, 5.0 and 25.0 amperes  **$[0.0306; 1.01515; 0.00301\ \Omega]$  (I.E.E. London)**

5. The following data refer to a moving-coil voltmeter: resistance =  $10,000\ \Omega$ , dimensions of coil =  $3\text{ cm} \times 3\text{ cm}$ ; number of turns on coil = 100, flux density in air-gap =  $0.08\text{ Wb/m}^2$ , stiffness of springs =  $3 \times 10^{-6}\text{ N-m per degree}$ . Find the deflection produced by 110 V.  **$[48.2^\circ]$  (London Univ.)**

6. A moving-coil instrument has a resistance of  $1.0\ \Omega$  and gives a full-scale deflection of 150 divisions with a p.d. of 0.15 V. Calculate the extra resistance required and show how it is connected to enable the instrument to be used as a voltmeter reading upto 15 volts. If the moving coil has a negligible temperature coefficient but the added resistance has a temperature coefficient of  $0.004\ \Omega$  per degree C, what reading will a p.d. of 10 V give at  $15^\circ\text{C}$ , assuming that the instrument reads correctly at  $0^\circ\text{C}$ .  **$[99\ \Omega, 9.45]$**

### 10.20. Electrodynamical or Dynamometer Type Instruments

An electrodynamic instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet but by another fixed coil. This instrument can be used either as an ammeter or a voltmeter but is generally used as a wattmeter.

As shown in Fig. 10.23, the fixed coil is usually arranged in two equal sections  $F$  and  $F$  placed close together and parallel to each other. The two fixed coils are air-cored to avoid hysteresis effects

when used on a.c. circuits. This has the effect of making the magnetic field in which moves the moving coil  $M$ , more uniform. The moving coil is spring-controlled and has a pointer attached to it as shown.

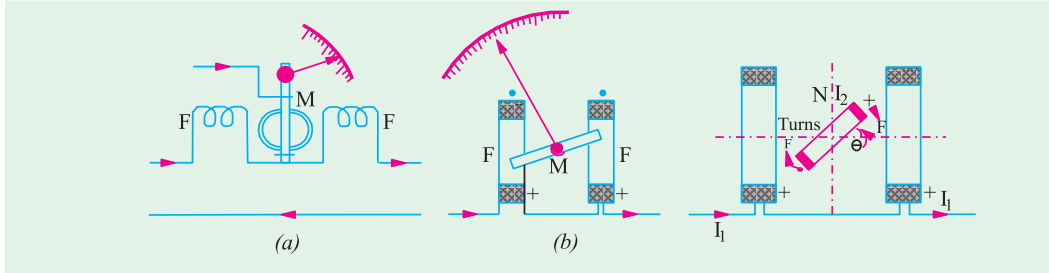


Fig. 10.23

Fig. 10.24

**Deflecting Torque\***

The production of the deflecting torque can be understood from Fig. 10.24. Let the current passing through the fixed coil be  $I_1$  and that through the moving coil be  $I_2$ . Since there is no iron, the field strength and hence the flux density is proportional to  $I_1$ .

$$\therefore B = KI_1 \text{ where } K \text{ is a constant}$$

Let us assume for simplicity that the moving coil is rectangular (it can be circular also) and of dimensions  $l \times b$ . Then, force on each side of the coil having  $N$  turns is  $(NBI_2l)$  newton.

The turning moment or deflecting torque on the coil is given by

$$T_d = NBI_2lb = NKI_1I_2lb \text{ N-m}$$

Now, putting  $NKlb = K_1$ , we have  $T_d = K_1I_1I_2$  where  $K_1$  is another constant.

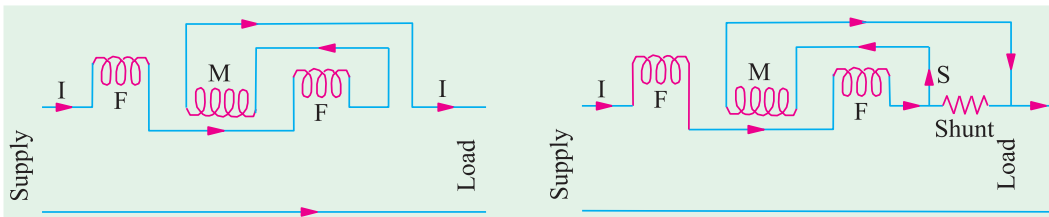


Fig. 10.25

Fig. 10.26

It shows that the deflecting torque is proportional to the product of the currents flowing in the fixed coils and the moving coil. Since the instrument is spring-controlled, the restoring or control torque is proportional to the angular deflection  $\theta$ .

i.e.  $T_c \propto K_2 \theta \therefore K_1I_1I_2 = K_2\theta \text{ or } \theta \propto I_1I_2$

\* As shown in Art. 10.12, the value of torque of a moving-coil instrument is

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} N - m$$

The equivalent inductance of the fixed and moving coils of the electrodynamic instrument is

$$L = L_1 + L_2 + 2M$$

where  $M$  is the mutual inductance between the two coils and  $L_1$  and  $L_2$  are their individual self-inductances.

Since  $L_1$  and  $L_2$  are fixed and only  $M$  varies,

$$\therefore \frac{dL}{d\theta} = 2 \frac{dM}{d\theta} \therefore T_d \frac{1}{2} I^2 \times 2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

If the currents in the fixed and moving coils are different, say  $I_1$  and  $I_2$  then

$$T_d = I_1 \cdot I_2 \cdot \frac{dM}{d\theta} \text{ N-m}$$

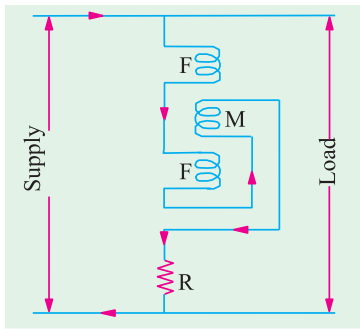


Fig. 10.27

When the instrument is used as an ammeter, the same current passes through both the fixed and the moving coils as shown in Fig. 10.25.

In that case  $I_1 = I_2 = I$ , hence  $\theta \propto I^2$  or  $I \propto \sqrt{\theta}$ . The connections of Fig. 10.24 are used when small currents are to be measured. In the case of heavy currents, a shunt  $S$  is used to limit current through the moving coil as shown in Fig. 10.26.

When used as a voltmeter, the fixed and moving coils are joined in series along with a high resistance and connected as shown in

Fig. 10.27. Here, again  $I_1 = I_2 = I$ , where  $I = \frac{V}{R}$  in d.c. circuits and

$I = V/Z$  in a.c. circuits.

$$\therefore \theta \propto V \times V \text{ or } \theta \propto V^2 \text{ or } V \propto \sqrt{\theta}$$

Hence, it is found that whether the instrument is used as an ammeter or voltmeter, its scale is uneven throughout the whole of its range and is particularly cramped or crowded near the zero.

**Damping** is pneumatic, since owing to weak operating field, eddy current damping is inadmissible. Such instruments can be used for both a.c. and d.c. measurements. But it is more expensive and inferior to a moving-coil instrument for d.c. measurements.

As mentioned earlier, the most important application of electrodynamic principle is the wattmeter and is discussed in detail in Art. 10.34.

### Errors

Since the coils are air-cored, the operating field produced is small. For producing an appreciable deflecting torque, a large number of turns is necessary for the moving coil. The magnitude of the current is also limited because two control springs are used both for leading in and for leading out the current. Both these factors lead to a heavy moving system resulting in frictional losses which are somewhat larger than in other types and so frictional errors tend to be relatively higher. The current in the field coils is limited for the fear of heating the coils which results in the increase of their resistance. A good amount of screening is necessary to avoid the influence of stray fields.

### Advantages and Disadvantages

1. Such instruments are free from hysteresis and eddy-current errors.
2. Since (torque/weight) ratio is small, such instruments have low sensitivity.

**Example 10.17.** The mutual inductance of a 25-A electrodynamic ammeter changes uniformly at a rate of  $0.0035 \mu\text{H/degree}$ . The torsion constant of the controlling spring is  $10^{-6} \text{ N-m per degree}$ . Determine the angular deflection for full-scale.

(Elect. Measurements, Poona Univ.)

**Solution.** By torsion constant is meant the deflecting torque per degree of deflection. If full-scale deflecting is  $\theta$  degree, then deflecting torque on full-scale is  $10^{-6} \times \theta \text{ N-m}$ .

Now,

$$T_d = I^2 dM/d\theta \quad \text{Also, } I = 25 \text{ A}$$

$$dM/d\theta = 0.0035 \times 10^{-6} \text{ H/degree} = 0.0035 \times 10^{-6} \times 180/\pi \text{ H/radian}$$

$$10^{-6} \times \theta = 25^2 \times 0.0035 \times 10^{-6} \times 180/\pi \quad \therefore \theta = 125.4^\circ$$

**Example 10.18.** The spring constant of a 10-A dynamometer wattmeter is  $10.5 \times 10^{-6} \text{ N-m per radian}$ . The variation of inductance with angular position of moving system is practically linear over the operating range, the rate of change being  $0.078 \text{ mH per radian}$ . If the full-scale deflection of the instrument is 83 degrees, calculate the current required in the voltage coil at full scale on d.c. circuit.

(Elect. Inst. and Means. Nagpur Univ. 1991)

**Solution.** As seen from foot-note of Art. 10.20,  $T_d = I_1 I_2 dM/d\theta \text{ N-m}$



$$\begin{aligned} \text{Spring constant} &= 10.5 \times 10^{-6} \text{ N/m/rad} = 10.5 \times 10^{-6} \times \pi/180 \text{ N-m/degree} \\ T_d = \text{spring constant} \times \text{deflection} &= (10.5 \times 10^{-6} \times \pi/180) \times 83 = 15.2 \times 10^{-6} \text{ N-m} \\ \therefore 15.2 \times 10^{-6} &= 10 \times I_2 \times 0.078 ; I_2 = \mathbf{19.5 \mu A}. \end{aligned}$$

### 10.21. Hot-wire Instruments

The working parts of the instrument are shown in Fig. 10.28. It is based on the heating effect of current. It consists of platinum-iridium (It can withstand oxidation at high temperatures) wire  $AB$  stretched between a fixed end  $B$  and the tension-adjusting screw at  $A$ . When current is passed through  $AB$ , it expands according to  $I^2R$  formula. This sag in  $AB$  produces a slack in phosphor-bronze wire  $CD$  attached to the centre of  $AB$ . This slack in  $CD$  is taken up by the silk fibre which after passing round the pulley is attached to a spring  $S$ . As the silk thread is pulled by  $S$ , the pulley moves, thereby deflecting the pointer. It would be noted that even a small sag in  $AB$  is magnified (Art. 10.22) many times and is conveyed to the pointer. Expansion of  $AB$  is magnified by  $CD$  which is further magnified by the silk thread.

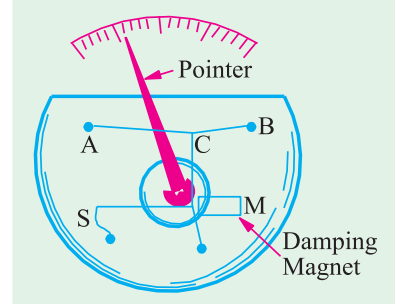


Fig. 10.28

It will be seen that the deflection of the pointer is proportional to the extension of  $AB$  which is itself proportional to  $I^2$ . Hence, deflection is  $\propto I^2$ . If spring control is used, then  $T_c \propto \theta$ .

Hence

$$\theta \propto I^2$$

So, these instruments have a 'square law' type scale. They read the r.m.s. value of current and their readings are independent of its form and frequency.

#### Damping

A thin light aluminium disc is attached to the pulley such that its edge moves between the poles of a permanent magnet  $M$ . Eddy currents produced in this disc give the necessary damping.

These instruments are primarily meant for being used as ammeters but can be adopted as voltmeters by connecting a high resistance in series with them. These instruments are suited both for a.c. and d.c. work.

#### Advantages of Hot-wire Instruments :

1. As their deflection depends on the r.m.s. value of the alternating current, they can be used on direct current also.
2. Their readings are independent of waveform and frequency.
3. They are unaffected by stray fields.

#### Disadvantages

1. They are sluggish owing to the time taken by the wire to heat up.
2. They have a high power consumption as compared to moving-coil instruments. Current consumption is 200 mA at full load.
3. Their zero position needs frequent adjustment.
4. They are fragile.

### 10.22. Magnification of the Expansion

As shown in Fig. 10.29 (a), let  $L$  be the length of the wire  $AB$  and  $dL$  its expansion after steady temperature is reached. The sag  $S$  produced in the wire as seen from Fig. 10.29 (a) is given by

$$S^2 = \left( \frac{L + dL}{2} \right)^2 - \left( \frac{L}{2} \right)^2 = \frac{2L \cdot dL + (dL)^2}{4}$$

$$\text{Neglecting } (dL)^2, \text{ we have } S = \sqrt{L \cdot dL/2}$$

$$\text{Magnification produced is } = \frac{S}{dL} = \frac{\sqrt{L \cdot dL/2}}{dL} = \sqrt{\frac{L}{2 \cdot dL}}$$

As shown in Fig. 10.29 (b), in the case of double-sag instruments, this sag is picked up by wire CD which is under the constant pull of the spring. Let  $L_1$  be the length of wire CD and let it be pulled at its center, so as to take up the slack produced by the sag  $S$  of the wire AB.

$$S_1^2 = \left(\frac{L_1}{2}\right)^2 - \left(\frac{L_1 - S}{2}\right)^2 = \frac{2L_1 S - S^2}{4}$$

Neglecting  $S^2$  as compared to  $2L_1 S$ , we have  $S_1 = \sqrt{L_1 S/2}$

Substituting the value of  $S$ , we get  $S_1 = \sqrt{\frac{L_1}{2}} \sqrt{\frac{L \cdot dL}{2}}$

$$\sqrt{\frac{L_1}{2}} \sqrt{\frac{L \cdot dL}{2}}$$

**Example 10.19.** The working wire of a single-sag hot wire instrument is 15 cm long and is made up of platinum-silver with a coefficient of linear expansion of  $16 \times 10^{-6}$ . The temperature rise of the wire is  $85^\circ\text{C}$  and the sag is taken up at the center. Find the magnification

(i) with no initial sag and (ii) with an initial sag of 1 mm.

(Elect. Meas and Meas. Inst., Calcutta Univ.)

**Solution.** (i) Length of the wire at room temperature = 15 cm

Length when heated through  $85^\circ\text{C}$  is =  $15 (1 + 16 \times 10^{-6} \times 85) = 15.02$  cm

Increase in length,  $dL = 15.02 - 15 = 0.02$  cm

Magnification  $\sqrt{\frac{L}{2 \cdot dL}} \sqrt{\frac{15}{0.04}} = 19.36$

(ii) When there is an initial sag of 1 mm, the wire is in the position ACB (Fig. 10.30). With rise in temperature, the new position becomes ADB. From the right-angled  $\triangle ADE$ ,

$$\text{we have } (S + 0.1)^2 = \left(\frac{L + dL}{2}\right)^2 - AE^2$$

$$\text{Now } AE^2 = AC^2 - EC^2 = \left(\frac{L}{2}\right)^2 - 0.1^2 = \frac{L^2}{4} - 0.1^2$$

$$\begin{aligned} \therefore (S + 0.1)^2 &= \frac{L^2}{4} - \frac{(dL)^2}{4} + \frac{L \cdot dL}{2} - \frac{L^2}{4} + 0.1^2 - \frac{L^2}{4} + \frac{(dL)^2}{4} - \frac{L \cdot dL}{2} + \frac{L^2}{4} + 0.01 \\ &= \frac{L \cdot dL}{2} + 0.01 \quad \dots \text{neglecting } (dL)^2/4 \\ &= \frac{15 \times 0.02}{2} + 0.01 = 0.16 \quad \therefore (S + 0.1) = 0.4 \quad \therefore S = 0.3 \text{ cm} \end{aligned}$$

$$\text{Magnification} = S/dL = 0.3/0.02 = 15$$

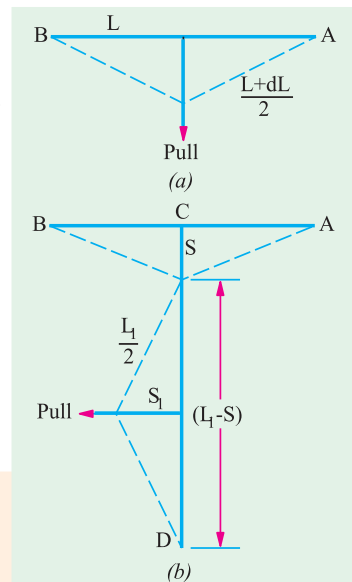


Fig. 10.29

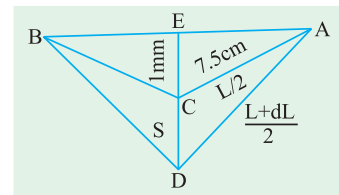


Fig. 10.30

### 10.23. Thermocouple Ammeter

The working principle of this ammeter is based on the Seebeck effect, which was discovered in 1821. A thermocouple, made of two dissimilar metals (usually bismuth and antimony) is used in the construction of this ammeter. The hot junction of the thermocouple is welded to a heater wire AB, both of which are kept in vacuum as shown in Fig. 10.31 (a). The cold junction of the thermocouple is connected to a moving-coil ammeter.

When the current to be measured is passed through the heater wire  $AB$ , heat is generated, which raises the temperature of the thermocouple junction  $J$ . As the junction temperature rises, the generated

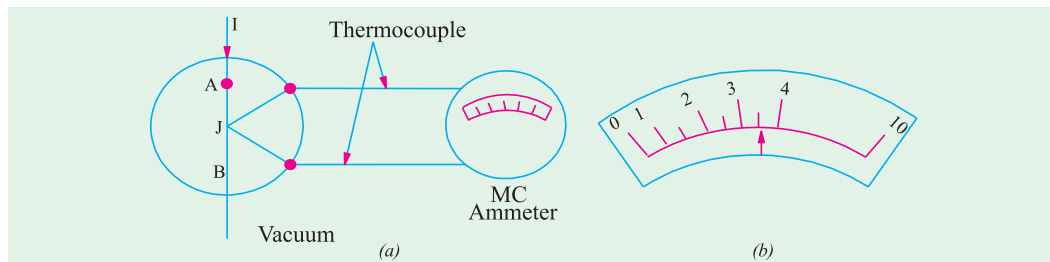


Fig. 10.31

thermoelectric EMF increases and drives a greater current through the moving-coil ammeter. The amount of deflection on the  $MC$  ammeter scale depends on the heating effect, since the amount of heat produced is directly proportional to the square of the current. The ammeter scale is non-linear so that, it is cramped at the low end and open at the high end as shown in Fig. 10.31 (b). This type of “current-squared” ammeter is suitable for reading both direct and alternating currents. It is particularly suitable for measuring radio-frequency currents such as those which occur in antenna systems of broadcast transmitters. Once calibrated properly, the calibration of this ammeter remains accurate from dc up to very high frequency currents.

### 10.24. Megger

It is a portable instrument used for testing the insulation resistance of a circuit and for measuring resistances of the order of megaohms which are connected across the outside terminals  $XY$  in Fig. 10.32 (b).



Megger

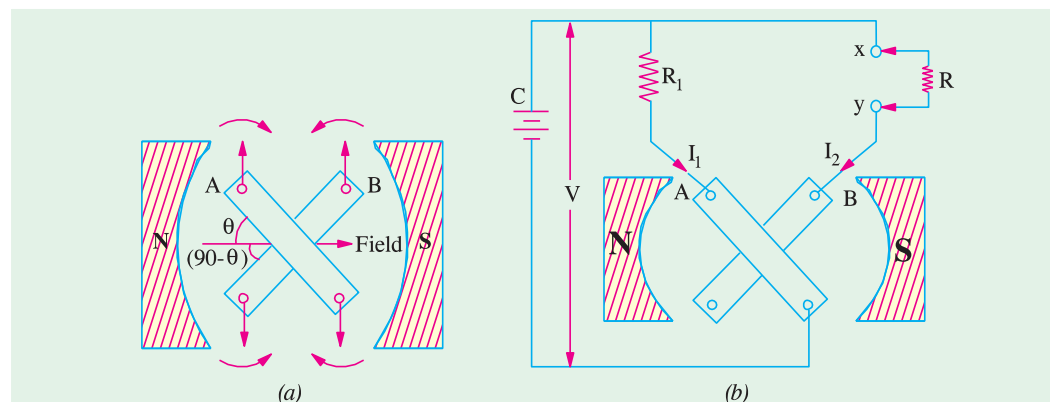


Fig. 10.32

#### 1. Working Principle

The working principle of a ‘cross-coil’ type megger may be understood from Fig. 10.32 (a) which shows two coils  $A$  and  $B$  mounted rigidly at right angles to each other on a common axis and free to rotate in a magnetic field. When currents are passed through them, the two coils are acted upon by torques which are in opposite directions. The torque of coil  $A$  is proportional to  $I_1 \cos \theta$  and that of  $B$  is proportional to  $I_2 \cos (90 - \theta)$  or  $I_2 \sin \theta$ . The two coils come to a position of equilibrium where the two torques are equal and opposite *i.e.* where

$$I_1 \cos \theta = I_2 \sin \theta \quad \text{or} \quad \tan \theta = I_1/I_2$$

In practice, however, by modifying the shape of pole faces and the angle between the two coils, the ratio  $I_1/I_2$  is made proportional to  $\theta$  instead of  $\tan \theta$  in order to achieve a linear scale.

Suppose the two coils are connected across a common source of voltage *i.e.* battery  $C$ , as shown in Fig. 10.32 (b). Coil  $A$ , which is connected directly across  $V$ , is called the voltage (or control) coil. Its current  $I_1 = V/R_1$ . The coil  $B$  called current or deflecting coil, carries the current  $I_2 = V/R$ , where  $R$  is the external resistance to be measured. This resistance may vary from infinity (for good insulation or open circuit) to zero (for poor insulation or a short-circuit). The two coils are free to rotate in the field of a permanent magnet. The deflection  $\theta$  of the instrument is proportional to  $I_1/I_2$  which is equal to  $R/R_1$ . If  $R_1$  is fixed, then the scale can be calibrated to read  $R$  directly (in practice, a current-limiting resistance is connected in the circuit of coil  $B$  but the presence of this resistance can be allowed for in scaling). The value of  $V$  is immaterial so long as it remains constant and is large enough to give suitable currents with the high resistance to be measured.

## 2. Construction

The essential parts of a megger are shown in Fig. 10.33. Instead of battery  $C$  of Fig. 10.32 (b), there is a hand-driven d.c. generator. The crank turns the generator armature through a clutch mechanism which is designed to slip at a pre-determined speed. In this way, the generator speed and voltage are kept constant and at their correct values when testing.

The generator voltage is applied across the voltage coil  $A$  through a fixed resistance  $R_1$  and across deflecting coil  $B$  through a current-limiting resistance  $R'$  and the external resistance is connected across testing terminal  $XY$ . The two coils, in fact, constitute a moving-coil voltmeter and an ammeter combined into one instrument.

(i) Suppose the terminals  $XY$  are open-circuited. Now, when crank is operated, the generator voltage so produced is applied across coil  $A$  and current  $I_1$  flows through it but no current flows through coil  $B$ . The torque so produced rotates the moving element of the megger until the scale points to 'infinity', thus indicating that the resistance of the external circuit is too large for the instrument to measure.

(ii) When the testing terminals  $XY$  are closed through a low resistance or are short-circuited, then a large current (limited only by  $R'$ ) passes through the deflecting coil  $B$ . The deflecting torque produced by coil  $B$  overcomes the small opposing torque of coil  $A$  and rotates the moving element until the needle points to 'zero', thus shown that the external resistance is too small for the instrument to measure.

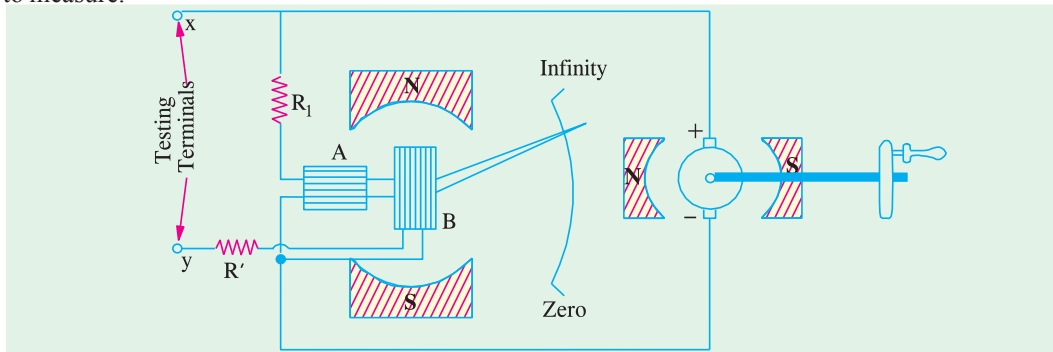


Fig. 10.33

Although, a megger can measure all resistance lying between zero and infinity, essentially it is a high-resistance measuring device. Usually, zero is the first mark and  $10\text{ k}\Omega$  is the second mark on its scale, so one can appreciate that it is impossible to accurately measure small resistances with the help of a megger.

The instrument described above is simple to operate, portable, very robust and independent of the external supplies.

## 10.25. Induction type Voltmeters and Ammeters

*Induction type instruments are used only for a.c. measurements and can be used either as ammeter,*

voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter. In such instruments, the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux. Before discussing the two types of most commonly-used induction instruments, we will first discuss the underlying principle of their operation.

**Principle**

The operation of all induction instruments depends on the production of torque due to the reaction between a flux  $\Phi_1$  (whose magnitude depends on the current or voltage to be measured) and eddy currents induced in a metal disc or drum by another flux  $\Phi_2$  (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depend on the flux producing them, the *instantaneous* value of torque is proportional to the square of current or voltage under measurement and the value of *mean* torque is proportional to the mean square value of this current or voltage.

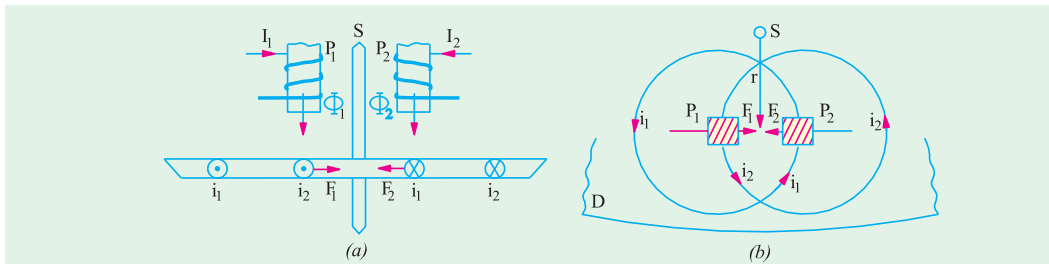


Fig. 10.34

Consider a thin aluminium or Cu disc  $D$  free to rotate about an axis passing through its centre as shown in Fig. 19.34. Two a.c. magnetic poles  $P_1$  and  $P_2$  produce alternating fluxes  $\Phi_1$  and  $\Phi_2$  respectively which cut this disc. Consider any annular portion of the disc around  $P_1$  with center on the axis of  $P_1$ . This portion will be linked by flux  $\Phi_1$  and so an alternating e.m.f.  $e_1$  be induced in it. This e.m.f. will circulate an eddy current  $i_1$  which, as shown in Fig. 10.34, will pass under  $P_2$ . Similarly,  $\Phi_2$  will induce an e.m.f.  $e_2$  which will further induce an eddy current  $i_2$  in an annular portion of the disc around  $P_2$ . This eddy current  $i_2$  flows under pole  $P_1$ .

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both  $\Phi_1$  and  $\Phi_2$  are increasing. By applying Lenz's law, the directions of the induced currents  $i_1$  and  $i_2$  can be found and are as indicated in Fig. 10.34.

The portion of the disc which is traversed by flux  $\Phi_1$  and carries eddy current  $i_2$  experiences a force  $F_1$  along the direction as indicated. As  $F = Bil$ , force  $F_1 \propto \Phi_1 i_2$ . Similarly, the portion of the disc lying in flux  $\Phi_2$  and carrying eddy current  $i_1$  experiences a force  $F_2 \propto \Phi_2 i_1$ .

$$\therefore F_1 \propto \Phi_1 i_2 = K \Phi_1 i_2 \text{ and } F_2 \propto \Phi_2 i_1 = K \Phi_2 i_1.$$

It is assumed that the constant  $K$  is the same in both cases due to the symmetrically positions of  $P_1$  and  $P_2$  with respect to the disc.

If  $r$  is the effective radius at which these forces act, the net instantaneous torque  $T$  acting on the disc begin equal to the difference of the two torques, is given by

$$T = r (K \Phi_1 i_2 - K \Phi_2 i_1) = K_1 (\Phi_1 i_2 - \Phi_2 i_1) \quad \dots(i)$$

Let the alternating flux  $\Phi_1$  be given by  $\Phi_1 = \Phi_{1m} \sin \omega t$ . The flux  $\Phi_2$  which is assumed to lag  $\Phi_1$  by an angle  $\alpha$  radian is given by  $\Phi_2 = \Phi_{2m} \sin (\omega t - \alpha)$

Induced e.m.f. 
$$e_1 = \frac{d\Phi_1}{dt} = \frac{d}{dt} (\Phi_{1m} \sin \omega t) = \omega \Phi_{1m} \cos \omega t$$

Assuming the eddy current path to be purely resistive and of value  $R^*$ , the value of eddy current is

\* If it has a reactance of  $X$ , then impedance  $Z$  should be taken, whose value is given by  $Z = \sqrt{R^2 + X^2}$ .

$$i_1 = \frac{e_1}{R} = \frac{1m}{R} \cos \omega t \quad \text{Similarly } e_2 = \frac{2m}{R} \cos(\omega t - \alpha) \quad \text{and } i_2 = \frac{2m}{R} \cos(\omega t - \alpha) *$$

Substituting these values of  $i_1$  and  $i_2$  in Eq. (i) above, we get

$$\begin{aligned} T &= \frac{K_1 \omega}{R} [\Phi_{1m} \sin \omega t \cdot \Phi_{2m} \cos(\omega t - \alpha) - \Phi_{2m} \sin(\omega t - \alpha) \Phi_{1m} \cos \omega t] \\ &= \frac{K_1 \omega}{R} \cdot \Phi_{1m} \Phi_{2m} [\sin \omega t \cdot \cos(\omega t - \alpha) - \cos \omega t \cdot \sin(\omega t - \alpha)] \\ &= \frac{K_1 \omega}{R} \cdot \Phi_{1m} \Phi_{2m} \sin \alpha = k_2 \omega \Phi_{1m} \Phi_{2m} \sin \alpha \quad (\text{putting } K_1/R = K_2) \end{aligned}$$

It is obvious that

(i) if  $\alpha = 0$  i.e. if two fluxes are in phase, then net torque is zero. If on the other hand,  $\alpha = 90^\circ$ , the net torque is maximum for given values of  $\Phi_{1m}$  and  $\Phi_{2m}$ .

(ii) the net torque is in such a direction as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(iii) since the expression for torque does not involve ' $t$ ', it is independent of time i.e. it has a steady value at all time.

(iv) the torque  $T$  is inversely proportional to  $R$ -the resistance of the eddy current path. Hence, for large torques, the disc material should have low resistivity. Usually, it is made of Cu or, more often, of aluminium.

### 10.26. Induction Ammeters

It has been shown in Art.10.22 above that the net torque acting on the disc is

$$T = K_2 \omega \Phi_{1m} \Phi_{2m} \sin \alpha$$

Obviously, if both fluxes are produced by the same alternating current (of maximum value  $I_m$ ) to be measured, then

$$T = K_3 \omega I_m^2 \sin \alpha$$

Hence, for a given frequency  $\omega$  and angle  $\alpha$ , the torque is proportional to the square of the current. If the disc has spring control, it will take up a steady deflected position where controlling torque becomes equal to the deflecting torque. By attaching a suitable pointer to the disc, the apparatus can be used as an ammeter.

There are three different possible arrangements by which the operational requirements of induction ammeters can be met as discussed below.

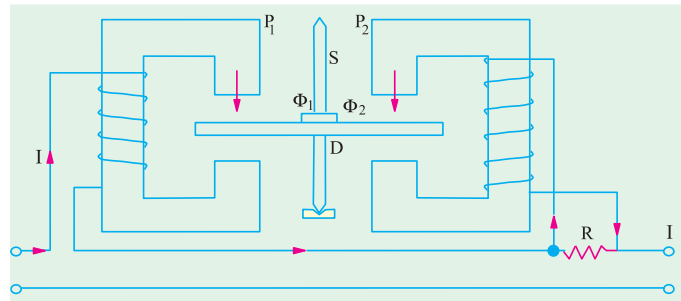


Fig. 10.35

#### (i) Disc Instrument with Split-phase Winding

In this arrangement, the windings on the two laminated a.c. magnets  $P_1$  and  $P_2$  are connected in series (Fig. 10.35). But, the winding of  $P_2$  is shunted by a resistance  $R$  with the result that the current in this winding lags with respect to the total line current. In this way, the necessary phase angle  $\alpha$  is produced between two fluxes  $\Phi_1$  and  $\Phi_2$  produced by  $P_1$  and  $P_2$  respectively. This angle is of the order of  $60^\circ$ . If the hysteresis effects etc. are neglected, then each flux would be proportional to the current to be measured i.e. line current  $I$

$$T_d \propto \Phi_{1m} \Phi_{2m} \sin \alpha$$

$$\text{or } T_d \propto I^2 \quad \text{where } I \text{ is the r.m.s. value.}$$

$$\text{If spring control is used, then } T_c \propto \theta$$

\* It being assumed that both paths have the same resistance.

In the final deflected position,  $T_c = T_d \therefore \theta \propto I^2$

Eddy current damping is employed in this instrument. When the disc rotates, it cuts the flux in the air-gap of the magnet and has eddy currents induced in it which provide efficient damping.

**(ii) Cylindrical type with Split-phase Winding**

The operating principle of this instrument is the same as that of the above instrument except that instead of a rotating disc, it employs a hollow aluminium drum as shown in Fig. 10.36. The poles  $P_1$  produce the alternating flux  $\Phi_1$  which produces eddy current  $i_1$  in those portions of the drum that lie under poles  $P_2$ . Similarly, flux  $F_2$  due to poles  $P_2$  produces eddy current  $i_2$  in those parts of the drum that lie under poles  $P_1$ . The force  $F_1$  which is  $\propto \Phi_1 i_2$  and  $F_2$  which is  $\propto \Phi_2 i_1$  are tangential to the surface of the drum and the resulting torque tends to rotate the drum about its own axis. Again, the winding of  $P_2$  is shunted by resistance  $R$  which helps to introduce the necessary phase difference  $\alpha$  between  $F_1$  and  $F_2$ .

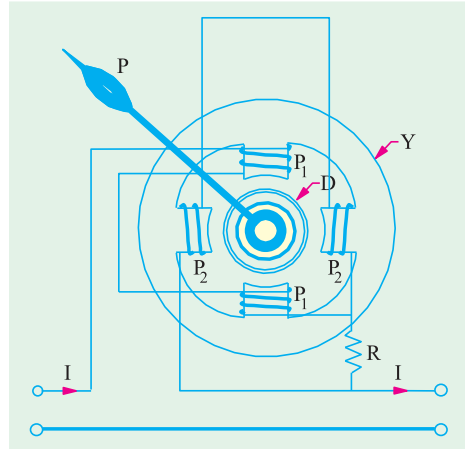


Fig. 10.36

The spiral control springs (not shown in the figure) prevent any continuous rotation of the drum and ultimately bring it to rest at a position where the deflecting torque becomes equal to the controlling torque of the springs. The drum has a pointer attached to it and is itself carried by a spindle whose two ends fit in jewelled bearings. There is a cylindrical laminated

core inside the hollow drum whose function is to strengthen the flux cutting the drum. The poles are laminated and magnetic circuits are completed by the yoke  $Y$  and the core.

Damping is by eddy currents induced in a separate aluminium disc (not shown in the figure) carried by the spindle when it moves in the air-gap flux of a horse-shoe magnet (also not shown in the figure).

**(iii) Shaded-pole Induction Ammeter**

In the shaded-pole disc type induction ammeter (Fig. 10.37) only single flux-producing winding is used. The flux  $F$  produced by this winding is split up into two fluxes  $\Phi_1$  and  $\Phi_2$  which are made to have the necessary phase difference of  $\alpha$  by the device shown in Fig. 10.37. The portions of the upper and lower poles near the disc  $D$  are divided by a slot into two halves one of which carries a closed 'shading' winding or ring. This shading winding or ring acts as a short-circuited secondary and the main winding as a primary. The current induced in the ring by transformer action retards the phase of flux  $\Phi_2$  with respect to that of  $\Phi_1$  by about  $50^\circ$  or so. The two fluxes  $\Phi_1$  and  $\Phi_2$  passing through the unshaded and shaded parts respectively, react with eddy currents  $i_2$  and  $i_1$  respectively and so produce the net driving torque whose value is

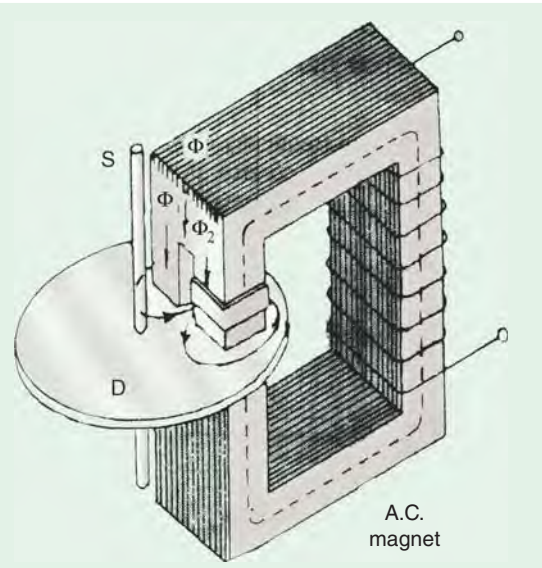


Fig. 10.37

$$T_d \propto \Phi_{1m} \Phi_{2m} \sin \alpha$$

Assuming that both  $\Phi_1$  and  $\Phi_2$  are proportional to the current  $I$ , we have

$$T_d \propto I^2$$

This torque is balanced by the controlling torque provided by the spiral springs.

The actual shaded-pole type induction instruments is shown in Fig. 10.38. It consists of a suitably-shaped aluminium or copper disc mounted on a spindle which is supported by jewelled bearings. The spindle carries a pointer and has a control spring attached to it. The edge or periphery of the disc moves in the air-gap of a laminated a.c. electromagnet which is energised either by the current to be measured (as ammeter) or by a current proportional to the voltage to be measured (as a voltmeter). Damping is by eddy currents induced by a permanent magnet embracing another portion

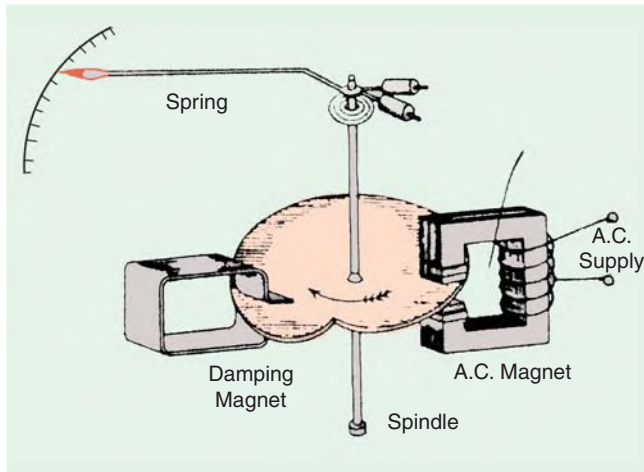


Fig. 10.38

of the **same** disc. As seen, the disc serves both for damping as well as operating purposes. The main flux is split into two component fluxes by shading one-half of each pole. These two fluxes have a phase difference of  $40^\circ$  to  $50^\circ$  between them and they induce two eddy currents in the disc. Each eddy current has a component in phase with the **other** flux, so that two torques are produced which are oppositely directed. The resultant torque is equal to the difference between the two. This torque **deflects** the disc-continuous rotation being prevented by the control spring and the deflection produced is proportional to the square of the current or voltage being measured.

As seen, for a given frequency,  $T_d \propto I^2 = KI^2$

For spring control  $T_c \propto \theta$  or  $T_c = K_1 \theta$

For steady deflection, we have  $T_c = T_d$  or  $\theta \propto I^2$

Hence, such instruments have uneven scales *i.e.* scales which are cramped at their lower ends. A more even scale can, however, be obtained by using a cam-shaped disc as shown in Fig. 10.38.

### 10.27. Induction Voltmeter

Its construction is similar to that of an induction ammeter except for the difference that its winding is wound with a large number of turns of fine wire. Since it is connected across the lines and carries very small current (5–10mA), the number of turns of its wire has to be large in order to produce an adequate amount of m.m.f. Split phase windings are obtained by connecting a high resistance  $R$  in series with the winding of one magnet and an inductive coil in series with the winding of the other magnet as shown in Fig. 10.39.

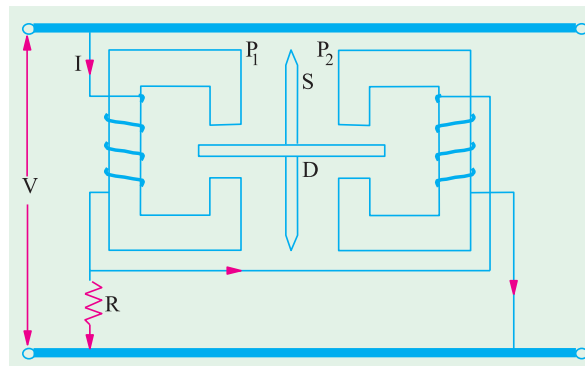


Fig. 10.39

### 10.28. Errors in Induction Instruments

There are two types of errors (*i*) **frequency** error and (*ii*) **temperature** error.

1. Since deflecting torque depends on frequency, hence unless the alternating current to be measured has same frequency with which the instrument was calibrated, there will be large



error in its readings. Frequency errors can be compensated for by the use of a non-inductive shunt in the case of ammeters. In voltmeters, such errors are not large and, to a great extent, are self-compensating.

2. Serious errors may occur due to the variation of temperature because the resistances of eddy current paths depends on the temperature. Such errors can, however, be compensated for by hunting in the case of ammeters and by combination of shunt and swamping resistances in the case of voltmeters.

### 10.29. Advantages and Disadvantages

1. A full-scale deflection of over  $200^\circ$  can be obtained with such instruments. Hence, they have long open scales.
2. Damping is very efficient.
3. They are not much affected by external stray fields.
4. Their power consumption is fairly large and cost relatively high.
5. They can be used for a.c. measurements only.
6. Unless compensated for frequency and temperature variations, serious errors may be introduced.

### 10.30. Electrostatic Voltmeters

Electrostatic instruments are almost always used as voltmeters and that too more as a laboratory rather than as industrial instruments. The underlying principle of their operation is the force of attraction between electric charges on neighboring plates between which a p.d. is maintained. This force gives rise to a deflecting torque. Unless the p.d. is sufficiently large, the force is small. Hence, such instruments are used for the measurement of very high voltages.

There are two general types of such instruments :

- (i) *the quadrant type*—used upto 20 kV. (ii) *the attracted disc type*— used upto 500 kV.

### 10.31. Attracted-disc Type Voltmeter

As shown in Fig. 10.40, it consists of two-discs or plates  $C$  and  $D$  mounted parallel to each other. Plate  $D$  is fixed and is earthed while  $C$  suspended by a coach spring, the support for which carries a micrometer head for adjustment. Plate  $C$  is connected to the positive end of the supply voltage. When a p.d. (whether direct or alternating) is applied between the two plates, then  $C$  is attracted towards  $D$  but may be returned to its original position by the micrometer head. The movement of this head can be made to indicate the force  $F$  with which  $C$  is pulled downwards. For this purpose, the instrument can be calibrated by placing known weights in turn on  $C$  and observing the movement of micrometer head necessary to bring  $C$  back to its original position. Alternatively, this movement of plate  $C$  is balanced by a control device which actuates a pointer attached to it that sweeps over a calibrated scale.

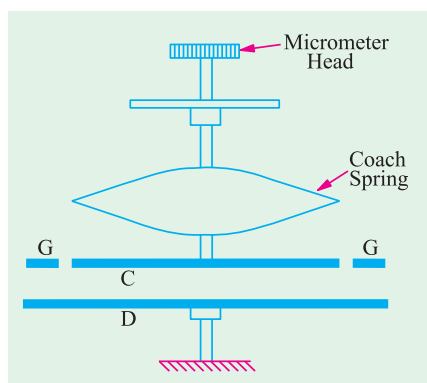


Fig. 10.40

There is a guard ring  $G$  surrounding the plate  $C$  and separated from it by a small air-gap. The ring is connected electrically to plate  $C$  and helps to make the field uniform between the two plates. The effective area of plate  $C$ , in that case, becomes equal to its actual area plus half the area of the air-gap.

#### Theory

In Fig. 10.41 are shown two parallel plates separated by a distance of  $x$  meters. Suppose the lower plate is fixed and carries a charge of  $-Q$  coulomb whereas the upper plate is movable and

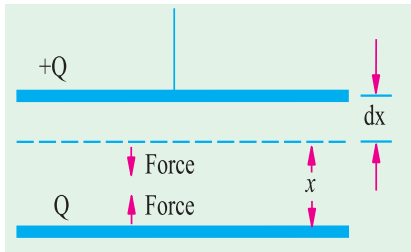


Fig. 10.41

carries a charge of  $+Q$  coulomb. Let the mutual force of attraction between the two plates be  $F$  newtons. Suppose the upper plate is moved apart by a distance  $dx$ . Then mechanical work done during this movement is  $F \times dx$  joule. Since charge on the plate is constant, no electrical energy can move into the system from outside. This work is done at the case of the energy stored in the parallel-plate capacitor formed by the two plates.

Before movement, let the capacitance of the capacitor be  $C$  farad. Then,

$$\text{Initial energy stored} = \frac{1}{2} \cdot \frac{Q^2}{C}$$

If the capacitance changes to  $(C + dC)$  because of the movement of plate, then

$$\text{Final energy stored} = \frac{1}{2} \frac{Q^2}{(C + dC)} = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{1}{\left(1 + \frac{dC}{C}\right)} = \frac{1}{2} \frac{Q^2}{C} \left(1 + \frac{dC}{C}\right)^{-1}$$

$$= \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) \text{ if } dC \leq C$$

$$\text{Change in stored energy} = \frac{1}{2} \frac{Q^2}{C} - \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C}$$

$$\therefore F \times dx = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C} \text{ or } F = \frac{1}{2} \frac{Q^2}{C^2} \cdot \frac{dC}{dx} = \frac{1}{2} V^2 \frac{dC}{dx}$$

$$\text{Now, } C = \frac{0.0001 A}{x} \quad \frac{dC}{dx} = \frac{0.0001 A}{x^2} \quad F = \frac{1}{2} V^2 \frac{0.0001 A}{x^2} \text{ N}$$

Hence, we find that force is directly proportional to the square of the voltage to be measured. The negative sign merely shows that it is a force of attraction.

### 10.32. Quadrant Type Voltmeters

The working principle and basic construction of such instruments can be understood from Fig. 10.42. A light aluminium vane  $C$  is mounted on a spindle  $S$  and is situated partially within a hollow metal quadrant  $B$ . Alternatively, the vane be suspended in the quadrant. When the vane and the quadrant are oppositely charged by the voltage under measurement, the vane is further attracted inwards into the quadrant thereby causing the spindle and hence the pointer to rotate. The amount of rotation and hence the deflecting torque is found proportional to  $V^2$ . The deflecting torque in the case of arrangement shown in Fig. 10.42 is very small unless  $V$  is extremely large.

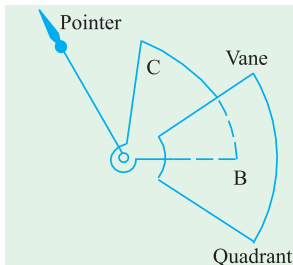


Fig. 10.42

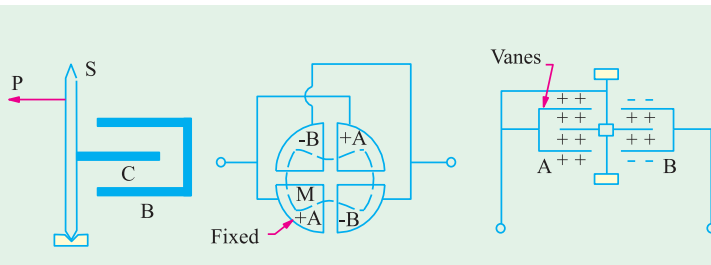


Fig. 10.43

The force on the vane may be increased by using a larger number of quadrants and a double-ended vane. In Fig. 10.43 are shown four fixed metallic double quadrants arranged so as to form a circular box with short air-gaps between the quadrants in which is suspended or pivoted as aluminium vane. Opposite quadrants  $AA$  and  $BB$  are joined together and each pair is connected to on terminal of the

a.c. or d.c. supply and at the same time, one pair is connected to the moving vane  $M$ . Under these conditions [Fig. 10.43.] the moving vane is recalled by quadrants  $AA$  and attracted by quadrants  $BB$ . Hence, a deflecting torque is produced which is proportional to  $(p.d.)^2$ . Therefore, such voltmeters have an uneven scale. Controlling torque is produced by torsion of the suspension spring or by the spring (used in pivoted type voltmeters). Damping is by a disc or vane immersed in oil in the case of suspended type or by air friction in the case of pivoted type instruments.

### Theory

With reference to Fig 10.42, suppose the quadrant and vane are connected across a source of  $V$  volts and let the resulting deflection be  $\theta$ . If  $C$  is a capacitance between the quadrant and vane in the deflected position, then the charge on the instrument will be  $CV$  coulomb. Suppose that the voltage is charged from  $V$  to  $(V + dV)$ , then as a result, let  $\theta$ ,  $C$  and  $Q$  change to  $(\theta + d\theta)$ ,  $(C + dC)$  and  $(Q + dQ)$  respectively. Then, the energy stored in the electrostatic field is increased by

$$dE = d \left[ \frac{1}{2} CV^2 \right] = \frac{1}{2} V^2 \cdot dC + CV \cdot dV \text{ joule}$$

If  $T$  is the value of controlling torque corresponding to a deflection of  $\theta$ , then the additional energy stored in the control will be  $T \times d\theta$  joule.

$$\text{Total increase in stored energy} = T \times d\theta + \frac{1}{2} V^2 \cdot dC + CVdV \text{ joule}$$

It is seen that during this charge, the source supplies a change  $dQ$  at potential  $V$ . Hence, the value of energy supplied is

$$= V \times dQ = V \times d(CV) = V^2 \times dC + CV \cdot dV$$

Since the energy supplied by the source must be equal to the extra energy stored in the field and the control

$$\therefore T \times d\theta + \frac{1}{2} V^2 \cdot dC + CV \cdot dV = V^2 \cdot dC + CVdV$$

or

$$T \cdot d \left[ \frac{1}{2} V^2 \cdot dC \right] = T \cdot \frac{1}{2} V^2 \frac{dC}{d} \text{ N-m}$$

The torque is found to be proportional to the square of the voltage to be measured whether that voltage is alternating or direct. However, in alternating circuits the scale will read r.m.s. values.

### 10.33. Kelvin's Multicellular Voltmeter

As shown in Fig. 10.44, it is essentially a quadrant type instrument, as described above, but with the difference that instead of four quadrants and one vane, it has a large number of fixed quadrants and vanes mounted on the same spindle. In this way, the deflecting torque for a given voltage is increased many times. Such voltmeters can be used to measure voltages as low as 30 V. As said above, this reduction in the minimum limit of voltage is due to the increasing operating force in proportion to the number of component units. Such an instrument has a torsion head for zero adjustment and a coach spring for protection against accidental fracture of suspension due to vibration etc. There is a pointer and scale of edgewise pattern and damping is by a vane immersed in an oil dashpot.

### 10.34. Advantages and Limitation of Electrostatic Voltmeters

Some of the main advantages and use of electrostatic voltmeters are as follows :

1. They can be manufactured with first grade accuracy.
2. They give correct reading both on d.c. and a.c. circuits. On a.c. circuits, the scale will, however, read r.m.s. values whatever the wave-form.
3. Since no iron is used in their construction, such instruments are free from hysteresis and eddy current losses and temperature errors.
4. They do not draw any continuous current on d.c. circuits and that drawn on a.c. circuits (due to the capacitance of the instrument) is extremely small. Hence, such voltmeters do not cause any disturbance to the circuits to which they are connected.

5. Their power loss is negligibly small.
6. They are unaffected by stray magnetic fields although they have to be guarded against any stray electrostatic field.
7. They can be used upto 1000 kHz without any serious loss of accuracy.

However, their main limitations are :

1. Low-voltage voltmeters (like Kelvin's Multicellular voltmeter) are liable to friction errors.
2. Since torque is proportional to the square of the voltage, their scales are not uniform although some uniformity can be obtained by suitably shaping the quadrants of the voltmeters.
3. They are expensive and cannot be made robust.

### 10.35. Range Extension of Electrostatic Voltmeters

The range of such voltmeters can be extended by the use of multipliers which are in the form of a resistance potential divider or capacitance potential divider. The former method can be used both for direct and alternating voltages whereas the latter method is useful only for alternating voltages.

#### (i) Resistance Potential Divider

This divider consists of a high non-inductive resistance across a small portion which is attached to the electrostatic voltmeter as shown in Fig. 10.45. Let  $R$  be the resistance of the whole of the potential divider across which is applied the voltage  $V$  under measurement. Suppose  $V$  is the maximum value of the voltage which the voltmeter can measure without the multiplier. If  $r$  is the resistance of the portion of the divider across which voltmeter is connected, then the multiplying factor is given by

$$\frac{V}{v} = \frac{R}{r}$$

The above expression is true for d.c. circuits but for a.c. circuits, the capacitance of the voltmeter (which is in parallel with  $r$ ) has to be taken into account. Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

#### (ii) Capacitance Potential Divider

In this method, the voltmeter may be connected in series with a single capacity  $C$  and put across the voltage  $V$  which is to be measured [Fig. 10.46 (a)] or a number of capacitors may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. 10.46 (b).

Consider the connection shown in Fig. 10.46 (a). It is seen that the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{reactance of total circuit}}{\text{reactance of voltmeter}}$$

Now, capacitance of the total circuit is  $\frac{CC_v}{C + C_v}$  and its reactance is

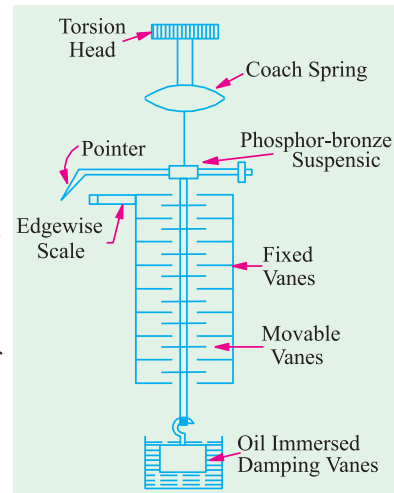


Fig. 10.44

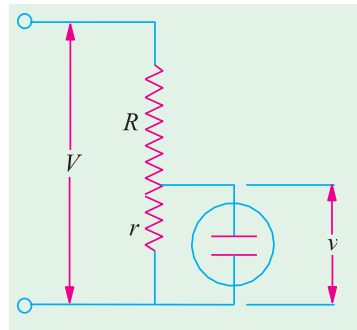


Fig. 10.45

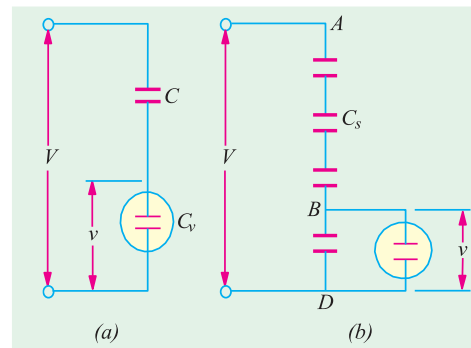


Fig. 10.46

$$= \frac{1}{\omega \times \text{capacitance}} = \frac{C + C_v}{\omega C C_v}$$

$$\text{Reactance of the voltmeter} = \frac{1}{\omega C_v}$$

$$\therefore \frac{V}{v} = \frac{(C + C_v)/C C_v}{1/C_v} = \frac{C + C_v}{C} \quad \text{Multiplying factor} = \frac{C + C_v}{C} = 1 + \frac{C_v}{C}$$

**Example 10.20.** The reading '100' of a 120-V electrostatic voltmeter is to represent 10,000 volts when its range is extended by the use of a capacitor in series. If the capacitance of the voltmeter at the above reading is 70  $\mu\text{F}$ , find the capacitance of the capacitor multiplier required.

**Solution.** Multiplying factor =  $\frac{V}{v} = 1 + \frac{C_v}{C}$

Here,  $V = 10,000$  volt,  $v = 100$  volts ;  $C_v =$  capacitance of the voltmeter = 70  $\mu\text{F}$

$$C = \text{capacitance of the multiplier} \quad \therefore \frac{10,000}{100} = 1 + \frac{70}{C}$$

or  $70/C = 99 \quad \therefore C = 70/99 \mu\text{F} = 0.707 \mu\text{F}$  (approx)

**Example 10.21 (a).** An electrostatic voltmeter is constructed with 6 parallel, semicircular fixed plates equal-spaced at 4 mm intervals and 5 interleaved semi-circular movable plates that move in planes midway between the fixed plates, in air. The movement of the movable plates is about an axis through the center of the circles of the plates system, perpendicular to the planes of the plates. The instrument is spring-controlled. If the radius of the movable plates is 4 cm, calculate the spring constant if 10 kV corresponds to a full-scale deflection of  $100^\circ$ . Neglect fringing, edge effects and plate thickness.

(Elect. Measurements, Bombay Univ.)

**Solution.** Total number of plates (both fixed and movable) is 11, hence there are 10 parallel plate capacitors.

Suppose, the movable plates are rotated into the fixed plates by an angle of  $\theta$  radian. Then, overlap area between one fixed and one movable semi-circular plate is

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 0.04^2 \times \theta = 8 \times 10^{-4} \theta \text{ m}^2; d = 4/2 = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

Capacitance of each of ten parallel-plate capacitors is

$$C = \frac{\epsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 8 \times 10^{-4} \theta}{2 \times 10^{-3}} = 3.54 \times 10^{-12} \theta \text{ F}$$

Total capacitance  $C = 10 \times 3.54 \times 10^{-12} \theta = 35.4 \times 10^{-12} \theta \text{ F} \therefore dC/d\theta = 35.4 \times 10^{-12}$  farad/radian

$$\text{Deflecting torque} = \frac{1}{2} V^2 \frac{dC}{d\theta} \text{ N-m} = \frac{1}{2} \times (10,000)^2 \times 35.4 \times 10^{-12} = 17.7 \times 10^{-4} \text{ N-m}$$

If  $S$  is spring constant i.e. torque per radian and  $\theta$  is the plate deflection, then control torque is

$$T_c = S\theta$$

Here,  $\theta = 100^\circ = 100 \times \pi/180 = 5\pi/9$  radian

$$\therefore S \times 5\pi/9 = 17.7 \times 10^{-4} \quad \therefore S = 10.1 \times 10^{-4} \text{ N-m/rad.}$$

**Example 10.21 (b).** A capacitance transducer of two parallel plates of overlapping area of  $5 \times 10^{-4} \text{ m}^2$  is immersed in water. The capacitance 'C' has been found to be 9.50 pF. Calculate the separation 'd' between the plates and the sensitivity,  $S = \partial C/\partial d$ , of this transducer, given :  $\epsilon_r$  water = 81 ;  $\epsilon_0 = 8.854 \text{ pF/m}$ .

(Elect. Measuer. A.M.I.E. Sec. B, 1992)

**Solution.** Since  $C = \epsilon_0 \epsilon_r A/d$ ,  $d = 3 \epsilon_0 \epsilon_r A/C$ .

Substituting the given values we get,  $d = 37.7 \times 10^{-3} \text{ m}$

\* It is helpful to compare it with a similar expression in Art. 10.17 for permanent magnet moving-coil instruments.

$$\text{Sensitivity } \frac{\partial C}{\partial d} = \frac{\partial}{\partial d} \left( \frac{\epsilon_0 \epsilon_r A}{d} \right) = - \frac{8.854 \times 10^{-12} \times 81 \times 5 \times 10^{-4}}{(37.7 \times 10^{-3})^2} = -0.025 \times 10^{-8} \text{ F/m}$$

### 10.36. Wattmeters

We will discuss the two main types of wattmeters in general use, that is, (i) the dynamometer or electrodynamic type and (ii) the induction type.

### 10.37. Dynamometer Wattmeter

The basic principle of dynamometer instrument has already been explained in detail in Art. 10.20. The connections of a dynamometer type wattmeter are shown in Fig. 10.47. The fixed

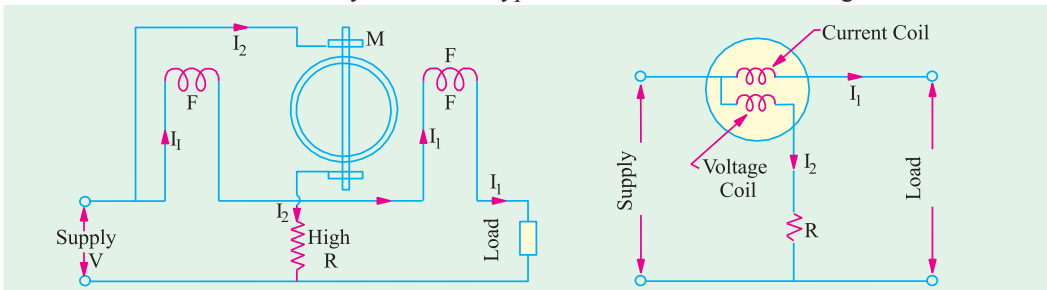


Fig. 10.47

Fig. 10.48

circular coil which carries the main circuit current  $I_1$  is wound in two halves positioned parallel to each other. The distance between the two halves can be adjusted to give a uniform magnetic field. The moving coil which is pivoted centrally carries a current  $I_2$  which is proportional to the voltage  $V$ . Current  $I_2$  is led into the moving coil by two springs which also supply the necessary controlling torque. The equivalent diagrammatic view is shown in Fig. 10.48.

#### Deflecting Torque

Since coils are air-cored, the flux density produced is directly proportional to the current  $I_1$ .

$$\therefore B \propto I_1 \text{ or } B = K_1 I_1; \text{ current } I_2 \propto V \text{ or } I_2 = K_2 V$$

$$\text{Now } T_d \propto B I_2 \propto I_1 V \quad \therefore T_d = K V I_1 = K \times \text{power}$$

In d.c. circuits, power is given by the product of voltage and current in amperes, hence torque is directly proportional to the power.

Let us see how this instrument indicates true power on a.c. circuits.

For a.c. supply, the value of instantaneous torque is given by  $T_{inst} \propto v i = K v i$

where  $v$  = instantaneous value of voltage across the moving coil  
 $i$  = instantaneous value of current through the fixed coils.

However, owing to the large inertia of the moving system, the instrument indicates the mean or average power.

$\therefore$  Mean deflecting torque  $T_m \propto$  average value of  $v i$

$$\begin{aligned} \text{Let } v &= V_{max} \sin \theta \quad \text{and } i = I_{max} \sin (\theta - \phi) \quad \therefore T_m \propto \frac{1}{2\pi} \int_0^{2\pi} V_{max} \sin \theta \times I_{max} \sin (\theta - \phi) d\theta \\ &\propto \frac{V_{max} I_{max}}{2} \int_0^{2\pi} \sin \theta \sin (\theta - \phi) d\theta = \frac{V_{max} I_{max}}{2} \int_0^{2\pi} \frac{\cos \phi - \cos (2\theta - \phi)}{2} d\theta \\ &\propto \frac{V_{max} I_{max}}{4\pi} \left[ \theta \cos \phi - \frac{\sin (2\theta - \phi)}{2} \right]_0^{2\pi} \propto \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cdot \cos \phi \propto V I \cos \phi \end{aligned}$$

where  $V$  and  $I$  are the r.m.s. values.  $\therefore T_m \propto V I \cos \phi \propto$  true power.

Hence, we find that in the case of a.c. supply also, the deflection is proportional to the true power in the circuit.

Scales of dynamometer wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection. Hence  $\theta \propto \text{power}$ . Damping is pneumatic with the help of a piston moving in an air chamber as shown in Fig. 10.49.

### Errors

The inductance of the moving or voltage coil is liable to cause error but the high non-inductive resistance connected in series with the coil swamps, to a great extent, the phasing effect of the voltage-coil inductance.

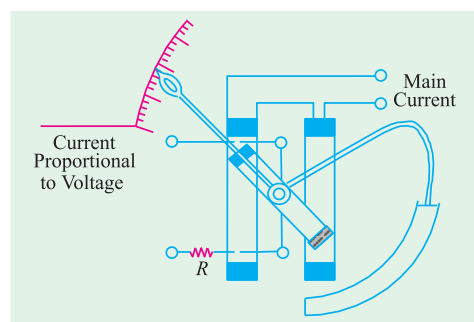


Fig. 10.49

Another possible error in the indicated power may be due to (i) some voltage drop in the circuit or (ii) the current taken by the voltage coil. In standard wattmeters, this defect is overcome by having an additional compensating winding which is connected in series with the voltage coil but is so placed that it produces a field in opposite direction to that of the fixed or current coils.

### Advantages and Disadvantages

By careful design, such instruments can be built to give a very high degree of accuracy. Hence they are used as a standard for calibration purposes. They are equally accurate on d.c. as well as a.c. circuits.

However, at low power factors, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect [Art. 10.38 (ii)].

## 10.38. Wattmeter Errors

### (i) Error Due to Different Connections

Two possible ways of connecting a wattmeter in a single-phase a.c. circuit are shown in Fig. 10.50 along with their phasor diagrams. In Fig. 10.50 (a), the pressure or voltage-coil current does not pass through the current coil of the wattmeter whereas in the connection of Fig. 10.50 (b) it does. A wattmeter is supposed to indicate the power consumed by the load but its actual reading is slightly higher due to power losses in the instrument circuits. The amount of error introduced depends on the connection.

(a) Consider the connection of Fig. 10.50 (a). If  $\cos \phi$  is the power factor of the load, then power in the load is  $VI \cos \theta$ .

Now, voltage across the pressure-coil of the wattmeter is  $V_1$  which is the phasor sum of the load voltage  $V$  and p.d. across current-coil of the instrument *i.e.*  $V'$  ( $= Ir$  where  $r$  is the resistance of the current coil).

Hence, power reading as indicated by the wattmeter is  $= V_1 I \cos \theta$

where  $\theta$  = phase difference between  $V_1$  and  $I$  as shown in the phasor diagram of Fig. 10.50 (a).

As seen from the phasor diagram,  $V_1 \cos \theta = (V \cos \phi + V')$

$$\begin{aligned} \therefore \text{wattmeter reading} &= V_1 \cos \theta \cdot I = (V \cos \phi + V') I \\ &= VI \cos \phi + V' I = VI \cos \phi + I^2 r = \text{power in load} + \text{power in current coil.} \end{aligned}$$

(b) Next, consider the connection of Fig. 10.50 (b). The current through the current-coil of the wattmeter is the phasor sum of load current  $I$  and voltage-coil current  $I' = V/R$ . The power reading indicated by the wattmeter is  $= VI_1 \cos \theta$ .

As seen from the phasor diagram of Fig. 10.50 (b),  $I_1 \cos \theta = (I \cos \phi + I')$

$$\begin{aligned} \therefore \text{wattmeter reading} &= V(I \cos \phi + I') = VI \cos \phi + VI' = VI \cos \phi + V^2/R \\ &= \text{power in load} + \text{power in pressure-coil circuit} \end{aligned}$$

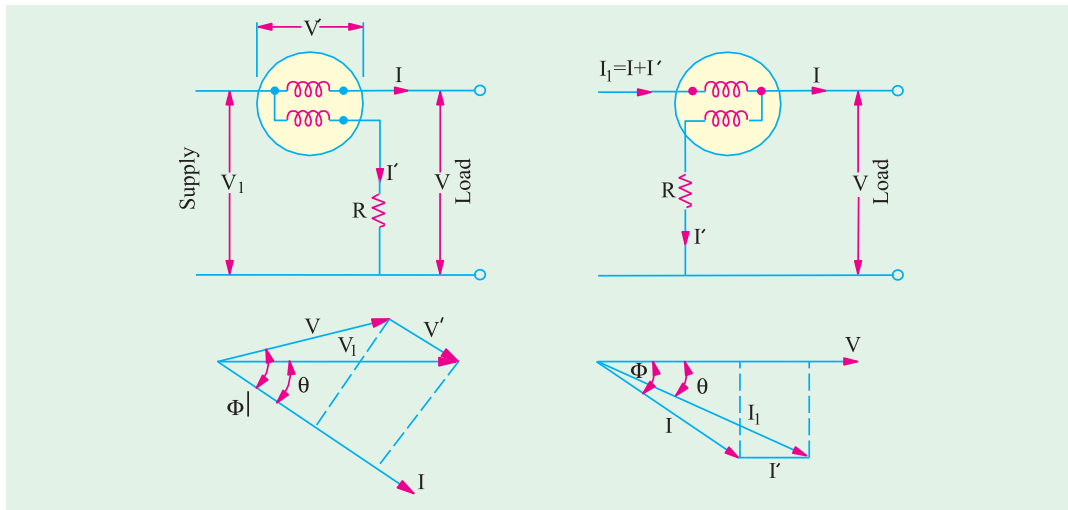


Fig. 10.50

**(ii) Error Due to Voltage-coil Inductance**

While developing the theory of electrodynamic instruments, it was assumed that pressure-coil does not possess any inductance (and hence reactance) so that current drawn by it was  $= V/R$ . The wattmeter reading is proportional to the mean deflecting torque, which is itself proportional to  $I_1 I_2 \cos \theta$ , where  $\theta$  is the angle between two currents (Fig. 10.52).

In case the inductance of the voltage-coil is neglected.

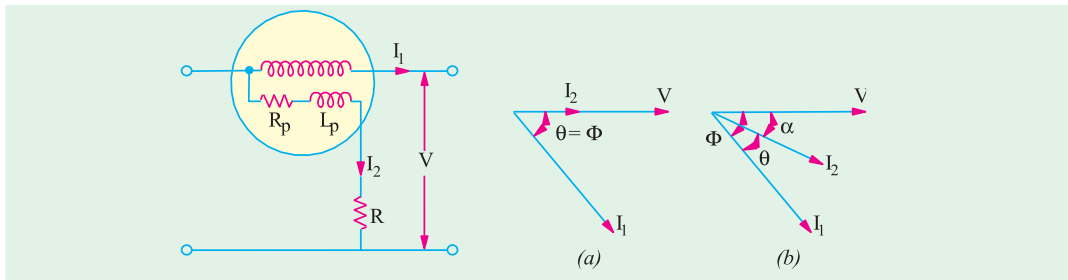


Fig. 10.51

Fig. 10.52

$$I_2 = V/(R + R_p) = V/R \text{ approximately}$$

and  $\theta = \phi$  as shown in the phasor diagram of 10.52 (a)

$$\therefore \text{wattmeter reading} \propto \frac{I_1 V}{R} \cos \phi \quad \dots(i)$$

In case, inductance of the voltage coil is taken into consideration, then

$$I_2 = \frac{V}{\sqrt{(R_p + R)^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_p}$$

It lags behind  $V$  by an angle  $\alpha$  [Fig. 10.52 (b)] such that

$$\tan \alpha = X_L/(R_p + R) = X_L/R \text{ (approx.)} = \omega L_p/R$$

$$\therefore \text{wattmeter reading} \propto \frac{I_1 V}{Z_p} \cos \theta \propto \frac{I_1 V}{Z_p} \cos (\phi - \theta)$$

$$\text{Now} \quad \cos \alpha = \frac{R_p + R}{Z_p} = \frac{R}{Z_p} \quad \therefore Z_p = \frac{R}{\cos \alpha}$$



$\therefore$  wattmeter reading in this case is  $\propto I_1 \frac{V}{R} \cos \alpha \cos (\phi - \alpha)$  ...**(ii)**

Eq. **(i)** above, gives wattmeter reading when inductance of the voltage coil is neglected and Eq. **(ii)** gives the reading when it is taken into account.

The correction factor which is given by the ratio of the true reading ( $W_t$ ) and the actual or indicated reading ( $W_a$ ) of the wattmeter is

$$\frac{W_t}{W_a} = \frac{\frac{VI_1}{R_1} \cos \phi}{\frac{VI_1}{R_1} \cos \alpha \cos (\phi - \alpha)} = \frac{\cos \phi}{\cos \alpha \cos (\phi - \alpha)}$$

Since, in practice,  $\alpha$  is very small,  $\cos \alpha = 1$ . Hence the correction factor becomes  $= \frac{\cos \phi}{\cos (\phi - \alpha)}$

$\therefore$  True reading  $= \frac{\cos \phi}{\cos \alpha \cos (\phi - \alpha)} \times \text{actual reading} \cong \frac{\cos \phi}{\cos (\phi - \alpha)} \times \text{actual reading}$

The error in terms of the actual wattmeter reading can be found as follows :

Actual reading – true reading

$$\begin{aligned} &= \text{actual reading} - \frac{\cos \phi}{\cos \alpha \cos (\phi - \alpha)} \times \text{actual reading} \\ &= 1 - \frac{\cos \phi}{\cos (\phi - \alpha)} \times \text{actual reading} = 1 - \frac{\cos \phi}{\cos \phi \sin \alpha} \times \text{actual reading} \\ &= \frac{\sin \phi \sin \alpha - \cos \phi}{\cos \phi \sin \alpha} \times \text{actual reading} = \frac{\sin \phi \sin \alpha - \cos \phi}{\cot \alpha \sin \alpha} \times \text{actual reading} \end{aligned}$$

The error, expressed as a fraction of the actual reading, is  $= \frac{\sin \phi \sin \alpha - \cos \phi}{\cot \alpha \sin \alpha}$

$$\text{Percentage error} = \frac{\sin \phi \sin \alpha - \cos \phi}{\cot \alpha \sin \alpha} \times 100$$

### **(iii) Error Due to Capacitance in Voltage-coil Circuit**

There is always present a small amount of capacitance in the voltage-coil circuit, particularly in the series resistor. Its effect is to reduce angle  $\alpha$  and thus reduce error due to the inductance of the voltage coil circuit. In fact, in some wattmeters, a small capacitor is purposely connected in parallel with the series resistor for obtaining practically non-inductive voltage-coil circuit. Obviously, over-compensation will make resultant reactance capacitive thus making  $\alpha$  negative in the above expressions.

### **(iv) Error Due to Stray Fields**

Since operating field of such an instrument is small, it is very liable to stray field errors. Hence, it should be kept as far away as possible from stray fields. However, errors due to stray fields are, in general, negligible in a properly-constructed instrument.

### **(v) Error Due to Eddy Currents**

The eddy current produced in the solid metallic parts of the instrument by the alternating field of the current coil changes the magnitude and strength of this operating field thus producing an error in the reading of the wattmeter. This error is not easily calculable although it can be serious if care is not taken to remove away solid masses of metal from the proximity of the current coil.

**Example 10.22.** A dynamometer type wattmeter with its voltage coil connected across the load side of the instrument reads 250 W. If the load voltage be 200 V, what power is being taken by load? The voltage coil branch has a resistance of 2,000  $\Omega$

**(Elect. Engineering, Madras Univ.)**

**Solution.** Since voltage coil is connected across the load side of the wattmeter (Fig. 10.53), the power consumed by it is also included in the meter reading.

$$\begin{aligned} \text{Power consumed by voltage coil is} \\ &= V^2/R = 200^2/2,000 = 20 \text{ W} \\ \therefore \text{Power being taken by load} &= 250 - 20 \\ &= 230 \text{ W} \end{aligned}$$

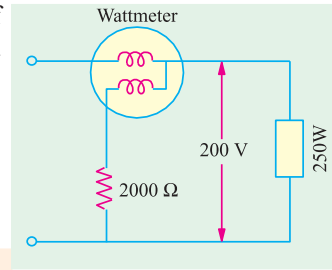


Fig. 10.53

**Example 10.23.** A 250-V, 10-A dynamometer type wattmeter has resistance of current and potential coils of 0.5 and 12,500 ohms respectively. Find the percentage error due to each of the two methods of connection when unity p.f. loads at 250 volts are of (a) 4A (b) 12 A.

Neglect the error due to the inductance of pressure coil.

(Elect. Measurements, Pune. Univ.)

**Solution.** (a) When  $I = 4 \text{ A}$

(i) Consider the type of connection shown in Fig. 10.50 (a)

$$\begin{aligned} \text{Power loss in current coil of wattmeter} &= I^2 r = 4^2 \times 0.5 = 8 \text{ W} \\ \text{Load power} &= 250 \times 4 \times 1 = 1000 \text{ W} ; \text{Wattmeter reading} = 1008 \text{ W} \\ \therefore \text{percentage error} &= (8/1008) \times 100 = 0.794\% \end{aligned}$$

(ii) Power loss in pressure coil resistance =  $V^2/R = 250^2/12,500 = 5 \text{ W}$   
 $\therefore$  Percentage error =  $5 \times 100/1005 = 0.497\%$

(b) When  $I = 12 \text{ A}$

$$\begin{aligned} \text{(i) Power loss in current coil} &= 12^2 \times 0.5 = 72 \text{ W} \\ \text{Load power} &= 250 \times 12 \times 1 = 3000 \text{ W} ; \text{wattmeter reading} = 3072 \text{ W} \\ \therefore \text{percentage error} &= 72 \times 100/3072 = 2.34\% \end{aligned}$$

(ii) Power loss in the resistance of pressure coil is  $250^2/12,500 = 5 \text{ W}$   
 $\therefore$  percentage error =  $5 \times 100/3005 = 0.166\%$

**Example 10.24.** An electrodynamic wattmeter has a voltage circuit of resistance of 8000  $\Omega$  and inductance of 63.6 mH which is connected directly across a load carrying a current of 8A at a 50-Hz voltage of 240-V and p.f. of 0.1 lagging. Estimate the percentage error in the wattmeter reading caused by the loading and inductance of the voltage circuit.

(Elect & Electronic Measu. & Instru. Nagpur, Univ. 1992)

**Solution.** The circuit connections are shown in Fig. 10.54.

$$\begin{aligned} \text{Load power} &= 240 \times 8 \times 0.1 = 192 \text{ W} \\ \cos \phi &= 0.1, \phi = \cos^{-1}(0.1) = 84^\circ 16' \\ \text{Power loss in voltage coil circuit is} &= V^2/R \\ &= 240^2/8000 = 7.2 \text{ W} \end{aligned}$$

Neglecting the inductance of the voltage coil, the wattmeter reading would be  
 $= 192 + 7.2 = 199.2 \text{ W}$

$$\begin{aligned} \text{Now, } X_p &= 2\pi \times 50 \times 63.3 \times 10^{-3} = 20 \Omega \\ \alpha &= \tan^{-1}(20/8000) = \tan^{-1}(0.0025) = 0^\circ 9' \end{aligned}$$

$$\text{Error factor due to inductance of the voltage coil} = \frac{\cos(\phi - \alpha)}{\cos \phi} = \frac{\cos 84^\circ 7'}{\cos 84^\circ 16'} = 1.026$$

$$\text{Wattmeter reading} = 1.026 \times 199.2 = 204.4 \text{ W}$$

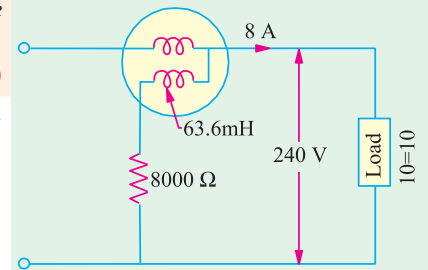


Fig. 10.54

$$\text{Percentage error} = \left( \frac{204.4 - 199.2}{199.2} \right) \times 100 = 2.6 \%$$

**Example 10.25.** The inductive reactance of the pressure-coil circuit of a dynamometer wattmeter is 0.4 % of its resistance at normal frequency and the capacitance is negligible.

Calculate the percentage error and correction factor due to reactance for load at (i) 0.707 p.f. lagging and (ii) 0.5 p.f. lagging. (Elect. Measurement, Bombay Univ.)

**Solution.** It is given that  $X_p/R = 0.4$   $R = 0.004$

$\tan \alpha = X_p/R = 0.004 \therefore \alpha = 0^\circ 14'$  and  $\sin \alpha = 0.004$

(i) When p.f. = 0.707 (i.e.  $\phi = 45^\circ$ )

$$\text{Correction factor} = \frac{\cos \phi}{\cos(\phi - \alpha)} = \frac{\cos 45^\circ}{\cos 44^\circ 46'} = 0.996$$

$$\begin{aligned} \text{Percentage error} &= \frac{\sin \alpha}{\cot \phi + \sin \alpha} \times 100 = \frac{\sin 0^\circ 14'}{\cot 45^\circ + \sin 0^\circ 14'} \times 100 \\ &= \frac{0.004 \times 100}{1 + 0.004} = \frac{0.4}{1.004} = 0.4 \text{ (approx)} \end{aligned}$$

(ii) When p.f. = 0.5 (i.e.,  $\phi = 60^\circ$ )

$$\text{Correction factor} = \frac{\cos 60^\circ}{\cos 59^\circ 46'} = 0.993$$

$$\text{Percentage error} = \frac{\sin 0^\circ 14'}{\cot 60^\circ + \sin 0^\circ 14'} \times 100 = \frac{0.004}{0.577} \frac{100}{0.004} = \frac{0.4}{0.581} = 0.7$$

**Example 10.26.** The current coil of wattmeter is connected in series with an ammeter and an inductive load. A voltmeter and the voltage circuit of the wattmeter are connected across a 400-Hz supply. The ammeter reading is 4.5 A and voltmeter and wattmeter readings are respectively 240 V and 29 W. The inductance of the voltage circuit is 5 mH and its resistance is 4 k $\Omega$ . If the voltage drops across the ammeter and current coil are negligible, what is the percentage error in wattmeter readings?

**Solution.** The reactance of the voltage-coil circuit is  $X_p = 2\pi \times 400 \times 5 \times 10^{-3} \pi$  ohm

$$\tan \alpha = X_p/R = 4\pi/4000 = 0.00314^2$$

$$\begin{aligned} \therefore \alpha &= 0.003142 \text{ radian (}\therefore \text{angle is very small)} \\ &= 0.18^\circ \text{ or } 0^\circ 11' \end{aligned}$$

$$\text{Now, true reading} = \frac{\cos \phi}{\cos \alpha \cos(\phi - \alpha)} \times \text{actual reading}$$

$$\text{or } VI \cos \phi = \frac{\cos \phi}{\cos \alpha \cos(\phi - \alpha)} \times \text{actual reading}$$

$$\text{or } VI = \frac{\text{actual reading}}{\cos(\phi - \alpha)}$$

taking  $(\cos \alpha = 1)$

$$\therefore \cos(\phi - \alpha) = 29/240 \times 4.5 = 0.02685$$

$$\therefore \phi - \alpha = 88^\circ 28' \text{ or } \phi = 88^\circ 39'$$

$$\begin{aligned} \therefore \text{Percentage error} &= \frac{\sin \alpha}{\cot \phi + \sin \alpha} \times 100 = \frac{\sin 11'}{\cot 88^\circ 39' + \sin 11'} \times 100 \\ &= \frac{0.0032}{0.235} \times 100 = 12 \% \end{aligned}$$

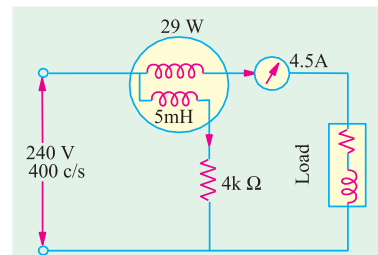


Fig. 10.55

### 10.39. Induction Wattmeters

Principle of induction wattmeters is the same as that of induction ammeters and voltmeters. They can be used on a.c. supply only in constant with dynamometer wattmeters, which can be used both on d.c. and a.c. supply. Induction wattmeters are useful only when the frequency and supply voltage are constant.

Since, both a current and a pressure element are required in such instrument, it is not essential to use the shaded-pole principle. Instead of this, two separate a.c. magnets are used, which produce two fluxes, which have the required phase difference.

### Construction

The wattmeter has two laminated electromagnets, one of which is excited by the current in the main circuit-exciting winding being joined in series with the circuit, hence it is also called a **series** magnet. The other is excited by current which is proportional to the voltage of the circuit. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as **shunt** magnet.

A thin aluminium disc is so mounted that it cuts the fluxes of both magnets. Hence, two eddy currents are produced in the disc. The deflection torque is produced due to the interaction of these eddy current and the inducing fluxes. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by  $90^\circ$ .

Two most common forms of the electromagnets are shown in Fig. 10.56 and 10.57. It is seen that in both cases, one magnet is placed above and the other below the disc. The magnets are so positioned and shaped that their fluxes are cut by the disc.

In Fig. 10.56, the two pressure coils are joined in series and are so wound that both send the flux through the central limb in the same direction. The series magnet carries two coils joined in series and so wound that they magnetise their respective cores in the same direction. Correct phase displacement between the shunt and series magnet fluxes can be obtained by adjusting the position of the copper shading bands as shown.

In the type of instrument shown in Fig. 10.57, there is only one pressure winding and one current winding. The two projecting poles of the shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with respect to the voltage.

Both types of induction wattmeters shown above, are spring-controlled, the spring being fitted to the spindle of the moving system which also carries the pointer. The scale is uniformly even and extends over  $300^\circ$ .

Currents upto 100 A can be handled by such wattmeters directly but for currents greater than this value, they are used in conjunction with current transformers. The pressure coil is purposely made as much inductive as possible in order that the flux through it should lag behind the voltage by  $90^\circ$ .

### Theory

The winding of one magnet carries line current  $I$  so that  $\Phi_1 \propto I$  and is in phase with  $I$  (Fig. 10.58).

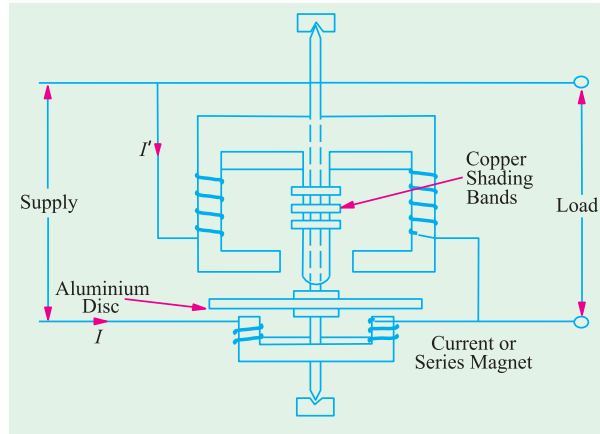


Fig. 10.56

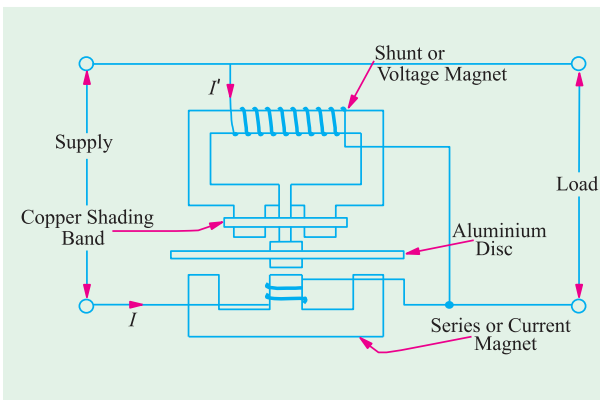


Fig. 10.57

The other coil *i.e.*, pressure or voltage coil is made highly inductive having an inductance of  $L$  and negligible resistance. This is connected across the supply voltage  $V$ . The current in the pressure coil is therefore, equal to  $V/\omega L$ . Hence,  $\Phi_2 \propto V/\omega L$  and lags behind the voltage by  $90^\circ$ . Let the load current  $I$  lag behind  $V$  by  $\phi$  *i.e.*, let the load power factor angle be  $\phi$ . As shown in Fig. 10.56, the phase angle between  $\Phi_1$  and  $\Phi_2$  is  $\alpha = (90 - \phi)$ .

The value of the torque acting on the disc is given by

$$T = k\omega \Phi_{1m} \Phi_{2m} \sin \alpha \quad \text{— Art. 10.25}$$

$$\text{or } T \propto 2 \omega I \cdot \frac{V}{\omega L} \cdot \sin (90 - \phi) \propto VI \cos \phi \propto \text{power}$$

Hence, the torque is proportional to the power in the load circuit. For spring control, the controlling torque  $T_c \propto \theta$ .  $\therefore \theta \propto \text{power}$ . Hence, the scale is even.

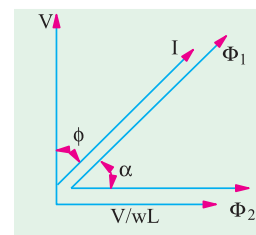


Fig. 10.58

### 10.40. Advantage and Limitations of Induction Wattmeters

These wattmeters possess the advantages of fairly long scales (extending over  $300^\circ$ ), are free from the effects of stray fields and have good damping. They are practically free from frequency errors. However, they are subject to (sometimes) serious temperature errors because the main effect of temperature is on the resistance of the eddy current paths.

### 10.41. Energy Meters

Energy meters are integrating instruments, used to measure quantity of electric energy supplied to a circuit in a given time. They give no direct indication of power *i.e.*, as to the rate at which energy is being supplied because their registrations are independent of the rate at which a given quantity of electric energy is being consumed. Supply or energy meters are generally of the following types :

- (i) **Electrolytic meters** - their operation depends on electrolytic action.
- (ii) **Motor meters** - they are really small electric motors.
- (iii) **Clock meters** - they function as clock mechanisms.

### 10.42. Electrolytic Meter

It is used on d.c. circuits\* only and is essentially an ampere-hour meter and not a true watt-hour meter. However, its registrations are converted into watt-hour by multiplying them by the voltage (assumed constant) of the circuits in which it is used. Such instruments are usually calibrated to read kWh directly at the declared voltage. Their readings would obviously be incorrect when used on any other voltage. **Because of the question of power factor, such instrument cannot be used on a.c. circuits.**

The advantages of simplicity, cheapness and of low power consumption of ampere-hour meters are, to a large extent, discounted by the fact that variations in supply voltage are not taken into account by them. As an example suppose that the voltage of a supply whose nominal value is 220 V, has an average value of 216 volts in one hour during which a consumer draws a current of 100 A. Quantity of electricity as measured by the instrument which is calibrated on 220 V, is  $220 \times 100/1000 = 22$  kWh. Actually, the energy consumed by the customer is only  $216 \times 100/1000 = 21.6$  kWh. Obviously, the consumer is being overcharged to the extent of the cost of  $22 - 21.6 = 0.4$  kWh of energy per hour. A true watt-hour-meter would have taken into account the decrease in the supply voltage and would have, therefore, resulted in a saving to the consumer. If the supply voltage would

\* Recently such instruments have been marketed for measurement of kilovoltampere-hours on a.c. supply, using a small rectifier unit, which consists of a current transformer and full-wave copper oxide rectifier.

have been higher by that amount, then the supply company would have been the loser (Ex. 10.27).

In this instrument, the operating current is passed through a suitable electrolyte contained in a voltmeter. Due to electrolysis, a deposit of mercury is given or a gas is liberated (depending on the type of meter) in proportion to the quantity of electricity passed (Faraday's Laws of Electrolysis). The quantity of electricity passed is indicated by the level of mercury in a graduated tube. Hence, such instruments are calibrated in amp-hour or if constancy of supply voltage is assumed, are calibrated in watt-hour or kWh.

Such instruments are cheap, simple and are accurate even at very small loads. They are not affected by stray magnetic fields and due to the absence of any moving parts are free from friction errors.

### 10.43. Motor Meters

Most commonly-used instruments of this type are :

(i) *Mercury motor* meters (ii) *Commutator motor* meters and (iii) *Induction motor* meters.

Of these, mercury motor meter is normally used on d.c. circuits whereas the induction type instrument is used only on a.c. circuits. However, the commutator type meter can be used both for d.c. as well as a.c. work.

Instruments used for d.c. work can be either in the form of a amp-hour meters or watt-hour meters. In both cases, the moving system is allowed to revolve continuously instead of being merely allowed to deflect or rotate through a fraction of a revolution as in indicating instruments. The speed of rotation is directly proportional to the current in the case of amp-hour meter and to power in the case of watt-hour meter. Hence the number of revolutions made in a given time is proportional, in the case of amp-hour meter, to the quantity of electricity ( $Q = i \times t$ ) and in the case of Wh meter, to the quantity of energy supplied to the circuit. The number of revolutions made are registered by a counting mechanism consisting of a train of gear wheels and dials.

The control of speed of the rotating system is brought about by a permanent magnet (known as braking magnet) which is so placed as to set up eddy currents in some parts of the rotating system. These eddy currents produce a retarding torque which is proportional to their magnitude-their magnitude itself depending on the speed of rotation of the rotating system. The rotating system attains a *steady speed* when the braking torque exactly balances the driving torque which is produced either by the current or power in the circuit.

The essential parts of motor meters are :

1. An operating system which produces an operating torque proportional to the current or power in the circuit and which causes the rotation of the rotating system.
2. A retarding or braking device, usually a permanent magnet, which produces a braking torque is proportional to the speed of rotation. Steady speed of rotation is achieved when braking torque becomes equal to the operating torque.
3. A registering mechanism for the revolutions of the rotating system. Usually, it consists of a train of wheels driven by the spindle of the rotating system. A worm which is cut on the spindle engages a pinion and so driven a wheel-train.

### 10.44. Errors in Motor Meters

The two main errors in such instruments are : (i) friction error and (ii) braking error. Friction error is of much more importance in their case than the corresponding error in indicating instruments because (a) it operates continuously and (b) it affects the speed of the rotor. The braking action in such meters corresponding to damping in indicating instruments. The braking torque directly affects the speed for a given driving torque and also the number of revolution made in a given time.

Friction torque can be compensated for providing a small constant driving torque which is applied to the moving system independent of the load.

As said earlier, steady speed of such instruments is reached when driving torque is equal to the braking torque. The braking torque is proportional to the flux of the braking magnet and the eddy current induced in the moving system due to its rotation in the field of the braking magnet

$$\therefore T_B \propto \Phi_i$$

where  $\Phi$  is the flux of the braking magnet and  $i$  the induced current. Now  $i = e/R$  where  $e$  is the induced e.m.f. and  $R$  the resistance of the eddy current path. Also  $e \propto \Phi n$  where  $n$  is the speed of the moving part of the instrument.

$$\therefore T_b \propto \frac{n}{R} \frac{n^2}{R}$$

The torque  $T_B'$  at the steady speed of  $N$  is given  $T_B' \propto \Phi^2 N/R$

Now  $T_B' = T_D$  – the driving torque

$$\therefore T_D \propto \Phi^2 N/R \quad \text{or} \quad N \propto T_D R / \Phi^2$$

Hence for a given driving torque, the steady speed is directly proportional to the resistance of the eddy current of path and inversely to the square of the flux.

Obviously, it is very important that the strength of the field of the brake magnet should be constant throughout the time the meter is in service. The constancy of field strengths can be assured by careful design treatment during the manufacturing of the brake magnet. Variations in temperature will affect the braking torque since the resistance of the eddy current path will change. This error is difficult to fully compensate for.

### 10.45. Quantity or ampere-hour Meters

The use of such meters is mostly confined to d.c. circuits. Their operation depends on the production of two torques (i) a driving torque which is proportional to the current  $I$  in the circuit and (ii) a braking torque which is proportional to the speed  $n$  of the spindle. This speed attains a steady value  $N$  when these two torques become numerically equal. In that case, speed becomes proportional to current *i.e.*,  $N \propto I$ . Over a certain period of time, the total number of revolutions  $\int N dt$  will be proportional to the quantity of electricity  $\int I dt$  passing through the meter. A worm cut in the spindle as its top engages gear wheels of the recording mechanism which has suitably marked dials reading directly in ampere hours. Since electric supply charges are based on watt-hour rather than ampere-hours, the dials of ampere-hour meters are frequently marked in corresponding watt-hour at the normal supply voltage. Hence, their indications of watt-hours are correct only when the supply voltages remains constant, otherwise reading will be wrong.

### 10.46. Ampere-hour Mercury Motor Meter

It is one of the best and most popular forms of mercury Ah meter used for d.c. work.

#### Construction

It consists of a thin Cu disc  $D$  mounted at the base of a spindle, working in jewelled cup bearings and revolving between a pair of **permanent** magnets  $M_1$  and  $M_2$ . One of the two magnets *i.e.*,  $M_2$  is used for driving purposes whereas  $M_1$  is used for braking. In between the poles of  $M_1$  and  $M_2$  is a hollow circular box  $B$  in which rotates the Cu disc and the rest of the space is filled up with mercury which exerts considerable upward thrust on the disc, thereby reducing the pressure on the bearings. The spindle is so weighted that it just sinks in the mercury bath. A worm cut in the spindle at its top engages the gear wheels of the recording mechanism as shown in Fig. 10.59.

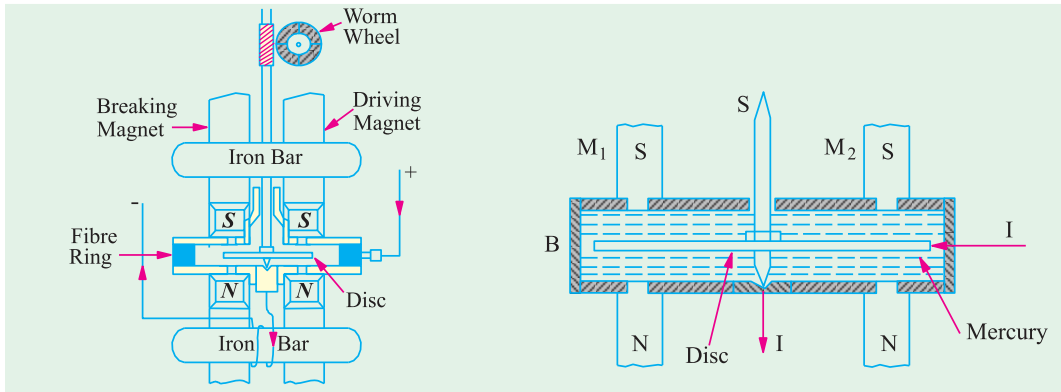


Fig. 10.59

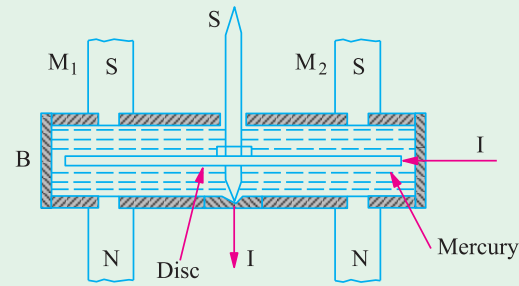


Fig. 10.60

### Principle of Action

Its principle of action can be understood from Fig. 10.61 which shows a separate line drawing of the motor element.

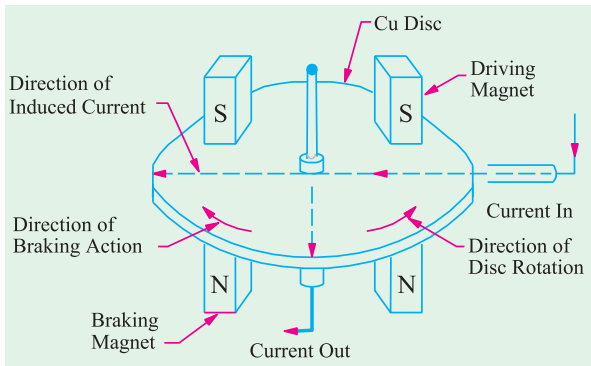


Fig. 10.61

The current to be measured is led into the disc through the mercury at a point at its circumference on the right-hand side. As shown by arrows, it flows radially to the centre of the disc where it passes out to the external circuit through the spindle and its bearings. It is worth noting **that current flows takes place only under the right-hand side magnet  $M_2$**  and not under the left-hand side magnet  $M_1$ . The field of  $M_2$  will, therefore, exert a force on the right-side portion of the disc which carries the current (motor action). The direction of the force, as found by Fleming's Left-hand rule,

is as shown by the arrow. The magnitude of the force depends on the flux density and current ( $\because F = BIl$ ). The driving or motoring torque  $T_d$  so produced is given by the product of the force and the distance from the spindle at which this force acts. When the disc rotates under the influence of this torque, it cuts through the field of left-hand side magnet  $M_1$  and hence eddy currents are produced in it which results in the production of braking torque. The magnitude of the retarding or braking torque is proportional to the speed of rotation of the disc.

### Theory

Driving torque  $T_d \propto \text{force on the disc} \times B I$

If the flux density of  $M_2$  remains constant, then  $T_d \propto I$ .

The braking torque  $T_B$  is proportional to the flux  $\Phi$  of braking magnet  $M_1$  and eddy current  $i$  induced in the disc due to its rotation in the field of  $M_1$ .

$\therefore$

$$T_B \propto \Phi_i$$

Now  $i = e/R$  where  $e$  is the induced e.m.f. and  $R$  the resistance of eddy current path.

Also  $e \propto \Phi_n$  - where  $n$  is the speed of the disc  $\therefore T_B \propto \Phi \times \frac{n}{R} = \frac{\Phi^2 n}{R}$

The speed of the disc will attain a steady value  $N$  when the driving and braking torques becomes equal. In that case,  $T_B \propto \Phi^2 N/R$ .

If  $\Phi$  and  $R$  are constant, then  $I \propto N$

The total number of revolution in any given time  $t$  i.e.,  $\int_0^t N dt$  will become proportional to



$\int_0^t I \cdot dt$  i.e., to the total quantity of electricity passed through the meter.

### 10.47. Friction Compensation

There are two types of frictions in this ampere-hour meter.

- (i) **Bearing Friction.** The effect of this friction is normally negligible because the disc and spindle float in mercury. Due to the upward thrust, the pressure on bearings is considerably reduced which results in freedom from wear as well as great reduction in the bearing friction.
- (ii) **Mercury Friction.** Since the disc revolves in mercury, there is friction between mercury and the disc, which gives rise to a torque, approximately proportional to the square of the speed of rotation. Hence, this friction causes the meter to run shown on heavy loads. It can be compensated for in the following two ways :
  - (a) a coil of few turns is wound on one of the poles of the driving magnet  $M_2$  and the meter current is passed through it in a suitable direction so as to increase the strength of  $M_2$ . The additional driving torque so produced can be made just sufficient to compensate for the mercury friction.
  - (b) in the other method, two iron bars are placed across the permanent magnets, one above and other one below the mercury chamber as shown in Fig. 10.62. The lower bar carries a small compensating coil through which is passed the load current. The local magnetic field set up by this coil strengthens the field of driving magnet  $M_2$  and weakens that of the braking magnet  $M_1$ , thereby compensating for mercury friction.

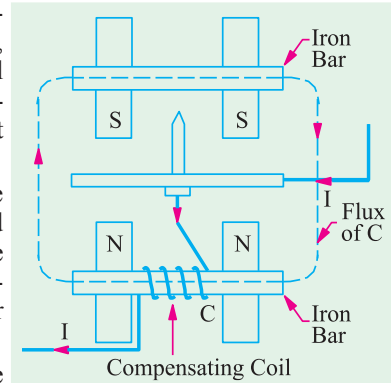


Fig. 10.62

### 10.48. Mercury Meter Modified as Watt-hour meter

If the permanent magnet  $M_2$  of the amp-hour meter, used for producing the driving torque, is replaced by a wound electromagnet connected across the supply, the result is a watt-hour meter. The exciting current of this electromagnet is proportional to the voltage of the supply. The driving torque is exerted on the aluminium disc immersed in the mercury chamber below which is placed this electromagnet. The aluminium disc has radial slots cut in it for ensuring the radial flow of current through it the current being led into and out of this disc through mercury contacts situated at diametrically opposite points. These radial slots, moreover, prevent the same disc being used for braking purposes. Braking is by a separate aluminium disc mounted on the same spindle and revolving in the air-gap of a separate braking magnet.

### 10.49. Commutator Motor Meters

These meters may be either ampere-hour or true watt-hour meters. In Fig. 10.63 is shown the principle of a common type of watt-hour meter known as Elihu-Thomson meter. It is based on the dynamometer principle (Art. 10.20) and is essentially an ironless motor with a wound armature having a commutator.

#### Construction

There are two fixed coils  $C_1$  and  $C_2$  each consisting of a few turns of heavy copper strip and joined in series with each other and with the supply circuit so that they carry the main current in

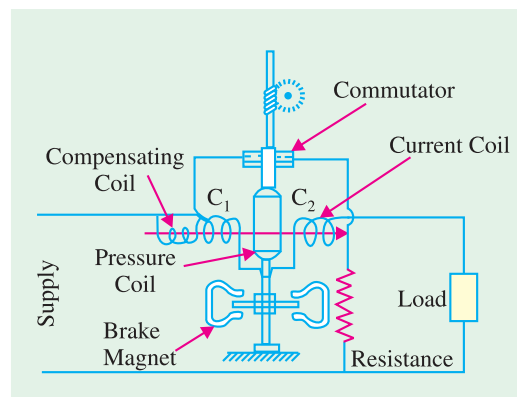


Fig. 10.63

the circuit (a shunt is used if the current is too heavy). The field produced by them is proportional to the current to be measured. In this field rotates an armature carrying a number of coils which are connected to the segments of a small commutator. The armature coils are wound on a former made of non-magnetic material and are connected through the brushes and in series with a large resistance across the supply lines. The commutator is made of silver and the brushes are silver tipped in order to reduce friction. Obviously, the current passing through the armature is proportional to the supply voltage.

The operating torque is produced due to the reaction of the field produced by the fixed coils and the armature coils. The magnitude of this torque is proportional to the product of the two currents *i.e.*,

$$T_d \propto \phi I_1 \quad \text{or} \quad T_d \propto I_1 \times I \quad (\because \Phi \propto I)$$

where

$$I = \text{main circuit current}$$

$$I_1 = \text{current in armature coils.}$$

since

$$I_1 \propto V \quad \therefore T_d \propto V I \quad \text{-power}$$

Brake torque is due to the eddy currents induced in an aluminium disc mounted on the *same* spindle and running in the air-gaps of two permanent magnets. As shown in Art, 10.44, this braking torque is proportional to the speed of the disc if the flux of the braking magnet and the resistance of the eddy current paths are assumed constant.

When steady speed of rotation is reached, then

$$T_B = T_d \quad \therefore N \propto VI \propto \text{power } W$$

Hence, steady number of revolutions in a given time is proportional to  $\int Wt =$  the energy in the circuit.

The friction effect is compensated for by means of a small compensating coil placed coaxially with the two currents coils and connected in series with the armature such that it strengthens the field of current coil. But its position is so adjusted that with zero line current the armature just fails to rotate. Such meters are now employed mainly for switchboard use, house service meters being invariably of the mercury ampere-hour type.

### 10.50. Induction Type Single-phase Watthour Meter

Induction type meters are, by far, the most common form of a.c. meters met with in every day domestic and industrial installations. These meters measure electric energy in kilo-watthours. The principle of these meters is practically the same as that of the induction wattmeters. Constructionally, the two are similar that the control spring and the pointer of the watt-meter are replaced, in the case of watthour meter, by a brake magnet and by a spindle of the meter.

The brake magnet induces eddy currents in the disc which revolves continuously instead of rotating through only a fraction of a revolution as in the case of wattmeters.

#### Construction

The meter consists of two a.c. electromagnets as shown in Fig. 10.64. (a), one of which *i.e.*,  $M_1$  is excited by the line current and is known as *series* magnet. The alternating flux  $\Phi_1$  produced by it is proportional to and in phase with the line current (provided effects of

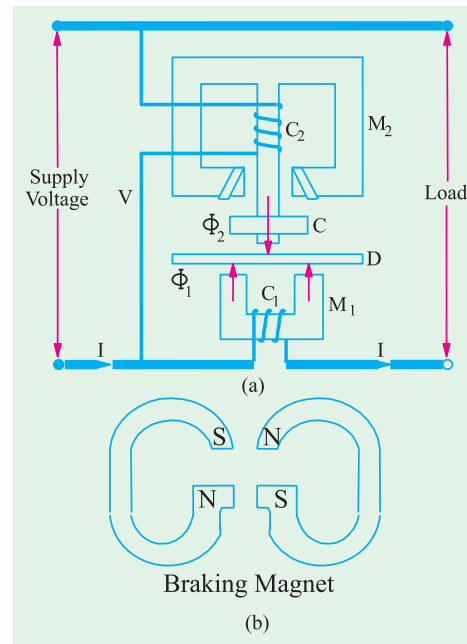


Fig. 10.64

hysteresis and iron saturation are neglected). The winding of the other magnet  $M_2$  called *shunt magnet*, is connected across the supply line and carries current proportional to the supply voltage  $V$ . The flux  $\Phi_2$  produced by it is proportional to supply voltage  $V$  and lags behind it by  $90^\circ$ . This phase displacement of exact  $90^\circ$  is achieved by adjustment of the copper shading band  $C$  (also known as power factor compensator) on the shunt magnet  $M_2$ . Major portion of  $\Phi_2$  crosses the narrow gap between the centre and side limbs of  $M_2$  but a small amount, which is the useful flux, passes through the disc  $D$ . The two fluxes  $\Phi_1$  and  $\Phi_2$  induce e.m.f.s in the disc which further produce the circulatory eddy currents. The reaction between these fluxes and eddy currents produces the driving torque on the disc in a manner similar to that explained in Art. 10.39. The braking torque is produced by a pair of magnets [Fig. 10.64 (b)] which are mounted diametrically opposite to the magnets  $M_1$  and  $M_2$ . The arrangements minimizes the interaction between the fluxes of  $M_1$  and  $M_2$ . This arrangements minimizes the interaction between the fluxes of  $M_1$  and  $M_2$  and that of the braking magnet. When the peripheral portion of the rotating disc passes through the air-gap of the braking magnet, the eddy currents are induced in it which give rise to the necessary torque. The braking torque  $T_B \propto \Phi^2 N/R$  where  $\Phi$  is the flux of braking magnet,  $N$  the speed of the rotating disc and  $R$  the resistance of the eddy current path. If  $\Phi$  and  $R$  are constant, then  $T_B \propto N$ .

The register mechanism is either of pointer type or cyclometer type. In the former type, the pinion on the rotor shaft drives, with the help of a suitable train of reduction gears, a series of five or six pointers rotating on dials marked with ten equal divisions. The gearing between different pointers is such that each pointer advances by 1/10th of a revolution for a complete revolution of the adjacent pointer on the main rotor disc in the train of gearing as shown in Fig. 10.65.

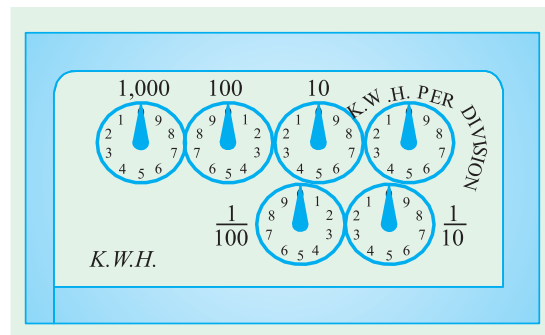


Fig. 10.65

**Theory**

As shown in Art. 10.39. and with reference to Fig. 10.58, the driving torque is given by  $T_d \propto \Phi_{1m}$  and  $\Phi_{2m}$  where  $\sin \alpha$ ,  $\Phi_{1m}$  and  $\Phi_{2m}$  are the maximum fluxes produced by magnets  $M_1$  and  $\alpha$  the angle between these fluxes. Assuming that fluxes are proportional to the current, we have

Current through the windings of  $M_1 = I$  –the line current

Current through the winding of  $M_2 = V/\omega L$

$\alpha = 90 - \phi$  where  $\phi$  is the load p.f. angle

$T_d \propto \frac{V}{L} \cdot I \cos (90 - \phi) = VI \cos \phi$  power

Also,  $T_b \propto N$

The disc achieves a steady speed  $N$  when the two torques are equal *i.e.*, when

$T_d = T_b \therefore N \propto \text{power } W$

Hence, in a given period of time, the total number of revolution  $\int_0^t N \cdot dt$  is proportional to  $\int_0^t W \cdot dt$  *i.e.*, the electric energy consumed.

**10.51. Errors in Induction Watthour Meters**

**1. Phase and speed errors**

Because ordinary the flux due to shunt magnet does not lag behind the supply voltage by exactly  $90^\circ$  owing to the fact that the coil has some resistance, the torque is not zero power factor. This is compensated for by means of an adjustable shading ring placed over the central limb of the sunt magnet. That is why this shading ring is known as *power factor compensator*.

An error in the speed of the meter, when tested on a non-inductive load, can be eliminated by correctly adjusting the position of the brake magnet. Movement of the poles of the braking magnet towards the centre of the disc reduces the braking torque and *vice-versa*.

The supply voltage, the full load current and the correct number of revolutions per kilowatthour are indicated on the name plate of the meter.

### 2. Friction compensation and creeping error

Frictional forces at the rotor bearings and in the register mechanism gives rise to unwanted braking torque on the disc rotor. This can be reduced to an unimportant level by making the ratio of the shunt magnet flux  $\Phi_2$  and series magnet flux  $\Phi_1$  large with the help of two shading bands. These bands embrace the flux contained in the two outer limbs of the shunt magnet and so eddy currents are induced in them which cause phase displacement between the enclosed flux and the main-gap flux. As a result of this, a small driving torque is exerted on the disc rotor solely by the pressure coil and independent of the main driving torque. The amount of this corrective torque is adjusted by the variation of the position of the two bands, so as to exactly compensate for frictional torque in the instrument. Correctness of friction compensation is achieved when the rotor does not run on no-load *with only the supply voltage connected*.

By '*creeping*' is meant the slow but continuous rotation of the rotor when only the pressure coils are excited but with no current flowing in the circuit. It may be caused due to various factors like incorrect friction compensation, to vibration, to stray magnetic fields or due to the voltage supply being in excess of the normal. In order to prevent creeping on no-load, two holes are drilled in the disc on a diameter *i.e.*, on the opposite sides of the spindle.

This causes sufficient distortion of the field to prevent rotation when one of the holes comes under one of the poles of the shunt magnet.

### 3. Errors due to temperature variations

The errors due to temperature variations of the instruments are usually small, because the various effects produced tend to neutralise one another.

**Example 10.27.** *An ampere-hour meter, calibrated at 210 V, is used on 230 V circuit and indicates a consumption of 730 units in a certain period. What is the actual energy supplied ?*

*If this period is reckoned as 200 hours, what is the average value of the current ?*

**(Elect. Technology, Utkal Univ.)**

**Solution.** As explained in Art. 10.42, ampere-hour meters are calibrated to read directly in kWh at the *declared voltage*. Obviously, their readings would be incorrect when used on any other voltage.

$$\text{Reading on 210 volt} = 730 \text{ kWh}$$

$$\text{Reading on 230 volt} = 730 \times 230/210 = \mathbf{800 \text{ kWh (approx.)}}$$

$$\text{Average current} = 800,000/230 \times 200 = \mathbf{17.4 \text{ A}}$$

**Example 10.28.** *In a test run of 30 min. duration with a constant current of 5 A, mercury-motor amp-hour meter, was found to register 0.51 kWh. If the meter is to be used in a 200-V circuit, find its error and state whether it is running fast or slow. How can the instrument be adjusted to read correctly ?*

**(Elect. Mean Inst. and Mean., Jadavpur Univ.)**

**Solution.** Ah passed in 30 minutes =  $5 \times 1/2 = 2.5$

$$\text{Assumed voltage} = 0.51 \times 1000/2.5 = 204 \text{ V}$$

When used on 200-V supply, it would obviously show higher values because actual voltage is less than the assumed voltage. It would be fast by  $4 \times 100/200 = \mathbf{2\%}$

**Example 10.29.** *An amp-hour meter is calibrated to read kWh on a 220-V supply. In one part of the gear train from the rotor to the first counting dial, there is a pinion driving a 75-tooth wheel. Calculate the number of teeth on a wheel which is required to replace 75-tooth wheel, in order to render the meter suitable for operation on 250-V supply.*

**Solution.** An amp-hour meter, which is calibrated on 220-V supply would run fast when operated on 250-V supply in the ratio 250/220 or 25/22. Hence, to neutralize the effect of increased

voltage, the number of teeth in the wheel should be reduced by the same ratio.

$$\therefore \text{Teeth on the new wheel} = 75 \times 22/25 = 66$$

**Example 10.30.** A meter, whose constant is 600 revolutions per kWh, makes five revolution in 20 seconds. Calculate the load in kW. (Elect. Meas. and Meas. Inst. Gujarat Univ.)

**Solution.** Time taken to make 600 revolution is =  $600 \times 20/5 = 2,400$  second

During this time, the load consumes 1 kWh of energy. If  $W$  is load in kW, then

$$W \times 2400(60) \times 60 = 1 \quad \text{or} \quad W = 1.5 \text{ kW}$$

**Example 10.31.** A current of 6 A flows for 20 minutes through a 220-V ampere hour meter. If during a test the initial and final readings on the meter are 3.53 and 4.00 kWh respectively, calculate the meter error as a percentage of the meter readings.

If during the test, the spindle makes 480 revolutions, calculate the testing constant in coulomb/rev and rev/kWh.

**Solution.** Energy actually consumed =  $6 \times \left(\frac{20}{60}\right) \times \frac{200}{1000} = 0.44$  kWh

Energy as registered by meter =  $4.00 - 3.53 = 0.47$  kWh

Error =  $0.47 - 0.44 = 0.03$  kWh ; % error =  $0.03 \times 100/0.47 = 6.38$  %

No. of coulombs passed through in 20 minutes =  $6 \times 20 \times 7,2000$  coulomb

Testing for 480 revolutions, only 0.44 kWh are consumed, hence testing constant =  $480/0.44 = 1091$  rev/kWh.

**Example 10.32.** A 230-V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter constant when operating at 230 V and 5 A for 4 hours. (Elect. Measure, A.M.I.E. Sec B, 1991)

**Solution.** Energy consumption in 6 hr =  $230 \times 4 \times 1 \times 6 = 5520$  W = 5.52 kW

Meter constant =  $2208/5.52 = 400$  rev/kWh.

Now, 1472 revolution represents energy consumption of  $1472/400 = 3.68$  kWh

$$\therefore VI \cos \phi \times \text{hours} = 3.68 \times 10^{10} \quad \text{or} \quad 230 \times 5 \times \cos \phi \times 4 = 3680, \quad \therefore \cos \phi = 0.8$$

**Example 10.33.** A 230 V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolution during this period, what is the constant in rev kWh? Calculate the power factor of the load if the No. of rev made by the meter are 1472 when operating at 230 V and 5 A for 4 hours. (Elect. Measuring, AMIE Sec. Winter 1991)

**Solution.** Energy supplied at unity p.f. =  $230 \times 4 \times 6 \times 1/1000 = 5.52$

$$\therefore 230 \times 5 \times 4 \times \cos \phi / 2000 = 3.68 \quad \therefore \cos \phi = 0.8.$$

**Example 10.34.** The testing constant of a supply meter of the amp-hour type is given as 60 coulomb/revolution. It is found that with a steady current of 50 A, the spindle makes 153 revolutions in 3 minutes. Calculate the factor by which dial indications of the meter must be multiplied to give the consumption. (City and Guilds, London)

**Solution.** Coulombs supplied in 3 min. =  $50 \times 3 \times 60$

At the rate of 60 C/rev., the correct of revolution should have been

$$= 50 \times 3 \times 60/60 = 150$$

Registered No. of revolutions = 153

Obviously, the meter is fact. The registered readings should be multiplied by  $150/153 = 0.9804$  for correction.

**Example 10.35.** A single phase kWhr meter makes 500 revolutions per kWh. It is found on testing as making 40 revolutions in 58.1 seconds at 5 kW full load. Find out the percentage error. (Elect. Measurement & Measuring Instrument Nagpur Univ. 1993)

**Solution.** The number of revolutions the meter will make in one hour on testing =  $40 \times 3600/58.1 = 2478.5$

These revolutions correspond to an energy of  $5 \times 5$  kWh

$\therefore$  No. of revolutions kWh =  $2478.5/5 = 495.7$

Percentage error =  $(500 - 495.7) \times 100/500 = 0.86 \%$

**Example 10.36.** An energy meter is designed to make 100 revolution of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 40 A at 230-V and 0.4 power factor for an hour. If it actually makes 360 revolutions, find the percentage error. (Elect. Engg - I Nagpur Univ. 1993)

**Solution.** Energy consumed in one hour =  $230 \times 40 \times 0.4 \times 1/1000 = 3.68$  kWh

No. of revolutions the meter should make if it is correct =  $3.68 \times 100 = 368$

No. of revolutions actually made = 360

$\therefore$  Percentage error =  $(369 - 360) \times 100/368 = 2.17 \%$

**Example 10.37.** The constant of a 25-ampere, 220-V meter is 500 rev/kWh. During a test at full load of 4.400 watt, the disc makes a 50 revolutions in 83 seconds. Calculate the meter error.

**Solution.** In one hour, at full-load the meter should make  $(4400 \times 1) \times 500/1000 = 2200$  revolutions. This corresponds to a speed of  $2200/60 = 36.7$  r.p.m.

Correct time for 50 rev. =  $(50 \times 60)/36.7 = 81.7$ s

Hence, meter is slow by  $83 - 81.7 = 1.3$  s

$\therefore$  Percentage error =  $1.3 \times 100/81.7 = 1.59 \%$

**Example 10.38.** A 16-A amp-hour meter with a dial marked in kWh, has an error of + 2.5 % when used on 250-V circuit. Find the percentage error in the registration of the meter if it is connected for an hour in series with a load taking 3.2 kW at 200.

**Solution.** In one hour, the reading given by the meter is =  $16 \times 250/1000 = 4$  kWh

Correct reading =  $4 + 2.5 \%$  of  $4 = 4.1$  kWh

Meter current on a 3.2 kW load at 200 V =  $3200/200 = 16$  A

Since on the given load, meter current is the same as the normal current of the meter, hence in one hour it would give a corrected reading of 4.1 kWh.

But actual load is 3.2 kWh.  $\therefore$  Error =  $4.1 - 3.2 = 0.9$  kWh

% error =  $0.9 \times 100/3.2 = 28.12 \%$

**Example 10.39.** The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.s if the load is on for 12 hours, how many units are recorded as error? (Measurs, Instru. Allahabad Univ. 1992)

**Solution.** Since load power is one kWh, energy actually consumed is

$$= 1 \times 12 = 12 \text{ kWh}$$

Total number of revolutions made by the disc during the period of 12 hours

$$= 10.2 \times 60 \times 12 = 7,344$$

since 600 revolutions record one kWh, energy recorded by the meter is

$$= 7,344/600 = 12.24 \text{ kWh}$$

Hence, 0.24 unit is recorded extra.

**Example 10.40.** A d.c. ampere-hour meter is rated at 5-A, 250-V. The declared constant is 5 A-s/rev. Express this constant in rev/kWh. Also calculate the full-load speed of the meter.

(Elect. Meas. Inst. and Meas., Jadavpur Univ.)

**Solution.** Meter constant = 5 A-s/rev

Now,  $1 \text{ kWh} = 10^3 \text{ Wh} = 10^3 \times 3600 \text{ volt-second}$

$$= 36 \times 10^5 \text{ volt} \times \text{amp} \times \text{second}$$

Hence, on a 250-V circuit, this corresponds to  $36 \times 10^5 / 250 = 14,400$  A-s

Since for every 5 A-s, there is one revolution, the number of revolution is one kWh is  
 $= 14,400 / 5 = 2,880$  revolutions

$\therefore$  Meter constant = **2,800 rev/kWh**

Since full-load meter current is 5 A and its constant is 5 A-s/rev, it is obvious that it makes one revolution every second.

$\therefore$  full load speed = 60 r.p.m.

**Example 10.41.** *The declared constant of a 5-A, 200-V amp-hour meter is 5 coulomb per revolution. Express the constant in rev/kWh and calculate the full-load speed of the meter.*

*In a rest at half load, the meter disc completed 60 revolutions in 119.5 seconds. Calculate the meter error.*

**Solution.** Meter constant = 5 C/rev, or 5 A-s/rev.

$$\begin{aligned} 1 \text{ kWh } 1000 \text{ Wh} &= 1000 \times 3600 \text{ watt-second} \\ &= 1000 \times 3600 \text{ volt} \times \text{amp} \times \text{second} \end{aligned}$$

Hence, on a 200-V circuit, this corresponds to

$$= 100 \times 3600 / 200 = 18,000 \text{ A-s}$$

Since for every 5 A-s, there is one revolution, hence number of revolution is one kWh =  $18,000 / 5 = 3600$  revolution  $\therefore$  Meter constant = **3600 rev/kWh**

Since full-load meter current is 5 A and its constant 5 A-s/rev, it is obvious that it makes one revolution every one second.

$\therefore$  its full-load speed = **60 r.p.m.**

**At half-load**

$$\text{Quantity passed in 60 revolutions} = 119.5 \times 2.5 \text{ A-s or } 298.75 \text{ C}$$

$$\text{Correct No. of revolutions} = 298.75 / 5 = 59.75$$

Obviously the meter is running fast because instead of making 59.75 revolutions, it is making 60 revolutions.

$$\text{Error} = 60 - 59.75 = 0.25 \therefore \% \text{ error} = 0.25 \times 100 / 59.75 = \mathbf{0.418 \%}$$

**Example 10.42. (a).** *A single-phase energy meter of the induction type is rated 230-V; 10-A, 50-Hz and has a meter constant of 600 rev/kWh when correctly adjusted. If 'quadrature' adjustment is slightly disturbed so that the lag is  $85^\circ$ , calculate the percentage error at full-load 0.8 p.f. lag.*

**(Measu & Instru. Nagpur Univ 1991)**

**Solution.** As seen from Art. 10.50, the driving torque,  $T_d$  depends among other factors, on  $\sin \alpha$  where  $\alpha$  is the angle between the two alternating fluxes.  $\therefore T_d \propto \sin \alpha$

If the voltage flux-lagging adjustment is disturbed so that the phase angle between the voltage flux and the voltage is less than  $90^\circ$  (instead of being exactly  $90^\circ$ ) the error is introduced.

$$\text{Now} \quad \cos \phi = 0.8, \quad \phi = \cos^{-1}(0.8) = 36^\circ 52'$$

$$\therefore \alpha = 85^\circ - 36^\circ 52' = 48^\circ 8' \text{ where it should be } = 90^\circ - 36^\circ 52' = 53^\circ 8'$$

$$\therefore \text{error} = \frac{\sin 53^\circ 8' - \sin 48^\circ 8'}{\sin 53^\circ 8'} \times 100 = \mathbf{7\%}$$

**Example 10.42 (b).** *A 50-A, 230-V energy meter is a full-load test makes 61 revolutions in 37 seconds if the normal speed of the disc is 520 revolutions/kWh, compute the percentage error.*

**[Nagpur University November 1999]**

**Solution.** Unity power-factor is assumed.

Energy consumed, in kWh, in 37 seconds

$$= \frac{50 \times 230}{1000} \times \frac{37}{3600} = 0.1182 \text{ kWh}$$

Number of revolutions corresponding to this energy =  $520 \times 0.1182 = 61.464$

The meter makes 61 revolutions

$$\therefore \quad \% \text{ Error} = \frac{61 - 61.464}{61.464} \times 100 \% = -0.755 \%$$

### 10.52. Ballistic Galvanometer

It is used principally for measuring small electric charges such as those obtained in magnetic flux measurements. Constructionally, it is similar to a moving-coil galvanometer except that (i) it has extremely small electromagnetic damping and (ii) has long period of undamped oscillation (several seconds). These conditions are necessary if the galvanometer is to measure electric charge. In fact, the moment of inertia of the coil is made so large that whole of the charge passes through the galvanometer before its coil has had time to move sufficiently. In that case, the first swing of the coil is proportional to the charge passing through the galvanometer. After this swing has been observed, the oscillating coil may be rapidly brought to rest by using eddy-current damping.



Ballistic Galvanometer

As explained above, the coil moves after the charge to be measured has passed through it. Obviously, during the movement of the coil, there is no current flowing through it. Hence, the equation of its motion is

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + C\theta = 0$$

where  $J$  is the moment of inertia,  $D$  is damping constant and  $C$  is restoring constant.

Since damping is extremely small, the approximate solution of the above equation is

$$0 = U e^{-(D/2J)t} \sin(\omega_0 t + \phi)$$

At the start of motion, where  $r = 0$ ,  $\theta = 0$ , hence  $\phi = 0 \quad \therefore \theta = U e^{-(D/2J)t} \sin \omega_0 t \quad \dots(i)$

During the passage of charge, at any instant, there will be a deflecting torque of  $Gi$  acting on the coil. If  $t$  is the time taken by the whole charge to pass through, the torque impulse due to this charge is

$$\int_0^t Gid t . \text{ Now } \int_0^t i dt = Q$$

Hence, torque impulse =  $GQ$ . This must be equal to the change of angular momentum produced i.e.,  $J\alpha$  where  $\alpha$  is the angular velocity of the coil at the end of the impulse period.

$$\therefore \quad GQ = J\alpha$$

$$\text{or} \quad \alpha = GQ/J$$

Differentiating Eq. (i) above, we get

$$\frac{d\theta}{dt} = U \left( e^{-(D/2J)t} \omega_0 t - \frac{D}{2} J e^{-(D/2J)t} \sin \omega_0 t \right)$$

Since duration of the passage of charge is very small, at the end of the passage,  $t \approx 0$ , so that from above,  $d\theta/dt = U\omega_0$ .

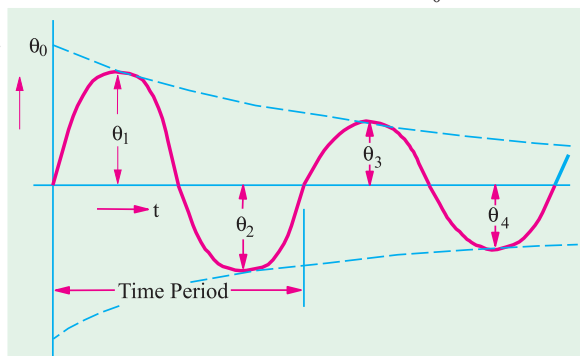


Fig. 10.66



However, at this time,  $\frac{d\theta}{dt} = \alpha = \frac{GQ}{J} \therefore \frac{GQ}{J} = U \omega_0$  or  $Q = \frac{J\omega_0}{G} U$

Now,  $U$  being the amplitude which the oscillations would have if the damping were zero, it may be called undamped swing  $\theta_0$ .

$$\therefore Q = \frac{J\omega_0}{G} \theta_0 \text{ or } Q \propto \theta_0 \quad \dots(ii)$$

However, in practice, due to the presence of small amount of damping, the successive oscillations diminish exponentially (Fig. 10.66). Even the first swing  $\theta_1$  is much less than  $\theta_0$ . Hence, it becomes necessary to obtain the value of  $\theta_0$  from the observed value of first maximum swing  $\theta_1$ .

As seen from Fig. 10.66, the successive peak value  $\theta_1, \theta_2, \theta_3$  etc. are  $\phi$  radian apart or  $\phi/\omega_0$  second apart. The ratio of the amplitude of any two successive peaks is

$$\begin{aligned} &= \frac{\theta_0 e^{-(D/2J)t} \sin \omega_0 t}{\theta_0 e^{-(D/2J)t + (\pi/\omega_0)} \sin (\omega_0 t + \pi)} = \frac{e^{-(D/2J)t} \sin \omega_0 t}{e^{-(D/2J)t} e^{-(D/2J)(\pi/\omega_0)} (-\sin \omega_0 t)} = -e^{(D/2J)(\pi/\omega_0)} \\ \therefore \frac{\theta_1}{\theta_2} &= \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \dots = e^{(D/2J)(\pi/\omega_0)} \end{aligned}$$

Let  $e^{(D/2J)(\pi/\omega_0)} = \Delta^2$  where  $\Delta$  is called the damping factor\*.

The time period of oscillation  $T_0 = 2\pi/\omega_0$ . If damping is very small  $\theta_0 = \theta_1, t = T_0/4 = \pi/2\omega_0$  as a very close approximation.

Hence, from Eq. (ii) above, putting  $t = \pi/2\omega_0$ , we have

$$\theta = \theta_0 e^{(D/2J)(\pi/2\omega_0)} \sin \pi/2 = \theta_0 \Delta - 1 \therefore \theta_0 = \Delta \theta_1 \quad \dots(iii)$$

or undamped swing = damping factor  $\times$  1st swing

Suppose, a steady current of  $I_s$  flowing through the galvanometer produces a steady deflection  $\phi_s$ , then

$$C\theta_s = GI_s \text{ or } G = C\theta_s/I_s$$

$$\text{Since damping is small, } \omega_0 = \sqrt{C/J} \therefore C = J\omega_0^2 = J \times \left(\frac{2\pi}{T_0}\right)^2 = 4\pi^2 \frac{J^2}{T_0^2} \therefore G = \frac{4\pi^2 J\theta_s}{T_0^2 I_s}$$

Substituting this value of  $G$  in Eq. (ii), we get

$$Q = \frac{J\omega_0 \theta_0}{4\pi^2 J\theta_s / T_0^2 I_s} \text{ or } Q_0 = \frac{T_0}{2\pi} \cdot \frac{I_s}{\theta_s} \cdot \theta_0 = \frac{T_0}{2\pi} \cdot \frac{I_s}{2\pi} \cdot \Delta \cdot \theta_1 \dots(iv)$$

Alternatively, let quantity  $(D/2J)(\pi/\omega_0)$  be called the logarithmic decrement  $\lambda$ . Since,  $\Delta^2 = e^\lambda$ , we have

$$\Delta = e^{\lambda/2} = 1 + (\lambda/2) + \frac{(\lambda/2)^2}{2!} + \dots \cong \left(1 + \frac{\lambda}{2}\right) \text{ when } \lambda \text{ is small}^{**}$$

$$\text{Hence, from Eq. (iv) above, we have } Q = \frac{T_0}{2\pi} \cdot \frac{I_s}{\theta_s} \left(1 + \frac{\lambda}{2}\right) \theta_1 \quad \dots(v)$$

In general, Eq. (iv) may be put as  $Q = k\theta_1$

\* Now,  $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_{n-1}}{\theta_n} = \Delta^2 \therefore \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \times \dots \times \frac{\theta_{n-1}}{\theta_n} = (\Delta^2)^{n-1} \therefore \frac{\theta_1}{\theta_2} = (\Delta^2)^{n-1}$  or  $\Delta = \left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)}$

Hence,  $\Delta$  may be obtained by observing the first and  $n$ th swing.

\*\* Since  $\Delta^2 = e^\lambda$ , taking logs, we have  $2 \log_e \Delta = \lambda \log_e e = \lambda$

$$\therefore \frac{\lambda}{2} = \log_e \Delta = \log_e \left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)} = \frac{1}{2(n-1)} \log_e \theta_1 / \theta_n \therefore \lambda = \frac{1}{(n-1)} \log_e \frac{\theta_1}{\theta_n}$$

**Example. 10.43.** A ballistic galvanometer has a free period of 10 seconds and gives a steady deflection of 200 divisions with a steady current of  $0.1 \mu\text{A}$ . A charge of  $121 \mu\text{C}$  is instantaneously discharged through the galvanometer giving rise to a first maximum deflection of 100 divisions. Calculate the 'decrement' of the resulting oscillations.

(Electrical Measurements, Bombay Univ.)

**Solution.** From Eq. (v) of Art 10.52, we have  $Q = \frac{T_0}{2} \cdot \frac{I_s}{s} \left(1 + \frac{\lambda}{2}\right)$

Here,  $Q = 121 \mu\text{C} = 121 \times 10^{-6} \text{C}$ ;  $T_0 = 10\text{s}$ ;  $I_s = 0.1 \text{mA} = 10^{-4} \text{A}$ ;  $\theta_s = 200$ ;  $\theta_1 = 100$

$$\therefore 121 \times 10^{-6} = \frac{10}{2\pi} \times \frac{10^{-4}}{200} \left(1 + \frac{\lambda}{2}\right) \times 100; \quad \therefore \lambda = 1.04$$

### 10.53. Vibration Galvanometer

Such galvanometers are widely used as null-point detectors in a.c. bridges.

#### Construction

As shown in Fig. 10.67. (a), it consists of a moving coil suspended between poles of a strong permanent magnet. The natural frequency of oscillation of the coil is very high, this being achieved by the use of a large value of control constant and a moving system of very small inertia. The suspension (which provides control) is either a phosphor-bronze strip or is a bifilar suspension in which case the two suspension wires carry the coil and a small piece of mirror (or in some cases the two suspension wires themselves from the coil.)

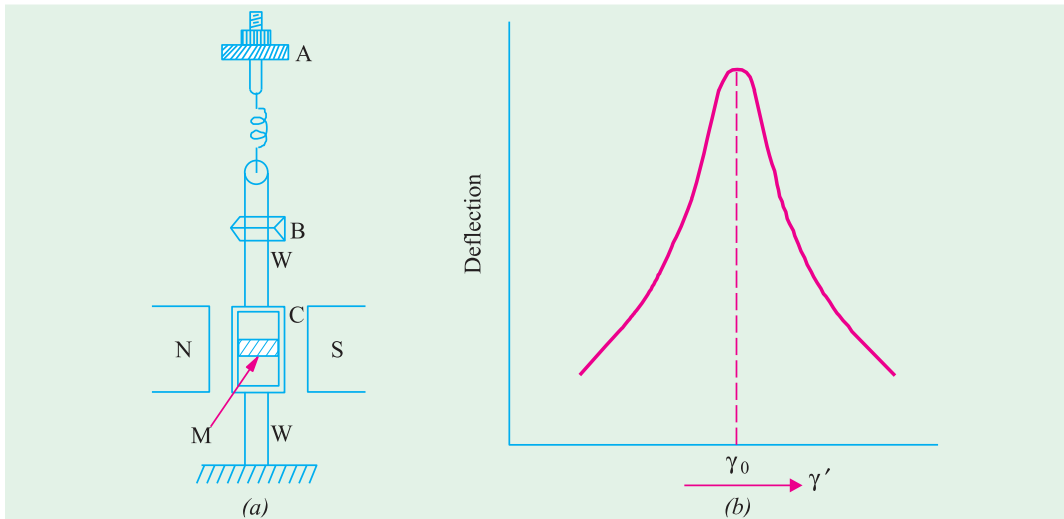


Fig. 10.67

As seen,  $W$  is the suspension,  $C$  is the moving coil and  $M$  the mirror on which is cast a beam of light. From mirror  $M$ , this beam is deflected on to a scale. When alternating current is passed through  $C$ , an alternating torque is applied to it so that the reflected spot of light on the scale is drawn out in the form of a band of light. The length of this band of light is maximum if the natural frequency of oscillation of  $C$  coincides with the supply frequency due to resonance. The turning of  $C$  may be done in the following two ways :

- (i) by changing the length of suspension  $W$ . This is achieved by raising or lowering bridge piece  $B$  against which the bifilar loop presses.
- (ii) by adjusting tension in the suspension. This is achieved by turning the knurled knob  $A$ .

By making the damping very small, the resonance curve of the galvanometer can be made sharply-peaked [Fig. 10.67 (b)]. In that case, the instrument discriminates sharply against frequencies other than its own natural frequency. In other words, its deflection becomes very small even when the frequency of the applied current differs by a very small amount from its resonance frequency.

### Theory

If the equation of the current passing through the galvanometer is  $i = I_m \sin \omega t$ , then the equation of motion of the coil is :

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + C\theta = Gi = GI_m \sin \omega t \quad \dots(i)$$

where  $J$ ,  $D$  and  $C$  have the usual meaning and  $G$  is the deflection constant.

The complementary function of the solution represents the transient motion, which in the case of vibration galvanometers, is of no practical importance. The particular integral is of the form

$$\theta = A \sin (\omega t - \phi)$$

where  $A$  and  $\phi$  are constant.

Now,  $d\theta/dt = \omega A \cos (\omega t - \phi)$  and  $d^2\theta/dt^2 = -\omega^2 A \sin (\omega t - \phi)$ . Substituting these values in Eq. (i) above, we get

$$\omega^2 JA \sin (\omega t - \phi) + \omega DA \cos (\omega t - \phi) + CA \sin (\omega t - \phi) = GI_m \sin \omega t$$

It must be true for all values of  $t$

$$\text{When } \omega t = \phi \quad DA \omega = G I_m \sin \phi \quad \dots(ii)$$

$$\text{When } (\omega t - \phi) = \pi/2 \quad \omega^2 JA + CA = G I_m \cos \phi \quad \dots(iii)$$

Since the phase angle  $\phi$  of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (ii) and (iii).

Since the phase angle  $\phi$  of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (ii) and (iii).

$$\therefore \omega^2 D^2 A^2 + A^2 (C - \omega^2 J)^2 = G^2 I_m^2 \quad \text{or} \quad A = \frac{G I_m}{\sqrt{[D^2 \omega^2 + (C - \omega^2 J)^2]}} \quad \dots(iv)$$

This represents the amplitude  $A$  of the resulting oscillation for a sinusoidally alternating current of peak value  $I_m$  flowing through the moving coil of the galvanometer.

## 10.54. The Vibrating-reed Frequency Meter

### 1. Working Principle

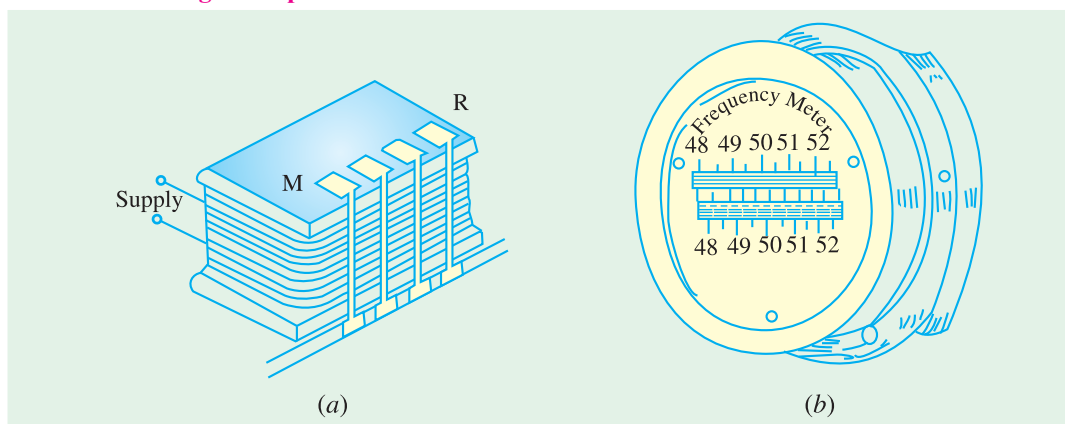


Fig. 10.68

The meter depends for its indication on the mechanical resonance of thin flat steel reeds arranged alongside and, close to, an electromagnet as shown in Fig. 10.68.

## 2. Construction

The electromagnet has a laminated armature and its winding, in series with a resistance, is connected across a.c. supply whose frequency is required. In that respect, the external connection of this meter is the same as that of a voltmeter.

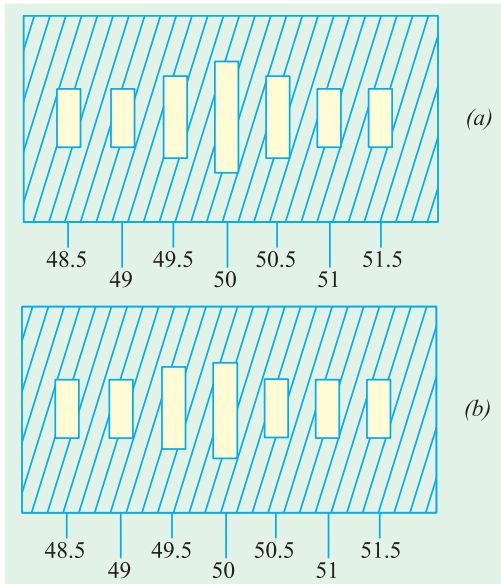


Fig. 10.69

The metallic reeds (about 4 mm wide and 0.5 mm thick) are arranged in a row and are mounted side by side on a common and slightly flexible base which also carries the armature of the electromagnet. The upper free ends of the reeds are bent over a right angles so as to serve as flags or targets and enamelled white for better visibility. The successive reeds are not exactly similar, their natural frequencies of vibration differing by  $\frac{1}{2}$  cycle. The reeds are arranged in ascending order of natural frequency.

## 3. Working

When the electromagnet is connected across the supply whose frequency is to be measured, its magnetism alternates with the same frequency. Hence the electromagnet exerts attracting force on each reed once every half cycle. All reeds tend to vibrate but only that whose natural frequency is exactly double the supply frequency vibrates with maximum amplitude due to mechanical resonance [Fig. 10.69 (a)]. The supply frequency is read directly by noting the scale mark opposite the white painted flag which is

vibrating the most ( $f = 50$  Hz). The vibrations of other reeds would be so small as to be almost unobservable. For a frequency exactly midway between the natural frequencies of the two reeds ( $f = 49.75$  Hz), both will vibrate with amplitudes which are equal but much less than when the supply frequency exactly coincides with that of the reeds.

## 4. Range

Such meters have a small range usually from 47 to 53 Hz or from 57 to 63 Hz etc.

The frequency range of a given set of reeds may be doubled by polarising the electromagnets as explained below. As seen from above description, each reed is attracted twice per cycle of the supply *i.e.*, once every half-cycle and the reeds whose natural frequency is twice that of the current is of the one which responds most. Suppose the electromagnet carries an additional winding carrying direct current whose steady flux is equal in magnitude to the alternating flux of the a.c. winding. The resultant flux would be zero in one half-cycle and double in the other half-cycle when the two fluxes reinforce each other so that the reeds would receive one impulse per cycle. Obviously, a reed will indicate the frequency of the supply if the electromagnet is polarised and half the supply frequency if it is unpolarised. The polarisation may be achieved by using an extra d.c. winding on the electromagnet or by using a permanent tangent which is then wound with an a.c. winding.

## 5. Advantages

One great advantage of this reed-type meter is that its indications are independent of the waveform of the applied voltage and of the magnitude of the voltage, except that the voltage should be high enough to provide sufficient amplitude for reed vibration so as to make its readings reliable.

However, its limitations are :

- (a) it cannot read closer than half the frequency difference between adjacent reeds.
- (b) its error is dependent upon the accuracy with which reeds can be turned to a given frequency.

### 10.55. Electrodynamic Frequency Meter

It is also referred to as moving-coil frequency meter and is a ratiometer type of instrument.

#### 1. Working Principle

The working principle may be understood from Fig. 10.70 which shows two moving coils rigidly fixed together with their planes at right angles to each other and mounted on the shaft or spindle situated in the field of a permanent magnet. There is no mechanical control torque acting on the two coils. If  $G_1$  and  $G_2$  are displacement constants of the two coils and  $I_1$  and  $I_2$  are the two currents, then their respective torques are  $T_1 = G_1 I_1 \cos \theta$ ,  $T_2 = G_2 I_2 \sin \theta$ . These torques act in the opposite directions.

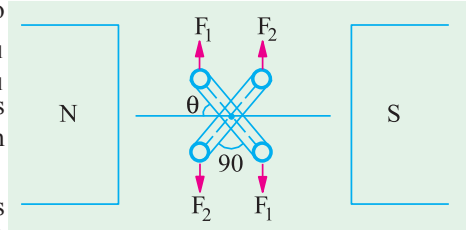


Fig. 10.70

Obviously,  $T_1$  decreases with  $\theta$  where as  $T_2$  increases but an equilibrium position is possible for same angle  $\theta$  for which

$$G_1 I_1 \cos \theta = G_2 I_2 \sin \theta \text{ or } \tan \theta = \frac{G_1}{G_2} \cdot \frac{I_1}{I_2}$$

By modifying the shape of pole faces and the angle between the planes of the two coils, the ratio  $I_1/I_2$  is made proportional to angle  $\theta$  instead of  $\tan \theta$ . In that case, for equilibrium  $\theta \propto I_1/I_2$

#### 2. Construction

The circuit connections are shown in Fig. 10.71. The two ratiometer coils  $X$  and  $Y$  are connected across the supply lines through their respective bridge rectifiers. The direct current  $I_1$  through

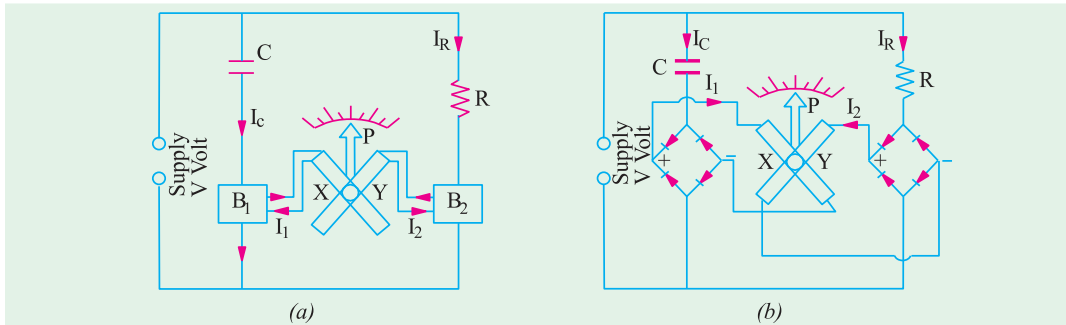


Fig. 10.71

coil  $X$  represents the R.M.S. value of capacitor current  $I_C$  as rectified by  $B_1$ . Similarly, direct current  $I_2$  flowing through  $Y$  is the rectified current  $I_R$  passing through series resistance  $R$ .

#### 3. Working

When the meter is connected across supply lines, rectified currents  $I_1$  and  $I_2$  pass through coils  $X$  and  $Y$  they come to rest at an angular position where their torques are equal but opposite. This angular position is dependent on the supply frequency which is read by a pointer attached to the coil.

As proved above,

$$\theta \propto I_1/I_2$$

Assuming sinusoidal waveform, mean values of  $I_1$  and  $I_2$  are proportional to the R.M.S. values of  $I_C$  and  $I_R$  respectively.

$$\therefore \theta \propto \frac{I_1}{I_2} \propto \frac{I_C}{I_R} \quad \text{Also } I_C \propto V_m \omega C \text{ and } I_R \propto V_m/R$$

where  $V_m$  is the maximum value of the supply voltage whose equation is assumed as  $v = V_m \sin \omega t$ .

$$\therefore \theta \propto \frac{V_m \omega C}{V_m/R} \propto \omega CR \propto \omega \quad \therefore \theta \propto f \quad (\because \omega = 2\pi f)$$

Obviously, such meters have linear frequency scales. Moreover, since their readings are independent of voltage, they can be used over a fairly wide range of voltage although at too low voltages, the distortions introduced by rectifier prevent an accurate indication of frequency.

It will be seen that the range of frequency covered by the meter depends on the value of  $R$  and  $C$  and these may be chosen to get ranges of 40–60 Hz, 1200–2000 Hz or 8000–12,000 Hz.

## 10.56. Moving-iron Frequency Meter

### 1. Working Principle

The action of this meter depends on the variation in current drawn by two parallel circuits – one inductive and the other non-inductive—when the frequency changes.

### 2. Construction

The construction and internal connections are shown in Fig. 10.72. The two coils  $A$  and  $B$  are so fixed that their magnetic axes are perpendicular to each other. At their centres is pivoted a long and thin soft-iron needle which aligns itself along the resultant magnetic field of the two coils. There is no control device used in the instrument.

It will be noted that the various circuit elements constitute a Wheatstone bridge which becomes balanced at the supply frequency. Coil  $A$  has a resistance  $R_A$  in series with it and a reactance  $L_A$  in parallel. Similarly  $R_B$  is in series with coil  $B$  and  $L_B$  is in parallel. The series inductance  $L$  helps to suppress higher harmonics in the current waveform and hence, tends to minimize the waveform errors in the indication of the instrument.

### 3. Working

On connecting the instrument across the supply, currents pass through coils  $A$  and  $B$  and produce opposing torques. When supply frequency is high, currents through coil  $A$  is more whereas that through coil  $B$  is less due to the increase in the reactance offered by  $L_B$ . Hence, magnetic field of coil  $A$  is stronger than that of coil  $B$ . Consequently, the iron needle lies more nearly to the magnetic axis of coil  $A$  than that of  $B$ . For low frequencies, coil  $B$  draws more current than coil  $A$  and, hence, the needle lies more nearly parallel to the magnetic axis of  $B$  than to that of coil  $A$ . The variations of frequency are followed by the needle as explained above.

The instrument can be designed to cover a broad or narrow range of frequencies determined by the parameters of the circuit.

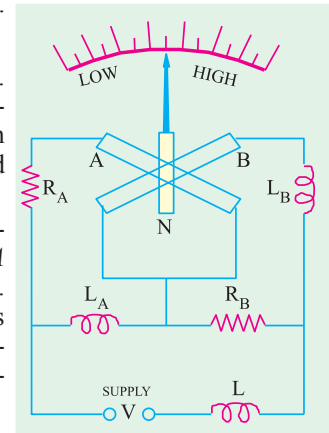


Fig. 10.72

## 10.57. Electrodynamic Power Factor Meter

### 1. Working Principle

The instrument is based on the dynamometer principle with spring control removed.

### 2. Construction

As shown in Fig. 10.73 and 10.74, the instrument has a stationary coil which is divided into two sections  $F_1$  and  $F_2$ . Being connected in series with the supply line, it carries the load current. Obviously, the uniform field produced by  $F_1$  and  $F_2$  is proportional to the line current. In this field are situated two moving coils  $C_1$  and  $C_2$  rigidly attached to each other and mounted on the same shaft or spindle. The two moving coils are ‘voltage’ coils but  $C_1$  has a series resistance  $R$  whereas  $C_2$  has

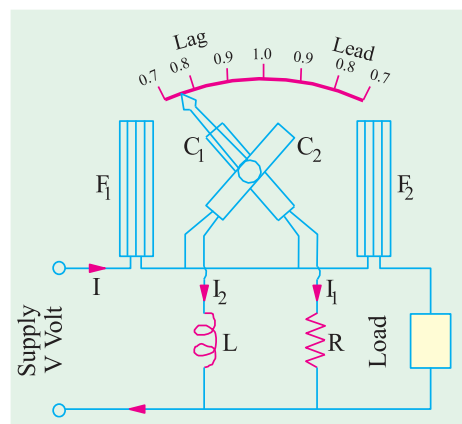


Fig. 10.73

a series inductance  $L$ . The values of  $R$  and  $L$  as well as turns on  $C_1$  and  $C_2$  are so adjusted that the ampere-turns of  $C_1$  and  $C_2$  are exactly equal. However,  $I_1$  is in phase with the supply voltage  $V$  whereas  $I_2$  lags behind  $V$  by nearly  $90^\circ$ . As mentioned earlier, there is no control torque acting on  $C_1$  and  $C_2$  – the currents being led into them by fine ligaments which exerts no control torque.

### 3. Working

Consider the case when load power factor is unity *i.e.*  $I$  is in phase with  $V$ . Then  $I_1$  is in phase with  $I$  whereas  $I_2$  lags behind by  $90^\circ$ . Consequently, a torque will act on  $C_1$  which will set its plane perpendicular to the common magnetic axis of coils  $F_1$  and  $F_2$  *i.e.* corresponding to the pointer position of unity p.f. However, there will be no torque acting on coil  $C_2$ .

Now, consider the case when load power factor is zero *i.e.*  $I$  lags behind  $V$  by  $90^\circ$  (like current  $I_2$ ). In that case,  $I_2$  will be in phase with  $I$  where as  $I_1$  will be  $90^\circ$  out of phase. As a result, there will be no torque on  $C_1$  but that acting on  $C_2$  will bring its plane perpendicular to the common magnetic axis of  $F_1$  and  $F_2$ . For intermediate values of power factor, the deflection of the pointer corresponds to the load power factor angle  $\phi$  or to  $\cos \phi$ , if the instrument has been calibrated to read to power factor directly.

For reliable readings, the instrument has to be calibrated at the frequency of the supply on which it is to be used. At any other frequency (or when harmonics are present), the reactance of  $L$  will change so that the magnitude and phase of current through  $C_2$  will be incorrect and that will lead to serious errors in the instrument readings.

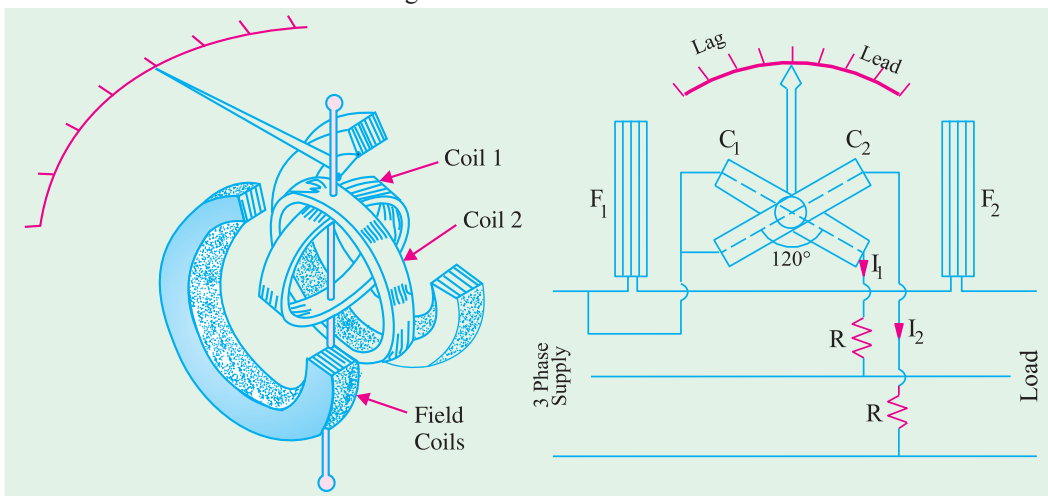


Fig. 10.74

Fig. 10.75

For use on balanced 3-phase load, the instrument is modified, so as to have  $C_1$  and  $C_2$  at  $120^\circ$  to each other, instead of  $90^\circ$ , as in 1-phase supply. As shown in Fig. 10.75,  $C_1$  and  $C_2$  are connected across two different phase of the supply circuit, the stationary coils  $F_1$  and  $F_2$  being connected in series with the third phase (so that it carries the line current). Since there is no need of phase splitting between the currents of  $C_1$  and  $C_2$ ,  $I_1$  and  $I_2$  are not determined by the phase-splitting circuit and consequently, the instrument is not affected by variations in frequency or waveform.

## 10.58. Moving-iron Power Factor Meter

### 1. Construction

One type of power factor meter suitable for 3-phase balanced circuits is shown in Fig. 10.76. It consists of three fixed coils  $R$ ,  $Y$  and  $B$  with axes mutually at  $120^\circ$  and intersecting on the centre line of the instrument. These coils are connected respectively in  $R$ ,  $Y$  and  $B$  lines of the 3-phase supply through current transformers. When so energised, the three coils produce a synchronously rotating flux.

There is a fixed coil  $B$  at the centre of three fixed coils and is connected in series with a high resistance across one of the pair of lines, say, across  $R$  and  $Y$  lines as shown. Coil  $B$  is threaded by the instrument spindle which carries an iron cylinder  $C$  [Fig. 10.76 (b)] to which are fixed sector-shaped iron vanes  $V_1$  and  $V_2$ . The same spindle also carries damping vanes and pointer (not shown in the figure) but **there are no control springs**. The moving system is shown separately in Fig. 10.76 (b).

### 2. Working

The alternating flux produced by coil  $B$  interacts with the fluxes produced by the three current coils and causes the moving system to take up a position determined by the power factor angle of the load. However, the instrument is calibrated to read the power factor  $\cos \phi$  directly instead of  $\phi$ . In other words, the angular deflection  $\phi$  of the iron vanes from the line  $MN$  in Fig. 10.76 (a) is equal to the phase angle  $\phi$ .

Because of the rotating field produced by coils  $R$ ,  $Y$  and  $B$ , there is a slight induction-motor action which tends to continuously turn the moving iron in the direction of the rotating flux. Hence, it becomes essential to design the moving iron as to make this torque negligibly small *i.e.* by using high-resistance metal for the moving iron in order to reduce eddy currents in it.

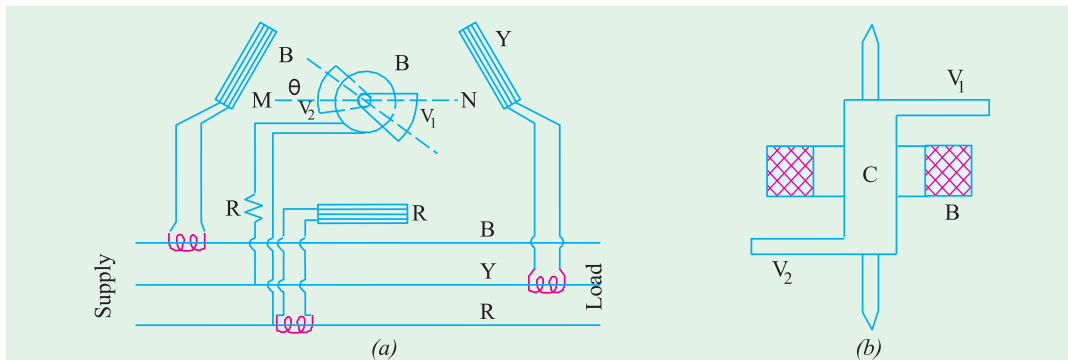


Fig. 10.76

### 3. Merits and Demerits

Moving iron p.f. meters are more commonly used as compared to the electrodynamic type because

- (i) they are robust and comparatively cheap
- (ii) they have scales upto  $360^\circ$  and
- (iii) in their case, all coils being fixed, there are no electrical connections to the moving parts.

On the other hand, they are not as accurate as the electrodynamic type of instruments and, moreover, suffer from errors introduced by the hysteresis and eddy-current losses in the iron parts—these losses varying with load and frequency.

## 10.59. Nalder-Lipman Moving-iron Power Factor Meter

### 1. Construction

The moving system of this instrument (Fig. 10.77) consists of three iron elements similar to the one shown in Fig. 10.76 (b). They are all mounted on a common shaft, one above the other, and are separated from one another by non-magnetic distance pieces  $D_1$  and  $D_2$ . The three pairs of sectors are displaced in space by  $120^\circ$  relative to each other. Each iron vane is magnetised by one of the three voltage coils  $B_1$ ,  $B_2$  and  $B_3$  which are connected (in series with a high resistance  $R$ ) in star across the supply lines. The whole system is free to move in the space between two parallel halves  $F_1$  and  $F_2$  of a single current coil connected in one line of the supply. The common spindle also carries the damping vanes (not shown) and the pointer  $P$ .



**2. Working**

The angular position of the moving system is determined by the phase angle  $\phi$  between the line current and the respective phase voltage. In other words, deflection  $\theta$  is equal to  $\phi$ , although, in practice, the instrument is calibrated to read the power factor directly.

**3. Advantages**

(i) Since no **rotating** magnetic field is produced, there is no tendency for the moving system to be dragged around continuously in one direction.

(ii) This instrument is not much affected by the type of variations of frequency, voltage and waveform as might be expected in an ordinary supply.

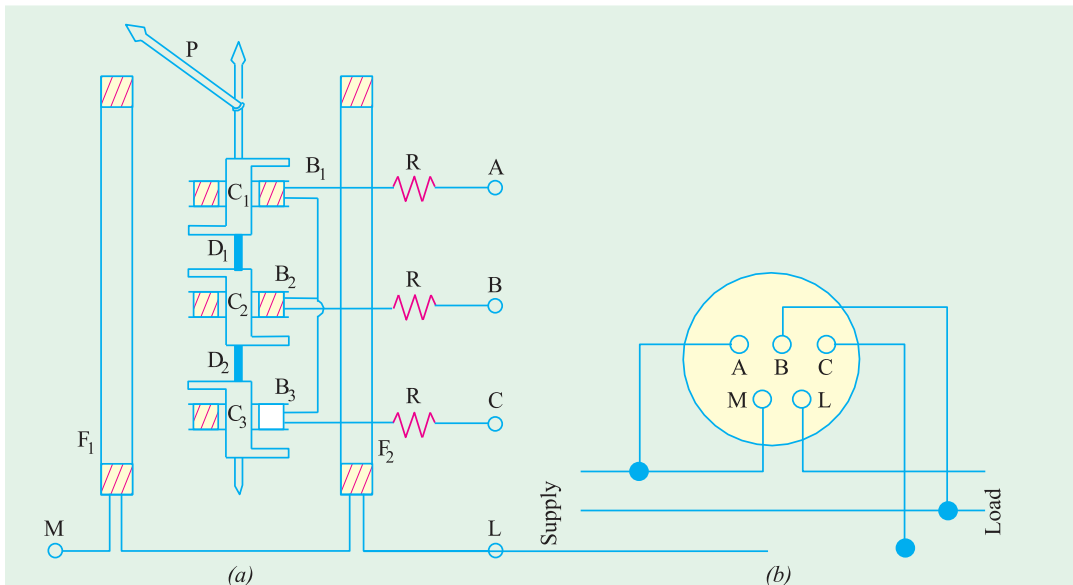


Fig. 10.77

**10.60. D.C. Potentiometer**



DC potentiometer

A potentiometer is used for measuring and comparing the e.m.f.s. of different cells and for calibrating and standardizing voltmeters, ammeters etc. In its simplest form, it consists of a German silver or manganin wire usually one meter long and stretched between two terminals as shown in Fig. 10.78.

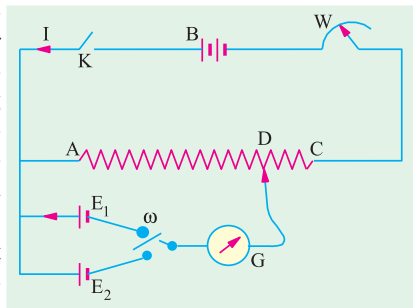


Fig. 10.78

This wire is connected in series with a suitable rheostat and battery  $B$  which sends a steady current through the resistance wire  $AC$ . As the wire is of uniform cross-section throughout, the fall in potential across it is uniform and the drop between any two points is proportional to the distance between them. As seen, the battery voltage is spread over the rheostat and the resistance wire  $AC$ . As we go along  $AC$ , there is a progressive fall of potential. If  $\rho$  is the resistance/cm of this wire,  $L$  its length, then for a current of  $I$  amperes, the fall of potential over the whole length of the wire is  $\rho LI$  volts.

The two cells whose e.m.fs are to be compared are joined as shown in Fig. 10.78, always remembering that **positive terminals of the cells and the battery must be joined together**. The cells can be joined with the galvanometer in turn through a two-way key. The other end of the galvanometer is connected to a movable contact on  $AC$ . By this movable contact, a point like  $D$  is found when there is no current in and hence no deflection of  $G$ . Then, it means that the e.m.f. of the cell just balances the potential fall on  $AD$  due to the battery current passing through it.

Suppose that the balance or null point for first cell of e.m.f.  $E_1$  occurs at a length  $L_1$  as measured from point  $A$ . The  $E_1 = \rho L_1 I$ .

Similarly, if the balance point is at  $L_2$  for the other cell, then  $E_2 = \rho L_2 I$ .

Dividing one equation by the other, we have  $\frac{E_1}{E_2} = \frac{\rho L_1 I}{\rho L_2 I} = \frac{L_1}{L_2}$

If one of the cells is a standard cell, the e.m.f. of the other cell can be found.

### 10.61. Direct-reading Potentiometer

The simple potentiometer described above is used for educational purposes only. But in its commercial form, it is so calibrated that the readings of the potentiometer give the voltage directly, thereby eliminating tedious arithmetical calculations and so saving appreciable time.

Such a direct-reading potentiometer is shown in Fig. 10.79. The resistance  $R$  consists of 14 equal resistances joined in series, the resistance of each unit being equal to that of the whole slide wire  $S$  (which is divided into 100 equal parts). The battery current is controlled by slide wire resistance  $W$ .

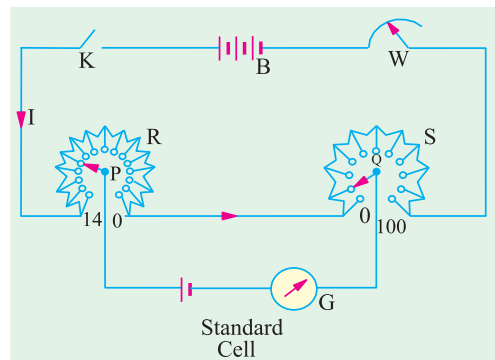


Fig. 10.79

### 10.62. Standardizing the Potentiometer

A standard cell *i.e.* Weston cadmium cell of e.m.f. 1.0183 V is connected to sliding contacts  $P$  and  $Q$  through a sensitive galvanometer  $G$ . First,  $P$  is put on stud No. 10 and  $Q$  on 18.3 division on  $S$  and then  $W$  is adjusted for zero deflection on  $G$ . In that case, potential difference between  $P$  and  $Q$  is equal to cell voltage *i.e.* 1.0183 V so that potential drop on each resistance of  $R$  is  $1/10 = 0.1$  V and every division of  $S$  represents  $0.1/100 = 0.001$  V. After standardizing this way, the **position of  $W$  is not to be changed in any case** otherwise the whole adjustment would go wrong. After this, the instrument becomes direct reading. Suppose in a subsequent experiment, for balance,  $P$  is moved to stud No. 7 and  $Q$  to 84 division, then voltage would be  $= (7 \times 0.1) + (84 \times 0.001) = 0.784$  V.

It should be noted that since most potentiometers have fourteen steps on  $R$ , it is usually not possible to measure p.d.s. exceeding 1.5 V. For measuring higher voltages, it is necessary to use a volt box.

### 10.63. Calibration of Ammeters

The ammeter to be calibrated is connected in series with a variable resistance and a standard resistance  $F$ , say, of  $0.1 \Omega$  across battery  $B_1$  of ample current capacity as shown in Fig. 10.80. Obviously, the resistance of  $F$  should be such that with maximum current flowing through the ammeter  $A$ , the potential drop across  $F$  should not exceed 1.5 V. Some convenient current, say

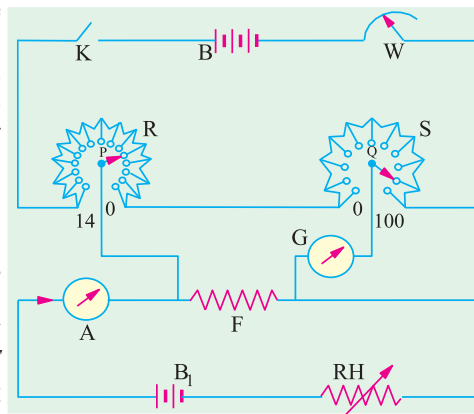


Fig. 10.80

6 amperes (as indicated by  $A$ ) is passed through the circuit by adjusting the rheostat  $RH$ .

The potential drop across  $F$  is applied between  $P$  and  $Q$  as shown. Next the sliding contacts  $P$  and  $Q$  are adjusted for zero deflection on  $G$ . Suppose  $P$  reads 5 and  $Q$  reads 86.7. Then it means that p.d. across  $F$  is 0.5867 V and since  $F$  is of  $0.1 \Omega$ , hence true value of current through  $F$  is  $0.5867/0.1 = 5.867$  amperes. Hence, the ammeter reads high by  $(6 - 5.867) = 0.133$  A. The test is repeated for various values of current over the entire ranges of the ammeter.

### 10.64. Calibration of Voltmeters

As pointed out in Art. 10.62, a voltage higher than 1.5 cannot be measured by the potentiometer directly, the limit being set by the standard cell and the type of the potentiometer (since it has only 14 resistances on  $R$  as in Fig. 10.79). However, with the help of a volt-box which is nothing else but a voltage reducer, measurements of voltage up to 150 V or 300 V can be made, the upper limit of voltage depending on the design of the volt-box.

The diagram of connections for calibration of voltmeters is shown in Fig. 10.81. By calibration is meant the determination of the extent of error in the reading of the voltmeter throughout its range. A high value resistor  $AB$  is connected across the supply terminals of high voltage battery  $B_1$  so that it acts as a voltage divider. The volt-box consists of a high resistance  $CD$  with tapings at accurately determined points like  $E$  and  $F$  etc. The resistance  $CD$  is usually 15,000 to 300  $\Omega$ . The two tapings  $E$  and  $F$  are such that the resistances of portions  $CE$  and  $CF$  are  $1/100$ th and  $1/10$ th the resistance  $CD$ . Obviously, whatever the potential drop across  $CD$ , the corresponding potential drop across  $CE$  is  $1/100$ th and that across  $CF$ ,  $1/10$ th of that across  $CD$ .

If supply voltage is 150 V, then p.d. across  $AB$  is also 150 V and if  $M$  coincides with  $B$ , then p.d. across  $CD$  is also 150 V, so across  $CF$  is 15 volts and across  $CE$  is 1.5 V. Then p.d. across  $CE$  can be balanced over the potentiometer as shown in Fig. 10.81. Various voltages can be applied across the voltmeter by moving the contact point  $M$  on the resistance  $AB$ .

Suppose that  $M$  is so placed by voltmeter  $V$  reads 70 V and p.d. across  $CE$  is balanced by adjusting  $P$  and  $Q$ . If the readings on  $P$  and  $Q$  to give balance are 7 and 8.4 respectively, then p.d. across  $CE$  is 0.7084 V.

Hence, the true p.d. across  $AM$  or  $CD$  or voltmeter is  $0.7048 \times 100 = 70.48$  V (because resistance of  $CD$  is 100 times greater than that of  $CE$ ). In other words, the reading of the voltmeter is low by 0.84 V.

By shifting the position of  $M$  and then balancing the p.d. across  $CE$  on the potentiometer, the voltmeter can be calibrated throughout its range. By plotting the errors on a graph, a calibration curve of the instrument can also be drawn.

### 10.65. A.C. Potentiometer

An A.C. potentiometer basically works on the same principle as a d.c. potentiometer. However,

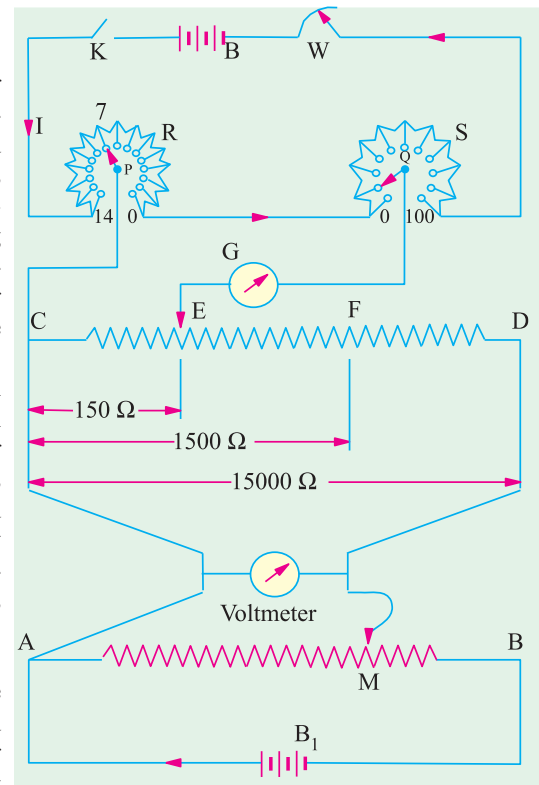


Fig. 10.81

there is one very important difference between the two. In d.c. potentiometer, only the **magnitudes** of the unknown e.m.f. and slide-wire voltage drop are made equal for obtaining balance. But in an a.c. potentiometer, not only the magnitudes but **phases** as well have to be equal for obtaining balance. Moreover, to avoid frequency and waveform errors, the a.c. supply for slide-wire must be taken from same source as the voltage or current to be measured.

A.C. potentiometers are of two general types differing in the manner in which the value of the unknown voltage is presented by the instrument dials or scales. The two types are :

- (i) Polar potentiometers in which the unknown voltage is measured in polar form *i.e.* in terms of magnitude and relative phase.
- (ii) Co-ordinate potentiometers which measure the rectangular co-ordinates of the voltage under test.

The two products are illustrated in Fig. 10.82. In Fig. 10.82 (a), vector  $OQ$  denotes the test voltage whose magnitude and phase are to be imitated. In polar potentiometer, the length  $r$  of the vector  $OP$  can be varied with the help of a sliding contact on the slidewire while its phase  $\phi$  is varied independently with the help of a phase-shifter. Drysdale potentiometer is of this type.

In co-ordinate type potentiometers, the unknown voltage vector  $OQ$  is copied by the adjustment of 'in phase' and 'quadrature' components  $X$  and  $Y$ . Their values are read from two scales of the potentiometer. The magnitude of the required vector is  $= \sqrt{X^2 + Y^2}$  and its phase is given by  $\phi = \tan^{-1}(X/Y)$ . Examples of this type are (i) Gall potentiometer and (ii) Campbell-Larsen potentiometer.

### 10.66. Drysdale Polar Potentiometer

As shown in Fig. 10.83 for a.c. measurements, the slide-wire  $MN$  is supplied from a phase shifting circuit so arranged that magnitude of the voltage supplied by it remains constant while its

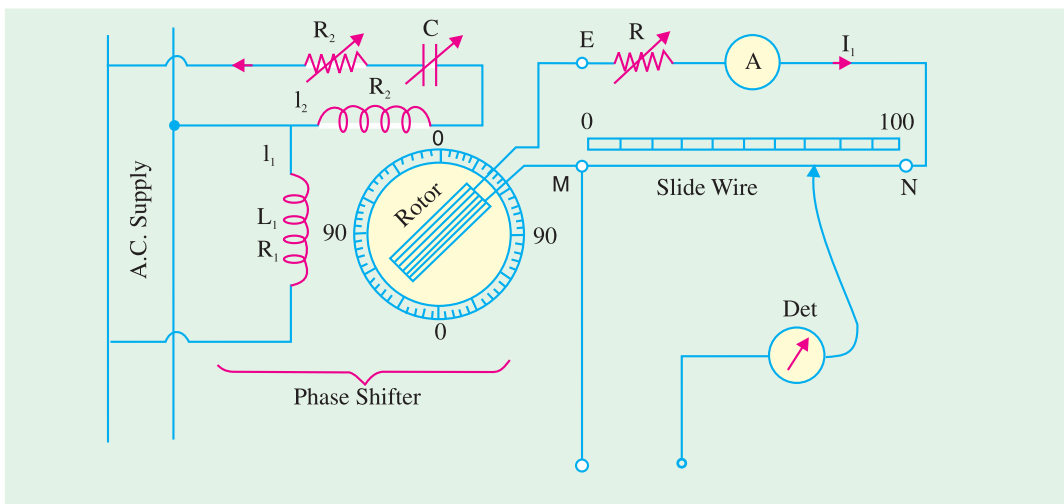


Fig. 10.83

phase can be varied through  $360^\circ$ . Consequently, slide-wire current  $I$  can be maintained constant in magnitude but varied in phase. The phase-shifting circuit consists of :

- (i) Two stator coils supplied from the same source in parallel. Their currents  $I_1$  and  $I_2$  are made to differ by  $90^\circ$  by using well-known phase-splitting technique.
- (ii) The two windings produce a rotating flux which induces a secondary e.m.f. in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position either manually or otherwise. The phase of the rotor e.m.f. is read from the circular graduated dial provided for the purpose.

The ammeter  $A$  in the slide-wire circuit is of electrodynamic or thermal type. Before using it for a.c. measurements, the potentiometer is first calibrated by using d.c. supply for slidewire and a standard cell for test terminals  $T_1$  and  $T_2$ .

The unknown alternating voltage to be measured is applied across test terminals  $I_2$  and  $T_2$ . Balance is effected by the alternate adjustment of the slide-wire contact and the position of phase-shifting rotor. The slide-wire reading represents the magnitude of the test voltage phase-shifter reading gives its phase with reference to an arbitrary reference vector.

### 10.67. Gall Co-ordinate Potentiometer

This potentiometer uses two slide-wires  $CD$  and  $MN$  with their currents  $I_1$  and  $I_2$  (Fig. 10.84) having a mutual phase difference of  $90^\circ$ . The two currents are obtained from the single phase supply through isolating transformers, the circuit for 'quadrature' slidewire  $MN$  incorporating a phase shifting arrangement.

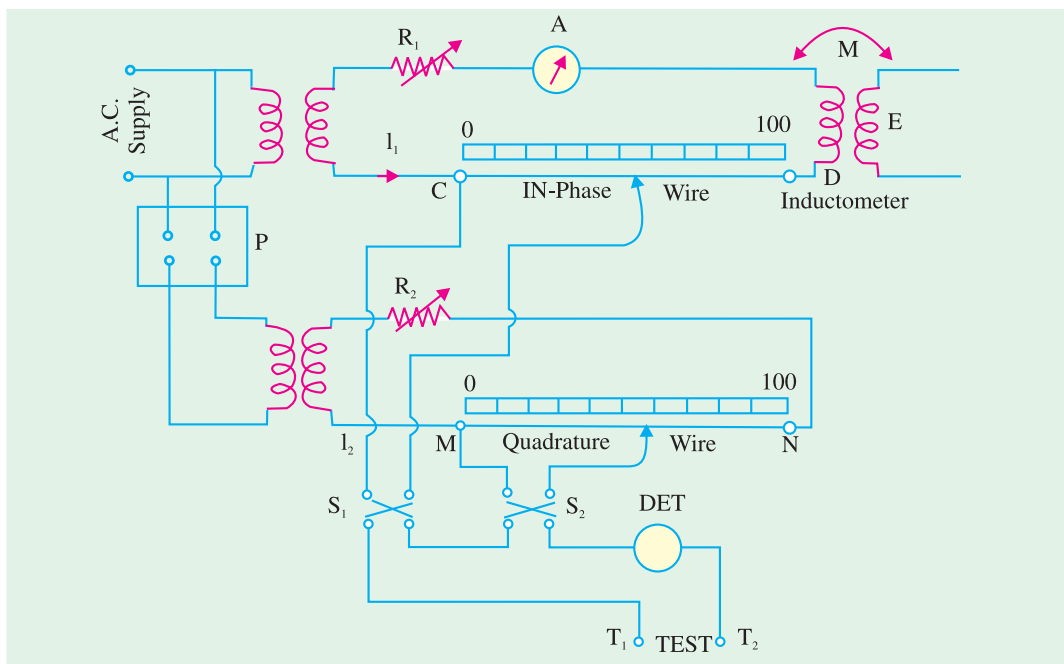


Fig. 10.84

Before use, then current  $I_1$  is first standardised as described for Drysdale potentiometer (Art. 10.66). Next, current  $I_2$  is standardised with the help of the mutually induced e.m.f.  $E$  in inductometer secondary. This e.m.f.  $E = \omega MI_1$  and is in quadrature phase with  $I_1$ . Now,  $E$  is balanced against the voltage drop on slide-wire  $MN$ . This balance will be obtained only when  $I_2$  is of correct magnitude and is in exact quadrature with  $I_1$ . Balance is achieved with the help of the phase-shifter and rheostat  $R_2$ .

The unknown voltage is applied across the test terminals  $T_1$  and  $T_2$ . Slide-wire  $MN$  measures that component of the unknown voltage which is in phase with  $I_2$ . Similarly, slide-wire  $CD$  measures that component of the unknown voltage which is in phase with  $I_1$ . Since  $I_1$  and  $I_2$  are in quadrature, the

two measured values are quadrature components of the unknown voltage. If  $V_1$  and  $V_2$  are these values, then

$$V = \sqrt{V_1^2 + V_2^2} \quad \text{and} \quad \phi = \tan^{-1} (V_2/V_1) \quad \text{—with respect to } I_1$$

Reversing switches  $S_1$  and  $S_2$  are used for measuring both positive and negative in-phase and quadrature components of the unknown-voltage.

### 10.68. Instrument Transformers

The d.c. circuits when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with high resistances connected in series with them. But it is neither convenient nor practical to use this method with alternating current and voltage instruments. For this purpose, specially constructed accurate-ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. Their purpose is to reduce the line current or supply voltage to a value small enough to be easily measured with meters of moderate size and capacity. In other words, they are used for extending the range of a.c. ammeters and voltmeters. Instrument transformers are of two types :

- (i) current transformers (*CT*) —for measuring large alternating currents.
- (ii) potential transformers (*VT*) —for measuring high alternating voltages.

Advantages of using instrument transformers for range extension of a.c. meters are as follows :

(1) the instrument is insulated from the line voltage, hence it can be grounded. (2) the cost of the instrument (or meter) together with the instrument transformer is less than that of the instrument alone if it were to be insulated for high voltages. (3) it is possible to achieve standardisation of instruments and meters at secondary ratings of 100–120 volts and 5 or 1 amperes (4) if necessary, several instruments can be operated from a single transformer and 5 power consumed in the measuring circuits is low.

In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage). These ratios give us the multiplying factor for finding the primary values from the instrument readings on the secondary side.

However, for energy or power measurements, it is essential to know not only the transformation ratio but also the phase angle between the primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

For range extension on a.c. circuits, instrument transformers are more desirable than shunts (for current) and multipliers (for voltage measurements) for the following reasons :

1. time constant of the shunt must closely match the time constant of the instrument. Hence, a different shunt is needed for each instrument.
2. range extension is limited by the current-carrying capacity of the shunt *i.e.* upto a few hundred amperes at the most.
3. if current is at high voltage, instrument insulation becomes a very difficult problem.
4. use of multipliers above 1000 becomes almost impracticable.
5. insulation of multipliers against leakage current and reduction of their distributed capacitance becomes not only more difficult but expensive above a few thousand volts.

### 10.69. Ratio and Phase-angle Errors

For satisfactory and accurate performance, it is necessary that the ratio of transformation of the instrument transformer should be constant within close limits. However, in practice, it is found that neither current transformation ratio  $I_1/I_2$  (in the case of current transformers) nor voltage transformation ratio  $V_1/V_2$  (in the case of potential transformers) remains constant. The transformation ratio is found to depend on the exciting current as well as the current and the power factor of the secondary circuit. This fact leads to an error called ratio error of the transformer which depends on the working component of primary.

It is seen from Fig. 1.85 (a) that the phase angle between the primary and secondary currents is not exactly  $180^\circ$  but slightly less than this value. This difference angle  $\beta$  may be found by reversing

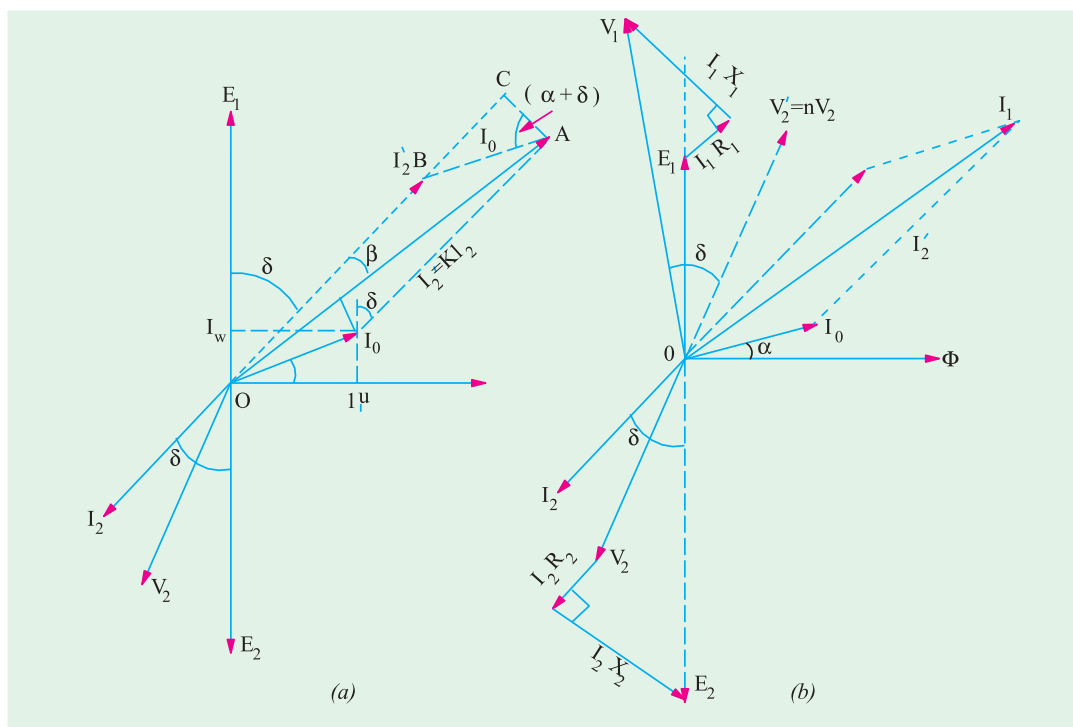


Fig. 10.85

vector  $I_2$ . The angular displacement between  $I_1$  and  $I_2$  reversed is called the phase angle error of the current transformer. This angle is reckoned positive if the reversed secondary current **leads** the primary current. However, on very low power factors, the phase angle may be negative. Similarly, there is an angle of  $\gamma$  between the primary voltage  $V_1$  and secondary voltage reversed—this angle represents the phase angle error of a voltage transformer. In either case, the phase error depends on the magnetising component  $I_\mu$  of the primary current. It may be noted that ratio error is primarily due to the reason that the **terminal** voltage transformation ratio of a transformer is not exactly equal to its turn ratio. The divergence between the two depends on the resistance and reactance of the transformer windings as well as upon the value of the exciting current of the transformer. Accuracy of voltage ratio is of utmost importance in a voltage transformer although phase angle error does not matter if it is to be merely connected **to a voltmeter**. Phase-angle error becomes important only when voltage transformer supplies the voltage coil of a wattmeter *i.e.* in power measurement. In that case, phase angle error causes the wattmeter to indicate on a wrong power factor.

In the case of current transformers, constancy of current ratio is of paramount importance. Again, phase angle error is of no significance if the current transformer is merely feeding an ammeter but it assumes importance when feeding the current coil of a wattmeter. While discussing errors, it is worthwhile to define the following terms :

(i) **Nominal transformation ratio ( $k_n$ )**. It is the ratio of the **rated** primary to the **rated** secondary current (or voltage).

$$k_n = \frac{\text{rated primary current } (I_1)}{\text{rated secondary current } (I_2)} \quad \text{—for CT}$$

$$= \frac{\text{rated primary voltage } (V_1)}{\text{rated secondary voltage } (V_2)} \quad \text{—for VT}$$

In the case of current transformers, it may be stated either as a fraction such as 500/5 or 100/1 or

simply as the number representing the numerator of the reduced fraction *i.e.* 100. It is also known as **marked ratio**.

(ii) **Actual transformation ratio ( $k$ )**. The actual transformation ratio or just ratio under any given condition of loading is

$$k = \frac{\text{primary current } (I_1)}{\text{corresponding secondary current } (I_2)}$$

In general,  $k$  differs from  $k_n$  except in the case of an ideal or perfect transformer when  $k = k_n$  for all conditions of loading.

(iii) **Ratio Error ( $\sigma$ )**. In most measurements it may be assumed that  $I_1 = k_n I_2$  but for very accurate work, it is necessary to correct for the difference between  $k$  and  $k_n$ . It can be done with the help of ratio error which is defined as

$$\sigma = \frac{k_n - k}{k} = \frac{\text{nominal ratio} - \text{actual ratio}}{\text{actual ratio}}$$

Also,

$$\sigma = \frac{k_n \cdot I_2 - k I_2}{k \cdot I_2} = \frac{k_n \cdot I_2 - I_1}{I_1}$$

Accordingly, ratio error may be defined as the difference **between the primary current reading (assuming the nominal ratio) and the true primary current divided by the true primary current**.

(iv) **Ratio Correction Factor (R.C.F.)**. It is given by

$$R.C.F. = \frac{\text{actual ratio}}{\text{nominal ratio}} = \frac{k}{k_n}$$

## 10.70. Current Transformer

A current transformer takes the place of shunt in d.c. measurements and enables heavy alternating current to be measured with the help of a standard 5-A range a.c. ammeter.

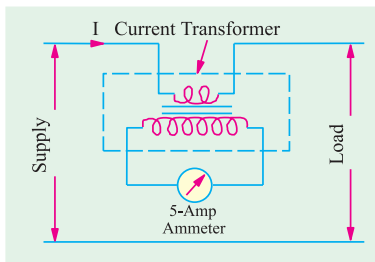


Fig. 10.86

As shown in Fig. 10.86, the current - or series-transformer has a primary winding of one or more turns of thick wire connected in series with the line carrying the current to be measured. The secondary consist of a large number of turns of fine wire and feeds a standard 5-A ammeter (Fig. 10.86) or the current coil of a watt-meter or wathour-meter (Fig. 10.87).

For example, a 1,000/5A current transformer with in single-turn primary will have 200 secondary turns. Obviously, it steps down the current in the 200 : 1 ratio whereas it steps up the voltage drop across the single-turn primary (an extremely small quantity) in the ratio 1 : 200. Hence if we know the current ratio of the transformer and the reading of the a.c. ammeter, the line current can be calculated.

It is worth noting that ammeter resistance being extremely low, a current transformer operates with its secondary under nearly short-circuit conditions. Should it be necessary to remove the ammeter of the current coils of the wattmeter or a relay, the secondary winding must, first of all, be short-circuited *before* the instrument is disconnected.

If it is not done then due to the absence of counter ampere-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating of and damage of the transformer insulation and a high voltage across the secondary

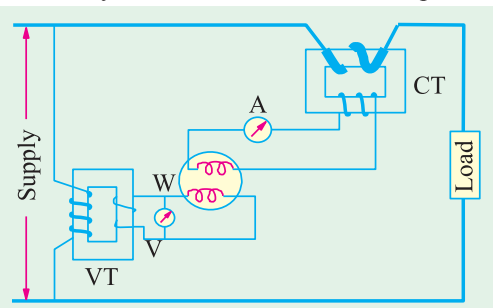


Fig. 10.87



terminals. This is not the case with the ordinary constant-potential transformers because their primary current is determined by the load on their secondary whereas in a current transformer, primary current is determined entirely by the load on the system and not by the load on its own secondary. Hence, the secondary of a current transformer should **never be left open under any circumstances**.

### 10.71. Theory of Current Transformer

Fig. 10.85 (b) represents the general phase diagram for a current transformer. Current  $I_0$  has been exaggerated for clarity.

**(a) Ratio Error.** For obtaining an expression for the ratio error, it will be assumed that the turn ratio  $n$  (= secondary turns,  $N_2$ /primary turns  $N_1$ ) is made equal to the nominal current ratio *i.e.*  $n = k_n$ . In other words, it will be assumed that  $I_1/I_2 = n$  although actually  $n = I_1/I_2'$ . As seen from Art. 10.63.

$$\begin{aligned}\sigma &= \frac{n I_2 - I_1}{I_1} = \frac{I_2' - I_1}{I_1} = \frac{OB - OA}{OA} && \text{--- } [\because n = k_n] \\ &\equiv \frac{OB - OC}{OA} && (\because \beta \text{ is very small angle}) \\ &= -\frac{BC}{OA} = -\frac{AB \sin(\alpha + \delta)}{OA} = -\frac{I_0 \sin(\alpha + \delta)}{I_1} = -\frac{I_0 \sin(\alpha + \delta)}{n I_2}\end{aligned}$$

For most instrument transformers, the power factor of the secondary burden is nearly unity so that  $\delta$  is very small. Hence, very approximately.

$$\sigma = \frac{I_0 \sin \alpha}{I_1} - \frac{I_\omega}{I_1}$$

where  $I_\omega$  is the iron-loss or working or wattful component of the exciting current  $I_0$

**Note.** The transformation ratio  $R$  may be found from Fig. 10.85 (a) as under :

$$I_1 = OA = OB + BC = n I_2 \cos \beta + I_0 \cos [90 - (\delta + \beta + \alpha)] = n I_2 \cos \beta + I_0 \sin (\delta + \beta + \alpha)$$

Now  $\beta = (\alpha + \delta)$  hence  $I_1 = n I_2 + I_0 \sin (\alpha + \delta)$  where  $n$  is the turn ratio of the transformer.

$$\therefore \text{ratio } R = \frac{I_1}{I_2} = \frac{n I_2 + I_0 \sin (\alpha + \delta)}{I_2} \text{ or } R = n + \frac{I_0 \sin (\alpha + \delta)}{I_2} \quad \dots(i)$$

$$\text{If } \delta \text{ is negligible small, then } R = n + \frac{I_0 \sin \alpha}{I_2} = n + \frac{I_\omega}{I_2}$$

It is obvious from (i) above that ratio error can be eliminated if secondary turn are reduced by a number

$$= I_0 \sin (\alpha + \delta) / I_2$$

#### (b) Phase angle ( $\beta$ )

Again from Fig. 10.85 (a), we find that

$$\beta \equiv \sin \beta = \frac{AC}{OA} = \frac{AB \cos (\alpha + \delta)}{OA} = \frac{I_0 \cos (\alpha + \delta)}{I_1} = \frac{I_0 \cos (\alpha + \delta)}{n I_2}$$

Again, if the secondary power factor is nearly unity, then  $\delta$  is very small, hence

$$\beta \equiv \frac{I_0 \cos \alpha}{I_1} = \frac{I_\mu}{I_1} \text{ or } \frac{I_\mu}{n I_2}$$

where  $I_\mu$  is the magnetising component of the exciting current  $I_0$ .

$$\therefore \beta = \frac{I_\mu}{I_1} \quad \text{---in radian; } = \frac{180}{\pi} \times \frac{I_\mu}{I_1} \quad \text{---in degrees}$$

$$\text{Note. As found above, } \beta = \frac{I_0 \cos ( \quad )}{I_1} \quad \frac{I_0 (\cos \quad \cos \quad \sin \quad \sin \quad )}{I_1}$$

$$= \frac{I_{\mu} \cos \delta - I_{\omega} \sin \delta}{I_1} = \frac{I_{\mu} \cos \delta - I_{\omega} \sin \delta}{nI_2} \text{ radian}$$

$$\therefore \beta = \frac{180}{\pi} \times \frac{I_{\mu} \cos \delta - I_{\omega} \sin \delta}{nI_2} \text{ degrees.}$$

Dependence of ratio error on working component of  $I_0$  and that of phase angle on the magnetising component is obvious. If  $R$  is to come closer to  $k$  and  $\beta$  is to become negligible small, then  $I_{\mu}$  and  $I_{\omega}$  and hence  $I_0$  should be very small.

### 10.72. Clip-on Type Current Transformer

It has a laminated core which is so arranged that it can be opened out at a hinged section by merely pressing a trigger-like projection (Fig. 10.88). When the core is thus opened, it permits the admission of very heavy current-carrying bus-bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current-carrying conductor of feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle itself.

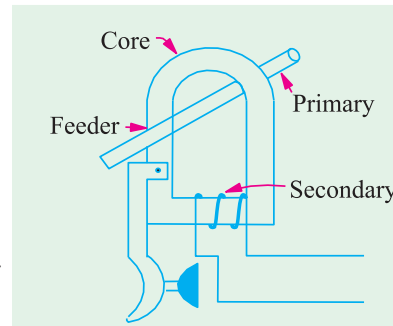


Fig. 10.88

### 10.73. Potential Transformers

These transformers are extremely accurate-ratio stepdown transformers and are used in conjunction with standard low-range voltmeters (100-120 V) whose deflection when divided by transformation ratio, gives the true voltage on the primary or high-voltage side. In general, they are of the shell type



Potential transformer

and do not differ much from the ordinary two-winding transformers except that their power rating is extremely small. Since their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100W. For safety, the secondary is completely insulated from the high voltage primary and is, in addition, grounded for affording protection to the operator. Fig. 10.89 shows the connection of such a transformer.

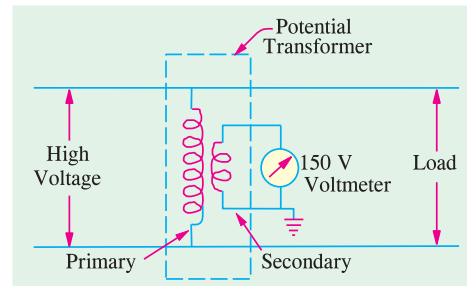


Fig. 10.89

### 10.74. Ratio and Phase-angle Errors

In the case of a potential transformer, we are interested in the ratio of the primary to the secondary terminal voltage and in the phase angle  $\gamma$  between the primary and reversed secondary terminal voltage  $V_2'$ .

The general theory of voltage transformer is the same as for the power transformers except that, as the current in the secondary burden is very small, the total primary current  $I_1$  is not much greater than  $I_0$ .

In the phasor diagram of Fig. 10.90, vectors  $AB$ ,  $BC$ ,  $CD$  and  $DE$  represent small voltage drops due to resistances and reactances of the transformer winding (they have been exaggerated for the sake of clarity). Since the drops as well as the phase angle  $\gamma$  are small, the top portion of diagram 10.90 (a) can be drawn with negligible loss of accuracy as in Fig. 10.90 (b) where  $V_2'$  vector has been drawn parallel to the vector for  $V_1$ .

In these diagrams,  $V_2'$  is the secondary terminal voltage as referred to primary assuming transformation without voltage drops. All actual voltage drops have been referred to the primary. Vector  $AB$  represents total resistive drop as referred to primary i.e.  $I_2' R_{01}$ . Similarly,  $BC$  represents total reactive drop as referred to primary i.e.  $I_2' X_{01}$ .

In a voltage transformer, the relatively large no-load current produces appreciable resistive drops which have been represented by vectors  $CD$  and  $DE$  respectively. Their values are  $I_0 R_1$  and  $I_0 X_1$  respectively.

**(a) Ratio Error**

In the following theory,  $n$  would be taken to represent the ratio of **primary turns to secondary turns** (Art. 10.69). Further, it would be assumed, as before, that  $n$  equals the nominal transformation ratio i.e.  $n = k_n$ .

In other words, it would be assumed that  $V_1/V_2 = n$ , although, actually,  $V_1/V_2' = n$ .

Then

$$\begin{aligned} \sigma &= \frac{k_n - k}{k} = \frac{k_n \cdot V_2 - k V_2}{k V_2} = \frac{V_2' - V_1}{V_1} = -\frac{EN}{OE} \quad \dots \text{Fig. 10.90 (a)} \\ &= -\frac{AG + FC + LD + EM}{OE} \quad \dots \text{Fig. 10.90 (b)} \\ &= -\frac{I_2' R_{02} \cos \delta + I_2' X_{02} \sin \delta + I_0 R_1 \sin \alpha + I_0 X_1 \cos \alpha}{V_1} \\ &= -\frac{I_2' R_{02} \cos \delta + I_2' X_{02} \sin \delta + I_\omega R_1 + I_\mu X_1}{V_1} \end{aligned}$$

where  $I_\omega$  and  $I_\mu$  are the iron-loss and magnetising components of the no-load primary current  $I_0$ .

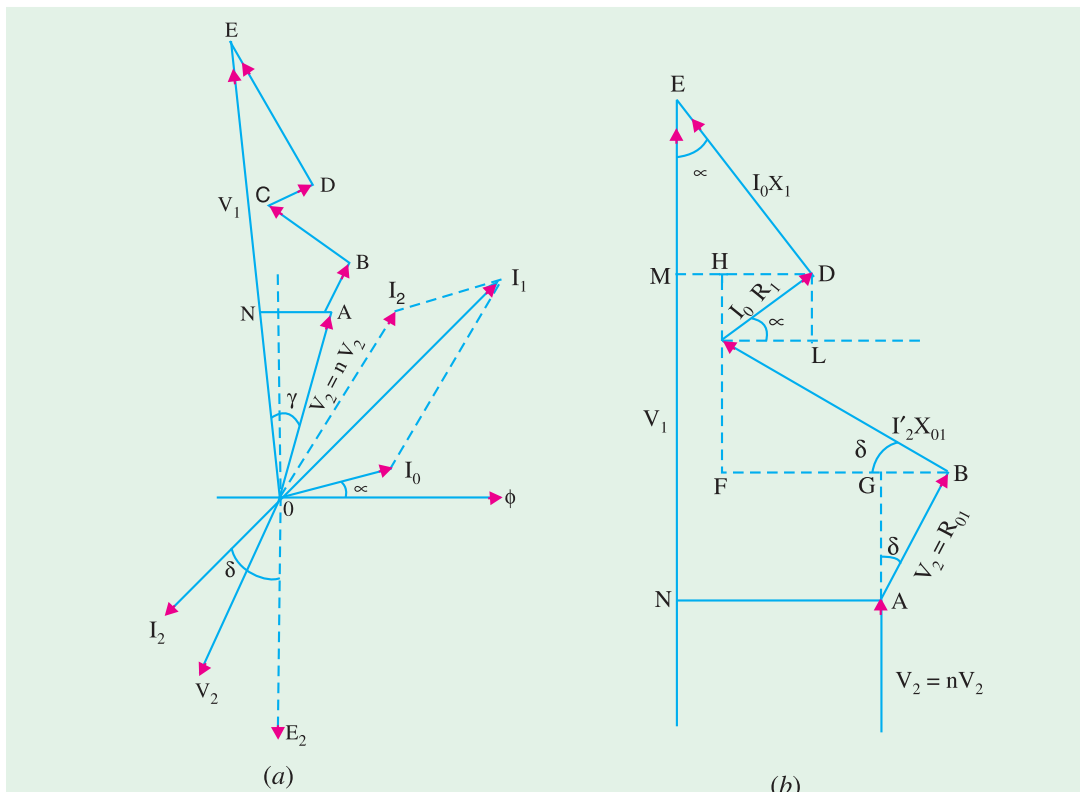


Fig. 10.90

**(b) Phase Angle ( $\gamma$ )**

To a very close approximation, value of  $\gamma$  is given by  $\gamma = AN/OA$  —in radian

Now,  $OA \cong OE$  provided ratio error is neglected. In that case,

$$\begin{aligned}\gamma &= \frac{AN}{OE} = -\frac{GF + HM}{OE} && \dots \text{Fig. 10.90 (b)} \\ &= -\frac{(BE - BG) + (DM - DH)}{OE} \\ &= -\frac{I_2' X_{01} \cos \delta - I_2' R_{01} \sin \delta + I_0 X_1 \sin \alpha - I_0 R_1 \cos \alpha}{V_1} \\ &= -\frac{I_2' X_{01} \cos \delta - I_2' R_{01} \sin \delta + I_\omega X_1 - I_\mu R_1}{V_1}\end{aligned}$$

The negative sign has been given because reversed secondary voltage *i.e.*  $V_2'$  lags behind  $V_1$ .

**Example 10.44.** A current transformer with 5 primary turns and a nominal ratio of 1000/5 is operating with a total secondary impedance of  $0.4 + j 0.3 \Omega$ . At rated load, the iron loss and magnetising components of no-load primary current are 1.5 A and 6 A respectively. Calculate the ratio error and phase angle at rated primary current if the secondary has (a) 1000 turns and (b) 990 turns.

**Solution.** Phasor diagram of Fig. 10.87 may please be referred to.

$$\begin{aligned}\text{(a)} \quad \tan \delta &= 0.3/0.4 \text{ or } \delta = \tan^{-1} (0.3/0.4) = 36^\circ 52' \\ \alpha &= \tan^{-1} (1.5/6) = 14^\circ 2' \quad \therefore (\alpha + \beta) = 50^\circ 54'\end{aligned}$$

$$I_0 = \sqrt{I_\omega^2 + I_\mu^2} = \sqrt{1.5^2 + 6^2} = 6.186 \text{ A}$$

$$BC = I_0 \sin (\alpha + \beta) = 6.186 \times \sin 50^\circ 54' = 4.8 \text{ A}$$

Since  $\beta$  is small,  $OC \cong OA = 1000 \text{ A} \therefore I_2' = OC - BC = 1000 - 4.8 = 995.2 \text{ A}$

$$\therefore \sigma = \frac{k_n I_2 - I_1}{I_1} = \frac{n I_2 - I_1}{I_1} \text{ because in this case } n (= N_2/N_1 = 1000/5)$$

is equal to nominal ratio  $k_n (= 1000/5 \text{ A})$ .

$$\text{or} \quad \sigma = \frac{I_2 - I_1}{I_1} = \frac{995.2 - 1000}{1000} = 0.0048 \text{ or } 48\%$$

$$\beta = \tan^{-1} (AC/OC)$$

$$\text{Now} \quad AC = I_0 \cos (\alpha + \beta) = 6.186 \cos 50^\circ 54' = 3.9 \text{ A}$$

$$\text{centre} \quad \beta = \tan^{-1} (AC/OC) = \tan^{-1} (3.9/1000) = 0^\circ 13'$$

$$\text{(b) In this case,} \quad I_2 = 995.2 \times \frac{5}{990}; k_n \cdot I_2 = \frac{1000}{5} \times 995.2 \times \frac{5}{990} = 1005.3 \text{ A}$$

$$\therefore \sigma = \frac{k_n I_2 - I_1}{I_1} = \frac{1005.3 - 1000}{1000} = 0.0053 \text{ or } 0.53\%$$

The value of phase angle  $\beta$  would not be significantly different from the value obtained in (a) above.

**Example 10.45.** A relay current-transformer has a bar primary and 200 secondary turns. The secondary burden is an ammeter of resistance  $1.2 \Omega$  and reactance of  $0.5 \Omega$  and the secondary winding has a resistance of  $0.2 \Omega$  and reactance of  $0.3 \Omega$ . The core requires the equivalent of 100 AT for magnetisation and 50 AT for core losses.

(i) Find the primary current and the ratio error when the secondary ammeter indicates 5.0 A.

(ii) By how many turns should the secondary winding be reduced to eliminate the ratio error for this condition? (Electrical Measurements, Bombay Univ.)

**Solution.** Total secondary impedance is

$$Z_2 = 1.4 + j 0.8 = 1.612 \angle 29^\circ 45' \quad \therefore \delta = 29^\circ 45'$$

$$I_0 = 100 + j50 = 111.8 \angle 26^\circ 34' \quad \therefore \alpha = 26^\circ 34'$$

Turn ratio,  $n = 200/I = 200$ ; Transformation ratio  $R = n + \frac{I_0 \sin(\alpha + \delta)}{I_2}$

$$\therefore R = 200 + \frac{111.8 \sin 56^\circ 19'}{5} = 218.6$$

(i) Primary current  $= 5 \times 218.6 = 1093 \text{ A}$

Ratio error  $\sigma = \frac{I_0 \sin(\quad)}{nI_2} = \frac{111.8 \cdot 0.8321}{200 \cdot 5} = 0.093 \text{ or } 9.3\%$

(ii) No. of secondary turns to be reduced  $= I_0 \sin(\alpha + \delta)/I_2 = 93/5 = 19$  (approx).

**Example 10.46.** A current transformer has 3 primary turns and 300 secondary turns. The total impedance of the secondary is  $(0.583 + j 0.25)$  ohm. The secondary current is 5 A. The ampere-turns required to supply excitation and iron losses are respectively 10 and 5 per volt induced in the secondary.

Determine the primary current and phase angle of the transformer.

(Elect Meas; M.S. Univ. Baroda)

**Solution.**  $Z_2 = 0.583 + j 0.25 = 0.6343 \angle 23^\circ 10' \therefore E_2 = I_2 Z_2 = 5 \times 0.6343 = 3.17 \text{ V}$

Now, there are 10 magnetising AT per secondary volt induced in secondary.

$$\therefore \text{total magnetising } AT = 3.17 \times 10 = 31.7; \text{ Similarly, iron-loss } AT = 3.17 \times 5 = 15.85$$

Remembering that there are 3 primary turns, the magnetising and iron-loss components of primary current are as under :

Magnetising current,  $I_\mu = 31.7/3 = 10.6 \text{ A}$ ; iron-loss current  $I_\omega = 15.85/3 = 5.28 \text{ A}$

$$I_0 = \sqrt{10.6^2 + 5.28^2} = 11.84 \text{ A}$$

Now,  $R = n \frac{I_0 \sin(\quad)}{I_2}$

Here,  $n = 300/3 = 100$ ;  $\alpha = \tan^{-1}(I_\omega/I_\mu) = \tan^{-1}(5.28/10.6)$   
 $= \tan^{-1}(0.498) = 26^\circ 30'$

$$\delta = \text{secondary load angle} = 23^\circ 10' \quad \text{—found earlier}$$

$$\therefore R = 100 + \frac{11.86}{5} (\sin 49^\circ 40') = 100 + 1.81 = 101.81$$

$$I_1 = R \times I_2 = 101.81 \times 5 = 509.05 \text{ A}$$

$$\beta = \frac{180}{\pi} \times \frac{I_\mu \cos \delta - I_\omega \sin \delta}{nI_2}$$

$$= \frac{180}{100 \cdot 5} \times \frac{10.6 \cos 23^\circ 10' - 5.28 \sin 23^\circ 10'}{5} = 0.88^\circ$$

**Example 10.47.** A current transformer with a bar primary has 300 turns in its secondary winding. The resistance and reactance of the secondary circuit are  $1.5 \Omega$  and  $1.0 \Omega$  respectively including the transformer winding. With 5 A flowing in the secondary circuit the magnetising ampere-turns required are 100 and iron loss is 1.2 W. Determine the ratio error at this condition.

(Elect. Measure, A.M.I.E. Sec. B, 1992)

**Solution.** Turn ratio  $n = 300/1 = 300$

Secondary impedance is  $Z_2 = 1.5 + j 1.0 = 1.8 \angle 33^\circ 42'$

Secondary induced e.m.f.  $E_2 = I_2 Z_2 = 5 \times 1.8 = 9 \text{ V}$

$$E_1 = E_2/n = 9/300 = 0.03 \text{ V}$$

Let us now find the magnetising and working components of primary no-load current  $I_0$

Magnetising  $AT = 100$ . Since there is one primary turn,  $\therefore I_\mu = 100/1 = 100$  A

Now,  $E_1 I_\omega = 1.2 \therefore I_\omega = 1.2/0.03 = 40$  A

$$I_0 = 100 + j40 = 107.7 \angle 21^\circ 48' ; \sigma = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

Now  $\alpha = 21^\circ 48'$  and  $\delta = 33^\circ 42'$

$$\therefore \sigma = \frac{107.7 \sin 55^\circ 30'}{300 \times 5} = 0.0592 \text{ or } 5.92\%$$

$$\begin{aligned} \text{Phase angle } \beta &= \frac{I_0 \cos(\alpha + \delta)}{nI_2} = \frac{107.7 \times \cos 55^\circ 30'}{1500} \\ &= \frac{180}{1500} \frac{107.7 \times 0.5664}{1500} = 2^\circ 20' \end{aligned}$$

### Tutorial Problems No. 10.4

1. A current transformer with 5 primary turns has a secondary burden consisting of a resistance of  $0.16 \Omega$  and an inductive reactance of  $0.12 \Omega$ . When primary current is 200 A, the magnetising current is 1.5 A and the iron-loss component is 0.4 A. Determine the number of secondary turns needed to make the current ratio 100/1 and also the phase angle under these conditions. **[407 :  $0.275^\circ$ ]**
2. A current transformer having a 1-turn primary is rated at 500/5 A, 50 Hz, with an output of 1.5 VA. At rated load with the non-inductive burden, the in-phase and quadrature components (referred to the flux) of the exciting ampere-turns are 8 and 10 respectively. The number of turns in the secondary is 98 and the resistance and leakage reactance of the secondary winding are  $0.35 \Omega$  and  $0.3 \Omega$  respectively. Calculate the current ratio and the phase angle error. **[501.95/5;  $0.533^\circ$ ]**  
*(Elect. Inst. and Meas, M.S. Univ. Baroda)*
3. A ring-core current transformer with a nominal ratio of 500/5 and a bar primary has a secondary resistance of  $0.5 \Omega$  and negligible secondary reactance. The resultant of the magnetising and iron-loss components of the primary current associated with a full-load secondary current of 5 A in a burden of  $1.0 \Omega$  (non-inductive) is 3 A at a power factor of 0.4. Calculate the true ratio and the phase-angle error of the transformer on full-load. Calculate also the total flux in the core, assuming that frequency is 50 Hz. **[501.2/5;  $0.314^\circ$ ;  $337 \mu\text{Wb}$ ]**
4. A current transformer has a single-turn primary and a 200-turn secondary winding. The secondary supplies a current of 5 A to a non-inductive burden of  $1 \Omega$  resistance, the requisite flux is set up in the core by 80 AT. The frequency is 50 Hz and the net cross-section of the core is  $10 \text{ cm}^2$ . Calculate the ratio and phase angle and the flux density in the core. **[200.64;  $4^\circ 35'$   $0.079 \text{ Wb/m}^2$ ]** *(Electrical Measurements, Osmania Univ.)*
5. A potential transformer, ratio 1000/100-V, has the following constants :  
 primary resistance =  $94.5 \Omega$  ; secondary resistance =  $0.86 \Omega$   
 primary reactance =  $66.2 \Omega$  ; equivalent reactance =  $66.2 \Omega$   
 magnetising current = 0.02 A at 0.4 p.f.  
 Calculate (i) the phase angle at no-load between primary and secondary voltages (ii) the load in VA at u.p.f. at which the phase angle would be zero. **[(i)  $0^\circ 4'$  (ii) 18.1 VA]**
6. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to  
 (i) 0.5 A range Ammeter (ii) 0.100 V range Voltmeter? **(Nagpur University, Summer 2002)**
7. What are the essential torques of an indicating instruments? Justify their necessity. **(Nagpur University, Winter 2002)**
8. Discuss the necessity of damping in Indicating instrument and explain eddy current damping. **(U.P. Technical University 2002) (Nagpur University, Summer 2003)**
9. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to (i) 0-5 A range Ammeter (ii) 0-100 V range Voltmeter? **(Nagpur University, Summer 2003)**

10. What are the essential requirements of indicating type instruments? Explain each of them. *(U.P. Technical University 2002) (Nagpur University, Winter 2003)*
11. What are the different operating systems required in an instrument? Explain damping system in detail. *(U.P. Technical University 2002) (Nagpur University, Summer 2004)*
12. If a shunt for a moving coil instrument is to have a multiplication factor  $m$ , show that its shunt resistance is given by
- $$R_{sh} = \frac{R_m}{m - 1}$$
- where  $R_m$  → resistance of the meter. *(Nagpur University, Summer 2004)*
13. Find the value of a series resistance to be connected to a basic d'Arsonval movement with internal resistance  $R_m = 100$  ohm and full scale deflection current is 1 mA, for conversion into 0–500 volt. *(Nagpur University, Summer 2004)*
14. What is the basic principle of Induction type instruments? *(Anna University, April 2002)*
15. What is damping torque in instruments? *(Anna University, April 2002)*
16. Why do MI instruments have non-linear scale? *(Anna University, April 2002)*
17. Explain how can you measure power by the ammeters. *(Anna University, April 2002)*
18. What is creeping in energy meter? *(Anna University, April 2002)*
19. What is the advantage of Induction type meters? *(Anna University, April 2002)*
20. What is a trivector meter? *(Anna University, April 2002)*
21. What is the difference between PMMC instrument and ballistic galvanometer? *(Anna University, April 2002)*
22. What is the basic principle of operation of reed type frequency meters? *(Anna University, April 2002)*
23. Why can't you measure low resistance with meggar? *(Anna University, April 2002)*
24. What are the advantages of electronic meters? *(Anna University, April 2002)*
25. Explain the principle of operation, construction and the expression for deflection of a moving iron instrument. *(Anna University, April 2002)*
26. Draw and explain the working of 2 element wattmeter. Explain how does the 2 element wattmeter read total power in a 3  $\phi$  circuit. *(Anna University, April 2002)*
27. Write the equations for wattmeter readings  $W_1$  and  $W_2$  in 3 phase power measurement and therefrom for power factor. *(Anna University, Winter 2002)*
28. Explain how power can be measured in a three phase circuit with the help of two wattmeters, for a balanced star connected load. Draw the phasor diagram. *(Anna University Winter 2002)*
29. State the working principle of a dynamometer type wattmeter and show its connections. *(V.T.U., Belgaum Karnataka University, February 2002)*
30. A D.C. milliammeter having a resistance of  $2\Omega$  gives full scale deflection when the current is 50 mA. How can it be used to measure (i) a current of 5 A (ii) a voltage of 500 V. *(V.T.U., Belgaum Karnataka University, February 2002)*
31. With the help of a neat diagram, explain the construction and principle of operation of single phase energy meter. *(V.T.U., Belgaum Karnataka University, Winter 2003)*
32. With a neat sketch explain the construction and working of a single phase induction type energy meter. *(V.T.U., Belgaum Karnataka University, Summer 2003)*

### OBJECTIVE TESTS—10

- The kWh meter can be classified as a/an-instrument :
  - deflecting
  - digital
  - recording
  - indicating
- The moving system of an indicating type of electrical instrument is subjected to :
  - a deflecting torque
  - a controlling torque
  - a damping torque
  - all of the above
- The damping force acts on the moving system of an indicating instrument only when it is :
  - moving
  - stationary
  - near its full deflection
  - just starting to move.

4. The most efficient form of damping employed in electrical instruments is :  
 (a) air friction  
 (b) fluid friction  
 (c) eddy currents  
 (d) none of the above.
5. Moving iron instruments can be used for measuring :  
 (a) direct currents and voltages  
 (b) alternating current and voltages  
 (c) radio frequency currents  
 (d) both (a) and (b).
6. Permanent-magnet moving-coil ammeters have uniform scales because :  
 (a) of eddy current damping  
 (b) they are spring-controlled  
 (c) their deflecting torque varies directly as current  
 (d) both (b) and (c).
7. The meter that is suitable for *only* direct current measurements is :  
 (a) moving-iron type  
 (b) permanent-magnet type  
 (c) electrodynamic type  
 (d) hot-wire type.
8. A moving coil voltmeters measures—  
 (a) only a.c. voltages  
 (b) only d.c. voltages  
 (c) both a.c. and d.c. voltages  
 (Principles of Elect. Engg. Delhi Univ.)
9. The reading of the voltmeter in Fig. 10.91 would be nearest to—volt :  
 (a) 80  
 (b) 120  
 (c) 200  
 (d) 0
10. The hot-wire ammeter :  
 (a) is used only for d.c. circuits  
 (b) is a high precision instrument  
 (c) is used only for a.c. circuits  
 (d) reads equally well on d.c. and/or a.c. circuits.
11. If an energy meter disc makes 10 revolutions in 100 seconds when a load of 450 w is connected to it, the meter constant (in rev/k Wh) is  
 (a) 1000  
 (b) 500  
 (c) 1600  
 (d) 800  
 (GATE 2001)
12. The inductance of a certain moving-iron ammeter is expressed as  $L = 10 + 3\theta - \frac{\theta^2}{4}$   $\mu\text{H}$ , where  $\theta$  is the deflection in radians from the zero position. The control spring torque in  $25 \times 10^{-6}$  Nm/radian. The deflection of the pointer in radian when the meter carries a current of 5A, is  
 (a) 2.4  
 (b) 2.0  
 (c) 1.2  
 (d) 1.0  
 (GATE 2003)
13. A dc potentiometer is designed to measure up to about 2 V with a slide wire of 800 mm. A standard cell of emf 1.18 V obtains balance at 600 mm. A test cell is seen to obtain balance at 680 mm. The emf of the test cell is  
 (a) 1.00 V  
 (b) 1.34 V  
 (c) 1.50 V  
 (d) 1.70 V  
 (GATE 2004)
14. A galvanometer with a full scale current of 10 mA has a resistance of 1000  $\Omega$ . The multiplying power (the ratio of measured current to galvanometer current) of a 100  $\Omega$  shunt with this galvanometer is  
 (a) 110  
 (b) 100  
 (c) 11  
 (d) 10  
 (GATE 2004)
15. A moving coil of a meter has 100 turns, and a length and depth of 10 mm and 20 mm respectively. It is positioned in a uniform radial flux density of 200mT. The coil carries a current of 50 mA. The torque on the coil is  
 (a) 200  $\mu\text{Nm}$   
 (b) 100  $\mu\text{Nm}$   
 (c) 2  $\mu\text{Nm}$   
 (d) 1  $\mu\text{Nm}$   
 (GATE 2004)
16. A dc A-h meter is rated for 15 A, 250 V. The meter constant is 14.4 A-sec/rev. The meter constant at rated voltage may be expressed as  
 (a) 3750 rev/kWh  
 (b) 3600 rev/kWh  
 (c) 1000 rev/kWh  
 (d) 960 rev/kWh  
 (GATE 2004)

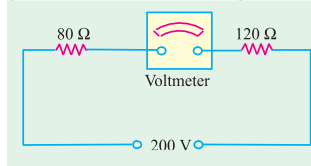


Fig. 10.91

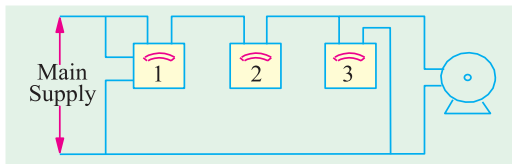


Fig. 10.92

## ANSWERS

1. c    2. d    3. a    4. c    5. d    6. d    7. b    8. b    9. c    10. d

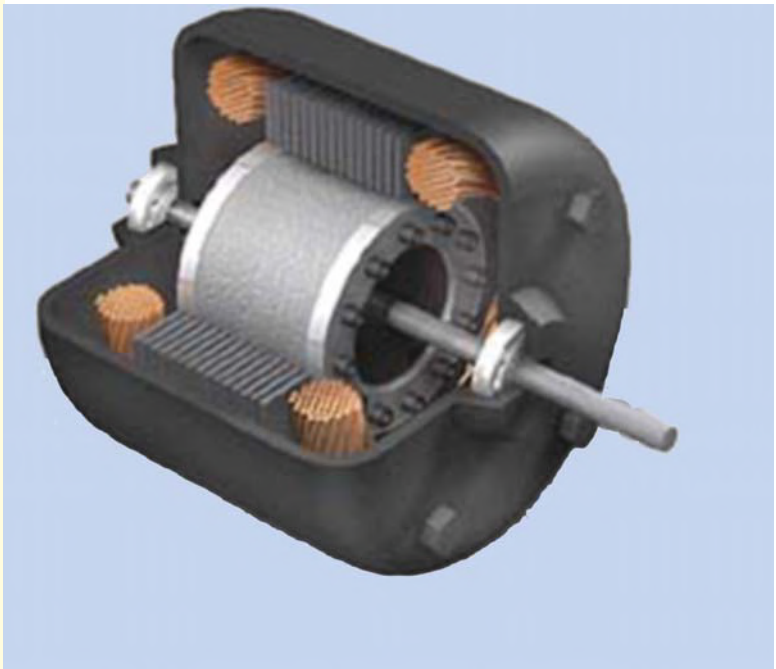


# CHAPTER 11

## Learning Objectives

- Generation of Alternating Voltages and Currents
- Alternate Method for the Equations of Alternating Voltages and currents
- Simple Waveforms
- Cycle
- Different Forms of E.M.F. Equation
- Phase
- Phase Difference
- Root Mean Square (R.M.S.) Value
- Mid-ordinate Method
- Analytical Method
- R.M.S. Value of a Complex Wave
- Average Value
- Form Factor
- Crest or Peak Factor
- R.M.S. Value of H.W. Rectified A.C.
- Average Value
- Form Factor of H.W. Rectified
- Vector Diagrams Using R.M.S. Values
- Vector Diagrams of Sine Waves of Same Frequency
- Addition and Subtraction of Vectors
- A.C. Through Resistance, Inductance and Capacitance
- A.C. through Pure Ohmic Resistance alone
- A.C. through Pure Inductance alone
- Complex Voltage Applied to Pure Inductance
- A.C. through Capacitance alone

## A.C. FUNDAMENTALS



Alternating current circuits improves the versatility and usefulness of electrical power system. Alternating current plays a vital role in today's energy generation

### 11.1. Generation of Alternating Voltages and Currents

Alternating voltage may be generated by rotating a coil in a magnetic field, as shown in Fig. 11.1 (a) or by rotating a magnetic field within a stationary coil, as shown in Fig. 11.1 (b).

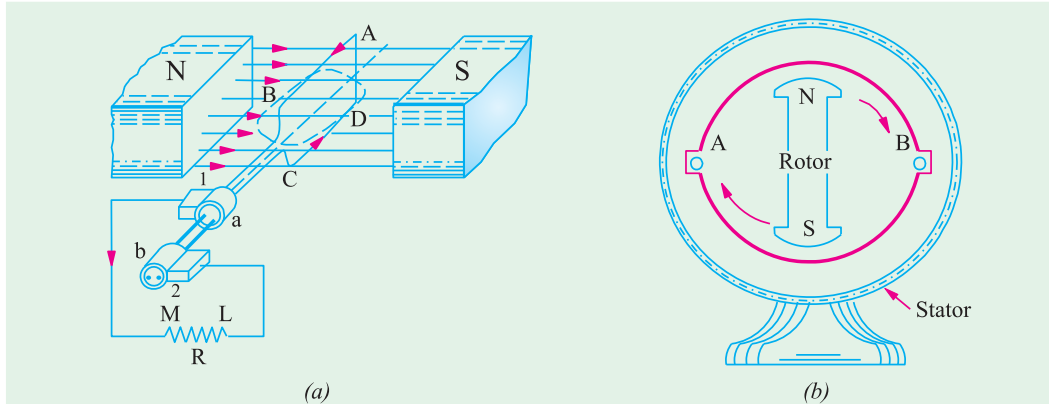


Fig. 11.1

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. Alternating voltage may be generated in either of the two ways shown above, but rotating-field method is the one which is mostly used in practice.

### 11.2. Equations of the Alternating Voltages and Currents

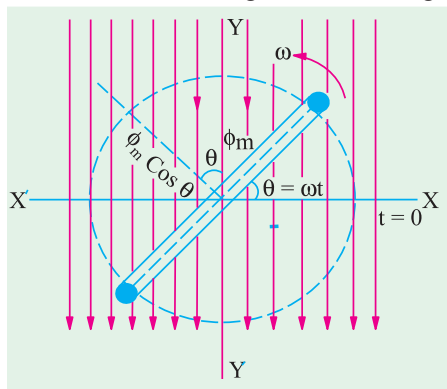


Fig. 11.2

Consider a rectangular coil, having  $N$  turns and rotating in a uniform magnetic field, with an angular velocity of  $\omega$  radian/second, as shown in Fig. 11.2. Let time be measured from the  $X$ -axis. Maximum flux  $\Phi_m$  is linked with the coil, when its plane coincides with the  $X$ -axis. In time  $t$  seconds, this coil rotates through an angle  $\theta = \omega t$ . In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is  $\Phi = \Phi_m \cos \omega t$ . Hence, **flux linkages** of the coil at any time are  $N \Phi = N \Phi_m \cos \omega t$ .

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence, the value of the induced e.m.f. at this instant (*i.e.* when  $\theta = \omega t$ ) or the instantaneous value of the induced e.m.f. is

$$e = -\frac{d}{dt}(N \Phi) \text{ volt} = -N \cdot \frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt} = -N \Phi_m \omega (-\sin \omega t) \text{ volt}$$

$$= \omega N \Phi_m \sin \omega t = \omega N \Phi_m \sin \theta \text{ volt} \quad \dots(i)$$

When the coil has turned through  $90^\circ$  *i.e.* when  $\theta = 90^\circ$ , then  $\sin \theta = 1$ , hence  $e$  has maximum value, say  $E_m$ . Therefore, from Eq. (i) we get

$$E_m = \omega N \Phi_m = \omega N B_m A = 2 \pi f N B_m A \text{ volt} \quad \dots(ii)$$

where

$$B_m = \text{maximum flux density in Wb/m}^2; A = \text{area of the coil in m}^2$$

$$f = \text{frequency of rotation of the coil in rev/second}$$

$$\text{Substituting this value of } E_m \text{ in Eq. (i), we get } e = E_m \sin \theta = E_m \sin \omega t \quad \dots(iii)$$

Similarly, the equation of induced alternating current is  $i = I_m \sin \omega t$  ...**(iv)**  
 provided the coil circuit has been closed through a resistive load.

Since  $\omega = 2\pi f$ , where  $f$  is the frequency of rotation of the coil, the above equations of the voltage and current can be written as

$$e = E_m \sin 2\pi ft = E_m \sin \left(\frac{2\pi}{T}\right)t \text{ and } i = I_m \sin 2\pi ft = I_m \sin \left(\frac{2\pi}{T}\right)t$$

where

$$T = \text{time-period of the alternating voltage or current} = 1/f$$

It is seen that the induced e.m.f. varies as sine function of the time angle  $\omega t$  and when e.m.f. is plotted against time, a curve similar to the one shown in Fig. 11.3 is obtained. This curve is known as sine curve and the e.m.f. which varies in this manner is known as **sinusoidal** e.m.f. Such a sine curve can be conveniently drawn, as shown in Fig. 11.4. A vector, equal in length to  $E_m$  is drawn. It rotates in the counter-clockwise direction with a velocity of  $\omega$  radian/second, making one revolution while the generated e.m.f. makes two loops or one cycle. The projection of this vector on  $Y$ -axis gives the instantaneous value  $e$  of the induced e.m.f. *i.e.*  $E_m \sin \omega t$ .

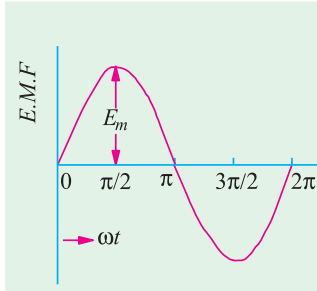


Fig. 11.3

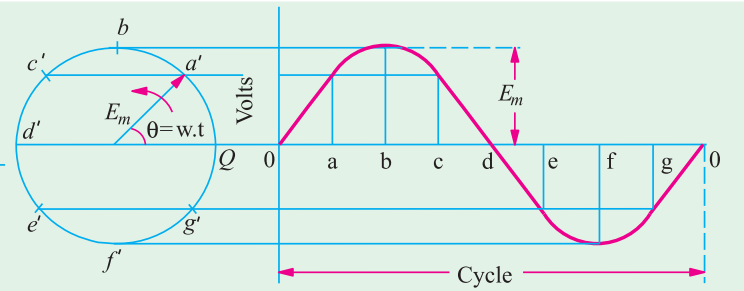


Fig. 11.4

To construct the curve, lay off along  $X$ -axis equal angular distance  $oa, ab, bc, cd$  etc. corresponding to suitable angular displacement of the rotating vector. Now, erect coordinates at the points  $a, b, c$  and  $d$  etc. (Fig. 11.4) and then project the free ends of the vector  $E_m$  at the corresponding positions  $a', b', c',$  etc to meet these ordinates. Next draw a curve passing through these intersecting points. The curve so obtained is the graphic representation of equation **(iii)** above.

### 11.3. Alternate Method for the Equations of Alternating Voltages and Currents

In Fig. 11.5 is shown a rectangular coil  $AC$  having  $N$  turns and rotating in a magnetic field of flux density  $B$   $\text{Wb./m}^2$ . Let the length of each of its sides  $A$  and  $C$  be  $l$  meters and their peripheral velocity  $v$  metre/second. Let angle be measured from the horizontal position *i.e.* from the  $X$ -axis. When in horizontal position, the two sides  $A$  and  $C$  move parallel to the lines of the magnetic flux. Hence, no flux is cut and so no e.m.f. is generated in the coil.

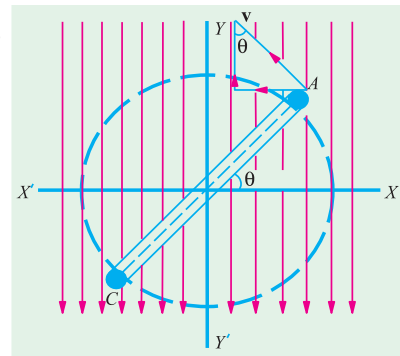


Fig. 11.5

When the coil has turned through angle  $\theta$ , its velocity can be resolved into two mutually perpendicular components **(i)**  $v \cos \theta$  component-parallel to the direction of the magnetic flux and **(ii)**  $v \sin \theta$  component-perpendicular to the direction of the magnetic flux. The e.m.f. is generated due entirely to the perpendicular component *i.e.*  $v \sin \theta$ .

Hence, the e.m.f. generated in one side of the coil which contains  $N$  conductors, as seen from Art. 7.7, is given by,  $e = N \times Bl v \sin \theta$ .

Total e.m.f. generated in both sides of the coil is

$$e = 2BNl v \sin \theta \text{ volt} \quad \dots (i)$$

Now,  $e$  has maximum value of  $E_m$  (say) when  $\theta = 90^\circ$ . Hence, from Eq. (i) above, we get,

$$E_m = 2BNl v \text{ volt. Therefore Eq. (i) can be rewritten as } e = E_m \sin \theta \quad \dots \text{as before}$$

If  $b$  = width of the coil in meters ;  $f$  = frequency of rotation of coil in Hz, then  $v = \pi b f$

$$\therefore E_m = 2BNl \times \pi b f = 2\pi fNB A \text{ volts} \quad \dots \text{as before}$$

**Example 11.1.** A square coil of 10 cm side and 100 turns is rotated at a uniform speed of 1000 revolutions per minute, about an axis at right angles to a uniform magnetic field of  $0.5 \text{ Wb/m}^2$ . Calculate the instantaneous value of the induced electromotive force, when the plane of the coil is (i) at right angles to the field (ii) in the plane of the field.

(Electromagnetic Theory, A.M.I.E. Sec B, 1992)

**Solution.** Let the magnetic field lie in the vertical plane and the coil in the horizontal plane. Also, let the angle  $\theta$  be measured from  $X$ -axis.

Maximum value of the induced e.m.f.,  $E_m = 2\pi fNB_m A$  volt.

Instantaneous value of the induced e.m.f.  $e = E_m \sin \theta$

Now  $f = 100/60 = (50/3)$  rps,  $N = 100$ ,  $B_m = 0.5 \text{ Wb/m}^2$ ,  $A = 10^{-2} \text{ m}^2$

(i) In this case,  $\theta = 0^\circ$

$$\therefore e = 0$$

(ii) Here  $\theta = 90^\circ$ ,  $\therefore e = E_m \sin 90^\circ = E_m$

Substituting the given values, we get

$$e = 2\pi \times (50/3) \times 100 \times 0.5 \times 10^{-2} = \mathbf{52.3 \text{ V}}$$



square coils

#### 11.4. Simple Waveforms

The shape of the curve obtained by plotting the instantaneous values of voltage or current as the ordinate against time as a abscissa is called its waveform or wave-shape.

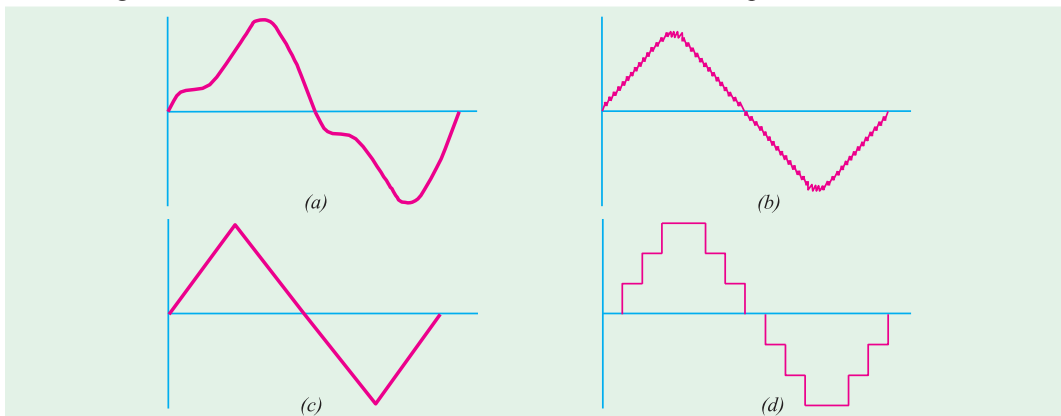


Fig. 11.6

An alternating voltage or current may not always take the form of a systematical or smooth wave such as that shown in Fig. 11.3. Thus, Fig. 11.6 also represents alternating waves. But while it is scarcely possible for the manufacturers to produce sine-wave generators or alternators, yet sine wave is the ideal form sought by the designers and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves.

In general, however, **an alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.**

### 11.5. Complex Waveforms

Complex waves are those which depart from the ideal sinusoidal form of Fig. 11.4. All alternating complex waves, which are periodic and have equal positive and negative half cycles can be shown to be made up of a number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the **fundamental** (or first harmonic). These waves of higher frequencies are called **harmonics**. If the fundamental frequency is 50 Hz, then the frequency of the second **harmonic** is 100 Hz and of the third is 150 Hz and so on. The complex wave may be composed of the fundamental wave (or first harmonic) and any number of other harmonics.

In Fig. 11.7 is shown a complex wave which is made up of a fundamental sine wave of frequency of 50 Hz and third harmonic of frequency of 150 Hz. It is seen that

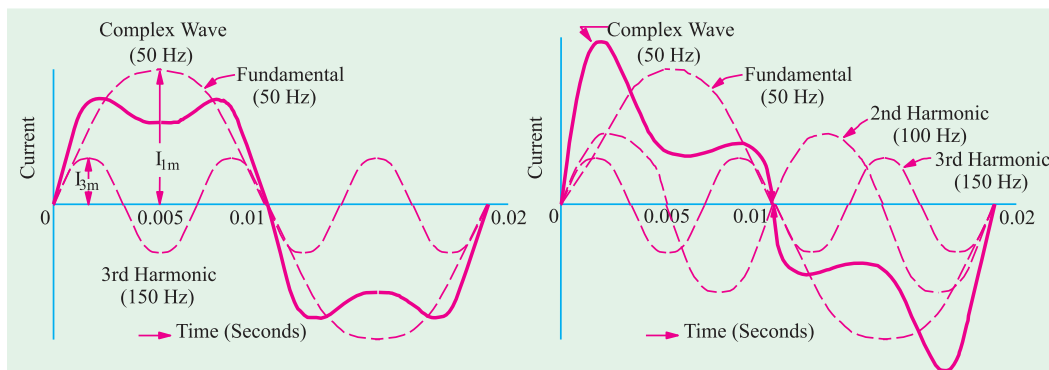


Fig. 11.7

Fig. 11.8

(i) the two halves of the complex wave are identical in shape. In other words, there is no distortion. This is always the case when only **odd** harmonic (3rd, 5th, 7th, 9th etc.) are present.

(ii) frequency of the complex wave is 50 Hz *i.e.* the same as that of the fundamental sine wave.

In Fig. 11.8 is shown a complex wave which is a combination of fundamental sine wave of frequency 50 Hz and 2nd harmonic of frequency 100 Hz and 3rd harmonic of frequency 150 Hz. It is seen that although the frequency of the complex wave even now remains 50 Hz, yet :

(i) the two halves of the complex wave are not identical. It is always so when **even** harmonics (2nd, 4th, 6th etc.) are present.

(ii) there is distortion and greater departure of the wave shape from the purely sinusoidal shape.

Sometimes, a combination of an alternating and direct current flows simultaneously through a circuit. In Fig. 11.9 is shown a complex wave (containing fundamental and third harmonic) combined with a direct current of value  $I_D$ . It is seen that the resultant wave remains undistorted in shape but is raised above the axis by an amount  $I_D$ . It is worth noting that with reference to the original axis, the two halves of the combined wave are not equal in area.

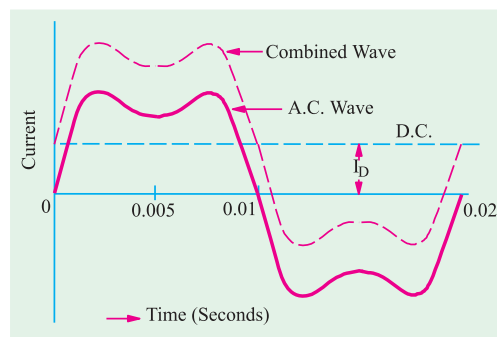


Fig. 11.9

### 11.6. Cycle

One complete set of positive and negative values of alternating quantity is known as cycle. Hence, each diagram of Fig. 11.6 represents one complete cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over  $360^\circ$  or  $2\pi$  radians.

### 11.7. Time Period

The time taken by an alternating quantity to complete one cycle is called its time period  $T$ . For example, a 50-Hz alternating current has a time period of  $1/50$  second.

### 11.8. Frequency

The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

In the simple 2-pole alternator of Fig. 24.1 (b), one cycle of alternating current is generated in one revolution of the rotating field. However, if there were 4 poles, then two cycles would have been produced in each revolution. In fact, the frequency of the alternating voltage produced is a function of the speed and the number of poles of the generator. The relation connecting the above three quantities is given as

$$f = PN/120 \text{ where } N = \text{revolutions in r.p.m. and } P = \text{number of poles}$$

For example, an alternator having 20 poles and running at 300 r.p.m. will generate alternating voltage and current whose frequency is  $20 \times 300/120 = 50$  hertz (Hz).

It may be noted that the frequency is given by the reciprocal of the time period of the alternating quantity.

$\therefore$

$$f = 1/T \text{ or } T = 1/f$$

### 11.9. Amplitude

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

### 11.10. Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already given in Art. 11.2, is

$$e = E_m \sin \theta = E_m \sin \omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi}{T} t$$

By closely looking at the above equations, we find that

(i) the maximum value or peak value or amplitude of an **alternating voltage is given by the coefficient of the sine of the time angle.**

(ii) **the frequency  $f$  is given by the coefficient of time divided by  $2\pi$ .**

For example, if the equation of an alternating voltage is given by  $e = 50 \sin 314t$  then its maximum value of 50 V and its frequency is  $f = 314/2\pi = 50$  Hz.

Similarly, if the equation is of the form  $e = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$ , then its maximum value is  $E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$  and the frequency is  $2\omega/2\pi$  or  $\omega/\pi$  Hz.

**Example 11.2.** *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A respectively at  $t = 0$  both increasing positively.*

(i) *Write down the expression for voltage and current at time  $t$ .*

(ii) *Determine the power consumed in the circuit.*

**(Elect. Engg. Pune Univ.)**

**Solution.** (i) In general, the expression for an a.c. voltage is  $v = V_m \sin(\omega t + \phi)$  where  $\phi$  is the phase difference with respect to the point where  $t = 0$ .

Now,  $v = 283 \text{ V}$  ;  $V_m = 400 \text{ V}$ . Substituting  $t = 0$  in the above equation, we get

$$283 = 400 (\sin \omega \times 0 + \phi) \therefore \sin \phi = 283/400 = 0.707 ; \therefore \phi = 45^\circ \text{ or } \pi/4 \text{ radian.}$$

Hence, general expression for voltage is

$$v = 400 (\sin 2\pi \times 50 \times t + \pi/4) \\ = 400 \sin (100 \pi t + \pi/4)$$

Similarly, at  $t = 0$ ,  $i = 20 \sin (\omega \times 0 + \phi)$

$$\therefore \sin \phi = 0.5 \therefore \phi = 30^\circ \text{ or } \pi/6 \text{ radian}$$

Hence, the general expression for the current is

$$i = 20 (\sin 100 \pi t + 30^\circ) = 20 \sin (100 \pi t + \pi/6)$$

(ii)  $P = VI \cos \theta$  where  $V$  and  $I$  are rms values and  $\theta$  is the phase difference between the voltage and current.

Now,  $V = V_m/\sqrt{2} = 400/\sqrt{2}$  ;  $I = 20/\sqrt{2}$  ;  $\theta = 45^\circ - 30^\circ = 15^\circ$  (See Fig. 11.10)

$$\therefore P = (400/\sqrt{2}) \times (20/\sqrt{2}) \times \cos 15^\circ = \mathbf{3864 \text{ W}}$$

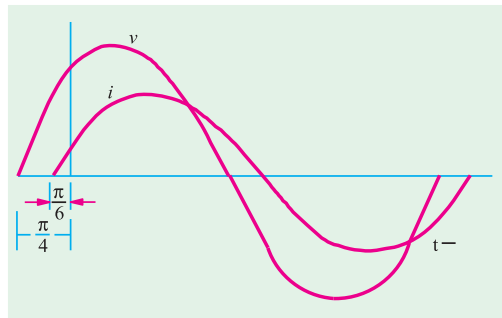


Fig. 11.10

**Example 11.3.** An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive, find (a) the instantaneous value after 1/360 second and (b) the time taken to reach 96 A for the first time.

**Solution.** The instantaneous current equation is

$$i = 120 \sin 2 \pi f t = 120 \sin 120 \pi t$$

Now when

$$t = 1/360 \text{ second, then}$$

(a)  $i = 120 \sin (120 \times \pi \times 1/360)$  ...angle in radians  
 $= 120 \sin (120 \times 180 \times 1/360)$  ...angle in degree  
 $= 120 \sin 60^\circ = 103.9 \text{ A}$

(b)  $96 = 120 \times \sin 2 \times 180 \times 60 \times t$  ...angle in degree  
 or  $\sin (360 \times 60 \times t) = 96/120 = 0.8 \therefore 360 \times 60 \times t = \sin^{-1} 0.8 = 53^\circ$  (approx)

$$\therefore t = \theta/2\pi f = 53/360 \times 60 = \mathbf{0.00245 \text{ second.}}$$

### 11.11. Phase

By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference. For example, the phase of current at point A is  $T/4$  second, where  $T$  is time period or expressed in terms of angle, it is  $\pi/2$  radians (Fig. 11.11). Similarly, the phase of the rotating coil at the instant shown in Fig. 11.1 is  $\omega t$  which is, therefore, called its phase angle.

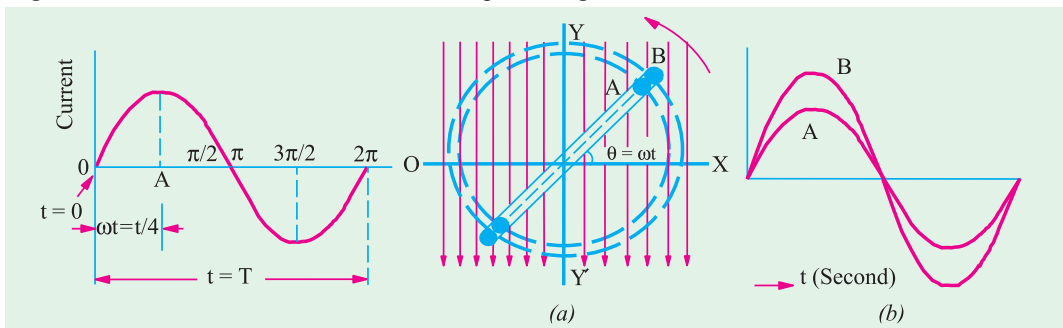


Fig. 11.11

Fig. 11.12

In electrical engineering, we are, however, more concerned with relative phases or phase differences between different alternating quantities, rather than with their absolute phases. Consider two single-turn coils of different sizes [Fig. 11.12 (a)] arranged radially in the same plane and rotating with the same angular velocity in a common magnetic field of uniform intensity. The e.m.fs. induced in both coils will be of the same frequency and of sinusoidal shape, although the values of instantaneous e.m.fs. induced would be different. However, the two alternating e.m.fs. would reach their maximum and zero values at the same time as shown in Fig. 11.12 (b). Such alternating voltages (or currents) are said to be in phase with each other. The two voltages will have the equations

$$e_1 = E_{m1} \sin \omega t \quad \text{and} \quad e_2 = E_{m2} \sin \omega t$$

### 11.12. Phase Difference

Now, consider three similar single-turn coils displaced from each other by angles  $\alpha$  and  $\beta$  and rotating in a uniform magnetic field with the same angular velocity [Fig. 11.13 (a)].

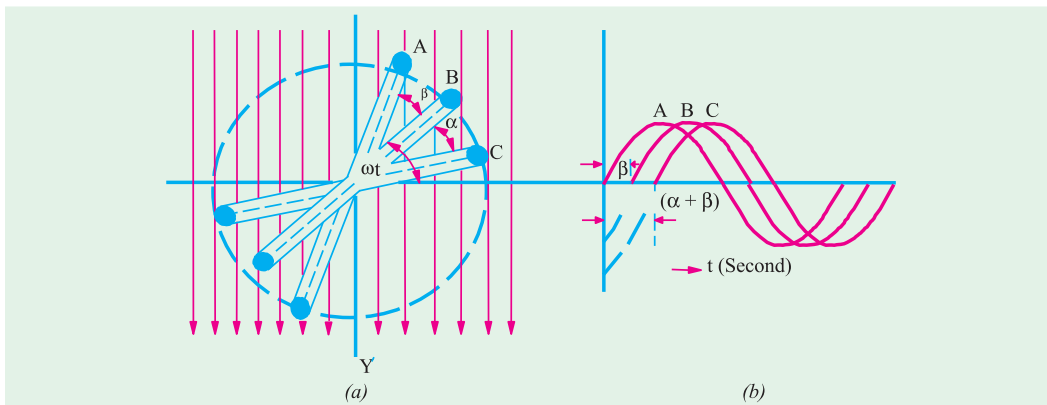


Fig. 11.13

In this case, the value of induced e.m.f.s. in the three coils are the same, but there is one important difference. The e.m.f.s. in these coils do not reach their maximum or zero values simultaneously but one after another. The three sinusoidal waves are shown in Fig. 11.13 (b). It is seen that curves *B* and *C* are displaced from curve *A* and angles  $\beta$  and  $(\alpha + \beta)$  respectively. Hence, it means that phase difference between *A* and *B* is  $\beta$  and between *B* and *C* is  $\alpha$  but between *A* and *C* is  $(\alpha + \beta)$ . The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the terms '*lag*' or '*lead*'.

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity. For example, in Fig. 11.13 (b), *B* lags behind *A* by  $\beta$  and *C* lags behind *A* by  $(\alpha + \beta)$  because they reach their maximum values later.

The three equations for the instantaneous induced e.m.f.s. are (Fig. 11.14)

$$e_A = E_m \sin \omega t \quad \dots \text{reference quantity}$$

$$e_B = E_m \sin (\omega t - \beta)$$

$$e_C = E_m \sin [\omega t - (\alpha + \beta)]$$

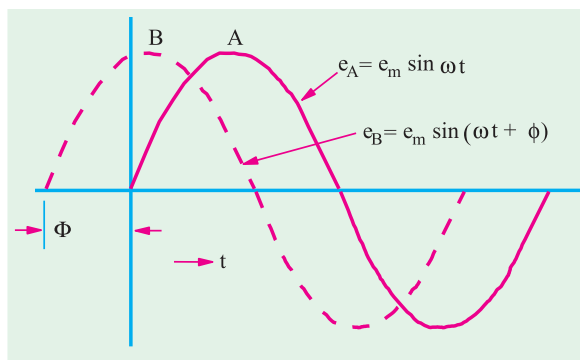


Fig. 11.14



In Fig. 11.14, quantity  $B$  leads  $A$  by an angle  $\phi$ . Hence, their equations are

$$e_A = E_m \sin \omega t \dots \text{reference quantity}$$

$$e_B = E_m \sin (\omega t - \phi)$$

A plus (+) sign when used in connection with phase difference denotes 'lead' whereas a minus (-) sign denotes 'lag'.

### 11.13. Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current *is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.*

It is also known as the *effective* or *virtual* value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid-ordinate method would be found more convenient.

A simple experimental arrangement for measuring the equivalent d.c. value of a sinusoidal current is shown in Fig. 11.15.

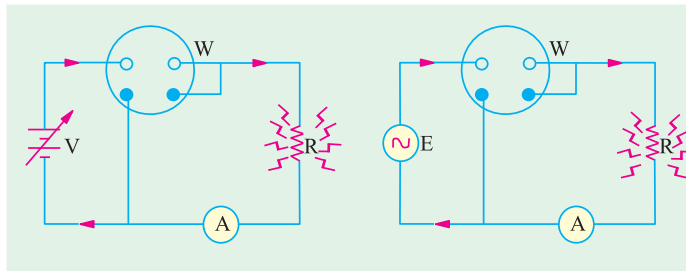


Fig. 11.15

The two circuits have

identical resistances but one is connected to battery and the other to a sinusoidal generator. Wattmeters are used to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that heat power production in each circuit is the same. In that case, the direct current will equal  $I_m/\sqrt{2}$  which is called r.m.s. value of the sinusoidal current.

### 11.14. Mid-ordinate Method

In Fig. 11.16 are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents. Divide time base ' $t$ ' into  $n$  equal intervals of time each of duration  $t/n$  seconds. Let the average values of instantaneous currents during these intervals be respectively  $i_1, i_2, i_3, \dots, i_n$  (i.e. mid-ordinates in Fig. 11.16). Suppose that this alternating current is passed through a circuit of resistance  $R$  ohms. Then,

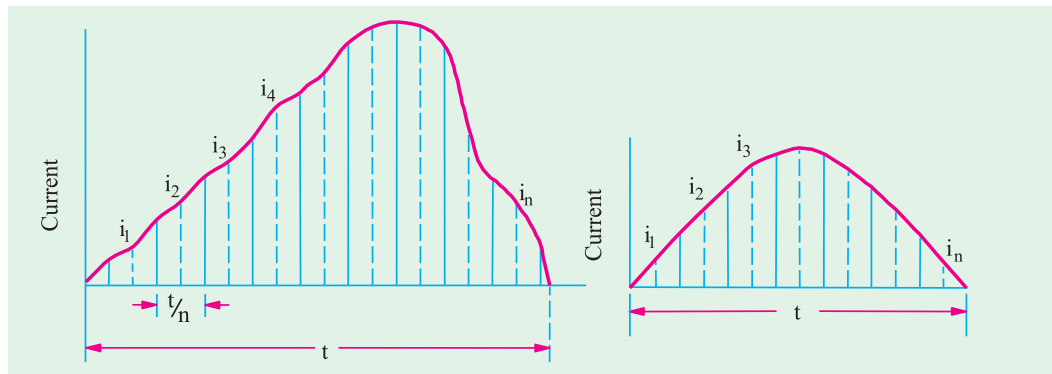


Fig. 11.16

$$\text{Heat produced in 1st interval} = 0.24 \times 10^{-3} i_1^2 Rt/n \text{ kcal} \quad (\because 1/J = 1/4200 = 0.24 \times 10^{-3})$$

$$\text{Heat produced in 2nd interval} = 0.24 \times 10^{-3} i_2^2 Rt/n \text{ kcal}$$

$$\begin{array}{ccccccc} \vdots & \vdots & \vdots & \vdots & \vdots & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \end{array}$$

$$\text{Heat produced in } n\text{th interval} = 0.24 \times 10^{-3} i_n^2 Rt/n \text{ kcal}$$

$$\text{Total heat produced in } t \text{ seconds is} = 0.24 \times 10^{-3} Rt \left( \frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right) \text{ kcal}$$

Now, suppose that a direct current of value  $I$  produces the same heat through the same resistance during the same time  $t$ . Heat produced by it is  $= 0.24 \times 10^{-3} I^2 Rt$  kcal. By definition, the two amounts of heat produced should be equal.

$$\therefore 0.24 \times 10^{-3} I^2 Rt = 0.24 \times 10^{-3} Rt \left( \frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right)$$

$$\begin{aligned} \therefore I^2 &= \frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \quad \therefore I = \sqrt{\left( \frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right)} \\ &= \text{square root of the mean of the squares of the instantaneous currents} \end{aligned}$$

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\left( \frac{v_1^2 + v_2^2 + \dots + v_n^2}{n} \right)}$$

### 11.15. Analytical Method

The standard form of a sinusoidal alternating current is  $i = I_m \sin \alpha t = I_m \sin \theta$ .

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2 d\theta}{(2\pi - 0)}$$

$$\text{The square root of this value is} = \sqrt{\left( \int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)}$$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left( \int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)} = \sqrt{\left( \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta \right)} \quad (\text{put } i = I_m \sin \theta)$$

$$\text{Now, } \cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\begin{aligned} \therefore I &= \sqrt{\left( \frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta \right)} = \sqrt{\left( \frac{I_m^2}{4\pi} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} \right)} \\ &= \sqrt{\frac{I_m^2}{4} \cdot 2} = \sqrt{\frac{I_m^2}{2}} \quad \therefore I = \frac{I_m}{\sqrt{2}} = 0.707 I_m \end{aligned}$$

Hence, we find that for a symmetrical sinusoidal current

$$\text{r.m.s. value of current} = 0.707 \times \text{max. value of current}$$

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In

electrical engineering work, **unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.**

It should be noted that the average heating effect produced during one cycle is

$$= I^2 R = (I_m/\sqrt{2})^2 R = \frac{1}{2} I_m^2 R$$

### 11.16. R.M.S. Value of a Complex Wave

In their case also, either the mid-ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known) may be used. Suppose a current having the equation  $i = 12 \sin \alpha t + 6 \sin (3\alpha t - \pi/6) + 4 \sin (5\alpha t + \pi/3)$  flows through a resistor of  $R$  ohm. Then, in the time period  $T$  second of the wave, the effect due to each component is as follows :

Fundamental .....  $(12/\sqrt{2})^2 RT$  watt

3rd harmonic .....  $(6/\sqrt{2})^2 RT$  watt

5th harmonic .....  $(4/\sqrt{2})^2 RT$  watt

∴ Total heating effect =  $RT[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$

If  $I$  is the r.m.s. value of the complex wave, then equivalent heating effect is  $I^2 RT$

∴  $I^2 RT = RT[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$

∴  $I = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$

Had there been a direct current of (say) 5 amperes flowing in the circuit also\*, then the r.m.s. value would have been

$$= \sqrt{(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2 + 5^2} = 10.93 \text{ A}$$

Hence, for complex waves the rule is as follows : **The r.m.s. value of a complex current wave is equal to the square root of the sum of the squares of the r.m.s. values of its individual components.**

### 11.17. Average Value

The average value  $I_a$  of an alternating current is expressed **by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.**

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. **But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.**

#### (i) Mid-ordinate Method

With reference to Fig. 11.16, 
$$I_{av} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

#### (ii) Analytical Method

The standard equation of an alternating current is,  $i = I_m \sin \theta$

$$I_{av} = \int_0^\pi \frac{id\theta}{(\pi - 0)} = \frac{I_m}{\pi} \int_0^\pi \sin \theta d\theta \quad \text{(putting value of } i \text{)}$$

---

\* The equation of the complex wave, in that case, would be,  
 $i = 5 + 12 \sin \alpha t + 6 \sin (3\alpha t - \pi/6) + 4 \sin (5\alpha t + \pi/3)$

$$= \frac{I_m}{\pi} \left| -\cos \theta \right|_{\pi}^0 = \frac{I_m}{\pi} \left| +1 - (-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$\therefore I_{av} = \frac{I_m}{\pi} \times \pi = 0.637 I_m \quad \therefore \text{average value of current} = 0.637 \times \text{maximum value}$$

**Note.** R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

### 11.18. Form Factor

It is defined as the ratio,  $K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707 I_m}{0.637 I_m} = 1.1$ . (for sinusoidal alternating currents only)

$$\text{In the case of sinusoidal alternating voltage also, } K_f = \frac{0.707 E_m}{0.637 E_m} = 1.11$$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice-versa*.

### 11.19. Crest or Peak or Amplitude Factor

It is defined as the ratio  $K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$  (for sinusoidal a.c. only)

$$\text{For sinusoidal alternating voltage also, } K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

**Example 11.4.** An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value (a) 0.0025 second (b) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A ?

(Elect. Science-I Allahabad Univ. 1992)

**Solution.**  $I_m = 20\sqrt{2} = 28.2$  A,  $\omega = 2\pi \times 50 = 100\pi$  rad/s.

The equation of the sinusoidal current wave with reference to point O (Fig. 11.17) as zero time point is

$$i = 28.2 \sin 100\pi t \text{ ampere}$$

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In the case, the equation becomes :

$$i = 28.2 \cos 100\pi t$$

(i) When  $t = 0.0025$  second

$$i = 28.2 \cos 100\pi \times 0.0025$$

...angle in radian

$$= 28.2 \cos 100 \times 180 \times 0.0025$$

...angle in degrees

$$= 28.2 \cos 45^\circ = 20 \text{ A ...point B}$$

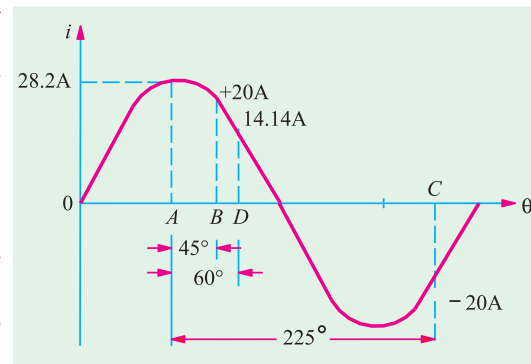


Fig. 11.17

(ii) When  $t = 0.0125$  second

$$\begin{aligned} i &= 28.2 \cos 100 \times 180 \times 0.0125 \\ &= 28.2 \cos 225^\circ = 28.2 \times (-1/\sqrt{2}) \\ &= -20 \text{ A} \end{aligned}$$

...point C

(iii) Here  $i = 14.14 \text{ A}$

$$\therefore 14.14 = 28.2 \cos 100 \times 180 t \quad \therefore \cos 100 \times 180 t = \frac{1}{2}$$

or  $100 \times 180 t = \cos^{-1}(0.5) = 60^\circ, t = 1/300 \text{ second}$

...point D

**Example 11.5.** An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate (a) its value 1/600 second after the instant the current is zero and its value decreasing thereafter (b) how many seconds after the instant the current is zero (increasing thereafter) will the current attain the value of 86.6 A ? (Elect. Technology, Allahabad Univ. 1991)

**Solution.** The equation of the alternating current (assumed sinusoidal) with respect to the origin O (Fig. 11.18) is

$$i = 100 \sin 2\pi \times 50t = 100 \sin 100 \pi t$$

(a) It should be noted that, in this case, time is being measured from point A (where current is zero and decreasing thereafter) and not from point O.

If the above equation is to be utilized, then, this time must be referred to point O. For this purpose, half time-period i.e. 1/100 second has to be added to 1/600 second. The given time as referred to point O becomes

$$= \frac{1}{100} + \frac{1}{600} = \frac{7}{600} \text{ second}$$

$$\begin{aligned} \therefore i &= 100 \sin 100 \times 180 \times 7/600 = 100 \sin 210^\circ \\ &= 100 \times -1/2 = -50 \text{ A} \end{aligned}$$

...point B

(b) In this case, the reference point is O.

$$\begin{aligned} \therefore 86.6 &= 100 \sin 100 \times 180 t \quad \text{or} \quad \sin 18,000 t = 0.866 \\ \text{or} \quad 18,000 t &= \sin^{-1}(0.866) = 60^\circ \quad \therefore t = 60/18,000 = 1/300 \text{ second} \end{aligned}$$

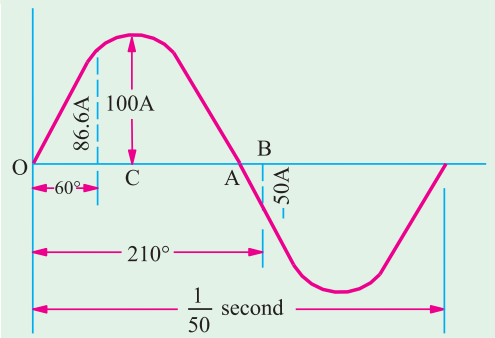


Fig. 11.18

**Example 11.6.** Calculate the r.m.s. value, the form factor and peak factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, -5, -10 V etc. What would be the r.m.s. value of sine wave having the same peak value ?

**Solution.** The waveform of the alternating voltage is shown in Fig. 11.19. Obviously, it is not sinusoidal but it is symmetrical. Hence, though r.m.s value may be full one cycle, the average value has necessarily to be considered for half-cycle only, otherwise the symmetrical negative and positive half-cycles will cancel each other out.

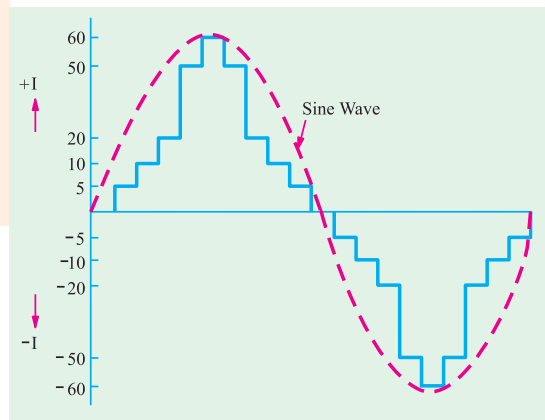


Fig. 11.19

$$\text{Mean value of } v^2 = \frac{0^2 + 5^2 + 10^2 + 20^2 + 50^2 + 60^2 + 50^2 + 20^2 + 10^2 + 5^2}{10} = 965 \text{ V}$$

$$\therefore \text{ r.m.s. value} = \sqrt{965} = 31 \text{ V (approx.)}$$

$$\text{Average value (half-cycle)} = \frac{0 + 5 + 10 + 20 + 50 + 60 + 50 + 20 + 10 + 5}{10} = 23 \text{ V}$$

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{31}{23} = 1.35. \text{ Peak factor} = 60/31 = 2 \text{ (approx.)}$$

$$\text{R.M.S. value of a sine wave of the same peak value} = 0.707 \times 60 = 42.2 \text{ V.}$$

#### Alternative Solution

If 't' be the regular time interval, then area of the half-cycle is

$$= (5t + 10t + 20t + 50t) 2 + 60t = 230t, \text{ Base} = 10t \therefore \text{Mean value} = 230t/10t = 23 \text{ V.}$$

$$\text{Area when ordinates are squared} = (25t + 100t + 400t + 2500t) 2 + 3600t = 9650t, \text{ Base} = 10t$$

$$\therefore \text{ Mean height of the squared curve} = 9650t/10t = 965$$

$$\therefore \text{ r.m.s. value} = \sqrt{965} = 31 \text{ V}$$

Further solution is as before.

**Example 11.7.** Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by

$$v = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$$

**Solution.** Since hot-wire voltmeter reads only r.m.s value, we will have to find the r.m.s. value of the given voltage. Considering one complete cycle,

$$\text{R.M.S. value} \quad V = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2 d\theta} \quad \text{where } \theta = \omega t$$

$$\begin{aligned} \text{or} \quad V^2 &= \frac{2}{2\pi} \int_0^{2\pi} (200 \sin \theta + 100 \sin 3\theta + 50 \sin 5\theta)^2 d\theta \\ &= \frac{1}{2\pi} \int_0^{2\pi} (200^2 \sin^2 \theta + 100^2 \sin^2 3\theta + 50^2 \sin^2 5\theta \\ &\quad + 2 \times 200 \cdot 100 \sin \theta \cdot \sin 3\theta + 2 \times 100 \cdot 50 \cdot \sin 3\theta \cdot \sin 5\theta \\ &\quad + 2 \times 50 \cdot 200 \cdot \sin 5\theta \cdot \sin \theta) d\theta \\ &= \frac{1}{2\pi} \left( \frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2} \right) 2\pi = 26,250 \end{aligned}$$

$$\therefore V = \sqrt{26,250} = 162 \text{ V}$$

#### Alternative Solution

The r.m.s. value of individual components are  $(200/\sqrt{2})$ ,  $(100/\sqrt{2})$  and  $(50/\sqrt{2})$ . Hence, as stated in Art. 11.16,

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(200/\sqrt{2})^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} = 162 \text{ V}$$

### 11.20. R.M.S. Value of H.W. Rectified Alternating Current

Half-wave (H.W.) rectified alternating current is one whose one half-cycle has been suppressed i.e. one which flows for half the time during one cycle. It is shown in Fig. 11.20 where suppressed half-cycle is shown dotted.

As said earlier, for finding r.m.s. value of such an alternating current, summation would be carried over the period for which current *actually* flows i.e. from 0 to  $\pi$ , though it would be averaged for the whole cycle i.e. from 0 to  $2\pi$ .

$\therefore$  R.M.S. current

$$\begin{aligned}
 I &= \sqrt{\left(\int_0^\pi \frac{i^2}{2\pi} d\theta\right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^\pi \sin^2 \theta d\theta\right)} \\
 &= \sqrt{\frac{I_m^2}{4\pi} \int_0^\pi (1 - \cos 2\theta) d\theta} \\
 &= \sqrt{\left(\frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi\right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \times \pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)} \quad \therefore I = \frac{I_m}{2} = 0.5I_m
 \end{aligned}$$

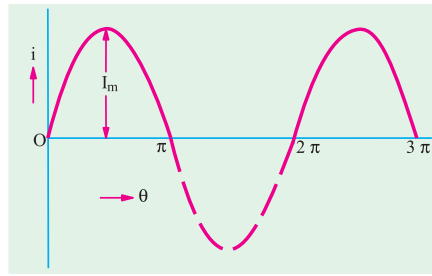


Fig. 11.20

### 11.21. Average Value of H.W. Rectified Alternating Current

For the same reasons as given in Art. 11.20, integration would be carried over from 0 to  $\pi$

$$\begin{aligned}
 \therefore I_{av} &= \int_0^\pi \frac{id\theta}{2\pi} = \frac{I_m}{2\pi} \int_0^\pi \sin \theta d\theta \quad (\because i = I_m \sin \theta) \\
 &= \frac{I_m}{2\pi} [-\cos \theta]_0^\pi = \frac{I_m}{2\pi} \times 2 = \frac{I_m}{\pi}
 \end{aligned}$$

### 11.22. Form Factor of H.W. Rectified Alternating Current

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

**Example 11.8.** An alternating voltage  $e = 200 \sin 314t$  is applied to a device which offers an ohmic resistance of  $20 \Omega$  to the flow of current in one direction, while preventing the flow of current in opposite direction. Calculate RMS value, average value and form factor for the current over one cycle. (Elect. Engg. Nagpur Univ. 1992)

**Solution.** Comparing the given voltage equation with the standard form of alternating voltage equation, we find that  $V_m = 200 \text{ V}$ ,  $R = 20 \Omega$ ,  $I_m = 200/20 = 10 \text{ A}$ . For such a half-wave rectified current, RMS value  $= I_m/2 = 10/2 = 5 \text{ A}$ .

Average current  $= I_m/\pi = 10/\pi = 3.18 \text{ A}$ ; Form factor  $= 5/3.18 = 1.57$

**Example 11.9.** Compute the average and effective values of the square voltage wave shown in Fig. 11.21.

**Solution.** As seen, for  $0 < t < 0.1$  i.e. for the time interval 0 to 0.1 second,  $v = 20 \text{ V}$ . Similarly, for  $0.1 < t < 0.3$ ,  $v = 0$ . Also time-period of the voltage wave is 0.3 second.

$$\begin{aligned}
 \therefore V_{av} &= \frac{1}{T} \int_0^T v dt = \frac{1}{0.3} \int_0^{0.1} 20 dt \\
 &= \frac{1}{0.3} (20 \times 0.1) = 6.67 \text{ V}
 \end{aligned}$$

$$V^2 = \frac{1}{T} \int_0^T v^2 dt = \frac{1}{0.3} \int_0^{0.1} 20^2 dt = \frac{1}{0.3} (400 \times 0.1) = 133.3; V = 11.5 \text{ V}$$

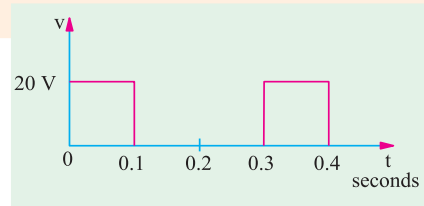


Fig. 11.21

**Example 11.10.** Calculate the RMS value of the function shown in Fig. 11.22 if it is given that for  $0 < t < 0.1$ ,  $y = 10(1 - e^{-100t})$  and  $0.1 < t < 0.2$ ,  $y = 10 e^{-50(t-0.1)}$

**Solution.**

$$\begin{aligned}
 Y^2 &= \frac{1}{0.2} \left\{ \int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt \right\} \\
 &= \frac{1}{0.2} \left\{ \int_0^{0.1} 10^2 (1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} (10e^{-50(t-0.1)})^2 dt \right\} \\
 &= \frac{1}{0.2} \left\{ \int_0^{0.1} 100 (1 + e^{-200t} - 2e^{-100t}) dt + \int_{0.1}^{0.2} 100 e^{-100(t-0.1)} dt \right\} \\
 &= 500 \left\{ \left[ t - 0.005e^{-200t} + 0.02 e^{-100t} \right]_0^{0.1} + \left[ -0.01 e^{-100(t-0.1)} \right]_{0.1}^{0.2} \right\} \\
 &= 500 \left\{ \left[ (0.1 - 0.005e^{-20} + 0.02e^{-10}) - (0 - 0.005 + 0.02) \right] + \left[ (-0.01e^{-10}) - (-0.01) \right] \right\} \\
 &= 500 \times 0.095 = 47.5 \quad \therefore Y = \sqrt{47.5} = 6.9
 \end{aligned}$$

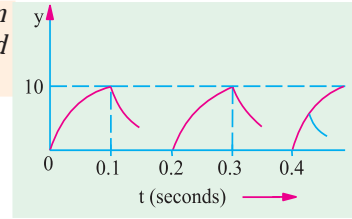


Fig. 11.22

**Example 11.11.** The half cycle of an alternating signal is as follows : It increases uniformly from zero at  $0^\circ$  to  $F_m$  at  $\alpha^\circ$ , remains constant from  $\alpha^\circ$  ( $180 - \alpha$ ) $^\circ$ , decreases uniformly from  $F_m$  at  $(180 - \alpha)^\circ$  to zero at  $180^\circ$ . Calculate the average and effective values of the signal.

(Elect. Science-I, Allahabad Univ. 1992)

**Solution.** For finding the average value, we would find the total area of the trapezium and divide it by  $\pi$  (Fig. 11.23).

$$\begin{aligned}
 \text{Area} &= 2 \times \Delta OAE + \text{rectangle } ABDE = 2 \times (1/2) \times F_m \alpha \\
 &\quad + (\pi - 2\alpha) F_m = (\pi - \alpha) F_m \\
 \therefore \text{average value} &= (\pi - \alpha) F_m / \pi
 \end{aligned}$$

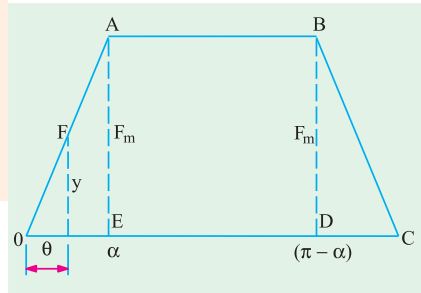


Fig. 11.23

**RMS Value** From similar triangles, we get  $\frac{y}{\theta} = \frac{F_m}{\alpha}$  or  $y^2 = \frac{F_m^2 \theta^2}{\alpha^2}$

This gives the equation of the signal over the two triangles  $OAE$  and  $DBC$ . The signal remains constant over the angle  $\alpha$  to  $(\pi - \alpha)$  i.e. over an angular distance of  $(\pi - \alpha) - \alpha = (\pi - 2\alpha)$

$$\text{Sum of the squares} = \frac{2F_m^2}{\alpha^2} \int_0^\alpha \theta^2 d\theta + F_m^2(\pi - 2\alpha) = F_m^2(\pi - 4\alpha/3).$$

$$\text{The mean value of the squares is} = \frac{1}{\pi} F_m^2 \left( \pi - \frac{4\alpha}{3} \right) = F_m^2 \left( 1 - \frac{4\alpha}{3\pi} \right)$$

$$\text{r.m.s. value} = F_m \sqrt{\left( 1 - \frac{4\alpha}{3\pi} \right)}$$

**Example 11.12.** Find the average and r.m.s values of the a.c. voltage whose waveform is given in Fig. 11.24 (a)

**Solution.** It is seen [Fig. 11.24 (a)] that the time period of the waveform is 5s. For finding the average value of the waveform, we will calculate the net area of the waveform over one period and then find its average value for one cycle.

$$A_1 = 20 \times 1 = 20 \text{ V-s}, \quad A_2 = -5 \times 2 = -10 \text{ V-s}$$



Net area over the full cycle =  $A_1 + A_2 = 20 - 10 = 10 \text{ V} \cdot \text{s}$ .

Average value =  $10 \text{ V} \cdot \text{s} / 5 \text{ s} = 2 \text{ V}$ .

Fig. 11.24 (b) shows a graph of  $v^2(t)$ . Since the negative voltage is also squared, it becomes positive.

Average value of the area =  $400 \text{ V}^2 \times 1 \text{ s} + 25 \text{ V}^2 \times 2 \text{ s} = 450 \text{ V}^2 \cdot \text{s}$ . The average value of the sum of the square =  $450 \text{ V}^2 \cdot \text{s} / 5 \text{ s} = 90 \text{ V}^2$  rms value =  $\sqrt{90 \text{ V}^2} = 9.49 \text{ V}$ .

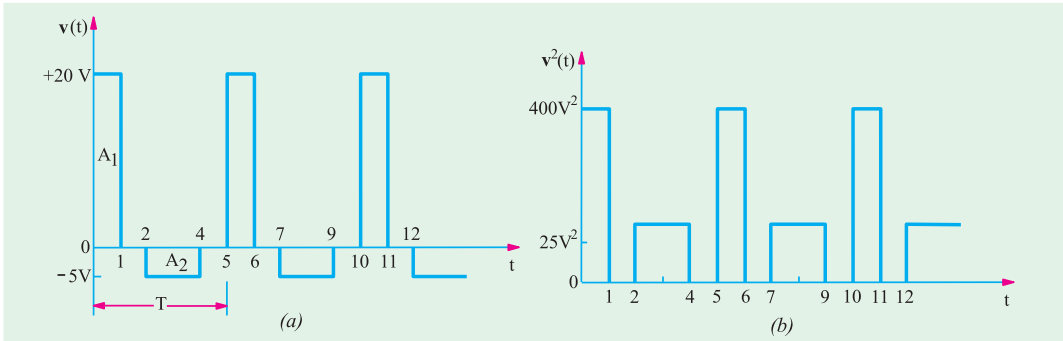


Fig. 11.24

**Example 11.13.** What is the significance of the r.m.s and average values of a wave? Determine the r.m.s. and average value of the waveform shown in Fig. 11.25

(Elect. Technology, Indore Univ.)

**Solution.** The slope of the curve AB is  $BC/AC = 20/T$ . Next, consider the function  $y$  at any time  $t$ . It is seen that  $DE/AE = BC/AC = 10/T$

$$\text{or } (y - 10)/t = 10/T$$

$$\text{or } y = 10 + (10/T)t$$

This gives us the equation for the function for one cycle.

$$Y_{av} = \frac{1}{T} \int_0^T y \, dt = \frac{1}{T} \int_0^T \left(10 + \frac{10}{T}t\right) dt$$

$$= \frac{1}{T} \int_0^T \left[10 \cdot dt + \frac{10}{T} \cdot t \cdot dt\right] = \frac{1}{T} \left[10t + \frac{5t^2}{T}\right]_0^T = 15$$

$$\text{Mean square value} = \frac{1}{T} \int_0^T y^2 \, dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T} \int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right) dt = \frac{1}{T} \left[100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right]_0^T = \frac{700}{3}$$

$$\text{or RMS value} = 10\sqrt{7/3} = 15.2$$

**Example 11.14.** For the trapezoidal current wave-form of Fig. 11.26, determine the effective value.

(Elect. Technology, Vikram Univ. Ujjain, Similar Example, Nagpur Univ. 1999)

**Solution.** For  $0 < t < 3T/20$ , equation of the current can be found from the relation

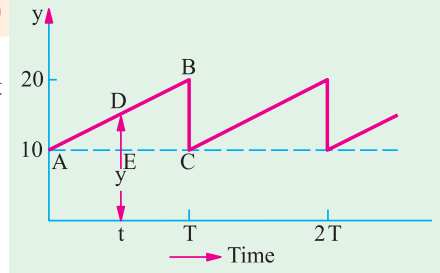


Fig. 11.25

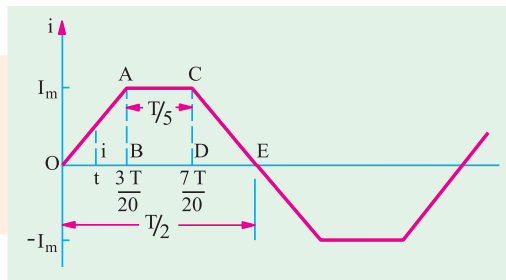


Fig. 11.26

$$\frac{i}{t} = \frac{I_m}{3T/20} \quad \text{or} \quad i = \frac{20I_m}{3T} \cdot t$$

When  $3T/20 < t < 7T/20$ , equation of the current is given by  $i = I_m$ . Keeping in mind the fact that  $\Delta OAB$  is identical with  $\Delta CDE$ ,

$$\begin{aligned} \text{RMS value of current} &= \sqrt{\frac{1}{T/2} \left[ 2 \int_0^{3T/20} i^2 dt + \int_{3T/20}^{7T/20} I_m^2 dt \right]} \\ &= \sqrt{\frac{2}{T} \left[ 2 \left( \frac{20I_m}{3T} \right)^2 \int_0^{3T/20} t^2 dt + I_m^2 \int_{3T/20}^{7T/20} dt \right]} = \frac{3}{5} I_m \end{aligned}$$

$$\therefore I = \sqrt{(3/5)} \cdot I_m = \mathbf{0.775 I_m}$$

Incidentally, the average value is given by

$$\begin{aligned} I_{ac} &= \frac{2}{T} \left\{ 2 \int_0^{3T/20} i dt + \int_{3T/20}^{7T/20} I_m dt \right\} = \frac{2}{T} \left\{ 2 \int_0^{3T/20} \left( \frac{20I_m}{3T} \right) t dt + I_m \int_{3T/20}^{7T/20} dt \right\} \\ &= \frac{2}{T} \left\{ 2 \left( \frac{20I_m}{3T} \right) \left[ \frac{t^2}{2} \right]_0^{3T/20} + I_m \left[ t \right]_{3T/20}^{7T/20} \right\} = \mathbf{\frac{7}{10} \cdot I_m} \end{aligned}$$

**Example 11.15.** A sinusoidal alternating voltage of 110 V is applied across a moving-coil ammeter, a hot-wire ammeter and a half-wave rectifier, all connected in series. The rectifier offers a resistance of 25  $\Omega$  in one direction and infinite resistance in opposite direction. Calculate (i) the readings on the ammeters (ii) the form factor and peak factor of the current wave.

(Elect. Engg.-I Nagpur Univ. 1992)

**Solution.** For solving this question, it should be noted that

(a) Moving-coil ammeter, due to the inertia of its moving system, registers the average current for the *whole* cycle.

(b) The reading of hot-wire ammeter is proportional to the average heating effect over the *whole* cycle. It should further be noted that in a.c. circuits, the given voltage and current values, unless indicated otherwise, always refer to r.m.s values.

$$E_m = 110/0.707 = 155.5 \text{ V (approx.)}; I_m/2 = 155.5/25 = 6.22 \text{ A}$$

Average value of current for positive half cycle =  $0.637 \times 6.22 = 3.96 \text{ A}$

Value of current in the negative half cycle is zero. But, as said earlier, due to inertia of the coil, M.C. ammeter reads the average value for the *whole* cycle.

(i) M.C. ammeter reading =  $3.96/2 = \mathbf{1.98 \text{ A}}$

Let  $R$  be the resistance of hot-wire ammeter. Average heating effect over the positive half cycle is  $\frac{1}{2} I_m^2 \cdot R$  watts. But as there is no generation of heat in the negative half cycle, the average heating effect over the whole cycle is  $\frac{1}{4} I_m^2 R$  watt.

Let  $I$  be the d.c. current which produces the same heating effect, then

$$I^2 R = \frac{1}{4} I_m^2 R \quad \therefore I = I_m/2 = 6.22/2 = 3.11 \text{ A}$$

Hence, hot-wire ammeter will read  $\mathbf{3.11 \text{ A}}$

(ii) Form factor =  $\frac{\text{r.m.s value}}{\text{average value}} = \frac{3.11}{1.98} = \mathbf{1.57}$ ; Peak factor =  $\frac{\text{max. value}}{\text{r.m.s. value}} = \frac{6.22}{3.11} = \mathbf{2}$

**Example 11.16.** Find the form-factor of the wave form given in fig.

[Nagpur University November 1991, Similar example, Sambalpur University]

**Solution.**

$$\text{Form-factor} = \frac{\text{RMS value}}{\text{Average value}}$$

$$\begin{aligned} \text{Average value of the current} \\ = 1/4 \times \int_0^4 (50/4) \times t \times dt = 25 \text{ amp} \end{aligned}$$

Let RMS value of the current be  $I$  amp

$$\begin{aligned} I^2 \times 4 &= \int_0^4 (12.5 \times t)^2 \cdot dt \\ &= \left[ \frac{12.5 \times 12.5 \times t^3}{3} \right]_0^4 = (1/3) \times (12.5 \times 12.5 \times 4 \times 4 \times 4) \end{aligned}$$

$$\text{Thus } I = \frac{50}{\sqrt{3}} = 28.87 \text{ amp, Hence, form factor } \frac{28.87}{25} = 1.1548$$

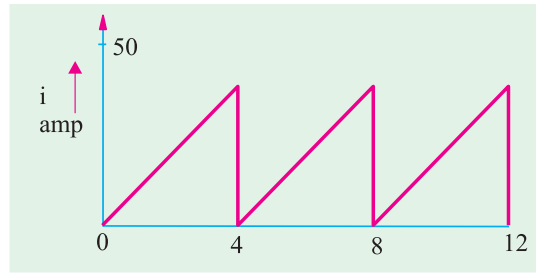


Fig. 11.27

**Example 11.17.** A half-wave rectifier which prevents current flowing in one direction is connected in series with an a.c. ammeter and a permanent-magnet moving-coil ammeter. The supply is sinusoidal. The reading on the a.c. ammeter is 10 A. Find the reading given by the other ammeter. What should be the readings on the ammeters, if the other half-wave were rectified instead of being cut off?

**Solution.** It should be noted that an a.c. ammeter reads r.m.s. value whereas the d.c. ammeter reads the average value of the rectified current.

As shown in Art. 11.20 from H.W. rectified alternating current,  $I = I_m/2$  and  $I_{av} = I_m/\pi$

As a.c. ammeter reads 10 A, hence r.m.s. value of the current is 10 A.

$$\therefore 10 = I_m/2 \text{ or } I_m = 20 \text{ A}$$

$$\therefore I_{av} = 20/\pi = \mathbf{6.365 \text{ A}}$$
 -reading of d.c. ammeter.

The full-wave rectified current wave is shown in Fig. 11.28. In this case mean value of  $i^2$  over a complete cycle is given as

$$\begin{aligned} &= 2 \int_0^\pi \frac{i^2 d\theta}{2\pi - 0} = \frac{1}{\pi} \int_0^\pi I_m^2 2 \sin^2 \theta d\theta \\ &= \frac{I_m^2}{2\pi} \int_0^\pi (1 - \cos 2\theta) d\theta = \frac{I_m^2}{2\pi} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^\pi = \frac{I_m^2}{2} \end{aligned}$$

$$\therefore I = I_m/\sqrt{2} = 20/\sqrt{2} = 14.14 \text{ A} \therefore \text{a.c. ammeter will read } \mathbf{14.14 \text{ A}}$$

Now, mean value of  $i$  over a complete cycle

$$= \frac{2}{2\pi} \int_0^\pi I_m \sin \theta d\theta = \frac{I_m}{\pi} \left[ -\cos \theta \right]_0^\pi = \frac{2I_m}{\pi} = \frac{2 \times 20}{\pi} = \mathbf{12.73 \text{ A}}$$

This value, as might have been expected, is twice the value obtained in the previous case.

$$\therefore \text{d.c. ammeter will read } \mathbf{12.73 \text{ A.}}$$

**Example 11.18.** A full-wave rectified sinusoidal voltage is clipped at  $1/\sqrt{2}$  of its maximum value. Calculate the average and RMS values of such a voltage.

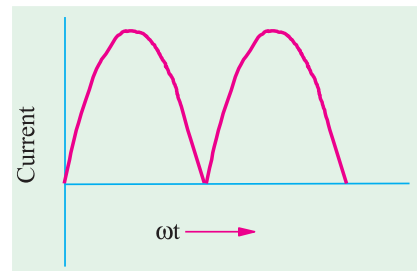


Fig. 11.28

**Solution.** As seen from Fig. 11.29, the rectified voltage has a period of  $\pi$  and is represented by the following equations during the different intervals.

$$0 < \theta < \pi/4 ; v = V_m \sin \theta$$

$$\pi/4 < \theta < 3\pi/4 ; v = V_m/\sqrt{2} = 0.707 V_m$$

$$3\pi/4 < \theta < \pi ; v = V_m \sin \theta$$

$$\therefore V_{av} = \frac{1}{\pi} \left\{ \int_0^{\pi/4} v d\theta + \int_{\pi/4}^{3\pi/4} v d\theta + \int_{3\pi/4}^{\pi} v d\theta \right\}$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi/4} V_m \sin \theta d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m d\theta + \int_{3\pi/4}^{\pi} V_m \sin \theta d\theta \right\}$$

$$= \frac{V_m}{\pi} \left\{ -\cos \theta \Big|_0^{\pi/4} + 0.707 \theta \Big|_{\pi/4}^{3\pi/4} + -\cos \theta \Big|_{3\pi/4}^{\pi} \right\} = \frac{V_m}{\pi} (0.293 + 1.111 + 0.293) = \mathbf{0.54 V_m}$$

$$V^2 = \frac{1}{\pi} \left\{ \int_0^{\pi/4} V_m^2 \sin^2 \theta d\theta + \int_{\pi/4}^{3\pi/4} (0.707 V_m)^2 d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2 \theta d\theta \right\} = 0.341 V_m^2$$

$$\therefore V = \mathbf{0.584 V_m}$$

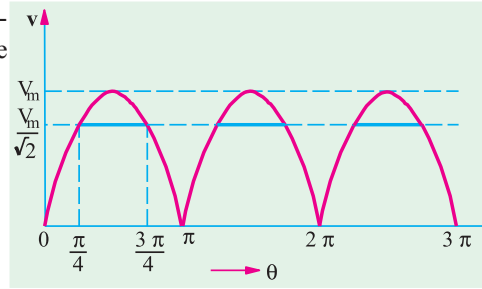


Fig. 11.29

**Example 11.19.** A delayed full-wave rectified sinusoidal current has an average value equal to half its maximum value. Find the delay angle  $\theta$ . **(Basic Circuit Analysis, Nagpur 1992)**

**Solution.** The current waveform is shown in Fig. 11.30.

$$I_{av} = \frac{1}{\pi} \int_{\theta}^{\pi} I_m \sin \theta d\theta = \frac{I_m}{\pi} (-\cos \pi + \cos \theta)$$

$$\text{Now, } I_{av} = I_m/2$$

$$\therefore \frac{I_m}{\pi} (-\cos \pi + \cos \theta) = \frac{I_m}{2}$$

$$\therefore \cos \theta = 0.57, \theta = \cos^{-1}(0.57) = \mathbf{55.25^\circ}$$

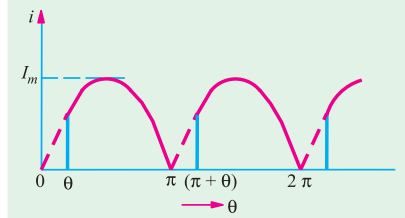


Fig. 11.30

**Example 11.20.** The waveform of an output current is as shown in Fig. 11.31. It consists of a portion of the positive half cycle of a sine wave between the angle  $\theta$  and  $180^\circ$ . Determine the effective value for  $\theta = 30^\circ$ .

**(Elect. Technology, Vikram Univ. 1984)**

**Solution.** The equation of the given delayed half-wave rectified sine wave is  $i = I_m \sin \alpha t = I_m \sin \theta$ . The effective value is given by

$$I = \sqrt{\frac{1}{2\pi} \int_{\pi/6}^{\pi} i^2 d\theta} \text{ or } I^2 = \frac{1}{2\pi} \int_{\pi/6}^{\pi} I_m^2 \sin^2 \theta d\theta$$

$$= \frac{I_m^2}{4\pi} \int_{\pi/6}^{\pi} (1 - \cos 2\theta) d\theta = \frac{I_m^2}{4\pi} \left[ \theta - \frac{\sin 2\theta}{2} \right]_{\pi/6}^{\pi}$$

$$= 0.242 I_m^2$$

$$\text{or } I = \sqrt{0.242 I_m^2} = \mathbf{0.492 I_m}$$

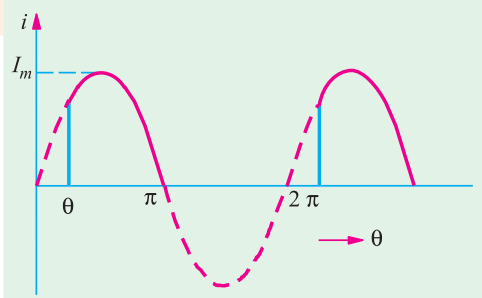


Fig. 11.31

**Example 11.21.** Calculate the “form factor” and “peak factor” of the sine wave shown in Fig. 11.32. **(Elect. Technology-I, Gwalior Univ.)**

**Solution.** For  $0 < \theta < \pi$ ,  $i = 100 \sin \theta$  and for  $\pi < \theta < 2\pi$ ,  $i = 0$ . The period is  $2\pi$ .

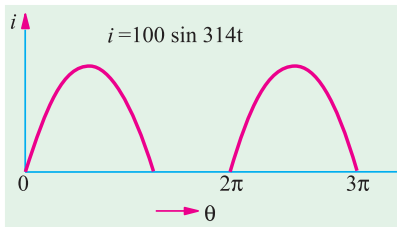


Fig. 11.32

$$\begin{aligned} \therefore 0 \quad I_{av} &= \frac{1}{2\pi} \left\{ \int_0^\pi i d\theta + \int_\pi^{2\pi} 0 d\theta \right\} \\ &= \frac{1}{2\pi} \left\{ 100 \int_0^\pi \sin \theta d\theta \right\} = 31.8 \text{ A} \end{aligned}$$

$$\begin{aligned} I^2 &= \frac{1}{2\pi} \int_0^\pi i^2 d\theta = \frac{100^2}{2\pi} \int_0^\pi \sin^2 \theta d\theta = \frac{100^2}{4} = 2500; I = 50 \text{ A} \\ \therefore \text{ form factor} &= 50/31.8 = \mathbf{1.57}; \text{ peak factor} = 100/50 = \mathbf{2} \end{aligned}$$

**Example 11.22.** Find the average and effective values of voltage of sinusoidal waveform shown in Fig. 11.33.

(Elect. Science-I Allahabad Univ. 1991)

**Solution.** Although, the given waveform would be integrated from  $\pi/4$  to  $\pi$ , it would be averaged over the whole cycle because it is unsymmetrical. The equation of the given sinusoidal waveform is  $v = 100 \sin \theta$ .

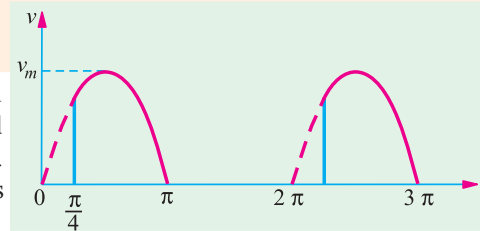


Fig. 11.33

$$\therefore V_{av} = \frac{1}{2\pi} \int_{\pi/4}^\pi 100 \sin \theta d\theta = \frac{100}{2\pi} \left[ -\cos \theta \right]_{\pi/4}^\pi = 27.2 \text{ V}$$

$$V^2 = \frac{1}{2\pi} \int_{\pi/4}^\pi 100^2 \sin^2 \theta d\theta = \frac{100^2}{4} \int_{\pi/4}^\pi (1 - \cos 2\theta) d\theta = \frac{100^2}{4} \left[ \theta - \frac{\sin 2\theta}{2} \right]_{\pi/4}^\pi = \frac{100^2}{4} \left[ \frac{3\pi}{4} - \frac{1}{2} \right]$$

$$\therefore V = \mathbf{47.7 \text{ V}}$$

**Example 11.23.** Find the r.m.s. and average values of the saw tooth waveform shown in Fig. 11.34 (a).

**Solution.** The required values can be found by using either graphical method or analytical method.

#### Graphical Method

The average value can be found by averaging the function from  $t = 0$  to  $t = 1$  in parts as given below :

$$\text{Average value of } (f) = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{T} \times (\text{net area over one cycle})$$

Now, area of a right-angled triangle =  $(1/2) \times (\text{base}) \times (\text{altitude})$ .

Hence, area of the triangle during  $t = 0$  to  $t = 0.5$  second is

$$A_1 = \frac{1}{2} \times (\Delta t) \times (-2) = \frac{1}{2} \times \frac{1}{2} \times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from  $t = 0.5$  to  $t = 1$  second is

$$A_2 = \frac{1}{2} \times (\Delta t) \times (+2) = \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$$

$$\text{Net area from } t = 0 \text{ to } t = 1.0 \text{ second is } A_1 + A_2 = -\frac{1}{2} + \frac{1}{2} = 0$$

Hence, average value of  $f(t)$  over one cycle is zero.

For finding the r.m.s. value, we will first square the ordinates of the given function and draw a new plot for  $f^2(t)$  as shown in Fig. 11.34 (b). It would be seen that the squared ordinates form a parabola.

Area under parabolic curve =  $\frac{1}{3} \times \text{base} \times \text{altitude}$ . The area under the curve from  $t = 0$  to

$$t = 0.5 \text{ second is ; } A_1 = \frac{1}{3}(\Delta t) \times 2^2 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

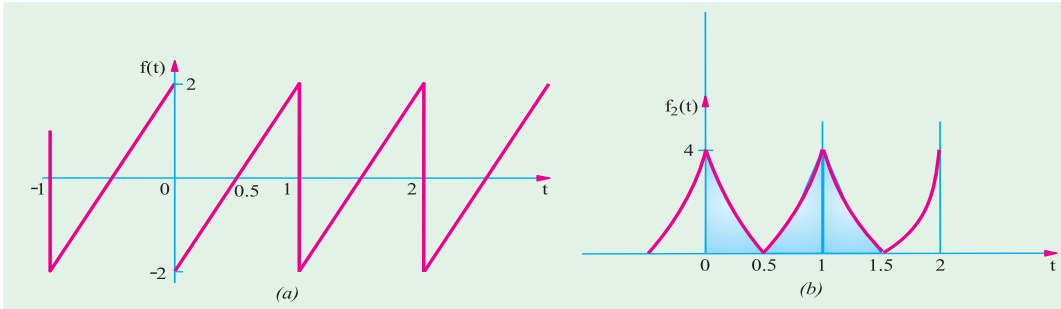


Fig. 11.34

$$\text{Similarly, for } t = 0.5 \text{ to } t = 1.0 \text{ second } A_2 = \frac{1}{3}(\Delta t) \times 4 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

$$\text{Total area} = A_1 + A_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}, \text{ r.m.s. value} = \sqrt{\frac{1}{T} \int_0^T f^2(t) dt} = \sqrt{\text{average of } f^2(t)}$$

$$\therefore \text{r.m.s. value} = \sqrt{4/3} = \mathbf{1.15}$$

#### Analytical Method

The equation of the straight line from  $t = 0$  to  $t = 1$  in Fig. 11.34 (a) is

$$f(t) = 4t - 2; \quad f^2(t) = 16t^2 - 16t + 4$$

$$\text{Average value} = \frac{1}{T} \int_0^T (4t - 2) dt = \frac{1}{T} \left[ \frac{4t^2}{2} - 2t \right]_0^T = 0$$

$$\text{r.m.s value} = \sqrt{\frac{1}{T} \int_0^T (16t^2 - 16t + 4) dt} = \sqrt{\frac{1}{T} \left[ \frac{16t^3}{3} - \frac{16t^2}{2} + 4t \right]_0^T} = \mathbf{1.15}$$

**Example 11.24.** A circuit offers a resistance of  $20 \Omega$  in one direction and  $100 \Omega$  in the reverse direction. A sinusoidal voltage of maximum value  $200 \text{ V}$  is applied to the above circuit in series with

- (a) a moving-iron ammeter      (b) a moving-coil ammeter  
(c) a moving-coil instrument with a full-wave rectifier      (d) a moving-coil ammeter.

Calculate the reading of each instrument.

**Solution. (a)** The deflecting torque of an MI instrument is proportional to  $(\text{current})^2$ . Hence, its reading will be proportional to the average value of  $I^2$  over the whole cycle. Therefore, the reading of such an instrument :

$$\begin{aligned} &= \sqrt{\left[ \frac{1}{2\pi} \left( \int_0^\pi 10^2 \sin^2 \theta d\theta + \int_\pi^{2\pi} 2^2 \sin^2 \theta d\theta \right) \right]} \\ &= \sqrt{\frac{1}{2} \left[ \frac{100}{2} \left| \frac{\sin 2\theta}{2} \right|_0^\pi + \frac{4}{2} \left| \frac{\sin 2\theta}{2} \right|_\pi^{2\pi} \right]} = \sqrt{26} \quad \mathbf{5.1 \text{ A}} \end{aligned}$$

(b) An MC ammeter reads the average current over the whole cycle.

$$\text{Average current over positive half-cycle is} = 10 \times 0.637 = 6.37 \text{ A}$$

$$\text{Average current over negative half-cycle is} = -2 \times 0.637 = -1.27 \text{ A}$$

$$\therefore \text{Average value over the whole cycle is} = (6.37 - 1.27)/2 = \mathbf{2.55 \text{ A}}$$

(c) In this case, due to the full-wave rectifier, the current passing through the operating coil of the instrument would flow in the positive direction during both the positive and negative half cycles.

$$\therefore \text{reading} = (6.37 + 1.27)/2 = \mathbf{3.82 \text{ A}}$$

$$(d) \text{Average heating effect over the positive half-cycle is} = \frac{1}{2} I_m^2 R$$

Average heating effect over the negative half-cycle is  $= \frac{1}{2} I_{m2}^2 R$

where  $I_{m1} = 200/20 = 10 \text{ A}$ ;  $I_{m2} = 200/100 = 2 \text{ A}$

Average heating effect over the whole cycle is  $= \left( \frac{1}{2} \times 10^2 R + \frac{1}{2} \times 2^2 \times R \right) / 2 = 26 R$

If  $I$  is the direct current which produces the same heating effect, then

$$I^2 R = 26 R \quad \therefore I = \sqrt{26} = 5.1 \text{ A}$$

**Example 11.25.** A moving coil ammeter, a hot-wire ammeter and a resistance of  $100 \Omega$  are connected in series with a rectifying device across a sinusoidal alternating supply of  $200 \text{ V}$ . If the device has a resistance of  $100 \Omega$  to the current in one direction and  $5.00 \Omega$  to current in opposite direction, calculate the readings of the two ammeters.

(Elect. Theory and Meas. Madras University)

**Solution.** R.M.S. current in one direction is  $= 200/(100 + 100) = 1 \text{ A}$

Average current in the first i.e. positive half cycle is  $= 1/1.11 = 0.9 \text{ A}$

Similarly, r.m.s. value in the negative half-cycle is  $= -200/(100 + 500) = -1/3 \text{ A}$

Average value  $= (-1/3)/1.11 = -0.3 \text{ A}$

Average value over the whole cycle is  $= (0.9 - 0.3)/2 = 0.3 \text{ A}$

Hence, M/C ammeter reads **0.3 A**

Average heating effect during the +ve half cycle  $= I_{rms}^2 \times R = I^2 \times R = R$

Similarly, average heating effect during the -ve half-cycle is  $= (-1/3)^2 \times R = R/9$

Here,  $R$  is the resistance of the hot-wire ammeter.

Average heating effect over the whole cycle is  $= \frac{1}{2} \left( R + \frac{R}{9} \right) = \frac{5R}{9}$

If  $I$  is the direct current which produces the same heating effect, then

$$I^2 R = 5R/9 \quad \therefore I = \sqrt{5/3} = 0.745 \text{ A}$$

Hence, hot-wire ammeter indicates **0.745 A**

**Example 11.26.** A resultant current wave is made up of two components : a  $5 \text{ A}$  d.c. component and a  $50\text{-Hz}$  a.c. component, which is of sinusoidal waveform and which has a maximum value of  $5 \text{ A}$ .

(i) Draw a sketch of the resultant wave.

(ii) Write an analytical expression for the current wave, reckoning  $t = 0$  at a point where the a.c. component is at zero value and when  $di/dt$  is positive.

(iii) What is the average value of the resultant current over a cycle?

(iv) What is the effective or r.m.s. value of the resultant current?

[Similar Problem: Bombay Univ. 1996]

**Solution.** (i) The two current components and resultant current wave have been shown in Fig. 11.35.

(ii) Obviously, the instantaneous value of the resultant current is given by  $i = (5 + 5 \sin \omega t) = (5 + 5 \sin \theta)$

(iii) Over one complete cycle, the average value of the alternating current is zero. Hence, the average value of the resultant current is equal to the value of d.c. component i.e. **5 A**

(iv) Mean value of  $i^2$  over complete cycle is

$$\begin{aligned} &= \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi} \int_0^{2\pi} (5 + 5 \sin \theta)^2 d\theta \\ &= \frac{1}{2\pi} \int_0^{2\pi} (25 + 50 \sin \theta + 25 \sin^2 \theta) d\theta \end{aligned}$$

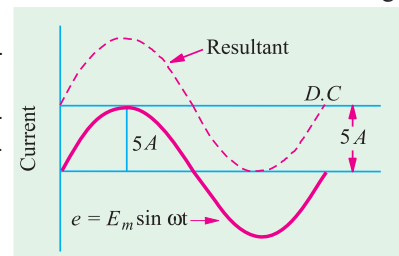


Fig. 11.35

$$\begin{aligned}
 &= \frac{1}{2\pi} \int_0^{2\pi} \left[ 25 + 50 \sin \theta + 25 \left( \frac{1 - \cos 2\theta}{2} \right) \right] d\theta = \frac{1}{2\pi} \int_0^{2\pi} (37.5 + 50 \sin \theta - 12.5 \cos 2\theta) d\theta \\
 &= \frac{1}{2} \left[ 37.5 \theta - 50 \cos \theta - \frac{12.5}{2} \sin 2\theta \right]_0^{2\pi} = \frac{75}{2} = 37.5 \text{ .R.M.S. value } I = \sqrt{37.5} = \mathbf{6.12 \text{ A}}
 \end{aligned}$$

**Note.** In general, let the combined current be given by  $i = A + B\sqrt{2} \sin \alpha t = A + B\sqrt{2} \sin \theta$  where  $A$  represents the value of direct current and  $B$  the r.m.s. value of alternating current.

The r.m.s. value of combined current is given by

$$\begin{aligned}
 I_{rms} &= \sqrt{\left( \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta \right)} \text{ or } I_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi} \int_0^{2\pi} (A + B\sqrt{2} \sin \theta)^2 d\theta \\
 &= \frac{1}{2\pi} \int_0^{2\pi} (A^2 + 2B^2 \sin^2 \theta + 2\sqrt{2} AB \sin \theta) d\theta = \frac{1}{2\pi} \int_0^{2\pi} (A^2 + B^2 - B^2 \cos 2\theta + 2\sqrt{2} AB \sin \theta) d\theta \\
 &= \frac{1}{2} \left[ A^2 \theta - \frac{B^2 \sin 2\theta}{2} - 2\sqrt{2} AB \cos \theta \right]_0^{2\pi} = \frac{1}{2} (2A^2 - 2B^2 - 2\sqrt{2} AB) \\
 &= A^2 + B^2 \quad \therefore I_{rms} = \sqrt{A^2 + B^2}
 \end{aligned}$$

The above example could be easily solved by putting  $A = 5$  and  $B = 5/\sqrt{2}$  (because  $B_{\max} = 5$ )

$$\therefore I_{rms} = \sqrt{5^2 + (5/\sqrt{2})^2} = \mathbf{6.12 \text{ A}}$$

**Example 11.27.** Determine the r.m.s. value of a semi-circular current wave which has a maximum value of  $a$ .

**Solution.** The equation of a semi-circular wave (shown in Fig. 11.36) is

$$x^2 + y^2 = a^2 \text{ or } y^2 = a^2 - x^2$$

$$\therefore I_{rms} = \sqrt{\frac{1}{2a} \int_{-a}^{+a} y^2 dx} \text{ or } I_{rms}^2 = \frac{1}{2a} \int_{-a}^{+a} (a^2 - x^2) dx$$

$$= \frac{1}{2a} \int_{-a}^{+a} (a^2 dx - x^2 dx) = \frac{1}{2a} \left[ a^2 x - \frac{x^3}{3} \right]_{-a}^{+a} = \frac{1}{2a} \left( a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3} \right) = \frac{2a^2}{3}$$

$$\therefore I_{rms} = \sqrt{2a^2/3} = \mathbf{0.816 a}$$

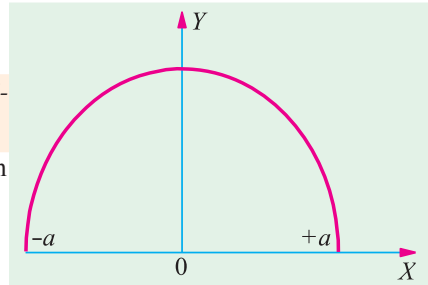


Fig. 11.36

**Example 11.28.** Calculate the r.m.s. and average value of the voltage wave shown in Fig. 11.37.

**Solution.** In such cases, it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 and an other form 1 to 2 millisecond.

For  $t$  lying between 0 and 1 ms,  $v_1 = 4$ , For  $t$  lying between 1 and 2 ms,  $v_2 = -4t + 4$

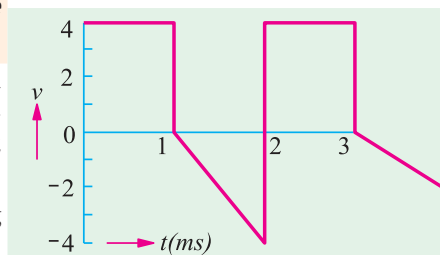


Fig. 11.37



$$\begin{aligned} \therefore V_{rms} &= \sqrt{\frac{1}{2} \left( \int_0^1 v_1^2 dt + \int_1^2 v_2^2 dt \right)} \\ V_{rms}^2 &= \frac{1}{2} \left[ \int_0^1 4^2 dt + \int_1^2 (-4t+4)^2 dt \right] \\ &= \frac{1}{2} \left[ 16t \Big|_0^1 + \left. \frac{16t^3}{3} \right|_1^2 + 16t \Big|_1^2 - \left. \frac{32t^2}{2} \right|_1^2 \right] \\ &= \frac{1}{2} \left[ 16 + \frac{16 \times 8}{3} - \frac{16}{3} + 16 \times 2 - 16 \times 1 - \frac{32 \times 4}{2} + \frac{32 \times 1}{2} \right] = \frac{32}{3} \quad \therefore V_{rms} = \sqrt{32/3} = \mathbf{3.265 \text{ volt}} \\ V_{av} &= \frac{1}{2} \left[ \int_0^1 v_1 dt + \int_1^2 v_2 dt \right] = \frac{1}{2} \left[ \int_0^1 4 dt + \int_1^2 (-4t+4) dt \right] = \frac{1}{2} \left[ 4t \Big|_0^1 + \left. -\frac{4t^2}{2} + 4t \right|_1^2 \right] = \mathbf{1 \text{ volt}} \end{aligned}$$

### Tutorial Problems No. 11.1

1. Calculate the maximum value of the e.m.f. generated in a coil which is rotating at 50 rev/s in a uniform magnetic field of 0.8 Wb/m<sup>3</sup>. The coil is wound on a square former having sides 5 cm in length and is wound with 300 turns. **[188.5 V]**

2. (a) What is the peak value of a sinusoidal alternating current of 4.78 r.m.s. amperes ?

(b) What is the r.m.s. value of a rectangular voltage wave with an amplitude of 9.87 V ?

(c) What is the average value of a sinusoidal alternating current of 31 A maximum value ?

(d) An alternating current has a periodic time of 0.03 second. What is its frequency ?

(e) An alternating current is represented by  $i = 70.7 \sin 520 t$ . Determine (i) the frequency (ii) the current 0.0015 second after passing through zero, increasing positively.

**[6.76 A ; 9.87 V ; 19.75 A ; 33.3 Hz ; 82.8 Hz ; 49.7 A]**

3. A sinusoidal alternating voltage has an r.m.s. value of 200 V and a frequency of 50 Hz. It crosses the zero axis in a positive direction when  $t = 0$ . Determine (i) the time when voltage first reaches the instantaneous value of 200 V and (ii) the time when voltage after passing through its maximum positive value reaches the value of 141.4 V.

**[(i) 0.0025 second (ii) 1/300 second]**

4. Find the form factor and peak factor of the triangular wave shown in Fig. 11.38 **[1.155; 1.732]**

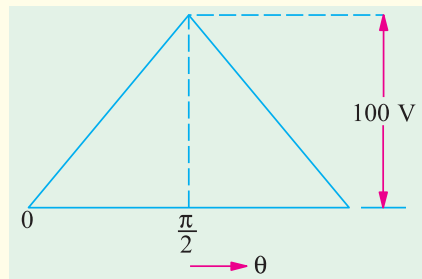


Fig. 11.38

5. An alternating voltage of  $200 \sin 471 t$  is applied to a h.w. rectifier which is in series with a resistance of 40  $\Omega$ . If the resistance of the rectifier is infinite in one direction and zero in the other, find the r.m.s. value of the current drawn from the supply source. **[2.5 A]**

6. A sinusoidally varying alternating current has an average value of 127.4 A. When its value is zero, then its rate change is 62,800 A/s. Find an analytical expression for the sine wave. **[ $i = 200 \sin 100 \pi t$ ]**

7. A resistor carries two alternating currents having the same frequency and phase and having the same value of maximum current *i.e.* 10 A. One is sinusoidal and the other is rectangular in waveform. Find the r.m.s. value of the resultant current. **[12.24 A]**

8. A copper-oxide rectifier and a non-inductive resistance of 20  $\Omega$  are connected in series across a sinusoidal a.c. supply of 230 V (r.m.s.). The resistance of the rectifier is 2.5  $\Omega$  in forward direction and 3,000  $\Omega$  in the reverse direction. Calculate the r.m.s. and average values of the current.

**[r.m.s. value = 5.1 A, average value = 3.22 A]**

9. Find the average and effective values for the waveshape shown in Fig. 11.39 if the curves are parts of a sine wave. **[27.2 V, 47.7V] (Elect. Technology, Indore Univ.)**

10. Find the effective value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 15 A.

[14.58A] (Elect. Technology, Vikram Univ. Ujjain)

11. Determine the r.m.s. value of the voltage defined by  $e = 5 + 5 \sin(314t + \pi/6)$

[6.12 V] (Elect. Technology, Indore Univ.)

12. Find the r.m.s. value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 10 A. [12.25 A] (Elect. Technology-I; Delhi Univ.)

13. An alternating voltage given by  $e = 150 \sin 100\pi t$  is applied to a circuit which offers a resistance of 50 ohms to the current in one direction and completely prevents the flow of current in the opposite direction. Find the r.m.s. and average values of this current and its form factor.

[1.5 A, 0.95 A, 1.57] (Elect. Technology, Indore Univ.)

14. Find the relative heating effects of three current waves of equal maximum value, one rectangular, the second semi-circular and the third sinusoidal in waveform [1: 2/2, 1/2] (Sheffield Univ. U.K.)

15. Calculate the average and root mean-square value, the form factor and peak factor of a periodic current wave have the following values for equal time intervals over half-cycle, changing suddenly from one value of the next. [0, 40, 60, 80, 100, 80, 60, 40, 0] (A.M.I.E.)

16. A sinusoidal alternating voltage of amplitude 100 V is applied across a circuit containing a rectifying device which entirely prevents current flowing in one direction and offers a resistance of 10 ohm to the flow of current in the other direction. A hot wire ammeter is used for measuring the current. Find the reading of instrument. (Elect. Technology, Punjab Univ.)

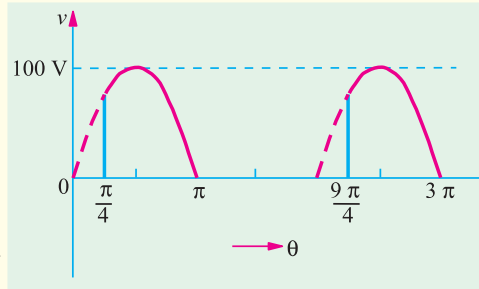


Fig. 11.39

### 11.23. Representation of Alternating Quantities

It has already been pointed out that an attempt is made to obtain alternating voltages and currents having sine waveform. In any case, a.c. computations are based on the assumption of sinusoidal voltages and currents. It is, however, cumbersome to continuously handle the instantaneous values in the form of equations of waves like  $e = E_m \sin \omega t$  etc. A conventional method is to employ vector method of representing these sine waves. These vectors may then be manipulated instead of the sine functions to achieve the desired result. **In fact, vectors are a shorthand for the representation of alternating voltages and currents and their use greatly simplifies the problems in a.c. work.**

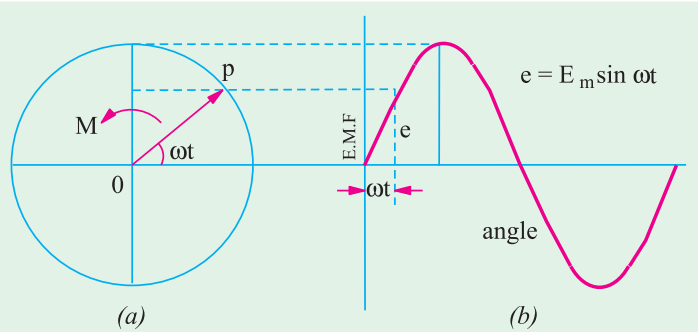


Fig. 11.40

A vector is a physical quantity which has magnitude as well as direction. Such vector quantities are completely known when particulars of their magnitude, direction and the sense in which they act, are given. They are graphically represented by straight lines called vectors. The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity. In Fig. 11.40 (a),  $OP$  is such a vector which represents the maximum value of the alternating current and its angle with  $X$  axis gives its phase. Let the alternating current be represented by the equation  $e = E_m \sin \omega t$ . It will be seen that the projection of  $OP$  and  $Y$ -axis at any instant gives the instantaneous value of that alternating current.

$$\therefore OM = OP \sin \omega t \text{ or } e = OP \sin \omega t = E_m \sin \omega t$$

It should be noted that a line like  $OP$  can be made to represent an alternating voltage or current if it satisfies the following conditions :

(i) Its length should be equal to the peak or maximum value of the sinusoidal alternating current to a suitable scale. (ii) It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing. (iii) Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

### 11.24. Vector Diagram using R.M.S. Values

Instead of using maximum values as above, it is very common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the  $Y$ -axis does not give the instantaneous value of that alternating quantity.

### 11.25. Vector Diagrams of Sine Waves of Same Frequency

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter-clockwise at the same frequency and maintain a fixed position relative to each other. This is illustrated in Fig. 11.41 where a voltage  $e$  and current  $i$  of the same frequency are shown. The current wave is supposed to pass upward through zero at the instant when  $t = 0$  while at the same time the voltage wave has already advanced an angle  $\alpha$  from its zero value. Hence, their equations can be written as

$$i = I_m \sin \omega t$$

and  $e = E_m \sin (\omega t + \alpha)$

*Sine wave of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.*

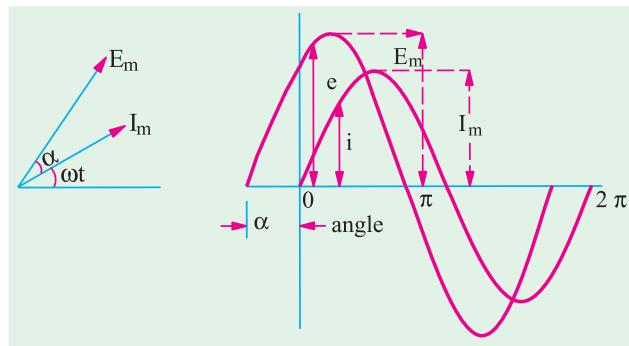


Fig. 11.41

### 11.26. Addition of Two Alternating Quantities

In Fig. 11.42 (a) are shown two rotating vectors representing the maximum values of two sinusoidal voltage waves represented by  $e_1 = E_{m1} \sin \omega t$  and  $e_2 = E_{m2} \sin (\omega t - \phi)$ . It is seen that the sum of the two sine waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. The value of

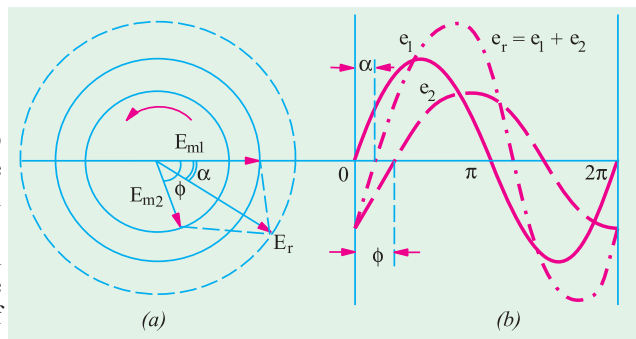


Fig. 11.42

the instantaneous resultant voltage  $e_r$  at any instant is obtained by algebraically adding the projections of the two vectors on the  $Y$ -axis. If these projections are  $e_1$  and  $e_2$ , then,  $e_r = e_1 + e_2$  at that time. The resultant curve is drawn in this way by adding the ordinates. It is found that the resultant wave is a sine wave of the same frequency as the component waves but lagging behind  $E_{m1}$  by an angle  $\alpha$ . The vector diagram of Fig. 11.42 (a) can be very easily drawn. Lay off  $E_{m2}$  lagging  $\phi$  behind  $E_{m1}$  and then complete the parallelogram to get  $E_r$ .

**Example 11.29.** Add the following currents as waves and as vectors.

$$i_1 = 7 \sin \omega t \text{ and } i_2 = 10 \sin (\omega t + \pi/3)$$

**Solution. As Waves**

$$\begin{aligned} i_r &= i_1 + i_2 = 7 \sin \omega t + 10 \sin (\omega t + 60^\circ) \\ &= 7 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ \\ &= 12 \sin \omega t + 8.66 \cos \omega t \end{aligned}$$

Dividing both sides by  $\sqrt{(12^2 + 8.66^2)} = 14.8$ , we get

$$\begin{aligned} i_r &= 14.8 \left( \frac{12}{14.8} \sin \omega t + \frac{8.66}{14.8} \cos \omega t \right) \\ &= 14.8 (\cos \alpha \sin \omega t + \sin \alpha \cos \omega t) \end{aligned}$$

where

$$\cos \alpha = 12/14.8 \text{ and } \alpha = 8.66/14.8$$

$\therefore$

$$i_r = 14.8 \sin (\omega t + \alpha)$$

where

$$\tan \alpha = 8.66/12 \text{ or } \alpha = \tan^{-1}(8.66/12) = 35.8^\circ$$

$\therefore$

$$i_r = 14.8 \sin (\omega t + 35.8^\circ)$$

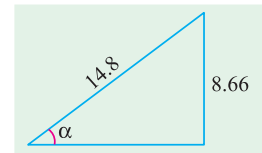


Fig. 11.43

—as shown in Fig. 11.43

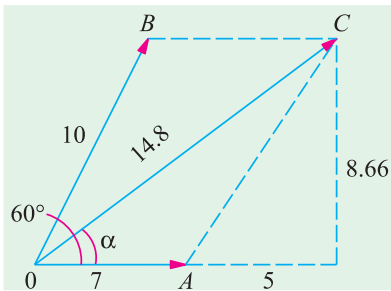


Fig. 11.44

**As Vectors**

Vector diagram is shown in Fig. 11.44. Resolving the vectors into their horizontal and vertical components, we have

$$X\text{-component} = 7 + 10 \cos 60^\circ = 12$$

$$Y\text{-component} = 0 + 10 \sin 60^\circ = 8.66$$

$$\text{Resultant} = \sqrt{(12^2 + 8.66^2)} = 14.8 \text{ A}$$

$$\text{and } \alpha = \tan^{-1}(8.66/12) = 35.8^\circ$$

Hence, the resultant equation can be written as

$$i_r = 14.8 \sin (\omega t + 35.8^\circ)$$

## 11.27. Addition and Subtraction of Vectors

**(i) Addition.** In a.c. circuit problems we may be concerned with a number of alternating voltages or currents of the same frequency but of different phases and it may be required to obtain the resultant voltage or current. As explained earlier (Art. 11.23) if the quantities are sinusoidal, they may be represented by a number of rotating vectors having a common axis of rotation and displaced from one another by fixed angles which are equal to the phase differences between the respective alternating quantities. The instantaneous value of the resultant voltage is given by the algebraic sum of the projections of the different vectors on  $Y$ -axis. The maximum value (or r.m.s. value if the vectors represent that value) is obtained by compounding the several vectors by using the parallelogram and polygon laws of vector addition.

However, another easier method is to resolve the various vectors into their  $X$ - and  $Y$ -components and then to add them up as shown in Example 11.30 and 31.

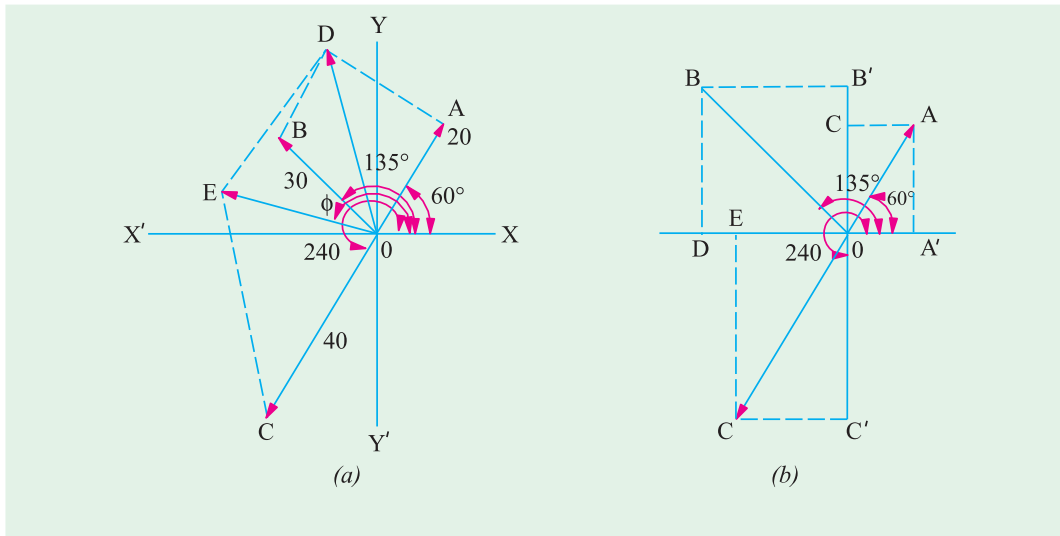


Fig. 11.45

Suppose we are given the following three alternating e.m.fs. and it is required to find the equation of the resultant e.m.f.

$$e_1 = 20 \sin (\omega t + \pi/3)$$

$$e_2 = 30 \sin (\omega t + 3\pi/4)$$

$$e_3 = 40 \sin (\omega t + 4\pi/3)$$

Then the vector diagram can be drawn as explained before and solved in any of the following three ways :

(i) By compounding according to parallelogram law as in Fig. 11.45 (a)

(ii) By resolving the various vectors into their X-and Y-components as in Fig. 11.45 (b).

(iii) By laying off various vectors end-on end at their proper phase angles and then measuring the closing vector as shown in Fig. 11.46.

Knowing the magnitude of the resultant vector and its inclination  $\phi$  with X axis, the equation of the resultant e.m.f. can be written as  $e = E_m \sin (\omega t + \phi)$ .

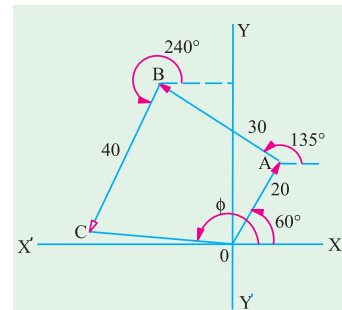


Fig. 11.46

**Example 11.30.** Represent the following quantities by vectors :

$$5 \sin (2\pi ft - 1) ; 3 \cos (2\pi ft + 1) ; 2 \sin (2\pi ft + 2.5) \text{ and } 4 \sin (2\pi ft - 1)$$

Add the vectors and express the result in the form :  $A \sin (2\pi ft \pm \Phi)$

**Solution.** It should be noted that all quantities have the same frequency  $f$  hence they can be represented vertically on the same vector diagram and added as outlined in Art. 11.27. But before doing this, it would be helpful to express all the quantities as sine functions. Therefore, the second expression  $3 \cos (2\pi ft + 1)$  can be written as

$$3 \sin \left( 2\pi ft + 1 + \frac{\pi}{2} \right) = 3 \sin (2\pi ft + 1 + 1.57) = 3 \sin (2\pi ft + 2.57)$$

The maximum value of each quantity, its phase with respect to the quantity of reference *i.e.*  $X \sin 2\pi ft$ , its horizontal and vertical components are given in the table on next page :

Quantity	Max. value	Phase		Horizontal component	Vertical component
		radians	angles		
(i) $5 \sin (2\pi ft - 1)$	5	-1	$-57.3^\circ$	$5 \times \cos (-57.3^\circ) = 2.7$	$5 \sin (-57.3^\circ) = -4.21$
(ii) $3 \sin (2\pi ft + 2.57)$	3	+2.57	$147.2^\circ$	$3 \times \cos 147.2^\circ = -2.52$	$3 \sin 147.2^\circ = 1.63$
(iii) $2 \sin (2\pi ft + 2.5)$	2	+2.5	$143.2^\circ$	$2 \cos 143.2^\circ = -1.6$	$2 \sin 143.2^\circ = 1.2$
(iv) $4 \sin (2\pi ft - 1)$	4	-1	$-57.3^\circ$	$4 \cos (-57.3^\circ) = 2.16$	$4 \sin (-57.3^\circ) = -3.07$
Total				0.74	-4.75

The vector diagram is shown in Fig. 11.47 in which  $OA$ ,  $OB$ ,  $OC$  and  $OD$  represent quantities (i), (ii), (iii) and (iv) given in the table.

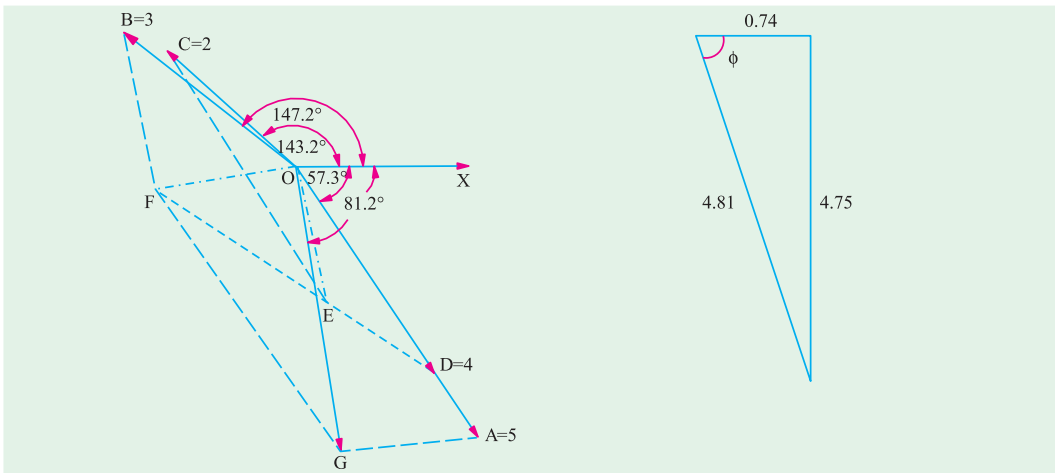


Fig. 11.47

Fig. 11.48

Their resultant is given by  $OG$  and the net horizontal and vertical components are shown in Fig. 11.48. Resultant

$$= \sqrt{[0.74^2 + (-4.75)^2]} = 4.81 \text{ and } \tan \theta = -4.75 / 0.74$$

$$\therefore \theta = \tan^{-1}(-4.75/0.74) = -81.2^\circ = -1.43 \text{ radians}$$

The equation of the resultant quantity is  **$4.81 \sin (2\pi ft - 1.43)$**

**Example 11.31.** Three voltages represented by

$$e_1 = 20 \sin \omega t; e_2 = 30 \sin (\omega t - \pi/4) \text{ and } e_3 = 40 \cos (\omega t + \pi/6)$$

act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors. **(Electro-technics Madras Univ.) (Elec. Circuit Nagpur Unvi. 1991)**

**Solution.** First, let us draw the three vectors representing the maximum values of the given alternating voltages.

$e_1 = 20 \sin \omega t$ —here phase angle with  $X$ -axis is zero, hence the vector will be drawn parallel to the  $X$ -axis

$e_2 = 30 \sin (\omega t - \pi/4)$  —its vector will be below  $OX$  by  $45^\circ$

$e_3 = 40 \cos (\omega t + \pi/6) = 40 \sin (90^\circ + \omega t + \pi/6)^*$   
 $= 40 \sin (\omega t + 120^\circ)$  —its vector will be at  $120^\circ$  with respect to  $OX$  in counter clock-wise direction.

These vectors are shown in Fig. 11.49 (a). Resolving them into  $X$ - and  $Y$ -components, we get

$$X\text{-component} = 20 + 30 \cos 45^\circ - 40 \cos 60^\circ = 21.2 \text{ V}$$

$$Y\text{-component} = 40 \sin 60^\circ - 30 \sin 45^\circ = 13.4 \text{ V}$$

\*  $\cos \theta = \sin (90^\circ + \theta)$

As seen from Fig. 11.49 (b), the maximum value of the resultant voltage is

$$OD = \sqrt{21.2^2 + 13.4^2} = 25.1 \text{ V}$$

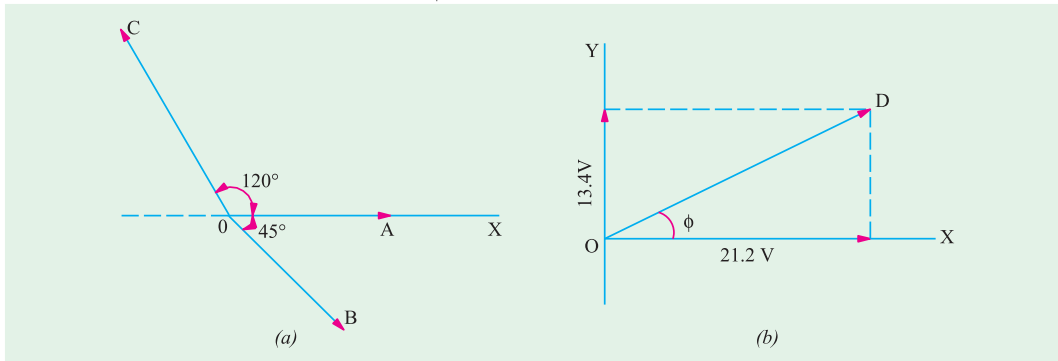


Fig. 11.49

The phase angle of the resultant voltage is given by  $\tan \phi = \frac{\text{Y-component}}{\text{X-component}} = \frac{13.4}{27.2} = 0.632$

$\therefore \phi = \tan^{-1} 0.632 = 32.3^\circ = 0.564 \text{ radian}$

The equation of the resultant voltage wave is  $e = 25.1 \sin(\omega t + 32.3^\circ)$  or  $e = 25.1 \sin(\omega t + 0.564)$

**Example 11.32.** Four circuits A, B, C and D are connected in series across a 240 V, 50-Hz supply. The voltages across three of the circuits and their phase angles relative to the current through them are,  $V_A$ , 80 V at  $50^\circ$  leading,  $V_B$ , 120 V at  $65^\circ$  lagging;  $V_C$ , 135 V at  $80^\circ$  leading. If the supply voltage leads the current by  $15^\circ$ , find from a vector diagram drawn to scale the voltage  $V_D$  across the circuit D and its phase angle.

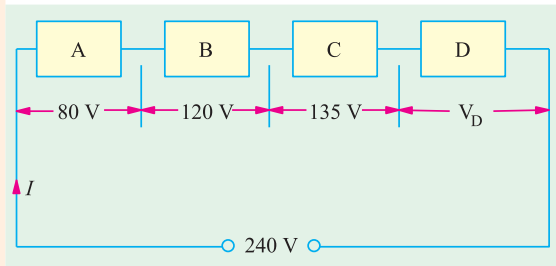


Fig. 11.50

**Solution.** The circuit is shown in Fig. 11.50.

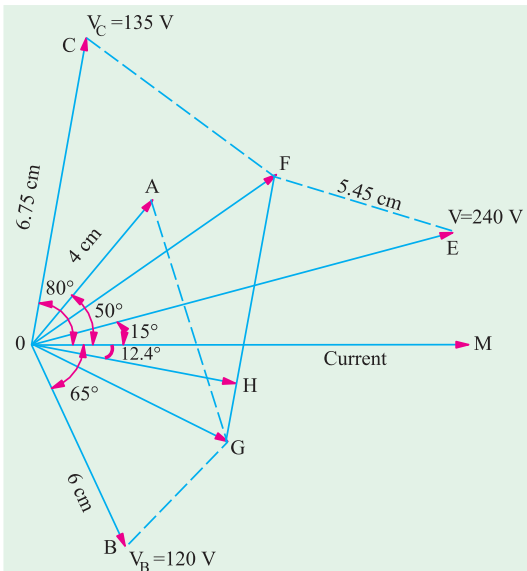


Fig. 11.51

(a) The vector diagram is shown in Fig. 11.50. The current vector  $OM$  is drawn horizontally and is taken as reference vector. Taking a scale of  $1 \text{ cm} = 20 \text{ V}$ , vector  $OA$  is drawn 4 cm in length and leading  $OM$  by an angle of  $50^\circ$ . Vector  $OB$  represents 120 V and is drawn lagging behind  $OM$  by  $65^\circ$ . Their vector sum, as found by Parallelogram Law of Vectors, is given by vector  $OG$ .

Next, vector  $OC$  is drawn ahead of  $OM$  by  $80^\circ$  representing 135 V. Vector  $OF$  represents the vector sum of  $OG$  and  $OC$ . Vector  $OE$  represents the applied voltage of 240 V and is drawn  $15^\circ$  ahead of current vector  $OM$ . The vector difference of  $OE$  and  $OF$  gives the required voltage  $V_D$ . It is equal to  $FE$ . It measures 5.45 cm which means that it represents  $20 \times 5.45 = 109 \text{ V}$ . This vector is transferred to position

$OH$  by drawing  $OH$  parallel to  $FE$ . It is seen that  $OH$  lags behind the current vector  $OM$  by  $12.4^\circ$ .

Hence,  $V_D' = 109$  volts lagging behind the current by  $12.4^\circ$ .

### (ii) Subtraction of Vectors

If difference of two vectors is required, then one of the vectors is reversed and this reversed vector is then compounded with the other vector as usual.

Suppose it is required to subtract vector  $OB$  from vector  $OA$ . Then  $OB$  is reversed as shown in Fig. 11.52 (a) and compounded with  $OA$  according to parallelogram law. The vector difference ( $A-B$ ) is given by vector  $OC$ .

Similarly, the vector  $OC$  in Fig. 11.52 (b) represents ( $B-A$ ) i.e. the subtraction of  $OA$  from  $OB$ .

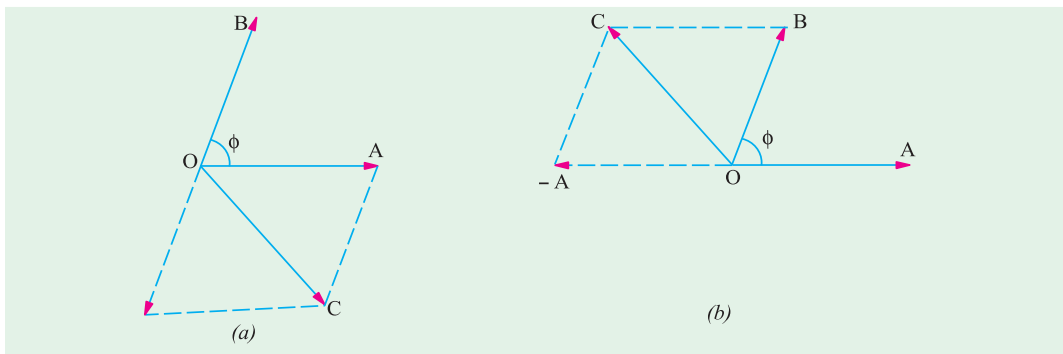


Fig 11.52

**Example 11.33.** Two currents  $i_1$  and  $i_2$  are given by the expressions

$$i_1 = 10 \sin (314t + \pi/4) \text{ amperes and } i_2 = 8 \sin (313t - \pi/3) \text{ amperes}$$

Find (a)  $i_1 + i_2$  and (b)  $i_1 - i_2$ . Express the answer in the form  $i = I_m \sin (314t \pm \phi)$

**Solution.** (a) The current vectors representing maximum values of the two currents are shown in Fig. 11.53 (a). Resolving the currents into their  $X$ - and  $Y$ -components, we get

$$X\text{-component} = 10 \cos 45^\circ + 8 \cos 60^\circ = 10/\sqrt{2} + 8/2 = 11.07 \text{ A}$$

$$Y\text{-component} = 10 \sin 45^\circ - 8 \sin 60^\circ = 0.14 \text{ A}$$

$$\therefore I_m = \sqrt{11.07^2 + 0.14^2} = 11.08 \text{ A}$$

$$\tan \phi = (0.14/11.07) = 0.01265 \quad \therefore \phi = 44'$$

Hence, the equation for the resultant current is  $i = 11.08 \sin (314t + 44')$  amperes

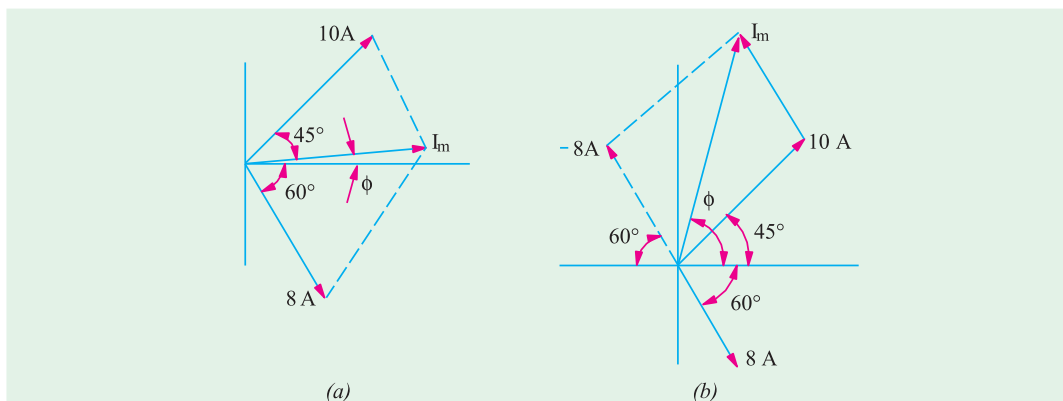


Fig. 11.53



$$(b) \quad X\text{-component} = 10 \cos 45^\circ - 8 \cos 60^\circ = 3.07 \text{ A}$$

$$Y\text{-component} = 10 \sin 45^\circ + 8 \sin 60^\circ = 14 \text{ A}$$

$$\therefore \quad I_m = \sqrt{3.07^2 + 14^2} = 14.33 \text{ A}$$

$$\phi = \tan^{-1} (14/3.07) = 77^\circ 38'$$

...Fig. 11.53 (b)

Hence, the equation of the resultant current is

$$i = 14.33 \sin (314 + 77^\circ 38') \text{ amperes}$$

**Example 11.34.** The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to a 50 Hz supply. The instantaneous values of voltage and current are 283 V and 10 A respectively at time  $t = 0$ , both increasing positively.

(i) Write down the expression for voltage and current at time  $t$ .

(ii) Determine the power consumed in the circuit.

Take the voltage and current to be sinusoidal

[Nagpur University]

**Solution.**  $V_m = 400$ ,  $I_m = 20$ ,  $\omega = 314$  rad./sec

(i) Let the expressions be as follows :

$$v(t) = V_m \sin (\omega t + \theta_1) = 400 \sin (314 t + \theta_1)$$

$$I(t) = I_m \sin (\omega t + \theta_2) = 20 \sin (314 t + \theta_2)$$

where  $\theta_1$  and  $\theta_2$  indicate the concerned phase-shifts with respect to some reference. Substituting the given instantaneous values at  $t = 0$ ,

$$\theta_1 = 45^\circ \text{ and } \theta_2 = 30^\circ$$

The required expressions are :

$$V(t) = 400 \sin (314 t + 45^\circ)$$

$$i(t) = 20 \sin (314 t + 30^\circ)$$

Thus, the voltage leads the current by  $15^\circ$ .

$$V = \text{RMS voltage} = 400/1.414 = 283 \text{ V}$$

$$I = \text{RMS voltage} = 20/1.414 = 14.14 \text{ A}$$

Power-factor,  $\cos \phi = \cos 15^\circ = 0.966$  lagging, since current lags behind the voltage.

$$(ii) \text{ Power} = V I \cos \phi = 3865 \text{ watts}$$

**Additional Hint :** Draw these two wave forms

**Example 11.35.** Voltage and current for a circuit with two elements in series are expressed as follows :

$$v(t) = 170 \sin (6280 t + \pi/3) \text{ Volts}$$

$$i(t) = 8.5 \sin (6280 t + \pi/2) \text{ Amps}$$

(i) Plot the two waveforms. (ii) Determine the frequency in Hz. (iii) Determine the power factor stating its nature. (iv) What are the values of the elements ?

[Nagpur University, April 1996]

**Solution.** (ii)  $\omega = 6280$  radiation/sec,  $f = \omega/2\pi = 1000$  Hz

(i) Two sinusoidal waveforms with a phase-difference of  $30^\circ$  ( $= \pi/2 - \pi/3$ ) are to be drawn. Each waveform completes a cycle in 1 milli-second, since  $f = 1000$  Hz.

The waveform for current leads that for the voltage by  $30^\circ$ . At  $\omega t = 0$ , the current is at its positive peak, while the voltage will be at its positive peak for  $\omega t = \pi/6 = 30^\circ$ . Peak value are 170 volts and 8.5 amp.

$$(iii) \text{ RMS value of voltage} = 170/\sqrt{2} = 120 \text{ volts}$$

$$\text{RMS value of current} = 8.5/\sqrt{2} = 6 \text{ amp.}$$

$$\text{Impedance} = V/I = 120/6 = 20 \text{ ohms}$$

$$\text{Power factor} = \cos 30^\circ, \text{ Leading} = 0.866,$$

Since the current leads the voltage, the two elements must be  $R$  and  $C$ .

$$R = Z \cos \phi = 20 \times 0.866 = 17.32 \text{ ohms}$$

$$X_c = Z \sin \phi = 20 \times 0.50 = 10 \text{ ohms}$$

$$C = 1/(\omega X_c) = \frac{1000 \times 1000}{6280 \times 10} = 15.92 \text{ mF}$$

**Example 11.36.** Three sinusoidally alternating currents of rms values 5, 7.5, and 10 A are having same frequency of 50 Hz, with phase angles of  $30^\circ$ ,  $-60^\circ$  and  $45^\circ$ .

(i) Find their average values, (ii) Write equations for their instantaneous values, (iii) Draw waveforms and phasor diagrams taking first current as the reference, (iv) Find their instantaneous values at 100 mSec from the original reference. [Nagpur University, Nov. 1996]

**Solution.** (i) Average value of alternating quantity in case of sinusoidal nature of variation = (RMS Value)/1.11

$$\text{Average value of first current} = 5/1.11 = 4.50 \text{ A}$$

$$\text{Average value of second current} = 7.5/1.11 = 6.76 \text{ A}$$

$$\text{Average value of third current} = 10/1.11 = 9.00 \text{ A}$$

(ii) Instantaneous Values :  $\omega = 2\pi \times 50 = 314 \text{ rad/sec}$

$$i_1(t) = 5\sqrt{2} \sin(314t - 30^\circ)$$

$$i_2(t) = 7.5\sqrt{2} \sin(314t - 60^\circ)$$

$$i_3(t) = 10\sqrt{2} \sin(314t - 45^\circ)$$

(iii) First current is to be taken as a reference, now. From the expressions, second current lags behind the first current by  $90^\circ$ . Third current leads the first current by  $15^\circ$ . Waveforms with this description are drawn in Fig. 11.54 (a) and the phasor diagrams, in Fig. 11.54 (b).

(iv) A 50 Hz a.c. quantity completes a cycle in 20 m sec. In 100 m sec, it completes five cycles. Original reference is the starting point required for this purpose. Hence, at 100 m sec from the reference.

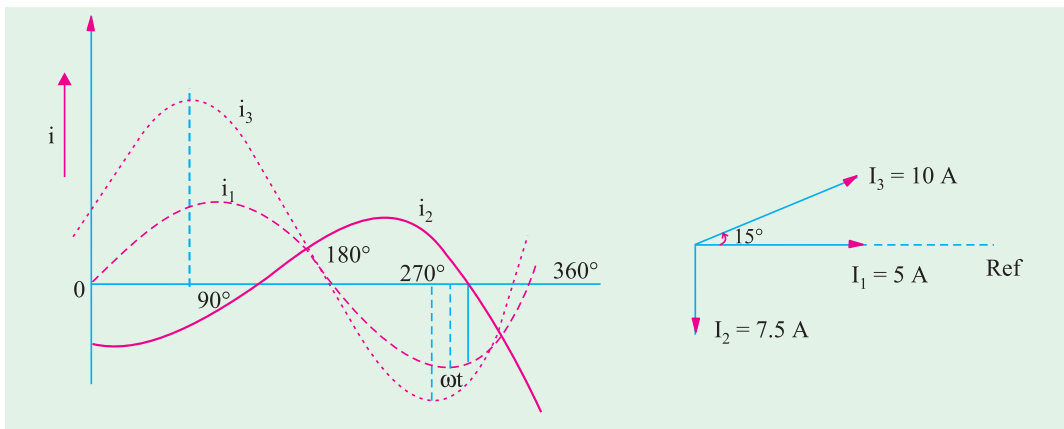


Fig. 11.54 (a)

Fig. 11.54 (b)

(v) instantaneous value of  $i_1(t) = 5\sqrt{2} \sin 30^\circ = 3.53 \text{ A}$

instantaneous value of  $i_2(t) = 7.5\sqrt{2} \sin(\omega t - 60^\circ) = 9.816 \text{ A}$

instantaneous value of  $i_3(t) = 10\sqrt{2} \sin(45^\circ) = 10 \text{ A}$

**Example 11.37.** Determine the form factor and peak factor for the unshaded waveform, in Fig. 11.55. **[Bombay University, 2000]**

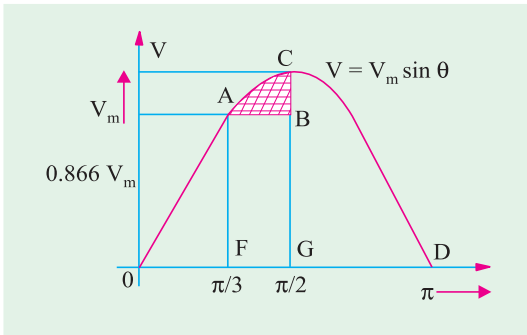


Fig. 11.55

**Solution.**  $v(\theta) = V_m \sin \theta$ , except for the region between

$\theta = 60^\circ$  to  $\theta = 90^\circ$ , wherein  $v = 0.866 V_m$ .

Area under the curve will be worked out first, for calculating the average value.

$$\text{Area OAF} = V_m \int_0^{\pi/3} \sin \theta d\theta = 0.5 V_m$$

$$\text{Area FABG} = 0.866 V_m (\pi/2 - \pi/3) = 0.4532 V_m$$

$$\text{Area GCD} = V_m \int_{\pi/2}^{\pi} \sin \theta d\theta = 0.5 V_m$$

Total area under the curve =  $V_m (1 + 0.432 + 0.5)$

$$\begin{aligned} \text{Average value, } V_{av} &= (1.9532 V_m) / 3.14 \\ &= 0.622 V_m \end{aligned}$$

For evaluating rms value, the square of the function is to be taken, its mean value calculated and square-root of the mean value found out.

Area under the squared function :

$$\text{For Portion OF : } V_m^2 \int_0^{\pi/2} \sin^2 \theta d\theta = V_m^2 / 2 \int_0^{\pi/2} (1 - \cos 2\theta) d\theta = 0.307 V_m^2$$

$$\text{For Portion FG : } (0.866 V_m)^2 \times \pi \times (1/2 - 1/3) = 0.3925 V_m^2$$

$$\text{For Portion GD : } V_m^2 \int_0^{\pi/2} \sin^2 \theta d\theta = 0.785 V_m^2$$

$$\text{Total area} = V_m^2 [0.307 + 0.3925 + 0.785] = 1.4845 V_m^2$$

Let R.M.S. Value be  $V_c$

$$V_m^2 \pi = 1.4845 V_m^2, \text{ or } V_c = 0.688 V_m$$

$$\text{Form factor} = \text{RMS Value} / \text{Average Value} = 0.688 / 0.622 = 1.106$$

$$\text{Peak factor} = \text{Peak Value} / \text{RMS Value} = 1.0 / 0.688 = 1.4535$$

### Tutorial Problems No. 11.2

- The values of the instantaneous currents in the branches of a parallel circuit are as follows :  
 $i_1 = 5 \sin 346 t$ ;  $i_2 = 10 \sin (346 t + \pi/4)$ ;  $i_3 = 7.5 \sin (346 t + \pi/2)$ ;  $i_4 = 8 \sin (346 t - \pi/3)$   
 Express the resultant line current in the same form as the original expression and determine the r.m.s. value and the frequency of this current. **[12.5 A; 55 Hz]**
- Four coils are connected in series. Each has induced in it a sinusoidal e.m.f. of 100 V, 50 Hz and there is a phase difference of 14 electrical degrees between one coil and the next. What is the total e.m.f. generated in the circuit? **[384 V]**
- The instantaneous voltage across each of the four coils connected in series is given by  
 $v_1 = 100 \sin 471 t$ ;  $v_2 = 250 \cos 471 t$ ;  $v_3 = 150 \sin (471 t + \pi/6)$ ;  $v_4 = 200 \sin (471 t - \pi/4)$   
 Determine the total p.d. expressed in similar form to those given. What will be the resultant p.d. if  $v_2$  is reversed in sign? **[v = 414 sin (471 t + 26.5°); v = 486 sin (471 t - 40°)]**
- An alternating voltage of  $v = 100 \sin 376.8 t$  is applied to a circuit consisting of a coil having a resistance of  $6\Omega$  and an inductance of 21.22 mH.

- (a) Express the current flowing in the circuit in the form  $i = I_m \sin (376.8 t \pm \phi)$   
 (b) If a moving-iron voltmeter, a wattmeter and a frequency meter are connected in the circuit, what would be the respective readings on the instruments ?  
**[ $i = 10 \sin (376.8 t - 53.1^\circ)$ ; 70.7 V; 300 W; 60 Hz]**
5. Three circuits  $A$ ,  $B$  and  $C$  are connected in series across a 200-V supply. The voltage across circuit  $A$  is 50 V lagging the supply voltage by  $45^\circ$  and the voltage across circuit  $C$  is 100 V leading the supply voltage by  $30^\circ$ . Determine graphically or by calculation, the voltage across circuit  $B$  and its phase displacement from the supply voltage.  
**[79.4 V;  $10^\circ 38'$  lagging]**
6. Three alternating currents are given by  
 $i_1 = 141 \sin (\omega t + \pi/4)$   $i_2 = 30 \sin (\omega t + \pi/2)$   $i_3 = 20 \sin (\omega t - \pi/6)$   
 and are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its r.m.s. value.  
**[ $i = 167.4 \sin (\omega t + 0.797)$ ,  $I_{rms} = 118.4$  A]**
7. Four e.m.f.s  $e_1 = 100 \sin \omega t$ ,  $e_2 = 80 \sin (\omega t - \pi/6)$ ,  $e_3 = 120 \sin (\omega t + \pi/4)$  and  $e_4 = 100 \sin (\omega t - 2\pi/3)$  are induced in four coils connected in series so that the vector sum of four e.m.f.s is obtained. Find graphically or by calculation the resultant e.m.f. and its phase difference with (a)  $e_1$  and (b)  $e_2$ . If the connections to the coil in which the e.m.f.  $e_2$  is induced are reversed, find the new resultant e.m.f.  
**[208 sin ( $\omega t - 0.202$ ) (a)  $11^\circ 34'$  lag (b)  $18^\circ 26'$  lead;  $76 \sin (\omega t + 0.528)$ ]**
8. Draw to scale a vector diagram showing the following voltages :  
 $v_1 = 100 \sin 500 t$   $v_2 = 200 \sin (500 t + \pi/3)$   $v_3 = -50 \cos 500 t$   $v_4 = 150 \sin (500 t - \pi/4)$   
 Obtain graphically or otherwise, their vector sum and express this in the form  $V_m \sin (500 t \pm \phi)$ , using  $v_1$  as the reference vector. Give the r.m.s. value and frequency of the resultant voltage.  
**[360.5 sin ( $500 t + 0.056$ ); 217 V; 79.6 Hz]**

### 11.28. A.C. Through Resistance, Inductance and Capacitance

We will now consider the phase angle introduced between an alternating voltage and current when the circuit contains resistance only, inductance only and capacitance only. **In each case, we will assume that we are given the alternating voltage of equation  $e = E_m \sin \omega t$**  and will proceed to find the equation and the phase of the alternating current produced in each case.

### 11.29. A.C. Through Pure Ohmic Resistance Alone

The circuit is shown in Fig. 11.56. Let the applied voltage be given by the equation.

$$v = V_m \sin \theta = V_m \sin \omega t \quad \dots (i)$$

Let  $R$  = ohmic resistance ;  $i$  = instantaneous current

Obviously, the applied voltage has to supply ohmic voltage drop only. Hence for equilibrium

$$v = iR,$$

Putting the value of 'v' from above, we get  $V_m \sin \omega t = iR$ ,  $i = \frac{V_m}{R} \sin \omega t$  ... (ii)

Current 'i' is maximum when  $\sin \omega t$  is unity  $\therefore I_m = V_m/R$  Hence, equation (ii) becomes,  
 $i = I_m \sin \omega t$  ... (iii)

Comparing (i) and (iii), we find that the alternating voltage and current are in phase with each other as shown in Fig. 11.57. It is also shown vectorially by vectors  $V_R$  and  $I$  in Fig. 11.54.

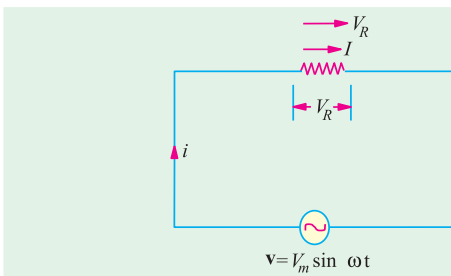


Fig. 11.56

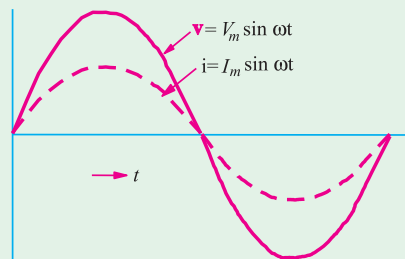


Fig. 11.57

**Power.** Instantaneous power,  $p = vi = V_m I_m \sin^2 \omega t$  ... (Fig. 11.58)

$$= \frac{V_m I_m}{2} (1 - \cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Power consists of a constant part  $\frac{V_m I_m}{2}$  and a fluctuating part  $\frac{V_m I_m}{2} \cos 2\omega t$  of frequency double that of voltage and current waves. For a complete cycle, the average value of  $\frac{V_m I_m}{2} \cos 2\omega t$  is zero.

Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or  $P = V \times I$  watt

where  $V =$  r.m.s. value of applied voltage.  
 $I =$  r.m.s. value of the current.

It is seen from Fig. 11.58 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

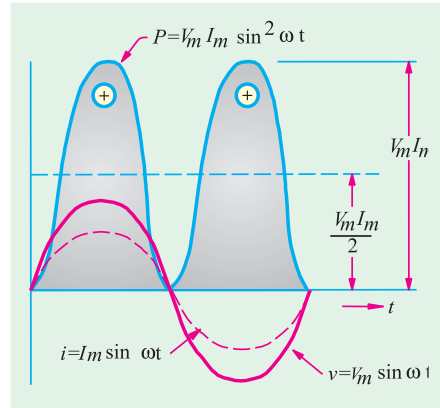


Fig. 11.58

**Example 11.38.** A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 ohm resistance :

(i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at  $t = 0$  (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram.

[Elect Technology, Hyderabad Univ. 1992, Similar Example, U.P. Technical Univ. 2001]

**Solution.** (i)  $V_{\max} = \sqrt{2} V = \sqrt{2} \times 115 = 163 \text{ V}$

$$I_{\max} = V_{\max} / R = 163 / 100 = 1.63 \text{ A}; \phi = 0; \omega = 2\pi f = 2\pi \times 60 = 377 \text{ rad/s}$$

The required equations are :  $v(t) = 1.63 \sin 377 t$  and  $i(t) = 1.63 \sin 377 t$

(ii) and (iii) These are similar to those shown in Fig. 11.56 and 11.57

### 11.30. A.C. Through Pure Inductance Alone

Whenever an alternating voltage is applied to a purely inductive coil\*, a back e.m.f. is produced due to the self-inductance of the coil. The back e.m.f., at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So at every step

$$v = L \frac{di}{dt}$$

Now  $v = V_m \sin \omega t$

$$\therefore V_m \sin \omega t = L \frac{di}{dt} \therefore di = \frac{V_m}{L} \sin \omega t dt$$

Integrating both sides, we get  $i = \frac{V_m}{L} \int \sin \omega t dt$

$$\frac{V_m}{L} (\cos t)$$

...(constant of integration = 0)

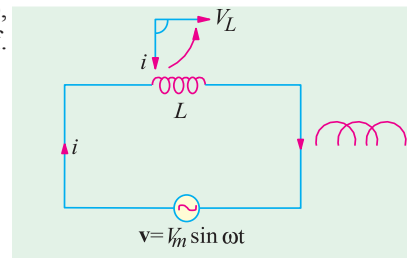


Fig. 11.59

\* By purely inductive coil is meant one that has no ohmic resistance and hence no  $I^2 R$  loss. Pure inductance is actually not attainable, though it is very nearly approached by a coil wound with such thick wire that its resistance is negligible. If it has some actual resistance, then it is represented by a separate equivalent resistance joined in series with it.

$$\therefore = \frac{V_m}{\omega L} \sin \left( \omega t - \frac{\pi}{2} \right) = \frac{V_m}{X_L} \sin \left( \omega t - \frac{\pi}{2} \right) \quad \dots (ii)$$

Max. value of  $i$  is  $I_m = \frac{V_m}{\omega L}$  when  $\sin \left( \omega t - \frac{\pi}{2} \right)$  is unity.

Hence, the equation of the current becomes  $i = I_m \sin (\omega t - \pi/2)$ .

So, we find that if applied voltage is represented by  $v = V_m \sin \omega t$ , then current flowing in a *purely* inductive circuit is given by  $i = I_m \sin \left( \omega t - \frac{\pi}{2} \right)$

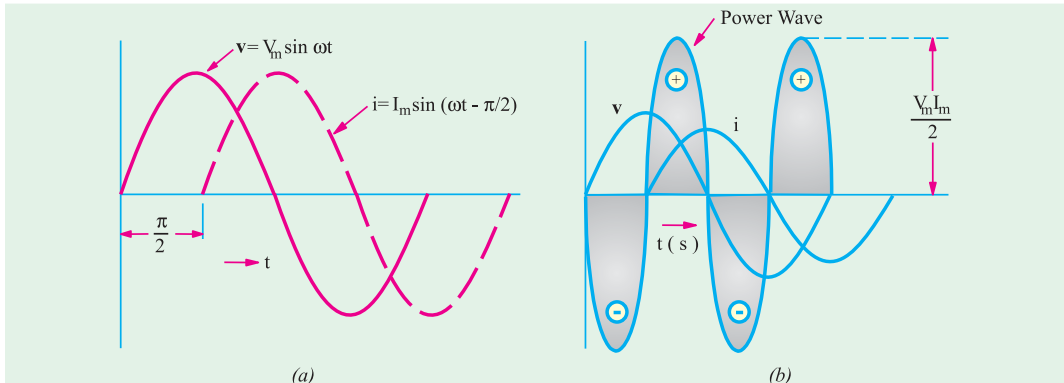


Fig. 11.60

Clearly, the current lags behind the applied voltage by a quarter cycle (Fig. 11.60) or the phase difference between the two is  $\pi/2$  with voltage leading. Vectors are shown in Fig. 11.59 where voltage has been taken along the reference axis. We have seen that  $I_m = V_m/\omega L = V_m/X_L$ . Here ' $\omega L$ ' plays the part of 'resistance'. It is called the (inductive) **reactance**  $X_L$  of the coil and is given in ohms if  $L$  is in henry and  $\omega$  is in radian/second.

Now,  $X_L = \omega L = 2\pi f L$  ohm. It is seen that  $X_L$  depends directly on frequency of the voltage. Higher the value of  $f$ , greater the reactance offered and **vice-versa**.

### Power

$$\text{Instantaneous power} = v_i = V_m I_m \sin \omega t \sin \left( \omega t - \frac{\pi}{2} \right) = V_m I_m \sin \omega t \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$$

$$\text{Power for whole cycle is } P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

It is also clear from Fig. 11.60 (b) that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is  $V_m I_m/2$ .

**Example 11.39.** Through a coil of inductance 1 henry, a current of the wave-form shown in Fig. 11.61 (a) is flowing. Sketch the wave form of the voltage across the inductance and calculate the r.m.s. value of the voltage. (Elect. Technology, Indore Univ.)

**Solution.** The instantaneous current  $i(t)$  is given by

(i)  $0 < t < 1$  second, here slope of the curve is  $1/1 = 1$ .

$$\therefore i = 1 \times t = t \text{ ampere}$$

(ii)  $1 < t < 3$  second, here slope is  $\frac{1 - (-1)}{2} = 1$

\* Or  $p = \frac{1}{2} E_m I_m [\cos 90^\circ - \cos (2\alpha - 90^\circ)]$ . The constant component  $= \frac{1}{2} E_m I_m \cos 90^\circ = 0$ . The pulsating component is  $-\frac{1}{2} E_m I_m \cos (2\alpha - 90^\circ)$  whose average value over one complete cycle is zero.

$\therefore i = 1 - (1)(t-1) = 1 - (t-1) = (2-t)$  ampere

(iii)  $3 < t < 4$  second, here slope is  $\frac{1-0}{1} = -1$

(b)  $i = -1 - (-1)(t-3) = (t-4)$  ampere

The corresponding voltage are (i)  $v_1 = Ldi/dt = 1 \times 1 = 1$  V (ii)  $v_2 = Ldi/dt = 1 \times \frac{d}{dt}(2-t) = -1$  V

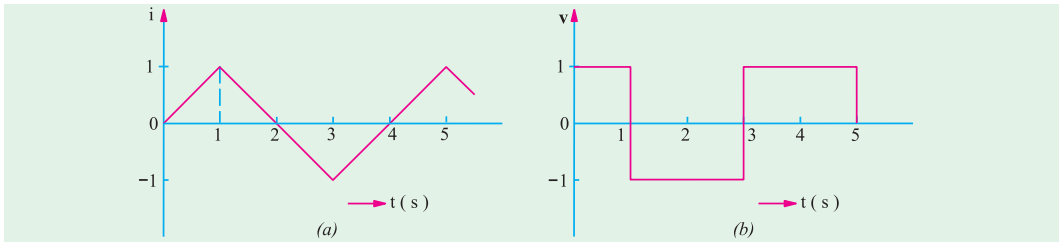


Fig. 11.61

(iii)  $v_3 = Ldi/dt = 1 \times \frac{d}{dt}(t-4) = 1$  V

The voltage waveform is sketched in Fig. 11.61. Obviously, the r.m.s. value of the symmetrical square voltage waveform is **1 V**.

**Example 11.40.** A 60-Hz voltage of 230-V effective value is impressed on an inductance of 0.265 H.

(i) Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at  $t = 0$ . (ii) Show the voltage and current on a phasor diagram. (iii) Find the maximum energy stored in the inductance. **(Elect. Engineering, Bhagalpur Univ.)**

**Solution.**  $V_{\max} = \sqrt{2} V = \sqrt{2} \times 230$  V,  $f = 60$  Hz

$\omega = 2\pi f = 2\pi \times 60 = 377$  rad/s,  $X_L = \omega L = 377 \times 0.265 = 100 \Omega$

(i) The time equation for voltage is  $v(t) = 230\sqrt{2} \sin 377 t$ ,

$I_{\max} = V_{\max}/X_L = 230\sqrt{2}/100 = 2.3\sqrt{2}$ , 90 (lag)

$\therefore$  Current equation is  $i(t) = 2.3\sqrt{2} \sin(377 t - \pi/2)$  or  $= 2.3\sqrt{2} \cos 377 t$ .

(ii) It is shown in Fig. 11.56. (iii)  $E_{\max} = \frac{1}{2} LI_{\max}^2 = \frac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2 = 1.4$  J

### 11.31. Complex Voltage Applied to Pure Inductance

In Art. 11.30, the applied voltage was a pure sine wave (i.e. without harmonics) given by

$$v = V_m \sin \alpha t.$$

The current was given by  $i = I_m \sin(\alpha t - \pi/2)$

Now, if the applied voltage has a complex form and is given by \*

$$v = V_{1m} \sin \alpha t + V_{3m} \sin 3\alpha t + V_{5m} \sin 5\alpha t$$

then the reactances offered to the fundamental voltage wave and the harmonics would be different.

For the fundamental wave,  $X_1 = \omega L$ . For 3rd harmonic;  $X_3 = 3\omega L$ . For 5th harmonic;  $X_5 = 5\omega L$ .

Hence, the current would be given by the equation.

$$i = \frac{V_{1m}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{3m}}{3\omega L} \sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{V_{5m}}{5\omega L} \sin\left(5\omega t - \frac{\pi}{2}\right)$$

\* It is assumed that the harmonics have no individual phase differences.

Obviously, the harmonics in the current wave are much smaller than in the voltage wave. For example, the 5th harmonic of the current wave is of only 1/5th of the harmonic in the voltage wave. It means that the self-inductance of a coil has the effect of 'smoothing' current waveform when the voltage waveform is complex *i.e.* contains harmonics.

**Example 11.41.** The voltage applied to a purely inductive coil of self-inductance 15.9 mH is given by the equation,  $v = 100 \sin 314 t + 75 \sin 942 t + 50 \sin 1570 t$ . Find the equation of the resulting current wave.

**Solution.** Here  $\omega = 314 \text{ rad/s}$   $\therefore X_1 = \omega L = (15.9 \times 10^{-3}) \times 314 = 5 \Omega$   
 $X_3 = 3\omega L = 3 \times 5 = 15 \Omega$   $X_5 = 5\omega L = 5 \times 5 = 25 \Omega$

Hence, the current equation is

$$i = (100/5) \sin (314 t - \pi/2) + (75/15) \sin (942 t - \pi/2) + (50/25) \sin (1570 t - \pi/2)$$

$$\text{or } i = 20 \sin (314 t - \pi/2) + 5 \sin (942 t - \pi/2) + 2 \sin (1570 t - \pi/2)$$

### 11.32. A.C. Through Pure Capacitance Alone

When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction. When reference to Fig. 11.62, let

$v$  = p.d. developed between plates at any instant

$q$  = Charge on plates at that instant.

Then

$q = Cv$  ...where  $C$  is the capacitance

$= C V_m \sin \alpha t$  ...putting the value of  $v$ .

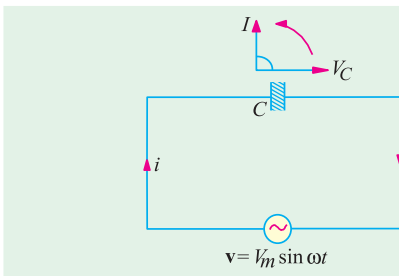


Fig. 11.62

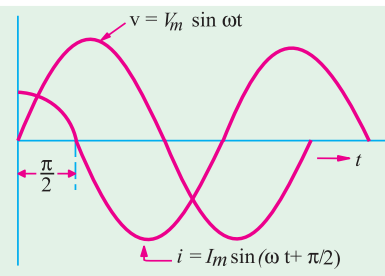


Fig. 11.63

Now, current  $i$  is given by the rate of flow of charge.

$$\therefore i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \alpha t) = CV_m \cos \alpha t \text{ or } i = \frac{V_m}{1/C} \cos \alpha t = \frac{V_m}{1/C} \sin \left( \alpha t + \frac{\pi}{2} \right)$$

Obviously,  $I_m = \frac{V_m}{1/C} = \frac{V_m}{X_C}$   $i = I_m \sin \left( \alpha t + \frac{\pi}{2} \right)$

The denominator  $X_C = 1/\omega C$  is known as capacitive reactance and is in ohms if  $C$  is in farad and  $\omega$  in radian/second. It is seen that if the applied voltage is given by  $v = V_m \sin \alpha t$ , then the current is given by  $i = I_m \sin (\alpha t + \pi/2)$ .

Hence, we find that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 11.63 or phase difference between its voltage and current is  $\pi/2$  with the current leading. Vector representation is given in Fig. 11.63. Note that  $V_c$  is taken along the reference axis.

**Power.** Instantaneous power

$$p = vi = V_m \sin \alpha t \cdot I_m \sin (\alpha t + 90^\circ)$$

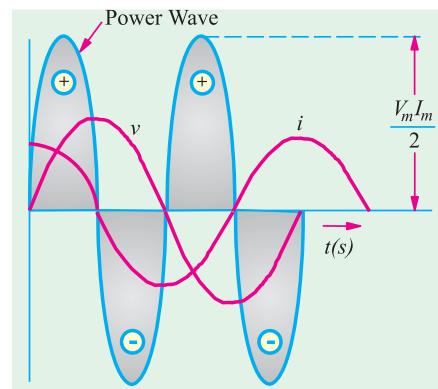


Fig. 11.64



$$= V_m I_m \sin t \cos t^* = \frac{1}{2} V_m I_m \sin 2t$$

Power for the whole cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

This fact is graphically illustrated in Fig. 11.64. We find that in a purely capacitive circuit \*\*, the average demand of power from supply is zero (as in a purely inductive circuit). Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is  $V_m I_m / 2$ .

**Example 11.42.** A 50-Hz voltage of 230 volts effective value is impressed on a capacitance of 26.5  $\mu\text{F}$ . (a) Write the time equations for the voltage and the resulting current. Let the zero axis of the voltage wave be at  $t = 0$ . (b) Show the voltage and current on a time diagram.

(c) Show the voltage and current on a phasor diagram. (d) Find the maximum energy stored in the capacitance. Find the relative heating effects of two current waves of equal peak value, the one sinusoidal and the other rectangular in waveform. (Elect. Technology, Allahabad Univ. 1991)

**Solution.**

$$V_{\max} = 230 \sqrt{2} = 325 \text{ V}$$

$$\omega = 2\pi \times 50 = 314 \text{ rad/s}; X_C = 1/\omega C = 10^6/314 \times 26.5 = 120 \, \Omega$$

$$I_{\max} = V_{\max}/X_C = 325/120 = 2.71 \text{ A}, \phi = 90^\circ \text{ (lead)}$$

(a)  $v(t) = 325 \sin 314 t$ ;  $i(t) = 2.71 \sin(314t + \pi/2) = 2.71 \cos 314 t$

(b) and (c) These are shown in Fig. 11.59.

(d)  $E_{\max} = \frac{1}{2} C V_{\max}^2 = \frac{1}{2} (26.5 \times 10^{-6}) \times 325^2 = 1.4 \text{ J}$

(e) Let  $I_m$  be the peak value of both waves.

For sinusoidal wave :  $H \propto I^2 R \propto (I_m/\sqrt{2})^2 R \propto I_m^2 R/2$ . For rectangular wave :  $H \propto I_m^2 R$  - Art. 12.15.

$$\frac{H \text{ rectangular}}{H \text{ sinusoidal}} = \frac{I_m^2 R}{I_m^2 R/2} = 2$$

**Example 11.43.** A 50- $\mu\text{F}$  capacitor is connected across a 230-V, 50-Hz supply. Calculate (a) the reactance offered by the capacitor (b) the maximum current and (c) the r.m.s. value of the current drawn by the capacitor.

**Solution.** (a)  $X_C = \frac{1}{C} = \frac{1}{2 \times 50 \times 10^{-6}} = 63.6 \, \Omega$

(c) Since 230 V represents the r.m.s. value,

$$\therefore I_{r.m.s.} = 230/X_C = 230/63.6 = 3.62 \text{ A} \quad (b) \quad I_m = I_{r.m.s.} \times \sqrt{2} = 3.62 \times \sqrt{2} = 5.11 \text{ A}$$

\* Or power  $p = \frac{1}{2} E_m I_m [\cos 90^\circ - \cos(2\omega t - 90^\circ)]$ . The constant component is again zero. The pulsating component averaged over one complete cycle is zero.

\*\* By pure capacitor is meant one that has neither resistance nor dielectric loss. If there is loss in a capacitor, then it may be represented by loss in (a) high resistance joined in parallel with the pure capacitor or (b) by a comparatively low resistance joined in series with the pure capacitor. But out of the two alternatives usually, (a) is chosen (Art. 13.8).

**Example 11.44.** The voltage applied across 3-branched circuit of Fig. 11.65 is given by  $v = 100 \sin(5000t + \pi/4)$ . Calculate the branch currents and total current.

**Solution.** The total instantaneous current is the vector sum of the three branch currents.

$$i_r = i_R + i_L + i_C$$

$$\begin{aligned} \text{Now } i_R &= v/R = 100 \sin(5000t + \pi/4)/25 \\ &= 4 \sin(5000t + \pi/4) \end{aligned}$$

$$\begin{aligned} i_L &= \frac{1}{L} \int v dt = \frac{10^3}{2} \int 100 \sin\left(5000t + \frac{\pi}{4}\right) dt \\ &= \frac{10^3 \times 100}{2} \left[ \frac{-\cos(5000t + \pi/4)}{5000} \right] = -10 \cos(5000t + \pi/4) \end{aligned}$$

$$\begin{aligned} i_C &= C \frac{dv}{dt} = C \frac{d}{dt} [100 \sin(5000t + \pi/4)] \\ &= 30 \times 10^{-6} \times 100 \times 5000 \times \cos(5000t + \pi/4) = 15 \cos(5000t + \pi/4) \\ i_r &= 4 \sin(5000t + \pi/4) - 10 \cos(5000t + \pi/4) + 15 \cos(5000t + \pi/4) \\ &= \mathbf{4 \sin(5000t + \pi/4) + 5 \cos(5000t + \pi/4)} \end{aligned}$$

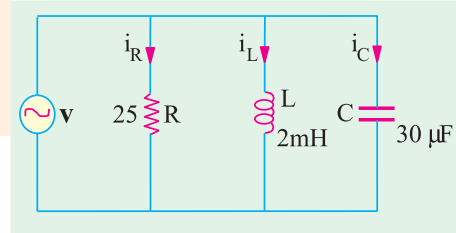


Fig. 11.65

### Tutorial Problems No. 11.2

- Define r.m.s. and average value as applied to ac voltage, prove that in pure inductive circuit, current lags behind applied voltage at an angle  $90^\circ$ .  
(Down waveform) (Nagpur University, Summer 2002)
- Find the rms value, average value, form factor and peak factor for the waveform shown in figure.  
(Nagpur University, Winter 2003)

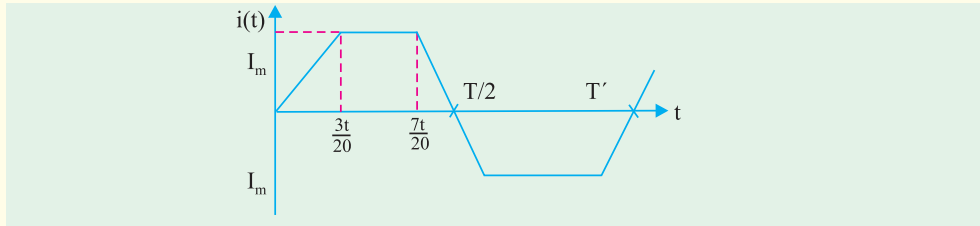


Fig. 11.66

- Derive an expression for the instantaneous value of alternating sinusoidal e.m.f. in terms of its maximum value, angular freq. and time. (Gujrat University, June/July 2003)
- Prove that average power consumption in pure inductor is zero when a.c. voltage is applied. (Gujrat University, June/July 2003)
- Define and explain the following :  
(i) time period (ii) amplitude (iii) phase difference. (Gujrat University, June/July 2003)
- What is the r.m.s value of an a.c. quantity? Obtain expression for the r.m.s. value of a sinusoidal current in terms of its maximum value. (V.T.U., Belgaum Karnataka University, February 2002)
- Deduce an expression for the average power in a single phase series R.L. circuit and therefrom explain the term power factor. (V.T.U., Belgaum Karnataka University, February 2002)

8. Derive an expression for the RMS value of a sine wave.  
(V.T.U., Belgaum Karnataka University, Summer 2002)
9. With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field.  
(V.T.U., Belgaum Karnataka University, Summer 2003)
10. Derive expressions for average value and RMS value of a sinusoidally varying AC voltage.  
(V.T.U., Belgaum Karnataka University, Summer 2003)
11. A circuit having a resistance of  $12\Omega$ , an inductance of  $0.15\text{ H}$  and a capacitance of  $100\mu\text{f}$  in series is connected across a  $100\text{V}$ ,  $50\text{Hz}$  supply. Calculate the impedance, current, the phase difference between the current and supply voltage.  
(V.T.U., Belgaum Karnataka University, Summer 2003)
12. Two circuits with impedances of  $Z_1 = 10 + j15\Omega$  and  $Z_2 = 6 - j8\Omega$  are connected in parallel. If the supply current is  $20\text{A}$ , what is the power dissipated in each branch?  
(V.T.U., Belgaum Karnataka University, Summer 2003)
13. Show that the power consumed in a pure inductance is zero.  
(U.P. Technical University 2002) (RGPV Bhopal 2002)
14. What do you understand by the terms power factor, active power and reactive power?  
(U.P. Technical University 2002) (RGPV Bhopal 2002)
15. Current flowing through each line.  
(RGPV Bhopal December 2002)
16. Distinguish between (i) apparent power (ii) active power and (iii) reactive power in A.C. circuits.  
(U.P. Technical University 2002) (RGPV Bhopal June 2003)

### OBJECTIVE TESTS – 11

1. An a.c. current given by  $i = 14.14 \sin(\omega t + \pi/6)$  has an r.m.s value of — amperes.  
(a) 10  
(b) 14.14  
(c) 1.96  
(d) 7.07  
and a phase of — degrees.  
(e) 180  
(f) 30  
(g)  $-30$   
(h) 210
2. If  $e_1 = A \sin \omega t$  and  $e_2 = B \sin(\omega t - \phi)$ , then  
(a)  $e_1$  lags  $e_2$  by  $\theta$   
(b)  $e_2$  lags  $e_1$  by  $\theta$   
(c)  $e_2$  leads  $e_1$  by  $\theta$   
(d)  $e_1$  is in phase with  $e_2$
3. From the two voltage equations  $e_A = E_m \sin 100\pi t$  and  $e_B = E_m \sin(100\pi t + \pi/6)$ , it is obvious that  
(a)  $A$  leads  $B$  by  $30^\circ$   
(b)  $B$  achieves its maximum value  $1/600$  second before  $A$  does.  
(c)  $B$  lags behind  $A$   
(d)  $A$  achieves its zero value  $1/600$  second before  $B$ .
4. The r.m.s. value of a half-wave rectified current is  $10\text{A}$ , its value for full-wave rectification would be — amperes.  
(a) 20  
(b) 14.14  
(c)  $20/\pi$   
(d)  $40/\pi$
5. A resultant current is made of two components : a  $10\text{ A}$  d.c. component and a sinusoidal component of maximum value  $14.14\text{ A}$ . The average value of the resultant current is — amperes.  
(a) 0  
(b) 24.14  
(c) 10  
(d) 4.14  
and r.m.s. value is — amperes.  
(e) 10  
(f) 14.14  
(g) 24.14  
(h) 100
6. The r.m.s. value of sinusoidal a.c. current is equal to its value at an angle of — degree  
(a) 60 (b) 45 (c) 30 (d) 90
7. Two sinusoidal currents are given by the equations :  $i_1 = 10 \sin(\omega t + \pi/3)$  and  $i_2 = 15 \sin(\omega t - \pi/4)$ . The phase difference between them is — degrees.  
(a) 105  
(b) 75  
(c) 15  
(d) 60

8. As sine wave has a frequency of 50 Hz. Its angular frequency is — radian/second.  
 (a)  $50/\pi$   
 (b)  $50/2 \pi$   
 (c)  $50 \pi$   
 (d)  $100 \pi$
9. An a.c. current is given by  $i = 100 \sin 100$ . It will achieve a value of 50 A after — second.  
 (a)  $1/600$   
 (b)  $1/300$   
 (c)  $1/1800$   
 (d)  $1/900$
10. The reactance offered by a capacitor to alternating current of frequency 50 Hz is  $10\Omega$ . If frequency is increased to 100 Hz reactance becomes—ohm.  
 (a) 20 (b) 5  
 (c) 2.5 (d) 40
11. A complex current wave is given by  $i = 5 + 5 \sin 100 \pi t$  ampere. Its average

value is — ampere.

- (a) 10 (b) 0  
 (c)  $\sqrt{50}$  (d) 5

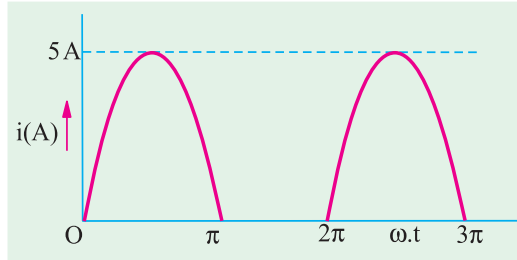


Fig. 11.67

12. The current through a resistor has a waveform as shown in Fig. 11.67. The reading shown by a moving coil ammeter will be—ampere.  
 (a)  $5/\sqrt{2}$  (b)  $2.5/\sqrt{2}$   
 (c)  $5/\pi$  (d) 5

(Principles of Elect. Engg. Delhi Univ.)

### ANSWERS

1. a, f 2. b 3. b 4. b 5. c, f 6. b 7. a 8. d 9. a 10. b  
 11. d 12. c

Top

## CHAPTER 12

**Learning Objectives**

- Mathematical Representation of Vectors
- Symbolic Notation
- Significance of Operator  $j$
- Conjugate Complex Numbers
- Trigonometrical Form of Vector
- Exponential Form of Vector
- Polar Form of Vector Representation
- Addition and Subtraction of Vector Quantities
- Multiplication and Division of Vector Quantities
- Power and Root of Vectors
- The  $120^\circ$  Operator

**COMPLEX NUMBERS**

In multi-phase generation of electricity, it is often convenient to express each phase as a complex number,  $Z$ , which has the form  $a + jb$ , where  $a$  and  $b$  are real numbers. This representation is known as Cartesian form of  $Z$ .

## 12.1. Mathematical Representation of Vectors

There are various forms or methods of representing vector quantities, all of which enable those operations which are carried out graphically in a phasor diagram, to be performed analytically. The various methods are :

(i) **Symbolic Notation.** According to this method, a vector quantity is expressed algebraically in terms of its rectangular components. Hence, this form of representation is also known as Rectangular or Cartesian form of notation or representation.

(ii) **Trigonometrical Form (iii) Exponential Form (iv) Polar Form.**

## 12.2. Symbolic Notation

A vector can be specified in terms of its  $X$ -component and  $Y$ -component. For example, the vector  $OE_1$  (Fig. 12.1) may be completely described by stating that its horizontal component is  $a_1$  and vertical component is  $b_1$ . But instead of stating this verbally, we may express symbolically

$$\mathbf{E}_1 = a_1 + jb_1$$

where symbol  $j$ , known as an operator, indicates that component  $b_1$  is perpendicular to component  $a_1$  and that the two terms are **not** to be treated like terms in any algebraic expression. The vector written in this way is said to be written in '**complex form**'. In Mathematics,  $a_1$  is known as real component and  $b_1$  as imaginary component but in electrical engineering, these are known as **in phase** (or active) and **quadrature** (or reactive) components respectively.

The other vectors  $OE_2$ ,  $OE_3$  and  $OE_4$  can similarly, be expressed in this form.

$$\mathbf{E}_2 = -a_2 + jb_2 ; \mathbf{E}_3 = -a_3 - jb_3 ; \mathbf{E}_4 = +a_4 - jb_4$$

It should be noted that in this book, a vector quantity would be represented by letters in heavy type and its numerical or scalar value by the same letter in ordinary type.\* Other method adopted for indicating a vector quantity is to put an arrow about the letter such as  $\vec{E}$ .

The numerical value of vector  $E_1$  is  $\sqrt{a_1^2 + b_1^2}$ . Its angle with  $X$ -axis is given by  $\phi = \tan^{-1} (b_1/a_1)$ .

## 12.3. Significance of Operator $j$

The letter  $j$  used in the above expression is a symbol of an operation. Just as symbols  $\times$ ,  $+$ ,  $\sqrt{\quad}$ ,  $\int$  etc. are used with numbers for indicating certain operations to be performed on those numbers, similarly, symbol  $j$  is used to indicate the counter-clockwise rotation of a vector through  $90^\circ$ . It is assigned a value of  $\sqrt{-1}$ \*\* . The double operation of  $j$  on a vector rotates it counter-clockwise through  $180^\circ$  and hence reverses its sense because

\* The magnitude of a vector is sometimes called 'modulus' and is represented by  $|E|$  or  $E$ .

\*\* In Mathematics,  $\sqrt{-1}$  is denoted by  $i$  but in electrical engineering  $j$  is adopted because letter  $i$  is reserved for representing current. This helps to avoid confusion.

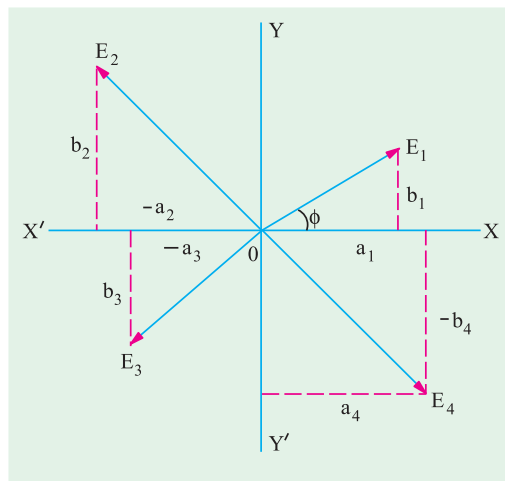


Fig. 12.1



$$jj = j^2 = \sqrt{(-1)^2} = -1$$

When operator  $j$  is operated on vector  $\mathbf{E}$ , we get the new vector  $j\mathbf{E}$  which is displaced by  $90^\circ$  in counter-clockwise direction from  $\mathbf{E}$  (Fig. 12.2). Further application of  $j$  will give  $j^2\mathbf{E} = -\mathbf{E}$  as shown.

If the operator  $j$  is applied to the vector  $j^2\mathbf{E}$ , the result is  $j^3\mathbf{E} = -j\mathbf{E}$ . The vector  $j^3\mathbf{E}$  is  $270^\circ$  counter-clockwise from the reference axis and is directly opposite to  $j\mathbf{E}$ . If the vector  $j^3\mathbf{E}$  is, turn, operated on by  $j$ , the result will be

$$j^4\mathbf{E} = [\sqrt{(-1)}]^4 \mathbf{E} = \mathbf{E}$$

Hence, it is seen that successive applications of the operator  $j$  to the vector  $\mathbf{E}$  produce successive  $90^\circ$  steps of rotation of the vector in the counter-clockwise direction without in anyway affecting the magnitude of the vector.

It will also be seen from Fig. 12.2 that the application of  $-j$  to  $\mathbf{E}$  yields  $-j\mathbf{E}$  which is a vector of identical magnitude but rotated  $90^\circ$  **clockwise** from  $\mathbf{E}$ .

Summarising the above, we have

$$j = 90^\circ \text{ ccw rotation} = \sqrt{(-1)}$$

$$j^2 = 180^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^2 = -1;$$

$$j^3 = 270^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$$

$$j^4 = 360^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^4 = +1;$$

$$j^5 = 450^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^5 = -\sqrt{(-1)} = j$$

It should also be noted that  $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

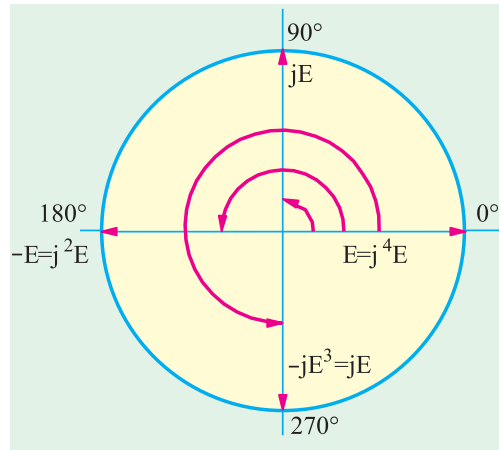


Fig. 12.2

## 12.4. Conjugate Complex Numbers

Two complex numbers are said to be conjugate if they differ only in the algebraic sign of their quadrature components. Accordingly, the numbers  $(a + jb)$  and  $(a - jb)$  are conjugate. The sum of two conjugate numbers gives in-phase (or active) component and their difference gives quadrature (or reactive) component.

## 12.5. Trigonometrical Form of Vector

From Fig. 12.3, it is seen that **X-component of  $\mathbf{E}$**  is  $E \cos \theta$  and **Y-component** is  $E \sin \theta$ . Hence, we can represent the vector  $\mathbf{E}$  in the form :  $\mathbf{E} = E(\cos \theta + j \sin \theta)$

This is equivalent to the rectangular form  $\mathbf{E} = a + jb$  because  $a = E \cos \theta$  and  $b = E \sin \theta$ . In general,  $\mathbf{E} = E(\cos \theta \pm j \sin \theta)$ .

## 12.6. Exponential Form of Vector

It can be proved that  $e^{\pm j\theta} = (\cos \theta \pm j \sin \theta)$

This equation is known as Euler's equation after the famous mathematician of 18th century : Leonard Euler.

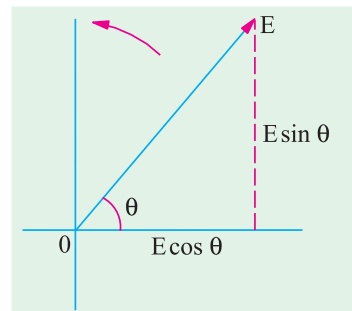


Fig. 12.3

This equation follows directly from an inspection of Maclaurin\* series expansions of  $\sin \theta$ ,  $\cos \theta$  and  $e^{j\theta}$ .

When expanded into series form :

$$\cos \theta = 1 - \frac{\theta^2}{L2} + \frac{\theta^4}{L4} - \frac{\theta^6}{L6} + \dots \quad \text{and} \quad \sin \theta = \theta - \frac{\theta^3}{L3} + \frac{\theta^5}{L5} - \frac{\theta^7}{L7} + \dots$$

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{L2} + \frac{(j\theta)^3}{L3} + \frac{(j\theta)^4}{L4} + \frac{(j\theta)^5}{L5} + \frac{(j\theta)^6}{L6} + \dots$$

Keeping in mind that  $j^2 = -1$ ,  $j^3 = -j$ ,  $j^4 = 1$ ,  $j^5 = j$ ,  $j^6 = -1$ , we get

$$e^{j\theta} = \left( 1 - \frac{\theta^2}{L2} + \frac{\theta^4}{L4} - \frac{\theta^6}{L6} + \dots \right) + j \left( \theta - \frac{\theta^3}{L3} + \frac{\theta^5}{L5} - \frac{\theta^7}{L7} + \dots \right)$$

$$\therefore e^{j\theta} = \cos \theta + j \sin \theta$$

Similarly, it can be shown that  $e^{-j\theta} = \cos \theta - j \sin \theta$

Hence  $\mathbf{E} = E(\cos \theta \pm j \sin \theta)$  can be written as  $\mathbf{E} = Ee^{\pm j\theta}$ . This is known as exponential form of representing vector quantities. It represents a vector of numerical value  $E$  and having phase angle of  $\pm \theta$  with the reference axis.

## 12.7. Polar Form of Vector Representation

The expression  $E(\cos \theta + j \sin \theta)$  is written in the simplified form of  $E \angle \theta$ . In this expression,  $E$  represents the magnitude of the vector and  $\theta$  its inclination (in ccw direction) with the  $X$ -axis. For angles in clockwise direction the expression becomes  $E \angle -\theta$ . In general, the expression is written as  $E \angle \pm \theta$ . **It may be pointed out here that  $E \angle \pm \theta$  is simply a short-hand or symbolic style of writing  $Ee^{\pm j\theta}$ .** Also, the form is purely conventional and does not possess the mathematical elegance of the various other forms of vector representation given above.

Summarizing, we have the following alternate ways of representing vector quantities

- (i) Rectangular form (or complex form)  $\mathbf{E} = a + jb$
- (ii) Trigonometrical form  $\mathbf{E} = E(\cos \theta \pm j \sin \theta)$
- (iii) Exponential form  $\mathbf{E} = Ee^{\pm j\theta}$
- (iv) Polar form (conventional)  $\mathbf{E} = E \angle \pm \theta$ .

**Example 12.1.** Write the equivalent exponential and polar forms of vector  $3 + j4$ . How will you illustrate the vector means of diagram?

**Solution.** With reference to Fig. 12.4., magnitude of the

vector is  $= \sqrt{3^2 + 4^2} = 5$ .  $\tan \theta = 4/3$ .

$$\therefore \theta = \tan^{-1} (4/3) = 53.1^\circ$$

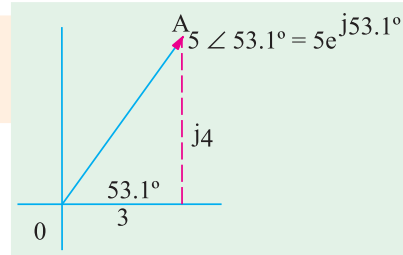


Fig. 12.4

\* Functions like  $\cos \theta$ ,  $\sin \theta$  and  $e^{j\theta}$  etc. can be expanded into series form with the help of Maclaurin's

Theorem. The theorem states :  $f(\theta) = f(0) + \frac{f'(0)\theta}{L1} + \frac{f''(0)\theta^2}{L2} + \frac{f'''(0)\theta^3}{L3} + \dots$  where  $f(\theta)$  is function of  $\theta$  which is to be expanded,  $f(0)$  is the value of the function when  $\theta = 0$ ,  $f'(0)$  is the value of first derivative of  $f(\theta)$  when  $\theta = 0$ ,  $f''(0)$  is the value of second derivative of function  $f(\theta)$  when  $\theta = 0$  etc.



$\therefore$  Exponential form =  $5 e^{j53.1^\circ}$

The angle may also be expressed in radians.

Polar form =  $5 \angle 53.1^\circ$ .

**Example 12.2.** A vector is represented by  $20 e^{j2\pi/3}$ . Write the various equivalent forms of the vector and illustrate by means of a vector diagram, the magnitude and position of the above vector.

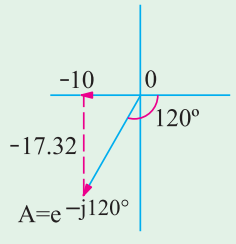


Fig. 12.5

**Solution.** The vector is drawn in a direction making an angle of  $2\pi/3 = 120^\circ$  in the clockwise direction (Fig. 12.5). The clockwise direction is taken because the angle is negative.

(i) **Rectangular Form**  $a = 20 \cos (-120^\circ) = -10$  ;

$b = 20 \sin (-120^\circ) = -17.32$

$\therefore$  Expression is  $(-10 - j17.32)$

(ii) **Polar Form** is  $20 \angle -120^\circ$

## 12.8. Addition and Subtraction of Vector Quantities

**Rectangular form is best suited for addition and subtraction of vector quantities.** Suppose we are given two vector quantities  $\mathbf{E}_1 = a_1 + jb_1$  and  $\mathbf{E}_2 = a_2 + jb_2$  and it is required to find their sum and difference.

**Addition.**  $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = a_1 + jb_1 + a_2 + jb_2 = (a_1 + a_2) + j(b_1 + b_2)$

The magnitude of resultant vector  $\mathbf{E}$  is  $\sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

The position of  $\mathbf{E}$  with respect to  $X$ -axis is  $\theta = \tan^{-1} \left( \frac{b_1 + b_2}{a_1 + a_2} \right)$

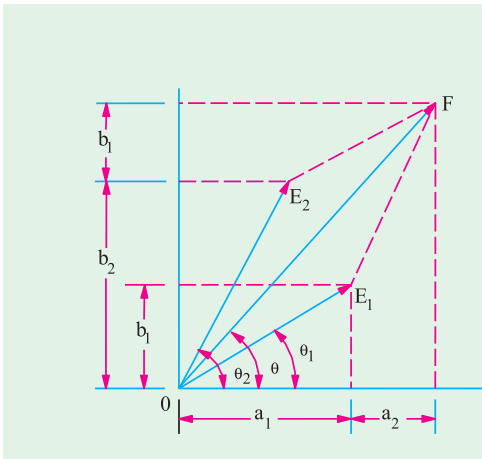


Fig. 12.6

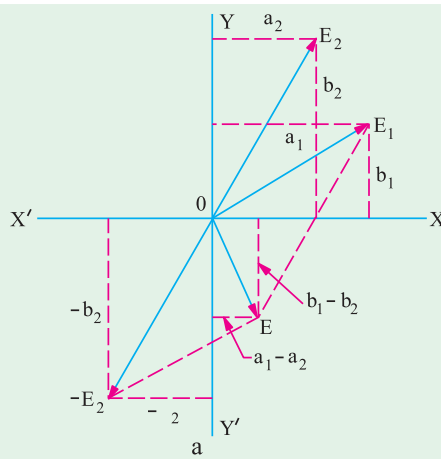


Fig. 12.7

A graphic representation of the addition process is shown in Fig. 12.6

**Subtraction.**  $\mathbf{E} = \mathbf{E}_1 - \mathbf{E}_2 = (a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$

Magnitude of  $\mathbf{E} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Its position with respect to  $x$ -axis is given by the angle  $\theta = \tan^{-1} \left( \frac{b_1 - b_2}{a_1 - a_2} \right)$ .

The graphic representation of the process of subtraction is shown in Fig. 12.7.

### 12.9. Multiplication and Division of Vector Quantities

Multiplication and division of vectors becomes very simple and easy *if they are represented in the polar or exponential form*. As will be shown below, the rectangular form of representation is not well-suited for this process.

#### (i) Multiplication – Rectangular form

Let the two vectors be given by  $\mathbf{A} = a_1 + jb_1$  and  $\mathbf{B} = a_2 + jb_2$

$$\begin{aligned} \therefore \mathbf{A} \times \mathbf{B} = \mathbf{C} &= (a_1 + jb_1)(a_2 + jb_2) = a_1a_2 + j^2b_1b_2 + j(a_1b_2 + b_1a_2) \\ &= (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2) \end{aligned} \quad (\because j^2 = -1)$$

The magnitude of  $\mathbf{C} = \sqrt{[(a_1a_2 - b_1b_2)^2 + (a_1b_2 + b_1a_2)^2]}$

In angle with respect to  $X$ -axis is given by  $\theta = \tan^{-1} \left( \frac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2} \right)$

#### (ii) Division – Rectangular Form : $\frac{\mathbf{A}}{\mathbf{B}} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$

Both the numerator and denominator have been multiplied by the conjugate of  $(a_2 + jb_2)$  i.e. by  $(a_2 - jb_2)$

$$\therefore \frac{\mathbf{A}}{\mathbf{B}} = \frac{(a_1a_2 + b_1b_2) + j(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + j \frac{b_1a_2 - a_1b_2}{a_2^2 + b_2^2}$$

The magnitude and the angle with respects  $X$ -axis can be found in the same way as given above.

As will be noted, both the results are somewhat awkward but unfortunately, there is no easier way to perform multiplication in rectangular form.

#### (iii) Multiplication – Polar Form

Let  $\mathbf{A} = a_1 + jb_1 = A \angle \alpha = A e^{j\alpha}$  where  $\alpha = \tan^{-1}(b_1/a_1)$

$\mathbf{B} = a_2 + jb_2 = B \angle \beta = B e^{j\beta}$  where  $\beta = \tan^{-1}(b_2/a_2)$

$$\therefore \mathbf{AB} = A \angle \alpha \times B \angle \beta = AB \angle (\alpha + \beta) \text{ or } AB = Ae^{j\alpha} \times Be^{j\beta} = AB e^{j(\alpha + \beta)}$$

Hence, product of any two vector  $\mathbf{A}$  and  $\mathbf{B}$  is given by another vector equal in length to  $\mathbf{A} \times \mathbf{B}$  and having a phase angle equal to the sum of the angles of  $\mathbf{A}$  and  $\mathbf{B}$ .

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A \angle \alpha}{B \angle \beta} = \frac{A}{B} \angle (\alpha - \beta)$$

Hence, the quotient  $\mathbf{A} \div \mathbf{B}$  is another vector having a magnitude of  $\frac{A}{B}$  and phase angle equal to angle of  $\mathbf{A}$  minus the angle of  $\mathbf{B}$ .

Also 
$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{Ae^{j\alpha}}{Be^{j\beta}} = \frac{A}{B} e^{j(\alpha - \beta)}$$

As seen, the division and multiplication become extremely simple if vectors are represented in their polar or exponential form.

**Example 12.3.** Add the following vectors given in rectangular form and illustrate the process graphically.

$$A = 16 + j12, \quad B = -6 + j10.4$$

\*  $\mathbf{A} = A(\cos \alpha + j \sin \alpha)$  and  $\mathbf{B} = B(\cos \beta + j \sin \beta)$

$$\begin{aligned} \therefore \mathbf{AB} &= AB(\cos \alpha \cos \beta + j \sin \alpha \cos \beta + j \cos \alpha \sin \beta + j^2 \sin \alpha \sin \beta) \\ &= AB[\cos \alpha \cos \beta - \sin \alpha \sin \beta + j(\sin \alpha \cos \beta + \cos \alpha \sin \beta)] \\ &= AB[\cos(\alpha + \beta) + j \sin(\alpha + \beta)] = AB \angle (\alpha + \beta) \end{aligned}$$

**Solution.**  $A + B = C = (16 + j12) + (-6 + j10.4) = 10 + j22.4$

$\therefore$  Magnitude of  $C = \sqrt{(10^2 + 22.4^2)} = 25.5$  units

$$\text{Slope of } C = \theta = \tan^{-1} \left( \frac{22.4}{10} \right) = 65.95^\circ$$

The vector addition is shown in Fig. 12.8.

$$\alpha = \tan^{-1} (12/16) = 36.9^\circ$$

$$\beta = \tan^{-1} (-10.4/6) = -240^\circ \text{ or } 120^\circ$$

The resultant vector is found by using parallelogram law of vectors

(Fig. 12.8).

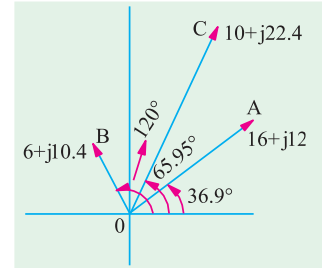


Fig. 12.8

**Example 12.4.** Perform the following operation and express the final result in the polar form :  
 $5 \angle 30^\circ + 8 \angle -30^\circ$ . (Elect. Engg. & Electronics Bangalore Univ. 1989)

**Solution.**  $5 \angle 30^\circ = 5 (\cos 30^\circ + j \sin 30^\circ) = 4.33 + j2.5$

$$8 \angle -30^\circ = 8 [\cos (-30^\circ) + j \sin (-30^\circ)] = 8 (0.866 - j0.5) = 6.93 - j4$$

$$\therefore 5 \angle 30^\circ + 8 \angle -30^\circ = 4.33 + j2.5 + 6.93 - j4 = 11.26 - j1.5 = \sqrt{11.26^2 + 1.5^2} \angle \tan^{-1} (-1.5/11.26) = 11.35$$

$$\tan^{-1} (-0.1332) = 11.35 \angle 7.6^\circ$$

**Example 12.5.** Subtract the following given vectors from one another :

$$A = 30 + j52 \text{ and } B = -39.5 - j14.36$$

**Solution.**  $A - B = C = (30 + j52) - (-39.5 - j14.36) = 69.5 + j66.36$

$\therefore$  Magnitude of  $C = \sqrt{(66.36^2 + 69.5^2)} = 96$

Slope of  $C = \tan^{-1} (66.36/69.5) = 43.6^\circ \therefore C = 96 \angle 43.6^\circ$ .

Similarly  $B - A = -69.5 - j66.36 = 96 \angle 223.6^\circ$ \* or  $= 96 \angle -136.4^\circ$

**Example 12.6.** Given the following two vectors :

$$A = 20 \angle 60^\circ \text{ and } B = 5 \angle 30^\circ$$

Perform the following indicated operations and illustrate graphically (i)  $A \times B$  and (ii)  $A/B$ .

**Solution.** (i)  $A \times B = C = 20 \angle 60^\circ \times 5 \angle 30^\circ = 100 \angle 90^\circ$

Vectors are shown in Fig. 12.9.

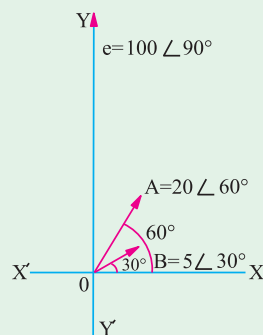


Fig. 12.9

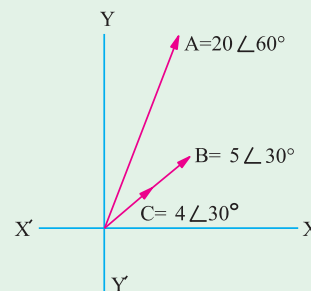


Fig. 12.10

\*  $\tan \theta = 66.36/69.5$  or  $\theta = \tan^{-1} (66.36/69.5) = 43.6^\circ$ . Since both components are negative, the vector lies in third quadrant. Hence, the angle measured from +ve direction of X-axis and in the CCW directions is  $= (180 + 43.6) = 223.6^\circ$ .

$$(ii) \quad \frac{\mathbf{A}}{\mathbf{B}} = \frac{20 \angle 60^\circ}{5 \angle 30^\circ} = 4 \angle 30^\circ \quad \text{—Fig. 12.10}$$

**Example 12.7.** Perform the following operation and the final result may be given in the polar form :  $(8 + j6) \times (-10 - j7.5)$  **(Elect. Engg. & Electronics Bangalore Univ. 1990)**

**Solution.** We will use the following two methods to solve the above question.

**Method No. 1**

We know that multiplication of  $(A + B)$  and  $(C + D)$  can be found as under :

$$\begin{array}{r} A + B \\ \times C + D \\ \hline CA + CB \\ + DA + DB \\ \hline CA + CD + DA + DB \end{array}$$

Similarly, the required multiplication can be carried out as follows :

$$\begin{array}{r} 8 + j6 \\ \times -10 - j7.5 \\ \hline -80 - j60 \\ -j60 - j^2 45 \\ \hline -80 - j120 + 45 \end{array}$$

$$\text{or } -35 - j120 = \sqrt{(-35)^2 + (-120)^2} \left[ \tan^{-1} (120/35) = 125 \tan^{-1} 3.42 = 125 \angle 73.8^\circ \right]$$

Since both the components of the vector are negative, it obviously lies in the third quadrant. As measured from the  $X$ -axis in the  $CCW$  direction, its angle is  $= 180^\circ + 73.8^\circ = 253.8^\circ$ . Hence, the product vector can be written as  $125 \angle 253.8^\circ$ .

**Method No. 2**

$$8 + j6 = 10 \angle 36.9^\circ, \quad -10 - j7.5 = 12.5 \tan^{-1} 0.75 = 12.5 \angle 36.9^\circ$$

Again as explained in Method 1 above, the actual angle of the vector is  $180^\circ + 36.9^\circ = 216.9^\circ$

$$\therefore -10 - j7.5 = 12.5 \angle 216.9^\circ \quad \therefore 10 \angle 36.9^\circ \times 12.5 \angle 216.9^\circ = 125 \angle 253.8^\circ$$

**Example 12.8.** The following three vectors are given :

$$A = 20 + j20, \quad B = 30 \angle -120^\circ \text{ and } C = 10 + j0$$

Perform the following indicated operations :

$$(i) \quad \frac{AB}{C} \quad \text{and} \quad (ii) \quad \frac{BC}{A}$$

**Solution.** Rearranging all three vectors in polar form, we get

$$\mathbf{A} = 28.3 \angle 45^\circ, \quad \mathbf{B} = 30 \angle -120^\circ, \quad \mathbf{C} = 10 \angle 0^\circ$$

$$(i) \quad \frac{AB}{C} = \frac{28.3 \angle 45^\circ \times 30 \angle -120^\circ}{10 \angle 0^\circ} = 84.9 \angle -75^\circ$$

$$(ii) \quad \frac{BC}{A} = \frac{30 \angle -120^\circ \times 10 \angle 0^\circ}{28.3 \angle 45^\circ} = 10.6 \angle -165^\circ$$

**Example 12.9.** Given two current  $i_1 = 10 \sin (\omega t + \pi/4)$  and  $i_2 = 5 \cos (\omega t - \pi/2)$ , find the r.m.s. value of  $i_1 + i_2$  using the complex number representation. **[Elect. Circuit Theory, Kerala Univ. ]**

**Solution.** The maximum value of first current is 10 A and it leads the reference quantity by  $45^\circ$ . The second current can be written as

$$i_2 = 5 \cos (\omega t - \pi/2) = 5 \sin [90 + (\omega t - \pi/2)] = 5 \sin \omega t$$

Hence, its maximum value is 5 A and is in phase with the reference quantity.

$$\therefore \mathbf{I}_{m1} = 10 (\cos 45^\circ + j \sin 45^\circ) = (7.07 + j7.07)$$

$$\mathbf{I}_{m2} = 5 (\cos 0^\circ + j \sin 0^\circ) = (5 + j0)$$

The maximum value of resultant current is

$$\mathbf{I}_m = (7.07 + j7.07) + (5 + j0) = 12.07 + j7.07 = 14 \angle 30.4^\circ$$

$$\therefore \text{R.M.S. value} = 14/\sqrt{2} = \mathbf{10 \text{ A}}$$

## 12.10. Power and Roots of Vectors

### (a) Powers

Suppose it is required to find the cube of the vector  $3 \angle 15^\circ$ . For this purpose, the vector has to be multiplied by itself three times.

$$\therefore (3 \angle 15^\circ)^3 = 3 \times 3 \times 3 \angle (15^\circ + 15^\circ + 15^\circ) = 27 \angle 45^\circ. \text{ In general, } \mathbf{A}^n = \mathbf{A}^n \angle n\alpha$$

Hence,  $n$ th power of vector  $\mathbf{A}$  is a vector whose magnitude is  $A^n$  and whose phase angle with respect to  $X$ -axis is  $n\alpha$ .

It is also clear that  $\mathbf{A}^n \mathbf{B}^n = A^n B^n \angle (n\alpha + n\beta)$

### (b) Roots

It is clear that  $\sqrt[3]{(8 \angle 45^\circ)} = 2 \angle 15^\circ$

In general,  $\sqrt[n]{\mathbf{A}} = \sqrt[n]{A} \angle \alpha/n$

Hence,  $n$ th root of a vector  $\mathbf{A}$  is a vector whose magnitude is  $n\sqrt[n]{A}$  and whose phase angle with respect to  $X$ -axis is  $\alpha/n$ .

## 12.11. The 120° Operator

In three-phase work where voltage vectors are displaced from one another by  $120^\circ$ , it is convenient to employ an operator which rotates a vector through  $120^\circ$  forward or backwards without changing its length. This operator is 'a'. Any operator which is multiplied by 'a' remains unchanged in magnitude but is rotated by  $120^\circ$  in the counter-clockwise (ccw) direction.

$$\therefore \alpha = 1 \angle 120^\circ$$

This, when expressed in the cartesian form, becomes

$$a = \cos 120^\circ + j \sin 120^\circ = -0.5 + j0.866$$

Similarly,  $a^2 = 1 \angle 120^\circ \times 1 \angle 120^\circ = 1 \angle 240^\circ = \cos 240^\circ + j \sin 240^\circ = -0.5 - j0.866$

Hence, operator ' $a^2$ ', will rotate the vector in ccw by  $240^\circ$ . This is the same as rotating the vector in *clockwise* direction by  $120^\circ$ .

$$\therefore a^2 = 1 \angle -120^\circ. \text{ Similarly, } a^3 = 1 \angle 360^\circ = 1^*$$

As shown in Fig. 12.11, the 3-phase voltage vectors with standard phase sequence may be represented as  $E$ ,  $a^2E$  and  $aE$  or as  $E$ ,  $E(-0.5 - j0.866)$  and  $E(-0.5 + j0.866)$

It is easy to prove that

$$(i) a^2 + a = -1 \quad (ii) a^2 + a + 1 = 0 \quad (iii) a^3 + a^2 + a = 0$$

**Note.** We have seen in Art. 12.3 that operator  $-j$  turns a vector through  $-90^\circ$  i.e. through  $90^\circ$  in *clockwise* direction. But it should be clearly noted that operator ' $-a$ ' does not turn a vector through  $-120^\circ$ . Rather ' $-a$ ' turns a vector through  $-60^\circ$  as shown below.

**Example 12.10.** Evaluate the following expressions in the polar form (i)  $a^2 - 1$ , (ii)  $1 - a - a^2$  (iii)  $2a^2 + 3 + a$  (iv)  $ja$ . [Elect. Meas and Meas. Inst., Madras Univ.]

\* Numerically,  $a$  is equivalent to the cube root of unity.

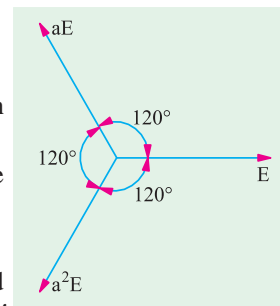


Fig. 12.11

**Solution. (i)**  $a^2 = a \times a = 1 \angle 120^\circ = 1 \angle 240^\circ = 1 \angle -120^\circ = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$\therefore a^2 - 1 = \frac{1}{2} + j\frac{\sqrt{3}}{2} - 1 - j\frac{\sqrt{3}}{2} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$       **180**       **$\sqrt{3}$**       **210**

**(ii)**  $a = 1 \angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ ;  $a^2 = 1 \angle 240^\circ = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$\therefore 1 - a - a^2 = 1 - \frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} - j\frac{\sqrt{3}}{2} = 0$

**(iii)**  $2a^2 = 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -1 - j\sqrt{3}$

$2a = 2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = -1 + j\sqrt{3}$

$\therefore 2a^2 + 3 + 2a = 3 - 1 - j\sqrt{3} - 1 + j\sqrt{3} = 1 \angle 0^\circ$

**(iv)**  $ja = j \times a = 1 \angle 90^\circ \times 1 \angle 120^\circ = 1 \angle 210^\circ$

### Tutorial Problem No. 12.1

- Perform the following indicated operations :  
 (a)  $(60 + j80) + (30 - j40)$  (b)  $(12 - j6) - (40 - j20)$  (c)  $(6 + j8)(3 - j4)$  (d)  $16 + j8 \div (3 - j4)$   
**[(a)  $90 + j40$  (b)  $-28 + j14$  (c)  $50 + j0$  (d)  $-0.56 + j1.92$ ]**
- Two impedances  $Z_1 = 2 + j6 \Omega$  and  $Z_2 = 6 - j12 \Omega$  are connected in a circuit so that they are additive. Find the resultant impedance in the polar form. **[ $10 \angle -36.9^\circ$ ]**
- Express in rectangular form and polar form a vector, the magnitude of which is 100 units and the phase of which with respect to reference axis is  
 (a)  $+30^\circ$  (b)  $+180^\circ$  (c)  $-60^\circ$  (d)  $+120^\circ$  (e)  $-120^\circ$  (f)  $-210^\circ$ .  
**[(a)  $86.6 + j50 \angle 30^\circ$  (b)  $-100 + j0, 100 \angle 180^\circ$  (c)  $50 - j86.6, 100 \angle -60^\circ$  (d)  $-50 + j86.6, 100 \angle -120^\circ$  (e)  $-50 - j86.6, 100 \angle -120^\circ$  (f)  $-50 + j86.6, 100 \angle -210^\circ$ ]**
- In the equation  $V_m = V - ZI$ ,  $V = 100 \angle 0^\circ$  volts,  $Z = 10 \angle 60^\circ \Omega$  and  $I = 8 \angle -30^\circ$  amperes. Express  $V_m$  in polar form. **[ $50.5 \angle -52^\circ$ ]**
- A voltage  $V = 150 + j180$  is applied across an impedance and the current flowing is found to be  $I = 5 - j4$ . Determine (i) scalar impedance (ii) resistance (iii) reactance (iv) power consumed. **[(i)  $3.73 \Omega$  (ii)  $0.75 \Omega$  (iii)  $36.6 \Omega$  (iv)  $30 \text{ W}$ ]**
- Calculate the following in polar form :  
 (i) Add  $(40 + j20)$  to  $(20 + j120)$  (ii) Subtract  $(10 + j30)$  from  $(20 - j20)$   
 (iii) Multiply  $(15 + j20)$  with  $(20 + j30)$  (iv) Divide  $(6 + j7)$  by  $(5 + j3)$   
**(Gujrat University, June/July 2003)**  
**(RGPV Bhopal December 2002)**
- Why is impedance represented by a complex number?

### OBJECTIVE TESTS – 12

- The symbol  $j$  represents counterclockwise rotation of a vector through—degrees.  
 (a) 180 (b) 90  
 (c) 360 (d) 270
- The operator  $j$  has a value of  
 (a) +1 (b) -1  
 (c)  $\sqrt{-1}$  (d)  $\sqrt{+1}$
- The vector  $j^5 E$  is the same as vector  
 (a)  $jE$  (b)  $j^2 E$   
 (c)  $j^3 E$  (d)  $j^4 E$
- The conjugate of  $(-a + jb)$  is  
 (a)  $(a - jb)$  (b)  $(-a - jb)$   
 (c)  $(a + jb)$  (d)  $(jb - a)$

### ANSWERS

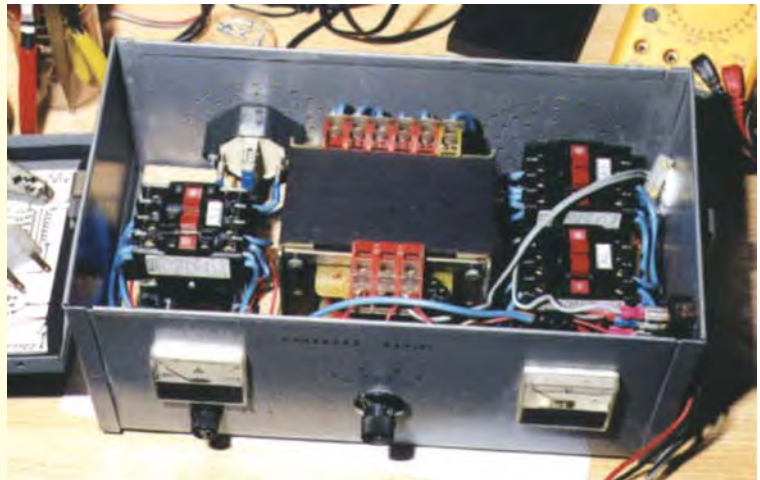
1. b      2. c      3. a      4. b

# CHAPTER 13

## Learning Objectives

- A.C. Through Resistance and Inductance
- Power Factor
- Active and Reactive Components of Circuit Current-I
- Active, Reactive and Apparent Power
- Q-factor of a Coil
- Power in an Iron-cored Choking Coil
- A.C. Through Resistance and Capacitance
- Dielectric Loss and Power Factor of a Capacitor
- Resistance, Inductance and Capacitance in Series
- Resonance in  $R-L-C$  Circuits
- Graphical Representation of Resonance
- Resonance Curve
- Half-power Bandwidth of a Resonant Circuit
- Bandwidth B at any Off-resonance Frequency
- Determination of Upper and Lower Half-Power Frequencies
- Values of Edge Frequencies
- Q-Factor of a Resonant Series Circuit
- Circuit Current at Frequencies Other than Resonant Frequencies
- Relation Between Resonant Power  $P_0$  and Off-resonant Power P

## SERIES A.C. CIRCUITS



This chapter discusses series AC circuits, and how they function

### 13.1. A.C. Through Resistance and Inductance

A pure resistance  $R$  and a pure inductive coil of inductance  $L$  are shown connected in series in Fig. 13.1.

Let  $V$  = r.m.s. value of the applied voltage,  $I$  = r.m.s. value of the resultant current  
 $V_R = IR$  – voltage drop across  $R$  (in phase with  $I$ ),  $V_L = I \cdot X_L$  – voltage drop across coil (ahead of  $I$  by  $90^\circ$ )

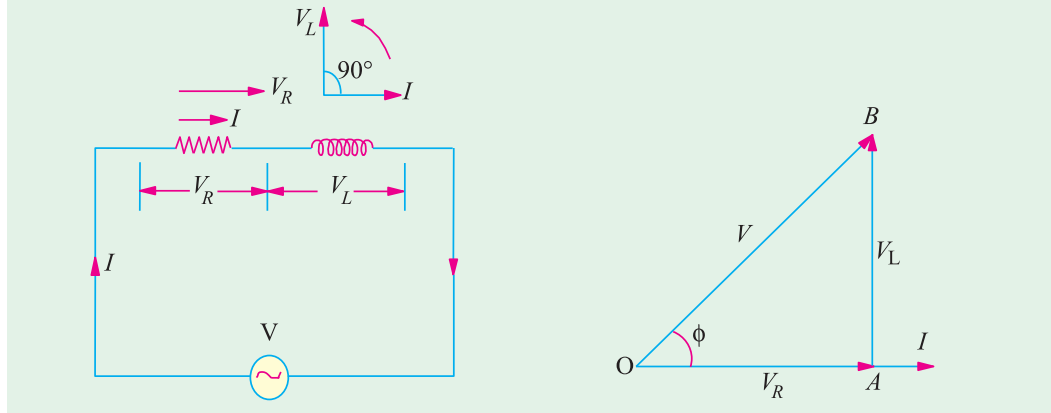


Fig. 13.1

Fig. 13.2

These voltage drops are shown in voltage triangle  $OAB$  in Fig. 13.2. Vector  $OA$  represents ohmic drop  $V_R$  and  $AB$  represents inductive drop  $V_L$ . The applied voltage  $V$  is the vector sum of the two *i.e.*  $OB$ .

$$\therefore V = \sqrt{(V_R^2 + V_L^2)} = \sqrt{[(IR)^2 + (I \cdot X_L)^2]} = I \sqrt{R^2 + X_L^2}, \frac{V}{\sqrt{(R^2 + X_L^2)}} = I$$

The quantity  $\sqrt{(R^2 + X_L^2)}$  is known as the **impedance** ( $Z$ ) of the circuit. As seen from the impedance triangle  $ABC$  (Fig. 13.3)  $Z^2 = R^2 + X_L^2$ .

$$\text{i.e. } (\mathbf{Impedance})^2 = (\mathbf{resistance})^2 + (\mathbf{reactance})^2$$

From Fig. 13.2, it is clear that the applied voltage  $V$  leads the current  $I$  by an angle  $\phi$  such that

$$\tan \phi = \frac{V_L}{V_R} = \frac{I \cdot X_L}{I \cdot R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}} \therefore \phi = \tan^{-1} \frac{X_L}{R}$$

The same fact is illustrated graphically in Fig. 13.4.

In other words, current  $I$  lags behind the applied voltage  $V$  by an angle  $\phi$ .

Hence, if applied voltage is given by  $v = V_m \sin \omega t$ , then current equation is

$$i = I_m \sin (\omega t - \phi) \text{ where } I_m = V_m / Z$$

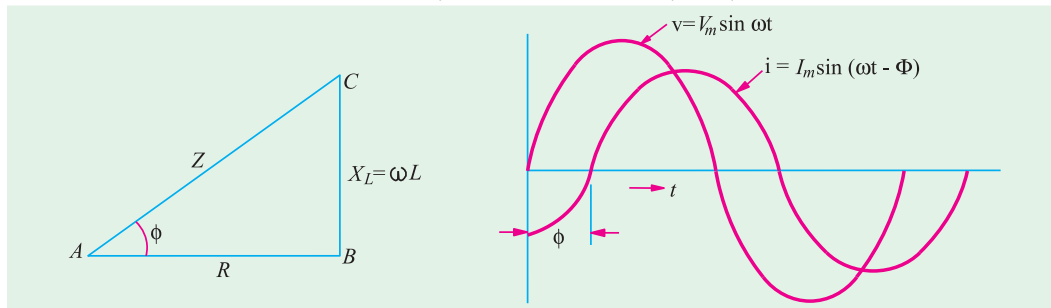


Fig. 13.3

Fig. 13.4





In Fig. 13.5,  $I$  has been resolved into its two mutually perpendicular components,  $I \cos \phi$  along the applied voltage  $V$  and  $I \sin \phi$  in quadrature (*i.e.* perpendicular) with  $V$ .

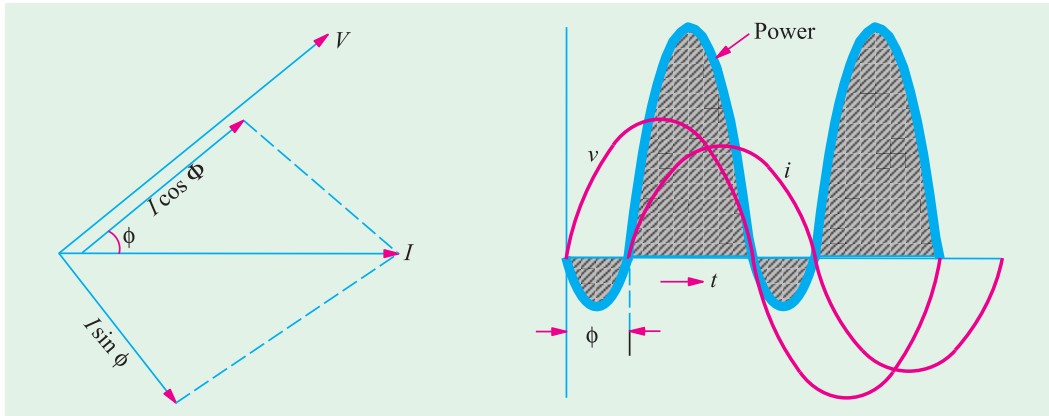


Fig. 13.5

Fig. 13.6

The mean power consumed by the circuit is given by the product of  $V$  and **that component of the current  $I$  which is in phase with  $V$** .

So  $P = V \times I \cos \phi = \text{r.m.s. voltage} \times \text{r.m.s. current} \times \cos \phi$

The term ' $\cos \phi$ ' is called the power factor of the circuit.

**Remember that in an a.c. circuit, the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (VA) and not true power in watts.** True power ( $W$ ) = volt-amperes ( $VA$ )  $\times$  power factor.

or

$$\text{Watts} = \text{VA} \times \cos \phi^*$$

It should be noted that power consumed is due to ohmic resistance only because pure inductance does not consume any power.

Now  $P = VI \cos \phi = VI \times (R/Z) = (V/Z) \times I \times R = I^2 R$  ( $\because \cos \phi = R/Z$ ) or  $P = I^2 R$  watt

Graphical representation of the power consumed is shown in Fig. 14.6.

Let us calculate power in terms of instantaneous values.

Instantaneous power is  $= v i = V_m \sin \omega t \times I_m \sin (\omega t - \phi) = V_m I_m \sin \omega t \sin (\omega t - \phi)$

$$= \frac{1}{2} V_m I_m [\cos \phi - \cos (2\omega t - \phi)]$$

Obviously, this power consists of two parts (Fig. 13.7).

(i) a constant part  $\frac{1}{2} V_m I_m \cos \phi$  which contributes to real power.

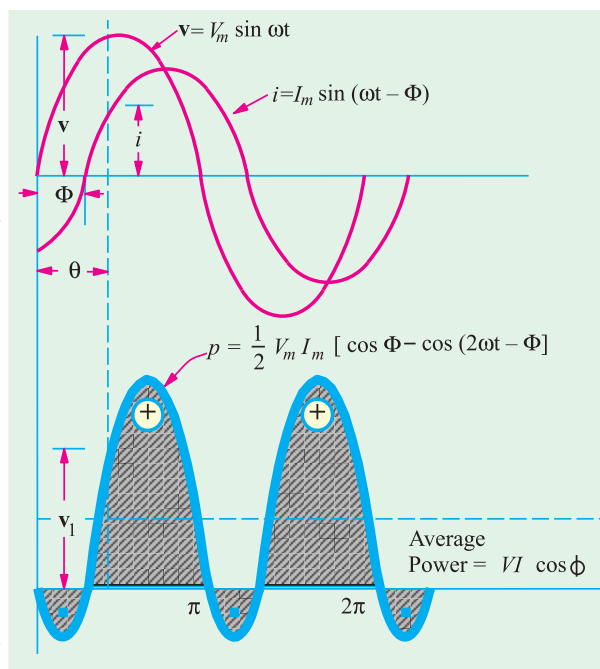


Fig. 13.7

\* While dealing with large supplies of electric power, it is convenient to use kilowatt as the unit  $\text{kW} = \text{kVA} \times \cos \phi$

(ii) a pulsating component  $\frac{1}{2} V_m I_m \cos (2\omega t - \phi)$  which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed  $= \frac{1}{2} V_m I_m \cos \phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi = VI \cos \phi$ , where  $V$  and  $I$  represent the r.m.s values.

**Symbolic Notation.**  $Z = R + jX_L$

Impedance vector has numerical value of  $\sqrt{(R^2 + X_L^2)}$ .

Its phase angle with the reference axis is  $\phi = \tan^{-1} (X_L/R)$

It may also be expressed in the polar form as  $Z = Z \angle \phi^\circ$

(i) Assuming  $V = V \angle 0^\circ$ ;  $I = \frac{V}{Z} = \frac{V \angle 0^\circ}{Z \angle \phi^\circ} = \frac{V}{Z} \angle -\phi^\circ$  (Fig. 13.8)

It shows that current vector is lagging behind the voltage vector by  $\phi^\circ$ . The numerical value of current is  $V/Z$ .

(ii) However, if we assumed that

$$I = I \angle 0, \text{ then}$$

$$V = IZ = I \angle 0^\circ \times Z \angle \phi^\circ$$

$$= IZ \angle \phi^\circ$$

It shows that voltage vector is  $\phi^\circ$  ahead of current vector in ccw direction as shown in Fig. 13.9.

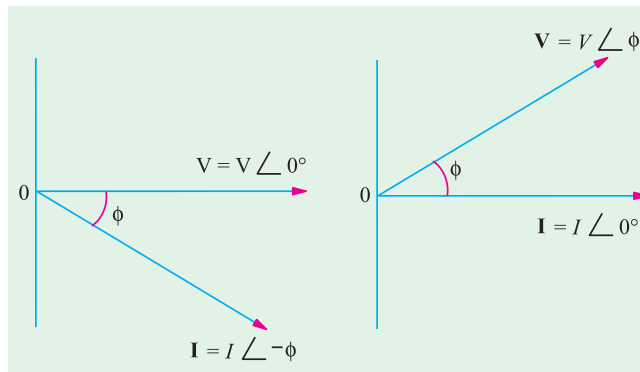


Fig. 13.8

Fig. 13.9

### 13.2. Power Factor

It may be defined as

(i) cosine of the angle of lead or lag

(ii) the ratio  $\frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$  (...Fig. 13.3) (iii) the ratio  $\frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt-amperes}} = \frac{W}{VA}$

### 13.3. Active and Reactive Components of Circuit Current I

Active component is that which is in phase with the applied voltage *V.i.e.*  $I \cos \phi$ . It is also known as ‘wattful’ component.

Reactive component is that which is in quadrature with *V.i.e.*  $I \sin \phi$ . It is also known as ‘wattless’ or ‘idle’ component.

It should be noted that the product of volts and amperes in an a.c. circuit gives voltamperes (VA). Out of this, the actual power is  $VA \cos \phi = W$  and reactive power is  $VA \sin \phi$ . Expressing the values in kVA, we find that it has two rectangular components :

(i) active component which is obtained by multiplying kVA by  $\cos \phi$  and this gives power in kW.

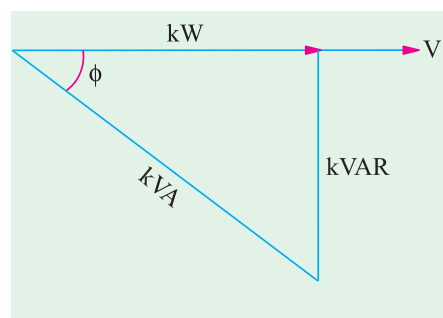


Fig. 13.10

(ii) the reactive component known as reactive kVA and is obtained by multiplying kVA by  $\sin \phi$ . It is written as kVAR (kilovar). The following relations can be easily deduced.

$$\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2}; \text{kW} = \text{kVA} \cos \phi \text{ and } \text{kVAR} = \text{kVA} \sin \phi$$

These relationships can be easily understood by referring to the kVA triangle of Fig. 13.10 where it should be noted that lagging kVAR has been taken as negative.

For example, suppose a circuit draws a current of 1000 A at a voltage of 20,000 V and has a power factor of 0.8. Then

$$\text{input} = 1,000 \times 20,000/1000 = 20,000 \text{ kVA}; \cos \phi = 0.8; \sin \phi = 0.6$$

$$\text{Hence kW} = 20,000 \times 0.8 = 16,000; \text{kVAR} = 20,000 \times 0.6 = 12,000$$

$$\text{Obviously, } \sqrt{16000^2 + 12000^2} = 20,000 \text{ i.e. } \text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2}$$

### 13.4. Active, Reactive and Apparent Power

Let a series  $R$ - $L$  circuit draw a current of  $I$  when an alternating voltage of r.m.s. value  $V$  is applied to it. Suppose that current lags behind the applied voltage by  $\phi$ . The three powers drawn by the circuit are as under :

**(i) apparent power ( $S$ )**

It is given by the product of r.m.s. values of applied voltage and circuit current.

$$\therefore S = VI = (IZ) \cdot I = I^2 Z \text{ volt-amperes (VA)}$$

**(ii) active power ( $P$  or  $W$ )**

It is the power which is actually dissipated in the circuit resistance.  $P = I^2 R = VI \cos \phi$  watts

**(iii) reactive power ( $Q$ )**

It is the power developed in the inductive reactance of the circuit.

$$Q = I^2 X_L = I^2 \cdot Z \sin \phi = I \cdot (IZ) \cdot \sin \phi = VI \sin \phi \text{ volt-amperes-reactive (VAR)}$$

These three powers are shown in the power triangle of Fig. 13.11 from where it can be seen that

$$S^2 = P^2 + Q^2 \text{ or } S = \sqrt{P^2 + Q^2}$$

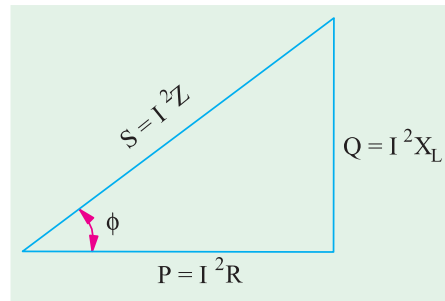


Fig. 13.11

### 13.5. Q-factor of a Coil

Reciprocal of power factor is called the  $Q$ -factor of a coil or its figure of merit. It is also known as quality factor of the coil.

$$Q \text{ factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos \phi} = \frac{Z}{R}$$

If  $R$  is small as compared to reactance, then  $Q$ -factor =  $Z/R = \omega L/R$

$$\text{Also, } Q = 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} \quad \text{---in the coil}$$

**Example 13.1.** In a series circuit containing pure resistance and a pure inductance, the current and the voltage are expressed as :

$$i(t) = 5 \sin(314t + 2\pi/3) \text{ and } v(t) = 15 \sin(314t + 5\pi/6)$$

(a) What is the impedance of the circuit? (b) What is the value of the resistance? (c) What is the inductance in henrys? (d) What is the average power drawn by the circuit? (e) What is the power factor?

[Elect. Technology, Indore Univ.]

**Solution.** Phase angle of current =  $2\pi/3 = 2 \times 180^\circ/3 = 120^\circ$  and phase angle of voltage =  $5\pi/6 = 5 \times 180^\circ/6 = 150^\circ$ . Also,  $Z = V_m/I_m = 3 \Omega$

Hence, current lags behind voltage by  $30^\circ$ . It means that it is an  $R$ - $L$  circuit. Also  $314 = 2\pi f$  or  $f = 50 \text{ Hz}$ . Now,  $R/Z = \cos 30^\circ = 0.866$ ;  $R = 2.6 \Omega$ ;  $X_L/Z = \sin 30^\circ = 0.5$

$$\therefore X_L = 1.5 \Omega \quad 314 L = 1.5, \quad L = 4.78 \text{ mH}$$

$$(a) Z = 3 \Omega \quad (b) R = 2.6 \Omega \quad (c) L = 4.78 \text{ mH}$$

$$(d) P = I^2 R = (5/\sqrt{2})^2 \times 2.6 = 32.5 \text{ W} \quad (e) \text{ p.f.} = \cos 30^\circ = 0.866 \text{ (lag)}.$$

**Example 13.2.** In a circuit the equations for instantaneous voltage and current are given by  $v =$

$$141.4 \sin \left( t - \frac{2\pi}{3} \right) \text{ volt and } i = 7.07 \sin \left( t - \frac{\pi}{2} \right) \text{ amp, where } \omega = 314 \text{ rad/sec.}$$

(i) Sketch a neat phasor diagram for the circuit. (ii) Use polar notation to calculate impedance with phase angle. (iii) Calculate average power & power factor. (iv) Calculate the instantaneous power at the instant  $t = 0$ .  
(F.Y. Engg. Pune Univ.)

**Solution.** (i) From the voltage equation, it is seen that the voltage lags behind the reference quantity by  $2\pi/3$  radian or  $2 \times 180/3 = 120^\circ$ . Similarly, current lags behind the reference quantity by  $\pi/2$  radian or  $180/2 = 90^\circ$ . Between themselves, voltage lags behind the current by  $(120 - 90) = 30^\circ$  as shown in Fig. 13.12 (b).

$$(ii) V = V_m/\sqrt{2} = 141.4/\sqrt{2} = 100 \text{ V}; I = I_m/\sqrt{2} = 7.07/\sqrt{2} = 5 \text{ A.}$$

$$\therefore V = 100 \angle -120^\circ \text{ and } I = 5 \angle -90^\circ \therefore Z = \frac{100 \angle -120^\circ}{5 \angle -90^\circ} = 20 \angle -30^\circ$$

$$(iii) \text{ Average power} = VI \cos \phi = 100 \times 5 \times \cos 30^\circ = 433 \text{ W}$$

$$(iv) \text{ At } t = 0; v = 141.4 \sin(0 - 120^\circ) = -122.45 \text{ V}; i = 7.07 \sin(0 - 90^\circ) = -7.07 \text{ A.}$$

$$\therefore \text{ instantaneous power at } t = 0 \text{ is given by } vi = (-122.45) \times (-7.07) = 865.7 \text{ W.}$$

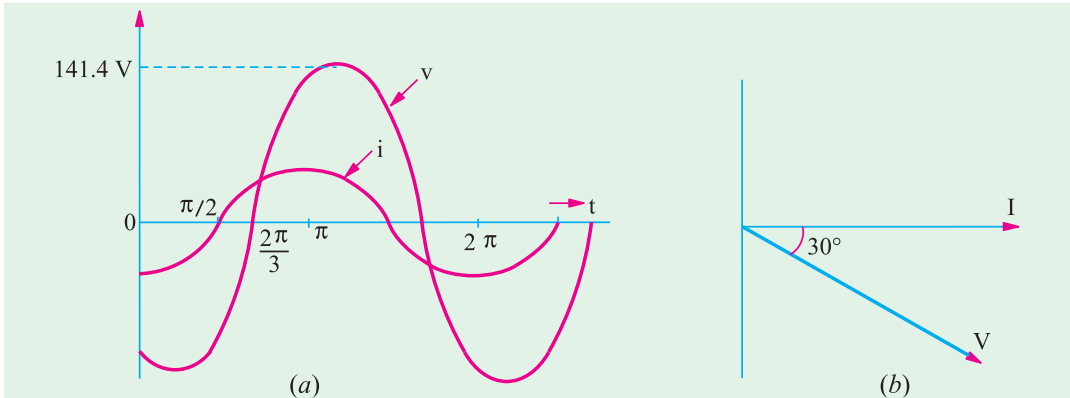


Fig. 13.12

**Example 13.3.** The potential difference measured across a coil is 4.5 V, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9 A at 25 Hz, the potential difference is 24 V. Find the current, the power and the power factor when it is supplied by 50 V, 50 Hz supply.  
(F.Y. Pune Univ.)

**Solution.** Let  $R$  be the d.c. resistance and  $L$  be the inductance of the coil.

$$\therefore R = V/I = 4.5/9 = 0.5 \Omega$$

$$\text{With a.c. current of 25 Hz, } Z = V/I = 24/9 = 2.66 \Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2} = 2.62 \Omega. \text{ Now } X_L = 2\pi \times 25 \times L; L = 0.0167 \Omega$$

**At 50 Hz**

$$X_L = 2.62 \times 2 = 5.24 \Omega; Z = \sqrt{0.5^2 + 5.24^2} = 5.26 \Omega$$

$$\text{Current } I = 50/5.26 = 9.5 \text{ A}; \text{ Power} = I^2 R = 9.5^2 \times 0.5 = 45 \text{ W.}$$

**Example 13.4.** In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 mA while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit? (Network Analysis A.M.I.E. Sec. B, S 1990)

**Solution.** (i)  $Z = \sqrt{R^2 + (2\pi \times 50 L)^2} = \sqrt{R^2 + 98696 L^2}$ ;  $V = IZ$  or  $10 = 700 \times 10^{-3} \sqrt{R^2 + 98696 L^2}$

$$\sqrt{(R^2 + 98696 L^2)} = 10/700 \times 10^3 = 100/7 \text{ or } R^2 + 98696 L^2 = 10000/49 \quad \dots (i)$$

(ii) In the second case  $Z = \sqrt{R^2 + (2\pi \times 75L)^2} = \sqrt{(R^2 + 222066 L^2)}$

$$\therefore 10 = 500 \times 10^{-3} \sqrt{(R^2 + 222066 L^2)} \text{ i.e. } \sqrt{(R^2 + 222066 L^2)} = 20 \text{ or } R^2 + 222066 L^2 = 400 \quad (ii)$$

Subtracting Eq. (i) from (ii), we get

$$222066 L^2 - 98696 L^2 = 400 - (10000/49) \text{ or } 123370 L^2 = 196 \text{ or } L = 0.0398 \text{ H} = \mathbf{40 \text{ mH.}}$$

Substituting this value of L in Eq. (ii), we get,  $R^2 + 222066 (0.398)^2 = 400 \therefore R = \mathbf{6.9 \Omega}$

**Example 13.5.** A series circuit consists of a resistance of 6  $\Omega$  and an inductive reactance of 8  $\Omega$ . A potential difference of 141.4 V (r.m.s.) is applied to it. At a certain instant the applied voltage is +100 V, and is increasing. Calculate at this current, (i) the current (ii) the voltage drop across the resistance and (iii) voltage drop across inductive reactance. (F.E. Pune Univ.)

**Solution.**  $Z = R + jX = 6 + j8 = 10 \angle 53.1^\circ$

It shows that current lags behind the applied voltage by  $53.1^\circ$ . Let V be taken as the reference quantity. Then  $v = (141.4 \times \sqrt{2}) \sin \omega t = 200 \sin \omega t$ ;  $i = (V_m/Z \sin \omega t) - 30^\circ = 20 \sin (\omega t - 53.1^\circ)$ .

(i) When the voltage is +100 V and increasing;  $100 = 200 \sin \omega t$ ;  $\sin \omega t = 0.5$ ;  $\omega t = 30^\circ$

At this instant, the current is given by  $i = 20 \sin (30^\circ - 53.1^\circ) = -20 \sin 23.1^\circ = \mathbf{-7.847 \text{ A.}}$

(ii) drop across resistor =  $iR = -7.847 \times 6 = -47 \text{ V.}$

(iii) Let us first find the equation of the voltage drop  $V_L$  across the inductive reactance. Maximum value of the voltage drop =  $I_m X_L = 20 \times 8 = 160 \text{ V.}$  It leads the current by  $90^\circ$ . Since current itself lags the applied voltage by  $53.1^\circ$ , the reactive voltage drop across the applied voltage by  $(90^\circ - 53.1^\circ) = 36.9^\circ$ . Hence, the equation of this voltage drop at the instant when  $\omega t = 30^\circ$  is

$$V_L = 160 \sin (30^\circ + 36.9^\circ) = 160 \sin 66.9^\circ = \mathbf{147.2 \text{ V.}}$$

**Example 13.6.** A 60 Hz sinusoidal voltage  $v = 141 \sin \omega t$  is applied to a series R-L circuit. The values of the resistance and the inductance are 3  $\Omega$  and 0.0106 H respectively.

(i) Compute the r.m.s. value of the current in the circuit and its phase angle with respect to the voltage.

(ii) Write the expression for the instantaneous current in the circuit.

(iii) Compute the r.m.s. value and the phase of the voltages appearing across the resistance and the inductance.

(iv) Find the average power dissipated by the circuit.

(v) Calculate the p.f. of the circuit.

(F.E. Pune Univ.)

**Solution.**  $V_m = 141 \text{ V}$ ;  $V = 141/\sqrt{2} = 100 \text{ V} \therefore V = 100 + j0$

$$X_L = 2\pi \times 60 \times 0.0106 = 4 \Omega. \quad Z = 3 + j4 = 5 \angle 53.1^\circ$$

(i)  $I = V/Z = 100 \angle 0^\circ / 5 \angle 53.1^\circ = 20 \angle -53.1^\circ$

Since angle is minus, the current lags behind the voltage by  $53.1^\circ$

(ii)  $I_m = \sqrt{2} \times 20 = 28.28$ ;  $\therefore i = 28.28 \sin (\omega t - 53.1^\circ)$

(iii)  $VR = IR = 20 \angle -53.1^\circ \times 3 = 60 \angle -53.1^\circ \text{ volt.}$

$$V_L = jIX_L = 1 \angle 90^\circ \times 20 \angle -53.1^\circ \times 4 = 80 \angle 36.9^\circ$$

(iv)  $P = VI \cos \phi = 100 \times 20 \times \cos 53.1^\circ = \mathbf{1200 \text{ W.}}$

(v) p.f. =  $\cos \phi = \cos 53.1^\circ = \mathbf{0.6.}$

**Example 13.7.** In a given R-L circuit,  $R = 3.5 \Omega$  and  $L = 0.1 \text{ H}$ . Find (i) the current through the circuit and (ii) power factor if a 50-Hz voltage  $V = 220 \angle 30^\circ$  is applied across the circuit.

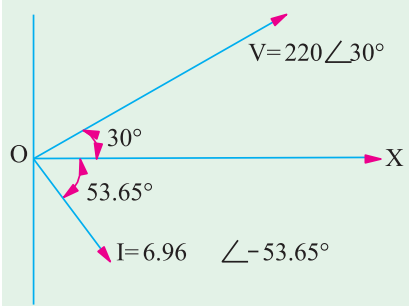


Fig. 13.13

**Solution.** The vector diagram is shown in Fig. 13.13.

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.42 \Omega$$

$$Z = \sqrt{(R^2 + X_L^2)} = \sqrt{3.5^2 + 31.42^2} = 31.6$$

$$\therefore Z = 31.6 \angle \tan^{-1}(31.42/3.5) = 31.6 \angle 83.65^\circ$$

$$(i) I = \frac{V}{Z} = \frac{220 \angle 30^\circ}{31.6 \angle 83.65^\circ} = 6.96 \angle -53.65^\circ$$

(ii) Phase angle between voltage and current is  
 $= 53.65^\circ + 30^\circ = 83.65^\circ$  with current lagging.

$$\text{p.f.} = \cos 83.65^\circ = \mathbf{0.11 \text{ (lag)}}.$$

**Example 13.8.** In an alternating circuit, the impressed voltage is given by  $V = (100 - j50)$  volts and the current in the circuit is  $I = (3 - j4)$  A. Determine the real and reactive power in the circuit.

(Electrical Engg., Calcutta Univ. 1991)

**Solution.** Power will be found by the conjugate method. Using current conjugate, we have

$$P_{VA} = (100 - j50)(3 + j4) = 300 + j400 - j150 + 200 = 500 + j250$$

Hence, real power is **500 W** and reactive power of VAR is **250**. Since the second term in the above expression is positive, the reactive volt-amperes of 250 are inductive.\*

**Example 13.9.** In the circuit of Fig. 14.14, applied voltage  $V$  is given by  $(0 + j10)$  and the current is  $(0.8 + j0.6)$  A. Determine the values of  $R$  and  $X$  and also indicate if  $X$  is inductive or capacitive.  
 (Elect. Technology, Nagpur Univ. 1991)

**Solution.**  $V = 0 + j10 = 10 \angle 90^\circ$ ;  $I = 0.8 + j0.6 = 1 \angle 36.9^\circ$

As seen,  $V$  leads the reference quantity by  $90^\circ$  whereas  $I$  leads by  $36.9^\circ$ . In other words,  $I$  lags behind the applied voltage by  $(90^\circ - 36.9^\circ) = 53.1^\circ$

Hence, the circuit of Fig. 13.14 is an R-L circuit.

$$\text{Now, } Z = V/I = 10 \angle 90^\circ / 1 \angle 36.9^\circ = 10 \angle 53.1^\circ = 6 + j8$$

Hence,  $R = 6 \Omega$  and  $X_L = 8 \Omega$

**Example 13.10.** A two-element series circuit is connected across an a.c. source  $e = 200\sqrt{2} \sin(\omega t + 20^\circ)$  V. The current in the circuit then is found to be  $i = 10\sqrt{2} \cos(314t - 25^\circ)$  A. Determine the parameters of the circuit.  
 (Electromechanic Allahabad Univ. 1991)

**Solution.** The current can be written as  $i = 10\sqrt{2} \sin(314t - 25^\circ + 90^\circ) = 10\sqrt{2} \sin(314t + 65^\circ)$ . It is seen that applied voltage leads by  $20^\circ$  and current leads by  $65^\circ$  with regards to the reference quantity, their mutual phase difference is  $= 65^\circ - (20^\circ) = 45^\circ$ . Hence, p.f.  $= \cos 45^\circ = 1/\sqrt{2}$  (lead).

$$\text{Now, } V_m = 200\sqrt{2} \text{ and } I_m = 10\sqrt{2} \quad \therefore Z = V_m/I_m = 200\sqrt{2}/10\sqrt{2} = 20 \Omega$$

$$R = Z \cos \phi = 20/\sqrt{2} \Omega = \mathbf{14.1 \Omega}; X_c = Z \sin \phi = 20/\sqrt{2} = \mathbf{14.1 \Omega}$$

$$\text{Now, } f = 314/2\pi = 50 \text{ Hz. Also, } X_c = 1/2\pi fC \quad \therefore C = 1/2\pi \times 50 \times 14.1 = \mathbf{226 \mu F}$$

Hence, the given circuit is an R-C circuit.

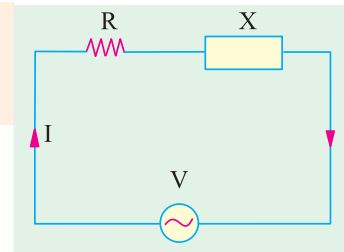


Fig. 13.14

\* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs are positive.

**Example 13.11.** Transform the following currents to the time domain : (i)  $6 - j8$  (ii)  $-6 + j8$  (iii)  $-j5$ .

**Solution.** (i) Now,  $(6 - j8)$  when expressed in the polar form is  $\sqrt{6^2 + 8^2} \angle -\tan^{-1} 8/6 = 10 \angle -53.1^\circ$ . The time domain representation of this current is  $i(t) = 10 \sin(\omega t - 53.1^\circ)$

$$(ii) -6 + j8 = \sqrt{6^2 + 8^2} \angle \tan^{-1} 8/-6 = 10 \angle 126.9^\circ$$

$$\therefore i(t) = 10 \sin(\omega t + 126.9^\circ)$$

$$(iii) -j5 = 10 \angle -90^\circ \therefore i(t) = 10 \sin(\omega t - 90^\circ)$$

**Example 13.12.** A choke coil takes a current of 2 A lagging  $60^\circ$  behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also, determine the power consumed when it is connected across 100-V, 25-Hz supply.

(Elect. Engg. & Electronics, Bangalore Univ.)

**Solution.** (i)  $Z_{\text{coil}} = 200/2 = 100 \Omega$ ;  $R = Z \cos \phi = 100 \cos 60^\circ = 50 \Omega$

$$X_L = Z \sin \phi = 100 \sin 60^\circ = 86.6 \Omega \quad X_L = 2\pi fL = 86.6 \therefore L = 86.6/2\pi \times 50 = 0.275 \text{ H}$$

(ii) Now, the coil will have different impedance because the supply frequency is different but its resistance would remain the same i.e.  $50 \Omega$ . Since the frequency has been halved, the inductive reactance of the coil is also halved i.e. it becomes  $86.6/2 = 43.3 \Omega$

$$Z_{\text{coil}} = \sqrt{50^2 + 43.3^2} = 66.1 \Omega$$

$$I = 100/66.1 = 1.5 \text{ A, p.f.} = \cos \phi = 50/66.1 = 0.75$$

$$\text{Power consumed by the coil} = VI \cos \phi = 100 \times 1.5 \times 0.75 = \mathbf{112.5 \text{ W}}$$



Choke coil

**Example 13.13.** An inductive circuit draws 10 A and 1 kW from a 200-V, 50 Hz a.c. supply. Determine :

(i) the impedance in cartesian form  $(a + jb)$  (ii) the impedance in polar form  $Z \angle \theta$  (iii) the power factor (iv) the reactive power (v) the apparent power.

**Solution.**  $Z = 200/10 = 20 \Omega$ ;  $P = I^2 R$  or  $1000 = 10^2 \times R$ ;  $R = 10 \Omega$ ;  $X_L = \sqrt{20^2 - 10^2} = 17.32 \Omega$

$$(i) Z = 10 + j17.32 \quad (ii) |Z| = \sqrt{10^2 + 17.32^2} = 20 \Omega; \tan \phi = 17.32/10 = 1.732; \phi = \tan^{-1}(1.732)$$

$$= 60^\circ \therefore Z = 20 \angle 60^\circ. \quad (iii) \text{ p.f.} = \cos \phi = \cos 60^\circ = \mathbf{0.5 \text{ lag}} \quad (iv) \text{ reactive power} = VI \sin \phi$$

$$= 200 \times 10 \times 0.866 = \mathbf{1732 \text{ VAR}} \quad (v) \text{ apparent power} = VI = 200 \times 10 = \mathbf{2000 \text{ VA.}}$$

**Example 13.14.** When a voltage of 100 V at 50 Hz is applied to a choking coil A, the current taken is 8 A and the power is 120 W. When applied to a coil B, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series?

(Elements of Elect. Engg., Bangalore Univ.)

**Solution.**  $Z_1 = 100/8 = 12.5 \Omega$ ;  $P = I^2 R_1$  or  $120 = 8^2 \times R_1$ ;  $R_1 = 15/8 \Omega$

$$X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{12.5^2 - (15/8)^2} = 12.36 \Omega$$

$$Z_2 = 100/10 = 10 \Omega; 500 = 10^2 \times R_2 \quad \text{or} \quad R_2 = 5 \Omega$$

$$X_2 = \sqrt{10^2 - 5^2} = 8.66 \Omega$$

**With Joined in Series**

$$R = R_1 + R_2 = (15/8) + 5 = 55/9 \Omega; X = 12.36 + 8.66 = 21.02 \Omega$$

$$Z = \sqrt{(55/8)^2 + (21.02)^2} = 22.1 \Omega, I = 100/22.1 = 4.52 \text{ A}, P = I^2 R = 4.52^2 \times 55/8 = \mathbf{140 \text{ W}}$$

**Example 13.15.** A coil takes a current of 6 A when connected to a 24-V d.c. supply. To obtain the same current with a 50-Hz a.c. supply, the voltage required was 30 V.

Calculate (i) the inductance of the coil (ii) the power factor of the coil.

(F.Y. Engg. Pune Univ.)

**Solution.** It should be kept in mind the coil offers only resistance to direct voltage whereas it offers impedance to an alternating voltage.

$$\therefore R = 24/6 = 4 \Omega; Z = 30/6 = 5 \Omega$$

$$(i) \therefore X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 4^2} = 3 \Omega \quad (ii) \text{ p.f.} = \cos \phi = R/Z = 4/5 = \mathbf{0.8 \text{ (lag)}}$$

**Example 13.16.** A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150  $\mu\text{F}$  are connected in series and are fed by a 230 V, 50 Hz supply. Find  $X_L$ ,  $X_C$ , Z, Y, p.f., active power and reactive power.

(Elect. Science-I, Allahabad Univ. 1992)

$$\text{Solution. } X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.8 \Omega; X_C = 1/2\pi fC$$

$$= 10^{-6} 2\pi \times 50 \times 150 = 21.2 \Omega; X = (X_L - X_C) = 41.6 \Omega;$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{20^2 + 41.6^2} = 46.2 \Omega; I = V/Z = 230/46.2 = 4.98 \text{ A}$$

$$\text{Also, } Z = R + jX = 20 + j41.6 = 46.2 \angle 64.3^\circ \text{ ohm}$$

$$\therefore Y = 1/Z = 1/46.2 \angle 64.3^\circ = 0.0216 \angle -64.3^\circ \text{ siemens}$$

$$\text{p.f.} = \cos 64.3^\circ = 0.4336 \text{ (lag)}$$

$$\text{Active power} = VI \cos \phi = 230 \times 4.98 \times 0.4336 = 497 \text{ W}$$

$$\text{Reactive power} = VI \sin \phi = 230 \times 4.98 \times \sin 64.3^\circ = 1031 \text{ VAR}$$

**Example 13.17.** A 120-V, 60-W lamp is to be operated on 220-V, 50-Hz supply mains. Calculate what value of (a) non-inductive resistance (b) pure inductance would be required in order that lamp is run on correct voltage. Which method is preferable and why?

**Solution.** Rated current of the bulb =  $60/120 = 0.5 \text{ A}$

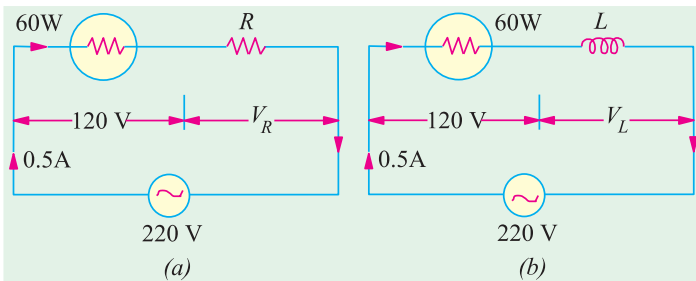


Fig. 13.15

(a) Resistor has been shown connected in series with the lamp in Fig. 13.15 (a).

$$\text{P.D. across } R \text{ is } V_R = 220 - 120 = 100 \text{ V}$$

It is in phase with the applied voltage,  $\therefore R = 100/0.5 = \mathbf{200 \Omega}$

(b) P.D. across bulb = 120 V  
P.D. across L is

$$V_L = \sqrt{(220^2 - 120^2)} = 184.4 \text{ V}$$

(Remember that  $V_L$  is in quadrature with  $V_R$ —the voltage across the bulb).

$$\text{Now, } V_L = 0.5 \times X_L \text{ or } 184.4 = 0.5 \times L \times 2\pi \times 50 \quad \therefore L = 184.4/0.5 \times 3.14 = \mathbf{1.17 \text{ H}}$$

Method (b) is preferable to (a) because in method (b), there is no loss of power. Ohmic resistance of  $200 \Omega$  itself dissipates large power (i.e.  $100 \times 0.5 = \mathbf{50 \text{ W}}$ ).

**Example 13.18.** A non-inductive resistor takes 8 A at 100 V. Calculate the inductance of a choke coil of negligible resistance to be connected in series in order that this load may be supplied from 220-V, 50-Hz mains. What will be the phase angle between the supply voltage and current?

(Elements of Elect. Engg.-I, Bangalore Univ.)



**Solution.** It is a case of pure resistance in series with pure inductance as shown in Fig. 13.16 (a).

$$\text{Here } V_R = 100 \text{ V,}$$

$$V_L = \sqrt{(220^2 - 100^2)} = 196 \text{ V}$$

$$\text{Now, } V_L = I \cdot X_L$$

$$\text{or } 196 = 8 \times 2\pi \times 50 \times L = \mathbf{0.078 \text{ H}}$$

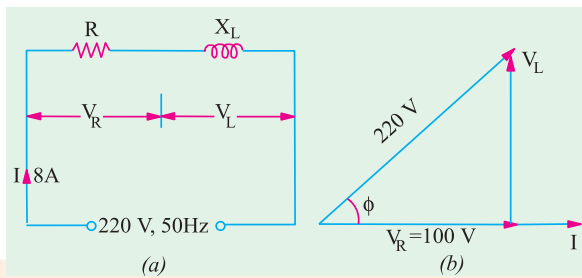


Fig. 13.16

**Example 13.19.** A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250-V, 50-Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate

(a) impedance, reactance and resistance of the coil (b) the power absorbed by the coil and (c) the total power. Draw the vector diagram. (Elect. Engg., Madras Univ.)

**Solution.** As seen from the vector diagram of Fig. 13.17 (b).

$$BC^2 + CD^2 = 200^2 \quad \dots(i) \quad (125 + BC)^2 + CD^2 = 250^2 \quad \dots(ii)$$

Subtracting Eq. (i) from (ii), we get,  $(125 + BC)^2 - BC^2 = 250^2 - 200^2$

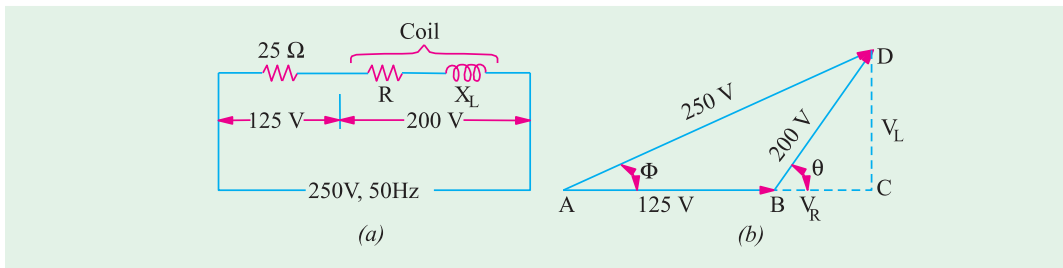


Fig. 13.17

$$\therefore BC = 27.5 \text{ V; } CD = \sqrt{200^2 - 27.5^2} = 198.1 \text{ V}$$

$$(i) \text{ Coil impedance} = 200/5 = \mathbf{40 \Omega}$$

$$V_R = IR = BC \quad \text{or } 5R = 27.5 \quad \therefore P = 27.5/5 = \mathbf{5.5 \text{ W}}$$

$$\text{Also } V_L = I \cdot X_L = CD = 198.1 \quad \therefore X_L = 198.1/5 = \mathbf{39.62 \Omega}$$

$$\text{or } X_L = \sqrt{40^2 - 5.5^2} = \mathbf{39.62 \Omega}$$

$$(ii) \text{ Power absorbed by the coil is } I^2 R = 5^2 \times 5.5 = 137.5 \text{ W}$$

$$\text{Also } P = 200 \times 5 \times 27.5/200 = \mathbf{137.5 \text{ W}}$$

$$(iii) \text{ Total power} = VI \cos \phi = 250 \times 5 \times AC/AD = 250 \times 5 \times 152.5/250 = \mathbf{762.5 \text{ W}}$$

The power may also be calculated by using  $I^2 R$  formula.

$$\text{Series resistance} = 125/5 = 25 \Omega$$

$$\text{Total circuit resistance} = 25 + 5.5 = 30.5 \Omega$$

$$\therefore \text{ Total power} = 5^2 \times 30.5 = \mathbf{762.5 \text{ W}}$$

**Example 13.20.** Two coils A and B are connected in series across a 240-V, 50-Hz supply. The resistance of A is 5  $\Omega$  and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.

(Elect. Technology Hyderabad Univ. 1991)

**Solution.** The kVA triangle is shown in Fig. 13.18 (b) and the circuit in Fig. 13.18(a). The circuit kVA is given by,  $\text{kVA} = \sqrt{(3^2 + 2^2)} = 3.606$  or  $VA = 3,606$  voltmeters

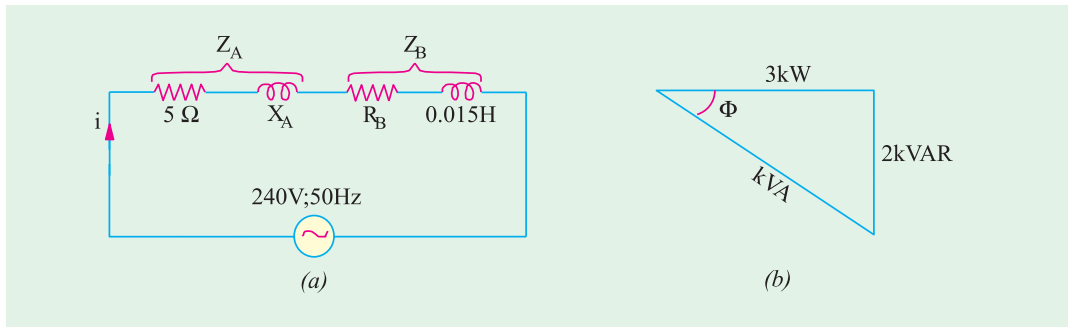


Fig. 13.18

Circuit current =  $3,606/240 = 15.03 \text{ A}$   $\therefore 15.03^2 (R_A + R_B) = 3,000$

$\therefore R_A + R_B = 3,000/15.03^2 = 13.3 \Omega$   $\therefore R_B = 13.3 - 5 = 8.3 \Omega$

Now, impedance of the whole circuit is given by  $Z = 240/15.03 = 15.97 \Omega$

$\therefore X_A + X_B = \sqrt{(Z^2 - (R_A + R_B)^2)} = \sqrt{15.97^2 - 13.3^2} = 8.84 \Omega$

Now  $X_B = 2\pi \times 50 \times 0.015 = 4.713 \Omega$   $\therefore X_A = 8.843 - 4.713 = 4.13 \Omega$

or  $2\pi \times 50 \times L_A = 4.13$   $\therefore L_A = 0.0132 \text{ H (approx)}$

Now  $Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.13^2} = 6.585 \Omega$

P.D. across coil A =  $I \cdot Z_A = 15.03 \times 6.485 = 97.5 \text{ V}$ ;  $Z_b = \sqrt{8.3^2 + 4.713^2} = 9.545 \Omega$

$\therefore$  p.d. across coil B =  $I \cdot Z_B = 15.03 \times 9.545 = 143.5 \text{ V}$

**Example 13.21.** An e.m.f.  $e_0 = 141.4 \sin(377t + 30^\circ)$  is impressed on the impedance coil having a resistance of  $4 \Omega$  and an inductive reactance of  $1.25 \Omega$  measured at  $25 \text{ Hz}$ . What is the equation of the current? Sketch the waves for  $i$ ,  $e_R$ ,  $e_L$  and  $e_0$ .

**Solution.** The frequency of the applied voltage is  $f = 377/2\pi = 60 \text{ Hz}$

Since coil reactance is  $1.25 \Omega$  at  $25 \text{ Hz}$ , its value at  $60 \text{ Hz} = 1.25 \times 60/25 = 3 \Omega$

Coil impedance,  $Z = \sqrt{4^2 + 3^2} = 5 \Omega$ ;  $\phi = \tan^{-1}(3/4) = 36^\circ 52'$

It means that circuit current lags behind the applied voltage by  $36^\circ 52'$ . Hence, equation of the circuit current is

$i = (141.4/5) \sin(377t + 30^\circ - 36^\circ 52') = 28.3 \sin(377t - 6^\circ 52')$

Since, resistance drop is in phase with current, its equation is  $e_R = iR = 113.2 \sin(377t - 6^\circ 52')$

The inductive voltage drop **leads the current** by  $90^\circ$ , hence its equation is

$e_L = iX_L = 3 \times 28.3 \sin(377t - 6^\circ 52' + 90^\circ) = 54.9 \sin(377t + 83^\circ 8')$

The waves for  $i$ ,  $e_R$ ,  $e_L$  and  $e_0$  have been drawn in Fig. 13.19.

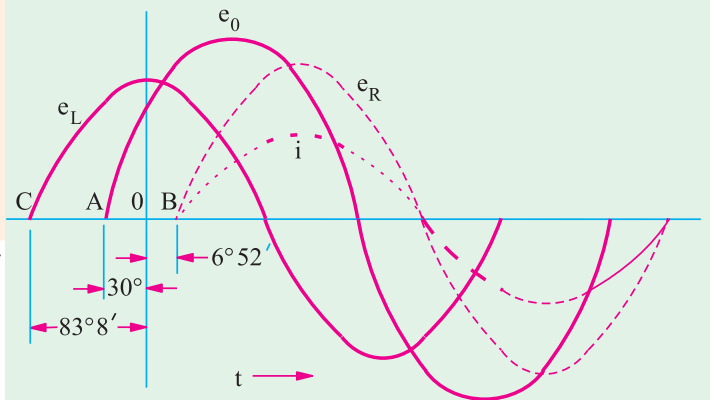


Fig. 13.19

**Example 13.22.** A single phase, 7.46 kW motor is supplied from a 400-V, 50-Hz a.c. mains. If its efficiency is 85% and power factor 0.8 lagging, calculate (a) the kVA input (b) the reactive components of input current and (c) kVAR.

**Solution.** Efficiency =  $\frac{\text{output in watts}}{\text{input in watts}}$   $\therefore 0.85 = \frac{7.46 \times 1000}{VI \cos \phi} = \frac{7,460}{VI \times 0.8}$

$\therefore VI = \frac{7460}{0.85 \times 0.8} = 10,970$  voltamperes

(a)  $\therefore$  Input =  $10,970/1000 = 10.97$  kVA

(b) Input current  $I = \frac{\text{voltamperes}}{\text{volts}} = \frac{10,970}{400} = 27.43$  A

Active component of current =  $I \cos \phi = 27.43 \times 0.8 = 21.94$  A

Reactive component of current =  $I \sin \phi = 27.43 \times 0.6 = 16.46$  A ( $\because \sin \phi = 0.6$ )

(Reactive component =  $\sqrt{27.43^2 - 21.94^2} = 16.46$  A

(c) kVAR = kVA  $\sin \phi = 10.97 \times 0.6 = 6.58$  (or kVAR =  $VI \sin \phi \times 10^{-3} = 400 \times 16.46 \times 10^{-3} = 6.58$ )

**Example 13.23.** Draw the phasor diagram for each of the following combinations :

(i) R and L in series and combination in parallel with C.

(ii) R, L and C in series with  $X_C > X_L$  when ac voltage source is connected to it.

[Nagpur University—Summer 2000]

**Solution. (i)**

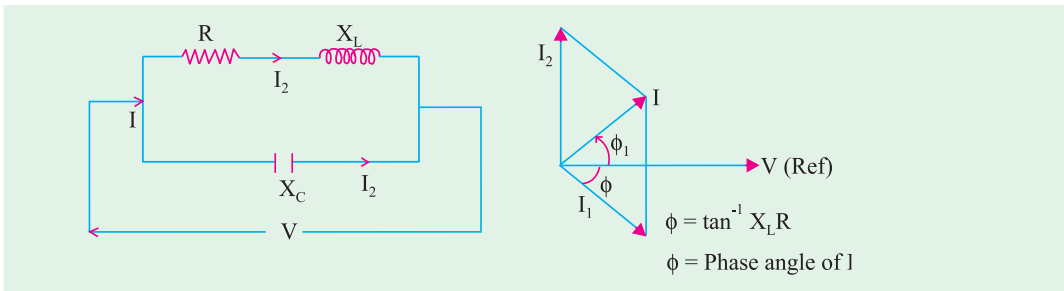


Fig. 13.20 (a) Circuit

Fig. 13.20 (b) Phasor diagram

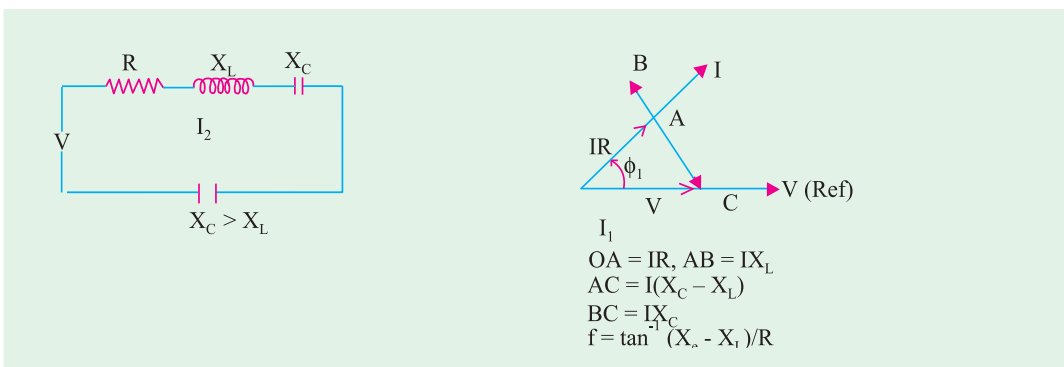


Fig. 13.20 (c) Circuit

Fig. 13.20 (d) Phasor diagram

**Example 13.24.** A voltage  $v(t) = 141.4 \sin(314t + 10^\circ)$  is applied to a circuit and the steady current given by  $i(t) = 14.14 \sin(314t - 20^\circ)$  is found to flow through it.

Determine

(i) The p.f. of the circuit

(ii) The power delivered to the circuit

(iii) Draw the phasor diagram.

[Nagpur University Summer 2000]

**Solution.**  $v(t) = 141.4 \sin(314t + 10^\circ)$

This expression indicates a sinusoidally varying alternating voltage at a frequency

$\omega = 314 \text{ rad/sec}, f = 50 \text{ Hz}$

$V = \text{RMS voltage (Peak voltage)} / \sqrt{2} = 100 \text{ volts}$

The expression for the current gives the following

data :

$I = \text{RMS value} = 14.14 / \sqrt{2} = 10 \text{ amp}$

frequency = 50 Hz, naturally.

Phase shift between  $I$  and  $V = 30^\circ$ ,  $I$  lags behind  $V$ .

(i) Power factor of the circuit =  $\cos 30^\circ = 0.866 \text{ lag}$

(ii)  $P = VI \cos \phi = 100 \times 10 \times 0.866 = 866 \text{ watts}$

(iii) Phasor diagram as drawn below, in Fig. 13.21 (a).

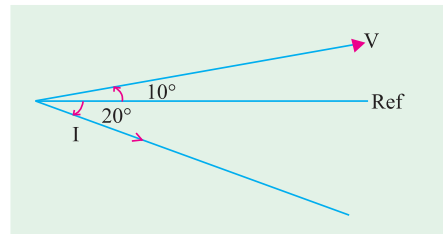


Fig. 13.21 (a) Phasor diagram

**Example 13.25.** A coil of 0.8 p.f. is connected in series with 110 micro-farad capacitor. Supply frequency is 50 Hz. The potential difference across the coil is found to be equal to that across the capacitor. Calculate the resistance and the inductance of the coil. Calculate the net power factor.

[Nagpur University, November 1997]

**Solution.**  $X_C = 1/(3.14 \times C) = 28.952 \text{ ohms}$

$\therefore$  Coil Impedance,  $Z = 28.952 \Omega$

Coil resistance =  $28.952 \times 0.8 = 23.162 \Omega$

Coil reactance = 17.37 ohms

Coil-inductance =  $17.37/314 = 55.32 \text{ milli-henrys}$

Total impedance,  $Z_T = 23.16 + j 17.37 - j 28.952 = 23.162 - j 11.582 = 25.9 \text{ ohms}$

Net power-factor =  $23.162/25.9 = 0.8943 \text{ leading}$

**Example 13.26.** For the circuit shown in Fig. 13.21 (c), find the values of  $R$  and  $C$  so that  $V_b = 3V_a$ , and  $V_b$  and  $V_a$  are in phase quadrature. Find also the phase relationships between  $V_s$  and  $V_b$ , and  $V_b$  and  $I$ .

[Rajiv Gandhi Technical University, Bhopal, Summer 2001]

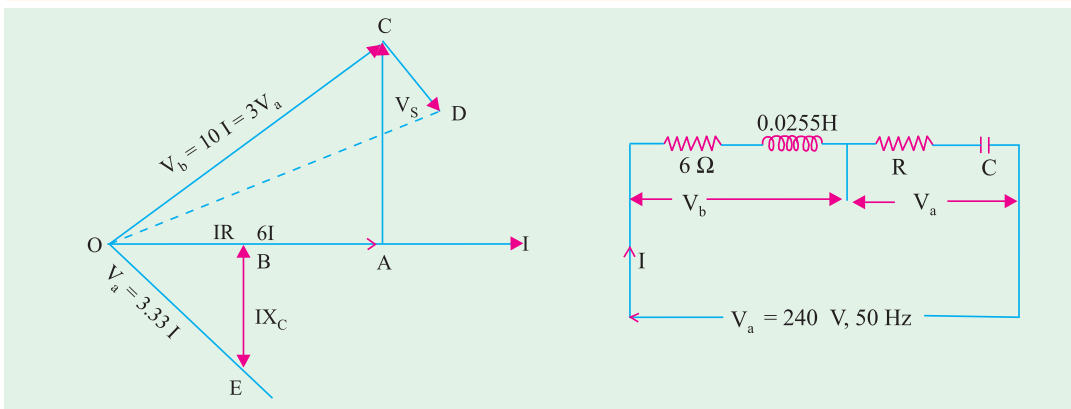


Fig. 13.21 (b)

Fig. 13.21 (c)

**Solution.**  $\angle COA = \phi = 53.13^\circ$   
 $\angle BOE = 90^\circ - 53.13^\circ = 36.87^\circ$   
 $\angle DOA = 34.7^\circ$  Angle between  $V$  and  $I$

Angle between  $V_s$  and  $V_b = 18.43^\circ$

$$X_L = 314 \times 0.0255 = 8 \text{ ohms}$$

$$Z_b = 6 + j8 = 10 \angle 53.13^\circ \text{ ohms}$$

$$V_b = 10 I = 3 V_a, \text{ and hence } V_a = 3.33 I$$

In phasor diagram,  $I$  has been taken as reference.  $V_b$  is in first quadrant. Hence  $V_a$  must be in the fourth quadrant, since  $Z_a$  consists of  $R$  and  $X_c$ . Angle between  $V_a$  and  $I$  is then  $36.87^\circ$ . Since  $Z_a$  and  $Z_b$  are in series,  $V$  is represented by the phasor  $OD$  which is at angle of  $34.7^\circ$ .

$$|V| = \sqrt{10} V_a = 10.53 I$$

Thus, the circuit has a total effective impedance of 10.53 ohms.

In the phasor diagram,  $OA = 6 I$ ,  $AC = 8 I$ ,  $OC = 10 I = V_b = 3 V_a$

Hence,  $V_a = OE = 3.33 I$ ,

Since  $BOE = 36.87^\circ$ ,  $OB = RI = OE \times \cos 36.87^\circ = 3.33 \times 0.8 \times I = 2.66 I$ .

Hence,  $R = 2.66$

And  $BE = OE \sin 36.87^\circ = 3.33 \times 0.6 \times I = 2 I$

Hence  $X_c = 2$  ohms. For  $X_c = 2$  ohms,  $C = 1/(314 \times 2) = 1592 \mu\text{F}$

Horizontal component of  $OD = OB + OA = 8.66 I$

Vertical component of  $OD = AC - BE = 6 I$

$$OD = 10.54 I = V_s$$

Hence, the total impedance = 10.54 ohms =  $8.66 + j 6$  ohms

Angle between  $V_s$  and  $I = \angle DOA = \tan^{-1}(6/8.66) = 34.7^\circ$

**Example 13.27.** A coil is connected in series with a pure capacitor. The combination is fed from a 10 V supply of 10,000 Hz. It was observed that the maximum current of 2 Amp flows in the circuit when the capacitor is of value 1 microfarad. Find the parameters ( $R$  and  $L$ ) of the coil.

[Nagpur University April 1996]

**Solution.** This is the situation of resonance in A.C. Series circuit, for which  $X_L = X_C$

$$Z = R = V/I = 10/2 = 5 \text{ ohms}$$

If  $\omega_0$  is the angular frequency, at resonance,  $L$  and  $C$  are related by  $\omega_0^2 = 1/(LC)$ ,

which gives  $L = 1/(\omega_0^2 C) = 2.5 \times 10^{-4} \text{ H} = 0.25 \text{ mH}$

**Example 13.28.** Two impedances consist of (resistance of 15 ohms and series-connected inductance of 0.04 H) and (resistance of 10 ohms, inductance of 0.1 H and a capacitance of 100  $\mu\text{F}$ , all in series) are connected in series and are connected to a 230 V, 50 Hz a.c. source. Find : (i) Current drawn, (ii) Voltage across each impedance, (iii) Individual and total power factor. Draw the phasor diagram.

[Nagpur University, Nov. 1996]

**Solution.** Let suffix 1 be used for first impedance, and 2 for the second one. At 50 Hz,

$$Z_1 = 15 + j(314 \times 0.04) = 15 + j 12.56 \text{ ohms}$$

$$Z_1 = \sqrt{15^2 + 12.56^2} = 19.56 \text{ ohms,}$$

Impedance-angle,  $\theta_1 = \cos^{-1}(15/19.56) = +40^\circ$ ,

$$Z_2 = 10 + j(314 \times 0.1) - j\{1/(314 \times 100 \times 10^{-6})\}$$

$$= 10 + j 31.4 - j 31.85 = 10 - j 0.45$$

$$= 10.01 \text{ ohms, Impedance angle, } \theta_2 = -2.56^\circ,$$

Total Impedance,  $Z = Z_1 + Z_2 = 15 + j 12.56 + 10 - j 0.85$

$$= 25 + j 12.11 = 27.78 \angle 25.85^\circ$$

For this, Phase-angle of  $+25.85^\circ$ , the power-factor of the total impedance  
 $= \cos 25.85^\circ = 0.90$ , Lag.

Current drawn  $= 230/27.78 = 8.28$  Amp, at 0.90 lagging p.f.

$$V_1 = 8.28 \times 19.56 = 162 \text{ Volts}$$

$$V_2 = 8.28 \times 10.01 = 82.9 \text{ Volts}$$

#### Individual Power-factor

$$\cos \theta_1 = \cos 40^\circ = 0.766 \text{ Lagging}$$

$$\cos \theta_2 = \cos 2.56^\circ = 0.999 \text{ leading}$$

**Phasor diagram :** In case of a series circuit, it is easier to treat the current as a reference. The phasor diagram is drawn as in Fig. 13.22.

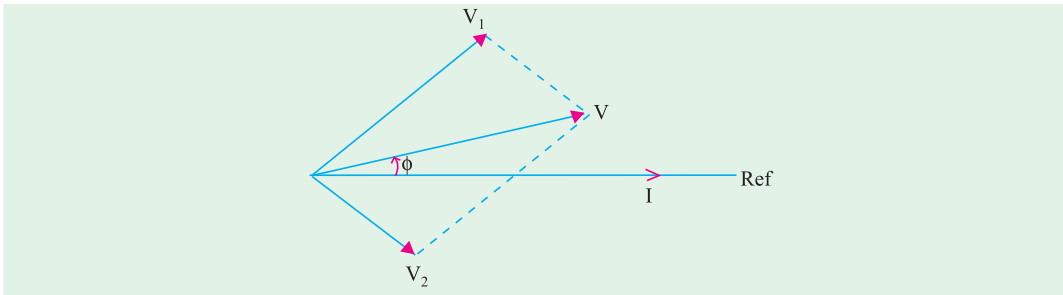


Fig. 13.22

**Example 13.29.** Resistor ( $= R$ ), choke-coil ( $r, L$ ), and a capacitor of  $25.2 \mu\text{F}$  are connected in series. When supplied from an A.C. source, it takes  $0.4 \text{ A}$ . If the voltage across the resistor is  $20 \text{ V}$ , voltage across the resistor and choke is  $45 \text{ volts}$ , voltage across the choke is  $35 \text{ volts}$ , and voltage across the capacitor is  $50 \text{ V}$ .

**Find :** (a) The values of  $r, L$  (b) Applied voltage and its frequency, (c) P.F. of the total circuit and active power consumed. Draw the phasor diagram. [Nagpur Univ. April 1998]

#### Solution.

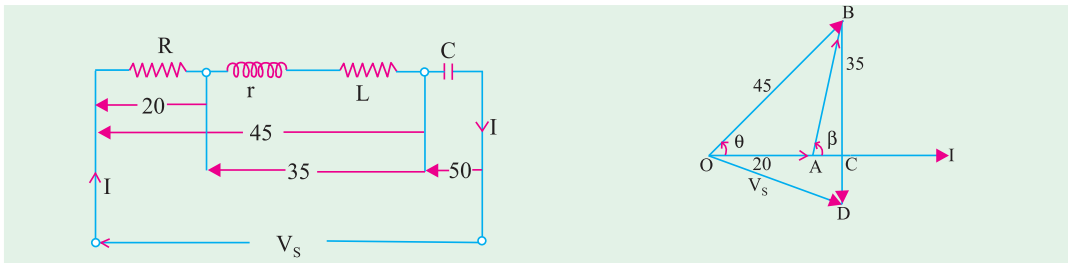


Fig. 13.23 (a)

Fig. 13.23 (b)

(b) Since the current  $I$  is  $0.4 \text{ amp}$ , and voltage drop across  $R$  is  $20 \text{ V}$ ,

$$R = 20/0.4 = 50 \text{ ohms}$$

Similarly, Impedance of the coil,  $Z_1 = 35/0.4 = 87.5 \text{ ohms}$

Capacitive reactance  $X_C = 50/0.4 = 125 \text{ ohms}$

With a capacitor of  $25.5 \mu\text{F}$ , and supply angular frequency of  $\omega$  radians/sec

$$\frac{1}{\omega} = X_C \cdot C = 125 \times 25.5 \times 10^{-6}, \text{ which gives } \omega = 314 \text{ ra./sec.}$$

The corresponding source frequency,  $f = 50 \text{ Hz}$

(c) The phasor diagram is drawn in Fig. 13.23 (b), taking  $I$  as the reference.

Solving triangle  $OAB$ ,

$$\cos \phi = \frac{45^2 + 20^2 - 35^2}{2 \times 45 \times 20} = 0.667, \text{ and hence } \phi = 48.2^\circ$$

$$\text{Similarly, } \cos (180^\circ - \beta) = \frac{400 + 1225 - 2025}{2 \times 20 \times 35} = 106.6^\circ$$

This gives  $\beta = 73.4^\circ$ . From the phasor diagram is Fig. 13.23 (b),

$$OC = OA + AC = 20 + 35 \cos \beta = 30$$

$$BC = 35 \sin \beta = 33.54. \text{ The capacitive-reactance drop is } BD.$$

Since  $BD = 50, CD = 16.46$  volts.

$$V_s = \sqrt{OC^2 + CD^2} = 34.22 \text{ volts}$$

$$\angle COD = \phi = \cos^{-1} (OC/OD) = 28.75^\circ$$

The power-factor of the total circuit =  $\cos \phi = 0.877$ , Leading,

Since  $I$  Leads  $V_s$  in Fig. 13.23 (b).

(a) For the coil,  $ACB$  part of the phasor diagram is to be observed.

$$r = AC/I = 10/0.4 = 25 \text{ ohms}$$

$$X_L = BC/I = 33.54/0.4 = 83.85 \text{ ohms}$$

Hence, coil-inductance,  $L = 83.85/314 = 267$  milli-henry.

$$P = \text{Active Power Consumed} = V_s I \cos \phi = 12 \text{ watts}$$

$$\text{or } P = (0.4)^2 \times (R + r) = 12 \text{ watts}$$

### 13.6. Power in an Iron-cored Choking Coil

Total power,  $P$  taken by an iron-cored choking coil is used to supply

(i) power loss in ohmic resistance *i.e.*  $I^2 R$ . (ii) iron-loss in core,  $P_i$

$\therefore P = I^2 R + P_i$  or  $\frac{P}{I^2} = R + \frac{P_i}{I^2}$  is known as the **effective resistance\*** of the choke.

$\therefore$  effective resistance = true resistance = equivalent resistance  $\frac{P_i}{I^2} \therefore R_{eff} = \frac{P}{I^2} = R + \frac{P_i}{I^2}$

**Example 13.30.** An iron-cored choking coil takes 5 A when connected to a 20-V d.c. supply and takes 5 A at 100 V a.c. and consumes 250 W. Determine (a) impedance (b) the power factor (c) the iron loss (d) inductance of the coil. **(Elect. Engg. & M.A.S.I. June, 1991)**

**Solution.** (a)  $Z = 100/5 = 20 \Omega$

(b)  $P = VI \cos \phi$  or  $250 = 100 \times 5 \times \cos \phi \therefore \cos \phi = 250/500 = 0.5$

(c) Total loss = loss in resistance + iron loss  $\therefore 250 = 20 \times 5 + P_i \therefore P_i = 250 - 100 = 150 \text{ W}$

(d) Effective resistance of the choke is  $\frac{P}{I^2} = \frac{250}{25} = 10 \Omega$

$\therefore X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(400 - 100)} = 17.32 \Omega$

**Example 13.31.** An iron-cored choking coil takes 5 A at a power factor of 0.6 when supplied at 100-V, 50 Hz. When the iron core is removed and the supply reduced to 15 V, the current rises to 6 A at power factor of 0.9.

Determine (a) the iron loss in the core (b) the copper loss at 5 A (c) the inductance of the choking coil with core when carrying a current of 5 A.

\* At higher frequencies like radio frequencies, there is skin-effect loss also.

**Solution.** When core is removed, then  $Z = 15/6 = 2.5 \Omega$

True resistance,  $R = Z \cos \phi = 2.5 \times 0.9 = 2.25 \Omega$

**With Iron Core**

Power input =  $100 \times 5 \times 0.6 = 300 \text{ W}$

Power wasted in the true resistance of the choke when current is  $5 \text{ A} = 5^2 \times 2.25 = 56.2 \text{ W}$

(a) Iron loss =  $300 - 56.2 = 244 \text{ W}$  (approx) (b) Cu loss at  $5 \text{ A} = 56.2 \text{ W}$

(c)  $Z = 100/5 = 20 \Omega$ ;  $X_L = Z \sin \phi = 20 \times 0.8 = 16 \Omega$ .  $\therefore 2\pi \times 50 \times L = 16 \therefore L = 0.0509 \text{ H}$

### Tutorial Problem No. 13.1

- The voltage applied to a coil having  $R = 200 \Omega$ ,  $L = 638 \text{ mH}$  is represented by  $e = 20 \sin 100 \pi t$ . Find a corresponding expression for the current and calculate the average value of the power taken by the coil. **[ $i = 0.707 \sin(100 \pi t - \pi/4)$ ; 50 W] (I.E.E. London)**
- The coil having a resistance of  $10 \Omega$  and an inductance of  $0.2 \text{ H}$  is connected to a  $100\text{-V}$ ,  $50\text{-Hz}$  supply. Calculate (a) the impedance of the coil (b) the reactance of the coil (c) the current taken and (d) the phase difference between the current and the applied voltage. **[(a) 63.5  $\Omega$  (b) 62.8  $\Omega$  (c) 1.575 A (d)  $80^\circ 57'$ ] (I.E.E. London)**
- An inductive coil having a resistance of  $15 \Omega$  takes a current of  $4 \text{ A}$  when connected to a  $100\text{-V}$ ,  $60 \text{ Hz}$  supply. If the coil is connected to a  $100\text{-V}$ ,  $50 \text{ Hz}$  supply, calculate (a) the current (b) the power (c) the power factor. Draw to scale the vector diagram for the  $50\text{-Hz}$  conditions, showing the component voltages. **[(a) 4.46 A (b) 298 W (c) 0.669] (I.E.E. London)**
- When supplied with current at  $240\text{-V}$ , single-phase at  $50 \text{ Hz}$ , a certain inductive coil takes  $13.62 \text{ A}$ . If the frequency of supply is changed to  $40 \text{ Hz}$ , the current increases to  $16.12 \text{ A}$ . Calculate the resistance and inductance of the coil. **[17.2 W, 0.05 H] (London Univ.)**
- A voltage  $v(t) = 141.4 \sin(314 t + 10^\circ)$  is applied to a circuit and a steady current given by  $i(t) = 14.4 \sin(314 t - 20^\circ)$  is found to flow through it. Determine (i) the p.f. of the circuit and (ii) the power delivered to the circuit. **[0.866 (lag); 866 W] (I.E.E. London)**
- A circuit takes a current of  $8 \text{ A}$  at  $100 \text{ V}$ , the current lagging by  $30^\circ$  behind the applied voltage. Calculate the values of equivalent resistance and reactance of the circuit. **[10.81  $\Omega$ ; 6.25  $\Omega$ ] (I.E.E. London)**
- Two inductive impedances  $A$  and  $B$  are connected in series.  $A$  has  $R = 5 \Omega$ ,  $L = 0.01 \text{ H}$ ;  $B$  has  $R = 3 \Omega$ ,  $L = 0.02 \text{ H}$ . If a sinusoidal voltage of  $230 \text{ V}$  at  $50 \text{ Hz}$  is applied to the whole circuit calculate (a) the current (b) the power factor (c) the voltage drops. Draw a complete vector diagram for the circuit. **[(a) 18.6 (b) 0.648 (c)  $V_A = 109.5 \text{ V}$ ,  $V_B = 129.5 \text{ V}$ ] (I.E.E. London)**
- A coil has an inductance of  $0.1 \text{ H}$  and a resistance of  $30 \Omega$  at  $20^\circ \text{C}$ . Calculate (i) the current and (ii) the power taken from  $100\text{-V}$ ,  $50\text{-Hz}$  mains when the temperature of the coil is  $60^\circ \text{C}$ , assuming the temperature coefficient of resistance to be  $0.4\% \text{ per}^\circ \text{C}$  from a basic temperature of  $20^\circ \text{C}$ . **[(i) 2.13 A (ii) 158.5 W] (London Univ.)**
- An air-cored choking coil takes a current of  $2 \text{ A}$  and dissipates  $200 \text{ W}$  when connected to a  $200\text{-V}$ ,  $50\text{-Hz}$  mains. In other coil, the current taken is  $3 \text{ A}$  and the power  $270 \text{ W}$  under the same conditions. Calculate the current taken and the total power consumed when the coils are in series and connected to the same supply. **[1.2, 115 W] (City and Guilds, London)**
- A circuit consists of a pure resistance and a coil in series. The power dissipated in the resistance is  $500 \text{ W}$  and the drop across it is  $100 \text{ V}$ . The power dissipated in the coil is  $100 \text{ W}$  and the drop across it is  $50 \text{ V}$ . Find the reactance and resistance of the coil and the supply voltage. **[9.168  $\Omega$ ; 4  $\Omega$ ; 128.5 V] (I.E.E. London)**
- A choking coil carries a current of  $15 \text{ A}$  when supplied from a  $50\text{-Hz}$ ,  $230\text{-V}$  supply. The power in the circuit is measured by a wattmeter and is found to be  $1300 \text{ watt}$ . Estimate the phase difference between the current and p.d. in the circuit. **[0.3768] (I.E.E. London)**
- An ohmic resistance is connected in series with a coil across  $230\text{-V}$ ,  $50\text{-Hz}$  supply. The current is  $1.8$



A and p.d.s. across the resistance and coil are 80 V and 170 V respectively. Calculate the resistance and inductance of the coil and the phase difference between the current and the supply voltage.

[61.1  $\Omega$ , 0.229 H, 34°20' ] (*App. Elect. London Univ.*)

13. A coil takes a current of 4 A when 24 V d.c. are applied and for the same power on a 50-Hz a.c. supply, the applied voltage is 40. Explain the reason for the difference in the applied voltage. Determine (a) the reactance (b) the inductance (c) the angle between the applied p.d. and current (d) the power in watts.  
[(a) 8  $\Omega$  (b) 0.0255 H (c) 53°7' (d) 96 W]
14. An inductive coil and a non-inductive resistance  $R$  ohms are connected in series across an a.c. supply. Derive expressions for the power taken by the coil and its power factor in terms of the voltage across the coil, the resistance and the supply respectively. If  $R = 12 \Omega$  and the three voltages are in order, 110 V, 180 V and 240 V, calculate the power and the power factor of the coil. [546 W; 0.331]
15. Two coils are connected in series. With 2 A d.c. through the circuit, the p.d.s. across the coils are 20 and 30 V respectively. With 2 A a.c. at 40 Hz, the p.d.s. across the coils are 140 and 100 V respectively. If the two coils in series are connected to a 230-V, 50-Hz supply, calculate (a) the current (b) the power (c) the power factor. [(a) 1.55 A (b) 60 W (c) 0.1684]
16. It is desired to run a bank of ten 100-W, 10-V lamps in parallel from a 230-V, 50-Hz supply by inserting a choke coil in series with the bank of lamps. If the choke coil has a power factor of 0.2, find its resistance, reactance and inductance. [ $R = 4.144 \Omega$ ,  $X = 20.35 \Omega$ ,  $L = 0.065$  H] (*London Univ.*)
17. At a frequency for which  $\omega = 796$ , an e.m.f. of 6 V sends a current of 100 mA through a certain circuit. When the frequency is raised so that  $\omega = 2866$ , the same voltage sends only 50 mA through the same circuit. Of what does the circuit consist? [ $R = 52 \Omega$ ,  $L = 0.038$  H in series] (*I.E.E. London*)
18. An iron-cored electromagnet has a d.c. resistance of 7.5  $\Omega$  and when connected to a 400-V 50-Hz supply, takes 10 A and consumes 2 kW. Calculate for this value of current (a) power loss in iron core (b) the inductance of coil (c) the power factor (d) the value of series resistance which is equivalent to the effect of iron loss. [1.25 kW, 0.11 H, 0.5; 12.5  $\Omega$ ] (*I.E.E. London*)

### 13.7. A.C. Through Resistance and Capacitance

The circuit is shown in Fig. 13.24 (a). Here  $V_R = IR =$  drop across  $R$  in phase with  $I$ .

$V_C = IX_C =$  drop across capacitor –lagging  $I$  by  $\pi/2$

As capacitive reactance  $X_C$  is taken negative,  $V_C$  is shown along negative direction of  $Y$ -axis in the voltage triangle [Fig. 13.24 (b)]

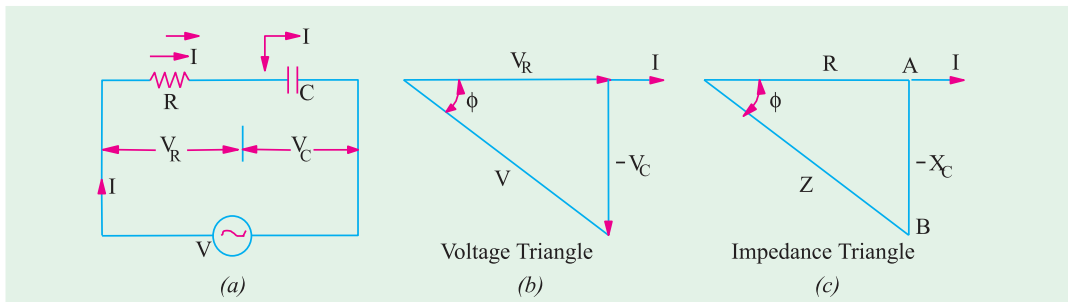


Fig. 13.24

$$\text{Now } V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I \sqrt{R^2 + X_C^2} \quad \text{or} \quad I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

The denominator is called the **impedance** of the circuit. So,  $Z = \sqrt{R^2 + X_C^2}$

Impedance triangle is shown in Fig. 13.24 (c)

From Fig. 13.24 (b) it is found that  $I$  leads  $V$  by angle  $\phi$  such that  $\tan \phi = -X_C/R$

Hence, it means that if the equation of the applied alternating voltage is  $v = V_m \sin \omega t$ , the equation of the resultant current in the  $R$ - $C$  circuit is  $i = I_m \sin (\omega t + \phi)$  so that current **leads** the applied voltage by an angle  $\phi$ . This fact is shown graphically in Fig. 13.25.

**Example 13.32.** An a.c. voltage  $(80 + j 60)$  volts is applied to a circuit and the current flowing is  $(-4 + j 10)$  amperes. Find (i) impedance of the circuit (ii) power consumed and (iii) phase angle. [Elect. Technology, Indore, Univ., Bombay Univ. 1999]

**Solution.**  $V = (80 + j 60) = 100 \angle 36.9^\circ$  ;

$I = -4 + j 10 = 10.77 \angle \tan^{-1}(-2.5) = 10.77 \angle (180^\circ - 68.2^\circ) = 10.77 \angle 111.8^\circ$

$$(i) \quad Z = V/I = 100 \angle 36.9^\circ / 10.77 \angle 111.8^\circ \\ = 9.28 \angle -74.9^\circ$$

$$= 9.28 (\cos 74.9^\circ - j \sin 74.9^\circ) = 2.42 - j 8.96 \Omega$$

Hence  $R = 2.42 \Omega$  and  $X_C = 8.96 \Omega$  capacitive

$$(ii) \quad P = I^2 R = 10.77^2 \times 2.42 = \mathbf{2.81 \text{ W}}$$

(iii) Phase angle between voltage and current =  $74.9^\circ$  with current **leading** as shown in Fig. 13.26.

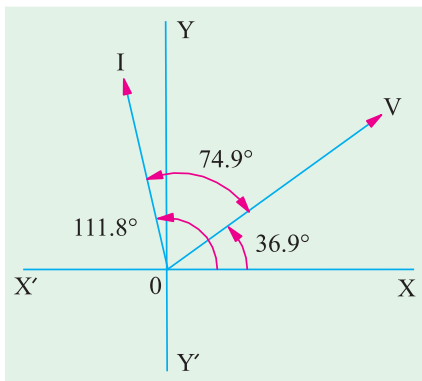


Fig. 13.26

### Alternative Method for Power

The method of conjugates will be used to determine the real power and reactive volt-ampere. It is a convenient way of calculating these quantities when both voltage and current are expressed in cartesian form. If the conjugate of current is multiplied by the voltage in cartesian form, the result is a complex quantity, the real part of which gives the real power  $j$  part of which gives the reactive volt-amperes ( $VAR$ ). It should, however, be noted that real power as obtained by this method of conjugates is the same regardless of whether  $V$  or  $I$  is reversed although sign of voltamperes will depend on the choice of  $V$  or  $I$ .\*

Using current conjugate, we get  $P_{VA} = (80 + j 60) (-4 - j 10) = 280 - j 1040$

$\therefore$  Power consumed =  $\mathbf{280 \text{ W}}$

**Example 13.33.** In a circuit, the applied voltage is  $100 \text{ V}$  and is found to lag the current of  $10 \text{ A}$  by  $30^\circ$ . (i) Is the p.f. lagging or leading? (ii) What is the value of p.f.?

(iii) Is the circuit inductive or capacitive? (iv) What is the value of active and reactive power in the circuit? (Basic Electricity, Bombay Univ.)

**Solution.** The applied voltage lags behind the current which, in other words, means that current leads the voltage.

(i)  $\therefore$  p.f. is **leading** (ii) p.f. =  $\cos \phi = \cos 30^\circ = \mathbf{0.866}$  (lead) (iii) Circuit is **capacitive**

(iv) Active power =  $VI \cos \phi = 100 \times 10 \times 0.866 = \mathbf{866 \text{ W}}$

$$\text{Reactive power} = VI \sin \phi = 100 \times 10 \times 0.5 = \mathbf{500 \text{ VAR (lead)}}$$

$$\text{or} \quad VAR = \sqrt{(VA)^2 - W^2} = \sqrt{(100 \times 10)^2 - 866^2} = \mathbf{500 \text{ (lead)}}$$

\* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs positive.

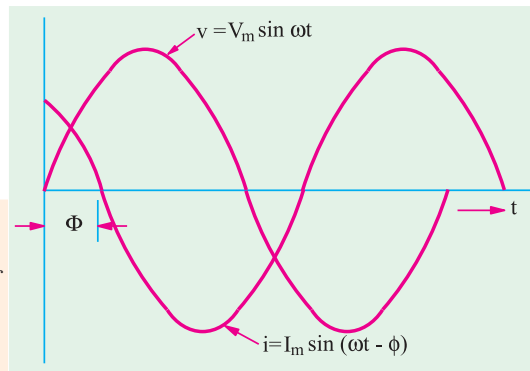


Fig. 13.25

**Example 13.34.** A tungsten filament bulb rated at 500-W, 100-V is to be connected to series with a capacitance across 200-V, 50-Hz supply. Calculate :

(a) the value of capacitor such that the voltage and power consumed by the bulb are according to the rating of the bulb. (b) the power factor of the current drawn from the supply. (c) draw the phasor diagram of the circuit.  
(Elect. Technology-1, Nagpur Univ. 1991)

**Solution.** The rated values for bulb are :  
voltage = 100 V and current  $I = W/V = 500/100 = 5\text{A}$ . Obviously, the bulb has been treated as a pure resistance :

$$(a) V_C = \sqrt{220^2 - 100^2} = 196\text{ V}$$

$$\text{Now, } IX_C = 196 \text{ or } 5 X_C = 196, X_C = 39.2\ \Omega$$

$$\therefore I/\omega C = 39.2 \text{ or } C = 1/314 \times 39.2 = \mathbf{81\ \mu\text{F}}$$

(b) p.f. =  $\cos \phi = V_R/V = 100/200 = \mathbf{0.455}$   
(lead)

(c) The phasor diagram is shown in Fig 13.27.

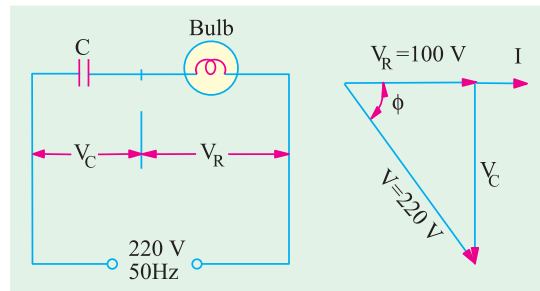


Fig 13.27

**Example 13.35.** A pure resistance of 50 ohms is in series with a pure capacitance of 100 microfarads. The series combination is connected across 100-V, 50-Hz supply. Find (a) the impedance (b) current (c) power factor (d) phase angle (e) voltage across resistor (f) voltage across capacitor. Draw the vector diagram.  
(Elect. Engg.-1, JNT Univ. Warrangel)

$$\text{Solution. } X_C = 10^6 \pi / 2\pi \times 50 \times 100 = 32\ \Omega; R = 50\ \Omega$$

$$(a) Z = \sqrt{50^2 + 32^2} = \mathbf{59.4\ \Omega} \quad (b) I = V/Z = 100/59.4 = \mathbf{1.684\ \text{A}}$$

$$(c) \text{ p.f.} = R/Z = 50/59.4 = \mathbf{0.842 \text{ (lead)}} \quad (d) \phi = \cos^{-1}(0.842) = \mathbf{32^\circ 36'}$$

$$(e) V_R = IR = 50 \times 1.684 = \mathbf{84.2\ \text{V}} \quad (f) V_C = IX_C = 32 \times 1.684 = \mathbf{53.9\ \text{V}}$$

**Example 13.36.** A 240-V, 50-Hz series R-C circuit takes an r.m.s. current of 20 A. The maximum value of the current occurs 1/900 second before the maximum value of the voltage. Calculate (i) the power factor (ii) average power (iii) the parameters of the circuit.  
(Elect. Engg.-I, Calcutta Univ.)

**Solution.** Time-period of the alternating voltage is 1/50 second. Now a time interval of 1/50 second corresponds to a phase difference of  $2\pi$  radian or  $360^\circ$ . Hence, a time interval of 1/900 second corresponds to a phase difference of  $360 \times 50/900 = 20^\circ$ .

Hence, current leads the voltage by  $20^\circ$ .

$$(i) \text{ power factor} = \cos 20^\circ = \mathbf{0.9397 \text{ (lead)}}$$

$$(ii) \text{ average power} = 240 \times 20 \times 0.9397 = \mathbf{4,510\ \text{W}}$$

$$(iii) Z = 240/20 = 12\ \Omega; R = Z \cos \Phi = 12 \times 0.9397 = \mathbf{11.28\ \Omega}$$

$$X_C = Z \sin \Phi = 12 \times \sin 20^\circ = 12 \times 0.342 = 4.1\ \Omega$$

$$C = 10^6 / 2\pi \times 50 \times 4.1 = \mathbf{775\ \mu\text{F}}$$

**Example 13.37.** A voltage  $v = 100 \sin 314 t$  is applied to a circuit consisting of a 25  $\Omega$  resistor and an 80  $\mu\text{F}$  capacitor in series. Determine : (a) an expression for the value of the current flowing at any instant (b) the power consumed (c) the p.d. across the capacitor at the instant when the current is one-half of its maximum value.

$$\text{Solution. } X_C = 1/(314 \times 80 \times 10^{-6}) = 39.8\ \Omega, Z = \sqrt{25^2 + 39.8^2} = 47\ \Omega$$

$$I_m = V_m/Z = 100/47 = 2.13\ \text{A}$$

$$\phi = \tan^{-1}(39.8/25) = 57^\circ 52' = 1.01 \text{ radian (lead)}$$

(a) Hence, equation for the instantaneous current

$i = 2.13 \sin(314t + 1.01)$  (b) Power =  $I^2 R = (2.13/\sqrt{2})^2 \times 25 = 56.7 \text{ W}$  (c) The voltage across the capacitor lags the circuit current by  $\pi/2$  radians. Hence, its equation is given by

$$v_c = V_{cm} \sin\left(314t + 1.01 - \frac{\pi}{2}\right) \text{ where } V_{cm} = I_m \times X_C = 2.13 \times 39.8 = 84.8 \text{ V}$$

Now, when  $i$  is equal to half the maximum current (say, in the positive direction) then

$$i = 0.5 \times 2.13 \text{ A}$$

$$\therefore 0.5 \times 2.13 = 2.13 \sin(314t + 1.01) \text{ or } 314t + 1.01 = \sin^{-1}(0.5) = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ radian}$$

$$\therefore v_c = 84.8 \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = 84.8 \sin(-\pi/3) = -73.5 \text{ V}$$

$$\text{or } v_c = 84.8 \sin\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) = 84.8 \sin \pi/3 = 73.5 \text{ V}$$

Hence, p.d. across the capacitor is **73.5 V**

**Example 13.38.** A capacitor and a non-inductive resistance are connected in series to a 200-V, single-phase supply. When a voltmeter having a non-inductive resistance of 13,500  $\Omega$  is connected across the resistor, it reads 132 V and the current then taken from the supply is 22.35 mA.

Indicate on a vector diagram, the voltages across the two components and also the supply current (a) when the voltmeter is connected and (b) when it is disconnected.

**Solution.** The circuit and vector diagrams are shown in Fig. 13.28 (a) and (b) respectively.

$$(a) \quad V_C = \sqrt{200^2 - 132^2} = 150 \text{ V}$$

It is seen that  $\phi = \tan^{-1}(150/132) = 49^\circ$  in Fig. 13.28 (b). Hence

(i) Supply voltage lags behind the current by  $49^\circ$ . (ii)  $V_R$  leads supply voltage by  $49^\circ$  (iii)  $V_C$  lags behind the supply voltage by  $(90^\circ - 49^\circ) = 41^\circ$

The supply current is, as given equal to **22.35 mA**. The value of unknown resistance  $R$  can be found as follows :

$$\text{Current through voltmeter} = 132/13,500 = 9.78 \text{ mA}$$

$$\therefore \text{Current through } R = 22.35 - 9.78 = 12.57 \text{ mA} \therefore R = 132/12.57 \times 10^{-3} = 10,500 \Omega ;$$

$$X_C = 150/22.35 \times 10^{-3} = 6,711 \Omega$$

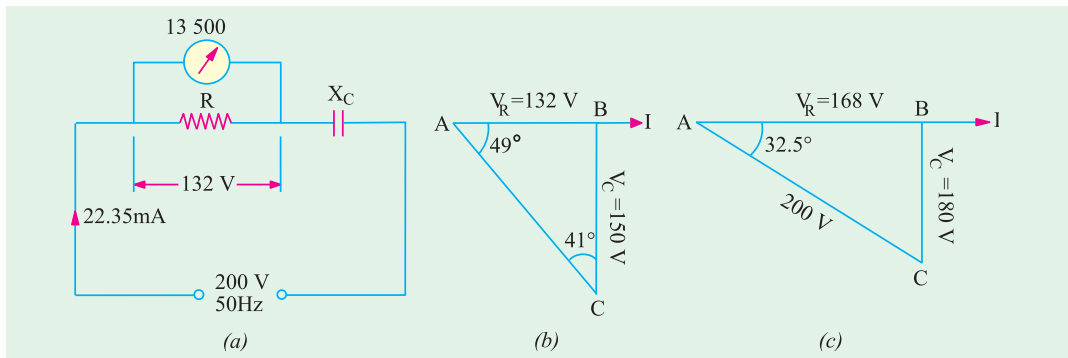


Fig. 13.28

(b) When voltmeter is disconnected,  $Z = \sqrt{R^2 + X_L^2} = \sqrt{10,500^2 + 6,730^2} = 12,500 \Omega$

Supply current =  $200/12,461 = 16.0 \text{ mA}$

In this case,  $V_R = 16.0 \times 10^{-3} \times 10,500 = 168 \text{ V}$

$V_C = 16.0 \times 10^{-3} \times 6711 = 107.4 \text{ V}$ ;  $\tan \phi = 107.4/168$

$$\therefore \phi = 32.5^\circ.$$

In this case, the supply voltage lags the circuit current by  $32.5^\circ$  as shown in Fig. 13.28 (c).

**Example 13.39.** It is desired to operate a 100-W, 120-V electric lamp at its current rating from a 240-V, 50-Hz supply. Give details of the simplest manner in which this could be done using (a) a resistor (b) a capacitor and (c) an inductor having resistance of  $10 \Omega$ . What power factor would be presented to the supply in each case and which method is the most economical of power.

(Principles of Elect. Engg.-I, Jadavpur Univ.)

**Solution.** Rated current of the bulb is  $= 100/120 = 5/6$  A

The bulb can be run at its correct rating by any one of the three methods shown in Fig. 13.29. (a) With reference to Fig. 13.29 (a), we have

$$\text{P.D. across } R = 240 - 120 = 120 \text{ V}$$

$$\therefore R = 120/(5/6) = 144 \Omega$$

Power factor of the circuit is **unity**. Power consumed  $= 240 \times 5/6 = 200 \text{ W}$

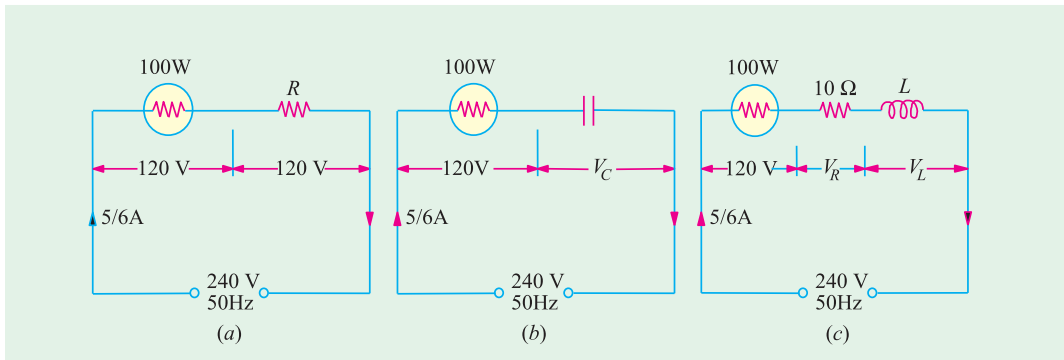


Fig. 13.29

(b) Referring to Fig. 13.29 (b), we have

$$V_C = \sqrt{240^2 - 120^2} = 207.5 \text{ V}; X_C = 207.5 (5/6) = 249 \Omega$$

$$\therefore 1/314 C = 249 \text{ or } C = 12.8 \mu\text{F}; \text{ p.f.} = \cos \phi = 120/240 = 0.5 \text{ (lead)}$$

$$\text{Power consumed} = 240 \times (5/6) \times 0.5 = 100 \text{ W}$$

(c) The circuit connections are shown in Fig 13.29 (c)

$$V_R = (5/6) \times 10 = 25/3 \text{ V} \quad \therefore V_L = \sqrt{240^2 - \left(120 + \frac{25}{3}\right)^2} = 203 \text{ V}$$

$$\therefore 314 L \times (5/6) = 203 \quad \therefore L = 0.775 \text{ H}$$

$$\text{Total resistive drop} = 120 + (25/3) = 128.3 \text{ V}; \cos \phi = 128.3/240 = 0.535 \text{ (lag)}$$

$$\text{Power consumed} = 240 \times (5/6) \times 0.535 = 107 \text{ W}$$

Method (b) is most economical because it involves least consumption of power.

**Example 13.40.** A two-element series circuit consumes 700 W and has a p.f. = 0.707 leading. If applied voltage is  $v = 141.1 \sin(314 t + 30^\circ)$ , find the circuit constants.

**Solution.** The maximum value of voltage is 141.4 V and it leads the reference quantity by  $30^\circ$ . Hence, the given sinusoidal voltage can be expressed in the phase form as

$$V = (141.4/\sqrt{2}) \angle 30^\circ = 100 \angle 30^\circ \text{ now, } P = VI \cos \phi \quad \therefore 700 = 100 \times I \times 0.707; I = 10 \text{ A.}$$

$$\text{Since p.f.} = 0.707 \text{ (lead); } \phi = \cos^{-1}(0.707) = 45^\circ \text{ (lead).}$$

It means that current leads the given voltage by  $45^\circ$  for it **leads the common reference quantity**

by  $(30^\circ + 45^\circ) = 75^\circ$ . Hence, it can be expressed as  $I = 10 \angle 75^\circ$

$$Z = \frac{V}{I} = \frac{100 \angle 30^\circ}{10 \angle 75^\circ} = 10 \angle -45^\circ = j 7.1 \quad R = 7.1 \Omega$$

Since  $X_C = 7.1 \therefore 1/314 C = 7.1; \therefore C = 450 \mu F$

### 13.8. Dielectric Loss and Power Factor of a Capacitor

An ideal capacitor is one in which there are no losses and whose current leads the voltage by  $90^\circ$  as shown in Fig. 13.30 (a). In practice, it is impossible to get such a capacitor although close approximation is achieved by proper design. In every capacitor, there is always some dielectric loss and hence it absorbs some power from the circuit. Due to this loss, the phase angle is somewhat less than  $90^\circ$  [Fig. 13.30 (b)]. In the case of a capacitor with a poor dielectric, the loss can be considerable and the phase angle much less than  $90^\circ$ . This dielectric loss appears as heat. By **phase difference** is meant the difference between the ideal and actual phase angles. As seen from Fig. 13.30 (b), the phase difference  $\psi$  is given by  $\psi = 90 - \phi$  where  $\phi$  is the actual phase angle,  $\sin \psi = \sin (90 - \phi) = \cos \phi$  where  $\cos \phi$  is the power factor of the capacitor.

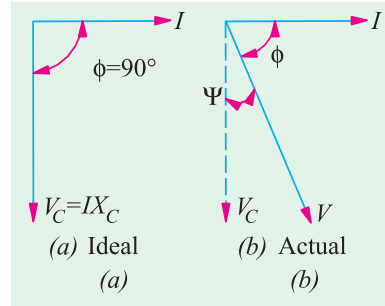


Fig. 13.30

Since  $\psi$  is generally small,  $\sin \psi = \psi$  (in radians)  $\therefore \tan \psi = \psi = \cos \phi$ .

It should be noted that dielectric loss increases with the frequency of the applied voltage. Hence phase difference increases with the frequency  $f$ .

The dielectric loss of an actual capacitor is allowed for by imagining it to consist of a pure capacitor having an equivalent resistance either in series or in parallel with it as shown in Fig. 13.31. These resistances are such that  $I^2 R$  loss in them is equal to the dielectric loss in the capacitor.

As seen from Fig. (13.27b),  $\tan \psi = \frac{IR_{se}}{IX_C} = \frac{R_{se}}{1/C} = CR_{se} \therefore R_{se} = \tan \psi / \omega C = \text{p.f.} / \omega C$

Similarly, as seen from Fig. 13.31 (d),  $\tan \psi = \frac{I_2}{I_1} = \frac{V/R_{sh}}{V/X_C} = \frac{X_C}{R_{sh}} = \frac{1}{CR_{sh}}$

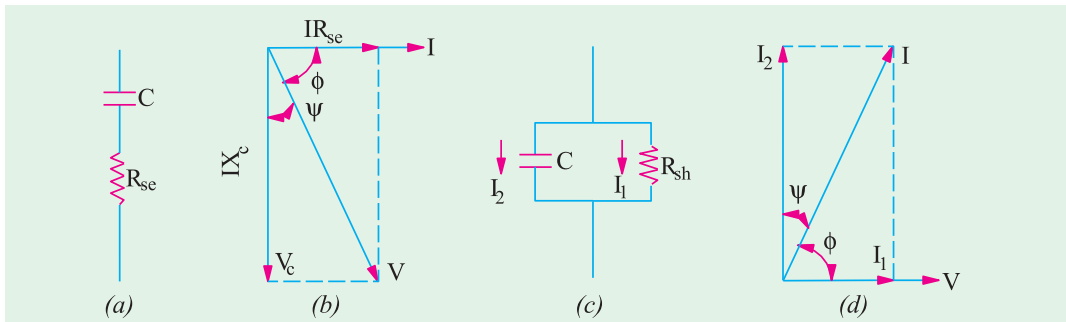


Fig. 13.31

$$R_{sh} = \frac{1}{\omega C \cdot \tan \psi} = \frac{1}{\omega C \times \text{power factor}} = \frac{1}{\omega C \times \text{p.f.}}$$

The power loss in these resistances is  $P = V^2/R_{sh} = \omega CV^2 \tan \psi = \omega CV^2 \times \text{p.f.}$

or  $= I^2 R_{se} = (I^2 \times \text{p.f.}) / \omega C$

where p.f. stands for the power factor of the capacitor.

**Note.** (i) In case,  $\psi$  is not small, then as seen from Fig. 13.41 (b)  $\tan \phi = \frac{X_C}{R_{se}}$  (Ex. 13.41)  $R_{se} = X_C / \tan \phi$

From Fig. 13.31 (d), we get  $\tan \phi = \frac{I_2}{I_1} = \frac{V/X_C}{V/R_{sh}} = \frac{R_{sh}}{X_C} \therefore R_{sh} = X_C \tan \phi = \tan \phi \omega C$

(ii) It will be seen from above that both  $R_{se}$  and  $R_{sh}$  vary inversely as the frequency of the applied voltage. In other words, the resistance of a capacitor decreases in proportion to the increase in frequency.

$$\frac{R_{se1}}{R_{se2}} = \frac{f_2}{f_1}$$

**Example 13.41.** A capacitor has a capacitance of  $10 \mu\text{F}$  and a phase difference of  $10^\circ$ . It is inserted in series with a  $100 \Omega$  resistor across a  $200\text{-V}$ ,  $50\text{-Hz}$  line, Find (i) the increase in resistance due to the insertion of this capacitor (ii) power dissipated in the capacitor and (iii) circuit power factor.

**Solution.**  $X_C = \frac{10^6}{2\pi \times 50 \times 10} = 318.3 \Omega$

The equivalent series resistance of the capacitor in Fig. 13.32 is  $R_{se} = X_C / \tan \phi$

Now  $\phi = 90 - \psi = 90^\circ - 10^\circ = 80^\circ$

$\tan \phi = \tan 80^\circ = 5.671$

$\therefore R_{se} = 318.3 / 5.671 = 56.1 \Omega$

(i) Hence, resistance of the circuit increases by  **$56.1 \Omega$**

(ii)  $Z = \sqrt{(R + R_{se})^2 + X_C^2} = \sqrt{156.1^2 + 318.3^2} = 354.4 \Omega$ ;  $I = 220 / 354 = 0.62 \text{ A}$

Power dissipated in the capacitor =  $I^2 R_{se} = 0.62^2 \times 56.1 = \mathbf{21.6 \text{ W}}$

(iii) Circuit power factor is  $= (R + R_{se}) / Z = 156.1 / 354.4 = \mathbf{0.44 \text{ (lead)}}$

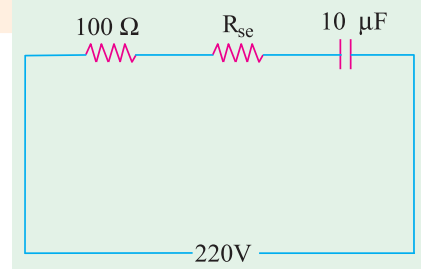


Fig. 13.32

**Example 13.42.** Dielectric heating is to be employed to heat a slab of insulating material  $2 \text{ cm}$  thick and  $150 \text{ sq. cm}$  in area. The power required is  $200 \text{ W}$  and a frequency of  $30 \text{ MHz}$  is to be used. The material has a relative permittivity of  $5$  and a power factor of  $0.03$ . Determine the voltage necessary and the current which will flow through the material. If the voltage were to be limited to  $600\text{-V}$ , to what would the frequency have to be raised?

[Elect. Engg. AMIETE (New Scheme) June 1992]

**Solution.** The capacitance of the parallel-plate capacitor formed by the insulating slab is

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 5 \times 150 \times 10^{-4}}{2 \times 10^{-2}} = 33.2 \times 10^{-12} \text{ F}$$

As shown in Art. 13.8  $R_{sh} = \frac{1}{\omega C \times p.f.} = \frac{1}{(2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 0.05} = 3196 \Omega$

Now,  $P = V^2 / R_{sh}$  or  $V = \sqrt{P \times R_{sh}} = \sqrt{200 \times 3196} = 800 \text{ V}$

Current  $I = V / X_C = \omega C V = (2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 800 = 5 \text{ A}$

Now, as seen from above  $P = \frac{V^2}{R_{sh}} = \frac{V^2}{1 / \omega C \times p.f.} = V^2 \omega C \times p.f.$  or  $P \propto V^2 f$

$\therefore 800^2 \times 30 = 600^2 \times f$  or  $f = \left(\frac{800}{600}\right)^2 \times 30 = \mathbf{53.3 \text{ MHz}}$

### Tutorial Problem No. 13.2

1. A capacitor having a capacitance of  $20 \mu\text{F}$  is connected in series with a non-inductive resistance of  $120 \Omega$  across a  $100\text{-V}$ ,  $50\text{-Hz}$  supply, Calculate (a) voltage (b) the phase difference between the current and the supply voltage (c) the power. Also draw the vector diagram.

[(a)  $0.501 \text{ A}$  (b)  $52.9^\circ$  (c)  $30.2 \text{ W}$ ]

2. A capacitor and resistor are connected in series to an a.c. supply of 50 V and 50 Hz. The current is 2 A and the power dissipated in the circuit is 80 W. Calculate the resistance of the resistor and the capacitance of the capacitor. [20  $\Omega$  ; 212  $\mu\text{F}$ ]
3. A voltage of 125 V at 50 Hz is applied to a series combination of non-inductive resistor and a lossless capacitor of 50  $\mu\text{F}$ . The current is 1.25 A. Find (i) the value of the resistor (ii) power drawn by the network (iii) the power factor of the network. Draw the phasor diagram for the network.  
[(i) 77.3  $\Omega$  (ii) 121 W (iii) 0.773 (lead)] (Electrical Technology-1, Osmania Univ.)
4. A black box contains a two-element series circuit. A voltage  $(40 - j30)$  drives a current of  $(40 - j3)$  A in the circuit. What are the values of the elements? Supply frequency is 50 Hz.  
[ $R = 1.05$  ;  $C = 4750 \mu\text{F}$ ] (Elect. Engg. and Electronics Bangalore Univ.)
5. Following readings were obtained from a series circuit containing resistance and capacitance :  
 $V = 150 \text{ V}$  ;  $I = 2.5 \text{ A}$  ;  $P = 37.5 \text{ W}$  ;  $f = 60 \text{ Hz}$ .  
 Calculate (i) Power factor (ii) effective resistance (iii) capacitive reactance and (iv) capacitance.  
[(i) 0.1 (ii) 6  $\Omega$  (iii) 59.7  $\Omega$  (iv) 44.4  $\mu\text{F}$ ]
6. An alternating voltage of 10 volt at a frequency of 159 kHz is applied across a capacitor of 0.01  $\mu\text{F}$ . Calculate the current in the capacitor. If the power dissipated within the dielectric is 100  $\mu\text{W}$ , calculate (a) loss angle (b) the equivalent series resistance (c) the equivalent parallel resistance.  
[0.A (a)  $10^{-4}$  radian (b) 0.01  $\Omega$  (c) 1 M $\Omega$ ]

### 13.9. Resistance, Inductance and Capacitance in Series

The three are shown in Fig. 13.33 (a) joined in series across an a.c. supply of r.m.s. voltage  $V$ .

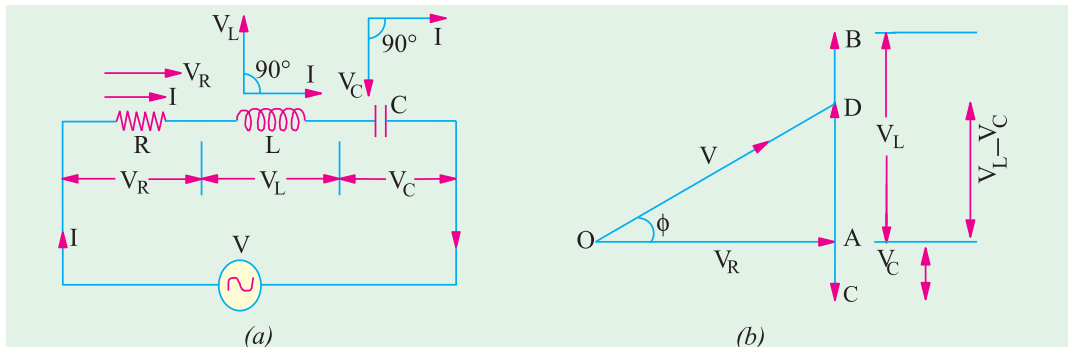


Fig. 13.33

Let  $V_R = IR =$  voltage drop across  $R$  —in phase with  $I$   
 $V_L = I.X_L =$  voltage drop across  $L$  —leading  $I$  by  $\pi/2$   
 $V_C = I.X_C =$  voltage drop across  $C$  —lagging  $I$  by  $\pi/2$

In voltage triangle of Fig. 13.33 (b),  $OA$  represents  $V_R$ ,  $AB$  and  $AC$  represent the inductive and capacitive drops respectively. It will be seen that  $V_L$  and  $V_C$  are  $180^\circ$  out of phase with each other *i.e.* they are in direct opposition to each other.

Subtracting  $BD (= AC)$  from  $AB$ , we get the net reactive drop  $AD = I(X_L - X_C)$

The applied voltage  $V$  is represented by  $OD$  and is the vector sum of  $OA$  and  $AD$

$$\therefore OD = \sqrt{OA^2 + AD^2} \text{ or } V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or } I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The term  $\sqrt{R^2 + (X_L - X_C)^2}$  is known as the impedance of the circuit. Obviously,

$$\text{(impedance)}^2 = \text{(resistance)}^2 + \text{(net reactance)}^2$$

$$\text{or } Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$$



where  $X$  is the net reactance (Fig. 13.33 and 13.34).

Phase angle  $\phi$  is given by  $\tan \phi = (X_L - X_C)/R = X/R = \text{net reactance/resistance}$

$$\text{Power factor is } \cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Hence, it is seen that if the equation of the applied voltage is  $v = V_m \sin \alpha$ , then equation of the resulting current in an  $R$ - $L$ - $C$  circuit is given by  $i = I_m \sin (\omega t \pm \phi)$

The +ve sign is to be used when current leads *i.e.*  $X_C > X_L$ .

The -ve sign is to be used when current lags *i.e.* when  $X_L > X_C$ .

In general, the current lags or leads the supply voltage by an angle  $\phi$  such that  $\tan \phi = X/R$

Using symbolic notation, we have (Fig. 13.35),  $Z = R + j(X_L - X_C)$

Numerical value of impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Its phase angle is  $\Phi = \tan^{-1} [X_L - X_C/R]$

$$Z = Z \angle \tan^{-1} [(X_L - X_C)/R] = Z \angle \tan^{-1} (X/R)$$

If  $V = V \angle 0$ , then,  $I = V/Z$

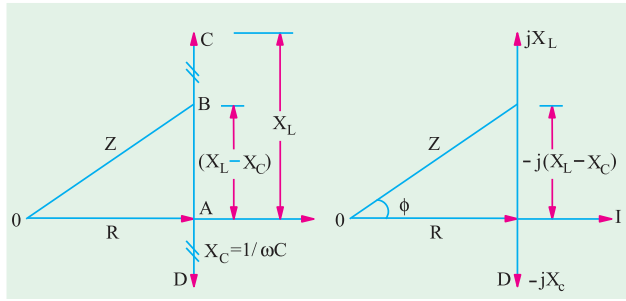


Fig. 13.34

Fig. 13.35

### Summary of Results of Series AC Circuits

Type of Impedance	Value of Impedance	Phase angle for current	Power factor
Resistance only	$R$	$0^\circ$	1
Inductance only	$\omega L$	$90^\circ$ lag	0
Capacitance only	$1/\omega C$	$90^\circ$ lead	0
Resistance and Inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90^\circ$ lag	$1 > \text{p.f.} > 0$ lag
Resistance and Capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90^\circ$ lead	$1 > \text{p.f.} > 0$ lead
$R$ - $L$ - $C$	$\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$	between $0^\circ$ and $90^\circ$ lag or lead	between 0 and unity lag or lead

**Example 13.43.** A resistance of  $20 \Omega$  an inductance of  $0.2 \text{ H}$  and a capacitance of  $100 \mu\text{F}$  are connected in series across  $220\text{-V}$ ,  $50\text{-Hz}$  mains. Determine the following (a) impedance (b) current (c) voltage across  $R$ ,  $L$  and  $C$  (d) power in watts and  $\text{VA}$  (e) p.f. and angle of lag.

(Elect. Engg. A.M.Ae S.I. 1992)

**Solution.**  $X_C = 0.2 \times 314 = 63 \Omega$ ,  $C = 10 \mu\text{F} = 100 \times 10^{-6} = 10^{-4}$  farad

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 10^{-4}} = 32 \Omega, X = 63 - 32 = 31 \Omega \text{ (inductive)}$$

(a)  $Z = \sqrt{(20^2 + 31)^2} = 37 \Omega$  (b)  $I = 220/37 = 6 \text{ A}$  (approx)

(c)  $V_R = I \times R = 6 \times 20 = 120 \text{ V}$ ;  $V_L = 6 \times 63 = 278 \text{ V}$ ,  $V_C = 6 \times 32 = 192 \text{ V}$

(d) Power in  $\text{VA} = 6 \times 220 = 1320$

Power in watts  $= 6 \times 220 \times 0.54 = 713 \text{ W}$

(e) p.f.  $= \cos \phi = R/Z = 20/37 = 0.54$ ;  $\phi = \cos^{-1} (0.54) = 57^\circ 18'$

**Example 13.44.** A voltage  $e(t) = 100 \sin 314 t$  is applied to series circuit consisting of 10 ohm resistance, 0.0318 henry inductance and a capacitor of 63.6  $\mu\text{F}$ . Calculate (i) expression for  $i(t)$  (ii) phase angle between voltage and current (iii) power factor (iv) active power consumed (v) peak value of pulsating energy. (Elect. Technology, Indore Univ.)

**Solution.** Obviously,  $\omega = 314 \text{ rad/s}$ ;  $X_L = \omega L = 314 \times 0.0318 = 10 \Omega$

$X_C = 1/\omega C = 1/314 \times 63.6 \times 10^{-6} = 50 \Omega$ ;  $X = X_L - X_C = (10 - 50) = -40 \Omega$  (capacitive)

$Z = 10 - j40 = 41.2 \angle -76^\circ$ ;

$$I = \frac{V}{Z} = \frac{(100/\sqrt{2})}{41.2} = 1.716$$

$$I_m = I \times \sqrt{2} = 1.716 \times \sqrt{2} = 2.43 \text{ A}$$

(i)  $i(t) = 2.43 \sin(314 t + 76^\circ)$

(ii)  $\phi = 76^\circ$  with current leading

(iii) p.f. =  $\cos \phi = \cos 76^\circ = 0.24$  (lead)

(iv) Active power,  $P = VI \cos \phi$

$$= (100/\sqrt{2})(2.43/\sqrt{2}) \times 0.24 = 29.16 \text{ W}$$

(v) As seen from Fig. 13.36, peak value of pulsating energy is  $\frac{V_m I_m}{2} + \frac{V_m I_m}{2} \cos \phi$

$$= \frac{V_m I_m}{2} (1 + \cos \phi) = \frac{100 \times 2.43}{2} (1 + 0.24) = 151 \text{ W}$$

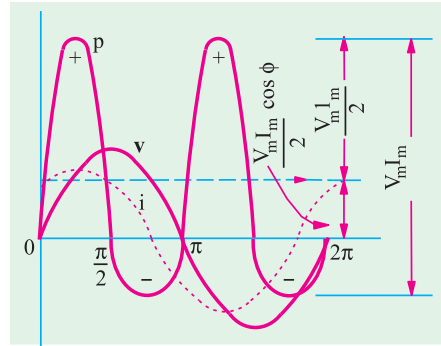


Fig. 13.36

**Example 13.45.** Two impedances  $Z_1$  and  $Z_2$  when connected separately across a 230-V, 50-Hz supply consumed 100 W and 60 W at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply, find :

(i) total power absorbed and overall p.f. (ii) the value of the impedance to be added in series so as to raise the overall p.f. to unity. (Elect. Circuits-I, Bangalore Univ.)

**Solution. Inductive Impedance**  $V_1 I_1 \cos \phi_1 = \text{power}$ ;  $230 \times I_1 \times 0.5 = 100$ ;  $I_1 = 0.87 \text{ A}$

Now,  $I_1^2 R_1 = \text{power}$  or  $0.87^2 R_1 = 100$ ;  $R_1 = 132 \Omega$ ;  $Z_1 = 230/0.87 = 264 \Omega$

$$X_L = \sqrt{Z_1^2 - R_1^2} = \sqrt{264^2 - 132^2} = 229 \Omega$$

**Capacitance Impedance**  $I_2 = 60/230 \times 0.6 = 0.434 \text{ A}$ ;  $R_2 = 60/0.434^2 = 318 \Omega$

$$Z_2 = 230/0.434 = 530 \Omega$$
;  $X_C = \sqrt{530^2 - 318^2} = 424 \Omega$  (capacitive)

**When  $Z_1$  and  $Z_2$  are connected in series**

$$R = R_1 + R_2 = 132 + 318 = 450 \Omega$$
;  $X = 229 - 424 = -195 \Omega$  (capacitive)

$$Z = \sqrt{R^2 + X^2} = \sqrt{450^2 + (-195)^2} = 490 \Omega$$
,  $I = 230/490 = 0.47 \text{ A}$

(i) Total power absorbed =  $I^2 R = 0.47^2 \times 450 = 99 \text{ W}$ ,  $\cos \phi = R/Z = 450/490 = 0.92$  (lead)

(ii) Power factor will become unity when the net capacitive reactance is neutralised by an equal inductive reactance. The reactance of the required series pure inductive coil is **195  $\Omega$**

**Example 13.46.** A resistance  $R$ , an inductance  $L = 0.01 \text{ H}$  and a capacitance  $C$  are connected in series. When a voltage  $v = 400 \cos(300 t - 10^\circ)$  volts is applied to the series combination, the current flowing is  $10\sqrt{2} \cos(3000 t - 55^\circ)$  amperes. Find  $R$  and  $C$ . (Elect. Circuits Nagpur Univ. 1992)

**Solution.** The phase difference between the applied voltage and circuit current is  $(55^\circ - 10^\circ) = 45^\circ$  with current lagging. The angular frequency is  $\omega = 3000$  radian/second. Since current lags,  $X_L > X_C$ .

Net reactance  $X = (X_L - X_C)$ . Also  $X_L = \omega L = 3000 \times 0.01 = 30 \Omega$

$$\tan \phi = X/R \quad \text{or} \quad \tan 45^\circ = X/R \quad \therefore X = R \quad \text{Now, } Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3 \Omega$$

$$Z^2 = R^2 + X^2 = 2R^2 \quad \therefore R = Z/\sqrt{2} = 28.3/\sqrt{2} = 20 \Omega; X = X_L - X_C = 30 - X_C = 20$$

$$X_C = 10 \Omega \quad \text{or} \quad \frac{1}{\omega C} = 10 \quad \text{or} \quad \frac{1}{3000 C} = 10 \quad \text{or} \quad C = 33 \mu\text{F}$$

**Example 13.47.** A non-inductive resistor is connected in series with a coil and a capacitor. The circuit is connected to a single-phase a.c. supply. If the voltages are as indicated in Fig. 13.37 when current flowing through the circuit is 0.345 A, find the applied voltage and the power loss in coil.

(Elect. Engg. Pune Univ.)

**Solution.** It may be kept in mind that the coil has not only inductance  $L$  but also some resistance  $r$  which produces power loss. In the voltage vector diagram,  $AB$  represents drop across  $R = 25 \text{ V}$ . Vector  $BC$  represents drop across coil which is due to  $L$  and  $r$ . Which value is  $40 \text{ V}$  and the vector  $BC$  is at any angle of  $\phi$  with the current vector.  $AD$  represents  $50 \text{ V}$  which is the drop across  $R$  and coil combined.  $AE$  represents the drop across the capacitor and leads the current by  $90^\circ$ .

It will be seen that the total horizontal drop in the circuit is  $AC$  and the vertical drop is  $AG$ . Their vector sum  $AF$  represents the applied voltage  $V$ .

From triangle  $ABD$ , we get  $50^2 = 40^2 + 25^2 + 2 \times 25 \times 40 \times \cos \phi$ ;  $\cos \phi = 0.1375$  and

$\sin \phi = 0.99$ . Considering the coil,  $IZ_L = 40 \quad \therefore Z_L = 40/0.345 = 115.94 \Omega$

Now  $r = Z_L \cos \phi = 115.94 \times 0.1375 = 15.94 \Omega$

Power loss in the coil =  $I^2 r = 0.345^2 \times 15.94 = 1.9 \text{ W}$

$BC = BD \cos \phi = 40 \times 0.1375 = 5.5 \text{ V}$ ;  $CD = BD \sin \phi = 40 \times 0.99 = 39.6 \text{ V}$

$AC = 25 + 5.5 = 30.5 \text{ V}$ ;  $AG = AE - DC = 55 - 39.6 = 15.4 \text{ V}$

$$AF = \sqrt{AC^2 + CF^2} = \sqrt{30.5^2 + 15.4^2} = 34.2 \text{ V}$$

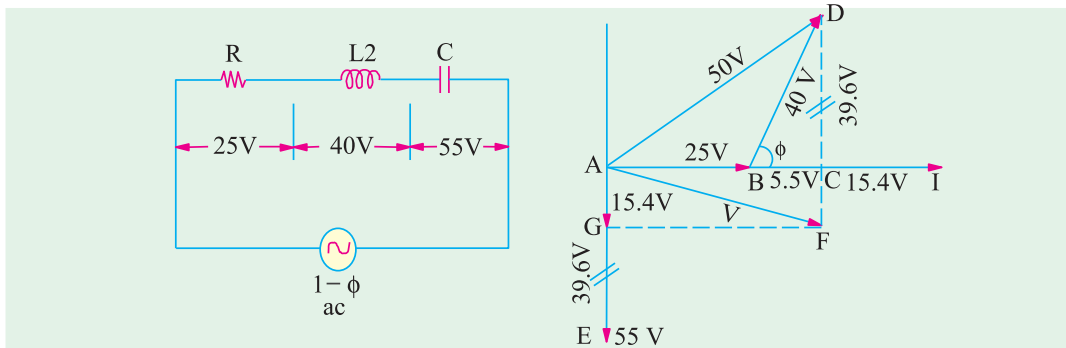


Fig. 13.37

**Example 13.48.** A  $4.7 \text{ H}$  inductor which has a resistance of  $20 \Omega$ , a  $4\text{-}\mu\text{F}$  capacitor and a  $100\text{-}\Omega$  non-inductive resistor are connected in series to a  $100\text{-V}$ ,  $50\text{-Hz}$  supply. Calculate the time interval between the positive peak value of the supply voltage and the next peak value of power.

**Solution.** Total resistance =  $120 \Omega$ ,  $X_L = 2\pi \times 50 \times 4.7 = 1477 \Omega$

$X_C = 10^6/2\pi \times 50 \times 4 = 796 \Omega$ ,  $X = 1477 - 796 = 681 \Omega$

$$Z = \sqrt{120^2 + 681^2} = 691.3 \Omega$$

$\cos \phi = R/Z = 120/691.3 = 0.1736$ ;  $\phi = 80^\circ$

Now, as seen from Fig. 13.38, the angular displace-

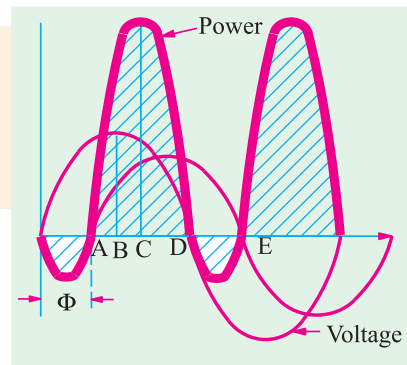


Fig. 13.38

ment between the peak values of supply voltage and power cycles is  $BC = \phi/2$  because  $AB = 90 - \phi$  and  $AD = 180 - \phi$ .

$$\text{Hence } AC = 90 - \phi/2$$

$$\therefore BC = AC - AB = (90 - \phi/2) - (90 - \phi) = \phi/2$$

$$\text{Angle difference} = \phi/2 = 80^\circ/2 = 40^\circ$$

Since a full cycle of  $360^\circ$  corresponds to a time interval of 1.50 second

$$\therefore 40^\circ \text{ angular interval} = \frac{40}{50 \times 360} = 2.22 \text{ ms.}$$

**Example 13.49.** A coil is in series with a  $20 \mu\text{F}$  capacitor across a  $230\text{-V}$ ,  $50\text{-Hz}$  supply. The current taken by the circuit is  $8 \text{ A}$  and the power consumed is  $200 \text{ W}$ . Calculate the inductance of the coil if the power factor of the circuit is (i) leading (ii) lagging.

Sketch a vector diagram for each condition and calculate the coil power factor in each case.

(Elect. Engg.-I Nagpur Univ. 1993)

**Solution.** (i) Since power factor is leading, net reactance  $X = (X_C - X_L)$  as shown in Fig. 13.39 (a).

$$I^2 R = 200 \text{ or } 8^2 \times R = 200; \therefore R = 200/64 = 25/8 \Omega = 3.125 \Omega$$

$$Z = V/I = 230/8 = 28.75 \Omega; X_C = 10^6/2\pi \times 50 \times 20 = 159.15 \Omega$$

$$R^2 + X^2 = 28.75^2 \therefore X = 28.58 \Omega \therefore (X_C - X_L) = 28.58 \text{ or } 159.15 - X_L = 28.58$$

$$\therefore X_L = 130.57 \Omega \text{ or } 2\pi \times 50 \times L = 130.57 \therefore L = 0.416 \text{ H}$$

If  $\theta$  is the p.f. angle of the coil, then  $\tan \theta = R/X_L = 3.125/130.57 = 0.024$ ;  $\theta = 1.37^\circ$ , p.f. of the coil = 0.9997

(ii) When power factor is lagging, net reactance is  $(X_L - X_C)$  as shown in Fig. 13.39 (b).

$$\therefore X_L - 159.15 = 28.58 \text{ or } X_L = 187.73 \Omega \therefore 187.73 = 2\pi \times 50 \times L \text{ or } L = 0.597 \text{ H.}$$

In this case,  $\tan \theta = 3.125/187.73 = 0.0167$ ;  $\theta = 0.954^\circ$ ;  $\therefore \cos \theta = 0.9998$ .

The vector diagrams for the two conditions are shown in Fig. 13.35.

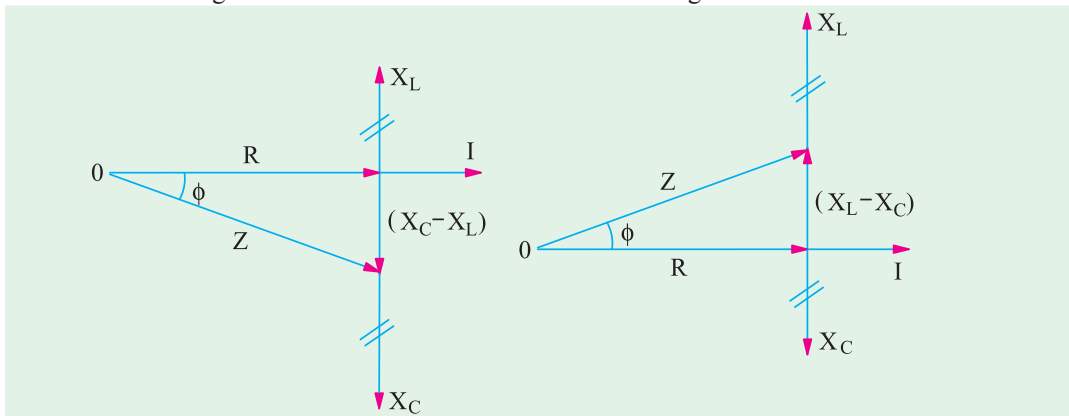


Fig. 13.39

**Example 13.50.** In Fig. 13.40, calculate (i) current (ii) voltage drops  $V_1$ ,  $V_2$ , and  $V_3$  and (iii) power absorbed by each impedance and total power absorbed by the circuit. Take voltage vector along the reference axis.

$$\text{Solution. } Z_1 = (4 + j3) \Omega; Z_2 = (6 - j8) \Omega; Z_3 = (4 + j0) \Omega$$

$$Z = Z_1 + Z_2 + Z_3 = (4 + j3) + (6 - j8) + (4 + j0) = (14 - j5) \Omega$$

$$\text{Taking } \mathbf{V} = V \angle 0^\circ = 100 \angle 0^\circ = (100 + j0)$$

$$\therefore \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{100}{(14 - j5)} = \frac{100(14 + j5)}{(14 - j5)(14 + j5)} = 6.34 + j2.26$$

(i) Magnitude of the current

$$= \sqrt{(6.34^2 + 2.26^2)} = \mathbf{6.73 \text{ A}}$$

(ii)  $V_1 = IZ_1 = (6.34 + j 2.26)(4 + j3) = 18.58 + j 28.06$

$$V_2 = IZ_2 = (6.34 + j 2.26)(6 - j8) = 56.12 - j 37.16$$

$$V_3 = IZ_3 = (6.34 + j 2.26)(4 + j0) = 25.36 + j 9.04$$

$$V = 100 + j 0 \text{ (check)}$$

(iii)  $P_1 = 6.73^2 \times 4 = 181.13 \text{ W}$ .

$$P_2 = 6.73^2 \times 6 = 271.74 \text{ W}, P_3 = 6.73^2 \times 4 = 181.13 \text{ W}$$

$$\text{Total} = 34 \text{ W}$$

Otherwise  $P_{VA} = (100 + j 0)(6.34 - j 2.26)$  (using current conjugate)

$$= 634 - j 226$$

$$\text{real power} = 634 \text{ W (as a check)}$$

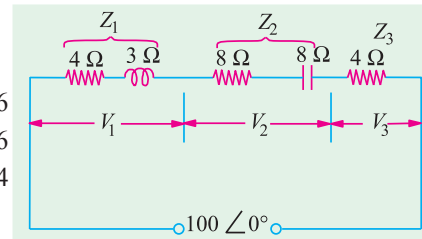


Fig. 13.40

**Example 13.51.** Draw a vector for the circuit shown in Fig. 13.41 indicating the resistance and reactance drops, the terminal voltages  $V_1$  and  $V_2$  and the current. Find the values of (i) the current  $I$  (ii)  $V_1$  and  $V_2$  and (iii) p.f. (Elements of Elect Engg-I, Bangalore Univ.)

**Solution.**  $L = 0.05 + 0.1 = 0.15 \text{ H}; X_L = 314 \times 0.5 = 47.1 \Omega$

$$X_C = 10^6/314 \times 50 = 63.7 \Omega, X = 47.1 - 63.7 = -16.6 \Omega, R = 30 \Omega$$

$$Z = \sqrt{30^2 + (-16.6)^2} = 34.3 \Omega$$

(i)  $I = V/Z = 200/34.3 = \mathbf{5.83 \text{ A}}$ ,

from Fig. 13.41 (a)

(ii)  $X_{L1} = 314 \times 0.05$

$$= 15.7 \Omega$$

$$Z_1 = \sqrt{10^2 + 15.7^2} = 18.6 \Omega$$

$$V_1 = IZ_1 = 5.83 \times 18.6$$

$$= \mathbf{108.4 \text{ V}}$$

$$\phi_1 = \cos^{-1}(10/18.6) = 57.5^\circ \text{ (lag)}$$

$$X_{L2} = 314 \times 0.1 = 31.4 \Omega, X_C = -63.7 \Omega, X = 31.4 - 63.7 = -32.2 \Omega, Z_2 = \sqrt{20^2 + (-32.2)^2} = \mathbf{221 \text{ V}}$$

$$\phi_2 = \cos^{-1}(20/38) = 58.2^\circ \text{ (lead)}$$

(iii) Combined p.f. =  $\cos \phi = R/Z = 30/34.3 = \mathbf{0.875 \text{ (lead)}}$ , from Fig. 13.41 (b).

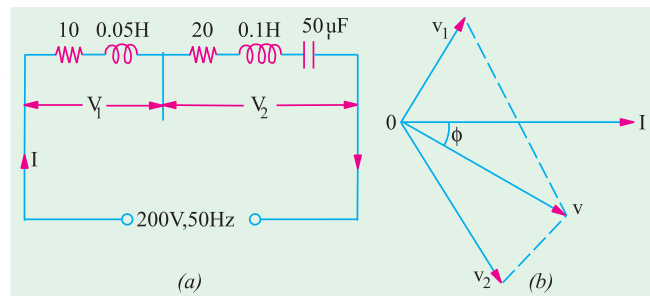


Fig. 13.41

**Example 13.52.** In a circuit, the applied voltage is found to lag the current by  $30^\circ$ .

(a) Is the power factor lagging or leading? (b) What is the value of the power factor? (c) Is the circuit inductive or capacitive?

In the diagram of Fig 13.42, the voltage drop across  $Z_1$  is  $(10 + j0)$  volts. Find out

(i) the current in the circuit (ii) the voltage drops across  $Z_2$  and  $Z_3$  (iii) the voltage of the generator:

(Elect. Engg.-I, Bombay Univ. 1991)

**Solution.** (a) Power factor is **leading** because current leads the voltage.

(b) p.f. =  $\cos 30^\circ = \mathbf{0.86 \text{ (lead)}}$  (c) The circuit is **capacitive**.

(i) Circuit current can be found by dividing voltage drop  $V_1$  by  $Z_1$

$$I = \frac{10 + j 0}{3 + j 4} = \frac{10 \angle 0^\circ}{5 \angle 53.1^\circ} = 2 \angle -53.1^\circ = 2(\cos 53.1^\circ - j \sin 53.1^\circ)$$

$$= 2(0.6 - j 0.8) = 1.2 - j 1.6$$

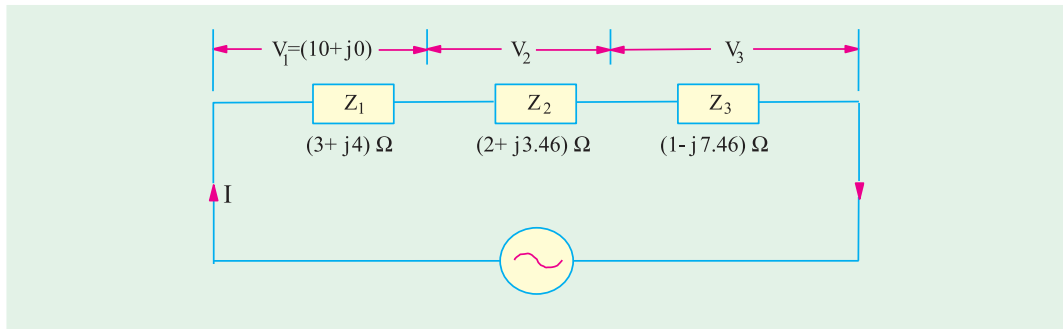


Fig. 13.42

$$\mathbf{Z}_2 = 2 + j 3.46; \mathbf{V}_2 = \mathbf{I}\mathbf{Z}_2 = (1.2 - j 1.6) (2 + j 3.46) = (7.936 + j 0.952) \text{ volt}$$

$$\mathbf{V}_3 = (1.2 - j 1.6) (1 - j 7.46) = (-10.74 - j 10.55) \text{ volt}$$

$$\begin{aligned} \text{(ii)} \quad \mathbf{V} &= \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = (10 + j 0) + (7.936 + j 0.952) + (-10.74 - j 10.55) \\ &= (7.2 - j 9.6) = 12 \angle - 53.1^\circ \end{aligned}$$

Incidentally, it shows that current  $\mathbf{I}$  and voltage  $\mathbf{V}$  are in phase with each other.

**Example 13.53.** A 230-V, 50-Hz alternating p.d. supplies a choking coil having an inductance of 0.06 henry in series with a capacitance of 6.8  $\mu\text{F}$ , the effective resistance of the circuit being 2.5  $\Omega$ . Estimate the current and the angle of the phase difference between it and the applied p.d. If the p.d. has a 10% harmonic of 5 times the fundamental frequency, estimate (a) the current due to it and (b) the p.d. across the capacitance. (Electrical Network Analysis, Nagpur Univ. 1993)

**Solution. Fundamental Frequency :** For the circuit in Fig. 13.43,

$$X_L = \omega L = 2\pi \times 50 \times 0.06 = 18.85 \Omega$$

$$X_C = \frac{10^6}{2\pi \times 50 \times 6.8} = 648 \Omega$$

$$\therefore X = 18.85 - 648 = -449.15 \Omega$$

$$Z = \sqrt{2.5^2 + (-449.15)^2} = 449.2 \Omega$$

$$\text{Current } I_f = 230/449.2 = 0.512 \text{ A, phase angle } \tan^{-1} \frac{449.2}{2.5} = 89.42^\circ$$

$\therefore$  current leads p.d. by  $89^\circ 42'$ .

$$\text{Fifth Harmonic Frequency } X_L = 18.85 \times 5 = 94.25 \Omega; X_C = 648/5 = 93.6 \Omega$$

$$X = 94.25 - 93.6 = 0.65 \Omega; Z = \sqrt{2.5^2 + 0.65^2} = 2.585 \Omega, \text{ Harmonic p.d.} = 230 \times 10/100 = 23 \text{ V}$$

$$\therefore \text{ Harmonic current } I_h = 23/2.585 = 8.893 \text{ A}$$

$$\text{P.D. across capacitor at harmonic frequency is, } V_h = 8.893 \times 93.6 = 832.6 \text{ V}$$

The total current flowing through the circuit, due to the complex voltage wave form, is found from the fundamental and harmonic components thus. Let,

$I$  = the r.m.s. value of total circuit current,

$I_f$  = r.m.s. value of fundamental current,

$I_h$  = r.m.s. value of fifth harmonic current,

$$\text{(a)} \quad \therefore I = \sqrt{I_f^2 + I_h^2} = \sqrt{0.512^2 + 8.893^2} = 8.9 \text{ A}$$

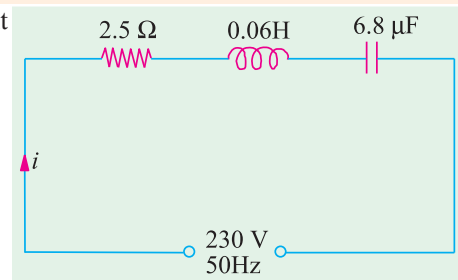


Fig. 13.43

- (b) The r.m.s. value of p.d. across capacitor is found in a similar way.

$$V_f = 0.512 \times 468 = 239.6 \text{ V}$$

$$\therefore V = \sqrt{(V_f^2 + V_h^2)} = \sqrt{239.6^2 + 832.6^2} = 866.4 \text{ V}$$

### Tutorial Problem No. 13.3

- An e.m.f. represented by  $e = 100 \sin 100 \pi t$  is impressed across a circuit consisting of  $40\text{-}\Omega$  resistor in series with a  $40\text{-}\mu\text{F}$  capacitor and a  $0.25 \text{ H}$  inductor. Determine (i) the r.m.s. value of the current (ii) the power supplied (iii) the power factor.  
[(i) 1.77 A (ii) 125 W (iii) 1.0] (London Univ.)
- A series circuit with a resistor of  $100 \Omega$  capacitor of  $25 \mu\text{F}$  and inductance of  $0.15 \text{ H}$  is connected across  $220\text{-V}$ ,  $60\text{-Hz}$  supply. Calculate (i) current (ii) power and (iii) power factor in the circuit.  
[(i) 1.97 A; (ii) 390 W (iii) 0.9 (lead)] (Elect. Engg. and Electronics Bangalore Univ.)
- A series circuit with  $R = 10 \Omega$   $L = 50 \text{ mH}$  and  $C = 100 \mu\text{F}$  is supplied with  $200 \text{ V}/50 \text{ Hz}$ . Find (i) the impedance (ii) current (iii) power (iv) power factor.  
[(i) 18.94  $\Omega$  (ii) 18.55 A (iii) 1966 W (iv) 0.53 (leading)] (Elect. Engg. & Electronics Bangalore Univ.)
- A coil of resistance  $10 \Omega$  and inductance  $0.1 \text{ H}$  is connected in series with a  $150\text{-}\mu\text{F}$  capacitor across a  $200\text{-V}$ ,  $50\text{-Hz}$  supply. Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the impedance (d) the current, (e) the power factor (f) the voltage across the coil and the capacitor respectively.  
[(a) 31.4  $\Omega$  (b) 21.2  $\Omega$  (c) 14.3  $\Omega$  (d) 14 A (e) 0.7 lag (f) 460 V, 297 V]
- A circuit is made up of  $10 \Omega$  resistance,  $12 \text{ mH}$  inductance and  $281.5 \mu\text{F}$  capacitance in series. The supply voltage is  $100 \text{ V}$  (constant). Calculate the value of the current when the supply frequency is (a)  $50 \text{ Hz}$  and (b)  $150 \text{ Hz}$ .  
[8 A leading; 8 A lagging]
- A coil having a resistance of  $10 \Omega$  and an inductance of  $0.2 \text{ H}$  is connected in series with a capacitor of  $59.7 \mu\text{F}$ . The circuit is connected across a  $100\text{-V}$ ,  $50\text{-Hz}$  a.c. supply. Calculate (a) the current flowing (b) the voltage across the capacitor (c) the voltage across the coil. Draw a vector diagram to scale.  
[(a) 10 A (b) 628 V (c) 635 V]
- A coil is in series with a  $20 \mu\text{F}$  capacitor across a  $230\text{-V}$ ,  $50\text{-Hz}$  supply. The current taken by the circuit is  $8 \text{ A}$  and the power consumed is  $200 \text{ W}$ . Calculate the inductance of the coil if the power factor of the circuit is (a) leading and (b) lagging. Sketch a vector diagram for each condition and calculate the coil power factor in each case.  
[0.415 H; 0.597 H; 0.0238; 0.0166]
- A circuit takes a current of  $3 \text{ A}$  at a power factor of  $0.6$  lagging when connected to a  $115\text{-V}$ ,  $50\text{-Hz}$  supply. Another circuit takes a current, of  $5 \text{ A}$  at a power factor of  $0.707$  leading when connected to the same supply. If the two circuits are connected in series across a  $230\text{-V}$ ,  $50\text{Hz}$  supply, calculate (a) the current (b) the power consumed and (c) the power factor.  
[(a) 5.5 A (b) 1.188 kW (c) 0.939 lag]
- A coil of insulated wire of resistance  $8 \text{ ohms}$  and inductance  $0.03 \text{ H}$  is connected to an a.c. supply at  $240 \text{ V}$ ,  $50\text{-Hz}$ . Calculate (a) the current, the power and power factor (b) the value of a capacitance which, when connected in series with the above coil, causes no change in the values of current and power taken from the supply.  
[(a) 19.4 A, 3012 W, 0.65 lag (b) 168.7  $\mu\text{F}$ ] (London Univ.)
- A series circuit, having a resistance of  $10 \Omega$ , an inductance of  $0.025 \text{ H}$  and a variable capacitance is connected to a  $100\text{-V}$ ,  $25\text{-Hz}$  single-phase supply. Calculate the capacitance when the value of the current is  $8 \text{ A}$ . At this value of capacitance, also calculate (a) the circuit impedance (b) the circuit power factor and (c) the power consumed.  
[556  $\mu\text{F}$  (a) 1.5  $\Omega$  (b) 0.8 leading (c) 640 W]
- An alternating voltage is applied to a series circuit consisting of a resistor and iron-cored inductor and a capacitor. The current in the circuit is  $0.5 \text{ A}$  and the voltages measured are  $30 \text{ V}$  across the resistor,  $48 \text{ V}$  across the inductor,  $60 \text{ V}$  across the resistor and inductor and  $90 \text{ V}$  across the capacitor. Find (a) the combined copper and iron losses in the inductor (b) the applied voltage.  
[(a) 3.3 W (b) 56 V] (City & Guilds, London)
- When an inductive coil is connected across a  $250\text{-V}$ ,  $50\text{-Hz}$  supply, the current is found to be  $10 \text{ A}$  and the power absorbed  $1.25 \text{ kW}$ . Calculate the impedance, the resistance and the inductance of the coil. A capacitor which has a reactance twice that of the coil, is now connected in series with the coil across the same supply. Calculate the p.d. across the capacitor.  
[25  $\Omega$ ; 12.5  $\Omega$ ; 68.7 mH; 433 V]

13. A voltage of 200 V is applied to a series circuit consisting on a resistor, an inductor and a capacitor. The respective voltages across these components are 170, 150 and 100 V and the current is 4 A. Find the power factor of the inductor and of the circuit. [0.16; 0.97]
14. A pure resistance  $R$ , a choke coil and a pure capacitor of  $50\mu F$  are connected in series across a supply of  $V$  volts, and carry a current of 1.57 A. Voltage across  $R$  is 30 V, across choke coil 50 V and across capacitor 100 V. The voltage across the combination of  $R$  and choke coil is 60 volt. Find the supply voltage  $V$ , the power loss in the choke, frequency of the supply and power factor of the complete circuit. Draw the phasor diagram. [60.7 V; 6.5 W; 0.562 lead] (F.E. Pune Univ.)

### 13.10. Resonance in R-L-C Circuits

We have seen from Art. 13.9 that net reactance in an  $R$ - $L$ - $C$  circuit of Fig. 13.40 (a) is

$$X = X_L - X_C \text{ and } Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{R^2 + X^2}$$

Let such a circuit be connected across an a.c. source of constant voltage  $V$  but of frequency varying from zero to infinity. There would be a certain frequency of the applied voltage which would make  $X_L$  equal to  $X_C$  in magnitude. In that case,  $X = 0$  and  $Z = R$  as shown in Fig. 13.40 (c). Under this condition, the circuit is said to be in electrical resonance.

As shown in Fig. 13.40 (c),  $V_L = I \cdot X_L$  and  $V_C = I \cdot X_C$  and the two are equal in magnitude but opposite in phase. Hence, they cancel each other out. The two reactances taken together act as a short-circuit since no voltage develops across them. Whole of the applied voltage drops across  $R$  so that  $V = V_R$ . The circuit impedance  $Z = R$ . The phasor diagram for series resonance is shown in Fig. 13.40 (d).

#### Calculation of Resonant Frequency

The frequency at which the net reactance of the series circuit is zero is called the resonant frequency  $f_0$ . Its value can be found as under :  $X_L - X_C = 0$  or  $X_L = X_C$  or  $\omega_0 L = 1/\omega_0 C$

$$\text{or } \omega_0^2 = \frac{1}{LC} \text{ or } (2\pi f_0)^2 = \frac{1}{LC} \text{ or } f_0 = \frac{1}{2\pi \sqrt{LC}}$$

If  $L$  is in henry and  $C$  in farad, then  $f_0$  is given in Hz.

When a series  $R$ - $L$ - $C$  circuit is in resonance, it possesses minimum impedance  $Z = R$ . Hence, circuit current is maximum, it being limited by value of  $R$  alone. The current  $I_0 = V/R$  and is in phase with  $V$ .

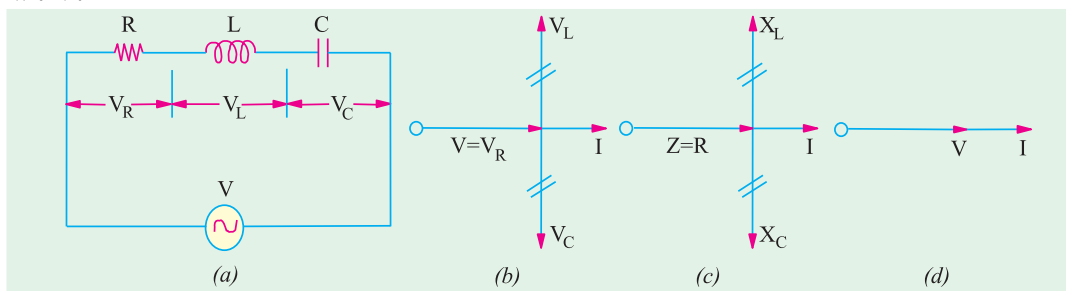


Fig. 13.44

Since circuit current is maximum, it produces large voltage drops across  $L$  and  $C$ . But these drops being equal and opposite, cancel each other out. Taken together,  $L$  and  $C$  form part of a circuit across which no voltage develops, however, large the current flowing. If it were not for the presence of  $R$ , such a resonant circuit would act like a short-circuit to currents of the frequency to which it resonates. Hence, a series resonant circuit is sometimes called **acceptor** circuit and the series resonance is often referred to as voltage resonance.

In fact, at resonance the series  $RLC$  circuit is reduced to a purely resistive circuit, as shown in Fig. 13.44.

Incidentally, it may be noted that if  $X_L$  and  $X_C$  are shown at any frequency  $f$ , that the value of the resonant frequency of such a circuit can be found by the relation  $f_0 = f \sqrt{X_C/X_L}$ .



**Summary**

When an  $R$ - $L$ - $C$  circuit is in resonance

1. net reactance of the circuit is zero *i.e.*  $(X_L - X_C) = 0$ . or  $X = 0$ .
2. circuit impedance is minimum *i.e.*  $Z = R$ . Consequently, circuit admittance is maximum.
3. circuit current is maximum and is given by  $I_0 = V/Z_0 = V/R$ .
4. power dissipated is maximum *i.e.*  $P_0 = I_0^2 R = V^2/R$ .
5. circuit power factor angle  $\theta = 0$ . Hence, power factor  $\cos \theta = 1$ .
6. although  $V_L = V_C$  yet  $V_{\text{coil}}$  is greater than  $V_C$  because of its resistance.
7. at resonance,  $\omega LC = 1$
8.  $Q = \tan \theta = \tan 0^\circ = 0^*$ .

**13.11. Graphical Representation of Resonance**

Suppose an alternating voltage of constant magnitude, but of varying frequency is applied to an  $R$ - $L$ - $C$  circuit. The variations of resistance, inductive reactance  $X_L$  and capacitive reactance  $X_C$  with frequency are shown in Fig. 13.45 (a).

(i) **Resistance** : It is independent of  $f$ , hence, it is represented by a straight line.

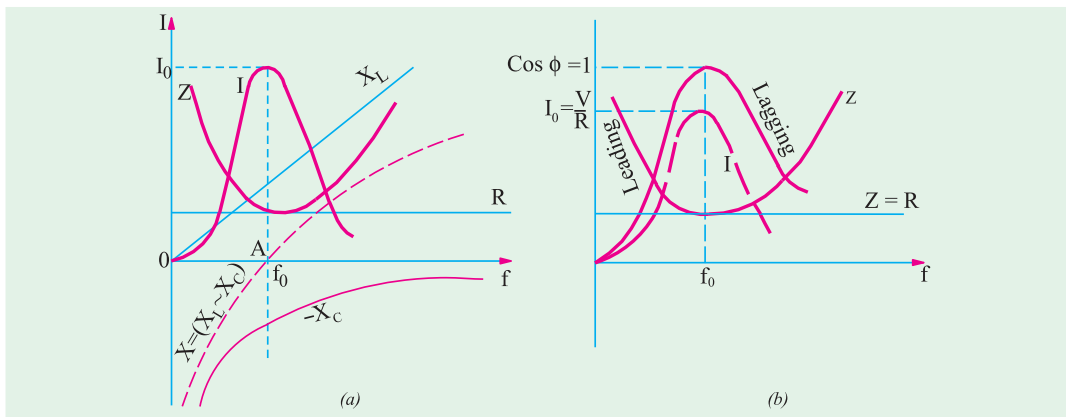
(ii) **Inductive Reactance** : It is given by  $X_L = \omega L = 2\pi fL$ . As seen,  $X_L$  is directly proportional to  $f$  *i.e.*  $X_L$  increases linearly with  $f$ . Hence, its graph is a straight line passing through the origin.

(iii) **Capacitive Reactance** : It is given by  $X_C = 1/\omega C = 1/2\pi fC$ . Obviously, it is inversely proportional to  $f$ . Its graph is a rectangular hyperbola which is drawn in the fourth quadrant because  $X_C$  is regarded negative. It is asymptotic to the horizontal axis at high frequencies and to the vertical axis at low frequencies.

(iv) **Net Reactance** : It is given by  $X = X_L \sim X_C$ . Its graph is a hyperbola (not rectangular) and crosses the  $X$ -axis at point  $A$  which represents resonant frequency  $f_0$ .

(v) **Circuit Impedance** : It is given by  $Z = \sqrt{[R^2 + (X_L \sim X_C)^2]} = \sqrt{R^2 + X^2}$

At low frequencies  $Z$  is large because  $X_C$  is large. Since  $X_C > X_L$ , the net circuit reactance  $X$  is capacitive and the p.f. is leading [Fig. 13.45 (b)]. At high frequencies,  $Z$  is again large (because  $X_L$  is large) but is inductive because  $X_L > X_C$ . Circuit impedance has minimum values at  $f_0$  given by  $Z = R$  because  $X = 0$ .



**Fig. 13.45**

(vi) **Current  $I_0$**  : It is the reciprocal of the circuit impedance. When  $Z$  is low,  $I_0$  is high and vice versa. As seen,  $I_0$  has low value on both sides of  $f_0$  (because  $Z$  is large there) but has maximum value of  $I_0 = V/R$  at resonance. Hence, maximum power is dissipated by the series circuit under resonant conditions. At frequencies below and above resonance, current decreases as shown in Fig. 13.45

\* However, value of  $Q_0$  is as given in Art 13.5, 13.9 and 13.17.

(b). Now,  $I_0 = V/R$  and  $I = V/Z = V/\sqrt{(R^2 + X^2)}$ . Hence  $I/I_0 = R/Z = V/\sqrt{(R^2 + X^2)}$  where  $X$  is the net circuit reactance at any frequency  $f$ .

(vii) **Power Factor**

As pointed out earlier,  $X$  is capacitive below  $f_0$ . Hence, current leads the applied voltage. However, at frequencies above  $f_0$ ,  $X$  is inductive. Hence, the current lags the applied voltage as shown in Fig. 13.45. The power factor has maximum value of unity at  $f_0$ .

### 13.12. Resonance Curve

The curve, between circuit current and the frequency of the applied voltage, is known as resonance curve. The shapes of such a curve, for different values of  $R$  are shown in Fig. 13.46. For smaller values of  $R$ , the resonance curve is sharply peaked and such a circuit is said to be sharply resonant or highly selective. However, for larger values of  $R$ , resonance curve is flat and is said to have poor selectivity. The ability of a resonant circuit to discriminate between one particular frequency and all others is called its selectivity. The selectivities of different resonant circuits are compared in terms of their half-power bandwidths (Art. 13.13).

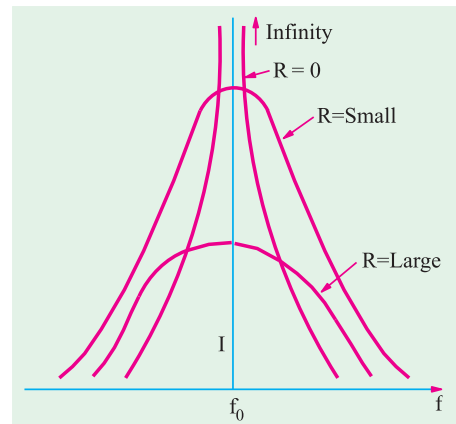


Fig. 13.46

### 13.13. Half-Power Bandwidth of a Resonant-Circuit

As discussed earlier, in an  $R$ - $L$ - $C$  circuit, the maximum current at resonance is solely determined by circuit resistance  $R$  ( $\because X = 0$ ) but at off-resonance frequencies, the current amplitude depends on  $Z$  (where  $X \neq 0$ ). The half-wave bandwidth of a circuit is given by the band of frequencies which lies between two points on either side of  $f_0$  where current falls to  $I_0/\sqrt{2}$ . Narrower the bandwidth, higher the selectivity of the circuit and vice versa. As shown in Fig. 13.47 the half-power bandwidth  $AB$  is given by

$$AB = \Delta f = f_2 - f_1 \text{ or } AB = \Delta \omega = \omega_2 - \omega_1 \text{ where } f_1 \text{ and } f_2 \text{ are the corner or edge frequencies.}$$

As seen,  $P_0 = I_0^2 R$ . However, power at either of the two points  $A$  and  $B$  is

$$\begin{aligned} P_1 = P_2 &= I^2 R \\ &= (I_0/\sqrt{2})^2 R = I_0^2 R/2 = \frac{1}{2} I_0^2 R = \frac{1}{2} \times \text{power at resonance} \end{aligned}$$

That is why the two points  $A$  and  $B$  on the resonance curve are known as half-power points\* and the corresponding value of the bandwidth is called half-power bandwidth  $B_{hp}$ . It is also called  $-3dB$ \* bandwidth. The following points regarding half-power point  $A$  and  $B$  are worth noting. At these points,

1. current is  $I_0/\sqrt{2}$
2. impedance is  $\sqrt{2} \cdot R$  or  $\sqrt{2} \cdot Z_0$
3.  $P_1 = P_2 = P_0/2$
4. the circuit phase angle is  $\theta = \pm 45^\circ$
5.  $Q = \tan \theta = \tan 45^\circ = 1$

\* The decibel power responses at these points, in terms of the maximum power at resonance, is

$$10 \log_{10} P/P_0 = 10 \log_{10} \frac{I_m^2 R/2}{I_m^2 R} = 10 \log_{10} \frac{1}{2} = -10 \log_{10} 2 = -3 \text{ dB}$$

Hence, the half-power points are also referred to as  $-3 \text{ dB}$  points.

$$6. B_{hp} = f_2 - f_1 = f_0/Q_0 = \sqrt{f_1 f_2}/Q_0 = R/2\pi L.$$

It is interesting to note that  $B_{hp}$  is independent of the circuit capacitance.

### 13.14. Bandwidth B at any Off-resonance Frequency

It is found that the bandwidth of a given  $R$ - $L$ - $C$  circuit at any off-resonance frequencies  $f_1$  and  $f_2$  is given by

$$B = f_0 Q/Q_0 = \sqrt{f_1 f_2}; \cdot Q/Q_0 = f_2 - f_1$$

where  $f_1$  and  $f_2$  are any frequencies (not necessarily half-power frequencies) below and above  $f_0$ .

$Q$  = tangent of the circuit phase angle at the off-resonance frequencies  $f_1$  and  $f_2$ .

$$Q_0 = \text{quality factor at resonance} = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

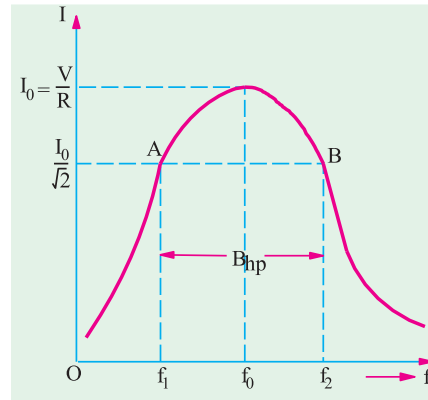


Fig. 13.47

### 13.15. Determination of Upper and Lower Half-power Frequencies

As mentioned earlier, at lower half-power frequencies,  $\omega_1 < \omega_0$  so that  $\omega_1 L < 1/\omega_1 C$  and  $\phi = -45^\circ$

$$\therefore \frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{or} \quad \omega_1^2 + \frac{R}{L} \omega_1 - \frac{1}{LC} = 0$$

Putting  $\frac{\omega_0}{Q_0} = \frac{R}{L}$  and  $\omega_0^2 = \frac{1}{LC}$  in the above equation, we get  $\omega_1^2 + \frac{\omega_0}{Q_0} \omega_1 - \omega_0^2 = 0$

The positive solution of the above equation is,  $\omega_1 = \omega_0 \left[ \sqrt{1 + \frac{1}{4Q_0^2}} - \frac{1}{2Q_0} \right]$

Now at the upper half-power frequency,  $\omega_2 > \omega_0$  so that  $\omega_2 > 1/\omega_2 C$  and  $\phi = +45^\circ$

$$\therefore \omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{or} \quad \omega_2^2 - \frac{\omega_0}{Q_0} \omega_2 - \omega_0^2 = 0$$

The positive solution of the above equation is  $\omega_2 = \omega_0 \left[ \sqrt{1 + \frac{1}{4Q_0^2}} + \frac{1}{2Q_0} \right]$

In case  $Q_0 > 1$ ; then the term  $1/4 Q_0^2$  is negligible as compared to 1.

Hence, in that case  $\omega_1 \cong \omega_0 \left( 1 - \frac{1}{2Q_0} \right)$  and  $\omega_2 \cong \omega_0 \left( 1 + \frac{1}{2Q_0} \right)$

Incidentally, it may be noted from above that  $\omega_2 - \omega_1 = \omega_0/Q_0$ .

### 13.16. Values of Edge Frequencies

Let us find the values of  $\omega_1$  and  $\omega_2$ ,  $I_0 = V/R$ ...at resonance

$$I = \frac{V}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}} \dots \text{at any frequency}$$

At points A and B,  $I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{V}{R}$

$$\therefore \frac{1}{\sqrt{2}} \cdot \frac{V}{R} = \frac{V}{[(R^2 + (\omega L - 1/\omega C)^2)^{1/2}]}$$

It shows that at half-power points, net reactance is equal to the resistance.

(Since resistance equals reactance, p.f. of the circuit at these points is  $= 1/\sqrt{2}$  i.e. 0.707, though leading at point *A* and lagging at point *B*).

$$\text{Hence } R^2 = (\omega L - 1/\omega C)^2 \quad \therefore \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = \pm \alpha \pm \sqrt{\alpha^2 + \omega_0^2}$$

$$\text{where } \alpha = \frac{R}{2L} \text{ and } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{Since } R^2/4L^2 \text{ is much less than } 1/\sqrt{LC} \quad \therefore \omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm \frac{R}{2L} \pm \omega_0$$

Since only positive values of  $\omega_0$  are considered,  $\omega = \omega_0 \pm R/2L = \omega_0 \pm \alpha$

$$\therefore \omega_1 = \omega_0 - \frac{R}{2L} \text{ and } \omega_2 = \omega_0 + \frac{R}{2L}$$

$$\therefore \Delta\omega = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/s and } f_2 - f_1 = \frac{R}{2L} \text{ Hz } \frac{f_0}{Q_0} \text{ Hz}$$

$$\text{Also } f_1 = f_0 - \frac{R}{4L} \text{ Hz and } f_2 = f_0 + \frac{R}{4L} \text{ Hz}$$

It is obvious that  $f_0$  is the *centre* frequency between  $f_1$  and  $f_2$ .

$$\text{Also, } \omega_1 = \omega_0 - \frac{1}{2} \Delta\omega \text{ and } \omega_2 = \omega_0 + \frac{1}{2} \Delta\omega$$

As stated above, bandwidth is a measure of circuits selectivity. Narrower the bandwidth, higher the selectivity and vice versa.

### 13.17. Q-Factor of a Resonant Series Circuit

The *Q*-factor of an *R-L-C* series circuit can be defined in the following different ways.

(i) it is given by the voltage magnification produced in the circuit at resonance.

We have seen that at resonance, current has maximum value  $I_0 = V/R$ . Voltage across either coil or capacitor  $= I_0 X_{L_0}$  or  $I_0 X_{C_0}$ , supply voltage  $V = I_0 R$

$$\therefore \text{Voltage magnification} = \frac{V_{L_0}}{V} = \frac{I_0 X_{L_0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

$$\text{or } = \frac{V_{C_0}}{V} = \frac{I_0 X_{C_0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{C_0}}{R} = \frac{\text{reactance}}{\text{resistance}} = \frac{1}{\omega_0 CR}$$

$$\therefore Q \text{ -factor, } Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \tan \phi \quad \dots (i)$$

where  $\phi$  is power factor of the coil.

(ii) The *Q*-factor may also be defined as under.

$$Q\text{-factor} = 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}} \quad \dots \text{in the circuit}$$

$$= 2\pi \frac{\frac{1}{2} L I_0^2}{I^2 R T_0} = 2\pi \frac{\frac{1}{2} L (\sqrt{2} I)^2}{I^2 R (1/f_0)} = \frac{I^2 2\pi f_0 L}{I^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \quad \dots (T_0 = 1/f_0)$$

$$\text{In other words, } Q_0 = \frac{\text{energy stored}^*}{\text{energy lost}} \quad \dots \text{in the circuit}$$

\* The author often jokingly tells students in his class that these days the quality of a person is also measured in terms of a quality factor given by

$$Q = \frac{\text{money earned}}{\text{money spent}}$$

Obviously, a person should try to have a high quality factor as possible by minimising the denominator and/or maximizing the numerator.

(iii) We have seen above that resonant frequency,  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  or  $2\pi f_0 = \frac{1}{\sqrt{LC}}$

Substituting this value in Eq. (i) above, we get the  $Q$ -factor,  $Q_0 = \frac{1}{R} \sqrt{\left(\frac{L}{C}\right)}$

(iv) In the case of series resonance, higher  $Q$ -factor means not only higher voltage magnification but also higher selectivity of the tuning coil. In fact,  $Q$ -factor of a resonant series circuit may be written as

$$Q_0 = \frac{\omega_0}{\text{bandwidth}} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \dots \text{as before}$$

Obviously,  $Q$ -factor can be increased by having a coil of large inductance but of small ohmic resistance.

(v) In summary, we can say that

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \sqrt{\frac{X_{L0} X_{C0}}{R}} = \frac{f_0}{B_{hp}} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

### 13.18. Circuit Current at Frequencies Other Than Resonant Frequencies

At resonance,  $I_0 = V/R$

At any other frequency above the resonant frequency, the current is given by  $I$

$$\frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

This current lags behind the applied voltage by a certain angle  $\phi$

$$\therefore \frac{I}{I_0} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \times \frac{R}{V} = \frac{1}{\sqrt{1 + \frac{1}{R^2} (\omega L - 1/\omega C)^2}} = \frac{1}{\left[1 + \left(\frac{\omega_0 L}{R}\right)^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2\right]^{1/2}}$$

$$\text{Now, } \omega_0 L/R = Q_0 \text{ and } \omega/\omega_0 = f/f_0 \text{ hence, } \frac{I}{I_0} = \frac{1}{\left[1 + Q_0^2 \left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2\right]^{1/2}}$$

### 13.19. Relation Between Resonant Power $P_0$ and Off-resonant Power $P$

In a series  $RLC$  resonant circuit, current is maximum i.e.  $I_0$  at the resonant frequency  $f_0$ . The maximum power  $P_0$  is dissipated by the circuit at this frequency where  $X_L$  equals  $X_C$ . Hence, circuit impedance  $Z_0 = R$ .

$$\therefore P_0 = I_0^2 R = (V/R)^2 \times R = V^2/R$$

At any other frequency either above or below  $f_0$  the power is (Fig. 13.48).

$$P = I^2 R = \left(\frac{V}{Z}\right)^2 \times R = \frac{V^2 R}{Z^2} = \frac{V^2 R}{R^2 + X^2} = \frac{V^2 R}{R^2 + X^2 R^2/R^2}$$

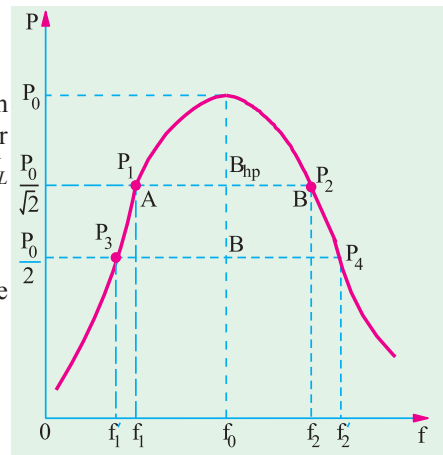


Fig. 13.48

$$= \frac{V^2 R}{R^2 + R^2 Q^2} = \frac{V^2 R}{R^2 (1 + Q^2)} = \frac{V^2}{R (1 + Q^2)} = \frac{P_0}{(1 + Q^2)}$$

The above equation shows that any frequency other than  $f_0$ , the circuit power  $P$  is reduced by a factor of  $(1 + Q^2)$  where  $Q$  is the tangent of the circuit phase angle (and not  $Q_0$ ). At resonance, circuit phase angle  $\theta = 0$ , and  $Q = \tan \theta = 0$ . Hence,  $P = P_0 = V^2/R$  (values of  $Q_0$  are given in Art.)

**Example 13.54.** For a series R.L.C circuit the inductor is variable. Source voltage is  $200\sqrt{2} \sin 100\pi t$ . Maximum current obtainable by varying the inductance is  $0.314 \text{ A}$  and the voltage across the capacitor then is  $300 \text{ V}$ . Find the circuit element values.

(Circuit and Field Theory, A.M.I.E. Sec B, 1993)

**Solution.** Under resonant conditions,  $I_m = V/R$  and  $V_L = V_C$ .

$$\therefore R = V/I_m = 200/0.314 = 637 \Omega, V_C = I_m \times X_{CD} = I_m/\omega_0 C$$

$$\therefore C = I_m/\omega_0 V_C = 0.314/100 \pi \times 300 = 3.33 \mu\text{F}.$$

$$V_L = I_m \times X_L = I_m \omega_0 L; L = V_L/\omega_0 I_m = 300/100 \pi \times 0.314 = \mathbf{3.03 \text{ H}}$$

**Example 13.55.** A coil having an inductance of  $50 \text{ mH}$  and resistance  $10 \Omega$  is connected in series with a  $25 \mu\text{F}$  capacitor across a  $200 \text{ V}$  ac supply. Calculate (a) resonance frequency of the circuit (b) current flowing at resonance and (c) value of  $Q_0$  by using different data.

(Elect. Engg. A.M.Ae. S.I, June 1991)

$$\text{Solution. (a)} \quad f_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{50 \times 10^{-3} \times 25 \times 10^{-6}}} = 142.3 \text{ Hz}$$

$$\text{(b)} \quad I_0 = V/R = 200/10 = 20 \text{ A}$$

$$\text{(c)} \quad Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi \times 142.3 \times 50 \times 10^{-3}}{10} = 4.47$$

$$Q_0 = \frac{1}{\omega_0 CR} = \frac{1}{2\pi \times 142.3 \times 25 \times 10^{-6} \times 10} = 4.47$$

$$Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{50 \times 10^{-3}}{25 \times 10^{-6}}} = 4.47$$

**Example 13.56.** A  $20\text{-}\Omega$  resistor is connected in series with an inductor, a capacitor and an ammeter across a  $25\text{-V}$  variable frequency supply. When the frequency is  $400\text{-Hz}$ , the current is at its maximum value of  $0.5 \text{ A}$  and the potential difference across the capacitor is  $150 \text{ V}$ . Calculate

(a) the capacitance of the capacitor

(b) the resistance and inductance of the inductor.

**Solution.** Since current is maximum, the circuit is in resonance.

$$X_C = V_C/I = 150/0.5 = 300 \Omega$$

$$\text{(a)} \quad X_C = 1/2\pi fC \text{ or } 300 = 1/2\pi \times 400 \times C$$

$$\therefore C = 1.325 \times 10^{-6} \text{ F} = \mathbf{1.325 \mu\text{F}}$$

$$\text{(b)} \quad X_L = X_C = 300 \Omega$$

$$\therefore 2\pi \times 400 \times L = 300 \quad \therefore L = \mathbf{0.119 \text{ H}}$$

(c) Now, at resonance,

$$\text{circuit resistance} = \text{circuit impedance or } 20 + R = V/I = 25/0.5 \quad \therefore R = 30 \Omega \quad \dots \text{Fig.13.49}$$

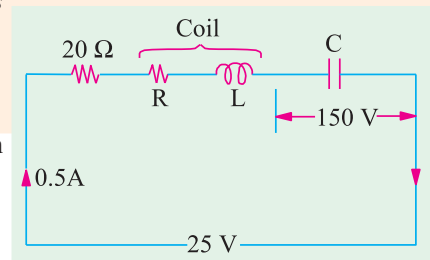


Fig. 13.49

**Example 13.57.** An R-L-C series circuit consists of a resistance of  $1000\ \Omega$ , an inductance of  $100\ \text{mH}$  and a capacitance of  $10\ \mu\text{F}$ . If a voltage of  $100\ \text{V}$  is applied across the combination, find (i) the resonance frequency (ii) Q-factor of the circuit and (iii) the half-power points.

(Elect. Circuit Analysis, Bombay Univ.)

**Solution. (i)**  $f_0 = \frac{1}{2\pi\sqrt{10^{-1} \times 10^{-11}}} = \frac{10^6}{2\pi} = 159\ \text{kHz}$

**(ii)**  $Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{1000} \times \sqrt{\frac{10^{-1}}{10^{-11}}} = 100$

**(iii)**  $f_1 = f_0 - \frac{R}{4\pi L} = 159 \times 10^3 - \frac{1000}{4\pi \times 10^{-1}} = 158.2\ \text{kHz}$

$f_2 = f_0 + \frac{R}{4\pi L} = 159 \times 10^3 + \frac{1000}{4\pi \times 10^{-1}} = 159.8\ \text{kHz}$

**Example 13.58.** A series R-L-C circuit consists of  $R = 1000\ \Omega$ ,  $L = 100\ \text{mH}$  and  $C = 10\ \text{picofarads}$ . The applied voltage across the circuit is  $100\ \text{V}$ .

- (i) Find the resonant frequency of the circuit.  
 (ii) Find the quality factor of the circuit at the resonant frequency.  
 (iii) At what angular frequencies do the half power points occur?  
 (iv) Calculate the bandwidth of the circuit.

(Networks-I, Delhi Univ. & U.P. Tech. Univ. 2001)

**Solution. (i)**  $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-12}}} = 159.15\ \text{kHz}$

**(ii)**  $Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{1000} \sqrt{\frac{100 \times 10^{-3}}{10 \times 10^{-12}}} = 100$

**(iii)**  $B_{hp} = \frac{R}{2\pi L} = \frac{1000}{2\pi \times 100 \times 10^{-3}} = 1591.5\ \text{Hz}$

Also,  $B_{hp} = f_0/Q_0 = 159.15\ \text{kHz}/100 = 1.5915\ \text{kHz} = 1591.5\ \text{Hz}$  ...as above

**(iv)**  $\omega_1 = \omega_0 \left(1 - \frac{1}{2Q_0}\right) = 2\pi \times 159.15 \times \left(1 - \frac{1}{2 \times 100}\right) = 994.969\ \text{rad/sec.}$

$\omega_2 = \omega_0 \left(1 + \frac{1}{2Q_0}\right) = 2\pi \times 159.15 \times \left(1 + \frac{1}{2 \times 100}\right) = 1004.969\ \text{rad/sec}$

**(v)** Band width =  $(\omega_2 - \omega_1) = 1004.969 - 994.969 = 10.00\ \text{rad/sec.}$

**Example 13.59.** An R-L-C series resonant circuit has the following parameters :  
 Resonance frequency =  $5000/2\pi\ \text{Hz}$ ; impedance at resonance =  $56\ \Omega$  and Q-factor = 25.  
 Calculate the capacitance of the capacitor and the inductance of the inductor.  
 Assuming that these values are independent of the frequency, find the two frequencies at which the circuit impedance has a phase angle of  $\pi/4$  radian.

**Solution.** Here  $\omega_0 = 2\pi f_0 = 2\pi \times 5000/2\pi = 5000\ \text{rad/s}$

Now,  $Q = \frac{\omega_0 L}{R}$  or  $25 = \frac{5000 L}{56}$  or  $L = 0.28\ \text{H}$

Also at resonance  $\omega_0 L = 1/\omega_0 C$  or  $5000 \times 0.28 = 1/5000 \times C \therefore C = 0.143\ \mu\text{F}$

The circuit impedance has a phase shift of  $45^\circ$  and the two half-power frequencies which can be found as follows :

$$BW = \frac{f_0}{Q} = \frac{5000/2\pi}{25} = 31.83\ \text{Hz}$$

Therefore lower half-power frequency =  $(f_0 - 31.83/2) = 5000/2\pi - 15.9 = 779.8 \text{ Hz}$ .

Upper half-power frequency =  $(f_0 + 31.83/2) = 5000/2\pi + 15.9 = 811.7 \text{ Hz}$ .

**Example 13.60.** An R-L-C series circuit is connected to a 20-V variable frequency supply. If  $R = 20 \Omega$ ,  $L = 20 \text{ mH}$  and  $C = 0.5 \mu\text{F}$ , calculate the following :

(a) resonant frequency  $f_0$  (b) resonant circuit  $Q_0$  using L/C ratio (c) half-power bandwidth using  $f_0$  and  $Q_0$  (d) half-power bandwidth using the general formula for any bandwidth (e) half-power bandwidth using the given component values (f) maximum power dissipated at  $f_0$ .

**Solution. (a)**  $f_0 = 1/2\pi \sqrt{LC} = 1/2\pi \sqrt{(20 \times 10^{-3} \times 0.5 \times 10^{-6})} = 159 \text{ Hz}$

(b)  $Q_0 = \frac{1}{R} \cdot \sqrt{\frac{L}{C}} = \frac{1}{20} \cdot \sqrt{\frac{20 \times 10^{-3}}{0.5 \times 10^{-6}}} = 10$

(c)  $B_{hp} = f_0/Q_0 = 159/10 = 15.9 \text{ Hz}$

(d)  $B_{hp} = f_0 Q/Q_0 = 159 \times \tan 45^\circ/10 = 15.9 \text{ Hz}$

It is so because the power factor angle at half-power frequencies is  $\pm 45^\circ$ .

(e)  $B_{hp} = R/2\pi L = 20/2\pi \times 20 \times 10^{-3} = 15.9 \text{ Hz}$

(f)  $f_0 = V^2/R = 20^2/20 = 20 \text{ W}$

**Example 13.61.** An inductor having a resistance of  $25 \Omega$  and a  $Q_0$  of 10 at a resonant frequency of 10 kHz is fed from a  $100 \angle 0^\circ$  supply. Calculate

(a) Value of series capacitance required to produce resonance with the coil

(b) the inductance of the coil (c)  $Q_0$  using the L/C ratio (d) voltage across the capacitor

(e) voltage across the coil.

**Solution. (a)**  $X_{L0} = Q_0 R = 10 \times 25 = 250 \Omega$

Now,  $X_{C0} = X_{L0} = 250 \Omega$

Hence,  $C = 1/2\pi f_0 \times X_{C0} = 1/2\pi \times 10^4 \times 250 = 63.67 \times 10^{-9} \text{ F} = 63.67 \mu\text{F}$

(b)  $L = X_{L0}/2\pi f_0 = 250/2\pi \times 10^4 = 3.98 \text{ mH}$

(c)  $Q_0 = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$

Now,  $\frac{L}{C} = \frac{3.98 \times 10^{-3}}{63.67 \times 10^{-9}} = 6.25 \times 10^4$

$\therefore Q_0 = \frac{1}{25} \sqrt{6.25 \times 10^4} = 10$  (verification)

(d)  $V_{C0} = -jQ_0 V = -j 10 \times 100 \angle 0^\circ = -j 1000 \text{ V} = -1000 \angle -90^\circ \text{ V}$

(e) Since  $V_{L0} = V_{C0}$  in magnitude, hence,  $V_{L0} = +j 1000 \text{ V}$   
 $= 1000 \angle 90^\circ \text{ V}$ ; Also,  $V_R = V = 100 \angle 0^\circ$

Hence,  $V_{coil} = V_R + V_{L0}$   
 $= 100 + 1000 \angle 90^\circ = 100 + j 1000 = 1005 \angle 84.3^\circ$

**Example 13.62.** A series L.C circuit has  $L = 100 \text{ pH}$ ,  $C = 2500 \mu\text{F}$  and  $Q = 70$ . Find (a) resonant frequency  $f_0$  (b) half-power points and (c) bandwidth.

**Solution. (a)**  $f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{10^9}{2\pi \sqrt{100 \times 2500}} = 318.3 \text{ kHz}$

(b)  $f_2 - f_1 = \Delta f = f_0/Q = 318.3/70 = 4.55 \text{ kHz}$

(c)  $f_2 f_1 = f_0^2 = 318.3^2$ ;  $f_2 - f_1 = 4.55 \text{ kHz}$



Solving for  $f_1$  and  $f_2$ , we get,  $f_1 = 3104 \text{ kHz}$  and  $f_2 = 320.59 \text{ kHz}$

**Note.** Since  $Q$  is very high, there would be negligible error in assuming that the half-power points are equidistant from the resonant frequency.

**Example 13.63.** A resistor and a capacitor are connected in series across a 150 V ac supply. When the frequency is 40 Hz, the circuit draws 5 A. When the frequency is increased to 50 Hz, it draws 6 A. Find the values of resistance and capacitance. Also find the power drawn in the second case. [Bombay University, 1997]

**Solution.** Suffix 1 for 40 Hz and 2 for 50 Hz will be given.

$$Z_1 = 150/5 = 30 \text{ ohms at } 40 \text{ Hz}$$

$$\text{or } R^2 + X_{C2}^2 = 900$$

$$\text{Similarly, } R^2 + X_{C2}^2 = 625 \text{ at } 50 \text{ Hz, since } Z_2 = 25 \Omega$$

Further, capacitive reactance is inversely proportional to the frequency.

$$X_{C1}/X_{C2} = 50/40 \text{ or } X_{C1} = 1.25 X_{C2}$$

$$X_{C1}^2 - X_{C2}^2 = 900 - 625 = 275$$

$$X_{C1}^2 (1.25^2 - 1) = 275 \text{ or } X_{C2}^2 = 488.9, X_{C2} = 22.11$$

$$X_{C1} = 1.25 \times 22.11 = 27.64 \text{ ohms}$$

$$R^2 = 900 - X_{C1}^2 = 900 - 764 = 136, R = 11.662 \text{ ohms}$$

$$C = \frac{1}{2 \times 40 \times 27.64} = 144 \text{ F}$$

$$\text{Power drawn in the second case} = 6^2 \times 11.662 = 420 \text{ watts}$$

**Example 13.64.** A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set 500 pF, the current has its maximum value while it is reduced to one-half when the capacitance is 600 pF. Find

(i) the resistance, (ii) the inductance, (iii) the  $Q$ -factor of the inductor.

[Bombay University, 1996]

**Solution.** Resonance takes place at 1 MHz, for  $C = 500 \text{ pF}$ .

$$LC = \frac{1}{\omega_0^2} = \frac{1}{(2\pi \times 10^6)^2} = \frac{10^{-12}}{4\pi^2}$$

$$L = 10^{-12}/(4 \times \pi^2 \times 500 \times 10^{-12}) = 1/(4 \times \pi^2 \times 500)$$

$$= 50.72 \text{ mH}$$

$$Z_1 = \text{Impedance with } 500 \text{ pF capacitor} = R + j \omega L - j 1/\omega C$$

$$= R + j (2\pi \times 10^6 \times 50.72 \times 10^{-6}) - j \frac{1}{2\pi \times 10^6 \times 500 \times 10^{-12}}$$

$$= R, \text{ since resonance occurs.}$$

$$Z_2 = \text{Impedance with } 600 \text{ pF capacitor}$$

$$|Z|_2 = R + j \omega L - j \frac{1}{\omega \times 600 \times 10^{-12}} = 2R, \text{ since current is halved.}$$

$$\omega L - 1/\omega C = \sqrt{3} R$$

$$\sqrt{3} R = 2\pi \times 10^6 \times 50.7 \times 10^{-6} - \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}}$$

$$= 2\pi \times 50.72 \frac{10^6}{2\pi \times 600}$$

$$= 318.52 - 265.4 = 53.12$$

$$R = 30.67 \text{ ohms}$$

$$Q \text{ Factor of coil} = (\omega_0 L)/R$$

$$= 50.72 \times 10^{-6} \times 2\pi \times 10^6 / 30.67 = 50.72 \times 2\pi / 30.67 = 10.38$$

**Example 13.65.** A large coil of inductance 1.405 H and resistance 40 ohms is connected in series with a capacitor of 20 microfarads. Calculate the frequency at which the circuit resonates.

If a voltage of 100 V at the corresponding frequency is applied to the circuit, calculate the current drawn from the supply and the voltages across the coil and across the capacitor.

[Nagpur University Nov. 1999]

**Solution.**  $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.405 \times 20 \times 10^{-6}}} = 188.65 \text{ radians/sec.}$

$$f_0 = \frac{188.65}{2\pi} = 30.04 \text{ Hz}$$

Reactance at 30.04 Hz have to be calculated for voltages across the coil and the capacitor

$$X_L = \omega_0 L = 188.65 \times 1.405 = 265 \Omega$$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{188.65 \times 20 \times 10^{-6}} = 265 \Omega$$

$$\text{Coil Impedance} = \sqrt{40^2 + 265^2} = 268 \Omega$$

$$\text{Impedance of the total circuit} = 40 + j 265 - j 265 = 40 \Omega$$

$$\text{Supply Current} = \frac{100}{40} = 2.5 \text{ amp, at unity p.f.}$$

$$\text{Voltage across the coil} = 2.5 \times 268 = 670 \text{ V}$$

$$\text{Voltage across the capacitor} = 2.5 \times 265 = 662.5 \text{ V}$$

The phasor diagram is drawn below; in Fig. 13.50 (a) for the circuit in Fig. 13.50 (b)

$$OB = V, BC = V_C$$

$$AB = IX_L$$

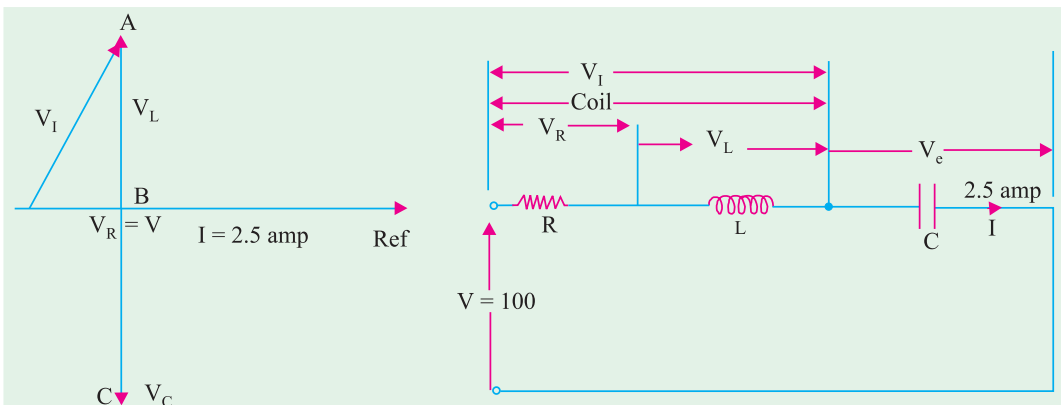


Fig. 13.50 (a) Phasor diagram

Fig. 13.50 (b) Series resonating circuit

**Example 13.66.** A series R-L-C circuit is excited from a constant-voltage variable frequency source. The current in the circuit becomes maximum at a frequency of  $600/2\pi$  Hz and falls to half the maximum value at  $400/2\pi$  Hz. If the resistance in the circuit is  $3 \Omega$ , find L and C.

(Grad. I.E.T.E. Summer 1991)

**Solution.** Current at resonance is  $I_0 = V/R$

Actual current at any other frequency is  $I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$\therefore \frac{I}{I_0} = \frac{\frac{V}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}{\frac{V}{R^2}} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Now  $Q = \frac{\omega_0 L}{R}$  and  $\frac{\omega}{\omega_0} = \frac{f}{f_0}$ , hence  $\frac{I}{I_0} = \frac{1}{\left[1 + Q^2 \left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2\right]^{1/2}}$

In the present case,  $f_0 = 600/2\pi$  Hz,  $f = 400/2\pi$  Hz and  $I/I_0 = 1/2$

$$\therefore \frac{1}{2} = \frac{1}{\left[1 + Q^2 \left(\frac{400}{600} - \frac{600}{400}\right)^2\right]^{1/2}} = \frac{1}{\left[1 + Q^2 \left(\frac{2}{2} - \frac{3}{2}\right)^2\right]^{1/2}}$$

or  $\frac{1}{4} = \frac{1}{1 + 25Q^2/36} \therefore Q = 2.08$

Now,  $Q = \frac{1}{\omega_0 RC}$  or  $2.08 = \frac{1}{600 \times 3 \times C} \therefore C = 267 \times 10^{-6} \text{ F} = \mathbf{267 \text{ mF}}$

Also  $Q = \omega_0 L/R \therefore 2.08 = \frac{600L}{R} = \frac{600L}{3} \therefore L = \mathbf{10.4 \text{ mH}}$

**Example 13.67.** Discuss briefly the phenomenon of electrical resonance in simple R-L-C circuits.

A coil of inductance  $L$  and resistance  $R$  in series with a capacitor is supplied at constant voltage from a variable-frequency source. Call the resonance frequency  $\omega_0$  and find, in terms of  $L$ ,  $R$  and  $\omega_0$ , the values of that frequency at which the circuit current would be half as much as at resonance.

(Basic Electricity, Bombay Univ.)

**Solution.** For discussion of resonance, please refer to Art. 13.10.

The current at resonance is maximum and is given by  $I_0 = V/R$ . Current at any other frequency is

$$I = \frac{V}{\left[R^2 + (\omega L - 1/\omega C)^2\right]^{1/2}}$$

$$\therefore \frac{I_0}{I} = \frac{\left[R^2 + (\omega L - 1/\omega C)^2\right]^{1/2}}{R}$$

or  $N = \left[1 + \frac{1}{R^2} \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$

Now  $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$

$\therefore R = \frac{\omega_0 L}{Q} = \frac{1}{\omega_0 CQ}$

Substituting this value in the above equation, we get

$$N = \left[1 + Q^2 \left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2\right]^{1/2}$$

or  $\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2 = \frac{N^2 - 1}{Q^2} \therefore \frac{\sqrt{N^2 - 1}}{Q} = \pm \left(\frac{f}{f_0} - \frac{f_0}{f}\right)$

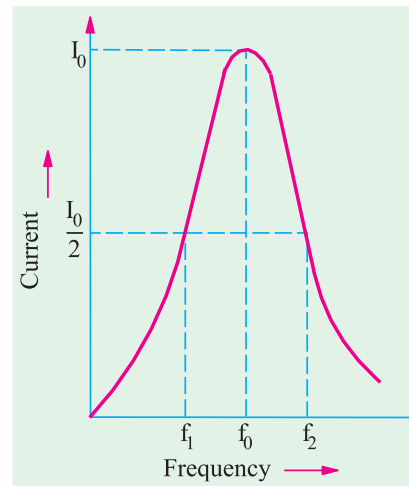


Fig. 13.51

$$\text{or } \frac{\sqrt{(N^2 - 1)}}{Q} = \frac{f_2 - f_0}{f_0} = \frac{f_0 - f_1}{f_1 - f_0}$$

where  $f_2 > f_0$  and  $f_1 < f_0$  are the two frequencies at which the current has fallen to  $1/N$  of the resonant value.

$$\text{In the present case, } N = 2 \text{ (Fig. 13.50)} \quad \therefore \frac{f_2 - f_0}{f_0} = \frac{\sqrt{3}}{Q} \text{ and } \frac{f_0 - f_1}{f_1 - f_0} = \frac{\sqrt{3}}{Q}$$

From these equations,  $f_1$  and  $f_2$  may be calculated.

**Example 13.68.** A coil of inductance  $9 \text{ H}$  and resistance  $50 \Omega$  in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current of  $1 \text{ A}$  occurs at  $75 \text{ Hz}$ , find the frequency when the current is  $0.5 \text{ A}$ .

(Principles of Elect. Engg. Delhi Univ.)

**Solution.** Here,  $N = I_0/I = 1/0.5 = 2$ ;  $Q = \omega_0 L/R = 2\pi \times 75 \times 9/50 = 84.8$

Let  $f_1$  and  $f_2$  be the frequencies at which current falls to half its maximum value at resonance frequency. Then, as seen from above

$$\frac{f_0 - f_1}{f_1 - f_0} = \frac{\sqrt{3}}{Q} \text{ or } \frac{75 - f_1}{f_1 - 75} = \frac{\sqrt{3}}{84.4}$$

$$\text{or } (75^2 - f_1^2)/75f_1 = 0.02 \text{ or } f_1^2 + 1.5f_1 - 5625 = 0 \text{ or } f_1 = \mathbf{74.25 \text{ Hz}}$$

$$\text{Also } \frac{f_2 - 75}{75 - f_2} = \frac{\sqrt{3}}{84.4} \text{ or } f_2^2 - 1.5f_2 - 5625 = 0 \text{ or } f_2 = \mathbf{75.75 \text{ Hz.}}$$

**Example 13.69.** Using the data given in Ex. 13.45 find the following when the power drops to  $4 \text{ W}$  on either side of the maximum power at resonance.

- (a) circuit  $Q$  (b) circuit phase angle  $\phi$  (c)  $4\text{-W}$  bandwidth  $B$   
 (d) lower frequency  $f_1$  (e) upper frequency  $f_2$ .  
 (f) resonance frequency using the value of  $f_1$  and  $f_2$ .

$$\text{(a) } P = \frac{P_0}{(1 + Q_0^2)} \quad \therefore Q = \sqrt{(P_0/P_1) - 1} = \sqrt{(20/4) - 1} = 2$$

$$\text{(b) } \tan(\pm\theta) = 2j \pm \theta / \tan^{-1} 2 = \pm \mathbf{63.4^\circ}$$

$$\text{(c) } B_{hp} = \frac{f_0 Q}{Q_0} = \frac{1591 \times 2}{10} = 318.2 \text{ Hz}$$

$$\text{(d) } f_1 = f_0 - B/2 = 1591 - (318.2/2) = 1431.9 \text{ Hz}$$

$$\text{(e) } f_2 = f_0 + B/2 = 1591 + (318.2/2) = 1750.1 \text{ Hz}$$

$$\text{(f) } f_0 = \sqrt{f_1 f_2} = \sqrt{1431.9 \times 1750.2} = \mathbf{1591 \text{ Hz.}}$$

It shows that regardless of the bandwidth magnitude,  $f_0$  is always the geometric mean of  $f_1$  and  $f_2$ .

**Example 13.70.** A constant e.m.f. source of variable frequency is connected to a series R.L.C. circuit of Fig. 13.51.

- (a) Shown in nature of the frequency  $-V_R$  graph  
 (b) Calculate the following (i) frequency at which maximum power is consumed in the  $2 \Omega$  resistor  
 (ii)  $Q$ -factor of the circuit at the above frequency (iii) frequencies at which the power consumed in  $2 \Omega$  resistor is one-tenth of its maximum value.

(Network Analysis A.M.I.E Sec. B.W.)

**Solution.** (a) The graph of angular frequency  $\omega$  versus voltage drop across  $R$  i.e.  $V_R$  is shown in Fig. 13.52. It is seen that as frequency of the applied voltage increases,  $V_R$  increases till it reaches its

maximum value when the given  $RLC$  circuit becomes purely reactive *i.e.* when  $X_L = X_C$  (Art. 13.10). (b) (i) maximum power will be consumed in the  $2\ \Omega$  resistor when maximum current flows in the circuit under resonant condition.

For resonance  $\omega_0 L = 1/\omega_0 C$  or

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{40 \times 10^{-6} \times 160 \times 10^{-12}} = 10^9/80 \text{ rad/s}$$

$$\therefore f_0 = \omega_0/2\pi = 10^9/2\pi \times 80 = \mathbf{1.989 \text{ MHz}}$$

$$(ii) \text{ } Q\text{-factor, } Q_0 = \frac{\omega_0 L}{R} = \frac{10^9 \times 40 \times 10^{-6}}{80 \times 2} = \mathbf{250}$$

(iii) Maximum current  $I_0 = V/R$  (Art. 13.10). Current at any other frequency is  $I = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

$$\text{Power at any frequency } I^2 R = \frac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \cdot R$$

$$\text{Maximum power } I_0^2 R = \left(\frac{V}{R}\right)^2 \cdot R$$

Hence, the frequencies at which power consume would be one-tenth of the maximum power will

$$\text{be given by the relation. } \frac{1}{10} \cdot \left(\frac{V}{R}\right)^2 \cdot R = \frac{V^2}{R^2 + (\omega L - 1/\omega C)^2} \cdot R$$

or cross multiplying, we get  $R^2 + (\omega L - 1/\omega C)^2 = 10R^2$  or  $(\omega L - 1/\omega C)^2 = 9R^2$

$$\therefore (\omega L - 1/\omega C) = \pm 3R \text{ and } \omega_1 L - 1/\omega_1 C = 3R \\ \text{and } \omega_2 L - 1/\omega_2 C = -3R$$

Adding the above two equations, we get

$$(\omega_1 - \omega_2)L - \frac{1}{C} \left( \frac{1}{\omega_1} - \frac{1}{\omega_2} \right) = 0 \text{ or } (\omega_1 - \omega_2) \left( L - \frac{1}{LC} \right) = 0$$

$$\text{Subtracting the same two equations, we have } L(\omega_1 - \omega_2) \left( \frac{1}{C} - \frac{1}{LC} \right) = 6R$$

$$(\omega_1 - \omega_2) \left( \frac{1}{LC} - \frac{1}{C} \right) = \frac{6R}{L}$$

Substituting the value of  $1/LC = \omega^2 = \omega_1 \omega_2$ , we get

$$(\omega_1 - \omega_2) + \omega_1 \omega_2 \left( \frac{\omega_1 - \omega_2}{\omega_1 \omega_2} \right) = \frac{6R}{L} \text{ or } (\omega_1 - \omega_2) = \Delta\omega = \frac{3R}{L}$$

$$\text{Now, } \omega_2 = \omega_0 + \frac{\Delta\omega}{2} = \omega_0 + \frac{1.5R}{L} \text{ and } \omega_1 = \omega_0 - \frac{1.5R}{L}$$

$$\therefore f_2 = f_0 + 1.5R/2\pi L = 1.989 \times 10^6 + 1.5 \times 2.2\pi \times 40 \times 10^{-6} = 1.989 \times 10^6 + 0.0119 \times 10^6 = \mathbf{2 \text{ MHz}}$$

$$f_1 = f_0 - 1.5R/2\pi L = 1.989 \times 10^6 - 0.0119 \times 10^6 = \mathbf{1.977 \text{ MHz}}$$

**Example 13.71.** Show that in  $R$ - $L$ - $C$  circuit, the resonant frequency  $\omega_0$  is the geometric mean of the lower and upper half-power frequencies  $\omega_1$  and  $\omega_2$  respectively.

**Solution.** As stated earlier, at lower half-power radiant frequency  $\omega_1$ ;  $X_C > X_L$  and at frequencies higher than half-power frequencies  $X_L > X_C$ . However, the difference between the two equals  $R$ .

$$\therefore \text{at } \omega_1, X_C - X_L = R \text{ or } 1/\omega_1 C - \omega_1 L = R \quad \dots(i)$$

$$\text{At } \omega_2, X_L - X_C = R \text{ or } \omega_2 L - 1/\omega_2 C = R$$

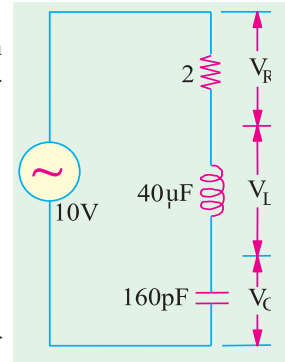


Fig. 13.52

Multiplying both sides of Eq. (i) by  $C$  and substituting  $\frac{1}{\omega_0} = 1/\sqrt{LC}$ , we get

$$\frac{1}{1} - \frac{1}{\frac{2}{0}} - \frac{2}{\frac{2}{0}} = \frac{1}{2} \text{ or } \frac{1}{1} - \frac{1}{\frac{1}{2}} = \frac{1}{\frac{2}{0}} \text{ or } 0 = \sqrt{1 - 2}$$

**Example 13.72.** Prove that in a series  $R$ - $L$ - $C$  circuit,  $Q_0 = \omega_0 L/R = f_0/\text{bandwidth} = f_0/BW$ .

**Solution.** As has been proved in Art. 14.13, at half power frequencies, net reactance equals resistance. Moreover, at  $\omega_1$ , capacitive reactance exceeds inductive reactance whereas at  $\omega_2$ , inductive reactance exceeds capacitive reactance.

$$\therefore \frac{1}{2\pi f_1 C} - 2\pi f_1 L = R \text{ or } f_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

$$\text{Also } 2\pi f_2 L - \frac{1}{2\pi f_2 C} = R \text{ or } f_2 = \frac{R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

$$\text{Now, } BW = f_2 - f_1 = R/2\pi L. \text{ Hence, } Q_0 = f_0/BW = 2\pi f_0 L/R = \omega_0 L/R.$$

### Tutorial Problem No. 13.4

- An a.c. series circuit has a resistance of  $10 \Omega$ , an inductance of  $0.2 \text{ H}$  and a capacitance of  $60 \mu\text{F}$ . Calculate  
(a) the resonant frequency (b) the current and (c) the power at resonance.  
Give that the applied voltage is  $200 \text{ V}$ . **[46 Hz; 20 A; 4 kW]**
- A circuit consists of an inductor which has a resistance of  $10 \Omega$  and an inductance of  $0.3 \text{ H}$ , in series with a capacitor of  $30 \mu\text{F}$  capacitance. Calculate  
(a) the impedance of the circuit to currents of  $40 \text{ Hz}$  (b) the resonant frequency (c) the peak value of stored energy in joules when the applied voltage is  $200 \text{ V}$  at the resonant frequency.  
**[58.31  $\Omega$ ; 53 Hz; 120 J]**
- A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a  $240\text{-V}$ ,  $50\text{-Hz}$  supply, the maximum current given by varying the inductance is  $0.5 \text{ A}$ . At this current, the voltage across the capacitor is  $250 \text{ V}$ . Calculate the values of  
(a) the resistance (b) the capacitance (c) the inductance. **[480  $\Omega$ , 6.36  $\mu\text{F}$ ; 1.59 H]**  
Neglect the resistance of the inductor
- A circuit consisting of a coil of resistance  $12\Omega$  and inductance  $0.15 \text{ H}$  in series with a capacitor of  $12\mu\text{F}$  is connected to a variable frequency supply which has a constant voltage of  $24 \text{ V}$ . Calculate (a) the resonant frequency (b) the current in the circuit at resonance (c) the voltage across the capacitor and the coil at resonance. **[(a) 153 Hz (b) 2 A (c) 224 V]**
- A resistance, a capacitor and a variable inductance are connected in series across a  $200\text{-V}$ ,  $50\text{-Hz}$  supply. The maximum current which can be obtained by varying the inductance is  $314 \text{ mA}$  and the voltage across the capacitor is then  $300 \text{ V}$ . Calculate the capacitance of the capacitor and the values of the inductance and resistance. **[3.33  $\mu\text{F}$ , 3.04 H, 637  $\Omega$  (I.E.E. London)]**
- A  $250\text{-V}$  circuit, consisting of a resistor, an inductor and a capacitor in series, resonates at  $50 \text{ Hz}$ . The current is then  $1 \text{ A}$  and the p.d. across the capacitor is  $500 \text{ V}$ . Calculate (i) the resistance (ii) the inductance and (iii) the capacitance. Draw the vector diagram for this condition and sketch a graph showing how the current would vary in a circuit of this kind if the frequency were varied over a wide range, the applied voltage remaining constant.  
**[(i) 250  $\Omega$  (ii) 0.798 H (iii) 12.72  $\mu\text{F}$ ] (City & Guilds, London)**
- A resistance of  $24 \Omega$ , a capacitance of  $150 \mu\text{F}$  and an inductance of  $0.16 \text{ H}$  are connected in series with each other. A supply at  $240 \text{ V}$ ,  $50 \text{ Hz}$  is applied to the ends of the combination. Calculate (a) the current in the circuit (b) the potential differences across each element of the circuit (c) the frequency to which the supply would need to be changed so that the current would be at unity power-factor and find the current at this frequency.  
**[(a) 6.37 A (b)  $V_R = 152.9 \text{ V}$ ,  $V_C = 320 \text{ V}$ ,  $V_L = 123.3 \text{ V}$  (c) 32 Hz; 10 A] (London Univ.)**
- A series circuit consists of a resistance of  $10 \Omega$ , an inductance of  $8 \text{ mH}$  and a capacitance of  $500 \mu\text{F}$ . A sinusoidal E.M.F. of constant amplitude  $5 \text{ V}$  is introduced into the circuit and its frequency varied

over a range including the resonant frequency.

At what frequencies will current be (a) a maximum (b) one-half the-maximum ?

**[(a) 79.6 kHz (b) 79.872 kHz, 79.528 kHz] (App. Elect. London Univ.)**

9. A circuit consists of a resistance of 12 ohms, a capacitance of 320  $\mu\text{F}$  and an inductance of 0.08 H, all in series. A supply of 240 V, 50 Hz is applied to the ends of the circuit. Calculate :

- (a) the current in the coil.  
 (b) the potential differences across each element of the circuit.  
 (c) the frequency at which the current would have unity power-factor.

**[(a) 12.4 A (b) 149 V, 311 V (c) 32 Hz] (London Univ.)**

10. A series circuit consists of a reactor of 0.1 henry inductance and 5 ohms resistance and a capacitor of 25.5  $\mu\text{F}$  capacitance.

Find the resonance frequency and the percentage change in the current for a divergence of 1 percent from the resonance frequency.

**[100 Hz, 1.96% at 99 Hz; 4.2% at 101 Hz] (City and Guilds, London)**

11. The equation of voltage and currents in two element series circuit are :

$$v(t) = 325 \cdot 3 \sin(6.28 \text{ kt} + \pi/3) \text{ volts}$$

$$i(t) = 14 \cdot 142 \sin(6.28 \text{ kt} + \pi/2) \text{ Amp.}$$

- (i) Plot the power  $p(t)$  on wave diagram. (ii) Determine power factor and its nature. (iii) Determine the elements value.

**(Nagpur University, Winter 2002)**

12. A pure capacitor is connected in series with practical inductor (coil). The voltage source is of 10 volts, 10,000 Hz. It was observed that the maximum current of 2 Amp. flows in the circuit when the value of capacitor is 1 micro-farad. Find the parameter (R & L) of the coil.

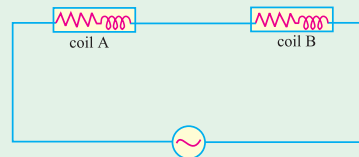
**(Nagpur University, Winter 2002)**

13. Draw the phasor diagram for each of the following combinations :

- (i) R and L in series and combination in parallel with C.  
 (ii) R and C in series and combination in parallel with L.  
 (iii) R, L and C in series, with  $X_L > X_C$ , when ac source is connected to it.

**(Nagpur University, Summer 2003)**

14. Two choke coils are connected in series as shown in Fig. 13.53. :



**Fig. 13.53**

Internal resistance and its inductive reactance of coil A is 4  $\Omega$  and 8  $\Omega$  resp. Supply voltage is 200 V. Total power consumed in the circuit is 2.2 kW and reactive power consumed is 1.5 kVAR. Find the internal resistance and inductive reactance of coil B. **(Nagpur University, Summer 2004)**

15. A series circuit having a resistance of 10  $\Omega$ , and inductance of  $(1/2\pi)$  H and a variable cap. is connected to 100 V, 50 Hz supply. Calculate the value of capacitor to form series resonance. Calculate resonant current, power and power factor. **(Gujrat University, June/July 2003)**

16. Derive expressions for impedance, current and power factor for an R-L-C series circuit when applied with a.c. voltage. Draw also the vector diagram. **(Gujrat University, June/July 2003)**

17. Explain the terms active power, reactive power and power factor. Also describe the series resonance of RLC circuit and list its important properties. **(R.G.P.V. Bhopal University, June 2004)**

18. A 60 Hz sinusoidal voltage  $v = 100 \sin \omega t$  is applied to a series R-L circuit. Assuming R = 10  $\Omega$ , L = 0.01 H, find the steady state current and relative phase angle. Also compute the effective magnitude and phase of voltage drops across each circuit elements.

**(R.G.P.V. Bhopal University, June 2004)**

19. With reference to Fig 13.54, find the values of R and X so that  $V_1 = 3V_2$  and  $V_1$  and  $V_2$  are in quadrature. Applied voltage across AB is 240V. **(Belgaum Karnataka University, February 2002)**

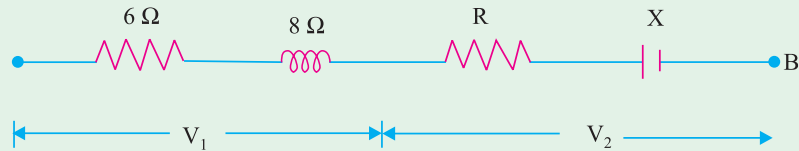


Fig. 13.54

20. A choke coil takes a current of 2 A lagging  $60^\circ$  behind the applied voltage of 220V at 50Hz. Calculate the inductance and resistance of the coil. (V.T.U., Belgaum Karnataka University Winter 2003)
21. The instantaneous values of the voltage across a two element series circuit and the current flowing through it are given by  $V = 100 \sin (314t - \pi/4)V$ ,  $i = 20 \sin (314t - 90^\circ)A$ . Find the frequency and the circuit elements. (V.T.U., Belgaum Karnataka University, Winter 2003)
22. Show that the power consumed in a pure inductance is zero. (U.P. Technical University 2003) (RGPV Bhopal 2002)
23. What do you understand by the terms power factor, active power and reactive power? (Mumbai University 2003) (RGPV Bhopal 2002)
24. Series R-L-C circuit (Mumbai University 2003) (RGPV Bhopal 2002)
25. Describe the properties of (i) Resistance (ii) Inductance and (iii) capacitance used in A.C. Circuit. (RGPV Bhopal June 2003)
26. Define Apparent Power and Power factor in a.c. circuit. Describe parallel resonance and list its important properties. (Mumbai University 2003) (RGPV Bhopal December 2003)

## OBJECTIVE TESTS -13

1. In a series R-L circuit,  $V_L - V_R$  by—degrees.
  - (a) lags, 45
  - (b) lags, 90
  - (c) leads, 90
  - (d) leads, 45
2. The voltage applied across an R-L circuit is equal to—of  $V_R$  and  $V_L$ .
  - (a) arithmetic sum
  - (b) algebraic sum
  - (c) phasor sum
  - (d) sum of the squares.
3. The power in an a.c. circuit is given by
  - (a)  $VI \cos \phi$
  - (b)  $VI \sin \phi$
  - (c)  $I^2 Z$
  - (d)  $I^2 X_L$
4. The p.f. of an R-C circuit is
  - (a) often zero
  - (b) between zero and 1
  - (c) always unity
  - (d) between zero and -1.0
5. In a series RLC circuit at resonance, the magnitude of the voltage developed across the capacitor
  - (a) is always zero
  - (b) can never be greater than the input voltage.
  - (c) can be greater than the input voltage, however, it is  $90^\circ$  out of phase with the input voltage
  - (d) can be greater than the input voltage, and in phase with the input voltage. (GATE 2001)
6. The total impedance Z ( $j\omega$ ) of the circuit shown above is
  - (a)  $(6 + j0) \Omega$
  - (b)  $(7 + j0) \Omega$
  - (c)  $(0 + j8) \Omega$
  - (d)  $(6 + j8) \Omega$  (ESE 2003)
7. The impedance of a parallel RC network is  $Z(s) = \frac{58}{s^2 + 0.5s + 100}$ . Then the values of R, L and C are, respectively
  - (a)  $10 \Omega$ ,  $\frac{1}{20} H$ ,  $\frac{1}{5} F$
  - (b)  $1 \Omega$ ,  $\frac{1}{2} H$ ,  $\frac{1}{5} F$
  - (c)  $10 \Omega$ ,  $\frac{1}{20} H$ ,  $\frac{1}{5} F$
  - (d)  $2 \Omega$ ,  $\frac{1}{20} H$ ,  $\frac{1}{5} F$  (Engineering Services Exam. 2003)

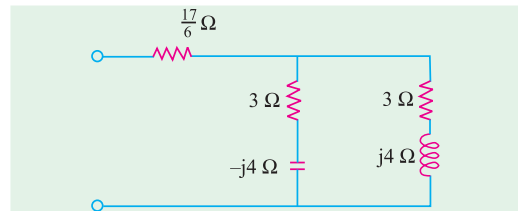


Fig. 13.55

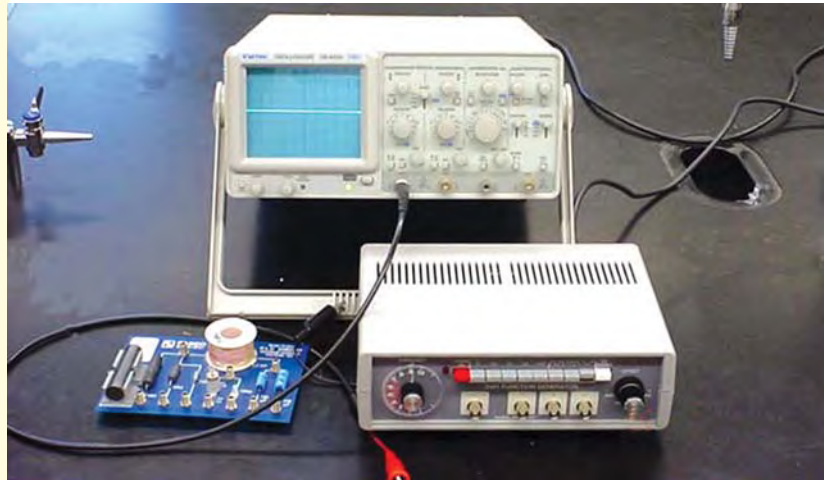
## ANSWERS

1. c 2. c 3. a 4. b



CHAPTER **14****Learning Objectives**

- Solving Parallel Circuits
- Vector or Phasor Method
- Admittance Method
- Application of Admittance Method
- Complex or Phasor Algebra
- Series-Parallel Circuits
- Series Equivalent of a Parallel Circuit
- Parallel Equivalent of a Series Circuit
- Resonance in Parallel Circuits
- Graphic Representation of Parallel Resonance
- Points to Remember
- Bandwidth of a Parallel Resonant Circuit
- Q-factor of a Parallel Circuit

**PARALLEL  
A.C.  
CIRCUITS**

- Parallel AC circuit combination is as important in power, radio and radar application as in series AC circuits

### 14.1. Solving Parallel Circuits

When impedances are joined in parallel, there are three methods available to solve such circuits:

(a) *Vector or phasor Method* (b) *Admittance Method* and (c) *Vector Algebra*

### 14.2. Vector or Phasor Method

Consider the circuits shown in Fig. 14.1. Here, two reactors *A* and *B* have been joined in parallel across an r.m.s. supply of *V* volts. The voltage across two parallel branches *A* and *B* is the same, but currents through them are different.

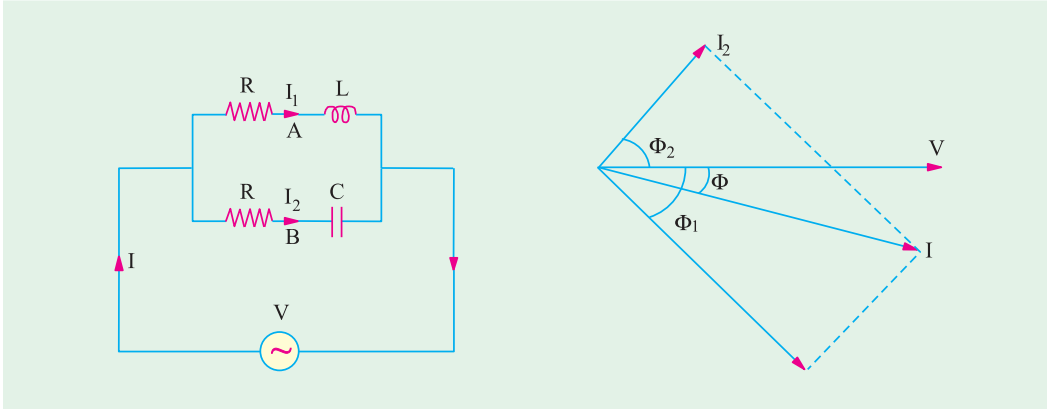


Fig. 14.1

Fig. 14.2

**For Branch A,**  $Z_1 = \sqrt{(R_1^2 + X_L^2)}$  ;  $I_1 = V/Z_1$  ;  $\cos \phi_1 = R_1/Z_1$  or  $\phi_1 = \cos^{-1} (R_1/Z_1)$

Current  $I_1$  lags behind the applied voltage by  $\phi_1$  (Fig. 14.2).

**For Branch B,**  $Z_2 = \sqrt{(R_2^2 + X_C^2)}$  ;  $I_2 = V/Z_2$  ;  $\cos \phi_2 = R_2/Z_2$  or  $\phi_2 = \cos^{-1} (R_2/Z_2)$

Current  $I_2$  leads  $V$  by  $\phi_2$  (Fig. 14.2).

#### Resultant Current *I*

The resultant circuit current  $I$  is the vector sum of the branch currents  $I_1$  and  $I_2$  and can be found by (i) using parallelogram law of vectors, as shown in Fig. 14.2. or (ii) resolving  $I_2$  into their  $X$ - and  $Y$ -components (or active and reactive components respectively) and then by combining these components, as shown in Fig. 14.3. Method (ii) is preferable, as it is quick and convenient.

With reference to Fig. 14.3. (a) we have

Sum of the active components of  $I_1$  and  $I_2$

$$= I_1 \cos \phi_1 + I_2 \cos \phi_2$$

Sum of the reactive components of  $I_1$  and  $I_2 = I_2 \sin \phi_2 - I_1 \sin \phi_1$

If  $I$  is the resultant current and  $\phi$  its phase, then its active and reactive components must be equal to these  $X$ - and  $Y$ -components respectively [Fig. 14.3. (b)]

$$I \cos \phi = I_1 \cos \phi_1 + I_2 \cos \phi_2 \quad \text{and} \quad I \sin \phi = I_2 \sin \phi_2 - I_1 \sin \phi_1$$

$$\therefore I = \sqrt{[(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (I_2 \sin \phi_2 - I_1 \sin \phi_1)^2]}$$

and

$$\tan \phi = \frac{I_2 \sin \phi_2 - I_1 \sin \phi_1}{I_1 \cos \phi_1 + I_2 \cos \phi_2} = \frac{Y \text{ component}}{X \text{ component}}$$

If  $\tan \phi$  is positive, then current leads and if  $\tan \phi$  is negative, then current lags behind the applied

voltage  $V$ . Power factor for the whole circuit is given by

$$\cos \phi = \frac{I_1 \cos \phi_1 + I_2 \cos \phi_2}{I} \quad \frac{X \text{ comp.}}{I}$$

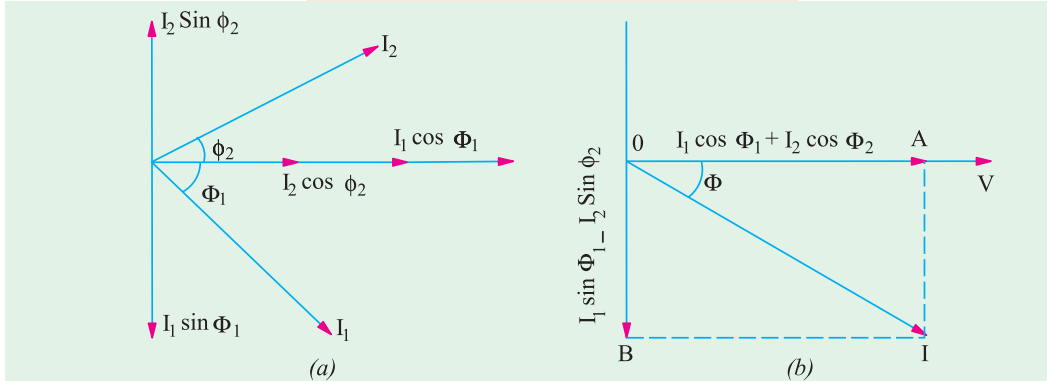


Fig. 14.3

### 14.3. Admittance Method

Admittance of a circuit is defined as *the reciprocal of its impedance*. Its symbol is  $Y$ .

$$\therefore Y = \frac{1}{Z} = \frac{I}{V} \text{ or } Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

Its unit is Siemens ( $S$ ). A circuit having an impedance of one ohm has an admittance of one Siemens. The old unit was mho (ohm spelled backwards).

As the impedance  $Z$  of a circuit has two components  $X$  and  $R$  (Fig. 14.4.), similarly, admittance  $Y$  also has two components as shown in Fig. 14.5. The  $X$ -component is known as *conductance* and  $Y$ -component as *susceptance*.

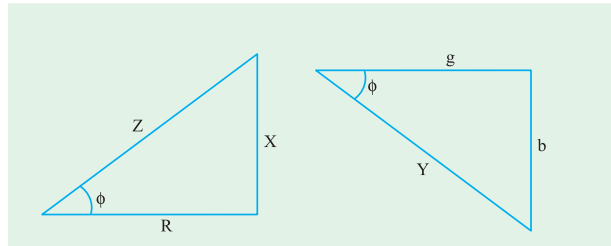


Fig. 14.4

Fig. 14.5

Obviously, conductance  $g = Y \cos \phi$

or  $g = \frac{1}{Z} \cdot \frac{R}{Z}$  (from Fig. 14.4)

$$\therefore g^* = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

Similarly, susceptance  $b = Y \sin \phi = \frac{1}{Z} \cdot \frac{X}{Z} \therefore b^{**} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}$  (from Fig. 14.5)

The admittance  $Y = \sqrt{(g^2 + b^2)}$  just as  $Z = \sqrt{(R^2 + X^2)}$

The unit of  $g$ ,  $b$  and  $Y$  is Siemens. We will regard the *capacitive susceptance as positive and inductive susceptance as negative*.

\* In the special case when  $X = 0$ , then  $g = 1/R$  i.e., conductance becomes reciprocal of resistance, not otherwise.

\*\* Similarly, in the special case when  $R = 0$ ,  $b = 1/X$  i.e., susceptance becomes reciprocal of reactance, not otherwise.

### 14.4. Application of Admittance Method

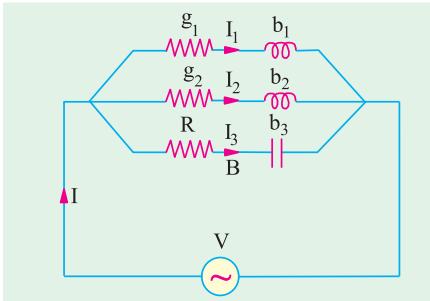


Fig. 14.6

Consider the 3-branched circuit of Fig. 14.6. Total conductance is found by merely adding the conductances of three branches. Similarly, total susceptance is found by **algebraically** adding the individual susceptances of different branches.

$$\text{Total conductance } G = g_1 + g_2 + g_3 \dots\dots\dots$$

$$\text{Total susceptance } B = (-b_1) + (-b_2) + b_3 \dots\dots$$

(algebraic sum)

$$\therefore \text{ Total admittance } Y = \sqrt{(G^2 + B^2)}$$

$$\text{Total current } I = VY; \text{ Power factor } \cos \phi = G/Y$$

### 14.5. Complex or Phasor Algebra

Consider the parallel circuit shown in Fig. 14.7. The two impedances,  $Z_1$  and  $Z_2$ , being in parallel, have the same p.d. across them.

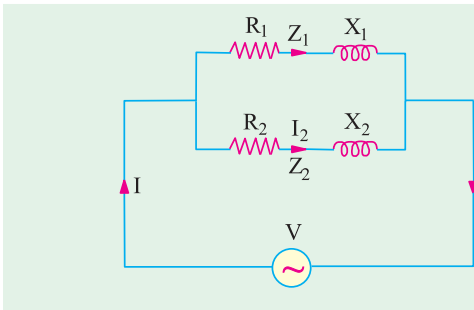


Fig. 14.7

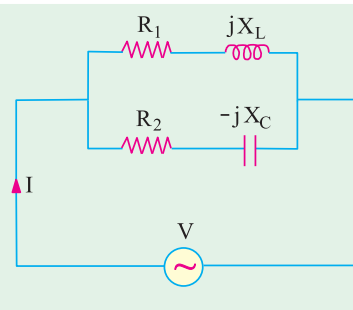


Fig. 14.8

Now 
$$I_1 = \frac{V}{Z_1} \text{ and } I_2 = \frac{V}{Z_2}$$

Total current 
$$I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) = V(Y_1 + Y_2) = VY$$

where

$$Y = \text{total admittance} = Y_1 + Y_2$$

It should be noted that **admittances are added** for parallel branches, whereas for branches in series, it is the **impedances** which are added. **However, it is important to remember that since both admittances and impedances are complex quantities, all additions must be in complex form.** Simple arithmetic additions must not be attempted.

Considering the two parallel branches of Fig. 14.8, we have

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} \\ &= \frac{R_1}{R_1^2 + X_L^2} - j \frac{X_L}{R_1^2 + X_L^2} \end{aligned}$$

where 
$$g_1 = \frac{R_1}{R_1^2 + X_L^2} - \text{conductance of upper branch,}$$

$$b_1 = -\frac{X_L}{R_1^2 + X_L^2} - \text{susceptance of upper branch}$$

$$\text{Similarly, } Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C}$$

$$= \frac{R_2 + jX_C}{(R_2 - jX_C)(R_2 + jX_C)} = \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j \frac{X_C}{R_2^2 + X_C^2} = g_2 + jb_2$$

$$\text{Total admittance } Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 + jb_2) = (g_1 + g_2) - j(b_1 - b_2) = G - jB$$

$$Y = \sqrt{[(g_1 + g_2)^2 + (b_1 - b_2)^2]}; \phi = \tan^{-1} \frac{b_1}{g_1} - \frac{b_2}{g_2}$$

The polar form for admittance is  $Y = Y \angle \phi^\circ$  where  $\phi$  is as given above.

$$Y = \sqrt{G^2 + B^2} \angle \tan^{-1} (B/G)$$

$$\text{Total current } I = VY; I_1 = VY_1 \text{ and } I_2 = VY_2$$

$$\text{If } V = V \angle 0^\circ \text{ and } Y = Y \angle \phi \text{ then } I = VY = V \angle 0^\circ \times Y \angle \phi = VY \angle \phi$$

$$\text{In general, if } V = V \angle \alpha \text{ and } Y = Y \angle \beta, \text{ then } I = VY = V \angle \alpha \times Y \angle \beta = VY \angle \alpha + \beta$$

Hence, it should be noted that when vector voltage is multiplied by admittance either in complex (rectangular) or polar form, the result is vector current in its proper phase relationship with respect to the voltage, **regardless of the axis to which the voltage may have been referred to.**

**Example 14.1.** Two circuits, the impedance of which are given by  $Z_1 = 10 + j15$  and  $Z_2 = 6 - j8$  ohm are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch? Find also the p.f. of individual circuits and of combination. Draw vector diagram.

(Elect. Technology, Vikram Univ, Ujjain)

$$\text{Solution. Let } I = 15 \angle 0^\circ; Z_1 = 10 + j15 = 18 \angle 57^\circ$$

$$Z_2 = 6 - j8 = 10 \angle -53.1^\circ$$

$$\text{Total impedance, } Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 + j15)(6 - j8)}{16 + j7}$$

$$= 9.67 - j3.6 = 10.3 \angle -20.4^\circ$$

Applied voltage is given by

$$V = IZ = 15 \angle 0^\circ \times 10.3 \angle -20.4^\circ = 154.4 \angle -20.4^\circ$$

$$I_1 = V/Z_1 = 154.4 \angle -20.4^\circ / 18 \angle 57^\circ = 8.58 \angle -77.4^\circ$$

$$I_2 = V/Z_2 = 154.4 \angle -20.4^\circ / 10 \angle -53.1^\circ$$

$$= 15.45 \angle 32.7^\circ$$

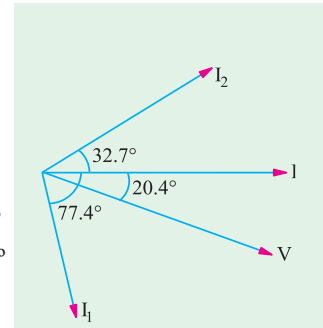


Fig. 14.9

We could also find branch currents as under :

$$I_1 = I \cdot Z_2 / (Z_1 + Z_2) \text{ and } I_2 = I \cdot Z_1 / (Z_1 + Z_2)$$

It is seen from phasor diagram of Fig. 14.9 that  $I_1$  lags behind  $V$  by  $(77.4^\circ - 20.4^\circ) = 57^\circ$  and  $I_2$  leads it by  $(32.7^\circ + 20.4^\circ) = 53.1^\circ$ .

$$\therefore P_1 = I_1^2 R_1 = 8.58^2 \times 10 = 736 \text{ W; p.f.} = \cos 57^\circ = 0.544 \text{ (lag)}$$

$$P_2 = I_2^2 R_2 = 15.45^2 \times 6 = 1432 \text{ W; p.f.} = \cos 53.1^\circ = 0.6$$

$$\text{Combined p.f.} = \cos 20.4^\circ = 0.937 \text{ (lead)}$$

**Example 14.2.** Two impedance  $Z_1 = (8 + j6)$  and  $Z_2 = (3 - j4)$  are in parallel. If the total current of the combination is 25 A, find the current taken and power consumed by each impedance.

(F.Y. Engg. Pune Univ.)

$$\text{Solution. } Z_1 = (8 + j6) = 10 \angle 36.87^\circ; Z_2 = (3 - j4) = 5 \angle -53.1^\circ$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 \angle 36.87^\circ)(5 \angle -53.1^\circ)}{(8 + j6) + (3 - j4)} = \frac{50 \angle -16.23^\circ}{11 + j2} = \frac{50 \angle -16.23^\circ}{11.18 \angle 10.3^\circ} = 4.47 \angle -26.53^\circ$$

$$\text{Let } I = 25 \angle 0^\circ; V = IZ = 25 \angle 0^\circ \times 4.47 \angle -26.53^\circ = 111.75 \angle -26.53^\circ$$

$$I_1 = V/Z_1 = 111.75 \angle -26.53^\circ / 10 \angle 36.87^\circ = 11.175 \angle -63.4^\circ$$

$$I_2 = 111.75 \angle -26.53^\circ / 5 \angle -53.1^\circ = 22.35 \angle 26.57^\circ$$

Now, the phase difference between  $V$  and  $I_1$  is  $63.4^\circ - 26.53^\circ = 36.87^\circ$  with current lagging. Hence,  $\cos \phi_1 = \cos 36.87^\circ = 0.8$ .

$$\text{Power consumed in } Z_1 = VI_1 \cos \phi = 111.75 \times 11.175 \times 0.8 = \mathbf{990 \text{ W}}$$

$$\text{Similarly, } \phi_2 = 26.57^\circ - (-26.53^\circ) = 53.1^\circ; \cos 53.1^\circ = 0.6$$

$$\text{Power consumed in } Z_2 = VI_2 \cos \phi_2 = 111.75 \times 22.35 \times 0.6 = \mathbf{1499 \text{ W}}$$

**Example 14.3.** Refer to the circuit of Fig. 14.10 (a) and determine the resistance and reactance of the lagging coil load and the power factor of the combination when the currents are as indicated.

(Elect. Engg. A.M.Ae. S.I.)

**Solution.** As seen from the  $\Delta ABC$  of Fig. 14.10 (b).

$$5.6^2 = 2^2 + 4.5^2 + 2 \times 2 \times 4.5 \times \cos \theta, \therefore \cos \theta = 0.395, \sin \theta = 0.919. Z = 300/4.5 = 66.67 \Omega$$

$$R = Z \cos \theta = 66.67 \times 0.395 = 61.3 \Omega$$

$$\text{p.f.} = \cos \phi = AC/AD = (2 + 4.5 \times 0.395)/5.6 = \mathbf{0.67 \text{ (lag)}}$$

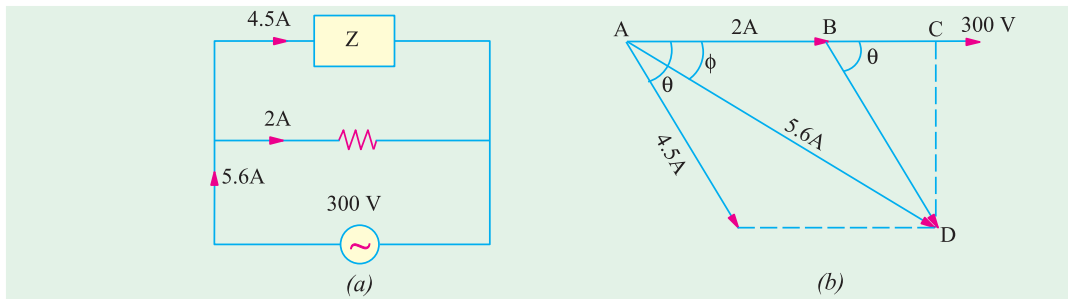


Fig. 14.10

**Example 14.4.** A mercury vapour lamp unit consists of a  $25\mu\text{f}$  condenser in parallel with a series circuit containing the resistive lamp and a reactor of negligible resistance. The whole unit takes  $400 \text{ W}$  at  $240 \text{ V}$ ,  $50 \text{ Hz}$  at unity p.f. What is the voltage across the lamp?

(F.Y. Engg. Pune Univ.)

$$\text{Solution. } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times (25 \times 10^{-6})} = 127.3 \Omega, \therefore I_C = \frac{240}{127.3} = 1.885 \text{ A}$$

$$W = VI \cos \phi = VI \therefore I = W/V = 400/240 = 1.667 \text{ A}$$

In the vector diagram of Fig. 14.10 (b)  $I_C$  leads  $V$  by  $90^\circ$  and current  $I_1$  in the series circuit lags  $V$  by  $\phi_1$  where  $\phi_1$  is the power factor angle of the series circuit. The vector sum of  $I_C$  and  $I_1$  gives the total current  $I$ . As seen  $\tan \phi_1 = I_C/I = 1.885/1.667 = 1.13077$ . Hence,  $\phi_1 = 48.5^\circ$  lag. The applied voltage  $V$  is the vector sum of the drop across the resistive lamp which is in phase with  $I_1$  and drop across the coil which leads  $I_1$  by  $90^\circ$ .

$$\text{Voltage across the lamp} = V \cos \Phi_1 = 240 \times \cos 48.5 = 240 \times 0.662 = \mathbf{159 \text{ V.}}$$

**Example 14.5.** The currents in each branch of a two-branched parallel circuit are given by the expression  $i_a = 7.07 \sin (314t - \pi/4)$  and  $i_b = 21.2 \sin (314t + \pi/3)$

The supply voltage is given by the expression  $v = 354 \sin 314t$ . Derive a similar expression for the supply current and calculate the ohmic value of the component, assuming two pure components in each branch. State whether the reactive components are inductive or capacitive.

(Elect. Engineering., Calcutta Univ.)

**Solution.** By inspection, we find that  $i_a$  lags the voltage by  $\pi/4$  radian or  $45^\circ$  and  $i_b$  leads it by  $\pi/3$  radian or  $60^\circ$ . Hence, branch A consists of a resistance in series with a pure inductive reactance. Branch B consists of a resistance in series with pure capacitive reactance as shown in Fig. 14.11 (a).

Maximum value of current in branch A is 7.07 A and in branch B is 21.2 A. The resultant current can be found vectorially. As seen from vector diagram.

$$X\text{-comp} = 21.2 \cos 60^\circ + 7.07 \cos 45^\circ = 15.6 \text{ A}$$

$$Y\text{-comp} = 21.2 \sin 60^\circ - 7.07 \sin 45^\circ = 13.36 \text{ A}$$

$$\text{Maximum value of the resultant current is} = \sqrt{15.6^2 + 13.36^2} = 20.55 \text{ A}$$

$$\phi = \tan^{-1} (13.36/15.6) = \tan^{-1} (0.856) = 40.5^\circ \text{ (lead)}$$

Hence, the expression for the supply current is  $i = 20.55 \sin (314 t + 40.5^\circ)$

$$Z_A = 354/7.07 = 50 \Omega; \cos \phi_A = \cos 45^\circ$$

$$= 1/\sqrt{2} \cdot \sin \phi_A = \sin 45^\circ = 1/\sqrt{2}$$

$$R_A = Z_A \cos \phi_A = 50 \times 1/\sqrt{2} = 35.4 \Omega$$

$$X_L = Z_A \sin \phi_A = 50 \times 1/\sqrt{2} = 35.4 \Omega$$

$$Z_B = 354/20.2 = 17.5 \Omega$$

$$R_B = 17.5 \times \cos 60^\circ = 8.75 \Omega$$

$$X_C = 17.5 \times \sin 60^\circ = 15.16 \Omega$$

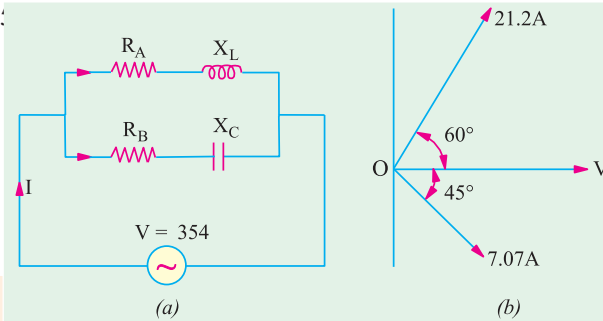


Fig. 14.11

**Example 14.5 (a).** A total current of 10 A flows through the parallel combination of three impedance :  $(2 - j5) \Omega$ ,  $(6 + j3) \Omega$  and  $(3 + j4) \Omega$ . Calculate the current flowing through each branch. Find also the p.f. of the combination.

(Elect. Engg., -I Delhi Univ.)

**Solution.** Let  $Z_1 = (2 - j5)$ ,  $Z_2 = (6 + j3)$ ,  $Z_3 = (3 + j4)$

$$Z_1 Z_2 = (2 - j5)(6 + j3) = 27 - j24. \quad Z_2 Z_3 = (6 + j3)(3 + j4) = 6 + j33$$

$$Z_3 Z_1 = (3 + j4)(2 - j5) = 26 - j7; \quad Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = 59 + j2$$

With reference to Art, 1.25

$$I_1 = I \cdot \frac{Z_2 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = (10 \angle 0^\circ) \frac{6 + j33}{59 + j2} = 1.21 - j5.55$$

$$I_2 = I \cdot \frac{Z_3 Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = (10 \angle 0^\circ) \frac{26 - j7}{59 + j2} = 4.36 + j1.33$$

$$I_3 = I \cdot \frac{Z_1 Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = (10 \angle 0^\circ) \frac{27 - j24}{59 + j2} = 4.43 - j4.22$$

$$\text{Now, } Z = \frac{Z_1 Z_2 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = \frac{(2 - j5)(6 + j3)(3 + j4)}{59 + j2} = 3.01 - j0.51$$

$$V = 10 \angle 0^\circ \times 3.05 \angle 9.6^\circ = 30.5 \angle 9.6^\circ$$

Combination p.f. =  $\cos 9.6^\circ = 0.986$  (lag)

**Example 14.6.** Two impedances given by  $Z_1 = (10 + j5)$  and  $Z_2 = (8 + j6)$  are joined in parallel and connected across a voltage of  $V = 200 + j0$ . Calculate the circuit current, its phase and the branch currents. Draw the vector diagram.

(Electrotechnics-I, M.S. Univ. Baroda)

**Solution.** The circuit is shown in Fig. 14.12

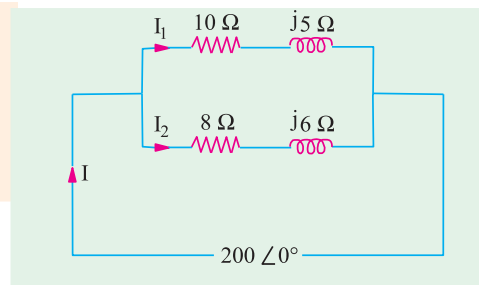


Fig. 14.12

$$\text{Branch A, } Y_1 = \frac{1}{Z_1} = \frac{1}{(10 + j5)}$$

$$= \frac{10 - j5}{(10 + j5)(10 - j5)} = \frac{10 - j5}{100 + 25}$$

$$= 0.08 - j0.04 \text{ Siemens}$$

$$\text{Branch B, } Y_2 = \frac{1}{Z_2} = \frac{1}{(8 + j6)}$$

$$= \frac{8 - j6}{(8 + j6)(8 - j6)} = \frac{8 - j6}{64 + 36} = 0.08 - j0.06 \text{ Siemens}$$

$$Y = (0.08 - j0.04) + (0.08 - j0.06) = 0.16 - j0.1 \text{ Siemens}$$

### Direct Method

We could have found total impedance straightway like this:  $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{Z_1 + Z_2}{Z_1 Z_2}$

$$\therefore Y = \frac{Z_1 + Z_2}{Z_1 Z_2} = \frac{(10 - j5) + (8 - j6)}{(10 - j5)(8 - j6)} = \frac{18 - j11}{50 - j100}$$

Rationalizing the above, we get

$$Y = \frac{(18 - j11)(50 + j100)}{(50 - j100)(50 + j100)} = \frac{200 - j1250}{12,500} = 0.16 - j0.1 \text{ (same as before)}$$

Now  $V = 200 \angle 0^\circ = 200 + j0$

$$\therefore I = VY = (200 + j0)(0.16 - j0.1)$$

$$= 32 - j20 = 37.74 \angle -32^\circ \dots \text{polar form}$$

Power factor =  $\cos 32^\circ = 0.848$

$$I_1 = VY_1 = (200 + j0)(0.08 - j0.04)$$

$$= 16 - j8 = 17.88 \angle -26^\circ 32'$$

It lags behind the applied voltage by  $26^\circ 32'$ .

$$I_2 = VY_2 = (200 + j0)(0.08 - j0.06)$$

$$= 16 - j12 = 20 \angle -36^\circ 46'$$

It lags behind the applied voltage by  $36^\circ 46'$ . The vector diagram is shown in Fig. 14.13.

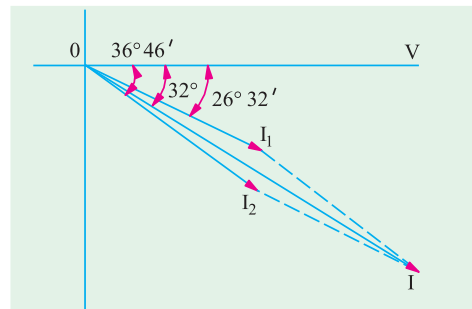


Fig. 14.13

**Example 14.7.** Explain the term admittance. Two impedance  $Z_1 = (6 - j8)$  ohm and  $Z_2 = (16 + j12)$  ohm are connected in parallel. If the total current of the combination is  $(20 + j10)$  amperes, find the complexor for power taken by each impedance. Draw and explain the complete phasor diagram. (Basic Electricity, Bombay Univ.)

**Solution.** Let us first find out the applied voltage,  $Y = Y_1 + Y_2 = \frac{1}{6 - j8} + \frac{1}{16 + j12}$

$$= (0.06 + j0.08) + (0.04 - j0.03) = 0.1 + j0.05 = 0.1118 \angle 26^\circ 34'$$

$$I = 20 + j10 = 22.36 \angle 26^\circ 34'$$

Now  $I = VY \therefore V = \frac{I}{Y} = \frac{22.36 \angle 26^\circ 34'}{0.1118 \angle 26^\circ 34'} = 200 \angle 0^\circ$

$$I_1 = VY_1 = (200 + j0)(0.06 + j0.08) = 12j + 16 \text{ A, } I_2 = 200(0.04 - j0.03) = 8 - j6 \text{ A}$$

Using the method of conjugates and taking voltage conjugate, the complexor power taken by each branch can be found as under:

$$P_1 = (200 - j0)(12 + j16) = 2400 + j3200; P_2 = (200 - j0)(8 - j6) = 100 - j1200$$

Drawing of phasor diagram is left to the reader.

**Note.** Total voltamperes =  $4000 + j2000$

As a check,  $P = VI = 200(20 + j10) = 4000 + j2000$



**Example 14.8.** A 15-mH inductor is in series with a parallel combination of an 80 Ω resistor and 20 μF capacitor. If the angular frequency of the applied voltage is  $\omega = 1000$  rad/s, find the admittance of the network. **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

**Solution.**  $X_L = \omega L = 1000 \times 15 \times 10^{-3} = 15\Omega$ ;  $X_C = 1/\omega C = 10^6/1000 \times 20 = 50 \Omega$   
 Impedance of the parallel combination is given by  
 $Z_p = 80 \parallel -j50 = -j4000/(80 - j50) = 22.5 - j36$ ,  
 Total impedance =  $j15 + 22.5 - j36 = 22.5 - j21$   
 Admittance  $Y = \frac{1}{Z} = \frac{1}{22.5 - j21} = 0.0238 + j0.022$  Siemens

**Example 14.9.** An impedance  $(6 + j8)$  is connected across 200-V, 50-Hz mains in parallel with another circuit having an impedance of  $(8 - j6)$  Ω. Calculate (a) the admittance, the conductance, the susceptance of the combined circuit (b) the total current taken from the mains and its p.f. **(Elect. Engg-AMIE, S.I. 1992)**

**Solution.**  $Y_1 = \frac{1}{6 + j8} = \frac{6 - j8}{6^2 + 8^2} = 0.06 - j0.08$  Siemens,  $Y = \frac{1}{8 - j6} = \frac{8 + j6}{100} = 0.08 + j0.06$

Siemens

(a) Combined admittance is  $Y = Y_1 + Y_2 = 0.14 - j0.02 = 0.1414 \angle -88^\circ$  Siemens  
 Conductance,  $G = 0.14$  Siemens ; Susceptance,  $B = -0.02$  Siemens (inductive)  
 (b) Let  $V = 200 \angle 0^\circ$  ;  $I = VY = 200 \times 0.1414 \angle -88^\circ$   $V = 28.3 \angle -88^\circ$   
 p.f. =  $\cos 88^\circ = 0.99$  (lag)

**Example 14.10.** If the voltmeter in Fig. 14.14 reads 60 V, find the reading of the ammeter.

**Solution.**  $I_2 = 60/4 = 15$  A. Taking it as reference quantity, we have  $I_2 = 15 \angle 0$ .

Obviously, the applied voltage is

$$V = 15 \angle 0^\circ \times (4 - j4) = 84.8 \angle 45^\circ$$

$$I_1 = 84.8 \angle 45^\circ / (6 + j3) = 84.8 \angle 45^\circ / 6.7 \angle 26.6^\circ = 12.6 \angle 18.4^\circ$$

$$I = I_1 + I_2 = (12.6 + j0) + (15 - j12) = 27.6 - j12 = 30.2 \angle -23.3^\circ$$

Hence, ammeter reads 22.47.

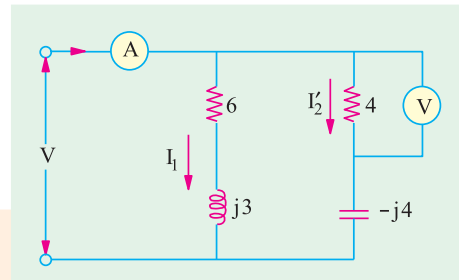


Fig. 14.14

**Example 14.11.** Find the reading of the ammeter when the voltmeter across the 3 ohm resistor in the circuit of Fig. 14.15 reads 45 V.

**(Elect. Engg. & Electronics Bangalore Univ. )**

**Solution.** Obviously  $I_1 = 45/3 = 15$  A. If we take it as reference quantity,  $I_1 = 15 \angle 0^\circ$

$$\text{Now, } Z_1 = 3 - j3 = 4.24 \angle -45^\circ$$

$$\text{Hence, } V = I_1 Z_1 = 15 \angle 0^\circ \times 4.24 \angle -45^\circ = 63.6 \angle -45^\circ$$

$$I_2 = \frac{V}{Z_2} = \frac{63.6 \angle -45^\circ}{5 + j2} = \frac{63.6 \angle -45^\circ}{5.4 \angle 21.8^\circ}$$

$$= 11.77 \angle -66.8^\circ = 4.64 - j10.8$$

$$I = I_1 + I_2 = 19.64 - j10.8 = 22.4 \angle -28.8^\circ$$

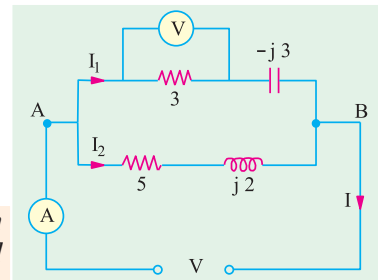


Fig. 14.15

**Example 14.12.** A coil having a resistance of 5 Ω and an inductance of 0.02 H is arranged in parallel with another coil having a resistance of 1 Ω and an inductance of 0.08 H. Calculate the current through the combination and the power absorbed when a voltage of 100 V at 50 Hz is applied. Estimate the resistance of a single coil which will take the same current at the same power factor.

**Solution.** The circuit and its phasor diagram are shown in Fig. 14.16.

**Branch No. 1**

$$X_1 = 314 \times 0.02 = 6.28 \Omega$$

$$Z_1 = \sqrt{5^2 + 6.28^2} = 8 \Omega$$

$$I_1 = 100/8 = 12.5 \text{ A}$$

$$\cos \phi_1 = R_1/Z_1 = 5/8$$

$$\sin \phi_1 = 6.28/8$$

**Branch No. 2**

$$X_2 = 314 \times 0.08 = 25.12 \Omega,$$

$$Z_2 = \sqrt{1^2 + 25.12^2} = 25.14 \Omega, I_2 = 100/25.14 = 4 \text{ A}$$

$$\cos \phi_2 = 1/25.14 \text{ and } \sin \phi_2 = 25.12/25.14$$

$$X\text{-components of } I_1 \text{ and } I_2 = I_1 \cos \phi_1 + I_2 \cos \phi_2 = (12.5 \times 5/8) + (4 \times 1/25.14) = 7.97 \text{ A}$$

$$Y\text{-components of } I_1 \text{ and } I_2 = I_1 \sin \phi_1 + I_2 \sin \phi_2 = (12.5 \times 6.28/8) + (4 \times 25.12/25.14) = 13.8 \text{ A}$$

$$I = \sqrt{7.97^2 + 13.8^2} = 15.94 \text{ A}$$

$$\cos \phi = 7.97/15.94 = 0.5 \text{ (lag)}$$

$$\phi = \cos^{-1}(0.5) = 60^\circ$$

Power absorbed

$$= 100 \times 15.94 \times 0.5 = 797 \text{ W}$$

The equivalent series circuit is shown in Fig. 14.17 (a).

$$V = 100 \text{ V}; I = 15.94 \text{ A}; \phi = 60^\circ$$

$$Z = 100/15.94 = 6.27 \Omega;$$

$$R = Z \cos \phi = 6.27 \times \cos 60^\circ = 3.14 \Omega$$

$$X = Z \sin \phi = 6.27 \times \sin 60^\circ = 5.43 \Omega$$

**Admittance Method For Finding Equivalent Circuit**

$$Y_1 = \frac{1}{5 + j6.28} = \frac{5 - j6.28}{5^2 + 6.28^2} = 0.078 - j0.098 \text{ S},$$

$$Y_2 = \frac{1}{1 + j25.12} = \frac{1 - j25.12}{1^2 + 25.12^2} = 0.00158 - j0.0397 \text{ S},$$

$$Y = Y_1 + Y_2 = 0.0796 - j0.138 = 0.159 \angle -60^\circ$$

Here

$$G = 0.0796 \text{ S}, B = -0.138 \text{ S}, Y = 0.159 \text{ S}$$

$$\therefore R_{eq} = G/Y^2 = 0.0796/0.159^2 = 3.14 \Omega, X_{eq} = B/Y^2 = 0.138/0.159^2 = 5.56 \Omega$$

**Example. 14.13.** A voltage of  $200 \angle 53^\circ 8'$  is applied across two impedances in parallel. The values of impedances are  $(12 + j16)$  and  $(10 - j20)$ . Determine the kVA, kVAR and kW in each branch and the power factor of the whole circuit. (Elect. Technology, Indore Univ.)

**Solution.** The circuit is shown in Fig. 14.18.

$$Y_A = 1/(12 + j16) = (12 - j16)/[(12 + j16)(12 - j16)] \\ = (12 - j16)/400 = 0.03 - j0.04 \text{ mho}$$

$$Y_B = 1/(10 - j20) = 10 + j20/[(10 - j20)(10 + j20)]$$

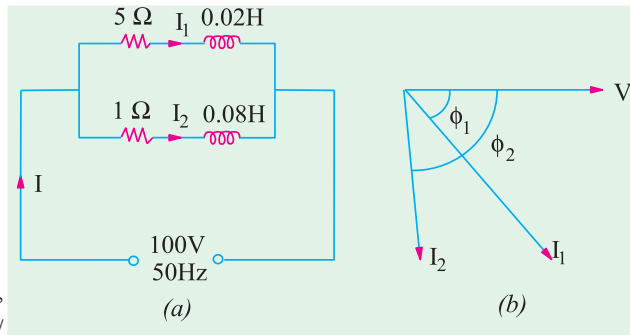


Fig. 14.16.

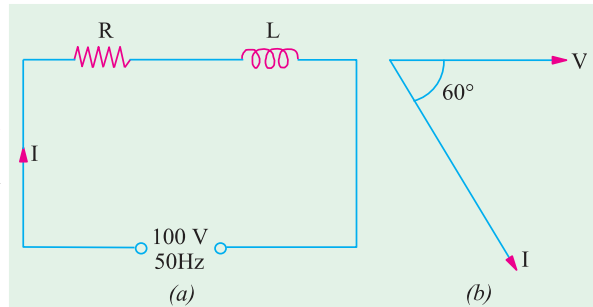


Fig. 14.17

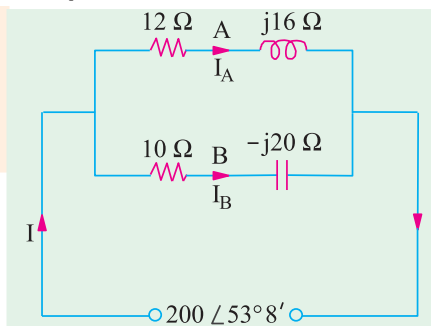


Fig. 14.18

$$= \frac{10 - j20}{500} = 0.02 - j0.04 \text{ mho}$$

$$\begin{aligned} \text{Now } V &= 200 \angle 53^\circ 8' = 200 (\cos 53^\circ 8' + j \sin 53^\circ 8') \\ &= 2000 (0.6 + j0.8) = 120 + j160 \text{ volt} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_A &= \mathbf{V} \mathbf{Y}_A = (120 + j160) (0.03 - j0.04) \\ &= (10 + j0) \text{ ampere (along the reference axis)} \end{aligned}$$

$$\begin{aligned} \therefore \mathbf{I}_B &= \mathbf{V} \mathbf{Y}_B = (120 + j160) (0.02 + j0.04) \\ &= -4.0 + j8 \text{ ampere (leading)} \end{aligned}$$

**Example 14.14.** Two circuits, the impedances of which are given by  $Z_1 = 15 + j12$  ohms and  $Z_2 = 8 - j5$  ohms are connected in parallel. If the potential difference across one of the impedance is  $250 + j0$  V, calculate.

- (i) total current and branch currents  
 (ii) total power and power consumed in each branch  
 and (iii) overall power-factor and power-factor of each branch.

(Nagpur University, November 1998)

**Solution.** (i)  $I_1 = (250 + j0)/(15 + j12) = 250 \angle 0^\circ / 19.21 \angle 38.6^\circ$   
 $= 13 \angle -38.6^\circ \text{ amp} = 13 (0.78 - j0.6247)$   
 $= 10.14 - j8.12 \text{ amp}$   
 $I_2 = (250 + j0)/(8 - j5) = 250 \angle 0^\circ / 9.434 \angle -32^\circ$   
 $= 26.5 \angle +32^\circ = 26.5 (0.848 + j0.530)$   
 $= 22.47 + j14.05 \text{ amp}$

$$I = I_1 + I_2 = 32.61 + j5.93 = 33.15 \angle +10.36^\circ$$

- (ii) Power in branch 1 =  $13^2 \times 15 = 2535$  watts  
 Power in branch 2 =  $26.5^2 \times 8 = 5618$  watts  
 Total power consumed =  $2535 + 5618 = 8153$  watts  
 (iii) Power factor of branch 1 =  $\cos 38.60^\circ = 0.78$  lag  
 Power factor of branch 2 =  $\cos 32^\circ = 0.848$  lead.  
 Overall power factor =  $\cos 10.36^\circ = 0.984$  lead.

**Additional hint :** Drawn phasor-diagram for these currents, in fig. 14.19, for the expressions written above,

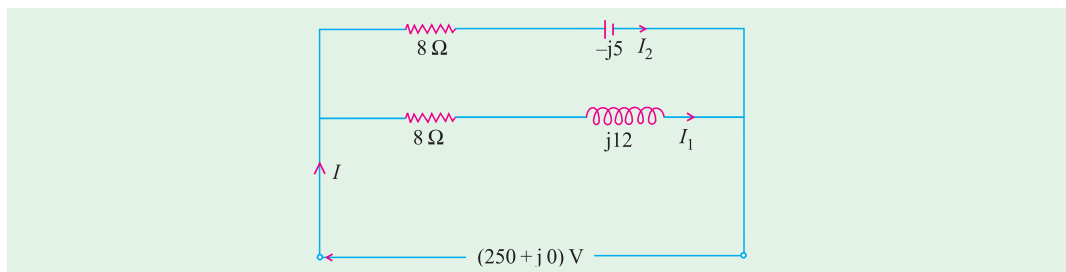


Fig. 14.19

**Example 14.15.** An inductive circuit, in parallel with a resistive circuit of 20 ohms is connected across a 50 Hz supply. The inductive current is 4.3 A and the resistive current is 2.7 A. The total current is 5.8 A Find : (a) Power factor of the combined circuit. Also draw the phasor diagram.

(Nagpur University, November 1997)

**Solution.**  $I_2$  ( $= 2.7$  A) is in phase with  $V$  which is  $54$  V in magnitude. The triangle for currents is drawn in the phasor diagram in fig. 14.20 (b)

Solving the triangle,  $\phi_1 = 180^\circ - \cos^{-1} [(2.7^2 = 4.3^2 - 5.8^2)/(2 \times 4.3 \times 2.7)] = 70.2^\circ$

Further,  $5.8 \sin \phi = 4.3 \sin \phi_1$ , giving  $\phi = 44.2^\circ$ .

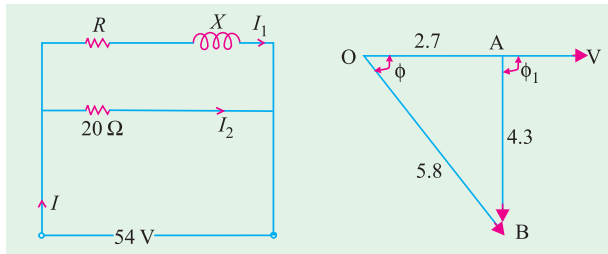


Fig. 14.20 (a)

Fig. 14.20 (b)

OA =  $I_2 = 2.7$  in phase with  $V$   
 AB =  $I_1 = 4.3$  lagging behind  $V$  by  $\phi_1$   
 OB =  $I = 5.8$  lagging behind  $V$  by  $\phi$

$$|Z_1| = 54/4.3 = 12.56 \text{ ohms}$$

$$R = Z_1 \cos \phi_1 = 4.25 \text{ ohms}$$

$$X = Z_1 \sin \phi_1 = 11.82 \text{ ohms, since } \phi_1 \text{ is the lagging angle}$$

(a) Power absorbed by the Inductive branch

$$= 4.3^2 \times 4.25 = 78.6 \text{ watts}$$

(b)  $L = 11.82/314 = 37.64 \text{ mH}$

(c) P.f. of the combined circuit =  $\cos \phi = 0.717$  lag

**Check :** Power consumed by 20 ohms resistor =  $2.7^2 \times 20 = 145.8 \text{ W}$

Total Power consumed in two branches =  $78.6 + 145.8 = 224.4 \text{ W}$

This figure must be obtained by input power =  $VI \cos \phi$

=  $54 \times 5.8 \times \cos 44.2^\circ = 224.5 \text{ W}$ . Hence checked.

**Example 14.16.** In a particular A.C. circuit, three impedances are connected in parallel, currents as shown in fig. 14.21 are flowing through its parallel branches.

(i) Write the equations for the currents in terms of sinusoidal variations and draw the wave-forms.

Find the total current supplied by the source.

[Nagpur University, April 1998]

**Solution.** In Fig. 14.21,  $V$  is taken as reference, and is very convenient for phasor diagrams for parallel circuits.

(i)  $I_1$  lags behind by  $30^\circ$ . Branch no. 1 must, therefore, have an R-L series combination. With 10-volt source, a current of 3 A in branch 1 means that its impedance  $Z_1$  is given by

$$Z_1 = 10/3 = 3.333 \text{ ohms}$$

The phase-angle for  $I_1$  is  $30^\circ$  lagging

$$R_1 = 3.333 \cos 30^\circ = 2.887 \text{ ohms}$$

$$X_{L1} = 3.333 \sin 30^\circ = 1.6665 \text{ ohms}$$

(ii)  $I_2$  is 2 amp and it leads the voltage by  $45^\circ$ . Branch 2 must, therefore, have R-C series combination.

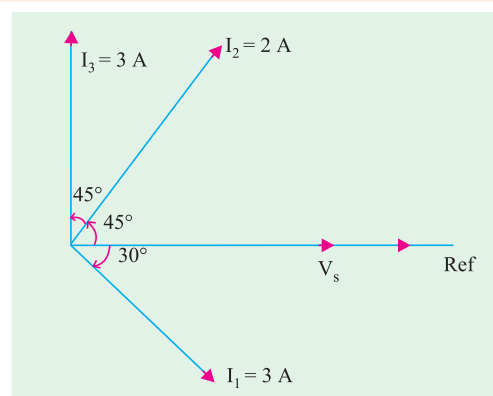


Fig. 14.21

$$Z_2 = 10/2 = 5 \text{ ohms}$$

$$R_2 = 5 \cos 45^\circ = 3.5355 \text{ ohms}$$

$$X_{c2} = 5 \sin 45^\circ = 3.5355 \text{ ohms}$$

(iii) Third branch draws a current of 3 amp which leads the voltage by  $90^\circ$ . Hence, it can only have a capacitive reactance.

$$|Z_3| = X_{c3} = 10/3 = 3.333 \text{ ohms}$$

Total current supplied by the source =  $I$  amp

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 3 [\cos 30^\circ - j \sin 30^\circ] + 2 [\cos 45^\circ + j \sin 45^\circ] + 3 [0 + j 1] \\ &= 4.0123 + j 2.9142 \end{aligned}$$

$$|I| = 4.96 \text{ amp, leading } V_s \text{ by } 36^\circ.$$

Expressions for currents : Frequency is assumed to be 50 Hz

$$v_s = 10\sqrt{2} \sin (314 t)$$

$$i_1 = 3\sqrt{2} \sin (314 t - 30^\circ)$$

$$i_2 = 2\sqrt{2} \sin (314 t + 45^\circ)$$

$$i_3 = 3\sqrt{2} \sin (314 t + 90^\circ)$$

$$\text{Total current, } i(t) = 4.96\sqrt{2} \sin (314 t + 36^\circ)$$

$$\begin{aligned} \text{Total power consumed} &= \text{Voltage} \times \text{active (or in phase-) component of current} \\ &= 10 \times 4.012 = 40.12 \text{ watts} \end{aligned}$$

**Example 14.17.** A resistor of 12 ohms and an inductance of 0.025 H are connected in series across a 50 Hz supply. What values of resistance and inductance when connected in parallel will have the same resultant impedance and p.f. Find the current in each case when the supply voltage is 230 V. (Nagpur University, Nov. 1996)

**Solution.** At 50 Hz, the series R-L circuit has an impedance of  $Z_s$  given by

$$Z_s = 12 + j(314 \times 0.025) = 12 + j 7.85 = 14.34 + \angle 33.2^\circ$$

$$I_s = (230 + j0) / (12 + j 7.85) = 16.04 - \angle 33.2^\circ$$

$$= 13.42 - j 8.8 \text{ amp}$$

Out of these two components of  $I_s$ , the in-phase component is 13.42 amp and quadrature component (lagging) is 8.8 amp. Now let the R-L parallel combination be considered. In Fig. 14.22 (b), R carries the in-phase component, and L carries the quadrature-component (lagging). For the two systems to be equivalent,

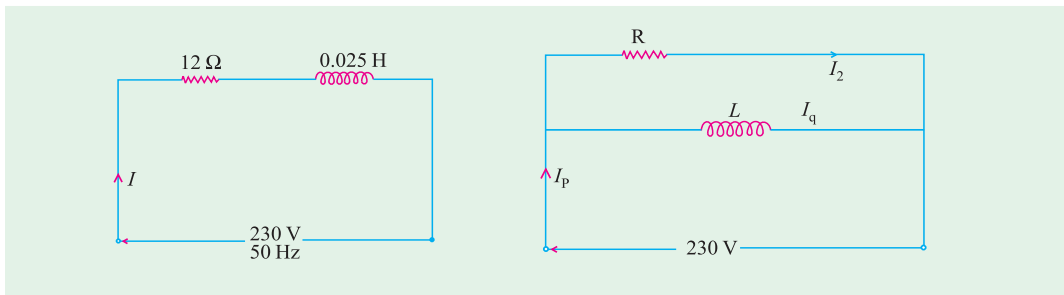


Fig. 14.22 (a)

Fig. 14.22 (b)

$$\begin{aligned} \text{It means} \quad I_s &= I_p \\ I_s &= 13.42 \text{ amp} \\ I_q &= 8.8 \text{ amp} \\ \text{Thus,} \quad R &= 230 / 13.42 = 17.14 \text{ ohms} \end{aligned}$$

$$X_L = 230 / 8.8 = 26.14 \text{ ohms}$$

$$L = 26.14/314 = 83.2 \text{ mH}$$

**Example 14.18.** An inductive coil of resistance 15 ohms and inductive reactance 42 ohms is connected in parallel with a capacitor of capacitive reactance 47.6 ohms. The combination is energized from a 200 V, 33.5 Hz a.c. supply. Find the total current drawn by the circuit and its power factor. Draw to the scale the phasor diagram of the circuit. (Bombay University, 2000)

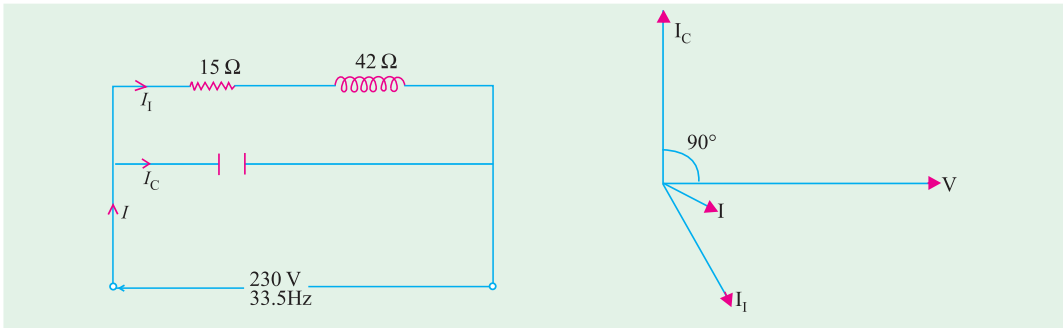


Fig. 14.23 (a)

Fig. 14.23 (b)

**Solution.**

$$Z_1 = 15 + j42, Z_1 = 44.6 \text{ ohms}, \cos \phi_1 = 15/44.6 = 0.3363$$

$$\phi_1 = 70.40 \text{ Lagging}, I_1 = 200/44.6 = 4.484 \text{ amp}$$

$$I_c = 200/47.6 = 4.2 \text{ amp}$$

$$I = 4.484 (0.3355 - j0.942) + j4.2 = 1.50 - j0.025 = 1.5002 \angle -1^\circ$$

For the circuit in Fig. 14.23 (a), the phasor diagram is drawn in Fig. 14.23 (b).

#### Power Calculation

Power etc. can be calculated by the method of conjugates as explained in Ex. 14.3

#### Branch A

The current conjugate of  $(10 + j0)$  is  $(10 - j0)$

$$\therefore \text{VIA} = (120 + j160)(10 - j0) = 1200 + j1600 \quad \therefore \text{kW} = 1200/1000 = 1.2$$

$$\therefore \text{kVAR} = 1600/1000 = 1.6. \text{ The fact that it is positive merely shows the reactive}$$

$$\text{volt-amperes are due to a lagging current}^* \text{ kVA} = \sqrt{(1.2^2 + 1.6^2)} = 2$$

#### Branch B

The current conjugate of  $(-4.0 + j8)$  is  $(-4.0 - j8)$

$$\therefore \text{VI}_B = (120 + j160)(-4 - j8) = 800 - j1600$$

$$\therefore \text{kW} = 800/1000 = 0.8 \quad \therefore \text{kVAR} = -1600/1000 = -1.6$$

The negative sign merely indicates that reactive volt-amperes are due to the leading current

$$\therefore \text{kVA} = \sqrt{[0.8^2 + (-1.6)^2]} = 1.788$$

$$\mathbf{Y} = \mathbf{Y}_A + \mathbf{Y}_B = (0.03 - j0.04) + (0.02 + j0.04) = 0.05 + j0$$

$$\mathbf{I} = \mathbf{VY} = (120 + j160)(0.05 + j0) = 6 + j8 = 10 \angle 53^\circ 8'$$

$$\text{or } \mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = (10 + j0) + (-4 + j8) = 6 + j8 \quad (\text{same as above})$$

$$\text{Circuit p.f.} = \cos 0^\circ = 1 \quad (\because \text{current is in phase with voltage})$$

\* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative.

**Example 14.19.** An impedance  $Z_1 = (8 - j5) \Omega$  is in parallel with an impedance  $Z_2 = (3 + j7) \Omega$ . If 100 V are impressed on the parallel combination, find the branch currents  $I_1$ ,  $I_2$  and the resultant current. Draw the corresponding phasor diagram showing each current and the voltage drop across each parameter. Calculate also the equivalent resistance, reactance and impedance of the whole circuit. (Elect. Technology-I, Gwalior Univ. 1998)

**Solution. Admittance Method**

$$Y_1 = 1/(8 - j5) = (0.0899 + j0.0562) S$$

$$Y_2 = 1/(3 + j7) = (0.0517 - j0.121) S, Y = Y_1 + Y_2 = (0.1416 - j0.065) S$$

$$\text{Let } V = (100 + j0); I_1 = VY_1 = 100(0.0899 + j0.0562) = 8.99 + j5.62$$

$$I_2 = VY_2 = 100(0.0517 - j0.121) = 5.17 - j12.1; I = VY = 100(0.1416 - j0.065) = 14.16 - j6.5$$

$$\text{Now, } G = 0.1416 S, B = -0.065 S \text{ (inductive)}$$

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.1416^2 + 0.065^2} = 0.1558 S$$

$$\text{Equivalent series resistance, } R_{eq} = G/Y^2 = 0.1416/0.1558^2 = 5.38 \Omega$$

$$\text{Equivalent series inductive reactance } X_{eq} = B/Y^2 = 0.065/0.1558^2 = 2.68 \Omega$$

$$\text{Equivalent series impedance } Z = 1/Y = 1/0.1558 = 6.42 \Omega$$

**Impedance Method**

$$I_1 = V/Z_1 = (100 + j0) / (8 - j5) = 8.99 + j5.62$$

$$I_2 = V/Z_2 = 100/(3 + j7) = 5.17 - j12.1$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(8 - j5)(3 + j7)}{(11 + j2)} = \frac{59 + j41}{11 + j2} = 5.848 + j2.664 = 6.426 \angle 24.5^\circ,$$

$$I = 100/6.426 \angle 24.5^\circ = 15.56 \angle -24.5^\circ = 14.16 - j6.54$$

As seen from the expression for  $Z$ , equivalent series resistance is **5.848**  $\Omega$  and inductive reactance is **2.664 ohm**.

**Example 14.20.** The impedances  $Z_1 = 6 + j8$ ,  $Z_2 = 8 - j6$  and  $Z_3 = 10 + j0$  ohms measured at 50 Hz, form three branches of a parallel circuit. This circuit is fed from a 100 volt, 50-Hz supply. A purely reactive (inductive or capacitive) circuit is added as the fourth parallel branch to the above three-branched parallel circuit so as to draw minimum current from the source. Determine the value of  $L$  or  $C$  to be used in the fourth branch and also find the minimum current. (Electrical Circuits, South Gujarat Univ.)

**Solution.** Total admittance of the 3-branched parallel circuit is

$$Y = \frac{1}{6 + j8} + \frac{1}{8 - j6} + \frac{1}{10 + j0} = 0.06 - j0.08 + 0.08 + j0.06 + 0.1 = 0.24 - j0.02$$

Current taken would be minimum when net susceptance is zero. Since combined susceptance is inductive, it means that we must add capacitive susceptance to neutralize it. Hence, we must connect a pure capacitor in parallel with the above circuit such that its susceptance equals  $+j0.02 S$

$$\therefore 1/X_C = 0.02 \text{ or } 2\pi/C = 0.02; C = 0.2/314 = 63.7 \mu F$$

$$\text{Admittance of four parallel branches} = (0.24 - j0.02) + j0.02 = 0.24 S$$

$$\therefore \text{Minimum current drawn by the circuit} = 100 \times 0.24 = 24 A$$

**Example 14.21.** The total effective current drawn by parallel circuit of Fig. 14.24 (a) is 20 A. Calculate (i) VA (ii) VAR and (iii) watts drawn by the circuit.

**Solution.** The combined impedance of the circuit is

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10(6 - j8)}{(16 - j8)} = (5 - j2.5) \text{ ohm}$$

(iii) Power =  $I^2 R = 20^2 \times 5 = 2000 \text{ W}$  (ii)  $Q = I^2 X = 20^2 \times 2.5 = 1000 \text{ VAR (leading)}$  (i)  $S = P + jQ = 2000 + j1000 = 2236 \angle 27^\circ$ ;  $S = 2236 \text{ VA}$

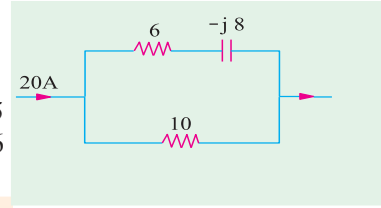


Fig. 14.24. (a)

**Example 14.22.** Calculate (i) total current and (ii) equivalent impedance for the four-branched circuit of Fig. 14.24 (b).

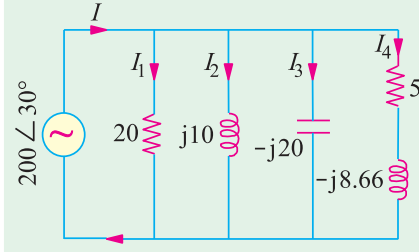


Fig. 14.24 (b)

**Solution.**  $Y_1 = 1/20 = 0.05 \text{ S}$ ,  $Y_2 = 1/j10 = -j0.1 \text{ S}$ ;  
 $Y_3 = 1/-j20 = j0.05 \text{ S}$ ;  $Y_4 = 1/5 - j8.66 = 1/10 \angle 60^\circ$   
 $= 0.1 \angle 60^\circ = (0.05 - j0.0866) \text{ S}$   
 $Y = Y_1 + Y_2 + Y_3 + Y_4 = (0.1 - j0.1366) \text{ S}$   
 $= 0.169 \angle 53.8^\circ \text{ S}$   
 (i)  $I = VY = 200 \angle 30^\circ \times 0.169 \angle 53.8^\circ = 33.8 \angle 23.8^\circ \text{ A}$   
 (ii)  $Z = 1/Y = 1/0.169 \angle 53.8^\circ = 5.9 \angle 53.8^\circ \Omega$

**Example 14.23.** The power consumed by both branches of the circuit shown in Fig. 14.23 is 2200 W. Calculate power of each branch and the reading of the ammeter.

**Solution.**  $I_1 = V/Z_1$   
 $= V/(6 + j8) = V/10 \angle 53.1^\circ$ ,  $I_2 = V/Z_2 = V/20$   
 $\therefore I_1/I_2 = 20/10 = 2$ ,  $P_1 = I_1^2 R_1$  and  $P_2 = I_2^2 R_2$   
 $\therefore \frac{P_1}{P_2} = \frac{I_1^2 R_1}{I_2^2 R_2} = 2^2 \times \left(\frac{6}{20}\right) = \frac{6}{5}$

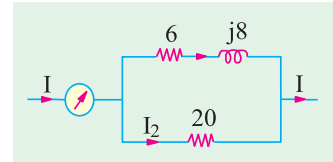


Fig. 14.25

Now,  $P = P_1 + P_2$  or  $\frac{P}{P_2} = \frac{P_1}{P_2} + 1 = \frac{6}{5} + 1 = \frac{11}{5}$

or  $P_2 = 2200 \times \frac{5}{11} = 1000 \text{ W}$   $\therefore P_1 = 2200 - 1000 = 1200 \text{ W}$

Since  $P_1 = I_1^2 R_1$  or  $1200 = I_1^2 \times 6$ ;  $I_1 = 14.14 \text{ A}$

If  $V = V \angle 0^\circ$ , then  $I_1 = 14.14 \angle 53.1^\circ = 8.48 - j11.31$

Similarly,  $P_2 = I_2^2 R_2$  or  $1000 = I_2^2 \times 20$ ;  $I_2 = 7.07 \text{ A}$  or  $I_2 = 7.07 \angle 0^\circ$

Total current  $I = I_1 + I_2 = (8.48 - j11.31) + 7.07 = 15.55 - j11.31 = 19.3 \angle 36^\circ$

Hence, ammeter reads **19.3 A**

**Example 14.24.** Consider an electric circuit shown in Figure 14.25 (a)

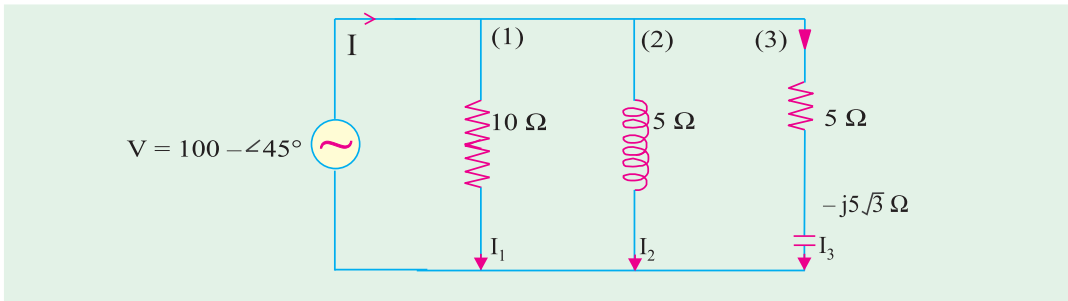


Fig. 14.25. (a)



**Determine :** (i) the current and power consumed in each branch.

(ii) the supply current and power factor. (U.P. Technical University, 2001)

**Solution.** Indicating branch numbers 1, 2, 3 as marked on the figure, and representing the source voltage by  $100 \angle 45^\circ$ ,

$$Z_1 = 10 + j0 = 10 \angle 0^\circ, I_1 = 100 \angle 45^\circ / 10 \angle 0^\circ = 10 \angle 45^\circ \text{ amp}$$

$$Z_2 = 5 + j5\sqrt{3} = 10 \angle 60^\circ, I_2 = 100 \angle 45^\circ / 10 \angle 60^\circ = 10 \angle -15^\circ \text{ amp}$$

$$Z_3 = 5 - j5\sqrt{3} = 10 \angle -60^\circ, I_3 = 100 \angle 45^\circ / 10 \angle -60^\circ = 10 \angle 105^\circ \text{ amp}$$

Phasor addition of these three currents gives the supply current,  $I$  which comes out to be  $I = 20 \angle 45^\circ$  amp.

This is in phase with the supply voltage.

(i) Power consumed by the branches :

Branch 1 :  $10^2 \times 10 = 1000$  watts

Branch 2 :  $10^2 \times 5 = 500$  watts

Branch 3 :  $10^2 \times 5 = 500$  watts

Total power consumed = 2000 watts

(ii) Power factor = 1.0 since  $V$  and  $I$  are in phase

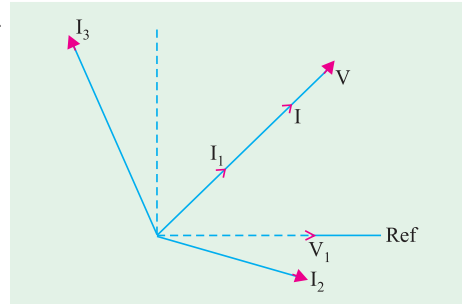


Fig. 14.25 (b)

### 14.6. Series-parallel Circuits

(i) **By Admittance Method**

In such circuits, the parallel circuit is first reduced to an equivalent series circuit and then, as usual, combined with the rest of the circuit. For a parallel circuit,

$$\text{Equivalent series resistance } R_{eq} = Z \cos \phi = \frac{1}{Y} \cdot \frac{G}{Y} = \frac{G}{Y^2}$$

- Sec Ex. 14.14

$$\text{Equivalent series reactance } X_{eq} = Z \sin \phi = \frac{1}{Y} \cdot \frac{B}{Y} = \frac{B}{Y^2}$$

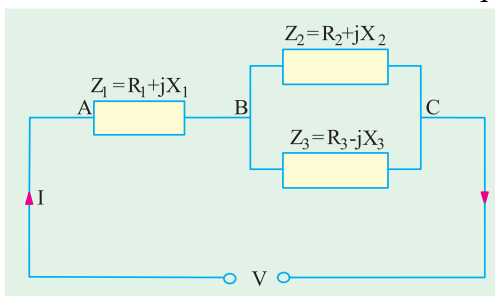


Fig. 14.26

(ii) **By Symbolic Method**

Consider the circuit of Fig. 14.26. First, equivalent impedance of parallel branches is calculated and it is then added to the series impedance to get the total circuit impedance. The circuit current can be easily found.

$$Y_2 = \frac{1}{R_2 - jX_2}; Y_3 = \frac{1}{R_3 + jX_3}$$

$$\therefore Y_{23} = \frac{1}{R_2 + jX_2} + \frac{1}{R_3 - jX_3}$$

$$\therefore Z_{23} = \frac{1}{Y_{23}}; Z_1 = R_1 + jX_1; Z = Z_{23} + Z_1$$

$$\therefore I = \frac{V}{Z} \quad \text{(Sec Ex. 14.21)}$$

### 14.7. Series Equivalent of a Parallel Circuit

Consider the parallel circuit of Fig. 14.27 (a). As discussed in Art. 14.5

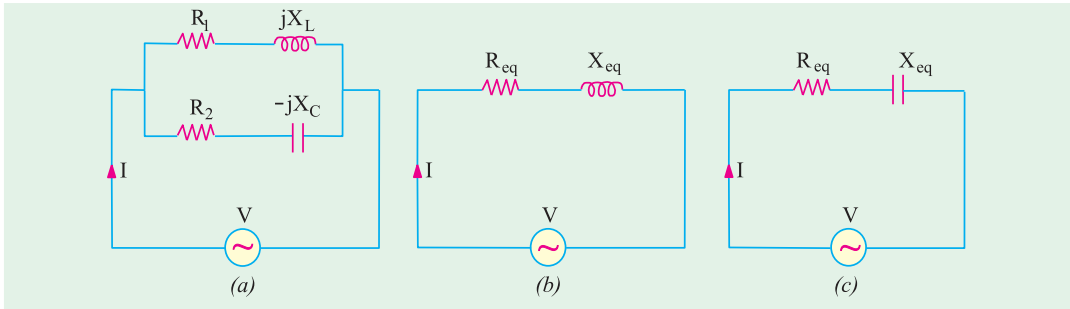


Fig. 14.27

$$Y_1 = \frac{R_1}{R_1^2 + X_L^2} + j \frac{X_L}{R_1^2 + X_L^2} \quad g_1 \quad jb_1; \quad Y_2 = \frac{R_2}{R_2^2 + X_c^2} + j \frac{X_c}{R_2^2 + X_c^2} \quad g_2 \quad jb_2$$

$$Y = Y_1 + Y_2 = g_1 - jb_1 + g_2 + jb_2 = (g_1 + g_2) + j(b_2 - b_1) = G + jB = \sqrt{G^2 + B^2} \angle \tan^{-1} (B/G)$$

As seen from Fig. 14.28.

$$R_{eq} = Z \cos \phi = \frac{1}{Y} \cdot \frac{G}{Y} = \frac{G}{Y^2}$$

$$X_{eq} = Z \sin \phi = \frac{1}{Y} \cdot \frac{B}{Y} = \frac{B}{Y^2}$$

Hence, equivalent series circuit is as shown in Fig. 14.27 (b) or (c) depending on whether net susceptance  $B$  is negative (inductive) or positive (capacitive). If  $B$  is negative, then it is an  $R$ - $L$  circuit of Fig. 14.27 (b) and if  $B$  is positive, then it is an  $R$ - $C$  circuit of Fig. 14.27 (c).

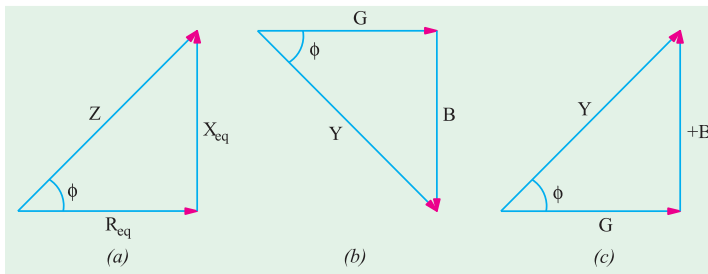


Fig. 14.28

### 14.8. Parallel Equivalent of a Series Circuit

The two circuits will be equivalent if  $Y$  of Fig. 14.29 (a) is equal to the  $Y$  of the circuit of Fig. 14.29 (b).

#### Series Circuit

$$Y_s = \frac{1}{R_s + jX_s}$$

$$= \frac{R_s - jX_s}{(R_s + jX_s)(R_s - jX_s)}$$

$$= \frac{R_s - jX_s}{R_s^2 + X_s^2} = \frac{R_s}{R_s^2 + X_s^2} - j \frac{X_s}{R_s^2 + X_s^2}$$

#### Parallel Circuit

$$Y_p = \frac{1}{R_p + j0} + \frac{1}{0 + jX_p} = \frac{1}{R_p} + \frac{1}{jX_p} = \frac{1}{R_p} - \frac{j}{X_p}$$

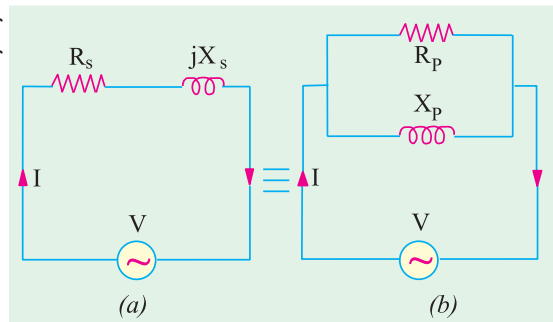


Fig. 14.29

$$\therefore \frac{R_s}{R_s^2 + X_s^2} - j \frac{X_s}{R_s^2 + X_s^2} = \frac{1}{R_p} - \frac{j}{X_p} \quad \therefore \frac{1}{R_p} = \frac{R_s}{R_s^2 + X_s^2} \quad \text{or } R_p = R_s + \frac{X_s^2}{R_s} = R_s \left( 1 + \frac{X_s^2}{R_s^2} \right)$$

$$\text{Similarly } X_p = X_s + \frac{R_s^2}{X_s} = X_s \left( 1 + \frac{R_s^2}{X_s^2} \right)$$

**Example 14.25.** The admittance of a circuit is  $(0.03 - j0.04)$  Siemens. Find the values of the resistance and inductive reactance of the circuit if they are joined (a) in series and (b) in parallel.

**Solution. (a)**  $Y = 0.03 - j0.04$

$$\therefore Z = \frac{1}{Y} = \frac{1}{0.03 - j0.04} = \frac{0.03 + j0.04}{0.03^2 + 0.04^2} = \frac{0.03 + j0.04}{0.0025} = 12 + j16$$

Hence, if the circuit consists of a resistance and inductive reactance in series, then resistance is  $12 \Omega$  and inductive reactance is  $16 \Omega$  as shown in Fig. 14.30.

(b) Conductance =  $0.03$  mho  $\therefore$  Resistance =  $1/0.03 = 33.3 \Omega$

Susceptance (inductive) =  $0.04$  S  $\therefore$  Inductive reactance =  $1/0.04 = 25 \Omega$

Hence, if the circuit consists of a resistance connected in parallel with an inductive reactance, then resistance is  $33.3 \Omega$  and inductive reactance is  $25 \Omega$  as shown in Fig. 14.31.

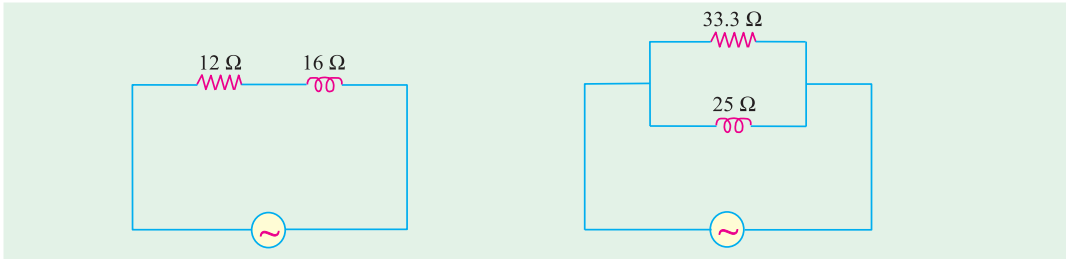


Fig. 14.30

Fig. 14.31

**Example 14.26.** A circuit connected to a  $115\text{-V}$ ,  $50\text{-Hz}$  supply takes  $0.8$  A at a power factor of  $0.3$  lagging. Calculate the resistance and inductance of the circuit assuming (a) the circuit consists of a resistance and inductance in series and (b) the circuit consists of a resistance and inductance in parallel. (Elect. Engg.-I, Sardar Patel Univ.)

**Solution. Series Combination**

$$Z = 115/0.8 = 143.7 \Omega; \cos \phi = R/Z = 0.3 \quad \therefore R = 0.3 \times 143.7 = 43.1 \Omega$$

$$\text{Now } X_L = \sqrt{Z^2 - R^2} = \sqrt{143.7^2 - 43.1^2} = 137.1 \Omega$$

$$\therefore L = 137.1/2\pi \times 50 = 0.436 \text{ H}$$

**Parallel Combination**

Active component of current (drawn by resistance)

$$= 0.8 \cos \phi = 0.8 \times 0.3 = 0.24 \text{ A}; \quad R = 115/0.24 = 479 \Omega$$

Quadrature component of current (drawn by inductance) =  $0.8 \sin \phi = 0.8 \sqrt{1 - 0.3^2} = 0.763 \text{ A}$

$$\therefore X_L = 115/0.763 \Omega \quad \therefore L = 115/0.763 \times 2\pi \times 50 = 0.48 \text{ H}$$

**Example 14.27.** The active and lagging reactive components of the current taken by an a.c. circuit from a  $250\text{-V}$  supply are  $50$  A and  $25$  A respectively. Calculate the conductance, susceptance, admittance and power factor of the circuit. What resistance and reactance would an inductive coil have if it took the same current from the same mains at the same factor?

(Elect. Technology, Sumbal Univ.)

**Solution.** The circuit is shown in Fig. 14.32.

$$\text{Resistance} = 250/50 = 5 \Omega; \quad \text{Reactance} = 250/25 = 10 \Omega$$

$\therefore$  Conductance  $g = 1/5 = 0.2 \text{ S}$ , Susceptance  $b = -1/10 = -0.1 \text{ S}$

Admittance  $Y = \sqrt{g^2 + b^2} = \sqrt{0.2^2 + (-0.1)^2} = \sqrt{0.05} = 0.224 \text{ S}$

$Y = 0.2 - j0.1 = 0.224 \angle -26^\circ 34'$ . Obviously, the total current lags the supply voltage by  $26^\circ 34'$ , p.f. =  $\cos 26^\circ 34' = 0.894$  (lag)

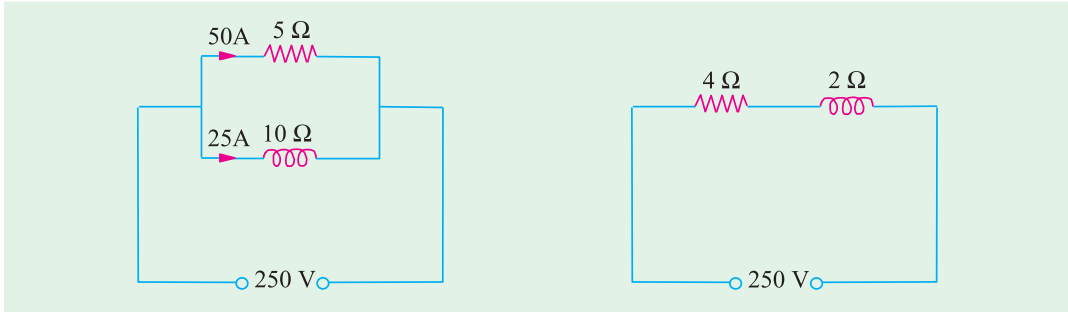


Fig. 14.32

Fig. 14.33

$$\text{Now } Z = \frac{1}{Y} = \frac{1}{0.2 - j0.1} = \frac{0.2 + j0.1}{0.05} = 4 + j2$$

Hence, resistance of the coil =  $4 \Omega$

Reactance of the coil =  $2 \Omega$  (Fig. 14.33)

**Example 14.28.** The series and parallel circuits shown in Fig. 14.34 have the same impedance and the same power factor. If  $R = 3 \Omega$  and  $X = 4 \Omega$ , find the values of  $R_1$  and  $X_1$ . Also, find the impedance and power factor. (Elect. Engg., Bombay Univ.)

**Solution. Series Circuit** [Fig. 14.34 (a)]

$$Y_S = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2}$$

**Parallel Circuit** [Fig. 14.34 (b)]

$$Y_P = \frac{1}{R_1 + j0} + \frac{1}{0 + jX_1} = \frac{1}{R_1} + \frac{1}{jX_1} = \frac{1}{R_1} - \frac{j}{X_1}$$

$$\therefore \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2} = \frac{1}{R_1} - \frac{j}{X_1}$$

$$\therefore R_1 = R + X^2/R \text{ and } X_1 = X + R^2/X$$

$$\therefore R_1 = 3 + (16/3) = 8.33 \Omega, \quad X_1 = 4 + (9/4) = 6.25 \Omega$$

Impedance =  $3 + j4 = 5 \angle 53.1^\circ$ ; Power factor =  $\cos 53.1^\circ = 0.6$  (lag)

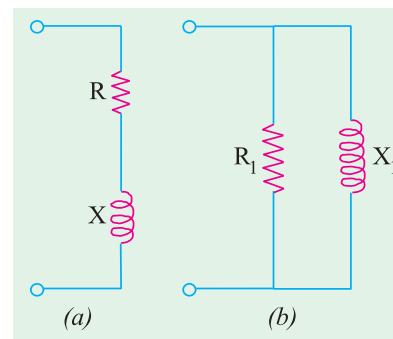


Fig. 14.34

**Example 14.29.** Find the value of the resistance  $R$  and inductance  $L$  which when connected in parallel will take the same current at the same power factor from 400-V, 50-Hz mains as a coil of resistance  $R_1 = 8 \Omega$  and an inductance  $L_1 = 0.2 \text{ H}$  from the same source of supply.

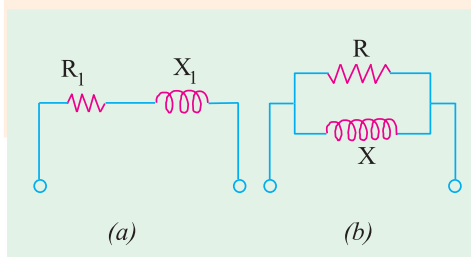


Fig. 14.35

Show that when the resistance  $R_1$  of the coil is small as compared to its inductance  $L_1$ , then  $R$  and  $L$  are respectively equal to  $\omega^2 L_1^2 / R_1$  and  $L_1$ .

(Elect. Technology, Utkal Univ.)

**Solution.** As seen from Art. 14.8 in Fig. 14.35.

$$R = R_1 + X_1^2 / R_1 \quad \dots (i)$$

$$X = X_1 + R_1^2 / X_1 \quad \dots (ii)$$

Now

$$R_1 = 8 \Omega, X_1 = 2\pi \times 50 \times 0.2 = 62.8 \Omega$$

$$\therefore R = 8 + (62.8^2/8) = 508 \Omega$$

$$X = 62.8 + (64/62.8) = 63.82 \Omega$$

From (i), it is seen that if  $R_1$  is negligible, then  $R = X_1^2/R_1 = \omega^2 L_1^2/R_1$

Similarly, from (ii) we find that the term  $R_1^2/X_1$  is negligible as compared to  $X_1$ ,

$$\therefore X = X_1 \text{ or } L = L_1$$

**Example 14.30.** Determine the current drawn by the following circuit [Fig. 14.36 (a)] when a voltage of 200 V is applied across the same. Draw the phasor diagram.

**Solution.** As seen from the figure

$$Z_2 = 10 - j12 = 15.6 \angle -50.2^\circ; Z_3 = 6 + j10 = 11.7 \angle 58^\circ$$

$$Z_1 = 4 + j6 = 7.2 \angle 56.3^\circ; Z_{BC} = \frac{(10 - j12)(6 + j10)}{16 - j2} = 10.9 + j3.1 = 11.3 \angle 15.9^\circ$$

$$Z = Z_1 + Z_{BC} = (4 + j6) + (10.9 + j3.1) = 14.9 + j9.1 = 17.5 \angle 31.4^\circ$$

$$\text{Assuming } V = 200 \angle 0^\circ; I = \frac{V}{Z} = \frac{200}{17.5 \angle 31.4^\circ} = 11.4 \angle -31.4^\circ$$

For drawing the phasor diagram, let us find the following quantities :

$$(i) V_{AB} = I Z_1 = 11.4 \angle -31.4^\circ \times 7.2 \angle 56.3^\circ = 82.2 \angle 24.9^\circ$$

$$V_{BC} = I Z_{BC} = 11.4 \angle -31.4^\circ \times 11.3 \angle 15.9^\circ = 128.8 \angle -15.5^\circ$$

$$I_2 = \frac{V_{BC}}{Z_2} = \frac{128.8}{15.6} \angle -15.5^\circ - 50.2^\circ = 8.25 \angle -65.7^\circ$$

$$I_3 = \frac{128.8}{11.7} \angle -15.5^\circ - 58^\circ = 11.0 \angle -73.5^\circ$$

Various currents and voltages are shown in their phase relationship in Fig. 14.36 (b).

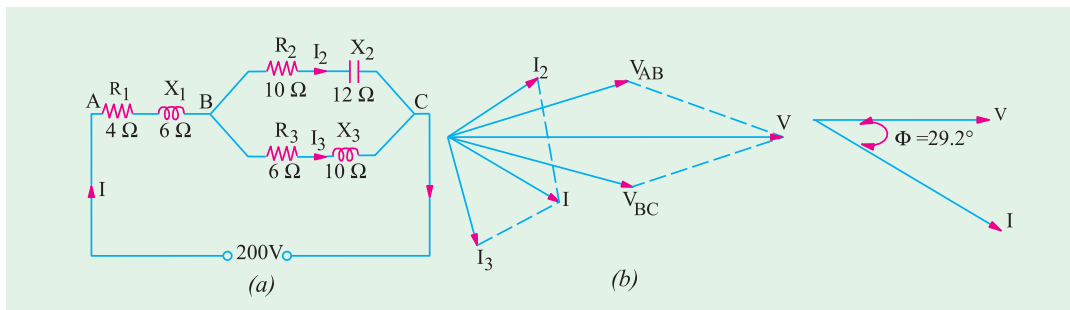


Fig. 14.36

Fig. 14.37 (a)

**Example 14.31.** For the circuit shown in Fig. 14.37 (a), find (i) total impedance (ii) total current (iii) total power absorbed and power-factor. Draw a vector diagram.

(Elect. Tech. Osmania Univ. Jan/Feb 1992)

$$\text{Solution. } Z_{BC} = (4 + j8) \parallel (5 - j8) = 9.33 + j0.89$$

$$(i) Z_{AC} = 3 + j6 + 9.33 + j0.89 = 12.33 + j6.89 = 14.13 \angle 29.2^\circ$$

$$(ii) I = 100/14.13 \angle 29.2^\circ, \text{ as drawn in Fig. 14.37 (b)}$$

$$= 7.08 \angle -29.2^\circ$$

$$(iii) \phi = 29.2^\circ; \cos \phi = 0.873; P = VI \cos \phi$$

$$= 100 \times 7.08 \times 0.873 = 618 \text{ W}$$

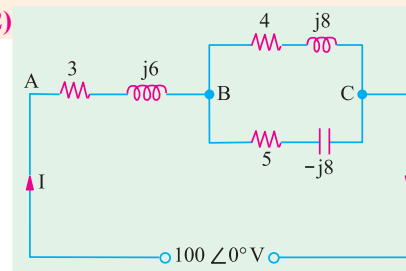


Fig. 14.37 (b)

**Example 14.32.** In a series-parallel circuit, the parallel branches A and B are in series with C. The impedances are :  $Z_A = (4 + j3)$  ;  $Z_B = (4 - j16/3)$  ;  $Z_C = (2 + j8)$  ohm.

If the current  $I_C = (25 + j0)$ , draw the complete phasor diagram determining the branch currents and voltages and the total voltage. Hence, calculate the complex power (the active and reactance powers) for each branch and the whole circuits. (Basic Electricity, Bombay Univ.)

**Solution.** The circuit is shown in Fig. 14.38 (a)

$$Z_A = (4 + j3) = 5 \angle 36^\circ 52' ; Z_B = (4 - j16/3) = 20/3 \angle -53^\circ 8' ; Z_C = (2 + j8) = 8.25 \angle 76^\circ$$

$$I_C = (25 + j0) = 25 \angle 0^\circ ; V_C = I_C Z_C = 206 \angle 76^\circ$$

$$Z_{AB} = \frac{(4 + j3)(4 - j16/3)}{(8 - j7/3)} = \frac{(32 - j28/3)}{(8 - j7/3)} = 4 + j0 = 4 \angle 0^\circ$$

$$V_{AB} = I_C Z_{AB} = 25 \angle 0^\circ \times 4 \angle 0^\circ = 100 \angle 0^\circ$$

$$Z = Z_C + Z_{AB} = (2 + j8) + (4 + j0) = (6 + j8) = 10 \angle 53^\circ 8' ; V = I_C Z = 25 \angle 0^\circ \times 10 \angle 53^\circ 8' = 250 \angle 53^\circ 8'$$

$$I_A = \frac{V_{AB}}{Z_A} = \frac{100}{5} = 20 \quad 52 ; I_B = \frac{V_{AB}}{Z_B} = \frac{100}{(20/3)} = 15 \quad 53 \quad 8$$

Various voltages and currents are shown in Fig. 14.38 (b). Powers would be calculated by using voltage conjugates.

$$\text{Power for whole circuit is } P = VI_C = 250 \angle 53^\circ 8' \times 25 \angle 0^\circ = 6,250 \angle 53^\circ 8'$$

$$= 6250 (\cos 53^\circ 8' - j \sin 53^\circ 8') = 3750 - j5000$$

$$P_C = 25 \times 206 \angle -76^\circ = 5150 (\cos 76^\circ - j \sin 76^\circ) = 1250 - j5000$$

$$P_A = 100 \times 20 \angle -36^\circ 52' = 2000 \angle -36^\circ 52' = 1600 - j1200$$

$$P_B = 100 \times 15 \angle 53^\circ 8' = (900 + j1200); \text{ Total} = 3,750 - j5000^\circ \text{ (as a check)}$$

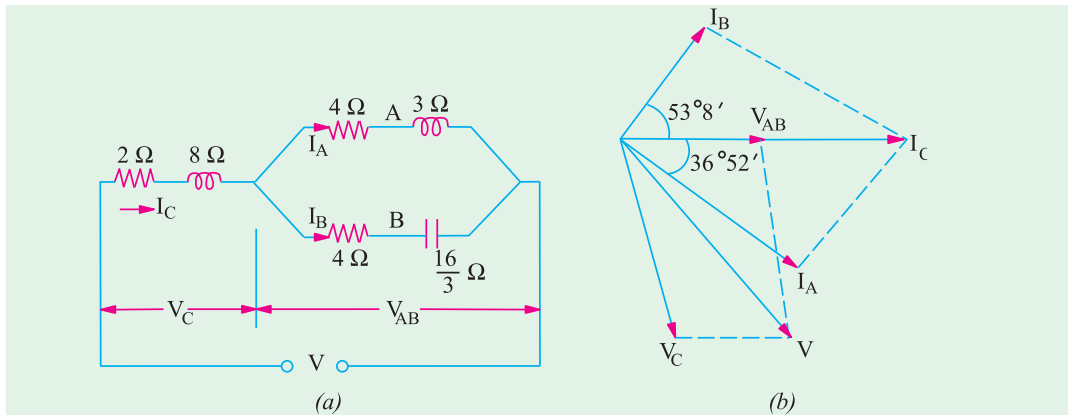


Fig. 14.38

**Example 14.33.** Find the value of the power developed in each arm of the series-parallel circuit shown in Fig. 14.39.

**Solution.** In order to find the circuit current, we must first find the equivalent impedance of the whole circuit.

$$Z_{AB} = (5 + j12) \parallel (-j20)$$

$$= \frac{(5 + j12)(-j20)}{5 + j12 - j20} = \frac{13 \angle 67.4^\circ \times 20 \angle -90^\circ}{9.43 \angle -58^\circ}$$

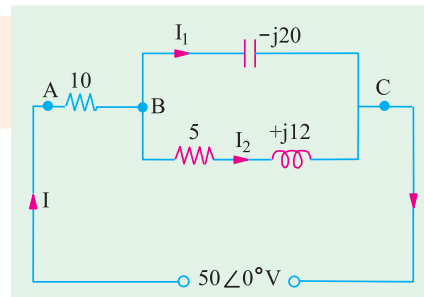


Fig. 14.39

$$= 27.57 \angle 35.4^\circ = (22.47 + j 15.97)$$

$$Z_{AC} = (10 + j0) + (22.47 + j15.97) = (32.47 + j14.97) \\ = 36.2 \angle 26.2^\circ$$

$$I = \frac{V}{Z} = \frac{50 \angle 0^\circ}{36.2 \angle 26.2^\circ} = 1.38 \angle -26.2^\circ \text{ A}$$

$$\text{Power developed in } 10 \Omega \text{ resistor} = I^2 R = 1.38^2 \times 10 = \mathbf{19 \text{ W}}$$

Potential difference across  $10 \Omega$  resistor is

$$IR = 1.38 \angle -26.2^\circ \times 10 = 13.8 \angle -26.2^\circ = (12.38 - j 6.1)$$

$$V_{BC} = \text{supply voltage} - \text{drop across } 10 \Omega \text{ resistor} \\ = (50 + j0) - (12.38 - j6.1) = (37.62 + j6.1) = 38.1 \angle 9.21^\circ$$

$$I_2 = \frac{V_{BC}}{(5 + j12)} = \frac{38.1 \angle 9.21^\circ}{13 \angle 67.4^\circ} = 2.93 \angle -58.2^\circ$$

$$\text{Power developed} = I_2^2 \times 5 = 2.93^2 \times 5 = \mathbf{43 \text{ W}}$$

No power is developed in the capacitor branch because it has no resistance.

**Example 14.34.** In the circuit shown in Fig. 14.40 determine the voltage at a frequency of 50 Hz to be applied across AB in order that the current in the circuit is 10 A. Draw the phasor diagram.

(Elect. Engg. & Electronics Bangalore Univ.)

$$\text{Solution. } X_{L1} = 2\pi \times 50 \times 0.05 = 15.71 \Omega; X_{L2} = -2\pi \times 50 \times 0.02 = 6.28 \Omega,$$

$$X_C = 1/2\pi \times 50 \times 400 \times 10^{-6} = 7.95 \Omega$$

$$Z_1 = R_1 + jX_{L1} = 10 + j15.71 = 18.6 \angle 57^\circ 33'$$

$$Z_2 = R_2 + jX_{L2} = 5 + j6.28 = 8 \angle 51^\circ 30'$$

$$Z_3 = R_3 - jX_C = 10 - j7.95 = 12.77 \angle -38^\circ 30'$$

$$Z_{BC} = Z_2 \parallel Z_3 = (5 + j6.28) \parallel (10 - j7.95) = 6.42 + j2.25 = 6.8 \angle 19^\circ 18'$$

$$Z = Z_1 + Z_{BC} = (10 + j15.71) + (6.42 + j2.25) = 16.42 + j17.96 = 24.36 \angle 47^\circ 36'$$

$$\text{Let } I = 10 \angle 0^\circ; \therefore V = IZ = 10 \angle 0^\circ \times 24.36 \angle 47^\circ 36' = 243.6 \angle 47^\circ 36'$$

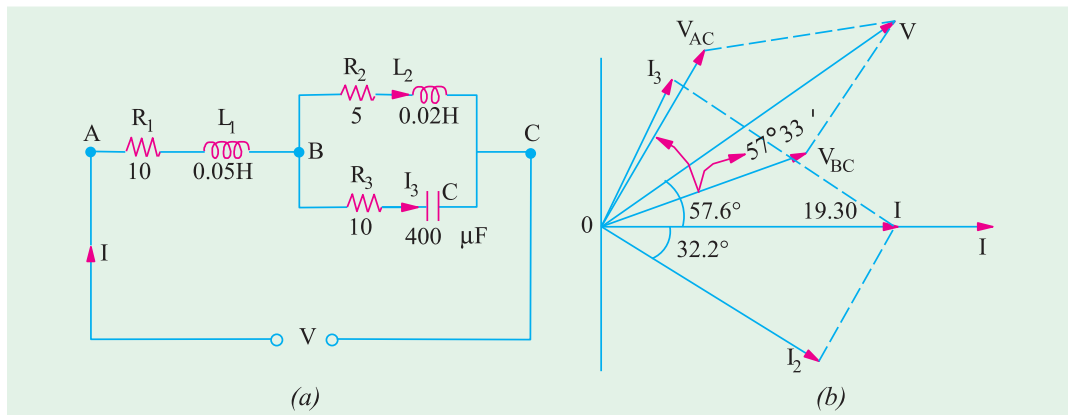


Fig. 14.40

$$V_{BC} = IZ_{BC} = 10 \angle 0^\circ \times 6.8 \angle 19^\circ 18' = 68 \angle 19^\circ 18'; I_2 = \frac{V_{BC}}{Z_2} = \frac{68 \angle 19^\circ 18'}{8 \angle 51^\circ 31'} = 8.5 \angle -32^\circ 12'$$

$$I_3 = \frac{V_{BC}}{Z_3} = \frac{68 \angle 19^\circ 18'}{12.77 \angle -38^\circ 30'} = 5.32 \angle 57^\circ 48'; V_{AC} = IZ_1 = 10 \angle 0^\circ \times 18.6 \angle 57^\circ 33' = 186 \angle 57^\circ 33'$$

The phasor diagram is shown in Fig. 14.36 (b).

**Example 14.35.** Determine the average power delivered to each of the three boxed networks in the circuit of Fig. 14.41. (Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)

**Solution.**  $Z_1 = 6 - j8 = 10 \angle -53.13^\circ$ ;  $Z_2 = 2 + j14 = 14.14 \angle 81.87^\circ$ ;  $Z_3 = 6 - j8 = 10 \angle -53.13^\circ$

$$Z_{23} = \frac{Z_2 Z_3}{Z_2 + Z_3} = 14.14 \angle -8.13^\circ = 14 - j2$$

Drop across two parallel impedances is given by

$$V_{23} = 100 \frac{14 - j2}{(6 - j8) + (14 - j2)} = 63.2 \angle 18.43^\circ = 60 + j20$$

$$V_1 = 100 \frac{10 \angle -53.13^\circ}{6 - j8 + (14 - j2)} = 44.7 \angle -26.57^\circ = 40 - j20$$

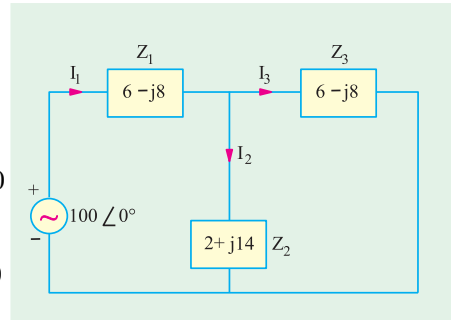


Fig. 14.41

$$I_1 = \frac{44.7 \angle -26.57^\circ}{10 \angle -53.13^\circ} = 4.47 \angle 26.56^\circ$$

$$I_2 = \frac{63.2 \angle 18.43^\circ}{14.14 \angle 81.87^\circ} = 4.47 \angle -63.44^\circ$$

$$I_3 = \frac{63.2 \angle 18.43^\circ}{10 \angle -53.13^\circ} = 6.32 \angle 71.56^\circ$$

$$P_1 = V_1 I_1 \cos \phi_1 = 44.7 \times 4.47 \times \cos 53.13^\circ = 120 \text{ W}$$

$$P_2 = V_2 I_2 \cos \phi_2 = 63.2 \times 4.47 \times \cos 81.87^\circ = 40 \text{ W};$$

$$P_3 = V_3 I_3 \cos \phi_3 = 63.2 \times 6.32 \times \cos 53.13^\circ = 240 \text{ W, Total} = 400 \text{ W}$$

As a check, power delivered by the 100-V source is,

$$P = VI \cos \phi = 100 \times 4.47 \times \cos 26.56^\circ = 400 \text{ W}$$

**Example 14.36.** In a series-parallel circuit of Fig. 14.42 (a), the parallel branches A and B are in series with C. The impedances are  $Z_A = (4 + j3)$ ,  $Z_B = (10 - j7)$  and  $Z_C = (6 + j5) \Omega$ .

If the voltage applied to the circuit is 200 V at 50 Hz, calculate : (a) current  $I_A$ ,  $I_B$  and  $I_C$ ; (b) the total power factor for the whole circuit.

Draw and explain complete vector diagram.

**Solution.**  $Z_A = 4 + j3 = 5 \angle 36.9^\circ$ ;  $Z_B = 10 - j7 = 12.2 \angle -35^\circ$ ;  $Z_C = 6 + j5 = 7.8 \angle 39.8^\circ$

$$Z_{AB} = \frac{Z_A Z_B}{Z_A + Z_B} = \frac{5 \angle 36.9^\circ \cdot 12.2 \angle -35^\circ}{14 \angle j4} = \frac{61 \angle 1.9^\circ}{14.56} = 4.19 \angle 1.3^\circ$$

$$Z = Z_C + Z_{AB} = (6 + j5) + (4 + j1.3) = 10 + j6.3 = 11.8 \angle 32.2^\circ$$

Let  $V = 200 \angle 0^\circ$ ;  $I_C = (V/Z) = (200/11.8) \angle -32.2^\circ = 16.35 \angle -32.2^\circ$

$$I_A = I_C \cdot \frac{Z_B}{Z_A + Z_B} = 16.35 \angle -32.2^\circ \cdot \frac{12.2 \angle -35^\circ}{14.56 \angle 1.3^\circ} = 13.7 \angle -68.5^\circ$$

$$I_B = I_C \cdot \frac{Z_A}{Z_A + Z_B} = 16.35 \angle -32.2^\circ \cdot \frac{5 \angle 36.9^\circ}{14.56} = 5.7 \angle 2.7^\circ$$



The phase angle between  $V$  and total circuit current  $I_C$  is  $32.2^\circ$ . Hence p.f. for the whole circuit is  $\cos 32.2^\circ = 0.846$  (lag)

For drawing the phasor diagram of Fig. 14.42 (b) following quantities have to be calculated :

$$V_C = I_C Z_C = 16.35 \angle -32.2^\circ \times 7.8 \angle 39.8^\circ = 127.53 \angle 7.6^\circ$$

$$V_{AB} = I_C Z_{AB} = 16.35 \angle -32.2^\circ \times 4.19 \angle 17.9^\circ = 18.5 \angle -14.3^\circ$$

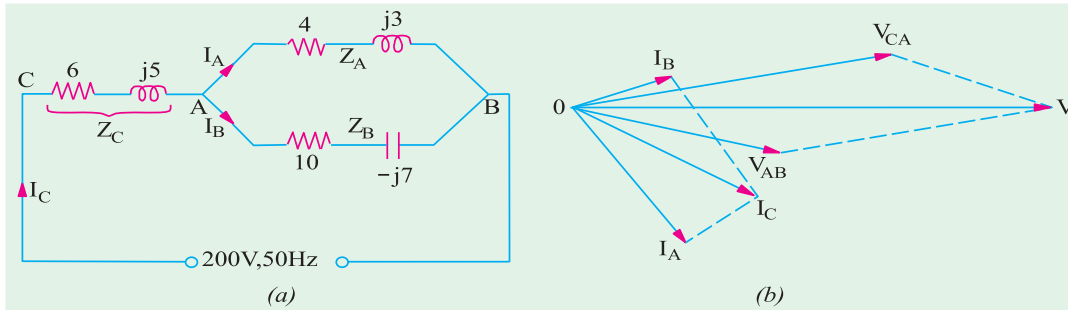


Fig. 14.42

The circuit and phasor diagrams are shown in Fig. 14.38.

**Example 14.37.** A fluorescent lamp taking 80 W at 0.7 power factor lagging from a 230-V 50-Hz supply is to be corrected to unity power factor. Determine the value of the correcting apparatus required.

**Solution.** Power taken by the 80-W lamp circuit can be found from the following equation,

$$230 \times I \times 0.7 = 80 \quad \therefore I = 80/230 \times 0.7 = 0.5 \text{ A}$$

$$\text{Reactive component of the lamp current is } I \sin \phi = 0.5 \sqrt{1 - 0.7^2} = 0.357 \text{ A}$$

The power factor of the lamp circuit may be raised to unity by connecting a suitable capacitor across the lamp circuit. The leading reactive current drawn by it should be just equal to 0.357 A. In that case, the two will cancel out leaving only the in-phase component of the lamp current.

$$I_C = 0.357 \text{ A}, \quad X_C = 230/0.357 = 645 \Omega$$

$$\text{Now } X_C = I/\omega C \quad \therefore 645 = 1/2\pi \times 50 \times C, \quad C = 4.95 \mu\text{F}$$

**Example 14.38.** For the circuit shown in Fig. 14.43, calculate  $I_1$ ,  $I_2$  and  $I_3$ . The values marked on the inductance and capacitance give their reactances. (Elect. Science-I Allahabad Univ. 1992)

$$\text{Solution. } Z_{BC} = Z_2 \parallel Z_3 = \frac{(4 + j2)(1 - j5)}{(3 + j2) + (1 - j5)} = \frac{14 - j18}{5 - j3} = \frac{(14 - j18)(5 + j3)}{5^2 + 3^2} = 3.65 - j1.41 = 3.9 \angle 21.2^\circ$$

$$Z = Z_1 + Z_{BC} = (2 + j3) + (3.65 - j1.41) = 5.65 + j1.59 = 5.82 \angle 74.3^\circ$$

$$\text{Let } V = 10 \angle 0^\circ; \quad I_1 = V/Z = 10 \angle 0^\circ / 5.82 \angle 74.3^\circ = 1.72 \angle -74.3^\circ$$

$$V_{BC} = I_1 Z_{BC} = 1.72 \angle -74.3^\circ \times 3.9 \angle 21.2^\circ = 6.7 \angle -53.1$$

$$\text{Now, } Z_2 = 4 + j2 = 4.47 \angle 63.4^\circ;$$

$$Z_3 = 1 - j5 = 5.1 \angle -11.3^\circ$$

$$I_2 = V_{BC}/Z_2 = 6.7 \angle -53.1^\circ / 4.47 \angle 63.4^\circ = 1.5 \angle 10.3^\circ$$

$$I_3 = V_{BC}/Z_3 = 6.7 \angle -53.1^\circ / 5.1 \angle -11.3^\circ = 1.3 \angle -41.8^\circ$$

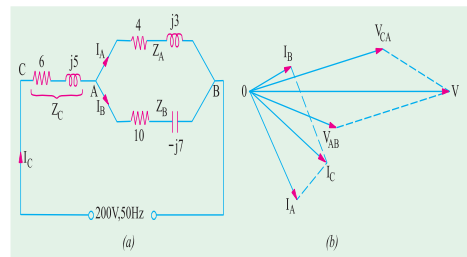


Fig. 14.43

**Example 14.39.** A workshop has four 240-V, 50-Hz single-phase motors each developing 3.73 kW having 85% efficiency and operating at 0.8 power factor. Calculate the values 0.9 lagging and (b) 0.9 leading. For each case, sketch a vector diagram and find the value of the supply current.

**Solution.** Total motor power input =  $4 \times 3730/0.85 = 17,550$  W

Motor current  $I_m = 17,550/240 \times 0.8 = 91.3$  A

Motor p.f. =  $\cos \phi_m = 0.8 \therefore \phi_m = \cos^{-1}(0.8) = 36^\circ 52'$

(a) Since capacitor does not consume any power, the power taken from the supply remains unchanged after connecting the capacitor. If  $I_s$  is current drawn from the supply, then  $240 \times I_s \times 0.9 = 17,550$

$\therefore I_s = 81.2$  A,  $\cos \phi_s = 0.9$ ;  $\phi_s = \cos^{-1}(0.9) = 25^\circ 50'$

As seen from vector diagram of Fig. 14.44 (a),  $I_s$  the vector sum of  $I_m$  and capacitor current  $I_C$ ,  $I_C = I_m \sin \phi_m - I_s \sin \phi_s = 91.3 \sin 36^\circ 52' - 81.2 \sin 25^\circ 50' = 54.8 - 35.4 = 19.4$  A

Now  $I_C = \omega VC$

or  $19.4 = 240 \times 2\pi \times 50 \times C$

$\therefore C = 257 \times 10^{-6} \text{ F} = 257 \mu\text{F}$

(b) In this case,  $I_s$  leads the supply voltage as shown in Fig. 14.44 (b)

$$I_C = I_m \sin \phi_m + I_s \sin \phi_s \\ = 54.8 + 35.4 = 90.2 \text{ A}$$

Now  $I_C = \omega VC$

$\therefore 90.2 = 240 \times 2\pi \times 50 \times C$

$\therefore C = 1196 \times 10^{-6} \text{ F} = 1196 \mu\text{F}$

The line or supply current is, as before, 81.2 A (leading)

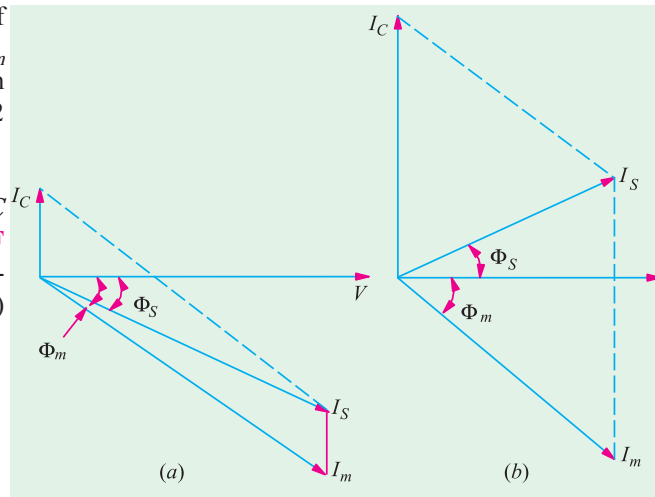


Fig. 14.44

**Example 14.40.** The load taken from a supply consists of (a) lamp load 10 kW a unity power factor (b) motor load of 80 kVA at 0.8 power factor (lag) and (c) motor load of 40 kVA at 0.7 power factor leading. Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.

**Solution.** Since it is more convenient to adopt the tabular method for such questions, we will use the same as illustrated below. We will tabulate the kW, kVA and kVAR (whether leading or lagging) of each load. The lagging kVAR will be taken as negative and leading kVAR as positive.

Load	kVA	$\cos \phi$	$\sin \phi$	kW	kVAR
(a)	10	1	0	10	0
(b)	80	0.8	0.6	64	-48
(c)	40	0.7	0.714	28	28/6
<b>Total</b>				<b>102</b>	<b>-19.2</b>

Total kW = 102 ; Total kVAR = -19.4 (lagging) ; kVA taken =  $\sqrt{102^2 + (-19.4)^2} = 103.9$

Power factor = kW/kVA =  $102/103.9 = 0.9822$  (lag)

**Example 14.41.** A 23-V, 50 Hz, 1-ph supply is feeding the following loads which are connected across it.

(i) A motor load of 4 kW, 0.8 lagging p.f.

(ii) A rectifier of 3 kW at 0.6 leading p.f.

(iii) A lighter-load of 10 kVA at unity p.f.

(iv) A pure capacitive load of 8 kVA

Determine : Total kW, Total kVAR, Total kVA

(I BE Nagpur University Nov. 1999)

**Solution.**

S. No.	Item	kW	P.f	kVA	kVAR	$I$	$I_x$	$I_r$
1	Motor	4	0.8 lag	5	3 -ve, Lag	21.74	17.4	13.04 Lag (-)
2	Rectifier	3	0.6 Lead	5	4 + ve, Lead	21.74	13.04	17.4 Lead (+)
3	Light-Load	10	1.0	10	zero	43.48	43.48	zero
4	Capacitive Load	Zero	0.0 Lead	8	8 + ve Lead	34.8	zero	34.8 Lead (+)
	Total	17	-	Phasor Addition required	+ 9 + ve Lead	Phasor addition required	73.92	39.16 Lead (+)

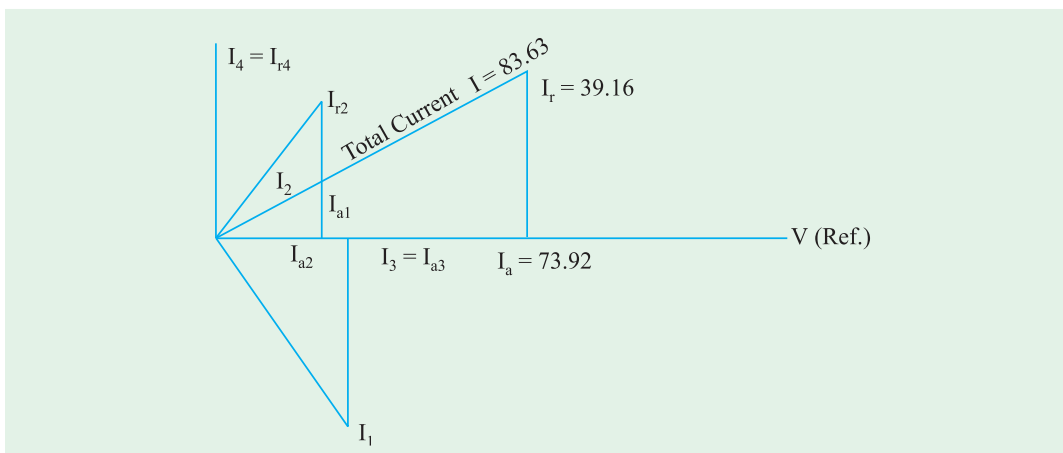
Performing the calculations as per the tabular entries above, following answers are obtained  
Total kW = 17

Total kVAR = + 9, leading

$$\text{Total } kVA = \sqrt{17^2 + 9^2} = 19.2354$$

$$\text{Overall circuit p.f.} = \frac{\text{kW}}{\text{kVA}} = \frac{17}{19.2354} = 0.884 \text{ leading}$$

$$\text{Overall Current} = \frac{19235.4}{230} = 83.63 \text{ amp}$$



**Fig. 14.45** Phasor diagram for currents corresponding to load

**Example 14.42.** A three phase induction Motor delivers an output of 15 h.p. at 83 % efficiency. The motor is  $\Delta$  (delta) connected and is supplied by 440 V, three phase, 50 Hz supply. Line current drawn by motor is 22.36 Amp. What is motor power factor ?

It is now decided to improve the power factor to 0.95 lag by connecting three similar capacitors in delta across the supply terminals. Determine the value of the capacitance of each capacitor.

[Note : 1 h.p. = 745 watts]

(Bombay University, 2000)

**Solution.** Power factor =  $\frac{15 \times 745}{0.83 \times 1.732 \times 440 \times 22.36} = 0.79$ , Lagging

$$\phi = \cos^{-1} 0.79 = 37.8^\circ$$

$$I_1 = I_{ph} = 22.36/1.732 = 12.91 \text{ amp}$$

Active Current  $I_a = I_1 \cos \phi_1 = 12.91 \times 0.79 = 10.2 \text{ amp}$

New Power-factor  $\cos \phi_2 = 0.95$ ,  $\phi_2 = 18.2^\circ$   
 $I_2 = 10.2/0.95 = 10.74$  amp  
 Capacitive current per phase  $= I_1 \sin \phi_1 - I_2 \sin \phi_2$   
 $= 4.563$   
 Capacitive reactance per phase  $= 440/4.563 = 96.43$  ohms  
 Capacitance per phase  $= 33 \mu\text{f}$   
 These have to be delta-connected

**Example 14.43.** Draw admittance triangle between the terminals AB of Fig 14.46 (a) labelling its sides with appropriate values and units in case of :

(i)  $X_L = 4$  and  $X_C = 8$     (ii)  $X_L = 10$  and  $X_C = 5$



Three phase induction motor

[Bombay University 1999]

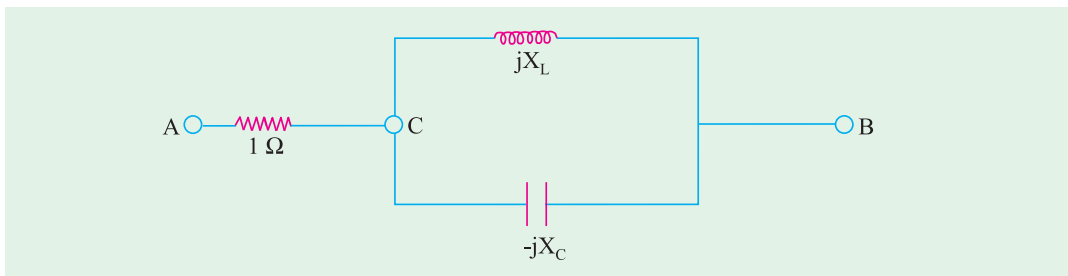


Fig. 14.46 (a)

**Solution. (i)**

$$X_L = 4 \Omega, X_C = 8 \Omega$$

$$Z_{CB} = \frac{jX_L (-jX_C)}{j(X_L - X_C)} = j8$$

$$Z_{AB} = 1 + j8 \text{ ohms}$$

$$Y_{AB} = 1/Z_{AB} = (1/65) - j(8/65) \text{ mho}$$

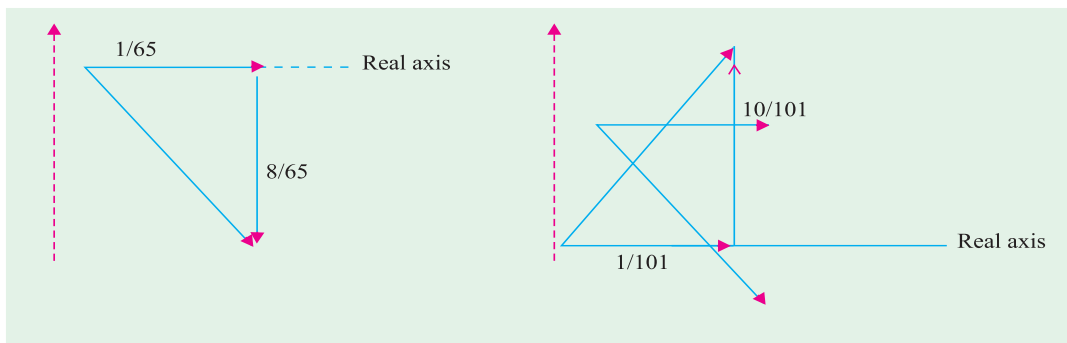
**(ii)**

$$X_L = 10 \Omega, X_C = 5 \Omega$$

$$Z_{CB} = \frac{j10 \times (-j5)}{j5} = -j10$$

$$Z_{AB} = 1 - j10 \text{ ohms}$$

$$Y_{AB} = (1/101) + j(10/101) \text{ mho}$$



(i) Admittance triangle for first case

Fig. 14.46 (b)

(ii) Admittance triangle for second case

Fig. 14.46 (c)

**Example 14.44.** For the circuit in Fig. 14.47 (a), given that  $L = 0.159 \text{ H}$

$$C = 0.3183 \text{ mf}$$

$$I_2 = 5 \angle 60^\circ \text{ A}$$

$$V_1 = 250 \angle 90^\circ \text{ volts.}$$

**Find :-**

- Impedance  $Z_1$  with its components.
- Source voltage in the form of  $V_m \cos(\omega t + \phi)$ .
- Impedance  $Z_2$  with its components so that source p.f. is unity, without adding to the circuit power loss.
- Power loss in the circuit
- Draw the phasor diagram.

(Bombay University 1997)

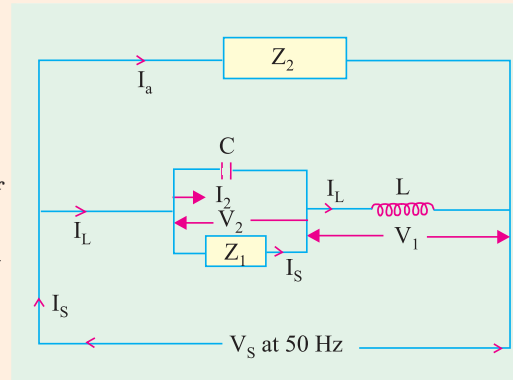


Fig. 14.47 (a)

**Solution.**  $X_L = 314 \times 0.159 = 50 \text{ ohms}$

$$X_C = 1/(314 \times 0.3183 \times 10^{-3}) = 10 \text{ ohms}$$

$$I_L = V_1/jX_L = (250 \angle 90^\circ) / (50 \angle 90^\circ) = 5 \angle 0^\circ \text{ amps}$$

$$V_2 = -jI_2 X_C = (5 \angle 60^\circ) \times (10 \angle -90^\circ) = 50 \angle -30^\circ \text{ volts} = 43.3 - j25 \text{ volts}$$

$$\begin{aligned} I_1 &= I_L - I_2 = 5 \angle 0^\circ - 5 \angle 60^\circ = 5 + j0 - 5(0.5 + j0.866) \\ &= 2.5 - j4.33 = 5 \angle -60^\circ \end{aligned}$$

$$\begin{aligned} \text{(a)} \quad Z_1 &= V_2/I_1 = (50 \angle -30^\circ) / 5 \angle -60^\circ = 10 \angle +30^\circ \\ &= 10(\cos 30^\circ + j \sin 30^\circ) = 8.66 + j5 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad V_s &= V_1 + V_2 = 0 + j250 + 43.3 - j25 = 43.3 + j225 \\ &= 229.1 \angle 79.1^\circ \text{ volts} \end{aligned}$$

$$V_s \text{ has a peak value of } (229.1 \times \sqrt{2}) = 324 \text{ volts}$$

$$V_s = 324 \cos(314t - 10.9^\circ), \text{ taking } V_1 \text{ as reference}$$

or  $V_s = 325 \cos(314t - 79.1^\circ), \text{ taking } I_L \text{ as reference.}$

**(c)** Source Current must be at unity P.f., with  $V_s$

$$\text{Component of } I_L \text{ in phase with } V_s = 5 \cos 79.1^\circ = 0.9455 \text{ amp}$$

$$\begin{aligned} \text{Component of } I_L \text{ in quadrature with } V_s \text{ (and is lagging by } 90^\circ) \\ = 5.00 \times \sin 79.1^\circ = 4.91 \text{ amp} \end{aligned}$$

$Z_2$  must carry  $I_a$  such that no power loss is there and  $I_s$  is at unity P.f. with  $V_s$ .

$I_a$  has to be capacitive, to compensate, in magnitude, the quadrature component of  $I_L$

$$|I_a| = 4.91 \text{ amp}$$

$$|Z_2| = V_s / |I_a| = 229.1/4.91 = 46.66 \text{ ohms}$$

$$\text{Corresponding capacitance, } C_2 = 1 / (46.66 \times 314) = 68.34 \mu\text{F}$$

**(d)** Power-loss in the circuit =  $I_1^2 \times 8.66 = 216.5 \text{ watts}$  or power =  $V_s \times \text{component of } I_L \text{ in phase with } V_s = 229.1 \times 0.9455 = 216.5 \text{ watts}$

**(e)** Phasor diagram is drawn in Fig. 14.47 (b)

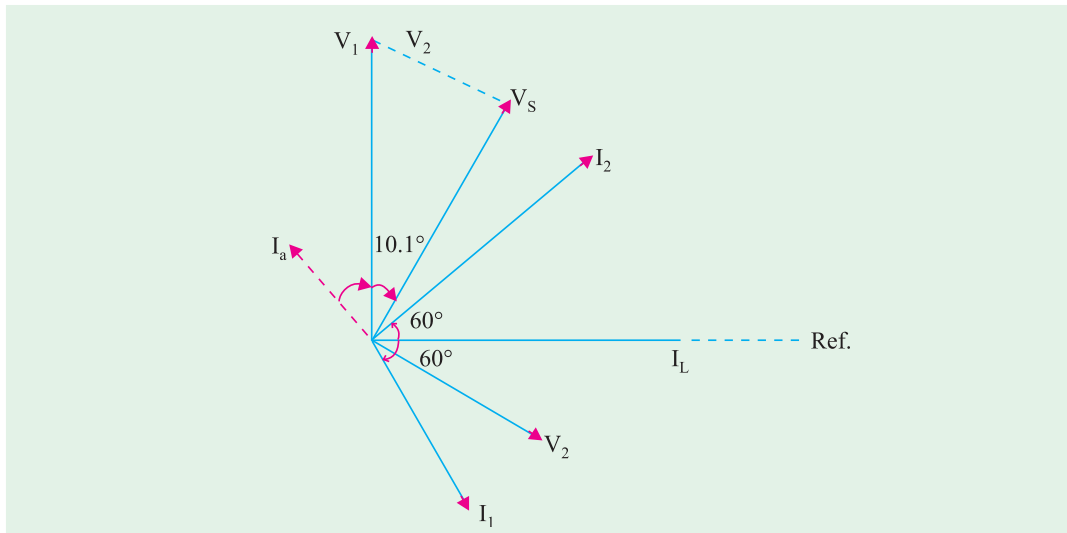


Fig. 14.47 (b) Phasor diagram

### Tutorial Problem No. 14.1

1. A capacitor of  $50 \mu\text{F}$  capacitance is connected in parallel with a reactor of  $22 \Omega$  resistance and  $0.07$  henry inductance across  $200\text{-V}$ ,  $50\text{-Hz}$  mains. Calculate the total current taken. Draw the vector diagram in explanation. **[4.76 A lagging,  $17^\circ 12'$ ] (City & Guilds, London)**
2. A non-inductive resistor is connected in series with a capacitor of  $100 \mu\text{F}$  capacitance across  $200\text{-V}$ ,  $50\text{-Hz}$  mains. The p.d. measured across the resistor is  $150 \text{ V}$ . Find the value of resistance and the value of current taken from the mains if the resistor were connected in parallel-with the capacitor instead of in series. **[R =  $36.1 \Omega$ ;  $8.37 \text{ A}$ ] (City & Guilds, London)**
3. An impedance of  $(10 + j15) \Omega$  is connected in parallel with an impedance of  $(6 - j8) \Omega$ . The total current is  $15 \text{ A}$ . Calculate the total power. **[2036 W] (City & Guilds, London)**
4. The load on a  $250\text{-V}$  supply system is :  $12 \text{ A}$  at  $0.8$  power factor lagging ;  $10 \text{ A}$  at  $0.5$  power factor lagging ;  $15 \text{ A}$  at unity power factor ;  $20 \text{ A}$  at  $0.6$  power factor leading. Find (i) the total load in kVA and (ii) its power factor. **[(i)  $10.4 \text{ kVA}$  (ii)  $1.0$ ] (City & Guilds, London)**
5. A voltage having frequency of  $50 \text{ Hz}$  and expressed by  $V = 200 + j100$  is applied to a circuit consisting of an impedance of  $50 \angle 30^\circ \Omega$  in parallel with a capacitance of  $10 \mu\text{F}$ . Find (a) the reading on a ammeter connected in the supply circuit (b) the phase difference between the current and the voltage. **[(a)  $4.52$  (b)  $26.6^\circ$  lag] (London University)**
6. A voltage of  $200 \angle 30^\circ \text{ V}$  is applied to two circuits  $A$  and  $B$  connected in parallel. The current in  $A$  is  $20 \angle 60^\circ \text{ A}$  and that in  $B$  is  $40 \angle 30^\circ \text{ A}$ . Find the kVA and kW in each branch circuit and the main circuit. Express the current in the main circuit in the form  $A + jB$ . **[kVA<sub>A</sub> = 4, kVA<sub>B</sub> = 8, kVA = 12, kW<sub>A</sub> = 3.46, kW<sub>B</sub> = 4, kW = 7.46, I =  $44.64 - j2.68$ ] (City & Guilds, London)**
7. A coil having an impedance of  $(8 + j6) \Omega$  is connected across a  $200\text{-V}$  supply. Express the current in the coil in (i) polar and (ii) rectangular co-ordinate forms. If a capacitor having a susceptance of  $0.1 \text{ S}$  is placed in parallel with the coil, find (iii) the magnitude of the current taken from the supply. **[(i)  $20 \angle 36.8^\circ \text{ A}$  (ii)  $16 - j12 \text{ A}$  (iii)  $17.9 \text{ A}$ ] (City & Guilds, London)**
8. A coil-A of inductance  $80 \text{ mH}$  and resistance  $120 \Omega$  is connected to a  $230\text{-V}$ ,  $50 \text{ Hz}$  single-phase supply. In parallel with it is a  $16 \mu\text{F}$  capacitor in series with a  $40 \Omega$  non-inductive resistor  $B$ . Determine (i) the power factor of the combined circuit and (ii) the total power taken from the supply. **[(i)  $0.945$  lead (ii)  $473 \text{ W}$ ] (London University)**
9. A choking coil of inductance  $0.08 \text{ H}$  and resistance  $12 \text{ ohm}$ , is connected in parallel with a capacitor

of  $120\ \mu\text{F}$ . The combination is connected to a supply at 240 V, 50 Hz. Determine the total current from the supply and its power factor. Illustrate your answers with a phasor diagram.

[3.94 A, 0.943 lag] (*London University*)

10. A choking coil having a resistance of  $20\ \Omega$  and an inductance of 0.07 henry is connected with a capacitor of  $60\ \mu\text{F}$  capacitance which is in series with a resistor of  $50\ \Omega$ . Calculate the total current and the phase angle when this arrangement is connected to 200-V, 50 Hz mains.

[7.15 A,  $24^\circ 39'$  lag] (*City & Guilds, London*)

11. A coil of resistance  $15\ \Omega$  and inductance 0.05 H is connected in parallel with a non-inductive resistance of  $20\ \Omega$ . Find (a) the current in each branch (b) the total current (c) the phase angle of whole arrangement for an applied voltage of 200 V at 50 Hz.

[9.22 A ; 10A ;  $22.1^\circ$ ]

12. A sinusoidal 50-Hz voltage of 200 V (r.m.s) supplies the following three circuits which are in parallel : (a) a coil of inductance 0.03 H and resistance  $3\ \Omega$  (b) a capacitor of  $400\ \mu\text{F}$  in series with a resistance of  $100\ \Omega$  (c) a coil of inductance 0.02 H and resistance  $7\ \Omega$  in series with a  $300\ \mu\text{F}$  capacitor. Find the total current supplied and draw a complete vector diagram.

[29.4 A] (*Sheffield Univ. U.K.*)

13. A 50-Hz, 250-V single-phase power line has the following loads placed across it in parallel : 4 kW at a p.f. of 0.8 lagging ; 6 kVA at a p.f. of 0.6 lagging; 5 kVA which includes 1.2 kVAR leading. Determine the overall p.f. of the system and the capacitance of the capacitor which, if connected across the mains would restore the power factor to unity.

[0.844 lag ; 336  $\mu\text{F}$ ]

14. Define the terms admittance, conductance and susceptance with reference to alternating current circuits. Calculate their respective values for a circuit consisting of resistance of  $20\ \Omega$ , in series with an inductance of 0.07 H when the frequency is 50 Hz.

[0.336 S, 0.0226 S, 0.0248 S]

(*City & Guilds, London*)

15. Explain the terms admittance, conductance, susceptance as applied to a.c. circuits. One branch A, of a parallel circuit consists of a coil, the resistance and inductance of which are  $30\ \Omega$  and 0.1 H respectively. The other branch B, consists of a  $100\ \mu\text{F}$  capacitor in series with a  $20\ \Omega$  resistor. If the combination is connected 240-V, Hz mains, calculate (i) the line current and (ii) the power. Draw to scale a vector diagram of the supply current and the branch-circuit currents.

[(i) 7.38 A (ii) 1740 W] (*City & Guilds, London*)

16. Find the value of capacitance which when placed in parallel with a coil of resistance  $22\ \Omega$  and inductance of 0.07 H, will make it resonate on a 50-Hz circuit.

[72.33  $\mu\text{F}$ ] (*City & Guilds, London*)

17. A parallel circuit has two branches. Branch A consists of a coil of inductance 0.2 H and a resistance of  $15\ \Omega$  ; branch B consists of a 30 mF capacitor in series with a  $10\ \Omega$  resistor. The circuit so formed is connected to a 230-V, 50-Hz supply. Calculate (a) current in each branch (b) line current and its power factor (c) the constants of the simplest series circuit which will take the same current at the same power factor as taken by the two branches in parallel.

[3.57 A, 2.16 A ; 1.67 A, 0.616 lag, 8.48  $\Omega$  0.345 H]

18. A 3.73 kW, 1-phase, 200-V motor runs at an efficiency of 75% with a power factor of 0.7 lagging. Find (a) the real input power (b) the kVA taken (c) the reactive power and (d) the current. With the aid of a vector diagram, calculate the capacitance required in parallel with the motor to improve the power factor to 0.9 lagging. The frequency is 50 Hz.

[4.97 kW ; 7.1 kVA ; 5.07 kVAR ; 35.5 A ; 212  $\mu\text{F}$ ]

19. The impedances of two parallel circuits can be represented by  $(20 + j15)$  and  $(1 - j60)\ \Omega$  respectively. If the supply frequency is 50 Hz, find the resistance and the inductance or capacitance of each circuit. Also derive a symbolic expression for the admittance of the combined circuit and then find the phase angle between the applied voltage and the resultant current. State whether this current is leading or lagging relatively to the voltage.

[20  $\Omega$  ; 0.0478 H ; 10  $\Omega$  ; 53  $\mu\text{F}$  ; (0.0347 - j 0.00778)S ;  $12^\circ 38'$  lag]

20. One branch A of a parallel circuit consists of a  $60\text{-}\mu\text{F}$  capacitor. The other branch B consists of a  $30\ \Omega$  resistor in series with a coil of inductance 0.2 H and negligible resistance. A  $140\ \Omega$  resistor is connected in parallel with the coil. Sketch the circuit diagram and calculate (i) the current in the  $30\ \Omega$  resistor and (ii) the line current if supply voltage is 230-V and the frequency 50 Hz.

[(i) 3.1  $\angle 44^\circ$  (ii) 3.1  $\angle 45^\circ$  A]

21. A coil having a resistance of  $45\ \Omega$  and an inductance of 0.4 H is connected in parallel with a capacitor having a capacitance of  $20\ \mu\text{F}$  across a 230-V, 50-Hz system. Calculate (a) the current taken from the

- supply (b) the power factor of the combination and (c) the total energy absorbed in 3 hours.  
**[(a) 0.615 (b) 0.951 (c) 0.402 kWh] (London University)**
22. A series circuit consists of a resistance of  $10\ \Omega$  and reactance of  $5\ \Omega$ . Find the equivalent value of conductance and susceptance in parallel.  
**[0.08 S, 0.04 S]**
23. An alternating current passes through a non-inductive resistance  $R$  and an inductance  $L$  in series. Find the value of the non-inductive resistance which can be shunted across the inductance without altering the value of the main current.  
 **$[\omega^2 L^2 / 2R]$  (Elec. Meas. London Univ.)**
24. A p.d. of 200 V at 50 Hz is maintained across the terminals of a series-parallel circuit, of which the series branch consists of an inductor having an inductance of 0.15 H and a resistance of  $30\ \Omega$ , one parallel branch consists of  $100\text{-}\mu\text{F}$  capacitor and the other consists of a  $40\text{-}\Omega$  resistor. Calculate (a) the current taken by the capacitor (b) the p.d. across the inductor and (c) the phase difference of each of these quantities relative to the supply voltage. Draw a vector diagram representing the various voltage and currents.  
**[(a) 29.5 A (b) 210 V (c) 7.25°, 26.25°] (City & Guilds, London)**
25. A coil (A) having an inductance of 0.2 H and resistance of  $3.5\ \Omega$  is connected in parallel with another coil (B) having an inductance of 0.01 H and a resistance of  $5\ \Omega$ . Calculate (i) the current and (ii) the power which these coils would take from a 100-V supply system having a frequency of 50-Hz. Calculate also (iii) the resistance and (iv) the inductance of a single coil which would take the same current and power.  
**[(i) 29.9 A (ii) 2116 W (iii) 2.365  $\Omega$  (iv) 0.00752 H] (London Univ.)**
26. Two coils, one (A) having  $R = 5\ \Omega$ ,  $L = 0.031\ \text{H}$  and the other (B) having  $R = 7\ \Omega$ ;  $L = 0.023\ \text{H}$ , are connected in parallel to an a.c. supply at 200 V, 50 Hz. Determine (i) the current taken by each coil and also (ii) the resistance and (iii) the inductance of a single coil which will take the same total current at the same power factor as the two coils in parallel.  
**[(i)  $I_A = 18.28\ \text{A}$ ,  $I_B = 19.9\ \text{A}$  (ii) 3.12  $\Omega$  (iii) 0.0137 H] (London Univ.)**
27. Two coils are connected in parallel across 200-V, 50-Hz mains. One coil takes 0.8 kW and 1.5 kVA and the other coil takes 1.0 kW and 0.6 kVAR. Calculate (i) the resistance and (ii) the reactance of a single coil which would take the same current and power as the original circuit.  
**[(i) 10.65  $\Omega$  (ii) 11.08  $\Omega$ ] (City & Guilds, London)**
28. An a.c. circuit consists of two parallel branches, one (A) consisting of a coil, for which  $R = 20\ \Omega$  and  $L = 0.1\ \text{H}$  and the other (B) consisting of a  $40\text{-}\Omega$  non-inductive resistor in series with  $60\text{-}\mu\text{F}$  capacitor. Calculate (i) the current in each branch (ii) the line current (iii) the power, when the circuit is connected to 230-V mains having a frequency of 50 Hz. Calculate also (iv) the resistance and (b) the inductance of a single coil which will take the same current and power from the supply.  
**[(i) 6.15 A, 3.46 A (ii) 5.89 (iii) 1235 W (iv) 35.7  $\Omega$  (b) 0.0509 H] (London Univ.)**
29. One branch (A) of a parallel circuit, connected to 230-V, 50-Hz mains consists of an inductive coil ( $L = 0.15\ \text{H}$ ,  $R = 40\ \Omega$ ) and the other branch (B) consists of a capacitor ( $C = 50\ \mu\text{F}$ ) in series with a  $45\ \Omega$  resistor. Determine (i) the power taken (ii) the resistance and (iii) the reactance of the equivalent series circuit.  
**[(i) 946 W (ii) 55.4  $\Omega$  (iii) 4.6  $\Omega$ ] (London Univ.)**

### 14.9. Resonance in Parallel Circuits

We will consider the practical case of a coil in parallel with a capacitor, as shown in Fig. 14.48. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as **resonant** frequency.

The vector diagram for this circuit is shown in Fig. 14.48 (b).

$$\begin{aligned} \text{Net reactive or wattless component} \\ = I_C - I_L \sin \phi_L \end{aligned}$$

As at resonance, its value is zero, hence

$$I_C - I_L \sin \phi_L = 0 \quad \text{or} \quad I_L \sin \phi_L = I_C$$

$$\text{Now, } I_L = V/Z; \sin \phi_L = X_L \text{ and } I_C = V/X_C$$

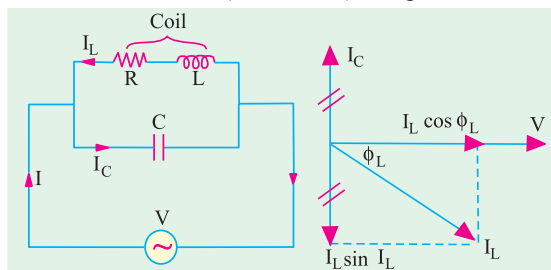


Fig. 14.48



Hence, condition for resonance becomes

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C} \quad \text{or} \quad X_L \times X_C = Z^2$$

Now,  $X_L = \omega L$ ,  $X_C = \frac{1}{\omega C}$

$$\therefore \frac{\omega L}{\omega C} = Z^2 \quad \text{or} \quad \frac{L}{C} = Z^2 \quad \dots (i)$$

or  $\frac{L}{C} = R^2 + X_L^2 = R^2 + (2\pi f_0 L)^2$

or  $(2\pi f_0 L)^2 = \frac{L}{C} - R^2 \quad \text{or} \quad 2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{or} \quad f_0 = \frac{1}{2} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

This is the resonant frequency and is given in Hz,  $R$  is in ohm,  $L$  is the henry and  $C$  is the farad.

If  $R$  is the negligible, then  $f_0 = \frac{1}{2\pi\sqrt{LC}}$

... same as for series resonance

### Current at Resonance

As shown in Fig. 14.41 (b), since wattless component of the current is zero, the circuit current is

$$I = I_L \cos \phi_L = \frac{V}{Z} \cdot \frac{R}{Z} \quad \text{or} \quad I = \frac{VR}{Z^2}$$

Putting the value of  $Z^2 = L/C$  from (i) above, we get  $I = \frac{VR}{L/C} = \frac{V}{L/CR}$

The denominator  $L/CR$  is known as the **equivalent** or **dynamic impedance** of the parallel circuit at resonance. It should be noted that impedance is ‘resistive’ only. Since current is minimum at resonance,  $L/CR$  must, therefore, represent the maximum impedance of the circuit. In fact, parallel resonance is a condition of maximum impedance or minimum admittance.

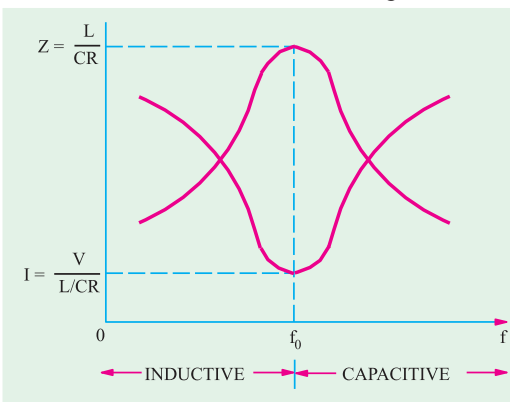


Fig. 14.49

Current at resonance is minimum, hence such a circuit (when used in radio work) is sometimes known as **rejector** circuit because it rejects (or takes minimum current of) that frequency to which it resonates. This resonance is often referred to as current resonance also because the current circulating **between** the two branches is many times greater than the line current taken from the supply.

The phenomenon of parallel resonance is of great practical importance because it forms the basis of tuned circuits in Electronics.

The variations of impedance and current with frequency are shown in Fig. 14.49. As seen, at resonant frequency, impedance is maximum and equals  $L/CR$ . Consequently, current at resonance is minimum and is  $V/(L/CR)$ . At off-resonance frequencies, impedance decreases and, as a result, current increases as shown.

### Alternative Treatment

$$Y_1 = \frac{1}{R + jX_L} = \frac{R}{R^2 + X_L^2} - j \frac{X_L}{R^2 + X_L^2}; \quad Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y = \frac{R}{R^2 + X_L^2} + j \left( \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right)$$

Now, circuit would be in resonance when  $j$ -component of the complex admittance is zero *i.e.*

$$\text{when } \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0 \text{ or } \frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$\text{or } X_L X_C = R^2 + X_L^2 = Z^2$$

–as before

Talking in terms of susceptance, the above relations can be put as under :

$$\text{Inductive susceptance } B_L = \frac{X_L}{R^2 + X_L^2}; \text{ capacitive susceptance } B_C = \frac{1}{X_C}$$

$$\text{Net susceptance } B = (B_C - B_L) \quad \therefore Y = G + j(B_C - B_L) = G + jB.$$

The parallel circuit is said to be in resonance when  $B = 0$ .

$$\therefore B_C - B_L = 0 \text{ or } \frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$$

The rest procedure is the same as above. It may be noted that at resonance, the admittance equals the conductance.

### 14.10. Graphic Representation of Parallel Resonance

We will now discuss the effect of variation of frequency on the susceptance of the two parallel branches. The variations are shown in Fig. 14.50.

**(i) Inductive susceptance ;**

$$b = -1/X_L = -1/2\pi f L$$

It is inversely proportional to the frequency of the applied voltage. Hence, it is represented by a rectangular hyperbola drawn in the fourth quadrant ( $\therefore$  it is assumed negative).

**(ii) Capacitive susceptance ;**

$$b = 1/X_C = \omega C = 2\pi f C$$

It increases with increase in the frequency of the applied voltage. Hence, it is represented by a straight line drawn in the first quadrant (it is assumed positive).

**(iii) Net Susceptance B**

It is the difference of the two susceptances and is represented by the dotted hyperbola. At point A, net susceptance is zero, hence admittance is minimum (and equal to  $G$ ). So at point A, line current is minimum.

Obviously, below resonant frequency (corresponding to point A), inductive susceptance predominates, hence line current lags behind the applied voltage. But for frequencies above the resonant frequency, capacitive susceptance predominates, hence line current leads.

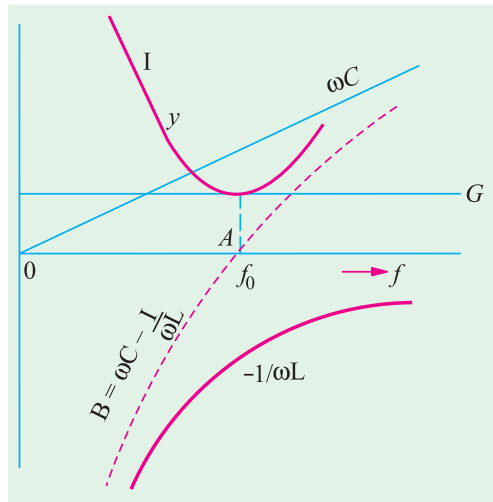


Fig. 14.50

### 14.11. Points to Remember

Following points about parallel resonance should be noted and compared with those about series resonance. At resonance.

1. net susceptance is zero *i.e.*  $1/X_C = X_L/Z^2$  or  $X_L \times X_C = Z^2$  or  $L/C = Z^2$
2. the admittance equals conductance
3. reactive or wattless component of line current is zero.
4. dynamic impedance =  $L/CR$  ohm.

5. line current at resonance is minimum and  $= \frac{V}{L/CR}$  but is in phase with the applied voltage.
6. power factor of the circuit is unity.

### 14.12. Bandwidth of a Parallel Resonant Circuit

The bandwidth of a parallel circuit is defined in the same way as that for a series circuit. This circuit also has upper and lower half-power frequencies where power dissipated is half of that at resonant frequency.

At bandwidth frequencies, the net susceptance  $B$  equals the conductance. Hence, at  $f_2$ ,  $B = B_{C2} - B_{L2} = G$ . At  $f_1$ ,  $B = B_{L1} - B_{C1} = G$ . Hence,  $Y = \sqrt{G^2 + B^2} = \sqrt{2}G$  and  $\phi = \tan^{-1}(B/G) = \tan^{-1}(1) = 45^\circ$ .

However, at off-resonance frequencies,  $Y > G$  and  $B_C \neq B_L$  and the phase angle is greater than zero.

#### Comparison of Series and Parallel Resonant Circuits

item	series circuit (R-L-C)	parallel circuit (R-L and C)
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum = $V/R$	Minimum = $V/(L/CR)$
Effective impedance	$R$	$L/CR$
Power factor at resonance	Unity	Unity
Resonant frequency	$1/2\pi\sqrt{LC}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$
It magnifies	Voltage	Current
Magnification is	$\omega L/R$	$\omega L/R$

### 14.13. Q-factor of a Parallel Circuit

It is defined as **the ratio of the current circulating between its two branches to the line current drawn from the supply or simply, as the current magnification**. As seen from Fig. 14.51, the circulating current between capacitor and coil branches is  $I_C$ .

Hence  $Q$ -factor =  $I_C/I$

Now  $I_C = V/X_C = V/(1/\omega C) = \omega CV$

and  $I_C = V/(L/CR)$

$$\therefore Q\text{-factor} = CV \frac{V}{L/CR} \frac{L}{R} \frac{2f_0 L}{R}$$

$$= \tan \phi \text{ (same as for series circuit)}$$

where  $\phi$  is the power factor angle of the coil.

Now, resonant frequency when  $R$  is negligible is,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Putting this value above, we get,  $Q\text{-factor} = \frac{2}{R} \frac{f_0 L}{2\sqrt{LC}} \frac{1}{R} \sqrt{\frac{L}{C}}$

It should be noted that in series circuits,  $Q$ -factor gives the voltage magnification, whereas in parallel circuits, it gives the current magnification.

Again,  $Q = 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated/cycle}}$

**Example 14.45.** A capacitor is connected in parallel with a coil having  $L = 5.52 \text{ mH}$  and  $R = 10 \Omega$ , to a 100-V, 50-Hz supply. Calculate the value of the capacitance for which the current taken from the supply is in phase with voltage. (Elect. Machines, A.M.I.E. Sec B, 1992)

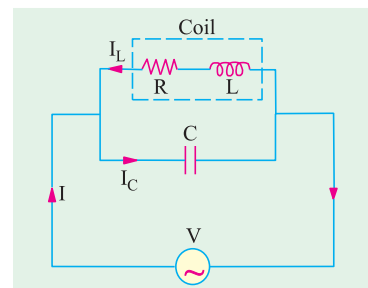


Fig. 14.51

**Solution.** At resonance,  $L/C = Z^2$  or  $C = L/Z^2$   
 $X_L = 2\pi \times 50 \times 5.52 \times 10^{-3} = 1.734 \Omega$ ,  $Z^2 = 10^2 + 1.734^2$ ,  $Z = 10.1 \Omega$   
 $C = 5.52 \times 10^{-3}/10.1 = 54.6 \mu\text{F}$

**Example 14.46.** Calculate the impedance of the parallel-tuned circuit as shown in Fig. 14.52 at a frequency of 500 kHz and for bandwidth of operation equal to 20 kHz. The resistance of the coil is  $5 \Omega$   
**(Circuit and Field Theory, A.M.I.E. Sec. B, 1993)**

**Solution.** At resonance, circuit impedance is  $L/CR$ . We have been given the value of  $R$  but that of  $L$  and  $C$  has to be found from the given data.

$$BW = \frac{R}{2L}, 20 \times 10^3 = \frac{5}{2L} \quad \text{or} \quad L = 39 \mu\text{H}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi} \sqrt{\frac{1}{39 \times 10^{-6} C} - \frac{5^2}{(39 \times 10^{-6})^2}}$$

$$\therefore C = 2.6 \times 10^{-9} \text{ F}, Z = L/CR = 39 \times 10^{-6}/2.6 \times 10^{-9} \times 5 = 3 \times 10^3 \Omega$$

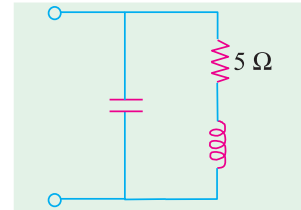


Fig. 14.52

**Example 14.47.** An inductive circuit of resistance 2 ohm and inductance 0.01 H is connected to a 250-V, 50-Hz supply. What capacitance placed in parallel will produce resonance?  
 Find the total current taken from the supply and the current in the branch circuits.  
**(Elect. Engineering, Kerala Univ.)**

**Solution.** As seen from Art. 14.9, at resonance  $C = L/Z^2$

$$\text{Now, } R = 2 \Omega, X_L = 314 \times 0.01 = 3.14 \Omega; Z = \sqrt{2^2 + 3.14^2} = 3.74 \Omega$$

$$C = 0.01/3.74^2 = 714 \times 10^{-6} \text{ F} = 714 \mu\text{F}; I_{RL} = 250/3.74 = 66.83 \text{ A}$$

$$\tan \phi_L = 3.14/2 = 1.57; \phi_L = \tan^{-1}(1.57) = 57.5^\circ$$

Hence, current in  $R$ - $L$  branch lags the applied voltage by  $57.5^\circ$

$$\therefore I_C = \frac{V}{X_C} = \frac{V}{1/\omega C} = \omega VC = 250 \times 314 \times 714 \times 10^{-6} = 56.1 \text{ A}$$

This current leads the applied voltage by  $90^\circ$ .

Total current taken from the supply under resonant condition is

$$I = I_{RL} \cos \phi_L = 66.83 \cos 57.5^\circ = 66.83 \times 0.5373 = 35.9 \text{ A} \quad \left(\text{or } I = \frac{V}{L/CR}\right)$$

**Example 14.48.** Find active and reactive components of the current taken by a series circuit consisting of a coil of inductance 0.1 henry and resistance  $8 \Omega$  and a capacitor of  $120 \mu\text{F}$  connected to a 240-V, 50-Hz supply mains. Find the value of the capacitor that has to be connected in parallel with the above series circuit so that the p.f. of the entire circuit is unity.  
**(Elect. Technology, Mysore Univ.)**

**Solution.**  $X_L = 2\pi \times 50 \times 0.1 = 31.4 \Omega$ ,  $X_C = 1/\omega C = 1/2\pi \times 50 \times 120 \times 10^{-6} = 26.5 \Omega$

$$X = X_L - X_C = 31.4 - 26.5 = 5 \Omega, Z = \sqrt{8^2 + 5^2} = 9.43 \Omega; I = V/Z = 240/9.43 = 25.45 \text{ A}$$

$$\cos \phi = R/Z = 8/9.43 = 0.848, \sin \phi = X/Z = 5/9.43 = 0.53$$

$$\text{active component of current} = I \cos \phi = 25.45 \times 0.848 = 21.58 \text{ A}$$

$$\text{reactive component of current} = I \sin \phi = 25.45 \times 0.53 = 13.49 \text{ A}$$

Let a capacitor of capacitance  $C$  be joined in parallel across the circuit.

$$Z_1 = R + jX = 8 + j5; Z_2 = -jX_C;$$

$$Y = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{8 + j5} + \frac{1}{-jX_C}$$

$$= \frac{8 - j5}{89} + \frac{j}{X_C} = 0.0899 - j0.056 + \frac{j}{X_C} = 0.0899 + j(1/X_C - 0.056)$$

For p.f. to be unit, the  $j$ -component of  $\mathbf{Y}$  must be zero.

$$\therefore \frac{1}{X_C} - 0.056 = 0 \text{ or } 1/X_C = 0.056 \text{ or } \omega C = 0.056 \text{ or } 2\pi \times 50 C = 0.056$$

$$\therefore C = 0.056/100\pi = 180 \times 10^{-6} \text{ F} = \mathbf{180 \mu F}$$

**Example 14.49.** A coil of resistance  $20 \Omega$  and inductance  $200 \mu\text{H}$  is in parallel with a variable capacitor. This combination is in series with a resistor of  $8000 \Omega$ . The voltage of the supply is  $200 \text{ V}$  at a frequency of  $10^6 \text{ Hz}$ . Calculate

- (i) the value of  $C$  to give resonance
- (ii) the  $Q$  of the coil
- (iii) the current in each branch of the circuit at resonance.

(Similar Question : Bombay Univ. 2000)

**Solution.** The circuit is shown in Fig. 14.53.

$$X_L = 2\pi fL = 2\pi \times 10^6 \times 200 \times 10^{-6} = 1256 \Omega$$

Since coil resistance is negligible as compared to its reactance, the resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore 10^6 = \frac{1}{2\pi\sqrt{200 \times 10^{-6} \times C}}$$

$$(i) \therefore C = \mathbf{125 \mu F}$$

$$(ii) Q = \frac{2\pi fL}{R} = \frac{2\pi \times 10^6 \times 200 \times 10^{-4}}{20} = \mathbf{62.8}$$

$$(iii) \text{ Dynamic resistance of the circuit is } = \frac{L}{CR} = \frac{200 \times 10^{-6}}{125 \times 10^{-12} \times 20} = 80,000 \Omega$$

Total equivalent resistance of the tuned circuit is  $80,000 + 8,000 = 88,000 \Omega$

$$\therefore \text{Current } I = 200/88,000 = 2.27 \text{ mA}$$

p.d. across tuned circuit = current  $\times$  dynamic resistance =  $2.27 \times 10^{-3} \times 80,000 = 181.6 \text{ V}$

$$\text{Current through inductive branch} = \frac{181.6}{\sqrt{10^2 + 1256^2}} = 0.1445 \text{ A} = \mathbf{144.5 \text{ mA}}$$

Current through capacitor branch

$$= \frac{V}{1/\omega C} = \omega VC = 181.6 \times 2\pi \times 10^6 \times 125 \times 10^{-12} = \mathbf{142.7 \text{ mA}}$$

**Note.** It may be noted in passing that current in each branch is nearly 62.8 (i.e. Q-factor) times the resultant current taken from the supply.

**Example 14.50.** Impedances  $Z_2$  and  $Z_3$  in parallel are in series with an impedance  $Z_1$  across a  $100\text{-V}$ ,  $50\text{-Hz}$  a.c. supply.  $Z_1 = (6.25 + j 1.25) \text{ ohm}$ ;  $Z_2 = (5 + j0) \text{ ohm}$  and  $Z_3 = (5 - j X_C) \text{ ohm}$ . Determine the value of capacitance of  $X_C$  such that the total current of the circuit will be in phase with the total voltage. When is then the circuit current and power?

(Elect. Engg-I, Nagpur Univ, 1992)

$$\text{Solution. } Z_{23} = \frac{5(5 - jX_C)}{(10 - jX_C)}, \text{ for the circuit in}$$

Fig. 14.59.

$$= \frac{25 - j5X_C}{(10 - jX_C)} \times \frac{10 + jX_C}{10 + jX_C} = \frac{250 + 5X_C^2}{100 + X_C^2} - j \frac{25X_C}{100 + X_C^2}$$

$$\mathbf{Z} = 6.25 + j1.25 + \frac{250 + 5X_C^2}{100 + X_C^2} - j \frac{25X_C}{100 + X_C^2}$$

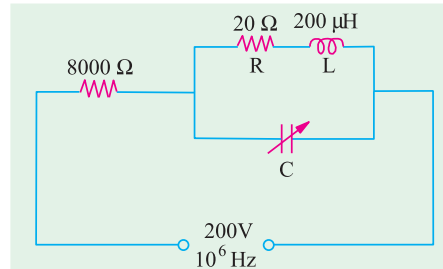


Fig. 14.53

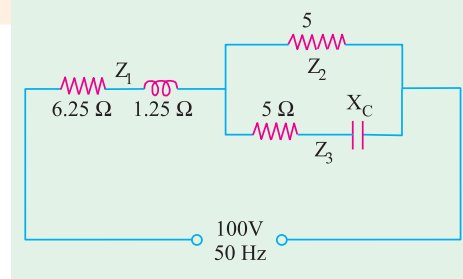


Fig. 14.54

$$= 6.25 \frac{250}{100} \frac{5X_C^2}{X_C^2} + j \frac{25X_C}{100} \frac{5}{X_C^2}$$

Power factor will be unity or circuit current will be in phase with circuit voltage if the  $j$  term in the above equation is zero.

$$\therefore \left( \frac{25X_C}{100 + X_C^2} - \frac{5}{4} \right) = 0 \text{ or } X_C = 10 \quad \therefore 1/\omega C = 10 \text{ or } C = 1/314 \times 10 = \mathbf{318 \mu F}$$

Substituting the value of  $X_C = 10 \Omega$  above, we get

$$Z = 10 - j0 = 10 \angle 0^\circ \text{ and } I = 100/10 = \mathbf{10 A}; \text{ Power} = I^2 R = 10^2 \times 10 = \mathbf{1000 W}$$

**Example 14.51.** In the circuit given below, if the value of  $R = \sqrt{L/C}$ , then prove that the impedance of the entire circuit is equal to  $R$  only and is independent of the frequency of supply. Find the value of impedance for  $L = 0.02 \text{ H}$  and  $C = 100 \mu\text{F}$ .

(Communication System, Hyderabad Univ. 1991)

**Solution.** The impedance of the circuit of Fig. 14.55 is

$$Z = \frac{(R + j\omega L)(R - j/\omega C)}{2R + j(\omega L - 1/\omega C)} = \frac{R^2 + (L/C) + jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}$$

If  $R^2 = L/C$  or  $R = \sqrt{L/C}$ , then

$$Z = \frac{R^2 + R^2 + jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)} \\ = R \left[ \frac{2R + j(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)} \right] \text{ or } \mathbf{Z = R}$$

$$\text{Now, } R = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.02}{100 \times 10^{-6}}} = \mathbf{14.14 \Omega}$$

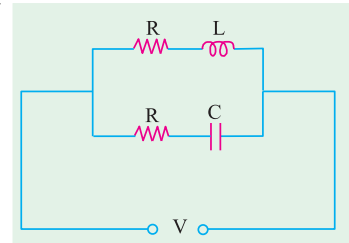


Fig. 14.55

**Example 14.52.** Derive an expression for the resonant frequency of the parallel circuit shown in Fig. 14.46.

(Electrical Circuit, Nagpur Univ, 1993)

**Solution.** As stated in Art. 14.9 for resonance of a parallel circuit, total circuit susceptance should be zero. Susceptance of the  $R$ - $L$  branch is

$$B_1 = -\frac{X_L}{R_1^2 + X_L^2}$$

Similarly, susceptance of the  $R$ - $C$  branch is

$$B_2 = \frac{X_C}{R_2^2 + X_C^2}$$

Net susceptance is  $B = -B_1 + B_2$

For resonance  $B = 0$  or  $0 = -B_1 + B_2 \therefore B_1 = B_2$

$$\text{or } \frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2} \text{ or } X_L(R_2^2 + X_C^2) = X_C(R_1^2 + X_L^2)$$

$$2 f L R_2^2 \frac{1}{2 f C} = \frac{1}{2 f C} [R_1^2 (2 f L)^2]; 4 f^2 L C R_2^2 = \frac{1}{2 f C} [R_1^2 (2 f L)^2]$$

$$\therefore 4\pi^2 f^2 L C R_2^2 + \frac{L}{C} = R_1^2 + 4\pi^2 f^2 L^2; 4\pi^2 f^2 [L(L - C R_2^2)] = \frac{L}{C} - R_1^2$$

$$\therefore f_0 = \frac{1}{2\pi} \sqrt{\left( \frac{L/C - R_1^2}{L(L - C R_2^2)} \right)} \quad \therefore f_0 = \frac{1}{2\pi} \sqrt{\left( \frac{L - C R_1^2}{LC(L - C R_2^2)} \right)}; \omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\left( \frac{L - C R_1^2}{L - C R_2^2} \right)}$$

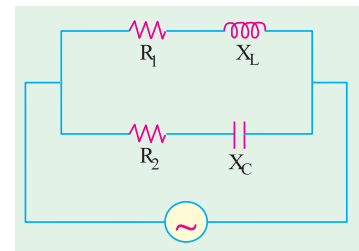


Fig. 14.56

**Note.** If both  $R_1$  and  $R_2$  are negligible, then  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  —as in Art. 14.9

**Example 14.53.** Calculate the resonant frequency of the network shown in Fig. 14.57.

**Solution.** Total impedance of the network between terminals A and B is

$$\begin{aligned} Z_{AB} &= (R_1 \parallel jX_L) + [R_2 \parallel (-jX_C)] = \frac{jR_1X_L}{R_1 + jX_L} + \frac{R_2(-jX_C)}{R_2 - jX_C} = \frac{jR_1\omega L}{R_1 + j\omega L} - \frac{jR_2/\omega C}{R_2 - j/\omega C} \\ &= \frac{R_1\omega^2 L^2}{R_1^2 + \omega^2 L^2} + \frac{R_2}{\omega C(R_2^2 + 1/\omega^2 C^2)} + j \left[ \frac{R_1^2 \omega L}{R_1^2 + \omega^2 L^2} - \frac{R_2^2}{\omega C(R_2^2 + 1/\omega^2 C^2)} \right] \end{aligned}$$

At resonance,  $\omega = \omega_0$  and the  $j$  term of  $Z_{AB}$  is zero.

$$\therefore \frac{R_1^2 \omega_0 L}{R_1^2 + \omega_0^2 L^2} - \frac{R_2^2}{\omega_0 C(R_2^2 + 1/\omega_0^2 C^2)} = 0$$

$$\text{or } \frac{R_1^2 \omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{R_2^2 \omega_0 C}{R_2^2 \omega_0^2 C^2 + 1}$$

Simplifying the above, we get

$$\omega_0^2 = \frac{G_2^2 - C/L}{LC(G_1^2 - C/L)} \text{ where } G_1 = \frac{1}{R_1} \text{ and } G_2 = \frac{1}{R_2}$$

The resonant frequency of the given network in Hz is

$$f_0 = \frac{0}{2} \frac{1}{2} \sqrt{\frac{G_2^2 - C/L}{LC(G_1^2 - C/L)}}$$

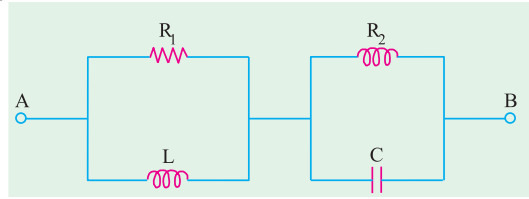


Fig. 14.57

**Example 14.54.** Compute the value of  $C$  which results in resonance for the circuit shown in Fig. 14.58 when  $f = 2500\pi$  Hz.

**Solution.**  $Y_1 = 1/(6 + j8)$

$Y_2 = 1/(4 - jX_C)$

$$\begin{aligned} Y &= Y_1 + Y_2 = \frac{1}{6 + j8} + \frac{1}{4 - jX_C} \\ &= \left( 0.06 + \frac{4}{16 + X_C^2} \right) + j \left( \frac{X_C}{16 + X_C^2} - 0.08 \right) \end{aligned}$$

For resonance,  $j$  part of admittance is zero, i.e. the complex admittance is real number.

$$\therefore X_C/(16 + X_C^2) - 0.08 = 0 \text{ or } 0.08 X_C^2 - X_C + 1.28 = 0$$

$$\therefore X_C = 11.05 \text{ or } 1.45 \therefore 1/\omega C = 11.05 \text{ or } 1.45$$

(i)  $1/5000C = 11.05$  or  $C = 18 \mu\text{F}$  (ii)  $1/5000C = 1.45$  or  $C = 138 \mu\text{F}$

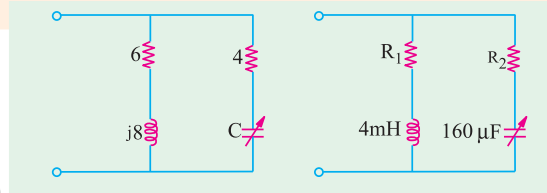


Fig. 14.58

Fig. 14.59

**Example 14.55.** Find the values of  $R_1$  and  $R_2$  which will make the circuit of Fig. 14.59 resonate at all frequencies.

**Solution.** As seen from Example 14.42, the resonant frequency of the given circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\left( \frac{L - CR_1^2}{L - CR_2^2} \right)}$$

Now,  $\omega_0$  can assume any value provided  $R_1^2 = R_2^2 = L/C$ .

In the present case,  $L/C = 4 \times 10^{-3}/60 \times 10^{-6} = 25$ . Hence,  $R_1 = R_2 = \sqrt{25} = 5 \text{ ohm}$ .

## Tutorial Problem No. 14.2

- A resistance of 20  $\Omega$  and a coil of inductance 31.8 mH and negligible resistance are connected in parallel across 230 V, 50 Hz supply. Find (i) The line current (ii) power factor and (iii) The power consumed by the circuit. **[(i) 25.73 A (ii) 0.44 T lag (iii) 246 W] (F. E. Pune Univ.)**
- Two impedances  $Z_1 = (150 + j157) \Omega$  and  $Z_2 = (100 + j110) \Omega$  are connected in parallel across a 220-V, 50-Hz supply. Find the total current and its power factor. **[24  $\angle -47^\circ$  A ; 0.68 (lag)] (Elect. Engg. & Electronics Bangalore Univ.)**
- Two impedances  $(14 + j5) \Omega$  and  $(18 + j10) \Omega$  are connected in parallel across a 200-V, 50-Hz supply. Determine (a) the admittance of each branch and of the entire circuit ; (b) the total current, power, and power factor and (c) the capacitance which when connected in parallel with the original circuit will make the resultant power factor unity. **[(a) (0.0634 -j0.0226), (0.0424 -j0.023) (0.1058 -j0.0462 S) (b) 23.1 A, 4.232 kW, 0.915 (c) 147  $\mu$ F]**
- A parallel circuit consists of two branches A and B. Branch A has a resistance of 10  $\Omega$  and an inductance of 0.1 H in series. Branch B has a resistance of 20  $\Omega$  and a capacitance of 100  $\mu$ F in series. The circuit is connected to a single-phase supply of 250 V, 50 Hz. Calculate the magnitude and the phase angle of the current taken from the supply. Verify your answer by measurement from a phasor diagram drawn to scale. **[6.05  $\angle -15.2^\circ$ ] (F. E. Pune Univ.)**
- Two circuits, the impedances of which are given by  $Z_1 = (10 + j15) \Omega$  and  $Z_2 = (6 - j8) \Omega$  are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch ? **[737 W ; 1430 W] (Elect. Engg. A.M.A.E. S.I.)**
- A voltage of 240 V is applied to a pure resistor, a pure capacitor, and an inductor in parallel. The resultant current is 2.3 A, while the component currents are 1.5, 2.0 and 1.1 A respectively. Find the resultant power factor and the power factor of the inductor. **[0.88 ; 0.5]**
- Two parallel circuits comprise respectively (i) a coil of resistance 20  $\Omega$  and inductance 0.07 H and (ii) a capacitance of 60  $\mu$ F in series with a resistance of 50  $\Omega$ . Calculate the current in the mains and the power factor of the arrangement when connected across a 200-V, 50-Hz supply. **[7.05 A ; 0.907 lag] (Elect. Engg. & Electronics, Bangalore Univ.)**
- Two circuits having the same numerical ohmic impedances are joined in parallel. The power factor of one circuit is 0.8 lag and that of other 0.6 lag. Find the power factor of the whole circuit. **[0.707] (Elect. Engg. Pune Univ.)**
- How is a current of 10 A shared by three circuits in parallel, the impedances of which are  $(2 - j5) \Omega$ ,  $(6 + j3) \Omega$  and  $(3 + j4) \Omega$  **[5.68 A ; 4.57 A, 6.12 A]**
- A piece of equipment consumes 2,000 W when supplied with 110 V and takes a lagging current of 25 A. Determine the equivalent series resistance and reactance of the equipment. If a capacitor is connected in parallel with the equipment to make the power factor unity, find its capacitance. The supply frequency is 100 Hz. **[3.2  $\Omega$ , 3.02  $\Omega$ , 248  $\mu$ F] (Sheffield Univ. U.K.)**
- A capacitor is placed in parallel with two inductive loads, one of 20 A at  $30^\circ$  lag and one of 40 $^\circ$  A at  $60^\circ$  lag. What must be current in the capacitor so that the current from the external circuit shall be at unity power factor ? **[44.5 A] (City & Guilds, London)**
- An air-cored choking coil is subjected to an alternating voltage of 100 V. The current taken is 0.1 A and the power factor 0.2 when the frequency is 50 Hz. Find the capacitance which, if placed in parallel with the coil, will cause the main current to be a minimum. What will be the impedance of this parallel combination (a) for currents of frequency 50 (b) for currents of frequency 40 ? **[3.14  $\mu$ F (a) 5000  $\Omega$  (b) 1940  $\Omega$ ] (London Univ.)**
- A circuit, consisting of a capacitor in series with a resistance of 10  $\Omega$ , is connected in parallel with a coil having  $L = 55.2$  mH and  $R = 10 \Omega$ , to a 100-V, 50-Hz supply. Calculate the value of the capacitance for which the current take from the supply is in phase with the voltage. Show that for the particular values given, the supply current is independent of the frequency. **[153  $\mu$ F] (London Univ.)**
- In a series-parallel circuit, the two parallel branches A and B are in series with C. The impedances are  $Z_A = (10 - j8) \Omega$ ,  $Z_B = (9 - j6) \Omega$  and  $Z_C = (100 + j0)$ . Find the currents  $I_A$  and  $I_B$  and the phase difference between them. Draw the phasor diagram. **[ $I_A = 12.71 \angle -30^\circ 58'$  ;  $I_B = 15 \angle -35^\circ 56'$  ;  $4^\circ 58'$ ] (Elect. Engg. & Electronics Bangalore Univ.)**
- Find the equivalent series circuits of the 4-branch parallel circuit shown in Fig. 14.60. **[(4.41 + j2.87)  $\Omega$ ] [A resistor of 4.415  $\Omega$  in series with a 4.57 mH inductor]**



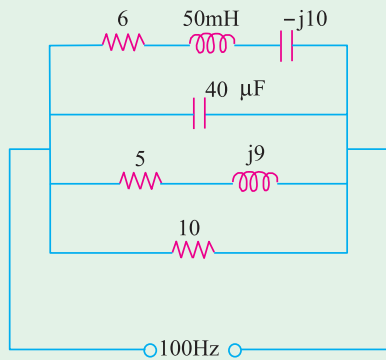


Fig. 14.60

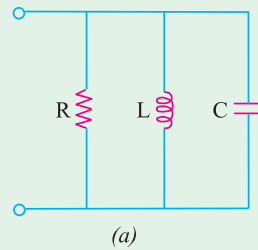
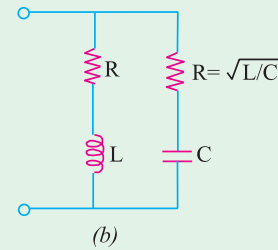


Fig. 14.61



16. A coil of  $20 \Omega$  resistance has an inductance of  $0.2 \text{ H}$  and is connected in parallel with a  $100\text{-}\mu\text{F}$  capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistance of  $R$  ohms. Find also the value of  $R$ . [31.8 Hz; 100  $\Omega$ ]
17. Calculate the resonant frequency, the impedance at resonance and the  $Q$ -factor at resonance for the two circuits shown in Fig. 14.61.

(a)  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  ;  $Z_0 = R$  ;  $Q_0 = \frac{R}{\sqrt{L/C}}$

(b) Circuit is resonant at all frequencies with a constant resistive impedance of  $(\sqrt{L/C})$  ohm,  $Q = 0$ .

18. Prove that the circuit shown in Fig. 14.62 exhibits both series and parallel resonances and calculate the frequencies at which two resonances occur.

Parallel  $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{(L_2 C_2)}}$  ; series  $f_0 = \frac{1}{2\pi} \sqrt{\frac{(L_1 + L_2)}{L_1 L_2 C_2}}$

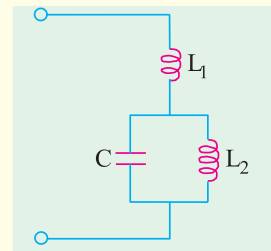


Fig. 14.62

19. Calculate the resonant frequency and the corresponding  $Q$ -factor for each of the networks shown in Fig. 14.63.

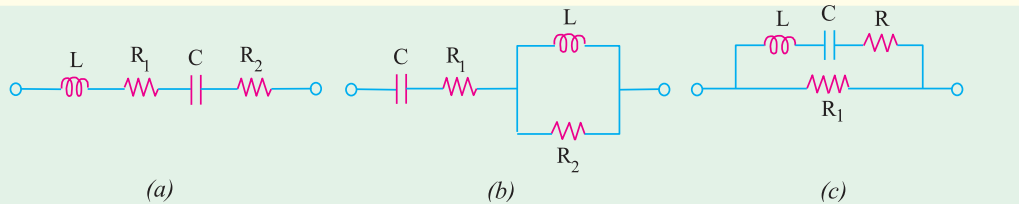


Fig. 14.63

(a)  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  ;  $Q = \frac{\omega_0 L}{R_1 + R_2} = \frac{1}{\omega_0 C(R_1 + R_2)} = \frac{1}{(R_1 + R_2)} \cdot \sqrt{\frac{L}{C}}$

(b)  $f_0 = \frac{1}{2} \sqrt{\frac{1}{LC} \frac{1}{L^2/R_2^2}}$  ;  $Q = \frac{R_2}{\omega_0(L + R_1 R_2 C)}$  (c)  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  ;  $Q = \frac{\omega_0 L}{R} \times \frac{R_1}{R + R_1} = \frac{R_1}{R(R + R_1)} \cdot \sqrt{\frac{L}{C}}$

20. A parallel  $R$ - $L$ - $C$  circuit is fed by a constant current source of variable frequency. The circuit resonates at  $100 \text{ kHz}$  and the  $Q$ -factor measured at this frequency is  $5$ . Find the frequencies at which the amplitude of the voltage across the circuit falls to (a)  $70.7\%$  (b)  $50\%$  of the resonant frequency amplitude. [(a) 90.5 kHz ; 110.5 kHz (b) 84.18 kHz ; 118.8 kHz]

21. Two impedance  $Z_1 = (6 + j8) \text{ ohm}$  and  $Z_2 = (8 - j6) \text{ ohm}$  are connected in parallel across  $100 \text{ V}$  supply. Determine : (i) Current and power factor of each branch. (ii) Overall current and power factor (iii) Power consumed by each branch and total power. (Nagpur University, Winter 2003)

22. The currents in each branch of a two branched parallel circuit is given as :

$$i_a = 8.07 \sin \left( 314 t - \frac{\pi}{4} \right)$$

$$i_b = 21.2 \sin \left( 314 t - \frac{\pi}{3} \right)$$

and supply voltage is  $v = 354 \sin 314 t$ .

Calculate :

- (i) Total current in the same form (ii) Calculate ohmic value of components in each branch.

*(Nagpur University, Summer 2004)*

23. Two coils are connected in parallel and a voltage of 200 V is applied between the terminals the total current taken by the circuit is 25 A and power dissipated in one of the coils is 1500 W. Calculate the resistance of each coil.

*(Gujrat University, June/July 2003)*

24. Compare the series and parallel resonance of R-L-C series and R-L-C parallel circuit.

*(Gujrat University, June/July 2003)*

25. Two circuits with impedances  $Z_1 = (10 + j15) \Omega$  and  $Z_2 = (6 - j8) \Omega$  are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch.

*(V.T.U., Belgaum Karnataka University, Winter 2003)*

26. Three impedances  $z_1 = 8 + j6 \Omega$ ,  $z_2 = 2 - j1.5 \Omega$  and  $z_3 = 2 \Omega$  are connected in parallel across a 50Hz supply. If the current through  $z_1$  is  $3 + j4$  amp, calculate the current through the other impedances and also power absorbed by this parallel circuit.

*(V.T.U. Belgaum Karnataka University, Winter 2004)*

27. Show that the power consumed in a pure inductance is zero.

*(RGPV Bhopal 2002)*

28. What do you understand by the terms power factor, active power and reactive power?

*(RGPV Bhopal 2002)*

29. Two circuits the impedances of which are given by  $Z_1 = (10 + j15) \Omega$  and  $Z_2 = (6 - j8) \Omega$  are connected in parallel. If the total current supplied is 15 A. What is the power taken by each branch?

*(RGPV Bhopal 2002)*

30. Does an inductance draw instantaneous power as well as average power?

*(RGPV Bhopal December 2002)*

31. Describe the properties of (i) Resistance (ii) Inductance and (iii) capacitance used in A.C. Circuit.

*(RGPV Bhopal June 2003)*

## OBJECTIVE TYPES – 14

- Fill in the blanks
  - unit of admittance is .....
  - unit of capacitive susceptance is .....
  - admittance equals the reciprocal of .....
  - admittance is given by the ..... sum of conductance and susceptance.
- An R-L circuit has  $Z = (6 + j8) \Omega$ . Its susceptance is -Siemens.
  - 0.06
  - 0.08
  - 0.1
  - 0.08
- The impedances of two parallel branches of a circuit are  $(10 + j10)$  and  $(10 - j10)$  respectively. The impedance of the parallel combination is
  - $20 + j0$
  - $10 + j0$
  - $5 - j5$
  - $0 - j20$

- The value of Z in Fig. 14.64 which is most appropriate to cause parallel resonance at 500 Hz is

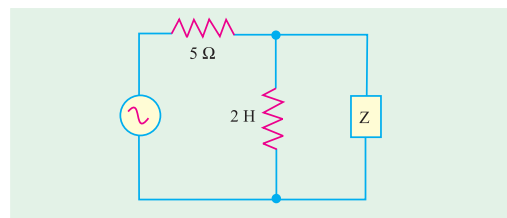


Fig. 14.64

- 125.00 mH
- 304.20  $\mu$ F
- 2.0  $\mu$ F
- 0.05  $\mu$ F

*(GATE 2004)*

## CHAPTER 15

**Learning Objectives**

- Introduction
- Kirchhoff's Laws
- Mesh Analysis
- Nodal Analysis
- Superposition Theorem
- Thevenin's Theorem
- Reciprocity Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem-General Case
- Maximum Power Transfer Theorem
- Millman's Theorem

**A.C.  
NETWORK  
ANAYLSIS**

↑ Engineers use AC network analysis in thousands of companies worldwide in the design, maintenance and operation of electrical power systems

### 15.1. Introduction

We have already discussed various d.c. network theorems in Chapter 2 of this book. The same laws are applicable to a.c. networks except that instead of resistances, we have impedances and instead of taking algebraic sum of voltages and currents we have to take the phasor sum.

### 15.2. Kirchhoff's Laws

The statements of Kirchhoff's laws are similar to those given in Art. 2.2 for d.c. networks except that instead of algebraic sum of currents and voltages, we take phasor or vector sums for a.c. networks.

**1. Kirchhoff's Current Law.** According to this law, in any electrical network, the phasor sum of the currents meeting at a junction is zero.

In other words,  $\sum I = 0$  ...at a junction

Put in another way, it simply means that in any electrical circuit the phasor sum of the currents flowing towards a junction is equal to the phasor sum of the currents going away from that junction.

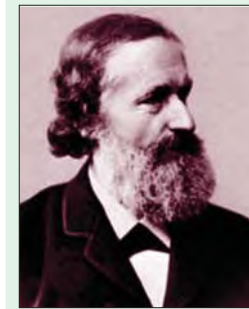
**2. Kirchhoff's Voltage Law.** According to this law, the phasor sum of the voltage drops across each of the conductors in any closed path (or mesh) in a network plus the phasor sum of the e.m.fs. connected in that path is zero.

In other words,  $\sum IR + \sum e.m.f. = 0$  ...round a mesh

**Example 15.1.** Use Kirchhoff's laws to find the current flowing in each branch of the network shown in Fig. 15.1.

**Solution.** Let the current distribution be as shown in Fig. 15.1 (b). Starting from point A and applying KVL to closed loop ABEFA, we get

$$-10(x + y) - 20x + 100 = 0 \quad \text{or} \quad 3x + y = 10 \quad \dots(i)$$



Gustav Kirchhoff  
(1824-1887)

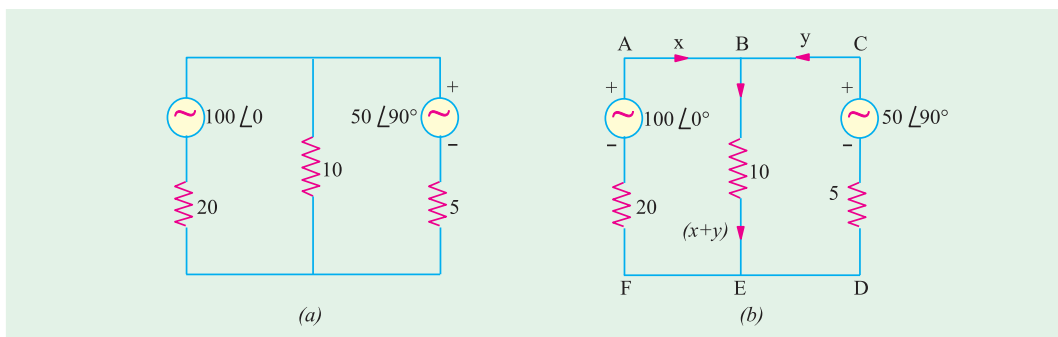
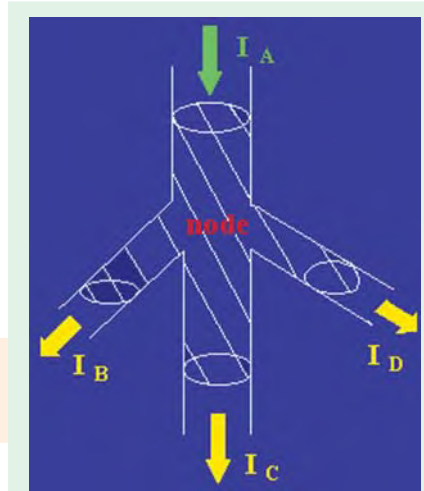


Fig. 15.1

Similarly, considering the closed loop BCDEB and starting from point B, we have

$$-50 \angle 90^\circ + 5y + 10(x + y) = 0 \quad \text{or} \quad 2x + 3y = j10 \quad \dots(ii)$$

Multiplying Eq. (i) by 3 and subtracting it from Eq. (ii), we get

$$7x = 30 - j10 \quad \text{or} \quad x = 4.3 - j1.4 = 4.52 \angle -18^\circ$$

Substituting this value of  $x$  in Eq. (i), we have

$$y = 10 - 3x = 5.95 \angle 119.15^\circ = -2.9 + j5.2$$

$$\therefore x + y = 4.3 - j1.4 - 2.9 + j5.2 = 1.4 + j3.8$$

### Tutorial Problem No. 15.1

- Using Kirchhoff's Laws, calculate the current flowing through each branch of the circuit shown in Fig. 15.2  
 $[I = 0.84 \angle 47.15^\circ \text{ A}; I_1 = 0.7 \angle -88.87^\circ \text{ A}; I_2 = 1.44 \angle 67.12^\circ \text{ A}]$

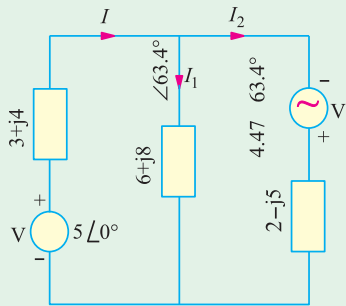


Fig. 15.2

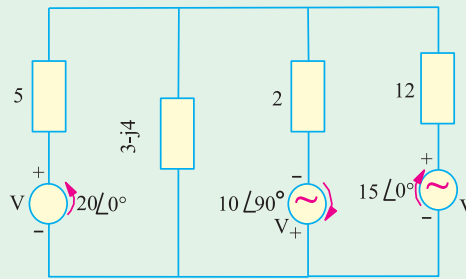


Fig. 15.3

- Use Kirchhoff's laws to find the current flowing in the capacitive branch of Fig. 15.3  
 $[5.87 \text{ A}]$

### 15.3. Mesh Analysis

It has already been discussed in Art. 2.3. Sign convention regarding the voltage drops across various impedances and the e.m.f.s is the same as explained in Art. 2.3. The circuits may be solved with the help of KVL or by use of determinants and Cramer's rule or with the help of impedance matrix  $[Z_m]$ .

**Example 15.2.** Find the power output of the voltage source in the circuit of Fig. 15.4. Prove that this power equals the power in the circuit resistors.

**Solution.** Starting from point A in the clockwise direction and applying KVL to the mesh ABEFA, we get.

$$-8 I_1 - j6 (I_1 - I_2) + 100 \angle 0^\circ = 0$$

$$\text{or } I_1 (8 - j6) + I_2 (j6) = 100 \angle 0^\circ \dots (i)$$

Similarly, starting from point B and applying KVL to mesh BCDEB, we get

$$-I_2 (3 + j4) - j6 (I_2 - I_1) = 0$$

$$\text{or } I_1 (j6) + I_2 (3 - j2) = 0 \dots (ii)$$

The matrix form of the above equation is

$$\begin{bmatrix} (8 - j6) & j6 \\ j6 & (3 - j2) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (8 - j6) & j6 \\ j6 & (3 - j2) \end{vmatrix} = (8 - j6)(3 - j2) - (j6)^2 = 62.5 \angle 39.8^\circ$$

$$\Delta_1 = \begin{vmatrix} 100 \angle 0^\circ & j6 \\ 0 & (3 - j2) \end{vmatrix} = (300 - j200) = 360 \angle -26.6^\circ$$

$$\Delta_2 = \begin{vmatrix} (8 - j6) & 100 \angle 0^\circ \\ j6 & 0 \end{vmatrix} = 600 \angle 90^\circ$$

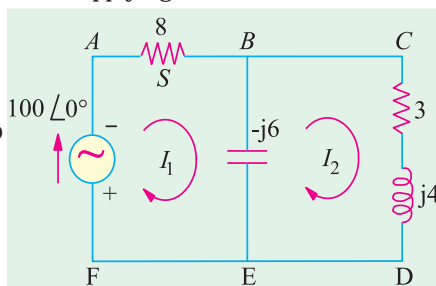


Fig. 15.4

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{360 \angle -26.6^\circ}{62.5 \angle -39.8^\circ} = 5.76 \angle 13.2^\circ; I_2 = \frac{\Delta_2}{\Delta} = \frac{600 \angle 90^\circ}{62.5 \angle -39.8^\circ} = 9.6 \angle 129.8^\circ$$

**Example 15.3.** Using Maxwell's loop current method, find the value of current in each branch of the network shown in Fig. 15.5 (a).

**Solution.** Let the currents in the two loops be  $I_1$  and  $I_2$  flowing in the clockwise direction as shown in Fig. 15.5 (b). Applying KVL to the two loops, we get

**Loop No. 1**

$$25 - I_1(40 + j50) - (-j100)(I_1 - I_2) = 0$$

$$\therefore 25 - I_1(40 - j50) - j100 I_2 = 0 \quad \dots(i)$$

**Loop No. 2**

$$-60 I_2 - (-j100)(I_2 - I_1) = 0$$

$$\therefore -j100 I_1 - I_2(60 - j100) = 0$$

$$\therefore I_2 = \frac{-j100 I_1}{(60 - j100)} = \frac{100 \angle -90^\circ I_1}{116.62 \angle -59^\circ} = 0.8575 \angle 31^\circ I_1 \quad \dots(ii)$$

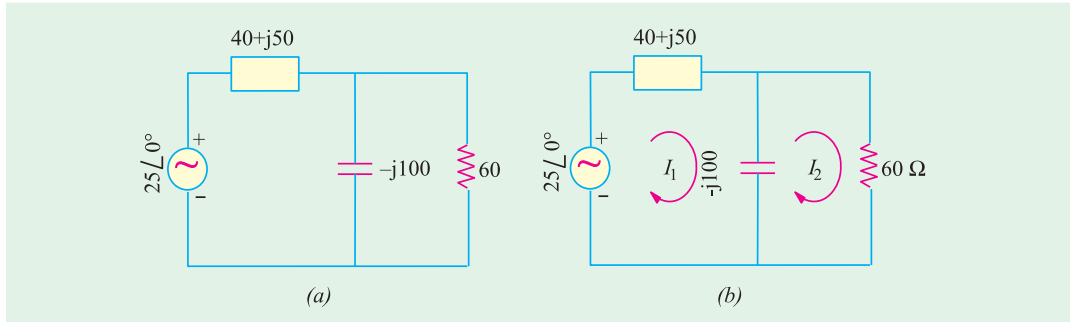


Fig. 15.5

Substituting this value of  $I_2$  in (i) above, we get  $25 - I_1(40 - j50) - j100 \times 0.8575 \angle 31^\circ I_1 = 0$

$$\text{or } 25 - 40 I_1 + j50 I_1 - 85.75 \angle 59^\circ I_1 = 0 \quad (j100 = 100 \angle 90^\circ)$$

$$\text{or } 25 - I_1(84.16 + j23.5) = 0.$$

$$\therefore I_1 = \frac{25}{(84.16 + j23.5)} = \frac{25}{87.38 \angle 15.6^\circ} = 0.286 \angle -15.6^\circ \text{ A}$$

$$\text{Also, } I_2 = 0.8575 \angle 31^\circ I_1 \times 0.286 \angle -15.6^\circ = 0.2452 \angle 46.6^\circ \text{ A}$$

Current through the capacitor =  $(I_1 - I_2) = 0.286 \angle -15.6^\circ - 0.2452 \angle 46.6^\circ = 0.107 + j0.1013 = 0.1473 \angle 43.43^\circ \text{ A}.$

**Example 15.4.** Write the three mesh current equations for network shown in Fig. 15.6.

**Solution.** While moving along  $I_1$ , if we apply KVL, we get

$$-(j10) I_1 - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$

$$\text{or } I_1(15 - j10) - 10 I_2 - 5 I_3 = 0 \quad \dots(i)$$

In the second loop, current through the a.c. source is flowing upwards indicating that its upper end is positive and lower is negative. As we move along  $I_2$ , we go from the positive terminal of the voltage source to its negative terminal. Hence, we

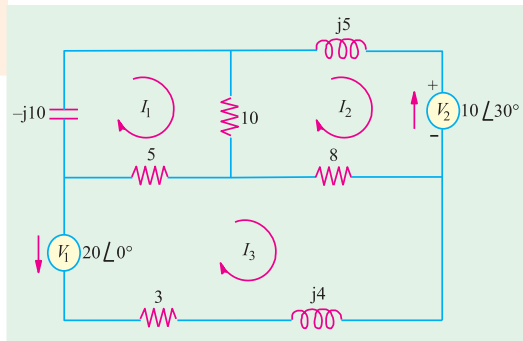


Fig. 15.6

experience a decrease in voltage which as per Art. would be taken as negative.

$$-j5 I_2 - 10 \angle 30^\circ - 8(I_2 - I_3) - 10(I_2 - I_1) = 0$$

$$\text{or } 10 I_1 - I_2 (18 + j5) + 8 I_3 = 10 \angle 30^\circ \quad \dots(ii)$$

Similarly, from third loop, we get

$$-20 \angle 0^\circ - 5(I_3 - I_1) - 8(I_3 - I_2) - I_3(3 + j4) = 0$$

$$\text{or } 5 I_1 + 8 I_2 - I_3 (16 + j4) = 20 \angle 0^\circ \quad \dots(iii)$$

The values of the three currents may be calculated with the help of Cramer's rule. However, the same values may be found with the help of mesh impedance  $[Z_m]$  whose different items are as under :

$$Z_{11} = -j10 + 10 + 5 = (15 - j10); Z_{22} = (18 + j5)$$

$$Z_{33} = (16 + j5); Z_{12} = Z_{21} = -10; Z_{23} = Z_{32} = -8$$

$$Z_{13} = Z_{31} = -5; E_1 = 0; E_2 = -10 \angle 30^\circ; E_3 = -20 \angle 0^\circ$$

Hence, the mesh equations for the three currents in the matrix form are as given below :

$$\begin{bmatrix} (15 - j10) & -10 & -5 \\ -10 & (18 + j5) & -8 \\ -5 & -8 & (16 + j5) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \angle 30^\circ \\ -20 \angle 0^\circ \end{bmatrix}$$

**Example 15.5.** For the circuit shown in Fig. 15.7 determine the branch voltage and currents and power delivered by the source using mesh analysis.

(Elect. Network Analysis Nagpur Univ. 1993)

**Solution.** Let the mesh currents be as shown in Fig. 15.7. The different items of the mesh resistance matrix  $[E_m]$  are :

$$Z_{11} = (2 + j1 + j2 - j1) = (2 + j2)$$

$$Z_{22} = (-j2 + 1 - j1 + j2) = (1 - j1)$$

$$Z_{12} = Z_{21} = -(j2 - j1) = -j1$$

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} (2 + j2) & -j1 \\ -j1 & (1 - j1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \angle 0^\circ \\ -5 \angle 30^\circ \end{bmatrix}$$

$$\therefore \Delta = (2 + j2)(1 - j1) + 1 = 5$$

$$\Delta_1 = \begin{bmatrix} 10 & -j1 \\ -(4.43 + j2.5) & (1 - j1) \end{bmatrix} = 10(1 - j1) - j1(4.43 + j2.5) = 12.5 - j14.43 = 19.1 \angle 49.1^\circ$$

$$\begin{aligned} \Delta_2 &= \begin{bmatrix} (2 + j2) & 10 \\ -j1 & -(4.43 + j2.5) \end{bmatrix} = (2 + j2)(4.43 + j2.5) + j10 = -3.86 - j3.86 \\ &= 5.46 \angle 135^\circ \text{ or } \angle 225^\circ \end{aligned}$$

$$I_1 = \Delta_1 / \Delta = 19.1 \angle 49.1^\circ / 5 = 3.82 \angle 49.1^\circ = 2.5 - j2.89$$

$$I_2 = \Delta_2 / \Delta = 5.46 \angle 135^\circ / 5 = 1.1 \angle 135^\circ = -0.78 - j0.78$$

Current through branch  $BC = I_1 - I_2 = 2.5 - j2.89 + 0.78 + j0.78 = 3.28 - j2.11 = 3.49 \angle 32.75^\circ$

Drop over branch  $AB = (2 + j1)(2.5 - j2.89) = 7.89 - j3.28$

Drop over branch  $BD = (1 - j2)(-0.78 - j0.78) = 2.34 + j0.78$

Drop over branch  $BC = j1(I_1 - I_2) = j1(3.28 - j2.11) = 2.11 + j3.28$

Power delivered by the sources would be found by using conjugate method. Using current conjugate, we get

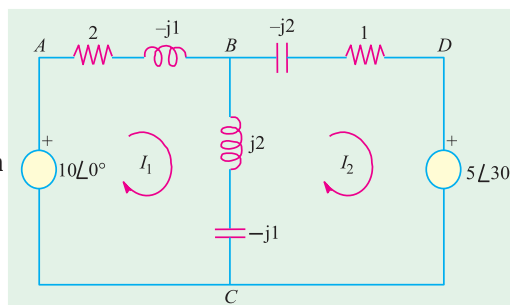


Fig. 15.7

$$VA_1 = 10(2.5 + j2.89) = 25 + j28.9 ; \quad \therefore W_1 = 25 \text{ W}$$

$VA_2 = V_2 \times I_2$  — because  $-I_2$  is the current coming out of the second voltage source. Again, using current conjugate, we have

$$VA_2 = (4.43 + j2.5)(0.78 - j0.78) \text{ or } W_2 = 4.43 \times 0.78 + 2.5 \times 0.78 = 5.4 \text{ W}$$

$$\therefore \text{total power supplied by the two sources} = 25 + 5.4 = 30.4 \text{ W}$$

Incidentally, the above fact can be verified by adding up the powers dissipated in the three branches of the circuit. It may be noted that there is no power dissipation in the branch  $BC$ .

$$\text{Power dissipated in branch } AB = 3.82^2 \times 2 = 29.2 \text{ W}$$

$$\text{Power dissipated in branch } BD = 1.1^2 \times 1 = 1.21 \text{ W}$$

$$\text{Total power dissipated} = 29.2 + 1.21 = 30.41 \text{ W}$$

### Tutorial Problems No. 15.2

1. Using mesh analysis, find current in the capacitor of Fig. 15.8.

[13.1  $\angle 70.12^\circ$  A]

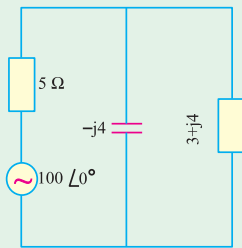


Fig. 15.8

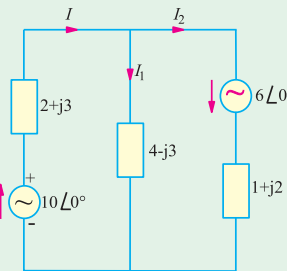


Fig. 15.9

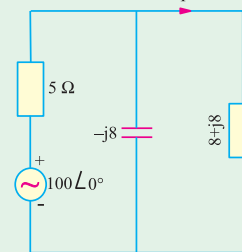


Fig. 15.10

2. Using mesh analysis or Kirchoff's laws, determine the values of  $I$ ,  $I_1$  and  $I_2$  (in Fig. 15.9)

[ $I = 2.7 \angle -58.8^\circ$  A ;  $I_1 = 0.1 \angle 97^\circ$  A ;  $I_2 = 2.8 \angle -59.6^\circ$  A]

3. Using mesh current analysis, find the value of current  $I$  and active power output of the voltage source in Fig. 15.10.

[7  $\angle -50^\circ$  A ; 645 W]

4. Find the mesh currents  $I_1$ ,  $I_2$  and  $I_3$  for the circuit shown in Fig. 15.11. All resistances and reactances are in ohms.

[ $I_1 = (1.168 + j1.281)$  ;  $I_2 = (0.527 - j0.135)$  ;  $I_3 = (0.718 + j0.412)$ ]

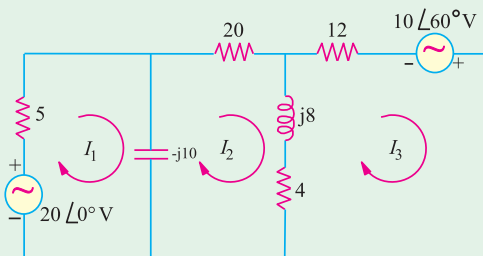


Fig. 15.11

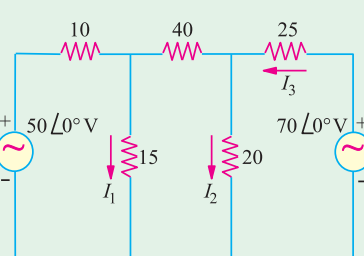


Fig. 15.12

5. Find the values of branch currents  $I_1$ ,  $I_2$  and  $I_3$  in the circuit shown in Fig. 15.12 by using mesh analysis. All resistances are in ohms.

[ $I_1 = 2.008 \angle 0^\circ$  ;  $I_2 = 1.545 \angle 0^\circ$  ;  $I_3 = 1.564 \angle 0^\circ$ ]

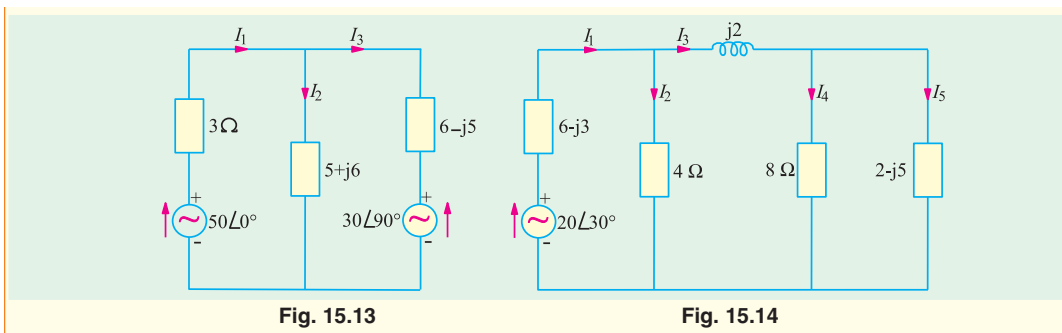
6. Using mesh-current analysis, determine the current  $I_1$ ,  $I_2$  and  $I_3$  flowing in the branches of the networks shown in Fig. 15.13.

[ $I_1 = 8.7 \angle -1.37^\circ$  A ;  $I_2 = 3 \angle -48.7^\circ$  A ;  $I_3 = 7 \angle 17.25^\circ$  A]

7. Apply mesh-current analysis to determine the values of current  $I_1$  to  $I_5$  in different branches of the circuit shown in Fig. 15.14.

[ $I_1 = 2.4 \angle 52.5^\circ$  A ;  $I_2 = 1.0 \angle 46.18^\circ$  A ;  $I_3 = 1.4 \angle 57.17^\circ$  A ;  $I_4 = 0.86 \angle 166.3^\circ$  A ;  $I_5 = 1.0 \angle 83.7^\circ$  A]





15.4. Nodal Analysis

This method has already been discussed in details Chapter 2. This technique is the same although we have to deal with circuit impedances rather than resistances and take phasor sum of voltages and currents rather than algebraic sum.

**Example 15.6.** Use Nodal analysis to calculate the current flowing in each branch of the network shown in Fig. 15.15.

**Solution.** As seen, there are only two principal nodes out of which node No. 2 has been taken as the reference node. The nodal equations are :

$$V_1 \left( \frac{1}{20} + \frac{1}{10} + \frac{1}{5} \right) - \frac{100\angle 0^\circ}{20} - \frac{50\angle 90^\circ}{5} = 0$$

$$\therefore 0.35 V_1 = 5 + j10; V_1 = \frac{5 + j10}{0.35} = 14.3 + j28.6 = 32\angle 63.4^\circ$$

$$\therefore I_1 = \frac{100\angle 0^\circ - V_1}{20} = \frac{100 - 14.3 - j28.6}{20} = 4.3 - j1.4 = 4.5 \angle -18^\circ \text{ flowing towards node No. 1}$$

(or  $4.5 \angle -18^\circ + 180^\circ = 4.5 \angle 162^\circ$  flowing away from node No. 1)

$$I_3 = \frac{V_1}{10} = \frac{32\angle 63.4^\circ}{10} = 3.2\angle 63.4^\circ = 1.4 + j2.9 \text{ flowing from node No. 1 to node No. 2}$$

$$I_2 = \frac{50\angle 90^\circ - V_1}{5} = \frac{j50 - 14.3 - j28.6}{5} = \frac{-14.3 + j21.4}{5} = -2.86 + j4.3 = 5.16\angle 123.6^\circ$$

flowing towards node No. 1  
 $\angle 123.6^\circ - 180^\circ = 5.16 \angle 56.4^\circ$  flowing away from node No. 1).

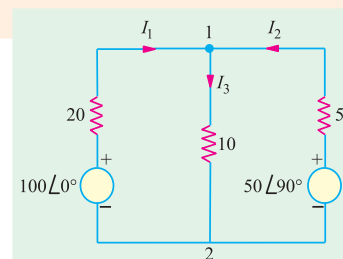


Fig. 15.15

**Example 15.7.** Find the current  $I$  in the  $j10 \Omega$  branch of the given circuit shown in Fig. 15.16 using the Nodal Method. (Principles of Elect. Engg. Delhi Univ.)

**Solution.** There are two principal nodes out of which node No. 2 has been taken as the reference node. As per Art.

$$V_1 \left( \frac{1}{6 + j8} + \frac{1}{6 - j8} + \frac{1}{j10} \right) - \frac{100\angle 0^\circ}{6 + j8} - \frac{100\angle -60^\circ}{6 - j8} = 0$$

$$V_1(0.06 - j0.08 + 0.06 + j0.08 - j0.1) = 6 - j8 + 9.93 - j1.2 = 18.4 \angle 30^\circ$$

$$\therefore V_1(0.12 - j0.1) = 18.4 \angle 30^\circ \text{ or}$$

$$V_1 \times 0.156 \angle 85.6^\circ = 18.4 \angle 30^\circ$$

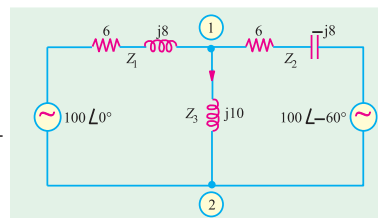


Fig. 15.16

$$\begin{aligned} \therefore V_1 &= 18.4 \angle 30^\circ / 0.156 \angle 85.6^\circ = 118 \angle 55.6^\circ \text{V} \\ \therefore V &= V_1 / j10 = 118 \angle 55.6^\circ / j10 = 11.8 \angle 34.4^\circ \text{A} \end{aligned}$$

**Example 15.8.** Find the voltage  $V_{AB}$  in the circuit of Fig. 15.17 (a). What would be the value of  $V_1$  if the polarity of the second voltage source is reversed as shown in Fig. 15.17 (b).

**Solution.** In the given circuit, there are no principle nodes. However, if we take point B as the reference node and point A as node 1, then using nodal method, we get

$$V_1 \left( \frac{1}{10} + \frac{1}{8 + j4} \right) - \frac{10 \angle 0^\circ}{10} - \frac{10 \angle 30^\circ}{8 + j4} = 0$$

$$V_1 \times 0.2 \angle 14.1^\circ = 1 + 1.116 + j0.066 = 4.48 \angle 1.78^\circ$$

$$\therefore V_1 = 4.48 \angle 1.78^\circ / 0.2 \angle 14.1^\circ = 22.4 \angle 15.88^\circ$$

**When source polarity is Reversed**

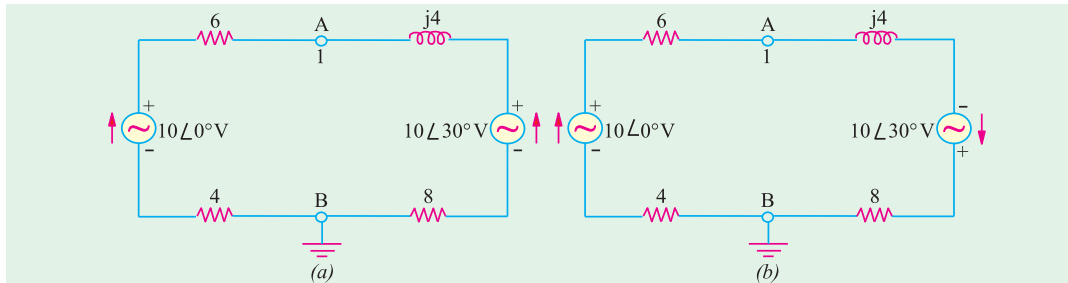


Fig. 15.17

$$V_1 \left( \frac{1}{10} + \frac{1}{8 + j4} \right) - \frac{10 \angle 0^\circ}{10} + \frac{10 \angle 30^\circ}{8 + j4} = 0 \text{ or } V_1 = 0.09 \angle 223.7^\circ$$

**Example 15.9.** Write the nodal equations for the network shown in Fig. 15.18.

**Solution.** Keeping in mind the guidance given in Art. 2.10, it would be obvious that since current of the second voltage source is flowing away from node 1, it would be taken as negative. Hence, the term containing this source will become positive because it has been reversed twice. As seen, node 3 has been taken as the reference node. Considering node 1, we have

$$V_1 \left( \frac{1}{10} + \frac{1}{4 + j4} + \frac{1}{j5} \right) - \frac{V_2}{4 + j4} - \frac{10 \angle 0^\circ}{10} + \frac{10 \angle 30^\circ}{j5} = 0$$

Similarly, considering node 2, we have

$$V_2 \left( \frac{1}{4 + j4} + \frac{1}{5} + \frac{1}{6 - j8} \right) - \frac{V_1}{4 + j4} - \frac{5 \angle 0^\circ}{5} = 0$$

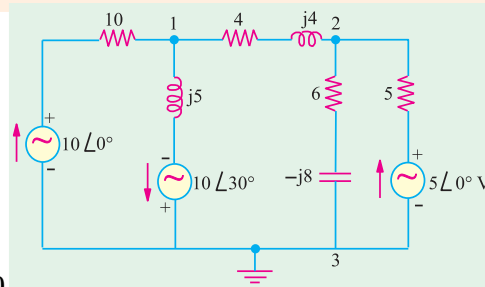


Fig. 15.18

**Example 15.10.** In the network of Fig. 15.19 determine the current flowing through the branch of  $4 \Omega$  resistance using nodal analysis. (Network Analysis Nagpur Univ. 1993)

**Solution.** We will find voltages  $V_A$  and  $V_B$  by using Nodal analysis and then find the current through  $4 \Omega$  resistor by dividing their difference by 4.

$$V_2 \left( \frac{1}{5} + \frac{1}{4} + \frac{1}{j2} \right) - \frac{V_B}{4} - \frac{50 \angle 30^\circ}{5} = 0 \quad \dots \text{for node A}$$

$$\therefore V_A(9 - j10) - 5V_B = 200 \angle 30^\circ \quad \dots (i)$$

Similarly, from node B, we have

$$V_B \left( \frac{1}{4} + \frac{1}{2} + \frac{1}{-j2} \right) - \frac{V_A}{4} - \frac{50 \angle 90^\circ}{2} = 0 \quad \therefore V_B(3 + j2) - V_A = 100 \angle 90^\circ = j100 \quad \dots (ii)$$

$V_A$  can be eliminated by multiplying Eq. (ii) by (9-j10) and adding the result.

$$\therefore V_B(42 - j12) = 1173 + j1000 \quad \text{or} \quad V_B = \frac{1541.4 \angle 40.40^\circ}{43.68 \angle -15.9^\circ} = 35.29 \angle 56.3^\circ = 19.58 + j29.36$$

Substituting this value of  $V_B$  in Eq. (i), we get

$$V_A = V_B(3 + j2) - j100 = (19.58 + j29.36)(3 + j2) - j100 = j27.26$$

$$\therefore V_A - V_B = j27.26 - 19.58 - j29.36 = -19.58 - j2.1 = 19.69 \angle 186.12^\circ$$

$$\therefore I_2 = (V_A - V_B)/4 = 19.69 \angle 186.12^\circ / 4 = 4.92 \angle 186.12^\circ$$

For academic interest only, we will solve the above question with the help of following two methods :

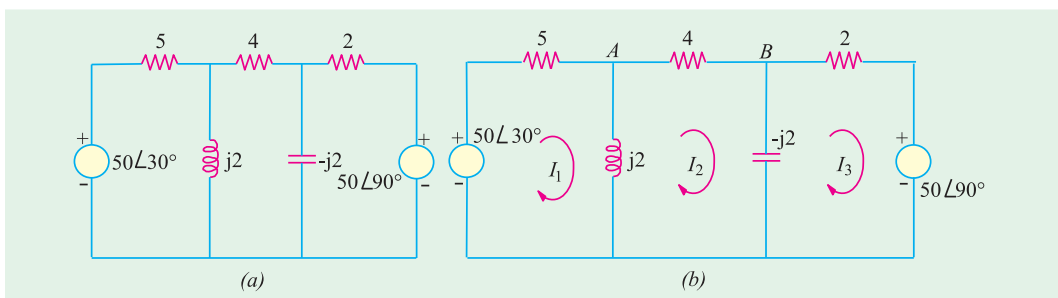


Fig. 15.19

### Solution by Using Mesh Resistance Matrix

Let the mesh currents  $I_1$ ,  $I_2$  and  $I_3$  be as shown in Fig. 15.19 (b). The different items of the mesh resistance matrix  $[R_m]$  are as under :

$$R_{11} = (5 + j2) ; R_{22} = 4 ; R_{33} = (2 - j2) ; R_{12} = R_{21} = -j2 ;$$

$$R_{23} = R_{32} = j2 ; R_{31} = R_{13} = 0$$

The mesh equations in the matrix form are :

$$\begin{bmatrix} (5 + j2) & -j2 & 0 \\ -j2 & 4 & j2 \\ 0 & j2 & (2 - j2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \angle 30^\circ \\ 0 \\ -j50 \end{bmatrix}$$

$$\Delta = (5 + j2) [4(2 - j2) - (j2 \times j2)] - (-j2) [(-j2)(2 - j2)] = 84 - j24 = 87.4 \angle -15.9^\circ$$

$$\Delta_2 = \begin{bmatrix} (5 + j2) & (43.3 + j25) & -0 \\ -j2 & 0 & j2 \\ 0 & -j50 & (2 - j2) \end{bmatrix} = (5 + j2) [-j2(-j50)] +$$

$$j2 [43.3 + j25)(2 - j2)] = -427 + j73 = 433 \angle 170.3^\circ$$

$$\therefore I_2 \Delta_2 / \Delta = 433 \angle 170.3^\circ / 87.4 \angle -15.9^\circ = 4.95 \angle 186.2^\circ$$

### Solution by using Thevenin's Theorem

When the  $4 \Omega$  resistor is disconnected, the given figure becomes as shown in Fig. 15.20 (a). The voltage  $V_A$  is given by the drop across  $j2$  reactance. Using the voltage-divider rule, we have

$$V_A = 50 \angle 30^\circ \times \frac{j2}{5 + j2} = 18.57 \angle 98.2^\circ = -2.65 + j18.38$$

$$\text{Similarly, } V_B = 50 \angle 90^\circ \frac{-j2}{2 - j2} = 35.36 \angle 45^\circ = 25 + j25$$

$$\therefore V_{th} = V_A - V_B = -2.65 + 18.38 - 25 - j25 = 28.43 \angle 193.5^\circ$$

$$R_{th} = 5 \parallel j2 + 2 \parallel (-j2) = \frac{j10}{5 + j2} + \frac{-j4}{2 - j2} = 1.689 + j0.72$$

The Thevenin's equivalent circuit consists of a voltage source of  $28.43 \angle 193.5^\circ$  V and an impedance of  $(1.689 + j0.72) \Omega$  as shown in Fig. 15.20 (c). Total resistance is  $4 + (1.689 + j0.72) = 5.689 + j0.72 = 5.73 \angle 7.2^\circ$ . Hence, current through the  $4 \Omega$  resistor is  $28.43 \angle 193.5^\circ / 5.73 \angle 7.2^\circ = 4.96 \angle 186.3^\circ$ .

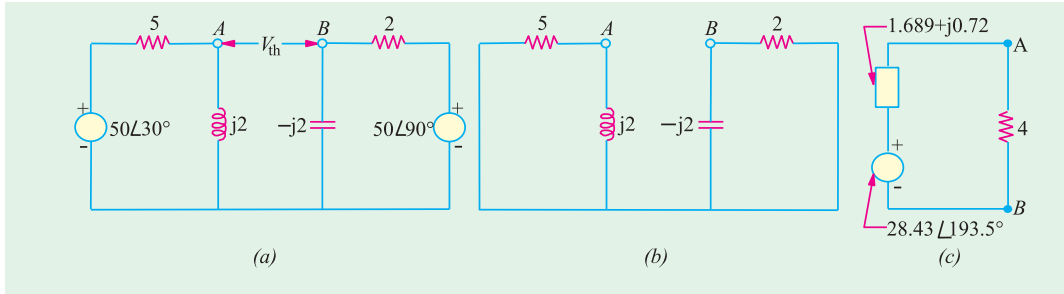


Fig. 15.20

**Note.** The slight variations in the answers are due to the approximations made during calculations.

**Example 15.11.** Using any suitable method, calculate the current through  $4 \Omega$  resistance of the network shown in Fig. 15.21. (Network Analysis AMIE Sec. B Summer 1990)

**Solution.** We will solve this question with the help of (i) Kirchhoff's laws (ii) Mesh analysis and (iii) Nodal analysis.

#### (i) Solution by using Kirchhoff's Laws

Let the current distribution be as shown in Fig. 15.21 (b). Using the same sign convention as given in Art. we have

$$\begin{aligned} \text{First Loop} & -10(I_1 + I_2 + I_3) - (j5) I_1 + 100 = 0 \\ \text{or} & I_1 (10 - j5) + 10 I_3 + 10 I_2 = 100 \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{Second Loop} & -5(I_2 + I_3) - 4I_2 + (j5) I_1 = 0 \\ \text{or} & j5 I_1 + 9 I_2 + 5 I_3 = 0 \quad \dots(ii) \end{aligned}$$

$$\begin{aligned} \text{Third Loop} & -I_3 (8 + j6) + 4 I_2 = 0 \\ \text{or} & 0 I_1 + 4 I_2 - I_3 (8 + j6) = 0 \quad \dots(iii) \end{aligned}$$

The matrix form of the above three equations is

$$\begin{bmatrix} (10-j5) & -10 & 10 \\ j5 & 9 & 5 \\ 0 & 4 & -(8+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Delta &= (10-j5) [-9(8+j6)-20] -j5 [-10(8+j6)-40] \\ &= -1490 + j520 = 1578 \angle 160.8^\circ \end{aligned}$$

Since we are interested in finding  $I_2$  only, we will calculate the value of  $\Delta_2$ .

$$\Delta_2 = \begin{bmatrix} (10-j5) & 100 & 10 \\ j5 & 0 & 5 \\ 0 & 0 & -(8+j6) \end{bmatrix}$$

$$= -j5(-800-j600) = -3000 + j4000 = 5000 \angle 126.9^\circ$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{5000 \angle 126.9^\circ}{1578 \angle 160.8^\circ} = 3.17 \angle -33.9^\circ \text{ A}$$

(ii) Solution by using Mesh Impedance Matrix

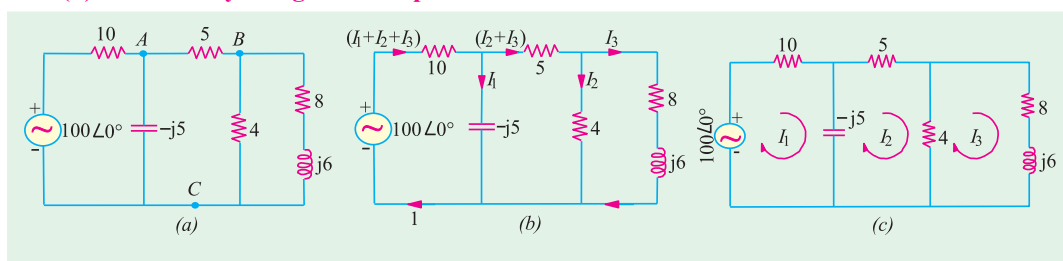


Fig. 15.21

Let the mesh currents  $I_1$ ,  $I_2$  and  $I_3$  be as shown in Fig. 15.21 (c). From the inspection of Fig. 15.21 (c), the different items of the mesh impedance matrix  $[Z_m]$  are as under :

$$Z_{11} = (10-j5); Z_{22} = (9-j5); Z_{33} = (12+j6)$$

$$Z_{21} = Z_{12} = -(-j5) = j5; Z_{23} = Z_{32} = -4; Z_{31} = Z_{13} = 0$$

Hence, the mesh equations in the matrix form are :

$$\begin{bmatrix} (10-j5) & j5 & 0 \\ j5 & (9-j5) & -4 \\ 0 & -4 & (12+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \therefore \Delta &= (10-j5) [(9-j5)(12+j6)-16] -j5(j60-30) \\ &= 1490 - j520 = 1578 \angle 19.2^\circ \end{aligned}$$

It should be noted that the current passing through  $4 \Omega$  resistance is the vector difference  $(I_2 - I_3)$ . Hence, we will find  $I_2$  and  $I_3$  only.

$$\Delta_2 = \begin{bmatrix} (10-j5) & 100 & 0 \\ j5 & 0 & -4 \\ 0 & 0 & (12+j6) \end{bmatrix} = j5(1200 + j600) = 3000 - j6000 = 6708 \angle -63.4^\circ$$

$$\Delta_3 = \begin{bmatrix} (10-j5) & j5 & 100 \\ j5 & (9-j5) & 0 \\ 0 & -4 & 0 \end{bmatrix} = -j5(400) = -j2000 = 2000 \angle -90^\circ$$

$$\therefore I_2 = \frac{\Delta_2}{\Delta} = \frac{6708 \angle -63.4^\circ}{1578 \angle -19.2^\circ} = 4.25 \angle -44.2^\circ = 3.05 - j2.96$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{2000 \angle -90^\circ}{1578 \angle -19.2^\circ} = 1.27 \angle -70.8^\circ = 0.42 - j1.2$$

$$\text{Current } (I_2 - I_3) = 2.63 - j1.76 = 3.17 \angle 33.9^\circ$$

**(iii) Solution by Nodal Analysis**

The current passing through  $4 \Omega$  resistance can be found by finding the voltage  $V_B$  of node  $B$  with the help of Nodal analysis. For this purpose point  $C$  in Fig. 15.21 (a) has been taken as the reference node. Using the Nodal technique as explained in Art. we have

$$V_A \left( \frac{1}{10} + \frac{1}{5} + \frac{1}{-j5} \right) - \frac{V_B}{5} - \frac{100 \angle 0^\circ}{10} = 0 \quad \dots \text{for node A}$$

$$V_A (3 + j2) - 2 V_B = 100 \quad \dots (i)$$

Similarly, for node  $B$ , we have

$$V_B \left( \frac{1}{5} + \frac{1}{4} + \frac{1}{(8 + j6)} \right) - \frac{V_A}{5} = 0 \quad \text{or} \quad V_B (53 - j6) - 20 V_A = 0 \quad \dots (ii)$$

Estimating  $V_A$  from Eq. (i) and (ii), we have

$$V_B (131 + j88) = 2000 \quad \text{or} \quad V_B = 12.67 \angle 33.9^\circ$$

Current through  $4 \Omega$  resistor  $12.67 \angle 33.9^\circ / 4 = 3.17 \angle 33.9^\circ$

### Tutorial Problems No. 15.3

- Apply nodal analysis to the network of Fig. 15.22 to determine the voltage at node A and the active power delivered by the voltage source. **[ $8 \angle 3.7^\circ$  V;  $9.85$  W]**
- Using nodal analysis, determine the value of voltages at models 1 and 2 in Fig. 15.23. **[ $V_1 = 88.1 \angle 33.88^\circ$  A;  $V_2 = 58.7 \angle 72.34^\circ$  A]**

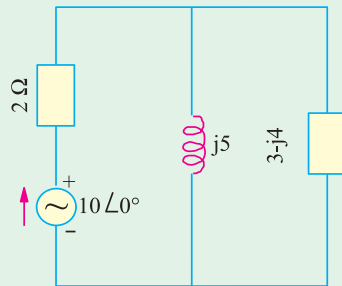


Fig. 15.22

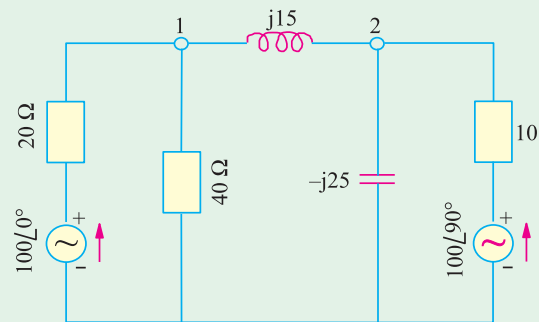


Fig. 15.23

- Using Nodal analysis, find the nodal voltages  $V_1$  and  $V_2$  in the circuit shown in Fig. 15.24. All resistances are given in terms of siemens **[ $V_1 = 1.64$  V;  $V_2 = 0.38$  V]**

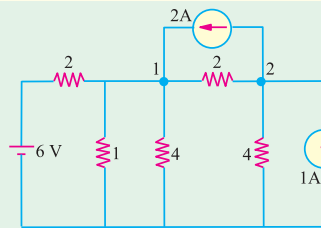


Fig. 15.24

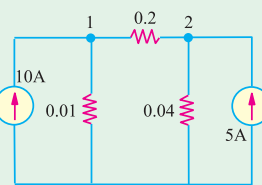


Fig. 15.25

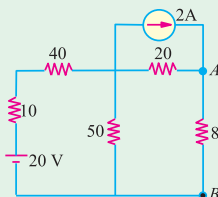


Fig. 15.26

- 4. Find the values of nodal voltages  $V_1$  and  $V_2$  in the circuit of Fig. 15.25. Hence, find the current going from node 1 to node 2. All resistances are given in siemens. [ $V_1 = 327 \text{ V}$ ;  $V_2 = 293.35 \text{ V}$  ;  $6.73 \text{ A}$ ]
- 5. Using Nodal analysis, find the voltage across points  $A$  and  $B$  in the circuit of Fig. 15.26: Check your answer by using mesh analysis. [ $32 \text{ V}$ ]

### 15.5. Superposition Theorem

As applicable to a.c. networks, it states as follows :

In any network made up of linear impedances and containing more than one source of e.m.f., the current flowing in any branch is the phasor sum of the currents that would flow in that branch if each source were considered separately, all other e.m.f. sources being replaced for the time being, by their respective internal impedances (if any).

**Note.** It may be noted that independent sources can be ‘killed’ i.e. removed leaving behind their internal impedances (if any) but dependent sources should not be killed.

**Example 15.12.** Use Superposition theorem to find the voltage  $V$  in the network shown in Fig. 15.27.

**Solution.** When the voltage source is killed, the circuit becomes as shown in the Fig. 15.27 (b) Using current-divider rule,

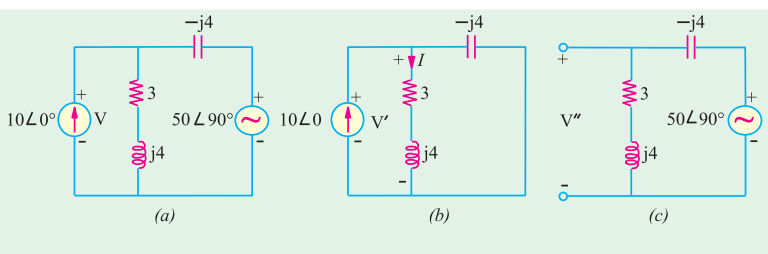


Fig. 15.27

$$I = 10\angle 0^\circ \times \frac{-j4}{(3+j4)-j4}, \text{ Now, } V = I(3+j4)$$

$$\therefore V = 10 \frac{-j4(3+j4)}{3} = 53.3 - j40$$

Now, when current source is killed, the circuit becomes as shown in Fig. 15.27 (c). Using the voltage-divider rule, we have

$$V' = 50\angle 90^\circ \times \frac{(3+j4)}{(3+j4)-j4} = -66.7 + j50$$

$$\therefore \text{ drop } V = V + V' = 53.3 - j40 + (-66.7 + j50) = -13.4 + j10 = 16.7 \angle 143.3^\circ \text{ V}$$

### Tutorial Problems No. 15.4

- 1. Using Superposition theorem to find the magnitude of the current flowing in the branch  $AB$  of the circuit shown in Fig. 15.28.

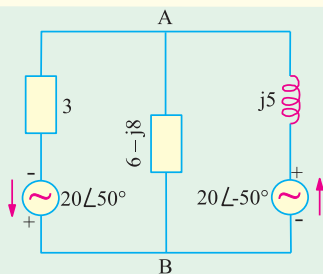


Fig. 15.28

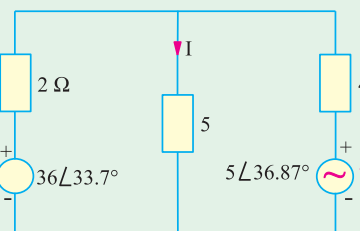


Fig. 15.29

- 2. Apply Superposition theorem to determine the circuit  $I$  in the circuit of Fig. 15.29. [ $0.53 \angle 5.7^\circ \text{ A}$ ]

### 15.6. Thevenin's Theorem

As applicable to a.c. networks, this theorem may be stated as follows :

The current through a load impedance  $Z_L$  connected across any two terminals  $A$  and  $B$  of a linear network is given by  $V_{th}/(Z_{th} + Z_L)$  where  $V_{th}$  is the open-circuit voltage across  $A$  and  $B$  and  $Z_{th}$  is the internal impedance of the network as viewed from the open-circuited terminals  $A$  and  $B$  with all voltage sources replaced by their internal impedances (if any) and current sources by infinite impedance.



Leon-Charles Thevenin

**Example 15.13.** In the network shown in Fig. 15.30.

$$Z_1 = (8 + j8) \Omega ; Z_2 = (8 - j8) \Omega ; Z_3 = (2 + j20) ; V = 10 \angle 0^\circ \text{ and } Z_L = j10 \Omega$$

Find the current through the load  $Z_L$  using Thevenin's theorem.

**Solution.** When the load impedance  $Z_L$  is removed, the circuit becomes as shown in Fig. 15.30 (b). The open-circuit voltage which appears across terminals  $A$  and  $B$  represents the Thevenin voltage  $V_{th}$ . This voltage equals the drop across  $Z_2$  because there is no current flow through  $Z_3$ .

Current flowing through  $Z_1$  and  $Z_2$  is

$$I = V/(Z_1 + Z_2) = 10 \angle 0^\circ / [(8 + j8) + (8 - j8)] = 10 \angle 0^\circ / 16 = 0.625 \angle 0^\circ$$

$$V_{th} = IZ_2 = 0.625 (8 - j8) = (5 - j5) = 7.07 \angle -45^\circ$$

The Thevenin impedance  $Z_{th}$  is equal to the impedance as viewed from open terminals  $A$  and  $B$  with voltage source shorted.

$$\therefore Z_{th} = Z_3 + Z_1 \parallel Z_2 = (2 + j20) + (8 + j8) \parallel (8 - j8) = (10 + j20)$$

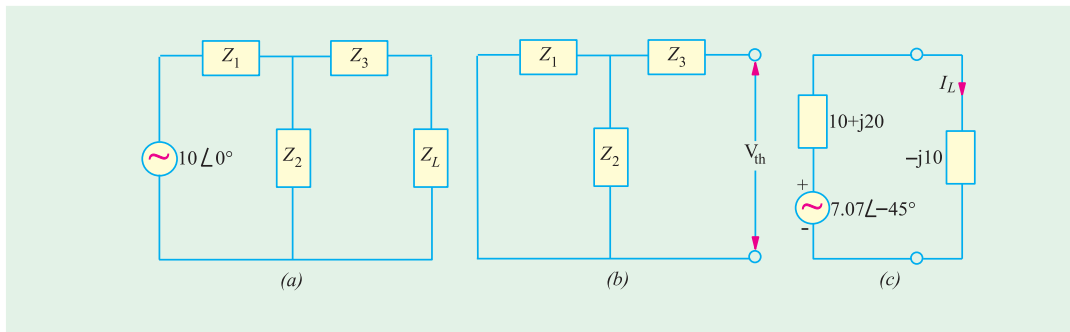


Fig. 15.30

The equivalent Thevenin circuit is shown in Fig. 15.30 (c) across which the load impedance has been reconnected. The load current is given by

$$\therefore I_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{(5 - j5)}{(10 + j20) + (-j10)} = \frac{-j}{2}$$

**Example 15.13 A.** Find the Thevenin equivalent circuit at terminals  $AB$  of the circuit given in Fig. 15.31 (a).

**Solution.** For finding  $V_{th} = V_{AB}$ , we have to find the phasor sum of the voltages available on the way as we go from point  $B$  to point  $A$  because  $V_{AB}$  means voltage of point  $A$  with respect to that of point  $B$  (Art.). The value of current  $I = 100 \angle 0^\circ / (6 - j8) = (6 + j8)A$ .



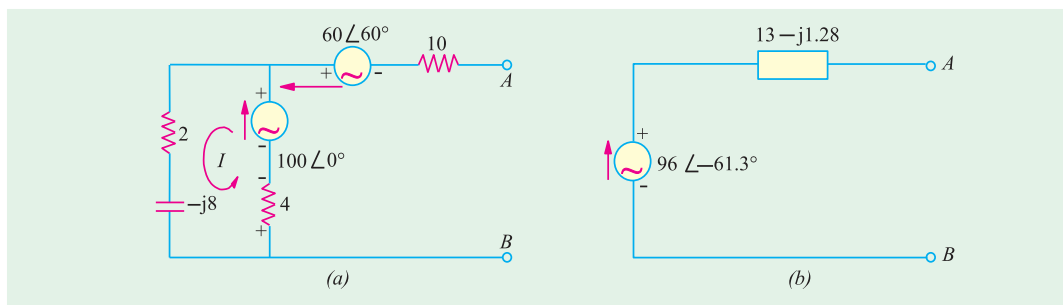


Fig. 15.31

Drop across  $4\ \Omega$  resistor  $= 4(6 + j8) = (24 + j32)$

$$\begin{aligned} \therefore V_{th} = V_{AB} &= -(24 + j32) + (100 + j0) - 60(0.5 + j0.866) \\ &= 46 - j84 = 96 \angle -61.3^\circ \end{aligned}$$

$$Z_{AB} = Z_{th} = [10 + [4 \parallel (2 - j8)]] = (13 - j1.28)$$

The Thevenin equivalent circuit is shown in Fig. 15.31 (b).

**Example 15.14.** Find the Thevenin's equivalent of the circuit shown in Fig. 15.32 and hence calculate the value of the current which will flow in an impedance of  $(6 + j30)\ \Omega$  connected across terminals A and B. Also calculate the power dissipated in this impedance.

**Solution.** Let us first find the value of  $V_{th}$  i.e. the Thevenin voltage across open terminals A and B. With terminals A and B open, there is no potential drop across the capacitor. Hence,  $V_{th}$  is the drop across the pure inductor  $j3\ \Omega$ .

$$\text{Drop across the inductor} = \frac{10 + j0}{4 + j3} \times j3 = \frac{j30}{4 + j3} = \frac{j30(4 - j3)}{4^2 + 3^2} = (3.6 + j4.8)\ \text{V}$$

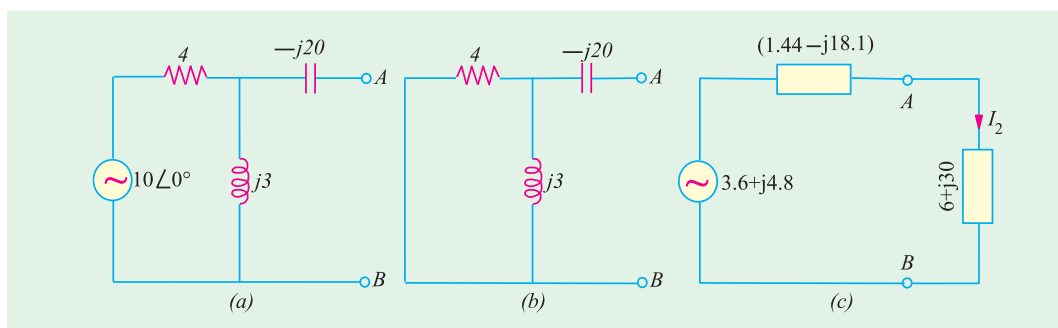


Fig. 15.32

Let us now find the impedance of the circuit as viewed from terminals A and B after replacing the voltage source by a short circuit as shown in Fig. 15.32 (a).

$$Z_{th} = -j20 + 4 \parallel j3 = -j20 + 1.44 + j1.92 = 1.44 - j18.1$$

The equivalent Thevenin circuit along with the load impedance of  $(6 + j30)$  is shown in Fig. 15.32 (c).

$$\text{Load current} = \frac{(3.6 + j4.8)}{(1.44 - j18.1) + (6 + j30)} = \frac{(3.6 + j4.8)}{(7.44 + j11.9)} = \frac{6 \angle 53.1^\circ}{14 \angle 58^\circ} = 0.43 \angle -4.9^\circ$$

The current in the load is 0.43 A and lags the supply voltage by  $4.9^\circ$

Power in the load impedance is  $0.43^2 \times 6 = 1.1\ \text{W}$

**Example 15.15.** Using Thevenin's theorem, calculate the current flowing through the load connected across terminals A and B of the circuit shown in Fig. 15.33 (a). Also calculate the power delivered to the load.

**Solution.** The first step is to remove the load from the terminals A and B.  $V_{th} = V_{AB}$  = drop across  $(10 + j10)$  ohm with A and B open.

$$\begin{aligned} \text{Circuit current } I &= \frac{100}{-j10 + 10 + j10} = 10 \angle 0^\circ \\ \therefore V_{th} &= 10(10 + j10) = 141.4 \angle 45^\circ \\ Z_{th} &= (-j10) \parallel (10 + j10) = (10 - j10) \end{aligned}$$

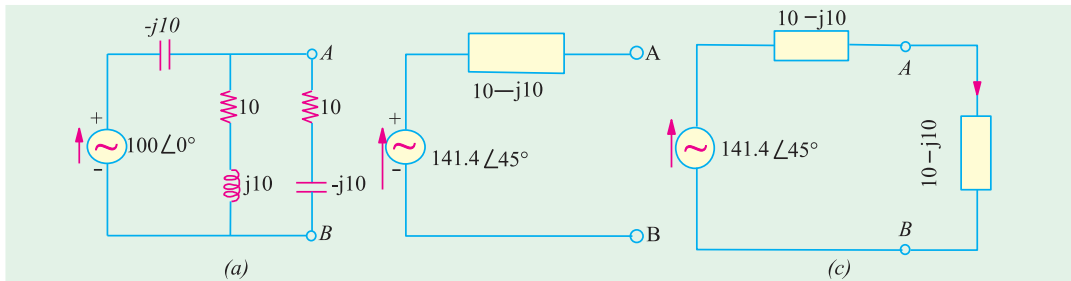


Fig. 15.33

The equivalent Thevenin's source is shown in Fig. 15.33 (b). Let the load be re-connected across A and B shown in Fig. 15.33 (c).

$$\begin{aligned} \therefore I_L &= \frac{141.4 \angle 45^\circ}{(10 - j10) + (10 - j10)} = \frac{141.4 \angle 45^\circ}{20 - j20} = \frac{141.4 \angle 45^\circ}{28.3 \angle -45^\circ} = 5 \angle 90^\circ \\ \text{Power delivered to the load} &= I_L^2 R_L = 5^2 \times 10 = 250 \text{ W} \end{aligned}$$

**Example 15.16.** Find the Thevenin's equivalent across terminals A and B of the networks shown in Fig. 15.34. (a).

**Solution.** The solution of this circuit involves the following steps :

(i) Let us find the equivalent Thevenin voltage  $V_{CD}$  and Thevenin impedance  $Z_{CD}$  as viewed from terminals C and D.

$$\begin{aligned} V_{CD} &= V \frac{Z_2}{Z_1 + Z_2} = \frac{100 \angle 0^\circ \times 20 \angle -30^\circ}{10 \angle 30^\circ + 20 \angle -30^\circ} = 75.5 \angle 19.1^\circ \text{ V} \\ Z_{CD} &= Z_1 \parallel Z_2 = \frac{10 \angle 30^\circ \times 20 \angle -30^\circ}{10 \angle 30^\circ + 20 \angle -30^\circ} = 7.55 \angle 10.9^\circ \text{ ohm} \end{aligned}$$

(ii) Using the source conversion technique (Art) we will replace the  $5 \angle 0^\circ$  current source by a voltage source as shown in Fig. 15.34 (b).

$$V_{EC} = 5 \angle 0^\circ \times 10 \angle -30^\circ = 50 \angle -30^\circ$$

Its series resistance is the same as  $Z_3 = 10 \angle -30^\circ$  as shown in Fig. 15.34 (b).

The polarity of the voltage source is such that it sends current in the direction EC, as before.

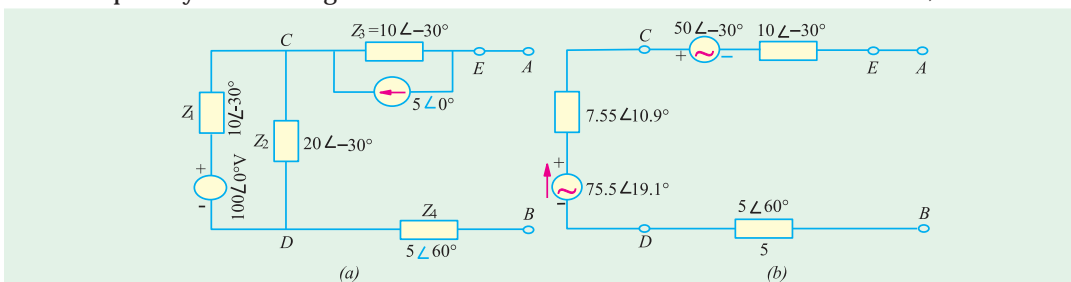


Fig. 15.34

(iii) From the above information, we can find  $V_{th}$  and  $Z_{th}$

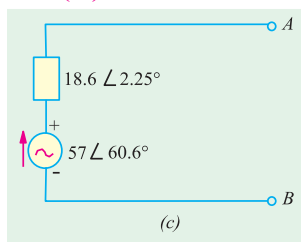


Fig. 15.34

$$V_{th} = V_{CD} = 75.5\angle 19.1^\circ - 50\angle 30^\circ = 57\angle 60.6^\circ$$

$$Z_{th} = Z_{CD} = 10\angle -30^\circ + 7.55\angle 10.9^\circ + 5\angle 60^\circ = 18.6\angle 2.25^\circ$$

The Thevenin equivalent with respect to the terminals  $A$  and  $B$  is shown in Fig. 15.34 (c).

For finding  $V_{AB}$  i.e. voltage at point  $A$  with respect to point  $B$ , we start from point  $B$  in Fig. 15.34 (b) and go to point  $A$  and calculate the phasor sum of the voltages met on the way.

$$\therefore V_{AB} = 75.1\angle 19.1^\circ - 50\angle 30^\circ = 57\angle 60.6^\circ$$

$$Z_{AB} = 10\angle 30^\circ + 7.55\angle 10.9^\circ + 5\angle 60^\circ = 18.6\angle 2.25^\circ$$

**Example 15.17.** For the network shown, determine using Thevenin's theorem, voltage across capacitor in Fig. 15.35. (Elect. Network Analysis Nagpur Univ. 1993)

$Z_{CD} = j5 \parallel (10 + j5) = 1.25 + j3.75$ . This impedance is in series with the  $10\Omega$  resistance. Using voltage divider rule, the drop over  $Z_{CD}$  is

**Solution.** When load of  $j5\Omega$  is removed the circuit becomes as shown in Fig. 15.35 (b). Thevenin voltage is given by the voltage drop produced by  $100\text{-V}$  source over  $(5 + j5)$  impedance. It can be calculated as under.

$$V_{CD} = 100 \frac{(1.25 + j3.75)}{10 + (1.25 + j3.75)} = \frac{125 + j375}{11.25 + j3.75}$$

This  $V_{CD}$  is applied across  $j5$  reactance as well as across the series combination of  $5\Omega$  and  $(5 + j5)\Omega$ . Again, using voltage-divider rule for  $V_{CD}$ , we get

$$V_{AB} = V_{th} = V_{CD} \times \frac{5 + j5}{10 + j5} = \frac{(125 + j375)}{11.25 + j3.75} \times \frac{5 + j5}{10 + j5} = 21.1\angle 71.57^\circ = 6.67 + j20$$

As looked into terminals  $A$  and  $B$ , the equivalent impedance is given by

$$R_{AB} = R_{th} = (5 + j5) \parallel (5 + 10 \parallel j5) = (5 + j5) \parallel (7 + j4) = 3 + j2.33$$

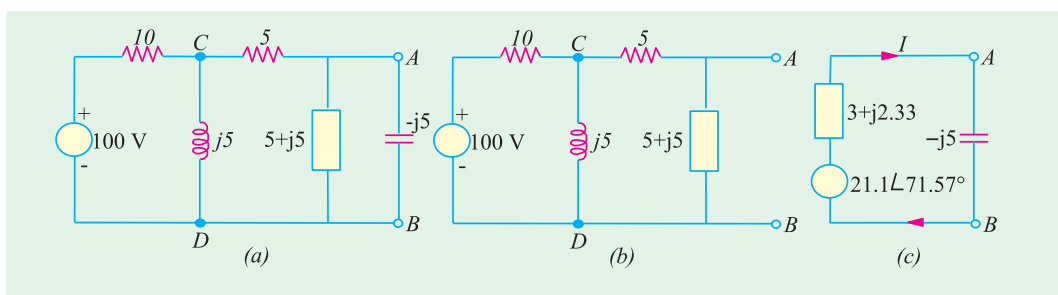


Fig. 15.35

The equivalent Thevenin's source along with the load is shown in Fig. 15.35 (c).

$$\text{Total impedance} = 3 + j2.33 - j5 = 3 - j2.67 = 4.02\angle -41.67^\circ$$

$$\therefore I = 21.1\angle 71.57^\circ / 4.02\angle -41.67^\circ = 5.25\angle 113.24^\circ$$

### Solution by Mesh Resistance Matrix

The different items of the mesh resistance matrix  $[R_m]$  are as under :

$$R_{11} = 10 + j5; R_{22} = 10 + j10; R_{33} = 5;$$

$$R_{12} = R_{21} = -j5;$$

$R_{23} = R_{32} = -(5 + j5)$ ;  $R_{31} = R_{13} = 0$ . Hence, the mesh equations in the matrix form are as given below :

$$\begin{bmatrix} (10 + j5) & -j5 & 0 \\ -j5 & (10 + j10) & -(5 + j5) \\ 0 & -(5 + j5) & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \Delta = (10 + j5) [5(10 + j10) - (5 + j5)(5 + j5)] + j5(-j25) = 625 + j250 = 673 \angle 21.8^\circ$$

$$\begin{bmatrix} (10 + j5) & -j5 & 100 \\ -j5 & (10 + j10) & 0 \\ 0 & -(5 + j5) & 0 \end{bmatrix} = j5(500 + j500) = 3535 \angle 135^\circ$$

$$\therefore I_3 = \Delta_3 / \Delta = 3535 \angle 135^\circ / 673 \angle 21.8^\circ = 5.25 \angle 113.2^\circ$$

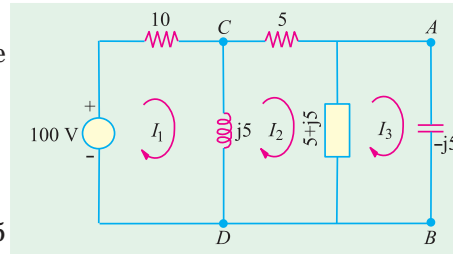


Fig. 15.36

### Tutorial Problems No. 15.5

1. Determine the Thevenin's equivalent circuit with respect to terminals  $AB$  of the circuit shown in Fig. 15.37. [ $V_{th} = 14.3 \angle 6.38^\circ$ ;  $Z_m = (4 + j0.55) \Omega$ ]

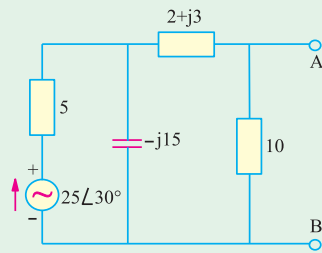


Fig. 15.37

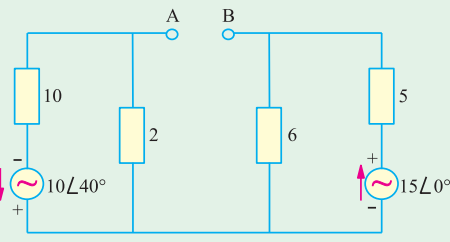


Fig. 15.38

2. Determine Thevenin's equivalent circuit with respect to terminals  $AB$  in Fig. 15.38. [ $V_{th} = 9.5 \angle 6.46^\circ$ ;  $Z_{th} = 4.4 \angle 0^\circ$ ]
3. The e.m.f.s. of two voltage source shown in Fig. 15.39 are in phase with each other. Using Thevenin's theorem, find the current which will flow in a  $16 \Omega$  resistor connected across terminals  $A$  and  $B$ . [ $V_{th} = 100 \text{ V}$ ;  $Z_{th} = (48 + j32)$ ;  $I = 1.44 \angle -26.56^\circ$ ]

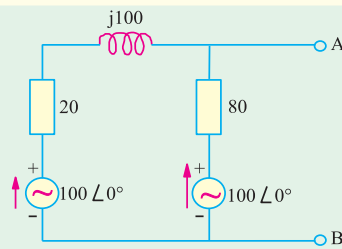


Fig. 15.39

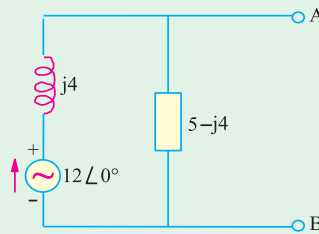


Fig. 15.40

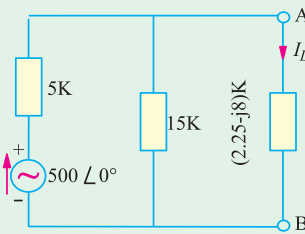


Fig. 15.41

4. Find the Thevenin's equivalent circuit for terminals  $AB$  for the circuit shown in Fig. 15.40. [ $V_{th} = 15.37 \angle 38.66^\circ$ ;  $Z_{th} = (3.2 + j4) \Omega$ ]
5. Using Thevenin's theorem, find the magnitude of the load current  $I_L$  passing through the load connected across terminals  $AB$  of the circuit shown in Fig. 15.41. [37.5 mA]

6. By using Thevenin's theorem, calculate the current flowing through the load connected across terminals A and B of circuit shown in Fig. 15.42. All resistances and reactances are in ohms.

$[V_{th} = 56.9 \angle 50.15^\circ ; 3.11 \angle 85.67^\circ]$

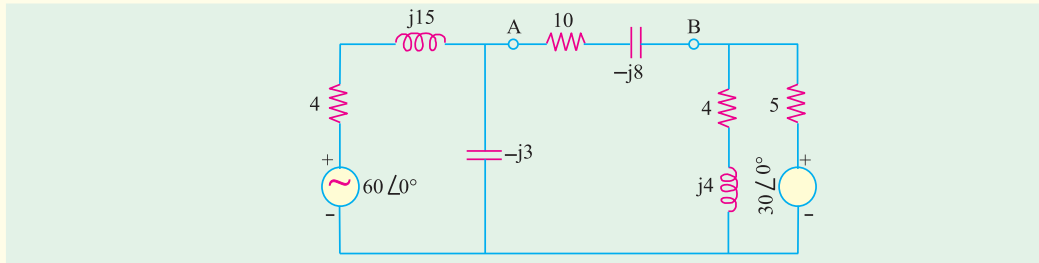


Fig. 15.42

7. Calculate the equivalent Thevenin's source with respect to the terminals A and B of the circuit shown in Fig. 15.43.  $[V_{th} = (6.34 + j2.93) \text{ V} ; Z_{th} = (3.17 - j5.07) \Omega]$
8. What is the Thevenin's equivalent source with respect to the terminals A and B of the circuit shown in Fig. 15.44 ?  $[V_{th} = (9.33 + j8) \text{ V} ; Z_{th} = (8 - j11) \Omega]$

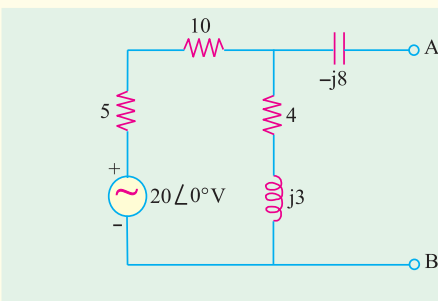


Fig. 15.43

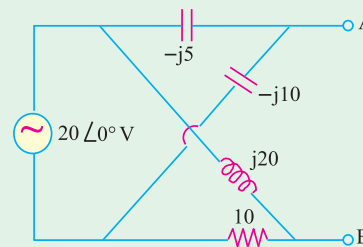


Fig. 15.44

9. What is the Thevenin's equivalent source with respect to terminals A and B the circuit shown in Fig. 15.45 ? Also, calculate the value of impedance which should be connected across AB for MPT. All resistances and reactances are in ohms.  $[V_{th} = (16.87 + j15.16) \text{ V} ; Z_{th} = (17.93 - j1.75) \Omega ; (17.93 + j1.75) \Omega]$

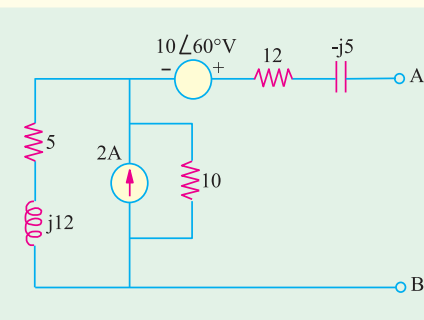


Fig. 15.45

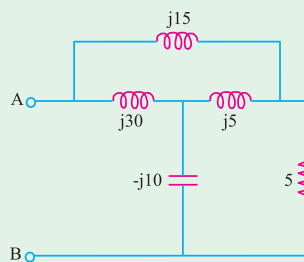


Fig. 15.46

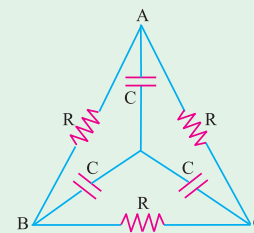


Fig. 15.47

10. Find the impedance of the network shown in Fig. 15.46, when viewed from the terminals A and B. All resistances and reactances are in ohms.  $[(4.435 + j6.878)]$
11. Find the value of the impedance that would be measured across terminals BC of the circuit shown in Fig. 15.47.

$$\left[ \frac{2R}{9 + \omega^2 C^2 R^2} (3 - j\omega CR) \right]$$

### 15.7. Reciprocity Theorem

This theorem applies to networks containing linear bilateral elements and a single voltage source or a single current source. This theorem may be stated as follows :

If a voltage source in branch  $A$  of a network causes a current of 1 branch  $B$ , then shifting the voltage source (but not its impedance) of branch  $B$  will cause the same current  $I$  in branch  $A$ .

It may be noted that currents in other branches will generally not remain the same. A simple way of stating the above theorem is that if an ideal voltage source and an ideal ammeter are inter-changed, the ammeter reading would remain the same. The ratio of the input voltage in branch  $A$  to the output current in branch  $B$  is called the transfer impedance.

Similarly, if a current source between nodes 1 and 2 causes a potential difference of  $V$  between nodes 3 and 4, shifting the current source (but not its admittance) to nodes 3 and 4 causes the same voltage  $V$  between nodes 1 and 2.

In other words, the interchange of an ideal current source and an ideal voltmeter in any linear bilateral network does not change the voltmeter reading.

However, the voltages between other nodes would generally not remain the same. The ratio of the input current between one set of nodes to output voltage between another set of nodes is called the transfer admittance.

**Example 15.18.** Verify Reciprocity theorem for  $V$  &  $I$  in the circuit shown in Fig. 15.48.

[Elect. Network Analysis, Nagpur Univ. 1993]

**Solution.** We will find the value of the current  $I$  as read by the ammeter first by applying series parallel circuits technique and then by using mesh resistance matrix (Art.)

#### 1. Series Parallel Circuit Technique

The total impedance as seen by the voltage source is

$$= 1 + [j1 \parallel (2 - j1)] = 1 + \frac{j1(2 - j1)}{2} = 1.5 + j1$$

$$\therefore \text{total circuit current } i = \frac{5 \angle 0^\circ}{1.5 + j1}$$

This current gets divided into two parts at point  $A$ , one part going through the ammeter and the other going along  $AB$ . By using current-divider rule. (Art), we have

$$I = \frac{5}{1.5 + j1} \times \frac{j1}{(2 + j1 - j1)} = \frac{j5}{3 + j2}$$

#### 2. Mesh Resistance Matrix

In Fig. 15.48 (b),  $R_{11} = (1 + j1)$ ,  $R_{22} = (2 + j1 - j1) = 2$ ;  $R_{12} = R_{21} = -j1$

$$\therefore \begin{vmatrix} (1+j1) & -j1 \\ -j1 & 2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 5 \\ j0 \end{vmatrix}; \Delta = 2(1+j1) - (j1)(-j1) = 3 + j2$$

$$\Delta_2 = \begin{vmatrix} (1+j1) & 5 \\ -j1 & 0 \end{vmatrix} = 0 + j5 = j5; I_2 = I = \frac{\Delta_2}{\Delta} = \frac{j5}{(3 + j2)}$$

As shown in Fig. 15.48 (c), the voltage source has been interchanged with the ammeter. The polarity of the voltage source should be noted in particular. It looks as if the voltage source has been pushed along the wire in the counterclockwise direction to its new position, thus giving the voltage polarity as shown in the figure. We will find the value of  $I$  in the new position of the ammeter by using the same two techniques as above.

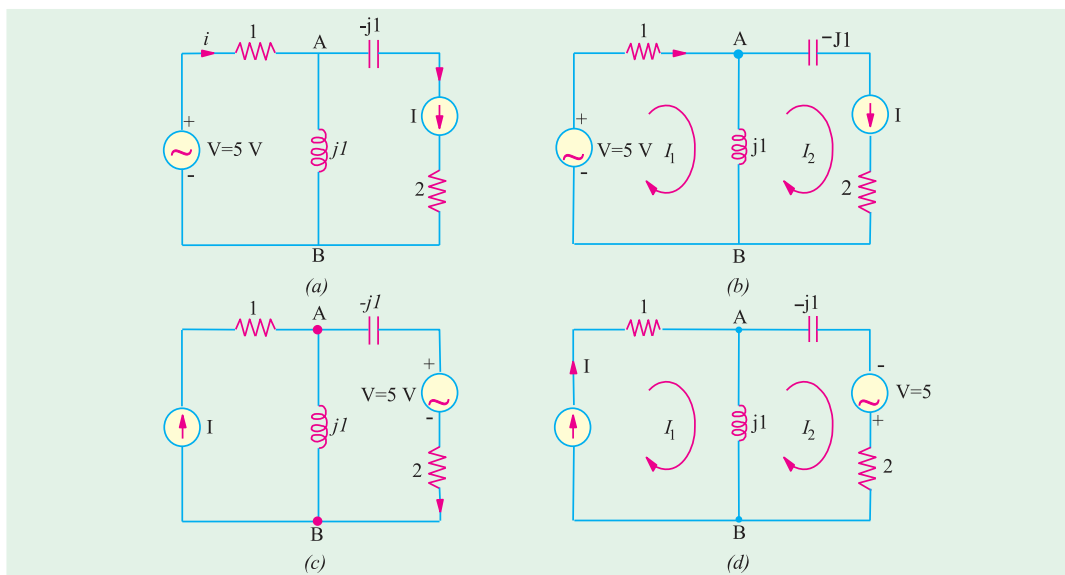


Fig. 15.48

### 1. Series Parallel Circuit Technique

As seen by the voltage source from its new position, the total circuit impedance is

$$= 2(2 - j1) + j1 \parallel 1 = \frac{3 + j2}{1 + j1}$$

The total circuit current  $i = 5 \times \frac{1 + j1}{3 + j2}$

This current  $i$  gets divided into two parts at point  $B$  as per the current-divider rule.

$$I = \frac{5(1 + j1)}{3 + j2} \times j \frac{1}{1 + j1} = \frac{j5}{3 + j2}$$

### 2. Mesh Resistance Matrix

As seen from Fig. 15.48 (d).

$$\begin{vmatrix} (1 + j1) & -j1 \\ -j1 & 2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 5 \end{vmatrix}; \Delta = 2(1 + j1) + 1 = 3 + j2$$

$$\Delta = \begin{vmatrix} 0 & -j1 \\ 5 & 2 \end{vmatrix} = j5; I = I_1 = \frac{\Delta_1}{\Delta} = \frac{j5}{3 + j2}$$

The reciprocity theorem stands verified from the above results.

### Tutorial problem No. 15.6

1. State reciprocity theorem. Verify for the circuit Fig. 15.49, with the help of any suitable current through any element.  
(Elect. Network Analysis Nagpur Univ. 1993)

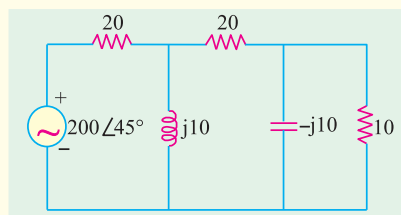


Fig. 15.49

### 15.8. Norton's Theorem

As applied to a.c. networks, this theorem can be stated as under :

Any two terminal active linear network containing voltage sources and impedances when viewed from its output terminals is equivalent to a constant current source and a parallel impedance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and the parallel impedance is the impedance of the network when viewed from open-circuited terminals after voltage sources have been replaced by their internal impedances (if any) and current sources by infinite impedance.

**Example 15.19.** Find the Norton's equivalent of the circuit shown in Fig. 15.50. Also find the current which will flow through an impedance of  $(10 - j20) \Omega$  across the terminals A and B.

**Solution.** As shown in Fig. 15.50 (b), the terminals A and B have been short-circuited.

$$\therefore I_{SC} = I_N = 25/(10 + j20) = 25/22.36 \angle 63.4^\circ = 1.118 \angle 63.4^\circ$$

When voltage source is replaced by a short, then the internal resistance of the circuit, as viewed from open terminals A and B, is  $R_N = (10 + j20)\Omega$ . Hence, Norton's equivalent circuit becomes as shown in Fig. 15.50(c).

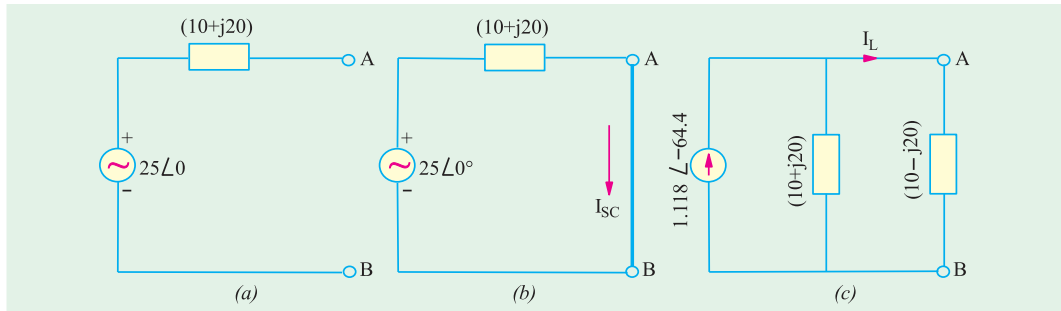


Fig. 15.50

When the load impedance of  $(10 - j20)$  is applied across the terminals A and B, current through it can be found with the help of current-divider rule.

$$\therefore I_L = 1.118 \angle 63.4^\circ \times \frac{(10 + j20)}{(10 + j20) + (10 - j20)} = 1.25 \text{ A}$$

**Example 15.20.** Use Norton's theorem to find current in the load connected across terminals A and B of the circuit shown in Fig. 15.51 (a).

**Solution.** The first step is to short-circuit terminals A and B as shown in Fig. 15.51 (a)\*. The short across A and B not only short-circuits the load but the  $(10 + j10)$  impedance as well.

$$I_N = 100 \angle 0^\circ / (j10) = j10 = 10 \angle 90^\circ$$

Since the impedance of the Norton and Thevenin equivalent circuits is the same,  $Z_N = 10 - j10$ .

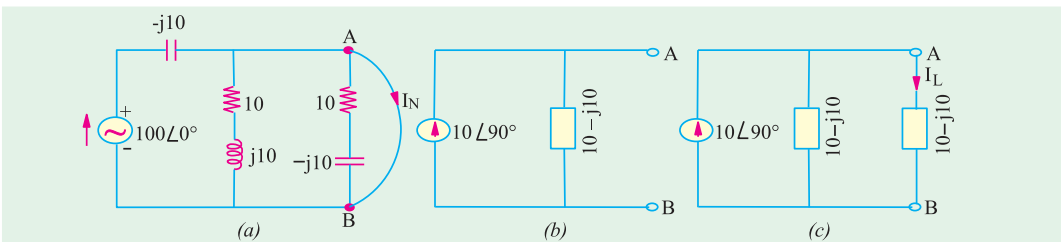


Fig. 15.51

\* For finding  $I_N$ , we may or may not remove the load from the terminals (because, in either case, it would be short-circuited) but for finding  $R_N$ , it has to be removed as in the case of Thevenin's theorem.



The Norton's equivalent circuit is shown in Fig. 15.51 (b). In Fig. 15.51 (c), the load has been reconnected across the terminals A and B. Since the two impedances are equal, current through each is half of the total current *i.e.*  $10\angle 90^\circ/2 = 5\angle 90^\circ$ .

### Tutorial Problems No. 15.7

1. Find the Norton's equivalent source with respect to terminals A and B of the networks shown in Fig. 15.51 (a) (b). All resistances and reactances are expressed in siemens in Fig. 15.51 (a) and in ohms in Fig. 15.52.

**[(a)  $I_N = -(2.1 - j3) \text{ A}$ ;  $1/Z_N = (0.39 + j0.3) \text{ S}$  (b)  $I_N = (6.87 + j0.5) \text{ A}$ ;  $1/Z_N = (3.17 + j1.46) \text{ S}$ ]**

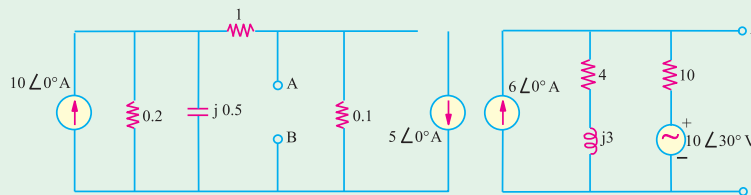


Fig. 15.52

Fig. 15.53

2. Find the Norton's equivalent source with respect to terminals A and B for the circuit shown in Fig. 15.54. Hence, find the voltage  $V_L$  across the  $100 \Omega$  load and check its result by using Millman's theorem. All resistances are in ohms.

**$[I_N = 9 \text{ A}$ ;  $Y_N = 0.15 \text{ S}$ ;  $V_L = 56.25 \angle 0^\circ]$**

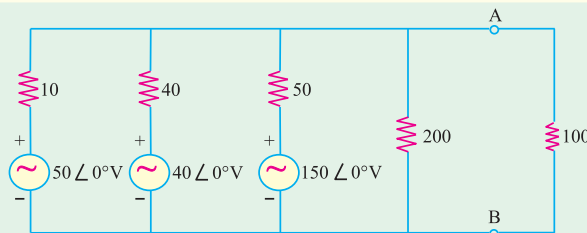


Fig. 15.54

3. Find the Norton's equivalent network at terminals AB of the circuit shown in Fig. 15.55.

**$[I_{SC} = 2.17 \angle 44^\circ \text{ A}$ ;  $Z_N = (2.4 + j1.47) \Omega]$**

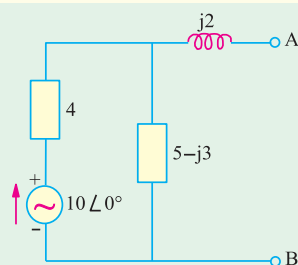


Fig. 15.55

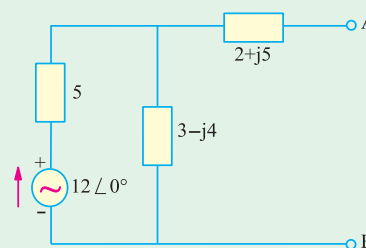


Fig. 15.56

4. What is the Norton equivalent circuit at terminals AB of the network shown in Fig. 15.56

**$[I_{SC} = 1.15 \angle -66.37^\circ \text{ A}$ ;  $Z_N = (4.5 + j3.75) \Omega]$**

### 15.9. Maximum Power Transfer Theorem

As explained earlier in Art. this theorem is particularly useful for analysing communication networks where the goal is transfer of maximum power between two circuits and not highest efficiency.

### 15.10. Maximum Power Transfer Theorems - General Case

We will consider the following maximum power transfer theorems when the source has a fixed complex impedance and delivers power to a load consisting of a variable resistance or a variable complex impedance.

**Case 1.** When load consists only for a variable resistance  $R_L$  [Fig. 15.57 (a)]. The circuit current is

$$I = \frac{V_g}{\sqrt{(R_g + R_L)^2 + X_g^2}}$$

$$\text{Power delivered to } R_L \text{ is } P_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2}$$

To determine the value of  $R_L$  for maximum transfer of power, we should set the first derivative  $dP_L/dR_L$  to zero.

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L} \left[ \frac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2} \right] = V_g^2 \left\{ \frac{[(R_g + R_L)^2 + X_g^2] - R_L(2)(R_g + R_L)}{[(R_g + R_L)^2 + X_g^2]^2} \right\} = 0$$

$$\text{or } R_g^2 + 2R_g R_L + R_L^2 + X_g^2 - 2R_L R_g - 2R_L^2 = 0 \text{ and } R_g^2 + X_g^2 = R_L^2$$

$$\therefore R_L = \sqrt{R_g^2 + X_g^2} = |Z_g|$$

It means that with a variable pure resistive load, maximum power is delivered across the terminals of an active network only when the load resistance is equal to the absolute value of the impedance of the active network. Such a match is called magnitude match.

Moreover, if  $X_g$  is zero, then for maximum power transfer  $R_L = R_g$

**Case 2.** Load impedance having both variable resistance and variable reactance [Fig. 15.57 (b)].

$$\text{The circuit current is } I = \frac{V_g}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$$

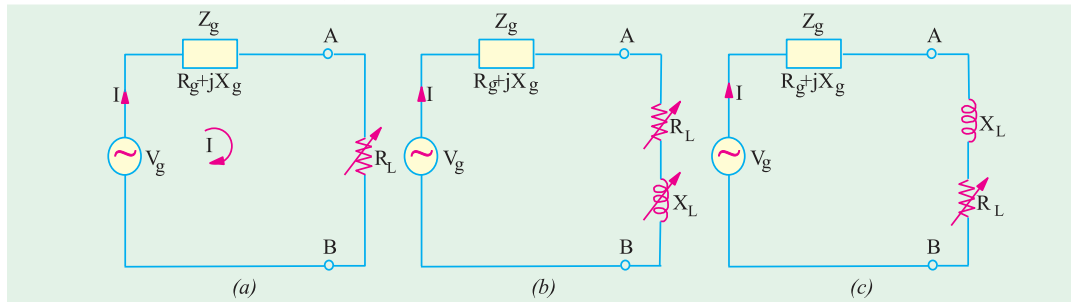


Fig. 15.57

$$\text{The power delivered to the load is } = P_L = I^2 R_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}$$

$$\text{Now, if } R_L \text{ is held fixed, } P_L \text{ is maximum when } X_g = -X_L. \text{ In that case } P_{Lmax} = \frac{V_g^2 R_L}{(R_g + R_L)^2}$$

If on the other hand,  $R_L$  is variable then, as in **Case 1** above, maximum power is delivered to the load when  $R_L = R_g$ . In that case if  $R_L = R_g$  and  $X_L = -X_g$ , then  $Z_L = Z_g^*$ . Such a match is called conjugate match.

From the above, we come to the conclusion that in the case of a load impedance having both variable resistance and variable reactance, maximum power transfer across the terminals of the active network occurs when  $Z_L$  equals the complex conjugate of the network impedance  $Z_g$  i.e. the two impedances are conjugately matched.

**Case 3.**  $Z_L$  with variable resistance and fixed reactance [Fig. 15.57 (c)]. The equations for current  $I$  and power  $P_L$  are the same as in **Case 2** above except that we will consider  $X_L$  to remain constant. When the first derivative of  $P_L$  with respect to  $R_L$  is set equal to zero, it is found that

$$R_L^2 = R_g^2 + (X_g + X_L)^2 \text{ and } R_L = |Z_g + jZ_L|$$

Since  $Z_g$  and  $X_L$  are both fixed quantities, these can be combined into a single impedance. Then with  $R_L$  variable, **Case 3** is reduced to **Case 1** and the maximum power transfer takes place when  $R_L$  equals the absolute value of the network impedance.

### Summary

The above facts can be summarized as under :

1. When load is purely resistive and adjustable, MPT is achieved when  $R_L = |Z_g| = \sqrt{R_g^2 + X_g^2}$ .
2. When both load and source impedances are purely resistive (i.e.  $X_L = X_g = 0$ ), MPT is achieved when  $R_L = R_g$ .
3. When  $R_L$  and  $X_L$  are both independently adjustable, MPT is achieved when  $X_L = -X_g$  and  $R_L = R_g$ .
4. When  $X_L$  is fixed and  $R_L$  is adjustable, MPT is achieved when  $R_L = \sqrt{[R_g^2 + (X_g + X_L)^2]}$

**Example 15.21.** In the circuit of Fig. 15.58, which load impedance of p.f. = 0.8 lagging when connected across terminals A and B will draw the maximum power from the source. Also find the power developed in the load and the power loss in the source.

**Solution.** For maximum power transfer  $|Z_L| = |Z_1| \sqrt{3^2 + 5^2} = 5.83 \Omega$

For p.f. = 0.8,  $\cos \phi = 0.8$  and  $\sin \phi = 0.6$ .

$$\therefore R_L = Z_L \cos \phi = 5.83 \times 0.8 = 4.66 \Omega$$

$$X_L = Z_L \sin \phi = 5.83 \times 0.6 = 3.5 \Omega$$

$$\text{Total circuit impedance } Z = \sqrt{[(R_1 + R_L)^2 + (X_1 + X_L)^2]}$$

$$= \sqrt{[(3 + 4.66)^2 + (5 + 3.5)^2]} = 11.44 \Omega$$

$$\therefore I = V/Z = 20/11.44 = 1.75 \text{ A.}$$

$$\text{Power in the load} = I^2 R_L = 1.75^2 \times 4.66 = 14.3 \text{ W}$$

$$\text{Power loss in the source} = 1.75^2 \times 3 = 9.2 \text{ W.}$$

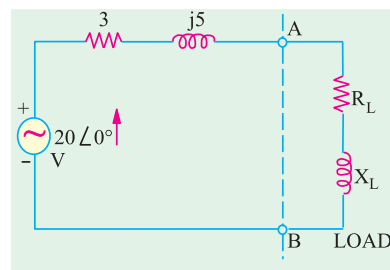


Fig. 15.58

**Example 15.22.** In the network shown in Fig. 15.59 find the value of load to be connected across terminals AB consisting of variable resistance  $R_L$  and capacitive reactance  $X_C$  which would result in maximum power transfer. (Network Analysis, Nagpur Univ. 1993)

**Solution.** We will first find the Thevenin's equivalent circuit between terminals A and B. When the load is removed, the circuit become as shown in Fig. 15.59 (b).

$$\begin{aligned} V_{th} &= \text{drop across } (2 + j10) = 50 \angle 45^\circ \times \frac{2 + j10}{7 + j10} \\ &= 41.8 \times 68.7^\circ = 15.2 + j38.9 \\ R_{th} &= 5 \parallel (2 + j10) = 4.1 \angle 23.7^\circ = 3.7 + j1.6 \end{aligned}$$

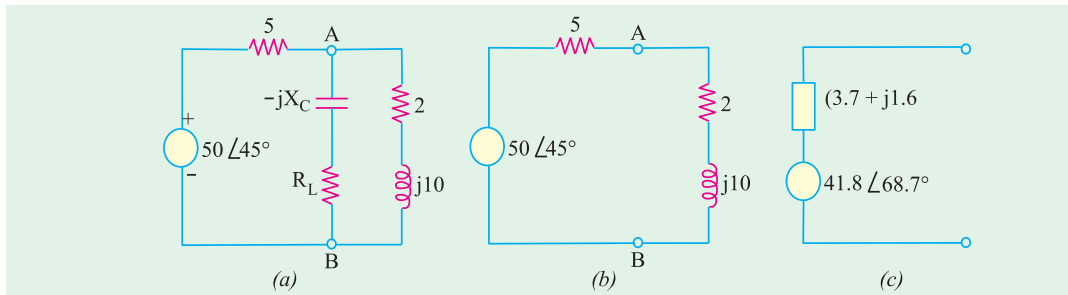


Fig. 15.59

The Thevenin's equivalent source is shown in Fig. 15.59 (c)

Since for MPT, conjugate match is required hence,  $X_C = 1.6 \Omega$  and  $R_L = 3.7 \Omega$

### Tutorial Problem No. 15.8

1. In the circuit of Fig. 15.60 the load consists of a fixed inductance having a reactance of  $j10 \Omega$  and a variable load resistor  $R_L$ . Find the value of  $R_L$  for MPT and the value of this power.

[58.3  $\Omega$  ; 46.2 W]

2. In the circuit of Fig. 15.61, the source resistance  $R_g$  is variable between  $5 \Omega$  and  $50 \Omega$  but  $R_L$  has a fixed value of  $25 \Omega$ . Find the value of  $R_g$  for which maximum power is dissipated in the load and the value of this power.

[5  $\Omega$  ; 250 W]

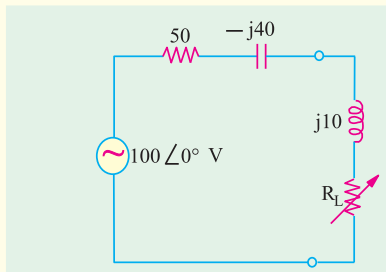


Fig. 15.60

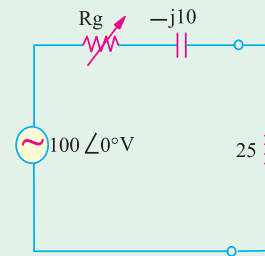


Fig. 15.61

### 15.11. Millman's Theorem

It permits any number of parallel branches consisting of voltage sources and impedances to be reduced to a single equivalent voltage source and equivalent impedance. Such multi-branch circuits are frequently encountered in both electronics and power applications.

**Example 15.23.** By using Millman's theorem, calculate node voltage  $V$  and current in the  $j6$  impedance of Fig. 15.62.

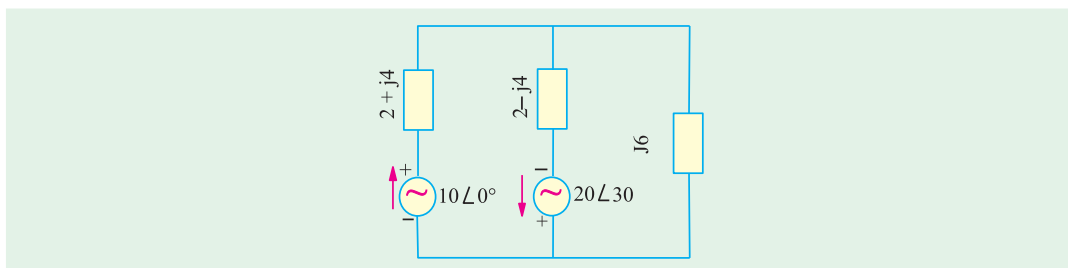


Fig. 15.62

**Solution.** According to Millman's theorem as applicable to voltage sources.

$$V = \frac{\pm V_1 Y_1 \pm V_2 Y_2 \pm V_3 Y_3 \pm \dots \pm V_n Y_n}{Y_1 + Y_2 + Y_3 + \dots + Y_n}$$

$$Y_1 = \frac{1}{2 + j4} = \frac{2 - j4}{20} = 0.01 - j0.02 = 0.022 \angle -63.4^\circ$$

$$Y_2 = \frac{1}{2 - j4} = \frac{2 + j4}{20} = 0.01 + j0.02 = 0.022 \angle 63.4^\circ$$

$$Y_3 = \frac{1}{j6} = 0.167 \angle -90^\circ = -j0.167$$

$$Y_1 + Y_2 + Y_3 = 0.02 - j0.167$$

In the present case,  $V_3 = 0$  and also  $V_2 Y_2$  would be taken as negative because current due to  $V_2$  flows away from the node.

$$\begin{aligned} \therefore V &= \frac{V_1 Y_1 - V_2 Y_2}{Y_1 + Y_2 + Y_3} = \frac{10 \angle 0^\circ \times 0.022 \angle -63.4^\circ - 20 \angle 30^\circ \times 0.022 \angle 63.4^\circ}{0.02 - j0.167} \\ &= 3.35 \angle 177^\circ \end{aligned}$$

$$\text{Current through } j6 \text{ impedance} = 3.35 \angle 177^\circ / 6 \angle 90^\circ = 0.56 \angle 87^\circ$$

### Tutorial Problems No. 15.9

1. With the help of Millman's theorem, calculate the voltage across the 1 K resistor in the circuit of Fig. 15.63.

[2.79 V]

2. Using Millman's theorem, calculate the voltage  $V_{ON}$  in the 3-phase circuit shown in Fig. 15.64. All load resistances and reactances are in milli-siemens.

[ $V_{ON} = 69.73 \angle 113.53^\circ$ ]

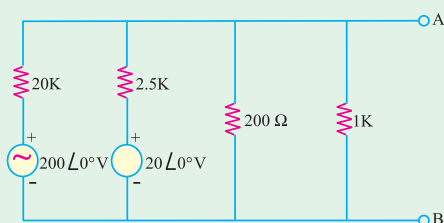


Fig. 15.63

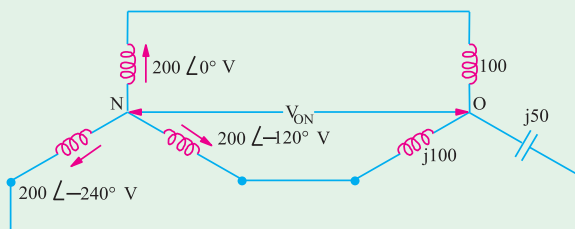


Fig. 15.64

3. Define mesh current and node voltage. *(Madras University, April 2002)*
4. State superposition theorem. *(Madras University, April 2002)*
5. State Millman's theorem. *(Madras University, April 2002)*
6. Two circuits the impedances of which are given by  $Z_1 = (10 + j15)\Omega$  and  $Z_2 = (6 - j8)\Omega$  are connected in parallel. If the total current supplied is 15 A. What is the power taken by each branch? *(RGPV Bhopal 2002)*

[RGPV Bhopal 2002]

## OBJECTIVE TYPES – 15

1. The Thevenin's equivalent resistance  $R_{th}$  for the given network is

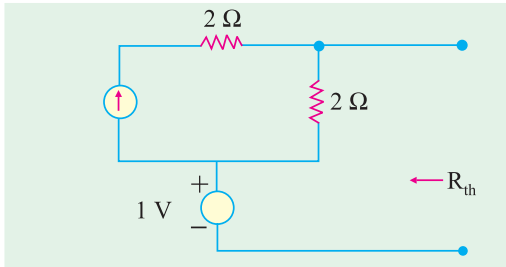


Fig. 15.65

- (a)  $1 \Omega$   
 (b)  $2 \Omega$   
 (c)  $4 \Omega$   
 (d) infinity

2. The Norton's equivalent of circuit shown in Figure 15.66 is drawn in the circuit shown in Figure 15.67. The values of  $I_{SC}$  and  $R_{eq}$  in Figure II are respectively

(ESE 2001)

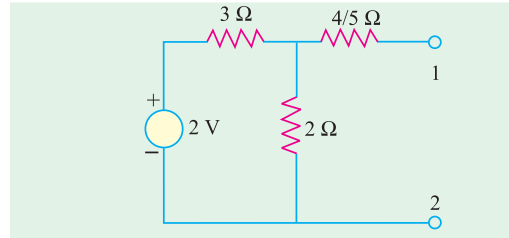


Fig. 15.66

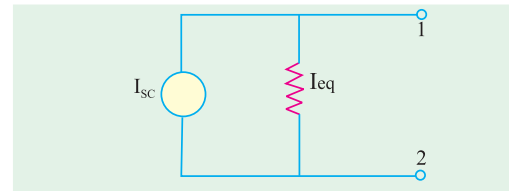


Fig. 15.67

- (a)  $\frac{5}{2}$  A and  $2 \Omega$     (b)  $\frac{2}{5}$  A and  $1 \Omega$   
 (c)  $\frac{4}{5}$  A and  $\frac{12}{5} \Omega$     (d)  $\frac{2}{5}$  A and  $2 \Omega$

# CHAPTER 16

## Learning Objectives

- A.C. Bridges
- Maxwell's Inductance Bridge
- Maxwell-Wien Bridge
- Anderson Bridge
- Hay's Bridge
- The Owen Bridge
- Heaviside Compbell Equal Ratio Bridge
- Capacitance Bridge
- De Sauty Bridge
- Schering Bridge
- Wien Series Bridge
- Wien Parallel Bridge

## A.C. BRIDGES



↑ A wide variety of AC bridge circuits (such as wheatstone) may be used for the precision measurement of AC resistance, capacitance and inductance

### 16.1. A.C. Bridges

Resistances can be measured by direct-current Wheatstone bridge, shown in Fig. 16.1 (a) for which the condition of balance is that

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} \text{ or } R_1 R_3 = R_2 R_4^*$$

Inductances and capacitances can also be measured by a similar four-arm bridge, as shown in Fig. 16.1 (b); instead of using a source of direct current, alternating current is employed and galvanometer is replaced by a vibration galvanometer (for commercial frequencies or by telephone detector if frequencies are higher (500 to 2000 Hz)).

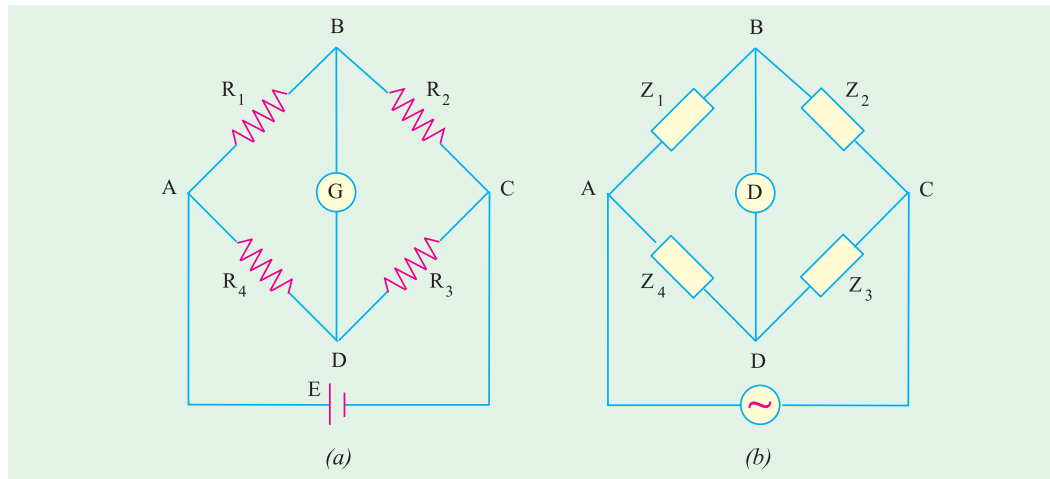


Fig. 16.1

The condition for balance is the same as before but instead of resistances, impedances are used *i.e.*

$$Z_1 / Z_2 = Z_4 / Z_3 \text{ or } Z_1 Z_3 = Z_2 Z_4$$

But there is one important difference *i.e.* not only should there be balance for the magnitudes of the impedances but also a phase balance. Writing the impedances in their polar form, the above condition becomes

$$Z_1 \angle \phi_1 \cdot Z_3 \angle \phi_3 = Z_2 \angle \phi_2 \cdot Z_4 \angle \phi_4 \text{ or } Z_1 Z_3 \angle \phi_1 + \phi_3 = Z_2 Z_4 \angle \phi_2 + \phi_4$$

Hence, we see that, in fact, there are two balance conditions which must be satisfied simultaneously in a four-arm a.c. impedance bridge.

(i)  $Z_1 Z_3 = Z_2 Z_4$  ... for magnitude balance

(ii)  $\phi_1 + \phi_3 = \phi_2 + \phi_4$  ... for phase angle balance

In this chapter, we will consider a few of the numerous bridge circuits used for the measurement of self-inductance, capacitance and mutual inductance, choosing as examples some bridges which are more common.

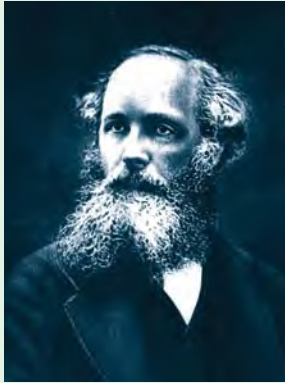
### 16.2. Maxwell's Inductance Bridge

The bridge circuit is used for medium inductances and can be arranged to yield results of considerable precision. As shown in Fig. 16.2, in the two arms, there are two pure resistances so

\* Products of opposite arm resistances are equal.



that for balance relations, the phase balance depends on the remaining two arms. If a coil of an unknown impedance  $Z_1$  is placed in one arm, then its positive phase angle  $\phi_1$  can be compensated for in either of the following two ways:



James Clark Maxwell

(i) A known impedance with an equal positive phase angle may be used in either of the *adjacent* arms (so that  $\phi_1 = \phi_3$  or  $\phi_2 = \phi_4$ ), remaining two arms have zero phase angles (being pure resistances). Such a network is known as Maxwell's a.c. bridge or  $L_1/L_4$  bridge.

(ii) Or an impedance with an equal *negative* phase angle (i.e. capacitance) may be used in *opposite* arm (so that  $\phi_1 + \phi_3 = 0$ ). Such a network is known as Maxwell-Wien bridge (Fig. 16.5) or Maxwell's L/C bridge.

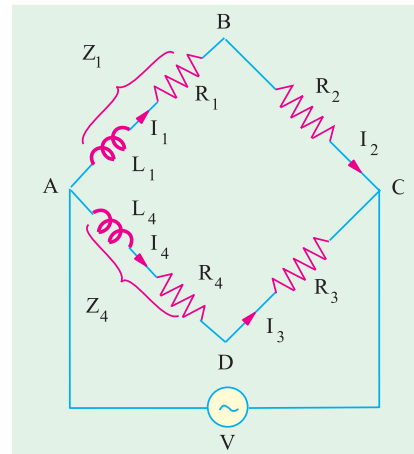


Fig. 16.2

Hence, we conclude that an inductive impedance may be measured in terms of another inductive impedance (of equal time constant) in either *adjacent* arm (Maxwell bridge) or the unknown inductive impedance may be measured in terms of a combination of resistance and capacitance (of equal time constant) in the *opposite* arm (Maxwell-Wien bridge). It is important, however, that in each case the time constants of the two impedances must be matched.

As shown in Fig. 16.2,

$$Z_1 = R_1 + jX_1 = R_1 + j\omega L_1 \dots \text{unknown}; Z_4 = R_4 + jX_4 = R_4 + j\omega L_4 \dots \text{known}$$

$R_2, R_3 =$  known pure resistances;  $D =$  detector

The inductance  $L_4$  is a variable self-inductance of constant resistance, its inductance being of the same order as  $L_1$ . The bridge is balanced by varying  $L_4$  and one of the resistances  $R_2$  or  $R_3$ .

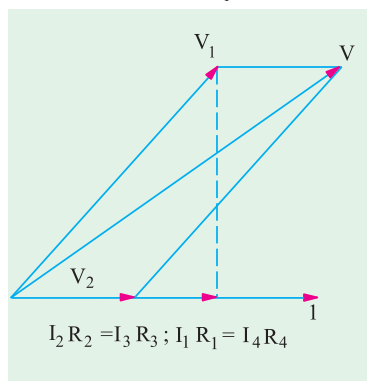


Fig. 16.3

Alternatively,  $R_2$  and  $R_3$  can be kept constant and the resistance of one of the other two arms can be varied by connecting an additional resistance in that arm (Ex. 16.1).

The balance condition is that  $Z_1 Z_3 = Z_2 Z_4$

$$\therefore (R_1 + j\omega L_1)R_3 = (R_4 + j\omega L_4)R_2$$

Equating the real and imaginary parts on both sides, we have

$$R_1 R_3 = R_2 R_4 \text{ or } R_1 / R_4 = R_2 / R_3 *$$

(i.e. products of the resistances of opposite arms are equal).

$$\text{and } \omega L_1 R_3 = \omega L_4 R_2 \text{ or } L_1 = L_4 \frac{R_2}{R_3}$$

We can also write that  $L_1 = L_4 \frac{R_1}{R_4}$

\* Or  $\frac{L_1}{R_1} = \frac{L_4}{R_4}$  i.e., the time constants of the two coils are matched.

Hence, the unknown self-inductance can be measured in terms of the known inductance  $L_4$  and the two resistors. Resistive and reactive terms balance independently and the conditions are independent of frequency. This bridge is often used for measuring the iron losses of the transformers at audio frequency.

The balance condition is shown vectorially in Fig. 16.3. The currents  $I_4$  and  $I_3$  are in phase with  $I_1$  and  $I_2$ . This is, obviously, brought about by adjusting the impedances of different branches, so that these currents lag behind the applied voltage  $V$  by the same amount. At balance, the voltage drop  $V_1$  across branch 1 is equal to that across branch 4 and  $I_3 = I_4$ . Similarly, voltage drop  $V_2$  across branch 2 is equal to that across branch 3 and  $I_1 = I_2$ .

**Example 16.1.** The arms of an a.c. Maxwell bridge are arranged as follows: AB and BC are non-reactive resistors of  $100\ \Omega$  each, DA is a standard variable reactor  $L_1$  of resistance  $32.7\ \Omega$  and CD comprises a standard variable resistor  $R$  in series with a coil of unknown impedance. Balance was obtained with  $L_1 = 47.8\ \text{mH}$  and  $R = 1.36\ \Omega$ . Find the resistance and inductance of the coil.

(Elect. Inst. & Meas. Nagpur Univ. 1993)

**Solution.** The a.c. bridge is shown in Fig. 16.4.

Since the products of the resistances of opposite arms are equal

$$\therefore 32.7 \times 100 = (1.36 + R_4) 100$$

$$\therefore 32.7 = 1.36 + R_4 \text{ or } R_4 = 32.7 - 1.36 = \mathbf{31.34\ \Omega}$$

$$\text{Since } L_1 \times 100 = L_4 \times 100 \therefore L_4 = L_1 = \mathbf{47.8\ \text{mH}}$$

or because time constants are the same, hence

$$L_1/32.7 = L_4/(31.34 + 1.36) \therefore L_4 = 47.8\ \text{mH}$$

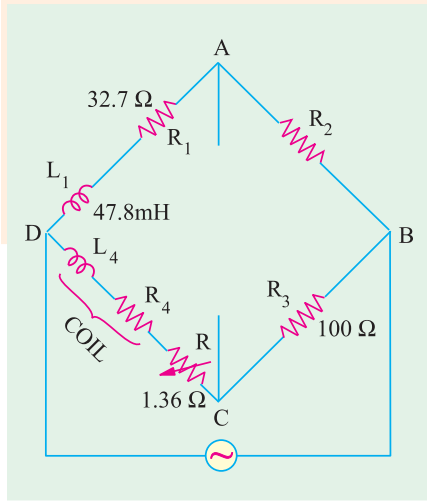


Fig. 16.4

### 16.3. Maxwell-Wien Bridge or Maxwell's L/C Bridge

As referred to in Art. 16.2, the *positive* phase angle of an inductive impedance may be compensated by the *negative* phase angle of a capacitive impedance put in the *opposite* arm. The unknown inductance then becomes known in terms of this capacitance.

Let us first find the combined impedance of arm 1.

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{-jX_C} = \frac{1}{R_1} + \frac{j}{X_C} = \frac{1}{R_1} + j\omega C = \frac{1 + j\omega CR_1}{R_1}$$

$$\therefore Z_1 = \frac{R_1}{1 + j\omega CR_1}; Z_2 = R_2$$

$$Z_3 = R_3 + j\omega L_3 \text{ and } Z_4 = R_4$$

Balance condition is  $Z_1 Z_3 = Z_2 Z_4$

$$\text{or } \frac{R_1(R_3 + jL_3)}{1 + j\omega CR_1} = R_2 R_4 \text{ or } R_1 R_3 + jL_3 R_1 = R_2 R_4 + j\omega CR_1 R_2 R_4$$

Separating the real and imaginaries, we get

$$R_1 R_3 = R_2 R_4 \text{ and } L_3 R_1 = \omega CR_1 R_2 R_4; R_3 = \frac{R_2 R_4}{R_1} \text{ and } L_3 = \omega CR_2 R_4$$

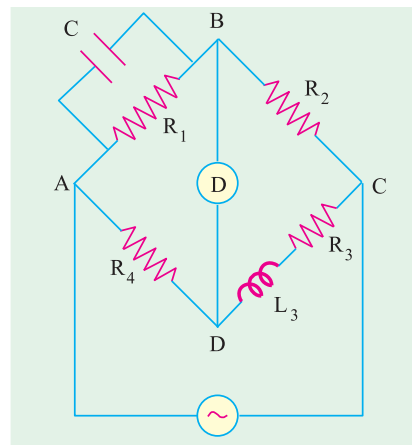


Fig. 16.5

**Example 16.2.** The arms of an a.c. Maxwell bridge are arranged as follows: AB is a non-inductive resistance of  $1,000 \Omega$  in parallel with a capacitor of capacitance  $0.5 \mu\text{F}$ , BC is a non-inductive resistance of  $600 \Omega$ , CD is an inductive impedance (unknown) and DA is a non-inductive resistance of  $400 \Omega$ . If balance is obtained under these conditions, find the value of the resistance and the inductance of the branch CD.

[Elect. & Electronic Meas, Madras Univ.]

**Solution.** The bridge is shown in Fig. 16.6. The conditions of balance have already been derived in Art. 16.3 above.

$$\text{Since } R_1 R_3 = R_2 R_4 \therefore R_3 = R_2 R_4 / R_1$$

$$\therefore R_3 = \frac{600 \times 400}{1000} = 240 \Omega$$

$$\begin{aligned} \text{Also } L_3 &= C R_2 R_4 \\ &= 0.5 \times 10^{-6} \times 400 \times 600 \\ &= 12 \times 10^{-2} = 0.12 \text{ H} \end{aligned}$$

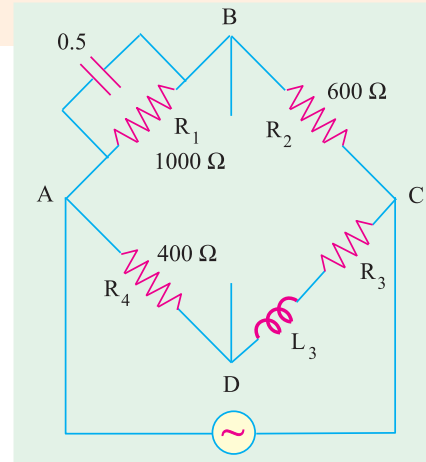


Fig. 16.6

#### 16.4. Anderson Bridge

It is a very important and useful modification of the Maxwell-Wien bridge described in Art. 16.3. In this method, the unknown inductance is measured in terms of a known capacitance and resistance, as shown in Fig. 16.7.

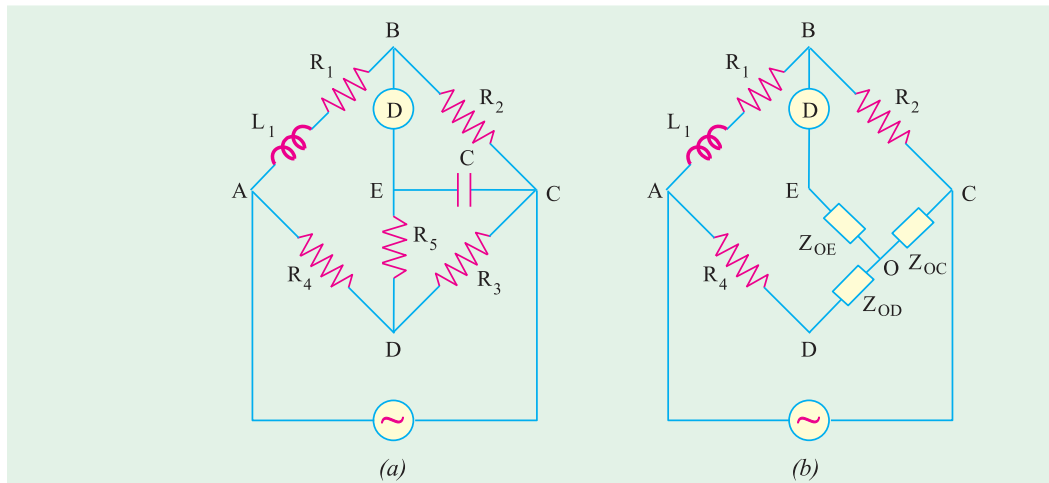


Fig. 16.7

The balance conditions for this bridge may be easily obtained by converting the mesh of impedances  $C$ ,  $R_5$  and  $R_3$  to an equivalent star with star point  $O$  by  $\Delta/Y$  transformation. As seen from Fig. 16.7 (b).

$$Z_{OD} = \frac{R_3 R_5}{(R_3 + R_5 + 1/jC)}; \quad Z_{OC} = \frac{R_3 / jC}{(R_3 + R_5 + 1/jC)} \quad Z_3$$

With reference to Fig. 16.7 (b) it is seen that

$$Z_1 = (R_1 + j\omega L_1) \quad Z_2 = R_2; \quad Z_3 = Z_{OC} \quad \text{and} \quad Z_4 = R_4 + Z_{OD}$$

For balance  $Z_1 Z_3 = Z_2 Z_4 \therefore (R_1 + j L_1) Z_{OC} = R_2 (R_4 + Z_{OD})$

$$\therefore (R_1 + j L_1) \frac{R_3 / j C}{(R_3 + R_5 - 1/j C)} = R_2 R_4 \frac{R_3 R_5}{R_3 + R_5 - 1/j C}$$

Further simplification leads to  $R_2 R_3 R_4 = R_2 R_4 R_5 + j \frac{R_2 R_4}{C} = R_2 R_3 R_5 + j \frac{R_1 R_3}{C} = \frac{R_3 L_1}{C}$

$$\therefore \frac{j R_2 R_4}{C} = \frac{j R_1 R_3}{C} \text{ or } R_1 = R_2 R_4 / R_3$$

$$\text{Also } \frac{R_3 L_1}{C} = R_2 R_3 R_4 = R_2 R_3 R_5 + R_2 R_4 R_5 \therefore L_1 = C R_2 (R_4 + R_5) + \frac{R_4 R_5}{R_3}$$

This method is capable of precise measurements of inductances over a wide range of values from a few micro-henrys to several henrys and is one of the commonest and the best bridge methods.

**Example 16.3.** An alternating current bridge is arranged as follows: The arms AB and BC consists of non-inductive resistances of 100-ohm each, the arms BE and CD of non-inductive variable resistances, the arm EC of a capacitor of 1  $\mu\text{F}$  capacitance, the arm DA of an inductive resistance. The alternating current source is connected to A and C and the telephone receiver to E and D. A balance is obtained when resistances of arms CD and BE are 50 and 2,500 ohm respectively. Calculate the resistance and inductance of arm DA.

Draw the vector diagram showing voltage at every point of the network.

(Elect. Measurements, Pune Univ.)

**Solution.** The circuit diagram and voltage vector diagram are shown in Fig. 16.8. As seen,  $I_2$  is vector sum of  $I_C$  and  $I_3$ . Voltage  $V_2 = I_2 R_2 = I_C X_C$ . Also, vector sum of  $V_1$  and  $V_2$  is  $V$  as well as that of  $V_3$  and  $V_4$ .  $I_C$  is at right angles to  $V_2$ .

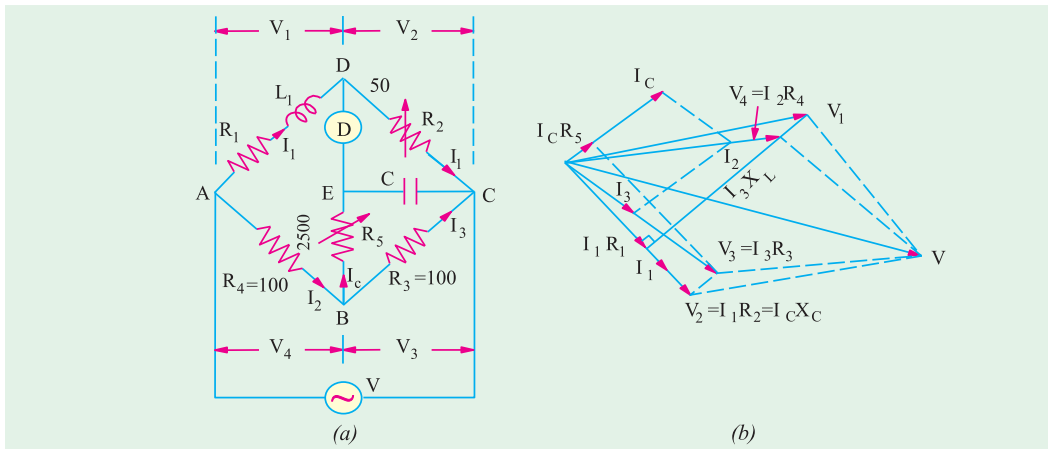


Fig. 16.8

Similarly,  $V_3$  is the vector sum of  $V_2$  and  $I_C R_5$ .

As shown in Fig. 16.8,  $R_1 = R_2$ .  $R_4/R_3 = 50 \times 100/100 = 50 \Omega$

The inductance is given by  $L = C R_2 (R_4 + R_5 + R_4 R_5 / R_3)$

$$\therefore L = 1 \times 10^{-6} (50(100 + 2500 + 100 \times 2500/100)) = 0.2505 \text{ H}$$

**Example 16.4.** Fig. 16.9 gives the connection of Anderson's bridge for measuring the inductance  $L_1$  and resistance  $R_1$  of a coil. Find  $R_1$  and  $L_1$  if balance is obtained when  $R_3 = R_4 = 2000$  ohms,  $R_2 = 1000$  ohms  $R_5 = 200$  ohms and  $C = 1\mu F$ . Draw the vector diagram for the voltages and currents in the branches of the bridge at balance.

(Elect. Measurements, AMIE Sec. B Summer 1990)

**Solution.**  $R_1 = R_2 R_4 / R_3 = 1000 \times 2000 / 2000 = 1000 \Omega$

$$L_1 = \frac{C R_2 R_4 R_5}{R_3} = 1 \times 10^{-6} \times 1000 \times 2000 \times 200 \times \frac{2000}{2000} = 2.4 \text{ H}$$

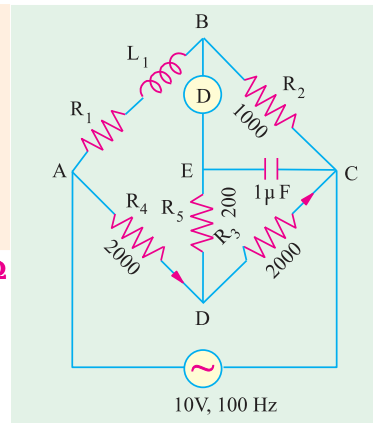


Fig. 16.9

### 16.5. Hay's Bridge

It is also a modification of the Maxwell-Wien bridge and is particularly useful if the phase angle of the inductive impedance  $\tan^{-1}(L/R)$  is large. The network is shown in Fig. 16.10. It is seen that, in this case, a comparatively smaller series resistance  $R_1$  is used instead of a parallel resistance (which has to be of a very large value).

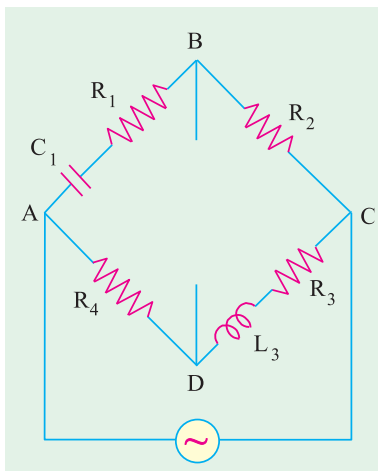


Fig. 16.10

Here  $Z_1 = R_1 - \frac{j}{C_1}$ ;  $Z_2 = R_2$

$Z_3 = R_3 + j L_3$ ;  $Z_4 = R_4$

Balance condition is  $Z_1 Z_3 = Z_2 Z_4$

or  $R_1 - \frac{j}{C_1} (R_3 + j L_3) = R_2 R_4$

Separating the reals and the imaginaries, we obtain

$R_1 R_3 - \frac{L_3}{C_1} = R_2 R_4$  and  $L_3 R_1 = \frac{R_3}{C_1}$

Solving these simultaneous equations, we get

$L_3 = \frac{C_1 R_2 R_4}{1 - \frac{R_3^2}{R_1^2 C_1^2}}$  and  $R_3 = \frac{R_1^2 C_1^2 R_2 R_4}{1 - \frac{R_3^2}{R_1^2 C_1^2}}$

The symmetry of expressions should help the readers to remember the results even when branch elements are exchanged, as in Ex. 16.5.

**Example 16.5.** The four arms of a Hay's a.c. bridge are arranged as follows: AB is a coil of unknown impedance; BC is a non-reactive resistor of  $1000 \Omega$ ; CD is a non-reactive resistor of  $833 \Omega$  in series with a standard capacitor of  $0.38 \mu F$ ; DA is a non-reactive resistor of  $16,800 \Omega$ . If the supply frequency is  $50 \text{ Hz}$ , determine the inductance and the resistance at the balance condition.

(Elect. Measu. A.M.I.E. Sec B, 1992)

**Solution.** The bridge circuit is shown in Fig. 16.11.

$277 \times 50 = 314.22 \text{ rad/s}$ ;  $0.38 \times 10^{-6} = 314.2^2 \times 98,721$

$$R_1 = \frac{98,721 (0.38 \times 10^{-6})^2}{1} \frac{833 \times 16,800 \times 1000}{98,721 \times 833^2 (0.38 \times 10^{-6})^2} = 210 \Omega$$

$$L_1 = \frac{16,800 \times 1000 \times 0.38 \times 10^{-6}}{1} \frac{833 \times 16,800 \times 1000}{98,721 \times 833^2 (0.38 \times 10^{-6})^2} = 6.38 \text{ H}$$

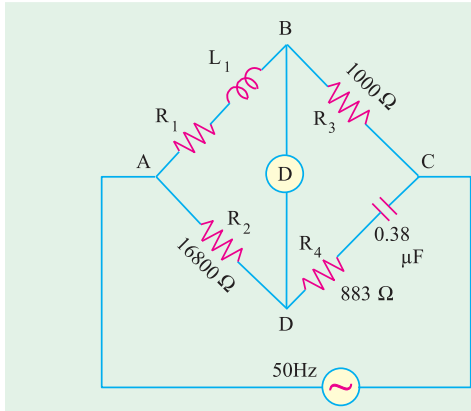


Fig. 16.11

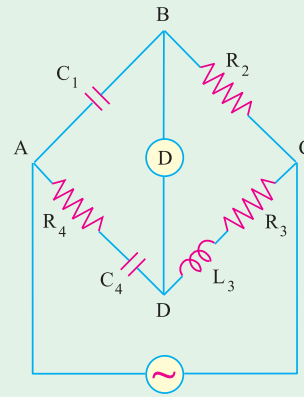


Fig. 16.12

### 16.6. The Owen Bridge

The arrangement of this bridge is shown in Fig. 16.12. In this method, also, the inductance is determined in terms of resistance and capacitance. This method has, however, the advantage of being useful over a very wide range of inductances with capacitors of reasonable dimensions.

Balance condition is  $Z_1 Z_3 = Z_2 Z_4$

$$\text{Here } Z_1 = -\frac{j}{\omega C_1}; \quad Z_2 = R_2; Z_3 = R_3 + j L_3; Z_4 = R_4 + \frac{j}{C_4}$$

$$\therefore -\frac{j}{C_1} (R_3 + j L_3) = R_2 R_4 + \frac{j}{C_4}$$

Separating the reals and imaginaries, we get  $R_3 = R_2 \frac{C_1}{C_4}$  and  $L_3 = C_1 R_2 R_4$ .

Since  $\omega$  does not appear in the final balance equations, hence the bridge is unaffected by frequency variations and wave-form.

### 16.7. Heaviside-Campbell Equal Ratio Bridge

It is a mutual inductance bridge and is used for measuring self-inductance over a wide range in terms of mutual inductometer readings. The connections for Heaviside's bridge employing a standard variable mutual inductance are shown in Fig. 16.13. The primary of the mutual inductometer is inserted in the supply circuit and the secondary having self-inductance  $L_2$  and resistance  $R_2$  is put in arm 2 of the bridge. The unknown inductive impedance having self-inductance of  $L_1$  and resistance  $R_1$  is placed in arm 1. The other two arms have pure resistances of  $R_3$  and  $R_4$ .

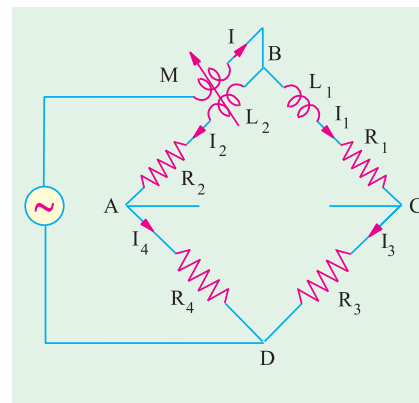


Fig. 16.13

Balance is obtained by varying mutual inductance  $M$  and resistances  $R_3$  and  $R_4$ .

$$\text{For balance, } I_1 R_3 = I_2 R_4 \quad \dots (i)$$

$$I_1 (R_1 + j L_1) = I_2 (R_2 + j L_2) + j M I \quad \dots (ii)$$

Since  $I = I_1 + I_2$ , hence putting the value of  $I$  in equation (ii), we get

$$I_1 [R_1 + j (L_1 + M)] = I_2 [R_2 + j (L_2 + M)] \quad \dots (iii)$$

$$\text{Dividing equation (iii) by (i), we have } \frac{R_1 + j (L_1 + M)}{R_3} = \frac{R_2 + j (L_2 + M)}{R_4}$$

$$\therefore R_3 [R_2 + j (L_2 + M)] = R_4 [R_1 + j (L_1 + M)]$$

$$\text{Equating the real and imaginaries, we have } R_2 R_3 = R_1 R_4 \quad \dots (iv)$$

$$\text{Also, } R_3 (L_2 + M) = R_4 (L_1 + M) \text{ . If } R_3 = R_4, \text{ then } L_2 + M = L_1 + M \therefore L_1 - L_2 = 2M \quad \dots (v)$$

This bridge, as modified by Campbell, is shown in Fig. 16.14. Here  $R_3 = R_4$ . A balancing coil or a test coil of self-inductance equal to the self-inductance  $L_2$  of the secondary of the inductometer and of resistance slightly greater than  $R_2$  is connected in series with the unknown inductive impedance ( $R_1$  and  $L_1$ ) in arm 1. A non-inductive resistance box along with a constant-inductance rheostat are also introduced in arm 2 as shown.

Balance is obtained by varying  $M$  and  $r$ . Two readings are taken; one when  $Z_1$  is in circuit and second when  $Z_1$  is removed or short-circuited across its terminals.

With unknown impedance  $Z_1$  still in circuit, suppose for balance the values of mutual inductance and  $r$  are  $M_1$  and  $r_1$ . With  $Z_1$  short-circuited, let these values be  $M_2$  and  $r_2$ . Then

$$L_1 = 2(M_1 - M_2) \text{ and } R_1 = r_1 - r_2$$

By this method, the self-inductance and resistance of the leads are eliminated.

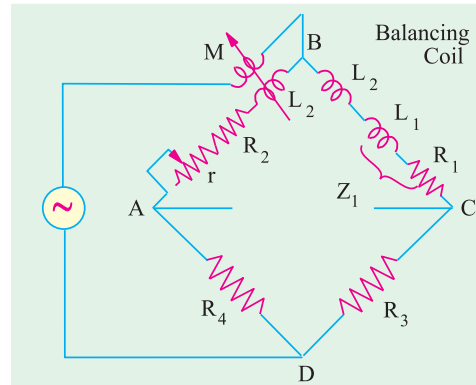


Fig. 16.14

**Example 16.6.** The inductance of a coil is measured by using the Heaviside-Campbell equal ratio bridge. With the test coil short-circuited, balance is obtained when adjustable non-reactive resistance is  $12.63 \Omega$  and mutual inductometer is set at  $0.1 \text{ mH}$ . When the test coil is in circuit, balance is obtained when the adjustable resistance is  $25.9 \Omega$  and mutual inductometer is set at  $15.9 \text{ mH}$ . What is the resistance and inductance of the coil?

**Solution.** With reference to Art. 16.7 and Fig. 16.14,  $r_1 = 25.9 \Omega$ ,  $M_1 = 15.9 \text{ mH}$

With test coil short-circuited  $r_2 = 12.63 \Omega$ ;  $M_2 = 0.1 \text{ mH}$

$$L_1 = 2 (M_1 - M_2) = 2 (15.9 - 0.1) = \mathbf{31.6 \text{ mH}}$$

$$R_1 = r_1 - r_2 = 25.9 - 12.63 = \mathbf{13.27 \Omega}$$

## 16.8. Capacitance Bridges

We will consider only De Sauty bridge method of comparing two capacitances and Schering bridge used for the measurement of capacitance and dielectric loss.

### 16.9. De Sauty Bridge

With reference to Fig. 16.15, let

$C_2$  = capacitor whose capacitance is to be measured

$C_3$  = a standard capacitor

$R_1, R_2$  = non-inductive resistors

Balance is obtained by varying either  $R_1$  or  $R_2$ .

For balance, points  $B$  and  $D$  are at the same potential.

$$\therefore I_1 R_1 = I_2 R_2 \text{ and } \frac{j}{C_2} \cdot I_1 = \frac{j}{C_3} \cdot I_2$$

Dividing one equation by the other, we get

$$\frac{R_1}{R_2} = \frac{C_2}{C_3}, \quad C_2 = C_3 \frac{R_1}{R_2}$$

The bridge has maximum sensitivity when  $C_2 = C_3$ . The simplicity of this method is offset by the impossibility of obtaining a perfect balance if both the capacitors are not free from the dielectric loss. A perfect balance can only be obtained if air capacitors are used.

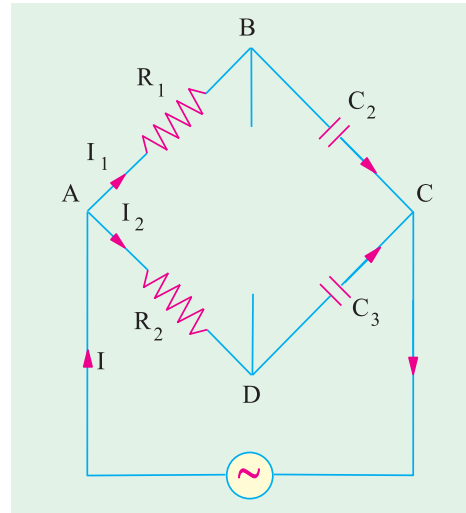


Fig. 16.15

### 16.10. Schering Bridge

It is one of the very important and useful methods of measuring the capacitance and dielectric loss of a capacitor. In fact, it is a device for comparing an imperfect capacitor  $C_2$  in terms of a loss-free standard capacitor  $C_1$  [Fig. 16.16 (a)]. The imperfect capacitor is represented by its equivalent loss-free capacitor  $C_2$  in series with a resistance  $r$  [Fig. 16.16 (b)].

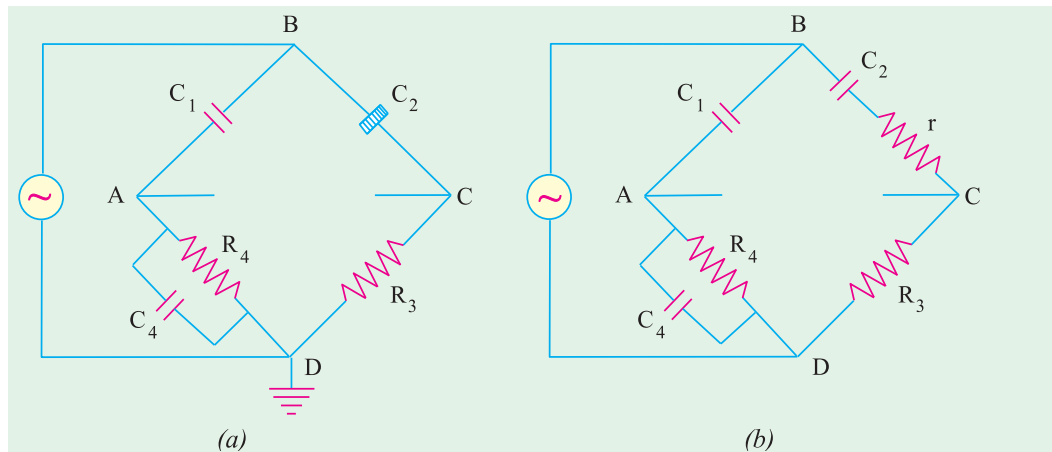


Fig. 16.16

For high voltage applications, the voltage is applied at the junctions shown in the figure. The junction between arms 3 and 4 is earthed. Since capacitor impedances at lower frequencies are much higher than resistances, most of the voltage will appear across capacitors. Grounding of the junction affords safety to the operator from the high-voltage hazards while making balancing adjustment in arms 3 and 4.

$$\text{Now } Z_1 = \frac{j}{C_1}; Z_2 = r + \frac{j}{C_2}; Z_3 = R_3; Z_4 = \frac{1}{(1/R_4) + j C_4} = \frac{R_4}{1 + j C_4 R_4}$$

For balance,  $Z_1 Z_3 = Z_2 Z_4$



$$\text{or } \frac{jR_3}{C_1} + r = \frac{j}{C_2} \frac{R_4}{1 - j C_4 R_4} \text{ or } \frac{jR_3}{C_1} (1 + j C_4 R_4) + R_4 + r = \frac{j}{C_2}$$

Separating the real and imaginaries, we have  $C_2 = C_1(R_4 / R_3)$  and  $r = R_3 \cdot (C_4 / C_1)$ .

The quality of a capacitor is usually expressed in terms of its phase defect angle or dielectric loss angle which is defined as the angle by **which current departs from exact quadrature from the applied voltage** i.e. the complement of the phase angle. If  $\phi$  is the actual phase angle and  $\delta$  the defect angle, then  $\phi + \delta = 90^\circ$ . For small values of  $\delta$ ,  $\tan \delta = \sin$

$\delta = \cos \phi$  (approximately).  $\tan \delta$  is usually called the *dissipation factor* of the  $R$ - $C$  circuit. For low power factors, therefore, dissipation factor is approximately equal to the power factor.

As shown in Fig. 16.17,

Dissipation factor = power factor =  $\tan \delta$

$$= \frac{r}{X_C} = \frac{r}{1/\omega C_2} = \omega r C_2$$

Putting the value of  $r C_2$  from above,

Dissipation factor =  $r C_2 = C_4 R_4 =$  power factor.

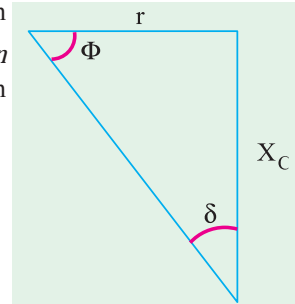


Fig. 16.17

**Example 16.7.** In a test on a bakelite sample at 20 kV, 50 Hz by a Schering bridge, having a standard capacitor of 106 pF, balance was obtained with a capacitance of 0.35  $\mu$ F in parallel with a non-inductive resistance of 318 ohms, the non-inductive resistance in the remaining arm of the bridge being 130 ohms. Determine the capacitance, the p.f. and equivalent series resistance of the specimen. Derive any formula used. Indicate the precautions to be observed for avoiding errors. (Elect. Engg. Paper I, Indian Engg. Services 1991)

**Solution.** Here  $C_1 = 106$  pF,  $C_4 = 0.35$   $\mu$ F,  $R_4 = 318$   $\Omega$ ,  $R_3 = 130$   $\Omega$ .

$$C_2 = C_1 \cdot (R_4 / R_3) = 106 \cdot 318 / 130 = \mathbf{259.3 \text{ pF}}$$

$$r = R_3 \cdot (C_4 / C_1) = 130 \cdot 0.35 \cdot 10^{-6} / 106 \cdot 10^{-12} = \mathbf{0.429 \text{ M}\Omega}$$

$$\text{p.f.} = r C_2 = (259.3 \cdot 10^{-12}) / (0.429 \cdot 10^6) = \mathbf{0.035}$$

**Example 16.8.** A lossy capacitor is tested with a Schering bridge circuit. Balance obtained with the capacitor under test in one arm, the succeeding arms being, a non-inductive resistor of 100  $\Omega$ , a non-reactive resistor of 309  $\Omega$  in parallel with a pure capacitor of 0.5  $\mu$ F and a standard capacitor of 109  $\mu$ F. The supply frequency is 50 Hz. Calculate from the equation at balance the equivalent series capacitance and power factor (at 50 Hz) of the capacitor under test. (Measu. & Instru., Nagpur Univ. 1992)

**Solution.** Here, we are given

$$C_1 = 109 \text{ pF}; R_3 = 100 \text{ } \Omega; C_4 = 0.5 \text{ } \mu\text{F}; R_4 = 309 \text{ } \Omega$$

Equivalent capacitance is  $C_2 = 109 \cdot 309 / 100 = \mathbf{336.8 \text{ pF}}$

$$\text{p.f.} = C_4 R_4 = 314 \cdot 0.5 \cdot 10^{-6} / 309 = \mathbf{0.0485}$$

### 16.11. Wien Series Bridge

It is a simple ratio bridge and is used for audio-frequency measurement of capacitors over a wide range. The bridge circuit is shown in Fig. 16.18.

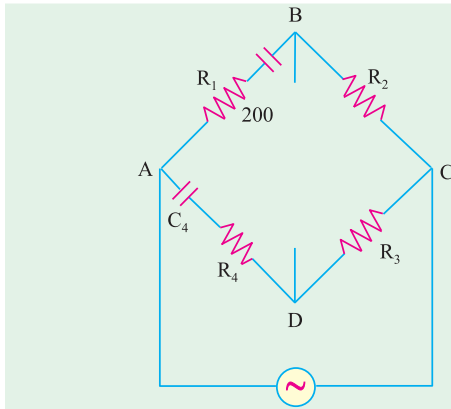


Fig. 16.18

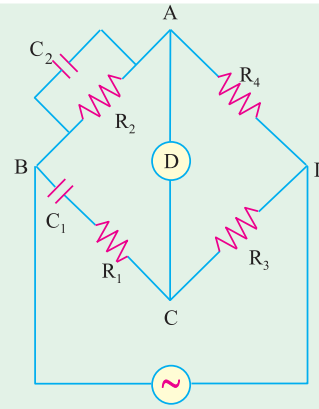


Fig. 16.19

The balance conditions may be obtained in the usual way. For balance

$$R_1 = R_2 R_4 / R_3 \text{ and } C_1 = C_4 (R_3 / R_2)$$

### 16.12. Wien Parallel Bridge

It is also a ratio bridge used mainly as the feedback network in the wide-range audio-frequency  $R$ - $C$  oscillators. It may be used for measuring audio-frequencies although it is not as accurate as the modern digital frequency meters.

The bridge circuit is shown in Fig. 16.19. In the simple theory of this bridge, capacitors  $C_1$  and  $C_2$  are assumed to be loss-free and resistances  $R_1$  and  $R_2$  are separate resistors.

The usual relationship for balance gives

$$R_4 R_1 \frac{j}{C_1} = R_3 \frac{R_2}{1 - j C_2 R_2} \text{ or } R_4 R_1 \frac{j}{C_1} (1 - j C_2 R_2) = R_2 R_3$$

Separating the real and imaginary terms, we have

$$R_1 R_4 = R_2 R_3 \frac{C_2}{C_1} \text{ or } \frac{C_2}{C_1} = \frac{R_3}{R_4} \frac{R_1}{R_2} \quad \dots (i)$$

$$\text{and } C_2 R_2 R_4 \frac{R_4}{C_1} = 0 \text{ or } \frac{1}{R_1 R_2 C_1 C_2} \quad \dots (ii)$$

or

$$f = \frac{1}{2 \sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

**Note.** Eq. (ii) may be used to find angular frequency  $\omega$  of the source if terms are known. For such purposes, it is convenient to make  $C_1 = 2C_2$ ,  $R_3 = R_4$  and  $R_2 = 2R_1$ . In that case, the bridge has equal ratio arms so that Eq. (i) will always be satisfied. The bridge is balanced simultaneously by adjusting  $R_2$  and  $R_1$  (though maintaining  $R_2 = 2R_1$ ). Then, as seen from Eq. (ii) above

$$\frac{1}{2 \sqrt{R_1 \cdot 2R_1 \cdot 2C_2 \cdot C_2}} \text{ or } \frac{1}{2 \sqrt{R_1 C_2}}$$

**Example 16.9.** The arms of a four-arm bridge ABCD, supplied with a sinusoidal voltage, have the following values:

AB : 200 ohm resistance in parallel with  $1 \mu\text{F}$  capacitor; BC : 400 ohm resistance; CD : 1000 ohm resistance and DA : resistance  $R$  in series with a  $2\mu\text{F}$  capacitor.

Determine (i) the value of  $R$  and (ii) the supply frequency at which the bridge will be balanced. (Elect. Meas. A.M.I.E. Sec. 1991)

**Solution.** The bridge circuit is shown in Fig. 16.20.

(i) As discussed in Art. 16.12, for balance we have

$$\frac{C_2}{C_1} \frac{R_3}{R_4} \frac{R_1}{R_2} \text{ or } \frac{2}{1} \frac{1000}{4000} \frac{R_1}{200}$$

$$\therefore R_1 = 200 \times 0.5 = \mathbf{100 \Omega}$$

(ii) The frequency at which bridge is balanced is given by

$$f = \frac{1}{2 \sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

$$= \frac{10^6}{2 \sqrt{100 \times 200 \times 1 \times 2}} = \mathbf{796 \text{ Hz}}$$

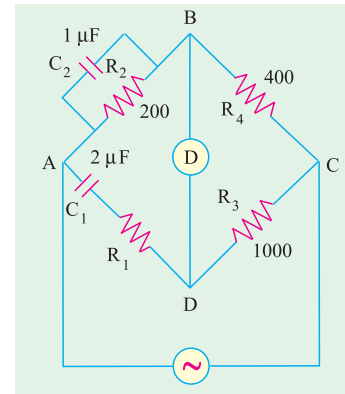


Fig. 16.20

### Tutorial Problems No. 16.1

- In Anderson a.c. bridge, an impedance of inductance  $L$  and resistance  $R$  is connected between  $A$  and  $B$ . For balance following data is obtained. An ohmic resistance of  $1000 \Omega$  each in arms  $AD$  and  $CD$ , a non-inductive resistance of  $500 \Omega$  in  $BC$ , a pure resistance of  $200 \Omega$  between points  $D$  and  $E$  and a capacitor of  $2 \mu\text{F}$  between  $C$  and  $E$ . The supply is 10 volt (A.C.) at a frequency of 100 Hz and is connected across points  $A$  and  $C$ . Find  $L$  and  $R$ . [**1.4 H; 500  $\Omega$** ]
- A balanced bridge has the following components connected between its five nodes,  $A, B, C, D$  and  $E$ :  
Between  $A$  and  $B$ : 1,000 ohm resistance;      Between  $B$  and  $C$ : 1,000 ohm resistance  
Between  $C$  and  $D$ : an inductor;              Between  $D$  and  $A$ : 218 ohm resistance  
Between  $A$  and  $E$ : 469 ohm resistance;      Between  $E$  and  $B$ : 10  $\mu\text{F}$  capacitance  
Between  $E$  and  $C$ : a detector;              Between  $B$  and  $D$ : a power supply (a.c.)  
Derive the equations of balance and hence deduce the resistance and inductance of the inductor.

[ **$R = 218 \Omega$ ,  $L = 7.89 \text{ H}$** ] (*Elect. Theory and Meas. London Univ.*)

- An a.c. bridge is arranged as follows: The arms  $AB$  and  $BC$  consist of non-inductive resistance of  $100 \Omega$ , the arms,  $BE$  and  $CD$  of non-inductive variable resistances, the arm  $EC$  of a capacitor of  $1 \mu\text{F}$  capacitance, the arm  $DA$  of an inductive resistance. The a.c. source is connected to  $A$  and  $C$  and the telephone receiver to  $E$  and  $D$ . A balance is obtained when the resistances of the arms  $CD$  and  $BE$  are  $50 \Omega$  and  $2500 \Omega$  respectively.

Calculate the resistance and the inductance of the arm  $DA$ . What would be the effect of harmonics in the waveform of the alternating current source? [ **$50 \Omega$ ; 0.25 H**]

- For the Anderson's bridge of Fig. 16.21, the values are underbalance conditions. Determine the values of unknown resistance  $R$  and inductance  $L$ . [ **$R = 500 \Omega$ ;  $L = 1.5 \text{ H}$** ]

(*Elect. Meas & Inst. Madras Univ. Nov. 1978*)

- An Anderson's bridge is arranged as under and balanced for the following values of the bridge components:  
Branch  $AB$  – unknown coil of inductance  $L$  and resistance  $R$   
Branch  $BC$  – non-inductive resistance of  $500 \Omega$   
Branches  $AD$  &  $CD$  – non-inductive resistance of  $100 \Omega$  each  
Branch  $DE$  – non-inductive resistance of  $200 \Omega$   
Branch  $EB$  – vibration galvanometer  
Branch  $EC$  –  $2.0 \mu\text{F}$  capacitance  
Between  $A$  and  $C$  is 10 V, 100-Hz a.c. supply. Find the values of  $R$  and  $L$  of the unknown coil.

[ **$R = 500 \Omega$ ;  $L = 0.5 \text{ H}$** ] (*Elect. Meas & Meas. Inst., Gujarat Univ.*)

- An a.c. Anderson bridge is arranged as follows:  
(i) branches  $BC$  and  $ED$  are variable non-reactive resistors  
(ii) branches  $CD$  and  $DA$  are non-reactive resistors of  $200 \text{ ohm}$  each  
(iii) branch  $CE$  is a loss-free capacitor of  $1 \mu\text{F}$  capacitance.

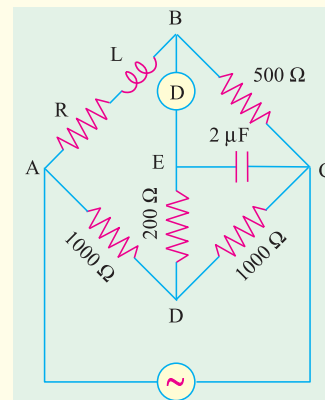


Fig. 16.21

The supply is connected across  $A$  and  $C$  and the detector across  $B$  and  $E$ . Balance is obtained when the resistance of  $BC$  is 400 ohm and that of  $ED$  is 500 ohm. Calculate the resistance and inductance of  $AB$ .

Derive the relation used and draw the vector diagram for balanced condition of the bridge.

[400  $\Omega$  ; 0.48 H] (*Elect. Measurements, Poona Univ.*)

7. In a balanced bridge network,  $AB$  is a resistance of 500 ohm in series with an inductance of 0.18 henry, the non-inductive resistances  $BC$  and  $DA$  have values of 1000 ohm and arm  $CD$  consists of a capacitance of  $C$  in series with a resistance  $R$ . A potential difference of 5 volts at a frequency  $5000/2\pi$  is the supply between the points  $A$  and  $C$ . Find out the values of  $R$  and  $C$  and draw the vector diagram.

[472  $\Omega$  ; 0.235  $\mu\text{F}$ ] (*Elect. Measurements, Poona Univ.*)

8. A sample of bakelite was tested by the Schering bridge method at 25 kV, 50-Hz. Balance was obtained with a standard capacitor of 106 pF capacitance, a capacitor of capacitance 0.4  $\mu\text{F}$  in parallel with a non-reactive resistor of 318  $\Omega$  and a non-reactive resistor of 120  $\Omega$ . Determine the capacitance, the equivalent series resistance and the power factor of the specimen. Draw the phase or diagram for the balanced bridge.

[281 pF ; 0.452 M $\Omega$  ; 0.04] (*Elect. Measurements-II; Bangalore Univ.*)

9. The conditions at balance of a Schering bridge set up to measure the capacitance and loss angle of a paper dielectric capacitor are as follows:

$$f = 500 \text{ Hz}$$

$$Z_1 = \text{a pure capacitance of } 0.1 \mu\text{F}$$

$$Z_2 = \text{a resistance of } 500 \Omega \text{ shunted by a capacitance of } 0.0033 \mu\text{F}$$

$$Z_3 = \text{pure resistance of } 163 \Omega$$

$$Z_4 = \text{the capacitor under test}$$

Calculate the approximate values of the loss resistance of the capacitor assuming—

(a) series loss resistance (b) shunt loss resistance. [5.37  $\Omega$  , 197,000  $\Omega$ ] (*London Univ.*)

10. Name and draw the bridge used for measurements of Inductance. (*Anna University, April 2002*)

11. A Wheat-stone bridge network has the following resistances :

$$AB = 10\Omega, BC = 15\Omega, CD = 25\Omega, DA = 20\Omega \text{ and } BD = 10\Omega$$

(*V.T.U., Belgaum Karnataka University, February 2002*)

### OBJECTIVE TESTS – 16

- Maxwell-Wien bridge is used for measuring
  - capacitance
  - dielectric loss
  - inductance
  - phase angle
- Maxwell's  $L/C$  bridge is so called because
  - it employs  $L$  and  $C$  in two arms
  - ratio  $L/C$  remains constant
  - for balance, it uses two opposite impedances in opposite arms
  - balance is obtained when  $L = C$
- ..... bridge is used for measuring an unknown inductance in terms of a known capacitance and resistance.
  - Maxwell's  $L/C$
  - Hay's
  - Owen
  - Anderson
- Anderson bridge is a modification of ..... bridge.
  - Owen
  - Hay's
  - De Sauty
  - Maxwell-Wien
- Hay's bridge is particularly useful for measuring
  - inductive impedance with large phase angle
  - mutual inductance
  - self inductance
  - capacitance and dielectric loss
- The most useful ac bridge for comparing capacitances of two air capacitors is ..... bridge.
  - Schering
  - De Sauty
  - Wien series
  - Wien parallel
- Heaviside-Campbell Equal Ratio bridge is used for measuring
  - self-inductance in terms of mutual inductance
  - capacitance in terms of inductance
  - dielectric loss of an imperfect capacitor
  - phase angle of a coil

### ANSWERS

1. c 2. c 3. d 4. d 5. a 6. b 7. a

## CHAPTER 17

**Learning Objectives**

- Introduction
- Applications
- Different Types of Filters
- Octaves and Decades of Frequency
- Decibel System
- Value of 1 dB
- Low-Pass RC Filter
- Other Types of Low-Pass Filters
- Low-Pass RL Filter
- High-Pass RC Filter
- High Pass R L Filter
- *R-C* Bandpass Filter
- *R-C* Bandstop Filter
- The-3 dB Frequencies
- Roll-off of the Response Curve
- Bandstop and Bandpass Resonant Filter Circuits
- Series-and Parallel-Resonant Bandstop Filters
- Parallel-Resonant Bandstop Filter
- Series-Resonant Bandpass Filter
- Parallel-Resonant Bandpass Filter

**A.C. FILTER NETWORKS**

By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies

### 17.1. Introduction

The reactances of inductors and capacitors depend on the frequency of the a.c. signal applied to them. That is why these devices are known as frequency-selective. By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies. These frequency-selective networks, which alter the amplitude and phase characteristics of the input a.c. signal, are called filters. Their performance is usually expressed in terms of how much attenuation a band of frequencies experiences by passing through them. Attenuation is commonly expressed in terms of decibels (dB).

### 17.2. Applications

A.C. filters find application in audio systems and television etc. Bandpass filters are used to select frequency ranges corresponding to desired radio or television station channels. Similarly, bandstop filters are used to reject undesirable signals that may contaminate the desirable signal. For example, low-pass filters are used to eliminate undesirable hum in d.c. power supplies.

No loudspeaker is equally efficient over the entire audible range of frequencies. That is why high-fidelity loudspeaker systems use a combination of low-pass, high-pass and bandpass filters (called crossover networks) to separate and then direct signals of appropriate frequency range to the different loudspeakers making up the system. Fig. 17.1 shows the output circuit of a high-fidelity audio amplifier, which uses three filters to separate, the low, mid-range and high frequencies, for feeding them to individual loudspeakers, best able to reproduce them.

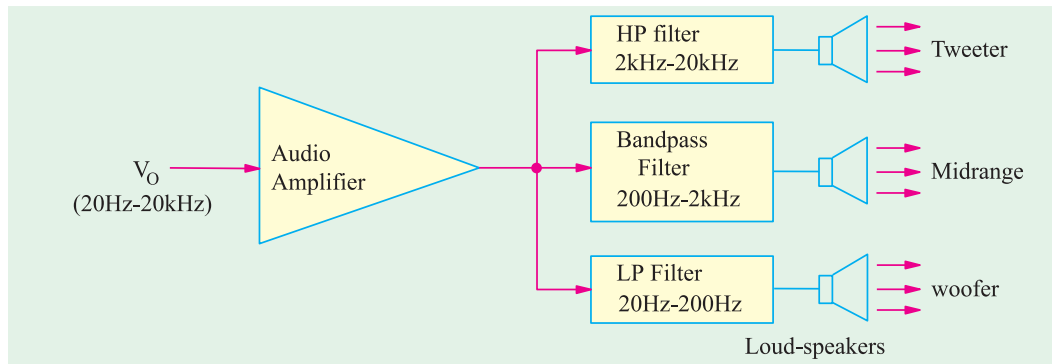
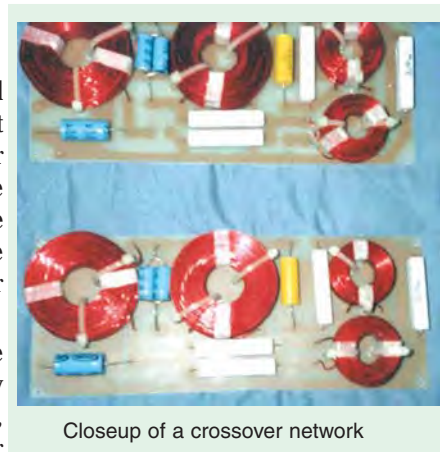


Fig. 17.1

### 17.3. Different Types of Filters

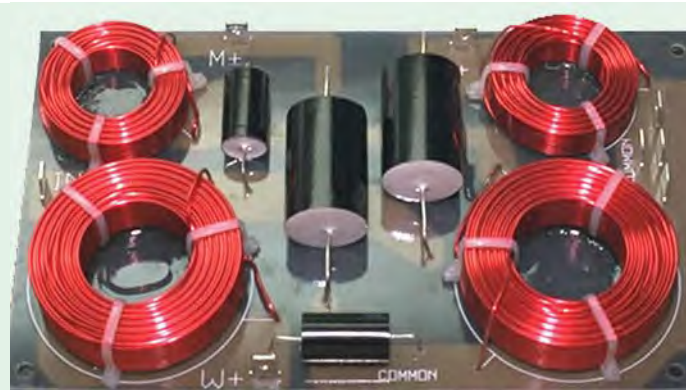
A.C. filter networks are divided into two major categories: **(i)** active networks and **(ii)** passive networks.

Active filter networks usually contain transistors and/or operational amplifiers in combination with  $R$ ,  $L$  and  $C$  elements to obtain the desired filtering effect. These will not be discussed in this book. We will consider passive filter networks only which usually consist of series-parallel combinations of  $R$ ,  $L$  and  $C$  elements. There are four types of such networks, as described below:



**1. Low-Pass Filter.** As the name shows, it allows only low frequencies to pass through, but attenuates (to a lesser or greater extent) all higher frequencies. The maximum frequency which it allows to pass through, is called cutoff frequency  $f_c$  (also called break frequency). There are  $R_L$  and  $R_C$  low-pass filters.

**2. High-Pass Filter.** It allows signals with higher frequencies to pass from input to output while rejecting lower frequencies.



Three way passive crossover network



High pass filter

The minimum frequency it allows to pass is called cutoff frequency  $f_c$ . There are  $R_L$  and  $R_C$  high-pass filters.

**3. Bandpass Filter.** It is a resonant circuit which is tuned to pass a certain band or range of frequencies while rejecting all frequencies below and above this range (called passband).

**4. Bandstop Filter.** It is a resonant circuit that rejects a certain band or range of frequencies while passing all frequencies below and above the rejected band. Such filters are also called wavetraps, notch filters or band-elimination, band-separation or band-rejection filters.

## 17.4. Octaves and Decades of Frequency

A filter's performance is expressed in terms of the number of decibels the signal is increased or decreased per frequency octave or frequency decade. An octave means a doubling or halving of a frequency whereas a decade means tenfold increase or decrease in frequency.

## 17.5. The Decibel System

This system of logarithmic measurement is widely used in audio, radio, TV and instrument industry for comparing two voltages, currents or power levels. These levels are measured in a unit called bel (B) or decibel (dB) which is  $1/10^{\text{th}}$  of a bel.

Suppose we want to compare the output power  $P_0$  of a filter with its input power  $P_i$ . The power level change is

$$= 10 \log_{10}^{\text{th}} P_0/P_i \text{ dB}$$

It should be noted that dB is the unit of power change (*i.e.* increase or decrease) and not of power itself. Moreover, 20 dB is not twice as much power as 10 dB.

However, when voltage and current levels are required, then the expressions are:

$$\text{Current level} = 20 \log_{10} (I_0/I_i) \text{ dB}$$

$$\text{Similarly, voltage level} = 20 \log V_0/V_i \text{ dB}$$

Obviously, for power, we use a multiplying factor of 10 but for voltages and currents, we use a multiplying factor of 20.

### 17.6. Value of 1 dB

It can be proved that 1 dB represents the log of two powers, which have a ratio of 1.26.

$$1 \text{ dB} = 10 \log_{10}(P_2/P_1) \text{ or } \log_{10}(P_2/P_1) = 0.1 \text{ or } \frac{P_2}{P_1} = 10^{0.1} = 1.26$$

Hence, it means that + 1 dB represents an increase in power of 26%.

**Example 17.1.** The input and output voltages of a filter network are 16 mV and 8 mV respectively. Calculate the decibel level of the output voltage.

**Solution.** Decibel level =  $20 \log_{10}(V_o/V_i) \text{ dB} = -20 \log_{10}(V_i/V_o) \text{ dB} = -20 \log_{10}(16/8) = -6 \text{ dB}$ .  
Whenever voltage ratio is less than 1, its log is negative which is often difficult to handle. In such cases, it is best to invert the fraction and then make the result negative, as done above.

**Example 17.2.** The output power of a filter is 100 mW when the signal frequency is 5 kHz. When the frequency is increased to 25 kHz, the output power falls to 50 mW. Calculate the dB change in power.

**Solution.** The decibel change in power is  
 $= 10 \log_{10}(50/100) = -10 \log_{10}(100/50) = -10 \log_{10} 2 = -10 \times 0.3 = -3 \text{ dB}$

**Example 17.3.** The output voltage of an amplifier is 10 V at 5 kHz and 7.07 V at 25 kHz. What is the decibel change in the output voltage?

**Solution.** Decibel change =  $20 \log_{10}(V_o/V_i) = 20 \log_{10}(7.07/10) = -20 \log_{10}(10/7.07)$   
 $= -20 \log_{10}(1.4/1) = -20 \times 0.15 = -3 \text{ dB}$

### 17.7. Low-Pass RC Filter

A simple low-pass RC filter is shown in Fig. 17.2 (a). As stated earlier, it permits signals of low frequencies upto  $f_c$  to pass through while attenuating frequencies above  $f_c$ . The range of frequencies upto  $f_c$  is called the passband of the filter. Fig. 17.2 (b) shows the frequency response curve of such a filter. It shows how the signal output voltage  $V_o$  varies with the signal frequency. As seen at  $f_c$ , output signal voltage is reduced to 70.7% of the input voltage. The output is said to be - 3 dB at  $f_c$ . Signal outputs beyond  $f_c$  roll-off or attenuate at a fixed rate of - 6 dB/octave or - 20 dB/decade. As seen from the frequency-phase response curve of Fig. 17.2 (c), the phase angle between  $V_o$  and  $V_i$  is  $45^\circ$  at cutoff frequency  $f_c$ .

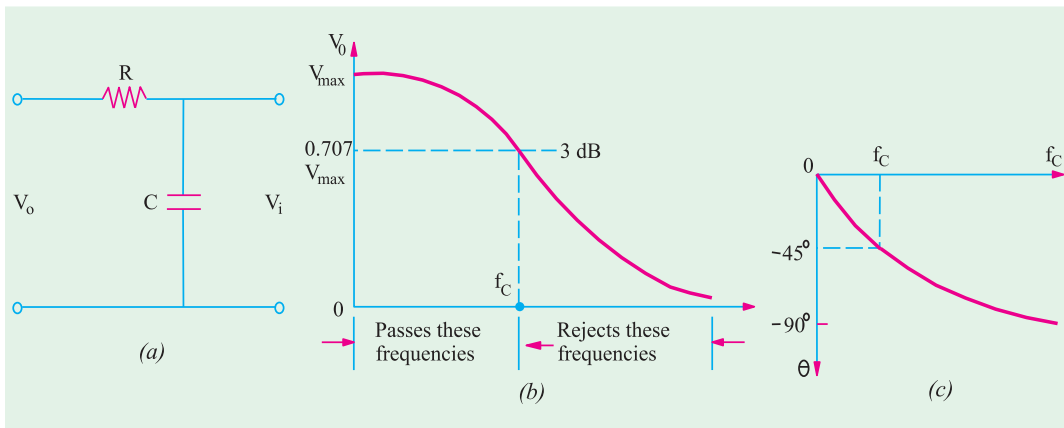


Fig. 17.2



By definition cutoff frequency  $f_c$  occurs where (a)  $V_0 = 70.7\% V_i$  i.e.  $V_0$  is  $-3$  dB down from  $V_i$  (b)  $R = X_c$  and  $V_R = V_C$  in magnitude. (c) The impedance phase angle  $\theta = -45^\circ$ . The same is the angle between  $V_0$  and  $V_i$ .

As seen, the output voltage is taken across the capacitor. Resistance  $R$  offers fixed opposition to frequencies but the reactance offered by capacitor  $C$  decreases with increase in frequency. Hence, low-frequency signal develops over  $C$  whereas high-frequency signals are grounded. Signal frequencies above  $f_c$  develop negligible voltage across  $C$ . Since  $R$  and  $C$  are in series, we can find the low-frequency output voltage  $V_0$  developed across  $C$  by using the voltage-divider rule.

$$V_0 = V_i \frac{jX_C}{R + jX_C} \text{ and } f_c = \frac{1}{2\pi CR}$$

### 17.8. Other Types of Low-Pass Filters

There are many other types of low-pass filters in which instead of pure resistance, series chokes are commonly used along with capacitors.

(i) **Inverted-L Type.** It is shown in Fig. 17.3 (a). Here, inductive reactance of the choke blocks higher frequencies and  $C$  shorts them to ground. Hence, only low frequencies below  $f_c$  (for which  $X$  is very low) are passed without significant attenuation.

(ii) **T-Type.** It is shown in Fig. 17.3 (b). In this case, a second choke is connected on the output side which improves the filtering action.

(iii)  **$\pi$ -Type.** It is shown in Fig. 17.3 (c). The additional capacitor further improves the filtering action by grounding higher frequencies.

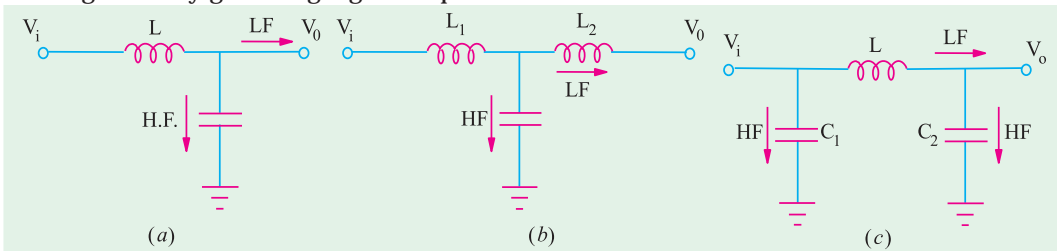


Fig. 17.3

It would be seen from the above figures that choke is always connected in series between the input and the output and capacitors are grounded in parallel. The output voltage is taken across the capacitor.

**Example 17.4.** A simple low-pass RC filter having a cutoff frequency of 1 kHz is connected to a constant ac source of 10 V with variable frequency. Calculate the following :

(a) value of  $C$  if  $R = 10\text{ k}\Omega$  (b) output voltage and its decibel level when

(i)  $f = f_c$  (ii)  $f = 2f_c$  and (iii)  $f = 10f_c$ .

**Solution.** (a) At  $f_c$ ,  $r = X_c = 1/2\pi f_c$  or  $C = 1/2\pi \times 1 \times 10^3 \times 10 \times 10^3 = 15.9 \times 10^{-9} = 15.9\text{ nF}$

(b) (i)  $f = f_c = 1\text{ kHz}$ . Now,  $-jX_c = R = -j10 = 10 \angle -90^\circ \Omega$

$$\therefore V_0 = V_i \frac{-jX_c}{R - jX_c} = 10 \frac{-j10}{10 - j10} = 7.07 \angle -45^\circ$$

Output decibel level =  $20 \log_{10} (V_0/V_i) = -20 \log_{10} (V_0/V_i) = -20 \log_{10} (10/7.07) = -3\text{ dB}$

(ii) Here,  $f = 2f_c = 2\text{ kHz}$  i.e. octave of  $f_c$ . Since capacitive reactance is inversely proportional to frequency,  $\therefore X_{c2} = C_{c1}(f_1/f_2) = -j10(1/2) = -j5 = 5 \angle -90^\circ\text{ k}\Omega$

$$\therefore V_0 = \frac{5 \angle -90^\circ}{10 - j5} = \frac{5^\circ \angle -90^\circ}{11.18 \angle -26.6^\circ} = 4.472 \angle -63.4^\circ$$

$$\text{Decibel level} = -20 \log_{10} (V_i/V_0) = -20 \log_{10} (10/4.472) = -6.98 \text{ dB}$$

$$\text{(iii) } X_{c3} = X_{c1}(f_1/f_3) = -j10 (1/10) = j1 = 1 \angle -90^\circ \text{ k}\Omega$$

$$\therefore V_0 = 10 \frac{10 \angle -90^\circ}{10 - j1} = 1 \angle -84.3^\circ$$

$$\text{Decibel level} = -20 \log_{10}(10/1) = -20 \text{ dB}$$

### 17.9. Low-Pass RL Filter

It is shown in Fig. 17.4 (a). Here, coil offers high reactance to high frequencies and low reactance to low frequencies. Hence, low frequencies upto  $f_c$  can pass through the coil without much opposition. The output voltage is developed across  $R$ . Fig 17.4 (b) shows the frequency-output response curve of the filter. As seen at  $f_c$ ,  $V_0 = 0.707 V_i$  and its attenuation level is  $-3 \text{ dB}$  with respect to  $V_0$  i.e. the voltage at  $f = 0$ .



A view of the inside of the low-pass filter assembly

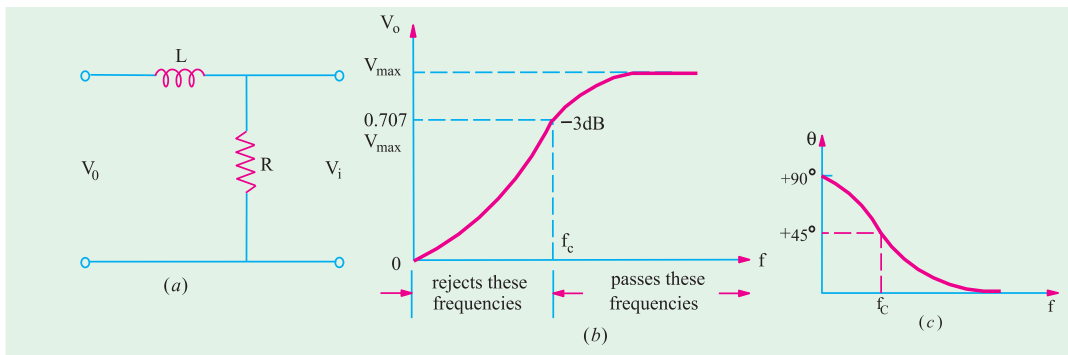


Fig. 17.4

However, it may be noted that being an  $RL$  circuit, the impedance phase angle is  $+45^\circ$  (and not  $-45^\circ$  as in low-pass  $RC$  filter). Again at  $f_c$ ,  $R = X_L$ .

Using the voltage-divider rule, the output voltage developed across  $R$  is given by

$$V_0 = V_i \frac{R}{R + jX_L} \text{ and } f_c = \frac{R}{2\pi L}$$

**Example 17.5.** An ac signal having constant amplitude of  $10 \text{ V}$  but variable frequency is applied across a simple low-pass  $RL$  circuit with a cutoff frequency of  $1 \text{ kHz}$ . Calculate (a) value of  $L$  if  $R = 1 \text{ k}\Omega$  (b) output voltage and its decibel level when (i)  $f = f_c$  and (iii)  $f = 10 f_c$ .

**Solution.** (a)  $L = R/2 f_c = 1 \times 10^3 / 2 \times 3.14 \times 10^3 = 159.2 \text{ mH}$

$$(b) (i) f = f_c = 1 \text{ kHz}; jX_L = R = j1; V_0 = 10 \frac{1}{(1 + j1)} = 7.07 \angle -45^\circ \text{ V}$$

$$\text{Decibel decrease} = -20 \log_{10} (V_i / V_0) = -20 \log_{10} 10/7.07 = -3 \text{ dB}$$

(ii)  $f = 2f_c = 2 \text{ kHz}$ . Since  $X_L$  varies directly with

$$f, X_{L2} = X_{L1} (f_2/f_1) = 1 \times 2/1 = 2 \text{ k}\Omega$$

$$\therefore V_0 = 10 \frac{1}{(1 + j2)} = \frac{10}{2.236 \angle 63.4^\circ} = 4.472 \angle -63.4^\circ$$

$$\text{Decibel decrease} = -20 \log_{10} (10/4.472) = -6.98 \text{ dB}$$

$$(iii) f = 10 f_c = 10 \text{ kHz}; X_{L3} = 1 \times 10/1 = 10 \Omega, V_0 = \frac{1}{(1 + j10)} = 1 \angle -84.3^\circ$$

$$\text{Decibel decrease} = -20 \log_{10}(10/1) = -20 \text{ dB}$$

### 17.10. High-Pass RC Filter

It is shown in Fig. 17.5 (a). Lower frequencies experience considerable reactance by the capacitor and are not easily passed. Higher frequencies encounter little reactance and are easily passed. The high frequencies passing through the filter develop output voltage  $V_0$  across  $R$ . As seen from the frequency response of Fig. 13.5 (b), all frequencies above  $f_c$  are passed whereas those below it are attenuated. As before,  $f_c$  corresponds to  $-3 \text{ dB}$  output voltage or half-power point. At  $f_c$ ,  $R = X_C$  and the phase angle between  $V_0$  and  $V_i$  is  $+45^\circ$  as shown in Fig. 17.5 (c). It may be noted that high-pass  $RC$  filter can be obtained merely by interchanging the positions of  $R$  and  $C$  in the low-pass  $RC$  filter of Fig. 17.5 (a).

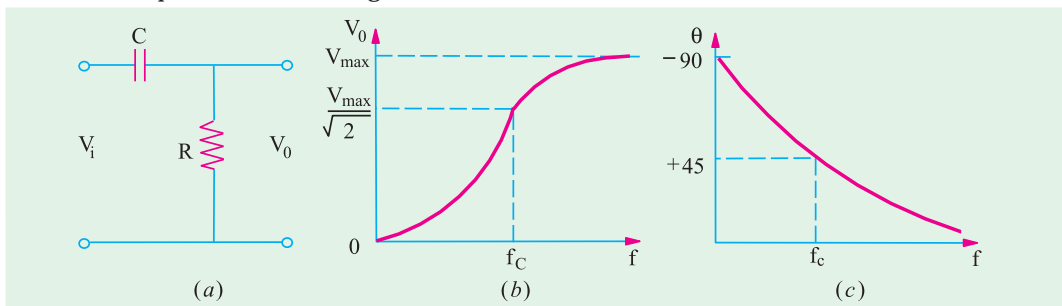


Fig. 17.5

Since  $R$  and  $C$  are in series across the input voltage, the voltage drop across  $R$ , as found by the voltage-divider rule, is

$$V_0 = V_i \frac{R}{R + jX_C} \text{ and } f_c = \frac{1}{2\pi CR}$$

A very common application of the series capacitor high-pass filter is a coupling capacitor between two audio amplifier stages. It is used for passing the amplified audio-signal from one stage to the next and simultaneously block the constant d.c. voltage.

Other high-pass  $RC$  filter circuits exist besides the one shown in Fig. 17.5 (a). These are shown in Fig 17.6.



High-pass filter

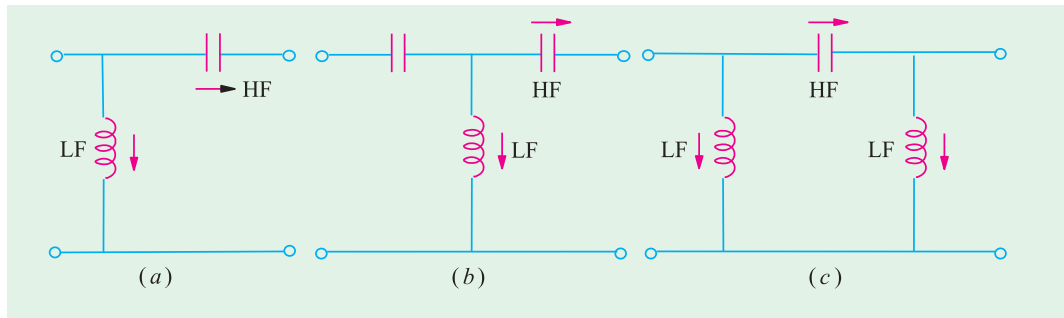


Fig. 17.6

**(i) Inverted-L Type.** It is so called because the capacitor and inductor form an upside down L. It is shown in Fig. 17.6 (a). At lower frequencies,  $X_C$  is large but  $X_L$  is small. Hence, most of the input voltage drops across  $X_C$  and very little across  $X_L$ . However, when the frequency is increased,  $X_C$  becomes less but  $X_L$  is increased thereby causing the output voltage to increase. Consequently, high frequencies are passed while lower frequencies are attenuated.

**(ii) T-Type.** It uses two capacitors and a choke as shown in Fig. 17.6 (b). The additional capacitor improves the filtering action.

**(iii)  $\pi$ -Type.** It uses two inductors which shunt out the lower frequencies as shown in Fig. 17.6 (c).

It would be seen that in all high-pass filter circuits, capacitors are in series between the input and output and the coils are grounded. In fact, capacitors can be viewed as shorts to high frequencies but as open to low frequencies. Opposite is the case with chokes.

### 17.11. High-Pass RL Filter

It is shown in Fig. 17.7 and can be obtained by 'swapping' position of  $R$  and  $L$  in the low-pass  $RL$  circuit of Fig. 17.4 (a). Its response curves are the same as for high-pass  $RC$  circuit and are shown in Fig. 17.5 (b) and (c).

As usual, its output voltage equals the voltage which drops across  $X_L$ . It is given by

$$V_0 = V_i \frac{jX_L}{R + jX_L} \quad \text{and} \quad f_c = \frac{R}{2L}$$

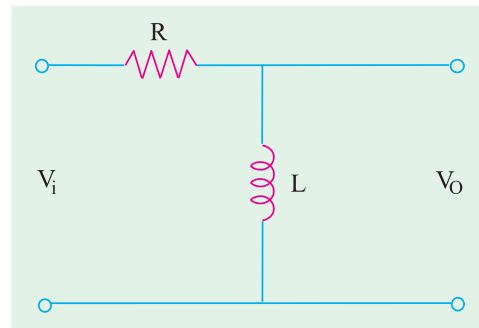


Fig. 17.7

**Example 17.6.** Design a high-pass  $RL$  filter that has a cutoff frequency of 4 kHz when  $R = 3 \text{ k}\Omega$ . It is connected to a  $10 \angle 0^\circ$  V variable frequency supply. Calculate the following :

**(a)** Inductor of inductance  $L$  but of negligible resistance **(b)** output voltage  $V_0$  and its decibel decrease at

**(i)**  $f = 0$  **(ii)**  $f = f_c$  **(iii)** 8 kHz and **(iv)** 40 kHz

**Solution.** **(a)**  $L = R/2\pi f_c = 3/2 \pi \times 4 = 119.4 \text{ mH}$

**(b)** **(i)** At  $f = 0$ ;  $X_L = 0$  i.e. inductor acts as a short-circuit across which no voltage develops. Hence,  $V_0 = 0$  V as shown in Fig. 17.74.

**(ii)**  $f = f_c = 4 \text{ kHz}$ ;  $X_L = R$ .  $\therefore jX_L = j3 = 3 \angle 90^\circ \text{ k}\Omega$

$$\therefore V_0 = V_i \frac{jX_L}{R + jX_L} = 10 \times 0 \frac{3 \angle 90^\circ}{3 + j3} = \frac{30 \angle 90^\circ}{4.24 \angle 45^\circ} = 7.07 \angle 45^\circ \text{ V}$$

Decibel decrease =  $-20 \log_{10}(10/7.07) = -3 \text{ dB}$

(iii)  $f = 2f_c = 8 \text{ kHz}$ .  $X_{L2} = 2 \times j3 = j6 \text{ k}\Omega$

$$\therefore V_0 = 10 \angle 0^\circ \frac{6 \angle 90^\circ}{3 + j6} = \frac{60 \angle 90^\circ}{6.7 \angle 63.4^\circ} = 8.95 \angle 26.6^\circ \text{ V}$$

Decibel decrease =  $-20 \log_{10}(10/8.95) = -0.96 \text{ dB}$

(iv)  $f = 10f_c = 40 \text{ kHz}$ ;  $X_{L3} = 10 \times j3 = j30 \text{ k}\Omega$

$$\therefore B_0 = 10 \frac{j30}{3 + j30} = \frac{300 \angle 90^\circ}{30.15 \angle 84.3^\circ} = 19.95 \angle 5.7^\circ \text{ V}$$

Decibel decrease =  $-20 \log_{10}(10/9.95) = 0.04 \text{ dB}$

As seen from Fig. 17.7, as frequency is increased,  $V_0$  is also increased.

### 17.12. R-C bandpass Filter

It is a filter that allows a certain band of frequencies to pass through and attenuates all other frequencies below and above the passband. This passband is known as the bandwidth of the filter. As seen, it is obtained by cascading a high-pass RC filter to a low-pass RC filter. It is shown in Fig. 17.8 along with its response curve. The passband of this filter is given by the band of frequencies lying between  $f_{c1}$  and  $f_{c2}$ . Their values are given by

$$f_{c1} = 1/2\pi C_1 R_1 \text{ and } f_{c2} = 1/2 C_2 R_2$$

The ratio of the output and input voltages is given by

$$\frac{V_0}{V_i} = \frac{R_1}{R_1 - jX_{C1}} \quad \dots \text{ from } f_1 \text{ to } f_{c1};$$

$$= \frac{-jX_{C2}}{R_2 - jX_{C2}} \quad \dots \text{ from } f_{c2} \text{ to } f_2$$



Bandpass filters

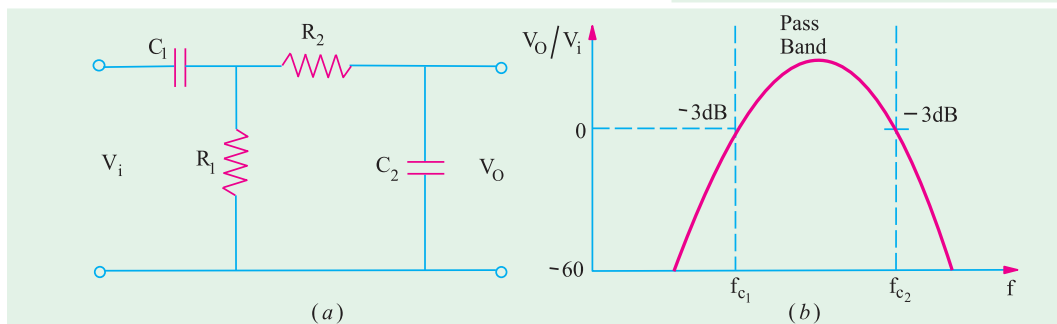


Fig. 17.8

### 17.13. R-C Bandstop Filter

It is a series combination of low-pass and high-pass RC filters as shown in Fig. 17.9 (a). In fact, it can be obtained by reversing the cascaded sequence of the RC bandpass filter. As stated earlier, this filter attenuates a single band of frequencies and allows those on either side to pass through. The stopband is represented by the group of frequencies that lie between  $f_1$  and  $f_2$  where response is below  $-60 \text{ dB}$ .

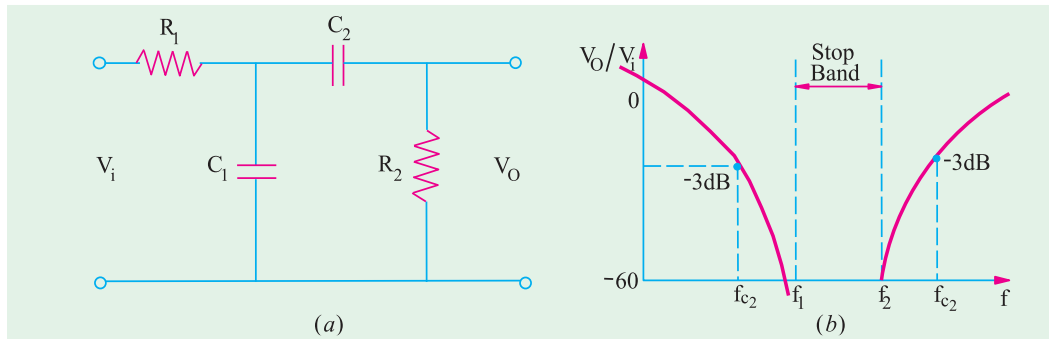


Fig. 17.9

For frequencies from  $f_{c1}$  to  $f_1$ , the following relationships hold good :

$$\frac{V_0}{V_i} = \frac{j X_{C1}}{(R_1 + j X_{C1})} \quad \text{and} \quad f_{c1} = \frac{1}{2 C_1 R_1}$$

For frequencies from  $f_2$  to  $f_{c2}$ , the relationships are as under :

$$\frac{V_0}{V_i} = \frac{R_2}{(R_2 + j X_{C2})} \quad \text{and} \quad f_{c2} = \frac{1}{2 C_2 R_2}$$

In practices, several low-pass  $RC$  filter circuits cascaded with several high-pass  $RC$  filter circuits which provide almost vertical roll-offs and rises. Moreover, unlike  $RL$  filters,  $RC$  filters can be produced in the form of large-scale integrated circuits. Hence, cascading is rarely done with  $RL$  circuits.



A window filter contains one band pass and one low-pass or one high-pass and is used for filtering out unwanted channels, in CATV reception systems or for application cablenet or other communication systems

### 17.14. The - 3 dB Frequencies

The output of an a.c. filter is said to be down 3 dB or -3 dB at the cutoff frequencies. Actually at this frequency, the output voltage of the circuit is 70.7% of the maximum input voltage as shown in Fig. 17.10 (a) for low-pass filter and in Fig. 17.10 (b) and (c) for high-pass and bandpass filters respectively. Here, maximum voltage is taken as the 0 dB reference.

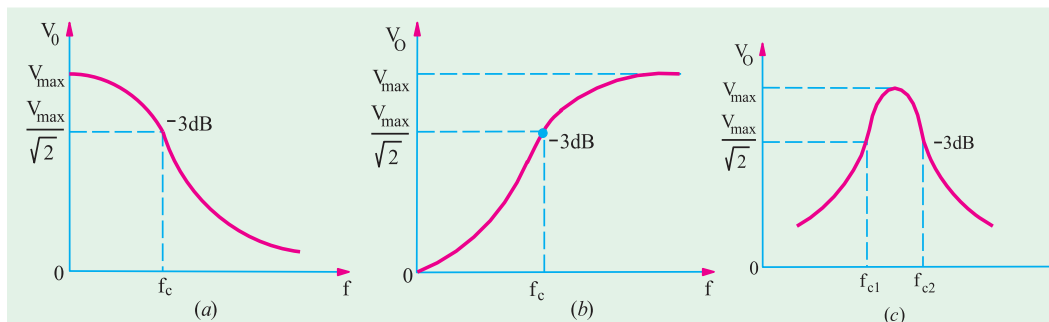


Fig. 17.10

It can also be shown that the power output at the cutoff frequency is 50% of that at zero frequency in the case of low-pass and high-pass filters and of that at  $f_0$  in case of resonant-circuit filter.

### 17.15. Roll-off of the Response Curve

Gradual decreasing of the output of an a.c. filter is called roll-off. The dotted curve in Fig. 17.11 (a) shows an actual response curve of a low-pass  $RC$  filter. The maximum output is defined to be zero dB as a reference. In other words, 0 dB corresponds to the condition when  $V_o = v_i$  because  $20 \log_{10} V_o/V_i = 20 \log 1 = 0$  dB. As seen, the output drops from 0 dB to  $-3$  dB at the cutoff frequency and then continues to decrease at a fixed rate. This pattern of decrease is known as the roll-off of the frequency response. The solid straight line in Fig. 17.11 (a) represents an ideal output response that is considered to be 'flat' and which cuts the frequency axis at  $f_c$ .

The roll-off for a basic IRC or IRL filter is 20 dB/decade or 6 dB/octave. Fig. 17.11 (b) shows the frequency response plot on a semi-log-scale where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is known as Bode plot. Fig. 17.11 (c) shown the Bode plot for a high-pass  $RC$  filter on a semi-log graph. The approximate actual response curve is shown by the dotted line. Here, the frequency is on the logarithmic scale and the filter output in decibel is along with the linear vertical scale. The filter output is flat beyond  $f_c$ . But as the frequency is reduced below  $f_c$ , the output drops at the rate of  $-20$  dB/decade.

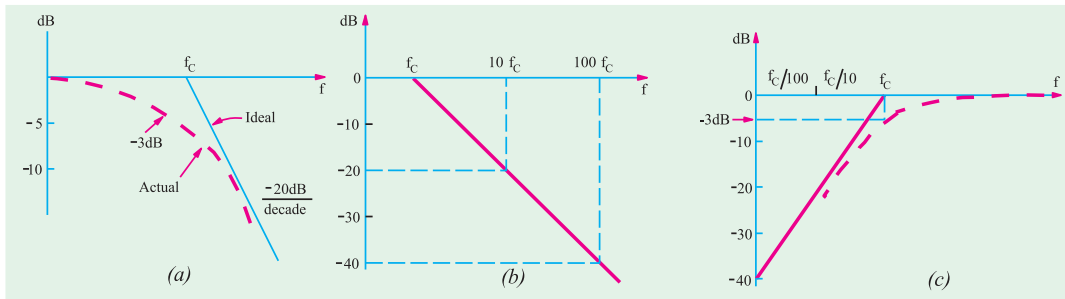


Fig. 17.11

### 17.16. Bandstop and Bandpass Resonant Filter Circuits

Frequency resonant circuits are used in electronic system to make either bandstop or bandpass filters because of their characteristic  $Q$ -rise to either current or voltage at the resonant frequency. Both series and parallel resonant circuits are used for the purpose. It has already been discussed in Chap. No. 7 that

(i) a series resonant circuit offers minimum impedance to input signal and provides maximum current. Minimum impedance equals  $R$  because  $X_L = X_C$  and maximum current  $I = V/R$ .

(ii) a parallel circuit offers maximum impedance to the input signal and provides minimum current. Maximum impedance offered is  $= L/CR$  and minimum current  $I = V/(L/CR)$ .

### 17.17. Series-and Parallel-Resonant Bandstop Filters

The series resonant bandstop filter is shown in Fig. 17.12 (a) where the output is taken across the series resonant circuit. Hence, at resonant frequency  $f_0$ , the output circuit 'sees' a very low resistance  $R$  over which negligible output voltage  $V_o$  is developed. That is why there is a shape resonant dip in the response curve of Fig. 17.12 (b). Such filters are commonly used to reject a particular frequency such as 50-cycle hum produced by transformers or inductors or turn table rumble in recording equipment.

For the series-resonant bandstop filter shown in Fig. 17.12 (a), the following relationships hold good :

$$\text{At } f_0, \frac{V_o}{V_i} = \frac{R_L}{(R_L + R_S)}; Q_0 = \frac{\omega_0 L}{(R + R_S)} \text{ and } B_{ph} = \frac{1/2\pi\sqrt{LC}}{Q_0}$$

At any other frequency  $f$ ,  $\frac{V_0}{V_i} = \frac{R_L + j(X_L - X_C)}{(R_L + R_S) + j(X_L - X_C)}$

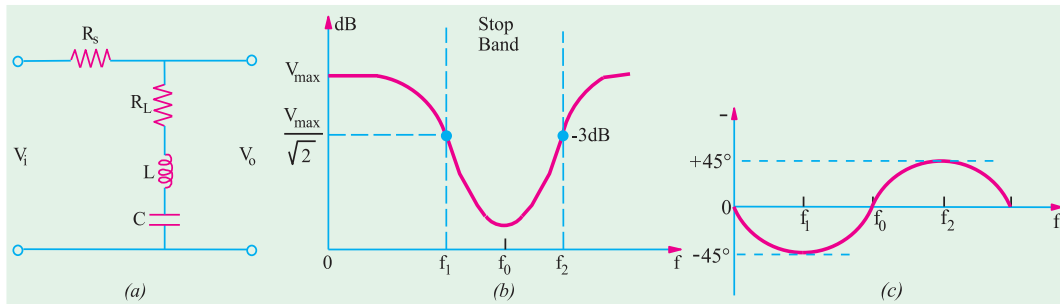


Fig. 17.12

### 17.18. Parallel-Resonant Bandstop Filter

In this filter, the parallel-resonant circuit is in series with the output resistor  $R$  as shown in Fig. 17.13. At resonance, the parallel circuit offers extremely high impedance to  $f_0$  (and nearby frequencies) as compared to  $R$ . Hence the output voltage  $V_0$  at  $f_0$  developed across  $R$  is negligibly small as compared to that developed across the parallel-resonant circuit. Following relationships hold good for this filter :

At  $f_0$ ;  $\frac{V_0}{V_i} = \frac{R_0}{R_0 + Z_{p0}}$  where  $Z_{p0} = Q_0^2 R_L$

At any frequency  $f$ ,  $\frac{V_0}{V_i} = \frac{R_0}{R_0 + Z_p}$

where  $Z_p = \frac{Z_L Z_C}{R_L + j(X_L - X_C)}$

Also  $Q_0 = \omega_0 L / R_L$  and  $B_{hp} = (1 / 2\pi\sqrt{LC}) / Q_0$

It should be noted that the same amplitude phase response curves apply both to the series resonant and parallel-resonant bandstop filters. Since  $X_C$  predominates at lower frequencies, phase angle  $\theta$  is negative below  $f_0$ , above  $f_0$ ,  $X_L$  predominates and the phase current leads. At cutoff frequency  $f_1$ ,  $\theta = -45^\circ$  and at other cutoff frequency  $f_2$ ,  $\theta = +45^\circ$  as in the case of any resonant circuit.

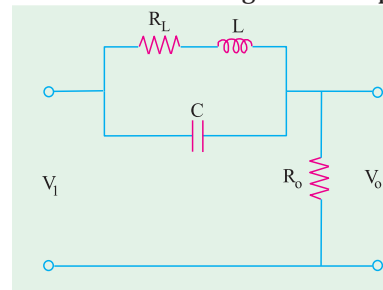


Fig. 17.13

**Example 17.7.** A series-resonant bandstop filter consist of a series resistance of  $2 \text{ k}\Omega$  across which is connected a series-resonant circuit consisting of a coil of resistance  $10 \text{ }\Omega$  and inductance  $350 \text{ mH}$  and a capacitor of capacitance  $181 \text{ pF}$ . If the applied signal voltage is  $10\angle 0^\circ$  of variable frequency, calculate

- (a) resonant frequency  $f_0$ ; (b) half-power bandwidth  $B_{hp}$ ; (c) edge frequencies  $f_1$  and  $f_2$ ; (d) output voltage at frequencies  $f_0$ ,  $f_1$  and  $f_2$ .

**Solution.** We are given that  $R_S = 2 \text{ k}\Omega$ ;  $R = 10 \text{ }\Omega$ ;  $L = 350 \text{ mH}$ ;  $C = 181 \text{ pF}$ .

(a)  $f_0 = 1 / 2\pi\sqrt{LC} = 1 / 2\pi\sqrt{350 \times 10^{-3} \times 181 \times 10^{-12}} = 20 \text{ kHz}$

(b)  $Q_0 = \omega_0 L / (R_S + R_L) = 2\pi \times 20 \times 10^3 \times 350 \times 10^{-3} / 2010 = 21.88$

$B_{hp} = f_0 / Q_0 = 0.914 \text{ kHz}$

(c)  $f_1 = f_0 - B_{hp}/2 = 20 - 0.457 = 19.543 \text{ kHz}$ ;  $f_2 = 20 + 0.457 = 20.457 \text{ kHz}$

(d) At  $f_0$ ,  $V_0 = V_i \frac{R_L}{(R_L + R_S)} = 10\angle 0^\circ \frac{10}{2110} = 0.05\angle 0^\circ \text{ V}$



$$\text{At } f_1, X_{L1} = 2\pi f_1 L = 2\pi \times 19.543 \times 10^3 \times 350 \times 10^{-3} = 42,977 \Omega$$

$$X_{C1} = 1/2\pi \times 19.543 \times 10^3 \times 181 \times 10^{-12} = 44,993 \Omega$$

$$\therefore (X_L - X_C) = (42,977 - 44,993) = -2016 \Omega$$

$$V_{01} = V_i \frac{R_L + j(X_L - X_C)}{(R_S + R_L) + j(X_L - X_C)} = 10 \angle 0^\circ \times \frac{f10 - j2016}{2010 - j2016}$$

$$= \frac{20160 \angle -89.7^\circ}{2847 \angle -45^\circ} = \mathbf{7.07 \angle -44.7^\circ V}$$

$$\text{At } f_2, X_{L2} = X_{L1} (f_2/f_1) = 42977 \times 20.457/19.453$$

$$= 44,987 \Omega; X_{C2} = X_{C1} (f_1/f_2) = 44,993 \times 19.543/20.457$$

$$= 42,983 \Omega; (X_{L2} - X_{C2}) = 44,987 - 42,983 = 2004 \Omega$$

$$\therefore V_0 = 10 \angle 0^\circ \frac{10 + j2004}{2010 + j2004} = \frac{2004 \angle 89.7^\circ}{2837 \angle 44.9^\circ} = \mathbf{7.07 \angle 44.8^\circ V}$$

### 17.19. Series-Resonant Bandpass Filter

As shown in Fig. 17.14 (a), it consists of a series-resonant circuit shunted by an output resistance  $R_0$ . It would be seen that this filter circuit can be produced by 'swapping' as series resonant bandstop filter. At  $f_0$ , the series resonant impedance is very small and equal  $R_L$  which is negligible as compared to  $R_0$ . Hence, output voltage is maximum at  $f_0$  and falls to 70.7% at cutoff frequency  $f_1$  and  $f_2$  and shown in the response curve of Fig. 17.14 (b). The phase angle is positive for frequencies above  $f_0$  and negative for frequencies below  $f_0$  as shown in Fig 17.14 (c) by the solid curve.

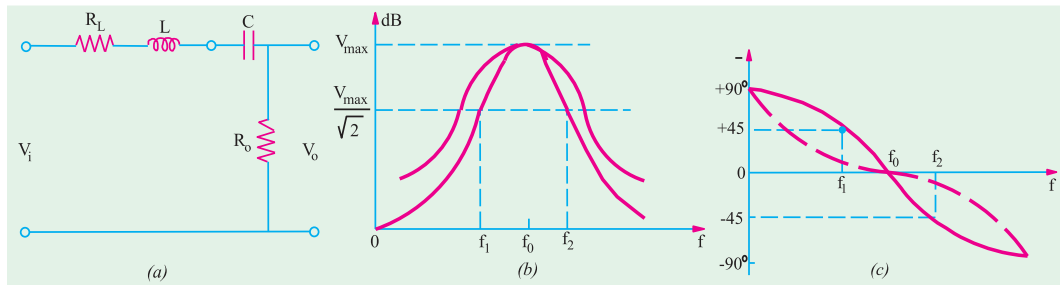


Fig. 17.14

Following relationships hold good for this filter circuit.

$$\text{At } f_0, \frac{V_0}{V_i} = \frac{R_0}{(R_L + R_0)}; Q_0 = \frac{\omega_0 L}{(R_L + R_0)} \text{ and } B_{hp} = \frac{1/2\pi\sqrt{LC}}{Q_0}$$

### 17.20. Parallel-Resonant Bandpass Filter

It can be obtained by transposing the circuit elements of a bandstop a parallel-resonant filter. As shown in Fig. 17.15, the output is taken across the two-branch parallel-resonant circuit. Since this circuit offers maximum impedance at resonance, this filter produces maximum output voltage  $V_0$  at  $f_0$ . The amplitude-response curve of this filter is similar to that of the series-resonant bandpass filter discussed above [Fig. 17.14 (b)]. The dotted curve in Fig. 17.14 (c) represents the phase relationship between the input and output voltages of this filter. The following relationships apply to this filter :

At  $f_0$ ,  $\frac{V_0}{V_i} = \frac{R_0}{(R_0 + Z_{p0})}$  where  $Z_{p0} = R_{p0} = Q_r^2 R_L$

and  $Q_0 = \frac{R_{p0}}{X_{C0}}$  and  $B_{hp} = \frac{1/2\pi\sqrt{LC}}{Q_0}$

At any frequency  $f$ ,

$\frac{V_0}{V_i} = \frac{Z_p}{R_0 + Z_p}$  where  $Z_p = \frac{Z_L(-jX_C)}{R_L + j(X_L - X_C)}$

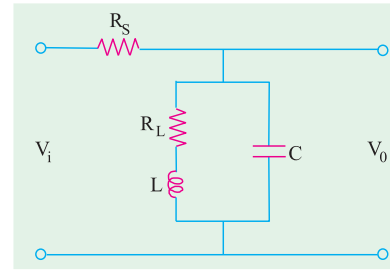


Fig. 17.15

### OBJECTIVE TEST – 17

- The decibel is a measure of
  - power
  - voltage
  - current
  - power level
- When the output voltage level of a filter decreases by  $-3$  dB, its absolute value changes by a factor of
  - $\sqrt{2}$
  - $1/\sqrt{2}$
  - 2
  - 1/2
- The frequency corresponding to half-power point on the response curve of a filter is known as —
  - cutoff
  - upper
  - lower
  - roll-off
- In a low-pass filter, the cutoff frequency is represented by the point where the output voltage is reduced to — per cent of the input voltage.
  - 50
  - 70.7
  - 63.2
  - 33.3
- In an  $RL$  low-pass filter, an attenuation of  $-12$  dB/octave corresponds to ..... dB/decade.
  - $-6$
  - $-12$
  - $-20$
  - $-40$
- A network which attenuate a single band of frequencies and allows those on either side to pass through is called ..... filter.
  - low-pass
  - high-pass
  - bandstop
  - bandpass
- In a simple high-pass RC filter, if the value of capacitance is doubled, the cutoff frequency is
  - doubled
  - halved
  - tripled
  - quadrupled
- In a simple high-pass RL filter circuit, the phase difference between the output and input voltages at the cutoff frequency is .... degrees.
  - $-90$
  - 45
  - $-45$
  - 90
- In a simple low-pass RC filter, attenuation is  $-3$  dB at  $f_c$ . At  $2f_c$  attenuation is  $-6$  dB. At  $10f_c$  the attenuation would be .... dB.
  - $-30$
  - $-20$
  - $-18$
  - $-12$
- An a.c. signal of constant voltage 10 V and variable frequency is applied to a simple high-pass RC filter. The output voltage at ten times the cutoff frequency would be ..... volt.
  - 1
  - 5
  - $10/\sqrt{2}$
  - $10\sqrt{2}$
- When two simple low-pass filters having same values of R and C are cascaded, the combined filter will have a roll-off of ..... dB/decade.
  - $-20$
  - $-12$
  - $-40$
  - $-36$
- An a.c. signal of constant voltage but with frequency varying from dc to 25 kHz is applied to a high-pass filter. Which of the following frequency will develop the greatest voltage at the output load resistance?
  - d.c.
  - 15 kHz
  - 10 kHz
  - 25 kHz
- A voltage signal source of constant amplitude with frequency varying from dc to 25 kHz is applied to a low-pass filter. Which frequency will develop greatest voltage across the output load resistance?
  - d.c.
  - 10 kHz
  - 15 kl
  - 25 kHz
- The output of a filter drops from 10 to 5 V as the frequency is increased from 1 to 2 kHz. The dB change in the output voltage is
  - $-3$  dB/decade
  - $-6$  dB/octave
  - 6 dB/octave
  - $-3$  dB/octave

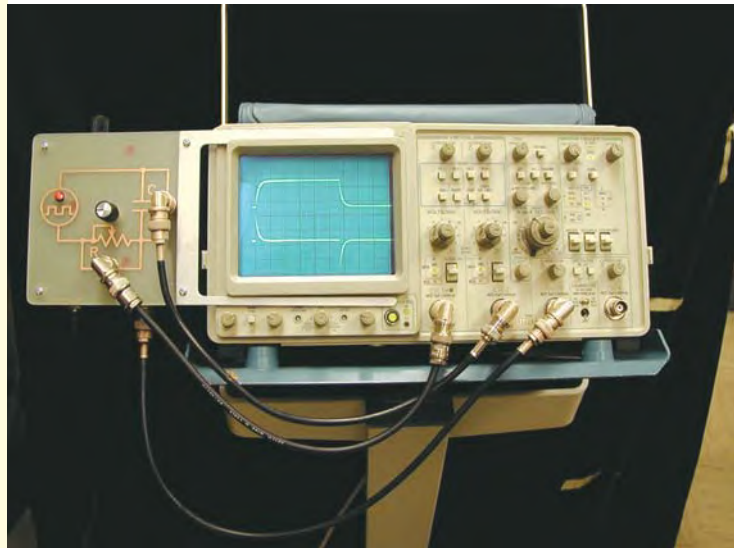
### ANSWERS

1. d 2. b 3. a 4. b 5. d 6. c 7. b 8. b 9. b 10. a 11. c 12. d 13. a 14. b

## CHAPTER 18

**Learning Objectives**

- Circle Diagram of a Series Circuit
- Rigorous Mathematical Treatment
- Constant Resistance but Variable Reactance
- Properties of Constant Reactance But Variable Resistance Circuit
- Simple Transmission Line Circuit

**CIRCLE  
DIAGRAMS**

Combinations of R and C circuits

### 18.1. Circle Diagram of a Series Circuit

Circle diagrams are helpful in analysing the operating characteristics of circuits, which, under some conditions, are used in representing transmission lines and a.c. machinery (like induction motor etc.)

Consider a circuit having a constant reactance but variable resistance varying from zero to infinity and supplied with a voltage of constant magnitude and frequency (Fig. 18.1).

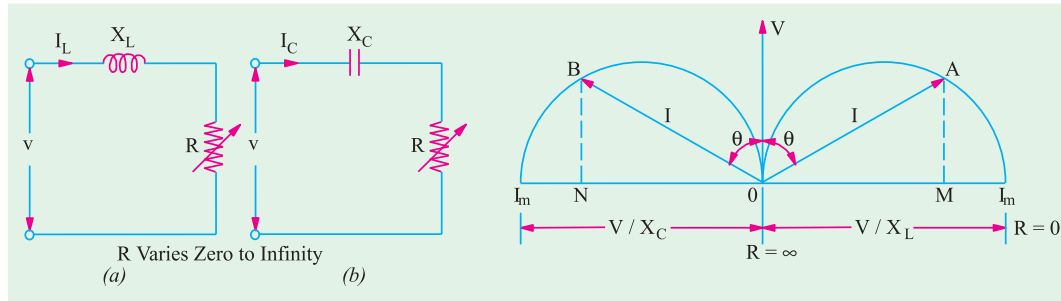


Fig. 18.1

Fig. 18.2

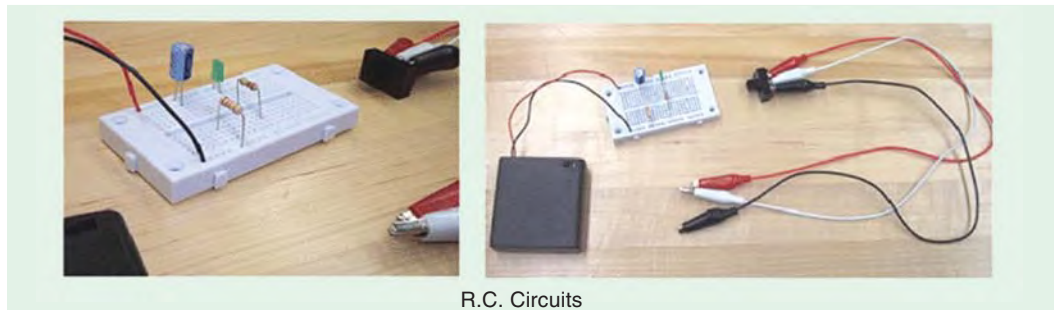
If  $R = 0$ , then  $I = V/X_L$  or  $V/X_C$  and has maximum value. It will lag or lead the voltage by  $90^\circ$  depending on whether the reactance is inductive or capacitive. In Fig. 18.2, angle  $\theta$  represents the phase angle. If  $R$  is now increased from its zero value, then  $I$  and  $\theta$  will both decrease. In the limiting case, when  $R = \infty$ , then  $I = 0$  and  $\theta = 0^\circ$ . It is found that the locus of end point of current vector  $OA$  or  $OB$  represents a semi-circle with diameter equal to  $V/X$  as shown in Fig. 18.2. It can be proved thus :

$$I = V/Z \text{ and } \sin \theta = X/Z \text{ or } Z = X/\sin \theta \therefore I = V \sin \theta / X$$

For constant value of  $V$  and  $X$ , the above is the polar equation of a circle of diameter  $V/X$ . This equation is plotted in Fig. 18.2. Here,  $OV$  is taken as reference vector. It is also seen that for inductive circuit, the current semi-circle is on the right-hand side of reference vector  $OV$  so that current vector  $OA$  lags by  $\theta^\circ$ . The current semi-circle for  $R-C$  circuit is drawn on the left hand side of  $OV$  so that current vector  $OB$  leads  $OV$  by  $\theta^\circ$ . It is obvious that  $AM = I \cos \theta$ , hence  $AM$  represents, on a suitable scale, the power consumed by the  $R-L$  circuit, Similarly,  $BN$  represents the power consumed by the  $R-C$  circuit.

### 18.2. Rigorous Mathematical Treatment

We will again consider both  $R-L$  and  $R-C$  circuits. The voltage drops across  $R$  and  $X_L$  (or  $X_C$ ) will be  $90^\circ$  out of phase with each other. Hence, for any given value of resistance, the vector diagram for the two voltage drops (*i.e.*  $IR$  and  $IX$ ) is a rightangled triangle having applied voltage as the hypotenuse.



R.C. Circuits



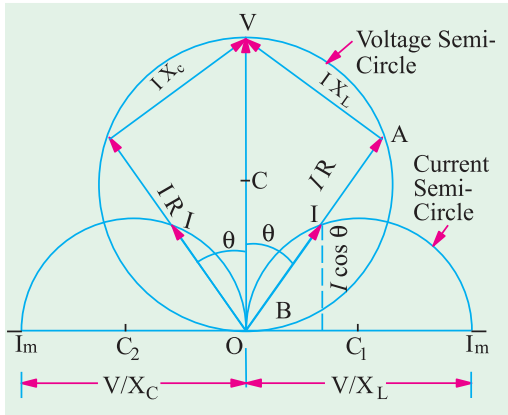


Fig. 18.3

For a constant applied voltage and reactance, the vector diagrams for different values of  $R$  are represented by a series of right-angled triangles having common hypotenuse as shown in Fig. 18.3. The locus of the apex of the right-angled voltage triangles is a semicircle described on the hypotenuse. The voltage semi-circle for  $R$ - $L$  circuit ( $OAV$ ) is on the right and for  $R$ - $C$  circuit ( $OBV$ ) on the left of the reference vector  $OV$  as shown in Fig. 18.3.

The foci of end points of current vectors are also semi-circles as shown but their centres lie on the opposite sides of and in an axis perpendicular to the reference vector  $OV$ .

**(ii) R-L Circuit** [Fig. 18.1 (a)]

The co-ordinates of point  $A$  with respect to the origin  $O$  are

$$y = I \cos \theta = \frac{V}{Z} \cdot \frac{R}{Z} = V \frac{R}{Z^2}; x = I \sin \theta = \frac{V}{Z} \cdot \frac{X_L}{Z} = V \frac{X_L}{Z^2} \quad \dots (i)$$

Squaring and adding, we get

$$x^2 + y^2 = \frac{VR}{R^2 + X_L^2} + \frac{VX_L}{R^2 + X_L^2} = \frac{V^2(R^2 + X_L^2)}{(R^2 + X_L^2)^2} = \frac{V^2}{R^2 + X_L^2}$$

From (i) above,  $\frac{VX_L}{x} = R^2 + X_L^2$   $x^2 + y^2 = \frac{V^2}{VX_L/x} = \frac{xV}{X_L}$

$$\therefore x^2 + y^2 = \frac{xV}{X_L} \quad \text{or} \quad y^2 = x \left[ \frac{V}{2X_L} - \frac{V^2}{4X_L^2} \right]$$

This is the equation of a circle, the co-ordinates of the centre of which are  $y = 0, x = V/2 X_L$  and whose radius is  $V/2X_L$ .

**(ii) R-C Circuit.** In this case it can be similarly proved that the locus of the end point of current vector is a semi-circle. The equation of this circle is

$$y^2 = x \left[ \frac{V}{2X_C} - \frac{V^2}{4X_C^2} \right]$$

The centre has co-ordinates of  $y = 0, x = -V/2X_C$ .



**18.3. Constant Resistance But Variable Reactance**

Fig. 18.4 shows two circuits having constant resistance but variable reactance  $X_L$  or  $X_C$  which vary from zero to infinity. When  $X_L = 0$ , current is maximum and equals  $V/R$ . For other values,  $I = V/\sqrt{R^2 + X_L^2}$ . Current becomes zero when  $X_L = \infty$ . As seen from Fig. 18.5, the end point of the current vector describes a semi-circle with radius  $OC = V/2R$  and centre lying in the reference sector *i.e.* voltage vector  $OV$ . For  $R$ - $C$  circuit, the semi-circle lies to the left of  $OV$ . As before, it may be proved that the equation of the circle shown in Fig. 18.5 is

$$x^2 + y^2 = \frac{V^2}{4R^2}$$

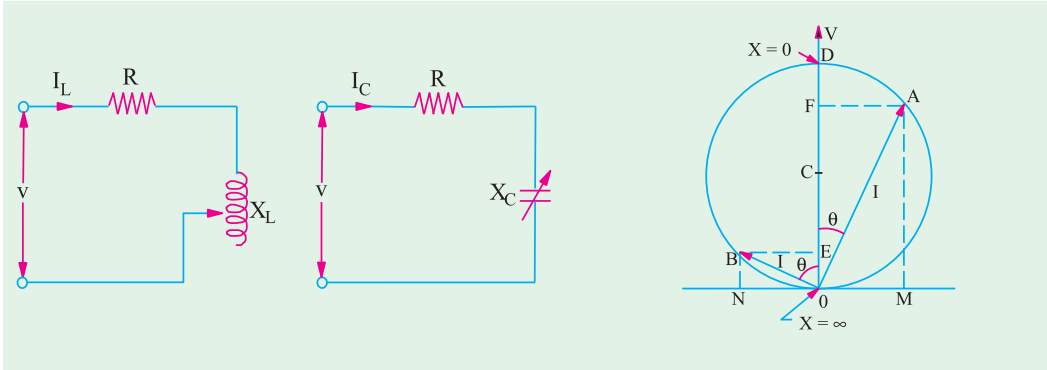


Fig. 18.4

Fig. 18.5

The co-ordinates of the centre are  $x = 0$ ,  $y = V/2R$  and radius  $= V/2R$ .

As before, power developed would be maximum when current vector makes an angle of  $45^\circ$  with the voltage vector  $OV$ . In that case, current is  $I_m / \sqrt{2}$  and  $P_m = VI_m / 2$ .

#### 18.4. Properties of Constant Reactance But Variable Resistance Circuit

From the circle diagram of Fig. 18.3, it is seen that circuits having a constant reactance but variable resistance or *vice-versa* have the following properties :

- (i) the current has limiting value
- (ii) the power supplied to the circuit has a limiting value also
- (iii) the power factor corresponding to maximum power supply is  $0.707 (= \cos 45^\circ)$

Obviously, the maximum current in the circuit is obtained when  $R = 0$ .

$$\therefore \begin{aligned} I_m &= V / X_L = V / \omega L \\ &= -V / X_C = -\omega VC \end{aligned}$$

... for R-L circuit

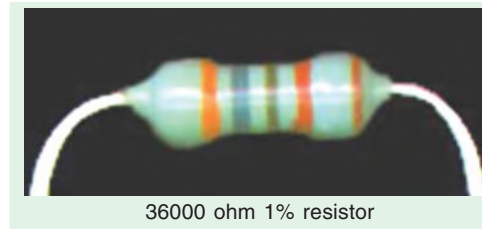
... for R-C circuit

Now, power  $P$  taken by the circuit is  $VI \cos \theta$  and if  $V$  is constant, then  $P \propto I \cos \theta$ . Hence, the ordinates of current semi-circles are proportional to  $I \cos \theta$ . The maximum ordinate possible in the semi-circle represents the maximum power possible in the circuit. The maximum ordinate passes through the centre of semi-circle so that current vector makes an angle of  $45^\circ$  with both the diameter and the voltage vector  $OV$ . Obviously, power factor corresponding to maximum power intake is  $\cos 45^\circ = 0.707$ .

Maximum power,

$$P_m = V \times AB = V \times \frac{I_m}{2} = \frac{1}{2} VI_m$$

Now, for R-L circuit,  $I_m = V/X_L$



36000 ohm 1% resistor

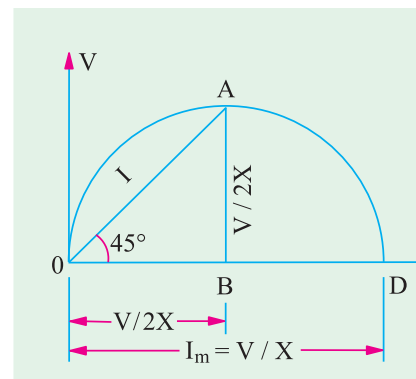


Fig. 18.6

$$\therefore P_m = \frac{V^2}{2X_L} = \frac{V^2}{2L}$$

For R-C circuit  $P_m = \frac{V^2}{2X_C} = \frac{V^2\omega C}{2}$

As said above, at maximum power,  $\theta = 45^\circ$ , hence vector triangle for voltages is an isosceles triangle which means that voltage drops across resistance and reactance are each equal to  $0.707$  of supply voltage *i.e.*  $V/\sqrt{2}$ . As current is the same, for maximum power, resistance equals reactance *i.e.*  $R = X_L$  (or  $X_C$ ).

Hence, the expression representing maximum power may be written as  $P_m = V^2/2R$ .

### 18.5. Simple Transmission Line Circuit

In Fig. 18.7 (a) is shown a simple transmission circuit having negligible capacitance and reactance.  $R$  and  $X_L$  represent respectively the resistance and reactance of the line and  $R_L$  represents load resistance.

If  $R$  and  $X_L$  are constant, then as  $R_L$  is varied, the current  $AM$  follows the equation  $I = (V/X) \sin \theta$  (Art. 18.1). The height  $AM$  in Fig. 18.7 (b) represents the power consumed by the circuit but, in the present case, this power is consumed both in  $R$  and  $R_L$ . The power absorbed by each resistance can be represented on the circle diagram.

In Fig. 18.7 (b),  $OB$  represents the line current when  $R_L = 0$ . The current  $OB = V/\sqrt{(R^2 + X_L^2)}$  and power factor is  $\cos \theta_1$ . The ordinate  $BN$  then represents on a different scale the power dissipated in  $R$  only.  $OA$  represents current when  $R_L$  has some finite value *i.e.*  $OA = V/\sqrt{(R + R_L)^2 + X_L^2}$ . The ordinate  $AM$  represents total power dissipated, out of which  $ME$  is consumed in  $R$  and  $AE$  in  $R_L$ .

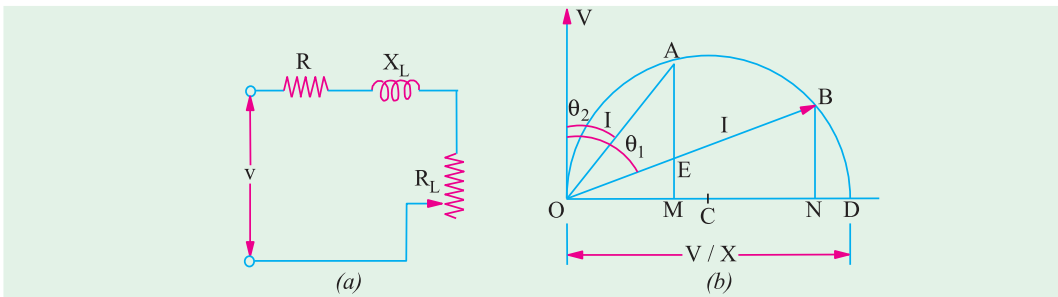


Fig. 18.7

In fact, if  $OA^2 \times R_L (= AE)$  is considered to be the output of the circuit (the power transmitted by the line), then

$$\eta = \frac{AE}{AM}$$

With  $R$  and  $X_L$  constant, the maximum power that can be transmitted by such a circuit occurs when the extremity of current vector  $OA$  coincides with the point of tangency to the circle of a straight line drawn parallel to  $OB$ . Obviously,  $V$  times  $AE$  under these conditions represents the maximum power and the power factor at that time is  $\cos \theta_2$ .

**Example 18.1.** A circuit consists of a reactance of  $5 \Omega$  in series with a variable resistance. A constant voltage of  $100 V$  is applied to the circuit. Show that the current locus is circular. Determine (a) the maximum power input to the circuit (b) the corresponding current, p.f. and value of the resistance. (Electrical Science II, Allahabad Univ. 1992)

**Solution.** For the first part, please refer to Art. 18.1

$$(a) I_m = V/X = 100/5 = 20 \text{ A}; P_m = \frac{1}{2} VI_m = \frac{1}{2} \times 100 \times 20 = \mathbf{1000 \text{ W}}$$

(b) At maximum power input, current is =  $OA$  (Fig. 18.6)

$$\therefore OA = I_m / \sqrt{2} = 20 / \sqrt{2} = 14.14 \text{ A}; p.f. = \cos 45^\circ = \mathbf{0.707}; R = X = \mathbf{5 \Omega}$$

**Example 18.2.** If a coil of unknown resistance and reactance is connected in series with a 100-V, 50-Hz supply, the current locus diagram is found to have a diameter of 5 A and when the value of series resistor is  $15 \Omega$ , the power dissipated is maximum. Calculate the reactance and resistance of the coil and the value of the maximum power in the circuit and the maximum current.

**Solution.** Let the unknown resistance and reactance of the coil be  $R$  and  $X$  respectively

$$\text{Diameter} = V/X \quad \therefore 5 = 100/X \quad \text{or} \quad X = \mathbf{20 \Omega}$$

Power is maximum when total resistance = reactance

$$\text{or } 15 + R = 20 \quad \therefore R = \mathbf{5 \Omega}$$

$$\text{Maximum power } P_m = V^2/2X = 100^2/2 \times 20 = \mathbf{250 \text{ W}}$$

$$\text{Maximum current } I_m = 100 / \sqrt{(20^2 + 5^2)} = \mathbf{4.85 \text{ A}}$$

**Example 18.3.** A constant alternating sinusoidal voltage at constant frequency is applied across a circuit consisting of an inductance and a variable resistance in series. Show that the locus diagram of the current vector is a semi-circle when the resistance is varied between zero and infinity.

If the inductance has a value of 0.6 henry and the applied voltage is 100 V at 25 Hz, calculate (a) the radius of the arc (in amperes) and (b) the value of variable resistance for which the power taken from the mains is maximum and the power factor of the circuit at the value of this resistance.

$$\text{Solution. } X_L = \omega L = 0.6 \times 2\pi \times 25 = 94.26 \Omega$$

$$(a) \text{ Radius} = V/2 X_L = 100/2 \times 4.26 = \mathbf{0.531 \text{ A}}$$

**Example 18.4.** A resistor of  $10 \Omega$  is connected in series with an inductive reactor which is variable between  $2 \Omega$  and  $20 \Omega$ . Obtain the locus of the current vector when the circuit is connected to a 250-V supply. Determine the value of the current and the power factor when the reactance is (i)  $5 \Omega$  (ii)  $10 \Omega$  (iii)  $15 \Omega$ . (Basic Electricity, Bombay Univ.)

**Solution.** As discussed in Art. 18.3, the end point of current vector describes a semi-circle whose diameter (Fig. 18.8) equals  $V/R = 250/10 = 25 \text{ A}$  and whose centre lies to right side of the vertical voltage vector  $OV$ .

$$I_{max} = 250 / \sqrt{10^2 + 2^2} = 24 \text{ A}; \theta = \tan^{-1} (2/10) = 11.3^\circ;$$

$$I_{min} = 250 / \sqrt{10^2 + 20^2} = 11.2 \text{ A}; \theta = \tan^{-1} (20/10) = 63.5^\circ$$

$$(i) \theta_1 = \tan^{-1} (5/10) = 26.7^\circ, p.f. = \cos 26.7^\circ = 0.89$$

$$I = OA = \mathbf{22.4 \text{ A}}$$

$$(ii) \theta_2 = \tan^{-1} (10/10) = 45^\circ, p.f. = \cos 45^\circ = \mathbf{1};$$

$$I = OB = \mathbf{17.7 \text{ A}}$$

$$(iii) \theta_3 = \tan^{-1} (15/10) = 56.3^\circ; p.f. = \cos 56.3^\circ = \mathbf{0.55};$$

$$I = OC = \mathbf{13.9 \text{ A}}.$$

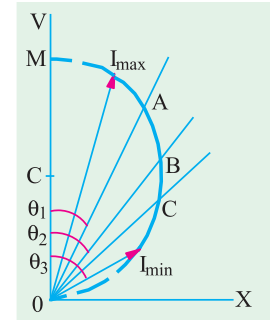


Fig. 18.8

**Example 18.5.** A voltage of  $100 \sin 10,000 t$  is applied to a circuit consisting of a  $1 \mu F$  capacitor in series with a resistance  $R$ . Determine the locus of the tip of the current phasor when  $R$  is varied from 0 to  $\infty$ . Take the applied voltage as the reference phasor.

(Network Theory and Design, AMIETE 1990)



**Solution.** As seen from Art. 18.2 the locus of the tip of the current phasor is a circle whose equation is

$$y^2 + x^2 - \frac{V}{2X_C}x = \frac{V^2}{4X_C^2}$$

We are given that  $V = V_m / \sqrt{2} = 100 / \sqrt{2} = 77.7 \text{ V}$

$$\omega = 10,000 \text{ rad/s}; X_C = 1/\omega \times C = 1/10,000 \times 1 \times 10^{-6} = 100 \text{ } \Omega \text{ C,}$$

$$(V/2 X_C)^2 = (77.7/2 \times 100)^2 = 0.151, \therefore y^2 + (x + 0.389)^2 = 0.151$$

**Example 18.6.** Prove that polar locus of current drawn by a circuit of constant resistance and variable capacitive reactance is circular when the supply voltage and frequency are constant.

If the constant resistance is  $10 \text{ } \Omega$  and the voltage is  $100 \text{ V}$ , draw the current locus and find the values of the current and p.f. when the reactance is (i)  $5.77 \text{ } \Omega$  (ii)  $10 \text{ } \Omega$  and (iii)  $17.32 \text{ } \Omega$ . Explain when the power will be maximum and find its value.

(Electromechanics, Allahabad Univ. 1992)

**Solution.** For the first part, please refer to Art. 18.3. The current semicircle will be drawn on the vertical axis with a radius  $OM = V/2 R = 100/2 \times 10 = 5 \text{ A}$  as shown in Fig. 18.9 (b)

(i)  $\theta_1 = \tan^{-1}(5.77/10) = 30^\circ$ ;  $\cos \theta_1 = 0.866$   
(lead) ; current =  $OA = 8.66 \text{ A}$

(ii)  $\theta_2 = \tan^{-1}(10/10) = 45^\circ$ ;  $\cos \theta_2 = 0.707$   
(lead) current =  $OB = 7.07 \text{ A}$

(iii)  $\theta_3 = \tan^{-1}(17.32/10) = 60^\circ$ ;  $\cos \theta_3 = 0.5$   
(lead) current =  $OC = 5 \text{ A}$

Power would be maximum for point B when  $\theta = 45^\circ$ ;  $I_m = V/R = 100/10 = 10 \text{ A}$

$$P_m = V \times OB \times \cos 45^\circ = V \times I_m \cos 45^\circ \times \cos$$

$$45^\circ = \frac{1}{2} VI_m = \frac{1}{2} \times 100 \times 10 = 500 \text{ W}$$

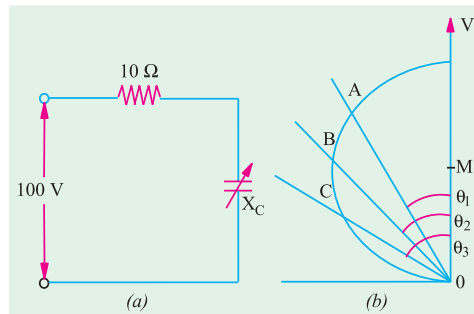


Fig. 18.9

**Example 18.7.** Prove that the polar locus of the current drawn by a circuit of constant reactance and variable resistance is circular when the supply voltage and frequency are constant.

If the reactance of such a circuit is  $25 \text{ } \Omega$  and the voltage  $250$ , draw the said locus and locate there on the point of maximum power and for this condition, find the power, current, power factor and resistance.

Locate also the point at which the power factor is  $0.225$  and for this condition, find the current, power and resistance.

(Basic Electricity, Bombay Univ.)

**Solution.** For the first part, please refer to Art. 18.3.

Radius of the current semi-circle is  $= V/2X = 250/2 \times 25 = 5 \text{ A}$ . As discussed in Art. 18.3, point A [Fig. 18.10 (a)] corresponds to maximum power.

$$\text{Now, } I_m = V/X = 250/25 = 10 \text{ A}; P_m = \frac{1}{2} VI_m = \frac{1}{2} \times 250 \times 10 = 1250 \text{ W}$$

$$\text{Current } OA = I_m/\sqrt{2} = 10/\sqrt{2} = 7.07 \text{ A}; \text{ p.f.} = \cos 45^\circ = 0.707.$$

$$\text{Under condition of maximum power, } R = X = 25 \text{ } \Omega.$$

$$\text{Now, } \cos \theta = 0.225;$$

$$\theta = \cos^{-1}(0.225) = 77^\circ$$

In Fig. 18.10 (b), current vector  $OA$  has been drawn at an angle of  $77^\circ$  with the vertical voltage vector  $OV$ .

By measurement, current

$$OA = 9.74 \text{ A}$$

By calculate,  $OA = I_m \cos$

$$13^\circ = 10 \times 0.974 = \mathbf{9.74 \text{ A}}$$

$$\text{Power} = VI \cos \theta = 250 \times 9.74 \times 0.225 = \mathbf{548 \text{ W}}$$

$$P = I^2 R; R = P/I^2 = 548/9.74^2 = \mathbf{5.775 \ \Omega}$$

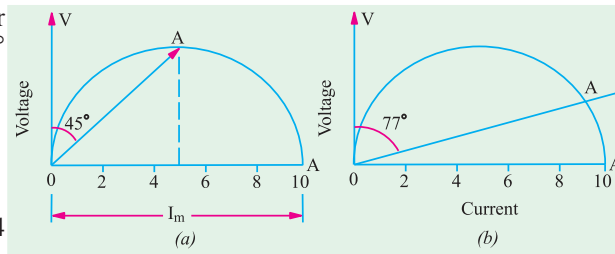


Fig. 18.10

**Example 18.8.** A non-inductive resistance  $R$ , variable between  $0$  and  $10 \ \Omega$ , is connected in series with a coil of resistance  $3 \ \Omega$  and reactance  $4 \ \Omega$  and the circuit supplied from a  $240\text{-V}$  a.c. supply. By means of a locus diagram, determine the current supplied to the circuit when  $R$  is (a) zero (b)  $5 \ \Omega$  and (c)  $10 \ \Omega$ . By means of the symbolic method, calculate the value of the current when  $R = 5 \ \Omega$ .

**Solution.** The locus of the current vector is a semi-circle whose centre is  $(0, V/2X)$  and whose radius is obviously equal to  $V/2X$ . Now,  $V/2X = 240/2 \times 4 = 30 \text{ A}$ .

Hence, the semi-circle is drawn as shown in Fig. 18.11 (b).

(a) Total resistance =  $3 \ \Omega$  and  $X = 4 \ \Omega$ .  $\therefore \tan \theta_1 = 4/2 \therefore \theta_1 = 53^\circ 8'$

Hence, current vector  $OA$  is drawn making an angle of  $53^\circ 8'$  with vector  $OV$ . Vector  $OA$  measures  $49 \text{ A}$ .

(b) Total resistance =  $3 + 5 = 8 \ \Omega$ .

Reactance =  $4 \ \Omega$ ;  $\tan \theta_2 = 4/8 = 0.5 \therefore \theta_2 = 26^\circ 34'$

Current vector  $OB$  is drawn at an angle of  $26^\circ 34'$  with  $OV$ . It measure  $27 \text{ A}$  (approx.)

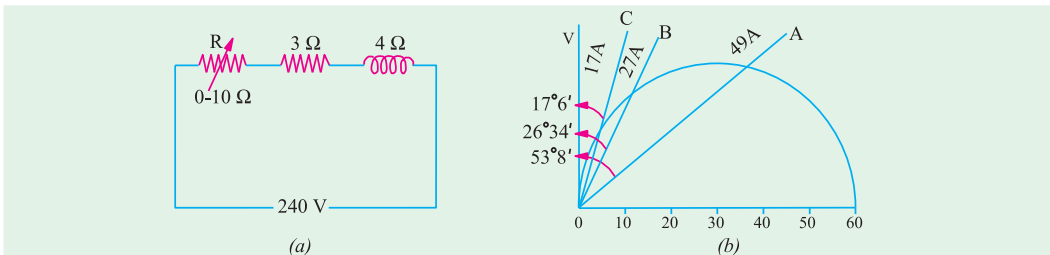


Fig. 18.11

(c) Total resistance =  $3 + 10 = 13 \ \Omega$ .

Reactance =  $4 \ \Omega$ ;  $\tan \theta_3 = 4/13 \therefore \theta_3 = 17^\circ 6'$

Current vector  $OC$  is drawn at an angle of  $17^\circ 6'$  with vector  $OA$ . It measures  $17 \text{ A}$ .

### Symbolic Method

$$I = \frac{240}{(5 + j3) + j4} = \frac{240}{8 + j4} = \frac{240}{8.96 \angle 26.5^\circ} = 26.7 \angle 26.5^\circ$$

**Note.** There is difference in the magnitudes of the currents and the angles as found by the two different methods. It is so because one has been found exactly by mathematical calculations, whereas the other has been measured from the graph.

**Example 18.9.** A circuit consisting of a  $50\text{-}\Omega$  resistor in series with a variable reactor is shunted by a  $100\text{-}\Omega$  resistor. Draw the locus of the extremity of the total current vector to scale and determine the reactance and current corresponding to the minimum overall power factor, the supply voltage being  $100 \text{ V}$ .

**Solution.** The parallel circuit is shown in Fig. 18.12 (a).

The resistive branch draws a fixed current  $I_2 = 100/100 = 1$  A. The current  $I_1$  drawn by the reactive branch is maximum when  $X_L = 0$  and its maximum value is  $= 100/50 = 2$  A and is in phase with voltage.

In the locus diagram of Fig. 18.12 (b), the diameter  $OA$  of the reactive current semi-circle is  $= 2$  A.  $OB$  is the value of  $I_1$  for some finite value of  $X_L$ .  $O'O$  represents  $I_2$ . Being in phase with voltage, it is drawn in phase with voltage vector  $OV$ . Obviously,  $O'B$  represents total circuit current, being the vector sum of  $I_1$  and  $I_2$ .

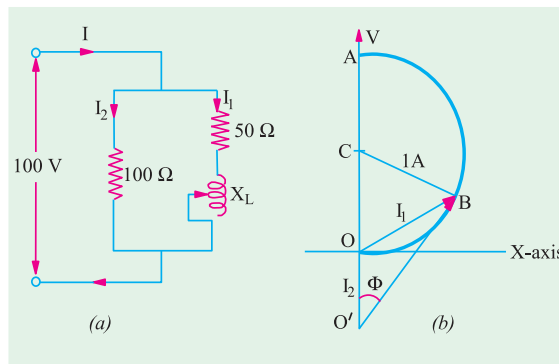


Fig. 18.12

The minimum power factor which corresponds to maximum phase difference between  $O'B$  and  $O'V$  occurs when  $O'B$  is tangential to the semi-circle. In that case,  $O'B$  is perpendicular to  $BC$ . It means that  $O'BC$  is a right-angled triangle.

Now,  $\sin \phi = BC / O'C = 1 / (1 + 1) = 0.5; \phi = 30^\circ$

$\therefore$  Minimum  $p.f. = \cos 30^\circ = 0.866$  (lag)

Current corresponding to minimum  $p.f.$  is  $O'B = O'C \cos \phi = 2 \times 0.866 = 1.732$  A.

Now,  $\Delta OBC$  is an equilateral triangle, hence  $I_1 = OB = 1$  A. Considering reactive branch,  $Z = 100/1 = 100 \Omega$ ,  $X_L = \sqrt{100^2 - 50^2} = 88.6 \Omega$

**Example 18.10.** A coil of resistance  $60 \Omega$  and inductance  $0.4$  H is connected in series with a capacitor of  $17.6 \mu F$  across a variable frequency source which is maintained at a fixed potential of  $120$  V. If the frequency is varied through a range of  $40$  Hz to  $80$  Hz, draw the complete current locus and calculate the following :

- (i) the resonance frequency, (ii) the current and power factor at  $40$  Hz and
- (iii) the current and power factor at  $80$  Hz.

(Elect. Circuits, South Gujarat Univ.)

**Solution.** (i)  $f_0 = 10^3 / 2\pi \sqrt{0.4 \times 17.6} = 60$  Hz.

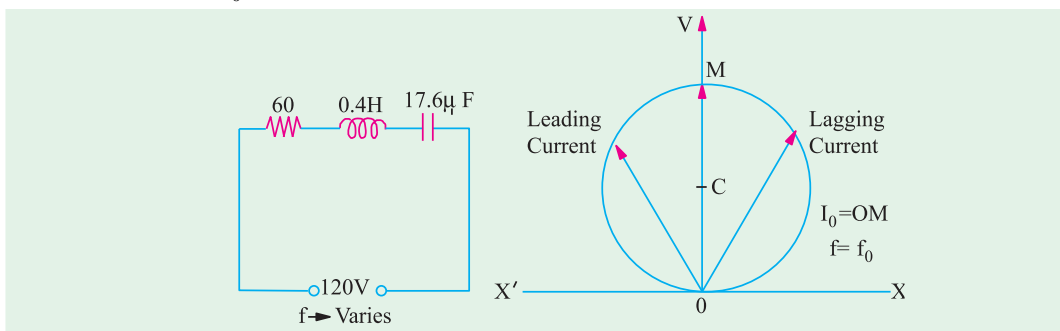


Fig. 18.13

(ii)  $f = 40$  Hz

$X_L = 2\pi \times 40 \times 0.4 = 100 \Omega$ ;  $X_C = 10^6 / 2\pi \times 40 \times 17.6 = 226 \Omega$

$X = 100 - 226 = -126 \Omega$  (capacitive);  $I = 120 / \sqrt{60^2 + (-126)^2} = 0.86$  A

$p.f. = \cos \theta = R/Z = 60/139.5 = 0.43$  (lead)

(iii)  $f = 80 \text{ Hz}$

$$X_L = 100 \times 2 = 200 \ \Omega ; X_C = 226/2 = 113 \ \Omega ; X = 200 - 113 = 87 \ \Omega \text{ (inductive)}$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{60^2 + 87^2} = 105.3 \ \Omega$$

$$I = 120/105.3 = \mathbf{1.14 \text{ A}} ; \text{p.f. } \cos \theta = 60/105.3 = \mathbf{0.57 \text{ (lag)}}$$

### Tutorial Problems No. 18.1

1. A circuit having a constant resistance of  $60 \ \Omega$  and a variable inductance of 0 to  $0.4 \text{ H}$  is connected across a  $100\text{-V}$ ,  $50\text{-Hz}$  supply. Derive from first principles the locus of the extremity of the current vector. Find (a) the power and (b) the inductance of the circuit when the power factor is  $0.8$ .

**[(a) 107 W (v) 0.143 H] (App. Elect. London Univ.)**

2. A constant reactance of  $10 \ \Omega$  is connected in series with a variable resistor and the applied voltage is  $100 \text{ V}$ . What is (i) the maximum power dissipated and (ii) at what value of resistance does it occur?

**[(a) 500 W (ii) 10  $\Omega$ ] (City & Guilds London)**

3. A variable capacitance and a resistance of  $300 \ \Omega$  are connected in series across a  $240\text{-V}$ ,  $50\text{-Hz}$  supply. Draw the complex or locus of impedance and current as the capacitance changes from  $5 \mu\text{F}$  to  $30 \mu\text{F}$ . From the diagram, find (a) the capacitance to give a current of  $0.7 \text{ A}$  and (b) the current when the capacitance is  $10 \mu\text{F}$ .

**[19.2  $\mu\text{F}$ , 0.55A] (London Univ.)**

4. An a.c. circuit consists of a variable resistor in series with a coil, for which  $R = 20 \ \Omega$  and  $L = 0.1 \text{ H}$ . Show that when this circuit is supplied at constant voltage and frequency and the resistance is varied between zero and infinity, the locus diagram of the current vector is a circular arc. Calculate when the supply voltage is  $100 \text{ V}$  and the frequency  $50 \text{ Hz}$  (i) the radius (in amperes) of the arc (ii) the value of the variable resistor in order that the power taken from the mains may be a maximum.

**[(i) 1.592 A (ii) 11.4  $\Omega$ ] (London Univ.)**

5. A circuit consists of an inductive coil ( $L = 0.2 \text{ H}$ ,  $R = 20 \ \Omega$ ) in series with a variable resistor ( $0 - 200 \ \Omega$ ). Draw to scale the locus of the current vector when the circuit is connected to  $230\text{-V}$ ,  $50\text{-Hz}$  supply mains and the resistor is varied between  $0$  and  $200 \ \Omega$ . Determine (i) the value of the resistor which will give maximum power in the circuit, (ii) the power when the resistor is  $150 \ \Omega$ .

**[(i) 42.8  $\Omega$  (ii) 275 W] (London Univ.)**

6. A  $15 \ \mu\text{F}$  capacitor, an inductive coil ( $L = 0.135 \text{ H}$ ,  $R = 50 \ \Omega$ ) and a variable resistor are in series and connected to a  $230\text{-V}$ ,  $50\text{-Hz}$  supply.

Draw to scale the vector locus of the current when the variable resistor is varied between  $0$  and  $500 \ \Omega$ .

Calculate (i) the value of the variable resistor when the power is a maximum (ii) the power under these conditions.

**[(i) 120  $\Omega$  (ii) 155.5 W] (London Univ.)**

7. As a.c. circuit supplied at  $100 \text{ V}$ ,  $50\text{-Hz}$  consists of a variable resistor in series with a fixed  $100 \ \mu\text{F}$  capacitor.

Show that the extremity of the current vector moves on a circle. Determine the maximum power dissipated in the circuit the corresponding power factor and the value of the resistor.

**[157 W ; 0.707 ; 131.8  $\Omega$ ]**

8. A variable non-inductive resistor  $R$  of maximum value  $10 \ \Omega$  is placed in series with a coil which has a resistance of  $3 \ \Omega$  and reactance of  $4 \ \Omega$ . The arrangement is supplied from a  $240\text{-V}$  a.c. supply. Show that the locus of the extremity of the current vector is a semi-circle. From the locus diagram, calculate the current supplied when  $R = 5 \ \Omega$ .

**[26.7 A]**

9. A  $20\text{-}\Omega$  reactor is connected in parallel with a series circuit consisting of a reactor of reactance  $10 \ \Omega$  and a variable resistance  $R$ . Prove that the extremity of the total current vector moves on a circle. If the supply voltage is constant at  $100 \text{ V}$  (r.m.s.), what is the maximum power factor? Determine also the value of  $R$  when the p.f. has its maximum value.

**[0.5 : 17.3  $\Omega$ ]**

# CHAPTER 19

## Learning Objectives

- Generation of Polyphase Voltages
- Phase Sequence
- Interconnection of Three Phases
- Star or Wye (Y) Connection
- Voltages and Currents in Y-Connection
- Delta (D) or Mesh Connection
- Balanced Y/D and D/Y Conversions
- Star and Delta Connected Lighting Loads
- Power Factor Improvement
- Parallel Loads
- Power Measurement in 3-phase Circuits
- Three Wattmeter Method
- Two Wattmeter Method
- *Balanced or Unbalanced load*
- Variations in Wattmeter Readings
- Leading Power Factor
- Power Factor-*Balanced Load*
- Reactive Voltamperes with One Wattmeter
- One Wattmeter Method
- Double Subscript Notation
- Unbalanced Loads
- Four-wire Star-connected Unbalanced Load
- Unbalanced Y-connected Load Without Neutral
- Millman's Theorem
- Application of Kirchhoff's Laws
- Delta/Star and Star/Delta Conversions
- Unbalanced Star-connected Non-inductive Load
- Phase Sequence Indicators

## POLYPHASE CIRCUITS



↑ This is a mercury arc rectifier 6-phase device, 150 A rating with grid control

### 19.1. Generation of Polyphase Voltage

The kind of alternating currents and voltages discussed in chapter 12 to 15 are known as single-phase voltage and current, because they consist of a single alternating current and voltage wave. A single-phase alternator was diagrammatically depicted in Fig. 11.1 (b) and it was shown to have one armature winding only. But if the number of armature windings is increased, then it becomes polyphase alternator and it produces as many independent voltage waves as the number of windings or phases. These windings are displaced from one another by equal angles, the values of these angles being determined by the number of phases or windings. In fact, the word 'poly-phase' means poly (*i.e.* many or numerous) and phases (*i.e.* winding or circuit).

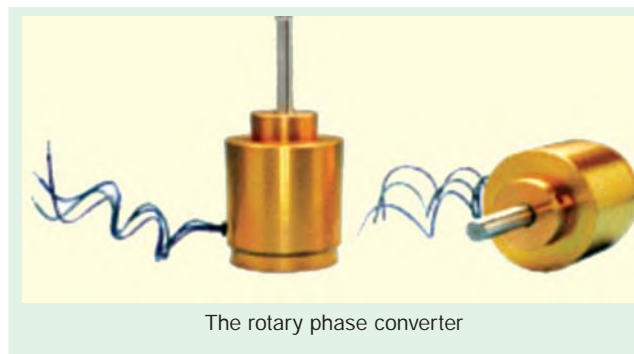
In a two-phase alternator, the armature windings are displaced 90 electrical degrees apart. A 3-phase alternator, as the name shows, has three independent armature windings which are 120 electrical degrees apart. Hence, the voltages induced in the three windings are 120° apart in time-phase. With the exception of two-phase windings, it can be stated that, in general, the electrical displacement between different phases is  $360/n$  where  $n$  is the number of phases or windings.

Three-phase systems are the most common, although, for certain special jobs, greater number of phases is also used. For example, almost all mercury-arc rectifiers for power purposes are either six-phase or twelve-phase and most of the rotary converters in use are six-phase. All modern generators are practically three-phase. For transmitting large amounts of power, three-phase is invariably used. The reasons for the immense popularity of three-phase apparatus are that (i) it is more efficient (ii) it uses less material for a given capacity and (iii) it costs less than single-phase apparatus etc.

In Fig. 19.1 is shown a two-pole, stationary-armature, rotating-field type three-phase alternator. It has three armature coils  $aa'$ ,  $bb'$  and  $cc'$  displaced 120° apart from one another. With the position and clockwise rotation of the poles as indicated in Fig. 19.1, it is found that the e.m.f. induced in conductor 'a' for coil  $aa'$  is maximum and its direction\* is away from the reader. The e.m.f. in conductor 'b' of coil  $bb'$  would be maximum and away from the reader when the N-pole has turned through 120° *i.e.* when  $N-S$  axis lies along  $bb'$ . It is clear that the induced e.m.f. in conductor 'b' reaches its maximum value 120° later than the maximum value in conductor 'a'. In the like manner, the maximum e.m.f. induced (in the direction away from the reader) in conductor 'c' would occur 120° later than that in 'b' or 240° later than that in 'a'.

Thus the three coils have three e.m.fs. induced in them which are similar in all respects except that they are 120° out of time phase with one another as pictured in Fig. 19.3. Each voltage wave is assumed to be sinusoidal and having maximum value of  $E_m$ .

In practice, the space on the armature is completely covered and there are many slots per phase per pole.



\* The direction is found with the help of Fleming's Right-hand rule. But while applying this rule, it should be remembered that the relative motion of the conductor with respect to the field is anticlockwise although the motion of the field with respect to the conductor is clockwise as shown. Hence, thumb should point to the left.

Fig. 19.2 illustrates the relative positions of the windings of a 3-phase, 4-pole alternator and Fig. 19.4 shows the developed diagram of its armature windings. Assuming full-pitched winding and the direction of rotation as shown, phase 'a' occupies the position under the centres of N and S-poles. It starts at  $S_a$  and ends or finishes at  $F_a$ .

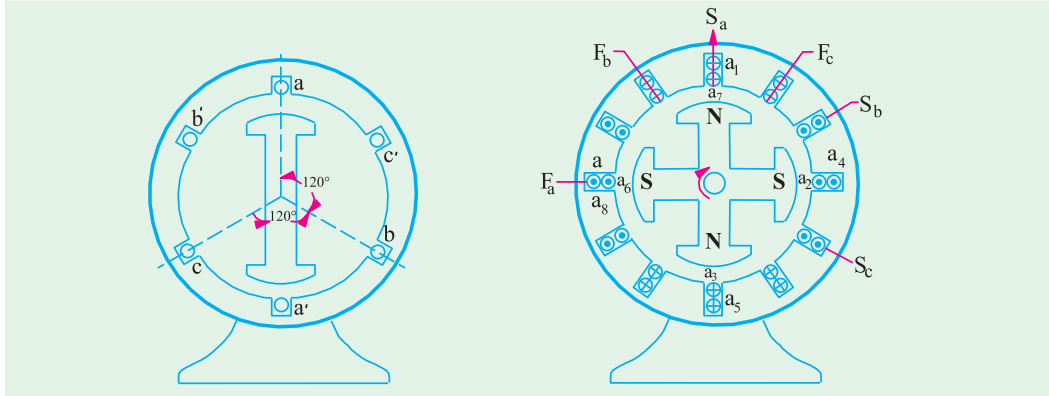


Fig. 19.1

Fig. 19.2

The second phase 'b' start at  $S_b$  which is 120 electrical degrees apart from the start of phase 'a', progresses round the armature clockwise (as does 'a') and finishes at  $F_b$ . Similarly, phase 'c' starts at  $S_c$  which is 120 electrical degrees away from  $S_b$ , progresses round the armature and finishes at  $F_c$ . As the three circuits are exactly similar but are 120 electrical degrees apart, the e.m.f. waves generated in them (when the field rotates) are displaced from each other by 120°. Assuming these waves to be sinusoidal and counting the time from the instant when the e.m.f. in phase 'a' is zero, the instantaneous values of the three e.m.fs. will be given by curves of Fig. 19.3.

Their equations are :

$$\begin{aligned}
 e_a &= E_m \sin \omega t && \dots (i) \\
 e_b &= E_m \sin(\omega t - 120^\circ) && \dots (ii) \\
 e_c &= E_m \sin(\omega t - 240^\circ) && \dots (iii)
 \end{aligned}$$

As shown in Art. 11.23, alternating voltages may be represented by revolving vectors which indicate their maximum values (or r.m.s. values if desired). The actual values of these voltages vary from peak positive to zero and to peak negative values in one revolution of the vectors. In Fig. 19.5 are shown the three vectors representing the r.m.s. voltages of the three phases  $E_a$ ,  $E_b$  and  $E_c$  (in the present case  $E_a = E_b = E_c = E$ , say).

It can be shown that the sum of the three phase e.m.fs. is zero in the following three ways :

(i) The sum of the above three equations (i), (ii) and (iii) is zero as shown below :

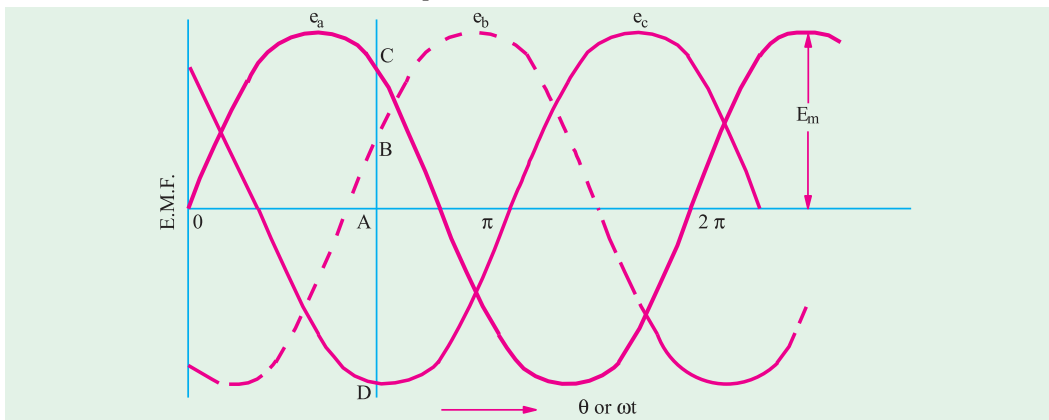


Fig. 19.3

$$\begin{aligned}
 \text{Resultant instantaneous e.m.f.} &= e_a + e_b + e_c \\
 &= E_m \sin t + E_m \sin(t - 120^\circ) + E_m \sin(t - 240^\circ) \\
 &= E_m [\sin t + 2 \sin(t - 180^\circ) \cos 60^\circ] \\
 &= E_m [\sin t + 2 \sin t \cos 60^\circ] = 0
 \end{aligned}$$

(ii) The sum of ordinates of three e.m.f. curves of Fig. 19.3 is zero. For example, taking ordinates  $AB$  and  $AC$  as positive and  $AD$  as negative, it can be shown by actual measurement that

$$AB + AC + (-AD) = 0$$

(iii) If we add the three vectors of Fig. 19.5 either vectorially or by calculation, the result is zero.

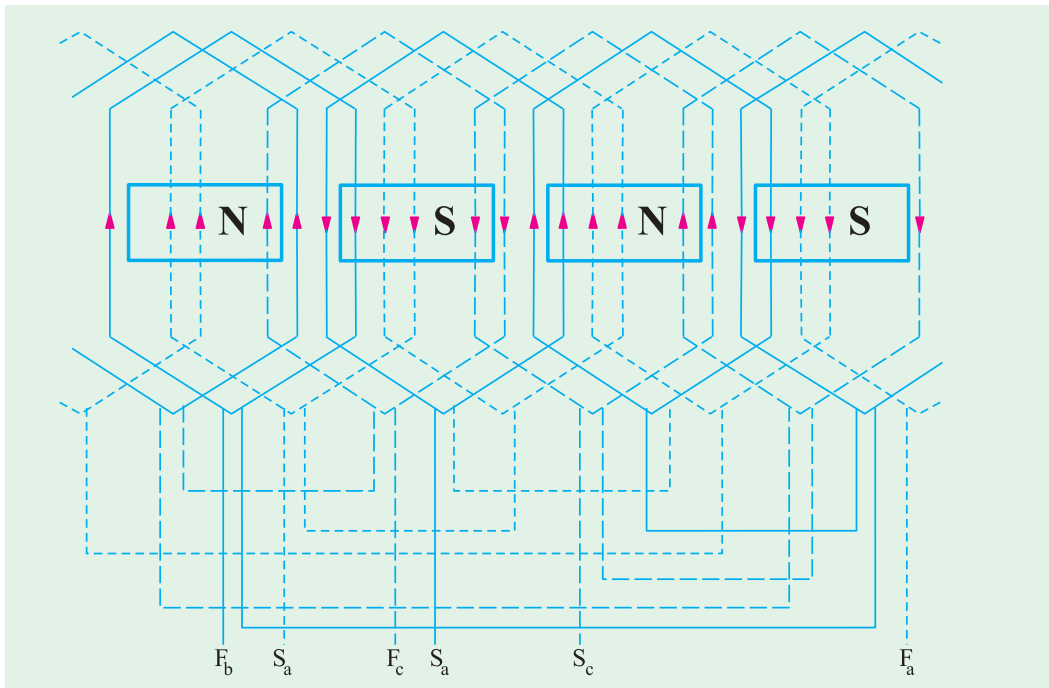


Fig. 19.4

### Vector Addition

As shown in Fig. 19.6, the resultant of  $E_a$  and  $E_b$  is  $E_r$  and its magnitude is  $2E \cos 60^\circ = E$  where  $E_a = E_b = E_c = E$ .

This resultant  $E_r$  is equal and opposite to  $E_c$ . Hence, their resultant is zero.

### By Calculation

Let us take  $E_a$  as reference voltage and assuming clockwise phase sequence

$$\begin{aligned}
 E_a &= E \angle 0^\circ = E \cdot j0 \\
 E_b &= E \angle -120^\circ = E (0.5 - j0.866) \\
 E_c &= E \angle 120^\circ = E (0.5 + j0.866) \\
 \therefore E_a + E_b + E_c &= (E \angle 0^\circ) + (E \angle -120^\circ) + (E \angle 120^\circ) = 0
 \end{aligned}$$



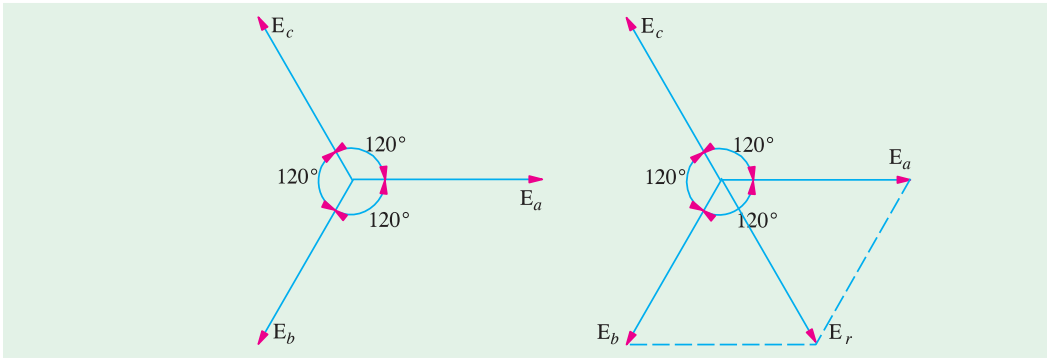


Fig. 19.5

Fig. 19.6

## 19.2. Phase Sequence

By phase sequence is meant the order in which the three phases attain their peak or maximum values. In the development of the three-phase e.m.fs. in Fig. 19.7, clockwise rotation of the field system in Fig. 19.1 was assumed. This assumption made the e.m.fs. of phase 'b' lag behind that

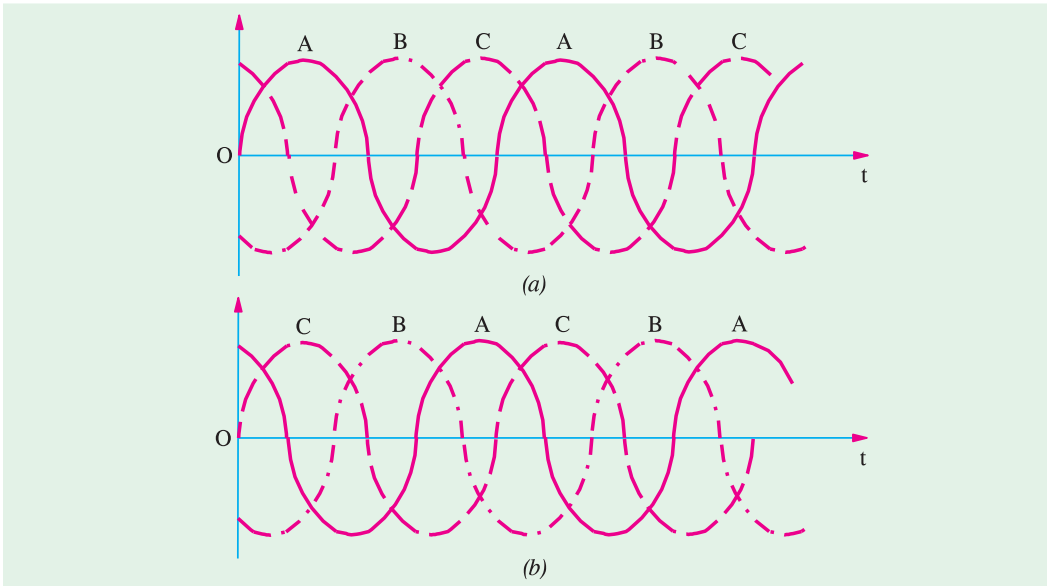


Fig. 19.7

of 'a' by  $120^\circ$  and in a similar way, made that of 'c' lag behind that of 'b' by  $120^\circ$  (or that of 'a' by  $240^\circ$ ). Hence, the order in which the e.m.fs. of phases *a*, *b* and *c* attain their maximum values is *a b c*. **It is called the phase order or phase sequence**  $a \rightarrow b \rightarrow c$  as illustrated in Fig. 19.7 (a).

If, now, the rotation of the field structure of Fig. 19.1 is reversed *i.e.* made anticlockwise, then the order in which the three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become  $a \rightarrow c \rightarrow b$ . This means that e.m.f. of phase 'c' would now lag behind that of phase 'a' by  $120^\circ$  instead of  $240^\circ$  as in the previous case as shown in Fig. 19.7 (b). By repeating the letters, this phase sequence can be written as **acb** which is the same thing as **cba**. Obviously, a three-phase system has only two possible sequences : **abc** and **cba** (*i.e.* **abc** read in the reverse direction).

### 19.3. Phase Sequence At Load

In general, the phase sequence of the voltages applied to load is determined by the order in which the 3-phase lines are connected. The phase sequence can be reversed by interchanging any pair of lines. In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation. In the case of 3-phase unbalanced loads, the effect of sequence reversal is, in general, to cause a completely different set of values of the currents. Hence, when working on such systems, it is essential that phase sequence be clearly specified otherwise unnecessary confusion will arise. Incidentally, reversing the phase sequence of a 3-phase generator which is to be paralleled with a similar generator can cause extensive damage to both the machines.

Fig. 19.8 illustrates the fact that by interchanging any two of the three

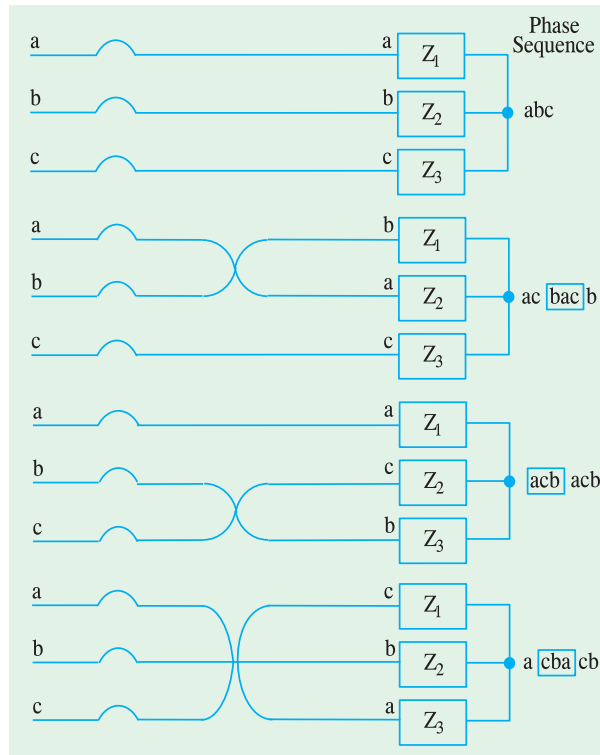


Fig. 19.8



Induction motor for drilling applications

cables the phase sequence **at the load** can be reversed though sequence of 3-phase supply remains the same *i.e.* **abc**. It is customary to define phase sequence at the load by reading repetitively from top to bottom. For example, load phase sequence in Fig. 19.8 (a) would be read as **abcabcabc**– or simply **abc**. The changes are as tabulated below :

Cables Interchanged	Phase Sequence
a and b	b a <span style="border: 1px solid black; padding: 2px;">c b a c b a</span> c – or c b a
b and c	a <span style="border: 1px solid black; padding: 2px;">c b a c b a</span> c b – or c b a
c and a	<span style="border: 1px solid black; padding: 2px;">c b a c b a</span> c b a – or c b a

### 19.4. Numbering of Phases

The three phases may be numbered 1, 2, 3 or a, b, c or as is customary, they may be given three colours. The colours used commercially are red, yellow (or sometimes white) and blue. In this case, the sequence is **RYB**.

Obviously, in any three-phase system, there are two possible sequences in which the three coil or phase voltages may pass through their maximum values *i.e.* red → yellow → blue (*RYB*) or red → blue → yellow (*RBV*). By convention, sequence *RYB* is taken as positive and *RBV* as negative.

### 19.5. Interconnection of Three Phases

If the three armature coils of the 3-phase alternator (Fig. 19.8) are not interconnected but are kept separate, as shown in Fig. 19.9, then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence, the three phases are generally interconnected which results in substantial saving of copper. The general methods of interconnection are

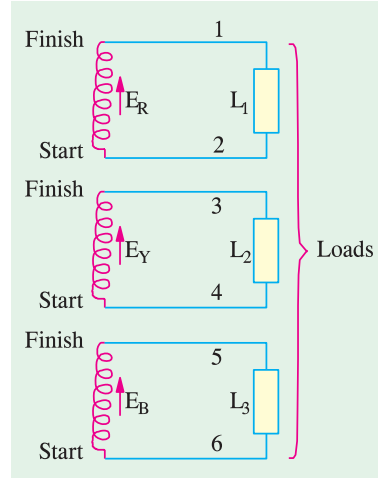


Fig. 19.9



3-phase alternator

- (a) Star or Wye (Y) connection and
- (b) Mesh or Delta (Δ) connection.

### 19.6. Star or Wye (Y) Connection

In this method of interconnection, the *similar\** ends say, 'star' ends of three coils (it could be 'finishing' ends also) are joined together at point *N* as shown in Fig. 19.10 (a).

The point *N* is known as *star point or neutral point*. The three conductors meeting at point *N* are replaced by a single conductor known as *neutral conductor* as shown in Fig. 19.10 (b). Such an interconnected system is known as four-wire, 3-phase system and is diagrammatically shown in Fig. 19.10 (b). If this three-phase voltage system is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120° out of phase with each other. Hence, their vector sum is zero.

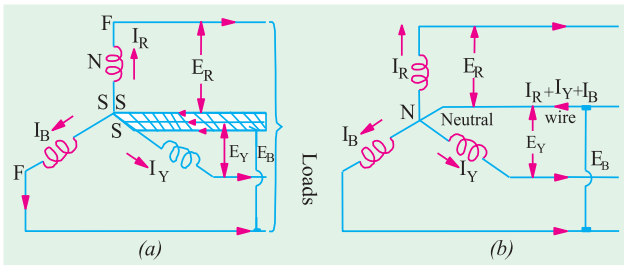


Fig. 19.10

*i.e.*  $I_R + I_Y + I_B = 0$  ... vectorially

The neutral wire, in that case, may be omitted although its retention is useful for supplying lighting loads at low voltages (Ex. 19.22). The p.d. between any terminal (or line) and neutral (or star) point gives the *phase* or *star* voltage. But the p.d. between any two lines gives the line-to-line voltage or simply line voltage.

### 19.7. Values of Phase Currents

When considering the distribution of current in a 3-phase system, it is extremely important to bear in mind that :

\* As an aid to memory, remember that first letter *S* of Similar is the same as that of Star.

(i) the arrow placed alongside the currents  $I_R$ ,  $I_Y$  and  $I_B$  flowing in the three phases [Fig. 19.10 (b)] indicate the directions of currents when they are assumed to be **positive** and not the directions at a particular instant. It should be clearly understood **that at no instant will all the three currents flow in the same direction either outwards or inwards**. The three arrows indicate that first the current flows outwards in phase  $R$ , then after a phase-time of  $120^\circ$ , it will flow outwards from phase  $Y$  and after a further  $120^\circ$ , outwards from phase  $B$ .

(ii) the current flowing outwards in one or two conductors is always equal to that flowing inwards in the remaining conductor or conductors. In other words, **each conductor in turn, provides a return path for the currents of the other conductors**.

In Fig. 19.11 are shown the three phase currents, having the same peak value of 20 A but displaced from each other by  $120^\circ$ . At instant 'a', the currents in phases  $R$  and  $B$  are each +10 A (i.e. flowing outwards) whereas the current in phase  $Y$  is -20 A (i.e. flowing inwards). In other words, at the instant 'a', phase  $Y$  is acting as return path for the currents in phases  $R$  and  $B$ . At instant  $b$ ,  $I_R = +15$  A and  $I_Y = +5$  A but  $I_B = -20$  A which means that now phase  $B$  is providing the return path.

At instant  $c$ ,  $I_Y = +15$  A and  $I_B = +5$  A and  $I_R = -20$  A.

Hence, now phase  $R$  carries current inwards whereas  $Y$  and  $B$  carry current outwards. Similarly at point  $d$ ,  $I_R = 0$ ,  $I_B = 17.3$  A and  $I_Y = -17.3$  A. In other words, current is flowing outwards from phase  $B$  and returning *via* phase  $Y$ .

In addition, it may be noted that although the distribution of currents between the three lines is continuously changing, yet at any instant the algebraic sum of the **instantaneous** values of the three currents is zero i.e.

$$i_R + i_Y + i_B = 0 \quad \text{— algebraically.}$$

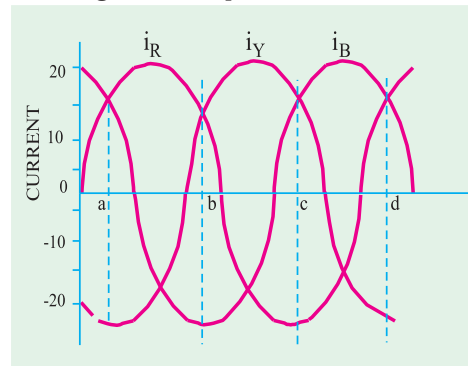


Fig. 19.11

## 19.8. Voltages and Currents in Y-Connection

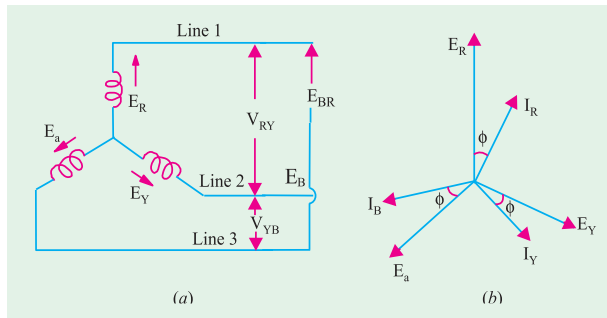


Fig. 19.12

are in opposition. Obviously, the **instantaneous** value of p.d. between any two terminals is the **arithmetic difference** of the two phase e.m.fs. concerned. However, the r.m.s. value of this p.d. is given by the **vector difference** of the two phase e.m.fs.

The vector diagram for phase voltages and currents in a star connection is shown in Fig. 19.12.

The voltage induced in each winding is called the **phase** voltage and current in each winding is likewise known as **phase** current. However, the voltage available between any pair of terminals (or outers) is called **line** voltage ( $V_L$ ) and the current flowing in each **line** is called **line** current ( $I_L$ ).

As seen from Fig. 19.12 (a), in this form of interconnection, there are two phase windings between each pair of terminals but since their **similar** ends have been joined together, they

(b) where a balanced system has been assumed.\* It means that  $E_R = E_Y = E_{ph}$  (phase e.m.f.).

Line voltage  $V_{RY}$  between line 1 and line 2 is the vector difference of  $E_R$  and  $E_Y$ .

Line voltage  $V_{YB}$  between line 2 and line 3 is the vector difference of  $E_Y$  and  $E_B$ .

Line voltage  $V_{BR}$  between line 3 and line 1 is the vector difference of  $E_B$  and  $E_R$ .

### (a) Line Voltages and Phase Voltages

The p.d. between line 1 and 2 is  $V_{RY} = E_R - E_Y$

Hence,  $V_{RY}$  is found by compounding  $E_R$  and  $E_Y$  reversed and its value is given by the diagonal of the parallelogram of Fig. 19.13. Obviously, the angle between  $E_R$  and  $E_Y$  reversed is  $60^\circ$ . Hence if  $E_R = E_Y = E_B =$  say,  $E_{ph}$  - the phase e.m.f., then

$$\begin{aligned} V_{RY} &= 2 \times E_{ph} \times \cos(60^\circ/2) \\ &= 2 \times E_{ph} \times \cos 30^\circ = 2 \times E_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3} E_{ph} \end{aligned}$$

Similarly,  $V_{YB} = E_Y - E_B = \sqrt{3} \cdot E_{ph}$  ...vector difference

and  $V_{BR} = E_B - E_R = \sqrt{3} \cdot E_{ph}$

Now  $V_{RY} = V_{YB} = V_{BR} =$  line voltage, say  $V_L$ . Hence, in

star connection  $V_L = \sqrt{3} \cdot E_{ph}$

It will be noted from Fig. 19.13 that

1. Line voltages are  $120^\circ$  apart.
2. Line voltages are  $30^\circ$  ahead of their respective *phase* voltages.

3. The angle between the line currents and the corresponding line voltages is  $(30 + \phi)$  with current lagging.

### (b) Line Currents and Phase Currents

It is seen from Fig. 19.12 (a) that each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 =  $I_R$ ; Current in line 2 =  $I_Y$ ; Current in line 3 =  $I_B$

Since  $I_R = I_Y = I_B =$  say,  $I_{ph}$  - the phase current

$\therefore$  line current  $I_L = I_{ph}$

### (c) Power

The total active or true power in the circuit is the sum of the three phase powers. Hence,

total active power =  $3 \times$  phase power or  $P = 3 \times V_{ph} I_{ph} \cos \phi$

Now  $V_{ph} = V_L / \sqrt{3}$  and  $I_{ph} = I_L$

Hence, in terms of line values, the above expression becomes

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi \text{ or } P = \sqrt{3} V_L I_L \cos \phi$$

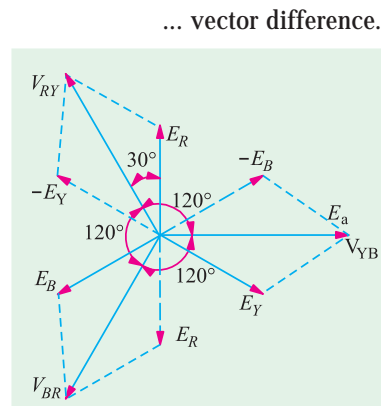


Fig. 19.13

\* A balanced system is one in which (i) the voltages in all phases are equal in magnitude and offer in phase from one another by equal angles, in this case, the angle =  $360/3 = 120^\circ$ , (ii) the currents in the three phases are equal in magnitude and also differ in phase from one another by equal angles.

A 3-phase balanced load is that in which the loads connected across three phases are identical.

It should be particularly noted that  $\phi$  is the angle between **phase** voltage and **phase** current and not between the line voltage and line current.

Similarly, total reactive power is given by  $Q = \sqrt{3} V_L I_L \sin \phi$

By convention, reactive power of a coil is taken as positive and that of a capacitor as negative.

The total apparent power of the three phases is

$$S = \sqrt{3} V_L I_L \quad \text{Obviously, } S = \sqrt{P^2 + Q^2} \quad \text{— Art. 13.4}$$

**Example 19.1.** A balanced star-connected load of  $(8 + j6) \Omega$  per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.

(Elect. Engg., Bhagalpur Univ.)

**Solution.**  $Z_{ph} = \sqrt{8^2 + 6^2} = 10 \Omega$

$$V_{ph} = 400 / \sqrt{3} = 231 \text{ V}$$

$$I_{ph} = V_{ph} / Z_{ph} = 231 / 10 = 23.1 \text{ A}$$

(i)  $I_L = I_{ph} = 23.1 \text{ A}$

(ii) p.f. =  $\cos \phi = R_{ph} / Z_{ph} = 8 / 10 = 0.8 \text{ (lag)}$

(iii) Power  $P = \sqrt{3} V_L I_L \cos \phi$

$$= \sqrt{3} \times 400 \times 23.1 \times 0.8 = 12,800 \text{ W} \quad [\text{Also, } P = 3 I_{ph}^2 R_{ph} = 3(23.1)^2 \times 8 = 12,800 \text{ W}]$$

(iv) Total volt-amperes,  $S = \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 23.1 = 16,000 \text{ VA}$

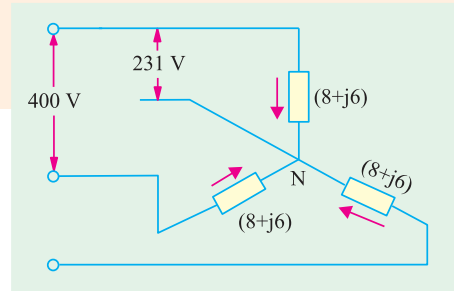


Fig. 19.14

**Example 19.2.** Phase voltages of a star connected alternator are  $E_R = 231 \angle 0^\circ \text{ V}$ ;  $E_Y = 231 \angle -120^\circ \text{ V}$ ; and  $E_B = 231 \angle +120^\circ \text{ V}$ . What is the phase sequence of the system? Compute the line voltages  $E_{RY}$  and  $E_{YB}$ .

(Elect. Machines AMIE Sec. B Winter 1990)

**Solution.** The phase voltage  $E_B = 231 \angle -120^\circ$  can be written as  $E_B = 231 \angle -240^\circ$ . Hence, the three voltages are:  $E_R = 231 \angle -0^\circ$ ,  $E_Y = 231 \angle -120^\circ$  and  $E_B = 231 \angle -240^\circ$ . It is seen that  $E_R$  is the reference voltage,  $E_Y$  lags behind it by  $120^\circ$  whereas  $E_B$  lags behind it by  $240^\circ$ . Hence, phase sequence is RYB. Moreover, it is a symmetrical 3-phase voltage system.

$$\therefore E_{RY} = E_{YB} = \sqrt{3} \times 231 = 400 \text{ V}$$

**Example 19.3** Three equal star-connected inductors take 8 kW at a power factor 0.8 when connected across a 460 V, 3-phase, 3-phase, 3-wire supply. Find the circuit constants of the load per phase.

(Elect. Machines AMIE Sec. B 1992)

**Solution.**  $P = \sqrt{3} V_L I_L \cos \phi$  or

$$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$$

$$\therefore I_L = 12.55 \text{ A} \quad \therefore I_{ph} = 12.55 \text{ A};$$

$$V_{ph} = V_L / \sqrt{3} = 460 / \sqrt{3} = 265 \text{ V}$$

$$I_{ph} = V_{ph} / Z_{ph}; \therefore Z_{ph} = V_{ph} / I_{ph} = 265 / 12.55 = 21.1 \Omega$$

$$R_{ph} = Z_{ph} \cos \phi = 21.1 \times 0.8 = 16.9 \Omega$$

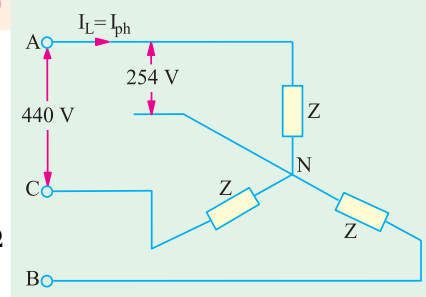


Fig. 19.15

$$X_{ph} = Z_{ph} \sin \phi = 21.1 \times 0.6 = 12.66 \Omega ;$$

The circuit is shown in Fig. 19.15.

**Example 19.4.** Given a balanced 3- $\phi$ , 3-wire system with Y-connected load for which line voltage is 230 V and impedance of each phase is  $(6 + j8)$  ohm. Find the line current and power absorbed by each phase. (Elect. Engg - II Pune Univ. 1991)

$$\text{Solution. } Z_{ph} = \sqrt{6^2 + 8^2} = 10 \Omega; V_{ph} = V_L / \sqrt{3} = 230 / \sqrt{3} = 133 \text{ V}$$

$$\cos \phi = R / Z = 6 / 10 = 0.6; I_{ph} = V_{ph} / Z_{ph} = 133 / 10 = 13.3 \text{ A}$$

$$\therefore I_L = I_{ph} = 13.3 \text{ A}$$

$$\text{Power absorbed by each phase} = I_{ph}^2 R_{ph} = 13.3^2 \times 6 = 1061 \text{ W}$$

#### Solution by Symbolic Notation

In Fig. 19.16 (b),  $V_R$ ,  $V_Y$  and  $V_B$  are the phase voltage whereas  $I_R$ ,  $I_Y$  and  $I_B$  are phase currents. Taking  $V_R$  as the reference vector, we get

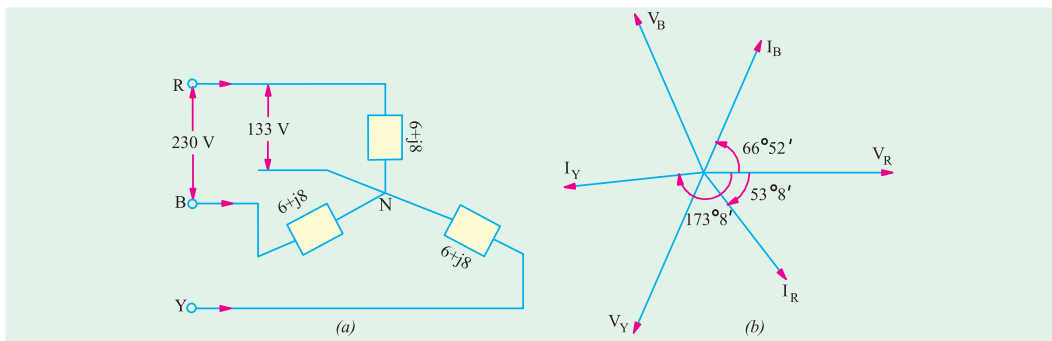


Fig 19.16

$$\mathbf{V}_R = 133 \angle 0^\circ = 133 + j0 \text{ volt}$$

$$\mathbf{V}_Y = 133 \angle 120^\circ = 133(0.5 + j0.866) = (66.5 + j115) \text{ volt}$$

$$\mathbf{V}_B = 133 \angle 240^\circ = 133(-0.5 + j0.866) = (-66.5 + j115) \text{ volt}$$

$$\mathbf{Z} = 6 + j8 = 10 \angle 53^\circ 8'; \mathbf{I}_R = \frac{\mathbf{V}_R}{\mathbf{Z}} = \frac{133 \angle 0^\circ}{10 \angle 53^\circ 8'} = 13.3 \angle -53^\circ 8'$$

This current lags behind the reference voltage by  $53^\circ 8'$  [Fig. 19.16 (b)]

$$\mathbf{I}_Y = \frac{\mathbf{V}_Y}{\mathbf{Z}} = \frac{133 \angle 120^\circ}{10 \angle 53^\circ 8'} = 13.3 \angle 66^\circ 52'$$

It lags behind the reference vector *i.e.*  $V_R$  by  $173^\circ 8'$  which amounts to lagging behind its phase voltage  $V_Y$  by  $53^\circ 8'$ .

$$\mathbf{I}_B = \frac{\mathbf{V}_B}{\mathbf{Z}} = \frac{133 \angle 240^\circ}{10 \angle 53^\circ 8'} = 13.3 \angle 173^\circ 8'$$

This current leads  $V_R$  by  $66^\circ 52'$  which is the same thing as *lagging* behind its phase voltage by  $53^\circ 8'$ . For calculation of power, consider R-phase

$$\mathbf{V}_R = (133 - j0); \mathbf{I}_R = 13.3(0.6 - j0.8) = (7.98 - j10.64)$$

Using method of conjugates, we get

$$\mathbf{P}_{VA} = (133 - j0)(7.98 - j10.64) = 1067 - j1415$$

$\therefore$  Real power absorbed/phase = 1067 W – as before

**Example 19.5.** When the three identical star-connected coils are supplied with 440 V, 50 Hz, 3- $\phi$  supply, the 1- $\phi$  wattmeter whose current coil is connected in line R and pressure coil across the phase R and neutral reads 6 kW and the ammeter connected in R-phase reads 30 Amp. Assuming RYB phase sequence find:

- (i) resistance and reactance of the coil, (ii) the power factor, of the load  
(iii) reactive power of 3- $\phi$  load.

(Elect. Engg.-I, Nagpur Univ. 1993)

**Solution.**  $V_{ph} = 440 / \sqrt{3} = 254 \text{ V}; I_{ph} = 30 \text{ A}$   
(Fig. 19.17.)

Now,  $V_{ph} I_{ph} \cos \phi = 6000$ ;  $254 \times 30 \times \cos \phi = 6000$

$\therefore \cos \phi = 0.787$ ;  $\phi = 38.06^\circ$  and  $\sin \phi = 0.616$ ;  $Z_{ph} = V_{ph} / I_{ph} = 254/30 = 8.47 \Omega$

(i) Coil resistance  $R = Z_{ph} \cos \phi = 8.47 \times 0.787 = 6.66 \Omega$

$X_L = Z_{ph} \sin \phi = 8.47 \times 0.616 = 5.22 \Omega$

(ii) p.f. =  $\cos \phi = 0.787$  (lag)

(iii) Reactive power =  $\sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 440 \times 30 \times 0.616 = 14,083 \text{ VA}$  **14.083 kVA**

**Example 19.6** Calculate the active and reactive components in each phase of Y-connected 10,000 V, 3-phase alternator supplying 5,000 kW at 0.8 p.f. If the total current remains the same when the load p.f. is raised to 0.9, find the new output.

(Elements of Elect. Engg.-I, Bangalore Univ.)

**Solution.**  $5000 \times 10^3 = \sqrt{3} \times 10,000 \times I_L \times 0.8$ ;  $I_L = 361 \text{ A}$

active component =  $I_L \cos \phi = 361 \times 0.8 = 288.8 \text{ A}$

reactive component =  $I_L \sin \phi = 361 \times 0.6 = 216.6 \text{ A}$

New power  $P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 10^4 \times 361 \times 0.9 = 5,625 \text{ kW}$

[or new power =  $5000 \times 0.9/0.8 = 5625 \text{ kW}$ ]

**Example 19.7.** Deduce the relationship between the phase and line voltages of a three-phase star-connected alternator. If the phase voltage of a 3-phase star-connected alternator be 200 V, what will be the line voltages (a) when the phases are correctly connected and (b) when the connections to one of the phases are reversed.

**Solution.** (a) When phases are correctly connected, the vector diagram is as shown in Fig. 19.12. (b). As proved in Art. 19.7

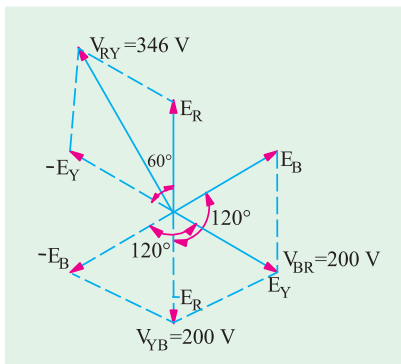


Fig. 19.18

$$V_{RY} = V_{YB} = V_{BR} = \sqrt{3} E_{ph}$$

$$\text{Each line voltage} = \sqrt{3} \times 200 = 346 \text{ V}$$

(b) Suppose connections to B-phase have been reversed. Then voltage vector diagram for such a case is shown in Fig. 19.18. It should be noted that  $E_B$  has been drawn in the reversed direction, so that angles between the three-phase voltages are  $60^\circ$  (instead of the usual  $120^\circ$ )

$$V_{RY} = E_R - E_Y \quad \dots \text{vector difference}$$

$$= 2 \times E_{ph} \times \cos 30^\circ = \sqrt{3} \times 200 = 346 \text{ V}$$

$$V_{YB} = E_Y - E_B \quad \dots \text{vector difference}$$



$$= 2 \times E_{ph} \times \cos 60^\circ = 2 \times 200 \times \frac{1}{2} = 200V$$

$$V_{BR} = E_B - E_R \dots \text{vector difference} = 2 \times E_{ph} \times \cos 60^\circ = 2 \times 200 \times \frac{1}{2} = 200V$$

**Example 19.8** In a 4-wire, 3-phase system, two phases have currents of 10A and 6A at lagging power factors of 0.8 and 0.6 respectively while the third phase is open-circuited. Calculate the current in the neutral and sketch the vector diagram.

**Solution.** The circuit is shown in Fig. 19.19 (a).

$$\phi_1 = \cos^{-1}(0.8) = 36^\circ 54'; \phi_2 = \cos^{-1}(0.6) = 53^\circ 6'$$

Let  $V_R$  be taken as the reference vector. Then

$$\mathbf{I}_R = 10 \angle -36^\circ 54' = (8 - j6) \quad \mathbf{I}_Y = 6 \angle -173^\circ 6' = (-6 - j0.72)$$

The neutral current  $\mathbf{I}_N$  as shown in Fig. 19.16 (b), is the sum of these two currents.

$$\therefore \mathbf{I}_N = (8 - j6) + (-6 - j0.72) = 2 - j6.72 = 7 \angle -73^\circ 26'$$

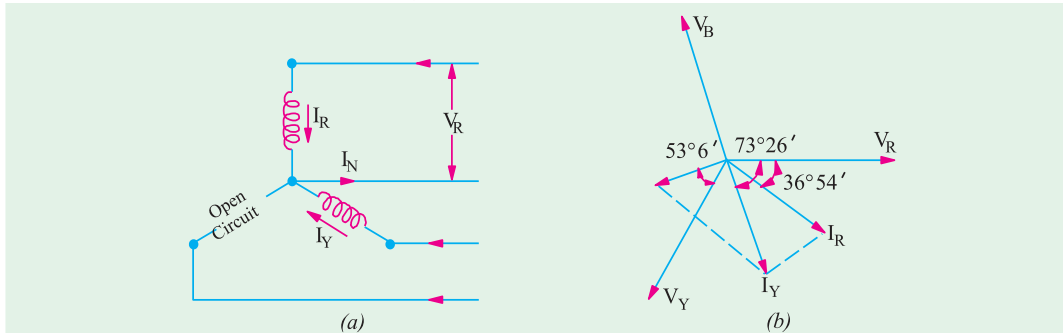


Fig. 19.19

**Example 19.9 (a).** Three equal star-connected inductors take 8 kW at power factor 0.8 when connected a 460-V, 3-phase, 3-wire supply. Find the line currents if one inductor is short-circuited.

**Solution.** Since the circuit is balanced, the three line voltages are represented by

$$V_{ab} = 460 \angle 0^\circ; V_{bc} = 460 \angle -120^\circ \text{ and } V_{ca} = 460 \angle 120^\circ$$

The phase impedance can be found from the given data :

$$8000 = \sqrt{3} \times 460 \times I_L \times 0.8 \therefore I_L = I_{ph} = 12.55 \text{ A}$$

$$Z_{ph} = V_{ph} / I_{ph} = 460 / \sqrt{3} \times 12.55 = 21.2 \Omega;$$

$$\therefore Z_{ph} = 21.2 \angle 36.9^\circ \text{ because } \phi = \cos^{-1}(0.8) = 36.9^\circ$$

As shown in the Fig. 19.20, the phase  $c$  has been short-circuited. The line current  $I_a = V_{ac}/Z_{ph} = -V_{ca}/Z_{ph}$  because the current enters at point  $a$  and leaves from point  $c$ .

$$\therefore I_a = -460 \angle 120^\circ / 21.2 \angle 36.9^\circ = 21.7 \angle 83.1^\circ$$

Similarly,  $I_b = V_{bc}/Z_{ph} = 460 \angle 120^\circ / 21.2 \angle 36.9^\circ = 21.7 \angle -156.9^\circ$ . The current  $I_c$  can be found by applying KVL to the neutral point N.

$$\therefore I_a + I_b + I_c = 0 \quad \text{or} \quad I_c = -I_a - I_b$$

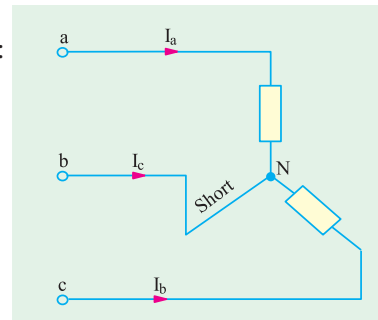


Fig. 19.20

$$\therefore I_C = 21.7 \angle 83.1^\circ - 21.7 \angle -156.9^\circ = 37.3 \angle 53.6^\circ$$

Hence, the magnitudes of the three currents are : 21.7 A;  
21.7 A; 37.3 A.

**Example 19.9 (b).** Each phase of a star-connected load consists of a non-reactive resistance of  $100 \Omega$  in parallel with a capacitance of  $31.8 \mu F$ .

Calculate the line current, the power absorbed, the total kVA and the power factor when connected to a 416-V, 3-phase, 50-Hz supply.

**Solution.** The circuit is shown in Fig. 14.20.

$$V_{ph} = (416 / \sqrt{3}) \angle 0^\circ = 240 \angle 0^\circ = (240 + j0)$$

Admittance of each phase is

$$\begin{aligned} Y_{ph} &= \frac{1}{R} + jC = \frac{1}{100} + j314 \times 31.8 \times 10^{-6} \\ &= 0.01 + j0.01 \end{aligned}$$

$$\begin{aligned} \therefore I_{ph} &= V_{ph} \cdot Y_{ph} = 240(0.01 + j0.01) \\ &= 2.4 + j2.4 = 3.39 \angle 45^\circ \end{aligned}$$

Since  $I_{ph} = I_L$  - for a star connection  $\therefore I_L = 3.39 \text{ A}$

Power factor =  $\cos 45^\circ = 0.707$  (leading)

Now  $V_{ph} = (240 + j0)$ ;  $I_{ph} = 2.4 + j2.4$

$$\therefore P_{VA} = (240 + j0)(2.4 + j2.4)$$

$$= 240 \times 2.4 - j2.4 \times 240 = 576 - j576 = 814.4 \angle -45^\circ \quad \dots \text{ per phase}$$

Hence, total power =  $3 \times 576 = 1728 \text{ W} = 1.728 \text{ kW}$

Total voltamperes =  $814.4 \times 3 = 2,443 \text{ VA}$ ; kilovolt amperes = **2.433 kVA**

**Example 19.10.** A three phase 400-V, 50 Hz, a.c. supply is feeding a three phase delta-connected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp. [Nagpur University, November 1999]

**Solution.** Impedance per phase  $r + jX_L - jX_C$

$$X_L = 2\pi \times 50 \times 0.15 = 47.1 \Omega$$

$$X_C = \frac{10^6}{32.37} = 26.54 \Omega$$

$\cos \phi = \frac{25}{32.37}$  Lagging, since inductive reactance is dominating.

$$\text{Phase Current} = \frac{400}{25 + j20.56} = 12.357$$

$$\text{Line Current} = \sqrt{3} \times 12.357 = 21.4 \text{ amp}$$

Since the power factor is 0.772 lagging,

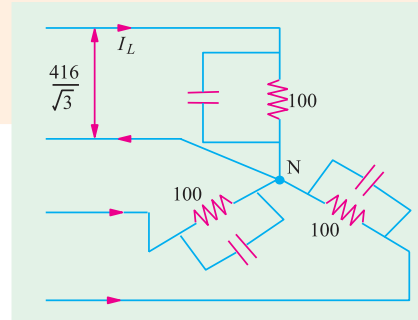


Fig. 19.21

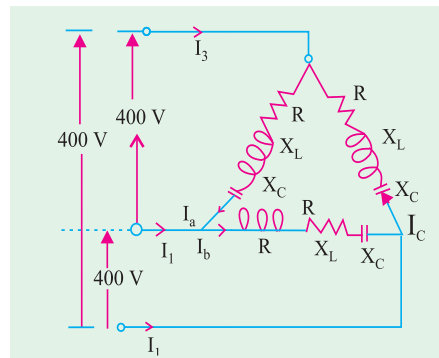


Fig. 19.22

$$P = \text{total three phase power} = \sqrt{3} V_L I_L \cos \phi \times 10^{-3} \text{ kW}$$

$$= \sqrt{3} \times 400 \times 21.4 \times 0.772 \times 10^{-3} = 11.446 \text{ kW}$$

$$S = \text{total 3 ph kVA} = \frac{11.446}{0.772} = 14.83 \text{ kVA}$$

$$Q = \text{total 3 ph "reactive kilo-volt-amp"} = \sqrt{3} (S^2 - P^2)^{0.50} = 9.43 \text{ kVAR lagging}$$

**Example 19.11.** Three phase star-connected load when supplied from a 400 V, 50 Hz source takes a line current of 10 A at 66.86° w.r. to its line voltage. Calculate : (i) Impedance-Parameters, (ii) P.f. and active-power consumed. Draw the phasor diagram.

[Nagpur University, April 1998]

**Solution.** Draw three phasors for phase-voltages.

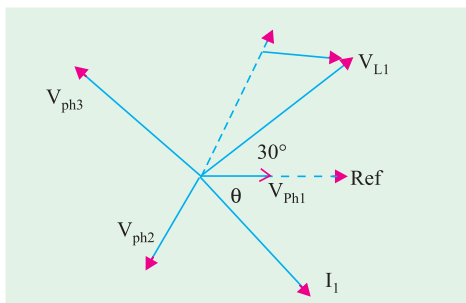


Fig. 19.23

These are  $V_{ph1}$ ,  $V_{ph2}$ ,  $V_{ph3}$  in Fig 19.23. As far as phase number 1 is concerned, its current is  $I_1$  and the associated line voltage is  $V_{L1}$ .  $V_{L1}$  and  $V_{ph1}$  differ in phase by  $30^\circ$ . A current differing in phase with respect to line voltage by  $66.86^\circ$  and associated with  $V_{ph1}$  can only be lagging, as shown in Fig. 19.23. This means  $\phi = 36.86^\circ$ , and the corresponding load power factor is 0.80 lagging.

$$Z = V_{ph}/I_{ph} = 231/10 = 23.1 \text{ ohms}$$

$$R = Z \cos \phi = 23.1 \times 0.8 = 18.48 \text{ ohms}$$

$$X_L = Z \sin \phi = 23.1 \times 0.6 = 13.86 \text{ ohms}$$

$$\text{Total active power consumed} = 3 V_{ph} I_{ph} \cos \phi$$

$$= 3 \times 231 \times 10 \times 0.8 \times 10^{-3} \text{ kW} = 5.544 \text{ kW}$$

$$\text{or total active power} = 3 \times I^2 R = 3 \times 10^2 \times 18.48 = 5544 \text{ watts}$$

For complete phasor diagram for three phases, the part of the diagram for Phase 1 in Fig 19.23 has to be suitably repeated for phase-numbers 2 and 3.

### 19.9. Delta ( $\Delta$ )\* or Mesh Connection

In this form, of interconnection the *dissimilar* ends of the three phase winding are joined together i.e. the 'starting' end of one phase is joined to the 'finishing' end of the other phase and

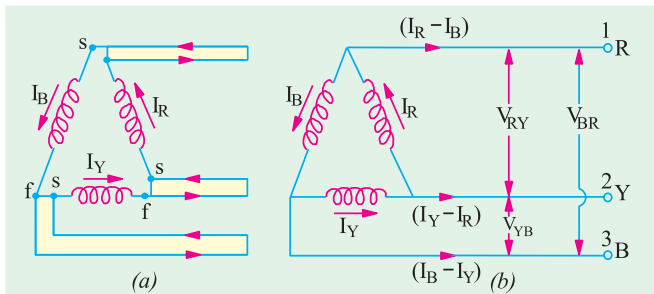


Fig. 19.24

so on as showing in Fig. 19.24 (a). In other words, the three windings are joined in series to form a closed mesh as shown in Fig. 19.24 (b).

Three leads are taken out from the three junctions as shown as outward directions are taken as positive.

It might look as if this sort of interconnection results in

\* As an aid to memory, remember that first letter *D* of Dissimilar is the same as that of Delta.

shortcircuiting the three windings. However, if the system is balanced then sum of the three voltages round the closed mesh is zero, hence no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.

This type of connection is also referred to as 3-phase, 3-wire system.

### (i) Line Voltages and Phase Voltages

It is seen from Fig. 19.24 (b) that there is only one phase winding completely included between any pair of terminals. Hence, in  $\Delta$ -connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is  $R Y B$ , the voltage having its positive direction from  $R$  to  $Y$  leads by  $120^\circ$  on that having its positive direction from  $Y$  to  $B$ . Calling the voltage between lines 1 and 2 as  $V_{RY}$  and that between lines 2 and 3 as  $V_{YB}$ , we find that  $V_{RY}$  lead  $V_{YB}$  by  $120^\circ$ . Similarly,  $V_{YB}$  leads  $V_{BR}$  by  $120^\circ$  as shown in Fig. 19.23. Let  $V_{RY} = V_{YB} = V_{BR} =$  line voltage  $V_L$ . Then, it is seen that  $V_L = V_{ph}$ .

### (ii) Line Currents and Phase Currents

It will be seen from Fig. 19.24 (b) that current in each line is the **vector difference** of the two phase currents flowing through that line. For example

$$\begin{aligned} \text{Current in line 1 is } I_1 &= I_R - I_B \\ \text{Current in line 2 is } I_2 &= I_Y - I_R \quad \text{vector difference} \\ \text{Current in line 3 is } I_3 &= I_B - I_Y \end{aligned}$$

Current in line No. 1 is found by compounding  $I_R$  and  $I_B$  reversed and its value is given by the diagonal of the parallelogram of Fig. 19.25. The angle between  $I_R$  and  $I_B$  reversed (*i.e.*  $-I_B$ ) is  $60^\circ$ . If  $I_R = I_Y =$  phase current  $I_{ph}$  (say), then

Current in line No. 1 is

$$I_1 = 2 \times I_{ph} \times \cos(60^\circ/2) = 2 \times I_{ph} \sqrt{3}/2 = \sqrt{3} I_{ph}$$

Current in line No. 2 is

$$I_2 = I_B - I_Y \dots \text{vector difference} = \sqrt{3} I_{ph} \text{ and current}$$

in line No. 3 is  $I_3 = I_B - I_Y \therefore$  Vector difference  $= \sqrt{3} \cdot I_{ph}$

Since all the line currents are equal in magnitude *i.e.*

$$I_1 = I_2 = I_3 = I_L$$

$$\therefore I_L = \sqrt{3} I_{ph}$$

With reference to Fig. 19.25, it should be noted that

1. line currents are  $120^\circ$  apart ;
2. line currents are  $30^\circ$  behind the respective phase currents ;
3. the angle between the line currents and the corresponding line voltages is  $(30 + \phi)$  with the current lagging.

### (iii) Power

Power/phase  $= V_{ph} I_{ph} \cos \phi$  ; Total power  $= 3 \times V_{ph} I_{ph} \cos \phi$ . However,  $V_{ph} = V_L$  and  $I_{ph} = I_L / \sqrt{3}$   
Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

where  $\phi$  is the phase power factor angle.

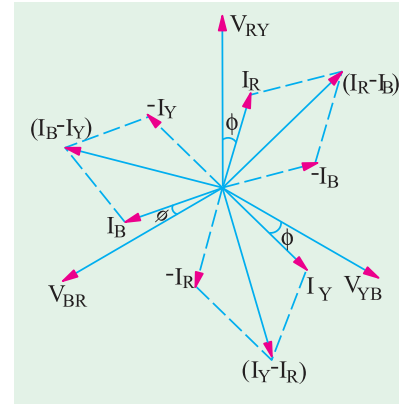


Fig. 19.25

**19.10. Balanced Y/Δ and Δ/Y Conversion**

In view of the above relationship between line and phase currents and voltages, any balanced Y-connected system may be completely replaced by an equivalent Δ-connected system. For example, a 3-phase, Y-connected system having the voltage of  $V_L$  and line current  $I_L$  may be replaced by a Δ-connected system in which phase voltage is  $V_L$  and phase current is  $I_L / \sqrt{3}$ .

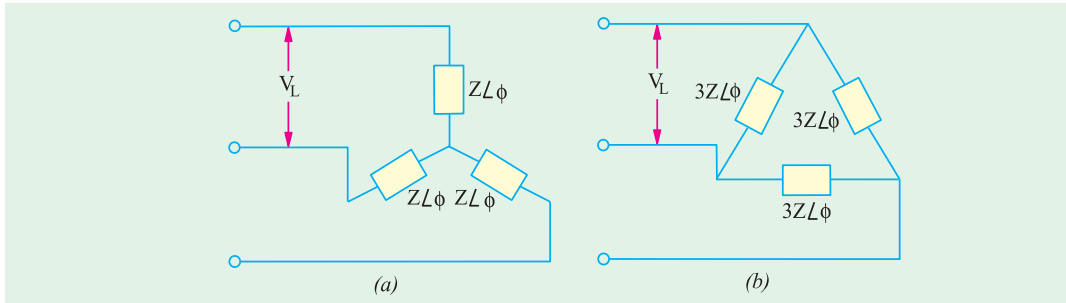


Fig. 19.26

Similarly, a balanced Y-connected load having equal branch impedances each of  $Z \angle \phi$  may be replaced by an equivalent Δ-connected load whose each phase impedance is  $3Z \angle \phi$ . This equivalence is shown in Fig. 19.26.

For a balanced star-connected load, let

$V_L$  = line voltage;  $I_L$  = line current ;  $Z_Y$  = impedance/phase

$$\therefore V_{ph} = V_L / \sqrt{3}; I_{ph} = I_L; Z_Y = V_L / (\sqrt{3} I_L)$$

Now, in the equivalent Δ-connected system, the line voltages and currents must have the same values as in the Y-connected system, hence we must have

$$V_{ph} = V_L, I_{ph} = I_L / \sqrt{3} \therefore Z_{\Delta} = V_L / (I_L / \sqrt{3}) = \sqrt{3} V_L / I_L = 3Z_Y$$

$$\therefore Z_{\Delta} \angle \phi = 3Z_Y \angle \phi \quad (V_L / I_L = \sqrt{3} Z_Y)$$

or  $Z_{\Delta} = 3Z_Y$  or  $Z_Y = Z_{\Delta} / 3$

The case of unbalanced load conversion is considered later. (Art. 19.34)

**Example 19.12.** A star-connected alternator supplies a delta connected load. The impedance of the load branch is  $(8 + j6)$  ohm/phase. The line voltage is 230 V. Determine (a) current in the load branch, (b) power consumed by the load, (c) power factor of load, (d) reactive power of the load. (Elect. Engg. A.M.Ae. S.I. June 1991)

**Solution.** Considering the Δ-connected load, we have  $Z_{ph} = \sqrt{8^2 + 6^2} = 10 \Omega$ ;  $V_{ph} = V_L = 230 \text{ V}$

(a)  $I_{ph} = V_{ph} / Z_{ph} = 230 / 10 = 23 \text{ A}$

(b)  $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 23 = 39.8 \text{ A}$ ;  $P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 230 \times 39.8 \times 0.8 = 12,684 \text{ W}$

(c) p.f.  $\cos \phi = R / Z = 8 / 10 = 0.8$  (lag)

(d) Reactive power  $Q = \sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 230 \times 39.8 \times 0.6 = 9513 \text{ W}$

**Example 19.13.** A 220-V, 3-φ voltage is applied to a balanced delta-connected 3-φ load of phase impedance  $(15 + j20) \Omega$

(a) Find the phasor current in each line. (b) What is the power consumed per phase ?

(c) What is the phasor sum of the three line currents ? Why does it have this value ?

(Elect. Circuits and Instruments, B.H.U.)

**Solution.** The circuit is shown in Fig. 19.27 (a).

$$V_{ph} = V_L = 220 \text{ V}; Z_{ph} = \sqrt{15^2 + 20^2} = 25 \Omega, I_{ph} = V_{ph} / Z_{ph} = 220 / 25 = 8.8 \text{ A}$$

$$(a) I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 8.8 = \mathbf{15.24 \text{ A}} \quad (b) P = I_{ph}^2 R_{ph} = 8.8^2 \times 15 = \mathbf{462 \text{ W}}$$

(c) Phasor sum would be zero because the three currents are equal in magnitude and have a mutual phase difference of  $120^\circ$ .

**Solution by Symbolic Notation**

Taking  $V_{RY}$  as the reference vector, we have [Fig. 19.27 (b)]

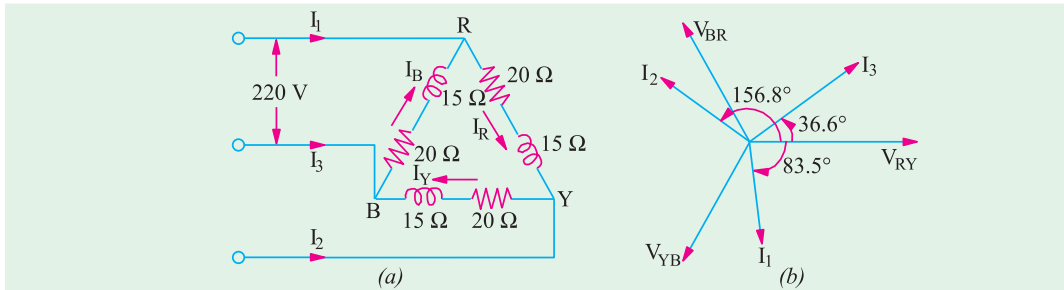


Fig. 19.27

$$\mathbf{V}_{RY} \quad 220 \quad 0^\circ;$$

$$\mathbf{V}_{YB} \quad 220 \quad 120^\circ$$

$$\mathbf{V}_{BR} \quad 220 \quad 120^\circ;$$

$$\mathbf{Z} \quad 15 \quad j20 \quad 125 \quad 53^\circ 8'$$

$$\mathbf{I}_R = \frac{\mathbf{V}_{RY}}{\mathbf{Z}} = \frac{220 \quad 0^\circ}{25 \quad 53^\circ 8'} = 8.8 \quad 53^\circ 8' \quad (5.28 \quad j7.04) \text{ A}$$

$$\mathbf{I}_Y = \frac{\mathbf{V}_{YB}}{\mathbf{Z}} = \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8'} = 8.8 \quad 173^\circ 8' \quad (-8.75 \quad j1.05) \text{ A}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_{BR}}{\mathbf{Z}} = \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8'} = 8.8 \quad 66^\circ 55' \quad (3.56 \quad j8.1)$$

(a) Current in line No. 1 is

$$\mathbf{I}_1 = \mathbf{I}_R - \mathbf{I}_B = (5.28 - j7.04) - (4.56 + j8.1) = (1.72 - j15.14) = 15.23 \angle -83.5^\circ$$

$$\mathbf{I}_2 = \mathbf{I}_Y - \mathbf{I}_R = (-8.75 - j1.05) - (5.28 - j7.04) = (-14.03 + j6.0) = 15.47 \angle -156.8^\circ$$

$$\mathbf{I}_3 = \mathbf{I}_B - \mathbf{I}_Y = (3.56 - j8.1) - (-8.75 - j1.05) = (12.31 + j9.15) = 15.26 \angle 36.8^\circ$$

(b) Using conjugate of voltage, we get for R-phase

$$\mathbf{P}_{VA} = \mathbf{V}_{RY} \cdot \mathbf{I}_R = (220 - j0) (5.28 - j7.04) = (1162 - j1550) \text{ voltampere}$$

Real power per phase = **1162 W**

(c) Phasor sum of three line currents

$$= \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = (1.72 - j15.14) + (-14.03 + j6.0) + (12.31 + j9.15) = 0$$

As expected, phasor sum of 3 line currents drawn by a balanced load is zero because these are equal in magnitude and have a phase difference of  $120^\circ$  amongst themselves.

**Example 19.14** A 3- $\phi$ ,  $\Delta$ -connected alternator drives a balanced 3- $\phi$  load whose each phase current is 10 A in magnitude. At the time when  $I_a = 10 \angle 30^\circ$ , determine the following, for a phase sequence of abc.

(i) Polar expression for  $I_b$  and  $I_c$  and (ii) polar expressions for the three line current. Show the phase and line currents on a phasor diagram.

**Solution. (i)** Since it is a balanced 3-phase system,  $I_b$  lags  $I_a$  by  $120^\circ$  and  $I_c$  lags  $I_a$  by  $240^\circ$  or leads it by  $120^\circ$ .

$$\therefore I_b = I_a \angle -120^\circ = 10 \angle (30^\circ - 120^\circ) = 10 \angle -90^\circ$$

$$I_c = I_a \angle 120^\circ = 10 \angle (30^\circ + 120^\circ) = 10 \angle 150^\circ$$

The 3-phase currents have been represented on the phasor diagram of Fig. 19.28 (b).

As seen from Fig. 19.28 (b), the line currents lag behind their nearest phase currents by  $30^\circ$ .

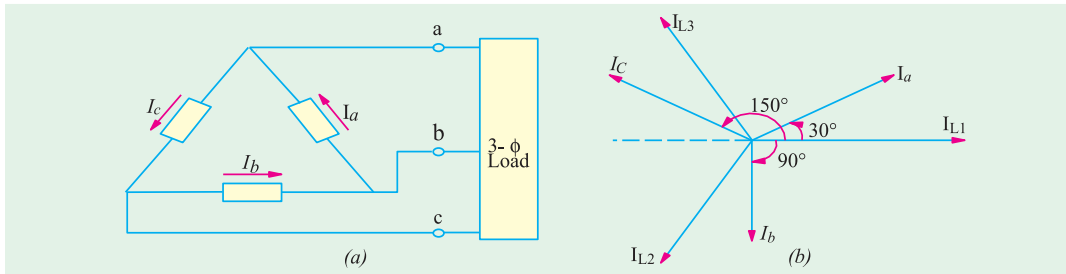


Fig. 19.28

$$\begin{aligned} \therefore I_{L1} &= \sqrt{3} I_a \quad (30^\circ \ 30^\circ) \quad \mathbf{17.3 \ 0^\circ} \\ I_{L2} &= \sqrt{3} I_b \quad (90^\circ \ 30^\circ) \quad \mathbf{17.3 \ 120^\circ} \\ I_{L3} &= \sqrt{3} I_c \quad (150^\circ \ 30^\circ) \quad \mathbf{17.3 \ 120^\circ} \end{aligned}$$

These line currents have also been shown in Fig. 19.28 (b).

**Example 19.15.** Three similar coils, each having a resistance of 20 ohms and an inductance of 0.05 H are connected in (i) star (ii) mesh to a 3-phase, 50-Hz supply with 400-V between lines. Calculate the total power absorbed and the line current in each case. Draw the vector diagram of current and voltages in each case. **(Elect. Technology, Punjab Univ. 1990)**

**Solution.**  $X_L = 2 \pi f L = 2 \pi \times 50 \times 0.05 = 15 \ \Omega$ ,  $Z_{ph} = \sqrt{15^2 + 20^2} = 25 \ \Omega$

**(i) Star Connection.** [Fig. 19.29 (a)]

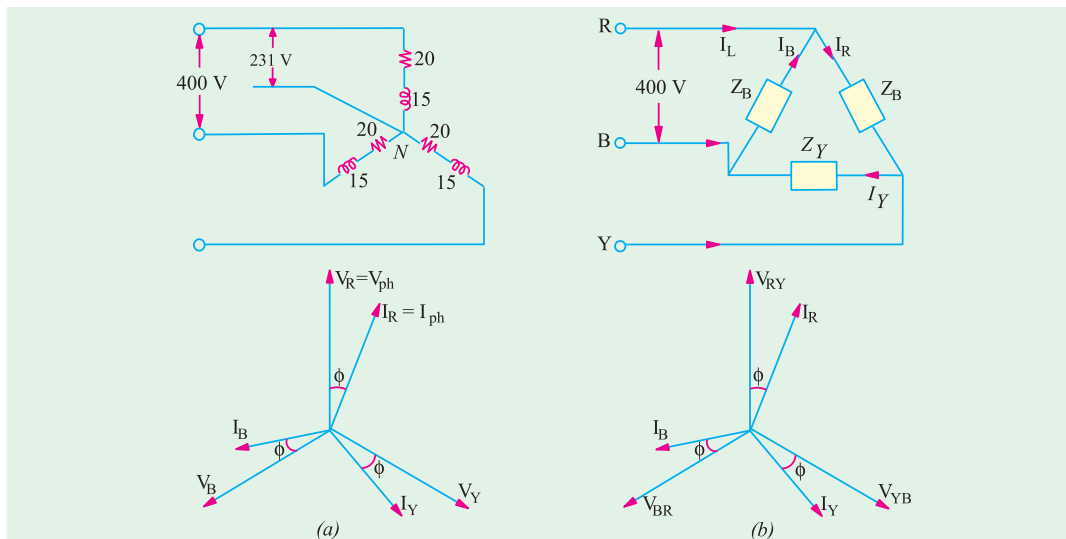


Fig. 19.29

$$V_{ph} = 400 / \sqrt{3} = 231 \text{ V}; I_{ph} = V_{ph} / Z_{ph} = 231 / 25 = \mathbf{9.24 \Omega}$$

$$I_L \quad I_{ph} \quad \mathbf{9.24 \text{ A}}; P = \sqrt{3} \quad 400 \quad 9.24 \quad (20/25) \quad \mathbf{5120 \text{ W}}$$

**(i) Delta Connection** [Fig. 19.29 (b)]

$$V_{ph} \quad V_L \quad 400 \text{ V}; I_{ph} \quad 400/25 \quad 16 \text{ A}; I_L \quad \sqrt{3} I_{ph} \quad \sqrt{3} \quad 16 \quad \mathbf{27.7 \text{ A}}$$

$$P = \sqrt{3} \times 400 \times 27.7 \times (20/25) = \mathbf{15,360 \text{ W}}$$

**Note.** It may be noted that line current as well as power are three times the star values.

**Example 19.16.** A  $\Delta$ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star. **(Electrical Machines, A.M.I.E. Sec. B. 1992)**

**Solution. (i) Delta Connection.**

$$V_{ph} = V_L = 400 \text{ V}; I_L = 20 \text{ A}; I_{ph} = 20 / \sqrt{3} \text{ A}$$

$$(i) \therefore Z_{ph} = \frac{400}{20/\sqrt{3}} = 20\sqrt{3} = \mathbf{34.64 \Omega}$$

$$\text{Now } P = \sqrt{3} V_L I_L \cos \phi \therefore \cos \phi = 10,000 / \sqrt{3} \times 400 \times 20 = \mathbf{0.7217}$$

**(ii) Star Connection**

$$V_{ph} = \frac{400}{\sqrt{3}}, I_{ph} = \frac{400/\sqrt{3}}{\sqrt{3}} = \frac{20}{3} \text{ A}, I_L = I_{ph} = \frac{20}{3} \text{ A}$$

Power factor remains the same since impedance is the same.

$$\text{Power consumed} = \sqrt{3} \times 400 \times (20/3) \times 0.7217 = \mathbf{3,330 \text{ W}}$$

**Note.** The power consumed is 1/3 of its value of  $\Delta$ -connection.

**Example 19.17.** Three similar resistors are connected in star across 400-V, 3-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors delta-connected.

**Solution. Star Connection**

$$I_L = I_{ph} = 5 \text{ A}; V_{ph} = 400 / \sqrt{3} = 231 \text{ V} \therefore R_{ph} = 231 / 5 = \mathbf{46.2 \Omega}$$

**Delta Connection**

$$I_L = 5 \text{ A... (given)}; I_{ph} = 5 / \sqrt{3} \text{ A}; R_{ph} = \mathbf{46.2 \Omega}$$

... found above

$$V_{ph} \quad I_{ph} \quad R_{ph} \quad 5 \quad 46.2/\sqrt{3} \quad \mathbf{133.3 \text{ V}}$$

**Note.** Voltage needed is 1/3rd the star value.

**Example 19.18.** A balanced delta connected load, consisting of three coils, draws  $10\sqrt{3}$  A at 0.5 power factor from 100 V, 3-phase supply. If the coils are re-connected in star across the same supply, find the line current and total power consumed.

**(Elect. Technology, Punjab Univ. Nov.)**

**Solution. Delta Connection**

$$V_{ph} \quad V_L \quad 100 \text{ V}; I_L \quad 10\sqrt{3} \text{ A}; I_{ph} \quad 10\sqrt{3}/\sqrt{3} \quad \mathbf{10 \text{ A}}$$



$$Z_{ph} = V_{ph} / I_{ph} = 100/10 = 10\Omega; \cos\phi = 0.5 \text{ (given); } \sin\phi = 0.866$$

$$\therefore R_{ph} = Z_{ph} \cos\phi = 10 \times 0.5 = 5\Omega; X_{ph} = Z_{ph} \sin\phi = 10 \times 0.866 = 8.66\Omega$$

$$\text{Incidentally, total power consumed} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 100 \times 10 \sqrt{3} \times 0.5 = 1500 \text{ W}$$

### Star Connection

$$V_{ph} = V_L / \sqrt{3} = 100 / \sqrt{3}; Z_{ph} = 10; I_{ph} = V_{ph} / Z_{ph} = 100 / \sqrt{3} \times 10 = 10 \sqrt{3} \text{ A}$$

$$\text{Total power absorbed} = \sqrt{3} \times 100 \times (10 \sqrt{3}) \times 0.5 = 500 \text{ W}$$

It would be noted that the line current as well as the power absorbed are one-third of that in the delta connection.

**Example 19.19.** Three identical impedances are connected in delta to a 3  $\phi$  supply of 400 V. The line current is 35 A and the total power taken from the supply is 15 kW. Calculate the resistance and reactance values of each impedance. *(Elect. Technology, Punjab Univ.,)*

$$\text{Solution. } V_{ph} = V_L = 400 \text{ V}; I_L = 35 \text{ A} \therefore I_{ph} = 35 / \sqrt{3} \text{ A}$$

$$Z_{ph} = V_{ph} / I_{ph} = 400 \times \sqrt{3} / 35 = 19.8 \text{ A}$$

Now,

$$\text{Power } P = \sqrt{3} V_L I_L \cos\phi \therefore \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{15,000}{\sqrt{3} \times 400 \times 35} = 0.619; \text{ But } \sin\phi = 0.786$$

$$\therefore R_{ph} = Z_{ph} \cos\phi = 19.8 \times 0.619 = 12.25\Omega; X_{ph} = Z_{ph} \sin\phi = 19.8 \times 0.786 = 15.5\Omega$$

**Example 19.20.** Three  $100\Omega$  non-inductive resistances are connected in (a) star (b) delta across a 400-V, 50-Hz, 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting open-circuited, what would be the value of total power taken from the mains in each of the two cases? *(Elect. Engg. A.M.Ae. S.I June, 1993)*

**Solution. (i) Star Connection [Fig. 19.30 (a)]**

$$V_{ph} = 400 / \sqrt{3} \text{ V}$$

$$P = \sqrt{3} V_L I_L \cos\phi \\ = \sqrt{3} \times 400 \times 4 \times 1 / \sqrt{3} = 1600 \text{ W}$$

**(ii) Delta Connection Fig. 19.30 (b)**

$$V_{ph} = 400 \text{ V}; R_{ph} = 100\Omega$$

$$I_{ph} = 400 / 100 = 4 \text{ A}$$

$$I_L = 4 \times \sqrt{3} \text{ A}$$

$$P = \sqrt{3} \times 400 \times 4 \times \sqrt{3} \times 1 = 4800 \text{ W}$$

**When one of the resistors is disconnected**

**(i) Star Connection [Fig. 19.28 (a)]**

The circuit no longer remains a 3-phase circuit but consists of two  $100\Omega$  resistors in series across a 400-V supply. Current in lines A and C is  $= 400/200 = 2 \text{ A}$

$$\text{Power absorbed in both} = 400 \times 2 = 800 \text{ W}$$

Hence, by disconnecting one resistor, the power consumption is reduced by half.

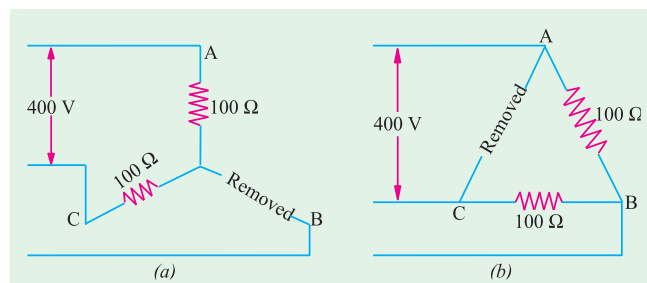


Fig. 19.30

**(ii) Delta Connection** [Fig. 19.28 (b)]

In this case, currents in  $A$  and  $C$  remain as usual  $120^\circ$  out of phase with each other.

Current in each phase =  $400/100 = 4$  A

Power consumption in both =  $2 \times 4^2 \times 100 = 3200$  W

(or  $P = 2 \times 4 \times 400 = 3200$  W)

In this case, when one resistor is disconnected, the power consumption is reduced by one-third.

**Example 19.21.** A 200-V, 3- $\phi$  voltage is applied to a balanced  $\Delta$ -connected load consisting of the groups of fifty 60-W, 200-V lamps. Calculate phase and line currents, phase voltages, power consumption of all lamps and of a single lamp included in each phase for the following cases :

(a) under normal conditions of operation

(b) after blowout in line  $R'R$  (c) after blowout in phase  $YB$

Neglect impedances of the line and internal resistances of the sources of e.m.f.

**Solution.** The load circuit is shown in Fig. 19.31 where each lamp group is represented by two lamps only. It should be kept in mind that lamps remain at the line voltage of the supply irrespective of whether the  $\Delta$ -connected load is balanced or not.

(a) **Normal operating conditions** [Fig. 19.31 (a)]

Since supply voltage equals the rated voltage of the bulbs, the power consumption of the lamps equals their rated wattage.

Power consumption/lamp = 60 W; Power consumption/phase =  $50 \times 60 = 3,000$  W

Phase current =  $3000/200 = 15$  A ; Line current =  $15 \times \sqrt{3} = 26$  A

(b) **Line Blowout** [Fig. 19.31 (b)]

When blowout occurs in line  $R$ , the lamp group of phase  $Y-B$  remains connected across line voltage  $V_{YB} = V_{YB}$ . However, the lamp groups of other two phases get connected in series across the same voltage  $V_{YB}$ . Assuming that lamp resistances remain constant, voltage drop across  $YR = V_{YB}/2 = 100$  V and that across  $RB = 100$  V.

Hence, phase currents are as under :

$$I_{YB} = 3000/200 = 15 \text{ A}, I_{YB} = I_{RB} = 15/2 = 7.5 \text{ A}$$

The line currents are :

$$I_{RR} = 0, I_{YY} = I_{BB} = I_{YB} = I_{YR} = 15 \quad 7.5 \quad 22.5 \text{ A}$$

Power in phase  $YR = 100 \times 7.5 = 750$  W; Power/lamp =  $750/50 = 15$  W

Power in phase  $YB = 200 \times 15 = 3000$  W ; Power/lamp =  $3000/50 = 60$  W

Power in phase  $RB = 100 \times 7.5 = 750$  W ; Power/lamp =  $750/50 = 15$  W

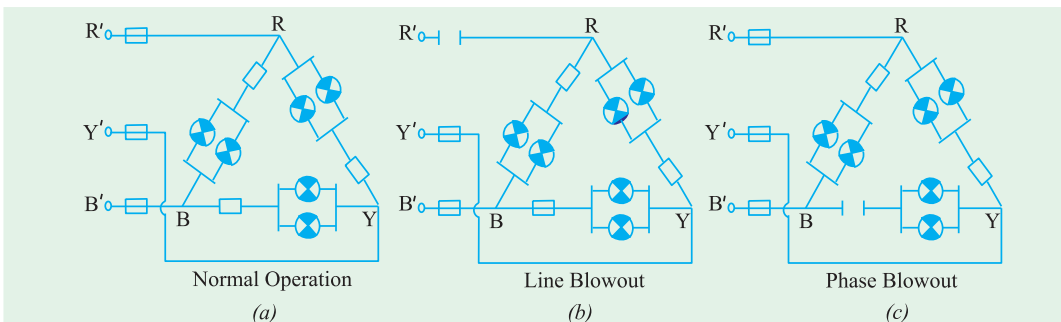


Fig. 19.31

**(c) Phase Blowout** [Fig. 19.31 (c)]

When fuse in phase *Y-B* blows out, the phase voltage becomes zero (though voltage across the open remains 200 V). However, the voltage across the other two phases remains the same as under normal operating conditions.

Hence, different phase currents are :

$$I_{RY} \quad 15 \text{ A}, I_{BR} \quad 15 \text{ A}, I_{YB} \quad 0$$

The line currents become

$$I_{RR} \quad 15\sqrt{3} \quad 26 \text{ A}; I_{YY} \quad 15 \text{ A}, I_{BB} \quad 15 \text{ A}$$

Power in phase *RY* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *RB* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *YB* = 0; power/lamp = 0.

**Example 19.22.** The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil.

If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

**Solution. Star Connection**

$$\cos \phi \quad k = W/kVA = 11/20 \quad I_L = 25 \text{ A} \quad P = 11 \text{ kW} = 11,000 \text{ W}$$

$$\text{Now } P = \sqrt{3} V_L I_L \cos \phi \quad \therefore 11,000 = \sqrt{3} \times V_L \times 25 \times 11/20$$

$$\therefore V_L \quad 462 \text{ V}; \quad V_{ph} = 462 / \sqrt{3} = 267 \text{ V}$$

$$kVAR \quad \sqrt{kVA^2 - kW^2} \quad \sqrt{20^2 - 11^2} \quad 16.7; Z_{ph} \quad 267/2 \quad 10.68$$

$$\therefore R_{ph} = Z_{ph} \times \cos \phi = 10.68 \times 11/20 = 5.87 \Omega$$

$$\therefore X_{ph} = Z_{ph} \sin \phi = 10.68 \times 0.838 = 8.97 \Omega$$

**Delta Connection**

$$V_{ph} = V_L = 462 \text{ V} \text{ and } Z_{ph} = 10.68 \Omega$$

$$\therefore I_{ph} = 462/10.68 \text{ A}, I_L = \sqrt{3} \times 462/10.68 = 75 \text{ A}$$

$$P = \sqrt{3} \times 462 \times 75 \times 11/20 = 33,000 \text{ W}$$

**Example 19.23.** A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 4, 5 and 6 Ω respectively in the three phases. Estimate the current flowing in each phase and the neutral current. Find the total power absorbed. (I.E.E. London)

**Solution.** Here,  $V_{ph} = 230 \text{ V}$  [Fig. 19.32 (a)]

$$\text{Current in 4-}\Omega \text{ resistor} = 230/4 = 57.5 \text{ A}$$

$$\text{Current in 5-}\Omega \text{ resistor} = 230/5 = 46 \text{ A}$$

$$\text{Current in 6-}\Omega \text{ resistor} = 230/6 = 38.3 \text{ A}$$

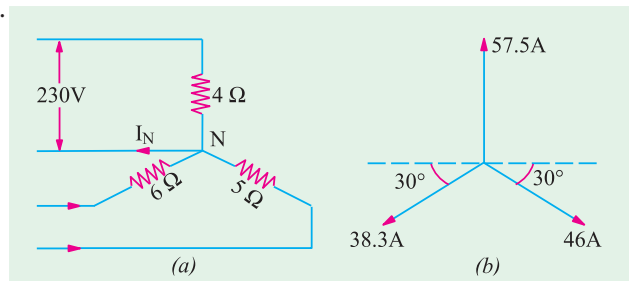


Fig. 19.32

These currents are mutually displaced by  $120^\circ$ . The neutral current  $I_N$  is the vector sum\* of these three currents.  $I_N$  can be obtained by splitting up these three phase currents into their  $X$ -components and  $Y$ -components and then by combining them together, in diagram 19.32 (b).

$$X\text{-component} = 46 \cos 30^\circ - 38.3 \cos 30^\circ = \mathbf{6.64 \text{ A}}$$

$$Y\text{-component} = 57.5 - 46 \sin 30^\circ - 38.3 \sin 30^\circ = 15.3 \text{ A} \quad I_N = \sqrt{6.64^2 + 15.3^2} = \mathbf{16.71 \text{ A}}$$

$$\text{The power absorbed} = 230 (57.5 + 46 + 38.3) = \mathbf{32.610 \text{ W}}$$

**Example 19.24.** A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps in use require 70, 84 and 33 A in each of the three lines, what should be the current in the neutral wire? If a 3-phase motor is now started, taking 200 A from the line at a power factor of 0.2, what would be the current in each line and the neutral current? Find also the total power supplied to the lamps and the motor. (Elect. Technology, Aligarh Univ.)

**Solution.** The lamp and motor connections are shown in Fig. 19.33.

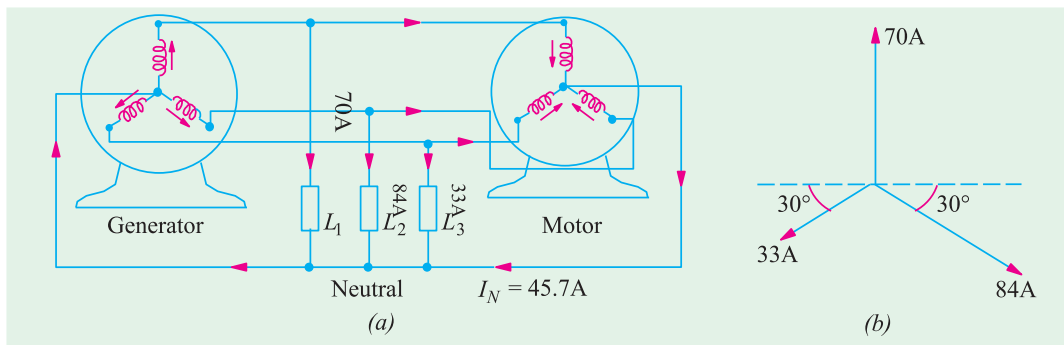


Fig. 19.33

#### When motor is not started

The neutral current is the vector sum of lamp currents. Again, splitting up the currents into their  $X$ - and  $Y$ -components, we get

$$X\text{-component} = 84 \cos 30^\circ - 33 \cos 30^\circ = 44.2 \text{ A}$$

$$Y\text{-component} = 70 - 84 \sin 30^\circ - 33 \sin 30^\circ = 11.5 \text{ A}$$

$$\therefore I_N = \sqrt{44.2^2 + 11.5^2} = \mathbf{45.7 \text{ A}}$$

#### When motor is started

A 3-phase motor is a balanced load. Hence, when it is started, it will change the line currents but being a balanced load, it contributes nothing to the neutral current. Hence, **the neutral current remains unchanged even after starting the motor.**

Now, the motor takes 200 A from the lines. It means that each line will carry motor current (which lags) as well as lamp current (which is in phase with the voltage). The current in each line would be the vector of sum of these two currents.

$$\text{Motor p.f.} = 0.2 ; \sin \phi = 0.9799 \dots \text{ from tables}$$

$$\text{Active component motor current} = 200 \times 0.2 = 40 \text{ A}$$

\* Some writers disagree with this statement on the ground that according to Kirchhoff's Current Law, at any junction,  $I_N + I_R + I_Y + I_B = 0 \therefore I_N = -(I_R + I_Y + I_B)$

Hence, according to them, numerical value of  $I_N$  is the same but its phase is changed by  $180^\circ$ .

Reactive component of motor current =  $200 \times 0.9799 = 196 \text{ A}$

(i) Current in first line =  $\sqrt{(40 + 70)^2 + 196^2} = \mathbf{224.8 \text{ A}}$

(ii) Current in second line =  $\sqrt{(40 - 84)^2 + 196^2} = \mathbf{232 \text{ A}}$

(iii) Current in third line =  $\sqrt{(40 + 33)^2 + 196^2} = \mathbf{210.6 \text{ A}}$

Power supplied to lamps =  $230 (33 + 84 + 70) = \mathbf{43,000 \text{ W}}$

Power supplied to motor =  $\sqrt{3} \times 200 \times 400 \times 0.2 = \mathbf{27,700 \text{ W}}$

### 19.11. Star and Delta connected Lighting Loads

In Fig. 19.34 (a) is shown a  $Y$ -connected lighting network in a three storey house. For such a load, it is essential to have neutral wire in order to ensure uniform distribution of load among the three phases despite random switching on and off or burning of lamps. It is seen from Fig. 19.34 (a),

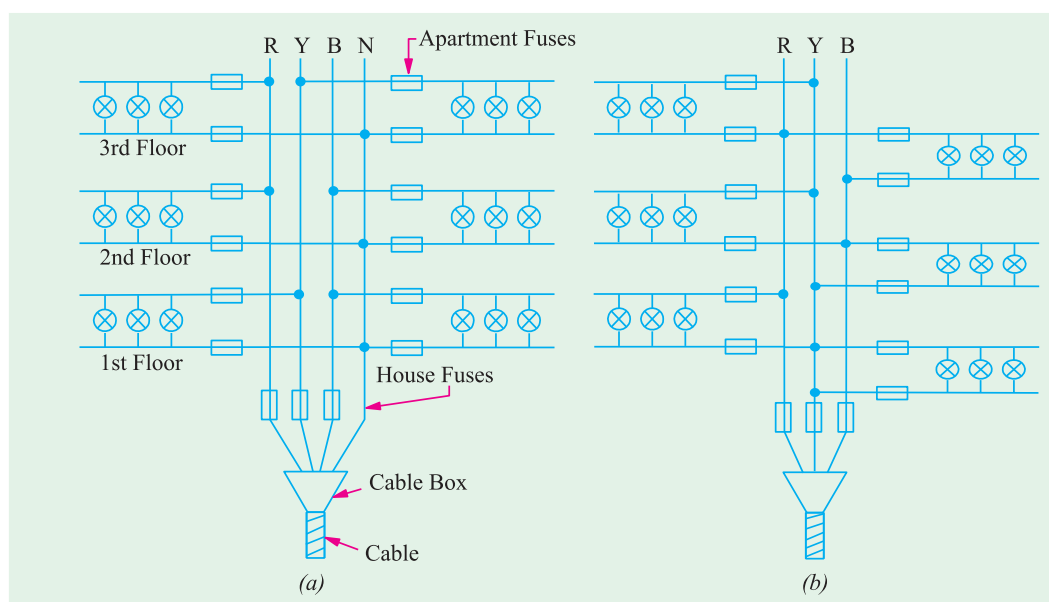


Fig. 19.34

that network supplies two flats on each floor of the three storey residence and there is balanced distribution of lamp load among the three phases. There are house fuses at the cable entry into the building which protect the two mains against short-circuits in the main cable. At the flat entry, there are apartment (or flat) fuses in the single-phase supply which protect the two mains and other flats in the same building from short-circuits in a given building. There is no fuse (or switch) on the neutral wire of the mains because blowing of such a fuse (or disconnection of such a switch) would mean a break in the neutral wire. This would result in unequal voltages across different groups of lamps in case they have different power ratings or number. Consequently, filaments in one group would burn dim whereas in other groups they would burn too bright resulting in their early burn-out.

The house-lighting wire circuit for  $\Delta$ -connected lamps is shown in Fig. 19.34 (b).

### 19.12. Power Factor Improvement

The heating and lighting loads supplied from 3-phase supply have power factors, ranging

from 0.95 to unity. But motor loads have usually low lagging power factors, ranging from 0.5 to 0.9. Single-phase motors may have as low power factor as 0.4 and electric welding units have even lower power factors of 0.2 or 0.3.

The power factor is given by  $\cos \frac{\text{kW}}{\text{kVA}}$  or  $\text{kVA} = \frac{\text{kW}}{\cos}$

In the case of single-phase supply,  $\text{kVA} = \frac{VI}{1000}$  or  $I = \frac{1000 \text{ kVA}}{V} \therefore I \propto \text{kVA}$

In the case of 3-phase supply  $\text{kVA} = \frac{\sqrt{3} V_L I_L}{1000}$  or  $I_L = \frac{1000 \text{ kVA}}{\sqrt{3} \times V_L} \therefore I \propto \text{kVA}$

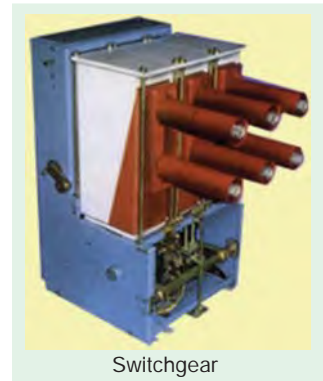
In each case, the kVA is directly proportional to current. The chief disadvantage of a low p.f. is that the current required for a given power, is very high. This fact leads to the following undesirable results.

**(i) Large kVA for given amount of power**

All electric machinery, like alternators, transformers, switchgears and cables are limited in their current-carrying capacity by the permissible temperature rise, which is proportional to  $I^2$ . Hence, they may all be fully loaded with respect to their rated kVA, without delivering their full power. Obviously, it is possible for an existing plant of a given kVA rating to increase its earning capacity (which is proportional to the power supplied in kW) if the overall power factor is improved *i.e.* raised.

**(ii) Poor voltage regulation**

When a load, having low lagging power factor, is switched on, there is a large voltage drop in the supply voltage because of the increased voltage drop in the supply lines and transformers. This drop in voltage adversely affects the starting torques of motors and necessitates expensive voltage stabilizing equipment for keeping the consumer's voltage fluctuations within the statutory limits. Moreover, due to this excessive drop, heaters take longer time to provide the desired heat energy, fluorescent lights flicker and incandescent lamps are not as bright as they should be. Hence, all supply undertakings try to encourage consumers to have a high power factor.



Switchgear

**Example 19.25.** A 50-MVA, 11-kV, 3- $\phi$  alternator supplies full load at a lagging power factor of 0.7. What would be the percentage increase in earning capacity if the power factor is increased to 0.95?

**Solution.** The earning capacity is proportional to the power (in MW or kW) supplied by the alternator.

MW supplied at 0.7 lagging =  $50 \times 0.7 = 35$

MW supplied at 0.95 lagging =  $50 \times 0.95 = 47.5$

increase in MW = 12.5

The increase in earning capacity is proportional to 12.5

$\therefore$  Percentage increase in earning capacity =  $(12.5/35) \times 100 = 35.7$

### 19.13. Power Correction Equipment

The following equipment is generally used for improving or correcting the power factor :

**(i) Synchronous Motors (or capacitors)**

These machines draw leading kVAR when they are over-excited and, especially, when they

are running idle. They are employed for correcting the power factor in bulk and have the special advantage that the amount of correction can be varied by changing their excitation.

**(ii) Static Capacitors**

They are installed to improve the power factor of a group of a.c. motors and are practically loss-free (*i.e.* they draw a current leading in phase by 90°). Since their capacitances are not variable, they tend to over-compensate on light loads, unless arrangements for automatic switching off the capacitor bank are made.

**(iii) Phase Advancers**

They are fitted with individual machines.

However, it may be noted that the economical degree of correction to be applied in each case, depends upon the tariff arrangement between the consumers and the supply authorities.

**Example 19.26.** A 3-phase, 37.3 kW, 440-V, 50-Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate the total kVA rating of capacitors required to raise the full-load power factor at 0.95 lagging. What will be the capacitance per phase if the capacitors are (a) delta-connected and (b) star-connected ?

**Solution.** It is helpful to approach such problems from the ‘power triangle’ rather than from vector diagram viewpoint.

Motor power input  $P = 37.3/0.89 = 41.191 \text{ kW}$

**Power Factor 0.85 (lag)**

$\cos \phi_1 = 0.85; \phi_1 = \cos^{-1} (0.85) = 31.8^\circ; \tan \phi_1 = \tan 31.8^\circ = 0.62$

Motor  $\text{kVAR}_1 = P \tan \phi_1 = 41.91 \times 0.62 = 25.98$

**Power Factor 0.95 (lag)**

Motor power input  $P = 41.91 \text{ kW}$  ... as before

It is the same as before because capacitors are loss-free *i.e.* they do not absorb any power.

$\cos \phi_2 = 0.95 \therefore \phi_2 = 18.2^\circ; \tan 18.2^\circ = 0.3288$

Motor  $\text{kVAR}_2 = P \tan \phi_2 = 41.91 \times 0.3288 = 13.79$

The difference in the values of kVAR is due to the capacitors which supply **leading** kVAR to partially neutralize the **lagging** kVAR of the motor.

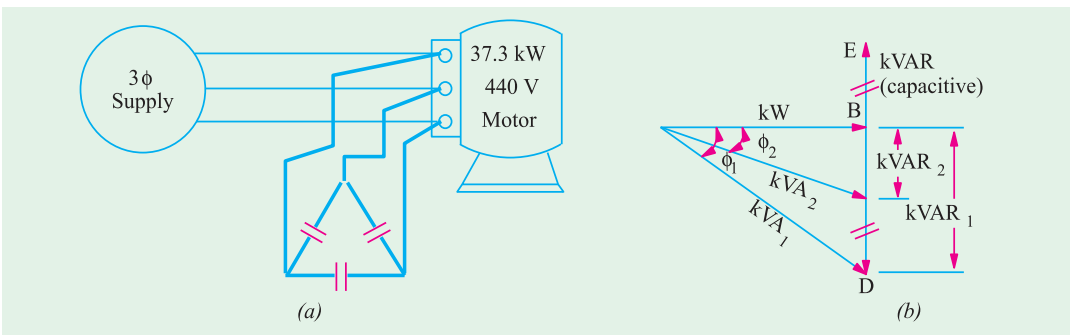


Fig. 19.35

$\therefore$  leading kVAR supplied by capacitors is  
 $= \text{kVAR}_1 - \text{kVAR}_2 = 25.98 - 13.79 = \mathbf{12.19}$  ... *CD* in Fig. 19.35 (b)  
 Since capacitors are loss-free, their kVAR is the same as kVA  
 $\therefore \text{kVA/capacitor} = 12.19/3 = 4.063 \therefore \text{VAR/capacitor} = 4,063$

(a) In  $\Delta$ -connection, voltage across each capacitor is 440 V

Current drawn by each capacitor  $I_c = 4063/440 = 9.23$  A

Now, 
$$I_c = \frac{V}{X_c} = \frac{V}{1/\omega C} = \omega VC$$

$\therefore C = I_c / \omega V = 9.23 / 2\pi \times 50 \times 440 = 66.8 \times 10^{-6} \text{ F} = \mathbf{66.8 \mu\text{F}}$

(b) In star connection, voltage across each capacitor is =  $440/\sqrt{3}$  volt

Current drawn by each capacitor,  $I_c = \frac{4063}{440/\sqrt{3}} = 16.0$  A

$$I_c = \frac{V}{X_c} = \omega VC \quad \text{or} \quad 16 = \frac{440}{\sqrt{3}} \times 2\pi \times 50 \times C$$

$\therefore C = 200.4 \times 10^{-6} \text{ F} = \mathbf{200.4 \mu\text{F}}$

**Note.** Star value is three times the delta value.

**Example 19.27.** If the motor of Example 19.24 is supplied through a cable of resistance  $0.04 \Omega$  per core, calculate

(i) the percentage reduction in cable Cu loss and

(ii) the additional balanced lighting load which the cable can supply when the capacitors are connected.

**Solution.** Original motor  $\text{kVA}_1 = P/\cos \phi_1 = 41.91/0.85 = 49.3$

Original line current,  $I_{L1} = \frac{\text{kVA}_1 \times 1000}{\sqrt{3} \times 440} = \frac{49.3 \times 1000}{\sqrt{3} \times 440} = 64.49$  A

$\therefore$  Original Cu loss/conductor =  $64.49^2 \times 0.04 = 167.4$  W

From Fig. 19.34, it is seen that the new kVA i.e.  $\text{kVA}_2$  when capacitors are connected is given by  $\text{kVA}_2 = \text{kW}/\cos \phi_2 = 41.91/0.95 = 44.12$

New line current  $I_{L2} = \frac{44,120}{\sqrt{3} \times 440} = 57.89$  A

New Cu loss =  $57.89^2 \times 0.04 = 134.1$  W

(i)  $\therefore$  percentage reduction =  $\frac{167.4 - 134.1}{167.4} \times 100 = \mathbf{19.9}$

The total kVA which the cable can supply is 49.3 kVA. When the capacitors are connected, the kVA supplied is 44.12 at a power factor of 0.94 lagging. The lighting load will be assumed at unity power factor. The kVA diagram is shown in Fig. 19.34. We will tabulate the different loads as follows. Let the additional lighting load be  $x$  kW.

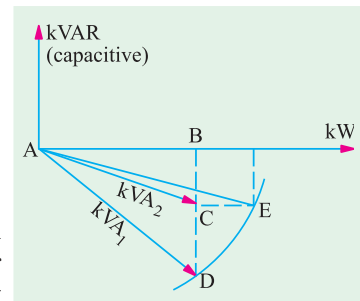


Fig. 19.36

Load	kVA	$\cos \phi$	kW	$\sin \phi$	kVAR
Motor	49.3	0.85 lag	41.91	0.527	-25.98
Capacitors	12.19	0 lead	0	1.0	+12.19
Lighting	-	1.0	$x$	0	0
			$(1.91 + x)$		-13.79



From Fig. 19.36 it is seen that

$$AF = 41.91 + x \text{ and } EF = 13.79 \quad AE = \text{resultant kVA} = 49.3$$

$$\text{Also } AF^2 + EF^2 = AE^2 \text{ or } (41.91 + x)^2 + 13.79^2 = 49.3^2 \quad \therefore x = \mathbf{5.42 \text{ kW}}$$

**Example 19.28.** Three impedance coils, each having a resistance of  $20\Omega$  and a reactance of  $15\Omega$ , are connected in star to a 400-V, 3- $\phi$ , 50-Hz supply. Calculate (i) the line current (ii) power supplied and (iii) the power factor.

If three capacitors, each of the same capacitance, are connected in delta to the same supply so as to form parallel circuit with the above impedance coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

$$\text{Solution. } V_{ph} = 400/\sqrt{3}V, Z_{ph} = \sqrt{20^2 + 15^2} = 25$$

$$\cos \phi_1 = R_{ph}/Z_{ph} = 20/25 = 0.8 \text{ lag; } \phi_1 = 0.6 \text{ lag}$$

where  $\phi_1$  is the power factor angle of the coils.

**When capacitors are not connected**

$$(i) I_{ph} = 400/25 \times \sqrt{3} = 9.24 \text{ A} \quad \therefore I_L = \mathbf{9.24 \text{ A}}$$

$$(ii) P = \sqrt{3} V_L I_L \cos \phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.8 = \mathbf{5.120 \text{ W}}$$

$$(iii) \text{ Power factor} = \mathbf{0.8 \text{ (lag)}}$$

$$\therefore \text{ Motor VAR}_1 = \sqrt{3} V_L I_L \sin \phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.6 = \mathbf{3,840}$$

**When capacitors are connected**

$$\text{Power factor, } \cos \phi_2 = 0.95, \phi_2 = 18.2^\circ; \tan 18.2^\circ = 0.3288$$

Since capacitors themselves do not absorb any power, power remains the same *i.e.* 5,120 W even when capacitors are connected. The only thing that changes is the VAR.

$$\text{Now } \text{VAR}_2 = P \tan \phi_2 = 5120 \times 0.3288 = 1684$$

**Leading VAR** supplied by the three capacitors is

$$= \text{VAR}_1 - \text{VAR}_2 = 3840 - 1684 = 2156 \text{ BD or CE in Fig 19.37 (b)}$$

$$\text{VAR/ Capacitor} = 2156/3 = 719$$

$$\text{For delta connection, voltage across each capacitor is } 400 \text{ V} \quad \therefore I_c = 719/400 = 1.798 \text{ A}$$

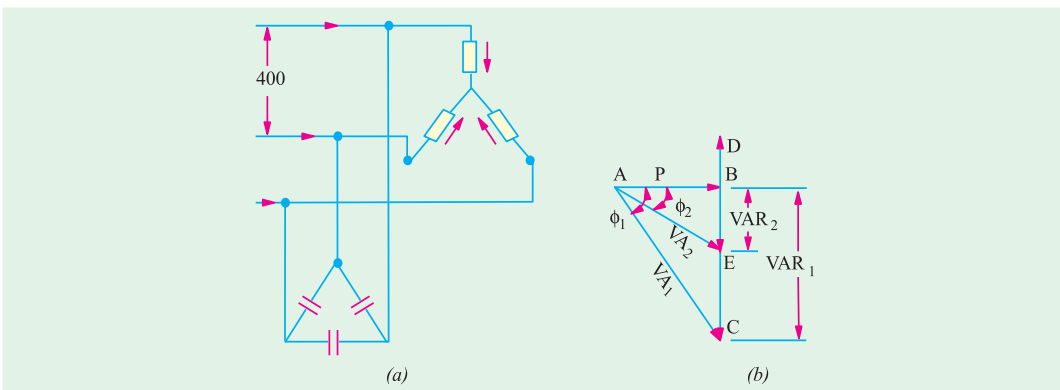


Fig. 19.37

$$\text{Also } I_c = \frac{V}{I/\omega C} = \omega VC \quad \therefore C = 1.798/\pi \times 50 \times 400 = 14.32 \times 10^{-6} \text{ F} = \mathbf{14.32 \mu\text{F}}$$

### 19.14. Parallel Loads

A combination of balanced 3-phase loads connected in parallel may be solved by any one of the following three methods :

1. All the given loads may be converted into equivalent  $\Delta$ -loads and then combined together according to the law governing parallel circuits.
2. All the given loads may be converted into equivalent  $Y$ -loads and treated as in (1) above.
3. The third method, which requires less work, is to work in terms of volt-amperes. The special advantage of this approach is that voltmeters can be added regardless of the kind of connection involved. The real power of various loads can be added arithmetically and VARs may be added algebraically so that total voltamperes are given by

$$VA = \sqrt{W^2 + VAR^2} \quad \text{or} \quad S = \sqrt{P^2 + Q^2}$$

where  $P$  is the power in watt and  $Q$  represents reactive voltamperes.

**Example 19.29.** For the power distribution system shown in Fig. 19.38, find

(a) total apparent power, power factor and magnitude of the total current  $I_T$  without the capacitor in the system

(b) the capacitive kVARs that must be supplied by  $C$  to raise the power factor of the system to unity ;

(c) the capacitance  $C$  necessary to achieve the power correction in part (b) above

(d) total apparent power and supply current  $I_T$  after the power factor correction.

**Solution.** (a) We will take the inductive i.e. lagging kVARs as negative and capacitive i.e. leading kVARs as positive.

$$\text{Total } Q = -16 + 6 - 12 = -22 \text{ kVAR (lag)}; \text{ Total } P = 30 + 4 + 36 = 70 \text{ kW}$$

$$\therefore \text{apparent power } S = \sqrt{(-22)^2 + 70^2} = \mathbf{73.4 \text{ kVA}}; \text{ p.f.} = \cos \phi = P/S = 70/73.4 = \mathbf{0.95}$$

$$S = VI_T \text{ or } 73.4 \times 10^3 = 400 \times I_T \therefore I_T = \mathbf{183.5 \text{ A}}$$

(b) Since total lagging kVARs are  $-22$ , hence, for making the power factor unity, 22 leading kVARs must be supplied by the capacitor to neutralize them. In that case, total  $Q = 0$  and  $S = P$  and p.f. is unity.

(c) If  $I_C$  is the current drawn by the capacitor, then  $22 \times 10^3 = 400 \times I_C$

$$\text{Now, } I_C = V/X_C = V\omega C$$

$$= 400 \times 2\pi \times 50 \times C$$

$$\therefore 22 \times 10^3 = 400 (2\pi \times 50 C);$$

$$\therefore C = \mathbf{483 \mu F}$$

(d) Since  $Q = 0$ ,

$$\text{hence, } S = \sqrt{10^2 + 70^2} = \mathbf{70 \text{ kVA}}$$

$$\text{Now, } VI_T = 70 \times 10^3;$$

$$I_T = 70 \times 10^3 / 400 = 175 \text{ A.}$$

It would be seen that after the power correction, lesser amount of current is required to deliver the same amount of real power to the system.

**Example 19.30.** A symmetrical 3-phase, 3-wire supply with a line voltage of 173 V supplies two balanced 3-phase loads; one  $Y$ -connected with each branch impedance equal to  $(6 + j8)$  ohm and the other  $\Delta$ -connected with each branch impedance equal to  $(18 + j24)$  ohm. Calculate

(i) the magnitudes of branch currents taken by each 3-phase load

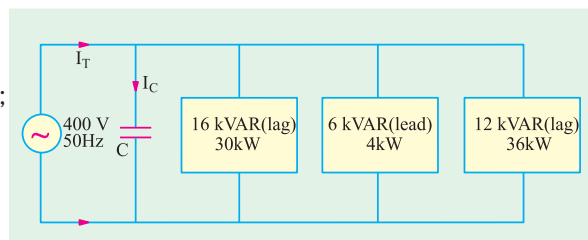


Fig. 19.38

(ii) the magnitude of the total line current and

(iii) the power factor of the entire load circuit

Draw the phasor diagram of the voltages and currents for the two loads.

(Elect. Engineering-I, Bombay Univ.)

**Solution.** The equivalent  $Y$ -load of the given  $\Delta$ -load (Art.19.10) is  $= (18 + j24)/3 = (6 + j8) \Omega$

With this, the problem now reduces to one of solving two equal  $Y$ -loads connected in parallel across the 3-phase supply as shown in Fig. 19.39 (a). Phasor diagram for the combined load for one phase only is given in Fig. 19.39 (b).

Combined load impedance

$$= (6 + j8) / 2 = 3 + j4$$

$$= 5 \angle 53.1^\circ \text{ ohm}$$

$$V_{ph} = 173 / \sqrt{3} = 100 \text{ V}$$

Let  $V_{ph} = 100 \angle 0^\circ$

$$\therefore I_{ph} = \frac{100 \angle 0^\circ}{5 \angle 53.1^\circ} = 20 \angle -53.1^\circ$$

Current in each load  $= 10 \angle -53.1^\circ \text{ A}$

(i) branch current taken by each load is **10 A**; (ii) line current is **20 A**;

(iii) combined power factor  $= \cos 53.1^\circ = \mathbf{0.6 \text{ (lag)}}$ .

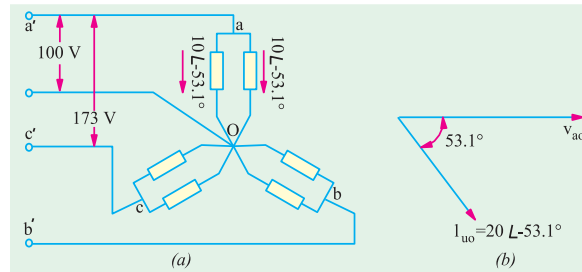


Fig. 19.39

**Example 19.31.** Three identical impedances of  $30 \angle 30^\circ$  ohms are connected in delta to a 3-phase, 3-wire, 208 V volt abc system by conductors which have impedances of  $(0.8 + j 0.63)$  ohm. Find the magnitude of the line voltage at the load end.

(Elect. Engg. Punjab Univ. May 1990)

**Solution.** The equivalent  $Z_y$  of the given  $Z_\Delta$  is  $30 \angle 30^\circ / 3 = 10 \angle 30^\circ = (8.86 + j5)$ . Hence, the load connections become as shown in Fig. 19.40.

$$Z_{an} = (0.8 + j0.6) + (8.86 + j5)$$

$$= 9.66 + j5.6 = 11.16 \angle 30.1^\circ$$

$$V_{an} = V_{ph} = 208 / \sqrt{3} = 120 \text{ V}$$

Let  $V_{an} = 120 \angle 0^\circ$

$$\therefore I_{an} = 120 \angle 0^\circ / 11.16 \angle 30.1^\circ = 10.75 \angle -30.1^\circ$$

Now,  $Z_{aa'} = 0.8 + j0.6 = 1 \angle 36.9^\circ$

Voltage drop on line conductors is

$$V'_{aa} = I_{an} Z'_{aa} = 10.75 \angle -30.1^\circ \times 1 \angle 36.9^\circ = 10.75 \angle 6.8^\circ = 10.67 + j1.27$$

$$\therefore V'_{an} = V_{an} - V'_{aa} = (120 + j0) - (10.67 + j1.27) = \mathbf{109.3 \quad 2.03^\circ}$$

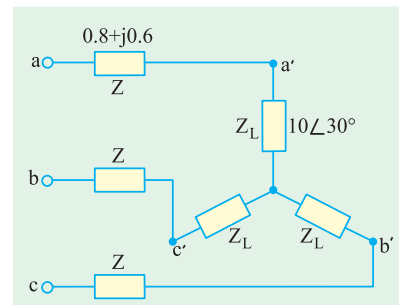


Fig. 19.40

**Example 19.32.** A balanced delta-connected load having an impedance  $Z_L = (300 + j210)$  ohm in each phase is supplied from 400-V, 3-phase supply through a 3-phase line having an impedance of  $Z_s = (4 + j8)$  ohm in each phase. Find the total power supplied to the load as well as the current and voltage in each phase of the load.

(Elect. Circuit Theory, Kerala Univ.)

**Solution.** The equivalent  $Y$ -load of the given  $\Delta$ -load is

$$= (300 + j210) / 3 = (100 + j70) \Omega$$

Hence, connections become as shown in Fig. 19.41

$$\mathbf{Z}_{a0} \quad (4 \quad j8) \quad (100 \quad j70) \quad 104 \quad j78 \quad 130 \quad 36.9$$

$$\mathbf{V}_{a0} \quad 400/\sqrt{3} \quad 231V,$$

$$\mathbf{I}_{a0} \quad 231 \quad 0 \quad /130 \quad 36.9 \quad 1.78 \quad 36.9$$

$$\text{Now, } \mathbf{Z}'_{a0} = (4 + j8) = 8.94 \angle 63.4^\circ$$

$$\text{Line drop } \mathbf{V}_{aa} \quad \mathbf{I}_{aa} \mathbf{Z}_{aa} \quad 1.78 \quad 36.9 \quad 8.94 \quad 63.4 \quad 15.9 \quad 26.5 \quad 14.2 \quad j7.1$$

$$\mathbf{V}_{a0} \quad \mathbf{V}_{a0} \quad \mathbf{V}_{aa} \quad (231 \quad j0) \quad (14.2 \quad j7.1)$$

$$= (216.8 - j7.1) = 216.9 \angle -1^\circ 52'$$

Phase voltage at load end,  $V_{a0} = 216.9 \text{ V}$

Phase current at load end,  $I_{a0} = 1.78 \text{ A}$

Power supplied to load =  $3 \times 1.782 \times 100 = 951 \text{ W}$

Incidentally, line voltage at load end  $V_{ac} = 216.9 \times \sqrt{3} = 375.7 \text{ V}^*$

**Example 19.33.** A star connected load having  $R = 42.6$  ohms/phase and  $X_L = 32$  ohms/phase is connected across 400 V, 3 phase supply, calculate:

- Line current, reactive power and power loss
- Line current when one of load becomes open circuited.

[Nagpur University, Summer 2001]

**Solution.**

$$(i) \quad \mathbf{Z} = 42.6 + j32$$

$$|\mathbf{Z}| = 53.28 \text{ ohms, Impedance angle, } \theta = \cos^{-1} \left( \frac{42.6}{53.28} \right) = \cos^{-1} 0.80$$

$$\theta = 36.9^\circ$$

Line Current = phase current, due to star-connection

$$= \frac{\text{Voltage/phase}}{\text{Impedance/phase}} = \frac{400/\sqrt{3}}{53.28} = 4.336 \text{ amp}$$

Due to the phase angle of  $36.9^\circ$  lagging,

Reactive Power for the three-phases

$$= 3 V_{ph} I_{ph} \sin \phi = 3 \times 231 \times 4.336 \times 0.6 = 1803 \text{ VAR}$$

$$\text{Total Power-loss} = 3 V_{ph} I_{ph} \cos \phi = 3 \times 231 \times 4.336 \times 0.8 = 2404 \text{ watts}$$

(ii) One of the Loads is open-circuited.

The circuit is shown in Fig. 19.42 (b).

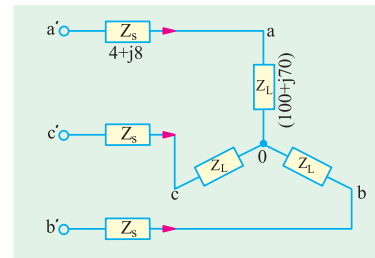


Fig. 19.41

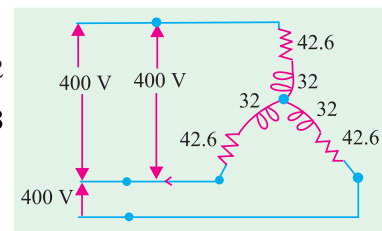


Fig. 19.42 (a)

\* It should be noted that total line drop is not the numerical sum of the individual line drops because they are  $120^\circ$  out of phase with each other. By a laborious process  $\mathbf{V}_{ac} = \mathbf{V}'_{ac} - \mathbf{V}'_{aa} - \mathbf{V}'_{ac}$ .

Between A and B, the Line voltage of 400 V drives a current through two “phase-impedances” in series.

Total Impedance between A and B =  $(42.6 + j 32) \times 2$  ohms

Hence, the line current I for the two Lines A and B

$$= \frac{400}{2 \times 53.25} = 3.754 \text{ amp}$$

**Note :** Third Line ‘C’ does not carry any current.

**Example 19.34.** Three non-inductive resistances, each of 100 ohms, are connected in star to a three-phase, 440-V supply. Three inductive coils, each of reactance 100 ohms connected in delta are also connected to the supply. Calculate: (i) Line-currents, and (ii) power factor of the system

[Nagpur University, November 1998]

**Solution. (a)** Three resistances are connected in star. Each resistance is of 100 ohms and 254 – V appears across it. Hence, a current of 2.54 A flows through the resistors and the concerned power-factor is unity. Due to star-connection,

Line-current = Phase-current = 2.54A

**(b)** Three inductive reactance are delta connected.

Line-Voltage = Phase – Voltage = 440 V

Phase Current =  $440/100 = 4.4$  A

Line current =  $1.732 \times 4.4 = 7.62$  A

The current has a zero lagging power-factor.

Total Line Current =  $2.54 - j 7.62$  A

= 8.032 A, in each of the lines.

Power factor =  $2.54/8.032 = 0.32$  Lag.

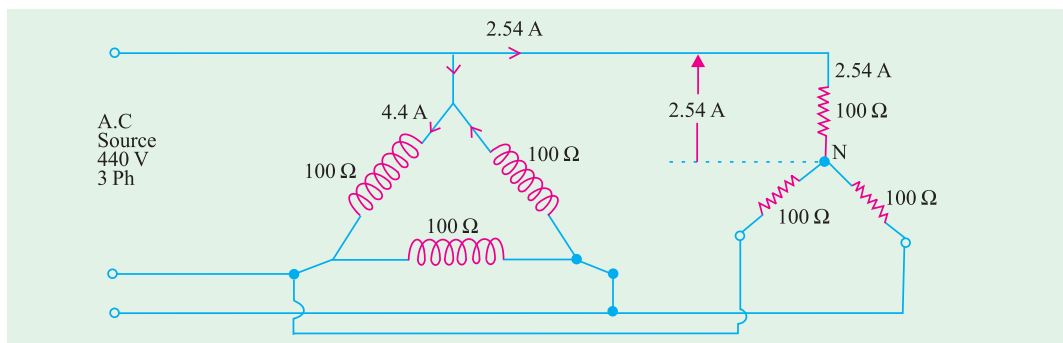


Fig. 19.43

**Example 19.35.** The delta-connected generator of Fig 19.44 has the voltage;  $V_{RY} = 220 \angle 0^\circ$ ,  $V_{YB} = 220 \angle -120^\circ$  and  $V_{BR} = 220 \angle -240^\circ$  Volts.

The load is balanced and delta-connected. Find:

(a) Impedance per phase, (b) Current per phase, (c) Other line – currents  $I_Y$  and  $I_B$

[Nagpur University, November 1997]

**Solution.** Draw phasors for voltages as mentioned in the data.  $V_{RY}$  naturally becomes a reference-phasor, along which the phasor  $I_R$  also must lie, as shown in Fig. 19.44 (b) & (c).  $I_R$  is the line voltage which is related to the phase-currents  $I_{RY}$  and  $-I_{BR}$ . In terms of magnitudes,

$$|I_{RY}| = |I_{BR}| = |I_R|/\sqrt{3} = 10/\sqrt{3} = 5.8 \text{ Amp}$$

Thus,  $I_{RY}$  leads  $V_{RY}$  by  $30^\circ$ . This can take place only with a series combination of a resistor and a capacitor, as the simplest impedance in each phase

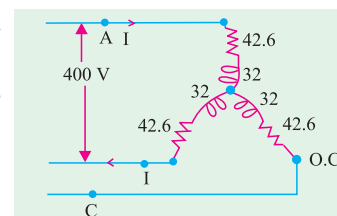


Fig. 19.42 (b) One phase open circuited

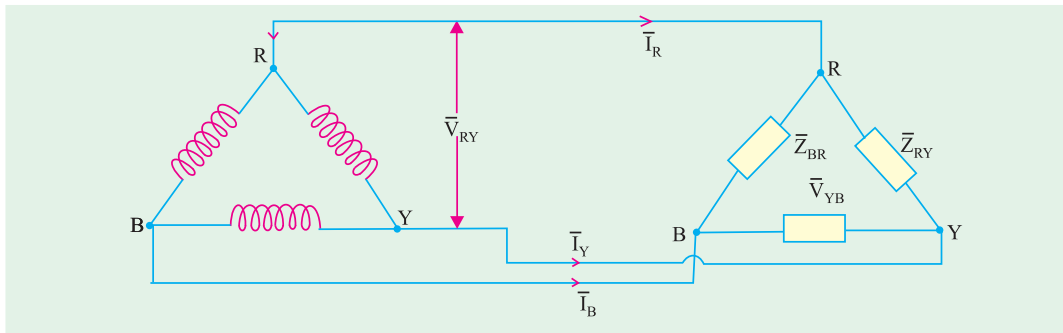


Fig 19.44 (a)

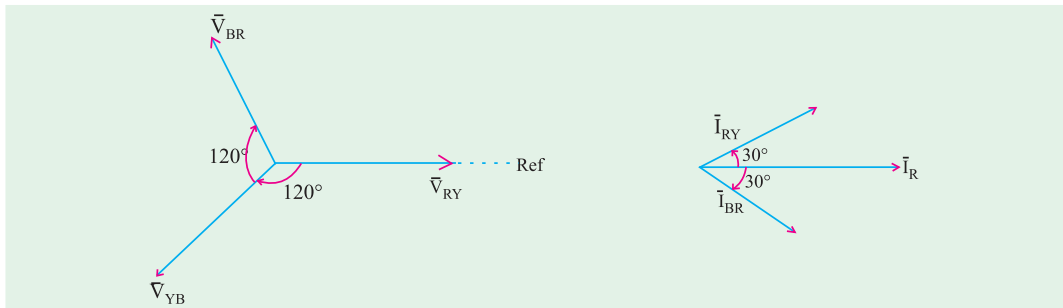


Fig. 19.44 (b)

Fig. 19.44 (c)

(a)  $|Z| = 220/5.8 = 38.1$  ohms

Resistance per phase =  $38.1 \times \cos 30^\circ = 33$  ohms

Capacitive Reactance/phase =  $38.1 \times \sin 30^\circ = 19.05$  ohms

(b) Current per phase = 5.8 amp, as calculated above.

(c) Otherline currents: Since a symmetrical three phase system is being dealt with, three currents have a mutual phase-difference of  $120^\circ$ . Hence

$I_R = 10 \angle 0^\circ$  as given,  $I_Y = 10 \angle -120^\circ$  amp;  $I_B = 10 \angle -240^\circ$  amp.

**Example 19.36.** A balanced 3-phase star-connected load of  $8 + j6$  ohms per phase is connected to a three-phase 230 V supply. Find the line-current, power-factor, active power, reactive-power, and total volt-amperes.

[Rajiv Gandhi Technical University, Bhopal, April 2001]

**Solution.** When a statement is made about three-phase voltage, when not mentioned otherwise, the voltage is the line-to-line voltage. Thus, 230 V is the line voltage, which means, in star-system, phase-voltage is  $230/1.732$ , which comes to 132.8 V.

$|Z| = \sqrt{8^2 + 6^2} = 10$  ohms

Line current = Phase current

=  $132.8/10 = 13.28$  amp

Power - factor =  $R/Z = 0.8$ , Lagging

Total Active Power =  $P = 1.732 \times$  Line Voltage  $\times$  Line Current  $\times$  P.f.

Or =  $3 \times$  Phase Voltage  $\times$  Phase-current  $\times$  P.f

=  $3 \times 132.8 \times 13.28 \times 0.8 = 4232$  watts

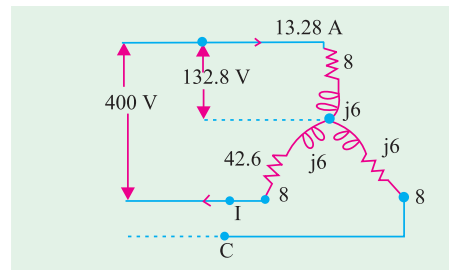


Fig. 19.45

Total Reactive Power =  $Q$

$$= 3 \times \text{Phase-voltage} \times \text{Phase-current} \times \sin \phi$$

$$= 3 \times 132.8 \times 13.28 \times 0.60 = 3174 \text{ VAR}$$

$$\text{Total Volt-amperes} = S = \sqrt{P^2 + Q^2} = 5290 \text{ VA}$$

$$\text{Or } S = \sqrt{3} \times 230 \times 13.28 = 5290 \text{ VA}$$

**Example 19.37.** A balanced three-phase star connected load of 100 kW takes a leading current of 80 amp, when connected across a three-phase 1100 V, 50 Hz, supply. Find the circuit constants of the load per phase. [Nagpur University, April 1996]

**Solution.** Voltage per phase =  $1100/1.732 = 635 \text{ V}$

Impedance =  $635/80 = 7.94 \text{ ohms}$ .

Due to the leading current, a capacitor exists.

Resistance  $R$  can be evaluated from current and power consumed

$$3 I^2 R = 100 \times 1000, \text{ giving } R = 5.21 \text{ ohms}$$

$$X_c = (7.94^2 - 5.21^2)^{0.5} = 6 \text{ ohms}$$

At 50 Hz,  $C = 1/(314 \times 6) = 531 \text{ microfarads}$ .

### Tutorial Problem No. 19.1

1. Each phase of a delta-connected load comprises a resistor of  $50 \Omega$  and capacitor of  $50 \mu\text{F}$  in series. Calculate (a) the line and phase currents (b) the total power and (c) the kilovoltamperes when the load is connected to a 440-V, 3-phase, 50-Hz supply. [(a) 9.46 A; 5.46 A (b) 4480 W (c) 7.24 kVA]

2. Three similar-coils,  $A$ ,  $B$  and  $C$  are available. Each coil has  $9 \Omega$  resistance and  $12 \Omega$  reactance. They are connected in delta to a 3-phase, 440-V, 50-Hz supply. Calculate for this load:

- (a) the line current (b) the power factor  
 (c) the total kilovolt-amperes (d) the total kilowatts

If the coils are reconnected in star, calculate for the new load the quantities named at (a), (b); (c) and (d) above. [50.7 A; 0.6; 38.6 kVA; 23.16 kW; 16.9 A; 0.6; 12.867 kVA; 7.72 kW]

3. Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.

[577.3 V; 333.3 V; 10.2 kVAR; 15.1  $\Omega$ ; 16.3  $\Omega$ ]

4. The load in each branch of a delta-connected balanced 3- $\phi$  circuit consists of an inductance of  $0.0318 \text{ H}$  in series with a resistance of  $10 \Omega$ . The line voltage is  $400 \text{ V}$  at  $50 \text{ Hz}$ . Calculate (i) the line current and (ii) the total power in the circuit. [(i) 49 A (ii) 24 kW] (London Univ.)

5. A 3-phase, delta-connected load, each phase of which has  $R = 10 \Omega$  and  $X = 8 \Omega$ , is supplied from a star-connected secondary winding of a 3-phase transformer each phase of which gives  $230 \text{ V}$ . Calculate

- (a) the current in each phase of the load and in the secondary windings of the transformer  
 (b) the total power taken by the load  
 (c) the power factor of the load. [(a) 31.1 A; 54 A (b) 29 kW (c) 0.78]

6. A 3-phase load consists of three similar inductive coils, each of resistance  $50 \Omega$  and inductance  $0.3 \text{ H}$ . The supply is  $415 \text{ V}$ ,  $50 \text{ Hz}$ , Calculate (a) the line current (b) the power factor and (c) the total power when the load is (i) star-connected and (ii) delta-connected.

[(i) 2.25 A, 0.47 lag, 762 W (ii) 6.75 A, 0.47 lag, 2280 W] (London Univ.)

7. Three  $20 \Omega$  non-inductive resistors are connected in star across a three phase supply the line voltage of which is  $480 \text{ V}$ . Three other equal non-inductive resistors are connected in delta across the same supply so as to take the same-line current. What are the resistance values of these other resistors and what is the current- flowing through each of them? [60  $\Omega$ ; 8A] (Sheffield Univ. U.K.)

**8.** A 415-V, 3-phase, 4-wire system supplies power to three non-inductive loads. The loads are 25 kW between red and neutral, 30 kW between yellow and neutral and 12 kW between blue and neutral. Calculate (a) the current in each-line wire and (b) the current in the neutral conductor.

**[(a) 104.2 A, 125 A, 50 A (b) 67 A] (London Univ.)**

**9.** Non-inductive loads of 10, 6 and 4 kW are connected between the neutral and the red, yellow and blue phases respectively of a three-phase, four-wire system. The line voltage is 400 V. Find the current in each line conductor and in the neutral.

**[(a) 43.3 A, 26A, 173. A, 22.9] (App. Elect. London Univ.)**

**10.** A three-phase, star-connected alternator supplies a delta-connected load, each phase of which has a resistance of  $20\ \Omega$  and a reactance of  $10\ \Omega$ . Calculate (a) the current supplied by the alternator (b) the output of the alternator in kW and kVA, neglecting the losses in the lines between the alternator and the load. The line voltage is 400 V.

**[(a) 30.95 A (b) 19.2 kW, 21.45 kVA]**

**11.** Three non-inductive resistances, each of  $100\ \Omega$ , are connected in star to 3-phase, 440-V supply. Three equal choking coils each of reactance  $100\ \Omega$  are also connected in delta to the same supply. Calculate:

(a) line current (b) p.f. of the system. **[(a) 8.04 A (b) 0.3156] (I.E.E. London)**

**12.** In a 3-phase, 4-wire system, there is a balanced 3-phase motor load taking 200 kW at a power factor of 0.8 lagging, while lamps connected between phase conductors and the neutral take 50, 70 and 100 kW respectively. The voltage between phase conductors is 430 V. Calculate the current in each phase and in the neutral wire of the feeder supplying the load.

**[512 A, 5.87 A, 699 A; 213.3 A] (Elect. Power; London Univ.)**

**13.** A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three  $\Delta$ -connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor.

**[11.45 kVA, 62.7  $\mu$ F] (I.E.E. London)**

**14.** Three resistances, each of  $500\ \Omega$  are connected in star to a 400-V, 50-Hz, 3-phase supply. If three capacitors, when connected in delta to the same supply, take the same line currents, calculate the capacitance of each capacitor and the line current.

**[2.123  $\mu$  F, 0.653 A] (London Univ.)**

**15.** A factory takes the following balanced loads from a 440-V, 3-phase, 50-Hz supply:

(a) a lighting load of 20 kW (b) a continuous motor load of 30 kVA at 0.5 p.f. lagging.  
(c) an intermittent welding load of 30 kVA at 0.5 p.f. lagging.

Calculate the kVA rating of the capacitor bank required to improve the power factor of loads (a) and (b) together to unity. Give also the value of capacitor required in each phase if a star-connected bank is employed.

What is the new overall p.f. if, after correction has been applied, the welding load is switched on.

**[30 kVAR; 490 $\mu$  F; 0.945 kg]**

**16.** A three-wire, three-phase system, with 400 V between the line wires, supplies a balanced delta-connected load taking a total power of 30 kW at 0.8 power factor lagging. Calculate (i) the resistance and (ii) the reactance of each branch of the load and sketch a vector diagram showing the line voltages and line currents. If the power factor of the system is to be raised to 0.95 lagging by means of three delta-connected capacitors, calculate (iii) the capacitance of each branch assuming the supply frequency to be 50 Hz.

**[(i) 10.24 A (ii) 7.68  $\Omega$  (iii) 83.2  $\mu$ F] (London Univ.)**

### 19.15. Power Measurement in 3-phase Circuits

Following methods are available for measuring power in a 3-phase load.

#### (a) Three Wattmeter Method

In this method, three wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.

#### (b) Two Wattmeter Method

(i) This method gives true power in the 3-phase circuit without regard to balance or wave form provided in the case of Y-connected load. The neutral of the load is isolated from the neutral of the source of power. Or if there is a neutral connection, the neutral wire should not carry any current. This is possible only if the load is perfectly balanced and there are no harmonics present of triple frequency or any other multiples of that frequency.



- (ii) This method can also be used for 3-phase, 4-wire system in which the neutral wire carries the neutral current. In this method, the current coils of the wattmeters are supplied from current transformers inserted in the principal line wires in order to get the correct magnitude and phase differences of the currents in the current coils of the wattmeter, because in the 3-phase, 4-wire system, the sum of the instantaneous currents in the principal line wires is not necessarily equal to zero as in 3-phase 3-wire system.

### (c) One Wattmeter Method

In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used only when the load is balanced.

## 19.16. Three Wattmeter Method

A wattmeter consists of (i) a low resistance current coil which is inserted in series with the line carrying the current and (ii) a high resistance pressure coil which is connected across the two points whose potential difference is to be measured.

A wattmeter shows a reading which is proportional to the product of the current through its current coil, the p.d. across its potential or pressure coil and cosine of the angle between this voltage and current.

As shown in Fig. 19.46 in this method three wattmeters are inserted in each of the three phases of the load whether  $\Delta$ -connected or  $Y$ -connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase-voltage of this phase. Hence, each wattmeter measures the power in a single phase. The algebraic sum of the readings of three wattmeters must give the total power in the load.

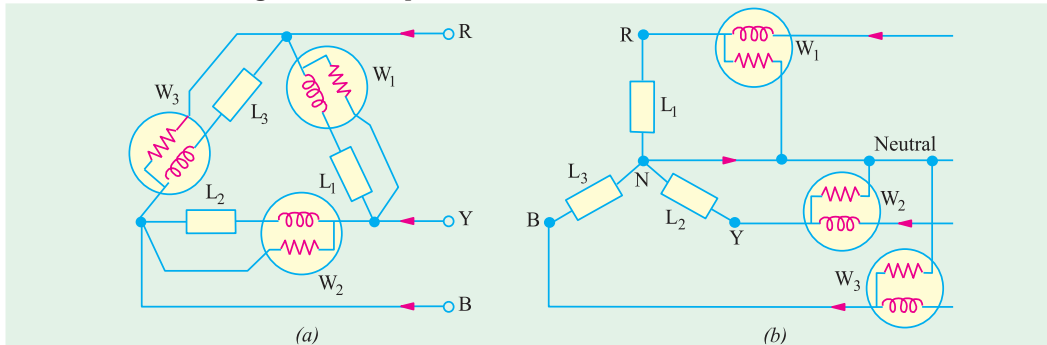


Fig. 19.46

The difficulty with this method is that under ordinary conditions it is not generally feasible to break into the phases of a delta-connected load nor is it always possible, in the case of a  $Y$ -connected load, to get at the neutral point which is required for connections as shown in Fig. 19.47 (b). However, it is not necessary to use three wattmeters to measure power, two wattmeters can be used for the purpose as shown below.

## 19.17. Two Wattmeter Method-Balanced or Unbalanced Load

As shown in Fig. 19.41, the current coils of the two wattmeters are inserted in *any two* lines and the potential coil of each joined to the third line. It can be proved that the sum of the instantaneous powers indicated by  $W_1$  and  $W_2$  gives the instantaneous power absorbed by the three loads  $L_1$ ,  $L_2$  and  $L_3$ . A star-connected load is considered in the following discussion although it can be equally applied to  $\Delta$ -connected loads because a  $\Delta$ -connected load can always be replaced by an equivalent  $Y$ -connected load.

Now, before we consider the currents through and p.d. across each wattmeter, it may be pointed out that *it is important to take the direction of the voltage through the circuit the same as that taken for the current when establishing the readings of the two wattmeters.*

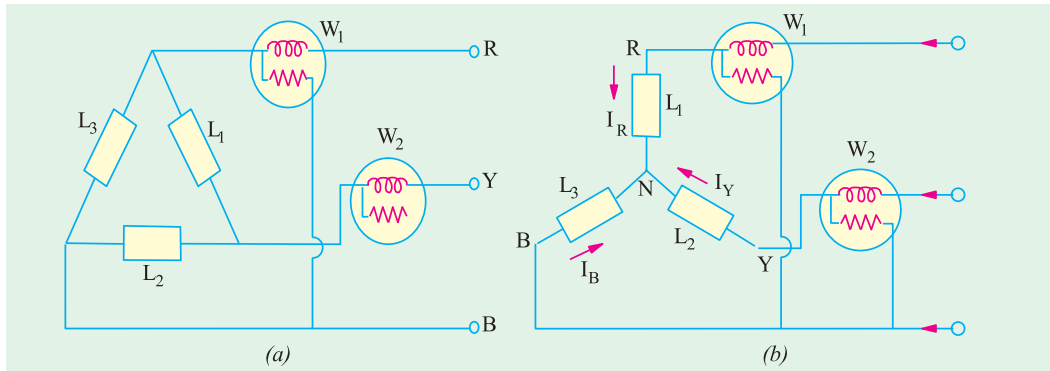


Fig. 19.47

**Instantaneous** current through  $W_1 = i_R$

p.d. across  $W_1 = e_{RB} = e_R - e_B$

p.d. across power read by  $W_1 = i_R (e_R - e_B)$

**Instantaneous** current through  $W_2 = i_Y$

**Instantaneous** p.d. across  $W_2 = e_{YB} = (e_Y - e_B)$

**Instantaneous** power read by  $W_2 = i_Y (e_Y - e_B)$

$$\therefore W_1 + W_2 = e_R(e_R - e_B) + i_Y(e_Y - e_B) = i_R e_R + i_Y e_Y - e_B(i_R + i_Y)$$

Now,  $i_R + i_Y + i_B = 0$  ... Kirchhoff's Current Law

$$\therefore i_R + i_Y = -i_B$$

$$\text{or } W_1 + W_2 = i_R \cdot e_R + i_Y \cdot e_Y + i_B \cdot e_B = p_1 + p_2 + p_3$$

where  $p_1$  is the power absorbed by load  $L_1$ ,  $p_2$  that absorbed by  $L_2$  and  $p_3$  that absorbed by  $L_3$

$$\therefore W_1 + W_2 = \text{total power absorbed}$$

The proof is true whether the load is balanced or unbalanced. If the load is Y-connected, it should have no neutral connection (i.e. 3- $\phi$ , 3-wire connected) and if it has a neutral connection (i.e. 3- $\phi$ , 4-wire connected) then it should be exactly balanced so that in each case there is no neutral current  $i_N$  otherwise Kirchhoff's current law will give  $i_N + i_R + i_Y + i_B = 0$ .

We have considered **instantaneous** readings, but in fact, the moving system of the wattmeter, due to its inertia, cannot quickly follow the variations taking place in a cycle, hence it indicates the **average** power.

$$\therefore W_1 = \frac{1}{T} \int_0^T i_R e_{RB} dt \quad W_2 = \frac{1}{T} \int_0^T i_Y e_{YB} dt$$

### 19.18. Two Wattmeter Method—Balanced Load

If the load is balanced, then power factor of the load can also be found from the two wattmeter readings. The Y-connected load in Fig. 19.47 (b) will be assumed inductive. The vector diagram for such a balanced Y-connected load is shown in Fig. 19.48. We will now consider the problem in terms of r.m.s. values instead of instantaneous values.

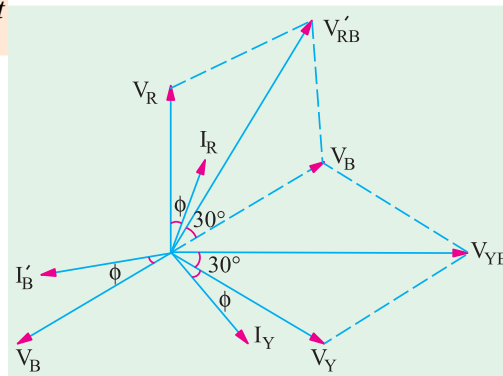


Fig. 19.48

Let  $V_R$ ,  $V_Y$  and  $V_B$  be the r.m.s. values of the three phase voltages and  $I_R$ ,  $I_Y$  and  $I_B$  the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by  $\phi$ .

Current through wattmeter  $W_1$  [Fig. 19.47 (b)] is  $I_R$

P.D. across voltage coil of  $W_1$  is

$$V_{RB} = V_R - V_B \quad \dots \text{vectorially}$$

This  $V_{RB}$  is found by compounding  $V_R$  and  $V_B$  reversed as shown in Fig. 19.42. It is seen that phase difference between  $V_{RB}$  and  $I_R = (30^\circ - \phi)$ .

$$\therefore \text{Reading of } W_1 = I_R V_{RB} \cos (30^\circ - \phi)$$

Similarly, as seen from Fig. 19.47 (b). Current through  $W_2 = I_Y$

$$\text{P.D. across } W_2 = V_{YB} = V_Y - V_B \quad \dots \text{vectorially}$$

Again,  $V_{YB}$  is found by compounding  $V_Y$  and  $V_B$  reversed as shown in Fig. 19.48. The angle between  $I_Y$  and  $V_{YB}$  is  $(30^\circ + \phi)$ . Reading of  $W_2 = I_Y V_{YB} \cos (30^\circ + \phi)$

Since load is balanced,  $V_{RB} = V_{YB} = \text{line voltage } V_L$ ;  $I_Y = I_R = \text{line current, } I_L$

$$\therefore W_1 = V_L I_L \cos (30^\circ - \phi) \text{ and } W_2 = V_L I_L \cos (30^\circ + \phi)$$

$$\therefore W_1 + W_2 = V_L I_L \cos (30^\circ - \phi) + V_L I_L \cos (30^\circ + \phi)$$

$$= V_L I_L [\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi + \cos 30^\circ \cos \phi - \sin 30^\circ \sin \phi]$$

$$= V_L I_L (2 \cos 30^\circ \cos \phi) = \sqrt{3} V_L I_L \cos \phi = \text{total power in the 3-phase load}$$

**Hence, the sum of the two wattmeter readings gives the total power consumption in the 3-phase load.**

It should be noted that phase sequence of  $RYB$  has been assumed in the above discussion. Reversal of phase sequence will interchange the readings of the two wattmeters.

### 19.19. Variations in Wattmeter Readings

It has been shown above that for a lagging power factor

$$W_1 = V_L I_L \cos (30^\circ - \phi) \text{ and } W_2 = V_L I_L \cos (30^\circ + \phi)$$

From this it is clear that individual readings of the wattmeters not only depend **on the load but upon its power factor also**. We will consider the following cases:

(a) When  $\phi = 0$  i.e. power factor is unity (i.e. resistive load) then,

$$W_1 = W_2 = V_L I_L \cos 30^\circ$$

Both wattmeters indicate equal and positive i.e. up-scale readings.

(b) When  $\phi = 60^\circ$  i.e. power factor = 0.5 (lagging)

Then  $W_2 = V_L I_L \cos (30^\circ + 60^\circ) = 0$ . Hence, the power is measured by  $W_1$  alone.

(c) When  $90^\circ > \phi > 60^\circ$  i.e.  $0.5 > \text{p.f.} > 0$ , then  $W_1$  is still positive but reading of  $W_2$  is reversed because the phase angle between the current and voltage is more than  $90^\circ$ . For getting the total power, the reading of  $W_2$  is to be subtracted from that of  $W_1$ .

Under this condition,  $W_2$  will read 'down scale' i.e. backwards. Hence, to obtain a reading on  $W_2$  it is necessary

$\phi$	$0^\circ$	$60^\circ$	$90^\circ$
$\cos \phi$	1	0.5	0
$W_1$	+ve	+ve	+ve
$W_2$	+ve	0	-ve
	$W_1 = W_2$		$W_1 = W_2$

to reverse either its pressure coil or current coil, usually the

*All readings taken after reversal of pressure coil are to be taken as negative.*

(d) When  $\phi = 90^\circ$  (i.e. pure inductive or capacitive load), then

$$W_1 = V_L I_L \cos(30^\circ - 90^\circ) = V_L I_L \sin 30^\circ;$$

$$W_2 = V_L I_L \cos(30^\circ + 90^\circ) = -V_L I_L \sin 30^\circ$$

As seen, the two readings are equal but of opposite sign.

$$\therefore W_1 + W_2 = 0$$

The above facts have been summarised in the above table for a lagging power factor.

### 19.20. Leading Power Factor\*

In the above discussion, lagging angles are taken positive. Now, we will see how wattmeter readings are changed if the power factor becomes leading. For  $\phi = +60^\circ$  (lag),  $W_2$  is zero. But for  $\phi = -60^\circ$  (lead),  $W_1$  is zero. So we find that for angles of lead, the reading of the two wattmeters are interchanged. Hence, for a *leading* power factor.

$$W_1 = V_L I_L \cos(30^\circ + \phi) \quad \text{and} \quad W_2 = V_L I_L \cos(30^\circ - \phi)$$

### 19.21. Power Factor–Balanced Load

In case the load is balanced (and currents and voltages are sinusoidal) and for a *lagging* power factor:

$$W_1 + W_2 = V_L I_L \cos(30^\circ - \phi) + V_L I_L \cos(30^\circ + \phi) = \sqrt{3} V_L I_L \cos \phi \quad \dots (i)$$

$$\text{Similarly } W_1 - W_2 = V_L I_L \cos(30^\circ - \phi) - V_L I_L \cos(30^\circ + \phi)$$

$$= V_L I_L (2 \times \sin \phi \times 1/2) = V_L I_L \sin \phi \quad \dots (ii)$$

$$\text{Dividing (ii) by (i), we have } \tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \quad \dots (iii)$$

Knowing  $\tan \phi$  and hence  $\phi$ , the value of power factor  $\cos \phi$  can be found by consulting the trigonometrical tables. It should, however, be kept in mind that if  $W_2$  reading has been taken after reversing the pressure coil i.e. if  $W_2$  is negative, then the above relation becomes

$$\tan \phi = -\sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \quad \dots \text{ Art 19.22}$$

$$\tan \phi = \sqrt{3} \frac{W_1 - (-W_2)}{W_1 + (-W_2)} = \sqrt{3} \frac{W_1 + W_2}{W_1 - W_2}$$

Obviously, in this expression, only *numerical* values of  $W_1$  and  $W_2$  should be substituted. We may express power factor in terms of the ratio of the two wattmeters as under:

$$\text{Let } \frac{\text{smaller reading}}{\text{larger reading}} = \frac{W_2}{W_1} = r$$

\* For a leading p.f., conditions are just the opposite of this. In that case,  $W_1$  reads negative (Art. 19.22).

\*\* For a leading power factor, this expression becomes

$$\tan \phi = -\sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \quad \dots \text{ Art 19.22}$$

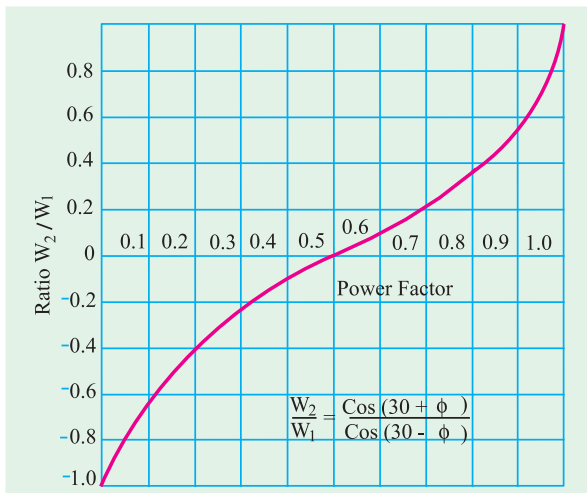


Fig. 19.49

If  $r$  is plotted against  $\cos \phi$ , then a curve called watt-ratio curve is obtained as shown in Fig. 19.49.

Then from equation (iii) above,

$$\tan \phi = \frac{\sqrt{3}[1 - (W_2 / W_1)]}{1 + (W_2 / W_1)} = \frac{\sqrt{3}(1-r)}{1+r}$$

$$\text{Now } \sec^2 \phi = 1 + \tan^2 \phi$$

$$\text{or } \frac{1}{\cos^2 \phi} = 1 + \tan^2 \phi$$

$$\therefore \cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}}$$

$$= \frac{1}{\sqrt{1 + 3 \left( \frac{1-r}{1+r} \right)^2}}$$

$$= \frac{1+r}{2\sqrt{1-r+r^2}}$$

### 19.22. Balanced Load – leading power factor

In this case, as seen from Fig. 19.50

$$W_1 = V_L I_L \cos(30 + \phi)$$

$$\text{and } W_2 = V_L I_L \cos(30 - \phi)$$

$$\therefore W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi - \text{as found above}$$

$$W_1 - W_2 = -V_L I_L \sin \phi$$

$$\therefore \tan \phi = -\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

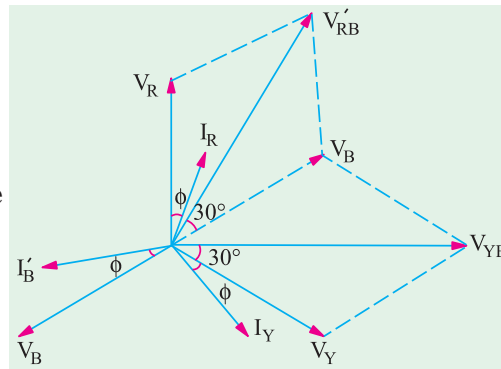


Fig. 19.50

Obviously, if  $\phi > 60^\circ$ , then phase angle between  $V_{RB}$  and  $I_R$  becomes more than  $90^\circ$ . Hence,  $W_1$  reads 'down-scale' *i.e.* it indicates negative reading. However,  $W_2$  gives positive reading even in the extreme case when  $\phi = 90^\circ$ .

### 19.23. Reactive Voltamperes with Two Wattmeters

$$\text{We have seen that } \tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

Since the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of the reactive power to the active power (in watts), it is clear that  $\sqrt{3}(W_1 - W_2)$  represents the reactive power (Fig. 19.51). Hence, for a balanced load, the reactive power is given by  $\sqrt{3}$  times the difference of the readings of the two wattmeters used to measure the power for a 3-phase circuit by the two wattmeter method. It may also be proved mathematically as follows:

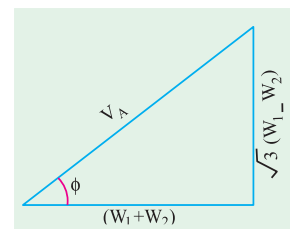


Fig. 19.51

$$\begin{aligned}
 &= \sqrt{3}(W_1 - W_2) = \sqrt{3}[V_L I_L \cos(30^\circ - \phi) - V_L I_L \cos(30^\circ + \phi)] \\
 &= \sqrt{3}V_L I_L (\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi - \cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi) \\
 &= \sqrt{3}V_L I_L \sin \phi
 \end{aligned}$$

### 19.24. Reactive Voltamperes with One Wattmeter

For this purpose, the wattmeter is connected as shown in Fig. 19.52 (a) and (b). The pressure coil is connected across Y and B lines whereas the current coil is included in the R line. In Fig.

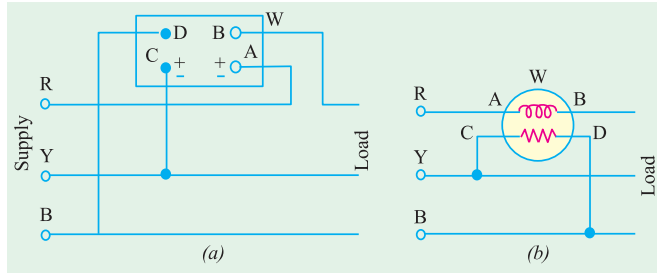


Fig. 19.52

19.48 (a), the current coil is connected between terminals A and B whereas pressure coil is connected between terminals C and D. Obviously, current flowing through the wattmeter is  $I_R$  and p.d. is  $V_{YB}$ . The angle between the two, as seen from vector diagram of Fig. 19.48, is  $(30 + 30 + 30 - \phi) = (90 - \phi)$

Hence, reading of the wattmeter is  $W = V_{YB} I_R \cos(90 - \phi) = V_{YB} I_R \sin \phi$

For a balanced load,  $V_{YB}$  equals the line voltage  $V_L$  and  $I_R$  equals the line current  $I_L$ , hence

$$W = V_L I_L \sin \phi$$

We know that the total reactive voltamperes of the load are  $Q = \sqrt{3}V_L I_L \sin \phi$ .

Hence, to obtain total VARs, the wattmeter reading must be multiplied by a factor of  $\sqrt{3}$ .

### 19.25. One Wattmeter Method

In this case, it is possible to apply two-wattmeter method by means of one wattmeter without breaking the circuit. The current coil is connected in any one line and the pressure coil is

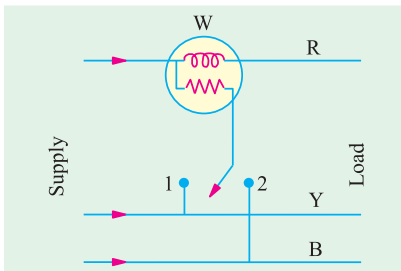


Fig. 19.53

connected alternately between this and the other two lines (Fig. 19.53). The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method. It should be kept in mind that this method is not of as much universal application as the two wattmeter method because it is restricted to fairly balanced loads only. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load upon the motor.

It may be pointed out here that the two wattmeters used in the two-wattmeter method (Art. 19.17) are usually combined into a single instrument in the case of switchboard wattmeter which is then known as a polyphase wattmeter. The combination is affected by arranging the two sets of coils in such a way as to operate on a single moving system resulting in an indication of the total power on the scale.

### 19.26. Copper Required for Transmitting Power under Fixed Conditions

The comparison between 3-phase and single-phase systems will be done on the basis of a fixed amount of power transmitted to a fixed distance with the same amount of loss and at the same maximum voltage between conductors. In both cases, the weight of copper will be directly

proportional to the number of wires (since the distance is fixed) and inversely proportional to the resistance of each wire. We will assume the same power factor and same voltage.

where

$$P_1 = VI_1 \cos \phi \text{ and } P_3 = \sqrt{3}VI_3 \cos \phi$$

$$I_1 = \text{r.m.s. value of current in 1-phase system}$$

$$I_3 = \text{r.m.s. value of line current in 3-phase system}$$

$$P_1 = P_2 \therefore VI_1 \cos \phi = \sqrt{3}VI_3 \cos \phi \therefore I_1 = \sqrt{3}I_3$$

also  $I_1^2 R_1 \times 2 = I_3^2 R_3 \times 3$  or  $\frac{R_1}{R_3} = \frac{3I_3^2}{2I_1^2}$

Substituting the value of  $I_1$ , we get  $\frac{R_1}{R_3} = \frac{3I_3^2}{3I_3^2 \times 2} = \frac{1}{2}$

$\therefore \frac{\text{copper 3-phase}}{\text{copper 1-phase}} = \frac{\text{No. of wires 3-phase}}{\text{No. of wires 1-phase}} \times \frac{R_1}{R_3} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

Hence, we find that for transmitting the same amount of power over a fixed distance with a fixed line loss, we need only three-fourths of the amount of copper that would be required for a single phase or to put it in another way, one-third more copper is required for a 1-phase system than would be necessary for a three-phase system.

**Example 19.38.** Phase voltage and current of a star-connected inductive load is 150 V and 25 A. Power factor of load is 0.707 (lag). Assuming that the system is 3-wire and power is measured using two wattmeters, find the readings of wattmeters.

(Elect. Instrument & Measurements, Nagpur Univ. 1993)

**Solution.**  $V_{ph} = 150\text{V}; V_L = 150 \sqrt{3}\text{V}; I_{ph} = I_L = 25 \text{ A}$

Total power =  $\sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707 = 7954 \text{ W}$

$\therefore W_1 + W_2 = 7954 \text{ W} \dots (i)$

$\cos \phi = 0.707; \phi = \cos^{-1}(0.707) = 45^\circ; \tan 45^\circ = 1$

Now, for a lagging power factor,  $\tan \phi = \sqrt{3} (W_1 - W_2) / (W_1 + W_2)$  or  $1 = \sqrt{3} (W_1 - W_2) / 7954$

$\therefore (W_1 - W_2) = 4592 \text{ W} \dots (ii)$

From (i) and (ii) above, we get,  $W_1 = 6273 \text{ W}; W_2 = 1681 \text{ W}.$

**Example 19.39.** In a balanced 3-phase 400-V circuit, the line current is 115.5 A. When power is measured by two wattmeter method, one meter reads 40 kW and the other zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?

**Solution.** Since  $W_2 = 0$ , the whole power is measured by  $W_1$ . As per Art. 19.18, in such a situation, p.f. = 0.5. However, it can be calculated as under.

Since total power is 40 kW,  $\therefore 40,000 = \sqrt{3} \times 400 \times 115.5 \cos \phi$ ;  $\cos \phi = 0.5$

If the power factor is unity with line currents remaining the same, we have

$$\tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = 0 \text{ or } W_1 = W_2$$

Also,  $(W_1 + W_2) = \sqrt{3} \times 400 \times 115.5 \times 1 = 80000 \text{ W} = 80 \text{ kW}$

As per Art. 19.19, at unity p.f.,  $W_1 = W_2$ . Hence, each wattmeter reads =  $80/2 = 40 \text{ kW}.$

**Example 19.40.** The input power to a three-phase motor was measured by two wattmeter method. The readings were 10.4 KW and - 3.4 KW and the voltage was 400 V. Calculate (a) the power factor (b) the line current.

(Elect. Engg. A.M.Ae, S.I. June 1991)

**Solution.** As given in Art. 19.21, when  $W_2$  reads negative, then we have

$\tan \phi = \sqrt{3}(W_1 + W_2) / (W_1 - W_2)$ . Substituting numerical values of  $W_1$  and  $W_2$ , we get

$$\tan \phi = \sqrt{3}(10.4 + 3.4) / (10.4 - 3.4) = 1.97; \phi = \tan^{-1}(1.97) = 63.1^\circ$$

$$(a) \text{ p.f.} = \cos \phi = \cos 63.1^\circ = \mathbf{0.45 \text{ (lag)}}$$

$$(b) W = 10.4 - 3.4 = 7 \text{ KW} = 7,000 \text{ W}$$

$$7000 = \sqrt{3}I_L \times 400 \times 0.45; I_L = \mathbf{22.4 \text{ A}}$$

**Example 19.41.** A three-phase, three-wire, 100-V, ABC system supplies a balanced delta connected load with impedance of  $20 \angle 45^\circ$  ohm.

(a) Determine the phase and line currents and draw the phase or diagram (b) Find the wattmeter readings when the two wattmeter method is applied to the system.

(Elect. Machines, A.M.I.E. Sec B.)

**Solution.** (a) The phasor diagram is shown in Fig. 19.54 (b).

Let  $V_{AB} = 100 \angle 0^\circ$ . Since phase sequence is ABC,  $V_{BC} = 100 \angle -120^\circ$  and  $V_{CA} = 100 \angle 120^\circ$

$$\text{Phase current } I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{100 \angle 0^\circ}{20 \angle 45^\circ} = 5 \angle -45^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100 \angle -120^\circ}{20 \angle 45^\circ} = 5 \angle -165^\circ, I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100 \angle 120^\circ}{20 \angle 45^\circ} = 5 \angle 75^\circ$$

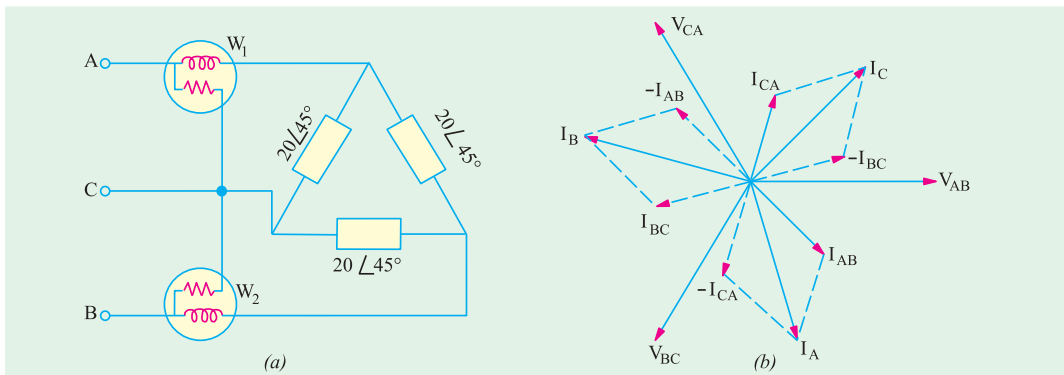


Fig. 19.54

Applying KCL to junction A, we have

$$I_A + I_{CA} - I_{AB} = 0 \text{ or } I_A = I_{AB} - I_{CA}$$

$$\therefore \text{Line current } I_A = 5 \angle -45^\circ - 5 \angle 75^\circ = 8.66 \angle -75^\circ$$

Since the system is balanced,  $I_B$  will lag  $I_A$  by  $120^\circ$  and  $I_C$  will lag  $I_A$  by  $240^\circ$ .

$$\therefore I_B = 8.66 \angle (75^\circ - 120^\circ) = 8.66 \angle -45^\circ; I_C = 8.66 \angle (-75^\circ - 240^\circ) = 8.66 \angle -315^\circ = 8.66 \angle 45^\circ$$

(b) As shown in Fig. 19.54 (b), reading of wattmeter  $W_1$  is  $W_1 = V_{AC} I_C \cos \phi$ . Phasor  $V_{AC}$  is the reverse of phasor  $V_{CA}$ . Hence,  $V_{AC}$  is the reverse of phasor  $V_{CA}$ . Hence,  $V_{AC}$  lags the reference vector by  $60^\circ$  whereas  $I_A$  lags by  $75^\circ$ . Hence, phase difference between the two is  $(75^\circ - 60^\circ) = 15^\circ$

$$\therefore W_1 = 100 \times 8.66 \times \cos 15^\circ = 836.5 \text{ W}$$

$$\text{Similarly } W_2 = V_{BC} I_B \cos \phi = 100 \times 8.66 \times \cos 75^\circ = 224.1 \text{ W}$$

$$\therefore W_1 + W_2 = 836.5 + 224.1 = 1060.6 \text{ W}$$

Resistance of each delta branch =  $20 \cos 45^\circ = 14.14 \Omega$

Total power consumed =  $3 I^2 R = 3 \times 5^2 \times 14.14 = 1060.6 \text{ W}$

Hence, it proves that the sum of the two wattmeter readings gives the total power consumed.

**Example 19.42.** A 3-phase, 500-V motor load has a power factor of 0.4. Two wattmeters connected to measure the power show the input to be 30 kW. Find the reading on each instrument.

(Electrical Meas., Nagpur Univ. 1991)



**Solution.** As seen from Art. 19.21

$$\tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \dots (i)$$

Now,  $\cos \phi = 0.4$ ;  $\phi = \cos^{-1}(0.4) = 66.6^\circ$ ;  $\tan 66.6^\circ = 2.311$

$$W_1 + W_2 = 30 \quad \dots (ii)$$

Substituting these values in equation (i) above, we get

$$2.311 = \frac{\sqrt{3}(W_1 - W_2)}{30} \quad \therefore W_1 - W_2 = 40 \quad \dots (iii)$$

From Eq. (ii) and (iii), we have  $W_1 = 45 \text{ kW}$  and  $W_2 = -5 \text{ kW}$

Since  $W_2$  comes out to be negative, second wattmeter reads 'down scale'. Even otherwise it is obvious that p.f. being less than 0.5,  $W_2$  must be negative (Art. 19.19)

**Example 19.43.** The power in a 3-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter? Give the connection diagram for the wattmeter circuit. For what p.f. will one of the wattmeter read zero?

**Solution.**  $\phi = \cos^{-1}(0.66) = 48.7^\circ$ ;  $\tan \phi = 1.1383$

Since p.f. is leading,

$$\therefore \tan \phi = -\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \therefore 1.1383 = \sqrt{3}(W_1 - W_2)/100$$

$$\therefore W_1 - W_2 = -65.7 \text{ and } W_1 + W_2 = 100 \quad \therefore W_1 = 17.14 \text{ kW}; W_2 = 82.85 \text{ kW}$$

Connection diagram is similar to that shown in Fig. 19.47 (b). One of the wattmeters will read zero when p.f. = 0.5

**Example 19.44.** Two wattmeters are used for measuring the power input and the power factor of an over-excited synchronous motor. If the readings of the meters are (-2.0 kW) and (+7.0 kW) respectively, calculate the input and power factor of the motor.

(Elect. Technology, Punjab Univ., June, 1991)

**Solution.** Since an over-excited synchronous motor runs with a leading p.f., we should use the relationship derived in Art. 19.22.

$$\tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

Moreover, as explained in the same article, it is  $W_1$  that gives negative reading and not  $W_2$ .

Hence,

$$W_1 = -2 \text{ kW}$$

$$\therefore \tan \phi = -\frac{\sqrt{3}(-2-7)}{-2+7} = \sqrt{3} \times \frac{9}{5} = 3.1176$$

$$\therefore \phi = \tan^{-1}(3.1176) = 71.2^\circ \text{ (lead)}$$

$$\therefore \cos \phi = \cos 71.2^\circ = 0.3057 \text{ (lead) and}$$

$$\text{Input} = W_1 + W_2 = -2 + 7 = 5 \text{ kW}$$

**Example 19.45.** A 440-V, 3-phase, delta-connected induction motor has an output of 14.92 kW at a p.f. of 0.82 and efficiency 85%. Calculate the readings on each of the two wattmeters connected to measure the input. Prove any formula used.

If another star-connected load of 10 kW at 0.85 p.f. lagging is added in parallel to the motor, what will be the current draw from the line and the power taken from the line?

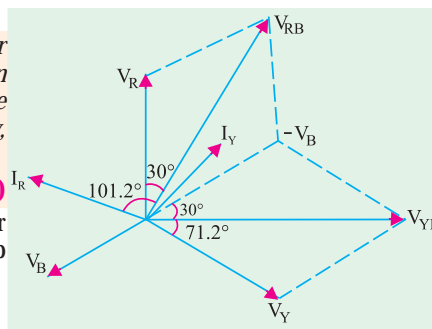


Fig. 19.55

(Elect. Technology-I, Bombay Univ.)

**Solution.** Motor input =  $14,920/0.85 = 17,600 \text{ W} \therefore W_1 + W_2 = 17.6 \text{ kW} \dots (i)$

$$\cos \phi = 0.82; \phi = 34.9^\circ, \tan 34.9^\circ = 0.6976; 0.6976 = \sqrt{3} \frac{W_1 - W_2}{17.6}$$

$$\therefore W_1 - W_2 = 7.09 \text{ kW} \dots (ii)$$

From (i) and (ii) above, we get  $W_1 = 12.35 \text{ kW}$  and  $W_2 = 5.26 \text{ kW}$

$$\text{Motor kVA, } S_m = \frac{\text{motor kW}}{\cos \phi_m} = \frac{17.6}{0.82} = 21.46 \therefore S_m = 21.46 \angle -34.9^\circ = (17.6 - j12.28) \text{ kVA}$$

$$\text{Load p.f.} = 0.85 \therefore \phi = \cos^{-1}(0.85) = 31.8^\circ; \text{ Load kVA, } S_Y = 10/0.85 = 11.76$$

$$\therefore S_Y = 11.76 \angle -31.8^\circ = (10 - j6.2) \text{ kVA}$$

$$\text{Combined kVA, } S = S_m + S_Y = (27.6 - j18.48) = 32.2 \angle -33.8^\circ \text{ kVA}$$

$$I = \frac{S}{\sqrt{3} \cdot V} = \frac{33.2 \times 10^3}{\sqrt{3} \times 440} = 43.56 \text{ A}$$

$$\text{Power taken} = 27.6 \text{ kW}$$

**Example 19.46.** The power input to a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the power factor of the motor be changed to 0.866 leading, determine the readings of the two wattmeters, the total input power remaining the same. Draw the vector diagram for the second condition of the load. (Elect. Technology, Nagpur Univ. 1992)

**Solution.** In the first case both wattmeters read equal and positive. Hence motor must be running at unity power (Art. 19.22).

**When p.f. is 0.866 leading**

$$\text{In this case; } W_1 = V_L I_L \cos(30^\circ + \phi);$$

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$\therefore W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$$

$$W_1 - W_2 = -V_L I_L \sin \phi$$

$$\therefore \tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \cos^{-1}(0.866) = 30^\circ$$

$$\tan \phi = 1/\sqrt{3}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{-\sqrt{3}(W_1 - W_2)}{100}$$

$$\therefore W_1 - W_2 = -100/3$$

$$\text{and } W_1 + W_2 = 100$$

$$\therefore 2W_1 = 200/3; W_1 = 33.33 \text{ kW}; W_2 = 66.67 \text{ kW}$$

For connection diagram, please refer to Fig. 19.47. The vector or phasor diagram is shown in Fig. 19.56.

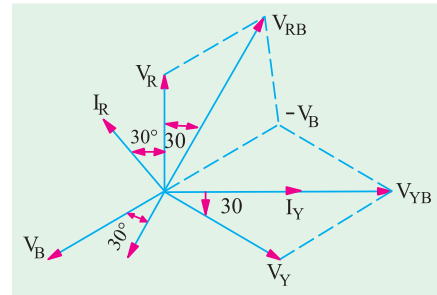


Fig. 19.56

**Example 19.47 (a).** A star-connected balanced load is supplied from a 3- $\phi$  balanced supply with a line voltage of 416 volts at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the reading on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate

(i) power factor of circuit, (ii) the line current, (iii) the capacitance of each capacitor.

(Elect. Engg. I, Nagpur Univ. 1993)

**Solution. (i)** As seen from Art. 19.21  $\tan \phi = -\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = -\frac{\sqrt{3}(782 - 1980)}{(782 + 1980)} = 0.75$ ;  
 $\phi = 36.9^\circ$ ,  $\cos \phi = 0.8$

$$(ii) \sqrt{3} \times 416 \times I_L \times 0.8 = 2762, I_L = 4.8 \text{ A}$$

$$(iii) Z_{ph} = V_{ph} / I_{ph} = (416\sqrt{3}) / 4.8 = 50\Omega, X_C = Z_{ph} \sin \phi = 50 \times 0.6 = 30\Omega$$

$$\text{Now, } X_C = 1 / 2\pi fC = 1 / 2\phi \times 50 \times C = 106 \times 10^{-6} \text{ F}$$

**Example 19.48.** Each phase of a 3-phase,  $\Delta$ -connected load consists of an impedance  $Z = 20 \angle 60^\circ$  ohm. The line voltage is 440 V at 50 Hz. Compute the power consumed by each phase impedance and the total power. What will be the readings of the two wattmeters connected?

(Elect. and Mech. Technology, Osmania Univ.)

$$\text{Solution. } Z_{ph} = 20\Omega; V_{ph} = V_L = 440\text{V}; I_{ph} = V_{ph} / Z_{ph} = 440/20 = 22 \text{ A}$$

$$\text{Since } \phi = 60^\circ; \cos \phi = \cos 60^\circ = 0.5; R_{ph} = Z_{ph} \times \cos 60^\circ = 20 \times 0.5 = 10\Omega$$

$$\therefore \text{Power/phase} = I_{ph}^2 R_{ph} = 22^2 \times 10 = \mathbf{4,840 \text{ W}}$$

$$\text{Total power} = 3 \times 4,840 = \mathbf{14,520 \text{ W}}$$
 [or  $P = \sqrt{3} \times 440 \times (\sqrt{3} \times 22) \times 0.5 = 14,520 \text{ W}$ ]

$$\text{Now, } W_1 + W_2 = 14,520.$$

$$\text{Also } \tan \phi = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2} \quad \therefore \tan 60^\circ = \sqrt{3} = \sqrt{3} \frac{W_1 - W_2}{14,520}$$

$$\therefore W_1 - W_2 = 14,520. \text{ Obviously, } W_2 = 0$$

Even otherwise it is obvious that  $W_2$  should be zero because p.f. =  $\cos 60^\circ = 0.5$  (Art. 19.19).

**Example 19.49.** Three identical coils, each having a reactance of  $20 \Omega$  and resistance of  $20 \Omega$  are connected in (a) star (b) delta across a 440-V, 3-phase line. Calculate for each method of connection the line current and readings on each of the two wattmeters connected to measure the power.

(Electro-mechanics, Allahabad Univ. 1992)

**Solution. (a) Star Connection**

$$Z_{ph} = \sqrt{20^2 + 20^2} = 20\sqrt{2} = 28.3\Omega; V_{ph} = 440 / \sqrt{3} = 254 \text{ V}$$

$$I_{ph} = 254 / 28.3 = 8.97 \text{ A}; I_L = \mathbf{8.97 \text{ A}}; \cos \phi = R_{ph} / Z_{ph} = 20/28.3 = 0.707$$

$$\text{Total power taken} = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 440 \times 8.97 \times 0.707 = 4830 \text{ W}$$

$$\text{If } W_1 \text{ and } W_2 \text{ are wattmeter readings, then } W_1 + W_2 = 4830 \text{ W} \quad \dots (i)$$

$$\text{Now, } \tan \phi = 20 / 20 = \sqrt{3} (W_1 - W_2) / (W_1 + W_2); (W_1 - W_2) = 2790 \text{ W} \quad \dots (ii)$$

$$\text{From (i) and (ii) above, } W_1 = \mathbf{3810 \text{ W}}; W_2 = \mathbf{1020 \text{ W}}$$

**(b) Delta Connection**

$$Z_{ph} = 28.3\Omega, V_{ph} = 440 \text{ V}, I_{ph} = 440 / 28.3 = 15.5 \text{ A}; I_L = 15.5 \times \sqrt{3} = 26.8 \text{ A}$$

$$P = \sqrt{3} \times 440 \times 26.8 \times 0.707 = 14,490 \text{ W} \quad (\text{it is 3 times the Y-power})$$

$$\therefore W_1 + W_2 = 14,490 \text{ W} \quad \dots (iii)$$

$$\tan \phi = 20 / 20 = \sqrt{3} (W_1 - W_2) / 14,490; W_1 - W_2 = 8370 \quad \dots (iv)$$

$$\text{From Eq. (iii) and (iv), we get, } W_1 = \mathbf{11,430 \text{ W}}; W_2 = \mathbf{3060 \text{ W}}$$

**Note:** These readings are 3-times the Y-readings.

**Example 19.50.** Three identical coils are connected in star to a 200-V, three-phase supply and each takes 500 W. The power factor is 0.8 lagging. What will be the current and the total power if the same coils are connected in delta to the same supply? If the power is measured by two wattmeters, what will be their readings? Prove any formula used.

(Elect. Engg. A.M. A. S.I. Dec. 1991)

**Solution.** When connected in star as shown in Fig. 19.57 (a),  $V_{ph} = 200/\sqrt{3} = 115.5 \text{ V}$

Now,  $V_{ph} I_{ph} \cos \phi = \text{power per phase or } 115.5 \times I_{ph} \times 0.8 = 500$

$$\therefore I_{ph} = 5.41 \text{ A}; Z_{ph} = V_{ph} / I_{ph} = 115.5 / 5.41 = 21.34 \Omega$$

$$R = Z_{ph} \cos \phi = 21.34 \times 0.8 = 17 \Omega; X_L = Z_{ph} \sin \phi = 21.34 \times 0.6 = 12.8 \Omega$$

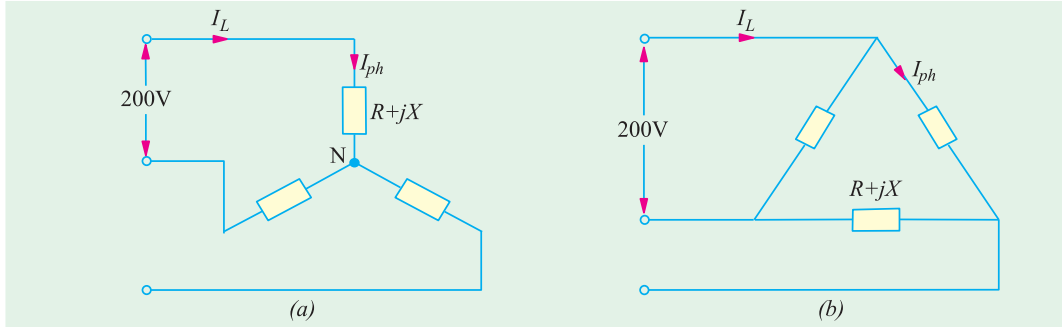


Fig. 19.57

The same three coils have been connected in delta in Fig. 19.57 (b). Here,  $V_{ph} = V_L = 200 \text{ V}$ .

$$I_{ph} = 200 / 21.34 = 9.37 \text{ A}; I_L = \sqrt{3} I_{ph} = 9.37 \times 1.732 = \mathbf{16.23 \text{ A}}$$

Total power consumed =  $\sqrt{3} \times 200 \times 16.23 \times 0.8 = \mathbf{4500 \text{ W}}$

It would be seen that when the same coils are connected in delta, they consume three times more power than when connected in star.

#### Wattmeter Readings

$$\text{Now, } W_1 + W_2 = 4500; \tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \cos^{-1}(0.8) = 36.87^\circ; \tan \phi = 0.75$$

$$0.75 = \frac{\sqrt{3}(W_1 - W_2)}{4500} \therefore (W_1 - W_2) = 1950 \text{ W}$$

$$\therefore W_1 = (4500 + 1950)/2 = 3225 \text{ W}; W_2 = \mathbf{1275 \text{ W}}$$

**Example 19.51.** A 3-phase, 3-wire, 415-V system supplies a balanced load of 20 A at a power factor 0.8 lag. The current coil of wattmeter 1 is in phase R and of wattmeter 2 in phase B. Calculate (i) the reading on 1 when its voltage coil is across R and Y (ii) the reading on 2 when its voltage coil is across B and Y and (iii) the reading on 1 when its voltage coil is across Y and B. Justify your answer with relevant phasor diagram. (Elect. Machines, A.M.I.E. Sec. B, 1991)

**Solution.** (i) As seen from phasor diagram of Fig. 19.57 (a)

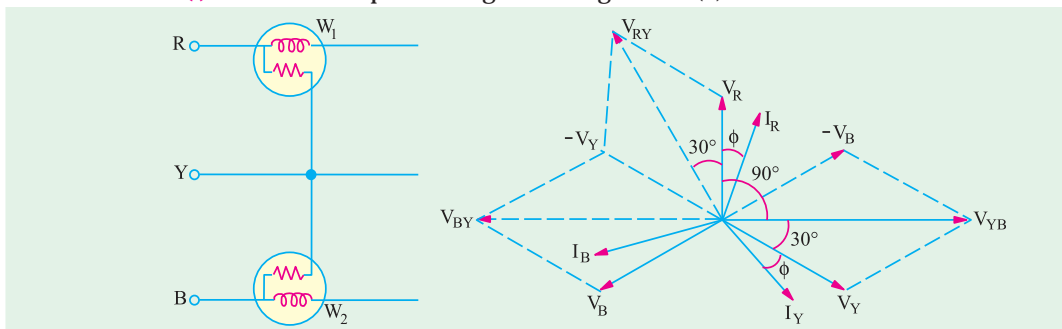


Fig. 19.57 (a)

$$W_1 = V_{RY} I_A \cos(30 + \phi) = \sqrt{3} \times 415 \times 20 \times \cos(36.87^\circ + 30^\circ) = 5647 \text{ W}$$

(ii) Similarly,  $W_2 = V_{BY} I_B \cos(30 - \phi)$

It should be noted that voltage across  $W_2$  is  $V_{BY}$  and not  $V_{YB}$ . Moreover,  $\phi = \cos^{-1}(0.8) = 36.87^\circ$ ,

$$\therefore W_2 = \sqrt{3} \times 415 \times 20 \times \cos(30^\circ - 36.87^\circ) = 14,275 \text{ W}$$

(iii) Now, phase angle between  $I_R$  and  $V_{YB}$  is  $(90^\circ - \phi)$

$$\therefore W_2 = V_{YB} I_R \cos(90^\circ - \phi) = \sqrt{3} \times 415 \times 20 \times \sin 36.87^\circ = 8626 \text{ VAR}$$

**Example 19.52.** A wattmeter reads 5.54 kW when its current coil is connected in R phase and its voltage coil is connected between the neutral and the R phase of a symmetrical 3-phase system supplying a balanced load of 30 A at 400 V. What will be the reading on the instrument if the connections to the current coil remain unchanged and the voltage coil be connected between B and Y phases? Take phase sequence RYB. Draw the corresponding phasor diagram. (Elect. Machines, A.M.I.E., Sec. B, 1992)

Fig. 19.57 (b)

**Solution.** As seen from Fig. 19.57 (b).

$$W_1 = V_R I_R \cos \phi \text{ or } 5.54 \times 10^3 = (400 / \sqrt{3}) \times 30 \times \cos \phi; \therefore \cos \phi = 0.8, \sin \phi = 0.6$$

In the second case (Fig. 19.57 (b))

$$W_2 = V_{YB} I_R \cos(90^\circ - \phi) = 400 \times 30 \times \sin \phi = 400 \times 30 \times 0.6 = 7.2 \text{ kW}$$

**Example 19.53.** A 3-phase, 3-wire balanced load with a lagging power factor is supplied at 400 V (between lines). A 1-phase wattmeter (scaled in kW) when connected with its current coil in the R-line and voltage coil between R and Y lines gives a reading of 6 kW. When the same terminals of the voltage coil are switched over to Y and B-lines, the current-coil connections remaining the same, the reading of the wattmeter remains unchanged. Calculate the line current and power factor of the load. Phase sequence is  $R \rightarrow Y \rightarrow B$ . (Elect. Engg-1, Bombay Univ. 1985)

**Solution.** The current through the wattmeter is  $I_R$  and p.d. across its pressure coil is  $V_{RY}$ . As seen from the phasor diagram of Fig. 19.58, the angle between the two is  $(30^\circ + \phi)$ .

$$\therefore W_1 = V_{RY} I_R \cos(30^\circ + \phi) = V_L I_L \cos(30^\circ + \phi) \quad \dots (i)$$

In the second case, current is  $I_R$  but voltage is  $V_{YB}$ . The angle between the two is  $(90^\circ - \phi)$

$$\therefore W_2 = V_{YB} I_R \cos(90^\circ - \phi) = V_L I_L \cos(90^\circ - \phi)$$

Since  $W_1 = W_2$  we have

$$V_L I_L \cos(30^\circ + \phi) = V_L I_L \cos(90^\circ - \phi)$$

$$\therefore 30^\circ + \phi = 90^\circ - \phi$$

$$\text{or } 2\phi = 60^\circ \therefore \phi = 30^\circ$$

$$\therefore \text{load power factor} = \cos 30^\circ = 0.866 \text{ (lag)}$$

Now  $W_1 = W_2 = 6 \text{ kW}$ .

Hence, from (i) above, we get

$$6000 = 400 \times I_L \cos 60^\circ; I_L = 30 \text{ A}$$

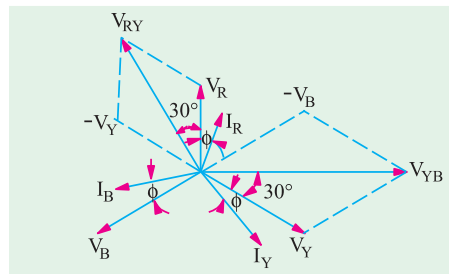


Fig. 19.58

**Example 19.54.** A 3-phase, 400 V circuit supplies a  $\Delta$ -connected load having phase impedances of  $Z_{AB} = 25\angle 0^\circ$ ;  $Z_{BC} = 25\angle 30^\circ$  and  $Z_{CA} = 25\angle -30^\circ$ .

Two wattmeters are connected in the circuit to measure the load power. Determine the wattmeter readings if their current coils are in the lines (a) A and B; (b) B and C; and (c) C and A. The phase sequence is ABC. Draw the connections of the wattmeter for the above three cases and check the sum of the two wattmeter readings against total power consumed.

**Solution.** Taking  $V_{AB}$  as the reference voltage, we have  $Z_{AB} = 400\angle 0^\circ$ ;  $Z_{BC} = 400\angle -120^\circ$  and  $Z_{CA} = 400\angle 120^\circ$ .

The three phase currents can be found as follows:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{400\angle 0^\circ}{25\angle 0^\circ} = 16\angle 0^\circ = (16 + j0)$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{400\angle -120^\circ}{25\angle 30^\circ} = 16\angle -150^\circ = (-13.8 - j8)$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -120^\circ}{25\angle 30^\circ} = 16\angle -150^\circ = (-13.8 - j8)$$

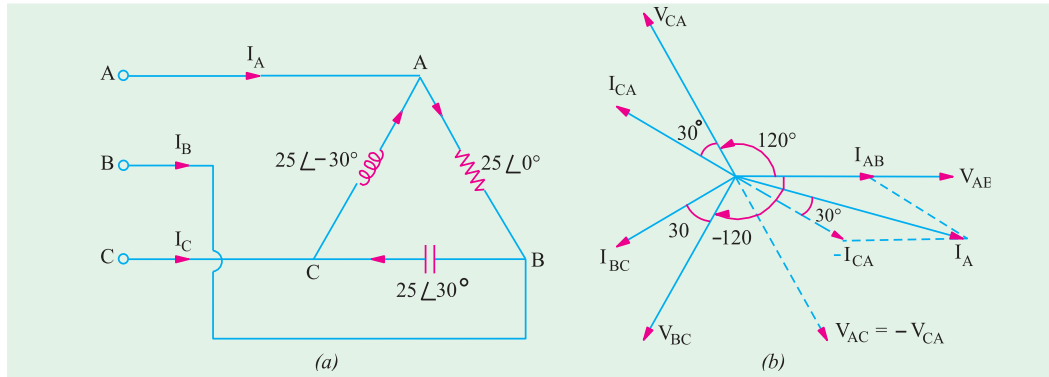


Fig. 19.59

The line currents  $I_A$ ,  $I_B$  and  $I_C$  can be found by applying KCL at the three nodes A, B and C of the load.

$$I_A = I_{AB} + I_{AC} = I_{AB} - I_{CA} = (16 + j0) - (-13.8 + j8) = 29.8 - j8 = 30.8\angle -15^\circ$$

$$I_B = I_{BC} - I_{AB} = (-13.8 - j8) - (16 + j0) = -29.8 - j8 = 30.8\angle -165^\circ$$

$$I_C = I_{CA} - I_{BC} = (-13.8 + j8) - (-13.8 - j8) = 0 + j16 = 16\angle 90^\circ$$

The phasor diagram for line and phase currents is shown in Fig. 19.59 (a) and (b).

(a) As shown in Fig. 19.60 (a), the current coils of the wattmeters are in the line A and B and the voltage coil of  $W_1$  is across the lines A and C and that of  $W_2$  is across the lines B and C. Hence, current through  $W_1$  is  $I_A$  and voltage across it is  $V_{AC}$ . The power indicated by  $W_1$  may be found in the following two ways:

$$(i) P_1 = |V_{AC}| \cdot |I_A| \times (\text{cosine of the angle between } V_{AC} \text{ and } I_A) \\ = 400 \times 30.8 \times \cos(30^\circ + 15^\circ) = 8710 \text{ W}$$

(ii) We may use current conjugate (Art.) for finding the power

$$P_{VA} = V_{AC} \cdot I_A = -400\angle 120^\circ \times 30.8\angle 15^\circ$$

$$\therefore P_1 = \text{real part of } P_{VA} = -400 \times 30.8 \times \cos 135^\circ = 8710 \text{ W}$$

$$P_2 = \text{real part of } [V_{BC} Z_B] = 400\angle 120^\circ \times 30.8\angle -165^\circ$$

$$= 400 \times 30.8 \times \cos(-45^\circ) = 8710 \text{ W}$$

$$\therefore P_1 + P_2 = 8710 + 8710 = 17,420 \text{ W.}$$

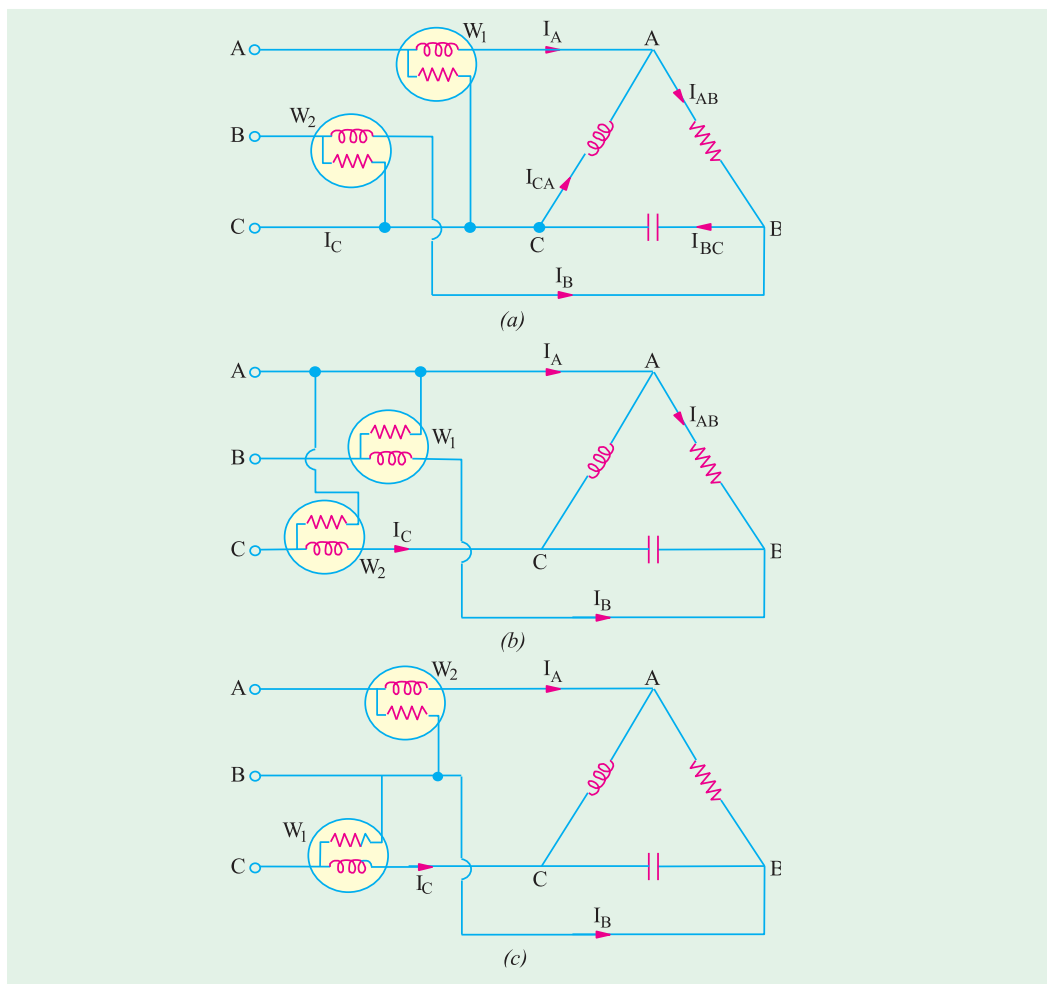


Fig. 19.60

(b) As shown in Fig. 19.60 (b), the current coils of the wattmeters are in the lines B and C whereas voltage coil of  $W_1$  is across the lines B and A and that of  $W_2$  is across lines C and A.

$$(i) \therefore P_1 = |V_{BA}| \cdot |I_B| \cos(\text{angle between } V_{BA} \text{ and } I_B) \\ = 400 \times 30.8 \times \cos 15^\circ = 11,900 \text{ W}$$

(ii) Using voltage conjugate (which is more convenient in this case), we have

$$P_{VA} = V_{BA}^* \cdot I_B = -400 \angle 0^\circ \times 30.8 \angle -165^\circ \\ \therefore P_1 = \text{real part of } P_{VA} = -400 \times 30.8 \times \cos(-165^\circ) = 11,900 \text{ W} \\ P_2 = \text{real part of } [V_{CA}^* I_C] = [400 \angle -120^\circ \times 16 \angle 90^\circ] = 400 \times 16 \times \cos(-30^\circ) = 5,540 \text{ W.}$$

$$\therefore P_1 + P_2 = 11,900 + 5,540 = 17,440 \text{ W.}$$

(c) As shown in Fig. 19.60 (c), the current coils of the wattmeters are in the lines C and A whereas the voltage coil of  $W_1$  is across the lines C and B and that of  $W_2$  is across the lines A and B.

$$(i) P_1 = \text{real part of } [V_{CB}^* I_C] = [(400 \angle 120^\circ) (16 \angle 90^\circ)] \\ = 400 \times 16 \times \cos 210^\circ = 5,540 \text{ W}$$

$$P_2 = \text{real part of } [V_{AB}^* I_A] = [400 \angle 0^\circ \times 30.8 \angle -15^\circ] = 400 \times 30.8 \times \cos \angle -15^\circ = 11,900 \text{ W}$$

$$\therefore P_1 + P_2 = 5,540 + 11,900 = 17,440 \text{ W}$$

Total power consumed by the phase load can be found directly as under :-

$$\begin{aligned}
 P_T &= \text{real part of } [V_{AB}I_{AB}^* + V_{BC}I_{BC}^* + V_{CA}I_{CA}^*] \\
 &= \text{real part of} \\
 &\quad [(400\angle 0^\circ)(16\angle -0^\circ) + (400\angle -120^\circ)(16\angle 150^\circ) + (400\angle 120^\circ)(16\angle -150^\circ)] \\
 &= 400 \times 16 \times \text{real part of } (1\angle 0^\circ + 1\angle 30^\circ + 1\angle -30^\circ) \\
 &= 400 \times 16 (\cos 0^\circ + \cos^\circ + \cos(-30^\circ)) = 17,485 \text{ W}
 \end{aligned}$$

**Note.** The slight variation in the different answers is due to the approximation made.

**Example 19.55.** In a balanced 3-phase system load 1 draws 60 kW and 80 leading kVAR whereas load 2 draws 160 kW and 120 lagging kVAR. If line voltage of the supply is 1000 V, find the line current supplied by the generator. (Fig. 19.61)

**Solution.** For load 1 which is a leading load,  $\tan \phi_1 = Q_1 / P_1 = 80/60 = -1.333$ ;  $\phi_1 = 53.1^\circ$ ,  $\cos \phi_1 = 0.6$ . Hence, line current of this load is

$$I_1 = 60,000 / \sqrt{3} \times 1000 \times 0.6 = 57.8 \text{ A}$$

For load 2,  $\tan \phi_2 = 120/160 = 0.75$ ;  $\phi_2 = 26.9^\circ$ ,  $\cos \phi_2 = 0.8$ . The line current drawn by this load is

$$I_2 = 160,000 / \sqrt{3} \times 1000 \times 0.8 = 115.5 \text{ A}$$

If we take the phase voltage as the reference voltage i.e.  $V_{ph} = (1000 / \sqrt{3}) \angle 0^\circ = 578 \angle 0^\circ$ ; then  $I_1$  leads this voltage by  $53.1^\circ$  whereas  $I_2$  lags it by  $36.9^\circ$ . Hence,  $I_1 = 57.8 \angle 53.1^\circ$  and  $I_2 = 115.5 \angle -36.9^\circ$

$$\begin{aligned}
 \therefore I_{L1} &= I_1 + I_2 = 57.8 \angle 53.1^\circ + 115.5 \\
 &\quad \angle -36.9^\circ = 171.7 \angle 42.3^\circ \text{ A}
 \end{aligned}$$

**Example 19.56.** A single-phase motor drawing 10A at 0.707 lagging power factor is connected across lines R and Y of a 3-phase supply line connected to a 3-phase motor drawing 15A at a lagging power factor of 0.8 as shown in Fig. 19.62(a). Assuming RYB sequence, calculate the three line currents.

**Solution.** In the phasor diagram of Fig. 19.61 (b) are shown the three phase voltages and the one line voltage  $V_{RY}$  which is ahead of its phase voltage  $V_R$ . The current  $I_1$  drawn by single-phase motor lags  $V_{RY}$  by  $\cos^{-1} 0.707$  or  $45^\circ$ . It lags behind the reference voltage  $V_R$  by  $15^\circ$  as shown. Hence,  $I_1 = 10 \angle -15^\circ = 9.6 - j2.6 \text{ A}$ . The 3-phase motor currents lag behind their respective phase voltages by  $\cos^{-1} 0.8$  or  $36.9^\circ$ . Hence,  $I_{R1} = 15 \angle -36.9^\circ = 12 - j9$ .

$$I_{Y1} = 15 \angle (-120^\circ - 36.9^\circ) = 15 \angle -156.9^\circ = -13.8 - j5.9$$

$$I_B = 15 \angle (120^\circ - 36.9^\circ) = 15 \angle 83.1^\circ$$

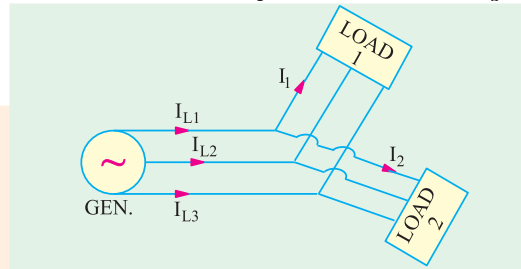


Fig. 19.61

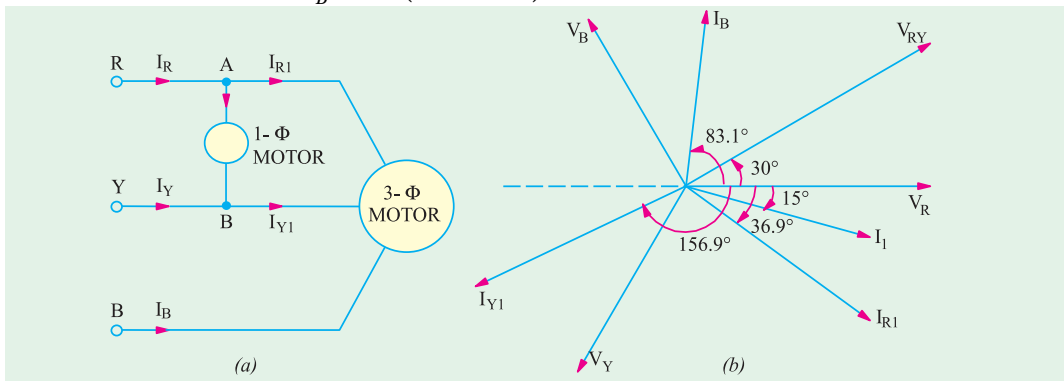


Fig. 19.62



Applying Kirchoff's laws to point A of Fig. 19.62 (a), we get

$$I_R = I_1 + I_{R1} = 9.6 - j2.6 + 12 - j9 = 21.6 - j11.6 = 24.5 \angle -28.2^\circ$$

Similarly, applying KCL to point B, we get

$$I_Y + I_1 = I_{Y1} \text{ or } I_Y = I_{Y1} - I_1 = -13.8 - j5.9 - 9.6 + j2.6 = -23.4 - j3.3 = 23.6 \angle -172^\circ.$$

**Example 19.57.** A 3- $\phi$ , 434-V, 50-Hz, supply is connected to a 3- $\phi$ , Y-connected induction motor and synchronous motor. Impedance of each phase of induction motor is  $(1.25 + j2.17) \Omega$ . The 3- $\phi$  synchronous motor is over-excited and it draws a current of 120 A at 0.87 leading p.f. Two wattmeters are connected in usual manner to measure power drawn by the two motors. Calculate (i) reading on each wattmeter (ii) combined power factor.

(Elect. Technology, Hyderabad Univ. 1992)

**Solution.** It will be assumed that the synchronous motor is Y-connected. Since it is over-excited it has a leading p.f. The wattmeter connections and phasor diagrams are as shown in Fig. 19.63.

$$Z_1 = 1.25 + j2.17 = 2.5 \angle 60^\circ$$

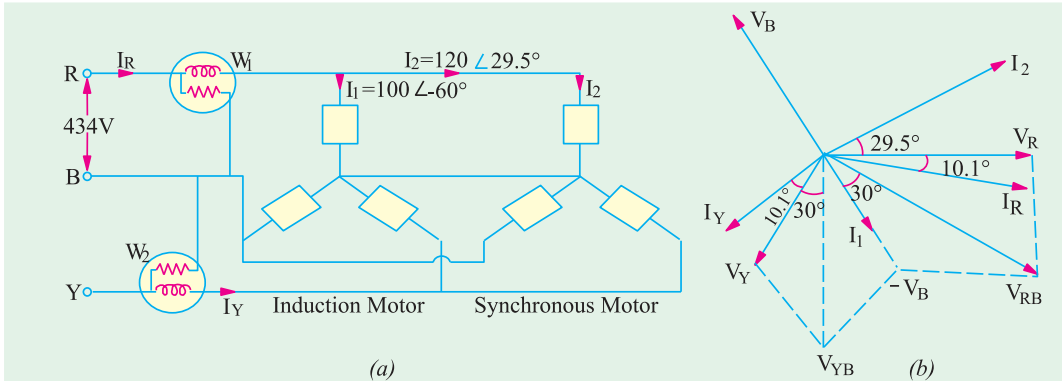


Fig. 19.63

Phase voltage in each case =  $434/\sqrt{3} = 250 \text{ V}$

$I_1 = 250/2.5 = 100 \text{ A}$  lagging the reference vector  $V_R$  by  $60^\circ$ . Current  $I_2 = 120 \text{ A}$  and leads  $V_R$  by an angle =  $\cos^{-1}(0.87) = 29.5^\circ$

$$\therefore \begin{aligned} I_1 &= 100 \angle 60^\circ = 50 + j86.6; & I_2 &= 120 \angle -29.5^\circ = 104.6 - j59 \\ I_R &= I_1 + I_2 = 154.6 - j27.6 = 156.8 \angle -10.1^\circ \end{aligned}$$

(a) As shown in Fig. 19.63 (b),  $I_R$  lags  $V_R$  by  $10.1^\circ$ . Similarly,  $I_Y$  lags  $V_Y$  by  $10.1^\circ$ .

As seen from Fig. 19.63 (a), current through  $W_1$  is  $I_R$  and voltage across it is  $V_{RB} = V_R - V_B$ . As seen,  $V_{RB} = 434 \text{ V}$  lagging by  $30^\circ$ . Phase difference between  $V_{RB}$  and  $I_R$  is =  $30 - 10.1 = 19.9^\circ$ .

$$\therefore \text{reading of } W_1 = 434 \times 156.8 \times \cos 19.9^\circ = \mathbf{63,970 \text{ W}}$$

Current  $I_Y$  is also (like  $I_R$ ) the vector sum of the line currents drawn by the two motors. It is equal to  $156.8 \text{ A}$  and lags behind its respective phase voltage  $V_Y$  by  $10.1^\circ$ . Current through  $W_2$  is  $I_Y$  and voltage across it is  $V_{YB} = V_Y - V_B$ . As seen,  $V_{YB} = 434 \text{ V}$ . Phase difference between  $V_{YB}$  and  $I_Y$  =  $30^\circ + 10.1^\circ = 40.1^\circ$  (lag).

$$\therefore \text{reading of } W_2 = 434 \times 156.8 \times \cos 40.1^\circ = \mathbf{52,050 \text{ W}}$$

$$(b) \text{ Combined p.f.} = \cos 10.1^\circ = \mathbf{0.9845 \text{ (lag)}}$$

**Example 19.58.** Power in a balanced 3-phase system is measured by the two-wattmeter method and it is found that the ratio of the two readings is 2 to 1. What is the power factor of the system? (Elect. Science-1, Allahabad Univ. 1991)

**Solution.** We are given that  $W_1 : W_2 = 2 : 1$ . Hence,  $W_1 / W_2 = r = 1/2 = 0.5$ . As seen from Art. 19.21.

$$\cos \phi = \frac{1+r}{2\sqrt{1-r+r^2}} = \frac{1+0.5}{2\sqrt{1-0.5+0.5^2}} = \mathbf{0.866 \text{ lag}}$$

**Example 19.59.** A synchronous motor absorbing 50 kW is connected in parallel with a factory load of 200 kW having a lagging power factor of 0.8. If the combination has a power factor of 0.9 lagging, find the kVAR supplied by the motor and its power factor.

(Elect. Machines, A.M.I.E. Sec B)

**Solution.** Load kVA =  $200/0.8 = 250$

Load kVAR =  $250 \times 0.6 = 150$  (lag) [ $\cos \phi = 0.8$   $\sin \phi = 0.6$ ]

Total combined load =  $50 + 200 = 250$  kW

kVA of combined load =  $250/0.9 = 277.8$

Combined kVAR =  $277.8 \times 0.4356 = 121$  (inductive) (combined  $\cos \phi = 0.9$ ,  $\sin \phi = 0.4356$ )

Hence, leading kVAR supplied by synch, motor =  $150 - 121 = 29$  (capacitive)

kVA of motor alone =  $\sqrt{(kW)^2 + (kVAR)^2} = \sqrt{50^2 + 29^2} = 57.8$

p.f. of motor =  $kW/kVA = 50/57.8 = 0.865$  (leading)

**Example 19.60.** A star-connected balanced load is supplied from a 3-phase balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of load consists of a resistance and a capacitor joined in series and the readings on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate (a) the power factor of the circuit (b) the line current and (c) the capacitance of each capacitor.

(Elect. Machinery-I, Bombay Univ.)

**Solution.**  $W_1 = 728$  and  $W_2 = 1980$

For a leading p.f.  $\tan \phi = -\sqrt{3} \frac{W_1 - W_2}{W_1 + W_2} \therefore \tan \phi = -\sqrt{3} \times \frac{(782 - 1980)}{782 + 1980} = 0.75$

From tables,  $\phi = 36^\circ 54'$

(a)  $\therefore \cos \phi = \cos 36^\circ 54' = 0.8$  (leading)

(b) power =  $\sqrt{3} V_L I_L \cos \phi$  or  $W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$

or  $(782 + 1980) = \sqrt{3} \times 416 \times I_L \times 0.8 \therefore I_L = I_{ph} = 4.8$  A

(c) Now  $V_{ph} = 416 / \sqrt{3}$  V  $\therefore Z_{ph} = 416 / \sqrt{3} \times 4.8 = 50 \Omega$

$\therefore$  In Fig. 19.64,  $Z_{ph} = 50 \angle -36^\circ 54' = 50(0.8 - j0.6) = 40 - j30$

Capacitive reactance  $X_C = 30$ ; or  $\frac{1}{2\pi \times 50 \times C} = 30 \therefore C = 106 \mu\text{F}$ .

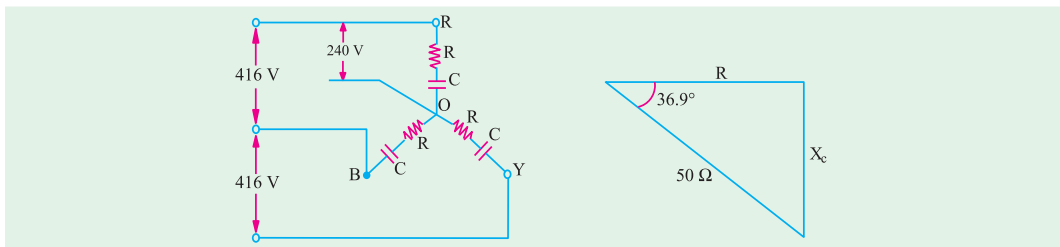


Fig. 19.64

**Example 19.61.** The two wattmeters A and B, give readings as 5000 W and 1000 W respectively during the power measurement of 3- $\phi$ , 3-wire, balanced load system. (a) Calculate the power and power factor if (i) both meters read direct and (ii) one of them reads in reverse. (b) If the voltage of the circuit is 400 V, what is the value of capacitance which must be introduced in each phase to cause the whole of the power to appear on A. The frequency of supply is 50 Hz.

(Elect. Engg-I, Nagpur Univ. 1992)

**Solution.** (a) (i) Both Meters Read Direct

$W_1 = 5000$  W;  $W_2 = 1000$  W;  $\therefore W_1 + W_2 = 6000$  W;  $W_1 - W_2 = 4000$  W

$$\tan \phi = \sqrt{3}(W_1 - W_2) / (W_1 + W_2) = \sqrt{3} \times 4000 / 6000 = 1.1547$$

$$\therefore \phi = \tan^{-1}(1.1547) = 49.1^\circ; \text{ p.f.} = \cos 49.1^\circ = 0.655 \text{ (lag)}$$

$$\text{Total power} = 5000 + 1000 = 6000 \text{ W}$$

**(ii) One Meter Reads in Reverse**

$$\text{In this case, } \tan \phi = \sqrt{3}(W_1 + W_2) / (W_1 - W_2) = \sqrt{3} \times 6000 / 4000 = 2.598$$

$$\therefore \phi = \tan^{-1}(2.598) = 68.95^\circ; \text{ p.f.} = \cos 68.95^\circ = 0.36 \text{ (lag)}$$

$$\text{Total power} = W_1 + W_2 = 5000 - 1000 = 4000 \text{ W}$$

...Art.

**(b)** The whole of power would be measured by wattmeter  $W_1$  if the load power factor is 0.5 (lagging) or less. It means that in the present case p.f. of the load will have to be reduced from 0.655 to 0.5. In other words, capacitive reactance will have to be introduced in each phase of the load in order to partially neutralize the inductive-reactance.

$$\text{Now, } \sqrt{3}V_L I_L \cos \phi = 6000 \text{ or } \sqrt{3} \times 400 I_L \times 0.655 = 6000$$

$$\therefore I_L = 13.2 \text{ A}; \therefore I_{ph} = 13.2 / \sqrt{3} = 7.63 \text{ A}$$

$$Z_{ph} = V_{ph} / I_{ph} = 400 / 7.63 = 52.4 \Omega$$

$$X_L = Z_{ph} \sin \phi = 52.4 \times \sin 49.1^\circ = 39.6 \Omega$$

**When p.f. = 0.5**

$$\sqrt{3} \times 400 \times I_L \times 0.5 = 6000; I_L = 17.32 \text{ A}; I_{ph} = 17.32 / \sqrt{3} = 10 \text{ A}; Z_{ph} = 400 / 10 = 40 \Omega$$

$$\cos \phi = 0.5; \phi = 60^\circ; \sin 60^\circ = 0.866; X = Z_{ph} \sin \phi = 40 \times 0.866 = 35.4 \Omega$$

$$\therefore X = X_L - X_C = 35.4 \text{ or } 39.6 - X_C = 35.4; \therefore X_C = 4.2 \Omega.$$

$$\text{If } C \text{ is the required capacitance, then } 4.2 = 1 / 2\pi \times 50 \times C; \therefore C = 758 \mu\text{F}.$$

### Tutorial Problems No. 19.2

**1.** Two wattmeters connected to measure the input to a balanced three-phase circuit indicate 2500 W and 500 W respectively. Find the power factor of the circuit (a) when both readings are positive and (b) when the latter reading is obtained after reversing the connections to the current coil of one instrument.

**[(a) 0.655 (b) 0.3591] (City & Guilds, London)**

**2.** A 400-V, 3-phase induction motor load takes 900 kVA at a power factor of 0.707. Calculate the kVA rating of the capacitor bank to raise the resultant power factor of the installation of 0.866 lagging.

Find also the resultant power factor when the capacitors are in circuit and the motor load has fallen to 300 kVA at 0.5 power factor.

**[296 kVA, 0.998 leading] (City & Guilds, London)**

**3.** Two wattmeters measure the total power in three-phase circuits and are correctly connected. One reads 4,800 W while other reads backwards. On reversing the latter, it reads 400 W. What is the total power absorbed by the circuit and the power factor?

**[4400 W; 0.49] (Sheffield Univ. U.K.)**

**4.** The power taken by a 3-phase, 400-V motor is measured by the two wattmeter method and the readings of the two wattmeters are 460 and 780 watts respectively. Estimate the power factor of the motor and the line current.

**[0.913, 1.96 A] (City & Guilds, London)**

**5.** Two wattmeters,  $W_1$  and  $W_2$  connected to read the input to a three-phase induction motor running unloaded, indicate 3 kW and 1 kW respectively. On increasing the load, the reading on  $W_1$  increases while that on  $W_2$  decreases and eventually reverses.

Explain the above phenomenon and find the unloaded power and power factor of the motor.

**[2 kW, 0.287 lag] (London Univ.)**

**6.** The power flowing in a 3- $\phi$ , 3-wire, balanced-load system is measured by the two wattmeter method. The reading on wattmeter A is 5,000 W and on wattmeter B is -1,000 W

(a) What is the power factor of the system?

(b) If the voltage of the circuit is 440, what is the value of capacitance which must be introduced into each phase to cause the whole of the power measured to appear on wattmeter A?

**[0.359; 5.43  $\Omega$ ] (Meters and Meas. Insts. A.M.I.E.E. London)**

**7.** Two wattmeters are connected to measure the input to a 400 V; 3-phase, connected motor outputting 24.4 kW at a power factor of 0.4 (lag) and 80% efficiency. Calculate the

(i) resistance and reactance of motor per phase

(ii) reading of each wattmeters.

**[(i) 2.55 Ω; 5.85 Ω; (ii) 34,915 W; - 4850 W] (Elect. Machines, A.M.I.E. Sec. B, 1993)**

**8.** The readings of the two instruments connected to a balanced three-phase load are 128 W and 56 W. When a resistor of about 25 Ω is added to each phase, the reading of the second instrument is reduced to zero. State, giving reasons, the power in the circuit before the resistors were added. **[72 W] (London Univ.)**

**9.** A balanced star-connected load, each phase having a resistance of 10 Ω and inductive reactance of 30 Ω is connected to 400-V, 50-Hz supply. The phase rotation is red, yellow and blue. Wattmeters connected to read total power have their current coils in the red and blue lines respectively. Calculate the reading on each wattmeter and draw a vector diagram in explanation. **[2190 W, - 583 W] (London Univ.)**

**10.** A 7.46 kW induction motor runs from a 3-phase, 400-V supply. On no-load, the motor takes a line current of 4 A at a power factor of 0.208 lagging. On full load, it operates at a power factor of 0.88 lagging and an efficiency of 89 per cent. Determine the readings on each of the two wattmeters connected to read the total power on (a) no load and (b) full load. **[1070 W, - 494 W; 5500 W; 2890 W]**

**11.** A balanced inductive load, connected in star across 415-V, 50-Hz, three-phase mains, takes a line current of 25A. The phase sequence is RYB. A single-phase wattmeter has its current coil connected in the R line and its voltage coil across the line YB. With these connections, the reading is 8 kW. Draw the vector diagram and find (i) the kW (ii) the kVAR (iii) the kVA and (iv) the power factor of the load.

**[(i) 11.45 kW (ii) 13.87 kVAR (iii) 18 kVA (iv) 0.637] (City & Guilds, London)**

## 19.27. Double Subscript Notation

In symmetrically-arranged networks, it is comparatively easier and actually more advantageous, to use single-subscript notation. But for unbalanced 3-phase circuits, it is essential to use double subscript notation, in order to avoid unnecessary confusion which is likely to result in serious errors.

Suppose, we are given two coils whose induced e.m.f.s. are 60° out of phase with each other [Fig. 19.65 (a)]. Next, suppose that it is required to connect these coils in additive series *i.e.* in such

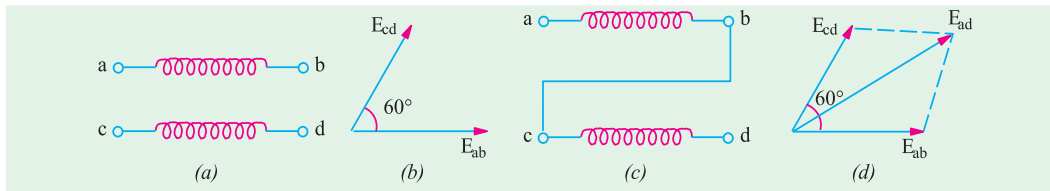


Fig. 19.65

a way that their e.m.f.s. add at an angle of 60°. From the information given, it is impossible to know whether to connect terminal 'a' to terminal 'c' or to terminal 'd'. But if additionally it were given that e.m.f. from terminal 'c' to terminal 'd' is 60° out of phase with that from terminal 'a' to terminal 'b', then the way to connect the coils is definitely fixed, as shown in Fig. 19.59 (b) and 19.60 (a). The double-subscript notation is obviously very convenient in such cases. The order in which these subscripts are written indicates the direction along which the voltage acts (or current flows). For example the e.m.f. 'a' to 'b' [Fig. 19.59 (a)], may be written as  $E_{ab}$  and that from 'c' to 'd' as  $E_{cd}$ .... The e.m.f. between 'a' and 'd' is  $E_{ad}$  where  $E_{ad} = E_{ab} + E_{cd}$  and is shown in Fig. 19.59 (b).

**Example 19.62.** If in Fig. 19.60 (a), terminal 'b' is connected to 'd', find  $E_{ac}$  if  $E = 100$  V.

**Solution.** Vector diagram is shown in Fig. 19.60 (b)

Obviously,  $E_{ac} = E_{ab} + E_{dc} = E_{ab} + (-E_{cd})$

Hence,  $E_{cd}$  is reversed and added to  $E_{ab}$  to get  $E_{ac}$  as shown in Fig. 19.60 (b). The magnitude of resultant vector is

$$E_{ac} = 2 \times 100 \cos 120^\circ / 2 = 100 \text{ V}; \quad E_{ac} = 100 \angle -60^\circ$$

**Example 19.62(a).** In Fig. 19.66 (a) with terminal 'b' connected to 'd', find  $E_{ca}$ .

**Solution.**  $E_{ca} = E_{cd} + E_{ba} = E_{cd} + (-E_{ab})$

As shown in Fig. 19.67, vector  $E_{ab}$  is reversed and then combined with  $E_{cd}$  to get  $E_{ca}$ .

Magnitude of  $E_{ca}$  is given by  $2 \times 100 \times \cos 60^\circ = 100$  V but it leads  $E_{ab}$  by  $120^\circ$ .

$\therefore E_{ca} = 100 \angle 120^\circ$

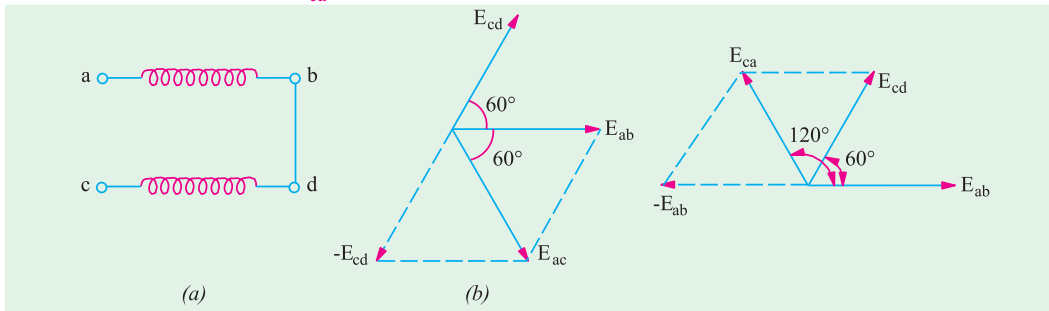


Fig. 19.66

Fig. 19.67

In Fig. 19.68 (b) is shown the vector diagram of the e.m.fs induced in the three phases 1, 2, 3 (or R, Y, B) of a 3-phase alternator [Fig.19.68 (a)]. According to double subscript notation, each phase e.m.f. may be written as  $E_{01}$ ,  $E_{02}$  and  $E_{03}$ , the order of the subscripts indicating the direction in which the e.m.fs. act. It is seen that while passing from phase 1 to phase 2 through the external circuit, we are in opposition to  $E_{02}$ .

$$E_{12} = E_{20} + E_{01} = (-E_{02}) + E_{01} = E_{01} - E_{02}$$

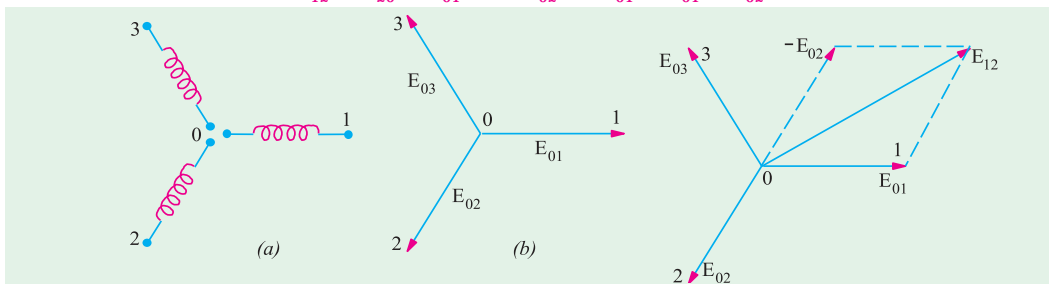


Fig. 19.68

Fig. 19.69

It means that for obtaining  $E_{12}$ ,  $E_{20}$  has to be reversed to obtain  $-E_{02}$  which is then combined with  $E_{01}$  to get  $E_{12}$  (Fig. 19.69). Similarly,

$$E_{23} = E_{30} + E_{02} = (-E_{03}) + E_{02} = E_{02} - E_{03}$$

$$E_{31} = E_{10} + E_{03} = (-E_{01}) + E_{03} = E_{03} - E_{01}$$

By now it should be clear that double-subscript notation is based on lettering every junction and terminal point of diagrams of connections and on the use of two subscripts with all vectors representing voltage or current. The subscripts on the vector diagram, taken from the diagram of connections, indicate that the positive direction of the current or voltage is from the first subscript to the second. For example, according to this notation  $I_{ab}$  represents a current whose +ve direction is from a to b in the branch ab of the circuit in the diagram of connections. In the like manner,  $E_{ab}$  represents the e.m.f. which produces this current. Further,  $I_{ba}$  will represent a current flowing from b to a, hence its vector will be drawn equal to but in a direction opposite to that of  $I_{ab}$ , i.e.  $I_{ab}$  and  $I_{ba}$  differ in phase by  $180^\circ$  although they do not differ in magnitude.

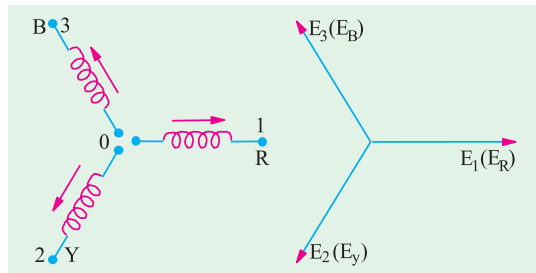


Fig. 19.69 (a)

In single subscript notation (i.e. the one in which single subscript is used) the +ve directions are fixed by putting arrows on the circuit diagrams as shown in Fig. 19.69 (a). According to this notation

$$E_{12} = -E_2 + E_1 = E_1 - E_2; E_{23} = -E_3 + E_2 = E_2 - E_3 \text{ and } E_{31} = -E_1 + E_3 = E_3 - E_1$$

$$\text{or } \mathbf{E}_{RY} = \mathbf{E}_R - \mathbf{E}_Y; \mathbf{E}_{YB} = \mathbf{E}_Y - \mathbf{E}_B; \mathbf{E}_{BR} = \mathbf{E}_B - \mathbf{E}_R$$

**Example 19.63.** Given the phasors  $V_{12} = 10\angle 30^\circ$ ;  $V_{23} = 5\angle 0^\circ$ ;  $V_{14} = 6\angle -60^\circ$ ;  $V_{45} = 10\angle 90^\circ$ . Find (i)  $V_{13}$  (ii)  $V_{34}$  and (iii)  $V_{25}$ .

**Solution.** Different points and the voltage between them have been shown in Fig. 19.70.

(i) Using KVL, we have

$$V_{12} + V_{23} + V_{31} = 0 \text{ or } V_{12} + V_{23} - V_{13} = 0$$

$$\text{or } V_{13} = V_{12} + V_{23} = 10\angle 30^\circ + 5\angle 0^\circ = 8.86 + j5 + 5 \\ = 13.86 + j5 = 14.7\angle 70.2^\circ$$

(ii) Similarly,  $V_{13} + V_{34} + V_{41} = 0$  or  $V_{13} + V_{34} - V_{14} = 0$

$$\text{or } V_{34} = V_{14} - V_{13} = 6\angle -60^\circ - 14.7\angle 70.2^\circ \\ = 3 - j5.3 - 13.86 - j5 = -10.86 - j10.3 = 15\angle 226.5^\circ$$

(iii) Similarly,  $V_{23} + V_{34} + V_{45} + V_{52} = 0$

$$\text{or } V_{23} + V_{34} + V_{45} - V_{52} = 0$$

$$\text{or } V_{52} = V_{23} + V_{34} + V_{45} = 5\angle 0^\circ + 15\angle 226.5^\circ + 10\angle 90^\circ \\ = 5 - 10.86 - j10.3 + j10 = -5.86 - j0.3 = 5.86\angle -2.9^\circ$$

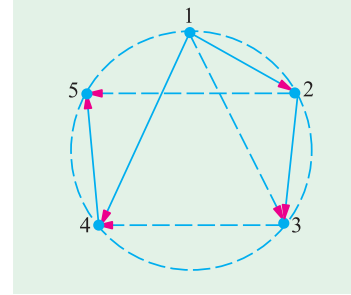


Fig. 19.70

**Example 19.64.** In a balanced 3-phase Y-connected voltage source having phase sequence abc,  $V_{an} = 230\angle 30^\circ$ . Calculate analytically (i)  $V_{bn}$  (ii)  $V_{cn}$  (iii)  $V_{ab}$  (iv)  $V_{bc}$  and (v)  $V_{ca}$ . Show the phase and line voltages on a phasor diagram.

**Solution.** It should be noted that  $V_{an}$  stands for the voltage of terminal a with respect to the neutral point n. The usual positive direction of the phase voltages are as shown in Fig. 19.71 (a). Since the 3-phase system is balanced, the phase differences between the different phase voltages are  $120^\circ$ .

$$(i) V_{bn} = \angle -120^\circ = 230 \angle (30^\circ - 120^\circ) = 230\angle -90^\circ$$

$$(ii) V_{cn} = V_{an}\angle 120^\circ = 230 \angle (30^\circ + 120^\circ) = 230\angle 150^\circ$$

... Fig. 19.71 (b)

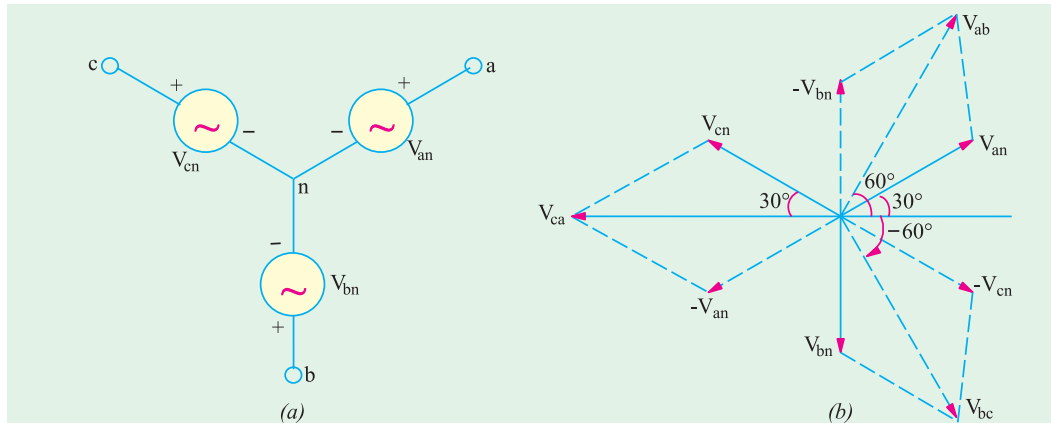


Fig. 19.71

(iii) It should be kept in mind that  $V_{ab}$  stands for the voltage of point a with respect to point b. For this purpose, we start from the reference point b in Fig. 19.71 (a) and go to point a and find the sum of the voltages met on the way. As per sign convention given in Art. 19.27 as we go from b to n, there is a fall in voltage of by an amount equal to  $V_{bn}$ . Next as we go from n to a, there is increase of voltage given by  $V_{an}$ .

$$\therefore V_{ab} = -V_{bn} + V_{an} = V_{an} - V_{bn} = 230\angle 30^\circ - 230\angle -90^\circ \\ = 230(\cos 30^\circ + j\sin 30^\circ) - 230(0 - j\sin 90^\circ)$$

$$= 230 \frac{\sqrt{3}}{2} j \frac{1}{2} j 230 \quad 230 \frac{\sqrt{3}}{2} j \frac{3}{2} = 230\sqrt{3} \frac{1}{2} j \frac{\sqrt{3}}{2} \quad 400 \quad 60$$

(iv)  $V_{bc} = V_{bn} - V_{cn} = 230 \angle -90^\circ - 230 \angle 150^\circ = -j230 - 230$

$$\frac{\sqrt{3}}{2} j \frac{1}{2} \quad 230\sqrt{3} \frac{1}{2} j \frac{\sqrt{3}}{2} \quad 400 \quad 60$$

(v)  $V_{ca} \quad V_{cn} \quad V_{an} \quad 230 \quad 150 \quad 230 \quad 30 \quad \frac{\sqrt{3}}{2} j \frac{1}{2} \quad 230 \frac{\sqrt{3}}{2} j \frac{1}{2} \quad 400 \quad 400 \quad 180$

These line voltages along with the phase voltages have been shown in the phasor diagram of Fig. 19.71 (b).

**Example 19.65.** Three non-inductive resistances, each of  $100 \Omega$  are connected in star to a 3-phase, 440-V supply. Three equal choking coils are also connected in delta to the same supply; the resistance of one coil being equal to  $100 \Omega$ . Calculate (a) the line current and (b) the power factor of the system. **(Elect. Technology-II, Sambal Univ.)**

**Solution.** The diagram of connections and the vector diagram of the Y- and  $\Delta$ -connected impedances are shown in Fig. 19.72.

The voltage  $E_{10}$  between line 1 and neutral is taken along the X-axis. Since the load is balanced, it will suffice to determine the current in one line only. Applying Kirchoff's Law to junction 1, we have

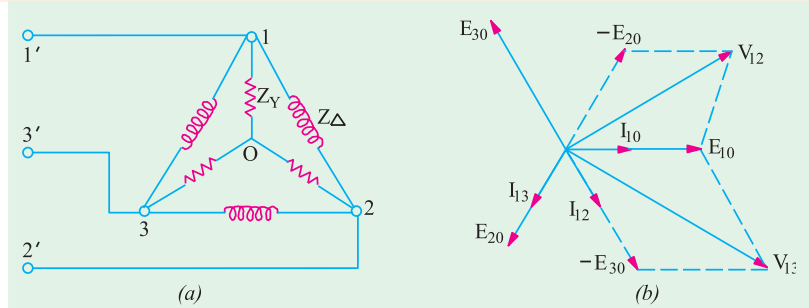


Fig. 19.72

$$I'_{11} = I_{10} + I_{12} + I_{13}$$

Let us first get the vector expressions for  $E_{10}$ ,  $E_{20}$  and  $E_{30}$

$$E_{10} = \frac{440}{\sqrt{3}} (1 + j0) = 254 + j0, \quad E_{20} = 254 \frac{1}{2} j \frac{\sqrt{3}}{2} = -127 - j220$$

$$E_{30} = 254 \frac{1}{2} j \frac{\sqrt{3}}{2} = -127 + j220$$

Let us now derive vector expressions for  $V_{12}$  and  $V_{31}$ :

$$V_{10} = E_{10} + E_{02} = E_{10} - E_{20} = (254 + j0) - (-127 - j220) = 381 + j220$$

$$V_{13} = E_{10} + E_{03} = E_{10} - E_{30} = (254 - j0) - (-127 + j220) = 381 - j220$$

$$I_{10} = \frac{E_{10}}{Z_Y} = \frac{254}{100} j0 = 2.54 \quad j0, \quad I_{12} = \frac{V_{13}}{Z} = \frac{381 - j220}{j100} = 2.2 \quad j3.81 \quad 4.4 \quad 60$$

$$I_{13} = \frac{V_{13}}{Z_{\Delta}} = \frac{381 - j220}{j100} = -2.2 - j3.81 = 4.4 \angle -120^\circ$$

(a)  $I'_{11} = (2.54 + j0) + (2.2 - j3.81) + (-2.2 - j3.81) = (2.54 - j7.62) = 8.03 \angle -71.6^\circ$

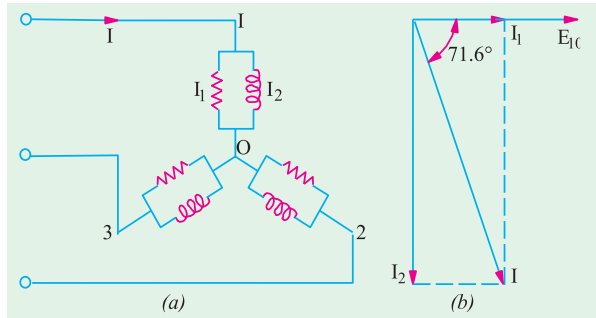


Fig. 19.73

$$(b) \text{ p.f.} = \cos 71.6^\circ = 0.316 \text{ (lag)}$$

#### Alternative Method

This question may be easily solved by  $\Delta/Y$  conversion. The star equivalent of the delta reactance is  $100/3 \Omega$  per phase.

As shown in Fig. 19.73, there are now two parallel circuits across each phase, one consisting of a resistance of  $100 \Omega$  and the other of a reactance of  $100/3 \Omega$

Taking  $E_{10}$  as the reference vector, we have

$$E_{10} = (254 + j0)$$

$$I_1 = \frac{254 - j0}{100} = 2.54 - j0; \quad I_2 = \frac{254 - j0}{j100/3} = j7.62$$

$$\text{Line current } I = (2.54 - j0) + (-j7.62) = (2.54 - j7.62) = 8.03 \angle -71.6^\circ \dots \text{ Fig. 19.73 (b)}$$

### 19.28. Unbalanced Loads

Any polyphase load in which the impedances in one or more phases differ from the impedances of other phases is said to be an unbalanced load. We will now consider different methods to handle unbalanced star-connected and delta-connected loads.

### 19.29. Unbalanced $\Delta$ -connected Load

Unlike unbalanced  $Y$ -connected load, the unbalanced  $\Delta$ -connected load supplied from a balanced 3-phase supply does not present any new problems because the voltage across each load phase is fixed. It is independent of the nature of the load and is equal to line voltage. In fact, the problem resolves itself into three independent single-phase circuits supplied with voltages which are  $120^\circ$  apart in phase.

The different phase currents can be calculated in the usual manner and the three line currents are obtained by taking the vector difference of phase currents in pairs.

If the load consists of three different pure resistances, then trigonometrical method can be used with advantage, otherwise symbolic method may be used.

**Example 19.66.** A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which  $Z_{AB} = 25 \angle 90^\circ$ ,  $Z_{BC} = 15 \angle 30^\circ$ ,  $Z_{CA} = 20 \angle 0^\circ$  ohms. Find the line currents and total power.

(Advanced Elect. Machines A.M.I.E. Sec. B, Summer 1991)

**Solution.** As explained in Art. 19.2, a 3-phase system has only two possible sequences :  $ABC$  and  $CBA$ . In the  $ABC$  sequence, the voltage of phase  $B$  lags behind voltage of phase  $A$  by  $120^\circ$  and that of phase  $C$  lags behind phase  $A$  voltage by  $240^\circ$ . In the  $CBA$  phase which can be written as  $A \rightarrow C \rightarrow B$ , voltage of  $C$  lags behind voltage  $A$  by  $120^\circ$  and that of  $B$  lags behind voltage  $A$  by  $240^\circ$ . Hence, the phase voltage which can be written as

$$E_{AB} = E \angle 0^\circ; \quad E_{BC} = E \angle -120^\circ$$

$$\text{and } E_{CA} = E \angle -240^\circ \text{ or } E_{CA} = E \angle 120^\circ$$

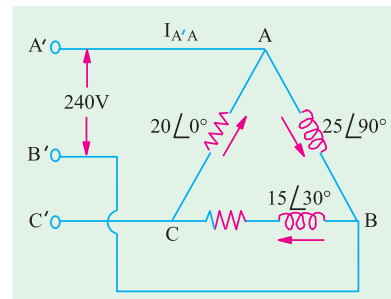


Fig. 19.74



$$\therefore I_{AB} = \frac{E_{AB}}{Z_{AB}} = \frac{240\angle 0^\circ}{25\angle 90^\circ} = 9.6\angle -90^\circ = -j9.6 \text{ A}$$

$$I_{BC} = \frac{E_{BC}}{Z_{BC}} = \frac{240\angle 120^\circ}{15\angle 30^\circ} = 16\angle 90^\circ = j16 \text{ A}$$

$$I_{CA} = \frac{E_{CA}}{Z_{CA}} = \frac{240\angle -120^\circ}{20\angle 0^\circ} = 12\angle -120^\circ = 12(0.5 - j0.866) = (-6 - j10.4) \text{ A}$$

The circuit is shown in Fig. 19.74.

$$\text{Line current } I_{AA} = I_{AB} + I_{AC} = I_{AB} - I_{CA} = -j9.6 - (-6 - j10.4) = 6 + j0.08$$

$$\text{Line current } I_{BB} = I_{BC} - I_{AB} = j16 - (-j9.6) = j25.6 \text{ A}$$

$$I_{CC} = I_{CA} - I_{BC} = (-6 - j10.4) - j16 = (-6 - j26.4) \text{ A}$$

$$\text{Now, } R_{AB} = 0; R_{BC} = 15 \cos 30 = 13 \Omega; R_{CA} = 20 \Omega$$

Power

$$W_{AB} = 0; W_{BC} = I_{BC}^2 R_{BC} = 16^2 \times 13 = 3328 \text{ W}; W_{CA} = I_{CA}^2 \times R_{CA} = 27^2 \times 20 = 14,580 \text{ W}$$

$$\text{Total Power} = 3328 + 14580 = 17,908 \text{ W}$$

**Example 19.67.** In the network of Fig. 19.75,  $E_{na} = 230\angle 0^\circ$  and the phase sequence is abc. Find the line currents  $I_a$ ,  $I_b$  and  $I_c$  as also the phase currents  $I_{AB}$ ,  $I_{BC}$  and  $I_{CA}$ .  $E_{na}$ ,  $E_{nb}$ ,  $E_{nc}$  is a balanced three-phase voltage system with phase sequence abc.

(Network Theory, Nagpur Univ. 1993)

**Solution.** Since the phase sequence is abc, the generator phase voltages are:

$$E_{na} = 320\angle 0^\circ; E_{nb} = 230\angle -120^\circ; E_{nc} = 230\angle 120^\circ$$

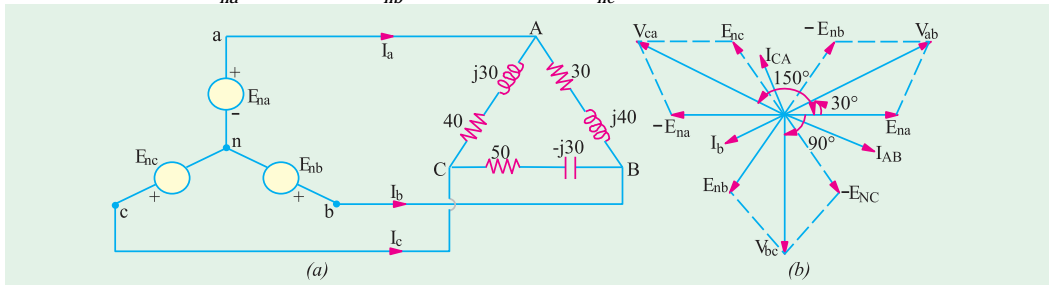


Fig. 19.75

As seen from the phasor diagram of Fig. 19.75 (b), the line voltages are as under :-

$$V_{ab} = E_{na} - E_{nb}; V_{bc} = E_{nb} - E_{nc}; V_{ca} = E_{nc} - E_{na}$$

$\therefore V_{ab} = \sqrt{3} \times 230\sqrt{30^\circ} = 400\angle 30^\circ$  i.e. it is ahead of the reference generator phase voltage  $E_{na}$  by  $30^\circ$ .

$$V_{bc} = \sqrt{3} \times 230\angle 90^\circ = 400\angle -90^\circ. \text{ This voltage is } 90^\circ \text{ behind } E_{na} \text{ but } 120^\circ \text{ behind } V_{ab}.$$

$V_{ca} = \sqrt{3} \times 230\angle 150^\circ = 400\angle 150^\circ$  or  $\angle -210^\circ$ . This voltage leads reference voltage  $E_{na}$  by  $150^\circ$  but leads  $V_{ab}$  by  $120^\circ$ .

These voltages are applied across the unbalanced  $\Delta$ -connected load as shown in Fig. 19.75 (a).

$$Z_{AB} = 30 + j40 = 50\angle 53.1^\circ; Z_{BC} = 50 - j30 = 58.3\angle -31^\circ,$$

$$Z_{CA} = 40 + j30 = 50\angle 36.9^\circ$$

$$I_{AB} = \frac{V_{ab}}{Z_{AB}} = \frac{400\angle 30^\circ}{50\angle 53.1^\circ} = 8\angle -23.1^\circ = 7.36 - j3.14$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{400\angle -90^\circ}{58.3\angle -31^\circ} = 6.86\angle -59^\circ = 3.53 - j5.88$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{400 \angle 150^\circ}{50 \angle 36.9^\circ} = 8 \angle 113.1^\circ = 3.14 + j7.36$$

$$I_a = I_{AB} - I_{CA} = 7.36 - j3.14 + 3.14 - j7.36 = 10.5 - j10.5 = 14.85 \angle -45^\circ$$

$$I_b = I_{BC} - I_{AB} = 3.53 - j5.88 - 7.36 + j3.14 = -3.83 - j2.74 = 4.71 \angle -215.6^\circ$$

$$I_c = I_{CA} - I_{BC} = -3.14 + j7.36 - 3.53 + j5.88 = -6.67 + j13.24 = 14.8 \angle 116.7^\circ$$

**Example 19.68.** For the unbalanced  $\Delta$ -connected load of Fig. 19.76 (a), find the phase currents, line currents and the total power consumed by the load when phase sequence is (a) abc and (b) acb.

**Solution.** (a) Phase sequence abc (Fig. 19.76).

Let  $V_{ab} = 100 \angle 0^\circ = 100 + j0$

$$V_{bc} = 100 \quad 120 \quad 100 \quad \frac{1}{2} j \frac{\sqrt{3}}{2} \quad 50 \quad j86.6$$

$$V_{ca} = 100 \quad 102 \quad 100 \quad \frac{1}{2} j \frac{\sqrt{3}}{2} \quad 50 \quad j86.6$$

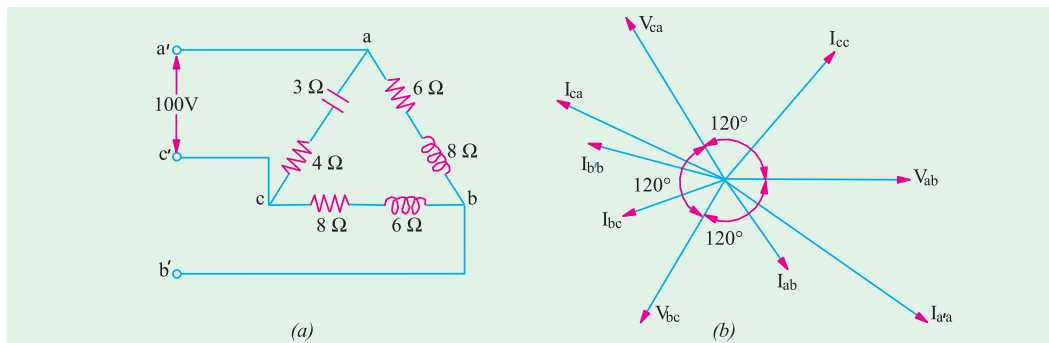


Fig. 19.76

(i) Phase currents

$$\text{Phase current, } I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{100 \quad j0}{6 \quad j8} = 6 \quad j8 \quad 10 \quad 53.8$$

$$\text{Similarly, } I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{30 \quad j86.6}{8 \quad j6} = 9.2 \quad j3.93 \quad 10 \quad 156.52$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{50 \quad j86.6}{4 \quad j3} = 18.39 \quad j7.86 \quad 20 \quad 156.52$$

(ii) Line Currents

$$\begin{aligned} \text{Line Current } I_a &= I_{ab} - I_{ca} = (6 \quad j8) - (18.39 \quad j7.86) \\ &= 24.39 - j15.86 = 29.1 \angle -33^\circ 2' \end{aligned}$$

$$\begin{aligned} \text{Similarly, } I_b &= I_{bc} - I_{ab} = (9.2 \quad j3.93) - (6 \quad j8) = 15.2 \quad j4.07 \\ &= 15.73 \angle 165.30^\circ \end{aligned}$$

$$\begin{aligned} I_c &= I_{ca} - I_{bc} = (18.39 \quad j7.86) - (9.2 \quad j3.93) \\ &= 9.19 + j11.79 = 14.94 \angle 52^\circ 3' \end{aligned}$$

Check  $\Sigma I = 0 + j0$

**(iii) Power**

$$W_{ab} = I_{ab}^2 R_{ab} = 10^2 \times 6 = 600 \text{ W}$$

$$W_{bc} = I_{bc}^2 R_{bc} = 10^2 \times 8 = 800 \text{ W}$$

$$W_{ca} = I_{ca}^2 R_{ca} = 20^2 \times 4 = 1600 \text{ W}$$

$$\text{Total} = 3000 \text{ W}$$

**(b) Phase sequence acb** (Fig. 19.77)

Here,

$$\mathbf{V}_{ab} \quad 100 \quad 0 \quad 100 \quad j9$$

$$\mathbf{V}_{bc} \quad 100 \quad 120 \quad 50 \quad 86.6$$

$$\mathbf{V}_{ca} \quad 100 \quad 120 \quad 50 \quad j86.6$$

**(i) Phase Currents**

$$\mathbf{I}_{ab} \quad \frac{100}{6 - j8} \quad 6 \quad j8 \quad 10 \quad 53 \quad 8$$

$$\mathbf{I}_{bc} \quad \frac{50}{8 - j6} \quad \frac{j86.6}{j6} = (1.2 + j9.93) = 10 \angle 83^\circ 8'$$

$$\mathbf{I}_{ca} \quad \frac{50}{7 - j3} \quad \frac{j86.6}{j3} = (2.4 - j19.86) = 20 \angle -83^\circ 8'$$

**(ii) Line Currents**

$$\mathbf{I}_{a'a} = \mathbf{I}_{ab} + \mathbf{I}_{ac} = \mathbf{I}_{ab} - \mathbf{I}_{ca}$$

$$= (6 - j8) - (2.4 - j19.86) = (3.6 + j11.86) = 12.39 \angle 73^\circ 6'$$

$$\mathbf{I}_{bb} \quad (1.2 \quad j9.93) \quad (6 \quad j8) \quad (4.8 \quad j17.93) \quad 18.56 \quad 105$$

$$\mathbf{I}_{cc} \quad (2.4 \quad j19.86) \quad (1.2 \quad j9.93) \quad (1.2 \quad j29.79) \quad 29.9 \quad 87 \quad 42$$

It is seen that  $\Sigma \mathbf{I} = 0 + j0$

**(iii) Power**

$$W_{ab} = 10^2 \times 6 = 600 \text{ W}$$

$$W_{bc} = 10^2 \times 8 = 800 \text{ W}$$

$$W_{ca} = 20^2 \times 4 = 1600 \text{ W}$$

$$\text{Total} = 3000 \text{ W}$$

- as before

It will be seen that the effect of phase reversal on an unbalanced  $\Delta$ -connected load is as under:

**(i)** phase currents change in angle only, their magnitudes remaining the same

**(ii)** consequently, phase powers remain unchanged

**(iii)** line currents change both in magnitude and angle.

The adjoining tabulation emphasizes the effect of phase sequence on the line currents drawn by an unbalanced 3-phase load.

Line	Ampere Sequence a b c	Sequence c b a
a	29.1 $\angle -33^\circ 2'$	12.39 $\angle 73.1^\circ$
b	15.73 $\angle 165^\circ$	18.56 $\angle 105^\circ$
c	14.94 $\angle 52^\circ 3'$	29.9 $\angle -87.7^\circ$

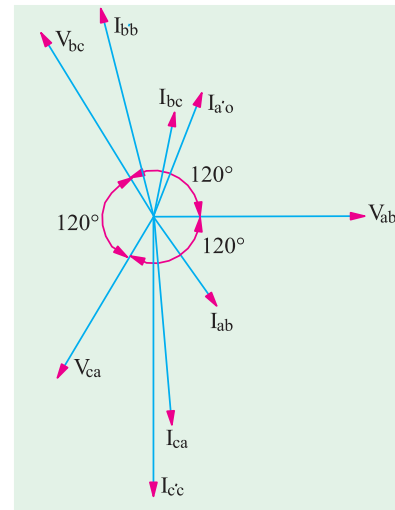


Fig. 19.77

**Example 19.69.** A balanced 3-phase supplies an unbalanced 3-phase delta-connected load made up of resistors  $100 \Omega$  and a reactor having an inductance of  $0.3 \text{ H}$  with negligible resistance.  $V_L = 100 \text{ V}$  at  $50 \text{ Hz}$ . Calculate (a) the total power in the system.

(Elect. Engineering-I, Madras Univ.)

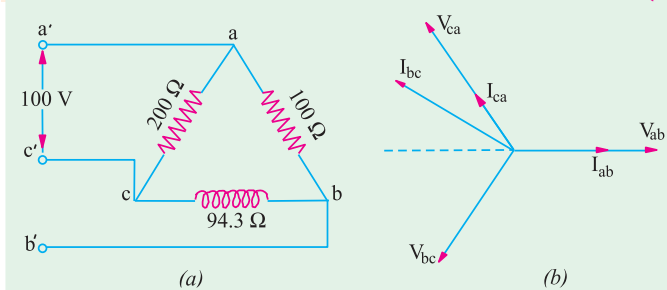


Fig. 19.78

**Solution.** The  $\Delta$ -connected load and its phasor diagram are shown in Fig. 19.78 (a).

$$X_L = \omega L = 314.2 \times 0.2 \\ = 94.3 \Omega$$

$$\text{Let } \mathbf{V}_{ab} \begin{bmatrix} 100 & 0 & 100 \\ 100 & 120 & -50 - j86.6 \end{bmatrix} \text{ j}0$$

$$\mathbf{V}_{ca} \begin{bmatrix} 100 & 120 & 50 \\ 100 & 0 & 130 \end{bmatrix} \text{ j}86.6$$

$$\mathbf{I}_{ab} \begin{bmatrix} \frac{V_{ab}}{Z_{ab}} & \frac{100}{100} & 0 \\ 130 & 0 & 1 \end{bmatrix} \text{ j}0$$

$$\mathbf{I}_{bc} \begin{bmatrix} \frac{100}{100} & 0 \\ 100 & 0 \end{bmatrix} \begin{bmatrix} 1.06 & 210 & 0.92 \\ 0 & 0 & j0.53 \end{bmatrix}$$

$$\mathbf{I}_{ca} \begin{bmatrix} \frac{100}{200} & \frac{120}{0} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 120 & 0.25 \\ 0 & 0 & j0.43 \end{bmatrix}$$

Watts in branch  $ab = V_{ab}^2 / R_{ab} = 100^2 / 100 = 100 \text{ W}$ ; VARs = 0

Watts in branch  $bc = 0$ ; VARs =  $100 \times 1.06 = 106 \text{ (lag)}$

Watts in branch  $ca = V_{ca}^2 / R_{ca} = 100^2 / 200 = 50 \text{ W}$ ; VARs = 0

(a) Total power =  $100 + 50 = 150 \text{ W}$ ; VARs = **106 (lag)**

### 19.30. Four-wire Star-connected Unbalanced Load

It is the simplest case of an unbalanced load and may be treated as three separate single-phase systems with a common return wire. It will be assumed that impedance of the line wires and source phase windings is zero. Should such an assumption be unacceptable, these impedances can be added to the load impedances. Under these conditions, source and load line terminals are at the same potential.

Consider the following two cases:

#### (i) Neutral wire of zero impedance

Because of the presence of neutral wire (assumed to behaving zero impedance), the star points of the generator and load are tied together and are at the same potential. Hence, the voltages across the three load impedances are **equalized** and each is equal to the voltage of the corresponding phase of the generator. In other words, due to the provision of the neutral, each phase voltage is a **forced** voltage so that the three phase voltages are balanced when line voltages are balanced even though phase impedances are unbalanced. However, it is worth noting that a break or open ( $Z_N = \infty$ ) in the neutral wire of a 3-phase, 2-wire system with **unbalanced** load always causes large (in most cases inadmissible) changes in currents and phase voltages. It is because of this reason that no fuses and circuit breakers are ever used in the neutral wire of such a 3-phase system.

The solution for currents follows a pattern similar to that for the unbalanced delta.

Obviously, the vector sum of the currents in the three lines is not zero but is equal to neutral current.

**(ii) Neutral wire with impedance  $Z_N$**

Such a case can be easily solved with the help of Node-pair Voltage method as detailed below. Consider the general case of a  $Y$ -to- $Y$  system with a neutral wire of impedance  $Z_N$  as shown in Fig. 19.79 (a). As before, the impedance of line wires and source phase windings would be assumed to be zero so that the line and load terminals,  $R, Y, B$  and  $R', Y', B'$  are the same respective potentials.

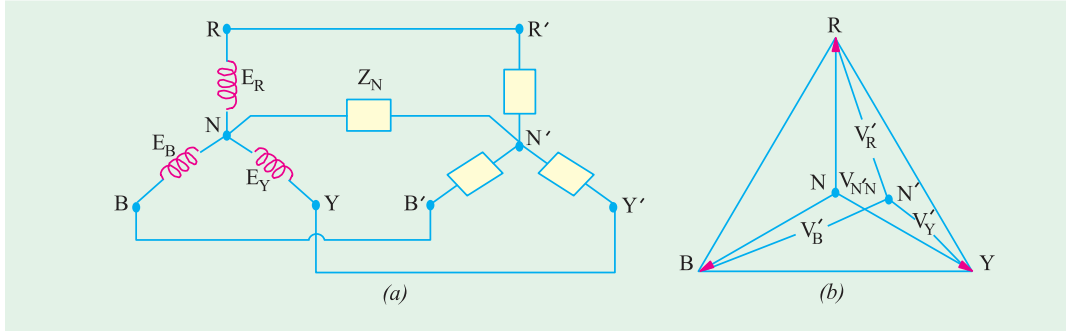


Fig. 19.79

According to Node-pair Voltage method, the above star-to-star system can be looked upon as multi-mesh network with a single pair of nodes *i.e.* neutral points  $N$  and  $N'$ . The node potential *i.e.* the potential difference between the supply and local neutrals is given by

$$V'_{NN} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B + Y_N}$$

where  $Y_R, Y_Y$  and  $Y_B$  represent the **load phase** admittances. Obviously, the load neutral  $N'$  does not coincide with source neutral  $N$ . Hence, load phase voltages are no longer equal to one another even when phase voltages are as seen from Fig. 19.79 (b).

The load phase voltage are given by

$$V'_R = E_R - V'_{NN}; V'_Y = E_Y - V'_{NN} \text{ and } V'_B = E_B - V'_{NN}$$

The phase currents are

$$I_R = V'_R Y_R, I'_Y = V'_Y Y_Y \text{ and } I_B = V'_B Y_B$$

The current in the neutral wire is  $I_N = V'_N Y_N$

**Note.** In the above calculations,  $I_R, I'_R, I_{RR}$

Similarly,  $I_Y = I'_Y = I'_{YY}$  and  $I'_B = I_{BB}$ .

**Example 19.70.** A 3-phase, 4-wire system having a 254-V line-to-neutral has the following loads connected between the respective lines and neutral;  $Z_R = 10 \angle 0^\circ \text{ ohm}$ ;  $Z_Y = 10 \angle 37^\circ \text{ ohm}$  and  $Z_B = 10 \angle -53^\circ \text{ ohm}$ . Calculate the current in the neutral wire and the power taken by each load when phase sequence is (i) RYB and (ii) RBY.

**Solution.** (i) Phase sequence RYB (Fig. 19.80)

$$V_{RN} = 254 \angle 0^\circ; V_{YN} = 254 \angle 120^\circ; V_{BN} = 254 \angle 240^\circ$$

$$I_R = I_{RN} = \frac{V_{RN}}{Z_R} = \frac{254 \angle 0^\circ}{10 \angle 0^\circ} = 25.4 \angle 0^\circ *$$

$$I'_Y = I_{YN} = \frac{254 \angle -120^\circ}{10 \angle 37^\circ} = 25.4 \angle -157^\circ = 25.4(-0.9205 - j0.3907) = -23.38 - j9.95$$

\* This method is similar to Millman's Theorem of Art. 19.32.

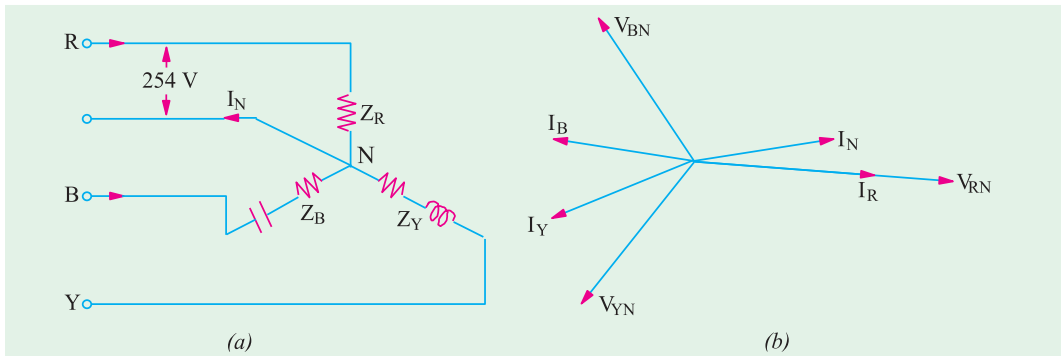


Fig. 19.80

$$I_B = I_{BN} = \frac{254 \angle 120^\circ}{10 \angle -53^\circ} = 25.4 \angle 173^\circ = 25.4(-0.9925 + j0.1219) = -25.2 + j3.1$$

$$I_N = -(I_R + I_Y + I_B) = -[25.4 + (-23.38 - j9.55) + (-25.21 + j3.1)] = 23.49 + j6.85$$

$$= \mathbf{24.46 \angle 16^\circ 15'}$$

Now  $R_R = 10 \Omega$ ;  $R_Y = 10 \cos 37^\circ = 8 \Omega$ ;  $R_B = 10 \cos 53^\circ = 6 \Omega$

$$W_R = 25.4^2 \cdot 10 = \mathbf{6,452 \text{ W}}; \quad W_Y = 25.4^2 \cdot 8 = \mathbf{5,162 \text{ W}}$$

$$W_B = 25.4^2 \cdot 6 = \mathbf{3,871 \text{ W}}$$

(ii) Phase sequence **RBV** [Fig. 19.81]

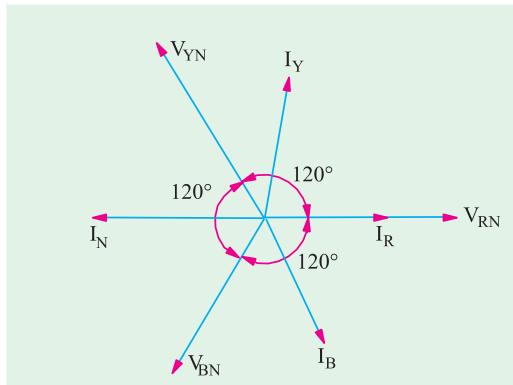


Fig. 19.81

$$V_{RN} = 254 \angle 0^\circ; \quad V_{YN} = 254 \angle 120^\circ$$

$$V_{BN} = 254 \angle 120^\circ$$

$$I_R = \frac{254 \angle 0^\circ}{10 \angle 0^\circ} = 25.4 \angle 0^\circ$$

$$I_Y = \frac{254 \angle 120^\circ}{10 \angle 37^\circ} = 25.4 \angle 83^\circ$$

$$= (3.1 + j25.2) I_B$$

$$I_B = \frac{254 \angle 120^\circ}{10 \angle 53^\circ} = 25.4 \angle 67^\circ = (9.95 + j23.4)$$

$$I_N = -(I_R + I_Y + I_B) = (38.45 - j1.8)$$

$$= -38.45 - j1.8 = 38.5 \angle -177.3^\circ$$

Obviously, power would remain the same because magnitude of branch currents is unaltered. From the above, we conclude that phase reversal in the case of a 4-wire unbalanced load supplied from a balanced voltage system leads to the following changes:

- (i) it changes the angles of phase currents but not their magnitudes.
- (ii) however, power remains unchanged.
- (iii) it changes the magnitude as well as angle of the neutral current  $I_N$

**Example 19.71.** A 3- $\phi$ , 4-wire, 380-V supply is connected to an unbalanced load having phase impedances of:  $Z_R = (8 + j6) \Omega$ ,  $Z_Y = (8 - j6) \Omega$  and  $Z_B = 5 \Omega$ . Impedance of the neutral wire is  $Z_N = (0.5 + j1) \Omega$ .

Ignoring the impedances of line wires and internal impedances of the e.m.f. sources, find the phase currents and voltages of the load.

**Solution.** This question will be solved by using Node-pair Voltage method discussed in Art. 19.30. The admittances of the various branches connected between nodes  $N$  and  $N$  in Fig. 19.82 (a).

$$\begin{aligned}
 Y_R &= 1/Z_R = 1/(8 + j6) = (0.08 - j0.06) \\
 Y_Y &= 1/Z_Y = 1/(8 - j6) = (0.08 + j0.06) \\
 Y_B &= 1/2Z_B = 1/(5 + j0) = 0.2 \\
 Y_N &= 1/Z_N = 1/(0.5 + j1) = (0.4 - j0.8)
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } E_R &= (380/\sqrt{3})\angle 0^\circ = 220\angle 0^\circ = 220 + j0 \\
 E_Y &= 220\angle -120^\circ = 220(-0.5 - j0.866) = -110 - j190 \\
 E_B &= 220\angle 120^\circ = 220(-0.5 + j0.866) = -110 + j190
 \end{aligned}$$

The node voltage between  $N$  and  $N'$  is given by

$$\begin{aligned}
 V'_{NN} &= \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B + Y_N} \\
 &= \frac{200(0.08 - j0.06) + (-110 - j190)(0.08 + j0.06) + (-110 + j190) \times 0.2}{(0.08 - j0.06) + (0.08 + j0.06) + 0.2 + (0.4 - j0.8)} \\
 &= \frac{-1.8 + j3}{0.76 - j0.8} = -3.41 + j0.76
 \end{aligned}$$

The three load phase voltages are as under:

$V_R$	$E_R$	$V'_{NN}$	220	3.41	$j0.76$	223.41	$j0.76$
$V_Y$	$E_Y$	$V'_{NN}$	( 110	$j90$ )	3.41	$j0.76$	106.59 $j190.76$
$V_B$	$E_B$	$V'_{NN}$	( 110	$j90$ )	3.41	$j0.76$	106.59 $j190.76$

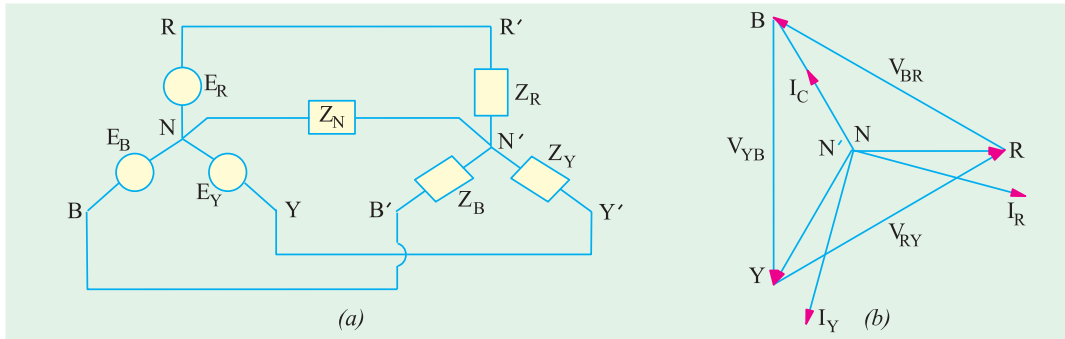


Fig 19.82

$I_R$	$V_R Y_R$	(223.41 $j0.76$ )	(0.08 $j0.06$ )	17.83	$j13.1$	22.1	36.3 A
$I_Y$	$V_Y Y_Y$	( 106.59 $j190.76$ )	(0.08 $j0.06$ )	2.92	$j21.66$	21.86	82.4 A
$I_B$	$V_B Y_B$	( 106.59 $j190.76$ )	0.2	21.33	$j37.85$	43.45	119.4 A
$I_N$	$V_N Y_N$	( 3.41 $j0.76$ )	(0.4 $j0.8$ )	0.76	$j3.03$	3.12	104.1 A

These voltage and currents are shown in the phasor diagram of Fig. 19.82 (b) where displacement of the neutral point has not been shown due to the low value of  $V'_{NN}$ .

**Note.** It can be shown that  $I_N = I'_R + I'_Y + I'_B$

### 19.31. Unbalanced Y-connected Load Without Neutral

When a star-connected load is unbalanced and it has no neutral wire. Then its star point is isolated from the star-point of the generator. The potential of the load star-point is different from that of the generator star-point. The potential of the former is subject to variations according to the imbalance of the load and under certain conditions of loading, the potentials of the two star-point may differ considerably. Such an isolated load star-point or neutral point is called 'floating' neutral point because its potential is always changing and is not fixed.

All  $Y$ -connected unbalanced loads supplied from polyphase systems have floating neutral points without a neutral wire. Any unbalancing of the load causes variations not only of the potential of the star-point but also of the voltages across the different branches of the load. **Hence, in that case, phase voltage of the load is not  $1/\sqrt{3}$  of the line voltage.**

There are many methods to tackle such unbalanced  $Y$ -connected loads having isolated neutral points.

1. By converting the  $Y$ -connected load to an equivalent unbalanced  $\Delta$ -connected load by using  $Y$ - $\Delta$  conversion theorem. The equivalent  $\Delta$ -connection can be solved in Fig. 19.80. The line currents so calculated are equal in magnitude and phase to those taken by the original unbalanced  $Y$ -connected load.

2. By applying Kirchhoff's Laws.

3. By applying Millman's Theorem.

4. By using Maxwell's Mesh or Loop Current Method.

### 19.32. Millman's Theorem

Fig. 19.83 shows a number of linear bilateral admittances,  $Y_1, Y_2, \dots$  connected to a common point or node  $O'$ . The voltages of the free ends of these admittances with respect to another common point  $O$  are  $V_{10}, V_{20}, \dots, V_{n0}$ . Then, according to this theorem, the voltage of  $O'$  with respect to  $O$  is given by

$$V'_{00} = \frac{V_{10}Y_1 + V_{20}Y_2 + V_{30}Y_3 + \dots + V_{n0}Y_n}{Y_1 + Y_2 + Y_3 + \dots + Y_n}$$

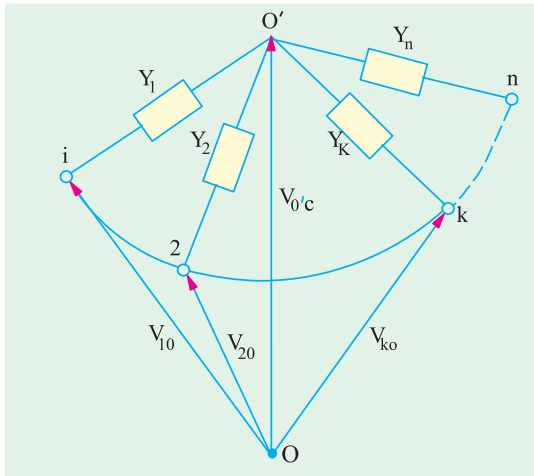


Fig. 19.83

$$(V'_{10} - V'_{00})Y_1 + (V'_{20} - V'_{00})Y_2 + \dots + (V'_{k0} - V'_{00})Y_k + \dots = 0$$

$$\text{or } V'_{10}Y_1 + V'_{20}Y_2 + \dots + V'_{k0}Y_k + \dots = V'_{00}(Y_1 + Y_2 + \dots + Y_k + \dots)$$

$$V'_{00} = \frac{V_{10}Y_1 + V_{20}Y_2 + \dots}{Y_1 + Y_2 + \dots}$$

$$\text{or } V'_{00} = \frac{\sum_{k=1}^n V_{k0}Y_k}{\sum_{k=1}^n Y_k}$$

**Proof.** Consider the closed loop  $O'Ok$ . The sum of p.d.s. around it is zero. Starting from  $O'$  and going anticlockwise, we have

$$V'_{00} + V_{ok} + V'_{ko} = 0$$

$$\therefore V'_{ko} = -V_{ok} - V'_{00} = V_{ko} - V'_{00}$$

$$\text{Current through } Y_k \text{ is } I'_{ko}$$

$$= V'_{ko}Y_k = (V_{ko} - V'_{00})Y_k =$$

By Kirchhoff's Current Law, sum of currents meeting at point  $O'$  is zero.

$$\therefore I'_{10} + I'_{20} + \dots + I'_{k0} + \dots + I'_{n0} = 0$$

### 19.33. Application of Kirchhoff's Laws

Consider the unbalanced  $Y$ -connected load of Fig. 19.84. Since the common point of the three load impedances is not at the potential of the neutral, it is marked  $O'$  instead of  $N^*$ . Let us assume the

\* For the sake of avoiding printing difficulties, we will take the load star point as  $O$  instead of  $O'$  for this article.



phase sequence  $V_{ab}, V_{bc}, V_{ca}$  i.e.  $V_{ab}$  leads  $V_{bc}$  and  $V_{bc}$  leads  $V_{ca}$ . Let the three branch impedances be  $Z_{ca}, Z_{ab}$  and  $Z_{ac}$ , however, since double subscript notation is not necessary for a Y-connected impedances in order to indicate to which phase it belongs, single-subscript notation may be used with advantage. Therefore  $Z_{oa}, Z_{ob}, Z_{oc}$  can be written as  $Z_a, Z_b, Z_c$  respectively. It may be pointed out that double-subscript notation is essential for mesh-connected impedances in order to make them definite.

From Kirchoff's laws, we obtain

$$V_{ab} = I_{ao}Z_a + I_{ab}Z_b \quad \dots (1)$$

$$V_{bc} = I_{bo}Z_b + I_{oc}Z_c \quad \dots (2)$$

$$V_{ca} = I_{co}Z_c + I_{oa}Z_a \quad \dots (3)$$

$$\text{and } I_{aa} + I_{ba} + I_{co} = 0 \text{ - point law} \quad \dots (4)$$

Equation (1), (3) and (4) can be used for finding  $I_{bo}$ .

Adding (1) and (3), we get

$$\begin{aligned} V_{ab} + V_{ca} &= I_{ao}Z_a + I_{ao}Z_b + I_{co}Z_c + I_{oc}Z_a \\ &= I_{ob}Z_b + I_{co}Z_c + I_{oa}Z_a - I_{oa}Z_a = I_{ob}Z_b + I_{co}Z_c \end{aligned} \quad \dots (5)$$

Substituting  $I_{oa}$  from equation (4) in equation (3), we get

$$V_{ca} = I_{co}Z_c + (I_{bo} + I_{co})Z_a = I_{co}(Z_c + Z_a) + I_{bo}Z_a \quad \dots (6)$$

Putting the value of  $I_{co}$  from equation (5) in equation (6), we have

$$V_{ca} = (Z_c + Z_a) \frac{(V_{ab} + V_{ca}) - I_{ob}Z_b}{Z_c} + I_{bo}Z_a$$

$$V_{ca}Z_c = -I_{ob}Z_bZ_c - I_{ob}Z_bZ_a + I_{bo}Z_aZ_c + V_{ab}Z_a + V_{ab}Z_c + V_{ca}Z_c + V_{ca}Z_a$$

$$\therefore I_{ob} = \frac{(V_{ab} + V_{ca})Z_a + V_{ab}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$$

$$\text{Since } V_{ab} + V_{bc} + V_{ca} = 0 \quad \therefore I_{ob} = \frac{V_{ab}Z_c - V_{bc}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \dots (7)$$

From the symmetry of the above equation, the expressions for the other branch currents are,

$$I_{oc} = \frac{V_{bc}Z_a - V_{ca}Z_b}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \dots (8) \quad I_{oa} = \frac{V_{ca}Z_b - V_{ab}Z_c}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \dots (9)$$

**Note.** Obviously, the three line currents can be written as

$$I_{ao} = -I_{oa} = \frac{V_{ab}Z_c - V_{ca}Z_b}{\sum Z_aZ_b} \quad \dots (10) \quad I_{bo} = -I_{ob} = \frac{V_{bc}Z_a - V_{ab}Z_c}{\sum Z_aZ_b} \quad \dots (11)$$

$$I_{co} = -I_{oc} = \frac{V_{ca}Z_b - V_{bc}Z_a}{\sum Z_aZ_b} \quad \dots (12)$$

**Example 19.72.** If in the unbalanced Y-connected load of Fig. 19.78,  $Z_a = (10 + j0)$ ,  $Z_b = (3 + j4)$  and  $Z_c = (0 - j10)$  and the load is put across a 3-phase, 200-V circuit with balanced voltages, find the three line currents and voltages across each branch impedance. Assume phase sequence of  $V_{ab}, V_{bc}, V_{ca}$ .

**Solution.** Take  $V_{ab}$  along the axis of reference. The vector expressions for the three voltages

are 
$$V_{ab} = 200 + j0$$

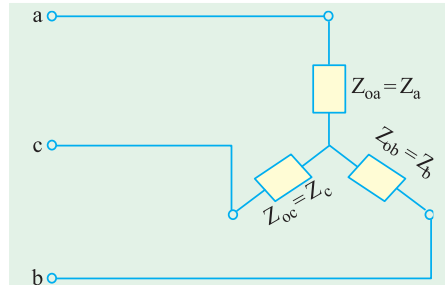


Fig. 19.84

$$V_{bc} = 200 \frac{1}{2} j \frac{\sqrt{3}}{2} = 100 j173.2; \quad V_{ca} = 200 \frac{1}{2} j \frac{\sqrt{3}}{2} = 100 j173.2$$

From equation (9) given above

$$\begin{aligned} I_{oa} &= \frac{(100 - j173.2)(3 - j4) - (200 - j0)(0 - j10)}{(0 - j10)(10 - j0) - (10 - j0)(3 - j4) - (3 - j4)(0 - j10)} \\ &= \frac{-992.8 + j2119.6}{70 - j90} = -20.02 + j4.54 \\ I_{ob} &= \frac{(200 - j0)(0 - j10) - (100 - j173.2)(10 - j0)}{(10 - j0)(3 - j4) - (3 - j4)(0 - j10) - (0 - j10)(10 - j0)} \\ &= \frac{1000 - j268}{70 - j90} = 7.24 + j5.48 \end{aligned}$$

Now,  $I_{oc}$  may also be calculated in the same way or it can be found easily from equation (4) of Art. 19.33.

$$\begin{aligned} \text{Now } I_{oc} &= I_{ao} + I_{bo} = -I_{oa} - I_{ob} = 20.02 - j4.54 - 7.24 - j5.48 = 12.78 - j10.02 \\ V_{oa} &= I_{oa} Z_a = (-20.02 + j4.54)(10 + j0) = 200.2 + j45.4 \\ V_{ob} &= I_{ob} Z_b = (7.24 + j5.48)(3 + j4) = -0.2 + j45.4 \\ V_{oc} &= I_{oc} Z_c = (12.78 - j10.02)(0 - j10) = -100.2 - j127.8 \end{aligned}$$

As a check, we may combine  $V_{oa}$ ,  $V_{ob}$  and  $V_{oc}$  to get the line voltages which should be equal to the applied line voltages. In passing from  $a$  to  $b$  through the circuit internally, we find that we are in opposition to  $V_{oa}$  but in the same direction as the positive direction of  $V_{ob}$ .

$$\begin{aligned} V_{ab} &= V_{ao} - V_{ob} = (200.2 - j45.4) - (-0.2 + j45.4) = 200 - j90.8 \\ V_{bc} &= V_{bo} - V_{oc} = (-0.2 + j45.4) - (-100.2 - j127.8) = 100 + j173.2 \\ V_{ca} &= V_{co} - V_{oa} = (-100.2 - j127.8) - (200.2 + j45.4) = -300.4 - j173.2 \end{aligned}$$

### 19.34. Delta/Star and Star/Delta Conversions

Let us consider the unbalanced  $\Delta$ -connected load of Fig. 19.85 (a) and  $Y$ -connected load of Fig. 19.85 (b). If the two systems are to be equivalent, then the impedances between corresponding pairs of terminals must be the same.

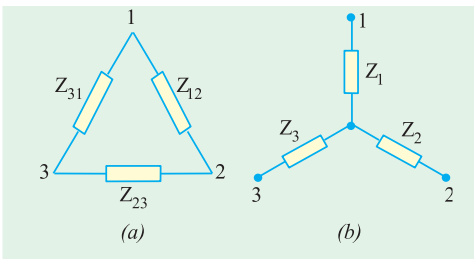


Fig. 19.85

#### (i) Delta/Star Conversion

For  $Y$ -load, total impedance between terminals 1 and 2 is  $Z_1 + Z_2$  (it should be noted that double subscript notation of  $Z_{01}$  and  $Z_{02}$  has been purposely avoided).

Considering terminals 1 and 2 of  $\Delta$ -load, we find that there are two parallel paths having impedances of  $Z_{12}$  and  $(Z_{31} + Z_{23})$ . Hence, the equivalent impedance between terminals 1 and 2 is given by

$$\frac{1}{Z} = \frac{1}{Z_{12}} + \frac{1}{Z_{23} + Z_{31}} \quad \text{or} \quad Z = \frac{Z_{12}(Z_{23} + Z_{31})}{Z_{12} + Z_{23} + Z_{31}}$$

$$\text{Therefore, for equivalence between the two systems } Z_1 + Z_2 = \frac{Z_{12}(Z_{23} + Z_{31})}{Z_{12} + Z_{23} + Z_{31}} \quad \dots (1)$$

$$\text{Similarly } \mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_{23}(\mathbf{Z}_{31} + \mathbf{Z}_{12})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \dots (2) \quad \mathbf{Z}_3 + \mathbf{Z}_1 = \frac{\mathbf{Z}_{31}(\mathbf{Z}_{12} + \mathbf{Z}_{23})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \dots (3)$$

Adding equation (3) to (1) and subtracting equation (2), we get

$$2\mathbf{Z}_1 = \frac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31}) + \mathbf{Z}_{31}(\mathbf{Z}_{12} + \mathbf{Z}_{23}) - \mathbf{Z}_{23}(\mathbf{Z}_{31} + \mathbf{Z}_{12})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} = \frac{2\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$$

$$\therefore \mathbf{Z}_1 = \frac{\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \dots (4)$$

The other two results may be written down by changing the subscripts cyclically

$$\therefore \mathbf{Z}_2 = \frac{\mathbf{Z}_{23}\mathbf{Z}_{12}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}; \quad \dots (5) \quad \mathbf{Z}_3 = \frac{\mathbf{Z}_{31}\mathbf{Z}_{23}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \dots (6)$$

The above expression can be easily obtained by remembering that (Art. 2.19)

$$\text{Star } \mathbf{Z} = \frac{\text{Product of } \Delta \mathbf{Z}'\text{'s connected to the same terminals}}{\text{Sum of } \Delta \mathbf{Z}'\text{'s}}$$

It should be noted that all  $\mathbf{Z}'$  are to be expressed in their complex form.

### (iii) Star/Delta Conversion

The equations for this conversion can be obtained by rearranging equations (4), (5) and (6), Rewriting these equations, we get

$$\mathbf{Z}_1(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{12}\mathbf{Z}_{31} \quad \dots (7)$$

$$\mathbf{Z}_2(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{23}\mathbf{Z}_{12} \quad \dots (8)$$

$$\mathbf{Z}_3(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{31}\mathbf{Z}_{23} \quad \dots (9)$$

$$\text{Dividing equation (7) by (9), we get } \frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_{12}}{\mathbf{Z}_{23}} \quad \therefore \mathbf{Z}_{23} = \mathbf{Z}_{12} \frac{\mathbf{Z}_3}{\mathbf{Z}_1}$$

$$\text{Dividing equation (8) by (9), we get } \frac{\mathbf{Z}_2}{\mathbf{Z}_3} = \frac{\mathbf{Z}_{12}}{\mathbf{Z}_{31}} \quad \therefore \mathbf{Z}_{31} = \mathbf{Z}_{12} \frac{\mathbf{Z}_3}{\mathbf{Z}_2}$$

$$\text{Substituting these values in equation (7), we have } \mathbf{Z}_1 \mathbf{Z}_{12} \left( 1 + \frac{\mathbf{Z}_3}{\mathbf{Z}_1} + \frac{\mathbf{Z}_{31}}{\mathbf{Z}_{12}} \right) = \mathbf{Z}_{12}\mathbf{Z}_{12} \frac{\mathbf{Z}_3}{\mathbf{Z}_2} \quad \therefore \mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \mathbf{Z}_{12} \times \mathbf{Z}_3$$

$$\text{or } \mathbf{Z}_1\mathbf{Z}_{12} \left( 1 + \frac{\mathbf{Z}_3}{\mathbf{Z}_1} + \frac{\mathbf{Z}_{31}}{\mathbf{Z}_{12}} \right) = \mathbf{Z}_{12}\mathbf{Z}_{12} \frac{\mathbf{Z}_3}{\mathbf{Z}_2} \quad \therefore \mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \mathbf{Z}_{12} \times \mathbf{Z}_3$$

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \quad \text{or } \mathbf{Z}_{12} = \mathbf{Z}_1 + \mathbf{Z}_2 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_3}$$

$$\text{Similarly, } \mathbf{Z}_{23} = \mathbf{Z}_2 + \mathbf{Z}_3 + \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_{31} = \mathbf{Z}_3 + \mathbf{Z}_1 + \frac{\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

As in the previous case, it is to be noted that all impedances must be expressed in their complex form.

Another point for noting is that *the line currents of this equivalent delta are the currents in the phases of the Y-connected load.*

**Example 19.73.** An unbalanced star-connected load has branch impedances of  $Z_1 = 10 \angle 30^\circ \Omega$ ,  $Z_2 = 10 \angle -45^\circ \Omega$ ,  $Z_3 = 20 \angle 60^\circ \Omega$  and is connected across a balanced 3-phase, 3-wire supply of 200 V. Find the line currents and the voltage across each impedance using Y/Δ conversion method.

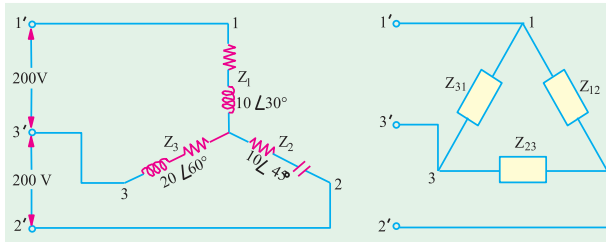


Fig. 19.86

**Solution.** The unbalanced Y-connected load and its equivalent  $\Delta$ -connected load are shown in Fig. 19.86.

$$\text{Now } \mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = (10 \angle 30^\circ)(10 \angle -45^\circ) + (10 \angle -45^\circ)(20 \angle 60^\circ) + (20 \angle 60^\circ)(10 \angle 30^\circ) = 100 \angle -15^\circ + 200 \angle 15^\circ + 200 \angle 90^\circ$$

Converting these into their cartesian

form, we get

$$= 100 [\cos(-15^\circ) - j \sin 15^\circ] + 200 (\cos 15^\circ + j \sin 15^\circ) + 200 (\cos 90^\circ + j \sin 90^\circ) \\ = 96.6 - j25.9 + 193.2 + j51.8 + 0 + j200 = 289.8 + j225.9 = 368 \angle 38^\circ$$

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2 \quad \mathbf{Z}_2\mathbf{Z}_3 \quad \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \quad \frac{368 \quad 38}{20 \quad 60} \quad 18.4 \quad 22 \quad 17.0 \quad j6.9$$

$$\mathbf{Z}_{23} = \frac{368 \quad 38}{10 \quad 30} \quad 36.8 \quad 8 \quad 36.4 \quad j5.1$$

$$\mathbf{Z}_{31} = \frac{368 \quad 38}{10 \quad 45} \quad 36.8 \quad 83 \quad 4.49 \quad j36.5$$

Assuming clockwise phase sequence of voltages  $\mathbf{V}_{12}$ ,  $\mathbf{V}_{23}$  and  $\mathbf{V}_{31}$ , we have

$$\mathbf{V}_{12} = 200 \quad 0, \quad \mathbf{V}_{23} = 200 \quad 120, \quad \mathbf{V}_{31} = 200 \quad 120$$

$$\mathbf{I}_{12} = \frac{\mathbf{V}_{12}}{\mathbf{Z}_{12}} = \frac{200 \quad 0}{18.4 \quad 22} \quad 10.86 \quad 22 \quad 10.07 \quad j4.06$$

$$\mathbf{I}_{23} = \frac{\mathbf{V}_{23}}{\mathbf{Z}_{23}} = \frac{200 \quad 120}{36.8 \quad 8} \quad 5.44 \quad 128 \quad 3.35 \quad j4.29$$

$$\mathbf{I}_{31} = \frac{\mathbf{V}_{31}}{\mathbf{Z}_{31}} = \frac{200 \quad 120}{36.8 \quad 83} \quad 5.44 \quad 37 \quad 4.34 \quad j3.2$$

$$\text{Line current} = \mathbf{I}'_{11} = \mathbf{I}_{12} + \mathbf{I}_{13} = \mathbf{I}_{12} - \mathbf{I}_{31} \\ = (10.07 + j4.06) - (4.34 + j3.2) = 5.73 + j0.86 = 5.76 \angle 8^\circ 32'$$

$$\mathbf{I}_{22} \quad \mathbf{I}_{23} \quad \mathbf{I}_{12} \quad (3.35 \quad j4.29) \quad (10.07 \quad j4.06) \quad 13.42 \quad j8.35 \quad 15.79 \quad 148 \quad 6$$

$$\mathbf{I}_{33} \quad \mathbf{I}_{31} \quad \mathbf{I}_{23} \quad (4.34 \quad j3.2) \quad (3.35 \quad j4.29) \quad 7.69 \quad j7.49 \quad 10.73 \quad 44 \quad 16$$

These are currents in the *phases of the Y-connected unbalanced load*. Let us find voltage drop across each star-connected branch impedance.

$$\text{Voltage drop across } \mathbf{Z}_1 = \mathbf{V}_{10} = \mathbf{I}_{11} \mathbf{Z}_1 \quad 5.76 \quad 8 \quad 32 \quad .10 \quad 30 \quad 57.6 \quad 38 \quad 32$$

$$\text{Voltage drop across } \mathbf{Z}_2 = \mathbf{V}_{20} = \mathbf{I}_{22} \mathbf{Z}_2 \quad 15.79 \quad 148 \quad 6.10 \quad 45 \quad 157.9 \quad 193 \quad 6$$

$$\text{Voltage drop across } \mathbf{Z}_3 = \mathbf{V}_{30} = \mathbf{I}_{33} \mathbf{Z}_3 \quad 10.73 \quad 44 \quad 16 \quad .20 \quad 60 \quad 214.6 \quad *104 \quad 16$$

**Example 19.74.** A 300-V (line) 3-phase supply feeds a star-connected load consisting of non-inductive resistors of 15, 6 and 10  $\Omega$  connected to the R, Y and B lines respectively. The phase sequence is RYB. Calculate the voltage across each resistor.

**Solution.** The Y-connected unbalanced load and its equivalent  $\Delta$ -connected load are shown in Fig. 19.87. Using Y/ $\Delta$  conversion method we have

$$\begin{aligned} Z_{12} &= \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} \\ &= \frac{90 + 60 + 150}{10} = 30 \Omega \end{aligned}$$

$$Z_{23} = 300/15 = 20$$

$$Z_{31} = 300/6 = 50$$

$$\text{Phase current } I_{RY} = V_{RY} / Z_{12} = 300 / 30 = 10 \text{ A}$$

$$\text{Similarly } I_{YB} = V_{YB} / Z_{23} = 300 / 20 = 15 \text{ A}$$

$$I_{BR} = V_{BR} / Z_{31} = 300 / 50 = 6 \text{ A}$$

Each current is in phase with its own voltage because the load is purely resistive.

The line currents for the delta connection are obtained by compounding these phase currents in pairs, either trigonometrically or by phasor algebra. Using phasor algebra and choosing  $V_{RY}$  as the reference axis, we get

$$I_{RY} = 10 \angle 0^\circ; I_{YB} = 15 \left( \frac{1}{2} - j\frac{\sqrt{3}}{2} \right) = 7.5 - j13.0; I_{BR} = 6 \left( \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = 3.0 + j5.2$$

Line currents for delta-connection [Fig. 19.66 (b)] are

$$I_R = I_{RY} - I_{BR} = (10 \angle 0^\circ) - (3.0 + j5.2) = 7.0 - j5.2 \text{ or } 14 \text{ A in magnitude}$$

$$I_Y = I_{YB} - I_{RY} = (7.5 - j13.0) - (10 \angle 0^\circ) = -2.5 - j13.0 \text{ or } 13.7 \text{ A in magnitude}$$

$$I_B = I_{BR} - I_{YB} = (3.0 + j5.2) - (7.5 - j13.0) = -4.5 + j18.2 \text{ or } 18.7 \text{ A in magnitude}$$

These line currents for  $\Delta$ -connection are the phase currents for Y-connection. Voltage drop across each limb of Y-connected load is

$$V_{RN} = I_R Z_1 = (7.0 - j5.2)(15 \angle 0^\circ) = 105 - j78 \text{ volt or } 131 \text{ V}$$

$$V_{YN} = I_Y Z_2 = (-2.5 - j13.0)(6 \angle 0^\circ) = -15 - j78 \text{ volt or } 131 \text{ V}$$

$$V_{BN} = I_B Z_3 = (-4.5 + j18.2)(10 \angle 0^\circ) = -45 + j182 \text{ volt or } 187 \text{ V}$$

As a check, it may be verified that the difference of phase voltages taken in pairs should give the three line voltages. Going through the circuit internally, we have

$$V_{RY} = V_{RN} - V_{YN} = (105 - j78) - (-15 - j78) = 120 \angle 0^\circ$$

$$V_{YB} = V_{YN} - V_{BN} = (-15 - j78) - (-45 + j182) = 30 - j260 = 300 \angle -120^\circ$$

$$V_{BR} = V_{BN} - V_{RN} = (-45 + j182) - (105 - j78) = -150 + j260 = 300 \angle 120^\circ$$

This question could have been solved by direct geometrical methods as shown in Ex. 19.52.

**Example 19.75** A Y-connected load is supplied from a 400-V, 3-phase, 3-wire symmetrical system RYB. The branch circuit impedances are

$$Z_R = 10\sqrt{3} + j10; Z_Y = 20 + j20\sqrt{3}; Z_B = 0 - j10$$

Determine the current in each branch. Phase sequence is RYB.

(Network Analysis, Nagpur Univ. 1993)

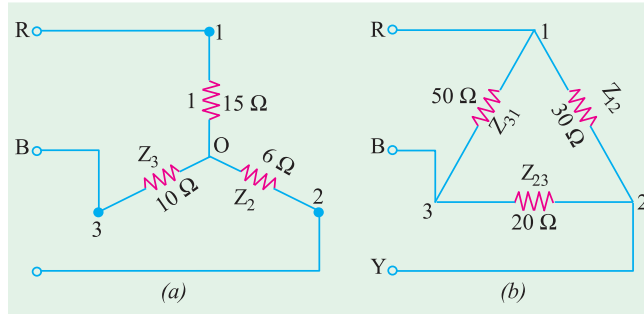


Fig. 19.87

**Solution.** The circuit is shown in Fig. 19.88. The problem will be solved by using all the four possible ways in which 3-wire unbalanced Y connected load can be handled.

$$\text{Now, } \mathbf{Z}_R \quad 20 \quad 30 \quad (17.32 \quad j10)$$

$$\mathbf{Z}_Y \quad 40 \quad 60 \quad (20 \quad j34.64)$$

$$\mathbf{Z}_B = 10 \angle -90^\circ = -j10$$

$$\text{Also, let } \mathbf{V}_{RY} \quad 400 \quad 0 \quad 400 \quad j0$$

$$\mathbf{V}_{RB} = 400 \angle -120^\circ = -200 - j346$$

$$\mathbf{V}_R = 400 \angle 120^\circ = -200 + j346$$

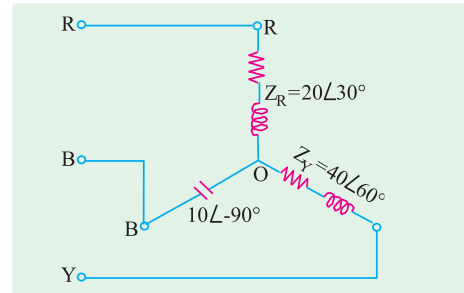


Fig. 19.88

**(a) By applying Kirchoff's Laws**

With reference to Art. 19.33, it is seen that

$$\mathbf{I}_{RO} = \mathbf{I}_R = \frac{\mathbf{V}_{RY}\mathbf{Z}_B - \mathbf{V}_{BR}\mathbf{Z}_Y}{\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R}; \mathbf{I}_{YO} = \mathbf{I}_Y = \frac{\mathbf{V}_{YB}\mathbf{Z}_R - \mathbf{V}_{RY}\mathbf{Z}_B}{\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R};$$

$$\mathbf{I}_{BO} = \mathbf{I}_B = \frac{\mathbf{V}_{BR}\mathbf{Z}_Y - \mathbf{V}_{YB}\mathbf{Z}_R}{\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R}$$

$$\text{Now, } \mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R$$

$$= 20 \angle 30^\circ \cdot 40 \angle 60^\circ + 40 \angle 60^\circ \cdot 10 \angle -90^\circ + 10 \angle -90^\circ \cdot 20 \angle 30^\circ$$

$$= 800 \angle 90^\circ + 400 \angle -30^\circ + 200 \angle -60^\circ = 446 + j426 = 617 \angle 43.7^\circ$$

$$\mathbf{V}_{RY}\mathbf{Z}_B \quad \mathbf{V}_{BR}\mathbf{Z}_Y \quad 400 \quad 10 \quad 90 \quad 400 \quad 120 \quad 40 \quad 60$$

$$= 16,000 - j4000 = 16,490 \angle -14^\circ 3'$$

$$\therefore \mathbf{I}_R \quad \frac{16,490 \quad 14 \quad 3}{617 \quad 43.7} \quad 26.73 \quad 57 \quad 45$$

$$\mathbf{V}_{YB}\mathbf{Z}_R \quad \mathbf{V}_{RY}\mathbf{Z}_B \quad 400 \quad 120 \quad 20 \quad 30 \quad 400 \quad 10 \quad 90 \quad j4000 \quad 4000 \quad 90$$

$$\mathbf{I}_Y \quad \frac{4000 \quad 90}{617 \quad 43.7} \quad 6.48 \quad 133.7$$

$$\mathbf{V}_{BR}\mathbf{Z}_Y \quad \mathbf{V}_{YB}\mathbf{Z}_R \quad 400 \quad 120 \quad 40 \quad 60 \quad 400 \quad 120 \quad 20 \quad 30$$

$$= -16,000 + j8,000 = 17,890 \angle 153^\circ 26'$$

$$\therefore \mathbf{I}_B \quad \frac{17,890 \quad 153 \quad 26}{617 \quad 43.7} \quad 29 \quad 109 \quad 45$$

**(b) By Star/Delta Conversion (Fig. 19.89)**

The given star may be converted into the equivalent delta with the help of equations given in Art. 19.34.

$$\mathbf{Z}_{RY} \quad \frac{\mathbf{Z}_R\mathbf{Z}_Y \quad \mathbf{Z}_Y\mathbf{Z}_B \quad \mathbf{Z}_B\mathbf{Z}_R}{\mathbf{Z}_B} \quad \frac{617 \quad 43.7}{10 \quad 90} \quad 61.73 \quad 133.7$$

$$\mathbf{Z}_{YB} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_R} \quad \frac{617 \quad 43.7}{20 \quad 30} \quad 30.87 \quad 13.7$$

$$\mathbf{Z}_{BR} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_Y} \quad \frac{617 \quad 43.7}{40 \quad 60} \quad 15.43 \quad 16.3$$

$$\mathbf{I}_{RY} \quad \frac{\mathbf{V}_{RY}}{\mathbf{Z}_{RY}} \quad \frac{400}{61.73 \quad 133.7} \quad 6.48 \quad 133.7 \quad (4.47 \quad j4.68)$$

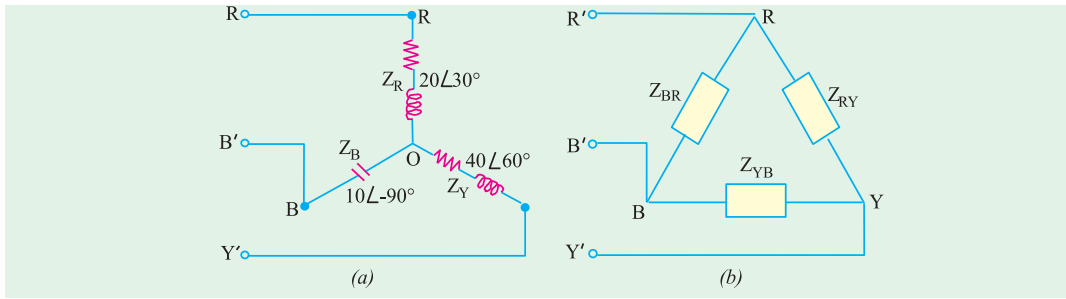


Fig. 19.89

$$\mathbf{I}_{YB} = \frac{\mathbf{V}_{YB}}{\mathbf{Z}_{YB}} = \frac{400}{30.87} \frac{120}{13.7} = 12.95 \angle 133.7^\circ = (8.95 - j9.35)$$

$$\mathbf{I}_{BR} = \frac{\mathbf{V}_{BR}}{\mathbf{Z}_{BR}} = \frac{400}{15.43} \frac{120}{16.3} = 25.9 \angle 136.3^\circ = (18.7 - j17.9)$$

$$\mathbf{I}_{RR} \quad \mathbf{I}_{RY} \quad \mathbf{I}_{BR} \quad 14.23 \quad j22.58 \quad 26.7 \quad 57.48$$

$$\mathbf{I}_{YY} \quad \mathbf{I}_{YB} \quad \mathbf{I}_{RY} \quad 4.48 \quad j4.67 \quad 6.47 \quad 134.6$$

$$\mathbf{I}_{YB} \quad \mathbf{I}_{BR} \quad \mathbf{I}_{YB} \quad 9.85 \quad j27.25 \quad 29 \quad 109.48$$

$$\mathbf{I} = (0 + j0) \quad \text{— as a check}$$

As explained in Art. 19.34, these line currents of the equivalent delta represent the phase currents of the star-connected load of Fig. 19.89 (a).

**Note.** Minor differences are due to accumulated errors.

**(c) By Using Maxwell's Loop Current Method**

Let the loop or mesh currents be as shown in Fig. 19.90. It may be noted that

$$\mathbf{I}_R = \mathbf{I}_1; \mathbf{I}_Y = \mathbf{I}_2 - \mathbf{I}_1 \text{ and } \mathbf{I}_B = -\mathbf{I}_2$$

Considering the drops across R and Y-arms, we get

$$\mathbf{I}_1 \mathbf{Z}_R + \mathbf{Z}_Y (\mathbf{I}_1 - \mathbf{I}_2) = \mathbf{V}_{RY} \quad \dots (i)$$

or

$$\mathbf{I}_1 (\mathbf{Z}_R + \mathbf{Z}_Y) - \mathbf{I}_2 \mathbf{Z}_Y = \mathbf{V}_{RY} \quad \dots (i)$$

Similarly, considering the legs Y and B, we have

$$\mathbf{Z}_Y (\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{Z}_B \mathbf{I}_2 = \mathbf{V}_{YB}$$

or

$$-\mathbf{I}_1 \mathbf{Z}_Y + \mathbf{I}_2 (\mathbf{Z}_B + \mathbf{Z}_Y) = \mathbf{V}_{YB} \quad \dots (ii)$$

Solving for  $\mathbf{I}_1$  and  $\mathbf{I}_2$ , we get

$$\mathbf{I}_1 = \frac{\mathbf{V}_{RY} (\mathbf{Z}_Y + \mathbf{Z}_B) + \mathbf{Z}_{YB} \mathbf{V}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y) (\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2};$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{YB} (\mathbf{Z}_R + \mathbf{Z}_Y) + \mathbf{V}_{RY} \mathbf{Z}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y) (\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2}$$

$$\begin{aligned} \mathbf{I}_1 &= \frac{400(20 - j24.64) + 400 \cdot 120 \cdot 40 \cdot 60}{(37.32 - j44.64)(20 + j24.64) - 1600 \cdot 120} \\ &= \frac{16,000 - j4,000}{448 + j427} = \frac{16,490 \angle -14^\circ 3'}{617 \angle 43.7^\circ} \\ &= 26 \angle -57^\circ 45' = (13.9 - j22) \end{aligned}$$

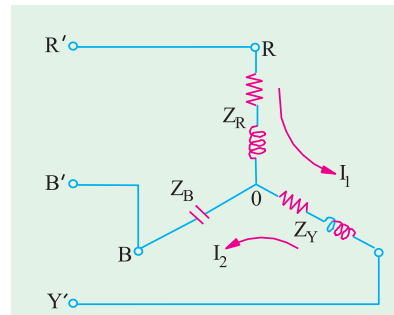


Fig. 19.90

$$\begin{aligned} \mathbf{I}_2 &= \frac{(200 - j346)(37.32 - j44.64) - 400(20 - j34.64)}{484 - j427} \\ &= \frac{16,000 - j8,000}{448 + j427} = \frac{17,890 \angle -26^\circ 34'}{617 \angle 43.7^\circ} = 28.4 \angle -70^\circ 16' \\ &= 28.4 \angle -70^\circ 16' = (9.55 - j26.7) \end{aligned}$$

$$\begin{aligned} \therefore \mathbf{I}_R &= \mathbf{I}_1 - \mathbf{I}_2 = 26 - 57 - j45 \\ \mathbf{I}_Y &= \mathbf{I}_2 - \mathbf{I}_1 = (9.55 - j26.7) - (13.9 - j22) = 4.35 - j4.7 = 6.5 \angle 134^\circ \\ \mathbf{I}_B &= \mathbf{I}_2 - \mathbf{I}_1 = 28.4 - 70 - j16 = 28.4 \angle 109.44^\circ \end{aligned}$$

**(d) By Using Millman's Theorem\***

According to this theorem, the voltage of the load star point  $O'$  with respect to the star point or neutral  $O$  of the generator or supply (normally zero potential) is given by

$$\mathbf{V}_{O'O} = \frac{\mathbf{V}_{RO}\mathbf{Y}_R + \mathbf{V}_{YO}\mathbf{Y}_Y + \mathbf{V}_{BO}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

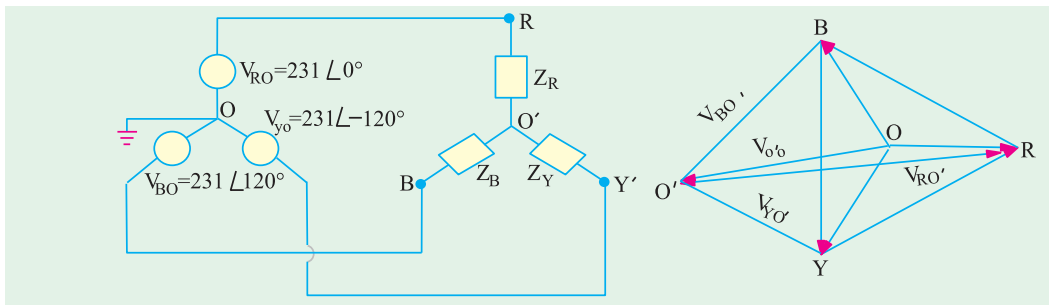


Fig. 19.91

where  $\mathbf{V}_{RO}$ ,  $\mathbf{V}_{YO}$  and  $\mathbf{V}_{BO}$  are the phase voltages of the generator or 3-phase supply.

As seen from Fig. 19.91, voltage across each phase of the load is

$$\mathbf{V}_{RO'} = \mathbf{V}_{RO} - \mathbf{V}_{O'O} \quad \mathbf{V}_{YO'} = \mathbf{V}_{YO} - \mathbf{V}_{O'O} \quad \mathbf{V}_{BO'} = \mathbf{V}_{BO} - \mathbf{V}_{O'O}$$

Obviously,  $\mathbf{I}_{RO'} = (\mathbf{V}_{RO} - \mathbf{V}_{O'O})\mathbf{Y}_R$ ;  $\mathbf{I}_{YO'} = (\mathbf{V}_{YO} - \mathbf{V}_{O'O})\mathbf{Y}_Y$  and

$$\mathbf{I}_{BO'} = (\mathbf{V}_{BO} - \mathbf{V}_{O'O})\mathbf{Y}_B$$

$$\text{Here } \mathbf{Y}_R = \frac{1}{20 - j30} = 0.05 + j0.03 \quad (0.0433 - j0.025)$$

$$\mathbf{Y}_Y = \frac{1}{40 - j60} = 0.025 + j0.0375 \quad (0.0125 - j0.0217)$$

$$\mathbf{Y}_B = \frac{1}{10 - j90} = 0.1 + j0.09 \quad (0.1 - j0.09)$$

\* Incidentally, it may be noted that the p.d. between load neutral and supply neutral is given by

$$\mathbf{V}_{O'O} = \frac{\mathbf{V}_{RO'} + \mathbf{V}_{YO'} + \mathbf{V}_{BO'}}{3}$$



$$\therefore \mathbf{Y}_R \ \mathbf{Y}_Y \ \mathbf{Y}_B \quad 0.0558 \quad j0.0533 \quad 0.077 \quad 43.7$$

$$\text{Let} \quad \mathbf{V}_{RO} = \frac{400}{\sqrt{3}} \quad 0 \quad (231 \quad j0)$$

$$\mathbf{V}_{BO} = 231 \quad 120 \quad 115.5 \quad j200$$

$$\mathbf{V}_{BO} = 231 \quad 120 \quad 115.5 \quad j200$$

$$\mathbf{V}_{OO} = \frac{231.0.05 \quad 30 \quad 231 \quad 120 \quad .025 \quad 60 \quad 231 \quad 120 \quad .01 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{-15.8 - j17.32}{0.077 \angle 43.7^\circ} = \frac{23.5 \angle -132.4^\circ}{0.077 \angle 43.7^\circ} = 305 \angle -176.1^\circ = (304.5 - j20.8)$$

$$\mathbf{V}_{RO} \quad \mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 231 \quad (304.5 \quad j20.8) \quad 535.5 \quad j20.8 \quad 536 \quad 2.2$$

$$\mathbf{V}_{YO} \quad (115.5 \quad j200) \quad (304.5 \quad j20.8) \quad 189 \quad j179 \quad 260 \quad 43 \quad 27$$

$$\mathbf{V}_{BO} \quad (115.5 \quad j200) \quad (304.5 \quad j20.8) \quad 189 \quad j221 \quad 291 \quad 49 \quad 27$$

$$\therefore \mathbf{I}_{RO} \quad 536 \quad 2.2 \quad 0.05 \quad 30 \quad 26.5 \quad 27.8$$

$$\mathbf{I}_{YO} \quad 260 \quad 43 \quad 27 \quad 0.025 \quad 60 \quad 6.5 \quad 103 \quad 27$$

$$\mathbf{I}_{BO} \quad 291 \quad 49 \quad 27 \quad 0.1 \quad 90 \quad 29.1 \quad 139 \quad 27$$

**Note.** As seen from above,  $\mathbf{V}'_{RO} = \mathbf{V}'_{RO} - \mathbf{V}'_{OO}$

Substituting the value of  $\mathbf{V}'_{OO}$ , we have

$$\begin{aligned} \mathbf{V}_{RO} &= \mathbf{V}_{RO} - \frac{\mathbf{V}_{RO} \mathbf{Y}_R + \mathbf{V}_{YO} \mathbf{Y}_Y + \mathbf{V}_{BO} \mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B} \\ &= \frac{(\mathbf{V}_{RO} - \mathbf{V}_{YO}) \mathbf{Y}_Y + (\mathbf{V}_{RO} - \mathbf{V}_{BO}) \mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B} \\ &= \frac{\mathbf{V}_{RY} \mathbf{Y}_Y + \mathbf{V}_{RB} \mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B} \end{aligned}$$

Since  $\mathbf{V}_{RO}$  is taken as the reference vector, then as seen from Fig. 19.92.

$$\mathbf{V}_{RY} \quad 400 \quad 30 \quad \text{and} \quad \mathbf{V}_{RB} \quad 400 \quad 30$$

$$\therefore \mathbf{V}_{RO} = \frac{400 \quad 30 \quad 0.025 \quad 60 \quad 400 \quad 30 \quad 0.1 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{28.6 + j29.64}{0.077 \angle 43.7^\circ} = \frac{41 \angle 46^\circ}{0.077 \angle 43.7^\circ} = 532.5 \angle 2.3^\circ$$

$$\mathbf{I}_{RO} \quad \mathbf{V}_{RO} \mathbf{Y}_R \quad 532.5 \quad 2.3 \quad 0.05 \quad 30 \quad 26.6 \quad 27.7$$

Similarly,  $\mathbf{V}_{YO}$  and  $\mathbf{V}_{BO}$  may be found and  $\mathbf{I}_Y$  and  $\mathbf{I}_B$  calculated therefrom.

**Example 19.76.** Three impedances,  $Z_R$ ,  $Z_Y$  and  $Z_B$  are connected in star across a 440-V, 3-phase supply. If the voltage of star-point relative to the supply neutral is  $200 \angle 150^\circ$  volt and Y and B line currents are  $10 \angle -90^\circ$  A and  $20 \angle 90^\circ$  A respectively, all with respect to the voltage between the supply neutral and the R line, calculate the values of  $Z_R$ ,  $Z_Y$  and  $Z_B$

(Elect Circuit; Nagpur Univ. 1991)

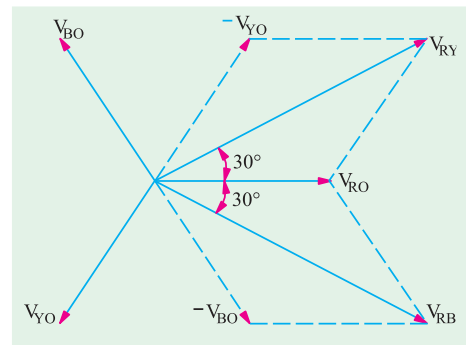


Fig. 19.92

**Solution.** Let  $O$  and  $O'$  be the supply and load neutrals respectively. Also, let,

$$\mathbf{V}_{RO} = \frac{440}{\sqrt{3}} \quad 0 \quad 254 \quad 0 \quad 254 \quad j0$$

$$\mathbf{V}_{YO} = 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{V}_{BO} = 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{I}_Y = 10 \angle -90^\circ = -j10; \quad \mathbf{I}_B = 20 \angle 90^\circ = j20$$

$$\mathbf{I}_R = -(\mathbf{I}_Y + \mathbf{I}_B) = -j10$$

$$\text{Also, } \mathbf{V}_{OO} = 200 \quad 150 \quad 173 \quad j100$$

$$\mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 254 \quad (173 \quad j100) \quad 427 \quad j100 \quad 438.5 \quad 13.2$$

$$\mathbf{V}_{YO} \quad \mathbf{V}_{OO} \quad (127 \quad j220) \quad (173 \quad j100) \quad 46 \quad j320 \quad 323 \quad 81.6$$

$$\mathbf{V}_{BO} \quad \mathbf{V}_{OO} \quad (127 \quad j220) \quad (173 \quad j100) \quad 46 \quad j120 \quad 128.6 \quad 69$$

As seen from Art. 19.32.

$$\mathbf{Z}_R = \frac{\mathbf{V}_{RO} \quad \mathbf{V}_{OO}}{\mathbf{I}_R} = \frac{438.5 \quad 13.2}{10 \quad 90} = 43.85 \quad 76.8$$

$$\mathbf{Z}_Y = \frac{\mathbf{V}_{YO} \quad \mathbf{V}_{OO}}{\mathbf{I}_Y} = \frac{323}{10 \quad 90} = 32.3 \quad 8.4$$

$$\mathbf{Z}_B = \frac{\mathbf{V}_{BO} \quad \mathbf{V}_{OO}}{\mathbf{I}_B} = \frac{128.6 \quad 69}{20 \quad 90} = 6.43 \quad 21$$

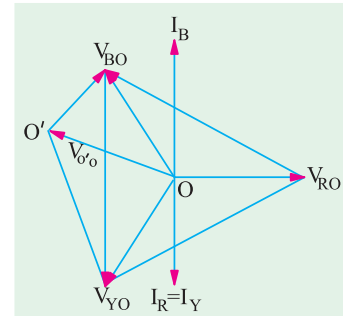


Fig. 19.93

### 19.35. Unbalanced Star-connected Non-inductive Load

Such a case can be easily solved by direct geometrical method. If the supply system is symmetrical, the line voltage vectors can be drawn in the form of an equilateral triangle  $RYB$  (Fig. 19.94). As the load is an unbalanced one, its neutral point will not, obviously, coincide with the centre of the gravity or centroid of the triangle. Let it lie at any other point like  $N$ . If point  $N$  represents the potential of the neutral point if the unbalanced load, then vectors drawn from  $N$  to points,  $R$ ,  $Y$  and  $B$  represent the voltages across the branches of the load. These voltages can be represented in their rectangular co-ordinates with respect to the rectangular axis drawn through  $N$ . It is seen that taking co-ordinates of  $N$  as  $(0, 0)$ , the co-ordinates of point  $R$  are  $[(V/2 - x), -y]$

of point  $Y$  are  $[-(V/2 + x), -y]$

and of point  $B$  are  $[-x, (\sqrt{3}V/2 - y)]$

$$\mathbf{V}_{RN} = \frac{V}{2} - x \quad -y; \quad \mathbf{V}_{YN} = \frac{V}{2} + x \quad -y$$

$$\mathbf{V}_{BN} = x \quad j \frac{\sqrt{3}V}{2} - y$$

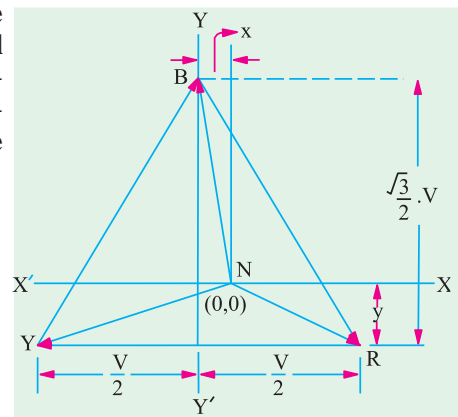


Fig. 19.94

Let  $R_1$ ,  $R_2$  and  $R_3$  be the respective branch impedances,  $Y_1$ ,  $Y_2$  and  $Y_3$  the respective admittances and  $\mathbf{I}_R$ ,  $\mathbf{I}_Y$  and  $\mathbf{I}_B$  the respective currents in them.

Then

$$\mathbf{I}_R = \mathbf{V}_{RN} / \mathbf{R}_1 = \mathbf{V}_{RN} \mathbf{Y}_1$$

Similarly,

$$\mathbf{I}_Y = \mathbf{V}_{YN} \mathbf{Y}_2 \text{ and } \mathbf{I}_B = \mathbf{V}_{BN} \mathbf{Y}_3. \text{ Since } \mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$$

$\therefore$

$$\mathbf{V}_{RN} \mathbf{Y}_1 + \mathbf{V}_{YN} \mathbf{Y}_2 + \mathbf{V}_{BN} \mathbf{Y}_3 = 0$$

$$\text{or } Y_1 \frac{V}{2} x - jy Y_2 \frac{V}{2} x - jy Y_3 x + j \frac{\sqrt{3}V}{2} y = 0$$

$$\text{or } x(Y_1 - Y_2 - Y_3) - \frac{V}{2}(Y_1 - Y_2) - j Y_3 \frac{\sqrt{3}V}{2} - y(Y_1 - Y_2 - Y_3) = 0$$

$$\therefore -x(Y_1 + Y_2 + Y_3) + \frac{V}{2}(Y_1 - Y_2) = 0 \quad \therefore x = \frac{V(Y_1 - Y_2)}{2Y_1 + Y_2 + Y_3}$$

$$\text{Also } Y_3 \frac{\sqrt{3}V}{2} - y(Y_1 + Y_2 + Y_3) = 0 \quad \therefore y = \frac{\sqrt{3}V}{2} \frac{Y_3}{(Y_1 + Y_2 + Y_3)}$$

Knowing the values of  $x$ , the values of  $\mathbf{V}_{RN}$ ,  $\mathbf{V}_{YN}$  and  $\mathbf{V}_{BN}$  and hence, of  $\mathbf{I}_R$ ,  $\mathbf{I}_Y$  and  $\mathbf{I}_B$  can be found as illustrated by Ex. 19.68.

**Example 19.77.** Three non-inductive resistances of 5, 10 and 15  $\Omega$  are connected in star and supplied from a 230-V symmetrical 3-phase system. Calculate the line currents (magnitudes). (Principles of Elect. Engg. Jadavpur Univ.)

**Solution.**

(a) Star/Delta Conversion Method

The Y-connected unbalanced load and its equivalent  $\Delta$ -connected load are shown in Fig. 19.95 (a) and (b) respectively. Using Y/ $\Delta$  conversion, we have

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1}{\mathbf{Z}_3} = \frac{50 + 150 + 75}{15} = \frac{55}{3}$$

$$\mathbf{Z}_{23} = \frac{275}{5} = 55 \text{ and } \mathbf{Z}_{31} = \frac{275}{10} = 27.5$$

$$\text{Phase current } \mathbf{I}_{RY} = \mathbf{V}_{RY} / \mathbf{Z}_{12} = 230 / (55/3) = 12.56 \text{ A}$$

$$\text{Similarly, } \mathbf{I}_{YB} = \mathbf{V}_{YB} / \mathbf{Z}_{23} = 230 / 55 = 4.18 \text{ A; } \mathbf{I}_{BR} = \mathbf{V}_{BR} / \mathbf{Z}_{31} = 230 / 27.5 = 8.36 \text{ A}$$

The line currents for  $\Delta$ -connection are obtained by compounding the above phase currents trigonometrically or vectorially. Choosing vector addition and taking  $\mathbf{V}_{RY}$  as the reference vector, we get;

$$\mathbf{I}_{RY} = (12.56 + j0)$$

$$\mathbf{I}_{YB} = 4.18 \frac{1}{2} j \frac{\sqrt{3}}{2} = -2.09 - j3.62$$

$$\mathbf{I}_{BR} = 8.36 \frac{1}{2} j \frac{\sqrt{3}}{2} = -4.18 + j7.24$$

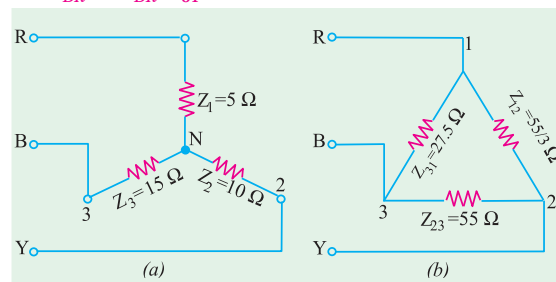


Fig. 19.95

Hence, line currents for  $\Delta$ -connection of Fig. 19.95 (b) are

$$\begin{aligned} \mathbf{I}_R &= \mathbf{I}_{RY} + \mathbf{I}_{RB} = \mathbf{I}_{RY} - \mathbf{I}_{BR} \\ &= (12.56 + j0) - (-4.18 + j7.24) = 16.74 - j7.24 \text{ or } 18.25 \text{ A - in magnitude} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_Y &= \mathbf{I}_{YR} + \mathbf{I}_{YB} = \mathbf{I}_{YB} - \mathbf{I}_{RY} \\ &= (-2.09 - j3.62) - (12.56 + j0) = -14.65 - j3.62 \text{ or } 15.08 \text{ A - in magnitude} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_B &= \mathbf{I}_{BR} + \mathbf{I}_{BY} = \mathbf{I}_{BR} - \mathbf{I}_{YB} \\ &= (-4.18 + j7.24) - (-2.09 - j3.62) = -2.09 + j10.86 \text{ or } 11.06 \text{ A - in magnitude} \end{aligned}$$

**(b) Geometrical Method**

Here,  $R_1 = 5 \Omega$ ,  $R_2 = 10 \Omega$ ; and  $R_3 = 15 \Omega$ ;  $Y_1 = 1/5S$ ;  $Y_2 = 1/10S$ ;  $Y_3 = 1/15S$

As found above in Art. 19.35  $x = \frac{V}{2}(Y_1 - Y_2)/(Y_1 + Y_2 + Y_3)$

$$= \frac{230}{2} \frac{1}{5} \frac{1}{10} \bigg/ \frac{1}{5} \frac{1}{10} \frac{1}{15} = 31.4$$

$$y = \frac{\sqrt{3}V}{2} \cdot Y_3 \bigg/ (Y_1 + Y_2 + Y_3) = \frac{(\sqrt{3} \cdot 115 \cdot 1/15)}{(11/30)} = 36.2$$

$$V_{RN} = \frac{V}{2} x - jy = (115 \cdot 31.4) - j36.2 = 83.6 - j36.2$$

$$V_{YN} = \frac{V}{2} x + jy = 146.4 + j36.2$$

$$V_{BN} = x + j \frac{\sqrt{3}V}{2} y = 31.4 + j163$$

$$I_R = \frac{V_{RN}}{R_1} = \frac{(83.6 - j36.2)}{5} = 16.72 - j7.24$$

$$I_Y = \frac{V_{YN}}{R_2} = \frac{(146.4 + j36.2)}{10} = 14.64 + j3.62$$

$$I_B = \frac{V_{BN}}{R_3} = \frac{(31.4 + j163)}{15} = 2.1 + j10.9$$

These are the same currents as found before.

**(c) Solution by Millman's Theorem**

$Y_R = 1/5 \Omega^{-1}$ ;  $Y_Y = 1/10 \Omega^{-1}$ ;  $Y_B = 1/15 \Omega^{-1}$  and  $Y_R + Y_Y + Y_B = 11/30 \Omega^{-1}$  Siemens

Let the supply voltages be represented (Fig. 19.96) by

$$V_{RO} = 230/\sqrt{3} \angle 0^\circ; V_{YO} = 133 \angle -120^\circ; V_{BO} = 133 \angle 120^\circ$$

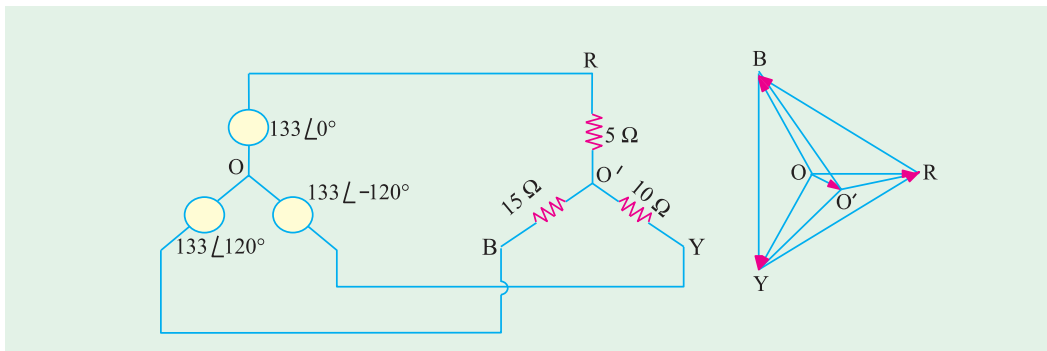


Fig. 19.96

The p.d. between load and supply neutral is

$$\begin{aligned} V_{OO'} &= \frac{133/5 \angle 0^\circ + (133/10) \angle -120^\circ + 120 \angle 120^\circ}{30/11} \\ &= 42.3 - j10.4 = 43.6 \angle -13.8^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{V}_{RO} &= 133 \quad (42.3 \quad j10.4) \quad 90.7 \quad j10.4 \\ \mathbf{V}_{YO} &= 133 \quad 120 \quad (42.3j \quad 10.4) \\ &= (-66.5 - j115) - (42.3 - j10.4) = -108.8 - j104.6 \\ \mathbf{V}_{BO} &= 133 \quad 120 \quad V_{OO} \quad (66.5 \quad j115) \quad (42.3 \quad j10.4) \quad 108.8 \quad j125.4 \\ \mathbf{I}_R &= \frac{\mathbf{V}_{RO}}{\mathbf{Z}_R} = \frac{1/5 \cdot (90.7 \quad j10.4)}{18.1} = j2.1 \text{ or } 18.22 \text{ A in magnitude} \\ \mathbf{I}_Y &= 10.88 \quad j10.5 \text{ or } 15.1 \text{ A in magnitude} \\ \mathbf{I}_B &= -7.5 + j8.4 \text{ or } 11.7 \text{ A in magnitude} \end{aligned}$$

**Example 19.78.** The unbalanced circuit of Fig. 19.97 (a) is connected across a symmetrical 3-phase supply of 400-V. Calculate the currents and phase voltages. Phase sequence is RYB.

**Solution.** The line voltages are represented by the sides of an equilateral triangle ABC in Fig. 19.97 (b). Since phase impedances are unequal, phase voltages are unequal and are represented by lengths, NA, NB and NC where N is the neutral point which is shifted from its usual position. CM and ND are drawn perpendicular to horizontal side AB. Let co-ordinates of point N be (0, 0). Obviously, AM = BM = 200 V, CM =  $\sqrt{3} \times 200$  V, CM =  $\sqrt{3} \times 200 = 346$  V. Let DM = x volts and ND = y volts.

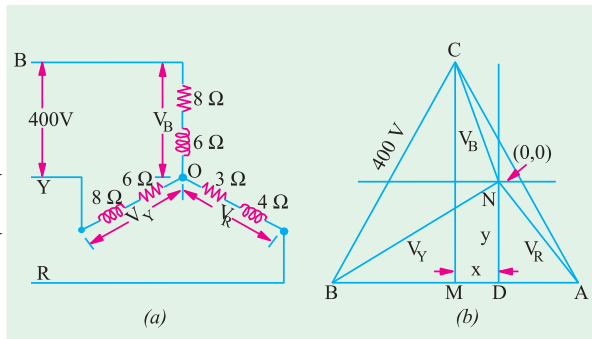


Fig. 19.97

Then, with reference to point N, the vector expressions for phase voltages are

$$\begin{aligned} \mathbf{V}_R &= (200 - x) - jy, \quad \mathbf{V}_Y = (200 - x) - jy, \quad \mathbf{V}_B = x - j(346 - y) \\ \mathbf{I}_R &= \frac{\mathbf{V}_R}{\mathbf{Z}_R} = \frac{(200 - x) - jy}{3 - j4} = \frac{3 + j4}{3 - j4} (24 - 0.12x - 0.16y) = j(32 - 0.16x - 0.12y) \\ \mathbf{I}_Y &= \frac{\mathbf{V}_Y}{\mathbf{Z}_Y} = \frac{(200 - x) - jy}{6 - j8} = \frac{6 + j8}{6 - j8} (-12 - 0.06x - 0.08y) + j(16 + 0.08x - 0.06y) \\ \mathbf{I}_B &= \frac{\mathbf{V}_B}{\mathbf{Z}_B} = \frac{x - j(346 - y)}{8 - j6} = \frac{8 + j6}{8 - j6} (20.76 - 0.08x - 0.06y) + j(27.68 + 0.06x - 0.08y) \end{aligned}$$

Now,  $\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$

$$\therefore (32.76 - 0.26x - 0.3y) + j(11.68 + 0.3x - 0.26y) = 0$$

Obviously, the real component as well as the *j*-component must be zero.

$$\therefore 32.76 - 0.26x - 0.3y = 0 \quad \text{and} \quad 11.68 + 0.3x - 0.26y = 0$$

Solving these equations for *x* and *y*, we have *x* = 31.9 V and *y* = 81.6 V

$$\begin{aligned} \mathbf{V}_R &= (200 - 31.9) - j81.6 = 168 - j81.6 = 186.7 \quad 25.9 \\ \mathbf{V}_Y &= (200 - 31.9) - j81.6 = 231.9 - j81.6 = 245.8 \quad 199.4 \\ \mathbf{V}_B &= 31.9 - j(346 - 81.6) = 31.9 - j264.4 = 266.3 \quad 83.1 \end{aligned}$$

Substituting these values of *x* and *y* in the expressions for currents, we get

$$\mathbf{I}_R \begin{pmatrix} 24 & 0.12 & 31.9 & 0.16 & 81.6 \end{pmatrix} j \begin{pmatrix} 32 & 0.16 & 31.9 & 0.12 & 81.6 \end{pmatrix}$$

$$= 7.12 - j36.7$$

$$\text{Similarly } \mathbf{I}_Y \begin{pmatrix} 20.44 & j13.65 \end{pmatrix}; \mathbf{I}_B \begin{pmatrix} 13.3 & j23.06 \end{pmatrix}$$

$$\Sigma \mathbf{I} = (0. + j0) \text{ - as a check}$$

**Example 19.79.** A 3- $\phi$ , 4-wire, 400-V symmetrical system supplies a Y-connected load having following branch impedances:

$$Z_R = 100\Omega, Z_Y = j10\Omega \text{ and } Z_B = -j10\Omega$$

Compute the values of load phase voltages and currents and neutral current. Phase sequence is RYB.

How will these values change in the event of an open in the neutral wire?

**Solution. (a) When Neutral Wire is Intact.** [Fig 19.98 (a)]. As discussed in Art. 19.30, the load phase voltages would be the same as supply phase voltages despite imbalance in the load. The three load phase voltages are:

$$\mathbf{V}_R \begin{pmatrix} 231 & 0 \end{pmatrix}, \mathbf{V}_Y \begin{pmatrix} 231 & 120 \end{pmatrix} \text{ and } \mathbf{V}_B \begin{pmatrix} 231 & 120 \end{pmatrix}$$

$$\mathbf{I}_R \begin{pmatrix} 231 & 0 \end{pmatrix} / 100 \begin{pmatrix} 0 \end{pmatrix} = 2.31 \begin{pmatrix} 0 \end{pmatrix} = 2.31 \begin{pmatrix} j0 \end{pmatrix}$$

$$\mathbf{I}_Y \begin{pmatrix} 231 & 120 \end{pmatrix} / 10 \begin{pmatrix} 90 \end{pmatrix} = 2.31 \begin{pmatrix} 210 \end{pmatrix} = 20 \begin{pmatrix} j11.5 \end{pmatrix}$$

$$\mathbf{I}_B \begin{pmatrix} 231 & 120 \end{pmatrix} / 10 \begin{pmatrix} 90 \end{pmatrix} = 23.1 \begin{pmatrix} 210 \end{pmatrix} = 20 \begin{pmatrix} j11.5 \end{pmatrix}$$

$$\mathbf{I}_N \begin{pmatrix} \mathbf{I}_R & \mathbf{I}_Y & \mathbf{I}_B \end{pmatrix} = \begin{pmatrix} 2.31 & 20 & j11.5 & 20 & j11.5 \end{pmatrix} = 37.7 \text{ A}$$

**(b) When Neutral is Open** [Fig.19.98

(b)]

In this case, the load phase voltages will be no longer equal. The node pair voltage method will be used to solve the question. Let the supply phase voltages be given by

$$\mathbf{E}_R \begin{pmatrix} 231 & 0 \end{pmatrix}, \mathbf{E}_Y \begin{pmatrix} 231 & 120 \end{pmatrix}$$

$$= -115.5 - j200$$

$$\mathbf{E}_B = 231 \angle 120^\circ = -115.5 + j200$$

$$\mathbf{Y}_R \begin{pmatrix} 1/100 & 0.01 \end{pmatrix}; \mathbf{Y}_Y \begin{pmatrix} 1/j10 & j0.1 \end{pmatrix} \text{ and } \mathbf{Y}_B \begin{pmatrix} 1/j10 & j0.1 \end{pmatrix}$$

$$\mathbf{V}_{NN'} = \frac{231 \times 0.01 + (-j0.1)(-115.5 - j200) + j0.1(-115.5 + j200)}{0.01 + (-j0.1) + j0.1} = -3769 + j0$$

The load phase voltages are given by

$$\mathbf{V}_R = \mathbf{E}_R - \mathbf{V}_{NN'} = \begin{pmatrix} 231 & j0 \end{pmatrix} - \begin{pmatrix} 3769 & j0 \end{pmatrix} = 4000 \text{ V}$$

$$\mathbf{V}_Y = \mathbf{E}_Y - \mathbf{V}_{NN'} = \begin{pmatrix} 115.5 & j200 \end{pmatrix} - \begin{pmatrix} 3769 & j0 \end{pmatrix} = \begin{pmatrix} 3653.5 & j200 \end{pmatrix}$$

$$\mathbf{V}_B = \mathbf{E}_B - \mathbf{V}_{NN'} = \begin{pmatrix} 115.5 & j200 \end{pmatrix} - \begin{pmatrix} 3769 & j0 \end{pmatrix} = \begin{pmatrix} 3653.5 & j0 \end{pmatrix}$$

$$\mathbf{I}_R = \mathbf{V}_R \mathbf{Y}_R = 4000 \begin{pmatrix} 0.01 \end{pmatrix} = 40 \text{ A}$$

$$\mathbf{I}_Y = \begin{pmatrix} j0.1 \end{pmatrix} \begin{pmatrix} 3653.5 & j200 \end{pmatrix} = 20 \begin{pmatrix} j3653.5 \end{pmatrix}$$

$$\mathbf{I}_B = \begin{pmatrix} j0.1 \end{pmatrix} \begin{pmatrix} 3653.5 & j200 \end{pmatrix} = 20 \begin{pmatrix} j3653.5 \end{pmatrix}$$

Obviously, the neutral current will just not exist.

**Note.** As hinted in Art. 19.30 (i), the load phase voltages and currents become abnormally high.

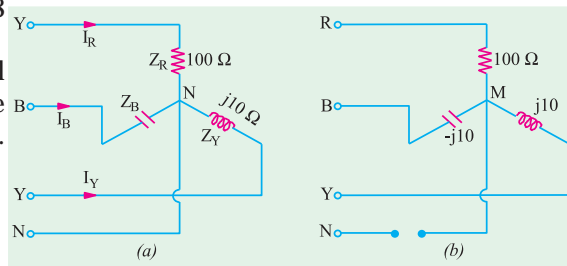


Fig. 19.98

**Example 19.80.** For the circuit shown in Fig. 19.99 find the readings on the two wattmeters  $W_a$  and  $W_c$ .

**Solution.** The three line currents for this problem have already been determined in Example 19.43.

$$I_{ao} = 20.02 - j4.54$$

$$I_{bo} = -7.24 - j5.48$$

$$I_{co} = 12.78 + j10.12$$

The line voltages are given by

$$V_{ab} = 200 \angle 0^\circ$$

$$V_{bc} = -100 - j173.2$$

$$V_{ca} = -100 + j173.2$$

Wattmeter  $W_a$  carries a current of using  $I_{ao} = 20.02 - j4.54$  and has voltage  $V_{ab}$  impressed across its pressure coil. Power can be found by using current conjugate.

$$P_{VA} = (200 + j0)(20.02 + j4.54) = (200)(20.02) + j(200)(4.54)$$

$$\text{Actual power} = 200 \times 20.02 = 4004 \text{ W} \therefore W_a = 4004 \text{ W}$$

The other wattmeter  $W_c$  carries current of  $I_{co} = 12.78 + j10.02$  and has a voltage  $V_{cb} = -V_{bc} = 100 + j173.2$  impressed across it. By the same method, wattmeter reading is

$$W_c = (100 \times -12.78) + (173.2 \times 10.02) = -1278 + 1735.5 = 457.5 \text{ W}$$

**Example 19.81.** Three resistors  $10, 20$  and  $20 \Omega$  are connected in star to the terminals  $A, B$  and  $C$  of a  $3-\phi, 3$  wire supply through two single-phase wattmeters for measurement of total power with current coils in lines  $A$  and  $C$  and pressure coils between  $A$  and  $B$  and  $C$  and  $B$ . Calculate (i) the line currents (ii) the readings of each wattmeter.

The line voltage is  $400\text{-V}$ .

(Electrical Engineering-I, Bombay Univ.)

**Solution.** Let  $V_{AB} = 400 \angle 0^\circ$ ;  $V_{BC} = 400 \angle -120^\circ$  and  $V_{CA} = 400 \angle 120^\circ$

As shown in Fig. 19.100, current through wattmeter  $W_1$  is  $I_{AO}$  or  $I_A$  and that through  $W_2$  is  $I_{CO}$  or  $I_C$  and the voltages are  $V_{AB}$  and  $V_{CB}$  respectively. Obviously,

$$Z_A = 10 \angle 0^\circ; Z_B = 20 \angle 0^\circ; Z_C = 20 \angle 0^\circ$$

The currents  $I_A$  and  $I_C$  may be found by applying either Kirchhoff's laws (Art. 19.33) or Maxwell's Mesh Method. Both methods will be used for illustration.

(a) From Eq. (10), (11) and (12) of Art. 19.33, we have

$$I_A = \frac{400 \times 20 - 20(-200 + j346)}{(10 \times 20) + (20 \times 20) + (20 \times 10)}$$

$$= \frac{12,000 - j6,920}{800} = 15 - j8.65 \text{ A}$$

$$I_C = \frac{20(200 - j346) - 10(200 - j346)}{800} = \frac{2000 - j10,380}{800} = 2.5 - j13$$

(b) From Eq. (i) and (ii) of solved example 17.48 (c) we get

$$I_A = I_1 = \frac{400 \times 40 - 20(200 - j346)}{30 \times 40 + 20^2} = 15 - j8.65 \text{ A}$$

$$I_C = I_2 = \frac{30(200 - j346) - 400 \times 20}{800} = 2.5 - j13$$

As seen, wattmeter  $W_1$  carries current  $I_A$  and has a voltage  $V_{AB}$  impressed across its pressure coil. Power may be found by using voltage conjugate.

$$P_{VA} = (400 \angle 0^\circ)(15 - j8.65) = 6000 - j3,460$$

$$\therefore \text{reading of } W_1 = 6000 \text{ W} = 6 \text{ kW}$$

Similarly,  $W_2$  carries  $I_C$  and has voltage  $V_{CB}$  impressed across its pressure coil.

Now,  $V_{CB} = V_{BC} = 400 \angle -120^\circ$ . Using voltage conjugate, we get

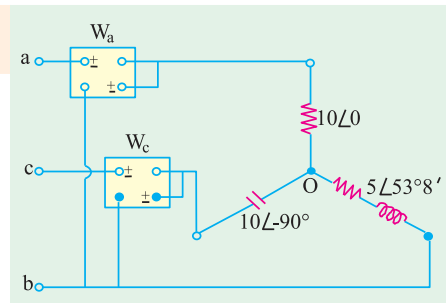


Fig. 19.99

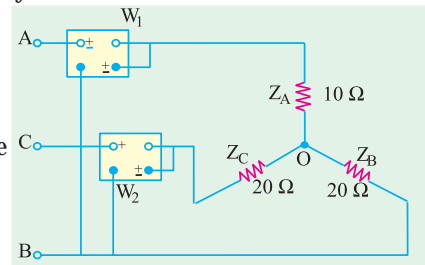


Fig. 19.100

$$P_{VA} = (200 - j346)(2.5 + j13)$$

$$\text{Real power} = (200 \times -2.5) + (13 \times 346) = 4000 \text{ W}$$

$$\therefore \text{reading of } W_2 = \mathbf{4kW}; \quad \text{Total power} = 10 \text{ kW}$$

**Example 19.82.** Three impedances  $Z_A$ ,  $Z_B$  and  $Z_C$  are connected in delta to a 200-V, 3-phase three-wire symmetrical system RYB.

$$Z_A = 10 \angle 60^\circ \text{ between lines R and Y}; \quad Z_B = 10 \angle 0^\circ \text{ between lines Y and B}$$

$$Z_C = 10 \angle 60^\circ \text{ between lines B and R}$$

The total power in the circuit is measured by means of two wattmeters with their current coils in lines R and B and their corresponding pressure coils across R and Y and B and Y respectively. Calculate the reading on each wattmeter and the total power supplied. Phase sequence RYB.

**Solution.** The wattmeter connections are shown in Fig. 19.101.

$$V_{RY} = 200 \angle 0^\circ = 200 + j0$$

$$V_{YB} = 200 \angle 120^\circ = 100 + j173.2$$

$$V_{BR} = 200 \angle 240^\circ = 100 - j173.2$$

$$I_{BR} = \frac{200 \angle 0^\circ}{10 \angle 60^\circ} = 20 \angle -60^\circ = 20 \cos 60^\circ - j20 \sin 60^\circ = 10 - j17.32$$

$$I_{YB} = \frac{200 \angle 120^\circ}{10 \angle 0^\circ} = 20 \angle 120^\circ = 20 \cos 120^\circ + j20 \sin 120^\circ = -10 + j17.32$$

$$= -10 - j17.32; \quad I_{BR} = \frac{200 \angle 120^\circ}{10 \angle 60^\circ} = 20 \angle 60^\circ = 10 + j17.32$$

As seen, current through  $W_1$  is  $I_R$  and voltage across its pressure coil is  $V_{RY}$

$$I_R = I_{RY} = I_{BR} = j34.64 \text{ A}$$

Using voltage conjugate, we have

$$P_{VA} = (200 - j0)(j34.64) = 0 + j6.928$$

Hence,  $W_1$  reads zero.

Current through  $W_2$  is  $I_B$  and voltage across its pressure coil is  $V_{BY}$

$$I_B = I_{BR} = I_{YB} = 20 \angle 34.64^\circ; \quad V_{BY} = V_{YB} = 100 + j173.2$$

Again using voltage conjugate, we get

$$P_{YA} = (100 + j173.2)(20 - j34.64)$$

$$= 8000 + j0$$

$$\therefore \text{reading of } W_2 = \mathbf{8000 \text{ W}}$$

### 19.36. Phase Sequence Indicators

In unbalanced 3-wire star-connected loads, phase voltages change considerably if the phase sequence of the supply is reversed. One or the other load phase voltage becomes dangerously large which may result in damage to the equipment. Some phase voltage becomes too

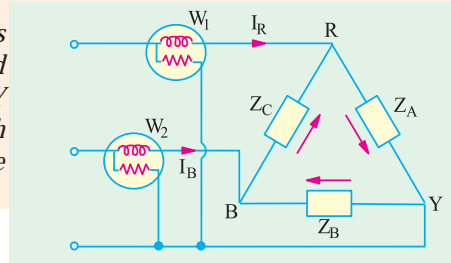


Fig. 19.101

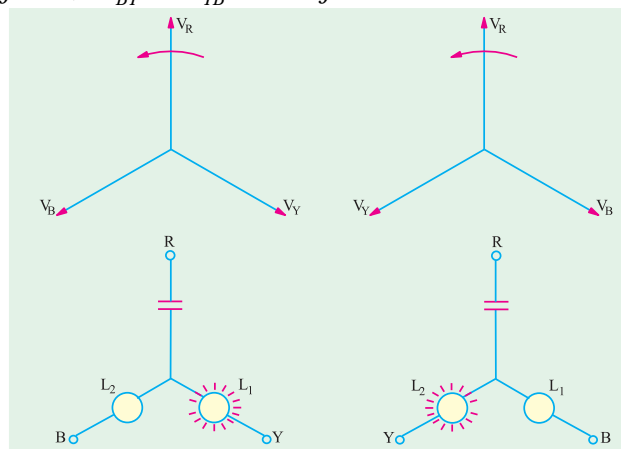


Fig. 19.102



small which is equally detrimental to some types of electrical equipment. Since phase voltage depends on phase sequence, this fact has been made the basis of several types of phase sequence indicators.\* A simple phase sequence indicator may be made by connecting two suitable incandescent lamps and a capacitor in a  $Y$ -connection as shown in Fig. 19.102. It will be found that for phase sequence  $RYB$ , lamp  $L_1$  will glow because its phase voltage will be large whereas  $L_2$  will not glow because of low voltage across it.

When, phase sequence is  $RBV$ , opposite conditions develop so that this time  $L_2$  glows but not  $L_1$ .

Another method of determining the phase sequence is by means of a small 3-phase motor. Once direction of rotation with a known sequence is found, the motor may be used thereafter for determining an unknown sequence.

### Tutorial Problem No. 19.3

1. Three impedances  $Z_1$ ,  $Z_2$  and  $Z_3$  are mesh-connected to a symmetrical 3-phase, 400-V, 50-Hz supply of phase sequence  $R \rightarrow Y \rightarrow B$ .

$$Z_1 = (10 + j0) \text{ ohm} \text{— between } R \text{ and } Y \text{ lines}$$

$$Z_2 = (5 + j6) \text{ ohm} \quad \text{— between } Y \text{ and } B \text{ lines}$$

$$Z_3 = (5 - j5) \text{ ohm} \quad \text{— between } B \text{ and } R \text{ lines}$$

Calculate the phase and line currents and total power consumed.

$$[40 \text{ A}, 40 \text{ A}, 56.6 \text{ A}; 95.7 \text{ A}, 78.4 \text{ A}, 35.2 \text{ A}; 44.8 \text{ kW}]$$

2. A symmetrical 3- $\phi$ , 380-V supply feeds a mesh-connected load as follows :

Load  $A$  : 19 kVA at p.f. 0.5 lag ; Load  $B$  : 20 kVA at p.f. 0.8 lag ; Load  $C$  : 10 kVA at p.f. 0.9 load  
Determine the line currents and their phase angles for  $RYB$  sequence.

$$[74.6 \angle -51^\circ \text{ A}, 98.6 \angle 172.7^\circ \text{ A}; 68.3 \angle 41.8^\circ \text{ A}]$$

3. Determine the line currents in an unbalanced  $Y$  connected load supplied from a symmetrical 3- $\phi$ , 440-V, 3-wire system. The branch impedances of the load are :  $Z_1 = 5 \angle 30^\circ$  ohm,  $Z_2 = 10 \angle 45^\circ$  ohm and  $Z_3 = 10 \angle 45^\circ$  ohm and  $Z_4 = 10 \angle 60^\circ$  ohm. The sequence is  $RYB$ .

$$[35.7 \text{ A}, 32.8 \text{ A}, 27.7 \text{ A}]$$

4. A 3- $\phi$ ,  $Y$ -connected alternator supplies an unbalanced load consisting of three impedances  $(10 + j20)$ ,  $(10 - j20)$  and  $10 \Omega$  respectively, connected in star. There is no neutral connection. Calculate the voltage between the star point of the alternator and that of the load. The phase voltage of the alternator is 230 V.

$$[-245.2 \text{ V}]$$

5. Non-reactive resistors of 10, 20 and 25  $\Omega$  are star-connected to the  $R$ ,  $Y$  and  $B$  phases of a 400-V, symmetrical system. Determine the current and power in each resistor and the voltage between star point and neutral. Phase sequence,  $RYB$ .

$$[16.5 \text{ A}, 2.72 \text{ kW}; 13.1 \text{ A}, 3.43 \text{ kW}; 11.2 \text{ A}, 3.14 \text{ kW}; 68 \text{ V}]$$

6. Determine the line current in an unbalanced, star-connected load supplied from a symmetrical 3-phase, 440-V system. The branch impedance of the load are  $Z_R = 5 \angle 30^\circ \Omega$ ,  $Z_Y = 10 \angle 45^\circ \Omega$  and  $Z_B = 10 \angle 60^\circ \Omega$ . The phase sequence is  $RYB$ .

$$[35.7 \text{ A}, 32.8 \text{ A}, 27.7 \text{ A}]$$

7. Three non-reactive resistors of 3, 4 and 5- $\Omega$  respectively are star-connected to a 3-phase, 400-V symmetrical system, phase sequence  $RYB$ . Find (a) the current in each resistor (b) the power dissipated in each resistor (c) the phase angles between the currents and the corresponding line voltages (d) the star-point potential. Draw to scale the complete vector diagram.

$$[(a) 66.5 \text{ A}, 59.5 \text{ A}, 51.8 \text{ A} (b) 13.2, 14.15, 13.4 \text{ kW} (c) 26^\circ 24', 38^\circ 10', 25^\circ 20' (d) 34 \text{ V}]$$

8. An unbalanced  $Y$ -connected load is supplied from a 400-V, 3- $\phi$ , 3-wire symmetrical system. The branch circuit impedances and their connection are  $(2 + j2) \Omega$ ,  $R$  to  $N$ ;  $(3 - j3) \Omega$ ,  $Y$  to  $N$  and  $(4 + j1) \Omega$ ,  $B$  to  $N$  of the load. Calculate (i) the value of the voltage between lines  $Y$  and  $N$  and (ii) the phase of this voltage relative to the voltage between line  $R$  and  $Y$ . Phase sequence  $RYB$ .

$$[(i) (-216 - j 135.2) \text{ or } 225.5 \text{ V} (ii) 2^\circ \text{ or } -178^\circ]$$

9. A star-connection of resistors  $R_a = 10 \Omega$ ;  $R_b = 20 \Omega$  is made to the terminals  $A$ ,  $B$  and  $C$  respectively of a symmetrical 400-V,  $\phi$  supply of phase sequence  $A \rightarrow B \rightarrow C$ . Find the branch voltages and currents and star-point voltage to neutral.

\* It may, however, be noted that phase sequence of currents in an unbalanced load is not necessarily the same as the voltage phase sequence. Unless indicated otherwise, voltage phase sequence is implied.

$$[V_A = 148.5 + j28.6 ; I_A = 14.85 + j2.86 ; V_B = -198 - j171.4 ; I_B = -9.9 - j8.57$$

$$V_C = -198 + j228.6 ; I_C = -4.95 + j5.71. V_N = 82.5 - j28.6 \text{ (to be subtracted from supply voltage)}]$$

10. Three non-reactive resistance of 5, 10 and 5 ohm are star-connected across the three lines of a 230-V 3-phase, 3-wire supply. Calculate the line currents.

$$[(18.1 + j21.1) \text{ A} ; (-10.9 - j10.45) \text{ A} ; (-7.3 + j8.4) \text{ A}]$$

11. A 3- $\phi$ , 400-V symmetrical supply feeds a star-connected load consisting of non-reactive resistors of 3, 4 and 5  $\Omega$  connected to the R, Y and B lines respectively. The phase sequence is RYB. Calculate (i) the load star point potential (ii) current in each resistor and power dissipated in each resistor.

$$[(i) 34.5 \text{ V} (ii) 66.4 \text{ A}, 59.7 \text{ A}, 51.8 \text{ A} (iii) 13.22 \text{ kW}, 14.21 \text{ kW}, 13.42 \text{ kW}]$$

12. A 20- $\Omega$  resistor is connected between lines R and Y, a 50- $\Omega$  resistor between lines Y and B and a 10- $\Omega$  resistor between lines B and R of a 415-V, 3-phase supply. Calculate the current in each line and the reading on each of the two wattmeters connected to measure the total power, the respective current coils of which are connected in lines R and Y. [(25.9 - j9); (-24.9 - j7.2); (-1.04 + j16.2); 8.6 kW; 7.75 kW]

13. A three-phase supply, giving sinusoidal voltage of 400 V at 50 Hz is connected to three terminals marked R, Y and B. Between R and Y is connected a resistance of 100  $\Omega$ , between Y and B an inductance of 318 mH and negligible resistance and between B and R a capacitor of 31.8  $\mu\text{F}$ . Determine (i) the current flowing in each line and (ii) the total power supplied. Determine (iii) the resistance of each phase of a balanced star-connected, non-reactive load, which will take the same total power when connected across the same supply. [(i) 7.73 A, 7.73 A, (ii) 1,600 W (iii) 100  $\Omega$  (London Univ.)]

14. An unbalanced, star-connected load is fed from a symmetrical 3-phase system. The phase voltages across two of the arms of the load are  $V_B = 295 \angle 97^\circ 30'$  and  $V_R = 206 \angle -25^\circ$ . Calculate the voltage between the star-point of the load and the supply neutral. [52.2  $\angle -49.54'$ ]

15. A symmetrical 440-V, 3-phase system supplies a star-connected load with the following branch impedances:  $Z_R = 100 \Omega$ ,  $Z_Y = j5 \Omega$ ,  $Z_B = -j5 \Omega$ . Calculate the voltage drop across each branch and the potential of the neutral point to earth. The phase sequence is RYB. Draw the vector diagram.

$$[8800 \angle -30^\circ, 8415 \angle -31.5^\circ, 8420 \angle -28.5^\circ, 8545 \angle 150^\circ]$$

16. Three star-connected impedances,  $Z_1 = (20 + j37.7) \Omega$  per phase are in parallel with three delta-connected impedances,  $Z_2 = (30 - j159.3) \Omega$  per phase. The line voltage is 398 V. Find the line current, power factor, power and reactive volt-amperes taken by the combination.

$$[3.37 \angle 10.4^\circ; 0.984 \text{ lag}; 2295 \text{ W}; 2295 \text{ VAR}; 420 \text{ VAR.}]$$

17. A 3-phase, 440-V, delta-connected system has the loads: branch RY, 20 kW at power factor 1.0; branch YB, 30 kVA at power factor 0.8 lagging; branch BR, 20 kVA at power factor 0.6 leading. Find the line currents and readings on watt-meters whose current coils are in phases R and B.

$$[90.5 \angle 176.5^\circ; 111.4 \angle 14^\circ; 36.7 \angle -119^\circ; 39.8 \text{ kW}; 16.1 \text{ kW}]$$

18. A 415 V, 50 Hz, 3-phase supply of phase sequence RYB is connected to a delta connected load in which branch RY consists of  $R_1 = 100 \Omega$ , branch YB consists of  $R_2 = 20 \Omega$  in series with  $X_2 = 60 \Omega$  and branch BR consists of a capacitor  $C = 30 \mu\text{F}$ . Take  $V_{RY}$  as the reference and calculate the line currents. Draw the complete phasor diagrams. (Elect. Machines, A.M.I.E. Sec. B, 1989)

$$[I_R = 7.78 \angle 14.54^\circ, I_Y = 10.66 \angle 172.92^\circ, I_B = 4.46 \angle -47^\circ]$$

19. Three resistances of 5, 10 and 15  $\Omega$  are connected in delta across a 3-phase supply. Find the values of the three resistors, which if connected in star across the same supply, would take the same line currents.

If this star-connected load is supplied from a 4-wire, 3-phase system with 260 V between lines, calculate the current in the neutral. [2.5  $\Omega$ , 1.67  $\Omega$ , 5  $\Omega$ ; 52 A] (London Univ.)

20. Show that the power consumed by three identical phase loads connected in delta is equal to three times the power consumed when the phase loads are connected in star.

(Nagpur University, Summer 2002)

21. Prove, that the power consumed in balanced three-phase Delta-connected load is three times the power consumed in star-connected load. (Nagpur University, Winter 2002)

22. A three-phase 230 volts system supplies a total load of 2000 watts at a line current of 6 Amp when three identical impedances are in star-connection across the line terminals of the systems. Determine

- the resistive and reactive components of each impedance. *(Nagpur University, Winter 2002)*
- 23.** Three similar coils each of impedance  $z = (8 + j10)$  ohms, are connected in star and supplied from 3-phase 400V, 50 Hz supply. Find the line current, power factor, power and total volt amperes. *(Nagpur University, Summer 2003)*
- 24.** Three similar each having a resistance of 20 ohm and an inductance of 0.05 H are connected in star to a 3-phase 50 Hz supply with 400 V between lines. Calculate power factor, total power absorbed and line current. If the same coil are reconnected in delta across the same supply what will be the power factor, total power absorbed and line current? *(Pune University 2003) (Nagpur University, Winter 2003)*
- 25.** A 3  $\phi$  star connected load when supplied from 440 V, 50 Hz source takes a line current of 12 amp lagging w.r.t. line voltage by  $70^\circ$ . Calculate : (i) impedance parameters (ii) Power factor and its nature (iii) Draw phasor diagram indicating all voltages and currents. *(Nagpur University, Summer 2004)*
- 26.** Derive the relationship between line current and phase current for Delta connected 3 phase load when supplied from 3 phase balanced supply. *(Nagpur University, Summer 2004)*
- 27.** Derive the relationship between line voltage, phase voltage, line current and phase current in a 3 phase star connected and delta connected circuit. *(Gujrat University, June/July 2003)*
- 28.** show that power input to a 3 phase circuit can be measured by two wattmeters connected properly in the circuit. Draw vector diagram. *(Gujrat University, June/July 2003)*
- 29.** A balanced 3 phase star connected load of 100 kW takes a leading current of 100 A when connected across a 3 phase, 1100 V, 50 Hz supply. Calculate the circuit constants of the load per phase. *(Mumbai University 2003) (Gujrat University, June/July 2003)*
- 30.** Establish relationship between line and phase voltages and currents in a balanced 3-phase star connection. Draw complete phasor diagram for voltages and currents. *(R.G.P.V. Bhopal University, June 2004)*
- 31.** A delta connected load has the following impedances :  
 $Z_{RY} = j 10 \Omega$ ,  $Z_{YB} = 10 \angle 0^\circ \Omega$  and  $Z_{BR} = -j 10 \Omega$  If the load is connected across 100 volt balanced 3-phase supply, obtain the line currents. *(R.G.P.V. Bhopal University, June 2004)*
- 32.** Two wattmeters  $\omega_1$  and  $\omega_2$  are used to measure power in a 3 phase balanced circuit. Mention the conditions under which (i)  $\omega_1 = \omega_2$  (ii)  $\omega_2 = 0$  (iii)  $\omega_1 = 2\omega_2$ . *(V.T.U. Belgaum Karnataka University, February 2002)*
- 33.** Three coils each of impedance  $20 \angle 60^\circ \Omega$  are connected across a 400V, 3 phase supply. Find the reading of each of the two wattmeters connected to measure the power when the coils are connected in (i) star (ii) Delta. *(V.T.U. Belgaum Karnataka University, February 2002)*
- 34.** The power input to a 3 phase circuit was measured by two wattmeter method and the readings were 3400 and - 1200 watts respectively. Calculate the total power and powerfactor. *(V.T.U. Belgaum Karnataka University, July/August 2002)*
- 35.** With the help of connection diagram and vector diagram, obtain expressions for the two wattmeter readings used to measure power in a 3 phase the DC generator is running. *(V.T.U. Belgaum Karnataka University, July/August 2002)*
- 36.** Obtain the relationship between line and phase values of current in a three phase, balanced, delta connected system. *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 37.** Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit. *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 38.** Each of the two wattmeters connected to measure the input to a three phase circuit, reads 20kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three phase power remaining unchanged in the altered condition? *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 39.** Two wattmeters connected to measure power in a 3 phase circuit read 5KW and 1KW, the latter reading being obtained after reversing current coil connections. Calculate power factor of the load and the total power consumed. *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 39.** Derive the relationship between phase and line values of voltages in a connected load. *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 40.** Three coils each of impedance  $20 \angle 60^\circ \Omega$  are connected in delta across a 400, 3 phase, 50Hz, 50Hz Acsupply. Calculate line current and total power. *(V.T.U. Belgaum Karnataka University, January/February 2003)*
- 41.** What are the advantages of a three phase system over a single phase system? *(V.T.U. Belgaum Karnataka University, July/August 2003)*
- 42.** With a neat circuit diagram and a vector diagram prove that two wattmeters are sufficient to measure total power in a 3 phase system. *(V.T.U. Belgaum Karnataka University, July/August 2003)*

- 43.** A balanced star connected load of  $(8 + jb)\Omega$  is connected to a 3 phase, 230 V supply. Find the line current, power factor, power, reactive voltamperes and total voltamperes.  
(V.T.U. Belgaum Karnataka University, July/August 2003)
- 44.** Two watt meters are used to measure the power delivered to a balance 3 phase load of power factor 0.281. One watt meter reads 5.2kW. Determine the reading of the second watt meter. What is the line current if the line voltage is 415 volt?  
(V.T.U. Belgaum Karnataka University, January/February 2004)
- 45.** Write the equations for wattmeter reading  $W_1$  and  $W_2$  in 3 phase power measurement and therefrom for power factor.  
(Anna University, October/November 2002)
- 46.** Show that the wattmeters will read equal in two wattmeter method under unity power factor loading condition.  
(Anna University, November/December 2003)
- 47.** A star connected 3-phase load has a resistance of  $6\Omega$  and an inductive reactance of  $8\Omega$  in each branch. Line voltage is 220 volts. Write the phasor expressions for voltage across each branch, line voltages and line currents. Calculate the total power.  
(Anna University, November/December 2003)
- 48.** Two wattmeters connected to measure the total power in a 3-phase balanced circuit. One measures 4,800 W, while the other reads backwards. On reversing the latter it is found to read 400 W. What is the total power and power and power factor? Draw the connection diagram and phasor diagram of the circuit.  
(Mumbai University 2003) (RGPV Bhopal 2001)
- 49.** A star-network in which N is star point made up as follows :  
AN =  $70\Omega$ , CN =  $90\Omega$  Find an equivalent delta network. If the above star-delta network are superimposed, what would be measured resistance between A and C?  
(Pune University, 2003) (RGPV Bhopal 2001)
- 50.** Explain with diagram the measurement of 3-phase power by two-wattmeter method.  
(RGPV Bhopal 2002)
- 51.** Show that the power taken by a 3-phase circuit can be measured by two wattmeters connected properly in the circuit.  
(RGPV Bhopal)
- 52.** With the aid of star-delta connection diagram, state the basic equation from which star-delta conversion equation can be derived.  
(Pune University, 2003) (RGPV Bhopal 2001)
- 53.** Star-delta connections in a 3-phase supply and their inter-relationship. (RGPV Bhopal 2001)
- 54.** Measurement of power in three-phase circuit in a balanced condition. (RGPV Bhopal 2001)
- 55.** Measurement of reactive power in three-phase circuit. (RGPV Bhopal 2001)
- 56.** Differentiate between balanced and unbalanced three-phase supply and balanced and unbalanced three-phase load. (RGPV Bhopal June 2002)
- 57.** A 3-phase 3 wire supply feeds a load consisting of three equal resistors. By how much is the load reduced if one of the resistors be removed ?  
(RGPV Bhopal June 2002)
- 58.** Establish relationship between line and phase voltages and currents in a balanced delta connection. Draw complete phasor diagram of voltages and currents. (RGPV Bhopal December 2003)

### OBJECTIVE TESTS – 19

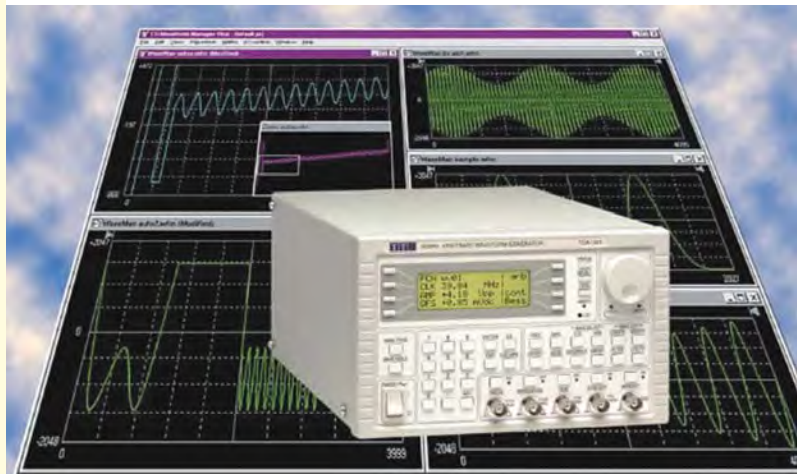
- The minimum number of wattmeter (s) required to measure 3-phase, 3-wire balanced or unbalanced power is  
(a) 1  
(b) 2  
(c) 3  
(d) 4  
(GATE 2001)
- A wattmeter reads 400 W when its current coil is connected in the R phase and its pressure coil is connected between this phase and the neutral of a symmetrical 3-phase system supplying a balanced star connected 0.8 p.f. inductive load. The phase sequence is RYB. What will be the reading of this wattmeter if its pressure coil alone is reconnected between the B and Y phases, all other connections remaining as before?  
(a) 33  $\frac{1}{3}$   
(b) 50  
(c) 66  $\frac{2}{3}$   
(d) 75 (GATE)
- Total instantaneous power supplied by a 3-phase ac supply to a balanced R-L load is  
(a) zero  
(b) constant  
(c) pulsating with zero average  
(d) pulsating with non-zero average  
(GATE 2004)
- A balanced 3-phase, 3-wire supply feeds balanced star connected resistors. If one of the resistors is disconnected, then the percentage reduction in the load will be  
(a) 33  $\frac{1}{3}$   
(b) 50  
(c) 66  $\frac{2}{3}$   
(d) 75 (GATE)

# CHAPTER 20

## Learning Objectives

- Fundamental Wave and Harmonics
- Different Complex Waveforms
- General Equation of a Complex Wave
- R.M.S. Value of a Complex Wave
- Form Factor of a Complex Wave
- Power Supplied by a Complex Wave
- Harmonics in Single-phase A.C. Circuits
- Selective Resonance Due to Harmonics
- Effect of Harmonics on Measurement of Inductance and Capacitance
- Harmonics in Different Three-phase Systems
- Harmonics in Single and 3-Phase Transformers

## HARMONICS



Harmonics are the multiples of a sine wave (the fundamental frequency)

## 20.1. Fundamental Wave and Harmonics

Upto this stage, while dealing with alternating voltages and currents, it has been assumed that they have sinusoidal waveform or shape. Such a waveform is an ideal one and much sought after by the manufacturers and designers of alternators. But it is nearly impossible to realize such a waveform in practice. All the alternating waveforms deviate, to a greater or lesser degree, from this ideal sinusoidal shape. Such waveforms are referred to as **non-sinusoidal** or **distorted** or **complex waveforms**.

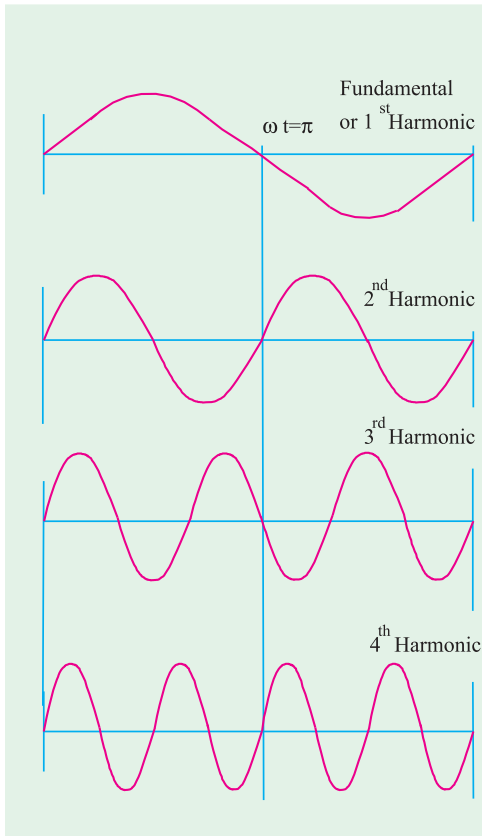


Fig. 20.1

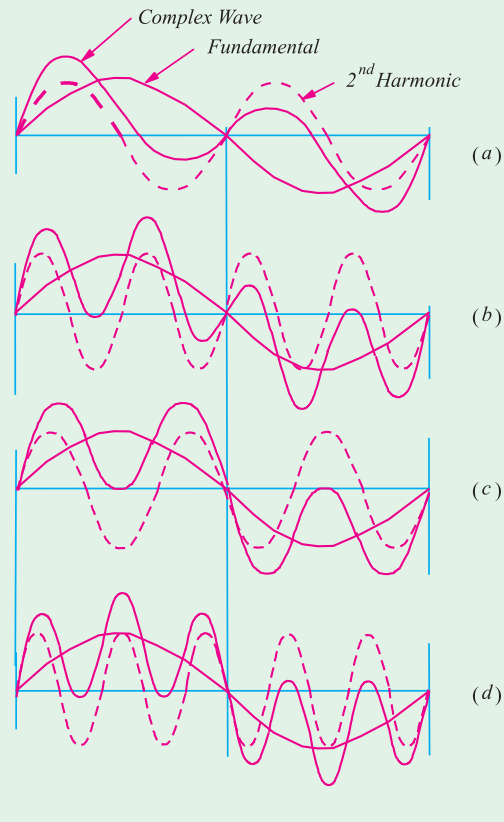


Fig. 20.2

Complex waveforms are produced due to the superposition of sinusoidal waves of different frequencies. Such waves occur in speech, music, TV, rectifier outputs and many other applications of electronics. On analysis, it is found that a complex wave essentially consists of

(a) a **fundamental** wave – it has the lowest frequency, say ' $f$ '

(b) a number of other sinusoidal waves whose frequencies are an integral multiple of the fundamental or basic frequency like  $2f$ ,  $3f$  and  $4f$  etc.

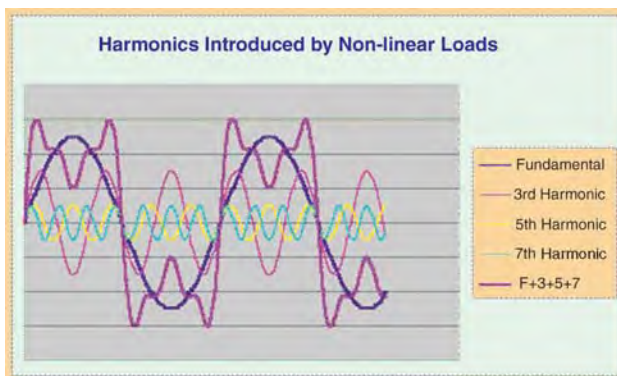
The fundamental and its higher multiples form a **harmonic** series.

As shown in Fig. 20.1, fundamental wave itself is called the **first** harmonic. The **second** harmonic has frequency **twice** that of the fundamental, the **third** harmonic has frequency **thrice** that of the fundamental and so on.

Waves having frequencies of  $2f$ ,  $4f$  and  $6f$  etc. are called **even harmonics** and those having frequencies of  $3f$ ,  $5f$  and  $7f$  etc. are called **odd harmonics**. Expressing the above in angular frequencies, we may say that successive odd harmonics have frequencies of  $3\omega$ ,  $5\omega$  and  $7\omega$  etc. and even harmonics have frequencies of  $2\omega$ ,  $4\omega$  and  $6\omega$  etc.



As mentioned earlier, harmonics are introduced in the output voltage of an alternator due to many reasons such as the irregularities of the flux distribution in it. Considerations of waveform and form factor are very important in the transmission of a.c. power but they are of much greater importance in radio work where the intelligibility of a signal is critically dependent on the faithful transmission of the harmonic structure of sound waves. In fact, it is only the rich harmonic content of the consonants and lesser at still plentiful harmonic content of vowels which helps the ear to distinguish a well regulated speech from a more rhythmical succession of musical sounds.



### 20.2. Different Complex Waveforms

Let us now find out graphically what the resultant shape of a complex wave is when we combine the fundamental with one of its harmonics. Two cases would be considered (i) when the fundamental and harmonic are in phase with each other and have equal or unequal amplitudes and (ii) when there is some phase difference between the two.

In Fig. 20.2 (a), the fundamental and second harmonic, both having the same amplitude, have been shown by the firm and broken line respectively. The resultant complex waveform is plotted out by algebraically adding the individual ordinates and is shown by thick line.

It may be noted that since the maximum amplitude of the harmonic is equal to the maximum amplitude of the fundamental, the complex wave is said to contain 100% of second harmonic.

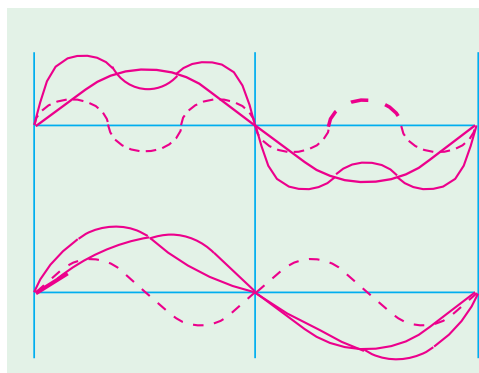


Fig. 20.3

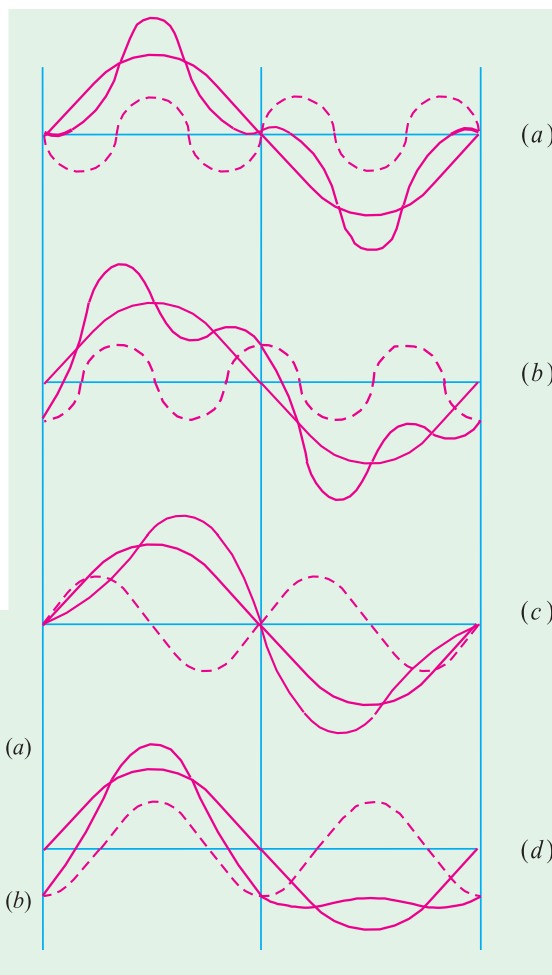


Fig. 20.4

The complex wave of Fig. 20.2 (b) is made up of the fundamental and 4th harmonic, that of Fig. 20.2 (c) consists of the fundamental and 3rd harmonic whereas that shown in Fig. 20.2 (d) is made up of the fundamental and 5th harmonic. Obviously, in all these cases, there is no phase difference between the fundamental and the harmonic.

Fig. 20.2 (a) and (c) have been reconstructed as Fig. 20.3 (a) and (b) respectively with the only difference that in this case, the amplitude of harmonic is half that of the fundamental *i.e.* the harmonic content is 50%.

The effect of phase difference between the fundamental and the harmonic on the shape of the resultant complex wave has been illustrated in Fig. 20.4.

Fig. 20.4 (a) shows the fundamental and second harmonic with phase difference of  $\pi/2$  and Fig. 20.4 (b) shows the same with a phase difference of  $\pi$ . In Fig. 20.4 (c) and (d) are shown the fundamental and third harmonic with a phase difference of  $\pi/2$  and  $\pi$  respectively. In all these figures, the amplitude of the harmonic has been taken equal to half that of the fundamental.

A careful examination of the above figures leads us to the following conclusions :

**1.** With **odd** harmonics, the positive and negative halves of the complex wave are symmetrical whatever the phase difference between the fundamental and the harmonic. In other words, the first and third quarters (*i.e.*  $\omega t$  from 0 to  $\pi/2$  and  $\omega t$  from  $3\pi/2$ ) and the second and fourth quarters (*i.e.*  $\omega t$  from  $\pi/2$  to  $\pi$  and  $\omega t$  from  $3\pi/2$  to  $2\pi$ ) are respectively similar.

**2. (i)** When **even** harmonics are present and their phase difference with the fundamental is 0 or  $\pi$ , then the first and fourth quarters of the complex wave are of the same phase but inverted and the same holds good for the second and third quarters.

**(ii)** When **even** harmonics are present and their phase difference with the fundamental is  $\pi/2$  or  $3\pi/2$ , then there is no symmetry as shown in Fig. 20 (a).

**3.** It may also be noted that the resultant displacement of the complex wave (whether containing odd or even harmonics) is zero at  $\omega t = 0$  only when the phase difference between the fundamental and the harmonics is either 0 to  $\pi$ .

The above conclusions are of great help in analysing a complex waveform into its harmonic constituents because a visual inspection of the complex wave enables us to rule out the presence of certain harmonics. For example, if the positive and negative half-cycles of a complex wave are symmetrical (*i.e.* the wave is symmetrical about  $\omega t = 0$ ), then we need not look for even harmonics. In some cases, we may be able to forecast the types of harmonics to be expected from their mode of production. For example, in alternators which are symmetrically designed, we should expect **odd** harmonics only.

### 20.3. General Equation of a Complex Wave

Consider a complex wave which is built up of the fundamental and a few harmonics, each of which has its own peak value of phase angle. The fundamental may be represented by

$$e_1 = E_{1m} \sin(\omega t + \Psi_1)$$

the second harmonic by  $e_2 = E_{2m} \sin(2\omega t + \Psi_2)$

the third harmonic by  $e_3 = E_{3m} \sin(3\omega t + \Psi_3)$  and so on.

The equation for the instantaneous value of the complex wave is given by

$$e = e_1 + e_2 + \dots + e_n = E_{1m} \sin(\omega t + \Psi_1) + E_{2m} \sin(2\omega t + \Psi_2) + \dots + E_{nm} \sin(n\omega t + \Psi_n)$$

when  $E_{1m}$ ,  $E_{2m}$  and  $E_{nm}$  etc. denote the maximum values or the amplitudes of the fundamental, second harmonic and  $n$ th harmonic etc. and  $\Psi_1$ ,  $\Psi_2$  and  $\Psi_n$  represent the phase differences with



respect to the complex wave\* (i.e. angle between the zero value of complex wave and the corresponding zero value of the harmonic).

The number of terms in the series depends on the shape of the complex wave. In relatively simple waves, the number of terms in the series would be less, in others, more.

Similarly, the instantaneous value of the complex wave is given by

$$i = I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \dots + I_{nm} \sin(n\omega t + \phi_n)$$

Obviously  $(\Psi_1 - \phi_1)$  is the phase difference between the harmonic voltage and current for the fundamental,  $(\Psi_2 - \phi_2)$  for the second harmonic and  $(\Psi_n - \phi_n)$  for the  $n$ th harmonic.

## 20.4. R.M.S. Value of a Complex Wave

Let the equation of the given complex current wave be

$$i = I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \dots + I_{nm} \sin(n\omega t + \phi_n)$$

Its r.m.s. value is given by  $I = \sqrt{\text{average value of } i^2 \text{ over whole cycle}}$

$$\begin{aligned} \text{Now } i^2 &= [I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \dots + I_{nm} \sin(n\omega t + \phi_n)]^2 \\ &= I_{1m}^2 \sin^2(\omega t + \phi_1) + I_{2m}^2 \sin^2(2\omega t + \phi_2) + \dots + I_{nm}^2 \sin^2(n\omega t + \phi_n) \\ &\quad + 2I_{1m}I_{2m} \sin(\omega t + \phi_1) \sin(2\omega t + \phi_2) + 2I_{1m}I_{3m} \sin(\omega t + \phi_1) \sin(3\omega t + \phi_3) + \dots \end{aligned}$$

The right-hand side of the above equation consists of two types of terms

- (i) harmonic self-products, the general expression for which is  $I_{pm}^2 \sin^2(p\omega t + \phi_p)$  for the  $p$ th harmonic and
- (ii) the products of different harmonics of the general form  $2I_{pm}I_{qm} \sin(p\omega t + \phi_p) \sin(q\omega t + \phi_q)$

The average value of  $i^2$  is the sum of the average values of these individual terms in the above equation. Let us now find the average value of the general term  $I_{pm}^2 \sin^2(p\omega t + \phi_p)$  over a whole cycle.

$$\begin{aligned} \text{Average value} &= \frac{1}{2\pi} \int_0^{2\pi} I_{pm}^2 \sin^2(p\omega t + \phi_p) d(\omega t) = \frac{I_{pm}^2}{2} \int_0^{2\pi} \sin^2(p\omega t + \phi_p) d\omega t \\ &= \frac{I_{pm}^2}{2} \int_0^{2\pi} \frac{1 - \cos 2(p\omega t + \phi_p)}{2} d\omega t = \frac{I_{pm}^2}{4} \left[ \omega t - \frac{\sin 2(p\omega t + \phi_p)}{2p} \right]_0^{2\pi} \\ &= \frac{I_{pm}^2}{4\pi} \times 2\pi = \frac{I_{pm}^2}{2} \end{aligned}$$

From this result, we can generalize that

$$\text{Average value of } I_{1m}^2 \sin^2(\omega t + \phi_1) = \frac{I_{1m}^2}{2}$$

$$\text{Average value of } I_{2m}^2 \sin^2(2\omega t + \phi_2) = \frac{I_{2m}^2}{2}$$

\* We could also express these phase angles with respect to the fundamentals wave instead of the complex wave.

Average value of  $I_{nm}^2 \sin^2(n\omega t + \phi_n) = \frac{I_{nm}^2}{2}$  and so on.

Now, the average value of the product terms is

$$\frac{1}{2} \int_0^{2\pi} I_{pm} I_{qm} \sin(p t + \phi_p) \sin(q t + \phi_q) d(t)$$

$$\frac{I_{pm} I_{qm}}{2} \int_0^{2\pi} \sin(p t + \phi_p) \sin(q t + \phi_q) d t = 0$$

$\therefore$  Average value of  $i^2 = \frac{I_{1m}^2}{2} + \frac{I_{2m}^2}{2} + \dots + \frac{I_{nm}^2}{2}$

$$\therefore \text{r.m.s. value, } I = \sqrt{\text{average value of } i^2} = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{2m}^2}{2} + \dots + \frac{I_{nm}^2}{2}}$$

... (i)

$$= 0.707 \sqrt{I_{1m}^2 + I_{2m}^2 + \dots + I_{nm}^2}$$

Equation (i) above may also be put in the form

$$I = \sqrt{\frac{I_{1m}^2}{\sqrt{2}^2} + \frac{I_{2m}^2}{\sqrt{2}^2} + \dots + \frac{I_{nm}^2}{\sqrt{2}^2}} = \sqrt{I_1^2 + I_2^2 + \dots + I_n^2}$$

where  $I_1 = I_{1m} / \sqrt{2}$  – r.m.s. value of fundamental

$I_2 = I_{2m} / \sqrt{2}$  – r.m.s. value of 2nd harmonic

$I_n = I_{nm} / \sqrt{2}$  – r.m.s. value of  $n$ th harmonic

Similarly, the r.m.s. value of a complex voltage wave is

$$E = 0.707 \sqrt{E_{1m}^2 + E_{2m}^2 + \dots + E_{nm}^2} = \sqrt{E_1^2 + E_2^2 + \dots + E_n^2}$$

Hence, the **rule is that the r.m.s. value of the complex current (or voltage) wave is given by the square-root of the sum of the squares of the r.m.s. values of its individual components.**

**Note.** If complex current wave contains a d.c. component of constant value  $I_D$  then its equation is given by

$$i = I_D + I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \dots + I_{nm} \sin(n\omega t + \phi_n)$$

$$\text{r.m.s. value, } I = \sqrt{I_D^2 + (I_{1m}/\sqrt{2})^2 + (I_{2m}/\sqrt{2})^2 + \dots + (I_{nm}/\sqrt{2})^2} = \sqrt{I_D^2 + I_1^2 + I_2^2 + \dots + I_n^2}$$

## 20.5. Form Factor of a Complex Wave

In general, it may be defined as  $k_f = \frac{\text{R.M.S. value}}{\text{average value}}$

A general expression for form factor in some simple cases may be found as under :

(i) **Sine Series.** Suppose the equation of a complex voltage wave is

$$v = V_{1m} \sin \omega t \pm V_{3m} \sin 3\omega t \pm V_{5m} \sin 5\omega t$$

$$= V_{1m} \sin \theta \pm V_{3m} \sin 3\theta \pm V_{5m} \sin 5\theta \quad \text{where } \omega = 2\pi / T.$$

Obviously, zeros occurs at  $t = 0$  or at  $\theta = 0^\circ$  and  $\theta = 180^\circ$  or  $t = T/2$ .

Mean value over half-cycle is

$$V_{av} = \frac{1}{\pi} \int_0^\pi v d\theta$$

$$\frac{1}{\sqrt{2}} \left( V_{1m} \sin \omega t + V_{3m} \sin 3\omega t + V_{5m} \sin 5\omega t \right) = \frac{2}{\sqrt{2}} \left( \frac{V_{1m}}{1} + \frac{V_{3m}}{3} + \frac{V_{5m}}{5} \right)$$

As found in Art. 20.4,

$$V = (V_1^2 + V_3^2 + V_5^2)^{1/2} = \frac{1}{\sqrt{2}} (V_{1m}^2 + V_{3m}^2 + V_{5m}^2)^{1/2}$$

$$\therefore k_f = \frac{(1/\sqrt{2})(V_{1m}^2 + V_{3m}^2 + V_{5m}^2)^{1/2}}{\frac{2}{\sqrt{2}} \left( \frac{V_{1m}}{1} + \frac{V_{3m}}{3} + \frac{V_{5m}}{5} \right)}$$

(ii) **Cosine Series.** Consider the following cosine series :

$$v = V_{1m} \cos \omega t + V_{3m} \cos 3\omega t + V_{5m} \cos 5\omega t$$

$$= V_{1m} \cos \theta + V_{3m} \cos 3\theta + V_{5m} \cos 5\theta$$

Obviously, in this case, zeros occur at  $\theta = \pm\pi/2$  or  $90^\circ$ . Moreover, positive and negative half-cycles are symmetrical.

$$\therefore V_{av} = \frac{1}{2} (V_{1m} \cos \theta + V_{3m} \cos 3\theta + V_{5m} \cos 5\theta) = \frac{2}{\sqrt{2}} \left( \frac{V_{1m}}{1} + \frac{V_{3m}}{3} + \frac{V_{5m}}{5} \right)$$

$$\therefore k_f = \frac{(1/\sqrt{2})(V_{1m}^2 + V_{3m}^2 + V_{5m}^2)^{1/2}}{\frac{2}{\sqrt{2}} \left( \frac{V_{1m}}{1} + \frac{V_{3m}}{3} + \frac{V_{5m}}{5} \right)}$$

**Example 20.1.** A voltage given by  $v = 50 + 24 \sin \omega t - 20 \sin 2\omega t$  is applied across the circuit shown in Fig. 20.5. What would be the readings of the instruments if  $\omega = 10,000$  rad/s.  $A_1$  is thermoelectric ammeter,  $A_2$  a moving-coil ammeter and  $V$  an electrostatic voltmeter.

**Solution.** It may be noted that the thermoelectric ammeter and the electrostatic voltmeter record the r.m.s. values of the current and voltage respectively. But the moving coil ammeter records the average values. Since the average values of the sinusoidal waves are zero, hence the moving coil ammeter reads the d.c. component of the current only. The d.c. will pass only through the inductive branch and not through the capacitive branch.

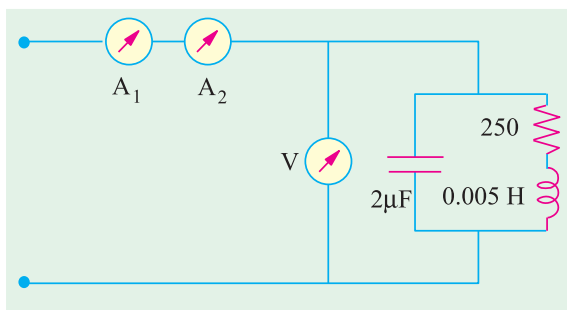


Fig. 20.5

$$\therefore I_{DC} = \frac{V_{DC}}{R} = \frac{50}{250} = 0.2 \text{ A}$$

Equivalent impedance of the circuit at fundamental frequency is

$$Z_1 = \frac{(R - jX_{L1})(jX_{C1})}{R - jX_{L1} + jX_{C1}} = \frac{(250 - j50)(j50)}{250 - j(50 - 50)} = \frac{2,500 + j12,500}{250} = 10 + j50 = 51 \angle 78^\circ 42'$$

$$\therefore \text{r.m.s. fundamental current } I_1 = I_{1m} / \sqrt{2} = 24 / 51 \times \sqrt{2} = 0.33 \text{ A}$$

Equivalent impedance of the circuit at the second harmonic is

$$Z_2 = \frac{(R - jX_{L2})(jX_{C2})}{R - jX_{L2} + jX_{C2}} = \frac{(250 - j100)(j25)}{200 - j75} = 31.5 \angle 88^\circ 43'$$

$\therefore$  r.m.s. value of second harmonic current

$$I_2 = I_{2m} / \sqrt{2} = 20 / 31.5 \times \sqrt{2} = 0.449 \text{ A}$$

r.m.s. current in the circuit is

$$I = \sqrt{I_{DC}^2 + I_1^2 + I_2^2} = \sqrt{0.2^2 + 0.33^2 + 0.449^2} = 0.593 \text{ A}$$

Hence, the reading of the thermoelectric ammeter is **0.593 A**

$$\text{The voltmeter reading } V = \sqrt{50^2 + (24/\sqrt{2})^2 + (20/\sqrt{2})^2} = \mathbf{54.66}$$

**Example 20.2.** Draw one complete cycle of the following wave

$$i = 100 \sin \omega t + 40 \sin 5 \omega t$$

Determine the average value, the r.m.s. value and form factor of the wave.

(Elect. Engineering, Osmania Univ.)

$$\text{Solution. } I_{av} = \frac{2}{1} \frac{I_{2m}}{5} = \frac{2}{1} \frac{100}{5} = \mathbf{68.7 \text{ A}}$$

$$I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{5m}^2}{2}} = \frac{1}{\sqrt{2}} (100^2 + 40^2)^{1/2} = \mathbf{76.2 \text{ A}}$$

$$\text{Form factor} = \frac{I}{I_{av}} = \frac{76.2}{68.7} = \mathbf{1.109}$$

## 20.6. Power Supplied by a Complex Wave

Let the complex voltage be represented by the equation

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + \dots + E_{nm} \sin n\omega t$$

be applied to a circuit. Let the equation of the resultant current wave be

$$i = I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \dots + I_{nm} \sin(n\omega t + \phi_n)$$

The instantaneous value of the power in the circuit is  $p = ei$  watt

For obtaining the value of this product, we will have to multiply every term of the voltage wave, in turn, by every term in the current wave. The average power supplied during a cycle would be equal to the sum of the average values over one cycle of each individual product term. However, as proved in Art. 20.4 earlier, the average value of all product terms involving harmonics of different frequencies will be zero over one cycle, so that we need consider only the products of current and voltage harmonics of the same frequency.

Let us consider a general term of this nature *i.e.*  $E_{nm} \sin n\omega t \times I_{nm} \sin(n\omega t - \phi_n)$  and find its average value over one cycle of the fundamental.

$$\text{Average value of power} = \frac{1}{2} \int_0^{2\pi} E_{nm} I_{nm} \sin n t \sin(n t - \phi_n) d(t)$$

$$= \frac{E_{nm} I_{nm}}{2} \int_0^{2\pi} \sin n t \sin(n t - \phi_n) d(t)$$

$$= \frac{E_{nm} I_{nm}}{2} \int_0^{2\pi} \frac{\cos \phi_n - \cos(2n t - \phi_n)}{2} d(t)$$

$$= \frac{E_{nm} I_{nm} \cos \phi_n}{2} = \frac{E_{nm}}{\sqrt{2}} \cdot \frac{I_{nm}}{\sqrt{2}} \cdot \cos \phi_n = E_n I_n \cos \phi_n$$

where  $E_n$  and  $I_n$  are the r.m.s. values of the voltage and current respectively. Hence, total average power supplied by a complex wave is **the sum of the average power supplied by each harmonic component acting independently.**

$$\therefore \text{Total power is } P = E_1 I_1 \cos \phi_1 + E_2 I_2 \cos \phi_2 + \dots + E_n I_n \cos \phi_n$$

The overall power factor is given by

$$\text{pf.*} = \frac{\text{total watts}}{\text{total voltamperes}} = \frac{E_1 I_1 \cos \phi_1 + E_2 I_2 \cos \phi_2 + \dots}{E I}$$

when

$E$  = r.m.s. value of the complex voltage wave

$I$  = r.m.s. value of the complex current wave

**Example 20.3.** A single-phase voltage source 'e' is given by

$$e = 141 \sin \omega t + 42.3 \sin 3\omega t + 28.8 \sin 5\omega t$$

The corresponding current in the load circuit is given by

$$i = 16.5 \sin(\omega t + 54.5^\circ) + 8.43 \sin(3\omega t - 38^\circ) + 4.65 \sin(5\omega t - 34.3^\circ)$$

Find the power supplied by the source.

(Electrical Circuits, Nagpur Univ. 1991)

**Solution.** In problems of such type, it is best to deal with each harmonic separately  
Power at fundamental

$$= E_1 I_1 \cos \phi_1 = \frac{E_{1m}}{\sqrt{2}} \cdot \frac{I_{1m}}{\sqrt{2}} \cos \phi_1 = \frac{E_{1m} I_{1m}}{2} \cos \phi_1 = \frac{141 \times 16.5}{2} \cos 54.5^\circ = 675.5 \text{ W}$$

$$\text{Power at 3rd harmonic} = \frac{E_{3m} I_{3m}}{2} \cos \phi_3 = \frac{42.3 \times 8.43}{2} \cos 38^\circ = 140.5 \text{ W}$$

$$\text{Power at 5th harmonic} = \frac{28.8 \times 4.65}{2} \cos 34.3^\circ = 55.5 \text{ W}$$

$$\text{Total power supplied} = 675.5 + 140.5 + 55.5 = \mathbf{871.5 \text{ W}}$$

**Example 20.4.** A complex voltage is given by  $e = 60 \sin \omega t + 24 \sin(3\omega t + \omega/6) + 12 \sin(5\omega t + \pi/3)$  is applied across a certain circuit and the resulting current is given by

$$i = 0.6 \sin(\omega t - 2\pi/10) + 0.12 \sin(\omega t - 2\pi/24) + 0.1 \sin(5\pi - 3\pi/4)$$

Find (i) r.m.s value of current and voltage (ii) total power supplied and (iii) the overall power factor.

**Solution.** In such problems where harmonics are involved, it is best to deal with each harmonic separately.

$$\text{Power at fundamental} = E_1 I_1 \cos \phi_1 = \frac{E_{1m} I_{1m}}{2} \cos \phi_1 = \frac{60 \times 0.6}{2} \times \cos 36^\circ = 14.56 \text{ W}$$

$$\text{Power at 3rd harmonic} = \frac{E_{3m} I_{3m}}{2} \cos 45^\circ = \frac{24 \times 0.12}{2} \times 0.707 = 1.02 \text{ W}$$

$$\text{Power at 5th harmonic} = \frac{E_{5m} I_{5m}}{2} \cos 75^\circ = \frac{12 \times 0.1}{2} \times 0.2588 = 0.16 \text{ W}$$

\* When harmonics are present, it is obvious that the overall p.f. of the circuit cannot be stated lagging or leading. It is simply the ratio of power in watts of voltamperes.

$$(i) \text{ R.M.S. current } I = \sqrt{I_1^2 + I_3^2 + I_5^2} = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2} + \frac{I_{5m}^2}{2}}$$

$$= \sqrt{\frac{0.6^2}{2} + \frac{0.12^2}{2} + \frac{0.1^2}{2}} = 0.438 \text{ A}$$

$$\text{R.M.S. volts, } E = \sqrt{\frac{60^2}{2} + \frac{24^2}{2} + \frac{12^2}{2}} = 46.5 \text{ V}$$

$$(ii) \text{ Total power} = 14.56 + 1.02 + 0.16 = 15.74 \text{ W}$$

$$(iii) \text{ Overall p.f.} = \frac{\text{watts}}{\text{voltamperes}} = \frac{15.74}{46.5 \times 0.438} = 0.773$$

## 20.7. Harmonics in Single-phase A.C. Circuits

If an alternating voltage, containing various harmonics, is applied to a single-phase circuit containing linear circuit elements, then the current so produced also contains harmonics. Each harmonic voltage will produce its own current independent of others. By the principle of superposition, the combined current can be found. We will now consider some of the well-known elements like pure resistance, pure inductance and pure capacitance and then various combinations of these. In each case, we will assume that the applied complex voltage is represented by

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + \dots + E_{nm} \sin n\omega t$$

### (a) Pure Resistance

Let the circuit have a resistance of  $R$  which is independent of frequency.

The instantaneous current  $i_1$  due to fundamental voltage is

$$i_1 = \frac{E_{1m} \sin \omega t}{R}$$

Similarly,  $i_2 = \frac{E_{2m} \sin 2\omega t}{R}$  for 2nd harmonic

and  $i_n = \frac{E_{nm} \sin n t}{R}$  ... for  $n$ th harmonic

total current  $i = i_1 + i_2 + \dots + i_n$

$$= \frac{E_{1m} \sin \omega t}{R} + \frac{E_{2m} \sin 2\omega t}{R} + \dots + \frac{E_{nm} \sin n\omega t}{R}$$

$$= I_{1m} \sin \omega t + I_{2m} \sin 2\omega t + \dots + I_{nm} \sin n\omega t$$

It shows that

- (i) the waveform of the resulting current is similar to that of the applied voltage *i.e.* the two waves are identical.
- (ii) the percentage of harmonic content in the current wave is the same as in the applied voltage.

### (b) Pure Inductance

Let the inductance of the circuit be  $L$  henry whose reactance varies directly as the frequency of the applied voltage. Its reactance for the fundamental would be  $X_1 = \omega L$ ; for the second harmonic,  $X_2 = 2\omega L$ , for the third harmonic,  $X_3 = 3\omega L$  and for the  $n$ th harmonic  $X_n = n\omega L$ .

However, for every harmonic term, the current will lag behind the voltage by  $90^\circ$ .

$$\text{Current due to fundamental, } i_1 = \frac{E_{1m}}{\omega L} \sin(\omega t - \pi/2)$$

$$\text{Current due to 2nd harmonic, } i_2 = \frac{E_{2m}}{2\omega L} \sin(2\omega t - \pi/2)$$

$$\text{Current due to 3rd harmonic, } i_3 = \frac{E_{3m}}{3\omega L} \sin(3\omega t - \pi/2)$$

$$\text{Current due to } n\text{th harmonic, } i_n = \frac{E_{nm}}{n\omega L} \sin(n\omega t - \pi/2)$$

$\therefore$  Total current  $i = i_1 + i_2 + \dots + i_n$

$$\frac{E_{1m}}{L} \sin(\omega t - \pi/2) + \frac{E_{2m}}{2L} \sin(2\omega t - \pi/2) + \dots + \frac{E_{nm}}{nL} \sin(n\omega t - \pi/2)$$

It can be seen from the above equation that

- (i) the waveform of the current differs from that of the applied voltage.
- (ii) for the  $n$ th harmonic, the percentage harmonic content in the current-wave is  $1/n$  of the corresponding harmonic content in the voltage wave. It means that in an inductive circuit, the current waveform shows less distortion than the voltage waveform. In this case, current more nearly approaches a sine wave than it does in a circuit containing resistance.

### (c) Pure Capacitance

In this case,

$$X_1 = \frac{1}{\omega C} \text{ - for fundamental ; } X_2 = \frac{1}{2\omega C} \text{ - for 2nd harmonic}$$

$$X_3 = \frac{1}{3\omega C} \text{ - for 3rd harmonic ; } X_n = \frac{1}{n\omega C} \text{ - for } n\text{th harmonic}$$

$$i_1 = \frac{E_{1m}}{1/\omega C} \sin(\omega t + \pi/2) = \omega C E_{1m} \sin(\omega t + \pi/2)$$

$$i_2 = \frac{E_{2m}}{1/2\omega C} \sin(2\omega t + \pi/2) = 2\omega C E_{2m} \sin(2\omega t + \pi/2)$$

$$i_n = \frac{E_{nm}}{1/n\omega C} \sin(n\omega t + \pi/2) = n\omega C E_{nm} \sin(n\omega t + \pi/2)$$

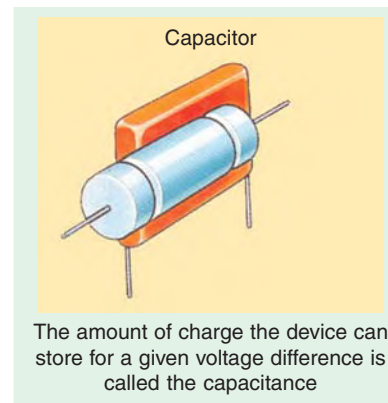
For every harmonic term, the current will lead the voltage by  $90^\circ$ .

Now  $i = i_1 + i_2 + \dots + i_n$

$$= \omega C E_{1m} \sin(\omega t + \pi/2) + 2\omega C E_{2m} \sin(2\omega t + \pi/2) + \dots + n\omega C E_{nm} \sin(n\omega t + \pi/2)$$

This equation shows that

- (i) the current and voltage waveforms are dissimilar.
- (ii) percentage harmonic content of the current is larger than that of the applied voltage wave. For example, for  $n$ th harmonic, it would be  $n$  time larger.
- (iii) as a result, the current wave is more distorted than the voltage wave.
- (iv) effect of capacitor on distortion is just the reverse of that of inductance.



**Example 20.5.** A complex wave of r.m.s. value 240 V has 20% 3rd harmonic content, 5% 5th harmonic content and 2% 7th harmonic content. Find the r.m.s. value of the fundamental and of each harmonic. **(Elect. Circuits, Gujarat Univ.)**

**Solution.** Let  $V_1$ ,  $V_3$ ,  $V_5$  and  $V_7$  be the r.m.s. values of the fundamental and harmonic voltages. Then

$$V_3 = 0.2 V_1; \quad V_5 = 0.05 V_1 \text{ and } V_7 = 0.02 V_1$$

$$240 = (V_1^2 + V_3^2 + V_5^2 + V_7^2)^{1/2}$$

$$\therefore 240 = [V_1^2 + (0.2 V_1)^2 + (0.05 V_1)^2 + (0.02 V_1)^2]^{1/2}$$

$$\therefore V_1 = \mathbf{235 \text{ V}}; \quad V_3 = 0.2 \times 235 = \mathbf{47 \text{ V}}$$

$$V_5 = 0.05 \times 235 = \mathbf{11.75 \text{ V}}; \quad V_7 = 0.02 \times 235 = \mathbf{4.7 \text{ V}}$$

**Example 20.6.** Derive an expression for the power, power factor and r.m.s. value for a complex wave.

A voltage  $e = 250 \sin \omega t + 50 \sin (3\omega t + \pi/3) + 2 \sin (\omega t + 5\pi/6)$  is applied to a series circuit of resistance  $20 \Omega$  and inductance  $0.05 \text{ H}$ . Derive (a) an expression for the current (b) the r.m.s. value of the current and for the voltage (c) the total power supplied and (d) the power factor. Take  $\omega = 314 \text{ rad/s}$ . **(Electrical Circuits, Nagpur Univ. 1991)**

**Solution. For Fundamental**

$$X_1 = \omega L = 314 \times 0.05 = 15.7 \Omega; \quad Z_1 = 20 + j15.7 = 25.4 \angle 38.1^\circ \Omega$$

**For Third Harmonic**

$$X_3 = 3\omega L = 3 \times 15.7 = 47.1 \Omega; \quad Z_3 = 20 + j47.1 = 51.2 \angle 67^\circ \Omega$$

**For Fifth Harmonic**

$$X_5 = 5\omega L = 5 \times 15.7 = 78.5 \Omega; \quad Z_5 = 20 + j78.5 = 81 \angle 75.7^\circ \Omega$$

(a) Expression for the current is

$$i = \frac{250}{25.4} \sin(\omega t - 38.1^\circ) + \frac{50}{51.2} \sin(3\omega t + 60^\circ - 67^\circ) + \frac{20}{81} \sin(5\omega t + 150^\circ - 75.7^\circ)$$

$$\therefore i = 9.84 \sin(\omega t - 38.1^\circ) + 0.9 \sin(3\omega t - 7^\circ) + 0.25 \sin(5\omega t + 74.3^\circ)$$

$$(b) \text{ R.M.S. current } I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2} + \frac{I_{5m}^2}{2}}$$

$$I^2 = \frac{9.84^2}{2} + \frac{0.9^2}{2} + \frac{0.25^2}{2} = 48.92$$

$$\therefore I = \sqrt{48.92} = \mathbf{6.99 \text{ A}}$$

$$\text{R.M.S. voltage } V = \sqrt{\frac{250^2}{2} + \frac{50^2}{2} + \frac{20^2}{2}} = \mathbf{180.8 \text{ V}}$$

$$(c) \text{ Total power } = I^2 R = 48.92 \times 20 = \mathbf{978 \text{ W}}$$

$$(d) \text{ Power factor } = \frac{\text{Watts}}{VI} = \frac{978}{180.8 \times 6.99} = \mathbf{0.773}$$



**Example 20.7.** An r.m.s. current of 5 A, which has a third harmonic content, is passed through a coil having a resistance of 1  $\Omega$  and an inductance of 10 mH. The r.m.s. voltage across the coil is 20 V. Calculate the magnitude of the fundamental and harmonic components of current if the fundamental frequency is  $300/2\pi$  Hz. Also, find the power dissipated.

**Solution. (i) Fundamental Frequency**

$$\omega = 300 \text{ rad/s}; X_L = 300 \times 10^{-2} = 3 \Omega \therefore Z_1 = 1 + j3 = 3.16 \angle 71.6^\circ \text{ ohm}$$

If  $V_1$  is the r.m.s. value of the fundamental voltage across the coil, then

$$V_1 = I_1 Z_1 = 3.16 I_1$$

**(ii) Third Harmonic**

$$X_3 = 3 \times 3 = 9 \Omega; Z_3 = 1 + j9 = 9.05 \angle 83.7^\circ \text{ ohm}; V_3 = I_3 Z_3 = 9.05 I_3$$

Since r.m.s. current of the complex wave is 5 A and r.m.s. voltage drop 20 V

$$5 = \sqrt{I_1^2 + I_3^2} \quad \text{and} \quad 20 = \sqrt{V_1^2 + V_3^2}$$

Substituting the values of  $V_1$  and  $V_3$ , we get,  $20 = [(3.16 I_1)^2 + (9.05 I_3)^2]^{1/2}$

Solving for  $I_1$  and  $I_3$ , we have  $I_1 = 4.8 \text{ A}$  and  $I_3 = 1.44 \text{ A}$

$$\text{Power dissipated} = I^2 R = 5^2 \times 1 = 25 \text{ W}$$

**Example 20.8.** An e.m.f. represented by the equation  $e = 150 \sin 314 t + 50 \sin 942 t$  is applied to a capacitor having a capacitance 20  $\mu\text{F}$ . What is the r.m.s. value of the charging current?

**Solution. For Fundamental**

$$X_{C1} = 1/\omega C = 10^6/20 \times 314 = 159; I_{1m} = E_{1m}/X_{C1} = 150/159 = 0.943 \text{ A}$$

**For Third Harmonic**

$$X_{C3} = 1/3\omega C = 159/3 = 53 \Omega \therefore I_{3m} = E_{3m}/X_{C3} = 50/53 = 0.943 \text{ A}$$

r.m.s. value of charging current,

$$I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2}} = \sqrt{\frac{0.943^2}{2} + \frac{0.943^2}{2}}$$

or  $I = 0.943 \text{ A}$

**Example 20.9.** The voltage given by  $v = 100 \cos 314 t + 50 \sin (1570t - 30^\circ)$  is applied to a circuit consisting of a 10  $\Omega$  resistance, a 0.02 H inductance and a 50  $\mu\text{F}$  capacitor. Determine the instantaneous current through the circuit. Also find the r.m.s. value of the voltage and current.

**Solution. For Fundamental**

$$\omega = 314 \text{ rad/s}; X_L = 314 \times 0.02 = 6.28 \Omega$$

$$X_C = 10^6/314 \times 50 = 63.8 \Omega; X = X_L - X_C = 6.28 - 63.8 = -57.32 \Omega$$

$$Z = \sqrt{10^2 + (-57.32)^2} = 58.3 \Omega; I_{1m} = 100/58.3 = 1.71 \text{ A}$$

$$\phi_1 = \tan^{-1}(-57.32/10) = -80.2^\circ \text{ (lead)}; i_1 = 1.71 \cos(314t + 80.2^\circ)$$

**For Fifth Harmonic**

$$\text{Inductive reactance} = 5 X_L = 5 \times 6.28 = 31.4 \Omega$$

$$\text{Capacitive reactance} = X_C/5 = 63.8/5 = 12.76 \Omega$$

$$\text{Net reactance} = 31.4 - 12.76 = 18.64 \Omega$$

$$Z = \sqrt{10^2 + 18.64^2} = 21.2 \Omega$$

$$I_{5m} = 50/21.2 = 2.36 \text{ A}; \quad \phi_5 = \tan^{-1}(18.64/10) = 61.8^\circ \text{ (lag)}$$

$$i_5 = 2.36 \sin(1570t - 30^\circ - 61.8^\circ) = 2.36 \sin(1570t - 91.8^\circ)$$

Hence, total instantaneous current is

$$i = i_1 + i_5 = 1.71 \cos(314t + 80.2^\circ) + 2.36 \sin(1570t - 91.8^\circ)$$

$$\text{R.M.S. voltage} = \sqrt{\frac{100^2}{2} + \frac{50^2}{5}} = 79.2 \text{ V}$$

$$\text{R.M.S. current} = \sqrt{\frac{1.71^2}{2} + \frac{2.36^2}{2}} = 2.06 \text{ A}$$

**Example 20.10.** A  $6.36 \mu\text{F}$  capacitor is connected in parallel with a resistance of  $500 \Omega$  and the combination is connected in series with a  $500\text{-}\Omega$  resistor. The whole circuit is connected across an a.c. voltage given by  $e = 300 \sin \omega t + 100 \sin(3\omega t + \pi/6)$ .

If  $\omega = 314 \text{ rad/s}$ , find

(i) power dissipated in the circuit

(ii) an expression for the voltage across the series resistor

(iii) the percentage harmonic content in the resultant current.

**Solution. For Fundamental**

$$X_{C1} = \frac{1}{\omega C} = \frac{10^6}{314 \times 6.36} = 500 \Omega$$

The impedance of the whole series-parallel circuit is given by

$$Z_1 = 500 + \frac{500(j500)}{500 + j500} = 750 + j250 = 791 \angle 18.4^\circ$$

**For Third Harmonic**

$$X_{C3} = 1/3\omega C = 500/3 = 167 \Omega$$

$$\therefore Z_3 = 500 + \frac{500(j167)}{500 + j167} = 550 + j150 = 570 \angle 15.3^\circ$$

$$\therefore i = \frac{300}{791} \sin(\omega t + 18.4^\circ) + \frac{100}{570} \sin(3\omega t + 45.3^\circ)$$

$$= 0.397 \sin(\omega t + 18.4^\circ) + 0.175 \sin(3\omega t + 45.3^\circ)$$

$$(i) \text{ Power dissipated} = \frac{E_{1m} I_{1m}}{2} \cos \phi_1 + \frac{E_{3m} I_{3m}}{2} \cos \phi_3$$

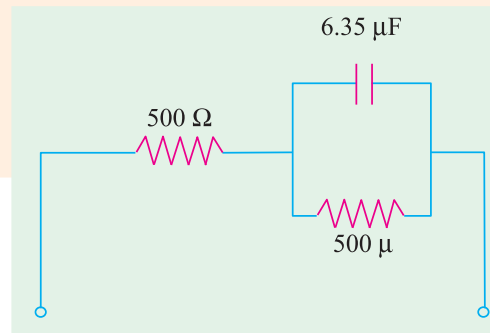


Fig. 20.6

$$= \frac{300 \times 0.379}{2} \times \cos 18.4^\circ + \frac{100 \times 0.175}{2} \cos 15.3^\circ = \mathbf{62.4 \text{ W}}$$

(ii) The voltage drop across the series resistor would be

$$E_R = iR = 500[0.379 \sin(\omega t + 18.4^\circ) + 0.175 \sin(3\omega t + 45.3^\circ)]$$

$$e_R = \mathbf{189.5 \sin(\omega t + 18.4^\circ) + 87.5 \sin(3\omega t + 45.3^\circ)}$$

(iii) The percentage harmonic content of the current is  $= 87.5/189.5 \times 100 = \mathbf{46.2\%}$

**Example 20.11.** An alternating voltage of  $v = 1.0 \sin 500t + 0.5 \sin 1500t$  is applied across a capacitor which can be represented by a capacitance of  $0.5 \mu\text{F}$  shunted by a resistance of  $4,000 \Omega$ . Determine

(i) the r.m.s. value of the current    (ii) the r.m.s. value of the applied voltage

(iii) the p.f. of the circuit.

(Circuit Theory and Components, Madras Univ.)

**Solution. For Fundamental** [Fig. 20.7 (a)]

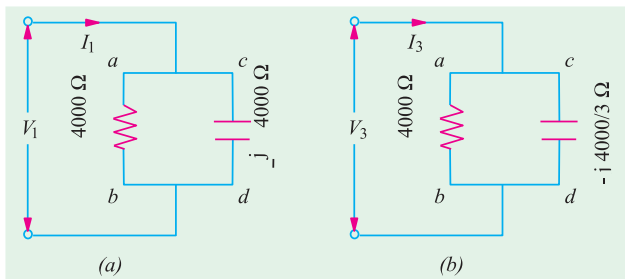


Fig. 20.7

$$V_1 = 1.0/\sqrt{2} = 0.707 \text{ V}$$

$$\text{Let, } V_1 = (0.707 + j0)$$

Capacitive reactance

$$= jX_{C1} = -j 10^6/500 \times 0.5$$

$$= j 4000 \Omega ; R = 4,000 \Omega$$

$$\therefore I_{ab1} = 0.707/4,000 = 0.177 \text{ mA}$$

$$I_{cd1} = 0.707/-j4,000 = j0.177 \text{ mA}$$

$$\therefore I_1 = 0.177 + j0.177$$

$$= 0.25 \angle 45^\circ \text{ mA}$$

Hence,  $I_1$  lead the fundamental voltage by  $45^\circ$ .

$$P_{ab1} = 0.707 \times 0.177 = 0.125 \text{ mW} ; P_{cd1} = 0$$

**For Third Harmonic** [Fig. 20.7 (b)]

$$V_3 = 0.5/\sqrt{2} = 0.3535 \angle 0^\circ ; R = 4,000 \Omega ; X_{C3} = -j4,000/3 \Omega$$

$$I_{ab3} = 0.3535/4000 = 0.0884 \angle 0^\circ \text{ mA} ; I_{cd3} = 0.3535/-j(4,000/3) = j 0.265 \text{ mA}$$

$$I_3 = 0.0884 + j0.265 = 0.28 \angle 71.6^\circ \text{ mA}$$

$$P_{ab3} = 0.3535 \times 0.0884 = 0.0313 \text{ mW} ; P_{cd3} = 0$$

$$(i) \text{ R.M.S. current} = \sqrt{I_1^2 + I_3^2} = \sqrt{0.25^2 + 0.28^2} = \mathbf{0.374 \text{ mA}}$$

$$(ii) \text{ R.M.S. voltage} = \sqrt{(1/\sqrt{2})^2 + (0.5/\sqrt{2})^2} = \mathbf{0.79 \text{ V}}$$

(iii) Power factor = watts/voltampere

$$\text{Wattage} = (0.125 + 0.0313) \times 10^{-3} = 0.1563 \times 10^{-3} \text{ W}$$

$$\text{Volt-amperes} = 0.79 \times 0.374 = 0.295 ; \text{p.f.} = 0.1563 \times 10^{-3}/0.295 = \mathbf{0.0005}$$

## 20.8. Selective Resonance due to Harmonics

When a complex voltage is applied across a circuit containing both inductance and capacitance, it may happen that the circuit resonates at one of the harmonic frequencies of the applied voltage. This phenomenon is known as **selective resonance**.

If it is a series circuit, then large currents would be produced at resonance, even though the applied voltage due to this harmonic may be small. Consequently, it would result in large harmonic voltage appearing across both the capacitor and the inductance.

If it is a parallel circuit, then at resonant frequency, the resultant current drawn from the supply would be minimum.

It is because of the possibility of such selective resonance happening that every effort is made to eliminate harmonics in supply voltage.

However, the phenomenon of selective resonance has been usefully employed in some wave analyses for determining the harmonic content of alternating waveforms. For this purpose, a variable inductance, a variable capacitor, a variable non-inductive resistor and a fixed non-inductive resistance or shunt for an oscillograph are connected in series and connected to show the wave-form of the voltage across the fixed non-inductive resistance. The values of inductance and capacitance are adjusted successively to give resonance for the first, third, first and seventh harmonics and a record of the waveform is obtained by the oscillograph. A quick inspection of the shape of the waveform helps to detect the presence or absence of a particular harmonic.

**Example 20.12.** An e.m.f.  $e = 200 \sin \omega t + 40 \sin 3 \omega t + 10 \sin 5 \omega t$  is impressed on a circuit comprising of a resistance of  $10 \Omega$ , a variable inductor and a capacitance of  $30 \mu\text{F}$ , all connected in series. Find the value of the inductance which will give resonance with triple frequency component of the pressure and estimate the effective p.f. of the circuit,  $\omega = 300$  radian/second. **(Elect. Engg. I, Bombay Univ.)**

**Solution.** For resonance at third harmonic

$$3\omega L = 1/3\omega C \quad \therefore L = 1/9\omega^2 C = 10^6/9 \times 300^2 \times 30 = \mathbf{0.041 \text{ H}}$$

$$Z_1 = 10 - j 300 \cdot 0.041 \frac{10^6}{300 \cdot 30} = 10 + j(12.3 - 111.1) = 10 - j98.8 = 99.3 \angle -84.2^\circ$$

$$Z_3 = 10 - j 3 L \frac{1}{3 C} = 10 - j(36.9 - 37.0) = 10 \angle 0^\circ$$

$$Z_5 = 10 - j 5 L \frac{1}{5 L} = 10 - j(61.5 - 22.2) = 10 - j39.3 = 40.56 \angle 75.7^\circ$$

$$I_{1m} = 200/99.3 = 2.015 \text{ A}; I_{3m} = 40/10 = 4 \text{ A}; I_{5m} = 10/40.56 = 0.246 \text{ A}$$

$$I = \sqrt{\frac{2.015^2}{2} + \frac{4^2}{2} + \frac{0.246^2}{2}} = \sqrt{10.06} = 3.172 \text{ A}$$

$$V = \sqrt{\frac{200^2}{2} + \frac{40^2}{2} + \frac{10^2}{2}} = 144.5 \text{ V}; \text{ Power} = I^2 R = 10.06 \times 10 = 100.6 \text{ W}$$

$$\text{Volt-amperes } VI = 144.5 \times 3.172 = 458 \text{ VA}; \text{ Power factor} = 100.6/458 = \mathbf{0.22}$$

**Example 20.13.** A coil having  $R = 100 \Omega$  and  $L = 0.1 \text{ H}$  is connected in series with a capacitor across a supply, the voltage of which is given by  $e = 200 \sin 314t + 5 \sin 3454t$ . What capacitance would be required to produce resonance with the 11th harmonic. Find (a) the equation of the current and (b) the r.m.s. value of the current, if this capacitance is in circuit.

**Solution.** For series resonance,  $X_L = X_C$

Since resonance is required for 11th harmonic whose frequency is 3454 rad/s, hence

$$3454L = \frac{1}{3454C}; C = \frac{1}{3454^2 \times 0.1} \text{ farad} = \mathbf{0.838 \mu\text{F}}$$

**(a) For Fundamental**

$$\text{Inductive reactance} = \omega L = 314 \times 0.1 = 31.4 \Omega$$

$$\text{Capacitive reactance} = 1/\omega C = 10^6 / 0.836 \times 314 = 3796 \Omega$$

$\therefore$  Net reactance =  $3796 - 31.4 = 3765 \ \Omega$ ; Resistance =  $100 \ \Omega$

$\therefore Z_1 = \sqrt{100^2 + 3765^2} = 3767 \ \Omega$ ;  $\tan \phi_1 = 3765/100 = 37.65$

$\therefore \phi_1 = 88^\circ 28'$  (leading) = 1.546 radian

Now  $E_{1m} = 200 \text{ V}$ ;  $Z_1 = 3767 \ \Omega \quad \therefore I_{1m} = 200/3767 = 0.0531 \text{ A}$

### Eleventh Harmonic

New reactance = 0; Impedance  $Z_{11} = 100 \ \Omega$

$\therefore$  Current  $I_{11m} = 5/100 = 0.05 \text{ A}$ ;  $\phi_{11} = 0$

... at resonance

Hence, the equation of the current is

$$i = \frac{200}{3767} (\sin 314t + 1.546) + \frac{5}{100} \sin(3454t + 0)$$

$$i = 0.0531 \sin(314t + 1.546) + 0.05 \sin 3454t$$

$$(b) I = \sqrt{(0.0531)^2/2 + (0.05)^2/2} = 0.052 \text{ A}$$

## 20.9. Effect of Harmonics on Measurement of Inductance and Capacitance

Generally, with the help of ammeter and voltmeter readings, the value of impedance, inductance and capacitance of a circuit can be calculated. But while dealing with complex voltages, the use of instrument readings does not, in general, give correct values of inductance and capacitance except in the case of a circuit containing only pure resistance. It is so because, in the case of resistance, the voltage and current waveforms are similar and hence the values of r.m.s. volts and r.m.s. amperes (as read by the voltmeter and ammeter respectively) would be the same whether they were sinusoidal or non-sinusoidal (*i.e.* complex).

### (i) Effect on Inductances

Let  $L$  be the inductance of a circuit and  $E$  and  $I$  the r.m.s. values of the applied voltage and current as read by the instruments connected in the circuit. For a complex voltage

$$E = 0.707 \sqrt{E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + \dots}$$

$$\text{Hence } I = 0.707 \sqrt{\frac{E_{1m}^2}{L} + \frac{E_{3m}^2}{3L} + \frac{E_{5m}^2}{5L} + \dots}$$

$$\frac{0.707}{L} \sqrt{E_{1m}^2 + \frac{1}{9}E_{3m}^2 + \frac{1}{25}E_{5m}^2 + \dots}$$

$$\therefore L = \frac{0.707}{I} \sqrt{E_{1m}^2 + \frac{1}{9}E_{3m}^2 + \frac{1}{25}E_{5m}^2 + \dots}$$

For calculating the value of  $L$  from the above expression, it is necessary to know the absolute value of the amplitudes of several harmonic voltages. But, in practice, it is more convenient to deal with relative values than with absolute values. For this purpose, let us multiply and divide the right-hand side of the above expression by  $E$  but write the  $E$  **in the denominator** in its form  $0.707 \sqrt{E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + \dots}$

$$\therefore L = \frac{0.707}{I} \sqrt{E_{1m}^2 + \frac{1}{9}E_{3m}^2 + \frac{1}{25}E_{5m}^2 + \dots} \times \frac{E}{0.707 \sqrt{E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + \dots}}$$

$$\text{or } L = \frac{E}{I} \sqrt{\frac{E_{1m}^2}{(E_{1m}^2)} + \frac{1/9 E_{3m}^2}{E_{3m}^2} + \frac{1/25 E_{5m}^2}{E_{5m}^2} + \dots} = \frac{E}{I} \sqrt{\frac{1}{1} + \frac{1/9 (E_{3m}/E_{1m})^2}{(E_{3m}/E_{1m})^2} + \frac{1/25 (E_{5m}/E_{1m})^2}{(E_{5m}/E_{1m})^2} + \dots}$$

If the effect of harmonics were to be neglected, then the value of the inductance would appear to be  $E/\omega I$  but the true or actual value is less than this. The apparent value has to be multiplied by the quantity under the radical to get the true value of inductance when harmonics are present.

The quantity under the radical is called the correction factor *i.e.*

True inductance ( $L$ ) = Apparent inductance ( $L'$ )  $\times$  correction factor

### (ii) Effect on Capacitance

Let the capacitance of the circuit be  $C$  farads and  $E$  and  $I$  the instrument readings for voltage and current. Since the instruments read r.m.s. values, hence, as before,

$$E = 0.707 \sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + \dots)}$$

$$\begin{aligned} \text{Hence } I &= 0.707 \sqrt{\frac{E_{1m}^2}{1/C} + \frac{E_{3m}^2}{1/3C} + \frac{E_{5m}^2}{1/5C} + \dots} \\ &= 0.707 \sqrt{(\omega C E_{1m})^2 + (3\omega C E_{3m})^2 + (5\omega C E_{5m})^2 + \dots} \\ &= 0.707 \omega C \sqrt{E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + \dots} \end{aligned}$$

$$\therefore C = \frac{I}{0.707 \omega \sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + \dots)}}$$

Again, we will multiply and divide the right-hand side  $E$  but in this case, we will write  $E$  in the numerator in its form  $[0.707 \sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + \dots)}]$

$$\therefore C = \frac{1}{0.707 E \sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + \dots)}} = \frac{1}{0.707 \sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + \dots)}} \times \frac{1}{E}$$

$$\frac{1}{E} \sqrt{\frac{E_{1m}^2}{E_{1m}^2} + \frac{E_{3m}^2}{9E_{3m}^2} + \frac{E_{5m}^2}{25E_{5m}^2} + \dots} = \frac{1}{E} \sqrt{\frac{1}{1} + \frac{(E_{3m}/E_{1m})^2}{9(E_{3m}/E_{1m})^2} + \frac{(E_{5m}/E_{1m})^2}{25(E_{5m}/E_{1m})^2} + \dots}$$

Again, if the effects of harmonics were neglected, the value of capacitance would appear to be  $I/\omega E$  but its true value is less than this. For getting the true value, this apparent value will have to be multiplied by the quantity under the radical (which, therefore, is referred to as correction factor).\*

$\therefore$  True capacitance ( $C$ ) = Apparent capacitance ( $C'$ )  $\times$  correction factor

**Example 20.14.** A current of 50-Hz containing first, third and fifth harmonics of maximum values 100, 15 and 12 A respectively, is sent through an ammeter and an inductive coil of negligibly small resistance. A voltmeter connected to the terminals shows 75 V. What would be the current indicated by the ammeter and what is the exact value of the inductance of the coil in henrys?

\* It may be noted that this correction factor is different from that in the case of pure inductance.

**Solution.** The r.m.s. current is

$$I = 0.707\sqrt{I_{1m}^2 + I_{3m}^2 + I_{5m}^2} = 0.707\sqrt{(100^2 + 15^2 + 12^2)} = 72 \text{ A}$$

Hence, current indicated by the ammeter is **72 A**

$$\text{Now } E = 0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2)}$$

$$\text{Also } I_{1m} = \frac{E_{1m}}{\omega L}; I_{3m} = \frac{E_{3m}}{3\omega L}; I_{5m} = \frac{E_{5m}}{5\omega L}$$

$$\therefore E_{1m} = I_{1m} \cdot \omega L; E_{3m} = I_{3m} \cdot 3\omega L; E_{5m} = I_{5m} \cdot 5\omega L$$

$$\therefore E = 0.707\sqrt{(I_{1m}\omega L)^2 + (I_{3m}3\omega L)^2 + (I_{5m}5\omega L)^2} = 0.707\omega L\sqrt{I_{1m}^2 + 9I_{3m}^2 + 25I_{5m}^2}$$

$$\therefore 75 = 0.707L \times 2\pi \times 50 \sqrt{100^2 + 9 \times 15^2 + 25 \times 12^2}$$

$$\therefore L = \mathbf{0.0027 \text{ H}}$$

$$\text{Note. Apparent inductance } L' = \frac{E}{\omega I} = \frac{75}{2\pi \times 50 \times 72} = 0.00331 \text{ H}$$

**Example 20.15.** The capacitance of a  $20 \mu\text{F}$  capacitor is checked by direct connection to an alternating voltage which is supposed to be sinusoidal, an electrostatic voltmeter and a dynamometer ammeter being used for measurement. If the voltage actually follows the law,

$$e = 100 \sin 250 t + 20 \sin (500 t - \phi) + 10 \sin (750 t - \phi)$$

Calculate the value of capacitance as obtained from the direct ratio of the instrument readings.

**Solution.** True value,  $C = 20 \mu\text{F}$

Apparent value  $C' =$  value read by the instruments

Now,  $C = C' \times$  correction factor.

Let us find the value of correction factor.

$$\text{Here } E_{1m} = 100; E_{2m} = 20 \text{ and } E_{3m} = 10$$

$$\therefore \text{Correction factor} = \sqrt{\frac{E_{1m}^2}{E_{1m}^2} \frac{E_{2m}^2}{4E_{2m}^2} \frac{E_{3m}^2}{9E_{3m}^2}} = \sqrt{\frac{100^2}{100^2} \frac{20^2}{4 \cdot 20^2} \frac{10^2}{9 \cdot 10^2}} = 0.9166$$

$$20 = C' \times 0.9166$$

$$\therefore C' = 20 / 0.9166 = \mathbf{21.82 \mu\text{F}}$$

## 21.10. Harmonics in Different Three-phase Systems

In three-phase systems, harmonics may be produced in the same way as in single-phase systems.

Hence, for all calculation they are treated in the same manner *i.e.* each harmonic is treated separately. Usually, even harmonics are absent in such systems. But care must be exercised when dealing with odd, especially, third harmonics and all multiples of 3rd harmonic (also called the triple- $n$  harmonics).

**(a) Expressions for Phase E.M.Fs.**

Let us consider a 3-phase alternator having identical phase windings ( $R$ ,  $Y$  and  $B$ ) in which harmonics are produced. The three phase e.m.fs. would be represented in their proper phase sequence by the equation.

$$e_R = E_{1m}(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5) + \dots$$

$$e_Y = E_{1m} \sin \left( t \frac{2}{3} \right) + E_{3m} \sin \left( 3 t \frac{2}{3} \right) + E_{5m} \sin \left( 5 t \frac{2}{3} \right) + \dots$$

$$e_B = E_{1m} \sin \left( t \frac{4}{3} \right) + E_{3m} \sin \left( 3 t \frac{4}{3} \right) + E_{5m} \sin \left( 5 t \frac{4}{3} \right) + \dots$$

On simplification, these become

$$e_R = E_{1m} \sin(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5) + \dots \quad \text{— as before}$$

$$e_Y = E_{1m} \sin \left( t \frac{2}{3} \right) + E_{3m} \sin \left( 3 t \frac{2}{3} \right) + E_{5m} \sin \left( 5 t \frac{10}{3} \right) + \dots$$

$$E_{1m} \sin \left( t \frac{2}{3} \right) + E_{3m} \sin \left( 3 t \frac{2}{3} \right) + E_{5m} \sin \left( 5 t \frac{4}{3} \right) + \dots$$

$$e_B = E_{1m} \sin \left( t \frac{4}{3} \right) + E_{3m} \sin \left( 3 t \frac{4}{3} \right) + E_{5m} \sin \left( 5 t \frac{2}{3} \right) + \dots$$

From these expressions, it is clear that

- (i) All third harmonics are equal in all phases of the circuit *i.e.* they are in time phase.
- (ii) Fifth harmonics in the three phases have a negative phase sequence of  $R, B, Y$  because the fifth harmonic of blue phase reaches its maximum value before that in the yellow phase.
- (iii) All harmonics which are not multiples of three, have a phase displacement of  $120^\circ$  so that they can be dealt with in the usual manner.
- (iv) At any instant, all the e.m.fs. have the same direction which means that in the case of a  $Y$ -connected system they are directed either away from or towards the neutral point and in the case of  $\Delta$ -connected system, they flow in the same direction.

Main points can be summarized as below :

- (i) all triple- $n$  harmonics *i.e.* 3rd, 9th, 15th etc. are in phase,
- (ii) the 7th, 13th and 19th harmonics have positive phase rotation of  $R, Y, B$ .
- (iii) the 5th, 11th and 17th harmonics have a negative phase sequence of  $R, B, Y$ .

**(b) Line Voltage for a Star-connected System**

In this system, the line voltages will be the **difference** between successive phase voltages and hence will contain no third harmonic terms because they, being identical in each phase, will cancel out. The fundamental will have a line voltage  $\sqrt{3}$  times the phase voltage. Also, fifth harmonic has line voltage  $\sqrt{3}$  its phase voltage.

But it should be noted that in this case the r.m.s. value of the line voltage will be less than  $\sqrt{3}$  times the r.m.s. value of the phase voltage due to the absence of third harmonic term from the line voltage. It can be proved that for any line voltage.

$$\text{Line value} = \sqrt{3} \sqrt{\frac{E_1^2 + E_5^2 + E_7^2}{E_1^2 + E_3^2 + E_5^2 + E_7^2}}$$



where  $E_1, E_3$  etc. are r.m.s. values of the phase e.m.fs.

**(c) Line Voltage for a  $\Delta$ -connected System**

If the winding of the alternator are delta-connected, then the resultant e.m.f. acting round the closed mesh would be the sum of the phase e.m.fs. The sum of these e.m.fs. is zero for fundamental, 5th, 7th, 11th etc. harmonics. Since the third harmonics are in phase, there will be a resultant third harmonic e.m.f. of three times the phase value acting round the closed mesh. It will produce a circulating current whose value will depend on the impedance of the windings at the third harmonic frequency. It means that the third harmonic e.m.f. would be short-circuited by the windings with the result that there will be no third harmonic voltage across the lines. The same is applicable to all triple- $n$  harmonic voltages. Obviously, the line voltage will be the phase voltage but without the triple- $n$  terms.

**Example 20.16.** A 3- $\phi$  generator has a generated e.m.f. of 230 V with 15 per cent third harmonic and 10 per cent fifth harmonic content. Calculate

(i) the r.m.s. value of line voltage for Y-connection.

(ii) the r.m.s. value of line voltage for  $\Delta$ -connection.

**Solution.** Let  $E_1, E_3, E_5$  be the r.m.s. values of the phase e.m.fs. Then

$$E_3 = 0.15 E_1 \text{ and } E_5 = 0.1 E_1$$

$$\therefore 230 = \sqrt{E_1^2 + (0.15E_1)^2 + (0.1E_1)^2}$$

$$E_1 = 226 \text{ V} \quad \therefore E_3 = 0.15 \times 226 = 34 \text{ V} \text{ and } E_5 = 0.1 \times 226 = 22.6 \text{ V}$$

(i) r.m.s. value of the fundamental line voltage =  $\sqrt{3} \times 226 = 392 \text{ V}$

r.m.s. value of third harmonic line voltage = 0

r.m.s. value of 5th harmonic line voltage  $\sqrt{3} \times 22.6 = 39.2 \text{ V}$

$$\therefore \text{r.m.s. value of line voltage } V_L = \sqrt{392^2 + 39.2^2} = 394 \text{ V}$$

(ii) In  $\Delta$ -connection, again the third harmonic would be absent from the line voltage

$$\therefore \text{r.m.s. value of line voltage } V_L = \sqrt{226^2 + 22.6^2} = 227.5 \text{ V}$$

**(d) Circulating Current in  $\Delta$ -connected Alternator**

Let the three symmetrical phase e.m.fs. of the alternator be represented by the equations,

$$e_R = E_{1m} \sin(\omega t + \Psi_1) + E_{3m} \sin(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5) + \dots$$

$$e_Y = E_{1m} \sin(\omega t + \Psi_1 - 2\pi/3) + E_{3m} \sin(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5 - 4\pi/3) + \dots$$

$$e_B = E_{1m} \sin(\omega t + \Psi_1 - 4\pi/3) + E_{3m} \sin(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5 - 2\pi/3) + \dots$$

The resultant e.m.f. acting round the  $\Delta$ -connected windings of the armature is the sum of these e.m.fs. Hence it is given by  $e = e_R + e_Y + e_B$

$$\therefore e = 3E_{3m} \sin(3\omega t + \Psi_3) + 3E_{9m} \sin(9\omega t + \Psi_9) + 3E_{15m} \sin(15\omega t + \Psi_{15}) + \dots$$

If  $R$  and  $L$  represent respectively the resistance and inductance per phase of the armature winding, then the circulating current due to the resultant e.m.f. is given by

$$i_c = \frac{3E_{3m} \sin(3\omega t + \Psi_3)}{3\sqrt{(R^2 + 9L^2\omega^2)}} + \frac{3E_{9m} \sin(9\omega t + \Psi_9)}{3\sqrt{(R^2 + 81L^2\omega^2)}} + \frac{3E_{15m} \sin(15\omega t + \Psi_{15})}{3\sqrt{(R^2 + 225L^2\omega^2)}} + \dots$$

$$= \frac{E_{3m} \sin(3\omega t + \Psi_3)}{\sqrt{(R^2 + 9L^2\omega^2)}} + \frac{E_{9m} \sin(9\omega t + \Psi_9)}{\sqrt{(R^2 + 81L^2\omega^2)}} + \frac{E_{15m} \sin(15\omega t + \Psi_{15})}{\sqrt{(R^2 + 225L^2\omega^2)}}$$

The r.m.s. value of the current is given by

$$I_C = 0.707 \sqrt{[E_{3m}^2 / (R^2 + 9L^2\omega^2) + E_{9m}^2 / (R^2 + 81L^2\omega^2) + (E_{15m}^2 / (R^2 + 225L^2\omega^2) + \dots)]}$$

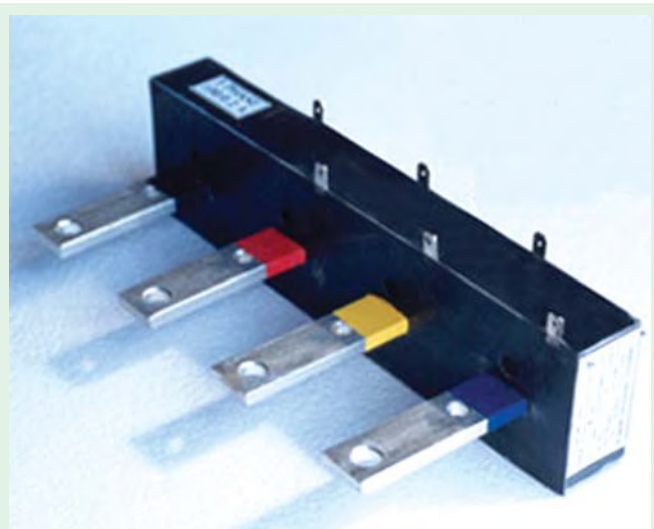
### (e) Three-phase four-wire System

In this case, there will be no third harmonic component in line voltage. For the 4-wire system, each phase voltage (*i.e.* line to neutral) may contain a third harmonic component. If it is actually present, then current will flow in the *Y*-connected load. In case load is balanced, the resulting third harmonic line currents will all be in phase so that neutral wire will have to carry three times the third harmonic line current. There will be no current in the neutral wire either at fundamental frequency, or any harmonic frequency other than the triple-*n* frequency.

## 20.11. Harmonics in Single and 3-phase Transformers

The flux density in transformer core is usually maintained at a fairly high value in order to keep the required volume of iron to the minimum. However, due to the non-linearity of magnetisation curve, some third harmonic distortion is always produced. Also, there is usually a small percentage of fifth harmonic. The magnetisation current drawn by the primary contains mainly third harmonic whose proportion depends on the size of the primary applied voltage. Hence, the flux is sinusoidal.

In the case of three-phase transformers, the production of harmonics will be affected by the method of connection and the type of construction employed.



3-phase current transformer

### (a) Primary Windings $\Delta$ -CONNECTED

Each primary phase can be considered as separately connected across the sinusoidal supply.

(i) The core flux will be sinusoidal which means that magnetizing current will contain 3rd harmonic component in addition to relatively small amounts of other harmonics of higher order.

(ii) In each phase, these third harmonic currents will be in phase and so produce a circulating current round the mesh with the result that there will be no third harmonic component in the line current.

### (b) Primary Winding Connected in 4-wire Star

Each phase of the primary can again be considered as separately connected across a sinusoidal supply.

- (j) The flux in the transformer core would be sinusoidal and so would be the output voltage.
- (ii) The magnetizing current will contain 3rd harmonic component. This component being in phase in each winding will, therefore, return through the neutral wire.

**(c) Primary Windings Connected 3-wire Star**

Since there is no neutral wire, there will be no return path for the 3rd harmonic component of the magnetizing current. Hence, there will exist a condition of forced magnetization so that core flux must contain third harmonic component which is in phase in each limb of the transformer core. Although there will be a magnetic path for these fluxes in the case of shell type 3-phase transformer, yet in the case of three-limb core type transformer, the third harmonic component of the flux must return *via* the air. Because of the high reluctance magnetic path in such transformers, the third harmonic flux is reduced to a very small value. However, if the secondary of the transformer is delta-connected, then a third harmonic circulating current would be produced. This current would be in accordance with Lenz's law tend to oppose the very cause producing it *i.e.* it would tend to minimize the third harmonic component of the flux.

Should the third and fifth secondary be *Y*-connected, then provision of an additional  $\Delta$ -connected winding, in which this current can flow, becomes necessary. This tertiary winding additionally served the purpose of preserving magnetic equilibrium of the transformer in the case of unbalanced loads. In this way, the output voltage from the secondary can be kept reasonably sinusoidal.

**Example 20.17.** Determine whether the following two waves are of the same shape

$$e = 10 \sin (\omega t + 30^\circ) - 50 \sin (3 \omega t - 60^\circ) + 25 \sin (5 \omega t + 40^\circ)$$

$$i = 1.0 \sin (\omega t - 60^\circ) + 5 \sin (3 \omega t - 150^\circ) + 2.5 \cos (5 \omega t - 140^\circ)$$

(Principles of Elect. Engg-II Jadavpur Univ.)

**Solution.** Two waves possess the same waveshape

- (i) if they contain the same harmonics
- (ii) if the ratio of the corresponding harmonics to their respective fundamentals is the same
- (iii) if the harmonics are similarly spaced with respect to their fundamentals.

In other words,

- (a) the ratio of the magnitudes of corresponding harmonics must be constant and
- (b) with fundamentals in phase, the corresponding harmonics of the two waves must be in phase.

The test is applied first by checking the ratio of the corresponding harmonics and then coinciding the fundamentals by shifting one wave. If the phase angles of the corresponding harmonics are the same, then the two waves have the same shape.

In the present case, condition (i) is fulfilled because the voltage and current waves contain the same harmonics, *i.e.* third and fifth.

Secondly, the ratio of the magnitude of corresponding current and voltage harmonics is the same *i.e.* 1/10.

Now, let the fundamental of the current wave be shifted ahead by  $90^\circ$  so that it is brought in phase with the fundamental of the voltage wave. It may be noted that the third and fifth harmonics of the current wave will be shifted by  $3 \times 90^\circ = 270^\circ$  and  $5 \times 90^\circ = 450^\circ$  respectively. Hence, the current wave becomes

$$i' = 1.0 \sin (\omega t - 60^\circ + 90^\circ) + 5 \sin (3\omega t - 150^\circ + 270^\circ) + 2.5 \cos (5\omega t - 140^\circ + 450^\circ)$$

$$= 1.0 \sin (\omega t + 30^\circ) + 5 \sin (3\omega t + 120^\circ) + 2.5 \cos (5\omega t + 310^\circ)$$

$$= 1.0 \sin(\omega t + 30^\circ) - 5 \sin(3\omega t - 60^\circ) + 2.5 \sin(5\omega t + 40^\circ)$$

It is seen that now the corresponding harmonics of the voltage and current waves are in phase. Since all conditions are fulfilled, the two waves are of the same waveshape.

### Tutorial Problem No. 20.1

1. A series circuit consist of a coil of inductance 0.1 H and resistance 25  $\Omega$  and a variable capacitor. Across this circuit is applied a voltage whose instantaneous value is given by

$$v = 100 \sin \omega t + 20 \sin(3\omega t - 45) + 5 \sin(5\omega t - 30) \text{ where } \omega = 314 \text{ rad / s}$$

Determine the value of  $C$  which will produce response at third harmonic frequency and with this value of  $C$ , find

(a) an expression for the current in the circuit (b) the r.m.s. value of this current (c) the total power absorbed.

$$[11.25 \mu\text{F}, (a) i = 0.398 \sin(\omega t + 84.3) + 0.8 \sin(3\omega t + 45) + 0.485 \sin(5\omega t + 106) (b) 0.633 \text{ A} (c) 10 \text{ W}]$$

2. A voltage given by  $v = 200 \sin 314 t + 520 \sin(942 t + 45^\circ)$  is applied to a circuit consisting of a resistance of 20  $\Omega$ , and inductance of 20 mH and a capacitance of 56.3  $\mu\text{F}$  all connected in series.

Calculate the r.m.s. values of the applied voltage and current. Find also the total power absorbed by the circuit.

$$[146 \text{ V} ; 3.16 \text{ A} ; 200 \text{ W}]$$

3. A voltage given by  $v = 100 \sin \omega t + 8 \sin 3\omega t$  is applied to a circuit which has a resistance of 1  $\Omega$ , an inductance of 0.02 H and a capacitance of 60  $\mu\text{F}$ . A hot-wire ammeter is connected in series with the circuit and a hot-wire voltmeter is connected to the terminals. Calculate the ammeter and voltmeter readings and the power supplied to the circuit.

$$[71 \text{ V} ; 5.18 \text{ A} ; 26.8 \text{ W}]$$

4. A certain coil has a resistance of 20  $\Omega$  and an inductance of 0.04 H. If the instantaneous current flowing in it is represented by  $i = 5 \sin 300 t + 0.8 \sin 900 t$  amperes, derive an expression for the instantaneous value of the voltage applied across the ends of the coil and calculate the r.m.s. value of that voltage.

$$[V = 117 \sin(300 t + 0.541) + 33 \sin(900 t + 1.06) ; 0.86 \text{ V}]$$

5. A voltage given by the equation  $v = \sqrt{2} 100 \sin 2\pi \times 50 t + \sqrt{2.20} \sin 2\pi \cdot 150 t$  is applied to the terminals of a circuit made up of a resistance of 5  $\Omega$ , an inductance of 0.0318 H and a capacitor of 12.5  $\mu\text{F}$  all in series. Calculate the effective current and the power supplied to the circuit.

$$[0.547 \text{ A} ; 1.5 \text{ W}]$$

6. An alternating voltage given by the expression  $v = 1,000 \sin 314 t + 100 \sin 942 t$  is applied to a circuit having a resistance of 100  $\Omega$  and an inductance of 0.5 H. Calculate r.m.s. value of the current and p.f.

$$[3.81 \text{ A} ; 0.535]$$

7. The current in a series circuit consisting of a 159  $\mu\text{F}$  capacitor, a reactor with a resistance of 10  $\Omega$  and an inductances of 0.0254 H is given by  $i = \sqrt{2}(8 \sin \omega t + 2 \sin 3\omega t)$  amperes. Calculate the power input and the power factor. Given  $\omega = 100\pi$  radian/second.

$$[680 \text{ W} ; 0.63]$$

8. If the terminal voltage of a circuit is  $100 \sin \omega t + 50 \sin(3\omega t + \pi/4)$  and the current is  $10 \sin(\omega t + \pi/3) + 5 \sin 3\omega t$ , calculate the power consumption.

$$[522.6 \text{ W}]$$

9. A single-phase load takes a current of  $4 \sin(\omega t + \pi/6) + 1.5 \sin(3\omega t + \pi/3)$  A from source represented by  $360 \sin \omega t$  volts. Calculate the power dissipated by the circuit and the circuit power factor.

$$[623.5 \text{ W} ; 0.837]$$

10. An e.m.f. given by  $e = 100 \sin \omega t + 40 \sin(\omega t - \pi/6) + 10 \sin(5\omega t - \pi/3)$  volts is applied to a series circuit having a resistance of  $100 \Omega$ , an inductance of  $40.6 \text{ mH}$  and a capacitor of  $10 \mu\text{F}$ . Derive an expression for the current in the circuit. Also, find the r.m.s. value of the current and the power dissipated in the circuit. Take  $\omega = 314 \text{ rad/second}$ .

[0.329 A, 10.8 W]

11. A p.d. of the form  $v = 400 \sin \omega t + 30 \sin 3\omega t$  is applied to a rectifier having a resistance of  $50 \Omega$  in one direction and  $200$  in the reverse direction. Find the average and effective values of the current and the p.f. of the circuit.

[1.96 A, 4.1 A, 0.51]

12. A coil having  $R = 2 \Omega$  and  $L = 0.01 \text{ H}$  carries a current given by  $i = 50 + 20 \sin 300 t$

A moving-iron ammeter, a moving-coil voltmeter and a dynamometer wattmeter, are used to indicate current, voltage and power respectively. Determine the readings of the instruments and equation for the p.d.

[121.1 V ; 52 A ; 5.4 kW,  $V = 100 + 72 \sin(300 t + 0.982)$ ]

13. Two circuits having impedances at  $50 \text{ Hz}$  of  $(10 + j6) \Omega$  and  $(10 - j6)$  respectively are connected in parallel across the terminals of an a.c. system, the waveform of which is represented by  $v = 100 \sin \omega t + 35 \sin 3\omega t + 10 \sin 5\omega t$ , the fundamental frequency being  $50 \text{ Hz}$ . Determine the ratio of the readings of two ammeters, of negligible resistance, connected one in each circuit.

[6.35 ; 6.72]

14. Explain what is meant by harmonic resonance in a.c. circuits.

A current having an instantaneous value of  $2(\sin \omega t + \sin 3\omega t)$  amperes is passed through a circuit which consists of a coil of resistance  $R$  and inductance  $L$  in series with a capacitor  $C$ . Derive an expression for the value of  $\omega$  at which the r.m.s. circuit voltage is a minimum. Determine the voltage if the coil has inductance  $0.1 \text{ H}$  and resistance  $150 \Omega$  and the capacitance is  $10 \mu\text{F}$ . Determine also the circuit voltage at the fundamental resonant frequency.

[ $\omega = 1/\sqrt{LC}$  ; 378 V ; 482 V]

15. An r.m.s. current of  $5 \text{ A}$  which has a third-harmonic content, is passed through a coil having a resistance of  $1 \Omega$  and an inductance of  $10 \text{ mH}$ . The r.m.s. voltage across the coil is  $20 \text{ V}$ . Calculate the magnitudes of the fundamental and harmonic components of current if the fundamental frequency is  $300/2\pi \text{ Hz}$ . Also, find the power dissipated.

[4.8 A ; 1.44 A ; 25 W]

16. Derive a general expression for the form factor of a complex wave containing only odd-order harmonics. Hence, calculate the form factor of the alternating current represented by

$$i = 2.5 \sin 157 t + 0.7 \sin 471 t + 0.4 \sin 785 t$$

[1.038]

## OBJECTIVE TESTS – 20

- Non-sinusoidal waveforms are made up of
  - different sinusoidal waveforms
  - fundamental and even harmonics
  - fundamental and odd harmonics
  - even and odd harmonics only.
- The positive and negative halves of a complex wave are symmetrical when
  - it contains even harmonics
  - phase difference between even harmonics and fundamental is  $0$  or  $\pi$
  - it contains odd harmonics
  - phase difference between even harmonics and fundamental is either  $\pi/2$  or  $3\pi/2$ .
- The r.m.s. value of the complex voltage given by  $v = 16\sqrt{2} \sin \omega t + 12\sqrt{2} \sin 3\omega t$  is

- (a)  $20\sqrt{2}$                       (b) 20  
(c)  $28\sqrt{2}$                       (d) 192
4. In a 3-phase system, \_\_\_th harmonic has negative phase sequence of RBY.  
(a) 9                                  (b) 13  
(c) 5                                  (d) 15
5. A complex current wave is given by the equation  $i = 14 \sin \omega t + 2 \sin 5\omega t$ . The r.m.s. value of the current is \_\_\_ ampere.  
(a) 16                                  (b) 12  
(c) 10                                  (d) 8
6. When a pure inductive coil is fed by a complex voltage wave, its current wave  
(a) has larger harmonic content  
(b) is more distorted  
(c) is identical with voltage wave  
(d) shows less distortion.
7. A complex voltage wave is applied across a pure capacitor. As compared to the fundamental voltage, the reactance offered by the capacitor to the third harmonic voltage would be  
(a) nine time                      (b) three times  
(c) one-third                      (d) one-ninth
8. Which of the following harmonic voltage components in a 3-phase system would be in phase with each other ?  
(a) 3rd, 9th, 15th etc.  
(b) 7th, 13th, 19th etc.  
(c) 5th, 11th, 17th etc.  
(d) 2nd, 4th, 6th etc.

**ANSWERS**

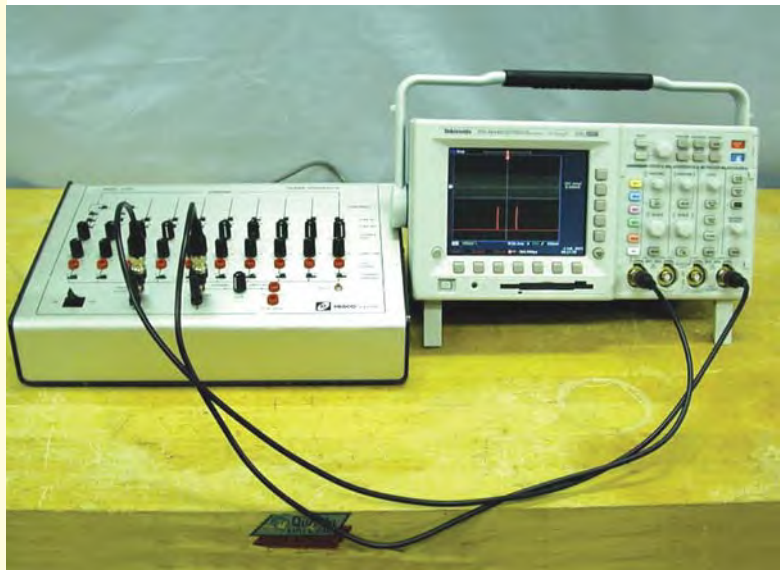
1. (a)   2. (c)   3. (b)   4. (c)   5. (c)   6. (d)   7. (c)   8. (d)

# CHAPTER 21

## Learning Objectives

- Harmonic Analysis
- Periodic Functions
- Trigonometric Fourier Series
- Alternate Forms of Trigonometric Fourier Series
- Certain Useful Integral Calculus Theorems
- Evaluation of Fourier Constants
- Different Types of Functional Symmetries
- Line or Frequency Spectrum
- Procedure for Finding the Fourier Series of a Given Function
- Wave Analyzer
- Spectrum Analyzer
- Fourier Analyzer
- Harmonic Synthesis

## FOURIER SERIES



With the help of Fourier Theorem, it is possible to determine the magnitude, order and phase of the several harmonics present in a complex periodic wave

### 21.1. Harmonic Analysis

By harmonic analysis is meant the process of determining the magnitude, order and phase of the several harmonics present in a complex periodic wave.

For carrying out this analysis, the following methods are available which are all based on Fourier theorem:

(i) Analytical Method—the standard Fourier Analysis  
 (ii) Graphical Method— (a) by Superposition Method (Wedgemore' Method) (b) Twenty four Ordinate Method

(iii) Electronic Method—by using a special instrument called 'harmonic analyser'

We will consider the first and third methods only.

### 21.2. Periodic Functions

A function  $f(t)$  is said to be periodic if  $f(t+T) = f(t)$  for all values of  $t$  where  $T$  is some positive number. This  $T$  is the interval between two successive repetitions and is called the period of  $f(t)$ . A sine wave having a period of  $T = 2\pi / \omega$  is a common example of periodic function.

### 21.3. Trigonometric Fourier Series

Suppose that a given function  $f(t)$  satisfies the following conditions (known as Dirichlet conditions):

1.  $f(t)$  is periodic having a period of  $T$ .
2.  $f(t)$  is single-valued everywhere.
3. In case it is discontinuous,  $f(t)$  has a finite number of discontinuities in any one period.
4.  $f(t)$  has a finite number of maxima and minima in any one period.

The function  $f(t)$  may represent either a voltage or current waveform. According to Fourier theorem, this function  $f(t)$  may be represented in the trigonometric form by the infinite series.

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + \dots + a_n \cos n\omega_0 t \\ + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + b_3 \sin 3\omega_0 t + \dots + b_n \sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

... (i)

Putting  $\omega_0 t = \theta$ , we can write the above equation as under

$$f(\theta) = a_0 + a_1 \cos \theta + a_2 \cos 2\theta + a_3 \cos 3\theta + \dots + a_n \cos n\theta + b_1 \sin \theta + b_2 \sin 2\theta + b_3 \sin 3\theta + \dots + b_n \sin n\theta$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

... (ii)

Since  $\omega_0 = 2\pi / T$ , Eq. (i) above can be written as

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{T} t \right)$$

... (iii)



where  $\omega_0$  is the fundamental angular frequency,  $T$  is the period and  $a_0, a_n$  and  $b_n$  are constants which depend on  $n$  and  $f(t)$ . The process of determining the values of the constants  $a_0, a_n$  and  $b_n$  is called Fourier Analysis. Also,  $\omega_0 = 2\pi / T = 2\pi f_0$  where  $f_0$  is the fundamental frequency.

It is seen from the above Fourier Series that the periodic function consists of sinusoidal components of frequency  $0, \omega_0, 2\omega_0, \dots, n\omega_0$ . This representation of the function  $f(t)$  is in the frequency domain. The first component  $a_0$  with zero frequency is called the *dc* component. The sine and cosine terms represent the harmonics. The number  $n$  represents the order of the harmonics.

When  $n = 1$ , the component  $(a_1 \cos \omega_0 t + b_1 \sin \omega_0 t)$  is called the first harmonic or the fundamental component of the waveform.

When  $n = 2$ , the component  $(a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t)$  is called the second harmonic of the waveform.

The  $n$ th harmonic of the waveform is represented by  $(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$ . It has a frequency of  $n\omega_0$  i.e.  $n$  times the frequency of the fundamental component.

### 21.4. Alternate Forms of Trigonometric Fourier Series

Eq. (j) given above can be written as

$$f(t) = a_0 + (a_1 \cos \omega_0 t + b_1 \sin \omega_0 t) + (a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t) + \dots + (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

Let,  $a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = A_n \cos(n\omega_0 t - \phi_n)$

$$= A_n \cos n\omega_0 t \cos \phi_n + A_n \sin n\omega_0 t \sin \phi_n$$

$$\therefore a_n = A_n \cos \phi_n \text{ and } b_n = A_n \sin \phi_n$$

$$\therefore A_n = \sqrt{a_n^2 + b_n^2} \text{ and } \phi_n = \tan^{-1} b_n / a_n$$

Similarly, let  $(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = A_n \sin(n\omega_0 t + \Psi_n))$

$$= A_n \sin n\omega_0 t \cos \Psi_n + A_n \cos n\omega_0 t \sin \phi_n$$

As seen from Fig. 21.1,  $b_n = A_n \cos \psi_n$  and  $a_n = A_n \sin \psi_n$

$$\therefore A_n = \sqrt{a_n^2 + b_n^2} \text{ and } \psi_n = \tan^{-1} a_n / b_n$$

The two angles  $\phi_n$  and  $\psi_n$  are complementary angles.

Hence, the Fourier series given in Art. 21.2 may be put in the following two alternate forms

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \phi_n)$$

or  $f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega_0 t + \psi_n)$

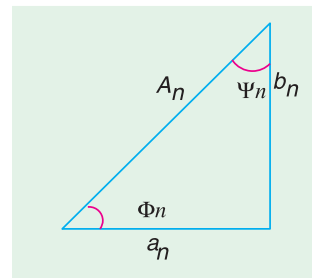


Fig. 21.1

### 21.5. Certain Useful Integral Calculus Theorems

The Fourier coefficients or constants  $a_0, a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_n$  can be evaluated by integration process for which purpose the following theorems will be used.

$$(i) \int_0^{2\pi} \sin n\theta d\theta = \frac{1}{n} \left| \cos n\theta \right|_0^{2\pi} = \frac{1}{n} (1-1) = 0$$

$$(ii) \int_0^{2\pi} \cos n\theta d\theta = -\frac{1}{n} \left| \sin n\theta \right|_0^{2\pi} = -\frac{1}{n} (0-0) = 0$$

$$(iii) \int_0^{2\pi} \sin^2 n\theta d\theta = \frac{1}{2} \int_0^{2\pi} (1 - \cos 2n\theta) d\theta = \frac{1}{2} \left| \theta - \frac{\sin 2n\theta}{2n} \right|_0^{2\pi} = \pi$$

$$(iv) \int_0^{2\pi} \cos^2 n\theta d\theta = \frac{1}{2} \int_0^{2\pi} (\cos 2n\theta + 1) d\theta = \frac{1}{2} \left| \frac{\sin 2n\theta}{2n} + \theta \right|_0^{2\pi} = \pi$$

$$(v) \int_0^{2\pi} \sin m\theta \cos n\theta d\theta = \frac{1}{2} \int_0^{2\pi} \{ \sin(m+n)\theta - \sin(m-n)\theta \} d\theta$$

$$= \frac{1}{2} \left| -\frac{1}{m+n} \cos(m+n)\theta - \frac{1}{m-n} \cos(m-n)\theta \right|_0^{2\pi} = 0$$

$$(vi) \int_0^{2\pi} \cos m\theta \cos n\theta d\theta = \frac{1}{2} \int_0^{2\pi} \{ \cos(m+n)\theta + \cos(m-n)\theta \} d\theta$$

$$= \frac{1}{2} \left| \frac{1}{m+n} \sin(m+n)\theta + \frac{1}{m-n} \sin(m-n)\theta \right|_0^{2\pi} = 0 \dots \text{for } n \neq m$$

$$(vii) \int_0^{2\pi} \sin m\theta \sin n\theta d\theta = \frac{1}{2} \int_0^{2\pi} \{ \cos(m-n)\theta - \cos(m+n)\theta \} d\theta$$

$$= \frac{1}{2} \left| \frac{1}{m-n} \sin(m-n)\theta - \frac{1}{m+n} \sin(m+n)\theta \right|_0^{2\pi} = 0 \dots \text{for } n \neq m$$

where  $m$  and  $n$  are any positive integers.

### 21.6. Evaluation of Fourier Constants

Let us now evaluate the constants  $a_0, a_n$  and  $b_n$  by using the above integral calculus theorems

#### (i) Value of $a_0$

For this purpose we will integrate both sides of the series given below over one period *i.e.* for  $\theta = 0$  to  $\theta = 2\pi$ .

$$f(\theta) = a_0 + a_1 \cos \theta + a_2 \cos 2\theta + \dots + a_n \cos n\theta + b_1 \sin \theta + b_2 \sin 2\theta + \dots + b_n \sin n\theta$$

$$\therefore \int_0^{2\pi} f(\theta) d\theta = \int_0^{2\pi} a_0 d\theta + a_1 \int_0^{2\pi} \cos \theta d\theta + a_2 \int_0^{2\pi} \cos 2\theta d\theta + \dots + a_n \int_0^{2\pi} \cos n\theta d\theta$$

$$+ b_1 \int_0^{2\pi} \sin \theta d\theta + b_2 \int_0^{2\pi} \sin 2\theta d\theta + \dots + b_n \int_0^{2\pi} \sin n\theta d\theta$$

$$= a_0 \int_0^{2\pi} 1 d\theta = 0 + 0 + \dots + 0 + 0 + \dots + 0 = 2\pi a_0$$

$$\therefore a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta \text{ or } = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) d\theta$$

= mean value of  $f(\theta)$  between the limits 0 to  $2\pi$  i.e. over one cycle or period.

$$\text{Also, } a_0 = \frac{1}{2} [\text{net area}]_0^{2\pi}$$

If we take the periodic function as  $f(t)$  and integrate over period  $T$  (which corresponds to  $2\pi$ ),

$$\text{we get } a_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt = \frac{1}{T} \int_{t_1}^{t_1+T} f(t) dt$$

where  $t_1$  can have any value.

### (ii) Value of $a_n$

For finding the value of  $a_n$ , multiply both sides of the Fourier Series by  $\cos n\theta$  and integrate between the limits 0 to  $2\pi$

$$\begin{aligned} \therefore \int_0^{2\pi} f(\theta) \cos n\theta d\theta &= a_0 \int_0^{2\pi} \cos n\theta d\theta + a_1 \int_0^{2\pi} \cos \theta \cos n\theta d\theta + a_2 \int_0^{2\pi} \cos 2\theta \cos n\theta d\theta \\ &+ a_n \int_0^{2\pi} \cos^2 n\theta d\theta + b_1 \int_0^{2\pi} \cos \theta \sin n\theta d\theta + b_2 \int_0^{2\pi} \sin 2\theta \cos n\theta d\theta + \dots + b_n \int_0^{2\pi} \sin n\theta \cos n\theta d\theta \\ &= 0 + 0 + 0 + \dots + a_n \int_0^{2\pi} \cos^2 n\theta d\theta + 0 + 0 + \dots + 0 = a_n \int_0^{2\pi} \cos^2 n\theta d\theta = \pi a_n \end{aligned}$$

$$\therefore a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = 2 \times \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta$$

= 2 × average value of  $f(\theta) \cos n\theta$  over one cycle of the fundamental.

$$\text{Also, } a_n = \frac{1}{2} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = \frac{1}{2} \int_0^{2\pi} f(\theta) \cos n\theta d\theta$$

If we take periodic function as  $f(t)$ , then different expressions for  $a_n$  are as under.

$$a_n = \frac{2}{T} \int_0^T f(t) \cos \frac{2\pi n}{T} t dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi n}{T} t dt$$

Giving different numerical values to  $n$ , we get

$$a_1 = 2 \times \text{average of } f(\theta) \cos \theta \text{ over one cycle} \quad \dots n = 1$$

$$a_2 = 2 \times \text{average value of } f(\theta) \cos 2\theta \text{ over one cycle etc.} \quad \dots n = 2$$

### (iii) Value of $b_n$

For finding its value, multiply both sides of the Fourier Series of Eq. (i) by  $\sin n\theta$  and integrate between limits  $\theta = 0$  to  $\theta = 2\pi$ .

$$\begin{aligned} \therefore \int_0^{2\pi} f(\theta) \sin n\theta d\theta &= a_0 \int_0^{2\pi} \sin n\theta d\theta + a_1 \int_0^{2\pi} \cos \theta \sin n\theta d\theta \\ &+ a_2 \int_0^{2\pi} \cos 2\theta \sin n\theta d\theta + \dots + a_n \int_0^{2\pi} \cos n\theta \sin n\theta d\theta \\ &+ b_1 \int_0^{2\pi} \sin \theta \sin n\theta d\theta + b_2 \int_0^{2\pi} \sin 2\theta \sin n\theta d\theta + \dots + b_n \int_0^{2\pi} \sin^2 n\theta d\theta \\ &= 0 + 0 + 0 + \dots + 0 + 0 + \dots + b_n \int_0^{2\pi} \sin^2 n\theta d\theta = b_n \int_0^{2\pi} \sin^2 n\theta d\theta = b_n \pi \end{aligned}$$

$$\therefore \int_0^{2\pi} f(\theta) \sin n\theta d\theta = b_n \times \pi$$

$$\therefore b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta = 2 \times \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta$$

= 2 × average value of  $f(\theta) \sin n\theta$  over one cycle of the fundamental.

$$\therefore b_1 = 2 \times \text{average value of } f(\theta) \sin \theta \text{ over one cycle} \quad \dots n = 1$$

$$b_2 = 2 \times \text{average value of } f(\theta) \sin 2\theta \text{ over one cycle} \quad \dots n = 2$$

$$\text{Also, } b_n = \frac{2}{T} \int_0^T f(t) \sin \frac{2\pi n}{T} t dt + \frac{2}{T} \int_{T/2}^{T/2} f(t) \sin \frac{2\pi n}{T} t dt$$

$$= \frac{2}{T} \int_0^T f(t) \sin n\omega_0 t dt = \frac{2}{T} \int_{T/2}^{T/2} f(t) \sin n\omega_0 t dt$$

Hence, for Fourier analysis of a periodic function, the following procedure should be adopted:

(i) Find the term  $a_0$  by integrating both sides of the equation representing the periodic function between limits 0 to  $2\pi$  or 0 to  $T$  or  $-T/2$  to  $T/2$  or  $t_1$  to  $(t_1 + T)$ .

$$\therefore a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt = \frac{1}{T} \int_{t_1}^{t_1+T} f(t) dt$$

= average value of the function over one cycle.

(ii) Find the value of  $a_n$  by multiplying both sides of the expression for Fourier series by  $\cos n\theta$  and then integrating it between limits 0 to  $2\pi$  or 0 to  $T$  or  $-T/2$  to  $T/2$  or  $t_1$  to  $(t_1 + T)$ .

$$\therefore a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = 2 \times \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta$$

Since  $\pi = T/2$ , we have

$$a_n = \frac{2}{T} \int_0^T f(t) \cos \frac{2\pi n}{T} t dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi n}{T} t dt = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \cos \frac{2\pi n}{T} t dt$$

$$= \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t dt = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \cos n\omega_0 t dt$$

= 2 × average value of  $f(\theta) \cos n\theta$  over one cycle of the fundamental.

Values of  $a_1, a_2, a_3$  etc. can be found from above by putting  $n = 1, 2, 3$  etc.

(iii) Similarly, find the value of  $b_n$  by multiplying both sides of Fourier series by  $\sin n\theta$  and integrating it between the limits 0 to  $2\pi$  or 0 to  $T$  or  $-T/2$  to  $T/2$  or  $t_1$  to  $(t_1 + T)$ .

$$\begin{aligned} \therefore b_n &= \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta = 2 \times \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta \\ &= \frac{2}{T} \int_0^T f(t) \sin \frac{2\pi n}{T} t dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin \frac{2\pi n}{T} t dt = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \sin \frac{2\pi n}{T} t dt \\ &= \frac{2}{T} \int_0^T f(t) \sin n\omega_0 t dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega_0 t dt = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \sin n\omega_0 t dt \\ &= 2 \times \text{average value of } f(\theta) \sin n\theta \text{ of } f(t) \sin \frac{2\pi n}{T} t \text{ or } f(t) \sin n\omega_0 t \text{ over one cycle of} \end{aligned}$$

the fundamental.

Values of  $b_1, b_2, b_3$  etc. can be found from above by putting  $n = 1, 2, 3$  etc.

## 21.7. Different Types of Functional Symmetries

A non-sinusoidal wave can have the following types of symmetry:

### 1. Even Symmetry

The function  $f(t)$  is said to possess even symmetry if  $f(t) = f(-t)$ .

It means that as we travel equal amounts in time to the left and right of the origin (*i.e.* along the  $+X$ -axis and  $-X$ -axis), we find the function to have the same value. For example in Fig. 21.2 (a), points  $A$  and  $B$  are equidistant from point  $O$ . Here the two function values are equal and positive. At points  $C$  and  $D$ , the two values of the function are again equal, though negative. Such a function is symmetric with respect to the vertical axis. Examples of even function are:  $t^2$ ,

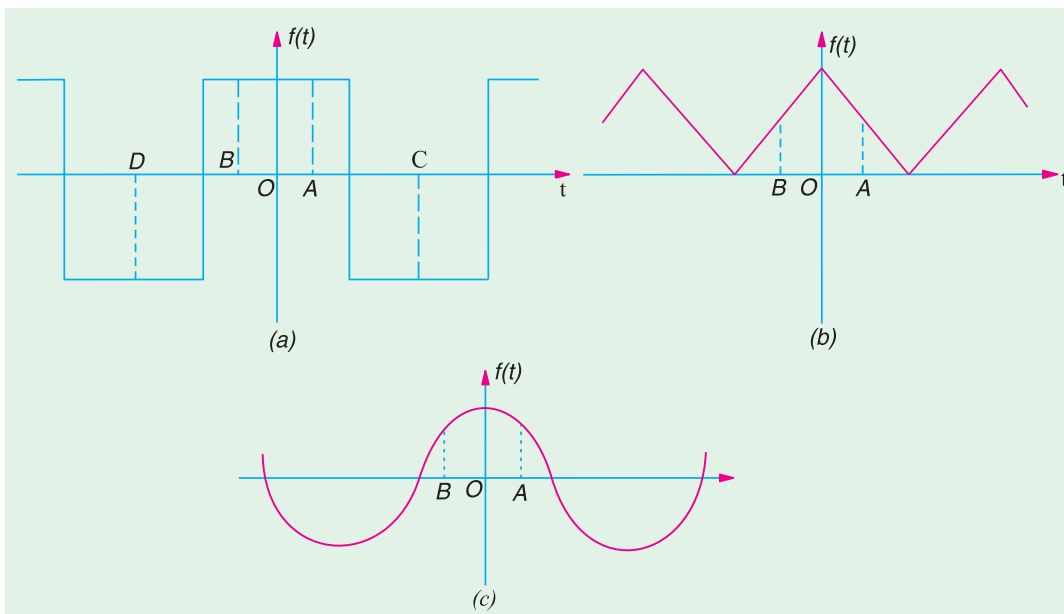


Fig. 21.2

$\cos 3t, \sin^2 5t (2 + t^2 + t^4)$  and a constant  $A$  because the replacement of  $t$  by  $(-t)$  does not change the value of any of these functions. For example,  $\cos \omega t = \cos(-\omega t)$ .

This type of symmetry can be easily recognised graphically because mirror symmetry exists about the vertical or  $f(t)$  axis. The function shown in Fig. 21.2 has even symmetry because when folded along vertical axis, the portions of the graph of the function for positive and negative time fit exactly, one on top of the other.

The effect of the even symmetry on Fourier series is that the constant  $b_n = 0$  i.e. the wave has no sine terms. In general,  $b_1, b_2, b_3 \dots b_n = 0$ . The Fourier series of an even function contains only a constant term and cosine terms i.e.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n}{T} t$$

The value of  $a_n$  may be found by integrating over any half-period.

$$\therefore a_n = \frac{2}{\pi} \int_0^{\pi} f(\theta) \cos n\theta d\theta = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega t dt$$

## 2. Odd Symmetry

A function  $f(t)$  is said to possess odd symmetry if  $f(-t) = -f(t)$ .

It means that as we travel an equal amount in time to the left or right from the origin, we find the function to be the same except for a reversal in sign. For example, in Fig. 21.3 the two points  $A$  and  $B$  are equidistant from point  $O$ . The two function values at  $A$  and  $B$  are equal in magnitude but opposite in sign. In other words, if we replace  $t$  by  $(-t)$ , we obtained the negative of the given function. The  $X$ -axis divides an odd function into two halves with equal areas above and below the  $X$ -axis. Hence,  $a_0 = 0$ .

Examples of odd functions are:  $t, \sin t, t \cos 50t, t(t+t^3+t^5)$  and  $t\sqrt{(1+t^2)}$

A sine function is an odd function because  $\sin(-\omega t) = -\sin \omega t$ .

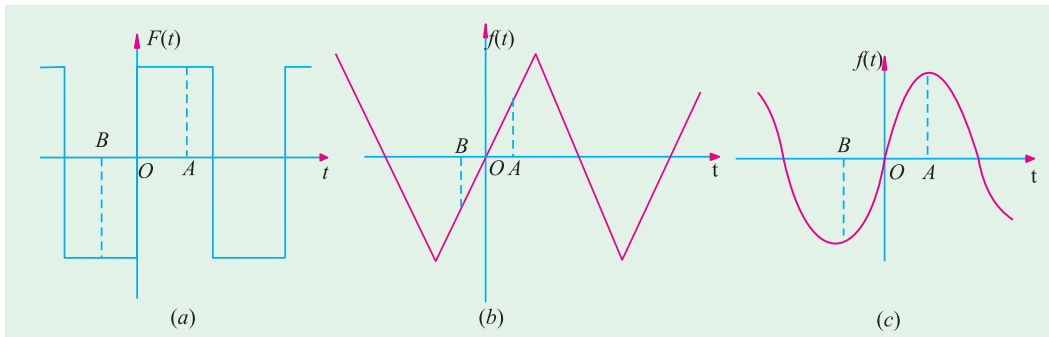


Fig. 21.3

An odd function has symmetry about the origin rather than about the  $f(t)$  axis which was the case for an even function. The effect of odd symmetry on a Fourier series is that it contains no constant term or cosine term. It means that  $a_0 = 0$  and  $a_n = 0$  i.e.  $a_1, a_2, a_3, \dots, a_n = 0$ . The Fourier series expansion contains only sine terms.

$$\therefore f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

The value of  $b_n$  may be found by integrating over any half-period.

$$\therefore b_n = \frac{2}{\pi} \int_0^{\pi} f(\theta) \sin n\theta d\theta = \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega t dt$$

### 3. Half-wave Symmetry or Mirror-Symmetry or Rotational Symmetry

A function  $f(t)$  is said to possess half-wave symmetry if  $f(t) = -f(t \pm T/2)$  or  $-f(t) = f(t \pm T/2)$ .

It means that the function remains the same if it is shifted to the left or right by half a period and then flipped over (*i.e.* multiplied by  $-1$ ) in respect to the  $t$ -axis or horizontal axis. It is called mirror symmetry because the negative portion of the wave is the mirror image of the positive portion of the wave displaced horizontally a distance  $T/2$ .

In other words, a waveform possesses half symmetry only when we invert its negative half-cycle and get an exact duplicate of its positive half-cycle. For example, in Fig. 21.4 (a) if we invert the negative half-cycle, we get the dashed  $ABC$  half-cycle which is exact duplicate of the positive half-cycle. Same is the case with the waveforms of Fig. 21.4 (b) and Fig. 21.4 (c). In case

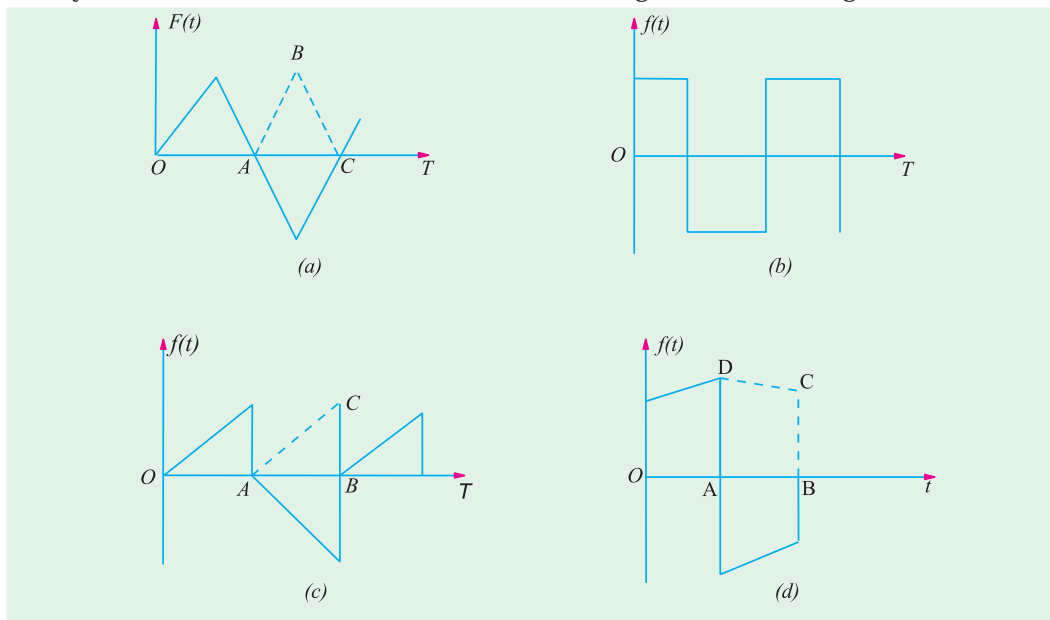


Fig. 21.4

of doubt, it is helpful to shift the inverted half-cycle by a half-period to the left and see if it superimposes the positive half-cycle. If it does so, there exists half-wave symmetry otherwise not. It is seen that the waveform of Fig. 21.4 (d) does not possess half-wave symmetry. It is so because when its negative half-cycle is inverted and shifted by half a period to the left it does not superimpose the positive half-cycle.

It may be noted that half-wave symmetry may be present in a waveform which also shows either odd symmetry or even symmetry:

For example, the square waveform shown in Fig. 21.4 (a) possesses even symmetry whereas the triangular waveform of Fig. 21.4 (b) has odd symmetry. All cosine and sine waves possess half-wave symmetry because

$$\cos \frac{2\pi}{T} \left( t \pm \frac{T}{2} \right) = \cos \left( \frac{2\pi}{T} t \pm \pi \right) = -\cos \frac{2\pi}{T} t; \sin \frac{2\pi}{T} \left( t \pm \frac{T}{2} \right) = \sin \left( \frac{2\pi}{T} t \pm \pi \right) = -\sin \frac{2\pi}{T} t$$

It is worth noting that the Fourier series of any function which possesses half-wave symmetry has zero average value and contains only odd harmonics and is given by

$$f(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \left( a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{T} t \right) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

$$\text{where, } a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos \frac{2\pi n}{T} t dt = \frac{2}{\pi} \int_0^{\pi} f(\theta) \cos n\theta d\theta \quad \dots n \text{ odd}$$

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin \frac{2\pi n}{T} t dt = \frac{2}{\pi} \int_0^{\pi} f(\theta) \sin n\theta d\theta \quad \dots n \text{ odd}$$

#### 4. Quarter-wave Symmetry

An odd or even function with rotational symmetry is said to possess quarter-wave symmetry.

Fig. 21.5 (a) possesses half-wave symmetry as well as odd symmetry. The wave shown in Fig. 21.5 (b) has both half-wave symmetry and even symmetry.

The mathematical test for quarter-wave symmetry is as under:

$$\text{Odd quarter-wave } f(t) = -f(t + T/2) \text{ and } f(-t) = -f(t)$$

$$\text{Even quarter-wave } f(t) = f(t + T/2) \text{ and } f(t) = f(-t)$$

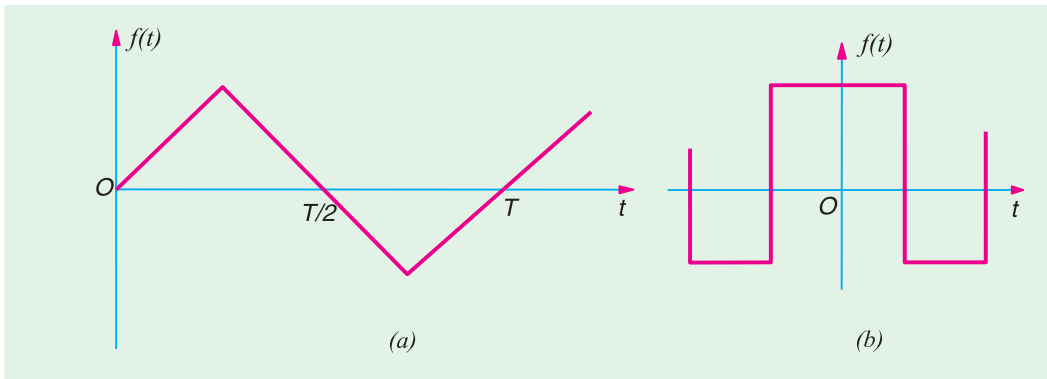


Fig. 21.5

Since each quarter cycle is the same in a way having quarter-wave symmetry, it is sufficient to integrate over one quarter period *i.e.* from 0 to  $T/4$  and then multiply the result by 4.

(i) If  $f(t)$  or  $f(\theta)$  is odd and has quarter-wave symmetry, then  $a_0$  is 0 and  $a_n$  is 0. Hence, the Fourier series will contain only odd sine terms.

$$\therefore f(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} b_n \sin \frac{2\pi n t}{T} \text{ or } f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} b_n \sin n\theta, \text{ where } b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta$$

It may be noted that in the case of odd quarter-wave symmetry, the integration may be carried over a quarter cycle.

$$\therefore a_n = \frac{4}{\pi} \int_0^{\pi/2} f(\theta) \cos n\theta d\theta \quad \dots n \text{ odd}$$

$$= \frac{8}{T} \int_0^{T/4} f(t) \sin n\omega t dt \quad \dots n \text{ odd}$$

(ii) If  $f(t)$  or  $f(\theta)$  is even and, additionally, has quarter-wave symmetry, then  $a_0$  is 0 and  $b_n$  is 0. Hence, the Fourier series will contain only odd cosine terms.



$$\therefore f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} a_n \cos n\theta d\theta = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} a_n \cos n\omega_0 t dt; \text{ where } a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta$$

In this case  $a_n$  may be found by integrating over any quarter period.

$$a_n = \frac{4}{\pi} \int_0^{\pi/2} f(\theta) \sin n\theta d\theta \quad \dots n \text{ odd}$$

$$= \frac{8}{T} \int_0^{T/4} f(t) \cos n\omega t dt \quad \dots n \text{ odd}$$

### 21.8. Line or Frequency Spectrum

A plot which shows the amplitude of each frequency component in a complex waveform is called the line spectrum or frequency spectrum (Fig. 21.6). The amplitude of each frequency component is indicated by the length of the vertical line located at the corresponding frequency. Since the spectrum represents frequencies of the harmonics as discrete lines of appropriate height, it is also called a discrete spectrum. The lines decrease rapidly for waves having convergent series. Waves with discontinuities such as the sawtooth and square waves have spectra whose amplitudes decrease slowly because their series have strong high harmonics. On the other hand, the line spectra of waveforms without discontinuities and with a smooth appearance have lines which decrease in height very rapidly.

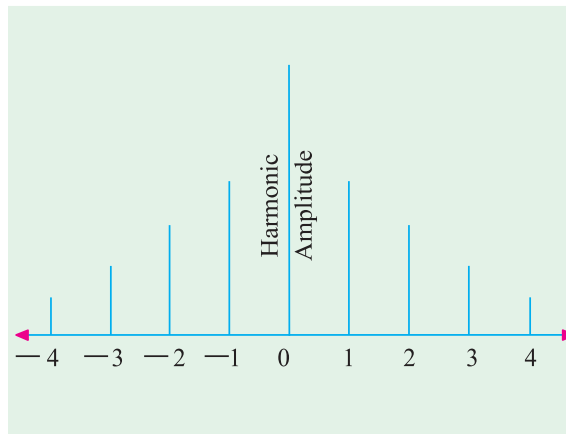


Fig. 21.6

The harmonic content and the line spectrum of a wave represent the basic nature of that wave and never change irrespective of the method of analysis. Shifting the zero axis changes the symmetry of a given wave and gives its trigonometric series a completely different appearance but the same harmonics always appear in the series and their amplitude remains constant.

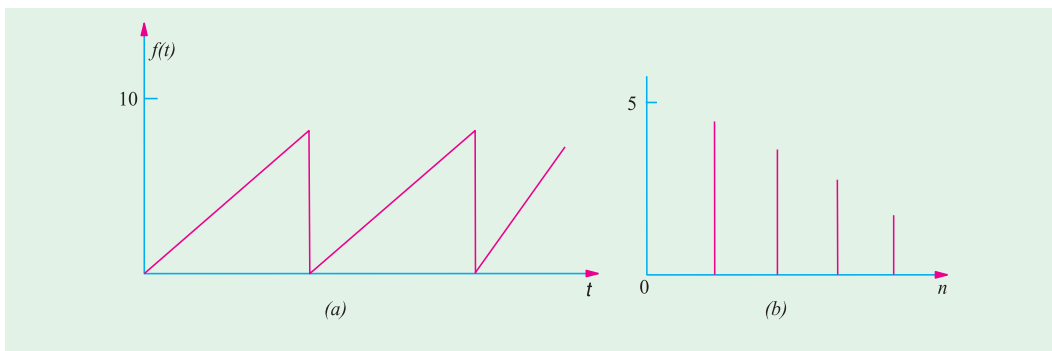


Fig. 21.7

Fig. 21.7 shows a smooth wave alongwith its line spectrum. Since there are only sine terms in its trigonometric series (apart from  $a_0 = \pi$ ), the harmonic amplitudes are given by  $b_n$ .

### 21.9. Procedure for Finding the Fourier Series of a Given Function

It is advisable to follow the following steps:

#### 1. Step No. 1

If the function is defined by a set of equations, sketch it approximately and examine for symmetry.

#### 2. Step No. 2

Whatever be the period of the given function, take it as  $2\pi$  (Ex. 20.6) and find the Fourier series in the form

$$f(\theta) = a_0 + a_1 \cos \theta + a_2 \cos 2\theta + \dots + a_n \cos n\theta + b_1 \sin \theta + b_2 \sin 2\theta + \dots + b_n \sin n\theta$$

#### 3. Step No. 3

The value of the constant  $a_0$  can be found in most cases by inspection. Otherwise it can be found as under:

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) d\theta$$

#### 4. Step No. 4

If there is no symmetry, then  $a_0$  is found as above whereas the other two Fourier constants can be found by the relation.

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) \cos n\theta d\theta$$

$$b_n = \int_0^{2\pi} f(\theta) \sin n\theta d\theta = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) \sin n\theta d\theta$$

#### 5. Step No. 5

If the function has even symmetry *i.e.*  $f(\theta) = f(-\theta)$ , then  $b_n = 0$  so that the Fourier series will have no sine terms. The series would be given by

$$f(\theta) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta \text{ where } a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta = \frac{2}{\pi} \int_0^{\pi} f(\theta) \cos n\theta d\theta$$

#### 6. Step No. 6

If the given function has odd symmetry *i.e.*  $f(-\theta) = -f(\theta)$  then  $a_0 = 0$  and  $a_n = 0$ . Hence, there would be no cosine terms in the Fourier series which accordingly would be given by

$$f(\theta) = \sum_{n=1}^{\infty} b_n \sin n\theta \text{ ; where } b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta = \frac{2}{\pi} \int_0^{\pi} f(\theta) \sin n\theta d\theta$$

#### 7. Step No. 7

If the function possesses half-wave symmetry *i.e.*  $f(\theta) = -f(\theta \pm \pi)$  or  $f(t) = -f(t \pm T/2)$ , then  $a_0$  is 0 and the Fourier series contains only odd harmonics. The Fourier series is given by

$$f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} a_n (\cos n\theta + b_n \sin n\theta)$$

where  $a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta \dots n \text{ odd}$ ,  $b_n = \frac{2}{\pi} \int_0^{\pi} f(\theta) \sin n\theta d\theta \dots n \text{ odd}$

**8. Step No. 8**

If the function has even quarter-wave symmetry then  $a_0 = 0$  and  $b_n = 0$ . It means the Fourier series will contain no sine terms but only odd cosine terms. It would be given by

$$f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}} a_n \cos n\theta ; \text{ where } a_n = \frac{2}{\pi} \int_0^{\pi/2} f(\theta) \cos n\theta d\theta = \frac{4}{\pi} \int_0^{\pi/4} f(\theta) \cos n\theta d\theta \dots n \text{ odd}$$

**9. Step No. 9**

If the function has odd quarter-wave symmetry, then  $a_0 = 0$  and  $a_n = 0$ . The Fourier series will contain only odd sine terms (but no cosine terms).

$$\therefore f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}} b_n \sin n\theta ; \text{ where } b_n = \frac{2}{\pi} \int_0^{\pi/2} f(\theta) \sin n\theta d\theta = \frac{4}{\pi} \int_0^{\pi/4} f(\theta) \sin n\theta d\theta \dots n \text{ odd}$$

**10. Step No. 10**

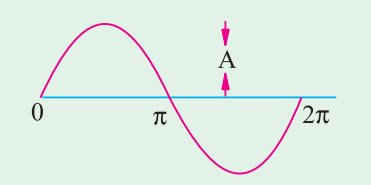
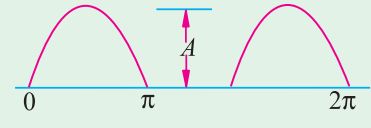
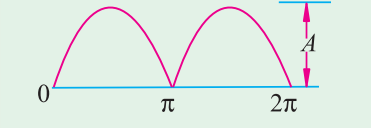
Having found the coefficients, the Fourier series as given in step No. 2 can be written down.

**11. Step No. 11**

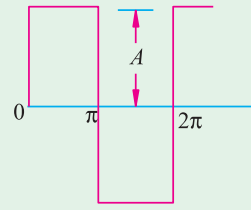
The different harmonic amplitudes can be found by combining similar sine and cosine terms

i.e.  $A_n = \sqrt{a_n^2 + b_n^2}$

where  $A_n$  is the amplitude of the  $n_{th}$  harmonic.

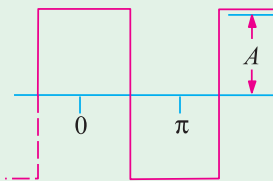
Table No. 21.1		
Wave form	Appearance	Equation
A. Sine wave		$f(t) = A = A \sin \omega t$
B. Half-wave rectified sine wave		$f(t) = A \left( \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t \right) - \frac{2}{15\pi} \cos 4\omega t - \frac{2}{35\pi} \cos 6\omega t - \frac{2}{63\pi} \cos 8\omega t \dots$
C. Full-wave rectified sine wave		$f(t) = \frac{2A}{\pi} \left( 1 - \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t - \frac{2}{35} \cos 6\omega t \dots \right)$

**D.** Rectangular or square wave



$$f(t) = \frac{4A}{\pi} \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \dots \right)$$

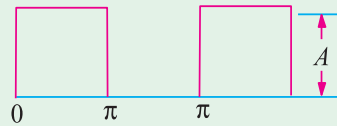
$$+ \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots$$



$$f(t) = \frac{4A}{\pi} \left( \cos \omega t - \frac{1}{3} \cos 3\omega t + \right.$$

$$\left. \frac{1}{5} \cos 5\omega t + \dots \right)$$

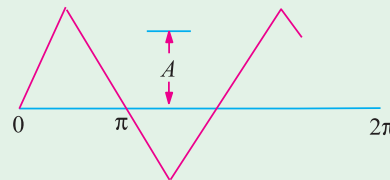
**E.** Rectangular or square wave pulse



$$f(t) = \frac{A}{2} + \frac{2A}{\pi} \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \right.$$

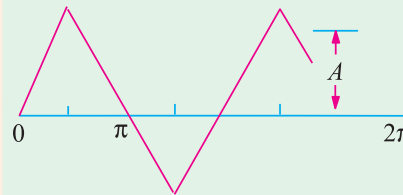
$$\left. \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots \right)$$

**F.** Triangular wave



$$f(t) = \frac{8A}{\pi^2} \left( \sin \omega t - \frac{1}{9} \sin 3\omega t + \frac{1}{25} \right.$$

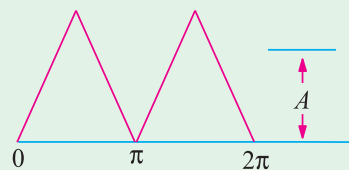
$$\left. \sin 5\omega t - \frac{1}{49} \sin 7\omega t + \dots \right)$$



$$f(t) = \frac{8A}{\pi^2} \left( \cos \omega t + \frac{1}{9} \cos 3\omega t + \right.$$

$$\left. \frac{1}{25} \cos 5\omega t + \frac{1}{49} \cos 7\omega t + \dots \right)$$

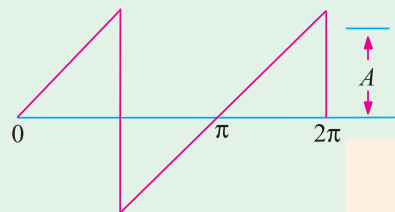
**G.** Triangular pulse



$$f(t) = \frac{A}{2} + \frac{4A}{\pi^2} \left( \sin \omega t - \frac{1}{9} \sin 3\omega t \right.$$

$$\left. + \frac{1}{25} \sin 5\omega t - \frac{1}{49} \sin 7\omega t + \dots \right)$$

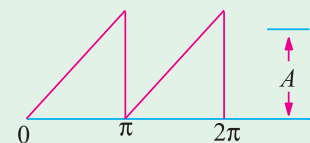
**H.** Sawtooth wave



$$f(t) = \frac{2A}{\pi} \left( \sin \omega t - \frac{1}{2} \sin 2\omega t + \right.$$

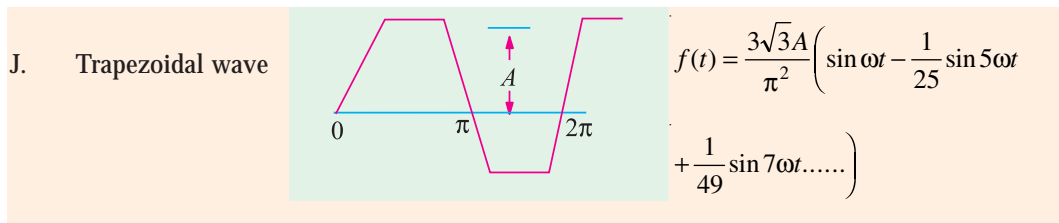
$$\left. \frac{1}{3} \sin 3\omega t - \frac{1}{4} \sin 4\omega t + \dots \right)$$

**I.** Sawtooth pulse



$$f(t) = \frac{A}{\pi} \left( \frac{\pi}{2} - \sin \omega t - \frac{1}{2} \sin 2\omega t - \frac{1}{3} \sin 3\omega t \right.$$

$$\left. - \frac{1}{4} \sin 4\omega t \right)$$



### 21.10. Wave Analyzer

A wave analyzer is an instrument designed to measure the individual amplitude of each harmonic

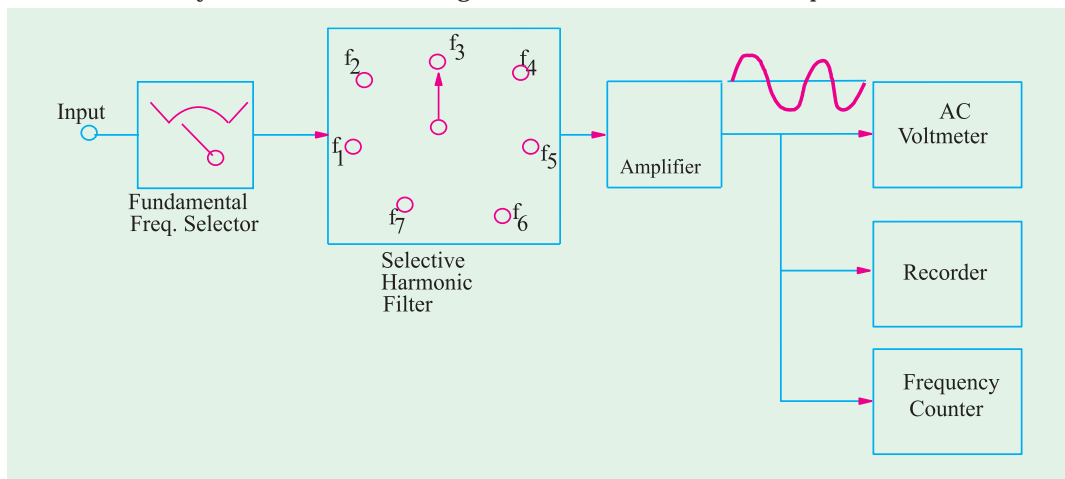


Fig. 21.8

component in a complex waveform. It is the simplest form of analysis in the frequency domain and can be performed with a set of tuned filters and a voltmeter. That is why such analyzers are also called frequency-selective voltmeters. Since such analyzers sample the frequency spectrum successively, *i.e.* one after the other, they are called non-real-time analyzers.

The block diagram of a simple wave analyzer is shown in Fig. 21.8. It consists of a tunable fundamental frequency selector that detects the fundamental frequency  $f_1$  which is the lowest frequency contained in the input waveform.

Once tuned to this fundamental frequency, a selective harmonic filter enables switching to multiples of  $f_1$ . After amplification, the output is fed to an a.c. voltmeter, a recorder and a frequency counter. The voltmeter reads the r.m.s amplitude of the harmonic wave, the recorder traces its waveform and the frequency counter gives its frequency. The line spectrum of the harmonic component can be plotted from the above data.

For higher frequencies (MHz) heterodyne wave analyzers are generally used. Here, the input complex wave signal is heterodyned to a higher intermediate frequency (IF) by an internal local oscillator. The output of the IF amplifier is rectified and is applied to a dc voltmeter called heterodyned tuned voltmeter.



Wave analyzer

The block diagram of a wave analyzer using the heterodyning principle is shown in Fig. 21.9.

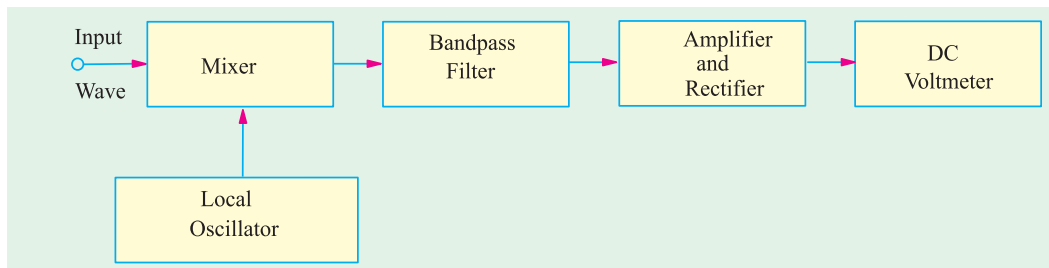


Fig. 21.9

The signal from the internal, variable-frequency oscillator heterodynes with the input signal in a mixer to produce output signal having frequencies equal to the sum and difference of the oscillator frequency  $f_0$  and the input frequency  $f_1$ . Generally, the bandpass filter is tuned to the 'sum frequency' which is allowed to pass through. The signal coming out of the filter is amplified, rectified and then applied to a dc voltmeter having a decibel-calibrated scale. In this way, the peak amplitudes of the fundamental component and other harmonic components can be calculated.

### 21.11. Spectrum Analyzer

It is a real-time instrument *i.e.* it simultaneously displays on a CRT, the harmonic peak

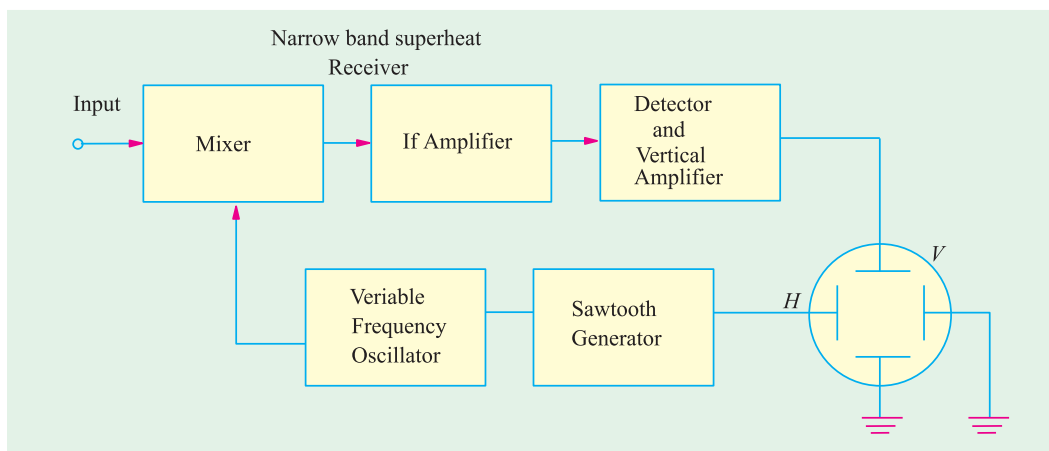
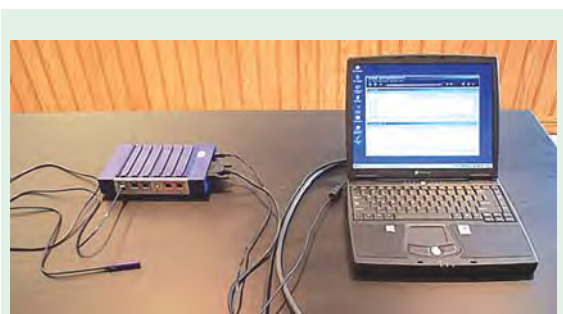


Fig. 21.10

values versus frequency of all wave components in the frequency range of the analyzer. The block diagram of such an analyzer is shown in Fig. 21.10.

As seen, the spectrum analyzer uses a CRT in combination with a narrow-band superheterodyne receiver. The receiver is tuned by varying the frequency of the voltage-tuned variable-frequency oscillator which also controls the sawtooth generator that sweeps the horizontal time base of the CRT deflection plates. As the oscillator is



Spectrum analyzer

swept through its frequency band by the sawtooth generator, the resultant signal mixes and beats with the input signal to produce an intermediate frequency (IF) signal in the mixer. The mixer output occurs only when there is a corresponding harmonic component in the input signal which matches with the sawtooth generator signal. The signals from the IF amplifier are detected and further amplified before applying them to the vertical deflection plates of the CRT. The resultant output displayed on the CRT represents the line spectrum of the input complex or nonsinusoidal waveform.

### 21.12. Fourier Analyzer

A Fourier analyzer uses digital signal processing technique and provides information regarding the contents of a complex wave which goes beyond the capabilities of a spectrum analyzer. These analyzers are based on the calculation of the discrete Fourier transform using an algorithm called the fast Fourier transformer. This algorithm calculates the amplitude and phase of each harmonic component from a set of time domain samples of the input complex wave signal.

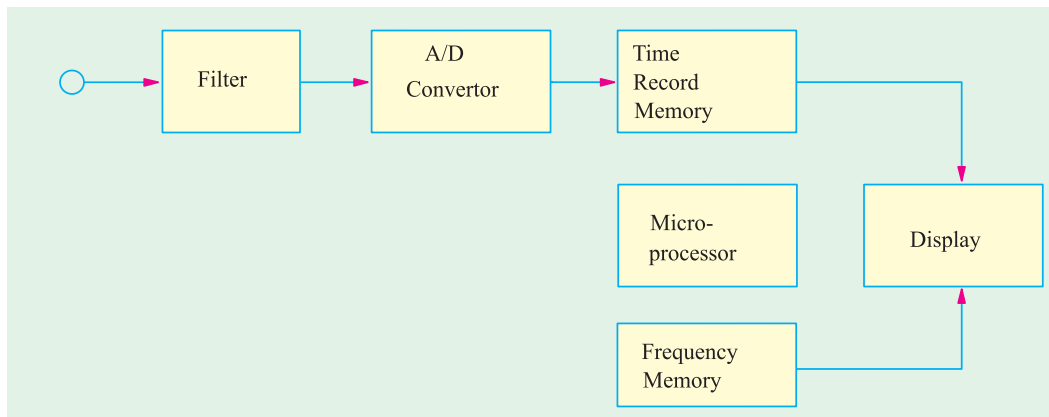


Fig. 21.11

A basic block diagram of a Fourier analyzer is shown in Fig. 21.11. The complex wave signal applied to the instrument is first filtered to remove out-of-band frequency components. Next, the signal is applied to an analog/digital (A/D) convertor which samples and digitizes it at regular time intervals until a full set of samples (called a time record) has been collected. The microprocessor then performs a series of computations on the time data to obtain the frequency-domain results *i.e.* amplitude versus frequency relationships. These results which are stored in memory can be displayed on a CRT or recorded permanently with a recorder or plotter.

Since Fourier analyzers are digital instruments, they can be easily interfaced with a computer or other digital systems. Moreover, as compared to spectrum analyzers, they provide a higher degree of accuracy, stability and repeatability.



Fourier Analyzer

### 21.13. Harmonic Synthesis

It is the process of building up the shape of a complex waveform by adding the instantaneous values of the fundamental and harmonics. It is a graphical procedure based on the knowledge of the different components of a complex waveform.

**Example 21.1.** A complex voltage waveform contains a fundamental voltage of r.m.s. value 220 V and frequency 50 Hz along with a 20% third harmonic which has a phase angle lagging by  $3\pi/4$  radian at  $t = 0$ . Find an expression representing the instantaneous complex voltage  $v$ . Using harmonic synthesis, also sketch the complex waveform over one cycle of the fundamental.

**Solution.** The maximum value of the fundamental voltage is  $= 200 \times \sqrt{2} = 310$  V. Its angular velocity is  $\omega = 2\pi \times 50 = 100\pi$  rad/s. Hence, the fundamental voltage is represented by  $310 \sin 100\pi t$ .

The amplitude of the third harmonic  $= 20\%$  of  $310 = 62$  V. Its frequency is  $3 \times 50 = 150$  Hz. Hence, its angular frequency is  $= 2\pi \times 150 = 300\pi$  rad/s. Accordingly, the third harmonic voltage can be represented by the equation  $62 \sin (300\pi t - 3\pi/4)$ . The equation of the complex voltage is given by  $v = 310 \sin 100\pi t + 62 \sin(300\pi t - 3\pi/4)$ .

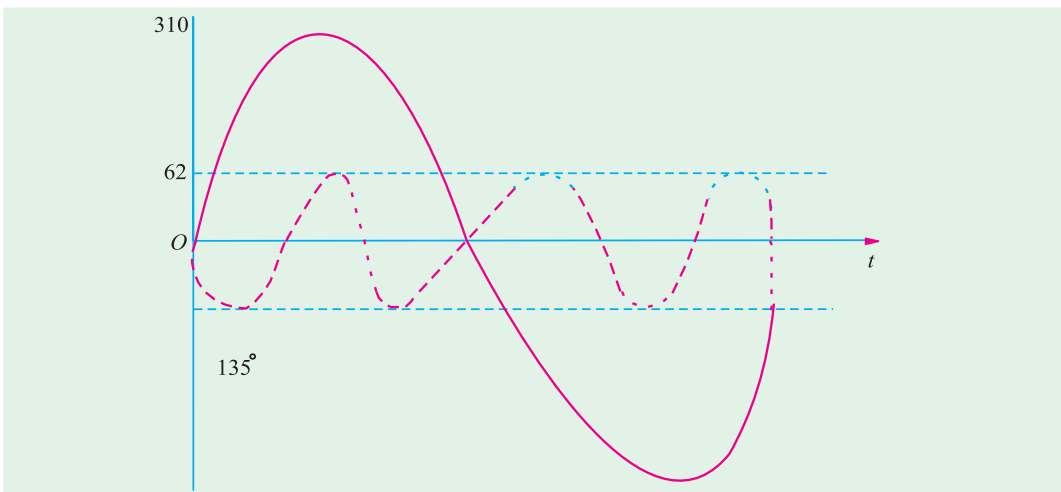


Fig 21.12

In Fig. 21.12 are shown one cycle of the fundamental and three cycles of the third harmonic component initially lagging by  $3\pi/4$  radian or  $135^\circ$ . By adding ordinates at different intervals, the complex voltage waveform is built up as shown.

Incidentally, it would be seen that if the negative half-cycle is reversed, it is identical to the positive half-cycle. This is a feature of waveforms possessing half-wave symmetry which contains the fundamental and odd harmonics.

**Example 21.2.** For the nonsinusoidal wave shown in Fig. 21.13, determine (a) Fourier coefficients  $a_0$ ,  $a_3$  and  $b_4$  and (b) frequency of the fourth harmonic if the wave has a period of 0.02 second.

**Solution.** The function  $f(\theta)$  has a constant value from  $\theta = 0$  to  $\theta = 4\pi/3$  radian and 0 value from  $\theta = 4\pi/3$  radian to  $\theta = 2\pi$  radian.

$$(a) a_0 = \frac{1}{2\pi} (\text{net area per cycle})_0^{2\pi} = \frac{1}{2\pi} \left( 6 \times \frac{4\pi}{3} \right) = 4$$

$$a_3 = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos 3\theta d\theta = \frac{1}{\pi} \int_0^{4\pi/3} 6 \cos 3\theta d\theta$$

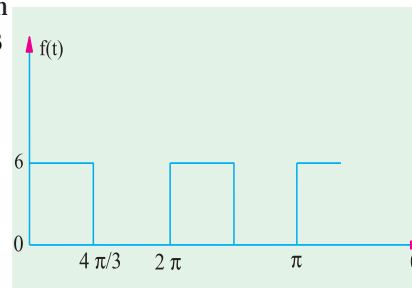


Fig. 21.13



$$= \frac{6}{\pi} \left| \frac{\sin 3\theta}{3} \right|_0^{4\pi/3} = \frac{2}{\pi} (\sin 4\pi) = 0$$

$$b_4 = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin 4\theta d\theta = \frac{1}{\pi} \int_0^{4\pi/3} 6 \sin 4\theta d\theta$$

$$= \frac{6}{\pi} \left| -\frac{\cos 4\theta}{4} \right|_0^{4\pi/3} = -\frac{3}{2\pi} (\cos 16\pi - \cos 0) = \frac{-3}{2\pi} (-0.5 - 1) = \frac{9}{4\pi}$$

(b) Frequency of the fourth harmonic =  $4f_0 = 4/T = 4/0.02 = 200$  Hz.

**Example 21.3.** Find the Fourier series of the "half sinusoidal" voltage waveform which represents the output of a half-wave rectifier. Sketch its line spectrum.

**Solution.** As seen from Fig. 21.14 (a),  $T = 0.2$  second,  $f_0 = 1/T = 1/0.2 = 5$  Hz and  $\omega_0 = 2\pi f_0 = 10\pi$  rad/s. Moreover, the function has even symmetry. Hence, the Fourier series will contain no sine terms because  $b_n = 0$ .

The limits of integration would not be taken from  $t = 0$  to  $t = 0.2$  second, but from  $t = -0.5$  to  $t = 0.15$  second in order to get fewer equations and hence fewer integrals. The function can be written as

$$f(t) = V_m \cos 10\pi t \quad -0.05 < t < 0.05$$

$$= 0 \quad 0.05 < t < 0.15$$

$$a_0 = \frac{1}{T} \int_{-0.05}^{0.15} f(t) dt = \frac{1}{0.2} \left[ \int_{-0.05}^{0.05} V_m \cos 10\pi t dt + \int_{0.05}^{0.15} 0 dt \right]$$

$$= \frac{V_m}{0.2} \left| \frac{\sin 10\pi t}{10\pi} \right|_{-0.05}^{0.05} = \frac{V_m}{\pi} a_n = \frac{2V_m}{0.2} \int_{-0.05}^{0.05} \cos 10\pi t \cdot \cos 10\pi n t dt$$

The expression we obtain after integration cannot be solved when  $n = 1$  although it can be solved when  $n$  is other than unity. For  $n = 1$ , we have

$$a_1 = 10V_m \int_{-0.05}^{0.05} \cos^2 10\pi t dt = \frac{V_m}{2}$$

$$\text{When } n \neq 1, \text{ then } a_n = 10V_m \int_{-0.05}^{0.05} \cos 10\pi t \cdot \cos 10\pi n t dt$$

$$= \frac{10V_m}{2} \int_{-0.05}^{0.05} [\cos 10\pi(1+n)t \cos 10\pi(1-n)t] dt = \frac{2V_m}{\pi} \cdot \frac{\cos(\pi n/2)}{(1-n^2)} \dots n \neq 1$$

$$a_2 = \frac{2V_m}{\pi} \cdot \frac{\cos \pi}{1-4} = \frac{2V_m}{\pi} \cdot \frac{-1}{-3} = \frac{2V_m}{2\pi}; a_3 = \frac{2V_m}{\pi} \cdot \frac{\cos 3\pi/2}{1-3^2} = 0; a_4 = \frac{2V_m}{\pi} \cdot \frac{\cos 2\pi}{1-4^2} = -\frac{2V_m}{15\pi}$$

$$a_5 = 0; a_6 = \frac{2V_m}{\pi} \cdot \frac{\cos 3\pi}{1-6^2} = \frac{2V_m}{35\pi} \text{ and so on}$$

Substituting the values of  $a_0, a_1, a_2, a_4$  etc. in the standard Fourier series expression given in Art. 20.3. we have

$$f(t) = a_0 + a_1 \cos 2\pi t + a_2 \cos 4\pi t + a_4 \cos 8\pi t + a_6 \cos 12\pi t + \dots$$

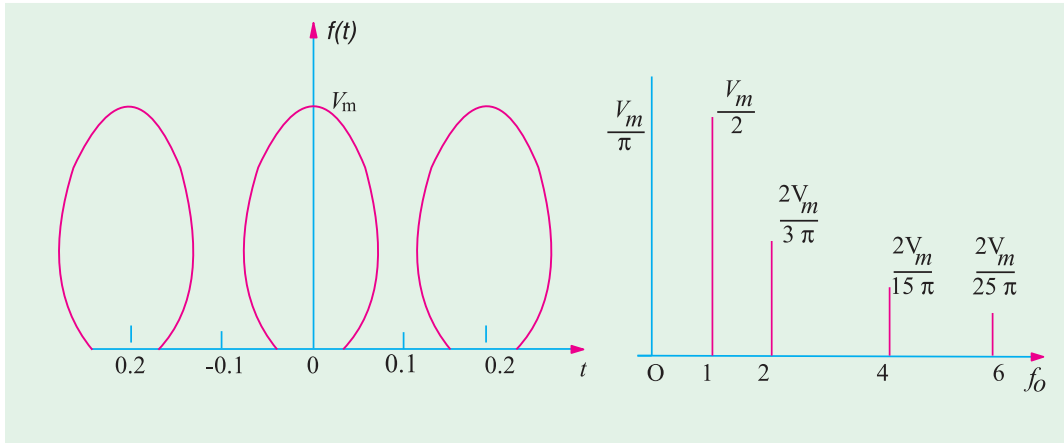


Fig. 21.14

$$\begin{aligned}
 &= \frac{V_m}{\pi} + \frac{V_m}{2} \cos 10\pi t + \frac{2V_m}{3\pi} \cos 20\pi t - \frac{2V_m}{15\pi} \cos 40\pi t + \frac{2V_m}{35} \cos 60\pi t \dots \\
 &= V_m \left( \frac{1}{\pi} + \frac{1}{2} \cos \omega_0 t + \frac{2}{3\pi} \cos 2\omega_0 t - \frac{2}{15\pi} \cos 4\omega_0 t + \frac{2}{35\pi} \cos 6\omega_0 t \dots \right)
 \end{aligned}$$

The line spectrum which is a plot of the harmonic amplitudes versus frequency is given in Fig. 21.14(b).

**Example 21.4.** Determine the Fourier series for the square voltage pulse shown in Fig. 21.15 (a) and plot its line spectrum. (Network Theory, Nagpur Univ. 1992)

**Solution.** The wave represents a periodic function of  $\theta$  or  $\omega t$  or  $\left(\frac{2\pi t}{T}\right)$  having a period extending over  $2\pi$  radians or  $T$  seconds. The general expression for this wave can be written as

$$\begin{aligned}
 f(\theta) &= a_0 + a_1 \cos \theta + a_2 \cos 2\theta + a_3 \cos 3\theta + \dots \\
 &\dots + b_1 \sin \theta + b_2 \sin 2\theta + b_3 \sin 3\theta + \dots
 \end{aligned}$$

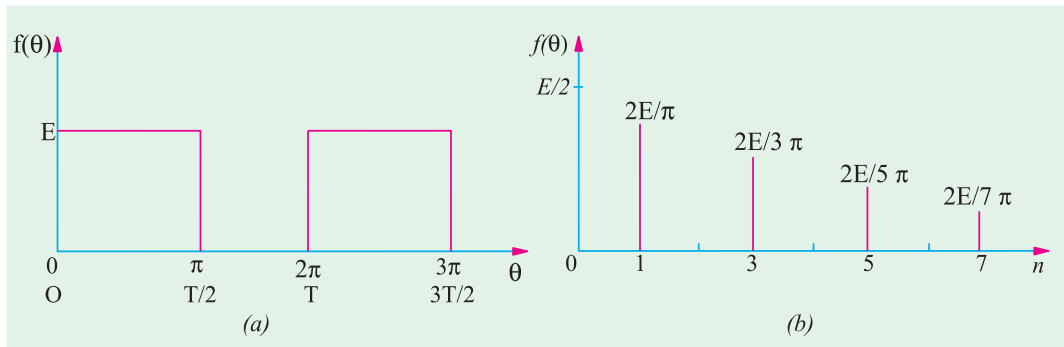


Fig. 21.15

(i) Now,  $f(\theta) = V$ ;  $\theta = 0$  to  $\theta = \pi$ ;  $f(\theta) = 0$ , from  $\theta = \pi$  to  $\theta = 2\pi$

$$\therefore a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta = \frac{1}{2\pi} \left\{ \int_0^{\pi} f(\theta) d\theta + \int_{\pi}^{2\pi} f(\theta) d\theta \right\}$$

$$\text{or } a_0 = \frac{1}{2\pi} \left\{ \int_0^\pi V d\theta + \int_\pi^{2\pi} 0 d\theta \right\} = \frac{V}{2\pi} [\theta]_0^\pi + 0 = V_{2\pi} \times \pi = \frac{V}{2}$$

$$\begin{aligned} \text{(ii) } a_n &= \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = \frac{1}{\pi} \left\{ \int_0^\pi V \cos n\theta d\theta + \int_\pi^{2\pi} 0 \times \cos n\theta d\theta \right\} \\ &= \frac{V}{\pi} \int_0^\pi \cos n\theta d\theta + 0 = \frac{V}{n\pi} [\sin n\theta]_0^\pi = 0 \quad \dots \text{ whether } n \text{ is odd or even} \end{aligned}$$

$$\begin{aligned} \text{(iii) } b_n &= \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta = \frac{1}{\pi} \left\{ \int_0^\pi V \sin n\theta d\theta + \int_\pi^{2\pi} 0 \times \sin n\theta d\theta \right\} \\ &= \frac{V}{\pi} \int_0^\pi \sin n\theta d\theta + 0 = \frac{V}{\pi} \left[ \frac{-\cos n\theta}{n} \right]_0^\pi = \frac{V}{n\pi} (-\cos n\pi + 1) \end{aligned}$$

Now, when  $n$  is odd,  $(1 - \cos n\pi) = 2$  but when  $n$  is even,  $(1 - \cos n\pi) = 0$ .

$$\therefore b_1 = \frac{2V}{\pi} \dots n = 1; b_2 = \frac{V}{2\pi} \times 0 = 0 \dots n = 2; b_3 = \frac{V}{3\pi} \times 2 = \frac{2V}{3\pi} \dots n = 3 \text{ and so on.}$$

Hence, substituting the values of  $a_0, a_1, a_2$  etc. and  $b_1, b_2$  etc. in the above given Fourier series, we get

$$f(\theta) = \frac{V}{2} + \frac{2V}{\pi} \sin \theta + \frac{2V}{3\pi} \sin 3\theta + \frac{2V}{5\pi} \sin 5\theta + \dots = \frac{E}{2} + \frac{2V}{\pi} \left( \sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t + \dots \right)$$

It is seen that the Fourier series contains a constant term  $V/2$  and odd harmonic components whose amplitudes are as under:

$$\text{Amplitude of fundamental or first harmonic} = \frac{2V}{\pi}$$

$$\text{Amplitude of second harmonic} = \frac{2V}{3\pi}$$

$$\text{Amplitude of third harmonic} = \frac{2V}{5\pi} \text{ and so on.}$$

The plot of harmonic amplitude versus the harmonic frequencies (called line spectrum) is shown in Fig. 21.15 (b).

**Example 21.5.** Obtain the Fourier series for the square wave pulse train indicated in Fig. 21.16. **(Network Theory and Design, AMIETE June, 1990)**

**Solution.** Here  $T = 2$  second,  $\omega_0 = 2\pi/T = \pi$  rad/s. The given function is defined by

$$\begin{aligned} f(t) &= 1 \quad 0 < t < 1 = 0 \text{ and} \\ &= 0; \quad 1 < t < 2 \end{aligned}$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{2} \int_0^2 1 dt = \frac{1}{2} \left[ \int_0^1 1 dt + \int_1^2 0 dt \right] = \frac{1}{2}$$

Even otherwise by inspection  $a_0 = (1 + 0)/2 = 1/2$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t dt = \frac{2}{2} \int_0^2 1 \cos n\pi t dt = \left[ \int_0^1 1 \cos n\pi t dt + \int_1^2 (0) \cos n\pi t dt \right]$$

$$= \int_0^1 \cos n\pi t dt = \left| \frac{\sin n\pi t}{n\pi} \right|_0^1 = 0$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega_0 t dt = \left[ \int_0^1 1 \cdot \sin n\pi t dt \right] + \int_1^2 (0) \sin n\pi t dt = \int_0^1 \sin n\pi t dt$$

$$= \left| \frac{-\cos n\pi t}{n\pi} \right|_0^1 = \frac{1 - \cos n\pi}{n\pi}$$

$$\therefore b_n = 2/n\pi \dots \text{when } n \text{ is odd; } = 0$$

... when  $n$  is even

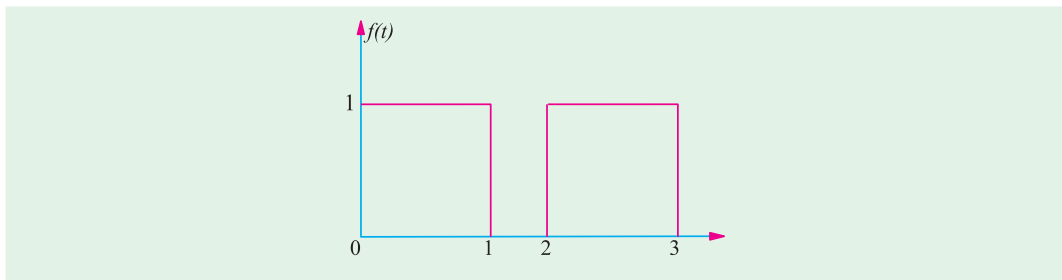


Fig. 21.16

$$\therefore f(t) = a_0 + \sum_{\substack{n=1 \\ \text{odd}}} \frac{2}{n} \sin 3n t + \frac{1}{2} - \frac{2}{2} \sin t + \frac{1}{3} \sin 3 t - \frac{1}{5} \sin 5 t, \text{ etc.}$$

**Example 21.6.** Find the trigonometric Fourier series for the square voltage waveform shown in Fig. 21.17(a) and sketch the line spectrum.

**Solution.** The function shown in Fig. 21.17 (a) is an odd function because at any time  $f(-t) = -f(t)$ . Hence, its Fourier series will contain only sine terms *i.e.*  $a_n = 0$ . The function also possesses half-wave symmetry, hence, it will contain only odd harmonics.

As seen from Art. 21.7 (2) the Fourier series for the above wave is given by

$$f(\theta) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} b_n \sin n\theta \text{ where } b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi} V \sin n\theta d\theta + \int_{\pi}^{2\pi} -V \sin n\theta d\theta \right\} = \frac{V}{\pi n} \left| -\cos n\theta \right|_0^{\pi} + \frac{V}{\pi n} \left| \cos n\theta \right|_{\pi}^{2\pi}$$

$$= \frac{V}{\pi n} \{ (-\cos n\pi + \cos 0) - \cos n\pi \}$$

$$= \frac{V}{\pi n} \{ (1 - \cos n\pi) + (1 - \cos n\pi) \} = \frac{2V}{\pi n} (1 - \cos n\pi)$$

Now,  $1 - \cos n\pi = 2$  when  $n$  is odd

and  $b_n = 0$  when  $n$  is even

$$\therefore b_1 = \frac{2V}{\pi} \times 2 = \frac{4V}{\pi} \dots \text{putting } n = 1; b_2 = 0 \dots \text{putting } n = 2$$

$$b_3 = \frac{2V}{\pi \cdot 3} \times 2 = \frac{4V}{3\pi} \dots \text{putting } n = 3; b_4 = 0 \dots \text{putting } n = 4$$

$$b_5 = \frac{2V}{\pi \cdot 5} \times 2 = \frac{4V}{5\pi} \dots \text{putting } n = 5 \text{ and so on.}$$

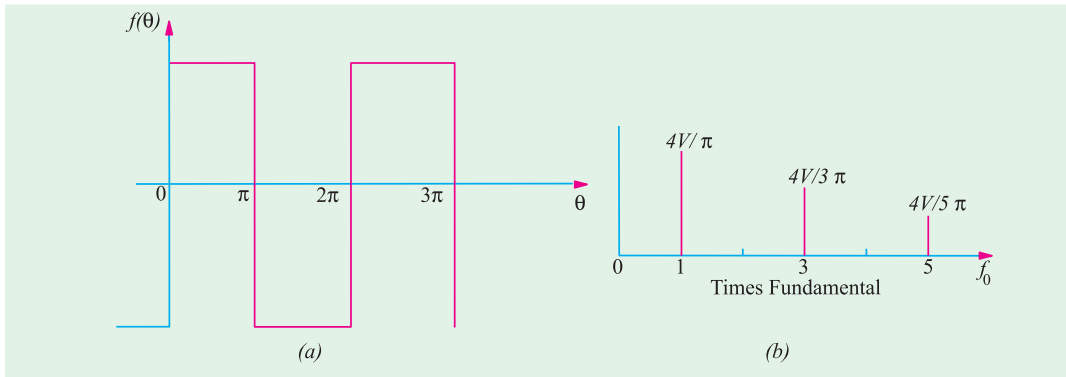


Fig. 21.17

Hence, the Fourier series for the given waveform is

$$f(\theta) = \frac{4V}{\pi} \sin \theta - \frac{4V}{3\pi} \sin 3\theta + \frac{4V}{5\pi} \sin 5\theta - \dots$$

$$= \frac{4V}{\pi} \left( \sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t + \dots \right) = \frac{4V}{\pi} \left( \sin \frac{2\pi}{T} t + \frac{1}{3} \sin \frac{6\pi}{T} t + \frac{1}{5} \sin \frac{10\pi}{T} t + \dots \right)$$

The line spectrum of the function is shown in Fig. 21.17 (b). It would be seen that the harmonic amplitudes decrease as  $1/n$ , that is, the third harmonic amplitude is  $1/3$  as large as the fundamental, the fifth harmonic is  $1/5$  as large and so on.

**Example 21.7.** Determine the Fourier series for the square voltage waveform shown in Fig. 21.17 (a). Plot its line spectrum.

**Solution.** This is the same question as given in Ex. 21.6 but has been repeated to illustrate a slightly different technique. As seen from Fig. 21.17(a)  $T = 2\pi$ , hence,  $\omega_0 = 2\pi f_0 = 2\pi / T = 2\pi / 2\pi = 1$ . Over one period the function can be defined as

$$f(t) = V \quad 0 < t < \pi$$

$$= -V, \quad \pi < t < 2\pi$$

$$b_n = \frac{1}{\pi} \int_0^\pi V \sin n t dt - \frac{1}{\pi} \int_\pi^{2\pi} V \sin n t dt = \frac{1}{\pi} \int_0^\pi V \sin n t dt - \frac{1}{\pi} \int_0^\pi V \sin n t dt$$

$$= \frac{1}{\pi} \int_0^\pi V \sin n t dt + \frac{1}{\pi} \int_\pi^{2\pi} (-V) \sin n t dt = \frac{V}{\pi} \left[ -\frac{\cos n t}{n} \right]_0^\pi + \frac{V}{\pi} \left[ \frac{\cos n t}{n} \right]_\pi^{2\pi}$$

$$= -\frac{V}{n\pi}(\cos n\pi - \cos 0) + \frac{V}{n\pi}(\cos 2n\pi - \cos n\pi)$$

Since  $\cos 0$  is 1 and  $\cos 2n\pi = 1 \therefore b_n = \frac{2V}{n\pi}(1 - \cos n\pi)$

When  $n$  is even,  $\cos n\pi = 1 \therefore b_n = 0$

When  $n$  is odd,  $\cos n\pi = -1 \therefore b_n = \frac{2V}{n}(1 - (-1)) = \frac{4V}{n}, n = 1, 3, 5, \dots$

Substituting the value of  $b_n$ , the Fourier series become

$$f(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{4V}{n\pi} \sin nt = \frac{4V}{\pi} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{1}{n} \sin nt = \frac{4V}{\pi} \left( \sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots \right)$$

Since  $\omega_0 = 1$ , the above expression in general terms becomes

$$f(t) = \frac{4V}{\pi} \left( \sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t + \dots \right)$$

The line spectrum is as shown in Fig. 21.17 (b).

**Example 21.8.** Determine the Fourier series of the square voltage waveform shown in Fig. 21.18.

**Solution.** As compared to Fig. 21.17 (a) given above, the vertical axis of figure has been shifted by  $\pi/2$  radians. Replacing  $t$  by  $(t + \pi/2)$  in the above equation, the Fourier series of the waveform shown in Fig. 21.18 becomes

$$\begin{aligned} f(t) &= \frac{4V}{\pi} \left[ \sin \left( t + \frac{\pi}{2} \right) + \frac{1}{3} \sin 3 \left( t + \frac{\pi}{2} \right) + \frac{1}{5} \sin 5 \left( t + \frac{\pi}{2} \right) + \dots \right] \\ &= \frac{4V}{\pi} \left( \cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t \dots \right) \end{aligned}$$

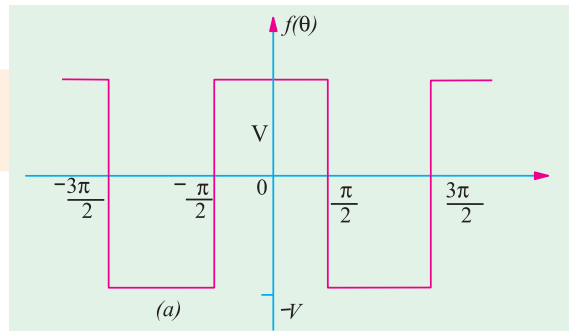


Fig. 21.18

**Example 21.9.** Determine the trigonometric Fourier series for the half-wave rectified sine wave form shown in Fig. 21.19 (a) and sketches line spectrum.

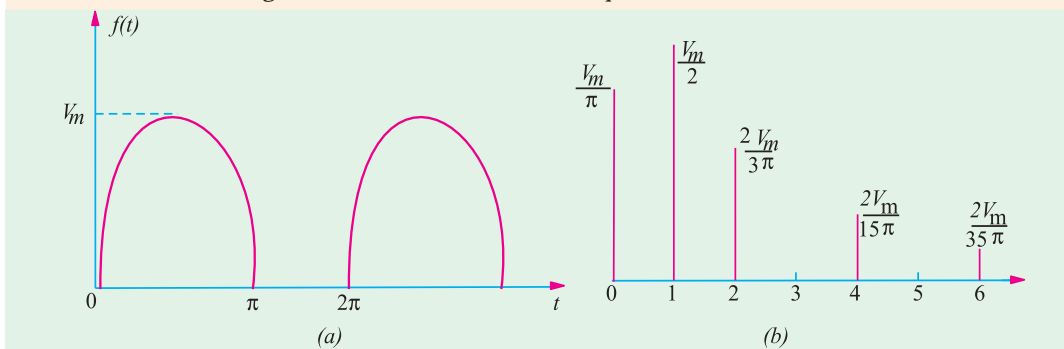


Fig. 21.19

**Solution.** The given waveform shows no symmetry, hence its series would contain both sine and cosine terms. Moreover, its average value is not obtainable by inspection, hence it will have to be found by integration.

Here,  $T = 2\pi$ . Hence, equation of the given waveform is  $V = V_m \sin \omega t = V_m \sin t$ .

The given waveform is defined by  $f(t) = V_m \sin t, 0 < t < \pi = 0, \pi < t < 2\pi$

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_0^{2\pi} f(t) dt = \frac{1}{2\pi} \left[ \int_0^{\pi} V_m \sin t dt + \int_{\pi}^{2\pi} 0 dt \right] \\ &= \frac{V_m}{2\pi} [-\cos t]_0^{\pi} = -\frac{V_m}{2\pi} (\cos \pi - \cos 0) = \frac{V_m}{\pi} \\ a_n &= \frac{1}{\pi} \int_0^{\pi} f(t) \cos nt dt = \frac{1}{\pi} \int_0^{\pi} V_m \sin t \cos nt dt \\ &= \frac{V_m}{2\pi} \int_0^{\pi} [\sin(n+1)t - \sin(n-1)t] dt = \frac{V_m}{2\pi} \left[ \frac{-\cos(n+1)t}{n+1} + \frac{\cos(n-1)t}{n-1} \right]_0^{\pi} \\ &= \frac{V_m}{2\pi} \left[ -\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} + \frac{1}{n+1} - \frac{1}{n-1} \right] \end{aligned}$$

when  $n$  is even,  $\cos(n+1)\pi = -1$  and  $\cos(n-1)\pi = -1$

$$\therefore a_n = \frac{V_m}{2\pi} \left[ \frac{1}{n+1} - \frac{1}{n-1} + \frac{1}{n+1} - \frac{1}{n-1} \right] = -\frac{2V_m}{\pi(n^2-1)} \quad \dots n = 2, 4, 6 \text{ etc.}$$

when  $n$  is odd and  $\neq 1$ ,  $\cos(n+1)\pi = 1$  and  $\cos(n-1)\pi = 1$

$$\therefore a_n = \frac{V_m}{2\pi} \left( -\frac{1}{n+1} + \frac{1}{n-1} + \frac{1}{n+1} - \frac{1}{n-1} \right) = 0 \quad \dots n = 3, 5, 7 \text{ etc.}$$

$$\text{when } n = 1, a_1 = \frac{1}{\pi} \int_0^{\pi} V_m \sin t \cos t dt = \frac{V_m}{2\pi} \int_0^{\pi} \sin 2t dt = 0$$

Hence,  $a_n = 0 \quad \dots n = 1, 3, 5 \dots$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin nt dt = \frac{1}{\pi} \left[ \int_0^{\pi} V_m \sin t \sin nt dt + \int_{\pi}^{2\pi} (0) \sin nt dt \right] \\ &= \frac{V_m}{\pi} \int_0^{\pi} \sin t \sin nt dt = 0 \text{ for } n = 2, 3, 4, 5 \text{ etc.} \end{aligned}$$

However, the expression indeterminate for  $n = 1$  so that  $b_1$  has to be evaluated separately.

$$b_1 = \frac{1}{\pi} \int_0^{\pi} V_m \sin t \cdot \sin t dt = \frac{V_m}{\pi} \int_0^{\pi} \sin^2 t dt = \frac{V_m}{2}$$

The required Fourier series for the half-wave rectified voltage waveform is

$$\begin{aligned} f(t) &= \frac{V_m}{\pi} + \frac{V_m}{2} \sin t - \frac{2V_m}{\pi} \sum_{\substack{n=1 \\ \text{even}}}^{\infty} \left( \frac{\cos nt}{n^2-1} \right) \\ &= \frac{V_m}{\pi} \left( 1 + \frac{\pi}{2} \sin t - \frac{2}{3} \cos 2t - \frac{2}{15} \cos 4t - \frac{2}{35} \cos 6t \dots \right) \end{aligned}$$

$$\begin{aligned}
 &= \frac{V_m}{\pi} \left( 1 + \frac{\pi}{2} \sin \omega_0 t - \frac{2}{3} \cos 2\omega_0 t - \frac{2}{15} \cos 4\omega_0 t - \frac{2}{35} \cos 6\omega_0 t - \dots \right) \\
 &= \frac{V_m}{T} \left( 1 + \frac{\pi}{2} \sin \theta - \frac{2}{3} \cos 2\theta - \frac{2}{15} \cos 4\theta - \frac{2}{35} \cos 6\theta - \dots \right)
 \end{aligned}$$

The line spectrum is shown in Fig. 21.19 (b) which has a strong fundamental term with rapidly decreasing amplitudes of the higher harmonics.

**Example 21.10.** Find the trigonometrical Fourier series for the full wave rectified voltage sine wave shown in Fig. 21.20.

**Solution.** Since the given function has even symmetry,  $b_n = 0$  i.e. it will contain no sine terms in its series.

The equation of the sinusoidal sine wave given by  $V = V_m \sin \theta$ . In other words,  $f(\theta) = V_m \sin \theta$ .

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta = \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \theta d\theta = \frac{2V_m}{2\pi} \int_0^{\pi} \sin \theta d\theta$$

It is so because the two parts  $0 - \pi$  and  $\pi - 2\pi$  are identical.

$$\therefore a_0 = \frac{V_m}{\pi} \left[ -\cos \theta \right]_0^{\pi} = \frac{2V_m}{\pi}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = \frac{2V_m}{\pi} \int_0^{\pi} \sin \theta \cos n\theta d\theta$$

$$\text{Now, } \sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\therefore a_n = \frac{2V_m}{\pi} \int_0^{\pi} [\sin(1+n)\theta + \sin(1-n)\theta] d\theta$$

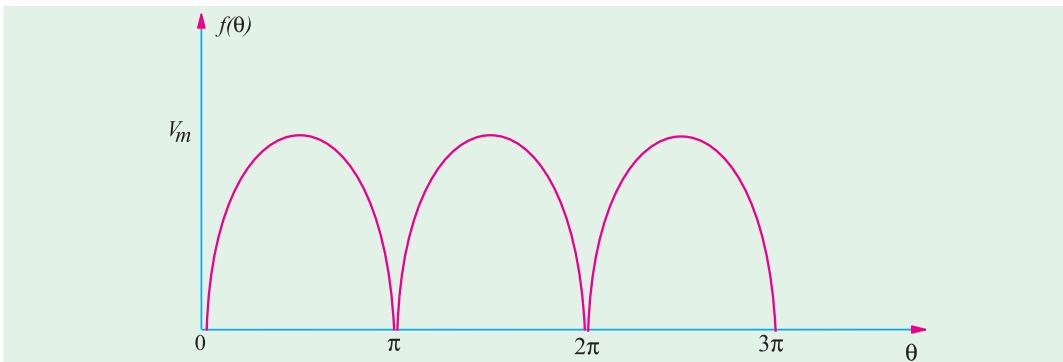


Fig. 21.20

$$= -\frac{V_m}{\pi} \frac{\cos(1+n)\theta}{(1+n)} + \frac{\cos(1-n)\theta}{(1-n)}$$

$$= 0 = \frac{-V_m}{\pi} \left[ \frac{\cos(1+n)\pi}{1+n} + \frac{\cos(1-n)\pi}{1-n} - \frac{1}{1+n} - \frac{1}{1-n} \right] \dots \text{ when } n \text{ is odd}$$



However, when  $n$  is even, then

$$a_n = \frac{V_m}{1} \frac{1}{n} \frac{1}{1} \frac{1}{n} \frac{1}{1} \frac{1}{n} \frac{1}{1} \frac{1}{n} \frac{2V_m}{1} \frac{1}{n} \frac{1}{1} \frac{1}{n} \frac{4V_m}{(n^2-1)}$$

$$\therefore f(\theta) = a_0 - \frac{4V_m}{\pi} \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \frac{\cos 2n\theta}{(n^2-1)}$$

$$\therefore f(\theta) = \frac{2V_m}{\pi} - \frac{4V_m}{\pi} \left( \frac{1}{3} \cos^2 \theta - \frac{1}{15} \cos 4\theta + \frac{1}{35} \cos 6\theta + \dots \right)$$

**Example 21.11.** Determine the Fourier series for the waveform shown in Fig. 21.21 (a) and sketch its line spectrum.

**Solution.** It is seen from Fig. 21.21 (a) that the waveform equation is  $f(\theta) = (V_m / \pi)\theta$ . The given function  $f(\theta)$  is defined by

$$f(\theta) = \begin{cases} \frac{V_m}{\pi} \theta, & 0 \leq \theta < \pi \\ 0, & \pi \leq \theta < 2\pi \end{cases}$$

Since the function possesses neither even nor odd symmetry, it will contain both sine and cosine terms.

Average value of the wave over one cycle is  $V_m/4$  or  $a_0 = V_m/4$ . It is so because the average value over the first half cycle is  $V_m/2$  and over the second half cycle is 0 hence, the average value

$$\text{for full cycle is } = \frac{(V_m/2) + 0}{2} = \frac{V_m}{4}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta = \frac{1}{\pi} \left[ \int_0^{\pi} (V_m/\pi)\theta \cos n\theta d\theta + \int_{\pi}^{2\pi} (0) \cos n\theta d\theta \right]$$

$$= \frac{V_m}{\pi^2} \int_0^{\pi} \theta \cos n\theta d\theta = \frac{V_m}{\pi^2} \left[ \frac{\cos n\theta}{n^2} + \frac{\theta}{n} \sin n\theta \right]_0^{\pi} = \frac{V_m}{\pi^2 n^2} (\cos n\pi - 1)$$

$$\therefore a_n = 0 \text{ when } n \text{ is odd because } \cos n\pi - 1 = 0$$

$$= -2V_m / \pi^2 n^2 \text{ when } n \text{ is even}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta = \frac{1}{\pi} \int_0^{\pi} (V_m/\pi)\theta \sin n\theta d\theta + \int_{\pi}^{2\pi} (0) \sin n\theta d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} \left( \frac{V_m}{\pi} \right) \theta \sin n\theta d\theta = \frac{V_m}{\pi^2} \left[ \frac{\sin n\theta}{n^2} - \frac{\theta}{n} \cos n\theta \right]_0^{\pi} = \frac{-V_m}{\pi n} \cos n\pi$$

$$\therefore b_n = -V_m / \pi n \text{ when } n \text{ is even } b_n = +V_m / \pi n \text{ when } n \text{ is odd}$$

Substituting the values of various constants in the general expression for Fourier series, we get

$$f(\theta) = \frac{V_m}{4} - \frac{2V_m}{\pi^2} \cos \theta - \frac{2V_m}{(3\pi)^2} \cos 3\theta - \frac{2V_m}{(5\pi)^2} \cos 5\theta \dots$$

$$+ \frac{V_m}{\pi} \sin \theta - \frac{V_m}{2\pi} \sin 2\theta + \frac{V_m}{3\pi} \sin 3\theta \dots$$

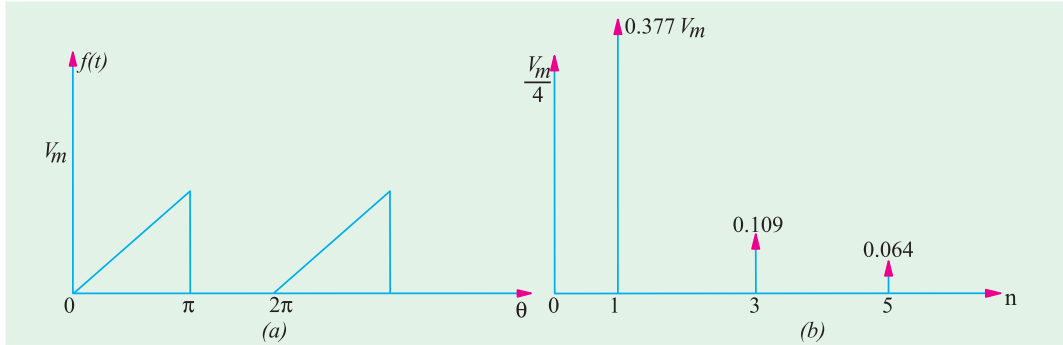


Fig. 21.21

The amplitudes of even harmonics are given directly by  $b_n$  but amplitudes of odd harmonics are given by  $A_n = \sqrt{a_n^2 + b_n^2}$  (Art. 21.4)

For example,

$$A_1 = \sqrt{(2V_m / \pi^2)^2 + (V_m / \pi)^2} = 0.377 V_m$$

$$A_3 = \sqrt{\left(\frac{2V_m}{(3\pi)^2}\right)^2 + \left(\frac{V_m}{2\pi}\right)^2} = 0.109 V_m$$

$$A_5 = \sqrt{\left(\frac{2V_m}{(5\pi)^2}\right)^2 + \left(\frac{V_m}{5\pi}\right)^2} = 0.064 V_m \text{ and so on.}$$

The line spectrum is as shown in Fig. 21.21 (b).

**Example 21.12.** Find the Fourier series for the sawtooth waveform shown in Fig. 21.22 (a). Sketch its line spectrum.

**Solution.** Using by the relation  $y = mx$ , the equation of the function becomes  $f(t) = 1, t$  or  $f(t) = t$ .

$$T = 2, \omega_0 = 2\pi / T = 2\pi / 2 = \pi$$

$$\text{By inspection it is clear that } a_0 = 2/2 = 1, a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t = \int_0^2 t \cdot \cos n\pi t \cdot dt$$

Since we have to find the integral of two functions, we use the technique of integration by parts *i.e.*

$$\int u v dx = u \int v dx - \int \left( \frac{du}{dx} \int v dx \right) dx$$

$$\begin{aligned} \therefore a_n &= t \int_0^2 \cos n\pi t dt - \int_0^2 \left( 1 \int_0^2 \cos n\pi t dt \right) dt \\ &= \left[ \frac{t \sin n\pi t}{n\pi} \right]_0^2 + \left[ \frac{\cos n\pi t}{(n\pi)^2} \right]_0^2 = 0 + \frac{1}{(n\pi)^2} (\cos 2n\pi - \cos 0) \end{aligned}$$

Since  $\cos 2n\pi = \cos 0$  for all values of  $n$ , hence  $a_n = 0$

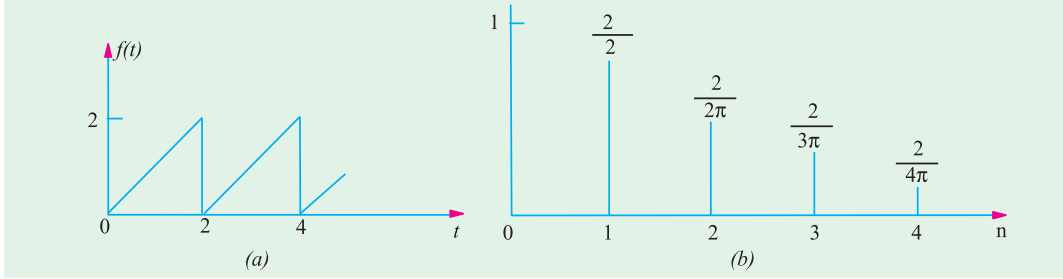


Fig. 21.22

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega_0 t dt = \int_0^2 t \sin n\pi t dt$$

Employing integration by parts, we get,

$$\begin{aligned} b_n &= t \int_0^2 \sin n\pi t dt - \int_0^2 \left( 1 \int_0^2 \sin n\pi t dt \right) dt \\ &= \left[ t \cdot \frac{-\cos n\pi t}{n\pi} \right]_0^2 - \int_0^2 \frac{-\cos n\pi t}{n\pi} dt = \left[ \frac{-t \cos n\pi t}{n\pi} \right]_0^2 + \left[ \frac{\sin n\pi t}{(n\pi)^2} \right]_0^2 = \frac{\sin 2n\pi}{(n\pi)^2} - \frac{2}{n\pi} \cos 2n\pi \end{aligned}$$

The sine term is 0 for all values of  $n$  because sign of any multiple of  $2\pi$  is 0. Since value of cosine term is 1 for any multiple of  $2\pi$ , we have,  $b_n = -2/n\pi$ .

$$\begin{aligned} \therefore f(t) &= a_0 + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t = a_0 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin n\pi t \\ &= 1 - \frac{2}{\pi} \left( \sin \pi t + \frac{1}{2} \sin 2\pi t + \frac{1}{3} \sin 3\pi t + \dots \right) \end{aligned}$$

The line spectrum showing the amplitudes of various harmonics is shown in Fig. 21.22 (b).

**Example 21.13.** Determine the trigonometric Fourier series of the triangular waveform shown in Fig. 21.23.

**Solution.** Since the waveform possesses odd symmetry, hence  $a_0 = 0$  and  $a_n = 0$  i.e. there would be  $n_o$  cosine terms in the series. Moreover, the waveform has half-wave symmetry. Hence, series will have only odd harmonics. In the present case, there would be only odd sine terms. Since the waveform possesses quarter-wave symmetry, it is necessary to integrate over only one quarter period of finding the Fourier coefficients.

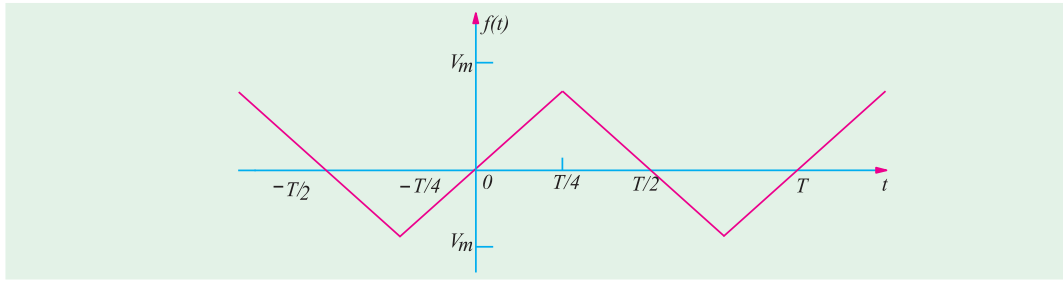


Fig. 21.23

$$f(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} b_n \sin n\omega_0 t$$

$$\text{where } b_n = \frac{8}{T} \int_0^{T/4} f(t) \sin n\omega_0 t$$

The quarter-wave of the given waveform can be represented by equation of a straight line. Slope of the straight line is  $V_m / (T/4) = 4V_m / T$ .

Hence, using  $Y = mx$ , we have

$$f(t) = \left( \frac{4V_m}{T} \right) t \quad 0 < t < T/4 \quad \therefore b_n = \frac{8}{T} \int_0^{T/4} \left( \frac{4V_m}{T} \right) t \sin n\omega_0 t = \frac{32V_m}{T^2} \int_0^{T/4} t \sin n\omega_0 t$$

Using the theorem of integration by parts, we have

$$b_n = \frac{32V_m}{T^2} \left[ t \int_0^{T/4} \sin n\omega_0 t \, dt - \int_0^{T/4} \left( 1 \cdot \int_0^{T/4} \sin n\omega_0 t \, dt \right) dt \right]$$

$$\frac{32V_m}{T^2} \left[ \frac{t}{4} \frac{\cos n\omega_0 T/4}{n\omega_0} - \left| \frac{\sin n\omega_0 t}{(n\omega_0)^2} \right|^{T/4} - \frac{32V_m}{T^2} \left[ \frac{T}{4} \frac{\cos n\omega_0 T/4}{n\omega_0} - \frac{\sin n\omega_0 T/4}{(n\omega_0)^2} \right] \right]$$

$$\text{Now, } \omega_0 = 2\pi/T \text{ or } \omega_0 T = 2\pi \quad \therefore n\omega_0 T/4 = n\pi/2$$

$$\therefore \cos n\omega_0 T/4 = \cos n\pi/2 = 0 \text{ when } n \text{ is odd}$$

$$\therefore b_n = \frac{32V_m}{n^2 \omega_0^2 T^2} \sin n\omega_0 \frac{T}{4} = \frac{32V_m}{n^2 (2\pi)^2} \sin \frac{n\pi}{2} = \frac{8V_m}{n^2 \pi^2} \sin \frac{n\pi}{2}$$

$$\therefore b_n = \frac{8V_m}{n^2 \pi^2} \dots n = 1, 5, 9, 13 \dots \quad b_n = \frac{-8V_m}{n^2 \pi^2} \dots n = 3, 7, 11, 15, \dots$$

Substituting this value of  $b_n$ , the Fourier series for the given waveform becomes

$$f(t) = \frac{8V_m}{2} \sin \omega_0 t - \frac{1}{3^2} \frac{8V_m}{2} \sin 3\omega_0 t + \frac{1}{5^2} \frac{8V_m}{2} \sin 5\omega_0 t - \frac{1}{7^2} \frac{8V_m}{2} \sin 7\omega_0 t \dots$$

**Example 21.14.** Determine the Fourier series of the triangular waveform shown in Fig. 21.24.

**Solution.** Since the function has even symmetry,  $b_n = 0$ . Moreover, it also has half-wave symmetry, hence,  $a_0 = 0$ . The Fourier series

can be written as  $f(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$  where

$$a_n = \frac{2}{T} \int_0^{T/2} f(t) \cos n \omega_0 t dt.$$

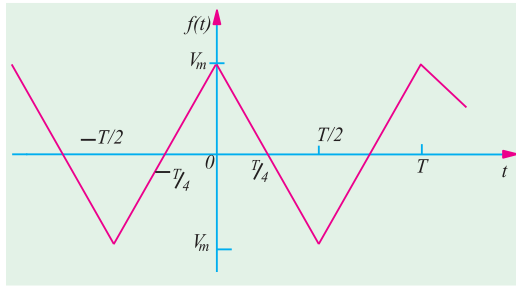


Fig. 21.24

The function is given by the relation  $f(t) = \frac{4V_m}{T} t - \frac{T}{4}$

It is so because for the interval  $0 \leq t \leq T/2$ , the slope of the line is  $4V_m/T$ .

$$\therefore a_n = \frac{2}{T} \int_0^{T/2} f(t) \cos n \omega_0 t dt = \frac{4}{T} \int_0^{T/4} f(t) \cos n \omega_0 t dt$$

$$= \frac{16V_m}{T^2} \int_0^{T/2} t \cos n \omega_0 t dt = \frac{16V_m}{T^2} \int_0^{T/2} t \cos n \omega_0 t dt = \frac{4V_m}{T} \int_0^{T/2} \cos n \omega_0 t dt$$

$$= \frac{-16V_m}{T^2} \left[ \frac{1}{4n^2\omega_0^2} \cdot \cos n\omega_0 t + \frac{t}{n\omega_0} \sin n\omega_0 t \right]_0^{T/2} + \frac{4V_m}{T} \left[ \frac{\sin n\omega_0 t}{n\omega_0} \right]_0^{T/2}$$

Substituting  $\omega = 2\pi/T$ , we get

$$a_n = \frac{16V_m}{T^2} \frac{T^2}{4n^2} (\cos n\pi - 1) - \frac{T^2}{4n} (\sin n\pi) - \frac{2V_m}{n} \sin n\pi$$

Now,  $\sin n\pi = 0$  for all values of  $n$ ,  $\cos n\pi = 1$  when  $n$  is even and  $-1$  when  $n$  is odd.

$$\therefore a_n = \frac{8V_m}{\pi^2 n^2}$$

$$\therefore f(t) = \sum_{\substack{n=0 \\ \text{odd}}}^{\infty} \frac{8V_m}{\pi^2 n^2} \cos n\omega_0 t = \frac{8V_m}{\pi^2} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{\cos n\omega_0 t}{n^2}$$

$$= \frac{8V_m}{2} \cos \omega_0 t - \frac{1}{9} \cos 3 \omega_0 t + \frac{1}{25} \cos 5 \omega_0 t - \frac{1}{49} \cos 7 \omega_0 t \dots$$

**Alternative Solution**

We can deduce the Fourier series from Fig. of Ex. 21.11 by shifting the vertical axis by  $\pi/2$  radians to the right. Replacing  $t$  by  $(t + \pi/2)$  in the Fourier series of Ex. 21.11, we get

$$f(t) = \frac{8V_m}{2} \sin \omega_0 t - \frac{1}{3^2} \sin 3 \omega_0 t + \frac{1}{5^2} \sin 5 \omega_0 t - \frac{1}{7^2} \sin 7 \omega_0 t \dots$$

$$= \frac{8V_m}{2} \cos \omega_0 t - \frac{1}{9} \cos 3 \omega_0 t + \frac{1}{25} \cos 5 \omega_0 t - \frac{1}{49} \cos 7 \omega_0 t \dots$$

**Example 21.15.** Obtain the Fourier series representation of the sawtooth waveform shown in Fig. 21.25 (a) and plot its spectrum.

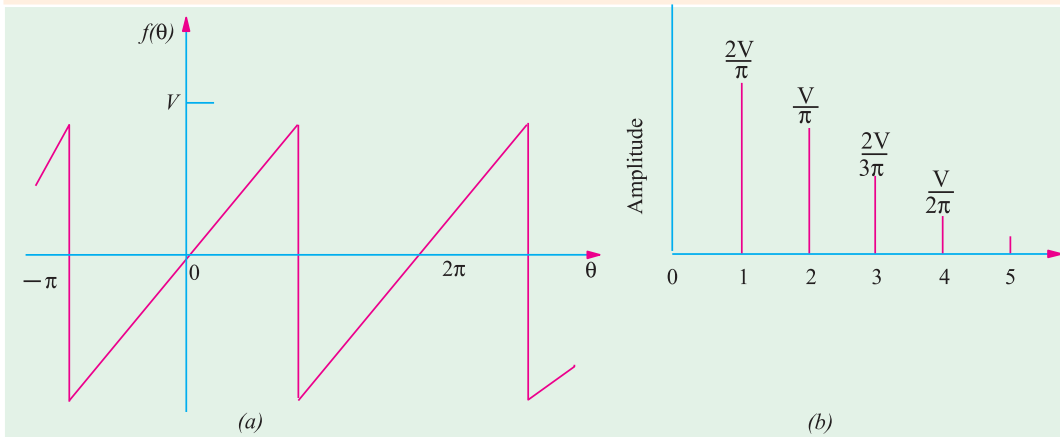


Fig. 21.25

**Solution.** By inspection, we know that the average value of the wave is zero over a cycle because the height of the curve below and above the  $X$ -axis is the same hence,  $a_0 = 0$ . Moreover, it has odd symmetry so that  $a_n = 0$  i.e. there would be no cosine terms. The series will contain only sine terms.

$$\therefore f(\theta) = \sum_{n=1}^{\infty} b_n \sin n\theta \quad \text{where } b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta \, d\theta$$

The slope of curve is  $m = V/\pi$

$\therefore$  we get,  $f(\theta) = (V/\pi)\theta$ .

If we are the limit of integration from  $-\pi$  to  $+\pi$  then

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{V}{\pi} \sin n\theta \, d\theta = \frac{V}{\pi^2} \left[ \frac{1}{n} \sin n\theta - \frac{\cos n\theta}{n} \right]_{-\pi}^{\pi} = \frac{2V}{n^2} \cos n\pi$$

The above result has been obtained by making use of integration by parts as explained earlier.  $\cos n\pi$  is positive when  $n$  is even and is negative when  $n$  is odd and thus the signs of the coefficients alternate. The required Fourier series is

$$f(\theta) = \frac{2V}{\pi} \sin \theta - \frac{1}{2} \sin 2\theta + \frac{1}{3} \sin 3\theta - \frac{1}{4} \sin 4\theta + \dots$$

$$\text{or } f(t) = \frac{2V}{\pi} \sin \omega_0 t - \frac{1}{2} \sin 2\omega_0 t + \frac{1}{3} \sin 3\omega_0 t - \frac{1}{4} \sin 4\omega_0 t + \dots$$

As seen, the coefficients decrease as  $1/n$  so that the series converges slowly as shown by the line spectrum of Fig. 21.25 (b).

The amplitudes of the fundamental of first harmonic, second harmonic, third harmonic and fourth harmonic are  $(2/\pi), (2V/2\pi), (2V/3\pi)$  and  $(2V/4\pi)$  respectively.

### Tutorial Problem. 21.1

1. Determine the Fourier series for the triangular waveform shown in Fig. 21.26 (a)

(Network Theory and Design, AMIETE June 1990)

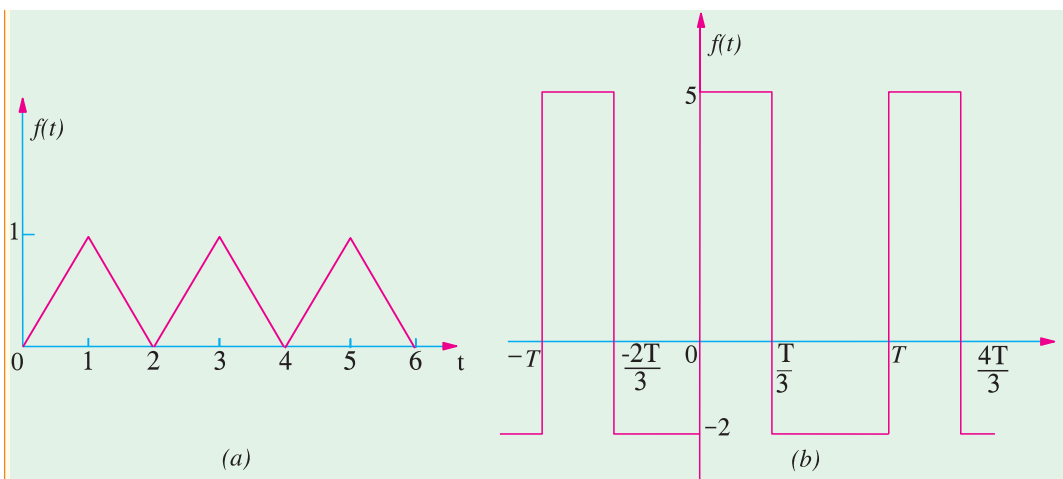


Fig. 21.26

$$\left[ \frac{1}{2} - \frac{4}{\pi^2} \cos \omega_0 t + \frac{1}{3^2} \cos 3 \omega_0 t + \frac{1}{5^2} \cos 5 \omega_0 t + \dots \right]$$

2. Find the values of the Fourier coefficients  $a_0$ ,  $a_n$  and  $b_n$  for the function given in fig. 21.26 (b).

$$\left[ a_0 = \frac{2}{3}; a_n = \frac{7}{n\pi} \sin \frac{2n\pi}{3}; b_n = \frac{7}{n\pi} \left( 1 - \cos \frac{2n\pi}{3} \right) \right]$$

3. Determine the trigonometric series of the triangular waveform shown in Fig. 21.27. Sketch its line spectrum.

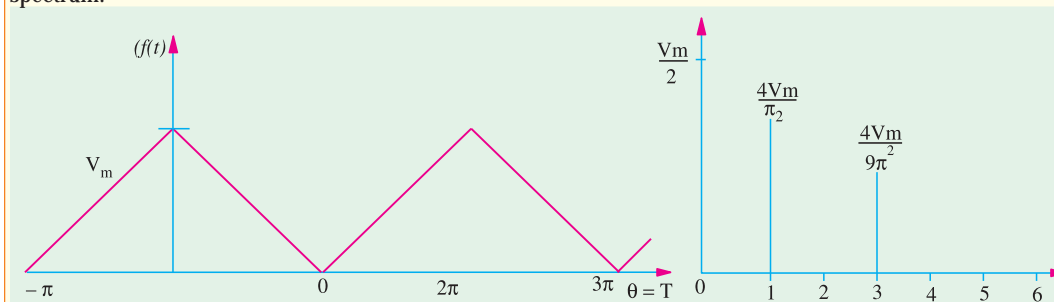


Fig. 21.27

$$\frac{V_m}{2} - \frac{4V_m}{\pi^2} \cos^2 \omega t + \frac{1}{3^2} \cos 3 \omega t - \frac{1}{5^2} \cos 5 \omega t + \dots$$

4. Determine the Fourier series for the sawtooth waveform shown in Fig. 21.28.

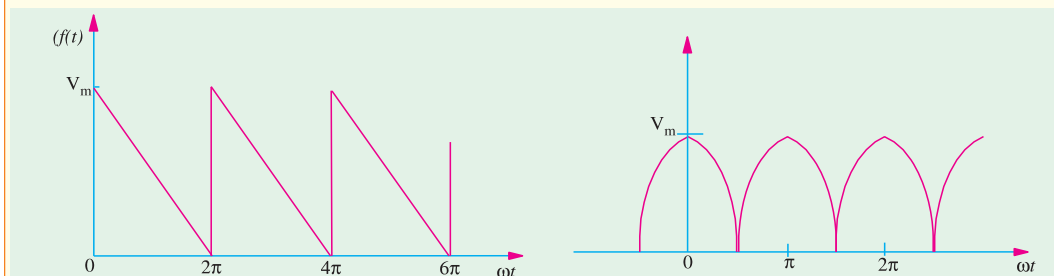


Fig. 21.28

Fig. 21.29

$$\left[ f(t) = \frac{V_m}{2} + \frac{V_m}{\pi} (\sin\omega_0 t + \frac{1}{2} \sin 2\omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \dots) \right]$$

5. Represent the full-wave rectified voltage sine waveform shown in Fig. 21.29 by a Fourier series.

$$\left[ f(t) = \frac{2V_m}{\pi} + \left( 1 + \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t + \frac{2}{35} \cos 6\omega t \dots \right) \right]$$

6. Obtain trigonometric Fourier series for the wave form shown in Fig. 21.30.

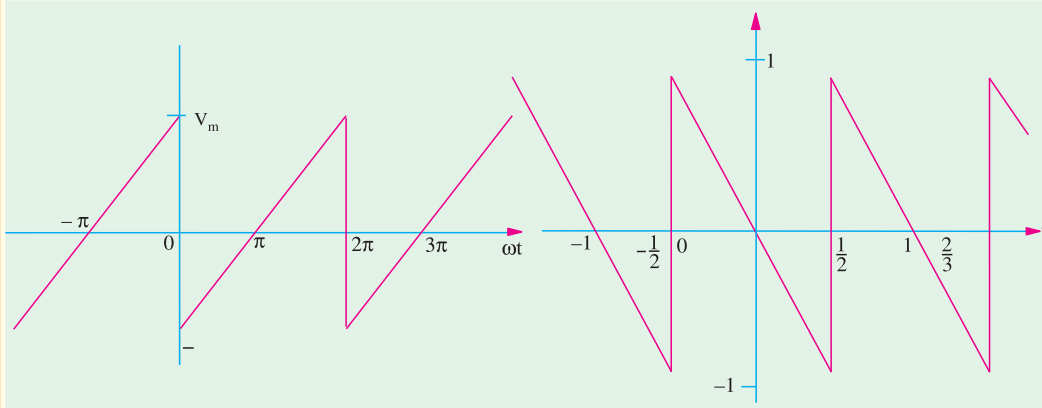


Fig. 21.30

Fig. 21.31

$$\left[ f(t) = -\frac{2V_m}{\pi} \left( \sin\omega_0 t + \frac{1}{2} \sin 2\omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{4} \sin 4\omega_0 t \dots \right) \right]$$

7. Find the Fourier series for the sawtooth waveform shown in Fig. 21.31.

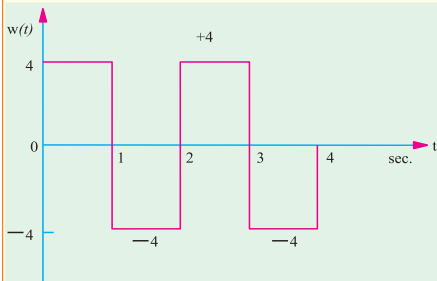


Fig. 21.32

$$\left[ f(t) = \frac{2}{\pi} \left( -\sin 2\pi t + \frac{1}{2} \sin 4\pi t - \frac{1}{3} \sin 6\pi t + \dots \right) \right]$$

8. For the waveform of Fig. 21.32, find the Fourier series terms up to the 5th harmonic.

(Network Theory Nagpur Univ. 1993)

$$\left[ V(t) = \frac{16}{\pi} \left( \sin t + \frac{2}{3} \sin 3t \right) + \frac{1}{5} \sin 5t + \dots \right]$$

9. Determine Fourier series of a repetitive triangular wave as shown in Fig. 21.33.

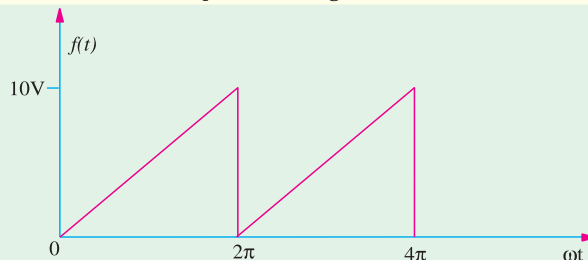


Fig. 21.33

(a) What is the magnitude of d.c. component?

(b) What is the fundamental frequency?



- (c) What is the magnitude of the fundamental?
- (d) Obtain its frequency spectrum.

(Network Theory Nagpur Univ.1993)

[(a) 5 V (b) 1 Hz (c) 10 / π volt]

10. Determine the Fourier series of voltage responses obtained at the o/p of a half wave rectifier shown in Fig. 21.34. Plot the discrete spectrum of the waveform.

(Elect. Network Analysis Nagpur Univ. 1993)

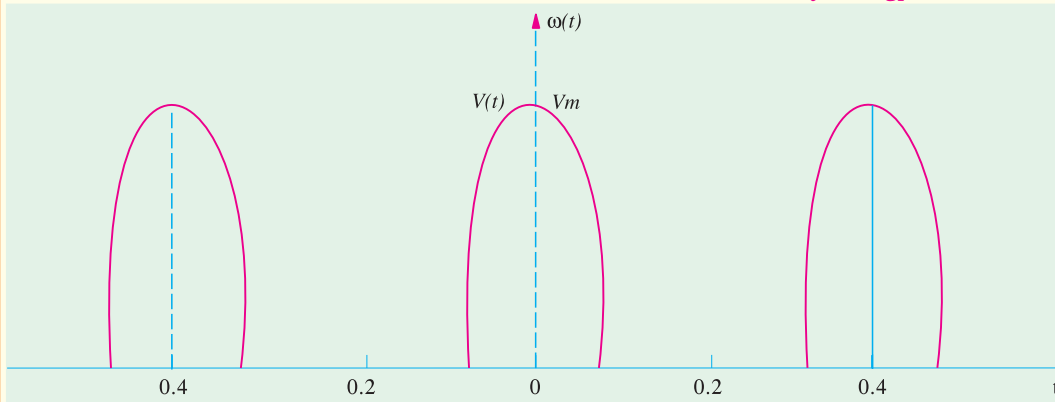


Fig. 21.34

$$\left[ V(t) = \frac{V_m}{\pi} + \frac{V_m}{2} \cos\omega_0 t + \frac{2V_m}{3\pi} \cos 10\pi t - \frac{2V_m}{15} \cos 20\pi t + \frac{2V_m}{35\pi} \cos 30\pi t \dots \right]$$

11. Determine the Fourier coefficients and plot amplitude and phase spectral.

(Network Analysis Nagpur Univ. 1993)

$$\left[ a_3 = 0, b_n = 0, a_1 = \frac{4V_m}{\pi}, a_2 = 0 \right]$$

$$\left[ a_3 = \frac{-4V_m}{3\pi}, a_4 = 0, a_5 = \frac{4V_m}{\pi} \right]$$

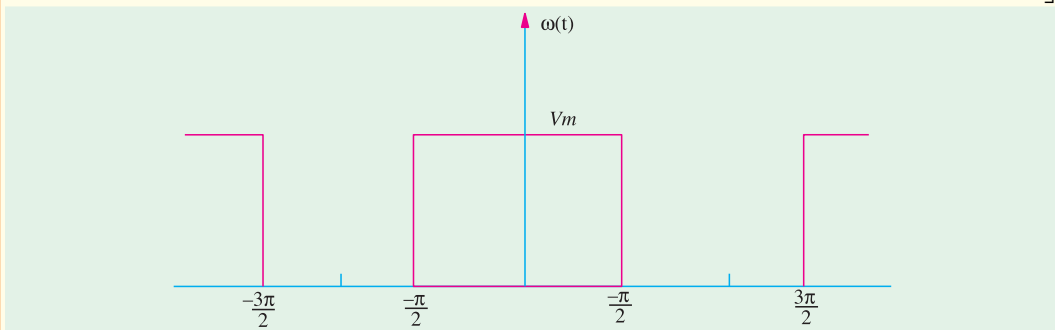


Fig. 21.35

### OBJECTIVE TESTS – 21

1. A given function  $f(t)$  can be represented by a Fourier series if it
  - (a) is periodic
  - (b) is single valued
  - (c) has a finite number of maxima and minima in any one period
  - (d) all of the above.
2. In a Fourier series expansion of a periodic function, the coefficient  $a_0$  represents its
  - (a) net area per cycle
  - (b) d.c value

- (c) average value over half cycle  
(d) average a.c value per cycle
3. If in the Fourier series of a periodic function, the coefficient  $a_0$  is zero, it means that the function has  
(a) odd symmetry  
(b) even quarter-wave symmetry  
(c) odd quarter-wave symmetry  
(d) any of the above.
4. A periodic function  $f(t)$  is said to possess odd quarter-wave symmetry if  
(a)  $f(t) = f(-t)$   
(b)  $f(-t) = -f(t)$   
(c)  $f(t) = -f(t + T/2)$   
(a) both (b) and (c).
5. If the average value of a periodic function over one period is zero and it consists of only odd harmonics then it must be possessing \_\_\_\_\_ symmetry.  
(a) half-wave  
(b) even quarter-wave  
(c) odd quarter-wave  
(d) odd.
6. If in the Fourier series of a periodic function, the coefficient  $a_0 = 0$  and  $a_n = 0$ , then it must be having \_\_\_\_\_ symmetry.  
(a) odd  
(b) odd quarter-wave  
(c) even  
(d) either (a) or (b).
7. In the case of a periodic function possessing half-wave symmetry, which Fourier coefficient is zero ?  
(a)  $a_n$  (b)  $b_n$   
(c)  $a_0$  (d) none of above.
8. A periodic function has zero average value over a cycle and its Fourier series consists of only odd cosine terms. What is the symmetry possessed by this function.  
(a) even  
(b) odd  
(c) even quarter-wave  
(d) odd quarter-wave
9. Which of the following periodic function possesses even symmetry ?  
(a)  $\cos 3t$  (b)  $\sin t$   
(c)  $t \cdot \cos 50t$  (d)  $(t + t^2 + t^3)$ .
10. If the Fourier coefficient  $b_n$  of a periodic function is zero, then it must possess \_\_\_\_\_ symmetry.  
(a) even  
(b) even quarter-wave  
(c) odd  
(d) either (a) and (b).
11. A complex voltage waveform is given by  $V = 120 \sin \omega t + 36(3\omega t + \pi/2) + 12 \sin(5\omega t + \pi)$ . It has a time period of  $T$  seconds. The percentage fifth harmonic contents in the waveform is  
(a) 12 (b) 10  
(c) 36 (d) 5
12. In the waveform of Q. 11 above, the phase displacement of the third harmonic represents a time interval of \_\_\_\_\_ seconds.  
(a)  $T/12$  (b)  $T/3$   
(c)  $3T$  (d)  $T/36$
13. When the negative half-cycle of a complex waveform is reversed, it becomes identical to its positive half-cycle. This feature indicates that the complex waveform is composed of  
(a) fundamental  
(b) odd harmonics  
(c) even harmonics  
(d) both (a) and (b)  
(e) both (a) and (c)
14. A periodic waveform possessing half-wave symmetry has no  
(a) even harmonics  
(b) odd harmonics  
(c) sine terms  
(d) cosine terms
15. The Fourier series of a wave form possessing even quarter-wave symmetry has only  
(a) even harmonics  
(b) odd cosine terms  
(c) odd sine terms  
(d) both (b) and (c).
16. The Fourier series of a waveform possessing odd quarter-wave symmetry contains only  
(a) even harmonics  
(b) odd cosine terms  
(c) odd cosine terms  
(d) none of above

## ANSWERS

1. (d) 2. (b) 3. (d) 4. (d) 5. (a) 6. (d) 7. (c) 8. (c) 9. (a) 10. (d)  
11. (b) 12. (a) 13. (d) 14. (a) 15. (b) 16. (c)

# C H A P T E R 22

## Learning Objectives

- Introduction
- Types of Transients
- Important Differential Equations
- Transients in R-L Circuits (D.C.),
- Short Circuit Current
- Time Constant
- Transients in R-L Circuits (A.C.)
- Transients in R-C Series Circuits (D.C.)
- Transients in R-C Series Circuits (A.C.)
- Double Energy Transients

## TRANSIENTS



↑ Transients are the surges or spikes in electric currents and voltages which are transmitted through power or data lines. The gadget shown in the above picture is claimed to protect the equipment against such surges or spikes

## 22.1. Introduction

It is quite an easy job to calculate the steady current, which flows in a circuit, when it is connected to a d.c. generator or a battery. Similarly, the alternating current which flows in a circuit when connected to an alternator can also be calculated by the various method discussed in Chapters 13 and 14. These currents are known as *steady* currents because in such cases, it is assumed that (i) the circuit components are constant and (ii) the circuit has been connected to the generator long enough for any disturbance produced on initial switching, to resolve itself.

In general, transients disturbances are produced whenever

- (a) an apparatus or circuit is *suddenly* connected to or disconnected from the supply,
- (b) a circuit is shorted and
- (c) there is a *sudden* change in the applied voltage from one finite value to another.

We will now discuss the transients produced whenever different circuits are suddenly switched on or off from the supply voltage. In each case, we will assume that the resultant current consists of two parts (i) a final steady-stage or normal current and (ii) a transient current superimposed on the steady-stage current.

It is essential to remember that the transient currents are not driven by any part of the applied voltage but are entirely associated with the changes in the stored energy in inductors and capacitors. Since there is no stored energy in resistors, *there are no transients in pure resistive circuits.*

## 22.2. Types of Transients

There are *single-energy* transients and *double-energy* transients. Single-energy transients are those in which only one form of energy, either electromagnetic or electrostatic is involved as in  $R-L$  and  $R-C$  circuits.

However, double-energy transients are those in which both electromagnetic or electrostatic is involved as in  $R-L-C$  circuits Transient disturbances may be further classified as follows :

- (a) **Initiation Transients** : These are produced when a circuit, which is originally dead, is energised.
- (b) **Subsidence Transients** : These are produced when an energised circuit is rapidly de-energised and reaches an eventual steady-stage of zero current or voltage, as in the case of short-circuiting an  $R-L$  or  $R-C$  circuit suddenly.
- (c) **Transition Transients** : These are due to sudden but energetic changes from one steady state to another.
- (d) **Complex Transients** : These are produced in a circuit which is simultaneously subjected to two transients due to two independent disturbances or when the disturbing force producing the transients is itself variable.
- (e) **Relaxation Transients** : In these transients, the transition occurs cyclically towards states, which when reached, become unstable themselves.

A distinction may also be made between free and forced transients which are produced due to the applied voltage being itself transient.

## 22.3. Important Differential Equations

Some of the important differential equations, used in the treatment of single and double energy transients, are given below. We will consider both first-order and second-order differential equations.

**1. First Order Equations**

(i) Let  $\frac{dy}{dx} + ay = 0$  where  $a$  is a constant.

Its solution is  $y = k e^{-ax}$  where  $k$  is the constant of integration whose value can be found from the boundary conditions *i.e.* conditions prevalent at the instant when the voltage to a circuit is applied or excluded.

(ii) If  $\frac{dy}{dx} + ay = b$  where  $a$  and  $b$  are constants, then solution is  $y = \frac{b}{a} + k e^{-ax}$

The value of  $k$  can again be found from boundary conditions.

(iii) If  $\frac{dy}{dx} + Ay = B$

where  $A$  and  $B$  are not constants but are functions of  $x$ , then the solution is given by

$$y e^{A dx} = \int B dx e^{A dx} + k e^{A dx}$$

If  $A = a = \text{constant}$ , then the above equation simplifies to

$$y e^{ax} = \int B dx e^{ax} + k e^{ax}$$

**2. Second Order Equations**

(i) Suppose  $\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = 0$  where  $a$  and  $b$  are constants, then the solution is

$$y = k_1 e^{\lambda_1 x} + k_2 e^{\lambda_2 x}$$

where  $\lambda_1$  and  $\lambda_2$  are constants of integration and whose values are,

$$\lambda_1 = \frac{-a}{2} \pm \sqrt{\frac{a^2}{4} - b} \quad \text{and} \quad \lambda_2 = \frac{-a}{2} \mp \sqrt{\frac{a^2}{4} - b}$$

- (a) If  $a^2/4 > b$ , the roots are real and the above solution can be applied without any difficulty.
- (b) If  $a^2/4 < b$ , the radicals contain a negative quantity. In that-case, the solution is given by

$$y = e^{-\frac{a}{2}x} (k_3 \sin \lambda_0 x + k_4 \cos \lambda_0 x)$$

where  $k_3$  and  $k_4$  are the new constants of integration and

$$\lambda_0 = \sqrt{b - \frac{a^2}{4}}$$

- (c) If  $a^2/4 = b$ , then both roots are equal and each is  $= -a/2$ .

Hence, in this case, the solution becomes  $y = k_5 e^{\lambda t} + k_6 t \cdot e^{\lambda t}$

(ii) Let  $\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = c$

where  $a$ ,  $b$  and  $c$  are constant. In this case also, the solution will again depend on the root as discussed above.

$$y = k_1 e^{\lambda_1 x} + k_2 e^{\lambda_2 x} + c/b$$

(iii) (a) Let the differential equation be given by

$$\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = u$$

where  $a, b$  are constants but  $u$  is a particular function of the variable  $x$ . The solution of such an equation consists of a **particular integral** and a complementary function.

(b) Let  $y$  be a sinusoidal function of  $x$ , then

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = c \sin \omega t$$

In this case, particular integral is

$$y_1 = \frac{c}{\sqrt{a^2 + b^2}} \cos x \tan^{-1} \frac{b}{a}$$

The complementary function is given by

$$y_2 = k_1 e^{p_1 x} + k_2 e^{p_2 x} \text{ where } p_1 = \frac{a}{2} + \sqrt{\frac{a^2}{4} - b} \text{ and } p_2 = \frac{a}{2} - \sqrt{\frac{a^2}{4} - b}$$

The complete solution for the above equation is  $y = y_1 + y_2$

Further treatment is the same as for case 2 (i) above.

### 22.4. Transients in R-L Circuits (D.C.)

If  $i, I_s$  and  $i_t$  be the resultant current, steady-state current and transient current respectively in R-L circuit of Fig. 22.1 (a), then by superimposition, the equation for the resultant current, for the duration of initiation transient, is

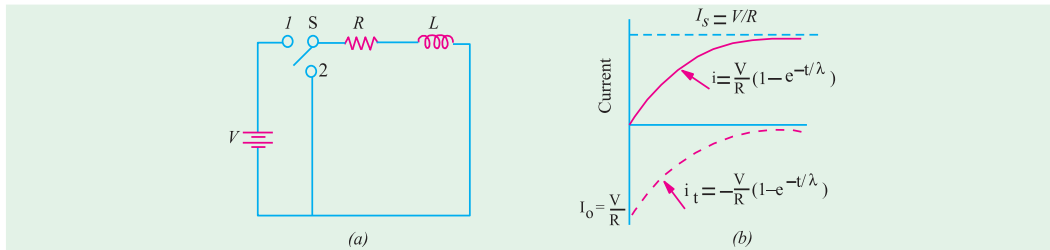


Fig. 22.1

$$i = I_s + i_t$$

... (i)

Since the applied voltage  $V$  drives the steady-state current, hence

$$I_s = V/R$$

Since the transient current  $i_t$  is not associated with any voltage,

$$\therefore i_t R + L \frac{di_t}{dt} = 0$$

... (ii)

or  $\frac{di_t}{dt} = -\frac{R}{L} i_t$  ... (iii)

or  $\frac{di_t}{i_t} = -\frac{R}{L} dt \therefore \log_e i_t = -\frac{R}{L} t + K^*$  ... (iv)

where  $K$  is the constant of integration whose value may be found from the initial conditions.

Now when  $t = 0, i_t = I_0$  (say). Then from Eq. (iv) above we get,  $\log_e I_0 = 0 + K$

\*  $\log_e x$  is written as  $\log x$ . Obviously,  $\log x = 2.3 \log_{10} x$ .

Putting this value of  $K$  in Eq. (iv), we have

$$\log i_t - \log I_0 = \frac{R}{L}t \text{ or } \log i_t / I_0 = \frac{R}{L}t$$

$$\therefore i_t = I_0 e^{-t/\lambda} \quad \dots (v)$$

where  $\lambda = L/R$  is called the **time-constant** of the circuit. Its reciprocal  $R/L$  is called the **damping coefficient** of the circuit. The current decreases exponentially as shown in Fig. 22.1 (b). From Eq. (i) and (v), we have

$$i = I_s + I_0 e^{-t/\lambda} \quad \dots (vi)$$

If the time is reckoned when the voltage  $V$  is applied, so that when  $t = 0, i = 0$ , then from equation (vi), we get

$$0 = I_s + i_0 e^{-0} = I_s + I_0 \quad \therefore I_0 = -I_s = -\frac{V}{R}$$

In that case, Eq. (vi) becomes

$$i = \frac{V}{R} - \frac{V}{R} e^{-t/\lambda} \quad \dots (vii)$$

$$= \frac{V}{R} (1 - e^{-t/\lambda}) \quad \dots (viii)$$

Curves for  $I_s$  and  $i_t$  have been plotted in Fig. 22.1 (b). The curve for resultant current has been obtained by the superposition of steady-state current  $I_s (= V/R)$  and transient current

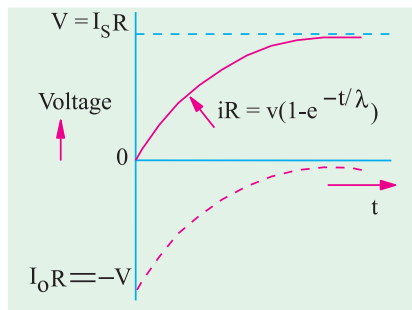


Fig. 22.2

$$i_t = \frac{V}{R} e^{-t/\lambda}$$

Theoretically, the transient current  $i_t$  takes infinite time to die off but, in practice, it disappears in a very short time.

The values of resultant, steady-state and transient voltages across the resistor can be found by multiplying Eq. (vii) by  $R$  and are shown in Fig. 22.2. The e.m.f. of self-induction  $-L di/dt$  is only transient in nature and equals  $i_t R$  as seen from Eq. (ii) above.

### 22.5. Short Circuit Current

After some time, the transient current would disappear and the only current flowing in the circuit would be the steady-state current  $I_s = V/R$ . Let the  $R-L$  circuit be closed upon itself *i.e.* be short-circuited by shifting the switch [Fig. 22.1 (a)] to position 2. Since the voltage  $V$  has been excluded from the circuit, the trapped current  $I_s$  will immediately cease to be a steady-state current, but on the other hand, will become the initial value  $I_0$  of a new subsidence transient current  $i_t$ .



Large-capacity short-circuit current generator

If time is measured at the instant of short-circuit, so that when  $t = 0$ , the current is  $I_s = V/R$ , then Eq. (v) becomes

$$i_t = \frac{V}{R} e^{-t/\lambda} \quad \dots (ix)$$

This equation has been plotted in Fig. 22.3. The only voltage acting in the circuit is that due to self-induction *i.e.*  $-L di/dt$  which equals  $iR$ .

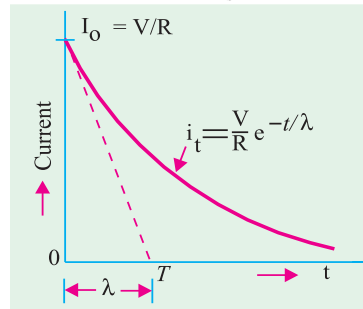


Fig. 22.3

## 22.6. Time Constant

The time constant of a circuit is defined as the time it would take for the transient current to decrease to zero, if the decrease were linear instead of being exponential.

In other words, it is the time during which the transient current would have decreased to zero, had it maintained its initial rate of decrease.

The initial rate of decrease can be found by differentiating Eq. (vi) and putting  $t = 0$

$$\therefore \frac{di_t}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda} \quad \frac{di_t}{dt} \Big|_{t=0} = \frac{I_0}{\lambda}$$

If the rate of decrease were constant throughout and equal to  $-I_0/\lambda$ , then the straight line showing the relation between  $i_t$  and  $t$  would be given by

$$i_t = -\frac{I_0}{\lambda} t$$

The time-period would be equal to the sub-tangent  $OT$  drawn to the exponential curve of Fig. 22.3 at  $i_t = I_0$  *i.e.* at the beginning of the curve.

If we put  $t = \lambda$  in Eq. (v), then  $i_t = I_0 e^{-1} = I_0/e = I_0/2.718 = 0.37 I_0$

Hence, time period of a circuit is the time during which the transient current decreases to 0.37 of its initial value.

**Example 22.1.** A coil having a resistance of  $30 \Omega$  and an inductance of  $0.09 \text{ H}$  is connected across a battery of  $20 \text{ V}$ . Plot the current and its two components. Assume that  $t = 0$  when the circuit is completed. (Electromechanic Allahabad Univ. 1992)

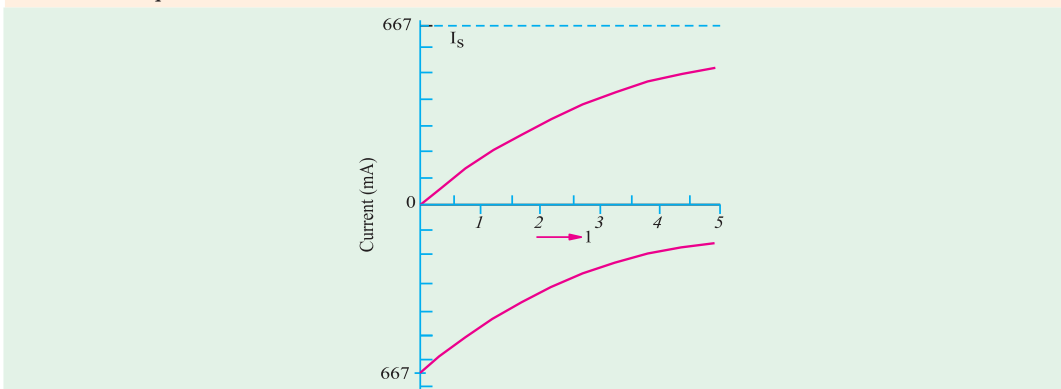


Fig. 22.4

**Solution.** The two components of the circuit current are (i) steady current  $I_s = V/R = 20/30 = 2/3 \text{ A} = 667 \text{ mA}$  and (ii) Transient current  $i_t = -(V/R)e^{-t/\lambda}$ .



Total current is  $i = I_s + i_t$ . Let us find the value of transient current after various time intervals. In the present case,  $\lambda = L/R = 0.09/30 = 0.003$  second = 3 millisecond.

The values of  $i_t$  and  $i$  at various times are tabulated below. Value of  $i = I_s + i_t$

$t$ (ms)	$e^{-t/\lambda}$	$i_t$ (mA)	$i$	$t$ (ms)	$e^{-t/\lambda}$	$i_t$ (mA)	$i$
0.0	1.000	-667	0	2.5	0.435	-290	377
0.5	0.847	-565	102	3.0	0.368	-244	423
1.0	0.716	-477	190	3.5	0.311	-208	459
1.5	0.606	-405	262	4.0	0.264	-176	491
2.0	0.514	-344	323	4.5	0.223	-148	519

It is seen that whereas transient current decreases exponentially, total circuit current increases exponentially as expected (Fig. 22.4).

**Example 22.2.** A circuit of resistance  $10 \Omega$  and inductance  $0.1 \text{ H}$  in series has a direct voltage of  $200 \text{ V}$  suddenly applied to it. Find the voltage drop across the inductance at the instant of switching on and at  $0.01$  second. Find also the flux-linkages at these instants.

(Basic Electricity, Bombay Univ.)

**Solution. (i) Switching instant**

At the instant of switching on,  $i = 0$ , so that  $iR = 0$  hence all applied voltage must drop across the inductance only. Therefore, voltage drop across inductance = **200 V**.

Since at this instant  $i = 0$ , there are no flux-linkages of the coil.

**(ii) When  $t = 0.01$  second**

As time passes, current grows so that the applied voltage is partly dropped across the resistance and partly across the coil. Let us first find  $iR$  drop for which purpose, we need the value of  $i$  at  $t = 0.01$  second.

Now, time period of the circuit is  $\lambda = L/R = 0.1/10 = 0.01$  second. Since the given time happens to be equal to time constant,

$$\therefore i = (200/10) \times 0.632 = 12.64 \text{ A}; iR = 12.64 \times 10 = \mathbf{126.4 \text{ V}}$$

$$\text{Drop across inductance} = \sqrt{200^2 - 126.4^2} = \mathbf{155 \text{ V}}$$

$$\text{Now, } L = N \Phi / i \quad \text{or} \quad N\Phi = Li$$

$$\therefore \text{Flux-linkages } Li = 0.1 \times 12.64 = \mathbf{1.264 \text{ Wb-turns.}}$$

**Example 22.3.** A coil of  $10 \text{ H}$  inductance and  $5 \Omega$  resistance is connected in parallel with a  $20 \Omega$  resistor across a  $100\text{-V}$  d.c. supply which is suddenly disconnected. Find

- the initial rate of change of current after switching.
- the voltage across the  $20 \Omega$  resistor initially and after  $0.3 \text{ s}$ .
- the voltage across the switch contacts at the instant of separation and
- the rate at which the coil is losing stored energy  $0.3$  second after switching.

**Solution. (a)** Since the steady-state current is zero,  $i = I_0 e^{-t/\lambda}$

Now, when  $t = 0$ , current is  $= 100/5 = 20 \text{ A}$ . It means the current flowing through the coil immediately before opening the switch is  $20 \text{ A}$ .

$$\therefore I_0 = 20 \text{ A}$$

Hence, the above equation becomes  $i = 20e^{-t/\lambda}$

$$\text{Now } \lambda = L/R = 10/25 = 1/2.5 \quad \therefore i = 20e^{-2.5t}$$

$$\frac{di}{dt} \bigg|_{t=0} (20 - 2.5e^{2.5t})_{t=0} = -50 \text{ A/s}$$

The negative sign merely shows that the current is decreasing.

(b) After the supply has been disconnected, the current through the  $20\text{-}\Omega$  resistor is  $i$  since it is in series with the coil.

$$\text{Initial p.d. across the } 20\Omega \text{ resistor} = (\text{current at } t=0) \times 20 = 20 \times 20 = \mathbf{400 \text{ V}}$$

$$\text{Current through the resistor after 0.3 second} = 20e^{-2.5 \times 0.3} = 9.45 \text{ A}$$

$$\therefore \text{Voltage across the resistor after 0.3 second} \\ = (\text{current at } t = 0.3 \text{ second}) \times 20 = 9.45 \times 20 = \mathbf{189 \text{ V}}$$

(c) The e.m.f. induced in the coil at break tends to maintain the current through it in the original direction. Hence, the direction of the current through  $20\text{-}\Omega$  resistor is upwards so that the p.d. across the switch contacts will be the *sum* of supply voltage and the voltage across  $20\text{-}\Omega$  resistor.

$$\therefore \text{Initial voltage across switch contacts} = 400 + 100 = \mathbf{500 \text{ V}}$$

(d) The rate of loss of energy = power = induced e.m.f. in coil  $\times$  current (after 0.3 s)

$$L \frac{di}{dt} \bigg|_{t=0.3} \text{ (after 0.3 second)}$$

Now, after 0.3 second,  $i = 9.45 \text{ A}$

$$\text{Value of } di/dt \text{ after 0.3 second} = -20 \times 2.5 \times e^{-0.75} = -23.6 \text{ A/second}$$

$$\therefore \text{Rate of loss of energy} = -10 \times 23.6 \times 9.45 = -2,230 \text{ joule/second}$$

## 22.7. Transients in R-L Circuits (A.C.)

Let a voltage given by  $v = V_m \sin(\omega t + \Psi)$  be *suddenly* applied across an  $R$ - $L$  circuit [Fig. 22.5 (a)] at a time when  $t = 0$ . It means that the voltage is applied when it is passing through the value  $V_m \sin \Psi$ . Since the contact may be closed at any point of the cycle, angle  $\Psi$  may have any value lying between zero and  $2\pi$  radians. The resultant current, as before, is given by

$$i = i_s + i_t$$

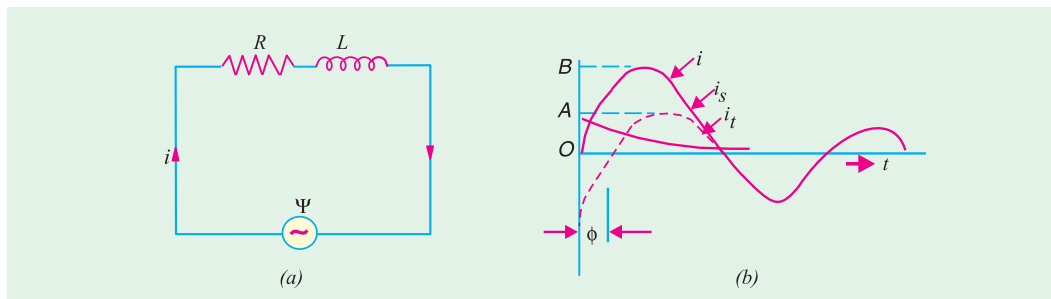


Fig. 22.5

The value of steady-state current is found by the normal circuit theory. The peak steady-state current is given by

$$I_m = \frac{V_m}{\sqrt{R^2 + X_L^2}} = \frac{V_m}{Z}$$

where  $\sqrt{R^2 + X_L^2}$  is the impedance of the circuit. This current lags behind the applied voltage by an angle  $\phi$  such that  $\tan \phi = X_L/R$  or  $\phi = \tan^{-1}(X_L/R)$

Hence, the equation for the instantaneous value of the steady-state current becomes

$$i_s = I_m \sin(\omega t + \Psi - \phi)$$

As before, the transient current is given by

$$i_t = I_0 e^{-t/\lambda} \quad \therefore i = I_m \sin(\omega t + \Psi - \phi) + I_0 e^{-t/\lambda} \quad \dots (i)$$

Now, when  $t = 0$ ,  $i = 0$ , hence putting these values in Eq. (i) above, we get

$$0 = I_m \sin(\Psi - \phi) + I_0 \quad \therefore I_0 = -I_m \sin(\Psi - \phi)$$

Hence Eq. (i) can be written as

$$i = I_m \sin(\omega t + \Psi - \phi) - I_m \sin(\Psi - \phi) e^{-t/\lambda} \quad \dots (ii)$$

From the above, it is seen that the value of  $I_0$  and hence the size of the transient current depends on angle  $\Psi$  i.e. it depends on the instant in the cycle at which the circuit is closed. We will consider the following three cases :

#### Case 1

When  $t = 0$ , let the voltage pass through its zero value and become positive i.e. let  $\Psi = 0$ . In that case, putting this value of  $\Psi$  in Eq. (ii), we get

$$i = I_m \sin(\omega t - \phi) - I_m \sin(-\phi) e^{-t/\lambda} = I_m [\sin(\omega t - \phi) + \sin \phi e^{-t/\lambda}]$$

This is shown in Fig. 22.5 (b). It is seen that maximum instantaneous peak current  $OB$  is larger than the normal peak current  $OA$ .

#### Case 2

Let  $t = 0$  when voltage is passing through its value  $V_m \sin(\phi)$  so that  $\Psi = \phi$  or  $\Psi - \phi = 0$

In that,  $I_0 = 0$ , there is no transient current at the time of switching on (i.e.  $i_t = 0$ ). It corresponds to the contacts closing at the instant when the steady state current itself is zero.

#### Case 3

When  $t = 0$ , let the voltage be passing through

$$V_m \sin \frac{\pi}{2} \quad \text{i.e.} \quad \frac{V_m}{2} \quad \text{and} \quad \frac{V_m}{2} / 2$$

In this case, the transient [as found from Eq. (ii)] would be given by

$$i_t = I_m \sin \frac{\pi}{2} \cdot e^{-t/\lambda} - I_m e^{-t/\lambda}$$

Under these conditions, the transient would have its maximum possible initial value.

**Example 22.4.** A 1.0 H choke has a resistance of 50  $\Omega$ . This choke is supplied with an a.c. voltage given by  $e = 141 \sin 314 t$ . Find the expression for the transient component of the current flowing through the choke after the voltage is suddenly switched on.

(Principles of Elect. Engg-II, Jadavpur Univ.)

**Solution.** The equation of the transient component of the current is (Art. 22.7 Case 1)

$$i_t = I_m \sin \phi e^{-t/\lambda}$$

Here,

$$\lambda = L/R = 1/50 = 0.02 \text{ second}; \mathbf{Z} = 50 + j314 = 318 \angle 80.95^\circ$$

$$I_m = V_m/Z = 141/318 = 0.443 \text{ A}; \quad \sin 80.95^\circ = 0.9875$$

$$\therefore i_t = 0.443 \times 0.9875 e^{-t/0.02} = 0.4376 e^{-t/0.02}$$

**Example 22.5.** A 50-Hz sinusoidal voltage of maximum value of 400 V is applied to a series circuit of resistance  $10 \Omega$  and inductance  $0.1 \text{ H}$ . Find an expression for the value of the current at any instant after the voltage is applied, assuming that voltage is zero at the instant of application. Calculate its value  $0.02$  second after switching on **(Electric Circuit, Punjab Univ. 1990)**

**Solution.** In such cases, as seen from Art. 22.7 (Case 1), the current consists of a steady-state component and a transient component. The equation of the resultant current is

$$i = \underbrace{I_m \sin(\omega t - \phi)}_{\text{steady-state current}} + \underbrace{I_m \sin \phi e^{t/\lambda}}_{\text{transient current}}$$

where  $I_m = V_m / Z$ ;  $\phi = \tan^{-1} (X_L / R)$ ;  $\lambda = L / R$  second

$$R = 10 \Omega; X_L = 314 \times 0.1 = 31.4 \Omega; \mathbf{Z} = 10 + j31.4 = 33 \angle 72.3^\circ$$

$$I_m = 400 / 33 = 12.1 \text{ A}; \quad \phi = 72.3^\circ = 1.26 \text{ rad.}$$

$$\sin \phi = \sin 72.3^\circ = 0.9527; \quad \lambda = 0.1 / 10 = 1 / 100 \text{ second}$$

$$i = 12.1 \{ \sin (314 t - 1.262) + 0.9527 e^{0.01 t} \}$$

Substituting  $t = 0.02$  second, we get

$$i = 12.1 \{ \sin (314 \times 0.02 - 1.262) + 0.9527 e^{-2} \}$$

$$= 12.1 \{ \sin 5.02 + 0.9527 e^{-2} \} = 12.1 \{ \sin 288^\circ + 0.9527 e^{-2} \}$$

$$= 12.1 \{ -\sin 72^\circ + 0.9527 \times 0.1353 \} = 12.1 \{ -0.9511 + 0.1289 \} = \mathbf{-9.95 \text{ A}}$$

**Example 22.6.** An alternating voltage  $v = 400 \sin (314 t + \Psi)$  is suddenly applied across a coil of resistance  $0.2 \Omega$  and inductance  $6.36 \text{ mH}$ . Determine the first peak value of the resultant current when the transient current has maximum value.

**Solution.** Obviously,  $\omega = 314 \text{ rad/s}$

$$X_L = \omega L = 314 \times 6.36 \times 10^{-3} = 2 \Omega$$

Coil impedance  $\mathbf{Z} = 0.2 + j2 \sim 2 \angle 84.3^\circ$

Max. value of steady-state current =  $400/2 = 200 \text{ A}$

As seen from Art. 22.7, the maximum value of transient current will occur when

$\Psi = \phi \pm \pi/2$  where  $\phi = 84.3^\circ$  i.e. the phase angle of the current w.r.t. voltage

$$\therefore \Psi = 84.3^\circ - 90^\circ = -5.7^\circ$$

$$\therefore \text{resultant current, } i = 400 \sin (314 t - 90^\circ) + I_0 e^{-31.4 t}$$

Now, at  $t = 0$ ,  $i = 0$

$$\therefore 0 = 400 \sin (-90^\circ) + I_0$$

$$\therefore I_0 = 400 \text{ A}$$

Hence, the above equation becomes

$$i = 400 \sin (\omega t - 90^\circ) + 400 e^{-31.4 t}$$

The procedure for determining an exact solution for the first peak of the resultant current is first to differentiate the above expression, next to equate the result to zero and then to solve the resulting expression graphically for  $t$ . However, sufficiently accurate result can be obtained by determining the instant at which steady-state current reaches its first positive peak value and then

to add to it the value of the transient current at this instant. The first peak value of steady-state current occurs when

$$(314 t - 90^\circ) = \pi/2 \text{ rad ; i.e. when } t = \pi/314 = 0.01 \text{ second}$$

At this time,  $i_t = 400 e^{-0.314} = 292 \text{ A}$

$\therefore$  resultant current  $i$  at this time =  $200 + 292 = \mathbf{492 \text{ A}}$

### 22.8. Transients in R-C Weeries Circuits (D.C.)

When a d.c. voltage  $V$  is **suddenly** applied to an  $R$ - $C$  series circuit (Fig. 22.6), the voltage  $v_c$  across the capacitor rises from zero value to the steady-state value  $V$ . If  $v_c$  is the voltage across capacitor,  $V_{ct}$  the transient voltage, then

$$v_c = V + v_{ct} \quad \dots (i)$$

The charging current is maximum at the beginning but then is reduced to zero so that there is no steady-state current but a transient one.

Since the transient current is not associated with any applied voltage, hence

$$i_t R + v_{ct} = 0 \quad \dots (ii)$$

Now, capacitor voltage  $v_{ct} = q_t/C$

Hence, Eq. (ii) becomes

$$i_t R + \frac{q_t}{C} = 0$$

or  $R \cdot \frac{di_t}{dt} + \frac{1}{C} \cdot \frac{dq_t}{dt} = 0$  or  $\frac{di_t}{dt} = -\frac{1}{CR} \frac{dq_t}{dt} = -\frac{1}{CR} i_t$  ( $dq_t/dt = i_t$ )

$$\therefore \frac{di_t}{i_t} = -\frac{dt}{CR}; \quad \text{As before } i_t = I_0 e^{-t/CR} = I_0 e^{-t/\lambda}$$

where  $CR = \lambda =$  time constant. The reciprocal  $1/CR$  is known as damping coefficient.

#### (i) Charging Current

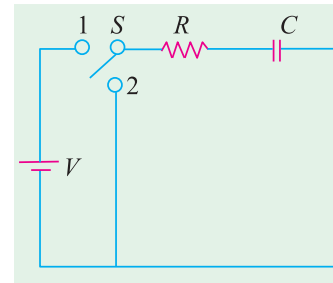


Fig. 22.6

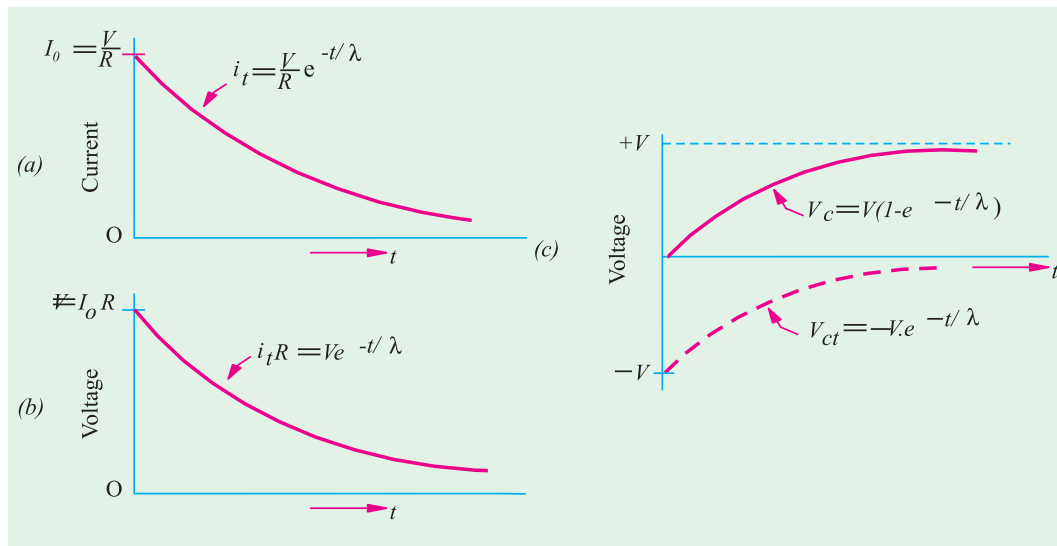


Fig. 22.7

When  $t = 0$ , transient current  $i_t = I_0$ , so that from Eq. (ii)  $v_{ct} = -I_0 R$ . Moreover, when  $t = 0$ ,  $v_c = 0$ , hence from Eq. (i),  $v_{ct} = -V$

Combining these results, we get

$$I_0 = V/R$$

$$\therefore i_t = I_0 e^{-t/\lambda} = \frac{V}{R} e^{-t/\lambda}$$

This is plotted in Fig. 22.7 (a)

The transient voltage across the resistor  $R$  is given by

$$i_t R = \frac{V}{R} e^{-t/\lambda} \times R = V e^{-t/\lambda} \dots \text{Fig. 19.7 (b)}$$

From Eq. (ii) the value of transient voltage across the capacitor is  $v_a = -i_t R$

Hence, Eq. (i) becomes

$$v_c = V - i_t R = V - V e^{-t/\lambda}$$

or

$$v_c = V(1 - e^{-t/\lambda}) \dots \text{(iii)}$$

The voltage across the capacitor  $v_c$  which is the sum of the transient voltage  $v_{ct}$  and steady-state  $V$  has been plotted in Fig. 22.7 (c).

The charge across the capacitor is given by

$$q = v_c = CV(1 - e^{-t/\lambda}) \text{ or } q = Q(1 - e^{-t/\lambda}) \quad (\because Q = CV)$$

### (ii) Discharge Current

When the capacitor has become fully charged so that charging current has ceased, then the  $R$ - $C$  circuit is short-circuited by shifting the switch  $S$  from position 1 to position 2 (Fig. 22.6). On doing so, a transient discharge current will start flowing immediately. If time is reckoned from the instant of short-circuit, then when  $t = 0$ ,  $i_t = I_0$ , hence from Eq. (ii) above  $v_{ct} = -I_0 R$ . Moreover, when  $t = 0$ ,  $v_c = V$ . However, since there is no steady-state voltage across the capacitor, from Eq. (i), we get  $v_c = v_{ct}$ . Combining these results, we get

$$I_0 = -V/R$$

$$\therefore i_t = -\frac{V}{R} e^{-t/\lambda}$$

It is plotted in Fig. 22.8 (a). The negative sign shows that discharge current flows in a direction opposite to that in which the charging current flows. That is why the curve has been drawn below the  $X$ -axis. It may be noted that the only voltage in the circuit is  $v_{ct}$  which equals  $-i_t R$ .

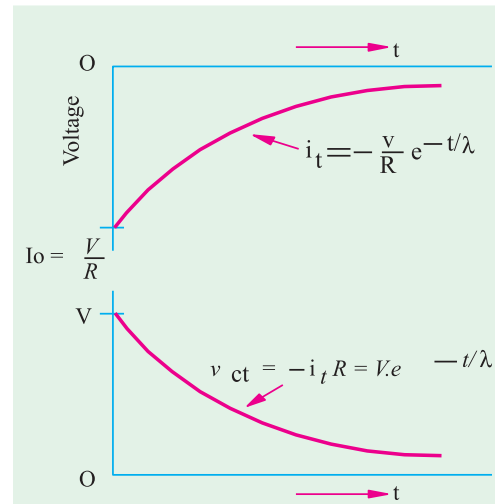


Fig. 22.8

**Example 22.7.** In a simple saw-tooth generator circuit with the thyatron switches on at 150 V and switches off at 10 V. If this circuit is supplied with 250 V d.c. source; find the time period of saw-tooth wave. The resistance and capacitance have the values of 10 k $\Omega$  and 1  $\mu$ F respectively.

(Principles of Elect. Engg-II, Jadavpur Univ.)

**Solution.** With reference to Fig. 22.9, let

$V =$  applied voltage

$v_{c1} =$  switching-off voltage of the thyatron = 10 V

$v_{c2} =$  switching-on voltage of the thyatron = 150 V

Now,  $v = V(1 - e^{-t/\lambda})$

$$\therefore v_{c1} = V(1 - e^{-t_1/\lambda}) \quad \dots (i)$$

$$V(1 - e^{-(t_1+T)/\lambda}) = v_{c2} \quad \dots (ii)$$

where  $T$  is the time-period of the saw-tooth wave. From Eq. (i) and (ii), we get

$$T = \lambda \log_e (V - v_{c1}) / (V - v_{c2})$$

Now  $\lambda = CR = 10^4 \times 10^{-6} = 10^{-2}$  second

$$V - v_{c1} = 250 - 10 = 240 \text{ V}; V - v_{c2} = 250 - 150 = 100 \text{ V}$$

$$\therefore \frac{V - v_{c1}}{V - v_{c2}} = \frac{240}{100} = 2.4 \quad \therefore T = 10^{-2} \log_c 2.4 = \mathbf{0.00875 \text{ second}}$$

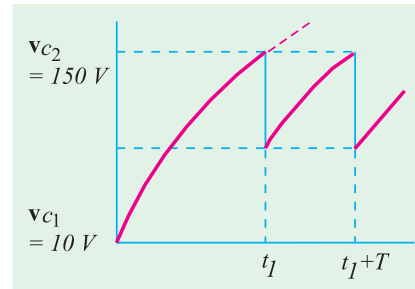


Fig. 22.9

**Example 22.8.** A simple neon-tube time base for a cathode-ray oscillography employs a  $300 \text{ k}\Omega$  and a  $0.016 \mu\text{F}$  capacitor. The striking and extinction voltages of the neon-tube are 170 V and 140 V respectively. Calculate the frequency of the time base if the supply voltage is 200 V.

**Solution.** The voltage across the capacitor increases according to the equation

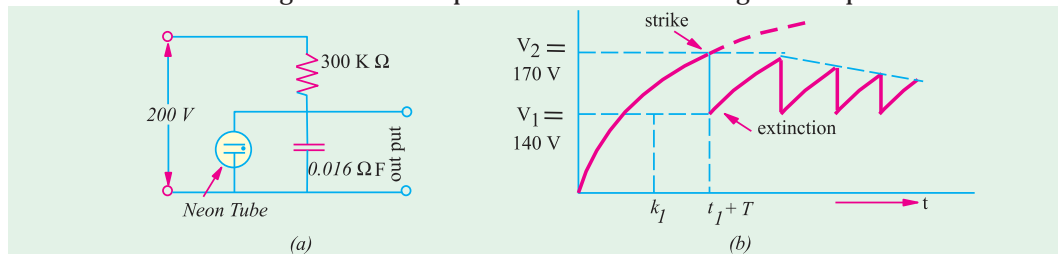


Fig. 22.10

$$v_c = V(1 - e^{-t/CR})$$

It is shown in Fig. 22.10 (b)

$$\therefore v_{c1} = V(1 - e^{-t_1/CR}) \quad \dots (i)$$

$$v_{c2} = V(1 - e^{-(t_1+T)/CR}) \quad \dots (ii)$$

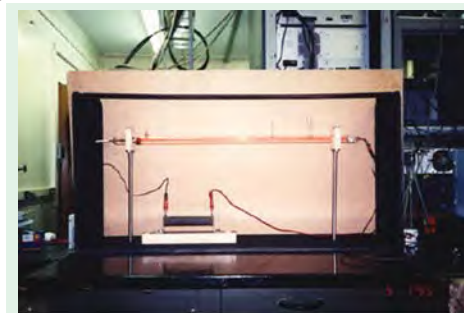
From eq. (i) and (ii), we get

$$T = CR \log_e (V - v_{c1}) / (V - v_{c2})$$

Now

$$\lambda = CR = 0.016 \times 10^{-6} \times 300 \times 10^3 = 4.8 \times 10^{-3} \text{ second}$$

$$V - v_{c1} = 200 - 140 = 60 \text{ V and } V - v_{c2} = 200 - 170 = 30 \text{ V}$$



A view of the neon tube experiment

$$\therefore T = 4.8 \times 10^{-3} \log 60/30 = 1/300 \text{ second}$$

$$\therefore \text{Frequency of time base} = 1/T = \mathbf{300 \text{ Hz}}$$

### 22.9. Transients in R-C Series Circuits (A.C.)

In this case, the resultant currents can be determined in the same way as for an  $R$ - $L$  circuit (Art. 22.7). It is given by

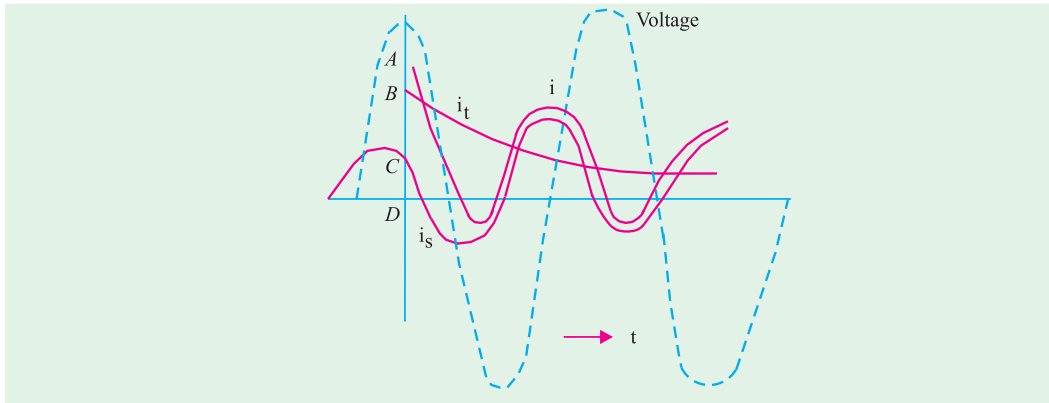


Fig. 22.11

$$i = i_s + i_t = I_m \sin(\omega t + \psi + \phi) + I_0 e^{-t/\lambda} \quad \text{where} \quad I_m = V_m / \sqrt{R^2 + X_C^2}$$

$$\text{and} \quad v = V_m \sin(\omega t + \psi)$$

The value of  $I_0$  as found from initial known conditions ( $t=0, i=0$ ) is given by  $I_0 = -I_m \sin(\psi + \phi)$

Hence, the resultant current becomes

$$i = I_m \sin(\omega t + \psi + \phi) - I_m \sin(\psi + \phi) e^{-t/\lambda}$$

As shown in Fig. 22.11, the resultant current at the moment of switch closing is  $OA$  and is made up of steady-state current  $OC$  and transient current  $OB$ .

### 22.10. Double Energy Transients

In an  $R$ - $L$ - $C$  circuit, both electromagnetic and electrostatic energies are involved, hence any sudden change in the conditions of the circuit involves the redistribution of these two forms of energy. The transient currents produced due to this redistribution are known as double-energy transients. The transient current produced may be unidirectional or a decaying oscillatory current.

In an  $R$ - $L$ - $C$  circuit, the transient voltages across the three circuit parameters are  $i_t R$ ,  $L di_t / dt$  and  $q_t / C$ . Hence, the equation of the transient voltage is

$$i_t R + L \frac{di_t}{dt} + \frac{q_t}{C} = 0 \quad \dots (i)$$

Differentiating the above equation and putting  $i_t$  for  $dq_t / dt$ , we get

$$\frac{d^2 i_t}{dt^2} + \frac{R}{L} \frac{di_t}{dt} + \frac{1}{LC} i_t = 0 \quad \dots (ii)$$

This is a linear differential equation of the second order with constant coefficient like 2 (i) given in Art. No. 22.3. Its solution is given by



$$i_t = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} \quad \dots \text{ (iii)}$$

where  $k_1$  and  $k_2$  are constants whose values are found from the boundary conditions. The values of  $\lambda_1$  and  $\lambda_2$  are given by

$$\lambda_1 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \quad \text{and} \quad \lambda_2 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Depending on the value of  $\lambda_1$  and  $\lambda_2$ , four different conditions of the circuit are distinguishable. We will now examine these four conditions in the case of an  $R$ - $L$ - $C$  circuit.

**Case 1. Loss-free Circuit,  $R = 0$  i.e. Undamped**

$$\text{In this case, } \lambda_1 = \sqrt{-\frac{1}{LC}} = j - 1\sqrt{LC} = -j\omega \quad \text{and} \quad \lambda_2 = -\sqrt{-\frac{1}{LC}} = -j - 1\sqrt{LC} = -j\omega$$

Hence, Eq. (iii) given above becomes

$$\begin{aligned} i_t &= k_1 e^{j\omega t} + k_2 e^{-j\omega t} = k_1 (\cos \omega t + j \sin \omega t) + k_2 (\cos \omega t - j \sin \omega t) \\ &= (k_1 + k_2) \cos \omega t + j(k_1 - k_2) \sin \omega t \end{aligned}$$

$$\text{or} \quad i_t = A \cos \omega t + B \sin \omega t \quad \dots \text{ (iv)}$$

where  $A = k_1 + k_2$  and  $B = j(k_1 - k_2)$

Eq. (iv) can be still further simplified to

$$i_t = I_m \sin(\omega t + \phi) \quad \dots \text{ (v)}$$

where  $I_m = \sqrt{A^2 + B^2}$  and  $\phi = \tan^{-1}(A/B)$

As seen from Eq. (v), the transient current in this case is sinusoidal wave of constant peak value and frequency  $f = 1/2\pi\sqrt{LC}$  as shown in Fig. 22.12 (a). The values of two constant terms  $I_m$  and  $\phi$  can be determined from any two known initial circuit conditions which are (i) the initial current in the inductance and (ii) the initial voltage across the capacitor.

**Case 2. Low-loss Circuit:  $\frac{R^2}{4L^2} < \frac{1}{LC}$  i.e. Under-damped**

In this case,  $\lambda_1$  and  $\lambda_2$  would be conjugate complex numbers because the term under the square root sign in each case would be negative.

$$\therefore \lambda_1 = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$\text{If } a = \frac{R}{2L} \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ then } \lambda_1 = -a + j\omega \text{ and } \lambda_2 = -a - j\omega$$

Putting these values in equation (v), we get

$$i_t = k_1 e^{(-a+j\omega)t} + k_2 e^{(-a-j\omega)t} = e^{-at} (k_1 e^{j\omega t} + k_2 e^{-j\omega t})$$

This equation can be reduced, as before, to the form

$$i_t = I_m e^{-at} \sin(\omega t + \phi) \quad \dots \text{ (iv)}$$

where  $I_m$  and  $\phi$  are constants as before. Equation (iv) represents damped transient oscillatory current as shown in Fig. 22.12 (b).

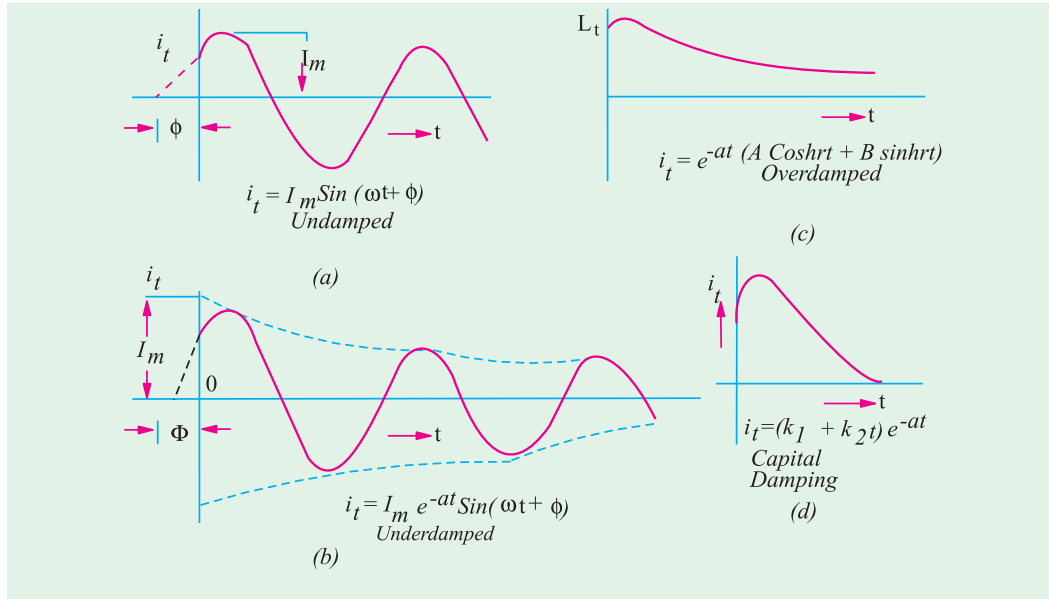


Fig. 22.12

The exponential term  $e^{-at}$  which accounts for the decay of oscillations, is called the decay or damping factor or merely **decrement**. It makes each current peak a definite fraction less than that preceding it. The logarithm to the Naperian base 'e' of the ratio of peaks one cycle apart in time is  $a/f = R/2fL$  and is referred to as **logarithmic decrement**. The frequency of damped oscillations is given by

$$f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ and is called the natural frequency of the circuit}$$

$$\text{If } \frac{R^2}{4L^2} < \frac{1}{LC}, \text{ then } f = \frac{1}{2\pi\sqrt{LC}}$$

**Case 3. High-loss Circuit:**  $\frac{R^2}{4L^2} > \frac{1}{LC}$  i.e. **overdamped**

In this case,  $\lambda_1$  and  $\lambda_2$  will be pure numbers.

$$\lambda_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -a + \gamma \text{ and } \lambda_2 = -a - \gamma$$

$$\therefore i_t = k_1 e^{(-a+\gamma)t} + k_2 e^{(-a-\gamma)t} = e^{-at} (k_1 e^{\gamma t} + k_2 e^{-\gamma t})$$

$$\text{Now } e^{\gamma t} = \sinh \gamma t + \cosh \gamma t$$

$$\text{and } e^{-\gamma t} = \cosh \gamma t - \sinh \gamma t$$

$$\therefore i_t = e^{-at} \{(k_1 + k_2) \cosh \gamma t + (k_1 - k_2) \sinh \gamma t\}$$

$$\text{or } i_t = e^{-at} (A \cosh \gamma t + B \sinh \gamma t)$$

A typical curve of this equation is shown in Fig. 22.12(c)

**Case 4.**  $\frac{R^2}{4L^2} = \frac{1}{LC}$  *i.e. Critical Damping*

In this case,  $\lambda_1 = \lambda_2 = -\frac{R}{2L}$

Hence, equation (iii) is reduced to

$$i_t = (k_1 + k_2 t)e^{-\frac{R}{2L}t} \text{ or } i_t = (k_1 + k_2 t)e^{-at}$$

It is a case of critical damping because current is reduced to almost zero in the shortest possible time. The above equation has been plotted in Fig. 22.12 (d).

Hence, we can summarize as follows:

1. Transient current is an undamped sine wave if  $R = 0$
2. Transient current is non-oscillatory if  $R < 2\sqrt{L/C}$
3. Transient current is non-oscillatory if  $R \geq 2\sqrt{L/C}$
4. Critical damping occurs if  $R = 2\sqrt{L/C}$

**Example 22.9.** A  $5\text{-}\mu\text{F}$  capacitor is discharged suddenly through a coil having an inductance of  $2\text{H}$  and a resistance of  $200\ \Omega$ . The capacitor is initially charged to a voltage of  $10\text{ V}$ . Find

(a) an expression for the current

(b) the additional resistance required to give critical damping.

**Solution.** Since there is no battery or generator in the circuit (Fig. 22.13), the steady-state current must be zero. It means that resultant current is simply the transient current.

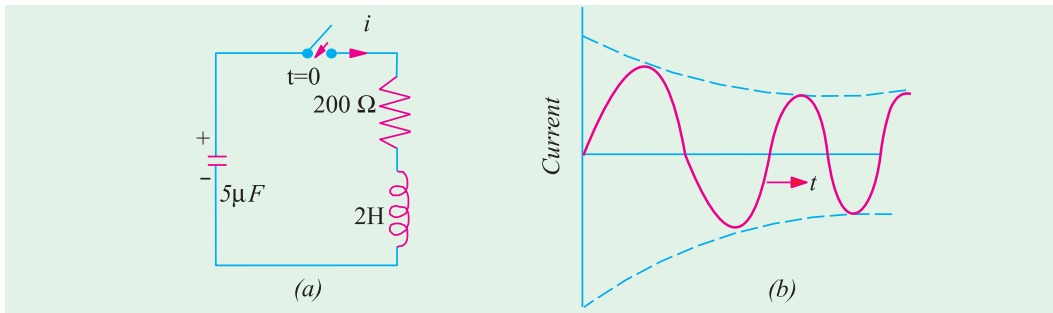


Fig. 22.13

$$\begin{aligned} \text{Value of } 2\sqrt{L/C} &= 2\sqrt{2/5 \times 10^{-6}} \\ &= 1265\ \Omega \end{aligned}$$

Since  $R < 2\sqrt{L/C}$ , the circuit is originally oscillatory.

(a) the expression for the transient current, therefore, is

$$i_t = I_m e^{-at} \sin(\omega t + \phi)$$

where  $a = R/2L = 200/2 \times 2 = 50$

$$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{100,000 - 2500} = 312.3 \text{ rad/s}$$

$$\therefore i_t = I_m e^{-50t} \sin(312.3t + \phi) = i \quad \dots (i)$$

Two initial conditions are known from which  $I_m$  and  $\phi$  can be found (a) at  $t = 0$ ;  $i = 0$  and (b) at  $t = 0$ ;  $v_c = 10$  V. Applying condition (a) to Eq. (i), we get

$$0 = I_m \sin \phi, \text{ hence } \phi = 0 \quad \therefore i = I_m e^{-50t} \sin 312.3t \quad \dots (ii)$$

Now, at  $t = 0$ , the voltage across the inductance must be 10 V because the current in the resistance is zero.

$$\text{i.e. } (L di / dt)_{t=0} = 10 \text{ V} \therefore (di / dt)_{t=0} = 10 / L = 5 \text{ A/s} \quad \dots (iii)$$

Now, from equation (ii), we have

$$\frac{di}{dt} = -50 I_m e^{-50t} \sin 312.3t + 312.3 I_m e^{-50t} \cos 312.3t \quad \dots (iv)$$

Putting  $t = 0$ , it becomes  $(di / dt)_{t=0} = 312.3 I_m$

From equation (iii), we have

$$312.3 I_m = 5 \quad \therefore I_m = 5 / 312.3 = 0.016 \text{ A}$$

Hence, the general expression for the current becomes

$$i = 0.016 e^{-50t} \sin 312.3t$$

It is roughly plotted (the first few cycles only) in Fig. 22.13 (b).

(b) Critical damping is achieved when  $R = 2\sqrt{L/C}$

$$\therefore R = 2\sqrt{2/5 \times 10^{-6}} = 1265 \Omega$$

$$\therefore \text{Additional resistance reqd.} = 1265 - 200 = 1065 \Omega$$

**Example 22.10.** A damped oscillation has the equation  $i = 50e^{-10t} \sin 628t$ . Find the number of oscillations which occurs before the amplitude of the oscillations decays to 1/10th of its undamped value.

**Solution.** Undamped amplitude = 50 A

$$1/10\text{th amplitude} = (1/10) \times 50 = 5 \text{ A}$$

Let the time required for this decay be  $t$ . Now, the decay of the peak of the oscillations is given by the term  $50e^{-10t}$

$$\therefore 5 = 50e_1^{-10t} \therefore e_1^{10t_1} = 10 \text{ or } 10t_1 = \log h^{10} = 2.3 \log_{10}^{10} = 2.3$$

$$\therefore t_1 = 0.23 \text{ second}$$

$$\text{Frequency of oscillations} = 628 / 2\pi = 100 \text{ Hz.}$$

Hence, the number of oscillations which occur before the amplitude falls to 1/10th of its undamped value is =  $0.23 \times 100 = 23$

**Example 22.11.** If, in Fig. 22.14, a break occurs at a point marked X, what would be the voltage across the break? It may be assumed that prior to the break, steady conditions existed in the circuit.

**Solution.** Steady-state current through the inductance =  $120/60 = 2 \text{ A}$

Energy stored in the inductor prior to the break

$$= \frac{1}{2} LI^2 = \frac{1}{2} \times 12 \times 10^{-3} \times 4 = 24 \times 10^{-3} \text{ J}$$

Energy initially stored in the capacitor

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times 10^{-8} \times 120^2 = 72 \times 10^{-6} \text{ J} = 0\text{-practically}$$

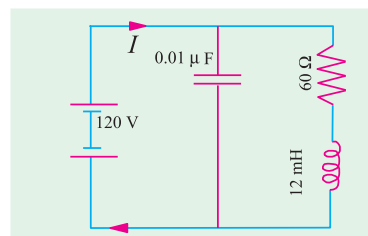


Fig. 22.14

When the break occurs, the energy stored in the inductor is transferred to the capacitor. If loss of energy during first transfer is neglected, then maximum energy stored in the capacitor is

$$= 20 \times 10^{-3} \text{ J} \therefore \frac{1}{2} CV_m^2 = 24 \times 10^{-3}$$

$$\therefore V_m = \sqrt{2 \times 24 \times 10^{-3} \times 10^8} = 2,190 \text{ V}$$

Maximum voltage across the break is = 2190 + 120 = 2310 V

The voltage would be oscillatory because the energy alternates between the inductor and capacitor.

Frequency of voltage oscillation is

$$f = 1/2\pi\sqrt{LC} = 10^5 / 2\pi \times \sqrt{1.2} = 14,530 \text{ Hz}$$

Decay or damping factor =  $e^{-at}$  Art. 22.10, Case 2

Here,  $a = R/2L = 60/2 \times 12 \times 10^{-3} = 2500$   $\therefore$  damping factor =  $e^{-2500t}$

Hence, voltage across the break is

$$= 120 + 2190 e^{-2500t} \sin 2\pi \times 14,500t = \mathbf{120 + 2190 e^{-2500t} \sin 91,290t}$$

### Tutorial Problem No. 22.1

1. Deduce an expression for the growth of current in an inductive circuit.

A 15-H inductance coil of 10  $\Omega$  resistance is suddenly connected to a 20 V d.c. supply. Calculate:

- the initial rate of change of current
- the current after 2 second
- the rate of change of current after 2 second
- the energy stored in the magnetic field in this time
- the energy lost as heat in this time
- the time constant.

**[(a) 1.33 A/s (b) 1.47 A (c) 0.352 A/s (d) 16.3 joules (e) 19.5 joules (f) 1.5 s]**

2. A circuit consisting of a 20  $\Omega$  resistor in series with a 0.2 H inductor is supplied from 200 V (r.m.s.) 50 Hz a.c. mains. Deduce equations showing how the current varies with time if the supply is suddenly switched on (a) at the instant when the voltage is zero (b) at the instant when the voltage is a maximum.

**[(a)  $4.11 e^{-100t} + 4.32 \sin(314t - 70^\circ 16')$  A (b)  $-1.245 e^{-100t} + 4.32 \cos(314t - 72^\circ 16')$  A]**

3. A circuit consisting of a 20  $\Omega$  resistor, 20 mH inductor and a 100  $\mu\text{F}$  capacitor in series is connected to a 200 V, d.c. supply. The capacitor is initially uncharged. Determine the equation relating the instantaneous current to the time and find the maximum instantaneous current.

**[( $20 e^{-500t} \sin 500t$ ) A; 6.44 A]**

4. Find an expression for the value of current at any instant after a sinusoidal voltage of amplitude 600 V at 50 Hz is applied to a series circuit of resistance 10  $\Omega$  and inductance 0.1 H, assuming that voltage is zero at the instant of switching. Also, find the value of transient current at  $t = 0.02$  second.

**[- 15.14 A; 2.17 A] (*Electric Circuits and Fields, Gujarat Univ.*)**

5. A 40  $\Omega$  resistor and a 50  $\mu\text{F}$  capacitor are connected in series and supplied with an alternating voltage  $v = 283 \sin 314t$ . The supply is switched on at the instant when the voltage is zero. Determine the expression for the instantaneous current at time  $t$ .

**[-  $3.18 e^{-500t} + 3.76 \sin(314t + 57^\circ 50')$ ]**

6. A d.c. voltage of 100 V is suddenly applied to a circuit consisting of a 100  $\Omega$  resistor, a 0.1 H inductor and a 100  $\mu\text{F}$  capacitor in series. The capacitor is initially uncharged. Obtain the equation which shows how the capacitor voltage varies with time.

**[ $100 - 115.3 e^{-500t} \sin(866t + \pi/3 \text{ V})$ ]**

7. The voltage  $v = 200 \sin 314t$  is suddenly applied at  $t = 0$  to a circuit consisting of a  $10 \Omega$  resistor in series with a  $0.1 \text{ H}$  inductor. Deduce an equation showing how the current varies with time.

$$[5.78 e^{-100t} + 6.06 \sin(314t - 72^\circ 20')]$$

8. A  $20 \Omega$  resistor, a  $0.01 \text{ H}$  inductor and a  $100 \mu\text{F}$  capacitor are connected in series. A d.c. voltage of  $100 \text{ V}$  is suddenly applied to the circuit. Obtain the equation showing how the current through the circuit varies with time. Find the maximum current and the time at which it occurs.

$$[10^4 e^{-100t}; 3.67 \text{ A}; 0.001 \text{ second}]$$

9. A  $4 \mu\text{F}$  capacitor is initially charged to  $300 \text{ V}$ . It is discharged through a  $100 \text{ mH}$  inductance and a resistor in series:

- find the frequency of the discharge if the resistance is zero.
- how many cycles at the above frequency will occur before the discharge oscillation decays to  $1/10$  of its initial value if the resistance is  $1 \Omega$ .
- find the value of the resistance which would just prevent oscillations.

$$[(a) 796 \text{ Hz} (b) 36.6 (c) 100 \Omega]$$

### OBJECTIVE TESTS – 22

- Transient disturbance is produced in a circuit whenever
  - it is suddenly connected or disconnected from the supply
  - it is shorted
  - its applied voltage is changed suddenly
  - all of the above.
- There are no transients in pure resistive circuits because they
  - offer high resistance
  - obey Ohm's law
  - have no stored energy
  - are linear circuits.
- Transient currents in electrical circuit are associated with
  - inductors
  - capacitors
  - resistors
  - both (a) and (b).
- The transients which are produced due to sudden but energetic changes from one steady state of a circuit to another are called ..... transients.
 

(a) initiation	(b) transition
(c) relaxation	(d) subsidence
- In an  $R-L$  circuit connected to an alternating sinusoidal voltage, size of transient current primarily depends on
  - the instant in the voltage cycle at which circuit is closed
  - the peak value of steady-state current
  - the circuit impedance
  - the voltage frequency.
- Double-energy transients are produced in circuits consisting of
  - two or more resistors
  - resistance and inductance
  - resistance and capacitance
  - resistance, inductance and capacitance.
- The transient current in a loss-free  $L-C$  circuit when excited from an ac source is  $a/$  an ..... sine wave.
 

(a) over damped	(b) undamped
(c) under damped	(d) critically damped.
- Transient current in an  $R-L-C$  circuit is oscillatory when
 

(a) $R = 0$	(b) $R > 2\sqrt{L/C}$
(c) $R < 2\sqrt{L/C}$	(d) $R = 2\sqrt{L/C}$

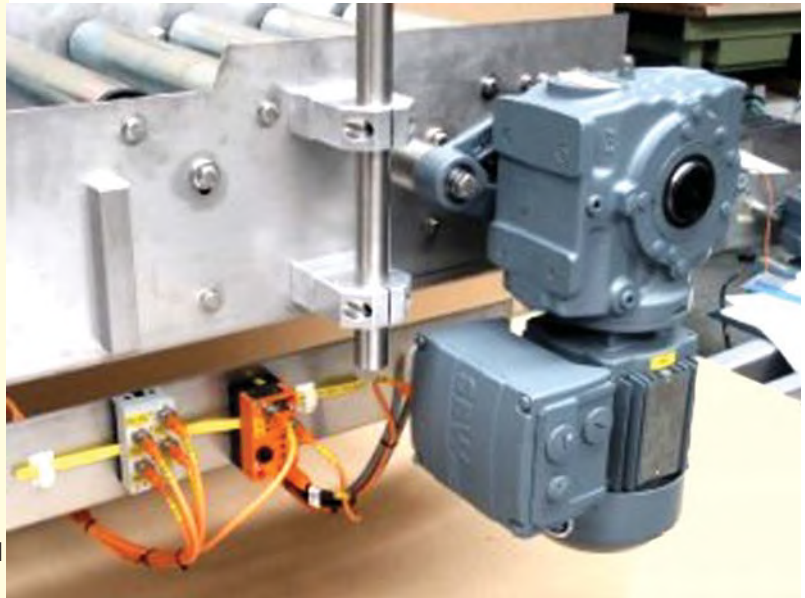
### ANSWERS

1. (d) 2. (c) 3. (d) 4. (b) 5. (a) 6. (d) 7. (b) 8. (c)

## CHAPTER 23

**Learning Objectives**

- Introduction
- The Positive-sequence Components
- The Negative-sequence Components
- The Zero-sequence Components
- Graphical Composition of Sequence Vectors
- Evaluation of  $V_{A1}$  or  $V_1$
- Evaluation of  $V_{A2}$  or  $V_2$
- Evaluation  $V_{A0}$  or  $V_0$
- Zero Sequence Components of Current and Voltage
- Unbalanced Star Load form Unbalanced Three-phase Three-Wire System
- Unbalanced Star Load Supplied from Balanced Three-phase Three-wire System
- Measurement of Symmetrical Components of Circuits
- Measurement of Positive and Negative-sequence Voltages
- Measurement of Zero-sequence Component of Voltage

**SYMMETRICAL COMPONENTS**

Any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three balanced systems of vectors which are called its 'symmetrical components'

### 23.1. Introduction

The method of symmetrical components was first proposed by C.L. Fortescue and has been found very useful in solving unbalanced polyphase circuits, for analytical determination of the

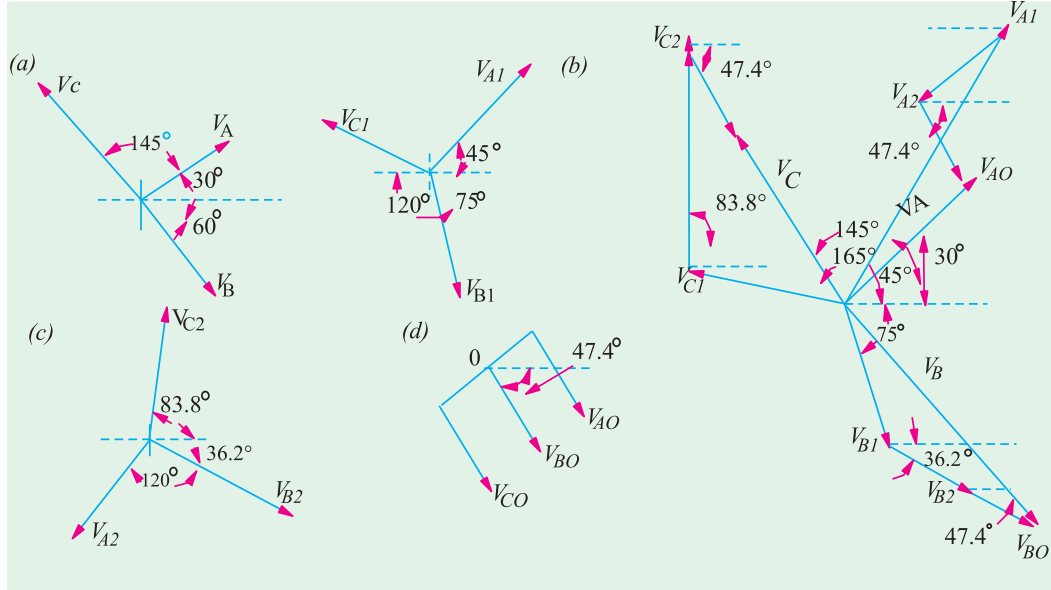


Fig. 23.1

Fig. 23.2

performance of polyphase electrical machinery when operated from a system of unbalanced voltages and for calculation of currents resulting from unbalanced faults. According to Fortescue's theorem, any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three **balanced** systems of vectors which are called its '**symmetrical components**'. In Fig. 23.1 (a) is shown a set of three unbalanced voltage vectors  $V_A$ ,  $V_B$  and  $V_C$  having phase sequence  $A \rightarrow B \rightarrow C$ . These can be regarded as made up of the following symmetrical components :-

(i) A balanced system of 3-phase vectors  $V_{A1}$ ,  $V_{B1}$  and  $V_{C1}$  having the phase sequence  $A \rightarrow B \rightarrow C$  as the original set of three unbalanced vectors. These vectors constitute the positive-sequence components [Fig. 23.1 (b)].

(ii) A balanced system of 3-phase vectors  $V_{A2}$ ,  $V_{B2}$  and  $V_{C2}$  having phase sequence  $A \rightarrow C \rightarrow B$  which is opposite to that of the original unbalanced vectors. These vectors constitute the negative-sequence components [Fig. 23.1 (c)].

(iii) A system of three vectors  $V_{A0}$ ,  $V_{B0}$  and  $V_{C0}$  which are equal in magnitude and are in phase with each other *i.e.*  $V_{A0} = V_{B0} = V_{C0}$ . These three co-phasal vectors form a uniphase system and are known as zero-sequence components [Fig. 23.1(d).]

Hence, it means that an unbalanced 3-phase system of voltages or current can be regarded as due to the superposition of two symmetrical 3-phase systems having opposite phase sequences and a system of zero phase sequence *i.e.* ordinary single-phase current or voltage system. In Fig. 23.2, each of the original vectors has been



3-phase monitor continuously monitors 3-phase wave lines for abnormal conditions



reconstructed by the vector addition of its positive-sequence, negative - sequence and zero-sequence components. It is seen that

$$V_A = \overset{-ve}{V_{A1}} + \overset{-ve}{V_{A2}} + \overset{zero}{V_{A0}} \quad \dots (i)$$

$$V_B = V_{B1} + V_{B2} + V_{B0} \quad \dots (ii)$$

$$V_C = V_{C1} + V_{C2} + V_{C0} \quad \dots (iii)$$

### 23.2. The Positive - sequence Components

As seen from above, the positive-sequence components have been designated as  $V_{A1}$ ,  $V_{B1}$  and  $V_{C1}$ . The subscript 1 is meant to indicate that the vector belongs to the positive-sequence system. The letter refers to the original vector of which the positive-sequence vector is a component part.

These positive-sequence vectors are completely determined when the magnitude and phase of any one of these is known. Usually, these vectors are related to each other with the help of the operator  $a$  (for details, please refer to Art. 12.11). As seen from Fig. 23.1 (b).

$$\mathbf{V}_{A1} = V_{A1}; \mathbf{V}_{B1} = a^2 V_{A1} = V_{A1} \angle -120^\circ; \mathbf{V}_{C1} = a V_{A1} = V_{A1} \angle 120^\circ$$

### 23.3. The Negative - sequence Components

This system has a phase sequence of  $A \rightarrow C \rightarrow B$ . Since this system is also balanced, it is completely determined when the magnitude and phase of one of the vectors becomes known. The suffix 2 indicates that the vector belongs to the negative-sequence system. Obviously, as seen from Fig. From 23.1 (c).

$$\mathbf{V}_{A2} = V_{A2}; \mathbf{V}_{B2} = a V_{A2} = V_{A2} \angle 120^\circ; \mathbf{V}_{C2} = a^2 V_{A2} = V_{A2} \angle -120^\circ$$

### 23.4. The Zero - sequence Components

These three vectors are equal in magnitude and phase and hence form what is known as uniphase system. They are designated as  $V_{A0}$ ,  $V_{B0}$  and  $V_{C0}$ . Since these are identical in magnitude

$$\therefore V_{A0} = V_{B0} = V_{C0}$$

### 23.5. Graphical Composition of Sequence Vectors

Fig. 23.2 Shows how the original vector  $V_A$  has been obtained by the addition of  $V_{A1}$ ,  $V_{A2}$  and  $V_{A0}$ . The same applies to other vectors  $V_B$  and  $V_C$ .

For simplicity, let us write  $V_{A1}$  as  $V_1$ ,  $V_{A2}$  as  $V_2$  and  $V_{A0}$  as  $V_0$ . Then

$$\mathbf{V}_A = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_0 \quad \dots (iv)$$

$$\mathbf{V}_B = a^2 \mathbf{V}_1 + a \mathbf{V}_2 + \mathbf{V}_0 \quad \dots (v)$$

$$\mathbf{V}_C = a \mathbf{V}_1 + a^2 \mathbf{V}_2 + \mathbf{V}_0 \quad \dots (vi)$$

### 23.6. Evaluation of $V_{A1}$ or $V_1$

The procedure for evaluating  $V_1$  is as follows :

Multiplying (v) by  $a$  and (vi) by  $a^2$ , we get

$$a \mathbf{V}_B = a^3 \mathbf{V}_1 + a^2 \mathbf{V}_2 + a \mathbf{V}_0; \quad a^2 \mathbf{V}_C = a^3 \mathbf{V}_1 + a^4 \mathbf{V}_2 + a^2 \mathbf{V}_0$$

Now  $a^3 = 1$  and  $a^4 = a$ , hence

$$a\mathbf{V}_B = \mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_0 \quad \dots \text{(vii)}$$

$$a^2\mathbf{V}_C = \mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_0 \quad \dots \text{(viii)}$$

Adding (iv), (vii) and (viii), we get

$$\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C = 3\mathbf{V}_1 + \mathbf{V}_2(1 + a + a^2) + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_1$$

$$\therefore \mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B \angle 120^\circ + \mathbf{V}_C \angle -210^\circ)$$

$$\frac{1}{3} V_A \quad V_B \quad \frac{1}{2} j \frac{\sqrt{3}}{2} \quad V_C \quad \frac{1}{2} j \frac{\sqrt{3}}{2}$$

This shows that, geometrically speaking,  $V_1$  is a vector one-third as large as the vector obtained by the vector addition of the three original vectors  $V_A$ ,  $V_B \angle 120^\circ$  and  $V_C \angle -120^\circ$ .

### 23.7. Evaluation of $V_{A2}$ or $V_2$

Multiplying (vi) by  $a$  and (v) by  $a^2$  and adding them to (iv) we get

$$a\mathbf{V}_C = a^2\mathbf{V}_1 + a^3\mathbf{V}_2 + a\mathbf{V}_0; a^2\mathbf{V}_B = a^4\mathbf{V}_1 + a^3\mathbf{V}_2 + a^2\mathbf{V}_0$$

$$\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + 3\mathbf{V}_2 + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_2 \quad \text{Now, } 1 + a + a^2 = 0$$

$$\therefore \mathbf{V}_2 = \frac{1}{3}(\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B \angle -120^\circ + \mathbf{V}_C \angle 120^\circ)$$

$$= \frac{1}{3} \left[ \mathbf{V}_A + \mathbf{V}_B \left( -\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) + \mathbf{V}_C \left( -\frac{1}{2} + j \frac{\sqrt{3}}{2} \right) \right]$$

### 23.8. Evaluation of $V_{A0}$ or $V_0$

Adding (iv), (v) and (vi), we get  $\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + \mathbf{V}_2(1 + a + a^2) + 3\mathbf{V}_0 = 3\mathbf{V}_0$

$$\therefore \mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

It shows that  $V_0$  is simply a vector one third as large as the vector obtained by adding the original vectors  $V_A$ ,  $V_B$  and  $V_C$ .

To summarize the above results, we have

$$\text{(i) } \mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C) \quad \text{(ii) } \mathbf{V}_2 = \frac{1}{3}(\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C)$$

$$\text{(iii) } \mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

**Note.** An unbalanced system of 3-phase currents can also be likewise resolved into its symmetrical components. Hence

$$\mathbf{I}_A = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_B = a^2\mathbf{I}_1 + a\mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_C = a\mathbf{I}_1 + a^2\mathbf{I}_2 + \mathbf{I}_0$$

$$\text{Also, as before } \mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_A + a\mathbf{I}_B + a^2\mathbf{I}_C); \mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_A + a^2\mathbf{I}_B + a\mathbf{I}_C); \mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C) \quad \dots \text{(ix)}$$

It shows that  $I_0$  is one-third of the neutral or earth-return current and is zero for an unearthed 3-wire system. It is seen from (ix) above that  $I_0$  is zero if the vector sum of the original current vectors is zero. This fact can be used with advantage in making numerical calculations because the original system of vectors can then be reduced to two balanced 3-phase systems having opposite phase sequences.

**Example 23.1.** Find out the positive, negative and zero-phase sequence components of the following set of three unbalanced voltage vectors:

$$V_A = 10\angle 30^\circ; V_B = 30\angle -60^\circ; V_C = 15\angle 145^\circ$$

Indicate on an approximate diagram how the original vectors and their different sequence components are located. **(Principles of Elect. Engg. – I, Jadavpur Univ.)**

**Solution. (i) Positive-sequence vectors**

As seen from Art. 23.6

$$\begin{aligned} \mathbf{V}_1 &= \frac{1}{3}(\mathbf{V}_A \ a\mathbf{V}_B \ a^2\mathbf{V}_C) = \frac{1}{3}(10 \ 30 \ a.30 \ 60 \ a^2.15 \ 145) \\ &= \frac{1}{3}(10\angle 30^\circ + 30\angle 60^\circ + 15\angle 25^\circ) = 12.42 + j12.43 = 17.6\angle 45^\circ \end{aligned}$$

$$\therefore \mathbf{V}_{A1} \ 17.6 \ 45^\circ; \mathbf{V}_{B1} \ 17.6 \ 45^\circ \quad 120^\circ \ 17.6 \ 75^\circ$$

$$\mathbf{V}_{C1} \ 17.6 \ 45^\circ \quad 120^\circ \ 17.6 \ 165^\circ$$

These are shown in Fig. 23.1 (b)

**(ii) Negative-sequence vectors**

As seen from Art. 23.7,

$$\begin{aligned} \mathbf{V}_2 &= \frac{1}{3}(\mathbf{V}_A \ a^2\mathbf{V}_B \ a\mathbf{V}_C) = \frac{1}{3}(10 \ 30 \ a^2.30 \ 60 \ a.15 \ 145) \\ &= \frac{1}{3}(10\angle 30^\circ + 30\angle -180^\circ + 15\angle 265^\circ) = -7.55 - j3.32 = 8.24\angle -156.2^\circ \end{aligned}$$

$$\mathbf{V}_{A2} \ 8.24 \ 156.2^\circ; \mathbf{V}_{B2} \ 8.24 \ 156.2^\circ \quad 120^\circ \ 8.24 \ 36.2^\circ$$

$$\mathbf{V}_{C2} \ 8.24 \ 156.2^\circ \quad 120^\circ \ 8.24 \ 276.2^\circ$$

These vectors are shown in Fig. 23.1 (c)

**(iii) Zero sequence vectors**

$$\begin{aligned} \mathbf{V}_0 &= \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C) \\ &= \frac{1}{3}(10\angle 30^\circ + 30\angle -60^\circ + 15\angle 145^\circ) = 3.8 - j4.12 = 5.6\angle -47.4^\circ \end{aligned}$$

These vectors are shown in Fig. 23.1 (d).

**Example 23.2.** Explain how an unsymmetrical system of 3-phase currents can be resolved into 3 symmetrical component systems.

Determine the values of the symmetrical components of a system of currents

$$I_R = 0 + j120A; I_Y = 50 - j100A; I_B = -100 - j50A$$

Phase sequence is RYB.

**(Elect. Engg.-I Bombay, Univ.)**

**Solution.**  $\mathbf{I}_R \ 0 \ j120 \ 120 \ 90$

$$\mathbf{I}_Y \ 50 \ j100 \ 111.8 \ 63.5^\circ; \mathbf{I}_B \ 100 \ j50 \ 111.8 \ 153.5$$

**(i) Positive-sequence Components**

$$\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_R \ a\mathbf{I}_Y \ a^2\mathbf{I}_B) = \frac{1}{3}(0 \ j120) + \frac{1}{2}j\frac{\sqrt{3}}{2}(50 \ j100) + \frac{1}{2}j\frac{\sqrt{3}}{2}(100 \ j50)$$

$$= 22.8 + j108.3 = 110.7 \angle 78.1^\circ \therefore \mathbf{I}_{R1} \quad 110.7 \quad 78.1; \mathbf{I}_{Y1} \quad 110.7 \quad 41.9; \mathbf{I}_{B1} \quad 110.7 \quad 198.1$$

(ii) Negative-sequence components

$$\mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_R - a^2\mathbf{I}_Y - a\mathbf{I}_B) = \frac{1}{3}(18.3 - j65.1 - 6.1 - j21.7 - 22.5 - 105.7)$$

$$\therefore \mathbf{I}_{R2} \quad 22.5 \quad 105.7; \mathbf{I}_{Y2} \quad 22.5 \quad 225; \mathbf{I}_{B2} \quad 22.5 \quad 14.3$$

(iii) Zero-sequence component

$$\mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B) = \frac{1}{3}[(0 - j120) + (50 - j100) + (100 - j50)] = 16.7 - j10$$

As a check, it may be found that

$$\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2} + \mathbf{I}_0; \mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_0; \mathbf{I}_B = \mathbf{I}_{B1} + \mathbf{I}_{B2} + \mathbf{I}_0$$

**Example 23.3.** In a 3-phase, 4-wire system, the currents in the R, Y and B lines under abnormal conditions of loading were as follows:

$$I_R = 100 \angle 30^\circ; I_Y = 50 \angle 300^\circ; I_B = 30 \angle 180^\circ$$

Calculate the positive, negative and zero-phase sequence currents in the R-line and the return current in the neutral conductor.

**Solution.** (i) The positive-sequence components of current in the R-line is

$$\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_R + a\mathbf{I}_Y + a^2\mathbf{I}_B)$$

$$\text{Now } \mathbf{I}_R = 100 \angle 30^\circ = 50(\sqrt{3} - j)$$

$$\mathbf{I}_Y = 50 \angle 300^\circ = 50 \left( \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = 25(1 + j\sqrt{3})$$

$$\mathbf{I}_B = 30 \angle 180^\circ = (-30 + j0)$$

$$\mathbf{I}_1 = \frac{1}{3} [50(\sqrt{3} - j) + 25(1 + j\sqrt{3}) - 30] = \frac{1}{2} \left( \frac{\sqrt{3}}{2} - j \right) + (30) \left( \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = 58 - j48.4$$

(ii) The negative-sequence components of the current in the R-line is

$$\mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_R + a^2\mathbf{I}_Y + a\mathbf{I}_B)$$

$$= \frac{1}{3} [50(\sqrt{3} - j) + 25(1 - j\sqrt{3}) - 30] = \frac{1}{2} \left( \frac{\sqrt{3}}{2} - j \right) - (30) \left( \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = 18.9 - j24.9$$

(iii) The zero-sequence component of current in the R-line is

$$\mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B) = \frac{1}{3}[50(\sqrt{3} - j) + 25(1 + j\sqrt{3}) - 30] = 27.2 - j4.7$$

The neutral current is

$$\mathbf{I}_N = \mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 3 \mathbf{I}_0 = 3(27.2 - j4.7) = 81.6 - j14.1$$

**Example 23.4.** A 3-phase, 4-wire system supplies loads which are unequally distributed on the three phases. An analysis of the currents flowing in the direction of the loads in the R, Y and B lines shows that in the R-line, the positive phase sequence current is  $200 \angle 0^\circ$  A and the

negative phase sequence current is  $100 \angle 60^\circ$ . The total observed current flowing back to the supply in the neutral conductor is  $300 \angle 300^\circ$  A. Calculate the currents in phase and magnitude in the three lines.

Assuming that the 3-phase supply voltages are symmetrical and that the power factor of the load on the R-phase is  $\sqrt{3}/2$  leading, determine the power factor of the loads on the two other phases.

**Solution.** It is given that in R-phase [Fig. 23.3 (a)]

$$\mathbf{I}_{R1} = 200 \angle 0^\circ = (200 + j0) \text{ A}; \mathbf{I}_{R2} = 100 \angle 60^\circ = (50 + j86.6) \text{ A}$$

$$\mathbf{I}_{R0} = \frac{1}{3} \mathbf{I}_N = (300/3) \angle 300^\circ = (50 - j86.6) \text{ A}$$

$$\mathbf{I}_R \quad \mathbf{I}_{R1} \quad \mathbf{I}_{R2} \quad \mathbf{I}_{R0} \quad (200 \quad j0) \quad (50 \quad j86.6) \quad (50 \quad j86.6) \quad (300 \quad j0) \quad 300 \quad 0$$

Similarly, as seen from Fig. 23.3 (b) for the Y-phase

$$\begin{aligned} \mathbf{I}_Y &= \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_{Y0} = a^2 \mathbf{I}_{R1} + a \mathbf{I}_{R2} + \mathbf{I}_{R0} \\ &= 200 \angle 0^\circ - 120^\circ + 100 \angle 60^\circ + 120^\circ \\ &\quad + 100 \angle 300^\circ = -100 - j173.2 - 100 + 50 - j86.6 = -150 - j259.8 = 300 \angle 240^\circ \text{ A} \end{aligned}$$

Similarly, as seen from Fig. 23-3 (c) for the B-phase

$$\mathbf{I}_B \quad \mathbf{I}_{B1} \quad \mathbf{I}_{B2} \quad \mathbf{I}_{B0} \quad a \mathbf{I}_{R1} \quad a^2 \mathbf{I}_{R2} \quad \mathbf{I}_{R0} \quad 200 \quad 0 \quad 120 \quad 100 \quad 60 \quad 120 \quad 100 \quad 300 \quad 0$$

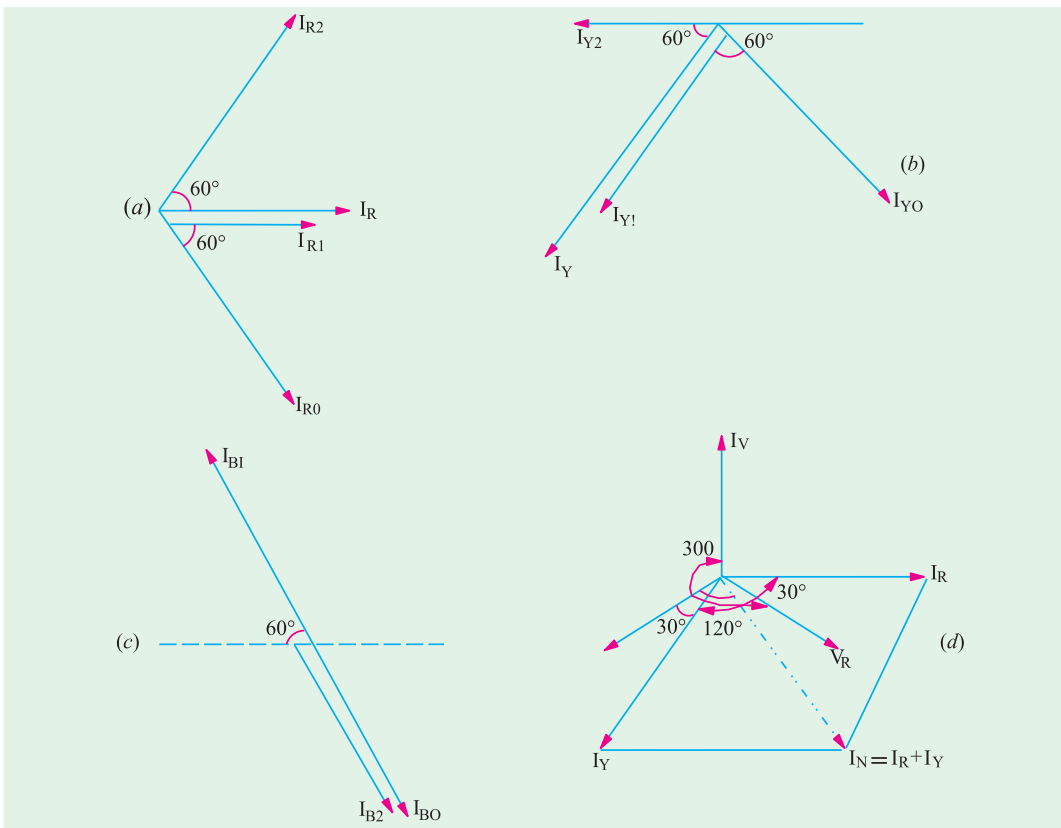


Fig. 23.3

Since the power factor of the  $R$ -phase is  $\sqrt{3}/2$  leading, the current  $I_R$  leads the voltage  $V_R$  by  $30^\circ$  [Fig. 23.3(d)]

(d) Now, phase angle of  $I_Y$  is  $240^\circ$  relative to  $I_R$  so that  $I_Y$  leads its voltage  $V_Y$  by  $30^\circ$ . Hence, power factor of  $Y$  phase is also  $\sqrt{3}/2$  leading. The power factor of  $B$  line is indeterminate because the current in this line is zero.

**Example 23.5.** Prove that in a 3-phase system if  $V_1$ ,  $V_2$  and  $V_3$  are the three balanced voltages whose phasor sum is zero, the positive and negative sequence components can be expressed as

$$V_{1p} = \left\{ \frac{1}{\sqrt{3}} (V_1 + V_2 \angle 60^\circ) \right\} \angle 30^\circ; V_{1N} = \left\{ \frac{1}{\sqrt{3}} (V_1 + V_2 \angle -60^\circ) \right\} \angle -30^\circ$$

Phase sequence is 1-2-3.

A system of 3-phase currents is given as  $I_1 = 10 \angle 180^\circ$ ,  $I_2 = 14.14 \angle -45^\circ$  and  $I_3 = 10 \angle 90^\circ$ . Determine phasor expression for the sequence components of these currents. Phase sequence is 1-2-3. **(Elect. Engg-I, Bombay Univ.)**

**Solution.** As seen from Art. 23.6

$$\mathbf{V}_{1P} = \frac{1}{3}(\mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_3); \text{ Now, } \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = 0 \therefore \mathbf{V}_3 = -(\mathbf{V}_1 + \mathbf{V}_2)$$

$$\mathbf{V}_{1P} = \frac{1}{3}[\mathbf{V}_1 + a\mathbf{V}_2 - a^2(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1 - a^2) + \mathbf{V}_2(a - a^2)]$$

$$\text{Now, } 1 - a^2 = \frac{2}{3} + j\frac{\sqrt{3}}{2} \text{ and } a - a^2 = j\sqrt{3}$$

$$\therefore \mathbf{V}_{1P} = \frac{1}{3} \mathbf{V}_1 \left[ \frac{2}{3} + j\frac{\sqrt{3}}{2} \right] + \frac{j\sqrt{3}}{3} \mathbf{V}_2 = \frac{1}{\sqrt{3}} \mathbf{V}_1 \left[ \frac{\sqrt{3}}{2} + j\frac{1}{2} \right] + j\mathbf{V}_2$$

$$= \frac{1}{\sqrt{3}} [\mathbf{V}_1 \angle 30^\circ + \mathbf{V}_2 \angle 90^\circ] = \left[ \frac{1}{\sqrt{3}} (\mathbf{V}_1 + \mathbf{V}_2 \angle 60^\circ) \right] \angle 30^\circ$$

Similarly, the negative-sequence component is given by

$$\mathbf{V}_{1N} = \frac{1}{3}(\mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_3) = \frac{1}{3}[\mathbf{V}_1 + a^2\mathbf{V}_2 - a(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1 - a) + \mathbf{V}_2(a^2 - a)]$$

$$\text{Now, } a^2 - a = -j\sqrt{3}$$

$$\begin{aligned} \therefore \mathbf{V}_{1N} &= \frac{1}{3} \left[ \mathbf{V}_1 \left( \frac{3}{2} - j\frac{\sqrt{3}}{2} \right) - j\sqrt{3}\mathbf{V}_2 \right] = \frac{1}{\sqrt{3}} \left[ \mathbf{V}_1 \left( \frac{\sqrt{3}}{2} - j\frac{1}{2} \right) - j\mathbf{V}_2 \right] \\ &= \frac{1}{\sqrt{3}} [\mathbf{V}_1 \angle -30^\circ + \mathbf{V}_2 \angle -90^\circ] = \left\{ \frac{1}{\sqrt{3}} (\mathbf{V}_1 + \mathbf{V}_2 \angle -60^\circ) \right\} \angle -30^\circ \end{aligned}$$

$$\text{Now } \mathbf{I}_1 = 10 \angle 180^\circ = -10 + j0; \mathbf{I}_2 = 14.14 \angle -45^\circ = 10 - j10$$

$$\mathbf{I}_3 = 10 \angle 90^\circ = j10$$

$$a\mathbf{I}_2 = 14.14 \cdot 75 \cdot 3.66 \cdot j13.66; a^2\mathbf{I}_2 = 14.14 \cdot 165 \cdot 13.66 \cdot j3.66$$

$$a\mathbf{I}_3 = 14.14 \cdot 21 \cdot 12.25 \cdot j7.07; a^2\mathbf{I}_3 = 14.14 \cdot 30 \cdot 12.25 \cdot j7.07$$

$$\mathbf{I}_{1P} = \frac{1}{3}(\mathbf{I}_1 + a\mathbf{I}_2 + a^2\mathbf{I}_3) = \frac{1}{3}(5.91 - j6.59) = 1.97 - j2.2$$

$$\mathbf{I}_{1N} = \frac{1}{3}(\mathbf{I}_1 + a^2\mathbf{I}_2 + a\mathbf{I}_3) = \frac{1}{3}(35.91 + j10.73) = 11.97 + j3.58$$

$$\mathbf{I}_{10} = \frac{1}{3}(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = 0$$

### Tutorial Problems No. 23.1

1. The following currents were recorded in the  $R$ ,  $Y$ , and  $B$  lines of a 3-phase system under abnormal conditions:

$$[\mathbf{I}_R = 300\angle 300^\circ \text{ A}; \mathbf{I}_Y = 500\angle 240^\circ \text{ A}; \mathbf{I}_B = 1,000\angle 60^\circ \text{ A}]$$

Calculate the values of the positive, negative and zero phase-sequence components.

$$[\mathbf{I}_1 = 536\angle -44^\circ 20' \text{ A}; \mathbf{I}_2 = 372\angle 171^\circ \text{ A}; \mathbf{I}_0 = 145\angle 23^\circ 20']$$

2. Determine the symmetrical components of the three currents  $I_0 = 10\angle 0^\circ$ ;  $\mathbf{I}_b = 100\angle 250^\circ$  and  $\mathbf{I}_c = 10\angle 110^\circ \text{ A}$

$$[\mathbf{I}_1 = (39.45 + j52.2) \text{ A}; \mathbf{I}_2 = (-20.24 + j22.98) \text{ A}; \mathbf{I}_0 = (-9.21 - j28.19) \text{ A}]$$

(*Elect. Meas & Measuring Instru., Madras Univ.*)

3. The three current vectors of a 3-phase, four-wire system have the following values;  $\mathbf{I}_A = 7 + j_0$ ,  $\mathbf{I}_B = -12 - j13$  and  $\mathbf{I}_C = -2 + j3$ . Find the symmetrical components. The phase sequence is  $A, B, C$

$$[\mathbf{I}_0 = -2.33 - j3.33; \mathbf{I}_{A1} = (27.75 - j3.67); \mathbf{I}_{B1} = (-17.05 - j22.22); \mathbf{I}_{C1} = (-10.7 + j25.9);$$

$$\mathbf{I}_{A2} = (0.25 + j13.67) \text{ A}; \mathbf{I}_{B2} = (-11.97 - j6.62) \text{ A}; \mathbf{I}_{C2} = (11.73 - j7.05) \text{ A}]$$

### 23.9. Zero-Sequence Components of Current and Voltage

Any circuit which allows the flow of positive-sequence currents will also allow the flow of negative-sequence currents because the two are similar. However, a fourth wire is necessary if zero-sequence components are to flow in the lines of the 3-phase system. It follows that the line currents of 3-phase 3-wire system can contain no zero sequence components whether it is delta - or star-connected. The zero sequence components of line-to-line voltages are non-existent regardless of the degree of imbalance in these voltages. It means that a set of unbalanced 3-phase, line-to-line voltages may be represented by a positive system and a negative system of balanced voltages. This fact is of considerable importance in the analysis of 3-phase rotating machinery. For example, the operation of an induction motor when supplied from an unbalanced system of 3-phase voltages, may be analysed on the basis of two balanced systems of voltages of opposite phase sequence.

Let us consider some typical 3-phase connections with reference to zero-sequence components of current and voltage.

(a) **Four-wire Star Connection.** Due to the presence of the fourth wire, the zero sequence currents may flow. The neutral wire carries only the zero-sequence current which is the sum of the zero-sequence currents in the three lines. Since the sum of line voltages is zero, there can be no zero sequence component of line voltages.

(b) **Three-wire Star Connection.** Since there is no fourth or return wire, zero-sequence components of current cannot flow. The absence of zero-sequence currents may be explained by considering that the impedance offered to these currents is infinite and that this impedance is situated between the star points of the generator and the load. If the two star points were joined by a neutral, only zero-sequence currents will flow through it so that only zero-sequence voltage can exist between the load and generator star points. Obviously, no zero-sequence component of voltage appears across the phase load.

(c) **Three-wire Delta Connection.** Due to the absence of fourth wire, zero-sequence components of currents cannot be fed into the delta-connected load. However, though line currents

have to sum up to zero (whereas phase currents need not do so) it is possible to have a zero-sequence current circulating in the delta-connected load.

Similarly, individual phase voltages will generally possess zero-sequence components though components are absent in the line-to-line voltage.\*

### 23.10. Unbalanced Star Load Supplied from Unbalanced Three-phase Three-wire System

In this case, line voltages and load currents will consist of only positive and negative-sequence components (but no zero-sequence component). But load voltages will consist of positive, negative and zero-sequence components.

Let the line voltages be denoted by  $V_{RY}$ ,  $V_{YB}$  and  $V_{BR}$ , line (and load) currents by  $I_R$ ,  $I_Y$  and  $I_B$ , the load voltages by  $V_{RN}$ ,  $V_{YN}$  and  $V_{BN}$  and load impedances by  $Z_R$ ,  $Z_Y$ , and  $Z_B$  (their values being the same for currents of any sequence).

Obviously,  $V_{RN} = I_R Z_R$ ;  $V_{YN} = I_Y Z_Y$  and  $V_{BN} = I_B Z_B$

If  $V_{RN1}$ ,  $V_{RN2}$  and  $V_0$  are the symmetrical components of  $V_{RN}$ , then we have

$$\begin{aligned} V_0 &= \frac{1}{3}(V_{RN} + V_{YN} + V_{BN}) = \frac{1}{3}(I_R Z_R + I_Y Z_Y + I_B Z_B) \\ &= \frac{1}{3}[Z_R(I_{R1} + I_{R2}) + Z_Y(I_{Y1} + I_{Y2}) + Z_B(I_{B1} + I_{B2})] \\ &= \frac{1}{3}[Z_R(I_{R1} + I_{R2}) + Z_Y(a^2 I_{R1} + a I_{R2}) + Z_B(a I_{R1} + a^2 I_{R2})] \\ &= I_{R1} \cdot \frac{1}{3}(Z_R + a^2 Z_Y + a Z_B) + I_{R2} \cdot \frac{1}{3}(Z_R + a Z_Y + a^2 Z_B) = I_{R1} Z_{R2} + I_{R2} Z_{R1} \end{aligned} \quad \dots (i)$$

$$\begin{aligned} V_{RN1} &= \frac{1}{3}(V_{RN} + a V_{YN} + a^2 V_{BN}) = \frac{1}{3}(I_R Z_R + a I_Y Z_Y + a^2 I_B Z_B) \\ &= \frac{1}{3}[Z_R(I_{R1} + I_{R2})] + a Z_Y(I_{Y1} + I_{Y2}) + a^2 Z_B(I_{B1} + I_{B2}) \\ &= I_{R1} \cdot \frac{1}{3}(Z_R + Z_Y + Z_B) + I_{R2} \cdot \frac{1}{3}(Z_R + a^2 Z_Y + a Z_B) = I_{R1} Z_0 + I_{R2} Z_{R2} \end{aligned} \quad \dots (ii)$$

$$V_{RN2} = I_{R2} Z_0 + I_{R1} Z_{R1}$$

Similarly,  $V_{YN1} = I_{Y1} Z_0 + I_{Y2} Z_{Y2} = a^2 (I_{R1} Z_0 + I_{R2} Z_{R2}) = a^2 V_{RN1}$

$$V_{BN1} = I_{B1} Z_0 + I_{B2} Z_{B2} = a (I_{R1} Z_0 + I_{R2} Z_{R2}) = a V_{RN1}$$

$$V_{YN2} = I_{Y2} Z_0 + I_{Y1} Z_{Y1} = a (I_{R2} Z_0 + I_{R1} Z_{R1}) = a V_{RN2}$$

$$V_{BN2} = I_{B2} Z_0 + I_{B1} Z_{B1} = a^2 (I_{R2} Z_0 + I_{R1} Z_{R1}) = a^2 V_{RN2}$$

Now,  $V_{RN1}$  and  $V_{RN2}$  may be determined from the relation between the line and phase voltages as given below:

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN} = V_0 + V_{RN1} + V_{RN2} - (V_0 + V_{RN1} + V_{YN2})$$

\* However, under balanced conditions, the phase voltages will possess no zero-sequence components.



$$= \mathbf{V}_{RN1}(1-a^2) + \mathbf{V}_{RN2}(1-a) = \frac{1}{2}\sqrt{3}[\mathbf{V}_{RN1}(\sqrt{3} + j1) + \mathbf{V}_{RN2}(\sqrt{3} - j1)] \quad \dots (iii)$$

$$\begin{aligned} \mathbf{V}_{YB} &= \mathbf{V}_{YN} + \mathbf{V}_{NB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = \mathbf{V}_0 + \mathbf{V}_{YN1} + \mathbf{V}_{YN2} - (\mathbf{V}_0 + \mathbf{V}_{BN1} + \mathbf{V}_{BN2}) \\ &= \mathbf{V}_{RN1}(a^2 - a) - \mathbf{V}_{RN2}(a - a^2) = -j\sqrt{3} \cdot \mathbf{V}_{RN1} + j\sqrt{3}\mathbf{V}_{RN2} \end{aligned}$$

$$\therefore \mathbf{V}_{RN2} = \mathbf{V}_{RN1} - j\mathbf{V}_{YB} / \sqrt{3}$$

Substituting this value of  $\mathbf{V}_{RN2}$  in Eq. (iii) above and simplifying, we have

$$\mathbf{V}_{RN1} = \frac{1}{3}[\mathbf{V}_{RY} + \frac{1}{2}\mathbf{V}_{YB}(1 + j\sqrt{3})] \quad \dots (iv)$$

$$\mathbf{V}_{RN2} = \frac{1}{3}[\mathbf{V}_{RY} + \frac{1}{2}\mathbf{V}_{YB}(1 - j\sqrt{3})] \quad \dots (v)$$

Having known  $\mathbf{V}_{RN1}$  and  $\mathbf{V}_{RN2}$ , the currents  $\mathbf{I}_{R1}$  and  $\mathbf{I}_{R2}$  can be determined from the following equations:

$$\mathbf{I}_{R1} = \frac{(\mathbf{V}_{RN1}\mathbf{Z}_0 - \mathbf{V}_{RN2}\mathbf{Z}_{R2})}{\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}} \quad \dots (vi)$$

$$\mathbf{I}_{R2} = \frac{(\mathbf{Z}_{RN2}\mathbf{Z}_0 - \mathbf{V}_{RN1}\mathbf{Z}_{R1})}{\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}} \quad \dots (vii)$$

Alternatively, when  $\mathbf{I}_{R1}$  becomes known,  $\mathbf{I}_{R2}$  may be found from the relation

$$\mathbf{I}_{R2} = \frac{\mathbf{V}_{RN2} - \mathbf{I}_{R1}\mathbf{Z}_{R1}}{\mathbf{Z}_0} \quad \dots (viii)$$

The symmetrical components of the currents in other phases can be calculated from  $\mathbf{I}_{R1}$  and  $\mathbf{I}_{R2}$  by using the relations given in Art. 23.8.

The phase voltages may be calculated by any one of the two methods given below:

(i) directly by calculating the products  $\mathbf{I}_R\mathbf{Z}_R$ ,  $\mathbf{I}_Y\mathbf{Z}_Y$  and  $\mathbf{I}_B\mathbf{Z}_B$ .

(ii) by first calculating the zero-sequence component  $\mathbf{V}_0$  of the phase or load voltages and then adding this to the appropriate positive and negative sequence components.

For example,  $\mathbf{V}_{RN} = \mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{RN2}$

### 23.11. Unbalanced Star Load Supplied from Balanced Three-phase, Three-wire System

It is a special case of the general case considered in Art. 23.10 above. In this case, the symmetrical components of the load voltages consist only of positive and zero-sequence components. This fact may be verified by substituting the value of  $\mathbf{V}_{YB}$  in Eq. (v) of Art. 23.10. Now, in a balanced or symmetrical system of positive-phase sequence

$$\mathbf{V}_{YB} = \mathbf{V}_{RY} \frac{1}{2} j\frac{\sqrt{3}}{2}$$

Substituting this value in Eq. (v) above, we have

$$\begin{aligned} \mathbf{V}_{RN2} &= \frac{1}{3}[\mathbf{V}_{RY} + \frac{1}{2}\mathbf{V}_{YB}(1 - j\sqrt{3})] \\ &= \frac{1}{3}\left[\mathbf{V}_{RY} + \frac{1}{2}\mathbf{V}_{RY}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(1 - j\sqrt{3})\right] = \frac{1}{3}(\mathbf{V}_{RY} - \mathbf{V}_{RY}) = 0 \end{aligned}$$

Hence, substituting this value of  $V_{RN2}$  in Eq. (iv) and (vi) of Art. 23.10, we get

$$V_{RN1} = jV_{YB}\sqrt{3} = \frac{1}{2}V_{RY} - j\frac{1}{2}V_{RY}/\sqrt{3} \quad \dots (i)$$

$$I_{R1} = V_{RN1}Z_0 / (Z_0^2 - Z_{R1}Z_{R2}) = V_{RN1} \frac{Z_R + Z_Y + Z_B}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R} \quad \dots (ii)$$

$$I_{R2} = -I_{R1} \cdot Z_{R1} / Z_0 = -I_{R1} = -I_{R1} \frac{Z_R + aZ_Y + a^2Z_B}{Z_R + Z_Y + Z_B} \quad \dots (iii)$$

Example 23.6 illustrates the procedure for calculating the current in an unbalanced star-connected load when supplied from a symmetrical three-wire system.

**Example 23.6.** A symmetrical 3-phase, 3-wire, 440-V system supplies an unbalanced Y-connected load of impedances  $Z_R = 5 \angle 30^\circ \Omega$ ;  $Z_Y = 10 \angle 45^\circ \Omega$ ,  $Z_B = 10 \angle 60^\circ \Omega$ . Phase sequence is  $R \rightarrow Y \rightarrow B$ . Calculate in the rectangular complex form the symmetrical components of the currents in the R-line. **(Elect. Engg.-I, Bombay Univ.)**

**Solution.** As shown in Fig. 23.4, the branch impedances are

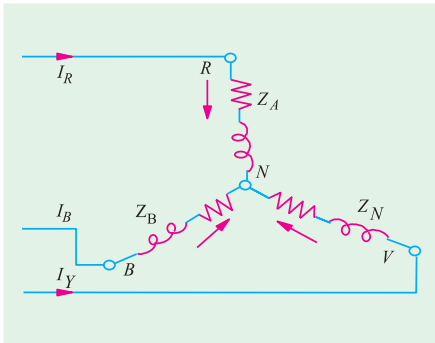


Fig. 23.4

$$Z_R \quad 5 \quad 30 \quad (4.33 \quad j2.5)$$

$$Z_Y \quad 10 \quad 45 \quad (7.07 \quad j7.07)$$

$$Z_B \quad 10 \quad 60 \quad (5 \quad j8.66)$$

$$aZ_Y = 10 \angle 165^\circ = 10(-0.966 + j0.259) = -9.66 + j2.59 \Omega$$

$$a^2Z_Y \quad 10 \quad 75 \quad 10(0.259 \quad j0.966) \quad 2.59 \quad j9.66$$

$$aZ_B \quad 10 \quad 180 \quad (10 \quad j0)$$

$$a^2Z_B \quad 10 \quad 60 \quad 10(0.5 \quad j0.866) \quad (5 \quad j8.66)$$

The symmetrical component impedances required for calculation purposes are as follows:

$$Z_0 = \frac{1}{3}(Z_R \quad Z_Y \quad Z_B) = \frac{1}{3}(16.4 \quad j18.23) \quad (5.47 \quad j6.08)$$

$$\begin{aligned} Z_{R1} &= \frac{1}{3}(Z_R + Z_Y + a^2Z_B) \\ &= \frac{1}{3}(4.33 + j2.5 - 9.66 + j2.59 + 5 - j8.66) \\ &= \frac{1}{3}(-0.33 - j3.57) = -0.11 - j1.19 \end{aligned}$$

$$\begin{aligned} Z_{R2} &= \frac{1}{3}(Z_R + aZ_Y + aZ_B) \\ &= \frac{1}{3}(4.33 + j2.5 + 2.59 - j9.66 - 10 + j0) = \frac{1}{3}(-3.08 - j7.16) = -1.03 - j2.55 \end{aligned}$$

Now

$$\begin{aligned} \mathbf{Z}_0^2 \mathbf{Z}_{R1} \mathbf{Z}_{R2} &= \frac{1}{3} (\mathbf{Z}_R \mathbf{Z}_Y + \mathbf{Z}_Y \mathbf{Z}_B + \mathbf{Z}_B \mathbf{Z}_R) \\ &= \frac{1}{3} [50 \ 75 \ 100 \ 105 \ 50 \ 90] \begin{pmatrix} 4.31 & \\ & j65 \end{pmatrix} \end{aligned}$$

Let  $\mathbf{V}_{RY}$  be taken as the reference vectors so that  $\mathbf{V}_{RY} = (440 + j0)$

$$\text{Then, } \mathbf{V}_{RN1} = \frac{1}{2} \mathbf{V}_{RY} - j \frac{1}{2} \mathbf{V}_{RY} / \sqrt{3} = \frac{1}{2} (440 + j0) - j \frac{1}{2} (440 + j0) / \sqrt{3} = (220 - j127)$$

$$\text{Now, } \mathbf{I}_{R1} = \frac{\mathbf{V}_{RN1} \cdot \mathbf{Z}_0}{(\mathbf{Z}_0^2 \mathbf{Z}_{R1} \mathbf{Z}_{R2})} = \frac{(220 \ j127) \begin{pmatrix} 5.47 & \\ & j6.08 \end{pmatrix}}{\begin{pmatrix} 4.31 & \\ & j65 \end{pmatrix}} = \frac{1986 \ j643}{\begin{pmatrix} 4.31 & \\ & j65 \end{pmatrix}}$$

$$= \frac{2088 \angle 18^\circ}{66.4 \angle 93.8^\circ} = 31.6 \angle -75.8^\circ = (7.75 - j30.6) \text{ A}$$

$$\mathbf{I}_{R2} = -\mathbf{I}_{R2} \mathbf{Z}_{R1} / \mathbf{Z}_0$$

$$= \frac{-(7.75 - j30.6)(-0.11 - j1.19)}{(5.47 + j6.08)} = \frac{(37.25 + j5.88)}{(5.47 + j6.08)}$$

$$= \frac{37.75 \angle 9^\circ}{8.18 \angle 48^\circ} = 4.61 \angle -39^\circ = (3.28 - j2.66) \text{ A}$$

Hence, the symmetrical components of  $\mathbf{I}_R$  are:-

Positive-sequence component =  $(7.75 - j30.6) \text{ A}$

Negative-sequence component =  $(3.28 - j2.66) \text{ A}$

**Note. (i)**  $\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2}$

**(ii)** Symmetrical components of other currents are

$$\mathbf{I}_Y = a^2 \mathbf{I}_{B1} \text{ and } \mathbf{I}_{Y2} = a \mathbf{I}_{R2}; \mathbf{I}_{B1} = a \mathbf{I}_{R1} \text{ and } \mathbf{I}_{B2} = a^2 \mathbf{I}_{R2}$$

**Example 23.7.** A balanced star-connected load takes 75 A from a balanced 3-phase, 4-wire supply. If the fuses in two of the supply lines are removed, find the symmetrical components of the line currents before and after the fuses are removed.

**Solution.** The circuit is shown in Fig. 23.5.

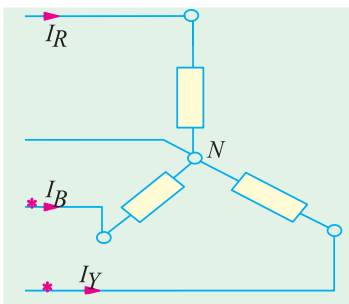


Fig. 23.5

**Before fuses are removed**

$$\mathbf{I}_R \ 75 \ 0; \mathbf{I}_Y \ 75 \ 120; \mathbf{I}_B \ 75 \ 120$$

$$\mathbf{I}_1 = \frac{1}{3} (\mathbf{I}_R \ a \mathbf{I}_Y \ a^2 \mathbf{I}_B) = \frac{1}{3} (75 \ 0 \ 75 \ 0 \ 75 \ 360) = 75 \ 0 \ \text{A}$$

$$\mathbf{I}_2 = \frac{1}{3} (\mathbf{I}_R \ a^2 \mathbf{I}_Y \ a \mathbf{I}_B) = \frac{1}{3} (75 \ 0 \ 75 \ 120 \ 75 \ 240) = 0$$

$$\mathbf{I}_0 = \frac{1}{3} (\mathbf{I}_R \ \mathbf{I}_Y \ \mathbf{I}_B) = \frac{1}{3} (75 \ 0 \ 75 \ 120 \ 75 \ 120) = 0.$$

After fuses are removed

$$\mathbf{I}_R = 75 \angle 0^\circ; \mathbf{I}_Y = \mathbf{I}_B = 0$$

$$\mathbf{I}_1 = \frac{1}{3} (75 \ 0 \ 0 \ 0) = 25 \ 0$$

$$\mathbf{I}_2 = 25\angle 0^\circ, \mathbf{I}_0 = 25\angle 0^\circ$$

**Example 23.8.** Explain the terms: positive-, negative- and zero-sequence components of a 3-phase voltage system. A star-connected load consists of three equal resistors, each of  $1 \Omega$  resistance. When the load is connected to an unsymmetrical 3-phase supply, the line voltages are 200 V, 346 V and 400 V.

Find the magnitude of the current in any one phase by the method of symmetrical components. **(Power Systems-II, A.M.I.E.)**

**Solution.** This question could be solved by using the following two methods:

(a) As shown in Fig. 23.6(a), the line voltages form a closed right-angled triangle with an angle =  $\tan^{-1}(346/200) = \tan^{-1}(1.732) = 60^\circ$

Hence, if  $\mathbf{V}_{AB} = 200 \angle 0^\circ$ , then  $\mathbf{V}_{BC} = 346 \angle -90^\circ = -j346$  and  $\mathbf{V}_{CA} = 400 \angle 120^\circ$ .

As seen from Eq. (iv) and (v) of Art. 23.10

$$\begin{aligned} \mathbf{V}_{AN1} &= \frac{1}{3} \left[ \mathbf{V}_{AB} + \frac{1}{2} \mathbf{V}_{BC} (1 + j\sqrt{3}) \right] \\ &= \frac{1}{3} \left[ 200 + \left( \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) (-j346) \right] = 166.7 - j57.7 \end{aligned}$$

$$\begin{aligned} \mathbf{V}_{AN2} &= \frac{1}{3} \left[ \mathbf{V}_{AB} + \mathbf{V}_{BC} \left( \frac{1}{2} - \frac{j\sqrt{3}}{2} \right) \right] \\ &= \frac{1}{3} \left[ 200 + \left( \frac{1}{2} - j\frac{\sqrt{3}}{2} \right) (-j346) \right] = -33.3 - j57.7 \end{aligned}$$

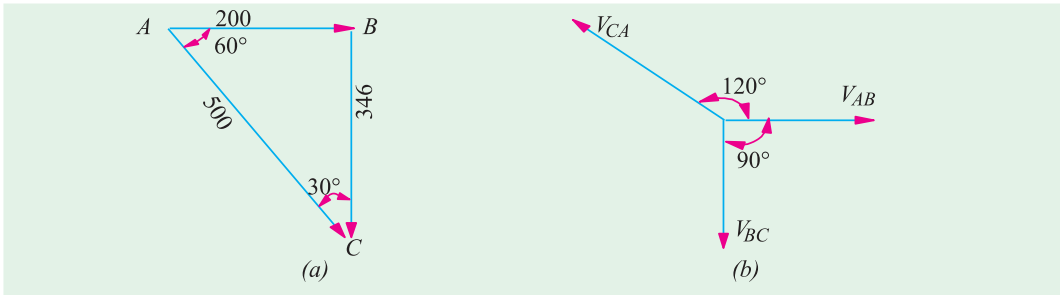


Fig. 23.6

Now,  $\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C = 3$

$$\mathbf{Z}_{A1} = \mathbf{Z}_A + a\mathbf{Z}_B + a^2\mathbf{Z}_C = 0 \text{ (because } \mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C \text{ and } 1 + a + a^2 = 0)$$

Similarly,  $\mathbf{Z}_{A2} = \mathbf{Z}_A + a^2\mathbf{Z}_B + a\mathbf{Z}_C = 0$

Hence, from Eq. (ii) of Art. 23.10 we have  $\mathbf{V}_{AN1} = \mathbf{I}_{A1}\mathbf{Z}_0 \therefore \mathbf{I}_{A1} = \mathbf{V}_{AN1}/\mathbf{Z}_0$

$$\text{Now, } \mathbf{Z}_0 = \frac{1}{3}(\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C) = 3/3 = 1 \therefore \mathbf{I}_{A1} = (166.7 - j57.7) \text{ A}$$

$$\mathbf{I}_{A2} = \mathbf{V}_{AN2}/\mathbf{Z}_0 = -33.3 - j57.7 \therefore \mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} \quad (\mathbf{I}_0 = 0)$$

$$\therefore \mathbf{I}_A = (166.7 - j57.7) + (-33.3 - j57.7) = 133.4 - j115.4 = 176.4 \angle 40.7^\circ \text{ A}$$

(b) Using Millman's theorem and taking the line terminal  $A$  as reference point, the voltage between  $A$  and the neutral point  $N$  is

$$\mathbf{V}_{NA} = \frac{\mathbf{V}_{BA} \mathbf{Y}_B + \mathbf{V}_{CA} \mathbf{Y}_C}{\mathbf{Y}_A + \mathbf{Y}_B + \mathbf{Y}_C} = \frac{200 \angle 180^\circ \cdot 1 + 400 \angle 120^\circ \cdot 1}{3} = 133.3 - j115.3 \text{ V}$$

$$\mathbf{V}_{AN} = 176.4 - j139.3 \text{ V} \quad \mathbf{I}_A = \mathbf{I}_{AN} / \mathbf{Z} = 176.4 - j40.7 \text{ A} \quad \dots \text{ as before}$$

**Example 23.9.** Three equal impedances of  $(8 + j6)$  are connected in star across a 3-phase, 3-wire supply. The phase voltages are  $V_A = (220 + j0)$ ,  $V_B = (-j220)$  and  $V_C = (-100 + j220)$  V. If there is no connection between the load neutral and the supply neutral, calculate the symmetrical components of A-phase current and the three line currents.

**Solution.** Since there is no fourth wire, there is no zero-component current. Moreover,  $\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$ . The symmetrical components of the A-phase voltages are

$$\mathbf{V}_0 = \frac{1}{3}(220 - j220 - 100 + j220) = 40 \text{ V}^*$$

$$\mathbf{V}_{A1} = \frac{1}{3}[220 + (0.5 - j0.866)(-j220) + (0.5 + j0.866)(-100 + j220)]$$

$$= \frac{1}{3}[(660 + j86.6)] = (220 + j28.9) \text{ V}$$

$$\mathbf{V}_{A2} = \frac{1}{3}[220 + (0.5 + j0.866)(-j220) + (0.5 - j0.866)(-100 + j220)]$$

$$= \frac{1}{3}(-120 - j86.6) = (-40 - j28.9) \text{ V}$$

The component currents in phase A are

$$\mathbf{I}_{A1} = \frac{\mathbf{V}_{A1}}{(8 + j6)} = \frac{220 + j28.9}{(8 + j6)} = (19.33 - j10.89) \text{ A}$$

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_{A2}}{(8 - j6)} = \frac{(-40 - j28.9)}{(8 - j6)} = (4.93 + j0.09) \text{ A}$$

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} = (14.4 - j10.8) \text{ A}; \mathbf{I}_B = \mathbf{I}_{B1} + \mathbf{I}_{B2} = a^2 \mathbf{I}_{A1} + a \mathbf{I}_{A2}$$

$$\mathbf{I}_B = \frac{1}{2} - j\frac{\sqrt{3}}{2} (19.33 - j10.89) + \frac{1}{2} + j\frac{\sqrt{3}}{2} (4.93 + j0.09)$$

**Example 23.10.** Two equal impedance arms  $AB$  and  $BC$  are connected to the terminals  $A$ ,  $B$ ,  $C$  of a 3-phase supply as shown in Fig. 23.7. Each capacitor has a reactance of  $X = \sqrt{3}R$ . A high impedance voltmeter  $V$  is connected to the circuit at points  $P$  and  $Q$  as shown. If the supply line voltages  $V_{AB}$ ,  $V_{BC}$ ,  $V_{CA}$  are balanced, determine the reading of the voltmeter (a) when the phase sequence of the supply voltages is  $A \rightarrow B \rightarrow C$  and (b) when the phase sequence is reversed. Hence, explain how this network could be employed to measure, respectively, the positive and negative phase sequence voltage components of an unbalanced 3-phase supply.

\* However, it is not required in the problem.

**Solution.** The various currents and voltages are shown in Fig. 23.7.

### Phase Sequence ABC

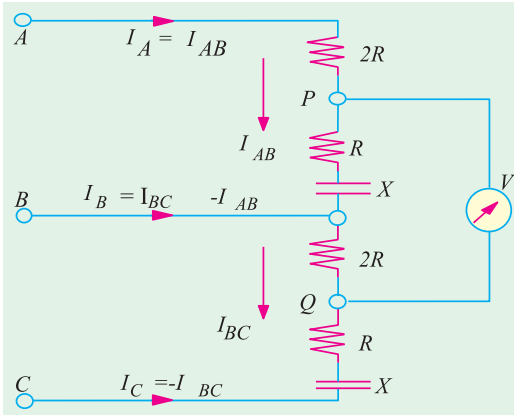


Fig. 23.7

Taking  $\mathbf{V}_{AB}$  as the reference vector, we have

$$\mathbf{V}_{AB} = V(1 + j0); \mathbf{V}_{BC} = a^2V; \mathbf{V}_{CA} = aV$$

As seen from the diagram,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{3R + jX} = \frac{V}{3R + j\sqrt{3}R} = \frac{V}{\sqrt{3}R(\sqrt{3} + j)}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{3R + jX} = \frac{a^2V}{\sqrt{3}R(\sqrt{3} + j)}$$

$$\text{Now, } \mathbf{V}_{PO} + \mathbf{V}_{OQ} = \mathbf{V}_{PQ}$$

$$\text{Also, } \mathbf{V}_{PQ} = \mathbf{I}_{AB}(R - jX) + \mathbf{I}_{BC}.2R$$

$$= \frac{V}{\sqrt{3}R(\sqrt{3} + j)}.R(1 - j\sqrt{3}) + \frac{a^2V}{\sqrt{3}R(\sqrt{3} + j)}.2R$$

$$= \frac{V}{\sqrt{3}(\sqrt{3} + j)}.(1 - j\sqrt{3} + 2a^2) \quad a^2 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}$$

$$= \frac{V}{\sqrt{3}(\sqrt{3} + j)}.(-j2\sqrt{3}) \quad \therefore V_{PQ} = \frac{V}{\sqrt{3}} \cdot \frac{1}{2} \cdot 3\sqrt{3} = V$$

Hence, the voltmeter which is not phase sensitive will read the line voltage when phase sequence is  $A \rightarrow B \rightarrow C$ .

### Phase Sequence ACB

$$\text{In this case, } \mathbf{V}_{AB} = V(1 + j0); \mathbf{V}_{BC} = aV; \mathbf{V}_{CA} = a^2V$$

$$\text{Also, } \mathbf{V}_{PQ} = \mathbf{V}_{PO} + \mathbf{V}_{OQ}$$

$$= \mathbf{I}_{AB}(R - jX) + \mathbf{I}_{BC}.2R = \frac{V(R - jX)}{\sqrt{3}.R(\sqrt{3} + j)} + \frac{aV.2R}{\sqrt{3}.R(\sqrt{3} + j)}$$

$$= \frac{V}{\sqrt{3}(\sqrt{3} + j)}(1 - j\sqrt{3} + 2a) = \frac{V}{\sqrt{3}(\sqrt{3} + j)}(1 - j\sqrt{3} - 1 + j\sqrt{3}) = 0$$

Hence, when the phase sequence is reversed, the voltmeter reads zero.

It can be proved that with sequence  $ABC$ , the voltmeter reads the positive sequence component ( $\mathbf{V}_1$ ) and with phase sequence  $ACB$ , it reads the negative sequence component ( $\mathbf{V}_2$ ) with phase sequence  $ABC$

$$\mathbf{V}_{PQ} = \frac{\mathbf{V}_{AB}}{3R + j\sqrt{3}R}(R - j\sqrt{3}R) + \frac{\mathbf{V}_{BC}}{3R + j\sqrt{3}R}.2R = \frac{1}{3} \frac{1}{j\sqrt{3}} [(\mathbf{V}_1 \quad \mathbf{V}_2)(1 \quad j\sqrt{3}) \quad 2(a^2\mathbf{V}_1 \quad a\mathbf{V}_2)]$$

$$= \frac{1}{3 - j\sqrt{3}} [\mathbf{V}_1(1 - j\sqrt{3} + 2a^2) + \mathbf{V}_2(1 - j\sqrt{3} + 2a)] = \frac{1}{3 - j\sqrt{3}} \mathbf{V}_1(-j2\sqrt{3}) \quad \therefore \mathbf{V}_{PQ} = \mathbf{V}_1$$

With phase sequence  $ACB$

$$\begin{aligned} \mathbf{V}_{PQ} &= \frac{1}{3-j\sqrt{3}} [(\mathbf{V}_1 \ \mathbf{V}_2) (1 \ j\sqrt{3}) \ 2(a \ \mathbf{V}_1 \ a^2 \ \mathbf{V}_2)] \\ &= \frac{1}{3-j\sqrt{3}} [\mathbf{V}_1(1-j\sqrt{3}+2a) + \mathbf{V}_2(1-j\sqrt{3}+2a^2)] \\ &= \frac{1}{3-j\sqrt{3}} [\mathbf{V}_2(-j2\sqrt{3})] \therefore V_{PQ} = V_2 \end{aligned}$$

### 23.12. Measurement of Symmetrical Components of Circuits

The apparatus consists of two identical current transformers, two impedances of the same ohmic value, one being more inductive than the other to the extent that its phase angle is  $60^\circ$  greater and two identical ammeters  $A_1$  greater and two identical ammeters  $A_1$  and  $A_2$  as shown in Fig. 23.8 (a). It can be shown that  $A_1$  reads positive-sequence current only while  $A_2$  reads negative-sequence current only. If the turn ratio of the current transformer is  $K$ , then keeping in mind that zero-sequence component is zero, we have

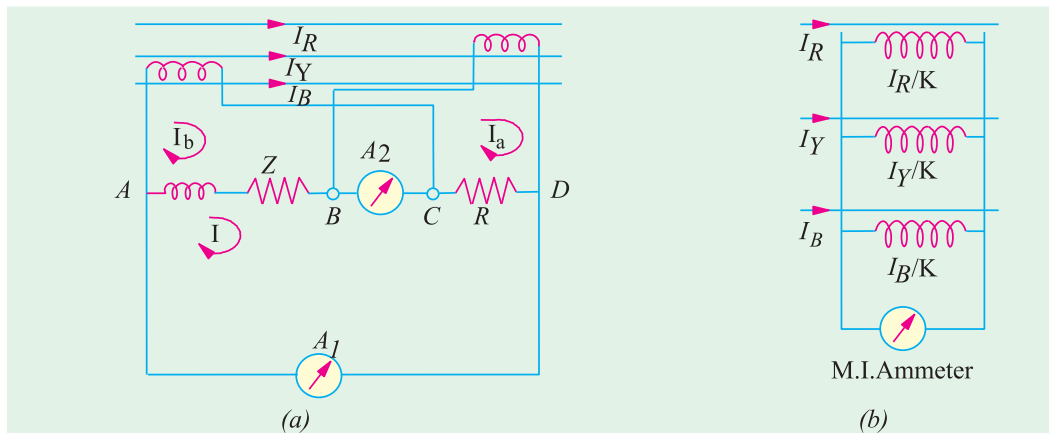


Fig. 23.8

$$\mathbf{I}_a = \mathbf{I}_R / K = (\mathbf{I}_1 + \mathbf{I}_2) / K$$

where  $\mathbf{I}_1$  and  $\mathbf{I}_2$  are the positive and negative-sequence components of the line current respectively.

Similarly,

$$\mathbf{I}_b = \mathbf{I}_Y / K = (a^2 \mathbf{I}_1 + a \mathbf{I}_2) / K$$

If the impedance of each ammeter is  $R_A + jX_A$ , then impedance between points  $B$  and  $D$  is

$$\mathbf{Z}_{BD} = (R \ R_A \ jX_A)$$

The value of  $\mathbf{Z}$  is so chosen that

$$\mathbf{Z}_{AC} = \mathbf{Z} \ R_A \ jX_A = (R \ R_A \ jX_A) \ 60 \ \mathbf{Z}_{BD} \ 60$$

For finding the current read by  $A_1$ , imagine a break at point  $X$ . Thevenin voltage across the break  $X$  is



Current transformers

$V_{th} = I_b Z_{AC} + I_a Z_{BD} = I_b Z_{BD} = I_b Z_{BD} \angle 60^\circ + I_a Z_{BD} = (I_b \angle 60^\circ + I_a) Z_{BD}$   
 Total impedance in series with this Thevenin voltage is

$$Z_T = Z_{AC} + Z_{BD} + Z_{BD} = 60 + Z_{BD} + \frac{3}{2} j \frac{\sqrt{3}}{2} Z_{BD}$$

The current flowing normally through the wire in which a break has been imagined is

$$\begin{aligned} I &= \frac{V_{th}}{Z_T} = \frac{I_b \angle 60^\circ + I_a}{(3/2) + j(\sqrt{3}/2)} = \frac{1}{K} \frac{(a^2 I_1 + a I_2) \angle 60^\circ + I_1 + I_2}{\sqrt{3} \angle 30^\circ} \\ &= \frac{1}{K} \frac{I_1 (1 \ 300 \ 1) \ I_2 (1 \ 180 \ 1)}{\sqrt{3} \ 30} = \frac{I_1}{K} \ 60 \end{aligned}$$

It means that  $A_1$  reads positive-sequence current only. The ammeter  $A_2$  reads current which is

$$\begin{aligned} &= I_a \ I_b \ I = \frac{1}{K} (I_1 + I_2 + I_1 \angle 240^\circ + I_2 \angle 120^\circ - I_2 \angle -60^\circ) \\ &= \frac{I_1}{K} \left( 1 - \frac{1}{2} - j \frac{\sqrt{3}}{2} - \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) + \frac{I_2}{K} (1 + 1 \angle 120^\circ) = \frac{I_2}{K} \angle 60^\circ \end{aligned}$$

In other words,  $A_2$  reads negative-sequence current only.

Now, it will be shown that the reading of the moving-iron ammeter of Fig. 23-8(b) is proportional to the zero-sequence component.

$$\begin{aligned} I_0 &= \frac{1}{3} (I_R + I_Y + I_B) = \frac{K}{3} \left( \frac{I_R}{K} + \frac{I_Y}{K} + \frac{I_B}{K} \right) \\ &= \frac{K}{3} \times (\text{current through the ammeter}) = \frac{K}{3} \times \text{ammeter reading.} \end{aligned}$$

It is obvious that for  $I_0$  to be present, the system must be 3-phase, 4-wire. However, when the fourth wire is available, then  $I_0$  may be found directly by finding the neutral current  $I_N$ . In that case  $I_0 = \frac{1}{3} I_N$ .

### 23.13. Measurement of Positive and Negative-sequence \*Voltages

With reference to Fig. 23.9 (a), it can be shown that the three voltmeters indicate only the positive-sequence component of the 3-phase system.

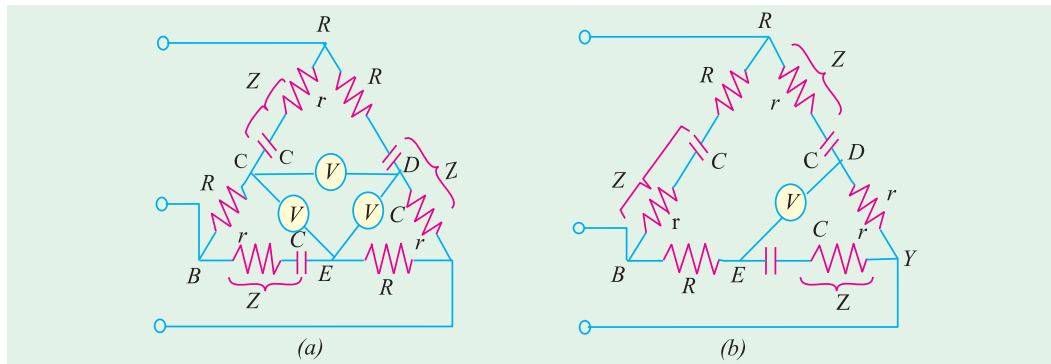


Fig. 23.9

\* It is supposed to possess infinite impedance.



The line-to-neutral voltage can be written (with reference to the red phase) as

$$\mathbf{V}_{RN} = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{YN} = a^2\mathbf{V}_1 + a\mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{BN} = a\mathbf{V}_1 + a^2\mathbf{V}_2 + \mathbf{V}_0$$

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = (1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2$$

$$\mathbf{V}_{YB} = \mathbf{V}_{YN} + \mathbf{V}_{NB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = (a^2-a)\mathbf{V}_1 + (a-a^2)\mathbf{V}_2$$

$$\therefore \mathbf{V}_{DY} = \frac{\mathbf{V}_{RY}(r \ 1/j \ C)}{(R \ r \ 1/j \ C)} \quad \mathbf{V}_{YE} = \frac{\mathbf{V}_{YB} \cdot R}{(R \ r \ 1/j \ C)}$$

$$\therefore \mathbf{V}_{DE} = \frac{(r \ 1/j \ C)}{(R \ r \ 1/j \ C)} \mathbf{V}_{RY} - \frac{R}{(r \ 1/j \ C)} \mathbf{V}_{YB}$$

The different equation of the bridge circuit are so chosen that

$$\frac{R}{r + j1/\omega C} = -a^2 = \frac{1}{2} + j \frac{\sqrt{3}}{2} \quad \text{or} \quad R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} + \frac{1}{j2\omega C} + j \frac{\sqrt{3}r}{2}$$

Equating the  $j$ -terms or quadrature terms on both sides, we have

$$0 = \frac{1}{j2\omega C} + j \frac{\sqrt{3}}{2} \quad \therefore \frac{1}{\omega C} = \sqrt{3}r \quad \dots (i)$$

Similarly, equating the reference or real terms, we have

$$R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} = \frac{r}{2} + \frac{3r}{2} = 2r \quad \dots (ii)$$

$$\therefore \frac{r + 1/j\omega C}{R + r + 1/\omega C} = \frac{r - j\sqrt{3}r}{3r - j\sqrt{3}r} = \frac{r(1 - j\sqrt{3})}{r(3 - j\sqrt{3})} = \frac{1}{\sqrt{3}} \angle -30^\circ$$

$$\text{and} \quad \mathbf{V}_{DE} = \frac{1 \angle -30^\circ}{\sqrt{3}} [(1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2 - a^2(a^2-a)\mathbf{V}_1 - a^2(a-a^2)\mathbf{V}_2]$$

$$= \frac{1 \angle -30^\circ}{\sqrt{3}} (1-a^2 - a^4 + a^3)\mathbf{V}_1$$

$$[1 - a = a^2(a - a^2)] = \sqrt{3}\mathbf{V}_1 \angle -30^\circ$$

Hence, the voltmeter connected between points  $D$  and  $E$  measures  $\sqrt{3}$  times the positive sequence component of the phase voltage. So do the other two voltmeters.

In Fig. 23.9 (b), the elements have been reversed. It can be shown that provided the same relation is maintained between the elements, the high impedance voltmeter measures  $\sqrt{3}$  times the negative-sequence component of phase voltage.

### 23.14. Measurement of Zero-sequence Component of Voltage

The zero-sequence voltage is given by

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN})$$

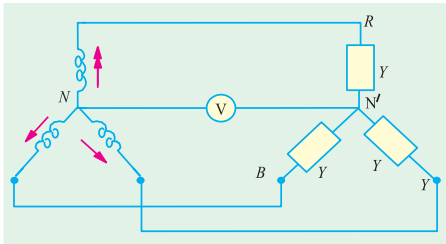


Fig. 23.10

Fig. 23.10 indicates one method of measuring  $V_0$ . As seen

$$V_{N'N} = \frac{V_{RN}Y + V_{YN}Y + V_{BN}Y}{3Y}$$

$$= \frac{1}{3}(V_{RN} + V_{YN} + V_{BN}) = V_0$$

Hence, voltmeter connected between the neutral points measures the zero-sequence component of the voltage.

### Tutorial Problem No. 23.2

1. Explain the essential features in the representation of an unsymmetrical three-phase system of voltages or currents by symmetrical components.

In a three-phase system, the three line currents are;  $I_R = (30 + j50)A$ ,  $I_Y = (15 - j45)A$ ,  $I_B = (-40 + j70)A$ .

A. Determine the values of the positive, negative and zero sequence components.

[52.2 A, 19.3 A, 25.1 A] (London Univ.)

2. The phase voltages of a three-phase, four-wire system are;  $V_{RN} = (200 + j0) V$ ,  $V_{YN} = (0 - j200) V$ ,  $V_{BN} = (-100 + j100) V$ . Show that these voltages can be replaced by symmetrical components of positive, negative and zero sequence and calculate the magnitude of each component.

Sketch vector diagrams representing the positive, negative and zero-sequence components for each of the three phases.

[176 V, 38 V, 47.1 V] (London Univ.)

3. Explain, with the aid of a diagram of connections, a method of measuring the symmetrical components of the currents in an unbalanced 3-phase, 3-wire system.

If in such a system, the line currents, in amperes, are

$$I_R = (10 - j2), I_Y = (-2 - j4), I_B = (-8 + j6)$$

Calculate their symmetrical components.

$$[I_{R0} = I_{Y0} = I_{B0} = 0; I_{R1} = 7.89 + j0.732; I_{Y1} = a^2 I_{R1}; I_{B1} = a I_{R1}; I_{R2} = 2.113 - j2.732; I_{Y2} = a I_{R2}; I_{B2} = a^2 I_{R2}]$$

### OBJECTIVE TESTS – 23

- The method of symmetrical components is very useful for
  - solving unbalanced polyphase circuits
  - analysing the performance of 3-phase electrical machinery
  - calculating currents resulting from unbalanced faults
  - all of the above
- An unbalanced system of 3-phase voltages having *RYB* sequence actually consists of
  - a positive-sequence component
  - a negative-sequence component
  - a zero-sequence component
  - all of the above.
- The zero-sequence component of the unbalanced 3-phase system of vectors  $V_A$ ,  $V_B$  and  $V_C$  is .... of their vector sum.
  - one-third
  - one-half
  - two-third
  - one-fourth
- In the case of an unbalanced star-connected load supplied from an unbalanced 3- $\phi$ , 3 wire system, load currents will consist of
  - positive-sequence components
  - negative-sequence components
  - zero-sequence components
  - only (a) and (b).

### ANSWERS

1. (d) 2. (d) 3. (a) 4. (d)

# CHAPTER 24

## Learning Objectives

- Preference for Electricity
- Comparison of Sources of Power
- Sources for Generation of Electricity
- Brief Aspects of Electrical Energy Systems
- Utility and Consumers
- Why is the Three-phase a.c. System Most Popular?
- Cost of Generation
- Staggering of Loads during peak-demand Hours
- Classifications of Power Transmission
- Selecting A.C. Transmission Voltage for a Particular Case
- Conventional Sources of Electrical Energy
- Steam Power Stations (Coal-fired)
- Nuclear Power Stations
- Advantages of Nuclear Generation
- Disadvantages
- Hydroelectric Generation
- Non-Conventional Energy Sources
- Photo Voltaic Cells (P.V. Cells or SOLAR Cells)
- Fuel Cells
- Principle of Operation
- Chemical Process (with Acidic Electrolyte)
- Schematic Diagram
- Array for Large outputs
- High Lights
- Wind Power
- Background
- Basic Scheme
- Indian Scenario

## INTRODUCTION TO ELECTRICAL ENERGY GENERATION



Electricity is generated at power plants and transmitted to different substations from where it will be distributed to individual customers

### 24.1. Preference for Electricity

Energy is vital for all living-beings on earth. Modern life-style has further increased its importance, since a faster life means faster transport, faster communication, and faster manufacturing processes. All these lead to an increase in energy required for all those modern systems.

Arising out of comparison of status of nations, the progress is related in terms of per capita consumption of electrical energy (i.e. kWh consumed per person per year). At present, this parameter for India is about 300, for UK it is 12 to 15 times more, and for USA, it is about 30 times more.

It simply means that Electrical energy is the most popular form of energy, whether we require it in the usable thermal form (= heating applications), in mechanical form (= electrical motor-applications in Industries), for lighting purposes (= illumination systems), or for transportation systems.

Following are the main reasons for its popularity.

1. Cleaner environments for user
2. Higher efficiency
3. Better controllability
4. Easier bulk-power, long-distance transportation of power using overhead transmission or underground cables
5. Most versatile devices of energy conversions from Electrical to other forms are available for different purposes, such as thermal, illumination, mechanical, sound, chemical, etc.

### 24.2. Comparison of Sources of Power

While selecting a method of generating electricity, following factors are taken into account for purposes of comparison:

- (a) Initial cost: For a given rating of a unit (in the minds of planners), investment must be known. Naturally, lower the initial cost, better it is.
- (b) Running Cost:- To produce a given amount of electrical energy, the cost of conversion process (including proportional cost of maintenance/repairs of the system) has to be known.
- (c) Limitations:- Whether a particular resource is available, whether a unit size of required rating is available from a single unit or from an array of large number of units, and whether a particular method of generation is techno-economically viable and is time-proven, are typical queries related to the limitations of the concerned method.
- (d) (1) perpetuity, (2) efficiency, (3) reliability, (4) cleanliness and (5) simplicity.

It is naturally desirable that the source must have perpetuity (= be of endless duration), high conversion efficiency, and reliability (in terms of availability in appropriate quantity). The energy conversion must be through a cleaner process (specially from the view- points of toxicity, pollution or any other hazardous side effects). Further, a simpler overall system is always preferred with regards to maintenance/repairs problems and is supposed to be more reliable.

### 24.3. Sources for Generation of Electricity

Following types of resources are available for generating electrical energy (No doubt, this list can be extended to include some more up-coming resources. The following list, however, gives the popular and potential resources).

#### (a) Conventional methods

- (a) **Thermal:** Thermal energy (from fossil fuels) or Nuclear Energy used for producing steam for turbines which drive the alternators (= rotating a.c. generators).
- (b) **Hydro-electric:** Potential of water stored at higher altitudes is utilized as it is passed through water-turbines which drive the alternators.

#### (b) Non-conventional methods

- (c) **Wind power:** High velocities of wind (in some areas) are utilized in driving wind turbines coupled to alternators. Wind power has a main advantage of having zero production cost. The cost of the equipment and the limit of generating-unit-rating is suitable for a particular



location (= geographically) are the important constraints. This method has exclusive advantages of being pollution free and renewable. It is available in plentiful quantity, at certain places. It suffers from the disadvantages of its availability being uncertain (since dependent on nature) and the control being complex (since wind-velocity has wide range of variation, as an input, and the output required is at constant voltage and constant frequency). Single large-power units cannot be planned due to techno-economic considerations.



Use of Wind Powder

- (d) **Fuel cells:** These are devices which enable direct conversion of energy, chemically, into electrical form. This is an up-coming technology and has a special merit of being pollution-free and noise-free. It is yet to become popular for bulk-power generation.
- (e) **Photo voltaic cells:** These directly convert solar energy into electrical energy through a chemical action taking place in solar cells. These operate based on the photo-voltaic effect, which develops an emf on absorption of ionizing radiation from Sun.

### Power Scenario In India:-

Following approximate statistical data give an idea about some aspects in this regard.

Total installed capacity	:	150,000 MW
Hydropower	:	50,000 MW
Nuclear	:	10,000 MW
Thermal (fossil fuels)	:	80,000 MW
Other methods	:	10,000 MW

Other methods include partly exploited Potential such as Wind, Solar, Co-generation, Methods using Bio-fuels, etc.

## 24.4. Brief Aspects of Electrical Energy Systems

### 24.4.1. Utility and Consumers

At generating stations, power is generated at the best locations. Load-centres are generally away from these. Generation-units and Loads are connected by transmission systems. Thus, the energy system is divided into two main parts.

- (a) Utility (including sources and transmission network) and,  
 (b) Consumers (who utilize the electrical energy)

### 24.4.2. Why is the three- phase a. c. system most popular?

- (a) It is well known that a.c. generation is simpler (than d.c. generation through electrical machines because of absence of commutators in a.c. machines). Further, mechanical commutation system in d.c. machines sets an upper limit of their size, while the rating of the individual generators in **modern power stations** is too large, say about 1500 times the rating of a single largest feasible d.c. machine. A.C. further facilitates in stepping down or stepping-up of a voltage to suit a particular requirement, with the help of a simple device, the well-known transformer.
- (b) Changing over from a.c. to d.c. is very easy these days due to the rectifiers of sufficiently high power ratings, so that a wide range of d.c. – applications can now be catered to.
- (c) As the number of phases goes on increasing the power-output (from a device using a given quantity of active material, namely, that used for core and for winding) increases, but the number of circuits (*i.e.* connecting lines/ wires, switches, etc) also increases. These two points are contradictory. A choice will be in favour of such a number (of phases) which will be high enough from power-output point of view but low enough from viewpoints

of complexity of connecting/controlling large number of phases. There is a Golden compromise. It has resulted into popularity of the three-phase a. c. systems over the entire time period. Now it has almost become a standard practice for all purposes.

#### 24.4.3. Cost of Generation

Cost of generation for one unit of electrical energy depends on the method of generation, formulae worked to assess its running cost under the specified conditions, and the cost of transmission line loss to transport power upto the load. These days, a modern utility (= electricity board) has a large number of generators sharing the responsibility of supplying power to all the customers connected to the Grid (= common supply-network). Then, for an increase in load-demand, at a known location, the most economical generating unit is to be identified and that unit should be monitored to meet the increased demand.

#### 24.4.4. Staggering of Loads during Peak-demand Hours

Incentives to consumers (by way of supplying at reduced rates during light load hours e.g. Late night 'hours, after noon hours) help in even demand throughout the day.

#### 24.4.5. Classifications of Power Transmission

- (a) Using underground cables or using overhead transmission lines.
- (b) Extra High Voltage A.C.-versus-Extra High Voltage D.C. transmission systems.

#### 24.4.6. Selecting A.C. Transmission Voltage for a Particular Case

In general, for transporting a given power of  $VI$  watts, either  $V$  can be high or  $V$  can be low. Accordingly, the current can be either low or high respectively. Higher voltage means higher cost of insulation, and larger clearances. Higher current means larger cross section of conductors. Considering these together, the most economical voltage has to be found out for a particular requirement.

Kelvin's Laws give a guideline for this. (These are discussed in Art. 47.21 in Vol. III, of this book).

### 24.5. Conventional Sources of Electrical Energy

Thermal (coal, gas, nuclear) and hydro-generations are the main conventional methods of generation of Electrical Energy. These enjoy the advantages of reaching perfections in technologies for these processes. Further, single units rated at large power-outputs can be manufactured along with main components, auxiliaries and switch-gear due to vast experiences during the past century. These are efficient and economical.

These suffer from the disadvantages listed below:

1. The fuels are likely to be depleted in near future, forcing us to conserve them and find alternative resources.
2. Toxic, hazardous fumes and residues pollute the environment.
3. Overall conversion efficiency is poor.
4. Generally, these are located at remote places with respect to main load centres, increasing the transmission costs and reducing the system efficiency.
5. Maintenance costs are high.

Out of these, only two such types will be dealt here, which have a steam turbine working as the prime mover. While remaining two use Internal Combustion Engines (I.C. engines) or Gas turbine as the prime mover, and these will *not* be dealt with, in this Introductory treatment.

The steam-turbine driven systems are briefly discussed below.

#### 24.5.1. Steam Power Stations (Coal-fired)

India has rich stocks of coal as a natural resource. Chemical energy stored within the coal is finally transformed into Electrical energy through the process of these stations. Heat released by the combustion of coal produces steam in a boiler at elevated temperatures and pressures. It is then passed through steam turbines, which drive the alternator, the output of which is the electrical energy.

Figure 24.1 shows a simple schematic diagram of a modern coal-fired thermal station.

In India, coal is generally of low grade containing ash upto 40 %. It poses two problems.

(i) Calorific value is low and hence system efficiency can be increased only by additional processes like pulverizing the coal and using oil firing to start with, (ii) large volumes of ash have to be handled after ensuring that ash is extracted to the maximum possible extent (upto 99 %) by using electrostatic precipitators, before flue gases from boiler are finally passed on to the atmosphere.

Coal is burnt in the boiler. This heat converts water into steam when passed through the boiler tubes. Modern plants have super heaters to raise the temperature and pressure of steam so that plant efficiency is increased. Condenser and cooling tower deal with steam coming out of turbine. Here, maximum heat is extracted from steam (which then takes the form of water) to pre-heat the incoming water and also to recycle the water for its best utilization.

Steam-turbine receives controlled steam from boiler and converts its energy into mechanical energy which drives the 3-ph a. c. generator (=alternator). The alternator delivers electrical energy, at its rated voltage (which may be between 11 to 30 kV). Through a circuit breaker, the step up transformer is supplied. Considering the bulk-power to be transmitted over long distances, the secondary rating

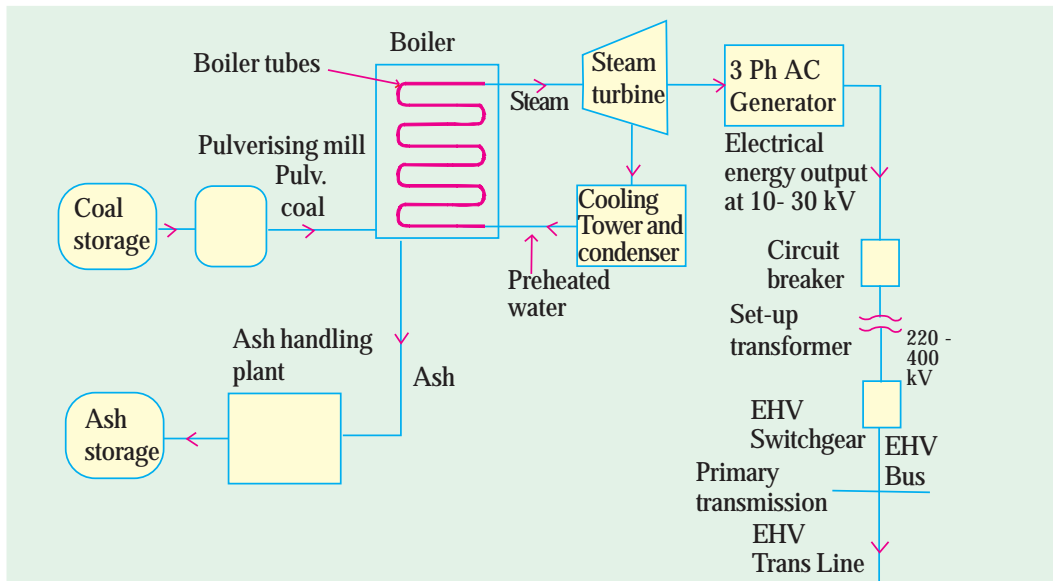


Fig. 24.1. Schematic Diagram of a Coal-fired Thermal Station

of this transformer may be 220 or 400 kV (as per figures for India). Through Extra-High Voltage Switch gear, the Bus is energized and the EHV primary transmission line can transport power to the Load centres connected to it.

A modern coal-fired thermal power station consumes about 10 % of its power for supplying to the Auxiliaries. These are mainly as follows.

- (a) Main-exciter for alternator.
- (b) Water pumps.
- (c) Fans: Forced draught and Induced draught fans for Pre-heaters and Chimney.
- (d) Coal handling plant including pulverising mill.
- (e) Ash handling plant including Electrolytic Precipitator.

Naturally, whenever such a station is to be brought into operation (either at commissioning or after repairs/maintenance schedule) the power required for the auxiliaries has to be supplied by the grid. Once the system is energized fully, it will look after supplying power to its own auxiliaries.

### Merits of Coal-fired thermal stations

1. Fuel (=coal) is cheap.
2. Less initial cost is required.
3. It requires less space.
4. As a combination of all above points, the cost of generating unit of electrical energy is less.

**Demerits**

1. Atmospheric pollution is considerable.
2. Coal may have to be transported over long distances, in some cases, after some years, and then the energy cost may be quite high.

**24.5.2. Nuclear Power Stations**

Nuclear energy is available as a result of fission reaction. In a typical system, Uranium 235 is bombarded with neutrons and Heat energy is released. In chain-reaction, these release more neutrons, since more Uranium 235 atoms are fissioned. Speeds of Neutrons must be reduced to critical speeds for the chain reaction to take place. Moderators (= speed-reducing agents like graphite, heavy water, etc). are used for this purpose. Nuclear fuel rods (of Uranium 235) must be embedded in speed-reducing agents. Further, control rods (made of cadmium) are required since they are strong neutron-absorbers and help in finely regulating this reaction so that power control of the generator is precisely obtainable. When control rods are pulled out and are away from fuel rods, intensity of chain reaction increases, which increases the power output of the system. While if they are pushed in and closer to the fuel rods, the power-output decreases. Thus, the electrical load demand on the generator decides (automatically) the control-rod positions through a very sophisticated control system.

Fig. 24.2 shows a basic scheme of such a Nuclear power-station.

**Advantages of Nuclear Generation**

1. Quantity of fuel required is small for generating a given amount of electrical energy, compared to that with other fuels.
2. It is more reliable, cheaper for running cost, and is efficient when operated at rated capacity.

**Disadvantages**

1. Fuel is expensive and not abundantly available everywhere.
2. It has high capital cost.
3. Maintenance charges are high.
4. Nuclear waste disposal is a problem.

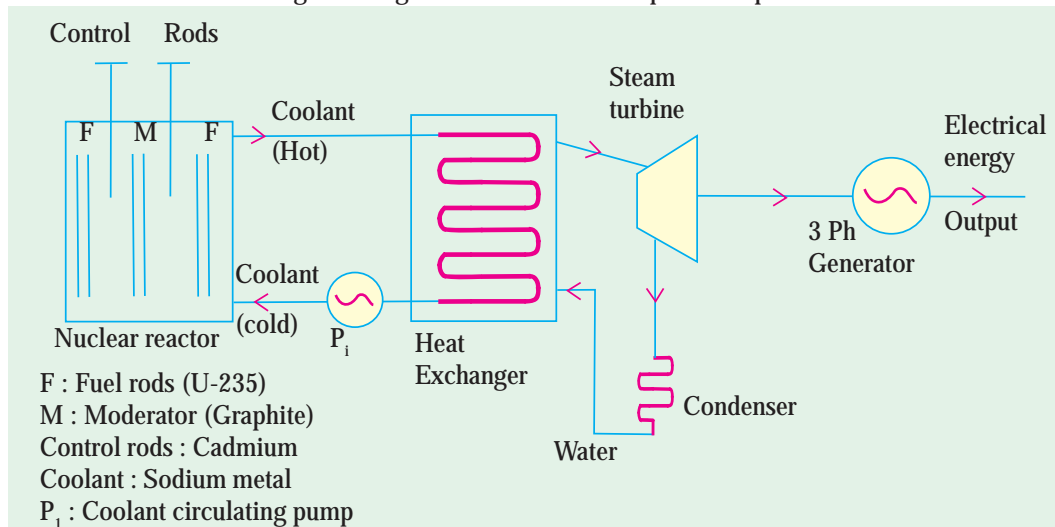


Fig. 24.2. Schematic Diagram of a Nuclear Power Station indicating Main Components

**24.5.3. Hydroelectric Generation**

Water-reservoir at higher altitudes is a pre-requisite for this purpose. Power-house is located at a lower level. The difference in these two levels is known as "Head."

Based on the "Heads", the Hydroelectric stations are categorized below:

1. Low head up to 60 metres.
2. Medium head between 60 and 300 metres.
3. High heads above 300 metres.



In this method of generation, water from higher height is passed through penstock as controlled in the valve-house, into the water turbine. Thus, potential energy of water stored at higher altitudes is first converted into Kinetic energy. As the water reaches the turbine, it gains speed after losing the Potential energy. Kinetic energy of this speedy water drives the water turbine, which converts this into mechanical output. It drives the coupled generator, which gives Electrical energy output.

A schematic diagram of such a system is shown in fig.

The valve house has a controlling valve (=main sluice valve) and a protecting valve (= an automatic, isolating, “butterfly” type valve). As is obvious, power control is done by the main sluice valve, while “butterfly” valve comes into action if water flows in opposite direction as a result of a sudden drop in load on the generator. Otherwise, the penstock is subjected to extreme strains and it has a tendency to burst due to pressure of water as a result of sudden load reduction.

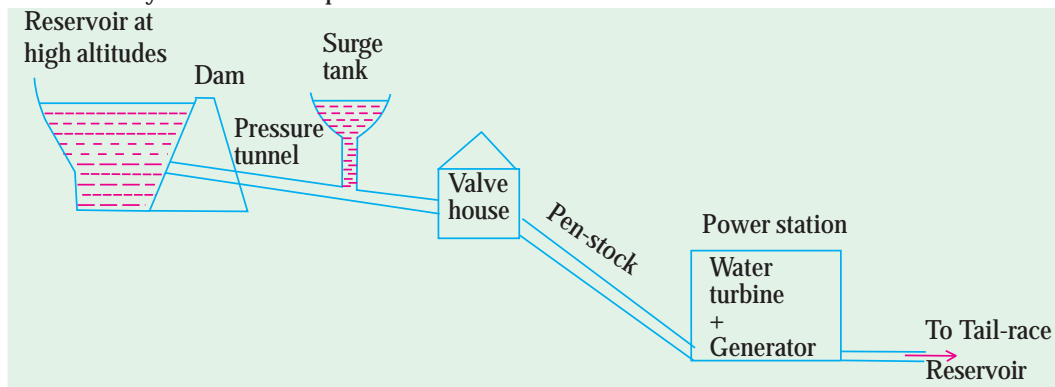


Fig. 24.3. Schematic of a typical Hydroelectric station

**Note:** Variation of Head (low-, medium-, or high-head) will affect the block-schematic

After doing the work (of imparting its energy to the water turbine), the water is allowed to pass into the tail-race reservoir.

The water turbines are essentially low-speed prime movers. In that, the best operating speed is dependent on the head. Alternators coupled to water turbines thus have large number of poles (since  $P = 120 f/N$ ). Such alternators have the Salient-Pole type rotor.

There are different types of water turbines suitable for different cases (i.e. Heads, Power rating, Load-variation curve, etc). Since, this is only an elementary treatment, these aspects will not be discussed here.

## 24.6. Non-Conventional Energy Sources

Considering the previously discussed thermal methods of conventional energy-generations, it is necessary to understand the non-conventional energy sources, since they have two points in their favour.

- (a) Non-polluting processes are used.
- (b) Perpetuity and renewability of the main source (which is a natural atmospheric resource) generally exists.

The non-conventional energy sources are further advantageous due to virtually zero running cost, since wind energy or solar energy is the input-source of power.

However, they are disadvantageous due to high initial cost (per MW of installed capacity) and due to uncertainty resulting out of weather changes. For example, dense clouds (or night hours) lead to non-availability of solar energy. Similarly, “still-air” condition means no possibility of wind power generation, and during stormy weathers, wind turbines cannot be kept in operation (due to dangerously high speeds they would attain if kept in operation).

### 24.6.1. Photo Voltaic Cells (P.V. Cells or SOLAR Cells)

When ionized solar radiation is incident on a semi-conductor diode, energy conversion can take place with a voltage of 0.5 to 1 volt (d.c.) and a current density of 20-40 mA/cm<sup>2</sup>, depending on the

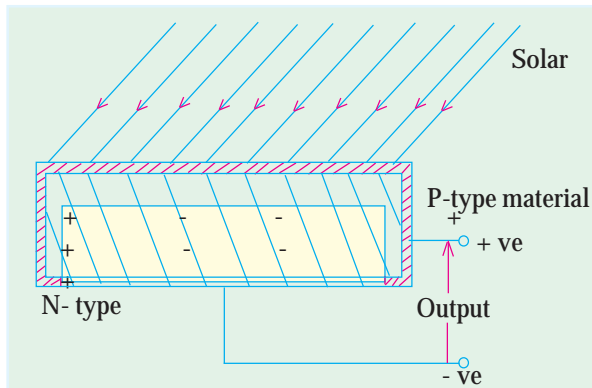


Fig. 24.4. Photo Voltaic (or Solar Cell)

materials used and the conditions of Sunlight. Area of these solar cells decides the current output. An array of large number of such diodes (i.e. Solar cells) results into higher d.c. output voltage.

Since, the final form of electrical energy required is generally an alternating current, it is realized from d.c. using inverters.

At quite a few locations in India, for realizing few hundred kilowatts of power-rating, huge arrays are accommodated in horizontal as well as vertical stacks, so that land area required is not too vast. Electrically, they are connected in series and in parallel

combinations of cells so that rated voltage and current are realized.

Just to understand the principle of operation of solar cells, let a semi-conductor diode receive ionized radiation from Sun, as in fig. 24.4.

Typical materials used for these cells are: material doped with boron, cadmium sulphide, gallium-arsenide, etc. Their choice is mainly decided by conversion efficiency. Best material may lead to the efficiency being typically 15%. Since solar energy is available free of cost, this low-efficiency does not matter.

This method suffers from the disadvantages of having high initial cost and uncertainty (since dependent on weather conditions) including non operative night periods. Main advantages are: **(i)** no running cost (however, replacements of components may be a botheration), **(ii)** no pollution, **(iii)** location can be near the load (hence transportation of power is not required over long distances), **(iv)** since natural source is involved, it is perpetual.

Individual stations using solar cells are in operation with ratings of the order of 250-1000 kW. With manufacturing costs of semi-conductor devices going down and with the advent of better and better quality of cells which will be available in future, this method of generation has bright prospects.

## 24.6.2. Fuel Cells

### 24.6.2.1. Principle of Operation

In Fuel cells, negative porous electrode is fed by hydrogen and the positive porous electrode is fed by oxygen. Both the electrodes are immersed in an electrolyte. The porous electrodes are made of such a conducting material that both the fuel (oxygen and hydrogen) and the electrolyte can pass through them. Such a material for electrodes is nickel. The electrolyte is a solution of sulphuric acid or potassium hydroxide. The electrodes have a catalyst (= platinum or sintered nickel) which break the fuel compound into more reactive atoms.

### 24.6.2.2. Chemical Process (with Acidic Electrolyte)



These hydrogen ions enter the solution (=electrolyte) leaving behind electrons which pass through external circuit to the positive electrode.



Thus, the combination of Hydrogen and Oxygen results into water at the positive electrode. Water is the waste-product of the cell, which is harmless. The process is, thus, pollution free. There is no source of energy required, since the process is basically “**chemical**” in nature.

### 24.6.2.3. Schematic Diagram.

It is given in fig 24.5

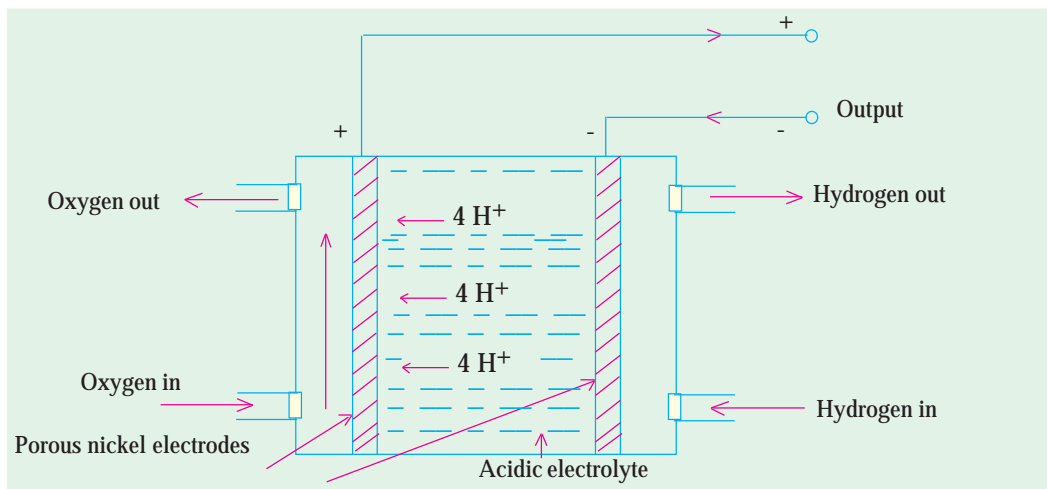


Fig. 24.5. Fuel Cell

### 24.6.2.4. Array for Large outputs

A fuel cell has a d. c. output voltage typically of 1.23 volts at normal atmospheric pressure and temperature. Raising pressure and temperature increases this voltage. To realize large output parameters (= voltage, current, and hence power), an array of a large number of fuel cells (connected in series as well as in parallel) is made. Voltage levels of 100 to 1000 V and power levels in kilowatts can be realized.

### 24.6.2.5. High Lights

1. Pollution-free, noiseless.
2. No outside source of energy is required.
3. Efficient.
4. No restriction on location
  - (a) High initial cost.
  - (b) Working life is short.

**Note:** Solar energy can also be used for generating electrical energy through an intermediate stage of producing steam, which is used later for driving an alternator. However, this method is not discussed here.

## 24.6.3. Wind Power

### 24.6.3.1. Background

Wind power has been in use for serving the mankind, since centuries through what has been popularly known as “Wind-mills.” There is no “electrical” stage of energy in old-styled uses where wind-velocity is directly used for performing the jobs such as wheat-grinding, pumping water for irrigation, sailing vessels, etc. It enjoys the advantages of being plentiful, inexhaustible, renewable and non-polluting, over and above being cheap for running costs. It suffers from disadvantages of being unreliable, and being economically un-viable for large power generation. In India, a large number of such units with small and medium power ratings (up to 100 kW) are already in operation mainly in coastal or hilly areas. With the modern system, it is now preferred to have suitable power-control circuits on the output side of wind-generators so that these can pump energy into low voltage lines of the grid over a wide range of variation of wind speeds.

### 24.6.3.2. Basic Scheme

A large variety of wind-turbines naturally exist arising out of large variation in wind-pattern

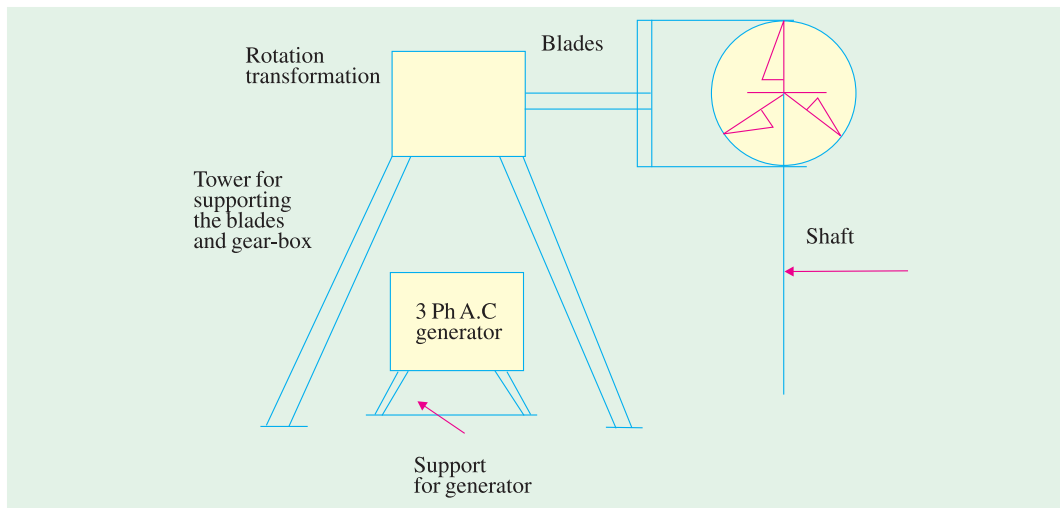


Fig. 24.6 (a) . Wind-generation, a Schematic view

Fig. 24.6 (b). Part-side-view to show a typical three bladed wind turbine

all over and out of different manufacturers producing systems with different designs. Since the aim here is to understand the basic system, only one type of system is presented here.

In Fig. 24.6 (a) an arrangement wherein, a horizontal three-bladed system is shown mounted on a tower. Through rotation-transformation using gears to step up the speed and to link the horizontal axis of turbine with vertical axis of generator.

The speed of wind varies, as such turbine speed also varies so that output frequency and voltage of three-phase alternator vary over a wide range. Further, its waveform is also a distorted one. To increase its utility, it is necessary to modulate (through proper power-control) to derive line-frequency constant voltage output and hook-up to local grid for pumping the available wind-energy into it. This is schematically represented in fig 24.7.

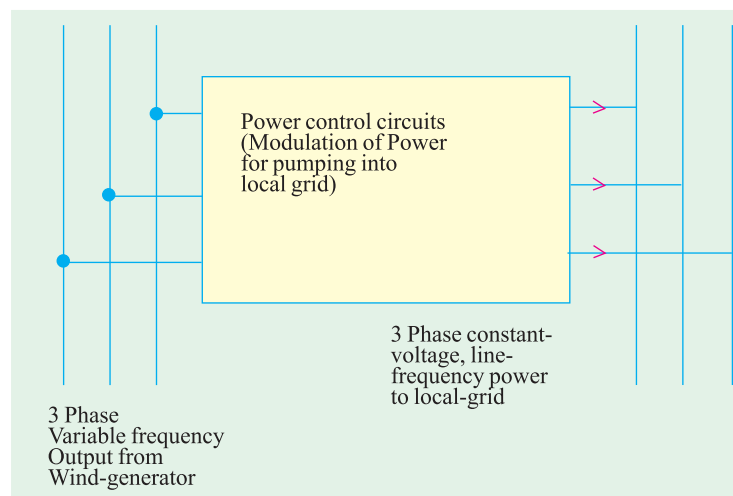


Fig. 24.7

### 24.6.3.3. Indian Scenario

Wind farms have been located where a large number of wind generators of ratings of few hundred kilowatts are in operation. For every unit, there is a safe wind - speed zone. If the wind-speed is below this, there is no appreciable power output, hence, it is better that the system is *not* brought into operation. If the wind speeds are too high, it is mechanically unsafe and hence it is *not* to be operated, even if the energy available is higher. This is decided by automated system. Such forms are located in coastal regions and in hilly areas. Because of the metering of energy received by the local grid, the investor can get a good return through payment from the grid-authorities. Hence, this has become a medium-level and attractive industrial investment.

# CHAPTER 25

## Learning Objectives

- Introduction
- Salient aspects of conversions
- Energy-balance
- Magnetic-field system: Energy and Co-energy
- Linear System
- A Simple Electro-mechanical System
- In terms of Field Energy
- In terms of field Co-energy
- Energy in terms of Electrical parameters
- Rotary Motion
- Description of Simple System
- Energy stored in the coils
- Different Categories
- One coil each on Stator and on Rotor
- Vital Role of Air-gap
- Statically induced emf and Dynamically induced emf.

## **ELEMENTS OF ELECTRO- MECHANICAL ENERGY CONVERSION**



An electric motor is a machine which converts electrical energy into mechanical (rotational or kinetic) energy

### 25.1. Introduction

“Energy can *neither* be created *nor* be destroyed”. We can only change its forms, using appropriate energy-conversion processes.

An interesting aspect about the energy in “Electrical form” is that neither it is so available *directly from nature* nor it is required to be finally consumed in that form. Still, it is the most popular form of Energy, since it can be transported at remote Load-locations, for optimum utilization of resources. Further, technological progress has now made it possible to devise Electrical-Power-modulation systems so flexible and controllable that modern systems tend to be energy-efficient, with increase in life-span of main equipment and the associated auxiliary components (like switches, connecting cables, contactors, etc.) since it is now



Generator

possible to avoid overstrain (= over-currents or over-voltage) on the system. This means a lot for the total production process (for which electrical energy is being used) since the quality of the production improves, and plant-maintenance is minimal. Energy-conversion systems then assume still higher importance.

Energy conversion takes place between well known pairs of forms of Energy: Electrical ↔ Chemical, Electrical ↔ Thermal, Electrical ↔ Optical, Electrical ↔ Sound, and Electrical ↔ Mechanical are the common forms with numerous varieties of engineering - applications. Electrical ↔ mechanical conversion is the focus of discussion in this chapter.

The elements of electro-mechanical energy conversion shall deal with basic principles and systems dealing with this aspect. Purpose of the study is to have a general approach to understand to design, and later to modify the system with the help of modern technologies, for overall improvisation.

It is necessary to be aware about:

- (a) basic conditions to be fulfilled by the conversion system.
- (b) methods for innovating the conversion systems.

Electromechanical energy-conversion finds applications in following categories of systems:

- (a) transducers: Devices for obtaining signals for measurement / control,
- (b) force-producing devices: Solenoid-actuators, relays, electromagnets,
- (c) devices for continuous-energy-conversion: Motors / Generators

These systems have different configurations. But the principles of their working are common. Understanding these principles enables us to analyze / design / improvise / innovate such systems. As a result of such development, newer types of motors and the associated modern power controllers have recently been manufactured and become popular. Controllers using power-

electronics switching devices offer energy-efficient, user-friendly, and high-performance drives. Their initial investment may be larger but two important parameters justify their use: *(i)* Considerable energy is saved, resulting into payback periods as short as 18-24 months, *(ii)* The controllers ensure to limit the currents to pre-set values under conditions of starting /overload/ unbalanced supply. Hence, the entire system enjoys longer life. Both these effects lead to better production-process and hence these are readily acceptable by industries.

## 25.2 Salient Aspects of Conversions

Purpose of electro-mechanical conversion device is to change the form of energy. Here, for simpler discussion, only rotary systems will be dealt with. When it is converting mechanical input to electrical output the device is “generating”. With electrical input, when mechanical output is obtained, the device is motoring.

Some simple aspects of an electrical machine (motor / generator) have to be noted at this place:

- (1) Electrical machine has a Stator, a Rotor, and an air-gap in between the two. For a flux path, the magnetic circuit has these three parts in series. In general, magnetic poles are established in Stator and in Rotor.
- (2) Magnetic effects of following types can be categorized:
  - (a) **Electromagnetic:** Due to currents passed through windings on Stator and / or Rotor, producing certain number of poles on these members.
  - (b) **Permanent Magnets:** One side (Stator or Rotor) can have permanent magnets.
  - (c) **Reluctance variation:** Surface of Rotor near the air-gap can be suitably shaped to have a particular pattern of Reluctance-variation so as to control the machine behaviour as per requirements.
- (3) Basic conditions which must be satisfied by such devices are:
  - (a) Equal number of poles must be created on the two sides.
  - (b) In some cases, reluctance-variation is primarily used for machine-action. The Stator side must accommodate a winding carrying current for the electromagnetic effect, when rotor surface is shaped so as to have the desired pattern of reluctance variation. Or, non-cylindrical rotor cannot have the current -carrying winding for machine action.
- (4) Out of stator, rotor and air-gap, maximum energy-storage at any angular position takes place in the air-gap, since its reluctance is highest out of the three members.
- (5) Stored energy must depend on rotor-position and the device tends to occupy that angular position which corresponds to maximum stored energy. If this position varies as a function of time, the device produces continuous torque.
- (6) Ideal output of a motor is a constant unidirectional torque with given currents through its windings. In some cases, the output torque (as a compromise) is an average value of a cyclically varying torque.
- (7) Where current-switching is done for motor-control, as in modern controllers, instantaneous effect has to be understood to conclude on any of the points mentioned above.
- (8) A device can work either as a generator or as a motor, provided pertinent conditions are satisfied for the concerned mode of operation.



Stator and Rotor

### 25.3. Energy – Balance

For an electro-mechanical system, following terms are important:

- (i) Electrical port (= armature terminals): receiving / delivering electrical energy.
- (ii) Mechanical port (= shaft): delivering / receiving mechanical energy.
- (iii) Coupling field: Magnetic field *or* Electric field.

Even though, theoretically, both the types of fields mentioned above are able to convert the energy, the magnetic medium is most popular since the voltage levels required are not very high, and the devices of given power rating are smaller in size and are economical. Hence, only those will be dealt with.

It is obvious that an electrical motor receives energy at the electrical port and delivers it at the mechanical port. While an electric generator receives the energy at the mechanical port and delivers it at the electrical port. It is also known

that the following losses take place in such systems and are dissipated away as heat: (i)  $i^2r$  losses in the windings of the machines, (ii) friction and windage losses, (iii) core-losses.

These can be either neglected or attached to electrical port, mechanical port and coupling magnetic field respectively, for simpler analysis. With this, the simple energy balance equation can be written as:

Change in Electrical Energy = Change in Mechanical Energy + Change in Field-Energy

$$dW_{elec} = dW_{mech} + dW_{fld} \quad \dots (25.1)$$

It is natural that this equation has +ve signs for electrical and mechanical-energy-terms when the device is motoring. For generating mode, however, both the terms assume –ve signs.

In case no mechanical work is done, eqn. (25.1) reduces to eqn. (25.2) below indicating that Electrical energy - input is stored in the magnetic field.

$$dW_{elec} = dW_{fld} \quad \dots (25.2)$$

### 25.4. Magnetic-field System: Energy and Co-energy

#### 25.4.1 Linear System

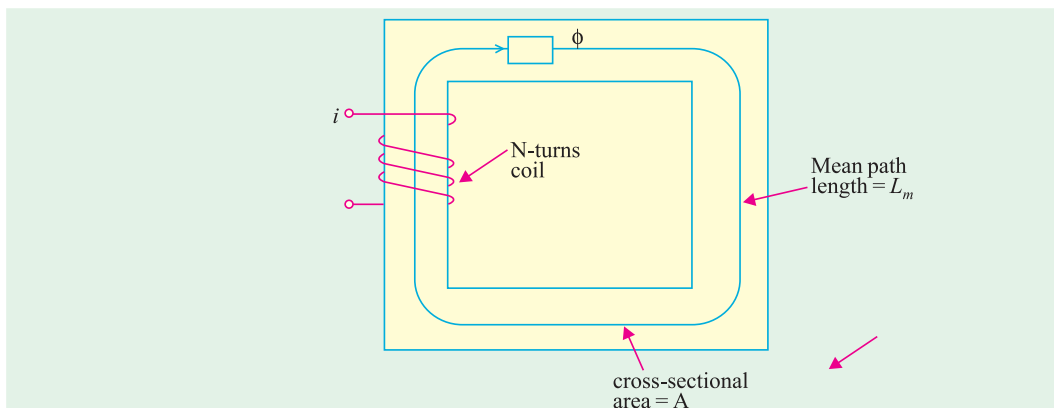


Fig. 25.1 (a) Magnetic circuit



Conversion of electrical energy into mechanical energy



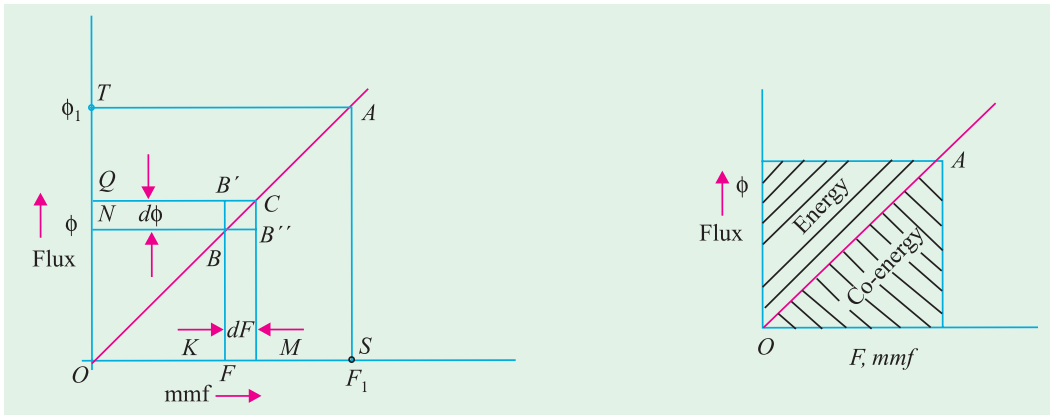


Fig. 25.1 (b) Characteristic of a magnetic circuit

Fig. 25.1 (c) Energy and co-energy

A simple magnetic circuit is shown in Fig 25.1 (a), with assumptions that air-gap length at the joints is negligible, and the magnetic medium is not saturated. With  $A$  as the cross-sectional area of the core and  $L_m$  as the mean length of the path, a coil with  $N$  turns carrying a current of  $i$  amp has an mmf of  $F$ , establishing a flux of  $\phi$ , related by

$$\phi = F \times \rho_m$$

where  $\rho_m$  = Permeance of the Magnetic circuit

$$= \mu_o \mu_r A / L_m$$

with  $\mu_r$  = relative permeability of the magnetic medium,

This corresponds to the following relationships:

Coil Inductance,  $L = N^2 \rho_m = N\phi / i = \lambda / i$

where  $\lambda$  = flux-linkage of the coil, in weber-turns

$$W_{fld} = \text{Energy stored in the coil} = \frac{1}{2} Li^2 = \frac{1}{2} N^2 \rho_m i^2 = \frac{1}{2} F^2 \rho_m$$

$$= \frac{1}{2} F(F\rho_m) = \frac{1}{2} F\phi \quad \dots (25.3)$$

In this eqn.,  $\rho_m$  is the slope of the characteristic in Fig 25.1 (b). Hence, the inductance is proportional to the slope of  $F$ - $\phi$  plot. In Fig., 25.1 (b), for the operating point  $A$ , the mmf is  $F_1$  and the flux  $\phi_1$ . At the point  $A$ , the energy stored in the field is given by eqn. below:

$$W_{fld} = \frac{1}{2} F_1 \phi_1$$

$F_1$  is due to the current  $i_1$ .  $W_{fld}$  is given by area  $OATO$  in Fig 25.1(b).

In Fig 25.1(b), the origin refers to the system without magnetization.

The system can reach the point  $A$ , starting from  $O$  as the current in the coil is increased from  $O$  to  $i_1$ .

Let us understand the intermediate events.

At point  $B$ , the flux is  $\phi$  due to the mmf  $F$ .

An increment in coil current results into increase in mmf by  $dF$ . This increases the core-flux by  $d\phi$ . New operating point is  $C$ .

Eqn. (25.3) is to be suitably re-written in terms of these incremental values.

$$W_{fld} = \int dW_{fld} = \int_0^{\phi_1} Fd\phi = \int_0^{\phi_1} (\text{area of the strip } NBB'Q) = \text{area of triangle } OAT$$

$$= (1/\rho_m) \int_0^{\phi_1} \phi \cdot d\phi = \frac{1}{2} \phi_1 \cdot F_1 \quad \dots (25.4)$$

Alternatively, we have the area of elemental strip  $kBB''M = \phi \cdot dF$

$$\begin{aligned} \text{Area of } \Delta OAS &= \int_0^{F_1} \phi \cdot dF \quad (\text{area of the strip } KBB''M) \\ &= \int_0^{F_1} \phi \cdot dF = \mathcal{O}_m \int_0^{F_1} F \, dF = \mathcal{O}_m \frac{F_1^2}{2} = \frac{1}{2} F_1 \phi_1 \quad \dots (25.5) \end{aligned}$$

In order to distinguish with respect to the terms in eqn. (25.4), this area is called as the “Co-energy of the field” and is represented by  $W'_{fld}$ . For a Linear system, however, for a given operating point, say A, the two energy-terms are equal. Hence,

$$W_{fld} = W'_{fld} = \frac{1}{2} F_1 \phi_1 = \frac{1}{2} Li_1^2 \quad \dots (25.6)$$

In order to have a simple and clear distinction between the two energy terms, it can be said that the differential variable “**Current**” (or mmf) is related to Co-energy and the differential variable “flux - linkage” (or flux) is related to Energy.

This energy stored in the magnetic field comes from the electrical source connected to the coil in Fig. 25.1(a).

### 25.4.2. A Simple Electromechanical System

A simple electro-mechanical system is shown in Fig. 25.2(a).

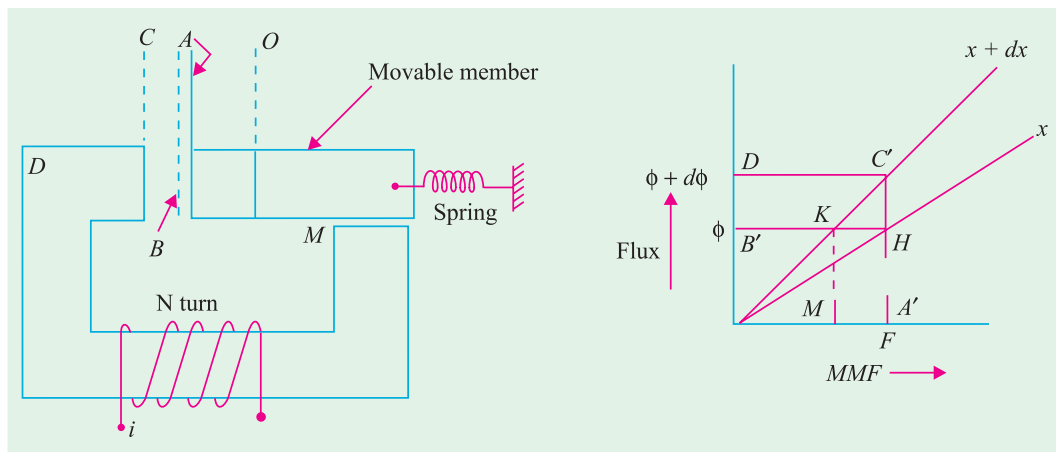


Fig. 25.2 (a)

Fig. 25.2 (b)

Reference point  $O$  corresponds to the unstretched spring. Energy stored in the spring is then Zero. In position  $A$  of the movable member, the spring is elongated by  $x$ , and the corresponding energy stored in the spring is  $\frac{1}{2} K_s x^2$ , where  $K_s$  is “spring-constant” of the linear system in  $Nw / m$ . In Fig 25.2 (a), the distance  $OA$  is  $x$ . The elemental distance  $AB$  is  $dx$ , so that  $OB$  is  $x + dx$ . For simpler analysis, it is assumed that magnetic material is highly permeable and that the clearance at point  $M$  (for movement of the member) is negligible. So that the mmf of the coil is required to drive the flux in the region  $OABC$  only. The flux-mmf



A simple electro-mechanical system

relationships are plotted for these two positions in Fig 25.2 (b). In position A, the movable member has moved a distance of  $x$  from its unstretched position or reference point. Let the operating point be  $H$ , so that the coil-mmfmf  $OA (=F)$  establishes a flux  $OB (=ϕ)$ . In this position, the movable member experiences a force in such a direction that the energy stored in the field tends to increase. It tends to reach  $B$ , so that an additional displacement of  $dx$  shifts the characteristic upwards and final operating point in position  $B$  is  $C'$ . From  $H$  to  $C'$ , the operating point can move in any one of the following ways:

(a)  $HC'$  vertically, if the mechanical movement is too slow so that change of flux is slow and induced emf in the coil is negligible. This corresponds to the coil-mmfmf remaining constant at  $F$  during the transition. Constant mmfmf means vertical travel of the operating point from  $H$  to  $C'$ .

(b)  $H$  to  $K$  horizontally and then  $K$  to  $C'$  along the characteristic corresponding to  $(x + dx)$  as the displacement of the movable part. This is possible when the motion is very fast, resulting into flux remaining constant till the operating point traverses from  $H$  to  $K$ . Then, from  $K$  to  $C'$ , the flux increases, an emf is induced in the coil and the mmfmf finally reaches its value of  $F$ , at the point  $C'$ .

(c) In reality, the transition from  $H$  to  $C'$  will be somewhere in between these two extremes mentioned above.

However, for simplicity, one of these extreme conditions has to be accepted. In (a) above, the mmfmf remains constant. In (b) above, the flux (and hence the flux-linkage) remains constant. Let us take the case of constant-mmfmf. If the process has taken a time of  $dt$ ,

$$\begin{aligned} \text{Electrical-energy input during the process} &= dW_{\text{elec}} \\ &= (\text{voltage applied to the coil}) \times \text{current} \times dt = e idt \\ &= d\lambda / dt \times i \times dt = id\lambda = i N d\phi = F d\phi = \text{area of rectangle } B'HC'D \end{aligned}$$

In this case, coil-resistance has been neglected.

### 25.4.2.1. In terms of Field Energy

At the previous operating point  $H$ , the energy stored in the magnetic field,

$$W_{\text{fld1}} = \text{area of } \Delta OHB'$$

At the new position corresponding to the operating point  $C'$ , the field energy stored is given

by  $W_{\text{fld2}} = \text{area of the } \Delta OC'D$

The difference of these two is the change in the energy stored in the

$$\begin{aligned} \text{magnetic field} \quad &= dW_{\text{fld}} = \frac{1}{2} [OA' \times A'C' - OA' \times A'H] \\ &= \frac{1}{2} OA' [A'C' \times A'H] = \frac{1}{2} OA' HC' \\ &= \frac{1}{2} \cdot F \cdot d\phi = \frac{1}{2} dW_{\text{elec}} \end{aligned}$$

Out of the energy delivered by the source, half is stored in the magnetic field. Where has the remaining half been utilized? Obviously, this must have been transformed into the mechanical work done. In this case, neglecting losses, it is finally stored in the stretched spring due to its elongation by  $dx$ .

Comparing this with the equation (25.1),

$$\begin{aligned} dW_{\text{elec}} &= dW_{\text{mech}} + dW_{\text{fld}} \\ &= dW_{\text{mech}} + \frac{1}{2} dW_{\text{elec}} \end{aligned}$$

or  $dW_{\text{mech}} = dW_{\text{fld}} = \frac{1}{2} dW_{\text{elec}}$

Consider that a force  $F$  is operative at the displacement of  $x$ . This force is in such a direction that  $x$  increases or the movable member is attracted towards  $D$ . In the same direction, a

displacement by  $dx$  results into the increase in the energy stored by the spring. Relating the concerned terms,

$$\begin{aligned}
 & F = k_s \cdot x \\
 dW_{\text{mech}} &= \text{mechanical work done against the force of the stretched spring} \\
 &= -F dx = dW_{\text{fld}} \\
 \text{or } F &= -dW_{\text{fld}}/dx, \text{ in this case} \\
 &= -\delta W_{\text{fld}}/\delta x, \text{ in general}
 \end{aligned}$$

Alternatively, the difference in the energy stored in the spring also gives a very useful relationship.

In the position corresponding to  $x + dx$ , the energy stored in the spring =  $\frac{1}{2} k_s (x + dx)^2$ . Similarly, at  $x$ , the energy =  $\frac{1}{2} k_s x^2$

$$\begin{aligned}
 \text{Difference} &= \frac{1}{2} k_s [(x + dx)^2 - (x)^2] \\
 &= \frac{1}{2} k_s [x^2 + 2 \cdot x \cdot dx + (dx)^2 - x^2] \\
 &= \frac{1}{2} k_s [2 \cdot x \cdot dx], \text{ neglecting } (dx)^2 \\
 &= k_s \cdot x \cdot dx = F dx
 \end{aligned}$$

This difference is nothing but  $dW_{\text{mech}}$ , which is equal in magnitude to  $dW_{\text{fld}}$  and confirms the relationship obtained earlier.

#### 25.4.2.2. In terms of field Co-energy

Proceeding along lines similar to those while dealing with field-energy above, following relationships exist. For simpler discussion, the transition is assumed to be along  $HKC'$ . Neglecting area of the small triangle  $HKC'$ , we have

$$\begin{aligned}
 \text{at } x, W'_{\text{fld1}} &= \text{area of } \Delta OA'H \\
 \text{at } x + dx, W'_{\text{fld2}} &= \text{area of } \Delta OA'C' \text{ (neglecting } \Delta HKC') \\
 dW'_{\text{fld}} &= W'_{\text{fld2}} - W'_{\text{fld1}} = \phi \cdot dF \text{ where } dF = MA' \\
 &= \text{Co-energy in the field } F dx
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, } F &= + dW'_{\text{fld}}/dx, \text{ in this case} \\
 &= + \delta W'_{\text{fld}}/\delta x, \text{ in general}
 \end{aligned}$$

### 25.5. Energy in Terms of Electrical Parameters

In the preceding article, the energy and force were related in terms of magnetic-system parameters, namely flux and mmf, through the third parameter, the permeance.

It is at times convenient to relate these things in terms of electrical-system-parameters, namely, the inductances and currents. That is being dealt with here only for linear systems. Let  $\rho$  be the permeance of the magnetic circuit and  $L$  be the coil-inductance.

$$\begin{aligned}
 W_{\text{fld}} = \text{Field-energy} &= \frac{1}{2} F \phi = \frac{1}{2} Ni (Ni \rho) = \frac{1}{2} (N^2 \rho) i^2 \\
 &= \frac{1}{2} Li^2 \\
 F &= -dW_{\text{fld}}/dx = -\frac{1}{2} \cdot i^2 \cdot dL/dx
 \end{aligned}$$

Thus, a force exists if the coil-inductance is dependent on  $x$ . Such analysis is more suitable when the system has more than one coils coupled through the magnetic circuit. If two such coils are considered, following data should be known for evaluation of the force, in case of linear displacement:

$$\begin{aligned}
 L_{11} &= \text{self inductance of coil - 1} \\
 L_{22} &= \text{self inductance of coil - 2}
 \end{aligned}$$

$L_{12}$  = Mutual inductance between two coils, 1 and 2

$i_1, i_2$  = currents through the two coils.

$W_{fld}$  = Total energy stored in the field =  $\frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2$

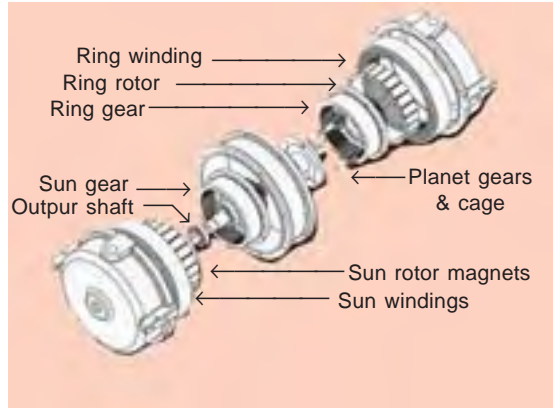
Magnitude of Force,  $F = dW_{fld}/dx = \frac{1}{2} i_1^2 dL_{11}/dx + \frac{1}{2} i_2^2 dL_{22}/dx + i_1 i_2 dL_{12}/dx$

From the right-hand side of this equation, it is noted that the inductance-term which is dependent on  $x$  contributes to the force.

### 25.6. Rotary Motion

Most popular systems for electro-mechanical energy are Generators and Motors. The preceding discussion dealt with the Linear motions, wherein  $x$  represented the displacement parameter, and force was being calculated.

Now we shall deal with the rotary systems, wherein angular displacement parameters (such as  $\theta$ ) and corresponding torque developed by the system will be correlated, through a systematic procedure for a typical rotary machine.



Rotary motion

#### 25.6.1. Description of Simple System

A simple rotary system has a 'stator' and a 'rotor'. Air-gap separates these two. Stator has two similar coils 'a' and 'b' located at  $90^\circ$  electrical, with respect to each other. Inner surface of the stator is cylindrical. Outer surface of the rotor is also cylindrical resulting into uniform air-gap length for the machine.

The diagram represents a two-pole machine. Axis of coil 'a' may be taken as reference, with respect to which the rotor-coil axis makes an angle of  $\theta$ , at a particular instant of time. For a continuous rotation of the rotor at  $\omega$  radians / sec,  $\theta = \omega t$ . Coil-'b'-axis is perpendicular to the reference, as shown. Due to the uniform air-gap length, and due to the perpendicularity between coils 'a' and 'b', inductance-parameters exhibit the following patterns:

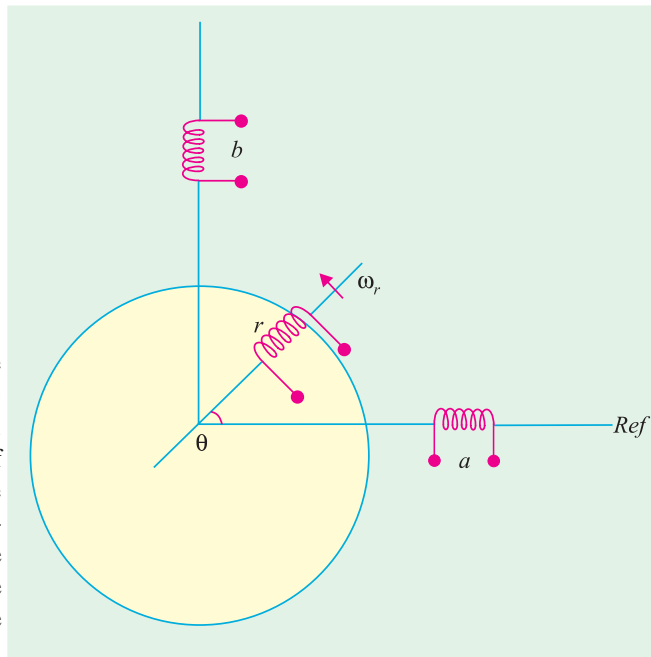


Fig. 25.3. Simple rotary system

Let  $x$  represents the inductance parameter as a function of  $\theta$ . The subscripts indicate the particular parameter.  $x_{aa}$  = self-inductance of coil 'a',  $x_{ab}$  = mutual inductance between coils a and b and so on.  $x$  represents value of the particular inductance parameter, which will help in knowing the variation of inductance with  $\theta$ .

(a) Self inductances of coils 'a' and 'b' are not dependent on rotor position.

$$x_{aa} = L_{aa} \text{ and } x_{bb} = L_{bb}, \text{ at all values of } \theta.$$

(b) Mutual inductance between stator coils 'a' and 'b' is zero, due to perpendicularity.

$$x_{ab} = \text{zero, for all values of } \theta.$$

(c) Self-inductance of rotor-coil is constant and not dependent on  $\theta$ .

$$x_{ra} = L_{rr} \text{ (= constant), at all values of } \theta.$$

(d) When 'r' and 'a' coils have their axes aligned, at  $\theta = 0$ , the mutual inductance between them is maximum, which is denoted by  $x_{ar}$ . At  $\theta = 90^\circ$ , their axes are perpendicular resulting into no coupling or zero mutual inductance. When  $\theta = 180^\circ$ , r and a coils are aligned in anti-parallel way and hence maximum mutual inductance exists between them with negative sign.

Further, whatever happens to coupling between r and a at a value of  $\theta$  happens to that between r and b with a delay of  $90^\circ$ . All these are mathematically represented as :

$$\begin{aligned} x_{ra} &= L_{ra} \cos \theta = L_{ra} \cos(\omega_r t) \\ x_{rb} &= L_{ra} \sin \theta = L_{ra} \sin(\omega_r t) = L_{ra} \cos(\omega_r t - 90^\circ) \end{aligned}$$

Since 'a' and 'b' coils are alike, the maximum mutual inductance is represented by the same term  $L_{ra}$ .

### 25.6.2. Energy stored in the coils

Energy stored in the magnetic field can either be expressed in terms of mmf and flux or be expressed in terms of inductance-terms and coil currents. If  $i_a$ ,  $i_b$  and  $i_r$  are the coil-currents, stored-energy-terms are as given below:

(i)  $W_1 =$  Energy in Self-ind. of coil 'a':  $\frac{1}{2} L_{aa} i_a^2$

(ii)  $W_2 =$  Energy in Self-ind. of coil 'b':  $\frac{1}{2} L_{bb} i_b^2$

(iii)  $W_3 =$  Energy in Self-ind. of coil 'r':  $\frac{1}{2} L_{rr} i_r^2$

(iv)  $W_4 =$  Energy in mutual inductance between 'a' and 'r':  $x_{ra} i_r i_a = x_{ra} i_r i_a = x_{ra} \cos \theta \cdot i_r \cdot i_a$

(v)  $W_5 =$  Energy in mutual inductance between 'b' and 'r':  $x_{rb} i_r \cdot i_b = x_{ra} \sin \theta \cdot i_r \cdot i_b$

$W =$  Total energy stored in the system = Sum of all the energy-terms cited above

$$= W_1 + W_2 + W_3 + W_4 + W_5$$

$T =$  Torque produced =  $\delta W / \delta \theta$

If  $i_a$ ,  $i_b$ ,  $i_r$  are assumed to be constant currents, for simplicity, so that their derivatives with respect to  $\theta$  (and hence with respect to time  $t$ ) are zero, the energy-terms which include constant inductances do not contribute to torque.  $W_1$ ,  $W_2$ , and  $W_3$  thus cannot contribute to torque.  $W_4$  and  $W_5$  contribute to torque related by:

$$\begin{aligned} T &= \delta W / \delta \theta = \delta / \delta \theta [W_4 + W_5] = \delta / \delta \theta [L_{ra} i_r i_a \cos \theta + L_{ra} i_r i_b \sin \theta] \\ &= L_{ra} i_r [-i_a \sin \theta + i_b \cos \theta] \end{aligned}$$

$$\text{If } i_a = i_b = i_s, T = L_{ra} i_r i_s [-\sin \theta + \cos \theta]$$

For such a system, the torque is zero at  $\theta = +45^\circ$ , and the torque is maximum at  $\theta = -45^\circ$ .

If one of the stator currents is reversed, the result differs. For this, let  $i_a = -i_s$  and  $i_b = +i_s$

$$T = L_{ra} i_r i_s [\sin \theta + \cos \theta]$$

And the maximum torque occurs at  $\theta = 45^\circ$ . This is a position for rotor, which is midway between the two stator coils.

25.6.2.1. Different Categories

From the torque expressions above, it is clear that the torque exists only when stator and rotor-coils carry currents. When only stator-coils (or only rotor coil) carry current, torque cannot be produced.

(a) One coil each on Stator and on Rotor

In the above mentioned case, let us excite only one stator-coil. Let  $i_a = i_s$ ,  $i_b = 0$  and  $i_r$  maintained as before.

$$T = -L_{ra} i_r i_s \sin \theta$$

Following observations are made for such a case:

- (i) At  $\theta = 0$ ,  $T = 0$ , Mutual inductance  $x_{ra}$  is maximum ( $= L_{ra}$ ) and hence the stored energy in mutual inductance is maximum, but torque is zero.
- (ii) At  $\theta = 90^\circ$ ,  $T$  is maximum.  $x_{ra}$  is zero, hence the concerned stored energy is zero.
- (iii) As seen earlier,  $W_4 = L_{ra} i_r i_a \cos \theta = L_{ra} i_r i_s \cos \theta$

Torque =  $dW_4 / d\theta = i_r i_s d(L_{ra} \cos \theta) / d\theta$

Contributed by  $W_4$ , the power is related as follows:

Power = rate of change of energy with time  
 $= dW_4 / dt = dW_4 / d\theta \cdot d\theta / dt = T \cdot \omega_r = i_r i_s L_{ra} (-\sin \theta) \omega_r$

Magnitudes of these terms are maximum for  $\theta = 90^\circ$ . If  $\theta$  can be set at  $90^\circ$ , at all instants of time, torque obtained is maximum. Such a situation does exist in a d. c. machine in which rotor carries an armature winding which is a lap- or wave-connected commutator winding. The brushes are so placed on the commutator that rotor-coil-axis satisfies the above-mentioned condition of  $\theta = 90^\circ$ , irrespective of the rotor-position or rotor speed. Such an equivalence of a **rotating** armature coil with such an **effectively stationary** coil is referred to as a **quasi-stationary** coil. It means that a rotating coil is being analyzed as a stationary coil due to its typical behaviour for electro-mechanical energy conversion purposes.



Permanent magnet synchronous motor for washing machine

(b) Two stator coils carrying two-phase currents and rotor-coil carrying d. c.:



Permanent magnet

When two stator coils carry two-phase alternating currents, a synchronously rotating mmf is established. If the rotor-coil carries direct current, and the rotor is run at same synchronous speed, a unidirectional constant torque is developed. Mathematically, similar picture can be visualized, with a difference that the total system is imagined to rotate at synchronous speed. Such a machine is Synchronous machine, (to be discussed in Later chapters). It can be understood through the simple system described here.

(c) Machines with Permanent Magnets.

With suitable interpretation, the field side of the simple system can be imagined to be with permanent magnets in place of coil-excited electromagnets. All the interpretations made above are

valid, except for the difference that in this case there is no scope for controlling the rotor-coil-current-magnitude.

**(d) Machines with no rotor coil, but with premeance variation.**

Smooth cylindrical rotor surfaces do not exist in such cases. There are no rotor-coils. Due to geometry of the rotor surface, stator-coil- self inductances vary with rotor position. Thinking on lines of relating energy terms and their derivatives for torque-calculations, the working principles can be understood. With simple construction, Reluctance motors belong to this category.

**(e) Switched currents in Stator Coils.**

In yet another type, stator coils are distributed and properly grouped. One group carries currents during certain time interval. Then, this current is switched off. Another group carries current in the next time interval and so on. The rotor surface is so shaped that it responds to this current switching and torque is produced. Even though stator-coil-inductances are complicated functions of rotor position, the method of analysis for such machines is same. Prominent types of machines of this type are: switched reluctance motors, stepper motors, etc.



Current in stator coils

### 25.6.2.2. Vital Role of Air-gap

Magnetic circuit of an electrical machine has a flux established due to coil-mmfs. This flux is associated with stator core, rotor core and air-gap. An important point for understanding is to know which out of these three stores major portion of the field energy. Through an illustrative case, it will be clear below, in example 25.1.

**Example 25.1.** Let a machine with following data be considered.

Calculate the energy stored in the air-gap and compare the same with that stored in the cores.

- Stator-core outer diameter = 15 cm
- Stator-core inner diameter = 10.05 cm
- Rotor-core outer diameter = 10.00 cm
- Rotor-core inner diameter = 5 cm
- Axial length of the machine = 8 cm

Effect of slotting is neglected. The core volumes and air-gap volume for the machine shown in Fig 25.4 have to be calculated.

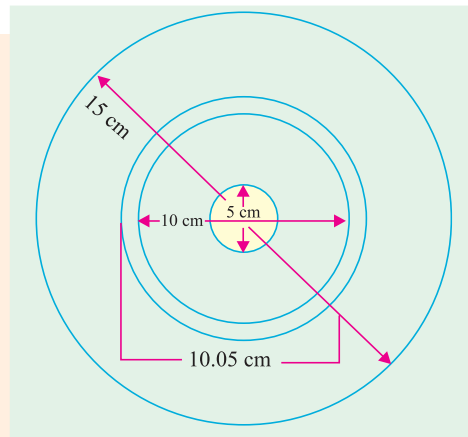


Fig. 25.4

**Solution.**

$$\text{Volume of Stator-core} = (\pi / 4) \times (15^2 - 10.05^2) \times 8 \text{ cm}^3 = 779 \text{ cm}^3$$

$$\text{Volume of Rotor-core} = (\pi / 4) \times (10^2 - 5^2) \times 8 \text{ cm}^3 = 471 \text{ cm}^3$$

$$\begin{aligned} \text{Volume of air-gap in the machine} &= (\pi / 4) \times (10.05^2 - 10^2) \times 8 \text{ cm}^3 \\ &= 6.3 \text{ cm}^3 \end{aligned}$$



Let the relative permeability of the core material be 1000. If the flux density is  $B$  Wb / m<sup>2</sup>, and  $\mu$  is the permeability, the energy-density is  $\frac{1}{2} \times B^2 / \mu$  Joules / m<sup>3</sup>. Let the flux density be 1.20 Wb / m<sup>2</sup>. Energy density in air-gap =  $\frac{1}{2} \times 1.20^2 / (4 \pi \times 10^{-7}) = 572350$  Joules / m<sup>3</sup>  
 = 0.573 Joules / cm<sup>3</sup>

Energy stored in air-gap =  $0.573 \times 6.3 = 3.6$  Joules

Energy-density in Magnetic medium =  $\frac{1}{2} \times 1.20^2 / (4\pi \times 10^{-7} \times 1000) = 573$  J / m<sup>3</sup>

It is assumed only for simplicity that the flux density is same for the entire core of stator and of rotor.

Energy stored in stator-core =  $573 \times 779 \times 10^{-6} = 0.45$  Joule

Energy stored in rotor-core =  $573 \times 471 \times 10^{-6} = 0.27$  Joule

It is worth noting that even though the ratio of volumes is 198, the ratio of energies is 0.2, since, for the present case,

$$K_v = \frac{\text{volume of (Stator - core + Rotor - core)}}{\text{Volume of air - gap}} = \frac{779+471}{6.3} = 198$$

$k_E = \text{Energy stored in cores} / \text{Energy stored in air-gap} = (0.45 + 0.27) / 3.6 = 0.2$

The ratio are like this due to  $\mu_r$  being 1000, and  $k_v / k_E = 198 / 0.2 = 1000$

Alternatively, an air-gap of volume 6.3 cm<sup>3</sup>, [surrounded by the magnetic medium of  $\mu_r = 1000$ ] is equivalent to the magnetic medium of volume  $6.3 \times 1000$  cm<sup>3</sup>.

$$\frac{\text{Converted equivalent volume of air-gap}}{\text{Volume of (Stator + Rotor)}} = \frac{6300}{779+471}$$

$$\frac{\text{Energy stored in air-gap}}{\text{Energy stored in (Stator + Rotor)}} = \frac{3.6}{(0.45+0.27)} = 5$$

This correlates the various parameters and confirms that the stored energy is maximum in the air-gap.

Or, one can now say that in the process of electro-mechanical energy-conversion, the air-gap plays a very vital role.

However, the stator-core and rotor-core help in completing the flux-path in a well defined manner for effective and efficient working of a rotary machine.

**Example 25.2.** An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross-sectional area of 5 cm × 5 cm. Neglect reluctance of the magnetic circuit and fringing.

(a) (i) Find the coil inductance if the air-gap length is 0.5 cm.

(ii) Find the field energy stored for a coil current of 1.25 amp.

(b) Coil-current remaining constant at 1.25 A, find the mechanical energy output based on field-energy changes when the armature moves to a position for which  $x = 0.25$  cm. Assume slow movement of armature.

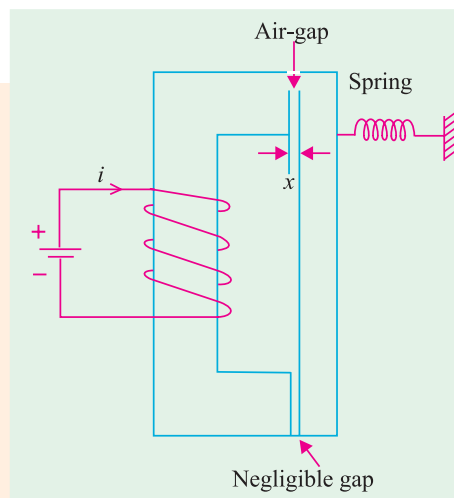


Fig. 25.5 Electro-magnetic relay

- (c) Repeat (b) above based on force-calculations and mechanical displacement.  
 (d) What will be change in above results of mechanical work done, if the mechanical movement is fast, keeping the flux initially constant ?

**Solution.**

$$(a) (i) \text{ Permeance at air-gap} = \frac{\mu_0 5 \times 5 \times 10^{-4}}{0.5 \times 10^{-2}} = 4\pi \times 10^{-7} \times 10^{-2} = 6.28 \times 10^{-7}$$

$$\text{Coil Inductance} = N_2^2 \mathcal{P}_m = 800 \times 800 \times 6.28 \times 10^{-7} = 0.402 \text{ H}$$

$$(ii) \text{ Energy stored in magnetic field} = 1/2 Li^2 = 1/2 \times 0.402 \times 1.25^2 = 0.314 \text{ joule}$$

$$(iii) W'_{fd} = 1/2 L(x)i^2 = 1/2 \left[ \frac{H^2 \mu_0 A}{l_x} \right] = \frac{1/2 \times 800 \times 800 \times 4\pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}}{x} \times i^2$$

$$= \frac{1.005 \times 10^{-3}}{x} \times i^2$$

$$F_f = \frac{\delta [i^2 \times (1.005 \times 10^{-3})]}{\delta_x} = [1.005 \times 10^{-3}] \times i^2 \times \frac{-1}{x^2}$$

This is to be evaluated at  $x = 0.5 \times 10^{-2}$

$$= \frac{-1.005 \times 10^{-3} \times 1.25 \times 1.25}{(0.5 \times 10^{-2})^2} = -62.8 \text{ NW}$$

This force has to be balanced by the spring-tension.

(b) **Energy-computations :** Inductance for  $x = 0.25 \text{ cm}$  is first calculated.  $L(x_2) = N^2 \mathcal{P}_{m_2}$   
 $= 800 \times 800 \times 2 \times 6.28 \times 10^{-7} = 0.804 \text{ Henry}$

If the mechanical movement is slow, net mmf remains unchanged and the operating point moves along  $HC$  vertically upwards and settles at  $C$ . Added Electrical Energy input during change-over of the operating point from  $H$  to  $C$ ,

$$= \text{area of rectangle } BDCH = (\phi_2 - \phi_1) F_1$$

$$\left[ \frac{L(x_2) \cdot i - L(x_1) \cdot i}{N} \right] Ni$$

$$= i^2 [L(x_2) - L(x_1)] = 1.25^2 \times [0.804 - 0.402]$$

$$= 0.628 \text{ joule}$$

Out of this, the additional stored energy in field,  $dW_{fd} = 1/2 [L(x_2) - L(x_1)]$

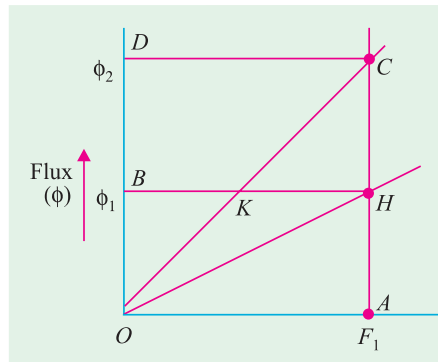
$$= 0.314 \text{ joule}$$

The remaining 0.314 joule is transformed into mechanical form and is related to the work-done. This is obtained when the force on moving member is multiplied by the displacement.

$$\therefore \int_{x_1}^{x_2} dW_{mech} = 0.314 \text{ joule}$$

(c) As in a (iii) above,

$$F(x) = [1.005 \times 10^{-3} \times 1.25^2] [-1/x^2]$$



**Fig. 25.6** Graphical correlation of energy-terms for the relay

$$\begin{aligned}
 dW_{mech} &= F(x).dx \\
 W_{mech} &= \int_{x_1}^{x_2} F(x)dx = k. \left[ \int_{x_1}^{x_2} -1/x^2 dx \right] = -k. \int_{x_1}^{x_2} x^{-2} dx \\
 &= 1.005 \times 10^{-3} \times 1.25^2 \times \left[ \frac{1}{0.25} - \frac{1}{0.5} \right] \times 10^2 = \frac{1.005 \times 1.25^2}{10} \times 2 = 0.314 \text{ joule}
 \end{aligned}$$

This agrees with answer obtained in (b) above.

With fast movement of armature, the operating point will move from H to K first, then follow the path *KC*.

This means that the energy represented by the area of the triangle *KHC* corresponds to the reduced consumption of energy.

$$OC \text{ has a slope of } \mathcal{P}_{m_2} = 12.56 \times 10^{-7}$$

$$OH \text{ has a slope of } \mathcal{P}_{m_1} = 6.28 \times 10^{-7}$$

*BK* = mmf required for establishing a flux of  $\phi_1$  with an air-gap of 0.25 cm

$$BK = \frac{1}{2} OA = \frac{1}{2} \times (800 \times 1.25) = 500 \text{ amp-turns} = HK \text{ in the present case.}$$

$$\begin{aligned}
 \text{Area of the triangle } KHC &= \frac{1}{2} \times KH \times HC \\
 &= \frac{1}{2} \times 500 \times [\phi_2 - \phi_1] \\
 &= \frac{1}{4} \text{th of area of rectangle } BDCH, \text{ in this case} \\
 &= \frac{1}{4} \times 0.628 = 0.157 \text{ joule.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, Electrical energy fed during this process} &= \text{area } BKCD \\
 &= \text{area } BDCH - \text{area } KHC = 0.628 - 0.157 = 0.471 \text{ joule}
 \end{aligned}$$

$$\begin{aligned}
 \text{Increase in field energy stored } \Delta W'_{fld} &= \text{area } OKH \\
 &= \text{area } OHC - \text{area } KHC = 0.314 - 0.157 = 0.157
 \end{aligned}$$

$$\text{Mechanical Energy output} = 0.471 - 0.157 = 0.314 \text{ Joule}$$

It indicates that with fast movement, the electrical energy-input and the field-stored energy have decreased by 0.157 J each but the mechanical-energy-term remains unaffected by fast or slow movements of armature.

## 25.7. Dynamic Equations and System-model of a Simple System

It is quite necessary to analyze electro-mechanical conversion system for predicting the performance and/or for monitoring the system. A simple system is being taken up here to deal with dynamic equations and a simple model with its components is being related to the system. The details will vary from system to system, and accordingly the equations will vary.

Fig. 25.7 shows different components of such a system meant for electrical to mechanical conversion. On one side, an electrical source feeds the device at the 'electrical port'. On the other side, a force  $f_e$  is developed at the 'mechanical port'. Mechanical load is connected to this port.

**(a) At Electrical Port :** A voltage source is shown to feed the device.  $r$  is its effective internal resistance. At the electrical port, the inputs are  $\lambda$  (= flux linkage with the coil) and  $i$ . From  $\lambda$ , the voltage induced in the coil can always be evaluated.

**(b) Role of the Conversion device:** With these inputs, the device converts the energy into mechanical form, and is available as a force  $f_e$  (in case of linear motions), and, displacement  $x$  measured from a suitable reference.

(c) **At the Mechanical Port:** The possible items are: spring, damper, mass and an applied mechanical force. Their natural and simple dependence on displacement  $x$  and its derivatives are indicated below:

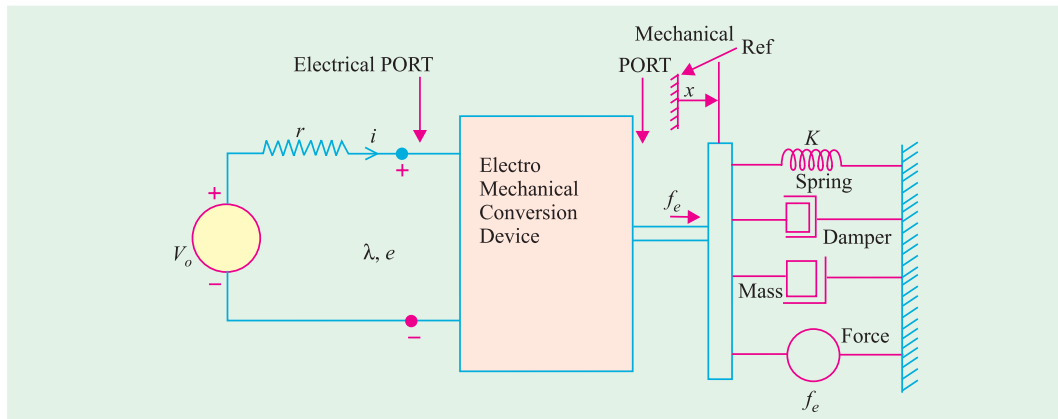


Fig. 25.7 Linear motion: MODEL

- (i) **Spring:** Force required to overcome spring elongation is proportional to the displacement  $x$ .
- (ii) **Damper:** Force required to overcome damping action in the system is proportional to derivative of  $x$ .
- (iii) **Mass:** Force required to overcome acceleration of mass is proportional to second derivative of  $x$ .
- (iv) **Applied force,  $f_o$ :** This has to be overcome by  $f_e$ . In terms of an equation, these terms are related as follows:

$$f_e = k_s (x - x_o) + B\dot{x} + M\ddot{x} + f_o$$

where

$k_s$  = spring constant

$x_o$  = value of  $x$  for unstretched spring

$B$  = damping constant

$M$  = Mass to be accelerated

$f_o$  = External mechanical force applied to the system.

### 25.8. Statically induced emf and Dynamically Induced emf :

In Fig. 25.7 source voltage is  $v_o$ . Let  $L(x)$  be the coil inductance as a function of displacement  $x$ . In a very general case,

$$\begin{aligned} v_o(t) &= ri + d\lambda/dt \\ &= ri + d/dt [L(x) \times i] \\ &= ri + L(x). di/dt + i. dL(x)/dx. dx/dt \end{aligned}$$

The second term on the right-hand side is statically induced emf (or transformer-emf), since change of current with time is responsible for it. This cannot produce any force (or torque) and hence cannot convert energy from electrical to mechanical form (or vice-versa).

The third term on the right hand side includes the speed ( $= dx/dt$ ) and dependence of  $L(x)$  on  $x$ . Any of these, if non-existent, will mean that third term reduces to zero. This term relates dynamically induced emf ( $=$  speed emf) and is the main indicator of the process of electro-

mechanical energy conversion. So, for conversion, there must be an inductance which varies with the system position, and a motion must be there. In addition, coil must carry a current.

Having understood the linear-motion-system, it is easier to understand the system with rotary motion, with due modifications.

**Example 25.3.** A doubly excited rotating machine has the following self and mutual inductances.

$$\begin{aligned} r_s &= 40 \Omega, & L_s &= 0.16 \text{ H} \\ r_r &= 2 \Omega, & L_r &= 0.04 + 0.02 \cos 2\theta \\ M_{sr} &= 0.08 \cos \theta \end{aligned}$$

where  $\theta$  is the space-angle between axes of rotor-coil and of stator-coil. The rotor is revolving at a speed of 100 radians/sec. For  $i_s = 10$  Amp d. c., and  $i_r = 2$  Amp d. c., obtain an expression for torque and corresponding electrical power.

[Rajiv Gandhi Technical University, Bhopal, Summer 2001]

**Solution.**  $W_{fld}$  = Total energy stored

$$\begin{aligned} &= \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + M_{sr} i_s i_r \\ &= \frac{1}{2} (0.16) i_s^2 + \frac{1}{2} [0.04 + 0.02 \cos 2\theta] i_r^2 \\ &\quad + [0.08 \cos \theta] i_s i_r \end{aligned}$$

since  $i_s$  and  $i_r$  are direct currents of constant magnitudes, there is no variation with  $\phi$  or with t. Relating torque with  $W_{fld}$  and substituting current-magnitudes,

$$\begin{aligned} \text{Torque, } T &= \frac{-dW_{fld}}{d\theta} = -[0 + \frac{1}{2} \times 0.02 \times 2^2 (-2 \sin 2\theta) + 0.08 (-\sin \theta) (10 \times 2)] \\ &= 0.08 \sin 2\theta + 1.6 \sin \theta \\ &= 1.6 \sin \theta + 0.08 \sin 2\theta \text{ Nw-m} \end{aligned}$$

On the right hand side, the first term is electromagnetic Torque which is dependent on both the currents. Second term is dependent only on one current, and is of the type categorized as Reluctance-torque which depends on non-cylindrical shape, in this case, on the stator side, as shown in Fig. 25.8

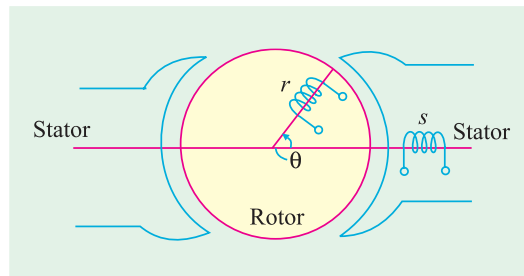


Fig. 25.8

Starting from  $W_{fld}$ , electrical power can be expressed, since it is well-known that

Power = time rate of change of energy

$$\begin{aligned} \text{Electrical power, } p &= \frac{dW_{fld}}{dt} \\ &= \frac{d}{dt} \left[ \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + M_{sr} i_s i_r \right] \\ &= \frac{1}{2} i_s^2 \frac{d}{dt} (L_s) + \frac{1}{2} i_r^2 \frac{d}{dt} (L_r) + i_s i_r \frac{d}{dt} (M_{sr}) \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} i_s^2 \left[ \frac{d}{d\theta} (L_s) \right] \frac{d\theta}{dt} + \frac{1}{2} i_r^2 \left[ \frac{d}{d\theta} (L_r) \right] \frac{d\theta}{dt} + i_s i_r \left[ \frac{d}{d\theta} (M_{sr}) \right] \frac{d\theta}{dt} \\
 &= \frac{1}{2} i_s^2 [\text{zero}] \times 100 + \frac{1}{2} i_r^2 [-0.02 \times 2 \times \sin 2\theta] \times 100 + i_s i_r [-0.08 \sin \theta] \times 100
 \end{aligned}$$

Substituting numerical values of currents, the electrical power is expressed as a function of  $\theta$ , as below :

$$\begin{aligned}
 p &= [\text{zero} + (-8) \sin 2\theta + (-160) \sin \theta] \text{ watts} \\
 &= [-160 \sin \theta - 8 \sin 2\theta] \text{ watts}
 \end{aligned}$$

Proper interpretation of sign of power (as dependent on  $\theta$ ) is important. Positive power is received by the coils, while negative power is received by the source.

**Example 25.4.** An inductor has an inductance which varies with displacement  $x$  as

$$L = 2 L_o / [1 + (x / x_o)]$$

Where  $L_o = 50 \text{ mh}$ ,  $x_o = 0.05 \text{ cm}$ ,  $x = \text{displacement in cm}$ ,

The coil resistance is  $0.5 \text{ ohm}$ .

- (a) The displacement  $x$  is held constant at  $0.075 \text{ cm}$ , and the current is increased from  $0$  to  $3 \text{ amp}$ . Find the resultant magnetic stored energy in the inductor.  
 (b) The current is then held constant at  $3 \text{ amp}$  and the displacement is increased to  $0.15 \text{ cm}$ . Find the corresponding change in the magnetic stored energy.

**Note.** Assume that all electrical transients are negligible.

**Solution.** (a) Inductance at  $x = 0.075 \text{ cm}$  is calculated first.

$$L_1 = \frac{2L_o}{1 + (0.075/0.05)} = 40 \text{ mH}$$

$\lambda_1 = L_1 \times \text{current} = 120 \times 10^{-3}$ , corresponding to point A in Fig. 25.9

$$W_{fld} = \frac{1}{2} L_1 (3)^2 = \frac{1}{2} (40 \times 10^{-3} \times 9) = 0.18 \text{ joule}$$

(b) Inductance for  $x = 0.15 \text{ cm}$  is to be calculated now.

$$L_2 = \frac{2L_o}{1 + (0.15/0.05)} = 25 \text{ mH}$$

With current held constant at  $3 \text{ amp}$ , the flux-Linkage is now  $\lambda_2 = (25 \times 10^{-3}) \times 3 = 75 \times 10^{-3}$

Since the current is constant at  $3 \text{ amp}$ , magnetic stored energy is reduced by the area of triangle  $OAB$ , in Fig. 25.9

Area of triangle  $OAB = \frac{1}{2} \times 3 \times (120 - 75) 10^{-3} = 0.0675 \text{ joule}$

**Check :** Stored-energy at B in terms of  $L_2$  and  $i$ , is given by

$$W_{fld} = \frac{1}{2} (25 \times 10^{-3}) \times 3^2 = 0.1125 \text{ joule}$$

$$\begin{aligned}
 \text{Alternatively, } W_{fld2} &= W_{fld1} - \text{area of } \Delta OAB \\
 &= 0.18 - 0.0675 = 0.1125 \text{ Joule}
 \end{aligned}$$

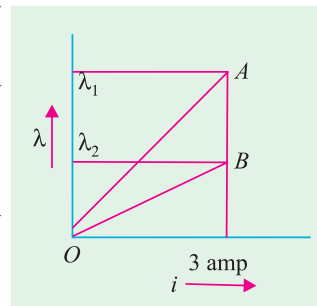


Fig. 25.9

**Example 25.5.** If the inductor in the previous case is connected to a voltage source which increases from 0 to 3 V [part (a)] and then is held constant at 3 V [part (b)], repeat the problem, assuming that electrical transients are negligible.

**Solution.** Coil resistance is 0.5 ohm. When the voltage reaches 3 V, the coil current is 6 amp. In part (a),  $L_1 = 40$  mH. Hence,  $W_{fld3} = \text{energy stored} = \frac{1}{2} L_1 i_1^2 = 0.72$  joule, at point C in Fig. 25.10. In part (b),  $L_2 = 25$  mH. The current is held constant at 6 amp. Working on similar lines,

$\Delta W_{fld} = \text{change in the field energy stored} = \text{area of triangle } ODC$  or  $\Delta W_{fld} = W_{fld3} - W_{fld4}$

$$W_{fld} = \frac{1}{2} \times 25 \times 10^{-3} \times 36 = 0.45 \text{ Joule, at point D}$$

$$\text{Change in energy stored in the field} = W_{fld3} - W_{fld4} = 0.72 - 0.45 = 0.27 \text{ joule}$$

Or  $\Delta W_{fld} = \text{area of } \triangle ODC = \frac{1}{2} \times 6 \times (\lambda_3 - \lambda_4)$

$$\text{Here } \lambda_3 = 40 \times 10^{-3} \times 6, \text{ and } \lambda_4 = 25 \times 10^{-3} \times 6$$

$$\Delta W_{fld} = \frac{1}{2} \times 6 \times 6 \times 10^{-3} (40 - 25) = 0.27 \text{ joule}$$

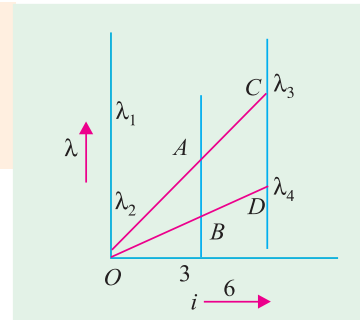


Fig. 25.10

**Example 25.6.** A coil of an electromagnetic relay is associated with a magnetic circuit whose reluctance is given by

$$= a + bx$$

where  $a$  and  $b$  are positive constants decided by the details of the magnetic circuit, in which  $x$  is the length of the air-gap between fixed and movable members. If the coil is connected to an A.C. source where voltage is described by

$$v = V_m \sin \omega t,$$

find the expression for the average force on armature, with air-gap held constant at  $x$ .

**Solution.** If  $\phi = \text{flux established, in Webers}$

$$N = \text{number of turns on the coil, } \lambda = \text{flux-linkage in Weber-turns}$$

$$W_{fld} = \frac{1}{2} \mathcal{R} \phi^2$$

$$\text{And force } F = \frac{\delta W_{fld}}{\delta x} = \frac{1}{2} \phi^2 \frac{\delta \mathcal{R}}{\delta x} = -\frac{1}{2} b \phi^2$$

The current in the coil is given by

$$v = R + L \frac{di}{dt}$$

for which, the steady-state solution for current with an a.c. voltage applied to the coil is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle -\theta \text{ where } \theta = \tan^{-1} \left( \frac{\omega L}{R} \right)$$

$$\text{RMS voltage, } V = (V_m) / \mathcal{R} \sqrt{2}$$

Instantaneous current  $i$  is expressed as

$$i = \frac{\sqrt{2}V}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

Further,  $L = N^2/\mathcal{R}$

$$\phi = Ni/\mathcal{R} = \frac{\sqrt{2}NV}{(R\mathcal{R})^2 + (N^2\omega)^2} \sin(\omega t - \theta)$$

$$\text{Force, } F_f = \frac{\mathcal{R} - bN^2V^2}{(R\mathcal{R})^2 + (N^2\omega)^2} \sin^2(\omega t - \theta)$$

The last term  $\sin^2(\omega t - \theta)$  has a time average (over a cycle) of  $1/2$ .

$$\text{Hence, average force, } F_f(\text{av}) = -\frac{1}{2} \frac{\mathcal{R}bN^2V^2}{(R\mathcal{R})^2 + (N^2\omega)^2}$$

Force is in such a direction that  $x$  will be reduced, or that the energy stored tends to increase.

**Example 25.7.** Two coupled coils have self- and mutual-inductances as expressed below:

$$L_{11} = 1 + (1/x),$$

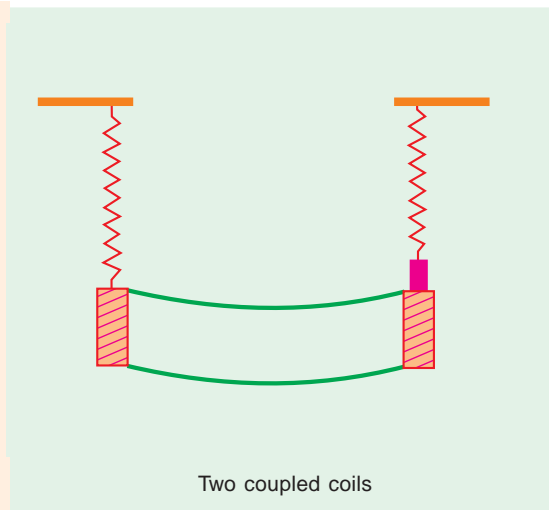
$$L_{22} = 0.5 + (1/x),$$

$$L_{12} = L_{21} = 1/x$$

These expressions are valid over a certain range of linear displacement  $x$ , in cms. The first coil is excited by a constant current of 20 A and the second one, by a constant current of  $-10$  A. Find

(a) mechanical work done if  $x$  changes from 0.5 to 1.0 cm

(b) energy supplied by the two electrical sources in (a) above.



**Solution.** With data given, substituting the values of currents,

$$\begin{aligned} W_{fld} &= \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2 \\ &= 225 + (50/x) \end{aligned}$$

$$F_f = -\delta W_{fld} / \delta x = 50/x^2$$

$$\begin{aligned} \text{(a) } \Delta W_{\text{mech}} &= \int_{0.5}^{1.0} (50/x^2) dx \\ &= 50[x^{-1} / -1]_{0.5}^{1.0} = +50 \text{ joules} \end{aligned}$$



At  $x = 0.5$ ,  $W_{fld} = 325$  joules, and

at  $x = 1.0$ ,  $W_{fld} = 275$  joules

Thus, increase in  $x$  from 0.50 to 1.0 cm decreases the stored energy in the field from 325 to 275 joules. The field-system, thus, releases an energy of 50 joules.

(b) Calculations of Energy input from electrical sources –

$$\begin{aligned} \lambda_1 &= L_{11} i_1 + L_{12} i_2 \\ &= 20 [1 + (1/x)] - 10 (1/x) = 20 + (10/x) \end{aligned}$$

At  $x = 0.5$ ,  $\lambda_1 = 20 + 20 = 40$  Wb-turns

$x = 1.0$ ,  $\lambda_1 = 20 + 10 = 30$  Wb-turns

$$\begin{aligned} \Delta W_{elec1} &= i_1 [\text{change in } \lambda_1 \text{ due to displacement}] \\ &= 20 \times (-10) = -200 \text{ Joules} \end{aligned}$$

Similarly,  $\lambda_2 = L_{12} i_1 + L_{22} i_2 = -5 + (10/x)$

At  $x = 0.5$ ,  $\lambda_2 = -5 + 20 = +15$  Wb-turns

$x = 1.0$ ,  $\lambda_2 = -5 + 10 = +5$  Wb-turns

$$\Delta W_{elec2} = (-10)(-10) = +100 \text{ Joules.}$$

Seeing the signs and numerical values, it can be seen that Source 1 receives an energy of 200 Joules, which comes from three constituents:

100 J from source 2,

50 J from field energy stored,

and 50 J from mechanical system.

### Tutorial Problems 25.1

(a) A magnetic circuit has a coil with 1000 turns. Its reluctance is expressed as

$$\mathcal{R} = [8.5 + 40g] \times 10^{-3} \text{ MKS units}$$

where  $g$  = air-gap length in mm, between fixed and movable parts. For a coil current of 2.0 amp held constant and with slow movement, calculate the change in the field energy stored, if the length of the air-gap changes from 0.20 to 0.15 cm. Calculate the mechanical force experienced by the system.

**Hint:**  $\Delta W_e = i(\lambda_2 - \lambda_1)$ ,  $\Delta W_{fld} = \frac{1}{2} \Delta W_e$

Force,  $F = -\Delta W_{fld} / \Delta x$

[Ans.  $\Delta W_{fld} = 6.60$  J, Force = 13200 Nw]

(b) An electro-magnetic relay with an air-gap of  $x$  cm has the current and flux-linkage relationship as

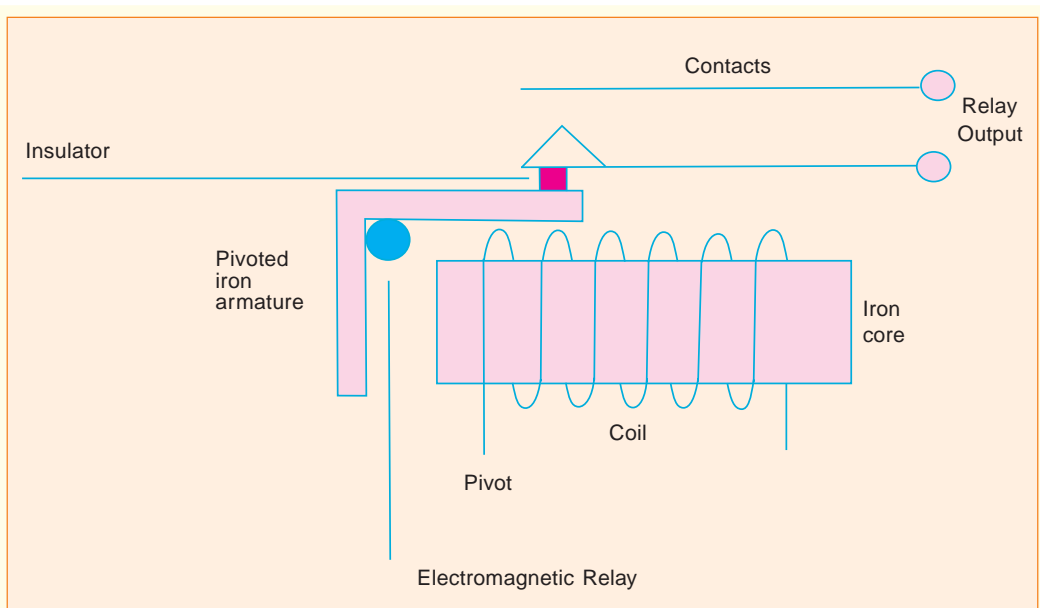
$$i = \lambda^2 + \lambda(0.5 - x)^2 \text{ amp, for } x < 0.5 \text{ cm}$$

Find the force on armature as a function of  $\lambda$  and  $x$ .

**Hint:**  $W_f(\lambda, x) = \int_{0.5}^{1.0} id \lambda$

And  $F_f = -\delta W_f / \delta x$

[Ans.  $W_f(\lambda, x) = (\lambda^3/3) + (\lambda^2/2)(0.5 - x)^2$   
 $F_f = \lambda^2(0.5 - x)$



(c) For a rotary system, the stator-coil and the rotor-coil have self and mutual-inductances as described below, with suffix 1 for stator and 2 for rotor:

$$L_{11} = L_{22} = 4 - (6\theta / \pi) \quad \text{for } 0 < \theta < \pi / 2$$

$$= 1 + (6 / \pi) (\theta - 0.5 \pi) \quad \text{for } \pi / 2 < \theta < \pi$$

(Note: Self inductances cannot be negative.)

$$L_{12} = L_{21} = 6 (1 - 2\theta / \pi) \quad \text{for } 0 < \theta < \pi$$

Evaluate the inductances and the torque for  $\theta = \pi / 4$  and the two coil currents of 5 amp constant in magnitude.

**Hint:**  $\delta L / \delta \theta$  contributes to torque.

$$[\text{Ans. } L_{11} = L_{22} = 2.5 \text{ H}$$

$$L_{12} = + 3 \text{ H}$$

$$T = 450 / \pi \text{ Nw}]$$

# CHAPTER 26

## Learning Objectives

- Generator Principal
- Simple Loop Generator
- Practical Generator
- Yoke
- Pole Cores and Pole Shoes
- Pole Coils
- Armature Core
- Armature Windings
- Bushes and Bearings
- Pole-pitch
- Conductor-Coil and Winding Element
- Coil-span or Coil-pitch
- Pitch of a Winding
- Back Pitch
- Front Pitch
- Resultant Pitch
- Commutator Pitch
- Single-layer Winding
- Two-layer Winding
- Degree of Re-entrancy of an Armature Winding
- Multiplex Winding
- Lap and Wave Winding
- Simplex-lap Winding
- Numbering of Coils and Commutator Segments
- Simplex Wave Winding
- Dummy or Idle Coils
- Uses of Lap and Wave Windings
- Types of Generators
- Brush Contact Drop
- Generated E.M.F. or E.M.F.
- Equation of a Generator
- Iron Loss in Armature
- Total loss in a D.C. Generator
- Stray Losses
- Constant or Standing Losses
- Power Stages
- Condition for Maximum Efficiency

## D.C. GENERATORS



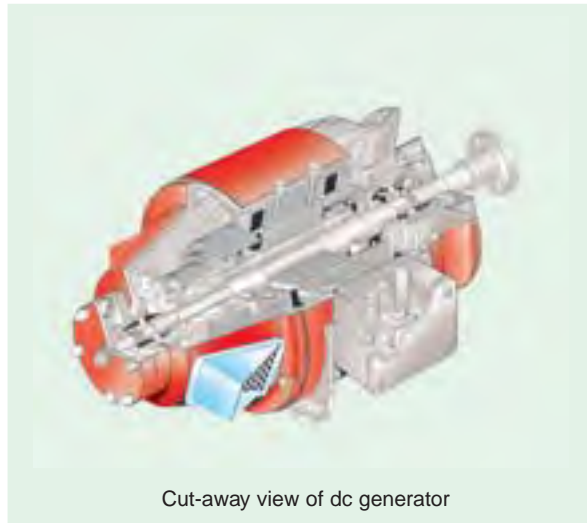
Generator converts mechanical energy into electrical energy using electromagnetic induction

### 26.1. Generator Principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power).

The energy conversion is based on the principle of the production of dynamically (or motionally) induced e.m.f. As seen from Fig. 26.1, whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic Induction. This e.m.f. causes a current to flow if the conductor circuit is closed.

Hence, two basic essential parts of an electrical generator are (i) a magnetic field and (ii) a conductor or conductors which can so move as to cut the flux.



### 26.2. Simple Loop Generator

#### Construction

In Fig. 26.1 is shown a single-turn rectangular copper coil  $ABCD$  rotating about its own axis in a magnetic field provided by either permanent magnet or electromagnets. The two ends of the coil

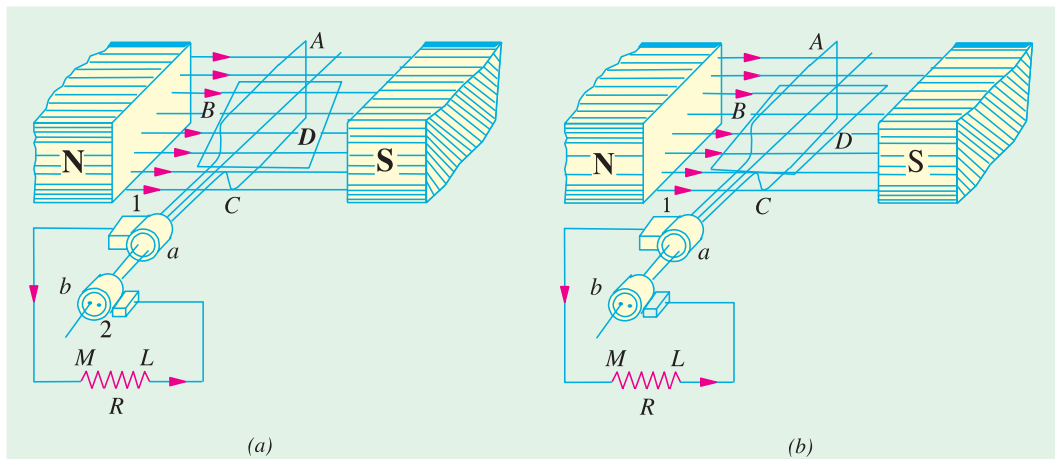


Fig. 26.1

are joined to two slip-rings 'a' and 'b' which are insulated from each other and from the central shaft. Two collecting brushes (of carbon or copper) press against the slip-rings. Their function is to collect the current induced in the coil and to convey it to the external load resistance  $R$ .

The rotating coil may be called 'armature' and the magnets as 'field magnets'.

#### Working

Imagine the coil to be rotating in clock-wise direction (Fig. 26.2). As the coil assumes successive positions in the field, the flux linked with it changes. Hence, an e.m.f. is induced in it which is

proportional to the rate of change of flux linkages ( $e = Nd\Phi/dt$ ). When the plane of the coil is at right angles to lines of flux *i.e.* when it is in position, 1, then flux linked with the coil is maximum but **rate of change of flux linkages is minimum.**

It is so because in this position, the coil sides  $AB$  and  $CD$  do not cut or shear the flux, rather they slide along them *i.e.* they move parallel to them. Hence, there is no induced e.m.f. in the coil. Let us take this no-e.m.f. or vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

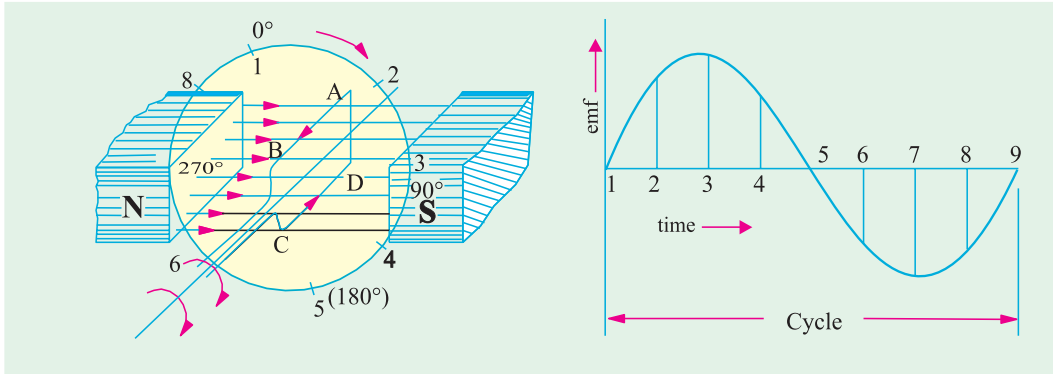


Fig. 26.2

Fig. 26.3

As the coil continues rotating further, the rate of change of flux linkages (and hence induced e.m.f. in it) increases, till position 3 is reached where  $\theta = 90^\circ$ . Here, the coil plane is horizontal *i.e.* parallel to the lines of flux. As seen, the flux linked with the coil is minimum but **rate of change of flux linkages is maximum.** Hence, maximum e.m.f. is induced in the coil when in this position (Fig. 26.3).

In the next quarter revolution *i.e.* from  $90^\circ$  to  $180^\circ$ , the flux linked with the coil gradually **increases** but the rate of change of flux linkages **decreases**. Hence, the induced e.m.f. decreases gradually till in position 5 of the coil, it is reduced to zero value.

So, we find that in the first half revolution of the coil, no (or minimum) e.m.f. is induced in it when in position 1, maximum when in position 3 and no e.m.f. when in position 5. The direction of this induced e.m.f. can be found by applying Fleming's Right-hand rule which gives its direction from  $A$  to  $B$  and  $C$  to  $D$ . Hence, the direction of current flow is **ABMLCD** (Fig. 26.1). The current through the load resistance  $R$  flows from  $M$  to  $L$  during the first half revolution of the coil.

In the next half revolution *i.e.* from  $180^\circ$  to  $360^\circ$ , the variations in the magnitude of e.m.f. are similar to those in the first half revolution. Its value is maximum when coil is in position 7 and minimum when in position 1. But it will be found that the direction of the induced current is from  $D$  to  $C$  and  $B$  to  $A$  as shown in Fig. 26.1 (b). Hence, the path of current flow is along **DCLMBA** which is just the reverse of the previous direction of flow.

Therefore, we find that the current which we obtain from such a simple generator reverses its direction after every half revolution. Such a current undergoing periodic reversals is known as alternating current. It is, obviously, different from a direct current which continuously flows in one and the same direction. It should be noted that alternating current not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two half-cycles may be called positive and negative half-cycles respectively (Fig. 26.3).

For making the flow of current unidirectional in the **external** circuit, the slip-rings are replaced by split-rings (Fig. 26.4). The split-rings are made out of a conducting cylinder which is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material (Fig. 26.5).

As before, the coil ends are joined to these segments on which rest the carbon or copper brushes.

It is seen [Fig. 26.6 (a)] that in the first half revolution current flows along **(ABMNLCD)** i.e. the brush No. 1 in contact with segment 'a' acts as the positive end of the supply and 'b' as the negative end. In the next half revolution [Fig. 26.6 (b)], the direction of the induced current in the coil has reversed. But at the same time, the positions of segments 'a' and 'b' have also reversed with the

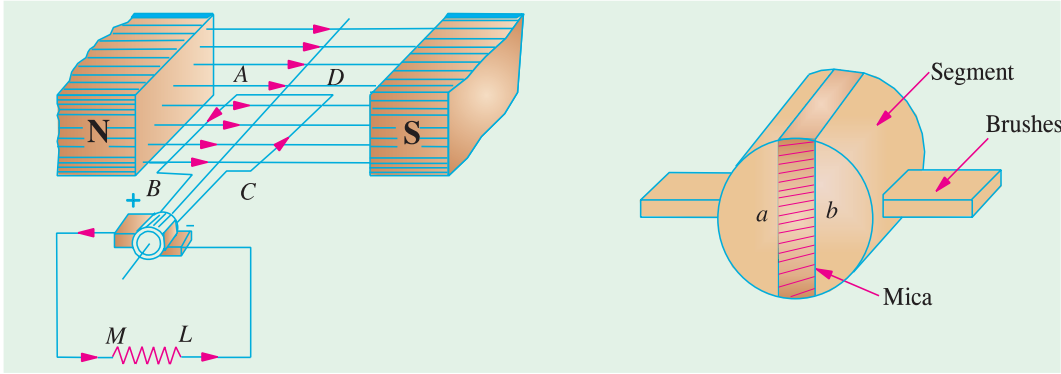


Fig. 26.4

Fig. 26.5

result that brush No. 1 comes in touch with the segment which is positive i.e. segment 'b' in this case. Hence, current in the load resistance again flows from **M to L**. The waveform of the current through the external circuit is as shown in Fig. 26.7. **This current is unidirectional but not continuous like pure direct current.**

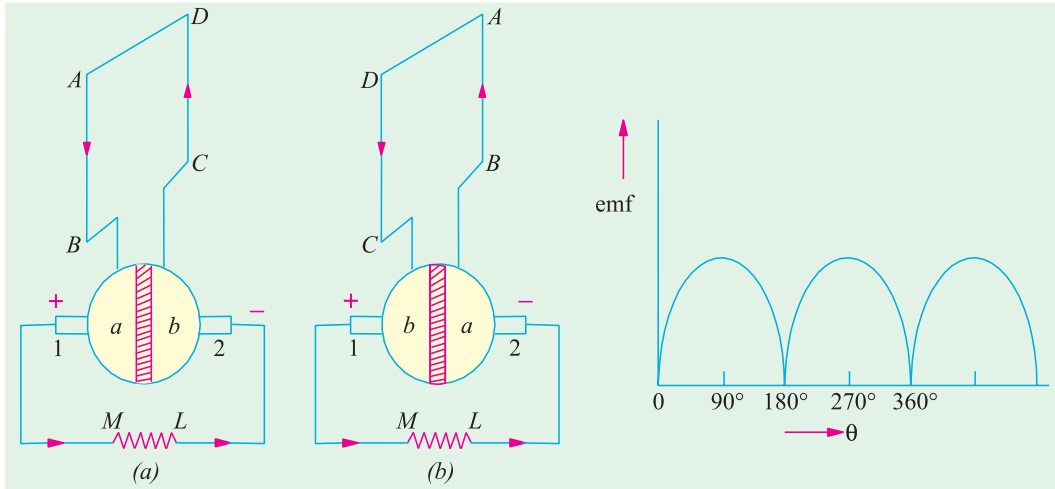


Fig. 26.6

Fig. 26.7

It should be noted that the position of brushes is so arranged that the change over of segments 'a' and 'b' from one brush to the other takes place when the plane of the rotating coil is at right angles to the plane of the lines of flux. It is so because in that position, the induced e.m.f. in the coil is zero.

Another important point worth remembering is that even now the current induced in the coil sides is alternating as before. It is only due to the rectifying action of the split-rings (also called commutator) that it becomes unidirectional in the external circuit. Hence, it should be clearly understood that even in the armature of a d.c. generator, the induced voltage is alternating.

### 26.3. Practical Generator

The simple loop generator has been considered in detail merely to bring out the basic principle

underlying construction and working of an actual generator illustrated in Fig. 26.8 which consists of the following essential parts :

- |                                    |                              |
|------------------------------------|------------------------------|
| 1. Magnetic Frame or Yoke          | 2. Pole-Cores and Pole-Shoes |
| 3. Pole Coils or Field Coils       | 4. Armature Core             |
| 5. Armature Windings or Conductors | 6. Commutator                |
| 7. Brushes and Bearings            |                              |

Of these, the yoke, the pole cores, the armature core and air gaps between the poles and the armature core or the magnetic circuit whereas the rest form the electrical circuit.

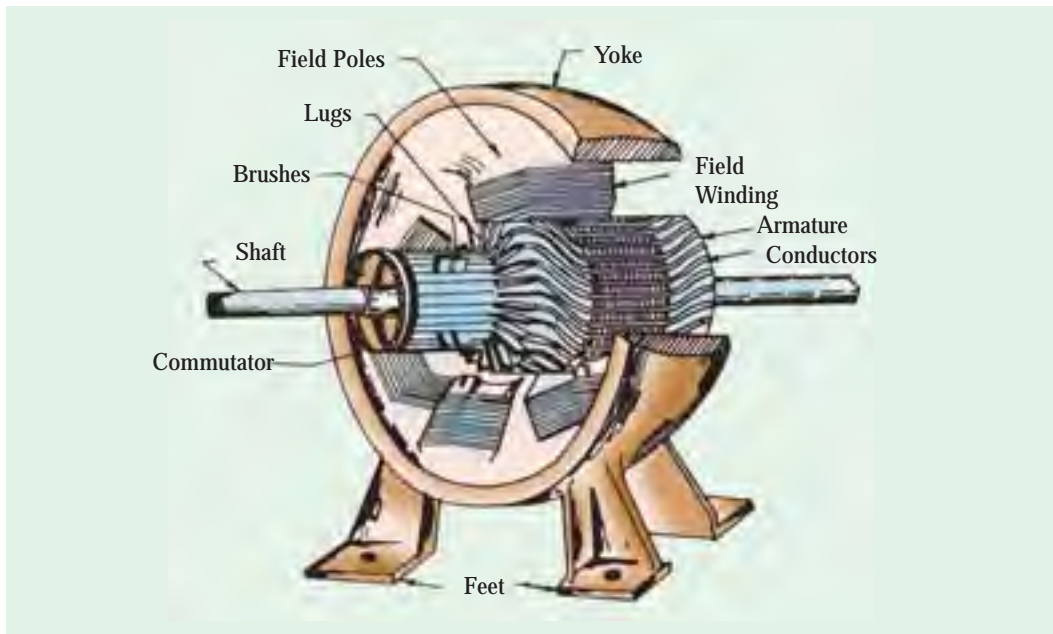


Fig. 26.8

#### 26.4. Yoke

The outer frame or yoke serves double purpose :

(i) It provides mechanical support for the poles and acts as a protecting cover for the whole machine and

(ii) It carries the magnetic flux produced by the poles.

In small generators where cheapness rather than weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed. The modern process of forming the yoke consists of rolling a steel slab round a cylindrical mandrel and then welding it at the bottom. The feet and the terminal box etc. are welded to the frame afterwards. Such yokes possess sufficient mechanical strength and have high permeability.



#### 26.5. Pole Cores and Pole Shoes

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes

(i) they spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path (ii) they support the exciting coils (or field coils) as shown in Fig. 26.14.

There are two main types of pole construction.

(a) The pole core itself may be a solid piece made out of either cast iron or cast steel but the pole shoe is laminated and is fastened to the pole face by means of counter sunk screws as shown in Fig. 24.10.

(b) In modern design, the complete pole cores and pole shoes are built of thin laminations of annealed steel which are rivetted together under hydraulic pressure (Fig. 26.11). The thickness of laminations varies from 1 mm to 0.25 mm. The laminated poles may be secured to the yoke in any of the following two ways :

- (i) Either the pole is secured to the yoke by means of screws bolted through the yoke and into the pole body or
- (ii) The holding screws are bolted into a steel bar which passes through the pole across the plane of laminations (Fig. 26.12).

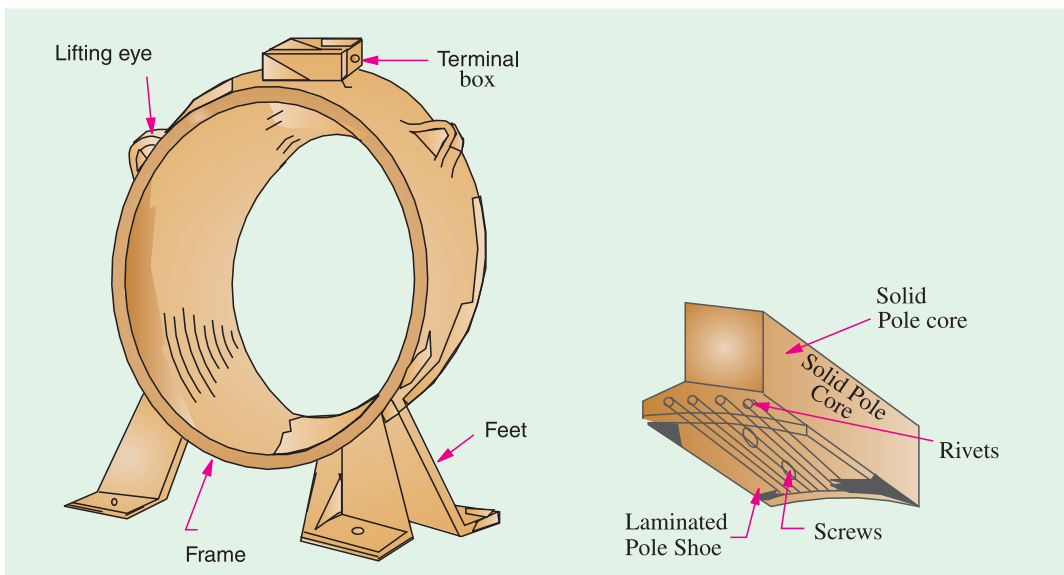


Fig. 26.9

Fig. 26.10

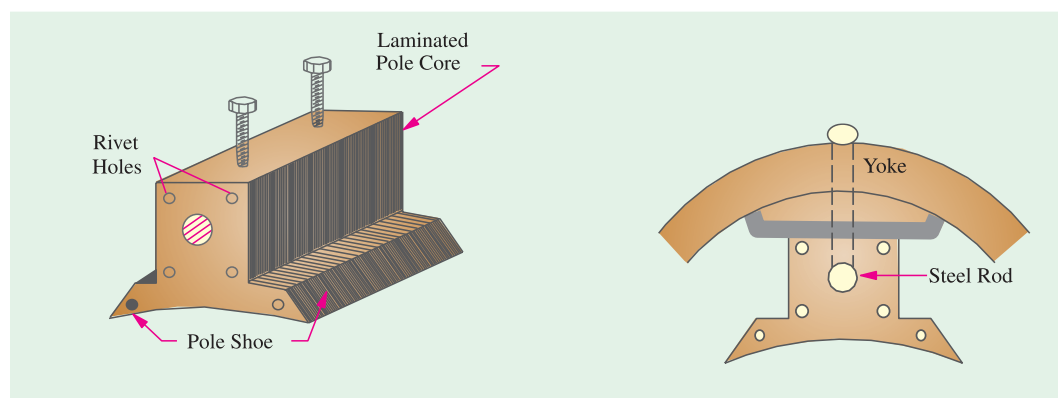


Fig. 26.11

Fig. 26.12



### 26.6. Pole Coils

The field coils or pole coils, which consist of copper wire or strip, are former-wound for the correct dimension (Fig. 26.13). Then, the former is removed and wound coil is put into place over the core as shown in Fig. 26.14.

When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by revolving armature conductors.

### 26.7. Armature Core

It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux through the armature from a  $N$ -pole to a  $S$ -pole.

It is cylindrical or drum-shaped and is built up of usually circular sheet steel discs or laminations approximately 0.5 mm thick (Fig. 26.15). It is keyed to the shaft.

The slots are either die-cut or punched on the outer periphery of the disc and the keyway is located on the inner diameter as shown. In small machines, the armature stampings are keyed directly to the shaft. Usually, these laminations are perforated for air ducts which permits axial flow of air through the armature for cooling purposes. Such ventilating channels are clearly visible in the laminations shown in Fig. 26.16 and Fig. 26.17.

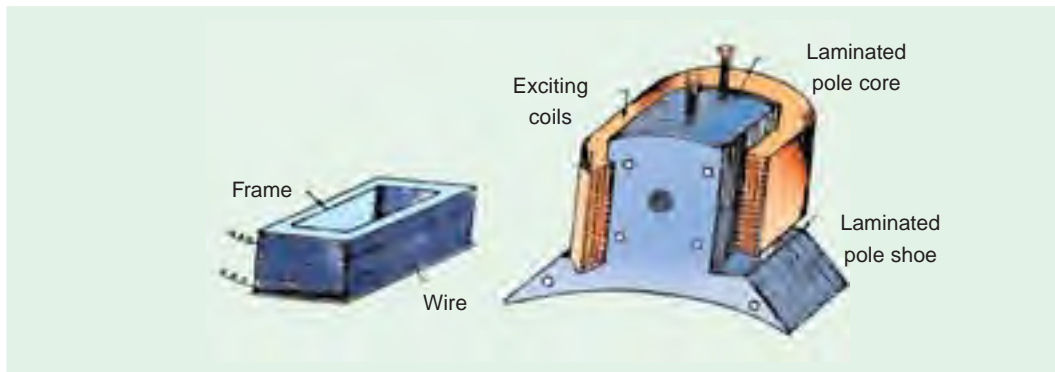


Fig. 26.13

Fig. 26.14

Up to armature diameters of about one metre, the circular stampings are cut out in one piece as shown in Fig. 26.16. But above this size, these circles, especially of such thin sections, are difficult to handle because they tend to distort and become wavy when assembled together. Hence, the circular laminations, instead of being cut out in one piece, are cut in a number of suitable sections or segments which form part of a complete ring (Fig. 26.17).

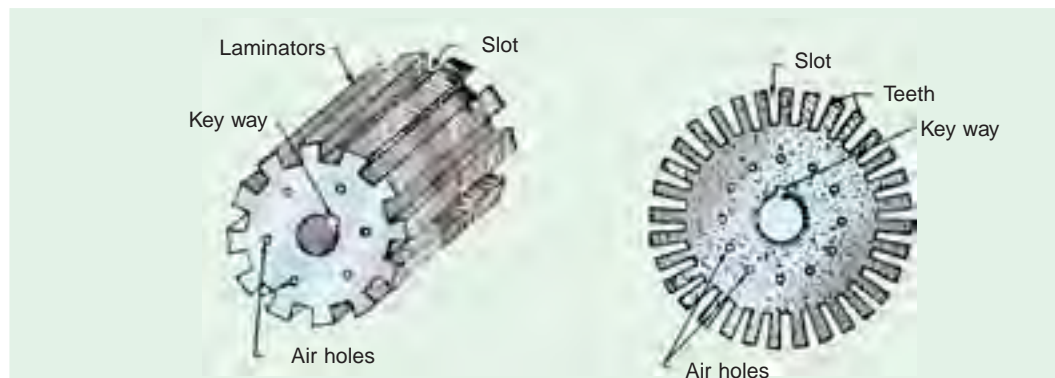


Fig. 26.15

Fig. 26.16

A complete circular lamination is made up of four or six or even eight segmental laminations. Usually, two keyways are notched in each segment and are dove-tailed or wedge-shaped to make the laminations self-locking in position.

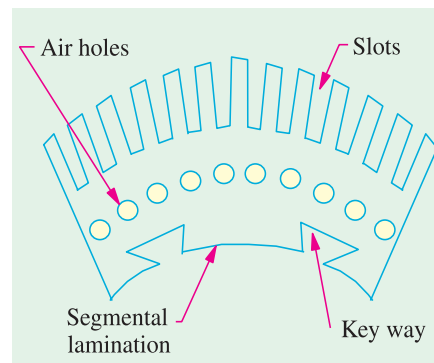
The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced e.m.f., smaller the current and hence lesser the  $I^2 R$  loss in the core.

### 26.8. Armature Windings

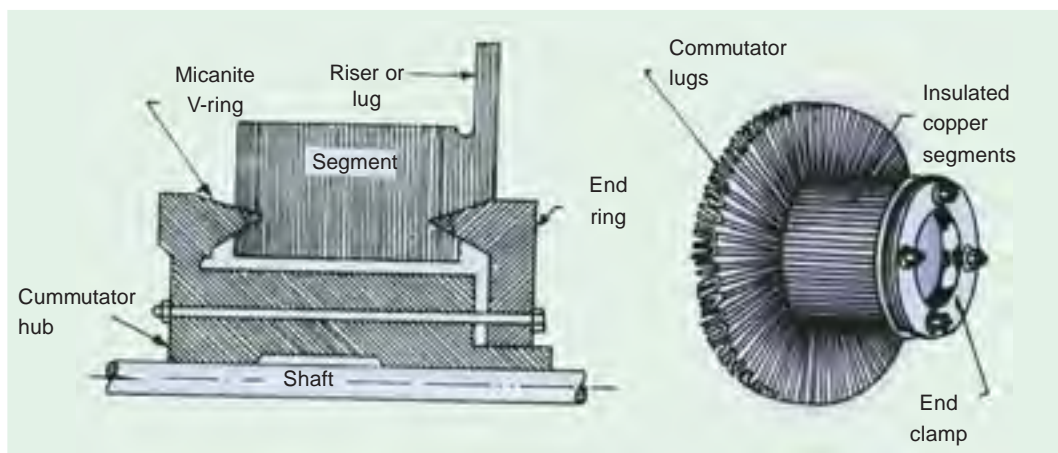
The armature windings are usually former-wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fibre wedges.

### 26.9. Commutator

The function of the commutator is to facilitate collection of current from the armature conductors. As shown in Art. 26.2, it rectified *i.e.* converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of cylindrical structure and is built up of wedge-shaped segments of high-conductivity hard-drawn or drop forged copper. These



**Fig. 26.17**



**Fig. 26.18**

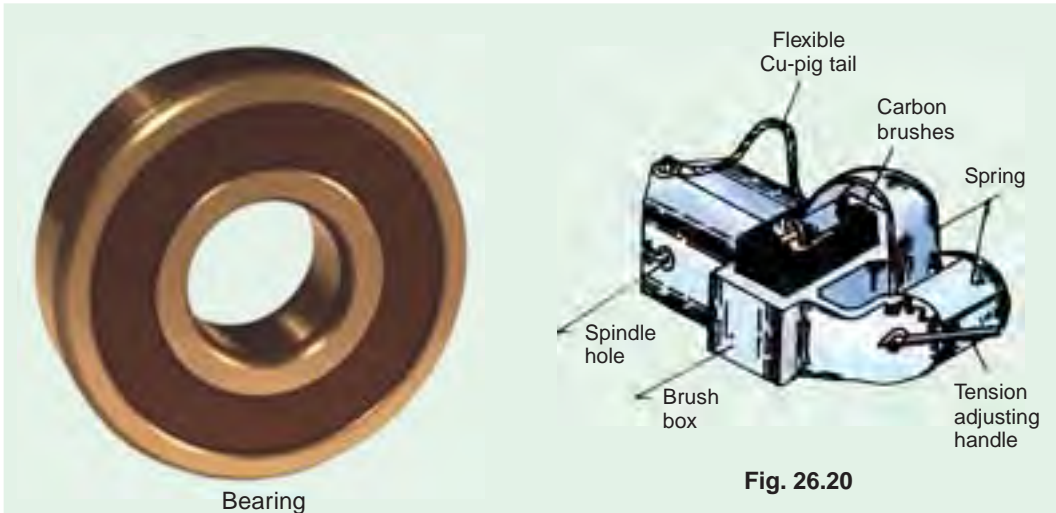
**Fig. 26.19**

segments are insulated from each other by thin layers of mica. The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor by means of a copper lug or strip (or riser). To prevent them from flying out under the action of centrifugal forces, the segments have V-grooves, these grooves being insulated by conical micanite rings. A sectional view of commutator is shown in Fig. 26.18 whose general appearance when completed is shown in Fig. 26.19.

### 26.10. Brushes and Bearings

The brushes whose function is to collect current from commutator, are usually made of carbon or

graphite and are in the shape of a rectangular block. These brushes are housed in brush-holders usually of the box-type variety. As shown in Fig. 26.20, the brush-holder is mounted on a spindle and the brushes can slide in the rectangular box open at both ends. The brushes are made to bear down on the commutator by a spring whose tension can be adjusted by changing the position of lever in the notches. A flexible copper pigtail mounted at the top of the brush conveys current from the brushes to the holder. The number of brushes per spindle depends on the magnitude of the current to be collected from the commutator.



Because of their reliability, ball-bearings are frequently employed, though for heavy duties, roller bearings are preferable. The ball and rollers are generally packed in hard oil for quieter operation and for reduced bearing wear, sleeve bearings are used which are lubricated by ring oilers fed from oil reservoir in the bearing bracket.

### 26.11. Armature Windings

Now, we will discuss the winding of an actual armature. But before doing this, the meaning of the following terms used in connection with armature winding should be clearly kept in mind.

### 26.12. Pole-pitch

It may be variously defined as :

- (i) The periphery of the armature divided by the number of poles of the generator *i.e.* the distance between two adjacent poles.
- (ii) It is equal to the number of armature conductors (or armature slots) per pole. If there are 48 conductors and 4 poles, the pole pitch is  $48/4 = 12$ .



### 26.13. Conductor

The length of a wire lying in the magnetic field and in which an e.m.f. is induced, is called a conductor (or inductor) as, for example, length *AB* or *CD* in Fig. 26.21.

### 26.14. Coil and Winding Element

With reference to Fig. 26.21, the two conductors  $AB$  and  $CD$  along with their end connections constitute one coil of the armature winding. The coil may be single-turn coil (Fig. 26.21) or multi-turn coil (Fig. 26.22). A single-turn coil will have two conductors. But a multi-turn coil may have many conductors per coil side. In Fig. 26.22, for example, each coil side has 3 conductors. The

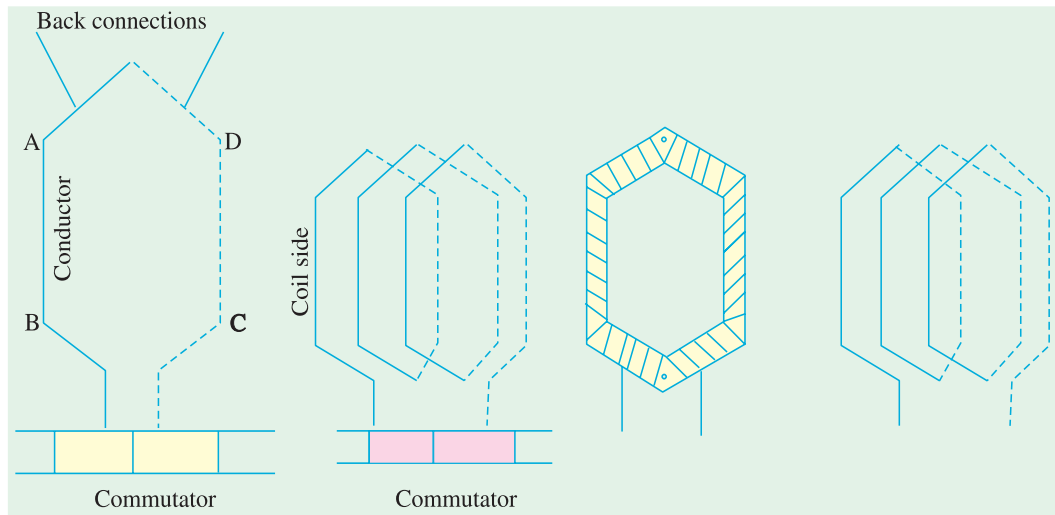


Fig. 26.21

Fig. 26.22

Fig. 26.23

group of wires or conductors constituting a coil side of a multi-turn coil is wrapped with a tape as a unit (Fig. 26.23) and is placed in the armature slot. It may be noted that since the beginning and the end of each coil must be connected to a commutator bar, there are as many commutator bars as coils for both the lap and wave windings (see Example 26.1).

The side of a coil (1-turn or multiturn) is called a winding element. Obviously, the number of winding elements is twice the number of coils.

### 26.15. Coil-span or Coil-pitch ( $Y_s$ )

It is the distance, measured in terms of armature slots (or armature conductors) between two sides of a coil. It is, in fact, the periphery of the armature spanned by the two sides of the coil.

If the pole span or coil pitch is equal to the pole pitch (as in the case of coil  $A$  in Fig. 26.24 where pole-pitch of 4 has been assumed), then winding is called **full-pitched**. It means that coil span is 180 electrical degrees. In this case, the coil sides lie under opposite poles, hence the induced e.m.f.s. in them are additive. Therefore, maximum e.m.f. is induced in the coil as a whole, it being the sum of the e.m.f.s induced in the two coil sides. For example, if there are 36 slots and 4 poles, then coil span is  $36/4 = 9$  slots. If number of slots is 35, then  $Y_s = 35/4 = 8$  because it is customary to drop fractions.

If the coil span is less than the pole pitch (as in coil  $B$  where coil pitch is  $3/4$ th of the pole pitch), then the

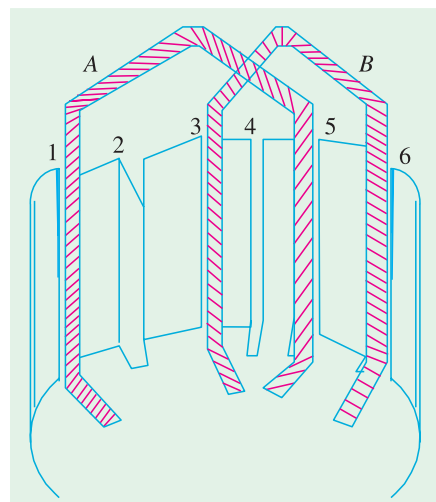


Fig. 26.24

winding is fractional-pitched. In this case, there is a phase difference between the e.m.fs. in the two sides of the coil. Hence, the total e.m.f. round the coil which is the vector sum of e.m.fs. in the two coil sides, is less in this case as compared to that in the first case.

**26.16. Pitch of a Winding ( $Y$ )**

In general, it may be defined as the distance round the armature between two successive conductors which are directly connected together. Or, it is the distance between the beginnings of two consecutive turns.

$$Y = Y_B - Y_F \text{ .....for lap winding}$$

$$= Y_B + Y_F \text{ .....for wave winding}$$

In practice, coil-pitches as low as eight-tenths of a pole pitch are employed without much serious reduction in the e.m.f. Fractional-pitched windings are purposely used to effect substantial saving in the copper of the end connections and for improving commutation.

**26.17. Back Pitch ( $Y_B$ )**

The distance, measured in terms of the armature conductors, which a coil advances on the back of the armature is called back pitch and is denoted by  $Y_B$ .

As seen from Fig. 26.28, element 1 is connected on the back of the armature to element 8. Hence,  $Y_B = (8 - 1) = 7$ .

**26.18. Front Pitch ( $Y_F$ )**

The number of armature conductors or elements spanned by a coil on the front (or commutator end of an armature) is called the front pitch and is designated by  $Y_F$ . Again in Fig. 26.28, element 8 is connected to element 3 on the front of the armature, the connections being made at the commutator segment. Hence,  $Y_F = 8 - 3 = 5$ .

Alternatively, the front pitch may be defined as the distance (in terms of armature conductors) between the second conductor of one coil and the first conductor of the next coil which are connected together at the front *i.e.* commutator end of the armature. Both front and back pitches for lap and wave-winding are shown in Fig. 26.25 and 26.26.

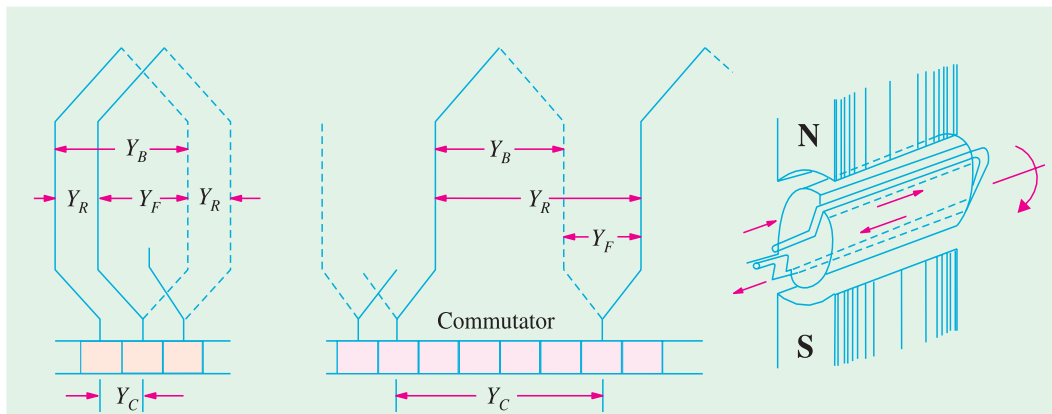


Fig. 26.25

Fig. 26.26

Fig. 26.27

**26.19. Resultant Pitch ( $Y_R$ )**

It is the distance between the beginning of one coil and the beginning of the next coil to which it is connected (Fig. 26.25 and 26.26).

As a matter of precaution, it should be kept in mind that all these pitches, though normally

stated in terms of armature conductors, are also sometimes given in terms of armature slots or commutator bars because commutator is, after all, an image of the winding.

### 26.20. Commutator Pitch ( $Y_C$ )

It is the distance (measured in commutator bars or segments) between the segments to which the two ends of a coil are connected. From Fig. 26.25 and 26.26 it is clear that for lap winding,  $Y_C$  is the **difference** of  $Y_B$  and  $Y_F$  whereas for wavewinding it is the **sum** of  $Y_B$  and  $Y_F$ . Obviously, commutator pitch is equal to the number of bars between coil leads. In general,  $Y_C$  equals the 'plex' of the lap-wound armature. Hence, it is equal to 1, 2, 3, 4 etc. for simplex-, duplex, triplex- and quadruplex etc. lap-windings.

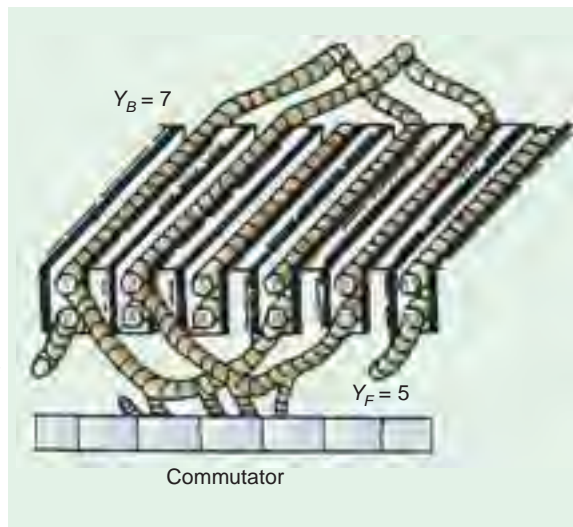


Fig. 26.28

### 26.21. Single-layer Winding

It is that winding in which one conductor or one coil side is placed in each armature slot as shown in Fig. 26.27. Such a winding is not much used.

### 26.22. Two-layer Winding

In this type of winding, there are two conductors or coil sides per slot arranged in two layers. Usually, one side of every coil lies in the upper half of one slot and other side lies in the lower half of some other slot at a distance of approximately one pitch away (Fig. 26.28). The transfer of the coil from one slot to another is usually made in a radial plane by means of a peculiar bend or twist at the back end as shown in Fig. 26.29. Such windings in which two coil sides occupy each slot are most commonly used for all medium-sized machines. Sometimes 4 or 6 or 8 coil sides are used in each slot in several layers because it is not practicable to have too many slots (Fig. 26.30). The coil sides lying at the upper half of the slots are numbered odd *i.e.* 1, 3, 5, 7 etc. while those at the lower half are numbered even *i.e.* 2, 4, 6, 8 etc.

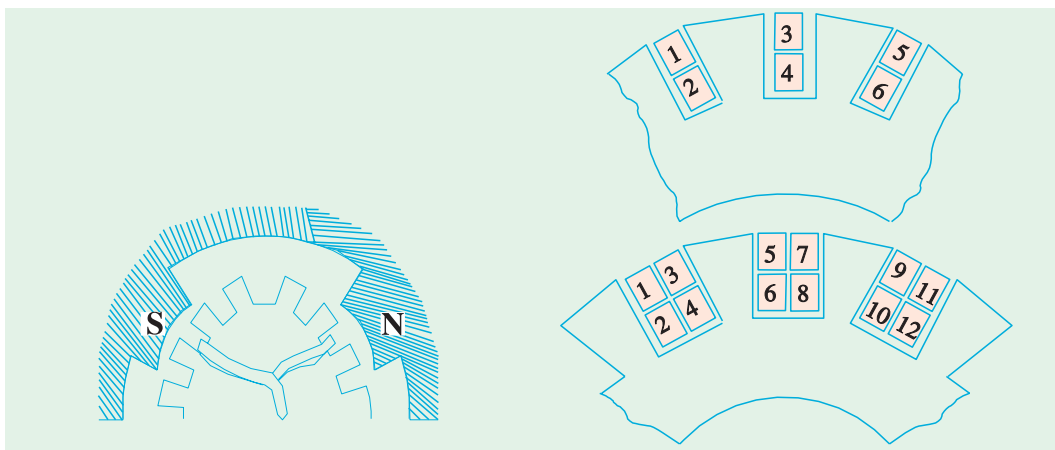


Fig. 26.29

Fig. 26.30

### 26.23. Degree of Re-entrant of an Armature Winding

A winding is said to be single re-entrant if on tracing through it once, all armature conductors are included on returning to the starting point. It is double re-entrant if only half the conductors are included in tracing through the winding once and so on.

### 26.24. Multiplex Winding

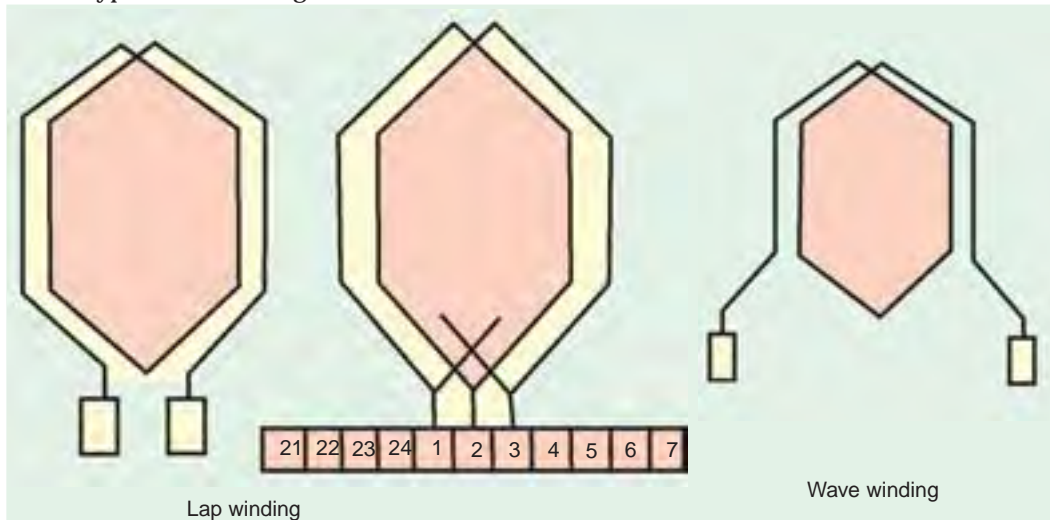
In such windings, there are several sets of completely closed and independent windings. If there is only one set of closed winding, it is called simplex wave winding. If there are two such windings on the same armature, it is called duplex winding and so on. The multiplicity affects a number of parallel paths in the armature. For a given number of armature slots and coils, as the multiplicity increases, the number of parallel paths in the armature increases thereby increasing the current rating but decreasing the voltage rating.



Multiplex Winding

### 26.25. Lap and Wave Windings

Two types of windings mostly employed for drum-type armatures are known as Lap Winding and Wave Winding. The difference between the two is merely due to the different arrangement of the end connections at the front or commutator end of armature. Each winding can be arranged progressively or retrogressively and connected in simplex, duplex and triplex. The following rules, however, apply to both types of the windings :



Lap winding

Wave winding

- (i) The front pitch and back pitch are each approximately equal to the pole-pitch *i.e.* windings should be full-pitched. This results in increased e.m.f. round the coils. For special purposes, fractional-pitched windings are deliberately used (Art. 26.15).
- (ii) Both pitches should be odd, otherwise it would be difficult to place the coils (which are former-wound) properly on the armature. For example, if  $Y_B$  and  $Y_F$  were both even, the *all* the coil sides and conductors would lie either in the upper half of the slots or in the lower

half. Hence, it would become impossible for one side of the coil to lie in the upper half. Hence, it would become impossible for one side of the coil to lie in the upper half of one slot and the other side of the same coil to lie in the lower half of some other slot.

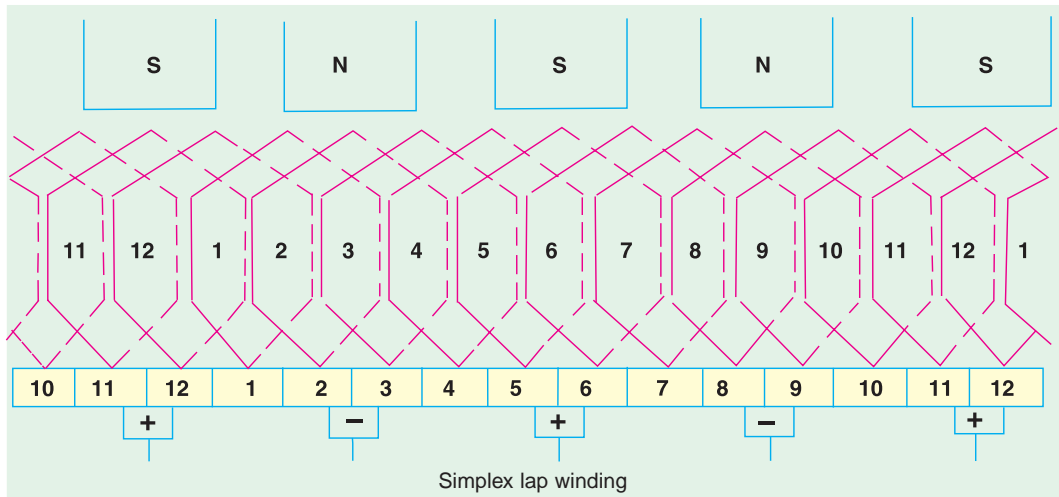
- (iii) The number of commutator segments is equal to the number of slots or coils (or half the number of conductors) because the front ends of conductors are joined to the segments in pairs.
- (iv) The winding must close upon itself *i.e.* if we start from a given point and move from one coil to another, then all conductors should be traversed and we should reach the same point again without a break or discontinuity in between.

### 26.26. Simplex Lap-winding\*

It is shown in Fig. 26.25 which employs single-turn coils. In lap winding, the finishing end of one coil is connected to a commutator segment and to the starting end of the adjacent coil situated under the same pole and so on, till all the coils have been connected. This type of winding derives its name from the fact it doubles or laps back with its succeeding coils.

Following points regarding simplex lap winding should be carefully noted :

1. The back and front pitches are odd and of opposite sign. But they cannot be equal. They differ by 2 or some multiple thereof.
2. Both  $Y_B$  and  $Y_F$  should be nearly equal to a pole pitch.
3. The average pitch  $Y_A = \frac{Y_B + Y_F}{2}$ . It equals pole pitch =  $\frac{Z}{P}$ .
4. Commutator pitch  $Y_C = \pm 1$ . (In general,  $Y_C = \pm m$ )
5. Resultant pitch  $Y_R$  is even, being the arithmetical difference of two odd numbers, *i.e.*,  $Y_R = Y_B - Y_F$
6. The number of slots for a 2-layer winding is equal to the number of coils (*i.e.* half the number of coil sides). The number of commutator segments is also the same.



\* However, where heavy currents are necessary, duplex or triplex lap windings are used. The duplex lap winding is obtained by placing two similar windings on the same armature and connecting the even-numbered commutator bars to one winding and the odd-numbered ones to the second winding. Similarly, in triplex lap winding, there would be three windings, each connected to one third of the commutator bars.



7. The number of parallel paths in the armature =  $mP$  where  $m$  is the multiplicity of the winding and  $P$  the number of poles.

Taking the first condition, we have  $Y_B = Y_F \pm 2$ .\*\*

(a) If  $Y_B > Y_F$  i.e.  $Y_B = Y_F + 2$ , then we get a progressive or right-handed winding i.e. a winding which progresses in the clockwise direction as seen from the commutator end. In this case, obviously,  $Y_C = +1$ .

(b) If  $Y_B < Y_F$  i.e.  $Y_B = Y_F - 2$ , then we get a retrogressive or left-handed winding i.e. one which advances in the anti-clockwise direction when seen from the commutator side. In this case,  $Y_C = -1$ .

(c) Hence, it is obvious that

$$\left. \begin{aligned} Y_F &= \frac{Z}{P} - 1 \\ Y_B &= \frac{Z}{P} + 1 \end{aligned} \right\} \text{for progressive winding and} \quad \left. \begin{aligned} Y_F &= \frac{Z}{P} + 1 \\ Y_B &= \frac{Z}{P} - 1 \end{aligned} \right\} \text{for retrogressive winding}$$

Obviously,  $Z/P$  must be even to make the winding possible.

### 26.27. Numbering of Coils and Commutator Segments

In the d.c. winding diagrams to follow, we will number the coils only (not individual turns). The upper side of the coil will be shown by a firm continuous line whereas the lower side will be shown by a broken line. The numbering of coil sides will be consecutive i.e. 1, 2, 3 ..... etc. and such that odd numbers are assigned to the top conductors and even numbers to the lower sides for a two-layer winding. The commutator segments will also be numbered consecutively, the number of the segments will be the same as that of the upper side connected to it.

**Example 26.1.** Draw a developed diagram of a simple 2-layer lap-winding for a 4-pole generator with 16 coils. Hence, point out the characteristics of a lap-winding.

(Elect. Engineering, Madras Univ. 1981)

**Solution.** The number of commutator segments = 16

Number of conductors or coil sides  $16 \times 2 = 32$  ; pole pitch =  $32/4 = 8$

Now remembering that (i)  $Y_B$  and  $Y_F$  have to be odd and (ii) have to differ by 2, we get for a progressive winding  $Y_B = 9$  ;  $Y_F = -7$  (retrogressive winding will result if  $Y_B = 7$  and  $Y_F = -9$ ). Obviously, commutator pitch  $Y_C = -1$ .

[Otherwise, as shown in Art. 26.26, for progressive winding

$$Y_F = \frac{Z}{P} - 1 = \frac{32}{4} - 1 = 7 \text{ and } Y_B = \frac{Z}{P} - 1 = \frac{32}{4} + 1 = 9]$$

The simple winding table is given as under :

<b>Back Connections</b>		<b>Front Connections</b>
1 to (1 + 9) = 10	—————>	10 to (10 - 7) = 3
3 to (3 + 9) = 12	—————>	12 to (12 - 7) = 5
5 to (5 + 9) = 14	—————>	14 to (14 - 7) = 7
7 to (7 + 9) = 16	—————>	16 to (16 - 7) = 9
9 to (9 + 9) = 18	—————>	18 to (18 - 7) = 11
11 to (11 + 9) = 20	—————>	20 to (20 - 7) = 13
13 to (13 + 9) = 22	—————>	22 to (22 - 7) = 15
15 to (15 + 9) = 24	—————>	24 to (24 - 7) = 17
17 to (17 + 9) = 26	—————>	26 to (26 - 7) = 19
19 to (19 + 9) = 28	—————>	28 to (28 - 7) = 21

\*\* In general,  $Y_B = Y_F \pm 2m$  where  $m = 1$  for simplex lap winding and  $m = 2$  for duplex lap winding etc.

21 to $(21 + 9) = 30$	—————>	30 to $(20 - 7) = 23$
23 to $(23 + 9) = 32$	—————>	32 to $(32 - 7) = 25$
25 to $(25 + 9) = 34 = (34 - 32) = 2$	—————>	2 to $(34 - 7) = 27$
27 to $(27 + 9) = 36 = (36 - 32) = 4$	—————>	4 to $(36 - 7) = 29$
29 to $(29 + 9) = 38 = (38 - 32) = 6$	—————>	6 to $(38 - 7) = 31$
31 to $(31 + 9) = 40 = (40 - 32) = 8$	—————>	8 to $(40 - 7) = 33 = (33 - 32) = 1$

The winding ends here because we come back to the conductor from where we started.

We will now discuss the developed diagram which is one that is obtained by imagining the armature surface to be removed and then laid out flat so that the slots and conductors can be viewed without the necessity of turning round the armature in order to trace out the armature windings. Such a developed diagram is shown in Fig. 26.31.

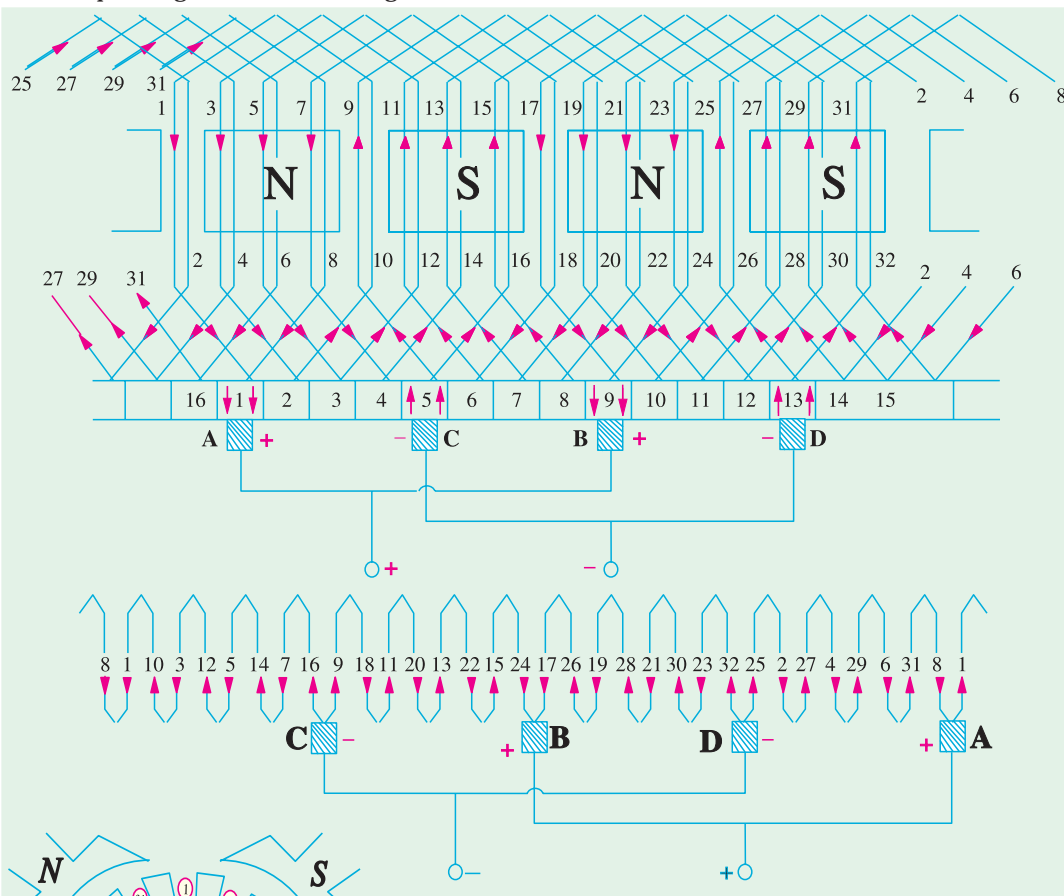


Fig. 26.31

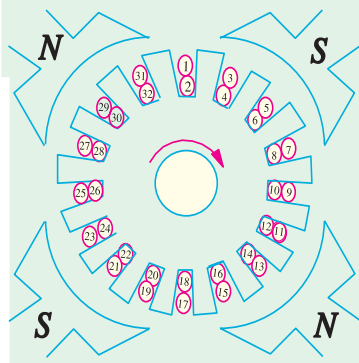


Fig. 26.32

The procedure of developing the winding is this :

Front end of the upper side of coil No. 1 is connected to a commutator segment (whose number is also 1). The back end is joined at the back to the  $1 + 9 = 10$ th coil side in the lower half of 5th slot. The front end of coil side 10 is joined to commutator segment 2 to which is connected the front end of  $10 - 7 = 3$  i.e. 3rd coil side lying in the upper half of second armature slot. In this way, by travelling 9 coil sides to the right at the back and 7 to the left at the

front we complete the winding, thus including every coil side once till we reach the coil side 1 from where we started. Incidentally, it should be noted that all upper coil sides have been given odd numbers, whereas lower ones have been given even numbers as shown in the polar diagram (Fig. 26.32) of the winding of Fig. 26.31.

Brush positions can be located by finding the direction of currents flowing in the various conductors. If currents in the conductors under the influence of a  $N$ -pole are assumed to flow downwards (as shown), then these will flow upwards in conductors under the influence of  $S$ -pole. By putting proper arrows on the conductors (shown separately in the equivalent ring diagram), it is found that commutator bars No. 1 and 9 are the meeting points of e.m.fs. and hence currents are flowing out of these conductors. The positive brushes should, therefore, be placed at these commutator bars. Similarly, commutator bars No. 5 and 13 are the separating points of e.m.fs. hence negative brushes are placed there. In all, there are four brushes, two positive and two negative. If brushes of the same polarity are connected together, then all the armature conductors are divided into four parallel paths.

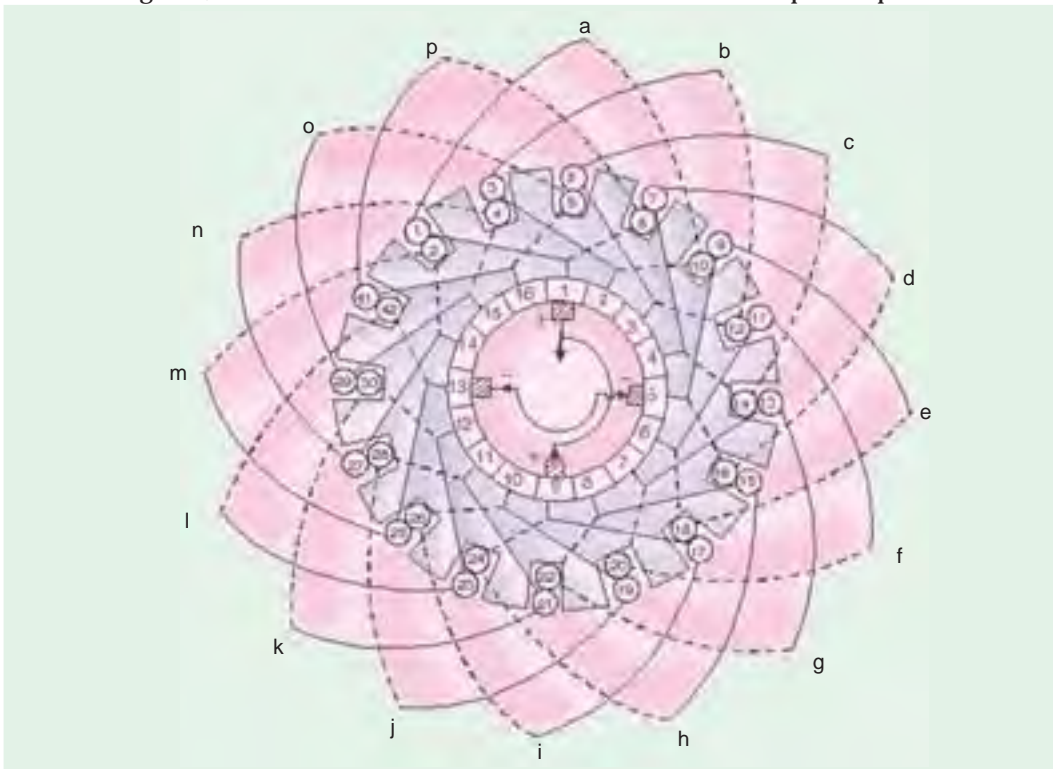


Fig. 26.33

Division of conductors into parallel paths is shown separately in the schematic diagram of Fig. 26.34. Obviously, if  $I_a$  is the total current supplied by the generator, then current carried by each parallel path is  $I_a/4$ .

Summarizing these conclusions, we have

1. The total number of brushes is equal to the number of poles.
2. There are as many parallel paths in the armature as the number of poles. That is why such a winding is sometimes known as 'multiple circuit' or 'parallel' winding. In general, number of parallel paths in armature =  $mP$  where  $m$  is the multiplicity (plex) of the lap winding. For example, a 6-pole duplex lap winding has  $(6 \times 2) = 12$  parallel paths in its armature.
3. The e.m.f. between the +ve and -ve brushes is equal to the e.m.f. generated in any one of the parallel paths. If  $Z$  is the total number of armature conductors and  $P$  the number of poles, then the number of armature conductors (connected in series) in any parallel path is  $Z/P$ .

$\therefore$  Generated e.m.f.  $E_g = (\text{Average e.m.f./conductor}) \times \frac{Z}{P} = e_{av} \times \frac{Z}{P}$

4. The total or equivalent armature resistance can be found as follows :

Let  $l$  = length of each armature conductor;  $S$  = its cross-section

$A$  = No. of parallel paths in armature =  $P$  – for simplex lap winding

$R$  = resistance of the whole winding then  $R = \frac{\rho l}{S} \times Z$

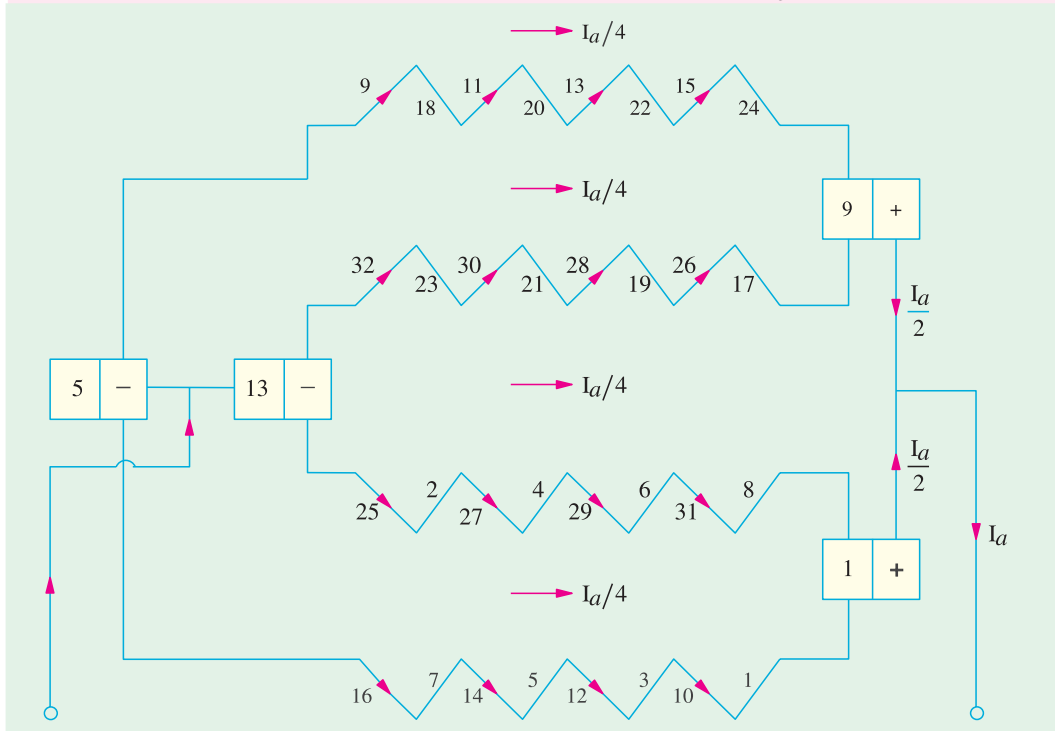


Fig. 26.34

$$\text{Resistance of each path} = \frac{\rho l Z}{S \times A}$$

There are  $P$  (or  $A$ ) such paths in parallel, hence equivalent resistance

$$= \frac{1}{A} \times \frac{\rho l Z}{SA} = \frac{\rho l Z}{SA^2}$$

5. If  $I_a$  is the total armature current, then current per parallel path (or carried by each conductor) is  $I_a/P$ .

### 26.28. Simplex Wave Winding\*

From Fig. 26.31, it is clear that in lap winding, a conductor (or coil side) under one pole is connected at the back to a conductor which occupies an almost corresponding position under the *next* pole of *opposite* polarity (as conductors 3 and 12). Conductor No. 12 is then connected to conductor No. 5 under the *original* pole but which is a little removed from the initial conductor No. 3. If, instead of returning to the same *N*-pole, the conductor No. 12 were taken *forward* to the next *N*-pole, it would make no difference so far as the direction and magnitude of the e.m.f. induced in the circuit are concerned.

\* Like lap winding, a wave winding may be duplex, triplex or may have any degree of multiplicity. A simplex wave winding has two paths, a duplex wave winding four paths and a triplex one six paths etc.

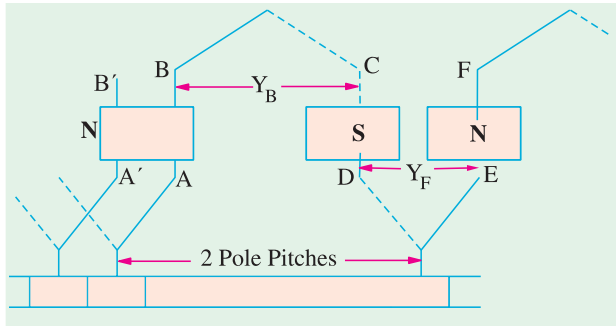


Fig. 26.35

As shown in Fig. 26.35, conductor  $AB$  is connected to  $CD$  lying under  $S$ -pole and then to  $EF$  under the next  $N$ -pole. In this way, the winding progresses, passing successively under every  $N$ -pole and  $S$ -pole till it returns to a conductor  $A'B'$  lying under the original pole. Because the winding progresses in one direction round the armature in a series of 'waves', it is known as wave winding.

If, after passing once round the armature, the winding falls in a slot to the left of its starting point (as  $A'B'$  in Fig. 26.35) then the winding is said to be retrogressive. If, however, it falls one slot to the right, then it is progressive.

Assuming a 2-layer winding and supposing that conductor  $AB$  lies in the upper half of the slot, then going once round the armature, the winding ends at  $A'B'$  which must be at the upper half of the slot at the left or right. Counting in terms of **conductors**, it means that  $AB$  and  $A'B'$  differ by two conductors (although they differ by one slot).

From the above, we can deduce the following relations. If  $P = \text{No. of poles}$ , then

$$\left. \begin{array}{l} Y_B = \text{back pitch} \\ Y_F = \text{front pitch} \end{array} \right\} \text{ nearly equal to pole pitch}$$

then 
$$Y_A = \frac{Y_B + Y_F}{2} = \text{average pitch}; Z = \text{total No. of conductors or coil sides}$$

$$\text{Then, } Y_A \times P = Z \pm 2 \quad Y_A = \frac{Z \pm 2}{P}$$

Since  $P$  is always even and  $Z = PY_A \pm 2$ , hence  $Z$  must always be even. Put in another way, it means that  $\frac{Z \pm 2}{P}$  must be an even integer.

The plus sign will give a progressive winding and the negative sign a retrogressive winding.

#### Points to Note :

1. Both pitches  $Y_B$  and  $Y_F$  are odd and of the same sign.
2. Back and front pitches are nearly equal to the pole pitch and may be equal or differ by 2, in which case, they are respectively one more or one less than the average pitch.
3. Resultant pitch  $Y_R = Y_F + Y_B$
4. Commutator pitch,  $Y_C = Y_A$  (in lap winding  $Y_C = \pm 1$ ).

$$\text{Also, } Y_C = \frac{\text{No. of Commutator bars} \pm 1}{\text{No. of pair of poles}}$$

5. The average pitch which must be an integer is given by

$$Y_A = \frac{Z \pm 2}{P} = \frac{\frac{Z}{2} + 1}{P/2} = \frac{\text{No. of Commutator bars} \pm 1}{\text{No. of pair of poles}}$$

It is clear that for  $Y_A$  to be an integer, there is a restriction on the value of  $Z$ . With  $Z = 32$ , this winding is impossible for a 4-pole machine (though lap winding is possible). Values of  $Z = 30$  or  $34$  would be perfectly alright.

6. The number of coils i.e.  $N_C$  can be found from the relation.

$$N_C = \frac{PY_A \pm 2}{2}$$

This relation has been found by rearranging the relation given in (5) above.

7. It is obvious from (5) that for a wave winding, the number of armature conductors with 2 either added or subtracted must be a multiple of the number of poles of the generator. This restriction eliminates many even numbers which are unsuitable for this winding.
8. The number of armature parallel paths =  $2m$  where  $m$  is the multiplicity of the winding.

**Example 26.2.** Draw a developed diagram of a simplex 2-layer wave-winding for a 4-pole d.c. generator with 30 armature conductors. Hence, point out the characteristics of a simple wave winding. (Elect. Engg-I, Nagpur Univ. 1991)

**Solution.** Here,  $Y_A = \frac{30 \pm 2}{4} = 8^*$  or 7. Taking  $Y_A = 7$ , we have  $Y_B = Y_F = 7$

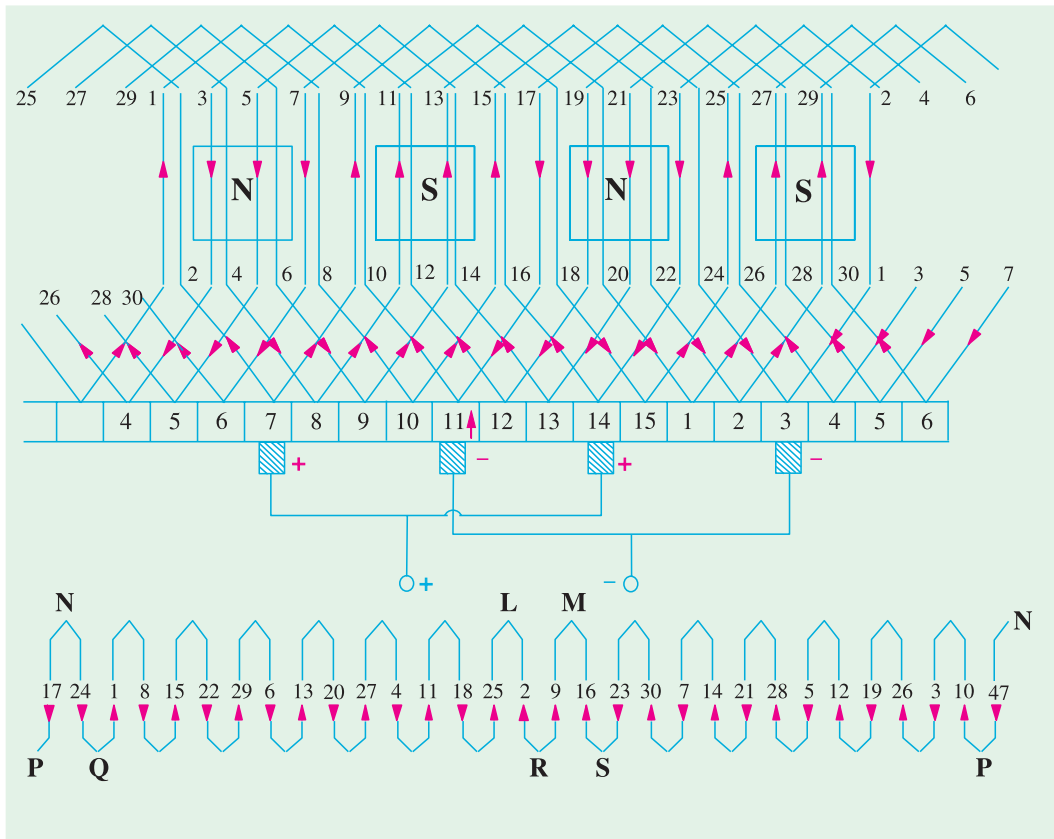


Fig. 26.36

As shown in Fig. 26.36 and 26.37, conductor No. 5 is taken to conductor No.  $5 + 7 = 12$  at the back and is joined to commutator segment 5 at the front. Next, the conductor No. 12 is joined to commutator segment  $5 + 7 = 12$  ( $\because Y_C = 7$ ) to which is joined conductor No.  $12 + 7 = 19$ . Continuing this way, we come back to conductor No. 5 from where we started. Hence, the winding closes upon itself.

\* If we take 8, then the pitches would be :  $Y_B = 9$  and  $Y_F = 7$  or  $Y_B = 7$  and  $Y_F = 9$ . Incidentally, if  $Y_A = Y_C$  is taken as 7, armature will rotate in one direction and if  $Y_C = 8$ , it will rotate in the opposite direction.

The simple winding table is as under :

<b>Back Connections</b>	→	<b>Front Connections</b>
1 to $(1 + 7) = 8$	→	8 to $(8 + 7) = 15$
15 to $(15 + 7) = 22$	→	22 to $(22 + 7) = 29$
29 to $(29 + 7) = 36 = (36 - 30) = 6$	→	6 to $(6 + 7) = 13$
13 to $(13 + 7) = 20$	→	20 to $(20 + 7) = 27$
27 to $(27 + 7) = 34 = (34 - 30) = 4$	→	4 to $(4 + 7) = 11$
11 to $(11 + 7) = 18$	→	18 to $(18 + 7) = 25$
25 to $(25 + 7) = 32 = (32 - 30) = 2$	→	2 to $(2 + 7) = 9$
9 to $(9 + 7) = 16$	→	16 to $(16 + 7) = 23$
23 to $(23 + 7) = 30$	→	30 to $(30 + 7) = 37 = (37 - 30) = 7$
7 to $(7 + 7) = 14$	→	14 to $(14 + 7) = 21$
21 to $(21 + 7) = 28$	→	28 to $(28 + 7) = 35 = (35 - 30) = 5$
5 to $(5 + 7) = 12$	→	12 to $(12 + 7) = 19$
19 to $(19 + 7) = 26$	→	26 to $(26 + 7) = 33 = (33 - 30) = 3$
3 to $(3 + 7) = 10$	→	10 to $(10 + 7) = 17$
17 to $(17 + 7) = 24$	→	24 to $(24 + 7) = 31 = (31 - 30) = 1$

Since we come back to the conductor No. 1 from where we started, the winding gets closed at this stage.

### Brush Position

Location of brush position in wave-winding is slightly difficult. In Fig. 26.36 conductors are supposed to be moving from left to right over the poles. By applying Fleming's Right-hand rule, the directions of the induced e.m.fs in various armature conductors can be found. The directions shown in the figure have been found in this manner. In the lower part of Fig. 26.36 is shown the equivalent ring or spiral diagram which is very helpful in understanding the formation of various parallel paths in the armature. It is seen that the winding is electrically divided into two portions. One portion consists of conductors lying between points  $N$  and  $L$  and the other of conductors lying between  $N$  and  $M$ . In the first portion, the general trend of the induced e.m.fs. is from left to right whereas in the second portion it is from right to left. Hence, in general, there are only two parallel paths through the winding, so that two brushes are required, one positive and one negative.

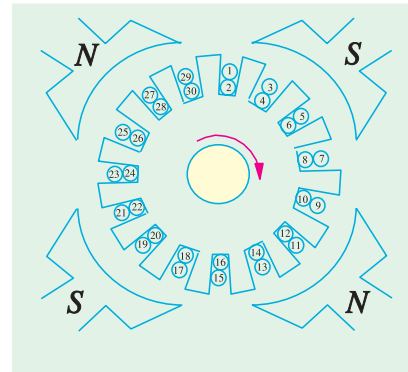


Fig. 26.37

From the equivalent ring diagram, it is seen that point  $N$  is the separating point of the e.m.fs. induced in the two portions of the winding. Hence, this fixes the position of the negative brush. But as it is at the back and not at the commutator end of the armature, the negative brush has two alternative positions *i.e.* either at point  $P$  or  $Q$ . These points on the equivalent diagram correspond to commutator segments No. 3 and 11.

Now, we will find the position of the positive brush. It is found that there are two meeting points of the induced e.m.fs. *i.e.* points  $L$  and  $M$  but both these points are at the back or non-commutator end of the armature. These two points are separated by one loop only, namely, the loop composed of conductors 2 and 9, hence the middle point  $R$  of this loop fixes the position of the positive brush, which should be placed in touch with commutator segment No. 7. We find that for one position of the +ve brush, there are two alternative positions for the -ve brush.

Taking the +ve brush at point  $R$  and negative brush at point  $P$ , the winding is seen to be divided into the following two paths.

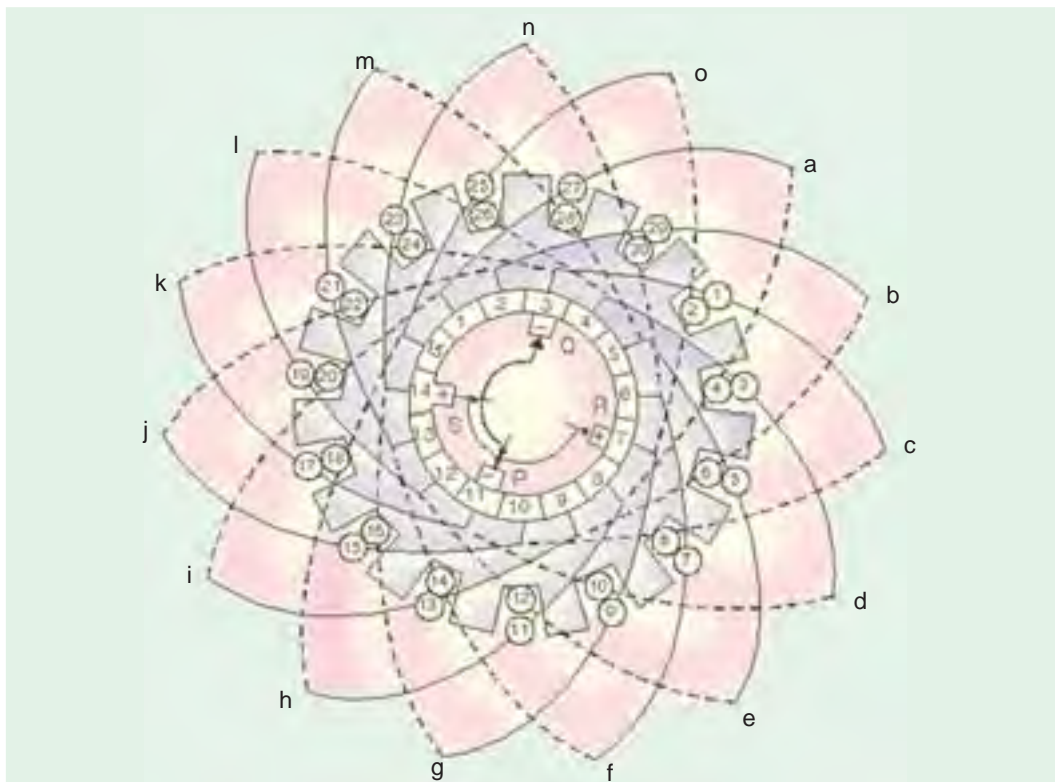


Fig. 26.38

In path 1 (Fig. 26.36) it is found that e.m.f. in conductor 9 is in opposition to the general trend of e.m.fs. in the other conductors comprising this path. Similarly, in path 2, the e.m.f. in conductor 2 is in position to the direction of e.m.fs. in the path as a whole. However, this will make no difference because these conductors lie almost in the interpolar gap and, therefore e.m.fs. in these conductors are negligible.

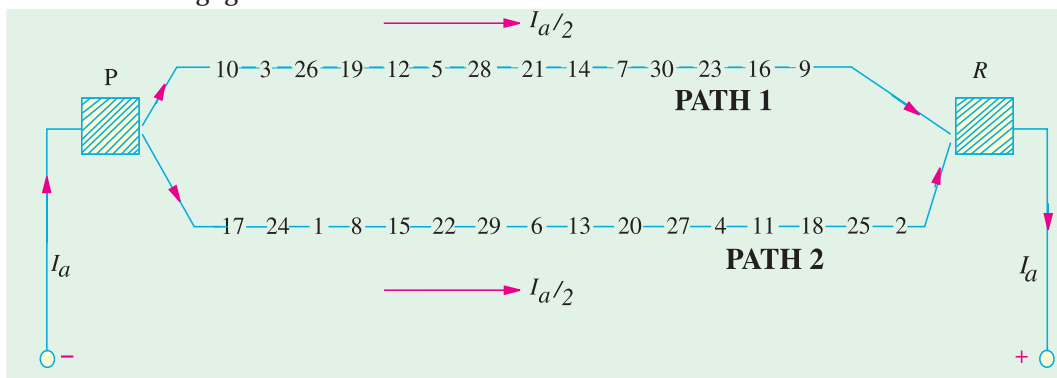


Fig. 26.39

Again, take the case of conductors 2 and 9 situated between points *L* and *M*. Since the armature conductors are in continuous motion over the pole faces, their positions as shown in the figure are only instantaneous. Keeping in this mind, it is obvious that conductor 2 is about to move from the influence of *S*-pole to that of the next *N*-pole. Hence, the e.m.f. in it is at the point of reversing. However, conductor 9 has already passed the position of reversal, hence its e.m.f. will not reverse,



rather it will increase in magnitude gradually. It means that in a very short interval, point  $M$  will

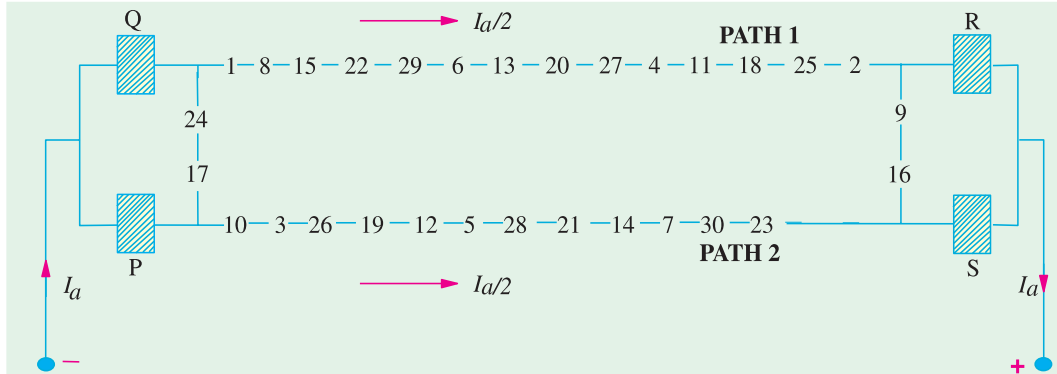


Fig. 26.40

become the meeting point of the e.m.f.s. But as it lies at the back of the armature, there are two alternative positions for the +ve brush *i.e.* either point  $R$  which has already been considered or point  $S$  which corresponds to commutator segment 14. This is the second alternative position of the positive brush. Arguing in the same way, it can be shown that after another short interval of time, the alternative position of the positive brush will shift from segment 14 to segment 15. Therefore, if one positive brush is in the contact with segment 7, then the second positive brush if used, should be in touch with both segments 14 and 15.

It may be noted that if brushes are placed in both alternative positions for both positive and negative (*i.e.* if in all, 4 brushes are used, two +ve and two -ve), then the effect is merely to short-circuit the loop lying between brushes of the same polarity. This is shown in Fig. 26.40 where it will also be noted that irrespective of whether only two or four brushes are used, the number of parallel paths through the armature winding is still two.

Summarizing the above facts, we get

1. Only two brushes are necessary, though their number may be equal to the number of poles.
2. The number of parallel paths through the armature winding is two irrespective of the number of generator poles. That is why this winding is sometimes called 'two-circuit' or 'series' winding.
3. The generator e.m.f. is equal to the e.m.f. induced in any one of the two parallel paths. If  $e_{av}$  is the e.m.f. induced/conductor, then generator e.m.f. is  $E_g = e_{av} \times Z/2$ .
4. The equivalent armature resistance is nearly one-fourth of the total resistance of the armature winding.
5. If  $I_a$  is the total armature current, then current carried by each path or conductor is obviously  $I_a/2$  whatever the number of poles.

### 26.29. Dummy or Idle Coils

These are used with wave-winding and are resorted to when the requirements of the winding are not met by the standard armature punchings available in armature-winding shops. These dummy coils do not influence the electrical characteristics of the winding because they are not connected to the commutator. They are exactly similar to the other coils except that their ends are cut short and taped. They are there simply to provide mechanical balance for the armature because an armature having some slots without windings would be out of balance mechanically. For example, suppose number of armature slots is 15, each containing 4 sides and the number of poles is 4. For a simplex wave-windings,



Dummy coils

$$Y_A = \frac{Z \pm 2}{P} = \frac{60 \pm 2}{4}$$

which does not come out to be an integer (Art. 26.28) as required by this winding. However, if we make one coil dummy so that we have 58 active conductors, then

$$Y_A = \frac{58 \pm 2}{4} = 14 \text{ or } 15$$

This makes the winding possible.

### 26.30. Uses of Lap and Wave Windings

The advantage of the wave winding is that, for a given number of poles and armature conductors, it gives more e.m.f. than the lap winding. Conversely, for the same e.m.f., lap winding would require large number of conductors which will result in higher winding cost and less efficient utilization of space in the armature slots. Hence, wave winding is suitable for small generators especially those meant for 500-600 V circuits.

Another advantage is that in wave winding, equalizing connections are not necessary whereas in a lap winding they definitely are. It is so because each of the two paths contains conductors lying under all the poles whereas in lap-wound armatures, each of the  $P$  parallel paths contains conductors which lie under one pair of poles. Any inequality of pole fluxes affects two paths equally, hence their induced e.m.fs. are equal. In lap-wound armatures, unequal voltages are produced which set up a circulating current that produces sparking at brushes.

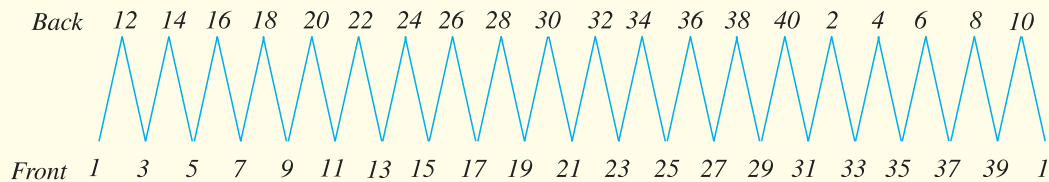
However, when large currents are required, it is necessary to use lap winding, because it gives more parallel paths.

Hence, lap winding is suitable for comparatively low-voltage but high-current generators whereas wave-winding is used for high-voltage, low-current machines.

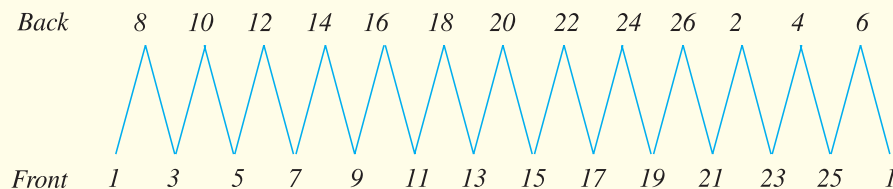
#### Tutorial Problem No. 26.1

1. Write down the winding table for a 2-layer simplex lap-winding for a 4-pole d.c. generator having (a) 20 slots and (b) 13 slots. What are the back and front pitches as measured in terms of armature conductors?

[Hint : (a) No. of conductors = 40 ;  $Y_B = 11$  and  $Y_F = -9$ ] (Elect. Engineering, Madras Univ. 1978)

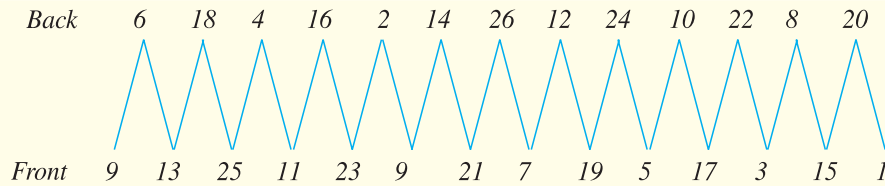


- (b) No. of conductors = 26 ;  $Y_B = 7$  ;  $Y_F = -5$



2. With a simplex 2-layer wave winding having 26 conductors and 4-poles, write down the winding table. What will be the front and back pitches of the winding?

[Hint :  $Y_F = 7$  and  $Y_B = 5$ ] (Electric Machinery-I, Madras Univ. Nov. 1979)



3. Is it possible to get simplex wave winding for a 4-pole d.c. machine with 28 conductors? Explain the reason for your answer. **[No, it would contain only 4 conductors]**

4. State for what type of winding each of the following armatures could be used and whether the winding must be four or six-pole if no dummy coils are to be used (a) 33 slots, 165 commutator segments (b) 64 slots, 256 commutator segments (c) 65 slots, 260 commutator segments.

**[(a) 4-pole lap with commutator pitch 82 or 83 or 6-pole lap.**

**(b) 4-pole lap or 6-pole wave with commutator pitch 85.**

**(c) 6-pole wave with commutator pitch 87.]**

### 26.31. Types of Generators

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into (a) separately-excited generators and (b) self-excited generators.

(a) **Separately-excited** generators are those whose field magnets are energised from an independent external source of d.c. current. It is shown diagrammatically in Fig. 26.41.

(b) **Self-excited generators** are those whose field magnets are energised by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

#### (i) Shunt wound

The field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them (Fig. 26.42).

#### (ii) Series Wound

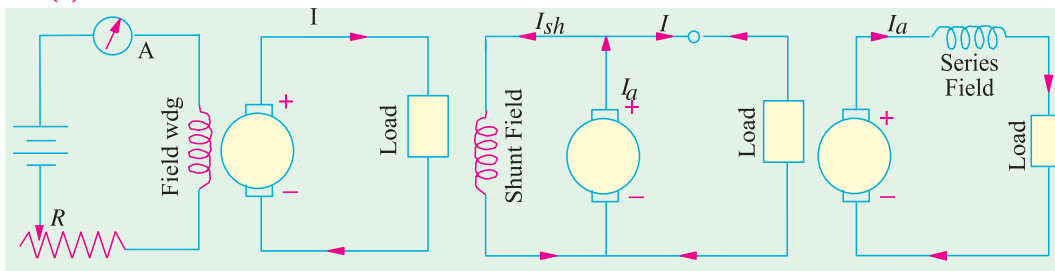


Fig. 26.41

Fig. 26.42

Fig. 26.43

In this case, the field windings are joined in series with the armature conductors (Fig. 26.43). As they carry full load current, they consist of relatively few turns of thick wire or strips. Such generators are rarely used except for special purposes *i.e.* as boosters etc.

#### (iii) Compound Wound

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig. 26.44 (a) and

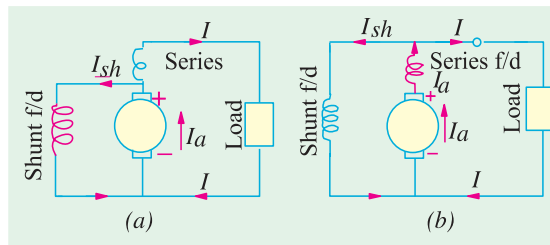


Fig. 26.44

(b) respectively. In a compound generator, the shunt field is stronger than the series field. When series field *aids* the shunt field, generator is said to be *cumulatively-compounded*. On the other hand if series field *opposes* the shunt field, the generator is said to be *differentially compounded*. Various types of d.c. generators have been shown separately in Fig. 26.45.

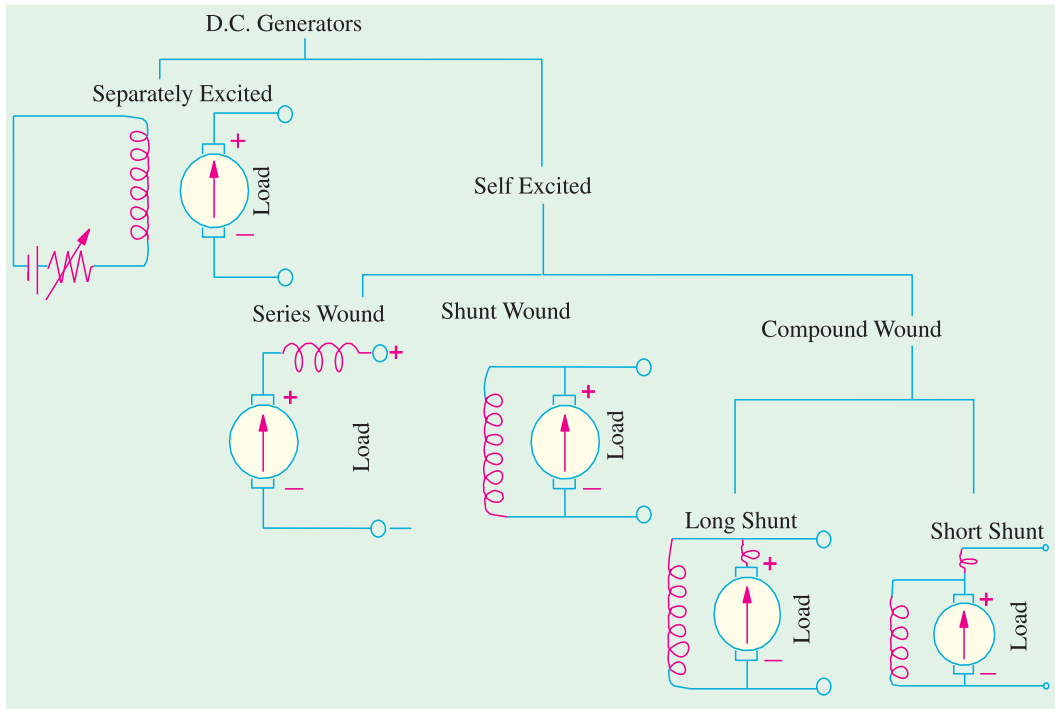


Fig. 26.45

### 26.32. Brush Contact Drop

It is the voltage drop over the brush contact resistance when current passes from commutator segments to brushes and finally to the external load. Its value depends on the amount of current and the value of contact resistance. This drop is usually small and includes brushes of both polarities. However, in practice, the brush contact drop is assumed to have following constant values for all loads.

0.5 V for metal-graphite brushes.

2.0 V for carbon brushes.

**Example 26.3.** A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.

**Solution.** Generator circuit is shown in Fig. 26.46.

Current through shunt field winding is

$$I_{sh} = 230/50 = 4.6 \text{ A}$$

Load current  $I = 450 \text{ A}$

$$\begin{aligned} \therefore \text{Armature current } I_a &= I + I_{sh} \\ &= 450 + 4.6 = 454.6 \text{ A} \end{aligned}$$

Armature voltage drop

$$I_a R_a = 454.6 \times 0.03 = \mathbf{13.6 \text{ V}}$$

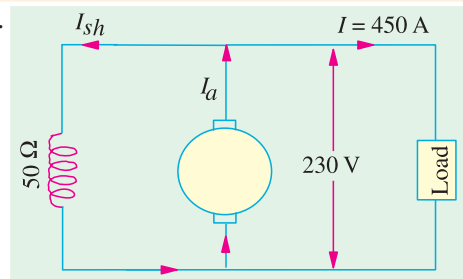


Fig. 26.46

Now  $E_g = \text{terminal voltage} + \text{armature drop}$   
 $= V + I_a R_a$

$\therefore$  e.m.f. generated in the armature

$$E_g = 230 + 13.6 = \mathbf{243.6 \text{ V}}$$

**Example 26.4.** A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05  $\Omega$ , 0.03  $\Omega$  and 250  $\Omega$  respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.

(Elect. Science 1, Allahabad Univ. 1992)

**Solution.** Generator circuit is shown in Fig. 26.47.

$$I_{sh} = 500/250 = 2 \text{ A}$$

Current through armature and series winding is  
 $= 50 + 2 = 52 \text{ A}$

Voltage drop on series field winding  
 $= 52 \times 0.03 = 1.56 \text{ V}$

Armature voltage drop  
 $I_a R_a = 52 \times 0.05 = 2.6 \text{ V}$

Drop at brushes =  $2 \times 1 = 2 \text{ V}$

Now,  $E_g = V + I_a R_a + \text{series drop} + \text{brush drop}$   
 $= 500 + 2.6 + 1.56 + 2 = \mathbf{506.16 \text{ V}}$

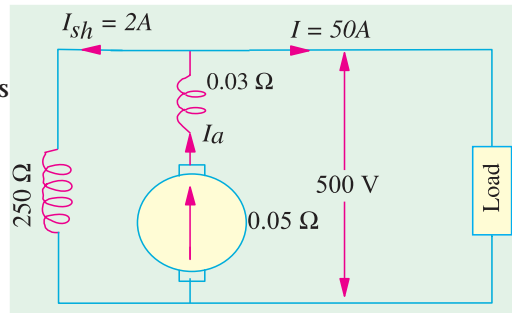


Fig. 26.47

**Example 26.5.** A short-shunt compound generator delivers a load current of 30 A at 220 V, and has armature, series-field and shunt-field resistances of 0.05  $\Omega$ , 0.30  $\Omega$  and 200  $\Omega$  respectively. Calculate the induced e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.

(AMIE Sec. B. Elect. Machines 1991)

**Solution.** Generator circuit diagram is shown in Fig. 26.48.

Voltage drop in series winding =  $30 \times 0.3 = 9 \text{ V}$

Voltage across shunt winding =  $220 + 9 = 229 \text{ V}$

$$I_{sh} = 229/200 = 1.145 \text{ A}$$

$$I_a = 30 + 1.145 = 31.145 \text{ A}$$

$$I_a R_a = 31.145 \times 0.05 = 1.56 \text{ V}$$

Brush drop =  $2 \times 1 = 2 \text{ V}$

$$E_g = V + \text{series drop} + \text{brush drop} + I_a R_a$$

$$= 220 + 9 + 2 + 1.56 = 232.56 \text{ V}$$

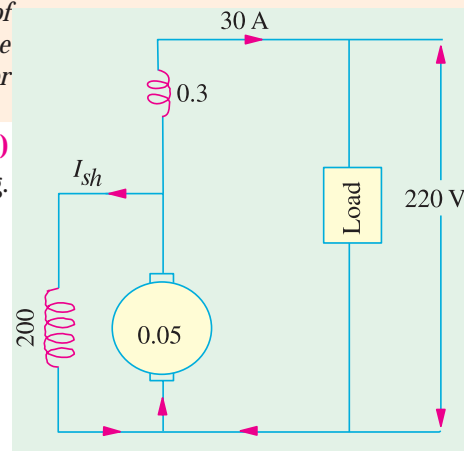


Fig. 26.48

**Example 26.6.** In a long-shunt compound generator, the terminal voltage is 230 V when generator delivers 150 A. Determine (i) induced e.m.f. (ii) total power generated and (iii) distribution of this power. Given that shunt field, series field, diverter and armature resistance are 92  $\Omega$ , 0.015  $\Omega$ , 0.03  $\Omega$  and 0.032  $\Omega$  respectively.

(Elect. Technology-II, Gwalior Univ. 1987)

**Solution.**  $I_{sh} = 230/92 = 2.5 \text{ A}$   
 $I_a = 150 + 2.5 = 152.5 \text{ A}$

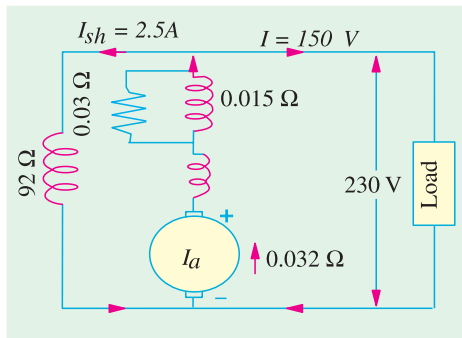


Fig. 26.49

Power lost in series field and diverter  
 Power dissipated in shunt winding  
 Power delivered to load

Since series field resistance and diverter resistances are in parallel (Fig. 26.49) their combined resistance is  
 $= 0.03 \times 0.015 / 0.045 = 0.01 \Omega$

Total armature circuit resistance is  
 $= 0.032 + 0.01 = 0.042 \Omega$

Voltage drop =  $152.5 \times 0.042 = 6.4 \text{ V}$

(i) Voltage generated by armature

$$E_g = 230 + 6.4 = 236.4 \text{ V}$$

(ii) Total power generated in armature

$$E_g I_a = 236.4 \times 152.5 = 36,051 \text{ W}$$

(iii) Power lost in armature

$$I_a^2 R_a = 152.5^2 \times 0.032 = 744 \text{ W}$$

$$= 152.5^2 \times 0.01 = 232 \text{ W}$$

$$= V I_{sh} = 230 \times 0.01 = 575 \text{ W}$$

$$= 230 \times 150 = 34500 \text{ W}$$

$$\text{Total} = 36,051 \text{ W.}$$

**Example 26.7.** The following information is given for a 300-kW, 600-V, long-shunt compound generator : Shunt field resistance = 75 Ω, armature resistance including brush resistance = 0.03 Ω, commutating field winding resistance = 0.011 Ω, series field resistance = 0.012 Ω, diverter resistance = 0.036 Ω. When the machine is delivering full load, calculate the voltage and power generated by the armature.

(Elect. Engg-II, Pune Univ. Nov. 1989)

**Solution.** Power output = 300,000 W

$$\text{Output current} = 300,000 / 600 = 500 \text{ A}$$

$$I_{sh} = 600 / 75 = 8 \text{ A,}$$

$$I_a = 500 + 8 = 508 \text{ A}$$

Since the series field resistance and diverter resistance are in parallel (Fig. 26.50) their combined resistance is

$$= \frac{0.012 \times 0.036}{0.048} = 0.009 \Omega$$

Total armature circuit resistance

$$= 0.03 - 0.011 + 0.009 = 0.05 \Omega$$

$$\text{Voltage drop} = 508 \times 0.05 = 25.4 \text{ V}$$

Voltage generated by armature

$$= 600 + 25.4 = 625.4 \text{ V}$$

$$\text{Power generated} = 625.4 \times 508 = 317,700$$

$$W = 317.7 \text{ kW}$$

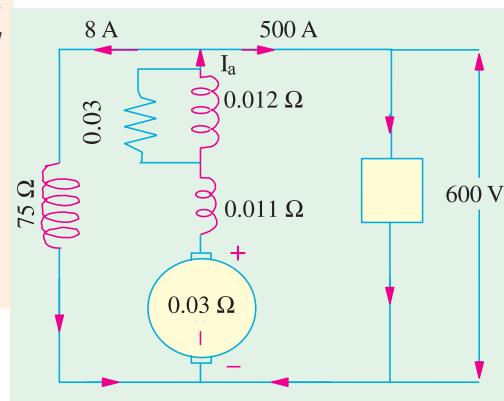


Fig. 26.50

### 26.33. Generated E.M.F. or E.M.F. Equation of a Generator

Let  $\Phi$  = flux/pole in weber

$Z$  = total number of armature conductors

- = No. of slots  $\times$  No. of conductors/slot
- $P$  = No. of generator poles
- $A$  = No. of parallel paths in armature
- $N$  = armature rotation in revolutions per minute (r.p.m.)
- $E$  = e.m.f. induced in any parallel path in armature

Generated e.m.f.  $E_g$  = e.m.f. generated in any one of the parallel paths *i.e.*  $E$ .

$$\text{Average e.m.f. generated/conductor} = \frac{d\Phi}{dt} \text{ volt } (\because n = 1)$$

Now, flux cut/conductor in one revolution  $d\Phi = \Phi PWb$

No. of revolutions/second =  $N/60$   $\therefore$  Time for one revolution,  $dt = 60/N$  second

Hence, according to Faraday's Laws of Electromagnetic Induction,

$$\text{E.M.F. generated/conductor} = \frac{d\Phi}{dt} = \frac{\Phi PN}{60} \text{ volt}$$

### For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors (in series) in one path =  $Z/2$

$$\therefore \text{E.M.F. generated/path} = \frac{\Phi PN}{60} \times \frac{Z}{2} = \frac{\Phi ZPN}{120} \text{ volt}$$

### For a simplex lap-wound generator

No. of parallel paths =  $P$

No. of conductors (in series) in one path =  $Z/P$

$$\therefore \text{E.M.F. generated/path} = \frac{\Phi PN}{60} \times \frac{Z}{P} = \frac{\Phi ZN}{60} \text{ volt}$$

$$\text{In general generated e.m.f. } E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

where

$$A = 2\text{-for simplex wave-winding}$$

$$= P\text{-for simplex lap-winding}$$

$$\text{Also, } E_g = \frac{1}{2\pi} \cdot \left(\frac{2\pi N}{60}\right) \Phi Z \left(\frac{P}{A}\right) = \frac{\omega \Phi Z}{2\pi} \left(\frac{P}{A}\right) \text{ volt} - \omega \text{ in rad/s}$$

For a given d.c. machine,  $Z$ ,  $P$  and  $A$  are constant. Hence, putting  $K_a = ZP/A$ , we get

$$E_g = K_a \Phi N \text{ volts--where } N \text{ is in r.p.s.}$$

**Example 26.8.** A four-pole generator, having wave-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rpm assuming the flux per pole to be 7.0 mWb? (**Elect. Machines-I, Allahabad Univ. 1993**)

$$\text{Solution. } E_g = \frac{\Phi ZN}{60} \left(\frac{P}{A}\right) \text{ volts}$$

$$\text{Here, } \Phi = 7 \times 10^{-3} \text{ Wb, } Z = 51 \times 20 = 1020, A = P = 4, N = 1500 \text{ r.p.m.}$$

$$\therefore E_g = \frac{7 \times 10^{-3} \times 1020 \times 1500}{60} \left(\frac{4}{2}\right) = \mathbf{178.5 \text{ V}}$$

**Example 26.9.** An 8-pole d.c. generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1200 rpm? What must be the speed at which it is to be driven produce the same e.m.f. if it is wave-wound?

(**U.P. Technical Univ. 2001**)

**Solution.** With lap-winding,  $P = a = 8$

$$E = \phi (N/60) (P/a)$$

$$= 0.05 \times 500 \times 20 \times 1,$$

$$= 500 \text{ volts}$$

for lap-winding

If it is wave-wound,  $P = 8, a = 2, P/a = 4$

and  $E = 0.05 \times 500 \times (N/60) \times 4$

For  $E = 500 \text{ volts}, N = 300 \text{ rpm}$

Hence, with wave-winding, it must be driven at 300 rpm to generate 500 volts.

**Additional Explanation.** Assume 1 amp as the current per conductor.

(a) Lap-wound, 1200 rpm : 500 V per coil-group, 8 groups in parallel

Net output current = 8 amp as in Fig. 26.51 (a).

Power output = 4 kW

(b) Wave-wound, 300 rpm : 2 groups in parallel, one group has four coils in series, as shown in Fig. 26.51 (b).

Total power-output is now

$$500 \times 2 = 1000 \text{ W.}$$

It is reduced to one fourth, being proportional to the speed.

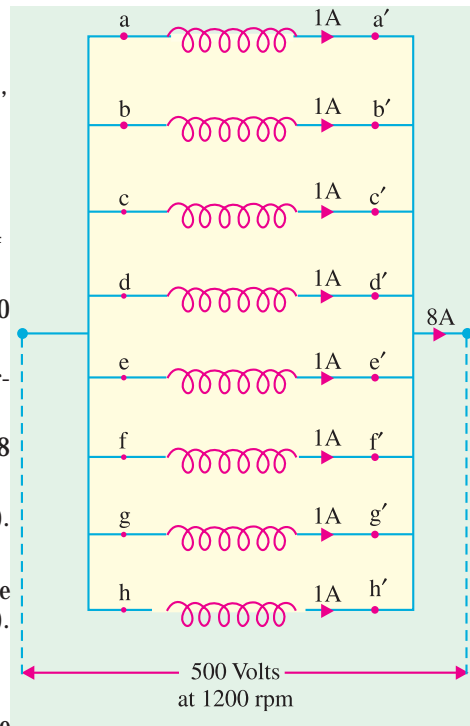


Fig. 26.51(a)

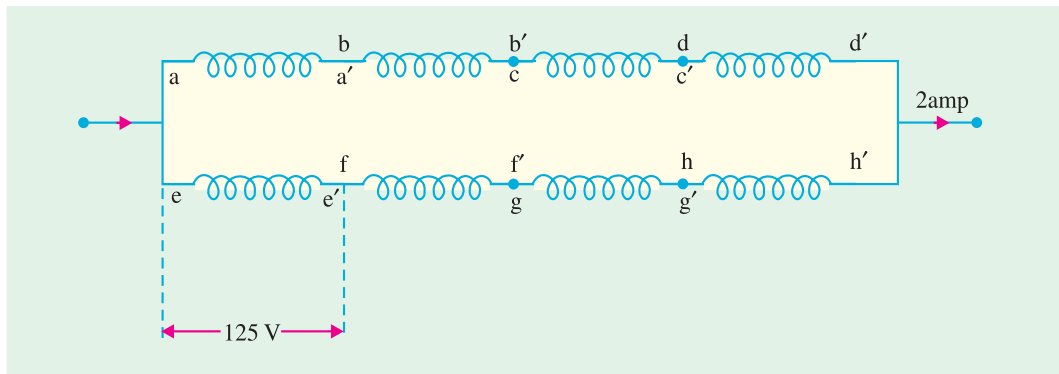


Fig. 26.51(b)

**Example 26.10.** A d.c. shunt generator has an induced voltage on open-circuit of 127 volts. When the machine is on load, the terminal voltage is 120 volts. Find the load current if the field-circuit resistance is 15 ohms and the armature-resistance is 0.02 ohm. Ignore armature reaction.

(Madras University April 1997, Bharathiar University Nov. 1997)

**Solution.**

**Note :** Even though the question does not specify some conditions, the solution given here is based on correct approach to deal with the case.

**Generator on no load :**

As shown in Fig. 26.52 (a), the machine is run at  $N_1$  rpm.

$$E_g = 127 + 8.47 \times 0.02 = 127.17 \text{ volts}$$

As in Fig. 26.52 (b),  $I_f = 8 \text{ amp}$



$E_g$  can be 127.17 volts, if the speed is increased to  $N_2$  rpm, such that

$$8.47 N_1 = 8N_2, \text{ or } N_2 = \frac{8.47}{8} N_1 = 1.05875 N_1$$

Thus the effect due to 5.875% decrease in flux is compensated by 5.875% increase in speed.

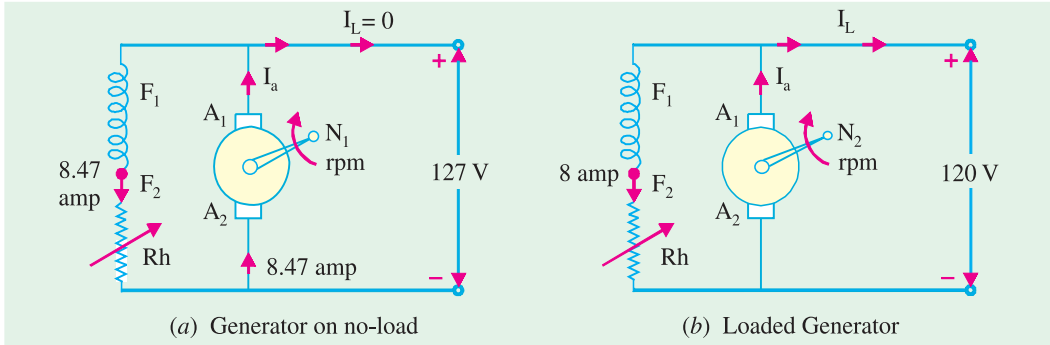


Fig. 26.52

If  $E_g$  is assumed to remain unaltered at 127.17 V,

$$I_a = \frac{127.17 - 120}{0.02} = 358.5 \text{ amp}$$

Hence,

$$I_L = 358.5 - 8 = 350.5 \text{ amp.}$$

**Example 26.11(a).** An 8-pole d.c. shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of  $12.5 \Omega$  resistance at terminal voltage of 50 V. The armature resistance is  $0.24 \Omega$  and the field resistance is  $250 \Omega$ . Find the armature current, the induced e.m.f. and the flux per pole.

(Electrical Engg-I, Bombay Univ. 1988)

**Solution.** The circuit is shown in Fig. 26.53

$$\text{Load current} = V/R = 250/12.5 = 20 \text{ A}$$

$$\text{Shunt current} = 250/250 = 1 \text{ A}$$

$$\text{Armature current} = 20 + 1 = \mathbf{21 \text{ A}}$$

$$\text{Induced e.m.f.} = 250 + (21 \times 0.24) = \mathbf{255.04 \text{ V}}$$

$$\text{Now, } E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right)$$

$$\therefore 255.04 = \frac{\Phi \times 778 \times 500}{60} \left(\frac{8}{2}\right)$$

$$\therefore \Phi = \mathbf{9.83 \text{ mWb}}$$

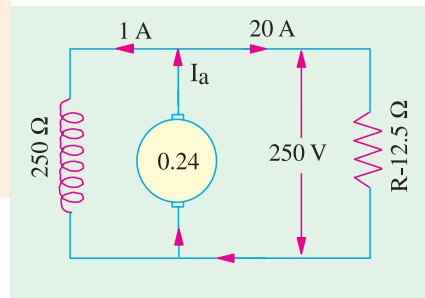


Fig. 26.53

**Example 26.11(b).** A 4-pole lap-connected armature of a d.c. shunt generator is required to supply the loads connected in parallel :

(1) 5 kW Geyser at 250 V, and

(2) 2.5 kW Lighting load also at 250 V.

The Generator has an armature resistance of 0.2 ohm and a field resistance of 250 ohms. The armature has 120 conductors in the slots and runs at 1000 rpm. Allowing 1 V per brush for contact drops and neglecting friction, find

(1) Flux per pole, (2) Armature-current per parallel path. (Nagpur University Nov. 1998)

**Solution.** Geyser current =  $5000/250 = 20$  A  
 Current for Lighting =  $2500/250 = 10$  A  
 Total current = 30 A  
 Field Current for Generator = 1 A  
 Hence, Armature Current = 31 A  
 Armature resistance drop =  $31 \times 0.2 = 6.2$  volts  
 Generated e.m.f. =  $250 + 6.2 + 2 = 258.2$  V,

since  $E = V_t + I_a r_a + \text{Total brush contact drop}$   
 For a 4-pole lap-connected armature,

Number of parallel paths = number of poles = 4

(1) The flux per pole is obtained from the emf equation

$$\begin{aligned} 258.2 &= [\phi Z N/60] \times (p/a) \\ &= [\phi \times 120 \times 1000/60] \times (4/4) \\ &= 2000 \phi \\ \phi &= 129.1 \text{ mWb} \end{aligned}$$

(2) Armature current per parallel path =  $31/4 = 7.75$  A.

**Example 26.12.** A separately excited generator, when running at 1000 r.p.m. supplied 200 A at 125 V. What will be the load current when the speed drops to 800 r.p.m. if  $I_f$  is unchanged? Given that the armature resistance = 0.04 ohm and brush drop = 2 V.

(Elect. Machines Nagpur Univ. 1993)

**Solution.** The load resistance  $R = 125/200 = 0.625 \Omega$ , in Fig. 26.54.

$$E_{g1} = 125 + 200 \times 0.04 + 2 = 135 \text{ V}; N_1 = 1000 \text{ r.p.m.}$$

$$\text{At } 800 \text{ r.p.m. } E_{g2} = 135 \times 800/1000 = 108 \text{ V}$$

If  $I$  is the new load current, then terminal voltage  $V$  is given by

$$V = 108 - 0.04 I - 2 = 106 - 0.04 I$$

$$\therefore I = V/R = (106 - 0.04 I)/0.635; I = 159.4 \text{ A}$$

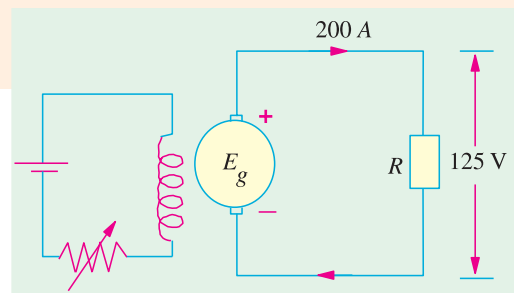


Fig. 26.54

**Example 26.13.** A 4-pole, 900 r.p.m. d.c. machine has a terminal voltage of 220 V and an induced voltage of 240 V at rated speed. The armature circuit resistance is  $0.2 \Omega$ . Is the machine operating as a generator or a motor? Compute the armature current and the number of armature coils if the air-gap flux/pole is 10 mWb and the armature turns per coil are 8. The armature is wave-wound.  
 (Elect. Machines AMIE Sec. B 1990)

**Solution.** Since the induced voltage  $E$  is more than the terminal voltage  $v$ , the machine is working as a generator.

$$E - V = I_a R_a \text{ or } 240 - 220 = I_a \times 0.2; I_a = 100 \text{ A}$$

$$\text{Now, } E_b = ZN(P/A) \text{ or } 240 = 10 \times 10^{-3} \times z \times (900/600) (4/2); Z = 8000$$

Since there are 8 turns in a coil, it means there are 16 active conductors/coil. Hence, the number of coils =  $8000/16 = 500$ .

**Example 26.14.** In a 120 V compound generator, the resistances of the armature, shunt and series windings are  $0.06 \Omega$ ,  $25 \Omega$  and  $0.04 \Omega$  respectively. The load current is 100 A at 120 V. Find the induced e.m.f. and the armature current when the machine is connected as (i) long-shunt and as (ii) short-shunt. How will the ampere-turns of the series field be changed in (i) if a diverter of  $0.1 \text{ ohm}$  be connected in parallel with the series winding? Neglect brush contact drop and ignore armature reaction.  
 (Elect. Machines AMIE Sec. B, 1992)

**Solution. (i) Long Shunt [Fig. 26.55 (a)]**

$$I_{sh} = 120/25 = 4.8 \text{ A} ; I = 100 \text{ A} ; I_a = 104.8 \text{ A}$$

Voltage drop in series winding =  $104.8 \times 0.04 = 4.19 \text{ V}$

Armature voltage drop =  $104.8 \times 0.06 = 6.29 \text{ V}$

$\therefore E_g = 120 + 3.19 + 6.29 = 130.5 \text{ V}$

**(ii) Short Shunt [Fig. 26.55 (c)]**

Voltage drop in series winding =  $100 \times 0.04 = 4 \text{ V}$

Voltage across shunt winding =  $120 + 4 = 124 \text{ V}$

$\therefore I_{sh} = 124/25 = 5 \text{ A} ; \therefore I_a = 100 + 5 = 105 \text{ A}$

Armature voltage drop =  $105 \times 0.06 = 6.3 \text{ V}$

$E_g = 120 + 5 + 4 = 129 \text{ V}$

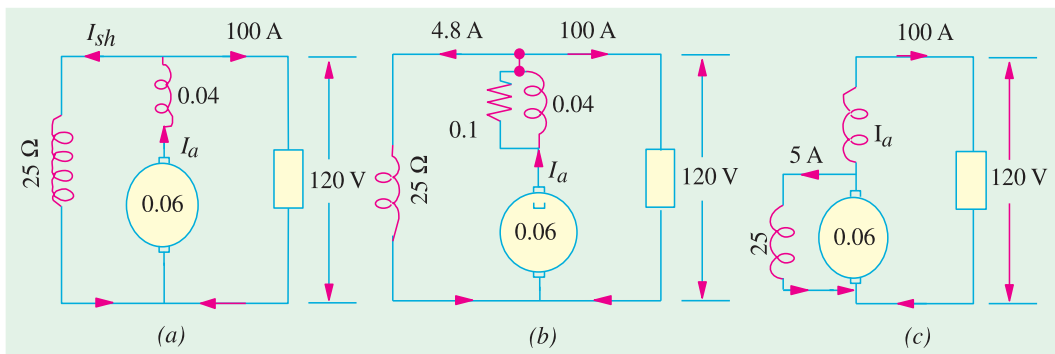


Fig. 26.55

When a diverter of  $0.1 \Omega$  is connected in parallel with the series winding, the diagram becomes as shown in Fig. 26.55 (b). As per current-divider rule, the current through the series winding is  $= 104.8 \times 0.1/(0.1 + 0.04) = 74.86 \text{ A}$ . It means that the series field current has decreased from an original value of  $104.8 \text{ A}$  to  $74.86 \text{ A}$ . Since No. of turns in the series winding remains the same, the change in series field ampere-turns would be the same as the change in the field current. Hence, the percentage decrease in the series field ampere-turns  $= (74.86 - 104.8) \times 100/104.8 = -28.6\%$ .

**Example 26.15.** A 4-pole, long-shunt lap-wound generator supplies 25 kW at a terminal voltage of 500 V. The armature resistance is 0.03 ohm, series field resistance is 0.04 ohm and shunt field resistance is 200 ohm. The brush drop may be taken as 1.0 V. Determine the e.m.f. generated.

Calculate also the No. of conductors if the speed is 1200 r.p.m. and flux per pole is 0.02 weber. Neglect armature reaction.

(Elec. Engineering-I, St. Patel Univ. 1986)

**Solution.**  $I = 25,000/500 = 50 \text{ A}$ ,  $I_{sh} = 500/200 = 2.5 \text{ A}$  (Fig. 26.56)

$$I_a = I + I_{sh} = 50 + 2.5 = 52.5 \text{ A}$$

Series field drop =  $52.5 \times 0.04 = 2.1 \text{ V}$

Armature drop =  $52.5 \times 0.03 = 1.575 \text{ V}$

Brush drop =  $2 \times 1 = 2 \text{ V}$

Generated e.m.f.,  $E_g = 500 + 2.1 + 1.575 + 2 = 505.67 \text{ V}$

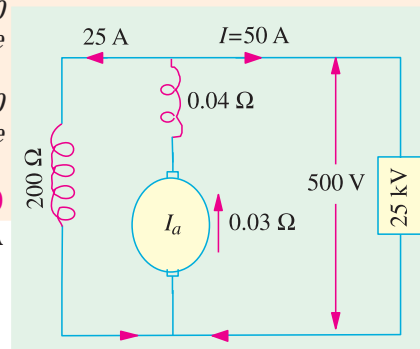


Fig. 26.56

Now, 
$$E_g = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right)$$
 or 
$$505.67 = \frac{0.02 \times Z \times 1200}{60} \left( \frac{4}{4} \right), Z = 1264$$

**Example 26.16.** A 4-pole d.c. generator runs at 750 r.p.m. and generates an e.m.f. of 240 V. The armature is wave-wound and has 792 conductors. If the total flux from each pole is 0.0145 Wb, what is the leakage coefficient ?

**Solution.** Formula used :

$$E = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \text{ volt} \quad \therefore 240 = \frac{\Phi \times 750 \times 792}{60} \times \frac{4}{2}$$

$\therefore$  Working flux/pole,  $\Phi = 0.0121 \text{ Wb}$  ; Total flux/pole = 0.0145 Wb

$\therefore$  Leakage coefficient  $\lambda = \frac{\text{total flux/pole}}{\text{working flux/pole}} = \frac{0.0145}{0.0121} = 1.2$

**Example 26.17.** A 4-pole, lap-wound, d.c. shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004  $\Omega$  resistance. Calculate the terminal voltage when running at 900 r.p.m. if the armature current is 50 A.

**Solution.** Since each turn has two sides,

$$Z = 220 \times 2 = 400 ; N = 900 \text{ r.p.m.} ; \Phi = 0.07 \text{ Wb} ; P = A = 4$$

$\therefore E_g = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) = \frac{0.07 \times 440 \times 900}{60} \times \left( \frac{4}{4} \right) = 462 \text{ volt}$

Total resistance of 220 turns (or 440 conductors) =  $220 \times 0.004 = 0.88 \Omega$

Since there are 4 parallel paths in armature,

$\therefore$  Resistance of each path =  $0.88/4 = 0.22 \Omega$

Now, there are four such resistances in parallel each of value 0.22  $\Omega$

$\therefore$  Armature resistance,  $R_a = 0.22/4 = 0.055 \Omega$

Armature drop =  $I_a R_a = 50 \times 0.055 = 2.75 \text{ V}$

Now, terminal voltage  $V = E_g - I_a R_a = 462 - 2.75 = 459.25 \text{ volt.}$

**Example 26.18.** A 4-pole, lap-wound, long-shunt, d.c. compound generator has useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns and the resistance per turn is 0.004 ohms. Calculate the terminal voltage if the resistance of shunt and series field are 100 ohms and 0.02 ohms respectively ; when the generator is running at 900 r.p.m. with armature current of 50 A. Also calculate the power output in kW for the generator.

(Basic Elect. Machine Nagpur Univ. 1993)

**Solution.** 
$$E_b = \frac{0.07 \times (220 \times 2) \times 900}{60} \times \left( \frac{4}{4} \right) = 462 \text{ V}$$

As found in Ex. 26.17,  $R_a = 0.055 \Omega$

Arm. circuit resistance =  $R_a + R_{se} = 0.055 + 0.02 = 0.075 \Omega$

Arm. circuit drop =  $50 \times 0.075 = 3.75 \text{ V}$

$V = 462 - 3.75 = 458.25 \text{ V}$ , in Fig. 26.57.

$I_{sh} = 458.25/100 = 4.58 \text{ A}$  ;  $I = 50 - 4.58 = 45.42 \text{ A}$

Output =  $VI = 458.25 \times 45.42 = 20,814 \text{ W} = 20.814 \text{ kW}$

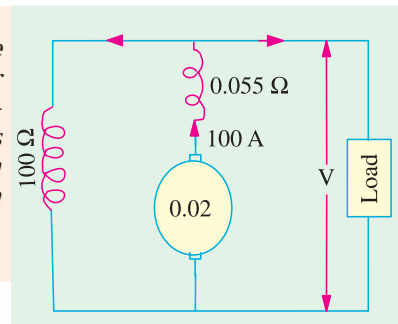


Fig. 26.57

**Example 26.19.** A separately excited d.c. generator, when running at 1200 r.p.m. supplies 200 A at 125 V to a circuit of constant resistance. What will be the current when the speed is dropped to 1000 r.p.m. and the field current is reduced to 80%? Armature resistance, 0.04 Ω and total drop at brushes, 2 V. Ignore saturation and armature reaction.

(Elect. Machines AMIE Sec. B, 1991)

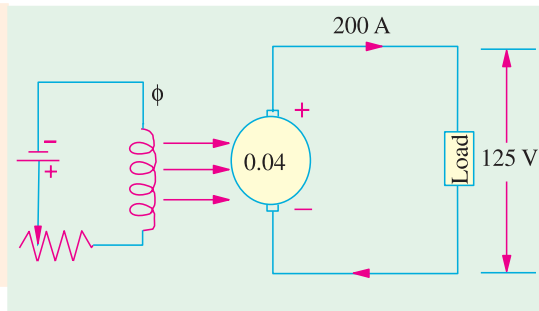


Fig. 26.58

**Solution.** We will find the generated e.m.f. when the load current is 200 A.

$$E_{g1} = V + \text{brush drop} + I_a R_a = 125 + 200 \times 0.04 = 135 \text{ V, in Fig. 26.58.}$$

$$\text{Now, } E_{g1} \propto \Phi_1 N_1 \text{ and } E_{g2} \propto \Phi_2 N_2$$

$$\therefore \frac{E_{g2}}{E_{g1}} = \frac{\Phi_2 N_2}{\Phi_1 N_1}$$

$$\text{or } \frac{E_{g2}}{135} = 0.8 \times \frac{1000}{1200} = 90 \text{ V}$$

**Example 26.20(a).** A 4-pole, d.c. shunt generator with a shunt field resistance of 100 Ω and an armature resistance of 1 Ω has 378 wave-connected conductors in its armature. The flux per pole is 0.02 Wb. If a load resistance of 10 Ω is connected across the armature terminals and the generator is driven at 1000 r.p.m., calculate the power absorbed by the load.

(Elect. Technology, Hyderabad Univ. 1991)

**Solution.** Induced e.m.f. in the generator is

$$E_s = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \text{ volt}$$

$$= \frac{0.02 \times 378 \times 1000}{60} \left( \frac{4}{2} \right) = 252 \text{ volt}$$

Now, let  $V$  be the terminal voltage i.e. the voltage available across the load as well as the shunt resistance (Fig. 26.59).

$$\text{Load current} = V/10 \text{ A and Shunt current} = V/100 \text{ A}$$

$$\text{Armature current} = \frac{V}{10} + \frac{V}{100} = \frac{11V}{100}$$

$$\text{Now, } V = E_g - \text{armature drop}$$

$$\therefore V = 252 - 1 \times \frac{11V}{100} \quad \therefore V = 227 \text{ volt}$$

$$\text{Load current} = 227/10 = 22.7 \text{ A, Power absorbed by the load is} = 227 \times 22.7 = 5,153 \text{ W}$$

**Example 26.20(b).** A four-pole, lap-wound shunt generator has 300 armature-conductors and a flux/pole of 0.1 Wb. It runs at 1000 r.p.m. The armature and field-resistances are 0.2 ohm and 125 ohms respectively. Calculate the terminal voltage when it is loaded to take a load current of 90 A. Ignore armature reaction.

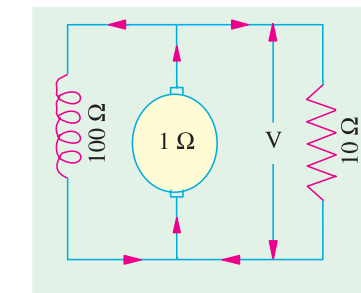


Fig. 26.59

**Solution.** First, the e.m.f. should be calculated

$$E = 0.1 \times 300 \times (1000/60) \times (4/4) = 500 \text{ volts}$$

The field current  $500/125 = 4$  amp

For the load current of 90 amp, armature current = 94 amp

$$I_a r_a = 94 \times 0.20 = 18.8 \text{ volts}$$

$$\text{Terminal voltage, } V = 500 - 18.8 = 481.2 \text{ volts}$$

**Note :** Due to the reduction in terminal voltage (as an effect of loading), the shunt field current tends to decrease, which will further reduce  $V$ . To compensate for this, either increase the speed slightly or decrease the shunt-field-circuit resistance slightly.

**Example 26.21(a).** A 6-pole dc generator runs at 1200 r.p.m. on no-load and has a generated e.m.f. of 250 V. Its armature diameter is 350 mm and the radial air-gap between the field poles and the armature is 3 mm. The axial length of the field poles is 260 mm and the field pole effective coverage is 80% including fringing. If the armature has 96 coils having 3 turns per coil and is wound duplex lap, calculate (a) flux per pole (b) effective pole arc length and (c) average air-gap flux density.

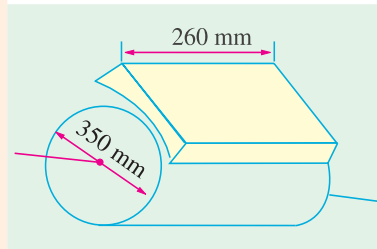


Fig. 26.60

**Solution. (a)**  $Z = (96 \times 3) \times 2 = 576$ ,  $P = 6$ ,  $A = P \times \text{plex} = 6 \times 2 = 12$ ,  $N = 1200$  r.p.m.

$$\therefore 250 = \frac{\Phi \times 576 \times 1200}{6} \left( \frac{6}{12} \right); \quad \therefore \Phi = 0.0434 \text{ Wb}$$

(b) Inner diameter of the pole shoe circle is  $= 350 + 6 = 356$  mm.

Since there are 6 poles, the net field pole flux coverage is 80% of one-sixth of the pole shoe circle. Hence, the effective pole arc length is

$$= \frac{1}{6} \times \pi d \times 0.8 = \frac{1}{6} \times \pi \times 356 \times 0.8 = 149 \text{ mm} = 0.149 \text{ m.}$$

(c) Pole surface area = pole shoe arc  $\times$  axial length of the pole (Fig. 26.60).

$$= 0.149 \times 0.260 = 0.03874 \text{ mm}^2$$

$\therefore$  Flux density  $B = 0.0434/0.03874 = 1.12$  T

**Example 26.21(b).** A 4-pole d.c. Generator with 1200 conductors generates 250 volts on open circuit, when driven at 500 rpm. The pole-shoes have a bore of 35 cm and the ratio of pole-arc to pole pitch is 0.7, while, the length of the pole shoe is 20 cm. Find the mean flux density in the airgap.

(Bharthiar Univ. Nov. 1972 & April 1998)

**Solution.** For a diameter of 35, 4-pole machine has a pole-pitch of  $(35\pi/4) = 27.5$  cm

Since pole-arc/pole pitch is 0.7, Pole-arc  $= 0.7 \times 27.5 = 19.25$  cm

Pole area  $= 19.25 \times 20 = 385$  sq. cm.

Substituting in the e.m.f. equation,  $250 = (\phi ZN/60) (p/a)$

For Lap-winding, in the case,  $p = a = 4$

Hence, flux/pole  $= (250 \times 60)/(1200 \times 500) = 0.025$  Wb

This flux is uniformly distributed over the pole-area.

Mean flux density in the air-gap  $= (0.025)/(385 \times 10^{-4}) = 0.65$  Wb/m<sup>2</sup>

**Example 26.21(c).** A four-pole lap-wound dc shunt generator having 80 slots with 10 conductors per slot generates at no-load an e.m.f. of 400 V, when run at 1000 rpm. How will you obtain a generated open-circuit voltage of 220 V?

(Nagpur University November 1996)

**Solution. (i)** Keeping operating speed at 1000 rpm only, **change the flux per pole**

The O.C. e.m.f. is given by  $E = (\phi ZN/60) \times (P/a)$

For the given operating conditions,

$$400 = \phi \times (80 \times 10) \times (1000/60) \times (4/4)$$

which gives  $\phi = 30$  mWb

When speed is kept constant at 1000 rpm only,

$$E \propto \phi$$

Or to get 220 V on O.C.,  $\phi_2 = (220/400) \times 30 \text{ mWb} = 16.5 \text{ mWb}$

Thus, by increasing the shunt-field-circuit resistance with the help of adding external rheostatic, the current in the field-circuit is decreased so as to decrease the flux to 16.5 mWb.

(ii) Keep same flux per pole, change the speed.

If  $\phi$  is held constant at 30 mWb, an O.C. e.m.f. of 220 V is obtained at a speed of  $N$  r.p.m., given by

$$220 = 30 \times 10^{-3} \times 800 \times N/60, \quad N = 550 \text{ rpm}$$

At 220 V, the flux can be maintained at 30 mWb provided the field **current is unchanged**.

$$400/R_{f1} = 200/R_{f2}$$

$$\text{or} \quad R_{f2} = 0.55 R_{f1}$$

Thus, the field circuit resistance must be reduced to the new value of  $0.55 R_A$  in order to obtain 30 mWb of flux per pole from a voltage of 220 V.

(iii) Any other combination of proper speed and flux/pole can be chosen and worked out on similar lines.

**Example 26.21 (d).** A short-shunt d.c. compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 ohms respectively. Find the emf generated. Also find the emf generated if same machine is connected as a long-shunt machine. (Nagpur University, April 1998)

**Solution.** With short-shunt connection, shown in Fig. 26.61 (a).

$$V_a = \text{armature terminal voltage} = 100 + (200 \times 0.03) = 106 \text{ V}$$

$$\text{Shunt field current} = 106/60 = 1.767 \text{ amp}$$

$$\text{Armature current} = I_a = 200 + 1.767 = 201.767 \text{ amp}$$

$$\text{Armature induced e.m.f.} = 106 + (201.767 \times 0.04) = 114.07 \text{ volts}$$

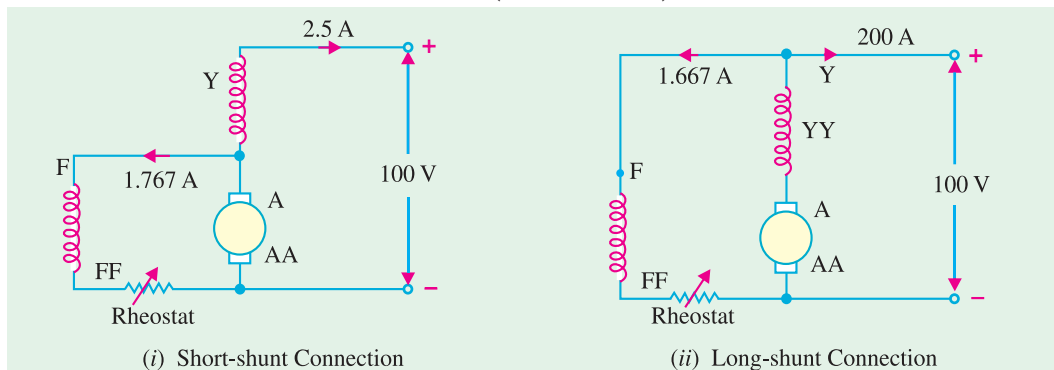


Fig. 26.61(a)

Now, with long-shunt connection shown in Fig. 26.61 (b),

$$\text{Shunt field current} = 100/60 = 1.667 \text{ amp}$$

$$\text{Armature current} = 201.667 \text{ amp}$$

Total voltage drop in armature and series field winding

$$= 201.667 (0.04 + 0.03) = 14.12 \text{ volts}$$

$$\text{Armature induced e.m.f.} = 100 + 14.12 = 114.12 \text{ volts}$$

**Note :** In case of long shunt connection, the generator has to develop the e.m.f. with shunt field current slightly reduced, compared to the case of short shunt connection. However, the series field winding carries a slightly higher current in latter case. Still, in practice, slight speed adjustment (or shunt field rheostatic variation) may be required to get this e.m.f., as per calculations done above.

**Example 26.22.** A long shunt dynamo running at 1000 r.p.m. supplies 20 kW at a terminal voltage of 220 V. The resistance of armature, shunt field, and series field are 0.04, 110 and 0.05 ohm respectively. Overall efficiency at the above load is 85%. Find :

- Copper loss,
- Iron and friction loss,
- Torque developed by the prime mover.

(Amravati University 1999)

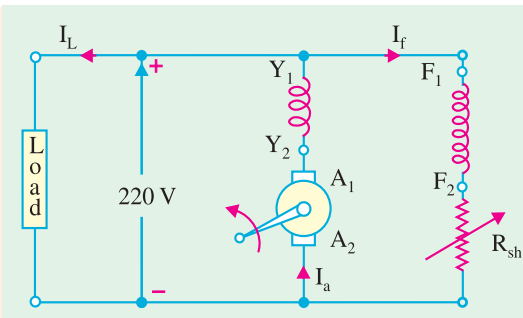


Fig. 26.61(b)

**Solution.**  $I_L$ , Load current =  $\frac{20,000}{220} = 90.91$  amp

Shunt field current,  $I_f = \frac{220}{110} = 2$  amp

Armature current,  $I_a = 92.91$  amp

Input power =  $20,000/0.85 = 23529$  watts

Total losses in the machine = Input – Output =  $23529 - 20,000 = 3529$  watts

**(i) Copper losses :**

Power loss in series field-winding + armature winding =  $92.91^2 \times 0.09$  watts = 777 watts

Power-loss in shunt field circuit :  $2^2 \times 110 = 440$  watts

Total copper losses =  $777 + 400 = 1217$  watts

**(ii) Iron and friction losses = Total losses – Copper losses**

=  $3529 - 1217 = 2312$  watts

**(iii) Let  $T$  = Torque developed by the prime-mover**

At 1000 r.p.m., angular speed,  $\omega = 2\pi \times 1000/60 = 104.67$  rad./sec

$T \times \omega =$  Input power

$\therefore T = 23529/104.67 = 224.8$  Nw-m

## 26.34. Iron Loss in Armature

Due to the rotation of the iron core of the armature in the magnetic flux of the field poles, there are some losses taking place continuously in the core and are known as Iron Losses or Core Losses. Iron losses consist of **(i) Hysteresis** loss and **(ii) Eddy Current** loss.

**(i) Hysteresis Loss ( $W_h$ )**

This loss is due to the reversal of magnetisation of the armature core. Every portion of the rotating core passes under  $N$  and  $S$  pole alternately, thereby attaining  $S$  and  $N$  polarity respectively. The core undergoes one complete cycle of magnetic reversal after passing under one pair of poles. If  $P$  is the number of poles and  $N$ , the armature speed in r.p.m., then frequency of magnetic reversals is  $f = PN/120$ .

The loss depends upon the volume and grade of iron, maximum value of flux density  $B_{max}$  and frequency of magnetic reversals. For normal flux densities (*i.e.* upto  $1.5 \text{ Wb/m}^2$ ), hysteresis loss is given by **Steinmetz** formula. According to this formula,

$$W_h = \eta B_{max}^{1.6} f V \text{ watts}$$

where

$V =$  volume of the core in  $\text{m}^3$

$\eta =$  Steinmetz hysteresis coefficient.



Value of  $\eta$  for :

Good dynamo sheet steel =  $502 \text{ J/m}^3$ , Silicon steel =  $191 \text{ J/m}^2$ , Hard Cast steel =  $7040 \text{ J/m}^3$ , Cast steel =  $750 - 3000 \text{ J/m}^3$  and Cast iron =  $2700 - 4000 \text{ J/m}^3$ .

### (ii) Eddy Current Loss ( $W_e$ )

When the armature core rotates, it also cuts the magnetic flux. Hence, an e.m.f. is induced in the body of the core according to the laws of electromagnetic induction. This e.m.f. though small, sets up large current in the body of the core due to its small resistance. This current is known as eddy current. The power loss due to the flow of this current is known as eddy current loss. This loss would be considerable if solid iron core were used. In order to reduce this loss and the consequent heating of the core to a small value, the core is built up of thin laminations, which are stacked and then riveted at right angles to the path of the eddy currents. These core laminations are insulated from each other by a thin coating of varnish.

The effect of laminations is shown in Fig. 26.62. Due to the core body being one continuous solid iron piece [Fig. 26.62 (a)], the magnitude of eddy currents is large. As armature cross-sectional area is large, its resistance is very small, hence eddy current loss is large. In Fig. 26.62 (b), the same core has been split up into thin circular discs insulated from each other. It is seen that now each current path, being of much less cross-section, has a very high resistance. Hence, magnitude of eddy currents is reduced considerably thereby drastically reducing eddy current loss.

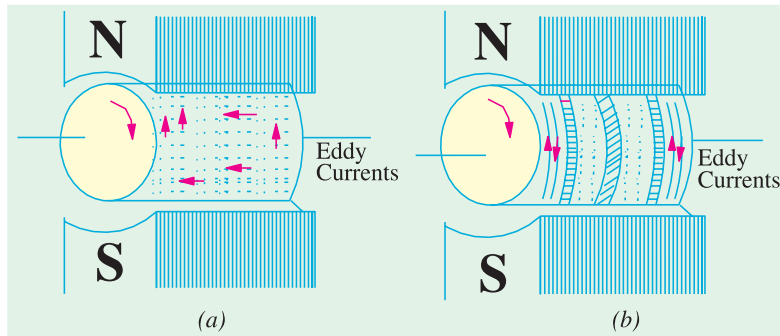


Fig. 26.62

It is found that eddy current loss  $W_e$  is given by the following relation :

$$W_e = KB_{\max}^2 f^2 t^2 V^2 \text{ watt}$$

where  $B_{\max}$  = maximum flux density  $f$  = frequency of magnetic reversals  
 $t$  = thickness of each lamination  $V$  = volume of armature core.

It is seen from above that this loss varies directly as the square of the thickness of laminations, hence it should be kept as small as possible. Another point to note is that  $W_h \propto f$  but  $W_e \propto f^2$ . This fact makes it possible to separate the two losses experimentally if so desired.

As said earlier, these iron losses if allowed to take place unchecked not only reduce the efficiency of the generator but also raise the temperature of the core. As the output of the machines is limited, in most cases, by the temperature rise, these losses have to be kept as small as is economically possible.

Eddy current loss is reduced by using laminated core but hysteresis loss cannot be reduced this way. For reducing the hysteresis loss, those metals are chosen for the armature core which have a low hysteresis coefficient. Generally, special silicon steels such as stalloys are used which not only have a low hysteresis coefficient but which also possess high electrical resistivity.

## 26.35. Total Loss in a D.C. Generator

The various losses occurring in a generator can be sub-divided as follows :

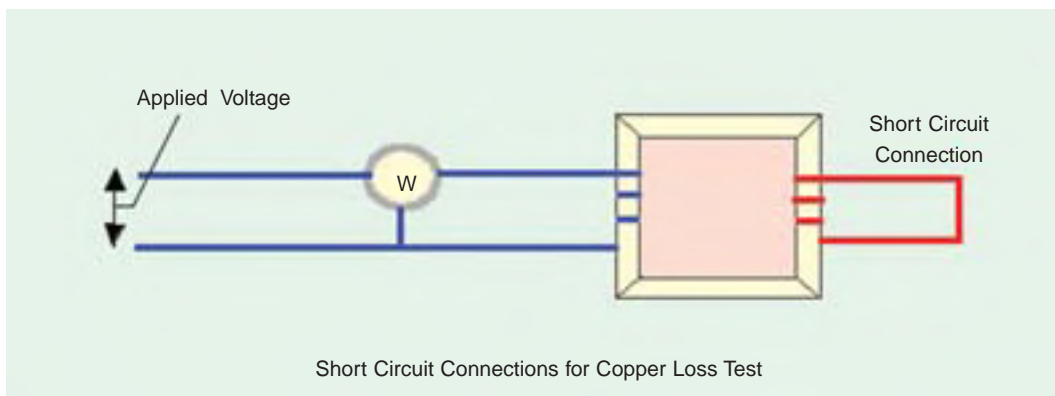
### (a) Copper Losses

(i) Armature copper loss =  $I_a^2 R_a$

[Note :  $E_g I_a$  is the power output from armature.]

where  $R_a$  = resistance of armature and interpoles and series field winding etc.

This loss is about 30 to 40% of full-load losses.



(ii) Field copper loss. In the case of shunt generators, it is practically constant and  $I_{sh}^2 R_{sh}$  (or  $VI_{sh}$ ). In the case of series generator, it is  $I_{se}^2 R_{se}$  where  $R_{se}$  is resistance of the series field winding.

This loss is about 20 to 30% of F.L. losses.

(iii) The loss due to brush contact resistance. It is usually included in the armature copper loss.

(b) **Magnetic Losses** (also known as iron or core losses),

(i) hysteresis loss,  $W_h \propto B_{\max}^{1.6} f$  and (ii) eddy current loss,  $W_e \propto B_{\max}^2 f^2$

These losses are practically constant for shunt and compound-wound generators, because in their case, field current is approximately constant.

Both these losses total up to about 20 to 30% of F.L. losses.

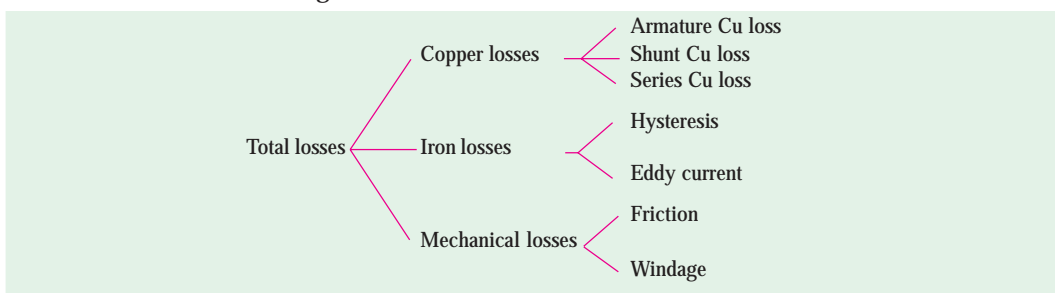
(c) **Mechanical Losses.** These consist of :

(i) friction loss at bearings and commutator.

(ii) air-friction or windage loss of rotating armature.

These are about 10 to 20% of F.L. Losses.

The total losses in a d.c. generator are summarized below :



### 26.36. Stray Losses

Usually, magnetic and mechanical losses are collectively known as **Stray Losses**. These are also known as rotational losses for obvious reasons.

### 26.37. Constant or Standing Losses

As said above, field Cu loss is constant for shunt and compound generators. Hence, stray losses and shunt Cu loss are constant in their case. These losses are together known as standing or constant losses  $W_c$ .

Hence, for shunt and compound generators,

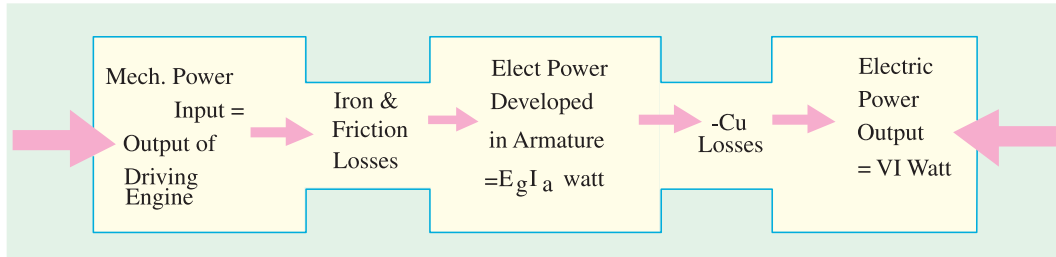
$$\text{Total loss} = \text{armature copper loss} + W_c = I_a^2 R_a + W_c = (I + I_{sh})^2 R_a + W_c$$

Armature Cu loss  $I_a^2 R_a$  is known as variable loss because it varies with the load current.

Total loss = variable loss + constant losses  $W_c$

### 26.38. Power Stages

Various power stages in the case of a d.c. generator are shown below :



Following are the three generator efficiencies :

#### 1. Mechanical Efficiency

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}} = \frac{E_g I_a}{\text{output of driving engine}}$$

#### 2. Electrical Efficiency

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{E_g I_a}$$

#### 3. Overall or Commercial Efficiency

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}}$$

It is obvious that overall efficiency  $\eta_c = \eta_m \times \eta_e$ . For good generators, its value may be as high as 95%.

**Note.** Unless specified otherwise, commercial efficiency is always to be understood.

### 26.39. Condition for Maximum Efficiency

Generator output =  $VI$

Generator input = output + losses

$$= VI + I_a^2 R_a + W_c = VI + (I + I_{sh})^2 R_a + W_c \quad (\because I_a = I + I_{sh})$$

However, if  $I_{sh}$  is negligible as compared to load current, then  $I_a = I$  (approx.)

$$\begin{aligned} \therefore \eta &= \frac{\text{output}}{\text{input}} = \frac{VI}{VI + I_a^2 R_a + W_c} = \frac{VI}{VI + I^2 R_a + W_c} \quad (\because I_a = I) \\ &= \frac{1}{1 + \left( \frac{IR_a}{V} + \frac{W_c}{VI} \right)} \end{aligned}$$

Now, efficiency is maximum when denominator is minimum *i.e.* when

$$\frac{d}{dI} \left( \frac{IR_a}{V} + \frac{W_c}{VI} \right) = 0 \text{ or } \frac{R_a}{V} - \frac{W_c}{VI^2} = 0 \text{ or } I^2 R_a = W_c$$

Hence, generator efficiency is maximum when

**Variable loss = constant loss.**

The load current corresponding to maximum efficiency is given by the relation.

$$I^2 R_a = W_c \text{ or } I = \sqrt{\frac{W_c}{R_a}}$$

Variation of  $\eta$  with load current is shown in Fig. 26.63.

**Example 26.23.** A 10 kW, 250 V, d.c., 6-pole shunt generator runs at 1000 r.p.m. when delivering full-load. The armature has 534 lap-connected conductors. Full-load Cu loss is 0.64 kW. The total brush drop is 1 volt. Determine the flux per pole. Neglect shunt current.

(Elect. Engg. & Electronics, M.S. Univ./Baroda 1987)

**Solution.** Since shunt current is negligible, there is no shunt Cu loss. The copper loss occurs in armature only.

$$I = I_a = 10,000/250 = 40 \text{ A}; I_a^2 R_a = \text{Arm. Cu loss} \text{ or } 40^2 \times R_a = 0.64 \times 10^3; R_a = 0.4 \Omega$$

$$I_a R_a \text{ drop} = 0.4 \times 40 = 16 \text{ V}; \text{Brush drop} = 2 \times 1 = 2 \text{ V}$$

$$\therefore \text{Generated e.m.f. } E_g = 250 + 16 + 1 = 267 \text{ V}$$

$$\text{Now, } E_g = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \text{ volt } \therefore 267 = \frac{\Phi \times 534 \times 1000}{60} \left( \frac{6}{6} \right) \therefore \Phi = 30 \times 10^{-3} \text{ Wb} = \mathbf{30 \text{ mWb}}$$

**Example 26.24(a).** A shunt generator delivers 195 A at terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0.02  $\Omega$  and 50  $\Omega$  respectively. The iron and friction losses equal 950 W. Find

- (a) E.M.F. generated (b) Cu losses (c) output of the prime motor  
(d) commercial, mechanical and electrical efficiencies.

(Elect. Machines-I, Nagpur Univ. 1991)

**Solution. (a)**  $I_{sh} = 250/50 = 5 \text{ A}; I_a = 195 + 5 = 200 \text{ A}$

Armature voltage drop =  $I_a R_a = 200 \times 0.02 = 4 \text{ V}$

$\therefore$  Generated e.m.f. =  $250 + 4 = \mathbf{254 \text{ V}}$

**(b)** Armature Cu loss =  $I_a^2 R_a = 200^2 \times 0.02 = 800 \text{ W}$

Shunt Cu loss =  $V.I_{sh} = 250 \times 5 = 1250 \text{ W}$

$\therefore$  Total Cu loss =  $1250 + 800 = \mathbf{2050 \text{ W}}$

**(c)** Stray losses = 950 W ; Total losses =  $2050 + 950 = 3000 \text{ W}$

Output =  $250 \times 195 = 48,750 \text{ W}$  ; Input =  $48,750 + 3000 = 51750 \text{ W}$

$\therefore$  Output of prime mover =  $\mathbf{51,750 \text{ W}}$

**(d)** Generator input = 51,750 W ; Stray losses = 950 W

Electrical power produced in armature =  $51,750 - 950 = 50,800$

$\eta_m = (50,800/51,750) \times 100 = \mathbf{98.2\%}$

Electrical or Cu losses = 2050 W

$\therefore \eta_e = \frac{48,750}{48,750 + 2,050} \times 100 = \mathbf{95.9\%}$

and  $\eta_c = (48,750/51,750) \times 100 = \mathbf{94.2\%}$

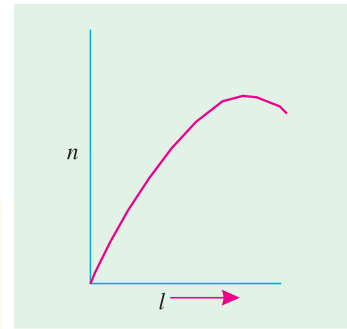


Fig. 26.63

**Example 26.24(b).** A 500 V, D.C. shunt motor draws a line current of 5 amps, on light load. If armature resistance is 0.15 ohm, and field resistance is 200 ohms, determine the efficiency of the machine running as a generator, delivering a load current of 40 Amp.

(Bharathiar Univ. Nov. 1997)

**Solution. (i)** As a motor, on Light load, out of 5 Amps of line current, 2.5 Amps are required for field circuit and 2.5 Amps are required for armature. Neglecting copper-loss in armature at no load (since it works out to be just one watt), the armature-power goes towards armature-core-loss and no-load mechanical loss at the rated speed. This amounts to  $(500 \times 2.5) = 1250$  watts.

**(ii)** As a generator, for a line current of 40 Amp, the total current for the armature is 42.5 amp. Output of generator =  $500 \times 40 \times 10^{-3} = 20$  kW

$$\begin{aligned} \text{Total losses as a generator} &= 1250 + \text{field copper-loss} + \text{arm. copper-loss} \\ &= (1250 + 1250 + 42.5^2 \times 0.15) \text{ watts} = 2.771 \text{ kW} \end{aligned}$$

$$\text{Efficiency} = \frac{20}{20 + 2.771} \times 100 = 87.83\%$$

**Example 26.25.** A shunt generator has a F.L. current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55  $\Omega$ . If it has a F.L. efficiency of 88%, find the armature resistance. Also, find the load current corresponding to maximum efficiency.

(Electrical Technology Punjab Univ. Nov. 1988)

$$\begin{aligned} \text{Solution.} \quad \text{Output} &= 220 \times 196 = 43,120 \text{ W} ; \eta = 88\% \text{ (overall efficiency)} \\ \text{Electrical input} &= 43,120/0.88 = 49,000 \text{ W} \\ \text{Total losses} &= 49,000 - 43,120 = 5,880 \text{ W} \\ \text{Shunt field current} &= 220/55 = 4 \text{ A} \quad \therefore I_a = 196 + 4 = 200 \text{ A} \\ \therefore \text{Shunt Cu loss} &= 220 \times 4 = 880 \text{ W} ; \text{Stray losses} = 720 \text{ W} \\ \text{Constant losses} &= 880 + 720 = 1,600 \\ \therefore \text{Armature Cu loss} &= 5,880 - 1,600 = 4,280 \text{ W} \\ \therefore I_a^2 R_a &= 4,280 \text{ W} \\ 200^2 R_a &= 4,280 \quad \text{or} \quad R_a = 4,280/200 \times 200 = \mathbf{0.107 \Omega} \end{aligned}$$

For maximum efficiency,

$$I^2 R_a = \text{constant losses} = 1,600 \text{ W} ; I = \sqrt{1,600/0.107} = \mathbf{122.34 \text{ A}}$$

**Example 26.26.** A long-shunt dynamo running at 1000 r.p.m. supplies 22 kW at a terminal voltage of 220 V. The resistances of armature, shunt field and the series field are 0.05, 110 and 0.06  $\Omega$  respectively. The overall efficiency at the above load is 88%. Find (a) Cu losses (b) iron and friction losses (c) the torque exerted by the prime mover.

(Elect. Machinery-I, Bangalore Univ. 1987)

**Solution.** The generator is shown in Fig. 26.64.

$$\begin{aligned} I_{sh} &= 220/110 = 2 \text{ A} \\ I &= 22,000/220 = 100 \text{ A}, \\ I_a &= 102 \text{ A} \end{aligned}$$

Drop in series field winding =  $102 \times 0.06 = 6.12$  V

$$\text{(a)} \quad I_a^2 R_a = 102^2 \times 0.05 = 520.2 \text{ W}$$

$$\text{Series field loss} = 102^2 \times 0.06 = 624.3 \text{ W}$$

$$\text{Shunt field loss} = 4 \times 110 = 440 \text{ W}$$

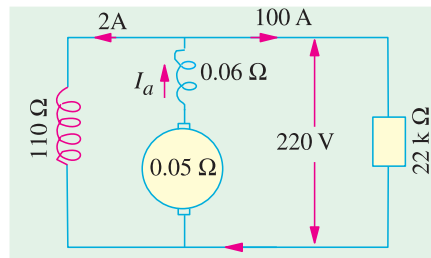


Fig. 26.64

$$\text{Total Cu losses} = 520.2 + 624.3 + 440 = \mathbf{1584.5 \text{ W}}$$

$$(b) \quad \text{Output} = 22,000 \text{ W}; \text{ Input} = 22,000/0.88 = 25,000 \text{ W}$$

$$\therefore \text{Total losses} = 25,000 - 22,000 = 3,000 \text{ W}$$

$$\therefore \text{Iron and friction losses} = 3,000 - 1,584.5 = \mathbf{1,415.5 \text{ W}}$$

$$\text{Now, } T \times \frac{2\pi N}{60} = 25,000; \quad T = \frac{25,000 \times 60}{1,000 \times 6.284} = \mathbf{238.74 \text{ N-m}}$$

**Example 26.27.** A 4-pole d.c. generator is delivering 20 A to a load of 10  $\Omega$ . If the armature resistance is 0.5  $\Omega$  and the shunt field resistance is 50  $\Omega$ , calculate the induced e.m.f. and the efficiency of the machine. Allow a drop of 1 V per brush.

(Electrical Technology-I, Osmania Univ., 1990)

**Solution.** Terminal voltage =  $20 \times 10 = 200 \text{ V}$

$$I_{sh} = 200/50 = 4 \text{ A}; I_a = 20 + 4 = 24 \text{ A}$$

$$I_a R_a = 24 \times 0.5 = 12 \text{ V}; \text{ Brush drop} = 2 \times 1 = 2 \text{ V}$$

$$\therefore E_g = 200 + 12 + 2 = \mathbf{214 \text{ V}}, \text{ as in Fig. 26.65.}$$

Since iron and friction losses are not given, only electrical efficiency of the machine can be found out.

Total power generated in the armature

$$= 214 \times 24 = 5,136 \text{ W}$$

$$\text{Useful output} = 200 \times 20 = 4,000 \text{ W}$$

$$\therefore \eta_e = 4,000/5,136 = 0.779 \text{ or } \mathbf{77.9\%}$$

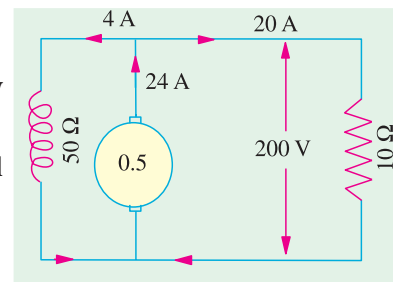


Fig. 26.65

**Example 26.28.** A long-shunt compound-wound generator gives 240 volts at F.L. output of 100 A. The resistances of various windings of the machine are : armature (including brush contact) 0.1  $\Omega$ , series field 0.02  $\Omega$ , interpole field 0.025  $\Omega$ , shunt field (including regulating resistance) 100  $\Omega$ . The iron loss at F.L. is 1000 W ; windage and friction losses total 500 W. Calculate F.L. efficiency of the machine.

(Electrical Machinery-I, Indore Univ. 1989)

**Solution.** Output =  $240 \times 100 = 24,000 \text{ W}$

Total armature circuit resistance =  $0.1 + 0.02 + 0.025 = 0.145 \Omega$

$$I_{sh} = 240/100 = 2.4 \text{ A} \quad \therefore I_a = 100 + 2.4 = 102.4 \text{ A}$$

$$\therefore \text{Armature circuit copper loss} = 102.4^2 \times 0.145 = 1,521 \text{ W}$$

$$\text{Shunt field copper loss} = 2.4 \times 240 = 576 \text{ W}$$

$$\text{Iron loss} = 1000 \text{ W}; \text{ Friction loss} = 500 \text{ W}$$

$$\text{Total loss} = 1,521 + 1,500 + 576 = 3,597 \text{ W}; \eta = \frac{24,000}{24,000 + 3,597} = 0.87 = \mathbf{87\%}$$

**Example 26.29.** In a d.c. machine the total iron loss is 8 kW at its rated speed and excitation. If excitation remains the same, but speed is reduced by 25%, the total iron loss is found to be 5 kW. Calculate the hysteresis and eddy current losses at (i) full speed (ii) half the rated speed.

(Similar Example, JNTU, Hyderabad, 2000)

**Solution.** We have seen in Art. 26.32 that

$$W_h \propto f \text{ and } W_e \propto f^2$$

Since  $f$ , the frequency of reversal of magnetization, is directly proportional to the armature speed,

$$W_h \propto N \text{ and } W_e \propto N^2$$

$\therefore$   $W_h = A \times N$  and  $W_e = BN^2$ , where  $A$  and  $B$  are constants.  
 Total loss  $W = W_h + W_e = AN + BN^2$

Let the full rated speed be 1.

$$\text{Then } 8 = A \times 1 + B \times 1^2 \quad \text{or} \quad 8 = A + B \quad \dots(i)$$

Now, when speed is 75% of full rated speed, then

$$5 = A \times (0.75) + B(0.75)^2 \quad \dots(ii)$$

Multiplying (i) by 0.75 and subtracting (ii) from it, we get

$$0.1875 B = 1 \quad \therefore B = 1/0.1875 = 5.33$$

**kW**

Substituting this value in (i) above

$$8 = 5.33 + A \quad \therefore A = 2.67 \text{ kW}$$

$$(i) \quad W_h \text{ at rated speed} = 2.67 \text{ kW}, \quad W_e \text{ at rated speed} = 5.33 \text{ kW}$$

$$(ii) \quad W_h \text{ at half the rated speed} = 2.67 \times 0.5 = 1.335 \text{ kW}$$

$$W_e \text{ at half the rated speed} = 5.33 \times 0.5^2 = 1.3325 \text{ kW}$$

**Example 26.30.** The hysteresis and eddy current losses in a d.c. machine running at 1000 r.p.m. are 250 W and 100 W respectively. If the flux remains constant, at what speed will be total iron losses be halved? **(Electrical Machines-I, Gujarat Univ. 1989)**

**Solution.** Total loss  $W = W_h + W_e = AN + BN^2$

$$\text{Now, } W_h = 250 \text{ W} \quad \therefore A \times (1000/60) = 250; \quad A = 15$$

$$W_e = 100 \text{ W} \quad \therefore B \times (1000/60)^2 = 100; \quad B = 9/25$$

Let  $N$  be the new speed in r.p.s. at which total loss is one half of the loss at 1000 r.p.m. New loss =  $(250 + 100)/2 = 175 \text{ W}$

$$\therefore 175 = 15N + (9/25)N^2 \quad \text{or} \quad 9N^2 + 375N - 4,375 = 0$$

$$\therefore N = \frac{-375 \pm \sqrt{375^2 + 36 \times 4,375}}{2 \times 9} = \frac{-375 \pm 546}{18} = 9.5 \text{ r.p.s} = 570 \text{ r.p.m.}^*$$

**Note.** It may be noted that at the new speed,  $W_h = 250 \times (570/100) = 142.5 \text{ W}$  and  $W_e = 100 \times (570/1000)^2 = 32.5 \text{ W}$ . Total loss =  $142.5 + 32.5 = 175 \text{ W}$ .

**Example 26.31.** A d.c. shunt generator has a full load output of 10 kW at a terminal voltage of 240 V. The armature and the shunt field winding resistances are 0.6 and 160 ohms respectively. The sum of the mechanical and core-losses is 500 W. Calculate the power required, in kW, at the driving shaft at full load, and the corresponding efficiency. **(Nagpur University November 99)**

**Solution.** Field current =  $\frac{240}{160} = 1.5 \text{ amp}$ , Load current =  $\frac{10,000}{240} = 41.67 \text{ amp}$

$$\text{Armature current} = 41.67 + 1.5 = 43.17 \text{ amp}$$

$$\text{Field copper losses} = 360 \text{ W}, \text{ Armature copper losses} = 43.17^2 \times 0.6 = 1118 \text{ W}$$

$$\text{Total losses in kW} = 0.36 + 1,118 + 0.50 = 1,978 \text{ kW}$$

Hence, Power input at the shaft = 11.978 kW

$$\text{Efficiency} = \frac{10}{11.978} \times 100\% = 83.5\%$$

\* The negative value has been rejected—being mathematically absurd.

**Example 26.32.** A long shunt d.c. compound generator delivers 110 kW at 220 V.

If  $r_a = 0.01$  ohm,  $r_{se} = 0.002$  ohm, and shunt field has a resistance of 110 ohms, calculate the value of the induced e.m.f. (Bharathithasan University Nov. 1997)

**Solution.** Load current =  $110 \times 1000/220$   
= 500 A

Shunt field current =  $220/110 = 2$  A

Armature current = 502 A

$$r_a + r_{se} = 0.012 \text{ ohm}$$

$$E_a = 220 + [502 \times (0.012)]$$

$$= 226.024 \text{ V}$$

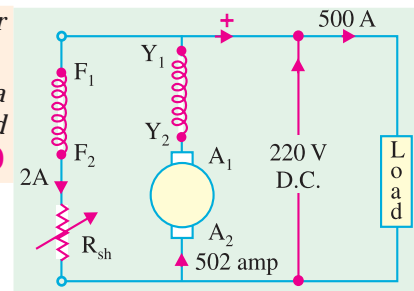


Fig. 26.66. Long-shunt d.c. compound generator

**Example 26.33.** The armature of a four-pole d.c. shunt generator is lap-wound and generates 216 V when running at 600 r.p.m. Armature has 144 slots, with 6 conductors per slot. If this armature is rewound, wave-connected, find the e.m.f. generated with the same flux per pole but running at 500 r.p.m. (Bharathithasan University April 1997)

**Solution.** Total number of armature conductors =  $Z = 144 \times 6 = 864$

For a Lap winding, number of parallel paths in armature = number of poles

In the e.m.f. equation,  $E = (\phi ZN/60) (P/a)$

Since  $P = a$

$$E = \phi ZN/60$$

$$216 = \phi \times 864 \times 600/60 = 8640\phi$$

Hence  $\phi = 25$  milli-webers

If the armature is rewound with wave-connection, number of parallel paths = 2.

Hence, at 500 r.p.m., with 25 mWb as the flux per pole.

$$\begin{aligned} \text{the armature emf} &= (25 \times 10^{-3} \times 864 \times 500/60) \times 4/2 \\ &= 25 \times 864 \times 0.50 \times 2/60 \\ &= 360 \text{ volts} \end{aligned}$$

**Additional note :**

**Extension to Que :** Comment on the armature output power in the two cases.

**Solution.** Assumption is that field side is suitably modified in the two cases.

**Case (i) :** Lap-wound Machine at 600 r.p.m.

Armature e.m.f. = 216 V

Let each armature-conductor be rated to carry a current of 10 amp.

In simple lap-wound machines, since a four-pole machine has four parallel paths in armature, the total armature output-current is 40 amp.

Hence, armature-output-power =  $216 \times 40 \times 10^{-3} = 8.64$  kW

**Case (ii) :** Wave-wound machine, at 500 r.p.m.

Armature e.m.f. = 360 V

Due to wave-winding, number of parallel paths in armature = 2



Hence, the total armature output current = 20 amp

Thus, Armature Electrical output-power =  $360 \times 20 \times 10^{-3} = 7.2 \text{ kW}$

**Observation.** With same flux per pole, the armature power outputs will be in the proportion of the speeds, as  $(7.2/8.64) = 5/6$ .

**Further Conclusion.** In case of common speed for comparing Electrical Outputs with same machine once lap-wound and next wave-wound, there is no difference in the two cases. Lap-wound machine has lower voltage and higher current while the wave-wound machine has higher voltage and lower current.

**Example 26.34.** A 4-pole, Lap-connected d.c. machine has an armature resistance of 0.15 ohm. Find the armature resistance of the machine is rewound for wave-connection.

(Bharthiar Univ. Nov. 1997)

**Solution.** A 4-pole lap-winding has 4 parallel paths in armature. If it is rewound for wave-connection, the resistance across the terminal becomes  $(4 \times 0.15) = 0.6 \text{ ohm}$ , as it obvious from Fig. 26.67.

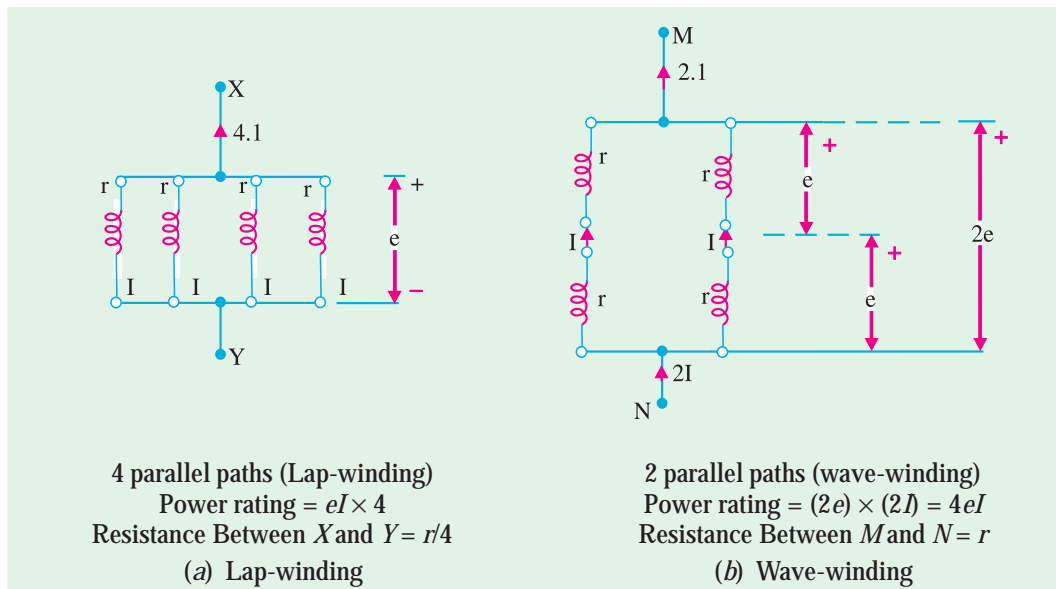


Fig. 26.67. Resistances for different methods

### Tutorial Problem No. 26.2

1. A 4-pole, d.c. generator has a wave-wound armature with 792 conductors. The flux per pole is 0.0121 Wb. Determine the speed at which it should be run to generate 240 V on no-load. [751.3 r.p.m.]

2. A 20 kW compound generator works on full-load with a terminal voltage of 230 V. The armature, series and shunt field resistances are 0.1, 0.05 and 115  $\Omega$  respectively. Calculate the generated e.m.f. when the generator is connected short-shunt. [243.25 V] (Elect. Engg. Madras Univ. April, 1978)

3. A d.c. generator generates an e.m.f. of 520 V. It has 2,000 armature conductors, flux per pole of 0.013 Wb, speed of 1200 r.p.m. and the armature winding has four parallel paths. Find the number of poles. [4] (Elect. Technology, Aligarh Univ. 1978)

4. When driven at 1000 r.p.m. with a flux per pole of 0.02 Wb, a d.c. generator has an e.m.f. of 200 V. If the speed is increased to 1100 r.p.m. and at the same time the flux per pole is reduced to 0.019 Wb per pole, what is then the induced e.m.f. ? [209 V]

5. Calculate the flux per pole required on full-load for a 50 kW, 400 V, 8-pole, 600 r.p.m. d.c. shunt generator with 256 conductors arranged in a lap-connected winding. The armature winding resistance is  $0.1 \Omega$ , the shunt field resistance is  $200 \Omega$  and there is a brush contact voltage drop of 1 V at each brush on full-load. **[0.162 Wb]**

6. Calculate the flux in a 4-pole dynamo with 722 armature conductors generating 500 V when running at 1000 r.p.m. when the armature is (a) lap connected (b) wave connected.

**[(a) 41.56 mWb (b) 20.78 mWb] (City & Guilds, London)**

7. A 4-pole machine running at 1500 r.p.m. has an armature with 90 slots and 6 conductors per slot. The flux per pole is 10 mWb. Determine the terminal e.m.f. as d.c. Generator if the coils are lap-connected. If the current per conductor is 100 A, determine the electrical power.

**[810 V, 324 kW] (London Univ.)**

8. An 8-pole lap-wound d.c. generator has 120 slots having 4 conductors per slot. If each conductor can carry 250 A and if flux/pole is 0.05 Wb, calculate the speed of the generator for giving 240 V on open circuit. If the voltage drops to 220 V on full load, find the rated output of the machine.

**[600 V, 440 kW]**

9. A 110-V shunt generator has a full-load current of 100 A, shunt field resistance of  $55 \Omega$  and constant losses of 500 W. If F.L. efficiency is 88%, find armature resistance. Assuming voltage to be constant at 110 V, calculate the efficiency at half F.L. And at 50% overload. Find the load current.

**[0.078  $\Omega$  ; 85.8% ; 96.2 A]**

10. A short-shunt compound d.c. Generator supplies a current of 100 A at a voltage of 220 V. If the resistance of the shunt field is  $50 \Omega$ , of the series field  $0.025 \Omega$ , of the armature  $0.05 \Omega$ , the total brush drop is 2 V and the iron and friction losses amount to 1 kW, find

(a) the generated e.m.f. (b) the copper losses (c) the output power of the prime-mover driving the generator and (d) the generator efficiency.

**[(a) 229.7 V (b) 1.995 kW (c) 24.99 kW (d) 88%]**

11. A 20 kW, 440-V, short-shunt, compound d.c. generator has a full-load efficiency of 87%. If the resistance of the armature and interpoles is  $0.4 \Omega$  and that of the series and shunt fields  $0.25 \Omega$  and  $240 \Omega$  respectively, calculate the combined bearing friction, windage and core-loss of the machine.

**[725 W]**

12. A long-shunt, compound generator delivers a load current of 50 A at 500 V and the resistances of armature, series field and shunt field are 0.05 ohm and 250 ohm respectively. Calculate the generated electromotive force and the armature current. Allow 1.0 V per brush for contact drop.

**[506.2 V ; 52 A] (Elect. Engg. Banaras Hindu Univ. 1977)**

13. In a 110-V compound generator, the resistances of the armature, shunt and the series windings are  $0.06 \Omega$ ,  $25 \Omega$  and  $0.04 \Omega$  respectively. The load consists of 200 lamps each rated at 55 W, 110 V.

Find the total electromotive force and armature current when the machine is connected (i) long shunt (ii) short shunt. Ignore armature reaction and brush drop.

**[(a) 1200.4, 104.4 A (b) 120.3 V, 104.6 A] (Electrical Machines-I, Bombay Univ. 1979)**

14. Armature of a 2-pole, 200-V generator has 400 conductors and runs at 300 r.p.m. Calculate the useful flux per pole. If the number of turns in each field coil is 1200, what is the average value of e.m.f. induced in each coil on breaking the field if the flux dies away completely in 0.1 sec ?

**(JNTU, Hyderabad, 2000)**

**Hint:** Calculate the flux per pole generating 200 V at 300 rpm. Calculate the e.m.f. induced in 1200-turn field coil due to this flux reducing to zero in 0.1 sec, from the rate of change of flux-linkage.

**[\(\phi\) = 0.1 Wb, e = 1200 V]**

**15.** A 1500 kW, 550-V, 16 pole generator runs at 150 rev. per min. What must be the useful flux if there are 2500 conductors lap-connected and the full-load copper losses are 25 kW? Calculate the area of the pole shoe if the gap density has a uniform value of  $0.9 \text{ wb/m}^2$  and find the no-load terminal voltage, neglecting armature reaction and change in speed.

(Rajiv Gandhi Techn. Univ., Bhopal, 2000) [0.09944 m<sup>2</sup>, 559.17 V]

### OBJECTIVE TESTS – 26

- The basic requirement of a d.c. armature winding is that it must be
  - a closed one
  - a lap winding
  - a wave winding
  - either (b) or (c)
- A wave winding must go at least ..... around the armature before it closes back where it started.
  - once
  - twice
  - thrice
  - four times
- The d.c. armature winding in which coil sides are a pole pitch apart is called ..... winding.
  - multiplex
  - fractional-pitch
  - full-pitch
  - pole-pitch
- For making coil span equal to a pole pitch in the armature winding of a d.c. generator, the back pitch of the winding must equal the number of
  - commutator bars per pole
  - winding elements
  - armature conductors per path
  - armature parallel paths.
- The primary reason for making the coil span of a d.c. armature winding equal to a pole pitch is to
  - obtain a coil span of  $180^\circ$  (electrical)
  - ensure the addition of e.m.fs. of consecutive turns
  - distribute the winding uniformly under different poles
  - obtain a full-pitch winding.
- In a 4-pole, 35 slot d.c. armature, 180 electrical-degree coil span will be obtained when coils occupy ..... slots.
  - 1 and 10
  - 1 and 9
  - 2 and 11
  - 3 and 12
- The armature of a d.c. generator has a 2-layer lap-winding housed in 72 slots with six conductors/slot. What is the minimum number of commutator bars required for the armature?
  - 72
  - 432
  - 216
  - 36
- The sole purpose of a commutator in a d.c. Generator is to
  - increase output voltage
  - reduce sparking at brushes
  - provide smoother output
  - convert the induced a.c. into d.c.
- For a 4-pole, 2-layer, d.c., lap-winding with 20 slots and one conductor per layer, the number of commutator bars is
  - 80
  - 20
  - 40
  - 160
- A 4-pole, 12-slot lap-wound d.c. armature has two coil-sides/slot. Assuming single turn coils and progressive winding, the back pitch would be
  - 5
  - 7
  - 3
  - 6
- If in the case of a certain d.c. armature, the number of commutator segments is found either one less or more than the number of slots, the armature must be having a simplex ..... winding.
  - wave
  - lap
  - frog leg
  - multielement
- Lap winding is suitable for ..... current, ..... voltage d.c. generators.
  - high, low
  - low, high
  - low, low
  - high, high

13. The series field of a short-shunt d.c. generator is excited by ..... currents.  
(a) shunt  
(b) armature  
(c) load  
(d) external
14. In a d.c. generator, the generated e.m.f. is directly proportional to the  
(a) field current  
(b) pole flux  
(c) number of armature parallel paths  
(d) number of dummy coils
15. In a 12-pole triplex lap-wound d.c. armature, each conductor can carry a current of 100 A. The rated current of this armature is ..... ampere.  
(a) 600  
(b) 1200  
(c) 2400  
(d) 3600
16. The commercial efficiency of a shunt generator is maximum when its variable loss equals ..... loss.  
(a) constant  
(b) stray  
(c) iron  
(d) friction and windage
17. In small d.c. machines, armature slots are sometimes not made axial but are skewed. Though skewing makes winding a little more difficult, yet it results in  
(a) quieter operation  
(b) slight decrease in losses  
(c) saving of copper  
(d) both (a) and (b)
18. The critical resistance of the d.c. generator is the resistance of  
(a) armature  
(b) field  
(c) load  
(d) brushes (Grad. I.E.T.E Dec. 1985)

## ANSWERS

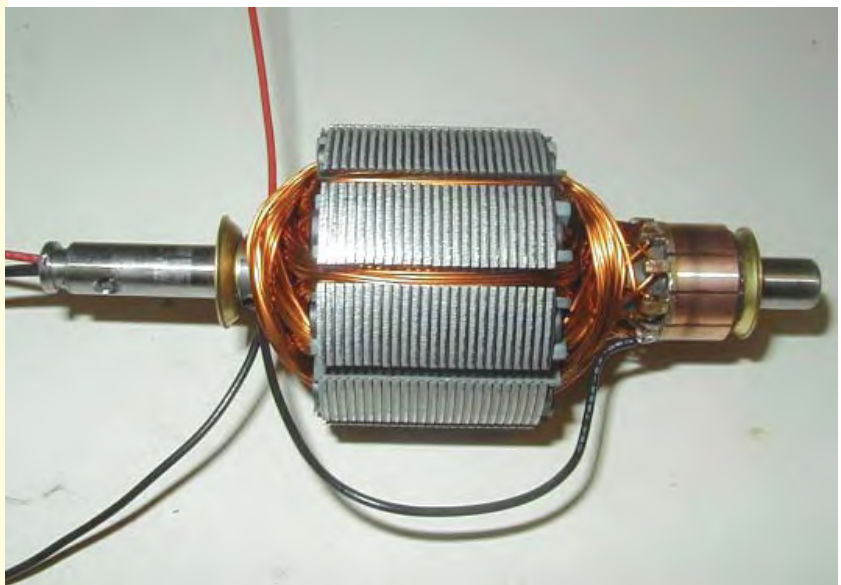
1. (a)    2. (b)    3. (c)    4. (a)    5. (b)    6. (b)    7. (c)    8. (d)    9. (b)    10. (b)  
11. (a)    12. (a)    13. (c)    14. (b)    15. (d)    16. (a)    17. (d)    18. (b)

# CHAPTER 27

## Learning Objectives

- Armature Reaction
- Demagnetising and Cross-magnetising Conductors
- Demagnetising AT per Pole
- Cross-magnetising AT per pole
- Compensating Windings
- No. of Compensating Windings
- Commutation
- Value of Reactance Voltage
- Methods of Improving Commutation
- Resistance Commutation
- E.M.F. Commutation
- Interpoles or Compoles
- Equalising Connections
- Parallel Operation of Shunt Generators
- Paralleling D.C. Generator
- Load Sharing
- Procedure for Paralleling D.C. Generators
- Compound Generators in Parallel
- Series Generators in Parallel

## ARMATURE REACTION AND COMMUTATION



↑ Armature reaction is the change in the neutral plane and the reaction of the magnetic field

### 27.1 Armature Reaction

By armature reaction is meant the effect of magnetic field set up by armature current on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :

- (i) *It demagnetises or weakens the main flux* and
- (ii) *It cross-magnetises or distorts it.*

The first effect leads to reduced generated voltage and the second to the sparking at the brushes.

These effects are well illustrated in Fig. 27.1 which shows the flux distribution of a bipolar generator when there is no current in the armature conductors. For convenience, only two poles have been considered, though the following remarks apply to multipolar fields as well. Moreover, the brushes are shown touching the armature conductors directly, although in practice, they touch commutator segments, It is seen that

- (a) the flux is distributed symmetrically with respect to the polar axis, which is the line joining the centres of *NS* poles.
- (b) The magnetic neutral axis or plane (*M.N.A.*) coincides with the geometrical neutral axis or plane (*G.N.A.*)

Magnetic neutral axis may be defined as the axis along which no e.m.f. is produced in the armature conductors because they then move parallel to the lines of flux.

Or *M.N.A.* is the axis which is perpendicular to the flux passing through the armature.

As hinted in Art. 27.2, brushes are always placed along *M.N.A.* Hence, *M.N.A.* is also called 'axis of commutation' because reversal of current in armature conductors takes place across this axis.

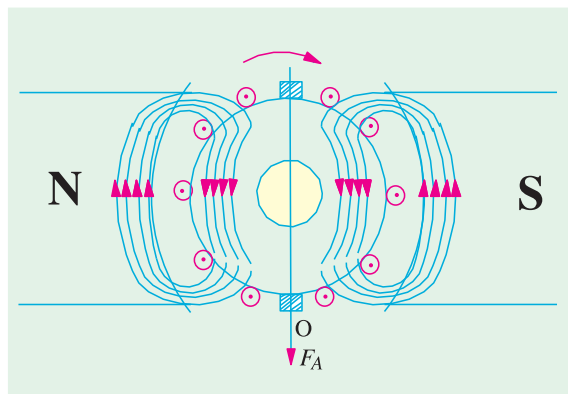


Fig. 27.2

As shown in Fig. 27.2, the m.m.fs. of the armature conductors combine to send flux downwards through the armature. The direction of the lines of force can be found by applying cork-screw rule. The armature m.m.f. (depending on the strength of the armature current) is shown separately both in magnitude and direction by the vector  $OF_A$  which is parallel to the brush axis.

So far, we considered the main m.m.f. and armature m.m.f. separately as if they existed independently, which is not the case in practice. Under actual load conditions, the two exist simultaneously in

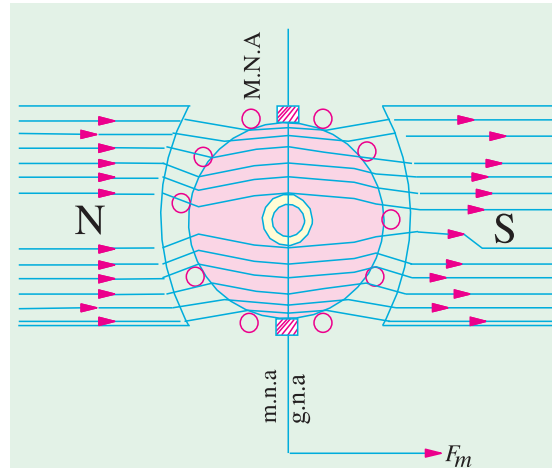


Fig. 27.1

In Fig. 27.1 is shown vector  $OF_m$  which represents, both in magnitude and direction, the m.m.f. producing the main flux and also *M.N.A.* which is perpendicular to  $OF_m$ .

In Fig. 27.2 is shown the field (or flux) set up by the armature conductors *alone* when carrying current, the field coils being unexcited. The direction of the armature current is the same as it would actually be when the generator is loaded. It may even be found by applying Fleming's Right-hand Rule. The current direction is downwards in conductors under *N*-pole and upwards in those under *S*-pole. The downward flow is represented by crosses and upward flow by dots.

the generator as shown in Fig. 27.3.

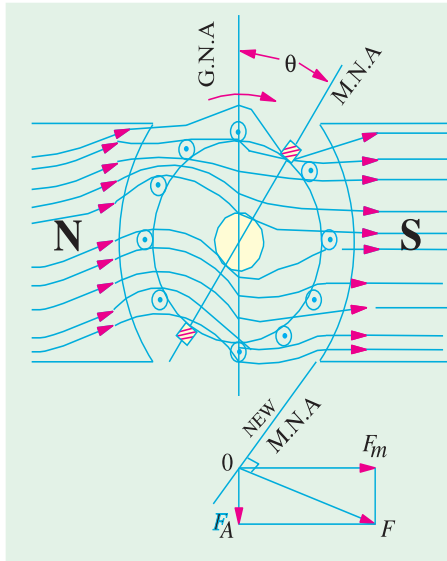


Fig. 27.3

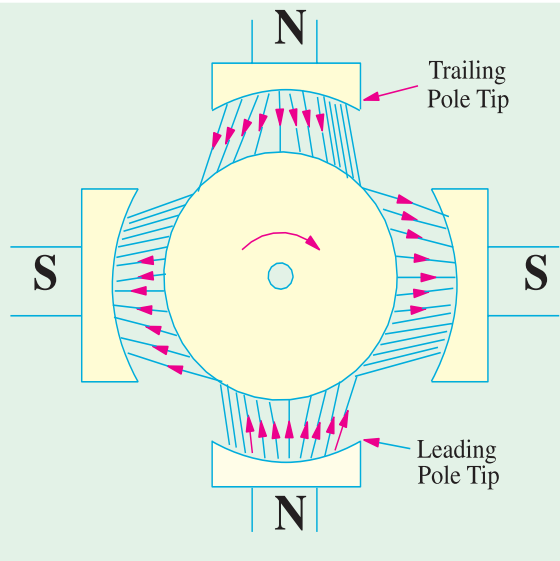
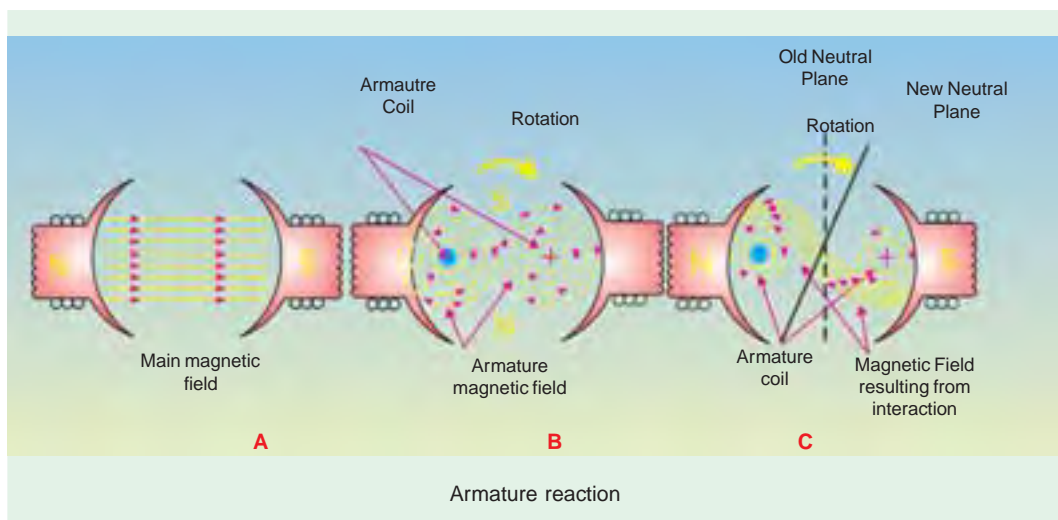


Fig. 27.4

It is seen that the flux through the armature is no longer uniform and symmetrical about the pole axis, rather it has been distorted. The flux is seen to be crowded at the trailing pole tips but weakened or thinned out at the leading pole tips (the pole tip which is first met during rotation by armature conductors is known as the leading pole tip and the other as trailing pole tip). The strengthening and weakening of flux is separately shown for a four-pole machine in Fig. 27.4. As seen, air-gap flux density under one pole half is greater than that under the other pole half.

If Fig. 27.3 is shown the resultant m.m.f.  $OF$  which is found by vectorially combining  $OF_m$  and  $OF_A$ .

The new position of  $M.N.A.$ , which is always perpendicular to the resultant m.m.f. vector  $OF$ , is also shown in the figure. With the shift of  $M.N.A.$ , say through an angle  $\theta$ , brushes are also shifted so as to lie along the new position of  $M.N.A.$  Due to this brush shift (or forward lead), the armature conductors and hence armature current is redistributed. Some armature conductors which were earlier



Armature reaction

under the influence of *N*-pole come under the influence of *S*-pole and *vice-versa*. This regrouping is shown in Fig. 27.5, which also shows the flux due to armature conductors. Incidentally, brush position shifts in the *same* direction as the direction of armature rotation.

All conductors to the left of new position of *M.N.A.* but between the two brushes, carry current downwards and those to the right carry current upwards. The armature m.m.f. is found to lie in the direction of the new position of *M.N.A.* (or brush axis). The armature m.m.f. is now represented by the vector  $OF_A$  which is not vertical (as in Fig 27.2) but is inclined by an angle  $\theta$  to the left. It can now be resolved into two rectangular components,  $OF_d$  parallel to polar axis and  $OF_C$  perpendicular to this axis. We find that

(i) component  $OF_C$  is at right angles to the vector  $OF_m$  (of Fig. 27.1) representing the main m.m.f. It produces distortion in the main field and is hence called the cross-magnetising or distorting component of the armature reaction.

(ii) The component  $OF_d$  is in direct opposition of  $OF_m$  which represents the main m.m.f. It exerts a demagnetising influence on the main pole flux. Hence, it is called the demagnetising or weakening component of the armature reaction.

It should be noted that both distorting and demagnetising effects will increase with increase in the armature current.

### 27.2. Demagnetising and Cross-magnetising Conductors

The exact conductors which produce these distorting and demagnetising effects are shown in Fig. 27.6 where the brush axis has been given a forward lead of  $\theta$  so as to lie along the new position of *M.N.A.* All conductors lying within angles  $AOC = BOD = 2\theta$  at the top and bottom of the armature, are carrying current in such a direction as to send the flux *through* the armature from right to left. This fact may be checked by applying crockscrew rule. It is these conductors which act in direct opposition to the main field and are hence called the demagnetising armature conductors.

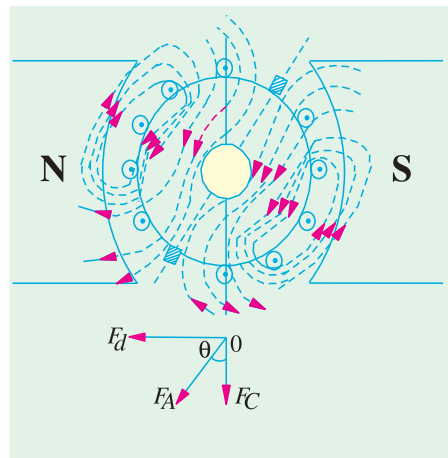


Fig. 27.5

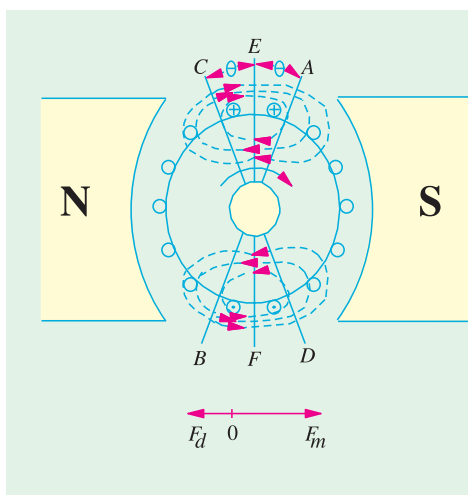


Fig. 27.6

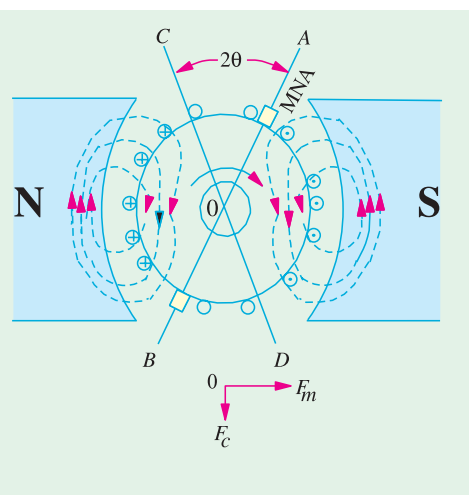


Fig. 27.7



Now consider the remaining armature conductors lying between angles  $AOD$  and  $COB$ . As shown in Fig. 27.7, these conductors carry current in such a direction as to produce a combined flux pointing vertically downwards *i.e.* at right angles to the main flux. This results in distortion of the main field. Hence, these conductors are known as cross-magnetising conductors and constitute distorting ampere-conductors.

### 27.3. Demagnetising AT per Pole

Since armature demagnetising ampere-turns are neutralized by adding extra ampere-turns to the main field winding, it is essential to calculate their number. But before proceeding further, it should be remembered that the number of turns is equal to half the number of conductors because two conductors-constitute one turn.

$$\begin{aligned} \text{Let } Z &= \text{total number of armature conductors} \\ I &= \text{current in each armature conductor} \\ &= I_d/2 \quad \dots \text{ for simplex wave winding} \\ &= I_d/P \quad \dots \text{ for simplex lap winding} \\ \theta_m &= \text{forward lead in mechanical or geometrical or angular degrees.} \end{aligned}$$

$$\text{Total number of armature conductors in angles } AOC \text{ and } BOD \text{ is } \frac{4\theta_m}{360} \times Z$$

As two conductors constitute one turn,

$$\therefore \text{Total number of turns in these angles} = \frac{2\theta_m}{360} \times ZI$$

$$\therefore \text{Demagnetising amp-turns per pair of poles} = \frac{2\theta_m}{360} \times ZI$$

$$\therefore \text{Demagnetising amp - turns/pole} = \frac{\theta_m}{360} \times ZI \quad \therefore AT_d \text{ per pole} = ZI \times \frac{\theta_m}{360}$$

### 27.4. Cross-magnetising AT per pole

The conductors lying between angles  $AOD$  and  $BOC$  constitute what are known as distorting or cross-magnetising conductors. Their number is found as under :

$$\text{Total armature-conductors/pole both cross and demagnetising} = Z/P$$

$$\text{Demagnetising conductors/pole} = Z \cdot \frac{2\theta_m}{360} \quad (\text{found above})$$

$$\therefore \text{Cross-magnetising conductors/pole} = \frac{Z}{P} - Z \times \frac{2\theta_m}{360} = Z \left( \frac{1}{P} - \frac{2\theta_m}{360} \right)$$

$$\therefore \text{Cross-magnetising amp-conductors/pole} = ZI \left( \frac{1}{P} - \frac{2\theta_m}{360} \right)$$

$$\therefore \text{Cross-magnetising amp-turns/pole} = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right)$$

(Remembering that two conductors make one turn)

$$\therefore AT_c/\text{pole} = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right)$$

**Note. (i)** For neutralizing the demagnetising effect of armature-reaction, an extra number of turns may be put on each pole.

$$\text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \quad \text{—for shunt generator}$$

$$= \frac{AT_d}{I_a} \quad \text{—for series generator}$$

If the leakage coefficient  $\lambda$  is given, then multiply each of the above expressions by it.

(ii) If lead angle is given in electrical degrees, it should be converted into mechanical degrees by the following relation.

$$\theta \text{ (mechanical)} = \frac{\theta \text{ (electrical)}}{\text{pair of poles}} \text{ or } \theta_m = \frac{\theta_e}{P/2} = \frac{2\theta_e}{P}$$

### 27.5. Compensating Windings

These are used for large direct current machines which are subjected to large fluctuations in load *i.e.* rolling mill motors and turbo-generators etc. Their function is to neutralize the cross magnetizing effect of armature reaction. In the absence of compensating windings, the flux will be suddenly shifting backward and forward with every change in load. This shifting of flux will induce *statically* induced e.m.f. in the armature coils. The magnitude of this e.m.f. will depend upon the rapidity of changes in load and the amount of change. It may be so high as to strike an arc between the consecutive commutator segments across the top of the mica sheets separating them. This may further develop into a flash-over around the whole commutator thereby short-circuiting the whole armature.

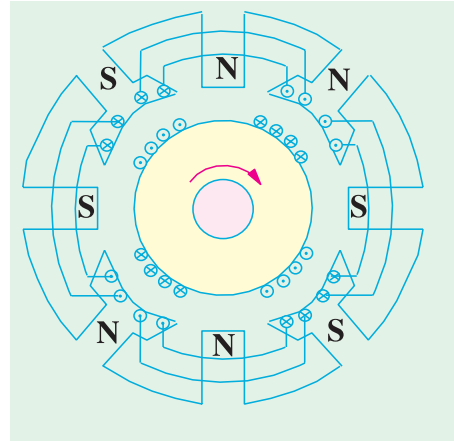
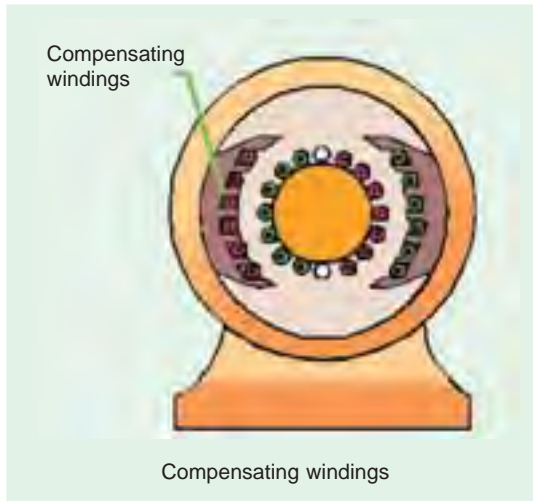


Fig. 27.8



These windings are embedded in slots in the pole shoes and are connected in series with armature in such a way that the current in them flows in opposite direction to that flowing in armature conductors directly below the pole shoes. An elementary scheme of compensating winding is shown in Fig. 27.8.

It should be carefully noted that compensating winding must provide sufficient m.m.f so as to counterbalance the armature m.m.f. Let

$$\begin{aligned} Z_c &= \text{No. of compensating conductors/pole face} \\ Z_a &= \text{No. of active armature conductors/pole,} \\ I_a &= \text{Total armature current} \\ I_a/A &= \text{current/armature conductor} \\ \therefore Z_c I_a &= Z_a (I_a/A) \text{ or } Z_c = Z_a/A \end{aligned}$$

Owing to their cost and the room taken up by them, the compensating windings are used in the case of large machines which are subject to violent fluctuations in load and also for generators which have to deliver their full-load output at considerable low induced voltage as in the Ward-Leonard set.

### 27.6. No. of Compensating Windings

$$\text{No. of armature conductors/pole} = \frac{Z}{P} \quad \text{No. of armature turns/pole} = \frac{Z}{2P}$$

∴ No. of armature-turns immediately under one pole

$$= \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7 \times \frac{Z}{2P} \text{ (approx.)}$$

∴ No. of armature amp-turns/pole for compensating winding

$$= 0.7 \times \frac{Z}{2P} = 0.7 \times \text{armature amp-turns/pole}$$

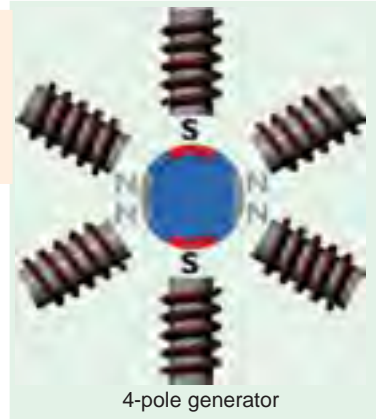
**Example 27.1.** A 4-pole generator has a wave-wound armature with 722 conductors, and it delivers 100 A on full load. If the brush lead is 8°, calculate the armature demagnetising and cross-magnetising ampere turns per pole.

(Advanced Elect. Machines AMIE Sec. B 1991)

**Solution.**  $I = I_a/2 = 100/2 = 50\text{A}$ ;  $Z = 722$ ;  $\theta_m = 8^\circ$

$$\text{AT}_d/\text{pole} = ZI \cdot \frac{\theta_m}{360} = 722 \times 50 \times \frac{8}{360} = 802$$

$$\begin{aligned} \text{AT}_c/\text{pole} &= ZI \cdot \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) \\ &= 722 \times 50 \left( \frac{1}{2 \times 4} - \frac{8}{360} \right) = 37/8 \end{aligned}$$



**Example 27.2** An 8-pole generator has an output of 200 A at 500 V, the lap-connected armature has 1280 conductors, 160 commutator segments. If the brushes are advanced 4-segments from the no-load neutral axis, estimate the armature demagnetizing and cross-magnetizing ampere-turns per pole.

(Electrical Machines-I, South Gujarat Univ. 1986)

**Solution.**  $I = 200/8 = 25\text{ A}$ ,  $Z = 1280$ ,  $\theta_m = 4 \times 360/160 = 9^\circ$ ;  $P = 8$

$$\text{AT}_d/\text{pole} = ZI\theta_m/360 = 1280 \times 25 \times 9/360 = \mathbf{800}$$

$$\text{AT}_c/\text{pole} = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) = 1280 \times 25 \left( \frac{1}{2 \times 8} - \frac{9}{360} \right) = \mathbf{1200}$$

**Note.** No. of coils = 160, No. of conductors = 1280. Hence, each coil side contains 1280/160 = 8 conductors.

**Example 27.3(a).** A 4-pole wave-wound motor armature has 880 conductors and delivers 120 A. The brushes have been displaced through 3 angular degrees from the geometrical axis. Calculate (a) demagnetising amp-turns/pole (b) cross- magnetising amp-turns/pole (c) the additional field current for neutralizing the demagnetisation of the field winding has 1100 turns/pole.

**Solution.**  $Z = 880$ ;  $I = 120/2 = 60\text{ A}$ ;  $\theta = 3^\circ$  angular

$$(a) \therefore \text{AT}_d = 880 \times 60 \times \frac{3}{360} = \mathbf{440\text{ AT}}$$

$$(b) \therefore \text{AT}_c = 880 \times 60 \left( \frac{1}{8} - \frac{3}{360} \right) = 880 \times \frac{7}{60} \times 60 = \mathbf{6,160}$$

or Total AT/pole = 440 × 60/4 = 6600

Hence,  $\text{AT}_c/\text{pole} = \text{Total AT/pole} - \text{AT}_d/\text{pole} = 6600 - 440 = 6160$

(c) Additional field current = 440/1100 = **0.4 A.**

**Example 27.3(b).** A 4-pole lap-wound Generator having 480 armature conductors supplies a current of 150 Amps. If the brushes are given an actual lead of 10°, calculate the demagnetizing and cross-magnetizing amp-turns per pole.

(Bharathiar Univ. April 1998)

**Solution.** 10° mechanical (or actual) shift = 20° electrical shift for a 4-pole machine.

Armature current = 150 amp

For 4-pole lap-wound armature, number of parallel paths = 4. Hence, conductor-current =  $150/4 = 37.5$  amps.

$$\text{Total armature amp-turns/pole} = \frac{1}{2} \times \frac{(480 \times 37.5)}{14} = 2250$$

$$\text{Cross - magnetizing amp turns/pole} = 2250 \times \left(1 - \frac{2 \times 20^\circ}{180^\circ}\right) = 1750$$

$$\text{Demagnetizing amp turns/pole} = 2250 \times (2 \times 20^\circ/180^\circ) = 500$$

**Example 27.4.** A 4-pole generator supplies a current of 143 A. It has 492 armature conductors (a) wave-wound (b) lap-wound. When delivering full load, the brushes are given an actual lead of  $10^\circ$ . Calculate the demagnetising amp-turns/pole. This field winding is shunt connected and takes 10 A. Find the number of extra shunt field turns necessary to neutralize this demagnetisation.

(Elect. Machines, Nagpur Univ. 1993 & JNTU Hyderabad, 2000 & RGPU, Bhopal, 2000)

**Solution.**  $Z = 492$  ;  $\theta_m = 10^\circ$  ;  $AT_d/\text{pole} = Z I \times \frac{\theta}{360}$   
 $I_a = 143 + 10 = 153 \text{ A}$  ;  $I = 153/2$  ... when wave-wound  
 $= 153/4$  ... when lap-wound

(a)  $\therefore AT_d/\text{pole} = 492 \times \frac{153}{2} \times \frac{10}{360} = 1046 \text{ AT}$

Extra shunt field turns =  $1046/10 = 105$  (approx.)

(b)  $AT_d/\text{pole} = 492 \times \frac{153}{2} \times \frac{10}{360} = 523$

Extra shunt field turns =  $523/10 = 52$  (approx.)

**Example 27.5.** A 4-pole, 50-kW, 250-V wave-wound shunt generator has 400 armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetisation amp-turns/pole if shunt field resistance is  $50 \Omega$ . Also, calculate extra shunt field turns/pole to neutralize the demagnetisation.

**Solution.** Load current supplied =  $50,000/250 = 200 \text{ A}$   
 $I_{sh} = 250/50 = 5 \text{ A}$   $\therefore I_a = 200 + 5 = 205 \text{ A}$

Current in each conductor  $I = 205/2 \text{ A}$

No. of commutator segments =  $N/A$  where  $A = 2$  ... for wave-winding

$\therefore$  No. of segments =  $\frac{400}{2} = 200$  ;  $\theta = \frac{4}{200} \times 360 = \frac{36}{5}$  degrees

$\therefore AT_d/\text{pole} = 400 \times \frac{205}{2} \times \frac{36}{5 \times 360} = 820 \text{ AT}$

Extra shunt turns/poles =  $\frac{AT_d}{I_{sh}} = \frac{820}{5} = 164$

**Example 27.6.** Determine per pole the number (i) of cross-magnetising ampere-turns (ii) of back ampere-turns and (iii) of series turns to balance the back ampere-turns in the case of a d.c. generator having the following data.

500 conductors, total current 200 A, 6 poles, 2-circuit wave winding, angle of lead =  $10^\circ$ , leakage coefficient = 1.3 (Electrical Machines-I, Bombay University, 1986)

**Solution.** Current/path,  $I = 200/2 = 100 \text{ A}$ ,  $\theta = 10^\circ$  (mech),  $Z = 500$

(a)  $AT_c/\text{pole} = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) = 500 \times 100 \left( \frac{1}{2 \times 6} - \frac{10}{360} \right) = 2,778$

(b)  $AT_d/\text{pole} = 500 \times 100 \times 10/360 = 1390$

(c) Series turns required to balance the demagnetising ampere-turns are

$$= \lambda \times \frac{AT_d}{I_a} = 1.3 \times \frac{1390}{200} = 9$$

**Example 27.7.** A 22.38 kW, 440-V, 4-pole wave-wound d.c. shunt motor has 840 armature conductors and 140 commutator segments. Its full-load efficiency is 88% and the shunt field current is 1.8 A. If brushes are shifted backward through 1.5 segments from the geometrical neutral axis, find the demagnetising and distorting amp-turns/pole. (Elect. Engg. Punjab Univ. 1991)

**Solution.** The shunt motor is shown diagrammatically in Fig. 27.9.

$$\begin{aligned} \text{Motor output} &= 22,380 \text{ W} ; \eta = 0.88 \\ \therefore \text{Motor input} &= 22,380/0.88 \text{ W} \\ \text{Motor input current} &= \frac{22,380}{0.88 \times 440} = 57.8 \text{ A} \\ I_{sh} = 1.8 \text{ A} ; \quad I_a &= 57.8 - 1.8 = 56 \text{ A} \\ \text{Current in each conductor} &= 56/2 = 28 \text{ A} \\ \theta &= 1.5 \times 360/140 \\ &= 27/7 \text{ degrees} \end{aligned}$$

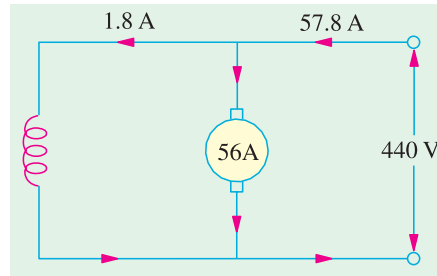


Fig. 27.9

$$\therefore AT_d/\text{pole} = 840 \times 28 \times \frac{27}{7 \times 360} = 252$$

$$AT_c/\text{pole} = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) = 840 \times 28 \left( \frac{1}{8} - \frac{27}{7 \times 360} \right) = 2,688$$

**Example 27.8.** A 400-V, 1000-A, lap-wound d.c. machine has 10 poles and 860 armature conductors. Calculate the number of conductors in the pole face to give full compensation if the pole face covers 70 % of pole span.

**Solution.** AT/pole for compensating winding

$$= \text{armature amp-turn/pole} \times \frac{\text{pole arc}}{\text{pole pitch}} = 0.7 \times \frac{ZI}{2P}$$

Here  $I = \text{current in each armature conductor} = 1,000/10 = 100 \text{ A}$   
 $Z = 860 ; P = 10$

$$\therefore AT/\text{pole for compensating winding} = 0.7 \times 860 \times 100/2 \times 10 = 3,010$$

### Tutorial Problem No. 27.1

1. Calculate the demagnetising amp-turns of a 4-pole, lap-wound generator with 720 turns, giving 50 A, if the brush lead is 10° (mechanical). (250 AT/pole)
2. A 250-V, 25-kW, 4-pole d.c. generator has 328 wave-connected armature conductors. When the machine is delivering full load, the brushes are given a lead of 7.2 electrical degrees. Calculate the cross-magnetising amp-turns/pole. (1886, 164)
3. An 8-pole lap-connected d.c. shunt generator delivers an output of 240 A at 500 V. The armature has 1408 conductors and 160 commutator segments. If the brushes are given a lead of 4 segments from the no-load neutral axis, estimate the demagnetising and cross-magnetising AT/pole. (1056, 1584) (Electrical Engineering, Bombay Univ. 1978)
4. A 500-V, wave-wound, 750 r.p.m. shunt generator supplies a load of 195 A. The armature has 720 conductors and shunt field resistance is 100 Ω. Find the demagnetising amp-turns/pole if brushes are advanced through 3 commutator segments at this load. Also, calculate the extra shunt field turns required to neutralize this demagnetisation. (600, 4800, 120)

5. A 4-pole, wave-wound generator has 320 armature conductors and carries an armature current of 400 A. If the pole arc/pole pitch ratio is 0.68, calculate the AT/pole for a compensating winding to give uniform flux density in the air gap. (5440)

6. A 500-kW, 500-V, 10 pole d.c. generator has a lap-wound armature with 800 conductors. Calculate the number of pole-face conductors in each pole of a compensating winding if the pole face covers 75 percent of the pitch.

(6 conductors/pole)

7. Three shunt generators, each having an armature resistance of 0.1 ohm are connected across a common bus feeding a two ohms load. Their generated voltages are 127 V, 120 V, and 119 V. Neglecting field currents, calculate the bus voltage and modes of operations of the three machines. (JNTU, Hyderabad, 200)

**Hint :** Solve the circuit from the data given. Since the voltages differ considerably, first machine with 127 V as the generated voltage with supply the largest current.

( $I_1 = 70$  amp, Generating mode,  $I_2 = 0$ ,  
Floating (= neither generating nor motoring).  
 $I_3 = -10$  amp, motoring mode  $I_L = 60$  amp.)

### 27.7. Commutation

It was shown in Art 26.2 that currents induced in armature conductors of a d.c. generator are alternating. To make their flow unidirectional in the external circuit, we need a commutator. Moreover, these currents flow in one direction when armature conductors are under  $N$ -pole and in the opposite direction when they are under  $S$ -pole. As conductors pass out of the influence of a  $N$ -pole and enter that of  $S$ -pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis or brush axis *i.e.* when the brush spans and hence short-circuits that particular coil undergoing reversal of current through it. This process by which current in the short-circuited coil is reversed while it crosses the  $M.N.A.$  is called commutation. The brief period during which coil remains short-circuited is known as commutation period  $T_c$ .

If the current reversal *i.e.* the change from  $+I$  to zero and then to  $-I$  is completed by the end of short circuit or commutation period, then the commutation is ideal. If current reversal is not complete by that time, then sparking is produced between the brush and the commutator which results in progressive damage to both.

Let us discuss the process of commutation or current reversal in more detail with the help of Fig. 27.10 where ring winding has been used for simplicity. The brush width is equal to the width of one commutator segment and one mica insulation. In Fig. 27.10 (a) coil  $B$  is about to be short circuited because brush is about to come in touch with commutator segment 'a'. It is assumed that each coil carries 20 A, so that brush current is 40 A. It is so because every coil meeting at the brush supplies half the brush current lap wound or wave wound. Prior to the beginning of short circuit, coil  $B$  belongs to the group of coils lying to the left of the brush and carries 20 A from left to right. In Fig. 27.10 (b) coil  $B$  has entered its period of short-circuit and is approximately at one-third of this period. The current through coil  $B$  has reduced down from 20 A to 10 A because the other 10 A flows *via* segment 'a'. As area of contact of the brush is more with segment 'b' than with segment 'a', it receives 30 A from the former, the total again being 40 A.



Commutation

Fig. 27.10 (c) shows the coil *B* in the middle of its short-circuit period. The current through it has decreased to zero. The two currents of value 20 A each, pass to the brush directly from coil *A* and *C* as shown. The brush contact areas with the two segments 'b' and 'a' are equal.

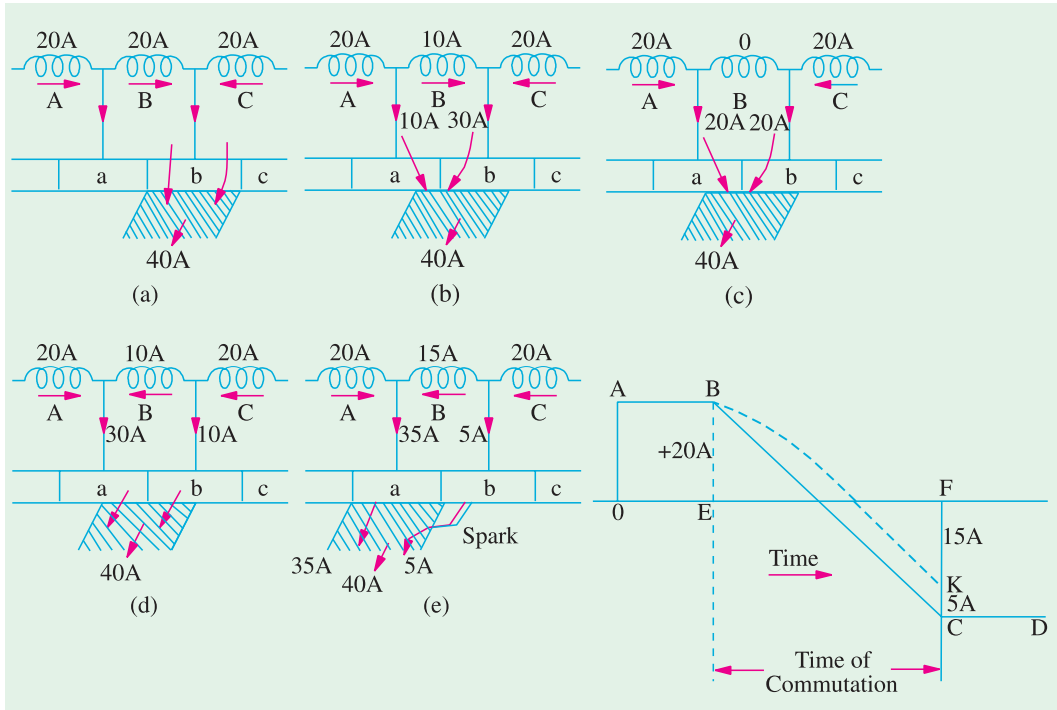


Fig. 27.10

Fig. 27.11

In Fig. 27.10 (d), coil *B* has become part of the group of coils lying to the right of the brush. It is seen that brush contact area with segment 'b' is decreasing rapidly whereas that with segment 'a' is increasing. Coil *B* now carries 10 A in the reverse direction which combines with 20 A supplied by coil *A* to make up 30 A that passes from segment 'a' to the brush. The other 10 A is supplied by coil *C* and passes from segment 'b' to the brush, again giving a total of 40 A at the brush.

Fig. 27.10 (e) depicts the moment when coil *B* is almost at the end of commutation or short-circuit period. For ideal commutation, current through it should have reversed by now but, as shown, it is carrying 15 A only (instead of 20 A). The difference of current between coils *C* and *B* i.e.  $20.15 = 5$  A in this case, jumps directly from segment *b* to the brush through air thus producing spark.

If the changes of current through coil *B* are plotted on a time base (as in Fig. 27.11) it will be represented by a horizontal line *AB* i.e. a constant current of 20 A up to the time of beginning of commutation. From the finish of commutation, the current will be represented by another horizontal line *CD*. Now, again the current value is  $FC = 20$  A, although in the reversed direction. The way in which current changes from its positive value of 20 A (= *BE*) to zero and then to its negative value of 20 A (= *CF*) depends on the conditions under which the coil *B* undergoes commutation. If the current varies at a uniform rate i.e. if *BC* is a straight line, then it is referred to as linear commutation. However, due to the production of self-induced e.m.f. in the coil (discussed below) the variations follow the dotted curve. It is seen that, in that case, current in coil *B* has reached only a value of *KF* = 15 A in the reversed direction, hence the difference of 5 A (20 A – 15 A) passes as a spark.

So, we conclude that sparking at the brushes, which results in poor commutation is due to the inability of the current in the short-circuited coil to reverse completely by the end of short-circuit period (which is usually of the order of 1/500 second).

At this stage, the reader might ask for the reasons which make this current reversal impossible in the specified period *i.e.* what factors stand in the way of our achieving ideal commutation. The main cause which retards or delays this quick reversal is the production of self-induced e.m.f. in the coil undergoing commutation. It may be pointed out that the coil possesses appreciable amount of self inductance because it lies embedded in the armature which is built up of a material of high magnetic permeability. This self-induced e.m.f. is known as **reactance voltage** whose value is found as given below. This voltage, even though of a small magnitude, produces a large current through the coil whose resistance is very low due to short circuit. It should be noted that if the brushes are set so that the coils undergoing short-circuit are in the magnetic neutral plane, where they are cutting no flux and hence have no e.m.f. induced in them due to armature rotation, there will still be the e.m.f. of self-induction which causes severe sparking at the brushes.

### 27.8. Value of Reactance Voltage

Reactance voltage = coefficient of self-inductance  $\times$  rate of change of current.

It should be remembered that the time of short-circuit or commutation is the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating plate of strip of mica.

$$\begin{aligned} \text{Let} \quad & W_b = \text{brush width in cm; } W_m = \text{width of mica insulation in cm} \\ & v = \text{peripheral velocity of commutator segments in cm/second} \\ \text{Then} \quad & T_c = \text{time of commutation or short-circuit} = \frac{W_b - W_m}{v} \text{ second} \end{aligned}$$

**Note.** If brush width etc. are given in terms of commutator segments, then commutator velocity should also be converted in terms of commutator segments per second.

If  $I$  is the current through a conductor, then total change during commutation =  $I - (-I) = 2I$ .

$$\begin{aligned} \therefore \text{ Self-induced or reactance voltage} \\ &= L \times \frac{2I}{T_c} \quad \text{-- if commutation is linear} \\ &= 1.11 L \times \frac{2I}{T_c} \quad \text{-- if commutation is sinusoidal} \end{aligned}$$

As said earlier, the reactance e.m.f. hinders the reversal of current. This means that there would be sparking at the brushes due to the failure of the current in short-circuited coil to reach its full value in the reversed direction by the end of short-circuit. This sparking will not only damage the brush and the commutator but this being a cumulative process, it may worsen and eventually lead to the short-circuit of the whole machine by the formation of an arc round the commutator from brush to brush.

**Example 27.9.** The armature of a certain dynamo runs at 800 r.p.m. The commutator consists of 123 segments and the thickness of each brush is such that the brush spans three segments. Find the time during which the coil of an armature remains short-circuited.

**Solution.** As  $W_m$  is not given, it is considered negligible.

$$W_b = 3 \text{ segments and } v = (800/60) \times 123 \text{ segments/second}$$

$$\therefore \text{ commutation time} = \frac{3 \times 60}{800 \times 123} = 0.00183 \text{ second} = \mathbf{1.83 \text{ millisecond}}$$

**Example 27.10.** A 4-pole, wave-wound, d.c. machine running at 1500 r.p.m. has a commutator of 30 cm diameter. If armature current is 150 A, thickness of brush 1.25 cm and the self-inductance of each armature coil is 0.07 mH, calculate the average e.m.f. induced in each coil during commutation. Assume linear commutation.



**Solution.** Formula :  $E = L \frac{2I}{T_c}$

Here  $L = 0.07 \times 10^{-3}$  H,  $I = 150/2 = 75$  A (It is wave-wound)

$W_b = 1.25$  cm,  $W_m = 0$  ...considered negligible

$v = \pi \times 30 \times (1500/60) = 2356$  cm/s ;  $T_c = W_b/v = 1.25/2356 = 5.3 \times 10^{-4}$  second

$E = L \times 2I/T_c = 0.07 \times 10^{-3} \times 2 \times 75/5.3 \times 10^{-4} = 19.8$  V

**Example 27.11.** Calculate the reactance voltage for a machine having the following particulars. Number of commutator segments = 55, Revolutions per minute = 900, Brush width in commutator segments = 1.74, Coefficient of self-induction =  $153 \times 10^{-6}$  henry, Current per coil = 27 A.

(Advanced Elect. Machines. AMIE Sec. Winter 1991)

**Solution.** Current per coil,  $I = 27$  A ;  $L = 153 \times 10^{-6}$  H

$v = 55 \times (900/60) = 825$  segments/second;  $T_c = W_b/v = 1.74/825 = 2.11 \times 10^{-3}$  second

Assuming linear commutation,  $E = L \times 2I/T_c$

$\therefore E = 153 \times 10^{-6} \times 2 \times 27/2.11 \times 10^{-3} = 3.91$  V

**Example 27.12.** A 4-pole, lap-wound armature running at 1500 r.p.m. delivers a current of 150 A and has 64 commutator segments. The brush spans 1.2 segments and inductance of each armature coil is 0.05 mH. Calculate the value of reactance voltage assuming (i) linear commutation (ii) sinusoidal commutation. Neglect mica thickness.

**Solution.** Formula :  $E = L \cdot \frac{2I}{T_c}$  Now,  $L = 0.05 \times 10^{-3}$  H;  $W_b = 1.2$  segments

$v = \frac{1500}{60} \times 64 = 1600$  segments/second

$\therefore T_c = \frac{1.2}{1600} = 7.5 \times 10^{-4}$  second ;  $I = \frac{150}{4}$  A = 37.5 A

$\therefore \frac{2I}{T_c} = \frac{2 \times 37.5}{7.5 \times 10^{-4}} = 10^5$  A/s

For linear commutation,  $E = 0.05 \times 10^{-3} \times 10^5 = 5$  V

For sinusoidal commutation,  $E = 1.11 \times 5 = 5.55$  V

### 27.9. Methods of Improving Commutation

There are two practical ways of improving commutation *i.e.* of making current reversal in the short-circuited coil as sparkless as possible. These methods are known as (i) resistance commutation and (ii) e.m.f. commutation (which is done with the help of either brush lead or interpoles, usually the later).

#### 27.10. Resistance Commutation

This method of improving commutation consists of replacing low-resistance Cu brushes by comparatively high-resistance carbon brushes.

From Fig. 27.12, it is seen that when current  $I$  from coil 'C' reaches the commutator segment  $b$ , it has two parallel paths open to it. The first part is straight from bar 'b' to the brush and the other parallel path is *via* the short-circuited coil B to bar 'a' and then to the brush. If the Cu brushes (which have low contact resistance) are used, then there is no inducement for the current to follow the sec-

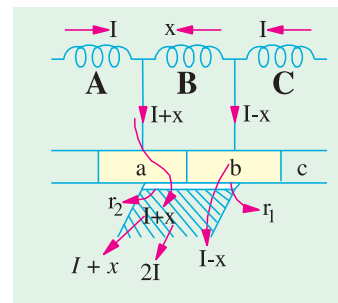


Fig. 27.12

ond longer path, it would preferably follow the first path. But when carbon brushes having high resistance are used, then current  $I$  coming from  $C$  will prefer to pass through the second path because (i) the resistance  $r_1$  of the first path will increase due to the diminishing area of contact of bar 'b' with the brush and because (ii) resistance  $r_2$  of second path will decrease due to rapidly increasing contact area of bar 'a' with the brush.

Hence, carbon brushes have, usually, replaced Cu brushes. However, it should be clearly understood that the main cause of sparking commutation is the self-induced e.m.f. (*i.e.* reactance voltage), so brushes alone do not give a sparkless commutation; though they do help in obtaining it.

The additional advantages of carbon brushes are that (i) they are to some degree self-lubricating and polish the commutator and (ii) should sparking occur, they would damage the commutator less than when Cu brushes are used.

But some of their minor disadvantages are : (i) Due to their high contact resistance (which is beneficial to sparkless commutation) a loss of approximately 2 volt is caused. Hence, they are not much suitable for small machines where this voltage forms an appreciable percentage loss. (ii) Owing to this large loss, the commutator has to be made some what larger than with Cu brushes in order to dissipate heat efficiently without greater rise of temperature. (iii) because of their lower current density (about 7-8 A/cm<sup>2</sup> as compared to 25-30 A/cm<sup>2</sup> for Cu brushes) they need larger brush holders.

### 27.11. E.M.F. Commutation

In this method, arrangement is made to neutralize the reactance voltage by producing a reversing e.m.f. in the short-circuited coil under commutation. This reversing e.m.f., as the name shows, is an e.m.f. in opposition to the reactance voltage and if its value is made equal to the latter, it will completely wipe it off, thereby producing quick reversal of current in the short-circuited coil which will result in sparkless commutation. The reversing e.m.f. may be produced in two ways : (i) either by giving the brushes a forward lead sufficient enough to bring the short-circuited coil under the influence of next pole of opposite polarity or (ii) by using interpoles.

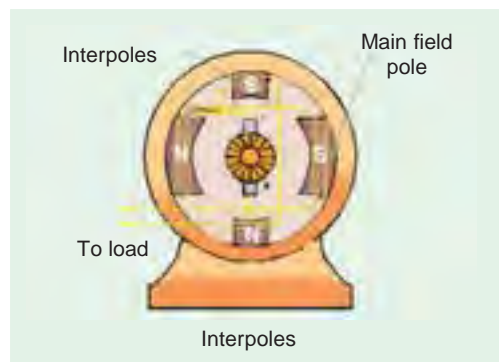
The first method was used in the early machines but has now been abandoned due to many other difficulties it brings along with.

### 27.12. Interpoles of Compoles

These are small poles fixed to the yoke and spaced in between the main poles. They are wound with comparatively few heavy gauge Cu wire turns and are connected in series with the armature so that they carry full armature current. Their polarity, in the case of a generator, is the same as that of the main pole *ahead* in the direction of rotation (Fig. 25.13).

The function of interpoles is two-fold :

(i) As their polarity is the same as that of the main pole ahead, they induce an e.m.f. in the coil (under commutation) which helps the reversal of current. The e.m.f. induced by the compoles is known as *commutating or reversing e.m.f.* The commutating e.m.f. neutralizes the reactance e.m.f. thereby making commutation sparkless. With interpoles, sparkless commutation can be obtained up to 20 to 30% overload with fixed brush position. In fact, interpoles raise sparking limit of a machine to *almost* the same value as heating limit. Hence, for a given output, an interpole machine can be made smaller and, therefore, cheaper than a non-interpolar machine.



As interpoles carry armature current, their commutating e.m.f. is proportional to the armature current. **This ensures automatic neutralization of reactance voltage which is also due to armature current.** Connections for a shunt generator with interpoles are shown in Fig. 27.14.

(ii) Another function of the interpoles is to neutralize the cross-magnetising effect of armature reaction. Hence, brushes are not to be shifted from the original position. In Fig 27.15,  $OF$  as before, represents the m.m.f. due to main poles.  $OA$  represents the cross-magnetising m.m.f. due to armature.  $BC$  which represents m.m.f. due to interpoles, is obviously in opposition to  $OA$ , hence they cancel each other out. **This cancellation of cross-magnetisation is automatic and for all loads because both are produced by the same armature current.**

The distinction between the interpoles and compensating windings should be clearly understood. Both are connected in series and their m.m.fs. are such as to neutralize armature reaction. But compoles additionally supply m.m.f. for counteracting the reactance voltage induced in the coil undergoing commutation. Moreover, the action of the compoles is localized, they have negligible effect on the armature reaction occurring on the remainder of the armature periphery.

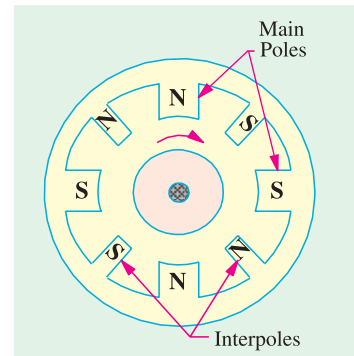


Fig. 27.13

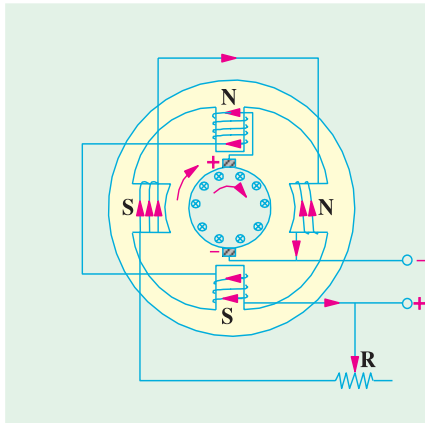


Fig. 27.14

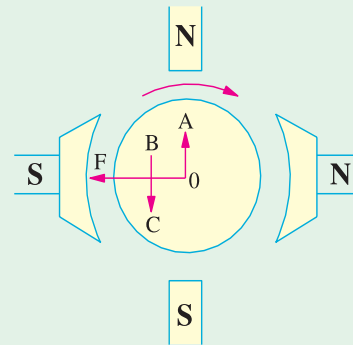


Fig. 27.15

**Example 27.13.** Determine the number of turns on each commutating pole of a 6-pole machine, if the flux density in the air-gap of the commutating pole =  $0.5 \text{ Wb/m}^2$  at full load and the effective length of the air-gap is 4 mm. The full-load current is 500 A and the armature is lap-wound with 540 conductors. Assume the ampere turns required for the remainder of the magnetic circuit to be one-tenth of that the air gap. **(Advanced Elect. Machines AMIE Sec.B, 1991)**

**Solution.** It should be kept in mind that compole winding must be sufficient to oppose the armature m.m.f. (which is directed along compole axis) and to provide the m.m.f. for compole air-gap and its magnetic circuit.

$$\therefore N_{CP} I_a = Z I_c / 2P + B_g l_g / \mu_0$$

where  $N_{CP}$  = No. of turns on the compole ;  $I_a$  = Armature current

$Z$  = No. of armature conductors ;  $I_c$  = Coil current

$P$  = No. of poles ;  $l_g$  = Air-gap length under the compole

$Z = 540$  ;  $I_a = 500 \text{ A}$  ;  $I_c = 500/6 \text{ A}$  ;  $P = 6$

$$\therefore \text{Arm. m.m.f.} = 540 \times (500/6) / 2 \times 6 = 3750$$

Compole air-gap m.m.f. =  $B_g \times l_g / \mu_0 = 0.5 \times 4 \times 10^{-3} / 4\pi \times 10^{-7} = 1591$   
 m.m.f. reqd. for the rest of the magnetic circuit = 10% of 1591 = 159  
 $\therefore$  Total compole air-gap m.m.f. = 1591 + 159 = 1750  
 Total m.m.f. reqd. = 3750 + 1750 = 5500  
 $\therefore N_{cp} I_a = 5500$  or  $N_{cp} = 5500/500 = 11$

### 27.13. Equalizing Connections

It is characteristic of lap -winding that all conductors in any parallel path lie under one pair of poles. If fluxes from all poles are exactly the same, then e.m.f. induced in each parallel path is the same and each path carries the same current. But despite best efforts, some inequalities in flux inevitably occur due either to slight variations in air-gap length or in the magnetic properties of steel. Hence, there is always a slight imbalance of e.m.f. in the various parallel paths. The result is that conductors under stronger poles generate greater e.m.f. and hence carry larger current. The current distribution at the brushes becomes unequal. Some brushes are overloaded *i.e.* carry more than their normal current whereas others carry less. Overloaded brushes spark badly whatever their position may be. This results in poor commutation and may even limit the output of the machine.

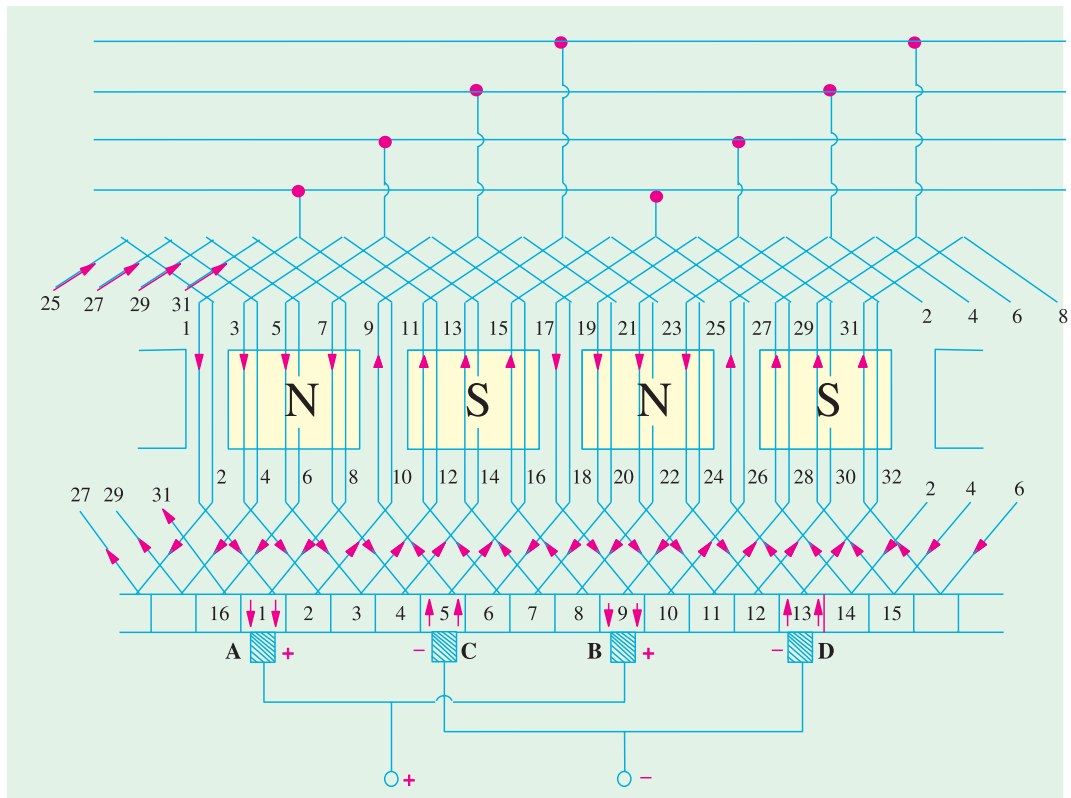


Fig. 27.16

By connecting together a number of symmetrical points on armature winding which would be at equal potential if the pole fluxes were equal, the difference in brush currents is diminished. This requires that there should be a whole number of slots per pair of poles so that, for example, if there is a slot under the centre of a *N*-pole, at some instant, then there would be one slot under the centre of every other *N*-pole. The equalizing conductors, which are in the form of Cu rings at the armature back and which connect such points are called *Equalizer Rings*. The circulating current due to the

slight difference in the e.m.fs. of various parallel paths, passes through these *equalizer rings* instead of passing through the brushes.

**Hence, the function of equalizer rings is to avoid unequal distribution of current at the brushes thereby helping to get sparkless commutation.**

One equalizer ring is connected to all conductors in the armature which are two poles apart (Fig. 27.17). For example, if the number of poles is 6, then the number of connections for each equalizer ring is 3 *i.e.* equal to the number of pair of poles.

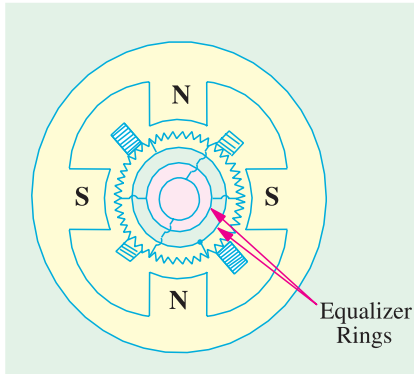


Fig. 27.17

Maximum number of equalizer rings is equal to the number of conductors under one pair of poles. Hence, number of rings is

$$= \frac{\text{No. of conductors}}{\text{No. of pair of poles}}$$

In practice, however, the number of rings is limited to 20 on the largest machines and less on smaller machines. In Fig. 27.16 is shown a developed armature winding. Here, only 4 equalizing bars have been used. It will be seen that the number of equalizing connections to each bar is two *i.e.* half the number of poles. Each alternate coil has been connected to the bar. In this case, the winding is said to be 50% equalized. If all conductors were connected to the equalizer rings, then the winding would have been 100% equalized.

Equalizer rings are not used in wave-wound armatures, because there is no imbalance in the e.m.fs. of the two parallel paths. This is due to the fact that armature conductors in either parallel path are not confined under one pair of poles as in lap-winding but are distributed under all poles. Hence, even if there are inequalities in the pole flux, they will affect each path equally.

#### 27.14. Parallel Operation of Shunt Generators

Power plants, whether in d.c. or a.c. stations, will be generally found to have several smaller generators running in parallel rather than large single units capable of supplying the maximum peak load. These smaller units can be run single or in various parallel combinations to suit the actual load demand. Such practice is considered extremely desirable for the following reasons :

##### (i) Continuity of Service

Continuity of service is one of the most important requirements of any electrical apparatus. This would be impossible if the power plant consisted only of a single unit, because in the event of breakdown of the prime mover or the generator itself, the entire station will be shut down. In recent years, the requirement of uninterrupted service has become so important especially in factories etc. that it is now recognized as an economic necessity.

##### (ii) Efficiency

Usually, the load on the electrical power plant fluctuates between its peak value sometimes during the day and its minimum value during the late night hours. Since generators operate most efficiently when delivering full load, it is economical to use a single small unit when the load is light. Then, as the load demand increases, a larger generator can be substituted for the smaller one or another smaller unit can be connected to run in parallel with the one already in operation.



Equalizer rings

**(iii) Maintenance and Repair**

It is considered a good practice to inspect generators carefully and periodically to forestall any possibility of failure or breakdown. This is possible only when the generator is at rest which means that there must be other generators to take care of the load. Moreover, when the generator does actually breakdown, it can be repaired with more care and not in a rush, provided there are other generators available to maintain service.

**(iv) Additions to Plant**

Additions to power plants are frequently made in order to deliver increasingly greater loads. Provision for future extension is, in fact, made by the design engineers from the beginning. It becomes easy to add other generators for parallel operation as the load demand increases.

**27.15. Paralleling DC Generator**

Whenever generators are in parallel, their +ve and -ve terminals are respectively connected to the +ve and -ve sides of the bus-bars. These bus-bars are heavy thick copper bars and they act as +ve and -ve terminals for the whole power station. If polarity of the incoming generator is not the same as the line polarity, a serious short-circuit will occur when  $S_1$  is closed.

Moreover, paralleling a generator with reverse polarity effectively short-circuits it and results in damaged brushes, a damaged commutator and a blacked-out plant. Generators that have been tripped off the bus-bar because of a heavy fault current should always be checked for reversed polarity before paralleling.

In Fig. 27.18 is shown a shunt generator No. 1 connected across the bus-bars  $BB$  and supplying some of the load. For putting generator No. 2 in parallel with it the following procedure is adopted.

The armature of generator No. 2 is speeded by the prime-mover up to its rated value and then switch  $S_2$  is closed and circuit is completed by putting a voltmeter  $V$  across the open switch  $S_1$ . The excitation of the incoming generator No. 2 is changed till  $V$  reads zero. Then it means that its terminal voltage is the same as that of generator No. 1 or bus-bar voltage. After this, switch  $S_1$  is closed and so the incoming machine is paralleled to the system. Under these conditions, however, generator No. 2 is not taking any load, because its induced e.m.f. is the same as bus-bar voltage and there can be no flow of current between two points at the same potential. The generator is said to be 'floating' on the bus-bar. If generator No. 2 is to deliver any current, then its induced e.m.f.  $E$  should be greater than the bus-bar voltage  $V$ . In that case, current supplied by it is  $I = (E - V)/R_a$  where  $R_a$  is the resistance of the armature circuit. The induced e.m.f. of the incoming generator is increased by strengthening its field till it takes its proper share of load. At the same time, it may be found necessary to weaken the field of generator No. 1 to maintain the bus-bar voltage  $V$  constant.

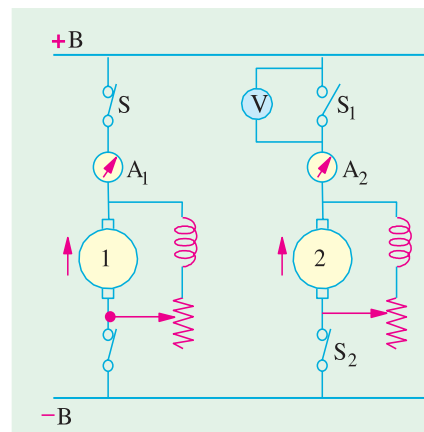


Fig. 27.18

**27.16. Load Sharing**

Because of their slightly drooping voltage characteristics, shunt generators are most suited for stable parallel operation. Their satisfactory operation is due to the fact that any tendency on the part of a generator to take more or less than its proper share of load results in certain changes of voltage in the system which immediately oppose this tendency thereby restoring the original division of load. Hence, once paralleled, they are automatically held in parallel.

Similarly, for taking a generator out of service, its field is weakened and that of the other generator is increased till the ammeter of the generator to be cleared reads zero. After that, its breaker and then the switch are opened thus removing the generator out of service. This method of connecting in and removing a generator from service helps in preventing any shock or sudden disturbance to the prime-mover or to the system itself.

It is obvious that if the field of one generator is weakened too much, then power will be delivered to it and it will run in its original direction as a motor, thus driving its prime-mover.

In Fig. 27.19. and 27.20 are shown the voltage characteristics of two shunt generators. It is seen that for a common terminal voltage  $V$ , the generator No. 1 delivers  $I_1$  amperes and generator No. 2,  $I_2$  amperes. It is seen that generator No. 1, having more drooping characteristic, delivers less current. It is found that two shunt generators will divide the load properly at all points if their characteristics are similar in form and each has the same voltage drop from no-load to full-load.

If it desired that two generators of different kW ratings automatically share a load in proportion to their ratings, then their external characteristics when plotted in terms of their percentage full-load currents (not actual currents) must be identical as shown in Fig. 27.21. If, for example, a 100-kW generator is working in parallel with a 200-kW generator to supply a total of 240-kW, then first generator will supply 80 kW and the other 160 kW.

When the individual characteristics of the generators are known, their combined characteristics can be drawn by adding the separate currents at a number of equal voltage (because generators are running in parallel). From this combined characteristic, the voltage for any combined load can be read off and from there, the current supplies by each generator can be found (Fig. 27.20).

If the generators have straight line characteristics, then the above result can be obtained by simple calculations instead of graphically.

Let us discuss the load sharing of two generators which have unequal no-load voltages.

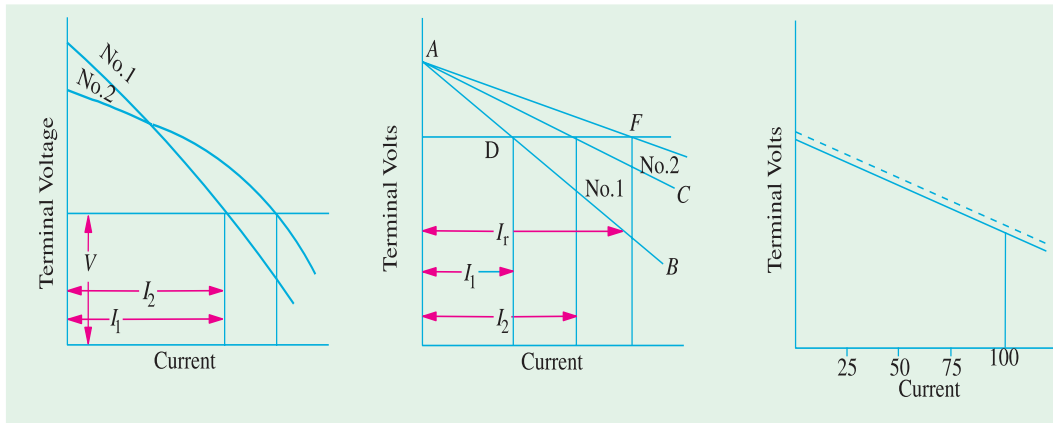


Fig. 27.19

Fig. 27.20

Fig. 27.21

Let  $E_1, E_2$  = no-load voltages of the two generators  
 $R_1, R_2$  = their armature resistances  
 $V$  = common terminal voltage

Then  $I_1 = \frac{E_1 - V}{R_1}$  and  $I_2 = \frac{E_2 - V}{R_2}$

$$\therefore \frac{I_2}{I_1} = \frac{E_2 - V}{E_1 - V} \cdot \frac{R_1}{R_2} = \frac{K_2 N_2 \Phi_2 - V}{K_1 N_1 \Phi_1 - V} \cdot \frac{R_1}{R_2}$$

From the above equation, it is clear that bus-bar voltage can be kept constant (and load can be transferred from 1 to 2) by increasing  $\Phi_2$  or  $N_2$  or by reducing  $N_1$  and  $\Phi_1$ .  $N_2$  and  $N_1$  are changed by changing the speed of driving engines and  $\Phi_1$  and  $\Phi_2$  are changed with the help of regulating shunt field resistances.

It should be kept in mind that

- (i) Two parallel shunt generators having equal no-load voltages share the load in such a ratio that the load current of each machine produces the same drop in each generator.
- (ii) In the case of two parallel generators having unequal no-load voltages, the load currents produce sufficient voltage drops in each so as to keep their terminal voltage the same.
- (iii) The generator with the least drop assumes greater share of the change in bus load.
- (iv) Paralleled generators with different power ratings but the same voltage regulation will divide any oncoming bus load in direct proportion to their respective power ratings (Ex. 27.14).

### 27.17. Procedure for Paralleling D.C. Generators

- (i) Close the disconnect switch of the incoming generator
- (ii) Start the prime-mover and adjust it to the rated speed of the machine
- (iii) Adjust the voltage of the incoming machine a few volts higher than the bus voltage
- (iv) Close the breaker of the incoming generator
- (v) Turn the shunt field rheostat of the incoming machine in the raise-voltage direction and that of the other machine(s) already connected to the bus in the lower-voltage direction till the desired load distribution (as indicated by the ammeters) is achieved.

### 27.18. Compound Generators in Parallel

In Fig. 27.22 are shown two compound generators (designated as No. 1 and No. 2) running in parallel. Because of the rising characteristics of the usual compounded generators, it is obvious that in the absence of any corrective devices, the parallel operation of such generators is unstable. Let us suppose that, to begin with, each generator is taking its proper share of load. Let us now assume that for some reason, generator No. 1 takes a slightly increased load. In that case, the current passing through its series winding increases which further strengthens its field and so raises its generated e.m.f. thus causing it to take still more load. Since the system load is assumed to be constant, generator No. 2 will drop some of its load, thereby weakening its series field which will result in its further dropping off its load. Since this effect is cumulative, Generator No. 1 will, therefore, tend to take the entire load and finally drive generator No. 2 as a motor. The circuit breaker of at least one of the two generators will open, thus stopping their parallel operation.

For making the parallel operation of over-compound and level-compound generators stable,\* they are always used with an equalizer bar (Fig. 27.22) connected to the armature ends of the series coils of the generators. The equalizer bar is a conductor of low resistance and its operation is as follows :

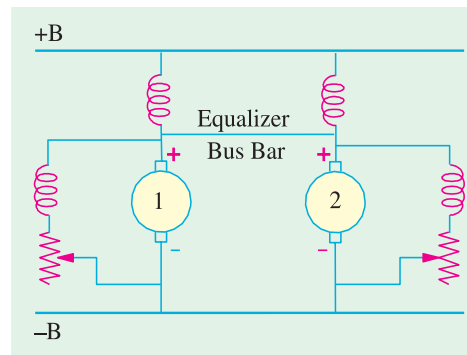


Fig. 27.22

\* Like shunt generators, the under-compound generators also do not need equalizers for satisfactory parallel operation.



Suppose that generator No. 1 starts taking more than its proper share of load. Its series field current is increased. But now this increased current passes partly through the series field coil of generator No. 1 and partly it flows via the equalizer bar through the series field winding of generator No. 2. Hence, the generators are affected in a similar manner with the result the generator No. 1 cannot take the entire load. For maintaining proper division of load from no-load to full-load, it is essential that

- (i) the regulation of each generator is the same.
- (ii) the series field resistances are inversely proportional to the generator rating.

### 27.19. Series Generators in Parallel

Fig 27.23 shows two identical series generators connected in parallel. Suppose  $E_1$  and  $E_2$  are initially equal, generators supply equal currents and have equal shunt resistances. Suppose  $E_1$  increases slightly so that  $E_1 > E_2$ . In that case,  $I_1$  becomes greater than  $I_2$ . Consequently, field of machine 1 is strengthened thus increasing  $E_1$  further whilst the field of machine 2 is weakened thus decreasing  $E_2$  further. A final stage is reached when machine 1 supplies not only the whole load but also supplies power to machine 2 which starts running as a motor.

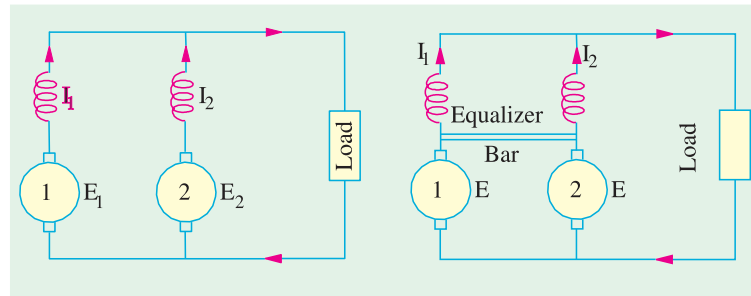


Fig. 27.23

Obviously, the two

machines will form a short-circuited loop and the current will rise indefinitely. This condition can be prevented by using equalizing bar because of which two similar machines pass approximately equal currents to the load, the slight difference between the two currents being confined to the loop made by the armatures and the equalizer bar.

**Example 27.14.** A 100-kW, 250-V generator is paralleled with a 300 kW, 250-V generator. Both generators have the same voltage regulation. The first generator is supplying a current of 200 A and the other 500 A. What would be the current supplied by each generator if an additional load of 600 A is connected to the bus ?

**Solution.** As explained in Art. 27.17, this additional load would be divided in direct proportion to the respective power ratings of the two generators.

$$\Delta I_1 = \left( \frac{100}{100 + 300} \right) \times 600 = 150 \text{ A ;}$$

$$\Delta I_2 = \left( \frac{300}{100 + 300} \right) \times 600 = 450 \text{ A}$$

**Example 27.15.** Two 220-V, d.c. generators, each having linear external characteristics, operate in parallel. One machine has a terminal voltage of 270 V on no-load and 220 V at a load current of 35 A, while the other has a voltage of 280 V at no-load and 220 V at 50 A. Calculate the output current of each machine and the bus-bar voltage when the total load is 60 A. What is the kW output of each machine under this condition ?

**Solution. Generator No. 1.**

Voltage drop for 35 A = 270 – 220 = 50 V

∴ Voltage drop/ampere = 50/35 = 10/7 V/A

**Generator No. 2**

Voltage drop/ampere =  $(280-220)/50 = 1.2 \text{ V/A}$

Let  $V =$  bus-bar voltage

$I_1 =$  current output of generator No.1

$I_2 =$  current output of generator No.2

then  $V = 270 - (10/7) I_1$  ...for generator No. 1  
 $= 280 - 1.2 I_2$  ...for generator No. 2

Since bus-bar voltage is the same.

$\therefore 270 - 10 I_1/7 = 280 - 1.2 I_2$  or  $4.2 I_2 - 5 I_1 = 35$  ...**(i)**

Also  $I_1 + I_2 = 60$  ...**(ii)**

Solving the two equations, we get  $I_1 = 23.6 \text{ A}; I_2 = 36.4 \text{ A}$

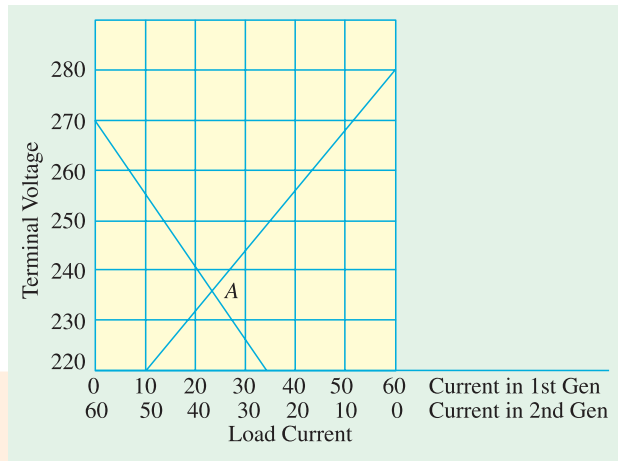
Now  $V = 280 - 1.2 I_2 = 280 - 1.2 \times 36.4$   
 $= 236.3 \text{ V}$

Output of 1st machine  
 $= 236.3 \times 23.6/1000$   
 $= 5.577 \text{ kW}$

Output of 2nd machine  
 $= 236.3 \times 36.4/1000$   
 $= 8.602 \text{ kW}$

**Graphical Solution.**

In Fig. 27.24, total load current of 60 A has been plotted along X-axis and the terminal voltage along Y-axis. The linear characteristics of the two generators are drawn from the given data. The common bus-bar voltage is given by the point of intersection of the two graphs. From the graph, it is seen that  $V = 236.3 \text{ V}; I_1 = 23.6 \text{ A}; I_2 = 36.4 \text{ A}$ .



**Fig. 27.24**

**Example 27.16.** Two shunt generators each with an armature resistance of  $20 \Omega$  and field resistance of  $0.01 \Omega$  run in parallel and supply a total load of 4000 A. The e.m.f.s are respectively 210 V and 220 V. Calculate the bus-bar voltage and output of each machine. **(Electrical Machines-1, South Gujarat Univ. 1988)**

**Solution.** Generators are shown in Fig. 27.25.

Let  $V =$  bus-bar voltage

$I_1 =$  output current of  $G_1$

$I_2 =$  output current of  $G_2$

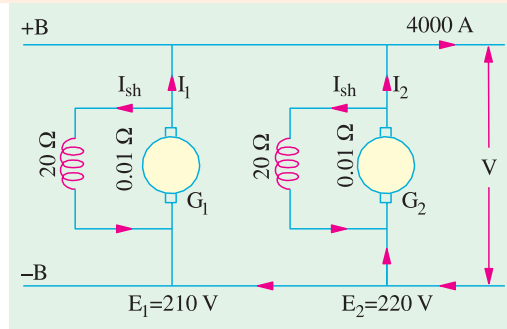
Now,  $I_1 + I_2 = 4000 \text{ A}, I_{sh} = V/20$ .

$I_{a1} = (I_1 + V/20); I_{a2} = (I_2 + V/20)$

In each machine,

$V +$  armature drop = induced e.m.f.

$\therefore V + I_{a1} R_a = E_1$



**Fig. 27.25**

$$\text{or } V + (I_1 + V/20) \times 0.01$$

$$= 210 \quad \dots 1\text{st machine}$$

$$\text{Also } V + I_{a2} R_a = E_2$$

$$\text{or } V + (I_2 + V/20) \times 0.01 = 220 \quad \dots 2\text{nd machine}$$

$$\text{Subtracting, we have } 0.01 (I_1 - I_2) = 10 \text{ or } I_1 - I_2 = 1000$$

$$\text{Also, } I_1 + I_2 = 4000 \text{ A } \therefore I_1 = \mathbf{2500 \text{ A}} ; I_2 = \mathbf{1500 \text{ A}}$$

Substituting the value of  $I_1$  above, we get

$$V + (2500 + V/20) \times 0.01 = 210 \quad \therefore V = \mathbf{184.9 \text{ V}}$$

$$\text{Output of 1st generator} = 184.9 \times 2500/1000 = \mathbf{462.25 \text{ kW}}$$

$$\text{Output of 2nd generator} = 184.9 \times 1500/1000 = \mathbf{277.35 \text{ kW}}$$

**Example 27.17.** Two shunt generators operating in parallel deliver a total current of 250 A. One of the generators is rated 50 kW and the other 100 kW. The voltage rating of both machine is 500 V and have regulations of 6 per cent (smaller one) and 4 per cent. Assuming linear characteristics, determine (a) the current delivered by each machine (b) terminal voltage.

(Elect. Machines, Nagpur Univ. 1991)

**Solution. 50 kW generator**

$$\text{F.L. voltage drop} = 500 \times 0.06 = 30 \text{ V} ; \text{F.L. current} = 50,000/500 = 100 \text{ A}$$

$$\text{Drop per ampere} = 30/100 = 3/10 \text{ V/A}$$

**100 kW generator**

$$\text{F.L. drop} = 500 \times 0.04 = 20 \text{ V} ; \text{F.L. current} = 100,000/5000 = 200 \text{ A}$$

$$\text{Drop per ampere} = 20/200 = 1/10 \text{ V/A}$$

If  $I_1$  and  $I_2$  are currents supplied by the two generators and  $V$  the terminal voltage, then

$$V = 500 - (3I_1/10) \quad \dots 1\text{st generator}$$

$$= 500 - (I_2/10) \quad \dots 2\text{nd generator}$$

$$\therefore 3I_1/10 = I_2/10 \text{ or } 3I_1 = I_2 ; \quad \text{Also } I_1 + I_2 = 250 \quad \dots \text{given}$$

$$\text{(a) Solving the above two equations, we get } I_1 = \mathbf{62.5 \text{ A}} ; I_2 = \mathbf{187.5 \text{ A}}$$

$$\text{(b) } V = 500 - (3 \times 62.5/10) = \mathbf{481.25 \text{ V}}$$

**Example 27.18.** Two shunt generators, each with a no-load voltage of 125 V are running parallel. Their external characteristics can be taken as straight lines over this operating ranges. Generator No. 1 is rated at 25 kW and its full-load voltage is 119 V, Generator No. 2 is rated at 200 kW at 116 V. Calculate the bus-bar voltage when the total load is 3500 A. How is the load divided between the two ?

(Elect. Machinery - I, Mysore Univ. 1988)

**Solution.** Let  $V$  = bus-bar voltage

$$x_1, x_2 = \text{load carried by each generator in terms of percentage of rated load}$$

$$P_1, P_2 = \text{load carried by each generator in watts}$$

$$V = 125 - [(125 - 119) (x_1/100)] \quad \dots \text{Generator No. 1}$$

$$V = 125 - [(125 - 116) (x_2/100)] \quad \dots \text{Generator No. 2}$$

$$\therefore 125 - \frac{6x_1}{100} = 125 - \frac{9x_2}{100} \quad x_2 = \frac{6x_1}{9} = \frac{2x_1}{3}$$

Since in d.c. circuits, power delivered is given by  $VI$  watt, the load on both generators is

$$\left(250 x_1 \times \frac{1000}{100}\right) + \left(200 x_2 \times \frac{1000}{100}\right) = V \times 3500$$

Now, replacing  $V$  and  $x_2$  by terms involving  $x_1$ , we get as a result

$$\left(250 x_1 \times \frac{1000}{100}\right) + \left(200 \times \frac{2x_1}{3} \times \frac{1000}{100}\right) = \left(125 - \frac{6x_1}{100}\right) \times 3500$$

$$x_1 = 108.2 \text{ per cent}$$

$$\therefore \text{Bus-bar voltage } V = 125 - (6 \times 108.2/100) = \mathbf{118.5 \text{ V}}$$

The division of load between the two generators can be found thus :

$$x_1 = \frac{P_1 \times 1000}{250,000} \quad \text{and} \quad x_2 = \frac{P_2 \times 1000}{200,000}$$

$$\therefore \frac{x_1}{x_2} = \frac{P_1 \times 200,000}{P_2 \times 250,000} = \frac{4P_1}{5P_2} = \frac{3}{2} \quad \therefore \frac{3}{2} = \frac{4P_1}{5P_2} = \frac{4VI_1}{5VI_2} = \frac{4I_1}{5I_2} \quad \dots(i)$$

$$\text{Since} \quad I_1 + I_2 = 3500 \quad \therefore I_2 = 3500 - I_1$$

$$\text{Hence (i) above becomes, } \frac{3}{2} = \frac{4I_1}{5(3500 - I_1)}$$

$$\therefore I_1 = \mathbf{2,283 \text{ A}} \quad \text{and} \quad I_2 = \mathbf{1,217 \text{ A}}$$

**Example 27.19.** Two shunt generators A & B operate in parallel and their load-characteristics may be taken as straight lines. The voltage of generator A falls from 240 V at no load to 220 V at 200 A, while that of B falls from 245 V at no load to 220 V to 150 A. Determine the currents supplied by each machine to a common load of 300 A and the bus-bar voltage.

(Bharathithasan Univ. April 1997)

**Solution.** Two graphs are plotted as shown in Fig. 27.26.

Their equations are :

$$240 - (20/200) I_A = 245 - (25/150) I_B$$

$$\text{Further, } I_A + I_B = 300$$

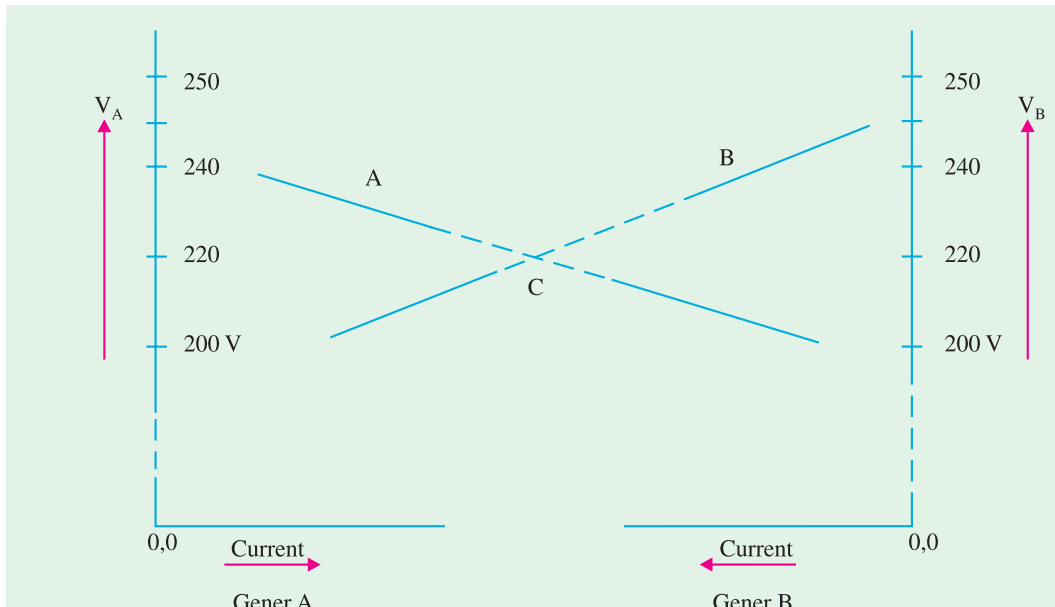


Fig. 27.26. Parallel operation of two D.C. Generators

This gives  $I_A = 168.75 \text{ A}$ ,  $I_B = 131.25 \text{ A}$

And common voltage of Bus-bar,  $V_{BUS}$

$$= 240 - (20/200) \times 168.75, \text{ or}$$

$$V_{BUS} = 245 - (25/150) \times 131.25 = 223.125 \text{ volts.}$$

It is represented by the point C, in graph, as an intersection, satisfying the condition that two currents ( $I_A$  and  $I_B$ ) add up to 300 amp.

**Example 27.20.** In a certain sub-station, there are 5 d.c. shunt generators in parallel, each having an armature resistance of  $0.1 \Omega$ , running at the same speed and excited to give equal induced e.m.f.s. Each generator supplies an equal share of a total load of 250 kW at a terminal voltage of 500 V into a load of fixed resistance. If the field current of one generator is raised by 4%, the others remaining unchanged, calculate the power output of each machine and their terminal voltages under these conditions. Assume that the speeds remain constant and flux is proportional to field current. (Elect. Technology, Allahabad Univ. 1991)

**Solution.** Generator connections are shown in Fig. 27.27.

Load supplied by each =  $250/5 = 50 \text{ kW}$

$\therefore$  Output of each =  $50,000/500 = 100 \text{ A}$

Terminal voltage of each = 500 V

Armature drop of each =  $0.1 \times 100 = 10 \text{ V}$

Hence, induced e.m.f. of each = 510 V

When field current of one is increased, its flux and hence its generated e.m.f. is increased by 4%. Now, 4% of 510 V = 20.4 V

$\therefore$  Induced e.m.f. of one =  $510 + 20.4 = 530.4 \text{ V}$

Let  $I_1$  = current supplied by one generator after increased excitation

$I_2$  = current supplied by each of the other 4 generators

$V$  = new terminal or bus-bar voltage

$$\therefore 530.4 - 0.1 I_1 = V \quad \dots(i)$$

$$510 - 0.1 I_2 = V \quad \dots(ii)$$

Now, fixed resistance of load =  $500/500 = 1 \Omega$ ; Total load current =  $I_1 + 4I_2$

$$\therefore 1 \times (I_1 + 4I_2) = V \text{ or } I_1 + 4I_2 = V \quad \dots(iii)$$

Subtracting (ii) from (i), we get,  $I_1 - I_2 = 20.4$  ... (iv)

Subtracting (iii) from (ii), we have  $I_1 + 4.1 I_2 = 510$  ... (v)

From (iv) and (v), we get  $I_2 = 3060/51 = 59.99 = 60 \text{ A (approx.)}$

From (iv)  $I_1 = 20.4 + 60 = 264 \text{ A}$

From (iii)  $V = 264 + 240 = 504 \text{ Volt}$

Output of 1st machine =  $504 \times 264 \text{ watt} = 133 \text{ kW}$

Output of each of other four generators =  $504 \times 60 \text{ W} = 30.24 \text{ kW}$

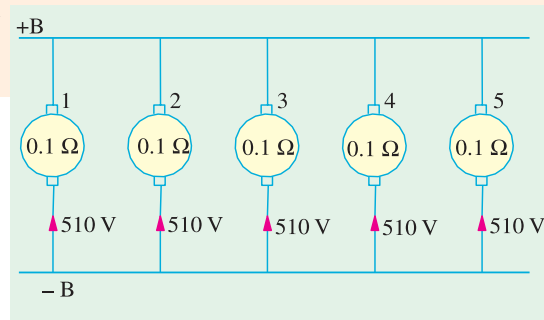


Fig. 27.27

**Example 27.21.** Two d.c. generators are connected in parallel to supply a load of 1500 A. One generator has an armature resistance of  $0.5 \Omega$  and an e.m.f. of 400 V while the other has an armature resistance of  $0.04 \Omega$  and an e.m.f. of 440 V. The resistances of shunt fields are  $100 \Omega$  and  $80 \Omega$  respectively. Calculate the currents  $I_1$  and  $I_2$  supplied by individual generator and terminal voltage  $V$  of the combination. (Power Apparatus-I, Delhi Univ. Dec. 1987)

**Solution.** Generator connection diagram is shown in Fig. 27.28.

Let  $V$  = bus-bar voltage

$I_1$  = output current of one generator

$$I_2 = \text{output current of other generator} \\ = (1500 - I_1)$$

$$\text{Now, } I_{sh1} = V/100 \text{ A ; } I_{sh2} = V/80 \text{ A}$$

$$I_{a1} = \left( I_1 + \frac{V}{100} \right) \text{ and } I_{a2} = \left( I_2 + \frac{V}{80} \right)$$

$$\text{or } I_{a2} = \left( 1500 - I_1 + \frac{V}{80} \right)$$

For each machine

$$E - \text{armature drop} = V \\ \therefore 400 - \left( I_1 + \frac{V}{100} \right) \times 0.5 = V$$

$$\text{or } 400 - 0.5 I_1 - 0.005 V = V \text{ or } 0.5 I_1 = 400 - 1.0005 V \quad \dots(i)$$

$$\text{Also } 440 - (1500 - I_1 + \frac{V}{80}) \times 0.04 = V \text{ or } 0.04 I_1 = 1.0005 V - 380 \quad \dots(ii)$$

Dividing Eq. (i) by (ii), we get

$$\frac{0.5 I_1}{0.04 I_2} = \frac{400 - 1.0005 V}{1.0005 V - 380} \therefore V = \mathbf{381.2 \text{ V}}$$

Substituting this value of  $V$  in Eq. (i), we get  $0.5 I_1 = 400 - 1.0005 \times 381.2$

$$\therefore I_1 = \mathbf{33.8 \text{ A}} ; I_2 = 1500 - 33.8 = \mathbf{1466.2 \text{ A}}$$

$$\text{Output of 1st generator} = 381.2 \times 33.8 \times 10^{-3} = \mathbf{12.88 \text{ kW}}$$

$$\text{Output of 2nd generator} = 381.2 \times 1466.2 \times 10^{-3} = \mathbf{558.9 \text{ kW}}$$

**Example 27.22.** Two shunt generators and a battery are working in parallel. The open circuit voltage, armature and field resistances of generators are 250 V, 0.24  $\Omega$ , 100  $\Omega$  are 248 V, 0.12  $\Omega$  and 100  $\Omega$  respectively. If the generators supply the same current when the load on the bus-bars is 40 A, calculate the e.m.f. of the battery if its internal resistance is 0.172  $\Omega$ .

**Solution.** Parallel combination is shown in Fig. 27.29.

Values of currents and induced e.m.fs. are shown in the diagram.

$$V + \left( I + \frac{V}{100} \right) \times 0.24 = 250 \quad \dots(i)$$

$$V + \left( I + \frac{V}{100} \right) \times 0.12 = 248 \quad \dots(ii)$$

$$\text{Also } I + I + I_b = 40$$

$$I_b + 2I = 40 \quad \dots(iii)$$

$$\text{Subtracting (iii) from (i), we get } \left( I + \frac{V}{100} \right) \times 0.12 = 2 \quad \dots(iv)$$

Putting this value in (ii) above,  $V = 246$  volt.

$$\text{Putting this value of } V \text{ in (iv), } \left( I + \frac{246}{100} \right) \times 0.12 = 2 \quad \dots(v)$$

$$\therefore I = 50/3 - 2.46 = 14.2 \text{ A}$$

From (iii), we have  $I_b = 40 - (2 \times 14.2) = 11.6 \text{ A}$

$$\text{Internal voltage drop in battery} = 11.6 \times 0.172 = 2 \text{ V } \therefore E_b = 246 + 2 = \mathbf{248 \text{ V}}$$

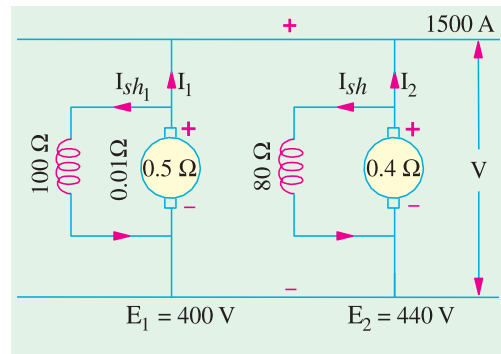


Fig. 27.28

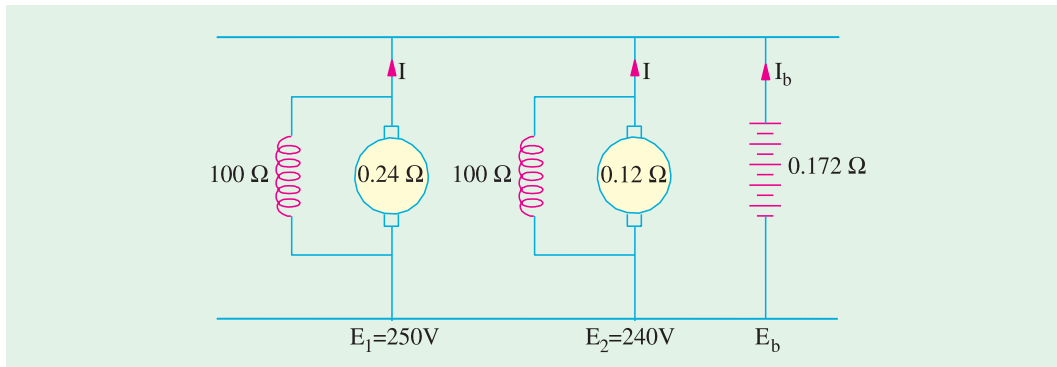


Fig. 27.29

**Example 27.23.** Two d.c. generators A and B are connected to a common load. A had a constant e.m.f. of 400 V and internal resistance of 0.25  $\Omega$  while B has a constant e.m.f. of 410 V and an internal resistance of 0.4  $\Omega$ . Calculate the current and power output from each generator if the load voltage is 390 V. What would be the current and power from each and the terminal voltage if the load was open-circuited ?  
(Elect. Engg; I, Bangalore Univ. 1987)

**Solution.** The generator connections are shown in Fig. 27.30 (a).

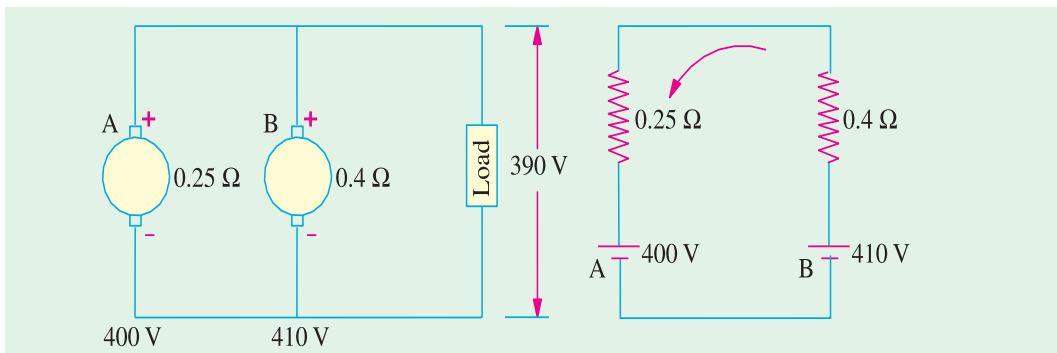


Fig. 27.30

Since the terminal or output voltage is 390 V, hence

$$\text{Load supplied by A} = (400 - 390)/0.25 = \mathbf{40 \text{ A}}$$

$$\text{Load supplied by B} = (410 - 390)/0.4 = \mathbf{50 \text{ A}}$$

$$\therefore \text{Power output from A} = 40 \times 390 = \mathbf{15.6 \text{ kW}}$$

$$\text{Power output from B} = 50 \times 390 = \mathbf{19.5 \text{ kW}}$$

If the load is open-circuited as shown in Fig. 27.30.(b), then the two generators are put in series with each other and a circulatory current is set up between them.

$$\text{Net voltage in the circuit} = 410 - 400 = 10 \text{ V}$$

$$\text{Total resistance} = 0.4 + 0.25 = 0.65 \Omega$$

$$\therefore \text{circulatory current} = 10/0.65 = 15.4 \text{ A}$$

$$\text{The terminal voltage} = 400 + (15.4 \times 0.25) = \mathbf{403.8 \text{ V}}$$

Obviously, machine B with a higher e.m.f. acts as a generator and drives machine A as a motor.

$$\text{Power taken by A from B} = 403.8 \times 15.4 = \mathbf{6,219 \text{ W}}$$

Part of this appears as mechanical output and the rest is dissipated as armature Cu loss.

Mechanical output =  $400 \times 15.4 = 6.16 \text{ kW}$  ; Armature Cu loss =  $3.8 \times 15.4 = 59 \text{ W}$

Power supplied by B to A =  $6,219 \text{ W}$  ; Armature Cu loss =  $6.16 \times 15.4 = 95 \text{ W}$

**Example 27.24.** Two compound generators A and B, fitted with an equalizing bar, supply a total load current of 500 A. The data regarding the machines are :

	A	B
Armature resistance (ohm)	0.01	0.02
Series field winding (ohm)	0.004	0.006
Generated e.m.fs. (volt)	240	244

Calculate (a) current in each armature (b) current in each series winding (c) the current flowing in the equalizer bar and (d) the bus-bar voltage. Shunt currents may be neglected.

**Solution.** The two generators (with their shunt windings omitted) are shown in Fig. 27.31.

Let  $V$  = bus-bar voltage ;  $v$  = voltage between equalizer bus-bar and the negative

tors  
 $i_1, i_2$  = armature currents of the two generators

$$\text{Now, } i_1 + i_2 = 500$$

$$\text{or } \frac{240-v}{0.01} + \frac{244-v}{0.02} = 500$$

Multiplying both sides by  $\frac{1}{100}$  we get

$$240 - v + 122 - (v/2) = 500$$

$$\therefore v = 238 \text{ volts}$$

$$(a) \therefore i_1 = \frac{240-238}{0.01} = 200 \text{ A}$$

$$i_2 = \frac{244-238}{0.02} = 300 \text{ A}$$

(b) The total current of 500 A divides between the series windings in the inverse ratio of their resistance i.e. in the ratio of  $\frac{1}{0.004} : \frac{1}{0.006}$  or in the ratio 3 : 2.

Hence, current in the series winding of generator A =  $500 \times \frac{3}{5} = 300 \text{ A}$

Similarly, current in the series winding of generator B =  $500 \times \frac{2}{5} = 200 \text{ A}$

(c) It is obvious that a current of 100 A flows in the equalizing bar from C to D. It is so because the armature current of generator A is 200 A only. It means that 100 A comes from the armature of generator, B, thus making 300 A for the series field winding of generator A.

(d)  $V = v - \text{voltage drop in one series winding} = 238 - (300 \times 0.004) = 236.8 \text{ V}$

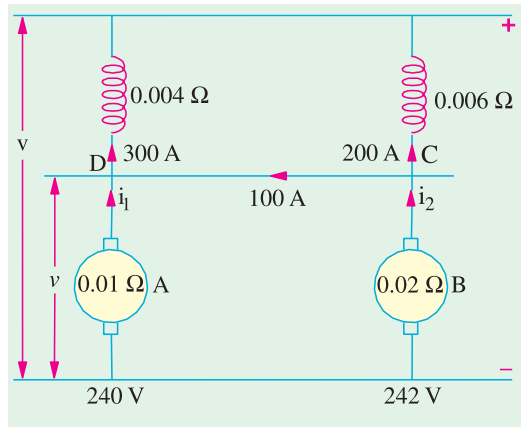


Fig. 27.31

### Tutorial Problem No. 27.2

1. Two separately-excited d.c. generators are connected in parallel supply a load of 200 A. The machines have armature circuit resistances of  $0.05 \Omega$  and  $0.1 \Omega$  and induced e.m.fs. 425 V and 440 V respectively. Determine the terminal voltage, current and power output of each machine. The effect of armature reaction is to be neglected. **(423.3 V ; 33.3 A ; 14.1 kW ; 166.7 A ; 70.6 kW)**

2. Two shunt generators operating in parallel given a total output of 600 A. One machine has an armature resistance of  $0.02 \Omega$  and a generated voltage of 455 V and the other an armature resistance of  $0.025 \Omega$  and a generated voltage of 460 V. Calculate the terminal voltage and the kilowatt output of each machine. Neglect field currents. **(450.56 V ; 100 kW ; 170.2 kW)**



3. The external characteristics of two d.c. shunt generators *A* and *B* are straight lines over the working range between no-load and full-load.

	<i>Generator A</i>		<i>Generator B</i>	
	<i>No-load</i>	<i>Full-load</i>	<i>No-load</i>	<i>Full-load</i>
Terminal P.D. (V)	400	360	420	370
Load current (A)	0	80	0	70

Determine the common terminal voltage and output current of each generator when sharing a total load of 100 A.

(57.7 A ; 42.3 A ; 378.8 V)

4. Two shunt generators operating in parallel have each an armature resistance of  $0.02 \Omega$ . The combined external load current is 2500 A. If the generated e.m.fs. of the machines are 560 V and 550 V respectively, calculate the bus-bar voltage and output in kW of each machine. (530 V ; 795 kW ; 530 kW)

5. Two shunt generators *A* and *B* operate in parallel and their load characteristics may be taken as straight lines. The voltage of *A* falls from 240 V at no-load to 220 V at 200 A, while that of *B* falls from 245 V at no-load to 220 V at 150 A. Determine the current which each machine supplies to a common load of 300 A and the bus-bar voltage at this load.

(169 A ; 131 A ; 223.1 V)

6. Two shunt-wound d.c. generators are connected in parallel to supply a load of 5,000 A. Each machine has an armature resistance of  $0.03 \Omega$  and a field resistance of  $60 \Omega$ , but the e.m.f. of one machine is 600 V and that of the other is 640 V. What power does each machine supply ?

(1,004 kW ; 1,730 kW including the fields)

7. Two shunt generators running in parallel share a load of 100 kW equally at a terminal voltage of 230 V. On no-load, their voltages rise to 240 V and 245 V respectively. Assuming that their volt-ampere characteristics are rectilinear, find how would they share the load when the total current is reduced to half its original value ? Also, find the new terminal voltage.

(20 kW ; 30 kW, 236 V)

8. Two generators, each having no-load voltage of 500 V, are connected in parallel to a constant resistance load consuming 400 kW. The terminal p.d. of one machine falls linearly to 470 V as the load is increased to 850 A while that of the other falls linearly to 460 V when the load is 600 A. Find the load current and voltage of each generator.

If the induced e.m.f. of one machine is increased to share load equally, find the new current and voltage.

( $I_1 = 626 \text{ A}$  ;  $I_2 = 313 \text{ A}$  ;  $V = 479 \text{ V}$  ;  $I = 469.5 \text{ A}$  ;  $V = 484.4 \text{ V}$ )

9. Estimate the number of turns needed on each interpole of a 6-pole generator delivering 200 kW at 200 V ; given : number of lap-connected armature conductors = 540 ; interpole air gap = 1.0 cm ; flux-density in interpole air-gap =  $0.3 \text{ Wb/m}^2$ . Ignore the effect of iron parts of the circuit and of leakage.

[10] (*Electrical Machines, B.H.U. 1980*)

## OBJECTIVE TEST – 27

- In d.c. generators, armature reaction is produced actually by
  - its field current
  - armature conductors
  - field pole winding
  - load current in armature
- In a d.c. generator, the effect of armature reaction on the main pole flux is to
  - reduce it
  - distort it
  - reverse it
  - both (a) and (b)
- In a clockwise-rotating loaded d.c. generator, brushes have to be shifted
  - clockwise
  - counterclockwise
  - either (a) or (b)
  - neither (a) nor (b).
- The primary reason for providing compensating windings in a d.c. generator is to
  - compensate for decrease in main flux
  - neutralize armature mmf
  - neutralize cross-magnetising flux
  - maintain uniform flux distribution.
- The main function of interpoles is to minimize ..... between the brushes and the commutator when the d.c. machine is loaded.
  - friction
  - sparking
  - current
  - wear and tear

6. In a 6-pole d.c. machine, 90 mechanical degrees correspond to ..... electrical degrees.  
 (a) 30 (b) 180  
 (c) 45 (d) 270
7. The most likely cause(s) of sparking at the brushes in a d.c. machine is /are  
 (a) open coil in the armature  
 (b) defective interpoles  
 (c) incorrect brush spring pressure  
 (d) all of the above
8. In a 10-pole, lap-wound d.c. generator, the number of active armature conductors per pole is 50. The number of compensating conductors per pole required is  
 (a) 5 (b) 50  
 (c) 500 (d) 10
9. The commutation process in a d.c. generator basically involves  
 (a) passage of current from moving armature to a stationary load  
 (b) reversal of current in an armature coil as it crosses MNA  
 (c) conversion of a.c. to d.c.  
 (d) suppression of reactance voltage
10. Point out the WRONG statement. In d.c. generators, commutation can be improved by  
 (a) using interpoles  
 (b) using carbon brushes in place of Cu brushes  
 (c) shifting brush axis in the direction of armature rotation  
 (d) none of the above
11. Each of the following statements regarding interpoles is true **except**  
 (a) they are small yoke-fixed poles spaced in between the main poles  
 (b) they are connected in parallel with the armature so that they carry part of the armature current  
 (c) their polarity, in the case of generators is the same as that of the main pole ahead  
 (d) they automatically neutralize not only reactance voltage but cross-magnetisation as well
12. Shunt generators are most suited for stable parallel operation because of their voltage characteristics.  
 (a) identical (b) dropping  
 (c) linear (d) rising
13. Two parallel shunt generators will divide the total load equally in proportion to their kilowatt output ratings only when they have the same  
 (a) rated voltage  
 (b) voltage regulation  
 (c) internal  $I_a R_a$  drops  
 (d) both (a) and (b)
14. The main function of an equalizer bar is to make the parallel operation of two over-compounded d.c. generators  
 (a) stable (b) possible  
 (c) regular (d) smooth
15. The *essential* condition for stable parallel operation of two d.c. generators having similar characteristics is that they should have  
 (a) same kilowatt output ratings  
 (b) dropping voltage characteristics  
 (c) same percentage regulation  
 (d) same no-load and full-load speed
16. The main factor which leads to unstable parallel operation of flat-and over-compound d.c. generators is  
 (a) unequal number of turns in their series field windings  
 (b) unequal series field resistances  
 (c) their rising voltage characteristics  
 (d) unequal speed regulation of their prime movers
17. The simplest way to shift load from one d.c. shunt generator running in parallel with another is to  
 (a) adjust their field rheostats  
 (b) insert resistance in their armature circuits  
 (c) adjust speeds of their prime movers  
 (d) use equalizer connections
18. Which one of the following types of generators does NOT need equalizers for satisfactory parallel operation ?  
 (a) series (b) over-compound  
 (c) flat-compound (d) under-compound.

## ANSWERS

1. (d) 2. (d) 3. (a) 4. (c) 5. (b) 6. (d) 7. (d) 8. (a) 9. (b) 10. (d) 11. (b)  
 12. (b) 13. (d) 14. (a) 15. (b) 16. (c) 17. (a) 18. (d)

# CHAPTER 28

## Learning Objectives

- Characteristics of D.C. Generators
- Separately-excited Generator
- No-load Curve for Self-excited Generator
- How to find Critical Resistance  $R_c$ ?
- How to draw O.C.C. at Different Speeds?
- Critical Speed
- Voltage Build up of a Shunt Generator
- Conditions for Build-up of a Shunt Generator
- Other factors Affecting Voltage Building of a D.C. Generator
- External Characteristic
- Voltage Regulation
- Internal or Total Characteristic
- Series Generator
- Compound-wound Generator
- How to calculate Required Series Turns?
- Uses of D.C. Generators

## GENERATOR CHARACTERISTICS



**C** Generator characteristics gives the relation between terminal voltage and load current. It is of great importance in judging the suitability of a generator for a particular purpose

### 28.1. Characteristics of D.C. Generators

Following are the three *most* important characteristics or curves of a d.c. generator :

#### 1. No-load saturation Characteristic ( $E_0/I_f$ )

It is also known as Magnetic Characteristic or Open-circuit Characteristic (*O.C.C.*). It shows the relation between the no-load generated e.m.f. in armature,  $E_0$  and the field or exciting current  $I_f$  at a given fixed speed. It is just the magnetisation curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately-excited or self-excited.

#### 2. Internal or Total Characteristic ( $E/I_a$ )

It gives the relation between the e.m.f.  $E$  actually induces in the armature (after allowing for the demagnetising effect of armature reaction) and the armature current  $I_a$ . This characteristic is of interest mainly to the designer and can be obtained as explained in Art. 28.12.

#### 3. External Characteristic ( $V/I$ )

It is also referred to as performance characteristic or sometimes voltage-regulating curve.

It gives relation between that terminal voltage  $V$  and the load current  $I$ . This curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The values of  $V$  are obtained by subtracting  $I_a R_a$  from corresponding values of  $E$ . This characteristic is of great importance in judging the suitability of a generator for a particular purpose. It may be obtained in two ways (i) by making simultaneous measurements with a suitable voltmeter and an ammeter on a loaded generator (Art. 28.10) or (ii) graphically from the *O.C.C.* provided the armature and field resistances are known and also if the demagnetising effect (under rated load conditions) or the armature reaction (from the short-circuit test) is known.

### 28.2. Separately-excited Generator

#### (a) (i) No-load Saturation Characteristic ( $E_0/I_f$ )

The arrangement for obtaining the necessary data to plot this curve is shown in Fig. 28.1. The exciting or field current  $I_f$  is obtained from an external independent d.c. source. It can be varied from zero upwards by a potentiometer and its value read by an ammeter  $A$  connected in the field circuit as shown.

Now, the voltage equation of a d.c. generator is,  $E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right)$  volt

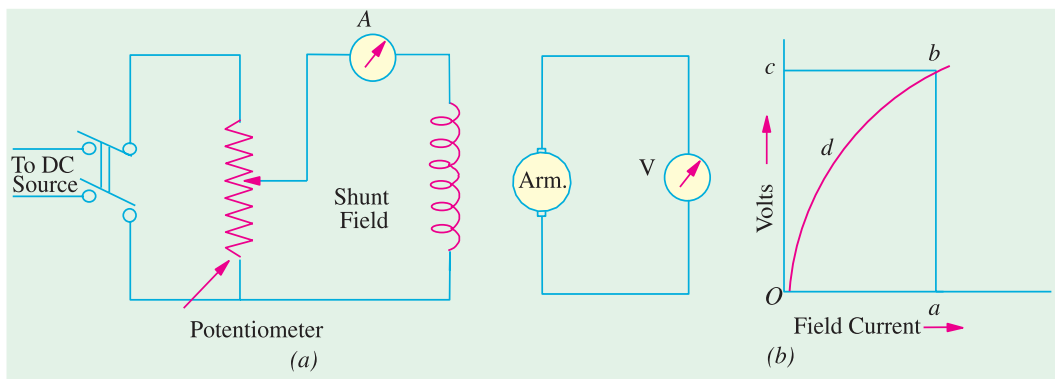


Fig. 28.1

Hence, if speed is constant, the above relation becomes  $E = k\Phi$

It is obvious that when  $I_f$  is increased from its initial small value, the flux  $\Phi$  and hence generated e.m.f.  $E_g$  increase directly as current so long as the poles are unsaturated. This is represented by the straight portion  $Od$  in Fig. 28.1 (b). But as the flux density increases, the poles become saturated, so a greater increase in  $I_f$  is required to produce a given increase in voltage than on the lower part of the curve. That is why the upper portion  $db$  of the curve  $Odb$  bends over as shown.

**(ii) Load Saturation Curve ( $V/I_f$ )**

The curve showing relation between the terminal voltage  $V$  and field current  $I_f$  when the generator is loaded, is known as Load Saturation Curve.

The curve can be deduced from the no-load saturation curve provided the values of armature reaction and armature resistance are known. While considering this curve, account is taken of the

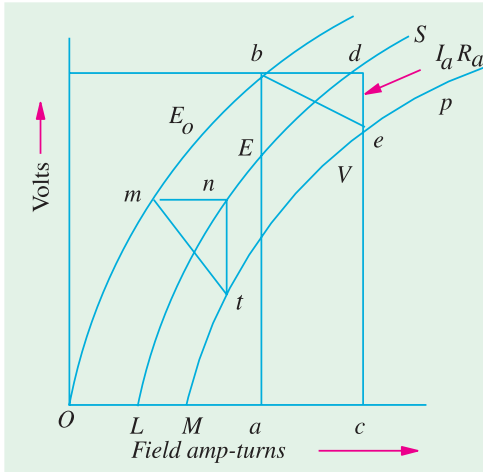


Fig. 28.2

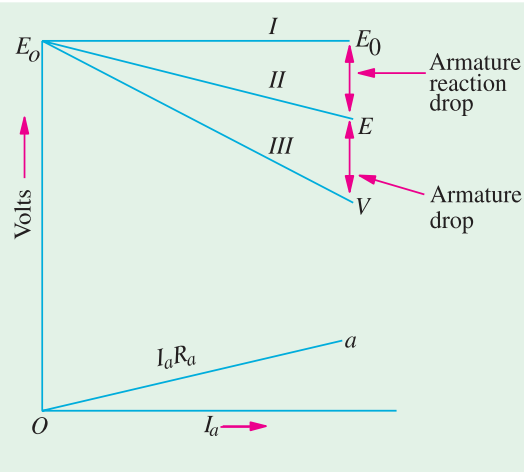


Fig. 28.3

demagnetising effect of armature reaction and the voltage drop in armature which are practically absent under no-load conditions. The no-load saturation curve of Fig. 28.1 has been repeated in Fig. 28.2 on a base of field amp-turns (and not current) and it is seen that at no-load, the field amp-turns required for rated no-load voltage are given by  $Oa$ . Under load conditions, the voltage will decrease due to demagnetising effect of armature reaction. This decrease can be made up by suitably increasing the field amp-turns. Let  $ac$  represent the equivalent demagnetising amp-turns per pole. Then, it means that in order to generate the same e.m.f. on load as at no-load, the field amp-turns/pole must be increased by an amount  $ac = bd$ . The point  $d$  lies on the curve  $LS$  which shows relation between the voltage  $E$  generated under **load** conditions and the field amp-turns. The curve  $LS$  is practically parallel to curve  $Ob$ . The terminal voltage  $V$  will be less than this generated voltage  $E$  by an amount  $= I_a R_a$  where  $R_a$  is the resistance of the armature circuit. From point  $d$ , a vertical line  $de = I_a R_a$  is drawn. The point  $e$  lies on the full-load saturation curve for the generator. Similarly, other points are obtained in the same manner and the full-load saturation curve  $Mp$  is drawn. The right-angled triangle  $bde$  is known as **drop reaction triangle**. Load saturation curve for half-load can be obtained by joining the mid-points of such lines as  $mn$  and  $bd$  etc. In the case of self-excited generators, load saturation curves are obtained in a similar way.

**(b) Internal and External Characteristics**

Let us consider a separately-excited generator giving its rated no-load voltage of  $E_0$  for a certain constant field current. If there were no armature reaction and armature voltage drop, then this voltage would have remained constant as shown in Fig. 28.3. by the dotted horizontal line  $I$ . But when the generator is loaded, the voltage falls due to these two causes, thereby giving slightly dropping characteristics. If we subtract from  $E_0$  the values of voltage drops due to armature reaction for different loads, then we get the value of  $E$ —the e.m.f. **actually** induced in the armature under load conditions. Curve  $II$  is plotted in this way and is known as the **internal characteristic**. The straight line  $Oa$  represents the  $I_a R_a$  drops corresponding to different armature currents. If we subtract from  $E$  the armature drop  $I_a R_a$ , we get terminal voltage  $V$ . Curve  $III$  represents the external characteristic and is obtained by subtracting ordinates the line  $Oa$  from those of curve  $II$ .

### 28.3. No-load Curve for Self-excited Generator

The *O.C.C.* or no-load saturated curves for self-excited generators whether shunt or series-connected, are obtained in a similar way.

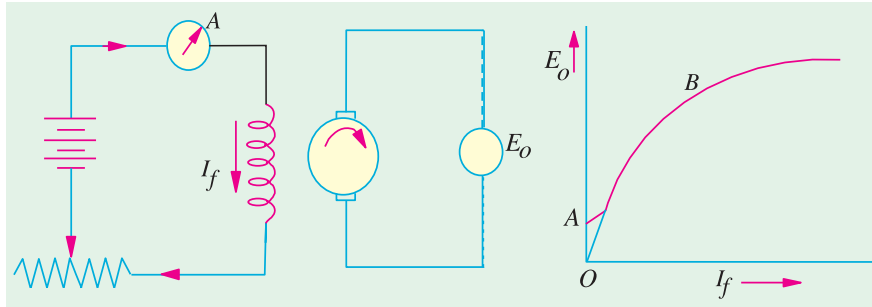


Fig. 28.4

Fig. 28.5

The field winding of the

generator (whether *shunt* or *series* wound) is disconnected from the machine and connected to an external source of direct current as shown in Fig. 28.4. The field or exciting current  $I_f$  is varied rheostatically and its value read on the ammeter *A*. The machine is driven at constant speed by the prime mover and the generated e.m.f. on no-load is measured by the voltmeter connected across the armature.  $I_f$  is increased by suitable steps (starting from zero) and the corresponding values of  $E_o$  are measured. On plotting the relation between  $I_f$  and  $E_o$ , a curve of this form shown in Fig. 28.5 is obtained.



Self Excited Generator

Due to residual magnetism in the poles, some e.m.f. (=  $OA$ ) is generated even when  $I_f = 0$ . Hence, the curve starts a little way up. The slight curvature at the lower end is due to magnetic inertia. It is seen that the first part of the curve is practically straight. This is due to the fact that at low flux densities, reluctance of iron path being negligible (due to high permeability), total reluctance is given by the air-gap reluctance which is constant. Hence, the flux and consequently, the generated e.m.f. is directly proportional to the exciting current. However, at high flux densities, where  $\mu$  is small, iron path

reluctance becomes appreciable and straight relation between  $E$  and  $I_f$  no longer holds good. In other words, after point *B*, saturation of poles starts. However, the initial slope of the curve is determined by air-gap width.

It should be noted that *O.C.C.* for a higher speed would lie above this curve and for a lower speed, would lie below it.

It should also be noted and the load-saturation curve for a shunt generator is similar to the one shown in Fig. 28.2.

#### Critical Resistance for Shunt Generator

Now, connect the field windings back to the armature and run the machine as a shunt generator. Due to residual magnetism in the poles, some e.m.f. and hence current, would be generated. This current while passing through the field coils will strengthen the magnetism of the poles (provided field coils are properly connected as regards polarity). This will increase the pole flux which will further increase the generated e.m.f. Increased e.m.f. means more current which further increases the flux and so on. This mutual reinforcement of e.m.f. and flux proceeds on till equilibrium is reached at some point like *P* (Fig. 28.6). The point lies on the resistance line *OA* of the field winding. Let  $R$  be the resistance of the field winding. Line *OA* is drawn such that its slope equals the field winding

resistance *i.e.* every point on this curve is such that volt/ampere =  $R$ .

The voltage  $OL$  corresponding to point  $P$  represents the maximum voltage to which the machine will build up with  $R$  as field resistance.  $OB$  represents smaller resistance and the corresponding voltage  $OM$  is slightly greater than  $OL$ . If field resistance is increased, then slope of the resistance line increased, and hence the maximum voltage to which the generator will build up at a given speed, decreases. If  $R$  is increased so much that the resistance line does not cut the *O.C.C.* at all (like  $OT$ ), then obviously the machine will fail to excite *i.e.* there will be no 'build up' of the voltage. If the resistance line just lies along the slope, then with that value of field resistance, the machine will *just* excite. The value of the resistance represented by the tangent to the curve, is known as **critical resistance**  $R_c$  for a *given speed*.

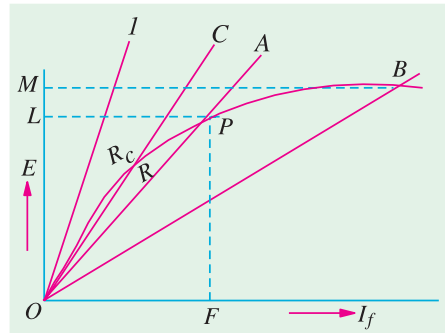


Fig. 28.6

**28.4 How to Find Critical Resistance  $R_c$  ?**

First, *O.C.C.* is plotted from the given data. Then, tangent is drawn to its initial portion. The slope of this curve gives the critical resistance for the speed at which the data was obtained.

**28.5 How to Draw *O.C.C.* at Different Speeds ?**

Suppose we are given the data for *O.C.C.* of a generator run at a fixed speed, say,  $N_1$ . It will be shown that *O.C.C.* at any other constant speed  $N_2$  can be deduced from the *O.C.C.* for  $N_1$ . In Fig. 28.7 the *O.C.C.* for speed  $N_1$  is shown.

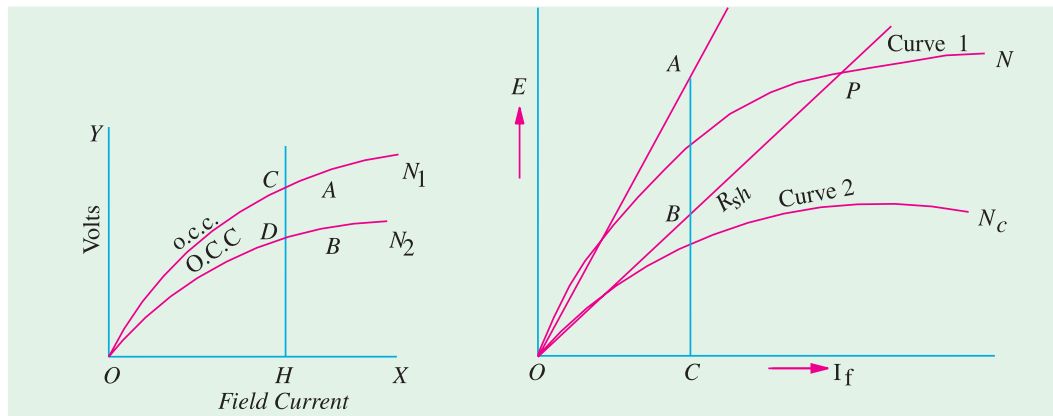


Fig. 28.7

Fig. 28.8

Since  $E \propto N$  for any fixed excitation, hence  $\frac{E_2}{E_1} = \frac{N_2}{N_1}$  or  $E_2 = E_1 \times \frac{N_2}{N_1}$

As seen, for  $I_f = OH$ ,  $E_1 = HC$ . The value of new voltage for the same  $I_f$  but at  $N_2$

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

In this way, point  $D$  is located. In a similar way, other such points can be found and the new *O.C.C.* at  $N_2$  drawn.

28.6. Critical Speed  $N_c$

Critical speed of a shunt generator is that speed for which the given shunt field resistance represents critical resistance. In Fig. 28.8, curve 2 corresponds to critical speed because  $R_{sh}$  line is tangential to it. Obviously

$$\frac{BC}{AC} = \frac{N_c}{\text{Full Speed}} = \frac{N_c}{N} \therefore N_c = \frac{BC}{AC} \times \text{Full speed } N$$

**Example 28.1.** The magnetization curve of a d.c. shunt generator at 1500 r.p.m. is :

$I_f$ (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_0$ (V) :	6	60	120	172.5	202.5	221	231	237	240

For this generator find (i) no load e.m.f. for a total shunt field resistance of  $100 \Omega$  (ii) the critical field resistance at 1500 r.p.m. and (iii) the magnetization curve at 1200 r.p.m. and therefrom the open-circuit voltage for a field resistance of  $100 \Omega$ .

(b) A long shunt, compound generator fitted with interpoles is cumulatively-compounded. With the supply terminals unchanged, the machine is now run as compound motor. Is the motor differentially or cumulatively compounded ? Explain. (Elect, Machines, A.M.I.E. Sec. B, 1990)

**Solution.** The magnetisation curve at 1500 r.p.m. is plotted in Fig. 28.9 from the given data. The  $100 \Omega$  resistance line  $OA$  is obtained by joining the origin (0, 0) with the point (1A, 100 V). The voltage corresponding to point A is 227.5 V. Hence, no-load voltage to which the generator will build-up is 227.5 V.

The tangent  $OT$  represents the critical resistance at 1500 r.p.m. considering point B,  $R_c = 225/1.5 = 150 \Omega$ .

For 1200 r.p.m., the induced voltages for different field currents would be  $(1200/1500) = 0.8$  of those for 1500 r.p.m. The values of these voltages are tabulated below :

$I_f$ (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_0$ (V) :	4.8	48	96	138	162	176.8	184.8	189.6	192

The new magnetisation curve is also plotted in Fig. 28.9. The  $100 \Omega$  line cuts the curve at point C which corresponds to an induced voltage of 166 V.

**Example 28.2.** A shunt generator is to be converted into a level compounded generator by the addition of a series field winding. From a test on the machine with shunt excitation only, it is found that the shunt current is 3.1 A to give 400 V on no load and 4.8 A to give the same voltage when the machine is supplying its full load of 200 A. The shunt winding has 1200 turns per pole. Find the number of series turns required per pole. (Elect. Machines, A.M.I.E. Sec. B, 1989)

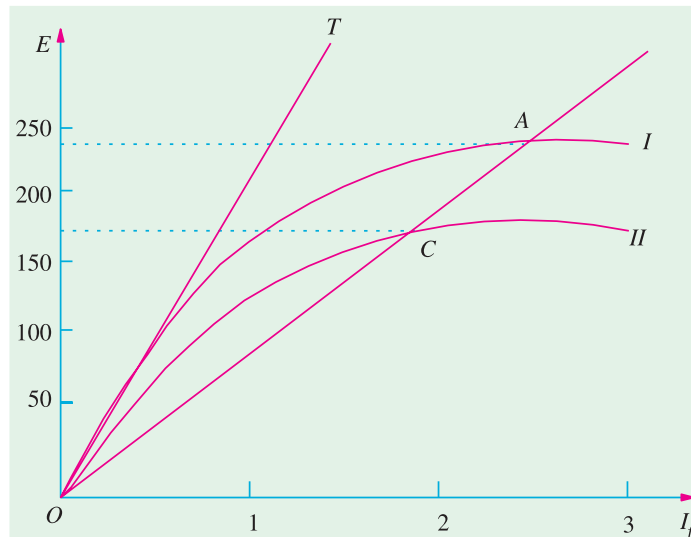


Fig. 28.9



**Solution.** At no-load the ampere turns required to produce 400 V =  $3.1 \times 1200 = 3720$   
 On full-load ampere turns required to produce the same voltage =  $4.8 \times 1200 = 5760$   
 Additional ampere turns required due to de-magnetising effect of load current =  $5760 - 3720 = 2040$ .

If  $N$  is a number of series turns required when load current is 200 A, then  
 $N \times 200 = 2040, N = 10.2$

**Example 28.3.** The open-circuit characteristic of a d.c. shunt generator driven at rated speed is as follows :

Field Amperes :	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
Induced Voltage :	60	120	138	145	149	151	152 V

If resistance of field circuit is adjusted to  $53 \Omega$ , calculate the open circuit voltage and load current when the terminal voltage is 100 V. Neglect armature reaction and assume an armature resistance of  $0.1 \Omega$ .  
**(Electrical Technology Punjab Univ. Dec. 1989)**

**Solution.** Take  $I_f = 3 \text{ A}, R_{sh} = 53 \Omega$ ,

Point A is (3A, 159V) point. Line OA is the  $53 \Omega$  line in Fig. 28.10. It cuts drop =  $3 \times 53 = 159 \text{ V}$ . O.C.C. at B. Line BM is drawn parallel to the base. OM represents the O.C. voltage which equals **150 V**.

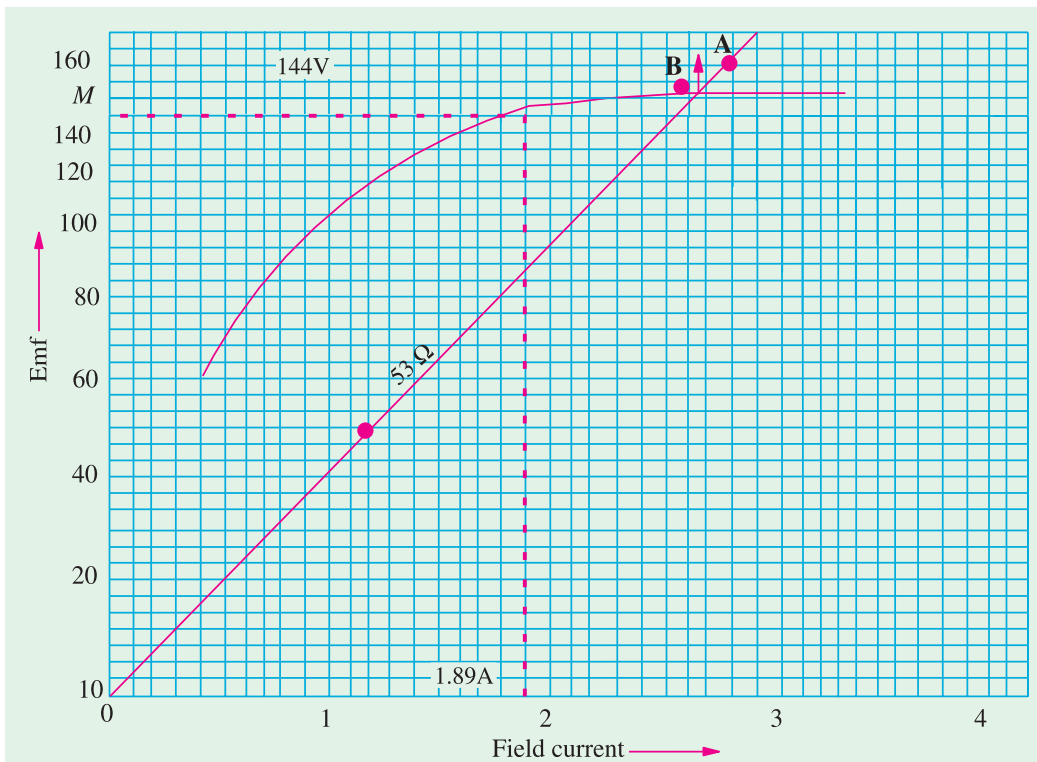


Fig. 28.10

Now, when  $V = 100 \text{ V}, I_{sh} = I_f = 100/53 = 1.89 \text{ A}$   
 Generated or O.C. voltage corresponding to this exciting current as seen from graph of Fig. 26.10 is 144 V.

Now  $E = V + I_a R_a$  or  $I_a R_a = 144 - 100 = 44 \text{ V}$   
 $\therefore 0.1 I_a = 44$  or  $I_a = 44/0.1 = 440 \text{ A}$

**Example 28.4.** The following figures give the O.C.C. of a d.c. shunt generator at 300 r.p.m.

Field amperes :	0	2	3	4	5	6	7
Armature volt :	7.5	92	132	162	183	190	212

Plot the O.C.C. for 375 r.p.m. and determine the voltage to which the machine will excite if field circuit resistance is  $40\ \Omega$ .

(a) What additional resistance would have to be inserted in the field circuit to reduce the voltage to 200 volts at 375 r.p.m.?

(b) Without this additional resistance, determine the load current supplied by the generator, when its terminal voltage is 200 V. Ignore armature reaction and assume speed to be constant. Armature resistance is  $0.4\ \Omega$ .  
(Elect. Machines - I, South Gujarat Univ. 1986)

**Solution.** The e.m.f. induced at 375 r.p.m. would be increased in the ratio 375/300 corresponding to different shunt field current values. A new table is given with the voltages multiplied by the above ratio.

Field amperes :	0	2	3	4	5	6	7
Armature volt :	9.4	115	165	202.5	228.8	248.8	265

The new O.C.C. at 375 r.p.m. is shown in Fig. 26.11. Line OA represents  $40\text{-}\Omega$  line.

The voltage corresponding to point A is 260 V. Hence machine will excite to 260 volt with  $40\ \Omega$  shunt field resistance.

(a) From Fig. 28.11, it is clear that for exciting the generator to 200 V, exciting current should be 3.8 A.

$$\therefore \text{Field circuit resistance} = 200/3.8 = 52.6\ \Omega$$

$$\therefore \text{Additional resistance required} = 52.6 - 40 = 12.6\ \Omega$$

(b) In this case, shunt field resistance =  $40\ \Omega$  ... (as above)

Terminal voltage = 200 V  $\therefore$  Field current =  $200/40 = 5\ \text{A}$

Generated e.m.f. for exciting current of 5 A = 228.8 V

For a generator  $E = V + I_a R_a \therefore I_a R_a = E - V$  or  $0.4 I_a = 228.8 - 20 = 208.8$

$\therefore I_a = 208.8/0.4 = 522\ \text{A} \therefore \text{Load current } I = 522 - 5 = 517\ \text{A}$

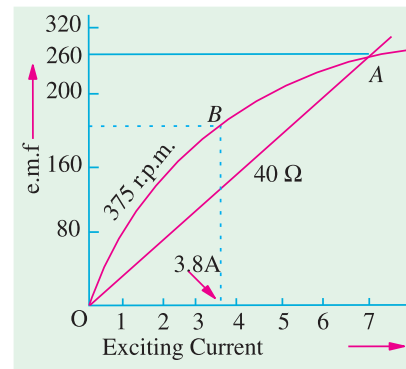


Fig. 28.11

**Example 28.5.** The open-circuit characteristic of a separately-excited d.c. generator driven at 1000 r.p.m. is as follows :

Field current :	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E.M.F. volts :	30.0	55.0	75.0	90.0	100.0	110.0	115.0	120.0

If the machine is connected as shunt generator and driven at 1,000 r.p.m. and has a field resistance of  $100\ \Omega$ , find (a) open-circuit voltage and exciting current (b) the critical resistance and (c) resistance to induce 115 volts on open circuit.

(Elect. Machines, Nagpur, Univ. 1993)

**Solution.** The O.C.C. has been plotted in Fig. 28.12. The shunt resistance line OA is drawn as usual.

(a) O.C. voltage = **100 V**; Exciting current = **1 A**

(b) Line OT is tangent to the initial part of the O.C.C. It represents critical resistance. As seen from point C, value of critical resistance is  $90/0.6 = 150\ \Omega$ .

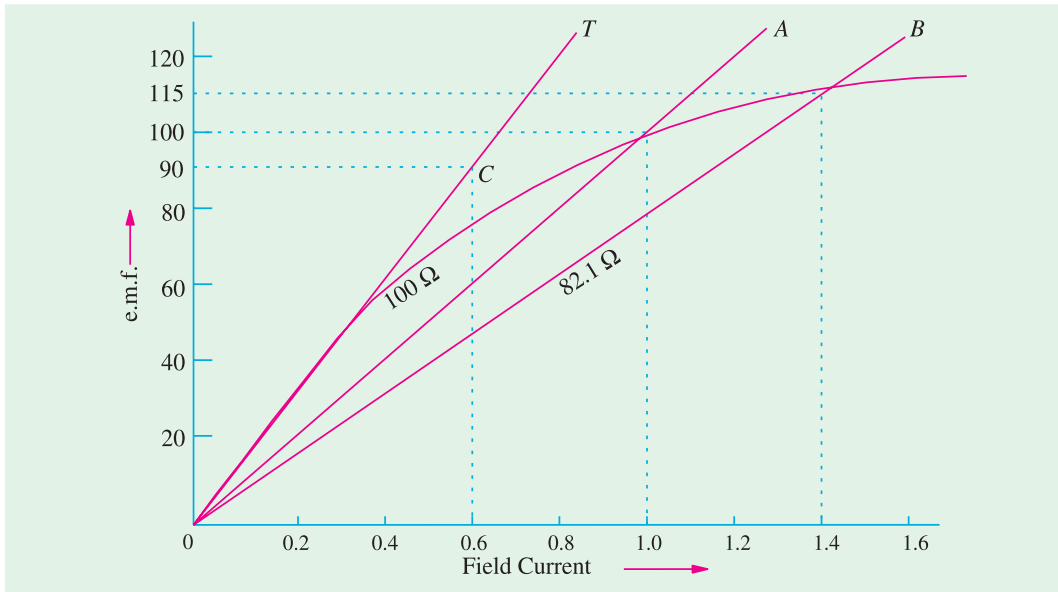


Fig. 28.12

(c) Line *OB* represents shunt resistance for getting 115 V on open-circuit. Its resistance =  $115/1.4 = 82.1 \Omega$ .

**Example 28.6.** A d.c. generator has the following magnetisation characteristics.

Field current (A) :	1	2	3	4	5	6	7	8
Generated e.m.f. (V) :	23	45	67	85	100	112	121	126

If the generator is shunt excited, determine the load current.

- (a) when terminal p.d. is 120 V, the field resistance is  $15 \Omega$  at a speed of 600 r.p.m. And
- (b) when terminal p.d. is 144 V, the field resistance is  $18 \Omega$  at a speed of 700 r.p.m.

**Solution.** (a) When terminal p.d. = 120 V, then field current  $I_f = V/R_{sh} = 120/15 = 8 \text{ A}$

From the given data, it is seen that the generated e.m.f. = 126 V\*

Voltage drop =  $126 - 120 = 6 \text{ V}$

Since drop due to armature reaction is neglected, this represents the armature drop.

$\therefore I_a R_a = 6$  or  $I_a = 6/0.02 = 300 \text{ A}$  Load current =  $300 - 8 = 292 \text{ A}$

(b) The O.C. data at 700 r.p.m. can be obtained by multiplying the given values of generated e.m.f. by a factor of  $700/600 = 7/6$ . Hence, the new data is :

Field current (A) :	1	2	3	4	5	6	7	8
Generated e.m.f. (V) :	26.8	52.5	78.2	99.2	116.6	131	141	146

When  $V = 144 \text{ V}$  and  $R_{sh} = 18 \Omega$ , field current =  $144/18 = 8 \text{ A}$

From the given data, the corresponding generated e.m.f. is 146 V.

Voltage drop  $I_a R_a = 146 - 144 = 2 \text{ V}$

$\therefore I_a = 2/0.02 = 100 \text{ A}$   $\therefore$  Load current =  $100 - 8 = 92 \text{ A}$

**Example 28.7.** The O.C.C. of a d.c. generator driven at 400 rev/min is as follows :

Field current (A) :	2	3	4	5	6	7	8	9
Terminal volts :	110	155	186	212	230	246	260	271

\* We need not plot the O.C.C. in this particular case.

Find :

(a) voltage to which the machine will excite when run as a shunt generator at 400 rev/min with shunt field resistance equal to  $34 \Omega$ .

(b) resistance of shunt circuit to reduce the O.C. voltage to 220 V.

(c) critical value of the shunt field circuit resistance.

(d) the critical speed when the field circuit resistance is  $34 \Omega$ .

(e) lowest possible speed at which an O.C. voltage of 225 V can be obtained.

(Electrical Technology, Bombay Univ. 1987)

**Solution.** The O.C.C. as plotted from the given data is shown in Fig. 28.13. The  $34\text{-}\Omega$  line  $OA$  is drawn as usual.

(a) The voltage to which machine will excite =  $OM = 255 \text{ V}$ .

(b) The horizontal line from  $N$  (220 V) is drawn which cuts the O.C.C. at point  $B$ . Resistance represented by line  $OB = 220/5.4 = 40.7 \Omega$ .

(c) Line  $OC$  has been drawn which is tangential at the origin to the O.C.C. This represents the value of critical resistance =  $140/2.25 = 62.2 \Omega$ .

(d) Take any convenient point  $D$  and erect a perpendicular which cuts both  $OA$  and  $OC$ .

$$\frac{DE}{DF} = \frac{N_C}{400} \quad \text{or} \quad \frac{110}{202} = \frac{N_C}{202}, \quad N_C = 218 \text{ r.p.m.}$$

(e) From point  $P$  (225 V) draw a horizontal line cutting  $OA$  at point  $G$ . From  $G$ , draw a perpendicular line  $GK$  cutting the O.C.C. at point  $H$ . If  $N'$  is the lowest speed possible for getting 225 volt with  $34 \Omega$  shunt circuit resistance, then

$$\frac{GK}{HK} = \frac{N'}{400} \quad \text{or} \quad \frac{225}{241} \text{ or } N' = 375 \text{ r.p.m.}$$

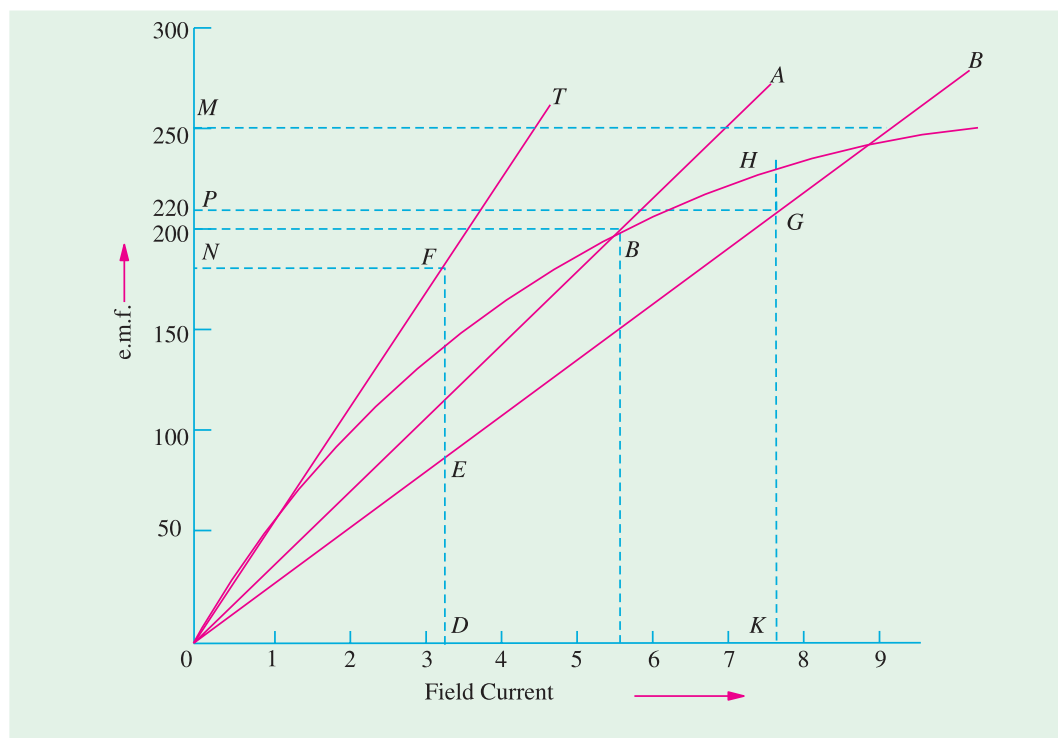


Fig. 28.13

**Example 28.8.** The magnetization characteristic for a 4-pole, 110-V, 1000 r.p.m. shunt generator is as follows :

Field current	0	0.5	1	1.5	2	2.5	3 A
O.C. voltage	5	50	85	102	112	116	120 V

Armature is lap-connected with 144 conductors. Field resistance is 45 ohms. Determine

- (i) voltage the machine will build up at no-load.
- (ii) the critical resistance.
- (iii) the speed at which the machine just fails to excite.
- (iv) residual flux per pole.

(Electrical Machinery- I, Mysore Unit. 1988)

**Solution.** In Fig. 28.14, OA represents the 45-Ω line which is drawn as usual.

(i) The voltage to which machine will build up = OM = **118 V**.

(ii) OT is tangent to the initial part of the O.C.C. It represents critical resistance. Take point B lying on this line. Voltage and exciting current corresponding to this point are 110 V and 1.1 A respectively.

$$\therefore R_c = 110/1.1 = \mathbf{100 \Omega}$$

(iii) From any point on OT, say point B, drop the perpendicular BD on X-axis.

$$\frac{CD}{BD} = \frac{N_c}{1000} \text{ or } \frac{49}{110} = \frac{N_c}{1000}$$

$$\therefore N_c = \mathbf{445 \text{ r.p.m.}}$$

(iv) As given in the table, induced e.m.f. due to residual flux (i.e. when there is no exciting current) is 5 V.

$$\therefore 5 = \frac{\Phi \times 144 \times 1000}{60} \left(\frac{4}{4}\right) \therefore \Phi = \mathbf{2.08 \text{ mWb.}}$$

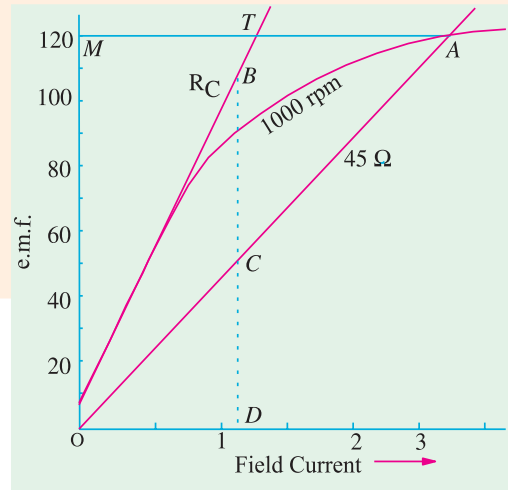


Fig. 28.14

**Example 28.9.** A shunt generator gave the following results in the O.C.C. test at a speed of r.p.m.

Field current (A) :	1	2	3	4	6	8	10
E.M.F. (volt) :	90	185	251	290	324	345	360

The field resistance is adjusted to 50 Ω and the terminal is 300 V on load. Armature resistance is 0.1 Ω and assuming that the flux is reduced by 5% due to armature reaction, find the load supplied by the generator. (Electromechanic, Allahabad Univ.; 1992)

**Solution.** When the terminal voltage is 300 V and  $R_{sh} = 50 \Omega$ , then field current is =  $300/40 = 6 \text{ A}$

With this shunt current, the induced e.m.f. as seen from the given table (we need not draw the O.C.C.) is 324 V.

Due to armature reaction, the flux and hence the induced e.m.f. is reduced to 0.95 of its no-load value.

Hence, induced e.m.f. when generator is no load =  $324 \times 0.95 = 307.8 \text{ V}$

Armature drop at the given load =  $307.8 - 300 = 7.8 \text{ V}$

$$I_a R_a = 7.8, \quad I_a = 7.8/0.1 = 78 \text{ A}$$

Load current =  $78 - 6 = 72 \text{ A}$ ; Generator output =  $72 \times 300/1000 = 21.6 \text{ kW}$

**Example 28.10.** A shunt generator gave the following open-circuit characteristic :

Field current :	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
O.C. e.m.f. :	54	107	152	185	210	230	245 V

The armature and field resistances are  $0.1 \Omega$  and  $80 \Omega$  respectively. Calculate (a) the voltage to which the machine will excite when run as a shunt generator at the same speed.

(b) The volts lost due to armature reaction when 100 A are passing in the armature at a terminal voltage of 175 V.

(c) The percentage reduction in speed for the machine to fail to excite on open circuit.

(Electrical Machines-I, Bombay Univ. 1988)

**Solution.** (a) O.C.C. is shown in Fig. 28.15. OA represents  $80 \Omega$  line. The maximum voltage to which the generator will build up is given by  $OM = 222 \text{ V}$ .

(b) With 175 V terminal p.d. on load

$$I_{sh} = 175/80 = 2.2 \text{ A}$$

Voltage corresponding to this field current is given by  $OC = 195 \text{ V}$ .

Voltage lost due to armature reaction and armature drop =  $195 - 175 = 20 \text{ V}$ .

Now, armature drop =  $0.1 \times 100 = 10 \text{ V}$

Let 'x' be the volts lost due to armature reaction.

Then  $10 + x = 20 \quad \therefore x = 10 \text{ V}$

(c) Line OT is drawn tangential to the curve. DFG is perpendicular to the base line.

$$\frac{N_c}{N} = \frac{FG}{DG} = \frac{160}{220} \quad \text{or} \quad \frac{N_e - N}{N} = \frac{-60}{220}$$

Percentage reduction in speed =  $\frac{-60}{220} \times 100 = -27.3 \%$

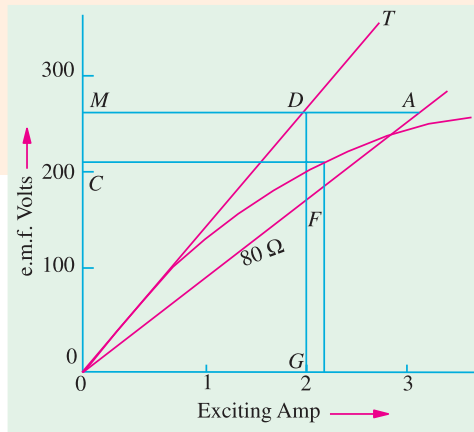


Fig. 28.15

**Example 28.11.** The O.C.C. of a shunt generator running at 800 r.p.m. is as follows :

Field current (amp.) :	1	2	3	4	5	6
Induced e.m.f. (volt).	82.5	180	225	252	273	282

If the shunt field resistance is  $60 \Omega$ , find

(i) the voltage to which the machine will build up running at the same speed (ii) the value of field regulating resistance if the machine is to build up to 120 V when its field coils are grouped in two parallel circuits and generator is running at half the speed.

(Electrical Technology, Hyderabad Univ. 1991)

**Solution.** O.C.C. is drawn from the given data and is shown in Fig. 28.16. OA represents  $60 \Omega$ . The voltage corresponding to point A is  $OM = 260 \text{ V}$ .

(i) Generator will build up to **260 volts**.

(ii) Lower curve represents the induced e.m.f. for different exciting current values at 400 r.p.m. From point B which represents 120 V, draw BC. From C draw a perpendicular CD which gives the

exciting current of 3.6 A. It means that current through *each* parallel path of shunt field coils is 3.6 A. Total current which passes through the field regulator resistance is  $3.6 \times 2 = 7.2$  A because it is in series with the field coils.

Hence, total shunt field resistance =  $120/7.2 = 16.67 \Omega$

Now, resistance of each shunt parallel path =  $60/2 = 30 \Omega$

Joint resistance of two parallel paths =  $30/2 = 15 \Omega$

$\therefore$  Shunt field regulator resistance =  $16.67 - 15 = 1.67 \Omega$

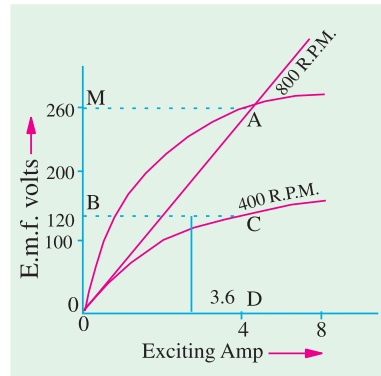


Fig. 28.16

**28.7. Voltage Build up of a Shunt Generator**

Before loading a shunt generator, it is allowed to build up its voltage. Usually, there is always present some residual magnetism in the poles, hence a small e.m.f. is produced initially.

This e.m.f. circulates a small current in the field circuit which increases the pole flux (provided field circuit is properly connected to armature, otherwise this current may wipe off the residual magnetism). When flux is increased, generated e.m.f. is increased which further increases the flux and so on. As shown in Fig. 28.17, *Oa* is the induced e.m.f. due to residual magnetism which appears across the field circuit and causes a field current *Ob* to flow. This current aids residual flux and hence produces, a larger induced e.m.f. *Oc*. In turn, this increased e.m.f. *Oc* causes an even larger current *Od* which creates more flux for a still larger e.m.f. and so on.

Now, the generated e.m.f. in the armature has

(a) to supply the ohmic drop  $I_f R_{sh}$  in the winding and (b) to overcome the opposing self-induced e.m.f. in the field coil *i.e.*  $L \cdot (dI_f/dt)$  because field coils have appreciable self-inductance.

$$e_g = I_f R_{sh} + L \cdot dI_f/dt$$

If (and so long as), the generated e.m.f. is in excess of the ohmic drop  $I_f R_{sh}$ , energy would continue being stored in the pole fields. For example, as shown in Fig. 28.17, corresponding to field current *OA*, the generated e.m.f. is *AC*.

Out of this, *AB* goes to supply ohmic drop  $I_f R_{sh}$  and *BC* goes to overcome self-induced e.m.f. in the coil. Corresponding to  $I_f = OF$ , whole of the generated e.m.f. is used to overcome the ohmic drop. None is left to overcome  $L \cdot dI_f/dt$ . Hence no energy is stored in the pole fields. Consequently, there is no further increase in pole flux and the generated e.m.f. With the given shunt field resistance represented by line *OP*, the maximum voltage to which the machine will build up is *OE*. If resistance is decreased, it will build up to a somewhat higher voltage. *OR* represents the resistance known as **critical resistance**. If shunt field resistance is greater than this value, the generator will fail to excite.

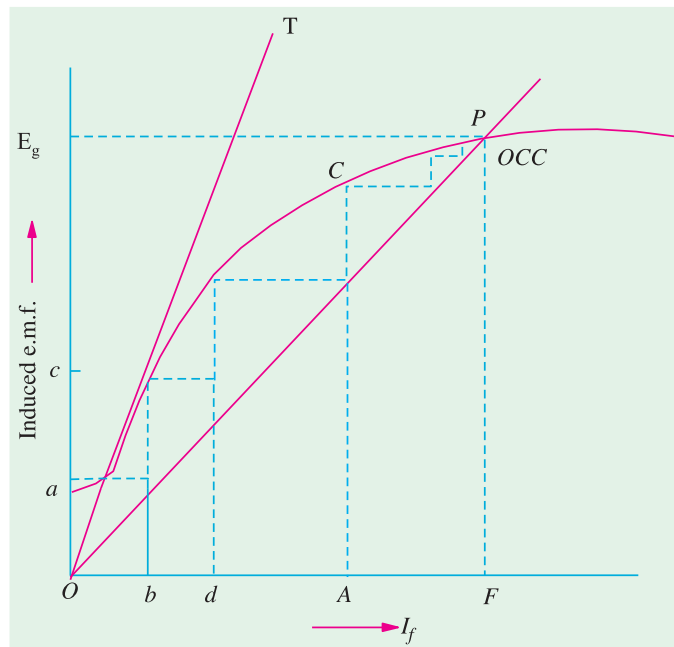


Fig. 28.17

### 28.8. Conditions for Build-up of a Shunt Generator

We may summarize the conditions necessary for the build-up of a (self-excited) shunt generator as follows :

1. There must be some residual magnetism in the generator poles.

2. For the given direction of rotation, the shunt field coils should be correctly connected to the armature *i.e.* they should be so connected that the induced current reinforces the e.m.f. produced initially due to residual magnetism.

3. If excited on open circuit, its shunt field resistance should be less than the critical resistance (which can be found from its O.C.C.)

4. If excited on load, then its shunt field resistance should be more than a certain minimum value of resistance which is given by internal characteristic (Art 28.11).

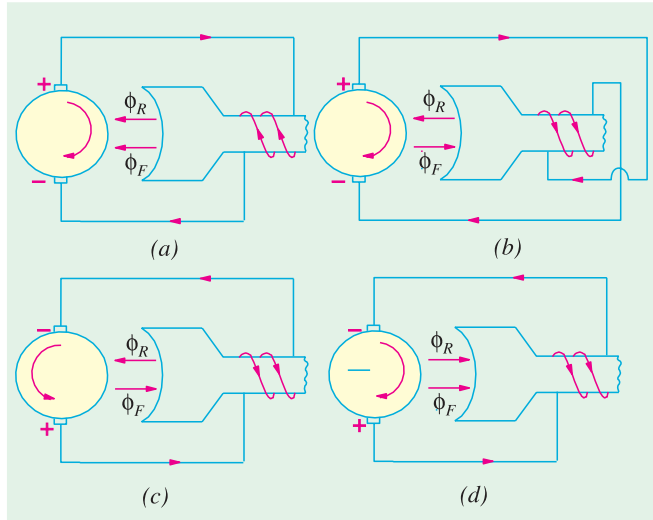


Fig. 28.18

### 28.9. Other Factors Affecting Voltage Building of a DC Generator

In addition to the factors mentioned above, there are some other factors which affect the voltage building of a self-excited d.c. generator. These factors are (i) reversed shunt field connection (ii) reversed rotation and (iii) reversed residual magnetism. These adverse effects would be explained with the help of Fig. 28.18 and the right-hand rule for finding the direction of the coil flux. For the sake of simplicity, only one field pole has been shown in the Fig. 28.18.

Fig. 28.18 (a) represents the normal operation, the prime mover rotation is clockwise and both the residual flux  $\Phi_R$  and the field flux  $\Phi_F$  are directed to the left.

Fig. 28.18 (b) shows reversed connection of the field circuit which causes  $\Phi_F$  to oppose  $\Phi_R$ . Consequently, the generator voltage builds down from its original residual value.

In Fig. 28.18 (c), reversed armature rotation causes the reversal of the voltage produced by the residual magnetism. Even though the field coil connections are correct, the reversed field current flow causes  $\Phi_F$  to oppose  $\Phi_R$  so that the voltage builds down from its original residual value.

Fig. 28.18 (d) shows the case when due to some reason the residual magnetism gets reversed. Hence, the armature voltage is also reversed which further reverses the field current. Consequently, both  $\Phi_F$  and  $\Phi_R$  are reversed but are directed to the right as shown. Under this condition, the voltage buildup is in the reversed direction. Obviously, the generator will operate at rated voltage but with reversed polarity.

If desired, the reversed polarity can be corrected by using an external d.c. source to remagnetise the field poles in the correct direction. This procedure is known as field flashing.

**Example 28.12.** The O.C.C. of a generator is given by the following :

Field current :	1.5	3.5	4.5	6	7.5	9	10.5
E.M.F. :	168	330	450	490	600	645	675

The speed at which data is obtained is 1000 r.p.m. Find the value of the shunt field resistance that will give a p.d. of 600 V with an armature current of 300 A at the same speed. Due to armature



reaction, the shunt field current is given by  $I_{sh}(\text{eff.}) = I_{sh} - 0.003 I_a$ . Armature resistance, including brush contact resistance, is  $0.1 \Omega$ . What will be the p.d. on open circuit at the same speed?

**Solution.** As shown in Fig. 28.19, the O.C.C. has been plotted from the given data.

Voltage drop due to armature resistance =  $300 \times 0.1 = 30 \text{ V}$ .

Reduction of field current due to armature reaction =  $0.003 \times 300 = 0.9 \text{ A}$ .

Any point A is taken on the O.C.C. A vertical distance  $AB = 30 \text{ V}$  is taken and then the horizontal line  $BC = 0.9 \text{ A}$  is drawn thus completing triangle  $ABC$  which is known as drop reaction triangle. Then, point C lies on the 300 ampere load saturation curve. This curve can be drawn by finding such similar points like  $C'$  etc.

From point L representing 600 V, a horizontal line is drawn cutting the load saturation curve at D. Join OD. Current corresponding to point D is 9.4 A. The slope of  $I_f$  the line OD gives the value of shunt resistance to give 600 V with 300 amperes of armature current.

$$\therefore R_{sh} = 600/9.4 = 63.8 \Omega$$

It is seen that e.m.f. on open circuit is 678 V.

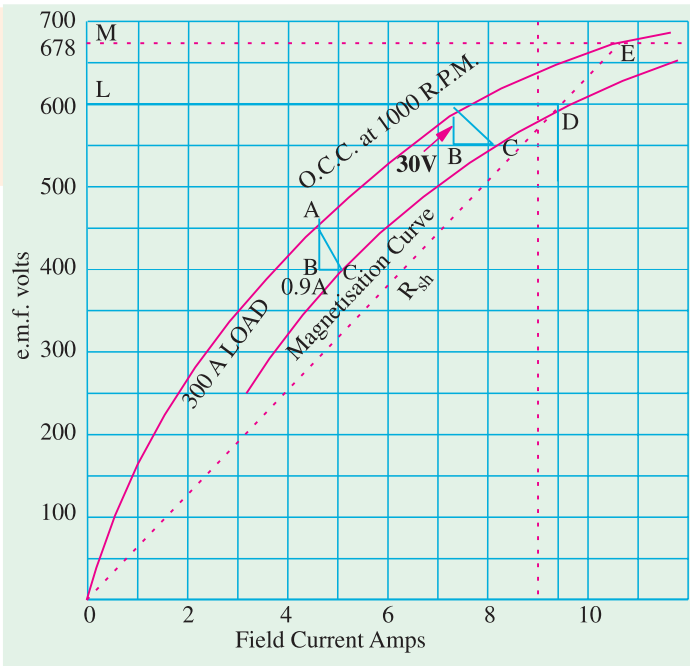


Fig. 28.19

### 28.10. External Characteristic

After becoming familiar with the no-load characteristic of a shunt generator, we will now proceed to find its external characteristic ( $V/I$ ) when loaded. It is found that if after building up, a shunt generator is loaded, then its terminal voltage  $V$  drops with increase in load current. Such a drop in voltage is undesirable especially when the generator is supplying current for light and power for which purpose it is desirable that  $V$  should remain practically constant and independent of the load. This condition of constant voltage is almost impossible to be fulfilled with a shunt generator unless the field current is being automatically adjusted by an automatic regulator. Without such regulation terminal voltage drops considerably as the load on the generator is increased. These are three main reasons for the drop in terminal voltage of a shunt generator when under load.

#### (i) Armature resistance drop :

As the load current increases, more and more voltage is consumed in the ohmic resistance of the armature circuit. Hence, the terminal voltage  $V = E - I_a R_a$  is decreased where  $E$  is the induced e.m.f. in the armature under load condition.

#### (ii) Armature reaction drop

Due to the demagnetising effect of armature reaction, pole flux is weakened and so the induced e.m.f. in the armature is decreased.

(iii) The drop in terminal voltage  $V$  due to (i) and (ii) results in a decreased field current  $I_f$  which further reduces the induced e.m.f.

For obtaining the relation between the terminal voltage and load current, the generator is connected as shown in Fig. 28.20 (a).

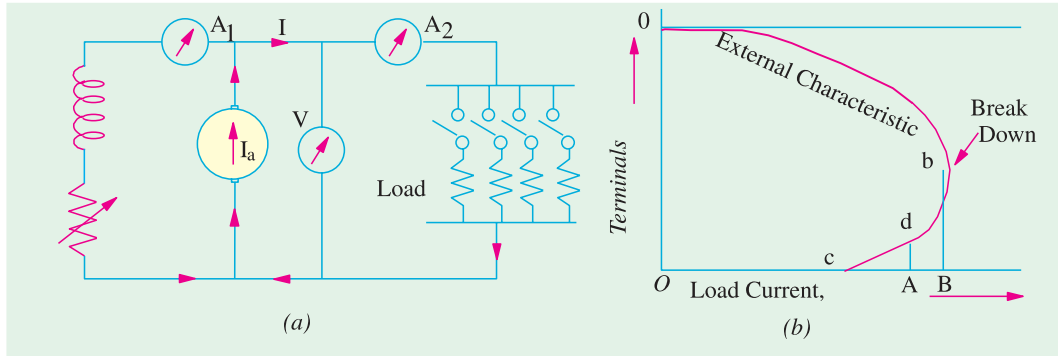


Fig. 28.20

The shunt generator is first excited on no-load so that it gives its full open circuit voltage =  $Oa$  [Fig. 28.20 (b)]. Then, the load is gradually applied and, at suitable intervals, the terminal voltage  $V$  (as read by the voltmeter) and the load current  $I$  (as read by the ammeter  $A_2$ ) are noted. The field current as recorded by ammeter  $A_1$  is kept constant by a rheostat (because during the test, due to heating, shunt field resistance is increased). By plotting these readings, the external characteristic of Fig. 28.20 (b) is obtained. The portion  $ab$  is the working part of this curve. Over this part, if the load resistance is decreased, load current is increased as usual, although this results in a comparatively small additional drop in voltage. These conditions hold good till point  $b$  is reached. This point is known as breakdown point. It is found that beyond this point (where load is maximum =  $OB$ ) **any effort to increase load current by further decreasing load resistance results in decreased load current (like  $OA$ ) due to a very rapid decrease in terminal voltage.**

We will discuss the reason for this unusual behaviour of the generator in more details. Over the earlier portion  $ab$  [Fig. 28.20 (b)] where the load current is comparatively small, when external load resistance is decreased, it results in increased load current as might be expected keeping Ohm's law in mind. It should not, however, be forgotten that due to increase in load current,  $V$  is also decreased somewhat due to the cause (iii) given above. But over the portion  $ab$ , the effect of decrease in load resistance predominates the effect of decrease in  $V$  because load current is relatively small.

At point  $b$ , generator is delivering a very large current *i.e.* current which is many times greater than its normal current. If load resistance is decreased at this point so as to be able to draw a load current greater than  $OB$ , the current is **increased** momentarily. But due to the severe armature reaction for this heavy current and increased  $I_a R_a$  drop, the terminal voltage  $V$  is drastically reduced. The effect of this drastic reduction in  $V$  results in less load current (=  $OA$ ). In other words, over the portion  $bdc$  of the curve, the terminal voltage  $V$  decreases more rapidly than the load resistance. Hence, any, further decrease in load resistance actually causes a **decrease** in load current (it may seem to contravene Ohm's law but this law is not applicable, here since  $V$  is not constant). As load resistance is decreased beyond point  $b$ , the curve turns back till when the generator is actually short-circuited, it cuts the current axis at point  $c$ . Here, terminal voltage  $V$  is reduced to zero, though there would be some value of  $E$  due to residual magnetism (Fig. 28.22).

### 28.11. Voltage Regulation

By voltage regulation of a generator is meant the change in its terminal voltage with the change in load current when it is running at a constant speed. If the change in voltage between no-load and full load is small, then the generator is said to have good regulation but if the change in voltage is large, then it has poor regulation. The **voltage regulation of a d.c. generator is the change in voltage when the load is reduced from rated value to zero, expressed as percentage of the rated load**

voltage.

If no-load voltage of a certain generator is 240 V and rated-load voltage is 220 V, then,  $\text{regn.} = (240 - 220)/220 = 0.091$  or 9.1 %

### 28.12. Internal or Total Characteristic

As defined before, internal characteristic gives the relation between  $E$  and  $I_a$ . Now in a shunt generator

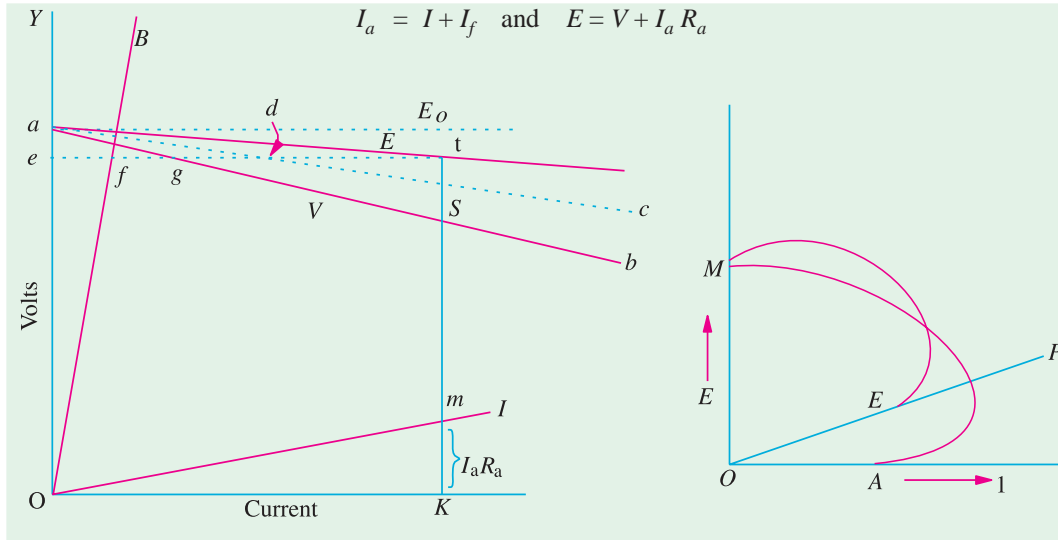


Fig. 28.21

Fig. 28.22

Hence,  $E/I_a$  curve can be obtained from  $V/I$  curve as shown in Fig. 28.21. In this figure,  $ab$  represents the external characteristic as discussed above. The field resistance line  $OB$  is drawn as usual. The horizontal distances from  $OY$  line to the line  $OB$  give the values of field currents for different terminal voltages. If we add these distances horizontally to the external characteristic  $ab$ , then we get the curve for the total armature current *i.e.* dotted curve  $ac$ . For example, point  $d$  on  $ac$  is obtained by making  $gd = ef$ . The armature resistance drop line  $Or$  is then plotted as usual. If brush contact resistance is assumed constant, then armature voltage drop is proportional to the armature current. For any armature current =  $OK$ , armature voltage drop  $I_a R_a = mK$ . If we add these drops to the ordinates of curve  $ac$ , we get the internal characteristic. For example,  $St = mK$ . The point  $t$  lies on the internal characteristic. Other points like  $t$  can be found similarly at different armature currents as the total characteristic can be drawn.

It may be noted here, in passing, that product  $E I_a$  gives the total power developed within the armature. Some of this power goes to meet  $I^2 R$  losses in armature and shunt field windings and the rest appears as output.

As explained in Art 28.10 if load resistance is decreased, the armature current increases up to a certain load current value. After that, any decrease in load resistance is not accompanied by increase in load current. Rather, it is decreased and the curve turns back as shown in Fig. 28.22. If the load resistance is too small, then the generator is short-circuited and there is no generated e.m.f. due to heavy demagnetisation of main poles.

Line  $OP$  is tangential to the internal characteristic  $MB$  and its slope gives the value of the **minimum** resistance with which the generator will excite if excited **on load**.

28.13. Series Generator



In this generator, because field windings are in series with the armature [Fig. 28.23 (a)], they carry full armature current  $I_a$ . As  $I_a$  is increased, flux and hence generated e.m.f. is also increased as shown by the curve. Curve  $Oa$  is the *O.C.C.* The extra exciting current necessary to neutralize the weakening effect of armature reaction at full load is given by the horizontal distance  $ab$ . Hence, point  $b$  is on the internal characteristic. If the ordinate  $bc = gh =$  armature voltage drop, then point  $c$  lies on the external characteristic [Fig. 28.23 (b)].

It will be noticed that a series generator has rising voltage characteristic *i.e.* with increase in load, its voltage is also increased.

But it is seen that at high loads, the voltage starts decreasing due to excessive demagnetising effects of armature reaction. In fact, terminal voltage starts decreasing as load current is increased as shown by the dotted curve. For a load current  $OC'$ , the terminal voltage is reduced to zero as shown.

**Example 28.13.** In a 220 V supply system, a series generator, working on a linear portion of its magnetisation characteristic is employed as a booster.

The generator characteristic is such that induced e.m.f. increases by 1 volt for every increase of 6 amperes of load current through the generator. The total armature resistance of the generator is 0.02 Ω. If supply voltage remains constant, find the voltage supplied to the consumer at a load current of 96 A. Calculate also the power supplied by the booster itself.

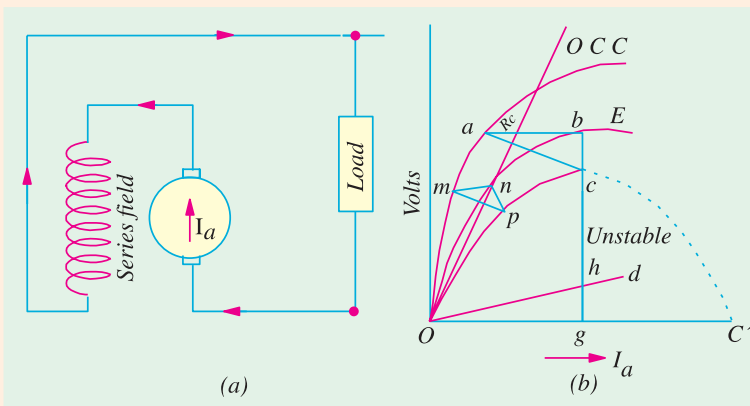


Fig. 28.23

**Solution.**

- Voltage increase for 6 amperes = 1 V ∴ Voltage increase for 96 A = 96/6 = 16 V
- Voltage drop in series coils = 96 × 0.02 = 1.9 V
- Net Voltage rise due to booster = 16 – 1.9 = 14.1 V
- Voltage at consumer end = 220 + 14.1 = **234.1 V**
- Power supplied by booster itself = 14.1 × 96 = 1354 W = **1.354 kW**

**Example 28.14.** A d.c. series generator, having an external characteristic which is a straight line through zero to 50 V at 200 A is connected as a booster between a station bus bar and a feeder of 0.3 ohm resistance. Calculate the voltage difference between the station bus-bar and the far end of the feeder at a current of (i) 200 A and (ii) 50 A.

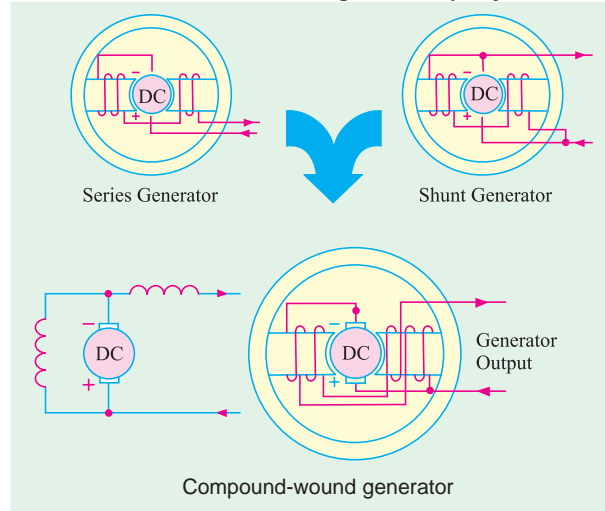
(AIME Sec. B Elect. Machine Summer 1991)

- Solution. (i)** Voltage drop = 200 × 0.3 = 60 V
- Booster voltage provided by series generator for 200 A current as given = 50 V.

$$\begin{aligned} \therefore \quad \text{Net voltage decrease} &= 60 - 50 = 10 \text{ V} \\ \text{(ii) Feeder drop} &= 50 \times 0.3 = 15 \text{ V} \\ \text{Booster voltage provided by series generator (by proportion) is} \\ &= 50 \times 50/200 = 12.5 \text{ V} \\ \therefore \quad \text{Net decrease in voltage} &= 15 - 12.5 = 2.5 \text{ V} \end{aligned}$$

### 28.14. Compound-wound Generator

A shunt generator is unsuitable where constancy of terminal voltage is essential, because its terminal voltage decreases as the load on it increases. This decrease in  $V$  is particularly objectionable for lighting circuit where even slight change in the voltage makes an appreciable change in the candle power of the incandescent lamps. A shunt generator may be made to supply substantially constant voltage (or even a rise in voltage as the load increases) by adding to it a few turns joined in series with either the armature or the load (Fig. 28.24). These turns are so connected as to aid to shunt turns when the generator supplies load. As the load current increases, the current through the series windings also increase thereby increasing the flux. Due to the increase in flux, induced e.m.f. is also increased. By adjusting the number of series turns (or series amp-turns), this increase in e.m.f. can be made to balance the combined voltage drop in the generator due to armature reaction and the armature drop. Hence,  $V$  remains practically constant which means that field current is also almost unchanged. We have already discussed the three causes which decrease the terminal voltage of a shunt generator (Art 28.10). Out of these three, the first two are neutralized by the series field amp-turns and the third one, therefore, does not occur.



If the series field amp-turns are such as to produce the same voltage at rated load as at no-load, then the generator is **flat-compounded**. It should be noted, however, that even in the case of a flat-compounded generator, the voltage is not constant from no-load to rated-load. At half the load, the voltage is actually greater than the rated voltage as seen from Fig. 28.24.

If the series field amp-turns are such that the rated-load voltage is greater than the no-load voltage, then generator is **over-compounded**. If rated-load voltage is less than the no-load voltage, then the generator is **under-compounded** but such generators are seldom used.

For short distances such as in hotels and office buildings, flat-compound generators are used because the loss of voltage over small lengths of the feeder is negligible. But when it is necessary to maintain a constant voltage then an over-compounded generator, which combines the functions of a generator and a booster, is invariably used.

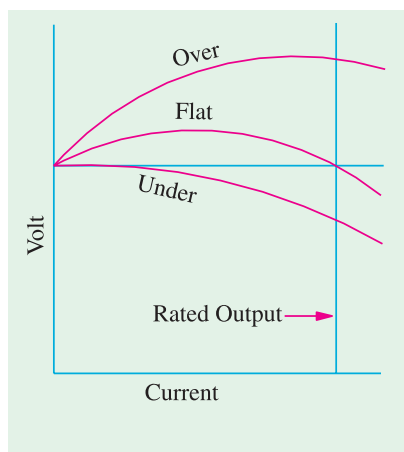


Fig. 28.24

### 28.15. How to Calculate Required Series Turns ?

Consider a 110-V, 250-ampere generator. Suppose it gives its rated no-load voltage with a field current of 5.8 A. If, now, the series windings are disconnected and the shunt field rheostat is left unchanged then the machine will act as shunt generator, hence its voltage will fall with increase in load current. Further, supply that the field current has to be increased to 6.3 A in order to maintain the rated terminal voltage at full load. If the number of turns of the shunt field winding is 2000, then  $2000 \times (6.3 - 5.8) = 1000$  amp-turns represent the additional excitation that has to be supplied by the series windings. As series turns will be carrying a full load current of 250 A, hence number of series turns =  $1000/250 = 4$ .

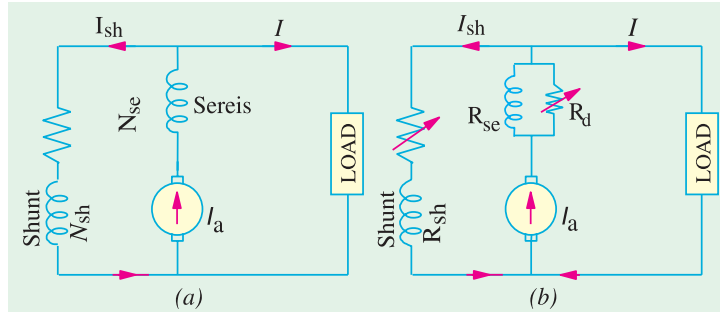


Fig. 28.25

In general, let

$\Delta I_{sh}$  = increase in shunt field current required to keep voltage constant from no-load to full-load

$N_{sh}$  = No. of shunt field turns per pole (or the total number of turns)

$N_{se}$  = No. of series turns per pole (or the total number of turns)

$I_{se}$  = current through series winding

= armature current  $I_a$  —for long-shunt

= load current  $I$  —for short-shunt

It is seen that while running as a simple generator, the increase in shunt field ampere-turns necessary for keeping its voltage constant from no-load to full-load is  $N_{sh} \cdot \Delta I_{sh}$ . This increase in field excitation can be alternatively achieved by adding a few series turns to the shunt generator [Fig. 28.25 (a)] thereby converting it into a compound generator.

$$\therefore N_{sh} \cdot \Delta I_{sh} = N_{se} I_{se}$$

If other things are known,  $N_{se}$  may be found from the above equation.

In practice, a few extra series amp-turns are taken in order to allow for the drop in armature. Any surplus amp-turns can be changed with the help of a diverter across the series winding as shown in Fig. 28.25 (b).

As said above, the degree of compounding can be adjusted with the help of a variable-resistance, diverter as shown in Fig. 28.25 (b). If  $I_d$  is the current through the diverter of resistance  $R_d$ , then remembering that series windings and diverter are in parallel,

$$\therefore I_{se} \cdot R_{se} = I_d R_d \quad \text{or} \quad R_d = I_{se} R_{se} / I_d$$

**Example 28.15.** A shunt generator is converted into a compound generator by addition of a series field winding. From the test on the machine with shunt excitation only, it is found that a field current of 5 A gives 440 V on no-load and that 6 A gives 440 V at full load current of 200 A. The shunt winding has 1600 turns per pole. Find the number of series turns required.

(Elect. Machines, A.M.I.E., Sec., B, 1991)

**Solution.** It would be assumed that shunt generator is converted into a short shunt compound generator. It is given that for keeping the voltage of shunt generator constant at 440 V both at no-load and full-load, shunt field ampere-turns per pole have to be increased from  $1600 \times 5 = 8000$  to

$(1600 \times 6) = 9600$  i.e. an increase of  $(9600 - 8000) = 1600$  AT. The same increase in field AT can be brought about by adding a few series turns.

Let  $n$  be the number of series turns required per pole. Since they carry 200 A,

$$\therefore n \times 200 = 1600; n = \mathbf{8 \text{ turns/pole}}$$

**Example 28.16.** A long shunt compound generator has a shunt field winding of 1000 turns per pole and series field winding of 4 turns per pole and resistance  $0.05 \Omega$ . In order to obtain the rated voltage both at no-load and full-load for operation as shunt generator, it is necessary to increase field current by 0.2 A. The full-load armature current of the compound generator is 80 A. Calculate the diverter resistance connected in parallel with series field to obtain flat compound operation.

(Elect. Machines A.M.I.E. Sec. B, 1993)

**Solution.**

Additional AT required to maintain rated voltage both at no-load and full-load (Fig. 28.26) =  $1000 \times 0.2 = 200$

No. of series turns/pole = 4

Current required to produce 200 AT by the series field =  $200/4 = 50$  A.

Since  $I_a = 80$  A, the balance of 30 A must pass through the parallel diverter resistance.

$$\therefore 30R = 50 \times 0.05, R = 0.0833 \Omega$$

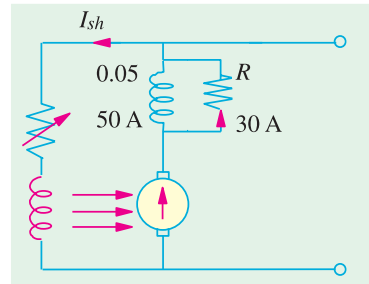


Fig. 28.26

**Example 28.17.** A 220-V compound generator is supplying a load of 100 A at 220 V. The resistances of its armature, shunt and series windings are  $0.1 \Omega$ ,  $50 \Omega$  and  $0.06 \Omega$  respectively. Find the induced e.m.f. and the armature current when the machine is connected (a) short shunt (b) long shunt (c) how will the series amp-turns be changed in (b) if a diverter of  $0.14 \Omega$  is connected in parallel with the series windings? Neglect armature reaction and brush contact drop.

**Solution. (a) Short-shunt** (Fig. 28.27).

$$\text{Voltage drop in series} = 100 \times 0.06 = 6 \text{ V}; I_{sh} = 220/50 = 4.4 \text{ A.}$$

$$\therefore I_a = 100 + 4.4 = \mathbf{104.4}$$

$$\text{Armature drop} = 104.4 \times 0.1 = 10.4 \text{ V}$$

$$\therefore \text{Induced e.m.f.} = 220 + 6 + 10.5 = \mathbf{236.4 \text{ V}}$$

**(b) Long-shunt** (Fig. 28.28)

$$I_{sh} = 220/50 = 4.4 \text{ A} \quad \therefore I_a = 100 + 4.4 = \mathbf{104.4 \text{ A}}$$

Voltage drop over armature and series field winding =  $104.4 \times 0.16 = 16.7$  V

$$\therefore \text{Induced e.m.f.} = 200 + 16.7 = \mathbf{216.7 \text{ V}}$$

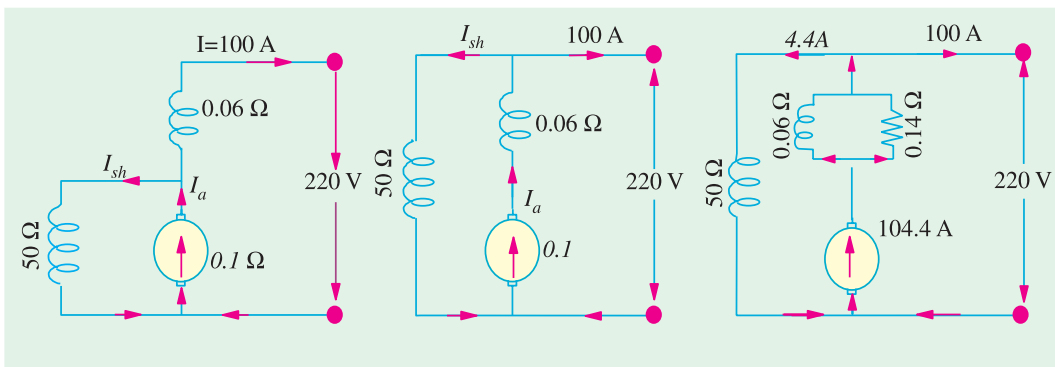


Fig. 28.27

Fig. 28.28

Fig. 28.29

(c) As shown in Fig. 28.29, a divertor of resistance  $0.14 \Omega$  is connected in parallel with the series field winding. Let  $n$  be the number of series turns.

Number of series amp-turns without divertor =  $n \times 104.4 = 104.4 n$

When divertor is applied, then current through series field is

$$= \frac{104.4 \times 0.14}{(0.14 + 0.06)} = 73.8 \text{ A}$$

$\therefore$  Series amp-turns =  $73.8 \times n$   $\therefore$  Series amp-turns are reduced to  $\frac{73.8 n}{104.4 n} \times 100 = 70 \%$

**Example 28.18.** A 250-kW, 240-V generator is to be compounded such that its voltage rises from 220 volts at no-load to 240 V at full load. When series field is cut out and shunt field is excited from an external source, then from the load test it is found that this rise in voltage can be obtained by increasing the exciting current from 7 A at no-load to 12 A at full-load. Given shunt turns/pole = 650, series turns/pole = 4 and resistance of series winding,  $0.006 \Omega$ . If the machine is connected long-shunt, find the resistance of the series amp-turns at no-load and drop in series winding resistance at full-load.

**Solution.** Full-load current =  $250 \times 10^3 / 240$   
= 1042 A

Increase in shunt field ampere-turns to over-compound the shunt generator

$$= 650 (12 - 7) = 3,250. \text{ As seen from Fig. 28.30}$$

$$4 \times I_{se} = 3250$$

$$I_{se} = 3250/4 = 812.5 \text{ A}$$

$$\therefore I_d = 1042 - 812.5 = 229.5 \text{ A}$$

It is so because no-load shunt current being negligible,  $I_a = I = 1042 \text{ A}$ .

Since series winding and divertor are in parallel,  $I_d R_d = I_{se} R_{se}$  or  $229.5 R_d = 812.5 \times 0.006$

$$\therefore R_d = 0.0212 \Omega$$

**Example 28.19.** A 60-kW d.c. shunt generator has 1600 turns/pole in its shunt winding. A shunt field current of 1.25 A is required to generate 125 V at no-load and 1.75 A to generate 150 V at full load. Calculate

(i) the minimum number of series turns/pole needed to produce the required no-load and full-load voltages as a short-shunt compound generator.

(ii) if the generator is equipped with 3 series turns/pole having a resistance of  $0.02 \Omega$ , calculate divertor resistance required to produce the desired compounding.

(iii) voltage regulation of the compound generator.

**Solution.** (i) Extra excitation ampere-turns required =  $1600 (1.75 - 1.25) = 800$

$$I_{se} = I = 60,000/150 = 400 \text{ A}$$

$$\therefore \text{No. of series turns/pole required} = 800/400 = 2$$

Hence, minimum number of series turns/pole required for producing the desired compound generator terminal voltage is 2.

(ii) Now, actual No. of series turns/pole is 3. Hence, current passing through it can be found from

$$3 \times I_{se} = 800; I_{se} = 800/3 \text{ A}$$

As shown in Fig. 28.31,  $I_d = 400 - (800/3) = 400/3 \text{ A}$

Also  $(800/3) \times 0.02 = (400/3) \times R_d$ ;  $R_d = 0.04 \Omega$

(iii) % regn. =  $(125 - 150) \times 100/150 = -16.7\%$

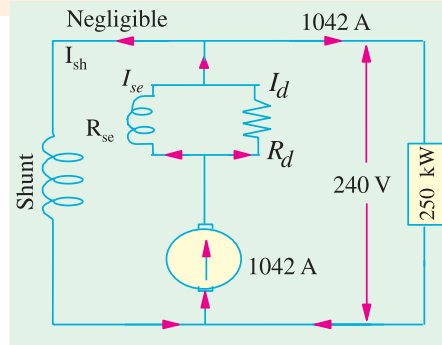


Fig. 28.30

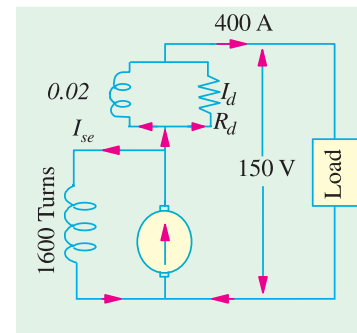


Fig. 28.31



**Example 28.20.** A D.C. generator having an external characteristics which is a straight line through zero to 50 V at 200 Amp, is connected as a booster between a station bus-bar and a feeder of 0.3 ohm resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (i) 160 A, (ii) 50 A. (Manomaniam Surdaranar Univ. Nov. 1998)

**Solution.** Due to the feeder resistance of 0.3 ohm there is a voltage drop of  $1 \times 0.3$  volts in the direction of current, where  $I$  refers to the current flowing. Due to booster, there is a rise in voltage given by  $I \times (50/200)$  according to the  $V$ - $I$  characteristic given for the series generator. As a sum total of these two, the net feeder drop will be  $I \times (0.3 - 0.25)$  or  $I$  volts, and that represents the voltage between far end of the feeder and the bus-bar.

(i) At 160 A, the net voltage drop in feeder =  $160 \times 0.05$  or  $48 - 40 = 9$  volts.

Since 48 volts drop is partially compensated by 40 volts boosted up.

(ii) At 50 A, the required answer = (15 volts drop 12.5 volts boosted up) = 2.5 volts drop.

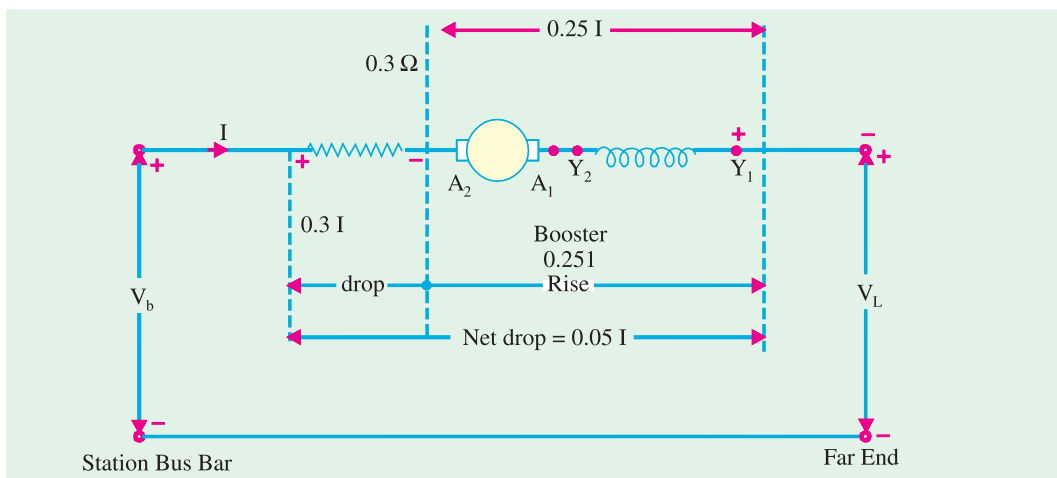


Fig. 28.32. Feeder and Booster

### 28.16. Uses of D.C. Generators

1. **Shunt generators** with field regulators are used for ordinary lighting and power supply purposes. They are also used for charging batteries because their terminal voltages are almost constant or can be kept constant.

2. **Series generators** are not used for power supply because of their rising characteristics. However, their rising characteristic makes them suitable for being used as boosters (Ex. 28.15) in certain types of distribution systems particularly in railway service.

#### 3. Compound generators

The cumulatively-compound generator is the most widely used d.c. generator because its external characteristic can be adjusted for compensating the voltage drop in the line resistance. Hence, such generators are used for motor driving which require d.c. supply at constant voltage, for lamp loads and for heavy power service such as electric railways.

The differential-compound generator has an external characteristic similar to that of a shunt generator but with large demagnetization armature reaction. Hence, it is widely used in arc welding where larger voltage drop is desirable with increase in current.



Compound generators are used in electric railways

## Tutorial Problem in 28.1

1. The *OC* curve of a d.c. shunt generator for a speed of 1000 r.p.m. is given by the following table.

<i>Field current</i> :	2.0	3.0	4.0	5.0	6.0	7.0
<i>E.M.F. volts</i> :	102	150	188	215	232	245

The shunt has a resistance of  $37 \Omega$ . Find the speed at which excitation may be expected to build up.

The armature resistance of  $0.04 \Omega$ . Neglecting the effects of brush drop and armature reaction, estimate the p.d. when the speed is 1000 r.p.m. and the armature delivers a current of 100 A. **(725 r.p.m.; 231 V)**

2. A d.c. shunt generator running at 850 r.p.m. gave the following *O.C.C.* data :

<i>Field current (A)</i> :	0	0.5	1	2	3	4	5
<i>E.M.F. (V)</i> :	10	60	120	199	232	248	258

If the resistance of the shunt field is  $50 \Omega$ , determine the additional resistance required in the shunt field circuit to give 240 V at a speed of 1000 r.p.m. **[64.3  $\Omega$ ]**

3. Sketch the load characteristic of a d.c. generator with (i) shunt (ii) series excitation. Give reasons for the particular shape in each case.

The *O.C.C.* at 700 r.p.m. of a series generator with separately-excited field is as follows :

<i>Field current (A)</i> :	20	40	50	60	75
<i>Armature e.m.f. (V)</i> :	190	360	410	450	480

Determine the current and terminal voltage as a self-excited series machine when running at 600 r.p.m. with a load of  $6 \Omega$  connected to the terminal. Resistance of armature and series winding is  $0.3 \Omega$ . Ignore effect of armature reaction. **[369 V; 61.5 A]**

4. The *O.C.C.* data for separately-excited generator when run at 130 r.p.m. on open circuit is

<i>E.M.F. (V)</i> :	12	44	73	98	113	122	127
<i>Exciting current</i> :	0	0.2	0.4	0.6	0.8	1.0	1.2

Deduce the curve of e.m.f. and excitation when the generator is running separately-excited at 1000 r.p.m. To what voltage will the generator build up on no-load when running at 1000 r.p.m.? To what voltage will the generator build up on no-load when running at 100 r.p.m. if the total field resistance is  $100 \Omega$ ? **[91 V]**

5. The following figures give the *O.C.C.* of d.c. Shunt generator driven at a constant speed of 700 r.p.m.

<i>Terminal voltage (V):</i>	10	20	40	80	120	160	200	240	260
<i>Field Current (A)</i> :	0	0.1	0.24	0.5	0.77	1.2	1.92	3.43	5.2

Determine the critical resistance at (a) 700 r.p.m. (b) 850 r.p.m. If resistance of field coils is  $50 \Omega$ , find the range of the field rheostat required to vary the voltage between the limits of 180 V and 250 V on open circuit at a speed of 700 r.p.m. **[160  $\Omega$  ; 194  $\Omega$ ; 70  $\Omega$  to 10  $\Omega$ ]**

6. The *O.C.C.* of a shunt generator when separately-excited and running at 1000 r.p.m. is given by :

<i>O.C.C. volt</i> :	56	112	150	180	200	216	230
<i>Field amp.</i> :	0.5	1.0	1.5	2.0	2.5	3.0	3.5

If the generator is shunt-connected and runs at 1100 r.p.m. with a total field resistance of  $80 \Omega$ , determine (a) no-load e.m.f.

(b) the output when the terminal voltage is 200 V if the armature resistance is  $0.1 \Omega$ .

(c) the terminal voltage of the generator when giving the maximum output current.

Neglect the effect of armature reaction and of brush contact drop.

**[236 V; 200 V; 460 V; 150 V (approx.)]**

7. A long-shunt compound d.c. generator with armature, series field and shunt field resistance of 0.5, 0.4 and  $250 \Omega$  respectively gave the following readings when run at constant speed :

<i>Load current (A)</i> :	0	10	20	30	40
<i>Terminal p.d. (V)</i> :	480	478	475	471	467

Plot the curve of internal generated e.m.f. against load current. Explain fully the steps by which this curve is obtained and tabulate the values from which it is plotted. **[For 40 A; E = 504.7 V (approx.)]**

8. A shunt generator has the following open-circuit characteristic at 800 r.p.m.

Field amperes	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5
E.M.F. Volt	:	54	107	152	185	210	230	245

Armature and shunt field resistances are respectively  $0.1 \Omega$  and  $80 \Omega$ . The terminal p.d. falls to 175 V when the armature current is 100 A. Find the O.C. volts and the volts lost due to (i) reduction in the field current (ii) armature resistance (iii) armature reaction. **[220 V (i) 27 V (ii) 120 V (iii) 8 V]**

9. The open-circuit characteristic of a shunt generator when driven at normal speed is as follows :

Field current	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
O.C. volts	:	54	107	152	185	210	230	240 V

The resistance of armature circuit is  $0.1 \Omega$ . Due to armature reaction the effective field current is given by the relation  $I_{sh}(\text{eff.}) = I_{sh} - 0.003 I_a$ . Find the shunt field circuit resistance that will give a terminal voltage of 220 V with normal speed (a) on open circuit (b) at a load current of 100 A. Also find (c) number of series turns for level compounding at 220 V with 100 A armature current ; take number of shunt turns per pole as 1200 and (d) No. of series turns for over-compounding giving a terminal voltage of 220 V at no-load and 230 V with 100 A armature current. **[(a)  $80 \Omega$  (b)  $66 \Omega$  (c)  $6.8 \Omega$  (d) 9.1 turns]**

10. Find how many series turns per pole are needed on a 500-kW compound generator required to give 450 V on no-load and 500 V on full-load, the requisite number of ampere-turns per pole being 9,000 and 6,500 respectively. The shunt winding is designed to give 450 V at no-load when its temperature is  $20^\circ\text{C}$ . The final temperature is  $60^\circ\text{C}$ . Take  $\alpha_0 = 1/234.5$  per  $^\circ\text{C}$ .

**[ 2.76] (Electrical Technology, Allahabad Univ. 1977)**

### OBJECTIVE TESTS – 28

- The external characteristic of a shunt generator can be obtained directly from its ——— characteristic.
  - internal
  - open-circuit
  - load-saturation
  - performance
- Load saturation characteristic of a d.c. generator gives relation between
  - V and  $I_a$
  - E and  $I_a$
  - $E_0$  and  $I_f$
  - V and  $I_f$
- The slight curvature at the lower end of the O.C.C. of a self-excited d.c. generator is due to
  - residual pole flux
  - high armature speed
  - magnetic inertia
  - high field circuit resistance.
- For the voltage built-up of a self-excited d.c. generator, which of the following is not an essential condition ?
  - There must be some residual flux
  - Field winding mmf must aid the residual flux
  - Total field circuit resistance must be less than the critical value
  - Armature speed must be very high.
- The voltage build-up process of a d.c. generator is
  - difficult
  - delayed
  - cumulative
  - infinite
- Which of the following d.c. generator cannot build up on open-circuit ?
  - shunt
  - series
  - short shunt
  - long shunt
- If a self-excited d.c. generator after being installed, fails to build up on its first trial run, the first thing to do is to
  - increase the field resistance
  - check armature insulation
  - reverse field connections
  - increase the speed of prime mover.
- If residual magnetism of a shunt generator is destroyed accidentally, it may be restored by connecting its shunt field
  - to earth
  - to an a.c. source
  - in reverse
  - to a d.c. source.
- The three factors which cause decrease in the terminal voltage of a shunt generator are
  - armature reactances
  - armature resistance
  - armature leakages
  - armature reaction
  - reduction in field current
- If field resistance of a d.c. shunt generator is increased beyond its critical value, the generator
  - output voltage will exceed its name-plate rating
  - will not build up

- (c) may burn out if loaded to its name-plate rating  
 (d) power output may exceed its name-plate rating
11. An ideal d.c. generator is one that has .....  
 (a) low (b) zero  
 (c) positive (d) negative
12. The ..... generator has poorest voltage regulation.  
 (a) series (b) shunt  
 (c) compound (d) high
13. The voltage regulation of an overcompound d.c. generator is always .....
- (a) positive  
 (b) negative  
 (c) zero  
 (d) high
14. Most commercial compound d.c. generator are normally supplied by the manufacturers as over compound machines because  
 (a) they are ideally suited for transmission of d.c. energy to remotely-located loads  
 (b) degree of compounding can be adjusted by using a divertor across series field  
 (c) they are more cost effective than shunt generators  
 (d) they have zero percent regulation.

## ANSWERS

1. (b) 2. (d) 3. (c) 4. (d) 5. (c) 6. (b) 7. (c) 8. (d) 9. (b,d,e) 10. (b) 11. (b) 12. (a) 13. (b) 14. (b).

## QUESTIONS AND ANSWERS ON D.C. GENERATORS

**Q. 1. How may the number of parallel paths in an armature be increased ?**

**Ans.** By increasing the number of magnetic poles.

**Q. 2. How are brushes connected in a d.c. generator ?**

**Ans.** Usually, all positive brushes are connected together and all the negative brushes together (Fig. 28.33).

**Q. 3. What is meant by armature reaction ?**

**Ans.** It is the effect of armature magnetic field on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :

- (i) It demagnetises or weakens the main flux and
- (ii) It cross-magnetises or distorts it.

**Q. 4. What is the effect of this distortion on the operation of the machine ?**

**Ans.** It acts as a magnetic drag on the armature which consequently requires more power to turn it.

**Q. 5. How can field distortion be remedied ?**

**Ans.** By using compensating windings which are embedded in the slots in the pole-shoe and are connected in series with the armature.

**Q. 6. What is meant by normal neutral plane ?**

**Ans.** It is a plane which passes through the axis of the armature perpendicular to the magnetic field of the generator when there is no flow of current through the armature.

**Q. 7. What is the importance of this plane in the working of the machine ?**

**Ans.** It gives the position where brushes would be placed to prevent sparking during the operation of the generator where the main pole field not distorted by armature field and were there no self-induction in the coils.

**Q. 8. How do you differentiate between normal neutral plane ?**

**Ans.** The *NNP* is the position of zero induction and hence minimum sparking assuming no field distortion *i.e.* on no-load. It is perpendicular to magnetic axis. *NA* is the position of zero induction and hence minimum sparking with distorted field *i.e.* **when generator is on load.**

**Q. 9. How do you define 'commutating plane' ?**

**Ans.** It is the plane which passes through the axis of the armature and through centre of contact of the brushes as shown in Fig. 28.34.

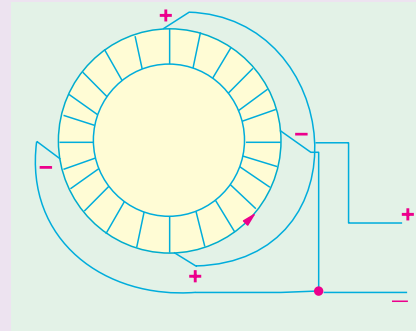


Fig. 28.33

**Q. 10. What is the angle of lead ?**

**Ans.** It is the angle between the *NNP* and the commutating plane.

**Q. 11. What affects this angle ?**

**Ans.** For sparkless commutation, the angle of lead varies directly with load. Its value can be kept small by making main pole field considerably more powerful than the armature field.

**Q. 12. What is the best way of minimizing eddy currents in an armature ?**

**Ans.** Lamination.

**Q. 13. How should the armature be laminated for the purpose ?**

**Ans.** It should be laminated at right angle to its axis.

**Q. 14. How does field distortion affect communication ?**

**Ans.** The neutral plane no longer coincides with the normal neutral plane but is advanced by a certain angle in the direction of rotation of the armature.

**Q. 15. Should the brushes of a loaded generator be placed in the neutral plane ?**

**Ans.** No.

**Q. 16. Why not ?**

**Ans.** The brushes must be advanced by a certain angle (called brush lead) beyond the neutral plane to prevent sparking.

**Q. 17. What causes sparking at the brushes ?**

**Ans.** It is due to the self-induction of the coil undergoing commutation.

**Q. 18. What is the standard direction of rotation of the d.c. generators ?**

**Ans.** Clockwise when viewed from the end opposite to the driven end.

**Q. 19. What is meant by build-up of a generator ?**

**Ans.** It means the gradual increase in the generator voltage to its maximum value after the generator is started from rest.

**Q. 20. How should a generator be started ?**

**Ans.** It is usually brought up to speed with the help of the driving engine called prime-mover.

**Q. 21. How should a shunt or compound generator be started ?**

**Ans.** Such machines excite best when all switches controlling the external circuit are open.

**Q. 22. How about a series generator ?**

**Ans.** In this case, the external circuit must be closed otherwise the generator will not build-up.

**Q. 23. What is the procedure for shunting down a generator ?**

**Ans.** First, the load should be gradually reduced, if possible, by easing down the driving engine, then when the generator is supplying little or no current, the main switch should be opened. When the voltmeter reads almost zero, then brushes should be raised from the commutator.

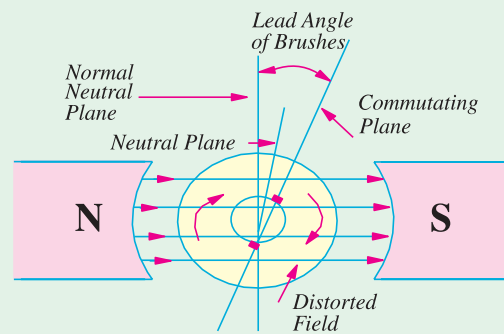
**Q. 24. What are the indications and causes of an overloaded generator ?**

**Ans.** A generator is said to be overloaded if a greater output is taken from it that it can safely carry. Overloading is indicated by (i) excessive sparking at brushes and (ii) overheating of the armature and other parts of the generator. Most likely causes of overloading are :

1. Excessive voltage—as indicated by the voltmeter or the increased brilliancy of the pilot lamp. This could be due to over-excitation of field magnets or too high speed of the engine.
2. Excessive current—which could be due to bad feeding of the load.
3. Reversal of polarity—this happens occasionally when the series or compound-wound generators are running in parallel. Polarity reversal occurs during stopping by the current from the machines at work.
4. Short-circuit or ground in the generator itself or in the external circuit.

**Q. 25. Mention and explain the various causes for the failure of the generator to build up.**

**Ans.** Principal causes due to which a generator may fail to excite are :



**Fig. 28.34**

1. Brushes not properly adjusted—if brushes are not in their proper positions, then whole of the armature voltage will not be utilized and so would be insufficient to excite the machine.
2. Defective contacts—unclean contacts may interpose large resistance in the path of the exciting current and reduce it to such a small value that it fails to excite the machine.
3. Incorrect adjustment of regulators—in the case of shunt and compound generators, it is possibly that the resistance of field regulator may be too high to permit the passage of sufficient current through the field windings.
4. Speed too low—in the case of shunt—and compound-wound generators, there is certain critical armature speed below which they will not excite.
5. Open-circuit—in the case of series machines.
6. Short-circuit—in the generator of external circuit.
7. Reversed field polarity—usually caused by the reversed connections of the field coils.
8. Insufficient residual magnetism—The trouble normally occurs when the generator is new. It can be remedied by passing a strong direct current through the field coils.

**Q. 26. How do we conclude that connections between field coils and armature are correct ?**

**Ans.** If the generator builds up when brought to full speed. If it does not, then connections are reversed.

**Q. 27. When a generator loses its residual magnetism either due to lighting or short circuit, how can it be made to build up ?**

**Ans.** By temporarily magnetising the main poles with the help of current from an external battery.

**Q. 28. Can a generator be reversed by reversing the connections between the armature and field coils ?**

**Ans.** No, because if these connections are reversed, the generator will not build up at all.

**Q. 29. Will a generator build up if it becomes reversed ?**

**Ans.** Yes.

**Q. 30. Then, what is the objection to a reversed generator ?**

**Ans.** Since the current of such a reversed generator is also reversed, serious trouble can occur if attempt is made to connect it in parallel with other machines which are not reversed.

**Q. 31. What are the two kinds of sparking produced in a generator ?**

**Ans.** One kind of sparking is due to bad adjustment of brushes and the other due to bad condition of the commutator. The sparking of the first are bluish whereas those of the other are reddish in colour.

**Q. 32. What is the probable reason if sparking does not disappear in any position when brushes are rocked around the commutator ?**

**Ans. (i)** The brushes may not be separated at correct distance.

**(ii)** The neutral plane may not be situated in the true theoretical position on the commutator due to faulty winding.

**Q. 33. What is the permissible rise of temperature in a well-designed generator ?**

**Ans.** 27°C above the surrounding air.

**Q. 34. What are the causes of hot bearings ?**

**Ans. (i)** lack of oil **(ii)** belt too tight **(iii)** armature not centered with respect of pole pieces **(iv)** bearing too tight or not in line.

**Q. 35. What causes heating of armature ?**

**Ans. 1.** Eddy currents.

**2.** Moisture which almost short-circuits the armature.

**3.** Unequal strength of magnetic poles.

**4.** Operation above rated voltage and below normal speed.

**Q. 36. What is the commutator pitch of a 4-pole d.c. armature having 49 commutator bars?**

**Ans.**  $Y_c = (49 \pm 1)/2 = 24$  or 25.

**Q. 37. Will it make any difference if lower figure of 24 is selected in preference to other.**

**Ans.** Yes. Direction of armature rotation would be reversed.

# C H A P T E R 29

## Learning Objectives

- Motor Principle
- Comparison of Generator and Motor Action
- Significance of the Back e.m.f.—Voltage Equation of a Motor
- Conditions for Maximum Power
- Torque
- Armature Torque of a Motor
- Shaft Torque
- Speed of a D.C. Motor
- Speed Regulation Torque and Speed of a D.C. Motor
- Motor Characteristics
- Characteristics of Series Motors
- Characteristics of Shunt Motors
- Compound Motors
- Performance Curves
- Comparison of Shunt and Series Motors
- Losses and Efficiency
- Power Stages

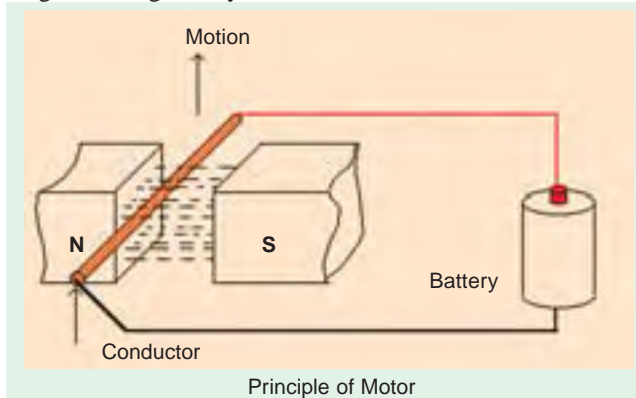
## D.C. MOTOR



**C** Design for optimum performance and durability in demanding variable speed motor applications. D.C. motors have earned a reputation for dependability in severe operating conditions

### 29.1. Motor Principle

An Electric motor is a machine which converts electric energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by  $F = BIl$  Newton.



Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 29.1 a part of multipolar d.c. motor is shown. When its field magnets are excited and its armature conductors

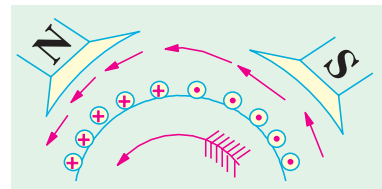


Fig. 29.1

are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under *N*-pole are assumed to carry current downwards (crosses) and those under *S*-poles, to carry current upwards (dots). By applying Fleming's Left-hand Rule, the direction of the force on each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor experiences a force  $F$  which tends to rotate the armature in anticlockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

### 29.2. Comparison of Generator and Motor Action

As said above, the same d.c. machine can be used, at least theoretically, interchangeably as a generator or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation.

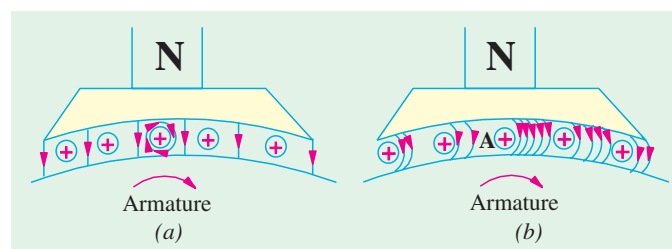


Fig. 29.2

Let us first consider its operation as a generator and see how exactly and through which agency, mechanical power is converted into electric power.

In Fig. 29.2 part of a generator whose armature is being driven clockwise by its prime mover is shown.

Fig. 29.2 (a) represents the fields set up independently by the main poles and the armature conductors like *A* in the figure. The resultant field or magnetic lines on flux are shown in Fig. 29.2 (b).



It is seen that there is a crowding of lines of flux on the right-hand side of A. These magnetic lines of flux may be likened to the rubber bands under tension. Hence, the bent lines of flux up a mechanical force on A much in the same way as the bent elastic rubber band of a catapult produces a mechanical force on the stone piece. It will be seen that this force is in a direction opposite to that of armature rotation. Hence, it is known as backward force or magnetic drag on the conductors. It is against this drag action on all armature conductor that the prime mover has to work. The work done in overcoming this opposition is converted into electric energy. Therefore, it should be clearly understood that it is only through the instrumentality of this magnetic drag that energy conversion is possible in a d.c. generator\*.

Next, suppose that the above d.c. machine is uncoupled from its prime mover and that current is sent through the armature conductors under a N-pole in the downward direction as shown in Fig. 29.3 (a). The conductors will again experience a force in the anticlockwise direction (Fleming's Left hand Rule). Hence, the machine will start rotating anticlockwise, thereby developing a torque which can produce mechanical rotation. The machine is then said to be motoring.

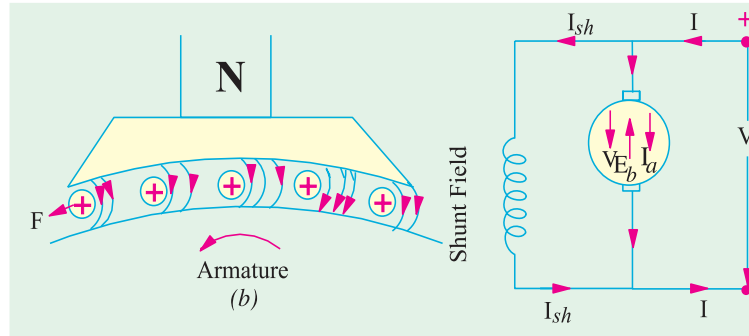


Fig. 29.3 (a)

Fig. 29.3 (b)

As said above, energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In the case of a generator, it was the magnetic drag which provided the necessary opposition. But what is the equivalent of that drag in the case of a motor? Well, it is the back e.m.f. It is explained in this manner:

As soon as the armature starts rotating, dynamically (or motionally) induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f. as found by Fleming's Right-hand Rule, is outwards *i.e.*, in direct opposition to the applied voltage (Fig. 29.3 (b)). This is why it is known as back e.m.f.  $E_b$  or counter e.m.f. Its value is the same as for the motionally induced e.m.f. in the generator *i.e.*  $E_b = (\Phi Z N) \times (P/A)$  volts. The applied voltage  $V$  has to be forced current through the armature conductors against this back e.m.f.  $E_b$ . The electric work done in overcoming this opposition is converted into mechanical energy developed in the armature. Therefore, it is obvious that but for the production of this opposing e.m.f. energy conversion would not have been possible.

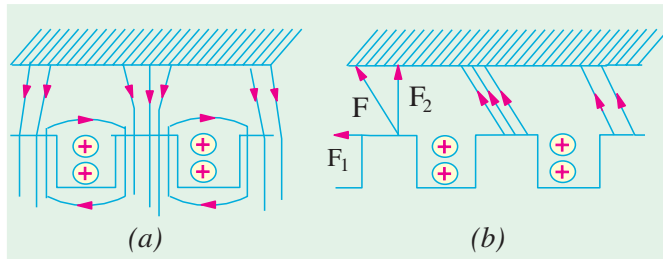


Fig. 29.4

Now, before leaving this topic, let it be pointed out that in an actual

motor with slotted armature, the torque is not due to mechanical force on the conductors themselves, but due to tangential pull on the armature teeth as shown in Fig. 29.4.

It is seen from Fig. 29.4 (a) that the main flux is concentrated in the form of tufts at the armature teeth while the armature flux is shown by the dotted lines embracing the armature slots. The effect of

\* In fact, it seems to be one of the fundamental laws of Nature that no energy conversion from one form to another is possible until there is some one to oppose the conversion. But for the presence of this opposition, there would simply be no energy conversion. In generators, opposition is provided by magnetic drag whereas in motors, back e.m.f. does this job. Moreover, it is only that part of the input energy which is used for overcoming this opposition that is converted into the other form.

armature flux on the main flux, as shown in Fig. 29.4 (b), is two-fold :

- (i) It increases the flux on the left-hand side of the teeth and decreases it on the right-hand side, thus making the distribution of flux density across the tooth section unequal.
- (ii) It inclines the direction of lines of force in the air-gap so that they are not radial but are disposed in a manner shown in Fig. 29.4 (b). The pull exerted by the poles on the teeth can now be resolved into two components. One is the tangential component  $F_1$  and the other vertical component  $F_2$ . The vertical component  $F_2$ , when considered for all the teeth round the armature, adds up to zero. But the component  $F_1$  is not cancelled and it is this tangential component which, acting on all the teeth, gives rise to the armature torque.

### 29.3. Significance of the Back e.m.f.

As explained in Art 29.2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's Right-hand Rule, is in opposition to the applied voltage (Fig. 29.5). Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f.  $E_b$ . The equivalent circuit of a motor is shown in Fig. 29.6. The rotating armature generating the back e.m.f.  $E_b$  is like a battery of e.m.f.  $E_b$  put across a supply mains of  $V$  volts. Obviously,  $V$  has to drive  $I_a$  against the opposition of  $E_b$ . The power required to overcome this opposition is  $E_b I_a$ .

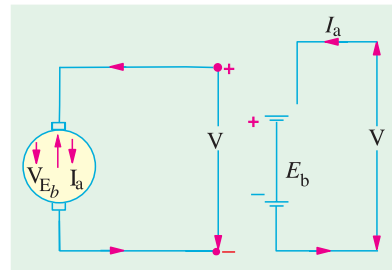


Fig. 29.5

In the case of a cell, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

$$\text{It will be seen that } I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{V - E_b}{R_a}$$

where  $R_a$  is the resistance of the armature circuit. As pointed out above,

$$E_b = \Phi ZN \times (P/A) \text{ volt where } N \text{ is in r.p.s.}$$

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high,  $E_b$  is large, hence armature current  $I_a$ , seen from the above equation, is small. If the speed is less, then  $E_b$  is less, hence more current flows which develops motor torque (Art 29.7). So, we find that  $E_b$  acts like a governor *i.e.*, it makes a motor self-regulating so that it draws as much current as is just necessary.

### 29.4. Voltage Equation of a Motor

The voltage  $V$  applied across the motor armature has to

- (i) overcome the back e.m.f.  $E_b$  and
- (ii) supply the armature ohmic drop  $I_a R_a$ .

$$\therefore V = E_b + I_a R_a$$

This is known as voltage equation of a motor.

Now, multiplying both sides by  $I_a$ , we get

$$V I_a = E_b I_a + I_a^2 R_a$$

As shown in Fig. 29.6,

$$V I_a = \text{Electrical input to the armature}$$

$$E_b I_a = \text{Electrical equivalent of mechanical power developed in the armature}$$

$$I_a^2 R_a = \text{Cu loss in the armature}$$

Hence, out of the armature input, some is wasted in  $I^2 R$  loss and the rest is converted into mechanical power *within* the armature.

It may also be noted that motor efficiency is given by the ratio of power developed by the arma-

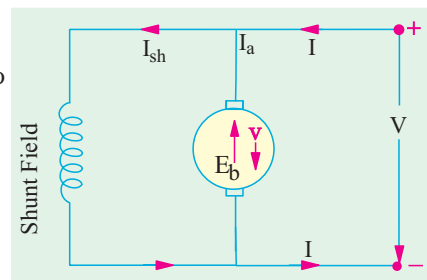


Fig. 29.6

ture to its input *i.e.*,  $E_b I_a / V I_a = E_b / V$ . Obviously, higher the value of  $E_b$  as compared to  $V$ , higher the motor efficiency.

### 29.5. Condition for Maximum Power

The gross mechanical power developed by a motor is  $P_m = V I_a - I_a^2 R_a$ .

Differentiating both sides with respect to  $I_a$  and equating the result to zero, we get

$$dP_m/dI_a = V - 2I_a R_a = 0 \quad \therefore I_a R_a = V/2$$

$$\text{As } V = E_b + I_a R_a \quad \text{and} \quad I_a R_a = V/2 \quad \therefore E_b = V/2$$

Thus gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

**Example 29.1.** A 220-V d.c. machine has an armature resistance of  $0.5 \Omega$ . If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as (i) generator (ii) motor.

(Electrical Technology-I, Bombay Univ. 1987)

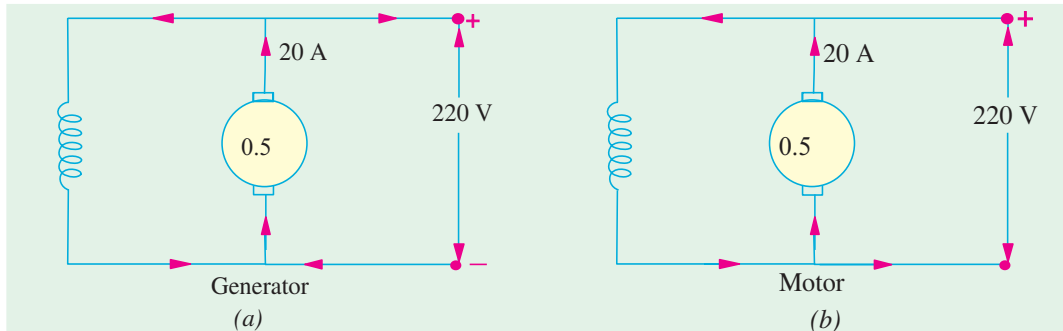


Fig. 29.7

**Solution.** As shown in Fig. 29.7, the d.c. machine is assumed to be shunt-connected. In each case, shunt current is considered negligible because its value is not given.

(a) As Generator [Fig. 29.7(a)]  $E_g = V + I_a R_a = 220 + 0.5 \times 20 = 230 \text{ V}$

(b) As Motor [Fig. 29.7 (b)]  $E_b = V - I_a R_a = 220 - 0.5 \times 20 = 210 \text{ V}$

**Example 29.2.** A separately excited D.C. generator has armature circuit resistance of  $0.1 \text{ ohm}$  and the total brush-drop is  $2 \text{ V}$ . When running at  $1000 \text{ r.p.m.}$ , it delivers a current of  $100 \text{ A}$  at  $250 \text{ V}$  to a load of constant resistance. If the generator speed drop to  $700 \text{ r.p.m.}$ , with field-current unaltered, find the current delivered to load. (AMIE, Electrical Machines, 2001)

**Solution.**  $R_L = 250/100 = 2.5 \text{ ohms.}$

$$E_{g1} = 250 + (100 \times 0.1) + 2 = 262 \text{ V.}$$

$$\text{At } 700 \text{ r.p.m., } E_{g2} = 262 \times 700/1000 = 183.4 \text{ V}$$

$$\text{If } I_a \text{ is the new current, } E_{g2} - 2 - (I_a \times 0.1) = 2.5 I_a$$

This gives  $I_a = 96.77 \text{ amp.}$

**Extension to the Question :** With what load resistance will the current be  $100 \text{ amp}$ , at  $700 \text{ r.p.m.}$ ?

**Solution.**  $E_{g2} - 2 - (I_a \times 0.1) = R_L \times I_a$

For  $I_a = 100 \text{ amp}$ , and  $E_{g2} = 183.4 \text{ V}$ ,  $R_L = 1.714 \text{ ohms.}$

**Example 29.3.** A  $440\text{-V}$ , shunt motor has armature resistance of  $0.8 \Omega$  and field resistance of  $200 \Omega$ . Determine the back e.m.f. when giving an output of  $7.46 \text{ kW}$  at  $85 \text{ percent}$  efficiency.

**Solution.** Motor input power =  $7.46 \times 10^3 / 0.85 \text{ W}$

Motor input current =  $7460/0.85 \times 440 = 19.95 \text{ A}$  ;  $I_{sh} = 440/200 = 2.2 \text{ A}$

$I_a = 19.95 - 2.2 = 17.75 \text{ A}$  ; Now,  $E_b = V - I_a R_a$

$\therefore E_b = 440 - (17.75 \times 0.8) = 425.8 \text{ V}$

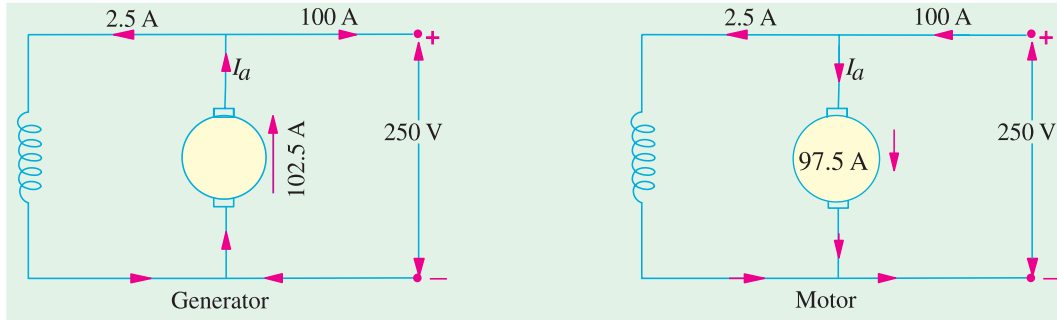


Fig. 29.8 (a)

Fig. 29.8 (b)

**Example 29.4.** A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of  $0.06 \Omega$  and  $100 \Omega$  respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.

(Electrical Technology, Punjab Univ., June 1991)

**Solution. As Generator** [Fig. 29.8 (a)]

Output current =  $25,000/250 = 100 \text{ A}$  ;  $I_{sh} = 250/100 = 2.5 \text{ A}$  ;  $I_a = 102.5 \text{ A}$

Generated e.m.f. =  $250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15 \text{ V}$

Power developed in armature =  $E_b I_a = \frac{256.15 \times 102.5}{1000} = 26.25 \text{ kW}$

**As Motor** [Fig 29.8 (b)]

Motor input current =  $100 \text{ A}$  ;  $I_{sh} = 2.5 \text{ A}$ ,  $I_a = 97.5 \text{ A}$

$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15 \text{ V}$

Power developed in armature =  $E_b I_a = 244.15 \times 97.5/1000 = 23.8 \text{ kW}$

**Example 29.5.** A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has  $r_a = 2$  and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine. [Sambalpur University, 1998]

**Solution.** Current distributions during two actions are indicated in Fig. 29.9 (a) and (b). As a generator,  $I_a = 13 \text{ amp}$

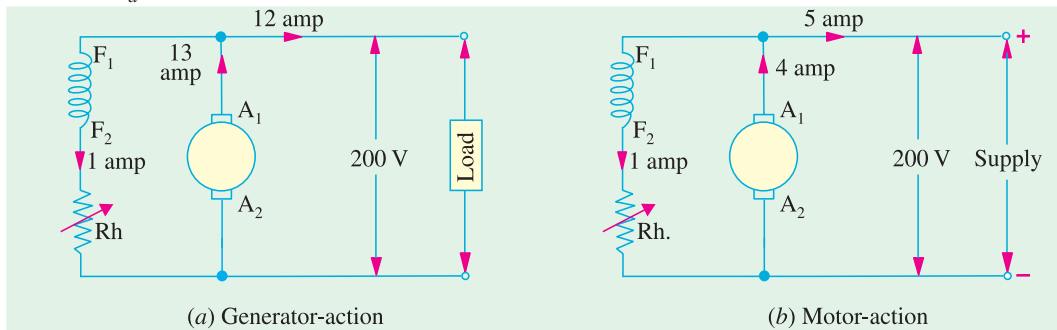


Fig. 29.9

$E_g = 200 + 13 \times 2 = 226 \text{ V}$

$$\phi \frac{ZN}{60} \times \frac{P}{a} = 226$$

For a Lap-wound armature,

$$P = a$$

$$\therefore \phi = \frac{226 \times 60}{1000 \times 32} = 0.42375 \text{ wb}$$

As a motor,

$$I_a = 4 \text{ amp}$$

$$E_b = 200 - 4 \times 2 = 192 \text{ V}$$

$$= \phi ZN/60$$

$$\text{Giving } N = \frac{60 \times 192}{0.42375 \times 32}$$

$$= 850 \text{ r.p.m.}$$

### Tutorial Problems 29.1

1. What do you understand by the term 'back e.m.f.'? A d.c. motor connected to a 460-V supply has an armature resistance of 0.15 Ω. Calculate

(a) The value of back e.m.f. when the armature current is 120 A.

(b) The value of armature current when the back e.m.f. is 447.4 V. **[(a) 442 V (b) 84 A]**

2. A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of 0.25 Ω, calculate the value of the back e.m.f. at this load. **[430 V]**

3. A 4-pole d.c. motor takes an armature current of 150 A at 440 V. If its armature circuit has a resistance of 0.15 Ω, what will be the value of back e.m.f. at this load? **[417.5 V]**

### 29.6. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius  $r$  metre acted upon by a circumferential force of  $F$  Newton which causes it to rotate at  $N$  r.p.m. (Fig. 29.10).

Then torque  $T = F \times r$  Newton-metre (N - m)

Work done by this force in one revolution

$$= \text{Force} \times \text{distance} = F \times 2\pi r \text{ Joule}$$

Power developed =  $F \times 2\pi r \times N$  Joule/second or Watt

$$= (F \times r) \times 2\pi N \text{ Watt}$$

Now  $2\pi N =$  Angular velocity  $\omega$  in radian/second and  $F \times r =$  Torque  $T$

$\therefore$  Power developed =  $T \times \omega$  watt or  $P = T \omega$  Watt

Moreover, if  $N$  is in r.p.m., then

$$\omega = 2\pi N/60 \text{ rad/s}$$

$$\therefore P = \frac{2\pi N}{60} \times T \text{ or } P = \frac{2\pi}{60} \cdot NT = \frac{NT}{9.55}$$

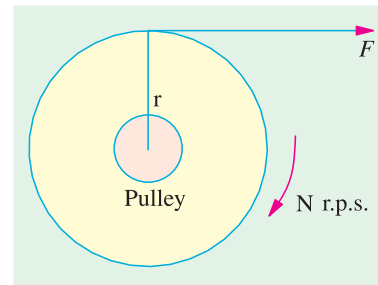


Fig. 29.10

### 29.7. Armature Torque of a Motor

Let  $T_a$  be the torque developed by the armature of a motor running at  $N$  r.p.s. If  $T_a$  is in  $N/M$ , then power developed =  $T_a \times 2\pi N$  watt ...(i)

We also know that electrical power converted into mechanical power in the armature (Art 29.4)

$$= E_b I_a \text{ watt} \quad \dots(ii)$$

Equating (i) and (ii), we get  $T_a \times 2\pi N = E_b I_a \quad \dots(iii)$

Since  $E_b = \Phi ZN \times (P/A)$  volt, we have

$$T_a \times 2\pi N = \Phi ZN \left(\frac{P}{A}\right) \cdot I_a \text{ or } T_a = \frac{1}{2\pi} \cdot \Phi Z I_0 \left(\frac{P}{A}\right) N\text{-m}$$

$$= 0.159 \text{ N newton metre}$$

$$\therefore T_a = 0.159 \Phi Z I_a \times (P/A) N\text{-m}$$

**Note.** From the above equation for the torque, we find that  $T_a \propto \Phi I_a$ .

(a) In the case of a series motor,  $\Phi$  is directly proportional to  $I_a$  (before saturation) because field windings carry full armature current  $\therefore T_a \propto I_a^2$

(b) For shunt motors,  $\Phi$  is practically constant, hence  $T_a \propto I_a$ .

As seen from (iii) above

$$T_a = \frac{E_b I_a}{2\pi N} \text{ N - m - N in r.p.s.}$$

If  $N$  is in r.p.m., then

$$T_a = \frac{E_b I_a}{2\pi N / 60} = 60 \frac{E_b I_a}{2\pi N} = \frac{60}{2\pi} \frac{E_b I_a}{N} = 9.55 \frac{E_b I_a}{N} \text{ N-m}$$

### 29.8. Shaft Torque ( $T_{sh}$ )

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque  $T_{sh}$ . It is so called because it is available at the shaft. The motor output is given by

Output =  $T_{sh} \times 2\pi N$  Watt provided  $T_{sh}$  is in N-m and  $N$  in r.p.s.

$$\therefore T_{sh} = \frac{\text{Output in watts}}{2\pi N} \text{ N-m -N in r.p.s}$$

$$= \frac{\text{Output in watts}}{2\pi N / 60} \text{ N-m -N in r.p.m.}$$

$$= \frac{60}{2\pi} \frac{\text{output}}{N} = 9.55 \frac{\text{Output}}{N} \text{ N-m.}$$

The difference ( $T_a - T_{sh}$ ) is known as lost torque and is due to iron and friction losses of the motor.

**Note.** The value of back e.m.f.  $E_b$  can be found from

(i) the equation,  $E_b = V - I_a R_a$

(ii) the formula  $E_b = \Phi ZN \times (P/A)$  volt

**Example 29.6.** A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2  $\Omega$ . The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate (i), the speed and (ii) the gross torque developed by the armature. **(Elect. Machines, A.M.I.E. Sec B, 1989)**

**Solution.**  $E_b = 480 - 110 \times 0.2 = 458 \text{ V}, \quad \Phi = 0.05 \text{ Wb}, Z = 864$

Now,  $E_b = \frac{\Phi ZN}{60} \left(\frac{P}{A}\right)$  or  $458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right)$

$$\therefore N = 636 \text{ r.p.m.}$$

$$T_a = 0.159 \times 0.05 \times 864 \times 110 (6/6) = 756.3 \text{ N-m}$$

**Example 29.7.** A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.

(Elect. Engg.-II, Pune Univ. 1991)

**Solution.**

$$E_b = \Phi ZN (P/A)$$

Now,

$$E_b = V - I_a R_a = 250 - 40 \times 0.75 = 220 \text{ V}$$

$$\therefore 220 = 25 \times 10^{-3} \times 782 \times N \times 0.75 = 220 \text{ V}$$

$$\therefore 220 = 0.159 \Phi Z I_a (P/A)$$

$$= 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) = \mathbf{249 \text{ N-m}}$$

**Example 29.8.** A d.c. shunt machine develops an a.c. e.m.f. of 250 V at 1500 r.p.m. Find its torque and mechanical power developed for an armature current of 50 A. State the simplifying assumptions.

(Basic Elect. Machine Nagpur Univ., 1993)

**Solution.** A given d.c. machine develops the same e.m.f. in its armature conductors whether running as a generator or as a motor. Only difference is that this armature e.m.f. is known as back e.m.f. when the machine is running as a motor.

Mechanical power developed in the arm =  $E_b I_a = 250 \times 50 = 12,500 \text{ W}$   
 $T_a = 9.55 E_b I_a / N = 9.55 \times 250 \times 50 / 1500 = \mathbf{79.6 \text{ N-m.}}$

**Example 29.9.** Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

(Elect. Machine AMIE Sec. B Winter 1991)

**Solution.** Developed torque or gross torque is the same thing as armature torque.

$$\therefore T_a = 0.159 \Phi Z A (P/A)$$

$$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45 (4/2) = 286.2 \text{ N-m}$$

$$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193 \text{ V}$$

Now,

$$E_b = \Phi ZN (P/A) \text{ or } 193 = 25 \times 10^{-3} \times 800 \times N \pi \times (4/2)$$

$$\therefore N = 4.825 \text{ r.p.s.}$$

Also,

$$2\pi N T_{sh} = \text{output or } 2\pi \times 4.825 T_{sh} = 8200 \quad \therefore T_{sh} = 270.5 \text{ N-m}$$

**Example 29.10.** A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that  $R_a = 0.2 \Omega$ .

(Electrical Technology-II, Gwalior Univ. 1985)

**Solution.** As seen from Art 27.7,  $T_a \propto \Phi I_a$ . Since  $\Phi$  is constant,  $T_a \propto I_a$

$$\therefore T_{a1} \propto I_{a1} \text{ and } T_{a2} \propto I_{a2} \quad \therefore T_{a2}/T_{a1} = I_{a2}/I_{a1}$$

$$\therefore 2 = I_{a2}/50 \text{ or } I_{a2} = 100 \text{ A}$$

Now,  $N_2/N_1 = E_{b2}/E_{b1}$  — since  $\Phi$  remains constant.

$$F_{b1} = 220 - (50 \times 0.2) = 210 \text{ V} \quad E_{b2} = 220 - (100 \times 0.2) = 200 \text{ V}$$

$$\therefore N_2/500 = 200/210 \quad \therefore N_2 = \mathbf{476 \text{ r.p.m.}}$$

**Example 29.11.** A 500-V, 37.3 kW, 1000 r.p.m. d.c. shunt motor has on full-load an efficiency of 90 percent. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine (i) full-load line current (ii) full load shaft torque in N-m and (iii) total resistance in motor starter to limit the starting current to 1.5 times the full-load current.

(Elect. Engg. I; M.S. Univ. Baroda 1987)

**Solution.** (i) Motor input =  $37,300/0.9 = 41,444 \text{ W}$   
 F.L. line current =  $41,444/500 = \mathbf{82.9 \text{ A}}$

$$(ii) \quad T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{37,300}{1000} = 356 \text{ N-m}$$

$$(iii) \quad \text{Starting line current} = 1.5 \times 82.9 = 124.3 \text{ A}$$

$$\text{Arm. current at starting} = 124.3 - 1.8 = 122.5 \text{ A}$$

If  $R$  is the starter resistance (which is in series with armature), then  
 $122.5 (R + 0.24) + 2 = 500 \quad \therefore R = 3.825 \Omega$

**Example 29.12.** A 4-pole, 220-V shunt motor has 540 lap-wound conductor. It takes 32 A from the supply mains and develops output power of 5.595 kW. The field winding takes 1 A. The armature resistance is  $0.09 \Omega$  and the flux per pole is 30 mWb. Calculate (i) the speed and (ii) the torque developed in newton-metre. (Electrical Technology, Nagpur Univ. 1992)

**Solution.**  $I_a = 32 - 1 = 31 \text{ A}$ ;  $E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2 \text{ V}$

Now, 
$$E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \quad \therefore 217.2 = \frac{30 \times 10^{-3} \times 540 \times N}{60} \left( \frac{4}{4} \right)$$

(i)  $\therefore N = 804.4 \text{ r.p.m.}$

(ii) 
$$T_{sh} = 9.55 \times \frac{\text{output in watts}}{N} = 9.55 \times \frac{5,595}{804.4} = 66.5 \text{ N-m}$$

**Example 29.13 (a).** Find the load and full-load speeds for a four-pole, 220-V, and 20-kW, shunt motor having the following data :

Field-current = 5 amp, armature resistance = 0.04 ohm,

Flux per pole = 0.04 Wb, number of armature-conductors = 160, Two-circuit wave-connection, full load current = 95 amp, No load current = 9 A. Neglect armature reaction.

(Bharathithasan Univ. April 1997)

**Solution.** The machine draws a supply current of 9 amp at no load. Out of this, 5 amps are required for the field circuit, hence the armature carries a no-load current of 4 amp.

At load, armature-current is 90 amp. The armature-resistance-drop increases and the back e.m.f. decreases, resulting into decrease in speed under load compared to that at No-Load.

At No Load :  $E_{ao} = 220 - 4 \times 0.04 = 219.84 \text{ volts}$

Substituting this,

$$0.04 \times 160 \times (N/60) \times (4/2) = 219.84$$

No-Load speed,  $N_0 = 1030.5 \text{ r.p.m.}$

At Full Load : Armature current = 90 A,  $E_a = 200 - 90 \times 0.04 = 216.4 \text{ V}$

$$N = (216.4/219.84) \times 1030.5 = 1014.4 \text{ r.p.m.}$$

**Example 29.13 (b).** Armature of a 6-pole, 6-circuit D.C. shunt motor takes 400 A at a speed of 350 r.p.m. The flux per pole is 80 milli-webers, the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron-loss. Calculate the brake-horse-power.

(Manonmaniam Sundaranar Univ. Nov. 1998)

**Solution.** Number of armature turns = 600

Therefore,  $Z = \text{Number of armature conductors} = 1200$

If electromagnetic torque developed is  $T \text{ Nw - m}$ ,

$$\begin{aligned} \text{Armature power} &= T \omega = T \times 2 \pi 350/60 \\ &= 36.67 T \text{ watts} \end{aligned}$$

To calculate armature power in terms of Electrical parameters,  $E$  must be known.

$$E = \phi Z (N/60) (P/A)$$



$$= 80 \times 10^{-3} \times 1200 \times (350/60) \times (6/6)$$

$$= 560 \text{ volts}$$

With the armature current of 400 A, Armature power = 560 × 400 watts

Equating the two,

$T = 560 \times 400/36.67 = 6108.5 \text{ Nw - m}$ . Since 3 % of this torque is required for overcoming different loss-terms,

$$\text{Net torque} = 0.97 \times 6108.5 = 5925 \text{ Nw - m}$$

For Brake-Horse-Power, net output in kW should be computed first. Then “kW” is to be converted to “BHP”, with 1 HP = 0.746 kW.

$$\text{Net output in kW} = 5925 \times 36.67 \times 10^{-3} = 217.27 \text{ kW}$$

Converting this to BHP, the output = 291.25 HP

**Example 29.13 (c).** Determine the torque established by the armature of a four-pole D.C. motor having 774 conductors, two paths in parallel, 24 milli-webers of pole-flux and the armature current is 50 Amps. (Bharathiar Univ. April 1998)

**Solution.** Expression for torque in terms of the parameters concerned in this problem is as follows :

$$T = 0.159 \phi Z I_a p/a \text{ Nw - m}$$

Two paths in parallel for a 4-pole case means a wave winding.

$$T = 0.159 \times (24 \times 10^{-3}) \times 774 \times 50 \times 4/2$$

$$= 295.36 \text{ Nw-m}$$

**Example 29.13 (d).** A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms, determine the efficiency of the machine running as a generator delivering a load current of 40 Amps. (Bharathiar Univ. April 1998)

**Solution. (i)** No Load, running as a motor :

$$\text{Input Power} = 500 \times 5 = 2500 \text{ watts}$$

$$\text{Field copper-loss} = 500 \times 2.5 = 1250 \text{ watts}$$

Neglecting armature copper-loss at no load (since it comes out to be  $2.5^2 \times 0.15 = 1$  watt), the balance of 1250 watts of power goes towards no load losses of the machine running at rated speed. These losses are mainly the no load mechanical losses and the core-loss.

**(ii)** As a Generator, delivering 40 A to load :

$$\text{Output delivered} = 500 \times 40 \times 10^{-3} = 20 \text{ kW}$$

Losses : **(a)** Field copper-loss = 1250 watts

**(b)** Armature copper-loss =  $42.5^2 \times 0.15 = 271$  watts

**(c)** No load losses = 1250 watts

Total losses = 2.771 kW

Generator Efficiency =  $(20/22.771) \times 100 \% = 87.83 \%$

**Extension to the Question :** At what speed should the Generator be run, if the shunt-field is not changed, in the above case ? Assume that the motor was running at 600 r.p.m. Neglect armature reaction.

**Solution.** As a motor on no-load,

$$E_{b0} = 500 - I_a r_a = 500 - 0.15 \times 2.5 = 499.625 \text{ V}$$

As a Generator with an armature current of 42.5 A,

$$E_{b0} = 500 + 42.5 \times 0.15 = 506.375 \text{ V}$$

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp. With armature reaction is ignored, the flux/pole remains same. The e.m.f. then becomes proportional to the speed. If the generator must be driven at  $N$  r.p.m.

$$N = (506.375/449.625) \times 600 = 608.1 \text{ r.p.m.}$$

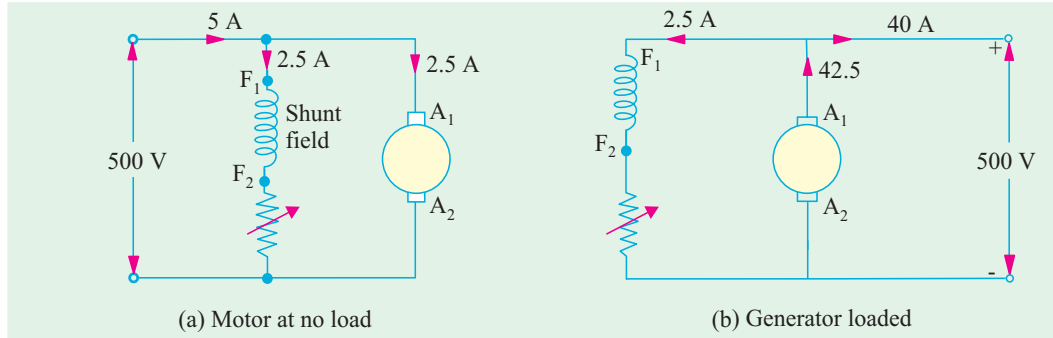


Fig. 29.11

**Note.** Alternative to this slight increase in the speed is to increase the field current with the help of decreasing the rheostatic resistance in the field-circuit.

**Example 29.13 (e).** A d.c. series motor takes 40 A at 220 V and runs at 800 r.p.m. If the armature and field resistance are 0.2 Ω and 0.1 Ω respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor ?

**Solution.** Armature torque is given by  $T_a = 9.55 \frac{E_b I_a}{N}$  N-m

Now  $E_b = V - I_a (R_a + R_{se}) = 220 - 40 (0.2 + 0.1) = 208 \text{ V}$

$\therefore T_a = 9.55 \times 208 \times 40/800 = 99.3 \text{ N-m}$

Cu loss in armature and series-field resistance =  $40^2 \times 0.3 = 480 \text{ W}$

Iron and friction losses = 500 W ; Total losses = 480 + 500 = 980 W

Motor power input =  $220 \times 40 = 8,800 \text{ W}$

Motor output =  $8,800 - 980 = 7,820 \text{ W} = 7.82 \text{ kW}$

**Example 29.14.** A cutting tool exerts a tangential force of 400 N on a steel bar of diameter 10 cm which is being turned in a simple lathe. The lathe is driven by a chain at 840 r.p.m. from a 220 V d.c. Motor which runs at 1800 r.p.m. Calculate the current taken by the motor if its efficiency is 80 %. What size is the motor pulley if the lathe pulley has a diameter of 24 cm ?

(Elect. Technology-II, Gwalior Univ. 1985)

**Solution.** Torque  $T_{sh} = \text{Tangential force} \times \text{radius} = 400 \times 0.05 = 20 \text{ N-m}$

Output power =  $T_{sh} \times 2\pi N \text{ watt} = 20 \times 2\pi \times (840/60) \text{ watt} = 1,760 \text{ W}$

Motor  $\eta = 0.8 \therefore$  Motor input =  $1,760/0.8 = 2,200 \text{ W}$

Current drawn by motor =  $2200/220 = 10 \text{ A}$

Let  $N_1$  and  $D_1$  be the speed and diameter of the driver pulley respectively and  $N_2$  and  $D_2$  the respective speed and diameter of the lathe pulley.

Then  $N_1 \times D_1 = N_2 \times D_2$  or  $1,800 \times D_1 = 840 \times 0.24$

$\therefore D_1 = 840 \times 0.24/1,800 = 0.112 \text{ m} = 11.2 \text{ cm}$

**Example 29.15.** The armature winding of a 200-V, 4-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3 Ω; the armature resistance 0.5 Ω and the iron and friction losses total 800 W. The pulley diameter is 0.41 m. Find the pull in newton at the rim of the pulley.

(Elect. Engg. AMIETE Sec. A. 1991)

**Solution.**

$$E_b = V - I_a R_a = 200 - 45(0.5 + 0.3) = 164 \text{ V}$$

Now

$$E_b = \frac{\Phi ZN}{60} \cdot \left(\frac{P}{A}\right) \text{ volt}$$

$$\therefore 164 = \frac{18 \times 10^{-3} \times 280 \times 4 \times N}{60} \times \frac{4}{4} \quad \therefore N = 488 \text{ r.p.m.}$$

$$\text{Total input} = 200 \times 45 = 9,000 \text{ W}; \text{ Cu loss} = I_a^2 R_a = 45^2 \times 0.8 = 1,620 \text{ W}$$

$$\text{Iron + Friction losses} = 800 \text{ W}; \text{ Total losses} = 1,620 + 800 = 2,420 \text{ W}$$

$$\text{Output} = 9,000 - 2,420 = 6,580 \text{ W}$$

$$\therefore T_{sh} = 9 \times 55 \times \frac{6580}{488} = 128 \text{ N-m}$$

Let  $F$  be the pull in newtons at the rim of the pulley.

$$\text{Then } F \times 0.205 = 128.8 \quad \therefore F = 128.8/0.205 \text{ N} = \mathbf{634 \text{ N}}$$

**Example 29.16.** A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1  $\Omega$ . Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) rotational losses and (e) efficiency.

**Solution.**

$$E_b = V - I_a R_a - \text{brush drop} = 240 - (50 \times 0.1) - 2 = 233 \text{ V}$$

Also

$$I_a = 50 \text{ A}$$

$$(a) \quad \text{Armature torque } T_a = 9.55 \frac{E_b I_a}{N} \text{ N-m} = 9.55 \times \frac{233 \times 50}{1000} = \mathbf{111 \text{ N-m}}$$

$$(b) \quad T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{11,190}{1000} = \mathbf{106.9 \text{ N-m}}$$

$$(c) \quad E_b = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

$$\therefore 233 = \frac{\Phi \times 540 \times 1000}{60} \times \left(\frac{4}{2}\right) \quad \therefore \Phi = \mathbf{12.9 \text{ mWb}}$$

$$(d) \quad \text{Armature input} = V I_a = 240 \times 50 = \mathbf{12,000 \text{ W}}$$

$$\text{Armature Cu loss} = I_a^2 R_a = 50^2 \times 0.1 = 250 \text{ W}; \text{ Brush contact loss} = 50 \times 2 = 100 \text{ W}$$

$$\therefore \text{Power developed} = 12,000 - 350 = 11,650 \text{ W}; \text{ Output} = 11.19 \text{ kW} = 11,190 \text{ W}$$

$$\therefore \text{Rotational losses} = 11,650 - 11,190 = \mathbf{460 \text{ W}}$$

$$(e) \quad \text{Total motor input} = VI = 240 \times 51 = 12,340 \text{ W}; \text{ Motor output} = 11,190 \text{ W}$$

$$\therefore \text{Efficiency} = \frac{11,190}{12,340} \times 100 = \mathbf{91.4 \%}$$

**Example 29.17.** A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8  $\Omega$ . Assume flux is proportional to the field current.

(Elect. Engg.-II, Kerala Univ. 1988)

**Solution.** Since  $\Phi \propto I_a$ , hence  $T \propto I_a^2$ 

$$\therefore T_1 \propto 40^2 \text{ and } T_2 \propto 30^2 \quad \therefore \frac{T_2}{T_1} = \frac{9}{16}$$

 $\therefore$  Percentage change in torque is

$$= \frac{T_1 - T_2}{T_1} \times 100 = \frac{7}{16} \times 100 = \mathbf{43.75 \%}$$

$$\text{Now } E_{b1} = 460 - (40 \times 0.8) = 428 \text{ V}; E_{b2} = 460 - (30 \times 0.8) = 436 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad \therefore \frac{N_2}{500} = \frac{436}{428} \times \frac{40}{30} \quad \therefore N_2 = \mathbf{679 \text{ r.p.m.}}$$

**Example 29.18.** A 460-V, 55.95 kW, 750 r.p.m. shunt motor drives a load having a moment of inertia of  $252.8 \text{ kg-m}^2$ . Find approximate time to attain full speed when starting from rest against full-load torque if starting current varies between 1.4 and 1.8 times full-load current.

**Solution.** Let us suppose that the starting current has a steady value of  $(1.4 + 1.8)/2 = 1.6$  times full-load value.

$$\text{Full-load output} = 55.95 \text{ kW} = 55,950 \text{ W} ; \text{ Speed} = 750 \text{ r.p.m.} = 12.5 \text{ r.p.s.}$$

$$\text{F.L. shaft torque } T = \text{power}/\omega = \text{power}/2\pi N = 55,950 / \pi \times (750/60) = 712.4 \text{ N-m}$$

During starting period, average available torque

$$= 1.6 T - T = 0.6 T = 0.6 \times 712.4 = 427.34 \text{ N-m}$$

This torque acts on the moment of inertia  $I = 252.8 \text{ kg-m}^2$ .

$$\therefore 427.4 = 252.8 \times \frac{d\omega}{dt} = 252.8 \times \frac{2\pi \times 12.5}{dt}, \quad \therefore dt = 46.4 \text{ s}$$

**Example 29.19.** A 14.92 kW, 400 V, 400 r.p.m. d.c. shunt motor draws a current of 40 A when running at full-load. The moment of inertia of the rotating system is  $7.5 \text{ kg-m}^2$ . If the starting current is 1.2 times full-load current, calculate

(a) full-load torque

(b) the time required for the motor to attain the rated speed against full-load.

(Electrical Technology, Gujarat Univ. 1988)

**Solution.** (a) F.L. output  $14.92 \text{ kW} = 14,920 \text{ W}$  ; Speed = 400 r.p.m. =  $20/3$  r.p.s

$$\text{Now, } T\omega = \text{output} \therefore T = 14,920 / 2\pi \times (20/3) = 356 \text{ N-m}$$

(b) During the starting period, the torque available for accelerating the motor armature is

$$= 1.2 T - T = 0.2 T = 0.2 \times 356 = 71.2 \text{ N-m}$$

$$\text{Now, torque} = I \frac{d\omega}{dt} \quad \therefore 71.2 = 7.5 \times \frac{2\pi \times (20/3)}{dt} \quad \therefore dt = 4.41 \text{ second}$$

### 29.9. Speed of a D.C. Motor

From the voltage equation of a motor (Art. 27.4), we get

$$E_b = V - I_a R_a \quad \text{or} \quad \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) = V - I_a R_a$$

$$\therefore N = \frac{V - I_a R_a}{\Phi} \times \left( \frac{60A}{ZP} \right) \text{ r.p.m.}$$

$$\text{Now} \quad V - I_a R_a = E_b \quad \therefore N = \frac{E_b}{\Phi} \times \left( \frac{60A}{ZP} \right) \text{ r.p.m. or } N = K \frac{E_b}{\Phi}$$

It shows that speed is directly proportional to back e.m.f.  $E_b$  and inversely to the flux  $\Phi$  on  $N \propto E_b/\Phi$ .

#### For Series Motor

Let  $N_1 =$  Speed in the 1st case ;  $I_{a1} =$  armature current in the 1st case

$\Phi_1 =$  flux/pole in the first case

$N_2, I_{a2}, \Phi_2 =$  corresponding quantities in the 2nd case.

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\Phi_1} \quad \text{where } E_{b1} = V - I_{a1} R_a ; N_2 \propto \frac{E_{b2}}{\Phi_2} \quad \text{where } E_{b2} = V - I_{a2} R_a$$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\text{Prior to saturation of magnetic poles ; } \Phi \propto I_a \quad \therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

**For Shunt Motor**

In this case the same equation applies,

$$i.e., \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{If } \Phi_2 = \Phi_1, \text{ then } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}.$$

**29.10. Speed Regulation**

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed-controlling devices.

The speed regulation is defined *as the change in speed when the load on the motor is reduced from rated value to zero, expressed as percent of the rated load speed.*

$$\therefore \quad \% \text{ speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100 = \frac{dN}{N} \times 100$$

**29.11. Torque and Speed of a D.C. Motor**

It will be proved that though torque of a motor is admittedly a function of flux and armature current, *yet it is independent of speed*. In fact, it is the speed which depends on torque and not *vice-versa*. It has been proved earlier that

$$N = K \frac{V - I_a R_a}{\phi} = \frac{KE_b}{\Phi} \quad \dots \text{Art. 27.9}$$

Also,  $T_a \propto \Phi I_a \quad \dots \text{Art. 27.7}$

It is seen from above that increase in flux would decrease the speed but increase the armature torque. It cannot be so because torque always tends to produce rotation. If torque increases, motor speed must *increase* rather than *decrease*. The apparent inconsistency between the above two equations can be reconciled in the following way :

Suppose that the flux of a motor is decreased by decreasing the field current. Then, following sequence of events take place :

1. Back e.m.f.  $E_b (= N\Phi/K)$  drops instantly (the speed remains constant because of inertia of the heavy armature).
2. Due to decrease in  $E_b$ ,  $I_a$  is increased because  $I_a = (V - E_b)/R_a$ . Moreover, a small reduction in flux produces a proportionately large increase in armature current.
3. Hence, the equation  $T_a \propto \Phi I_a$ , a small decrease in  $\phi$  is more than counterbalanced by a *large* increase in  $I_a$  with the result that there is a net *increase* in  $T_a$ .
4. This increase in  $T_a$  produces an increase in motor speed.

It is seen from above that with the applied voltage  $V$  held constant, motor speed varies inversely as the flux. However, it is possible to increase flux and, at the same time, increase the speed provided  $I_a$  is held constant as is actually done in a d.c. servomotor.

**Example 29.20.** A 4-pole series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.

(Elect. Engg. AMIETE Sec. A Part II June 1991)

**Solution.**

$$\begin{aligned} T_a &= 0.159 \phi Z I_a \text{ (P/A) N-m} \\ \therefore 209 &= 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a (4/2); I_a = 20.1 \text{ A} \\ E_a &= V - I_a R_a = 500 - 20.1 \times 3 = 439.7 \text{ V} \end{aligned}$$

Now, speed may be found either by using the relation for  $E_b$  or  $T_a$  as given in Art.

$$E_b = \Phi ZN \times (P/A) \text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2$$

$$\therefore N = 6.73 \text{ r.p.s. or } \mathbf{382.2 \text{ r.p.m.}}$$

**Example 29.21.** A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2  $\Omega$  and 250  $\Omega$ . Calculate the speed when loaded and taking 50 A. Assume the flux to be constant. (Elect. Engg. A.M.Ae. S.I. June 1991)

**Solution.** Formula used :  $\frac{N}{N_0} = \frac{E_b}{E_{b0}} \times \frac{\Phi_0}{\Phi}$ ; Since  $\Phi_0 = \Phi$  (given);  $\frac{N}{N_0} = \frac{E_b}{E_{b0}}$

$$I_{sh} = 250/250 = 1 \text{ A}$$

$$E_{b0} = V - I_{a0} R_a = 250 - (7 \times 0.2) = 248.6 \text{ V}; E_b = V - I_a R_a = 250 - (49 \times 0.2) = 240.2 \text{ V}$$

$$\therefore \frac{N}{1000} = \frac{240.2}{248.6}; N = \mathbf{9666.1 \text{ r.p.m.}}$$

**Example 29.22.** A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15  $\Omega$  and its field resistance 0.1  $\Omega$ . Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A. (Electrical Machinery - I, Bangalore Univ. 1986)

**Solution.**  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ ;  $\Phi_2 = 0.45 \Phi_1$  or  $\frac{\Phi_1}{\Phi_2} = \frac{1}{0.45}$

$$E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ V}; E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ V}$$

$$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{1}{0.45}; N_2 = \mathbf{1940 \text{ r.p.m.}}$$

**Example 29.23.** A 230-V d.c. shunt motor has an armature resistance of 0.5  $\Omega$  and field resistance of 115  $\Omega$ . At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load. (Elect. Technology, Kerala Univ. 1988)

**Solution.**  $N_1 = 1200 \text{ r.p.m.}, E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V}$

$$N_2 = 1120 \text{ r.p.m.}, E_{b2} = 230 - 0.5 I_{a2}$$

Now,  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \therefore \frac{1120}{1200} = \frac{230 - 0.5 I_{a2}}{228.75}; I_{a2} = \mathbf{33 \text{ A}}$

Line current drawn by motor =  $I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35 \text{ A}}$

Power input at rated load =  $230 \times 35 = \mathbf{8,050 \text{ W}}$

**Example 29.24.** A belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V bus-bars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed? Given armature resistance = 0.025  $\Omega$ , field resistance = 60  $\Omega$  and contact drop under each brush = 1 V, Ignore armature reaction. (Elect. Machines (E-3) AMIE Sec.C Winter 1991)

**Solution. As Generator** [Fig. 29.12 (a)]

Load current,  $I = 100,000/220 = 454.55 \text{ A}; I_{sh} = 220/60 = 3.67 \text{ A}$

$$I_a = I + I_{sh} = 458.2 \text{ A}; I_a R_a = 458.2 \times 0.025 = 11.45$$

$$E_b = 220 + 11.45 + 2 \times 1 = 233.45 \text{ V}; N_1 = 300 \text{ r.p.m.}$$

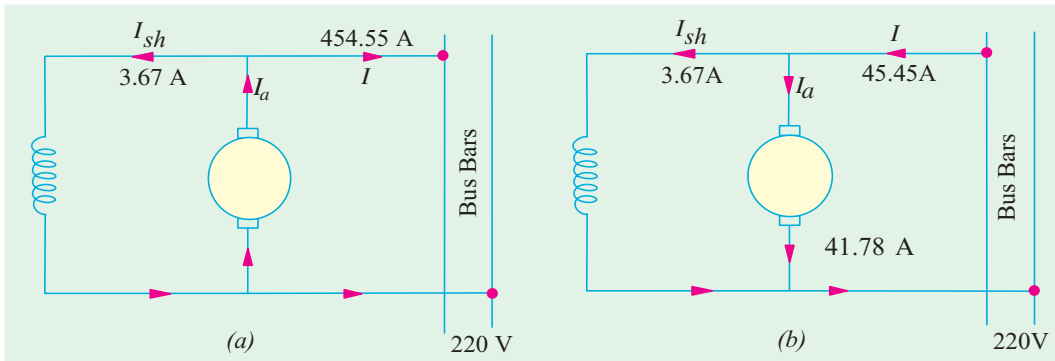


Fig. 29.12

**As Motor** [Fig. 29.12 (b)]

$$\begin{aligned} \text{Input line current} &= 100,000/220 = 45.45 \text{ A}; I_{sh} = 220/60 = 3.67 \text{ A} \\ I_a &= 45.45 - 3.67 = 41.78 \text{ A}; I_a R_a = 41.78 \times 0.025 = 1.04 \text{ V}; E_{b2} = 220 - 1.04 - 2 \times 1 = 216.96 \text{ V} \\ \frac{N_2}{N_1} &= \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \text{ since } \Phi_1 = \Phi_2 \text{ because } I_{sh} \text{ is constant} \\ \therefore \frac{N_2}{300} &= \frac{216.96}{233.45}; N_2 = 279 \text{ r.p.m.} \end{aligned}$$

**Example 29.25.** A d.c. shunt machine generates 250-V on open circuit at 1000 r.p.m. Effective armature resistance is 0.5 Ω, field resistance is 250 Ω, input to machine running as a motor on no-load is 4 A at 250 V. Calculate speed of machine as a motor taking 40 A at 250 V. Armature reaction weakens field by 4%. **(Electrical Machines-I, Gujarat Univ. 1987)**

**Solution.** Consider the case when the machine runs as a motor on no-load.

$$\text{Now, } I_{sh} = 250/250 = 1 \text{ A}; \text{ Hence, } I_{a0} = 4 - 1 = 3 \text{ A}; E_{b0} = 250 - 0.5 \times 3 = 248.5 \text{ V}$$

It is given that when armature runs at 1000 r.p.m., it generates 250 V. When it generates 248.5 V, it must be running at a speed =  $1000 \times 248.5/250 = 994$  r.p.m.

$$\text{Hence, } N_0 = 994 \text{ r.p.m.}$$

**When Loaded**

$$I_a = 40 - 1 = 39 \text{ A}; E_b = 250 - 39 \times 0.5 = 230.5 \text{ V} \text{ Also, } \Phi_0/\Phi = 1/0.96$$

$$\frac{N}{E} = \frac{E_b}{E_{b0}} \therefore \frac{N}{994} = \frac{230.5}{248.5} \times \frac{1}{0.96} \quad N = 960 \text{ r.p.m.}$$

**Example 29.26.** A 250-V shunt motor giving 14.92 kW at 1000 r.p.m. takes an armature current of 75 A. The armature resistance is 0.25 ohm and the load torque remains constant. If the flux is reduced by 20 percent of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of the armature current and speed. **(Elect. Engg. AMIETE (New Scheme) 1990)**

**Solution.**  $E_{b1} = 250 - 75 \times 0.25 = 231.25 \text{ V}$ , as in Fig. 29.13.

When flux is reduced by 20%, the back e.m.f. is also reduced instantly by 20% because speed remains constant due to inertia of the heavy armature (Art. 29.11).

$$\therefore \text{Instantaneous value of back e.m.f. } (E_b)_{inst} = 231.25 \times 0.8 = 185 \text{ V}$$

$$(I_a)_{inst} = [V - (E_b)_{inst}]/R_a = (250 - 185)/0.25 = 260 \text{ A}$$

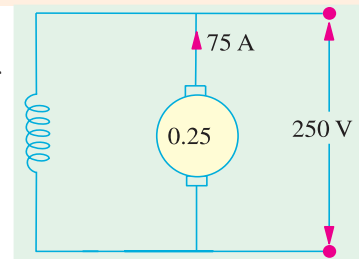


Fig. 29.13

$$\text{Instantaneous value of the torque} = 9.55 \times \frac{(E_b)_{inst} \times (I_a)_{inst}}{N \text{ (in r.p.m.)}}$$

or  $(T_a)_{inst} = 9.55 \times 185 \times 260/1000 = 459 \text{ N-m}$

**Steady Conditions**

Since torque remains constant,  $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$I_{a2} = \Phi_1 I_{a1} / \Phi_2 = 75 \times \Phi_1 / 0.8 \Phi_1 = \mathbf{93.7 \text{ A}}$$

$\therefore E_{b2} = 250 - 93.7 \times 0.25 = 226.6 \text{ V}$

Now,  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{226.6}{231.25} \times \frac{1}{0.8}; N_2 = 1225 \text{ r.p.m.}$

**Example 29.27.** A 220-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100 Ω. The resistance of armature at standstill gives a drop of 6 volts across armature terminals when 10 A were passed through it. Calculate (a) speed on load (b) torque in N-m and (c) efficiency. The normal input of the motor is 8 kW.

(Electrotechnics-II; M.S. Univ. Baroda 1988)

**Solution. (a)**  $I_{sh} = 200/100 = 2 \text{ A}$

F.L. Power input = 8,000 W; F.L. line current = 8,000/200 = 40 A

$$I_a = 40 - 2 = 38 \text{ A}; R_a = 6/10 = 0.6 \Omega$$

$$E_{b0} = 200 - 2 \times 0.6 = 198.8 \text{ V}; E_b = 200 - 38 \times 0.6 = 177.2 \text{ V}$$

Now,  $\frac{N}{N_0} = \frac{E_b}{E_{b0}}$  or  $\frac{N}{700} = \frac{177.2}{198.8}; N = \mathbf{623.9 \text{ r.p.m.}}$

(b)  $T_a = 9.55 E_b I_a / N = 9.55 \times 177.2 \times 38 / 623.9 = \mathbf{103 \text{ N-m}}$

(c) N.L. power input = 200 × 4 = 800 W; Arm. Cu loss =  $I_a^2 R_a = 2^2 \times 0.6 = 2.4 \text{ W}$

Constant losses = 800 - 2.4 = 797.6 W; F.L. arm. Cu loss =  $38^2 \times 0.6 = 866.4 \text{ W}$

Total F.L. losses = 797.6 + 866.4 = 1664 W; F.L. output = 8,000 - 1664 = 6336 W

F.L. Motor efficiency = 6336/8,000 = 0.792 or **79.2 %**

**Example 29.28.** The input to 230-V, d.c. shunt motor is 11kW. Calculate (a) the torque developed (b) the efficiency (c) the speed at this load. The particulars of the motor are as follows :

No-load current = 5 A; No-load speed = 1150 r.p.m.

Arm. resistance = 0.5 Ω; shunt field resistance = 110 Ω.

(Elect. Technology ; Bombay University 1988)

**Solution.** No-load input = 220 × 5 = 1,100 W;  $I_{sh} = 220/110 = 2\text{A}; I_{ao} = 5 - 2 = 3\text{A}$

No-load armature Cu loss =  $3^2 \times 0.5 = 4.5 \text{ W}$

$\therefore$  Constant losses = 1,100 - 4.5 = 1,095.5 W

**When input is 11 kW.**

Input current = 11,000/220 = 50A ; Armature current = 50 - 2 = **48 A**

Arm. Cu loss =  $48^2 \times 0.5 = 1,152 \text{ W};$

Total loss = Arm. Cu loss + Constant losses = 1152 + 1095.5 = 2248 W

Output = 11,000 - 2,248 = 8,752 W

(b) Efficiency =  $8,752 \times 100 / 11,000 = \mathbf{79.6\%}$

(c) Back e.m.f. at no-load = 220 - (3 × 0.5) = 218.5 V

Back e.m.f. at given load = 220 - (48 × 0.5) = 196 V

$\therefore$  Speed  $N = 1,150 \times 196 / 218.5 = \mathbf{1,031 \text{ r.p.m.}}$



(a) 
$$T_a = 9.55 \times \frac{196 \times 48}{1031} = 87.1 \text{ N-m}$$

**Example 29.29.** The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.

(Elect. Machines, A.M.I.E. Sec. B, 1993)

**Solution.**

$$E_{b1} = 250 - 80(0.1 + 0.05) - 3 = 235 \text{ V}$$

$$E_{b2} = 250 - 100(0.1 + 0.05) - 3 = 232 \text{ V}$$

Since  $\Phi \propto I_a$ , hence,  $\Phi_1 \propto 80$ ,  $\Phi_2 \propto 100$ ,  $\Phi_1/\Phi_2 = 80/100$

Now 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100}; N_2 = 474 \text{ r.p.m.}$$

**Example 29.30.** A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.

(Elect. Machines ; A.M.I.E. Sec. B, 1991)

**Solution.**

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad (\because \Phi \propto I_a)$$

Since field is unsaturated,  $T_a \propto \Phi I_a \propto I_a^2$ .  $(\because T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2)$

or 
$$T_2/T_1 = (I_{a2}/I_{a1})^2 \text{ or } 1/2 = (I_{a2}/I_{a1})^2; I_{a1} = I_{a2}/\sqrt{2} = 70.7 \text{ A}$$

$$E_{b1} = 220 - 100 \times 0.1 = 210 \text{ V}; E_{b2} = 220 - 0.1 \times 70.7 = 212.9 \text{ V}$$

$\therefore$  
$$\frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7}; N_2 = 1147 \text{ r.p.m.}$$

**Example 29.31.** A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.

$$\Phi = (1.7 \times 10^{-2} \times I^{0.5}) \text{ weber}$$

where I is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** Let us first find  $R_a$

Now 
$$N = \frac{E_b}{Z\Phi} \left( \frac{60 A}{P} \right) \text{ r.p.m.}$$

$\therefore$  
$$600 = \frac{E_b}{1.7 \times 10^{-2} \times 25^{0.5}} \times \frac{60 \times 4}{500 \times 4}$$

$\therefore$  
$$E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425 \text{ V}$$

$$I_a R_a = 450 - 425 = 25 \text{ V}; R_a = 25/25 = 1.0 \Omega$$

**Now in the Ist Case**

$$T_1 \propto \Phi_1 I_1 \quad \therefore T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25$$

Similarly 
$$T_2 \propto 1.7 \times 10^{-2} \times \sqrt{1 \times I}; \text{ Now } T_1 = 2T_2$$

$\therefore$  
$$1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I^{3/2} \times 2 \quad \therefore I = (125/2)^{2/3} = 15.75 \text{ A}$$

$$E_{b1} = 425 \text{ V}; E_{b2} = 225 - (15.75 \times 1) = 209.3 \text{ V}$$

Using the relation 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \text{ we have}$$

$$\frac{N_2}{600} = \frac{209.3}{425} \times \frac{1.7 \times 10^{-2} \times 5}{1.7 \times 10^{-2} \times \sqrt{15.75}}; \quad N_2 = 372 \text{ r.p.m.}$$

### Tutorial Problems 29.2

1. Calculate the torque in newton-metre developed by a 440-V d.c. motor having an armature resistance of  $0.25 \Omega$  and running at 750 r.p.m. when taking a current of 60 A. **[325 N-m]**
2. A 4-pole, lap-connected d.c. motor has 576 conductors and draws an armature current of 10 A. If the flux per pole is 0.02 Wb, calculate the armature torque developed. **[18.3 N-m]**
3. (a) A d.c. shunt machine has armature and field resistances of  $0.025 \Omega$  and  $80 \Omega$  respectively. When connected to constant 400-V bus-bars and driven as a generator at 450 r.p.m., it delivers 120 kW. Calculate its speed when running as a motor and absorbing 120 kW from the same bus-bars.  
(b) Deduce the direction of rotation of this machine when it is working as a motor assuming a clockwise rotation as a generator. **[(a) 435 r.p.m. (b) Clockwise]**
4. The armature current of a series motor is 60 A when on full-load. If the load is adjusted to that this current decreases to 40-A, find the new torque expressed as a percentage of the full-load torque. The flux for a current of 40 A is 70% of that when current is 60 A. **[46%]**
5. A 4-pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap-wound with 720 conductors. The shunt field resistance is  $240 \Omega$  and the armature resistance is  $0.2 \Omega$ . Brush contact drop is 1V per brush. Determine the speed of the machine when running (a) as a motor taking 60 A and (b) as a generator supplying 120 A. The terminal voltage in each case is 480 V. **[972 r.p.m. ; 1055 r.p.m.]**
6. A 25-kW shunt generator is delivering full output to 400-V bus-bars and is driven at 950 r.p.m. by belt drive. The belt breaks suddenly but the machine continues to run as a motor taking 25 kW from the bus-bars. At what speed does it run? Take armature resistance including brush contact resistance as  $0.5 \Omega$  and field resistance as  $160 \Omega$ . **[812.7 r.p.m.] (Elect. Technology, Andhra Univ. Apr. 1977)**
7. A 4-pole, d.c. shunt motor has a wave-wound armature with 65 slots each containing 6 conductors. The flux per pole is 20 mWb and the armature has a resistance of  $0.15 \Omega$ . Calculate the motor speed when the machine is operating from a 250-V supply and taking a current of 60 A. **[927 r.p.m.]**
8. A 500-V, d.c. shunt motor has armature and field resistances of  $0.5 \Omega$  and  $200 \Omega$  respectively. When loaded and taking a total input of 25 kW, it runs at 400 r.p.m. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. **[442 r.p.m.]**
9. A d.c. shunt motor runs at 900 r.p.m. from a 400 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230 V supply when taking an armature current of 15 A. The resistance of the armature circuit is  $0.8 \Omega$ . Assume the flux per pole at 230 V to have decreased to 75% of its value at 400 V. **[595 r.p.m.]**
10. A shunt machine connected to 250-A mains has an armature resistance of  $0.12 \Omega$  and field resistance of  $100 \Omega$ . Find the ratio of the speed of the machine as a generator to the speed as a motor, if line current is 80 A in both cases. **[1.08] (Electrical Engineering-II, Bombay Univ. April. 1977, Madras Univ. Nov. 1978)**
11. A 20-kW d.c. shunt generator delivering rated output at 1000 r.p.m. has a terminal voltage of 500 V. The armature resistance is  $0.1 \Omega$ , voltage drop per brush is 1 volt and the field resistance is  $500 \Omega$ . Calculate the speed at which the machine will run as a motor taking an input of 20 kW from a 500 V d.c. supply. **[976.1 r.p.m.] (Elect. Engg-I Bombay Univ. 1975)**
12. A 4-pole, 250-V, d.c. shunt motor has a lap-connected armature with 960 conductors. The flux per pole is  $2 \times 10^{-2}$  Wb. Calculate the torque developed by the armature and the useful torque in newton-metre when the current taken by the motor is 30A. The armature resistance is  $0.12 \text{ ohm}$  and the field resistance is  $125 \Omega$ . The rotational losses amount to 825 W. **[85.5 N-m ; 75.3 N-m] (Electric Machinery-I, Madras Univ. Nov. 1979)**

### 29.12. Motor Characteristics

The characteristic curves of a motor are those curves which show relationships between the following quantities.

1. **Torque and armature current** *i.e.*  $T_a/I_a$  characteristic. It is known as **electrical characteristic**.
2. **Speed and armature current** *i.e.*  $N/I_a$  characteristic.
3. **Speed and torque** *i.e.*  $N/T_a$  characteristic. It is also known as **mechanical characteristic**. It can be found from (1) and (2) above.

While discussing motor characteristics, the following two relations should always be kept in mind :

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

### 29.13. Characteristics of Series Motors

1.  **$T_a/I_a$  Characteristic.** We have seen that  $T_a \propto \Phi I_a$ . In this case, as field windings also carry the armature current,  $\Phi \propto I_a$  up to the point of magnetic saturation. Hence, before saturation,

$$T_a \propto \Phi I_a \quad \text{and} \quad \therefore T_a \propto I_a^2$$

At light loads,  $I_a$  and hence  $\Phi$  is small. But as  $I_a$  increases,  $T_a$  increases as the square of the current. Hence,  $T_a/I_a$  curve is a parabola as shown in Fig. 29.14. After saturation,  $\Phi$  is almost independent of  $I_a$  hence  $T_a \propto I_a$  only. So the characteristic becomes a straight line. The shaft torque  $T_{sh}$  is less than armature torque due to stray losses. It is shown dotted in the figure. So we conclude that (prior to magnetic saturation) on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in cases where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc., series motors are used.

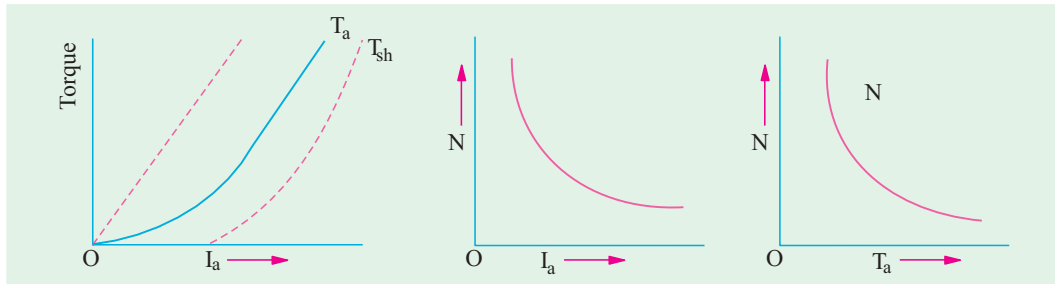


Fig. 29.14

Fig. 29.15

Fig. 29.16

2.  **$N/I_a$  Characteristics.** Variations of speed can be deduced from the formula :

$$N \propto \frac{E_b}{\Phi}$$

Change in  $E_b$ , for various load currents is small and hence may be neglected for the time being. With increased  $I_a$ ,  $\Phi$  also increases. Hence, speed varies inversely as armature current as shown in Fig. 29.15.

When load is heavy,  $I_a$  is large. Hence, speed is low (this decreases  $E_b$  and allows more armature current to flow). But when load current and hence  $I_a$  falls to a small value, speed becomes dangerously high. Hence, a series motor should never be started without some mechanical (not belt-driven) load on it otherwise it may develop excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

3.  **$N/T_a$  or mechanical characteristic.** It is found from above that when speed is high, torque is low and **vice-versa**. The relation between the two is as shown in Fig. 29.16.

## 29.14. Characteristics of Shunt Motors

### 1. $T_a/I_a$ Characteristic

Assuming  $\Phi$  to be practically constant (though at heavy loads,  $\phi$  decreases somewhat due to increased armature reaction) we find that  $T_a \propto I_a$ .

Hence, the electrical characteristic as shown in Fig. 29.17, is practically a straight line through the origin. Shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on (heavy) load.

### 2. $N/I_a$ Characteristic

If  $\Phi$  is assumed constant, then  $N \propto E_b$ . As  $E_b$  is also practically constant, speed is, for most purposes, constant (Fig. 29.18).

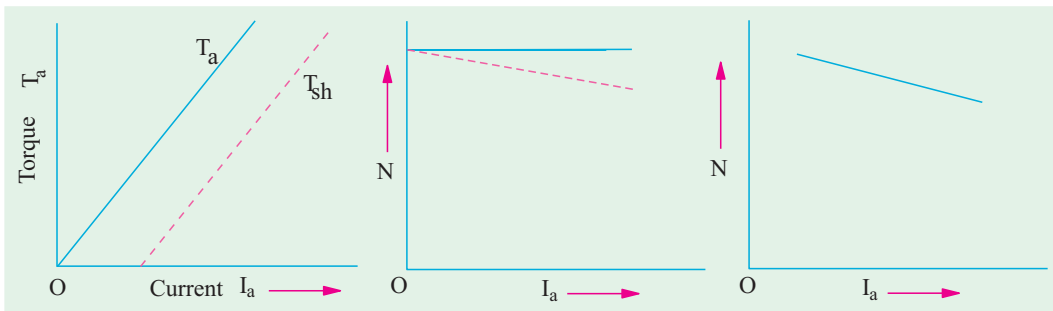


Fig. 29.17

Fig. 29.18

Fig. 29.19

But strictly speaking, both  $E_b$  and  $\Phi$  decrease with increasing load. However,  $E_b$  decreases slightly more than  $\phi$  so that on the whole, there is some decrease in speed. The drop varies from 5 to 15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly drooping as shown by the dotted line in Fig. 29.18. But, for all practical purposes, shunt motor is taken as a constant-speed motor.

Because there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

**3.  $N/T_a$  Characteristic** can be deduced from (1) and (2) above and is shown in Fig. 29.19.

## 29.15. Compound Motors

These motors have both series and shunt windings. If series excitation helps the shunt excitation *i.e.* series flux is in the **same** direction (Fig. 29.20); then the motor is said to be cumulatively compounded. If on the other hand, series field opposes the shunt field, then the motor is said to be differentially compounded.

The characteristics of such motors lie in between those of shunt and series motors as shown in Fig. 29.21.

### (a) Cumulative-compound Motors

Such machines are used where series characteristics are required and where, in addition,



Compound Motors

the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to take sudden cuts quite often. Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy loads. In conjunction with fly-wheel (functioning as load equalizer), it is employed where there are sudden temporary loads as in rolling mills. The fly-wheel supplies its stored kinetic energy when motor slows down due to sudden heavy load. And when due to the removal of load motor speeds up, it gathers up its kinetic energy.

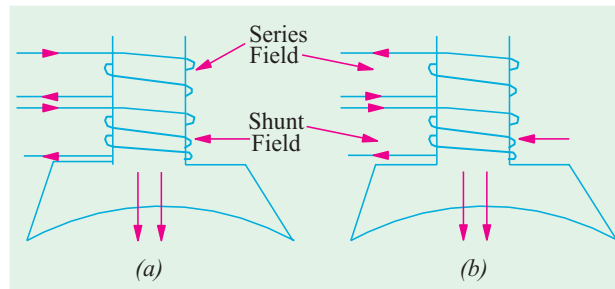


Fig. 29.20

Compound-wound motors have greatest application with loads that require high starting torques or pulsating loads (because such motors smooth out the energy demand required of a pulsating load). They are used to drive electric shovels, metal-stamping machines, reciprocating pumps, hoists and compressors etc.

**(b) Differential-compound Motors**

Since series field opposes the shunt field, the flux is decreased as load is applied to the motor. This results in the motor speed remaining almost constant or even increasing with increase in load (because,  $N \propto E_b/(\Phi)$ ). Due to this reason, there is a decrease in the rate at which the motor torque increases with load. Such motors are not in common use. But because they can be designed to give an accurately constant speed under all conditions, they find limited application for experimental and research work.

One of the biggest drawback of such a motor is that due to weakening of flux with increases in load, there is a tendency towards speed instability and motor running away unless designed properly.

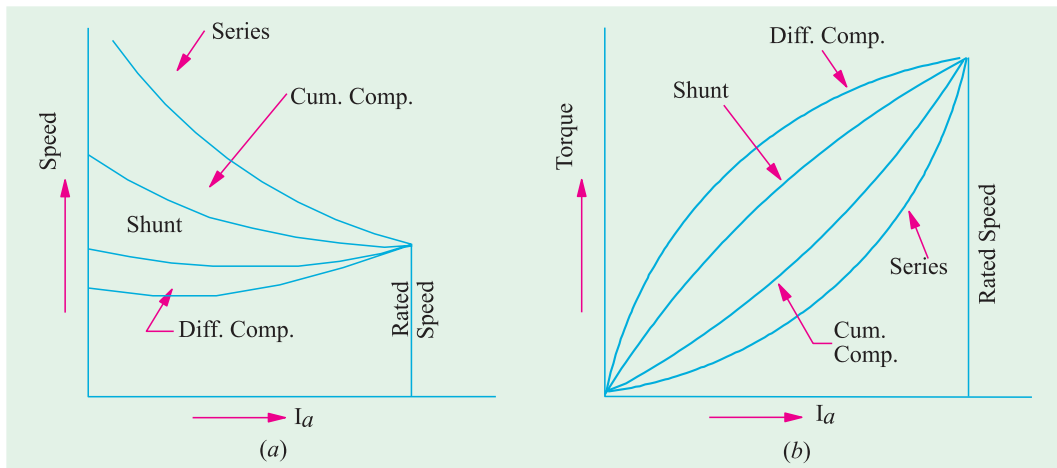


Fig. 29.21

**Example 29.32.** The following results were obtained from a static torque test on a series motor :

Current (A) :	20	30	40	50
Torque (N - m) :	128.8	230.5	349.8	46.2

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 Ω. Ignore iron and friction losses.

**Solution.** Taking the case when input current is 20 A, we have

$$\text{Motor input} = 460 \times 20 = 9,200 \text{ W}$$

$$\begin{aligned} \text{Field and armature Cu loss} \\ = 20^2 \times 0.5 = 200 \text{ W} \end{aligned}$$

Ignoring iron and friction losses,

$$\text{output} = 9,200 - 200 = 9,000 \text{ W}$$

$$\text{Now, } T_{sh} \times 2\pi N = \text{Output in watts.}$$

$$\therefore 128.8 \times 2\pi \times N = 9,000$$

$$\therefore N = 9,000 / 2\pi \times 128.8$$

$$= 11.12 \text{ r.p.s.} = 667 \text{ r.p.m.}$$

Similar calculations for other values of current are tabulated below :

Current (A)	20	30	40	50
Input (W)	9,200	13,800	18,400	23,000
$I^2R$ loss (W)	200	450	800	1,250
Output (W)	9,200	13,350	17,600	21,850
Speed (r.p.m.)	667	551	480	445
Torque (N-m)	128.8	230.5	349.8	469.2

From these values, the speed/torque curve can be drawn as shown in Fig. 29.22.

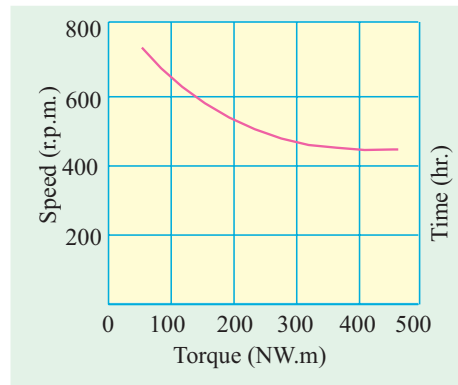


Fig. 29.22

**Example 29.33.** A fan which requires 8 h.p. (5.968 kW) at 700 r.p.m. is coupled directly to a d.c. series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the cube of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as follows :

Load current (A)	:	7	10.5	14	27.5
Terminal voltage (V)	:	347	393	434	458

The resistance of both the armature and field windings of the motor is 3.5 Ω and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage. (I.E.E. London)

**Solution.** Let us, by way of illustration, calculate the speed and output when motor is running off a 500-V supply and taking a current of 7A.

$$\text{Series voltage drop} = 7 \times 3.5 = 24.5 \text{ V}$$

$$\text{Generated or back e.m.f. } E_b = 500 - 24.5 = 475.5 \text{ V}$$

The motor speed is proportional to  $E_b$  for a given current. For a speed of 600 r.p.m. and a current of 7A, the generated e.m.f. is 347 V. Hence,

$$N = 600 \times 475.5 / 347 = 823 \text{ r.p.m.}$$

$$\text{Power to armature} = E_b I_a = 475.5 \times 7 = 3,329 \text{ W}$$

$$\text{Output} = \text{Armature power} - 450 = 3,329 - 450 = 2,879 \text{ W} = \mathbf{2.879 \text{ kW}}$$

$$\text{Power required by the fan at 823 r.p.m. is } = 5.968 \times 823^2 / 700^2 = 9.498 \text{ kW}$$

These calculations are repeated for the other values of current in the following table.

Input current (A)	7	10.5	14	27.5
Series drop (V)	24.5	36.7	49	96.4
Back e.m.f. (V)	475.5	463.3	451	403.6

E.M.F. at 600 r.p.m. (V)	347	393	434	458
Speed $N$ (r.p.m.)	823	707	623	528
Armature power (W)	3329	4870	6310	11,100
Motor output (kW)	2.879	4.420	5.860	10.65
Power required by fan (kW)	9.698	6.146	4.222	2.566

In Fig. 29.23 (i) the motor output in kW and (ii) power required by fan in kW against input current is plotted. Since motor output equals the input to fan, hence the intersection point of these curves gives the value of motor input current under the given conditions.

Input current corresponding to intersection point = 12 A

$\therefore$  Motor input =  $500 \times 12 = 6,000 \text{ W}$

### 29.16. Performance Curves

#### (a) Shunt Motor

In Fig. 29.24 the four essential characteristics of a shunt motor are shown *i.e.* torque, current speed and efficiency, each plotted as a function of motor output power. These are known as the **performance curves** of a motor.

It is seen that shunt motor has a definite no-load speed. Hence, it does not ‘run away’ when load is suddenly thrown off provided the field circuit remains closed. The drop in speed from no-load to full-load is small, hence this motor is usually referred to as **constant speed** motor. The speed for any load within the operating range of the motor can be readily obtained by varying the field current by means of a field rheostat.

The efficiency curve is usually of the same shape for all electric motors and generators. The shape of efficiency curve and the point of maximum efficiency can be varied considerably by the designer, though it is advantageous to have an efficiency curve which is fairly flat, so that there is little change in efficiency between load and 25% overload and to have the maximum efficiency as near to the full load as possible.

It will be seen from the curves, that a certain value of current is required even when output is zero. The motor input under no-load conditions goes to meet the various losses occurring within the machine.

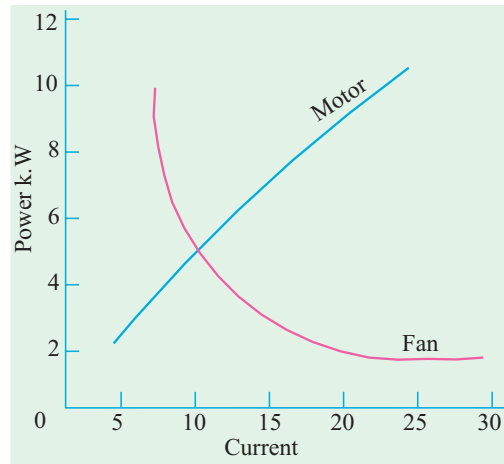


Fig. 29.23

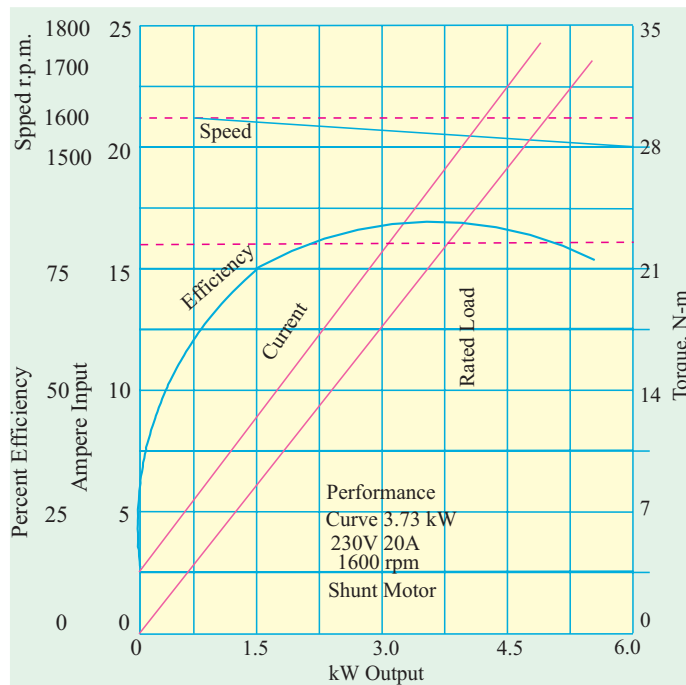


Fig. 29.24

As compared to other motors, a shunt motor is said to have a lower starting torque. But this should not be taken to mean that a shunt motor is incapable of starting a heavy load. Actually, it means that series and compound motors are capable of starting heavy loads with less excess of current inputs over normal values than the shunt motors and that consequently the depreciation on the motor will be relatively less. For example, if twice full load torque is required at start, then shunt motor draws twice the full-load current ( $T_a \propto I_a$  or  $I_a \propto \sqrt{T_a}$ ) whereas series motor draws only approximately one and a half times the full load current ( $T_a \propto I_a^2$  or  $I_a \propto \sqrt{T_a}$ ).

The shunt motor is widely used with loads that require essentially constant speed but where high starting torques are not needed. Such loads include centrifugal pumps, fans, winding reels conveyors and machine tools etc.

**(b) Series Motor**

The typical performance curves for a series motor are shown in Fig. 29.25.

It will be seen that drop in speed with increased load is much more prominent in series motor than in a shunt motor. Hence, a series motor is not suitable for applications requiring a substantially constant speed.

For a given current input, the starting torque developed by a series motor is greater than that developed by a shunt motor. Hence, series motors are used where huge starting torques are necessary *i.e.* for street cars, cranes, hoists and for electric-railway operation. In addition to the huge starting torque, there is another unique characteristic of series motors which makes them especially desirable for traction work *i.e.* when a load comes on a series motor, it responds by decreasing its speed (and hence,  $E_b$ ) and supplies the increased torque with a small increase in current. On the other hand a shunt motor under the same conditions would hold its speed nearly constant and would supply the required increased torque with a large increase of input current. Suppose that instead of a series motor, a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed for the car at approximately the same value it had on the level ground, but the motor tends to take an excessive current. A series motor, however, automatically slows down on such a grade because of increased current demand, and so it develops more torque at reduced speed. The drop in speed permits the motor to develop a large torque with but a moderate increase of power. Hence, under the same load conditions, rating of the series motor would be less than for a shunt motor.

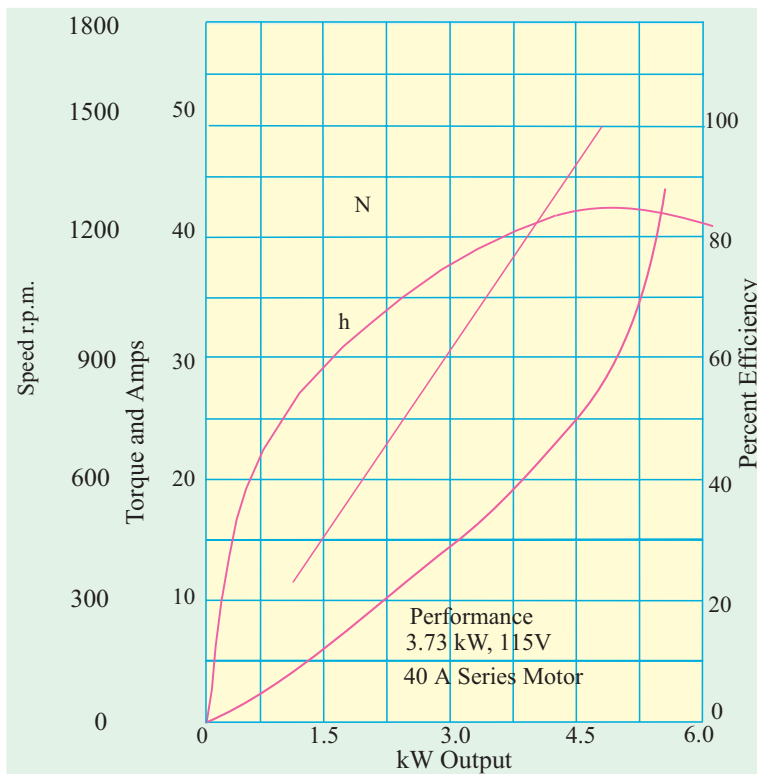


Fig. 29.25



### 29.17. Comparison of Shunt and Series Motors

#### (a) Shunt Motors

The different characteristics have been discussed in Art. 29.14. It is clear that

- (a) speed of a shunt motor is sufficiently constant.
- (b) for the same current input, its starting torque is not a high as that of series motor. Hence, it is used.
  - (i) When the speed has to be maintained approximately constant from N.L. to F.L. *i.e.* for driving a line of shafting etc.
  - (ii) When it is required to drive the load at various speeds, any one speed being kept constant for a relatively long period *i.e.* for individual driving of such machines as lathes. The shunt regulator enables the required speed control to be obtained easily and economically.



Shunt Motors

#### Summary of Applications

Type of motor	Characteristics	Applications
Shunt	Approximately constant speed Adjustable speed Medium starting torque (Up to 1.5 F.L. torque)	For driving constant speed line shafting Lathes Centrifugal pumps Machine tools Blowers and fans Reciprocating pumps
Series	Variable speed Adjustable varying speed High Starting torque	For traction work <i>i.e.</i> Electric locomotives Rapid transit systems Trolley, cars etc. Cranes and hoists Conveyors
Comulative Compound	Variable speed Adjustable varying speed High starting torque	For intermittent high torque loads For shears and punches Elevators Conveyors Heavy planers Heavy planers Rolling mills; Ice machines; Printing presses; Air compressors

#### (b) Series Motors

The operating characteristics have been discussed in Art 29.13. These motors

1. have a relatively huge starting torques.
2. have good accelerating torque.
3. have low speed at high loads and dangerously high speed at low loads.

Hence, such motors are used

1. when a large starting torque is required *i.e.* for driving hoists, cranes, trams etc.

2. when the motor can be directly coupled to a load such as a fan whose torque increases with speed.

3. if constancy of speed is not essential, then, in fact, the decrease of speed with increase of load has the advantage that the power absorbed by the motor does not increase as rapidly as the torque. For instance, when torque is doubled, the power approximately increases by about 50 to 60% only. ( $\therefore I_a \propto \sqrt{T_a}$ ).

4. a series motor should not be used where there is a possibility of the load decreasing to a very small value. Thus, it should not be used for driving centrifugal pumps or for a belt-drive of any kind.



### 29.18. Losses and Efficiency

The losses taking place in the motor are the same as in generators. These are (i) Copper losses (ii) Magnetic losses and (iii) Mechanical losses.

The condition for maximum power developed by the motor is

$$I_a R_a = V/2 = E_b$$

The condition for maximum efficiency is that armature Cu losses are equal to constant losses. (Art. 26.39).

### 29.19. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Fig. 29.26.

Overall or commercial efficiency  $\eta_c = \frac{C}{A}$ , Electrical efficiency  $\eta_e = \frac{B}{A}$ , Mechanical efficiency  $\eta_m = \frac{C}{B}$ .

The efficiency curve for a motor is similar in shape to that for a generator (Art. 24.35).

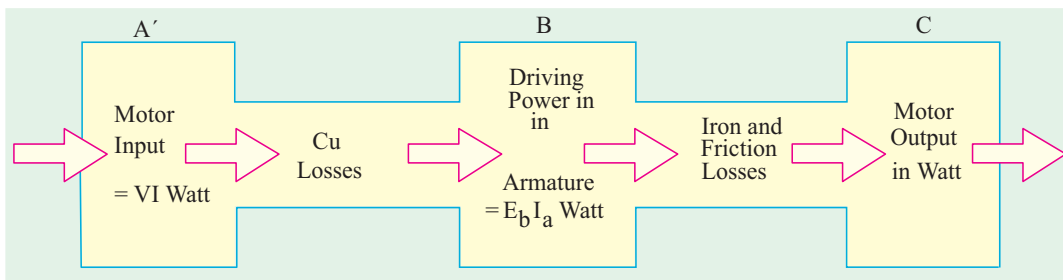


Fig. 29.26

It is seen that  $A - B =$  copper losses and  $B - C =$  iron and friction losses.

**Example 29.34.** One of the two similar 500-V shunt machines A and B running light takes 3 A. When A is mechanically coupled to B, the input to A is 3.5 A with B unexcited and 4.5 A when B is separately-excited to generate 500 V. Calculate the friction and windage loss and core loss of each machine. (Electric Machinery-I, Madras Univ. 1985)

**Solution.** When running light, machine input is used to meet the following losses (i) armature Cu loss (ii) shunt Cu loss (iii) iron loss and (iv) mechanical losses i.e. friction and windage losses. Obviously, these no-load losses for each machine equal  $500 \times 3 = 1500$  W.

**(a) With B unexcited**

In this case, only mechanical losses take place in  $B$ , there being neither Cu loss nor iron-loss because  $B$  is unexcited. Since machine  $A$  draws 0.5 A more current.

Friction and windage loss of  $B = 500 \times 0.5 = 250 \text{ W}$

**(b) With B excited**

In this case, both iron losses as well as mechanical losses take place in machine  $B$ . Now, machine  $A$  draws,  $4.5 - 3 = 1.5 \text{ A}$  more current.

Iron and mechanical losses of  $B = 1.5 \times 500 = 750 \text{ W}$

Iron losses of  $B = 750 - 250 = 500 \text{ W}$

**Example 29.35.** A 220 V shunt motor has an armature resistance of 0.2 ohm and field resistance of 110 ohm. The motor draws 5 A at 1500 r.p.m. at no load. Calculate the speed and shaft torque if the motor draws 52 A at rated voltage. **(Elect. Machines Nagpur Univ. 1993)**

**Solution.**

$$\begin{aligned} I_{sh} &= 220/110 = 2 \text{ A}; I_{a1} = 5 - 2 = 3 \text{ A}; I_{a2} = 52 - 2 = 50 \text{ A} \\ E_{b1} &= 220 - 3 \times 0.2 = 219.4 \text{ V}; E_{b2} = 220 - 50 \times 0.2 = 210 \text{ V} \\ \frac{N_2}{1500} &= \frac{210}{219.4}; N_2 = 1436 \text{ r.p.m. } (\Phi_1 = \Phi_2) \end{aligned}$$

For finding the shaft torque, we will find the motor output when it draws a current of 52 A. First we will use the no-load data for finding the constant losses of the motor.

$$\text{No load motor input} = 220 \times 5 = 1100 \text{ W}; \text{Arm. Cu loss} = 3^2 \times 0.2 = 2 \text{ W}$$

$$\therefore \text{Constant or standing losses of the motor} = 1100 - 2 = 1098$$

$$\text{When loaded, arm. Cu loss} = 50^2 \times 0.2 = 500 \text{ W}$$

$$\text{Hence, total motor losses} = 1098 + 500 = 1598 \text{ W}$$

$$\text{Motor input on load} = 220 \times 52 = 11,440 \text{ W}; \text{output} = 11,440 - 1598 = 9842 \text{ W}$$

$$\therefore T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 9842/1436 = 65.5 \text{ N-m}$$

**Example 29.36.** 250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 amperes. Armature and shunt field resistances are 0.2 and 250 ohms respectively. Calculate the speed when loaded taking a current of 50 A. The armature reaction weakens the field by 3%. **(Elect. Engg.-I Nagpur Univ. 1993)**

**Solution.**

$$\begin{aligned} I_{sh} &= 250/250 = 1 \text{ A}; I_{a1} = 5 - 1 = 4 \text{ A}; I_{a2} = 50 - 1 = 49 \text{ A} \\ E_{b1} &= 250 - 4 \times 0.2 = 249.2 \text{ V}; E_{b2} = 250 - 49 \times 0.2 = 240.2 \text{ V} \\ \frac{N_2}{1000} &= \frac{240.2}{249.2} \times \frac{\Phi_1}{0.97 \Phi_1}; N_2 = 944 \text{ r.p.m.} \end{aligned}$$

**Example 29.37.** A 500 V d.c. shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuit are 0.22 ohm and 250 ohm respectively. Find (a) the efficiency when loaded and taking a current of 100 A (b) the percentage change of speed. State precisely the assumptions made. **(Elect. Engg.-I, M.S. Univ. Baroda 1987)**

**Solution. No-Load condition**

$$I_{sh} = 500/250 = 2 \text{ A}; I_{a0} = 5 - 2 = 3 \text{ A}; E_{b0} = 500 - (3 \times 0.22) = 499.34 \text{ V}$$

$$\text{Arm. Cu loss} = 3^2 \times 0.22 = 2 \text{ W}; \text{Motor input} = 500 \times 5 = 2500 \text{ W}$$

$$\text{Constant losses} = 2500 - 2 = 2498 \text{ W}$$

It is assumed that these losses remain constant under all load conditions.

**Load condition**

$$(a) \text{ Motor current} = 100 \text{ A}; I_a = 100 - 2 = 98 \text{ A}; E_b = 500 - (98 \times 0.22) = 478.44 \text{ V}$$

$$\begin{aligned} \text{Arm. Cu loss} &= 98^2 \times 0.22 = 2110 \text{ W, Total losses} = 2110 + 2498 = 4608 \text{ W} \\ \text{Motor input} &= 500 \times 100 = 50,000 \text{ W, Motor output} = 50,000 - 4,608 = 45,392 \text{ W} \\ \text{Motor } \eta &= 45,392/50,000 = 0.908 \text{ or } \mathbf{90.8\%} \\ \frac{N}{N_0} &= \frac{E_b}{E_{b0}} = \frac{478.44}{499.34} \text{ or } \frac{N - N_0}{N_0} = \frac{-20.9}{499.34} = -0.0418 \text{ or } \mathbf{-4.18\%} \end{aligned}$$

**Example 29.38.** A 250 V d.c. shunt motor runs at 1000 r.p.m. while taking a current of 25 A. Calculate the speed when the load current is 50 A if armature reaction weakens the field by 3%. Determine torques in both cases.

$$R_a = 0.2 \text{ ohm} ; R_f = 250 \text{ ohms}$$

Voltage drop per brush is 1 V. (Elect. Machines Nagpur Univ. 1993)

**Solution.**

$$\begin{aligned} I_{sh} &= 250/250 = 1 \text{ A}; I_{a1} = 25 - 1 = 24 \text{ A} \\ E_{bh} &= 250 - \text{arm. drop} - \text{brush drop} \\ &= 250 - 24 \times 0.2 - 2 = 243.2 \text{ V} \\ I_{a2} &= 50 - 1 = 49 \text{ A}; E_{b2} = 250 - 49 \times 0.2 - 2 = 238.2 \text{ V} \\ \frac{N_2}{1000} &= \frac{238.2}{243.2} \times \frac{\Phi_1}{0.97 \Phi_1}; N_2 = 1010 \text{ r.p.m.} \\ T_{a1} &= 9.55 E_{b1} I_{a1}/N_1 = 9.55 \times 243.2 \times 24/1000 = 55.7 \text{ N-m} \\ T_{a2} &= 9.55 \times 238.2 \times 49/1010 = 110.4 \text{ r.p.m.} \end{aligned}$$

**Example 29.39.** A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5 Ω and field resistance of 250 Ω. When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.

**Solution.**

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back e.m.f. of 250 V produced in its armature. Hence  $N_1 = 1000$  r.p.m. and  $E_{b1} = 250$  V.

When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is

$$E_{b2} = 250 - (40 - 1) \times 0.5 = 230.5 \text{ V; Also } \Phi_2 = 0.96 \Phi_1, N_2 = ?$$

Using the above equation we have

$$\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96 \Phi_1}; N_2 = \mathbf{960 \text{ r.p.m.}}$$

**Efficiency**

No-load input represents motor losses which consists of

- (a) armature Cu loss =  $I_a^2 R_a$  which is variable.
- (b) constant losses  $W_c$  which consists of (i) shunt Cu loss (ii) magnetic losses and (iii) mechanical losses.

No-load input or total losses =  $250 \times 4 = 1000$  W

Arm. Cu loss =  $I_a^2 R_a = 3^2 \times 0.5 = 4.5$  W,  $\therefore W_c = 1000 - 4.5 = 995.5$  W

When motor draws a line current of 40 A, its armature current is  $(40 - 1) = 39$  A

Arm. Cu loss =  $39^2 \times 0.5 = 760.5$  W; Total losses =  $760.5 + 955.5 = 1756$  W

Input =  $250 \times 40 = 10,000$  W; output =  $10,000 - 1756 = 8,244$  W

$\therefore \eta = 8,244 \times 100/10,000 = \mathbf{82.44\%}$

**Example 29.40.** The armature winding of a 4-pole, 250 V d.c. shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125  $\Omega$  respectively. If the rotational losses amount to be 810 W find,

(i) gross torque (ii) useful torque and (iii) efficiency. (Elect. Machines Nagpur Univ. 1993)

**Solution.**  $I_{sh} = 250/125 = 2$  A;  $I_a = 25 - 2 = 23$  A;  $E_b = 250 - (23 \times 0.1) = 247.7$  V

$$\text{Now, } E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \quad \therefore 247.7 = \frac{20 \times 10^{-3} \times 960 \times N}{60} \left( \frac{4}{4} \right); N = 773 \text{ r.p.m.}$$

(i) Gross torque or armature torque  $T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \times \frac{247.7 \times 23}{773} = 70.4 \text{ N-m}$

(ii) Arm Cu loss =  $23^2 \times 0.1 = 53$  W; Shunt Cu loss =  $250 \times 2 = 500$  W

Rotational losses = 810 W; Total motor losses =  $810 + 500 + 53 = 1363$  W

Motor input =  $250 \times 25 = 6250$  W; Motor output =  $6250 - 1363 = 4887$  W

$T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 4887/773 = 60.4 \text{ N-m}$

(iii) Efficiency =  $4887/6250 = 0.782 = 78.2\%$

**Example 29.41.** A 20-hp (14.92 kW); 230-V, 1150-r.p.m. 4-pole, d.c. shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature circuit resistance of 0.2  $\Omega$ . When it delivers rated power at rated speed, it draws a line current of 74.8 A and a field current of 3 A. Calculate (i) the flux per pole (ii) the torque developed (iii) the rotational losses (iv) total losses expressed as a percentage of power. (Electrical Machinery-I, Bangalore Univ. 1987)

**Solution.**  $I_a = 74.8 - 3 = 71.8$  A;  $E_b = 230 - 71.8 \times 0.2 = 215.64$  V

(i) Now,  $E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right); 215.64 = \frac{\Phi \times 620 \times 1150}{60} \left( \frac{4}{2} \right); \Phi = 9 \text{ mWb}$

(ii) Armature Torque,  $T_a = 9.55 \times 215.64 \times 71.8/1150 = 128.8 \text{ N-m}$

(iii) Driving power in armature =  $E_b I_a = 215.64 \times 71.8 = 15,483$  W

Output = 14,920 W; Rotational losses =  $15,483 - 14,920 = 563$  W

(iv) Motor input =  $VI = 230 \times 74.8 = 17,204$  W; Total loss =  $17,204 - 14,920 = 2,284$  W

Losses expressed as percentage of power input =  $2284/17,204 = 0.133$  or 13.3%

**Example 29.42.** A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5  $\Omega$  and 250  $\Omega$  respectively. At what output power will the efficiency be maximum? Is it possible to obtain this output from the machine? (Electrotechnics-II, M.S. Univ. Baroda 1985)

**Solution. When loaded lightly**

Total motor input (or total no-load losses) =  $250 \times 5 = 1,250$  W

$$I_{sh} = 250/250 = 1 \text{ A} \quad \therefore I_a = 5 - 1 = 4 \text{ A}$$

$$\text{Field Cu loss} = 250 \times 1 = 250 \text{ W; Armature Cu loss} = 4^2 \times 0.5 = 8 \text{ W}$$

$\therefore$  Iron losses and friction losses =  $1250 - 250 - 8 = 992$  W

These losses would be assumed constant.

Let  $I_a$  be the full-load armature current, then armature input is =  $(250 \times I_a)$  W

$$\text{F.L. output} = 7.46 \times 1000 = 7,460 \text{ W}$$

The losses in the armature are :

(i) Iron and friction losses = 992 W

(ii) Armature Cu loss =  $I_a^2 \times 0.05$  W  $\therefore 250 I_a = 7,460 + 992 + I_a^2 \times 0.5$

or  $0.5 I_a^2 - 250 I_a + 8,452 = 0 \quad \therefore I_a = 36.5 \text{ A}$   
 $\therefore$  F.L. input current =  $36.5 + 1 = 37.5 \text{ A}$  ; Motor input =  $250 \times 37.5 \text{ W}$   
 F.L. output =  $7,460 \text{ W}$   
 $\therefore$  F.L. efficiency =  $7460 \times 100 / 250 \times 37.5 = 79.6\%$   
 Now, efficiency is maximum when armature Cu loss equals constant loss.  
*i.e.*  $I_a^2 R_d = I_a^2 \times 0.5 = (1,250 - 8) = 1,242 \text{ W}$  or  $I_a = 49.84 \text{ A}$   
 $\therefore$  Armature input =  $250 \times 49.84 = 12,460 \text{ W}$   
 Armature Cu loss =  $49.84^2 \times 0.5 = 1242 \text{ W}$ ; Iron and friction losses =  $992 \text{ W}$   
 $\therefore$  Armature output =  $12,460 - (1,242 + 992) = 10,226 \text{ W}$   
 $\therefore$  Output power =  $10,226 \text{ W} = 10.226 \text{ kW}$

As the input current for maximum efficiency is beyond the full-load motor current, it is never realised in practice.

**Example 29.43.** A d.c. series motor drives a load, the torque of which varies as the square of the speed. Assuming the magnetic circuit to be remain unsaturated and the motor resistance to be negligible, estimate the percentage reduction in the motor terminal voltage which will reduce the motor speed to half the value it has on full voltage. What is then the percentage fall in the motor current and efficiency? Stray losses of the motor may be ignored.

(Electrical Engineering-III, Pune Univ. 1987)

**Solution.**  $T_a \propto \Phi I_a \propto I_a^2$ . Also,  $T_a \propto N^2$ . Hence  $N^2 \propto I_a^2$  or  $N \propto I_a$

$\therefore N_1 \propto I_{a1}$  and  $N_2 \propto I_{a2}$  or  $N_2/N_1 = I_{a2}/I_{a1}$

Since,  $N_2/N_1 = 1/2 \quad \therefore I_{a2}/I_{a1} = 1/2$  or  $I_{a2} = I_{a1}/2$

Let  $V_1$  and  $V_2$  be the voltages across the motor in the two cases. Since motor resistance is negligible,  $E_{b1} = V_1$  and  $E_{b2} = V_2$ . Also  $\Phi_1 \propto I_{a1}$  and  $\Phi_2 \propto I_{a2}$  or  $\Phi_1/\Phi_2 = I_{a1}/I_{a2} = I_{a1} \times 2/I_{a1} = 2$

Now,  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$  or  $\frac{1}{2} = \frac{V_2}{V_1} \times 2$  or  $\frac{V_2}{V_1} = \frac{1}{4}$

$\therefore \frac{V_1 - V_2}{V_1} = \frac{4 - 1}{4} = 0.75$

$\therefore$  Percentage reduction in voltage =  $\frac{V_1 - V_2}{V_1} \times 100 = 0.75 \times 100 = 75\%$

Percentage change in motor current =  $\frac{I_{a1} - I_{a2}}{I_{a1}} \times 100 = \frac{I_{a1} - I_{a1}/2}{I_{a1}} \times 100 = 50\%$

**Example 29.44.** A 6-pole, 500-V wave-connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. The armature and field resistance are  $0.5 \Omega$  and  $250 \Omega$  respectively. What will be the speed and torque developed by the motor when it draws 20 A from the supply mains? Neglect armature reaction. If magnetic and mechanical losses amount to 900 W, find (i) useful torque (ii) output in kW and (iii) efficiency at this load.

**Solution. (i)**  $I_{sh} = 500/250 = 2 \text{ A} \quad \therefore I_a = 20 - 2 = 18 \text{ A}$

$\therefore E_b = 500 - (18 \times 0.5) = 491 \text{ V}$ ; Now,  $E_b = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right)$  volt

$\therefore 491 = \frac{20 \times 10^{-3} \times 1200 \times N}{60} \times \left(\frac{6}{2}\right)$ ;  $N = 410 \text{ r.p.m. (approx.)}$

Now  $T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \frac{491 \times 18}{410} = 206 \text{ N - m}$

$$\begin{aligned} \text{Armature Cu loss} &= 18^2 \times 0.5 = 162 \text{ W}; \text{ Field Cu loss} = 500 \times 2 = 1000 \text{ W} \\ \text{Iron and friction loss} &= 900 \text{ W}; \text{ Total loss} = 162 + 1000 + 900 = 2,062 \text{ W} \\ \text{Motor input} &= 500 \times 20 = 10,000 \text{ W} \end{aligned}$$

- (i)  $T_{sh} = 9.55 \times \frac{7938}{410} = 184.8 \text{ N - m}$   
 (ii) Output = 10,000 – 2062 = **7,938 kW**  
 (iii)  $\% \eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{7,938 \times 100}{10,000} = 0.794 = 79.4\%$

**Example 29.45.** A 50-h.p. (37.3 kW), 460-V d.c. shunt motor running light takes a current of 4 A and runs at a speed of 660 r.p.m. The resistance of the armature circuit (including brushes) is 0.3 Ω and that of the shunt field circuit 270 Ω.

Determine when the motor is running at full load

- (i) the current input (ii) the speed. Determine the armature current at which efficiency is maximum. Ignore the effect of armature reaction. **(Elect. Technology Punjab, Univ. 1991)**

**Solution.**  $I_{sh} = 460/270 = 1.7 \text{ A}$ ; Field Cu loss = 460 × 1.7 = 783 W

**When running light**

$$I_a = 4 - 1.7 = 2.3 \text{ A}; \text{ Armature Cu loss} = 2.3^2 \times 0.3 = 1.5 \text{ W (negligible)}$$

$$\text{No-load armature input} = 460 \times 2.3 = 1,058 \text{ W}$$

As armature Cu loss is negligible, hence 1,058 W represents iron, friction and windage losses which will be assumed to be constant.

Let full-load armature input current be  $I_a$ . Then

$$\text{Armature input} = 460 I_a \text{ W}; \text{ Armature Cu loss} = I_a^2 \times 0.3 \text{ W}$$

$$\text{Output} = 37.3 \text{ kW} = 37,300 \text{ W}$$

$$\therefore 460 I_a = 37,300 + 1,058 + 0.3 I_a^2 \text{ or } 0.3 I_a^2 - 460 I_a + 38,358 = 0$$

$$\therefore I_a = 88.5 \text{ A}$$

(i) Current input = 88.5 + 1.7 = **90.2 A**

(ii)  $E_{b1} = 460 - (2.3 \times 0.3) = 459.3 \text{ V}$ ;  $E_{b2} = 460 - (88.5 \times 0.3) = 433.5 \text{ V}$

$$\therefore N_2 = 660 \times 433.5/459.3 = \mathbf{624 \text{ r.p.m.}}$$

For maximum efficiency,  $I_a^2 R_a = \text{constant losses}$  (Art. 24.37)

$$\therefore I_a^2 \times 0.3 = 1058 + 783 = 1,841 \therefore I_a = (1841/0.3)^{1/2} = \mathbf{78.33 \text{ A}}$$

### Tutorial Problems 29.3

- A 4-pole 250-V, d.c. series motor has a wave-wound armature with 496 conductors. Calculate
  - the gross torque
  - the output torque and
  - the speed
  - the efficiency, if the motor current is 50 A

The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19 Ω, field resistance = 0.14 Ω.

[(a) **173.5 N-m** (b) **642 r.p.m.** (c) **161.4 N-m** (d) **86.9%**]

- On no-load, a shunt motor takes 5 A at 250 V, the resistances of the field and armature circuits are 250 Ω and 0.1 Ω respectively. Calculate the output power and efficiency of the motor when the total supply current is 81 A at the same supply voltage. State any assumptions made.

[**18.5 kW; 91%**. It is assumed that windage, friction and eddy current losses are independent of the current and speed]

3. A 230 V series motor is taking 50 A. Resistance of armature and series field windings is 0.2  $\Omega$  and 0.1  $\Omega$  respectively. Calculate :
- (a) brush voltage (b) back e.m.f.  
 (c) power wasted in armature (d) mechanical power developed
- [(a) 215 V (b) 205 V (c) 500 W (d) 13.74 h.p.] (10.25 kW)
4. Calculate the shaft power of a series motor having the following data; overall efficiency 83.5%, speed 550 r.p.m. when taking 65 A; motor resistance 0.2  $\Omega$ , flux per pole 25 mWb, armature winding lap with 1200 conductor.  
 (15.66 kW)
5. A shunt motor running on no-load takes 5 A at 200 V. The resistance of the field circuit is 150  $\Omega$  and of the armature 0.1  $\Omega$ . Determine the output and efficiency of motor when the input current is 120 A at 200 V. State any conditions assumed. (89.8%)
6. A d.c. shunt motor with interpoles has the following particulars :  
 Output power ; 8,952 kW, 440-V, armature resistance 1.1  $\Omega$ , brush contact drop 2 V, interpole winding resistance 0.4  $\Omega$  shunt resistance 650  $\Omega$ , resistance in the shunt regulator 50  $\Omega$ . Iron and friction losses on full-load 450 W. Calculate the efficiency when taking the full rated current of 24 A.  
 (85%)
7. A d.c. series motor on full-load takes 50 A from 230 V d.c. mains. The total resistance of the motor is 0.22  $\Omega$ . If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 A. (10.275 kW)
8. A 2-pole d.c shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500  $\Omega$  and the armature has a resistance of 0.6  $\Omega$ . Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm. Neglect temperature rise.  
 [Rajiv Gandhi Tech. Univ. Bhopal,2000] (82.63%)

### OBJECTIVE TESTS – 29

- In a d.c. motor, unidirectional torque is produced with the help of  
 (a) brushes (b) commutator  
 (c) end-plates (d) both (a) and (b)
- The counter e.m.f. of a d.c. motor  
 (a) often exceeds the supply voltage  
 (b) aids the applied voltage  
 (c) helps in energy conversion  
 (d) regulates its armature voltage
- The normal value of the armature resistance of a d.c. motor is  
 (a) 0.005 (b) 0.5  
 (c) 10 (d) 100  
 (Grad. I.E.T.E. June 1987)
- The  $E_b/V$  ratio of a d.c. motor is an indication of its  
 (a) efficiency (b) speed regulation  
 (c) starting torque (d) Running Torque  
 (Grad. I.E.T.E. June 1987)
- The mechanical power developed by the armature of a d.c. motor is equal to  
 (a) armature current multiplied by back e.m.f.  
 (b) power input minus losses  
 (c) power output multiplied by efficiency  
 (d) power output plus iron losses
- The induced e.m.f. in the armature conductors of a d.c. motor is  
 (a) sinusoidal (b) trapezoidal  
 (c) rectangular (d) alternating
- A d.c. motor can be looked upon as d.c. generator with the power flow  
 (a) reduced (b) reversed  
 (c) increased (d) modified
- In a d.c. motor, the mechanical output power actually comes from  
 (a) field system  
 (b) air-gap flux  
 (c) back e.m.f.  
 (d) electrical input power
- The maximum torque of d.c. motors is limited by  
 (a) commutation (b) heating



- (c) speed (d) armature current
10. Which of the following quantity maintains the same direction whether a d.c. machine runs as a generator or as a motor ?  
 (a) induced e.m.f. (b) armature current  
 (c) field current (d) supply current
11. Under constant load conditions, the speed of a d.c. motor is affected by  
 (a) field flux (b) armature current  
 (c) back e.m.f. (d) both (b) and (c)
12. It is possible to increase the field flux and, at the same time, increase the speed of a d.c. motor provided its ..... is held constant.  
 (a) applied voltage  
 (b) torque  
 (c) Armature circuit resistance  
 (d) armature current
13. The current drawn by a 120 - V d.c. motor of armature resistance  $0.5 \Omega$  and back e.m.f. 110 V is ..... ampere.  
 (a) 20 (b) 240  
 (c) 220 (d) 5
14. The shaft torque of a d.c. motor is less than its armature torque because of ..... losses.  
 (a) copper (b) mechanical  
 (c) iron (d) rotational
15. A d.c. motor develops a torque of 200 N-m at 25 rps. At 20 rps it will develop a torque of ..... N-m.  
 (a) 200 (b) 160  
 (c) 250 (d) 128
16. Neglecting saturation, if current taken by a series motor is increased from 10 A to 12 A, the percentage increase in its torque is ..... percent  
 (a) 20 (b) 44  
 (c) 30.5 (d) 16.6
17. If load on a d.c. shunt motor is increased, its speed is decreased due primarily to  
 (a) increase in its flux  
 (b) decrease in back e.m.f.  
 (c) increase in armature current  
 (d) increase in brush drop
18. If the load current and flux of a d.c. motor are held constant and voltage applied across its armature is increased by 10 per cent, its speed will  
 (a) decrease by about 10 per cent  
 (b) remain unchanged  
 (c) increase by about 10 per cent  
 (d) increase by 20 per cent.
19. If the pole flux of a d.c. motor approaches zero, its speed will  
 (a) approach zero  
 (b) approach infinity  
 (c) no change due to corresponding change in back e.m.f.  
 (d) approach a stable value somewhere between zero and infinity.
20. If the field circuit of a loaded shunt motor is suddenly opened  
 (a) it would race to almost infinite speed  
 (b) it would draw abnormally high armature current  
 (c) circuit breaker or fuse will open the circuit before too much damage is done to the motor  
 (d) torque developed by the motor would be reduced to zero.
21. Which of the following d.c. motor would be suitable for drives requiring high starting torque but only fairly constant speed such as crushers ?  
 (a) shunt (b) series  
 (c) compound (d) permanent magnet
22. A d.c. shunt motor is found suitable to drive fans because they require  
 (a) small torque at start up  
 (b) large torque at high speeds  
 (c) practically constant voltage  
 (d) both (a) and (b)
23. Which of the following load would be best driven by a d.c. compound motor ?  
 (a) reciprocating pump  
 (b) centrifugal pump  
 (c) electric locomotive  
 (d) fan
24. As the load is increased, the speed of a d.c. shunt motor  
 (a) increases proportionately  
 (b) remains constant  
 (c) increases slightly  
 (d) reduces slightly
25. Between no-load and full-load, ..... motor develops the *least* torque  
 (a) series  
 (b) shunt  
 (c) cumulative compound  
 (d) differential compound

26. The  $T_d/I_a$  graph of a d.c. series motor is a  
 (a) parabola from no-load to overload  
 (b) straight line throughout  
 (c) parabola throughout  
 (d) parabola upto full-load and a straight line at overloads.
27. As compared to shunt and compound motors, series motor has the highest torque because of its comparatively ..... at the start.  
 (a) lower armature resistance  
 (b) stronger series field  
 (c) fewer series turns  
 (d) larger armature current
28. Unlike a shunt motor, it is difficult for a series motor to stall under heavy loading because  
 (a) it develops high overload torque  
 (b) its flux remains constant  
 (c) it slows down considerably  
 (d) its back e.m.f. is reduced to almost zero.
29. When load is removed, ..... motor will run at the **highest** speed.  
 (a) shunt  
 (b) cumulative-compound  
 (c) differential compound  
 (d) series
30. A series motor is best suited for driving  
 (a) lathes  
 (b) cranes and hoists  
 (c) shears and punches  
 (d) machine tools
31. A 220 V shunt motor develops a torque of 54 N-m at armature current of 10 A. The torque produced when the armature current is 20 A, is  
 (a) 54 N-m (b) 81 N-m  
 (c) 108 N-m (d) None of the above  
*(Elect. Machines, A.M.I.E. Sec. B, 1993)*
32. The d.c. series motor should never be switched on at no load because  
 (a) the field current is zero  
 (b) The machine does not pick up  
 (c) The speed becomes dangerously high  
 (d) It will take too long to accelerate.  
*(Grad. I.E.T.E. June 1988)*
33. A shunt d.c. motor works on a.c. mains  
 (a) unsatisfactorily (b) satisfactorily  
 (c) not at all (d) none of the above  
*(Elect. Machines, A.M.I.E. Sec. B, 1993)*
34. A 200 V, 10 A motor could be rewound for 100 V, 20 A by using ..... as many turns per coil of wire, having ..... the cross-sectional area.  
 (a) twice, half  
 (b) thrice, one third  
 (c) half, twice  
 (d) four times, one-fourth

### ANSWERS

1. (d) 2. (c) 3. (b) 4. (a) 5. (a) 6. (a) 7. (b) 8. (d) 9. (a) 10. (a) 11. (a)  
 12. (d) 13. (a) 14. (d) 15. (a) 16. (b) 17. (b) 18. (c) 19. (b) 20. (c) 21. (c) 22. (d)  
 23. (a) 24. (d) 25. (a) 26. (d) 27. (b) 28. (a) 29. (d) 30. (b) 31. (c) 32. (c) 33. (a)  
 34. (c).

# CHAPTER 30

## Learning Objectives

- Factors Controlling Motor Speed
- Speed Control of Shunt Motors
- Speed Control of Series Motors
- Merits and Demerits of Rheostatic Control Method
- Series-Parallel Control
- Electric Braking
- Electric Braking of Shunt Motor
- Electric Braking of Series Motors
- Electronic Speed control Method for D.C. Motors
- Uncontrolled Rectifiers
- Controlled Rectifiers
- Thyristor Choppers
- Thyristor Inverters
- Thyristor Speed Control of Separately-excited D.C. Motor
- Thyristor Speed Control of D.C. Series Motor
- Full-wave Speed Control of a Shunt Motor
- Thyristor Control of a Shunt Motor
- Thyristor Speed Control of a Series D.C. Motor
- Necessity of a Starter
- Shunt Motor Starter
- Three-point Starter
- Four-point Starter
- Starting and Speed Control of Series Motors
- Grading of Starting Resistance
- Shunt Motors
- Series Motor Starters
- Thyristor Controller Starters

## SPEED CONTROL OF D.C. MOTORS



DC motor speed controller control the speed of any common dc motor rated upto 100 V. It operates on 5V to 15 V.

### 30.1. Factors Controlling Motor Speed

It has been shown earlier that the speed of a motor is given by the relation

$$N = \frac{V - I_a R_a}{Z \Phi} \cdot \left(\frac{A}{P}\right) = K \frac{V - I_a R_a}{\Phi} \text{ r.p.s.}$$

where

$R_a$  = armature circuit resistance.

It is obvious that the speed can be controlled by varying (i) flux/pole,  $\Phi$  (Flux Control) (ii) resistance  $R_a$  of armature circuit (Rheostatic Control) and (iii) applied voltage  $V$  (Voltage Control). These methods as applied to shunt, compound and series motors will be discussed below.

### 30.2. Speed Control of Shunt motors

#### (i) Variation of Flux or Flux Control Method

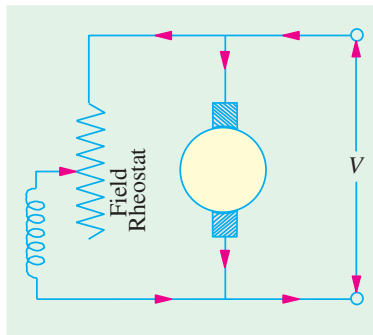


Fig. 30.1

It is seen from above that  $N \propto 1/\Phi$ . By decreasing the flux, the speed can be increased and *vice versa*. Hence, the name **flux** or **field control** method. The flux of a d.c. motor can be changed by changing  $I_{sh}$  with help of a shunt field rheostat (Fig. 30.1). Since  $I_{sh}$  is relatively small, shunt field rheostat has to carry only a small current, which means  $I^2 R$  loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpole machine, the speed can be increased by this method in the ratio 2 : 1. Any further weakening of flux  $\Phi$  adversely affects the commutation and hence puts a limit to the maximum speed obtainable with the method. In machines fitted with interpoles, a ratio of maximum to minimum speed of 6 : 1 is fairly common.

**Example 30.1.** A 500 V shunt motor runs at its normal speed of 250 r.p.m. when the armature current is 200 A. The resistance of armature is 0.12 ohm. Calculate the speed when a resistance is inserted in the field reducing the shunt field to 80% of normal value and the armature current is 100 ampere. (Elect. Engg. A.M.A.E. S.I. June 1992)

**Solution.**  $E_{b1} = 500 - 200 \times 0.12 = 476 \text{ V}$ ;  $E_{b2} = 500 - 100 \times 0.12 = 488 \text{ V}$

$$\Phi_2 = 0.8 \Phi_1 ; N_1 = 250 \text{ rpm} ; N_2 = ?$$

Now, 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{250} = \frac{488}{476} \times \frac{\Phi_1}{0.8 \Phi_1}, N_2 = 320 \text{ r.p.m.}$$

**Example 30.2.** A 250 volt d.c. shunt motor has armature resistance of 0.25 ohm, on load it takes an armature current of 50 A and runs at 750 r.p.m. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor. (Elect. Eng-II, Pune Univ. 1987)

**Solution.** 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now,  $T_a \propto \Phi I_a$ . Hence  $T_{a1} \propto \Phi_1 I_{a1}$  and  $T_{a2} \propto \Phi_2 I_{a2}$ .

Since  $T_{a1} = T_{a2} \therefore \Phi_1 I_{a1} = \Phi_2 I_{a2}$

Now,  $\Phi_2 = 0.9 \Phi_1 \therefore 50 \Phi_1 = 0.9 \Phi_2 I_{a2} = 0.9 \Phi_1 I_{a2}$ ,  $I_{a2} = 55.6 \text{ A}$

$\therefore E_{b1} = 250 - (50 \times 0.25) = 237.5 \text{ V}$  ;  $E_{b2} = 250 - (55.6 \times 0.25) = 231.1 \text{ V}$

$\therefore \frac{N_2}{750} = \frac{231.1}{237.5} \times \frac{\Phi_1}{0.9 \Phi_1}$  ;  $N_2 = 811 \text{ r.p.m.}$

**Example 30.3.** Describe briefly the method of speed control available for dc motors.

A 230 V d.c. shunt motor runs at 800 r.p.m. and takes armature current of 50 A. Find resistance

to be added to the field circuit to increase speed to 1000 r.p.m. at an armature current of 80 A. Assume flux proportional to field current. Armature resistance = 0.15  $\Omega$  and field winding resistance = 250  $\Omega$ .  
(Elect. Technology, Hyderabad Univ. 1991)

**Solution.**  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$  since flux  $\propto$  field current

$E_{b1} = 230 - (50 \times 0.15) = 222.5 \text{ V}; E_{b2} = 230 - (80 \times 0.15) = 218 \text{ V}$

Let  $R_t =$  total shunt resistance = (250 + R) where R is the additional resistance

$I_{sh1} = 230/250 = 0.92 \text{ A}, I_{sh2} = 230/R_t; N_1 = 800 \text{ r.p.m.}; N = 1000 \text{ r.p.m.}$

$\therefore \frac{1000}{800} = \frac{218}{222.5} \times \frac{0.92}{230/R_t}; R_t = 319 \Omega \quad \therefore R = 319 - 250 = 69 \Omega,$

$I_{sh2} = \frac{230}{319} = 0.721$

Ratio of torque in two cases =  $\frac{T_2}{T_1} = \frac{I_{sh2} I_{a2}}{I_{sh1} I_{a1}} = \frac{0.721 \times 80}{0.92 \times 50} = 1.254$

**Example 30.4.** A 250 V, d.c. shunt motor has shunt field resistance of 250  $\Omega$  and an armature resistance of 0.25  $\Omega$ . For a given load torque and no additional resistance included in the shunt field circuit, the motor runs at 1500 r.p.m. drawing an armature current of 20 A. If a resistance of 250  $\Omega$  is inserted in series with the field, the load torque remaining the same, find out the new speed and armature current. Assume the magnetisation curve to be linear.

(Electrical Engineering-I, Bombay Univ. 1987)

**Solution.** In this case, the motor speed is changed by changing the flux.

Now,  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since it is given that magnetisation curve is linear, it means that flux is directly proportional to shunt current. Hence  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$  where  $E_{b2} = V - I_{a2} R_a$  and  $E_{b1} = V - I_{a1} R_a$ .

Since load torque remains the same  $\therefore T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2}$  or  $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

$\therefore I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$

Now,  $I_{sh1} = 250/250 = 1 \text{ A}; I_{sh2} = 250/(250 + 250) = 1/2 \text{ A}$

$\therefore I_{a2} = 20 \times \frac{1}{1/2} = 40 \text{ A} \quad \therefore E_{b2} = 250 - (40 \times 0.25) = 240 \text{ V}$  and

$E_{b1} = 250 - (20 \times 0.25) = 245 \text{ V} \quad \therefore \frac{N_2}{1500} = \frac{240}{245} \times \frac{1}{1/2}$

$\therefore N_2 = 2,930 \text{ r.p.m.}$

**Example 30.5.** A 250 V, d.c. shunt motor has an armature resistance of 0.5  $\Omega$  and a field resistance of 250  $\Omega$ . When driving a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance should be inserted in the shunt field circuit? Assume that the magnetic circuit is unsaturated.

(Elect. Engg. AMIETE, June 1992)

**Solution.**  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since the magnetic circuit is unsaturated, it means that flux is directly proportional to the shunt current.

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}} \quad \text{where } E_{b2} = V - I_{a2} R_a \text{ and } E_{b1} = V - I_{a1} R_a$$

Since motor is driving at load of constant torque,

$$T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \therefore \quad \Phi_2 I_{a2} = \Phi_1 I_{a1} \quad \text{or} \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$$

Now,  $I_{sh1} = 250/250 = 1 \text{ A}$ ;  $I_{sh2} = 250/R_t$   
 where  $R_t$  is the total resistance of the shunt field circuit.

$$\therefore \quad I_{a2} = 20 \times \frac{1}{250/R_t} = \frac{2R_t}{25}; \quad E_{b1} = 250 - (20 \times 0.5) = 240 \text{ V}$$

$$E_{b2} = 250 - \left( \frac{2R_t}{25} \times 0.5 \right) = 250 - (R_t/25) \quad \therefore \quad \frac{800}{600} = \frac{250 - (R_t/25)}{240} \times \frac{1}{250/R_t}$$

$$0.04 R_t^2 - 250 R_t + 80,000 = 0$$

$$\text{or} \quad R_t = \frac{250 \pm \sqrt{62,500 - 12,800}}{0.08} = \frac{27}{0.08} = 337.5 \Omega$$

Additional resistance required in the shunt field circuit =  $337.5 - 250 = 87.5 \Omega$ .

**Example 30.6.** A 220 V shunt motor has an armature resistance of 0.5 Ω and takes a current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant? (Elect. Machines, AMIE, Sec B, 1991)

**Solution.** Formula used is  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ . Since torque remains constant,

$$\text{Hence} \quad \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \therefore \quad I_{a2} = I_{a1} \cdot \frac{\Phi_1}{\Phi_2} = 40x \text{ where } x = \frac{\Phi_1}{\Phi_2}$$

$$E_{b1} = 220 - (40 \times 0.5) = 200 \text{ V}; \quad E_{b2} = 220 - (40x \times 0.5) = (220 - 20x) \text{ V}$$

$$\frac{N_2}{N_1} = \frac{3}{2} \quad \dots \text{ given} \quad \therefore \quad \frac{3}{2} = \frac{(220 - 20x)}{200} \times x \quad \therefore \quad x^2 - 11x + 15 = 0$$

$$\text{or} \quad x = \frac{11 \pm \sqrt{121 - 60}}{2} = \frac{11 \pm 7.81}{2} = 9.4^* \quad \text{or} \quad 1.6$$

$$\therefore \quad \frac{\Phi_1}{\Phi_2} = 1.6 \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{1}{1.6}$$

$$\therefore \quad \frac{\Phi_1 - \Phi_2}{\Phi_1} = \frac{1.6 - 1}{1.6} = \frac{3}{8} \quad \therefore \quad \text{percentage change in flux} = \frac{3}{8} \times 100 = 37.5\%$$

**Example 30.7.** A 220-V, 10-kW, 2500 r.p.m. shunt motor draws 41 A when operating at rated conditions. The resistances of the armature, compensating winding, interpole winding and shunt field winding are respectively 0.2 Ω, 0.05 Ω, 0.1 Ω and 110 Ω. Calculate the steady-state values of armature current and motor speed if pole flux is reduced by 25%, a 1 Ω resistance is placed in series with the armature and the load torque is reduced by 50%.

**Solution.**  $I_{sh} = 220/110 = 2 \text{ A}$ ;  $I_{a1} = 41 - 2 = 39 \text{ A}$  (Fig. 30.2)

$$T_1 \propto \Phi_1 I_{a1} \text{ and } T_2 \propto \Phi_2 I_{a2}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{\Phi_2}{\Phi_1} \times \frac{I_{a2}}{I_{a1}}$$

\* This figure is rejected as it does not give the necessary increase in speed.

or  $\frac{1}{2} = \frac{3}{4} \times \frac{I_{a2}}{39} \therefore I_{a2} = 26 \text{ A}$

$E_{b1} = 220 - 39(0.2 + 0.1 + 0.05) = 206.35 \text{ V}$

$E_{b2} = 220 - 26(1 + 0.35) = 184.9 \text{ V}$

Now,  $\frac{N_2}{2500} = \frac{184.9}{206.35} \times \frac{4}{3}; N_2 = 2987 \text{ r.p.m.}$

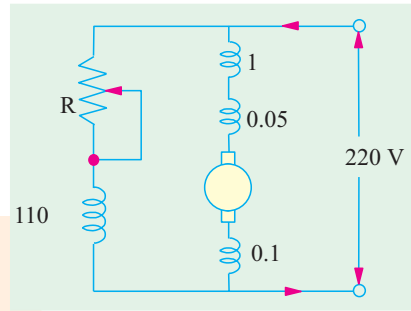


Fig. 30.2

**Example 30.8.** A 220 V, 15 kW, 850 r.p.m. shunt motor draws 72.2 A when operating at rated condition. The resistances of the armature and shunt field are 0.25 Ω and 100 Ω respectively. Determine the percentage reduction in field flux in order to obtain a speed of 1650 r.p.m. when armature current drawn is 40 A.

**Solution.**

$I_{sh} = 220/100 = 2.2 \text{ A}; I_{a1} = 72.2 - 2.2 = 70 \text{ A}$

$E_{b1} = 220 - 70 \times 0.25 = 202.5 \text{ V}, E_{b2} = 220 - 40 \times 0.25 = 210 \text{ V.}$

Now,  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$  or  $\frac{1650}{850} = \frac{210}{202.5} \times \frac{\Phi_1}{\Phi_2}$

$\therefore \Phi_2 = 0.534 \Phi_1;$

$\therefore$  Reduction in field flux =  $\frac{\Phi_1 - 0.534 \Phi_1}{\Phi_1} \times 100 = 46.6\%.$

**Example 30.9.** A 220 V shunt motor has an armature resistance of 0.5 ohm and takes an armature current of 40 A on a certain load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant? Neglect saturation and armature reaction.

(Elect. Machines, AMIE, Sec B, 1991)

**Solution.**

$T_1 \propto \Phi_1 \times \Phi_1 \rightarrow \Phi_1 I_{a1}$ , and  $T_2 \propto \Phi_2 I_{a2}$  Since,  $T_1 = T_2$

$\therefore \Phi_2 I_{a2} = \Phi_1 I_{a1}$  or  $\Phi_2/\Phi_1 = I_{a1}/I_{a2} = 40/I_{a2}$

$E_{b1} = 220 - 40 \times 0.5 = 200 \text{ V}; E_{b2} = 220 - 0.5 I_{a2}$

Now,  $\frac{E_{b2}}{E_{b1}} = \frac{N_2 \Phi_2}{N_1 \Phi_1}$  or  $\frac{220 - 0.5 I_{a2}}{200} = \frac{1.5 N_1}{N_1} \times \frac{40}{I_{a2}}$

$\therefore I_{a2} - 40 I_{a2} + 24,000 = 0$  or  $I_{a2} = 63.8 \text{ A}$

$\therefore \frac{\Phi_2}{\Phi_1} = \frac{40}{63.8} = 0.627$  or  $\Phi_2 = 0.627 \Phi_1 = 62.7\% \text{ of } \Phi_1$

**Example 30.10.** A d.c. shunt motor takes an armature current of 20 A from a 220 V supply. Armature circuit resistance is 0.5 ohm. For reducing the speed by 50%, calculate the resistance required in the series, with the armature, if

(a) the load torque is constant

(b) the load torque is proportional to the square of the speed. (Sambalpur Univ., 1998)

**Solution.**

$E_{b1} = V - I_a r_a = 220 - 20 \times 0.5 = 210 \text{ V}$

$210 \propto N_1$

(a) Constant Load torque

In a shunt motor, flux remains constant unless there is a change in terminal voltage or there is a change in the field-circuit resistance.

If torque is constant, armature-current then must remain constant.  $I_a = 20 \text{ amp}$

With an external armature-circuit resistor of R ohms,  $20 \times (R + 0.5) = 220 - E_{b2}$

The speed required now is  $0.5 N_1$ .

With constant flux,  $E_b \propto \text{speed}$ . Hence,  $210 \propto N_1$

$$E_{b2} \propto 0.5 N_1, E_{b2} = 105$$

$$R + 0.5 = (220 - 105)/20 = 5.75, \text{ giving } R = \mathbf{5.25 \text{ ohms}}$$

**(b) Load torque is proportional to the square of speed.**

With constant flux, Developed Torque at  $N_1$  r.p.m.  $\propto I_{a1}$

$$T_{m1} \propto 20$$

From the Load Side,  $T_{L1} \propto N_1^2$

Since  $T_{m1} = T_{L1}$

$$20 \propto N_1^2 \quad \dots(a)$$

At 50% speed, Load Torque,  $T_{L2} \propto (0.5 N_1)^2$

For motor torque,  $T_{m2} \propto I_{a2}$

Since  $T_{L2} = T_{m2}, I_{a2} \propto (0.5 N_1)^2 \quad \dots(b)$

From eqn. (a) and (b) above

$$\frac{I_{a2}}{I_{a1}} = 0.25, I_{a2} = 5 \text{ amp}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{220 - I_{a2}(R + 0.5)}{210} = 0.5 N_1$$

$$220 - I_{a2}(R + 0.5) = 0.5 \times 210$$

$$R + 0.5 = \frac{220 - 105}{5} = 23, R = \mathbf{22.5 \text{ ohms}}$$

**Check :** With the concept of armature power output : (applied here for part (b) only as an illustration).

Armature power-output =  $E_b \times I_a = T \times \omega$

When  $T \propto (\text{speed})^2, E_b I_a = k_1 T \omega = K_2 \omega^3$

At  $N_1$  r.p.m.  $210 \times 20 = K_3 N_1^3 \quad \dots(c)$

With constant flux, at half speed

$$E_{b2} = 105$$

$$105 \times I_{a2} = K_3 (0.5 N_1)^3 \quad \dots(d)$$

From eqs. (c) and (d),

$$\frac{105 \times I_{a2}}{210 \times 20} = \frac{0.125 \times N_1^3}{N_1^3}, \text{ giving } I_{a2} = 5 \text{ amp}$$

This gives  $R = \mathbf{22.5 \text{ ohms.}}$

**Example 30.11.** A 250 V shunt motor runs at 1000 r.p.m. at no-load and takes 8 A. The total armature and shunt field resistances are respectively 0.2 ohm, 250 ohm. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant. (Nagpur Univ. Summer 2000)

**Solution.** The current distribution is shown in Fig. 30.3.

At no load,  $I_L = 8 \text{ amp}, I_f = 1 \text{ amp},$

Hence,  $I_a = 7 \text{ amp}$

$$E_{b0} = 250 - 7 \times 0.2 = 248.6 \text{ volts,}$$

$$= K \phi \times 1000$$

$\therefore K \phi = 0.2486$

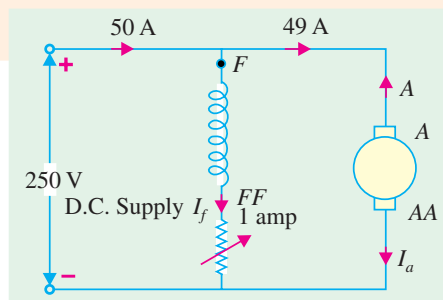


Fig. 30.3



$$\begin{aligned} \text{At load, } I_a &= 49 \text{ amp} \\ E_{b1} &= 250 - 49 \times 0.2 = 240.2 \\ N_1 &= \frac{240.2}{0.2486} = 966.2 \text{ r.p.m.} \end{aligned}$$

**Notes :** (i) The assumption of constant flux has simplified the issue. Generally, armature reaction tends to weaken the flux and then the speed tends to increase slightly.

(ii) The no load armature current of 7 amp is required to overcome the mechanical losses of motor as well as driven load, at about 1000 r.p.m.

**Example 30.12.** A 240 V d.c. shunt motor has an armature-resistance of 0.25 ohm, and runs at 1000 r.p.m., taking an armature current 40 A. It is desired to reduce the speed to 800 r.p.m.

(i) If the armature current remains the same, find the additional resistance to be connected in series with the armature-circuit.

(ii) If, with the above additional resistance in the circuit, armature current decreases to 20 A, find the speed of the motor. (Bhartiar Univ., November 1997)

**Solution.**  $E_b = 240 - 0.25 \times 40 = 230 \text{ V}, 230 \propto 1000 \quad \dots(a)$

(i) For 800 r.p.m.,  $E_{b2} \propto 800 \quad \dots(b)$

From (a) and (b),  $E_{b2} = \frac{800}{1000} \times 230 = 184$

$240 - (R + 0.25) \times 40 = 184, R = 1.15 \text{ ohm}$

(ii)  $E_{b3} = 240 - 20 (1.40) = 212$

$E_{b3} = \frac{N_3}{1000} \times 230 = 212, N_3 = \frac{212}{230} \times 1000 = 922 \text{ r.p.m.}$

**Example 30.13.** A 7.48 kW, 220 V, 990 r.p.m. shunt motor has a full load efficiency of 88%, the armature resistance is 0.08 ohm and shunt field current is 2 A. If the speed of this motor is reduced to 450 r.p.m. by inserting a resistance in the armature circuit, find the motor output, the armature current, external resistance to be inserted in the armature circuit and overall efficiency. Assume the load torque to remain constant. (Nagpur Univ., November 1998)

**Solution.** With an output of 7.48 kW and an efficiency of 88%, the input power is 8.50 kW. Losses are 1.02 kW.

Input Current =  $85000/220 = 38.64 \text{ A}$

Armature-Current =  $38.64 - 2.00 = 36.64 \text{ A}$

Power Loss in the shunt field circuit =  $220 \times 2 = 440 \text{ W}$

Copper-Loss in armature-circuit =  $36.64^2 \times 0.08 = 107.4 \text{ W}$

No-Load-Loss at 900 r.p.m. =  $1020 - 107.4 - 440 = 473 \text{ W}$

At 900 r.p.m. Back-emf =  $E_{b1} = 200 - (36.64 \times 0.08) = 217.3 \text{ V}$

Motor will run at 450 r.p.m. with flux per pole kept constant, provided the back-emf =  $E_{b2}$

=  $(459/900) \times 217.1 \text{ V} = 108.5 \text{ V}$

There are two simplifying assumptions in this case, which must be stated before further calculations :

1. Load-torque is constant,
2. No Load Losses are constant.

(These statements can be different which leads to variations in the next steps of calculations.)

For constant load-torque, the condition of constant flux per pole results into constant armature current, which is 36.64 A.

With an armature current of 36.64 A, let the external resistance required for this purpose be  $R$ .

$36.64 R = 217.1 - 108.5 = 108.6 \text{ V}, R = 2.964 \text{ ohms}$

**Total Losses**

$$\begin{aligned} \text{Total } i^2r \text{ - loss in armature} &= 36.64^2 \times (2.964 + 0.08) = 4086 \text{ W} \\ \text{Field-copper-loss} + \text{No load loss} &= 440 + 473 = 913 \\ \text{Total Loss} &= 4999 \text{ W} \\ \text{Total Output} &= 8500 - 4999 = 3501 \text{ W} \end{aligned}$$

Hence, Efficiency =  $(3501/8500) \times 100 = 41.2\%$

**(Note :** Because of missing data and clarification while making the statements in the question, there can be variations in the assumption and hence in the final solutions.)

**Example 30.14.** A d.c. shunt motor supplied at 230 V runs at 990 r.p.m. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 r.p.m. assuming that armature current is 25 amp. (Nagpur Univ., November 1997)

**Solution.** It is assumed that armature resistance is to

(a) At 900 r.p.m. :  $E_{b1} = 230 = K \times 900$

(b) At 500 r.p.m. :  $E_{b2} = K \times 500$

Therefore,  $E_{b2} = E_{b1} \times 500/900 = 127.8$  volts

The difference between  $E_{b1}$  and  $E_{b2}$  must be the drop in the external resistance to be added to the armature circuit for the purpose of reducing the speed to 500 r.p.m.

$$\begin{aligned} E_{b1} - E_{b2} &= 25 \times R \\ R &= (230 - 127.8)/25 = 4.088 \text{ ohms.} \end{aligned}$$

**Example 30.15.** A 220 V d.c. shunt motor has an armature resistance of 0.4 ohm and a field circuit resistance of 200 ohms. When the motor is driving a constant-torque load, the armature-current is 20 A, the speed being 600 r.p.m. It is desired to run the motor at 900 r.p.m. by inserting a resistance in the field circuit. Find its value, assuming that the magnetic circuit is not saturated. (Nagpur Univ., November 1996)

**Solution. (i)** At 600 r.p.m.  $i_{f1} = 220/200 = 1.1$  amp

$$E_{b1} = 220 - (20 \times 0.4) = 212 \text{ volts}$$

$$T_L = K_1 \times 1.1 \times 20$$

$$\text{The back e.m.f.} = 212 = K_2 \times 1.1 \times 600$$

or  $K_2 = 212/660 = 0.3212$

(ii) At 900 r.p.m. :  $T_L = K_1 \times i_{f2} \times I_{a2}$

Due to constant load torque,

$$i_{f2} \times I_{a2} = 1.1 \times 20 = 22$$

$$E_{b2} = 220 - (0.4 I_{a2}) = K_2 \times i_{f2} \times 900 = 289 i_{f2}$$

$$220 - (0.4 \times 22/i_{f2}) = 289 i_{f2}$$

Guess for approximate value of  $i_{f2}$  : Neglecting armature -resistance drop and saturation, 50% rise in speed is obtained with proportional decrease in  $i_{f2}$  related by

$$\frac{600}{900} \cong \frac{i_{f2}}{i_{f1}} \text{ giving } i_{f2} \cong 0.73 \text{ amp}$$

[In place of  $i_{f2}$ ,  $I_{a2}$  can be evaluated first. Its guess-work will give  $I_{a2} \cong 1.5 \times 20 \cong 30$  amp]

Continuing with the solution of the equation to evaluate a value of  $i_{f2}$ , accepting that value which is near 0.73 amp, we have  $i_{f2} = 0.71865$ . [Note that other value of  $i_{f2}$ , which is 0.04235, is not acceptable.]. Corresponding  $I_{a2} = 30.6$  amp. Previous shunt field current,  $i_{f1} = 1.1$ ,  $R_{f1} = 200 \Omega$ . New shunt field current,  $i_{f2} = 0.71865$ ,  $R_{f2} = 220/0.71865 = 306 \Omega$ . Final answer is that a resistor of 106 ohms is to be added to the field circuit to run the motor at 900 r.p.m. at constant torque.

**Example 30.16.** A 220 V d.c. shunt motor has an armature resistance of 0.40 ohm and field-resistance of 200 ohms. It takes an armature current of 22 A and runs at 600 r.p.m. It drives a load whose torque is constant. Suggest a suitable method to raise the speed to 900 r.p.m. Calculate the value of the controllable parameter. (Nagpur Univ., April 1998)

**Solution.** At 600 r.p.m.  $I_{a1} = 22$  amp,  $N_1 = 600$  r.p.m.,  $i_{f1} = 220/200 = 1.1$  amp.  
 $E_{b1} = 220 - (22 \times 0.40) = 211.2$  volts.

Let the Load-torque be denoted by  $T_L$ ,  $k_1$  and  $k_2$  in the equations below represent machine constants appearing in the usual emf-equation and torque-equation for the d.c. shunt motor.

$$E_{b1} = 211.2 = k_1 \times i_{f1} \times N_1 = k_1 \times 1.10 \times 600 \quad \text{or} \quad k_1 = 211.2/660 = 0.32$$

$$T_L = k_2 \times i_{f1} \times I_{a1} = k_2 \times 1.10 \times 22$$

Since the load torque will remain constant at 900 r.p.m. also, the corresponding field current ( $= i_{f2}$ ) and armature current ( $= I_{a2}$ ) must satisfy the following relationships :

$$T_L = k_2 i_{f2} I_{a2} = k_2 \times 1.10 \times 22$$

or  $i_{f2} \times I_{a2} = 24.2$

And  $E_{b2} = 220 - (I_{a2} \times 0.40) = k_1 \times i_{f2} \times 900$

$$220 - (I_{a2} \times 0.40) = 0.32 \times (24.2 \times I_{a2}) \times 900$$

(Alternatively, the above equation can also lead to a quadratic in  $i_{f2}$ .)

This leads to a quadratic equation in  $I_{a2}$ .

Guess for  $i_{f2}$  : Approximately, speed of a d.c. shunt motor is inversely proportional to the field current. Comparing the two speeds of 600 and 900 r.p.m., the value of  $i_{f2}$  should be approximately given by

$$i_{f2} \cong i_{f1} \times (600/900) = 0.733 \text{ amp}$$

Guess for  $I_{a2}$  : For approximate conclusions, armature-resistance drop can be ignored. With constant load-torque, armature-power must be proportional to the speed.

$$\frac{\text{Armature-power at 900 r.p.m.}}{\text{Armature-power at 600 r.p.m.}} = \frac{900}{600} = 1.5$$

$$E_{b2} I_{a2} = 1.5 \times E_{b1} I_{a1}$$

Neglecting armature-resistance drops,  $E_{b1} = V$  and  $E_{b2} = V$ .

This gives  $I_{a2} = 1.5 \times 22 = 33$  amps

Thus, out of the two roots for  $i_{f2}$ , that which is close to 0.733 is acceptable. If quadratic equation for  $I_{a2}$  is being handled, that root which is near 33 amp is acceptable.

Continuing with the solution to quadratic equation for  $I_{a2}$ , we have

$$220 - 0.40 I_{a2} = 0.32 \times (24.2 \times I_{a2}) \times 900$$

$$220 - 0.40 I_{a2} = 6969/I_{a2}$$

$$I_{a2}^2 - 550 I_{a2} + 17425 = 0$$

This gives  $I_{a2}$  as **either** 33.75 amp **or** 791.25 amp.

From the reasoning given above, acceptable root corresponds to  $I_{a2} = 33.75$  amp.

Corresponding field current,  $i_{f2} = 24.2/33.75 = 0.717$  amp

Previous field circuit resistance = 200 ohms

New field circuit resistance =  $220/0.717 = 307$  ohms

Hence, additional resistance of 107 ohms must be added to the shunt field circuit to run the motor at 900 r.p.m. under the stated condition of constant Load torque.

**Additional Check :** Exact calculations for proportions of armature-power in two cases will give the necessary check.

$$E_{b2} = 220 - (33.75 \times 0.40) = 206.5$$

As mentioned above, while guessing the value of  $I_{a2}$ , the proportion of armature-power should be 1.5.

$$\frac{E_{b2} I_{a2}}{E_{b1} I_{a1}} = \frac{206.56 \times 33.75}{211.2 \times 22} = 1.50$$

Thus, the results obtained are confirmed.

**Example 30.17.** A 250 V, 25 kW d.c. shunt motor has an efficiency of 85% when running at 1000 r.p.m. on full load. The armature resistance is 0.1 ohm and field resistance is 125 ohms. Find the starting resistance required to limit the starting current to 150% of the rated current.

(Amravati Univ., 1999)

**Solution.**

Output power = 25 kW, at full-load.

$$\text{Input power} = \frac{25,000}{0.85} = 29412 \text{ watts}$$

At Full load, Input Current =  $29412/250 = 117.65$  amp

Field Current =  $250/125 = 2$  amp

F.L. Armature Current =  $117.65 - 2 = 115.65$  amp

Limit of starting current =  $1.50 \times 115.65 = 173.5$  amp

Total resistance in armature circuit at starting

$$= \frac{250}{173.5} = 1.441 \text{ ohms}$$

External resistance to be added to armature circuit

$$= 1.441 - 0.1 = 1.341 \text{ ohm.}$$

### Tutorial Problems 30.1

- A d.c. shunt motor runs at 900 r.p.m. from a 460 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230-V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8  $\Omega$ . Assume the flux per pole at 230 V to have decreased to 75% of its value at 460 V. [595 r.p.m.]
- A 250 V shunt motor has an armature resistance of 0.5  $\Omega$  and runs at 1200 r.p.m. when the armature current is 80 A. If the torque remains unchanged, find the speed and armature current when the field is strengthened by 25%. [998 r.p.m. ; 64 A]
- When on normal full-load, a 500 V, d.c. shunt motor runs at 800 r.p.m. and takes an armature current 42 A. The flux per pole is reduced to 75% of its normal value by suitably increasing the field circuit resistance. Calculate the speed of the motor if the total torque exerted on the armature is (a) unchanged (b) reduced by 20%.  
The armature resistance is 0.6  $\Omega$  and the total voltage loss at the brushes is 2 V. [(a) 1,042 r.p.m. (b) 1,061 r.p.m.]
- The following data apply to d.c. shunt motor.  
Supply voltage = 460 V ; armature current = 28 A ; speed = 1000 r.p.m. ; armature resistance = 0.72  $\Omega$ . Calculate (i) the armature current and (ii) the speed when the flux per pole is increased to 120% of the initial value, given that the total torque developed by the armature is unchanged. [(i) 23.33 A (ii) 840 r.p.m.]
- A 100-V shunt motor, with a field resistance of 50  $\Omega$  and armature resistance of 0.5  $\Omega$  runs at a speed of 1,000 r.p.m. and takes a current of 10 A from the supply. If the total resistance of the field circuit is reduced to three quarters of its original value, find the new speed and the current taken from the supply. Assume that flux is directly proportional to field current. [1,089 r.p.m. ; 8.33 A]

6. A 250 V d.c. shunt motor has armature circuit resistance of  $0.5 \Omega$  and a field circuit resistance of  $125 \Omega$ . It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to  $150 \Omega$ . If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current. **[1186 r.p.m. 33.6 A]**
7. A 250 V, shunt motor with an armature resistance of  $0.5 \Omega$  and a shunt field resistance of  $250 \Omega$  drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be affected in the shunt field resistance? Assume that the magnetization curve of the motor is a straight line. **[88  $\Omega$ ]**
8. A 240 V, d.c. shunt motor runs at 800 r.p.m. with no extra resistance in the field or armature circuit, on no-load. Determine the resistance to be placed in series with the field so that the motor may run at 950 r.p.m. when taking an armature current of 20 A. Field resistance =  $160 \Omega$ . Armature resistance =  $0.4 \Omega$ . It may be assumed that flux per pole is proportional to field current. **[33.6  $\Omega$ ]**
9. A shunt-wound motor has a field resistance of  $400 \Omega$  and an armature resistance of  $0.1 \Omega$  and runs off 240 V supply. The armature current is 60 A and the motor speed is 900 r.p.m.; Assuming a straight line magnetization curve, calculate (a) the additional resistance in the field to increase the speed to 1000 r.p.m. for the same armature current and (b) the speed with the original field current of 200 A. **[(a) 44.4  $\Omega$  (b) 842.5 r.p.m.]**
10. A 230 V d.c. shunt motor has an armature resistance of  $0.5 \Omega$  and a field resistance of  $76 \frac{2}{3} \Omega$ . The motor draws a line current of 13 A while running light at 1000 r.p.m. At a certain load, the field circuit resistance is increased by  $38 \frac{1}{3} \Omega$ . What is the new speed of the motor if the line current at this load is 42 A? **[1400 r.p.m.] (Electrical Engg. ; Grad I.E.T.E. Dec. 1986)**
11. A 250 V d.c. shunt motor runs at 1000 r.p.m. and takes an armature current of 25 amp. Its armature resistance is  $0.40 \text{ ohm}$ . Calculate the speed with increased load with the armature current of 50 amp. Assume that the increased load results into flux-weakening by 3%, with respect to the flux in previous loading condition. **(Nagpur Univ., April 1996)**

Hint : (i) First Loading condition :

$$E_{b1} = 250 - 25 \times 0.40 = K_1 \times 1000$$

(ii) Second Loading condition :

$$E_{b2} = 250 - 50 \times 0.40 = 230 K_1 \times (0.97 \phi) \times N_2. \text{ This gives } N_2. \quad \text{[988 r.p.m.]}$$

**(ii) Armature or Rheostatic Control Method**

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 30.4 (a). As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed. For a load constant torque, speed is approximately proportional to the p.d. across the armature. From the speed/armature current characteristic [Fig. 30.4 (b)], it is seen that greater the resistance in the armature circuit, greater is the fall in the speed.

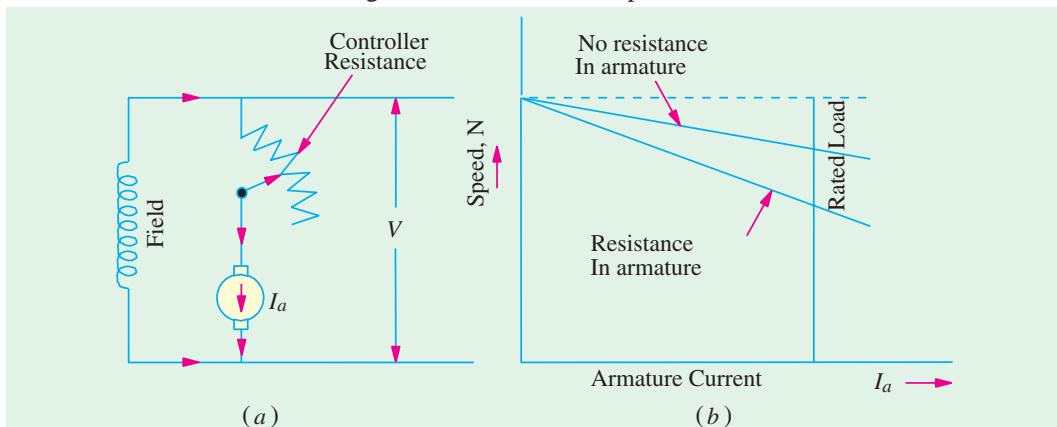


Fig. 30.4

Let  $I_{a1}$  = armature current in the first case  
 $I_{a2}$  = armature current in the second case  
 (If  $I_{a1} = I_{a2}$ , then the load is of constant torque.)  
 $N_1, N_2$  = corresponding speeds,  $V$  = supply voltage

Then  $N_1 \propto V - I_{a1} R_a \propto E_{b1}$

Let some controller resistance of value  $R$  be added to the armature circuit resistance so that its value becomes  $(R + R_a) = R_r$

Then  $N_2 \propto V - I_{a2} R_r \propto E_{b2} \quad \therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$

(In fact, it is a simplified form of relation given in Art. 27.9 because here  $\Phi_1 = \Phi_2$ .)

Considering no-load speed, we have  $\frac{N}{N_0} = \frac{V - I_a R_t}{V - I_{a0} R_a}$

Neglecting  $I_{a0} R_a$  with respect to  $V$ , we get

$$N = N_0 \left( 1 - \frac{I_a R_t}{V} \right)$$

It is seen that for a given resistance  $R_t$  the speed is a linear function of armature current  $I_a$  as shown in Fig. 30.5 (a).

The load current for which the speed would be zero is found by putting  $N = 0$  in the above relation.

$$\therefore 0 = N_0 \left( 1 - \frac{I_a R_t}{V} \right) \quad \text{or} \quad I_a = \frac{V}{R_t}$$

This is the maximum current and is known as **stalling** current.

As will be shown in Art. 30.5 (a), this method is very wasteful, expensive and unsuitable for rapidly changing loads because for a given value of  $R_r$ , speed will change with load. A more stable operation can be obtained by using a diverter across the armature in addition to armature control resistance (Fig. 30.5 (b)). Now, the changes in armature current (due to changes in the load torque) will not be so effective in changing the p.d. across the armature (and hence the armature speed).

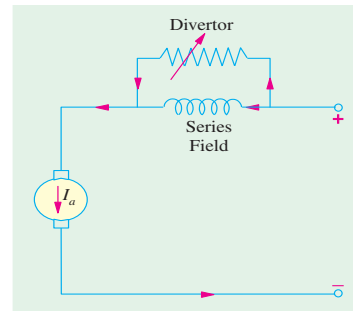


Fig. 30.5 (a)

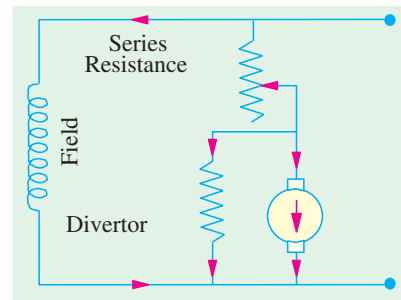


Fig. 30.5 (b)

**Example 30.18.** A 200 V d.c. shunt motor running at 1000 r.p.m. takes an armature current of 17.5 A. It is required to reduce the speed to 600 r.p.m. What must be the value of resistance to be inserted in the armature circuit if the original armature resistance is 0.4 Ω? Take armature current to be constant during this process. (Elect. Engg. I Nagpur Univ. 1993)

**Solution.**  $N_1 = 1000$  r.p.m. ;  $E_{b1} = 200 - 17.5 \times 0.4 = 193$  V  
 $R_t =$  total arm. circuit resistance ;  $N_2 = 600$  r.p.m. ;  $E_{b2} = (200 - 17.5 R_t)$

Since  $I_{sh}$  remains constant ;  $\Phi_1 = \Phi_2$   
 $\therefore \frac{600}{1000} = \frac{(200 - 17.5 R_t)}{193}$  ;  $R_t = 4.8 \Omega$

$\therefore$  Additional resistance reqd.  $R = R_t - R_a = 4.8 - 0.4 = 4.4 \Omega$ .

It may be noted that brush voltage drop has not been considered.

**Example 30.19.** A 500 V d.c. shunt motor has armature and field resistances of 1.2 Ω and 500 Ω respectively. When running on no-load, the current taken is 4 A and the speed is 1000 r.p.m. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 26 A. Estimate the speed at this load if (a) a resistance of 2.3 Ω is connected in series with the armature and (b) the shunt field current is reduced by 15%. (Electrical Engg. I, Sd. Patel Univ. 1985)

**Solution.**  $I_{sh} = 500/500 = 1 \text{ A}$  ;  $I_{a1} = 4 - 1 = 3 \text{ A}$   
 $E_{b1} = 500 - (3 \times 1.2) = 496.4 \text{ V}$   $I_{a2} = 26 - 1 = 25 \text{ A}$   
 $E_{b2} = 500 - (25 \times 1.2) = 470 \text{ V}$   $\therefore \frac{N_2}{1000} = \frac{470}{496.4}$  ;  $N_2 = 947 \text{ r.p.m.}$

(a) In this case, total armature circuit resistance =  $1.2 + 2.3 = 3.5 \Omega$

$\therefore E_{b2} = 500 - (25 \times 3.5) = 412.5 \text{ V}$   $\therefore \frac{N_2}{1000} = \frac{412.5}{496.4}$  ;  $N_2 = 831 \text{ r.p.m.}$

(b) When shunt field is reduced by 15%,  $\Phi_2 = 0.85 \Phi_1$  assuming straight magnetisation curve.

$\frac{N_2}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85}$  ;  $N_2 = 977.6 \text{ r.p.m.}$

**Example 30.20.** A 250-V shunt motor (Fig. 30.6) has an armature current of 20 A when running at 1000 r.p.m. against full load torque. The armature resistance is 0.5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 r.p.m. at the same torque and what will be the speed if the load torque is halved with this resistance in the circuit? Assume the flux to remain constant throughout and neglect brush contact drop.

(Elect. Machines AMIE Sec. B Summer 1991)

**Solution.**  $E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240 \text{ V}$

Let  $R_t$  to be total resistance in the armature circuit i.e.  $R_t = R_a + R$ , where  $R$  is the additional resistance.

$\therefore E_{b2} = V - I_{a2} R_t = 250 - 20 R_t$

It should be noted that  $I_{a1} = I_{a2} = 20 \text{ A}$  because torque remains the same and  $\Phi_1 = \Phi_2$  in both cases.

$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}}$  or  $\frac{500}{1000} = \frac{250 - 20 R_t}{240}$

$\therefore R_t = 6.5 \Omega$  ; hence,  $R = 6.5 - 0.5 = 6 \Omega$

Since the load is halved, armature current is also halved because flux remains constant. Hence,  $I_{a3} = 10 \text{ A}$ .

$\therefore \frac{N_3}{1000} = \frac{250 - 10 \times 6.5}{240}$  or  $N_3 = 771 \text{ r.p.m.}$

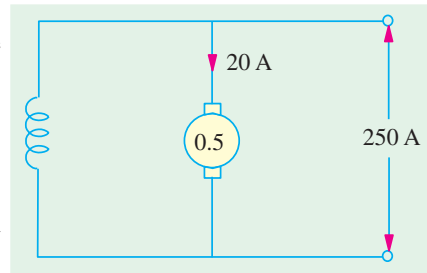


Fig. 30.6

**Example 30.21.** A 250-V shunt motor with armature resistance of 0.5 ohm runs at 600 r.p.m. on full-load and takes an armature current of 20 A. If resistance of 1.0 ohm is placed in the armature circuit, find the speed at (i) full-load torque (ii) half full-load torque.

(Electrical Machines-II, Punjab Univ. May 1991)

**Solution.** Since flux remains constant, the speed formula becomes  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$ .

(i) In the first case, full-load torque is developed.

$N_1 = 600 \text{ r.p.m.}$ ;  $E_{b1} = V - I_{a1} R_{a1} = 250 - 20 \times 0.5 = 240 \text{ V}$

Now,  $T \propto \Phi I_a \propto I_a$  ( $\because \Phi$  is constant)

$\therefore \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$  Since  $T_2 = T_1$  ;  $I_{a2} = I_{a1} = 20 \text{ A}$

$E_{b2} = V - I_{a2} R_{a2} = 250 - 20 \times 1.5 = 220 \text{ V}$ ,

$N_2 = \frac{N_1 E_{b2}}{E_{b1}} = \frac{600 \times 220}{240}$  ;  $N_2 = 550 \text{ r.p.m.}$

(ii) In this case, the torque developed is half the full-load torque.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{T_1/2}{T_1} = \frac{I_{a2}}{20}; I_{a2} = 10 \text{ A}; E_{b2} = 250 - 10 \times 1.5 = 235 \text{ V}$$

$$\frac{N_2}{600} = \frac{235}{240}; N_2 = 600 \times \frac{235}{240} = \mathbf{587.5 \text{ r.p.m.}}$$

**Example 30.22.** A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs of 500 rev. per minute and takes an armature current of 30 A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at (a) full-load torque (b) double full-load torque. (Elect. Machines-I, Nagpur Univ. 1993)

**Solution.** Since flux remains constant, the speed formula becomes  $N_2/N_1 = E_{b2}/E_{b1}$ .

(a) Full-load torque

With no additional resistance in the armature circuit,

$$N_1 = 500 \text{ r.p.m.}; I_{a1} = 30 \text{ A}; E_{b1} = 220 - 30 \times 0.5 = 205 \text{ V}$$

Now,  $T \propto I_a$  (since  $\Phi$  is constant.)  $\therefore \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$  Since  $T_2 = T_1; I_{a2} = I_{a1} = 30 \text{ A}$

When additional resistance of 1  $\Omega$  is introduced in the armature circuit,

$$E_{b2} = 220 - 30(1 + 0.5) = 175 \text{ V}; N_2 = ? \quad \frac{N_2}{500} = \frac{175}{205}; N_2 = \mathbf{427 \text{ r.p.m.}}$$

(b) Double Full-load Torque

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{2T_1}{T_1} = \frac{I_{a2}}{30}; I_{a2} = 60 \text{ A}$$

$$\therefore E_{b2} = 220 - 60(1 + 0.5) = 130 \text{ V}$$

$$\therefore \frac{N_2}{500} = \frac{130}{205}; N_2 = \mathbf{317 \text{ r.p.m.}}$$

**Example 30.23.** The speed of a 50 h.p (37.3 kW) series motor working on 500 V supply is 750 r.p.m. at full-load and 90 per cent efficiency. If the load torque is made 350 N-m and a 5 ohm resistance is connected in series with the machine, calculate the speed at which the machine will run. Assume the magnetic circuit to be unsaturated and the armature and field resistance to be 0.5 ohm. (Electrical Machinery I, Madras Univ. 1986)

**Solution.** Load torque in the first case is given by

$$T_1 = 37,300/2\pi (750/60) = 474.6 \text{ N-m}$$

$$\text{Input current, } I_{a1} = 37,300/0.9 \times 500 = 82.9 \text{ A}$$

$$\text{Now, } T_2 = 250 \text{ N-m}; I_{a2} = ?$$

In a series motor, before magnetic saturation,

$$T \propto \Phi I_a \propto I_{a2} \quad \therefore T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2$$

$$\therefore \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \frac{T_2}{T_1} \quad \therefore I_{a2} = 82.9 \times \sqrt{250/474.6} = 60.2 \text{ A}$$

$$\text{Now, } E_{b1} = 500 - (82.9 \times 0.5) = 458.5 \text{ V};$$

$$E_{b2} = 500 - 60.2(5 + 0.5) = 168.9 \text{ V}$$

$$\text{Using } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}, \text{ we get } \frac{N_2}{750} = \frac{168.9}{458.5} \times \frac{82.9}{60.2} \quad \therefore N_2 = \mathbf{381 \text{ r.p.m.}}$$

**Example 30.24.** A 7.46 kW, 220 V, 900 r.p.m. shunt motor has a full-load efficiency of 88 per cent, an armature resistance of 0.08  $\Omega$  and shunt field current of 2 A. If the speed of this motor is



reduced to 450 r.p.m. by inserting a resistance in the armature circuit, the load torque remaining constant, find the motor output, the armature current, the external resistance and the overall efficiency.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** Full-load motor input current  $I = 7460/220 \times 0.88 = 38.5$  A

$$\therefore I_{a1} = 38.5 - 2 = 36.5 \text{ A}$$

Now,  $T \propto \Phi I_a$ . Since flux remains constant.

$$\therefore T \propto I_a \quad \therefore T_{a1} \propto I_{a1} \text{ and } T_{a2} \propto I_{a2} \quad \text{or} \quad \frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}}$$

It is given that  $T_{a1} = T_{a2}$ ; hence  $I_{a1} = I_{a2} = 36.5$  A

$$E_{b1} = 220 - (36.5 \times 0.08) = 217.1 \text{ V}$$

$$E_{b2} = 220 - 36.5 R_t; N_1 = 900 \text{ r.p.m.}; N_2 = 450 \text{ r.p.m.}$$

$$\text{Now, } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \quad (\because \Phi_1 = \Phi_2)$$

$$\therefore \frac{450}{900} = \frac{220 - 36.5 R_t}{217.1}; R_t = 3.05 \text{ } \Omega$$

$$\therefore \text{External resistance } R = 3.05 - 0.08 = 2.97 \text{ } \Omega$$

For calculating the motor output, it will be assumed that all losses except copper losses vary directly with speed.

Since motor speed is halved, stray losses are also halved in the second case. Let us find their value.

In the first case, motor input =  $220 \times 38.5 = 8,470$  W; Motor output = 7,460 W

Total Cu losses + stray losses =  $8470 - 7460 = 1010$  W

Arm. Cu loss =  $I_{a1}^2 R_a = 36.5^2 \times 0.08 = 107$  W; Field Cu loss =  $220 \times 2 = 440$  W

Total Cu loss =  $107 + 440 = 547$  W  $\therefore$  Stray losses in first case =  $1010 - 547 = 463$  W

Stray losses in the second case =  $463 \times 450/900 = 231$  W

Field Cu loss = 440 W, as before; Arm. Cu loss =  $36.5^2 \times 3.05 = 4,064$  W

Total losses in the 2nd case =  $231 + 440 + 4,064 = 4,735$  W

Input = 8,470 W – as before

Output in the second case =  $8,470 - 4,735 = 3,735$  W

$$\therefore \text{Overall } \eta = 3,735/8,470 = 0.441 \text{ or } 44.1 \text{ per cent}^*$$

**Example 30.25.** A 240 V shunt motor has an armature current of 15 A when running at 800 r.p.m. against F.L. torque. The arm. resistance is 0.6 ohms. What resistance must be inserted in series with the armature to reduce the speed to 400 r.p.m., at the same torque ?

What will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout.

(Elect. Machines-I Nagpur Univ. 1993)

**Solution.** Here,  $N_1 = 800$  r.p.m.,  $E_{b1} = 240 - 15 \times 0.6 = 231$  V

Flux remaining constant,  $T \propto I_a$ . Since torque is the same in both cases,  $I_{a2} = I_{a1} = 15$  A. Let  $R$  be the additional resistance inserted in series with the armature.  $E_{b2} = 240 - 15(R + 0.6)$ ;  $N_2 = 400$  r.p.m.

$$\therefore \frac{400}{800} = \frac{240 - 15(R + 0.6)}{231}; R = 7.7 \text{ } \Omega$$

\* It may be noted that efficiency is reduced almost in the ratio of the two speeds.

**When load torque is halved :**

With constant flux when load torque is halved,  $I_a$  is also halved. Hence,  $I_{a3} = I_{a1}/2 = 15/2 = 7.5$  A

$$\therefore E_{b3} = 240 - 7.5(7.7 + 0.6) = 177.75 \text{ V}; N_3 = ?$$

$$\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b1}} \text{ or } \frac{N_3}{800} = \frac{177.75}{231}; N_3 = \mathbf{614.7 \text{ r.p.m.}}$$

**Example 30.26. (a)** A 400 V shunt connected d.c. motor takes a total current of 3.5 A on no load and 59.5 A at full load. The field circuit resistance is 267 ohms and the armature circuit resistance is 0.2 ohms (excluding brushes where the drop may be taken as 2 V). If the armature reaction effect at 'full-load' weakens the flux per pole by 2 percentage change in speed from no-load to full-load.

**(b)** What resistant must be placed in series with the armature in the machine of (a) if the full-load speed is to be reduced by 50 per cent with the gross torque remaining constant? Assume no change in the flux. **(Electrical Machines, AMIE Sec. B, 1989)**

**Solution. (a)** Shunt current  $I_{sh} = 400/267 = 1.5$  A. At no load,  $I_{a1} = 3.5 - 1.5 = 2$  A,  $E_{b1} = V - I_{a1}R_{ai} - \text{brush drop} = 400 - 2 \times 0.2 - 2 = 397.6$  V. On full-load,  $I_{a2} = 59.5 - 1.5 = 58$  A,  $E_{b2} = 400 - 58 \times 0.2 - 2 = 386.4$  V.

$$\therefore \frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \text{ or } \frac{397.6}{386.4} = \frac{\Phi_1 N_1}{0.98 \Phi_1 N_2}; \frac{N_1}{N_2} = 1.0084$$

$$\begin{aligned} \% \text{ change in speed} &= \frac{N_1 - N_2}{N_1} \times 100 \\ &= \left(1 - \frac{1}{1.0084}\right) \times 100 = 0.833 \end{aligned}$$

**(b)** Since torque remains the same,  $I_a$  remains the same, hence  $I_{a3} = I_{a2}$ . Let  $R$  be the resistance connected in series with the armature.

$$\begin{aligned} E_{b3} &= V - I_{a2}(R_a + R) - \text{brush drop} \\ &= 400 - 58(0.2 + R) - 2 = 386.4 - 58R \end{aligned}$$

$$\therefore \frac{E_{b2}}{E_{b3}} = \frac{\Phi_2 N_2}{\Phi_3 N_3} = \frac{N_2}{N_3} \quad (\text{as } \Phi_2 = \Phi_3)$$

$$\frac{386.4}{386.4 - 58R} = \frac{1}{0.5}; R = \mathbf{3.338 \Omega}$$

**Example 30.27.** A d.c. shunt drives a centrifugal pump whose torque varies as the square of the speed. The motor is fed from a 200 V supply and takes 50 A when running at 1000 r.p.m. What resistance must be inserted in the armature circuit in order to reduce the speed to 800 r.p.m.? The armature and field resistance of the motor are 0.1  $\Omega$  and 100  $\Omega$  respectively.

**(Elect. Machines, Allahabad Univ. 1992)**

**Solution.** In general,  $T \propto \Phi I_a$

For shunt motors whose excitation is constant,

$$T \propto I_a \propto N^2, \text{ as given.}$$

$$\therefore I_a \propto N^2. \text{ Now } I_{sh} = 200/100 = 2 \text{ A} \quad \therefore I_{a1} = 50 - 2 = 48 \text{ A}$$

Let  $I_{a2}$  = new armature current at 800 r.p.m.

then  $48 \propto N_1^2 \propto 1000^2$  and  $I_{a2} \propto N_2^2 \propto 800^2$

$$\therefore \frac{I_{a2}}{48} = \left(\frac{800}{1000}\right)^2 = 0.8^2 \quad \therefore I_{a2} = 48 \times 0.64 = 30.72 \text{ A}$$

$$\therefore E_{b1} = 200 - (48 \times 0.1) = 195.2 \text{ V}; E_{b2} = (200 - 30.72 R_t) \text{ V}$$

Now, 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \therefore \frac{800}{1000} = \frac{200 - 30.72 R_t}{195.2}, R_t = 1.42 \Omega$$

Additional resistance =  $1.42 - 0.1 = 1.32 \Omega$

**Example 30.28.** A 250 V, 50 h.p. (373 kW) d.c. shunt motor has an efficiency of 90% when running at 1,000 r.p.m. on full-load. The armature and field resistances are 0.1 Ω and 115 Ω respectively. Find

- (a) the net and developed torque on full-load.
- (b) the starting resistance to have the line start current equal to 1.5 times the full-load current.
- (c) the torque developed at starting. (Elect. Machinery-I, Kerala Univ. 1987)

**Solution.** (a)  $T_{sh} = 9.55 \times 37,300/1000 = 356.2 \text{ N-m}$

Input current =  $\frac{37,300}{250 \times 0.9} = 165.8 \text{ A}; I_{sh} = \frac{250}{125} = 2 \text{ A}$

$\therefore I_a = 165.8 - 2 = 163.8 \text{ A}; E_b = 250 - (163.8 \times 0.1) = 233.6 \text{ V}$

$\therefore T_a = 9.55 \frac{233.6 \times 163.8}{1000} = 365.4 \text{ N-m}$

- (b) F.L. input line  $I = 165.8 \text{ A}$ ; Permissible input  $I = 165.8 \times 1.5 = 248.7 \text{ A}$
- Permissible armature current =  $248.7 - 2 = 246.7 \text{ A}$
- Total armature resistance =  $250/246.7 = 1.014 \Omega$
- $\therefore$  Starting resistance required =  $1.014 - 0.1 = 0.914 \Omega$

- (c) Torque developed with 1.5 times the F.L. current would be practically 1.5 times the F.L. torque.
- i.e.  $1.5 \times 365.4 = 548.1 \text{ N-m.}$

**Example 30.29.** A 200 V shunt motor with a shunt resistance of 40 Ω and armature resistance of 0.02 Ω takes a current of 55 A and runs at 595 r.p.m. when there is a resistance of 0.58 Ω in series with armature. Torque remaining the same, what change should be made in the armature circuit resistance to raise the speed to 630 r.p.m. ? Also find

- (i) At what speed will the motor run if the load torque is reduced such that armature current is 15 A.
- (ii) Now, suppose that a diverter of resistance 5 Ω is connected across the armature and series resistance is so adjusted that motor speed is again 595 r.p.m., when armature current is 50 A. What is the value of this series resistance ? Also, find the speed when motor current falls of 15 A again.

**Solution.** The circuit is shown in Fig. 30.7.

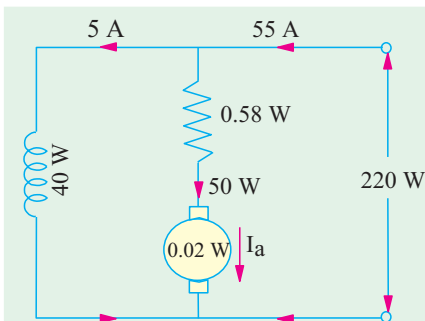


Fig. 30.7

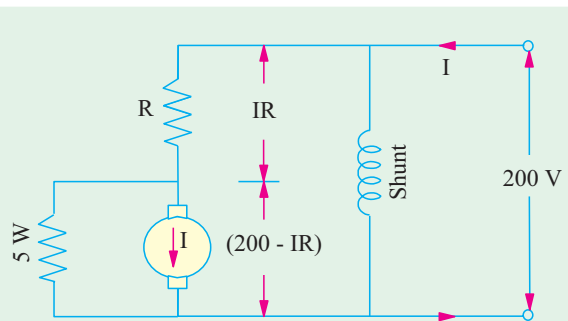


Fig. 30.8

$I_{sh} = 200/40 = 5 \text{ A} \quad \therefore I_{a1} = 55 - 5 = 50 \text{ A}$   
 Armature circuit resistance =  $0.58 + 0.02 = 0.6 \Omega$   
 $\therefore E_{b1} = 200 - (50 \times 0.6) = 170 \text{ V}$   
 Since torque is the same in both cases,  $I_{a1} \Phi_1 = I_{a2} \Phi_2$   
 Moreover,  $\Phi_1 = \Phi_2 \quad \therefore I_{a1} = I_{a2} \quad \therefore I_{a2} = 50 \text{ A}$   
 Now  $E_{b1} = 170 \text{ V}, N_1 = 595 \text{ r.p.m.}, N_2 = 630 \text{ r.p.m.}, E_{b2} = ?$   
 Using  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad (\because \Phi_1 = \Phi_2)$   
 we get  $E_{b2} = 170 \times (630/595) = 180 \text{ V}$   
 Let  $R_2$  be the new value of armature circuit resistance for raising the speed from 595 r.p.m. to 630 r.p.m.

$\therefore 180 = 200 - 50 R_2 \quad \therefore R = 0.4 \Omega$   
 Hence, armature circuit resistance should be reduced by  $0.6 - 0.4 = 0.2 \Omega$ .

(i) We have seen above that

$I_{a1} = 50 \text{ A}, E_{b1} = 170 \text{ V}, N_1 = 595 \text{ r.p.m.}$   
 If  $I_{a2} = 15 \text{ A}, E_{b2} = 200 - (15 \times 0.6) = 191 \text{ V}$   
 $\therefore \frac{N_2}{595} = \frac{191}{170} \quad \therefore N_2 = 668.5 \text{ r.p.m.}$

(ii) When armature divertor is used (Fig. 30.8).

Let  $R$  be the new value of series resistance

$\therefore E_{b3} = 200 - IR - (50 \times 0.02) = 199 - IR$   
 Since speed is 595 r.p.m.,  $E_{b3}$  must be equal to 170 V  
 $\therefore 170 = 199 - IR \quad \therefore IR = 29 \text{ V}; \text{ P.D. across divertor} = 200 - 29 = 171 \text{ V}$   
 Current through divertor  $I_d = 171/5 = 34.2 \text{ A} \quad \therefore I = 50 + 34.2 = 84.2 \text{ A}$   
 As  $IR = 29 \text{ V} \quad \therefore R = 29/84.2 = 0.344 \text{ W}$   
 When  $I_a = 15 \text{ A}$ , then  $I_d = (I - 15) \text{ A}$   
 P.D. across divertor =  $5(I - 15) = 200 - 0.344 I \quad \therefore I = 51.46 \text{ A}$   
 $E_{b4} = 200 - 0.344 I - (15 \times 0.02)$   
 $= 200 - (0.344 \times 51.46) - 0.3 = 182 \text{ V}$   
 $\therefore \frac{N_4}{N_1} = \frac{E_{b4}}{E_{b1}} \quad \text{or} \quad \frac{N_4}{595} = \frac{182}{170} \quad \therefore N_4 = 637 \text{ r.p.m.}$

The effect of armature divertor is obvious. The speed without divertor is 668.5 r.p.m. and with armature divertor, it is 637 r.p.m.

(iii) Voltage Control Method

(a) Multiple Voltage Control

In this method, the shunt field of the motor is connected permanently to a fixed exciting voltage, but the armature is supplied with different voltages by connecting it across one of the several different voltages by means of suitable switchgear. The armature speed will be approximately proportional to these different voltages. The intermediate speeds can be obtained by adjusting the shunt field regulator. The method is not much used, however.

(b) Ward-Leonard System

This system is used where an unusually wide (upto 10 : 1) and very sensitive speed control is required as for colliery winders, electric excavators, elevators and the main drives in steel mills and blooming and paper mills. The arrangement is illustrated in Fig. 30.9.

$M_1$  is the main motor whose speed control is required. The field of this motor is permanently connected across the d.c. supply lines. By applying a variable voltage across its armature, any desired

speed can be obtained. This variable voltage is supplied by a motor-generator set which consists of either a d.c. or an a.c. motor  $M_2$  directly coupled to generator  $G$ .

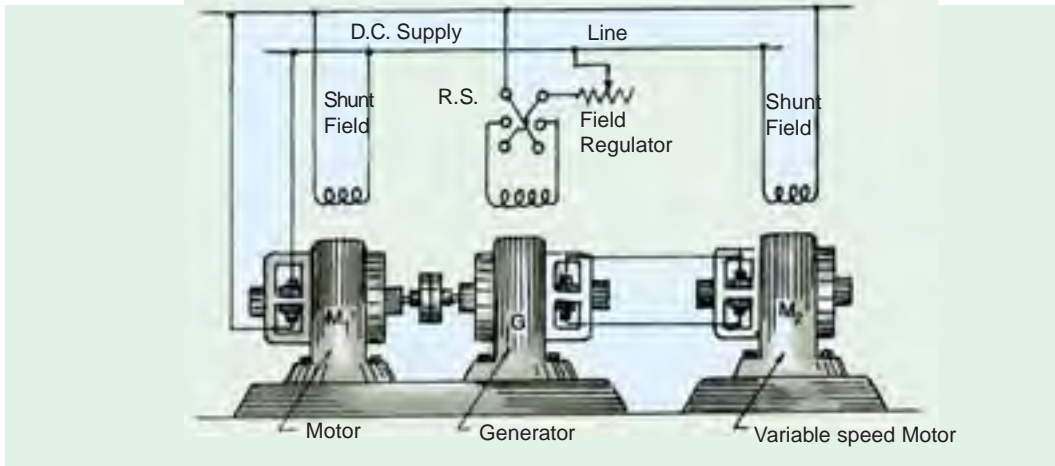


Fig. 30.9

The motor  $M_2$  runs at an approximately constant speed. The output voltage of  $G$  is directly fed to the main motor  $M_1$ . The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator. By reversing the direction of the field current of  $G$  by means of the reversing switch  $RS$ , generated voltage can be reversed and hence the direction of rotation of  $M_1$ . It should be remembered that motor generator set always runs in the same direction.

Despite the fact that capital outlay involved in this system is high because (i) a large output machine must be used for the motor generator set and (ii) that two extra machines are employed, still it is used extensively for elevators, hoist control and for main drive in steel mills where motor of ratings 750 kW to 3750 kW are required. The reason for this is that the almost unlimited speed control in either direction of rotation can be achieved entirely by field control of the generator and the resultant economies in steel production outweigh the extra expenditure on the motor generator set.

A modification of the Ward-Leonard system is known as Ward-Leonard-Ilgner system which uses a smaller motor-generator set with the addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit. When main motor  $M_1$  becomes suddenly overloaded, the driving motor  $M_2$  of the motor generator set slows down, thus allowing the inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor  $M_1$ , then  $M_2$  speeds up, thereby again storing energy in the flywheel.

When the Ilgner system is driven by means of an a.c. motor (whether induction or synchronous) another refinement in the form of a 'slip regulator' can be usefully employed, thus giving an additional control.

The chief disadvantage of this system is its low overall efficiency especially at light loads. But as said earlier, it has the outstanding merit of giving wide speed control from maximum in one direction through zero to the maximum in the opposite direction and of giving a smooth acceleration.

**Example 30.30.** The O.C.C. of the generator of a Ward-Leonard set is

Field amps :	1.4	2.2	3	4	5	6	7	8
Armature volts :	212	320	397	472	522	560	586	609

The generator is connected to a shunt motor, the field of which is separately-excited at 550 V. If the speed of motor is 300 r.p.m. at 550 V, when giving 485 kW at 95.5% efficiency, determine the excitation of the generator to give a speed of 180 r.p.m. at the same torque. Resistance of the motor

armature circuit = 0.01 Ω, resistance of the motor field = 60 Ω, resistance of generator armature circuit = 0.01 Ω. Ignore the effect of armature reaction and variation of the core factor and the windage losses of the motor.

**Solution.** Motor input =  $485 \times 10^3 / 0.955 = 509,300 \text{ W}$

Motor to motor field =  $550 / 60 = 55 / 6 \text{ A}$

Input to motor field =  $550 \times 55 / 6 = 5,040 \text{ W}$

∴ Motor armature input =  $509,300 - 5,040 = 504,260 \text{ W}$

∴ Armature current =  $504,260 / 550 = 917 \text{ A}$

Back e.m.f.  $E_{b1}$  at 300 r.p.m. =  $550 - (917 \times 0.01) = 540.83 \text{ V}$

Back e.m.f.  $E_{b2}$  at 180 r.p.m. =  $540.83 \times 180 / 300 = 324.5 \text{ V}$

Since torque is the same, the armature current of the main motor is also the same i.e. 917 A because its excitation is independent of its speed.

∴  $V = 324.5 + (917 \times 0.01) = 333.67 \text{ V}$

Generated e.m.f. =  $V + I_a R_a$  ...for generator

$333.67 + (917 \times 0.011) = 343.77 \text{ V}$

If O.C.C. is plotted from the above given data, then it would be found that the excitation required to give 343.77 V is 2.42 A.

∴ Generator exciting current = **2.42 A**

### 30.3. Speed Control of Series Motors

#### 1. Flux Control Method

Variations in the flux of a series motor can be brought about in any one of the following ways :

##### (a) Field Divertors

The series winding are shunted by a variable resistance known as field diverter (Fig. 30.10). Any desired amount of current can be passed through the diverter by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.

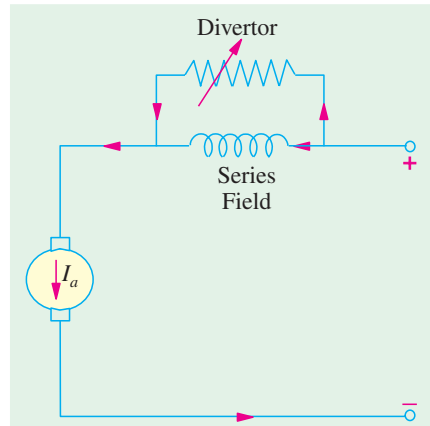


Fig. 30.10

##### (b) Armature Divertor

A diverter across the armature can be used for giving speeds lower than the normal speed (Fig. 30.11). For a **given constant load torque**, if  $I_a$  is reduced due to armature diverter, the  $\Phi$  must increase.

(∵  $T_a \propto \Phi I_a$ ). This results in an increase in current taken from the supply (which increases the flux and a fall in speed ( $N \propto I/\Phi$ )). The variation in speed can be controlled by varying the diverter resistance.

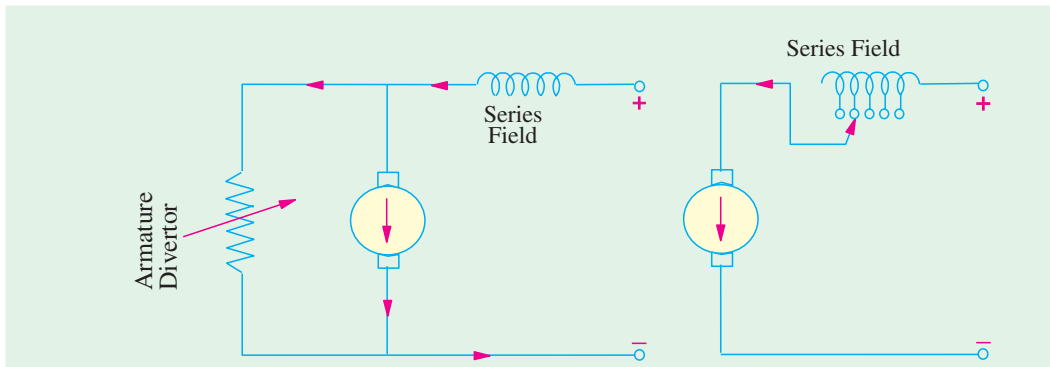


Fig. 30.11

Fig. 30.12

**(c) Trapped Field Control Field**

This method is often used in electric traction and is shown in Fig. 30.12.

The number of series filed turns in the circuit can be changed at will as shown. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

**(d) Paralleling Field coils**

In this method, used for fan motors, several speeds can be obtained by regrouping the field coils as shown in Fig. 30.13. It is seen that for a 4-pole motor, three speeds can be obtained easily. (Ex.30.35)

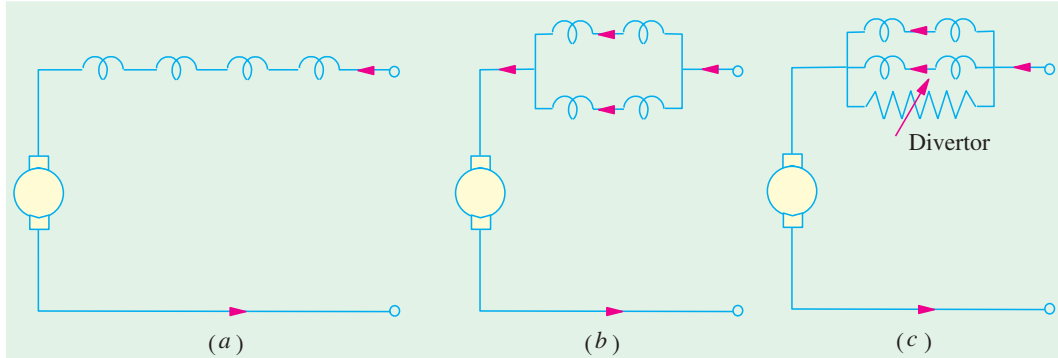


Fig. 30.13

**2. Variable Resistance in Series with Motor**

By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

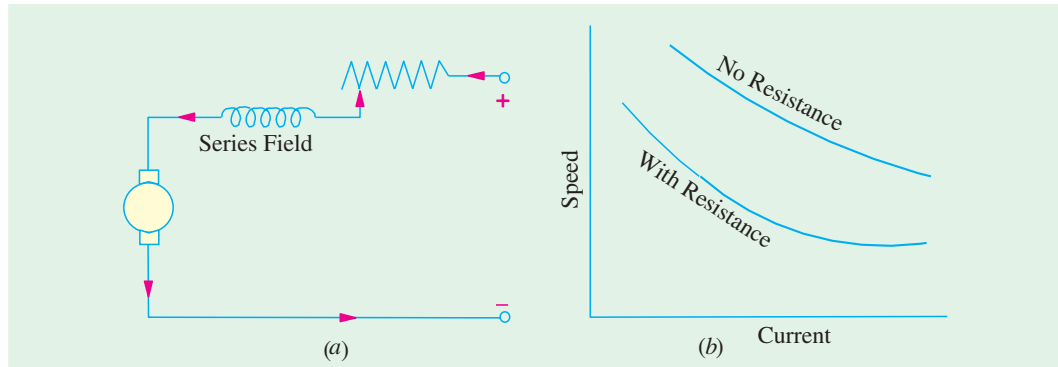


Fig. 30.14

**Example 30.31.** A d.c. series motor drives a load the torque of which varies as the square of the speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and the current when the motor field winding is shunted by a diverter of the same resistance as that of the field winding. Mention the assumptions made, if any. (Elect. Machines, AMIE Sec B, 1993)

**Solution.**

$$T_{a1} \propto N_1^2, T_{a2} \propto N_2^2 \quad \therefore T_{a2}/T_{a1} = N_2^2/N_1^2$$

Also,

$$T_{a1} \propto \Phi_1 I_{a1} \propto I_{a1}^2, T_{a2} \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$$

It is so because in the second case, field current is half the armature current.

$$\therefore \frac{N_2^2}{N_1^2} = \frac{I_{a2}^2/2}{I_{a1}^2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{I_{a2}}{\sqrt{2} I_{a1}} \quad \dots (i)$$

Now, 
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

If we neglect the armature and series winding drops as well as brush drop, then  $E_{b1} = E_{b2} = V$

$$\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{\tau_1}{\tau_2} = \frac{2\tau_1}{\tau_2} \quad \dots(ii)$$

From (i) and (ii), 
$$\frac{I_{a2}}{\sqrt{2} I_{a1}} = \frac{2 I_{a1}}{I_{a2}} \quad \text{or} \quad I_{a2}^2 = 2\sqrt{2} I_{a1}^2 = 2\sqrt{2} \times 15^2 \quad \text{or} \quad I_{a2} = 25.2 \text{ A}$$

From (ii), we get, 
$$N_2 = 600 \times 2 \times 15/25.2 = 714 \text{ r.p.m.}$$

**Example 30.32.** A 2-pole series motor runs at 707 r.p.m. when taking 100 A at 85 V and with the field coils in series. The resistance of each field coil is 0.03 Ω and that of the armature 0.04 Ω. If the field coils are connected in parallel and load torque remains constant, find (a) speed (b) the additional resistance to be inserted in series with the motor to restore the speed to 707 r.p.m.

**Solution.** Total armature circuit resistance = 0.04 + (2 × 0.03) = 0.1 Ω

$$I_{a1} = 100 \text{ A} ; E_{b1} = 85 - (100 \times 0.1) = 75 \text{ V}$$

When series field windings are placed in parallel, the current through each is half the armature current.

If  $I_{a2}$  = new armature current ; then  $\Phi_2 \propto I_{a2}/2$ .

As torque is the same in the two cases,

$$\therefore \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \text{or} \quad I_{a1}^2 = \frac{I_{a2}}{2} \times I_{a2} = \frac{I_{a2}^2}{2}$$

$$\therefore 100^2 = \frac{I_{a2}^2}{2} \quad \therefore I_{a2} = 100 \sqrt{2} = 141.4 \text{ A}$$

In this case, series field resistance = 0.03/2 = 0.015 Ω

$$\therefore E_{b2} = 85 - 141.4 (0.04 + 0.015) = 77.22 \text{ V}$$

$$\frac{N_2}{707} = \frac{77.22}{75} \times \frac{100}{141.4/2} \quad (\because \Phi_2 \propto I_{a2}/2)$$

(a)  $\therefore N_2 = 707 \times \frac{77.22}{75} \times \frac{200}{141.4} = 1029 \text{ r.p.m.}$

(b) Let the total resistance of series circuit be  $R_t$ .

Now,  $E_{b1} = 77.22 \text{ V}, N_1 = 1029 \text{ r.p.m.} ; E_{b2} = 85 - 141.4 R_t, N_2 = 707 \text{ r.p.m.}$

$$\frac{707}{1029} = \frac{85 - 141.4 R_t}{77.22} \quad \therefore R_t = 0.226 \Omega$$

$\therefore$  Additional resistance = 0.226 - 0.04 - 0.015 = 0.171 Ω

**Example 30.33.** A 240 V series motor takes 40 amperes when giving its rated output at 1500 r.p.m. Its resistance is 0.3 ohm. Find what resistance must be added to obtain rated torque (i) at starting (ii) at 1000 r.p.m. (Elect. Engg., Madras Univ. 1987)

**Solution.** Since torque remains the same in both cases, it is obvious that current drawn by the motor remains constant at 40 A. ( $\because T_a \propto I_a^2$ )

(i) If  $R$  is the series resistance added, then  $40 = 240/(R + 0.3) \quad \therefore R = 5.7 \Omega$

(ii) Current remaining constant,  $T_a \propto E_b/N$  – Art. 29.7

$$\frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2}$$

Now,  $E_{b1} = 240 - 40 \times 0.3 = 228 \text{ V} ; N_1 = 1500 \text{ r.p.m.}$

$$E_{b2} = 240 - 40 (R + 0.3) \text{ V} ; N_2 = 1000 \text{ r.p.m.}$$



$$\therefore \frac{228}{1500} = \frac{240 - 40(R + 0.3)}{1000}; R = 1.9 \Omega$$

**Example 30.34.** A 4-pole, series-wound fan motor runs normally at 600 r.p.m. on a 250 V d.c. supply taking 20 A. The field coils are connected in series. Estimate the speed and current taken by the motor if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses. **(Elect. Machines, AMIE, Sec B. 1990)**

**Solution.** When coils are connected in two parallel groups, current through each becomes  $I_{a2}/2$  where  $I_{a2}$  is the new armature current.

Hence,  $\Phi_2 \propto I_{a2}/2$   
 Now  $T_a \propto \Phi I_a$   
 $\propto N^2$  - given

$$\therefore \Phi_1 I_{a1} \propto N_1^2 \text{ and } \Phi_2 I_{a2} \propto N_2^2 \quad \therefore \left(\frac{N_2}{N_1}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \dots (i)$$

Since losses are negligible, field coil resistance as well as armature resistance are negligible. It means that armature and series field voltage drops are negligible. Hence, back e.m.f. in each case equals the supply voltage.

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ becomes } \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} \quad \dots (ii)$$

Putting this value in (i) above, we get

$$\left(\frac{\Phi_1}{\Phi_2}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \text{ or } \frac{I_{a2}}{I_{a1}} = \left(\frac{\Phi_1}{\Phi_2}\right)^3$$

Now,  $\Phi_1 \propto 20$  and  $\Phi_2 \propto I_{a2}/2$   $\therefore \frac{I_{a2}}{20} = \left(\frac{20}{I_{a2}/2}\right)^3$  or  $I_{a2} = 20 \times 2^{3/4} = 33.64 \text{ A}$

From (ii) above, we get  $\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}/2} = \frac{2 I_{a1}}{I_{a2}}$ ;  $N_2 = 600 \times 2 \times 20/33.64 = 714 \text{ r.p.m.}$

**Example 30.35.** A d.c. series motor having a resistance of 1 Ω drives a fan for which the torque varies as the square of the speed. At 220 V, the set runs at 350 r.p.m. and takes 25 A. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated. **(Electrical Engg., Banaras Hindu Univ. 1998)**

**Solution.** Since  $\Phi \propto I_a$ , hence  $T_a \propto \Phi I_a \propto I_a^2$ . Also  $T_a \propto N^2$  ... (given)

$$\therefore I_a^2 \propto N^2 \text{ or } I_a \propto N \text{ or } I_{a1} \propto N_1 \text{ and } I_{a2} \propto N_2$$

$$\therefore \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{500}{350}; I_{a2} = 25 \times \frac{500}{350} = \frac{250}{7} \text{ A}$$

$$E_{b1} = 220 - 25 \times 1 = 195 \text{ V}; E_{b1} = V - (250/7) \times 1, \frac{\Phi_1}{\Phi_2} = \frac{25}{250/7} = \frac{7}{10}$$

Now  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \frac{500}{350} = \frac{V - (250/7)}{195} \times \frac{7}{10}; V = 433.7 \text{ V}$

**Example 30.36.** A d.c. series motor runs at 1000 r.p.m. when taking 20 A at 200 V. Armature resistance is 0.5 Ω. Series field resistance is 0.2 Ω. Find the speed for a total current of 20 A when a diverter of 0.2 Ω resistance is used across the series field. Flux for a field current of 10 A is 70 per cent of that for 20 A.

**Solution.**  $E_{b1} = 200 - (0.5 + 0.2) \times 20 = 186 \text{ V}$  ;  $N_1 = 1000 \text{ r.p.m.}$

Since diverter resistance equals series field resistance, the motor current of 20 A is divided equally between the two. Hence, a current of 10 A flows through series field and produces flux which is 70% of that corresponding to 20 A. In other words,  $\Phi_2 = 0.7$  or  $\Phi_1/\Phi_2 = 1/0.7$

Moreover, their combined resistance =  $0.2/2 = 0.1 \Omega$

Total motor resistance becomes =  $0.5 + 0.1 = 0.6 \Omega$

$$\therefore E_{b2} = 200 - 0.6 \times 20 = 188 \text{ V} ; N_2 = ?$$

$$\therefore \frac{N_2}{1000} = \frac{188}{186} \times \frac{1}{0.7} ; N_2 = \mathbf{1444 \text{ r.p.m.}}$$

**Example 30.37.** A 200 V, d.c. series motor takes 40 A when running at 700 r.p.m. Calculate the speed at which the motor will run and the current taken from the supply if the field is shunted by a resistance equal to the field resistance and the load torque is increased by 50%.

Armature resistance =  $0.15 \Omega$ , field resistance =  $0.1 \Omega$

It may be assumed that flux per pole is proportional to the field.

**Solution.** In a series motor, prior to magnetic saturation

$$T \propto \Phi I_a \propto I_a^2 \quad \therefore T_1 \propto I_{a1}^2 \propto 40^2 \quad \dots(i)$$

If  $I_{a2}$  is the armature current (or motor current) in the second case when diverter is used, then only  $I_{a2}/2$  passes through the series field winding.

$$\therefore \Phi_2 \propto I_{a2}/2 \text{ and } T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) \times I_{a2} \propto I_{a2}^2/2 \quad \dots(ii)$$

From (i) and (ii), we get  $\frac{T_2}{T_1} = \frac{I_{a2}^2}{2 \times 40^2}$

Also  $T_2/T_1 = 1.5 \quad \therefore 1.5 = I_{a2}^2/2 \times 40^2$

$$\therefore I_{a2} = \sqrt{1.5 \times 2 \times 40^2} = 69.3 \text{ A} ;$$

Now  $E_{b1} = 220 - (40 \times 0.25) = 210 \text{ V}$

$$E_{b2} = 220 - (69.3 \times 0.2^*) = 206.14 \text{ V} ; N_1 = 700 \text{ r.p.m.} ; N_2 = ?$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \therefore \frac{N_2}{700} = \frac{206.14}{210} \times \frac{40}{69.3/2} \quad \therefore N_2 = \mathbf{794 \text{ r.p.m.}}$$

**Example 30.38.** A 4-pole, 250 V d.c. series motor takes 20 A and runs at 900 r.p.m. Each field coil has resistance of 0.025 ohm and the resistance of the armature is 0.1 ohm. At what speed will the motor run developing the same torque if :

(i) a diverter of 0.2 ohm is connected in parallel with the series field

(ii) rearranging the field coils in two series and parallel groups

Assume unsaturated magnetic operation.

(Electric Drives and their Control, Nagpur Univ. 1993)

**Solution.** The motor with its field coils all connected in series is shown in Fig. 30.15 (a). Here,  $N_1 = 900 \text{ r.p.m.}$ ,  $E_{b1} = 250 - 20 \times (0.1 + 4 \times 0.025) = 246 \text{ V}$ .

In Fig. 30.15 (b), a diverter of resistance  $0.2 \Omega$  has been connected in parallel with the series field coils. Let  $I_{a2}$  be the current drawn by the motor under this condition. As per current-divider rule, part of the current passing through the series fields is  $I_{a2} \times 0.2 / (0.1 + 0.2) = 2 I_{a2} / 3$ . Obviously,  $\Phi_2 \propto 2 I_{a2} / 3$ .

\* The combined resistance of series field winding and the diverter is  $0.1/2 = 0.05 \Omega$ . Hence, the total resistance =  $0.15 + 0.05 = 0.2 \Omega$ , in example 30.37.

Now,  $T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2$ ;  $T_2 \propto \Phi_2 I_{a2} \propto (2 I_{a2}/3) I_{a2} \propto 2 I_{a2}^2/3$   
 Since  $T_1 = T_2$ ;  $\therefore I_{a1}^2 = 2 I_{a2}^2/3$  or  $20^2 = 2 I_{a2}^2/3$ ;  $\therefore I_{a2} = 24.5$  A.  
 Combined resistance of the field and diverter  $= 0.2 \times 0.1/0.3 = 0.667 \Omega$ ; Arm. circuit resistance  $= 0.1 + 0.0667 = 0.1667 \Omega$ ;  $E_{b2} = 250 - 24.5 \times 0.1667 = 250 - 4.1 = 245.9$  V  
 $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ ;  $\frac{N_2}{900} = \frac{245.9}{246} \times \frac{20}{(2/3) 24.5}$ ;  $N_2 = 1102$  r.p.m.  $\dots(\because \Phi_2 \propto 2 I_{a2}/3)$

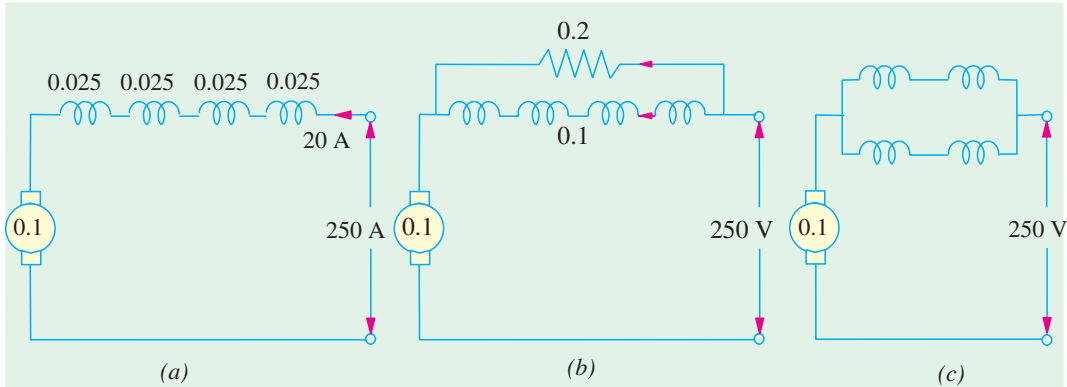


Fig. 30.15

(ii) In Fig. 30.15 (c), the series field coils have been arranged in two parallel groups. If the motor current is  $I_{a2}$ , then it is divided equally between the two parallel paths. Hence,  $\Phi_2 \propto I_{a2}/2$ .

Since torque remains the same,

$$T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \propto 20^2$$

$$T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$$

Since  $T_1 = T_2$ ;  $\therefore 20^2 = I_{a2}^2/2$ ;  $I_{a2} = 28.28$  A

Combined resistance of the two parallel paths  $= 0.05/2 = 0.025 \Omega$

Total arm. circuit resistance  $= 0.1 + 0.025 = 0.125 \Omega$

$\therefore E_{b2} = 250 - 28.28 \times 0.125 = 246.5$  V

$$\frac{N_2}{900} = \frac{246.5}{246} \times \frac{20}{28.28/2}$$

$$N_2 = 1275 \text{ r.p.m.}$$

**Example 30.39.** A 4-pole, 230 V series motor runs at 1000 r.p.m., when the load current is 12 A. The series field resistance is 0.8  $\Omega$  and the armature resistance is 1.0  $\Omega$ . The series field coils are now regrouped from all in series to two in series with two parallel paths. The line current is now 20 A. If the corresponding weakening of field is 15%, calculate the speed of the motor.

(Electrotechnology-I, Guwahati Univ. 1987)

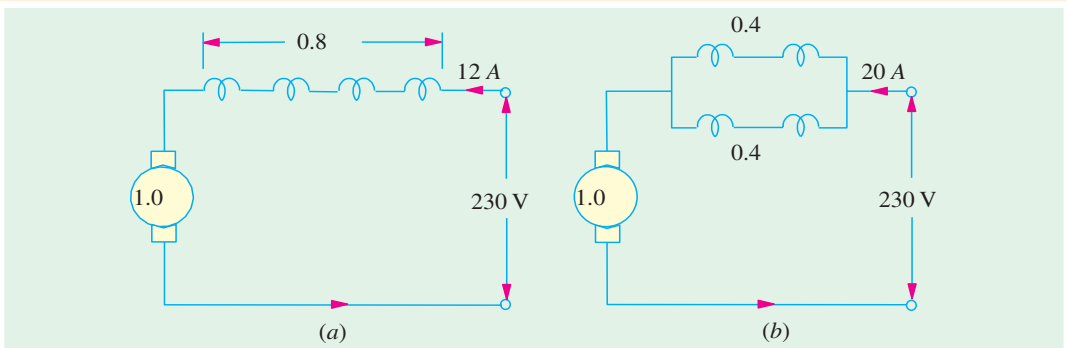


Fig. 30.16

**Solution.**  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ ,  $E_{b1} = 230 - 12 \times 1.8 = 208.4$  V, as in Fig. 30.16(a)

For circuit in Fig. 30.16 (b),

$$E_{b2} = 230 - 20(1 + 0.4/2) = 206 \text{ V ;}$$

$$\Phi_2 = 0.85 \Phi_1 \text{ or } \Phi_1/\Phi_2 = 1/0.85$$

$$\therefore \frac{N_2}{1000} = \frac{206}{208.4} \times \frac{1}{0.85} \quad \therefore N_2 = \mathbf{1163 \text{ r.p.m.}}$$

**Example 30.40.** A 200 V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0.2 Ω and that of the series field 0.6 Ω. At what speed will it run when developing the same torque when armature diverter of 10 Ω is used ? Assume a straight line magnetisation curve. (D.C. Machines, Jadavpur Univ. 1988)

**Solution.** Resistance of motor = 0.2 + 0.6 = 0.8 Ω

$$\therefore E_{b1} = 200 - (25 \times 0.8) = 180 \text{ V}$$

Let the motor input current be  $I_2$ , when armature diverter is used, as shown in Fig. 30.17.

$$\text{Series field voltage drop} = 0.6 I_2$$

$$\therefore \text{P.D. at brushes} = 200 - 0.6 I_2$$

$$\therefore \text{Arm. diverter current} = \left( \frac{200 - 0.6 I_2}{10} \right) \text{ A}$$

$$\therefore \text{Armature current} = I_2 \left( \frac{200 - 0.6 I_2}{10} \right)$$

$$\therefore I_{a2} = \frac{10.6 I_2 - 200}{10}$$

As torque in both cases is the same,  $\therefore \Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$\therefore 25 \times 25 = I_2 \left( \frac{10.6 I_2 - 200}{10} \right) \text{ or } 6,250 = 10.6 I_2^2 - 200 I_2$$

$$\text{or } 10.6 I_2^2 - 200 I_2 - 6250 = 0 \text{ or } I_2 = 35.6 \text{ A}$$

$$\text{P.D. at brushes in this case} = 200 - (35.6 \times 0.6) = 178.6 \text{ V}$$

$$\therefore I_{a2} = \frac{10.6 \times 35.6 - 200}{10} = 17.74 \text{ A;}$$

$$E_{b2} = 178.6 - (17.74 \times 0.2) = 175 \text{ V}$$

$$\text{Now } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_1}{I_2} \quad (\because \Phi \propto I)$$

$$\therefore \frac{N_2}{500} = \frac{175}{180} \times \frac{25}{35.6} \quad \therefore N_2 = \mathbf{314 \text{ r.p.m.}}$$

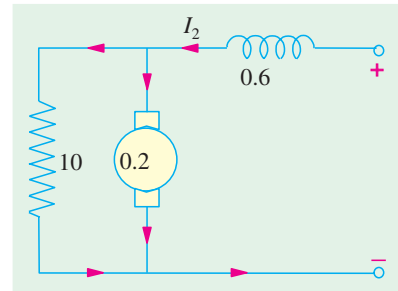


Fig. 30.17

**Example 30.41.** A series motor is running on a 440 V circuit with a regulating resistance of  $R$  Ω for speed adjustment. The armature and field coils have a total resistance of 0.3 Ω. On a certain load with  $R = \text{zero}$ , the current is 20 A and speed is 1200 r.p.m. With another load and  $R = 3$  Ω, the current is 15 A. Find the new speed and also the ratio of the two values of the power outputs of the motor. Assume the field strength at 15 A to be 80% of that at 20 A.

**Solution.**  $I_{a1} = 20 \text{ A}, R_a = 0.3 \text{ } \Omega$  ;  $E_{b1} = 440 - (20 \times 0.3) = 434 \text{ V}$

$$I_{a2} = 15 \text{ A}, R_a = 3 + 0.3 = 3.3 \text{ } \Omega \therefore E_{b2} = 440 - (3.3 \times 15) = 390.5 \text{ V}$$

$$\Phi_2 = 0.8 \Phi_1, N_1 = 1200 \text{ r.p.m.}$$

Using  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ , we get  $N_2 = 1200 \times \frac{390.5}{434} \times \frac{1}{0.8} = 1350 \text{ r.p.m.}$

Now, in a series motor,

$$T \propto \Phi I_a \text{ and power } P \propto T \times N \text{ or } P \propto \Phi N I_a$$

$$\therefore P_1 \propto \Phi_1 \times 1200 \times 20 \text{ and } P_2 \propto 0.8 \Phi_1 \times 1350 \times 15$$

$$\therefore \frac{P_1}{P_2} = \frac{1200 \times 20 \Phi_1}{1350 \times 15 \times 0.8 \Phi_1} = 1.48$$

Hence, power in the first case is 1.48 times the power in the second case.

**Example 30.42.** A d.c. series with an unsaturated field and negligible resistance, when running at a certain speed on a given load takes 50 A at 460 V. If the load-torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.

(Nagpur, Univ. November 1999, Madras Univ. 1987)

**Solution.** Let the speed be  $\omega$  radians/sec and the torque,  $T_{em}$  Nw-m developed by the motor. Hence power handled =  $T \cdot \omega$  watts

Let load torque be  $T_L$ ,  $T_L \propto \omega^3$   
 $T_L = T_{em}$ ,  $T_{em} \propto (50)^2$

Hence Load power =  $T_L \omega$

Since no losses have to be taken into account,  $50^2 \propto \omega^3$

Armature power,  $460 \times 50 \propto \omega^4$

Based on back e.m.f. relationship,  $E_b \propto \omega I_a$

$$460 \propto \omega \times 50$$

To reduce the speed by 25%, operating speed =  $0.75 \omega$  rad/sec

Let the new current be  $I$ .

From Load side torque  $\propto (0.75 \omega)^3$

From electro-mech side,  $T \propto I^2$   
 $I^2 \propto (0.75 \omega)^3$

Comparing similar relationship in previous case,

$$\frac{I^2}{50^2} = \frac{(0.75 \omega)^3}{\omega^3} = 0.75^3$$

$$I^2 = 50^2 \times 0.422 = 1055$$

$$I = 32.48 \text{ amp}$$

$$E_{b2} \propto I \times \text{speed}$$

$$\propto I \times 0.75 \omega$$

$$\propto 32.48 \times 0.75 \omega$$

$$\frac{E_{b2}}{460} = \frac{32.48 \times 0.75}{50}$$

$$E_{b2} = 224 \text{ volts}$$

If  $R$  is the resistance externally connected in series with the motor,

$$E_{b2} = 460 - 32.48 \times R = 224$$

$$\therefore R = 7.266 \text{ ohms}$$

$$\text{Previous armature power} = 460 \times 50 \times 10^{-3} = 23 \text{ kW}$$

New armature power should be

$$23 \times (0.75)^4 = 7.28 \text{ kW}$$

With  $E_b$  as 224 V and current as 32.48 amp

$$\begin{aligned} \text{Armature power} &= 224 \times 32.48 \text{ watts} \\ &= 7.27 \text{ kW} \end{aligned}$$

Thus, the final answer is checked by this step, since the results agree.

**Example 30.43.** A d.c. series motor drives a load, the torque of which varies as the square of speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and current when the motor-field-winding is shunted by a diverter of equal resistance as that of the field winding. Neglect all motor losses and assume the magnetic circuit as unsaturated.

(Bharathithasan Univ. April 1997)

**Solution.** Let the equations governing the characteristics of series motor be expressed as follows, with no losses and with magnetic circuit unsaturated.

Torque developed by motor = Load torque

$$k_m \times i_{se} \times I_a = k_L \times (600)^2$$

where  $k_m, k_L$  are constants

$I_{se}$  = series field current

$I_a$  = Armature current

In the first case

$$i_{se} = I_a$$

With diverter,

$$i_{se} = i_d = 0.5 I_a,$$

Since, the resistances of diverter and series field are equal.

$$\text{In the first case, } k_m \times 15 \times 15 = k_L \times (600)^2 \quad \dots(a)$$

Let the supply voltage be  $V_L$  volts.

Since Losses are to be neglected, armature receives a power of  $(V_L \times 15)$  watts.

Case (ii) Let the new speed =  $N_2$  r.p.m. and the new armature current =  $I_{a2}$  amp

So that new series-field current =  $0.5 I_{a2}$

Torque developed by motor = Load torque

$$k_m \times (0.5 I_{a2}) \times I_{a2} = k_L \times (N_2)^2 \quad \dots(b)$$

From equations (a) and (b) above,

$$\frac{0.5 I_{a2}^2}{225} = \frac{N_2^2}{600^2}$$

$$\text{or } \left( \frac{N_2}{I_{a2}} \right)^2 = \frac{600^2}{450} = 800 \quad \dots(c)$$

$$\text{or } N_2/I_{a2} = 28.28 \quad \dots(d)$$

Now armature receives a power of  $I_{a2} V_L$  watts. Mechanical outputs in the two cases have to be related with these electrical-power-terms.

$$k_L = (600)^2 \times 2600/60 = 15 V_L \quad \dots(e)$$

$$k_L = N^2 \times 2 N/60 = I_{a2} V_L \quad \dots(f)$$

From these two equations,

$$N^3/600^3 = I_{a2}/15 \quad \dots(g)$$

$$\text{From (c) and (g), } N_2 I_{a2} = 18,000 \quad \dots(h)$$

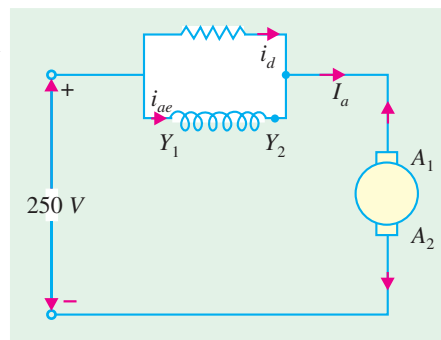


Fig. 30.18 (Diverter for speed control)

From (d) and (h),  $N_2 = 713.5$  r.p.m.

And  $I_{a2} = 25.23$  amp

**Additional Correlation :** Since the load-torque is proportional to the square of the speed, the mechanical output power is proportional to the cube of the speed. Since losses are ignored, electrical power (input) must satisfy this proportion.

$$(15 V_L) / (25.23 V_L) = (600/713.5)^3$$

$$\text{L.H.S.} = 0.5945, \text{R.H.S.} = 0.5947$$

Hence, correlated and checked.

### 30.4. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also. This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.
2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.
3. Maximum power developed is diminished in the same ratio as speed.
4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.
5. It gives speeds *below* the normal, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method is, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where motor is continually started and stopped.

#### Advantages of Field Control Method

This method is economical, more efficient and convenient though it can give speeds *above* (not below) the normal speed. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

### 30.5. Series-parallel Control

In this system of speed control, which is widely used in electric traction, two or more similar mechanically-coupled series motors are employed. At low speeds, the motors are joined in series Fig. 30.19 (a) and for high speeds, are joined in parallel Fig. 30.19 (b).

When in series, the two motors have the same current passing through them, although the voltage across each motor is  $V/2$  i.e., half the supply voltage. When joined in parallel, voltage across each machines is  $V$ , though current drawn by each motor is  $I/2$ .

#### When in Parallel

Now speed  $\propto E_b / \phi \propto E_b / \text{current}$   
(being series motors)

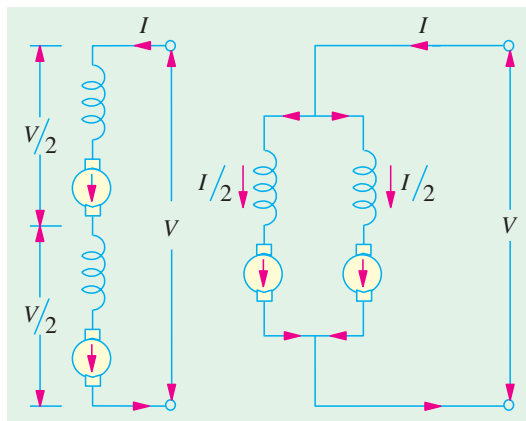


Fig. 30.19 (a)

Fig. 30.19 (b)

Since  $E_b$  is approximately equal to the applied voltage  $V$  :

$$\therefore \text{speed} \propto \frac{V}{I/2} \propto \frac{2V}{I}$$

Also,  $\text{torque} \propto \Phi I \propto I^2$  ( $\because \Phi \propto I$ ) ...**(i)**

$\therefore T \propto (I/2)^2 \propto I^2/4$  ...**(ii)**

**When in Series**

Here  $\text{speed} \propto \frac{E_b}{\Phi} \propto \frac{V/2}{I} \propto \frac{V}{2I}$  ...**(iii)**

*This speed is one-fourth of the speed of the motors when in parallel.*

Similarly  $T \propto \Phi I \propto I^2$

*The torque is four times that produced by motors when in parallel.*

This system of speed control is usually combined with the variable resistance method of control described in Art. 30.3 (2).

The two motors are started up in series with each other and with variable resistance which is cut out in sections to increase the speed. When all the variable series resistance is cut out, the motors are connected in parallel and at the same time, the series resistance is reinserted. The resistance is again reduced gradually till full speed is attained by the motors. The switching sequence is shown in Fig.30.20. As the variable series controller resistance is not continuously rated, it has to be cut out of the circuit fairly quickly although in the four running positions A, B, C and D, it may be left in circuit for any length of time.

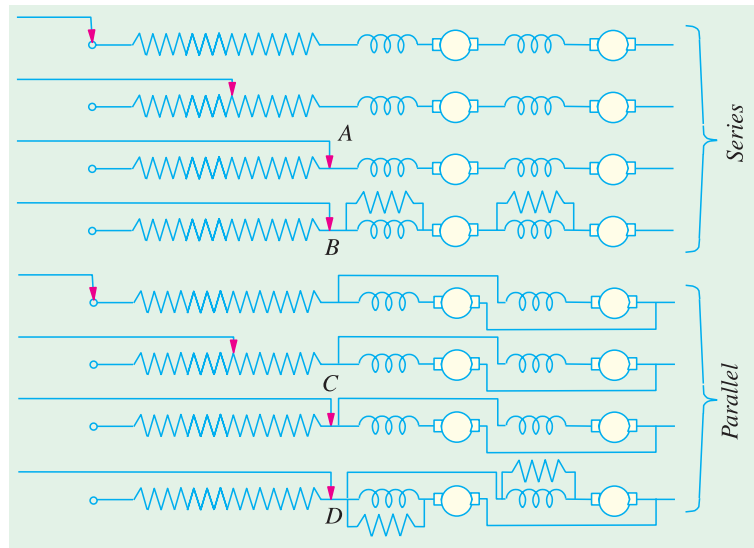


Fig. 30.20

**Example 30.44.** Two series motors run at a speed of 500 r.p.m. and 550 r.p.m. respectively when taking 50 A at 500 V. The terminal resistance of each motor is 0.5 Ω. Calculate the speed of the combination when connected in series and coupled mechanically. The combination is taking 50 A on 500 V supply. **(Electrical Machinery-I, Mysore Univ. 1985)**

**Solution. First Motor**

$$E_{b1} = 500 - (50 \times 0.5) = 475 \text{ V} ; I = 50 \text{ A}$$

Now,  $N_1 \propto E_{b1}/\Phi_1$  or  $E_{b1} \propto N_1 \Phi_1$  or  $E_{b1} = k N_1 \Phi_1$

$\therefore 475 = k \times 500 \times \Phi_1 \therefore k \Phi_1 = 475/500$

**Second Motor**  $E_{b2} = 500 - (50 \times 0.5) = 475 \text{ V}$ . Similarly,  $k \Phi_2 = 475/550$

**When both motors are in series**

$$E_b' = 500 - (50 \times 2 \times 0.5) = 450 \text{ V}$$



Now,  $E_b' = E_{b1} + E_{b2} = k \Phi_1 N + k \Phi_2 N$   
 where  $N$  is the common speed when joined in series.

$$\therefore 450 = \frac{475}{500} N + \frac{475}{550} N \quad \therefore N = 248 \text{ r.p.m.}$$

**Example 30.45.** Two similar 20 h.p. (14.92 kW), 250 V, 1000 r.p.m. series motors are connected in series with each other across a 250 V supply. The two motors drive the same shaft through a reduction gearing 5 : 1 and 4 : 1 respectively. If the total load torque on the shaft is 882 N-m, calculate (i) the current taken from the supply main (ii) the speed of the shaft and (iii) the voltage across each motor. Neglect all losses and assume the magnetic circuits to be unsaturated.

(Elect. Machines, Punjab Univ., 1991)

**Solution.** (i) Rated current of each motor =  $14,920/250 = 59.68 \text{ A}$

Back e.m.f.  $E_b = 250 \text{ V}$  (neglecting  $I_a R_a$  drop)

Now,  $E_b \propto N \Phi$  As  $\Phi \propto I \therefore E_b \propto NI$  or  $E_b = kNI$   
 $250 = k \times (1000/60) \times 59.68 \quad \therefore k = 0.25$

Let  $N_{sh}$  be the speed of the shaft.

Speed of the first motor  $N_1 = 5 N_{sh}$ ; Speed of the second motor  $N_2 = 4 N_{sh}$

Let  $I$  be the new current drawn by the series set, then

$$E_b' = E_{b1} + E_{b2} = kI \times 5 N_1 + kI \times N_2 = kI \times 5 N_{sh} + kI \times 4 N_{sh} \quad \dots(i)$$

$$250 = 9 \times kI N_{sh}$$

Now, torque  $T = 0.159 \frac{E_b I}{N} = 0.159 \times \frac{kI N_{sh} \times I}{N_{sh}} = 0.159 k I^2$

Shaft torque due to gears of 1st motor =  $5 \times 0.159 k I^2$

Shaft torque due to gears of 2nd motor =  $4 \times 0.159 k I^2$

$$\therefore 882 = k I^2 (5 \times 0.159 + 4 \times 0.159) = 1.431 k I^2$$

$$\therefore I^2 = 882/1.431 \times 0.25 = 2,449 \text{ A} \quad \therefore I = 49.5 \text{ A}$$

(ii) From equation (i), we get

$$250 = 9 \times 0.25 \times 49.5 \times N_{sh} \quad \therefore N_{sh} = 2.233 \text{ r.p.s.} = 134 \text{ r.p.m.}$$

(iii) Voltage across the armature of 1st motor is

$$E_{b1}' = 5 kI N_{sh} = 5 \times 0.25 \times 49.5 \times 2.233 = 139 \text{ V}$$

Voltage across the armature of 2nd motor

$$E_{b2}' = 4 kI N_{sh} = 4 \times 0.25 \times 49.5 \times 2.233 = 111 \text{ V}$$

Note that  $E_{b1}$  and  $E_{b2}$  are respectively equal to the applied voltage across each motor because  $I_a R_a$  drops are negligible.

### 30.6. Electric Braking

A motor and its load may be brought to rest quickly by using either (i) Friction Braking or (ii) Electric Braking. The commonly-used mechanical brake has one drawback : it is difficult to achieve a smooth stop because it depends on the condition of the braking surface as well as on the skill of the operator.

The excellent electric braking methods are available which eliminate the need of brake lining levers and other mechanical gadgets. Electric braking, both for shunt and series motors, is of the following three types: (i) rheostatic or dynamic braking



Strong electric brake for airwheels

- (ii) plugging *i.e.*, reversal of torque so that armature tends to rotate in the opposite direction and
- (iii) regenerative braking.

Obviously, friction brake is necessary for holding the motor even after it has been brought to rest.

### 30.7. Electric Braking of Shunt Motors

#### (a) Rheostatic or Dynamic Braking

In this method, the armature of the shunt motor is disconnected from the supply and is connected across a variable resistance  $R$  as shown in Fig. 30.21 (b). The field winding is, however, left connected across the supply undisturbed. The braking effect is controlled by varying the series resistance  $R$ . Obviously, this method makes use of generator action in a motor to bring it to rest.\* As seen from Fig. 30.21 (b), armature current is given by

$$I_a = \frac{E_b}{R + R_a} = \frac{\Phi Z N (P/A)}{R + R_a}$$

$$= \frac{k_1 \Phi N}{R + R_a}$$

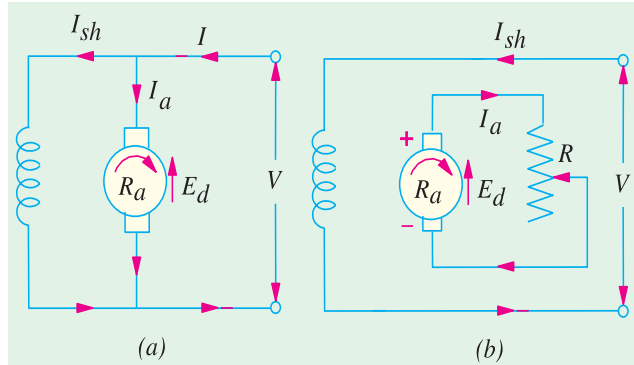


Fig. 30.21

Braking torque is given by

$$T_B = \frac{1}{2\pi} \Phi Z I_a \left( \frac{P}{A} \right) \text{ N-m}$$

$$= \frac{1}{2\pi} \Phi Z \left( \frac{P}{A} \right) \cdot \frac{\Phi Z N (P/A)}{R + R_a} = \frac{1}{2\pi} \left( \frac{ZP}{A} \right)^2 \frac{\Phi^2 N}{R + R_a} = k_2 \Phi^2 N \therefore T_B \propto N$$

Obviously,  $T_B$  decreases as motor slows down and disappear altogether when it comes to a stop.

#### (b) Plugging or Reverse Current Braking

This method is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

In this method, connections to the armature terminals are reversed so that motor tends to run in the opposite direction (Fig. 30.22). Due to the reversal of armature connections, applied voltage  $V$  and  $E_b$  start acting **in the same direction** around the circuit. In order to limit the armature current to a reasonable value, it is necessary to insert a resistor in the circuit while reversing armature connections.

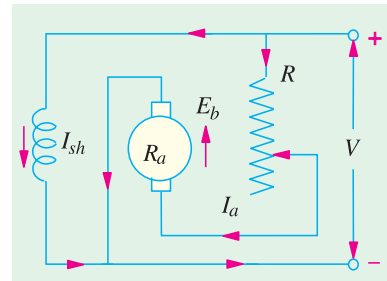


Fig. 30.22

$$I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{E_b}{R + R_a}$$

$$= \frac{V}{R + R_a} + \frac{\Phi Z N (P/A)}{R + R_a} = \frac{V}{R + R_a} + \frac{k_1 \Phi Z}{R + R_a}$$

$$T_B = \frac{1}{2\pi} \cdot \Phi Z I_a \left( \frac{P}{A} \right) = \frac{1}{2\pi} \cdot \left( \frac{\Phi Z P}{A} \right) \cdot I_a = \frac{1}{2\pi} \left( \frac{\Phi Z P}{A} \right) \left[ \frac{V}{R + R_a} + \frac{\Phi Z N (P/A)}{R + R_a} \right]$$

\* The motor while acting as a generator feeds current to the resistor dissipating heat at the rate of  $I_a^2 R$ . The current  $I_a$  produced by dynamic braking flow in the opposite direction, thereby producing a counter torque that slows down the machine.

$$= \frac{1}{2\pi} \left( \frac{ZP}{A} \right) \left( \frac{V}{R+R_a} \right) \cdot \Phi + \frac{1}{2\pi} \cdot \left( \frac{ZP}{A} \right)^2 \cdot \frac{\Phi^2 N}{R+R_a} = k_2 \Phi + k_3 \Phi^2 N$$

or  $T_b = k_4 + k_5 N$ , where  $k_4 = \frac{1}{2\pi} \left( \frac{\Phi ZP}{A} \right) \left( \frac{V}{R+R_a} \right)$

and  $k_5 = \frac{1}{2\pi} \left( \frac{\Phi ZP}{A} \right)^2 \times \frac{1}{(R+R_a)}$ .

Plugging gives greater braking torque than rheostatic braking. Obviously, during plugging, power is drawn from the supply and is dissipated by  $R$  in the form of heat. It may be noted that even when motor is reaching zero speed, there is some braking torque  $T_B = k_4$  (see Ex. 30.47).

**(c) Regenerative Braking**

This method is used when the load on the motor has overhauling characteristic as in the lowering of the cage of a hoist or the downgrade motion of an electric train. Regeneration takes place when  $E_b$  becomes greater than  $V$ . This happens when the overhauling



Regenerative braking demonstrations

load acts as a prime mover and so drives the machines as a generator. Consequently, direction of  $I_a$  and hence of armature torque is reversed and speed falls until  $E_b$  becomes lower than  $V$ . It is obvious that during the slowing down of the motor, power is returned to the line which may be used for supplying another train on an upgrade, thereby relieving the powerhouse of part of its load (Fig. 30.23).

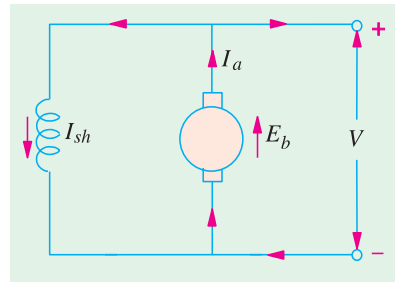


Fig. 30.23

For protective purposes, it is necessary to have some type of mechanical brake in order to hold the load in the event of a power failure.

**Example 30.46.** A 220 V compensated shunt motor drives a 700 N-m torque load when running at 1200 r.p.m. The combined armature compensating winding and interpole resistance is 0.008 Ω and shunt field resistance is 55 Ω. The motor efficiency is 90%. Calculate the value of the dynamic braking resistor that will be capable of 375 N-m torque at 1050 r.p.m. The windage and friction losses may be assumed to remain constant at both speeds.

**Solution.** Motor output =  $\omega T = 2\pi NT = 2\pi (1200/60) \times 700 = 87,965 \text{ W}$

Power drawn by the motor =  $87,965/0.9 = 97,740 \text{ W}$

Current drawn by the motor =  $97,740/220 = 444 \text{ A}$ .

$I_{sh} = 220/55 = 4 \text{ A}$  ;  $I_{a1} = 444 - 4 = 440 \text{ A}$

$E_{b1} = 220 - 440 \times 0.008 = 216.5 \text{ V}$

Since field flux remains constant,  $T_1$  is proportional to  $I_{a1}$  and  $T_2$  to  $I_{a2}$ .

$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$  or  $I_{a2} = 440 \times \frac{375}{100} = 2650 \text{ A}$

$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$  or  $\frac{1050}{1200} = \frac{E_{b2}}{216.5}$  ;  $E_{b2} = 189.4 \text{ V}$

With reference to Fig. 30.23, we have

$189.4 = 2650 (0.008 + R)$  ;  $R = 0.794 \text{ } \Omega$

30.8. Electric Braking of Series Motor

The above-discussed three methods as applied to series motors are as follows :

(a) Rheostatic (or Dynamic) Braking

The motor is disconnected from the supply, the field connections **are reversed** and the motor is connected in series with a variable resistance  $R$  as shown in Fig. 30.24. Obviously, now, the machine is running as a generator.

The field connections are reversed to make sure that current through field winding is in the *same* direction as before (i.e., from  $M$  to  $N$ ) in order to assist residual magnetism. In practice, the variable resistance employed for starting purpose is itself used for braking purposes. As in the case of shunt motors,

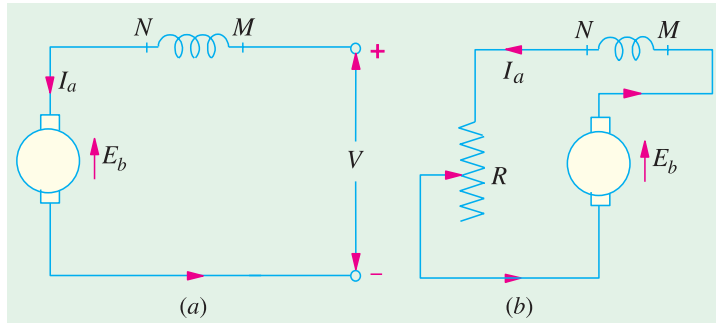


Fig. 30.24

$$T_B = k_2 \Phi^2 N = k_3 I_a^2 N \quad \dots \text{prior to saturation}$$

(b) Plugging or Reverse Current Braking

As in the case of shunt motors, in this case also the connections of the armature are reversed and a variable resistance  $R$  is put in series with the armature as shown in Fig. 30.25. As found in Art. 30.7 (b),

$$T_B = k_2 \Phi + k_3 \Phi^2 N$$

(c) Regenerative Braking

This type of braking of a series motor is not possible without modification because reversal of  $I_a$  would also mean reversal of the field and hence of  $E_b$ . However, this method is sometimes used with traction motors, special arrangements being necessary for the purpose.

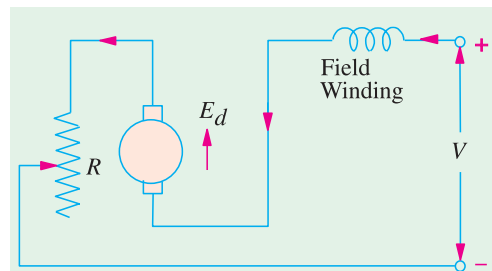


Fig. 30.25

**Example 30.47.** A 400 V, 25 h.p. (18.65 kW), 45 r.p.m., d.c. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full-load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2  $\Omega$ . (Electrical Technology, M.S. Univ. Baroda, 1988)

**Solution.**

F.L. Motor input current	$I = 18,650/0.746 \times 400 = 62.5 \text{ A}$
$I_a = 62.5 \text{ A}$ (neglecting $I_{sh}$ );	$E_b = 400 - 62.5 \times 0.2 = 387.5 \text{ V}$
Total voltage around the circuit is	$= 400 + 387.5 = 787.5 \text{ V}$
Max. braking current	$= 2 \times 62.5 = 125 \text{ A}$
Total resistance required in the circuit	$= 787.5/125 = 6.3 \Omega$
Braking resistance	$R = 6.3 - 0.2 = 6.1 \Omega$

Maximum braking torque will be produced initially when the motor speed is maximum i.e., 450 r.p.m. or 7.5 r.p.s.

Maximum value of  $T_B = k_4 + k_5 N$  – Art. 25.7(b)

Now, 
$$k_4 = \frac{1}{2\pi} \left( \frac{\Phi Z P}{A} \right) \left( \frac{V}{R + R_a} \right) \text{ and } k_5 = \frac{1}{2\pi} \left( \frac{\Phi Z P}{A} \right)^2 \cdot \frac{1}{(R + R_a)}$$

Now,  $E_b = \Phi ZN (P/A)$  ; also  $N = 450/60 = 7.5$  r.p.s.  
 $\therefore 387.5 = 7.5 (\Phi ZP/A)$  or  $(\Phi ZP/A) = 51.66$   
 $\therefore k_4 = \frac{1}{2\pi} \times 51.66 \times \frac{400}{6.3} = 522$  and  $k_5 = \frac{1}{2\pi} \times (51.66)^2 \times \frac{1}{6.3} = 67.4$   
 $\therefore$  Maximum  $T_B = 522 + 67.4 \times 7.5 = 1028$  N-m

When speed is also zero *i.e.*,  $N = 0$ , the value of torque is  $T_B = K_4 = 522$  N-m.

### 30.9. Electronic Speed Control Method for DC Motors

Of late, solid-state circuits using semiconductor diodes and thyristors have become very popular for controlling the speed of a.c. and d.c. motors and are progressively replacing the traditional electric power control circuits based on thyratrons, ignitrons, mercury arc rectifiers, magnetic amplifiers and motor-generator sets etc. As compared to the electric and electromechanical systems of speed control, the electronic methods have higher accuracy, greater reliability, quick response and also higher efficiency as there are no  $I^2R$  losses and moving parts. Moreover, full 4-quadrant speed control is possible to meet precise high-speed standards.

All electronic circuits control the motor speed by adjusting either (i) the voltage applied to the motor armature or (ii) the field current or (iii) both.

DC motors can be run from d.c. supply if available or from a.c. supply after it has been converted into d.c. supply with the help of rectifiers which can be either half-wave or full-wave and either controlled (by varying the conduction angle of the thyristors used) or uncontrolled.

AC motors can be run on the a.c. supply or from d.c. supply after it has been converted into a.c. supply with the help of inverters (opposite of rectifiers).

As stated above, the average output voltage of a thyristor-controlled rectifier can be changed by changing its conduction angle and hence the armature voltage of the d.c. motor can be adjusted to control its speed.

When run on a d.c. supply, the armature d.c. voltage can be changed with the help of a thyristor chopper circuit which can be made to interrupt d.c. supply at different rates to give different average values of the d.c. voltage. If d.c. supply is not available, it can be obtained from the available a.c. supply with the help of uncontrolled rectifiers (using only diodes and not thyristors). The d.c. voltages so obtained can be then chopped with the help of a thyristor chopper circuit.

A brief description of rectifiers, inverters\* and d.c. choppers would now be given before discussing the motor speed control circuits.

### 30.10. Uncontrolled Rectifiers

As stated earlier, rectifiers are used for a.c. to d.c. conversion *i.e.*, when the supply is alternating but the motor to be controlled is a d.c. machine.

Fig. 30.26 (a) shows a half-wave uncontrolled rectifier. The diode  $D$  conducts only during positive half-cycles of the single-phase a.c. input *i.e.*, when its anode  $A$  is positive with respect to its cathode  $K$ . As shown, the average voltage available across the load (or motor) is  $0.45 V$  where  $V$  is the r.m.s. value of the a.c. voltage (in fact,  $V = V_m / \sqrt{2}$ ). As seen it is a pulsating d.c. voltage.

In Fig. 30.26 (b) a single-phase, full-wave bridge rectifier which uses four semiconductor diodes and provides double the voltage *i.e.*,  $0.9 V$  is shown. During positive input half-cycles when end  $A$  is positive with respect to end  $B$ , diodes  $D_1$  and  $D_4$  conduct (*i.e.* opposite diodes) whereas during negative input half-cycles,  $D_2$  and  $D_3$  conduct. Hence, current flows through the load during both half-cycles in the *same* direction. As seen, the d.c. voltage supplied by a bridge rectifier is much less pulsating than the one supplied by the half-wave rectifier.

\* Rectifiers convert a.c. power into d.c. power, whereas inverters convert d.c. power into a.c. power. However, converter is a general term embracing both rectifiers and inverters.

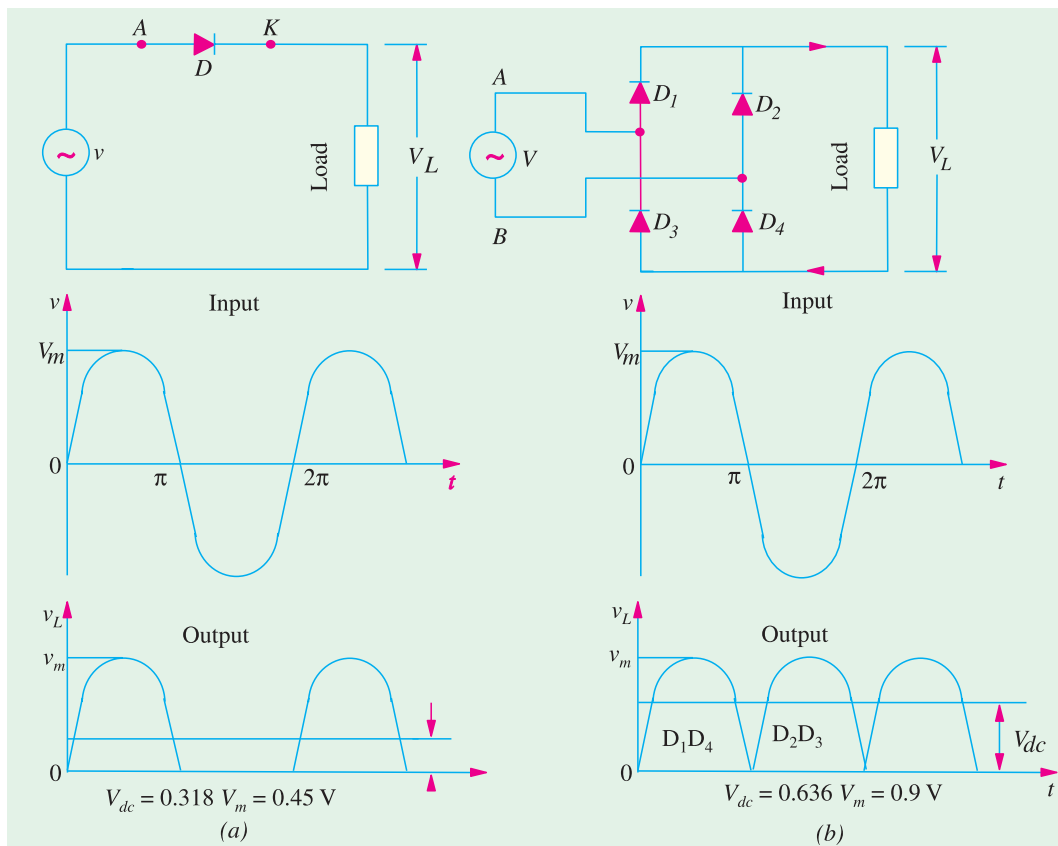


Fig. 30.26

### 30.11. Controlled Rectifiers

In these rectifiers, output load current (or voltage) can be varied by controlling the point in the input a.c. cycle at which the thyristor is turned ON with the application of a suitable low-power gate pulse. Once triggered (or fired) into conduction, the thyristor remains in the conducting state for the rest of the half-cycle *i.e.*, upto  $180^\circ$ . The firing angle  $\alpha$  can be adjusted with the help of a control circuit. When conducting, it offers no resistance *i.e.*, it acts like a short-circuit.

Fig. 30.27 (a) shows an elementary half-wave rectifier in which thyristor triggering is delayed by angle  $\alpha$  with the help of a phase-control circuit. As shown, the thyristor starts conducting at point A and not at point O because its gate pulse is applied after a delay of  $\alpha$ . Obviously, the conduction angle is reduced from  $180^\circ$  to  $(180^\circ - \alpha)$  with a consequent decrease in output voltage whose value is given by

$$V_L = \frac{V_m}{2\pi} (1 + \cos \alpha) = 0.16 V_m (1 + \cos \alpha) = 0.32 V_m \cos^2 \frac{\alpha}{2}$$

where  $V_m$  is the peak value of a.c. input voltage. Obviously,  $V_L$  is maximum when  $\alpha = 0$  and is zero when  $\alpha = 180^\circ$ .

Fig. 30.27 (b) shows the arrangement where a thyristor is used to control current through a load connected in series with the a.c. supply line.

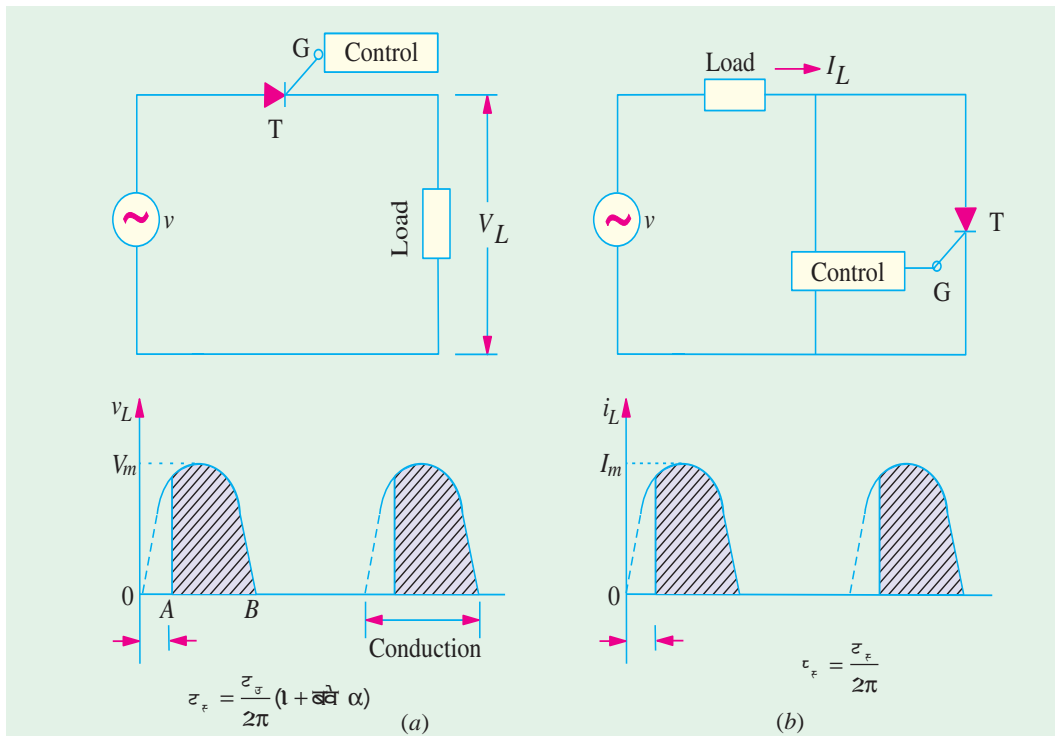


Fig. 30.27

The load current is given by

$$I_L = \frac{V_L}{R_L} = \frac{V_m}{2\pi R_L} (1 + \cos \alpha) = \frac{V_m}{\pi R_L} \cos^2 \frac{\alpha}{2}$$

Fig. 30.28 (a) shows a single-phase, full-wave half-controlled rectifier. It is called half-controlled because it uses two thyristors and two diodes instead of four thyristors. During positive input half-cycle when A is positive, conduction takes place via  $T_1$ , load and  $D_1$ . During the negative half-cycle when B becomes positive, conduction route is via  $T_2$ , load and  $D_2$ .

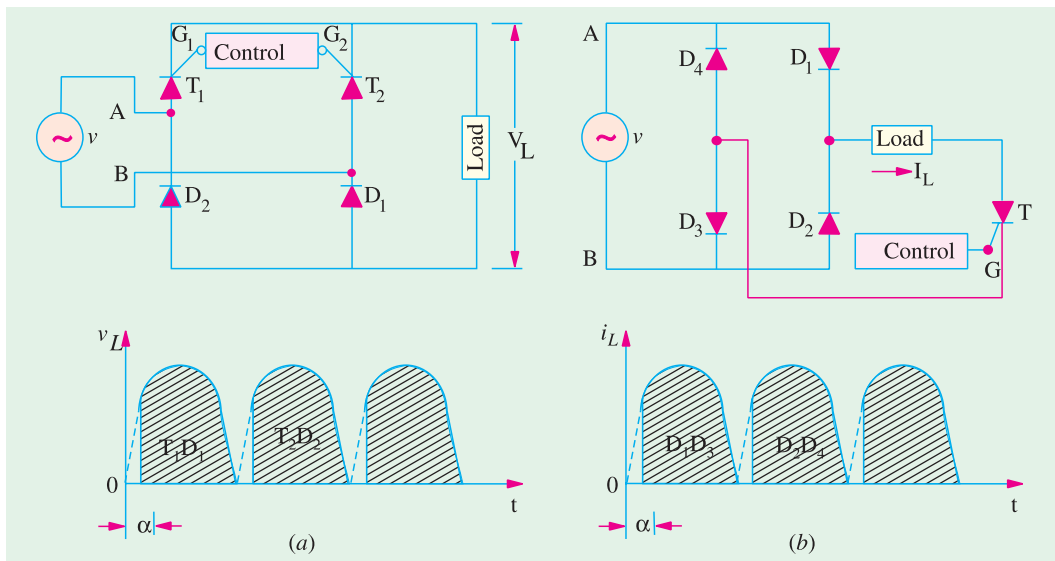


Fig. 30.28

The average output voltage  $V_L$  or  $V_{dc}$  is given by  $V_L = 2 \times$  half-wave rectifier output

$$\therefore V_L = 2 \times \frac{V_m}{2\pi} (1 + \cos \alpha) = \frac{V_m}{\pi} (1 + \cos \alpha) = \frac{2V_m}{\pi} \cos^2 \frac{\alpha}{2}$$

Similarly, Fig. 30.28 (b) shows a 4-diode bridge rectifier controlled by a single thyristor. The average load current through the series load is given by

$$I_L = \frac{V_m}{\pi R_L} (1 + \cos \alpha) = \frac{2V_m}{\pi R_L} \cos^2 \frac{\alpha}{2}$$

As seen from the figure, when  $A$  is positive,  $D_1$  and  $D_3$  conduct provided  $T$  has been fired. In the negative half-cycle,  $D_2$  and  $D_4$  conduct via the load.

### 30.12. Thyristor Choppers

Since thyristors can be switched ON and OFF very rapidly, they are used to interrupt a d.c. supply at a regular frequency in order to produce a lower (mean) d.c. voltage supply. In simple words, they can produce low-level d.c. voltage from a high-voltage d.c. supply as shown in Fig. 30.29.

The mean value of the output voltage is given by

$$V_{dc} = V_L = V \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

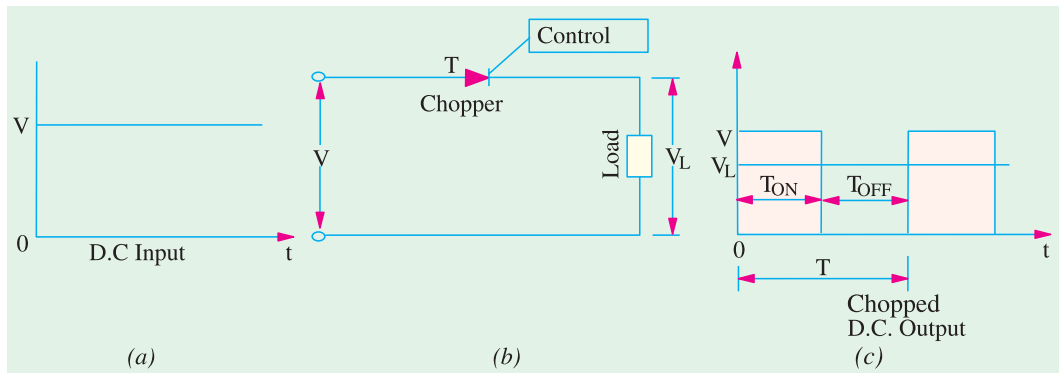


Fig. 30.29

Fig. 30.30 (a) shows a simple thyristor chopper circuit along with extra commutating circuitry for switching  $T_1$  OFF. As seen,  $T_1$  is used for d.c. chopping, whereas  $R$ ,  $T_2$  and  $C$  are used for commutation purposes as explained below.

When  $T_1$  is fired into conduction by its control circuit (not shown), current is set up through the load and commutation capacitor  $C$  gets charged via  $R$  with the polarity shown in the figure during this ON period.

For switching  $T_1$  OFF, second thyristor  $T_2$  is triggered into conduction allowing  $C$  to discharge through it (since it acts as a short-circuit while conducting) which reverse-biases  $T_1$  thus turning it OFF. The discharge from  $C$  leaves  $T_2$  with reverse polarity so that it is turned OFF, whereas  $T_1$  is triggered into conduction again.

Depending upon the frequency of switching ON and OFF, the input d.c. voltage is cut into d.c. pulses as shown in Fig. 30.30 (c).

\* For a fully-controlled bridge rectifier, its value is

$$V_L = \frac{2V_m}{\pi} \cos \alpha .$$



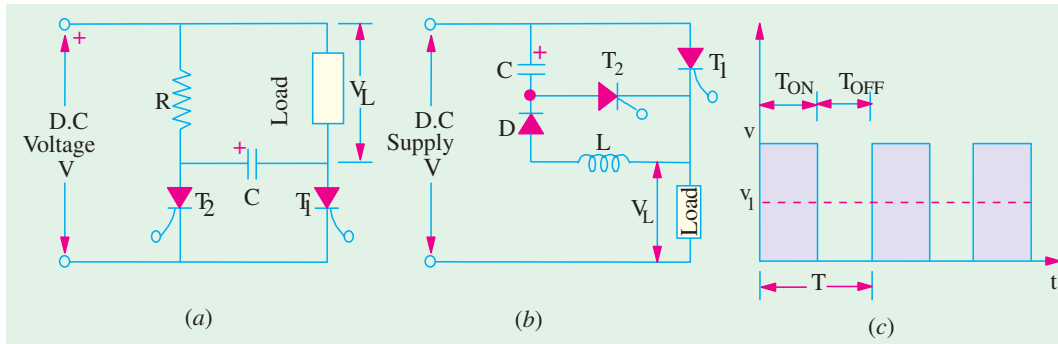


Fig. 30.30

In Fig. 30.30 (b),  $T_1$  is the chopping thyristor, whereas  $C, D, T_2$  and  $L$  constitute the commutation circuitry for switching  $T_1$  OFF and ON at regular intervals.

When  $T_2$  is fired,  $C$  becomes charged via the load with the polarity as shown. Next, when  $T_1$  is fired,  $C$  reverse-biases  $T_2$  to OFF by discharging via  $T_1, L$  and  $D$  and then recharges with reverse polarity.  $T_2$  is again fired and the charge on  $C$  reverse-biases  $T_1$  to non-conducting state.

It is seen that output (or load) voltage is present only when  $T_1$  is ON and is absent during the interval it is OFF. The mean value of output d.c. voltage depends on the relative values of  $T_{ON}$  and  $T_{OFF}$ . In fact, output d.c. voltage is given by

$$V_{dc} = V_L \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

Obviously, by varying thyristor ON/OFF ratio,  $V_L$  can be made any percentage of the input d.c. voltage  $V$ .

**Example 30.48.** The speed of a separately excited d.c. motor is controlled by a chopper. The supply voltage is 120 V, armature circuit resistance = 0.5 ohm, armature circuit inductance = 20 mH and motor constant = 0.05 V/r.p.m. The motor drives a constant load torque requiring an average current of 20 A. Assume motor current is continuous. Calculate (a) the range of speed control (b) the range of duty cycle. (Power Electronics-I, Punjab Univ. Nov. 1990)

**Solution.** The minimum speed is zero when  $E_b = 0$

$$V_t = E_b + I_a R_a = I_a \times R_a = 200 \times 0.5 = 10 \text{ V}$$

Now, 
$$V_t = \frac{V \alpha}{1 - \alpha} \Rightarrow 10 = \frac{120 \alpha}{1 - \alpha} \Rightarrow \alpha = \frac{1}{12}$$

Maximum speed corresponds to  $\alpha = 1$  when  $V_t = V = 120 \text{ V}$

$\therefore E_b = 120 - 20 \times 0.5 = 110 \text{ V}$

Now, 
$$N = E_b / K_a \Phi = 110 / 0.05 = 2200 \text{ r.p.m.}$$

(a) Hence, speed range is from 0 to 2200 r.p.m.

(b) Range of duty cycle is from  $\frac{1}{12}$  to 1.

### 30.13. Thyristor Inverters

Such inverters provide a very efficient and economical way of converting direct current (or voltage) into alternating current (or voltage). In this application, a thyristor serves as a controlled switch alternately opening and closing a d.c. circuit. Fig. 30.31 (a), shows a basic inverter circuit where an a.c. output is obtained by alternately opening and closing switches  $S_1$  and  $S_2$ . When we

replace the mechanical switches by two thyristors (with their gate triggering circuits), we get the thyristor inverter in Fig. 30.31 (b).

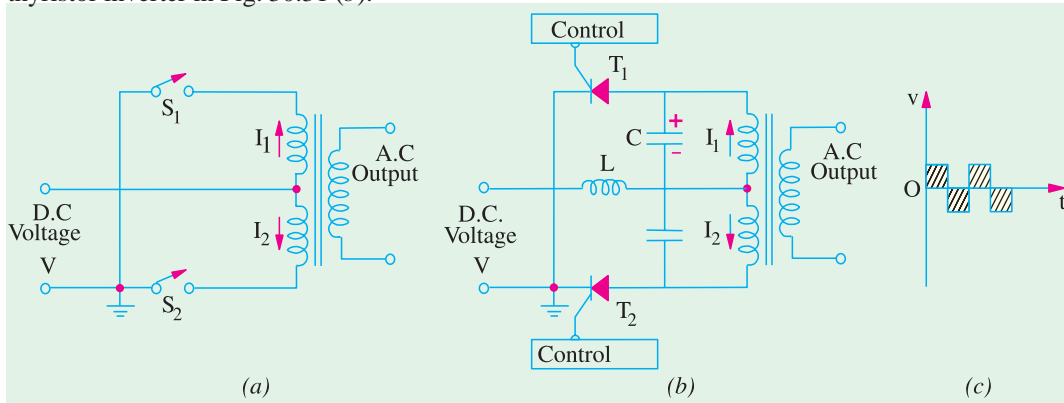


Fig. 30.31

Before discussing the actual circuit, it is worthwhile to recall that thyristor is a latching device which means that once it starts conducting, gate loses control over it and cannot switch it OFF whatever the gate signal. A separate **commutating** circuitry is used to switch the thyristor OFF and thus enable it to perform ON-OFF switching function.

Suppose  $T_1$  is fired while  $T_2$  is still OFF. Immediately  $I_1$  is set up which flows through  $L$ , one half of transformer primary and  $T_1$ . At the same time,  $C$  is charged with the polarity as shown.

Next when  $T_2$  is fired into condition,  $I_2$  is set up and  $C$  starts discharging through  $T_1$  thereby reverse-biasing it to CUT-OFF.

When  $T_1$  is again pulsed into condition,  $I_1$  is set up and  $C$  starts discharging thereby reverse-biasing  $T_2$  to OFF and the process just described repeats. As shown in Fig. 30.31 (c), the output is an alternating voltage whose frequency depends on the switching frequency to thyristors  $T_1$  and  $T_2$ .



Second generation thyristor inverter

### 30.14. Thyristor Speed Control of Separately-excited D.C. Motor

In Fig. 30.32, the bridge rectifier converts a voltage into d.c. voltage which is then applied to the armature of the separately-excited d.c. motor  $M$ .

As we know, speed of a motor is given by

$$N = \frac{V - I_a R_a}{\Phi} \left( \frac{A}{ZP} \right)$$

If  $\Phi$  is kept constant and also if  $I_a R_a$  is neglected, then,  $N \propto V \propto$  voltage across the armature. The value of this voltage furnished by the rectifier can be changed by varying the firing angle  $\alpha$  of the thyristor  $T$  with the help of its control circuit. As  $\alpha$  is increased *i.e.*, thyristor firing is delayed

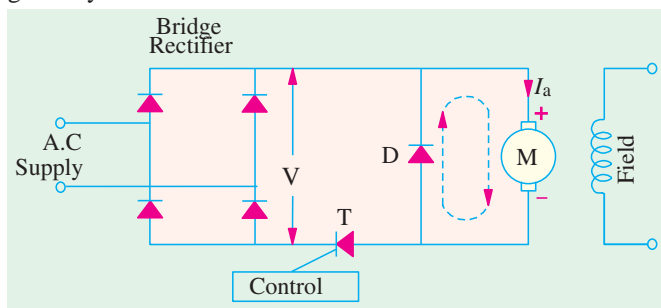


Fig. 30.32

more, its conduction period is reduced and, hence, armature voltage is decreased which, in turn, decreases the motor speed. When  $\alpha$  is decreased *i.e.*, thyristor is fired earlier, conduction period is increased which increases the mean value of the voltage applied across the motor armature. Consequently, motor speed is increased. In short, as  $\alpha$  increases,  $V$  decreases and hence  $N$  decreases. Conversely, as  $\alpha$  decreases,  $V$  increases and so,  $N$  increases. The free-wheeling diode  $D$  connected across the motor provides a circulating current path (shown dotted) for the energy stored in the inductance of the armature winding at the time  $T$  turns OFF. Without  $D$ , current will flow through  $T$  and bridge rectifier, prohibiting  $T$  from turning OFF.

### 30.15. Thyristor Speed Control of a D.C. Series Motor

In the speed control circuit of Fig. 30.33, an  $RC$  network is used to control the diac voltage that triggers the gate of a thyristor. As the a.c. supply is switched ON, thyristor  $T$  remains OFF but the capacitor  $C$  is charged through motor armature and  $R$  towards the peak value of the applied a.c. voltage. The time it takes for the capacitor voltage  $V_C$  to reach the breakover voltage of the diac\* depends on the setting of the variable resistor  $T$ . When  $V_C$  becomes equal to the breakover voltage of diac, it conducts and a triggering pulse is applied to the thyristor gate  $G$ . Hence,  $T$  is turned ON and allows current to pass through the motor. Increasing  $R$  delays the rise of  $V_C$  and hence the breakover of diac so that thyristor is fired later in each positive half cycle of the a.c. supply. It reduces the conduction angle of the thyristor which, consequently, delivers less power to the motor. Hence, motor speed is reduced.

If  $R$  is reduced, time-constant of the  $RC$  network is decreased which allows  $V_C$  to rise to the breakover voltage of diac more quickly. Hence, it makes the thyristor fire early in each positive input half-cycle of the supply. Due to increase in the conduction angle of the thyristor, power delivered to the motor is increased with a subsequent increase in its speed. As before  $D$  is the free-wheeling diode which provides circulating current path for the energy stored in the inductance of the armature winding.

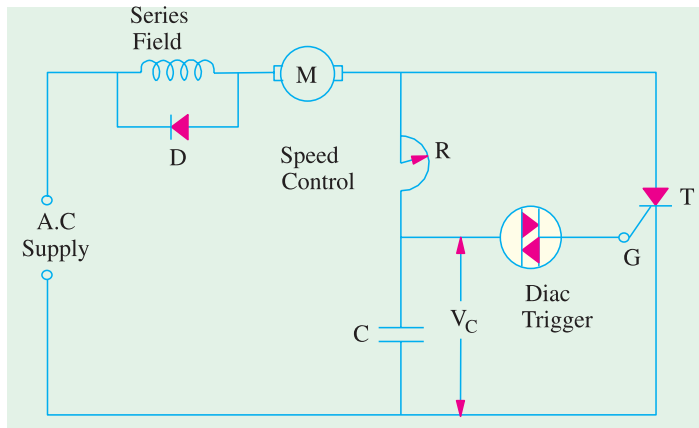


Fig. 30.33

### 30.16. Full-wave Speed Control of a Shunt Motor

Fig. 30.34 shows a circuit which provides a wide range of speed control for a fractional kW shunt d.c. motor. The circuit uses a bridge circuit for full-wave rectification of the a.c. supply. The shunt field winding is permanently connected across the d.c. output of the bridge circuit. The armature voltage is supplied through thyristor  $T$ . The magnitude of this

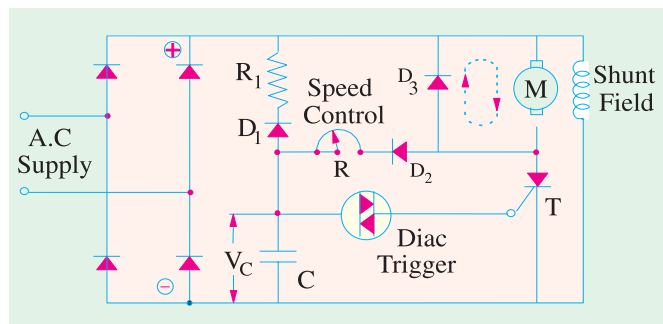


Fig. 30.34

\* Like a triac, it has directional switching characteristics.

voltage (and hence, the motor speed) can be changed by turning  $T_{ON}$  at different points in each half-cycle with the help of  $R$ . The thyristor turns OFF only at the end of each half-cycle. Free-wheeling diode  $D_3$  provides a circulating current path (shown dotted) for the energy stored in the armature winding at the time  $T$  turns OFF. Without  $D_3$ , This current would circulate through  $T$  and the bridge rectifier thereby prohibiting  $T$  from turning OFF.

At the beginning of each half-cycle,  $T$  is the OFF state and  $C$  starts charging up via motor armature, diode  $D_2$  and speed-control variable resistor  $R$  (it cannot charge through  $R_1$  because of reverse-biased diode  $D_1$ ). When voltage across  $C$  i.e.,  $V_C$  builds up to the breakover voltage of diac, diac conducts and applies a sudden pulse to  $T$  thereby turning it ON. Hence, power is supplied to the motor armature for the remainder of that half-cycle. At the end of each half-cycle,  $C$  is discharged through  $D_1, R_1$  and shunt field winding. The delay angle  $\alpha$  depends on the time it takes  $V_C$  to become equal to the breakover voltage of the diac. This time, in turn, depends on the time-constant of the  $R$ - $C$  circuit and the voltage available at point  $A$ . By changing  $R$ ,  $V_C$  can be made to build-up either slowly or quickly and thus change the angle  $\alpha$  at will. In this way, the average value of the d.c. voltage across the motor armature can be controlled. It further helps to control the motor speed because it is directly proportional to the armature voltage.

Now, when load is increased, motor tends to slow down. Hence,  $E_b$  is reduced. The voltage of point  $A$  is increased because it is equal to the d.c. output voltage of the bridge rectifier *minus* back e.m.f.  $E_b$ . Since  $V_A$  increases i.e., voltage across the  $R$ - $C$  charging circuit increases, it builds up  $V_C$  more quickly thereby decreasing which leads to early switching ON of  $T$  in each half-cycle. As a result, power supplied to the armature is increased which increases motor speed thereby compensating for the motor loading.

### 30.17. Thyristor Speed Control of a Shunt Motor

The speed of a shunt d.c. motor (upto 5 kW) may be regulated over a wide range with the help of the full-wave rectifier using only one main thyristor (or SCR)  $T$  as shown in Fig. 30.35. The firing angle  $\alpha$  of  $T$  is adjusted by  $R_1$  thereby controlling the motor speed. The thyristor and  $SUS$  (silicon unilateral switch) are reset (i.e., stop conduction) when each half-wave of voltage drops to zero. Before switching on the supply,  $R_1$  is increased by turning it in the counter-clockwise direction. Next, when supply is switched ON,  $C$  gets charged via motor armature and diode  $D_1$  (being forward biased). It means that it takes much longer for  $V_C$  to reach the breakdown voltage of  $SUS^*$  due to large time constant of  $R_1$ - $C$  network. Once  $V_C$  reaches that value,  $SUS$  conducts suddenly and triggers  $T$  into conduction. Since thyristor starts conducting late (i.e., its  $\alpha$  is large), it furnishes low voltage to start the motor. As speed selector  $R_1$  is turned clockwise (for less resistance),  $C$  charges up more rapidly (since time constant is decreased) to the breakover voltage of  $SUS$  thereby firing  $T$  into conduction earlier. Hence, average value of the d.c. voltage across the motor armature increases thereby increasing its speed.

While the motor is running at the speed set by  $R_1$ , suppose that load on the motor is increased. In that case, motor will tend to slow down thereby decreasing armature back e.m.f. Hence, potential of point 3 will rise which will charge  $C$  faster to the breakover voltage of  $SUS$ . Hence, thyristor will be fired earlier thereby applying greater armature voltage which will return the motor speed to its desired value. As seen, the speed is automatically regulated to offset changes in load.

The function of free-wheeling diode  $D_2$  is to allow dissipation of energy stored in motor

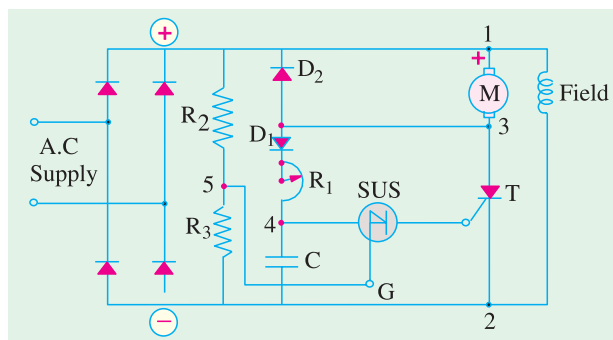


Fig. 30.35

\* It is a four-layer semiconductor diode with a gate terminal. Unlike diac, it conducts in one direction only.

armature during the time the full-wave rectified voltage drops to zero between half-cycles. If  $D_2$  is not there, then decreasing armature current during those intervals would be forced to flow through  $T$  thereby preventing its being reset. In that case,  $T$  would not be ready to be fired in the next half-cycle.

Similarly, towards the end of each half cycle as points 1 and 5 decrease towards zero potential, the negative going gate  $G$  turns  $SUS$  on thereby allowing  $C$  to discharge completely through  $SUS$  and thyristor gate-cathode circuit so that it can get ready to be charged again in the next half-cycle.

### 30.18. Thyristor Speed Control of a Series D.C. Motor

Fig. 30.36 shows a simple circuit for regulating the speed of a d.c. motor by changing the average value of the voltage applied across the motor armature by changing the thyristor firing angle  $\alpha$ . The trigger circuit  $R_1 - R_2$  can give a firing range of almost  $180^\circ$ . As the supply is switched on, full d.c. voltage is applied across  $R_1 - R_2$ . By changing the variable resistance  $R_2$ , drop across it can be made large enough to fire the SCR at any desired angle from  $0^\circ - 180^\circ$ . In this way, output voltage of the bridge rectifier can be changed considerably, thus enabling a wide-range control of the motor speed. The speed control can be made somewhat smoother by joining a capacitor  $C$  across  $R_2$  as shown in the figure.

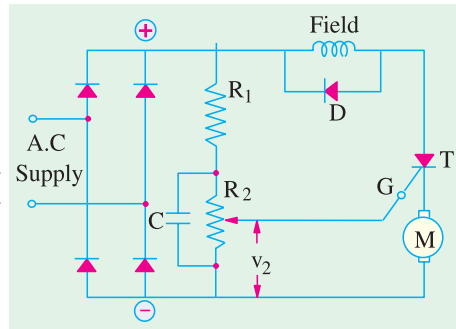


Fig. 30.36

### 30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = (V - E_b)/R_a$$

where  $V$  is the supply voltage,  $E_b$  the back e.m.f. and  $R_a$  the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resistance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of  $0.25 \Omega$  and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of  $440/0.25 = 1760 \text{ A}$  which is  $1760/50 = 35.2$  times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

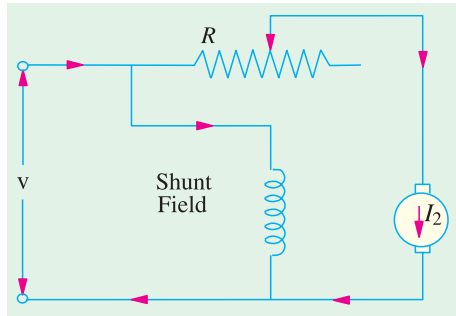


Fig. 30.37

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons :

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance  $R$  used for starting a shunt motor is shown. It will be seen that the starting resistance  $R$  is in series with the **armature** and not with the **motor** as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the

resistance  $R$ . If  $R$  was introduced in the motor circuit, then  $I_{sh}$  will be small at the start, hence starting torque  $T_{st}$  would be small ( $\because T_a \propto \Phi I_a$ ) and there would be experienced some difficulty in starting the motor. Such a simple starter is shown diagrammatically in Fig. 30.38.

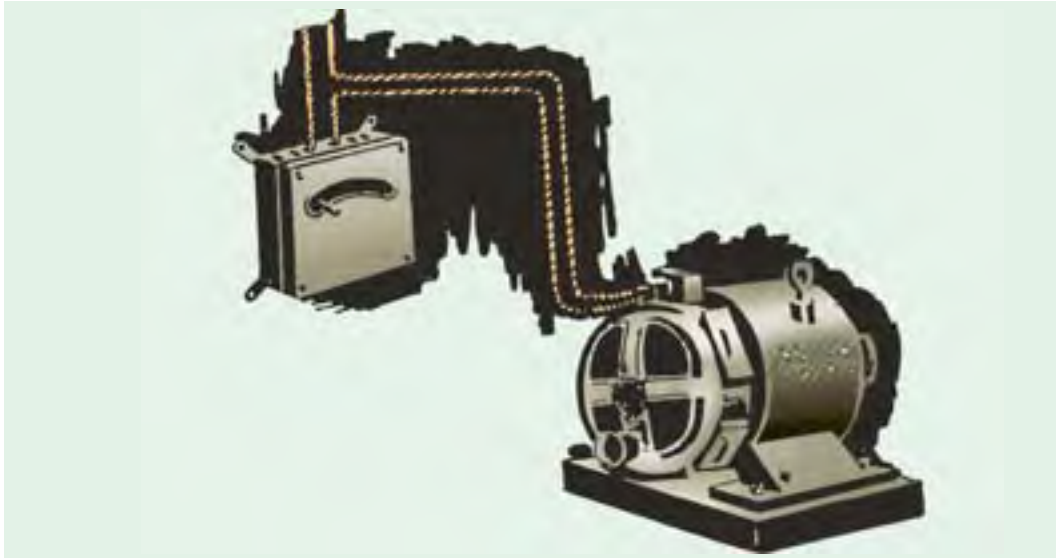


Fig. 30.38

### 30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

### 30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked  $A$ ,  $B$  and  $C$ . One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point  $A$  which is further connected to the starting arm  $L$ , through the overcurrent (or overload) release  $M$ .

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance  $R$ , is placed in series with the armature. The starting current drawn by the armature =  $V/(R_a + R_s)$  where  $R_s$  is the starting

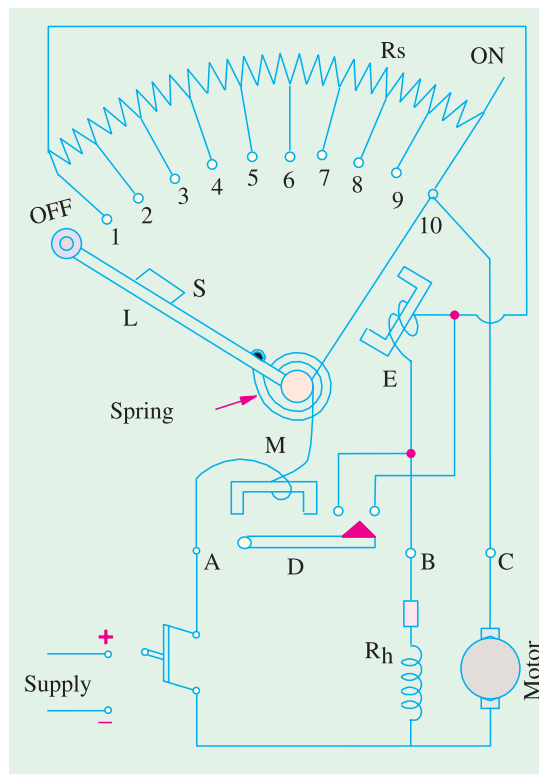


Fig. 30.39

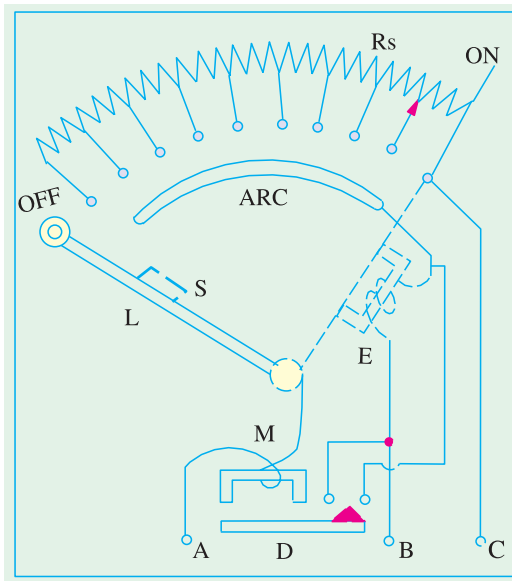


Fig. 30.40

which is connected to stud No. 1 (Fig. 30.40). The field circuit is completed through the starting resistance as it did in Fig. 30.39.

Now, we will discuss the action of the two protective devices shown in Fig. 30.39. The normal function of the HOLD-ON coil is to hold on the arm in the full running position when the motor is in normal operation. But, in the case of failure or disconnection of the supply or a break in the field circuit, it is de-energised, thereby releasing the arm which is pulled back by the spring to the OFF position. This prevents the stationary armature from being put across the lines again when the supply is restored after temporary shut down. This would have happened if the arm were left in the full ON position. One great advantage of connecting the HOLD-ON coil in series with the shunt field is that, should the field circuit become open, the starting arm immediately springs back to the OFF position thereby preventing the motor from running away.

The overcurrent release consists of an electromagnet connected in the supply line. If the motor becomes overloaded beyond a certain predetermined value, then *D* is lifted and short circuits the electromagnet. Hence, the arm is released and returns to OFF position.

The form of overload protection described above is becoming obsolete, because it cannot be made either as accurate or as reliable as a separate well-designed circuit breaker with a suitable time element attachment. Many a times a separated magnetic contactor with an overload relay is also used.

Often the motors are protected by thermal overload relays in which a bimetallic strip is heated by the motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contactor, thereby isolating the motor from the supply.

If it is desired to control the speed of the motor in addition, then a field rheostat is connected in the field circuit as indicated in Fig. 30.39. The motor speed can be increased by weakening the flux ( $\mathcal{E} N \propto I/\Phi$ ). Obviously, there is a limit to the speed increase obtained in this way, although speed ranges of three to four are possible. The connections of a starter and speed regulator with the motor are shown diagrammatically in Fig. 30.41. But there is one difficulty with such an arrangement for speed control. If too much resistance is 'cut in' by the field rheostat, then field current is reduced very much so that it is unable to create enough electromagnetic pull to overcome the spring tension. Hence,

resistance. As the arm is further moved, the starting resistance is gradually cut out till, when the arm reaches the running position, the resistance is all cut out. The arm moves over the various studs against a strong spring which tends to restore it to OFF position. There is a soft iron piece *S* attached to the arm which in the full 'ON' or running position is attracted and held by an electromagnet *E* energised by the shunt current. It is variously known as 'HOLD-ON' coil, LOW VOLTAGE (or NO-VOLTAGE) release.

It will be seen that as the arm is moved from stud NO. 1 to the last stud, the field current has to travel back through that portion of the starting resistance that has been cut out of the armature circuit. This results in slight decrease of shunt current. But as the value of starting resistance is very small as compared to shunt field resistance, this slight decrease in  $I_{sh}$  is negligible. This defect can, however, be remedied by using a brass arc

the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-speed motors. This has resulted in widespread application of four-point starter discussed below.

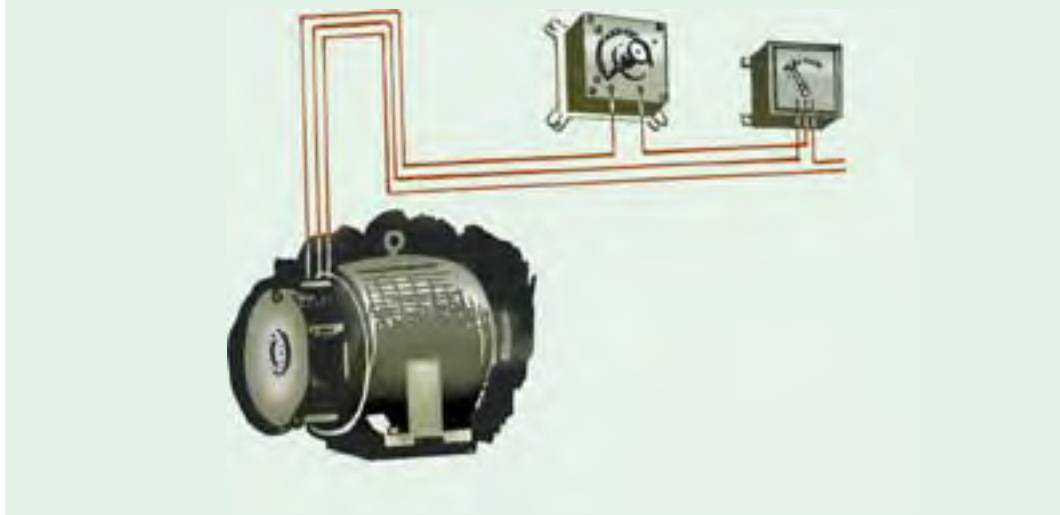


Fig. 30.41

### 30.22. Four-point Starter

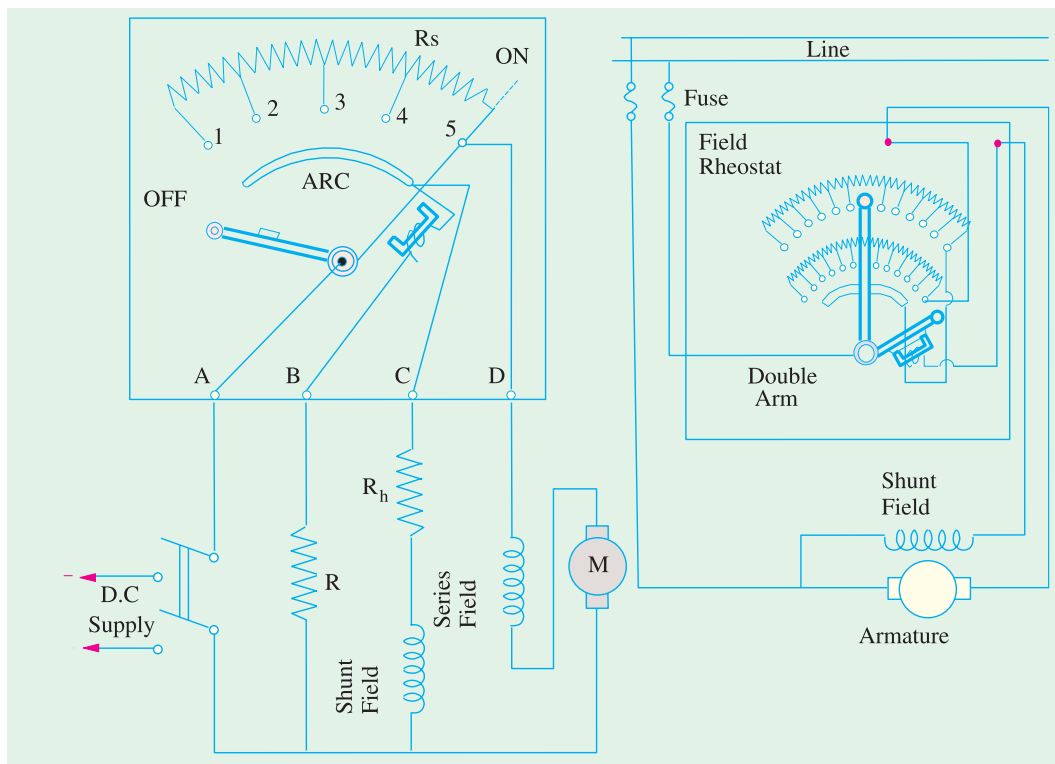


Fig. 30.42

Fig. 30.43



Such a starter with its internal wiring is shown, connected to a long-shunt compound motor in Fig. 30.42. When compared to the three-point starter, it will be noticed that one important change has been made *i.e.*, the HOLD-ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a protecting resistance as shown. When the arm touches stud No. 1, then the line current divides into three parts (*i*) one part passes through starting resistance  $R_s$ , series field and motor armature (*ii*) the second part passes through the shunt field and its field rheostat  $R_f$  and (*iii*) the third part passes through the HOLD-ON coil and current-protecting resistance  $R$ . It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not at all affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means that the electromagnetic pull exerted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting arm to OFF position no matter how the field rheostat or regulator is adjusted.

### 30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor,



Speed-control Rheostats

whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The

field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

### 30.24. Starting and Speed Control of Series Motor

For starting and speed control of series motor either a face-plate type or drum-type controller is used which usually has the reversing feature also. A face-plate type of reversing controller is shown in Fig. 30.44.

Except for a separate overload circuit, no inter-locking or automatic features are required because the operator watches the performance continuously.

As shown, the regulating lever consists of three pieces separated by strips of insulation. The outside parts form the electrical connections and the middle one is insulated from them. By moving the regulating lever, resistance can be cut in and out of the motor circuit. Reversing is obtained by moving the lever in the opposite direction as shown, because in that case, connections to the armature are reversed. Such an arrangement is employed where series motors are used as in the case of cranes, hoists and streetcars etc.

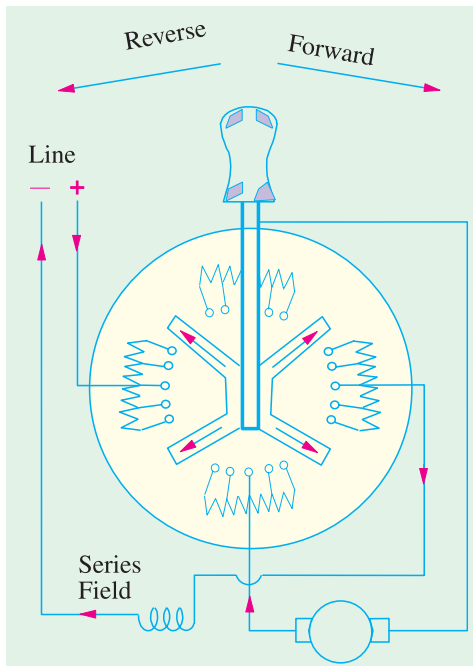


Fig. 30.44

circuit at this stage, then when resistance in the armature circuit is completely cut out, further rotation of the handle inserts resistance into the field circuit. Turning of the handle in the opposite direction starts and speeds up the motor in the reverse direction.

### 30.25. Grading of Starting Resistance for Shunt Motors

$T_{st}$  would be small in designing shunt motor starters, it is usual to allow an overload of 50% for starting and to advance the starter a step when armature current has fallen to definite lower value. Either this lower current limit may be fixed or the number of starter steps may be fixed. In the former case, the number of steps are so chosen as to suit the upper and lower current limits whereas in the latter case, the lower current limit will depend on the number of steps specified. It can be shown that the resistances in the circuit on successive studs from geometrical progression, having a common ratio equal to lower current limit/upper current limit i.e.,  $I_2/I_1$ .

In Fig. 30.45 the starter connected to a shunt motor is shown. For the sake of simplicity, four live studs have been taken. When arm A makes contact with stud No. 1, full shunt field is

However, for adjustable speed service in connection with the operation of machine tools, a drum controller is preferred. It is called 'controller' because in addition to accelerating the motor to its normal speed, it provides the means for reversing the direction of the motor. Other desirable features such as safety protection against an open field or the temporary failure of power supply and overloads are frequently provided in this type of controller.

The controller consists of armature resistance grids of cross-section sufficient to carry the full-load operating current continuously and are used for adjusting the motor speed to values lower than the base speed obtained with no external resistance in the armature of field circuit. As the operating handle is gradually turned, the resistance is cut out of the armature circuit—there being as yet no resistance in the field cir-

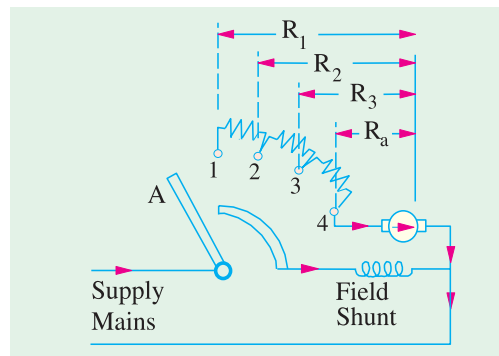


Fig. 30.45

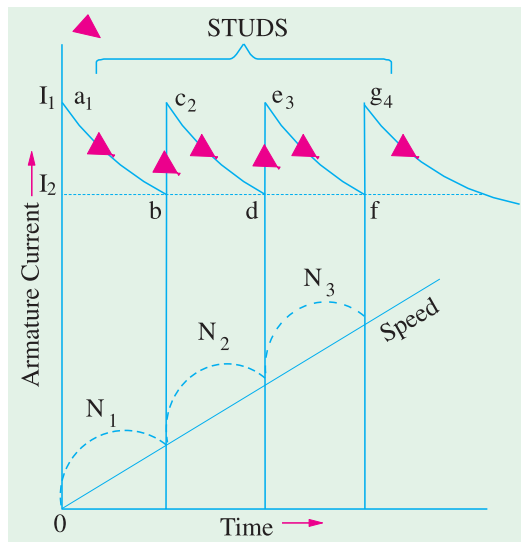


Fig. 30.46

established and at the same time the armature current immediately jumps to a maximum value  $I_1$  given by  $I_1 = V/R_1$  where  $R_1 =$  armature and starter resistance (Fig. 30.45).

$I_1$  the maximum permissible armature current at the start ( $I_{max}$ ) and is, as said above, usually limited to 1.5 times the full-load current of the motor. Hence, the motor develops 1.5 times its full-load torque and accelerates very rapidly. As the motor speeds up, its back e.m.f. grows and hence decreases the armature current as shown by curve  $ab$  in Fig. 30.46.

When the armature current has fallen to some predetermined value  $I_2$  (also called  $I_{min}$ ) arm  $A$  is moved to stud No. 2. Let the value of back e.m.f. be  $E_{b1}$  at the time of leaving stud No. 1. Then

$$I_2 = \frac{V - E_{b1}}{R_1} \quad \dots(i)$$

It should be carefully noted that  $I_1$  and  $I_2$  [ $(I_{max})$  and  $(I_{min})$ ] are respectively the maximum and minimum currents of the motor. When arm  $A$  touches stud No. 2, then due to diminution of circuit resistance, the current again jumps up to its previous value  $I_1$ . Since speed had no time to change, the back e.m.f. remains the same as initially.

$$\therefore I_1 = \frac{V - E_{b1}}{R_2} \quad \dots(ii)$$

From (i) and (ii), we get  $\frac{I_1}{I_2} = \frac{R_1}{R_2} \quad \dots(iii)$

When arm  $A$  is held on stud No. 2 for some time, then speed and hence the back e.m.f. increases to a value  $E_{b2}$ , thereby decreasing the current to previous value  $I_2$ , so that

$$I_2 = \frac{V - E_{b2}}{R_2} \quad \dots(iv)$$

Similarly, on first making contact with stud No. 3, the current is

$$I_1 = \frac{V - E_{b2}}{R_3} \quad \dots(v)$$

From (iv) and (v), we again get  $\frac{I_1}{I_2} = \frac{R_2}{R_3} \quad \dots(vi)$

When arm  $A$  is held on stud No. 3 for some time, the speed and hence back e.m.f. increases to a new value  $E_{b3}$ , thereby decreasing the armature current to a value  $I_2$  such that

$$I_2 = \frac{V - E_{b3}}{R_3} \quad \dots(vii)$$

On making contact with stud No. 4, current jumps to  $I_1$  given by

$$I_1 = \frac{V - E_{b3}}{R_a} \quad \dots(viii)$$

From (vii) and (viii), we get  $\frac{I_1}{I_2} = \frac{R_3}{R_a} \quad \dots(ix)$

From (iii), (vi) and (ix), it is seen that

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K \text{ (say)} \quad \dots(x)$$

Obviously,

$$R_3 = KR_a; \quad R_2 = KR_3 = K^2R_a$$

$$R_1 = KR_2 = K.K^2R_a = K^3R_a$$

In general, if  $n$  is the number of live studs and therefore  $(n - 1)$  the number of sections in the starter resistance, then

$$R_1 = K^{n-1}.R_a \text{ or } \frac{R_1}{R_a} = K^{n-1} \text{ or } \left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_a} \quad \dots\text{from (x)}$$

Other variations of the above formula are

$$(a) \quad K^{n-1} = \frac{R_1}{R_a} = \frac{V}{I_1 R_a} = \frac{V}{I_{max} R_a}$$

$$(b) \quad K^n = \frac{V}{I_1 R_a} \cdot \frac{I_1}{I_2} = \frac{V}{I_2 R_a} = \frac{V}{I_{min} R_a} \text{ and}$$

$$(c) \quad n = 1 + \frac{\log(V / R_a I_{max})}{\log K} \text{ from (a) above.}$$

Since  $R_1 = V/I_1$  and  $R_a$  are usually known and  $K$  is known from the given values of maximum and minimum currents (determined by the load against which motor has to start), the value of  $n$  can be found and hence the value of different starter sections.

**When Number of Sections is Specified.**

Since  $I_1$  would be given,  $R_1$  can be found from  $R_1 = V/I_1$ .

Since  $n$  is known,  $K$  can be found from  $R_1/R_a = K^{n-1}$  and the lower current limit  $I_2$  from  $I_1/I_2 = K$ .

**Example 30.49.** A 10 b.h.p. (7.46 kW) 200-V shunt motor has full-load efficiency of 85%. The armature has a resistance of 0.25 Ω. Calculate the value of the starting resistance necessary to limit the starting current to 1.5 times the full-load current at the moment of first switching on. The shunt current may be neglected. Find also the back e.m.f. of the motor, when the current has fallen to its full-load value, assuming that the whole of the starting resistance is still in circuit.

**Solution.** Full-load motor current =  $7,460/200 \times 0.85 = 43.88$  A  
 Starting current,  $I_1 = 1.5 \times 43.88 = 65.83$  A  
 $R_1 = V/I_1 = 200/65.83 = 3.038 \Omega$ ;  $R_a = 0.25 \Omega$   
 $\therefore$  Starting resistance =  $R_1 - R_a = 3.038 - 0.25 = 2.788 \Omega$   
 Now, full-load current  $I_2 = 43.88$  A  
 Now,  $I_2 = \frac{V - E_{b1}}{R_1}$   
 $\therefore E_{b1} = V - I_2 R_1 = 200 - (43.88 \times 3.038) = 67$  V

**Example 30.50.** A 220-V shunt motor has an armature resistance of 0.5 Ω. The armature current at starting must not exceed 40 A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter. **(Elect. Machines, AMIE Sec. B, 1992)**

**Solution.** Since the number of starter sections is specified, we will use the relation.

$R_1/R_a = K^{n-1}$  or  $R_1 = R_a K^{n-1}$   
 Now,  $R_1 = 220/40 = 5.5 \Omega$ ,  $R_a = 0.4 \Omega$ ;  $n-1 = 6$ ,  $n = 7$   
 $\therefore 5.5 = 0.4 K^6$  or  $K^6 = 5.5/0.4 = 13.75$   
 $6 \log_{10} K = \log_{10} 13.75 = 1.1383$ ;  $K = 1.548$   
 Now,  $R_2 = R_1/K = 5.5/1.548 = 3.553 \Omega$   
 $R_3 = R_2/K = 3.553/1.548 = 2.295 \Omega$   
 $R_4 = 2.295/1.548 = 1.482 \Omega$   
 $R_5 = 1.482/1.548 = 0.958 \Omega$   
 $R_6 = 0.958/1.548 = 0.619 \Omega$   
 Resistance of 1st section =  $R_1 - R_2 = 5.5 - 3.553 = 1.947 \Omega$   
 ,, 2nd ,, =  $R_2 - R_3 = 3.553 - 2.295 = 1.258 \Omega$   
 ,, 3rd ,, =  $R_3 - R_4 = 2.295 - 1.482 = 0.813 \Omega$   
 ,, 4th ,, =  $R_4 - R_5 = 1.482 - 0.958 = 0.524 \Omega$

5th " =  $R_5 - R_6 = 0.958 - 0.619 = 0.339 \Omega$   
 6th " =  $R_6 - R_a = 0.619 - 0.4 = 0.219 \Omega$

**Example 30.51.** Find the value of the step resistance in a 6-stud starter for a 5 h.p. (3.73 kW), 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The total Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is found to be 88%. (D.C. Machines, Jadavpur Univ. 1988)

**Solution.** Output = 3,730 W  
 Total loss = 4,238 - 3,730 = 508 W  
 Armature Cu loss alone = 508/2 = 254 W  
 Input current = 4,238/200 = 21.19 A  
 Armature current = 21.19 - 0.6 = 20.59 A  
 $\therefore 20.59^2 R_a = 254 \quad \therefore R_a = 254/20.59^2 = 0.5989 \Omega$   
 Permissible input current = 21.19  $\times$  2 = 42.38 A  
 Permissible armature current = 42.38 - 0.6 = 41.78 A  
 $\therefore R_1 = 200/41.78 = 4.787 \Omega; n = 6; n - 1 = 5$   
 $\therefore 4.787 = K^5 \times 0.5989 \quad \therefore K^5 = 4.787/0.5989 = 7.993$   
 or  $5 \log K = \log 7.993 = 0.9027$   
 $\therefore \log K = 0.1805; K = 1.516$   
 Now  $R_2 = R_1/K = 4.789/1.516 = 3.159 \Omega$   
 $R_3 = 2.084 \Omega; R_4 = 1.376 \Omega; R_5 = 0.908 \Omega$   
 Resistance in 1st step =  $R_1 - R_2 = 4.787 - 3.159 = 1.628 \Omega$   
 Resistance in 2nd step =  $R_2 - R_3 = 3.159 - 2.084 = 1.075 \Omega$   
 Resistance in 3rd step =  $R_3 - R_4 = 2.084 - 1.376 = 0.708 \Omega$   
 Resistance in 4th step =  $R_4 - R_5 = 1.376 - 0.908 = 0.468 \Omega$   
 Resistance in 5th step =  $R_5 - R_a = 0.908 - 0.5989 = 0.309 \Omega$

The various sections are shown in Fig. 28.47.

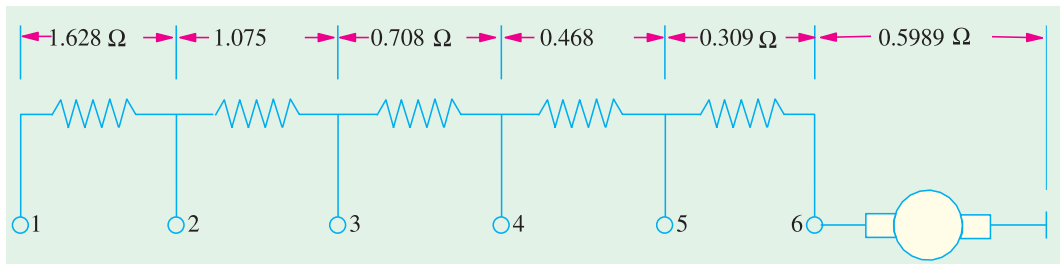


Fig. 30.47

**Example 30.52.** Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, d.c. shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.

(Elec. Machines, Gujarat Univ. 1987)

**Solution.** Output = 36,775 W ; Input = 36,775/0.92 = 39,980 W  
 Total Cu loss = 0.05  $\times$  39,980 = 1,999 W  
 Shunt Cu loss =  $V^2/R_{sh} = 400^2/200 = 800 \text{ W}$

Armature Cu loss = 1,999 – 800 = 1199 W

F.L. input current = 39,980/400 = 99.95 A

$I_{sh} = 400/200 = 2A ; I_a = 99.95 - 2 = 97.95 A$

$\therefore 97.95^2 R_a = 1199 W \text{ or } R_a = 0.125 \Omega$

Now, minimum armature current equals full-load current i.e.  $I_a = 97.95 A$ . As seen from Art. 30.25 in its formula given in (b), we have

$K^n = \frac{z}{\tau \cdot \tau}$

or  $K^7 = 400/97.95 \times 0.125 = 32.68$

$\therefore K = 32.68^{1/7} = 1.645$

$I_1 = \text{maximum permissible armature current}$

$= KI_2 = 1.645 \times 97.94 = 161 A$

$\therefore R_1 = V/I_1 = 400/161 = 2.483 \Omega$

$R_2 = R_1/K = 2.483/1.645 = 1.51 \Omega$

$R_3 = 1.51/1.645 = 0.917 \Omega$

$R_4 = 0.917/1.645 = 0.557 \Omega$

$R_5 = 0.557/1.645 = 0.339 \Omega$

$R_6 = 0.339/1.645 = 0.206 \Omega$

$R_7 = 0.206/1.645 = 0.125 \Omega$

Resistance in 1st step =  $R_1 - R_2 = 0.973 \Omega$

Resistance in 2nd step =  $R_2 - R_3 = 0.593 \Omega$

Resistance in 3rd step =  $R_3 - R_4 = 0.36 \Omega$

Resistance in 4th step =  $R_4 - R_5 = 0.218 \Omega$

Resistance in 5th step =  $R_5 - R_6 = 0.133 \Omega$

Resistance in 6th step =  $R_6 - R_a = 0.081 \Omega$

The various starter sections are shown in Fig 30.48.

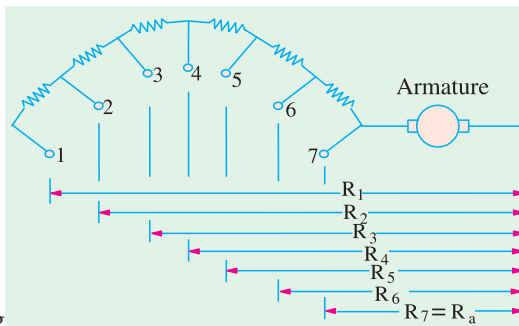


Fig. 30.48

**Example 30.53.** Calculate the resistance steps for the starter of a 250-V, d.c. shunt motor having an armature resistance of 0.125 Ω and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A

(Elect.Engineering-I, Bombay Univ. 1989)

**Solution.** As the motor is to start against its full-load, the minimum current is its F.L. current i.e. 150A. We will use the formula given in Art. 30.25.

$(I_1/I_2^{n-1}) = R_1 / R_a$

Here  $I_1 = 200 A ; I_2 = 150 A ; R_1 = 250/200 = 1.25 \Omega$

$R_a = 0.125 \Omega ; n = \text{No. of live studs}$

$\therefore (200/150)^{n-1} = 1.25/0.125 = 10 \text{ or } (4/3)^{n-1} = 10$

$(n - 1) \log 4/3 = \log 10 \text{ or } (n - 1) \times 0.1249 = 1$

$\therefore (n - 1) = 1/0.1249 = 8$

Hence, there are 9 studs and 8 steps.

Now  $R_2 = R_1 \times I_2/I_1 = 1.25 \times 3/4 = 0.938 \Omega$

$R_3 = 0.938 \times 3/4 = 0.703 \Omega$

$R_4 = 0.703 \times 3/4 = 0.527 \Omega$

$$\begin{aligned}
 R_5 &= 0.527 \times 3/4 = 0.395 \, \Omega \\
 R_6 &= 0.395 \times 3/4 = 0.296 \, \Omega \\
 R_7 &= 0.296 \times 3/4 = 0.222 \, \Omega \\
 R_8 &= 0.222 \times 3/4 = 0.167 \, \Omega \\
 R_a &= 0.167 \times 3/4 = 0.125 \, \Omega \\
 \therefore \quad \text{Resistance of 1st element} &= 1.25 - 0.938 = \mathbf{0.312 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{2nd} \quad \quad \quad \text{,,} &= 0.938 - 0.703 = \mathbf{0.235 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{3rd} \quad \quad \quad \text{,,} &= 0.703 - 0.527 = \mathbf{0.176 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{4th} \quad \quad \quad \text{,,} &= 0.527 - 0.395 = \mathbf{0.132 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{5th} \quad \quad \quad \text{,,} &= 0.395 - 0.296 = \mathbf{0.099 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{6th} \quad \quad \quad \text{,,} &= 0.296 - 0.222 = \mathbf{0.074 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{7th} \quad \quad \quad \text{,,} &= 0.222 - 0.167 = \mathbf{0.055 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{8th} \quad \quad \quad \text{,,} &= 0.167 - 0.125 = \mathbf{0.042 \, \Omega}
 \end{aligned}$$

**Example 30.54.** The 4-pole, lap-wound armature winding of a 500-V, d.c. shunt motor is housed in a total number of 60 slots each slot containing 20 conductors. The armature resistance is 1.31  $\Omega$ . If during the period of starting, the minimum torque is required to be 218 N-m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.

(Elect. Machinery-II, Bangalore Univ. 1991)

**Solution.** From the given minimum torque, we will be able to find the minimum current required during starting. Now

$$\begin{aligned}
 T_a &= 0.159 \, \Phi \, Z I_a (P/A) \\
 \therefore \quad 218 &= 0.159 \times 23 \times 10^{-3} \times (60 \times 20) I_a \times (4/4) \quad \therefore I_a = 50 \text{ A (approx.)} \\
 \text{Maximum current} &= 50 \times 1.5 = 75 \text{ A} \\
 \therefore \quad I_1 &= 75 \text{ A} ; I_2 = 50 \text{ A} \quad \therefore I_1/I_2 = 75/50 = 1.5 \\
 R_1 &= 500/75 = 6.667 \, \Omega
 \end{aligned}$$

If  $n$  is the number of stator studs, then

$$\begin{aligned}
 (I_1/I_2)^{n-1} &= R_1/R_a \quad \text{or} \quad 1.5^{n-1} = 6.667/1.31 = 5.09 \\
 \therefore (n-1) \log_{10} 1.5 &= \log_{10} 5.09 \quad \therefore (n-1) \times 0.1761 = 0.7067 \quad \therefore (n-1) = 4 \text{ or } n = 5
 \end{aligned}$$

Hence, there are five studs and four sections.

$$\begin{aligned}
 R_2 &= R_1 \times I_2/I_1 = 6.667 \times 2/3 = 4.44 \, \Omega \\
 R_3 &= 4.44 \times 2/3 = 2.96 \, \Omega ; \quad R_4 = 2.96 \times 2/3 = 1.98 \, \Omega \\
 \text{Resistance of 1st section} &= R_1 - R_2 = 6.67 - 4.44 = \mathbf{2.23 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{2nd} \quad \quad \quad \text{,,} &= R_2 - R_3 = 4.44 - 2.96 = \mathbf{1.48 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{3rd} \quad \quad \quad \text{,,} &= R_3 - R_4 = 2.96 - 1.98 = \mathbf{0.98 \, \Omega} \\
 \quad \quad \quad \text{,,} \quad \quad \quad \text{4th} \quad \quad \quad \text{,,} &= R_4 - R_a = 1.98 - 1.31 = \mathbf{0.67 \, \Omega}
 \end{aligned}$$

### 30.26. Series Motor Starters

The basic principle employed in the design of a starter for series motor is the same as for a shunt motor *i.e.*, the motor current is not allowed to exceed a certain upper limit as the starter arm moves from one stud to another. However, there is one significant difference. In the case of a series motor, the flux does not remain constant but varies with the current because armature current is also the exciting current. The determination of the number of steps is rather complicated as illustrated in Example 30.55. It may however, be noted that the section resistances form a geometrical progression.

The face-plate type of starter formerly used for d.c. series motor has been almost entirely replaced by automatic starter in which the resistance steps are cut out automatically by means of a contactor operated by electromagnets. Such starters are well-suited for remote control.

However, for winch and crane motors where frequent starting, stopping, reversing and speed variations are necessary, drum type controllers are used. They are called controllers because they can be left in the circuit for any length of time. In addition to serving their normal function of starters, they also used as speed controllers.

**Example 30.55.** (a) Show that, in general, individual resistances between the studs for a rheostat starter for a series d.c. motor with constant ratio of maximum to minimum current at starting, are in geometrical progression, stating any assumptions made.

(b) Assuming that for a certain d.c. series motor the flux per pole is proportional to the starting current, calculate the resistance of the each rheostat section in the case of a 50 b.h.p. (37.3 kW) 440-V motor with six sections.

The total armature and field voltage drop at full-load is 2% of the applied voltage, the full-load efficiency is 95% and the maximum starting current is 130% of full-load current.

**Solution.** (a) Let  $I_1$  = maximum current,  $I_2$  = minimum current  
 $\Phi_1$  = flux/pole for  $I_1$ ;  $\Phi_2$  = flux/pole for  $I_2$   
 $\frac{I_1}{I_2} = K$  and  $\frac{\Phi_1}{\Phi_2} = \alpha$ .

Let us now consider the conditions when the starter arm is on the  $n$ th and  $(n + 1)$ th stud. When the current is  $I_2$ , then  $E_b = V - I_2 R_n$ .

If, now, the starter is moved up to the  $(n + 1)$ th stud, then

$$E_b' = \frac{\Phi_1}{\Phi_2} \cdot E_b = \alpha E_b$$

$$\therefore R_{n+1} = \frac{z - r'_{a1}}{c_1} = \frac{z - \alpha r_{a2}}{c_1} = \frac{z - \alpha (z - r_{a2})}{c_1} = \frac{z}{c_1} (1 - \alpha) + \alpha \frac{r_{a2}}{c_1}$$

Now,  $V/I_1 = R_1$ —the total resistance in the circuit when the starter arm is on the first stud.

$$\therefore R_{n+1} = R_1 (1 - \alpha) + \frac{\alpha}{K} R_n$$

Similarly, by substituting  $(n - 1)$  for  $n$ , we get  $R_n = R_1 (1 - \alpha) + \frac{\alpha}{K} R_{n-1}$

Therefore, the resistance between the  $n$ th and  $(n + 1)$ th studs is

$$r_n = R_n - R_{n+1} = \frac{\alpha}{K} R_{n-1} - \frac{\alpha}{K} R_n = \frac{\alpha}{K} (R_{n-1} - R_n) = \frac{\alpha}{K} r_{n-1}$$

$$\therefore \frac{r_n}{r_{n-1}} = \frac{\alpha}{K} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = b \quad \text{— constant}$$

Obviously, the resistance elements form a geometrical progression series.

(b) Full-load input current =  $\frac{37,300}{440 \times 0.95} = 89.2$  A

Max. starting current  $I_1 = 1.3 \times 89.2 = 116$  A

Arm. and field voltage drop on full-load = 2% of 440 =  $0.02 \times 440 = 8.8$  V

Resistance of motor =  $8.8/89.2 = 0.0896 \Omega$

Total circuit resistance on starting,  $R_1 = V/I_1 = 440/116 = 3.79 \Omega$



Assuming straight line magnetisation, we have  $I_1 \propto \Phi_1$  and  $I_2 \propto \Phi_2$

$$\therefore I_1/I_2 = \Phi_1/\Phi_2 \quad \therefore \alpha = K \text{ and } b = \alpha/K = 1 \quad \therefore r_n = b \times r_{n-1}$$

In other words, all sections have the same resistance.

$$\therefore r = \frac{R_1 - R_{motor}}{\text{No. of sections}} = \frac{3.79 - 0.0896}{6} = 0.6176 \Omega$$

**Example 30.56.** A 75 h.p. (55.95 kW) 650-V, d.c. series traction motor has a total resistance of 0.51 Ω. The starting current is to be allowed to fluctuate between 140 A and 100 A the flux at 140 A being 20 % greater than at 100 A. Determine the number of steps required in the controller and the resistance of each step.

**Solution.** Let  $R_1$  = total resistance on the first stud = 650/140 = 4.65 Ω

When motor speeds up, then back e.m.f. is produced and current falls to  $I_2$ .

$$V = E_{b1} + I_2 R_1 \quad \dots(i)$$

When the starter moves to the next stud, the speed is still the same, but since current rises to  $I_1$  for which flux is 1.2 times greater than for  $I_2$ , hence back e.m.f. becomes  $1.2 E_{b1}$ . Since resistance in the circuit is now  $R_2$ .

$$\therefore V = I_1 R_2 + 1.2 E_{b1} \quad \dots(ii)$$

From (i) and (ii) we get,  $0.2 V = 1.2 I_2 R_1 - I_1 R_2$

$$\therefore R_2 = (1.2 I_2 / I_1) R_1 - 0.2 V / I_1 = (1.2 I_2 / I_1) R_1 - 0.2 R_1 = (1.2 I_2 / I_1 - 0.2) R_1$$

$$\text{Similarly } R_3 = (1.2 I_2 / I_1) R_2 - 0.2 R_1$$

In this way, we continue till we reach the value of resistance equal to the armature resistance. Hence, we obtain  $R_1, R_2$  etc. and also the number of steps.

In the present case,  $I_1 = 140$  A,  $I_2 = 100$  A,  $V = 650$  V and  $I_1/I_2 = 100/140 = 1.14$

$$\therefore R_1 = 4.65 \Omega; R_2 = (1.2/1.4 - 0.2) 4.65 = 3.07 \Omega$$

$$R_3 = (1.2/1.4) \times 3.07 - 0.2 \times 4.65 = 1.70 \Omega$$

$$R_4 = (1.2/1.4) \times 1.7 - 0.2 \times 4.65 = 0.53 \Omega$$

We will stop here because  $R_4$  is very near the value of the motor resistance. Hence, there are 4 studs and 3 sections or steps.

$$R_1 - R_2 = 4.65 - 3.07 = 1.58 \Omega, R_2 - R_3 = 3.07 - 1.7 = 1.37 \Omega$$

$$R_3 - R_4 = 1.70 - 0.53 = 1.17 \Omega$$

**Note.** It will be seen that

$$R_2 - R_3 = (1.2 I_2 / I_1) (R_1 - R_2) \text{ and } R_3 - R_4 = (1.2 I_2 / I_1) (R_2 - R_3) \text{ and so on.}$$

or 
$$\frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} = 1.2 \frac{I_2}{I_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = \frac{\alpha}{K} = b$$

It is seen that individual resistances of various sections decrease in the ratio of  $\alpha/K = b$ .

### 30.27. Thyristor Controller Starters

The moving parts and metal contacts etc., of the resistance starters discussed in Art. 30.21 can be eliminated by using thyristors which can short circuit the resistance sections one after another. A thyristor can be switched on to the conducting state by applying a suitable signal to its gate terminal. While conducting, it offers zero resistance in the forward (i.e., anode-to-cathode) direction and thus acts as a short-circuit for the starter resistance section across which it is connected. It can be switched off (i.e., brought back to the non-conducting state) by reversing the polarity of its anode-cathode voltage. A typical thyristor-controlled starter for d.c. motors is shown in Fig. 30.49.

After switching on the main supply, when switch  $S_1$  is pressed, positive signal is applied to gate G of thyristor  $T_1$  which is, therefore, turned ON. At the same time, shunt field gets established since it is directly connected across the d.c. supply. Consequently, motor armature current  $I_a$  flows via  $T_1$ ,

$R_2$ ,  $R_3$  and  $R_4$  because  $T_2$ ,  $T_3$  and  $T_4$  are, as yet in the non-conducting state. From now onwards, the starting procedure is automatic as detailed below :

1. As  $S_1$  is closed, capacitor  $C$  starts charging up with the polarity as shown when  $I_a$  starts flowing.
2. The armature current and field flux together produce torque which accelerates the motor and load.

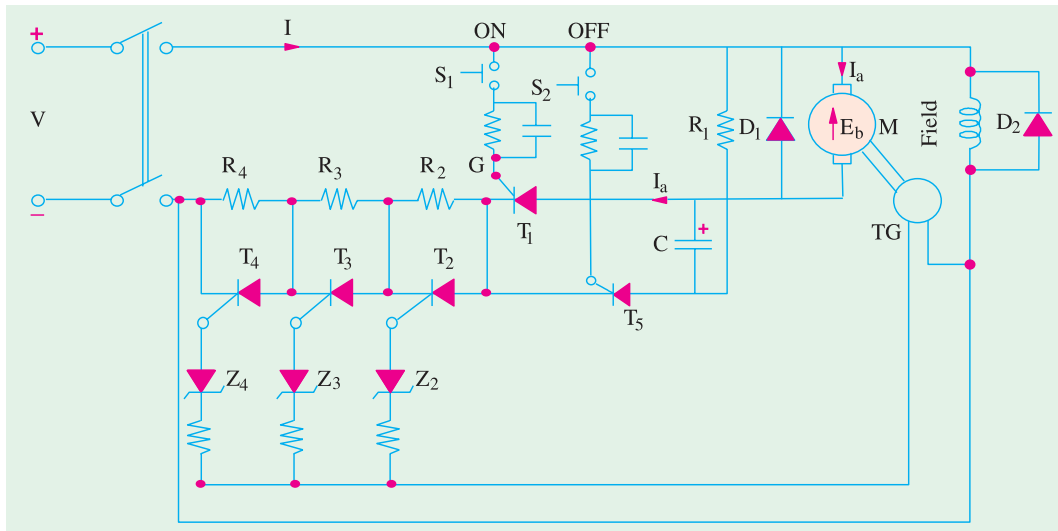


Fig. 30.49

3. As motor speeds up, voltage provided by tachogenerator ( $TG$ ) is proportionately increased because it is coupled to the motor.
4. At some motor speed, the voltage provided by  $TG$  becomes large enough to breakdown Zener diode  $Z_2$  and hence trigger  $T_2$  into conduction. Consequently,  $R_2$  is shorted out and now  $I_a$  flows via motor armature,  $T_1$ ,  $T_2$ ,  $R_3$  and  $R_4$  and back to the negative supply terminal.
5. As  $R_2$  is cut out,  $I_a$  increases, armature torque increases, motor speed increases which further increases the voltage output of the tachogenerator. At some speed,  $Z_3$  breaks down, thereby triggering  $T_3$  into conduction which cuts out  $R_3$ .
6. After sometime,  $R_4$  is cut out as  $Z_4$  breaks down and triggers  $T_4$  into conduction. In fact, Zener diodes  $Z_2$ ,  $Z_3$  and  $Z_4$  can be rated for  $1/3$ ,  $1/2$  and  $3/4$  full speed respectively.

For stopping the motor, switch  $S_2$  is closed which triggers  $T_5$  into conduction, thereby establishing current flow via  $R_1$ . Consequently, capacitor  $C$  starts discharging thereby reverse-biasing  $T_1$  which stops conducting. Hence  $I_a$  ceases and, at the same time,  $T_2$ ,  $T_3$  and  $T_4$  also revert back to their non-conducting state.

Incidentally, it may be noted that the function of  $C$  is to switch  $T_1$ , ON and OFF. Hence, it is usually called **commutating capacitor**.

The function of the diodes  $D_1$  and  $D_2$  is to allow the decay of inductive energy stored in the motor armature and field when supply is disconnected. Supply failure will cause the thyristors to block because of this current decay, thereby providing protection usually given by no-voltage release coil.

Recently, thyristor starting circuits have been introduced which use no starting resistance at all, thereby making the entire system quite efficient and optimized as regards starting time. These are based on the principle of 'voltage chopping' (Art. 30.12). By varying the chopping frequency, the ratio of the time the voltage is ON to the time it is OFF can be varied. By varying this ratio, the average voltage applied to the motor can be changed. A low average voltage is needed to limit the

armature current while the motor is being started and gradually the ratio is increased to reach the maximum at the rated speed of the motor.

### Tutorial Problems 32.2

1. A shunt-wound motor runs at 600 r.p.m. from a 230-V supply when taking a line current of 50 A. Its armature and field resistances are  $0.4 \Omega$  and  $104.5 \Omega$  respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (a) the no-load speed if the no-load line current is 5A (b) the resistance to be placed in armature circuit in order to reduce the speed to 500 r.p.m. when motor is taking a line current of 50 A (c) the percentage reduction in the flux per pole in order that the speed may be 750 r.p.m. when the armature current is 30 A with no added resistance in the armature circuit. [(a) 652 r.p.m. (b)  $0.73 \Omega$  (c) 1.73 %]
2. The resistance of the armature of a 250-V shunt motor is  $0.3 \Omega$  and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run? Neglect armature reaction. [ $0.94 \Omega$ ; 932 r.p.m.]
3. A 230-V d.c. shunt motor takes an armature current of 20 A on a certain load. Resistance of the armature is  $0.5 \Omega$ . Find the resistance required in series with the armature to half the speed if (a) the load torque is constant (b) the load torque is proportional to the square of the speed. [(a)  $5.5 \Omega$  (b)  $23.5 \Omega$ ]
4. A 230-V series motor runs at 1200 r.p.m. at a quarter full-load torque, taking a current of 16 A. Calculate its speed at half and full-load torques. The resistance of the armature brushes, and field coils is  $0.25 \Omega$ . Assume the flux per pole to be proportional to the current. Plot torque/speed graph between full and quarter-load. [842 r.p.m. ; 589 r.p.m.]
5. A d.c. series motor drives a load the torque of which is proportional to the square of the speed. The motor current is 20 A when speed is 500 r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistance of the same value as the field winding. Neglect all motor losses and assume that the magnetic field is unsaturated. [595 r.p.m. ; 33.64 A]  
(Electrical Machines-I, Aligarh Muslim Univ. 1979)
6. A d.c. series motor, with unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load takes 50 A at 500 V. If the load torque varies as the cube of the speed, find the resistance which should be connected in series with machine to reduce the speed by 25 per cent. [7.89  $\Omega$ ]  
(Electrical Engg-I, M.S. Univ. Baroda 1980)
7. A series motor runs at 500 r.p.m. on a certain load. Calculate the resistance of a diverter required to raise the speed to 650 r.p.m. with the same load current, given that the series field resistance is  $0.05 \Omega$  and the field is unsaturated. Assume the ohmic drop in the field and armature to be negligible. [0.1665  $\Omega$ ]
8. A 230-V d.c. series motor has armature and field resistances of  $0.5 \Omega$  and  $0.3 \Omega$  respectively. The motor draws a line current of 40 A while running at 400 r.p.m. If a diverter of resistance  $0.15 \Omega$  is used, find the new speed of the motor for the same armature current.  
It may be assumed that flux per pole is directly proportional to the field current. [1204 r.p.m.]  
(Electrical Engineering Grad. I.E.T.E. June 1986)
9. A 250-V, d.c. shunt motor runs at 700 r.p.m. on no-load with no extra resistance in the field and armature circuit. Determine :
  - (i) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 A.
  - (ii) the resistance to be placed in series with the field to produce a speed of 1,000 r.p.m. when taking an armature current of 18 A.

Assume that the useful flux is proportional to the field. Armature resistance =  $0.35 \Omega$ , field resistance =  $125 \Omega$ . [(i)  $7.3 \Omega$  (ii)  $113 \Omega$ ] (Elect. Engg. Grad. I.E.T.E., June 1984)

10. A d.c. series motor is operating from a 220-V supply. It takes 50 A and runs at 1000 r.p.m. The resistance of the motor is  $0.1 \Omega$ . If a resistance of  $2 \Omega$  is placed in series with the motor, calculate the resultant speed if the load torque is constant. [534 r.p.m.]
11. A d.c. shunt motor takes 25 A when running at 1000 r.p.m. from a 220-V supply. Calculate the current taken from the supply and the speed if the load torque is halved, a resistance of  $5 \Omega$  is placed in the armature circuit and a resistance of  $50 \Omega$  is placed in the field circuit. Armature resistance =  $0.1 \Omega$ ; field resistance =  $100 \Omega$ . Assume that the field flux per pole is directly proportional to the field current. [17.1 A; 915 r.p.m.] (Elect. Technology, Gwalior Univ. Nov. 1977)
12. A 440-V shunt motor takes an armature current of 50 A and has a flux/pole of 50 mWb. If the flux is suddenly decreased to 45 mWb, calculate (a) instantaneous increase in armature current (b) percentage increase in the motor torque due to increase in current (c) value of steady current which motor will take eventually (d) the final percentage increase in motor speed. Neglect brush contact drop and armature reaction and assume an armature resistance of  $0.6 \Omega$ . [(a)  $118 \text{ A}$  (b)  $112 \%$  (c)  $5.55 \text{ A}$  (d)  $10\%$ ]
13. A 440-V shunt motor while running at 1500 r.p.m. takes an armature current of 30 A and delivers a mechanical output of 15 h.p. (11.19 kW). The load torque varies as the square of the speed. Calculate the value of resistance to be connected in series with the armature for reducing the motor speed to 1300 r.p.m. and the armature current at that speed. [2.97  $\Omega$ , 22.5 A]
14. A 460-V series motor has a resistance of  $0.4 \Omega$  and takes a current of 25 A when there is no additional controller resistance in the armature circuit. Its speed is 1000 r.p.m. The control resistance is so adjusted as to reduce the field flux by 5%. Calculate the new current drawn by the motor and its speed. Assume that the load torque varies as the square of the speed and the same motor efficiency under the two conditions of operation. [22.6 A; 926 r.p.m.] (Elect. Machines, South Gujarat Univ. Oct. 1977)
15. A 460-V, series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change and torque if the load is reduced so that the motor is taking 30 A. Total resistance of armature and field circuit is  $0.8 \Omega$ . Assume flux proportional to the field current. [680 r.p.m. 43.75%]
16. A 440-V, 25 h.p (18.65 kW) motor has an armature resistance of  $1.2 \Omega$  and full-load efficiency of 85%. Calculate the number and value of resistance elements of a starter for the motor if maximum permissible current is 1.5 times the full-load current. [1.92  $\Omega$ , 1.30  $\Omega$ , 0.86  $\Omega$ ; 0.59  $\Omega$ ] (Similar example in JNTU, Hyderabad, 2000)
17. A 230-V, d.c. shunt motor has an armature resistance of  $0.3 \Omega$ . Calculate (a) the resistance to be connected in series with the armature to limit the armature current to 75 A at starting and (b) value of the generated e.m.f. when the armature current has fallen to 50 A with this value of resistance still in circuit. [(a)  $2.767 \Omega$  (b)  $76.7 \text{ A}$ ]
18. A 200-V, d.c. shunt motor takes full-load current of 12 A. The armature circuit resistance is  $0.3 \Omega$  and the field circuit resistance is  $100 \Omega$ . Calculate the value of 5 steps in the 6-stud starter for the motor. The maximum starting current is not to exceed 1.5 times the full-load current. [6.57  $\Omega$ , 3.12  $\Omega$ , 1.48  $\Omega$ , 0.7  $\Omega$ , 0.33  $\Omega$ ]
19. The resistance of a starter for a 200-V, shunt motor is such that maximum starting current is 30 A. When the current has decreased to 24 A, the starter arm is moved from the first to the second stud. Calculate the resistance between these two studs if the maximum current in the second stud is 34 A. The armature resistance of the motor is  $0.4 \Omega$ . [1.334  $\Omega$ ]
20. A totally-enclosed motor has thermal time constant of 2 hr. and final temperature rise at no-load and  $40^\circ$  on full load.

Determine the limits between which the temperature fluctuates when the motor operates on a load cycle consisting of alternate period of 1 hr. on full-load and 1 hr. on no-load, steady state conditions having been established. [28.7° C, 21.3° C]

21. A motor with a thermal time constant of 45 min. has a final temperature rise of 75°C on continuous rating (a) What is the temperature rise after one hour at this load ? (b) If the temperature rise on one-hour rating is 75°C, find the maximum steady temperature at this rating (c) When working at its one-hour rating, how long does it take the temperature to increase from 60°C to 75°C ? [(a) 55 °C (b) 102 °C (c) 20 min]

(Electrical Technology, M.S. Univ. Baroda. 1976)

### OBJECTIVE TESTS – 30

- The speed of a d.c. motor can be controlled by varying
  - its flux per pole
  - resistance of armature circuit
  - applied voltage
  - all of the above
- The most efficient method of increasing the speed of a 3.75 kW d.c. shunt motor would be the .....method.
  - armature control
  - flux control
  - Ward-Leonard
  - tapped-field control
- Regarding Ward-Leonard system of speed control which statement is **false** ?
  - It is usually used where wide and very sensitive speed control is required.
  - It is used for motors having ratings from 750 kW to 4000 kW
  - Capital outlay involved in the system is right since it uses two extra machines.
  - It gives a speed range of 10 : 1 but in one direction only.
  - It has low overall efficiency especially at light loads.
- In the rheostatic method of speed control for a d.c. shunt motor, use of armature diverter makes the method
  - less wasteful
  - less expensive
  - unsuitable for changing loads
  - suitable for rapidly changing loads
- The chief advantage of Ward-Leonard system of d.c. motor speed control is that it
  - can be used even for small motors
  - has high overall efficiency at all speeds
  - gives smooth, sensitive and wide speed control
  - uses a flywheel to reduce fluctuations in power demand
- The flux control method using paralleling of field coils when applied to a 4-pole series d.c. motor can give ..... speeds.
  - 2
  - 3
  - 4
  - 6
- The series-parallel system of speed control of series motors widely used in traction work gives a speed range of about
  - 1 : 2
  - 1 : 3
  - 1 : 4
  - 1 : 6
- In practice, regenerative braking is used when
  - quick motor reversal is desired
  - load has overhauling characteristics
  - controlling elevators, rolling mills and printing presses etc.
  - other methods can not be used.
- Statement 1.** A direct-on-line (DOL) starter is used to start a small d.c. motor because **Statement 2.** it limits initial current drawn by the armature circuit.
  - both statement 1 and 2 are incorrect
  - both statement 1 and 2 are correct
  - statement 1 is correct but 2 is wrong
  - statement 2 is correct but 1 is wrong
- Ward-Leonard system of speed control is **NOT** recommended for
  - wide speed range
  - constant-speed operation
  - frequent motor reversals
  - very low speeds
- Thyristor chopper circuits are employed for
  - lowering the level of a d.c. voltage
  - rectifying the a.c. voltage
  - frequency conversion
  - providing commutation circuitry

- 12.** An inverter circuit is employed to convert  
 (a) a.c. voltage into d.c. voltage  
 (b) d.c. voltage into a.c. voltage  
 (c) high frequency into low frequency  
 (d) low frequency into high frequency
- 13.** The phase-control rectifiers used for speed of d.c. motors convert fixed a.c. supply voltage into  
 (a) variable d.c. supply voltage  
 (b) variable a.c. supply voltage  
 (c) full-rectified a.c. voltage  
 (d) half-rectified a.c. voltage
- 14.** If some of the switching devices in a convertor are controlled devices and some are diodes, the convertor is called  
 (a) full convertor    (b) semiconvertor  
 (c) solid-state chopper  
 (d) d.c. convertor
- 15.** A solid-state chopper converts a fixed-voltage d.c. supply into a  
 (a) variable-voltage a.c. supply  
 (b) variable-voltage d.c. supply  
 (c) higher-voltage d.c. supply  
 (d) lower-voltage a.c. supply
- 16.** The d.c. motor terminal voltage supplied by a solid-state chopper for speed control purposes varies.....with the duty ratio of the chopper  
 (a) inversely      (b) indirectly  
 (c) linearly      (d) parabolically

**ANSWERS**

- 1. d    2. b    3. d    4. d    5. c    6. b    7. c    8. b    9. c    10. b    11. a**  
**12. b    13. a    14. b    15. b    16. c**

# CHAPTER 31

## Learning Objectives

- Brake Test
- Swinburne's Test
- Advantages of Swinburne's Test
- Main Disadvantages
- Regenerative or Hopkinson's Test
- Alternative Connections for Hopkinson's Test
- Merits of Hopkinson's Test
- Retardation or Running Down Test
- Field's Test for Series Motors
- Objective Test
- Questions and Answers on D.C. Motors.

## TESTING OF D.C. MACHINES



Testing is performed on D.C. machines to determine efficiency and power losses

### 31.1 Brake Test

It is a direct method and consists of applying a brake to a water-cooled pulley mounted on the motor shaft as shown in Fig. 31.1. The brake band is fixed with the help of wooden blocks gripping the pulley. One end of the band is fixed to earth via a spring balance  $S$  and the other is connected to a suspended weight  $W_1$ . The motor is running and the load on the motor is adjusted till it carries its full load current.

Let  $W_1$  = suspended weight in kg  
 $W_2$  = reading on spring balance in kg-wt

The net pull on the band due to friction at the pulley is  $(W_1 - W_2)$  kg. wt. or  $9.81 (W_1 - W_2)$  newton.

If  $R$  = radius of the pulley in metre  
 and  $N$  = motor or pulley speed in r.p.s.

Then, shaft torque  $T_{sh}$  developed by the motor  
 $= (W_1 - W_2) R$  kg-m =  $9.81 (W_1 - W_2) R$  N-m

Motor output power =  $T_{sh} \times 2\pi N$  watt  
 $= 2\pi \times 9.81 N (W_1 - W_2) R$  watt  
 $= 61.68 N (W_1 - W_2) R$  watt

Let  $V$  = supply voltage ;  $I$  = full-load current taken by the motor.  
 Then, input power =  $VI$  watt

$$\therefore \eta = \frac{\text{Output}}{\text{Input}} = \frac{61.68 N(W_1 - W_2)R}{VI}$$

The simple brake test described above can be used for small motors only, because in the case of large motors, it is difficult to dissipate the large amount of heat generated at the brake.

Another simple method of measuring motor output is by the use of poney brake one form of which is shown in Fig. 31.2 (a). A rope is wound round the pulley and its two ends are attached to two spring balances  $S_1$  and  $S_2$ . The tension of the rope can be adjusted with the help of swivels. Obviously, the force acting tangentially on the pulley is equal to the difference between the readings of the two spring balances. If  $R$  is the pulley radius, the torque at the pulley is  $T_{sh} = (S_1 - S_2)R$ . If  $\omega (= 2\pi N)$  is the angular velocity of the pulley, then

motor output =  $T_{sh} \times \omega = 2\pi N (S_1 - S_2)R$  m-kg. wt. =  $9.81 \times 2\pi N (S_1 - S_2) R$  watt.

The motor input may be measured as shown in Fig. 31.2 (b). Efficiency may, as usual, be found by using the relation  $\eta = \text{output/input}$ .

**Example 31.1.** In a brake test the effective load on the branch pulley was 38.1 kg, the effective diameter of the pulley 63.5 cm and speed 12 r.p.s. The motor took 49 A at 220 V. Calculate the output power and the efficiency at this load.

**Solution.** Effective load  $(W_1 - W_2) = 38.1$  kg. wt ; radius =  $0.635/2 = 0.3175$  m

Shaft torque =  $38.1 \times 0.3175$  kg-m =  $9.81 \times 38.1 \times 0.3175 = 118.6$  N-m

Power output = torque  $\times$  angular velocity in rad/s =  $118.6 \times 2\pi \times 12 = 8,945$  W

Now, motor input =  $49 \times 220$  W  $\therefore$  Motor  $\eta = \frac{8,945}{49 \times 220} = 0.83$  or **83%**

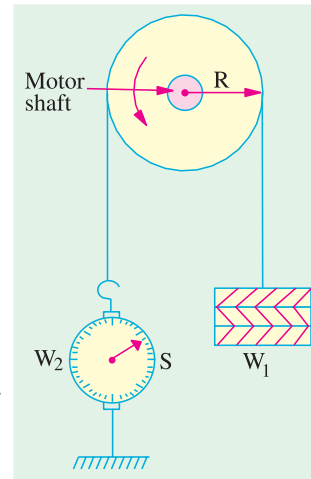


Fig. 31.1



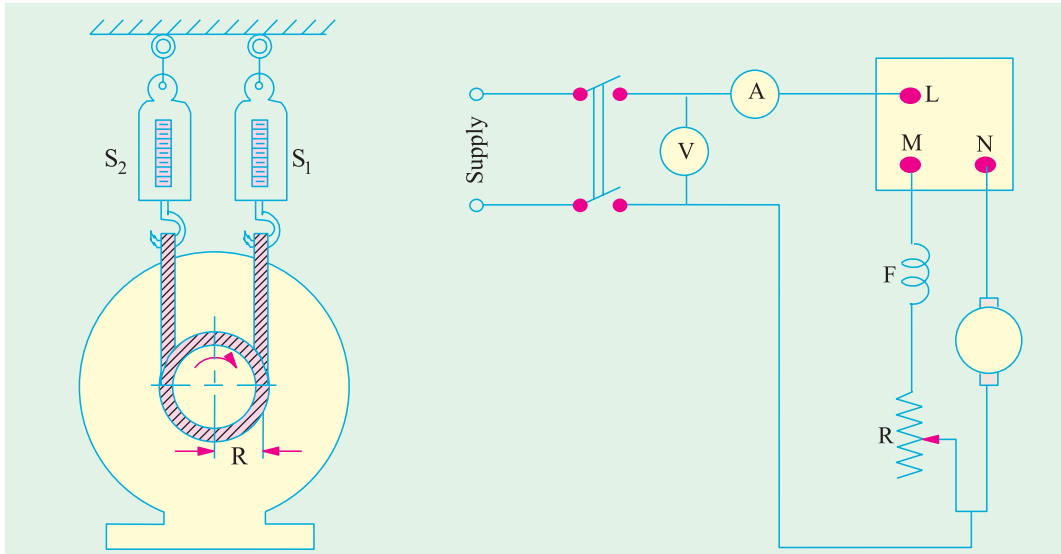


Fig. 31.2 (a)

Fig. 31.2 (b)

**Example 31.2(a).** The following readings are obtained when doing a load test on a d.c. shunt motor using a brake drum :

Spring balance reading	10 kg and 35 kg	Diameter of the drum	40 cm
Speed of the motor	950 r.p.m.	Applied voltage	200 V
Line current	30 A		

Calculate the output power and the efficiency. (Electrical Engineering, Madras Univ. 1986)

**Solution.** Force on the drum surface  $F = (35 - 10) = 25 \text{ kg wt} = 25 \times 9.8 \text{ N}$

Drum radius  $R = 20 \text{ cm} = 0.2 \text{ m}$  ; Torque  $T_{sh} = F \times R = 25 \times 9.8 \times 0.2 = 49 \text{ N}$

$N = 950/60 = 95/6 \text{ r.p.s.}$  ;  $\omega = 2\pi (95/6) = 99.5 \text{ rad/s}$

Motor output =  $T_{sh} \times \omega \text{ watt} = 49 \times 99.5 = 4,876 \text{ W}$

Motor input =  $200 \times 30 = 6000 \text{ W}$  ;  $\eta = 4876/6000 = 0.813$  or **81.3%**

**Example 31.2(b).** In a brake-test, on a d.c. shunt motor, the tensions on the two sides of the brake were 2.9 kg and 0.17 kg. Radius of the pulley was 7 cm. Input current was 2 amp at 230 volts. The motor speed was 1500 rpm. Find the torque, power-output and efficiency.

(Bharathiar Univ. April 1998)

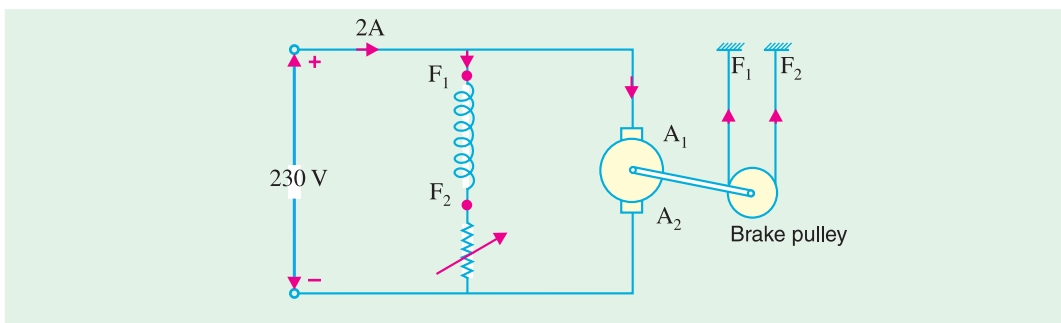


Fig. 31.3. D.C. Shunt Motor Brake Test

**Solution.** Net force on pulley =  $2.90 - 0.17 = 2.73 \text{ kg}$   
 =  $2.73 \times 9.81 = 26.78 \text{ Nw}$

$$\begin{aligned} \text{Net torque} &= \text{Force} \times \text{Radius} = 26.78 \times 7/100 \\ &= 1.8746 \text{ Nw-m} \\ \text{Power output} &= \text{Torque} \times \text{Radians/sec.} \\ &= 1.8746 \times 2\pi \times 1500/60 \\ &= 294 \text{ watts} \\ \text{Efficiency} &= 294/(230 \times 2) = 0.639 \\ \% \text{ efficiency} &= 63.9\% \end{aligned}$$

### 31.2. Swinburne's\* Test (or No-load Test or Losses Method)

It is a simple method in which losses are measured separately and from their knowledge, efficiency at any desired load can be predetermined in advance. The only running test needed is no-load test. *However, this test is applicable to those machines in which flux is practically constant i.e. shunt and compound-wound machines.*



Shunt regulator

The machine is running as a motor on no-load at its rated voltage *i.e.* voltage stamped on the name-plate. The speed is adjusted to the rated speed with the help of shunt regulator as shown in Fig. 31.4.

The no-load current  $I_0$  is measured by the ammeter  $A_1$  whereas shunt field current  $I_{sh}$  is given by ammeter  $A_2$ . The no-load armature current is  $(I_0 - I_{sh})$  or  $I_{a0}$ .

$$\begin{aligned} \text{Let, supply voltage} &= V & \text{no-load input} &= VI_0 \text{ watt} \\ \therefore \text{Power input to armature} &= V(I_0 - I_{sh}); & \text{Power input to shunt} &= VI_{sh} \end{aligned}$$

No-load power input to armature supplies the following :

- (i) Iron losses in core
- (ii) friction loss
- (iii) windage loss and
- (iv) armature Cu loss,  $(I_0 - I_{sh})^2 R_a$  or  $I_{a0}^2 R_a$

In calculating armature Cu loss, 'hot' resistance of armature should be used. A stationary measurement of armature circuit resistance at the room-temperature of, say, 15°C is made by passing current through the armature from a low voltage d.c. supply [Fig. 31.5 (a)].

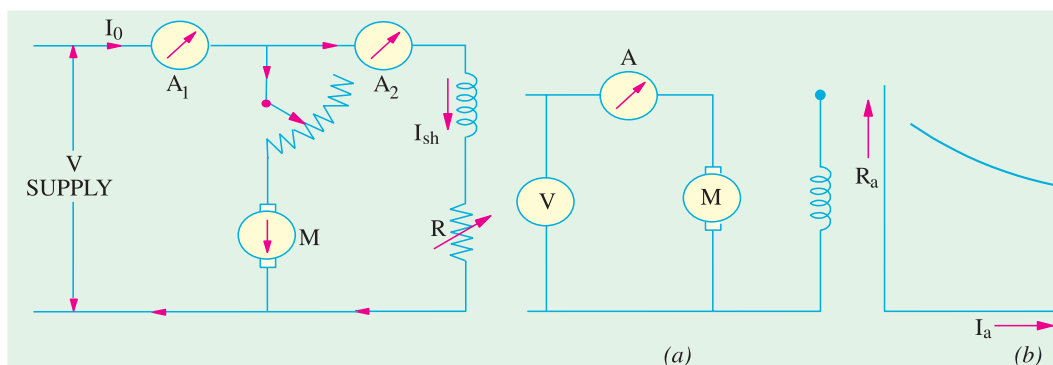


Fig. 31.4

Fig. 31.5

\* Sir James Swinburne (1858-1958) made outstanding contributions to the development of electric lamps, electric machines and synthetic resins.

Then, the 'hot' resistance, allowing a temperature rise of 50°C is found thus :

$$R_{15} = R_0(1 + 15\alpha_0) ; R_{65} = (1 + 65\alpha_0), R_{65} = R_{15} \times \frac{1 + 65\alpha_0}{1 + 15\alpha_0}$$

Taking  $\alpha_0 = 1/234.5$ , we have  $R_{65} = R_{15} \times \frac{234.5 + 65}{234.5 + 15} = 1.2 R_{15}$  (approx. \*)

If we subtract from the total input the no-load armature Cu loss, then we get constant losses.

$$\therefore \text{Constant losses} \quad W_c = VI_0 - (I_0 - I_{sh})^2 R_a$$

Knowing the constant losses of the machine, its efficiency at any other load can be determined as given below. Let  $I$  = load current at which efficiency is required.

Then, armature current is  $I_a = I - I_{sh}$  ...if machine is motoring  
 $= I + I_{sh}$  ...if machine is generating

### Efficiency when running as a motor

Input =  $VI$ , Armature Cu loss =  $I_a^2 R_a = (I - I_{sh})^2 R_a$  ...found above  
 Constant losses =  $W_c$

$$\therefore \text{Total losses} = (I - I_{sh})^2 R_a + W_c ; \eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{VI - (I - I_{sh})^2 R_a - W_c}{VI}$$

### Efficiency when running as a generator

Output =  $VI$ ; Armature, Cu loss =  $(I + I_{sh})^2 R_a$ ; Constant loss =  $W_c$  ...found above

$$\therefore \text{Total losses} = (I + I_{sh})^2 R_a + W_c ; \eta_g = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + W_c}$$

### 31.3. Advantages of Swinburne's Test

1. It is convenient and economical because power required to test a large machine is small *i.e.* only no-load input power.
2. The efficiency can be predetermined at any load because constant-losses are known.

### 31.4. Main Disadvantages

1. No account is taken of the change in iron losses from no-load to full-load. At full-load, due to armature reaction, flux is distorted which increases the iron losses in some cases by as much as 50%.
2. As the test is on no-load, it is impossible to know whether commutation would be satisfactory at full-load and whether the temperature rise would be within the specified limits.

**Example 31.3.** A 220 V, d.c. shunt motor at no load takes a current of 2.5 A. The resistances of the armature and shunt field are 0.8 Ω and 200 Ω respectively. Estimate the efficiency of the motor when the input current is 20 A. State precisely the assumptions made.

(Electrical Technology, Kerala Univ. 1986)

**Solution.** No-load input =  $220 \times 2.5 = 550$  W

This input meets all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

$I_{sh} = 220/200 = 1.1$  A. No-load arm current,  $I_{a0} = 2.5 - 1.1 = 1.4$  A

\* The armature resistance is found to decrease slightly with increasing armature current as shown in Fig. 31.5 (b). This is due to the fact that brush contact resistance is inversely proportional to the armature current.

$$\text{No-load armature Cu loss} = I_{a0}^2 R_a = 1.4^2 \times 0.8 = 1.6 \text{ W}$$

$$\text{Constant losses} = 550 - 1.6 = 548.4 \text{ W}$$

**When input current is 20 A**

$$I_a = 32 - 1.1 = 30.9 \text{ A ; Armature Cu loss} = 30.9^2 \times 0.8 = 764 \text{ W}$$

$$\text{Total loss} = 764 + 548.4 = 1312 \text{ W (approx.) ; Input} = 220 \times 20 = 4,400 \text{ W}$$

$$\text{Output} = 4,400 - 1,312 = 3,088 \text{ W ; Efficiency} = (3088/4400) \times 100 = \mathbf{70.2\%}$$

In the above calculations, it has been assumed that :

**1.** mechanical losses remain constant even through motor speed changes from no-load to the given load.

**2.** effect of armature reaction on main pole flux with a consequent change in iron losses has been neglected.

**3.** decrease in flux due to increase in shunt resistance by heating has been neglected.

**Example 31.4.** When running on no-load, a 400-V shunt motor takes 5 A. Armature resistance is 0.5  $\Omega$  and field resistance 200  $\Omega$ . Find the output of the motor and efficiency when running on full-load and taking a current of 50 A. Also, find the percentage change in speed from no-load to full-load. **(Electro Mechanics, Allahabad Univ. 1991)**

**Solution.** No-load input =  $400 \times 5 = 2,000 \text{ W}$

This input goes to meet all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

$$I_{sh} = 400/200 = 2 \text{ A ; No-load } I_a = 5 - 2 = 3 \text{ A}$$

$$\text{No-load arm. Cu loss} = 3^2 \times 0.5 = 4.5 \text{ W ; Constant losses} = 2,000 - 4.5 = \mathbf{1,995.5 \text{ W}}$$

**When line current is 50 A**

$$I_a = 50 - 2 = 48 \text{ A ; Arm. Cu loss} = 48^2 \times 0.5 = 1,152 \text{ W}$$

$$\text{Total loss on F.L.} = 1,152 + 1,995.5 = 3,147.5 \text{ W ; Input} = 50 \times 400 = 20,000 \text{ W}$$

$$\text{Output} = 20,000 - 3,147.5 = \mathbf{16,852.5 \text{ W} = 16.8 \text{ kW}}$$

$$\text{F.L. efficiency} = 16,852.5/20,000 = 0.8426 \text{ or } \mathbf{84.26\%}$$

$$\text{Now, } E_{b1} = 400 - (3 \times 0.5) = 398.5 \text{ V ; } E_{b2} = 400 - (48 \times 0.5) = 376 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} = \frac{398.5}{376} \quad \therefore \frac{N_1 - N_2}{N_2} = \frac{22.5}{376} = 0.0598$$

$\therefore$  percentage change in speed = **5.98**

**Example 31.5.** The no-load test of a 44.76 kW, 220-V, d.c. shunt motor gave the following figures:

Input current = 13.25 A ; field current = 2.55 A ; resistance of armature at 75°C = 0.032  $\Omega$  and brush drop = 2 V. Estimate the full-load current and efficiency.

**(Electrical Engineering, Madras Univ. 1987)**

**Solution. No-load Condition**

$$\text{No-load input} = 220 \times 13.25 = 2915 \text{ W ; Armature current} = 13.25 - 2.55 = 10.7 \text{ A}$$

$$\text{Armature Cu loss} = 10.7^2 \times 0.032 = 3.6 \text{ W}$$

$$\text{Loss due to brush drop} = 2 \times 10.7 = 21.4 \text{ W}$$

$$\text{Variable loss} = 21.4 + 3.6 = 25 \text{ W}, \text{ Constant losses } W_c = 2915 - 25 = 2890 \text{ W}$$

**Full-load Condition**

If  $I_a$  is the full-load armature current, then full-load motor input current is  $(I_a + 2.55) \text{ A}$ .

$$\text{F.L. motor power input} = 220 (I_a + 2.55) \text{ W}$$

This input must be equal to the sum of

- (i) output = 44.76 kW = 44,760 W
- (ii) brush loss =  $2I_a$  watt
- (iii)  $W_c = 2,890 \text{ W}$
- (iv) Arm. Cu loss =  $0.032 I_a^2$

$$\begin{aligned} \therefore 220(I_a + 2.55) &= 44,750 + 2,890 + 2I_a + 0.032I_a^2 \\ \text{or } 0.032I_a^2 - 218I_a + 47,090 &= 0 \\ \text{or } I_a &= \frac{218 \pm \sqrt{218^2 - 4 \times 0.032 \times 47,090}}{2 \times 0.032} = 223.5 \text{ A} \\ \text{Line input current } I &= I_a + I_{sh} = 223.5 + 2.55 = \mathbf{226 \text{ A}} \\ \text{F.L. power input} &= 226 \times 220 = 49,720 \text{ W} \\ \therefore \text{F.L. efficiency} &= 44,760/49,720 = 0.9 \text{ or } \mathbf{90\%}. \end{aligned}$$

**Example 31.6.** A 200-V, shunt motor develops an output of 17.158 kW when taking 20.2 kW. The field resistance is 50 Ω and armature resistance 0.06 Ω. What is the efficiency and power input when the output is 7.46 kW? **(Elect. Machines-I, Aligarh Muslim Univ. 1989)**

**Solution. In the first case :**

$$\begin{aligned} \text{Output} &= 17,158 \text{ W} & \text{Input} &= 20,200 \text{ W} \\ \text{Total losses} &= 20,200 - 17,158 = 3,042 \text{ W} ; \text{ Input current} &= 20,200/200 = 101 \text{ A} \\ I_{sh} &= 200/50 = 4 \text{ A} ; I_a = 101 - 4 = 97 \text{ A} \\ \therefore \text{Armature Cu loss} &= 97^2 \times 0.06 = 564.5 \text{ W} \\ \therefore \text{Constant losses} &= 3,042 - 564.5 = 2,477.5 = 2478 \text{ W (approx.)} \end{aligned}$$

**In the second case :**

$$\begin{aligned} \text{Let, } I_a &= \text{armature current} & \text{Input current} &= (I_a + 4) \text{ A} \\ \text{Now, input power} &= \text{output} + I_a^2 R_a + \text{constant losses} \\ \therefore 200(I_a + 4) &= 7,460 + 0.06 I_a^2 + 2,478 \\ \text{or } 0.06I_a^2 - 200I_a + 9,138 &= 0 \\ \therefore I_a &= \frac{200 \pm \sqrt{200^2 - 4 \times 0.06 \times 9,138}}{2 \times 0.06} = \frac{200 \pm 194}{0.12} = 3,283.3 \text{ A or } 46 \text{ A} \end{aligned}$$

We will reject the larger value because it corresponds to unstable operation of the motor. Hence, take  $I_a = 46 \text{ A}$ .

$$\begin{aligned} \therefore \text{Input current } I &= I_a + I_{sh} = 46 + 4 = 50 \text{ A} \\ \text{Power input} &= \frac{50 \times 200}{1000} = \mathbf{10 \text{ kW}} \quad \therefore \eta = \frac{7,460 \times 100}{10,000} = \mathbf{74.6\%} \end{aligned}$$

**Example 31.7.** A 200-V, 14.92 kW dc shunt motor when tested by the Swinburne method gave the following results :

*Running light : armature current was 6.5 A and field current 2.2 A. With the armature locked, the current was 70 A when a potential difference of 3 V was applied to the brushes. Estimate the efficiency of the motor when working under full-load conditions.*

**(Electrical Engg.-I, Bombay Univ. 1985)**

**Solution.** No-load input current = 6.5 + 2.2 = 8.7 A

$$\text{No-load power input} = 200 \times 8.7 = 1,740 \text{ W}$$

No-load input equals Cu losses and stray losses.

$$\text{Field Cu loss} = 200 \times 2.2 = 440 \text{ W}$$

$$\text{Armature Cu loss} = 6.5^2 \times 0.04286 = 1.8 \text{ W} \quad (\because R_a = 3/70 = 0.04286 \Omega)$$

$$\therefore \text{Constant losses} = 1,740 - 1.8 = 1738 \text{ W}$$

We will assume that constant losses are the same at full-load also.

$$\text{Let, } I_a = \text{full-load armature current}$$

$$\begin{aligned} \text{F.L. armature Cu loss} &= 0.04286 I_a^2 \text{ W ; Constant losses} = 1,738 \text{ W} \\ \text{F.L. total loss} &= 1,738 + 0.04286 I_a^2 \\ \text{F.L. output} &= 14,920 \text{ W ; F.L. input} = 200 (I_a + 2.2) \text{ W} \\ \text{We know,} & \quad \text{input} = \text{output} + \text{losses} \\ \text{or} & \quad 200 I_a + 440 = 14,920 + 1,738 + 0.04286 I_a^2 \\ \text{or} & \quad 0.04286 I_a^2 - 200 I_a + 16,218 = 0 \quad \therefore I_a = 82.5 \text{ A} \\ \therefore & \quad \text{Input current} = 82.5 + 2.2 = 84.7 \text{ A} \\ & \quad \text{F.L. power input} = 200 \times 84.7 \text{ A} = \mathbf{16,940 \text{ W}} \\ \therefore & \quad \eta = 14,920 \times 100 / 16,940 = \mathbf{88\%} \end{aligned}$$

**Example 31.8.** In a test on a d.c. shunt generator whose full-load output is 200 kW at 250 V, the following figures were obtained :

(a) When running light as a motor at full speed, the line current was 36 A, the field current 12 A, and the supply voltage 250.

(b) With the machine at rest, a p.d. of 6 V produced a current of 400 A through the armature circuit. Explain how these results may be utilised to obtain the efficiency of generator at full-load and half-load. Neglect brush voltage drop.

**Solution. At no-load :**

$$\begin{aligned} I_a &= 36 - 12 = 24 \text{ A ; } R_a = 6/400 = 0.015 \Omega \\ \therefore \text{Armature Cu loss} &= 24^2 \times 0.015 = 8.64 \text{ watt} \\ \text{No-load input} &= \text{total losses in machine} = 250 \times 36 = 9,000 \text{ W} \\ \text{Constant losses} &= 9,000 - 8.64 = \mathbf{8,991.4 \text{ W}} \end{aligned}$$

**At full-load :**

$$\begin{aligned} \text{Output} &= 200,000 \text{ W ; Output current} = 200,000/250 = 800 \text{ A ; } I_{sh} = 12 \text{ A} \\ \therefore \text{F.L. armature current} &= 800 + 12 = 812 \text{ A} \\ \therefore \text{F.L. armature Cu losses} &= 812^2 \times 0.015 = 9890 \text{ W} \\ \therefore \text{F.L. total losses} &= 9890 + 8,991.4 = 18881 \text{ W} \quad \therefore \eta = \frac{200,000 \times 100}{200,000 + 18,881} = \mathbf{91.4\%} \end{aligned}$$

**At half-load :**

$$\begin{aligned} \text{Output} &= 100,000 \text{ W ; Output current} = 100,000/250 = 400 \text{ A} \\ I_a &= 400 + 12 = 412 \text{ A} \quad \therefore I_a^2 R_a = 412^2 \times 0.015 = 2,546 \text{ W} \\ \text{Total losses} &= 8,991.4 + 2,546 = 11,537 \text{ W} \quad \therefore \eta = \frac{100,000 \times 100}{100,000 + 11,537} = \mathbf{89.6\%} \end{aligned}$$

**Example 31.9.** A 250-V, 14.92 kW shunt motor has a maximum efficiency of 88% and a speed of 700 r.p.m. when delivering 80% of its rated output. The resistance of its shunt field is 100 Ω. Determine the efficiency and speed when the motor draws a current of 78 A from the mains.

**Solution.** Full-load output = 14,920 W  
 80% of F.L. output = 0.8 × 14,920 = 11,936 W ; η = 0.88  
 Input = 11,936/0.88 = 13,564 W  
 Total losses = 13,564 - 11,936 = 1,628 W

As efficiency is maximum at this load, the variable loss is equal to constant losses.

$$\begin{aligned} \therefore W_c &= I_a^2 R_a = 1,628/2 \quad \therefore I_a^2 R_a = 814 \text{ W} \\ \text{Now, input current} &= 13,564/250 = 54.25 \text{ A} \\ \therefore I_{sh} &= 250/100 = 2.5 \text{ A} \quad \therefore I_a = 54.25 - 2.5 = 51.75 \text{ A} \end{aligned}$$

$$\therefore 51.75^2 R_a = 814 \quad \therefore R_a = 814/51.75^2 = 0.3045 \Omega$$

**When input current is 78 A**

$$I_a = 78 - 2.5 = 75.5 \text{ A} \quad \therefore I_a^2 R_a = 75.5^2 \times 0.3045 = 1,736 \text{ W}$$

$$\text{Total losses} = 1,736 + 814 = 2,550 \text{ W} ; \text{Input} = 250 \times 78 = 19,500 \text{ W}$$

$$\eta = \frac{19,500 - 2,550}{19,500} \times 100 = \mathbf{86.9\%}$$

**Speed :**

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{N_2}{700} = \frac{250 - (75.5 \times 0.3045)}{250 - (51.75 \times 0.3045)} = \frac{227}{234.25} ; N_2 = \mathbf{680 \text{ r.p.m.}}$$

### 31.5. Regenerative or Hopkinson's Test (Back-to-Back Test)

By this method, **full-load test** can be carried out on two shunt machines, preferably identical ones, without wasting their outputs. The two machines are mechanically coupled and are so adjusted electrically that one of them runs as a motor and the other as a generator. The mechanical output of the motor drives the generator and the electrical output of generator is used in supplying the greater part of input to the motor. If there were no losses in the machines, they would have run without any external power supply. But due to these losses, generator output is not sufficient to drive the motor and **vice-versa**. The losses are supplied either by an extra motor which is belt-connected to the motor-generator set or as suggested by Kapp, electrically from the supply mains.

Essential connections for the test are shown in Fig. 31.6. The two shunt machines are connected in parallel. They are, to begin with, started as unloaded motors. Then, the field of one is weakened and that of the other is strengthened so that the former runs as a motor and the latter as a generator. The usual method of procedure is as follows :

Machine *M* is started up from the supply mains with the help of a starter (not shown) whereas main switch *S* of the other machine is kept open. Its speed is adjusted to normal value by means of its shield regulator. Machine *M* drives machine *G* as a

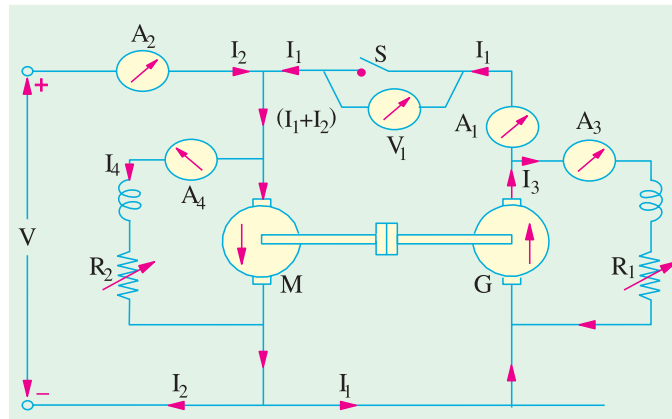


Fig. 31.6



Motor-cum generator set

generator and its voltage is read on voltmeter *V*<sub>1</sub>. The voltage of *G* is adjusted by its field regulator until voltmeter *V*<sub>1</sub> reads zero, thereby showing that its voltage is the same, both in polarity and magnitude as that of the main supply. Thereafter, *S* is closed to parallel the machines. By adjusting the respective field regulators, any load can now be thrown on to the machines. Generator current *I*<sub>1</sub> can be adjusted to any desired value by increasing the excitation of *G* or by reducing the excitation of *M* and the corresponding values of different ammeters are read.

ing the excitation of *M* and the corresponding values of different ammeters are read.

The electrical output of the generator **plus** the small power taken from the supply, is taken by the motor and is given out as a mechanical power **after** supplying the motor losses.

If supply voltage is  $V$ , then

$$\text{Motor input} = V(I_1 + I_2), \text{ where } I_2 \text{ is the current taken from the supply.}$$

$$\text{Generator output} = VI_1 \quad \dots(i)$$

**Assuming that both machines have the same efficiency  $\eta$ ,**

$$\text{Output of motor} = \eta \times \text{input} = \eta V(I_1 + I_2) = \text{generator input}$$

$$\text{Output of generator} = \eta \times \text{input} = \eta \times \eta V(I_1 + I_2) = \eta^2 V(I_1 + I_2) \quad \dots(ii)$$

Hence, from (i) and (ii), we get

$$\eta^2 V(I_1 + I_2) = VI_1 \quad \text{or} \quad \eta = \sqrt{\frac{I_1}{I_1 + I_2}}$$

However, it is not quite correct to assume equal efficiencies for two machines because their armature currents as well as excitations are different. We will not find the efficiencies separately.

Let  $R_a$  = armature resistance of each machine

$I_3$  = exciting current of the generator

$I_4$  = exciting current of the motor

Armature Cu loss in generator =  $(I_1 + I_3)^2 R_a$ ; Armature Cu loss in motor =  $(I_1 + I_2 - I_4)^2 R_a$

Shunt Cu loss in generator =  $VI_3$ ; Shunt Cu loss in motor =  $VI_4$

But total motor and generator losses are equal to the power supplied by the mains.

Power drawn from supply =  $VI_2$

If we subtract the armature and shunt Cu losses from this, we get the stray losses of *both* machines.

$\therefore$  Total stray losses for the set

$$= VI_2 - [(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4] = W \text{ (say)}$$

Making one assumption that stray losses are equally divided between the two machines, we have

Stray loss per machine =  $W/2$

**For Generator**

$$\text{Total losses} = (I_1 + I_3)^2 R_a + VI_3 + W/2 = W_g \text{ (say)}$$

$$\text{Output} = VI_1 \quad \therefore \quad \eta_g = \frac{VI_1}{VI_1 + W_g}$$

$$\text{Total losses} = (I_1 + I_2 - I_4)^2 R_a + VI_4 + W/2 = W_m \text{ (say)}$$

$$\text{Input} = V(I_1 + I_2) \quad \therefore \quad \eta_m = \frac{V(I_1 + I_2) - W_m}{V(I_1 + I_2)}$$

### 31.6. Alternative Connections for Hopkinson's Test

In Fig. 31.7 is shown in slightly different method of connecting the two machines to the supply. Here, the main difference is that the shunt windings are directly connected across the lines. Hence, the line input current is  $I_1$  **excluding the field currents**. The efficiencies can be calculated as detailed below :

Motor armature Cu loss =  $(I_1 + I_2)^2 R_a$ ; Generator armature Cu loss =  $I_2^2 R_a$

Power drawn from the supply =  $VI_1$

$\therefore$  Total stray losses *i.e.* iron, friction and windage losses for the two machines are

$$= VI_1 - [(I_1 + I_2)^2 R_a - I_2^2 R_a] = W \text{ (say)}$$



∴ stray loss for each machine =  $W/2$

**Motor Efficiency**

$$\begin{aligned} \text{Motor input} &= \text{armature input} + \text{shunt field input} = V(I_1 + I_2) + VI_3 + W_{input} \\ \text{Motor losses} &= \text{armature Cu loss} + \text{shunt Cu loss} + \text{stray losses} \\ &= (I_1 + I_2)^2 R_a + VI_3 + W/2 = W_m \text{ (say)} \end{aligned}$$

$$\text{Motor } \eta = \frac{W_{input} - W_m}{W_{input}} \times 100$$

**Generator Efficiency**

$$\text{Generator output} = VI_2 ; \text{Generator losses} = I_2^2 R_a + VI_4 + W/2 = W_g \text{ (say)}$$

$$\therefore \text{Generator } \eta = \frac{VI_2}{VI_2 + W_g}$$

**31.7. Merits of Hopkinson's Test**

1. Power required for the test is small as compared to the full-load powers of the two machines.
2. As machines are being tested under full-load conditions, the temperature rise and the commutation qualities of the machines can be observed.

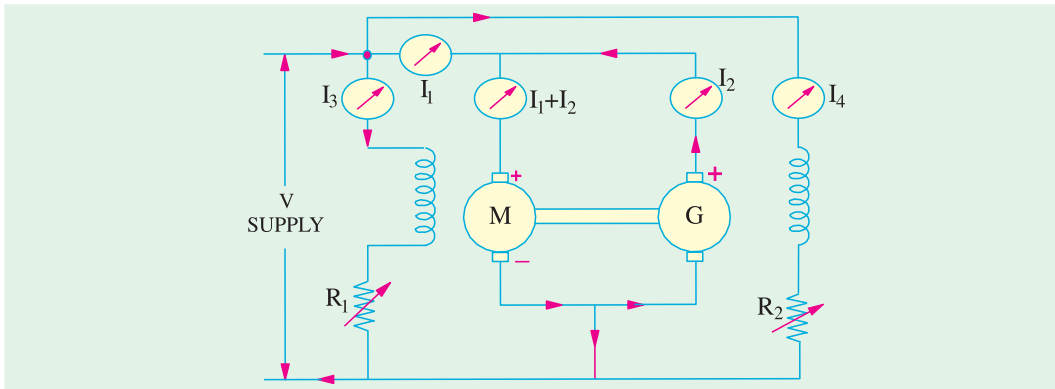


Fig. 31.7

3. Because of full-load conditions, any change in iron loss due to flux distortion at full-load, is being taken into account.

The only disadvantage is with regard to the availability of two identical machines.

**Example 31.10 (a).** In a Hopkinson's test on two 220-V, 100-kW generators, the circulating current is equal to the full-load current and, in addition, 90 A are taken from the supply. Obtain the efficiency of each machine.

**Solution.** Output current of the generator

$$I_1 = \frac{100,000}{220} = \frac{5,000}{11} = 454.4 \text{ A}, I_2 = 90 \text{ A}$$

Assuming equal efficiencies, from Art. 29.5, we have

$$\eta = \sqrt{\frac{I_1}{I_1 + I_2}} = \sqrt{\frac{454.5}{454.5 + 90}} = 0.914 \text{ or } \mathbf{91.4\%}$$

**Example 31.10 (b).** In the Hopkinson's test on two d.c. machines, machine A has a field current of 1.4 A and machine B has a field current of 1.3 A. Which machine acts as a generator ?

(Bharathithasan University April 1997)

**Solution.** In Hopkinson's test (on two identical d.c. shunt machines), since the two machines are coupled, the speed is common and is decided by the field current of the motor. The field windings of both the machines are in parallel with a separate D.C. source.

Since the machines are identical and are running at the **same speed**, their e.m.fs are in proportion to their field currents.

$$\frac{\text{E.M.F. induced in the armature of machine A}}{\text{E.M.F. induced in the armature of machine B}} = \frac{1.4}{1.3}$$

$$E_A = (1.4/1.3) \times E_B = 1.077 E_B$$

Since  $E_A$  is larger than  $E_B$ , Machine A supplies power to Machine B. It means, A is working as a generator, and B is motoring.

**Example 31.11.** Two shunt machines loaded for the Hopkinson's test take 15 A at 200 V from the supply. The motor current is 100 A and the shunt currents are 3 A and 2.5 A. If the armature resistance of each machine is 0.05 ohm, calculate the efficiency of each machine for this particular load-condition. (Bharathithasan Univ. April 1997)

**Solution.** Line current into armature circuits = 15 A, Motor armature copper-loss = 500 W

Motor-armature-current = 100 A, Generator armature copper loss = 361 W

Hence generator-armature-current = 85 A

For each machine, No load Mechanical losses + Core-loss + Stray losses

$$= \frac{1}{2}(VI_a - I_{am}^2 r_{am} - I_{ag}^2 r_{ag})$$

$$= \frac{1}{2}(200 \times 15 - 100^2 \times 0.05 - 85^2 \times 0.05)$$

$$= \frac{1}{2}(3000 - 500 - 361) = 1069.5 \text{ W}$$

Motor field copper-loss =  $200 \times 3 = 600 \text{ W} \cong 1.07 \text{ kW}$

Generator field copper-loss =  $200 \times 2.5 = 500 \text{ W}$

Total Losses in motor =  $600 + 1069.5 + 500 = 2169.5 \text{ W}$

Total Losses in Generator =  $500 + 1069.5 + 361 = 1931 \text{ W}$

$$\text{Efficiency of motor} = \frac{\text{Motor output}}{\text{Motor input}} \times 100\%$$

Motor Input : (a)  $200 \times 100 = 20 \text{ kW}$  to armature

(b)  $0.6 \text{ kW}$  to field winding

Total Input to motor =  $20.6 \text{ kW}$

From armature side, losses to be catered are :

(i) Stray losses + No Load Mech. Losses + Core Losses =  $1.07 \text{ kW}$

(ii) Armature copper-loss =  $0.5 \text{ kW}$

Motor Output from armature =  $20 - 0.5 - 1.07 = 18.43 \text{ kW}$

$$\text{Motor efficiency} = \frac{18.43}{20.6} \times 100\% = 89.47\%$$

Generator armature output =  $200 \times 85 \times 10^{-3} = 17 \text{ kW}$

Generator losses : (a) Field wdg :  $0.5 \text{ kW}$

(b) Total no-load-losses :  $1.07 \text{ kW}$

(c) armature copper-loss =  $0.36 \text{ kW}$

Total losses in Generator =  $1.93 \text{ kW}$

$$\text{Generator efficiency} = \frac{17}{17 + 1.93} \times 100\% = 89.80\%$$

**Special Note:** 15 A current for d.c. supply is related here to armature-input for two machines which are under back-to-back regenerative tests. There are different variations in handling and giving the test data. It is always desirable to draw the circuit diagram according to which the calculations are being related.

**Example 31.12.** The Hopkinson's test on two similar shunt machines gave the following full-load data :

Line voltage	= 110 V	Field currents are 3 A and 3.5 A
Line current	= 48 A	Arm. resistance of each is 0.035 Ω
Motor arm. current	= 230 A	

Calculate the efficiency of each machine assuming a brush contact drop of 1 volt per brush.

(Electrical Machines, Nagpur Univ. 1992)

**Solution.** The motor-generator set is shown in Fig. 31.8. It should also be noted that the machine with lesser excitation is motoring. We will find the total armature Cu losses and brush contact loss for both machines.

**Motor**

$$\begin{aligned} \text{Arm. Cu loss} &= 230^2 \times 0.035 = 1,851.5 \text{ W} \\ \text{Brush contact loss} &= 230 \times 2 = 460 \text{ W} \\ \text{Total arm. Cu loss} &= 1851.5 + 460 = 2,312 \text{ W} \\ \text{Shunt Cu loss} &= 110 \times 3 = 330 \text{ W} \\ \text{Total Cu loss} &= 2,312 + 330 = 2,642 \text{ W} \end{aligned}$$

**Generator**

$$\begin{aligned} \text{Generator arm. current} &= 233 - 48 + 3.5 \\ &= 188.5 \text{ W} \\ \text{Arm. Cu loss} &= 188.5^2 \times 0.035 = 1,244 \text{ W} \\ \text{Brush contact Cu loss} &= 188.5 \times 2 = 377 \text{ W} \\ \text{Total arm. Cu loss} &= 1,244 + 377 = 1,621 \text{ W} \\ \text{Shunt Cu loss} &= 110 \times 3.5 = 385 \text{ W} ; \\ \text{Total Cu loss} &= 1,621 + 385 = 2,006 \text{ W} \end{aligned}$$

**For the Set**

$$\begin{aligned} \text{Total arm. and shunt Cu loss for the set} &= 2,642 + 2,006 = 4,648 \text{ W} \\ \text{Total input} &= 110 \times 48 = 5,280 \text{ W} ; \text{ Stray losses for the set} = 5,280 - 4,648 = 632 \text{ W} \\ \text{Stray losses per machine} &= 632/2 = 316 \text{ W} \end{aligned}$$

**Motor Efficiency**

$$\begin{aligned} \text{Arm. Cu + brush drop loss} &= 2,312 \text{ W} & \text{Shunt Cu loss} &= 330 \text{ W} \\ \text{Stray losses} &= 316 \text{ W} & \text{Total loss} &= 2,312 + 330 + 316 = 2,958 \text{ W} \\ \text{Motor input} &= 110 \times 233 = 25,630 \text{ W} ; \text{ Motor output} &= 25,630 - 2,958 = 22,672 \end{aligned}$$

$$\therefore \eta = 22,672 \times 100/25,630 = \mathbf{88.8\%}$$

**Generator Efficiency**

$$\begin{aligned} \text{Total losses} &= 2,006 + 316 = 2,322 \text{ W} ; \text{ Output} = 110 \times 185 = 20,350 \text{ W} \\ \text{Generator input} &= 20,350 + 2,322 = 22,672 \text{ W} = \text{motor input} \\ \eta &= 20,350/22,672 = 0.894 \text{ or } \mathbf{89.4\%} \end{aligned}$$

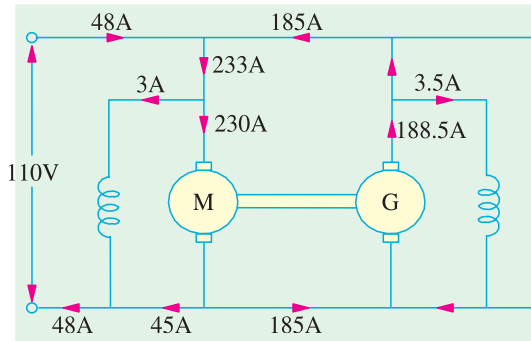


Fig. 31.8

**Example 31.13.** In a Hopkinson's test on a pair of 500-V, 100-kW shunt generators, the following data was obtained :

Auxiliary supply, 30 A at 500 V : Generator output current, 200 A  
 Field currents, 3.5 A and 1.8 A  
 Armature circuit resistances, 0.075Ω each machine. Voltage drop at brushes, 2 V (each machine).  
 Calculate the efficiency of the machine acting as a generator.

(Elect. Technology-1, Gwalior Univ. 1986)

**Solution.** Motor arm. current = 200 + 30 = 230 A, as shown in Fig. 31.9.

$$\text{Motor arm. Cu loss} = 230^2 \times 0.075 + 230 \times 2 = 4,428 \text{ W}$$

$$\text{Motor field Cu loss} = 500 \times 1.8 = 900 \text{ W}$$

$$\text{Generator arm. Cu loss} = 200^2 \times 0.075 + 200 \times 2 = 3,400 \text{ W}$$

$$\text{Generator field Cu loss} = 500 \times 3.5 = 1,750 \text{ W}$$

Total Cu loss for two machines

$$= 4,428 + 900 + 3400 + 1750$$

$$= 10,478 \text{ W}$$

Power taken from auxiliary supply

$$= 500 \times 30 = 15,000 \text{ W}$$

Stray losses for the two machines

$$= 15,000 - 10,478 = 4,522 \text{ W}$$

Stray loss per machine = 4,522/2 = 2,261 W

Total losses in generator = 3400 + 1750 + 2261 = 7,411 W

$$\text{Generator output} = 500 \times 200 = 100,000 \text{ W}$$

$$\therefore \eta_g = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{100,000}{107,411} \times 100 = \mathbf{93.09\%}$$

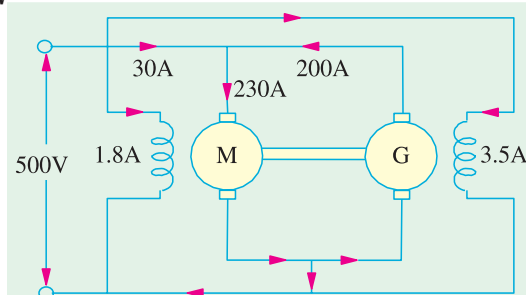


Fig. 31.9

**Example 31.14.** Explain the Hopkinson's test on a pair of shunt motors.

In such a test on 250-V machines, the line current was 50 A and the motor current 400 A not including the field currents of 6 A and 5 A. The armature resistance of each machine was 0.015 Ω. Calculate the efficiency of each machine. (Adv. Elect. Machines, A.M.I.E. Sec. B, 1991)

**Solution.** The connections are shown in Fig. 31.10.

$$\text{Motor armature Cu loss} = 400^2 \times 0.015 = 2,400 \text{ W}$$

$$\text{Generator armature Cu loss} = 350^2 \times 0.015 = 1,838 \text{ W}$$

$$\text{Power drawn from supply} = 250 \times 50 = 12,500 \text{ W}$$

∴ Iron, friction and windage losses for the two machines

$$= 12,500 - (2,400 + 1,838)$$

$$= 8,262 \text{ W}$$

Iron, friction and windage loss per machine

$$= 8,262/2 = 4,130 \text{ W* (approx.)}$$

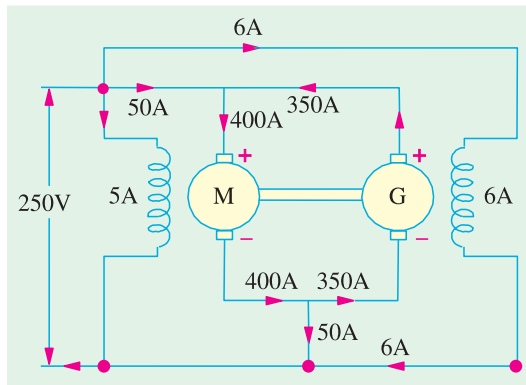


Fig. 31.10

### Motor Losses and Efficiency

$$\text{Motor arm. Cu loss} = 2,400 \text{ W} ; \text{ Motor field Cu loss} = 250 \times 5 = 1,250 \text{ W}$$

$$\text{Iron, friction and windage losses} = 4,130 \text{ W}$$

$$\text{Total motor losses} = 2,400 + 1,250 + 4,130 = 7,780 \text{ W}$$

$$\text{Motor input} = 250 \times 400 + 250 \times 5 = 101,250 \text{ W}$$

$$\therefore \text{ Motor efficiency} = (101,250 - 7,780)/101,250 = 0.923 \text{ or } \mathbf{92.3\%}$$

\* We could also get this value as follows :

$$\text{Total supply input} = 250 \times 61 = 15,250 \text{ W} ; \text{ Gen. and motor field Cu loss} = 250 \times 6 + 250 \times 5 = 2,750 \text{ W}$$

Iron, friction and windage losses for both machines

$$= 15,250 - (2,400 + 1,838 + 2,750) = \mathbf{8,262 \text{ W}} \quad \text{-- as before}$$

### Generator Losses and Efficiency

Generator arm. Cu loss = 1,838 W ; Generator field Cu loss =  $250 \times 6 = 1,500$  W  
 Iron, friction and windage loss = 4,130 W  
 Total losses =  $1,838 + 1,500 + 4,130 = 7,468$  W  
 Generator output =  $250 \times 350 = 87,500$  W  
 $\therefore$  Generator efficiency =  $(87,500 - 7,468)/87,500 = 0.915$  or **91.5%**

**Example 31.15.** The Hopkinson's test on two shunt machines gave the following results for full-load :

Line voltage = 250 V ; current taken from supply system excluding field currents = 50 A ; motor armature current = 380 A ; field currents 5 A and 4.2 A. Calculate the efficiency of the machine working as a generator. Armature resistance of each machine is 0.2  $\Omega$ .

(Electrical Machinery-I Mysore Univ. 1988)

**Solution.** The connections are shown in Fig. 31.11.

Motor arm. Cu loss =  $380^2 \times 0.02 = 2,888$  W  
 Generator arm. Cu loss =  $330^2 \times 0.02 = 2,178$  W  
 Power drawn from supply =  $250 \times 50 = 12,500$  W  
 Stray losses for the two machines  
 =  $12,500 - (2,888 + 2,178) = 7,434$  W  
 Stray losses per machine =  $7,434/2 = 3,717$  W

### Motor Efficiency

Arm. Cu loss = 2,888 W  
 Field Cu loss =  $250 \times 4.2 = 1050$  W  
 Stray losses = 3,717 W  
 Total loss =  $2,888 + 1050 + 3,717 = 7,655$  W  
 Motor input =  $250 \times 380 + 250 \times 4.2 = 96,050$  W  
 Motor output =  $96,050 - 7,655 = 88,395$  W  
 $\therefore \eta = 88,395/96,050 = 0.9203$  or **92.03%**

### Generator Efficiency

Arm. Cu loss = 2,178 W ; Field Cu loss =  $250 \times 5 = 1250$  W  
 Stray losses = 3,717 W ; Total losses = 7,145 W  
 Generator output =  $250 \times 330 = 82,500$  W  
 Generator input =  $82,500 + 7,145 = 89,645$  W  
 $\eta = 82,500/89,645 = 0.9202$  or **92.02%**

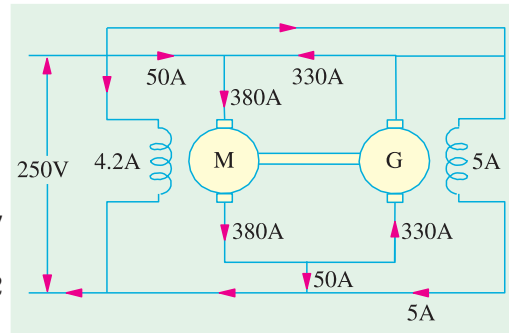


Fig. 31.11

## 31.8. Retardation or Running Down Test

This method is applicable to shunt motors and generators and is **used for finding stray losses**. Then, knowing the armature and shunt Cu losses at a given load current, efficiency can be calculated.

The machine under test is speeded up slightly beyond its normal speed and then supply is cut off from the armature while keeping the field excited. Consequently, the armature slows down and its kinetic energy is used to meet the **rotational losses i.e.** friction, windage and iron losses.\*

Kinetic energy of the armature is  $K.E. = \frac{1}{2} I\omega^2$

where  $I$  = moment of inertia of the armature and  $\omega$  = angular velocity

$\therefore$  Rotational losses,  $W$  = Rate of loss of  $K.E.$

\* If armature slows down with no excitation, then energy of the armature is used to overcome mechanical losses only, there being no iron losses (see Ex. 31.19).

$$\therefore W = \frac{d}{dt} \left( \frac{1}{2} I \omega^2 \right) = I \omega \cdot \frac{d\omega}{dt}$$

Two quantities need be known (i) moment of inertia ( $I$ ) of the armature and (ii)  $\frac{d\omega}{dt}$  or  $\frac{dN}{dt}$  because  $\omega \propto N$ . These are found as follows :

**(a) Finding  $\frac{d\omega}{dt}$**

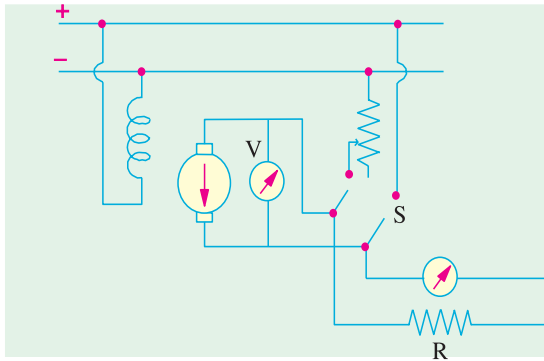


Fig. 31.12

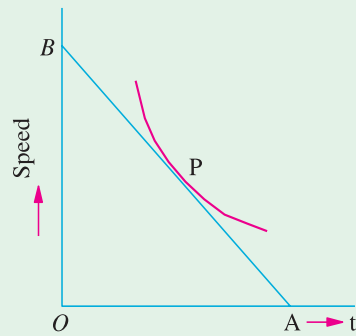


Fig. 31.13

As shown in Fig. 31.12, a voltmeter  $V$  is connected across the armature. This voltmeter is used as a speed indicator by suitably graduating it, because  $E \propto N$ . When supply is cut off, the armature speed and hence voltmeter reading falls. By noting different amounts of voltage fall in different amounts of time, a curve is drawn between time and the speed (obtained from voltage values) as shown in Fig. 31.13.

From any point  $P$  which corresponds to normal speed, a tangent  $AB$  is drawn.

Then 
$$\frac{dN}{dt} = \frac{OB \text{ (in r.p.m.)}}{OA \text{ (in seconds)}}$$

From (i), above 
$$W = I \omega \frac{d\omega}{dt}$$

Now 
$$\omega = \frac{2\pi N}{60} \quad \dots (N \text{ in r.p.m.})$$

$$W = I \left( \frac{2\pi N}{60} \right) \times \frac{d}{dt} \left( \frac{2\pi N}{60} \right); W = \left( \frac{2\pi}{60} \right)^2 I.N. \frac{dN}{dt} = 0.011 I.N. \frac{dN}{dt} \quad \dots (ii)$$

**(ii) Finding Moment of Inertia (I)**

**(a) First Method—where  $I$  is calculated.**

First, slowing down curve is drawn with armature alone. Next, a fly-wheel of known moment of inertia  $I_1$  is keyed onto the shaft and slowing down curve is drawn again. Obviously, slowing down time will be longer due to combined increased moment of inertia of the two. For any given speed,  $(\frac{dN}{dt}_1)$  and  $(\frac{dN}{dt}_2)$  are determined as before. It should be noted that the losses in both cases would be almost the same, because addition of a fly-wheel will not make much difference to the losses.

Hence, from equation (ii) above

In the first case, 
$$W = \left( \frac{2\pi}{60} \right)^2 I N \left( \frac{dN}{dt}_1 \right)$$



Shunt wound generator

In the second case,  $W = \left(\frac{2\pi}{60}\right)^2 (I + I_1) N \times \left(\frac{dN}{dt_2}\right)$

$$\therefore (I + I_1) \left(\frac{dN}{dt_2}\right) = I \left(\frac{dN}{dt_1}\right) \text{ or } \left(\frac{I + I_1}{I}\right) = \left(\frac{dN}{dt_1}\right) / \left(\frac{dN}{dt_2}\right)$$

$$\therefore I = I_1 \times \frac{(dN/dt_2)}{(dN/dt_1) - (dN/dt_2)} = I_1 \frac{dt_1}{dt_2 - dt_1}; I = I_1 \frac{t_1}{t_2 - t_1}$$

**(b) Second Method—where  $I$  is eliminated.**

In this method, first, time taken to slow down, say by 5%, is noted with armature alone. Next, a retarding torque—mechanical or preferably electrical, is applied to the armature and again time is noted. The method using electrical torque is shown in Fig. 31.12. The double-throw switch  $S$  while cutting off the armature from supply, automatically joins it to a non-inductive resistance  $R$  as shown. The power drawn by this resistance acts as a retarding torque on the armature, thereby making it slow down comparatively quickly. The additional loss is  $I_a^2 (R_a + R)$  or  $VI_a$ ,

where  $I_a =$  average current through  $R$ ;  $V =$  average voltage across  $R$ .

Let  $W'$  be this power. Then from (i) above

$$\left. \begin{aligned} W &= \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt_1} \\ W + W' &= \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt_2} \end{aligned} \right\} \text{If } dN \text{ is the same}$$

$$\therefore \frac{W + W'}{W} = \frac{dt_1}{dt_2} \quad \therefore W = W' \times \frac{dt_2}{dt_1 - dt_2} \text{ or } W = W' \times \frac{t_2}{t_1 - t_2}$$

where  $\frac{dN}{dt_1} =$  rate of change of speed **without** extra load

$\frac{dN}{dt_2} =$  rate of change of speed **with** extra electrical load.

**Example 31.16.** In a retardation test on a separately-excited motor, the induced e.m.f. in the armature falls from 220 V to 190 V in 30 seconds on disconnecting the armature from the supply. The same fall takes place in 20 seconds if, immediately after disconnection, armature is connected to a resistance which takes 10 A (average) during this fall. Find stray losses of the motor.

(Adv. Elect. Machines, A.M.I.E. Sec. B, 1992)

**Solution.** Let  $W =$  stray losses (mechanical and magnetic losses)

Average voltage across resistance =  $(200 + 190)/2 = 195$  V, Average current = 10 A

$\therefore$  Power absorbed  $W' = 1950$  W

Using the relation  $\frac{W}{W'} = \frac{t_2}{t_1 - t_2}$ ; we get  $W = 1950 \times \frac{20}{30 - 20} = 3,900$  watt

**Example 31.17.** In a retardation test on a d.c. motor, with its field normally excited, the speed fell from 1525 to 1475 r.p.m. in 25 seconds. With an average load of 1.0 kW supplied by the armature, the same speed drop occurred in 20 seconds. Find out the moment of inertia of the rotating parts in  $\text{kg.m}^2$ .

(Electrical Machines-III, Gujarat Univ. 1984)

**Solution.** As seen from Art. 31.8 (ii) (b).

$$W' = \left(\frac{2\pi}{60}\right)^2 I.N. \frac{dN}{dt}. \text{ Also } W = W' \times \frac{t_2}{t_1 - t_2}$$

Here,  $W' = 1$  kW = 1000 W,  $t_1 = 25$  second,  $t_2 = 20$  second

$$\begin{aligned} \therefore W &= 1000 \times 20 / (25 - 20) = 4000 \text{ W} \\ \text{Now, } N &= 1500 \text{ r.p.m. (average speed) ; } dN = 1525 - 1475 = 50 \text{ r.p.m. ; } dt = 25 \\ \therefore 4000 &= (2\pi/60)^2 I \cdot 1500 \times 50 / 25 \quad \therefore I = \mathbf{121.8 \text{ kg.m}^2}. \end{aligned}$$

**Example 31.18.** A retardation test is made on a separately-excited d.c. machine as a motor. The induced voltage falls from 240 V to 225 V in 25 seconds on opening the armature circuit and 6 seconds on suddenly changing the armature connection from supply to a load resistance taking 10 A (average). Find the efficiency of the machines when running as a motor and taking a current of 25 A on a supply of 250 V. The resistance of its armature is 0.4 Ω and that of its field winding is 250 Ω.

(Elect. Technology, Allahabad Univ. 1991)

**Solution.** Average voltage across load

$$= (240 + 225) / 2 = 232.5 \text{ V ; } I_{av} = 10 \text{ A}$$

$$\therefore \text{ Power absorbed } W' = 232.5 \times 10 = 2,325 \text{ W}$$

and  $t_1 = 30 \text{ second, } t_2 = 6 \text{ second ; } W = \text{stray loss}$

Using  $\frac{W}{W'} = \frac{t_2}{t_1 - t_2} = \mathbf{734.1 \text{ W}}$ , we get

Stray losses  $W = 2325 \times \frac{6}{25 - 6} = \mathbf{734.1 \text{ W}}$

$$\text{Input current} = 25 \text{ A ; } I_{sh} = 250 / 250 = 1 \text{ A ; } I_a = 25 - 1 = 24 \text{ A}$$

$$\text{Armature Cu loss} = 24^2 \times 0.4 = 230.4 \text{ W ; Shunt Cu loss} = 250 \times 1 = 250 \text{ W}$$

$$\therefore \text{ Total losses} = 734.1 + 230.4 + 250 = 1,215 \text{ W (approx.)}$$

$$\text{Input} = 250 \times 25 = 6,250 \text{ W ; Output} = 6,250 - 1,215 = 5,035 \text{ W}$$

$$\therefore \eta = 5,035 / 6,250 = 0.806 \text{ or } \mathbf{80.6\%}$$

**Example 31.19.** A retardation test is carried out on a 1000 r.p.m. d.c. machine. The time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m. is :

(a) 36 seconds with no excitation

(b) 15 seconds with full excitation and

(c) 9 seconds with full excitation and the armature supplying an extra load of 10 A at 219 V.

Calculate (i) the moment of inertia of the armature in kg. m<sup>2</sup> (ii) iron losses and (iii) the mechanical losses at the mean speed of 1000 r.p.m.

**Solution.** It should be noted that

(i) when armature slows down with no excitation, its kinetic energy is used to overcome mechanical losses only ; because due to the absence of flux, there is no iron loss.

(ii) with excitation, kinetic energy is used to supply mechanical and iron losses collectively known as stray losses.

(iii) If  $I$  is taken in kg-m<sup>2</sup> unit, then rate of loss of energy is in watts.

$$\text{Mechanical loss} \quad W_m = \left(\frac{2\pi}{60}\right)^2 I \cdot N \cdot \frac{dN}{dt} \quad \dots \text{Art. 31.8}$$

Here  $dN = 1030 - 970 = 60 \text{ r.p.m., } dt = 36 \text{ seconds, } N = 1000 \text{ r.p.m.}$

$$W_m = \left(\frac{2\pi}{60}\right)^2 I \cdot N \cdot \frac{60}{36} \quad \dots \text{(i)}$$

Similarly  $W_s = \left(\frac{2\pi}{60}\right)^2 I \cdot N \cdot \frac{60}{15} \quad \dots \text{(ii)}$

$$\text{Also } W_s = W' \frac{t_2}{t_1 - t_2} = 219 \times 10 \times \frac{9}{15 - 9} = 3,285 \text{ W}$$



Using equation (ii), we get

$$3,285 = \left(\frac{2\pi}{60}\right)^2 \times I \times 1000 \times \frac{60}{15}$$

(i) ∴  $I = 75 \text{ kg.m}^2$

Dividing (i) by (ii), we get  $\frac{W_m}{W_s} = \frac{15}{36}$

(ii) ∴  $W_m = 3,285 \times 15/36 = 1,369 \text{ W}$

(iii) ∴ Iron losses =  $W_s - W_m = 3,285 - 1,369 = 1,916 \text{ W}$

### 31.9. Field's Test for Series Motor

This test is applicable to two similar **series motors**. Series motors which are mainly used for traction work are easily available in pairs. The two machines are coupled mechanically.

One machine runs normally as a motor and drives generator whose output is wasted in a variable load  $R$  (Fig. 31.14). Iron and friction losses of two machines are made equal (i) by joining the series field winding of the generator in the motor armature circuit so that both

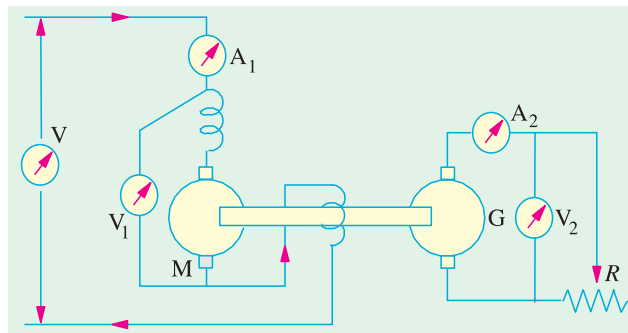


Fig. 31.14



Micro series motors

machines are equally excited and (ii) by running them at equal speed. Load resistance  $R$  is varied till the motor current reaches its full-load value indicated by ammeter  $A_1$ . After this adjustment for full-load current, different ammeter and voltmeter readings are noted.

Let  $V$  = supply voltage ;  $I_1$  = motor current ;  $V_2$  = terminal p.d. of generator ;  $I_2$  = load current.

∴ Intake of the whole set =  $V I_1$  ; output =  $V_2 I_2$ .

Total losses in the set,  $W_t = V I_1 - V_2 I_2$

Armature and field Cu losses  $W_{cu} = (R_a + 2R_{se}) I_1^2 + I_2^2 R_a$

where  $R_a$  = hot armature resistance of each machine

$R_{se}$  = hot series field resistance of each machine

∴ Stray losses for the set =  $W_t - W_{cu}$

Stray losses per machine  $W_s = \frac{W_t - W_{cu}}{2}$

Stray losses are equally divided between the machines because of their equal excitation and speed.

#### Motor Efficiency

Motor input =  $V_1 I_1$

Motor losses = armature + field Cu losses + stray losses

=  $(R_a + R_{se}) I_1^2 + W_s = W_m$  (say)

$$\eta_m = \frac{V_1 I_1 - W_m}{V_1 I_1}$$

**Generator Efficiency**

The generator efficiency will be of little use because it is running under abnormal conditions of *separate* excitation. However, the efficiency under these unusual conditions can be found if desired.

$$\begin{aligned} \text{Generator output} &= V_2 I_2 \\ \text{Field Cu loss} &= I_1^2 R_{se} \quad (\because \text{Motor current is passing through it.}) \\ \text{Armature Cu loss} &= I_2^2 R_a; \text{ Stray losses} = W_s \\ \text{Total losses} &= I_1^2 R_{se} + I_2^2 R_a + W_s = W_g \text{ (say)} \end{aligned}$$

$$\eta_g = \frac{V_2 I_2}{V_2 I_2 + W_g}$$

It should be noted that although the two machines are mechanically coupled yet it is not a regenerative method, because the generator output is wasted instead of being fed back into the motor as in Hopkinson’s (back-to-back) test.

**Example 31.20.** A test on two coupled similar tramway motors, with their fields connected in series, gave the following results when one machine acted as a motor and the other as a generator:

Motor : Armature current = 56 A ; Armature voltage = 590 V  
Voltage drop across field winding = 40 V

Generator : Armature current = 44 A ; Armature voltage = 400 V  
Field voltage drop = 40 V ; Resistance of each armature = 0.3 Ω

Calculate the efficiency of the motor and gearing at this load.

(Elect. Machinery-II, Nagpur Univ. 1992 & JNTU, Hyderabad, 2000)

**Solution.** The connection for the two machines are shown in Fig. 31.15.

$$\text{Total input} = 630 \times 56 = 35,280 \text{ W}$$

$$\text{Output} = 400 \times 44 = 17,600 \text{ W}$$

Total losses in the two machines are

$$= 35,280 - 17,600 = 17,680 \text{ W}$$

Series field resistance

$$\begin{aligned} R_{se} &= 40/56 \\ &= 0.714 \text{ } \Omega \end{aligned}$$

$$\begin{aligned} \text{Total Cu loss} &= (0.3 + 2 \times 0.714) \times 56^2 + 44^2 \\ &\times 0.3 = 5,425 + 581 = 6,006 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Stray losses of the set} &= 17,680 - 6,006 \\ &= 11,674 \text{ W} \end{aligned}$$

$$\therefore \text{ Stray losses/machine} = 11,674/2 = 5,837 \text{ W}$$

**Motor Efficiency**

$$\begin{aligned} \text{Motor armature input} &= \text{arm. voltage} \times \text{motor} \\ \text{current} &= 590 \times 56 = 33,040 \text{ W} \end{aligned}$$

Armature circuit Cu loss

$$= (0.3 + 0.714) \times 56^2 = 3,180 \text{ W}$$

$$\text{Stray loss} = 5,837 \text{ W} - \text{found above}$$

$$\text{Total losses} = 3,180 + 5,837 = 9,017 \text{ W, Output} = 33,040 - 9,017 = 24,023 \text{ W}$$

$$\therefore \eta_m = 24,023/33,040 = 0.727 \text{ or } 72.7\%$$

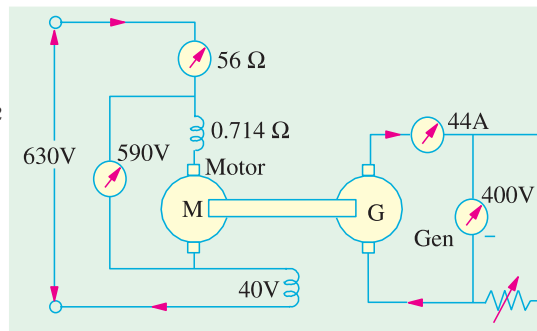


Fig. 31.15

**Generator Efficiency**

$$\text{Armature Cu loss} = 44^2 \times 0.3 = 581 \text{ W, Series field Cu loss} = 40 \times 56 = 2,240 \text{ W}$$

$$\text{Stray losses} = 5,837 \text{ W ; Total losses} = 581 + 2,240 + 5,837 = 8,658 \text{ W}$$

$$\text{Output} = 400 \times 44 = 17,600 \text{ W}$$

$$\therefore \eta_g = 17,600 / (17,600 + 8,658) = 0.67 \text{ or } 67\%$$

**Tutorial Problem No. 31.1**

1. A 500-V, shunt motor takes a total current of 5 A when running unloaded. The resistance of armature circuit is  $0.25 \Omega$  and the field resistance is  $125 \Omega$ . Calculate the efficiency and output when the motor is loaded and taking a current of 100 A. **[90.4% ; 45.2 kW]**

2. A d.c. shunt motor rated at 12.5 kW output runs at no-load at 1000 r.p.m. from a 250-V supply consuming an input current of 4 A. The armature resistance is  $0.5 \Omega$  and shunt field resistance is  $250 \Omega$ . Calculate the efficiency of the machine when delivering full-load output of 12.5 kW while operating at 250 V. **[81.57%] (Elect. Technology-I Madras Univ. 1979)**

3. The following results were obtained during Hopkinson's test on two similar 230-V machines; armature currents 37 A and 30 A; field currents 0.85 A and 0.8 A. Calculate the efficiencies of machines if each has an armature resistance of  $0.33 \Omega$ . **[Generator 87.9%, Motor 87.7%]**

4. In a Field's test on two 230-V, 1.492 kW mechanically-coupled similar series motors, the following figures were obtained. Each has armature and compole winding resistance of  $2.4 \Omega$ , series field resistance of  $1.45 \Omega$  and total brush drop of 2 V. The p.d. across armature and field was 230 V with a motor current of 10.1 A. The generator supplied a current of 8.9 A at a terminal p.d. of 161 V. Calculate the efficiency and output of the motor for this load. **[76.45%, 1.775 kW]**

5. Describe the Hopkinson's test for obtaining the efficiency of two similar shunt motors. The readings obtained in such a test were as follows; line voltage 100 V; motor current 30 A; generator current 25 A; armature resistance of each machine  $0.25 \Omega$ . Calculate the efficiency of each machine from these results, ignoring the field currents and assuming that their iron and mechanical losses are the same.

**[Motor 90.05%, Generator 92.5%]**

6. The Hopkinson's test on two similar d.c. shunt machines gave the following results :

Line voltage = 220 V ; line current excluding field currents = 40 A ; the armature current of motoring machine = 200 A ; field currents 6 A and 7 A. Calculate the efficiency of each of the machines at the given load conditions. The armature resistance of each of the machines is  $0.05 \Omega$ .

**[\eta\_m = 86.58% ; \eta\_g = 86.3%] (Electrical Engg-I, M.S. Univ. Baroda 1980)**

**OBJECTIVE TEST – 31**

- One of the main advantages of Swinburne's test is that it
  - is applicable both to shunt and compound motors
  - needs one running test
  - is very economical and convenient
  - ignores any change in iron loss
- The main disadvantage of Hopkinson's test for finding efficiency of shunt d.c. motors is that it
  - requires full-load power
  - ignores any change in iron loss
  - needs one motor and one generator
  - requires two identical shunt machines
- The most economical method of finding no-load losses of a large d.c. shunt motor is—test.
  - Hopkinson's
  - Swinburne's
  - retardation
  - Field's
- Retardation test on a d.c. shunt motor is used for finding—losses.
  - stray
  - copper
  - friction
  - iron

5. The main thing common between Hopkinson's test and Field's test is that both
- require two electrically-coupled series motors
  - need two similar mechanically-coupled motors
  - use negligible power
  - are regenerative tests
6. The usual test for determining the efficiency of a traction motor is the ..... test.
- Field's
  - retardation
  - Hopkinson's
  - Swinburne's

## ANSWERS

1. c 2. d 3. b 4. a 5. b 6. a

## QUESTIONS AND ANSWERS ON D.C. MOTORS

**Q. 1. How may the direction of rotation of a d.c. motor be reversed ?**

**Ans.** By reversing either the field current or current through the armature. Usually, reversal of current through the armature is adopted.

**Q. 2. What will happen if both currents are reversed ?**

**Ans.** The motor will run in the original direction.

**Q. 3. What will happen if the field of a d.c. shunt motor is opened ?**

**Ans.** The motor will achieve dangerously high speed and may destroy itself.

**Q. 4. What happens if the direction of current at the terminals of a series motor is reversed ?**

**Ans.** It does not reverse the direction of rotation of motor because current flows through the armature in the same direction as through the field.

**Q. 5. Explain what happens when a d.c. motor is connected across an a.c. supply ?**

**Ans. 1.** Since on a.c. supply, reactance will come into the picture, the a.c. supply will be offered impedance (not resistance) by the armature winding. Consequently, with a.c. supply, current will be much less. The motor will run but it would not carry the same load as it would on d.c. supply.

**2.** There would be more sparking at the brushes.

**3.** Though motor armature is laminated as a rule, the field poles are not. Consequently, eddy currents will cause the motor to heat up and eventually burn on a.c. supply.

**Q. 6. What will happen if a shunt motor is directly connected to the supply line ?**

**Ans.** Small motors up to 1 kW rating may be line-started without any adverse results being produced. High rating motors must be started through a suitable starter in order to avoid the huge starting current which will

(i) damage the motor itself and (ii) badly affect the voltage regulation of the supply line.

**Q. 7. What is the function of interpoles and how are interpole windings connected ?**

**Ans.** Interpoles are small poles placed in between the main poles. Their function is to assist commutation by producing the auxiliary or commutating flux. Consequently, brush sparking is practically eliminated. Interpole windings are connected in series with the armature windings.

**Q. 8. In rewinding the armature of a d.c. motor, progressive connections are changed to retrogressive ones. Will it affect the operation in any way ?**

**Ans.** Yes. Now, the armature will rotate in the opposite direction.

**Q. 9. A d.c. motor fails to start when switched on. What could be the possible reasons and remedies ?**

**Ans.** Any one of the following reasons could be responsible :

1. Open-circuit in controller—should be checked for open starting resistance or open switch or open fuse.
2. Low terminal voltage—should be adjusted to name-plate value.
3. Overload—should be reduced if possible otherwise larger motor should be installed.
4. Excessive friction—bearing lubrication should be checked.

**Q. 10. A d.c. motor is found to stop running after a short period of time. What do you think could be the reasons ? How would you remedy each ?**

**Ans.** Possible causes are as under :

1. Motor not getting enough power—check voltage at motor terminals as well as fuses, clups and overload relay.
2. Weak or no field—in the case of adjustable-speed motors, check if rheostat is correctly set. Also, check field winding for any 'open'. Additionally, look for any loose winding or broken connection.
3. Motor torque insufficient for driving the given load—check line voltage with name-plate voltage. If necessary, use larger motor to match the load.

**Q. 11. What are the likely causes if a d.c. motor is found to run too slow under load ? And the remedy ?**

- Ans.**
1. Supply line voltage too low—remove any excessive resistance in supply line, connections or controller.
  2. Brushes ahead of neutral—set them on neutral.
  3. Overload—reduce it to allowable value or use larger motor.

**Q. 12. Why does a d.c. motor sometime run too fast when under load ? Give different possible causes and their remedies.**

**Ans.** Different possible causes are as under :

1. Weak field—remove any extra resistance in shunt field circuit. Also, check for 'grounds'.
2. Line voltage too high—reduce it to name-plate value.
3. Brushes back of neutral—set them on neutral.

**Q. 13. Under what conditions is sparking produced at the brushes of a d.c. motor ? How would you remedy it ?**

- Ans.**
1. Commutator in bad condition—clean and reset brushes.
  2. commutator either eccentric or rough—grind and true the commutator. Also, undercut mica.
  3. Excessive vibration—balance armature. Make sure that brushes ride freely in holders.
  4. Brush-holding spring broken or sluggish—replace spring and adjust pressure to recommended value.
  5. Motor overloaded—reduce load or install motor of proper rating.
  6. Short-circuit in armature circuit—remove any metallic particles between commutator segments and check for short between adjacent commutator risers. Locate and repair internal armature short if any.

**Q. 14. Sometimes a hissing noise (or brush chatter) is heard to emanate from the commutator end of a running d.c. motor. What could it be due to and how could it be removed ?**

**Ans.** Any one of the following causes could produce brush chatter :

1. Excessive clearance of brush holders—adjust properly
2. Incorrect angle of brushes—adjust to correct value
3. Unsuitable brushes—replace them
4. High mica—undercut it
5. Wrong brush spring pressure—adjust to correct value.

**Q. 15. What are the possible causes of excessive sparking at brushes in a d.c. motor ?**

- Ans.**
1. Poor brush fit on commutator—sand-in the brushes and polish commutator.
  2. Brushes binding in the brush holders—clean holders and brushes and remove any irregularities on surfaces of brush holders or rough spots on brushes.
  3. Excessive or insufficient pressure on brushes—adjust pressure.
  4. Brushes off neutral—set them on neutral.

**Q. 16. Why does a d.c. motor sometime spark on light load ?**

**Ans.** Due to the presence of paint spray, chemical, oil or grease etc. on commutator.

**Q. 17. When is the armature of a d.c. motor likely to get over-heated ?**

- Ans.**
1. When motor is over-loaded.
  2. When it is installed at a place having restricted ventilation.
  3. When armature winding is shorted.

**Q. 18. What causes are responsible for over-heating of commutator in a d.c. motor ?**

**Ans.** It could be due either to the brushes being off neutral or being under excessive spring pressure. Accordingly, brushes should be adjusted properly and the spring pressure should be reduced but not to the point where sparking is introduced.

### Learning Objectives

- Working Principle of Transformer
- Transformer Construction
- Core-type Transformers
- Shell-type Transformers
- E.M.F. Equation of Transformer
- Voltage Transformation Ratio
- Transformer with losses
- Equivalent Resistance
- Magnetic Leakage
- Transformer with Resistance and Leakage Reactance
- Total Approximate Voltage Drop in Transformer
- Exact Voltage Drop
- Separation of Core Losses
- Short-Circuit or Impedance Test
- Why Transformer Rating in KVA?
- Regulation of a Transformer
- Percentage Resistance, Reactance and Impedance
- Kapp Regulation Diagram
- Sumpner or Back-to-back-Test
- Efficiency of a Transformer
- Auto-transformer

## TRANSFORMER



To overcome losses, the electricity from a generator is passed through a step-up transformer, which increases the voltage. Throughout the distribution system, the voltages are changed using step-down transformers to voltages suitable to the applications at industry and homes.

### 32.1. Working Principle of a Transformer

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is *mutual induction* between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig. 32.1. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction  $e = Mdi/dt$ ). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called *primary* winding and the other from which energy is drawn out, is called *secondary* winding. In brief, a transformer is a device that

1. transfers electric power from one circuit to another
2. it does so without a change of frequency
3. it accomplishes this by electromagnetic induction and
4. where the two electric circuits are in mutual inductive influence of each other.

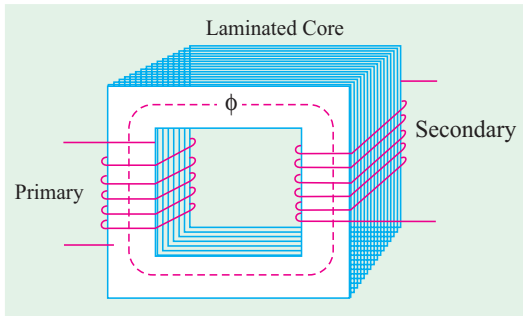


Fig. 32.1

### 32.2. Transformer Construction

The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are : some suitable container for assembled core and windings ; a suitable medium for insulating the core and its windings from its container ; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.

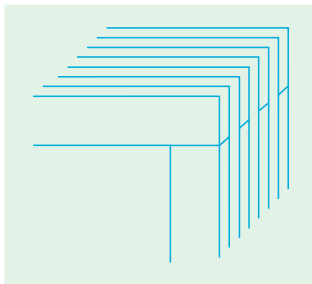
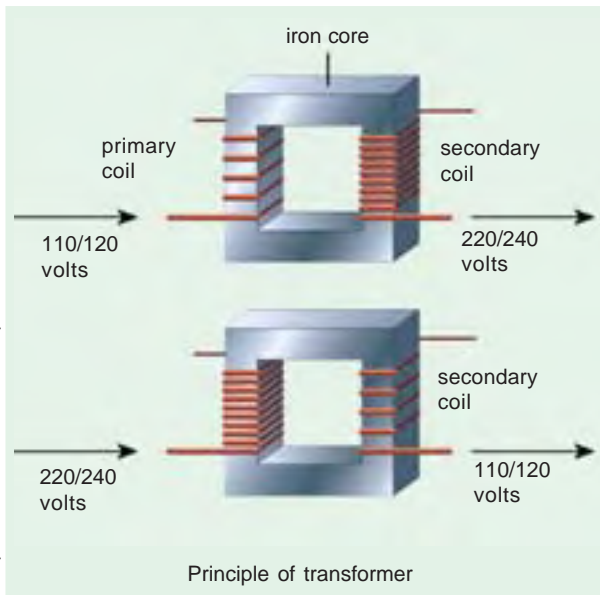


Fig. 32.2

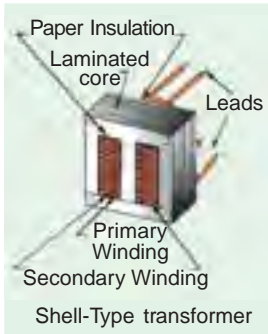
In all types of transformers,

the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum of air-gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the





usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz. The core laminations (in the form of strips) are joined as shown in Fig. 32.2. It is seen that the joints in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross-section of the core. Such staggered joints are said to be 'imbricated'.



Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core. The two types are known as (i) core-type and (ii) shell-type. Another recent development is spiral-core or wound-core type, the trade name being spirakore transformer.

In the so-called core type transformers, the windings surround a considerable part of the core whereas in shell-type transformers, the core surrounds a considerable portion of the windings as shown schematically in Fig. 32.3 (a) and (b) respectively.

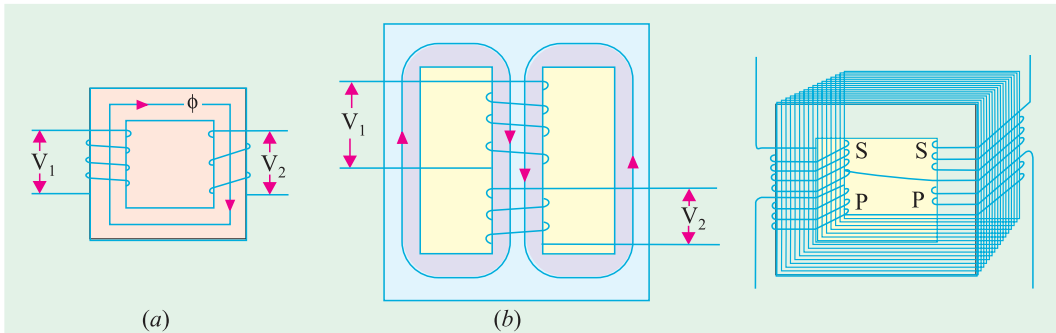
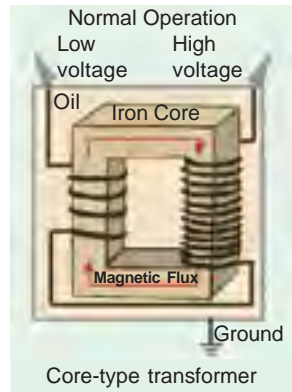


Fig. 32.3

Fig. 32.4

In the simplified diagram for the core type transformers [Fig. 32.3 (a)], the primary and secondary winding are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux. As shown in Fig. 32.4, half the primary and half the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or leg) and the secondary on the other.

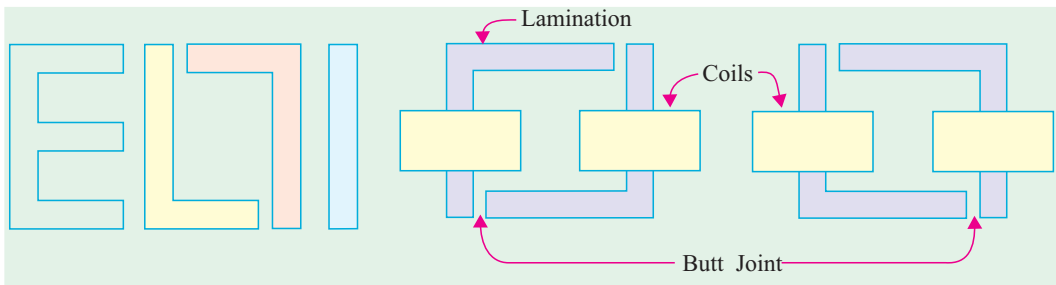


Fig. 32.5

Fig. 32.6

In both core and shell-type transformers, the individual laminations are cut in the form of long strips of L's, E's and I's as shown in Fig. 32.5. The assembly of the complete core for the two types of transformers is shown in Fig. 32.6 and Fig. 32.7.

As said above, in order to avoid high reluctance at the joints where the laminations are butted against each other, the alternate layers are stacked differently to eliminate these joints as shown in Fig. 32.6 and Fig. 32.7.

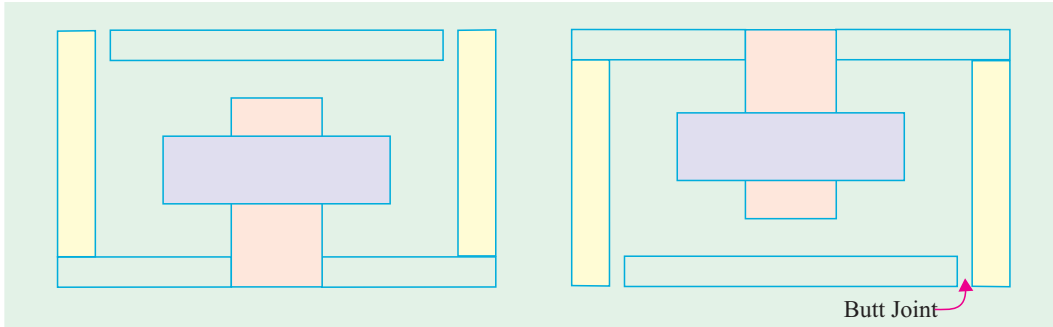


Fig. 32.7

### 32.3. Core-type Transformers

The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval or rectangular. In small size core-type transformers, a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large-size core-type transformers, round

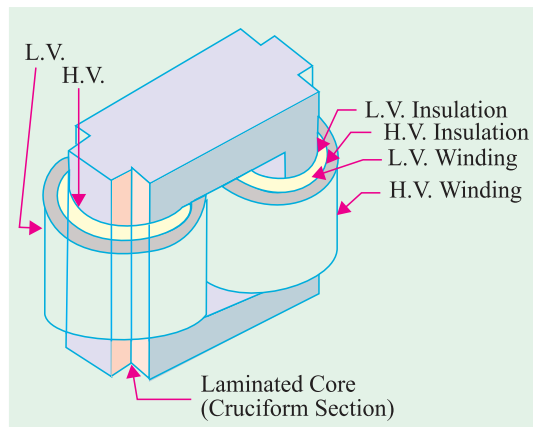
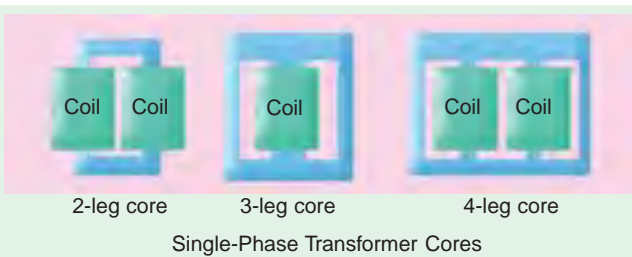


Fig. 32.8 (a)

or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 32.8(a). The circular cylindrical coils are used in most of the core-type transformers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from each other by paper, cloth, micarta board or cooling ducts. Fig. 32.8(c) shows the general arrangement of these coils with respect to the core. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. Since the low-voltage (LV) winding is easiest to insulate, it is placed nearest to the core (Fig. 32.8).

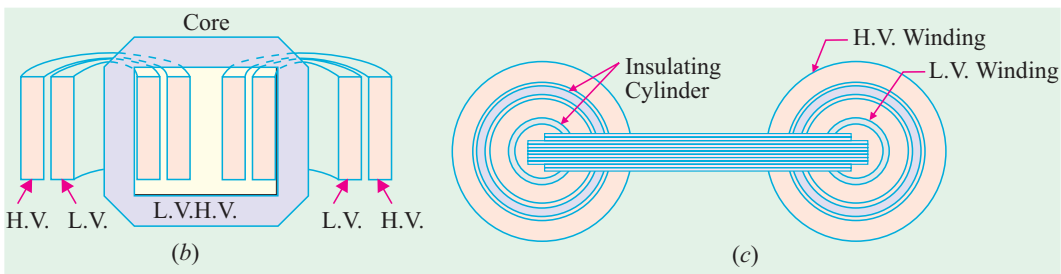


Fig. 32.8

Because of laminations and insulation, the net or effective core area is reduced, due allowance for which has to be made (Ex. 32.6). It is found that, in general, the reduction in core sectional area due to the presence of paper, surface oxide etc. is of the order of 10% approximately.

As pointed out above, rectangular cores with rectangular cylindrical coils can be used for small-size core-type transformers as shown in Fig. 32.9 (a) but for large-sized transformers, it becomes wasteful to use rectangular cylindrical coils and so circular cylindrical coils are preferred. For such purposes, square cores may be used as shown in Fig. 32.9 (b) where circles represent the tubular former carrying the coils. Obviously, a considerable amount of useful space is still wasted. A common improvement on square core is to employ cruciform core as in Fig. 32.9 (c) which demands, at least, two sizes of core strips. For very large transformers, further core-stepping is done as in Fig. 32.9 (d) where at least three sizes of core plates are necessary. Core-stepping not only gives high space factor but also results in reduced length of the mean turn and the consequent  $I^2R$  loss. Three stepped core is the one most commonly used although more steps may be used for very large transformers as in Fig. 32.9 (e). From the geometry of Fig. 32.9, it can be shown that maximum gross core section for Fig. 32.9 (b) is  $0.5 d^2$  and for Fig. 32.9 (c) it is  $0.616 d^2$  where  $d$  is the diameter of the cylindrical coil.

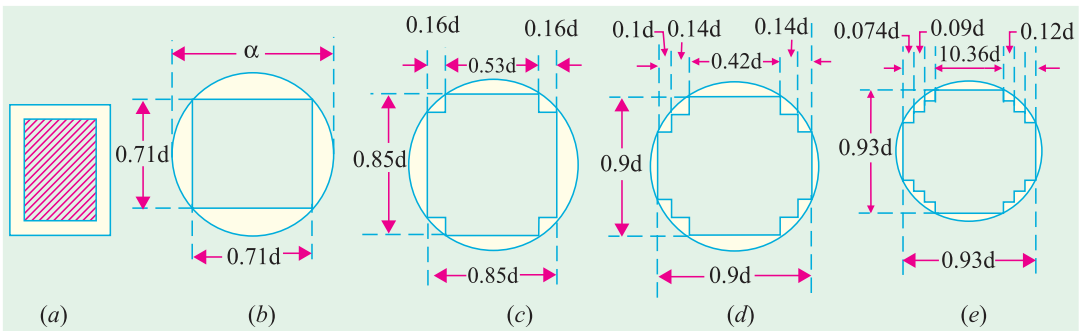


Fig. 32.9

### 32.4. Shell-type Transformers

In these case also, the coils are form-wound but are multi-layer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper. The complete winding consists of stacked discs with insulation space between the coils—the spaces forming horizontal cooling and insulating ducts. A shell-type transformer may have a simple rectangular form as shown in Fig. 32.10 or it may have distributed form as shown in Fig. 32.11.

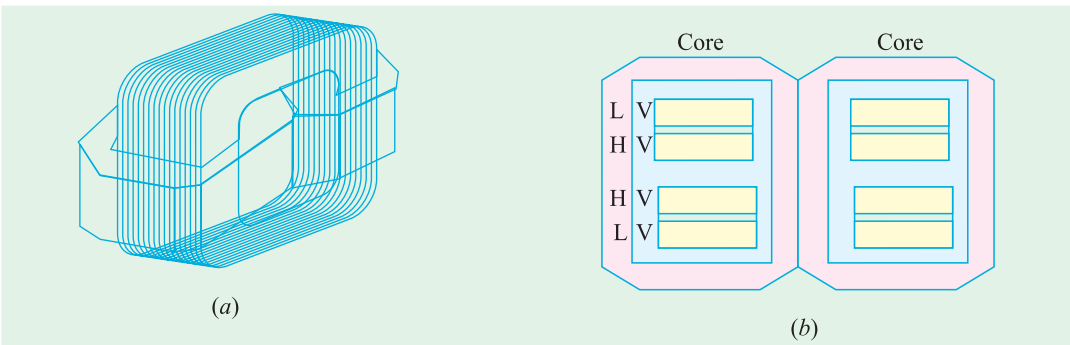


Fig. 32.10

A very commonly-used shell-type transformer is the one known as Berry Transformer—so called after the name of its designer and is cylindrical in form. The transformer core consists of laminations arranged in groups which radiate out from the centre as shown in section in Fig. 32.12.

It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise—a humming sound—during operation.

The spiral-core transformer employs the newest development in core construction. The core is assembled of a continuous strip or ribbon of transformer steel wound in the form of a circular or elliptical cylinder. Such construction allows the core flux to follow the grain of the iron. Cold-rolled steel of high silicon content enables the designer to use considerably higher operating flux densities with lower loss per kg. The use of higher flux density reduces the weight per kVA. Hence, the advantages of such construction are (i) a relatively more rigid core (ii) lesser weight and size per kVA rating (iii) lower iron losses at higher operating flux densities and (iv) lower cost of manufacture.

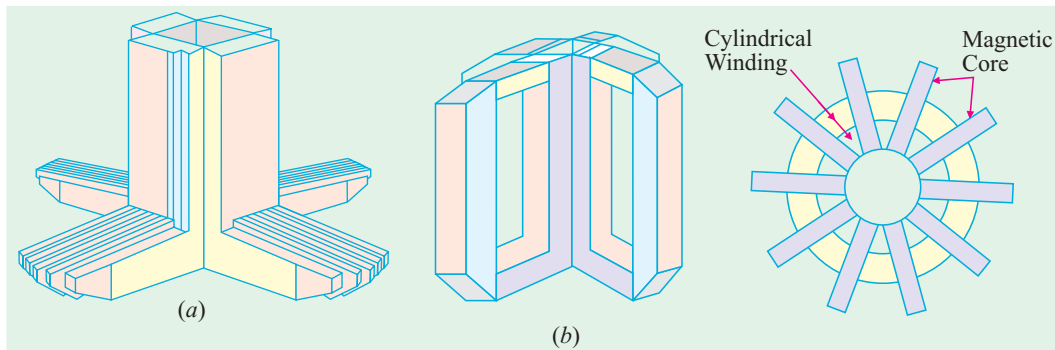


Fig. 32.11

Fig. 32.12

Transformers are generally housed in tightly-fitted sheet-metal tanks filled with special insulating oil\*. This oil has been highly developed and its function is two-fold. By circulation, it not only keeps the coils reasonably cool, but also provides the transformer with additional insulation not obtainable when the transformer is left in the air.

In cases where a smooth tank surface does not provide sufficient cooling area, the sides of the tank are corrugated or provided with radiators mounted on the sides. Good transformer oil should be absolutely free from alkalies, sulphur and particularly from moisture. The presence of even an extremely small percentage of moisture in the oil is highly detrimental from the insulation viewpoint because it lowers the dielectric strength of the oil considerably. The importance of avoiding moisture in the transformer oil is clear from the fact that even an addition of 8 parts of water in 1,000,000 reduces the insulating quality of the oil to a value generally recognized as below standard. Hence, the tanks are sealed air-tight in smaller units. In the case of large-sized transformers where complete air-tight construction is impossible, chambers known as *breathers* are provided to permit the oil inside the tank to expand and contract as its temperature increases or decreases. The atmospheric moisture is entrapped in these breathers and is not allowed to pass on to the oil. Another thing to avoid in the oil is slogging which is simply the decomposition of oil with long and continued use. Slogging is caused principally by exposure to oxygen during heating and results in the formation of large deposits of dark and heavy matter that eventually clogs the cooling ducts in the transformer.

No other feature in the construction of a transformer is given more attention and care than the insulating materials, because the life on the unit almost solely depends on the quality, durability and handling of these materials. All the insulating materials are selected on the basis of their high quality and ability to preserve high quality even after many years of normal use.

\* Instead of natural mineral oil, now-a-days synthetic insulating fluids known as ASKARELS (trade name) are used. They are non-inflammable and, under the influence of an electric arc, do not decompose to produce inflammable gases. One such fluid commercially known as PYROCLOR is being extensively used because it possesses remarkable stability as a dielectric and even after long service shows no deterioration through slogging, oxidation, acid or moisture formation. Unlike mineral oil, it shows no rapid burning.

All the transformer leads are brought out of their cases through suitable bushings. There are many designs of these, their size and construction depending on the voltage of the leads. For moderate voltages, porcelain bushings are used to insulate the leads as they come out through the tank. In general, they look almost like the insulators used on the transmission lines. In high voltage installations, oil-filled or capacitor-type bushings are employed.

The choice of core or shell-type construction is usually determined by cost, because similar characteristics can be obtained with both types. For very high-voltage transformers or for multiwinding design, shell-type construction is preferred by many manufacturers. In this type, usually the mean length of coil turn is longer than in a comparable core-type design. Both core and shell forms are used and the selection is decided by many factors such as voltage rating, kVA rating, weight, insulation stress, heat distribution etc.

Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use :

- (a) *oil-filled self-cooled*      (b) *oil-filled water-cooled*      (c) *air-blast type*

Small and medium size distribution transformers—so called because of their use on distribution systems as distinguished from line transmission—are of type (a). The assembled windings and cores of such transformers are mounted in a welded, oil-tight steel tank provided with steel cover. After putting the core at its proper place, the tank is filled with purified, high quality insulating oil. The oil serves to convey the heat from the core and the windings to the case from where it is radiated out to the surroundings. For small size, the tanks are usually smooth-surfaced, but for larger sizes, the cases are frequently corrugated or fluted to get greater heat radiation area without increasing the cubical capacity of the tank. Still larger sizes are provided with radiators or pipes.

Construction of very large self-cooled transformers is expensive, a more economical form of construction for such large transformers is provided in the oil-immersed, water-cooled type. As before, the windings and the core are immersed in the oil, but there is mounted near the surface of oil, a cooling coil through which cold water is kept circulating. The heat is carried away by this water. The largest transformers such as those used with high-voltage transmission lines, are constructed in this manner.

Oil-filled transformers are built for outdoor duty and as these require no housing other than their own, a great saving is thereby effected. These transformers require only periodic inspection.

For voltages below 25,000 V, transformers can be built for cooling by means of an air-blast. The transformer is not immersed in oil, but is housed in a thin sheet-metal box open at both ends through which air is blown from the bottom to the top by means of a fan or blower.

### 32.5. Elementary Theory of an Ideal Transformer

An ideal transformer is one which has no losses *i.e.* its windings have no ohmic resistance, there is no magnetic leakage and hence which has no  $I^2R$  and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted *that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer.*

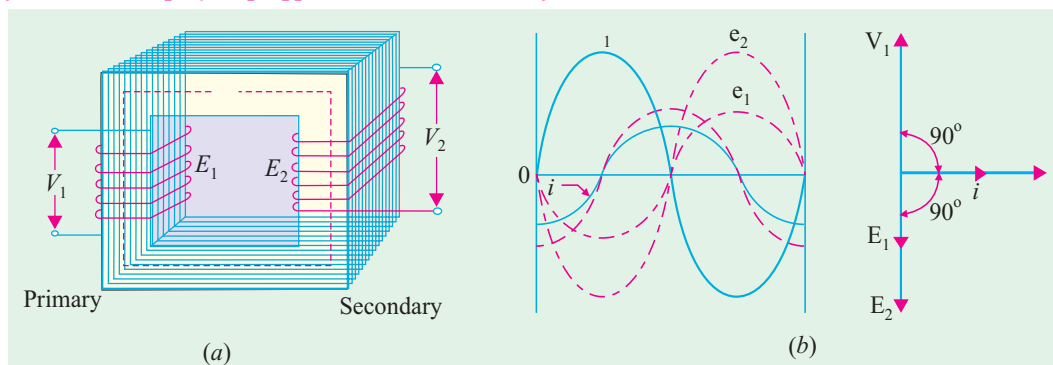
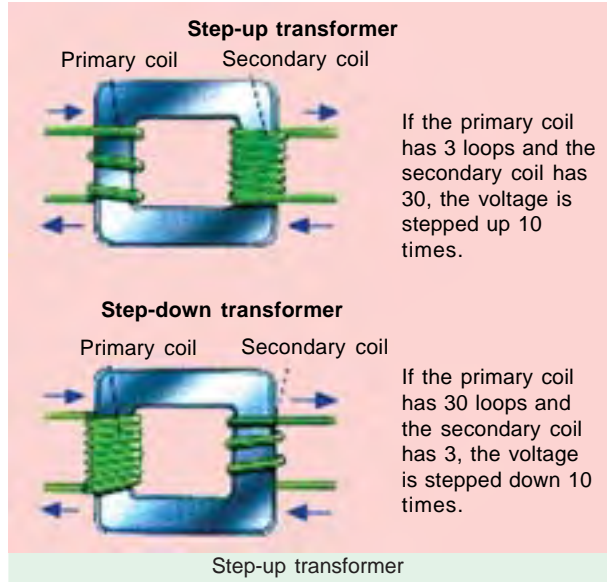


Fig. 32.13

Consider an ideal transformer [Fig. 32.13 (a)] whose secondary is open and whose primary is connected to sinusoidal alternating voltage  $V_1$ . This potential difference causes an alternating current to flow in the primary. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current  $I_\mu$  only. The function of this current is merely to magnetise the core, it is small in magnitude and lags  $V_1$  by  $90^\circ$ . This alternating current  $I_\mu$  produces an alternating flux  $\phi$  which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary. This *self-induced* e.m.f.  $E_1$  is, at every instant, equal to and in opposition to  $V_1$ . It is also known as counter e.m.f. or back e.m.f. of the primary.

Similarly, there is produced in the secondary an induced e.m.f.  $E_2$  which is known as *mutually* induced e.m.f. This e.m.f. is antiphase with  $V_1$  and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

The instantaneous values of applied voltage, induced e.m.fs, flux and magnetising current are shown by sinusoidal waves in Fig. 32.13 (b). Fig. 32.13 (c) shows the vectorial representation of the effective values of the above quantities.



### 32.6. E.M.F. Equation of a Transformer

- Let  $N_1$  = No. of turns in primary
- $N_2$  = No. of turns in secondary
- $\Phi_m$  = Maximum flux in core in webers  
 $= B_m \times A$
- $f$  = Frequency of a.c. input in Hz

As shown in Fig. 32.14, flux increases from its zero value to maximum value  $\Phi_m$  in one quarter of the cycle *i.e.* in  $1/4f$  second.

$$\therefore \text{Average rate of change of flux} = \frac{\Phi_m}{1/4f}$$

$$= 4f\Phi_m \text{ Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\therefore \text{Average e.m.f./turn} = 4f\Phi_m \text{ volt}$$

If flux  $\Phi$  varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$$\therefore \text{r.m.s. value of e.m.f./turn} = 1.11 \times 4f\Phi_m = 4.44f\Phi_m \text{ volt}$$

Now, r.m.s. value of the induced e.m.f. in the whole of primary winding

$$= (\text{induced e.m.f./turn}) \times \text{No. of primary turns}$$

$$E_1 = 4.44fN_1\Phi_m = 4.44fN_1B_mA \quad \dots(i)$$

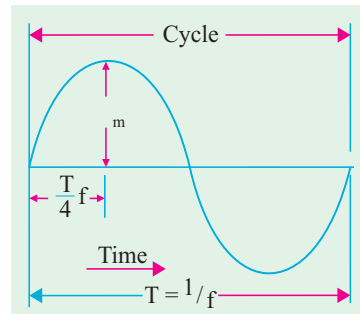


Fig. 32.14

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E_2 = 4.44 f N_2 \Phi_m = 4.44 f N_2 B_m A \quad \dots (ii)$$

It is seen from (i) and (ii) that  $E_1/N_1 = E_2/N_2 = 4.44 f \Phi_m$ . It means that e.m.f./turn is the same in both the primary and secondary windings.

In an ideal transformer on no-load,  $V_1 = E_1$  and  $E_2 = V_2$  where  $V_2$  is the terminal voltage (Fig. 32.15).

### 32.7 Voltage Transformation Ratio (K)

From equations (i) and (ii), we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This constant  $K$  is known as voltage transformation ratio.

(i) If  $N_2 > N_1$  i.e.  $K > 1$ , then transformer is called **step-up** transformer.

(ii) If  $N_2 < N_1$  i.e.  $K < 1$ , then transformer is known as **step-down** transformer.

Again, for an ideal transformer, input VA = output VA.

$$V_1 I_1 = V_2 I_2 \text{ or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

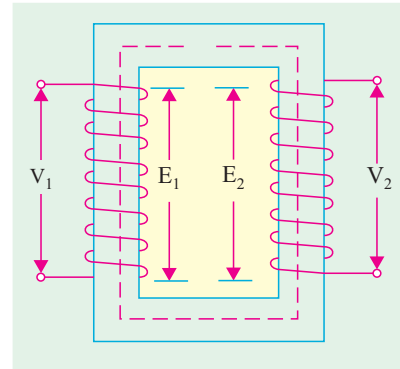


Fig. 32.15

**Example 32.1.** The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is  $1.2 \text{ Wb/m}^2$ . If the e.m.f. per turn is 8 volt, determine

- (i) primary and secondary turns (ii) area of the core.

(Electrical Engg.-I, Nagpur Univ. 1991)

**Solution. (i)**

$$E_1 = N_1 \times \text{e.m.f. induced/turn}$$

$$N_1 = 250/8 = 32; N_2 = 3000/8 = 375$$

(ii) We may use

$$E_2 = -4.44 f N_2 B_m A$$

$$\therefore 3000 = 4.44 \times 50 \times 375 \times 1.2 \times A; A = 0.03\text{m}^2.$$

**Example 32.2.** The core of a 100-kVA, 11000/550 V, 50-Hz, 1-ph, core type transformer has a cross-section of  $20 \text{ cm} \times 20 \text{ cm}$ . Find (i) the number of H.V. and L.V. turns per phase and (ii) the e.m.f. per turn if the maximum core density is not to exceed 1.3 Tesla. Assume a stacking factor of 0.9.

What will happen if its primary voltage is increased by 10% on no-load ?

(Elect. Machines, A.M.I.E. Sec. B, 1991)

**Solution. (i)**

$$B_m = 1.3 \text{ T}, A = (0.2 \times 0.2) \times 0.9 = 0.036 \text{ m}^2$$

$$\therefore 11,000 = 4.44 \times 50 \times N_1 \times 1.3 \times 0.036, N_1 = 1060$$

$$550 = 4.44 \times 50 \times N_2 \times 1.3 \times 0.036; N_2 = 53$$

or,  $N_2 = K N_1 = (550/11,000) \times 1060 = 53$

(ii) e.m.f./turn =  $11,000/1060 = 10.4 \text{ V}$  or  $550/53 = 10.4 \text{ V}$

Keeping supply frequency constant, if primary voltage is increased by 10%, magnetising current will increase by much more than 10%. However, due to saturation, flux density will increase only marginally and so will the eddy current and hysteresis losses.

**Example 32.3.** A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is  $60 \text{ cm}^2$ . If the primary winding be connected to a 50-Hz supply at 520 V, calculate (i) the peak value of flux density in the core (ii) the voltage induced in the secondary winding.

(Elect. Engg.-I, Pune Univ. 1989)

**Solution.**

$$K = N_2/N_1 = 1000/400 = 2.5$$

$$(i) \quad E_2/E_1 = K \quad \therefore \quad E_2 = KE_1 = 2.5 \times 520 = \mathbf{1300 \text{ V}}$$

$$(ii) \quad E_1 = 4.44 f N_1 B_m A$$

$$\text{or} \quad 520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4}) \quad \therefore \quad B_m = \mathbf{0.976 \text{ Wb/m}^2}$$

**Example 32.4.** A 25-kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core. Neglect leakage drops and no-load primary current. **(Elect. & Electronic Engg., Madras Univ. 1985)**

**Solution.**

$$K = N_2/N_1 = 50/500 = 1/10$$

$$\text{Now, full-load} \quad I_1 = 25,000/3000 = \mathbf{8.33 \text{ A}}. \quad \text{F.L. } I_2 = I_1/K = 10 \times 8.33 = \mathbf{83.3 \text{ A}}$$

$$\text{e.m.f. per turn on primary side} = 3000/500 = 6 \text{ V}$$

$$\therefore \quad \text{secondary e.m.f.} = 6 \times 50 = \mathbf{300 \text{ V}} \quad (\text{or } E_2 = KE_1 = 3000 \times 1/10 = 300 \text{ V})$$

$$\text{Also,} \quad E_1 = 4.44 f N_1 \Phi_m; \quad 3000 = 4.44 \times 50 \times 500 \times \Phi_m \quad \therefore \quad \Phi_m = \mathbf{27 \text{ mWb}}$$

**Example 32.5.** The core of a three phase, 50 Hz, 11000/550 V delta/star, 300 kVA, core-type transformer operates with a flux of 0.05 Wb. Find

(i) number of H.V. and L.V. turns per phase. (ii) e.m.f. per turn

(iii) full load H.V. and L.V. phase-currents. **(Bharathithasan Univ. April 1997)**

**Solution.** Maximum value of flux has been given as 0.05 Wb.

$$(ii) \quad \text{e.m.f. per turn} = 4.44 f \phi_m \\ = 4.44 \times 50 \times 0.05 = 11.1 \text{ volts}$$

(i) Calculations for number of turns on two sides :

Voltage per phase on delta-connected primary winding = 11000 volts

Voltage per phase on star-connected secondary winding =  $550/1.732 = 317.5$  volts

$$T_1 = \text{number of turns on primary, per phase} \\ = \text{voltage per phase/e.m.f. per turn} \\ = 11000/11.1 = 991$$

$$T_2 = \text{number of turns on secondary, per phase} \\ = \text{voltage per phase/e.m.f. per turn} \\ = 317.5/11.1 = 28.6$$

**Note :** (i) Generally, Low-voltage-turns are calculated first, the figure is rounded off to next higher even integer. In this case, it will be 30. Then, number of turns on primary side is calculated by turns-ratio.

$$\text{In this case,} \quad T_1 = T_2 (V_1/V_2) = 30 \times 11000/317.5 = 1040$$

This, however, reduces the flux and results into less saturation. This, in fact, is an elementary aspect in Design-calculations for transformers. (Explanation is added here only to overcome a doubt whether a fraction is acceptable as a number of L.V. turns).

(ii) Full load H.V. and L.V. phase currents :

$$\text{Output per phase} = (300/3) = 100 \text{ kVA}$$

$$\text{H.V. phase-current} = \frac{100 \times 1000}{11,000} = 9.1 \text{ Amp}$$

$$\text{L.V. phase-current} = (100 \times 1000/317.5) = 315 \text{ Amp}$$

**Example 32.6.** A single phase transformer has 500 turns in the primary and 1200 turns in the secondary. The cross-sectional area of the core is 80 sq. cm. If the primary winding is connected to a 50 Hz supply at 500 V, calculate (i) Peak flux-density, and (ii) Voltage induced in the secondary.

**(Bharathiar University November 1997)**



**Solution.** From the e.m.f. equation for transformer,

$$500 = 4.44 \times 50 \times \phi_m \times 500$$

$$\phi_m = 1/222 \text{ Wb}$$

(i) Peak flux density,  $B_m = \phi_m / (80 \times 10^{-4}) = 0.563 \text{ wb/m}^2$

(ii) Voltage induced in secondary is obtained from transformation ratio or turns ratio

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

or  $V_2 = 500 \times 1200/500 = 1200 \text{ volts}$

**Example 32.7.** A 25 kVA, single-phase transformer has 250 turns on the primary and 40 turns on the secondary winding. The primary is connected to 1500-volt, 50 Hz mains. Calculate (i) Primary and Secondary currents on full-load, (ii) Secondary e.m.f., (iii) maximum flux in the core.

(Bharathiar Univ. April 1998)

**Solution. (i)** If  $V_2 =$  Secondary voltage rating, = secondary e.m.f.,

$$\frac{V_2}{1500} = \frac{40}{250}, \text{ giving } V_2 = 240 \text{ volts}$$

(ii) Primary current =  $25000/1500 = 16.67 \text{ amp}$

Secondary current =  $25000/240 = 104.2 \text{ amp}$

(iii) If  $\phi_m$  is the maximum core-flux in Wb,

$$1500 = 4.44 \times 50 \times \phi_m \times 250, \text{ giving } \phi_m = 0.027 \text{ Wb or } 27 \text{ mWb}$$

**Example 32.8.** A single-phase, 50 Hz, core-type transformer has square cores of 20 cm side. Permissible maximum flux-density is  $1 \text{ Wb/m}^2$ . Calculate the number of turns per Limb on the High and Low-voltage sides for a 3000/220 V ratio. (Manonmaniam Sundaranar Univ. April 1998)

**Solution.** E.M.F. equation gives the number of turns required on the two sides. We shall first calculate the L.V.-turns, round the figure off to the next higher even number, so that given maximum flux density is not exceeded. With the corrected number of L.V. turns, calculate H.V.-turns by transformation ratio. Further, there are two Limbs. Each Limb accommodates half-L.V. and half H.V. winding from the view-point of reducing leakage reactance.

Starting with calculation for L.V. turns,  $T_2$ ,

$$4.44 \times 50 \times [(20 \times 20 \times 10^{-4}) \times 1] \times T_2 = 220$$

$$T_2 = 220/8.88 = 24.77$$

Select  $T_2 = 26$

$$T_1/T_2 = V_1/V_2$$

$$T_1 = 26 \times 3000/220 = 354, \text{ selecting the nearest even integer.}$$

Number of H.V. turns on each Limb = 177

Number of L.V. turns on each Limb = 13

## 32.8. Transformer with Losses but no Magnetic Leakage

We will consider two cases (i) when such a transformer is on no load and (ii) when it is loaded.

## 32.9. Transformer on No-load

In the above discussion, we assumed an ideal transformer *i.e.* one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing

theory. When an *actual* transformer is put on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Even when the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply (i) iron losses in the core *i.e.* hysteresis loss and eddy current loss and (ii) a very small amount of copper loss in primary (there being no Cu loss in secondary as it is open). Hence, the no-load primary input current  $I_0$  is not at  $90^\circ$  behind  $V_1$  but lags it by an angle  $\phi_0 < 90^\circ$ . No-load input power

$$W_0 = V_1 I_0 \cos \phi_0$$

where  $\cos \phi_0$  is primary power factor under no-load conditions. No-load condition of an actual transformer is shown vectorially in Fig. 32.16.

As seen from Fig. 32.16, primary current  $I_0$  has two components :

(i) One in phase with  $V_1$ . This is known as *active* or *working* or *iron* loss component  $I_w$  because it mainly supplies the iron loss plus small quantity of primary Cu loss.

$$I_w = I_0 \cos \phi_0$$

(ii) The other component is in quadrature with  $V_1$  and is known as *magnetising* component  $I_\mu$  because its function is to sustain the alternating flux in the core. It is wattless.

$$I_\mu = I_0 \sin \phi_0$$

Obviously,  $I_0$  is the vector sum of  $I_w$  and  $I_\mu$ , hence  $I_0 = \sqrt{I_\mu^2 + I_w^2}$ .

The following points should be noted carefully :

1. The no-load primary current  $I_0$  is very small as compared to the full-load primary current. It is about 1 per cent of the full-load current.
2. Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting or magnetising current, the wave of the exciting or magnetising current is not truly sinusoidal. As such it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors. But, in practice, it makes no appreciable difference.
3. As  $I_0$  is very small, the no-load primary Cu loss is negligibly small which means *that no-load primary input is practically equal to the iron loss in the transformer.*
4. As it is principally the core-loss which is responsible for shift in the current vector, angle  $\phi_0$  is known as *hysteresis angle of advance.*

**Example 32.9.** (a) A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetising and iron loss currents.

(b) A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetising and working components of no-load primary current.

**Solution.** (a) Iron-loss current

$$= \frac{\text{no-load input in watts}}{\text{primary voltage}} = \frac{400}{2,200} = \mathbf{0.182 \text{ A}}$$

Now 
$$I_0^2 = I_w^2 + I_\mu^2$$

Magnetising component 
$$I_\mu = \sqrt{(0.6^2 - 0.182^2)} = \mathbf{0.572 \text{ A}}$$

The two components are shown in Fig. 29.15.

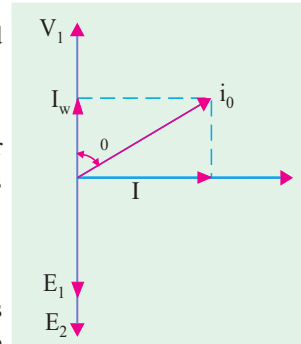


Fig. 32.16

$$(b) \quad I_0 = 0.5 \text{ A}, \cos \phi_0 = 0.3 \therefore I_w = I_0 \cos \phi_0 = 0.5 \times 0.3 = 0.15 \text{ A}$$

$$I_\mu = \sqrt{0.5^2 - 0.15^2} = 0.476 \text{ A}$$

**Example 32.10.** A single-phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the iron core is 150 cm and the joints are equivalent to an air-gap of 0.1 mm. When a p.d. of 3,000 V is applied to the primary, maximum flux density is  $1.2 \text{ Wb/m}^2$ . Calculate (a) the cross-sectional area of the core (b) no-load secondary voltage (c) the no-load current drawn by the primary (d) power factor on no-load. Given that AT/cm for a flux density of  $1.2 \text{ Wb/m}^2$  in iron to be 5, the corresponding iron loss to be 2 watt/kg at 50 Hz and the density of iron as  $7.8 \text{ gram/cm}^3$ .

**Solution.** (a)  $3,000 = 4.44 \times 50 \times 500 \times 1.2 \times A \therefore A = 0.0225 \text{ m}^2 = 225 \text{ cm}^2$

This is the net cross-sectional area. However, the gross area would be about 10% more to allow for the insulation between laminations.

(b)  $K = N_2/N_1 = 40/500 = 4/50$   
 $\therefore$  N.L. secondary voltage =  $KE_1 = (4/50) \times 3000 = 240 \text{ V}$

(c) AT per cm = 5  $\therefore$  AT for iron core =  $150 \times 5 = 750$

$$AT \text{ for air-gap} = Hl = \frac{B}{\mu_0} \times l = \frac{1.2}{4\pi \times 10^{-7}} \times 0.0001 = 95.5$$

Total AT for given  $B_{max} = 750 + 95.5 = 845.5$

Max. value of magnetising current drawn by primary =  $845.5/500 = 1.691 \text{ A}$

Assuming this current to be sinusoidal, its r.m.s. value is  $I_\mu = 1.691/\sqrt{2} = 1.196 \text{ A}$

Volume of iron = length  $\times$  area =  $150 \times 225 = 33,750 \text{ cm}^3$

Density =  $7.8 \text{ gram/cm}^3 \therefore$  Mass of iron =  $33,750 \times 7.8/1000 = 263.25 \text{ kg}$

Total iron loss =  $263.25 \times 2 = 526.5 \text{ W}$

Iron loss component of no-load primary current  $I_0$  is  $I_w = 526.5/3000 = 0.176 \text{ A}$

$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{1.196^2 + 0.176^2} = 0.208 \text{ A}$$

(d) Power factor,  $\cos \phi_0 = I_w/I_0 = 0.176/1.208 = 0.1457$

**Example 32.11.** A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 amp. at a p.f. of 0.2 lagging. Calculate the primary current and power-factor when the secondary current is 280 Amp at a p.f. of 0.80 lagging.

(Nagpur University, November 1997)

**Solution.**  $V_2$  is taken as reference.  $\cos^{-1} 0.80 = 36.87^\circ$

$$I_2 = 280 \angle -36.87^\circ \text{ amp}$$

$$I'_2 = (280/5) \angle -36.87^\circ \text{ amp}$$

$$\phi = \cos^{-1} 0.20 = 78.5^\circ, \sin \phi = 0.98$$

$$I_1 = I_0 + I'_2 = 3(0.20 - j0.98) + 56(0.80 - j0.60)$$

$$= 0.6 - j2.94 + 44.8 - j33.6$$

$$= 45.4 - j2.94 + 44.8 - j33.6$$

$$= 45.4 - j36.54 = 58.3 \angle 38.86^\circ$$

Thus  $I$  lags behind the supply voltage by an angle of  $38.86^\circ$ .

## Tutorial Problems 32.1

1. The number of turns on the primary and secondary windings of a 1- $\phi$  transformer are 350 and 35 respectively. If the primary is connected to a 2.2 kV, 50-Hz supply, determine the secondary voltage on no-load. [220 V] (Elect. Engg.-II, Kerala Univ. 1980)
2. A 3000/200-V, 50-Hz, 1-phase transformer is built on a core having an effective cross-sectional area of 150 cm<sup>2</sup> and has 80 turns in the low-voltage winding. Calculate
  - (a) the value of the maximum flux density in the core
  - (b) the number of turns in the high-voltage winding. [(a) 0.75 Wb/m<sup>2</sup> (b) 1200]
3. A 3,300/230-V, 50-Hz, 1-phase transformer is to be worked at a maximum flux density of 1.2 Wb/m<sup>2</sup> in the core. The effective cross-sectional area of the transformer core is 150 cm<sup>2</sup>. Calculate suitable values of primary and secondary turns. [830; 58]
4. A 40-kVA, 3,300/240-V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine
  - (a) the number of turns on the secondary
  - (b) the maximum value of flux in the core
  - (c) the approximate value of primary and secondary full-load currents.
 Internal drops in the windings are to be ignored. [(a) 48 (b) 22.5 mWb (c) 12.1 A; 166.7 A]
5. A double-wound, 1-phase transformer is required to step down from 1900 V to 240 V, 50-Hz. It is to have 1.5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively.
 

The peak value of flux density is required to be not more than 1.2 Wb/m<sup>2</sup>. Calculate the required cross-sectional area of the steel core. If the output is 10 kVA, calculate the secondary current.

[1,267; 160; 56.4 cm<sup>2</sup>; 41.75 A]
6. The no-load voltage ratio in a 1-phase, 50-Hz, core-type transformer is 1,200/440. Find the number of turns in each winding if the maximum flux is to be 0.075 Wb. [24 and 74 turns]
7. A 1-phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is 75 cm<sup>2</sup>. If the primary winding be connected to a 400-V, 50 Hz supply, calculate.
  - (i) the peak value of flux density in the core and (ii) voltage induced in the secondary winding. [0.48 Wb/m<sup>2</sup>; 60 V]
8. A 10-kVA, 1-phase transformer has a turn ratio of 300/23. The primary is connected to a 1500-V, 60 Hz supply. Find the secondary volts on open-circuit and the approximate values of the currents in the two windings on full-load. Find also the maximum value of the flux. [115 V; 6.67 A; 87 A; 11.75 mWb]
9. A 100-kVA, 3300/400-V, 50 Hz, 1 phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns.
 

How does the core flux vary with load ? [30.3 A; 250 A; 16.4 mWb; 907]
10. The no-load current of a transformer is 5.0 A at 0.3 power factor when supplied at 230-V, 50-Hz. The number of turns on the primary winding is 200. Calculate (i) the maximum value of flux in the core (ii) the core loss (iii) the magnetising current. [5.18 mWb; 345 W; 4.77 A]
11. The no-load current of a transformer is 15 at a power factor of 0.2 when connected to a 460-V, 50-Hz supply. If the primary winding has 550 turns, calculate
  - (a) the magnetising component of no-load current
  - (b) the iron loss
  - (c) the maximum value of the flux in the core. [(a) 14.7 A (b) 1,380 W (c) 3.77 mWb]
12. The no-load current of a transformer is 4.0 A at 0.25 p.f. when supplied at 250-V, 50 Hz. The number of turns on the primary winding is 200. Calculate
  - (i) the r.m.s. value of the flux in the core (assume sinusoidal flux)
  - (ii) the core loss
  - (iii) the magnetising current. [(i) 3.96 mWb (ii) 250 W (iii) 3.87 A]

13. The following data apply to a single-phase transformer:  
 output : 100 kVA, secondary voltage; 400 V; Primary turns: 200; secondary turns: 40; Neglecting the losses, calculate: (i) the primary applied voltage (ii) the normal primary and secondary currents (iii) the secondary current, when the load is 25 kW at 0.8 power factor.  
 (Rajiv Gandhi Technical University, Bhopal 2000) [(i) 2000 V, (ii) 50 amp, (iii) 78.125 amp]

### 32.10. Transformer on Load

When the secondary is loaded, the secondary current  $I_2$  is set up. The magnitude and phase of  $I_2$  with respect to  $V_2$  is determined by the characteristics of the load. Current  $I_2$  is in phase with  $V_2$  if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. ( $=N_2I_2$ ) and hence its own flux  $\Phi_2$  which is in opposition to the main primary flux  $\Phi$  which is due to  $I_0$ . The secondary ampere-turns  $N_2 I_2$  are known as **demagnetising** amp-turns. The opposing secondary flux  $\Phi_2$  weakens the primary flux  $\Phi$  momentarily, hence primary back e.m.f.  $E_1$  tends to be reduced. For a moment  $V_1$  gains the upper hand over  $E_1$  and hence causes more current to flow in primary.

Let the additional primary current be  $I_2'$ . It is known as **load component of primary current**. This current is antiphase with  $I_2$ . The additional primary m.m.f.  $N_1 I_2'$  sets up its own flux  $\Phi_2'$  which is in opposition to  $\Phi_2$  (but is in the same direction as  $\Phi$ ) and is equal to it in magnitude. Hence, the two cancel each other out. So, we find that the magnetic effects of secondary current  $I_2$  are immediately neutralized by the additional primary current  $I_2'$  which is brought into existence exactly at the same instant as  $I_2$ . The whole process is illustrated in Fig. 32.17.

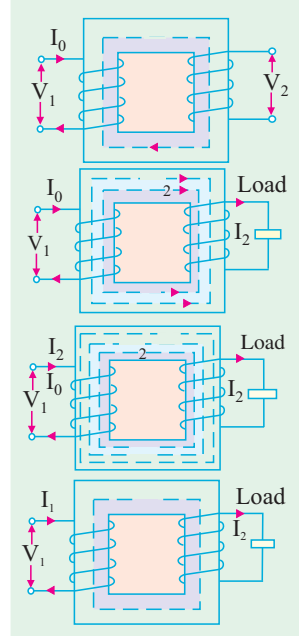


Fig. 32.17

Hence, whatever the load conditions, **the net flux passing through the core is approximately the same as at no-load**. An important deduction is that due to the constancy of core flux at all loads, **the core loss is also practically the same under all load conditions**.

As  $\Phi_2 = \Phi_2' \therefore N_2 I_2 = N_1 I_2' \therefore I_2' = \frac{N_2}{N_1} \times I_2 = K I_2$

Hence, when transformer is on load, the primary winding has two currents in it; one is  $I_0$  and the other is  $I_2'$  which is anti-phase with  $I_2$  and  $K$  times in magnitude. **The total primary current is the vector sum of  $I_0$  and  $I_2'$ .**

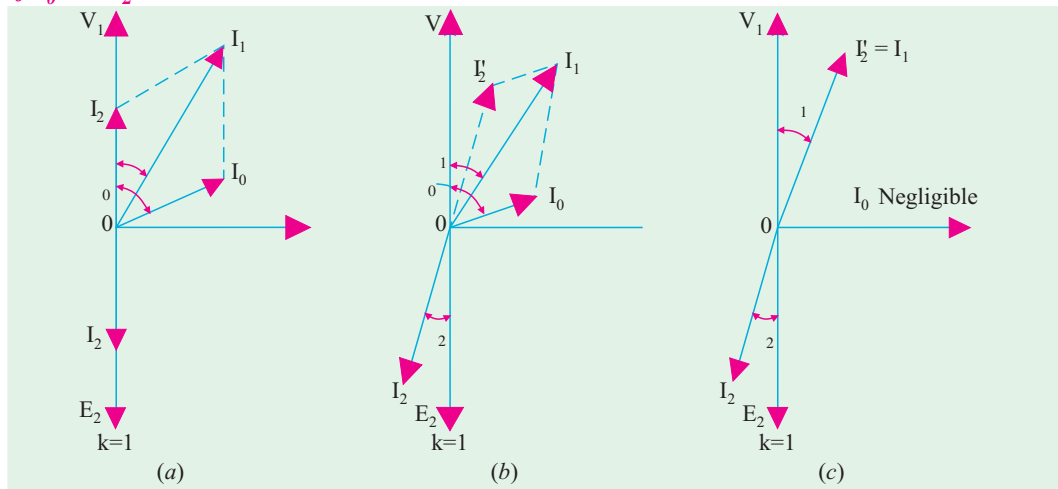


Fig. 32.18

In Fig. 32.18 are shown the vector diagrams for a load transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Fig. 32.18 (a),  $I_2$  is secondary current in phase with  $E_2$  (strictly speaking it should be  $V_2$ ). It causes primary current  $I_2'$  which is anti-phase with it and equal to it in magnitude ( $\because K = 1$ ). Total primary current  $I_1$  is the vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by an angle  $\phi_1$ .

In Fig. 32.18 (b) vectors are drawn for an inductive load. Here  $I_2$  lags  $E_2$  (actually  $V_2$ ) by  $\phi_2$ . Current  $I_2'$  is again antiphase with  $I_2$  and equal to it in magnitude. As before,  $I_1$  is the vector sum of  $I_2'$  and  $I_0$  and lags behind  $V_1$  by  $\phi_1$ .

It will be observed that  $\phi_1$  is slightly greater than  $\phi_2$ . But if we neglect  $I_0$  as compared to  $I_2'$  as in Fig. 32.18 (c), then  $\phi_1 = \phi_2$ . Moreover, under this assumption

$$N_1 I_2' = N_2 I_1 = N_1 I_2 \quad \therefore \quad \frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant. This important relationship is made the basis of current transformer—a transformer which is used with a low-range ammeter for measuring currents in circuits where the direct connection of the ammeter is impracticable.

**Example 32.12.** A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary. (Elect. Engg. Punjab Univ. 1991)

**Solution.**  $\cos \phi_2 = 0.8, \phi_2 = \cos^{-1}(0.8) = 36^\circ 54'$   
 $\cos \phi_0 = 0.2 \quad \therefore \quad \phi_0 = \cos^{-1}(0.2) = 78^\circ 30'$   
 Now  $K = V_2/V_1 = 110/440 = 1/4$   
 $\therefore \quad I_2' = K I_2 = 120 \times 1/4 = 30 \text{ A}$   
 $I_0 = 5 \text{ A.}$

Angle between  $I_0$  and  $I_2'$   
 $= 78^\circ 30' - 36^\circ 54' = 41^\circ 36'$

Using parallelogram law of vectors (Fig. 32.19) we get

$$I_1 = \sqrt{(5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41^\circ 36')} = 34.45 \text{ A}$$

The resultant current could also have been found by resolving  $I_2'$  and  $I_0$  into their X and Y-components.

**Example 32.13.** A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage.

**Solution.** Here  $K = 200/800 = 1/4; I_2' = 80 \times 1/4 = 20 \text{ A}$   
 $\phi_2 = \cos^{-1}(0.8) = 36.9^\circ; \phi_1 = \cos^{-1}(0.707) = 45^\circ$

As seen from Fig. 32.20,  $I_1$  is the vector sum of  $I_0$  and  $I_2'$ . Let  $I_0$  lag behind  $V_1$  by an angle  $\phi_0$ .

$$I_0 \cos \phi_0 + 20 \cos 36.9^\circ = 25 \cos 45^\circ$$

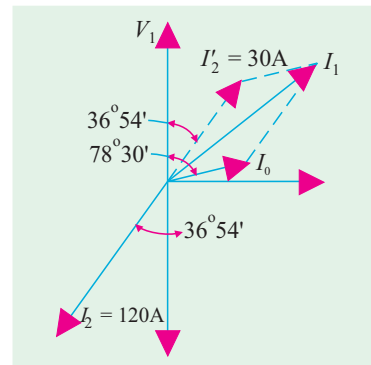


Fig. 32.19

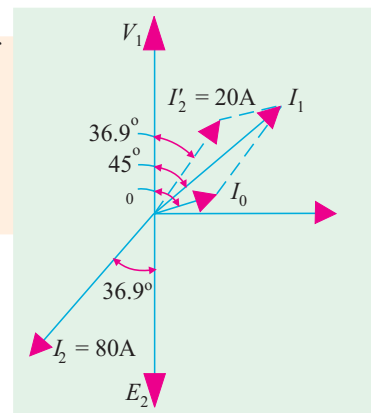


Fig. 32.20

$$\begin{aligned} \therefore I_0 \cos \phi_2 &= 25 \times 0.707 - 20 \times 0.8 \\ &= 1.675 \text{ A} \\ I_0 \sin \phi_0 + 20 \sin 36.9^\circ &= 25 \sin 45^\circ \\ \therefore I_0 \sin \phi_0 &= 25 \times 0.707 - 20 \times 0.6 \\ &= 5.675 \text{ A} \\ \therefore \tan \phi_0 &= 5.675/1.675 = 3.388 \\ \therefore \phi_0 &= 73.3^\circ \\ \text{Now, } I_0 \sin \phi_0 &= 5.675 \\ \therefore I_0 &= 5.675/\sin 73.3^\circ = 5.93 \text{ A} \end{aligned}$$

**Example 32.14.** A single phase transformer takes 10 A on no load at p.f. of 0.2 lagging. The turns ratio is 4 : 1 (step down). If the load on the secondary is 200 A at a p.f. of 0.85 lagging. Find the primary current and power factor.

Neglect the voltage-drop in the winding.

(Nagpur University November 1999)

**Solution.** Secondary load of 200 A, 0.85 lag is reflected as 50 A, 0.85 lag in terms of the primary equivalent current.

$$\begin{aligned} I_0 &= 10 \angle -\phi_0, \text{ where } \phi_0 = \cos^{-1} 0.20 = 78.5^\circ \text{ lagging} \\ &= 2 - j9.8 \text{ amp} \end{aligned}$$

$$\begin{aligned} I_2' &= 50 \angle -\phi_L \text{ where } \phi_L = \cos^{-1} 0.85 = 31.8^\circ, \text{ lagging} \\ I_2' &= 42.5 - j26.35 \end{aligned}$$

Hence primary current  $I_1$

$$\begin{aligned} &= I_0 + I_2' \\ &= 2 - j9.8 + 42.5 - j26.35 \\ &= 44.5 - j36.15 \end{aligned}$$

$$|I_1| = 57.333 \text{ amp, } \phi = 0.776 \text{ Lag.}$$

$$\phi = \cos^{-1} \frac{44.5}{57.333} = 39.10^\circ \text{ lagging}$$

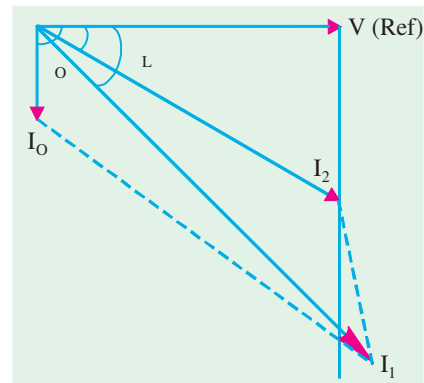


Fig. 32.21

The phasor diagram is shown in Fig. 32.21.

### Tutorial Problems 32.2

- The primary of a certain transformer takes 1 A at a power factor of 0.4 when it is connected across a 200-V, 50-Hz supply and the secondary is on open circuit. The number of turns on the primary is twice that on the secondary. A load taking 50 A at a lagging power factor of 0.8 is now connected across the secondary. What is now the value of primary current ? [25.9 A]
- The number of turns on the primary and secondary windings of a single-phase transformer are 350 and 38 respectively. If the primary winding is connected to a 2.2 kV, 50-Hz supply, determine
  - the secondary voltage on no-load,
  - the primary current when the secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging,
  - the power factor of the primary current. [239 V; 25-65 A; 0.715 lag]
- A 400/200-V, 1-phase transformer is supplying a load of 25 A at a p.f. of 0.866 lagging. On no-load the current and power factor are 2 A and 0.208 respectively. Calculate the current taken from the supply. [13.9 A lagging V1 by 36.1°]
- A transformer takes 10 A on no-load at a power factor of 0.1. The turn ratio is 4 : 1 (step down). If

a load is supplied by the secondary at 200 A and p.f. of 0.8, find the primary current and power factor (internal voltage drops in transformer are to be ignored). **[57.2 A; 0.717 lagging]**

5. A 1-phase transformer is supplied at 1,600 V on the h.v. side and has a turn ratio of 8 : 1. The transformer supplies a load of 20 kW at a power factor of 0.8 lag and takes a magnetising current of 2.0 A at a power factor of 0.2. Calculate the magnitude and phase of the current taken from the h.v. supply. **[17.15 A ; 0.753 lag] (Elect. Engg. Calcutta Univ. 1980)**

6. A 2,200/200-V, transformer takes 1 A at the H.T. side on no-load at a p.f. of 0.385 lagging. Calculate the iron losses.

If a load of 50 A at a power of 0.8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its power factor. **[847 W; 5.44 A; 0.74 lag]**

7. A 400/200-V, I-phase transformer is supplying a load of 50 A at a power factor of 0.866 lagging. The no-load current is 2 A at 0.208 p.f. lagging. Calculate the primary current and primary power factor. **[26.4 A; 0.838 lag] (Elect. Machines-I, Indore Univ. 1980)**

### 32.11. Transformer with Winding Resistance but No Magnetic Leakage

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that :

(i) The secondary terminal voltage  $V_2$  is vectorially less than the secondary induced e.m.f.  $E_2$  by an amount  $I_2 R_2$  where  $R_2$  is the resistance of the secondary winding. Hence,  $V_2$  is equal to the vector difference of  $E_2$  and resistive voltage drop  $I_2 R_2$ .

∴  $V_2 = E_2 - I_2 R_2$  ...vector difference

(ii) Similarly, primary induced e.m.f.  $E_1$  is equal to the vector difference of  $V_1$  and  $I_1 R_1$  where  $R_1$  is the resistance of the primary winding.

$E_1 = V_1 - I_1 R_1$  ...vector difference

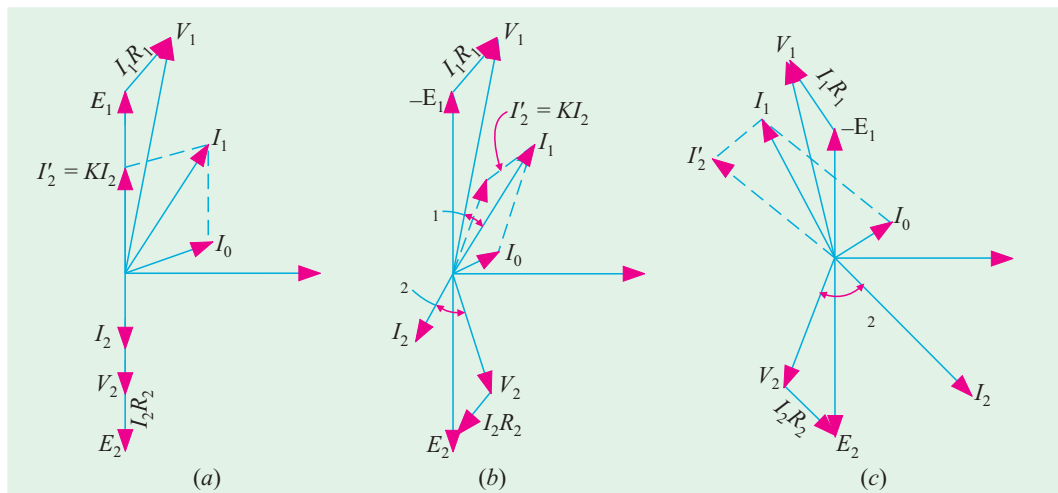


Fig. 32.22

The vector diagrams for non-inductive, inductive and capacitive loads are shown in Fig. 32.22 (a), (b) and (c) respectively.

### 32.12. Equivalent Resistance

In Fig. 32.23 a transformer is shown whose primary and secondary windings have resistances of  $R_1$  and  $R_2$  respectively. The resistances have been shown external to the windings.



It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of  $R_2$  in secondary is equivalent to  $R_2/K^2$  in primary. The value  $R_2/K^2$  will be denoted by  $R_2'$ —the *equivalent secondary resistance as referred to primary*.

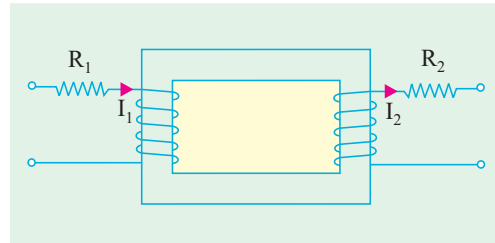


Fig. 32.23

The copper loss in secondary is  $I_2^2 R_2$ . This loss is supplied by primary which takes a current of  $I_1$ . Hence if  $R_2'$  is the *equivalent resistance in primary which would have caused the same loss* as  $R_2$  in secondary, then

$$I_1^2 R_2' = I_2^2 R_2 \text{ or } R_2' = (I_2/I_1)^2 R_2$$

Now, if *we neglect no-load current*  $I_0$ , then  $I_2/I_1 = 1/K$ \*. Hence,  $R_2' = R_2/K^2$

Similarly, equivalent primary resistance as referred to secondary is  $R_1' = K^2 R_1$

In Fig. 32.24, secondary resistance has been transferred to primary side leaving secondary circuit resistanceless. The resistance  $R_1 + R_2' = R_1 + R_2/K^2$  is known as the *equivalent or effective resistance of the transformer as referred to primary* and may be designated as  $R_{01}$ .

∴  $R_{01} = R_1 + R_2' = R_1 + R_2/K^2$

Similarly, the *equivalent resistance of the transformer as referred to secondary* is

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

This fact is shown in Fig. 32.25 where all the resistances of the transformer has been concentrated in the secondary winding.

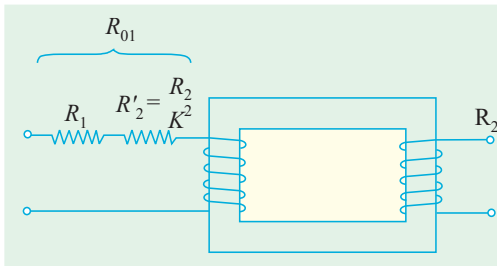


Fig. 32.24

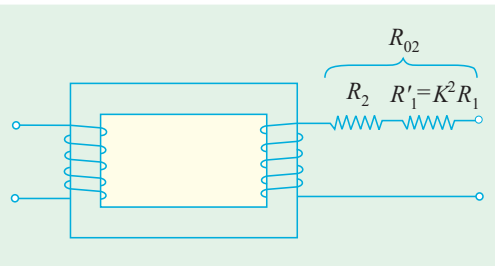


Fig. 32.25

It is to be noted that

1. a resistance of  $R_1$  in primary is equivalent to  $K^2 R_1$  in secondary. Hence, it is called *equivalent resistance as referred to secondary* i.e.  $R_1'$ .
2. a resistance of  $R_2$  in secondary is equivalent to  $R_2/K^2$  in primary. Hence, it is called the *equivalent secondary resistance as referred to primary* i.e.  $R_2'$ .

3. Total or effective resistance of the transformer as referred to primary is

$$R_{01} = \text{primary resistance} + \text{equivalent secondary resistance as referred to primary} \\ = R_1 + R_2' = R_1 + R_2/K^2$$

4. Similarly, total transformer resistance as referred to secondary is,

$$R_{02} = \text{secondary resistance} + \text{equivalent primary resistance as referred to secondary} \\ = R_2 + R_1' = R_2 + K^2 R_1$$

\* Actually  $I_2 \neq 2/I_2' = 1/K$  and not  $I_2 \neq 2/I_1$ . However, if  $I_0$  is neglected, then  $I_2' = I_1$ .

**Note :** It is important to remember that

- (a) when shifting any primary resistance to the secondary, **multiply** it by  $K^2$  i.e. (transformation ratio)<sup>2</sup>.
- (b) when shifting secondary resistance to the primary, **divide** it by  $K^2$ .
- (c) however, when shifting any voltage from one winding to another only  $K$  is used.

### 32.13. Magnetic Leakage

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition. It is found, however, that all the flux linked with primary does not link the secondary but part of it i.e.  $\Phi_{L1}$  completes its magnetic circuit by passing through air rather than around the core, as shown in Fig. 32.26. This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points  $a$  and  $b$ , acts along the leakage paths. Hence, this flux is known as **primary leakage flux** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of  $\Phi_{L1}$ . The flux  $\Phi_{L1}$  is in time phase with  $I_1$ . It induces an e.m.f.  $e_{L1}$  in primary but not in secondary.

Similarly, secondary ampere-turns (or m.m.f.) acting across points  $c$  and  $d$  set up leakage flux  $\Phi_{L2}$  which is linked with secondary winding alone (and not with primary turns). This flux  $\Phi_{L2}$  is in time phase with  $I_2$  and produces a self-induced e.m.f.  $e_{L2}$  in secondary (but not in primary).

At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.s are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux linking with each winding, produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux. In other words, **a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits** as shown in Fig. 32.27 such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

$$X_1 = e_{L1}/I_1 \text{ and } X_2 = e_{L2}/I_2$$

The terms  $X_1$  and  $X_2$  are known as primary and secondary **leakage reactances** respectively.

Following few points should be kept in mind :

1. The leakage flux links one or the other winding but **not both**, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.
2. The primary voltage  $V_1$  will have to supply reactive drop  $I_1 X_1$  in addition to  $I_1 R_1$ . Similarly  $E_2$  will have to supply  $I_2 R_2$  and  $I_2 X_2$ .
3. In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 32.27 because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimised by sectionalizing and interleaving the primary and secondary windings as in Fig. 32.6 or Fig. 32.8.

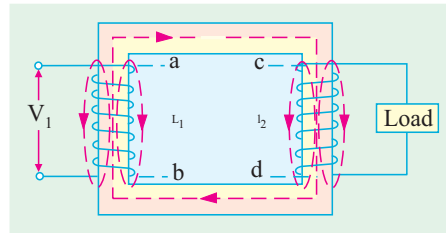


Fig. 32.26

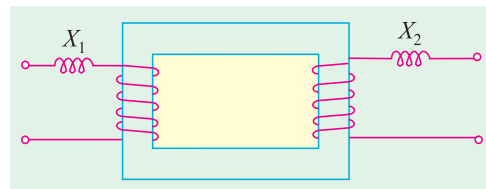


Fig. 32.27

### 32.14. Transformer with Resistance and Leakage Reactance

In Fig. 32.28 the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{R_1^2 + X_1^2}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is  $I_1 X_1$  (usually 1 or 2% of  $V_1$ ). Hence

$$V_1 = E_1 + I_1(R_1 + jX_1) = E_1 + I_1 Z_1$$

Similarly, there are  $I_2 R_2$  and  $I_2 X_2$  drops in secondary which combine with  $V_2$  to give  $E_2$ .

$$E_2 = V_2 + I_2(R_2 + jX_2) = V_2 + I_2 Z_2$$

The vector diagram for such a transformer for different kinds of loads is shown in Fig. 32.29. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle  $\phi_1$  between  $V_1$  and  $I_1$  gives the power factor angle of the transformer.

It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

$$\begin{aligned} \therefore X_2' &= X_2/K^2 \quad \text{and} \quad X_1' = K^2 X_2 \\ \text{and} \quad X_{01} &= X_1 + X_2' = X_1 + X_2/K^2 \quad \text{and} \quad X_{02} = X_2 + X_1' = X_2 + K^2 X_1 \end{aligned}$$

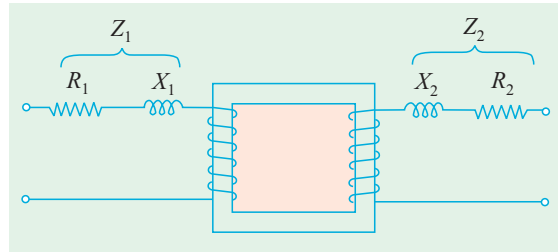


Fig. 32.28

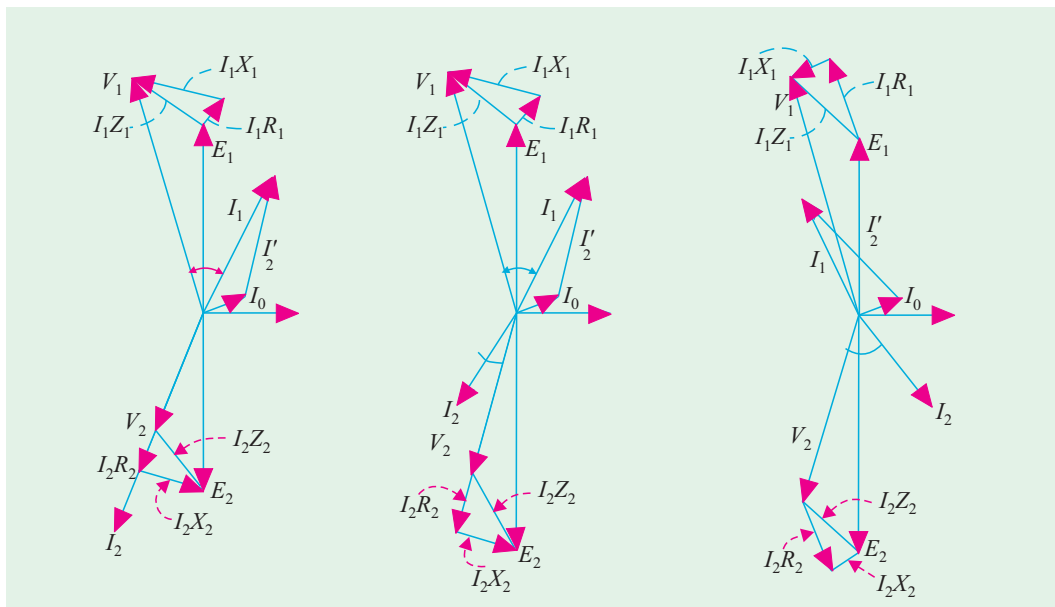


Fig. 32.29

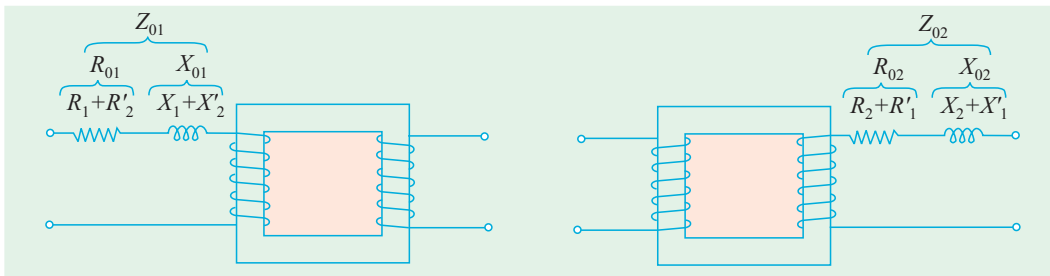


Fig. 32.30 (a)

Fig. 32.30 (b)

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} \quad \dots \text{Fig. 32.30 (a)}$$

and

$$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} \quad \dots \text{Fig. 32.30 (b)}$$

**Example 32.15.** A 30 kVA, 2400/120-V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance referred to the (i) high voltage side and (ii) the low-voltage side.

(Electrical Machines-I, Bangalore Univ. 1987)

**Solution.**

$$K = 120/2400 = 1/20; R_1 = 0.1 \Omega, X_1 = 0.22 \Omega$$

$$R_2 = 0.035 \Omega \quad \text{and} \quad X_2 = 0.012 \Omega$$

(i) Here, high-voltage side is, obviously, the primary side. Hence, values as referred to primary side are

$$R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + 0.035/(1/20)^2 = \mathbf{14.1 \Omega}$$

$$X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 0.22 + 0.012/(1/20)^2 = \mathbf{5.02 \Omega}$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15 \Omega}$$

(ii)

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = 0.035 + (1/20)^2 \times 0.1 = \mathbf{0.03525 \Omega}$$

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.012 + (1/20)^2 \times 0.22 = \mathbf{0.01255 \Omega}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.03525^2 + 0.01255^2} = \mathbf{0.0374 \Omega}$$

$$\text{(or } Z_{02} = K^2 Z_{01} = (1/20)^2 \times 15 = 0.0375 \Omega \text{)}$$

**Example 32.16.** A 50-kVA, 4,400/220-V transformer has  $R_1 = 3.45 \Omega$ ,  $R_2 = 0.009 \Omega$ . The values of reactances are  $X_1 = 5.2 \Omega$  and  $X_2 = 0.015 \Omega$ . Calculate for the transformer (i) equivalent resistance as referred to primary (ii) equivalent resistance as referred to secondary (iii) equivalent reactance as referred to both primary and secondary (iv) equivalent impedance as referred to both primary and secondary (v) total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.

(Elect. Engg.-I, Nagpur Univ. 1993)

**Solution.** Full-load

$$I_1 = 50,000/4,400 = 11.36 \text{ A (assuming 100\% efficiency)}$$

Full-load

$$I_2 = 50,000/220 = 227 \text{ A; } K = 220/4,400 = 1/20$$

(i)

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(1/20)^2} = 3.45 + 3.6 = \mathbf{7.05 \Omega}$$

(ii)

$$R_{02} = R_2 + K^2 R_1 = 0.009 + (1/20)^2 \times 3.45 = 0.009 + 0.0086 = \mathbf{0.0176 \Omega}$$

Also,

$$R_{02} = K^2 R_{01} = (1/20)^2 \times 7.05 = 0.0176 \Omega \text{ (check)}$$

$$\begin{aligned}
 \text{(iii)} \quad X_{01} &= X_1 + X_2' = X_1 + X_2/K^2 = 5.2 + 0.015/(1/20)^2 = \mathbf{11.2 \Omega} \\
 X_{02} &= X_2 + X_1' = X_2 + K^2 X_1 = 0.015 + 5.2/20^2 = \mathbf{0.028 \Omega} \\
 \text{Also } X_{02} &= K^2 X_{01} = 11.2/400 = 0.028 \Omega \text{ (check)} \\
 \text{(iv)} \quad Z_{01} &= \sqrt{(R_{01}^2 + X_{01}^2)} = \sqrt{(7.05^2 + 11.2)^2} = \mathbf{13.23 \Omega} \\
 Z_{02} &= \sqrt{(R_{02}^2 + X_{02}^2)} = \sqrt{(0.0176^2 + 0.028)^2} = \mathbf{0.03311 \Omega} \\
 \text{Also } Z_{02} &= K^2 Z_{01} = 13.23/400 = 0.0331 \Omega \text{ (check)} \\
 \text{(v)} \quad \text{Cu loss} &= I_1^2 R_2 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = \mathbf{910 W} \\
 \text{Also Cu loss} &= I_1^2 R_{01} = 11.36^2 \times 7.05 = \mathbf{910 W} \\
 &= I_2^2 R_{02} = 227^2 \times 0.0176 = \mathbf{910 W}
 \end{aligned}$$

**Example 32.17.** A transformer with a 10 : 1 ratio and rated at 50-kVA, 2400/240-V, 50-Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V.

(a) What load impedance connected to low-tension side will be loading the transformer fully at 0.8 power factor (lag) ?

(b) What is the value of this impedance referred to high tension side ?

(c) What is the value of the current referred to the high tension side ?

(Elect. Engineering-I, Bombay Univ. 1987)

**Solution. (a)** F. L.  $I_2 = 50,000/240 = 625/3$  A;  $Z_2 = \frac{240}{(625/3)} = \mathbf{1.142 \Omega}$

(b)  $K = 240/2400 = 1/10$

The secondary impedance referred to primary side is

$$Z_2' = Z_2/K^2 = 1.142/(1/10)^2 = \mathbf{114.2 \Omega}$$

(c) Secondary current referred to primary side is  $I_2' = KI_2 = (1/10) \times 625/3 = \mathbf{20.83 A}$

**Example 32.18.** The full-load copper loss on the h.v. side of a 100-kVA, 11000/317-V, 1-phase transformer is 0.62 kW and on the L.V. side is 0.48 kW.

(i) Calculate  $R_1$ ,  $R_2$  and  $R_3$  in ohms (ii) the total reactance is 4 per cent, find  $X_1$ ,  $X_2$  and  $X_3$  in ohms if the reactance is divided in the same proportion as resistance.

(Elect. Machines A.M.I.E., Sec. B, 1991)

**Solution. (i)** F.L.  $I_1 = 100 \times 10^3/11000 = 9.1$  A. F.L.  $I_2 = 100 \times 10^3/317 = 315.5$  A

Now,  $I_1^2 R_1 = 0.62$  kW or  $R_1 = 620/9.1^2 = 7.5 \Omega$

$$I_2^2 R_2 = 0.48 \text{ kW}, R_2 = 480/315.5^2 = 0.00482 \Omega$$

$$R_2' = R_2/K^2 = 0.00482 \times (11,000/317)^2 = 5.8 \Omega$$

$$\% \text{ reactance} = \frac{I_1 \times X_{01}}{V_1} \times 100 \text{ or } 4 = \frac{9.1 \times X_{01}}{11000} \times 100, X_{01} = 48.4 \Omega$$

$$X_1 + X_2' = 48.4 \Omega. \text{ Given } R_1/R_2' = X_1/X_2'$$

$$\text{or } (R_1 + R_2')/R_2' = (X_1 + X_2')/X_2' \quad (7.5 + 5.8)/5.8 = 48.4/X_2' \quad \therefore X_2' = 21.1 \Omega$$

$$\therefore X_1 = 48.4 - 21.1 = 27.3 \Omega, X_2 = 21.1 \times (317/11000)^2 = \mathbf{0.175 \Omega}$$

**Example 32.19.** The following data refer to a 1-phase transformer :

Turn ratio 19.5 : 1 ;  $R_1 = 25 \Omega$  ;  $X_1 = 100 \Omega$  ;  $R_2 = 0.06 \Omega$  ;  $X_2 = 0.25 \Omega$ . No-load current = 1.25 A leading the flux by  $30^\circ$ .

The secondary delivers 200 A at a terminal voltage of 500 V and p.f. of 0.8 lagging. Determine

by the aid of a vector diagram, the primary applied voltage, the primary p.f. and the efficiency.

(Elect. Machinery-I, Madras Univ. 1989)

**Solution.** The vector diagram is similar to Fig. 30.28 which has been redrawn as Fig. 32.31. Let us take  $V_2$  as the reference vector.

$$\begin{aligned} \therefore \quad V_2 &= 500 \angle 0^\circ = 500 + j0 \\ I_2 &= 200(0.8 - j0.6) = 160 - j120 \\ Z_2 &= (0.06 + j0.25) \\ E_2 &= V_2 + I_2 Z_2 \\ &= (500 + j0) + (160 - j120)(0.06 + j0.25) \\ &= 500 + (39.6 + j32.8) = 539.6 + j32.8 = 541 \angle 3.5^\circ \end{aligned}$$

Obviously,  $\beta = 3.5^\circ$

$$\begin{aligned} E_1 &= E_2/K = 19.5 E_2 = 19.5(539.6 + j32.8) \\ &= 10,520 + j640 \end{aligned}$$

$$\therefore -E_1 = -10,520 - j640 = 10,540 \angle 183.5^\circ$$

$$\begin{aligned} I_2' &= -I_2 K = (-160 + j120)/19.5 \\ &= -8.21 + j6.16 \end{aligned}$$

As seen from Fig. 32.31,  $I_0$  leads  $V_2$  by an angle  $= 3.5^\circ + 90^\circ + 30^\circ = 123.5^\circ$

$$\begin{aligned} \therefore I_0 &= 1.25 \angle 123.5^\circ \\ &= 1.25(\cos 123.5^\circ + j \sin 123.5^\circ) \\ &= 1.25(-\cos 56.5^\circ + j \sin 56.5^\circ) \\ &= -0.69 + j1.04 \end{aligned}$$

$$\begin{aligned} I_1 &= I_2' + I_0 = (-8.21 + j6.16) + (-0.69 + j1.04) \\ &= -8.9 + j7.2 = 11.45 \angle 141^\circ \end{aligned}$$

$$\begin{aligned} V_2 &= -E_1 + I_1 Z_1 \\ &= -10,520 - j640 + (-8.9 + j7.5)(25 + j100) \\ &= -10,520 - j640 - 942 - j710 \\ &= -11,462 - j1350 \\ &= 11,540 \angle 186.7^\circ \end{aligned}$$

Phase angle between  $V_1$  and  $I_1$  is  $= 186.7^\circ - 141^\circ = 45.7^\circ$

$$\therefore \text{primary p.f.} = \cos 45.7^\circ = \mathbf{0.698 \text{ (lag)}}$$

$$\begin{aligned} \text{No-load primary input power} &= V_1 I_0 \sin \phi_0 \\ &= 11,540 \times 1.25 \times \cos 60^\circ = 7,210 \text{ W} \end{aligned}$$

$$R_{02} = R_2 + K^2 R_1 = 0.06 + 25/19.5^2 = 0.1257 \Omega$$

$$\text{Total Cu loss as referred to secondary} = I_2^2 R_{02} = 200^2 \times 0.1257 = 5,030 \text{ W}$$

$$\text{Output} = V_2 I_2 \cos \phi_2 = 500 \times 200 \times 0.8 = 80,000 \text{ W}$$

$$\text{Total losses} = 5030 + 7210 = 12,240 \text{ W}$$

$$\text{Input} = 80,000 + 12,240 = 92,240 \text{ W}$$

$$\eta = 80,000/92,240 = 0.8674 \text{ or } \mathbf{86.74\%}$$

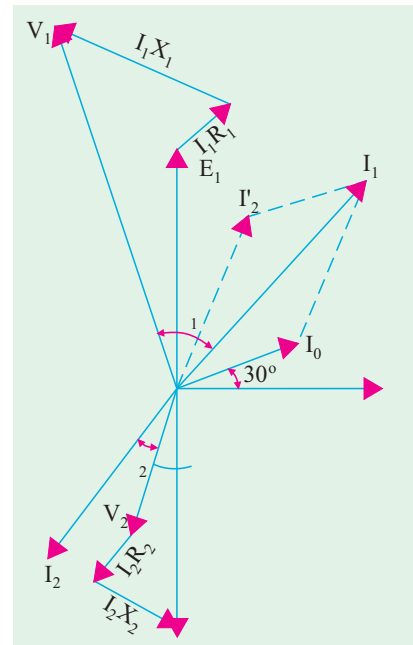


Fig. 32.31

**Example 32.20.** A 100 kVA, 1100/220 V, 50 Hz, single-phase transformer has a leakage impedance of  $(0.1 + j0.4)$  ohm for the H.V. winding and  $(0.006 + j0.015)$  ohm for the L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

(Bharathiar Univ. Nov. 1997)

**Solution.** Turns ratio =  $(N_1/N_2) = (V_1/V_2) = 1100/220 = 5$

(i) Referred to H.V. side :

Resistance =  $r_1 + r_2 = 0.1 + (25 \times 0.006) = 0.25$  ohm  
 Reactance =  $x_1 + x_2' = 0.4 + (25 \times 0.015) = 0.775$  ohm  
 Impedance =  $(0.25^2 + 0.775^2)^{0.5} = 0.8143$  ohm

(ii) Referred to L.V. side :

Resistance =  $0.25/25 = 0.01$   
 (or resistance =  $0.006 + (0.1/25) = 0.01$  ohm)  
 Reactance =  $0.775/25 = 0.031$  ohm  
 Impedance =  $0.8143/25 = 0.0326$  ohm

### 32.15. Simplified Diagram

The vector diagram of Fig. 32.29 may be considerably simplified if the no-load current  $I_0$  is neglected. Since  $I_0$  is 1 to 3 per cent of full-load primary current  $I_1$ , it may be neglected without serious error. Fig. 32.32 shows the diagram of Fig. 32.29 with  $I_0$  omitted altogether.

In Fig. 32.32,  $V_2, V_1, \phi_2$  are known, hence  $E_2$  can be found by adding vectorially  $I_2 R_2$  and  $I_2 X_2$  to  $V_2$ . Similarly,  $V_1$  is given by the vector addition of  $I_1 R_1$  and  $I_1 X_1$  to  $E_1$ . All the voltages on the primary side can be transferred to the secondary side as shown in figure, where the upper part of the diagram has been rotated through  $180^\circ$ . However, it should be noted that each voltage or voltage drop should be multiplied by transformation ratio  $K$ .

The lower side of the diagram has been shown separately in Fig. 32.34 laid horizontally where vector for  $V_2$  has been taken along X-axis.

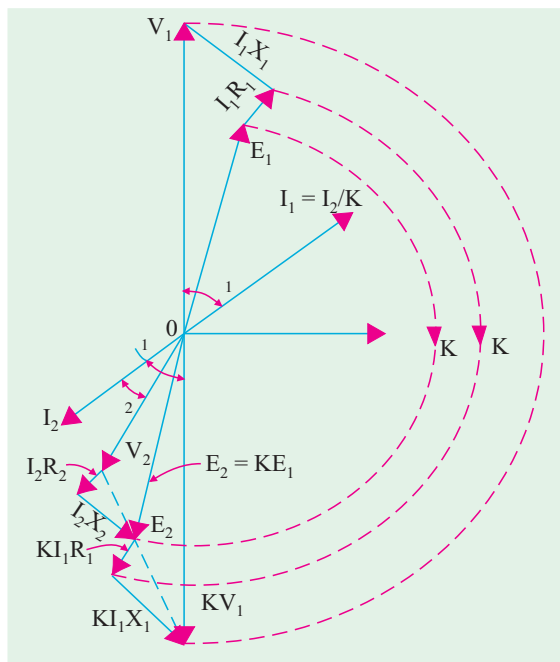


Fig. 32.32

It is a simple matter to find transformer regulation as shown in Fig. 32.34 or Fig. 32.35.

It may be noted that  $V_2 = KV_1 - I_2(R_{02} + jX_{02}) = KV_1 - I_2Z_{02}$ .

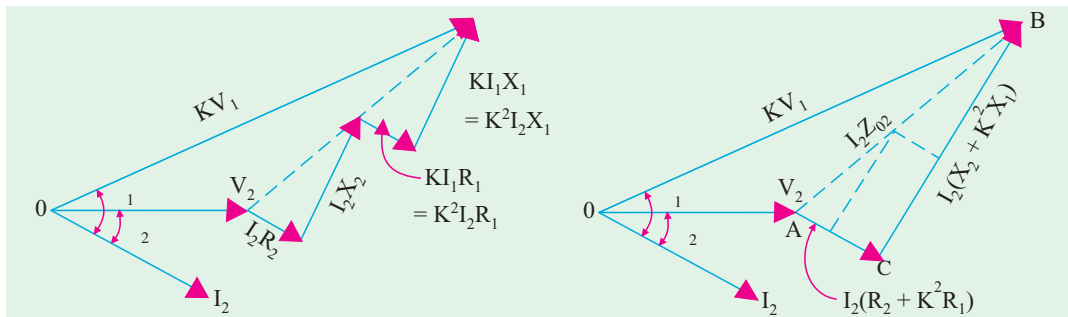


Fig. 32.33

Fig. 32.34

\* Also,  $V_1 = (V_2 + I_2 Z_{02}) \#3/K$

**32.16. Total Approximate Voltage Drop in a Transformer**

When the transformer is on *no-load*, then  $V_1$  is approximately equal to  $E_1$ . Hence  $E_2 = KE_1 = KV_1$ . Also,  $E_2 = {}_0V_2$  where  ${}_0V_2$  is secondary terminal voltage on *no-load*, hence no-load secondary terminal voltage is  $KV_1$ . The secondary voltage on load is  $V_2$ . The difference between the two is  $I_2 Z_{02}$  as shown in Fig. 32.35. The approximate voltage drop of the transformer *as referred to secondary* is found thus :

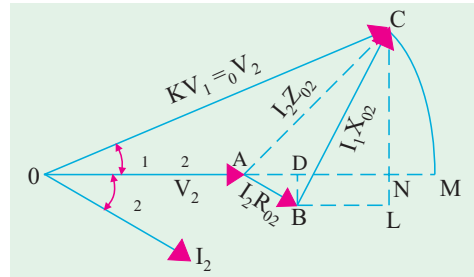


Fig. 32.35

With  $O$  as the centre and radius  $OC$  draw an arc cutting  $OA$  produced at  $M$ . The total voltage drop  $I_2 Z_{02} = AC = AM$  which is approximately equal to  $AN$ . From  $B$  draw  $BD$  perpendicular on  $OA$  produced. Draw  $CN$  perpendicular to  $OM$  and draw  $BL$  parallel to  $OM$ .

Approximate voltage drop  
 $= AN = AD + DN$   
 $= I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi$

where  $\phi_1 = \phi_2 = \phi$  (approx).

This is the value of approximate voltage drop for a *lagging* power factor.

The different figures for unity and leading power factors are shown in Fig. 32.36 (a) and (b) respectively.

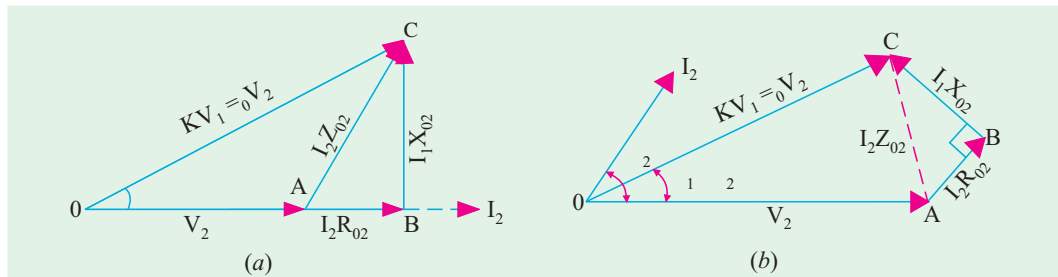


Fig. 32.36

The approximate voltage drop for *leading* power factor becomes

$$(I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi)$$

In general, approximate voltage drop is  $(I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi)$

It may be noted that approximate voltage drop as referred to primary is

$$(I_1 R_{01} \cos \phi \pm I_1 X_{01} \sin \phi)$$

$$\% \text{ voltage drop in secondary is} = \frac{I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi}{{}_0V_2} \times 100$$

$$= \frac{100 \times I_2 R_{02}}{{}_0V_2} \cos \phi \pm \frac{100 I_2 X_{02}}{{}_0V_2} \sin \phi$$

$$= v_r \cos \phi \pm v_x \sin \phi$$

where  $v_r = \frac{100 I_2 R_{02}}{{}_0V_2} = \text{percentage resistive drop} = \frac{100 I_1 R_{01}}{V_1}$

$$v_x = \frac{100 I_2 X_{02}}{{}_0V_2} = \text{percentage reactive drop} = \frac{100 I_1 X_{01}}{V_1}$$



### 32.17. Exact Voltage Drop

With reference to Fig. 32.35, it is to be noted that exact voltage drop is  $AM$  and not  $AN$ . If we add the quantity  $NM$  to  $AN$ , we will get the exact value of the voltage drop.

Considering the right-angled triangle  $OCN$ , we get

$$NC^2 = OC^2 - ON^2 = (OC + ON)(OC - ON) = (OC + ON)(OM - ON) = 2 OC \times NM$$

$$\therefore NM = NC^2 / 2 \cdot OC \quad \text{Now, } NC = LC - LN = LC - BD$$

$$\therefore NC = I_2 X_{02} \cos \phi - I_2 R_{02} \sin \phi \quad \therefore NM = \frac{(I_2 X_{02} \cos \phi - I_2 R_{02} \sin \phi)^2}{2_0 V_2}$$

$\therefore$  For a **lagging** power factor, exact voltage drop is

$$= AN + NM = (I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi) + \frac{(I_2 X_{02} \cos \phi - I_2 R_{02} \sin \phi)^2}{2_0 V_2}$$

For a **leading** power factor, the expression becomes

$$= (I_2 R_{02} \cos \phi - I_2 X_{02} \sin \phi) + \frac{(I_2 X_{02} \cos \phi + I_2 R_{02} \sin \phi)^2}{2_0 V_2}$$

In general, the voltage drop is

$$= (I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi) + \frac{(I_2 X_{02} \cos \phi \pm I_2 R_{02} \sin \phi)^2}{2_0 V_2}$$

Percentage drop is

$$= \frac{(I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi) \times 100}{_0 V_2} + \frac{(I_2 X_{02} \cos \phi \mp I_2 R_{02} \sin \phi)^2 \times 100}{2_0 V_2^2}$$

$$= (v_r \cos \phi \pm v_x \sin \phi) + (1/200) (v_x \cos \phi \mp v_r \sin \phi)^2$$

The upper signs are to be used for a **lagging** power factor and the lower ones for a **leading** power factor.

**Example 32.21.** A 230/460-V transformer has a primary resistance of  $0.2 \Omega$  and reactance of  $0.5 \Omega$  and the corresponding values for the secondary are  $0.75 \Omega$  and  $1.8 \Omega$  respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging.

(Electric Machines-II, Bangalore Univ. 1991)

**Solution.**

$$K = 460/230 = 2; R_{02} = R_2 + K^2 R_1 = 0.75 + 2^2 \times 0.2 = 1.55 \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 1.8 + 2^2 \times 0.5 = 3.8 \Omega$$

$$\text{Voltage drop} = I_2 (R_{02} \cos \phi + X_{02} \sin \phi) = 10 (1.55 \times 0.8 + 3.8 \times 0.6) = 35.2 \text{V}$$

$$\therefore \text{Secondary terminal voltage} = 460 - 35.2 = \mathbf{424.8 \text{ V}}$$

**Example 32.22.** Calculate the regulation of a transformer in which the percentage resistance drops is 1.0% and percentage reactance drop is 5.0% when the power factor is (a) 0.8 lagging (b) unity and (c) 0.8 leading.

(Electrical Engineering, Banaras Hindu Univ. 1988)

**Solution.** We will use the approximate expression of Art 30.16.

$$(a) \text{ p.f.} = \cos \phi = 0.8 \text{ lag} \quad \mu = v_r \cos \phi + v_x \sin \phi = 1 \times 0.8 + 5 \times 0.6 = \mathbf{3.8\%}$$

$$(b) \text{ p.f.} = \cos \phi = 1 \quad \mu = 1 \times 1 + 5 \times 0 = \mathbf{1\%}$$

$$(c) \text{ p.f.} = \cos \phi = 0.8 \text{ lead} \quad \mu = 1 \times 0.8 - 5 \times 0.6 = \mathbf{-2.2\%}$$

**Example 32.23.** A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.

(Elect. Engg. Punjab Univ. 1991)

**Solution.** The percentage voltage regulation ( $\mu$ ) is given by

$$\mu = v_r \cos \phi + v_x \sin \phi$$

where  $v_r$  is the percentage resistive drop and  $v_x$  is the percentage reactive drop.

Differentiating the above equation, we get  $\frac{d\mu}{d\phi} = -v_r \sin \phi + v_x \cos \phi$

For regulation to be maximum,  $d\mu/d\phi = 0 \therefore -v_r \sin \phi + v_x \cos \phi = 0$

or  $\tan \phi = v_x/v_r = 5/2.5 = 2 \therefore \phi = \tan^{-1}(2) = 63.5^\circ$  Now,  $\cos \phi = 0.45$  and  $\sin \phi = 0.892$

Maximum percentage regulation =  $(2.5 \times 0.45) + (5 \times 0.892) = 5.585$

Maximum percentage regulation is **5.585** and occurs at a power factor of **0.45** (lag).

**Example 32.24.** Calculate the percentage voltage drop for a transformer with a percentage resistance of 2.5% and a percentage reactance of 5% of rating 500 kVA when it is delivering 400 kVA at 0.8 p.f. lagging. **(Elect. Machinery-I, Indore Univ. 1987)**

**Solution.** 
$$\% \text{ drop} = \frac{(\%R) I \cos \phi}{I_f} + \frac{(\%X) I \sin \phi}{I_f}$$

where  $I_f$  is the full-load current and  $I$  the actual current.

$$\therefore \% \text{ drop} = \frac{(\%R)kW}{kVA \text{ rating}} + \frac{(\%X) kVAR}{kVA \text{ rating}}$$

In the present case,  $kW = 400 \times 0.8 = 320$  and  $kVAR = 400 \times 0.6 = 240$

$$\therefore \% \text{ drop} = \frac{2.5 \times 320}{500} + \frac{5 \times 240}{500} = 4\%$$

### 32.18. Equivalent Circuit

The transformer shown diagrammatically in Fig. 32.37 (a) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage (Fig. 32.37 (b)). The no-load

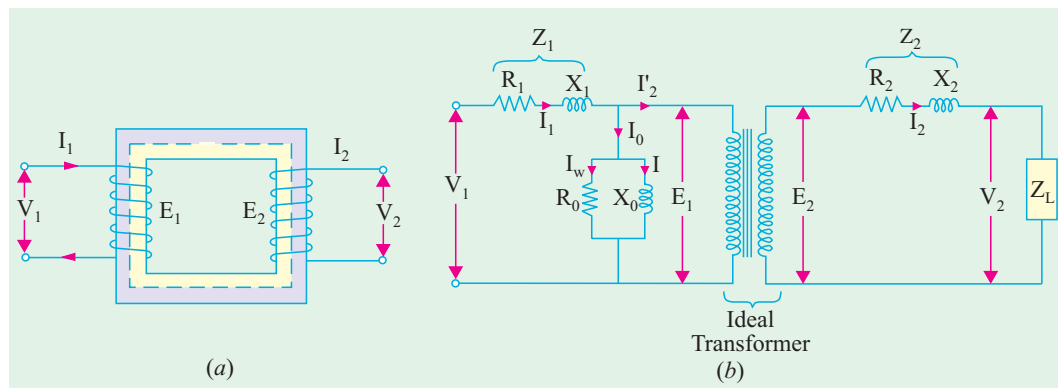


Fig. 32.37

current  $I_0$  is simulated by pure inductance  $X_0$  taking the magnetising component  $I_\mu$  and a non-inductive resistance  $R_0$  taking the working component  $I_w$  connected in parallel across the primary circuit. The value of  $E_1$  is obtained by subtracting vectorially  $I_1 Z_1$  from  $V_1$ . The value of  $X_0 = E_1/I_0$  and of  $R_0 = E_1/I_w$ . It is clear that  $E_1$  and  $E_2$  are related to each other by expression

$$E_2/E_1 = N_2/N_1 = K.$$

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance

either to the primary or to the secondary. In that case, we would have to work in one winding only which is more convenient.

The primary equivalent of the secondary induced voltage is  $E_2' = E_2/K = E_1$ .

Similarly, primary equivalent of secondary terminal or output voltage is  $V_2' = V_2/K$ .

Primary equivalent of the secondary current is  $I_2' = KI_2$ .

For transferring secondary impedance to primary  $K^2$  is used.

$$R_2' = R_2/K^2, X_2' = X_2/K^2, Z_2' = Z_2/K^2$$

The same relationship is used for shifting an external load impedance to the primary.

The secondary circuit is shown in Fig. 32.38 (a) and its equivalent primary values are shown in Fig. 32.38 (b).

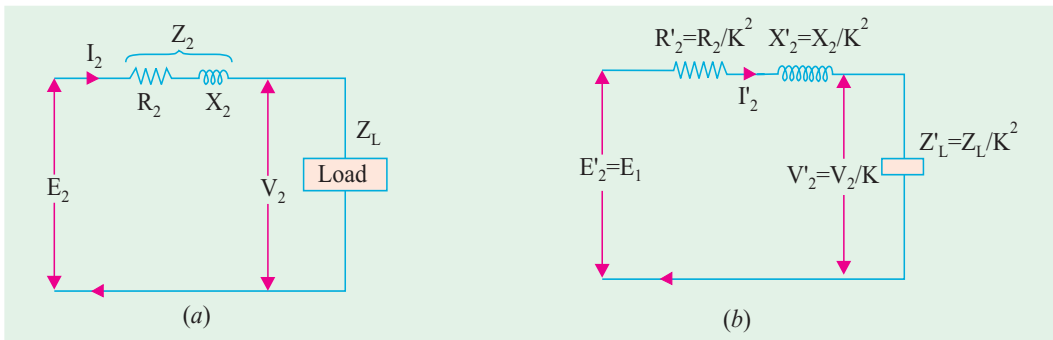


Fig. 32.38

The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in Fig. 32.39. This is known as the exact equivalent circuit but it presents a somewhat harder circuit problem to solve. A simplification can be made by transferring the exciting circuit across the terminals as in Fig. 32.40 or in Fig. 32.41 (a). It should be noted that in this case  $X_0 = V_1/I_\mu$ .

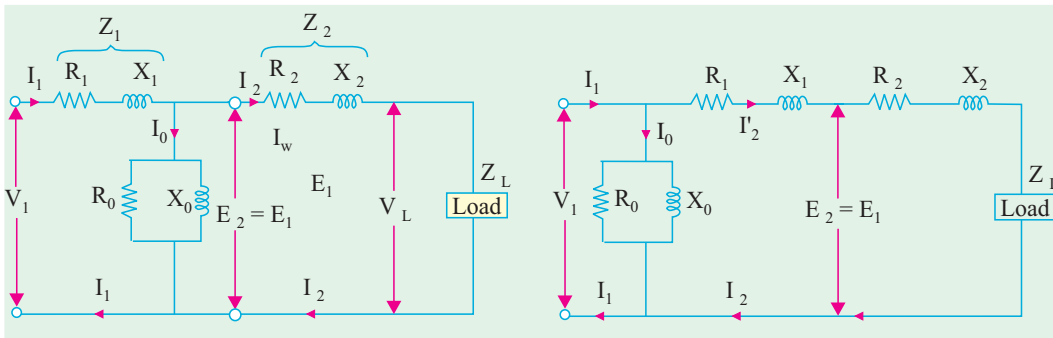


Fig. 32.39

Fig. 32.40

Further simplification may be achieved by omitting  $I_0$  altogether as shown in Fig. 32.41 (b).

From Fig. 32.39 it is found that total impedance between the input terminal is

$$Z = Z_1 + Z_m || (Z_2' + Z_L') = \left( Z_1 + \frac{Z_m (Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')} \right)$$

where  $Z_2' = R_2' + jX_2'$  and  $Z_m$  = impedance of the exciting circuit.

This is so because there are two parallel circuits, one having an impedance of  $Z_m$  and the other having  $Z_2'$  and  $Z_L'$  in series with each other.

$$\therefore V_1 = I_1 \left[ Z_1 + \frac{Z_m (Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')} \right]$$

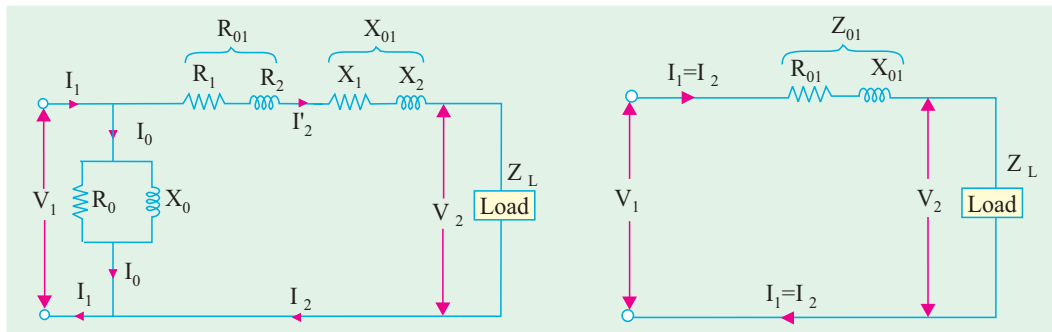


Fig. 32.41 (a)

Fig. 32.41 (b)

**Example 32.25.** The parameters of a 2300/230 V, 50-Hz transformer are given below :

$$R_1 = 0.286 \Omega \quad R_2' = 0.319 \Omega \quad R_0 = 250 \Omega$$

$$X_1 = 0.73 \Omega \quad X_2' = 0.73 \Omega \quad X_0 = 1250 \Omega$$

The secondary load impedance  $Z_L = 0.387 + j 0.29$ . Solve the exact equivalent circuit with normal voltage across the primary.

**Solution.**

$$K = 230/2300 = 1/10; \quad Z_L = 0.387 + j0.29$$

$$Z_L' = Z_L/K^2 = 100(0.387 + j0.29) = 38.7 + j29 = 48.4 \angle 36.8^\circ$$

$$\therefore Z_2' + Z_L' = (38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = 49.0 \angle 37.3^\circ$$

$$Y_m = (0.004 - j0.0008); \quad Z_m = 1/Y_m = 240 + j48 = 245 \angle 11.3^\circ$$

$$Z_m + (Z_2' + Z_L') = (240 + j48) + (39 + j29.7) = 279 \angle 15.6^\circ$$

$$\therefore I_1 = \frac{V_1}{Z_1 + \frac{Z_m(Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')}} = \left[ \frac{2300 \angle 0^\circ}{0.286 + j0.73 + 41.4 \angle 33^\circ} \right]$$

$$= \frac{2300 \angle 0^\circ}{42 \angle 33.7^\circ} = 54.8 \angle -33.7^\circ$$

Now 
$$I_2' = I_1 \times \frac{Z_m}{(Z_2' + Z_L') + Z_m} = 54.8 \angle -33.7^\circ \times \frac{245 \angle 11.3^\circ}{290 \angle 15.6^\circ}$$

$$= 54.8 \angle -33.7^\circ \times 0.845 \angle -4.3^\circ = 46.2 \angle -38^\circ$$

$$I_0 = I_1 \times \frac{(Z_2' + Z_L')}{Z_m + (Z_2' + Z_L')} = 54.8 \angle -33.7^\circ \times \frac{49 \angle 37.3^\circ}{290 \angle 15.6^\circ}$$

$$= 54.8 \angle -33.7^\circ \times 0.169 \angle 21.7^\circ = 9.26 \angle -12^\circ$$

Input power factor

$$= \cos 33.7^\circ = 0.832 \text{ lagging}$$

Power input

$$= V_1 I_1 \cos \phi_1 = 2300 \times 54.8 \times 0.832 = 105 \text{ kW}$$

Power output

$$= 46.2^2 \times 38.7 = 82.7 \text{ kW}$$

Primary Cu loss

$$= 54.8^2 \times 0.286 = 860 \text{ W}$$

Secondary Cu loss

$$= 46.2^2 \times 0.319 = 680 \text{ W}; \quad \text{Core loss} = 9.26^2 \times 240 = 20.6 \text{ kW}$$

$$\eta = (82.7/105) \times 100 = 78.8\%; \quad V_2' = I_2' Z_L' = 46.2 \times 48.4 = 2,240 \text{ V}$$

$$\therefore \text{Regulation} = \frac{2300 - 2240}{2240} \times 100 = 2.7\%$$

**Example 32.26.** A transformer has a primary winding with a voltage-rating of 600 V. Its secondary-voltage rating is 1080 V with an additional tap at 720 V. An 8 kW resistive load is connected across 1080-V output terminals. A purely inductive load of 10kVA is connected across the tapping point and common secondary terminal so as to get 720 V. Calculate the primary current and its power-factor. Correlate it with the existing secondary loads. Neglect losses and magnetizing current. (Nagpur University, Winter 1999)

**Solution.** Loads are connected as shown in Fig. 32.42.

$$I_{r_2} = \frac{8000}{1080} = 7.41 \text{ at unity p.f.}$$

$$I_{L_2} = 10000/720 = 13.89 \text{ at zero lagging p.f.}$$

These are reflected on to the primary sides with appropriate ratios of turns, with corresponding power-factors. If the corresponding transformed currents are represented by the above symbols modified by dashed superscripts,

$$I'_{r_2} = 7.41 \times 1080/600 = 13.34 \text{ A at unity p.f.}$$

$$I'_{L_2} = 13.89 \times 720/600 = 16.67 \text{ A at zero lag. p.f.}$$

Hence,

$$I_{r_2} = [I'^2_{r_2} + I'^2_{L_2}]^{0.5} = 21.35 \text{ A, at } 0.625 \text{ lag p.f.}$$

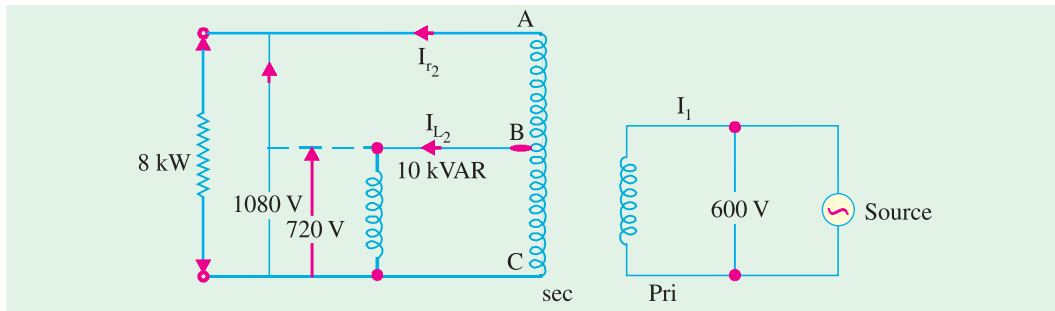


Fig. 32.42

**Correlation :** Since losses and magnetizing current are ignored, the calculations for primary current and its power-factor can also be made with data pertaining to the two Loads (in kW/kVAR), as supplied by the 600 V source.

$S$  = Load to be supplied : 8 kW at unity p.f. and 10 kVAR lagging

Thus,

$$S = P + jQ = 8 - j 10 \text{ kVA}$$

$$S = (8^2 + 10^2)^{0.5} = 12.8 \text{ kVA}$$

$$\text{Power - factor} = \cos \phi = 8/12.8 = 0.625 \text{ lag}$$

$$\text{Primary current} = 12.8 \times 1000/600 = 21.33 \text{ A}$$

### 32.19. Transformer Tests

As shown in Ex 32.25, the performance of a transformer can be calculated on the basis of its equivalent circuit which contains (Fig. 32.41) four main parameters, the equivalent resistance  $R_{01}$  as referred to primary (or secondary  $R_{02}$ ), the equivalent leakage reactance  $X_{01}$  as referred to primary (or secondary  $X_{02}$ ), the core-loss conductance  $G_0$  (or resistance  $R_0$ ) and the magnetising susceptance  $B_0$  (or reactance  $X_0$ ). These constants or parameters can be easily determined by two tests (i) **open-circuit test** and (ii) **short-circuit test**. These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of very large a.c. machinery consists of running two tests similar to the open and short-circuit tests of a transformer.



Small transformer

32.20. Open-circuit or No-load Test

The purpose of this test is to determine no-load loss or core loss and no-load  $I_0$  which is helpful in finding  $X_0$  and  $R_0$ .

One winding of the transformer – whichever is convenient but usually high voltage winding – is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter  $W$ , voltmeter  $V$  and an ammeter  $A$  are connected in the low-voltage winding *i.e.* primary winding in the present case.

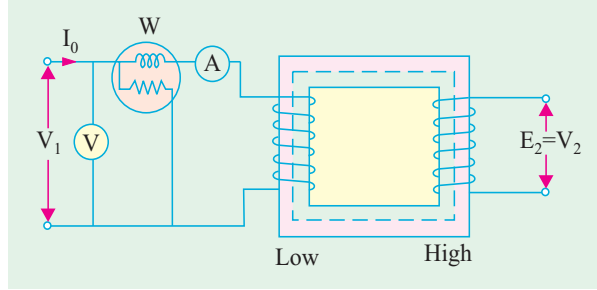


Fig. 32.43

With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which are recorded by the wattmeter. As the primary no-load current  $I_0$  (as measured by ammeter) is small (usually 2 to 10% of rated load current), Cu loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition (and which is the same for all loads as pointed out in Art. 32.9).

It should be noted that since  $I_0$  is itself very small, the pressure coils of the wattmeter and the voltmeter are connected such that the current in them does not pass through the current coil of the wattmeter.

Sometimes, a high-resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced e.m.f. in the secondary winding. This helps to find transformation ratio  $K$ .

The no-load vector diagram is shown in Fig. 32.16. If  $W$  is the wattmeter reading (in Fig. 32.43), then

$$W = V_1 I_0 \cos \phi_0 \quad \therefore \quad \cos \phi_0 = W / V_1 I_0$$

$$\therefore \quad I_\mu = I_0 \sin \phi_0, \quad I_w = I_0 \cos \phi_0 \quad \therefore \quad X_0 = V_1 / I_\mu \quad \text{and} \quad R_0 = V_1 / I_w$$

Or since the current is practically all-exciting current when a transformer is on no-load (*i.e.*  $I_0 \cong I_\mu$ ) and as the voltage drop in primary leakage impedance is small\*, hence the exciting admittance  $Y_0$  of the transformer is given by  $I_0 = V_1 Y_0$  or  $Y_0 = I_0 / V_1$ .

$$\text{The exciting conductance } G_0 \text{ is given by } W = V_1^2 G_0 \text{ or } G_0 = W / V_1^2.$$

$$\text{The exciting susceptance } B_0 = \sqrt{(Y_0^2 - G_0^2)}$$

**Example. 32.27.** In no-load test of single-phase transformer, the following test data were obtained :

Primary voltage : 220 V ; Secondary voltage : 110 V ;

Primary current : 0.5 A ; Power input : 30 W.

Find the following :

(i) The turns ratio (ii) the magnetising component of no-load current (iii) its working (or loss) component (iv) the iron loss.

Resistance of the primary winding = 0.6 ohm.

Draw the no-load phasor diagram to scale.

(Elect. Machine A.M.I.E. 1990)

**Solution.** (i) Turn ratio,  $N_1 / N_2 = 220 / 110 = 2$

(ii)  $W = V_1 I_0 \cos \phi_0$ ;  $\cos \phi_0 = 30 / 220 \times 0.5 = 0.273$ ;  $\sin \phi_0 = 0.962$

$$I_\mu = I_0 \sin \phi_0 = 0.5 \times 0.962 = 0.48 \text{ A}$$

\* If it is not negligibly small, then  $I_0 = E_1 Y_0$  *i.e.* instead of  $V_1$  we will have to use  $E_1$ .

$$\begin{aligned} \text{(iii)} \quad I_w &= I_0 \cos \phi_0 = 0.5 \times 0.273 = 0.1365 \text{ A} \\ \text{(iv)} \quad \text{Primary Cu loss} &= I_0^2 R_1 = 0.5^2 \times 0.6 = 0.15 \text{ W} \\ \therefore \quad \text{Iron loss} &= 30 - 0.15 = 29.85 \text{ W} \end{aligned}$$

**Example 32.28.** A 5 kVA 200/1000 V, 50 Hz, single-phase transformer gave the following test results :

O.C. Test (L.V. Side) : 2000 V, 1.2 A, 90 W

S.C. Test (H.V. Side) : 50 V, 5A, 110 W

- (i) Calculate the parameters of the equivalent circuit referred to the L.V. side.  
(ii) Calculate the output secondary voltage when delivering 3 kW at 0.8 p.f. lagging, the input primary voltage being 200 V. Find the percentage regulation also.

(Nagpur University, November 1998)

**Solution.** (i) Shunt branch parameters from O.C. test (L.V. side) :

$$\begin{aligned} R_0 &= V^2/P_i = 200^2/90 = 444 \text{ ohms}, \quad I_{\omega} = 200/444 = 0.45 \text{ amp} \\ I_{\mu} &= (1.2^2 - 0.45^2)^{0.5} = 1.11 \text{ amp}, \quad X_m = 200/1.11 = 180.2 \text{ ohms} \end{aligned}$$

All these are referred to L.V. side.

(ii) Series-branch Parameters from S.C test (H.V side) :

Since the S.C. test has been conducted from H.V. side, the parameters will refer to H.V. side.

They should be converted to the parameters referred to L.V. side by transforming them suitably.

From S.C. Test readings,  $Z = 50/5 = 10 \text{ ohms}$

$$R = 110/25 = 4.40 \text{ ohms}, \quad X = (10^2 - 4.4^2)^{0.5} = 8.9 \text{ ohms}$$

These are referred to H.V. side.

For referring these to L.V. side, transform these using the ratio of turns, as follows :

$$\begin{aligned} r_1 &= 4.40 \times (200/1000)^2 = 0.176 \text{ ohm} \\ x_1 &= 8.98 \times (200/1000)^2 = 0.36 \text{ ohm} \end{aligned}$$

Equivalent circuit can be drawn with  $R_0$  and  $X_m$  calculated above and  $r_1$  and  $x_1$  as above.

L.V. Current at rated load =  $5000/200 = 25 \text{ A}$

L.V. Current at 3 kW at 0.8 lagging p.f. =  $(3000/0.8)/200 = 18.75 \text{ A}$

$$\begin{aligned} \text{Regulation at this load} &= 18.75 (r_1 \cos \phi + x_1 \sin \phi) \\ &= 18.75 (0.176 \times 0.80 + 0.36 \times 0.6) \\ &= +6.69 \text{ Volts} = + (6.69/200) \times 100\% = +3.345\% \end{aligned}$$

This is referred to L.V. side, and positive sign means voltage drop.

Regulation in volts ref. to H.V. side =  $6.69 \times 1000/200 = 33.45 \text{ V}$

With 200 V across primary (i.e. L.V. side), the secondary (i.e. H.V. side)

$$\text{terminal voltage} = 1000 - 33.45 = 966.55 \text{ V}$$

**Note:** Since approximate formula for voltage regulation has been used, the procedure is simpler, and faster.

### 32.21. Separation of Core Losses

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts (i) hysteresis loss  $W_h = PB_{\max}^{1.6} f$  as given by Steinmetz's empirical relation and (ii) eddy current loss  $W_e = QB_{\max}^2 f^2$  where  $Q$  is a constant. The total core-loss is given by

$$W_i = W_h + W_e = PB_{\max}^{1.6} f^2 + QB_{\max}^2 f^2$$

If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants  $P$  and  $Q$  and hence calculate hysteresis and eddy current losses separately.

**Example 32.29.** In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** Since the flux density is the same in both cases, we can use the relation

$$\text{Total core loss } W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

$$\therefore 52/40 = A + 40B \quad \text{and} \quad 90/60 = A + 60B; \quad \therefore A = 0.9 \quad \text{and} \quad B = 0.01$$

At 50 Hz, the two losses are

$$W_h = Af = 0.9 \times 50 = 45 \text{ W}; \quad W_e = Bf^2 = 0.01 \times 50^2 = 25 \text{ W}$$

**Example 32.30.** In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:

Frequency (Hz)	25	40	50	60	80
Total loss (watt)	18.5	36	50	66	104

Calculate the eddy current loss per kg at a frequency of 50 Hz.

(Elect. Measur. A.M.I.E. Sec B, 1991)

**Solution.** When flux density and wave form factor remain constant, the expression for iron loss can be written as

$$W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

The values of  $W_i/f$  for different frequencies are as under :

$f$	25	40	50	60	80
$W_i/f$	0.74	0.9	1.0	1.1	1.3

The graph between  $f$  and  $W_i/f$  has been plotted in Fig. 32.44. As seen from it,  $A = 0.5$  and  $B = 0.01$

$$\therefore \text{Eddy current loss at 50 Hz} = Bf^2 = 0.01 \times 50^2 = 25 \text{ W}$$

$$\therefore \text{Eddy current loss/kg} = 25/10 = 2.5 \text{ W}$$

**Example 32.31.** In a test for the determination of the losses of a 440-V, 50-Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850 W. Calculate the eddy-current loss at normal voltage and frequency.

(Elect. Inst. and Meas. Punjab Univ. 1991)

**Solution.** The flux density in both cases is the same because in second case voltage as well as frequency are halved. Flux density remaining the same, the eddy current loss is proportional to  $f^2$  and hysteresis loss  $\propto f$ .

$$\text{Hysteresis loss } \propto f = Af \quad \text{and} \quad \text{eddy current loss } \propto f^2 = Bf^2$$

where  $A$  and  $B$  are constants.

$$\text{Total iron loss} \quad W_i = Af + Bf^2 \quad \therefore \quad \frac{W_i}{f} = A + Bf \quad \dots(i)$$

Now, when  $f = 50 \text{ Hz} : W_i = 2500 \text{ W}$

and when  $f = 25 \text{ Hz} ; W_i = 850 \text{ W}$

Using these values in (i) above, we get, from Fig. 32.44

$$2,500/50 = A + 50B \quad \text{and} \quad 850/25 = A + 25B \quad \therefore \quad B = 16/25 = 0.64$$

Hence, at normal p.d. and frequency

$$\text{eddy current loss} = Bf^2 = 0.64 \times 50^2 = \mathbf{1600 \text{ W}}$$

$$\text{Hysteresis loss} = 2500 - 1600 = 900 \text{ W}$$

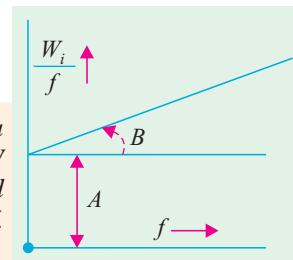


Fig. 32.44



**Example 32.32.** When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.

**Solution.** Hysteresis loss  $W_h \propto B_{\max}^{1.6} f = PB_{\max}^{1.6} f$

Eddy current loss  $W_e \propto B_{\max}^2 f^2 = QB_{\max}^2 f^2$

From the relation  $E = 4.44 fNB_{\max}$  A volt, we get  $B_{\max} \propto E/f$

Putting this value of  $B_{\max}$  in the above equations, we have

$$W_h = P \left( \frac{E}{f} \right)^{1.6} f = PE^{1.6} f^{-0.6} \quad \text{and} \quad W_e = Q \left( \frac{E}{f} \right)^2 f^2 = QE^2$$

In the first case,

$$E = 1000 \text{ V}, f = 50 \text{ Hz}, W_h = 650 \text{ W}, W_e = 350 \text{ W}$$

$$\therefore 650 = P \times 1000^{1.6} \times 50^{-0.6} \quad \therefore P = 650 \times 1000^{-1.6} \times 50^{0.6}$$

Similarly,

$$350 = Q \times 1000^2 \quad \therefore Q = 350 \times 1000^{-2}$$

Hence, constants  $P$  and  $Q$  are known.

Using them in the second case, we get

$$W_h = (650 \times 1000^{-1.6} \times 50^{0.6}) \times 2000^{1.6} \times 100^{-0.6} = 650 \times 2 = 1,300 \text{ W}$$

$$W_e = (350 \times 1000^{-2}) \times 2,000^2 = 350 \times 4 = 1,400 \text{ W}$$

$$\therefore \text{Core loss under new condition is} = 1,300 + 1,400 = \mathbf{2700 \text{ W}}$$

#### Alternative Solution

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

#### With 1000 V at 50 Hz

$$W_h = Af \text{ or } 650 = 50A; A = 13$$

$$W_e = Bf^2 \text{ or } 350 = B \times 50^2; B = 7/50$$

#### With 2000 V at 100 Hz

$$W_h = Af = 13 \times 100 = 1300 \text{ W and}$$

$$W_e = Bf^2 = (7/50) \times 100^2 = 1400 \text{ W}$$

$$\therefore \text{New core loss} = 1300 + 1400 = \mathbf{2700 \text{ W}}$$

**Example 32.33.** A transformer with normal voltage impressed has a flux density of 1.4 Wb/m<sup>2</sup> and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions ?

- increasing the applied voltage by 10% at rated frequency.
- reducing the frequency by 10% with normal voltage impressed.
- increasing both impressed voltage and frequency by 10 per cent.

(Electrical Machinery-I, Madras Univ. 1985)

**Solution.** As seen from Ex. 32.32

$$W_h = PE^{1.6} f^{-0.6} \quad \text{and} \quad W_e = QE^2$$

$$\text{From the given data, we have } 3000 = PE^{1.6} f^{-0.6} \quad \dots(i)$$

$$\text{and } 1000 = QE^2 \quad \dots(ii)$$

where  $E$  and  $f$  are the normal values of primary voltage and frequency.

$$(a) \text{ Here voltage becomes } = E + 10\% E = 1.1 E$$

$$\text{The new hysteresis loss is } W_h = P(1.1 E)^{1.6} f^{-0.6} \quad \dots(iii)$$

$$\text{Dividing Eq. (iii) by (i), we get } \frac{W_h}{3000} = 1.1^{1.6}; W_h = 3000 \times 1.165 = \mathbf{3495 \text{ W}}$$

The new eddy-current loss is

$$W_e = Q (1.1 \cdot E)^2 \quad \therefore \quad \frac{W_e}{1000} = 1.1^2$$

$$\therefore \quad W_e = 1000 \times 1.21 = \mathbf{1210 \text{ W}}$$

(b) As seen from Eq. (i) above eddy-current loss would not be effected. The new hysteresis loss is

$$W_h = PE^{1.6} (0.9 f)^{-0.6} \dots (iv)$$

From (i) and (iv), we get  $\frac{W_h}{3000} = 0.9^{-0.6}$ ,  $W_h = 3000 \times 1.065 = \mathbf{3,196 \text{ W}}$

(c) In this case, both  $E$  and  $f$  are increased by 10%. The new losses are as under :

$$W_h = P (1.1 E)^{1.6} (1.1 f)^{-0.6}$$

$$\therefore \quad \frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6} = 1.165 \times 0.944$$

$$\therefore \quad W_h = 3000 \times 1.165 \times 0.944 = \mathbf{3,299 \text{ W}}$$

As  $W_e$  is unaffected by changes in  $f$ , its value is the same as found in (a) above i.e. **1210 W**

**Example 32.34.** A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core-loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.

(Bharathiar Univ. Nov. 1997)

**Solution.** For constant flux density (i.e. constant  $V/f$  ratio), which is fulfilled by 2200/40 or 3300/60 figures in two cases,

$$\text{Core-loss} = Af + Bf^2$$

First term on the right-hand side represents hysteresis-loss and the second term represents the eddy-current loss.

At 40 Hz,  $800 = 600 + \text{eddy current loss}$ .

Thus,  $Af = 600$ , or  $A = 15$

$$Bf^2 = 200, \text{ or } B = 200/1600 = 0.125$$

At 60 Hz,  $\text{core-loss} = 15 \times 60 + 0.125 \times 60^2$

$$= 900 + 450$$

$$= 1350 \text{ watts}$$

### 32.22 Short-Circuit or Impedance Test

This is an economical method for determining the following :

(i) Equivalent impedance ( $Z_{01}$  or  $Z_{02}$ ), leakage reactance ( $X_{01}$  or  $X_{02}$ ) and total resistance ( $R_{01}$  or  $R_{02}$ ) of the transformer as referred to the winding in which the measuring instruments are placed.

(ii) Cu loss at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer.

(iii) Knowing  $Z_{01}$  or  $Z_{02}$ , the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

In this test, one winding, usually the low-voltage winding, is solidly short-circuited by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current) as shown in Fig. 32.45.

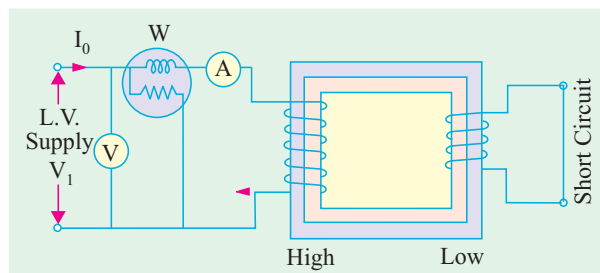


Fig. 32.45

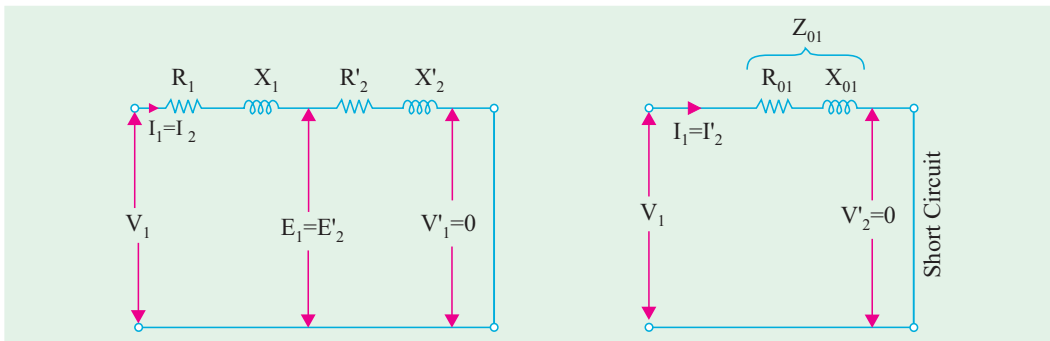


Fig. 32.46

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency (though for Cu losses it is not essential) is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary (as indicated by the respective ammeters).

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux  $\Phi$  produced is also a small percentage of its normal value (Art. 32.6). Hence, core losses are very small with the result that the wattmeter reading represent the full-load Cu loss or  $I^2R$  loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss. The equivalent circuit of the transformer under short-circuit condition is shown in Fig. 32.46. If  $V_{sc}$  is the voltage required to circulate rated load currents, then  $Z_{01} = V_{sc}/I_1$

Also  $W = I_1^2 R_{01}$

$\therefore R_{01} = W/I_1$

$\therefore X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$

In Fig. 32.47 (a) the equivalent circuit vector diagram for the short-circuit test is shown. This diagram is the same as shown in Fig. 32.34 except that all the quantities are referred to the primary side. It is obvious that the entire voltage  $V_{sc}$  is consumed in the impedance drop of the two windings.

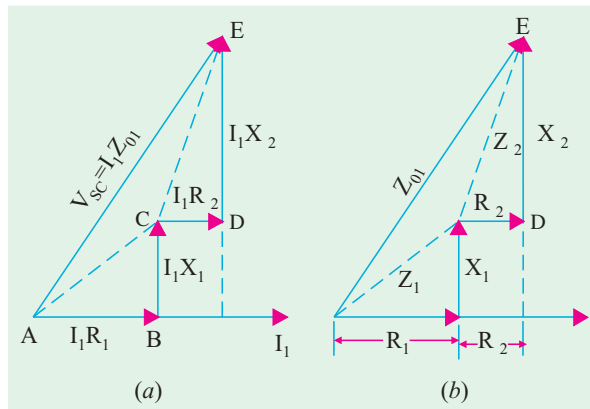


Fig. 32.47

If  $R_1$  can be measured, then knowing

$R_{01}$ , we can find  $R_2' = R_{01} - R_1$ . The impedance triangle can then be divided into the appropriate equivalent triangles for primary and secondary as shown in Fig. 32.47 (b).

### 32.23. Why Transformer Rating in kVA ?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current *i.e.* it is independent of load power factor. That is why rating of transformers is in kVA and not in kW.

**Example 32.35.** The primary and secondary windings of a 30 kVA 76000/230, V, 1-phase transformer have resistance of 10 ohm and 0.016 ohm respectively. The reactance of the transformer referred to the primary is 34 ohm. Calculate the primary voltage required to circulate full-load current when the secondary is short-circuited. What is the power factor on short circuit ?

(Elect. Machines AMIE Sec. B 1991)

**Solution.**

$$K = 230/6000 = 23/600, X_{01} = 34 \Omega,$$

$$R_{01} = R_1 + R_2/K^2 = 10 + 0.016 (600/23)^2 = 20.9 \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{20.9^2 + 34^2} = 40 \Omega$$

$$\text{F.L., } I_1 = 30,000/6000 = 5 \text{ A ; } V_{SC} = I_1 Z_{01} = 5 \times 40 = 200 \text{ V}$$

$$\text{Short circuit p.f.} = R_{01}/Z_{01} = 20.9/40 = 0.52$$

**Example 32.36.** Obtain the equivalent circuit of a 200/400-V, 50-Hz, 1-phase transformer from the following test data :

O.C test : 200 V, 0.7 A, 70 W – on L.V. side

S.C. test : 15 V, 10 A, 85 W – on H.V. side

Calculate the secondary voltage when delivering 5 kW at 0.8 p.f. lagging, the primary voltage being 200V. (Electrical Machinery-I, Madras Univ. 1987)

**Solution. From O.C. Test**

$$V_1 I_0 \cos \phi_0 = W_0$$

$$\therefore 200 \times 0.7 \times \cos \phi_0 = 70$$

$$\cos \phi_0 = 0.5 \text{ and } \sin \phi_0 = 0.866$$

$$I_w = I_0 \cos \phi_0 = 0.7 \times 0.5 = 0.35 \text{ A}$$

$$I_\mu = I_0 \sin \phi_0 = 0.7 \times 0.866 = 0.606 \text{ A}$$

$$R_0 = V_1/I_w = 200/0.35 = 571.4 \Omega$$

$$X_0 = V_1/I_\mu = 200/0.606 = 330 \Omega$$

As shown in Fig. 32.48, these values refer to primary i.e. low-voltage side.

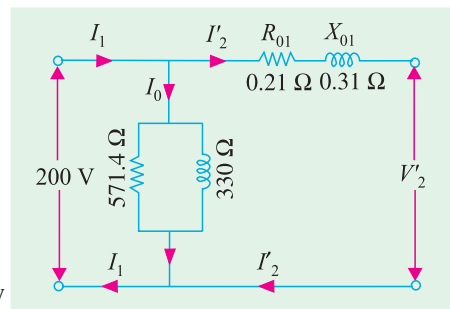


Fig. 32.48

**From S.C. Test**

It may be noted that in this test, instruments have been placed in the secondary i.e. high-voltage winding whereas the low-voltage winding i.e. primary has been short-circuited.

Now, as shown in Art. 32.32

$$Z_{02} = V_{sc}/I_2 = 15/10 = 1.5 \Omega ; K = 400/200 = 2$$

$$Z_{01} = Z_{02}/K^2 = 1.5/4 = 0.375 \Omega$$

Also  $I_2^2 R_{02} = W ; R_{02} = 85/100 = 0.85 \Omega$

$$R_{01} = R_{02}/K^2 = 0.85/4 = 0.21 \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31 \Omega$$

$$\text{Output kVA} = 5/0.8 ; \text{ Output current } I_2 = 5000/0.8 \times 400 = 15.6 \text{ A}$$

This value of  $I_2$  is approximate because  $V_2$  (which is to be calculated as yet) has been taken equal to 400 V (which, in fact, is equal to  $E_2$  or  $V_2$ ).

Now,  $Z_{02} = 1.5 \Omega, R_{02} = 0.85 \Omega \therefore X_{02} = \sqrt{1.5^2 - 0.85^2} = 1.24 \Omega$

Total transformer drop as referred to secondary

$$= I_2 (R_{02} \cos \phi_2 + X_{02} \sin \phi_2) = 15.6 (0.85 \times 0.8 + 1.24 \times 0.6) = 22.2 \text{ V}$$

$$\therefore V_2 = 400 - 22.2 = \mathbf{377.8 \text{ V}}$$

**Example 32.37.** Starting from the ideal transformer, obtain the approximate equivalent circuit of a commercial transformer in which all the constants are lumped and represented on one side.

A 1-phase transformer has a turn ratio of 6. The resistance and reactance of primary winding are 0.9  $\Omega$  and 5  $\Omega$  respectively and those of the secondary are 0.03  $\Omega$  and 0.13  $\Omega$  respectively. If 330 -V at 50-Hz be applied to the high voltage winding with the low-voltage winding short-circuited, find the current in the low-voltage winding and its power factor. Neglect magnetising current.

**Solution.**

$$\text{Here } K = 1/6; R_{01} = R_1 + R'_2 = 0.9 + (0.03 \times 36) = 1.98 \Omega$$

$$X_{01} = X_1 + X'_2 = 5 + (0.13 \times 36) = 9.68 \Omega$$

$$\therefore Z_{01} = \sqrt{(9.68^2 + 1.98^2)} = 9.9 \Omega; V_{SC} = 330 \text{ V}$$

$$\therefore \text{Full-load primary current } I_1 = V_{sc}/Z_{01} = 330/9.9 = 100/3 \text{ A}$$

$$\text{As } I_0 \text{ is negligible, hence } I_1 = I'_2 = 100/3 \text{ A. Now, } I'_2 = KI_2$$

$$\text{F.L. secondary current } I_2 = I'_2 K = (100/3) \times 6 = 200 \text{ A}$$

$$\text{Now, Power input on short-circuit} = V_{SC} I_1 \cos \phi_{SC} = \text{Cu loss} = I_1^2 R_{01}$$

$$\therefore (100/3)^2 \times 1.98 = 330 \times (100/3) \times \cos \phi_{SC}; \cos \phi_{SC} = \mathbf{0.2}$$

**Example 32.38.** A 1-phase, 10-kVA, 500/250-V, 50-Hz transformer has the following constants:

Reactance : primary 0.2  $\Omega$ ; secondary 0.5  $\Omega$

Resistance : primary 0.4  $\Omega$ ; secondary 0.1  $\Omega$

Resistance of equivalent exciting circuit referred to primary,  $R_0 = 1500 \Omega$

Reactance of equivalent exciting circuit referred to primary,  $X_0 = 750 \Omega$

What would be the reading of the instruments when the transformer is connected for the open-circuit and short-circuit tests ?

**Solution.** While solving this question, reference may please be made to Art. 30.20 and 30.22.

**O.C. Test**

$$I_\mu = V_1/X_0 = 500/750 = 2/3 \text{ A}; I_w = V_1/R_0 = 500/1500 = 1/3 \text{ A}$$

$$\therefore I_0 = \sqrt{[(1/3)^2 + (2/3)^2]} = 0.745 \text{ A}$$

$$\text{No-load primary input} = V_1 I_w = 500 \times 1/3 = 167 \text{ W}$$

Instruments used in primary circuit are : voltmeter, ammeter and wattmeter, their readings being 500 V, 0.745 A and 167 W respectively.

**S.C. Test**

Suppose S.C. test is performed by short-circuiting the l.v. winding i.e. the secondary so that all instruments are in primary.

$$R_{01} = R_1 + R'_2 = R_1 + R_2/K^2; \text{ Here } K = 1/2 \quad \therefore R_{01} = 0.2 + (4 \times 0.5) = 2.2 \Omega$$

$$\text{Similarly, } X_{01} = X_1 + X'_2 = 0.4 + (4 \times 0.1) = 0.8 \Omega$$

$$Z_{01} = \sqrt{(2.2^2 + 0.8^2)} = 2.341 \Omega$$

Full-load primary current

$$I_1 = 10,000/500 = 20 \text{ A} \quad \therefore V_{SC} = I_1 Z_{01} = 20 \times 2.341 = 46.8 \text{ V}$$

$$\text{Power absorbed} = I_1^2 R_{01} = 20^2 \times 2.2 = 880 \text{ W}$$

Primary instruments will read : **46.8 V, 20 A, 880 W.**

**Example 32.39.** The efficiency of a 1000-kVA, 110/220 V, 50-Hz, single-phase transformer, is 98.5 % at half full-load at 0.8 p.f. leading and 98.8 % at full-load unity p.f. Determine (i) iron loss (ii) full-load copper loss and (iii) maximum efficiency at unity p.f.

(Elect. Engg. AMIETE Sec. A Dec. 1991)

**Solution.** Output at F.L. unity p.f. = 1000  $\times$  1 = 1000 kW

$$\text{F.L. input} = 1000/0.988 = 1012.146 \text{ kW}$$

$$\text{F.L. losses} = 1012.146 - 1000 = 12.146 \text{ kW}$$

If F.L. Cu and iron losses are  $x$  and  $y$  respectively then

$$x + y = 12.146 \text{ kW} \quad \dots(i)$$

Input at half F.L. 0.8 p.f. =  $500 \times 0.8/0.985 = 406.091$  kW

Total losses at half F.L. =  $406.091 - 400 = 6.091$  kW

Cu loss at half-load =  $x (1/2)^2 = x/4$

$\therefore x/4 + y = 6.091$  ... (ii)

From Eqn. (i) and (ii), we get (i)  $x = 8.073$  kW and (ii)  $y = 4.073$  kW

(iii) kVA for  $\eta_{max} = 1000 \times \sqrt{4.073/8.073} = 710.3$  kVA

Output at u.p.f. =  $710.3 \times 1 = 710.3$  kW

Cu loss = iron loss =  $4.037$  kW ; Total loss =  $2 \times 4.037 = 8.074$  kW

$\therefore \eta_{max} = 710.3/(710.3 + 8.074) = 0.989$  or 98.9 %

**Example 32.40.** The equivalent circuit for a 200/400-V step-up transformer has the following parameters referred to the low-voltage side.

Equivalent resistance =  $0.15 \Omega$  ; Equivalent reactance =  $0.37 \Omega$

Core-loss component resistance =  $600 \Omega$  ; Magnetising reactance =  $300 \Omega$

When the transformer is supplying a load at 10 A at a power factor of 0.8 lag, calculate (i) the primary current (ii) secondary terminal voltage. (Electrical Machinery-I, Bangalore Univ. 1989)

**Solution.** We are given the following :

$R_{01} = 0.15 \Omega, X_{01} = 0.37 \Omega ; R_0 = 600 \Omega, X_0 = 300 \Omega$

Using the approximate equivalent circuit of Fig. 32.41, we have,

$I_\mu = V_1/X_0 = 200/300 = (2/3)$  A

$I_w = V_1/R_0 = 200/600 = (1/3)$  A

$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(2/3)^2 + (1/3)^2} = 0.745$  A

As seen from Fig. 32.49

$\tan \theta = \frac{I_w}{I_\mu} = \frac{1/3}{2/3} = \frac{1}{2} ; \theta = 26.6^\circ$

$\therefore \phi_0 = 90^\circ - 26.6^\circ = 63.4^\circ$  ; Angle between  $I_0$  and

$I'_2 = 63.4^\circ - 36.9^\circ = 26.5^\circ ; K = 400/200 = 2$

$I'_2 = KI_2 = 2 \times 10 = 20$  A

(i)  $I_1 = (0.745^2 + 20^2 + 2 \times 0.745 \times 20 \times \cos 26.5^\circ)^{1/2}$   
= **20.67 A**

(ii)  $R_{02} = K^2 R_{01} = 2^2 \times 0.15 = 0.6 \Omega$

$X_{02} = 2^2 \times 0.37 = 1.48 \Omega$

Approximate voltage drop

=  $I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$

=  $10(0.6 \times 0.8 + 1.48 \times 0.6) = 13.7$  V

$\therefore$  Secondary terminal voltage =  $400 - 13.7$

= **386.3 V**

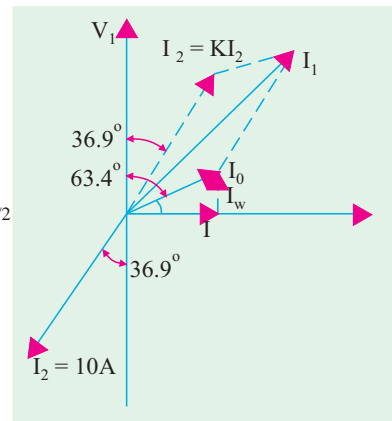


Fig. 32.49

**Example 32.41.** The low voltage winding of a 300-kVA, 11,000/2500-V, 50-Hz transformer has 190 turns and a resistance of 0.06. The high-voltage winding has 910 turns and a resistance of 1.6  $\Omega$ . When the l.v. winding is short-circuited, the full-load current is obtained with 550-V applied to the h.v. winding. Calculate (i) the equivalent resistance and leakage reactance as referred to h.v. side and (ii) the leakage reactance of each winding.

**Solution.** Assuming a full-load efficiency of 0.985, the full-load primary current is

$$= 300,000/0.985 \times 11,000 = 27.7\text{A}$$

$$\therefore Z_{01} = 550/27.7 = 19.8 \Omega ; R'_2 = R^2/K^2 = 0.06 (910/190)^2 = 1.38 \Omega$$

$$\therefore R_{01} = R_1 + R'_2 = 1.6 + 1.38 = 2.98 \Omega$$

$$X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)} = \sqrt{(19.8^2 - 2.98^2)} = 19.5 \Omega$$

Let us make another assumption that for each winding the ratio (reactance/resistance) is the same, then

$$X_1 = 19.5 \times 1.6/2.98 = 10.5 \Omega$$

$$X'_2 = 19.5 \times 1.38/2.98 = 9.0 \Omega ; X_2 = 9(190/910)^2 = 0.39 \Omega$$

(a)  $R_{01} = 2.98 \Omega ; X_{01} = 19.5 \Omega$  (b)  $X_1 = 10.5 \Omega ; X_2 = 0.39 \Omega$

**Example 32.42.** A 230/115 volts, single phase transformer is supplying a load of 5 Amps, at power factor 0.866 lagging. The no-load current is 0.2 Amps at power factor 0.208 lagging. Calculate the primary current and primary power factor.  
(Nagpur University Summer 2000)

**Solution.** L.V. current of 5 amp is referred to as a 2.5 amp current on the primary (= H.V.) side, at 0.866 lagging p.f. To this, the no load current should be added, as per the phasor diagram in Fig. 32.50. The phase angle of the load-current is 30° lagging. The no load current has a phase angle of 80° lagging. Resultant of these two currents has to be worked out. Along the reference, active components are added.

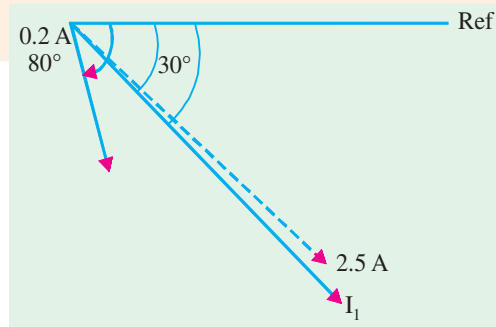


Fig. 32.50. Phasor diagram for Currents

$$\text{Active components of currents} = 2.5 \times 0.866 + 0.2 \times 0.208$$

$$= 2.165 + 0.0416$$

$$= 2.2066 \text{ amp}$$

Along the perpendicular direction, the reactive components get added up.

$$\text{Reactive component} = 2.5 \times 0.5 + 0.2 \times 0.9848$$

$$= 1.25 + 0.197 = 1.447 \text{ amp.}$$

$$I_1 = 2.2066 - j 1.447$$

$$\phi = \tan^{-1} \frac{1.447}{2.2066} = 33.25^\circ ; \text{ as shown}$$

### Tutorial Problems 32.3

- The S.C. test on a 1-phase transformer, with the primary winding short-circuited and 30 V applied to the secondary gave a wattmeter reading of 60 W and secondary current of 10 A. If the normal applied primary voltage is 200, the transformation ratio 1 :2 and the full-load secondary current 10 A, calculate the secondary terminal p.d. at full-load current for (a) unity power factor (b) power factor 0.8 lagging. If any approximations are made, they must be explained. [394 V, 377.6 V]
- A single-phase transformer has a turn ratio of 6, the resistances of the primary and secondary windings are 0.9 Ω and 0.025 Ω respectively and the leakage reactances of these windings are 5.4 Ω and 0.15 Ω respectively. Determine the voltage to be applied to the low-voltage winding to obtain a current of 100 A in the short-circuited high voltage winding. Ignore the magnetising current. [82 V]
- Draw the equivalent circuit for a 3000/400-V, I-phase transformer on which the following test results were obtained. Input to high voltage winding when 1.v. winding is open-circuited : 3000 V, 0.5 A, 500

W. Input to l.v. winding when h.v. winding is short-circuited : 11 V, 100 A, 500 W. Insert the appropriate values of resistance and reactance.

$$[R_0 = 18,000 \Omega, X_0 = 6,360 \Omega, R_{01} = 2.81 \Omega, X_{01} = 5.51 \Omega] \text{ (I.E.E. London)}$$

4. The iron loss in a transformer core at normal flux density was measured at frequencies of 30 and 50 Hz, the results being 30 W and 54 W respectively. Calculate (a) the hysteresis loss and (b) the eddy current loss at 50 Hz. [44 W, 10 W]
  5. An iron core was magnetised by passing an alternating current through a winding on it. The power required for a certain value of maximum flux density was measured at a number of different frequencies. Neglecting the effect of resistance of the winding, the power required per kg of iron was 0.8 W at 25 Hz and 2.04 W at 60 Hz. Estimate the power needed per kg when the iron is subject to the same maximum flux density but the frequency is 100 Hz. [3.63 W]
  6. The ratio of turns of a 1-phase transformer is 8, the resistances of the primary and secondary windings are  $0.85 \Omega$  and  $0.012 \Omega$  respectively and leakage reactances of these windings are  $4.8 \Omega$  and  $0.07 \Omega$  respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary circuit when the secondary terminals are short-circuited. Ignore the magnetising current. [176.4 W]
  7. A transformer has no-load losses of 55 W with a primary voltage of 250 V at 50 Hz and 41 W with a primary voltage of 200 V at 40 Hz. Compute the hysteresis and eddy current losses at a primary voltage of 300 volts at 60 Hz of the above transformer. Neglect small amount of copper loss at no-load. [43.5 W ; 27 W] (Elect. Machines AMIE Sec. B. (E-3) Summer 1992)
  8. A 20 kVA, 2500/250 V, 50 Hz, 1-phase transformer has the following test results :  
O.C. Test (l.v. side) : 250 V, 1.4 A, 105 W  
S.C. Test (h.v. side) : 104 V, 8 A, 320 W  
Compute the parameters of the approximate equivalent circuit referred to the low voltage side and draw the circuit. ( $R_0 = 592.5 \Omega$  ;  $X_0 = 187.2 \Omega$  ;  $R_{02} = 1.25 \Omega$  ;  $X_{12} = 3 \Omega$ )  
(Elect. Machines A.M.I.E. Sec. B Summer 1990)
  9. A 10-kVA, 2000/400-V, single-phase transformer has resistances and leakage reactances as follows :  
 $R_1 = 5.2 \Omega$ ,  $X_1 = 12.5 \Omega$ ,  $R_2 = 0.2 \Omega$ ,  $X_2 = 0.5 \Omega$   
Determine the value of secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with power factor 0.8 lag. The no-load current can be neglected. Draw the phasor diagram. [376.8 V] (Elect. Machines, A.M.I.E. Sec B, 1989)
  10. A 1000-V, 50-Hz supply to a transformer results in 650 W hysteresis loss and 400 W eddy current loss. If both the applied voltage and frequency are doubled, find the new core losses. [ $W_h = 1300 \text{ W}$  ;  $W_e = 1600 \text{ W}$ ] (Elect. Machine, A.M.I.E. Sec. B, 1993)
  11. A 50 kVA, 2200/110 V transformer when tested gave the following results :  
O.C. test (L.V. side) : 400 W, 10 A, 110 V.  
S.C. test (H.V. side) : 808 W, 20.5 A, 90 V.  
Compute all the parameters of the equivalent ckt. referred to the H.V. side and draw the resultant ckt. (Rajiv Gandhi Technical University, Bhopal 2000)
- [Shunt branch :  $R_0 = 12.1 \text{ k-ohms}$ ,  $X_m = 4.724 \text{ k-ohms}$  Series branch :  $r = 1.923 \text{ ohms}$ ,  $x = 4.39 \text{ ohms}$ ]

### 32.24. Regulation of a Transformer

1. When a transformer is loaded with a *constant primary voltage*, the secondary voltage decreases\* because of its internal resistance and leakage reactance.

Let  ${}_0V_2 =$  secondary terminal voltage at *no-load*.

---

\* Assuming lagging power factor. It will increase if power factor is leading.



=  $E_2 = EK_1 = K V_1$  because at no-load the impedance drop is negligible.

$V_2$  = secondary terminal voltage on full-load.

The change in secondary terminal voltage from no-load to full-load is  $= {}_0V_2 - V_2$ . This change divided by  ${}_0V_2$  is known as regulation 'down'. If this change is divided by  $V_2$ , i.e., full-load secondary terminal voltage, then it is called regulation 'up'.

$$\therefore \quad \% \text{ regn 'down'} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100 \quad \text{and} \quad \% \text{ regn 'up'} = \frac{{}_0V_2 - V_2}{V_2} \times 100$$

In further treatment, unless stated otherwise, regulation is to be taken as regulation 'down'.

We have already seen in Art. 32.16 (Fig. 32.35) that the change in secondary terminal voltage from no-load to full-load, expressed as a percentage of no-load secondary voltage is,

$$= v_r \cos \phi \pm v_x \sin \phi \quad (\text{approximately})$$

Or more accurately

$$= (v_r \cos \phi \pm v_x \sin \phi) + \frac{1}{200} (v_x \cos \phi \mp v_r \sin \phi)^2$$

$$\therefore \quad \% \text{ regn} = v_r \cos \phi \pm v_x \sin \phi \quad \dots \text{approximately.}$$

The lesser this value, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load.

(2) The regulation may also be explained in terms of primary values.

In Fig. 32.51 (a) the approximate equivalent circuit of a transformer is shown and in Fig. 32.51 (b), (c) and (d) the vector diagrams corresponding to different power factors are shown.

The secondary *no-load* terminal voltage as referred to primary is  $E'_2 = E_2/K = E_1 = V_1$  and if the secondary full-load voltage as referred to primary is  $V'_2 (= V_2/K)$  then

$$\% \text{ regn} = \frac{V_1 - V'_2}{V_1} \times 100$$

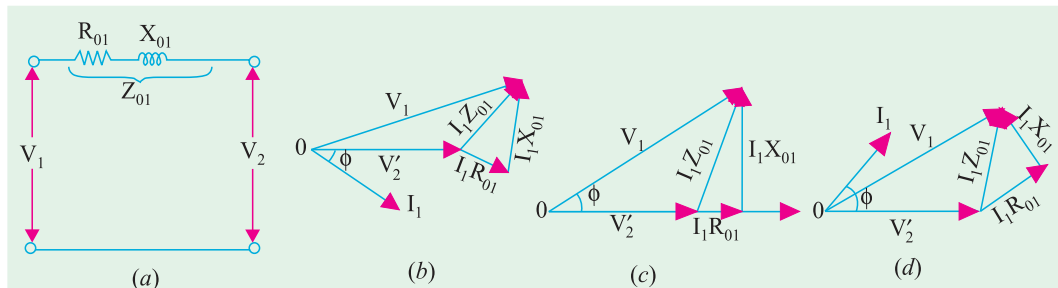


Fig. 32.51

From the vector diagram, it is clear that if angle between  $V_1$  and  $V'_2$  is neglected, then the value of numerical difference  $V_1 - V'_2$  is given by  $(I_1 R_{01} \cos \phi + I_1 X_{01} \sin \phi)$  for lagging p.f.

$$\therefore \quad \% \text{ regn} = \frac{I_1 R_{01} \cos \phi + I_1 X_{01} \sin \phi}{V_1} \times 100 = v_r \cos \phi + v_x \sin \phi$$

where  $\frac{I_1 R_{01} \times 100}{V_1} = v_r$  and  $\frac{I_1 X_{01} \times 100}{V_1} = v_x$  ...Art. 32.16

As before, if angle between  $V_1$  and  $V'_2$  is not negligible, then

$$\% \text{ regn} = (v_r \cos \phi \pm v_x \sin \phi) + \frac{1}{200} (v_x \cos \phi \mp v_r \sin \phi)^2$$

(3) In the above definitions of regulation, *primary voltage was supposed to be kept constant* and the changes in secondary terminal voltage were considered.

As the transformer is loaded, the secondary terminal voltage falls (for a lagging p.f.). Hence, to keep the output voltage constant, the primary voltage must be increased. The rise in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer.

Suppose primary voltage has to be raised from its rated value  $V_1$  to  $V_1'$ , then

$$\% \text{ regn.} = \frac{V_1' - V_1}{V_1} \times 100$$

**Example 32.43.** A-100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3  $\Omega$  and 0.01  $\Omega$  respectively and the corresponding leakage reactances are 1.1 and 0.035  $\Omega$  respectively. The supply voltage is 2200 V. Calculate (i) equivalent impedance referred to primary and (ii) the voltage regulation and the secondary terminal voltage for full load having a power factor of 0.8 leading.

(Elect. Machines, A.M.I.E. Sec. B, 1989)

**Solution.**  $K = 80/400 = 1/5$ ,  $R_1 = 0.3 \Omega$ ,  $R_{01} = R_1 + R_2/K^2 = 0.3 + 0.01/(1/5)^2 = 0.55 \Omega$

$$X_{01} = X_1 + X_2/K^2 = 1.1 + 0.035/(1/5)^2 = 1.975 \Omega$$

(i)  $Z_{01} = 0.55 + j 1.975 = 2.05 \angle 74.44^\circ$

(ii)  $Z_{02} = K^2 Z_{01} = (1/5)^2 (0.55 + j 1.975) = (0.022 + j 0.079)$

No-load secondary voltage =  $K V_1 = (1/5) \times 2200 = 440 \text{ V}$ ,  $I_2 = 10 \times 10^3/440 = 227.3 \text{ A}$

Full-load voltage drop as referred to secondary

$$= I_2 (R_{02} \cos \phi - X_{02} \sin \phi)$$

$$= 227.3 (0.022 \times 0.8 - 0.079 \times 0.6) = -6.77 \text{ V}$$

$$\% \text{ regn.} = -6.77 \times 100/440 = -1.54$$

Secondary terminal voltage on load =  $440 - (-6.77) = 446.77 \text{ V}$

**Example 32.44.** The corrected instrument readings obtained from open and short-circuit tests on 10-kVA, 450/120-V, 50-Hz transformer are :

O.C. test :  $V_1 = 120 \text{ V}$ ;  $I_1 = 4.2 \text{ A}$ ;  $W_1 = 80 \text{ W}$ ;  $V_1$ ,  $W_1$  and  $I_1$  were read on the low-voltage side.

S.C. test :  $V_1 = 9.65 \text{ V}$ ;  $I_1 = 22.2 \text{ A}$ ;  $W_1 = 120 \text{ W}$  - with low-voltage winding short-circuited

Compute :

- (i) the equivalent circuit (approximate) constants,
- (ii) efficiency and voltage regulation for an 80% lagging p.f. load,
- (iii) the efficiency at half full-load and 80% lagging p.f. load.

(Electrical Engineering-I, Bombay Univ. 1988)

**Solution.** It is seen from the O.C. test, that with primary open, the secondary draws a no-load current of 4.2 A. Since  $K = 120/450 = 4/15$ , the corresponding no-load primary current  $I_0 = 4.2 \times 4/15 = 1.12 \text{ A}$ .

(i) Now,  $V_1 I_0 \cos \phi_0 = 80 \therefore \cos \phi_0 = 80/450 \times 1.12 = 0.159$

$\therefore \phi_0 = \cos^{-1}(0.159) = 80.9^\circ$ ;  $\sin \phi_0 = 0.987$

$I_w = I_0 \cos \phi_0 = 1.12 \times 0.159 = 0.178 \text{ A}$  and  $I_\mu = 1.12 \times 0.987 = 1.1 \text{ A}$

$\therefore R_0 = 450/0.178 = 2530 \Omega$  and  $X_0 = 450/1.1 = 409 \Omega$

During S.C. test, instruments have been placed in primary.

$\therefore Z_{01} = 9.65/22.2 = 0.435 \Omega$

$R_{01} = 120/22.2^2 = 0.243 \Omega$

$X_{01} = \sqrt{0.435^2 - 0.243^2} = 0.361 \Omega$

The equivalent circuit is shown in Fig. 32.52.

(ii) Total approximate voltage drop as referred to primary is  $I_1 (R_{01} \cos \phi + X_{01} \sin \phi)$ .

\* Assuming  $\phi_1 = \phi_2 = \cos^{-1}(0.8)$ .

Now, full-load  $I_1 = 10,000/450 = 22.2 \text{ A}$   
 $\therefore \text{Drop} = 22.2 (0.243 \times 0.8 + 0.361 \times 0.6) = 9.2 \text{ V}$   
 Regulation  $= 9.2 \times 100/450 = 2.04\%$   
 F.L. losses  $= 80 + 120 = 200 \text{ W};$   
 F.L. output  $= 10,000 \times 0.8 = 8000 \text{ W}$   
 $\eta = 8000/8200 = 0.9757$  or **97.57%**

(iii) Half-load

Iron loss  $= 80 \text{ W}; \text{Cu loss} = (1/2)^2 \times 120 = 30 \text{ W}$   
 Total losses  $= 110 \text{ W}; \text{Output} = 5000 \times 0.8 = 4000 \text{ W}$   
 $\therefore \eta = 4000/4110 = 0.9734$  or **97.34%**

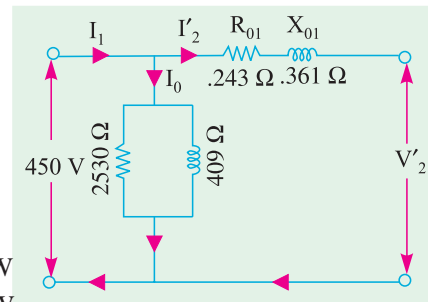


Fig. 32.52

**Example 32.45.** Consider a 20 kVA, 2200/220 V, 50 Hz transformer. The O.C./S.C. test results are as follows :

O.C. test : 220 V, 4.2 A, 148 W (l.v. side)

S.C. test : 86 V, 10.5 A, 360 W (h.v. side)

Determine the regulation at 0.8 p.f. lagging and at full load. What is the p.f. on short-circuit ?

(Elect. Machines Nagpur Univ. 1993)

**Solution.** It may be noted that O.C. data is not required in this question for finding the regulation. Since during S.C. test instruments have been placed on the h.v. side i.e. primary side.

$$\therefore Z_{01} = 86/10.5 = 8.19 \Omega; R_{01} = 360/10.5^2 = 3.26 \Omega$$

$$X_{01} = \sqrt{8.19^2 - 3.26^2} = 7.5 \Omega$$

F.L. primary current,  $I_1 = 20,000/2200 = 9.09 \text{ A}$

Total voltage drop as referred to primary  $= I_1 (R_{01} \cos \phi + X_{01} \sin \phi)$

Drop  $= 9.09 (3.26 \times 0.8 + 7.5 \times 0.6) = 64.6 \text{ V}$

% age regn.  $= 64.6 \times 100/2200 = 2.9\%$ , p.f. on short-circuit  $= R_{01}/Z_{01} = 3.26/8.19 = 0.4 \text{ lag}$

**Example 32.46.** A short-circuit test when performed on the h.v. side of a 10 kVA, 2000/400 V single phase transformer, gave the following data ; 60 V, 4 A, 100 W.

If the l.v. side is delivering full load current at 0.8 p.f. lag and at 400 V, find the voltage applied to h.v. side.

(Elect. Machines-I, Nagpur Univ. 1993)

**Solution.** Here, the test has been performed on the h.v. side i.e. primary side.

$$Z_{01} = 60/4 = 15 \Omega; R_{01} = 100/4^2 = 6.25 \Omega; X_{01} = \sqrt{15^2 - 6.25^2} = 13.63 \Omega$$

FL.  $I_1 = 10,000/2000 = 5 \text{ A}$

Total transformer voltage drop as referred to primary is

$$I_1 (R_{01} \cos \phi + X_{01} \sin \phi) = 5 (6.25 \times 0.8 + 13.63 \times 0.6) = 67 \text{ V}$$

Hence, primary voltage has to be raised from 2000 V to 2067 V in order to compensate for the total voltage drop in the transformer. In that case secondary voltage on load would remain the same as on no-load.

**Example 32.47.** A 250/500-V transformer gave the following test results :

Short-circuit test : with low-voltage winding short-circuited :

20 V; 12 A, 100 W

Open-circuit test : 250 V, 1 A, 80 W on low-voltage side.

Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.

(Elect. Machines, Nagpur Univ. 1993)

**Solution. Open-circuit Test :**

$$V_1 I_0 \cos \phi_0 = 80 \quad \therefore \cos \phi_0 = 80/250 \times I = 0.32$$

$$I_w = I_0 \cos \phi_0 = I \times 0.32 = 0.32 \text{ A}, I_\mu = \sqrt{(1^2 - 0.32^2)} = 0.95 \text{ A}$$

$$R_0 = V_1 / I_w = 250/0.32 = 781.3 \Omega, X_0 = V_1 / I_\mu = 250/0.95 = 263.8 \Omega$$

The circuit is shown in Fig. 32.53 (a).

**Short-circuit Test :**

As the primary is short-circuited, all values refer to secondary winding.

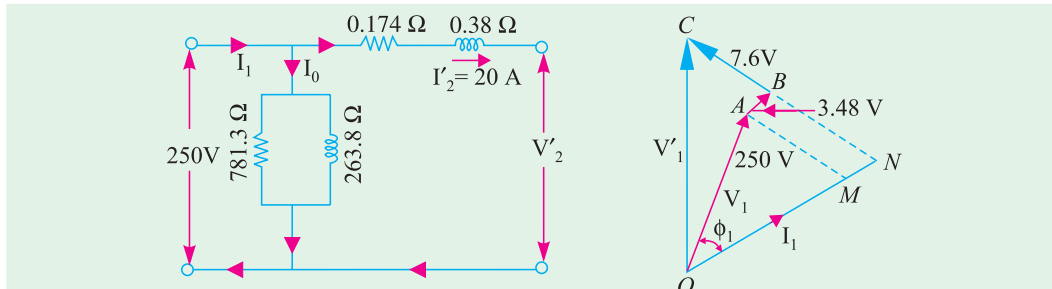


Fig. 32.53 (a)

Fig. 32.53 (b)

$$\therefore R_{02} = \frac{\text{short-circuit power}}{\text{F.L. secondary current}} = \frac{100}{12^2} = 0.694 \Omega$$

$$Z_{02} = 20/12 = 1.667 \Omega; X_{02} = \sqrt{(1.667^2 - 0.694^2)} = 1.518 \Omega$$

As  $R_0$  and  $X_0$  refer to primary, hence we will transfer these values to primary with the help of transformation ratio.

$$K = 500/250 = 2 \quad \therefore R_{01} = R_{02}/K^2 = 0.694/4 = 0.174 \Omega$$

$$X_{01} = X_{02}/K^2 = 1.518/4 = 0.38 \Omega; Z_{01} = Z_{02}/K^2 = 1.667/4 = 0.417 \Omega$$

The equivalent circuit is shown in Fig. 32.53 (a).

**Efficiency**

$$\text{Total Cu loss} = I_2^2 R_{02} = 100 \times 0.694 = 69.4 \text{ W}; \text{Iron loss} = 80 \text{ W}$$

$$\text{Total loss} = 69.4 + 80 = 149.4 \text{ W} \quad \therefore \eta = \frac{5000 \times 0.8 \times 100}{4000 + 149.4} = 96.42\%$$

The applied voltage  $V_1'$  is the vector sum of  $V_1$  and  $I_1 Z_{01}$  as shown in Fig. 32.53 (b).

$$I_1 = 20 \text{ A}; I_1 R_{01} = 20 \times 0.174 = 3.84 \text{ V}; I_1 X_{01} = 20 \times 0.38 = 7.6 \text{ V}$$

Neglecting the angle between  $V_1$  and  $V_1'$ , we have

$$\begin{aligned} V_1'^2 &= OC^2 = ON^2 + NC^2 = (OM + MN)^2 + (NB + BC)^2 \\ &= (250 \times 0.8 + 3.48)^2 + (250 \times 0.6 + 7.6)^2 \\ V_1'^2 &= 203.5^2 + 157.6^2 \quad \therefore V_1' = 257.4 \text{ V} \end{aligned}$$

**Example 32.48.** A 230/230 V, 3 kVA transformer gave the following results :

O.C. Test : 230 V, 2 amp, 100 W

S.C. Test : 15 V, 13 amp, 120 W

Determine the regulation and efficiency at full load 0.80 p.f. lagging.

(Sambalpur University, 1998)

**Solution.** This is the case of a transformer with turns ratio as 1 : 1. Such a transformer is mainly required for isolation.

$$\begin{aligned} \text{Rated Current} &= \frac{3000}{230} = 13 \text{ amp} \\ \text{Cu-losses at rated load} &= 120 \text{ watts, from S.C. test} \\ \text{Core losses} &= 100 \text{ watts, from O.C. test} \\ \text{At full load, VA output} &= \mathbf{3000} \\ \text{At 0.8 lag p.f., Power output} &= 3000 \times 0.8 = 2400 \text{ watts} \\ \text{Required efficiency} &= \frac{2400}{2400 + 220} \times 100\% = 91.6\% \\ \text{From S.C. test, } Z &= \frac{15}{13} = 1.154 \text{ ohms} \\ R &= \frac{120}{15 \times 15} = 0.53 \text{ ohm, } X = \sqrt{1.154^2 - 0.53^2} = 1.0251 \text{ ohm} \\ \text{Approximate voltage regulation} &= IR \cos \phi + IX \sin \phi = 13[0.53 \times 0.8 + 1.0251 \times 0.6] \\ &= 13[0.424 + 0.615] = 13.51 \text{ volts} \end{aligned}$$

$$\text{In terms of \% , the voltage regulation} = \frac{13.51}{230} \times 100\% = \mathbf{5.874\%}$$

**Example 32.49.** A 10 kVA, 500/250 V, single-phase transformer has its maximum efficiency of 94% when delivering 90% of its rated output at unity p.f. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging. (Nagpur University, November 1998)

**Solution.** Rated output at unity p.f. = 10000 W. Hence, 90% of rated output = 9,000 W

$$\text{Input with 94\% efficiency} = 9000/0.94 \text{ W}$$

$$\text{Losses} = 9000((1/0.94) - 1) = 574 \text{ W}$$

At maximum efficiency, variable copper-loss = constant = Core loss = 574/2 = 287 W

At rated current, Let the copper-loss =  $P_c$  watts

At 90% load with unity p.f., the copper-loss is expressed as  $0.90^2 \times P_c$ .

$$\text{Hence, } P_c = 287/0.81 = 354 \text{ W}$$

(b) Output at full-load, 0.8 lag p.f. = 10,000 × 0.80 = 8000 W

At the corresponding load, Full Load copper-loss = 354 W

$$\text{Hence, efficiency} = 8000/(8000 + 354 + 287) = 0.926 = \mathbf{92.6\%}$$

**Example 32.50.** Resistances and Leakage reactance of 10 kVA, 50 Hz, 2300/230 V single phase distribution transformer are  $r_1 = 3.96$  ohms,  $r_2 = 0.0396$  ohms,  $x_1 = 15.8$  ohms,  $x_2 = 0.158$  ohm. Subscript 1 refers to HV and 2 to LV winding (a) transformer delivers rated kVA at 0.8 p.f. Lagging to a load on the L.V. side. Find the H.V. side voltage necessary to maintain 230 V across Load-terminals. Also find percentage voltage regulation. (b) Find the power-factor of the rated load-current at which the voltage regulation will be zero, hence find the H.V. side voltage.

(Nagpur University, November 1997)

**Solution.** (a) Rated current on L.V. side = 10,000/230 = 43.5 A. Let the total resistance and total leakage reactance be referred to L.V. side. Finally, the required H.V. side voltage can be worked out after transformation.

$$\begin{aligned} \text{Total resistance, } r &= r_1' + r_2 = 3.96 \times (230/2300)^2 + 0.0396 \\ &= 0.0792 \text{ ohms} \end{aligned}$$

$$\begin{aligned} \text{Total leakage-reactance, } x &= x_1' + x_2 = 15.8 \times (230/2300)^2 + 0.158 \\ &= 0.316 \text{ ohm} \end{aligned}$$

For purpose of calculation of voltage-magnitudes, approximate formula for voltage regulation can be used. For the present case of 0.8 lagging p.f.

$$\begin{aligned} V_1' &= V_2 + I [r \cos \phi + x \sin \phi] \\ &= 230 + 43.5 [(0.0792 \times 0.8) + (0.316 \times 0.6)] \\ &= 230 + 43.5 [0.0634 + 0.1896] = 230 + 11 = 241 \text{ volts} \end{aligned}$$

Hence,  $V_1 = 241 \times (2300/230) = 2410 \text{ volts.}$

It means that H.V. side terminal voltage must be 2410 for keeping 230 V at the specified load.

(b) Approximate formula for voltage regulation is :  $V_1' - V_2 = I [r \cos \phi \pm x \sin \phi]$

With Lagging p.f., +ve sign is retained. With leading power-factor, the -ve sign is applicable. For the voltage-regulation to be zero, only leading P.f. condition can prevail.

Thus,  $r \cos \phi - x \sin \phi = 0$

or  $\tan \phi = r/x = 0.0792/0.316 = 0.25$

or  $\phi = 14^\circ, \cos \phi = 0.97 \text{ leading}$

Corresponding  $\sin \phi = \sin 14^\circ = 0.243$

H.V. terminal voltage required is 2300 V to maintain 230 V at Load, since Zero regulation condition is under discussion.

**Example 32.51.** A 5 kVA, 2200/220 V, single-phase transformer has the following parameters.

H.V. side :  $r_1 = 3.4 \text{ ohms}, x_1 = 7.2 \text{ ohms}$

L.V. side :  $r_2 = 0.028 \text{ ohms}, x_2 = 0.060 \text{ ohms}$

Transformer is made to deliver rated current at 0.8 lagging P.f. to a load connected on the L.V. side. If the load voltage is 220 V, calculate the terminal voltage on H.V. side

(Neglect the exciting current). (Rajiv Gandhi Technical University, Bhopal, Summer 2001)

**Solution.** Calculations may be done referring all the parameters the L.V. side first. Finally, the voltage required on H.V. side can be obtained after transformation.

Rated current ref. to L.V. side =  $5000/220 = 22.73 \text{ A}$

Total winding resistance ref. to L.V. side =  $r_1' + r_2 = (220/2200)^2 \times 3.4 + 0.028$

Total winding-leakage-reactance ref. to L.V. side =  $x_1' + x_2$   
 $= (220/2200)^2 \times 7.2 + 0.060 = 0.132 \text{ ohm}$

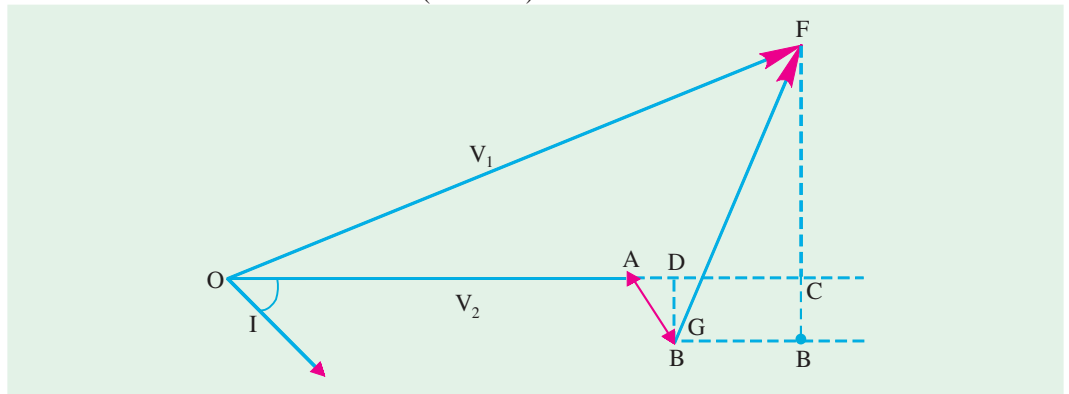


Fig. 32.53(c)

In the phasor diagram of Fig. 32.53 (c).

$QA = V_2 = 220 \text{ volts}, I = 22.73 \text{ A at lagging phase angle of } 36.87^\circ$

$AB = Ir, AD = Ir \cos \phi = 22.73 \times 0.062 \times 0.80 = 1.127 \text{ V}$

$DC = Ix \sin \phi = 22.73 \times 0.132 \times 0.60 = 1.80 \text{ V}$

$$\begin{aligned}
 OC &= 220 + 1.127 + 1.80 = 222.93 \text{ volts} \\
 BD &= Ir \sin \phi = 0.85 \text{ V} \\
 B'F &= x \cos \phi = 2.40 \text{ V} \\
 CF &= 2.40 - 0.85 = 1.55 \text{ V} \\
 V_1' &= OF = (222.93^2 + 1.55^2)^{0.50} = 222.935 \text{ volts}
 \end{aligned}$$

Required terminal voltage of H.V. side =  $V_1 = 222.935 \times (2200/220) = 2229.35$  volts

[Note. In approximate and fast calculations,  $CF$  is often ignored for calculation of magnitude of  $V_1'$ . The concerned expression is :  $V_1' = V_2 + Ir \cos \phi + Ix \sin \phi$ , for lagging P.f.]

**Example 32.52.** A 4-kVA, 200/400 V, single-phase transformer takes 0.7 amp and 65 W on Open-circuit. When the low-voltage winding is short-circuited and 15 V is applied to the high-voltage terminals, the current and power are 10 A and 75 W respectively. Calculate the full-load efficiency at unity power factor and full-load regulation at 0.80 power-factor lagging.

(Nagpur University April 1999)

**Solution.** At a load of 4 kVA, the rated currents are :

$$\begin{aligned}
 \text{L.V. side:} & \quad 4000/200 = 20 \text{ amp} \\
 \text{And H.V. side:} & \quad 4000/400 = 10 \text{ amp} \\
 \text{From the test data, full-load copper-loss} &= 75 \text{ W} \\
 \text{And constant core-loss} &= 65 \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 \text{From S.C. test,} & \quad Z = 15/10 = 1.5 \text{ ohms} \\
 & \quad R = 75/100 = 0.75 \text{ ohm}
 \end{aligned}$$

$$\text{Hence} \quad x = \sqrt{1.5^2 - 0.75^2} = 1.30 \text{ ohms}$$

All these series-parameters are referred to the H.V. side, since the S.C. test has been conducted from H.V. side.

$$\begin{aligned}
 \text{Full-load efficiency at unity p.f.} &= 4000 / (4000 + 65 + 75) \\
 &= 0.966 = 96.6\%
 \end{aligned}$$

$$\begin{aligned}
 \text{Full load voltage regulation at 0.80 lagging p.f.} & \\
 &= Ir \cos \phi + Ix \sin \phi \\
 &= 10 (0.75 \times 0.80 + 1.30 \times 0.60) = 16.14 \text{ Volts}
 \end{aligned}$$

Thus, due to loading, H.V. side voltage will drop by 16.14 volts (*i.e.* terminal voltage for the load will be 383.86 volts), when L.V. side is energized by 200-V source.

### 32.25. Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

(i) Percentage resistance at full-load

$$\begin{aligned}
 \% R &= \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100 \\
 &= \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \% \text{ Cu loss at full-load} \\
 \% R &= \% \text{ Cu loss} = v_r \qquad \dots \text{Art.32.16}
 \end{aligned}$$

(ii) Percentage reactance at full-load

$$\% X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{I_2 X_{02}}{V_2} \times 100 = v_x$$

(iii) Percentage impedance at full-load

$$\% Z = \frac{I_1 Z_{01}}{V_1} \times 100 = \frac{I_2 Z_{02}}{V_2} \times 100$$

(iv) 
$$\% Z = \sqrt{(\% R^2 + \% X^2)}$$

It should be noted from above that the reactances and resistances in ohm can be obtained thus :

$$R_{01} = \frac{\% R \times V_1}{100 \times I_1} = \frac{\% \text{ Cu loss} \times V_1}{100 \times I_1} ; \text{ Similarly } R_{02} = \frac{\% R \times V_2}{100 \times I_2} = \frac{\% \text{ Cu loss} \times V_2}{100 \times I_2}$$

$$X_{01} = \frac{\% X \times V_1}{100 \times I_1} = \frac{v_z \times V_1}{100 \times I_1} ; \text{ Similarly } X_{02} = \frac{\% X \times V_2}{100 \times I_2} = \frac{v_x \times V_2}{100 \times I_2}$$

It may be noted that percentage resistance, reactance and impedance have the same value whether referred to primary or secondary.

**Example 32.53.** A 3300/230 V, 50-kVA, transformer is found to have impedance of 4% and a Cu loss of 1.8% at full-load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant ?

**Solution.** 
$$\% X = \sqrt{(\% Z^2 - \% R^2)} = \sqrt{(4^2 - 1.8^2)} = 3.57\% (\because \text{Cu loss} = \% R)$$

Full load  $I_1 = 50,000/3300 = 15.2 \text{ A}$  (assuming 100% efficiency). Considering primary winding, we have

$$\% R = \frac{R_{01} I_1 \times 100}{V_L} = 1.8 \quad \therefore R_{01} = \frac{1.8 \times 3300}{100 \times 15.2} = 3.91 \Omega$$

Similarly 
$$\% X = \frac{X_{01} I_1 \times 100}{V_1} = 3.57 \quad \therefore X_{01} = \frac{3.57 \times 3300}{100 \times 15.2} = 7.76 \Omega$$

Similarly 
$$Z_{01} = \frac{4 \times 3300}{100 \times 15.2} = 8.7 \Omega$$

Now 
$$\frac{\text{Short-circuit current}^*}{\text{Full load current}} = \frac{100}{4} \quad \therefore \text{S.C. current} = 15.2 \times 25 = 380 \text{ A}$$

**Example 32.54.** A 20-kVA, 2200/220-V, 50-Hz distribution transformer is tested for efficiency and regulation as follows :

O.C. test : 220 V      4.2 A, 148 W      -l.v side

S.C. test : 86 V      10.5 A, 360 W      -l.v side

Determine (a) core loss (b) equivalent resistance referred to primary (c) equivalent resistance referred to secondary (d) equivalent reactance referred to primary (e) equivalent reactance referred to secondary (f) regulation of transformer at 0.8 p.f. lagging current (g) efficiency at full-load and half the full-load at 0.8 p.f. lagging current.

**Solution.** (a) As shown in Art 32.9, no-load primary input is practically equal to the core loss. Hence, core loss as found from no-load test, is **148 W**.

(b) From S.C. test, 
$$R_{01} = 360/10.5^2 = 3.26 \Omega$$

(c) 
$$R_{02} = K^2 R_{01} = (220/2200)^2 \times 3.26 = 0.0326 \Omega$$

\* Short circuit

$$I_{SC} = \frac{V_1}{Z_{01}} \quad \text{Now, } Z_{01} = \frac{V_1 \times \% Z}{100 \times I_1}$$

$$I_{SC} = \frac{V_1 \times 100 \times I_1}{V_1 \times \% Z} = \frac{100 \times I_1}{\% Z} \quad \therefore \frac{I_{SC}}{I_1} = \frac{100}{\% Z}$$



$$(d) \quad Z_{10} = \frac{V_{SC}}{I_{SC}} = \frac{86}{10.5} = 8.19 \, \Omega$$

$$X_{01} = \sqrt{(8.19^2 - 3.26^2)} = 7.51 \, \Omega$$

$$(e) \quad X_{02} = K^2 X_{01} = (220/2200)^2 \times 7.51 = 0.0751 \, \Omega$$

(f) We will use the definition of regulation as given in Art. 32.24 (3).

We will find the rise in primary voltage necessary to maintain the output terminal voltage constant from no-load to full-load.

$$\text{Rated primary current} = 20,000/2200 = 9.1 \, \text{A}$$

$$V_1' = \sqrt{[2200 \times 0.8 + 9.1 \times 3.26]^2 + (2200 \times 0.6 + 9.1 \times 7.51)^2} = 2265 \, \text{V}$$

$$\therefore \quad \% \text{ regn} = \frac{2265 - 2200}{2200} \times 100 = 2.95\%$$

We would get the same result by working in the secondary. Rated secondary current = 91 A.

$${}_0V_2 = \sqrt{(220 \times 0.8 + 91 \times 0.0326)^2 + (220 \times 0.6 + 91 \times 0.0751)^2} = 226.5 \, \text{V}$$

$$\therefore \quad \% \text{ regns.} = \frac{226.5 - 220}{220} \times 100 = 2.95\%$$

(g) Core loss = 1.48 W. It will be the same for all loads.

Cu loss at full load =  $I_1^2 R_{01} = 9.1^2 \times 3.26 = 270 \, \text{W}$

Cu loss at half full-load =  $4.55^2 \times 3.26 = 67.5 \, \text{W}$  (or F.L. Cu loss/4)

$$\therefore \quad \eta \text{ at full-load} = \frac{22,000 \times 0.8 \times 100}{20,000 \times 0.8 + 148 + 270} = 97.4\%$$

$$\therefore \quad \eta \text{ at half-load} = \frac{10,000 \times 0.8 \times 100}{10,000 \times 0.8 + 148 + 67.5} = 97.3\%$$

**Example 32.55.** Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop is 5% of the voltage, when the power factor is (i) 0.80 Lag (ii) unity (iii) 0.80 Leading. (Madras University, 1997)

**Solution.** When 1% of output is the ohmic loss, p.u. resistance of the transformer,  $\epsilon_r = 0.01$

When 5% is the reactance drop, p.u. reactance of the transformer  $\epsilon_x = 0.05$

(i) Per Unit regulation of the transformer at full-load, 0.8 Lagging p.f.

$$= 0.01 \times \cos \phi + 0.05 \times \sin \phi = 0.01 \times 0.8 + 0.05 \times 0.6 = 0.038 \text{ or } 3.8\%$$

(ii) Per Unit regulation at unity p.f. =  $0.01 \times 1 = 0.01$  or 1%

(iii) Per Unit regulation at 0.8 Leading p.f. =  $0.01 \times 0.8 - 0.05 \times 0.6 = -0.022$  or -2.2%

**Example 32.56.** The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, single phase transformer is 97% and occurs at 3/4<sup>th</sup> full-load u.p.f. If the impedance is 10% calculate the regulation at fullload, 0.8 p.f. Lag. (Madurai Kamraj University, November 1997)

**Solution.** At unity p.f. with 3/4<sup>th</sup> full load, the output of the transformer

$$= 500 \times 0.75 \times 1 \, \text{kW} = 375 \, \text{kW}$$

$$0.97 = \frac{375}{375 + 2 P_i}$$

where  $P_i$  = core loss in kW, at rated voltage.

At maximum efficiency,  $x^2 P_c = P_i$

$$(0.75)^2 P_c = P_i$$

where  $x = 0.75$ , i.e. 3/4<sup>th</sup> which is the fractional loading of the transformer

$P_c$  = copper losses in kW, at rated current

$$P_i = \frac{1}{2} \left\{ (375) \times \left( \frac{1}{0.97} - 1 \right) \right\} = \frac{1}{2} \times 375 \times \frac{3}{97} = 5.8 \text{ kW}$$

$$P_c = 5.8 / (0.75)^2 = 10.3 \text{ kW}$$

$$\text{Full load current in primary (H.V.) winding} = \frac{500 \times 1000}{3300} = \mathbf{151.5 \text{ amp}}$$

Total winding resistance ref. to primary

$$= \frac{10.3 \times 1000}{(151.5)^2} = 0.44876 \text{ ohm}$$

$$\epsilon_r = \% \text{ resistance} = \frac{151.5 \times 0.44876}{3300} \times 100 \% = \mathbf{2.06\%}$$

$$\epsilon_z = \% \text{ Impedance} = 10\%$$

$$\epsilon_x = \% \text{ reactance} = \sqrt{100 - 4.244} = \mathbf{9.7855\%}$$

By Approximate formula at 0.8 p.f. lag

$$\begin{aligned} \% \text{ regulation} &= \epsilon_r \cos \phi + \epsilon_x \sin \phi \\ &= 2.06 \times 0.8 + 9.7855 \times 0.6 \\ &= 1.648 + 5.87 = \mathbf{7.52\%} \end{aligned}$$

**Example 32.57.** A transformer has copper-loss of 1.5% and reactance-drop of 3.5% when tested at full-load. Calculate its full-load regulation at (i) u.p.f. (ii) 0.8 p.f. Lagging and (iii) 0.8 p.f. Leading. (Bharathithasan Univ. April 1997)

**Solution.** The test-data at full-load gives following parameters :

$$\text{p.u. resistance} = 0.015, \quad \text{p.u. reactance} = 0.035$$

(i) Approximate Voltage – Regulation at unity p.f. full load

$$\begin{aligned} &= 0.015 \cos \phi + 0.035 \sin \phi \\ &= 0.015 \text{ per unit} = \mathbf{1.5\%} \end{aligned}$$

(ii) Approximate Voltage – Regulation at 0.80 Lagging p.f.

$$= (0.015 \times 0.8) + (0.035 \times 0.6) = 0.033 \text{ per unit} = \mathbf{3.3\%}$$

(iii) Approximate Voltage Regulation at 0.8 leading p.f.

$$\begin{aligned} &= I_r \cos \phi - I_x \sin \phi \\ &= (0.015 \times 0.8) - (0.035 \times 0.6) = -0.009 \text{ per unit} = \mathbf{-0.9\%} \end{aligned}$$

### 32.26. Kapp Regulation Diagram

It has been shown that secondary terminal voltage falls as the load on the transformer is increased when p.f. is lagging and it increases when the power factor is leading. In other words, secondary terminal voltage not only depends on the load but on power factor also (Art. 32.16). For finding the voltage drop (or rise) which is further used in determining the regulation of the transformer, a graphical construction is employed which was proposed by late Dr. Kapp.

For drawing Kapp regulation diagram, it is necessary to know the equivalent resistance and reactance as referred to secondary *i.e.*  $R_{02}$  and  $X_{02}$ . If  $I_2$  is the secondary load current, then secondary terminal voltage on load  $V_2$ , is obtained by subtracting  $I_2 R_{02}$  and  $I_2 X_{02}$  voltage drops vectorially from secondary no-load voltage  ${}_0V_2$ .

Now,  ${}_0V_2$  is constant, hence it can be represented by a circle of constant radius  $OA$  as in Fig. 32.54. This circle is known as no-load or open-circuit e.m.f. circle. For a given load,  $OI_2$  represents the load current and is taken as the reference vector,  $CB$  represents  $I_2 R_{02}$  and is parallel to  $OI_2$ ,  $AB$  represents  $I_2 X_{02}$  and is drawn at right angles to  $CB$ . Vector  $OC$  obviously represents  $I_2 X_{02}$  and is drawn at right

angles to  $CB$ . Vector  $OC$  obviously represents secondary terminal voltage  $V_2$ . Since  $I_2$  is constant, the drop triangle  $ABC$  remains constant in size. It is seen that end point  $C$  of  $V_2$  lies on another circle whose centre is  $O'$ . This point  $O'$  lies at a distance of  $I_2 X_{02}$  vertically below the point  $O$  and a distance of  $I_2 R_{02}$  to its left as shown in Fig. 32.54.

Suppose it is required to find the voltage drop on full-load at a lagging power factor of  $\cos \phi$ , then a radius  $OLP$  is drawn inclined at an angle of  $\phi$  with  $OX$ .  $LM = I_2 R_{02}$  and is drawn horizontal  $MN = I_2 X_{02}$  and is drawn perpendicular to  $LM$ . Obviously,  $ON$  is no-load voltage  ${}_0V_2$ . Now,  $ON = OP = {}_0V_2$ . Similarly,  $OL$  is  $V_2$ . The voltage drop =  $OP - OL = LP$ .

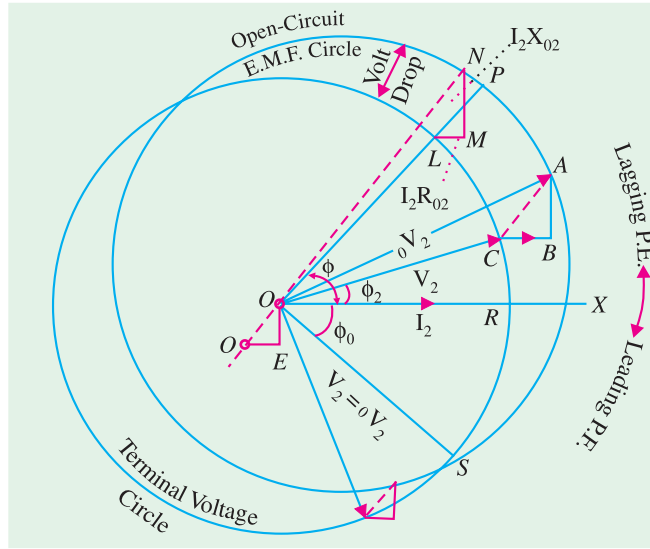


Fig. 32.54

$$\text{Hence, percentage regulation 'down' is} = \frac{OP - OL}{OP} \times 100 = \frac{LP}{OP} \times 100$$

It is seen that for finding voltage drop, triangle  $LMN$  need not be drawn, but simply the radius  $OLP$ .

The diagram shows clearly how the secondary terminal voltage falls as the angle of lag increases. Conversely, for a leading power factor, the fall in secondary terminal voltage decreases till for an angle of  $\phi_0$  leading, the fall becomes zero; hence  $V_2 = {}_0V_2$ . For angles greater than  $\phi_0$ , the secondary terminal voltage  $V_2$  becomes greater than  ${}_0V_2$ .

The Kapp diagram is very helpful in determining the variation of regulation with power factor but it has the disadvantage that since the lengths of the sides of the impedance triangle are very small as compared to the radii of the circles, the diagram has to be drawn on a very large scale, if sufficiently accurate results are desired.

### 32.27. Sumpner of Back-to-Back Test

This test provides data for finding the regulation, efficiency and heating under load conditions and is employed only when two similar transformers are available. One transformer is loaded on the other and both are connected to supply. The power taken from the supply is that necessary for supplying the losses of both transformers and the negligibly small loss in the control circuit.

As shown in Fig. 32.55, primaries of the two transformers are connected in parallel across the same a.c. supply. With switch  $S$  open, the wattmeter  $W_1$  reads the core loss for the two transformers.

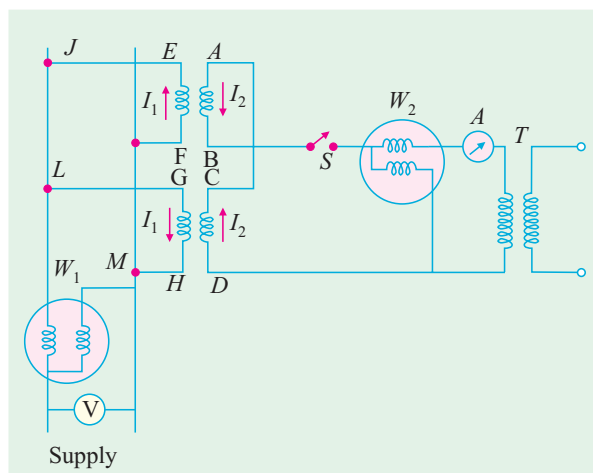


Fig. 32.55

The secondaries are so connected that their potentials are in opposition to each other. This would so if  $V_{AB} = V_{CD}$  and  $A$  is joined to  $C$  whilst  $B$  is joined to  $D$ . In that case, there would be no secondary current flowing around the loop formed by the two secondaries.  $T$  is an auxiliary low-voltage transformer which can be adjusted to give a variable voltage and hence current in the secondary loop circuit. By proper adjustment of  $T$ , full-load secondary current  $I_2$  can be made to flow as shown. It is seen, that  $I_2$  flows from  $D$  to  $C$  and then from  $A$  to  $B$ . Flow of  $I_1$  is confined to the loop  $FEJLGHMF$  and it does not pass through  $W_1$ . Hence,  $W_1$  continues to read the core loss and  $W_2$  measures full-load Cu loss (or at any other load current value  $I_2$ ). Obviously, the power taken in is twice the losses of a single transformer.

**Example 32.58.** Two similar 250-kVA, single-phase transformers gave the following results when tested by back-to-back method :

Mains wattmeter,  $W_1 = 5.0 \text{ kW}$

Primary series circuit wattmeter,  $W_2 = 7.5 \text{ kW}$  (at full-load current).

Find out the individual transformer efficiencies at 75% full-load and 0.8 p.f. lead.

(Electrical Machines-III, Gujarat Univ. 1986)

**Solution.** Total losses for both transformers =  $5 + 7.5 = 12.5 \text{ kW}$

F.L. loss for each transformer =  $12.5/2 = 6.25 \text{ kW}$

$$\text{Copper-loss at 75\% load} = \left(\frac{3}{4}\right)^2 \times \frac{7.5}{2} \text{ kW} = 2.11 \text{ kW}$$

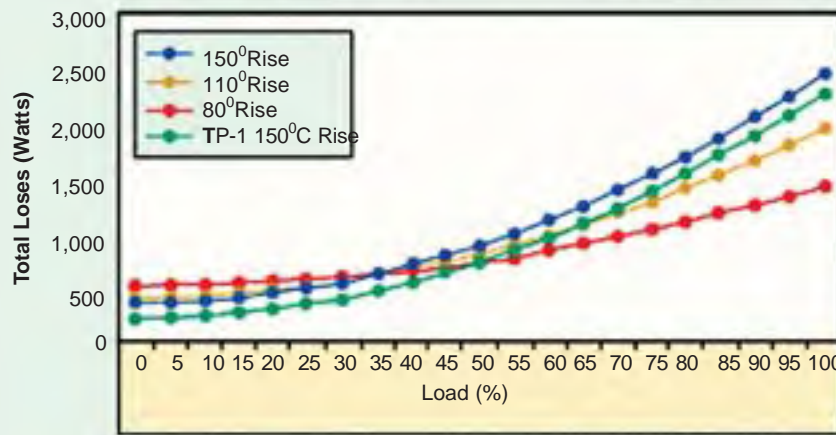
Output of each transformer at 75% F.L. and 0.8 p.f. =  $(250 \times 0.75) \times 0.8 = 150 \text{ kW}$

$$\eta = \frac{150}{150 + 2.5 + 2.11} = 97\%$$

### 32.28. Losses in a Transformer

In a static transformer, there are no friction or windage losses. Hence, the only losses occurring are :

(i) **Core or Iron Loss :** It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being



Typical 75kVA Transformer Losses vs. Load

1 to 3% from no-load to full-load). The core loss is practically the same at all loads.

Hysteresis loss  $W_h = \eta B_{\text{max}}^{1.6} f V$  watt; eddy current loss  $W_e = PB_{\text{max}}^2 f^2 t^2$  watt

These losses are minimized by using steel of high silicon content for the core and by using very thin

laminations. Iron or core loss is found from the *O.C.* test. *The input of the transformer when on no-load measures the core loss.*

(ii) **Copper loss.** This loss is due to the ohmic resistance of the transformer windings. Total Cu loss =  $I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} + I_2^2 R_{02}$ . It is clear that Cu loss is proportional to (current)<sup>2</sup> or kVA<sup>2</sup>. In other words, Cu loss at half the full-load is one-fourth of that at full-load.

The value of Cu loss is found from the short-circuit test (Art. 32.22).

### 32.29. Efficiency of a Transformer

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input—the two being measured in the same units (either watts or kilowatts).

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

But a transformer being a highly efficient piece of equipment, has very small loss, hence it is impractical to try to measure transformer efficiency by measuring input and output. These quantities are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from ;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{\text{Output}}{\text{Output} + \text{Cu loss} + \text{iron loss}}$$

or

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$$

It may be noted here that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to VA. Hence, at any volt-ampere load, the efficiency depends on power factor, being maximum at a power factor of unity.

Efficiency can be computed by determining core loss from no-load or open-circuit test and Cu loss from the short-circuit test.

### 32.30. Condition for Maximum Efficiency

$$\text{Cu loss} = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02} = W_{cu}$$

$$\text{Iron loss} = \text{Hysteresis loss} + \text{Eddy current loss} = W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1 I_1 \cos \phi_1$$

$$\begin{aligned} \eta &= \frac{V_1 I_1 \cos \phi_1 - \text{losses}}{V_1 I_1 \cos \phi_1} = \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi_1} \\ &= 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1} \end{aligned}$$

Differentiating both sides with respect to  $I_1$ , we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi_1} + \frac{W_i}{V_1 I_1^2 \cos \phi_1}$$

For  $\eta$  to be maximum,  $\frac{d\eta}{dI_1} = 0$ . Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos \phi_1} = \frac{W_i}{V_1 I_1^2 \cos \phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02}$$

or

$$\text{Cu loss} = \text{Iron loss}$$

The output current corresponding to maximum efficiency is  $I_2 = \sqrt{(W_i/R_{02})}$ .

It is this value of the output current which will make the Cu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

**Note. (i)** If we are given iron loss and full-load Cu loss, then the load at which two losses would be equal (i.e. corresponding to maximum efficiency) is given by

$$= \text{Full load} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L. Cu loss}}\right)}$$

In Fig. 32.56, Cu losses are plotted as a percentage of power input and the efficiency curve as deduced from these is also shown. It is obvious that the point of intersection of the Cu and iron loss curves gives the point of maximum efficiency. It would be seen that the efficiency is high and is practically constant from 15% full-load to 25% overload.

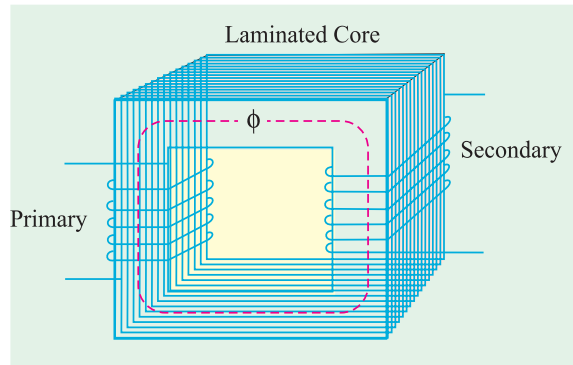


Fig. 32.56

(ii) The efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} + W_i} \times 100$$

where  $x$  = ratio of actual to full-load kVA

$W_i$  = iron loss in kW ;  $W_{cu}$  = Cu loss in kW.

**Example 32.59.** In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on (i) full load (ii) half full-load.

(Elect. Engg. & Electronic, Bangalore Univ. 1990 and Similar example in U.P. Technical University 2001)

**Solution. (i) Full-load Unity p.f.**

$$\text{Total loss} = 350 + 400 = 750 \text{ W}$$

$$\text{F.L. output at u.p.f.} = 25 \times 1 = 25 \text{ kW} ; \eta = 25/25.75 = 0.97 \text{ or } 97\%$$

(ii) Half F.L. Unity p.f.

Cu loss =  $400 \times (1/2)^2 = 100 \text{ W}$ . Iron loss remains constant at 350 W, Total loss =  $100 + 350 = 450 \text{ W}$ .

Half-load output at u.p.f. = 12.5 kW

$$\therefore \eta = 12.5/(12.5 + 0.45) = 96.52\%$$

**Example 32.60.** If  $P_1$  and  $P_2$  be the iron and copper losses of a transformer on full-load, find the ratio of  $P_1$  and  $P_2$  such that maximum efficiency occurs at 75% full-load.

(Elect. Machines AMIE Sec. B, Summer 1992)

**Solution.** If  $P_2$  is the Cu loss at full-load, its value at 75% of full-load is  $= P_2 \times (0.75)^2 = 9P_2/16$ . At maximum efficiency, it equals the iron loss  $P_1$  which remains constant throughout. Hence, at maximum efficiency.

$$P_1 = 9P_2/16 \text{ or } P_1/P_2 = 9/16.$$

**Example 32.61.** A 11000/230 V, 150-kVA, 1-phase, 50-Hz transformer has core loss of 1.4 kW and F.L. Cu loss of 1.6 kW. Determine

(i) the kVA load for max. efficiency and value of max. efficiency at unity p.f.

(ii) the efficiency at half F.L. 0.8 p.f. leading (Basic Elect. Machine, Nagpur Univ. 1993)

**Solution. (i)** Load kVA corresponding to maximum efficiency is

$$= \text{F.L. kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}} = 250 \times \sqrt{\frac{1.6}{1.4}} = 160 \text{ kVA}$$

Since Cu loss equals iron loss at maximum efficiency, total loss = 1.4 + 1.4 = 2.8 kW ;  
output = 160 × 1 = 160 kW

$$\eta_{max} = 160/162.8 = 0.982 \text{ or } \mathbf{98.2\%}$$

**(ii)** Cu loss at half full-load =  $1.6 \times (1/2)^2 = 0.4 \text{ kW}$  ; Total loss = 1.4 + 0.4 = 1.8 kW

Half F.L. output at 0.8 p.f. =  $(150/2) \times 0.8 = 60 \text{ kW}$

∴ Efficiency =  $60/(60 + 1.8) = 0.97$  or **97%**

**Example 32.62.** A 5-kVA, 2,300/230-V, 50-Hz transformer was tested for the iron losses with normal excitation and Cu losses at full-load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs :

1.25                  2.5                  3.75                  5.0                  6.25                  7.5

Plot efficiency vs kVA output curve.

**(Elect. Engg. -I, Bombay Univ. 1987)**

**Solution.** F.L. Cu loss = 112 W ; Iron loss = 40 W

**(i)** Cu loss at 1.25 kVA =  $112 \times (1.25/5)^2 = 7 \text{ W}$

Total loss = 40 + 7 = 47 W    Output =  $1.25 \times 0.8 = 1 \text{ kW} = 1,000 \text{ W}$

$$\eta = 100 \times 1,000/1,047 = \mathbf{95.51\%}$$

**(ii)** Cu loss at 2.5 kVA =  $112 \times (2.5/5)^2 = 28 \text{ W}$

Total loss = 40 + 28 = 68 W

Output =  $2.5 \times 0.8 = 2 \text{ kW}$

$$\eta = 2,000 \times 100/2,068 = \mathbf{96.71\%}$$

**(iii)** Cu loss at 3.75 kVA

$$= 112 \times (3.75/5)^2 = 63 \text{ W}$$

Total loss = 40 + 63 = 103 W

$$\eta = 3,000 \times 100/3,103 = \mathbf{96.68\%}$$

**(iv)** Cu loss at 5 kVA

$$= 112 \text{ W}$$

Total loss = 152 W = 0.152 kW

Output =  $5 \times 0.8 = 4 \text{ kW}$

$$\eta = 4 \times 100/4.142 = \mathbf{96.34\%}$$

**(v)** Cu loss at 6.25 kVA

$$= 112 \times (6.25/5)^2 = 175 \text{ W}$$

Total loss = 125 W = 0.125 kW ; Output =  $6.25 \times 0.8 = 5 \text{ kW}$

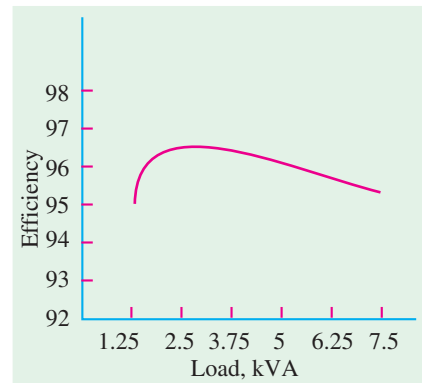
$$\eta = 5 \times 100/5.215 = \mathbf{95.88\%}$$

**(vi)** Cu loss at 7.5 kVA =  $112 \times (7.5/5)^2 = 252 \text{ W}$

Total loss = 292 W = 0.292 kW ; Output =  $7.5 \times 0.8 = 6 \text{ kW}$

$$\eta = 6 \times 100/6.292 = \mathbf{95.36\%}$$

The curve is shown in Fig. 32.57.



**Fig. 32.57**

**Example 32.63.** A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume negligible magnetizing current and p.f. 0.8 at all loads. **(Elect. Technology Punjab Univ. Jan. 1991)**

**Solution.** As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

$$\text{F.L. output} = 200 \times 0.8 = 160 \text{ kW} ; \text{F.L. input} = 160/0.98 = 163.265 \text{ kW}$$

$$\text{F.L. losses} = 163.265 - 160 = 3.265 \text{ kW}$$

This loss consists of F.L. Cu loss  $x$  and iron loss  $y$ .

$$\therefore x + y = 3.265 \text{ kW} \quad \dots(i)$$

It is also given that  $\eta_{max}$  occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

$$\therefore \text{Cu loss at 75 \% of F.L.} = x (3/4)^2 = 9x/16$$

$$\text{Since } y \text{ remains constant, hence } 9x/16 = y \quad \dots(ii)$$

$$\text{Substituting the value of } y \text{ in Eqn. (i), we get } x + 9x/16 = 3265 \text{ or } x = 2090 \text{ W; } y = 1175 \text{ W}$$

**Half-load Unity p.f.**

$$\text{Cu loss} = 2090 \times (1/2)^2 = 522 \text{ W} ; \text{total loss} = 522 + 1175 = 1697 \text{ W}$$

$$\text{Output} = 100 \times 0.8 = 80 \text{ kW} ; \eta = 80/81.697 = 0.979 \text{ or } \mathbf{97.9 \%}$$

**Example 32.64.** A 25-kVA, 1-phase transformer, 2,200 volts to 220 volts, has a primary resistance of 1.0  $\Omega$  and a secondary resistance of 0.01  $\Omega$ . Find the equivalent secondary resistance and the full-load efficiency at 0.8 p.f. if the iron loss of the transformer is 80% of the full-load Cu loss.

(Elect. Technology, Utkal Univ. 1998)

**Solution.**  $K = 220/2,200 = 1/10$  ;  $R_{02} = R_2 + K^2 R_1 = 0.01 + 1/100 = \mathbf{0.02 \Omega}$

$$\text{Full-load } I_2 = 25,000/220 = 113.6 \text{ A} ; \text{F.L. Cu loss} = I_2^2 R_{02} = 113.6^2 \times 0.02 = 258 \text{ W.}$$

$$\text{Iron loss} = 80\% \text{ of } 258 = 206.4 \text{ W} ; \text{Total loss} = 258 + 206.4 = 464.4 \text{ W}$$

$$\text{F.L. output} = 25 \times 0.8 = 20 \text{ kW} = 20,000 \text{ W}$$

$$\text{Full-load } \eta = 20,000 \times 100 / (20,000 + 464.4) = \mathbf{97.7 \%}$$

**Example 32.65.** A 4-kVA, 200/400-V, 1-phase transformer has equivalent resistance and reactance referred to low-voltage side equal to 0.5  $\Omega$  and 1.5  $\Omega$  respectively. Find the terminal voltage on the high-voltage side when it supplies 3/4th full-load at power factor of 0.8, the supply voltage being 220 V. Hence, find the output of the transformer and its efficiency if the core losses are 100 W.

(Electrical Engineering ; Bombay Univ. 1985)

**Solution.** Obviously, primary is the low-voltage side and the secondary, the high voltage side.

Here,  $R_{01} = 0.5 \Omega$  and  $X_{01} = 1.5 \Omega$ . These can be transferred to the secondary side with the help of the transformation ratio.

$$K = 400/200 = 2 ; R_{02} = K^2 R_{01} = 2^2 \times 0.5 = 2 \Omega ; X_{02} = K^2 X_{01} = 4 \times 1.5 = 6 \Omega$$

$$\text{Secondary current when load is 3/4 the, full-load is} = (1,000 \times 4 \times 3/4) / 400 = 7.5 \text{ A}$$

Total drop as referred to transformer secondary is

$$= I_2 (R_{02} \cos \phi + X_{02} \sin \phi)^* = 7.5 (2 \times 0.8 + 6 \times 0.6) = 39 \text{ V}$$

$\therefore$  Terminal voltage on high-voltage side under given load condition is

$$= 400 - 39 = \mathbf{361 \text{ V}}$$

$$\text{Cu loss} = I_2^2 R_{02} = 7.5^2 \times 2 = 112.5 \text{ W}$$

$$\text{Iron loss} = 100 \text{ W}$$

$$\text{Total loss} = 212.5 \text{ W}$$

$$\text{output} = (4 \times 3/4) \times 0.8 = \mathbf{2.4 \text{ kW}}$$

$$\text{Input} = 2,400 + 212.5 = 2,612.5 \text{ W}$$

$$\eta = 2,400 \times 100 / 2,612.5 = \mathbf{91.87 \%}$$

**Example 32.66.** A 20-kVA, 440/220 V, I- $\phi$ , 50 Hz transformer has iron loss of 324 W. The Cu loss is found to be 100 W when delivering half full-load current. Determine (i) efficiency when

\* Assuming a lagging power factor



delivering full-load current at 0.8 lagging p.f. and (ii) the percent of full-load when the efficiency will be maximum. **(Electrotechnique-II, M.S. Univ., Baroda 1987)**

**Solution.** F.L. Cu loss =  $2^2 \times 100 = 400$  W ; Iron loss = 324 W

$$(i) \quad \text{F.L. efficiency at 0.8 p.f.} = \frac{20 \times 0.8}{(20 \times 0.8) + 0.724} \times 100 = \mathbf{95.67\%}$$

$$(ii) \quad \frac{\text{kVA for maximum}}{\text{F.L. kVA}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}} = \sqrt{\frac{324}{400}} = 0.9$$

Hence, efficiency would be maximum at **90%** of F.L.

**Example 32.67.** Consider a 4-kVA, 200/400 V single-phase transformer supplying full-load current at 0.8 lagging power factor. The O.C./S.C. test results are as follows :

O.C. test : 200 V, 0.8 A, 70 W (I.V. side)

S.C. test : 20 V, 10 A, 60 (H.V. side)

Calculate efficiency, secondary voltage and current into primary at the above load.

Calculate the load at unity power factor corresponding to maximum efficiency.

**(Elect. Machines Nagpur Univ. 1993)**

**Solution.** Full-load,  $I_2 = 4000/400 = 10$  A

It means that S.C. test has been carried out with full secondary flowing. Hence, 60 W represents full-load Cu loss of the transformer.

Total F.L. losses =  $60 + 70 = 130$  W ; F.L. output =  $4 \times 0.8 = 3.2$  kW

F.L.  $\eta = 3.2/3.33 = 0.96$  or **96%**

**S.C. Test**

$$Z_{02} = 20/10 = 2 \Omega ; I_2^2 R_{02} = 60 \text{ or } R_{02} = 60/10^2 = 0.6 \Omega ; X_{02} = \sqrt{2^2 - 0.6^2} = 1.9 \Omega$$

Transformer voltage drop as referred to secondary

$$= I_2 (R_{02} \cos \phi + X_{02} \sin \phi) = 10 (0.6 \times 0.8 + 1.9 \times 0.6) = 16.2 \text{ V}$$

$$\therefore V_2 = 400 - 16.2 = 383.8 \text{ V}$$

Primary current =  $4000/200 = 20$  A

kVA corresponding to  $\eta_{max} = 4 \times \sqrt{70/60} = 4.32$  kVA

$\therefore$  Load at u.p.f. corresponding to  $\eta_{max} = 4.32 \times 1 = \mathbf{4.32}$  kW

**Example 32.68.** A 600 kVA, 1-phase transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full-load at 0.8 power factor lag.

**(Elect. Machines, A.M.I.E. Sec. B, 1992)**

**Solution.**

$$\eta = \frac{x \times kVA \times \cos \phi}{(x \times kVA) \times \cos \phi + W_i + x^2 W_{Cu}} \times 100$$

where  $x$  represents percentage of full-load

$W_i$  is iron loss and  $W_{Cu}$  is full-load Cu loss.

**At F.L. u.p.f.** Here  $x = 1$

$$\therefore 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + 1^2 W_{Cu}} \times 100, W_i + W_{Cu} = 52.174 \text{ kW} \quad \dots(i)$$

At half F.L. UPF. Here  $x = 1/2$

$$92 = \frac{1/2 \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{Cu}} \times 100 ;$$

$$\therefore W_i + 0.25 W_{Cu} = 26.087 \text{ kW} \quad \dots(ii)$$

From (i) and (ii), we get,  $W_i = 17.39 \text{ kW}$ ,  $W_{Cu} = 34.78 \text{ kW}$

**60 % F.L. 0.8 p.f. (lag)** Here,  $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8 \times 100}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2 34.78} = \mathbf{85.9 \%}$$

**Example 32.69.** A 600-kVA, 1-ph transformer when working at u.p.f. has an efficiency of 92 % at full-load and also at half-load. Determine its efficiency when it operates at unity p.f. and 60 % of full-load. **(Electric Machines, Kerala Univ. 1987)**

**Solution.** The fact that efficiency is the same i.e. 92 % at both full-load and half-load will help us to find the iron and copper losses.

**At full-load**

Output = 600 kW ; Input =  $600/0.92 = 652.2 \text{ kW}$  ; Total loss =  $652.2 - 600 = 52.2 \text{ kW}$

Let  $x =$  Iron loss – It remains constant at all loads.  
 $y =$  F.L. Cu loss – It is  $\propto (\text{kVA})^2$ .  $\therefore x + y = 52.2 \quad \dots(i)$

**At half-load**

Output = 300 kW ; Input =  $300/0.92$   $\therefore$  Losses =  $(300/0.92 - 300) = 26.1 \text{ kW}$

Since Cu loss becomes one-fourth of its F.L. value, hence

$$x + y/4 = 26.1$$

Solving for  $x$  and  $y$ , we get  $x = \mathbf{17.4 \text{ kW}}$  ;  $y = 34.8 \text{ kW} \quad \dots(ii)$

**At 60 % full-load**

Cu loss =  $0.62 \times 34.8 = 12.53 \text{ kW}$  ; Total loss =  $17.4 + 12.53 = 29.93 \text{ kW}$

Output =  $600 \times 0.6 = 360 \text{ kW}$   $\therefore \eta = 360/389.93 = 0.965$  or **96.5 %**

**Example 32.70.** The maximum efficiency of a 100-kVA, single phase transformer is 98% and occurs at 80% of full load at 8 p.f. If the leakage impedance of the transformer is 5 %, find the voltage regulation at rated load of 0.8 power factor lagging.

**(Elect. Machines-I, Nagpur Univ. 1993)**

**Solution.** Since maximum efficiency occurs at 80 percent of full-load at 0.8 p.f.,

Output at  $\eta_{max} = (100 \times 0.8) \times 0.8 = 64 \text{ kW}$  ; Input =  $64/0.98 = 65.3 \text{ kW}$

$\therefore$  Total loss =  $65.3 - 64 = 1.3 \text{ kW}$ . This loss is divided equally between Cu and iron.

$\therefore$  Cu loss at 80 % of full-load =  $1.3/2 = 0.65 \text{ kW}$

Cu loss at full-load =  $0.65/0.8^2 = 1 \text{ kW}$

$$\% R = \frac{\text{Cu loss}}{V_2 I_2} \times 100 = 1 \times \frac{100}{100} = 1 \% = v_r ; v_x = 5 \%$$

$$\therefore \% \text{ age regn.} = (1 \times 0.8 + 5 \times 0.6) + \frac{1}{200} (5 \times 0.8 - 1 \times 0.6)^2 = \mathbf{0.166 \%}$$

**Example 32.71.** A 10 kVA, 5000/440-V, 25-Hz single phase transformer has copper, eddy current and hysteresis losses of 1.5, 0.5 and 0.6 per cent of output on full load. What will be the percentage losses if the transformer is used on a 10-kV, 50-Hz system keeping the full-load current constant ? Assume unity power factor operation. Compare the full load efficiencies for the two cases.

**(Elect. Machines, A.M.I.E., Sec. B, 1991)**

**Solution.** We know that  $E_1 = 4.44 f N_1 B_1 A$ . When both excitation voltage and frequency are doubled, flux remains unchanged.

F.L. output at upf =  $10 \text{ kVA} \times 1 = 10 \text{ kW}$

$$\begin{aligned} \text{F.L. Cu loss} &= 1.5 \times 10/100 = 0.15 \text{ kW} ; \text{ Eddy current loss} \\ &= 0.5 \times 10/100 = 0.05 \text{ kW} ; \text{ Hysteresis loss} = 0.6 \times 10/100 = 0.06 \text{ kW} \end{aligned}$$

Now, full-load current is kept constant but voltage is increased from 5000 V to 10,000 V. Hence, output will be doubled to 20 kW. Due to constant current, Cu loss would also remain constant.

$$\text{New Cu loss} = 0.15 \text{ kW}, \% \text{ Cu loss} = (0.15/20) \times 100 = 0.75 \%$$

Now, eddy current loss  $\propto f^2$  and hysteresis loss  $\propto f$ .

$$\text{New eddy current loss} = 0.05 (50/25)^2 = 0.2 \text{ kW}, \% \text{ eddy current loss} = (0.2/20) \times 100 = 1 \%$$

$$\text{Now, } W_h = 0.06 \times (50/25) = 0.12 \text{ kW}, \% W_h = (0.12/20) \times 100 = 0.6 \%$$

$$\eta_1 = \frac{10}{10 + 0.15 + 0.05 + 0.06} \times 100 = \mathbf{87.4 \%$$

$$\eta_2 = \frac{20}{20 + 0.15 + 0.2 + 0.12} \times 100 = \mathbf{97.7 \%$$

**Example 32.72.** A 300-kVA, single-phase transformer is designed to have a resistance of 1.5 % and maximum efficiency occurs at a load of 173.2 kVA. Find its efficiency when supplying full-load at 0.8 p.f. lagging at normal voltage and frequency. (Electrical Machines-I, Gujarat Univ. 1985)

$$\text{Solution. } \% R = \frac{\text{F.L. Cu loss}}{\text{Full-load } V_2 I_2} \times 100 ; 1.5 = \frac{\text{F.L. Cu loss}}{300 \times 1000} \times 100$$

$$\therefore \text{F.L. Cu loss} = 1.5 \times 300 \times 1000/100 = 4500 \text{ W}$$

$$\text{Also, } 173.2 = 300 \sqrt{\frac{\text{Iron Loss}}{4500}} ; \text{ Iron loss} = 1500 \text{ W}$$

$$\text{Total F.L. loss} = 4500 + 1500 = 6 \text{ kW}$$

$$\text{F.L. } \eta \text{ at } 0.8 \text{ p.f.} = \frac{300 \times 0.8}{(300 \times 0.8) + 6} \times 100 = \mathbf{97.6 \%$$

**Example 32.73.** A single phase transformer is rated at 100-kVA, 2300/230-V, 50 Hz. The maximum flux density in the core is 1.2 Wb/m<sup>2</sup> and the net cross-sectional area of the core is 0.04 m<sup>2</sup>. Determine

(a) The number of primary and secondary turns needed.

(b) If the mean length of the magnetic circuit is 2.5 m and the relative permeability is 1200, determine the magnetising current. Neglect the current drawn for the core loss.

(c) On short-circuit with full-load current flowing, the power input is 1200 W and an open-circuit with rated voltage, the power input was 400 W. Determine the efficiency of the transformer at 75 % of full-load with 0.8 p.f. lag.

(d) If the same transformer is connected to a supply of similar voltage but double the frequency (i.e., 100 Hz). What is the effect on its efficiency? (Elect. Engg., Bombay Univ. 1988)

**Solution.** (a) Applying e.m.f. equation of the transformer to the primary, we have

$$2300 = 4.44 \times 50 \times N_1 \times (1.2 \times 0.04) \quad \therefore N_1 = \mathbf{216}$$

$$K = 230/2300 = 1/10 \quad N_2 = KN_1 = 216/10 = 21.6 \text{ or } \mathbf{22}$$

$$(b) \quad AT = H \times l = \frac{B}{\mu_0 \mu_r} \times l = \frac{1.2 \times 2.5}{4\pi \times 10^{-7} \times 1200} = 1989 \quad \therefore I = \frac{1989}{216} = \mathbf{9.21 \text{ A}}$$

$$(c) \quad \text{F.L. Cu loss} = 1200 \text{ W} - \text{S.C. test} ; \text{ Iron loss} = 400 \text{ W} - \text{O.C. test}$$

$$\text{Cu loss at } 75 \% \text{ of F.L.} = (0.75)^2 \times 1200 = 675 \text{ W}$$

$$\text{Total loss} = 400 + 675 = 1075 \text{ kW}$$

$$\text{Output} = 100 \times (3/4) \times 0.8 = 60 \text{ kW} ; \eta = (60/61.075) \times 100 = \mathbf{98.26 \%$$

(d) When frequency is doubled, iron loss is increased because

(i) hysteresis loss is doubled –  $W_h \propto f$

(ii) eddy current loss is quadrupled –  $W_e \propto f^2$

Hence, efficiency will be decreased.

**Example 32.74.** A transformer has a resistance of 1.8 % and a reactance of 5.4 %. (a) At full load, what is the power-factor at which the regulation will be : (i) Zero, (ii) positive-maximum ? (b) If its maximum efficiency occurs at full-load (at unity p.f.), what will be the efficiency under these conditions ?

**Solution :** Approximate percentage regulation is given, in this case, by the relationship

$$1.8 \cos \phi \pm 5.4 \sin \phi.$$

(a) **Regulation :**

(i) If regulation is zero, negative sign must be applicable. This happens at leadings p.f.

$$\text{Corresponding p.f.} = \tan \phi = 1.8/5.4 = 0.333 \text{ leading}$$

$$\phi = 18.44^\circ \text{ leading}$$

(ii) For maximum positive regulation, lagging p.f. is a must. From phasor diagram, the result can be obtained.

$$\text{Corresponding } \tan \phi = 5.4/1.8 = 3, \phi = 71.56^\circ \text{ lagging}$$

$$\% \text{ Voltage regulation} = 1.8 \cos \phi + 5.4 \sin \phi = 5.7\%$$

(b) **Efficiency :** Maximum efficiency occurs at such a load when

$$\text{Iron losses} = \text{Copper losses}$$

This means Iron-losses are 1.8 %.

$$\text{Efficiency} = 100/(100 + 1.8 + 1.8) = 96.52\%$$

**Example 32.75.** A 10 kVA, 1 phase, 50 Hz, 500/250 V transformer gave following test results :

OC test (LV) side : 250 V, 3.0 A, 200 W

SC test (LV) side : 15 V, 30 A, 300 W.

Calculate efficiency and regulation at full load, 0.8 p.f. lagging.

(Nagpur University, Summer 2000)

**Solution.** For efficiency calculations, full load current should be calculated, on the L.V. side in this case,

$$\text{F.L. Current} = \frac{10,000}{250} = 40 \text{ amp}$$

Short-circuit test data have been given at 30 A current on the L.V. side.

$$i^2 r \text{ losses at 40 A L.V. side} = \left(\frac{40}{30}\right)^2 \times 300 \text{ Watts} = 533.3 \text{ Watts}$$

At rated voltage, iron losses (from O.C. test) = 200 Watts

F.L. Output at 0.8 P.F. = 10,000 × 0.8 = 8000 Watts

$$\text{Hence, } \eta = \frac{8000}{8000 + 4733.3} \times 100\% = 91.6\%$$

For regulation, series resistance and reactance parameters of the equivalent circuit have to be evaluated, from the S.C. test.

$$\text{Series Impedance, } Z = \frac{15}{30} = 0.5 \text{ ohm}$$

$$\text{Series resistance, } r = \frac{300}{30 \times 30} = 0.333 \text{ ohm}$$

$$\text{Series reactance, } x = \sqrt{0.5^2 - 0.333^2} = 0.373 \text{ ohm}$$

By Approximate formula,

p.u. regulation at full load, 0.8 p.f. lagging

$$= \frac{40}{250} [0.333 \times 0.8 - 0.373 \times 0.6] = 6.82 \times 10^{-3} \text{ p.u.}$$

When converted into volts, this is  $6.82 \times 10^{-3} \times 250 = 1.70 \text{ volt}$

**Example 32.76.** A 40 kVA, 1-ph, transformer has an iron loss of 400 W, and full copper loss of 800 W. Find the load at which maximum efficiency is achieved at unity power factor.

(Amravati University, Winter 1999)

**Solution.** If  $x$  = fraction of rated load at which the efficiency is maximum.

$$P_i = \text{Iron-loss} = 400 \text{ W}$$

$$P_c = \text{F.L. copper-loss} = 800 \text{ W}$$

Then

$$x^2 P_c = P_i$$

On substitution of numerical values of  $P_i$  and  $P_c$ , we get

$$x = 0.707$$

Hence, the efficiency is maximum, at unity p.f. and at 70.7 % of the rated Load. At this load, copper-loss = Iron-loss = 0.40 kW

$$\begin{aligned} \text{Corresponding output} &= 40 \times 0.707 \times 1 \\ &= 28.28 \text{ kW} \end{aligned}$$

$$\text{Corresponding efficiency} = \frac{28.28}{28.28 + 0.4 + 0.4} = 97.25 \%$$

**Extension to Question : (a)** At what load (s) at unity p.f. the efficiency will be 96.8 % ?

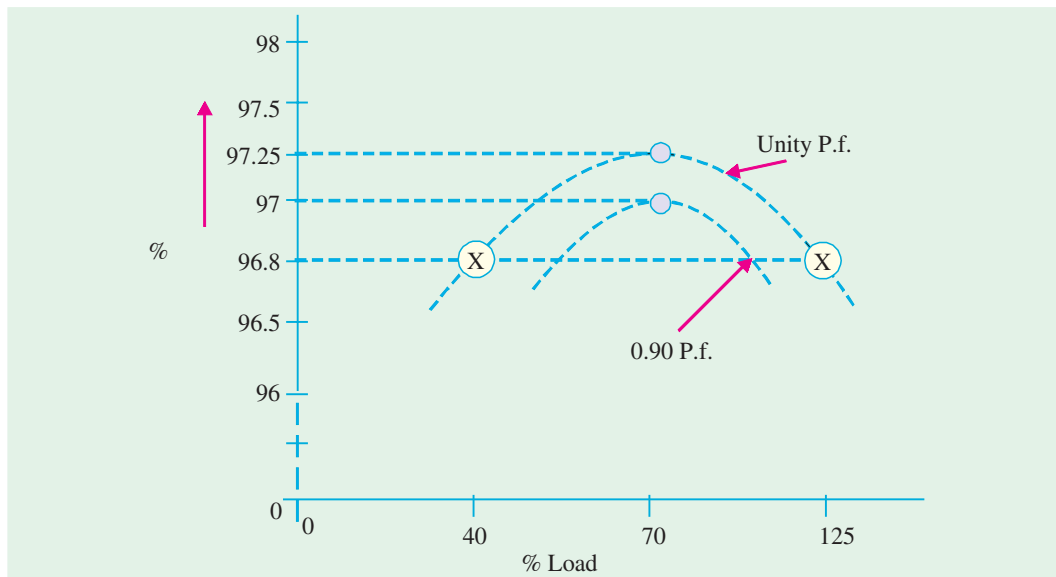


Fig. 32.58. Efficiency variation with load

**Solution.** Let  $x$  = Fractional load at which the concerned efficiency occurs, at unity p.f.

$$\frac{40x}{40x + 0.8x^2 + 0.40} = 0.968$$

This gives the following values of  $x$  :

$$x_1 = 1.25$$

$$x_2 = 0.40$$

Thus, at 40 % and at 125 % of the rated load, the efficiency will be **96.8 %** as marked on the graph, in Fig. 32.58.

**(b) How will maximum-efficiency condition be affected if the power factor is 0.90 lagging ?**

**Solution.**

The condition for efficiency-variation-statement is that the power factor remains constant. Thus, for 0.90 lagging p.f., another curve (Lower curve in Fig. 32.58) will be drawn for which the maximum efficiency will occur at the same value of  $x (= 0.707)$ , but

$$\begin{aligned} \text{Maximum efficiency} &= \frac{40 x \cos \phi}{40 \cos \phi + 0.80 x^2 + 0.40} \\ &= \frac{28.28 \times 0.90}{(28.28 \times 0.90) + 0.80} = \mathbf{97 \%} \end{aligned}$$

**Example 32.77.** A 10 kVA, 500/250 V, single-phase transformer gave the following test results:  
 S.C. Test (H.V. side) : 60 V, 20 A, 150 W  
 The maximum efficiency occurs at unity power factor and at 1.20 times full-load current. Determine full-load efficiency at 0.80 p.f. Also calculate the maximum efficiency.

**(Rajiv Gandhi Technical University, Bhopal, Summer 2001)**

**Solution.** Full-load current on H.V. side = 10,000/500 = 20 Amp

S.C. test has been conducted from H.V. side only. Hence, full-load copper-loss, at unity p.f. = 150 watts

**(a)** Maximum efficiency occurs at 1.2 times full-load current, at unity p.f. corresponding copper-loss =  $(1.2)^2 \times 150 = 216$  watts

At maximum efficiency, copper-loss = core-loss = 216 watts

Corresponding Power-output =  $1.2 \times 10,000 \times 1.0 = 12$  kW

Hence, maximum efficiency at unity P.f. =  $(12)/(12 + 0.216 + 0.2160) = 0.9653 = 96.53 \%$

**(b)** Full-load efficiency at 0.80 P.f.

Output Power at full-load, 0.80 P.f. =  $10,000 \times 0.8 = 8000$  W, constant core-loss = 216 W

Corresponding copper-loss = 150 W

Total losses = 366 W

Hence, efficiency =  $(8000/8366) \times 100 \% = 95.63 \%$ .

### 32.31. Variation of Efficiency with Power Factor

The efficiency of a transformer is given by

$$\begin{aligned} \eta &= \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \\ &= 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{(V_2 I_2 \cos \phi + \text{losses})} \end{aligned}$$

Let, losses/ $V_2 I_2 = x$

$$\begin{aligned} \therefore \eta &= 1 - \frac{\text{losses}/V_2 I_2}{\cos \phi + (\text{losses}/V_2 I_2)} \\ &= 1 - \frac{x}{(\cos \phi + x)} = 1 - \frac{x/\cos \phi}{1 + (x/\cos \phi)} \end{aligned}$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Fig. 32.59.

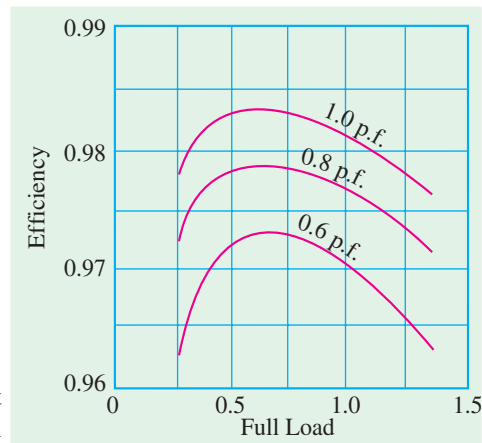


Fig. 32.59

## Tutorial Problems 32.4

1. A 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate (a) iron loss at F.L. (b) Cu loss at F.L. (c) efficiency at half-load. Ignore magnetising current and assume a p.f. of 0.8 at all loads.  
 [(a) 1.777 kW (b) 2.09 kW (c) 97.92%]
2. A 600 kVA, 1-ph transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full load at 0.8 power factor lag.  
 [90.59%] (*Elect. Machines, A.M.I.E. Sec. B, 1992*)
3. Find the efficiency of a 150 kVA transformer at 25 % full load at 0.8 p.f. lag if the copper loss at full load is 1600 W and the iron loss is 1400 W. Ignore the effects of temperature rise and magnetising current.  
 [96.15%] (*Elect. Machines, A.M.I.E. Sec. B, 1991*)
4. The F.L. Cu loss and iron loss of a transformer are 920 W and 430 W respectively. (i) Calculate the loading of the transformer at which efficiency is maximum (ii) what would be the losses for giving maximum efficiency at 0.85 of full-load if total full-load losses are to remain unchanged ?  
 [(a) 68.4% of F.L. (ii)  $W_i = 565 \text{ W}$  ;  $W_{cu} = 785 \text{ W}$ ]
5. At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 2.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8.  
 [93.58%] (*City & Guilds London*)
6. A transformer, when tested on full-load, is found to have Cu loss 1.8% and reactance drop 3.8%. Calculate its full-load regulation (i) at unity p.f. (ii) 0.8 p.f. lagging (iii) 0.8 p.f. leading.  
 [(i) 1.80% (ii) 3.7 % (iii) -0.88%]
7. With the help of a vector diagram, explain the significance of the following quantities in the open-circuit and short-circuit tests of a transformer (a) power consumed (b) input voltage (c) input current. When a 100-kVA single-phase transformer was tested in this way, the following data were obtained : On open circuit, the power consumed was 1300 W and on short-circuit the power consumed was 1200 W. Calculate the efficiency of the transformer on (a) full-load (b) half-load when working at unity power factor.  
 [(a) 97.6% (b) 96.9%] (*London Univ.*)
8. An 11,000/230-V, 150-kVA, 50-Hz, 1-phase transformer has a core loss of 1.4 kW and full-load Cu loss of 1.6 kW. Determine (a) the kVA load for maximum efficiency and the minimum efficiency (b) the efficiency at half full-load at 0.8 power factor lagging.  
 [140.33 kVA, 97.6% ; 97%]
9. A single-phase transformer, working at unity power factor has an efficiency of 90 % at both half-load and a full-load of 500 kW. Determine the efficiency at 75 % of full-load. [90.5%] (*I.E.E. London*)
10. A 10-kVA, 500/250-V, single-phase transformer has its maximum efficiency of 94 % when delivering 90 % of its rated output at unity power factor. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging.  
 [92.6%] (*Elect. Machinery, Mysore Univ, 1979*)
11. A single-phase transformer has a voltage ratio on open-circuit of 3300/660-V. The primary and secondary resistances are  $0.8 \Omega$  and  $0.03 \Omega$  respectively, the corresponding leakage reactance being  $4 \Omega$  and  $0.12 \Omega$ . The load is equivalent to a coil of resistance  $4.8 \Omega$  and inductive reactance  $3.6 \Omega$ . Determine the terminal voltage of the transformer and the output in kW.  
 [636 V, 54 kW]
12. A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (a) 100 kVA output at 0.8 p.f. lagging (b) 50 kVA output at unity power factor.  
 [(a) 97.44% (b) 98.09%]
13. A 10-kVA, 440/3300-V, 1-phase transformer, when tested on open circuit, gave the following figures on the primary side : 440V ; 1.3 A ; 115 W. When tested on short-circuit with full-load current flowing, the power input was 140 W. Calculate the efficiency of the transformer at (a) full-load unity p.f. (b) one quarter full-load 0.8 p.f.  
 [(a) 97.51% (b) 94.18%] (*Elect. Engg-I, Sd. Patel Univ. June 1977*)
14. A 150-kVA single-phase transformer has a core loss of 1.5 kW and a full-load Cu loss of 2 kW. Calculate the efficiency of the transformer (a) at full-load, 0.8 p.f. lagging (b) at one-half full-load

unity p.f. Determine also the secondary current at which the efficiency is maximum if the secondary voltage is maintained at its rated value of 240 V. **[(a) 97.17% (b) 97.4% ; 541 A]**

15. A 200-kVA, 1-phase, 3300/400-V transformer gave the following results in the short-circuit test. With 200 V applied to the primary and the secondary short-circuited, the primary current was the full-load value and the input power 1650 W. Calculate the secondary p.d. and percentage regulation when the secondary load is passing 300 A at 0.707 p.f. lagging with normal primary voltage. **[380 V ; 480%]**
16. The primary and secondary windings of a 40-kVA, 6600/250-V, single-phase transformer have resistances of  $10\ \Omega$  and  $0.02\ \Omega$  respectively. The leakage reactance of the transformer referred to the primary is  $35\ \Omega$ . Calculate
- the primary voltage required to circulate full-load current when the secondary is short-circuited.
  - the full-load regulations at (i) unity (ii) 0.8 lagging p.f. Neglect the no-load current.
- [(a) 256 V (b) (i) 2.2% (ii) 3.7%] (Elect. Technology, Kerala Univ. 1979)**
17. Calculate :
- F.L. efficiency at unity p.f.
  - The secondary terminal voltage when supplying full-load secondary current at p.f. (i) 0.8 lag (ii) 0.8 lead for the 4-kVA, 200/400 V, 50 Hz, 1-phase transformer of which the following are the test figures :
- Open circuit with 200 V supplied to the primary winding-power 60 W. Short-circuit with 16 V applied to the h.v. winding-current 8 A, power 40 W. **[0.97 ; 383 V ; 406 V]**
18. A 100-kVA, 6600/250-V, 50-Hz transformer gave the following results :
- O.C. test : 900 W, normal voltage.  
S.C. test (data on h.v. side) : 12 A, 290 V, 860 W
- Calculate
- the efficiency and percentage regulation at full-load at 0.8 p.f. lagging.
  - the load at which maximum efficiency occurs and the value of this efficiency at p.f. of unity, 0.8 lag and 0.8 lead. **[(a) 97.3%, 4.32% (b) 81 kVA, 97.8%, 97.3% ; 97.3%]**
19. The primary resistance of a 440/110-V transformer is  $0.5\ \Omega$  and the secondary resistance is  $0.04\ \Omega$ . When 440 V is applied to the primary and secondary is left open-circuited, 200 W is drawn from the supply. Find the secondary current which will give maximum efficiency and calculate this efficiency for a load having unity power factor. **[53 A ; 93.58%] (Basic Electricity & Electronics. Bombay Univ. 1981)**
20. Two tests were performed on a 40-kVA transformer to predetermine its efficiency. The results were:
- Open circuit : 250 V at 500 W  
Short circuit : 40 V at F.L. current, 750 W both tests from primary side.
- Calculate the efficiency at rated kVA and 1/2 rated kVA at (i) unity p.f. (ii) 0.8 p.f. **[96.97% ; 96.68% ; 96.24% ; 95.87%]**
21. The following figures were obtained from tests on a 30-kVA, 3000/110-V transformer :
- O.C. test : 3000 V 0.5 A 350 W ; S.C. test ; 150 V 10 A 500 W
- Calculate the efficiency of the transformer at
- full-load, 0.8 p.f.
  - half-load, unity p.f.
- Also, calculate the kVA output at which the efficiency is maximum. **[96.56% ; 97% ; 25.1 kVA]**
22. The efficiency of a 400 kVA, 1-phase transformer is 98.77 % when delivering full load at 0.8 power factor, and 99.13 % at half load and unity power factor. Calculate (a) the iron loss, (b) the full load copper loss. **[(a) 1012 W (b) 2973 W] (Rajiv Gandhi Technical University, 2000)**



### 32.32. All-day Efficiency

The ordinary or commercial efficiency of a transformer is given by the ratio

$$\frac{\text{Output in watts}}{\text{Input in watts}}$$

But there are certain types of transformers whose performance cannot be judged by this efficiency. Transformers used for supplying lighting and general network *i.e.*, distribution transformers have their primaries energised all the twenty-four hours, although their secondaries supply little or no-load much of the time during the day except during the house lighting period. It means that whereas core loss occurs throughout the day, the Cu loss occurs only when the transformers are loaded. Hence, it is considered a good practice to design such transformers so that core losses are very low. The Cu losses are relatively less important, because they depend on the load. The performance of such is compared on the basis of *energy* consumed during a certain time period, usually a day of 24 hours.

$$\therefore \eta_{\text{all-day}} = \frac{\text{Output in kWh (For 24 hours)}}{\text{Input in kWh}}$$

This efficiency is always less than the commercial efficiency of a transformer.

To find this all-day efficiency or (as it is also called) energy efficiency, we have to know the load cycle on the transformer *i.e.*, how much and how long the transformer is loaded during 24 hours. Practical calculations are facilitated by making use of a **load factor**.

**Example 32.78.** Find the all-day efficiency of 500-kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW respectively. During a day of 24 hours, it is loaded as under :

No. of hours	Loading in kW	Power factor
6	400	0.8
10	300	0.75
4	100	0.8
4	0	–

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** It should be noted that a load of 400 kW at 0.8 p.f. is equal to  $400/0.8 = 500$  kVA. Similarly, 300 kW at 0.75 p.f. means  $300/0.75 = 400$  kVA and 100 kW at 0.8 p.f. means  $100/0.8 = 125$  kVA *i.e.*, one-fourth of the full-load.

$$\text{Cu loss at F.L. of 500 kVA} = 4.5 \text{ kW}$$

$$\text{Cu loss at 400 kVA} = 4.5 \times (400/500)^2 = 2.88 \text{ kW}$$

$$\text{Cu loss at 125 kVA} = 4.5 \times (125/500)^2 = 0.281 \text{ kW}$$

$$\begin{aligned} \text{Total Cu loss in 24 hrs} &= (6 \times 4.5) + (10 \times 2.88) + (4 \times 0.281) + (4 \times 0) \\ &= 56.924 \text{ kWh} \end{aligned}$$

The iron loss takes place throughout the day irrespective of the load on the transformer because its primary is energized all the 24 hours.



The world first 5,000 KVA amorphous transformer commissioned in August 2001 in Japan

$$\begin{aligned} \therefore \text{Iron loss in 24 hours} &= 24 \times 3.5 = 84 \text{ kWh} \\ \text{Total transformer loss} &= 56.924 + 84 = 140.924 \text{ kWh} \\ \text{Transformer output in 24 hrs} &= (6 \times 400) + (10 \times 300) + (4 \times 100) = 5800 \text{ kWh} \\ \therefore \eta_{\text{all-day}} &= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{5800}{5800 + 140.924} = 0.976 \text{ or } \mathbf{97.6\%} \end{aligned}$$

**Example 32.79.** A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency. (Elect. Engg. Punjab Univ. 1990)

**Solution.** It should be noted that lighting transformers are taken to have a load p.f. of unity.

$$\begin{aligned} \text{Iron loss for 24 hour} &= 1.5 \times 24 = 36 \text{ kWh ; F.L. Cu loss} = 1.5 \text{ kW} \\ \therefore \text{Cu loss for 3 hours on F.L.} &= 1.5 \times 3 = 4.5 \text{ kWh} \\ \text{Cu loss at half full-load} &= 1.5/4 \text{ kW} \\ \text{Cu loss for 4 hours at half the load} &= (1.5/4) \times 4 = 1.5 \text{ kWh} \\ \text{Total losses} &= 36 + 4.5 + 1.5 = 42 \text{ kWh} \\ \text{Total output} &= (100 \times 3) + (50 \times 4) = 500 \text{ kWh} \\ \therefore \eta_{\text{all-day}} &= 500 \times 100/542 = \mathbf{92.26\%} \\ \text{Incidentally, ordinary or commercial efficiency of the transformer is} & \\ &= 100/(100 + 3) = 0.971 \text{ or } \mathbf{97.1\%} \end{aligned}$$

**Example 32.80.** Two 100-kW transformers each has a maximum efficiency of 98 % but in one the maximum efficiency occurs at full-load while in the other, it occurs at half-load. Each transformer is on full-load for 4 hours, on half-load for 6 hours and on one-tenth load for 14 hours per day. Determine the all-day efficiency of each transformer. (Elect. Machines-I, Vikram Univ. 1988)

**Solution.** Let  $x$  be the iron loss and  $y$  the full-load Cu loss. If the ordinary efficiency is a maximum at  $1/m$  of full-load, then  $x = y/m^2$ .

$$\begin{aligned} \text{Now,} \quad \text{output} &= 100 \text{ kW ; Input} = 100/0.98 \\ \therefore \text{Total losses} &= 100/0.98 - 100 = 2.04 \text{ kW} \\ \therefore y + x/m^2 &= 2.04 \end{aligned}$$

**1st Transformer**

$$\begin{aligned} \text{Here } m = 1 ; \quad y + y &= 2.04 ; y = 1.02 \text{ kW and } x = 1.02 \text{ kW} \\ \text{Iron loss for 24 hours} &= 1.02 \times 24 = 24.48 \text{ kWh} \\ \text{Cu loss for 24 hours} &= 4 \times 1.02 + 6 \times (1.02/4) + 14 (1.02/10^2) = 5.73 \text{ kWh} \\ \text{Total loss} &= 24.48 + 5.73 = 30.21 \text{ kWh} \\ &= 4 \times 100 + 6 \times 50 + 14 \times 10 = 840 \text{ kWh} \\ \therefore \eta_{\text{all-day}} &= 840/870.21 = 0.965 \text{ or } \mathbf{96.5\%} \end{aligned}$$

**2nd Transformer**

$$\begin{aligned} \text{Here} \quad 1/m &= 1/2 \text{ or } m = 2 \therefore y + y/4 = 2.04 \\ \text{or} \quad y &= 1.63 \text{ kW ; } x = 0.14 \text{ kW} \\ \text{Output} &= 840 \text{ kWh} \quad \dots\text{as above} \\ \text{Iron loss for 24 hours} &= 0.14 \times 24 = 3.36 \text{ kWh} \\ \text{Cu loss for 24 hours} &= 4 \times 1.63 + 6 (1.63/4) + 14 (1.63/10^2) = 9.19 \text{ kWh} \\ \text{Total loss} &= 3.36 + 9.19 = 12.55 \text{ kWh} \\ \therefore \eta_{\text{all-day}} &= 840/852.55 = 0.974 \text{ or } \mathbf{97.4\%} \end{aligned}$$

**Example 32.81.** A 5-kVA distribution transformer has a full-load efficiency at unity p.f. of 95%, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows :

No load for 10 hours      Quarter load for 7 hours  
Half load for 5 hours      Full load for 2 hours

Assume load p.f. of unity.

(Power Apparatus-I, Delhi Univ. 1987)

**Solution.** Let us first find out the losses from the given commercial efficiency of the transformer.

Output =  $5 \times 1 = 5$  kW ; Input =  $5/0.95 = 5.264$  kW

Losses =  $(5.264 - 5.000) = 0.264$  kW = 264W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$\therefore$  Cu loss at F.L. of 5 kVA =  $264/2 = 132$  W ; Iron loss = 132 W

Cu loss at one-fourth F.L. =  $(1/4)^2 \times 132 = 8.2$  W

Cu loss at one-half F.L. =  $(1/2)^2 \times 132 = 33$  W

Quarter load Cu loss for 7 hours =  $7 \times 8.2 = 57.4$  Wh

Half-load Cu loss for 5 hours =  $5 \times 33 = 165$  Wh

F.L. Cu loss for 2 hours =  $2 \times 132 = 264$  Wh

Total Cu loss during one day =  $57.4 + 165 + 264 = 486.4$  Wh = 0.486 kWh

Iron loss in 24 hours =  $24 \times 132 = 3168$  Wh = 3.168 kWh

Total losses in 24 hours =  $3.168 + 0.486 = 3.654$  kWh

Since load p.f. is to be assumed as unity.

F.L. output =  $5 \times 1 = 5$  kW ; Half F.L. output =  $(5/2) \times 1 = 2.5$  kW

Quarter load output =  $(5/4) \times 1 = 1.25$  kW

Transformer output in a day of 24 hours =  $(7 \times 1.25) + (5 \times 2.5) + (2 \times 5) = 31.25$  kWh

$$\eta_{\text{all-day}} = \frac{31.25}{(31.25 + 3.654)} \times 100 = 89.53 \%$$

**Example 32.82.** Find “all day” efficiency of a transformer having maximum efficiency of 98% at 15 kVA at unity power factor and loaded as follows :

12 hours – 2 kW at 0.5 p.f. lag

6 hours – 12 kW at 0.8 p.f. lag

6 hours – at no load.

(Elect. Machines-I, Nagpur Univ. 1993)

**Solution.**

Output =  $15 \times 1 = 15$  kW, input =  $15/0.98$

Losses =  $(15/0.98 - 15) = 0.306$  kW = 306 W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$\therefore$  Cu loss at 15 kVA =  $306/2 = 153$  W, Iron loss = 153 W

2 kW at 0.5 p.f. =  $2/0.5 = 4$  kVA, 12 kW at 0.8 p.f. =  $12/0.8 = 15$  kVA

Cu loss at 4 kVA =  $153 (4/15)^2 = 10.9$  W ; Cu loss at 15 kVA = 153 W.

Cu loss in 12 hrs =  $12 \times 10.9 = 131$  Wh ; Cu loss in 6 hr =  $6 \times 153 = 918$  Wh.

Total Cu loss for 24 hr =  $131 + 918 = 1050$  Wh = 1.05 kWh

Iron loss for 24 hrs =  $24 \times 153 = 3,672$  Wh = 3.672 kWh

Output in 24 hrs =  $(2 \times 12) + (6 \times 12) = 96$  kWh

Input in 24 hrs =  $96 + 1.05 + 3.672 = 100.72$  kWh

$\therefore$   $\eta_{\text{all-day}} = 96 \times 100/100.72 = 95.3 \%$

**Example 32.83.** A 150-kVA transformer is loaded as follows :

Load increases from zero to 100 kVA in 3 hours from 7 a.m. to 10.00 a.m., stays at 100 kVA from 10 a.m. to 6 p.m. and then the transformer is disconnected till next day. Assuming the load to be resistive and core-loss equal to full-load copper loss of 1 kW, determine the all-day efficiency and the ordinary efficiency of the transformer. **(Electrical Machines-II, Indore Univ. 1990)**

**Solution.** Since load is resistive, its p.f. is unity.

Average load from 7 a.m. to 10 a.m. =  $(0 + 100)/2 = 50$  kVA *i.e.*, one-third F.L.

Load from 10 a.m. to 6 p.m. = 100 kVA *i.e.*, 2/3 of F.L.

#### Ordinary Efficiency

In this case, load variations are not relevant.

Output =  $150 \times 1 = 150$  kW ; Iron loss = Cu loss = 1 kW ; Total loss = 2 kW.

$\therefore$  Ordinary  $\eta = 150/(150 + 2) = 0.9868$  or **98.68 %**

#### All-day Efficiency

Cu loss from 7-10 a.m. =  $3 \times (1/3)^2 \times 1 = 0.333$  kWh

Cu loss from 10 a.m. to 6 p.m. =  $8 \times (2/3)^2 \times 1 = 3.555$  kWh

Total Cu loss for 24 hrs =  $0.333 + 3.555 = 3.888$  kWh

Total iron loss for 24 hrs =  $24 \times 1 = 24$  kWh

Losses for a day of 24 hrs = 27.888 kWh

Output for 24 hrs =  $3 \times (50 \times 1) + 8 (100 \times 1) = 950$  kWh

$\therefore \eta_{\text{all-day}} = \frac{950 \times 100}{(950 + 27.888)} = \mathbf{97.15 \%}$

**Example 32.84.** Find the all-day efficiency of a 50 kVA distribution transformer having full load efficiency of 94 % and full-load copper losses are equal to the constant iron losses. The loading of the transformer is as follows, the power factor being 1.0.

(i) No load for 10 hours

(ii) Half load for 5 hours

(iii) 25 % load for 6 hours

(iv) Full load for 3 hours.

**(Sambalpur University, 1998)**

**Solution.** At full load unity p.f.

$$\text{efficiency} = 94 \% = \frac{50,000}{50,000 + 2P_i}$$

$$2P_i = \left[ \frac{50,000}{0.94} - 50,000 \right], \quad \text{or} \quad P_i = \frac{1}{2} \times 50,000 \left[ \frac{1 - 0.94}{0.94} \right],$$

$$P_i = 25,000 \times \frac{0.06}{0.94} = 1596 \text{ Watts}$$

Hence, full load Cu-losses = 1596 Watts

(a) Energy required in overcoming Cu-losses, during 24 hours

(i) No load for 10 hours : zero

(ii) At half load, Cu-losses =  $(0.5)^2 \times 1596$  Watts = 399

$$\text{Energy in 5 hours} = \frac{399 \times 5}{1000} \text{ kWh} = 1.995 \text{ kWh}$$

(iii) At 25 % load, Cu-loss =  $(0.25)^2 \times 1596 = 99.75$  Watts

$$\text{Energy in 6 hours} = \frac{6 \times 99.75}{1000} = 0.5985 \text{ kWh}$$

(iv) Energy lost during 3 hours of full load =  $\frac{1596 \times 3}{1000} = 4.788$  kWh

(b) Energy lost in constant core-losses for 24 hours =  $\frac{1596}{1000} \times 24 = 38.304$  kWh

(c) Energy required by the load =  $25 \times 5 + 12.5 \times 6 + 50 \times 3 = 125 + 75 + 150 = 350 \text{ kWh}$

$$\text{All-day efficiency} = \frac{350}{350 + 38.304 + 7.3815} \times 100 = 88.454 \%$$

**Example 32.85.** A 10 kVA, 1-ph transformer has a core-loss of 40 W and full load ohmic loss of 100 W. The daily variation of load on the transformer is as follows :

6 a.m. to 1 p.m. 3 kW at 0.60 p.f.  
 1 p.m. to 5 p.m. - 8kW at 0.8 p.f.  
 5 p.m. to 1 a.m. full load at u.p.f.  
 1 a.m. to 6 a.m. no load

Determine all day efficiency of the transformer

(Amravati University, 1999)

**Solution.** Fractional loading (=  $x$ ) and the output kWh corresponding to load variations can be worked out in tabular form, as below :

S.N.	Number of hours	$x = \frac{\text{load kVA}}{\text{Xmer Rating}}$	$x^2 P_c$ in kW	Output in kWh	Copper Loss in kWh
1	7	$\frac{3/0.6}{10} = 0.5$	$0.50^2 \times 0.10 = 0.025$	$3 \times 7 = 21$	$0.025 \times 7 = 0.175$
2	4	$\frac{8/0.8}{10} = 1.0$	0.10	$8 \times 4 = 32$	$0.1 \times 4 = 0.40$
3	8	$\frac{10/1}{10} = 1.0$	0.10	$10 \times 8 = 80$	$0.1 \times 8 = 0.8$
4	5	Zero	Zero	Zero	Zero
Output in kWh				$21 + 32 + 80 = 133$	
Ohmic Loss, in kWh					$0.175 + 0.40 + 0.80 = 1.375$
Core loss during 24 Hrs = $\frac{400}{1000} \times 24 = 0.96 \text{ kWh}$					

Hence, Energy efficiency (= All day Efficiency) =  $\frac{133}{133 + 1.375 + 0.96} \times 100 = 98.3 \%$

**Example 32.86.** A transformer has its maximum efficiency of 0.98 at 15 kVA at unity p.f. During a day, it is loaded as follows :

12 hours : 2 kW at 0.8 p.f.  
 6 hours : 12 kW at 0.8 p.f.  
 6 hours : 18 kW at 0.9 p.f.

Find the all day efficiency.

(Manomaniam Sundaranar Univ. April 1998)

**Solution.** Let 15 kVA be treated as full load.

Output at maximum efficiency =  $15000 \times 1$  watts

Input =  $15000/0.98$  watts

Losses = Input - Output =  $15000 (1/0.98 - 1) = 15000 \times 2/98 = 306$  Watts

At maximum efficiency, since the variable copper-loss and constant core-loss are equal.

Full load copper-loss = Constant core-loss =  $306/2 = 153$  Watts

Let the term  $x$  represents the ratio of required Load/Full load.

Output =  $15 \times \cos \phi$

Following tabular entries simplify the calculations for all-day efficiency.

S.N.	$x$	Hrs	$x^2 P_c$ in kW	Energy in Copper-loss in kWh	Output during the period in kWh
1	$\frac{2/0.5}{15} = \frac{4}{15}$	12	0.01088	0.131	24
2	$\frac{12/0.8}{15} = 1.0$	6	0.153	0.918	72
3	$\frac{18/0.9}{15} = 4/3$	6	0.272	1.632	108

Total output during the day = 204 kWh

Total copper-loss during the day = 2.681 kWh

Total core-loss during the day =  $0.153 \times 24 = 3.672$

All day efficiency =  $(204/210.353) \times 100 = 96.98\%$

### Tutorial Problems 32.5

1. A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.

The transformer undergoes a daily load cycle as follows :

Load	Power factor	Load duration
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

Determine its all-day efficiency.

(Electrical Engineering, MS Univ. Baroda 1979)

2. What is meant by energy efficiency of a transformer ?

A 20-kVA transformer has a maximum efficiency of 98 percent when delivering three-fourth full-load at u.p.f. If during the day, the transformer is loaded as follows :

12 hours	No load
6 hours	12 kWh, 0.8 p.f.
6 hours	20 kW, u.p.f.

Calculate the energy efficiency of the transformer.

(Electrical Technology-III, Gwalior Univ., 1980)

### 32.33. Auto-transformer

It is a transformer with one winding only, part of this being common to both primary and secondary. Obviously, in this transformer the primary and secondary are not electrically *isolated* from each other as is the case with a 2-winding transformer. But its theory and operation are similar to those of a two-winding transformer. Because of one winding, it uses less copper and hence is cheaper. It is used where transformation ratio differs little from unity. Fig. 32.60 shows both step down and step-up auto-transformers.

As shown in Fig. 32.60 (a),  $AB$ , is primary winding having  $N_1$  turns and  $BC$  is secondary winding having  $N_2$  turns. Neglecting iron losses and no-load current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

The current in section *CB* is vector difference\* of  $I_2$  and  $I_1$ . But as the two currents are practically in phase opposition, the resultant current is  $(I_2 - I_1)$  where  $I_2$  is greater than  $I_1$ .

As compared to an ordinary 2-winding transformer of same output, an auto-transformer has higher efficiency but smaller size. Moreover, its voltage regulation is also superior.

**Saving of Cu**

Volume and hence weight of Cu, is proportional to the length and area of the cross-section of the conductors.

Now, length of conductors is proportional to the number of turns and cross-section depends on current. Hence, weight is proportional to the product of the current and number of turns.

With reference to Fig. 32.60,

Wt. of Cu in section *AC* is  $\propto (N_1 - N_2) I_1$ ; Wt. of Cu in section *BC* is  $\propto N_2 (I_2 - I_1)$ .

$\therefore$  Total Wt. of Cu in auto-transformer  $\propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$

If a two-winding transformer were to perform the same duty, then

Wt. of Cu on its primary  $\propto N_1 I_1$ ; Wt. of Cu on secondary  $\propto N_2 I_2$

Total Wt. of Cu  $\propto N_1 I_1 + N_2 I_2$

$$\begin{aligned} \therefore \frac{\text{Wt. of Cu in auto-transformer}}{\text{Wt. of Cu in ordinary transformer}} &= \frac{(N_1 - N_2) I_1 + N_2 (I_2 - I_1)}{N_1 I_1 + N_2 I_2} \\ &= 1 - \frac{2 \frac{N_2}{N_1}}{1 + \frac{N_2}{N_1} \times \frac{I_2}{I_1}} = 1 - \frac{2K}{2} = 1 - K \left( \because \frac{N_1}{N_2} = K ; \frac{I_2}{I_1} = \frac{1}{K} \right) \end{aligned}$$

Wt. of Cu in auto-transformer ( $W_a$ ) =  $(1 - K) \times$  (Wt. of Cu in ordinary transformer  $W_0$ )

$$\begin{aligned} \therefore \text{Saving} &= W_0 - W_a \\ &= W_0 - (1 - K) W_0 = K W_0 \end{aligned}$$

$\therefore$  Saving =  $K \times$  (Wt. of Cu in ordinary transformer)

Hence, saving will increase as  $K$  approaches unity.

It can be proved that power transformed **inductively** is input  $(1 - K)$ .

The rest of the power =  $(K \times$  input) is **conducted** directly from the source to the load *i.e.*, it is transferred **conductively** to the load.

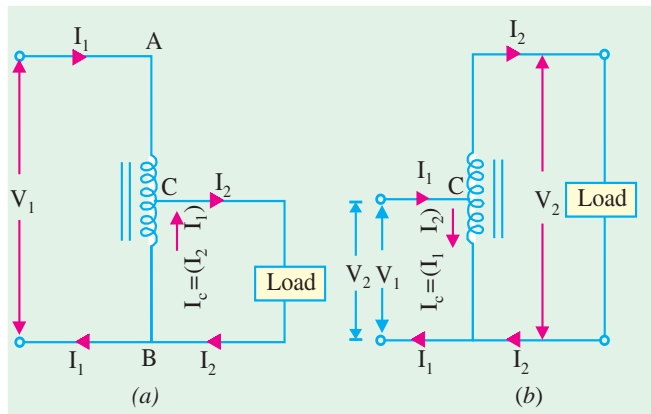


Fig. 32.60



Step up auto-transformer

\* In fact, current flowing in the common winding of the auto-transformer is always equal to the difference between the primary and secondary currents of an ordinary transformer.

**Uses**

As said earlier, auto-transformers are used when  $K$  is nearly equal to unity and where there is no objection to electrical connection between primary and secondary. Hence, such transformers are used :

1. to give small boost to a distribution cable to correct the voltage drop.
2. as auto-starter transformers to give upto 50 to 60 % of full voltage to an induction motor during starting.
3. as furnace transformers for getting a convenient supply to suit the furnace winding from a 230-V supply
4. as interconnecting transformers in 132 kV/330 kV system.
5. in control equipment for 1-phase and 3-phase electrical locomotives.

**Example 32.87.** An auto-transformer supplies a load of 3 kW at 115 volts at a unity power factor. If the applied primary voltage is 230 volts, calculate the power transferred to the load

(a) inductively and (b) conductively. (Basic Elect. Machines, Nagpur Univ, 1991)

**Solution.** As seen from Art 32.33

Power transferred inductively = Input  $(1 - K)$

Power transferred conductively = Input  $\times K$

Now,  $K = 115/230 = 1/2$ , input  $\cong$  output = 3 kW

$\therefore$  Inductively transferred power =  $3(1 - 1/2) = 1.5 \text{ kW}$

Conductively transferred power =  $(1/2) \times 3 = 1.5 \text{ kW}$

**Example 32.88.** The primary and secondary voltages of an auto-transformer are 500 V and 400 V respectively. Show with the aid of diagram, the current distribution in the winding when the secondary current is 100 A and calculate the economy of Cu in this particular case.

**Solution.** The circuit is shown in Fig. 30.61.

$$K = V_2/V_1 = 400/500 = 0.8$$

$$\therefore I_1 = KI_2 = 0.8 \times 100 = 80 \text{ A}$$

The current distribution is shown in Fig. 32.61.

Saving =  $K W_0 = 0.8 W_0$  - Art 32.33

$$\therefore \text{Percentage saving} = 0.8 \times 100 = 80$$

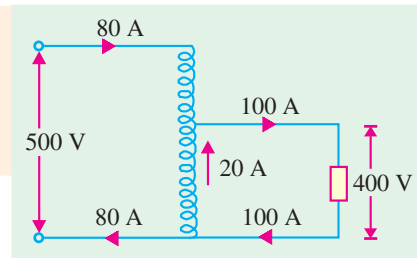


Fig. 32.61

**Example 32.89.** Determine the core area, the number of turns and the position of the tapping point for a 500-kVA, 50-Hz, single-phase, 6,600/5,000-V auto-transformer, assuming the following approximate values : e.m.f. per turn 8 V. Maximum flux density 1.3 Wb/m<sup>2</sup>.

**Solution.**

$$E = 4.44f\Phi_m N \text{ volt}$$

$$\Phi_m = \frac{E/N}{4.44f} = \frac{8}{4.44 \times 50} = 0.03604 \text{ Wb}$$

$$\text{Core area} = 0.03604/1.3 = 0.0277 \text{ m}^2 = 277 \text{ cm}^2$$

$$\text{Turns of h.v. side} = 6600/8 = 825 ; \text{Turns of L.V. side} = 5000/8 = 625$$

Hence, tapping should be 200 turns from high voltage end or 625 turns from the common end.

**32.34. Conversion of 2-Winding Transformer into Auto-transformer**

Any two-winding transformer can be converted into an auto-transformer either step-down or step-up. Fig. 32.62 (a) shows such a transformer with its polarity markings. Suppose it is a 20-kVA,



2400/240 V transformer. If we employ *additive* polarity between the high-voltage and low-voltage sides, we get a step-up auto-transformer. If, however, we use the *subtractive* polarity, we get a step-down auto-transformer.

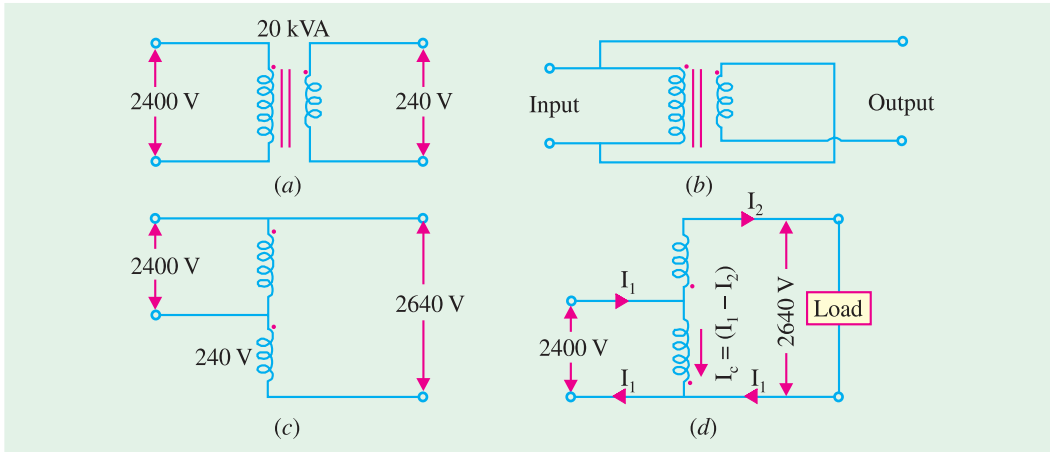


Fig. 32.62

**(a) Additive Polarity**

Connections for such a polarity are shown in Fig. 32.62 (b). The circuit is re-drawn in Fig. 32.62 (c) showing common terminal of the transformer at the top whereas Fig. 32.62 (d) shows the same circuit with common terminal at the bottom. Because of additive polarity,  $V_2 = 2400 + 240 = 2640$  V and  $V_1$  is 2400 V. There is a marked increase in the kVA of the auto-transformer (Ex. 32.90). As shown in Fig. 32.62 (d), common current flows *towards* the common terminal. The transformer acts as a step-up transformer.

**(b) Subtractive Polarity**

Such a connection is shown in Fig. 32.63 (a). The circuit has been re-drawn with common polarity at top in Fig. 32.63 (b) and at bottom in Fig. 32.63 (c). In this case, the transformer acts as a step-down auto-transformer.

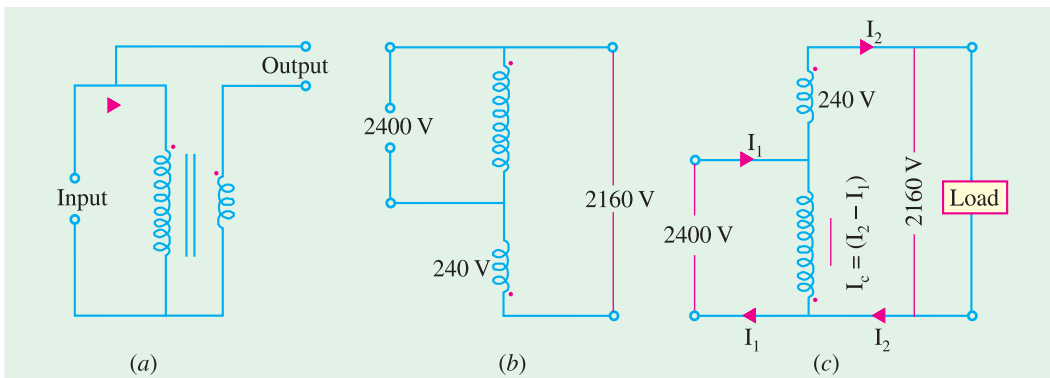


Fig. 32.63

The common current flows *away* from the common terminal. As will be shown in Example 32.91, in this case also, there is a very large increase in kVA rating of the auto-transformer though not as marked as in the previous case. Here,  $V_2 = 2400 - 240 = 2160$  V.

**Example 32.90.** For the 20-kVA, 2400/240-V two-winding step-down transformer shown in Fig. 32.63 (a) connected as an auto-transformer with additive polarity as shown in Fig. 30.61 (d), compute

- (i) original current capacity of HV-windings.
- (ii) original current capacity of LV-windings.
- (iii) kVA rating of auto-transformer using current capacity of current LV winding as calculated in (ii) above.
- (iv) per cent increase in kVA capacity of auto-transformer as compared to original two-winding transformer.
- (v) values of  $I_1$  and  $I_c$  in Fig. 30.61 (d) from value of  $I_2$  used in (iii) above.
- (vi) per cent overload of 2400-V winding when used as an auto-transformer.
- (vii) comment on the results obtained.

**Solution.** (i)  $I_1 = 20 \times 10^3 / 2400 = 8.33 \text{ A}$  (ii)  $I_2 = I_1 / K = 8.33 \times 10 = 83.3 \text{ A}$   
 (iii) kVA rating of auto-transformer  $V_2 I_2 = 2640 \times 83.3 \times 10^{-3} = 220 \text{ kVA}$   
 (iv) Per cent increase in kVA rating =  $\frac{220}{20} \times 100 = 1100\%$   
 (v)  $I_1 = 220 \times 10^3 / 2400 = 91.7 \text{ A}$ ,  $I_c = I_1 - I_2 = 91.7 - 83.3 = 8.4 \text{ A}$   
 (vi) Per cent overload of 2400 V winding =  $8.4 \times 100 / 8.33 = 100.8\%$   
 (vii) As an auto-transformer, the kVA has increased tremendously to 1100% of its original value with LV coil at its rated current capacity and HV coil at negligible overload i.e.  $1.008 \times$  rated load.

**Example 32.91.** Repeat Example 30.64 for subtractive polarity as shown in Fig. 32.62 (c).

**Solution.** (i)  $I_1 = 8.33 \text{ A}$  (ii)  $I_2 = 83.3 \text{ A}$   
 (iii) New kVA rating of auto-transformer is  $2160 \times 83.3 \times 10^{-3} = 180 \text{ kVA}$   
 (iv) Per cent increase in kVA rating =  $\frac{180}{20} \times 100 = 900\%$   
 (v)  $I_1 = 180 \times 10^3 / 2400 = 75 \text{ A}$ ,  $I_c = I_2 - I_1 = 83.3 - 75 = 8.3 \text{ A}$   
 (vi) Per cent overload of 2400V winding =  $8.3 \times 100 / 8.33 = 100\%$   
 (vii) In this case, kVA has increased to 900% of its original value as a two-winding transformer with both low-voltage and high-voltage windings carrying their rated currents.

The above phenomenal increase in kVA capacity is due to the fact that in an auto-transformer energy transfer from primary to secondary is by both conduction as well as induction whereas in a 2-winding transformer it is by induction only. This extra conductive link is mainly responsible for the increase in kVA capacity.

**Example 32.92.** A 5-kVA, 110/110-V, single-phase, 50-Hz transformer has full-load efficiency of 95% and an iron loss of 50 W. The transformer is now connected as an auto-transformer to a 220-V supply. If it delivers a 5-kW load at unity power factor to a 110-V circuit, calculate the efficiency of the operation and the current drawn by the high-voltage side.

(Electric Machinery-II, Bangalore Univ. 1991)

**Solution.** Fig. 32.64 (a) shows the normal connection for a 2-winding transformer. In Fig. 32.64 (b) the same unit has been connected as an auto transformer. Since the two windings are connected in series, voltage across each is 110 V.

The iron loss would

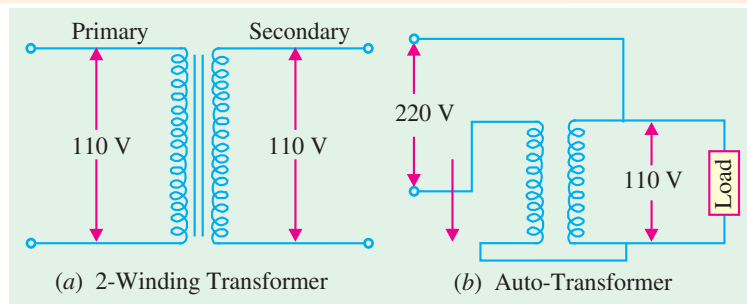


Fig. 32.64

remain the same in both connections. Since the auto-transformer windings will each carry but half the current as compared to the conventional two-winding transformer, the copper loss will be one-fourth of the previous value.

**Two-winding Transformer**

$$\eta = 0.95 \quad \therefore 0.95 = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{5,000}{5,000 + 50 + \text{Cu loss}}$$

$$\therefore \text{Cu loss} = 212 \text{ W}$$

**Auto-transformer**

$$\text{Cu loss} = 212/4 = 53 \text{ W}; \text{ Iron loss} = 50 \text{ W} \quad \therefore \eta = \frac{5,000}{5,000 + 53 + 50} = 0.9797 \text{ or } \mathbf{97.97\%}$$

$$\text{Current of the h.v. side} = 5103/220 = \mathbf{23.2 \text{ A}}$$

**Example 32.93.** A transformer has a primary voltage rating of 11500 volts and secondary voltage rating of 2300 volts. Two windings are connected in series and the primary is connected to a supply of 11500 volts, to act as a step-up auto transformer. Determine the voltage output of the transformer.

*Question extended :* If the two winding transformer is rated at 115 kVA, what will be the kVA rating of the auto-transformer ? **(Madras University, 1997)**

**Solution.** As in Fig. 32.65 (a), 115 kVA, 11500/2300 V, transformer has the current ratings of 10 A and 50 A.

Referring to Fig. 32.65 (b), the step-up connections have been shown. Winding currents have to be at the same rated values. As in Fig. 32.65 (b), the voltage obtainable at  $B_1 - B_2$  is 13800 V, and from  $b_1$  a load-current of 50 A can be supplied.

$$\text{kVA rating} = 13800 \times 50 \times 10^{-3} = 690$$

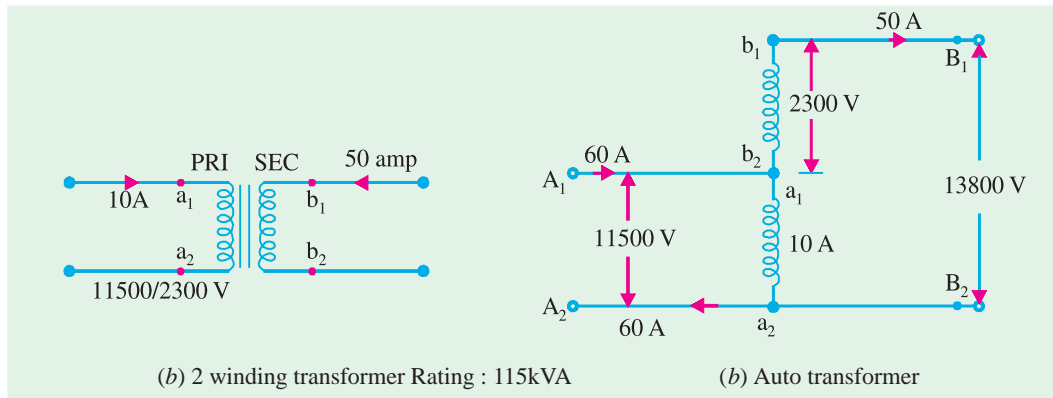


Fig. 32.65

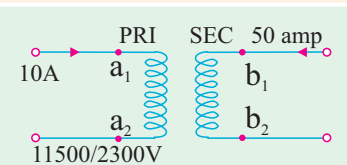
**Example 32.94.** An 11500/2300 V transformer is rated at 100 kVA as a 2 winding transformer. If the two windings are connected in series to form an auto-transformer, what will be the possible voltage ratios ? **(Manonmaniam Sundaranar Univ. April 1998)**

**Solution.** Fig. 32.66 (a) shows this 2-winding transformer with rated winding currents marked.

Rated current of 11.5 kV winding =  $100 \times 100/11500 = 8.7$  Amp

Rated current of 2300 V winding = 43.5 Amp

Fig. 32.66 (b) and Fig. 32.66 (c) show autotransformer **Fig. 32.66 (a).** 2 winding transformer



connections. On H.V. side, they have a rating of 13.8 kV. On L.V. side, with connection as in Fig. 32.66 (b), the rating is 2300 V. On L.V. side of Fig. 32.66 (c), the output is at 11.5 kV.

Thus, possible voltage ratios are : 13800/2300 V and 13800/11500 V.

With both the connections, step-up or step-down versions are possible.

**Extension of Question :** Calculate kVA ratings in the two cases.

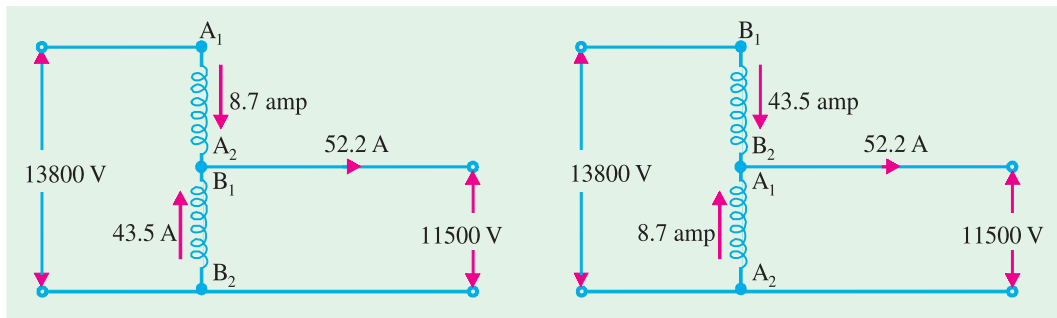


Fig. 32.66

Windings will carry the rated currents, while working out kVA outputs.

In Fig. 32.66 (b), Input current (into terminal  $A_1$  of windings  $A_1-A_2$ ) can be 8.7 Amp with H.V.-side-voltage ratings as 13.8 kV. Transformation ratio =  $13800/2300 = 6$

$$\text{Hence, kVA rating} = 13.8 \times 8.7 = 120$$

$$\text{Output current} = 120 \times 1000/2300 = 52.2 \text{ Amp}$$

$$\text{Current in the winding } B_1-B_2$$

$$= \text{Difference of Output current and Input current}$$

$$= 52.2 - 8.7 = 43.5 \text{ A, which is the rated current of the winding } B_1-B_2.$$

In Fig. 32.66 (c). Similarly, transformation ratio =  $13800/11500 = 1.2$

$$\text{kVA rating} = 13800 \times 43.5 \times 10^{-3} = 600$$

$$\text{Output current} = 600 \times 1000/11500 = 52.2 \text{ Amp}$$

Current carried by common winding =  $52.2 - 43.5 = 8.7 \text{ A}$ , which is rated current for the winding  $A_1-A_2$ . Thus, with the same two windings give, a transformation ratio closer to unity gives higher kVA rating as an auto transformer.

Thus, a 100 kVA two winding transformer is reconnected as an autotransformer of 120 kVA with transformation ratio as 6, and becomes a 600 kVA autotransformer with transformation ratio as 1.2.

**Example 32.95.** A two-winding transformer is rated at 2400/240 V, 50-kVA. It is re-connected as a step-up auto-transformer, with 2400 V input. Calculate the rating of the auto-transformer and the inductively and conductively transferred powers while delivering the rated output at unity power-factor.

(Nagpur University, Winter 1999)

**Solution.** With 50 kVA as the rating, the rated currents on the two sides are 20.8 A (2400-V side) and 208 A (240-V side). With the required re-connection, the 2400-V winding will work as a common winding. As shown in Fig. 32.67, the winding common to input and output can carry 20.8 A, the output current can be 208 A with a voltage of 2640 V, which means that the output of this auto

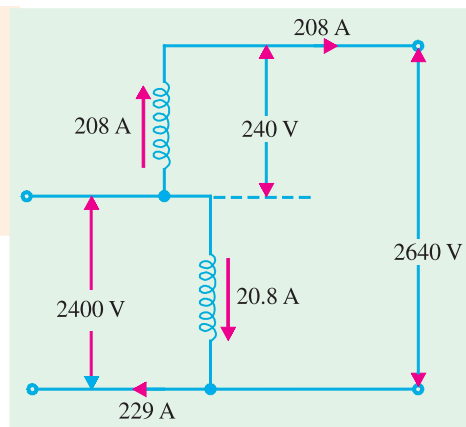


Fig. 32.67

transformer is  $(2640 \times 208) = 550 \text{ kVA}$ .

The corresponding input-current is  
 $(208 \times 2640/2400) = 229 \text{ A}$ .

The ratio of turns in this case is given by

$$k = 2640/2400 = 1.1$$

With the step-up job, and  $k = 1.1$

$$\frac{\text{Rating of Auto-transformer}}{\text{Rating as two-winding transformer}} = \frac{k}{k-1} = \frac{1.1}{0.1}$$

This gives the rating as auto-transformer of 550 kVA.

At unity power-factor, the rated load = 550 kW.

Out of this, the “*inductively*” transferred power

$$\begin{aligned} &= \text{Power handled by the common winding} \\ &= (2400 \text{ V}) \times (20.8 \text{ A}) \times 10^{-3} = 50 \text{ kW.} \\ &= \text{Rated output as a two-winding transformer.} \end{aligned}$$

$$\text{Remaining Power} = 550 \text{ kW} - 50 \text{ kW} = 500 \text{ kW.}$$

This power of 500 kW is “*conductively*” transferred as is clear from the division of currents at the input node. Out of the total current of 229 A from the source, 208 A goes straight to the output. The remaining current of 20.8 A is through the “*common*” and “*inductive*” path, as marked in the Fig. 32.67.

### 32.35. Parallel Operation of Single-phase Transformers

For supplying a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it as shown in Fig. 32.68. It is seen that primary windings are connected to the supply bus bars and secondary windings are connected to the load bus-bars. In connecting two or more than two transformers in parallel, it is essential that their terminals of similar polarities are joined to the same bus-bars as in Fig. 32.68. If this is not done, the two e.m.fs. induced in the secondaries which are paralleled with incorrect polarities, will act together in the local secondary circuit even when supplying no load and will hence produce the equivalent of a dead short-circuit as shown in Fig. 32.69.

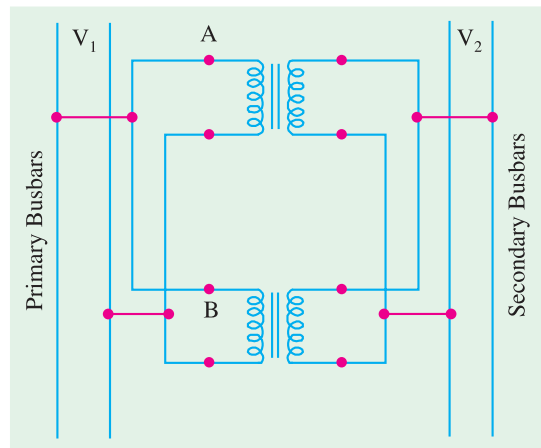


Fig. 32.68

There are certain definite conditions which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their kVA ratings. The conditions are :

1. Primary windings of the transformers should be suitable for the supply system voltage and frequency.
2. The transformers should be properly connected with regard to polarity.
3. The voltage ratings of both primaries and secondaries should be identical. In other words, the transformers should have the same turn ratio *i.e.* transformation ratio.
4. The percentage impedances should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factors.

- 5. With transformers having different kVA ratings, the equivalent impedances should be inversely proportional to the individual kVA rating if circulating currents are to be avoided.

Of these conditions, (1) is easily comprehended ; condition (2) is absolutely essential (otherwise paralleling with incorrect polarities will result in dead short-circuit). There is some latitude possible with conditions (3) and (4). If condition (3) is not exactly satisfied *i.e.* the two transformers have slightly different transformation or voltage ratios, even then parallel operation is possible. But due to inequality of induced e.m.fs. in secondaries, there will be even on no-load, some circulating current between them (and therefore between the primary windings also) when secondary terminals are connected in parallel. When secondaries are loaded, this localized circulating current will tend to produce unequal loading condition. Hence, it may be impossible to take full kVA output from the parallel connected group without one of the transformers becoming over-heated.

If condition (4) is not exactly satisfied *i.e.* impedance triangles are not identical in shape and size, parallel operation will still be possible, but the power factors at which the two transformers operate will be different from the power factor of the common load. Therefore, in this case, the two transformers will not share the load in proportion to their kVA ratings.

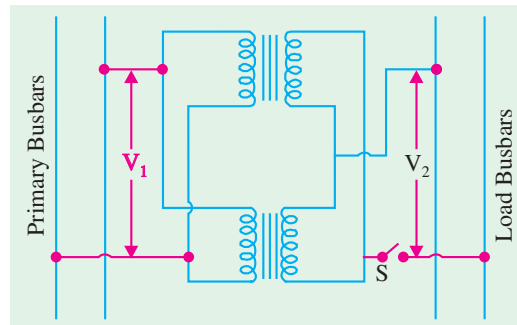


Fig. 32.69

It should be noted that the impedances of two transformers may differ in magnitude and in quality (*i.e.* ratio of equivalent resistance to reactance). It is worthwhile to distinguish between the **percentage** and **numerical** value of an impedance. For example, consider two transformers having ratings in the ratio 1 : 2. It is obvious that to carry double the current, the latter must have half the impedance of the former for the same regulation. For parallel operation, the regulation must be the same, this condition being enforced by the very fact of their being connected in parallel. It means that the currents carried by the two transformers are proportional to their ratings provided their **numerical** impedances are inversely proportional to these ratings and their **percentage** impedances are identical.

If the **quality** of the two percentage impedances is different (*i.e.* ratio of percentage resistance to reactance is different), then this will result in divergence of phase angle of the two currents, with the result that one transformer will be operating with a higher and the other with a lower power factor than that of the combined load.

(a) Case 1. Ideal Case

We will first consider the ideal case of two transformers having the same voltage ratio and having impedance voltage triangles identical in size and shape.

Let  $E$  be the no-load secondary voltage of each transformer and  $V_2$  the terminal voltage ;  $I_A$  and  $I_B$  the currents supplied by them and  $I$  the total current, lagging behind  $V_2$  by an angle  $\phi$  (Fig. 32.70(a))

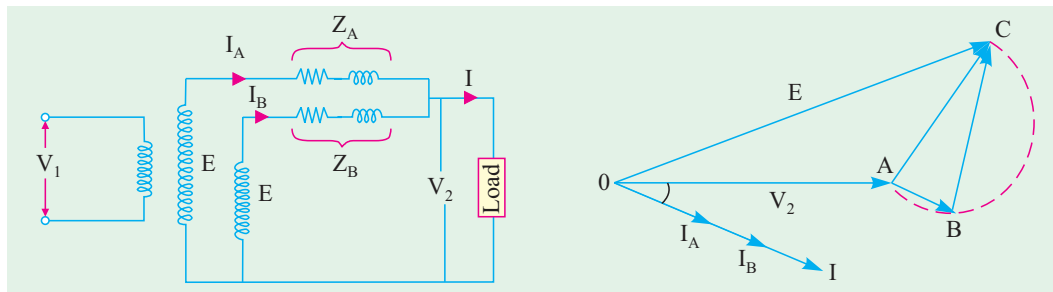


Fig. 32.70 (a)

Fig. 32.70 (b)

In Fig. 32.70 (b) a single triangle  $ABC$  represents the identical impedance voltage triangles of both the transformers. The currents  $I_A$  and  $I_B$  of the individual transformers are in phase with the load current  $I$  and are inversely proportional to the respective impedances. Following relations are obvious.

Also 
$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B; \mathbf{V}_2 = \mathbf{E} - \mathbf{I}_A \mathbf{Z}_A = \mathbf{E} - \mathbf{I}_B \mathbf{Z}_B = \mathbf{E} - \mathbf{I} \mathbf{Z}_{AB}$$

$$\mathbf{I}_A \mathbf{Z}_A = \mathbf{I}_B \mathbf{Z}_B \text{ or } \mathbf{I}_A / \mathbf{I}_B = \mathbf{Z}_B / \mathbf{Z}_A$$

$$\therefore \mathbf{I}_A = \mathbf{I} \mathbf{Z}_B / (\mathbf{Z}_A + \mathbf{Z}_B) \text{ and } \mathbf{I}_B = \mathbf{I} \mathbf{Z}_A / (\mathbf{Z}_A + \mathbf{Z}_B)$$

(b) Case 2. Equal Voltage Ratios

Let us assume that no-load voltages of both secondaries is the same i.e.  $E_A = E_B = E$ , and that the two voltages are coincident i.e. there is no phase difference between  $E_A$  and  $E_B$ , which would be true if the magnetising currents of the two transformers are not much different from each other. Under these conditions, both primaries and secondaries of the two transformers can be connected in parallel and there will circulate no current between them on *on-load*.

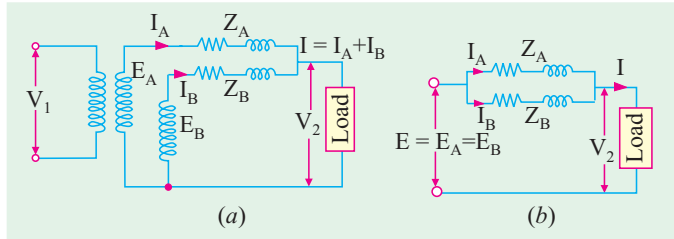


Fig. 32.71

However, if we neglect the magnetising admittances, the two transformers can be connected as shown by their equivalent circuits in Fig. 32.71. The vector diagram is shown in Fig. 32.72.

From Fig. 32.71 (a) or (b) it is seen that it represents two impedances in parallel. Considering all values *consistently with reference to secondaries*, let

- $Z_A, Z_B$  = impedances of the transformers
- $I_A, I_B$  = their respective currents
- $V_2$  = common terminal voltage
- $I$  = combined current

It is seen that  $I_A Z_A = I_B Z_B = I Z_{AB}$  ... (i)  
 where  $Z_{AB}$  is the combined impedance of  $Z_A$  and  $Z_B$  in parallel.

$1/Z_{AB} = 1/Z_A + 1/Z_B$

Hence  $Z_{AB} = Z_A Z_B / (Z_A + Z_B)$  ... (ii)

From equation (i), we get

$Z_A = I Z_{AB} Z_A = I Z_B / (Z_A + Z_B)$  and  $I_B = I Z_{AB} / Z_B = I Z_A / (Z_A + Z_B)$

Multiplying both sides by common terminal voltage  $V_2$ , we have

$V_2 I_A = V_2 I \frac{Z_B}{Z_A + Z_B}$ ; Similarly  $V_2 I_B = V_2 I \frac{Z_A}{Z_A + Z_B}$

Let  $V_2 I \times 10^{-3} = S$  - the combined load kVA. Then, the kVA carried by each transformer is

$S_A = S \frac{Z_B}{Z_A + Z_B} = S \frac{1}{1 + Z_A / Z_B}$  and  $S_B = S \frac{Z_A}{Z_A + Z_B} = S \frac{1}{1 + Z_B / Z_A}$  ... (iii)

Hence,  $S_A$  and  $S_B$  are obtained in magnitude as well as in phase from the above vectorial expressions.

The above problem may be solved graphically, although somewhat more laboriously. As shown in Fig. 32.72, drawn  $I_A$  and  $I_B$  with an angular difference of  $(\phi_A - \phi_B)$  and magnitude (according to some suitable scale) inversely proportional to the respective impedances. Vector sum of  $I_A$  and  $I_B$  gives total combined current  $I$ . The phase angle and magnitude of  $I$  will be known from the conditions of loading, so that angle  $\phi$

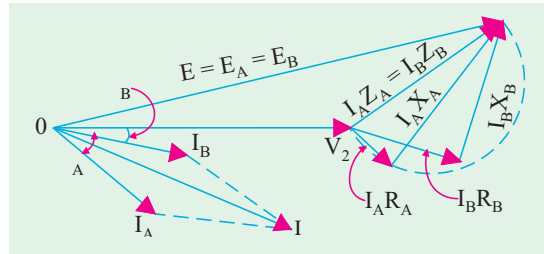


Fig. 32.72

between  $V_2$  and  $I$  will be known. Inserting this, the transformer currents  $I_A$  and  $I_B$  become known in magnitude and phase with respect to  $V_2$ .

**Note. (a)** In equation (iii) above, it is not necessary to use the ohmic values of resistances and reactances, because only impedance ratios are required.

**(b)** The two percentage impedances must be adjusted to the same kVA in the case of transformers of different rating as in Ex. 32.101.

**(c)** From equation (iii) above, it is seen that if two transformers having the same rating and the same transformation ratio are to share the load equally, then their impedances should be equal *i.e.* equal resistances and reactances and not numerical equality of impedances. In general, for transformers of different ratings but same transformation ratio, their equivalent impedances must be inversely proportional to their ratings if each transformer is to assume a load in proportion to its rating. For example, as said earlier, a transformer operating in parallel with another of twice the rating, must have an impedance twice that of the large transformer in order that the load may be properly shared between them.

**Example 32.96.** Two 1-phase transformers with equal turns have impedances of  $(0.5 + j3)$  ohm and  $(0.6 + j10)$  ohm with respect to the secondary. If they operate in parallel, determine how they will share a total load of 100 kW at p.f. 0.8 lagging? (Electrical Technology, Madras Univ. 1987)

**Solution.**  $Z_A = 0.5 + j3 = 3.04 \angle 80.6^\circ$      $Z_B = 0.6 + j10 = 10.02 \angle 86.6^\circ$

$$Z_A + Z_B = 1.1 + j13 = 13.05 \angle 85.2^\circ$$

Now, a load of 100 kW at 0.8 p.f. means a kVA of  $100/0.8 = 125$ . Hence,

$$S = 125 \angle -36.9^\circ$$

$$S_A = S \frac{Z_B}{Z_A + Z_B} = \frac{125 \angle -36.9^\circ \times 10.02 \angle 86.6^\circ}{13.05 \angle 85.2^\circ} = 96 \angle -35.5^\circ$$

= a load of  $96 \times \cos 35.5^\circ = 78.2$  kW

$$S_B = S \frac{Z_A}{Z_A + Z_B} = \frac{125 \angle -36.9^\circ \times 3.04 \angle 80.6^\circ}{13.05 \angle 85.2^\circ} = 29.1 \angle -41.5^\circ$$

= a load of  $29.1 \times \cos 41.5^\circ = 21.8$  kW

**Note.** Obviously, transformer A is carrying more than its due share of the common load.

**Example 32.97.** Two single-phase transformers A and B are connected in parallel. They have same kVA ratings but their resistances are respectively 0.005 and 0.01 per unit and their leakage reactances 0.05 and 0.04 per unit. If A is operated on full-load at a p.f. of 0.8 lagging, what will be the load and p.f. of B? (A.C. Machines-I, Jadavpur Univ. 1985)

**Solution.** In general,  $S_A = S \frac{Z_B}{Z_A + Z_B}$  and  $S_B = S \frac{Z_A}{Z_A + Z_B}$

where  $S$  is the total kVA supplied and  $Z_A$  and  $Z_B$  are the percentage impedances of these transformers.

$$\frac{S_B}{S_A} = \frac{Z_A}{Z_B}$$

Now,  $Z_A = 0.005 + j0.05$  per unit ; %  $Z_A = 0.5 + j5$  and %  $Z_B = 1 + j4$

Let  $S_A = S_A \angle -36.87^\circ$

where  $S_A$  represents the rating of transformer A (and also of B).

$$\begin{aligned} S_B &= S_A \angle -36.87^\circ \times \frac{(0.5 + j5)}{(1 + j4)} \\ &= S_A \angle -36.87^\circ \times \frac{20.7}{17} \angle 8.3^\circ = 1.22 S_A \angle -28.57^\circ \end{aligned}$$



It is obvious that transformer *B* is working **22%** over-load and its power factor is

$$\cos 28.57^\circ = \mathbf{0.878 \text{ (lag)}}$$

**Example 32.98.** Two 1-phase transformers *A* and *B* rated at 250 kVA each are operated in parallel on both sides. Percentage impedances for *A* and *B* are  $(1 + j6)$  and  $(1.2 + j4.8)$  respectively. Compute the load shared by each when the total load is 500 kVA at 0.8 p.f. lagging.

(Electrical Machines-II, Indore Univ. 1989)

**Solution.**

$$\frac{Z_A}{Z_A + Z_B} = \frac{1 + j6}{2.2 + j10.8} = 0.55 \angle 2.1^\circ; \quad \frac{Z_B}{Z_A + Z_B} = \frac{1.2 + j4.8}{2.2 + j10.8} = 0.45 \angle -2.5^\circ$$

$$S_A = S \frac{Z_B}{Z_A + Z_B} = 500 \angle -36.9^\circ \times 0.45 \angle -2.5^\circ = \mathbf{225 \angle -39.4^\circ}$$

$$S_B = S \frac{Z_A}{Z_A + Z_B} = 500 \angle -36.9^\circ \times 0.55 \angle 2.1^\circ = \mathbf{275 \angle -34.8^\circ}$$

Obviously, transformer *B* is overloaded to the extent of  $(275 - 250) \times 100/250 = 10\%$ . It carries  $(275/500) \times 100 = 55\%$  of the total load.

**Example 32.99.** Two 100-kW, single-phase transformers are connected in parallel both on the primary and secondary. One transformer has an ohmic drop of 0.5% at full-load and an inductive drop of 8% at full-load current. The other has an ohmic drop of 0.75% and inductive drop of 2%. Show how will they share a load of 180 kW at 0.9 power factor.

(Elect. Machines-I, Calcutta Univ. 1988)

**Solution.** A load of 180 kW at 0.9 p.f. means a kVA of  $180/0.9 = 200$

$$\therefore \text{Load } S = 200 \angle -25.8^\circ$$

$$\begin{aligned} \frac{Z_1}{Z_1 + Z_2} &= \frac{(0.5 + j8)}{(1.25 + j12)} = \frac{(0.5 + j8)(1.25 - j12)}{1.25^2 + 12^2} \\ &= \frac{96.63 + j4}{145.6} = \frac{96.65 \angle 2.4^\circ}{145.6} = 0.664 \angle 2.4^\circ \end{aligned}$$

$$\begin{aligned} \frac{Z_2}{Z_1 + Z_2} &= \frac{(0.75 + j4)(1.25 - j12)}{145.6} \\ &= \frac{48.94 - j4}{145.6} = \frac{49.1 \angle -5^\circ}{145.6} \\ &= 0.337 \angle -5^\circ \end{aligned}$$

$$S_1 = S \frac{Z_2}{Z_1 + Z_2} = 200 \angle -25.8^\circ \times 0.337 \angle -5^\circ = 67.4 \angle -30.8^\circ$$

$$\therefore \text{kW}_1 = 67.4 \times \cos 30.8^\circ = 67.4 \times 0.859 = 57.9 \text{ kW}$$

$$S_2 = 200 \angle -25.8^\circ \times 0.664 \angle 2.4^\circ = 132.8 \angle -23.4^\circ$$

$$\text{kW}_2 = 132.8 \times \cos 23.4^\circ = 132.8 \times 0.915 = 121.5 \text{ kW}$$

**Note.** Second transformer is working 21.5% over-load. Also, it shares 65.7% of the total load.

**Example 32.100.** A load of 200 kW at 0.85 power factor lagging is to be shared by two transformers *A* and *B* having the same ratings and the same transformation ratio. For transformer *A*, the full-load resistive drop is 1% and reactance drop 5% of the normal terminal voltage. For transformer *B* the corresponding values are : 2% and 6%. Calculate the load kVA supplied by each transformer.

**Solution.**  $Z_A = 1 + j5; Z_B = Z + j6$

$$\frac{Z_A}{Z_B} = \frac{1+j5}{2+j6} = 0.8 + j0.1; \quad \frac{Z_B}{Z_A} = \frac{2+j6}{1+j5} = 1.23 - j0.514$$

Load

$$\text{kVA} = \text{kW}/\text{p.f.} = 200/0.85 = 235$$

$$\begin{aligned} \text{kVA} = S &= 235(0.85 - j0.527) \\ &= 200 - j123.8 \quad (\because \cos \phi = 0.85; \sin \phi = 0.527) \end{aligned}$$

$$\begin{aligned} S_A &= S \frac{Z_B}{Z_A + Z_B} = S \frac{1}{1 + (Z_A/Z_B)} \\ &= \frac{200 - j123.8}{1 + (0.8 + j0.1)} = \frac{200 - j123.8}{1.8 + j0.1} = 107.3 - j74.9 \end{aligned}$$

$$S_A = \sqrt{(107.3^2 + 74.9^2)} = 131; \quad \cos \phi_A = 107.3/131 = 0.82 \text{ (lag)}$$

Similarly

$$\begin{aligned} S_B &= S \frac{Z_A}{Z_A + Z_B} = S \frac{1}{1 + (Z_B/Z_A)} \\ &= \frac{200 - j123.8}{1 + (1.23 - j0.514)} = \frac{200 - j123.8}{2.23 - j0.154} = 93 - j49 \end{aligned}$$

$$\therefore S_B = \sqrt{(93^2 + 49^2)} = 105; \quad \cos \phi_B = 93/105 = 0.888 \text{ (lag)}$$

(As a check,  $S = S_A + S_B = 131 + 105 = 236$ . The small error is due to approximations made in calculations.)

**Example 32.101.** Two 2,200/110-V, transformers are operated in parallel to share a load of 125 kVA at 0.8 power factor lagging. Transformers are rated as below :

A : 100 kVA ; 0.9% resistance and 10% reactance

B : 50 kVA ; 1.0% resistance and 5% reactance

Find the load carried by each transformer.

(Elect. Technology, Utkal Univ. 1989)

**Solution.** It should be noted that the percentages given above refer to different ratings. As pointed out in Art. 32.35, these should be adjusted to the same basic kVA, say, 100 kVA.

$$\begin{aligned} \% Z_A &= 0.9 + j10; \quad \% Z_B = (100/50)(1 + j5) = (2 + j10) \\ \frac{Z_A}{Z_A + Z_B} &= \frac{0.9 + j10}{(2.9 + j20)} = \frac{(0.9 + j10)(2.9 - j20)}{2.9^2 + 20^2} \\ &= \frac{(202.6 + j11)}{408.4} = \frac{202.9 \angle 3.1^\circ}{408.4} = 0.4968 \angle 3.1^\circ \\ \frac{Z_B}{Z_A + Z_B} &= \frac{(2 + j10)(2.9 - j20)}{408.4} = \frac{206 - j11}{408.4} \\ &= \frac{206.1 \angle -3.1^\circ}{408.4} = 0.504 \angle -3.1^\circ \end{aligned}$$

$$\text{Also} \quad \cos \phi = 0.8, \phi = \cos^{-1}(0.8) = 36.9^\circ$$

$$S_A = S \frac{Z_B}{Z_A + Z_B} = 125 \angle -36.9^\circ \times 0.504 \angle -3.1^\circ = 63 \angle -40^\circ$$

$$S_B = S \frac{Z_A}{Z_A + Z_B} = 125 \angle -36.9^\circ \times 0.4968 \angle 3.1^\circ = 62.1 \angle -33.8^\circ$$

**Example 32.102.** A 500-kVA transformer with 1% resistance and 5% reactance is connected in parallel with a 250-kVA transformer with 1.5% resistance and 4% reactance. The secondary voltage of each transformer is 400 V on no-load. Find how they share a load of 750-kVA at a p.f. of 0.8 lagging. **(Electrical Machinery-I, Madras Univ. 1987)**

**Solution.** It may be noted that percentage drops given above refer to different ratings. These should be adjusted to the same basic kVA i.e. 500 kVA.

$$\begin{aligned} \% Z_A &= 1 + j5 = 5.1 \angle 78.7^\circ; \% Z_B = \left(\frac{500}{250}\right)(1.5 + j4) = 3 + j8 = 8.55 \angle 69.4^\circ \\ \% (Z_A + Z_B) &= 4 + j13 = 13.6 \angle 72.9^\circ; S = 750 \angle -36.9^\circ \\ S_A &= S \frac{Z_B}{Z_A + Z_B} = \frac{750 \angle -36.9^\circ \times 8.55 \angle 69.4^\circ}{13.6 \angle 72.9^\circ} = 470 \angle -40.4^\circ \\ &= 470 \text{ kVA at p.f. of } 0.762 \text{ lagging} \\ S_B &= S \frac{Z_A}{Z_A + Z_B} = \frac{750 \angle -36.9^\circ \times 5.1 \angle 78.7^\circ}{13.6 \angle 72.9^\circ} = 280 \angle -31.1^\circ \\ &= 280 \text{ kVA at p.f. } 0.856 \text{ lagging} \end{aligned}$$

**Note.** The above solution has been attempted vectorially, but in practice, the angle between  $I_A$  and  $I_B$  is so small that if instead of using vectorial expressions, arithmetic expressions were used, the answer would not be much different. In most cases, calculations by both vectorial and arithmetical methods generally yield results that do not differ sufficiently to warrant the more involved procedure by the vector solution. The above example will now be attempted arithmetically :

$$\begin{aligned} Z_A &= 5.1 \Omega, Z_B = 8.55 \Omega; I_A/I_B = Z_B/Z_A = 8.55/5.1 = 1.677 \therefore I_A = 1.677 I_B \\ \text{Total current} &= 750,000/400 = 1875 \text{ A} \\ I &= I_A + I_B; 1875 = 1.677 I_B + I_B = 2.677 I_B \therefore I_B = 1875/2.677 \\ \therefore S_B &= 400 \times 1875/2.677 \times 100 = 280 \text{ kVA} \\ I_A &= 1.677 \times 1875/2.677 \\ S_A &= 400 \times 1.677 \times 1875/2.677 \times 1000 = 470 \text{ kVA} \end{aligned}$$

**Example 32.103.** Two single-phase transformers A and B of equal voltage ratio are running in parallel and supply a load of 1000 A at 0.8 p.f. lag. The equivalent impedances of the two transformers are  $(2 + j3)$  and  $(2.5 + j5)$  ohms respectively. Calculate the current supplied by each transformer and the ratio of the kW output of the two transformers. **(Electrical Machines-I, Bombay Univ. 1986)**

**Solution.**  $Z_A = (2 + j3), Z_B = (2.5 + j5)$

Now,  $\frac{I_A}{I_B} = \frac{Z_B}{Z_A} = \frac{2.5 + j5}{2 + j3} = (1.54 + j0.2); I_A = I_B (1.54 + j0.2)$

Taking secondary terminal voltage as reference vector, we get

$$\begin{aligned} I &= 1000(0.8 - j0.6) = 800 - j600 = 200(4 - j3) \\ \text{Also, } I &= I_A + I_B = I_B(1.54 + j0.2) + I_B(2.54 + j0.2) \\ \therefore 200(4 - j3) &= I_B(2.54 + j0.2); I_B = 294.6 - j259.5 = 392.6 \angle -41.37^\circ \\ I_A = I_B(1.54 + j0.2) &= (294.6 - j259.5)(1.54 + j0.2) = 505.6 - j340.7 \\ &= 609.7 \angle -33.95^\circ \end{aligned}$$

The ratio of the kW output is given by the ratio of the in-phase components of the two currents.

$$\frac{\text{output of } A}{\text{output of } B} = \frac{505.6}{294.6} = \frac{1.7}{1}$$

**Note.** Arithmetic solution mentioned above could also be attempted.

**Example 32.104.** Two transformers A and B, both of no-load ratio 1,000/500-V are connected in parallel and supplied at 1,000 V. A is rated at 100 kVA, its total resistance and reactance being 1% and 5% respectively, B is rated at 250 kVA, with 2% resistance and 2% reactance. Determine the load on each transformer and the secondary voltage when a total load of 300 kVA at 0.8 power factor lagging is supplied.

**Solution.** Let the percentage impedances be adjusted to the common basic kVA of 100.

Then  $\%Z_A = (1 + j5)$ ;  $\%Z_B = (100/250)(2 + j2) = (0.8 + j0.8)$

$$\frac{Z_A}{Z_A + Z_B} = \frac{1 + j5}{(1.8 + j5.8)} = 0.839 \angle 5.9^\circ$$

$$\frac{Z_B}{Z_A + Z_B} = \frac{0.8 + j0.8}{(1.8 + j5.8)} = 0.1865 \angle -27.6^\circ$$

Now,  $S = 300 \angle -36.9^\circ = 240 - j180$

$$S_A = S \cdot \frac{Z_B}{Z_A + Z_B} = 300 \angle -36.9^\circ \times 0.1865 \angle -27.6^\circ = 55.95 \angle -64.5^\circ$$

$$S_B = S \cdot \frac{Z_A}{Z_A + Z_B} = 300 \angle -36.9^\circ \times 0.839 \angle 5.9^\circ = 251.7 \angle -31^\circ$$

Since  $Z_A$  and  $Z_B$  are in parallel, their combined impedance on 100 kVA basis is

$$Z_{AB} = \frac{Z_A Z_B}{Z_A + Z_B} = \frac{(1 + j5)(0.8 + j0.8)}{(1.8 + j5.8)} = 0.6 + j0.738$$

Percentage drop over  $Z_{AB}$  is  $= (0.6 \times 240/100) + (0.738 \times 180/100) = 1.44 + 1.328 = 2.768\%$

$$\therefore V_2 = \left( 500 - \frac{500 \times 2.768}{100} \right) = \mathbf{486.12 \text{ V}}$$

**Example 32.105.** Two 1- $\phi$  transformers are connected in parallel at no-load. One has a turn ratio of 5,000/440 and a rating of 200 kVA, the other has a ratio of 5,000/480 and a rating of 350 kVA. The leakage reactance of each is 3.5%.

What is the no-load circulation current expressed as a percentage of the nominal current of the 200 kVA transformer.

**Solution.** The normal currents are

$$200 \times 10^3 / 440 = 455 \text{ A and } 350 \times 10^3 / 480 = 730 \text{ A}$$

Reactances seen from the secondary side are

$$\frac{3.5}{100} \times \frac{440}{455} = 0.034 \Omega, \quad \frac{3.5}{100} \times \frac{480}{730} = 0.023 \Omega$$

The difference of induced voltage is 40 V. The circulating current is

$$I_C = 40 / 0.057 = 704 \text{ A} = \mathbf{1.55 \text{ times}}$$
 the normal current of 200 kVA unit.

### Tutorial Problems 32.6

- Two single-phase transformers A and B of equal voltage ratio are running in parallel and supplying a load requiring 500 A at 0.8 power factor lagging at a terminal voltage of 400 V. The equivalent impedances of the transformers, as referred to secondary windings, are  $(2 + j3)$  and  $(2.5 + j5)$  ohm. Calculate the current supplied by each transformer.

**(Note.** The student is advised to try by arithmetic method also).  $[I_A = 304 \text{ A} ; I_B = 197 \text{ A}]$

- Two single-phase transformers A and B are operating in parallel and supplying a common load of 1000 kVA at 0.8 p.f. lagging. The data regarding the transformers is as follows :

Transformer	Rating	%Resistance	% Reactance
A	750 kVA	3	5
B	500 kVA	2	4

Determine the loading of each transformer.  $[S_A = 535 \angle -34.7^\circ ; S_B = 465 \angle -39.3^\circ]$

3. Two transformers *A* and *B* give the following test results. With the low-tension side short-circuited, *A* takes a current of 10 A at 200 V, the power input being 1000 W. Similarly, *B* takes 30 A at 200 V ; the power input being 1,500 W. On open circuit, both transformers give a secondary voltage of 2200 when 11,000 volts are applied to the primary terminals. These transformers are connected in parallel on both high tension and low tension sides. Calculate the current and power in each transformer when supplying a load of 200 A at 0.8 power factor lagging. The no-load currents may be neglected.

(Hint : Calculate  $Z_A$  and  $Z_B$  from S.C. test data)

$[I_A = 50.5 \text{ A}, P_A = 100 \text{ kW} ; I_B = 151 \text{ A}, P_B = 252 \text{ kW}]$  (London University)

4. Two 6600/250-V transformers have the following short-circuit characteristics : Applied voltage 200 V, current 30 A, power input 1,200 W for one of the transformers ; the corresponding data for the other transformer being 120 V, 20 A and 1,500 W. All values are measured on the H.V. side with the L.V. terminals short circuited. Find the approximate current and the power factor of each transformer when working in parallel with each other on the high and low voltage sides and taking a total load of 150 kW at a p.f. of 0.8 lagging from the high voltage bus-bars.

$[I_A = 13.8 \text{ A}, \cos \phi_A = 0.63 ; I_B = 15.35 \text{ A}, \cos \phi_B = 0.91]$  (Electrical Engg-IV, Baroda Univ. 1978)

5. Two 11,000/2,200-V, 1-phase transformers are connected in parallel to supply a total load of 200 at 0.8 p.f. lagging at 2,200 V. One transformer has an equivalent resistance of 0.4  $\Omega$  and equivalent reactance of 0.8  $\Omega$  referred to the low-voltage side. The other has equivalent resistance of 0.1  $\Omega$  and a reactance of 0.3  $\Omega$ . Determine the current and power supplied by each transformer.

$[52 \text{ A} ; 148 \text{ A} ; 99 \text{ A} ; 252 \text{ kW}]$

6. A 2,000-kVA transformer (*A*) is connected in parallel with a 4,000-kVA transformer (*B*) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage volt drops in the windings at the rated loads are as follows :

Transformer *A* : resistance 2% ; reactance 8%

Transformer *B* : resistance 1.6% ; reactance 3%

$[A : 860 \text{ kVA}, 0.661 \text{ lag} ; B : 4170 \text{ kVA}, 0.824 \text{ lag}]$  (A.C. Machines-I, Jadavpur Univ. 1979)

7. Two single-phase transformers work in parallel on a load of 750 A at 0.8 p.f. lagging. Determine secondary voltage and the output and power factor of each transformer. Test data are :

Open circuit : 11,00/13,300 V for each transformer

Short circuit : with h.v. winding short-circuit

Transformer *A* : secondary input 200 V, 400 A, 15 kW

Transformer *B* : secondary input 100 V, 400 A, 20 kW

$[3,190 \text{ VA} ; 80 \text{ kVA}, 0.65 \text{ lag} ; B : 1,615 \text{ kVA} ; 0.86 \text{ lag}]$

(c) Case 3. Unequal Voltage Ratios

In this case, the voltage ratios (or transformation ratios) of the two transformers are different. It means that their no-load secondary voltages are unequal. Such cases can be more easily handled by phasor algebra than graphically.

Let  $E_A, E_B$  = no-load secondary e.m.f.s of the two transformers.

$Z_L$  = load impedance across the secondary.

The equivalent circuit and vector diagram are also shown in Fig. 32.73 and 32.74.

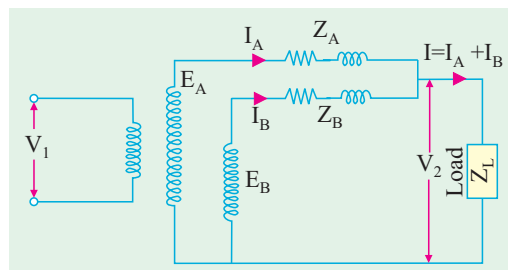


Fig. 32.73

It is seen that even when secondaries are on no-load, there will be some cross-current in them because of inequality in their induced e.m.fs. This circulating current  $I_C$  is given by

$$I_C = (E_A - E_B)/(Z_A + Z_B) \quad \dots(i)$$

As the induced e.m.fs. of the two transformers are equal to the total drops in their respective circuits.

$$\therefore Z_A = I_A Z_A + V_2 ; E_B = I_B Z_B + V_2$$

Now,  $V_2 = I Z_L = (I_A + I_B) Z_L$

where  $Z_L =$  load impedance

$$E_A = I_A Z_A + (I_A + I_B) Z_L \quad \dots(ii)$$

$$E_B = I_B Z_B + (I_A + I_B) Z_L \quad \dots(iii)$$

$$\therefore E_A - E_B = I_A Z_A - I_B Z_B \quad \dots(iv)$$

$$I_A = [(E_A - E_B) + I_B Z_B]/Z_A$$

Substituting this value of  $I_A$  in equation (iii), we get

$$\begin{aligned} E_B &= I_B Z_B + [(E_A - E_B) + I_B Z_B]/Z_A + I_B Z_L \\ I_B &= [E_B Z_A - (E_A - E_B) Z_L]/[Z_A Z_B + Z_L(Z_A + Z_B)] \end{aligned} \quad \dots(v)$$

From the symmetry of the expression, we get

$$I_A = [E_A Z_B + (E_A - E_B) Z_L]/[Z_A Z_B + Z_L(Z_A + Z_B)] \quad \dots(vi)$$

Also,  $I = I_A + I_B = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)}$

By multiplying the numerator and denominator of this equation by  $1/Z_A Z_B$  and the result by  $Z_L$  we get

$$V_2 = I Z_L = \frac{E_A/Z_A + E_B/Z_B}{1/Z_A + 1/Z_B + 1/Z_L}$$

The two equations (v) and (vi) then give the values of secondary currents. The primary currents may be obtained by the division of transformation ratio *i.e.*  $K$  and by addition (if not negligible) of the no-load current. Usually,  $E_A$  and  $E_B$  have the same phase (as assumed above) but there may be some phase difference between the two due to some difference of internal connection in parallel of a star/star and a star/delta 3-phase transformers.

If  $Z_A$  and  $Z_B$  are small as compared to  $Z_L$  *i.e.* when the transformers are not operated near short-circuit conditions, then equations for  $I_A$  and  $I_B$  can be put in a simpler and more easily under-standable form. Neglecting  $Z_A Z_B$  in comparison with the expression  $Z_L(Z_A + Z_B)$ , we have

$$I_A = \frac{E_A Z_B}{Z_L(Z_A + Z_B)} + \frac{E_A - E_B}{Z_A + Z_B} \quad \dots(vii)$$

$$I_B = \frac{E_B Z_A}{Z_L(Z_A + Z_B)} - \frac{E_A - E_B}{Z_A + Z_B} \quad \dots(viii)$$

The physical interpretation of the second term in equations (vii) and (viii) is that it represents the cross-current between the secondaries. The first term shows how the actual load current divides between the loads. The value of current circulating in transformer secondaries (even when there is no-load) is given by\*  $I_C = (E_A - E_B)/(Z_A + Z_B)$  assuming that  $E_A > E_B$ . It lags behind  $E_A$  by an angle  $\alpha$  given by  $\tan \alpha = (X_A + X_B)/(R_A + R_B)$ . If  $E_A + E_B$  the ratios of the currents are inversely as the impedances (numerical values).

\* Under load conditions, the circulating current is

$$I_C = \frac{E_A - E_B}{Z_A + Z_B + Z_A Z_B/Z_L}$$

If  $Z_L = \infty$  *i.e.* on open-circuit, the expression reduces to that given above.

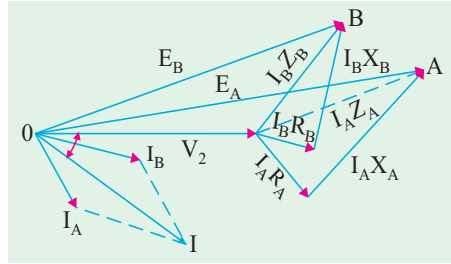


Fig. 32.74

If in Eq. (iv) we substitute  $\mathbf{I}_B = \mathbf{I} - \mathbf{I}_A$  and simplify, we get

$$\mathbf{I}_A = \frac{\mathbf{I}Z_B}{Z_A + Z_B} + \frac{\mathbf{E}_A - \mathbf{E}_B}{Z_A + Z_B} \quad \dots(ix)$$

Similarly, if we substitute  $\mathbf{I}_A = \mathbf{I} - \mathbf{I}_B$  and simplify, then

$$\mathbf{I}_B = \frac{\mathbf{I}Z_A}{Z_A + Z_B} + \frac{\mathbf{E}_A - \mathbf{E}_B}{Z_A + Z_B} \quad \dots(x)$$

In a similar manner, value of terminal voltage  $V_2$  is given by

$$V_2 = \mathbf{I}Z_L = \frac{\mathbf{E}_A Z_B + \mathbf{E}_B Z_A - \mathbf{I}Z_A Z_B}{Z_A + Z_B} \quad \dots(xi)$$

These expressions give the values of transformer currents and terminal voltage in terms of the load current. The value of  $V_2$  may also be found as under :

As seen from Fig. 30.70.

$$\begin{aligned} \mathbf{I}_A &= (\mathbf{E}_A - V_2)/Z_A = (\mathbf{E}_A - V_2)\mathbf{Y}_A; \mathbf{I}_B = (\mathbf{E}_B - V_2)\mathbf{Y}_B \\ \mathbf{I} &= V_2\mathbf{Y}_L = \mathbf{I}_A + \mathbf{I}_B \text{ or } V_2\mathbf{Y}_L = (\mathbf{E}_A - V_2)\mathbf{Y}_A + (\mathbf{E}_B - V_2)\mathbf{Y}_B \end{aligned}$$

$$\therefore V_2(\mathbf{Y}_L + \mathbf{Y}_A + \mathbf{Y}_B) = \mathbf{E}_A\mathbf{Y}_A + \mathbf{E}_B\mathbf{Y}_B \text{ or } V_2 = \frac{\mathbf{E}_A\mathbf{Y}_A + \mathbf{E}_B\mathbf{Y}_B}{\mathbf{Y}_L + \mathbf{Y}_A + \mathbf{Y}_B} \quad \dots(xii)$$

Eq. (xi) gives  $V_2$  in terms of load current. But if only load kVA is given, the problem becomes more complicated and involves the solution of a quadratic equation in  $V_2$ .

Now,  $\mathbf{S} = V_2\mathbf{I}$ . When we substitute this value of  $\mathbf{I}$  in Eq. (xi), we get

$$V_2 = \frac{\mathbf{E}_A Z_B + \mathbf{E}_B Z_A - \mathbf{S}Z_A Z_B / V_2}{(Z_A + Z_B)}$$

$$\text{or } V_2^2(Z_A + Z_B) - V(\mathbf{E}_A Z_B + \mathbf{E}_B Z_A) + \mathbf{S}Z_A Z_B = 0$$

When  $V_2$  becomes known, then  $\mathbf{I}_A$  and  $\mathbf{I}_B$  may be directly found from

$$V_2 = \mathbf{E}_A - \mathbf{I}_A Z_A \text{ and } V_2 = \mathbf{E}_B - \mathbf{I}_B Z_B.$$

**Note.** In the case considered above, it is found more convenient to work with numerical values of impedances instead of % values.

**Example 32.106.** Two transformers A and B are joined in parallel to the same load. Determine the current delivered by each transformer having given : open-circuit e.m.f. 6600 V for A and 6,400 V for B. Equivalent leakage impedance in terms of the secondary =  $0.3 + j3$  for A and  $0.2 + j1$  for B. The load impedance is  $8 + j6$ . (Elect. Machines-I, Indore Univ. 1987)

**Solution.** 
$$\mathbf{I}_A = \frac{\mathbf{E}_A Z_B + (\mathbf{E}_A - \mathbf{E}_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

Here  $\mathbf{E}_A = 6,600 \text{ V}$  ;  $\mathbf{E}_B = 6,400 \text{ V}$  ;  $Z_L = 8 + j6$  ;  $Z_A = 0.3 + j3$  ;  $Z_B = 0.2 + j1$

$$\mathbf{I}_A = \frac{6600(0.2 + j1) + (6600 - 6400)(8 + j6)}{(0.3 + j3)(0.2 + j1) + (8 + j6)(0.3 + j3 + 0.2 + j1)}$$

$$117 - j156 = 195 \text{ A in magnitude}$$

Similarly,

$$\begin{aligned} \mathbf{I}_B &= \frac{\mathbf{E}_B Z_A - (\mathbf{E}_A - \mathbf{E}_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)} \\ &= \frac{6400(0.3 + j3) - (6600 - 6400)(8 + j6)}{(0.3 + j3) + (0.2 + j1)(8 + j6) + (0.5 + j4)} \\ &= 349 - j231 = 421 \text{ A (in magnitude)} \end{aligned}$$

**Example 32.107.** Two 1- $\phi$  transformers, one of 100 kVA and the other of 50 kVA are connected in parallel to the same bus-bars on the primary side, their no-load secondary voltages being 1000 V and 950 V respectively. Their resistances are 2.0 and 2.5 per cent respectively and their reactances 8 and 6 per cent respectively. Calculate no-load circulating current in the secondaries.

(Adv. Elect Machines, A.M.I.E. Sec. B, 1991)

**Solution.** The circuit connections are shown in Fig. 32.75.

**1st transformer**

Normal secondary current =  $100,000/1000 = 100$  A

$$R_A = \frac{1000 \times 2.0}{100 \times 100} = 0.2 \Omega ; X_A = \frac{1000 \times 8}{100 \times 100} = 0.8 \Omega$$

**2nd Transformer**

Normal secondary current =  $50,000/950 = 52.63$  A

$$R_B = 950 \times 2.5/100 \times 52.63 = 0.45 \Omega$$

$$X_B = 850 \times 6/100 \times 52.63 = 1.08 \Omega$$

Combined impedance of the two secondaries is

$$Z = \sqrt{(R_A + R_B)^2 + (X_A + X_B)^2}$$

$$= \sqrt{0.65^2 + 1.88^2} = 1.99 \Omega$$

$$\therefore I_c = (1000 - 950)/1.99 = 25.1 \text{ A} ; \alpha = \tan^{-1}(1.88/0.65) = 71^\circ$$

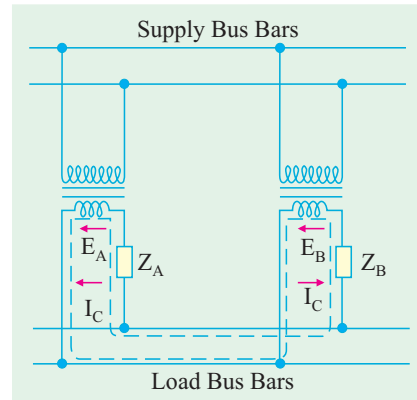


Fig. 32.75

**Example 32.108.** Two single-phase transformers, one of 1000-kVA and the other of 500-kVA are connected in parallel to the same bus-bars on the primary side ; their no-load secondary voltages being 500 V and 510 V respectively. The impedance voltage of the first transformer is 3% and that of the second 5%. Assuming that ratio of resistance to reactance is the same and equal to 0.4 in each. What will be the cross current when the secondaries are connected in parallel ?

(Electrical Machines-I, Madras Univ. 1985)

**Solution.** Let us first determine the ohmic value of the two impedances. Also, let the secondary voltage be 480 V\*.

Full-load  $I_A = 1000 \times 1000/480 = 2083$  A ; F.L.  $I_B = 500 \times 1000/480 = 1042$  A

$$Z_A = \frac{\%Z_A \times E_A}{100 \times I_A} = \frac{9 \times 500}{100 \times 2083} = 0.0072 \Omega$$

$$Z_B = \frac{5 \times 510}{100 \times 1042} = 0.0245 \Omega$$

$$I_C = \frac{E_B - E_A}{Z_A + Z_B} = \frac{510 - 500}{(0.0072 + 0.0245)} = 315.4 \text{ A}$$

**Note.** Since the value of X/R is the same for the two transformers, there is no phase difference between  $E_A$  and  $E_B$ .

**Example 32.109.** Two transformers A and B of ratings 500 kVA and 250 kVA are supplying a load kVA of 750 at 0.8 power factor lagging. Their open-circuit voltages are 405V and 415 V respectively. Transformer A has 1% resistance and 5% reactance and transformer B has 1.5% resistance and 4% reactance. Find (a) cross-current in the secondaries on no-load and (b) the load shared by each transformer.

\* Though it is chosen arbitrarily, its value must be less than either of the two no-load e.m.fs.



**Solution.** As said earlier, it is more convenient to work with ohmic impedances and for that purpose, we will convert percentage value into numerical values by assuming 400 volt as the terminal voltage (this value is arbitrary but this assumption will not introduce appreciable error).

$$\text{Now } I_A R_A = 1\% \text{ of } 400 \quad \therefore R_A = \frac{1}{100} \times \frac{400}{1250} = 0.0032 \Omega$$

$$\text{where } I_A = 500,000/400 = 1250 \text{ A}$$

$$I_A X_A = 5\% \text{ of } 400; X_A = \frac{5}{100} \times \frac{400}{1250} = 0.016 \Omega \text{ (i.e. } X_A = 5R_A)$$

In a similar way, we can find  $R_B$  and  $X_B$ ;  $R_B = 0.0096 \Omega$ ;  $X_B = 0.0256 \Omega$

$$\therefore \mathbf{Z}_A = 0.0032 + j0.016 = 0.0163 \angle 78.5^\circ; \mathbf{Z}_B = 0.0096 + j0.0256 = 0.0275 \angle 69.4^\circ$$

$$\mathbf{Z}_A + \mathbf{Z}_B = 0.0128 + j0.0416 = 0.0436 \angle 72.9^\circ$$

Next step is to calculate load impedance. Let  $\mathbf{Z}_L$  be the load impedance and  $V_2$  the terminal voltage which has been assumed as 400 V.

$$\therefore (V_2^2 / \mathbf{Z}_L) = 750 \angle -36.9^\circ$$

$$\therefore \mathbf{Z}_L = 400^2 \times 10^{-3} / 750 \angle -36.9^\circ = 0.214 \angle 36.9^\circ = (0.171 + j0.128) \Omega$$

$$(a) \quad \mathbf{I}_C = \frac{\mathbf{E}_A - \mathbf{E}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{(405 - 415)}{0.0436 \angle 72.9^\circ} = -230 \angle -72.9^\circ$$

$$(b) \quad \mathbf{I}_A = \frac{405 \times 0.0275 \angle 69.4^\circ + (405 - 415) \times 0.214 \angle 36.9^\circ}{0.0163 \angle 78.5^\circ \times 0.0275 \angle 69.4^\circ + 0.214 \angle 36.9^\circ \times 0.0436 \angle 72.9^\circ} \\ = 970 \angle -35^\circ$$

$$\text{Similarly, } \mathbf{I}_B = \frac{415 \times 0.0163 \angle 78.5^\circ - (405 - 415) \times 0.214 \angle 36.9^\circ}{0.0163 \angle 78.5^\circ \times 0.0275 \angle 69.4^\circ + 0.214 \angle 36.9^\circ \times 0.0436 \angle 72.9^\circ}$$

$$\therefore \mathbf{S}_A = 400 \times 970 \times 10^{-3} \angle -35^\circ = 388 \angle -35^\circ \text{ kVA}; \cos \phi_A = \cos 35^\circ = 0.82 \text{ (lag)}$$

$$\mathbf{S}_B = 400 \times 875 \times 10^{-3} \angle 42.6^\circ = 350 \angle -42.6^\circ \text{ kVA}$$

$$\cos \phi_B = \cos 42.6^\circ = 0.736 \text{ (lag)}$$

**Example 32.110.** Two transformers A and B are connected in parallel to a load of  $(2 + j1.5)$  ohms. Their impedances in secondary terms are  $\mathbf{Z}_A = (0.15 + j0.5)$  ohm and  $\mathbf{Z}_B = (0.1 + j0.6)$  ohm. Their no-load terminal voltages are  $E_A = 207 \angle 0^\circ$  volt and  $E_B = 205 \angle 0^\circ$  volt. Find the power output and power factor of each transformer. **(Elect. Machines-I, Punjab Univ. 1991)**

**Solution.** Using the equations derived in Art. 30.34 (c), we have

$$\mathbf{I}_A = \frac{\mathbf{E}_A \mathbf{Z}_B + (\mathbf{E}_A - \mathbf{E}_B) \mathbf{Z}_L}{\mathbf{Z}_A \mathbf{Z}_B + \mathbf{Z}_L (\mathbf{Z}_A + \mathbf{Z}_B)}$$

$$\mathbf{Z}_A = (0.15 + j0.5) \Omega; \mathbf{Z}_B = (0.1 + j0.6) \Omega; \mathbf{Z}_L = (2 + j1.5) = 2.5 \angle 36.9^\circ$$

$$\therefore \mathbf{I}_A = \frac{207(0.1 + j0.6) + (207 - 205)(2 + j1.5)}{(0.15 + j0.5)(0.1 + j0.6) + (2 + j1.5)(0.25 + j1.1)} \\ = \frac{24.7 + j127.2}{-1.435 + j2.715} = \frac{129.7 \angle 79^\circ}{3.07 \angle 117.9^\circ} = 42.26 \angle -38.9^\circ \text{ A} = (32.89 - j26.55) \text{ A}$$

$$\mathbf{I}_B = \frac{\mathbf{E}_B \mathbf{Z}_A - (\mathbf{E}_A - \mathbf{E}_B) \mathbf{Z}_L}{\mathbf{Z}_A \mathbf{Z}_B + \mathbf{Z}_L (\mathbf{Z}_A + \mathbf{Z}_B)} = \frac{205(0.15 + j0.5) - 2(2 + j1.5)}{-1.435 + j2.715} = \frac{103 \angle 75^\circ}{3.07 \angle 117.9^\circ} \\ = 33.56 \angle -42.9^\circ = (24.58 - j22.84) \text{ A}$$

$$\text{Now } \mathbf{V}_2' = \mathbf{I} \mathbf{Z}_L = (\mathbf{I}_A + \mathbf{I}_B) \mathbf{Z}_L \\ = (57.47 - j49.39)(2 + j1.5) = 189 - j12.58 = 189.4 \angle -3.9^\circ$$

p.f. angle of transformer A =  $-3.9^\circ - (-38.9^\circ) = 35^\circ$

$\therefore$  p.f. of A =  $\cos 35^\circ = 0.818$  (lag) ; p.f. of B =  $\cos [-3.9^\circ - (-42.9^\circ)] = 0.776$  (lag)

Power output of transformer A is  $P_A = 189.4 \times 42.26 \times 0.818 = 6,548 \text{ W}$

Similarly,  $P_B = 189.4 \times 33.56 \times 0.776 = 4,900 \text{ W}$

**Example 32.111.** Two transformers have the following particulars :

	Transformer A	Transformer B
Rated current	200 A	600 A
Per unit resistance 0.02	0.025	
Per unit reactance	0.05	0.06
No-load e.m.f.	245 V	240 V

Calculate the terminal voltage when they are connected in parallel and supply a load impedance of  $(0.25 + j 0.1) \Omega$ . (Elect. Machines-I, Sd. Patel Univ. 1981)

**Solution.** Impedance in ohms =  $Z_{pu} \times \text{N.L. e.m.f./full-load current}$

$$Z_A = (245/200)(0.02 + j 0.05) = 0.0245 + j 0.0613 \Omega = 0.066 \angle 68.2^\circ$$

$$Z_B = (240/600)(0.025 + j 0.06) = 0.01 + j 0.024 \Omega = 0.026 \angle 67.3^\circ$$

$$Z = (0.25 + j 0.1) = 0.269 \angle 21.8^\circ ; Z_A + Z_B = 0.0345 + j 0.0853 = 0.092 \angle 68^\circ$$

$$Z_L(Z_A + Z_B) = 0.269 \times 0.092 \angle 89.8^\circ = 0.0247 \angle 89.8^\circ = (0 + j 0.0247)$$

$$Z_A Z_B = 0.066 \times 0.026 \angle 135.5^\circ = (-0.001225 + j 0.001201)$$

$$\therefore Z_A Z_B + Z_L(Z_A + Z_B) = (-0.00125 + j 0.259) = 0.0259 \angle 92.7^\circ$$

Let us take  $E_A$  as reference quantity.

Also  $E_B$  is in phase with  $E_A$  because transformers are in parallel on both sides.

$$E_A Z_B = 245(0.01 + j 0.0245) = 2.45 + j 5.87$$

$$E_B Z_A = 240(0.0245 + j 0.0613) = 5.88 + j 14.7$$

$$E_A Z_B + E_B Z_A = 8.33 + j 20.57 = 22.15 \angle 67.9^\circ$$

$$\text{Now, } I = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)} = \frac{22.15 \angle 67.9^\circ}{0.0259 \angle 92.7^\circ} = 855 \angle -24.8^\circ$$

$$\therefore V_2 = I Z_L = 885 \angle -24.8^\circ \times 0.269 \angle 21.8^\circ = 230 - 3^\circ$$

### Tutorial Problems 29.1

1. A 1000-kVA and a 500-kVA, 1-phase transformers are connected to the same bus-bars on the primary side. The secondary e.m.fs. at no-load are 500 and 510 V respectively. The impedance voltage of the first transformer is 3.4% and of the second 5%. What cross-current will pass between them when the secondaries are connected together in parallel ? Assuming that the ratio of resistance to reactance is the same in each, what currents will flow in the windings of the two transformers when supplying a total load of 1200 kVA.

[(i) 290 A (ii) 1577 and 900 A] (City & Guilds, London)

2. Two transformers A and B are connected in parallel to supply a load having an impedance of  $(2 + j 1.5\Omega)$ . The equivalent impedances referred to the secondary windings are  $0.15 + j 0.5 \Omega$  and  $0.1 + j 0.6 \Omega$  respectively. The open-circuit e.m.f. of A is 207 V and of B is 205 V. Calculate (i) the voltage at the load (ii) the power supplied to the load (iii) the power output of each transformer and (iv) the kVA input to each transformer.

[(i) 189  $\angle -3.8^\circ$  V (ii) 11.5 kW (iii) 6.5 kW, 4.95 kW (iv) 8.7 kVA, 6.87 kVA]

## QUESTIONS AND ANSWERS ON TRANSFORMERS

**Q.1. How is magnetic leakage reduced to a minimum in commercial transformers ?**

**Ans.** By interleaving the primary and secondary windings.

**Q.2. Mention the factors on which hysteresis loss depends ?**

**Ans.** (i) Quality and amount of iron in the core (ii) Flux density and (iii) Frequency.

**Q.3. How can eddy current loss be minimised ?**

**Ans.** By laminating the core.

**Q.4. In practice, what determines the thickness of the laminae or stampings ?**

**Ans.** Frequency.

**Q.5. Does the transformer draw any current when its secondary is open ?**

**Ans.** Yes, no-load primary current.

**Q.6. Why ?**

**Ans.** For supplying no-load iron and copper losses in primary.

**Q.7. Is Cu loss affected by power factor ?**

**Ans.** Yes, Cu loss varies inversely with power factor.

**Q.8. Why ?**

**Ans.** Cu loss depends on current in the primary and secondary windings. It is well-known that current required is higher when power factor is lower.

**Q.9. What effects are produced by change in voltage ?**

- Ans.**
1. Iron loss.....varies approximately as  $V^2$ .
  2. Cu loss.....it also varies as  $V^2$  but decreases with an increase in voltage *if constant kVA output is assumed.*
  3. Efficiency.....for distribution transformers, efficiency at fractional loads decreases with increase in voltage while at full load or overload it increases with increase in voltage and *vice-versa.*
  4. Regulation.....it varies as  $V^2$  but decreases with increase in voltage *if constant kVA output is assumed.*
  5. Heating.....for constant kVA output, iron temperatures increase whereas Cu temperatures decrease with increase in voltages and *vice-versa.*

**Q. 10. How does change in frequency affect the operation of a given transformer ?**

- Ans.**
1. Iron loss .....increases with a decrease in frequency. A 60-Hz transformer will have nearly 11% higher losses when worked on 50Hz instead of 60 Hz. However, when a 25-Hz transformer is worked on 60 Hz, iron losses are reduced by 25%.
  2. Cu loss.....in distribution transformers, it is independent of frequency.
  3. Efficiency.....since Cu loss is unaffected by change in frequency, a given transformer efficiency is less at a lower frequency than at a higher one.
  4. Regulation.....regulation at unity power factor is not affected because *IR* drop is independent of frequency. Since reactive drop is affected, regulation at low power factors decreases with a decrease in frequency and *vice-versa.* For example, the regulation of a 25-Hz transformer when operated at 50-Hz and low power factor is much poorer.
  5. Heating.....since total loss is greater at a lower frequency, the temperature is increased with decrease in frequency.

## OBJECTIVE TESTS – 32

1. A transformer transforms
  - (a) frequency
  - (b) voltage
  - (c) current
  - (d) voltage and current.
2. Which of the following is not a basic element of a transformer ?
  - (a) core
  - (b) primary winding
  - (c) secondary winding
  - (d) mutual flux.
3. In an ideal transformer,
  - (a) windings have no resistance
  - (b) core has no losses
  - (c) core has infinite permeability
  - (d) all of the above.
4. The main purpose of using core in a transformer is to
  - (a) decrease iron losses
  - (b) prevent eddy current loss
  - (c) eliminate magnetic hysteresis
  - (d) decrease reluctance of the common magnetic circuit.
5. Transformer cores are laminated in order to
  - (a) simplify its construction
  - (b) minimise eddy current loss
  - (c) reduce cost
  - (d) reduce hysteresis loss.
6. A transformer having 1000 primary turns is connected to a 250-V a.c. supply. For a secondary voltage of 400 V, the number of secondary turns should be
 

(a) 1600	(b) 250
(c) 400	(d) 1250
7. The primary and secondary induced e.m.fs.  $E_1$  and  $E_2$  in a two-winding transformer are **always**
  - (a) equal in magnitude
  - (b) antiphase with each other
  - (c) in-phase with each other
  - (d) determined by load on transformer secondary.
8. A step-up transformer increases
 

(a) voltage	(b) current
(c) power	(d) frequency.
9. The primary and secondary windings of an ordinary 2-winding transformer **always** have
  - (a) different number of turns
  - (b) same size of copper wire
  - (c) a common magnetic circuit
  - (d) separate magnetic circuits.
10. In a transformer, the leakage flux of each winding is proportional to the current in that winding because
  - (a) Ohm's law applies to magnetic circuits
  - (b) leakage paths do not saturate
  - (c) the two windings are electrically isolated
  - (d) mutual flux is confined to the core.
11. In a two-winding transformer, the e.m.f. per turn in secondary winding is **always**.....the induced e.m.f. per turn in primary.
  - (a) equal to  $K$  times
  - (b) equal to  $1/K$  times
  - (c) equal to
  - (d) greater than.
12. In relation to a transformer, the ratio 20 : 1 indicates that
  - (a) there are 20 turns on primary one turn on secondary
  - (b) secondary voltage is  $1/20$ th of primary voltage
  - (c) primary current is 20 times greater than the secondary current.
  - (d) for every 20 turns on primary, there is one turn on secondary.
13. In performing the short circuit test of a transformer
  - (a) high voltage side is usually short circuited
  - (b) low voltage side is usually short circuited
  - (c) any side is short circuited with preference
  - (d) none of the above.

*(Elect. Machines, A.M.I.E. Sec. B, 1993)*
14. The equivalent resistance of the primary of a transformer having  $K = 5$  and  $R_1 = 0.1$  ohm when referred to secondary becomes.....ohm.
  - (a) 0.5
  - (b) 0.02
  - (c) 0.004
  - (d) 2.5
15. A transformer has negative voltage regulation when its load power factor is
  - (a) zero
  - (b) unity
  - (c) leading
  - (d) lagging.

- 16.** The primary reason why open-circuit test is performed on the low-voltage winding of the transformer is that it
- draws sufficiently large on-load current for convenient reading
  - requires least voltage to perform the test
  - needs minimum power input
  - involves less core loss.
- 17.** No-load test on a transformer is carried out to determine
- copper loss
  - magnetising current
  - magnetising current and no-load loss
  - efficiency of the transformer.
- 18.** The main purpose of performing open-circuit test on a transformer is to measure its
- Cu loss
  - core loss
  - total loss
  - insulation resistance.
- 19.** During short-circuit test, the iron loss of a transformer is negligible because
- the entire input is just sufficient to meet Cu losses only
  - flux produced is a small fraction of the normal flux
  - iron core becomes fully saturated
  - supply frequency is held constant.
- 20.** The iron loss of a transformer at 400 Hz is 10 W. Assuming that eddy current and hysteresis losses vary as the square of flux density, the iron loss of the transformer at rated voltage but at 50 Hz would be..... watt.
- 80
  - 640
  - 1.25
  - 100
- 21.** In operating a 400 Hz transformer at 50 Hz
- only voltage is reduced in the same proportion as the frequency
  - only kVA rating is reduced in the same proportion as the frequency
  - both voltage and kVA rating are reduced in the same proportion as the frequency
  - none of the above.
- 22.** The voltage applied to the h.v. side of a transformer during short-circuit test is 2% of its rated voltage. The core loss will be.....percent of the rated core loss.
- 4
  - 0.4
  - 0.25
  - 0.04
- 23.** Transformers are rated in kVA instead of kW because
- load power factor is often not known
  - kVA is fixed whereas kW depends on load p.f.
  - total transformer loss depends on volt-ampere
  - it has become customary.
- 24.** When a 400-Hz transformer is operated at 50 Hz its kVA rating is
- reduced to 1/8
  - increased 8 times
  - unaffected
  - increased 64 times.
- 25.** At relatively light loads, transformer efficiency is low because
- secondary output is low
  - transformer losses are high
  - fixed loss is high in proportion to the output
  - Cu loss is small.
- 26.** A 200 kVA transformer has an iron loss of 1 kW and full-load Cu loss of 2kW. Its load kVA corresponding to maximum efficiency is ..... kVA.
- 100
  - 141.4
  - 50
  - 200
- 27.** If Cu loss of a transformer at 7/8th full load is 4900 W, then its full-load Cu loss would be .....watt.
- 5600
  - 6400
  - 375
  - 429
- 28.** The ordinary efficiency of a given transformer is maximum when
- it runs at half full-load
  - it runs at full-load
  - its Cu loss equals iron loss
  - it runs slightly overload.
- 29.** The output current corresponding to maximum efficiency for a transformer having core loss of 100 W and equivalent resistance referred to secondary of 0.25  $\Omega$  is ..... ampere.
- 20
  - 25
  - 5
  - 400
- 30.** The maximum efficiency of a 100-kVA transformer having iron loss of 900 kW and F.L. Cu loss of 1600 W occurs at ..... kVA.
- 56.3
  - 133.3
  - 75
  - 177.7

31. The all-day efficiency of a transformer depends primarily on  
 (a) its copper loss  
 (b) the amount of load  
 (c) the duration of load  
 (d) both (b) and (c).
32. The marked increase in kVA capacity produced by connecting a 2 winding transformer as an autotransformer is due to  
 (a) increase in turn ratio  
 (b) increase in secondary voltage  
 (c) increase in transformer efficiency  
 (d) establishment of conductive link between primary and secondary.
33. The kVA rating of an ordinary 2-winding transformer is increased when connected as an autotransformer because  
 (a) transformation ratio is increased  
 (b) secondary voltage is increased  
 (c) energy is transferred both inductively and conductivity  
 (d) secondary current is increased.
34. The saving in Cu achieved by converting a 2-winding transformer into an autotransformer is determined by  
 (a) voltage transformation ratio  
 (b) load on the secondary  
 (c) magnetic quality of core material  
 (d) size of the transformer core.
35. An autotransformer having a transformation ratio of 0.8 supplies a load of 3 kW. The power transferred conductively from primary to secondary is.....kW.  
 (a) 0.6 (b) 2.4  
 (c) 1.5 (d) 0.27
36. The essential condition for parallel operation of two 1- $\phi$  transformers is that they should have the same  
 (a) polarity  
 (b) kVA rating  
 (c) voltage ratio  
 (d) percentage impedance.
37. If the impedance triangles of two transformers operating in parallel are not identical in shape and size, the two transformers will  
 (a) share the load unequally  
 (b) get heated unequally  
 (c) have a circulatory secondary current even when unloaded  
 (d) run with different power factors.
38. Two transformers A and B having equal outputs and voltage ratios but unequal percentage impedances of 4 and 2 are operating in parallel. Transformer A will be running over-load by ..... percent.  
 (a) 50 (b) 66  
 (c) 33 (d) 25

### ANSWERS

1. d 2. d 3. d 4. d 5. b 6. a 7. c 8. a 9. c 10. b 11. c 12. d 13. b 14. d  
 15. c 16. a 17. c 18. b 19. b 20. b 21. b 22. d 23. c 24. a 25. c 26. b 27. b 28. c 29. a 30. c  
 31. d 32. d 33. c 34. a 35. b 36. a 37. d 38. c

### Learning Objectives

- Three-phase Transformers
- Three-phase Transformer Connections
- Star/Star or Y/Y Connection
- Delta-Delta or Connection
- Wye/Delta or Y/ Connection
- Delta/Wye or /Y Connection
- Open-Delta or V-V Connection
- Power Supplied by V-V Bank
- Scott Connection or T-T Connection
- Three-phase to Two-Phase Conversion and vice-versa
- Parallel Operation of 3-phase Transformers
- Instrument Transformers
- Current Transformers
- Potential Transformers

## TRANSFORMER: THREE PHASE



Three phase transformers are used throughout industry to change values of three phase voltage and current. Three phase power is the most common way in which power is produced.

33.1. Three-Phase Transformer

Large scale generation of electric power is usually 3-phase at generated voltages of 13.2 kV or somewhat higher. Transmission is generally accomplished at higher voltages of 110, 132, 275, 400 and 750 kV for which purpose 3-phase transformers are necessary to step up the generated voltage to that of the transmission line. Next, at load centres, the transmission voltages are reduced to distribution voltages of 6,600, 4,600 and 2,300 volts. Further, at most of the consumers, the distribution voltages are still reduced to utilization voltages of 440, 220 or 110 volts. Years ago, it was a common

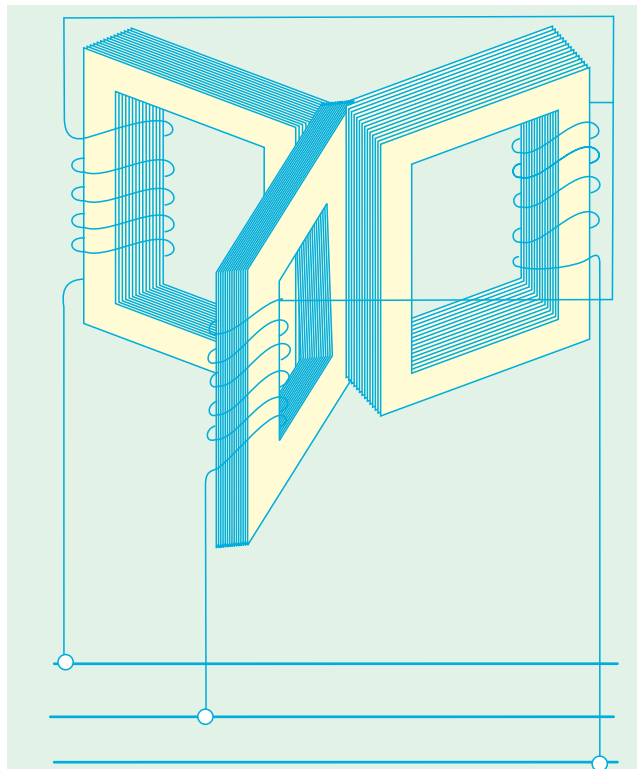
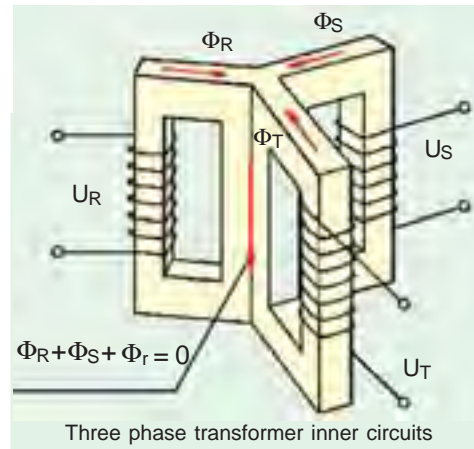


Fig. 33.1

practice to use suitably interconnected three single-phase transformers instead of a single 3-phase transformer. But these days, the latter is gaining popularity because of improvement in design and manufacture but principally because of better acquaintance of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a 3 phase transformer are that it occupies less floor space for equal rating, weighs less, costs about 15% less and further, that only one unit is to be handled and connected.

Like single-phase transformers, the three-phase transformers are also of the core type or shell type. The basic principle of a 3-phase transformer is illustrated in Fig. 33.1 in which only primary windings have been shown interconnected in star and put across 3-phase supply. The three cores are 120° apart and their empty legs are shown in

contact with each other. The centre leg, formed by these three, carries the flux produced by the three-phase currents  $I_R$ ,  $I_Y$  and  $I_B$ . As at any instant  $I_R + I_Y + I_B = 0$ , hence the sum of three fluxes is also zero. Therefore, it will make no difference if the common leg is removed. In that case any two legs will act as the return for the third just as in a 3-phase system any two conductors act as the return for the current

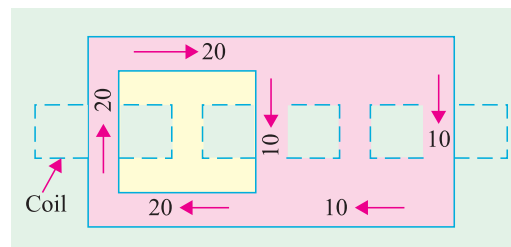


Fig. 33.2 (a)



in the third conductor. This improved design is shown in Fig. 33.2 (a) where dotted rectangles indicate the three windings and numbers in the cores and yokes represent the directions and magnitudes of fluxes at a particular instant. It will be seen that at any instant, the amount of 'up' flux in any leg is equal to the sum of 'down' fluxes in the other two legs. The core type transformers are usually wound with circular cylindrical coils.

In a similar way, three single-phase shell type transformers can be combined together to form a 3-phase shell type unit as shown in Fig. 33.2(b). But some saving in iron can be achieved in

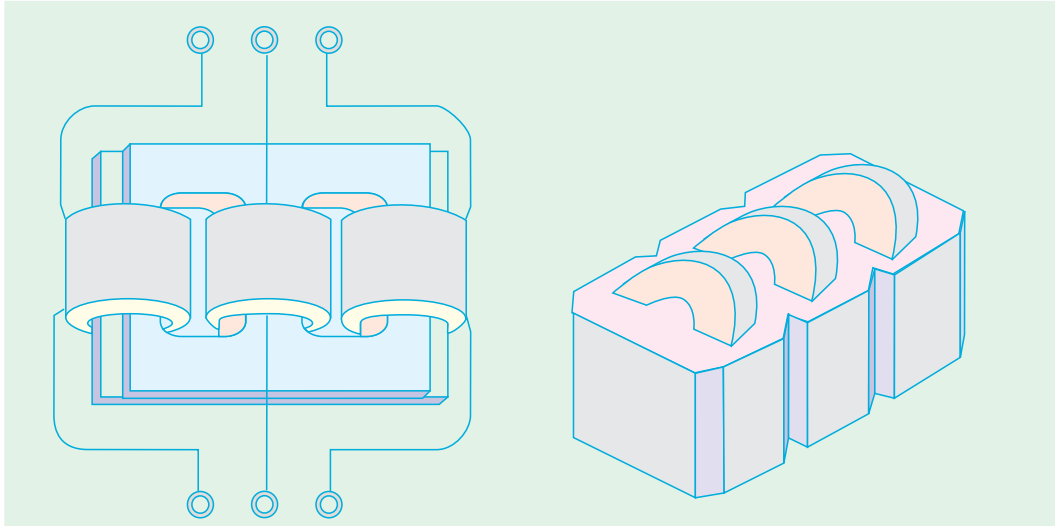
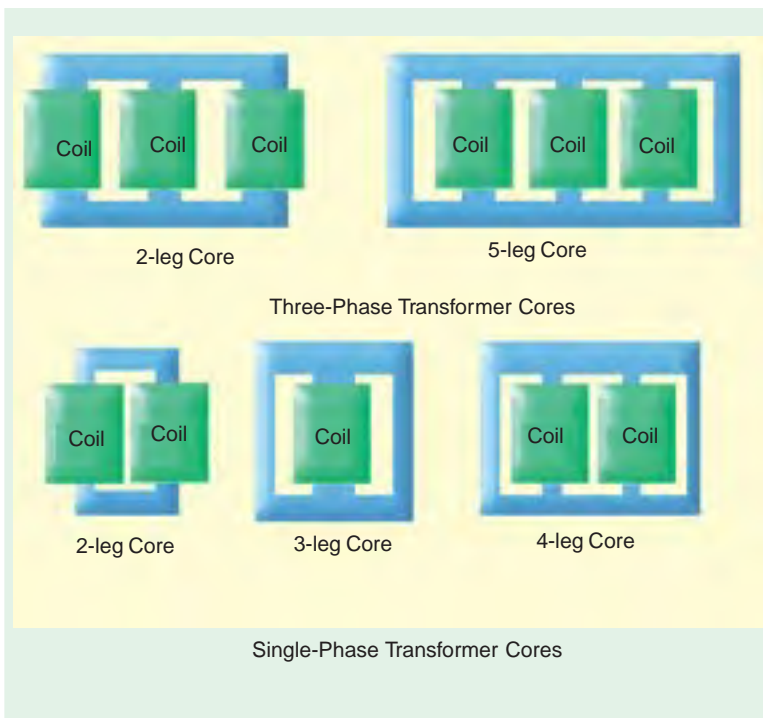


Fig. 33.2 (b)

Fig. 33.3



constructing a single 3-phase transformer as shown in Fig. 33.3. It does not differ from three single-phase transformers put side by side. Saving in iron is due to the joint use of the magnetic paths between the coils. The three phases, in this case, are more independent than they are in the core type transformers, because each phase has a magnetic circuit independent of the other.

One main drawback in a 3-phase transformer is that if any one phase becomes disabled, then the whole transformer has to be ordinarily removed from service for repairs (the shell type may be operated open

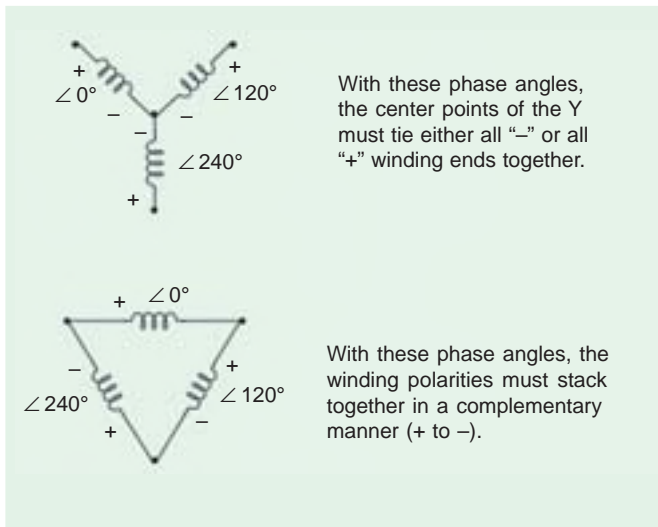
$\Delta$  or Vee but this is not always feasible). However, in the case of a 3-phase bank of single-phase transformers, if one transformer goes out of order, the system can still be run open- $\Delta$  at reduced capacity or the faulty transformer can be readily replaced by a single spare.

### 33.2. Three-phase Transformer Connections

There are various methods available for transforming 3-phase voltages to higher or lower 3-phase voltages *i.e.* for handling a considerable amount of power. The most common connections are (i)  $Y-Y$  (ii)  $\Delta-\Delta$  (iii)  $Y-\Delta$  (iv)  $\Delta-Y$  (v) open-delta or  $V-V$  (vi) Scott connection or  $T-T$  connection.

### 33.3. Star/Star or Y/Y Connection

This connection is most economical for small, high-voltage transformers because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only  $1/\sqrt{3}$  of line voltage). In Fig. 33.4 a bank of 3 transformers connected in  $Y$  on both the primary and the secondary sides is shown. The ratio of line voltages on the primary and secondary sides is the same as the transformation ratio of each transformer. However, there is a phase shift of  $30^\circ$  between the phase voltages and line voltages both on the primary and secondary sides. Of course, line voltages on both sides as well as primary voltages are respectively in phase with each other.



respectively in phase with each other. **This connection works**

**satisfactorily only if the load is balanced.** With the unbalanced load to the neutral, the neutral point shifts thereby making the three line-to-neutral (*i.e.* phase) voltages unequal. The effect of unbalanced loads can be illustrated by placing a single load between phase (or coil)  $a$  and the neutral on the secondary side. The power to the load has to be supplied by primary phase (or coil)  $A$ . This primary coil  $A$  cannot supply the required power because it is in series with primaries  $B$  and  $C$  whose secondaries are open. Under these conditions, the primary coils  $B$  and  $C$  act as very high impedances so that primary coil  $A$  can obtain but very little current through them from the line. Hence, secondary coil  $a$  cannot supply any appreciable power. In fact, a very low resistance approaching a short-circuit may be connected between point  $a$  and the neutral and only a very small amount of current will flow. This, as said above, is due to the reduction of voltage  $E_{an}$  because of neutral shift. In other words, under short-circuit conditions, the neutral is pulled too much towards coil  $a$ . This reduces  $E_{an}$  but increases  $E_{bn}$  and  $E_{cn}$  (however line voltage  $E_{AB}$ ,  $E_{BC}$  and  $E_{CA}$  are unaffected). On the primary side,  $E_{AN}$  will be

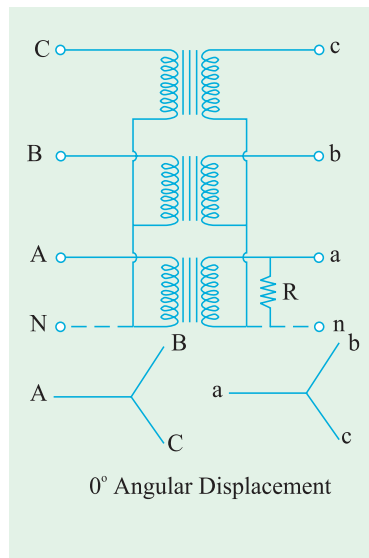


Fig. 33.4

practically reduced to zero whereas  $E_{BN}$  and  $E_{CN}$  will rise to nearly full primary line voltage. This difficulty of shifting (or floating) neutral can be obviated by connecting the primary neutral (shown dotted in the figure) back to the generator so that primary coil  $A$  can take its required power from between its line and the neutral. It should be noted that if a single phase load is connected between the lines  $a$  and  $b$ , there will be a similar but less pronounced neutral shift which results in an overvoltage on one or more transformers.

Another advantage of stabilizing the primary neutral by connecting it to neutral of the generator is that it eliminates distortion in the secondary phase voltages. This is explained as follows. For delivering a sine wave of voltage, it is necessary to have a sine wave of flux in the core, but on account of the characteristics of iron, a sine wave of flux requires a third harmonic component in the exciting current. As the frequency of this component is thrice the frequency of the circuit, at any given instant, it tends to flow either towards or away from the neutral point in all the three transformers. If the primary neutral is isolated, the triple frequency current cannot flow. Hence, the flux in the core cannot be a sine wave and so the voltages are distorted. But if the primary neutral is earthed *i.e.* joined to the generator neutral, then this provides a path for the triple-frequency currents and e.m.fs. and the difficulty is overcome. Another way of avoiding this trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in  $\Delta$  and provides a circuit in which the triple-frequency component of the magnetising current can flow (with an isolated neutral, it could not). In that case, a sine wave of voltage applied to the primary will result in a sine wave of phase voltage in the secondary. As said above, the advantage of this connection is that insulation is stressed only to the extent of line to neutral voltage *i.e.* 58% of the line voltage.

### 33.4. Delta-Delta or $\Delta - \Delta$ Connection

This connection is economical for large, low-voltage transformers in which insulation problem is not so urgent, because it increases the number of turns/phase. The transformer connections and voltage triangles are shown in Fig. 33.5. The ratio of transformation between primary and secondary line voltage is exactly the same as that of each transformer. Further, the secondary voltage triangle  $abc$  occupies the same relative position as the primary voltage triangle  $ABC$  *i.e.* there is no angular displacement between the two. Moreover, there is no internal phase shift between phase and line voltages on either side as was the case in  $Y-Y$  connection. This connection has the following advantages :

1. As explained above, in order that the output voltage be sinusoidal, it is necessary that the magnetising current of the transformer must contain a third harmonic component. In this case, the third harmonic component of the magnetising current can flow in the  $\Delta$ -connected transformer primaries without flowing in the line wires. The three phases are  $120^\circ$  apart which is  $3 \times 120 = 360^\circ$  with respect to the third harmonic, hence it merely circulates in the  $\Delta$ . Therefore, the flux is sinusoidal which results in sinusoidal voltages.
2. No difficulty is experienced from unbalanced loading as was the case in  $Y-Y$  connection. The three-phase voltages remain practically constant regardless of load imbalance.
3. An added advantage of this connection is that if one transformer becomes disabled, the system can continue to operate in open-delta or in  $V-V$  although with reduced available capacity. The reduced capacity is 58% and not 66.7% of the normal value, as explained in Art. 33.7.

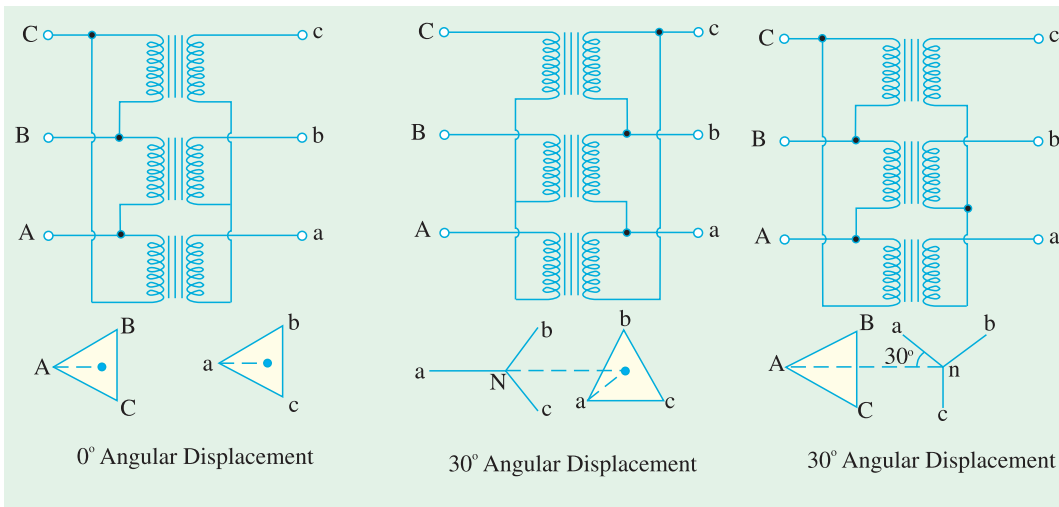


Fig. 33.5

Fig. 33.6

Fig. 33.7

### 33.5. Wye/Delta or Y/Δ Connection

The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is Y-connected with grounded neutral as shown in Fig. 33.6. The ratio between the secondary and primary line voltage is  $1/\sqrt{3}$  times the transformation ratio of each transformer. There is a  $30^\circ$  shift between the primary and secondary line voltages which means that a Y-Δ transformer bank cannot be paralleled with either a Y-Y or a Δ-Δ bank. Also, third harmonic currents flows in the Δ to provide a sinusoidal flux.

### 33.6. Delta/Wye or Δ/Y Connection

This connection is generally employed where it is necessary to step up the voltage as for example, at the beginning of high tension transmission system. The connection is shown in Fig. 33.7. The neutral of the secondary is grounded for providing 3-phase 4-wire service. In recent years, this connection has gained considerable popularity because it can be used to serve both the 3-phase power equipment and single-phase lighting circuits.

This connection is not open to the objection of a floating neutral and voltage distortion because the existence of a Δ-connection allows a path for the third-harmonic currents. It would be observed that the primary and secondary line voltages and line currents are out of phase with each other by  $30^\circ$ . Because of this  $30^\circ$  shift, it is impossible to parallel such a bank with a Δ-Δ or Y-Y bank of transformers even though the voltage ratios are correctly adjusted. The ratio of secondary to primary voltage is  $\sqrt{3}$  times the transformation ratio of each transformer.

**Example 33.1.** A 3-phase, 50-Hz transformer has a delta-connected primary and star-connected secondary, the line voltages being 22,000 V and 400 V respectively. The secondary has a star-connected balanced load at 0.8 power factor lagging. The line current on the primary side is 5 A. Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in kW ?

**Solution.** It should be noted that in three-phase transformers, the *phase* transformation ratio is equal to the turn ratio but the terminal or line voltages depend upon the method of connection employed. The Δ/Y connection is shown in Fig. 33.8.

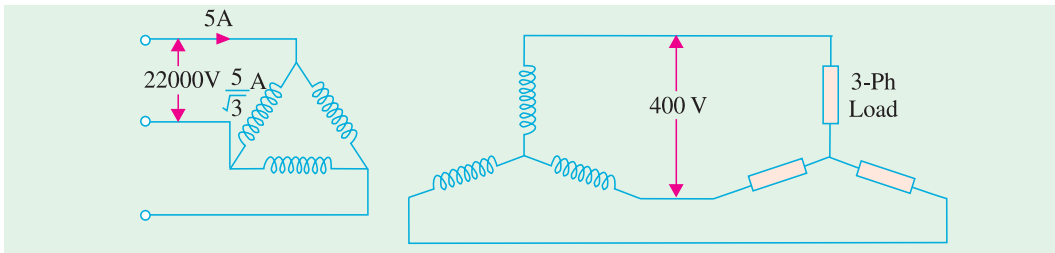


Fig. 33.8

Phase voltage on primary side = 22,000 V  
 Phase voltage on secondary side =  $400/\sqrt{3}$   
 $\therefore K = 400/22,000 \times \sqrt{3} = 1/55\sqrt{3}$   
 Primary phase current =  $5/\sqrt{3}$  A  
 Secondary phase current =  $\frac{5}{\sqrt{3}} \div \frac{1}{55\sqrt{3}} = 275$  A  
 Secondary line current = **275 A**  
 $\therefore$  Output =  $\sqrt{3}V_L I_L \cos \phi = \sqrt{3} \times 400 \times 275 \times 0.8 = \mathbf{15.24 \text{ kW}}$ .

**Example 33.2.** A 500-kVA, 3-phase, 50-Hz transformer has a voltage ratio (line voltages) of 33/11-kV and is delta/star connected. The resistances per phase are : high voltage 35  $\Omega$ , low voltage 0.876  $\Omega$  and the iron loss is 3050 W. Calculate the value of efficiency at full-load and one-half of full-load respectively (a) at unity p.f. and (b) 0.8 p.f. **(Electrical Machinery, Madras Univ. 1985)**

**Solution.** Transformation ratio  $K = \frac{11,000}{\sqrt{3} \times 33,000} = \frac{1}{3\sqrt{3}}$

Per phase  $R_{02} = 0.876 + (1/3\sqrt{3})^2 \times 35 = 2.172 \Omega$

Secondary phase current =  $\frac{500,000}{\sqrt{3} \times 11,000} = \frac{500}{11\sqrt{3}}$  A

**Full-load condition**

Full load total Cu loss =  $3 \times (500/11\sqrt{3})^2 \times 2.172 = 4,490$  W; Iron loss = 3,050 W

Total full-load losses = 4,490 + 3,050 = 7,540 W; Output at unity p.f. = 500 kW

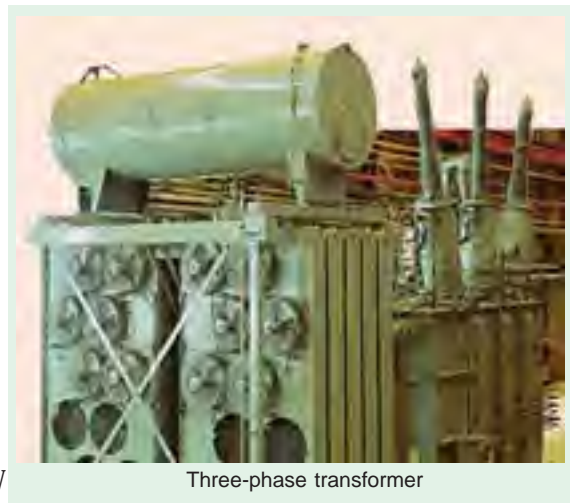
$\therefore$  F.L. efficiency =  $500,000/507,540 = 0.9854$  or **98.54%**; Output at 0.8 p.f. = 400 kW

$\therefore$  Efficiency =  $400,000/407,540 = 0.982$  or **98.2%**

**Half-load condition**

Output at unity p.f. = 250 kW  
 Cu losses =  $(1/2)^2 \times 4,490 = \mathbf{1,222 \text{ W}}$

Total losses = 3,050 + 1,122 = 4,172 W



Three-phase transformer

$$\therefore \eta = 250,000/254,172 = 0.9835 = \mathbf{98.35\%}$$

Output at 0.8 p.f. = 200 kW  $\therefore \eta = 200,000/204,172 = \mathbf{0.98}$  or **98%**

**Example 33.3.** A 3-phase, 6,600/415-V, 2,000-kVA transformer has a per unit resistance of 0.02 and a per unit leakage reactance of 0.1. Calculate the Cu loss and regulation at full-load 0.8 p.f. lag. **(Electrical Machines-I, Bombay Univ. 1987)**

**Solution.** As seen from Art. 27 – 16, %  $R = \% \text{ Cu loss} = \frac{\text{Cu loss}}{VA} \times 100$

Now, %  $R = 0.02 \times 100 = 2\%$   $\therefore 2 = \frac{\text{Cu loss}}{2,000} \times 100 \therefore \text{Cu loss} = \mathbf{40 \text{ kW}}$

Now, percentage leakage reactance =  $0.1 \times 100 = 10\%$

$$\text{Regn.} = v_r \cos \phi + v_x \sin \phi = 2 \times 0.8 + 10 \times 0.6 = \mathbf{7.6\%}$$

**Example 33.4.** A 120-kVA, 6,000/400-V, Y/Y 3-ph, 50-Hz transformer has an iron loss of 1,600 W. The maximum efficiency occurs at 3/4 full load.

Find the efficiencies of the transformer at

- (i) full-load and 0.8 power factor (ii) half-load and unity power factor  
 (iii) the maximum efficiency. **(Elect. Technology Utkal Univ. 1987)**

**Solution.** Since maximum efficiency occurs at 3/4 full-load, Cu loss at 3/4 full-load equals iron loss of 1,600 W.

Cu loss at 3/4 F.L. = 1,600 W; Cu loss at F.L. =  $1,600 \times (4/3)^2 = 2,845 \text{ W}$

(i) F.L. output at 0.8 p.f. =  $120 \times 0.8 = 96 \text{ kW} = 96,000 \text{ W}$

Total loss =  $1,600 + 2,845 = 4,445 \text{ W}$

$$\therefore \eta = \frac{96,000}{100,445} \times 100 = \mathbf{95.57\%}$$

(ii) Cu loss at 1/2 full-load =  $(1/2)^2 \times 2,845 = 710 \text{ W}$

Total loss =  $710 + 1,600 = 2,310 \text{ W}$

Output at 1/2 F.L. and u.p.f. is =  $60 \text{ kW} = 60,000 \text{ W}$ ;  $\eta = \frac{60,000}{62,310} \times 100 = \mathbf{96.57\%}$

(iii) Maximum efficiency occurs at 3/4 full-load when iron loss equals Cu loss.

Total loss =  $2 \times 1,600 = 3,200 \text{ W}$

Output at u.p.f. =  $(3/4) \times 120 = 90 \text{ kW} = 90,000 \text{ W}$

Input =  $90,000 + 3,200 = 93,200 \text{ W} \therefore \eta = \frac{90,000}{93,200} \times 100 = \mathbf{96.57\%}$

**Example 33.5.** A 3-phase transformer, ratio 33/6.6-kV,  $\Delta/Y$ , 2-MVA has a primary resistance of 8  $\Omega$  per phase and a secondary resistance of 0.08 ohm per phase. The percentage impedance is 7%. Calculate the secondary voltage with rated primary voltage and hence the regulation for full-load 0.75 p.f. lagging conditions. **(Elect. Machine-I, Nagpur, Univ. 1993)**

**Solution.** F.L. secondary current =  $\frac{2 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 175 \text{ A}$

$$K = 6.6 / \sqrt{3} \times 33 = 1/8.65; R_{02} = 0.08 + 8/8.65^2 = 0.1867 \Omega \text{ per phase}$$

Now, secondary impedance drop per phase =  $\frac{7}{100} \times \frac{6,600}{\sqrt{3}} = 266.7 \text{ V}$

$$\therefore Z_{02} = 266.7/175 = 1.523 \Omega \text{ per phase}$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.523^2 - 0.1867^2} = 1.51 \Omega / \text{phase}$$

Drop per phase =  $I_2 (R_{02} \cos \phi + X_{02} \sin \phi) = 175 (0.1867 \times 0.75 + 1.51 \times 0.66) = 200 \text{ V}$

Secondary voltage/phase =  $6,600 / \sqrt{3} = 3,810 \text{ V} \quad \therefore V_2 = 3,810 - 200 = 3,610 \text{ V}$

$\therefore$  Secondary line voltage =  $3,610 \times \sqrt{3} = \mathbf{6,250 \text{ V}}$

%regn. =  $200 \times 100 / 3,810 = \mathbf{5.23 \%}$

**Example 33.6.** A 100-kVA, 3-phase, 50-Hz 3,300/400 V transformer is  $\Delta$ -connected on the h.v. side and Y-connected on the l.v. side. The resistance of the h.v. winding is 3.5  $\Omega$  per phase and that of the l.v. winding 0.02  $\Omega$  per phase. Calculate the iron losses of the transformer at normal voltage and frequency if its full-load efficiency be 95.8% at 0.8 p.f. (lag).

(A.C. Machines-I, Jadavpur Univ. 1989)

**Solution.** F.L. output =  $100 \times 0.8 = 80 \text{ kW}$ ; Input =  $80 / 0.958 = 83.5 \text{ kW}$

Total loss = Input – Output =  $83.5 - 80 = 3.5 \text{ kW}$

Let us find full-load Cu losses for which purpose, we would first calculate  $R_{02}$ .

$$K = \frac{\text{secondary voltage/phase}}{\text{primary voltage/phase}} = \frac{400 / \sqrt{3}}{3,300} = \frac{4}{33\sqrt{3}}$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (4 / \sqrt{3} \times 33)^2 \times 3.5 = 0.037 \Omega$$

Full-load secondary phase current is  $I_2 = 100,000 / \sqrt{3} \times 400 = 144.1 \text{ A}$

Total Cu loss =  $3 I_2^2 R_{02} = 3 \times 144.1^2 \times 0.037 = 2,305 \text{ W}$

Iron loss = Total loss – F.L. Cu loss =  $3,500 - 2,305 = \mathbf{1,195 \text{ W}}$

**Example 33.7.** A 5,000-kVA, 3-phase transformer, 6.6/33-kV,  $\Delta/Y$ , has a no-load loss of 15 kW and a full-load loss of 50 kW. The impedance drop at full-load is 7%. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV.

**Solution.** Full-load  $I_2 = 5 \times 10^6 / \sqrt{3} \times 33,000 = 87.5 \text{ A}$

Impedance drop/phase = 7% of  $(33 / \sqrt{3}) = 7\%$  of 19 kV = 1,330 V

$\therefore Z_{02} = 1,330 / 87.5 = 15.3 \Omega / \text{phase}$ ; F.L. Cu loss =  $50 - 15 = 35 \text{ kW}$

$\therefore 3 I_2 R_{02} = 35,000$ ;  $R_{02} = 35,000 / 3 \times 8.75^2 = 1.53 \Omega / \text{phase}$

$\therefore X_{02} = \sqrt{15.3^2 - 1.53^2} = \mathbf{15.23 \Omega}$

**When load is 3,200 kW at 0.8 p.f.**

$I_2 = 3,200 / \sqrt{3} \times 33 \times 0.8 = 70 \text{ A}$ ; drop =  $70 (1.53 \times 0.8 + 15.23 \times 0.6) = 725 \text{ V/phase}$

$\therefore$  % regn. =  $\frac{725 \times 100}{19,000} = 3.8 \%$

Primary voltage will have to be increased by 3.8%.

$\therefore$  Primary voltage =  $6.6 + 3.8\%$  of 6.6 = 6.85 kV =  $\mathbf{6,850 \text{ V}}$

**Example 33.8.** A 3-phase transformer has its primary connected in  $\Delta$  and its secondary in Y. It has an equivalent resistance of 1% and an equivalent reactance of 6%. The primary applied voltage is 6,600 V. What must be the ratio of transformation in order that it will deliver 4,800 V at full-load current and 0.8 power factor (lag) ?

(Elect. Technology-II, Magadh Univ. 1991)

**Solution.** Percentage regulation

$$= v_r \cos \phi + v_x \sin \phi$$

$$= 1 \times 0.8 + 6 \times 0.6 = 4.4\%$$

Induced secondary e.m.f. (line value)

$$= 4,800 + 4.4\% \text{ of } 4,800 = 5,010 \text{ V, as in Fig. 33.9.}$$

Secondary phase voltage

$$= 5,010 / \sqrt{3} = 2,890 \text{ V}$$

Transformation ratio

$$K = 2,890 / 6,600 = \mathbf{0.437.}$$

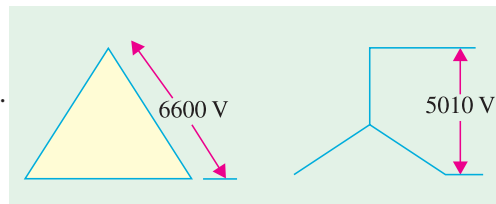


Fig. 33.9

**Example 33.9.** A 2000-kVA, 6,600/400-V, 3-phase transformer is delta-connected on the high voltage side and star-connected on the low-voltage side. Determine its % resistance drops, % efficiency and % regulation on full load 0.8 p.f. leading given the following data :  
 S.C. test ; H.V. data : 400 V, 175 A and 17 kW  
 O.C. test; L.V. data : 400 V, 150 A and 15 kW **(Basic Elect., Machines Nagpur Univ. 1993)**

**Solution.** From S.C. test data, we have

Primary voltage/phase = 400 V; Primary current/phase =  $175 / \sqrt{3} = 100 \text{ A}$

$$\therefore Z_{01} = \frac{400}{101} = 3.96 \Omega$$

$$I_1^2 R_{01} = \frac{17000}{3} \text{ or } R_{01} = 0.555 \Omega; X_{01} = \sqrt{3.96^2 - 0.555^2} = 3.92 \Omega$$

$$\% R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{101 \times 0.555}{6,600} \times 100 = \mathbf{0.849}$$

$$\% X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{101 \times 3.92}{6,600} \times 100 = \mathbf{6}$$

$$\% \text{ regn} = v_r \cos \phi - v_x \sin \phi = 0.49 \times 0.8 - 6 \times 0.6 = -2.92\%$$

Full-load primary line current can be found from

$$\sqrt{3} \times 6,600 \times I_1 = 2000 \times 1,000; I_1 = 175 \text{ A}$$

It shows that S.C. test has been carried out under full-load conditions.

Total losses = 17 + 15 = 32 kW; F.L. output =  $2,000 \times 0.8 = 1600 \text{ kW}$

$$\eta = 1,600 / 1,632 = 0.98 \text{ or } \mathbf{98\%}$$

**Example 33.10.** A 3-ph, delta/star connected 11,000/440 V, 50 Hz transformer takes a line current of 5 amp, when secondary Load of 0.8 Lagging p.f. is connected. Determine each coil-current and output of transformer. **(Amravati Univ. 1999)**

**Solution.** Due to delta/star connections the voltage ratings of the two sides on per phase basis are :

Primary coil rating = 11,000 V, Secondary coil rating =  $\frac{440}{\sqrt{3}} = 254 \text{ volts}$

Primary coil-current =  $5 / \sqrt{3} = 2.887 \text{ amp}$

Each coil is delivering equal volt. amps.

Since three phase volt – amps =  $3 \times 11,000 \times 5 / \sqrt{3}$   
 = 95266

Volts amps/phase = 31755

This corresponds to the secondary coil-current of  $I_2$ , given by

$$I_2 = \frac{31755}{254} = 125 \text{ amp. This is shown in Fig. 33.10.}$$



Total Output of transformer, in kVA = 31,755

Since, the p.f. given is 0.8 lagging.

The total output power in kW =  $31,755 \times 0.80 = 25.4 \text{ kW}$

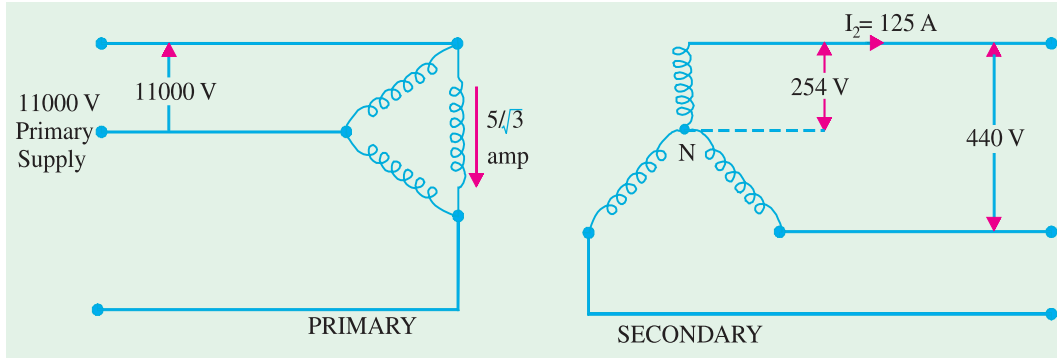


Fig. 33.10. Transformer coil currents

**Example 33.11.** A load of 1000 kVA at 0.866 p.f. lagging is supplied by two 3 phase transformers of 800 kVA capacity operating in parallel. Ratio of transformation is same : 6600/400 V, delta/star. If the equivalent impedances referred to secondary are  $(0.005 + j.015)$  ohm and  $(0.012 + j 0.030)$  ohm per phase respectively. Calculate load and power factor of each transformer.

(Amravati Univ. 1999)

**Solution.** Total load = 1000 kVA  
 $\cos \phi = 0.866$  Lag,  $\phi = 30^\circ$  lag  
 Total power output = 866 kW  
 Secondary current with star connection,

$$I_2 = \frac{1000}{\sqrt{3} \times 440} \times 1000 = 1312.2 \text{ amp}$$

If the two transformers are identified as A and B, with their parameters with subscripts of a and b, we have :

$$\begin{aligned} \dot{Z}_a &= 0.005 + j0.015 = 0.0158 \angle 71.56^\circ \text{ ohm} \\ \dot{Z}_b &= 0.012 + j0.030 = 0.0323 \angle 68.2^\circ \text{ ohm} \\ \dot{Z}_a + \dot{Z}_b &= 0.017 + j0.045 = 0.0481 \angle 69.3^\circ \text{ ohm} \\ \dot{i}_{2a} &= \text{secondary current of transformer A} \\ \dot{i}_{2b} &= \text{secondary current of transformer B} \\ \dot{i}_{2a} &= \frac{\dot{Z}_b}{\dot{Z}_b + \dot{Z}_a} \times \dot{I}_2 = \frac{0.0323 \angle 68.2^\circ}{0.0481 \angle 69.3^\circ} \times 1312 \angle -30^\circ \\ &= 88.1 \angle -31.1^\circ \\ \dot{i}_{2b} &= \frac{\dot{Z}_a}{\dot{Z}_a + \dot{Z}_b} \times \dot{I}_2 = \frac{0.0158 \angle 71.56^\circ}{0.0481 \angle 69.3^\circ} \times 1312 \angle -30^\circ \\ &= 43.1 \angle -27.74^\circ \end{aligned}$$

**For Transformer A**

Load =  $3 \times 254 \times 881 \times 10^{-3} = 671.3 \text{ kVA}$

Power factor =  $\cos 31.1^\circ \text{ lag} = 0.856 \text{ lag}$

**For Transformer B**

Load =  $3 \times 254 \times 431 \times 10^{-3} = 328.4 \text{ kVA}$

Power factor =  $\cos 23.74^\circ \text{ lag} = 0.885 \text{ lag}$

**Check :** Total kW gives a check.

1000 kVA at 0.866 lag means 866 kW.

Output, in kW, of transformer A =  $671.3 \times 0.856 = 574.6 \text{ kW}$

Output in kW of transformer B =  $328.4 \times 0.885 = 290.6 \text{ kW}$

Sum of these two outputs =  $574.6 + 290.6 = 865.2 \text{ kW}$

**Note.** Total kVAR also gives a check.

Depending on leading or lagging p.f., appropriate sign (+ ve or - ve) must be assigned to the kVAR-term.

**Tutorial Problem No. 33.1**

1. A 3-phase, star-connected alternator generates 6,360 V per phase and supplies 500 kW at a p.f. 0.9 lagging to a load through a step-down transformer of turns 40 : 1. The transformer is delta connected on the primary side and star-connected on the secondary side. Calculate the value of the line volts at the load. Calculate also the currents in (a) alternator windings (b) transformer primary windings (c) transformen secondary windings. **[476 V (a) 29.1 A (b) 16.8 A (c) 672 A]**

2. A 11,000/6,600 V, 3- $\phi$ , transformer has a star-connected primary and a delta-connected secondary. It supplies a 6.6 kV motor having a star-connected stator, developing 969.8 kW at a power factor of 0.9 lagging and an efficiency of 92 per cent. Calculate (i) motor line and phase currents (ii) transformer secondary current and (iii) transformer primary current.

**[(a) Motor ;  $I_L = I_{ph} = 126.3 \text{ A}$  (b) phase current 73 A (c) 75.8 A]**

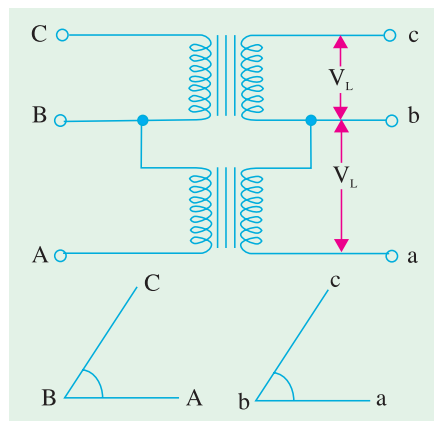
**33.7. Open-Delta or V – V connection**

If one of the transformers of a  $\Delta - \Delta$  is removed and 3-phase supply is connected to the primaries as shown in Fig. 33.11, then three equal 3-phase voltages will be available at the secondary terminals on no-load. This method of transforming 3-phase power by means of only two transformers is called the open -  $\Delta$  or V – V connection.

It is employed :

1. when the three-phase load is too small to warrant the installation of full three-phase transformer bank.
2. when one of the transformers in a  $\Delta - \Delta$  bank is disabled, so that service is continued although at reduced capacity, till the faulty transformer is repaired or a new one is substituted.
3. when it is anticipated that in future the load will increase necessitating the closing of open delta.

One important point to note is that the total load that can be carried by a V – V bank is *not* two-third of the capacity of a  $\Delta - \Delta$  bank but it is only 57.7% of it. That is a reduction of 15% (strictly, 15.5%) from its normal rating. Suppose there is  $\Delta - \Delta$  bank of three 10-kVA transformers. When one transformer is removed, then it runs in V – V. The total rating of the two transformers is 20 kVA. But the capacity of the V – V bank is not the sum of the transformer kVA ratings but only 0.866 of it i.e.  $20 \times 0.866 = 17.32$  (or  $30 \times 0.57 = 17.3 \text{ kVA}$ ). The fact that the ratio of V-capacity to  $\Delta$ -capacity is  $1/\sqrt{3} = 57.7\%$  (or nearly 58%) instead of  $66\frac{2}{3}\%$  per cent can be proved as follows :



**Fig. 33.11**

As seen from Fig. 33.12 (a)

$$\Delta - \Delta \text{ capacity} = \sqrt{3} \cdot V_L \cdot I_L = \sqrt{3} \cdot V_L (\sqrt{3} \cdot I_S) = 3 V_L I_S$$

In Fig. 33.12 (b), it is obvious that when  $\Delta - \Delta$  bank becomes  $V - V$  bank, the secondary line current  $I_L$  becomes equal to the secondary phase current  $I_S$

$$\therefore V - V \text{ capacity} = \sqrt{3} \cdot V_L I_L = \sqrt{3} V_L \cdot I_S$$

$$\therefore \frac{V - V \text{ capacity}}{\Delta - \Delta \text{ capacity}} = \frac{\sqrt{3} \cdot V_L I_S}{3 V_L I_S} = \frac{1}{\sqrt{3}} = 0.577 \text{ or } 58 \text{ per cent}$$

It means that the 3-phase load which can be carried **without exceeding the ratings** of the transformer is 57.7 per cent of the original load rather than the expected 66.7%.

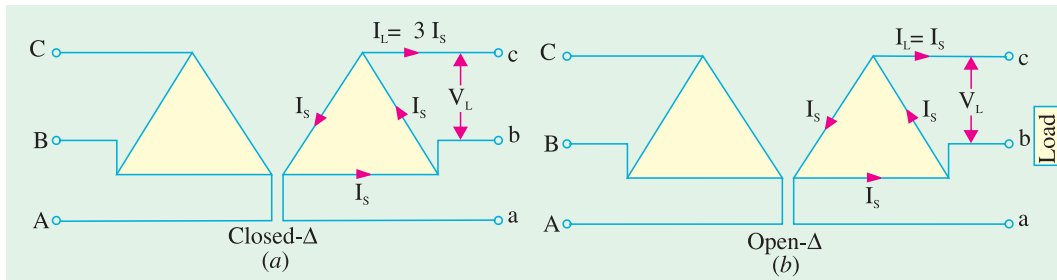


Fig. 33.12

It is obvious from above that when one transformer is removed from a  $\Delta - \Delta$  bank.

1. the bank capacity is reduced from 30 kVA to  $30 \times 0.577 = 17.3$  kVA and not to 20 kVA as might be thought off-hand.
2. only 86.6% of the rated capacity of the two remaining transformers is available (*i.e.*  $20 \times 0.866 = 17.3$  kVA). In other words, ratio of *operating* capacity to *available* capacity of an open- $\Delta$  is 0.866. This factor of 0.866 is sometimes called the **utility factor**.
3. each transformer will supply 57.7% of load and not 50% when operating in  $V - V$  (Ex. 33.13).

However, it is worth noting that if three transformers in a  $\Delta - \Delta$  bank are delivering their *rated load*\* and one transformer is removed, the overload on *each* of the two remaining transformers is 73.2% because

$$\frac{\text{total load in } V - V}{VA/\text{transformer}} = \frac{\sqrt{3} \cdot V_L I_S}{V_L I_S} = \sqrt{3} = 1.732$$

This over-load may be carried temporarily but some provision must be made to reduce the load if overheating and consequent breakdown of the remaining two transformers is to be avoided.

The disadvantages of this connection are :

1. The average power factor at which the  $V$ -bank operates is less than that of the load. **This power factor is actually 86.6% of the balanced load power factor**. Another significant point to note is that, except for a balanced unity power factor load, the two transformers in the  $V - V$  bank operate at different power factors (Art. 33.8).
2. Secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

It may, however, be noted that if two transformers are operating in  $V - V$  and loaded to rated capacity (in the above example, to 17.3 kVA), the addition of a third transformer increases the total capacity by  $\sqrt{3}$  or 173.2% (*i.e.* to 30 kVA). It means that for an increase in cost of 50% for the third transformer, the increase in capacity is 73.2% when converting from a  $V - V$  system to a  $\Delta - \Delta$  system.

\* In Ex. 33.13, the three transformers are not supplying their rated load of  $20 \times 3 = 60$  kVA but only 40 kVA.

### 33.8. Power Supplied by V – V Bank

When a V–V bank of two transformers supplies a balanced 3-phase load of power factor  $\cos \phi$ , then one transformer operates at a p.f. of  $\cos (30^\circ - \phi)$  and the other at  $\cos (30^\circ + \phi)$ . Consequently, the two transformers will not have the same voltage regulation.

$$\therefore P_1 = \text{kVA} \cos (30^\circ - \phi) \text{ and } P_2 = \text{kVA} \cos (30^\circ + \phi)$$

**(i) When  $\phi = 0$  i.e. load p.f. = 1**

Each transformer will have a p.f. =  $\cos 30^\circ = 0.866$

**(ii) When  $\phi = 30^\circ$  i.e. load p.f. = 0.866.**

In this case, one transformer has a p.f. of  $\cos (30^\circ - 30^\circ) = 1$  and the other of  $\cos (30^\circ + 30^\circ) = 0.866$ .

**(iii) When  $\phi = 60^\circ$  i.e. load p.f. = 0.5**

In this case, one transformer will have a p.f. =  $\cos (30^\circ - 60^\circ) = \cos (-30^\circ) = 0.866$  and the other of  $\cos (30^\circ + 60^\circ) = 0$ . It means that one of the transformers will not supply any load whereas the other having a p.f. = 0.866 will supply the entire load.

**Example 33.12.** What should be the kVA rating of each transformer in a V–V bank when the 3-phase balanced load is 40 kVA? If a third similar transformer is connected for operation, what is the rated capacity? What percentage increase in rating is affected in this way?

**Solution.** As pointed out earlier, the kVA rating of each transformer has to be 15% greater.

$$\therefore \text{kVA/transformer} = (40/2) \times 1.15 = 23$$

$$\Delta - \Delta \text{ bank rating} = 23 \times 3 = 69; \text{ Increase} = [(69 - 40)/40] \times 100 = 72.5\%$$

**Example 33.13.** A  $\Delta - \Delta$  bank consisting of three 20-kVA, 2300/230-V transformers supplies a load of 40 kVA. If one transformer is removed, find for the resulting V–V connection

- kVA load carried by each transformer
- per cent of rated load carried by each transformer
- total kVA rating of the V–V bank
- ratio of the V–V bank to  $\Delta - \Delta$  bank transformer ratings.
- per cent increase in load on each transformer when bank is converted into V–V bank.

**Solution. (i)** As explained earlier in Art. 33.7,  $\frac{\text{total kVA load in V – V bank}}{\text{VA/transformer}} = \sqrt{3}$

$$\therefore \text{kVA load supplied by each of the two transformers} = 40 / \sqrt{3} = 23.1 \text{ kVA}$$

Obviously, each transformer in V–V bank does not carry 50% of the original load but 57.7%.

$$\text{(ii) per cent of rated load} = \frac{\text{kVA load/transformer}}{\text{kVA rating/transformer}} = \frac{23.1}{20} = 115.5\%$$

carried by each transformer.

Obviously, in this case, each transformer is overloaded to the extent of 15.5 per cent.\*

$$\text{(iii) kVA rating of the V – V bank} = (2 \times 20) \times 0.866 = 34.64 \text{ kVA}$$

$$\text{(iv) } \frac{\text{V – V rating}}{\Delta - \Delta \text{ rating}} = \frac{34.64}{60} = 0.577 \text{ or } 57.7\%$$

\* Overloading becomes 73.2% only when full rated load is supplied by the  $\Delta - \Delta$  bank (i.e.  $3 \times 20 = 60$  kVA in this case) before it becomes V–V bank.

As seen, the rating is reduced to 57.7% of the original rating.

(v) Load supplied by each transformer in  $\Delta - \Delta$  bank =  $40/3 = 13.33$  kVA

$\therefore$  Percentage increase in load supplied by each transformer

$$= \frac{\text{kVA load/transformer in } V - V \text{ bank}}{\text{kVA load/transformer in } \Delta - \Delta \text{ bank}} = \frac{23.1}{13.3} = 1.732 = \mathbf{173.2\%}$$

It is obvious that each transformer in the  $\Delta - \Delta$  bank supplying 40 kVA was running underloaded (13.33 vs 20 kVA) but runs overloaded (23.1 vs 20 kVA) in  $V - V$  connection.

**Example 33.14.** A balanced 3-phase load of 150 kW at 1000 V, 0.866 lagging power factor is supplied from 2000 V, 3-phase mains through single-phase transformers (assumed to be ideal) connected in (i) delta-delta (ii) Vee-Vee. Find the current in the windings of each transformer and the power factor at which they operate in each case. Explain your calculations with circuit and vector diagrams.

**Solution. (i) Delta-Delta Connection**

$$\sqrt{3} V_L I_L \cos \phi = 150,000$$

$$\sqrt{3} \times 1000 \times I_L \times 0.866 = 150,000 \therefore I_L = 100 \text{ A}$$

$\therefore$  Secondary line current = 100 A; secondary phase current =  $100/\sqrt{3} = \mathbf{57.7 \text{ A}}$

Transformation ratio =  $1000/2000 = 1/2$

$\therefore$  Primary phase current =  $57.7/2 = \mathbf{28.85 \text{ A}}$

**(ii) Vee-Vee Connection**

Let  $I$  be the secondary line current which is also the phase current in  $V - V$  connection. Then

$$\sqrt{3} \times 1000 \times I \times 0.866 = 150,000 \therefore I = 100 \text{ A}$$

$\therefore$  Secondary phase current =  $\mathbf{100 \text{ A}}$ ; primary phase current =  $100 \times 1/2 = \mathbf{50 \text{ A}}$

Transformer power factor = 86.6 per cent of 0.866 =  $\mathbf{0.75 \text{ (lag)}}$ .

**Example 33.15.** (a) Two identical 1-phase transformers are connected in open-delta across 3-phase mains and deliver a balanced load of 3000 kW at 11 kV and 0.8 p.f. lagging. Calculate the line and phase currents and the power factors at which the two transformers are working.

(b) If one more identical unit is added and the open delta is converted to closed delta, calculate the additional load of the same power factor that can now be supplied for the same temperature rise. Also calculate the phase and line currents. **(Elect. Machinery-I, Madras Univ. 1987)**

**Solution. (a)** If  $I$  is the line current, then

$$\sqrt{3} \times 11,000 \times I \times 0.8 = 3,000,000 \qquad \mathbf{I = 197 \text{ A}}$$

Since, this also represents the phase current,

$\therefore$  Secondary phase current =  $\mathbf{197 \text{ A}}$ ; Transformer p.f. = 86.6 per cent of 0.8 =  $\mathbf{0.693}$

**(b)** Additional load = 72.5 per cent of 3000 =  $\mathbf{2175 \text{ kW}}$

Total load =  $3000 + 2175 = 5175 \text{ kW}$

Now,  $\sqrt{3} \times V_L I_L \cos \phi = 5,175,000$  or  $\sqrt{3} \times 11,000 \times I_L \times 0.8 = 5,175,000$

$\therefore I_L = \mathbf{340 \text{ A}}$ ; phase current =  $340/\sqrt{3} = \mathbf{196 \text{ A}}$

**Example 33.16.** Two transformers connected in open delta supply a 400-kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V. What is the (a) kVA supplied by each transformer? (b) kW supplied by each transformer? **(Elect. Machines-I, Gwalior Univ. 1991)**

**Solution.** As stated in Art 33.7, the ratio of operating capacity to available capacity in an open- $\Delta$  is 0.866. Hence, kVA of each transformer is one-half of the total kVA load divided by 0.866.

(a) kVA of each transformer =  $\frac{(400/2)}{0.866} = 231 \text{ kVA}$

(b) As stated in Art 33.8, the two transformers have power factors of  $\cos(30^\circ - \phi)$  and  $\cos(30^\circ + \phi)$ .

$\therefore P_1 = \text{kVA} \cos(30^\circ - \phi)$  and  $P_2 = \text{kVA} \cos(30^\circ + \phi)$

Now, load p.f. =  $\cos \phi = 0.866$ ;  $\phi = \cos^{-1}(0.866) = 30^\circ$

$\therefore P_1 = 231 \times \cos 0^\circ = 231 \text{ kW}$ ;  $P_2 = 231 \times \cos 60^\circ = 115.5 \text{ kW}$

Obviously,  $P_1 + P_2$  must equal  $400 \times 0.86 = 346.5 \text{ kW}$

**Tutorial Problem No. 33.2**

1. Three 1100/110-V transformers connected delta-delta supply a lighting load of 100 kW. One of the transformers is damaged and removed for repairs. Find

- (a) What currents were flowing in each transformer when the three transformers were in service ?
- (b) What current flows in each transformer when the third is removed ? and
- (c) The output kVA of each transformer if the transformers connected in open  $\Delta$  supply the full-load with normal heating ?

[(a) primary = 30.3 A; secondary 303 A (b) primary =  $30.3 \sqrt{3} \text{ A}$ ; secondary =  $303\sqrt{3} \text{ A}$  (c) 33.33 kVA] (Elect. Machines-I, Gwalior Univ. Apr. 1977)

**33.9. Scott Connection or T - T Connection**

This is a connection by which 3-phase to 3-phase transformation is accomplished with the help of two transformers as shown in Fig. 33.13. Since it was first proposed by Charles F. Scott, it is frequently referred to as Scott connection. This connection can also be used for 3-phase to 2-phase transformation as explained in Art. 33.10.

One of the transformers has centre taps both on the primary and secondary windings (Fig. 33.13) and is known as the **main** transformer. It forms the horizontal member of the connection (Fig. 33.14).

The other transformer has a 0.866 tap and is known as **teaser** transformer. One end of both the primary and secondary of the teaser transformer is joined to the centre taps both primary and secondary of the main transformer respectively as shown in Fig. 33.14 (a). The other end A of the teaser primary and the two ends B and C of the main transformer primary are connected to the 3-phase supply.

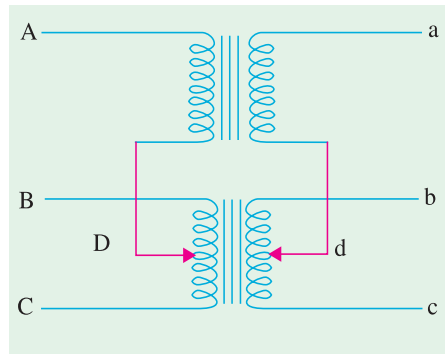


Fig. 33.13

The voltage diagram is shown in Fig. 33.14 (a) where the 3-phase supply line voltage is assumed to be 100 V and a transformation ratio of unity. For understanding as to how 3-phase transformation results from this arrangement, it is desirable to think of the primary and secondary vector voltages as forming geometrical  $T_s'$  (from which this connection gets its name).

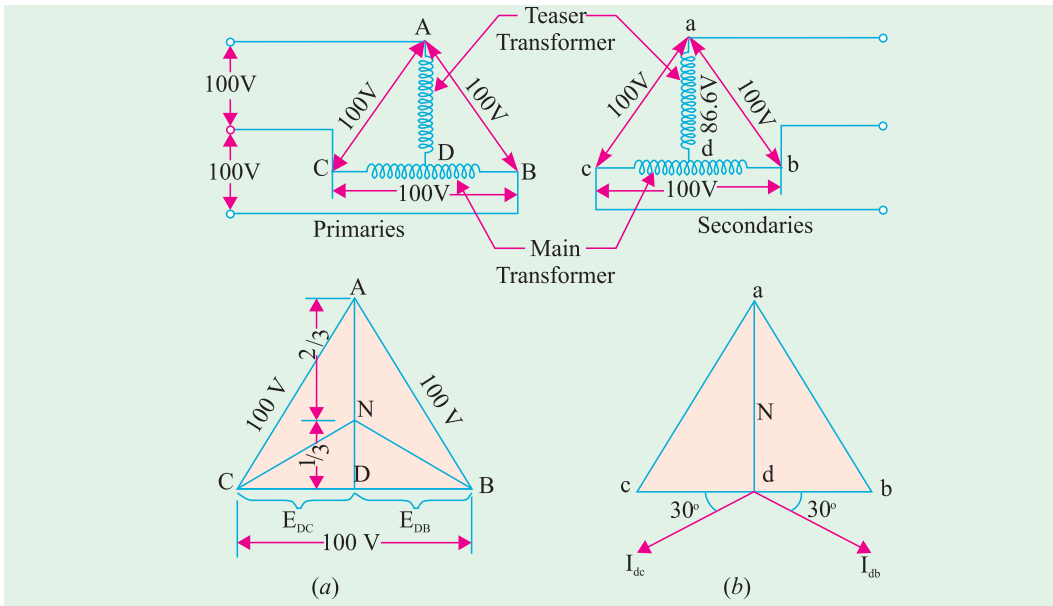


Fig. 33.14

In the primary voltage  $T$  of Fig. 33.14 (a),  $E_{DC}$  and  $E_{DB}$  are each 50 V and differ in phase by  $180^\circ$ , because both coils  $DB$  and  $DC$  are on the same magnetic circuit and are connected in opposition. Each side of the equilateral triangle represents 100 V. The voltage  $E_{DA}$  being the altitude of the equilateral triangle is equal to  $(\sqrt{3}/2) \times 100 = 86.6$  V and lags behind the voltage across the main by  $90^\circ$ . The same relation holds good in the secondary winding so that  $abc$  is a symmetrical 3-phase system.

With reference to the secondary voltage triangle of Fig. 33.14 (b), it should be noted that for a load of unity power factor, current  $I_{db}$  lags behind voltage  $E_{db}$  by  $30^\circ$  and  $I_{dc}$  leads  $E_{dc}$  by  $30^\circ$ . In other words, the teaser transformer and each half of the main transformer, all operate at different power factors.

Obviously, the full rating of the transformers is not being utilized. The teaser transformer operates at only 0.866 of its rated voltage and the main transformer coils operate at  $\cos 30^\circ = 0.866$  power factor, which is equivalent to the main transformer's coils working at 86.6 per cent of their kVA rating. Hence the capacity to rating ratio in a  $T-T$  connection is 86.6% — the same as in  $V-V$  connection if two identical units are used, although heating in the two cases is not the same.

If, however, both the teaser primary and secondary windings are designed for 86.6 volts only, then they will be operating at full rating, hence the combined rating of the arrangement would become  $(86.6 + 86.6)/(100 + 86.6) = 0.928$  of its total rating.\* In other words, ratio of kVA utilized to that available would be 0.928 which makes this connection more economical than open- $\Delta$  with its ratio of 0.866.

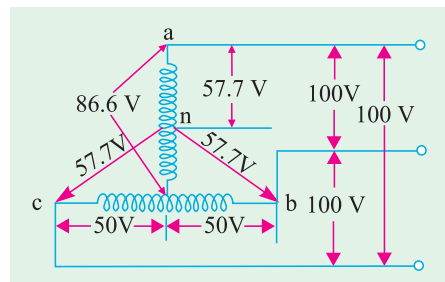


Fig. 33.15

\* Alternatively, VA capacity available is  $= V_L I_L + (0.866 V_L) I_L = 1.866 V_L I_L$  where  $I_L$  is the primary line current. Since 3-phase power is supplied, volt-amperes actually utilized  $= 1.732 V_L I_L$ . Hence, ratio of kVA actually utilized to those available is  $= 1.732 V_L I_L / 1.866 V_L I_L = 0.928$ .

Fig. 33.15 shows the secondary of the  $T-T$  connection with its different voltages based on a nominal voltage of 100 V. As seen, the neutral point  $n$  is one third way up from point  $d$ . If secondary voltage and current vector diagram is drawn for load power factor of unity, it will be found that

1. current in teaser transformer is in phase with the voltage.
2. in the main transformer, current leads the voltage by  $30^\circ$  across one half but lags the voltage by  $30^\circ$  across the other half as shown in Fig. 33.14 (b).

Hence, when a balanced load of p.f. =  $\cos \phi$ , is applied, the teaser current will lag or lead the voltage by  $\Phi$  while in the two halves of the main transformer, the angle between current and voltage will be  $(30^\circ - \Phi)$  and  $(30^\circ + \Phi)$ . The situation is similar to that existing in a  $V-V$  connection.

**Example 33.17.** Two  $T$ -connected transformers are used to supply a 440-V, 33-kVA balanced load from a balanced 3-phase supply of 3300 V. Calculate (a) voltage and current rating of each coil (b) kVA rating of the main and teaser transformer.

**Solution.** (a) Voltage across main primary is 3300 V whereas that across teaser primary is  $= 0.866 \times 3300 = 2858$  V.

The current is the same in the teaser and the main and equals the line current.

$$\therefore I_{LP} = 33,000 / \sqrt{3} \times 3300 = 5.77 \text{ A} \quad \text{---Fig. 33.16}$$

The secondary main voltage equals the line voltage of 440 V whereas teaser secondary voltage  $= 0.866 \times 440 = 381$  V.

The secondary line current,  $I_{LS} = I_{LP} / k = 5.77 / (440 / 3300) = 43.3$  A as shown in Fig. 33.16.

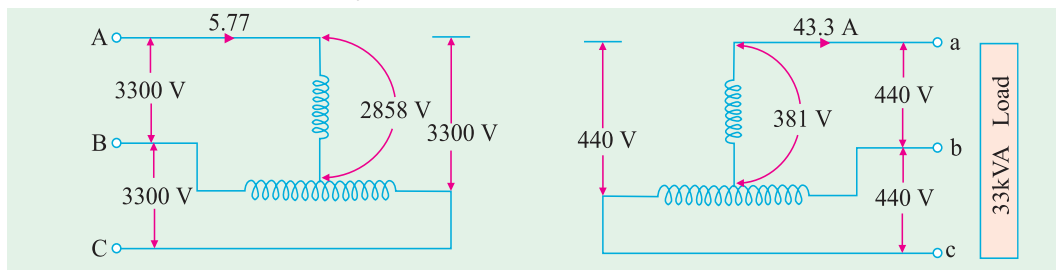


Fig. 33.16

- (b) Main kVA  $= 3300 \times 5.77 \times 10^{-3} = 19 \text{ kVA}$   
 Teaser kVA  $= 0.866 \times \text{main kVA} = 0.866 \times 19 = 16.4 \text{ kVA}$

### 33.10. Three-phase to Two-phase Conversion and vice-versa

This conversion is required to supply two-phase furnaces, to link two-phase circuit with 3-phase system and also to supply a 3-phase apparatus from a 2-phase supply source. For this purpose, Scott connection as shown in Fig. 33.17 is employed. This connection requires two transformers of different ratings although for interchangeability and provision of spares, both transformers may be identical but having suitable tapplings.

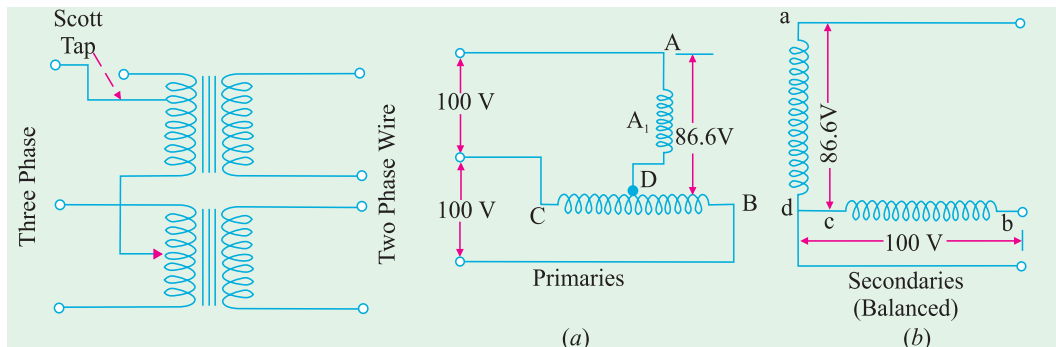


Fig. 33.17

Fig. 33.18



If, in the secondaries of Fig. 33.14 (b), points *c* and *d* are connected as shown in Fig. 33.18 (b), then a 2-phase, 3-wire system is obtained. The voltage  $E_{dc}$  is 86.6 V but  $E_{Cb} = 100$  V, hence the

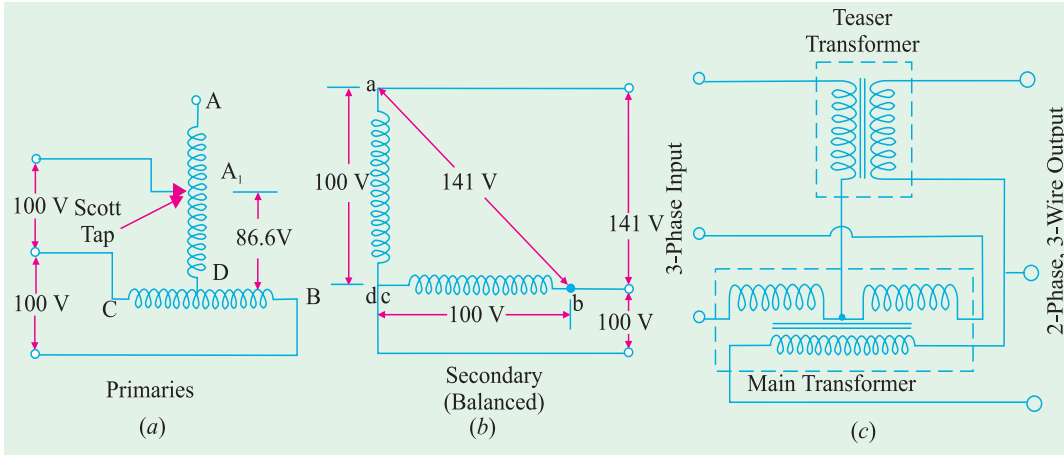


Fig. 33.19

resulting 2-phase voltages will be unequal. However, as shown in Fig. 33.19 (a) if the 3-phase line is connected to point  $A_1$ , such that  $DA_1$  represents 86.6% of the teaser primary turns (which are the same as that of main primary), then this will increase the volts/turn in the ratio of 100 : 86.6, because now 86.6 volts are applied across 86.6 per cent of turns and not 100% turns. In other words, this will make volts/turn the same both in primary of the teaser and that of the main transformer. If the secondaries of both the transformers have the same number of turns, then secondary voltage will be equal in magnitude as shown, thus resulting in a symmetrical 2-phase, 3-wire system.

Consider the same connection drawn slightly differently as in Fig. 33.20. The primary of the main transformer having  $N_1$  turns is connected between terminals *CB* of a 3-phase supply. If supply line voltage is  $V$ , then obviously  $V_{AB} = V_{BC} = V_{CA} = V$  but voltage between *A* and *D* is  $V \times \sqrt{3}/2$ . As said

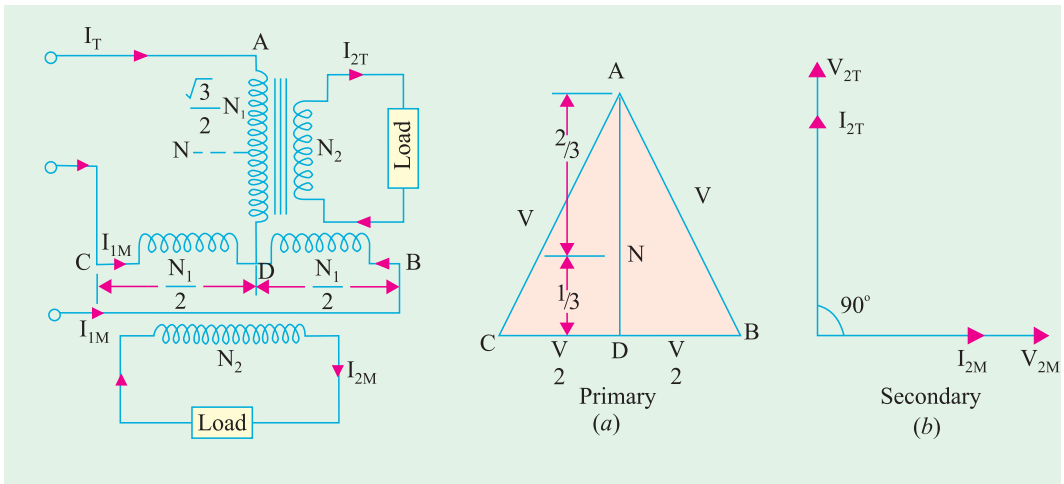


Fig. 33.20

Fig. 33.21

above, the number of turns between *A* and *D* should be also  $(\sqrt{3}/2) N_1$  for making volt/turn the same in both primaries. If so, then for secondaries having equal turns, the secondary terminal voltages will be equal in magnitude although in phase quadrature.

It is to be noted that point  $D$  is not the neutral point of the primary supply because its voltage with respect to any line is not  $V/\sqrt{3}$ . Let  $N$  be the neutral point. Its position can be determined as follows. Voltage of  $N$  with respect to  $A$  must be  $V/\sqrt{3}$  and since  $D$  to  $A$  voltage is  $V \times \sqrt{3}/2$ , hence  $N$  will be  $(\sqrt{3}V/2 - V/\sqrt{3}) = 0.288V$  or  $0.29V$  from  $D$ . Hence,  $N$  is above  $D$  by a number of turns equal to 29% of  $N_1$ . Since 0.288 is one-third of 0.866, hence  $N$  divides the teaser winding  $AD$  in the ratio 2 : 1.

Let the teaser secondary supply a current  $I_{2T}$  at unity power factor. If we neglect the magnetizing current  $I_0$ , then teaser primary current is  $I_{1T} = I_{2T} \times$  transformation ratio.

$\therefore I_{1T} = I_{2T} \times N_2/(\sqrt{3}N_1/2) = (2/\sqrt{3}) \times (N_2/N_1) \times I_{2T} = 1.15 (N_2/N_1) I_{2T} = 1.15 KI_{2T}$  where  $K = N_2/N_1 =$  transformation ratio of **main** transformer. The current is in phase with star voltage of the primary supply (Fig. 33.21).

The total current  $I_{1M}$  in each half of the primary of the **main** transformer consists of two parts :

(i) One part is that which is necessary to balance the main secondary current  $I_{2M}$ . Its value is

$$= I_{2M} \times \frac{N_2}{N_1} = KI_{2M}$$

(ii) The second part is equal to one-half of

the teaser primary current i.e.  $\frac{1}{2} I_{1T}$ . This is so because the main transformer primary forms a return path for the teaser primary current which divides itself into two halves at mid-point  $D$  in either direction. The value of each half is  $= I_{1T}/2 = 1.15 KI_{2T}/2 = 0.58 KI_{2T}$ .

Hence, the currents in the lines  $B$  and  $C$  are obtained vectorially as shown in Fig. 33.22. It should be noted that as the two halves of the teaser primary current flow in opposite directions from point  $D$ , they have no magnetic effect on the core and play no part at all in balancing the secondary ampere-turns of the main transformer.

The line currents thus have rectangular components of  $KI_{2M}$  and  $0.58 KI_{2T}$  and, as shown in Fig. 33.22, are in phase with the primary star voltages  $V_{NB}$  and  $V_{NC}$  and are equal to the teaser primary current. Hence, the three-phase side is balanced when the two-phase load of unity power factor is balanced.

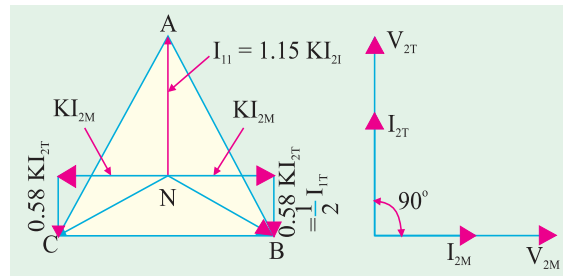


Fig. 33.22

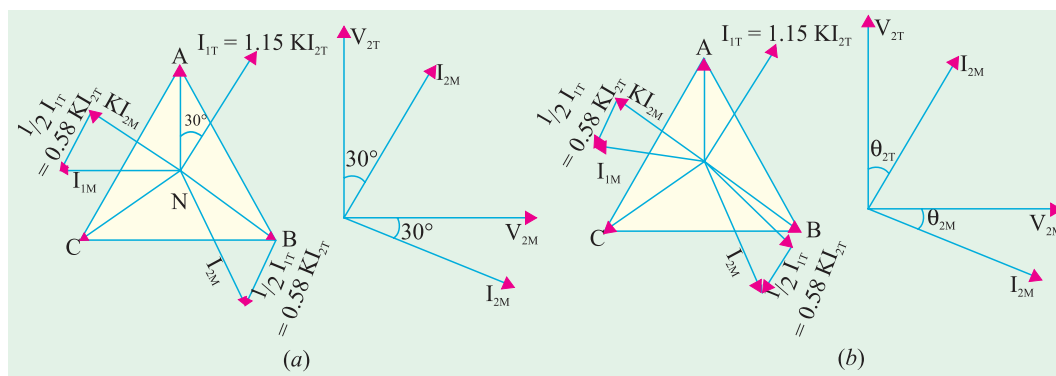


Fig. 33.23

Fig. 33.23 (a) illustrates the condition corresponding to a balanced two-phase load at a lagging power factor of 0.866. The construction is the same as in Fig. 33.22. It will be seen that the 3-phase side is again balanced. But under these conditions, the main transformer rating is 15% greater than that of the teaser, because its voltage is 15% greater although its current is the same.

Hence, **we conclude that if the load is balanced on one side, it would always be balanced on the other side.**

The conditions corresponding to an unbalanced two-phase load having different currents and power factors are shown in Fig. 33.23 (b). The geometrical construction is similar to those explained in Fig. 33.22 and 33.23 (a).

Summarizing the above we have :

1. Teaser transformer primary has  $\sqrt{3}/2$  times the turns of main primary. But volt/turn is the same. Their secondaries have the same turns which results in equal secondary terminal voltages.

2. If main primary has  $N_1$  turns and main secondary has  $N_2$  turns, then main transformation ratio is  $N_2/N_1$ . However, the transformation ratio of teaser is

$$N_2/(\sqrt{3}N_1/2) = 1.15 N_2/N_1 = 1.15 K$$

3. If the load is balanced on one side, it is balanced on the other side as well.

4. Under balanced load conditions, main transformer rating is 15% greater than that of the teaser.

5. The currents in either of the two halves of main primary are the vector sum of  $KI_{2M}$  and  $0.58 KI_{2T}$  (or  $\frac{1}{2} I_{1T}$ ).

**Example 33.18.** Two transformers are required for a Scott connection operating from a 440-V, 3-phase supply for supplying two single-phase furnaces at 200 V on the two-phase side. If the total output is 150 kVA, calculate the secondary to primary turn ratio and the winding currents of each transformer.

**Solution. Main Transformer**

Primary volts = 440 V ; secondary volts = 200 V  $\therefore \frac{N_2}{N_1} = \frac{200}{440} = \frac{1}{2.2}$

Secondary current =  $150,000/2 \times 200 = 375 \text{ A}$

$\therefore$  Primary current =  $375 \times 1/2.2 = 171 \text{ A}$

**Teaser Transformer**

Primary volts =  $(\sqrt{3}/2 \times 440) = 381 \text{ V}$  ; Secondary volts = 200 V

$$\frac{\text{secondary turns}}{\text{primary turns}} = \frac{200}{381} = \frac{1}{1.905} \text{ (also teaser ratio} = 1.15 \times 1/2.2 = 1/1.905)$$

**Example 33.19.** Two single-phase furnaces working at 100 V are connected to 3300-V, 3-phase mains through Scott-connected transformers. Calculate the current in each line of the 3-phase mains when the power taken by each furnace is 400-kW at a power factor of 0.8 lagging. Neglect losses in the transformers. **(Elect. Machines-III, South Gujarat Univ. 1988)**

**Solution.** Here  $K = 100/3,300 = 1/33$  (main transformer)

$$I_2 = \frac{400,000}{0.8 \times 100} = 5,000 \text{ A (Fig. 33.24); Here } I_{2T} = I_{2M} = I_2 = 5,000 \text{ A}$$

As the two-phase load is balanced, the 3-phase side is also balanced.

Primary phase currents are =  $1.15 KI_2 = 1.15 \times (1/33) \times 5,000 = 174.3 \text{ A}$

Since for a star-connection, phase current is equal to line current,

$\therefore$  Line current = **174.3 A**

**Note.** We have made use of the fact that since secondary load is balanced, primary load is also balanced. If necessary,  $I_{1M}$  can also be found.

$I_{1M}$  is the vector sum of (i)  $KI_{2M}$  and (ii)  $\frac{1}{2} I_{1T}$  or  $0.58 KI_{2T}$ .

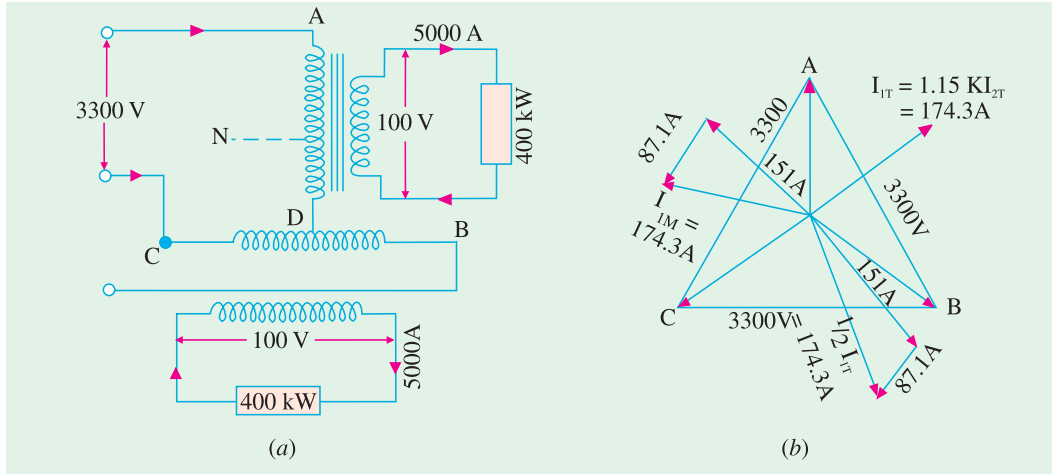


Fig. 33.24

Now,  $KI_{2M} = (1/33) \times 5,000 = 151 \text{ A}$  and  $0.58 KI_2 = \frac{1}{2} I_{1T} = 174.3/2 = 87.1 \text{ A}$

$$\therefore I_{1M} = \sqrt{151^2 + 87.1^2} = 174.3 \text{ A}$$

**Example 33.20.** In a Scott-connection, calculate the values of line currents on the 3-phase side if the loads on the 2-phase side are 300 kW and 450 kW both at 100 V and 0.707 p.f. (lag) and the 3-phase line voltage is 3,300 V. The 300-kW load is on the leading phase on the 2-phase side. Neglect transformer losses. **(Elect. Technology, Allahabad Univ. 1991)**

**Solution.** Connections are shown in Fig. 33.25 (a) and phasor diagram in Fig. 33.25 (b).

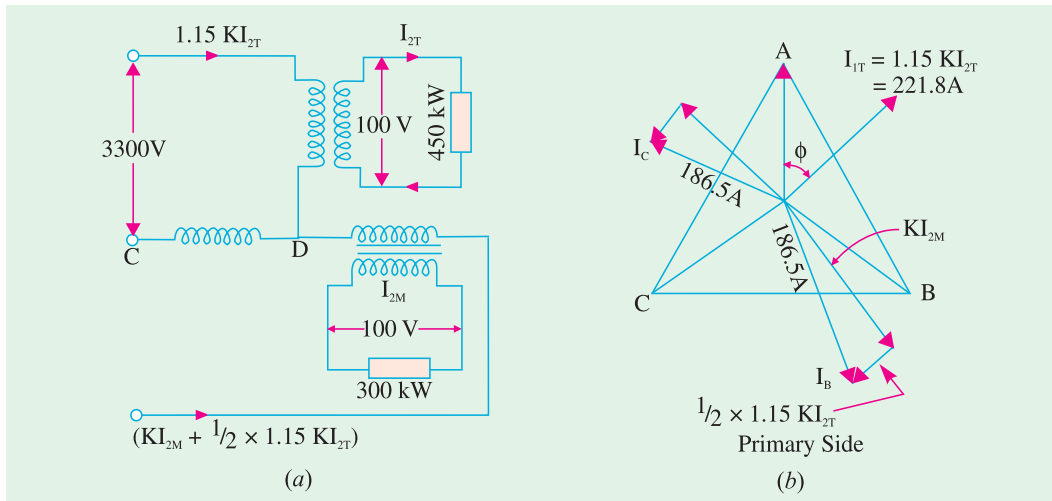


Fig. 33.25

Here,  $K = 100/3,300 = 1/33$

Teaser secondary current is  $I_{2T} = 450,000/100 \times 0.707 = 6360 \text{ A}$

Teaser primary current is  $I_{1T} = 1.15 KI_{2T} = 1.5 \times (1/33) \times 6360 = 221.8 \text{ A}$

As shown in Fig. 33.25 (b), main primary current  $I_{1M}$  has two rectangular components.

(i)  $KI_{2M}$  where  $I_{2M}$  is the secondary current of the main transformer and

(ii) Half of the teaser primary current  $\frac{1}{2} I_{1T} = \frac{1}{2} \times 1.15 KI_{2T} = 0.577 KI_{2T}$

Now  $KI_{2M} = \frac{1}{33} \times \frac{300,000}{100 \times 0.707} = 128.58 \text{ A}$ ; Also  $\frac{1}{2} I_{1T} = \frac{1}{2} \times 221.8 = 110.9 \text{ A}$

Main Primary current =  $\sqrt{128.58^2 + 110.9^2} = 169.79 \text{ A}$

Hence, the 3-phase line currents are **221.8 A** in one line and **169.79 A** in each of the other two.

**Example 33.21.** Two electric furnaces are supplied with 1-phase current at 80 V from a 3-phase, 11,000 V system by means of two single-phase Scott-connected transformers with similar secondary windings. When the load on one furnace is 500 kW (teaser secondary) and on the other 800 kW (secondary of main transformer) what current will flow in each of the 3-phase lines (a) at unity power factor and (b) at 0.5 power factor? Neglect phase displacement in and efficiency of, the transformers. **(Electrical Engineering, Madras Univ. 1987)**

**Solution.** The connections are shown in Fig. 33.26 and the phasor diagrams for unity and 0.5 p.f. are shown in Fig. 33.27 (a) and (b) respectively.

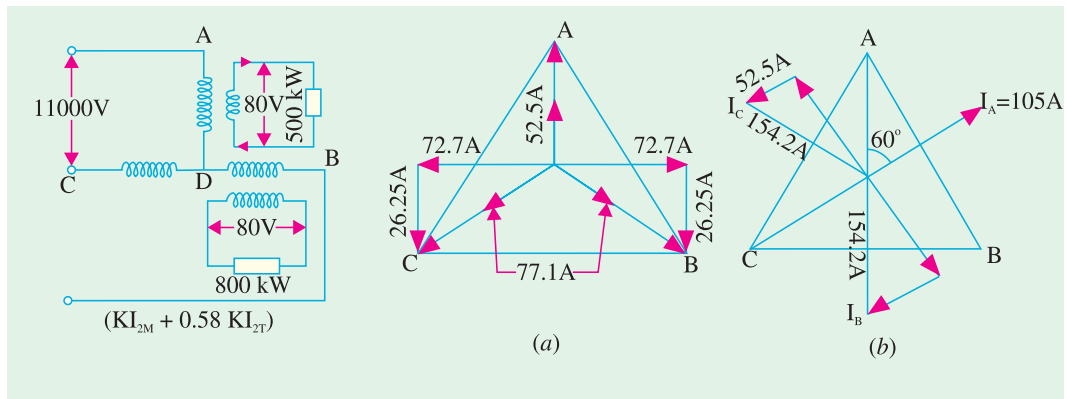


Fig. 33.26

Fig. 33.27

Here,

$$K = 80/11,000 = 2/275$$

(a) **Unity p.f.**

With reference to Fig. 33.27 (a), we have  $I_{2T} = 500,000/80 \times 1 = 6,250 \text{ A}$

Teaser primary current  $I_{1T} = 1.15 KI_{2T} = 1.15 \times (2/275) \times 6,250 = 52.5 \text{ A}$

For the main transformer primary

(i)  $KI_{2M} = \frac{2}{275} \times \frac{800,000}{80 \times 1} = 72.7 \text{ A}$  (ii)  $\frac{1}{2} \times I_{1T} = 52.5/2 = 26.25 \text{ A}$

Current in the primary of the main transformer is  $= \sqrt{72.7^2 + 26.25^2} = 77.1 \text{ A}$

Hence, one 3-phase line carries **52.5 A** whereas the other 2 carry **77.1 A** each [Fig. 33.27 (a)].

(b) **0.5 p.f.**

With reference to Fig. 33.27 (b) we have  $I_{2T} = 500,000/80 \times 0.5 = 12,500 \text{ A}$

Teaser primary current  $I_{1T} = 1.15 \times (2/275) \times 12,500 = 105 \text{ A}$

For the main transformer primary

(i) 
$$KI_{2M} = \frac{2}{275} \times \frac{800,000}{80 \times 0.5} = 145.4 \text{ A} \quad (ii) \quad \frac{1}{2} I_{1T} = 105/2 = 52.5 \text{ A}.$$

Current in the primary of the main transformer is  $= \sqrt{145.4^2 + 52.5^2} = 154.2 \text{ A}.$

Hence, one 3-phase line carries **105 A** and the other two carry **154.2 A** each.

**Note :** Part (b) need not be worked out in full because at 0.5 p.f., each component current and hence the resultant are doubled. Hence, in the second case, answers can be found by multiplying by a factor of 2 the line currents found in (a).

**Example 33.22.** Two furnaces are supplied with 1-phase current at 50 V from a 3-phase, 4.6 kV system by means of two 1-phase, Scott-connected transformers with similar secondary windings. When the load on the main transformer is 350 kW and that on the other transformer is 200 kW at 0.8 p.f. lagging, what will be the current in each 3-phase line ? Neglect phase displacement and losses in transformers. **(Electrical Machinery-II, Bangalore Univ. 1991)**

**Solution.** Connections and vector diagrams are shown in Fig. 33.28.

$$K = 50/4,600 = 1/92; \quad I_{2T} = 200,000/50 \times 0.8 = 5,000 \text{ A}$$

$$I_{1T} = 1.15 KI_{2T} = 1.1 \times (1/92) \times 5,000 = 62.5 \text{ A}$$

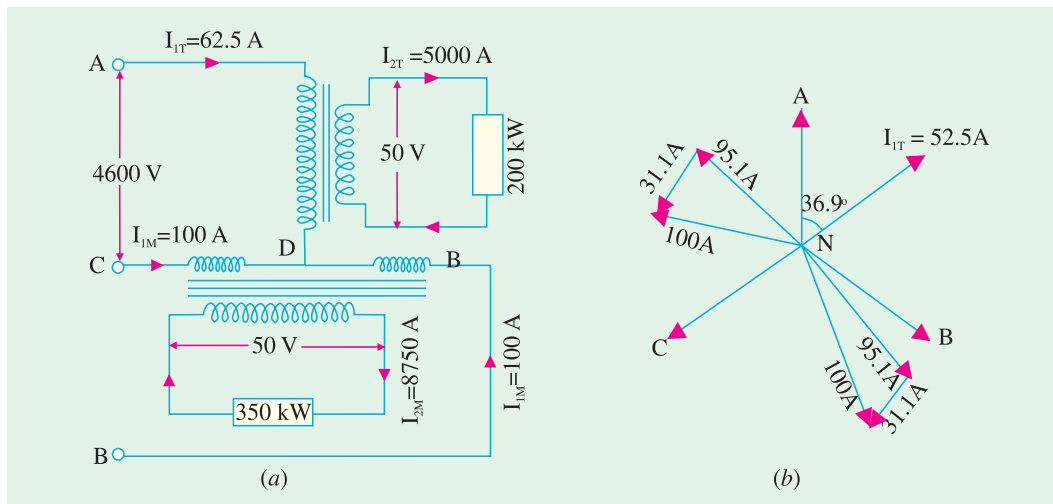


Fig. 33.28

As shown in Fig. 33.28 (b), main primary current  $I_{1M}$  has two rectangular components.

(i)  $KI_{2M}$  where  $I_{2M} = 350,000/50 \times 0.8 = 8,750 \text{ A} \therefore KI_{2M} = 8,750/92 = 95.1 \text{ A}$

(ii)  $(1/2) I_{1T} = 62.5/2 = 31.3 \text{ A} \quad \therefore I_{1M} = \sqrt{95.1^2 + 31.3^2} = 100 \text{ A}$

$\therefore$  Current in line A = **62.5 A**; Current in line B = **100 A**; Current in line C = **100 A**.

**Example 33.23.** Two single-phase Scott-connected transformers supply a 3-phase four-wire distribution system with 231 volts between lines and the neutral. The h.v. windings are connected to a two-phase system with a phase voltage of 6,600 V. Determine the number of turns in each section of the h.v. and l.v. winding and the position of the neutral point if the induced voltage per turn is 8 volts.

**Solution.** As the volt/turn is 8 and the h.v. side voltage is 6,600 V, the h.v. side turns are  $= 6,600/8 = 825$  on both transformers.

Now, voltage across points *B* and *C* of main winding  
 = line voltage =  $231 \times \sqrt{3} = 400 \text{ V}$

No. of turns on the l.v. side of the main transformer =  $400/8 = 50$

No. of turns on the l.v. side of teaser transformer =  $\sqrt{(3/2)} \times \text{mains turns}$   
 =  $\sqrt{3} \times 50 / 2 = 43$  (whole number)

The neutral point on the 3-phase side divides teaser turns in the ratio 1 : 2.

$\therefore$  Number of turns between *A* and *N* =  $(2/3) \times AD$   
 =  $(2/3) \times 43 = 29$

Hence, neutral point is located on the 29th turn from *A* downwards (Fig. 33.29).

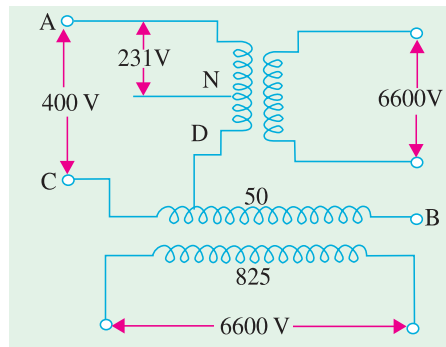


Fig. 33.29

**Example 33.24.** A Scott-connected (2 to 3-phase) transformer links a 6,000 V, 2-phase system with a 440 V, 3-phase system. The frequency is 50 Hz, the gross core area is  $300 \text{ cm}^2$ , while the maximum flux density is to be about  $1.2 \text{ Wb/m}^2$ . Find the number of turns on each winding and the point to be tapped for the neutral wire on the 3-phase side. If the load is balanced on the one side of such a transformer, find whether it will also be balanced on the other side. (London Univ.)

**Solution.** Use the transformer voltage equation 1,

$$E = 4.44 f N \Phi_x \text{ volt}$$

$$\text{Gross core area} = 300 \text{ cm}^2$$

Assuming net iron = 0.9 of gross area, and considering the h.v. side, we have

$$6000 = 4.44 \times 50 \times N_1 \times 1.2 \quad (300 \times 0.9 \times 10^{-4})$$

$$N_1 = 834$$

Hence, h.v. sides of both transformers have 834 turns each.

$$\text{Now } K = 440/6000 = 11/150$$

$\therefore$  Turns on the l.v. side of main transformer

$$N_2 = 834 \times 11/150 = 61$$

$$\text{Turns on the l.v. side of teaser} = (\sqrt{3}/2) \times 61 = 53$$

$$\text{With reference to Fig. 33.30, number of turns in } AN = 53 \times 2/3 = 35$$

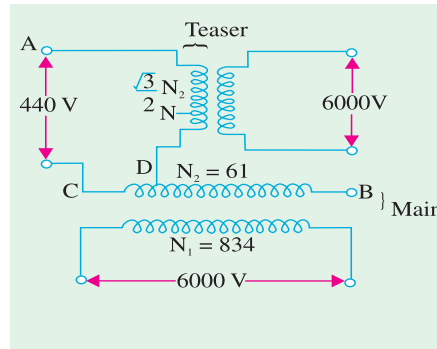


Fig. 33.30

**Example 33.25.** A 2-phase, 4-wire, 250 V system is supplied to a plant which has a 3-phase motor load of 30 kVA. Two Scott-connected transformers supply the 250 V motors. Calculate (a) voltage (b) kVA rating of each transformer. Draw the wiring connection diagram.

**Solution. (a)** Both the main and the teaser have the same voltage rating as the supply voltage i.e. 250 V. The current in the main and the teaser coils is the same as the supply current and is

$$= \frac{\text{Total kVA}}{2 \times \text{Line voltage}} = \frac{30,000}{2 \times 250} = 60 \text{ A arcsin } \theta$$

On the three-phase side, current is the same in all coils and is equal to the load line current  
 =  $30,000 / \sqrt{3} \times 250 = 69.3 \text{ A}$

Load voltage on main secondary = line voltage = 250 V

Load voltage on teaser secondary =  $0.866 \times 250 = 216.5 \text{ V}$

Hence, voltage rating of main transformer is **250/250** whereas that of teaser transformer is **250/216.5**.

The current rating of main transformer is **60/69.3** and it is the same for the teaser transformer.

(b) The volt-amp rating of the teaser primary as well as secondary is the same *i.e.*  $60 \times 250 \times 10^{-3} = 69.3 \times 216.5 \times 10^{-3} = \mathbf{15 \text{ kVA}}$

The main volt-ampere rating of secondary is  $= 250 \times 69.3 \times 10^{-3} = \mathbf{17.3 \text{ kVA}}$

Incidentally, if two identical transformers are used for providing inter-changeability, then both must be rated at 17.3 kVA. In that case, a total capacity of 34.6 kVA would be required to provide a 30 kVA load.

The wiring connections are shown in Fig. 33.31.

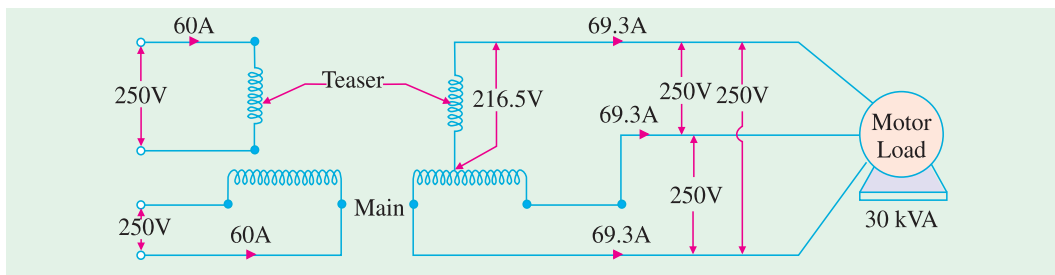


Fig. 33.31

### Tutorial Problem No. 33.3

1. A Scott-connected transformer is fed from a 6,600-V, 3-phase network and supplies two single-phase furnaces at 100 V. Calculate the line currents on the 3-phase side when the furnaces take 400 kW and 700 kW respectively at 0.8 power factor lagging. (Elect. Machines II, Indore Univ. 1977)

[With 400 kW on teaser, line currents are 87.2 A; 139 A; 139 A]

2. Two 220-V, 1-phase electrical furnaces take loads of 350 kW and 500 kW respectively at a power factor of 0.8 lagging. The main supply is at 11-kV, 3-phase, 50 Hz. Calculate current in the 3-phase lines which energise a Scott-connected transformer combination. (Elect. Machines, Madras Univ. 1978)

[With 350 kW on teaser line currents are : 45.7 A ; 61.2 A ; 61.2 A]

3. Two electric furnaces are supplied with 1-phase current at 80 V from 3-phase, 11,000-V supply mains by means of two Scott-connected transformers with similar secondary windings. Calculate the current flowing kW respectively in each of the 3-phase lines at U.P.P. when the loads on the two transformers are 550 kW of 800 kW.

[With 550 kW on teaser, line currents are : 57.5 A; 78.2; 78.2 A] (Electrical Machines-I, Madras University, 1977)

### 33.11. Parallel Operation of 3-phase Transformers

All the conditions which apply to the parallel operation of single-phase transformers also apply to the parallel running of 3-phase transformers but with the following additions :

1. The voltage ratio must refer to the terminal **voltage of primary and secondary**. It is obvious that this ratio may not be equal to the ratio of the number of turns per phase. For example, if  $V_1$ ,  $V_2$  are the primary and secondary terminal voltages, then for  $Y/\Delta$  connection, the turn ratio is  $V_2 / (V_1 / \sqrt{3})$

$$= \sqrt{3} V_2 / V_1.$$

2. The phase displacement between primary and secondary voltages must be the same for all transformers which are to be connected for parallel operation.

3. The phase sequence must be the same.



4. All three transformers in the 3-phase transformer bank will be of the same construction either core or shell.

**Note. (i)** In dealing with 3-phase transformers, calculations are made for one phase only. The value of equivalent impedance used is the equivalent impedance per phase referred to secondary.

**(ii)** In case the impedances of primary and secondary windings are given separately, then primary impedance must be referred to secondary by multiplying it with (transformation ratio)<sup>2</sup>.

**(iii)** For Y/Δ or Δ/Y transformers, it should be remembered that the voltage ratios as given in the questions, refer to terminal voltages and are quite different from turn ratio.

**Example 33.26.** A load of 500 kVA at 0.8 power factor lagging is to be shared by two three-phase transformers A and B of equal ratings. If the equivalent delta impedances as referred to secondary are  $(2 + j6) \Omega$  for A and  $(2 + j5) \Omega$  for B, calculate the load supplied by each transformer.

**Solution.** 
$$S_A = S \frac{Z_B}{Z_A + Z_B} = S \frac{1}{1 + (Z_A/Z_B)}$$

Now 
$$S = 500 (0.8 - j0.6) = (400 - j300)$$

$$Z_A/Z_B = (2 + j6)/(2 + j5) = 1.17 + j0.07; Z_B/Z_A = (2 + j5)/(2 + j6) = 0.85 - j0.05$$

$$S_A = (400 - j300)/(2.17 + j0.07) = 180 - j144.2 = 230.7 \angle -38.7^\circ$$

$$\cos \phi_A = 0.78 \text{ lagging}$$

$$S_B = (400 - j300)/(1.85 - j0.05) = 220.1 - j156 = 270 \angle -40^\circ 28' \therefore \cos \phi_B = 0.76 \text{ lagging.}$$

**Example 33.27.** State (i) the essential and (ii) the desirable conditions to be satisfied so that two 3-phase transformers may operate successfully in parallel.

A 2,000-kVA transformer (A) is connected in parallel with a 4,000 kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage voltage drops in the windings at their rated loads are as follows :

Transformer A	resistance 2% ;	reactance 8 %
Transformer B	resistance 1.6 % ;	reactance 3 %

(Elect. Engineering-II, Bombay Univ. 1987)

**Solution.** On the basis of 4,000 kVA

$$\% Z_A = (4,000/2,000) (2 + j8) = (4 + j16) = 16.5 \angle 76^\circ$$

$$\% Z_B = (1.6 + j3); \% Z_A + \% Z_B = (5.6 + j16) = 19.8 \angle 73.6^\circ$$

$$S = 5,000 \angle -36.9^\circ = (4,000 - j3,000)$$

Now 
$$S_B = S \frac{Z_A}{Z_A + Z_B} = 5,000 \angle -36.9^\circ \times \frac{16.5 \angle 76^\circ}{19.8 \angle 73.6^\circ}$$

$$= 5,000 \angle -36.9^\circ \times 0.832 \angle 2.4^\circ = 4,160 \angle -34.5^\circ = (3,425 - j2,355)$$

$$S_A = S - S_B = (4,000 - j3,000) - (3,425 - j2,355)$$

$$= (575 - j645) = 864 \angle -48.3^\circ$$

$$\cos \phi_B = \cos 34.5^\circ = 0.824 \text{ (lag)}; \cos \phi_A = \cos 48.3^\circ = 0.665 \text{ (lag).}$$

**Example 33.28.** A load of 1,400 kVA at 0.866 p.f. lagging is supplied by two 3-phase transformers of 1,000 kVA and 500 kVA capacity operating in parallel. The ratio of transformation is the same in both : 6,600/400 delta-star. If the equivalent secondary impedances are  $(0.001 + j 0.003)$  ohm and  $(0.0028 + j0.005)$  ohm per phase respectively, calculate the load and power factor of each transformer.

(Elect. Engg-I, Nagpur Univ. 1993)

**Solution.** On the basis of 1000 kVA,  $Z_A = (0.001 + j0.003) \Omega$

$$Z_B = (1000/500) (0.0028 + j0.005) = (0.0056 + j0.01) \Omega$$

$$\frac{Z_A}{Z_A + Z_B} = \frac{(0.001 + j0.003)}{(0.0066 + j0.013)} = \frac{3.162 \times 10^{-3} \angle 71.6^\circ}{14.57 \times 10^{-3} \angle 63.1^\circ} = 0.2032 \angle 8.5^\circ$$

$$S = 1400 \angle \cos^{-1}(0.866) = 1400 \angle -30^\circ = (1212 - j700)$$

$$S_B = S \cdot \frac{Z_A}{Z_A + Z_B} = 1400 \angle -30^\circ \times 0.2032 \angle 8.5^\circ$$

$$= 284.5 \angle -21.5^\circ = 265 - j104$$

$$S_A = S - S_B = (1212 - j700) - (265 - j104) = (947 - j596) = 1145 \angle -32.2^\circ$$

$$\cos \phi_A = \cos 32.3^\circ = 0.846 \text{ (lag)}; \cos \phi_B = \cos 21.5^\circ = \mathbf{0.93 \text{ (lag)}}.$$

**Example 33.29.** Two 3-phase transformers A and B having the same no-load line voltage ratio 3,300/400-V supply a load of 750 kVA at 0.707 lagging when operating in parallel. The rating of A is 500 kVA, its resistance is 2% and reactance 3%. The corresponding values for B are 250 kVA; 1.5% and 4% respectively. Assuming that both transformers have star-connected secondary windings, calculate

- the load supplied by each transformer,
- the power factor at which each transformer is working,
- the secondary line voltage of the parallel circuit.

**Solution.** On the basis of 500 kVA,

$$\% Z_A = 2 + j3, \% Z_B = (500/200) (1.5 + j4) = (3 + j8)$$

$$\frac{Z_A}{Z_A + Z_B} = \frac{2 + j3}{5 + j1} = 0.3 \angle -9.3^\circ; \frac{Z_B}{Z_A + Z_B} = \frac{3 + j8}{5 + j11} = 0.711 \angle 3.8^\circ$$

Now,  $S = 750 \angle -45^\circ$

$$(a) \quad S_A = S \frac{Z_B}{Z_A + Z_B} = 750 \angle -45^\circ \times 0.711 \angle 3.8^\circ$$

$$= 533 \angle -41.2^\circ = (400 - j351)$$

$$S_B = 750 \angle -45^\circ \times 0.3 \angle 9.3^\circ = 225 \angle -54.3^\circ$$

$$(b) \quad \cos \phi_A = \cos 41.2^\circ = 0.752 \text{ (lag)}; \cos \phi_B = \cos 54.3^\circ = \mathbf{0.5835 \text{ (lag)}}$$

(c) Since voltage drop of each transformer is the same, its value in the case of transformer A would only be calculated. Now, for transformer A, kW = 400 for the active component of the current and kVAR = 351 for the reactive component.

$$\therefore \% \text{ resistive drop} = 2 \times 400/500$$

$$= 1.6\%; \% \text{ reactive drop} = 3 \times 351/500 = 2.1\%$$

$$\text{Total percentage drop} = 1.6 + 2.1 = 3.7$$

$$\text{Secondary line voltage} = 400 - (3.7 \times 400/100) = \mathbf{385.2 \text{ V}}.$$

### 33.12. Instrument Transformers

In d.c. circuit when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with a high resistance connected

in series with them. But it is not convenient to use these methods with alternating current and voltage instruments. For this purpose, specially constructed accurate ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. These instrument transformers are of two kinds : (i) current transformers for measuring large alternating currents and (ii) potential transformers for measuring high alternating voltages.

### 33.13. Current Transformers

These transformers are used with low-range ammeters to measure currents in high-voltage alternating-current circuits where it is not practicable to connect instruments and meters directly to the lines. In addition to insulating the instrument from the high voltage line, they step down the current in a known ratio. The current (or series) transformer has a primary coil of one or more turns of thick wire connected in **series** with the line whose current is to be measured as shown in Fig. 33.32. The secondary consists of a large number of turns of fine wire and is connected across the ammeter terminals (usually of 5-ampere bracket should be removed or 1-ampere range).



Current transformers

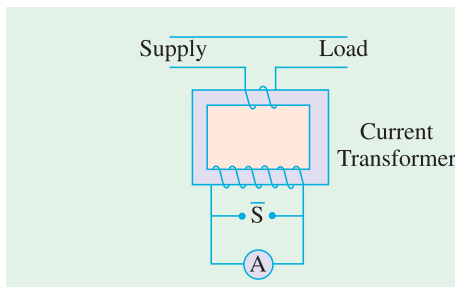


Fig. 33.32

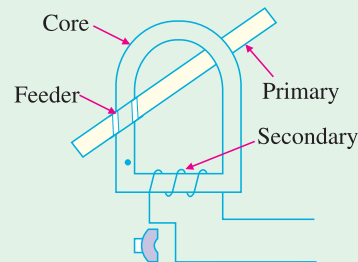


Fig. 33.33

As regards voltage, the transformers is of step-up variety but it is obvious that current will be stepped down. Thus, if the current transformer has **primary to secondary** current ratio of 100 : 5, then it steps up the voltage 20 times whereas it steps down the current to 1/20th of its actual value. Hence, if we know current ratio ( $I_1/I_2$ ) of the transformer and the reading of the a.c. ammeter, the line current can be calculated. In fact, line current is given by the current transformation ratio times the reading on the ammeter. One of the most commonly used current transformer is the one known as **clamp-on or clip-on type**. It has a laminated core which is so arranged that it can be opened out at hinged section by merely pressing a trigger-like projection (Fig. 33.33). When the core is thus opened, it permits the admission of very heavy current-carrying bus bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current carrying conductor or feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle.

It should be noted that, since the ammeter resistance is very low, the current transformer normally works short circuited. If for any reason, the ammeter is taken out of the secondary winding, then this winding



Small current transformer

must be short-circuited with the help of short-circuiting switch  $S$ . If this is not done, then due to the absence of counter amp-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating and a high voltage across the secondary terminals. This is not the case with ordinary constant-potential transformers, because their primary current is determined by the load in their secondary whereas in a current transformer, the primary current is determined entirely by the load on the system and not by the load on its own secondary.

Hence, **the secondary of a current transformer should never be left open under any circumstances.**

**Example 33.30.** A 100 : 5 transformer is used in conjunction with a 5-amp ammeter. If the latter reads 3.5 A, find the line current.

**Solution.** Here, the ratio 100 : 5 stands for the ratio of primary-to-secondary currents *i.e.*  $I_1/I_2 = 100/5$

$\therefore$  Primary (or line) current =  $3.5 \times (100/5) = 70 \text{ A}$

**Example 33.31.** It is desired to measure a line current of the order of 2,000 A to 2,500 A. If a standard 5-amp ammeter is to be used along with a current transformer, what should be the turn ratio of the latter? By what factor should the ammeter reading be multiplied to get the line current in each case?

**Solution.**  $I_1/I_2 = 2000/5 = 400$  or  $2500/5 = 500$ . Since  $I_1/I_2 = N_2/N_1$  hence  $N_2/N_1 = 400$  in the first case and 500 in the second case. It means that  $N_1 : N_2 :: 1 : 400$  or  $1 : 500$ .

Ratio or multiplication factor in the first case is 400 and in the second in the second case 500.

### 33.14. Potential Transformers

These transformers are extremely accurate-ratio step-down transformers and are used in conjunction with standard low-range voltmeters (usually 150-V) whose deflection when divided by voltage transformation ratio, gives the true voltage on the high voltage side. In general, they are of the shell-type and do not differ much from the ordinary two-winding transformers discussed so far, except that their power rating is extremely small. Upto voltages of 5,000, potential transformers are usually of the dry type, between 5,000 and 13,800 volts, they may be either dry type or oil immersed type, although for voltages above 13,800 they are always oil immersed type. Since their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100 W. For safety, the secondary should be completely insulated from the high-voltage primary and should be, in addition, grounded for affording protection to the operator. Fig. 33.34 shows the connections of such a transformer.



Small Potential transformer

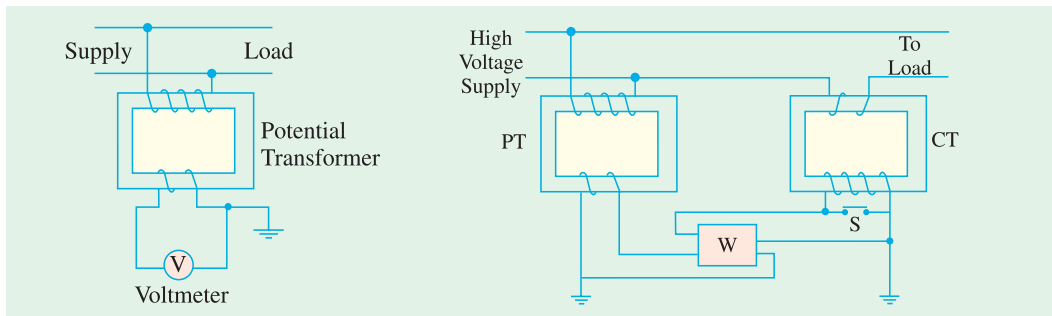


Fig. 33.34

Fig. 33.35

Fig. 33.35 shows the connections of instrument transformers to a wattmeter. While connecting the wattmeter, the relative polarities of the secondary terminals of the transformers with respect to their primary terminals must be known for connections of the instruments.

OBJECTIVE TEST – 33

- Which of the following connections is best suited for 3-phase, 4-wire service ?  
 (a)  $\Delta - \Delta$  (b) Y - Y  
 (c)  $\Delta - Y$  (d) Y -  $\Delta$
- In a three-phase Y-Y transformer connection, neutral is fundamental to the  
 (a) suppression of harmonics  
 (b) passage of unbalanced currents due to unbalanced loads  
 (c) provision of dual electric service  
 (d) balancing of phase voltages with respect to line voltages.
- As compared to  $\Delta - \Delta$  bank, the capacity of the V - V bank of transformers is ..... percent.  
 (a) 57.7 (b) 66.7  
 (c) 50 (d) 86.6
- If three transformers in a  $\Delta - \Delta$  are delivering their rated load and one transformer is removed, then overload on *each* of the remaining transformers is ..... percent.  
 (a) 66.7 (b) 173.2  
 (c) 73.2 (d) 58
- When a V - V system is converted into a  $\Delta - \Delta$  system, increase in capacity of the system is ..... percent.  
 (a) 86.6 (b) 66.7  
 (c) 73.2 (d) 50
- For supplying a balanced 3 -  $\phi$  load of 40-kVA, rating of each transformer in V - V bank should be nearly ..... kVA.  
 (a) 20 (b) 23  
 (c) 34.6 (d) 25
- When a closed -  $\Delta$  bank is converted into an open -  $\Delta$  bank, each of the two remaining transformers supplies ..... percent of the original load.  
 (a) 66.7 (b) 57.7  
 (c) 50 (d) 73.2
- If the load p.f. is 0.866, then the average p.f. of the V - V bank is  
 (a) 0.886 (b) 0.75  
 (c) 0.51 (d) 0.65
- A T - T connection has higher ratio of utilization than a V - V connection only when  
 (a) identical transformers are used  
 (b) load power factor is leading  
 (c) load power factor is unity  
 (d) non-identical transformers are used.
- The biggest advantage of T - T connection over the V - V connection for 3-phase power transformation is that it provides  
 (a) a set of balanced voltages under load  
 (b) a true 3-phase, 4-wire system  
 (c) a higher ratio of utilization  
 (d) more voltages.
- Of the following statements concerning parallel operation of transformers, the one which is **not** correct is  
 (a) transformers must have equal voltage ratings  
 (b) transformers must have same ratio of transformation  
 (c) transformers must be operated at the same frequency  
 (d) transformers must have equal kVA ratings.

**12. Statement**

An auto-transformer is more efficient in transferring energy from primary to secondary circuit.

**Reason**

Because it does so both inductively and conductively.

**Key**

- (a) statement is false, reason is correct and relevant  
 (b) statement is correct, reason is correct but irrelevant  
 (c) both statement and reason are correct and are connected to each other as cause and effect  
 (d) both statement and reason are false.
- 13.** Out of the following given choices for poly phase transformer connections which one will you select for three-to-two phase conversion ?

- (a) Scott (b) star/star  
 (c) double Scott (d) star/double-delta

- 14.** A T – T transformer cannot be paralleled with ..... transformer.

- (a) V – V (b) Y – Δ  
 (c) Y – Y (d) Δ – Δ

- 15.** Instrument transformers are used on a.c. circuits for extending the range of

- (a) ammeters (b) voltmeters  
 (c) wattmeters (d) all of the above.

- 16.** Before removing the ammeter from a current transformer, its secondary must be short-circuited in order to avoid

- (a) excessive heating of the core  
 (b) high secondary e.m.f.  
 (c) increase in iron losses  
 (d) all of the above.

**ANSWERS**

1. c 2. a 3. a 4. c 5. c 6. b 7. b 8. b 9. d 10. b 11. d 12. c 13. a 14. b 15. d 16. d

# CHAPTER 34

## Learning Objectives

- Classification of AC Motors
- Induction Motor: General Principal
- Construction
- Phase-wound Rotor
- Mathematical Proof
- Relation between Torque and Rotor Power Factor
- Condition for Maximum Starting Torque
- Rotor E.M.F and Reactance under Running Conditions
- Condition for Maximum Torque Under Running Conditions
- Rotor Torque and Breakdown Torque
- Relation between Torque and Slip
- Full-load Torque and Maximum Torque
- Starting Torque and Maximum Torque
- Torque/Speed Characteristic Under Load
- Complete Torque/Speed Curve of a Three-phase Machine
- Power Stages in an Induction Motor
- Torque Developed by an Induction Motor
- Induction Motor Torque Equation
- Variation in Rotor Current
- Sector Induction Motor
- Magnetic Levitation
- Induction Motor as a Generalized Transformer
- Power Balance Equation
- Maximum Power Output

## INDUCTION MOTOR



↑ The high-speed magnetic levitation trains employ the principle of linear induction motor

### 34.1. Classification of A.C. Motors

With the almost universal adoption of a.c. system of distribution of electric energy for light and power, the field of application of a.c. motors has widened considerably during recent years. As a result, motor manufactures have tried, over the last few decades, to perfect various types of a.c. motors suitable for all classes of industrial drives and for both single and three-phase a.c. supply. This has given rise to bewildering multiplicity of types whose proper classification often offers considerable difficulty. Different a.c. motors may, however, be classified and divided into various groups from the following different points of view :

#### 1. AS REGARDS THEIR PRINCIPLE OF OPERATION

##### (A) Synchronous motors

(i) plain and (ii) super—

##### (B) Asynchronous motors

##### (a) Induction motors

(i) Squirrel cage { single  
double

(ii) Slip-ring (external resistance)

##### (b) Commutator motors

(i) Series { single phase  
universal

(ii) Compensated { conductively  
inductively

(iii) shunt { simple  
compensated

(iv) repulsion { straight  
compensated

(v) repulsion-start induction

(vi) repulsion induction

#### 2. AS REGARDS THE TYPE OF CURRENT

(i) single phase

(ii) three phase

#### 3. AS REGARDS THEIR SPEED

(i) constant speed

(ii) variable speed

(iii) adjustable speed

#### 4. AS REGARDS THEIR STRUCTURAL FEATURES

(i) open

(ii) enclosed

(iii) semi-enclosed

(iv) ventilated

(v) pipe-ventilated

(vi) riverted frame eye etc.



Three phase high voltage asynchronous motors



Fig. 34.1 Squirrel cage AC induction motor opened to show the stator and rotor construction, the shaft with bearings, and the cooling fan.

### 34.2. Induction Motor : General Principle

As a general rule, conversion of electrical power into mechanical power takes place in the **rotating** part of an electric motor. In d.c. motors, the electric power is **conducted** directly to the armature (*i.e.* rotating part) through brushes and commutator (Art. 29.1). Hence, in this sense, a d.c. motor can be called a **conduction** motor. However, in a.c. motors, the rotor does not receive electric power by conduction but by **induction** in exactly the same way as the secondary of a 2-winding transformer receives its power from



the primary. That is why such motors are known as *induction* motors. In fact, an induction motor can be treated as a *rotating transformer* i.e. one in which primary winding is stationary but the secondary is free to rotate (Art. 34.47).

Of all the a.c. motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages and also some dis-advantages:

**Advantages:**

1. It has very simple and extremely rugged, almost unbreakable construction (especially squirrel-cage type).
2. Its cost is low and it is very reliable.
3. It has sufficiently high efficiency. In normal running condition, no brushes are needed, hence frictional losses are reduced. It has a reasonably good power factor.
4. It requires minimum of maintenance.
5. It starts up from rest and needs no extra starting motor and has not to be synchronised. Its starting arrangement is simple especially for squirrel-cage type motor.

**Disadvantages:**

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of a d.c. shunt motor.

**34.3. Construction**

An induction motor consists essentially of two main parts :

(a) a stator and (b) a rotor.

**(a) Stator**

The stator of an induction motor is, in principle, the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings [Fig.34.2 (a)]. The stator carries a 3-phase winding [Fig.34.2 (b)] and is fed from a 3-phase supply. It is wound for a definite number of poles\*, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and *vice versa*. It will be shown in Art. 34.6 that the stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by  $N_s = 120 / P$ ). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.



**Fig. 34.2 (a)** Unwound stator with semi-closed slots. Laminations are of high-quality low-loss silicon steel. (Courtesy : Gautam Electric Motors)

**Fig. 34.2 (b)** Completely wound stator for an induction motor. (Courtesy : Gautam Electric Motors)

\* The number of poles  $P$ , produced in the rotating field is  $P = 2n$  where  $n$  is the number of stator slots/pole/phase.

**(b) Rotor**

**(i) Squirrel-cage rotor:** Motors employing this type of rotor are known as squirrel-cage induction motors.

**(ii) Phase-wound or wound rotor:** Motors employing this type of rotor are variously known as 'phase-wound' motors or 'wound' motors or as 'slip-ring' motors.

**34.4. Squirrel-cage Rotor**

Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, it should be



**Fig. 34.3 (a)** Squirrel-cage rotor with copper bars and alloy brazed end-rings  
(Courtesy : Gautam Electric Motors)

**Fig. 34.3 (b)** Rotor with shaft and brings  
(Courtesy : Gautam Electric Motors)

noted clearly, are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded or bolted to two heavy and stout short-circuiting end-rings, thus giving us, what is so picturesquely called, a squirrel-cage construction (Fig. 34.3).

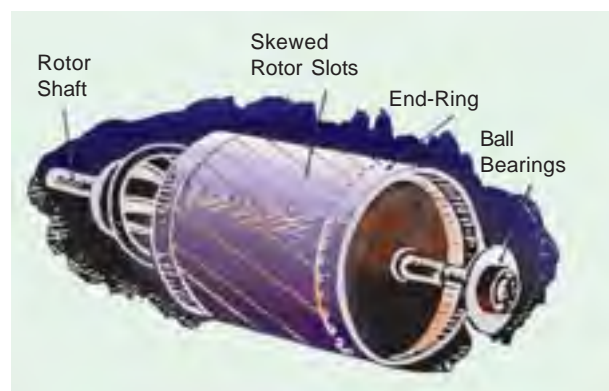
It should be noted that the **rotor bars are permanently short-circuited on themselves**, hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes.

The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (Fig. 34.4). This is useful in two ways :

- (i)** it helps to make the motor run quietly by reducing the magnetic hum and
- (ii)** it helps in reducing the locking tendency of the rotor *i.e.* the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction between the two.\*

In small motors, another method of construction is used. It consists of placing the entire rotor core in a mould and casting all the bars and end-rings in one piece. The metal commonly used is an aluminium alloy.

Another form of rotor consists of a solid cylinder of steel without any conductors or slots at all. The motor operation depends upon the production of eddy currents in the steel rotor.



**Fig. 34.4**

\* Other results of skew which may or may not be desirable are (i) increase in the effective ratio of transformation between stator and rotor (ii) increased rotor resistance due to increased length of rotor bars (iii) increased impedance of the machine at a given slip and (iv) increased slip for a given torque.

### 34.5. Phase-wound Rotor

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase even **when the stator is wound two-phase**.

The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them [Fig. 34.5 (b)]. These three brushes are further externally connected to a 3-phase star-connected rheostat [Fig. 34.5 (c)]. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor, as shown in Fig. 34.6 (a) (Ex.34.7 and 34.10) and for changing its speed-torque/current characteristics. When running under normal conditions, the **slip-rings are automatically short-circuited** by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-case rotor.



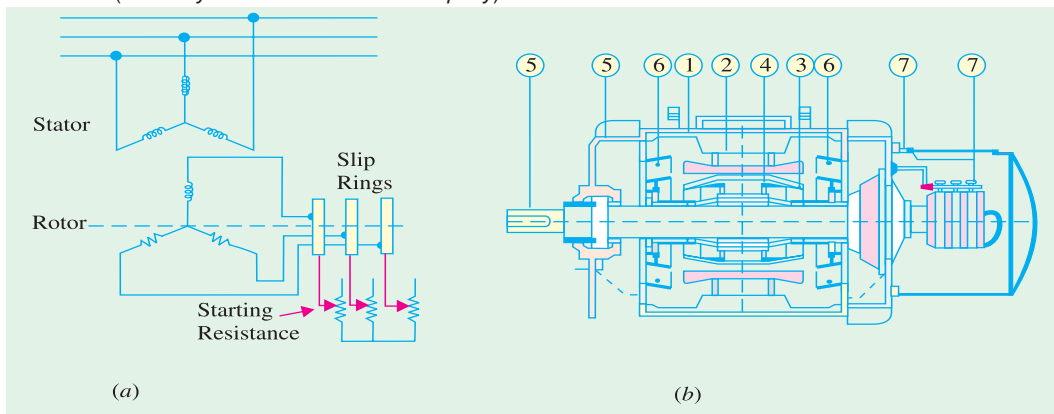
Fig. 34.5 (a)

Fig. 34.6 (b) shows the longitudinal section of a slip-ring motor, whose structural details are as under :



Fig. 34.5 (b) Slip-ring motor with slip-rings brushes and short-circuiting devices  
(Courtesy : Kirloskar Electric Company)

Fig. 34.5 (c)



(a)

(b)

Fig. 34.6 (a)

Fig. 34.6 (b) Longitudinal section of a Jyoti splash-proof slip-ring motor  
(Courtesy : Jyoti Colour-Emag Ltd.)

1. **Frame.** Made of close-grained alloy cast iron.
2. **Stator and Rotor Core.** Built from high-quality low-loss silicon steel laminations and flash-enamelled on both sides.
3. **Stator and Rotor Windings.** Have moisture proof tropical insulation embodying mica and high quality varnishes. Are carefully spaced for most effective air circulation and are rigidly braced to withstand centrifugal forces and any short-circuit stresses.
4. **Air-gap.** The stator rabbets and bore are machined carefully to ensure uniformity of air-gap.
5. **Shafts and Bearings.** Ball and roller bearings are used to suit heavy duty, trouble-free running and for enhanced service life.
6. **Fans.** Light aluminium fans are used for adequate circulation of cooling air and are securely keyed onto the rotor shaft.
7. **Slip-rings and Slip-ring Enclosures.** Slip-rings are made of high quality phosphor-bronze and are of moulded construction.

Fig. 34.6 (c) shows the disassembled view of an induction motor with squirrel-cage rotor. According to the labelled notation (a) represents stator (b) rotor (c) bearing shields (d) fan (e) ventilation grill and (f) terminal box.

Similarly, Fig. 34.6 (d) shows the disassembled view of a slip-ring motor where (a) represents stator (b) rotor (c) bearing shields (d) fan (e) ventilation grill (f) terminal box (g) slip-rings (h) brushes and brush holders.

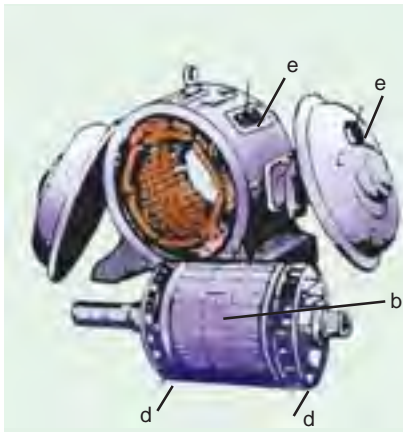


Fig. 34.6 (c)

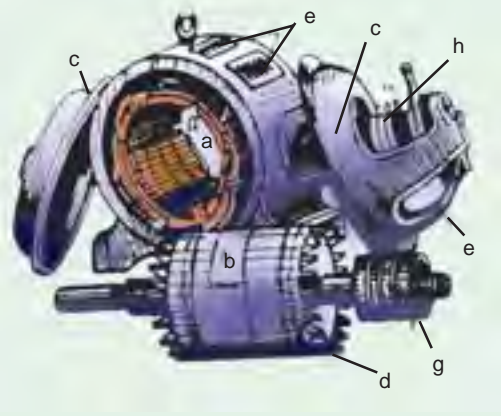


Fig. 34.6 (d)

### 34.6. Production of Rotating Field

It will now be shown that when stationary coils, wound for two or three phases, are supplied by two or three-phase supply respectively, a uniformly-rotating (or revolving) magnetic flux of constant value is produced.

#### Two-phase Supply

The principle of a 2- $\phi$ , 2-pole stator having two identical windings, 90 space degrees apart, is illustrated in Fig. 34.7.



Induction motor

The flux due to the current flowing in each phase winding is assumed sinusoidal and is represented in Fig. 34.9. The assumed positive directions of fluxes are those shown in Fig. 34.8.

Let  $\Phi_1$  and  $\Phi_2$  be the instantaneous values of the fluxes set up by the two windings. The resultant flux  $\Phi_r$  at any time is the vector sum of these two fluxes ( $\Phi_1$  and  $\Phi_2$ ) at that time. We will consider conditions at intervals of  $1/8$ th of a time period *i.e.* at intervals corresponding to angles of  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$  and  $180^\circ$ . It will be shown that resultant flux  $\Phi_r$  is constant in magnitude *i.e.* equal to  $\Phi_m$  - the maximum flux due to either phase and is making one revolution/cycle. In other words, it means that the resultant flux rotates synchronously.

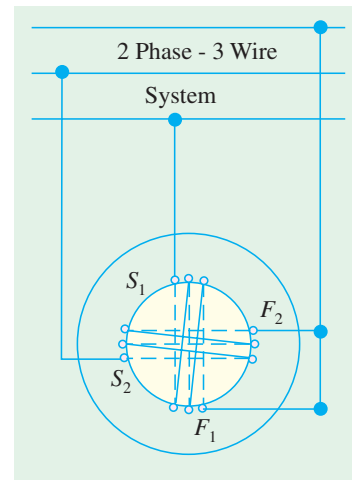


Fig. 34.7

- (a) When  $\theta = 0^\circ$  *i.e.* corresponding to point 0 in Fig. 34.9,  $\Phi_1 = 0$ , but  $\Phi_2$  is maximum *i.e.* equal to  $\Phi_m$  and negative. Hence, resultant flux  $\Phi_r = \Phi_m$  and, being negative, is shown by a vector pointing downwards [Fig. 34.10 (i)].
- (b) When  $\theta = 45^\circ$  *i.e.* corresponding to point 1 in Fig. 34.9. At this instant,  $\Phi_1 = \Phi_m / \sqrt{2}$  and is positive;  $\Phi_2 = \Phi_m / \sqrt{2}$  but is still negative. Their resultant, as shown in Fig. 34.10 (ii), is  $\Phi_r = \sqrt{[(\Phi_m / \sqrt{2})^2 + (\Phi_m / \sqrt{2})^2]} = \Phi_m$  although this resultant has shifted  $45^\circ$  clockwise.
- (c) When  $\theta = 90^\circ$  *i.e.* corresponding to point 2 in Fig. 34.9. Here  $\Phi_2 = 0$ , but  $\Phi_1 = \Phi_m$  and is positive. Hence,  $\Phi_r = \Phi_m$  and has further shifted by an angle of  $45^\circ$  from its position in (b) or by  $90^\circ$  from its original position in (a).
- (d) When  $\theta = 135^\circ$  *i.e.* corresponding to point 3 in Fig. 34.9. Here,  $\Phi_1 = \Phi_m / \sqrt{2}$  and is positive,  $\Phi_2 = \Phi_m / \sqrt{2}$  and is also positive. The resultant  $\Phi_r = \Phi_m$  and has further shifted clockwise by another  $45^\circ$ , as shown in Fig. 34.10 (iv).

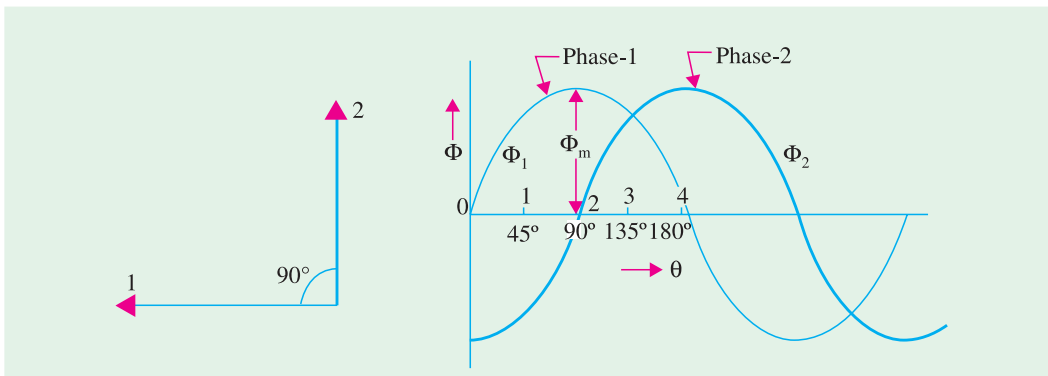


Fig. 34.8

Fig. 34.9

- (e) When  $\theta = 180^\circ$  *i.e.* corresponding to point 4 in Fig. 34.9. Here,  $\Phi_1 = 0$ ,  $\Phi_2 = \Phi_m$  and is positive. Hence,  $\Phi_r = \Phi_m$  and has shifted clockwise by another  $45^\circ$  or has rotated through an angle of  $180^\circ$  from its position at the beginning. This is shown in Fig. 34.10 (v).

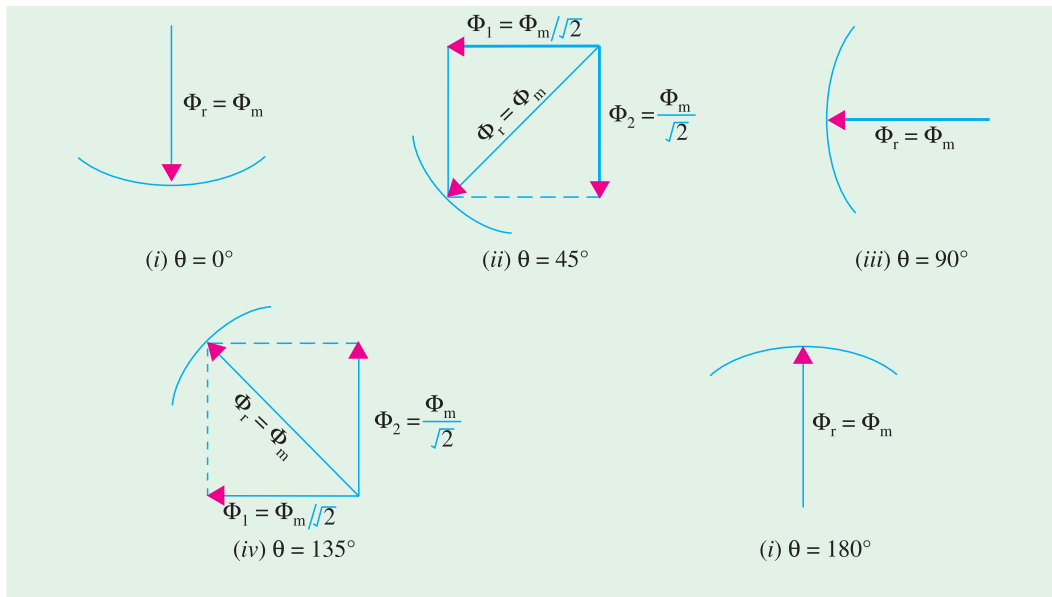


Fig. 34.10

Hence, we conclude

1. that the magnitude of the resultant flux is constant and is equal to  $\Phi_m$  — the maximum flux due to either phase.
2. that the resultant flux rotates at synchronous speed given by  $N_s = 120 f/P$  rpm.

However, it should be clearly understood that in this revolving field, there is no actual revolution of the flux. The flux due to each phase changes periodically, according to the changes in the phase current, but the magnetic flux itself does not move around the stator. It is only the seat of the resultant flux which keeps on shifting synchronously around the stator.

**Mathematical Proof**

Let  $\Phi_1 = \Phi_m \sin \omega t$  and  $\Phi_2 = \Phi_m \sin (\omega t - 90^\circ)$

$\therefore \Phi_r^2 = \Phi_1^2 + \Phi_2^2$

$\Phi_r^2 = (\Phi_m \sin \omega t)^2 + [\Phi_m \sin (\omega t - 90^\circ)]^2 = \Phi_m^2 (\sin^2 \omega t + \cos^2 \omega t) = \Phi_m^2$

$\therefore \Phi_r = \Phi_m$

It shows that the flux is of constant value and does not change with time.

**34.7. Three-phase Supply**

It will now be shown that when three-phase windings displaced in space by  $120^\circ$ , are fed by three-phase currents, displaced in time by  $120^\circ$ , they produce a resultant magnetic flux, which rotates in space as if actual magnetic poles were being rotated mechanically.

The principle of a 3-phase, two-pole stator having three identical windings placed  $120$  space degrees apart is shown in Fig. 34.11. The flux (assumed sinusoidal) due to three-phase windings is shown in Fig 34.12.

The assumed positive directions of the fluxes are shown in Fig 34.13. Let the maximum value of flux due to any one of the three phases be  $\Phi_m$ . The resultant flux  $\Phi_r$ , at any instant, is given by the vector sum of the individual fluxes,  $\Phi_1$ ,  $\Phi_2$  and  $\Phi_3$  due to three phases. We will consider values of  $\Phi_r$  at four instants  $1/6$ th time-period apart corresponding to points marked 0, 1, 2 and 3 in Fig. 34.12.

(i) When  $\theta = 0^\circ$  i.e. corresponding to point 0 in Fig. 34.12.

Here  $\Phi_1 = 0$ ,  $\Phi_2 = -\frac{\sqrt{3}}{2} \Phi_m$ ,  $\Phi_3 = \frac{\sqrt{3}}{2} \Phi_m$ . The vector for  $\Phi_2$  in Fig. 34.14 (i) is drawn in a direction opposite to the direction assumed positive in Fig. 34.13.

$$\therefore \Phi_r = 2 \times \frac{\sqrt{3}}{2} \Phi_m \cos \frac{60^\circ}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2} \Phi_m = \frac{3}{2} \Phi_m$$

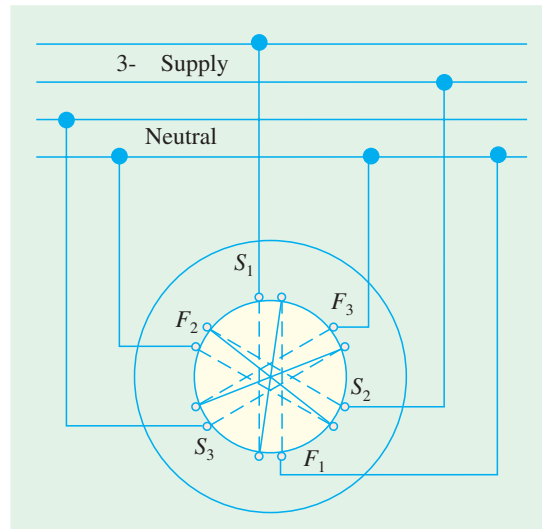


Fig. 34.11

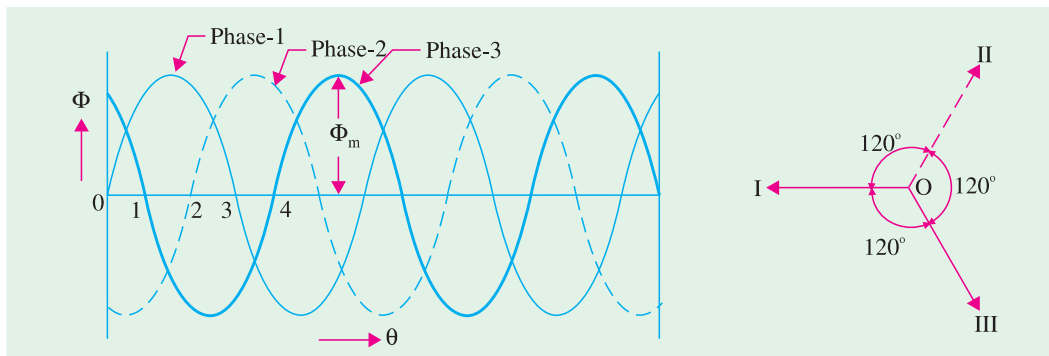


Fig. 34.12

Fig. 34.13

(ii) when  $\theta = 60^\circ$  i.e. corresponding to point 1 in Fig. 34.12.

Here  $\Phi_1 = \frac{\sqrt{3}}{2} \Phi_m$  ...drawn parallel to  $OI$  of Fig. 34.13 as shown in Fig. 34.14 (ii)

$\Phi_2 = -\frac{\sqrt{3}}{2} \Phi_m$  ...drawn in opposition to  $OII$  of Fig. 34.13.

$\Phi_3 = 0$

$$\therefore \Phi_r = 2 \times \frac{\sqrt{3}}{2} \Phi_m \times \cos 30^\circ = \frac{3}{2} \Phi_m \quad \text{[Fig. 34.14 (ii)]}$$

It is found that the resultant flux is again  $\frac{3}{2} \Phi_m$  but has rotated clockwise through an angle of  $60^\circ$ .

(iii) When  $\theta = 120^\circ$  i.e. corresponding to point 2 in Fig. 34.12.

Here,  $\Phi_1 = \frac{\sqrt{3}}{2} \Phi_m$ ,  $\Phi_2 = 0$ ,  $\Phi_3 = -\frac{\sqrt{3}}{2} \Phi_m$

It can be again proved that  $\Phi_r = \frac{3}{2} \Phi_m$

So, the resultant is again of the same value, but has further rotated clockwise through an angle of  $60^\circ$  [Fig. 34.14 (iii)].

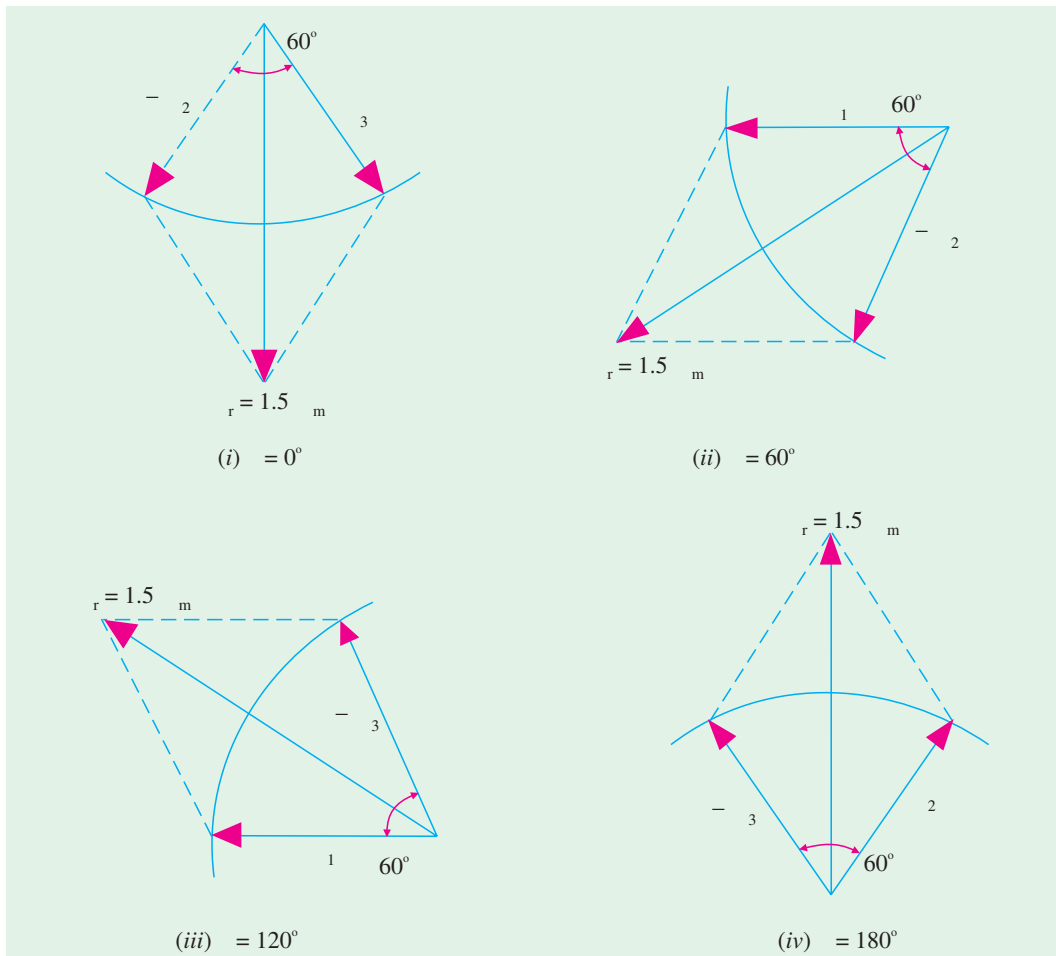


Fig. 34.14

(iv) When  $\theta = 180^\circ$  i.e. corresponding to point 3 in Fig. 34.12.

$$\Phi_1 = 0, \Phi_2 = \frac{\sqrt{3}}{2} \Phi_m, \Phi_3 = -\frac{\sqrt{3}}{2} \Phi_m$$

The resultant is  $\frac{3}{2} \Phi_m$  and has rotated clockwise through an additional angle  $60^\circ$  or through an angle of  $180^\circ$  from the start.

Hence, we conclude that

1. the resultant flux is of constant value =  $\frac{3}{2} \Phi_m$  i.e. **1.5 times the maximum value of the flux due to any phase.**
2. **the resultant flux rotates around the stator at synchronous speed given by  $N_s = 120 f/P$ .**

Fig. 34.15 (a) shows the graph of the rotating flux in a simple way. As before, the positive directions of the flux phasors have been shown separately in Fig. 34.15 (b). Arrows on these flux phasors are reversed when each phase passes through zero and becomes negative.



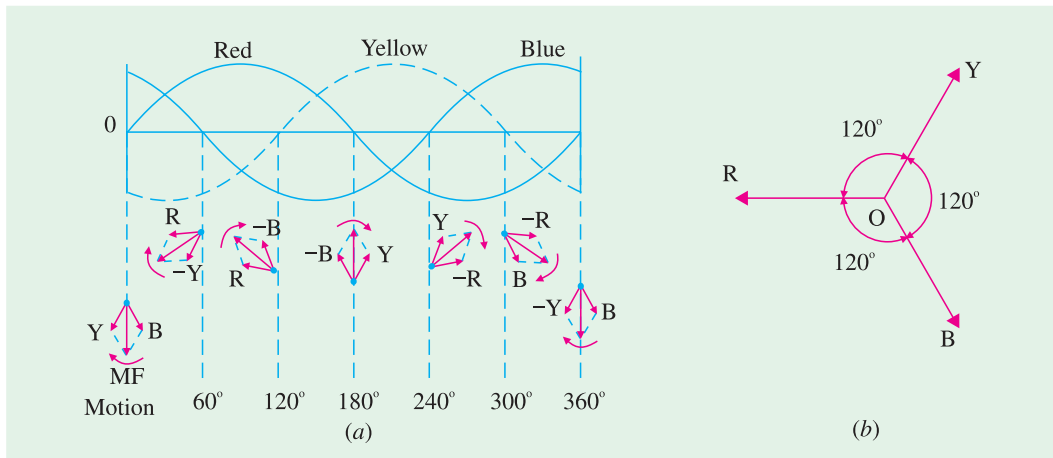


Fig. 34.15

As seen, positions of the resultant flux phasor have been shown at intervals of 60° only. The resultant flux produces a field rotating in the clockwise direction.

### 34.8. Mathematical Proof

Taking the direction of flux due to phase 1 as reference direction, we have

$$\begin{aligned} \Phi_1 &= \Phi_m (\cos 0^\circ + j \sin 0^\circ) \sin \omega t \\ \Phi_2 &= \Phi_m (\cos 240^\circ + j \sin 240^\circ) \sin (\omega t - 120^\circ) \\ \Phi_3 &= \Phi_m (\cos 120^\circ + j \sin 120^\circ) \sin (\omega t - 240^\circ) \end{aligned}$$

Expanding and adding the above equations, we get

$$\Phi_r = \frac{3}{2} \Phi_m (\sin \omega t + j \cos \omega t) = \frac{3}{2} \Phi_m \angle 90^\circ - \omega t$$

The resultant flux is of constant magnitude and does not change with time 't'.

### 34.9. Why Does the Rotor Rotate ?

The reason why the rotor of an induction motor is set into rotation is as follow:

When the 3-phase stator windings, are fed by a 3-phase supply then, as seen from above, a magnetic flux of constant magnitude, but rotating at synchronous speed, is set up. The flux passes through the air-gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter, according to Faraday's laws of electro-magnetic induction. **The frequency of the induced e.m.f. is the same as the supply frequency.** Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is given by Fleming's Right-hand rule. Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the **same** direction as that of the flux and tries to catch up with the rotating flux.



Windings of induction electric motor

The setting up of the torque for rotating the rotor is explained below :

In Fig 34.16 (a) is shown the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is **anticlockwise**. By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current **alone**, is as shown in Fig. 34.16 (b). Now, by applying the Left-hand rule, or by the effect of combined field [Fig. 34.16(c)] it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).

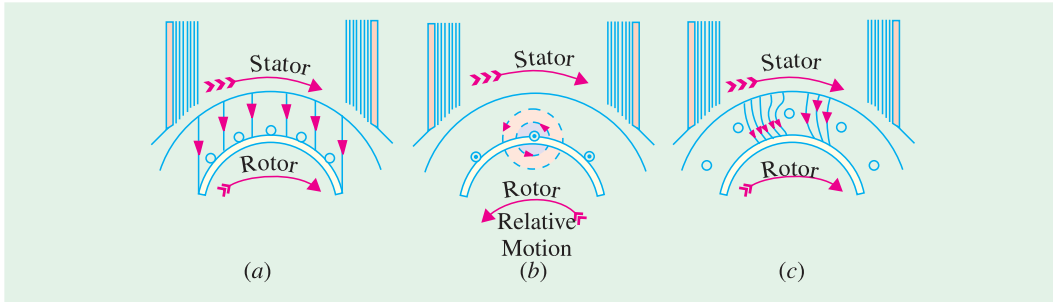


Fig. 34.16

### 34.10. Slip

In practice, the rotor never succeeds in ‘catching up’ with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor.\*

The difference between the synchronous speed  $N_s$  and the actual speed  $N$  of the rotor is known as **slip**. Though it may be expressed in so many revolutions/second, yet it is usual to express it as a percentage of the synchronous speed. Actually, the term ‘**slip**’ is descriptive of the way in which the rotor ‘slips back’ from synchronism.

$$\% \text{ slip } s = \frac{N_s - N}{N_s} \times 100$$

Sometimes,  $N_s - N$  is called the **slip speed**.

Obviously, rotor (or motor) speed is  $N = N_s (1 - s)$ .

It may be kept in mind that revolving flux is rotating synchronously, relative to the stator (*i.e.* stationary space) but at slip speed relative to the rotor.

### 34.11. Frequency of Rotor Current

When the rotor is stationary, the frequency of rotor current is **the same as the supply frequency**. But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip-speed. Let at any slip-speed, the frequency of the rotor current be  $f'$ . Then

$$N_s - N = \frac{120 f'}{P} \quad \text{Also,} \quad N_s = \frac{120 f}{P}$$

Dividing one by the other, we get,  $\frac{f'}{f} = \frac{N_s - N}{N_s} = s \qquad \therefore f' = sf$

As seen, rotor currents have a frequency of  $f' = sf$  and when flowing through the individual

\* It may be noted that as the load is applied, the natural effect of the load or braking torque is to slow down the motor. Hence, slip increases and with it increases the current and torque, till the driving torque of the motor balances the retarding torque of the load. This fact determines the speed at which the motor runs on load.

phases of rotor winding, give rise to rotor magnetic fields. These individual rotor magnetic fields produce a combined rotating magnetic field, whose speed relative to rotor is

$$= \frac{120 f'}{P} = \frac{120 sf}{P} = sN_s$$

However, the rotor itself is running at speed  $N$  with respect to space. Hence, speed of rotor field in **space** = speed of rotor magnetic field relative to rotor + speed of rotor relative to space

$$= sN_s + N = sN_s + N_s (1 - s) = N_s$$

It means that no matter what the value of slip, rotor currents and stator currents each produce a sinusoidally distributed magnetic field of constant magnitude and constant space speed of  $N_s$ . In other words, both the rotor and stator fields rotate synchronously, which means that they are stationary with respect to each other. These two synchronously rotating magnetic fields, in fact, superimpose on each other and give rise to the actually existing rotating field, which corresponds to the magnetising current of the stator winding.

**Example 34.1.** A slip-ring induction motor runs at 290 r.p.m. at full load, when connected to 50-Hz supply. Determine the number of poles and slip.

(Utilisation of Electric Power AMIE Sec. B 1991)

**Solution.** Since  $N$  is 290 rpm;  $N_s$  has to be somewhere near it, say 300 rpm. If  $N_s$  is assumed as 300 rpm, then  $300 = 120 \times 50/P$ . Hence,  $P = 20$ .  $\therefore s = (300 - 290)/300 = 3.33\%$

**Example 34.2.** The stator of a 3- $\phi$  induction motor has 3 slots per pole per phase. If supply frequency is 50 Hz, calculate

- (i) number of stator poles produced and total number of slots on the stator
- (ii) speed of the rotating stator flux (or magnetic field).

**Solution. (i)**  $P = 2n = 2 \times 3 = 6$  poles  
 Total No. of slots = 3 slots/pole/phase  $\times$  6 poles  $\times$  3 phases = 54  
**(ii)**  $N_s = 120 f/P = 120 \times 50/6 = 1000$  r.p.m.

**Example 34.3.** A 4-pole, 3-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate :

- (i) the speed at which the magnetic field of the stator is rotating.
- (ii) the speed of the rotor when the slip is 0.04.
- (iii) the frequency of the rotor currents when the slip is 0.03.
- (iv) the frequency of the rotor currents at standstill.

(Electrical Machinery II, Bangalore Univ. 1991)

**Solution. (i)** Stator field revolves at synchronous speed, given by

$$N_s = 120 f/P = 120 \times 50/4 = 1500 \text{ r.p.m.}$$

- (ii)** rotor (or motor) speed,  $N = N_s(1 - s) = 1500(1 - 0.04) = 1440$  r.p.m.
- (iii)** frequency of rotor current,  $f' = sf = 0.03 \times 50 = 1.5$  r.p.s = 90 r.p.m
- (iv)** Since at standstill,  $s = 1$ ,  $f' = sf = 1 \times f = f = 50$  Hz

**Example 34.4.** A 3- $\phi$  induction motor is wound for 4 poles and is supplied from 50-Hz system. Calculate (i) the synchronous speed (ii) the rotor speed, when slip is 4% and (iii) rotor frequency when rotor runs at 600 rpm.

(Electrical Engineering-I, Pune Univ. 1991)

**Solution. (i)**  $N_s = 120 f/P = 120 \times 50/4 = 1500$  rpm  
**(ii)** rotor speed,  $N = N_s(1 - s) = 1500(1 - 0.04) = 1440$  rpm

(iii) when rotor speed is 600 rpm, slip is

$$s = (N_s - N)/N_s = (1500 - 600)/1500 = 0.6$$

rotor current frequency,  $f' = sf = 0.6 \times 50 = \mathbf{30 \text{ Hz}}$

**Example 34.5.** A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor, at full-load is 3%, calculate the full-load speed of the motor

**Solution.** Let  $N$  = actual motor speed; Supply frequency,  $f = 12 \times 500/120 = 50 \text{ Hz}$ . Synchronous speed  $N_s = 120 \times 50/8 = 750 \text{ r.p.m.}$

$$\% \text{ slip } s = \frac{N_s - N}{N} \times 100; \quad 3 = \frac{750 - N}{750} \times 100 \quad \therefore N = \mathbf{727.5 \text{ r.p.m.}}$$

**Note.** Since slip is 3%, actual speed  $N$  is less than  $N_s$  by 3% of  $N_s$  i.e. by  $3 \times 750/100 = 22.5 \text{ r.p.m.}$

### 34.12. Relation Between Torque and Rotor Power Factor

In Art. 29.7, it has been shown that in the case of a d.c. motor, the torque  $T_a$  is proportional to the product of armature current and flux per pole i.e.  $T_a \propto \phi I_a$ . Similarly, in the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current. However, there is one more factor that has to be taken into account i.e. the power factor of the rotor.

$$\therefore T \propto \phi I_2 \cos \phi_2 \quad \text{or} \quad T = k \phi I_2 \cos \phi_2$$

where  $I_2$  = rotor current at standstill

$\phi_2$  = angle between rotor e.m.f. and rotor current

$k$  = a constant

Denoting rotor e.m.f. at **standstill** by  $E_2$ , we have that  $E_2 \propto \phi$

$$\therefore T \propto E_2 I_2 \cos \phi_2$$

or  $T = k_1 E_2 I_2 \cos \phi_2$

where  $k_1$  is another constant.

The effect of rotor power factor on rotor torque is illustrated in Fig. 34.17 and Fig. 34.18 for various values of  $\phi_2$ . From the above expression for torque, it is clear that as  $\phi_2$  increases (and hence,  $\cos \phi_2$  decreases) the torque decreases and **vice versa**.

In the discussion to follow, the stator flux distribution is assumed sinusoidal. This revolving flux induces in each rotor conductor or bar an e.m.f. whose value depends on the flux density, in which the conductor is lying at the instant considered ( $\because e = Blv$  volt). Hence, the induced e.m.f. in the rotor is also sinusoidal.

#### (i) Rotor Assumed Non-inductive (or $\phi_2 = 0$ )

In this case, the rotor current  $I_2$  is in phase with the e.m.f.  $E_2$  induced in the rotor (Fig. 34.17). The instantaneous value of the torque acting on each rotor conductor is given by the product of instantaneous value of the flux and the rotor current ( $\because F \propto BI_2l$ ). Hence, torque curve is obtained by plotting the products of flux  $\phi$  (or flux density  $B$ ) and  $I_2$ . It is seen that the torque is always positive i.e. unidirectional.

#### (ii) Rotor Assumed Inductive

This case is shown in Fig. 34.18. Here,  $I_2$  lags behind  $E_2$  by an angle  $\phi_2 = \tan^{-1} X_2/R_2$  where  $R_2$  = rotor resistance/phase;  $X_2$  = rotor reactance/phase at **standstill**.

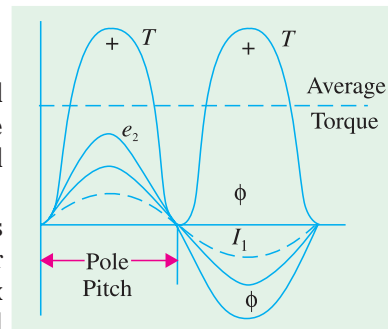


Fig. 34.17

It is seen that for a portion 'ab' of the pole pitch, the torque is negative *i.e.* reversed. Hence, the total torque which is the difference of the forward and the backward torques, is considerably reduced. If  $\phi_2 = 90^\circ$ , then the total torque is zero because in that case the backward and the forward torques become equal and opposite.

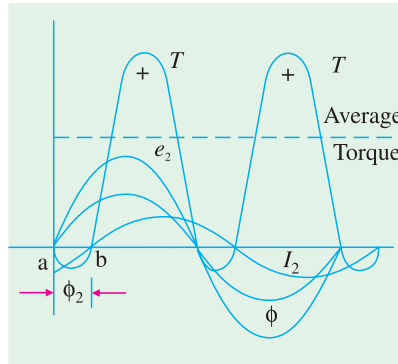


Fig. 34.18

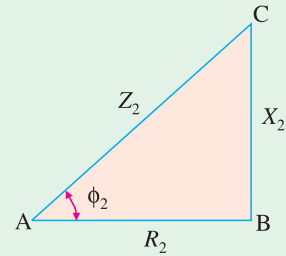


Fig. 34.19

### 34.13. Starting Torque

The torque developed by the motor at the instant of starting is called starting torque. In some cases, it is greater than the normal running torque, whereas in some other cases it is somewhat less.

Let  $E_2 =$  rotor *e.m.f.* per phase at **standstill**;  
 $R_2 =$  rotor resistance/phase  
 $X_2 =$  rotor reactance/phase at **standstill**

$$\therefore Z_2 = \sqrt{(R_2^2 + X_2^2)} = \text{rotor impedance/phase at standstill} \quad \dots \text{Fig. 34.19}$$

Then, 
$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}}; \quad \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

Standstill or starting torque  $T_{st} = k_1 E_2 I_2 \cos \phi_2 \quad \dots \text{Art. 34.12}$

or 
$$T_{st} = k_1 E_2 \cdot \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \quad \dots (i)$$

If supply voltage  $V$  is constant, then the flux  $\Phi$  and hence,  $E_2$  both are constant.

$\therefore T_{st} = k_2 \frac{R_2}{R_2^2 + X_2^2} = k_2 \frac{R_2}{Z_2^2}$  where  $k_2$  is some other constant.

Now,  $k_1 = \frac{3}{2\pi N_s}$ ,  $\therefore T_{st} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$

Where  $N_s \rightarrow$  synchronous speed in rps.

### 34.14. Starting Torque of a Squirrel-cage Motor

The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor currents equals the supply frequency. Hence, the starting current  $I_2$  of the rotor, though very large in magnitude, lags by a very large angle behind  $E_2$ , with the result that the starting torque per ampere is very poor. It is roughly 1.5 times the full-load torque, although the starting current is 5 to 7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.

### 34.15. Starting Torque of a Slip-ring Motor

The starting torque of such a motor is increased by improving its power factor by adding external resistance in the rotor circuit from the star-connected rheostat, the rheostat resistance being progres-

sively cut out as the motor gathers speed. Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

### 34.16. Condition for Maximum Starting Torque

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

$$\text{Now } T_{st} = \frac{k_2 R_2}{R_2^2 + X_2^2} \quad \therefore \frac{dT_{st}}{dR_2} = k_2 \left[ \frac{1}{R_2^2 + X_2^2} - \frac{R_2 (2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\text{or } R_2^2 + X_2^2 = 2R_2^2 \quad \therefore R_2 = X_2.$$

### 34.17. Effect of Change in Supply Voltage on Starting Torque

We have seen in Art. 34.13 that  $T_{st} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$ . Now  $E_2 \propto$  supply voltage  $V$

$$\therefore T_{st} = \frac{k_3 V^2 R_2}{R_2^2 + X_2^2} = \frac{k_3 V^2 R_2}{Z_2^2} \quad \text{where } k_3 \text{ is yet another constant. Hence } T_{st} \propto V^2.$$

Clearly, the torque is very sensitive to any changes in the supply voltage. A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque. This fact is of importance in star-delta and auto transformer starters (Art. 33-11).

**Example 34.6.** A 3-6 induction motor having a star-connected rotor has an induced e.m.f. of 80 volts between slip-rings at standstill on open-circuit. The rotor has a resistance and reactance per phase of 1  $\Omega$  and 4  $\Omega$  respectively. Calculate current/phase and power factor when (a) slip-rings are short-circuited (b) slip-rings are connected to a star-connected rheostat of 3  $\Omega$  per phase.

(Electrical Technology, Bombay Univ. 1987, and similar example: Rajiv Gandhi Techn. Univ. Bhopal, Dec. 2000)

**Solution.** Standstill e.m.f./rotor phase =  $80/\sqrt{3} = 46.2$  V

$$\begin{aligned} \text{(a) Rotor impedance/phase} &= \sqrt{(1^2 + 4^2)} = 4.12 \Omega \\ \text{Rotor current/phase} &= 46.2/4.12 = \mathbf{11.2} \text{ A} \\ \text{Power factor} &= \cos \phi = 1/4.12 = \mathbf{0.243} \end{aligned}$$

As p.f. is low, the starting torque is also low.

$$\begin{aligned} \text{(b) Rotor resistance/phase} &= 3 + 1 = 4 \Omega \\ \text{Rotor impedance/phase} &= \sqrt{(4^2 + 4^2)} = 5.66 \Omega \\ \therefore \text{Rotor current/phase} &= 46.2/5.66 = \mathbf{8.16} \text{ A}; \quad \cos \phi = 4/5.66 = \mathbf{0.707}. \end{aligned}$$

Hence, the starting torque is increased due to the improvement in the power factor. It will also be noted that improvement in p.f. is much more than the decrease in current due to increased impedance.

**Example 34.7.** A 3-phase, 400-V, star-connected induction motor has a star-connected rotor with a stator to rotor turn ratio of 6.5. The rotor resistance and standstill reactance per phase are 0.05  $\Omega$  and 0.25  $\Omega$  respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to obtain maximum torque at starting and what will be rotor starting current with this resistance?

**Solution.** Here  $K = \frac{1}{6.5}$  because transformation ratio  $K$  is defined as

$$= \frac{\text{rotor turns/phase}}{\text{stator turns/phase}}$$

$$\text{Standstill rotor e.m.f./phase, } E_2 = \frac{400}{\sqrt{3}} \times \frac{1}{6.5} = 35.5 \text{ volt}$$

It has been shown in Art. 34.16 that starting torque is maximum when  $R_2 = X_2$  i.e. when  $R_2 = 0.25 \Omega$  in the present case

$$\therefore \text{ External resistance/phase required} = 0.25 - 0.05 = \mathbf{0.2 \Omega}$$

$$\text{Rotor impedance/phase} = \sqrt{(0.25^2 + 0.25^2)} = 0.3535 \Omega$$

$$\text{Rotor current/phase, } I_2 = 35.5/0.3535 = \mathbf{100 \text{ A (approx)}}$$

**Example 34.8.** A 1100-V, 50-Hz delta-connected induction motor has a star-connected slip-ring rotor with a phase transformation ratio of 3.8. The rotor resistance and standstill leakage reactance are 0.012 ohm and 0.25 ohm per phase respectively. Neglecting stator impedance and magnetising current determine.

- (i) the rotor current at start with slip-rings shorted
- (ii) the rotor power factor at start with slip-rings shorted
- (iii) the rotor current at 4% slip with slip-rings shorted
- (iv) the rotor power factor at 4% slip with slip-rings shorted
- (v) the external rotor resistance per phase required to obtain a starting current of 100 A in the stator supply lines. (Elect. Machines AMIE Sec. B 1992)

**Solution.** It should be noted that in a  $\Delta/Y$  connection, primary phase voltage is the same as the line voltage. The rotor phase voltage can be found by using the phase transformation ratio of 3.8 i.e.  $K = 1/3.8$ .

$$\text{Rotor phase voltage at standstill} = 1100 \times 1/3.8 = 289.5 \text{ V}$$

$$(i) \text{ Rotor impedance/phase} = \sqrt{0.012^2 + 0.25^2} = 0.2503 \Omega$$

$$\text{Rotor phase current at start} = 289.5/0.2503 = \mathbf{1157 \text{ A}}$$

$$(ii) \text{ p.f.} = R_2/Z_2 = 0.012/0.2503 = \mathbf{0.048 \text{ lag}}$$

$$(iii) \text{ at 4\% slip, } X_r = sX_2 = 0.04 \times 0.25 = 0.01 \Omega$$

$$\therefore Z_r = \sqrt{0.012^2 + 0.01^2} = \mathbf{0.0156 \Omega}$$

$$E_r = sE_2 = 0.04 \times 289.5 = 11.58 \text{ V; } I_2 = 11.58/0.0156 = \mathbf{742.3 \text{ A}}$$

$$(iv) \text{ p.f.} = 0.012/0.0156 = \mathbf{0.77}$$

$$(v) I_2 = I_1/K = 100 \times 3.8 = 380 \text{ A; } E_2 \text{ at standstill} = 289.5 \text{ V}$$

$$Z_2 = 289.5/380 = 0.7618 \Omega; R_2 = \sqrt{Z_2^2 - X_2^2} = \sqrt{0.7618^2 - 0.25^2} = 0.7196 \Omega$$

$$\therefore \text{ External resistance reqd./phase} = 0.7196 - 0.012 = \mathbf{0.707 \Omega}$$

**Example 34.9.** A 150-kw, 3000-V, 50-Hz, 6-pole star-connected induction motor has a star-connected slip-ring rotor with a transformation ratio of 3.6 (stator/rotor). The rotor resistance is 0.1 $\Omega$ /phase and its per phase leakage reactance is 3.61 mH. The stator impedance may be neglected. Find the starting current and starting torque on rated voltage with short-circuited slip rings.

(Elect. Machines, A.M.I.E. Sec. B, 1989)

$$\text{Solution. } X_2 = 2\pi \times 50 \times 3.61 \times 10^{-3} = 1.13 \Omega$$

$$K = 1/3.6, R_2' = R_2/K^2 = (3.6)^2 \times 0.1 = 1.3 \Omega$$

$$X_2 = 2\pi \times 50 \times 3.61 \times 10^{-3} = 1.13 \Omega; X_2' = (3.6)^2 \times 1.13 = 14.7 \Omega$$

$$I_{st} = \frac{V}{(R_2')^2 + (X_2')^2} \cdot \frac{3000/\sqrt{3}}{\sqrt{(1.3)^2 + (14.7)^2}} = 117.4 \text{ A}$$

Now,  $N_s = 120 \times 50/6 = 1000 \text{ rpm} = (50/3) \text{ rps}$

$$T_{st} = \frac{3}{2\pi N_s} \cdot \frac{V^2 R_2'}{(R_2')^2 + (X_2')^2} = \frac{3}{2\pi (50/3)} \times \frac{(3000/\sqrt{3})^2 \times 1.3}{(1.3^2 + 14.7^2)} = 513 \text{ N-m}$$

### Tutorial Problem No. 34.1

- In the case of an 8-pole induction motor, the supply frequency was 50-Hz and the shaft speed was 735 r.p.m. What were the magnitudes of the following: **(Nagpur Univ., Summer 2000)**
  - synchronous speed
  - speed of slip
  - per unit slip
  - percentage slip

**[750 r.p.m. ; 15 r.p.m.; 0.02 ; 2%]**
- A 6-pole, 50-Hz squirrel-cage induction motor runs on load at a shaft speed of 970 r.p.m. Calculate:-
  - the percentage slip
  - the frequency of induced current in the rotor.

**[3% ; 1.5 Hz]**
- An 8-pole alternator runs at 750. r.p.m. and supplies power to a 6-pole induction motor which has at full-load a slip of 3%. Find the full-load speed of the induction motor and the frequency of its rotor e.m.f. **[970 r.p.m. ; 1.5 Hz]**
- A 3-phase, 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between the slip-rings on open-circuit. Calculate the current and power factor at standstill when the rotor winding is joined to a star-connected external circuit, each phase of which has a resistance of 10  $\Omega$  and an inductance of 0.04 H. The resistance per phase of the rotor winding is 0.2  $\Omega$  and its inductance is 0.04 H.  
Also, calculate the current and power factor when the slip-rings are short-circuited and the motor is running with a slip of 5 per cent. Assume the flux to remain constant. **[10.67 A; 0.376; 21.95 A; 0.303]**
- Obtain an expression for the condition of maximum torque of an induction motor. Sketch the torque-slip curves for several values of rotor circuit resistance and indicate the condition for maximum torque to be obtained at starting.  
If the motor has a rotor resistance of 0.02  $\Omega$  and a standstill reactance of 0.1  $\Omega$ , what must be the value of the total resistance of a starter for the rotor circuit for maximum torque to be exerted at starting? **[0.08  $\Omega$ ] (City and Guilds, London)**
- The rotor of a 6-pole, 50-Hz induction motor is rotated by some means at 1000 r.p.m. Compute (i) rotor voltage (ii) rotor frequency (iii) rotor slip and (iv) torque developed. Can the rotor rotate at this speed by itself? **[(i) 0 (ii) 0 (iii) 0 (iv) 0; No] (Elect. Engg. Grad I.E.T.E. June 1985)**
- The rotor resistances per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.024 ohm and 0.12 ohm respectively. Find the speed at maximum torque. Also find the value of the additional rotor resistance per phase required to develop 80% of maximum torque at starting. **[1200 r.p.m. 0.036  $\Omega$ ] (Elect. Machines, A.M.I.E. Sec. B, 1990)**
- The resistance and reactance per phase of the rotor of a 3-phase induction motor are 0.6 ohm and 5 ohms respectively. The induction motor has a star-connected rotor and when the stator is connected to a supply of normal voltage, the induced e.m.f. between the slip rings at standstill is 80 V. Calculate the current in each phase and the power factor at starting when (i) the slip-rings are shorted, (ii) slip-rings are connected to a star-connected resistance of 4 ohm per phase. **[(i) 9.17 amp, 0.1194 lag (ii) 6.8 amp, 0.6765 lag] [Rajiv Gandhi Technical University, Bhopal, 2000]**



### 34.18. Rotor E.M.F. and Reactance Under Running Conditions

Let  $E_2 =$  **standstill** rotor induced e.m.f./phase  
 $X_2 =$  **standstill** rotor reactance/phase,  $f_2 =$  rotor current frequency at **standstill**

When rotor is stationary i.e.  $s = 1$ , the frequency of rotor e.m.f. is the same as that of the stator supply frequency. The value of e.m.f. induced in the rotor at standstill is maximum because the relative speed between the rotor and the revolving stator flux is maximum. In fact, the motor is equivalent to a 3-phase transformer with a short-circuited rotating secondary.

When rotor starts running, the relative speed between it and the rotating stator flux is decreased. Hence, the rotor induced e.m.f. which is directly proportional to this relative speed, is also decreased (and may disappear altogether if rotor speed were to become equal to the speed of stator flux). Hence, for a slip  $s$ , the rotor induced e.m.f. will be  $s$  times the induced e.m.f. at standstill.

Therefore, under **running conditions**  $E_r = sE_2$

The frequency of the induced e.m.f. will likewise become  $f_r = sf_2$

Due to decrease in frequency of the rotor e.m.f., the rotor reactance will also decrease.

$$\therefore X_r = sX_2$$

where  $E_r$  and  $X_r$  are rotor e.m.f. and reactance under **running** conditions.

### 34.19. Torque Under Running Conditions

$$T \propto E_r I_r \cos \phi_2 \text{ or } T \propto \phi I_r \cos \phi_2 \quad (\because E_r \propto \phi)$$

where  $E_r =$  rotor e.m.f./phase under **running conditions**  
 $I_r =$  rotor current/phase under **running conditions**

Now  $E_r = sE_2$

$$\therefore I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos \phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad \text{—Fig. 34.20}$$

$$\therefore T \propto \frac{s \Phi E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{k \Phi \cdot s \cdot E_2 R_2}{R_2^2 + (sX_2)^2}$$

Also  $T = \frac{k_1 \cdot s E_2^2 R_2}{R_2^2 + (sX_2)^2} \quad (\because E_2 \propto \phi)$

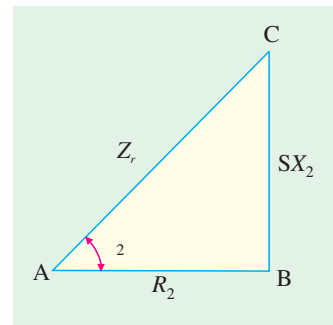


Fig. 34.20

where  $k_1$  is another constant. Its value can be proved to be equal to  $3/2 \pi N_s$  (Art. 34.38). Hence, in that case, expression for torque becomes

$$T = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{Z_r^2}$$

At standstill when  $s = 1$ , obviously

$$T_{st} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \left( \text{or } = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \right) \text{ the same as in Art. 34.13.}$$

**Example 34.10.** The star connected rotor of an induction motor has a standstill impedance of  $(0.4 + j4)$  ohm per phase and the rheostat impedance per phase is  $(6 + j2)$  ohm.

The motor has an induced emf of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find

- (i) rotor current at standstill with the rheostat is in the circuit.
- (ii) when the slip-rings are short-circuited and motor is running with a slip of 3%.

(Elect. Engg. I, Nagpur Univ. 1993)

**Solution. (1) Standstill Conditions**

Voltage/rotor phase =  $80/\sqrt{3} = 46.2$  V; rotor and starter impedance/phase =  $(6.4 + j6) = 8.77 \angle 43.15^\circ$   
 Rotor current/phase =  $46.2/8.77 = 5.27$  A (p.f. =  $\cos 43.15^\circ = 0.729$ )

**(2) Running Conditions.** Here, starter impedance is cut out.

Rotor voltage/phase,  $E_r = sE_2 = 0.03 \times 46.2 = 1.386$  V  
 Rotor reactance/phase,  $X_r = 0.03 \times 4 = 0.12 \Omega$   
 Rotor impedance/phase,  $Z_r = 0.4 + j0.12 = 0.4176 \angle 16.7^\circ$   
 Rotor current/phase =  $1.386/0.4176 = 3.32$  A (p.f. =  $\cos 16.7^\circ = 0.96$ )

**Note.** It has been assumed that flux across the air-gap remains constant

**34.20. Condition for Maximum Torque Under Running Conditions**

The torque of a rotor under **running** conditions is

$$T = \frac{k\Phi s E_2 R_2}{R_2^2 + (sX_2)^2} = k_1 \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \dots (i)$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip  $s$  and then putting it equal to zero. However, it is simpler to put  $Y = \frac{1}{T}$  and then differentiate it.

$$\therefore Y = \frac{R_2^2 + (sX_2)^2}{k\Phi s E_2 R_2} = \frac{R_2}{k\Phi s E_2} + \frac{sX_2^2}{k\Phi E_2 R_2}; \frac{dY}{ds} = \frac{-R_2}{k\Phi s^2 E_2} + \frac{X_2^2}{k\Phi E_2 R_2} = 0$$

$$\therefore \frac{R_2}{k\Phi s^2 E_2} = \frac{X_2^2}{k\Phi E_2 R_2} \quad \text{or} \quad R_2^2 = s^2 X_2^2 \quad \text{or} \quad R_2 = sX_2$$

Hence, torque under **running condition** is maximum at that value of the slip  $s$  which makes rotor reactance per phase equal to rotor resistance per phase. This slip is sometimes written as  $s_b$  and the maximum torque as  $T_b$ .

Slip corresponding to maximum torque is  $s = R_2/X_2$

Putting  $R_2 = sX_2$  in the above equation for the torque, we get

$$T_{\max} = \frac{k\Phi s^2 E_2 X_2}{2 s^2 X_2^2} \left( \text{or} \frac{k\Phi s E_2 R_2}{2 R_2^2} \right) \quad \text{or} \quad T_{\max} = \frac{k\Phi E_2}{2 X_2} \left( \text{or} \frac{k\Phi s E_2}{2 R_2} \right) \quad \dots (ii)$$

Substituting value of  $s = R_2/X_2$  in the other equation given in (i) above, we get

$$T_{\max} = k_1 \frac{(R_2 / X_2) \cdot E_2^2 \cdot R_2}{R_2^2 + (R_2 / X_2)^2 \cdot X_2^2} = k_1 \frac{E_2^2}{2 X_2}$$

Since,  $k_1 = 3/2\pi N_s$ , we have  $T_{\max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2}$  N-m

From the above, it is found

1. that the maximum torque is independent of rotor resistance as such.
2. however, the speed or slip at which maximum torque occurs is determined by the rotor

resistance. As seen from above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).

3. maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.
4. maximum torque varies directly as the square of the applied voltage.
5. for obtaining maximum torque at starting ( $s = 1$ ), rotor resistance must be equal to rotor reactance.

**Example 34.11.** A 3-phase, slip-ring, induction motor with star-connected rotor has an induced e.m.f. of 120 volts between slip-rings at standstill with normal voltage applied to the stator. The rotor winding has a resistance per phase of 0.3 ohm and standstill leakage reactance per phase of 1.5 ohm.

Calculate (i) rotor current/phase when running short-circuited with 4 percent slip and (ii) the slip and rotor current per phase when the rotor is developing maximum torque.

(Elect. Engg.-II, Pune Univ. 1989)

**Solution.** (i) induced e.m.f./rotor phase,  $E_r = sE_2 = 0.04 \times (120/\sqrt{3}) = 2.77\text{V}$

rotor reactance/phase,  $X_r = sX_2 = 0.04 \times 1.5 = 0.06 \Omega$

rotor impedance/phase  $= \sqrt{0.3^2 + 0.06^2} = 0.306 \Omega$

rotor current/phase  $= 2.77/0.306 = 9\text{A}$

(ii) For developing maximum torque,

$$R_2 = sX_2 \quad \text{or} \quad s = R_2/X_2 = 0.3/1.5 = 0.2$$

$$X_r = 0.2 \times 1.5 = 0.3 \Omega, Z_r = \sqrt{0.3^2 + 0.3^2} = 0.42 \Omega$$

$$E_r = sE_2 = 0.2 \times (120/\sqrt{3}) = 13.86 \text{ V}$$

$$\therefore \text{Rotor current/phase} = 13.86/0.42 = 33\text{A}$$

### 34.21. Rotor Torque and Breakdown Torque

The rotor torque at any slip  $s$  can be expressed in terms of the maximum (or breakdown) torque  $T_b$  by the following equation

$$T = T_b \left[ \frac{2}{(s_b/s) + (s/s_b)} \right] \quad \text{where } s_b \text{ is the breakdown or pull-out slip.}$$

**Example 34.12.** Calculate the torque exerted by an 8-pole, 50-Hz, 3-phase induction motor operating with a 4 per cent slip which develops a maximum torque of 150 kg-m at a speed of 660 r.p.m. The resistance per phase of the rotor is 0.5  $\Omega$ . (Elect. Machines, A.M.I.E. Sec. B, 1989)

**Solution.**  $N_s = 120 \times 50/8 = 750 \text{ r.p.m.}$

Speed at maximum torque  $= 660 \text{ r.p.m.}$  Corresponding slip  $s_b = \frac{750 - 660}{750} = 0.12$

For maximum torque,  $R_2 = s_b X_2$

$$\therefore X_2 = R_2/s_b = 0.5/0.12 = 4.167 \Omega$$

As seen from Eq. (ii) of Art. 34.20,

$$T_{\max} = k \Phi E_2 \cdot \frac{s_b}{2R_2} = k \Phi E_2 \cdot \frac{0.12}{2 \times 0.5} = 0.12 k \Phi E_2 \quad \dots (i)$$

**When slip is 4 per cent**

As seen from Eq. (i) of Art. 34.20

$$T = k \Phi E_2 \frac{sR_2}{R_2^2 + (sX_2)^2} = k \Phi E_2 \frac{0.04 \times 0.5}{0.5^2 + (0.04 \times 4.167)^2} = \frac{0.02 k \Phi E_2}{0.2778}$$

$$\therefore \frac{T}{T_{\max}} = \frac{T}{150} = \frac{0.02}{0.2778 \times 0.12} \quad \therefore T = 90 \text{ kg-m}$$

**Alternative Solution**

$$T_b = 150 \text{ kg.m; } s_b = 0.12, s = 4\% = 0.04, T = ?$$

$$T = T_b \left( \frac{2}{(s_b/s) + (s/s_b)} \right) \quad \dots \text{Art 34.21}$$

$$= 150 \left( \frac{2}{(0.12/0.04) + (0.04/0.12)} \right) = 90 \text{ kg-m}$$

**34.22. Relation Between Torque and Slip**

A family of torque/slip curves is shown in Fig. 34.21 for a range of  $s = 0$  to  $s = 1$  with  $R_2$  as the parameter. We have seen above in Art. 34.19 that

$$T = \frac{k \Phi s E_2 R_2}{R_2^2 + (sX_2)^2}$$

It is clear that when  $s = 0, T = 0$ , hence the curve starts from point  $O$ .

At normal speeds, close to synchronism, the term  $(sX_2)$  is small and hence negligible w.r.t.  $R_2$ .

$$\therefore T \propto \frac{s}{R_2}$$

or  $T \propto s$  if  $R_2$  is constant.

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load on the motor), the torque also increases and becomes maximum when  $s = R_2/X_2$ . This torque is known as 'pull-out' or 'breakdown' torque  $T_b$  or stalling torque. As the slip further increases (*i.e.* motor speed falls) with further increase in motor load, then  $R_2$  becomes negligible as compared to  $(sX_2)$ . Therefore, for large values of slip

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

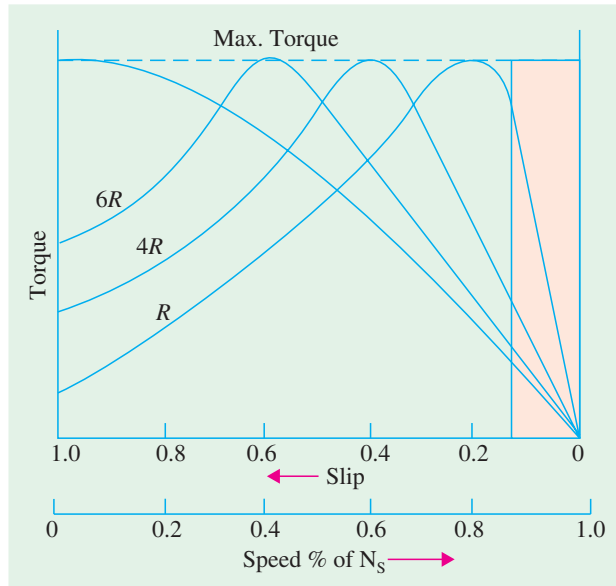


Fig. 34.21

Hence, the torque/slip curve is a rectangular hyperbola. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of  $s = 0$  and that corresponding to maximum torque. The operating range is shown shaded in Fig. 34.21.

It is seen that although maximum torque does not depend on  $R_2$ , yet the exact location of  $T_{\max}$  is dependent on it. Greater the  $R_2$ , greater is the value of slip at which the maximum torque occurs.

### 34.23. Effect of Change in Supply Voltage on Torque and Speed

As seen from Art. 34.19,  $T = \frac{k\Phi sE_2R_2}{R_2^2 + (sX_2)^2}$

As  $E_2 \propto \phi \propto V$  where  $V$  is supply voltage  $\therefore T \propto sV^2$

Obviously, torque at any speed is proportional to the square of the applied voltage. If stator voltage decreases by 10%, the torque decreases by 20%. Changes in supply voltage not only affect the starting torque  $T_{st}$  but torque under running conditions also. If  $V$  decreases, then  $T$  also decreases. Hence, for maintaining the same torque, slip increases *i.e.* speed falls.

Let  $V$  change to  $V'$ ,  $s$  to  $s'$  and  $T$  to  $T'$ ; then  $\frac{T}{T'} = \frac{sV^2}{s'V'^2}$

### 34.24. Effect of Changes in supply Frequency on Torque and Speed

Hardly any important changes in frequency take place on a large distribution system except during a major disturbance. However, large frequency changes often take place on isolated, low-power systems in which electric energy is generated by means of diesel engines or gas turbines. Examples of such systems are : emergency supply in a hospital and the electrical system on a ship etc.

The major effect of change in supply frequency is on motor **speed**. If frequency drops by 10%, then motor speed also drops by 10%. Machine tools and other motor-driven equipment meant for 50 Hz causes problem when connected to 60-Hz supply. Everything runs  $(60 - 50) \times 100/50 = 20\%$  faster than normal and this may not be acceptable in all applications. In that case, we have to use either gears to reduce motor speed or an expensive 50-Hz source.

A 50-Hz motor operates well on a 60-Hz line provided its terminal voltage is raised to  $60/50 = 6/5$  (*i.e.* 120%) of the name-plate rating. In that case, the new breakdown torque becomes equal to the original breakdown torque and the starting torque is only slightly reduced. However, power factor, efficiency and temperature rise remain satisfactory.

Similarly, a 60-Hz motor can operate satisfactorily on 50-Hz supply provided its terminal voltage is reduced to  $5/6$  (*i.e.* 80%) of its name-plate rating.

### 34.25. Full-load Torque and Maximum Torque

Let  $s_f$  be the slip corresponding to full-load torque, then

$$T_f \propto \frac{s_f R_2}{R_2^2 + (s_f X_2)^2} \quad \text{and} \quad T_{\max} \propto \frac{1}{2 \times X_2} \quad \dots \text{Art 34.20}$$

$$\therefore \frac{T_f}{T_{\max}} = \frac{2s_f R_2 X_2}{R_2^2 + (s_f X_2)^2}$$

Dividing both the numerator and the denominator by  $X_2^2$ , we get

$$\frac{T_f}{T_{\max}} = \frac{2s_f \cdot R_2 / X_2}{(R_2 / X_2)^2 + s_f^2} = \frac{2as_f}{a^2 + s_f^2}$$

where  $a = R_2/X_2 = \text{resistance/standstill reactance}^*$

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\* In fact  $a = s_m$ —slip corresponding to maximum torque. In that case, the relation becomes

$$\frac{T_f}{T_{\max}} = \frac{2s_m s_f^2}{s_m^2 + s_f^2} \quad \text{— where } s_f = \text{full-load slip.}$$

In general, 
$$\frac{\text{operating torque at any slip } s}{\text{maximum torque}} = \frac{2as}{a^2 + s^2}$$

**34.26. Starting Torque and Maximum Torque**

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2} \quad \dots \text{Art 34.13}$$

$$T_{max} \propto \frac{1}{2X_2} \quad \dots \text{Art. 34.20}$$

$$\therefore \frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2/X_2}{1 + (R_2/X_2)^2} = \frac{2a}{1 + a^2}$$

where 
$$a = \frac{R_2}{X_2} = \frac{\text{rotor resistance}}{\text{stand still reactance}} \text{ per phase*}$$

**Example. 34.13(a).** A 3- $\phi$  induction motor is driving full-load torque which is independent of speed. If line voltage drops to 90% of the rated value, find the increase in motor copper losses.

**Solution.** As seen from Art. 34.23, when  $T$  remains constant,  $s_1 V_1^2 = s_2 V_2^2$

$$\therefore \frac{s_2}{s_1} = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{1}{0.9}\right)^2 = 1.23$$

Again from Art. 34-19,  $I_2 \propto sV \therefore \frac{I_2'}{I_2} = \frac{s_2 V_2}{s_1 V_1} = 1.2 \times 0.9 = 1.107$

Now, Cu losses are nearly proportional to  $I_2^2$

$$\therefore \frac{\text{Cu loss in the 2nd case}}{\text{Cu loss in the 1st case}} = \frac{(I_2')^2}{I_2^2} = 1.107^2 = 1.23$$

Thus a reduction of 10% in line voltage causes about 23% increase in Cu losses.

**Example. 34.13 (b).** A 230-V, 6-pole, 3- $\phi$ , 50-Hz, 15-kW induction motor drives a constant torque load at rated frequency, rated voltage and rated kW output and has a speed of 980 rpm and an efficiency of 93%. Calculate (i) the new operating speed if there is a 10% drop in voltage and 5% drop in frequency and (ii) the new output power. Assume all losses to remain constant.

**Solution. (i)**  $V_2 = 0.9 \times 230 = 207$  V;  $f_2 = 0.95 \times 50 = 47.5$  Hz;  $N_{s1} = 120 \times 50/6 = 1000$  rpm;  $N_{s2} = 120 \times 47.5/6 = 950$  rpm;  $s_1 = (1000 - 980)/1000 = 0.02$

Since the load torque remains constant, the product  $(sV^2/f)$  remains constant.

$$\therefore s_1 V_1^2 / f_1 = \frac{s_2 V_2^2}{f_2} \text{ or } s_2 = s_1 \left(\frac{V_1}{V_2}\right)^2 \cdot f_2 / f_1 = 0.02 (230/207)^2 \times 47.5/50 = 0.2234$$

$$\therefore N_2 = N_{s2} (1 - s_2) = 950 (1 - 0.0234) = 928 \text{ rpm}$$

**(ii)**  $P \propto TN$ . Since torque remains constant,  $P \propto N$

$$\therefore P_1 \propto N_1; P_2 \propto N_2; \text{ or } P_2 = P_1 \times N_2 / N_1 = 15 \times 928 / 980 = \mathbf{14.2 \text{ kW}}$$

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\* Similarly, the relation becomes 
$$\frac{T_f}{T_{max}} = \frac{2s_m}{1 + s_m^2}$$

**Example 34.14 (a).** A 3-phase, 400/200-V, Y-Y connected wound-rotor induction motor has  $0.06\Omega$  rotor resistance and  $0.3\Omega$  standstill reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque of the motor.

(Electrical Technology, Bombay Univ. 1990)

**Solution.**

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}; \text{ Since } T_{st} = T_{max}$$

$$\therefore 1 = \frac{2a}{1+a^2} \text{ or } a = 1 \text{ Now, } a = \frac{R_2 + r}{X_2}$$

where  $r =$  external resistance per phase added to the rotor circuit

$$\therefore 1 = \frac{0.06 + r}{0.3} \therefore r = 0.3 - 0.06 = \mathbf{0.24\Omega}$$

**Example 34.14 (b).** 3-phase, 50-Hz, 8-pole, induction motor has full-load slip of 2%. The rotor-resistance and stand still rotor-reactance per phase are 0.001 ohm and 0.005 ohm respectively. Find the ratio of the maximum to full-load torque and the speed at which the maximum torque occurs.

(Amravati University, 1999)

**Solution.** Synchronous speed,  $N_s = 120 \times 50 / 8 = 750$  rpm  
 Slip at maximum torque,  $s_{mT} = r_2/x_2$

Thus, let a  $= \frac{r_2}{x_2} = \frac{0.001}{0.005} = 0.2$

Corresponding speed  $= (1 - 0.2) \times 750 = 600$  rpm

$$\frac{\text{Full-load torque}}{\text{Maximum torque}} = \frac{2 s_{mT} s_{fL}^2}{s_{mT}^2 + s_{fL}^2} \therefore \frac{T_{fL}}{T_{max}} = \frac{2 \times 0.2 \times 0.02^2}{0.20^2 + 0.02^2} = \frac{1.6 \times 10^{-4}}{0.0404}$$

$$\therefore \frac{T_{max}}{T_f} = 252.5$$

$$= \mathbf{3.96 \times 10^{-3}}$$

**Example 34.14 (c).** A 12-pole, 3-phase, 600-V, 50-Hz, star-connected, induction motor has rotor-resistance and stand-still reactance of 0.03 and 0.5 ohm per phase respectively. Calculate:

(a) Speed of maximum torque. (b) ratio of full-load torque to maximum torque, if the full-load speed is 495 rpm.

(Nagpur University, April 1999)

**Solution.** For a 12-pole, 50 Hz motor,  
 Synchronous speed  $= 120 \times 50 / 12 = 500$  rpm  
 For  $r = 0.03$  and  $x = 0.5$  ohm, the slip for maximum torque is related as :

$$s_{mT} = a = r/x = 0.03/0.5 = 0.06$$

(a) Corresponding speed  $= 500 (1 - s_{mT}) = 470$  rpm  
 (b) Full-load speed  $= 495$  rpm, slip  $s = 0.01$ , at full load.

$$\frac{\text{Full-load torque}}{\text{Maximum torque}} = \frac{2 as}{a^2 + s^2} = \frac{2 \times 0.06 \times 0.01}{0.06^2 + 0.01^2} = \mathbf{0.324}$$

**Example 34.15.** A 746-kW, 3-phase, 50-Hz, 16-pole induction motor has a rotor impedance of  $(0.02 + j 0.15) \Omega$  at standstill. Full-load torque is obtained at 360 rpm. Calculate (i) the ratio of maximum to full-load torque (ii) the speed of maximum torque and (iii) the rotor resistance to be added to get maximum starting torque.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** Let us first find out the value of full-load slip  $s_f$

$$N_s = 120 \times 50 / 16 = 375 \text{ rpm.}; \text{ F.L. Speed} = 360 \text{ rpm.}$$

$$s_f = (375 - 360) / 375 = 0.04; a = R_2/X_2 = 0.02/0.15 = 2/15$$

$$(i) \quad \frac{T_f}{T_{\max}} = \frac{2as_f}{a^2 + s_f^2} = \frac{2 \times (2/15) \times 0.04}{(2/15)^2 + (0.04)^2} = 0.55 \text{ or } \frac{T_{\max}}{T_f} = \frac{1}{0.55} = \mathbf{1.818}$$

$$(ii) \text{ At maximum torque, } a = s_m = R_2/X_2 = 0.02/0.15 = 2/15$$

$$N = N_s(1 - s) = 375(1 - 2/15) = \mathbf{325 \text{ r.p.m.}}$$

$$(iii) \text{ For maximum starting torque, } R_2 = X_2. \text{ Hence, total rotor resistance per phase} = \mathbf{0.15\Omega}$$

$$\therefore \text{ external resistance required/phase} = 0.15 - 0.02 = \mathbf{0.13\Omega}$$

**Example 34.16.** The rotor resistance and reactance per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.025 ohm and 0.12 ohm respectively. Make simplifying assumptions, state them and :

(i) find speed at maximum torque

(ii) find value of additional rotor resistance per phase required to give three-fourth of maximum torque at starting. Draw the equivalent circuit of a single-phase induction motor.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** (i) At maximum torque,  $s = R_2/X_2 = 0.025/0.12 = 0.208$ .

$$N_s = 120 \times 50/4 = 1500 \text{ rpm} \quad \therefore N = 1500(1 - 0.208) = \mathbf{1188 \text{ rpm}}$$

$$(ii) \text{ It is given that } T_{st} = 0.75 T_{\max} \quad \text{Now, } \frac{T_{st}}{T_{\max}} = \frac{2a}{1 + a^2} = \frac{3}{4}$$

$$\therefore 3a^2 - 8a + 3 = 0; \quad a = \frac{8 \pm \sqrt{64 - 36}}{6} = 0.45 \Omega^*$$

Let,

$r$  = additional rotor resistance reqd., then

$$a = \frac{R_2 + r}{R_2} \quad \text{or} \quad 0.45 = \frac{0.025 + r}{0.12} \quad \therefore r = \mathbf{0.029\Omega}$$

**Example 34.17.** A 50-Hz, 8-pole induction motor has F.L. slip of 4%. The rotor resistance/phase = 0.01 ohm and standstill reactance/phase = 0.1 ohm. Find the ratio of maximum to full-load torque and the speed at which the maximum torque occurs.

$$\text{Solution.} \quad \frac{T_f}{T_{\max}} = \frac{2as_f}{a^2 + s_f^2}$$

$$\text{Now,} \quad a = R_2/X_2 = 0.01/0.1 = 0.1, \quad s_f = 0.04$$

$$\therefore \quad \frac{T_f}{T_{\max}} = \frac{2 \times 0.1 \times 0.04}{0.1^2 + 0.04^2} = \frac{0.008}{0.0116} = 0.69 \quad \therefore \frac{T_{\max}}{T_f} = \frac{1}{0.69} = \mathbf{1.45}$$

$$N_s = 120 \times 50/8 = 750 \text{ rpm, } s_m = 0.1$$

$$N = (1 - 0.1) \times 750 = 675 \text{ rpm}$$

**Example 34.18.** For a 3-phase slip-ring induction motor, the maximum torque is 2.5 times the full-load torque and the starting torque is 1.5 times the full-load torque. Determine the percentage reduction in rotor circuit resistance to get a full-load slip of 3%. Neglect stator impedance.

(Elect. Machines, A.M.I.E. Sec. B, 1992)

**Solution.** Given,  $T_{\max} = 2.5 T_f$ ;  $T_{st} = 1.5 T_f$ ;  $T_{st}/T_{\max} = 1.5/2.5 = 3/5$ .

\* The larger value of 2.214  $\Omega$  has been rejected.



Now, 
$$\frac{T_{st}}{T_f} = \frac{3}{5} = \frac{2a}{1+a^2} \quad \text{or} \quad 3a^2 - 10a + 3 = 0 \quad \text{or} \quad a = 1/3$$

Now, 
$$a = R_2/X_2 \text{ or } R_2 = X_2/3$$

When F.L. slip is 0.03

$$\frac{T_f}{T_{st}} = \frac{2as}{a^2 + s^2} \quad \text{or} \quad \frac{2}{2.5} = \frac{2a \times 0.03}{a^2 + 0.03^2}$$

$$a^2 - 0.15a + 0.009 = 0 \quad \text{or} \quad a = 0.1437$$

If  $R'_2$  is the new rotor circuit resistance, then  $0.1437 = R'_2/X_2$  or  $R'_2 = 0.1437 X_2$

% reduction in rotor resistance is

$$= \frac{(X_2/3) - 0.1437 \times X_2}{(X_2/3)} \times 100 = \mathbf{56.8\%}$$

**Example 34.19.** An 8-pole, 50-Hz, 3-phase slip-ring induction motor has effective rotor resistance of 0.08 Ω/phase. Stalling speed is 650 r.p.m. How much resistance must be inserted in the rotor phase to obtain the maximum torque at starting? Ignore the magnetising current and stator leakage impedance. (Elect. Machines-I, Punjab Univ. 1991)

**Solution.** It should be noted that stalling speed corresponds to maximum torque (also called stalling torque) and to maximum slip **under running conditions**.

$$N_s = 120 \times 50/8 = 750 \text{ r.p.m.}; \text{ stalling speed is } = 650 \text{ r.p.m.}$$

$$s_b = (750 - 650)/750 = 2/15 = 0.1333 \text{ or } 13.33\%$$

Now, 
$$s_b = R_2/X_2 \quad \therefore \quad X_2 = 0.08 \times 15/2 = 0.6 \Omega$$

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}. \text{ Since } T_{st} = T_{max} \quad \therefore \quad 1 = \frac{2a}{1+a^2} \text{ or } a = 1$$

Let  $r$  be the external resistance per phase added to the rotor circuit. Then

$$a = \frac{R_2 + r}{X_2} \quad \text{or} \quad 1 = \frac{0.08 + r}{0.6} \quad \therefore \quad \mathbf{r = 0.52 \Omega \text{ per phase.}}$$

**Example 34.20.** A 4-pole, 50-Hz, 3-φ induction motor develops a maximum torque of 162.8 N-m at 1365 r.p.m. The resistance of the star-connected rotor is 0.2 Ω/phase. Calculate the value of the resistance that must be inserted in series with each rotor phase to produce a starting torque equal to half the maximum torque.

**Solution.** 
$$N_s = 120 \times 50/4 = 1500 \text{ r.p.m. } N = 1365 \text{ r.p.m.}$$

∴ Slip corresponding to maximum torque is

$$s_b = (1500 - 1365)/1500 = 0.09 \quad \text{But } s_b = R_2/X_2 \quad \therefore \quad X_2 = 0.2/0.09 = 2.22 \Omega$$

Now, 
$$T_{max} = \frac{k \Phi E_2}{2 X_2} = \frac{K}{2 X_2} \quad (\text{where } K = k \Phi E_2) \quad \dots \text{Art 34.20}$$

$$= \frac{K}{2 \times 2.22} = 0.225 K$$

Let ' $r$ ' be the external resistance introduced per phase in the rotor circuit, then

Starting torque 
$$T_{st} = \frac{k \Phi E_2 (R_2 + r)}{(R_2 + r)^2 + (X_2)^2} = \frac{K (0.2 + r)}{(0.2 + r)^2 + (0.2/0.09)^2}$$

∴ 
$$T_{st} = \frac{1}{2} \cdot T_{max} \quad \therefore \quad \frac{K (0.2 + r)}{(0.2 + r)^2 + (2.22)^2} = \frac{0.225 K}{2}$$

Solving the quadratic equation for ' $r$ ', we get  $r = \mathbf{0.4 \Omega}$

**Example 34.21.** A 4-pole, 50-Hz, 7.46.kW motor has, at rated voltage and frequency, a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Determine (i) full-load speed (ii) speed at maximum torque. **(Electrical Technology-I, Osmania Univ. 1990)**

**Solution.**  $\frac{T_{st}}{T_f} = 1.6$  and  $\frac{T_{max}}{T_f} = 2 \quad \therefore \frac{T_{st}}{T_{max}} = \frac{1.6}{2} = 0.8$

Now,  $\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2} \quad \therefore \frac{2a}{1+a^2} = 0.8$

or  $0.8a^2 - 2a + 0.8 = 0 \quad a = 0.04 \quad \therefore a = R_2/X_2 = 0.04$  or  $R_2 = 0.04X_2$

Also,  $\frac{T_f}{T_{max}} = \frac{2as_f}{a^2 + s_f^2} = \frac{1}{2}$  or  $\frac{2 \times 0.04 s_f}{0.0016 + s_f^2} = \frac{1}{2}$  or  $s_f = 0.01$

(i) full-load speed occurs at a slip of 0.01 or 1 per cent. Now,

$$N_s = 120 \times 50/4 = 1500 \text{ r.p.m.}; N = 1500 - 15 = \mathbf{1485 \text{ r.p.m.}}$$

(ii) Maximum torque occurs at a slip given by  $s_b = R_2/X_2$ . As seen from above slip corresponding to maximum torque is 0.04.

$$\therefore N = 1500 - 1500 \times 0.04 = \mathbf{1440 \text{ r.p.m.}}$$

**Example 34.22.** A 3-phase induction motor having a 6-pole, star-connected stator winding runs on 240-V, 50-Hz supply. The rotor resistance and standstill reactance are 0.12 ohm and 0.85 ohm per phase. The ratio of stator to rotor turns is 1.8. Full load slip is 4%.

Calculate the developed torque at full load, maximum torque and speed at maximum torque.

**(Elect. Machines, Nagpur Univ. 1993)**

**Solution.** Here,  $K = \frac{\text{rotor turns/phase}}{\text{stator turns/phase}} = \frac{1}{1.8}$

$$E_2 = KE_1 = \frac{1}{1.8} \times \frac{240}{\sqrt{3}} = 77 \text{ V}; s = 0.04;$$

$$N_s = 120 \times 50/6 = 1000 \text{ rpm} = 50/3 \text{ rps}$$

$$T_f = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \dots \text{Art. 34.19}$$

$$= \frac{3}{2\pi (50/3)} \cdot \frac{0.04 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 52.4 \text{ N-m}$$

For maximum torque,  $s = R_2/X_2 = 0.12/0.85 = 0.14$

$$\therefore T_{max} = \frac{3}{2\pi (50/3)} \cdot \frac{0.14 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 99.9 \text{ N-m}$$

Alternatively, as seen from Art 34.20.

$$T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2}$$

$$\therefore T_{max} = \frac{3}{2\pi (50/3)} \cdot \frac{77^2}{2 \times 0.85} = 99.9 \text{ N-m}$$

Speed corresponding to maximum torque,  $N = 1000(1 - 0.14) = \mathbf{860 \text{ rpm}}$

**Example 34.23.** The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015  $\Omega$  and 0.09  $\Omega$  per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at half full-load speed. Also, calculate the power factor. **(Adv. Elect. Machines, A.M.I.E. 1989)**

**Solution.** Let  $N_s = 100$  r.p.m. F.L. speed =  $(1 - 0.03)100 = 97$  r.p.m.

Let the normal voltage be  $V_1$  volts.

Speed in second case =  $97/2 = 48.5$  r.p.m.

$\therefore$  slip =  $(100 - 48.5)/100 = 0.515$  or 51.5%

Now, 
$$T = \frac{k \Phi s E_2 R_2}{R_2^2 + (s X_2)^2} = \frac{k s V^2 R_2}{R_2^2 + (s X_2)^2} \quad (\because E_2 \propto \Phi \propto V)$$

Since torque is the same in both cases,

$$\frac{k V_1^2 s_1 R_2}{R_2^2 + (s_1 X_2)^2} = \frac{k V_2^2 s_2 R_2}{R_2^2 + (s_2 X_2)^2} \quad \text{where } V_2 = \text{stator voltage in second case}$$

$$\begin{aligned} \therefore \left(\frac{V_1}{V_2}\right)^2 &= \frac{s_2}{s_1} \cdot \frac{R_2^2 + (s_1 X_2)^2}{R_2^2 + (s_2 X_2)^2} \\ &= \frac{51.3}{3} \cdot \frac{0.015^2 + (0.03 \times 0.09)^2}{0.015^2 + (0.515 \times 0.09)^2} = 1.68 \end{aligned}$$

$$\therefore \frac{V_1}{V_2} = \sqrt{1.68} = 1.296 \quad \text{or} \quad \frac{V_1 - V_2}{V_1} = \frac{0.296}{1.296}$$

Hence, percentage, reduction in stator (or supply voltage) is

$$= \frac{V_1 - V_2}{V_1} \times 100 = \frac{0.296 \times 100}{1.296} = \mathbf{22.84\%}$$

In the second case,  $\tan \phi = s_2 X_2 / R_2 = 0.515 \times 0.09 / 0.015 = 3.09$

$\therefore \phi = \tan^{-1}(3.09) = 72^\circ 4'$  and p.f. =  $\cos \phi = \cos 72^\circ 4' = \mathbf{0.31}$

### 34.27. Torque/Speed Curve

The torque developed by a conventional 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easier to show the relationship in the form of a curve (Fig. 34.22). In this diagram,  $T$  represents the nominal full-load torque of the motor. As seen, the starting torque (at  $N=0$ ) is  $1.5 T$  and the maximum torque (also called breakdown torque) is  $2.5 T$ .

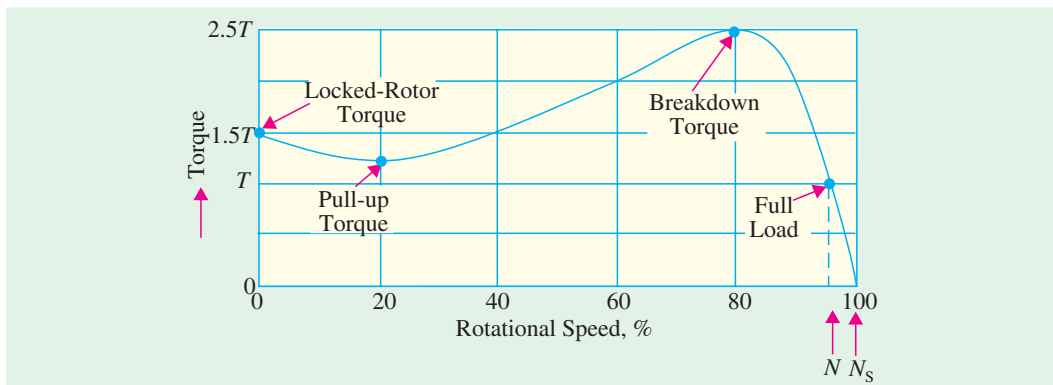


Fig. 34.22

At full-load, the motor runs at a speed of  $N$ . When mechanical load increases, motor speed decreases till the motor torque again becomes equal to the load torque. As long as the two torques are in balance, the motor will run at constant (but lower) speed. However, if the load torque exceeds  $2.5 T$ , the motor will suddenly stop.

### 34.28. Shape of Torque/Speed Curve

For a squirrel-cage induction motor (SCIM), shape of its torque/speed curve depends on the voltage and frequency applied to its stator. If  $f$  is fixed,  $T \propto V^2$  (Art 34.22). Also, synchronous speed

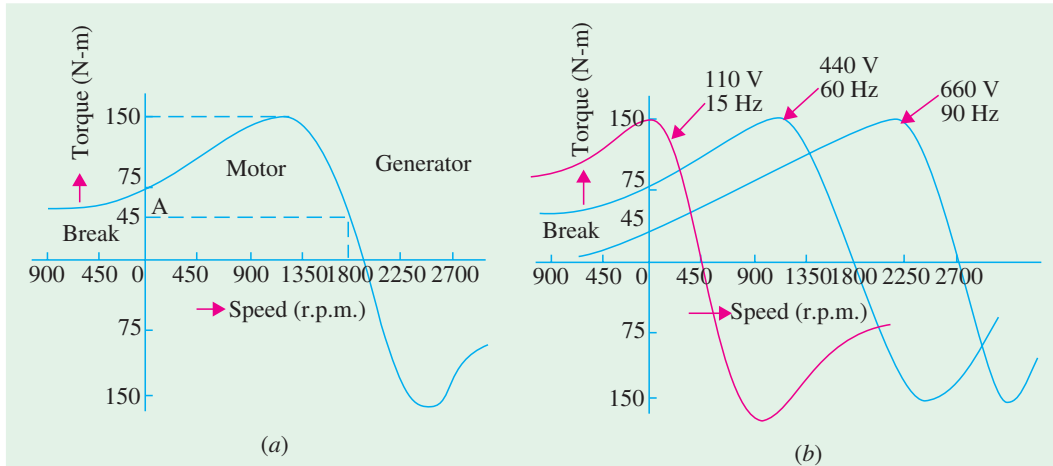


Fig. 34.23

depends on the supply frequency. Now, let us see what happens when *both* stator voltage and frequency are changed. In practice, supply voltage and frequency are varied in the **same proportion** in order to maintain a constant flux in the air-gap. For example, if voltage is doubled, then frequency is also doubled. Under these conditions, shape of the torque/speed curve remains the same but its position along the  $X$ -axis (*i.e.* speed axis) shifts with frequency.

Fig. 34.23 (a) shows the torque/speed curve of an 11.kW, 440-V, 60-Hz 3- $\phi$  SCIM. As seen, full-load speed is 1728 rpm and full-load torque is 45 N-m (point-A) whereas breakdown torque is 150 N-m and locked-rotor torque is 75 N-m.

Suppose, we now reduce both the voltage and frequency to **one-fourth** their original values *i.e.* to 110 V and 15 Hz respectively. As seen in Fig. 34.23 (b), the torque/speed curve shifts to the left. Now, the curve crosses the  $X$ -axis at the synchronous speed of  $120 \times 15/4 = 450$  rpm (*i.e.*  $1800/4 = 450$  rpm). Similarly, if the voltage and frequency are increased by 50% (660 V 90 Hz), the curve shifts to the right and cuts the  $X$ -axis at the synchronous speed of 2700 rpm.

Since the *shape* of the torque/speed curve remains the same at all **frequencies**, it follows that torque developed by a SCIM is the same **whenever slip-speed is the same**.

**Exampel 34.26.** A 440-V, 50-Hz, 4-pole, 3-phase SCIM develops a torque of 100 N-m at a speed of 1200 rpm. If the stator supply frequency is reduced by half, calculate

- the stator supply voltage required for maintaining the same flux in the machine.
- the new speed at a torque of 100 N-m.

**Solution.** (a) The stator voltage must be reduced in proportion to the frequency. Hence, it should also be reduced by half to  $440/2 = 220$  V.

(b) Synchronous speed at 50 Hz frequency =  $120 \times 50/4 = 1500$  rpm. Hence, slip speed for a torque of 100 N-m =  $1500 - 1200 = 300$  rpm.

Now, synchronous speed at 25 Hz =  $1500/2 = 750$  rpm.

Since slip-speed has to be the same for the same torque irrespective of the frequency, the new speed at 100 N-m is =  $750 + 300 = 1050$  rpm.

### 34.29. Current/Speed Curve of an Induction Motor

It is a  $V$ -shaped curve having a minimum value at synchronous speed. This minimum is equal to

the magnetising current which is needed to create flux in the machine. Since flux is purposely kept constant, it means that magnetising current is the same at all synchronous speeds.

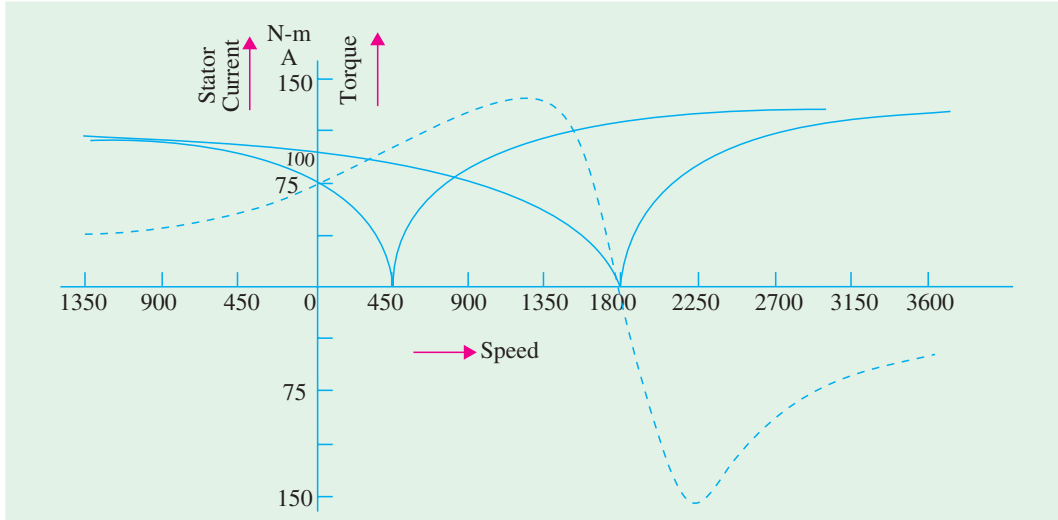


Fig. 34.24

Fig. 34.24. shows the current/speed curve of the SCIM discussed in Art. 34.28 above. Refer Fig. 34.23(b) and Fig. 34.24. As seen, locked rotor current is 100 A and the corresponding torque is 75 N-m. If stator voltage and frequency are varied in the same proportion, current/speed curve has the same shape, but shifts along the speed axis. Suppose that voltage and frequency are reduced to one-fourth of their previous values *i.e.* to 110 V, 15 Hz respectively. Then, locked rotor current **decreases** to 75 A but corresponding torque **increases** to 150 N-m which is equal to full breakdown torque (Fig. 34.25).

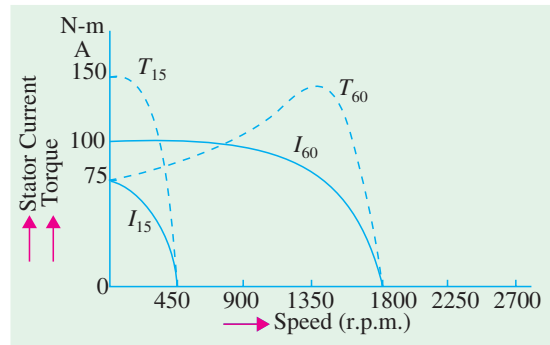


Fig. 34.25

It means that by reducing frequency, we can obtain **a larger torque with a reduced current**. This is one of the big advantages of frequency control method. By progressively increasing the voltage and current during the start-up period, a SCIM can be made to develop close to its breakdown torque all the way from zero to rated speed.

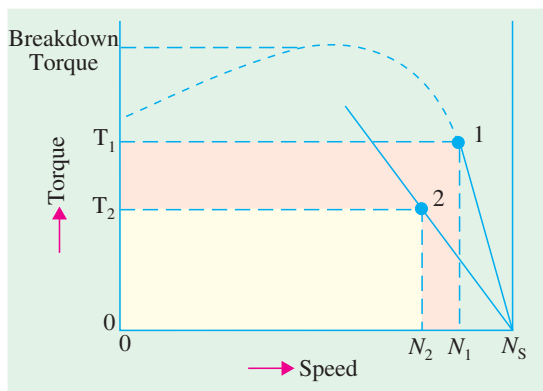


Fig. 34.26

Another advantage of frequency control is that it permits regenerative braking of the motor. In fact, the main reason for the popularity of frequency-controlled induction motor drives is their ability to develop high torque from zero to full speed together with the economy of regenerative braking.

### 34.30. Torque/Speed Characteristic Under Load

As stated earlier, stable operation of an induction motor lies over the linear portion of its torque/speed curve. The slope of this straight line depends

mainly on the rotor resistance. Higher the resistance, sharper the slope. This linear relationship between torque and speed (Fig. 34.26) enables us to establish a very simple equation between different parameters of an induction motor. The parameters under two different load conditions are related by the equation

$$s_2 = s_1 \cdot \frac{T_2}{T_1} \cdot \frac{R_2}{R_1} \left( \frac{V_1}{V_2} \right)^2 \quad \dots(i)$$

The only restriction in applying the above equation is that the new torque  $T_2$  must not be greater than  $T_1$  ( $V_2/V_1$ )<sup>2</sup>. In that case, the above equation yields an accuracy of better than 5% which is sufficient for all practical purposes.

**Example 34.24.** A 400-V, 60-Hz, 8-pole, 3-φ induction motor runs at a speed of 1140 rpm when connected to a 440-V line. Calculate the speed if voltage increases to 550V.

**Solution.** Here,  $s_1 = (1200 - 1140)/1200 = 0.05$ . Since everything else remains the same in Eq. (i) of Art. 34.30 except the slip and voltage, hence

$$S_2 = s_1 (V_1/V_2)^2 = 0.05 \times (440/550)^2 = 0.032 \quad \dots\text{Art. 34.23}$$

$$\therefore N_2 = 1200(1 - 0.032) = \mathbf{1161.6\text{ rpm.}}$$

**Example 34.25.** A 450-V, 60-Hz, 8-Pole, 3-phase induction motor runs at 873 rpm when driving a fan. The initial rotor temperature is 23° C. The speed drops to 864 rpm when the motor reaches its final temperature. Calculate (i) increase in rotor resistance and (ii) approximate temperature of the hot rotor if temperature coefficient of resistance is 1/234 per ° C.

**Solution.**  $s_1 = (900 - 873)/900 = 0.03$  and  $s_2 = (900 - 864)/900 = 0.04$

Since voltage and frequency etc. are fixed, the change in speed is entirely due to change in rotor resistance.

$$(i) \quad s_2 = s_1 (R_2/R_1) \quad \text{or} \quad 0.04 = 0.03 (R_2/R_1); \quad R_2 = 1.33 R_1$$

Obviously, the rotor resistance has increased by **33 percent**.

(ii) Let  $t_2$  be temperature of the rotor. Then, as seen from Art.1-11,

$$R_2 = R_1 [1 + \alpha (t_2 - 23)] \quad \text{or} \quad 1.33 R_1 = R_1 \left[ 1 + \frac{1}{234} (t_2 - 23) \right] \quad \therefore t_2 = \mathbf{100.2^\circ\text{C}}$$

### 34.31. Plugging of an Induction Motor

An induction motor can be quickly stopped by simply inter-changing any of its two stator leads. It reverses the direction of the revolving flux which produces a torque in the reverse direction, thus

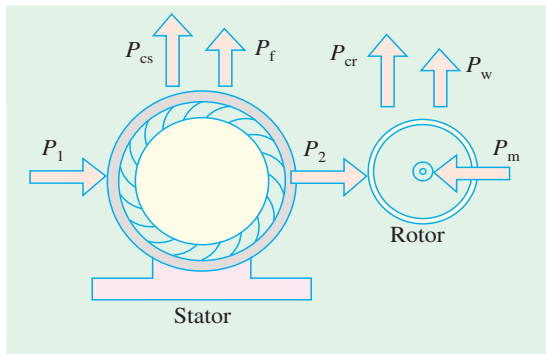


Fig. 34.27



Induction asynchronous motor

applying brake on the motor. Obviously, during this so-called plugging period, **the motor acts as a brake**. It absorbs kinetic energy from the still

revolving load causing its speed to fall. The associated Power  $P_m$  is dissipated as heat in the rotor. At the same time, the rotor also continues to receive power  $P_2$  from the stator (Fig. 34.27) which is also dissipated as heat. Consequently, plugging produces rotor  $I^2R$  losses which even exceed those when the rotor is locked.

### 34.32. Induction Motor Operating as a Generator

When run *faster than* its synchronous speed, an induction motor runs as a generator called a **Induction generator**. It converts the mechanical energy it receives into electrical energy and this energy is released by the stator (Fig. 34.29). Fig. 34.28 shows an ordinary squirrel-cage motor which is driven by a petrol engine and is connected to a 3-phase line. As soon as motor speed exceeds its synchronous speed, it starts delivering **active** power  $P$  to the 3-phase line. However, for creating its own magnetic field, it absorbs **reactive** power  $Q$  from the line to which it is connected. As seen,  $Q$  flows in the **opposite** direction to  $P$ .

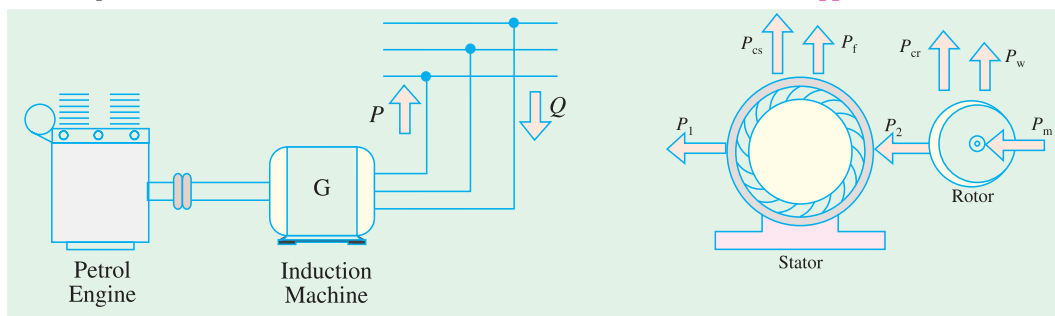


Fig. 34.28

Fig. 34.29

The active power **is directly proportional to the slip** above the synchronous speed. The reactive power required by the machine can also be supplied by a group of capacitors connected across its terminals (Fig. 34.30). This arrangement can be used to supply a 3-phase load without using an external source. The frequency generated is slightly less than that corresponding to the speed of rotation.

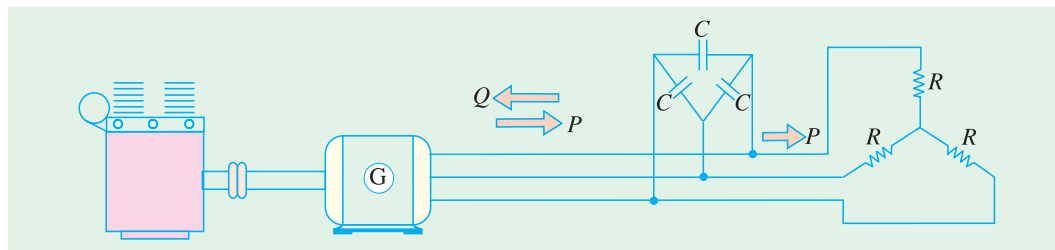


Fig. 34.30

The terminal voltage increases with capacitance. If capacitance is insufficient, the generator voltage will not build up. Hence, capacitor bank must be large enough to supply the reactive power normally drawn by the motor.

**Example 34.26.** A 440-V, 4-pole, 1470 rpm, 30-kW, 3-phase induction motor is to be used as an asynchronous generator. The rated current of the motor is 40 A and full-load power factor is 85%. Calculate

- (a) capacitance required per phase if capacitors are connected in delta.
- (b) speed of the driving engine for generating a frequency of 50 Hz.

**Solution. (i)** 
$$S = \sqrt{3} \cdot VI = 1.73 \times 440 \times 40 = 30.4 \text{ kVA}$$

$$P = S \cos \phi = 30.4 \times 0.85 = 25.8 \text{ kW}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{30.4^2 - 25.8^2} = 16 \text{ kVAR}$$

Hence, the  $\Delta$ -connected capacitor bank (Fig. 32.31) must provide  $16/3 = 5.333 \text{ kVAR}$  per phase.

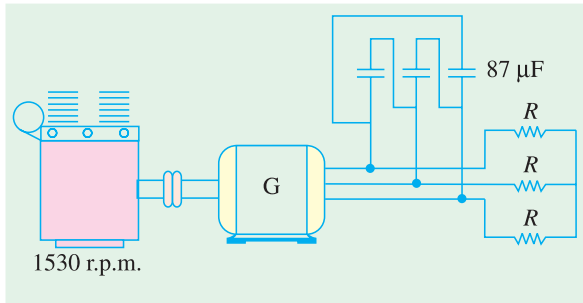


Fig. 34.31

Capacitor current per phase is  $= 5,333/440 = 12 \text{ A}$ . Hence  $X_c = 440/12 = 36.6 \Omega$ . Now,

$$C = \frac{1}{2\pi f X_c} = 1/2\pi \times 50 \times 36.6 = \mathbf{87\mu F}$$

(ii) The driving engine must run at slightly more than synchronous speed. The slip speed is usually the same as that when the machine runs as a motor *i.e.* 30 rpm.

Hence, engine speed is  $= 1500 + 30 = \mathbf{1530 \text{ rpm}}$ .

### 34.33. Complete Torque/Speed Curve of a Three-Phase Machine

We have already seen that a 3-phase machine can be run as a **motor**, when it takes electric power and supplies mechanical power. The directions of torque and rotor rotation are in the **same** direction. The same machine can be used as an **asynchronous generator** when driven at a speed **greater** than the synchronous speed. In this case, it receives mechanical energy in the rotor and supplies electrical energy from the stator. The torque and speed are **oppositely-directed**.

The same machine can also be used as a **brake** during the plugging period (Art. 34.31). The three modes of operation are depicted in the torque/speed curve shown in Fig. 34.32.

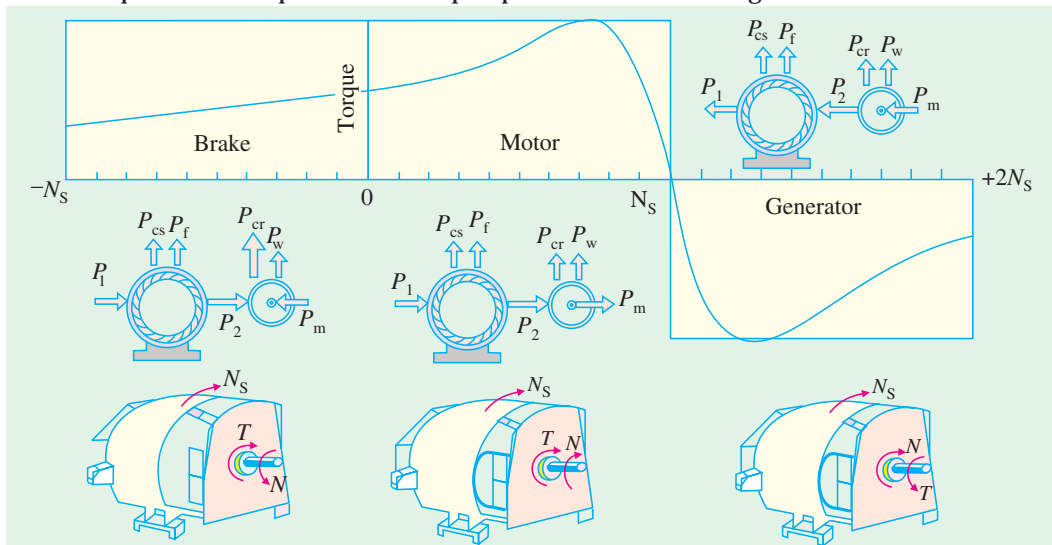


Fig. 34.32

### Tutorial Problem No. 34.2

1. In a 3-phase, slip-ring induction motor, the open-circuit voltage across slip-rings is measured to be 110 V with normal voltage applied to the stator. The rotor is star-connected and has a resistance of  $1 \Omega$  and reactance of  $4 \Omega$  at standstill condition. Find the rotor current when the machine is (a) at standstill with slip-rings joined to a star-connected starter with a resistance of  $2\Omega$  per phase and negligible reactance (b) running normally with 5% slip. State any assumptions made.

[12.7 A ; 3.11 A] (Electrical Technology-I, Bombay Univ. 1978)



2. The star-connected rotor of an induction motor has a standstill impedance of  $(0.4 + j4)$  ohm per phase and the rheostat impedance per phase is  $(6 + j2)$  ohm. The motor has an induced e.m.f. of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find (a) rotor current at standstill with the rheostat in the circuit (b) when the slip-rings are short-circuited and the motor is running with a slip of 3%. **[5.27 A ; 3.3 A]**
3. A 4-pole, 50-Hz induction motor has a full-load slip of 5%. Each rotor phase has a resistance of 0.3  $\Omega$  and a standstill reactance of 1.2  $\Omega$ . Find the ratio of maximum torque to full-load torque and the speed at which maximum torque occurs. **[2.6 ; 1125 r.p.m.]**
4. A 3-phase, 4-pole, 50-Hz induction motor has a star-connected rotor. The voltage of each rotor phase at standstill and on open-circuit is 121 V. The rotor resistance per phase is 0.3  $\Omega$  and the reactance at standstill is 0.8  $\Omega$ . If the rotor current is 15 A, calculate the speed at which the motor is running. Also, calculate the speed at which the torque is a maximum and the corresponding value of the input power to the motor, assuming the flux to remain constant. **[1444 r.p.m.; 937.5 r.p.m.]**
5. A 4-pole, 3-phase, 50 Hz induction motor has a voltage between slip-rings on open-circuit of 520 V. The star-connected rotor has a standstill reactance and resistance of 2.0 and 0.4  $\Omega$  per phase respectively. Determine :
  - (a) the full-load torque if full-load speed is 1,425 r.p.m.
  - (b) the ratio of starting torque to full-load torque
  - (c) the additional rotor resistance required to give maximum torque at standstill**[(a) 200 N-m (b) 0.82 (c) 1.6  $\Omega$ ] (Elect. Machines-II, Vikram Univ. Ujjain 1977)**
6. A 50-Hz, 8-pole induction motor has a full-load slip of 4 per cent. The rotor resistance is 0.001  $\Omega$  per phase and standstill reactance is 0.005  $\Omega$  per phase. Find the ratio of the maximum to the full-load torque and the speed at which the maximum torque occurs. **[2.6; 600 r.p.m.] (City & Guilds, London)**
7. A 3- $\phi$ , 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between slip-rings on open circuit. Calculate the current and power factor in each phase of the rotor windings at standstill when joined to a star-connected circuit, each limb of which has a resistance of 10  $\Omega$  and an inductance of 0.03 H. The resistance per phase of the rotor windings is 0.2  $\Omega$  and inductance 0.03 H. Calculate also the current and power factor in each rotor phase when the rings are short-circuited and the motor is running with a slip of 4 per cent. **[13.6 A, 0.48; 27.0 A, 0.47] (London University)**
8. A 4-pole, 50-Hz, 3-phase induction motor has a slip-ring rotor with a resistance and standstill reactance of 0.04  $\Omega$  and 0.2  $\Omega$  per phase respectively. Find the amount of resistance to be inserted in each rotor phase to obtain full-load torque at starting. What will be the approximate power factor in the rotor at this instant ? The slip at full-load is 3 per cent. **[0.084  $\Omega$ , 0.516 p.f.] (London University)**
9. A 3- $\phi$  induction motor has a synchronous speed of 250 r.p.m. and 4 per cent slip at full-load. The rotor has a resistance of 0.02  $\Omega$ /phase and a standstill leakage reactance of 0.15  $\Omega$ /phase. Calculate (a) the ratio of maximum and full-load torque (b) the speed at which the maximum torque is developed. Neglect resistance and leakage of the stator winding. **[(a) 1.82 (b) 217 r.p.m.] (London University)**
10. The rotor of an 8-pole, 50-Hz, 3-phase induction motor has a resistance of 0.2  $\Omega$ /phase and runs at 720 r.p.m. If the load torque remains unchanged. Calculate the additional rotor resistance that will reduce this speed by 10% **[0.8  $\Omega$ ] (City & Guilds, London)**
11. A 3-phase induction motor has a rotor for which the resistance per phase is 0.1  $\Omega$  and the reactance per phase when stationary is 0.4  $\Omega$ . The rotor induced e.m.f. per phase is 100 V when stationary. Calculate the rotor current and rotor power factor (a) when stationary (b) when running with a slip of 5 per cent. **[(a) 242.5 A; 0.243 (b) 49 A; 0.98]**
12. An induction motor with 3-phase star-connected rotor has a rotor resistance and standstill reactance of 0.1  $\Omega$  and 0.5  $\Omega$  respectively. The slip-rings are connected to a star-connected resistance of 0.2  $\Omega$  per phase. If the standstill voltage between slip-rings is 200 volts, calculate the rotor current per phase when the slip is 5%, the resistance being still in circuit. **[19.1 A]**
13. A 3-phase, 50-Hz induction motor has its rotor windings connected in star. At the moment of starting

the rotor, induced e.m.f. between each pair of slip-rings is 350 V. The rotor resistance per phase is  $0.2 \Omega$  and the standstill reactance per phase is  $1 \Omega$ . Calculate the rotor starting current if the external starting resistance per phase is  $8 \Omega$  and also the rotor current when running with slip-rings short-circuited, the slip being 3 per cent. **[24.5 A ; 30.0 A]**

14. In a certain 8-pole, 50-Hz machine, the rotor resistance per phase is  $0.04 \Omega$  and the maximum torque occurs at a speed of 645 r.p.m. Assuming that the air-gap flux is constant at all loads, determine the percentage of maximum torque (a) at starting (b) when the slip is 3%.

**[(a) 0.273 (b) 0.41] (London University)**

15. A 6-pole, 3-phase, 50-Hz induction motor has rotor resistance and reactance of  $0.02 \Omega$  and  $0.1 \Omega$  respectively per phase. At what speed would it develop maximum torque ? Find out the value of resistance necessary to give half of maximum torque at starting.

**[800 rpm;  $0.007 \Omega$ ] (Elect.Engg. Grad I.E.T.E. June 1988)**

### 34.34. Measurement of Slip

Following are some of the methods used for finding the slip of an induction motor whether squirrel-cage or slip-ring type.

**(i) By actual measurement of motor speed**

This method requires measurement of actual motor speed  $N$  and calculation of synchronous speed  $N_s$ .  $N$  is measured with the help of a speedometer and  $N_s$  calculated from the knowledge of supply frequency and the number of poles of the motor.\* Then slip can be calculated by using the equation.

$$s = (N_s - N) \times 100 / N_s$$

**(ii) By comparing rotor and stator supply frequencies**

This method is based on the fact that  $s = f_r / f$

Since  $f$  is generally known,  $s$  can be found if frequency of rotor current can be measured by some method. In the usual case, where  $f$  is 50 Hz,  $f_r$  is so low that individual cycles can be easily counted. For this purpose, a d.c. moving-coil millivoltmeter, preferably of centre-zero, is employed as described below :

**(a)** In the case of a slip-ring motor, the leads of the millivoltmeter are lightly pressed against the adjacent slip-rings as they revolve (Fig. 34.33). Usually, there is sufficient voltage drop in the brushes and their short-circuiting strap to provide an indication on the millivoltmeter. The current in the millivoltmeter follows the variations of the rotor current and hence the pointer oscillates about its mean zero position. The number of complete cycles made by the pointer per second can be easily counted (it is worth remembering that one cycle consists of a movement from zero to a maximum to the right, back to zero and on to a maximum to the left and then back to zero).

As an example, consider the case of a 4-pole motor fed from a 50-Hz supply and running at 1,425 r.p.m. Since

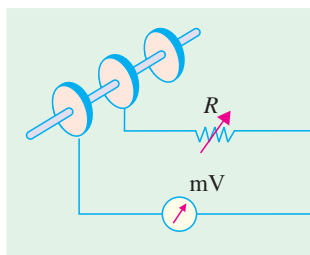
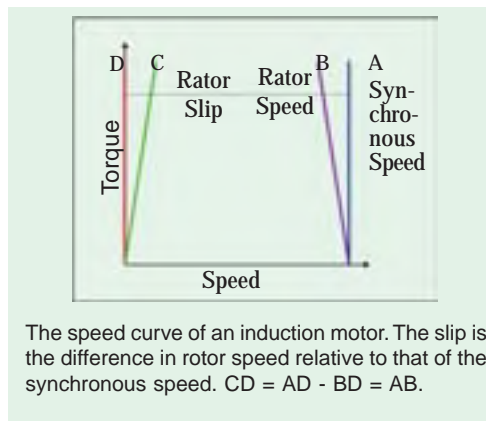


Fig. 34.33

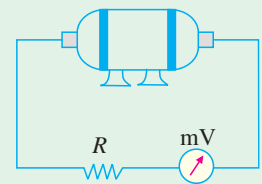


Fig. 34.34

\* Since an induction motor does not have salient poles, the number of poles is usually inferred from the no-load speed or from the rated speed of the motor.

$N_s = 1,500$  r.p.m., its slip is 5% or 0.05. The frequency of the rotor current would be  $f_r = s_f = 0.05 \times 50 = 2.5$  Hz which (being slow enough) can be easily counted.

(b) For squirrel-cage motors (which do not have slip-rings) it is not possible to employ the millivoltmeter so **directly**, although it is sometimes possible to pick up some voltage by connecting the millivoltmeter across the ends of the motor shaft (Fig. 34.34)

Another method, sometime employed, is as follows :

A large flat search coil of many turns is placed centrally against the end plate on the non-driving end of the motor. Quite often, it is possible to pick up sufficient voltage by induction from the leakage fluxes to obtain a reading on the millivoltmeter. Obviously, a large 50-Hz voltage will also be induced in the search coil although it is too rapid to affect the millivoltmeter. Commercial slip-indicators use such a search coil and, in addition, contain a low-pass filter amplifier for eliminating fundamental frequency and a bridge circuit for comparing stator and rotor current frequencies.

**(iii) Stroboscopic Method**

In this method, a circular metallic disc is taken and painted with alternately black and white segments. The number of segments (both black and white) is equal to the number of poles of the motor. For a 6-pole motor, there will be six segments, three black and three white, as shown in Fig. 34.35(a). The painted disc is mounted on the end of the shaft and illuminated by means of a neon-filled stroboscopic lamp, which may be supplied preferably with a combined d.c. and a.c. supply although only a.c. supply will do\*. The connections for combined supply are shown in Fig. 34.36 whereas Fig. 34.35 (b) shows the connection for a.c. supply only. It must be noted that with combined d.c. and a.c. supply, the lamp will flash once per cycle\*\*. But with a.c. supply, it will flash twice per cycle.

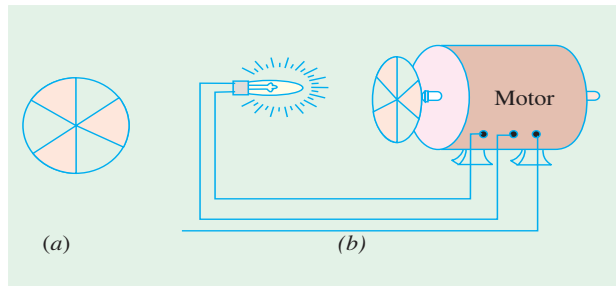


Fig. 34.35

Consider the case when the revolving disc is seen in the flash light of the bulb which is fed by the combined d.c. and a.c. supply.

If the disc were to rotate at synchronous speed, it would appear to be stationary. Since, in actual practice, its speed is slightly less than the synchronous speed, it appears to rotate slowly backwards. The reason for this apparent backward movement is as follows :

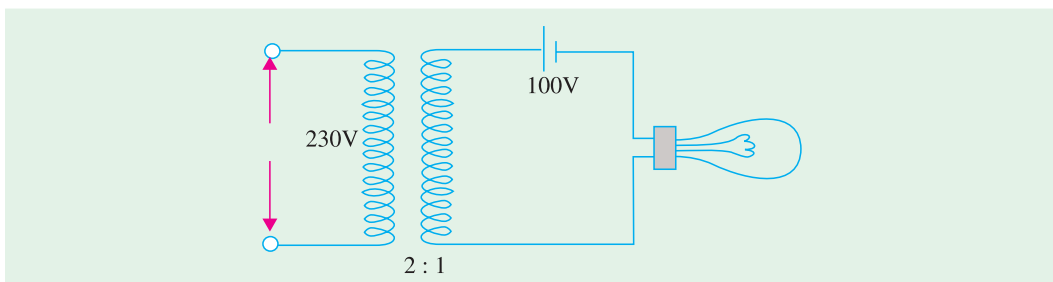


Fig. 34.36

\* When combined d.c. and a.c. supply is used, the lamp should be tried both ways in its socket to see which way it gives better light.

\*\* It will flash only when the two voltages add and remain extinguished when they oppose.

Let Fig. 34.37(a) represent the position of the white lines when they are illuminated by the first flash. When the next flash comes, they have **nearly** reached positions 120° ahead (but not quite), as shown in Fig. 34.37(b). Hence, line No. 1 has **almost** reached the position previously occupied by line No. 2 and one flash still later [Fig. 34.37 (c)] it has **nearly** reached the position previously occupied by line No. 3 in Fig. 34.37(a).

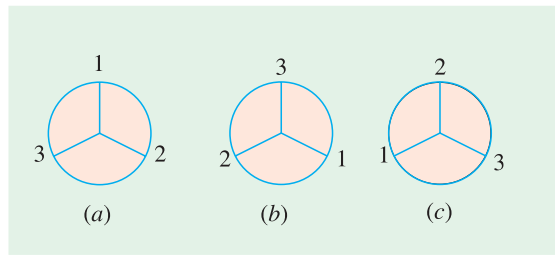


Fig. 34.37

By counting the number of lines passing a fixed point in, say, a minute and dividing by the number of lines seen (*i.e.* three in the case of a 6-pole motor and so on) the apparent backward speed in r.p.m. can be found. This gives slip-speed in r.p.m. *i.e.*  $N_s - N$ . The slip may be found from the

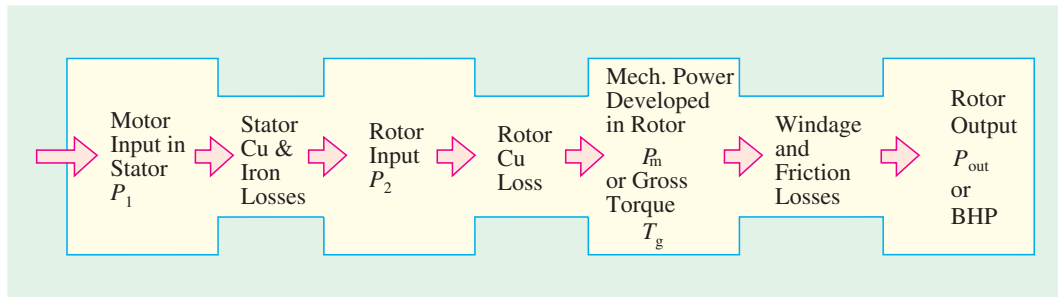
$$\text{relation } s = \frac{N_s - N}{N_s} \times 100$$

**Note.** If the lamp is fed with a.c. supply alone, then it will flash twice per cycle and twice as many lines will be seen rotating as before.

### 34.35. Power Stages in an Induction Motor

Stator iron loss (consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core. It is practically constant. The iron loss of the rotor is, however, negligible because frequency of rotor currents under normal running conditions is always small. Total rotor Cu loss =  $3 I_2^2 R_2$ .

Different stages of power development in an induction motor are as under :



A better visual for power flow, within an induction motor, is given in Fig. 34.38.

### 34.36. Torque Developed by an Induction Motor

An induction motor develops gross torque  $T_g$  due to gross rotor output  $P_m$  (Fig 34.38). Its value can be expressed either in terms of rotor input  $P_2$  or rotor gross output  $P_m$  as given below.

$$T_g = \frac{P_2}{\omega_s} = \frac{P_m}{2\pi N_s} \quad \dots \text{in terms of rotor input}$$

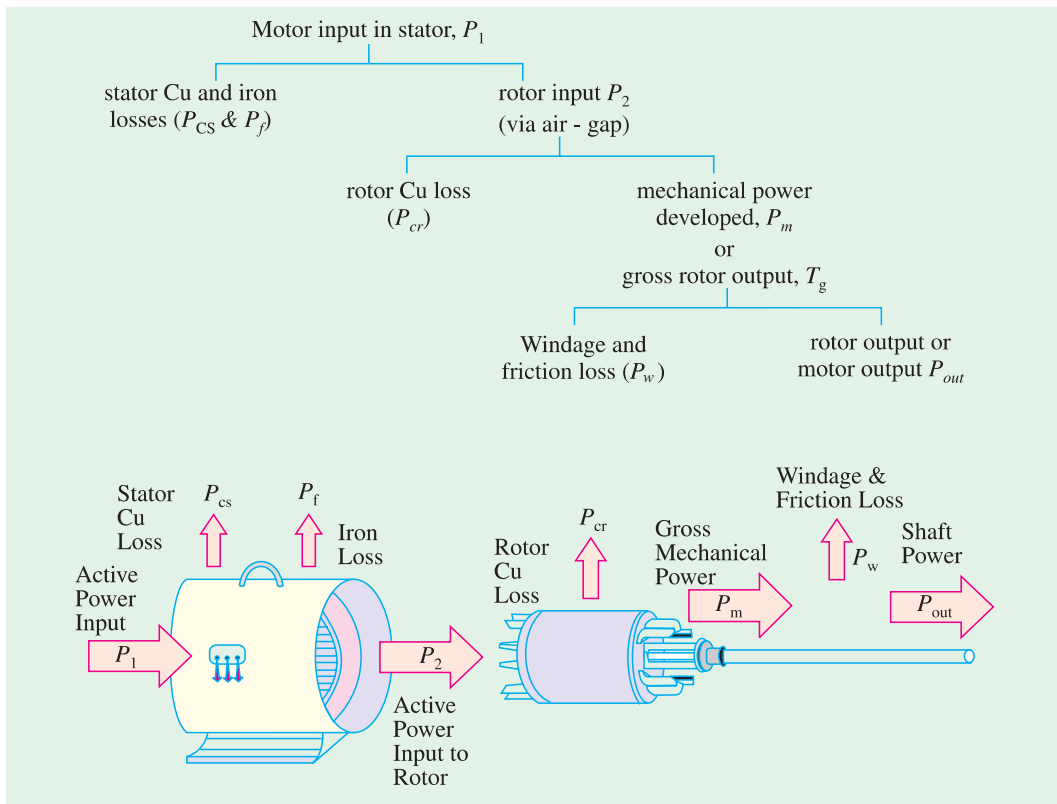


Fig. 34.38

$$T_g = \frac{P_m}{\omega} = \frac{P_m}{2\pi N} \quad \dots \text{ in terms of rotor output}$$

The shaft torque  $T_{sh}$  is due to output power  $P_{out}$  which is less than  $P_m$  because of rotor friction and windage losses.

$$\therefore T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

The difference between  $T_g$  and  $T_{sh}$  equals the torque lost due to friction and windage loss in the motor.

In the above expressions,  $N$  and  $N_s$  are in r.p.s. However, if they are in r.p.m., the above expressions for motor torque become

$$\begin{aligned} T_g &= \frac{P_2}{2\pi N_s/60} = \frac{60}{2\pi} \cdot \frac{P_2}{N_s} = 9.55 \frac{P_2}{N_s} \text{ N-m} \\ &= \frac{P_m}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_m}{N} = 9.55 \frac{P_m}{N} \text{ N-m} \\ T_{sh} &= \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_{out}}{N} = 9.55 \frac{P_{out}}{N} \text{ N-m} \end{aligned}$$

### 34.37. Torque, Mechanical Power and Rotor Output

Stator input  $P_1 = \text{stator output} + \text{stator losses}$   
 The stator output is transferred entirely inductively to the rotor circuit.

Obviously, rotor input  $P_2 =$  stator output  
 Rotor gross output,  $P_m =$  rotor input  $P_2$  — rotor Cu losses



The type DF200 diesel electric locomotive is the first motor-driven train equipped with diesel generators since 1958. It adopted induction motors to realize high acceleration, high speed, and large torque, which resulted in a quick-response generator brake system.

This rotor output is converted into mechanical energy and gives rise to gross torque  $T_g$ . Out of this gross torque developed, some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque  $T_{shr}$ .

Let  $N$  r.p.s. be the actual speed of the rotor and if  $T_g$  is in N-m, then

$$T_g \times 2\pi N = \text{rotor gross output in watts, } P_m$$

$$\therefore T_g = \frac{\text{rotor gross output in watts, } P_m}{2\pi N} \text{ N-m}^* \quad \dots(1)$$

If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed.

$$\therefore T_g = \frac{\text{rotor input } P_2}{2\pi N_s} \quad \dots(2)$$

From (1) and (2), we get,

Rotor gross output  $P_m = T_g \omega = T_g \times 2\pi N$

Rotor input  $P_2 = T_g \omega_s = T_g \times 2\pi N_s$  ... (3)

The difference of two equals rotor Cu loss.

$$\therefore \text{rotor Cu loss} = P_2 - P_m = T_g \times 2\pi (N_s - N) \quad \dots(4)$$

From (3) and (4),

$$\frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \text{rotor Cu loss} = s \times \text{rotor input} = s \times \text{power across air-gap} = s P_2 \quad \dots(5)$$

Also, rotor input = rotor Cu loss/s

Rotor gross output,  $P_m = \text{input } P_2 - \text{rotor Cu loss} = \text{input} - s \times \text{rotor input}$

$$= (1 - s) \text{ input } P_2 \quad \dots(6)$$

$$\therefore \text{rotor gross output } P_m = (1 - s) \text{ rotor input } P_2$$

or  $\frac{\text{rotor gross output, } P_m}{\text{rotor input, } P_2} = 1 - s = \frac{N}{N_s}; \quad \frac{P_m}{P_2} = \frac{N}{N_s}$

$$\therefore \text{rotor efficiency} = \frac{N}{N_s} \quad \text{Also, } \frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1 - s}$$

**Important Conclusion**

If some power  $P_2$  is delivered to a rotor, then a part  $sP_2$  is lost in the rotor itself as copper loss (and appears as heat) and the remaining  $(1 - s)P_2$  appears as gross mechanical power  $P_m$  (including friction and windage losses).

$$\therefore P_2 : P_m : P_{cr} :: 1 : (1 - s) : s \quad \text{or} \quad P_2 : P_m : P_{cr} :: 1 : (1 - s) : s$$

\* The value of gross torque in kg-m is given by

$$T_g = \frac{\text{rotor gross output in watts}}{9.81 \times 2\pi N} \text{ kg-m.} = \frac{P_m}{9.81 \times 2\pi N} \text{ kg-m}$$

The rotor input power will always divide itself in this ratio, hence it is advantageous to run the motor with as small a slip as possible.

**Example 34.27.** The power input to the rotor of 440 V, 50 Hz, 6-pole, 3-phase, induction motor is 80 kW. The rotor electromotive force is observed to make 100 complete alterations per minute. Calculate (i) the slip, (ii) the rotor speed, (iii) rotor copper losses per phase. [Madras University, 1997]

**Solution.** 100 alterations/minute =  $\frac{100}{60}$  cycles/sec

$$1.6667 \text{ Hz} = sf$$

Hence, the slip,  $s = \frac{1.6667}{50} = 0.3333 \text{ P.u. or } 3.333\%$

(ii) rotor speed,  $N = (1 - s) N_s = (1 - 0.03333) \times 1000$

Since  $N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$ ,  $N = 966.67 \text{ rpm}$

(iii) rotor copper losses/phase =  $\frac{1}{3} \times (s \times \text{rotor input})$

$$\text{total rotor power input} = 80 \text{ kW}$$

$$\text{rotor power input per phase} = 80/3 \text{ kW}$$

rotor copper losses per phase

$$= \frac{0.0333 \times 80}{3} \text{ kW} = 0.8888 \text{ kW}$$

**Example 34.28.** A 440-V, 3- $\phi$ , 50-Hz, 4-pole, Y-connected induction motor has a full-load speed of 1425 rpm. The rotor has an impedance of  $(0.4 + j4)$  ohm and rotor/stator turn ratio of 0.8. Calculate (i) full-load torque (ii) rotor current and full-load rotor Cu loss (iii) power output if windage and friction losses amount to 500 W (iv) maximum torque and the speed at which it occurs (v) starting current and (vi) starting torque.

**Solution.**  $N_s = 120 \times 50/4 = 1500 \text{ rpm} = 25 \text{ rps}$ ,  $s = 75/1500 = 0.05$   
 $E_1 = 440/1.73 = 254 \text{ V/phase}$

(i) Full-load  $T_f = \frac{3}{2\pi \times 25} \times \frac{0.05 (0.8 \times 254)^2 \times 0.4}{(0.4)^2 + (0.05 \times 4)^2}$   
 $= 78.87 \text{ N-m}$

(ii)  $I_r = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{sKE_1}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.05 \times (0.8 \times 254)}{\sqrt{(0.4)^2 + (0.05 \times 4)^2}}$   
 $= 22.73 \text{ A}$

Total Cu loss =  $3I_r^2 R = 3 \times 22.73^2 \times 0.4 = 620 \text{ W}$

(iii) Now,  $P_m = 2\pi NT = 2\pi \times (1425/60) \times 78.87 = 11,745 \text{ W}$

$\therefore P_{\text{out}} = P_m - \text{windage and friction loss} = 11,745 - 500 = 11,245 \text{ W}$

(iv) For maximum torque,  $s = R_2/X_2 = 0.4/4 = 0.1$

$\therefore T_{\text{max}} = \frac{3}{2\pi \times 25} \times \frac{0.1 \times (0.8 \times 254)^2 \times 0.4}{(0.4)^2 + (0.1 \times 4)^2} = 98.5 \text{ N-m}$

Since  $s = 0.1$ , slip speed =  $sN_s = 0.1 \times 1500 = 150 \text{ rpm}$ .

$\therefore$  Speed for maximum torque =  $1500 - 150 = 1350 \text{ rpm}$ .

$$\begin{aligned}
 \text{(v) starting current} &= \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{KE_1}{\sqrt{R_2^2 + X_2^2}} = \frac{0.8 \times 254}{\sqrt{0.4^2 + 4^2}} = \mathbf{50.5 \text{ A}} \\
 \text{(vi) At start,} & \quad s = 1, \text{ hence} \\
 T_{st} &= \frac{3}{2\pi \times 25} \times \frac{(0.8 \times 254)^2 \times 0.4}{(0.4)^2 + 4^2} = \mathbf{19.5 \text{ N}\cdot\text{m}}
 \end{aligned}$$

It is seen that as compared to full-load torque, the starting torque is much less—almost 25 per cent.

### 34.38. Induction Motor Torque Equation

The gross torque  $T_g$  developed by an induction motor is given by

$$\begin{aligned}
 T_g &= P_2 / 2\pi N_s && -N_s \text{ in r.p.s.} \\
 &= 60 P_2 / 2\pi N_s = 9.55 P_2 / N_s && -N_s \text{ in r.p.m.}
 \end{aligned}$$

Now,

$$P_2 = \text{rotor Cu loss}/s = 3I_2^2 R_2 / s$$

$$\text{As seen from Art. 34.19, } I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{sKE_1}{\sqrt{R_2^2 + (sX_2)^2}}$$

where  $K$  is rotor/stator turn ratio per phase.

$$\therefore P_2 = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{1}{s} = \frac{3s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Also,

$$P_2 = 3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{1}{s} = \frac{3s K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore T_g = \frac{P_2}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{— in terms of } E_2$$

$$\text{or} \quad T_g = \frac{3}{2\pi N_s} \times \frac{s K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{— in terms of } E_1$$

Here,  $E_1$ ,  $E_2$ ,  $R_2$  and  $X_2$  represent phase values.

In fact,  $3K^2/2\pi N_s = k$  is called the constant of the given machine. Hence, the above torque equation may be simplified to

$$T_g = k \frac{s E_1^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{— in terms of } E_1$$

### 34.39. Synchronous Watt

It is clear from the above relations that torque is proportional to rotor input. By defining a new unit of torque (instead of the force-at-radius unit), we can say that the rotor torque *equals* rotor input. The new unit is synchronous watt. When we say that a motor is developing a torque of 1,000 synchronous watts, we mean that the rotor input is 1,000 watts and that the torque is such that power developed would be 1,000 watts provided the rotor were running synchronously and developing the same torque.

Or

Synchronous watt is that torque which, at the synchronous speed of the machine under consideration, would develop a power of 1 watt.

$$\text{rotor input} = T_{sw} \times 2\pi N_s \quad \therefore T_{sw} = \frac{\text{rotor input, } P_2}{2\pi \times \text{synch. speed}}$$



$$= \frac{1}{\omega_s} \cdot \frac{N_s}{N} \cdot P_g = \frac{1}{\omega_s} \cdot \frac{N_s}{N} \cdot P_m$$

**Synchronous wattage of an induction motor equals the power transferred across the air-gap to the rotor.**

$$\begin{aligned} \therefore \text{torque in synchronous watt} \\ = \text{rotor input} = \frac{\text{rotor Cu loss}}{s} = \frac{\text{gross output power, } P_m}{(1-s)} \end{aligned}$$

Obviously, at  $s = 1$ , torque in synchronous watt equals the total rotor Cu loss because at standstill, entire rotor input is lost as Cu loss.

Suppose a 23-kW, 4-pole induction motor has an efficiency of 92% and a speed of 1440 r.p.m. at rated load. If mechanical losses are assumed to be about 25 per cent of the total losses, then

$$\text{motor input} = 23/0.92 = 25 \text{ kW, total loss} = 25 - 23 = 2 \text{ kW.}$$

$$\text{Friction and windage loss} = 2/4 = 0.5 \text{ kW}$$

$$\therefore P_m = 23 + 0.5 = 23.5 \text{ kW}$$

$$\text{Power in synchronous watts } P_{sw} = P_2 = 23.5 \times 1500/1440 = 24.5 \text{ kW}$$

$$\text{synchronous speed } \omega_s = 2\pi (1500/60) = 157 \text{ rad/s}$$

$$\therefore \text{synchronous torque, } T_{sw} = 24.5 \times 10^3 / 157 = 156 \text{ N-m}$$

$$\begin{aligned} \text{or } T_{sw} &= P_m \frac{N_s}{N} \cdot \frac{1}{\omega_s} = 23.5 \times \frac{1500}{1440} \times \frac{1}{2\pi (1500/60)} \\ &= 156 \text{ N-m} \end{aligned} \quad \text{---Art 34.39}$$

### 34.40. Variations in Rotor Current

The magnitude of the rotor current varies with load carried by the motor.

As seen from Art. 34.37

$$\frac{\text{rotor output}}{\text{rotor input}} = \frac{N}{N_s} \text{ or rotor output} = \text{rotor input} \times \frac{N}{N_s}$$

$$\therefore \text{rotor input} = \text{rotor output} \times N_s / N$$

$$\text{Also, rotor output} \propto 2\pi N T = k N T$$

$$\therefore \text{rotor input} = k N T \times N_s / N = k N_s T$$

$$\text{Now, } \frac{\text{rotor Cu loss}}{\text{rotor input}} = s \text{ or } \frac{3 I_2^2 R_2}{s} = \text{rotor input}$$

$$\therefore 3 I_2^2 R_2 / s = k N_s T \text{ or } T \propto I_2^2 R_2 / s$$

$$\begin{aligned} T_{st} &\propto I_{2st}^2 R_2 && \text{--- since } s = 1 \\ T_f &\propto I_{2f}^2 R_2 / s_f && \text{--- } s_f = \text{full-load slip} \end{aligned}$$

$$\therefore \frac{T_{st}}{T_f} = s_f \left( \frac{I_{2st}}{I_{2f}} \right)^2$$

where  $I_{2st}$  and  $I_{2f}$  are the rotor currents for starting and full-load running conditions.

### 34.41. Analogy with a Mechanical Clutch

We have seen above that, rotor Cu loss = slip  $\times$  rotor input

This fact can be further clarified by considering the working of a mechanical clutch (though it is

not meant to be a proof for the above) similar to the one used in automobiles. A plate clutch is shown in Fig. 34.39. It is obvious that the torque on the driving shaft must exactly equal the torque on the driven shaft. In fact, these two torques are actually one and the same torque, because the torque is caused by friction between the discs and it is true whether the clutch is slipping or not. Let  $\omega_1$  and  $\omega_2$  be the angular velocities of the shaft when the clutch is slipping.

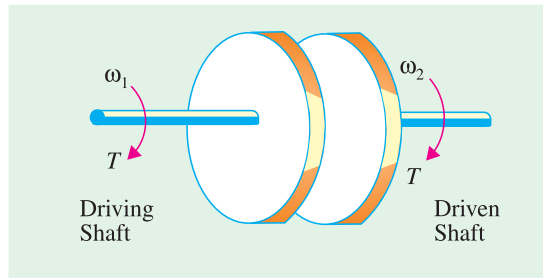


Fig. 34.39

Then, input =  $T \omega_1$  and output =  $T \omega_2 = T \omega_1(1 - s)$  : [  $\because \omega_2 = \omega_1(1 - s)$  ]

$$\text{loss} = T \omega_1 - T \omega_2 = T \omega_1 - T \omega_1(1 - s) = sT \omega_1 = \text{slip} \times \text{input}$$

### 34.42. Analogy with a D.C. Motor

The above relations could also be derived by comparing an induction motor with a d.c. motor. As shown in Art 29.3, in a d.c. shunt motor, the applied voltage is always opposed by a back e.m.f.  $E_b$ . The power developed in the motor armature is  $E_b I_a$  where  $I_a$  is armature current. This power, as we know, is converted into mechanical power in the armature of the motor.

Now, in an induction motor, it is seen that the induced e.m.f in the rotor decreases from its standstill value of  $E_2$  to  $sE_2$  when in rotation. Obviously, the difference  $(1 - s) E_2$  is the e.m.f. called forth by the rotation of the rotor similar to the back e.m.f. in a d.c. motor. Hence, gross power  $P_m$  developed in the rotor is given by the product of the back e.m.f., armature current and rotor power factor.

$$P_m = (1 - s)E_2 \times I_2 \cos \phi_2 ; \quad \text{Now } I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \text{ and } \cos \phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\therefore P_m = (1 - s)E_2 \times \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \times \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} = \frac{s(1 - s)E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Multiplying the numerator and the denominator by  $s$ , we get

$$P_m = \left( \frac{1 - s}{s} \right) R_2 \times \frac{s^2 E_2^2}{[R_2^2 + (sX_2)^2]} = \left( \frac{1 - s}{s} \right) I_2^2 R_2 \quad \left( \because I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \right)$$

Now,  $I_2^2 R_2 = \text{rotor Cu loss/phase} \quad \therefore \frac{\text{Cu loss}}{\text{rotor output}} = \frac{s}{1 - s}$

This is the same relationship as derived in Art. 34-37.

**Example 34.30.** The power input to a 3-phase induction motor is 60 kW. The stator losses total 1 kW. Find the mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.

(Elect. Machines AMIE Sec. E Summer 1991)

**Solution.** Rotor input,  $P_2 = \text{stator input} - \text{stator losses} = 60 - 1 = 59 \text{ kW}$

$$P_m = (1 - s)P_2 = (1 - 0.03) \times 59 = 57.23 \text{ kW}$$

$$\text{Total rotor Cu loss} = sP_2 = 0.03 \times 59 = 1.77 \text{ kW} = 1770 \text{ W}$$

$$\text{Rotor Cu loss/phase} = 1770/3 = 590 \text{ W}$$

**Example 34.31.** The power input to the rotor of a 400 V, 50-Hz, 6-pole, 3-phase induction motor is 20 kW. The slip is 3%. Calculate (i) the frequency of rotor currents (ii) rotor speed (iii) rotor copper losses and (iv) rotor resistance per phase if rotor current is 60 A.

(Elect. Engg. Punjab Univ. 1991)

- Solution.** (i) Frequency of rotor current =  $sf = 0.03 \times 50 = 1.5$  Hz  
 (ii)  $N_s = 120 \times 50/6 = 1000$  rpm;  $N = 1000(1 - 0.03) = 700$  rpm  
 (iii) rotor Cu loss =  $s \times$  rotor input =  $0.03 \times 20 = 0.6$  kW = 600 W  
 (iv) rotor Cu loss/phase = 200 W;  $\therefore 60^2 R_2 = 200$ ;  $R_2 = 0.055 \Omega$

**Example 34.32.** A 3-phase, 6-pole, 50-Hz induction motor develops 3.73 kW at 960 rpm. What will be the stator input if the stator loss is 280 W?

(Madurai Kamraj Univ. 1999)

**Solution.** As seen from Art. 34.37,  $\frac{\text{power developed in rotor}}{\text{rotor input}} = \frac{N}{N_s}$

Now, mechanical power developed in rotor = 3.73 kW.,  $N_s = 120 \times 50/6 = 1000$  r.p.m.

$\therefore 3,730/\text{rotor input} = 960/1000 \quad \therefore \text{rotor input} = 3,885$  W

Stator input = rotor input + stator losses =  $3885 + 280 = 4,156$  W

**Example 34.33.** The power input to the rotor of a 400 V, 50-Hz, 6-pole, 3- $\phi$  induction motor is 75 kW. The rotor electromotive force is observed to make 100 complete alteration per minute. Calculate:

(i) slip (ii) rotor speed (iii) rotor copper losses per phase (iv) mechanical power developed.

(Elect. Engg. I, Nagpur Univ. 1993)

**Solution.** Frequency of rotor emf,  $f' = 100/60 = 5/3$  Hz

(i) Now,  $f' = sf$  for  $5/3 = s \times 50$ ;  $s = 1/30 = 0.033$  or **3.33%**

(ii)  $N_s = 120 \times 50/6 = 1000$  rpm;  $N = N_s(1 - s) = 1000(1 - 1/30) = 966.7$  rpm

(iii)  $P_2 = 75$  kW; total rotor Cu loss =  $sP_2 = (1/30) \times 75 = 2.5$  kW  
 rotor Cu loss/phase =  $2.5/3 = 0.833$  kW

(iv)  $P_m = (1 - s)P_2 = (1 - 1/30) \times 75 = 72.5$  kW

**Example 34.34.** The power input to a 500 V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate: (i) the slip (ii) the rotor copper loss (iii) shaft power and (iv) the efficiency.

(Elect. Engg. - II, Pune Univ. 1989)

**Solution.** (i)  $N_s = 120 \times 50/6 = 1000$  rpm;  $s = (100 - 975)/1000 = 0.025$  or 2.5%

(ii) Motor input  $P_1 = 40$  kW; stator loss = 1 kW; rotor input  $P_2 = 40 - 1 = 39$  kW

$\therefore$  rotor Cu loss =  $s \times$  rotor input =  $0.025 \times 39 = 0.975$  kW

(iii)  $P_m = P_2 -$  rotor Cu loss =  $39 - 0.975 = 38.025$  kW

$P_{out} = P_m -$  friction and windage loss =  $38.025 - 2 = 36.025$  kW

(iv)  $\eta = P_{out}/P_1 = 36.025/40 = 0.9$  or **90%**

**Example 34.35.** A 100-kW (output), 3300-V, 50-Hz, 3-phase, star-connected induction motor has a synchronous speed of 500 r.p.m. The full-load slip is 1.8% and F.L. power factor 0.85. Stator copper loss = 2440 W. Iron loss = 3500 W. Rotational losses = 1200 W. Calculate (i) the rotor copper loss (ii) the line current (iii) the full-load efficiency.

(Elect. Machines, Nagpur Univ. 1993)

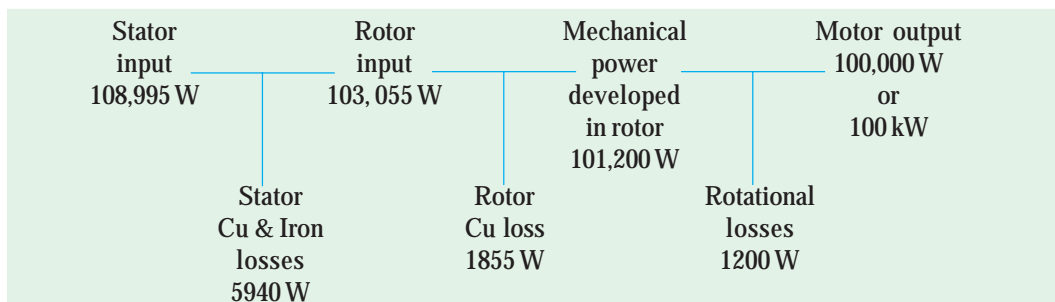
**Solution.**  $P_m = \text{output} + \text{rotational loss} = 100 + 1.2 = 101.2 \text{ kW}$

(i) rotor Cu loss =  $\frac{s}{1-s} \times P_m = \frac{0.018}{1-0.018} \times 101.2 = 1.855 \text{ kW}$

(ii) rotor input,  $P_2 = P_m + \text{rotor Cu loss} = 101.2 + 1.855 = 103.055 \text{ kW}$   
 Stator input =  $P_2 + \text{stator Cu and iron losses}$   
 =  $103.055 + 2.44 + 3.5 = 108.995 \text{ kW}$

$\therefore 108,995 = \sqrt{3} \times 3300 \times I_L \times 0.85; \quad I_L = 22.4 \text{ A}$

The entire power flow in the motor is given below.



(iii) F.L. efficiency =  $100,000/108,995 = 0.917$  or **91.7%**

**Example 34.36.** The power input to the rotor of a 440 V, 50-Hz, 6-pole, 3-phase induction motor is 100 kW. The rotor electromotive force is observed to make 120 cycles per minute. Calculate (i) the slip (ii) the rotor speed (iii) mechanical power developed (iv) the rotor copper loss per phase and (v) speed of stator field with respect to rotor. (Elect. Engg. AMIETE Sec. A June 1991)

**Solution.** (i)  $f' = sf$  or  $(120/60) = s \times 50; s = 0.01$

(ii)  $N_s = 120 \times 50/6 = 1000 \text{ rpm}; N = 1000(1 - 0.01) = 990 \text{ rpm}$

(iii)  $P_m (1 - s) P_2 = (1 - 0.01) \times 100 = 99 \text{ kW}$

(iv) total rotor Cu loss =  $sP_2 = 0.01 \times 100 = 1 \text{ kW}; \text{ Cu loss/phase} = 1/3 \text{ kW}$

(v)  $N_s = 1000 \text{ rpm}; N = 990 \text{ rpm}$ . Hence, speed of stator field with respect to rotor is  $= 1000 - 990 = 10 \text{ rpm}$ .

**Example 34.37.** An induction motor has an efficiency of 0.9 when delivering an output of 37 kW. At this load, the stator Cu loss and rotor Cu loss each equals the stator iron loss. The mechanical losses are one-third of the no-load loss. Calculate the slip.

(Adv. Elect. Machines, A.M.I.E. Sec. B Winter 1993)

**Solution.** Motor input =  $37,000/0.9 = 41,111 \text{ W}$

$\therefore \text{total loss} = 41,111 - 37,000 = 4,111 \text{ W}$

This includes (i) stator Cu and iron losses (ii) rotor Cu loss (its iron loss being negligibly small) and (iii) rotor mechanical losses.

Now, no-load loss of an induction motor consists of (i) stator iron loss and (ii) mechanical losses provided we neglect the small amount of stator Cu loss under no-load condition. Moreover, these two losses are independent of the load on the motor.

no-load loss =  $W_i + W_m = 3W_m$

$\therefore W_m = W_i/2$

where  $W_i$  is the stator iron loss and  $W_m$  is the rotor mechanical losses.

Let, stator iron loss =  $x$ ; then stator Cu loss =  $x$ ; rotor Cu loss =  $x$ ; mechanical loss =  $x/2$

$$\therefore 3x + x/2 = 4,111 \text{ or } x = 1175 \text{ W}$$

$$\begin{aligned} \text{Now, rotor input} &= \text{gross output} + \text{mechanical losses} + \text{rotor Cu loss} \\ &= 37,000 + (1175/2) + 1175 = 38,752 \text{ W} \end{aligned}$$

$$s = \frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{1175}{38,752} = 0.03 \text{ or } 3\%.$$

**Example 34.38.** A 400 V, 50-Hz, 6-pole,  $\Delta$ -connected, 3- $\phi$  induction motor consumes 45 kW with a line current of 75 A and runs at a slip of 3%. If stator iron loss is 1200 kW, windage and friction loss is 900 W and resistance between two stator terminals is 0.12  $\Omega$ , calculate (i) power supplied to the rotor  $P_2$  (ii) rotor Cu loss  $P_{cr}$  (iii) power supplied to load  $P_{out}$  (iv) efficiency and (v) shaft torque developed.

**Solution.**  $\cos \phi = \frac{45 \times 1000}{\sqrt{3} \times 400 \times 75} = 0.866 \text{ lag}$

A line current of 75 amp means a phase-current of  $75/\sqrt{3}$  i.e. 43.3 amp

Next, winding resistance has to be worked out

Refer to Fig. 34.40.

$r$  and  $2r$  in parallel have an equivalent resistance measured at  $a$  and  $b$  in delta connected motor as  $r \times 2r/3r = 2r/3$  ohms

From the data given  $\frac{2r}{3} = 0.12, r = 0.18$

Total stator copper loss =  $3 \times 43.3^2 \times 0.18 = 1012$

Watts

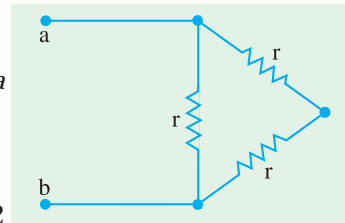


Fig. 34.40

$$\begin{aligned} \text{Total input to stator} &= 45,000 \text{ Watts} \\ \text{Stator copper loss} &= 1012 \text{ Watts, stator core loss} = 1200 \text{ Watts} \\ \text{Stator output} &= \text{Rotor input} = 42,788 \text{ Watts} \\ \text{Rotor copper loss} &= \text{Slip} \times \text{Rotor input} = 0.03 \times 42,788 = 1284 \text{ Watts} \\ \text{Rotor mech. output power} &= 42,788 - 1284 = 41,504 \text{ Watts} \\ \text{Shaft output} &= \text{Mech. output of rotor} - \text{Mech losses} \\ &= 41504 - 900 = 40604 \text{ Watts} \\ \text{Efficiency} &= \frac{40604}{45000} \times 100\% = 92.23\% \\ \text{Shaft output torque, } T &= \frac{40604 \times 60}{2\pi \times 970} = 400 \text{ Nw-m} \end{aligned}$$

**Example 34.39(a).** A 3-phase induction motor has a 4-pole, star-connected stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance per phase is 0.1  $\Omega$  and reactance 0.9  $\Omega$ . The ratio of stator to rotor turns is 1.75. The full-load slip is 5%. Calculate for this load:

- (a) the load torque in kg-m
- (b) speed at maximum torque
- (c) rotor e.m.f. at maximum torque. (Electrical Machines-I, South Gujarat Univ. 1985)

**Solution.** (a)  $K = \text{rotor turns/stator turns} = 1/1.75$

stator voltage/phase,  $E_1 = 220/\sqrt{3}$  V

$\therefore$  standstill rotor e.m.f./phase,  $E_2 = K E_1 = \frac{220}{\sqrt{3}} \times \frac{1}{1.75} = 72.6 \text{ V}$

$$Z_r = \sqrt{R_2^2 + (sX_2^2)} = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.11 \Omega$$

$$I_2 = sE_2 / Z_r = 0.05 \times 72.6 / 0.11 = 33 \text{ A}$$

$$\text{Rotor Cu loss } P_{cr} = 3I_2^2 R_2 = 3 \times 33^2 \times 0.1 = 327 \text{ W}$$

$$\frac{\text{rotor Cu loss}}{\text{mech. power developed}} = \frac{s}{1-s}; \quad \frac{327}{P_m} = \frac{0.05}{1-0.05}; \quad P_m = 6213 \text{ W}$$

$$T_g = 9.55 P_m / N; \quad N = N_s (1-s) = 1500 (1-0.05) = 1425 \text{ rpm}$$

$$\therefore T_g = 9.55 \times 6213 / 1425 = 41.6 \text{ N-m} = 41.6 / 9.81 = \mathbf{4.24 \text{ kg.m}}$$

(b) For maximum torque,  $s_m = R_2 / X_2 = 0.1 / 0.81 = \mathbf{1/9}$

$$\therefore N = N_s (1-s) = 1500 (1 - 1/9) = \mathbf{1333 \text{ r.p.m.}}$$

(c) rotor e.m.f./phase at maximum torque =  $(1/9) \times 72.6 = \mathbf{8.07 \text{ V}}$

**Example 34.39 (b).** A 400 V, 3-phase, 50 Hz, 4-pole, star-connected induction-motor takes a line current of 10 A with 0.86 p. f. lagging. Its total stator losses are 5 % of the input. Rotor copper losses are 4 % of the input to the rotor, and mechanical losses are 3 % of the input of the rotor. Calculate (i) slip and rotor speed, (ii) torque developed in the rotor, and (iii) shaft-torque.

[Nagpur University, April 1998]

**Solution.** Input to motor =  $\sqrt{3} \times 400 \times 10 \times 0.86 = 5958 \text{ Watts}$

Total stator losses = 5% of 5958 = 298 Watts

Rotor input = Stator Output = 5958 – 298 = 5660 Watts

Rotor Copper-loss = 4 % of 5660 = 226.4 Watts

Mechanical losses = 3% of 5660 = 169.8 Watts

Shaft output = 5660 – 226.4 – 169.8 = 5264 Watts

(i) slip,  $s = \text{Rotor-copper-loss} / \text{rotor-input} = 4 \%$ , as given

Synchronous speed,  $N_s = 120 \times f / P = 1500 \text{ rpm}$

Rotor speed,  $N = N_s (1-s) = 1500 (1 - 0.04) = 1440 \text{ rpm}$

(ii) Let the torque developed in the rotor =  $T_r$

$\omega_r$  Angular speed of rotor =  $2\pi N / 60 = 150.72 \text{ radians/sec}$

Rotor output = Rotor input – Rotor-copper-loss

$$= 5660 - 226.4 = 5433.6 \text{ watts}$$

$$T_r \omega_r = 5433.6, \text{ giving } T_r = 5433.6 / 150.72 = 36.05 \text{ Nw-m}$$

(Alternatively, synchronous angular speed =  $\omega_s = 2\pi \times 1500 / 60 = 157 \text{ rad / sec}$

$$T_r \omega_s = \text{rotor input in Watts} = 5660$$

$$T_r = 5660 / 157 = 36.05 \text{ Nw - m}$$

(iii) Shaft-torque =  $T_m = \text{Shaft output in Watts} / \omega_r$

$$= 5264 / 150.72 = 34.93 \text{ Nw - m.}$$

**Example 34.40.** A 3-phase, 440-V, 50-Hz, 40-pole, Y-connected induction motor has rotor resistance of 0.1  $\Omega$  and reactance 0.9  $\Omega$  per phase. The ratio of stator to rotor turns is 3.5. Calculate:

- (a) gross output at a slip of 5%
- (b) the maximum torque in synchronous watts and the corresponding slip.

**Solution. (a)** Phase voltage =  $\frac{440}{\sqrt{3}} \text{ V}$ ;  $K = \frac{\text{rotor turns}}{\text{stator turns}} = \frac{1}{3.5}$

Standstill e.m.f. per rotor phase is  $E_2 = KE_1 = \frac{440}{\sqrt{3}} \times \frac{1}{3.5} = 72.6 \text{ V}$

$$E_r = s.E_2 = 0.05 \times 72.6 = 3.63 \text{ V}; Z_r = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.1096 \Omega$$

$$I_2 = \frac{E_r}{Z_r} = \frac{3.63}{0.1096} = 33.1 \text{ A}; \quad \text{Total Cu loss} = 3 I_2^2 R = 3 \times (33.1)^2 \times 0.1 = 330 \text{ W}$$

$$\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s} \quad \therefore \text{rotor gross output} = \frac{330 \times 0.95}{0.05} = \mathbf{7,250 \text{ W}}$$

**(b) For Maximum Torque**

$$R_2 = s_m \cdot X_2 \quad \therefore s_m = R_2 / X_2 = 0.1/0.9 = \mathbf{1/9}; \quad E_r = (1/9) \times 72.6 = 8.07 \text{ V}$$

$$Z_r = \sqrt{0.1^2 + (0.9/9)^2} = 0.1414 \Omega; \quad I_2 = 8.07/0.1414 = 57.1 \text{ A}$$

$$\text{Total rotor Cu loss} = 3 \times 57.1^2 \times 0.1 = 978 \text{ W}$$

$$\text{rotor input} = \frac{\text{rotor Cu loss}}{s} = \frac{978}{1/9} = 8,802 \text{ W}$$

$\therefore$  Maximum torque in synchronous watts = rotor input = **8,802 W**

**Example 34.41.** An 18.65-kW, 4-pole, 50-Hz, 3-phase induction motor has friction and windage losses of 2.5 per cent of the output. The full-load slip is 4%. Compute for full load **(a)** the rotor Cu loss **(b)** the rotor input **(c)** the shaft torque **(d)** the gross electromagnetic torque.

**(Elect. Machines-II, Indore Univ. 1987)**

**Solution.** Motor output  $P_{out} = 18,650 \text{ W}$

Friction and windage loss  $P_w = 2.5\% \text{ of } 18,650 = 466 \text{ W}$

Rotor gross output  $P_m = 18,650 + 466 = 19,116 \text{ W}$

**(a)**  $\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s}$ , rotor Cu loss =  $\frac{0.04}{(1-0.04)} \times 19,116 = \mathbf{796.6 \text{ W}}$

**(b)** Rotor input  $P_2 = \frac{\text{rotor Cu loss}}{s} = \frac{796.6}{0.04} = \mathbf{19,912.5 \text{ W}}$

(or rotor input = 19,116 + 796.6 = 19,913 W)

**(c)**  $T_{sh} = 9.55 P_{out} / N$ ;  $N_s = 120 \times 50/4 = 1500 \text{ r.p.m.}$

$$N = (1 - 0.04) \times 1500 = 1440 \text{ r.p.m.}$$

$\therefore$   $T_{sh} = 9.55 \times 18,650/1440 = \mathbf{123.7 \text{ N-m}}$

**(d)** Gross torque  $T_g = 9.55 P_m / N = 9.55 \times 19,116/1440 = \mathbf{126.8 \text{ N-m}}$

(or  $T_g = P_2 / 2 \pi N_s = 19,913/2 \pi \times 25 = 127 \text{ N-m}$ )

**Example 34.42.** An 8-pole, 3-phase, 50 Hz, induction motor is running at a speed of 710 rpm with an input power of 35 kW. The stator losses at this operating condition are known to be 1200 W while the rotational losses are 600 W. Find **(i)** the rotor copper loss, **(ii)** the gross torque developed, **(iii)** the gross mechanical power developed, **(iv)** the net torque and **(v)** the mechanical power output.

**(Elect. Engg. AMIETE Sec. A 1991 & Rajiv Gandhi Techn. Univ., Bhopal, 2000)**

**Solution.** **(i)**  $P_2 = 35 - 1.2 = 33.8 \text{ kW}$ ;  $N_s = 120 \times 50/8 = 750 \text{ rpm}$ ;  $N = 710 \text{ rpm}$ ;  $s = (750 - 710)/750 = 0.0533$

$\therefore$  Rotor Cu loss =  $sP_2 = \mathbf{1.8 \text{ kW}}$

**(ii)**  $P_m = P_2 - \text{rotor Cu loss} = 33.8 - 1.8 = \mathbf{32 \text{ kW}}$

**(iii)**  $T_g = 9.55 P_m / N = 9.55 \times 32000/710 = 430.42 \text{ N-m}$

**(iv)**  $P_{out} = P_m - \text{rotational losses} = 32000 - 600 = \mathbf{31400 \text{ W}}$

(v)  $T_{sh} = 9.55 P_{out} / N = 9.55 \times 31400 / 710 = 422.35 \text{ N-m}$

**Example 34.43.** A 6-pole, 50-Hz, 3-phase, induction motor running on full-load with 4% slip develops a torque of 149.3 N-m at its pulley rim. The friction and windage losses are 200 W and the stator Cu and iron losses equal 1,620 W. Calculate (a) output power (b) the rotor Cu loss and (c) the efficiency at full-load. (Elect. Technology, Mysore Univ. 1989)

**Solution.**  $N_s = 120 \times 50 / 6 = 1,000 \text{ r.p.m.}; N = (1 - 0.04) \times 1,000 = 960 \text{ r.p.m.}$

Out put power =  $T_{sh} \times 2 \pi N = 2 \pi \times (960/60) \times 149.3 = 15 \text{ kW}$

Now, output = 15,000 W

Friction and windage losses = 200 W ; Rotor gross output = 15,200 W

$\frac{P_m}{P_2} = \frac{N}{N_s} \therefore \text{rotor input } P_2 = 15,200 \times 1,000 / 960 = 15,833 \text{ W}$

(b)  $\therefore$  rotor Cu loss = 15,833 – 15,200 = 633 W

( rotor Cu loss is given by :  $\frac{\text{rotor Cu loss}}{\text{rotor output}} = \frac{s}{1-s}$  )

(c) stator output = rotor input = 15,833 W

Stator Cu and iron losses = 1,620 W

$\therefore$  Stator input  $P_1 = 15,833 + 1,620 = 17,453 \text{ W}$

overall efficiency,  $\eta = 15,000 \times 100 / 17,453 = 86\%$

**Example 34.44.** An 18.65-kW, 6-pole, 50-Hz, 3- $\phi$  slip-ring induction motor runs at 960 r.p.m. on full-load with a rotor current per phase of 35 A. Allowing 1 kW for mechanical losses, find the resistance per phase of 3-phase rotor winding. (Elect. Engg-I, Nagpur Univ. 1992)

**Solution.** Motor output = 18.65 kW; Mechanical losses = 1 kW

$\therefore$  Mechanical power developed by rotor,  $P_m = 18.65 + 1 = 19.65 \text{ kW}$

Now,  $N_s = 120 \times 50 / 6 = 1000 \text{ r.p.m.}; s = (1000 - 960) / 1000 = 0.04$

rotor Cu loss =  $\frac{s}{1-s} \times P_m = \frac{0.04}{(1-0.04)} \times 19.65 = 0.819 \text{ kW} = 819 \text{ W}$

$\therefore 3I_2^2 R_2 = 819 \text{ or } 3 \times 35^2 \times R_2 = 819 \quad R_2 = 0.023 \Omega / \text{phase}$

**Example 34.45.** A 400 V, 4-pole, 3-phase, 50-Hz induction motor has a rotor resistance and reactance per phase of 0.01  $\Omega$  and 0.1  $\Omega$  respectively. Determine (a) maximum torque in N-m and the corresponding slip (b) the full-load slip and power output in watts, if maximum torque is twice the full-load torque. The ratio of stator to rotor turns is 4.

**Solution.** Applied voltage/phase  $E_1 = 400 / \sqrt{3} = 231 \text{ V}$

Standstill e.m.f. induced in rotor,  $E_2 = KE_1 = 231 / 4 = 57.75 \text{ V}$

(a) Slip for maximum torque,  $s_m = R_2 / X_2 = 0.01 / 0.1 = 0.1$  or 10%

$T_{max} = \frac{3}{2\pi N_s} \times \frac{E_2^2}{2 X_2} \quad \dots \text{ Art. 34.20}$

$N_s = 120 \times 50 / 4 = 1500 \text{ r.p.m.} = 25 \text{ r.p.s.}$

$\therefore T_{max} = \frac{3}{2\pi \times 25} \times \frac{57.75^2}{2 \times 0.1} = 320 \text{ N-m}$

(b)  $\frac{T_f}{T} = \frac{2as_f}{a^2 + s^2} = \frac{1}{2}$ ; Now,  $a = R_2 / X_2 = 0.01 / 0.1 = 0.1$



$$\therefore 2 \times 0.1 \times s_f / (0.1^2 + s_f^2) = 1/2 \quad \therefore s_f = 0.027 \text{ or } 0.373$$

Since  $s_f = 0.373$  is not in the operating region of the motor, we select  $s_f = 0.027$ .

Hence,  $s_f = \mathbf{0.027}$ .  $N = 1500 (1 - 0.027) = 1459.5 \text{ r.p.m.}$

Full-load torque  $T_f = 320/2 = 160 \text{ N-m}$

F.L. Motor output  $= 2 \pi N T_f / 60 = 2 \pi \times 1459.5 \times 160 / 60 = \mathbf{24,454 \text{ W}}$

**Example 34.46.** A 3-phase induction motor has a 4-pole, Y-connected stator winding. The motor runs on 50-Hz supply with 200V between lines. The motor resistance and standstill reactance per phase are 0.1 Ω and 0.9 Ω respectively. Calculate

(a) the total torque at 4% slip (b) the maximum torque

(c) the speed at maximum torque if the ratio of the rotor to stator turns is 0.67. Neglect stator impedance. (Elect. Machinery, Mysore Univ. 1987)

**Solution. (a)** Voltage/phase  $= \frac{200}{\sqrt{3}} \text{ V}$ ;  $K = \frac{\text{rotor turns}}{\text{stator turns}} = 0.67$

Standstill rotor e.m.f. per phase is  $E_2 = \frac{200}{\sqrt{3}} \times 0.67 = 77.4 \text{ V}$  and  $s = 0.04$

$$Z_r = \sqrt{R_2^2 + (sX_2^2)} = \sqrt{0.1^2 + (0.04 \times 0.9)^2} = 0.106 \Omega$$

$$I_2 = \frac{sE_2}{Z_r} = \frac{77.4 \times 0.04}{0.106} = 29.1 \text{ A}$$

Total Cu loss in rotor  $= 3 I_2^2 R_2 = 3 \times 29.1^2 \times 0.1 = 255 \text{ W}$

Now,  $\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s} \quad \therefore P_m = 255 \times 0.96 / 0.04 = 6,120 \text{ W}$

$N_s = 120 \times 50 / 4 = 1,500 \text{ r.p.m.}$ ;  $N = (1 - 0.04) \times 1,500 = 1440 \text{ r.p.m.}$

Gross torque developed,  $T_g = 9.55 P_m / N = 9.55 \times 6120 / 1440 = \mathbf{40.6 \text{ N-m}}$

(b) For maximum torque,  $s_m = R_2 / X_2 = 0.1 / 0.9 = 1/9$  and  $E_r = sE_2 = 77.4 \times 1/9 = 8.6 \text{ V}$

$$Z_r = \sqrt{0.1^2 + (0.9 \times 1/9)^2} = 0.1414 \Omega; \quad I_2 = 8.6 / 0.1414 = 60.8 \text{ A}$$

Total rotor Cu loss  $= 3 \times 60.8^2 \times 0.1 = 1,110 \text{ W}$

Rotor gross output  $= \frac{1,110 \times (1 - 1/9)}{1/9} = 8,800 \text{ W}$

$$N = (1 - 1/9) \times 1,500 = 1,333 \text{ rpm}$$

$\therefore T_{max} = 9.55 \times 8,800 / 1333 = \mathbf{63 \text{ N-m}}$

(c) Speed at maximum torque, as found above, is **1333.3 r.p.m.**

**Example 34.47.** The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase.

(i) What is the p.f. of the motor at start?

(ii) What is the p.f. at a slip of 4 percent ?

(iii) If the number of poles is 4, the supply frequency is 50-Hz and the standstill e.m.f. per rotor phase is 110 V, find out the full-load torque. Take full-load slip as 4 per cent.

(Electrical Technology-I, Osmania Univ. 1990)

**Solution. (i)** rotor impedance/phase  $= \sqrt{0.015^2 + 0.09^2} = 0.0912 \Omega$

p.f.  $= 0.015 / 0.0912 = \mathbf{0.164}$

(ii) reactance/phase  $= sX_2 = 0.04 \times 0.09 = 0.0036 \Omega$

$$\text{rotor impedance/phase} = \sqrt{0.015^2 + 0.0036^2} = 0.0154 \Omega$$

$$\text{p.f.} = 0.015/0.0154 = \mathbf{0.974}$$

$$\text{(iii)} \quad N_s = 120 \times 50/4 = 1,500 \text{ r.p.m.}; N = 1,500 - (0.04 \times 1,500) = 1,440 \text{ r.p.m.}$$

$$E_r = sE_2 = 0.04 \times 110 = 4.4 \text{ V}; Z_r = 0.0154 \Omega \quad \dots \text{found above}$$

$$I_2 = 4.4/0.0154 = 286 \text{ A}$$

$$\text{Total rotor Cu loss} = 3I_2^2 R_2 = 3 \times 286^2 \times 0.015 = 3,650 \text{ W}$$

$$\text{Now,} \quad \frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1-s}$$

$$\therefore \text{Rotor gross output } P_m = 3,650 \times 0.96/0.04 = 87,600 \text{ W}$$

If  $T_g$  is the gross torque developed by the rotor, then

$$T_g = 9.55 P_m / N = 9.55 \times 87,600/1440 = \mathbf{581 \text{ N-m}}$$

**Example 34.48.** The useful full load torque of 3-phase, 6-pole, 50-Hz induction motor is 162.84 N-m. The rotor e.m. f. is observed to make 90 cycles per minute. Calculate (a) motor output (b) Cu loss in rotor (c) motor input and (d) efficiency if mechanical torque lost in windage and friction is 20.36 N-m and stator losses are 830 W. (Elect. Machines-II, Indore Univ. 1988)

**Solution.**  $N_s = 120 \times 50/6 = 1,000 \text{ r.p.m.}$

Frequency of rotor e.m.f. = 90/60 = 1.5 Hz;  $s = f_r/f = 1.5/50 = 0.03$

Rotor speed = 1,000(1 - 0.03) = 970 r.p.m.; Useful F.L. torque = 162.84 N-m

$$\text{(a)} \quad \text{motor output} = T_{sh} \frac{2\pi N}{60} = \frac{2\pi \times 970 \times 162.84}{60} = \mathbf{16,540 \text{ W}}$$

$$\text{(b)} \quad \text{gross torque} \quad T_g = 162.84 + 20.36 = 183.2 \text{ N-m}$$

$$\text{Now,} \quad T_g = 9.55 \frac{P_2}{N_s} \quad \therefore 183.2 = 9.55 \times \frac{P_2}{1000}$$

$$\therefore \text{rotor input,} \quad P_2 = 183.2 \times 1000/9.55 = 19,170 \text{ W}$$

$$\therefore \text{rotor Cu loss} = s \times \text{rotor input} = 0.03 \times 19,170 = \mathbf{575.1 \text{ W}}$$

$$\text{(c)} \quad \text{Motor input,} \quad P_1 = 19,170 + 830 = \mathbf{20,000 \text{ W}}$$

$$\text{(d)} \quad \eta = (16,540/20,000) \times 100 = \mathbf{82.27\%}$$

**Example 34.49.** Estimate in kg-m the starting torque exerted by an 18.65-kW, 420-V, 6-pole, 50-Hz, 3-phase induction motor when an external resistance of 1  $\Omega$  is inserted in each rotor phase.

stator impedance : (0.25 + j 0.75)  $\Omega$

rotor impedance : (0.173 + j 0.52)  $\Omega$

stator/rotor voltage ratio: 420/350

connection: Star-Star

**Solution.**  $K = E_2/E_1 = 350/420 = 5/6$

Equivalent resistance of the motor as referred to rotor is

$$R_{02} = R_2 + K^2 R_1 = 0.173 + (5/6)^2 \times 0.25 = 0.346 \Omega / \text{phase}$$

Similarly,  $X_{02} = 0.52 + (5/6)^2 \times 0.75 = 1.04 \Omega / \text{phase}$

When an external resistance of 1 ohm/phase is added to the rotor circuit, the equivalent motor impedance as referred to rotor circuit is

$$Z_{02} = \sqrt{(1 + 0.346)^2 + 1.04^2} = 1.7 \Omega$$

$$\text{Short-circuit rotor current is } I_2 = \frac{350/\sqrt{3}}{1.7} = 119 \text{ A}$$

$$\text{Rotor Cu loss per phase on short-circuit} = 119^2 \times 1.173 = 16,610 \text{ W}$$

Now, rotor power input =  $\frac{\text{rotor Cu loss}}{s}$  ; On short-circuit,  $s = 1$   
 $\therefore$  rotor power input = rotor Cu loss on short-circuit  
 = 16,610 W/phase = 49,830 W for 3 phases  
 $N_s = 120 \times 50/6 = 1000$  r.p.m.

If  $T_{st}$  is the starting torque in newton-metres, then  
 $T_{st} = 9.55 P_2 / N_s = 9.55 \times 49,830/1000 = 476$  N-m =  $476/9.81 = 46.7$  kg-m.

**Example 34.50.** An 8-pole, 37.3-kW, 3-phase induction motor has both stator and rotor windings connected in star. The supply voltage is 280 V per phase at a frequency of 50 Hz. The short-circuit current is 200 A per phase at a short-circuit power factor of 0.25. The stator resistance per phase is 0.15 Ω. If transformation ratio between the stator and rotor windings is 3, find

- (i) the resistance per phase of the rotor winding
- (ii) the starting torque of the motor.

**Solution.** Under short circuit, all the power supplied to the motor is dissipated in the stator and rotor winding resistances. Short-circuit power supplied to the motor is

$$W_{sc} = 3 V_1 I_{sc} \cos \phi_{sc} = 3(280 \times 200 \times 0.25) \text{ W}$$

Power supplied/phase =  $280 \times 200 \times 0.25 = 14,000$  W

Let  $r_2'$  be the rotor resistance per phase as referred to stator. If  $r_1$  is the stator resistance per phase, then

$$I_{sc}^2 (r_1 + r_2') = 14,000 \quad \therefore r_1 + r_2' = 14,000 / 200^2 = 0.35 \Omega$$

$\therefore r_2' = 0.35 - 0.15 = 0.2 \Omega$  ; Now,  $r_2' = r_2 / K^2$  where  $K = 1/3$

(i)  $\therefore r_2 = K^2 r_2' = 0.2/9 = 0.022 \Omega$  per phase

(ii) Power supplied to the rotor circuit is  
 =  $3 I_{sc}^2 r_2' = 3 \times 200^2 \times 0.2 = 24,000$  W  
 $N_s = 120 \text{ } f/P = 120 \times 50/8 = 750$  r.p.m. = 12.5 r.p.s.

$\therefore$  Starting torque =  $9.55 P_2 / N_s = 9.55 \times 24,000/750 = 305.6$  N-m

**Example 34.51.** A 3-phase induction motor, at rated voltage and frequency has a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Neglecting stator resistance and rotational losses and assuming constant rotor resistance, determine

- (a) the slip at full-load
- (b) the slip at maximum torque
- (c) the rotor current at starting in terms of F.L. rotor current.

(Electrical Machine - II, Bombay Univ. 1987)

**Solution.** As seen from Example 34.22 above,

- (a)  $s_f = 0.01$  or 1 %
- (b) From the same example it is seen that at maximum torque,  $a = s_b = 0.04$  or 4 %
- (c) As seen from Art. 34.40.

$$\frac{I_{2st}}{I_{2f}} = \sqrt{\frac{T_{st}}{s_f \cdot T_f}} = \sqrt{\frac{1.6}{0.01}} = 12.65$$

$\therefore$  Starting rotor current =  $12.65 \times$  F.L. rotor current

### 34.43. Sector Induction Motor

Consider a standard 3-phase, 4-pole, 50-Hz, Y-connected induction motor. Obviously, its  $N_s = 1500$  rpm. Suppose we cut the stator in half i.e. we remove half the stator winding with the result that only two complete  $N$  and  $S$  poles are left behind. Next, let us star the three phases without making any other changes in the existing connections. Finally, let us mount the original rotor above this **sector stator** leaving a small air-gap between the two. When this stator is energised from a 3-phase 50-Hz source, the rotor is found to run at almost 1500 rpm. In order to prevent saturation, the stator voltage should be reduced to half its original value because the sector stator winding has only half the original number of turns. It is found that under these conditions, this half-truncated sector motor still develops about 30% of its original rated power.

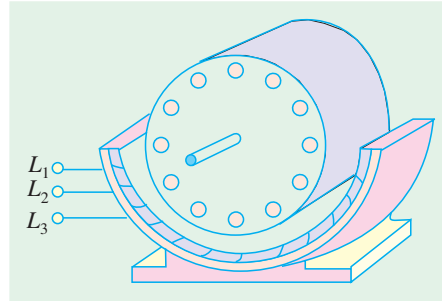


Fig. 34.41

The stator flux of the sector motor revolves at the same peripheral speed as the flux in the original motor. But instead of making a complete round, the flux in the sector motor simply travels **continuously from one end of the stator to the other**.

### 34.44. Linear Induction Motor

If, in a sector motor, the sector is laid out flat and a flat squirrel-cage winding is brought near to it, we get a linear induction motor (Fig. 34.42). In practice, instead of a flat squirrel-cage winding, an aluminium or copper or iron plate is used as a 'rotor'. The flat stator produces a flux that moves in a straight line from its one end to the other at a linear synchronous speed given by

where

$$v_s = 2 w f$$

$v_s$  = linear synchronous speed (m/s)  
 $w$  = width of one pole-pitch (m)  
 $f$  = supply frequency (Hz)

It is worth noting that speed **does not depend on the number of poles**, but only on the pole-pitch and stator supply frequency. As the flux moves linearly, it drags the rotor plate along with it in the **same direction**. However, in many practical applications, the 'rotor' is stationary, while the stator moves.

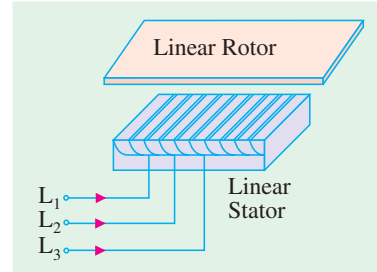


Fig. 34.42

For example, in high-speed trains, which utilize magnetic levitation (Art. 34.46), the rotor is composed of thick aluminium plate that is fixed to the ground and extends over the full length of the track. The linear stator is bolted to the undercarriage of the train.

### 34.45. Properties of a Linear Induction Motor

These properties are almost identical to those of a standard rotating machine.

1. **Synchronous speed.** It is given by  $v_s = 2 w f$

2. **Slip.** It is given by  $s = (v_s - v) / v_s$

where  $v$  is the actual speed.

3. **Thrust or Force.** It is given by  $F = P_2 / v_s$

where  $P_2$  is the active power supplied to the rotor.

4. **Active Power Flow.** It is similar to that in a rotating motor.

(i)  $P_{cr} = s P_2$  and

(ii)  $P_m = (1 - s) P_2$

**Example 34.52.** An electric train, driven by a linear motor, moves with 200 km/h, when stator frequency is 100 Hz. Assuming negligible slip, calculate the pole-pitch of the linear motor.

**Solution.**  $v_s = 2 \omega f$       $\omega = \frac{(200 \times 5/18)}{2 \times 100} = 277.8 \text{ mm}$

**Example 34.53.** An overhead crane in a factory is driven horizontally by means of two similar linear induction motors, whose 'rotors' are the two steel I-beams, on which the crane rolls. The 3-phase, 4-pole linear stators, which are mounted on opposite sides of the crane, have a pole-pitch of 6 mm and are energised by a variable-frequency electronic source. When one of the motors was tested, it yielded the following results:

Stator frequency = 25 Hz;     Power to stator = 6 kW  
 Stator Cu and iron loss = 1.2 kW;     crane speed = 2.4 m/s

Calculate (i) synchronous speed and slip (ii) power input to rotor (iii) Cu losses in the rotor (iv) gross mechanical power developed and (v) thrust.

**Solution. (i)**      $v_s = 2 \omega f = 2 \times 0.06 \times 25 = 3 \text{ m/s}$   
 $s = (v_s - v)/v_s = (3 - 2.4)/3 = 0.2$  or **20%**

**(ii)**      $P_2 = 6 - 1.2 = 4.8 \text{ kW}$

**(iii)**      $P_{cr} = sP_2 = 0.2 \times 4.8 = 0.96 \text{ kW}$

**(iv)**      $P_m = P_2 - P_{cr} = 4.8 - 0.96 = 3.84 \text{ kW}$

**(v)**      $F = P_2 / v_s = 4.8 \times 10^3 / 3 = 1600 \text{ N} = 1.60 \text{ kN}$

### 34.46. Magnetic Levitation

As shown in Fig. 34.43 (a), when a moving permanent magnet sweeps across a conducting ladder, it tends to drag the ladder along with, because it applies a horizontal tractive force  $F = BIl$ . It will now be shown that this horizontal force is also accompanied by a **vertical** force (particularly, at high magnet speeds), which tends to push the magnet away from the ladder in the upward direction.



Magnetic levitation

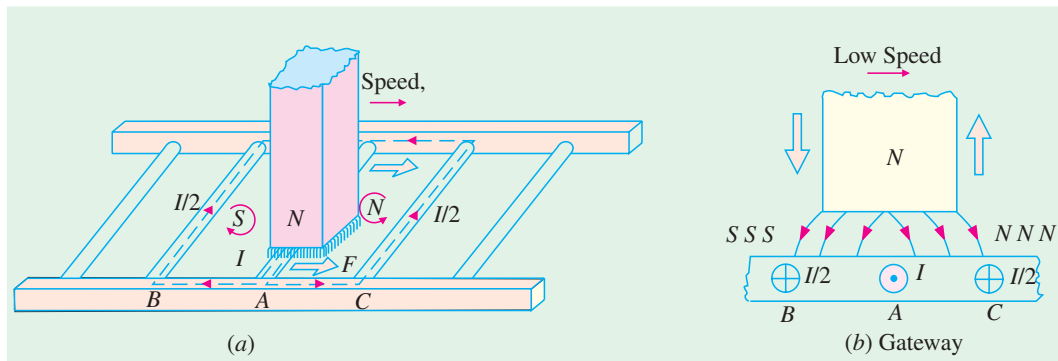


Fig. 34.43

A portion of the conducting ladder of Fig. 34.43 (a) has been shown in Fig. 34.43 (b). The voltage induced in conductor (or bar) A is maximum because flux is greatest at the centre of the N pole. If the magnet speed is very low, the induced current reaches its maximum value in A at virtually the **same time** (because delay due to conductor inductance is negligible). As this current flows via conductors B and C, it produces induced SSS and NNN poles, as shown. Consequently, the front half of

the magnet is pushed upwards while the rear half is pulled downwards. Since distribution of *SSS* and *NNN* pole is symmetrical with respect to the centre of the magnet, the vertical forces of attraction and repulsion, being equal and opposite, cancel each other out, leaving behind only horizontal tractive force.

Now, consider the case when the magnet sweeps over the conductor *A* with a *very high speed*, as shown in Fig. 34.44. Due to conductor inductance, current in *A* reaches its maximum value a fraction of a second ( $\Delta t$ ) after voltage reaches its maximum value. Hence, by the time *I* in conductor *A* reaches its maximum value, the centre of the magnet is already ahead of *A* by a distance =  $v \cdot \Delta t$  where  $v$  is the magnet velocity. The induced poles *SSS* and *NNN* are produced, as before, by the currents returning via conductors *B* and *C* respectively. But, by now, the *N* pole of the permanent magnet lies over the induced *NNN* pole, which pushes it upwards with a strong vertical force. \* This forms the basis of **magnetic levitation** which literally means 'floating in air'.

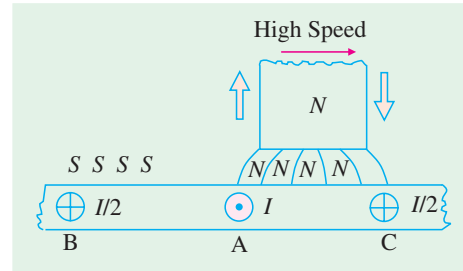


Fig. 34.44

Magnetic levitation is being used in ultrahigh speed trains (upto 300 km/h) which float in the air about 100 mm to 300 mm above the metallic track. They do not have any wheels and do not require the traditional steel rail. A powerful electromagnet (whose coils are cooled to about 4 °K by liquid helium) fixed underneath the train moves across the conducting rail, thereby inducing current in the rail. This gives rise to vertical force (called force of levitation) which keeps the train pushed up in the air above the track. Linear motors are used to propel the train.

A similar magnetic levitation system of transit is being considered for connecting Vivek Vihar in East Delhi to Viaspuri in West Delhi. The system popularly known as Magneto-Bahn (*M*-Bahn) completely eliminates the centuries-old 'steel-wheel-over steel rail' traction. The *M*-Bahn train floats in the air through the principle of magnetic levitation and propulsion is by linear induction motors. There is 50% decrease in the train weight and 60% reduction in energy consumption for propulsion purposes. The system is extraordinarily safe (even during an earthquake) and the operation is fully automatic and computer-based.

### Tutorial Problem No. 34.3

1. A 500-V, 50-Hz, 3-phase induction motor develops 14.92 kW inclusive of mechanical losses when running at 995 r.p.m., the power factor being 0.87. Calculate (a) the slip (b) the rotor Cu losses (c) total input if the stator losses are 1,500 W (d) line current (e) number of cycles per minute of the rotor e.m.f. **[(a) 0.005 (b) 75 W (c) 16.5 kW (d) 22 A (e) 15] (City & Guilds, London)**
2. The power input to a 3-phase induction motor is 40 kW. The stator losses total 1 kW and the friction and winding losses total 2 kW. If the slip of the motor is 4%, find (a) the mechanical power output (b) the rotor Cu loss per phase and (c) the efficiency. **[(a) 37.74 kW (b) 0.42 kW (c) 89.4%]**
3. The rotor e.m.f. of a 3-phase, 440-V, 4-pole, 50-Hz induction motor makes 84 complete cycles per minute when the shaft torque is 203.5 newton-metres. Calculate the h.p. of the motor. **[41.6 h.p. (31.03 kW)] (City & Guilds, London)**
4. The input to a 3-phase induction motor, is 65 kW and the stator loss is 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the slip is 3%. Calculate also in

\* The induced current is always delayed (even at low magnet speeds) by an interval of time  $\Delta t$  which depends on the  $L/R$  time-constant of the conductor circuit. This delay is so brief at slow speed that voltage and the current reach their maximum value virtually at the same *time* and *place*. But at high speed, the same delay  $\Delta t$  is sufficient to produce large shift *in space* between the points where the voltage and current achieve their maximum values.

- terms of the mechanical power developed the input to the rotor when the motor yields full-load torque at half speed. **[83.2 h.p. (62.067 kW) : 640 W, Double the output] (City & Guilds. London)**
5. A 6-pole, 50-Hz, 8-phase induction motor, running on full-load, develops a useful torque of 162 N-m and it is observed that the rotor electromotive force makes 90 complete cycles per min. Calculate the shaft output. If the mechanical torque lost in friction be 13.5 Nm, find the copper loss in the rotor windings, the input to the motor and the efficiency. Stator losses total 750 W.  
**[16.49 kW; 550 W; 19.2 kW; 86%]**
  6. The power input to a 500-V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate (a) the slip (b) the rotor copper loss (c) shaft output (d) the efficiency. **[(a) 0.025 (b) 975 W (c) 36.1 kW (d) 90%]**
  7. A 6-pole, 3-phase induction motor develops a power of 22.38kW, including mechanical losses which total 1.492 kW at a speed of 950 rpm on 550-V, 50-Hz mains. The power factor is 0.88. Calculate for this load (a) the slip (b) the rotor copper loss (c) the total input if the stator losses are 2000 W (d) the efficiency (e) the line current (f) the number of complete cycles of the rotor electromotive force per minute. **[(a) 0.05 (b) 1175 (c) 25.6 kW (d) 82% (e) 30.4 A (f) 150]**
  8. A 3-phase induction motor has a 4-pole, star-connected stator winding. The motor runs on a 50-Hz supply with 200 V between lines. The rotor resistance and standstill reactance per phase are 0.1  $\Omega$  and 0.9  $\Omega$  respectively. The ratio of rotor to stator turns is 0.67. Calculate (a) total torque at 4% slip (b) total mechanical power at 4% slip (c) maximum torque (d) speed at maximum torque (e) maximum mechanical power. Prove the formulae employed, neglecting stator impedance.  
**[(a) 40 Nm (b) 6 kW (c) 63.7 Nm (d) 1335 rpm (e) 8.952 kW.]**
  9. A 3-phase induction motor has a 4-pole, star-connected, stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance is 0.1  $\Omega$  and reactance 0.9. The ratio of stator to rotor turns is 1.75. The full load slip is 5%. Calculate for this load (a) the total torque (b) the shaft output. Find also (c) the maximum torque (d) the speed at maximum torque.  
**[(a) 42 Nm (b) 6.266 kW (c) 56 Nm (d) 1330 rpm ]**
  10. A 3000-V, 24-pole, 50-Hz 3-phase, star-connected induction motor has a slip-ring rotor of resistance 0.016  $\Omega$  and standstill reactance 0.265  $\Omega$  per phase. Full-load torque is obtained at a speed of 247 rpm. Calculate (a) the ratio of maximum to full-load torque (b) the speed at maximum torque. Neglect stator impedance. **[(a) 2.61 (b) 235 rpm ]**
  11. The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015  $\Omega$  and 0.09  $\Omega$  per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at one-half of full-load speed. What is then the power factor ? **[22.5%; 0.31 ]**
  12. The power input to a 3-phase, 50-Hz induction motor is 60 kW. The total stator loss is 1000 W. Find the total mechanical power developed and rotor copper loss if it is observed that the rotor e.m.f. makes 120 complete cycles per minute.  
**[56.64 kW; 2.36 kW] (AMIE Sec. B Elect. Machine (E-3) Summer 1990)**
  13. A balanced three phase induction motor has an efficiency of 0.85 when its output is 44.76 kW. At this load both the stator copper loss and the rotor copper loss are equal to the core losses. The mechanical losses are one-fourth of the no-load loss. Calculate the slip.  
**[4.94%] (AMIE Sec. B Elect. Machines (E-3) Winter 1991)**
  14. An induction motor is running at 20% slip, the output is 36.775 kW and the total mechanical losses are 1500 W. Estimate Cu losses in the rotor circuit. If the stator losses are 3 kW, estimate efficiency of the motor. **[9,569 W, 72.35%] (Electrical Engineering-II, Bombay Univ. 1978)**
  15. A 3- $\phi$ , 50-Hz, 500-V, 6-pole induction motor gives an output of 37.3 kW at 955 r.p.m. The power factor is 0.86, frictional and windage losses total 1.492 kW; stator losses amount to 1.5 kW. Determine (i) line current (ii) the rotor Cu loss for this load.  
**[(i) 56.54 A (ii) 88.6% (iii) 1.828 kW] (Electrical Technology, Kerala Univ. 1977)**
  16. Determine the efficiency and the output horse-power of a 3-phase, 400-V induction motor running

on load with a slip of 4 per cent and taking a current of 50 A at a power factor of 0.86. When running light at 400 V, the motor has an input current of 15 A and the power taken is 2,000 W, of which 650 W represent the friction, windage and rotor core loss. The resistance per phase of the stator winding (delta-connected) is 0.5 Ω.

[85.8 per cent; 34.2 h.p. (25.51 kW)] (Electrical Engineering-II, M.S. Univ. Baroda 1977)

17. The power input to the rotor of a 440-V, 50-Hz, 3-phase, 6-pole induction motor is 60 kW. It is observed that the rotor e.m.f. makes 90 complete cycles per minute. Calculate (a) the slip (b) rotor speed (c) rotor Cu loss per phase (c) the mechanical power developed and (e) the rotor resistance per phase if rotor current is 60 A. [(a) 0.03 (b) 970 r.p.m. (c) 600 W (d) 58.2 kW (e) 0.167 Ω]
18. An induction motor is running at 50% of the synchronous speed with a useful output of 41.03 kW and the mechanical losses total 1.492 kW. Estimate the Cu loss in the rotor circuit of the motor. If the stator losses total 3.5 kW, at what efficiency is the motor working? [42.52 kW; 46.34%] (Electrical Engineering-II, Bombay Univ. 1975)
19. Plot the torque/speed curve of a 6-pole, 50-Hz, 3-phase induction motor. The rotor resistance and reactance per phase are 0.02 Ω and 0.1 Ω respectively. At what speed is the torque a maximum? What must be the value of the external rotor resistance per phase to give two-third of maximum torque at starting? [(a) 800 rpm (b) 0.242 Ω or 0.018 Ω]

### 34.47. Induction Motor as a Generalized Transformer

The transfer of energy from stator to the rotor of an induction motor takes place entirely *inductively*, with the help of a flux mutually linking the two. Hence, an induction motor is essentially a transformer with stator forming the primary and rotor forming (the short-circuited) rotating secondary (Fig. 34.45). The vector diagram is similar to that of a transformer (Art. 32.15).

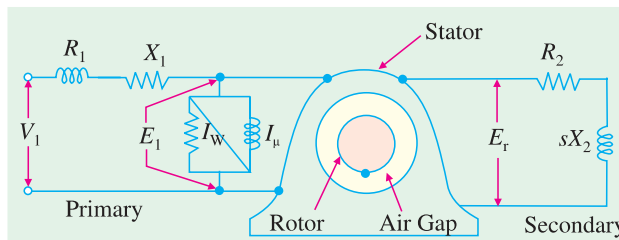


Fig. 34.45

In the vector diagram of Fig. 34.46,  $V_1$  is the applied voltage per stator phase,  $R_1$  and  $X_1$  are stator resistance and leakage reactance per phase respectively, shown external to the stator winding in Fig. 34.45. The applied voltage  $V_1$  produces a magnetic flux which links both primary and secondary thereby producing a counter e.m.f. of self-induction  $E_1$  in primary (i.e. stator) and a mutually-induced e.m.f.  $E_r (= sE_2)$  in secondary (i.e. rotor). There is no secondary terminal voltage  $V_2$  in secondary because whole of the induced e.m.f.  $E_r$  is used up in circulating the rotor current as the rotor is closed upon itself (which is equivalent to its being short-circuited).

Obviously 
$$\mathbf{V}_1 = \mathbf{E}_1 + I_1 R_1 + j I_1 X_1$$

The magnitude of  $E_r$  depends on voltage transformation ratio  $K$  between stator and rotor and the slip. As it is wholly absorbed in the rotor impedance.

∴ 
$$\mathbf{E}_r = \mathbf{I}_2 \mathbf{Z}_2 = I_2 (R_2 + j s X_2)$$

In the vector diagram,  $I_0$  is the no-load primary current. It has two components (i) the working or iron loss components  $I_w$  which supplies the no-load motor losses and (ii) the magnetising component  $I_\mu$  which sets up magnetic flux in the core and the air-gap.

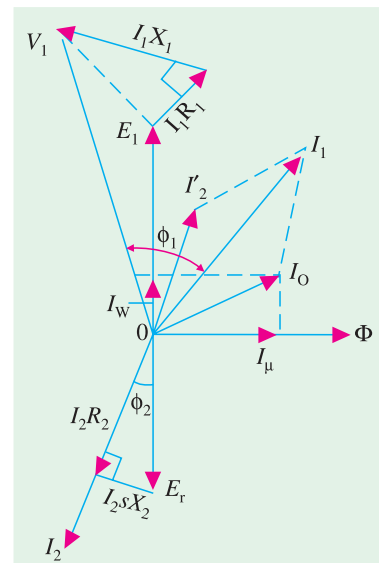


Fig. 34.46



Obviously  $I_0 = \sqrt{(I_{\omega}^2 + I_{\mu}^2)}$  Fig. 34.45,  $I_{\omega}$  and  $I_{\mu}$  are taken care of by an exciting circuit containing  $R_0 = E_1/I_{\omega}$  and  $X_0 = E_1/I_{\mu}$  respectively.

It should be noted here, in passing that in the usual two-winding transformer,  $I_0$  is quite small (about 1% of the full-load current). The reason is that the magnetic flux path lies almost completely in the steel core of low reluctance, hence  $I_{\mu}$  is small, with the result that  $I_0$  is small. But in an induction motor, the presence of an air-gap (of high reluctance) necessitates a large  $I_{\mu}$  hence  $I_0$  is very large (approximately 40 to 50% of the full-load current).

In the vector diagram,  $I_2'$  is the equivalent load current in primary and is equal to  $KI_2$ . Total primary current is the **vector** sum of  $I_0$  and  $I_2'$ .

At this place, a few words may be said to justify the representation of the stator and rotor quantities on the same vector diagram, even though the frequency of rotor current and e.m.f. is only a fraction of that of the stator. We will now show that even though the frequencies of stator and rotor currents are different, yet magnetic fields due to them are synchronous with each other, when seen by an observer stationed in space—both fields rotate at synchronous speed  $N_s$  (Art. 34.11).

The current flowing in the short-circuited rotor produces a magnetic field, which revolves round the rotor in the same direction as the stator field. The speed of rotation of the rotor field is

$$= \frac{120 f_r}{P} = \frac{120 sf}{P} = sN_s = N_s \times \frac{N_s - N}{N_s} = (N_s - N)$$

Rotor speed  $N = (1 - s) N_s$

Hence, speed of the rotating field of the rotor with respect to the stationary stator or space is

$$= sN_s + N = (N_s - N) + N = N_s$$

### 34.48. Rotor Output

Primary current  $I_1$  consists of two parts,  $I_0$  and  $I_2'$ . It is the latter which is transferred to the rotor, because  $I_0$  is used in meeting the Cu and iron losses in the stator itself. Out of the applied primary voltage  $V_1$ , some is absorbed in the primary itself ( $= I_1 Z_1$ ) and the remaining  $E_1$  is transferred to the rotor. If the angle between  $E_1$  and  $I_2'$  is  $\phi$ , then

Rotor input / phase =  $E_1 I_2' \cos \phi$ ; Total rotor input =  $3E_1 I_2' \cos \phi$

The electrical input to the rotor which is wasted in the form of heat is

$$= 3I_2 E_r \cos \phi \quad (\text{or } = 3 I_2^2 R_2)$$

Now  $I_2' = KI_2$  or  $I_2 = I_2' / K$

$$E_r = s E_2 \quad \text{or} \quad E_2 = KE_1$$

$\therefore E_r = sKE_1$

$\therefore$  electrical input wasted as heat

$$= 3 \times (I_2' / K) \times sK E_1 \times \cos \phi = 3E_1 I_2' \cos \phi \times s = \text{rotor input} \times s$$

Now, rotor output = rotor input – losses =  $3E_1 I_2' \cos \phi - 3E_1 I_2' \cos \phi \times s$

$$= (1 - s) 3E_1 I_2' \cos \phi = (1 - s) \times \text{rotor input}$$

$\therefore \frac{\text{rotor output}}{\text{rotor input}} = 1 - s \quad \therefore \text{rotor Cu loss} = s \times \text{rotor input}$

$$\text{rotor efficiency} = 1 - s = \frac{N}{N_s} = \frac{\text{actual speed}}{\text{synchronous speed}}$$

In the same way, other relation similar to those derived in Art. 34.37 can be found.

### 34.49. Equivalent Circuit of the Rotor

When motor is loaded, the rotor current  $I_2$  is given by

$$I_2 = s \frac{E_2}{\sqrt{[R_2^2 + (sX_2)^2]}} = \frac{E_2}{\sqrt{[(R_2/s)^2 + X_2^2]}}$$

From the above relation it appears that the rotor circuit which actually consists of a fixed resistance  $R_2$  and a variable reactance  $sX_2$  (proportional to slip) connected across  $E_r = sE_2$  [Fig. 34.47 (a)] can be looked upon as equivalent to a rotor circuit having a fixed reactance  $X_2$  connected in series with a variable resistance  $R_2/s$  (inversely proportional to slip) and supplied with constant voltage  $E_2$ , as shown in Fig. 34.47 (b).

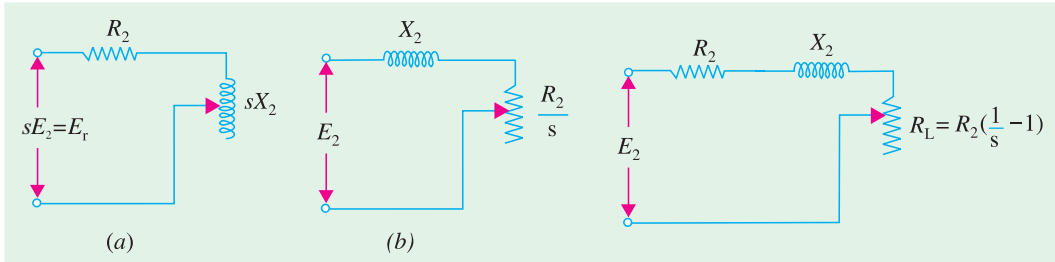


Fig. 34.47

Fig. 34.48

Now, the resistance  $\frac{R_2}{s} = R_2 + R_2 \left(\frac{1}{s} - 1\right)$ . It consists of two parts :

- (i) the first part  $R_2$  is the rotor resistance itself and represents the rotor Cu loss.
- (ii) the second part is  $R_2 \left(\frac{1}{s} - 1\right)$

This is known as the load resistance  $R_L$  and is the electrical equivalent of the mechanical load on the motor. In other words, the mechanical load on an induction motor can be represented by a non-inductive resistance of the value  $R_2 \left(\frac{1}{s} - 1\right)$ . The equivalent rotor circuit along with the load resistance  $R_L$  may be drawn as in Fig. 34.48.

### 34.50. Equivalent Circuit of an Induction Motor

As in the case of a transformer (Fig. 32.14), in this case also, the secondary values may be transferred to the primary and *vice versa*. As before, it should be remembered that when shifting impedance or resistance from secondary to primary, it should be **divided** by  $K^2$  whereas current should be **multiplied** by  $K$ . The equivalent circuit of an induction motor where all values have been referred to primary *i.e.* stator is shown in Fig. 34.49.

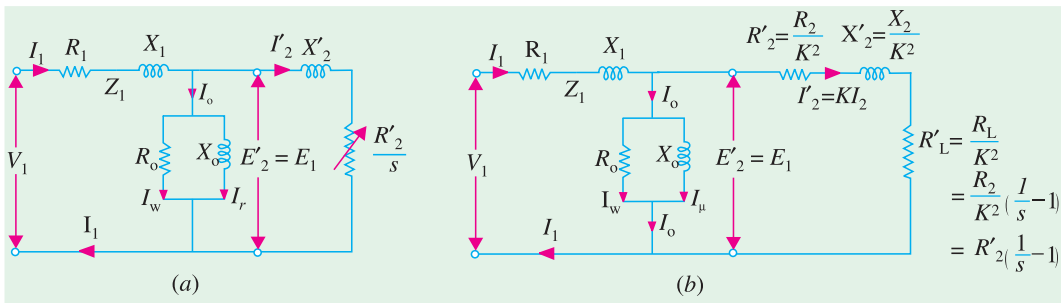


Fig. 34.49

As shown in Fig. 34.50, the exciting circuit may be transferred to the left, because inaccuracy involved is negligible but the circuit and hence the calculations are very much simplified. This is

known as the **approximate equivalent** circuit of the induction motor.

If transformation ratio is assumed unity i.e.  $E_2/E_1 = 1$ , then the equivalent circuit is as shown in Fig. 34.51 instead of that in Fig. 34.49.

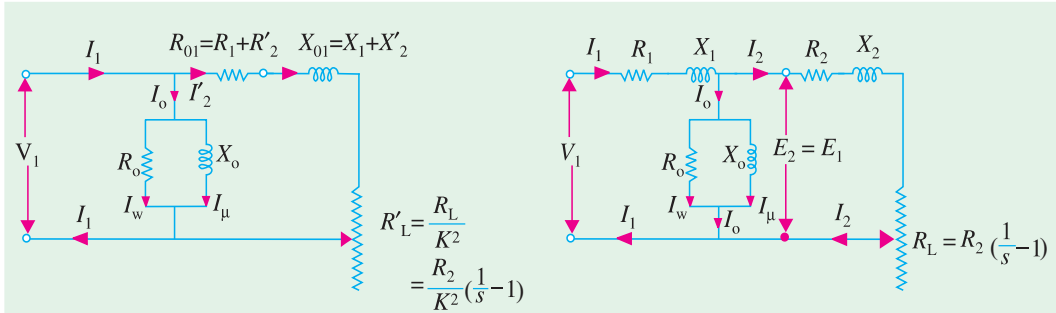


Fig. 34.50

Fig. 34.51

### 34.51. Power Balance Equations

With reference to Fig. 34.49 (a), following power relations in an induction motor can be deduced:

Input power =  $3 V_1 I_1 \cos \phi_1$ ; stator core loss =  $I_w^2 R_0$ ; stator Cu loss =  $3 I_1^2 R_1$

Power transferred to rotor =  $3 I_2'^2 R_2'/s$ ; Rotor Cu loss =  $3 I_2'^2 R_2'$

Mechanical power developed by rotor ( $P_m$ ) or gross power developed by rotor ( $P_g$ ) = rotor input – rotor Cu losses

$$= 3 I_2'^2 R_2' / s - 3 I_2'^2 R_2' = 3 I_2'^2 R_2' \left( \frac{1-s}{s} \right) \text{ Watt}$$

If  $T_g$  is the gross torque\* developed by the rotor, then

$$T_g \times \omega = T_g \times 2\pi \frac{N}{60} = 3 I_2'^2 R_2' \left( \frac{1-s}{s} \right)$$

$$\therefore T_g = \frac{3 I_2'^2 R_2' \left( \frac{1-s}{s} \right)}{2\pi N / 60} \text{ N-m}$$

Now,

$N = N_s(1 - s)$ . Hence gross torque becomes

$$T_g = \frac{3 I_2'^2 R_2' / s}{2\pi N_s / 60} \text{ N-m} = 9.55 \times \frac{3 I_2'^2 R_2' / s}{N_s} \text{ N-m}$$

Since gross torque in **synchronous watts** is equal to the power transferred to the rotor across the air-gap.

$$\therefore T_g = 3 I_2'^2 R_2' / s \text{ synch. watt.}$$

It is seen from the approximate circuit of Fig. 34.50 that

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$

$$T_g = \frac{3}{2\pi N_s / 60} \times \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s} \text{ N-m}$$

\* It is different from shaft torque, which is less than  $T_g$  by the torque required to meet windage and frictional losses.

### 34.52. Maximum Power Output

Fig. 34.52 shows the approximate equivalent circuit of an induction motor with the simplification that :

- (i) exciting circuit is omitted i.e.  $I_0$  is neglected and
- (ii)  $K$  is assumed unity.

As seen, gross power output for 3-phase induction motor is

$$P_g = 3 I_1^2 R_L$$

Now, 
$$I_1 = \frac{V_1}{\sqrt{[(R_{01} + R_L)^2 + X_{01}^2]}} \quad \therefore P_g = \frac{3 V_1^2 R_L}{(R_{01} + R_L)^2 + X_{01}^2}$$

The condition for maximum power output can be found by differentiating the above equation and by equating the first derivative to zero. If it is done, it will be found that

$$R_L^2 = R_{01}^2 + X_{01}^2 = Z_{01}^2$$

where  $Z_{01}$  = leakage impedance of the motor as referred to primary

$$\therefore R_L = Z_{01}$$

Hence, the power output is maximum **when the equivalent load resistance is equal to the stand-still leakage impedance of the motor.**

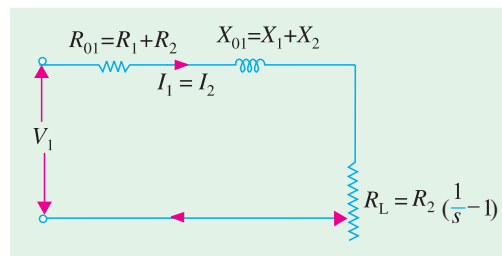


Fig. 34.52

### 34.53. Corresponding Slip

Now  $R_L = R_2 [(1/s) - 1] \quad \therefore Z_{01} = R_L = R_2 [(1/s) - 1] \text{ or } s = \frac{R_2}{R_2 + Z_{01}}$

This is the slip corresponding to maximum gross power output. The value of  $P_{gmax}$  is obtained by substituting  $R_L$  by  $Z_{01}$  in the above equation.

$$\therefore P_{gmax} = \frac{3 V_1^2 Z_{01}}{(R_{01} + Z_{01})^2 + X_{01}^2} = \frac{3 V_1^2 Z_{01}}{R_{01}^2 + Z_{01}^2 + 2 R_{01} Z_{01} + X_{01}^2} = \frac{3 V_1^2}{2 (R_{01} + Z_{01})}$$

It should be noted that  $V_1$  is voltage/phase of the motor and  $K$  has been taken as unity.

**Example 34.54.** The maximum torque of a 3-phase induction motor occurs at a slip of 12%. The motor has an equivalent secondary resistance of 0.08  $\Omega$ /phase. Calculate the equivalent load resistance  $R_L$ , the equivalent load voltage  $V_L$  and the current at this slip if the gross power output is 9,000 watts.

**Solution.**  $R_L = R_2 [(1/s) - 1] = 0.08 [(1/0.12) - 1] = 0.587 \Omega$  /phase.

As shown in the equivalent circuit of the rotor in Fig. 34.53,  $V$  is a fictitious voltage drop equivalent to that consumed in the load connected to the secondary i.e. rotor. The value of  $V = I_2 R_L$

Now, gross power  $P_g = 3 I_2^2 R_L = 3 V^2 / R_L$

$$V = \sqrt{(P_g \times R_L / 3)} = \sqrt{(0.587 \times 9000 / 3)} = 42 \text{ V}$$

Equivalent load current =  $V / R_L = 42 / 0.587 = 71.6 \text{ A}$

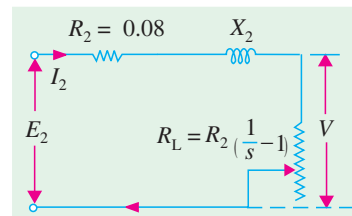


Fig. 34.53

**Example 34.55.** A 3-phase, star-connected 400 V, 50-Hz, 4-pole induction motor has the following per phase parameters in ohms, referred to the stators.

$$R_1 = 0.15, X_1 = 0.45, R_2' = 0.12, X_2' = 0.45, X_m = 28.5$$

Compute the stator current and power factor when the motor is operated at rated voltage and frequency with  $s = 0.04$ . **(Elect. Machines, A.M.I.E. Sec. B, 1990)**

**Solution.** The equivalent circuit with all values referred to stator is shown in Fig. 34.54.

$$R_L' = R_2' \left( \frac{1}{s} - 1 \right)$$

$$= 0.12 \left( \frac{1}{0.04} - 1 \right) = 2.88 \Omega$$

$$I_2' = \frac{V}{(R_{01} + R_L') + j X_{01}}$$

$$= \frac{400/\sqrt{3}}{(0.15 + 0.12 + 2.88) + j(0.45 + 0.45)}$$

$$= 67.78 - j 19.36$$

$$I_0 = \frac{400/\sqrt{3}}{X_m} = \frac{400}{\sqrt{3} \times j 28.5} = -j 8.1$$

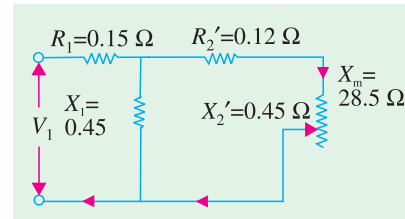


Fig. 34.54

Stator current,  $I_1 = I_0 + I_2' = 67.78 - j 19.36 - j 8.1 = 73.13 \angle - 22^\circ$   
 p.f. =  $\cos \phi = \cos 22^\circ = 0.927$  (lag)

**Example 34.56.** A 220-V, 3- $\phi$ , 4-pole, 50-Hz, Y-connected induction motor is rated 3.73 kW. The equivalent circuit parameters are:

$$R_1 = 0.45 \Omega, X_1 = 0.8 \Omega; R_2' = 0.4 \Omega, X_2' = 0.8 \Omega, B_0 = -1/30 \text{ mho}$$

The stator core loss is 50 W and rotational loss is 150 W. For a slip of 0.04, find (i) input current (ii) p.f. (iii) air-gap power (iv) mechanical power (v) electro-magnetic torque (vi) output power and (vii) efficiency.

**Solution.** The exact equivalent circuit is shown in Fig. 34.55. Since  $R_0$  (or  $G_0$ ) is negligible in determining  $I_1$ , we will consider  $B_0$  (or  $X_0$ ) only

$$Z_{AB} = \frac{j X_m [(R_2'/s) + j X_2']}{(R_2'/s) + j (X_2' + X_m)} = \frac{j 30 (10 + j 0.8)}{10 + j 30.8} = 8.58 + j 3.56 = 9.29 \angle 22.5^\circ$$

$$Z_{01} = Z_1 + Z_{AB} = (0.45 + j 0.8) + (8.58 + j 3.56) = 9.03 + j 4.36 = 10 \angle 25.8^\circ$$

$$V_{ph} = \frac{220}{\sqrt{3}} \angle 0^\circ = 127 \angle 0^\circ$$

(i)  $\therefore I_1 = V_1 / Z_{01} = 127 \angle 0^\circ / 10 \angle 25.8^\circ = 12.7 \angle - 25.8^\circ \text{ A}$

(ii) p.f. =  $\cos 25.8^\circ = 0.9$

(iii) air-gap power,  $P_2 = 3 I_2'^2 (R_2'/s) = 3 I_1^2 R_{AB}$   
 $= 3 \times 12.7^2 \times 8.58 = 4152 \text{ W}$

(iv)  $P_m = (1 - s) P_2 = 0.96 \times 4152 = 3,986 \text{ W}$

(v) Electromagnetic torque (i.e. gross torque)

$$T_g = \frac{P_m}{2\pi N / 60} = 9.55 \frac{P_m}{N} \text{ N-m}$$

Now,  $N_s = 1500 \text{ r.p.m.}, N = 1500 (1 - 0.04) = 1440 \text{ r.p.m.}$

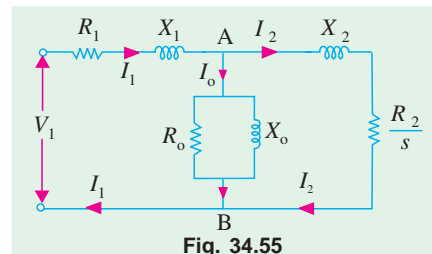


Fig. 34.55

$$T_g = 9.55 \times 3986/1440 = \mathbf{26.4 \text{ N-m}}$$

$$\text{(or } T_g = 9.55 \frac{P_2}{N_s} = 9.55 \times \frac{4152}{1500} = \mathbf{26.4 \text{ N-m}}$$

(vi) output power = 3986 – 150 = **3836 W**

(vii) stator core loss = 50 W; stator Cu loss =  $3 I_1^2 R_1 = 3 \times 12.7^2 \times 0.45 = 218 \text{ W}$

Rotor Cu loss =  $3 I_2'^2 R_2' = sP_2 = 0.04 \times 4152 = 166 \text{ W}$ ; Rotational losses = 150 W

Total loss = 50 + 218 + 166 + 150 = 584 W;  $\eta = 3836/(3836 + 584) = 0.868$  or **86.8%**

**Example 34.57.** A 440-V, 3- $\phi$  50-Hz, 37.3 kW, Y-connected induction motor has the following parameters:

$$R_1 = 0.1 \Omega, X_1 = 0.4 \Omega, R_2' = 0.15 \Omega, X_2' = 0.44 \Omega$$

Motor has stator core loss of 1250 W and rotational loss of 1000 W. It draws a no-load line current of 20 A at a p.f. of 0.09 (lag). When motor operates at a slip of 3%, calculate (i) input line current and p.f. (ii) electromagnetic torque developed in N-m (iii) output and (iv) efficiency of the motor. **(Elect. Machines -II, Nagpur Univ. 1992)**

**Solution.** The equivalent circuit of the motor is shown in Fig. 34.49 (a). Applied voltage per phase =  $440/\sqrt{3} = 254 \text{ V}$ .

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')} = \frac{254 \angle 0^\circ}{(0.1 + 0.15/0.03) + j(0.4 + 0.44)}$$

$$= \frac{254 \angle 0^\circ}{5.1 + j0.84} = \frac{254 \angle 0^\circ}{5.17 \angle 9.3^\circ} = 49.1 \angle -9.3^\circ = 48.4 - j7.9$$

For all practical purposes, no-load motor current may be taken as equal to magnetising current  $I_0$ . Hence,  $I_0 = 20 \angle -84.9^\circ = 1.78 - j19.9$ .

(i)  $I_1 = I_0 + I_2' = (1.78 - j19.9) + (48.4 - j7.9) = 50.2 - j27.8 = \mathbf{57.4 \angle -29^\circ}$

$\therefore$  p.f. =  $\cos 29^\circ = \mathbf{0.875 \text{ (lag)}}$ .

(ii)  $P_2 = 3 I_2'^2 (R_2'/s) = 3 \times 49.1^2 \times (0.15/0.03) = 36,160 \text{ W}$

$N_s = 1500 \text{ r.p.m.}$

$\therefore T_g = 9.55 \times 36,160/1500 = \mathbf{230 \text{ N-m}}$

(iii)  $P_m = (1 - s) P_2 = 0.97 \times 36,160 = 35,075 \text{ W}$

Output power = 35,075 – 1000 = **34,075 W**

Obviously, motor is delivering less than its rated output at this slip.

(iv) Let us total up the losses.

Core loss = 1250 W, stator Cu loss =  $3 I_1^2 R_1 = 3 \times 57.4^2 \times 0.1 = 988 \text{ W}$

Rotor Cu loss =  $3 I_2'^2 R_2' = sP_2 = 0.03 \times 36,160 = 1085 \text{ W}$

rotational i.e. friction and windage losses = 1000 W

Total losses = 1250 + 988 + 1085 + 1000 = 4323 W

$\eta = 34,075/(34,075 + 4323) = 0.887$  or **88.7%**

or input =  $\sqrt{3} \times 440 \times 57.4 \times 0.875 = 38,275 \text{ W}$

$\therefore \eta = 1 - (4323/38,275) = 0.887$  or **88.7%**

**Example 34.58.** A 400 V, 3- $\phi$ , star-connected induction motor has a stator exciting impedance of  $(0.06 + j 0.2) \Omega$  and an equivalent rotor impedance of  $(0.06 + j 0.22) \Omega$ . Neglecting exciting current, find the maximum gross power and the slip at which it occurs. **(Elect. Engg.-II, Bombay Univ. 1987)**

**Solution.** The equivalent circuit is shown in Fig. 34.56.

$$R_{01} = R_1 + R_2' = 0.06 + 0.06 = 0.12 \Omega$$

$$X_{01} = X_1 + X_2' = 0.2 + 0.22 = 0.42 \Omega$$

$$\therefore Z_{01} = \sqrt{(0.12^2 + 0.42^2)} = 0.44 \Omega$$

As shown in Art. 34.53, slip corresponding to maximum gross power output is given by

$$s = \frac{R_2}{R_2 + Z_{01}} = \frac{0.06}{0.06 + 0.44} = 0.12 \text{ or } 12\%$$

Voltage/phase,  $V_1 = 400/\sqrt{3} \text{ V}$

$$P_{g \max} = \frac{3V_1^2}{2(R_{01} + Z_{01})} = \frac{3(400/\sqrt{3})^2}{2(0.12 + 0.44)} = 142,900 \text{ W.}$$

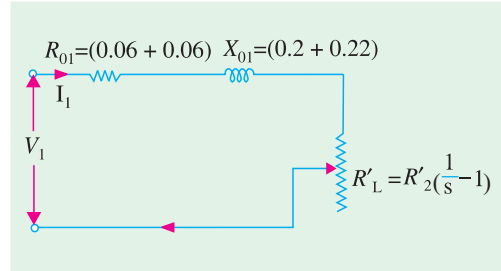


Fig. 34.56

**Example 34.59.** A 115V, 60-Hz, 3-phase, Y-Connected, 6-pole induction motor has an equivalent T-circuit consisting of stator impedance of  $(0.07 + j 0.3) \Omega$  and an equivalent rotor impedance at standstill of  $(0.08 + j 0.3) \Omega$ . Magnetising branch has  $G_o = 0.022 \text{ mho}$ ,  $B_o = 0.158 \text{ mho}$ . Find (a) secondary current (b) primary current (c) primary p.f. (d) gross power output (e) gross torque (f) input (g) gross efficiency by using approximate equivalent circuit. Assume a slip of 2% .

**Solution.** The equivalent circuit is shown in Fig. 34.57  $R_L' = R_2' [(1/s) - 1]$

$$= 0.88 \left( \frac{1}{0.02} - 1 \right) = 3.92 \Omega / \text{phase}$$

The impedance to the right of terminals c and d is

$$\begin{aligned} Z_{cd} &= R_{01} + R_L' + j X_{01} \\ &= (0.07 + 0.08) + 3.92 + j 0.6 \\ &= 4.07 + j 0.6 \\ &= 4.11 \angle 8.4^\circ \Omega / \text{phase} \end{aligned}$$

$$V = 115/\sqrt{3} = 66.5 \text{ V}$$

(a) Secondary current  $I_2' = I_2$

$$= \frac{66.5}{4.11 \angle 8.4^\circ} = 16.17 \angle -j 8.4^\circ$$

$$= 16 - j 2.36 \text{ A}$$

The exciting current  $I_0 = V(G_o - j B_o) = 66.5 (0.022 - j 0.158) = 1.46 - j 10.5 \text{ A}$

(b)  $I_1 = I_0 + I_2' = I_0 + I_2 = (1.46 - j 10.5) + (16 - j 2.36)$

$$= 17.46 - j 12.86 = 21.7 \angle -36.5^\circ$$

(c) Primary p.f. =  $\cos 36.5^\circ = 0.804$

(d)  $P_g = 3 I_2'^2 R_L' = 3 \times 16.17^2 \times 3.92 = 3,075 \text{ W}$

(e) Synchronous speed  $N_s = 120 \times 60/6 = 1,200 \text{ r.p.m.}$

Actual rotor speed  $N = (1 - s) N_s = (1 - 0.02) \times 1200 = 1,176 \text{ r.p.m.}$

$\therefore T_g = 9.55 \cdot \frac{P_m}{N} = 9.55 \times \frac{3075}{1176} = 24.97 \text{ N - m}$

(f) Primary power input =  $\sqrt{3} V_1 I_1 \cos \phi = \sqrt{3} \times 115 \times 21.7 \times 0.804 = 3,450 \text{ W}$

(g) Gross efficiency =  $3,075 \times 100/3,450 = 89.5 \%$

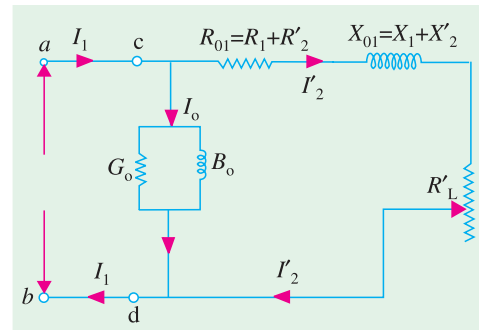


Fig. 34.57

**Alternative Solution**

Instead of using the equivalent circuit of Fig. 34.57, we could use that shown in Fig. 34.49 which is reproduced in Fig. 34.58.

$$\begin{aligned}
 (a) \quad I_2' &= \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')} \\
 &= \frac{66.5 \angle 0^\circ}{(0.07 + 0.08/0.02) + j(0.3 + 0.3)} \\
 &= \frac{66.5}{4.07 + j0.6} \\
 &= 16 - j2.36 = 16.17 \angle -8.4^\circ
 \end{aligned}$$

(b)  $I_1 = I_0 + I_2 = 21.7 \angle -36.5^\circ$  ...as before

(c) primary p.f. = **0.804** ...as before

(d) gross power developed,  $P_g = 3 I_2'^2 R_2' \left(\frac{1-s}{s}\right)$

$$= 3 \times 16.17^2 \times 0.08 \left(\frac{1-0.02}{0.02}\right) = \mathbf{3075 \text{ W}}$$

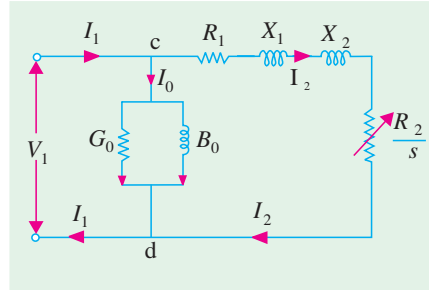


Fig. 34.58

The rest of the solution is the same as above.

**Example 34.60.** The equivalent circuit of a 400 V, 3-phase induction motor with a star-connected winding has the following impedances per phase referred to the stator at standstill:

Stator :  $(0.4 + j 1)$  ohm; Rotor :  $(0.6 + j 1)$  ohm; Magnetising branch :  $(10 + j 50)$  ohm.

Find (i) maximum torque developed (ii) slip at maximum torque and (iii) p.f. at a slip of 5%. Use approximate equivalent circuit. **(Elect. Machinery-III, Bangalore Univ. 1987)**

**Solution.** (i) Gap power transferred and hence the mechanical torque developed by rotor would be maximum when there is maximum transfer of power to the resistor  $R_2'/s$  shown in the approximate equivalent circuit of the motor in Fig. 34.59. It will happen when  $R_2'/s$  equals the impedance looking back into the supply source. Hence,

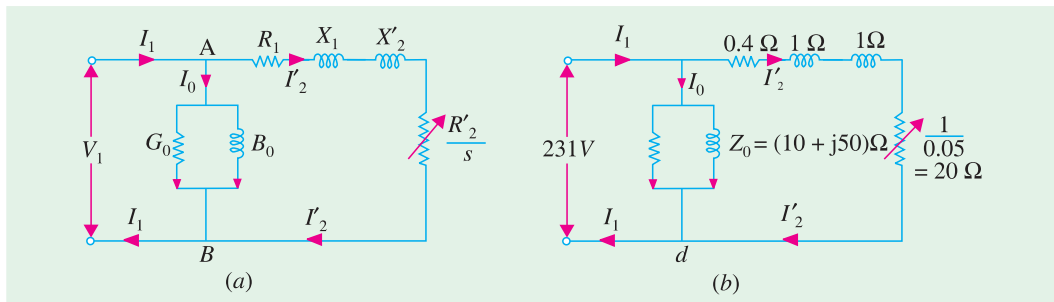


Fig. 34.59

$$\frac{R_2'}{s_m} = \sqrt{R_1^2 + (X_1 + X_2')^2}$$

or

$$s_m = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.6}{\sqrt{0.4^2 + 2^2}} = 0.29 \text{ or } \mathbf{29\%}$$

(i) Maximum value of gross torque developed by rotor

$$T_{g \text{ max}} = \frac{P_{g \text{ max}}}{2\pi N_s/60} = \frac{3 I_2'^2 R_2'/s_m}{2\pi N_s/60} \text{ N - m}$$



Now, 
$$I_2' = \frac{V_1}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \frac{400/\sqrt{3}}{\sqrt{(0.4 + 0.6)^2 + (1 + 1)^2}} = 103.3 \text{ A}$$

$$\therefore T_{g-max} = \frac{3 \times 103.3^2 \times 1/0.29}{2\pi \times 1500/60} = 351 \text{ N-m} \quad \dots \text{ assuming } N_s = 1500 \text{ r.p.m.}$$

(iii) The equivalent circuit for one phase for a slip of 0.05 is shown in Fig. 32.59 (b).

$$I_2' = 231 / [(20 + 0.4) + j 2] = 11.2 - j 1.1$$

$$I_0 = 231 / (10 + j 50) = 0.89 - j 4.4$$

$$I_1 = I_0 + I_2' = 12.09 - j 5.5 = 13.28 \angle - 24.4^\circ; \text{ p.f.} = \cos 24.4^\circ = 0.91 \text{ (lag)}$$

**Tutorial Problem No. 34.4.**

- A 3-phase, 115-volt induction motor has the following constants :  $R_2 = 0.07 \Omega$  ;  $R_2' = 0.08 \Omega$  ,  $X_1 = 0.4 \Omega$  and  $X_2' = 0.2 \Omega$ . All the values are for one phase only. At which slip the gross power output will be maximum and the value of the gross power output ? **[11.4% ; 8.6 kW]**
- A 3-phase, 400-V, Y-connected induction motor has an equivalent T-circuit consisting of  $R_1 = 1 \Omega$  ,  $X_1 = 2 \Omega$  , equivalent rotor values are  $R_2' = 1.2 \Omega$  ,  $X_2' = 1.5 \Omega$ . The exciting branch has an impedance of  $(4 + j 40) \Omega$ . If slip is 5% find (i) current (ii) efficiency (iii) power factor (iv) output. Assume friction loss to be 250 W. **[ (i) 10.8 A (ii) 81% (iii) 0.82 (iv) 5 kW]**
- A 50 HP, 440 Volt, 3-phase, 50 Hz Induction motor with star-connected stator winding gave the following test results:  
 (i) No load test: Applied line voltage 440 V, line current 24 A, wattmeter reading 5150 and 3350 watts.  
 (ii) Blocked rotor test: applied line voltage 33.6 volt, line current 65 A, wattmeter reading 2150 and 766 watts.  
 Calculate the parameters of the equivalent circuit.

**[Rajiv Gandhi Technical University, Bhopal, 2000]**

**[ (i) Shunt branch :  $R_o = 107.6$  ohms,  $X_m = 10.60$  ohms (ii) Series branch :  $r = 0.23$  ohm,  $x = 0.19$  ohm ]**

**OBJECTIVE TESTS – 34**

- Regarding skewing of motor bars in a squirrel-cage induction motor, (SCIM) which statement is false ?  
 (a) it prevents cogging  
 (b) it increases starting torque  
 (c) it produces more uniform torque  
 (d) it reduces motor 'hum' during its operation.
- The principle of operation of a 3-phase. Induction motor is most similar to that of a  
 (a) synchronous motor  
 (b) repulsion-start induction motor  
 (c) transformer with a shorted secondary  
 (d) capacitor-start, induction-run motor.
- The magnetising current drawn by transformers and induction motors is the cause of their .....power factor.  
 (a) zero (b) unity  
 (c) lagging (d) leading.
- The effect of increasing the length of air-gap in an induction motor will be to increase the  
 (a) power factor  
 (b) speed  
 (c) magnetising current  
 (d) air-gap flux.  
**(Power App-II, Delhi Univ. Jan. 1987)**
- In a 3-phase induction motor, the relative speed of stator flux with respect to .....is zero.  
 (a) stator winding (b) rotor  
 (c) rotor flux (d) space.
- An eight-pole wound rotor induction motor operating on 60 Hz supply is driven at 1800 r.p.m. by a prime mover in the opposite direction of revolving magnetic field. The frequency of rotor current is  
 (a) 60 Hz (b) 120 Hz  
 (c) 180 Hz (d) none of the above.

**(Elect. Machines, A.M.I.E. Sec. B, 1993)**

7. A 3-phase, 4-pole, 50-Hz induction motor runs at a speed of 1440 r.p.m. The rotating field produced by the rotor rotates at a speed of .....r.p.m. with respect to the rotor.  
 (a) 1500 (b) 1440  
 (c) 60 (d) 0.
8. In a 3- $\phi$  induction motor, the rotor field rotates at synchronous speed with respect to  
 (a) stator (b) rotor  
 (c) stator flux (d) none of the above.
9. Irrespective of the supply frequency, the torque developed by a SCIM is the same whenever ..... is the same.  
 (a) supply voltage (b) external load  
 (c) rotor resistance (d) slip speed.
10. In the case of a 3- $\phi$  induction motor having  $N_s = 1500$  rpm and running with  $s = 0.04$   
 (a) revolving speed of the stator flux is space is .....rpm  
 (b) rotor speed is .....rpm  
 (c) speed of rotor flux relative to the rotor is .....rpm  
 (d) speed of the rotor flux with respect to the stator is .....rpm.
11. The number of stator poles produced in the rotating magnetic field of a 3- $\phi$  induction motor having 3 slots per pole per phase is  
 (a) 3 (b) 6  
 (c) 2 (d) 12
12. The power factor of a squirrel-cage induction motor is  
 (a) low at light loads only  
 (b) low at heavy loads only  
 (c) low at light and heavy loads both  
 (d) low at rated load only.
- (Elect. Machines, A.M.I.E. Sec. B, 1993)**
13. Which of the following rotor quantity in a SCIM does NOT depend on its slip ?  
 (a) reactance (b) speed  
 (c) induced emf (d) frequency.
14. A 6-pole, 50-Hz, 3- $\phi$  induction motor is running at 950 rpm and has rotor Cu loss of 5 kW. Its rotor input is .....kW.  
 (a) 100 (b) 10  
 (c) 95 (d) 5.3.
15. The efficiency of a 3-phase induction motor is approximately proportional to  
 (a)  $(1-s)$  (b)  $s$   
 (c)  $N$  (d)  $N_s$
16. A 6-pole, 50-Hz, 3- $\phi$  induction motor has a full-load speed of 950 rpm. At half-load, its speed would be .....rpm.  
 (a) 475 (b) 500  
 (c) 975 (d) 1000
17. If rotor input of a SCIM running with a slip of 10% is 100 kW, gross power developed by its rotor is ..... kW.  
 (a) 10 (b) 90  
 (c) 99 (d) 80
18. Pull-out torque of a SCIM occurs at that value of slip where rotor power factor equals  
 (a) unity (b) 0.707  
 (c) 0.866 (d) 0.5
19. Fill in the blanks.  
 When load is placed on a 3-phase induction motor, its  
 (i) speed .....  
 (ii) slip .....  
 (iii) rotor induced emf .....  
 (iv) rotor current .....  
 (v) rotor torque .....  
 (vi) rotor continues to rotate at that value of slip at which developed torque equals ..... torque.
20. When applied rated voltage per phase is reduced by one-half, the starting torque of a SCIM becomes ..... of the starting torque with full voltage.  
 (a)  $1/2$  (b)  $1/4$   
 (c)  $1/\sqrt{2}$  (d)  $\sqrt{3}/2$
21. If maximum torque of an induction motor is 200 kg-m at a slip of 12%, the torque at 6% slip would be ..... kg-m.  
 (a) 100 (b) 160  
 (c) 50 (d) 40
22. The fractional slip of an induction motor is the ratio  
 (a) rotor Cu loss/rotor input  
 (b) stator Cu loss/stator input  
 (c) rotor Cu loss/rotor output  
 (d) rotor Cu loss/stator Cu loss
23. The torque developed by a 3-phase induction motor depends on the following three factors:  
 (a) speed, frequency, number of poles  
 (b) voltage, current and stator impedance  
 (c) synchronous speed, rotor speed and frequency

- (d) rotor emf, rotor current and rotor p.f.
24. If the stator voltage and frequency of an induction motor are reduced proportionately, its
- (a) locked rotor current is reduced
  - (b) torque developed is increased
  - (c) magnetising current is decreased
  - (d) both (a) and (b)
25. The efficiency and p.f. of a SCIM increases in proportion to its
- (a) speed
  - (b) mechanical load
  - (c) voltage
  - (d) rotor torque
26. A SCIM runs at *constant* speed only so long as
- (a) torque developed by it remains constant
  - (b) its supply voltage remains constant
  - (c) its torque exactly equals the mechanical load
  - (d) stator flux remains constant
27. The synchronous speed of a linear induction motor does NOT depend on
- (a) width of pole pitch
  - (b) number of poles
  - (c) supply frequency
  - (d) any of the above
28. Thrust developed by a linear induction motor depends on
- (a) synchronous speed
  - (b) rotor input
  - (c) number of poles
  - (d) both (a) and (b)

### ANSWERS

1. b 2. c 3. c 4. c 5. c 6. c 7. c 8. a 9. d 10. (i) 1500 (ii) 1440 (iii) 60 (iv) 1500 11. b 12. a 13. b 14. a 15. a 16. c 17. b 18. b 19. (i) decreases (ii) increases (iii) increases (iv) increases (v) increases (vi) applied 20. b 21. b 22. a 23. d 24. d 25. b 26. c 27. b 28. d

## ROUGHWORK

# CHAPTER 35

## Learning Objectives

- General
- Circle Diagram for a Series Circuit
- Circle Diagram of the Approximate Equivalent Circle
- Determination of  $G_0$  and  $B_0$
- No-load Test
- Blocked Rotor Test
- Construction of the Circle Diagram
- Maximum Quantities
- Starting of Induction Motors
- Direct-Switching or Line Starting of Induction Motors
- Squirrel-cage Motors
- Starting of Slip-ring Motors
- Starter Steps
- Crawling
- Cogging or Magnetic Locking
- Double Squirrel-cage Motor
- Equivalent circuit
- Speed Control of Induction Motor
- Three-Phase A.C. Commutator Motors
- Schrage Motor
- Motor Enclosures
- Standard type of Squirrel-cage Motors
- Class A Motors
- Class B Motors
- Class C Motors
- Class D Motors
- Class E Motors
- Class F Motors
- Questions and Answer on Induction Motors

## COMPUTATIONS AND CIRCLE DIAGRAMS



This chapter explains you how to derive performance characteristics of induction motors using circular diagrams

35.1. General

In this chapter, it will be shown that the performance characteristics of an induction motor are derivable from a circular locus. The data necessary to draw the circle diagram may be found from no-load and blocked-rotor tests, corresponding to the open-circuit and short-circuit tests of a transformer. The stator and rotor Cu losses can be separated by drawing a torque line. The parameters of the motor, in the equivalent circuit, can be found from the above tests, as shown below.

35.2. Circle Diagram for a Series Circuit

It will be shown that the end of the current vector for a series circuit with constant reactance and voltage, but with a variable resistance is a circle. With reference to Fig. 35.1, it is clear that

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R^2 + X^2)}}$$

$$= \frac{V}{X} \times \frac{X}{\sqrt{(R^2 + X^2)}} = \frac{V}{X} \sin \phi$$

$$\therefore \sin \phi = \frac{X}{\sqrt{(R^2 + X^2)}} \quad \text{— Fig. 35.2}$$

$\therefore I = (V/X) \sin \phi$

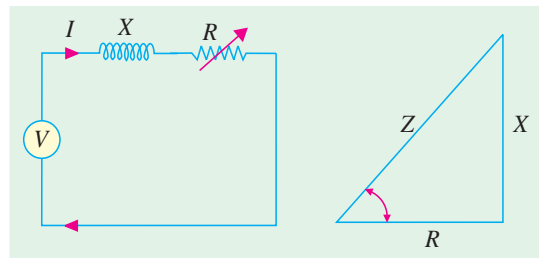


Fig. 35.1

Fig. 35.2

It is the equation of a circle in polar co-ordinates, with diameter equal to  $V/X$ . Such a circle is drawn in Fig. 35.3, using the magnitude of the current and power factor angle  $\phi$  as polar co-ordinates of the point A. In other words, as resistance R is varied (which means, in fact,  $\phi$  is changed), the end of the current vector lies on a circle with diameter equal to  $V/X$ . For a lagging current, it is usual to orientate the circle of Fig. 35.3 (a) such that its diameter is horizontal and the voltage vector takes a vertical position, as shown in Fig. 35.3 (b). There is no difference between the two so far as the magnitude and phase relationships are concerned.

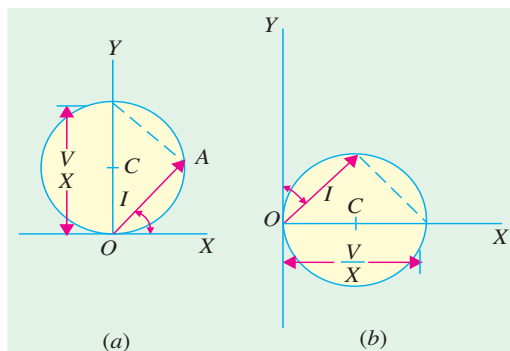


Fig. 35.3

35.3. Circle Diagram for the Approximate Equivalent Circuit

The approximate equivalent diagram is redrawn in Fig. 35.4. It is clear that the circuit to the right of points  $ab$  is similar to a series circuit, having a constant voltage  $V_1$  and reactance  $X_{01}$  but variable resistance (corresponding to different values of slip  $s$ ).

Hence, the end of current vector for  $I_2'$  will lie on a circle with a diameter of  $V/X_{01}$ . In Fig. 35.5,  $I_2'$  is the rotor current referred to stator,  $I_0$  is no-load current (or exciting current) and  $I_1$  is the total stator current and is the vector sum of the first two. When  $I_2'$  is lagging and  $\phi_2 = 90^\circ$ , then the position of vector for  $I_2'$  will be along  $OC$  i.e. at right angles to the voltage vector  $OE$ . For any other value of  $\phi_2$ , point A will move along the circle shown dotted. The exciting current  $I_0$  is drawn lagging  $V$  by an angle  $\phi_0$ . If conductance  $G_0$  and susceptance

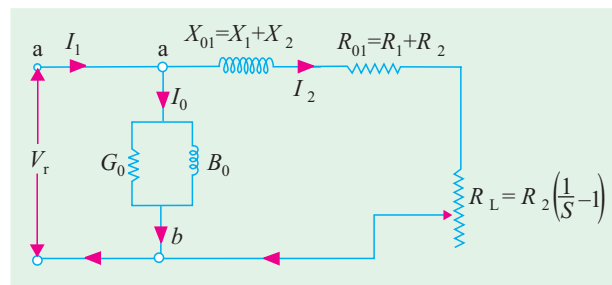


Fig. 35.4

$B_0$  of the exciting circuit are assumed constant, then  $I_0$  and  $\phi_0$  are also constant. The end of current vector for  $I_1$  is also seen to lie on another circle which is displaced from the dotted circle by an amount  $I_0$ . Its diameter is still  $V/X_{01}$  and is parallel to the horizontal axis  $OC$ . Hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.

### 35.4. Determination of $G_0$ and $B_0$

If the total leakage reactance  $X_{01}$  of the motor, exciting conductance  $G_0$  and exciting susceptance  $B_0$  are found, then the position of the circle  $O'BC'$  is determined uniquely. One method of finding  $G_0$  and  $B_0$  consists in running the motor synchronously so that slip  $s = 0$ . In practice, it is impossible for an induction motor to run at synchronous speed, due to the inevitable presence of friction and windage losses. However, the induction motor may be run at synchronous speed by

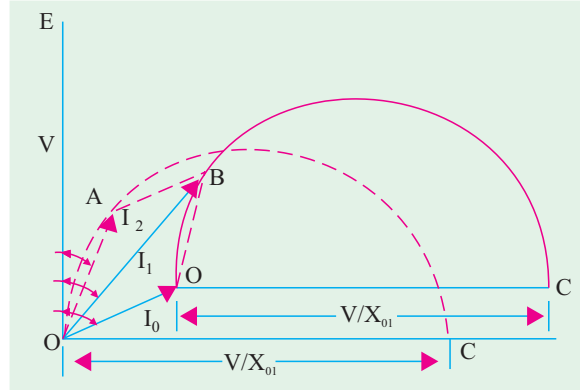


Fig. 35.5

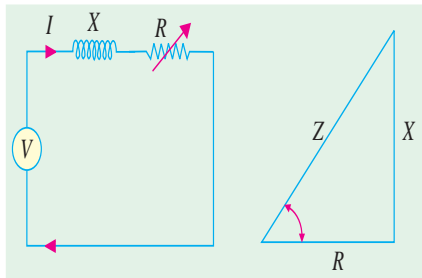


Fig. 35.6

another machine which supplies the friction and windage losses. In that case, the circuit to the right of points  $ab$  behaves like an open circuit, because with  $s = 0$ ,  $R_L = \infty$  (Fig. 35.6). Hence, the current drawn by the motor is  $I_0$  only. Let

$V$  = applied voltage/phase;  $I_0$  = motor current / phase

$W$  = wattmeter reading *i.e.* input in watt ;  $Y_0$  = exciting admittance of the motor. Then, for a 3-phase induction motor

$$W = 3G_0V^2 \quad \text{or} \quad G_0 = \frac{W}{3V^2} \quad \text{Also, } I_0 = V Y_0 \quad \text{or} \quad Y_0 = I_0/V$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{[(I_0/V)^2 - G_0^2]}$$

Hence,  $G_0$  and  $B_0$  can be found.

### 35.5. No-load Test

In practice, it is neither necessary nor feasible to run the induction motor synchronously for getting  $G_0$  and  $B_0$ . Instead, the motor is run without any external mechanical load on it. The speed of the rotor would not be synchronous, but very much near to it ; so that, for all practical purposes, the speed may be assumed synchronous. The no load test is carried out with different values of applied voltage, below and above the value of normal voltage. The power input is measured by two wattmeters,

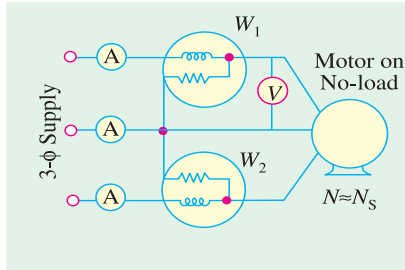


Fig. 35.7

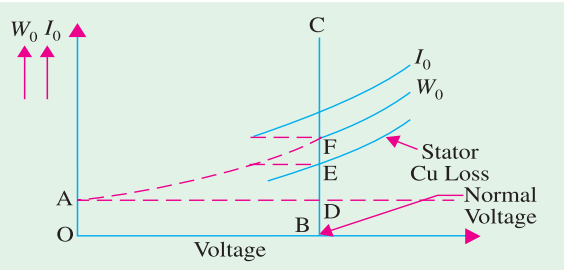


Fig. 35.8

$I_0$  by an ammeter and  $V$  by a voltmeter, which are included in the circuit of Fig. 35.7. As motor is running on light load, the p.f. would be low *i.e.* less than 0.5, hence total power input will be the difference of the two wattmeter readings  $W_1$  and  $W_2$ . The readings of the total power input  $W_0$ ,  $I_0$  and voltage  $V$  are plotted as in Fig. 35.8. If we extend the curve for  $W_0$ , it cuts the vertical axis at point  $A$ .  $OA$  represents losses due to friction and windage. If we subtract loss corresponding to  $OA$  from  $W_0$ , then we get the no-load electrical and magnetic losses in the machine, because the no-load input  $W_0$  to the motor consists of

- (i) small stator Cu loss  $3 I_0^2 R_1$
- (ii) stator core loss  $W_{CL} = 3G_0 V^2$
- (iii) loss due to friction and windage.

The losses (ii) and (iii) are collectively known as fixed losses, because they are independent of load.  $OB$  represents normal voltage. Hence, losses at normal voltage can be found by drawing a vertical line from  $B$ .

$$BD = \text{loss due to friction and windage} \quad DE = \text{stator Cu loss} \quad EF = \text{core loss}$$

Hence, knowing the core loss  $W_{CL}$ ,  $G_0$  and  $B_0$  can be found, as discussed in Art. 35.4.

$$\text{Additionally, } \phi_0 \text{ can also be found from the relation } W_0 = \sqrt{3} V_L I_0 \cos \phi_0$$

$$\therefore \cos \phi_0 = \frac{W_0}{\sqrt{3} V_L I_0} \quad \text{where } V_L = \text{line voltage and } W_0 \text{ is no-load stator input.}$$

**Example 35.1.** In a no-load test, an induction motor took 10 A and 450 watts with a line voltage of 110 V. If stator resistance/phase is 0.05  $\Omega$  and friction and windage losses amount to 135 watts, calculate the exciting conductance and susceptance/phase.

**Solution.** stator Cu loss =  $3 I_0^2 R_1 = 3 \times 10^2 \times 0.05 = 15 \text{ W}$

$\therefore$  stator core loss =  $450 - 135 - 15 = 300 \text{ W}$

Voltage/phase  $V = 110/\sqrt{3} \text{ V}$ ; Core loss =  $3 G_0 V^2$

$$300 = 3 G_0 \times (110/\sqrt{3})^2 ; G_0 = \frac{300}{3 \times (110/\sqrt{3})^2} = \mathbf{0.025 \text{ siemens/phase}}$$

$Y_0 = I_0 / V = (10 \times \sqrt{3}) / 110 = 0.158 \text{ siemens/phase}$

$$B_0 = \sqrt{Y_0^2 - G_0^2} = \sqrt{(0.158^2 - 0.025^2)} = \mathbf{0.156 \text{ siemens/phase.}}$$

### 35.6. Blocked Rotor Test

It is also known as locked-rotor or short-circuit test. This test is used to find—

1. short-circuit current with *normal* voltage applied to stator
2. power factor on short-circuit

Both the values are used in the construction of circle diagram

3. total leakage reactance  $X_{01}$  of the motor as referred to primary (*i.e.* stator)
4. total resistance of the motor  $R_{01}$  as referred to primary.

In this test, the rotor is locked (or allowed very slow rotation)



This vertical test stand is capable of absorbing up to 10,000 N-m of torque at continuous load rating (max 150.0 hp at 1800 rpm). It helps to develop speed torque curves and performs locked rotor testing



and the rotor windings are short-circuited at slip-rings, if the motor has a wound rotor. Just as in the case of a short-circuit test on a transformer, a reduced voltage (up to 15 or 20 per cent of normal value) is applied to the stator terminals and is so adjusted that full-load current flows in the stator. As in this case  $s = 1$ , the equivalent circuit of the motor is exactly like a transformer, having a short-circuited secondary. The values of current, voltage and power input on short-circuit are measured by the ammeter, voltmeter and wattmeter connected in the circuits as before. Curves connecting the above quantities may also be drawn by taking two or three additional sets of readings at progressively reduced voltages of the stator.

(a) It is found that relation between the short-circuit current and voltage is approximately a straight line. Hence, if  $V$  is normal stator voltage,  $V_s$  the short-circuit voltage (a fraction of  $V$ ), then short-circuit or standstill rotor current, if normal voltage were applied to stator, is found from the relation

$$I_{SN} = I_s \times V/V_s$$

where

$$I_{SN} = \text{short-circuit current obtainable with normal voltage}$$

$$I_s = \text{short-circuit current with voltage } V_s$$

(b) Power factor on short-circuit is found from

$$W_s = \sqrt{3} V_{SL} I_{SL} \cos \phi_s; \quad \therefore \cos \phi_s = W_s / (\sqrt{3} V_{SL} I_{SL})$$

where

$$W_s = \text{total power input on short-circuit}$$

$$V_{SL} = \text{line voltage on short-circuit}$$

$$I_{SL} = \text{line current on short-circuit.}$$

(c) Now, the motor input on short-circuit consists of

(i) mainly stator and rotor Cu losses

(ii) core-loss, which is small due to the fact that applied voltage is only a small percentage of the normal voltage. This core-loss (if found appreciable) can be calculated from the curves of Fig. 35.8.

$$\therefore \begin{aligned} \text{Total Cu loss} &= W_s - W_{CL} \\ 3 I_s^2 R_{01} &= W_s - W_{CL} : \quad R_{01} = (W_s - W_{CL}) / 3 I_s^2 \end{aligned}$$

(d) With reference to the approximate equivalent circuit of an induction motor (Fig. 35.4), motor leakage reactance per phase  $X_{01}$  as referred to the stator may be calculated as follows :

$$Z_{01} = V_s / I_s \quad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$$

Usually,  $X_1$  is assumed equal to  $X_2'$  where  $X_1$  and  $X_2$  are stator and rotor reactances per phase respectively as referred to stator.  $X_1 = X_2' = X_{01} / 2$

If the motor has a wound rotor, then stator and rotor resistances are separated by dividing  $R_{01}$  in the ratio of the d.c. resistances of stator and rotor windings.

In the case of squirrel-cage rotor,  $R_1$  is determined as usual and after allowing for 'skin effect' is subtracted from  $R_{01}$  to give  $R_2'$  – the effective rotor resistance as referred to stator.

$$\therefore R_2' = R_{01} - R_1$$

**Example 35.2.** A 110-V, 3- $\phi$ , star-connected induction motor takes 25 A at a line voltage of 30 V with rotor locked. With this line voltage, power input to motor is 440 W and core loss is 40 W. The d.c. resistance between a pair of stator terminals is 0.1  $\Omega$ . If the ratio of a.c. to d.c. resistance is 1.6, find the equivalent leakage reactance/phase of the motor and the stator and rotor resistance per phase. (Electrical Technology, Madras Univ. 1987)

**Solution.** S.C. voltage/phase,  $V_s = 30 / \sqrt{3} = 17.3 \text{ V} ; I_s = 25 \text{ A per phase}$   
 $Z_{01} = 17.3/25 = 0.7 \Omega$  (approx.) per phase

Stator and rotor Cu losses = input – core loss = 440 – 40 = 400 W

$$\therefore 3 \times 25^2 \times R_{01} = 400 \quad \therefore R_{01} = 400/3 \times 625 = \mathbf{0.21 \Omega}$$

where  $R_{01}$  is equivalent resistance/phase of motor as referred to stator.

Leakage reactance/phase  $X_{01} = \sqrt{(0.7^2 - 0.21^2)} = 0.668 \Omega$

d.c. resistance/phase of stator =  $0.1/2 = 0.05 \Omega$

a.c. resistance/phase  $R_1 = 0.05 \times 1.6 = 0.08 \Omega$

Hence, effective resistance/phase of rotor as referred to stator

$$R_2' = 0.21 - 0.08 = \mathbf{0.13 \Omega}$$

### 35.7. Construction of the Circle Diagram

Circle diagram of an induction motor can be drawn by using the data obtained from (1) no-load (2) short-circuit test and (3) stator resistance test, as shown below.

#### Step No. 1

From no-load test,  $I_0$  and  $\phi_0$  can be calculated. Hence, as shown in Fig. 35.9, vector for  $I_0$  can be laid off lagging  $\phi_0$  behind the applied voltage  $V$ .

#### Step No. 2

Next, from blocked rotor test or short-circuit test, short-circuit current  $I_{SN}$  corresponding to normal voltage and  $\phi_s$  are found. The vector  $OA$  represents  $I_{SN} = (I_S V/V_S)$  in magnitude and phase. Vector  $O'A$  represents rotor current  $I_2'$  as referred to stator.

Clearly, the two points  $O'$  and  $A$  lie on the required circle. For finding the centre  $C$  of this circle, chord  $O'A$  is bisected at right angles—its bisector giving point  $C$ . The diameter  $O'D$  is drawn perpendicular to the voltage vector.



Windings inside a motor

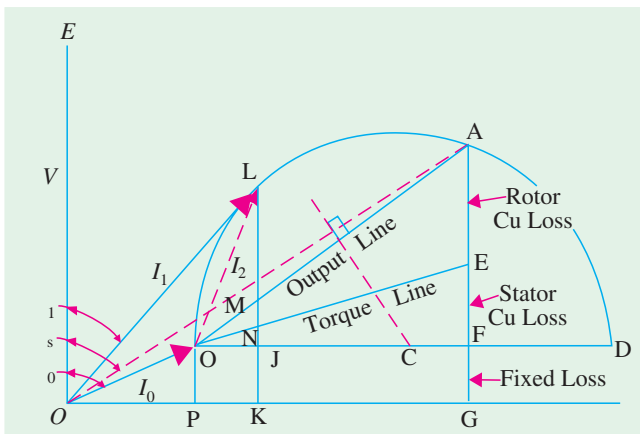


Fig. 35.9

For example, the vertical component  $O'P$  of no-load current  $OO'$  represents the no-load input, which supplies core loss, friction and windage loss and a negligibly small amount of stator  $I^2R$  loss. Similarly, the vertical component  $AG$  of short-circuit current  $OA$  is proportional to the motor input on short-circuit or if measured to a proper scale, may be said to equal power input.

#### Step No. 3

**Torque line.** This is the line which separates the stator and the rotor copper losses. When the

As a matter of practical contingency, it is recommended that the scale of current vectors should be so chosen that the diameter is more than 25 cm, in order that the performance data of the motor may be read with reasonable accuracy from the circle diagram. With centre  $C$  and radius =  $CO'$ , the circle can be drawn. The line  $O'A$  is known as **out-put line**.

It should be noted that as the voltage vector is drawn vertically, all vertical distances represent the active or power or energy components of the currents.

rotor is locked, then all the power supplied to the motor goes to meet core losses and Cu losses in the stator and rotor windings. The power input is proportional to  $AG$ . Out of this,  $FG (= O'P)$  represents fixed losses *i.e.* stator core loss and friction and windage losses.  $AF$  is proportional to the sum of the stator and rotor Cu losses. The point  $E$  is such that

$$\frac{AE}{EF} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

As said earlier, line  $O'E$  is known as torque line.

**How to locate point E ?**

(i) **Squirrel-cage Rotor.** Stator resistance/phase *i.e.*  $R_1$  is found from stator-resistance test. Now, the short-circuit motor input  $W_s$  is approximately equal to motor Cu losses (neglecting iron losses).

$$\text{Stator Cu loss} = 3I_s^2 R_1 \quad \therefore \text{rotor Cu loss} = W_s - 3I_s^2 R_1 \quad \therefore \frac{AE}{EF} = \frac{W_s - 3I_s^2 R_1}{3I_s^2 R_1}$$

(ii) **Wound Rotor.** In this case, rotor and stator resistances per phase  $r_2$  and  $r_1$  can be easily computed. For any values of stator and rotor currents  $I_1$  and  $I_2$  respectively, we can write

$$\frac{AE}{EF} = \frac{I_2^2 r_2}{I_1^2 r_1} = \frac{r_2}{r_1} \left( \frac{I_2}{I_1} \right)^2; \quad \text{Now, } \frac{I_1}{I_2} = K = \text{transformation ratio}$$

$$\frac{AE}{EF} = \frac{r_2}{r_1} \times \frac{1}{K^2} = \frac{r_2 / K^2}{r_1} = \frac{r_2'}{r_1} = \frac{\text{equivalent rotor resistance per phase}}{\text{stator resistance per phase}}$$

Value of  $K$  may be found from short-circuit test itself by using two ammeters, both in stator and rotor circuits.

Let us assume that the motor is running and taking a current  $OL$  (Fig. 35.9). Then, the perpendicular  $JK$  represents fixed losses,  $JN$  is stator Cu loss,  $NL$  is the rotor input,  $NM$  is rotor Cu loss,  $ML$  is rotor output and  $LK$  is the total motor input.

From our knowledge of the relations between the above-given various quantities, we can write :

$$\begin{aligned} \sqrt{3} \cdot V_L \cdot LK &= \text{motor input} & \sqrt{3} \cdot V_L \cdot JK &= \text{fixed losses} \\ \sqrt{3} \cdot V_L \cdot JN &= \text{stator copper loss} & \sqrt{3} \cdot V_L \cdot MN &= \text{rotor copper loss} \\ \sqrt{3} \cdot V_L \cdot MK &= \text{total loss} & \sqrt{3} \cdot V_L \cdot ML &= \text{mechanical output} \\ \sqrt{3} \cdot V_L \cdot NL &= \text{rotor input} \propto \text{torque} \end{aligned}$$

1.  $ML / LK = \text{output/input} = \text{efficiency}$
2.  $MN / NL = (\text{rotor Cu loss}) / (\text{rotor input}) = \text{slip, } s.$
3.  $\frac{ML}{NL} = \frac{\text{rotor output}}{\text{rotor input}} = 1 - s = \frac{N}{N_s} = \frac{\text{actual speed}}{\text{synchronous speed}}$
4.  $\frac{LK}{OL} = \text{power factor}$

Hence, it is seen that, at least, theoretically, it is possible to obtain all the characteristics of an induction motor from its circle diagram. As said earlier, for drawing the circle diagram, we need (a) stator-resistance test for separating stator and rotor Cu losses and (b) the data obtained from (i) no-load test and (ii) short-circuit test.

**35.8. Maximum Quantities**

It will now be shown from the circle diagram (Fig. 35.10) that the maximum values occur at the positions stated below :

(i) **Maximum Output**

It occurs at point M where the tangent is parallel to output line  $O'A$ . Point  $M$  may be located by

drawing a line  $CM$  from point  $C$  such that it is perpendicular to the output line  $O'A$ . Maximum output is represented by the vertical  $MP$ .

**(ii) Maximum Torque or Rotor Input**

It occurs at point  $N$  where the tangent is parallel to torque line  $O'E$ . Again, point  $N$  may be found by drawing  $CN$  perpendicular to the torque line. Its value is represented by  $NQ$ . Maximum torque is also known as stalling or pull-out torque.

**(iii) Maximum Input Power**

It occurs at the highest point of the circle *i.e.* at point  $R$  where the tangent to the circle is horizontal. It is proportional to  $RS$ . As the point  $R$  is beyond the point of maximum torque, the induction motor will be unstable here. However, the maximum input is a measure of the size of the circle and is an indication of the ability of the motor to carry short-time over-loads. Generally,  $RS$  is twice or thrice the motor input at rated load.

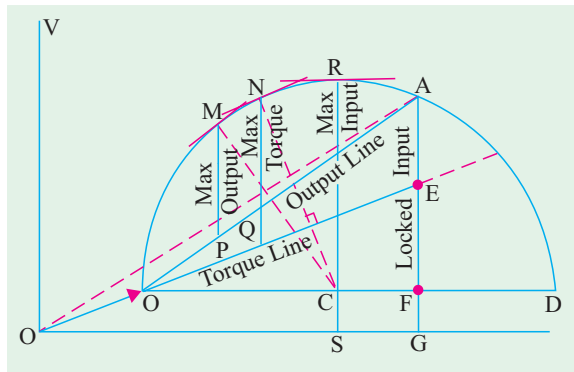


Fig. 35.10

**Example 35.3.** A 3-ph, 400-V induction motor gave the following test readings;

No-load : 400 V, 1250 W, 9 A, Short-circuit : 150 V, 4 kW, 38 A

Draw the circle diagram.

If the normal rating is 14.9 kW, find from the circle diagram, the full-load value of current, p.f. and slip.

(Electrical Machines-I, Gujarat Univ. 1985)

**Solution.**

$$\cos \phi_0 = \frac{1250}{\sqrt{3} \times 400 \times 9} = 0.2004 ; \quad \phi_0 = 78.5^\circ$$

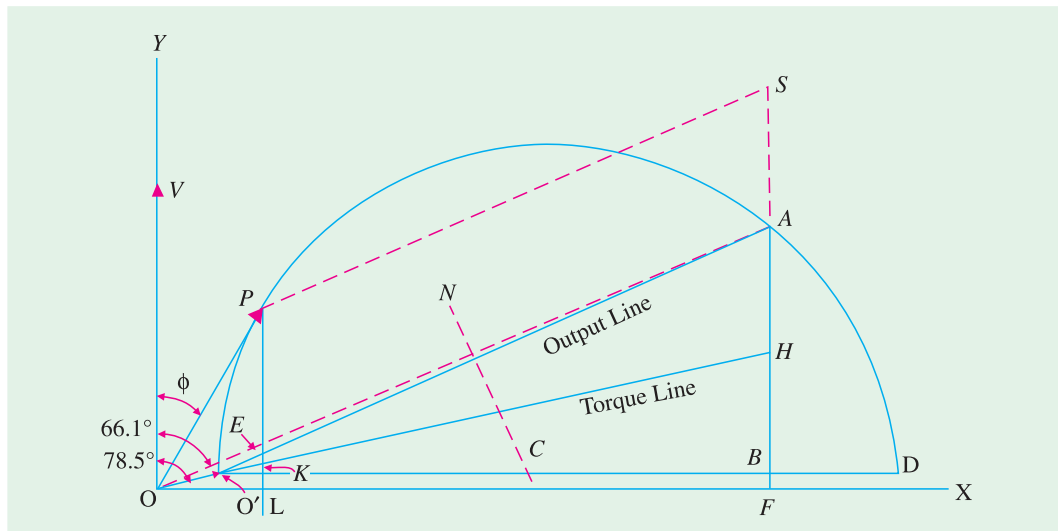


Fig. 35.11

$$\cos \phi_S = \frac{4000}{\sqrt{3} \times 150 \times 38} = 0.405 ; \quad \phi_S = 66.1^\circ$$

Short-circuit current with normal voltage is  $I_{SN} = 38 (400/150) = 101.3$  A. Power taken would be  $= 4000 (400/150)^2 = 28,440$  W. In Fig. 33.11,  $OO'$  represents  $I_0$  of 9 A. If current scale is 1 cm = 5 A,

then vector  $OO' = 9/5 = 1.8 \text{ cm}^*$  and is drawn at an angle of  $\phi_0 = 78.5^\circ$  with the vertical  $OV$  (which represents voltage). Similarly,  $OA$  represents  $I_{SN}$  (S.C. current with normal voltage applied) equal to 101.3 A. It measures  $101.3/5 = 20.26^*$  cm and is drawn at an angle of  $66.1^\circ$ , with the vertical  $OV$ .

Line  $O'D$  is drawn parallel to  $OX$ .  $NC$  is the right-angle bisector of  $O'A$ . The semi-circle  $O'AD$  is drawn with  $C$  as the centre. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit. Now,  $AF$  represents 28,440 W and measures 8.1 cm. Hence, power scale becomes : 1 cm = 28,440/8.1 = 3,510 W. Now, full-load motor output =  $14.9 \times 10^3 = 14,900 \text{ W}$ . According to the above calculated power scale, the intercept between the semi-circle and output line  $O'A$  should measure =  $14,900/3510 = 4.25 \text{ cm}$ . For locating full-load point  $P$ ,  $BA$  is extended.  $AS$  is made equal to 4.25 cm and  $SP$  is drawn parallel to output line  $O'A$ .  $PL$  is perpendicular to  $OX$ .

Line current =  $OP = 6 \text{ cm} = 6 \times 5 = 30 \text{ A}$ ;  $\phi = 30^\circ$  (by measurement)

p.f. =  $\cos 30^\circ = 0.886$  (or  $\cos \phi = PL/OP = 5.2/6 = 0.865$ )

Now, slip =  $\frac{\text{rotor Cu loss}}{\text{rotor input}}$

In Fig. 35.11,  $EK$  represents rotor Cu loss and  $PK$  represents rotor input.

$\therefore$  slip =  $\frac{EK}{PK} = \frac{0.3}{4.5} = 0.067$  or **6.7%**

**Example 35.4.** Draw the circle diagram for a 3.73 kW, 200-V, 50-Hz, 4-pole, 3- $\phi$  star-connected induction motor from the following test data :

No-load : Line voltage 200 V, line current 5 A; total input 350 W

Blocked rotor : Line voltage 100 V, line current 26 A; total input 1700 W

Estimate from the diagram for full-load condition, the line current, power factor and also the maximum torque in terms of the full-load torque. The rotor Cu loss at standstill is half the total Cu loss. **(Electrical Engineering, Bombay Univ. 1987)**

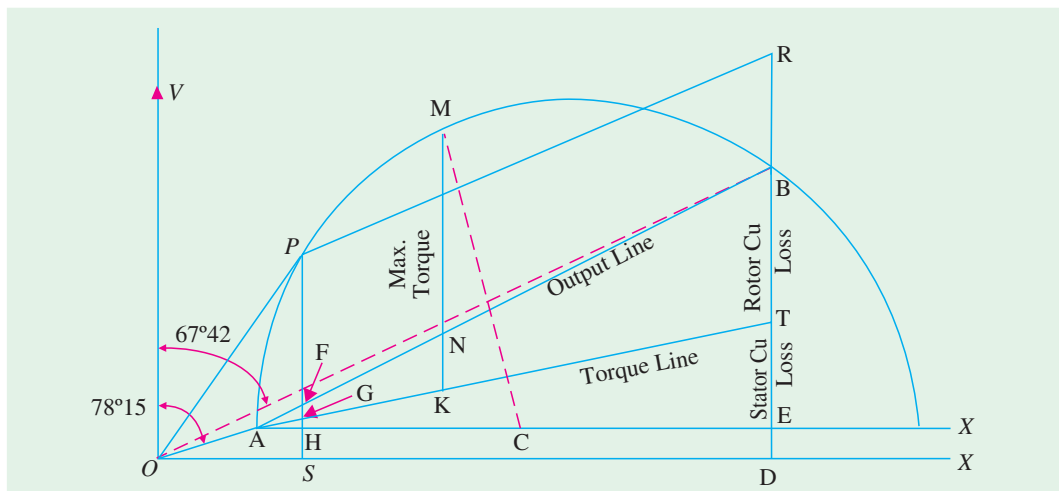


Fig. 35.12

**Solution. No-load test**

$$I_0 = 5 \text{ A, } \cos \phi_0 = \frac{350}{\sqrt{3} \times 200 \times 5} = 0.202 ; \phi_0 = 78^\circ 15'$$

\* The actual lengths are different from these values, due to reduction in block making.

**Blocked-rotor test :**

$$\cos \phi_s = \frac{1700}{\sqrt{3} \times 100 \times 26} = 0.378 ; \phi_s = 67^\circ 42'$$

Short-circuit current with normal voltage,  $I_{SN} = 26 \times 200/100 = 52 \text{ A}$

Short-circuit/blocked rotor input with normal voltage =  $1700(52/26)^2 = 6,800 \text{ W}$

In the circle diagram of Fig. 35.12, voltage is represented along  $OV$  which is drawn perpendicular to  $OX$ . Current scale is  $1 \text{ cm} = 2 \text{ A}$

Line  $OA$  is drawn at an angle of  $\phi_0 = 78^\circ 15'$  with  $OV$  and  $2.5 \text{ cm}$  in length. Line  $A X'$  is drawn parallel to  $OX$ . Line  $OB$  represents short-circuit current with normal voltage *i.e.*  $52 \text{ A}$  and measures  $52/2 = 26 \text{ cm}$ .  $AB$  represents output line. Perpendicular bisector of  $AB$  is drawn to locate the centre  $C$  of the circle. With  $C$  as centre and radius =  $CA$ , a circle is drawn which passes through points  $A$  and  $B$ . From point  $B$ , a perpendicular is drawn to the base.  $BD$  represents total input of  $6,800 \text{ W}$  for blocked rotor test. Out of this,  $ED$  represents no-load loss of  $350 \text{ W}$  and  $BE$  represents  $6,800 - 350 = 6,450 \text{ W}$ . Now  $BD = 9.8 \text{ cm}$  and represents  $6,800 \text{ W}$

$$\therefore \text{power scale} = 6,800/9.8 = 700 \text{ watt/cm} \quad \text{or} \quad 1 \text{ cm} = 700 \text{ W}$$

$BE$  which represents total copper loss in rotor and stator, is bisected at point  $T$  to separate the two losses.  $AT$  represents torque line.

Now, motor output =  $3,730 \text{ watt}$ . It will be represented by a line =  $3,730/700 = 5.33 \text{ cm}$

The output point  $P$  on the circle is located thus :

$DB$  is extended and  $BR$  is cut =  $5.33 \text{ cm}$ . Line  $RP$  is drawn parallel to output line  $AB$  and cuts the circle at point  $P$ . Perpendicular  $PS$  is drawn and  $P$  is joined to origin  $O$ .

Point  $M$  corresponding to maximum torque is obtained thus :

From centre  $C$ , a line  $CM$  is drawn such that it is perpendicular to torque line  $AT$ . It cuts the circle at  $M$  which is the required point. Point  $M$  could also have been located by drawing a line parallel to the torque line.  $MK$  is drawn vertical and it represents maximum torque.

Now, in the circle diagram,  $OP =$  line current on full-load =  $7.6 \text{ cm}$ . Hence,  $OP$  represents  $7.6 \times 2 = 15.2 \text{ A}$

$$\text{Power factor on full-load} = \frac{SP}{OP} = \frac{6.45}{7.6} = 0.86$$

$$\frac{\text{Max. torque}}{\text{F.L. torque}} = \frac{MK}{PG} = \frac{10}{5.6} = 1.8$$

$$\therefore \text{Max. torque} = 180\% \text{ of full-load torque.}$$

**Example. 35.5.** Draw the circle diagram from no-load and short-circuit test of a 3-phase.  $14.92 \text{ kW}$ ,  $400\text{-V}$ ,  $6\text{-pole}$  induction motor from the following test results (line values).

No-load :  $400\text{-V}$ ,  $11 \text{ A}$ ,  $p.f. = 0.2$

Short-circuit :  $100\text{-V}$ ,  $25 \text{ A}$ ,  $p.f. = 0.4$

Rotor  $\text{Cu}$  loss at standstill is half the total  $\text{Cu}$  loss.

From the diagram, find (a) line current, slip, efficiency and *p.f.* at full-load (b) the maximum torque. **(Electrical Machines-I, Gujarat Univ. 1985)**

**Solution.** No-load *p.f.* =  $0.2$ ;  $\phi_0 = \cos^{-1}(0.2) = 78.5^\circ$

Short-circuit *p.f.* =  $0.4$ ;  $\phi_s = \cos^{-1}(0.4) = 66.4^\circ$

S.C. current  $I_{SN}$  if normal voltage were applied =  $25 (400/100) = 100 \text{ A}$

S.C. power input with this current =  $\sqrt{3} \times 400 \times 100 \times 0.4 = 27,710 \text{ W}$

Assume a current scale of 1 cm = 5 A.\* The circle diagram of Fig. 35.13 is constructed as follows :

- (i) No-load current vector  $OO'$  represents 11 A. Hence, it measures  $11/5 = 2.2$  cm and is drawn at an angle of  $78.5^\circ$  with  $OY$ .
- (ii) Vector  $OA$  represents 100 A and measures  $100/5 = 20$  cm. It is drawn at an angle of  $66.4^\circ$  with  $OY$ .
- (iii)  $O'D$  is drawn parallel to  $OX$ .  $NC$  is the right angle bisector of  $O'A$ .
- (iv) With  $C$  as the centre and  $CO'$  as radius, a semicircle is drawn as shown.
- (v)  $AF$  represents power input on short-circuit with normal voltage applied. It measures 8 cm and (as calculated above) represents 27,710 W. Hence, power scale becomes

$$1 \text{ cm} = 27,710/8 = 3,465 \text{ W}$$

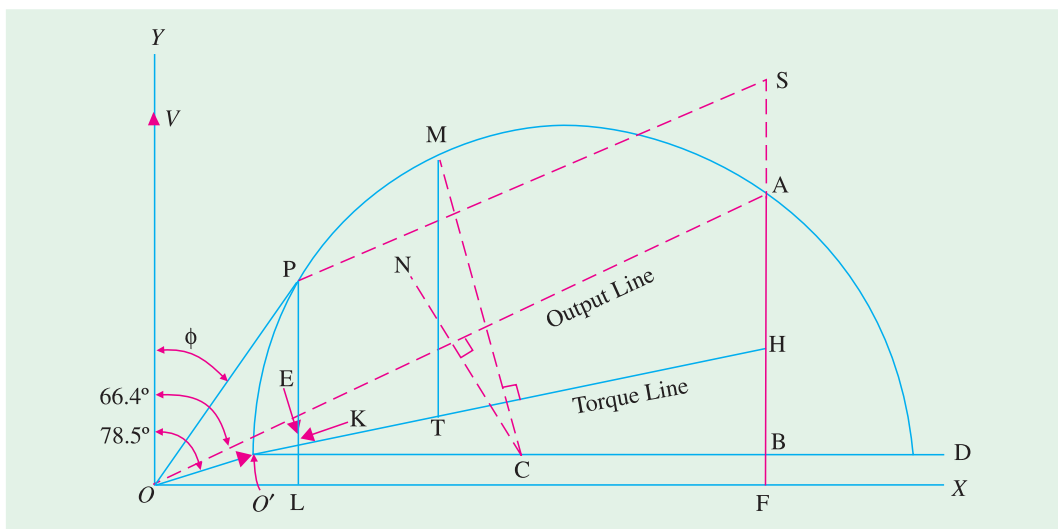


Fig. 35.13

(a) F.L motor output = 14,920 W. According to the above power scale, the intercept between the semicircle and the output line  $O'A$  should measure  $= 14,920/3,465 = 4.31$  cm. Hence, vertical line  $PL$  is found which measures 4.31 cm. Point  $P$  represents the full-load operating point.\*\*

(a) Line current =  $OP = 6.5$  cm which means that full-load line current  $= 6.5 \times 5 = 32.5 \text{ A}$ .  $\phi = 32.9^\circ$  (by measurement)

$\therefore \cos 32.9^\circ = 0.84$  (or  $\cos \phi = PL/OP = 5.4/6.5 = 0.84$ )

$$\text{slip} = \frac{EK}{PK} = \frac{0.3}{5.35} = 0.056 \text{ or } 5.6\% ; \eta = \frac{PE}{PL} = \frac{4.3}{5.4} = 0.8 \text{ or } 80\%$$

(b) For finding maximum torque, line  $CM$  is drawn  $\perp$  to torque line  $O'H$ .  $MT$  is the vertical intercept between the semicircle and the torque line and represents the maximum torque of the motor in synchronous watts

Now,  $MT = 7.8$  cm (by measurement)  $\therefore T_{max} = 7.8 \times 3465 = 27,030$  synch. watt

**Example 35.6.** A 415-V, 29.84 kW, 50-Hz, delta-connected motor gave the following test data :

No-load test	: 415 V,	21 A,	1,250 W
Locked rotor test	: 100 V,	45 A,	2,730 W

Construct the circle diagram and determine

\* The actual scale of the book diagram is different because it has been reduced during block making.

\*\* The operating point may also be found by making  $AS = 4.31$  cm and drawing  $SP$  parallel to  $O'A$ .

(a) the line current and power factor for rated output (b) the maximum torque.  
 Assume stator and rotor Cu losses equal at standstill. (A.C. Machines-I, Jadavpur Univ. 1990)

**Solution.** Power factor on no-load is  $= \frac{1250}{\sqrt{3} \times 415 \times 21} = 0.0918$

$\therefore \phi_0 = \cos^{-1}(0.0918) = 84^\circ 44'$

Power factor with locked rotor is  $= \frac{2,730}{\sqrt{3} \times 100 \times 45} = 0.3503$

$\therefore \phi_s = \cos^{-1}(0.3503) = 69^\circ 30'$

The input current  $I_{SN}$  on short-circuit if normal voltage were applied  $= 45 (415/100) = 186.75$  A and power taken would be  $= 2,730 (415/100)^2 = 47,000$  W.

Let the current scale be 1 cm = 10 A. The circle diagram of Fig. 35.14 is constructed as follows :

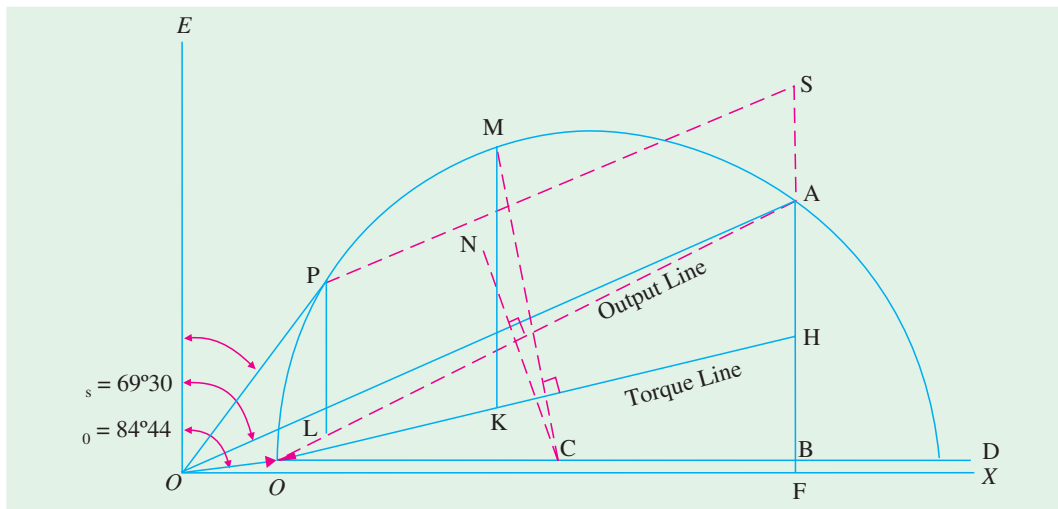


Fig. 35.14

- (i) Vector  $OO'$  represents 21 A so that it measures 2.1 cm and is laid at an angle of  $84^\circ 44'$  with  $OE$  (which is vertical i.e. along  $Y$ -axis).
- (ii) Vector  $OA$  measures  $186.75/10 = 18.675$  cm and is drawn at an angle of  $69^\circ 30'$  with  $OE$ .
- (iii)  $O'D$  is drawn parallel to  $OX$ .  $NC$  is the right-angle bisector of  $O'A$
- (iv) With  $C$  as the centre and  $CO'$  as radius, a semi-circle is drawn as shown. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit.
- (v) The vertical  $AF$  represents power input on short-circuit with normal voltage applied.  $AF$  measures 6.6 cm and (as calculated above) represents 47,000 W. Hence, power scale becomes, 1 cm =  $47,000/6.6 = 7,120$  W
- (a) Full-load output = 29,840 W. According to the above power scale, the intercept between the semicircle and output line  $O'A$  should measure  $29,840/7,120 = 4.19$  cm. Hence, line  $PL$  is found which measures 4.19 cm. Point  $P$  represents the full-load operating point.\*

Phase current  $= OP = 6$  cm =  $6 \times 10 = 60$  A; Line current  $= \sqrt{3} \times 60 = 104$  A

Power factor  $= \cos \angle POE = \cos 35^\circ = 0.819$

- (b) For finding the maximum torque, line  $CM$  is drawn  $\perp$  to the torque line  $O'H$ . Point  $H$  is such that

$$\frac{AH}{BH} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

\* The operating point may also be found by making  $AS = 4.19$  cm and drawing  $SP$  parallel to  $O'A$ .



Since the two Cu losses are equal, point  $H$  is the mid-point of  $AB$ .

Line  $MK$  represents the maximum torque of the motor in synchronous watts

$$MK = 7.3 \text{ cm (by measurement)} = 7.3 \times 7,120 = \mathbf{51,980 \text{ synch. watt.}}$$

**Example 35.7.** Draw the circle diagram for a 5.6 kW, 400-V, 3- $\phi$ , 4-pole, 50-Hz, slip-ring induction motor from the following data :

No-load readings : 400 V, 6 A,  $\cos \phi_0 = 0.087$  : Short-circuit test : 100 V, 12 A, 720 W.

The ratio of primary to secondary turns = 2.62, stator resistance per phase is  $0.67 \Omega$  and of the rotor is  $0.185 \Omega$ . Calculate

- (i) full-load current
- (ii) full-load slip
- (iii) full-load power factor
- (iv)  $\frac{\text{maximum torque}}{\text{full - load torque}}$
- (v) maximum power.

**Solution. No-load condition**

$$\phi_0 = \cos^{-1}(0.087) = 85^\circ$$

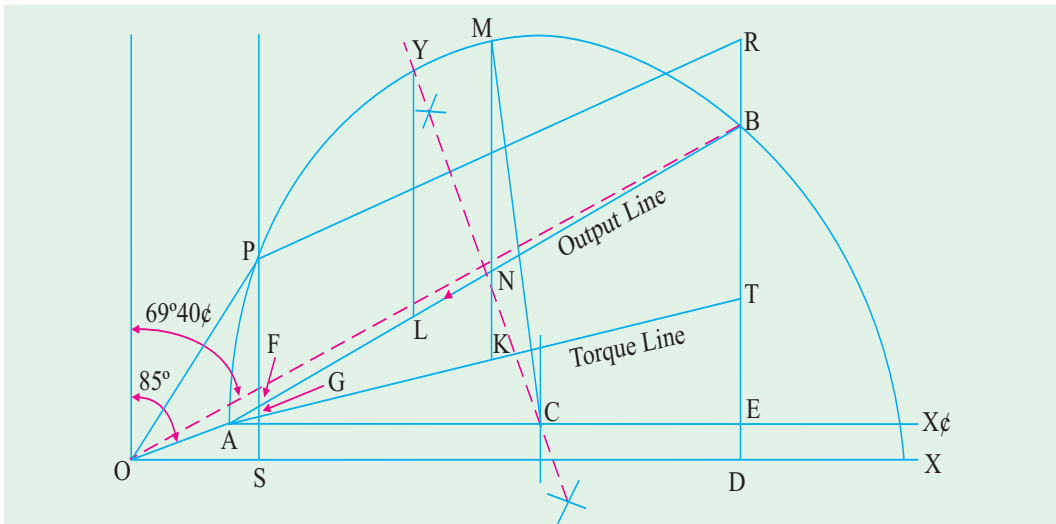


Fig. 35.15

**Short-circuit condition**

Short-circuit current with normal voltage =  $12 \times 400/100 = 48 \text{ A}$

$$\text{Total input} = 720 \times (48/12)^2 = 11.52 \text{ kW}$$

$$\cos \phi_s = \frac{720}{\sqrt{3} \times 100 \times 12} = 0.347 \quad \text{or} \quad \phi_s = 69^\circ 40'$$

Current scale is,  $1 \text{ cm} = 2 \text{ A}$

In the circle diagram of Fig. 35.15,  $OA = 3 \text{ cm}$  and inclined at  $85^\circ$  with  $OV$ . Line  $OB$  represents short-circuit current with normal voltage. It measures  $48/2 = 24 \text{ cm}$  and represent 48 A.  $BD$  is perpendicular to  $OX$ .

**For Drawing Torque Line**

$$K = 2.62 \qquad R_1 = 0.67 \Omega \qquad R_2 = 0.185 \Omega$$

(in practice, an allowance of 10% is made for skin effect)

$$\therefore \frac{\text{rotor Cu loss}}{\text{stator Cu loss}} = 2.62^2 \times \frac{0.185}{0.67} = 1.9 \qquad \therefore \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{1.9}{2.9} = 0.655$$

Now  $BD = 8.25 \text{ cm}$  and represents 11.52 kW  
 power scale =  $11.52/8.25 = 1.4 \text{ kW/cm}$

∴  $1 \text{ cm} = 1.4 \text{ kW}$   
 BE represents total Cu loss and is divided at point T in the ratio 1.9 : 1.  
 $BT = BE \times 1.9/2.9 = 0.655 \times 8 = 5.24 \text{ cm}$

AT is the torque line  
 Full-load output = 5.6 kW  
 It is represented by a line =  $5.6/1.4 = 4 \text{ cm}$

DB is produced to R such that BR = 4 cm. Line RP is parallel to output line and cuts the circle at P. OP represents full-load current.

PS is drawn vertically. Points M and Y represent points of maximum torque and maximum output respectively.

(i) F.L. current =  $OP = 5.75 \text{ cm} = 5.75 \times 2 = \mathbf{11.5 \text{ A}}$

(ii) F.L. slip =  $\frac{FG}{PG} = \frac{0.2}{4.25} = 0.047$  or **4.7%**

(iii) p.f. =  $\frac{SP}{OP} = \frac{4.6}{5.75} = \mathbf{0.8}$

(iv)  $\frac{\text{max. torque}}{\text{full-load torque}} = \frac{MK}{PG} = \frac{10.05}{4.25} = \mathbf{2.37}$

(v) Maximum output is represented by  $YL = 7.75 \text{ cm}$ .

∴ Max. output =  $7.75 \times 1.4 = \mathbf{10.8 \text{ kW}}$

**Example 35.8.** A 440-V, 3- $\phi$ , 4-pole, 50-Hz slip-ring motor gave the following test results :

No-load reading : 440 V, 9 A, p.f. = 0.2

Blocked rotor test : 110 V, 22 A, p.f. = 0.3

The ratio of stator to rotor turns per phase is 3.5/1. The stator and rotor Cu losses are divided equally in the blocked rotor test. The full-load current is 20 A. Draw the circle diagram and obtain the following :

- (a) power factor, output power, efficiency and slip at full-load
- (b) standstill torque or starting torque.
- (c) resistance to be inserted in the rotor circuit for giving a starting torque 200 % of the full-load torque. Also, find the current and power factor under these conditions.

**Solution.** No-load p.f. = 0.2 ∴  $\phi_0 = \cos^{-1}(0.2) = 78.5^\circ$

Short-circuit p.f. = 0.3 ∴  $\phi_s = 72.5^\circ$

Short-circuit current at normal voltage =  $22 \times 440/110 = 88 \text{ A}$

S.C. input =  $\sqrt{3} \times 440 \times 88 \times 0.3 = 20,120 \text{ W} = 20.12 \text{ kW}$

Take a current scale of  $1 \text{ cm} = 4 \text{ A}$

In the circle diagram of Fig. 35.16, OA = 2.25 cm drawn at an angle of  $78.5^\circ$  behind OV. Similarly, OB =  $88/4 = 22 \text{ cm}$  and is drawn at an angle of  $72.5^\circ$  behind OV. The semi-circle is drawn as usual. Point T is such that BT = TD. Hence, torque line AT can be drawn. BC represents 20.12 kW. By measurement BC = 6.6 cm.

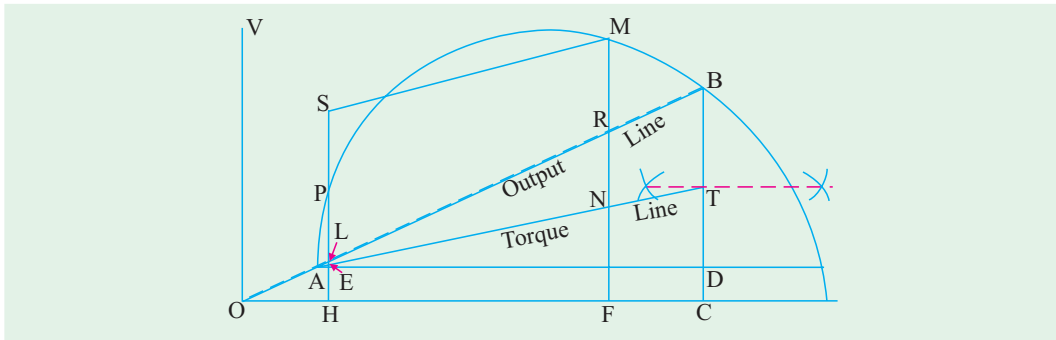


Fig. 35.16

∴ power scale = 20.12/6.6 = 3.05  
 ∴ 1 cm = 3.05 kW

Full-load current = 20 A. Hence, it is represented by a length of 20/4 = 5 cm. With *O* as centre and 5 cm as radius, an arc is drawn which cuts the semi-circle at point *P*. This point represents full-load condition. *PH* is drawn perpendicular to the base *OC*.

(a) (i) p.f. = cos φ = *PH/OP* = 4.05/5 = **0.81**

(ii) Torque can be found by measuring the input.

Rotor input = *PE* = 3.5 cm = 3.5 × 3.05 = 10.67 kW

Now  $N_s = 120 \times 50/4 = 1500$  r.p.m.

∴  $T_g = 9.55 P_2/N_s = 9.55 \times 10,670/1500 = \mathbf{61 \text{ N-m}}$

(iii) output = *PL* = 3.35 × 3.05 = **10.21 kW**

(iv) efficiency =  $\frac{\text{output}}{\text{input}} = \frac{PL}{PH} = \frac{3.35}{4.05} = 0.83$  or **83%**

(v) slip *s* =  $\frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{LE}{PE} = \frac{0.1025}{3.5} = 0.03$  or **3%**

(b) Standstill torque is represented by *BT*.

$BT = 3.1 \text{ cm} = 3.1 \times 3.05 = 9.45 \text{ kW}$  ∴  $T_{st} = 9.55 \times \frac{9.45 \times 10^3}{1500} = \mathbf{60.25 \text{ N-m}}$

(c) We will now locate point *M* on the semi-circle which corresponds to a starting torque twice the full-load torque *i.e.* 200% of F.L. torque.

Full-load torque = *PE*. Produce *EP* to point *S* such that *PS* = *PE*. From point *S* draw a line parallel to torque line *AT* cutting the semi-circle at *M*. Draw *MN* perpendicular to the base.

At starting when rotor is stationary, *MN* represents total rotor copper losses.

*NR* = Cu loss in rotor itself as before ; *RM* = Cu loss in external resistance

*RM* = 4.5 cm = 4.5 × 3.05 = 13.716 kW = 13,716 watt.

Cu loss/phase = 13,716/3 = 4,572 watt

Rotor current *AM* = 17.5 cm = 17.5 × 4 = 70 A

Let  $r_2'$  be the additional external resistance in the rotor circuit (as referred to stator) then

$r_2' \times 70^2 = 4,572$  or  $r_2' = 4,572/4,900 = 0.93 \Omega$

Now  $K = 1/3.5$

∴ rotor resistance/phase,  $r_2 = r_2' \times K^2 = 0.93/3.5^2 = \mathbf{0.076 \Omega}$

Stator current = *OM* = 19.6 × 4 = **78.4 A** ; power factor =  $\frac{MF}{OM} = \frac{9.75}{18.7} = \mathbf{0.498}$

**Example 35.9.** Draw the circle diagram of a 7.46 kW, 200-V, 50-Hz, 3-phase slip-ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of 0.38 ohm/phase and a rotor resistance of 0.24 ohm/phase. The following are the test readings ;

No-load : 200 V, 7.7 A,  $\cos \phi_0 = 0.195$

Short-circuit : 100 V, 47.6 A,  $\cos \phi_s = 0.454$

Find (a) starting torque and (b) maximum torque, both in synchronous watts

(c) the maximum power factor (d) the slip for maximum torque

(e) the maximum output (Elect. Tech.-II, Madras Univ. 1989)

**Solution.**  $\phi_0 = \cos^{-1}(0.195) = 78^\circ 45'$ ;  $\phi_s = \cos^{-1}(0.454) = 63^\circ$

The short-circuit  $I_{SN}$  with normal voltage applied is = 47.6 × (200/100) = 95.2 A

The circle diagram is drawn as usual and is shown in fig. 35.17.

With a current scale of 1 cm = 5 A, vector *OO'* measures 7.7/5 = 1.54 cm and represents the no-load current of 7.7 A.

Similarly, vector  $OA$  represents  $I_{SN}$  i.e. short-circuit current with normal voltage and measures  $95.2/5 = 19.04$  cm

Both vectors are drawn at their respective angles with  $OE$ .

The vertical line  $AF$  measures the power input on short-circuit with normal voltage and is  $= \sqrt{3} \times 200 \times 95.2 \times 0.454 = 14,970$  W.

Since  $AF$  measures 8.6 cm, the power scale is  $1 \text{ cm} = 14,970/8.6 = 1740$  W

The point  $H$  is such that

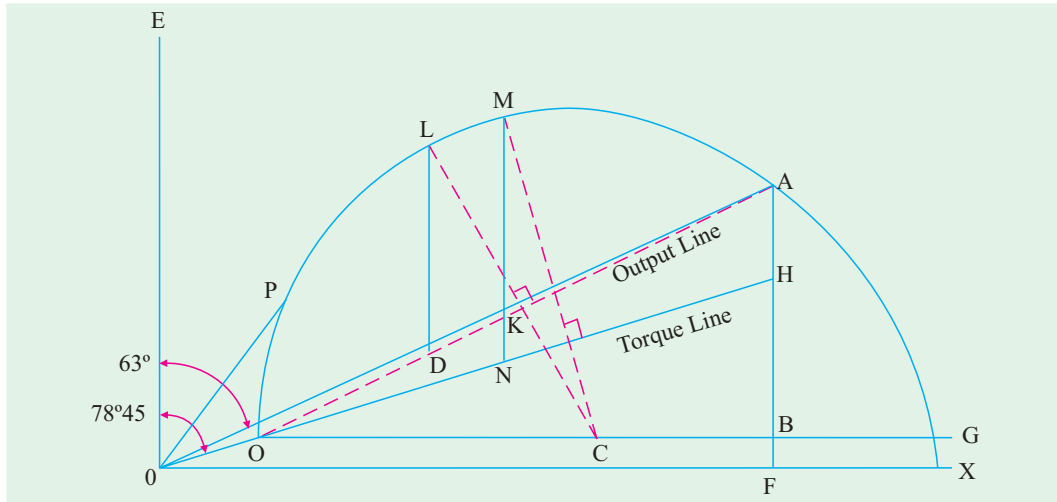


Fig. 35.17

$$\frac{AH}{AB} = \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{\text{rotor resistance}^*}{\text{rotor + stator resistance}} = \frac{0.24}{0.62}$$

Now  $AB = 8.2$  cm (by measurement)  $\therefore AH = 8.2 \times 0.24/0.62 = 3.2$  cm

(a) Starting torque =  $AH = 3.2$  cm =  $3.2 \times 1740 = 5,570$  synchron. watt.

(b) Line  $CM$  is drawn perpendicular to the torque line  $O'H$ . The intercept  $MN$  represents the maximum torque in synchronous watts.

Maximum torque =  $MN = 7.15$  cm =  $7.15 \times 1740 = 12,440$  synchron. watts.

(c) For finding the maximum power, line  $OP$  is drawn tangential to the semi-circle.

$$\angle POE = 28.5^\circ$$

$$\therefore \text{maximum p.f.} = \cos 28.5^\circ = 0.879$$

(d) The slip for maximum torque is =  $KN/MN = 1.4/7.15 = 0.195$

(e) Line  $CL$  is drawn perpendicular to the output line  $O'A$ . From  $L$  is drawn the vertical line  $LD$ . It measures 5.9 cm and represents the maximum output.

$$\therefore \text{maximum output} = 5.9 \times 1740 = 10,270 \text{ W}$$

### Tutorial Problems 35.1

1. A 300 h.p. (223.8 kW), 3000-V, 3- $\phi$ , induction motor has a magnetising current of 20 A at 0.10 p.f. and a short-circuit (or locked) current of 240 A at 0.25 p.f. Draw the circuit diagram, determine the p.f. at full-load and the maximum horse-power. [**0.85 p.f. 621 h.p. (463.27 kW)**] (I.E.E. London)
2. The following are test results for a 18.65 kW, 3- $\phi$ , 440.V slip-ring induction motor :  
Light load : 440-V, 7.5 A, 1350 W (including 650 W friction loss).

\* Because  $K = 1$ , otherwise it should be  $R_2' = R_2/K^2$ .

S.C. test : 100 V, 32 A, 1800 W

Draw the locus diagram of the stator current and hence obtain the current, p.f. and slip on full-load. On short-circuit, the rotor and stator copper losses are equal. **[30 A, 0.915, 0.035] (London Univ)**

3. Draw the circle diagram for 20 h.p. (14.92 kW), 440-V, 50-Hz, 3- $\phi$  induction motor from the following test figures (line values) :

No-load : 440 V, 10A, p.f. 0.2 Short-circuit : 200 V, 50 A, p.f. 0.4

From the diagram, estimate (a) the line current and p.f. at full-load (b) the maximum power developed (c) the starting torque. Assume the rotor and stator  $I^2R$  losses on short-circuit to be equal.

**[(a) 28.1 A at 0.844 p.f. (b) 27.75 kW (c) 11.6 synchronous kW/phase] (London Univ.)**

4. A 40 h.p. (29.84 kW), 440-V, 50-Hz, 3-phase induction motor gave the following test results

No. load : 440 V, 16 A, p.f. = 0.15 S.C. test : 100 V, 55 A, p.f. = 0.225

Ratio of rotor to stator losses on short-circuit = 0.9. Find the full-load current and p.f., the pull-out torque and the maximum output power developed.

**[49 A at 0.88 p.f. ; 78.5 synch. kW or 2.575 times F.L. torque ; 701.2 kW] (I.E.E. London)**

5. A 40 h.p. (29.84 kW), 50-Hz, 6-pole, 420-V, 3- $\phi$ , slip-ring induction motor furnished the following test figures :

No-load : 420 V, 18 A, p.f. = 0.15 S.C. test : 210 V, 140 A, p.f. = 0.25

The ratio of stator to rotor Cu losses on short-circuit was 7 : 6. Draw the circle diagram and find from it (a) the full-load current and power factor (b) the maximum torque and power developed.

**[(a) 70 A at 0.885 p.f. (b) 89.7 kg.m ; 76.09 kW] (I.E.E. London)**

6. A 500 h.p. (373 kW), 8-pole, 3- $\phi$ , 6,000-V, 50-Hz induction motor gives on test the following figures :

Running light at 6000 V, 14 A/phase, 20,000 W ; Short-circuit at 2000 V, 70 A/phase, 30,500 W

The resistance/phase of the star-connected stator winding is  $1.1 \Omega$ , ratio of transformation is 4 : 1. Draw the circle diagram of this motor and calculate how much resistance must be connected in each phase of the rotor to make it yield full-load torque at starting. **[0.138  $\Omega$ ] (London Univ.)**

7. A 3-phase induction motor has full-load output of 18.65 kW at 220 V, 720 r.p.m. The full-load p.f. is 0.83 and efficiency is 85%. When running light, the motor takes 5 A at 0.2 p.f. Draw the circle diagram and use it to determine the maximum torque which the motor can exert (a) in N-m (b) in terms of full-load torque and (c) in terms of the starting torque.

**[(a) 268.7 N-m (b) 1.08 (c) 7.2 approx.] (London Univ.)**

8. A 415-V, 40 h.p. (29.84 kW), 50.Hz,  $\Delta$ -connected motor gave the following test data :

No-load test : 415 V, 21 A, 1250 W ; Locked rotor test : 100 V, 45 A, 2,730 W

Construct the circle diagram and determine

(a) the line current and power factor for rated output (b) the maximum torque. Assume stator and rotor Cu losses equal at standstill.

**[(a) 104 A : 0.819 (b) 51,980 synch watt] (A.C. Machines-I, Jadavpur Univ. 1978)**

9. Draw the no-load and short circuit diagram for a 14.92 kW., 400-V, 50-Hz, 3-phase star-connected induction motor from the following data (line values) :

No load test : 400 V, 9 A,  $\cos \phi = 0.2$

Short circuit test : 200 V, 50 A,  $\cos \phi = 0.4$

From the diagram find (a) the line current and power factor at full load, and (b) the maximum output power. **[(a) 32.0 A, 0.85 (b) 21.634 kW]**

### 35.9. Starting of Induction Motors

It has been shown earlier that a plain induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary. Therefore, if normal supply voltage is applied to the stationary motor, then, as in the case of a transformer, a very large initial current is taken by the primary, at least, for a short while. It would be remembered that exactly similar conditions exist in the case of a d.c. motor, if it is thrown directly across the supply lines, because at the time of starting it, there is no back e.m.f. to oppose the initial inrush of current.

Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque. This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines. Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

It was seen in Art. 34.15 that the starting torque of an induction motor can be improved by increasing the resistance of the rotor circuit. This is easily feasible in the case of slip-ring motors but not in the case of squirrel-cage motors. However, in their case, the initial in-rush of current is controlled by applying a reduced voltage to the stator during the starting period, full normal voltage being applied when the motor has run up to speed.

### 35.10. Direct-switching or Line starting of Induction Motors

It has been shown earlier that

$$\text{Rotor input} = 2 \pi N_s T = k T \quad \text{---Art. 34.36}$$

Also, rotor Cu loss =  $s \times$  rotor input

$$\therefore 3 I_2^2 R_2 = s \times k T \quad \therefore T \propto I_2^2 / s \quad (\text{if } R_2 \text{ is the same})$$

$$\text{Now } I_2 \propto I_1 \quad \therefore T \propto I_1^2 / s \quad \text{or } T = K I_1^2 / s$$

$$\text{At starting moment } s = 1 \quad \therefore T_{st} = K I_{st}^2 \quad \text{where } I_{st} = \text{starting current}$$

$$\text{If } I_f = \text{normal full-load current and } s_f = \text{full-load slip}$$

$$\text{then } T_f = K I_f^2 / s_f \quad \therefore \frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 \cdot s_f$$

When motor is direct-switched onto normal voltage, then starting current is the short-circuit current  $I_{sc}$ .

$$\therefore \frac{T_{st}}{T_f} = \left( \frac{I_{sc}}{I_f} \right)^2 \cdot s_f = a^2 \cdot s_f \quad \text{where } a = I_{sc} / I_f$$

$$\text{Suppose in a case, } I_{sc} = 7 I_f, s_f = 4\% = 0.04, \text{ the } T_{st} / T_f = 7^2 \times 0.04 = 1.96$$

$$\therefore \text{starting torque} = 1.96 \times \text{full-load torque}$$

Hence, we find that with a current as great as seven times the full-load current, the motor develops a starting torque which is only 1.96 times the full-load value.

Some of the methods for starting induction motors are discussed below :

#### Squirrel-cage Motors

- (a) Primary resistors (or rheostat) or reactors
- (b) Auto-transformer (or autostarter)
- (c) Star-delta switches

In all these methods, terminal voltage of the squirrel-cage motor is reduced during starting.

#### Slip-ring Motors

- (a) Rotor rheostat

### 35.11. Squirrel-cage Motors

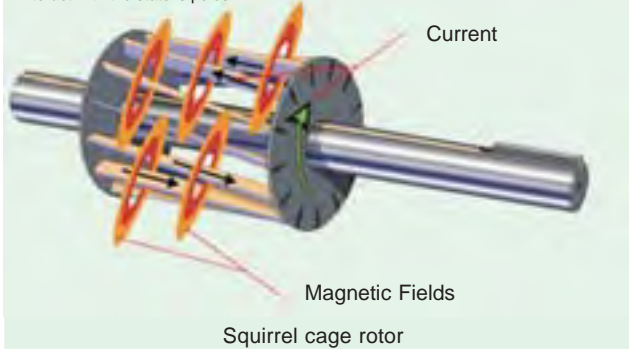
#### (a) Primary resistors

Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way, the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as the voltage, the torque varies as square of applied voltage\*

\* When applied voltage is reduced, the rotating flux  $\Phi$  is reduced which, in turn, decreases rotor e.m.f. and hence rotor current  $I_2$ . Starting torque, which depends both on  $\Phi$  and  $I_2$  suffers on two counts when impressed voltage is reduced.

**Squirrel Cage Rotor**

When the stator's moving magnetic field cuts across the rotor's conductor bars, it induces voltage in them. This voltage produces current, which circulates through the bars and around the rotor end ring. This current in turn produces magnetic fields around each rotor bar. The continuously changing stator magnetic field results in a continuously changing rotor field. The rotor becomes an electromagnet with continuously alternating poles, which interact with the stator's poles.



(Art 34.17). If the voltage applied across the motor terminals is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

By using primary resistors (Fig. 35.18), the applied voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x I_{sc} \quad \text{and} \quad T_{st} = x^2 T_{sc}$$

As seen from Art 35.10, above,

$$\begin{aligned} \frac{T_{st}}{T_f} &= \left( \frac{I_{st}}{I_f} \right)^2 \cdot s_f = \left( \frac{x I_{sc}}{I_f} \right)^2 s_f \\ &= x^2 \left( \frac{I_{sc}}{I_f} \right)^2 s_f = x^2 \cdot a^2 \cdot s_f \end{aligned}$$

It is obvious that the ratio of the starting torque to full-load torque is  $x^2$  of that obtained with direct switching or across-the-line starting. This method is useful for the smooth starting of small machines only.

**(b) Auto-transformers**

Such starters, known variously as *auto-starters* or *compensators*, consist of an auto-transformer, with necessary switches. We may use either two auto-transformers connected as usual [Fig. 35.19 (b)] or 3 auto-transformers connected in open delta [Fig. 35.19 (a)]. **This method can be used both for star-and delta-connected motors.** As shown in Fig. 35.20

with starting connections, a reduced voltage is applied across the motor terminals. When the motor has run up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor. The switch making these changes from 'start' to 'run' may be airbreak (for small motors) or may be oil-immersed (for large motors) to reduce sparking. There is also provision for no-voltage and over-load protection, along with a time-delay device, so that momentary interruption of voltage or momentary over-load do not disconnect the motor from supply line. Most of the auto-starters are provided with 3 sets of taps, so as to reduce voltage to 80, 65 or 50 per cent of the line voltage, to suit the local conditions of supply. The

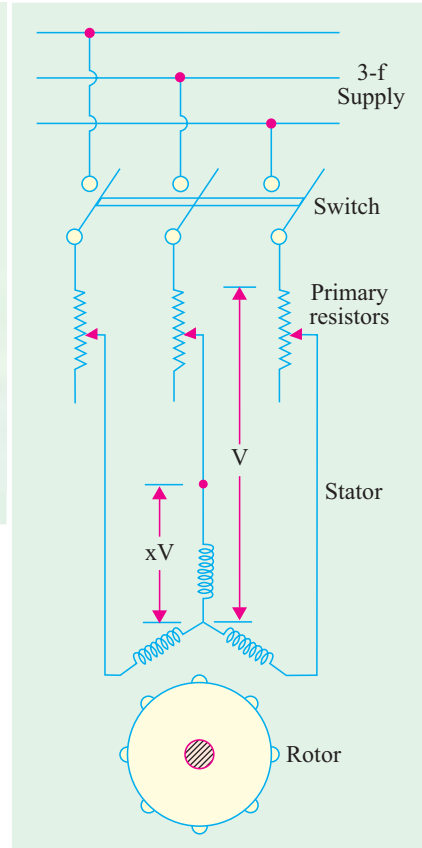


Fig. 35.18



V-connected auto-transformer is commonly used, because it is cheaper, although the currents are unbalanced during starting period. This is, however, not much objectionable firstly, because the current imbalance is about 15 per cent and secondly, because balance is restored as soon as running conditions are attained.

The quantitative relationships between the motor current, line current, and torque developed can be understood from Fig.35.20.

In Fig 35.20 (a) is shown the case when the motor is direct-switched to lines. The motor current is, say, 5 times the full-load current. If  $V$  is the line voltage, then voltage/phase across motor is  $V/\sqrt{3}$ .

$$\therefore I_{sc} = 5 I_f = \frac{V}{\sqrt{3}Z} \text{ where } Z \text{ is stator impedance/phase.}$$

In the case of auto-transformer, if a tapping of transformation ratio  $K$  is used, then phase voltage across motor is  $KV/\sqrt{3}$ , as marked in Fig. 35.20 (b).

$$\therefore \text{ motor current at starting } I_2 = \frac{KV}{\sqrt{3}Z} = K \cdot \frac{V}{\sqrt{3}Z} = K \cdot I_{sc} = K \cdot 5 I_f$$

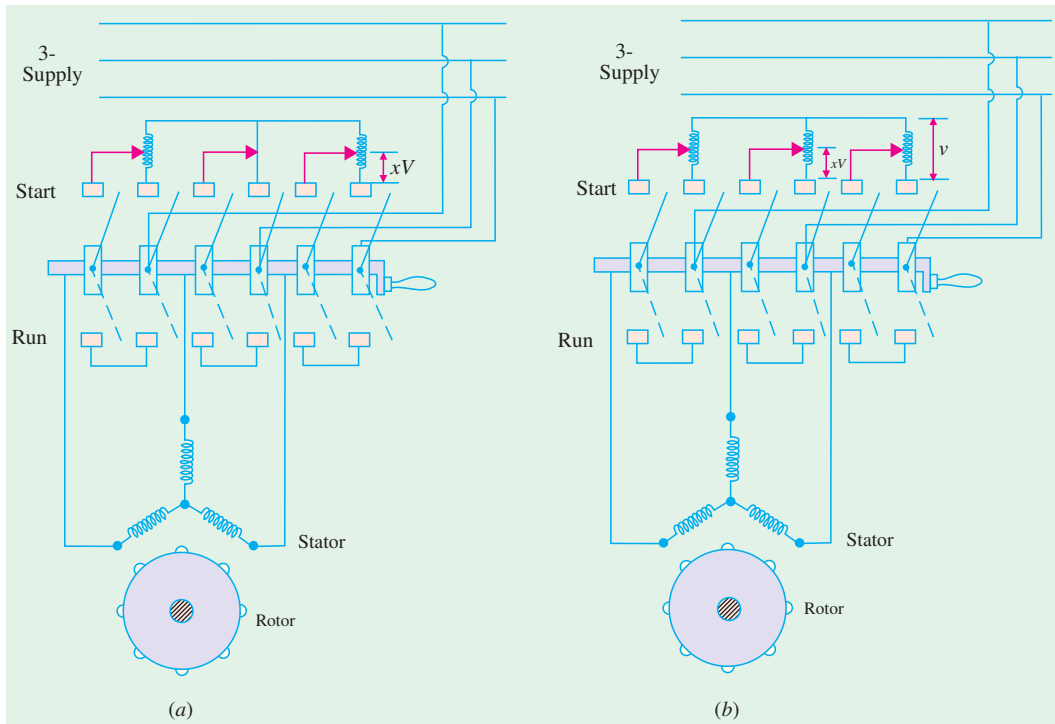


Fig. 35.19

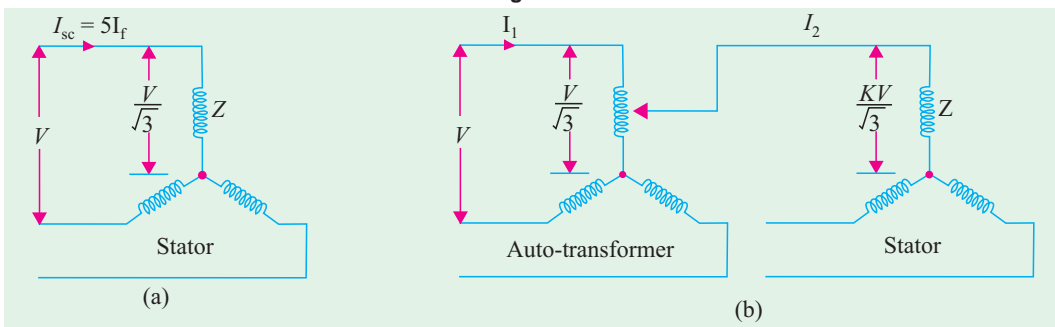


Fig. 35.20



The current taken from supply or by auto-transformer is  $I_1 = K I_2 = K^2 \times 5 I_f = K^2 I_{sc}$  if magnetising current of the transformer is ignored. Hence, we find that although motor current per phase is reduced only  $K$  times the direct-switching current ( $\because K < 1$ ), the current taken by the line is reduced  $K^2$  times.

Now, remembering that torque is proportional to the square of the voltage, we get

$$\begin{aligned} \text{With direct-switching, } T_1 &\propto (V/\sqrt{3})^2; & \text{With auto-transformer, } T_2 &\propto (KV/\sqrt{3})^2 \\ \therefore T_2/T_1 &= (KV/\sqrt{3})^2/(V/\sqrt{3})^2 & \text{or } T_2 &= K^2 T_1 & \text{or } T_{st} &= K^2 \cdot T_{sc} \\ \therefore \text{torque with auto-starter} & & & & & = K^2 \times \text{torque with direct-switching.} \end{aligned}$$

**Relation Between Starting and F.L. Torque**

It is seen that voltage across motor phase on direct-switching is  $V/\sqrt{3}$  and starting current is  $I_{st} = I_{sc}$ . With auto-starter, voltage across motor phase is  $KV/\sqrt{3}$  and  $I_{st} = K I_{sc}$ .

$$\begin{aligned} \text{Now, } T_{st} &\propto I_{st}^2 (s=1) & \text{and } T_f &\propto \frac{I_f^2}{s_f} \\ \therefore \frac{T_{st}}{T_f} &= \left(\frac{I_{st}}{I_f}\right)^2 s_f & \text{or } \frac{T_{st}}{T_f} &= K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = K^2 \cdot a^2 \cdot s_f & (\because I_{st} = K I_{sc}) \end{aligned}$$

Note that this expression is similar to the one derived in Art. 34.11. (a) except that  $x$  has been replaced by transformation ratio  $K$ .

**Example 35.10.** Find the percentage tapping required on an auto-transformer required for a squirrel-cage motor to start the motor against 1/4 of full-load torque. The short-circuit current on normal voltage is 4 times the full-load current and the full-load slip is 3%.

**Solution.**

$$\frac{T_{st}}{T_f} = \frac{1}{4}, \quad \frac{I_{sc}}{I_f} = 4, \quad s_f = 0.03$$

$$\therefore \text{Using } \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f, \text{ we get } \frac{1}{4} = K^2 \times 4^2 \times 0.03$$

$$\therefore K^2 = \frac{1}{64 \times 0.03} \quad \therefore K = 0.722 \text{ or } K = 72.2\%$$

**Example 35.11.** A 20 h.p. (14.92 kW), 400-V, 950 r.p.m., 3- $\phi$ , 50-Hz, 6-pole cage motor with 400 V applied takes 6 times full-load current at standstill and develops 1.8 times full-load running torque. The full-load current is 30 A.

- (a) what voltage must be applied to produce full-load torque at starting ?
  - (b) what current will this voltage produce ?
  - (c) if the voltage is obtained by an auto-transformer, what will be the line current ?
  - (d) if starting current is limited to full-load current by an auto-transformer, what will be the starting torque as a percentage of full-load torque ?
- Ignore the magnetising current and stator impedance drops.

**Solution.** (a) Remembering that  $T \propto V^2$ , we have

In the first case,  $1.8 T_f \propto 400^2$ ; ; In the second case,  $T_f \propto V^2$

$$\therefore \left(\frac{V}{400}\right)^2 = \frac{1}{1.8} \text{ or } V = \frac{400}{\sqrt{1.8}} = 298.1 \text{ V}$$

(b) Currents are proportional to the applied voltage.

$$\therefore 6 I_f \propto 400; I \propto 298.1 \quad \therefore I = 6 \times \frac{298.1}{400}, I_f = \frac{6 \times 298.1 \times 30}{400} = 134.2 \text{ A}$$

(c) Here  $K = 298.1/400$   
 Line current =  $K^2 I_{sc} = (298.1/400)^2 \times 6 \times 30 = 100 \text{ A}$

(d) We have seen in Art. 33.11 (b) that line current =  $K^2 I_{sc}$   
 Now, line current = full-load current  $I_f$  (given)  
 $\therefore 30 = K^2 \times 6 \times 30 \quad \therefore K^2 = 1/6$

Now, using  $\frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$  we get  $\frac{T_{st}}{T_f} = \frac{1}{6} \times \left( \frac{6I_f}{I_f} \right)^2 \times 0.05 = 0.3$

Here  $N_s = 120 \times 50/6 = 1000 \text{ r.p.m.}$   $N = 950 \text{ r.p.m.}; s_f = 50/1000 = 0.05$   
 $\therefore T_{st} = 0.3 T_f$  or **30% F.L. torque**

**Example 35.12.** Determine the suitable auto-transformation ratio for starting a 3-phase induction motor with line current not exceeding three times the full-load current. The short-circuit current is 5 times the full-load current and full-load slip is 5%.

Estimate also the starting torque in terms of the full-load torque.

(Elect. Engg.II, Bombay Univ. 1987)

**Solution.** Supply line current =  $K^2 I_{sc}$

It is given that supply line current at start equals  $3 I_f$  and short-circuit current  $I_{sc} = 5 I_f$  where  $I_f$  is the full-load current

$\therefore 3 I_f = K^2 \times 5 I_f$  or  $K^2 = 0.6 \quad \therefore K = 0.775$  or **77.5%**

In the case of an auto starter,

$\frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f \quad \therefore \frac{T_{st}}{T_f} = 0.6 \times \left( \frac{5 I_f}{I_f} \right)^2 \times 0.05 = 0.75$

$\therefore T_{st} = 0.75 T_f = \mathbf{75\%}$  of full-load torque.

**Example 35.13.** The full-load slip of a 400-V, 3-phase cage induction motor is 3.5% and with locked rotor, full-load current is circulated when 92 volt is applied between lines. Find necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque.

(Elect. Machines, Bangalore Univ. 1991)

**Solution.** Short-circuit current with full normal voltage applied is

$I_{sc} = (400/92) I_f = (100/23) I_f$

Supply line current =  $I_{st} = 2 I_f$

Now, line current  $I_{st} = K^2 I_{sc}$

$\therefore 2 I_f = K^2 \times (100/23) I_f \quad \therefore K^2 = 0.46 ; K = 0.678$  or **67.8%**

Also,  $\frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f = 0.46 \times (100/23)^2 \times 0.035 = 0.304$

$\therefore T_{st} = \mathbf{30.4\%}$  of full-load torque

### Tutorial Problems 35.2

1. A 3- $\phi$  motor is designed to run at 5% slip on full-load. If motor draws 6 times the full-load current at starting at the rated voltage, estimate the ratio of starting torque to the full-load torque.

[1.8] (Electrical Engineering Grad, I.E.T.E. Dec. 1986)

2. A squirrel-cage induction motor has a short-circuit current of 4 times the full-load value and has a full-load slip of 5%. Determine a suitable auto-transformer ratio if the supply line current is not to

exceed twice the full-load current. Also, express the starting torque in terms of the full-load torque. Neglect magnetising current. [70.7%, 0.4]

3. A 3- $\phi$ , 400-V, 50-Hz induction motor takes 4 times the full-load current and develops twice the full-load torque when direct-switched to 400-V supply. Calculate in terms of full-load values (a) the line current, the motor current and starting torque when started by an auto-starter with 50% tap and (b) the voltage that has to be applied and the motor current, if it is desired to obtain full-load torque on starting. [(a) 100%, 200%, 50% (b) 228 V, 282%]

(c) Star-delta Starter

This method is used in the case of motors which are built to run normally with a delta-connected stator winding. It consists of a two-way switch which connects the motor in star for starting and then in delta for normal running. The usual connections are shown in Fig. 35.21. When star-connected, the applied voltage over each motor phase is reduced by a factor of  $1/\sqrt{3}$  and hence the torque developed becomes  $1/3$  of that which would have been developed if motor were directly connected in delta. The line current is reduced to  $1/3$ . Hence, during starting period when motor is Y-connected, it takes  $1/3$ rd as much starting current and develops  $1/3$ rd as much torque as would have been developed were it directly connected in delta.

Relation Between Starting and F.L. Torque

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} I_{sc} \text{ per phase}$$

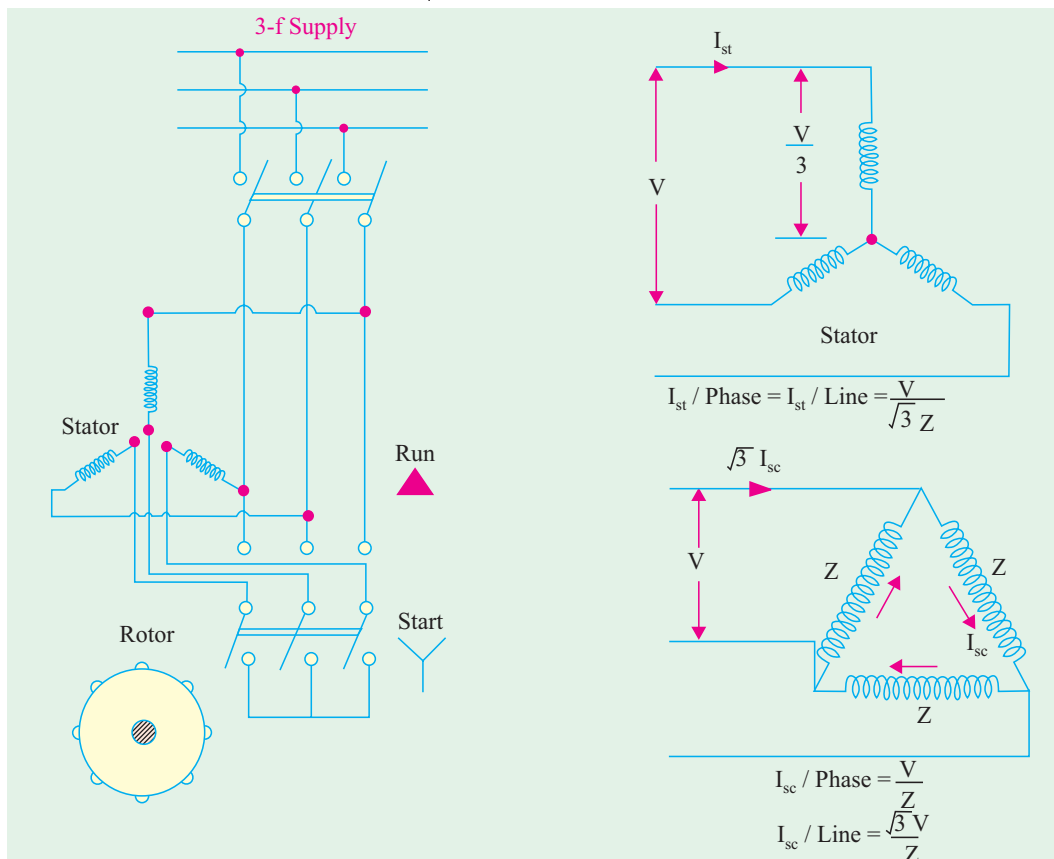


Fig. 35.21

where  $I_{sc}$  is the current/phase which  $\Delta$ -connected motor would have taken if switched on to the supply directly (however, line current at start =  $1/3$  of line  $I_{sc}$ )

$$\begin{aligned} \text{Now} \quad T_{st} &\propto I_{st}^2 & (s = 1) & & \text{--- Art. 35.10} \\ T_f &\propto I_f^2/s_f \end{aligned}$$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{I_{sc}}{\sqrt{3}I_f}\right)^2 s_f = \frac{1}{3} \left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} a^2 s_f$$

Here,  $I_{st}$  and  $I_{sc}$  represent phase values.

*It is clear that the star-delta switch is equivalent\* to an auto-transformer of ratio  $1/\sqrt{3}$  or 58% approximately.*

This method is cheap and effective provided the starting torque is required not to be more than 1.5 times the full-load torque. Hence, it is used for machine tools, pumps and motor-generators etc.

**Example 35.14.** The full-load efficiency and power factor of a 12-kW, 440-V, 3-phase induction motor are 85% and 0.8 lag respectively. The blocked rotor line current is 45 A at 220 V. Calculate the ratio of starting to full-load current, if the motor is provided with a star-delta starter. Neglect magnetising current. (Elect. Machines, A.M.I.E. Sec. B, 1991)

**Solution.** Blocked rotor current with full voltage applied

$$I_{sc} = 45 \times 440/220 = 90 \text{ A}$$

$$\text{Now, } \sqrt{3} \times 440 \times I_f \times 0.8 = 12,000/0.85, \quad \therefore I_f = 23.1 \text{ A}$$

$$\text{In star-delta starter, } I_{st} = I_{sc}/\sqrt{3} = 90/\sqrt{3} = 52 \text{ A}$$

$$\therefore I_{st}/I_f = 52/23.1 = \mathbf{2.256}$$

**Example 35.15.** A 3-phase, 6-pole, 50-Hz induction motor takes 60 A at full-load speed of 940 r.p.m. and develops a torque of 150 N-m. The starting current at rated voltage is 300 A. What is the starting torque? If a star/delta starter is used, determine the starting torque and starting current. (Electrical Machinery-II, Mysore Univ. 1988)

**Solution.** As seen from Art. 33.10, for direct-switching of induction motors

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f \quad \text{Here, } I_{st} = I_{sc} = 300 \text{ A (line value)} ; I_f = 60 \text{ A (line value),}$$

$$s_f = (1000 - 940)/1000 = 0.06 ; T_f = 150 \text{ N-m}$$

$$\therefore T_{st} = 150(300/60)^2 \times 0.06 = \mathbf{225 \text{ N-m}}$$

**When star/delta starter is used**

$$\text{Starting current} = 1/3 \times \text{starting current with direct starting} = 300/3 = \mathbf{100 \text{ A}}$$

$$\text{Starting torque} = 225/3 = \mathbf{75 \text{ N-m}} \quad \text{--- Art 35-11 (c)}$$

**Example 35.16.** Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (a) a star-delta switch (b) an auto-transformer with 70.7% tapping. The short-circuit current of the motor at normal voltage is 6 times the full-load current and the full-load slip is 4%. Neglect the magnetising current. (Electrotechnics, M.S. Univ. Baroda 1986)

$$\text{Solution. (a)} \quad \frac{T_{st}}{T_f} = \frac{1}{3} \left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} \times 6^2 \times 0.04 = 0.48$$

$$\therefore T_{st} = 0.48 T_f \quad \text{or } \mathbf{48\% \text{ of F.L. value}}$$

$$\text{(b) Here } K = 0.707 = 1/\sqrt{2} ; K^2 = 1/2$$

\* By comparing it with the expression given in Art. 35.11 (b)

Now, 
$$\frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 s_f = \frac{1}{2} \times 6^2 \times 0.04 = 0.72$$

$\therefore T_{st} = 0.72 T_f$  or **72% of  $T_f$**

**Example 35.17.** A 15 h.p. (11.2 kW), 3- $\phi$ , 6-pole, 50-HZ, 400-V,  $\Delta$ -connected induction motor runs at 960 r.p.m. on full-load. If it takes 86.4 A on direct starting, find the ratio of starting torque to full-load torque with a star-delta starter. Full-load efficiency and power factor are 88% and 0.85 respectively.

**Solution.** Here,  $I_{sc}/\text{phase} = 86.4/\sqrt{3}$  A

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} \cdot I_{sc} \text{ per phase} = \frac{86.4}{\sqrt{3} \times \sqrt{3}} = 28.8 \text{ A}$$

Full-load input line current may be found from

$$\sqrt{3} \times 400 \times I_L \times 0.85 = 11.2 \times 10^3/0.88 \quad \therefore \text{Full-load } I_L = 21.59 \text{ A}$$

$$\text{F.L. } I_{ph} = 21.59/\sqrt{3} \text{ A}; \quad I_f = 21.59/\sqrt{3} \text{ A per phase}$$

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m.}, \quad N = 950; s_f = 0.05$$

$$\frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 s_f = \left( \frac{28.8 \times \sqrt{3}}{21.59} \right)^2 \times 0.05 \quad \therefore T_{st} = 0.267 T_f \text{ or } \mathbf{26.7\% \text{ F.L. torque}}$$

**Example 35.18.** Find the ratio of starting to full-load current in a 10 kW (output), 400-V, 3-phase induction motor with star/delta starter, given that full-load p.f. is 0.85, the full-load efficiency is 0.88 and the blocked rotor current at 200 V is 40 A. Ignore magnetising current.

(Electrical Engineering, Madras Univ. 1985)

**Solution.** F.L. line current drawn by the  $\Delta$ -connected motor may be found from

$$\sqrt{3} \times 400 \times I_L \times 0.85 = 10 \times 1000/0.88 \quad \therefore I_L = 19.3 \text{ A}$$

Now, with 200 V, the line value of S.C. current of the  $\Delta$ -connected motor is 40 A. If full normal voltage were applied, the line value of S.C. current would be =  $40 \times (400/200) = 80$  A.

$$\therefore I_{sc} \text{ (line value)} = 80 \text{ A}; \quad I_{sc} \text{ (phase value)} = 80/\sqrt{3} \text{ A}$$

When connected in star across 400 V, the starting current per phase drawn by the motor stator during starting is

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} \times I_{sc} \text{ per phase} = \frac{1}{\sqrt{3}} \times \frac{80}{\sqrt{3}} = \frac{80}{3} \text{ A} \quad \text{— Art. 35.10}$$

Since during starting, motor is star-connected,  $I_{st}$  per phase = line value of  $I_{sc} = 80/3$  A

$$\therefore \frac{\text{line value of starting current}}{\text{line value of F.L. current}} = \frac{80/3}{19.3} = \mathbf{1.38}$$

**Example 35.19.** A 5 h.p. (3.73 kW), 400-V, 3- $\phi$ , 50-Hz cage motor has a full-load slip of 4.5%. The motor develops 250% of the rated torque and draws 650% of the rated current when thrown directly on the line. What would be the line current, motor current and the starting torque if the motor were started (i) by means of a star/delta starter and (ii) by connecting across 60% taps of a starting compensator. (Elect. Machines-II, Indore Univ. 1989)

**Solution. (i)** Line current =  $(1/3) \times 650 = \mathbf{216.7\%}$

Motor being star-connected, line current is equal to phase current.

$$\therefore \text{motor current} = 650/3 = \mathbf{216.7\%}$$

As shown earlier, starting torque developed for star-connection is one-third of that developed on direct switching with delta-connection  $\therefore T_{st} = 250/3 = \mathbf{83.3\%}$

**(ii)** Line current =  $K^2 \times I_{sc} = (60/100)^2 \times 650 = \mathbf{234\%}$

$$\begin{aligned}\text{Motor current} &= K \times I_{sc} = (60/100) \times 650 = \mathbf{390\%} \\ T_{st} &= K^2 \times T_{sc} = (60/100)^2 \times 250 = \mathbf{90\%}\end{aligned}$$

**Example. 35.20.** A squirrel-cage type induction motor when started by means of a star/delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculate the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed. (Utilization of Elect. Power, A.M.I.E. 1987)

**Solution.** With star-delta starter,  $\frac{T_{st}}{T_f} = \frac{1}{3} \left( \frac{I_{sc}}{I_f} \right)^2 s_f$

Line current on line-start  $I_{sc} = 3 \times 180\%$  of  $I_f = 3 \times 1.8 I_f = 5.4 I_f$

Now,  $T_{st}/T_f = 0.35$  (given);  $I_{sc}/I_f = 5.4$

$\therefore 0.35 = (1/3) \times 5.4^2 s_f$  or  $5.4^2 s_f = 1.05$

**Autostarter :** Here,  $K = 0.75$

Line starting current  $= K^2 I_{sc} = (0.75)^2 \times 5.4 I_f = \mathbf{3.04 I_f = 304\% \text{ of F.L. current}}$

$$\begin{aligned}\frac{T_{st}}{T_f} &= K^2 \left( \frac{I_{sc}}{I_f} \right)^2 s_f; \quad \frac{T_{st}}{T_f} = (0.75)^2 \times 5.4^2 s_f \\ &= (0.75)^2 \times 1.05 = 0.59\end{aligned}$$

$T_{st} = 0.59 T_f = \mathbf{59\% \text{ F.L. torque}}$

**Example. 35.21.** A 10 h.p. (7.46 kW) motor when started at normal voltage with a star-delta switch in the star position is found to take an initial current of  $1.7 \times$  full-load current and gave an initial starting torque of 35% of full-load torque. Explain what happens when the motor is started under the following conditions : (a) an auto-transformer giving 60% of normal voltage (b) a resistance in series with the stator reducing the voltage to 60% of the normal and calculate in each case the value of starting current and torque in terms of the corresponding quantities at full-load. (Elect. Machinery-III, Kerala Univ. 1987)

**Solution.** If the motor were connected in delta and direct-switched to the line, then it would take a line current three times that which it takes when star-connected.

$\therefore$  line current on line start or  $I_{sc} = 3 \times 1.7 I_f = 5.1 I_f$

We know  $\frac{T_{st}}{T_f} = \frac{1}{3} \left( \frac{I_{sc}}{I_f} \right)^2 s_f$

Now  $\frac{T_{st}}{T_f} = 0.35$  ...given;  $\frac{I_{sc}}{I_f} = 5.1$  ...calculated

$\therefore 0.35 = (1/3) \times 5.1^2 \times s_f$

We can find  $s_f$  from  $5.1^2 \times s_f = 1.05$

(a) When it is started with an auto-starter, then  $K = 0.6$

Line starting current  $= K^2 \times I_{sc} = 0.6^2 \times 5.1 I_f = \mathbf{0.836 I_f}$

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.6^2 \times 1.05 = 0.378$   $\therefore T_{st} = \mathbf{37.8\%}$  of F.L. torque

(b) Here, voltage across motor is reduced to 60% of normal value. In this case motor current is the same as line current but it decreases in proportion to the decrease in voltage.

As voltage across motor  $= 0.6$  of normal voltage

$\therefore$  line starting current  $= 0.6 \times 5.1 I_f = \mathbf{3.06 I_f}$

Torque at starting would be the same as before.

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.378$   $\therefore T_{st} = \mathbf{37.8\%}$  of F.L. torque.

**Tutorial Problems 35.3**

1. A 3-phase induction motor whose full-load slip is 4 per cent, takes six times full-load current when switched directly on to the supply. Calculate the approximate starting torque in terms of the full-load torque when started by means of an auto-transformer starter, having a 70 percent voltage tap. [0.7  $T_f$ ]
2. A 3-phase, cage induction motor takes a starting current at normal voltage of 5 times the full-load value and its full-load slip is 4 per cent. What auto-transformer ratio would enable the motor to be started with not more than twice full-load current drawn from the supply ?  
What would be the starting torque under these conditions and how would it compare with that obtained by using a stator resistance starter under the same limitations of line current ?  
[63.3% tap ; 0.4  $T_f$  ; 0.16  $T_f$ ]
3. A 3-phase, 4-pole, 50-Hz induction motor takes 40 A at a full-load speed of 1440 r.p.m. and develops a torque of 100 N-m at full-load. The starting current at rated voltage is 200 A. What is the starting torque ? If a star-delta starter is used, what is the starting torque and starting current ? Neglect magnetising current.  
[100 N-m; 33.3 N-m; 66.7 A] (Electrical Machines-IV, Bangalore Univ. Aug. 1978)
4. Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (a) a star-delta switch (b) an auto-transformer with 50% tapping. Ignore magnetising current. The short-circuit current of the motor at normal voltage is 5 times the full-load current and the full-load slip is 4 per cent. [(a) 0.33 (b) 0.25] (A.C. Machines, Madras Univ. 1976)
5. Find the ratio of starting to full-load current for a 7.46 kW, 400-V, 3-phase induction motor with star/delta starter, given that the full-load efficiency is 0.87, the full-load p.f. is 0.85 and the short-circuit current is 15 A at 100 V. [1.37] (Electric Machinery-II, Madras Univ. April 1978)
6. A four-pole, 3-phase, 50-Hz, induction motor has a starting current which is 5 times its full-load current when directly switched on. What will be the percentage reduction in starting torque if (a) star-delta switch is used for starting (b) auto-transformer with a 60 per cent tapping is used for starting ?  
(Electrical Technology-III, Gwalior Univ. Nov. 1917)
7. Explain how the performance of induction motor can be predicted by circle diagram. Draw the circle diagram for a 3-phase, mesh-connected, 22.38 kW, 500-V, 4-pole, 50-Hz induction motor. The data below give the measurements of line current, voltage and reading of two wattmeters connected to measure the input :
 

No load	500 V	8.3 A	2.85 kW	- 1.35 kW
Short circuit	100 V	32 A	- 0.75 kW	2.35 kW

 From the diagram, find the line current, power factor, efficiency and the maximum output.  
[83 A, 0.9, 88%, 50.73 kW] (Electrical Machines-II, Vikram Univ. Ujjan 1977)

**35.12. Starting of Slip-ring Motors**

These motors are practically always started with full line voltage applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. The controlling resistance is in the form of a rheostat, connected in star (Fig. 35.22), the resistance being gradually cut-out of the rotor circuit, as the motor gathers speed. It has been already shown that by increasing the rotor resistance, not only is the rotor (and hence stator) current reduced at starting, but at the same time, the starting torque is also increased due to improvement in power factor.

The controlling rheostat is either of stud or contactor type and



Slip-ring electric motor



Rheostat

may be hand-operated or automatic. The starter unit usually includes a line switching contactor for the stator along with no-voltage (or low-voltage) and over-current protective devices. There is some form of interlocking to ensure proper sequential operation of the line contactor and the starter. This interlocking prevents the closing of stator contactor unless the starter is 'all in'.

As said earlier, the introduction of additional external resistance in the rotor circuit enables a slip-ring motor to develop a high starting torque with reasonably moderate starting current. Hence, such motors can be started under load. This additional resistance is for starting purpose only. It is gradually cut out as the motor comes up to speed.

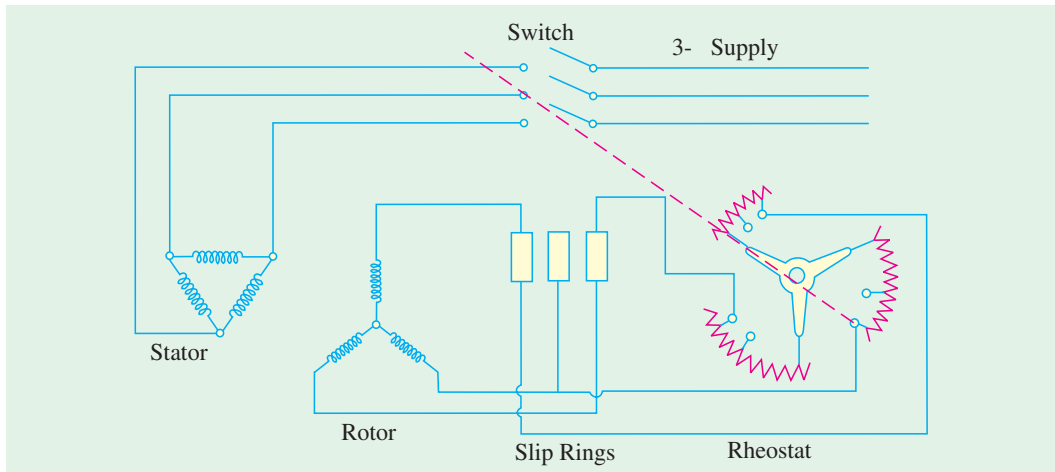


Fig. 35.22

The rings are, later on, short-circuited and brushes lifted from them when motor runs under normal conditions.

### 35.13. Starter Steps

Let it be assumed, as usually it is in the case of starters, that (i) the motor starts against a constant torque and (ii) that the rotor current fluctuates between fixed maximum and minimum values of  $I_{2max}$  and  $I_{2min}$  respectively.

In Fig. 35.23 is shown one phase of the 3-phase rheostat  $AB$  having  $n$  steps and the rotor circuit. Let  $R_1, R_2, \dots$  etc. be the total resistances of the rotor circuit on the first, second step...etc. respectively. The resistances  $R_1, R_2, \dots$ , etc. consist of rotor resistance per phase  $r_2$  and the external resistances  $\rho_1, \rho_2, \dots$  etc. Let the corresponding values of slips be  $s_1, s_2, \dots$  etc. at stud No.1, 2...etc. At the commencement of each step, the current is  $I_{2max}$  and at the instant of leaving it, the current is  $I_{2min}$ . Let  $E_2$  be the standstill e.m.f. induced in each phase of the rotor. When the handle touches first stud, the current rises to a maximum value  $I_{2max}$ , so that

$$I_{2max} = \frac{s_1 E_2}{\sqrt{[R_1^2 + (s_1 X_2)^2]}} = \frac{E_2}{\sqrt{[(R_1 / s_1)^2 + X_2^2]}}$$

where

$s_1 =$  slip at starting *i.e.* unity and  $X_2 =$  rotor reactance/phase

Then, *before* moving to stud No. 2, the current is reduced to  $I_{2min}$  and slip changes to  $s_2$  such that



$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}}$$

As we now move to stud No. 2, the speed momentarily remains the same, but current rises to  $I_{2max}$  because some resistance is cut out.

$$\text{Hence, } I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}}$$

After some time, the current is again reduced to  $I_{2min}$  and the slip changes to  $s_3$  such that

$$I_{2min} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}}$$

As we next move over to stud No.3, again current rises to  $I_{2max}$  although speed remains momentarily the same.

$$\therefore I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} \quad \text{Similarly } I_{2min} = \frac{E_2}{\sqrt{[(R_3/s_4)^2 + X_2^2]}}$$

At the last stud *i.e.* nth stud,  $I_{2max} = \frac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$  where  $s_{max}$  = slip under normal running conditions, when external resistance is completely cut out.

It is found from above that

$$I_{2max} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}} = \dots = \frac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$$

$$\text{or } \frac{R_1}{s_1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = \dots = \frac{R_{n-1}}{s_{n-1}} = \frac{R_n}{s_n} = \frac{r_2}{s_{max}} \quad \dots(i)$$

Similarly,

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} = \dots = \frac{E_2}{\sqrt{[(R_{n-1}/s_{max})^2 + X_2^2]}}$$

$$\text{or } \frac{R_1}{s_2} = \frac{R_2}{s_3} = \frac{R_3}{s_4} = \dots = \frac{R_{n-1}}{s_{max}} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = \dots = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \dots = \frac{r_2}{R_{n-1}} = K \text{ (say)} \quad \dots(iii)$$

Now, from (i) it is seen that  $R_1 = \frac{s_1 \times r_2}{s_{max}}$ .

Now,  $s_1 = 1$  at starting, when rotor is stationary.

$\therefore R_1 = r_2/s_{max}$ . Hence,  $R_1$  becomes known in terms of rotor resistance/phase and normal slip. From (iii), we obtain

$$R_2 = KR_1; R_3 = KR_2 = K^2R_1; R_4 = KR_3 = K^3R_1 \text{ and } r_2 = KR_{n-1} = K^{n-1} \cdot R_1$$

$$\text{or } r_2 = K^{n-1} \cdot \frac{r_2}{s_{max}} \quad \text{(putting the value of } R_1)$$

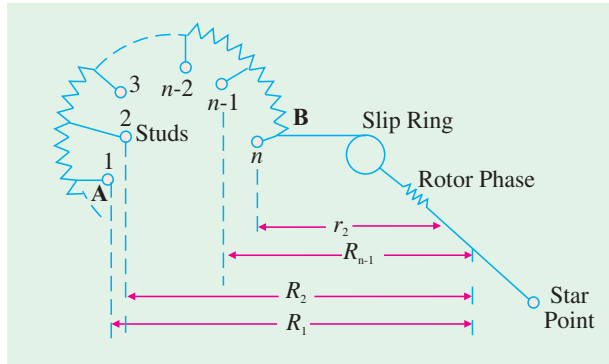


Fig. 35.23

$K = (s_{max})^{1/n-1}$  where  $n$  is the number of starter studs.

The resistances of various sections can be found as given below :

$$\rho_1 = R_1 - R_2 = R_1 - KR_1 = (1 - K) R_1 ; \rho_2 = R_2 - R_3 = KR_1 - K^2R_1 = K\rho_1$$

$$\rho_3 = R_3 - R_4 = K^2 \rho_1 \text{ etc.}$$

Hence, it is seen from above that if  $s_{max}$  is known for the assumed value  $I_{2max}$  of the starting current, then  $n$  can be calculated.

**Example 35.22.** Calculate the steps in a 5-step rotor resistance starter for a 3 phase induction motor. The slip at the maximum starting current is 2% with slip-ring short-circuited and the resistance per rotor phase is 0.02 Ω.

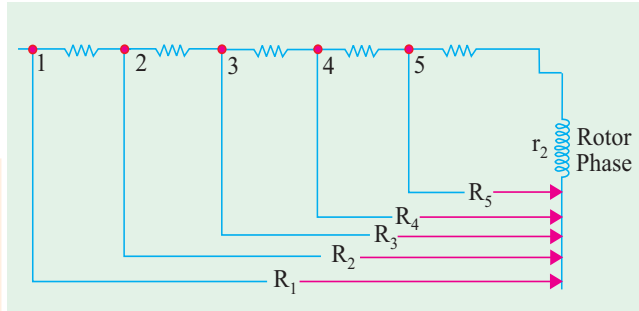


Fig. 35.24

**Solution.** Here,  $s_{max} = 2\%$   
 $= 0.02; r_2 = 0.02 \Omega, n = 6$

$$R_1 = \text{total resistance in rotor circuit/phase on first stud}$$

$$= r_2/s_{max} = 0.02/0.02 = 1 \Omega$$

Now,  $K = (s_{max})^{1/n-1} = (0.02)^{1/5} = 0.4573$

$$R_1 = 1 \Omega; R_2 = KR_1 = 0.4573 \times 1 = 0.4573 \Omega$$

$$R_3 = KR_2 = 0.4573 \times 0.4573 = 0.2091 \Omega; R_4 = KR_3 = 0.4573 \times 0.2091 = 0.0956 \Omega$$

$$R_5 = KR_4 = 0.4573 \times 0.0956 = 0.0437 \Omega; r_2 = KR_5 = 0.4573 \times 0.0437 = 0.02 \Omega \text{ (as given)}$$

The resistances of various starter sections are as found below :

$$\rho_1 = R_1 - R_2 = 1 - 0.4573 = \mathbf{0.5427 \Omega}; \quad \rho_2 = R_2 - R_3 = 0.4573 - 0.2091 = \mathbf{0.2482 \Omega}$$

$$\rho_3 = R_3 - R_4 = 0.2091 - 0.0956 = \mathbf{0.1135 \Omega}; \quad \rho_4 = R_4 - R_5 = 0.0956 - 0.0437 = \mathbf{0.0519 \Omega}$$

$$\rho_5 = R_5 - r_2 = 0.0437 - 0.02 = \mathbf{0.0237 \Omega}$$

The resistances of various sections are shown in Fig. 35.24.

### 35.14. Crawling

It has been found that induction motors, particularly the squirrel-cage type, sometimes exhibit a tendency to run stably at speeds as low as one-seventh of their synchronous speed  $N_s$ . This phenomenon is known as crawling of an induction motor.

This action is due to the fact that the a.c. winding of the stator produces a flux wave, which is not a pure sine wave. It is a complex wave consisting of a fundamental wave, which revolves synchronously and odd harmonics like 3rd, 5th, and 7th etc. which rotate either in the forward or backward direction at  $N_s / 3, N_s / 5$  and  $N_s / 7$  speeds respectively. As a result, in addition to the fundamental torque, harmonic torques are also developed, whose synchronous speeds are  $1/n$ th of the speed for the fundamental torque i.e.  $N_s / n$ , where  $n$  is the order of the harmonic torque. Since 3rd harmonic currents are absent in a balanced 3-phase system, they produce no rotating field and, therefore, no torque. Hence, total motor torque has three components : (i) the fundamental torque, rotating with the synchronous speed  $N_s$  (ii) 5th harmonic torque\* rotating at  $N_s / 5$  speed and (iii) 7th harmonic torque, having a speed of  $N_s / 7$ .

\* The magnitude of the harmonic torques is  $1/n^2$  of the fundamental torque.

Now, the 5th harmonic currents have a phase difference of  $5 \times 120^\circ = 600^\circ = -120^\circ$  in three stator windings. The revolving field, set up by them, rotates in the *reverse* direction at  $N_s / 5$ . The forward speed of the rotor corresponds to a slip greater than 100%. The small amount of 5th harmonic reverse torque produces a braking action and may be neglected.

The 7th harmonic currents in the three stator windings have a phase difference of  $7 \times 120^\circ = 2 \times 360^\circ + 120^\circ = 120^\circ$ . They set up a forward rotating field, with a synchronous speed equal to 1/7th of the synchronous speed of the fundamental torque.

If we neglect all higher harmonics, the resultant torque can be taken as equal to the sum of the fundamental torque and the 7th harmonic torque, as shown in Fig. 35.25. It is seen that the 7th harmonic torque reaches its maximum positive value just before 1/7th synchronous speed  $N_s$ , beyond which it becomes negative in value. Consequently, the resultant torque characteristic shows a dip which may become very pronounced with certain slot combinations. If the mechanical load on the shaft involves a constant load torque, it is possible that the torque developed by the motor may fall below this load torque. When this happens, the motor will not accelerate upto its normal speed but will remain running at a speed, which is nearly 1/7th of its full-speed. This is referred to as crawling of the motor.

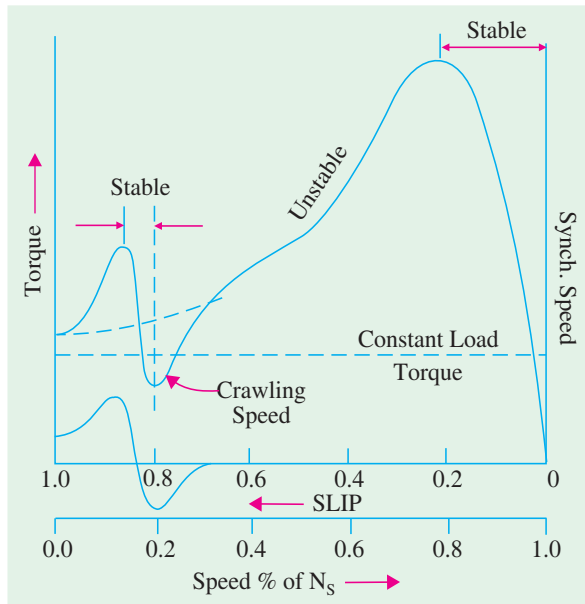


Fig. 35.25

35.15. Cogging or Magnetic Locking

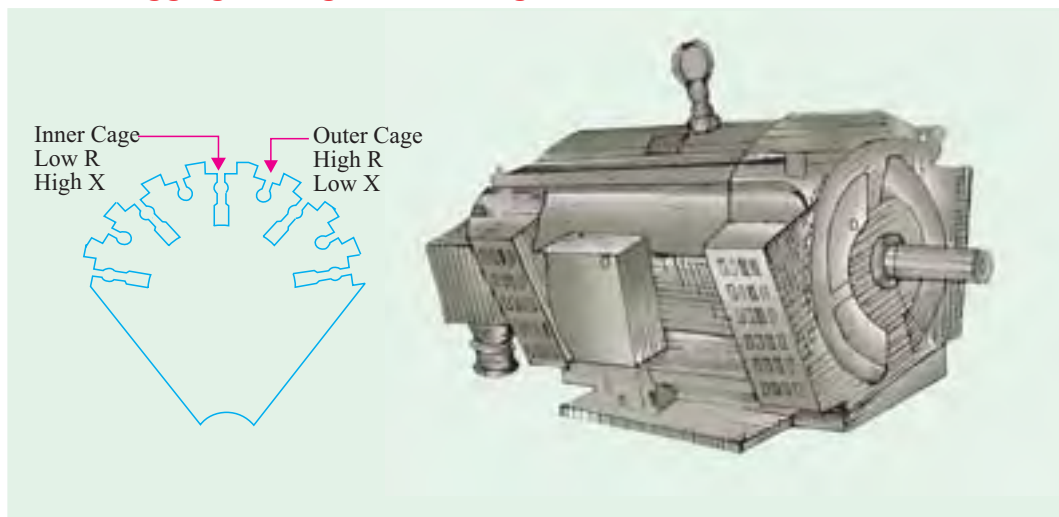


Fig. 35.26

Double-cage, 30-kW, 400/440-V, 3-f, 960 r.p.m.

Fig. 35.27

squirrel-cage motor. (Courtesy : Jyoti Ltd., Baroda)

The rotor of a squirrel-cage motor sometimes refuses to start at all, particularly when the voltage is low. This happens when the number of stator teeth  $S_1$  is equal to the number of rotor teeth  $S_2$  and is due to the magnetic locking between the stator and rotor teeth. That is why this phenomenon is sometimes referred to as teeth-locking.

It is found that the reluctance of the magnetic path is minimum when the stator and rotor teeth face each other rather than when the teeth of one element are opposite to the slots on the other. It is in such positions of minimum reluctance, that the rotor tends to remain fixed and thus cause serious trouble during starting. Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

### 35.16. Double Squirrel Cage Motor

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance. The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions (because there will be more rotor Cu losses). The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period. Many efforts have been made to build a squirrel-cage motor which should have a high starting torque without sacrificing its electrical efficiency, under normal running conditions. The result is a motor, due to Boucheort, which has two independent cages on the same rotor, one inside the other. A punching for such a double cage rotor is shown in Fig. 35.26.

The **outer** cage consists of bars of a **high-resistance** metal, whereas the inner cage has low-resistance copper bars.

Hence, outer cage has high resistance and low ratio of reactance-to-resistance, whereas the inner cage has low resistance but, being situated deep in the rotor, has a large ratio of reactance-to-resistance. Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

As said earlier, at starting and at large slip values, frequency of induced e.m.f in the rotor is high. So the reactance of the inner cage ( $= 2\pi f L$ ) and therefore, its impedance are both high. Hence, very little current flows in it. Most of the starting current is confined to outer cage, despite its high resistance. Hence, the motor develops a high starting torque due to high-resistance outer cage. Double squirrel-cage motor is shown in Fig. 35.27.

As the speed increases, the frequency of the rotor e.m.f. decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions. Most of the current then flows through it and hence it develops the greater part of the motor torque.

In fact, when speed is normal, frequency of rotor e.m.f. is so small that the reactance of both cages is practically negligible. The current is carried by two cages in parallel, giving a low combined resistance.

Hence, it has been made possible to construct a single machine, which has a good starting torque

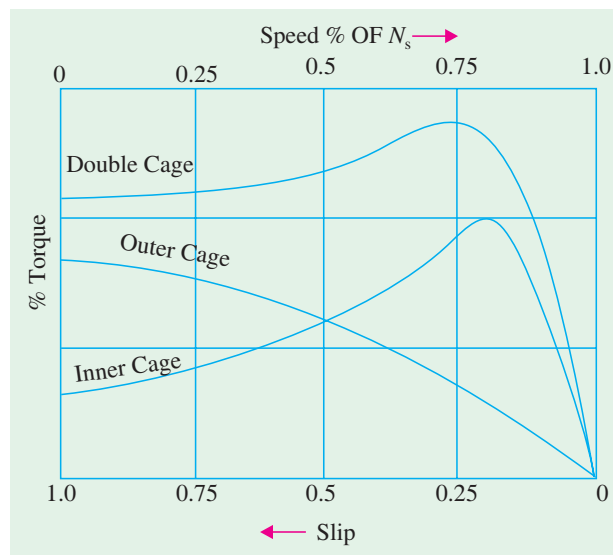


Fig. 35.28

with reasonable starting current and which maintains high efficiency and good speed regulation, under normal operating conditions.

The torque-speed characteristic of a double cage motor may be approximately taken to be the sum of two motors, one having a high-resistance rotor and the other a low-resistance one (Fig. 35.28).

Such motors are particularly useful where frequent starting under heavy loads is required.

### 35.17. Equivalent Circuit

The two rotor cages can be considered in parallel, if it is assumed that both cages completely link the main flux. The equivalent circuit for one phase of the rotor, as referred to stator, is shown in Fig. 35.29. If the magnetising current is neglected, then the figure is simplified to that shown in Fig. 35.30. Hence,  $R'_0/s$  and  $R'_i/s$  are resistances of outer and inner rotors as referred to stator respectively and  $X'_0$  and  $X'_i$  their reactances

Total impedance as referred to primary is given by

$$Z_{01} = R_1 + jX_1 + \frac{1}{1/Z'_1 + 1/Z'_0} = R_1 + jX_1 + \frac{Z'_i Z'_0}{Z'_0 + Z'_i}$$

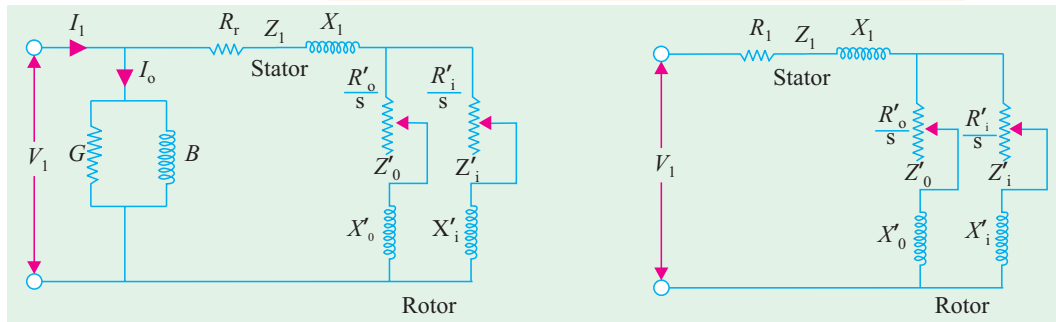


Fig. 35.29

Fig. 35.30

**Example 35.23.** A double-cage induction motor has the following equivalent circuit parameters, all of which are phase values referred to the primary :

Primary	$R_1 = 1 \Omega$	$X_1 = 3 \Omega$
Outer cage	$R'_0 = 3 \Omega$	$X'_0 = 1 \Omega$
Inner cage	$R'_i = 0.6 \Omega$	$X'_i = 5 \Omega$

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4%. The magnetising current may be neglected.

**Solution.**

Refer to Fig. 35.31

(i) At start,  $s = 1$

$$Z_{01} = 1 + j3 + \frac{1}{1/(3 + j1) + 1/(0.6 + j5)}$$

$$= 2.68 + j4.538 \Omega$$

Current / phase =  $440 / (2.68^2 + 4.538^2)^{1/2} = 83.43 \text{ A}$

Torque =  $3 \times 83.43^2 \times (2.68 - 1)$

= **35,000 synch watt.**

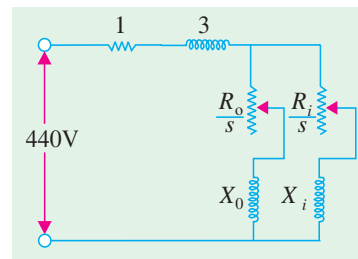


Fig. 35.31

(ii) when  $s = 4$  per cent

In this case, approximate torque may be found by neglecting the outer cage impedance altogether. However, it does carry some current, which is almost entirely determined by its resistance.

$$Z_{01} = 1 + j3 + \frac{1}{1/(3/0.04) + 1/(0.6/0.04 + j5)} = 13.65 + j6.45$$

Current/phase =  $440/(13.65^2 + 6.45^2)^{1/2} = 29.14 \text{ A}$

Torque =  $3 \times 29.14^2 \times (13.65 - 1) = 32,000 \text{ synch watt.}$

**Example 35.24.** At standstill, the equivalent impedance of inner and outer cages of a double-cage rotor are  $(0.4 + j2) \Omega$  and  $(2 + j0.4) \Omega$  respectively. Calculate the ratio of torques produced by the two cages (i) at standstill (ii) at 5% slip. (Elect. Machines-II, Punjab Univ. 1989)

**Solution.** The equivalent circuit for one phase is shown in Fig. 35.32.

(i) At standstill,  $s = 1$

Impedance of inner cage =  $Z_i = \sqrt{0.4^2 + 2^2} = 2.04 \Omega$

Impedance of outer cage =  $Z_o = \sqrt{2^2 + 0.4^2} = 2.04 \Omega$

If  $I_o$  and  $I_i$  are the current inputs of the two cages, then

power input of inner cage,  $P_i = I_i^2 R_i = 0.4 I_i^2$  watt

power input of outer cage,  $P_o = I_o^2 R_o = 2 I_o^2$

$$\begin{aligned} \therefore \frac{\text{torque of outer cage, } T_o}{\text{torque of inner cage, } T_i} &= \frac{P_o}{P_i} = \frac{2 I_o^2}{0.4 I_i^2} = 5 \left( \frac{I_o}{I_i} \right)^2 \\ &= 5 \left( \frac{Z_i}{Z_o} \right)^2 = 5 \left( \frac{2.04}{2.04} \right)^2 = 5 \quad \therefore T_o : T_i :: 5 : 1 \end{aligned}$$

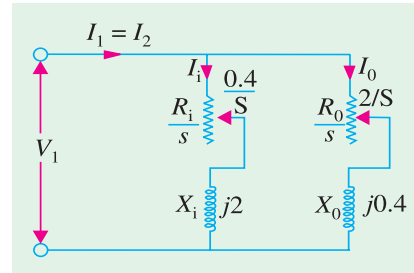


Fig. 35.32

(ii) When  $s = 0.05$

$$Z_o = \sqrt{[(R_o/s)^2 + X_o^2]} = \sqrt{(2/0.05)^2 + 0.4^2} = 40 \Omega$$

$$Z_i = \sqrt{[(R_i/s)^2 + X_i^2]} = \sqrt{(0.4/0.05)^2 + 2^2} = 8.25 \Omega$$

$$\therefore \frac{I_o}{I_i} = \frac{Z_i}{Z_o} = \frac{8.25}{40} = 0.206$$

$$P_o = I_o^2 R_o / s = 40 I_o^2; P_i = I_i^2 R_i / s = 8 I_i^2$$

$$\therefore \frac{T_o}{T_i} = \frac{P_o}{P_i} = \frac{40 I_o^2}{8 I_i^2} = 5 \left( \frac{I_o}{I_i} \right)^2 = 5 (0.206)^2 = 0.21$$

$\therefore T_o : T_i :: 0.21 : 1$

It is seen from above, that the outer cage provides maximum torque at starting, whereas inner cage does so later.

**Example 35.25.** A double-cage rotor has two independent cages. Ignoring mutual coupling between cages, estimate the torque in synchronous watts per phase (i) at standstill and at 5 per cent slip, given that the equivalent standstill impedance of the inner cage is  $(0.05 + j0.4)$  ohm per phase and of the outer cage  $(0.5 + j0.1)$  ohm per phase and that the rotor equivalent induced e.m.f. per phase is 100 V at standstill.

**Solution.** The equivalent circuit of the double-cage rotor is shown in Fig. 35.33.

(i) At standstill,  $s = 1$

The combined impedance of the two cages is

$$Z = \frac{Z_o Z_i}{Z_o + Z_i}$$

where  $Z_0$  = impedance of the outer cage  
 $Z_i$  = impedance of the inner cage  
 $Z = \frac{(0.5 + j 0.1)(0.05 + j 0.4)}{(0.55 + j 0.5)}$

$$= 0.1705 + j 0.191 \text{ ohm}$$

$$\therefore Z = \sqrt{0.1705^2 + 0.191^2} = 0.256 \Omega$$

Rotor current  $I_2 = 100/0.256$  A; Combined resistance  $R_2 = 0.1705 \Omega$

Torque at standstill in synchronous watts per phase is

$$= I_2^2 R_2 = \left(\frac{100}{0.256}\right)^2 \times 0.1705 = \mathbf{26,000 \text{ synch watts}}$$

(ii) Here  $s = 0.05$

$$Z = \frac{\left(\frac{0.5}{0.05} + j 0.1\right)\left(\frac{0.05}{0.05} + j 0.4\right)}{\left(\frac{0.05}{0.5} + \frac{0.5}{0.05} + j 0.5\right)} = 1.01 + j 0.326 \text{ ohm}$$

$$Z = \sqrt{(1.01)^2 + 0.326^2} = 1.06 \Omega$$

Combined resistance  $R_2 = 1.01 \Omega$  ; rotor current =  $100/1.06$  A

Torque in synchronous watts per phase is

$$= I_2^2 R_2 = (100/1.06)^2 \times 1.1 = \mathbf{9,000 \text{ synch.watt.}}$$

**Example 35.26.** In a double-cage induction motor, if the outer cage has an impedance at standstill of  $(2 + j 1.2)$  ohm, determine the slip at which the two cages develop equal torques if the inner cage has an impedance of  $(0.5 + j 3.5)$  ohm at standstill.

(Electric Machines, Osmania Univ. 1991)

**Solution.** Let  $s$  be the slip at which two cage develop equal torques.

Then  $Z_1 = \sqrt{(2/s)^2 + 1.2^2}$  and  $Z_2 = \sqrt{(0.5/s)^2 + 3.5^2}$

$$\therefore \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{Z_2}{Z_1}\right)^2 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44}$$

Power input to outer cage  $P_1 = I_1^2 R_1/s$

$$\therefore P_1 = I_1^2 \times \frac{2}{s}; \quad P_2 = I_2^2 \times \frac{0.5}{s}$$

$$\therefore \frac{T_1}{T_2} = \frac{P_1}{P_2} = \left(\frac{I_1}{I_2}\right)^2 \times 4 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44} \times 4$$

As  $T_1 = T_2$

$$\therefore \frac{4}{s^2} + 1.44 = \left(\frac{0.25}{s^2} + 12.25\right) \times 4 \quad \therefore s = 0.251 = \mathbf{25.1\%}$$

**Example 35.27.** The resistance and reactance (equivalent) values of a double-cage induction motor for stator, outer and inner cage are 0.25, 1.0 and 0.15 ohm resistance and 3.5, zero and 3.0 ohm reactance respectively. Find the starting torque if the phase voltage is 250 V and the synchronous speed is 1000 r.p.m. (I.E.E. London)

**Solution.** The equivalent circuit is shown in Fig. 35.34 where magnetising current has been neglected. At starting,  $s = 1$

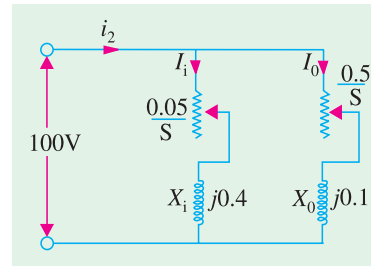


Fig. 35.33

Impedance of outer cage  $Z_0' = (1 + j 0)$   
 Impedance of inner cage  $Z_i' = (0.15 + j 3)$

The two impedances are in parallel. Hence, their equivalent impedance

$$Z_2' = \frac{Z_0'Z_i'}{Z_0' + Z_i'} = \frac{(1 + j 0)(0.15 + j 3)}{(1 + j 0) + (0.15 + j 3)}$$

$$= 0.889 + j 0.29$$

Stator impedance =  $(0.25 + j 3.5)$

$\therefore$  total impedance  $Z_{01} = (0.889 + j 0.29) + (0.25 + j 3.5)$   
 $= (1.14 + j 3.79) \Omega$  (approx.)

$$\text{Current } I = \frac{\text{phase voltage}}{\text{total phase impedance}} = \frac{250 + j 0}{1.14 + j 3.79} = 18.2 - j 60.5 = 66.15 \text{ A}$$

Rotor Cu loss/phase = (current)<sup>2</sup> × total resistance/phase of two rotors  
 $= 66.15^2 \times 0.889 = 3,890 \text{ W}$

Total Cu loss in 3-phases =  $3 \times 3890 = 11,670 \text{ W}$

Now, rotor input =  $\frac{\text{rotor Cu loss}}{s}$ ; At starting,  $s = 1 \therefore$  rotor input = 11,670 W

$\therefore T_{start} = 11,670$  synchronous watts

Also  $T_{start} \times 2\pi N_s = 11,670$  or  $T_{start} = \frac{11,670}{2\pi \times (1000/60)} = 111.6 \text{ N-m (approx.)}$

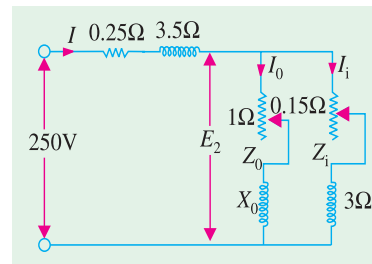


Fig. 35.34

**Note.** If torques developed by the two rotors separately are required, then find  $E_2$  (Fig. 35.34), then  $I_1$  and  $I_2$ . Knowing these values,  $T_1$  and  $T_2$  can be found as given in previous example.

**Example 35.28.** A double-cage induction motor has the following equivalent circuit parameters all of which are phase values referred to the primary :

Primary :  $R_1 = 1 \text{ ohm } X_1 = 2.8 \text{ ohm}$

Outer cage :  $R_0' = 3 \text{ ohm } X_0' = 1.0 \text{ ohm}$

Inner cage :  $R_i' = 0.5 \text{ ohm } X_i' = 5 \text{ ohm}$

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4 per cent. The magnetizing branch can be assumed connected across the primary terminals.

(Electrical Machines-II, South Gujarat Univ. 1987)

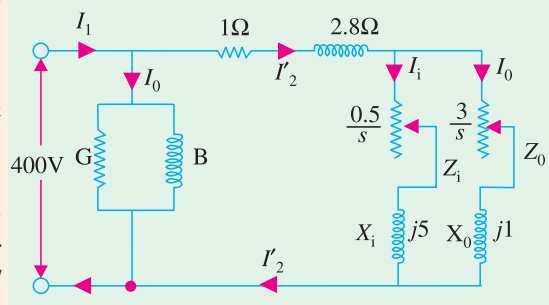


Fig. 35.35

**Solution.** The equivalent circuit for one phase is shown in Fig. 35.35. It should be noted that magnetising impedance  $Z_0$  has no bearing on the torque and speed and hence, can be neglected so far as these two quantities are concerned

(i) At standstill  $s = 1$

$$Z_2' = \frac{Z_0'Z_i'}{Z_0' + Z_i'} = \frac{(3 + j1.0)(0.5 + j5)}{(3.5 + j6)}$$

$$= 1.67 + j 1.56$$

$$Z_{01} = Z_1 + Z_2' = (1 + j 2.8) + (1.67 + j 1.56)$$

$$= (2.67 + j 4.36) = 5.1 \angle 58.5^\circ$$



Voltage per phase,  $V_1 = 440$  V. Since stator is delta-connected.

$$I_2' = V_1/Z_{01} = 440/5.1 \angle 58.5^\circ = 86.27 \angle -58.5^\circ$$

Combined resistance  $R_2 = 1.6 \Omega$

$$\therefore \text{starting torque per phase} = I_2'^2 R_2 = 86.27^2 \times 1.67 = \mathbf{12,430 \text{ synch. watt.}}$$

(ii) when  $s = 0.04$

$$Z_0' = (3/0.04) + j 1.0 = 75 + j 1.0; \quad Z_i' = (0.5/0.04) + j 5 = 12.5 + j 5$$

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(75 + j 1.0)(12.5 + j 5)}{(87.5 + j 6)} = 10.3 + j 3.67$$

$$Z_{01} = Z_1 + Z_2' = (1 + j 2.8) + (10.3 + j 6.47) = 11.3 + j 6.47 = 13.03 \angle 29.8^\circ$$

$$I_2' = V_1/Z_{01} = 440/13.03 \angle 29.8^\circ = 33.76 \angle -29.8^\circ$$

combined resistance  $R_2 = 10.3 \Omega$

$$\therefore \text{full-load torque per phase} = I_2'^2 R_2 = 33.76^2 \times 10.3 = \mathbf{11,740 \text{ synch. watts}}$$

Obviously, starting torque is higher than full-load torque.

### Tutorial Problems 35.4

1. Calculate the steps in a 5-section rotor starter of a 3-phase induction motor for which the starting current should not exceed the full-load current, the full-load slip is 0.018 and the rotor resistance is  $0.015 \Omega$  per phase

$$\rho_1 = 0.46 \Omega; \rho_2 = 0.206 \Omega; \rho_3 = 0.092 \Omega; \rho_4 = 0.042 \Omega; \rho_5 = 0.0185 \Omega$$

(Electrical Machinery-III, Kerala Univ. Apr. 1976)

2. The full-load slip of a 3-phase double-cage induction motor is 6% and the two cages have impedances of  $(3.5 + j 1.5) \Omega$  and  $(0.6 + j 7.0) \Omega$  respectively. Neglecting stator impedances and magnetising current, calculate the starting torque in terms of full-load torque. [79%]
3. In a double-cage induction motor, if the outer cage has an impedance at standstill of  $(2 + j 2)$  ohm and the inner cage an impedance of  $(0.5 + j 5) \Omega$ , determine the slip at which the two cages develop equal torques. [17.7%]
4. The two independent cages of a rotor have the respective standstill impedance of  $(3 + j 1)$  ohm and  $(1 + j 4)$  ohm. What proportion of the total torque is due to the outer cage (a) at starting and (b) at a fractional slip of 0.05? [(a) 83.6% (b) 25.8%] (Principle of Elect. Engg.I, Jadavpur Univ. 1975)
5. An induction motor has a double cage rotor with equivalent impedance at standstill of  $(1.0 + j 1.0)$  and  $(0.2 + j 4.0)$  ohm. Find the relative value of torque given by each cage at a slip of 5%. [(a) 40.1 (b) 0.4 : 1]

(Electrical Machines-I, Gwalior Univ. Nov. 1977)

### 35.18. Speed Control of Induction Motors\*

A 3-phase induction motor is practically a constant-speed machine, more or less like a d.c. shunt motor. The speed regulation of an induction motor (having low resistance) is usually less than 5% at full-load. However, there is one difference of practical importance between the two. Whereas d.c. shunt motors can be made to run at any speed within wide limits, with good efficiency and speed regulation, merely by manipulating a simple field rheostat, the same is not possible with induction motors. In their case, speed reduction is accompanied by a corresponding loss of efficiency and good speed regulation. That is why it is much easier to build a good adjustable-speed d.c. shunt motor than an adjustable speed induction motor.

Different methods by which speed control of induction motors is achieved, may be grouped under two main headings :

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\* For Electronic Control of AC Motors, please consult the relevant chapter of this book, in vol. III.

**1. Control from stator side**

- (a) by changing the applied voltage                      (b) by changing the applied frequency  
 (c) by changing the number of stator poles

**2. Control from rotor side**

- (d) rotor rheostat control  
 (e) by operating two motors in concatenation or cascade  
 (f) by injecting an e.m.f. in the rotor circuit.

A brief description of these methods would be given below :

**(a) Changing Applied Voltage**

This method, though the cheapest and the easiest, is rarely used because

- (i) a large change in voltage is required for a relatively small change in speed  
 (ii) this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

**(b) Changing the Applied Frequency**

This method is also used very rarely. We have seen that the synchronous speed of an induction motor is given by  $N_s = 120f/P$ . Clearly, the synchronous speed (and hence the running speed) of an induction motor can be changed by changing the supply frequency  $f$ . However, this method could only be used in cases where the induction motor happens to be the only load on the generators, in which case, the supply frequency could be controlled by controlling the speed of the prime movers of the generators. But, here again the range over which the motor speed may be varied is limited by the economical speeds of the prime movers. This method has been used to some extent on electrically-driven ships.

**(c) Changing the Number of Stator Poles**

This method is *easily applicable to squirrel-cage motors* because the squirrel-cage rotor adopts itself to any reasonable number of stator poles.

From the above equation it is also clear that the synchronous (and hence the running) speed of an induction motor could also be changed by changing the number of stator poles. This change of number of poles is achieved by having two or more entirely independent stator windings in the same slots. Each winding gives a different number of poles and hence different synchronous speed. For example, a 36-slot stator may have two 3- $\phi$  windings, one with 4 poles and the other with 6-poles. With a supply frequency of 50-Hz, 4-pole winding will give  $N_s = 120 \times 50/4 = 1500$  r.p.m. and the 6-pole winding will give  $N_s = 120 \times 50/6 = 1000$  r.p.m. Motors with four independent stator winding are also in use and they give four different synchronous (and hence running) speeds. Of course, one winding is used at a time, the others being entirely disconnected.

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Speeds in the ratio of 2:1 can be produced by a single winding if wound on the consequent-pole principle. In that case, each of the two stator windings can be connected by a simple switch to give two speeds, each, which means four speeds in all. For example, one stator winding may give 4 or 8-poles and the other 6 or 12-poles. For a supply frequency of 50-Hz, the four speeds will be 1500, 750, 1000 and 500 r.p.m. Another combination, commonly used, is to group 2- and 4-pole winding with a 6- and 12-pole winding, which gives four synchronous speeds of 3000, 1500, 1000 and 500 r.p.m.

**(d) Rotor Rheostat Control**

In this method (Fig. 35.36), which is applicable to slip-ring motors alone, the motor speed is reduced by introducing an external resistance in the rotor circuit. For this purpose, the rotor starter

may be used, provided it is continuously rated. *This method is, in fact, similar to the armature rheostat control method of d.c. shunt motors.*

It has been shown in Art 34.22 that near synchronous speed (i.e. for very small slip value),  $T \propto s/R_2$ .

It is obvious that for a given torque, slip can be increased i.e. speed can be decreased by increasing the rotor resistance  $R_2$ .

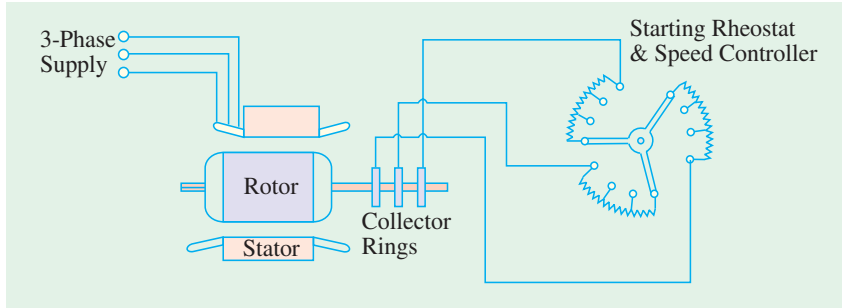


Fig. 35. 36

One serious disadvantage of this method is that with increase in rotor resistance,  $I^2R$  losses also increase which decrease the operating efficiency of the motor. In fact, the loss is directly proportional to the reduction in the speed.

The second disadvantage is the double dependence of speed, not only on  $R_2$  but on load as well.

Because of the wastefulness of this method, it is used where speed changes are needed for short periods only.

**Example 35.29.** The rotor of a 4-pole, 50-Hz slip-ring induction motor has a resistance of 0.30  $\Omega$  per phase and runs at 1440 rpm. at full load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before.

(Advanced Elect. Machines AMIE Sec.E1992)

**Solution.** The motor torque is given by  $T = \frac{K s R_2}{R_2^2 + (s X_2)^2}$

Since,  $X_2$  is not given,  $T = \frac{K s R_2}{R_2^2} = \frac{K s}{R_2}$

In the first case,  $T_1 = K s_1 / R_2$ ; in the second case,  $T_2 = K s_2 / (R_2 + r)$

where  $r$  is the external resistance per phase, added to the rotor circuit

Since  $T_1 = T_2 \quad \therefore K s_1 / R_2 = K s_2 / (R_2 + r)$  or  $(R_2 + r) / R_2 = s_2 / s_1$

Now,  $N_s = 120 \times 50 / 4 = 1500$  rpm;  $N_1 = 1440$  rpm;  $N_2 = 1320$  rpm

$\therefore s_1 = (1500 - 1440) / 1500 = 0.04$ ;  $s_2 = (1500 - 1320) / 1500 = 0.12$

$\therefore \frac{0.3 + r}{0.3} = \frac{0.12}{0.04} \quad \therefore r = 0.6 \Omega$

**Example 35.30.** A certain 3-phase, 6-pole, 50-Hz induction motor when fully-loaded, runs with a slip of 3%. Find the value of the resistance necessary in series per phase of the rotor to reduce the speed by 10%. Assume that the resistance of the rotor per phase is 0.2 ohm.

(Electrical Engineering-II (M), Bangalore Univ. 1989)

**Solution.**  $T = \frac{K s R_2}{R_2^2 + (s X_2)^2} = \frac{K_s R_2}{R_2^2} = \frac{K s}{R_2}$  - neglecting  $(s X_2)$

$\therefore T_1 = K s_1 / R_2$  and  $T_2 = K s_2 / (R_2 + r)$  where  $r$  is the external resistance per phase added to the rotor circuit.

Since  $T_1 = T_2, K s_1 / R_2 = K s_2 / (R_2 + r)$  or  $(R_2 + r) / R_2 = s_2 / s_1$

Now,  $N_s = 120 \times 50 / 6 = 1000$  rpm.,  $s_1 = 0.03, N_1 = 1000 (1 - 0.030) = 970$  rpm.

$$N_2 = 970 - 10\% \text{ of } 970 = 873 \text{ rpm.}, s_2 = (1000 - 873)/1000 = 0.127$$

$$\therefore \frac{0.2 + r}{0.2} = \frac{0.127}{0.03}; \quad r = \mathbf{0.65 \Omega}$$

**(e) Cascade or Concatenation or Tandem Operation**

In this method, two motors are used (Fig. 35.37) and are ordinarily mounted on the same shaft, *so that both run at the same speed* (or else they may be geared together).

The stator winding of the main motor A is connected to the mains in the usual way, while that of the auxiliary motor B is fed from the rotor circuit of motor A. For satisfactory operation, the main motor A should be phase-wound *i.e.* of slip-ring type with stator to rotor winding ratio of 1 : 1, so that, in addition to concatenation, each motor may be run from the supply mains separately.

There are at least three ways (and sometimes four ways) in which the combination may be run.

1. Main motor A may be run separately from the supply. In that case, the synchronous speed is  $N_{sa} = 120 f / P_a$  where  $P_a$  = Number of stator poles of motor A.

2. Auxiliary motor B may be run separately from the mains (with motor A being disconnected). In that case, synchronous speed is  $N_{sb} = 120 \times f / P_b$  where  $P_b$  = Number of stator poles of motor B.

3. The combination may be connected in cumulative cascade *i.e.* in such a way that the phase rotation of the stator fields of both motors is in the same direction. The synchronous speed of the cascaded set, in this case, is  $N_{sc} = 120 f / (P_a + P_b)$ .

**Proof**

Let  $N$  = actual speed of concatenated set ;  
 $N_{sa}$  = synchronous speed of motor A, it being independent of  $N$ .

Clearly, the relative speed of rotor A with respect to its stator field is  $(N_{sa} - N)$ . Hence, the frequency  $f'$  of the induced e.m.f. in rotor A is given by

$$f' = \frac{N_{sa} - N}{N_{sa}} \times f$$

*This is also the frequency of the e.m.f. applied to the stator of motor B.* Hence, the synchronous speed of motor B with this input frequency is

$$N' = 120 \frac{f'}{P_b} = \frac{120 (N_{sa} - N) f}{P_b \times N_{sa}} \quad \dots(i)$$

(Note that  $N'$  is not equal to  $N_{sb}$  which is the synchronous speed of motor B with supply frequency  $f$ ).

This will induce an e.m.f. of frequency, say,  $f''$  in the rotor B. Its value is found from the fact that the stator and rotor frequencies are proportional to the speeds of stator field and the rotor

$$\therefore f'' = \frac{N' - N}{N'} f$$

Now, on no-load, the speed of rotor B is almost equal to its synchronous speed, so that the frequency of induced e.m.f. is, to a first approximation, zero.

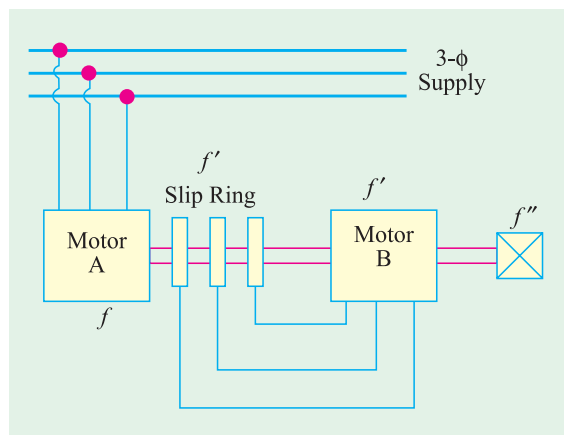


Fig. 35. 37

$$f'' = 0, \text{ or } \frac{N' - N}{N'} f = 0 \text{ or } N' = N \quad \dots(ii)$$

From (i) above 
$$N' = \frac{120 f (N_{sa} - N)}{P_b \times N_{sa}} = \frac{120 f}{P_b} \left( 1 - \frac{N}{N_{sa}} \right)$$

Hence, from (ii) above,

$$\frac{120 f}{P_b} \left( 1 - \frac{N}{N_{sa}} \right) = N \text{ or } \frac{120 f}{P_b} = N \left( 1 + \frac{1}{N_{sa}} \times \frac{120 f}{P_b} \right)$$

Putting  $N_{sa} = 120 f / P_a$ , we get

$$\frac{120 f}{P_b} = N \left( 1 + \frac{P_a}{120 f} \times \frac{120 f}{P_b} \right) = N \left( 1 + \frac{P_a}{P_b} \right) \therefore N = \frac{120 f}{(P_a + P_b)}$$

Concatenated speed of the set =  $120 f / (P_a + P_b)$

**How the Set Starts ?**

When the cascaded set is started, the voltage at frequency  $f$  is applied to the stator winding of machine  $A$ . An induced e.m.f. of the same frequency is produced in rotor  $A$  which is supplied to auxiliary motor  $B$ . Both the motors develop a forward torque. As the shaft speed rises, the rotor frequency of motor  $A$  falls and so does the synchronous speed of motor  $B$ . The set settles down to a stable speed when the shaft speed becomes equal to the speed of rotating field of motor  $B$ .

Considering load conditions, we find that the electrical power taken in by stator  $A$  is partly used to meet its  $I^2R$  and core losses and the rest is given to its rotor. The power given to rotor is further divided into two parts : one part, proportional to the speed of set *i.e.*  $N$  is converted into mechanical power and the other part proportional to  $(N_{sa} - N)$  is developed as electrical power at the slip frequency, and is passed on to the auxiliary motor  $B$ , which uses it for producing mechanical power and losses. Hence, approximately, the mechanical outputs of the two motors are in the ratio  $N : (N_{sa} - N)$ . **In fact, it comes to that the mechanical outputs are in the ratio of the number of poles of the motors.**

It may be of interest to the reader to know that it can be proved that

(i)  $s = f'' / f$  where  $s$  = slip of the set referred to its synchronous speed  $N_{sc}$ .  
 $= (N_{sc} - N) / N_{sc}$

(ii)  $s = s_a s_b$

where  $s_a$  and  $s_b$  are slips of two motors, referred to their respective stators *i.e.*

$$s_a = \frac{N_{sa} - N}{N_{sa}} \text{ and } s_b = \frac{N' - N}{N}$$

**Conclusion**

We can briefly note the main conclusions drawn from the above discussion :

(a) the mechanical outputs of the two motors are in the ratio of their number of poles.

(b)  $s = f'' / f$  (c)  $s = s_a s_b$

4. The fourth possible connection is the **differential cascade**. In this method, the phase rotation of stator field of the motor  $B$  is opposite to that of the stator of motor  $A$ . This reversal of phase rotation of stator of motor  $B$  is obtained by interchanging any of its two leads. It can be proved in the same way as above, that for this method of connection, the synchronous speed of the set is

$$N_{sc} = 120 f / (P_a - P_b)$$

As the differentially-cascaded set has a very small or zero starting torque, this method is rarely used. Moreover, the above expression for synchronous speed becomes meaningless for  $P_a = P_b$ .

**Example 35.31.** Two 50-Hz, 3- $\phi$  induction motors having six and four poles respectively are cumulatively cascaded, the 6-pole motor being connected to the main supply. Determine the

frequencies of the rotor currents and the slips referred to each stator field if the set has a slip of 2 per cent. (Elect. Machinery-II Madras Univ. 1987)

**Solution.** Synchronous speed of set  $N_{sc} = 120 \times 50/10 = 600$  r.p.m.

Actual rotor speed  $N = (1 - s) N_{sc} = (1 - 0.02) 600 = 588$  r.p.m.

Synchronous speed of the stator field of 6-pole motor,  $N_{sa} = 120 \times 50/6 = 1000$  r.p.m.

Slip referred to this stator field is

$$s_a = \frac{N_{sa} - N}{N_{sa}} = \frac{1000 - 588}{1000} = 0.412 \quad \text{or} \quad \mathbf{41.2\%}$$

Frequency of the rotor currents of 6-pole motor  $f' = s_a f = 0.412 \times 50 = \mathbf{20.6 \text{ Hz}}$

This is also the frequency of stator currents of the four pole motor. The synchronous speed of the stator of 4-pole motor is

$$N' = 120 \times 20.6/4 = 618 \text{ r.p.m.}$$

This slip, as referred to the 4-pole motor, is  $s_b = \frac{N' - N}{N'} = \frac{618 - 588}{618}$

$$= 0.0485 \text{ or } \mathbf{4.85\%}$$

The frequency of rotor current of 4-pole motor is

$$f'' = s_b f' = 0.0485 \times 20.6 = \mathbf{1.0 \text{ Hz (approx)}}$$

As a check,

$$f'' = s f = 0.02 \times 50 = 1.0 \text{ Hz}$$

**Example 35.32.** A 4-pole induction motor and a 6-pole induction motor are connected in cumulative cascade. The frequency in the secondary circuit of the 6-pole motor is observed to be 1.0 Hz. Determine the slip in each machine and the combined speed of the set. Take supply frequency as 50 Hz. (Electrical Machinery-II, Madras Univ. 1986)

**Solution.** With reference to Art. 35.18 (e) and Fig. 35.38

$$N_{sc} = 120 \times 50/(4 + 6) = 600 \text{ r.p.m.}$$

$$s = f''/f = 1/50 = 0.02$$

$N$  = actual speed of the concatenated set

$$\therefore 0.02 = \frac{600 - N}{600} \quad \text{or} \quad N = \mathbf{588 \text{ r.p.m.}}$$

$$N_{sa} = 120 \times 50/4 = 1500 \text{ r.p.m.}$$

$$s_a = (1500 - 588)/1500$$

$$= 0.608 \quad \text{or} \quad \mathbf{60.8\%}$$

$$f' = s_a f = 0.608 \times 50 = 30.4 \text{ Hz}$$

$N'$  = synchronous speed of 6-pole motor with frequency  $f'$

$$= 120 \times 30.4/6 = 608 \text{ r.p.m.}$$

$$s_b = \frac{N' - N}{N'} = \frac{608 - 588}{608} = 0.033 \quad \text{or} \quad \mathbf{3.3\%}$$

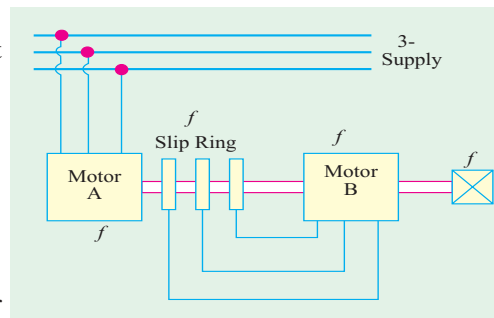


Fig. 35.38

**Example 35.33.** The stator of a 6-pole motor is joined to a 50-Hz supply and the machine is mechanically coupled and joined in cascade with a 4-pole motor, Neglecting all losses, determine the speed and output of the 4-pole motor when the total load on the combination is 74.6 kW.

**Solution.** As all losses are neglected, the actual speed of the rotor is assumed to be equal to the synchronous speed of the set.

Now, 
$$N_{sc} = 120 \times 50/10 = \mathbf{600 \text{ r.p.m.}}$$

As said earlier, mechanical outputs are in the ratio of the number of poles of the motors.

∴ output of 4-pole motor =  $74.6 \times 4/10 = 29.84 \text{ kW}$

**Example 35.34.** A cascaded set consists of two motors A and B with 4 poles and 6 poles respectively. The motor A is connected to a 50-Hz supply. Find

(i) the speed of the set

(ii) the electric power transferred to motor B when the input to motor A is 25 kW. Neglect losses. (Electric Machines-I, Utkal Univ. 1990)

**Solution.** Synchronous speed of the set is\*

$$N_{sc} = 120 f / (P_a + P_b) = 120 \times 50 / (6 + 4) = 600 \text{ r.p.m.}$$

(ii) The outputs of the two motors are proportional to the number of their poles.

∴ output of 4-pole motor B =  $25 \times 4/10 = 10 \text{ kW}$

**(f) Injecting an e.m.f. in the Rotor Circuit**

In this method, the speed of an induction motor is controlled by injecting a voltage in the rotor circuit, it being of course, necessary for the injected voltage to have the same frequency as the slip frequency. There is, however, no restriction as to the phase of the injected e.m.f.

When we insert a voltage which is in *phase opposition* to the induced rotor e.m.f., it amounts to increasing the rotor resistance, whereas inserting a voltage which is in phase with the induced rotor e.m.f., is equivalent to *decreasing* its resistance. Hence, by changing the phase of the injected e.m.f. and hence the rotor resistance, the speed can be controlled.

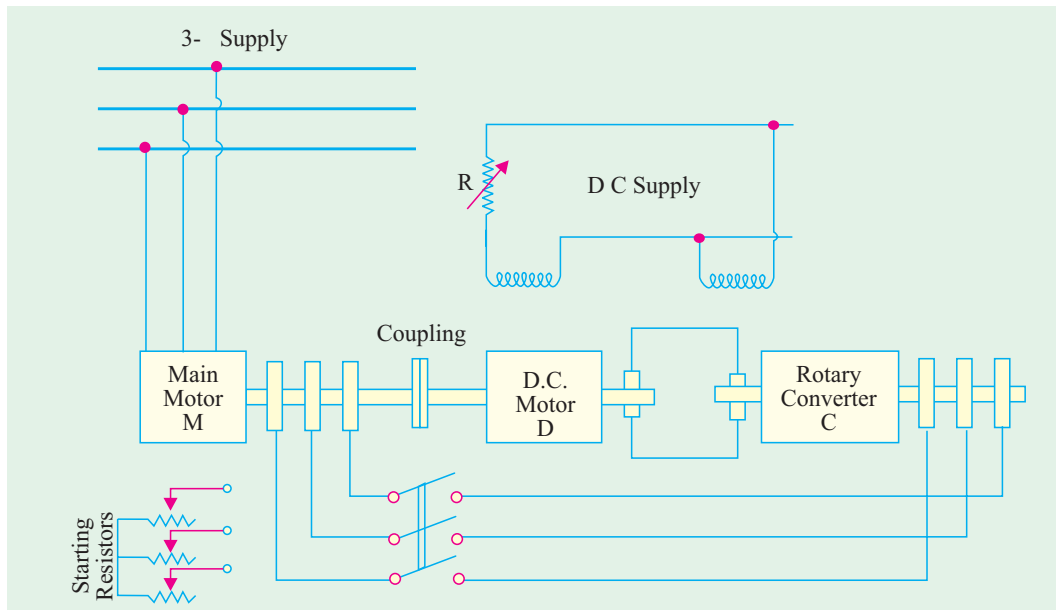


Fig. 35.39

One such practical method of this type of speed control is Kramer system, as shown in Fig. 35.39, which is used in the case of large motors of 4000 kW or above. It consists of a rotary converter C which converts the low-slip frequency a.c. power into d.c. power, which is used to drive a d.c. shunt motor D, mechanically coupled to the main motor M.

The main motor is coupled to the shaft of the d.c. shunt motor D. The slip-rings of M are connected to those of the rotary converter C. The d.c. output of C is used to drive D. Both C and D

\* It is assumed that the two motors are connected in cumulative cascade.

are excited from the d.c. bus-bars or from an exciter. There is a field regulator which governs the back e.m.f.  $E_b$  of  $D$  and hence the d.c. potential at the commutator of  $C$  which further controls the slip-ring voltage and therefore, the speed of  $M$ .

One big advantage of this method is that any speed, within the working range, can be obtained instead of only two or three, as with other methods of speed control.

Yet another advantage is that if the rotary converter is over-excited, it will take a leading current which compensates for the lagging current drawn by main motor  $M$  and hence improves the power factor of the system.

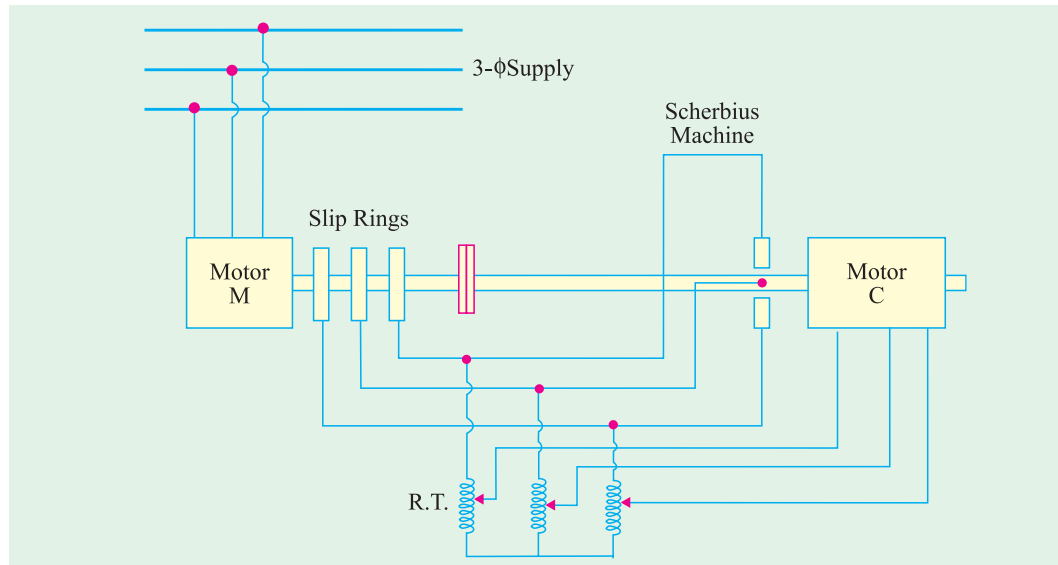


Fig. 35.40

In Fig. 35.40 is shown another method, known as Scherbius system, for controlling the speed of large induction motors. The slip energy is not converted into d.c. and then fed to a d.c. motor, rather it is fed directly to a special 3-phase (or 6-phase) a.c. commutator motor-called a, Scherbius machine.

The polyphase winding of machine  $C$  is supplied with the low-frequency output of machine  $M$  through a regulating transformer  $RT$ . The commutator motor  $C$  is a variable-speed motor and its speed (and hence that of  $M$ ) is controlled by either varying the tapings on  $RT$  or by adjusting the position of brushes on  $C$ .

### Tutorial Problems 35.5

1. An induction motor has a double-cage rotor with equivalent impedances at standstill of  $(1.0 + j 1.0)$  and  $(0.2 + j 4.0) \Omega$ . Find the relative values of torque given by each cage (a) at starting and (b) at 5 % slip [(a) 40:1 (b) 0.4:1] *(Adv. Elect. Machines AMIE Sec. B 1991)*
2. The cages of a double-cage induction motor have standstill impedances of  $(3.5 + j 1.5) \Omega$  and  $(0.6 + j 7.0) \Omega$  respectively. The full-load slip is 6%. Find the starting torque at normal voltage in terms of full-load torque. Neglect stator impedance and magnetizing current. [300%] *(Elect. Machines-I, Nagpur Univ. 1993)*
3. The rotor of a 4 pole, 50 Hz, slip ring induction motor has a resistance of 0.25 ohm per phase and runs at 1440 rpm at full-load. Calculate the external resistance per phase, which must be added to lower the speed to 1200 rpm, the torque being same as before. [1  $\Omega$ ] *(Utilisation of Electric Power (E-8) AMIE Sec. B Summer 1992)*



### 35.19. Three-phase A.C. Commutator Motors

Such motors have shunt speed characteristics *i.e.* change in their speed is only moderate, as compared to the change in the load. They are ideally suited for drives, requiring a uniform accelerating torque and continuously variable speed characteristics over a wide range. Hence, they find wide use in high-speed lifts, fans and pumps and in the drives for cement kilns, printing presses, pulverised fuel plants, stokers and many textile machines. Being more complicated, they are also more expensive than single-speed motors. Their efficiency is high over the whole speed range and their power factor varies from low value at synchronous speed to unity at maximum (supersynchronous) speed.

The speed control is obtained by injecting a variable voltage at **correct** frequency into the secondary winding of the motor via its commutator. If injected voltage assists the voltage induced in the secondary winding, the speed is increased but if it is in the opposing direction, then motor speed is reduced. The commutator acts as a frequency changer because it converts the supply frequency of the regulating voltage to the slip frequency corresponding to the speed required.

Following are the two principal types of such motors :

- (i) Schrage or rotor-fed or brush shift motor and (ii) stator-fed or induction-regulator type motor.

### 35.20. Schrage Motor\*

It is a rotor-fed, shunt-type, brush-shifting, 3-phase commutator induction motor which has built-in arrangement both for **speed control and power factor improvement**. In fact, it is an induction motor with a built-in slip-regulator. It has **three** windings: two in rotor and one in stator as shown in Fig. 35.41 and 35.42 (a) The three windings are as under:

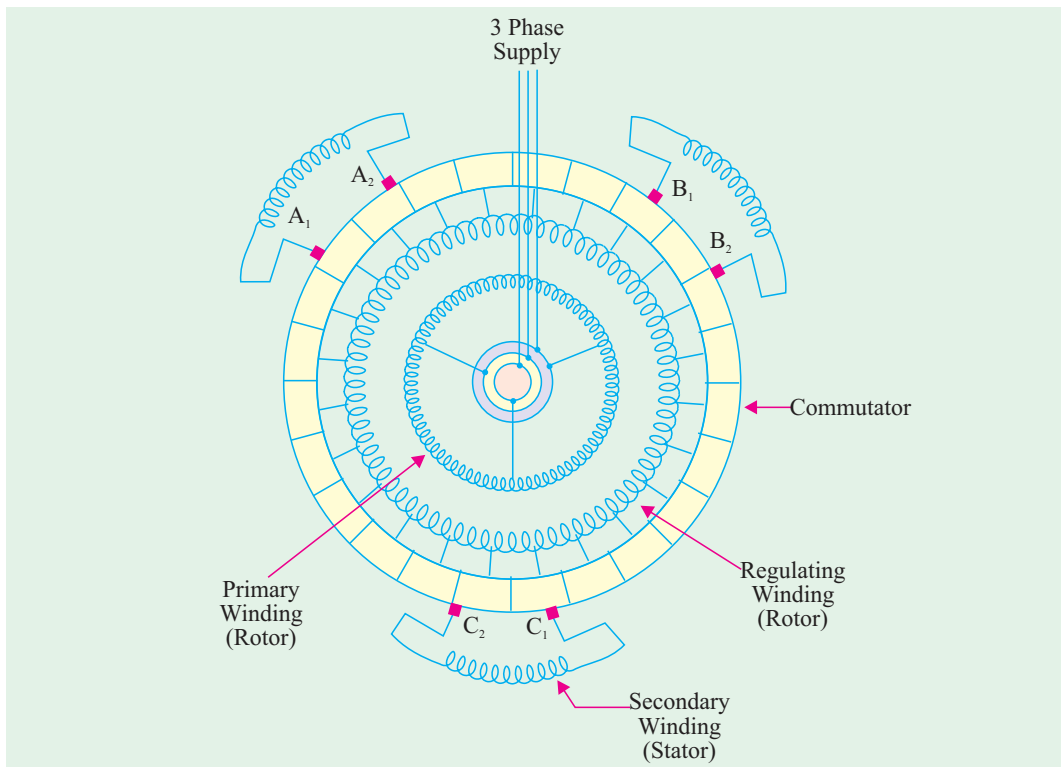


Fig. 35.41

- (i) **Primary winding.** It is housed in the lower part of the rotor slots (not stator) and is supplied through slip-rings and brushes at **line** frequency. It generates the working flux in the machine.

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\* After the name of its inventor K.H. Schrage of Sweden.

(ii) **Regulating winding.** It is variously known as **compensating** winding or **tertiary** winding. It is also housed in **rotor slots** (in the upper part) and is connected to the commutator in a manner similar to the armature of a d.c. motor.

(iii) **Secondary winding.** It is contained in the **stator slots**, but end of each phase winding is connected to one of the pair of brushes arranged on the commutator. These brushes are mounted on two separate brush rockers, which are designed to move in opposite directions relative to the centre line of the corresponding stator phase (usually by a rack and pinion mechanism). Brushes  $A_1, B_1$ , and  $C_1$  move together and are 120 electrical degrees apart. Similarly, brushes  $A_2, B_2$  and  $C_2$  move together and are also 120 electrical degrees apart. A sectional drawing of the motor is shown in Fig. 35.44.

#### (a) Working

When primary is supplied at line frequency, there is transformer action between primary and regulating winding and normal induction motor action between primary and secondary winding. Hence, voltage at line frequency is induced in the regulating winding by transformer action. The commutator, acting as a frequency changer, converts this line-frequency voltage of the regulating winding to the slip frequency for feeding it into the secondary winding on the stator. The voltage across brush pairs  $A_1A_2, B_1B_2$  and  $C_1C_2$  increases as brushes are separated. In fact, magnitude of the voltage injected into the secondary winding depends on the angle of separation of the brushes  $A_1$  and  $A_2, B_1$  and  $B_2$  and  $C_1$  and  $C_2$ . How slip-frequency e.m.f. is induced in secondary winding is detailed below:

When 3- $\phi$  power is connected to slip-rings, synchronously rotating field is set up in the rotor core. Let us suppose that this field revolves in the clockwise direction. Let us further suppose that brush pairs are on **one commutator segment**, which means that secondary is short-circuited. With rotor still at rest, this field cuts the secondary winding, thereby inducing voltage and so producing currents in it which react with the field to produce **clockwise (CW)** torque in the **stator**. Since stator cannot rotate, as a reaction, it makes the rotor rotate in the **counterclockwise (CCW)** direction.

Suppose that the rotor speed is  $N$  rpm. Then

1. rotor flux is still revolving with synchronous speed relative to the primary and regulating winding.
2. however, this rotor flux will rotate at slip speed  $(N_s - N)$  relative to the stator. It means that the revolving rotor flux will rotate at slip speed in space.
3. if rotor could rotate at synchronous speed *i.e.* if  $N = N_s$ , then flux would be stationary in space (*i.e.* relative to stator) so that there would be no cutting of the secondary winding by the flux and, consequently, no torque would be developed in it.

As seen from above, in a Schrage motor, the flux rotates at synchronous speed, relative to rotor but with slip speed relative to space (*i.e.* stator), whereas in a normal induction motor, flux rotates synchronously relative to stator (*i.e.* space) but with slip speed relative to the rotor. (Art. 34.11).

Another point worth noting is that since at **synchronous** speed, magnetic field is stationary in space, the regulating winding acts as a d.c. armature and the direct current taken from the commutator flows in the secondary winding. Hence, Schrage motor then operates like a synchronous motor.

#### (b) Speed Control

It is quite easy to obtain speeds above as well as below synchronism in a Schrage motor. As shown in Fig. 35.42 (b) (i) when brush pairs are together on the same commutator segment (*i.e.* are electrically connected via commutator), the secondary winding is short-circuited and the machine

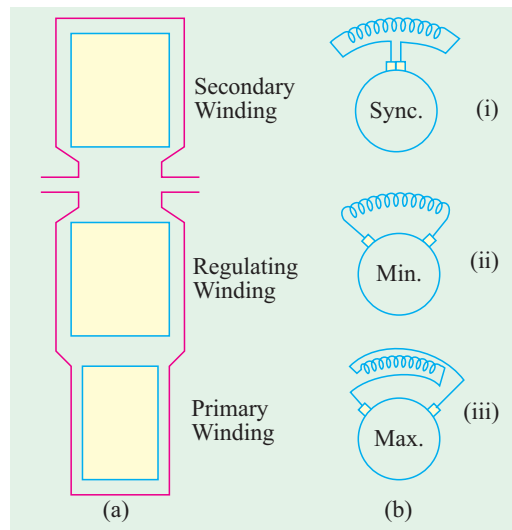


Fig. 35.42

operates as an inverted\* plain squirrel-cage induction motor, running with a small positive slip. Parting the brushes in one direction, as shown in Fig. 35.42 (b) (ii) produces subsynchronous speeds, because in this case, regulating voltage injected into the secondary winding *opposes* the voltage induced in it from primary winding. However, when movement of brushes is reversed and they are parted in opposite directions, the direction of the regulating voltage is reversed and so motor speed *increases* to super synchronous (maximum) value, as shown in Fig. 35.41 (b) (iii) The commutator provides maximum voltage when brushes are separated by one pole pitch.

No-load motor speed is given by  $N \cong N_s (1 - K \sin 0.5\beta)$  where  $\beta$  is brush separation in electrical degrees and  $K$  is a constant whose value depends on turn ratio of the secondary and regulating windings.

Maximum and minimum speeds are obtained by changing the magnitude of the regulating voltage. Schrage motors are capable of speed variations from zero to nearly twice the synchronous speed, though a speed range of 3:1 is sufficient for most applications. It is worth noting that Schrage motor is essentially a shunt machine, because for a particular brush separation, speed remains approximately constant as the load torque is increased as happens with dc shunt motors (Art 29.14).

**(c) Power Factor Improvement**

Power factor improvement can be brought about by changing the phase angle of the voltage injected into the secondary winding. As shown in Fig. 35.43, when one set of brushes is advanced more rapidly than the other is retarded, then injected voltage has a quadrature component which leads the rotor induced voltage. Hence, it results in the improvement of motor power factor. This differential movement of brush sets is obtained by coupling the brush rockers to the hand wheel with gears having differing ratios. In Schrage motor, speed depends on angular distance between the individual brush sets ( $A_1$  and  $A_2$  in Fig. 35.41) but p.f. depends on the angular positions of the brushes as a whole.

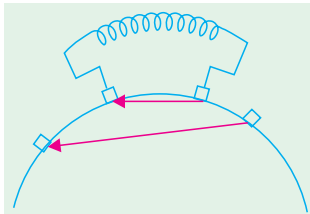


Fig. 35.43

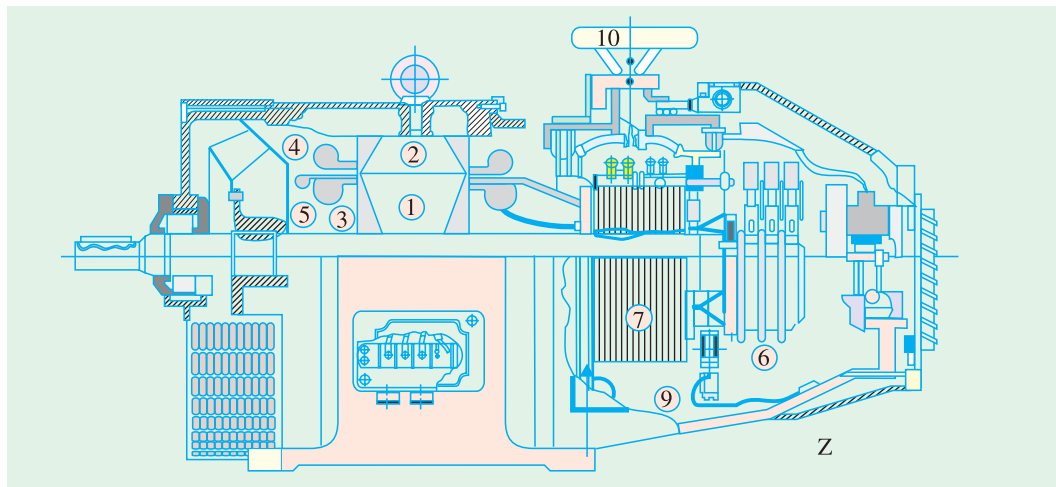


Fig. 35.44. Sectional drawing of a Schrage motor (Courtesy : Elektra Faurandou, Germany)

**(d) Starting**

Schrage motors are usually started with brushes in the lowest speed position by direct-on contactor starters. Usually, interlocks are provided to prevent the contactor getting closed on the line when brushes are in any other position. One major disadvantage of this motor is that its operating voltage is limited to about 700 V because a.c. power has to be fed through slip-rings. It is available in sizes upto 40 kW and is designed to operate on 220, 440 and 550 V. It is ordinarily wound for four or six poles.

\* Inverted in the sense that primary is in the rotor and secondary in the stator - just opposite of that in the normal induction motor (Art. 34.3).



Fig. 35.44 shows a sectional drawing of a Schrage motor. The details of different parts labelled in the diagram are as under:

1. rotor laminations 2. stator laminations 3. primary winding  
4. secondary winding 5. regulating winding 6. slip-ring unit  
7. commutator 8. cable feed for outer brush yoke 9. cable feed for inner brush yoke 10. hand wheel.

### 35.21. Motor Enclosures

Enclosed and semi-enclosed motors are practically identical with open motors in mechanical construction and in their operating characteristics. Many different types of frames or enclosures are available to suit particular requirements. Some of the common type enclosures are described below:

#### (i) Totally-enclosed, Non-ventilated Type

Such motors have solid frames and end-shields, but no openings for ventilation. They get cooled by surface radiation only (Fig. 35.45). Such surface-cooled motors are seldom furnished in sizes above two or three kW, because higher ratings require frames of much larger sizes than fan-cooled motors of corresponding rating.

**Fig. 35.45.** Totally-enclosed surface cooled 750 W, 750 r.p.m. high torque induction motor (Courtesy : Jyoti Limited)



**Fig. 35.46.** Totally-enclosed, fan-cooled 10-kW 440/400-V, 1000 r.p.m. 50-Hz induction motor (Courtesy : Jyoti Limited)

**Fig. 35.47.** Squirrel-cage motor, showing cowl over the external fan. (Courtesy : General Electric Co. of India)

#### (ii) Splash-proof Type

In the frames of such motors, the ventilating openings are so constructed that the liquid drops or dust particles falling on the motor are coming towards it in a straight line at any angle not greater than  $100^\circ$  from the vertical are not able to enter the motor either directly or by striking and running along the surface.

#### (iii) Totally-enclosed, Fan-cooled Type

In such motors (Fig. 35.46), cooling air is drawn into the motor by a fan mounted on the shaft. This air is forced through the motor between the inner fully-enclosed frame and an outer shell, over the end balls and the stator laminations and is then discharged through openings in the opposite side. An internal fan carries the generated heat to the totally enclosing frame, from where it is conducted to

the outside. Because of totally enclosing frame, all working parts are protected against corrosive or abrasive effects of fumes, dust, and moisture.

**(iv) Cowl-covered Motor**

These motors are simplified form of fan-cooled motors (Fig. 35.47). These consist of totally-enclosed frame with a fan and cowl mounted at the end opposite to the driving end. The air is drawn into the cowl with the help of fan and is then forced over the frame. The contours of the cowl guide the cooling air in proper directions. These motors are superior to the usual fan-cooled motors for operation in extremely dusty atmosphere *i.e.* gas works, chemical works, collieries and quarries etc. because there are no air passages which will become clogged with dust.



Fig. 35.48. Squirrel cage motor

**(v) Protected Type**

This construction consists of perforated covers for the openings in both end shields



Fig. 35.49. Protected slip-ring motor with totally enclosed slip-rings. (Courtesy : General Electric Co. of India)

**(vi) Drip-proof Motors**

The frames of such motors are so constructed that liquid drops or dust particles, falling on the machine at any angle greater than 15° from the vertical, cannot enter the motor, either directly or by striking and running along a horizontal or inwardly inclined smooth surface (Fig. 35.51).

**(vii) Self (Pipe) Ventilated Type**

The construction of such motors consists of enclosed shields with provision for pipe connection on both the shields. The motor fan circulates sufficient air through pipes which are of ample section.

**(viii) Separately (Forced) Ventilated Type**

These motors are similar to the self-ventilated type except that ventilation is provided by a separated blower.

**35.22. Standard Types of Squirrel-cage Motors**

Different types of 3-phase squirrel-cage motors have been standardized, according to their electric characteristics, into six types, designated as design A, B, C, D, E and F respectively. The original commercial squirrel-cage induction motors which were of shallow-slot type are designated as class A. For this reason, Class A motors are used as a reference and are referred to as ‘normal starting-torque, normal starting-current, normal slip’ motors.

**(i) Class A** - Normal starting torque, normal starting current, normal slip

**(ii) Class B** - Normal starting torque, low starting current, normal slip

**(iii) Class C** - High starting torque, low starting current, normal slip



Fig. 35.50. Squirrel cage A C induction motor

(iv) Class *D* - High starting torque, low starting current, high slip

(v) Class *E* - Low starting torque, normal starting current, low slip

(vi) Class *F* - Low starting torque, low starting current, normal slip

### 35.23. Class A Motors

It is the most popular type and employs squirrel cage having relatively low resistance and reactance. Its locked-rotor current with full voltage is generally more than 6 times the rated full-load current. For smaller sizes and number of poles, the starting torque with full voltage is nearly twice the full-load torque whereas for larger sizes and number of poles, the corresponding figure is 1.1 times the full-load torque. The full-load slip is less than 5 per cent. The general configuration of slot construction of such motors is shown in Fig. 35.52. As seen, the rotor bars are placed close to the surface so as to reduce rotor reactance.

Such motors are used for fans, pumps, compressors and conveyors etc. which are started and stopped in frequently and have low inertia loads so that the motor can accelerate in a few seconds.

### 35.24. Class B Motors

These motors are so built that they can be started at full-load while developing normal starting torque with relatively low starting current. Their locked-rotor current with full voltage applied is generally 5 to 5 1/2 times the full-load current. Their cages are of high reactance as seen from Fig. 35.53. The rotor is constructed with deep and narrow bars so as to obtain high reactance during starting.

Such motors are well-suited for those applications where there is limitation on the starting current or if the starting current is still in excess of what can be permitted, then reduced voltage starting is employed. One of the common applications of such motors is large fans most of which have high moment of inertia. It also finds wide use in many machine tool applications, for pumps of centrifugal type and for driving electric generators.

### 35.25. Class C Motors

Such motors are usually of double squirrel-cage type (Fig. 35.54) and combine high starting torque with low starting current. Their locked-rotor currents and slip with full voltage applied are nearly the same as for class *B* motors. Their starting torque with full voltage applied is usually 2.75 times the full-load torque.

For those applications where reduced voltage starting does not give sufficient torque to start the load with either class *A* or *B* motor, class *C* motor, with its high inherent starting torque along with reduced starting current supplied by reduced-voltage starting may be used. Hence, it is frequently used for crushers, compression pumps, large refrigerators, conveyor equipment, textile machinery, boring mills and wood-working equipment etc.



Fig. 35.51. Drip-proof slip-ring 50 up, 440/400-V, 50HZ, 1000 r.p.m. motor (courtesy : Jyoti Limited)

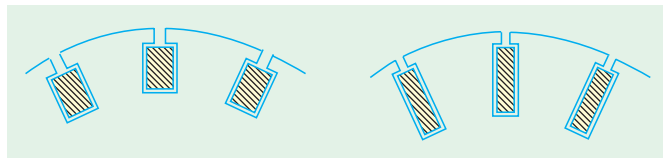


Fig. 35.52

Fig. 35.53

**35.26. Class D Motors**

Such motors are provided with a high-resistance squirrel cage giving the motor a high starting torque with low starting current. Their locked-rotor currents with full voltage applied are of the same order as for class C motors. Their full-load slip varies from 5% to 20 per cent depending on the application. Their slot structure is shown in Fig. 35.55. For obtaining high starting torque with low starting current, thin rotor bars are used which make the leakage flux of the rotor low and the useful flux high.

Since these motors are used where extremely high starting torque is essential, they are usually used for bulldozers, shearing machines, punch presses, foundry equipment, stamping machines, hoists, laundry equipment and metal drawing equipment etc.

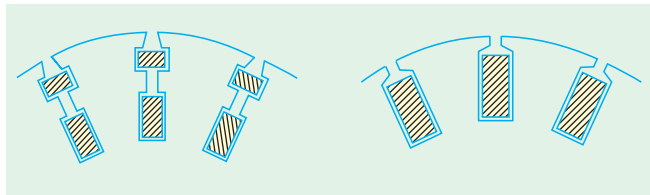


Fig. 35.54

Fig. 35.55

**35.27. Class E Motors**

These motors have a relatively low slip at rated load. For motors above 5 kW rating, the starting current may be sufficiently high as to require a compensator or resistance starter. Their slot structure is shown in Fig. 35.56 (a).

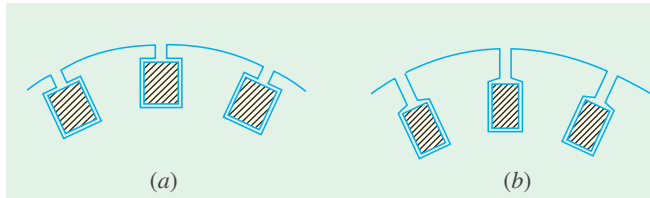


Fig. 35.56

**35.28. Class F Motors**

Such motors combine a low starting current with a low starting torque and may be started on full voltage. Their low starting current is due to the design of rotor which has high reactance during starting [Fig. 35.56 (b)]. The locked rotor currents with full voltage applied and the full-load slip are in the same range as those for class B and C motors. The starting torque with full voltage applied is nearly 1.25 times the full-torque.

**QUESTIONS AND ANSWERS ON THREE-PHASE INDUCTION MOTORS**

**Q. 1. How do changes in supply voltage and frequency affect the performance of an induction motor ?**

**Ans.** High voltage decreases both power factor and slip, but increases torque. Low voltage does just the opposite. Increase in frequency increases power factor but decreases the torque. However, per cent slip remains unchanged. Decrease in frequency decreases power factor but increases torque leaving per cent slip unaffected as before.

**Q. 2. What is, in brief, the basis of operation of a 3-phase induction motor ?**

**Ans.** The revolving magnetic field which is produced when a 3-phase stator winding is fed from a 3-phase supply.

**Q. 3. What factors determine the direction of rotation of the motor ?**

**Ans.** The phase sequence of the supply lines and the order in which these lines are connected to the stator winding.

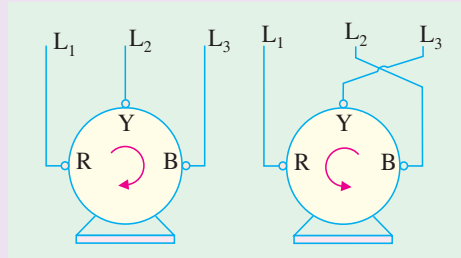


Fig. 35.57

**Q. 4. How can the direction of rotation of the motor be reversed ?**

**Ans.** By transposing or changing over any two line leads, as shown in Fig. 35.57.

**Q. 5. Why are induction motors called asynchronous ?**

**Ans.** Because their rotors can never run with the synchronous speed.

**Q. 6. How does the slip vary with load ?**

**Ans.** The greater the load, greater is the slip or slower is the rotor speed.

**Q. 7. What modifications would be necessary if a motor is required to operate on voltage different from that for which it was originally designed ?**

**Ans.** The number of conductors per slot will have to be changed in the same ratio as the change in voltage. If the voltage is doubled, the number of conductors per slot will have to be doubled.

**Q. 8. Enumerate the possible reasons if a 3-phase motor fails to start.**

**Ans.** Any one of the following reasons could be responsible :

1. one or more fuses may be blown.
2. voltage may be too low.
3. the starting load may be too heavy.
4. worn bearings due to which the armature may be touching field laminae, thus introducing excessive friction.

**Q. 9. A motor stops after starting *i.e.* it fails to carry load. What could be the causes ?**

**Ans.** Any one of the following:

1. hot bearings, which increase the load by excessive friction.
2. excessive tension on belt, which causes the bearings to heat.
3. failure of short cut-out switch.
4. single-phasing on the running position of the starter.

**Q. 10. Which is the usual cause of blow-outs in induction motors ?**

**Ans.** The commonest cause is single-phasing.

**Q. 11. What is meant by 'single-phasing' and what are its causes ?**

**Ans.** By single-phasing is meant the opening of one wire (or leg) of a three-phase circuit whereupon the remaining leg at once becomes single-phase. When a three-phase circuit functions normally, there are three distinct currents flowing in the circuit. As is known, any two of these currents use the third wire as the return path *i.e.* one of the three phases acts as a return path for the other two. Obviously, an open circuit in one leg kills two of the phases and there will be only one current or phase working, even though two wires are left intact. The remaining phase attempts to carry all the load. The usual cause of single-phasing is, what is generally referred to as **running fuse**, which is a fuse whose current-carrying capacity is equal to the full-load current of the motor connected in the circuit. This fuse will blow-out whenever there is overload (either momentary or sustained) on the motor.

**Q. 12. What happens if single-phasing occurs when the motor is running ? And when it is stationary ?**

**Ans.** (i) If already running and carrying half load or less, the motor will continue running as a single-phase motor on the remaining single-phase supply, without damage because half loads do not blow normal fuses.

(ii) If motor is very heavily loaded, then it will stop under single-phasing and since it can neither restart nor blow out the remaining fuses, the burn-out is very prompt.

A stationary motor will not start with one line broken. In fact, due to heavy standstill current, it is likely to burn-out quickly unless immediately disconnected.

**Q. 13. Which phase is likely to burn-out in a single-phasing delta-connected motor, shown in Fig. 35.58.**

**Ans.** The *Y*-phase connected across the live or operative lines carries nearly three times its normal current and is the one most likely to burn-out.

The other two phases *R* and *B*, which are in series across  $L_2$  and  $L_3$  carry more than their full-load currents.



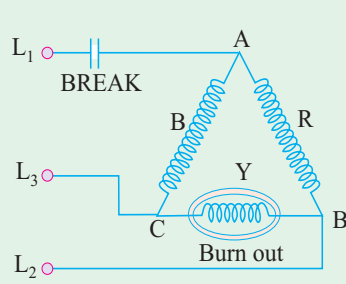


Fig. 35.58

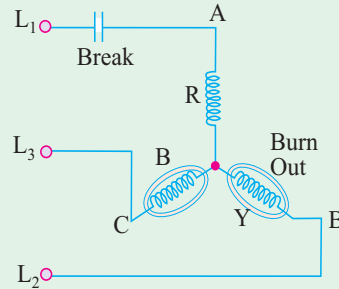


Fig. 35.59

**Q. 14. What currents flow in single-phasing star-connected motor of Fig. 35.59.**

**Ans.** With  $L_1$  disabled, the currents flowing in  $L_2$  and  $L_3$  and through phases  $Y$  and  $B$  in series will be of the order of 250 per cent of the normal full-load current, 160 per cent on 3/4 load and 100 per cent on 1/2 load.

**Q. 15. How can the motors be protected against single-phasing ?**

**Ans.** (i) By incorporating a combined overload and single-phasing relay in the control gear.  
 (ii) By incorporating a phase-failure relay in the control gear. The relay may be either voltage or current-operated.

**Q. 16. Can a 3-phase motor be run on a single-phase line ?**

**Ans.** Yes, it can be. But a phase-splitter is essential.

**Q. 17. What is meant by a phase-splitter ?**

**Ans.** It is a device consisting of a number of capacitors so connected in the motor circuit that it produces, from a single input wave, three output waves which differ in phase from each other.

**Q. 18. What is the standard direction of rotation of an induction motor ?**

**Ans.** Counterclockwise, when looking from the front end *i.e.* non-driving end of the motor.

**Q. 19. Can a wound-motor be reversed by transposing any two leads from the slip-rings ?**

**Ans.** No. There is only one way of doing so *i.e.* by transposing any two line leads.

**Q. 20. What is jogging ?**

**Ans.** It means inching a motor *i.e.* make it move a little at a time by constant starting and stopping.

**Q. 21. What is meant by plugging ?**

**Ans.** It means stopping a motor by instantaneously reversing it till it stops.

**Q. 22. What are the indications of winding faults in an induction motor ?**

**Ans.** Some of the indications are as under:  
 (i) excessive and unbalanced starting currents  
 (ii) some peculiar noises and (iii) overheating.

### OBJECTIVE TESTS – 35

- In the circle diagram for a 3- $\phi$  induction motor, the diameter of the circle is determined by
  - rotor current
  - exciting current
  - total stator current
  - rotor current referred to stator.
- Point out the WRONG statement.  
 Blocked rotor test on a 3- $\phi$  induction motor helps to find
  - short-circuit current with normal voltage
  - short-circuit power factor
  - fixed losses
  - motor resistance as referred to stator.
- In the circle diagram of an induction motor, point of maximum input lies on the tangent drawn parallel to
  - output line
  - torque line
  - vertical axis
  - horizontal axis.

4. An induction motor has a short-circuit current 7 times the full-load current and a full-load slip of 4 per cent. Its line-starting torque is ..... times the full-load torque.  
(a) 7 (b) 1.96  
(c) 4 (d) 49
5. In a SCIM, torque with autostarter is ..... times the torque with direct-switching.  
(a)  $K^2$  (b)  $K$   
(c)  $1/K^2$  (d)  $1/K$   
where  $K$  is the transformation ratio of the autostarter.
6. If stator voltage of a SCIM is reduced to 50 per cent of its rated value, torque developed is reduced by ..... per cent of its full-load value.  
(a) 50 (b) 25  
(c) 75 (d) 57.7
7. For the purpose of starting an induction motor, a Y- $\Delta$  switch is equivalent to an auto-starter of ratio.....per cent.  
(a) 33.3 (b) 57.7  
(c) 73.2 (d) 60.
8. A double squirrel-cage motor (DSCM) scores over SCIM in the matter of  
(a) starting torque  
(b) high efficiency under running conditions  
(c) speed regulation under normal operating conditions  
(d) all of the above.
9. In a DSCM, outer cage is made of high resistance metal bars primarily for the purpose of increasing its  
(a) speed regulation  
(b) starting torque  
(c) efficiency  
(d) starting current.
10. A SCIM with 36-slot stator has two separate windings : one with 3 coil groups/ phase/pole and the other with 2 coil groups/phase/pole. The obtainable two motor speeds would be in the ratio of  
(a) 3 : 2 (b) 2 : 3  
(c) 2 : 1 (d) 1 : 2
11. A 6-pole 3- $\phi$  induction motor taking 25 kW from a 50-Hz supply is cumulatively-cascaded to a 4-pole motor. Neglecting all losses, speed of the 4-pole motor would be ..... r.p.m.  
(a) 1500 (b) 1000  
(c) 600 (d) 3000.  
and its output would be ..... kW.  
(e) 15 (f) 10  
(g) 50/3 (h) 2.5.
12. Which class of induction motor will be well suited for large refrigerators?  
(a) Class E (b) Class B  
(c) Class F (d) Class C
13. In a Schrage motor operating at supersynchronous speed, the injected emf and the standstill secondary induced emf  
(a) are in phase with each other  
(b) are at  $90^\circ$  in time phase with each other  
(c) are in phase opposition  
(d) none of the above.  
**(Power App.-III, Delhi Univ. July 1987)**
14. For starting a Schrage motor, 3- $\phi$  supply is connected to  
(a) stator  
(b) rotor via slip-rings  
(c) regulating winding  
(d) secondary winding via brushes.
15. Two separate induction motors, having 6 poles and 5 poles respectively and their cascade combination from 60 Hz, 3-phase supply can give the following synchronous speeds in rpm  
(a) 720, 1200, 1500 and 3600  
(b) 720, 1200 1800  
(c) 600, 1000, 15000  
(d) 720 and 3000  
**(Power App.-II, Delhi Univ. Jan 1987)**
16. Mark the WRONG statement.  
A Schrage motor is capable of behaving as a/an  
(a) inverted induction motor  
(b) slip-ring induction motor  
(c) shunt motor  
(d) series motor  
(e) synchronous motor.
17. When a stationary 3-phase induction motor is switched on with one phase disconnected  
(a) it is likely to burn out quickly unless immediately disconnected  
(b) it will start but very slowly  
(c) it will make jerky start with loud growing noise  
(d) remaining intact fuses will be blown out due to heavy inrush of current
18. If single-phasing of a 3-phase induction motor occurs under running conditions, it  
(a) will stall immediately  
(b) will keep running though with slightly increased slip  
(c) may either stall or keep running depending on the load carried by it  
(d) will become noisy while it still keeps running.

## ANSWERS

1. c 2. c 3. d 4. b 5. a 6. c 7. b 8. d 9. b 10. a 11. c, f 12. d 13. a 14. b 15. a  
16. d 17. a 18. c

# CHAPTER 36

## Learning Objectives

- Types of Single-phase Motors
- Single-phase Induction Motor
- Double-field Revolving Theory
- Making Single-phase Induction Motor Self-starting
- Equivalent Circuit of Single-phase Induction Motor without Core Loss
- Equivalent Circuit With Core Loss
- Types of Capacitors Start Motors
- Capacitor Start-and-Run Motor
- Shaded-pole Single-phase Motor
- Repulsion Type Motors
- Repulsion Motor
- Repulsion Principle
- Compensated Repulsion Motor
- Repulsion-start Induction-Run Motor
- Repulsion Induction Motor
- A.C. Series Motors
- Universal Motor
- Speed control of Universal Motors
- Reluctance Motor
- Hysteresis Motor

## SINGLE-PHASE MOTORS



↑ Single phase motor

### 36.1. Types of Single-Phase Motors

Such motors, which are designed to operate from a single-phase supply, are manufactured in a large number of types to perform a wide variety of useful services in home, offices, factories, workshops and in business establishments etc. Small motors, particularly in the fractional kilo watt sizes are better known than any other. In fact, most of the new products of the manufacturers of space vehicles, aircrafts, business machines and power tools etc. have been possible due to the advances made in the design of fractional-kilowatt motors. Since the performance requirements of the various applications differ so widely, the motor-manufacturing industry has developed many different types of such motors, each being designed to meet specific demands.



Split-phase motor. Switch mechanism disconnects start winding when motor reached three-fourths of rated speed

Single-phase motors may be classified as under, depending on their construction and method of starting :

1. **Induction Motors** (split-phase, capacitor and shaded-pole etc.)
2. **Repulsion Motors** (sometime called Inductive-Series Motors)
3. **A.C. Series Motor**
4. **Un-excited Synchronous Motors**

### 36.2. Single-phase Induction Motor

Constructionally, this motor is, more or less, similar to a polyphase induction motor, except that (i) its stator is provided with a single-phase winding and (ii) a centrifugal switch is used in some types of motors, in order to cut out a winding, used only for starting purposes. It has distributed stator winding and a squirrel-cage rotor. When fed from a single-phase supply, its stator winding produces a flux (or field) which is only *alternating i.e.* one which alternates along one space axis only. It is not a synchronously revolving (or rotating) flux, as in the case of a two- or a three-phase stator winding, fed from a 2- or 3-phase supply. Now, an alternating or pulsating flux acting on a *stationary* squirrel-cage rotor cannot produce rotation (only a revolving flux can). That is why a single-phase motor is *not self-starting*.



Single-phase induction motor

However, if the rotor of such a machine is given an initial start by hand (or small motor) or otherwise, *in either* direction, then immediately a torque arises and the motor accelerates to its final speed (unless the applied torque is too high).

This peculiar behaviour of the motor has been explained in two ways : (i) by two -field or double-field revolving theory and (ii) by cross-field theory. Only the first theory will be discussed briefly.

### 36.3. Double-field Revolving Theory

This theory makes use of the idea that an alternating uni-axial quantity can be represented by two oppositely-rotating vectors of half magnitude. Accordingly, an *alternating* sinusoidal flux can be

represented by two **revolving** fluxes, each equal to half the value of the alternating flux and each rotating synchronously ( $N_s = 120f/P$ ) in opposite direction\*.

As shown in Fig. 36.1 (a), let the alternating flux have a maximum value of  $\Phi_m$ . Its component fluxes A and B will each be equal to  $\Phi_m/2$  revolving in anticlockwise and clockwise directions respectively.

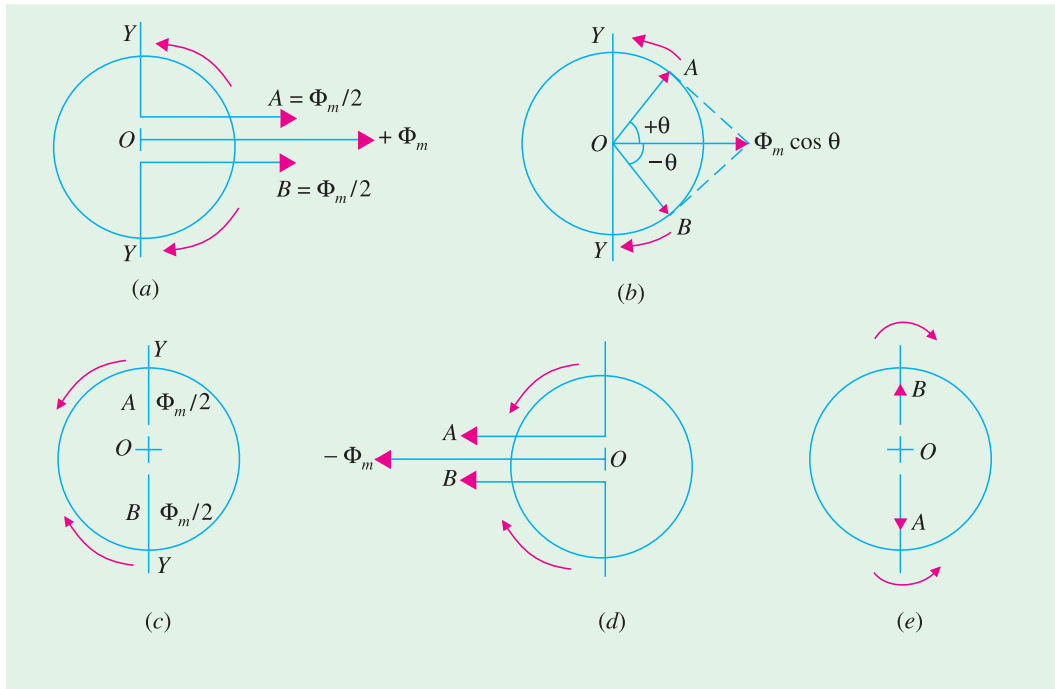


Fig. 36.1

After some time, when A and B would have rotated through angle  $+\theta$  and  $-\theta$ , as in Fig. 36.1 (b), the resultant flux would be

$$= 2 \times \frac{\Phi_m}{2} \cos \frac{2\theta}{2} = \Phi_m \cos \theta$$

After a quarter cycle of rotation, fluxes A and B will be oppositely-directed as shown in Fig. 36.1 (c) so that the resultant flux would be zero.

After half a cycle, fluxes A and B will have a resultant of  $-2 \times \Phi_m/2 = -\Phi_m$ . After three-quarters of a cycle, again the resultant is

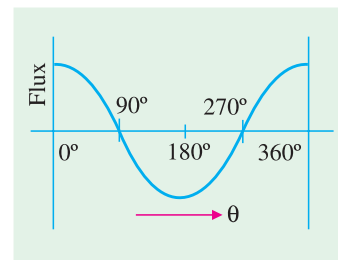


Fig. 36.2

\* For example, a flux given by  $\Phi = \Phi_m \cos 2\pi ft$  is equivalent to two fluxes revolving in opposite directions, each with a magnitude of  $1/2\Phi$  and an angular velocity of  $2\pi f$ . It may be noted that Euler's expressions for  $\cos \theta$  provides interesting justification for the decomposition of a pulsating flux. His expression is

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

The term  $e^{j\theta}$  represents a vector rotated clockwise through an angle  $\theta$  whereas  $e^{-j\theta}$  represents rotation in anticlockwise direction. Now, the above given flux can be expressed as

$$\Phi_m \cos 2\pi ft = \frac{\Phi_m}{2} (e^{j2\pi ft} + e^{-j2\pi ft})$$

The right-hand expression represents two oppositely-rotating vectors of half magnitude.

zero, as shown in Fig. 36.1 (e) and so on. If we plot the values of resultant flux against  $\theta$  between limits  $\theta = 0^\circ$  to  $\theta = 360^\circ$ , then a curve similar to the one shown in Fig. 36.2 is obtained. That is why an alternating flux can be looked upon as composed of two revolving fluxes, each of half the value and revolving synchronously in opposite directions.

It may be noted that if the slip of the rotor is  $s$  with respect to the forward rotating flux (*i.e.* one which rotates in the same direction as rotor) then its slip with respect to the backward rotating flux is  $(2 - s)^*$ .

Each of the two component fluxes, while revolving round the stator, cuts the rotor, induces an e.m.f. and this produces its own torque. Obviously, the two torques (called forward and backward torques) are oppositely-directed, so that the net or resultant torque is equal to their difference as shown in Fig. 36.3.

Now, power developed by a rotor is  $P_g = \left(\frac{1-s}{s}\right) I_2^2 R_2$

If  $N$  is the rotor r.p.s., then torque is given by  $T_g = \frac{1}{2\pi N} \cdot \left(\frac{1-s}{s}\right) I_2^2 R_2$

Now,  $N = N_s(1-s) \quad \therefore T_g = \frac{1}{2\pi N_s} \cdot \frac{I_2^2 R_2}{s} = k \cdot \frac{I_2^2 R_2}{s}$

Hence, the forward and backward torques are given by

$$T_f = K \frac{I_2^2 R_2}{s} \quad \text{and} \quad T_b = -K \cdot \frac{I_2^2 R_2}{(2-s)}$$

or  $T_f = \frac{I_2^2 R_2}{s}$  synch.watt and  $T_b = -\frac{I_2^2 R_2}{(2-s)}$  synch. watt

Total torque  $T = T_f + T_b$

Fig. 36.3 shows both torques and the resultant torque for slips between zero and +2. At standstill,  $s = 1$  and  $(2 - s) = 1$ . Hence,  $T_f$  and  $T_b$  are numerically equal but, being oppositely directed, produce no resultant torque. That explains why there is no **start-ing** torque in a single-phase induction motor.

However, if the rotor is started somehow, say, in the clockwise direction, the clockwise torque starts increasing and, at the same time, the anticlockwise torque starts decreasing. Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

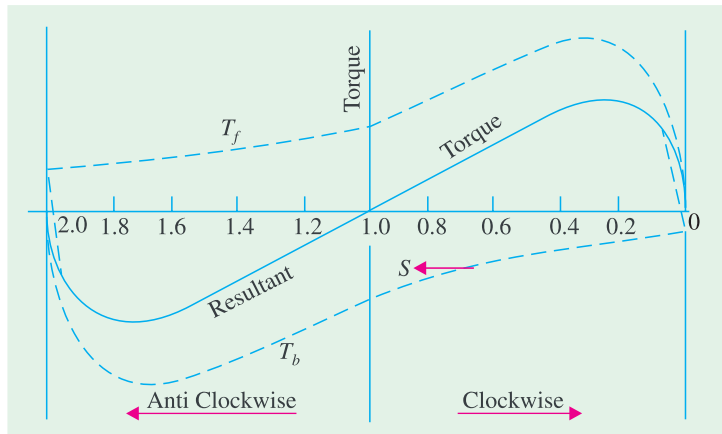


Fig. 36.3

\* It may be proved thus : If  $N$  is the r.p.m. of the rotor, then its slip with respect to forward rotating flux is

$$s = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s} \quad \text{or} \quad \frac{N}{N_s} = 1 - s$$

Keeping in mind the fact that the backward rotating flux rotates opposite to the rotor, the rotor slip with respect to this flux is

$$s_b = \frac{N_s - (-N)}{N_s} = 1 + \frac{N}{N_s} = 1 + (1 - s) = (2 - s)$$

### 36.4. Making Single-phase Induction Motor Self-starting

As discussed above, a single-phase induction motor is not self-starting. To overcome this drawback and make the motor self-starting, it is temporarily converted into a two-phase motor during starting period. For this purpose, the stator of a single-phase motor is provided with an extra winding, known as **starting** (or auxiliary) winding, in addition to the **main or running winding**. The two windings are spaced  $90^\circ$  electrically apart and are connected in *parallel* across the single-phase supply as shown in Fig. 36.4.

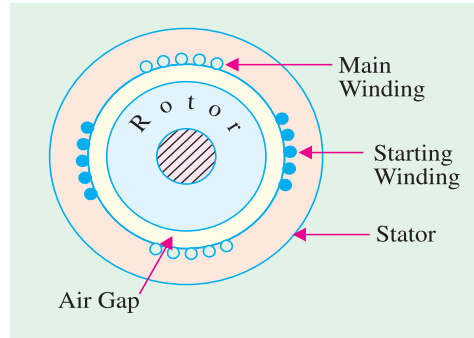


Fig. 36.4

It is so arranged that the phase-difference between the currents in the two stator windings is very large (ideal value being  $90^\circ$ ). Hence, the motor behaves like a two-phase motor. These two currents produce a revolving flux and hence make the motor self-starting.

There are many methods by which the necessary phase-difference between the two currents can be created.

(i) In *split-phase machine*, shown in Fig. 36.5 (a), the main winding has low resistance but high reactance whereas the starting winding has a high resistance, but low reactance. The resistance of the starting winding may be increased either by connecting a high resistance  $R$  in series with it or by choosing a high-resistance fine copper wire for winding purposes.



Single-phase motor.

Hence, as shown in Fig. 36.5 (b), the current  $I_s$  drawn by the starting winding lags behind the applied voltage  $V$  by a small angle whereas current  $I_m$  taken by the main winding lags behind  $V$  by a very large angle. Phase angle between  $I_s$  and  $I_m$  is made as large as possible because the starting torque of a split-phase motor is proportional to  $\sin \alpha$ . A centrifugal switch  $S$  is connected in series with the starting winding and is located inside the motor. Its function is to automatically disconnect the starting winding from the supply when the motor has reached 70 to 80 per cent of its full-load speed.

In the case of split-phase motors that are hermetically sealed in refrigeration units, instead of internally-mounted centrifugal switch, an electromagnetic type of relay is used. As shown in Fig. 36.6, the relay coil is connected in series with main winding and the pair of contacts which are normally open, is included in the starting winding.

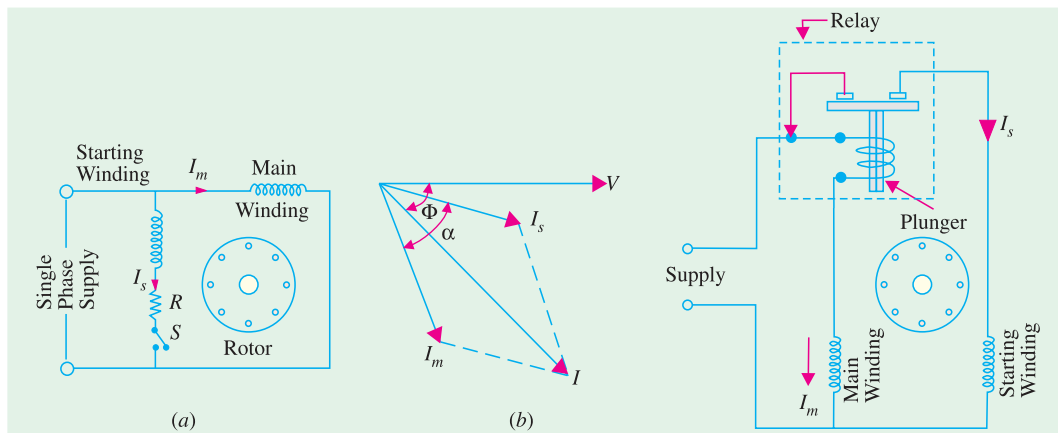


Fig. 36.5

Fig. 36.6

During starting period, when  $I_m$  is large, relay contacts close thereby allowing  $I_s$  to flow and the motor starts as usual. After motor speeds up to 75 per cent of full-load speed,  $I_m$  drops to a value that is low enough to cause the contacts to open.

A typical torque/speed characteristic of such a motor is shown in Fig. 36.7. As seen, the starting torque is 150 to 200 per cent of the full-load torque with a starting current of 6 to 8 times the full-load current. These motors are often used in preference to the costlier capacitor-start motors. Typical applications are : fans and blowers, centrifugal pumps and separators, washing machines, small machine tools, duplicating machines and domestic refrigerators and oil burners etc. Commonly available sizes range from 1/20 to 1/3 h.p. (40 to 250 W) with speeds ranging from 3,450 to 865 r.p.m.

As shown in Fig. 36.8, the direction of rotation of such motors can be reversed by reversing the connections of one of the two stator windings (not both). For this purpose, the four leads are brought outside the frame.

As seen from Fig. 36.9, the connections of the starting winding have been reversed.

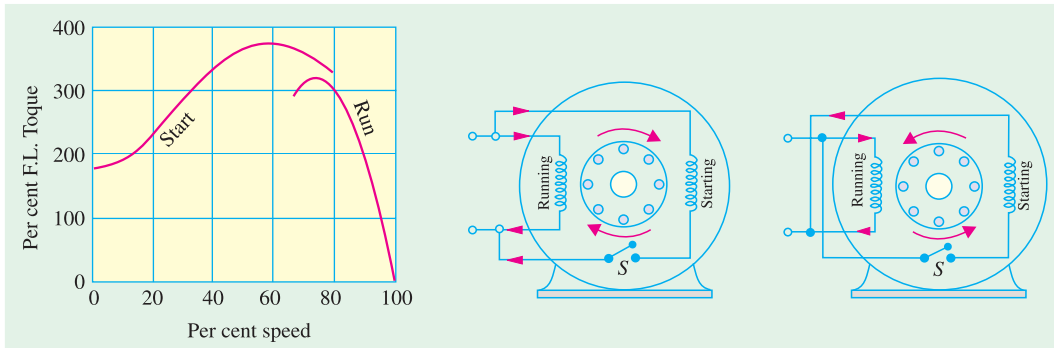


Fig. 36.7

Fig. 36.8

Fig. 36.9

The speed regulation of standard split-phase motors is nearly the same as of the 3-phase motors. Their speed varies about 2 to 5% between no load and full-load. For this reason such motors are usually regarded as practically constant-speed motors.

**Note.** Such motors are sometimes referred to as resistance-start split-phase induction motors in order to distinguish them from capacitor-start induction run and capacitor start-and-run motors described later.

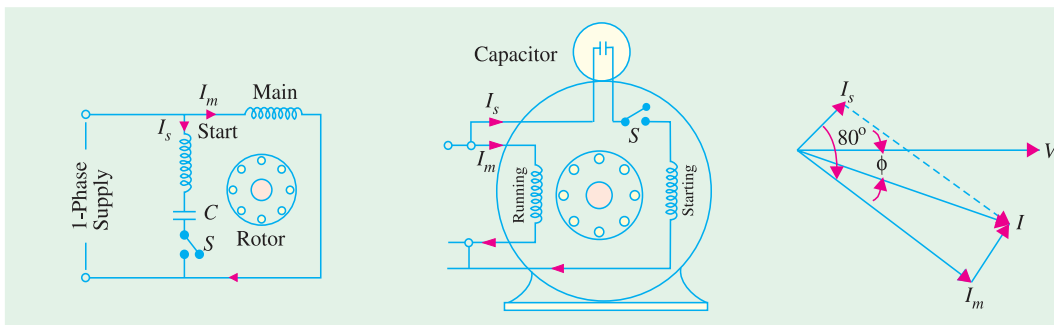


Fig. 36.10

Fig. 36.11

Fig. 36.12

**(ii) Capacitor-start Induction-run motors.** In these motors, the necessary phase difference between  $I_s$  and  $I_m$  is produced by connecting a capacitor in series with the starting winding as shown in Fig. 36.10. The capacitor is generally of the electrolytic type and is usually mounted on the outside of the motor as a separate unit (Fig. 36.11).

The capacitor is designed for extremely short-duty service and is guaranteed for not more than 20 periods of operation per hour, each period not to exceed 3 seconds. When the motor reaches about



75 per cent of full speed, the centrifugal switch  $S$  opens and cuts out both the starting winding and the capacitor from the supply, thus leaving only the running winding across the lines. As shown in Fig.



Capacitor start/capacitor-run motor.

36.12, current  $I_m$  drawn by the main winding lags the supply voltage  $V$  by a large angle whereas  $I_s$  leads  $V$  by a certain angle. The two currents are out of phase with each other by about  $80^\circ$  (for a 200-W 50-Hz motor) as compared to nearly  $30^\circ$  for a split-phase motor. Their resultant current  $I$  is small and is almost in phase with  $V$  as shown in Fig. 36.12.

Since the torque developed by a split-phase motor is proportional to the sine of the angle between  $I_s$  and  $I_m$ , it is obvious that the

increase in the angle (from  $30^\circ$  to  $80^\circ$ ) alone increases the starting torque to nearly twice the value developed by a standard split-phase induction motor. Other improvements in motor design have made it possible to increase the starting torque to a value as high as 350 to 450 per cent.

Typical performance curve of such a motor is shown in Fig. 36.13.

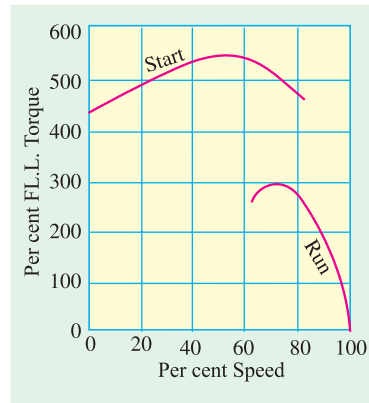


Fig. 36.13

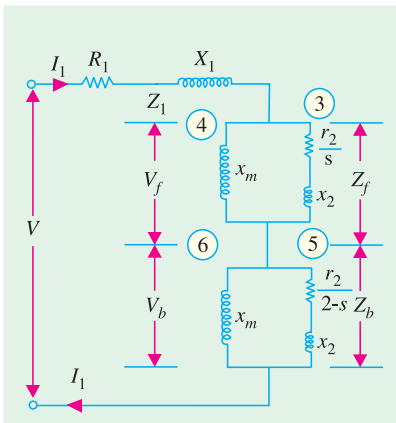


Fig. 36.14

### 36.5. Equivalent Circuit of a Single-phase Induction Motor-Without Core Loss

A single-phase motor may be looked upon as consisting of two motors, having a common stator winding, but with their respective rotors revolving in opposite directions. The equivalent circuit of such a motor based on double-field revolving theory is shown in Fig. 36.14. Here, the single-phase motor has been imagined to be made-up of (i) one stator winding and (ii) two imaginary rotors. The stator impedance is  $Z = R_1 + j X_1$ . The impedance of each rotor is  $(r_2 + jx_2)$  where  $r_2$  and  $x_2$  represent half the actual rotor values in stator terms (i.e.  $x_2$  stands for half the standstill reactance of the rotor, as referred to stator). Since iron loss has been neglected, the exciting branch is shown consisting of exciting reactance only. Each rotor has been assigned half the magnetising reactance\* (i.e.  $x_m$  represents half the actual reactance). The

impedance of 'forward running' rotor is

$$Z_f = \frac{j x_m \left( \frac{r_2}{s} + j x_2 \right)}{\frac{r_2}{s} + j (x_m + x_2)}$$

and it runs with a slip of  $s$ . The impedance of 'backward' running rotor is

\* In fact, full values are shown by capital letters and half values by small letters.

$$Z_b = \frac{j x_m \left( \frac{r_2}{2-s} + j x_2 \right)}{\frac{r_2}{2-s} + j (x_m + x_2)}$$

and it runs with a slip of  $(2 - s)$ . Under standstill conditions,  $V_f = V_b$ , but under running conditions  $V_f$  is almost 90 to 95% of the applied voltage.

The forward torque in synchronous watts is  $T_f = I_3^2 r_2 / s$ . Similarly, backward torque is  $T_b = I_5^2 r_2 / (2 - s)$

The total torque is  $T = T_f - T_b$ .

### 36.6. Equivalent Circuit-With Core Loss

The core loss can be represented by an equivalent resistance which may be connected either in parallel or in series with the magnetising reactance as shown in Fig. 36.15.

Since under running conditions  $V_f$  is very high (and  $V_b$  is correspondingly, low) most of the iron loss takes place in the ‘forward motor’ consisting of the common stator and forward-running rotor. Core-loss current  $I_w = \text{core loss} / V_f$ . Hence, half value of core-loss equivalent resistance is  $r_c = V_f / I_w$ . As shown in Fig. 36.15 (a),  $r_c$  has been connected in parallel with  $x_m$  in each rotor.

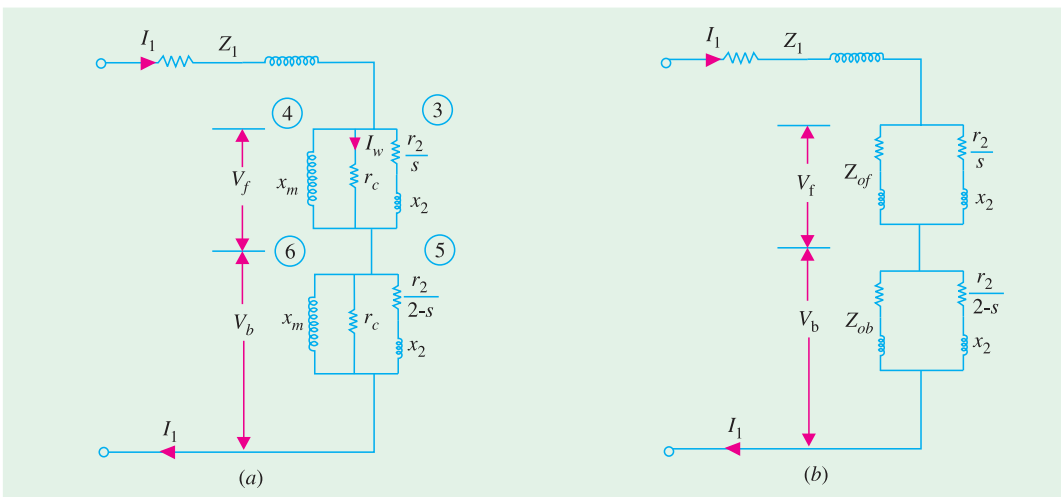


Fig. 36.15

**Example 36.1.** Discuss the revolving field theory of single-phase induction motors. Find the mechanical power output at a slip of 0.05 of the 185-W, 4-pole, 110-V, 60-Hz single-phase induction motor, whose constants are given below:

Resistance of the stator main winding	$R_1 = 1.86 \text{ ohm}$
Reactance of the stator main winding	$X_1 = 2.56 \text{ ohm}$
Magnetizing reactance of the stator main winding	$X_m = 53.5 \text{ ohm}$
Rotor resistance at standstill	$R_2 = 3.56 \text{ ohm}$
Rotor reactance at standstill	$X_2 = 2.56 \text{ ohm}$

(Elect. Machines, Nagpur Univ. 1991)

**Solution.** Here,  $X_m = 53.5 \Omega$ , hence  $x_m = 53.5/2 = 26.7 \Omega$

Similarly,  $r_2 = R_2 / 2 = 3.56 / 2 = 1.78 \Omega$  and  $x_2 = X_2 / 2 = 2.56 / 2 = 1.28 \Omega$

$$\therefore Z_f = \frac{j x_m \left( \frac{r_2}{s} + j x_2 \right)}{\frac{r_2}{s} + j (x_2 + x_m)} = x_m \frac{\frac{r_2}{s} \cdot x_m + j \left[ (r_2/s)^2 + x_2 x_0 \right]}{(r_2/s)^2 + x_0^2} \quad \text{where } x_0 = (x_m + x_2)$$

$$\therefore Z_f = 26.7 \frac{(1.78/0.05) \times 26.7 + j [(1.78/0.05)^2 + 1.28 \times 27.98]}{(1.78/0.05)^2 + (27.98)^2}$$

$$= 12.4 + j 17.15 = 21.15 \angle 54.2^\circ$$

Similarly,  $Z_b = \frac{j x_m \left( \frac{r_2}{2-s} + j x_2 \right)}{\frac{r_2}{2-s} + j (x_2 + x_m)} = x_m \frac{\left( \frac{r_2}{2-s} \right) x_m + j \left[ \left( \frac{r_2}{2-s} \right)^2 + x_0 x_2 \right]}{\left( \frac{r_2}{2-s} \right)^2 + x_0^2}$

$$= 26.7 \frac{(1.78/1.95) \times 26.7 + j [(1.78/1.95)^2 + 1.28 \times 27.98]}{(1.78/1.95)^2 + (27.98)^2}$$

$$= 0.84 + j 1.26 = 1.51 \angle 56.3^\circ$$

$$\mathbf{Z_1} = R_1 + j X_1 = 1.86 + j 2.56 = 3.16 \angle 54^\circ$$

Total circuit impedance is

$$\mathbf{Z_{01}} = \mathbf{Z_1} + \mathbf{Z_f} + \mathbf{Z_b} = (1.86 + j 2.56) + (12.4 + j 17.15) + (0.84 + j 1.26)$$

$$= 15.1 + j 20.97 = 25.85 \angle 54.3^\circ$$

Motor current  $I_1 = 110/25.85 = 4.27 \text{ A}$

$$V_f = I_1 Z_f = 4.27 \times 21.15 = 90.4 \text{ V}; V_b = I_1 Z_b = 4.27 \times 1.51 = 6.44 \text{ V}$$

$$Z_3 = \sqrt{\left( \frac{r_2}{s} \right)^2 + x_2^2} = 35.7 \Omega, Z_5 = \sqrt{\left( \frac{r_2}{2-s} \right)^2 + x_2^2} = 1.57 \Omega$$

$$I_3 = V_f / Z_3 = 90.4/35.7 = 2.53 \text{ A}, I_5 = V_b / Z_5 = 6.44/1.57 = 4.1 \text{ A}$$

$$T_f = I_3^2 R_2 / s = 228 \text{ synch. watts}, T_5 = I_5^2 r_2 / (2-s) = 15.3 \text{ synch. watts.}$$

$$T = T_f - T_b = 228 - 15.3 = 212.7 \text{ synch. watts}$$

$$\text{Output} = \text{synch. watt} \times (1-s) = 212.7 \times 0.95 = \mathbf{202 \text{ W}}$$

Since friction and windage losses are not given, this also represents the net output.

**Example 36.2.** Find the mechanical power output of 185-W, 4 pole, 110-V, 50-Hz single-phase induction motor, whose constants are given below at a slip of 0.05.

$$R_1 = 1.86 \Omega \quad X_1 = 2.56 \Omega \quad X_\phi = 53.5 \Omega \quad R_2 = 3.56 \Omega \quad X_2 = 2.56 \Omega$$

$$\text{Core loss} = 3.5 \text{ W, Friction and windage loss} = 13.5 \text{ W.}$$

(Electrical Machines-III, Indore Univ. 1987)

**Solution.** It would be seen that major part of this problem has already been solved in Example 36.1. Let us, now, assume that  $V_f = 82.5\%$  of 110 V = 90.7 V. Then the core-loss current  $I_c = 35/90.7 = 0.386 \text{ A}$ ;  $r_c = 90.7/0.386 = 235 \Omega$ .

#### Motor I

$$\text{conductance of core-loss branch} = 1/r_c = 1/235 = 0.00426 \text{ S}$$

$$\text{susceptance of magnetising branch} = -j/x_m = -j/26.7 = -j 0.0374 \text{ S}$$

$$\text{admittance of branch 3} = \frac{(r_2/s) - j x_2}{(r_2/s)^2 + x_2^2} = 0.028 - j 0.00101 \text{ S}$$

admittance of 'motor' I is  $Y_f = 0.00426 - j 0.0374 + 0.028 - j 0.00101$   
 $= 0.03226 - j 0.03841 \text{ S}$

impedance  $Z_f = 1/Y_f = 12.96 + j 15.2 \text{ or } 19.9 \Omega$

**Motor II**

admittance of branch 5  $= \frac{\frac{r_2}{2-s} - j x_2}{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} = \frac{0.91 - j 1.28}{2.469} = 0.369 - j 0.517$

admittance of 'motor' II,  $Y_b = 0.00426 - j 0.0374 + 0.369 - j 0.517$   
 $= 0.3733 - j 0.555 \text{ S}$

Impedance of 'motor' II,  $Z_b = 1/Y_b = 0.836 + j 1.242 \text{ or } 1.5 \Omega$

Impedance of entire motor (Fig. 36.16)  $Z_{01} = Z_1 + Z_f + Z_b = 15.66 + j 19 \text{ or } 24.7 \Omega$

$I_1 = V/Z_{01} = 110/24.7 = 4.46 \text{ A}$

$V_f = I_1 Z_f = 4.46 \times 19.9 = 88.8 \text{ V}$

$V_b = 4.46 \times 1.5 = 6.69 \text{ V}$

$I_3 = 88.8/35.62 = 2.5 \text{ A}$

$I_5 = 6.69/1.57 = 4.25 \text{ A}$

$T_f = I_3^2 (r_2/s) = 222 \text{ synch. watt}$

$T_b = I_5^2 \left(\frac{r_2}{2-s}\right) = 16.5 \text{ synch. watt}$

$T = T_f - T_b = 205.5 \text{ synch. watt}$

Watts converted = synch. watt (1 - s)  
 $= 205.5 \times 0.95 = 195 \text{ W}$

Net output =  $195 - 13.5 = 181.5 \text{ W}$ .

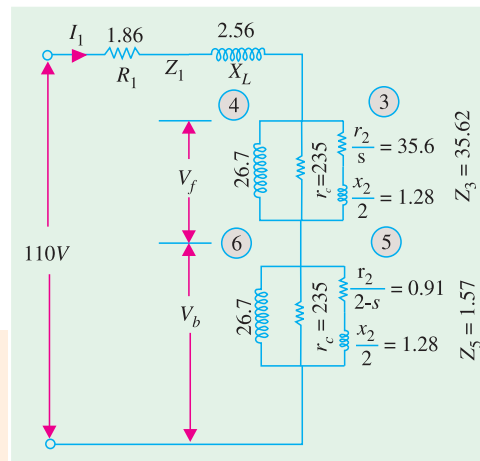


Fig. 36.16

**Example 36.3.** A 250-W, 230-V, 50-Hz capacitor-start motor has the following constants for the main and auxiliary windings: Main winding,  $Z_m = (4.5 + j 3.7) \text{ ohm}$ . Auxiliary winding  $Z_a = (9.5 + j 3.5) \text{ ohm}$ . Determine the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.

(Electrical Machines-III, South Gujarat Univ. 1985)

**Solution.** Let  $X_C$  be the reactance of the capacitor connected in the auxiliary winding.

Then  $Z_a = 9.5 + j 3.5 - j X_C = (9.5 + jX)$   
 where  $X$  is the net reactance.

Now,  $Z_m = 4.5 + j 3.5 = 5.82 \angle 39.4^\circ \text{ ohm}$

Obviously,  $I_m$  lags behind  $V$  by  $39.4^\circ$

Since, time phase angle between  $I_m$  and  $I_a$  has to be  $90^\circ$ ,  $I_a$  must lead  $V$  by

$(90^\circ - 39.4^\circ) = 50.6^\circ$ .

For auxiliary winding,  $\tan \phi_a = X/R$  or  $\tan 50.6^\circ = X/9.5$

or  $X = 9.5 \times 1.217 = 11.56 \Omega$  (capacitive)

$\therefore X_C = 11.56 + 3.5 = 15.06 \Omega \quad \therefore 15.06 = 1/314 C; C = 211 \mu\text{F}$

### Tutorial Problem No. 36.1.

1. A 1- $\phi$ , induction motor has stator windings in space quadrature and is supplied with a single-phase voltage of 200 V at 50 Hz. The standstill impedance of the main winding is  $(5.2 + j 10.1)$  and of the auxiliary winding is  $(19.7 + j 14.2)$ . Find the value of capacitance to be inserted in the auxiliary winding for maximum starting torque. *(Electrical Machines-III, Indore Univ. July, 1977)*

2. A 230-V, 50-Hz, 6-pole, single-phase induction motor has the following constants.

$$r_1 = 0.12 \Omega, r_2 = 0.14 \Omega, x_1 = x_2 = 0.25 \Omega, x_m = 15 \Omega.$$

If the core loss is 250 W and friction and windage losses are 500 W, determine the efficiency and torque at  $s = 0.05$ . *(Electrical Machines-IV, Bangalore Univ. Aug. 1978)*

3. Explain how the pulsating mmf of a single-phase induction motor may be considered equivalent to two oppositely-rotating fields. Develop an expression for the torque of the motor.

A 125-W, 4-pole, 110-V, 50-Hz single-phase induction motor has the no-load rotational loss of 25 watts and total rotor copper loss at rated load of 25 watts at a slip of 0.06. The rotor  $I^2R$  loss may be neglected.

At a slip  $s = 0.06$ , what is the power input to the machine ?

*(Electrical Machines-III, Indore Univ. July 1977)*

### 36.7. Types of Capacitor-start Motors

Some of the important types of such motors are given below :

#### 1. Single-voltage, externally-reversible type

In this motor, four leads are brought outside its housing; two from the main winding and two from the starting-winding circuit. These four leads are necessary for external reversing. As usual, internally, the starting winding is connected in series with the electrolytic capacitor and a centrifugal switch. The direction of rotation of the motor can be easily reversed externally by reversing the starting winding leads with respect to the running winding leads.

#### 2. Single-voltage, non-reversible type

In this case, the starting winding leads are connected internally to the leads of the running winding. Consequently, there are only two external leads in such motors. Obviously, direction of rotation cannot be reversed unless the motor is taken apart and leads of the starting winding reversed.

#### 3. Single-voltage reversible and with thermostat type

Many motors are fitted with a device called thermostat which provides protection against overload, overheating, and short-circuits etc. The thermostat usually consists of a bimetallic element that is connected in series with the motor and is often mounted on the outside of the motor.

The wiring diagram of a capacitor-start motor fitted with this protective device is shown in Fig. 36.17. When due to some reasons, excessive current flows through the motor, it produces abnormal heating of the bimetallic strip with the result that it bends and opens the contact points thus disconnecting the motor from the supply lines. When the thermostat element cools, it automatically closes the contacts\*.

In the case of capacitor-start motors used for refrigerators, generally a terminal block is attached to the motor. Three out of the four block terminals are marked  $T$ ,  $TL$  and  $L$  as shown in Fig. 36.18. Thermostat is connected to  $T$  and  $TL$ , capacitor between  $L$  and the unmarked terminal and the supply lines to  $TL$  and  $L$ .

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\* However, in some thermal units, a reset button has to be operated manually to restore the motor to operation. In certain types of thermal units, a heating element is used for heating the bimetallic strip. In that case, the heating element is connected in the line and the element or bimetallic strip is placed either inside the heating unit or besides it.

**4. Single-voltage, non-reversible with magnetic switch type**

Such motors are commonly used in refrigerators where it is not possible to use a centrifugal switch. The circuit diagram is similar to that shown in Fig. 36.6. Since their application requires just one direction of rotation, these motors are not connected for reversing.

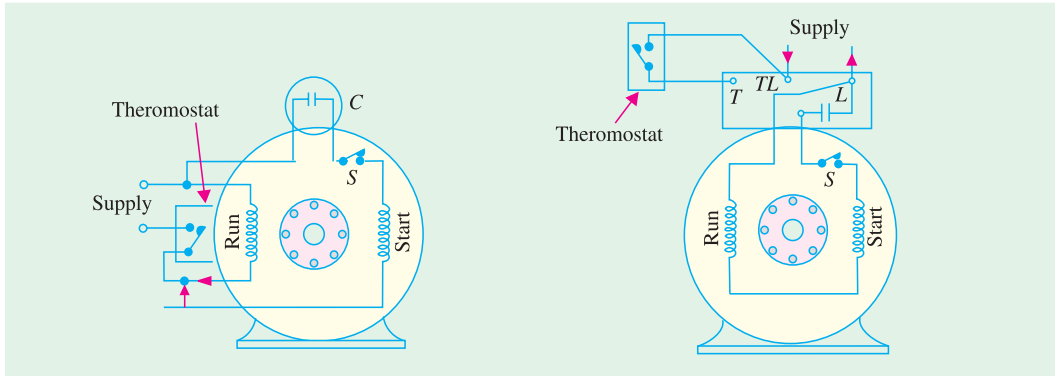


Fig. 36.17

Fig. 36.18

One disadvantage of a capacitor-start motor having magnetic switch lies in the possibility that slight overloads may operate the plunger thereby connecting the starting winding circuit to the supply. Since this winding is designed to operate for very short periods (3 seconds or less) it is likely to be burnt out.

**5. Two-voltage, non-reversible Type**

These motors can be operated from two a.c. voltage either 110 V and 220 V or 220 V and 440 V. Such motors have two main windings (or one main winding in two sections) and one starting winding with suitable number of leads brought out to permit changeover from one voltage to another.

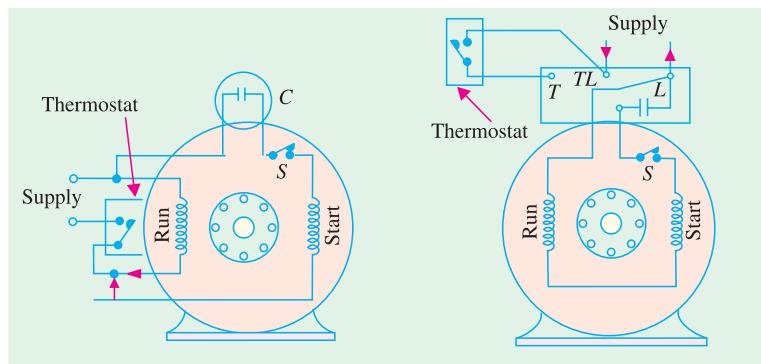


Fig. 36.19

Fig. 36.20

When the motor is to operate from lower voltage, the two main windings are connected in parallel (Fig. 36.19). Whereas for higher voltage, they are connected in series (Fig. 36.20). As will be seen from the above circuit diagrams, the starting winding is always operated on the low-voltage for which purpose it is connected across one of the main windings.

**6. Two-voltage, reversible type**

External reversing is made possible by means of two additional leads that are brought out from the starting winding.

Fig. 36.21 and 36.22 show connections for clockwise and anticlockwise rotations respectively when motor is operated from lower voltage. Similar wiring diagram can be drawn for higher voltage supply.

**7. Single-voltage, three-lead reversible type**

In such motors, a two-section running winding is used. The two sections  $R_1$  and  $R_2$  are internally connected in series and one lead of the starting winding is connected to the mid-point of  $R_1$  and  $R_2$ .

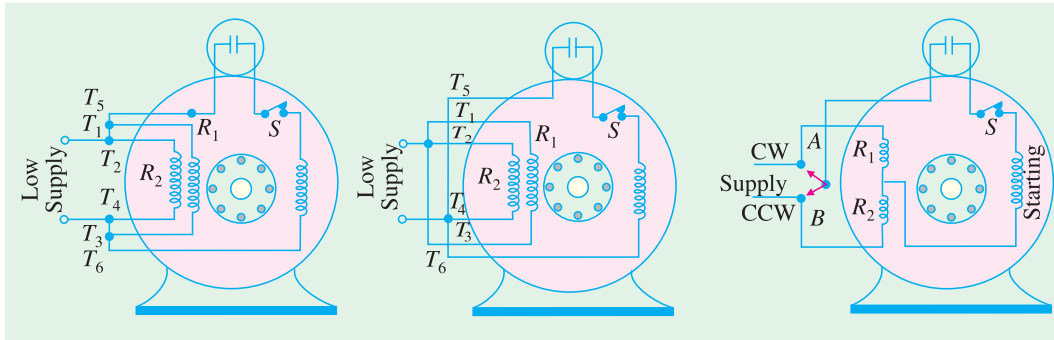


Fig. 36.21

Fig. 36.22

Fig. 36.23

The second lead of the starting winding and both leads of the running winding are brought outside as shown in Fig. 36.23. When the external lead of the starting winding is connected to point A, the winding is connected across  $R_1$  and the motor runs clockwise. When the lead of starter winding is connected to point B, it is connected across  $R_2$ . Since current flowing through starting winding is reversed, the motor runs in counter-clockwise direction.

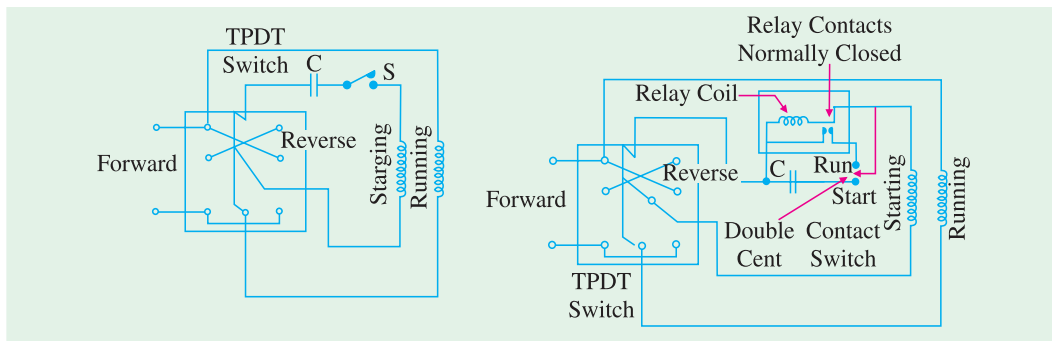


Fig. 36.24

Fig. 36.25

**8. Single-voltage, instantly-reversible type**

Normally, a motor must be brought to complete rest before it can be started in the reverse direction. It is so because the centrifugal switch cannot close unless the motor has practically stopped. Since starting winding is disconnected from supply when the motor is running, reversal of starting winding leads will not affect the operation of the motor. This reversal is achieved by a triple-pole, double-throw (TPDT) switch as shown in Fig. 36.24. The switch consists of three blades or poles which move together as one unit in either of the two positions. In one position of the switch (shown in one figure) motor runs clockwise and in the other, in counter-clockwise direction. Obviously, in this type of arrangement, it is necessary to wait till motor stops.

In certain applications where instant reversal is necessary while the motor is operating at full speed, a relay is fitted in the circuit to short-circuit the centrifugal switch and connect the starting winding in the circuit in the reversed direction (Fig. 36.25).

It will be seen that when at rest, the double-contact centrifugal switch is in the 'start' position. In this position, two connections are made :

- (i) the starting winding and capacitor  $C$  are placed in series across the supply line and
- (ii) the coil of the normally-closed relay is connected across  $C$

With the manual *TPDT* switch in the ‘forward’ position (a) running winding is connected across the line (b) starting winding and  $C$  are in series across the line and (c) relay coil is connected across  $C$ . The voltage developed across  $C$  is applied across the relay coil which results in opening of the relay contacts. With increase in the speed of the motor, the centrifugal switch is thrown in the ‘running’ position. This cuts out  $C$  from the circuit and leaves starting winding in series with the relay coil. Since relay coil has high resistance, it permits only enough current through the starting winding as to keep the relay contacts open.

During the fraction-of-a-second interval while *TPDT* switch is shifted from ‘forward’ to ‘reverse’ position, no current flows through the relay coil as a result of which the relay contacts close. When *TPDT* switch reaches the ‘reverse’ position, current flows through the now-closed relay contacts to the starting winding but in opposite direction. This produces a torque which is applied in a direction opposite to the rotation. Hence, (i) rotor is immediately brought to rest and (ii) centrifugal switch falls to the ‘start’ position. As before,  $C$  is put in series with the starter winding and the motor starts rotating in the opposite direction.

**9. Two-speed type**

Speed can be changed by changing the number of poles in the winding for which purpose two separate running windings are placed in the slots of the stator, one being 6-pole winding and the other, 8-pole winding. Only one starting winding is used which always acts in conjunction with the higher-speed running winding. The double-action or transfer type centrifugal switch  $S$  has two contact

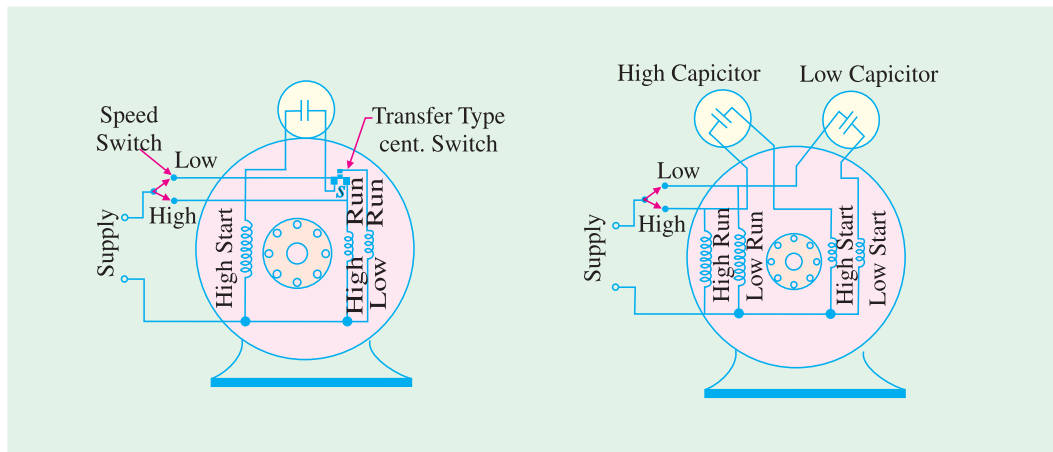


Fig. 36.26

Fig. 36.27

points on the ‘start’ side and one on the ‘run’ side. As shown in Fig. 36.26, an external speed switch is used for changing the motor speed. The motor will always start on high speed irrespective of whether the speed switch is on the ‘high’ or ‘low’ contact. If speed switch is set on ‘low’, then as soon as the motor comes up to speed, the centrifugal switch

- (a) cuts out the starting winding and high-speed running winding and
- (b) cuts in the low-speed running winding.

**10. Two-speed with two-capacitor type**

As shown in Fig. 36.27, this motor has two running windings, two starting windings and two capacitors. One capacitor is used for high-speed operation and the other for low-speed operation. A double centrifugal switch  $S$  is employed for cutting out the starting winding after start.



### 36.8. Capacitor Start-and-Run Motor

This motor is similar to the capacitor-start motor [Art.36.4 (ii)] except that the starting winding and capacitor are connected in the circuit at **all times**. The advantages of leaving the capacitor permanently in circuit are (i) improvement of over-load capacity of the motor (ii) a higher power factor (iii) higher efficiency and (iv) quieter running of the motor which is so much desirable for small power drives in offices and laboratories. Some of these motors which start and run with one value of capacitance in the circuit are called **single-value** capacitor-run motors. Other which start with high value of capacitance but run with a low value of capacitance are known as **two-value** capacitor-run motors.



Capacitor starts and run motor

#### (i) Single-value capacitor-Run Motor

It has one running winding and one starting winding in series with a capacitor as shown in Fig. 36.28. Since capacitor remains in the circuit permanently, this motor is often referred to as permanent-

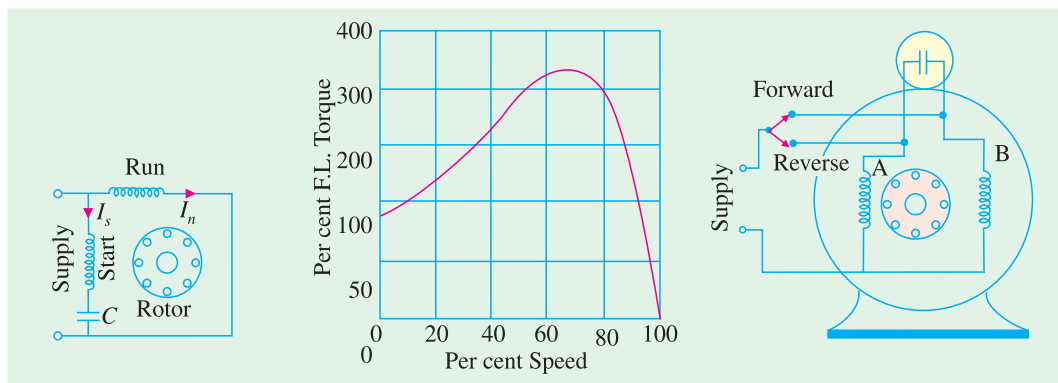


Fig. 36.28

Fig. 36.29

Fig. 36.30

split capacitor-run motor and behaves practically like an unbalanced 2-phase motor. Obviously, there is no need to use a centrifugal switch which was necessary in the case of capacitor-start motors. Since the same capacitor is used for starting and running, it is obvious that neither optimum starting nor optimum running performance can be obtained because value of capacitance used must be a compromise between the best value for starting and that for running. Generally, capacitors of 2 to 20  $\mu\text{F}$  capacitance are employed and are more expensive oil or pyranol-insulated foil-paper capacitors because of continuous-duty rating. The low value of the capacitor results in small starting torque which is about 50 to 100 per cent of the rated torque (Fig. 36.29). Consequently, these motors are used where the required starting torque is low such as air - moving equipment *i.e.* fans, blowers and voltage regulators and also oil burners where quiet operation is particularly desirable.

One unique feature of this type of motor is that it can be easily reversed by an external switch provided its **running and starting windings are identical**. One serves as the running winding and the other as a starting winding for one direction of rotation. For reverse rotation, the one that previously served as a running winding becomes the starting winding while the former starting winding serves as the running winding. As seen from Fig. 36.30 when the switch is in the forward position, winding B serves as running winding and A as starting winding. When switch is in 'reverse' position, winding A becomes the running winding and B the starting winding.

Such reversible motors are often used for operating devices that must be moved back and forth very frequently such as rheostats, induction regulations, furnace controls, valves and arc-welding controls.

### (ii) Two-value capacitor-Run Motor

This motor starts with a high capacitor in series with the starting winding so that the starting torque is high. For running, a lower capacitor is substituted by the centrifugal switch. Both the running and starting windings remain in circuit.

The two values of capacitance can be obtained as follows:

1. by using two capacitors in parallel at the start and then switching out one for low-value run. (Fig. 36.31) or
2. by using a step-up auto-transformer in conjunction with one capacitor so that effective capacitance value is increased for starting purposes.

In Fig. 36.31, *B* is an electrolytic capacitor of high capacity (short duty) and *A* is an oil capacitor of low value (continuous duty). Generally, starting capacitor *B* is 10 to 15 times the running capacitor *A*. At the start, when the centrifugal switch is closed, the two capacitors are put in parallel, so that their combined capacitance is the sum of their individual capacitances. After the motor has reached 75 per cent full-load speed, the switch opens and only capacitor *A* remains in the starting winding circuit. In this way, both optimum starting and running performance is achieved in such motors. If properly designed, such motors have operating characteristics very closely resembling those displayed by two-phase motors. Their performance is characterised by

1. ability to start heavy loads  
 2. extremely quiet operation  
 3. higher efficiency and power factor  
 4. ability to develop 25 per cent overload capacity

Hence, such motors are ideally suited where load requirements are severe as in the case of compressors and fire strokers etc.

The use of an auto-transformer and single oil-type capacitor is illustrated in Fig.36.32. The transformer and capacitor are sealed in a rectangular iron box and mounted on top of the motor. The idea behind using this combination is that a capacitor of value *C* connected to the secondary of a step-up transformer, appears to the primary as though it had a value of  $K^2C$  where *K* is voltage transformation ratio. For example, if actual value of *C* = 4  $\mu$ F and *K* = 6, then low-voltage primary acts as if it had a 144  $\mu$ F (=  $6^2 \times 4$ ) capacitor connected across its terminals. Obviously, effective value of capacitance has increased 36 times. In the 'start' position of the switch, the connection is made to the

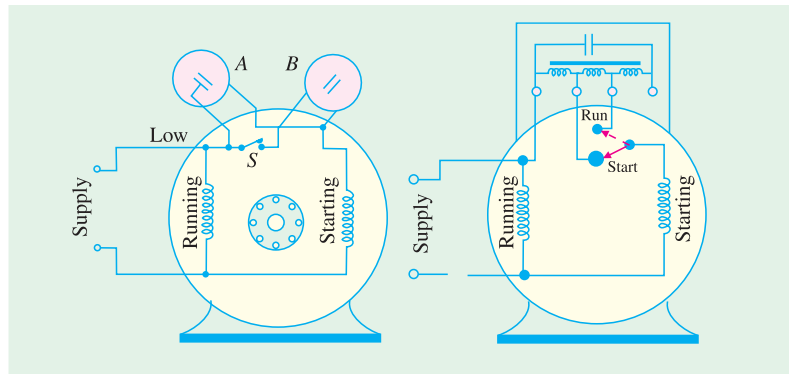
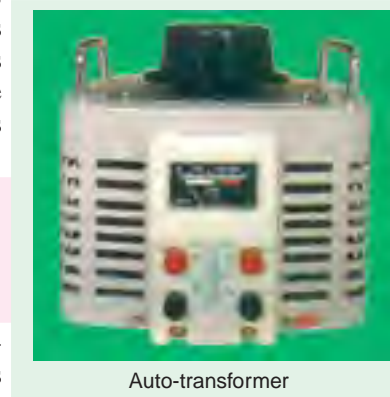


Fig. 36.31

Fig.36.32



Auto-transformer

mid-tap of the auto-transformer so that  $K = 2$ . Hence, effective value of capacitance at start is 4 times the running value and is sufficient to give a high starting torque. As the motor speeds up, the centrifugal switch shifts the capacitor from one voltage tap to another so that the voltage transformation ratio changes from higher value at starting to a lower value for running. The capacitor which is actually of the paper-tin-foil construction is immersed in a high grade insulation like wax or mineral oil.

### 36.9. Shaded-pole Single-phase Motor

In such motors, the necessary phase-splitting is produced by induction. These motors have salient poles on the stator and a squirrel-cage type rotor Fig. 36.33 shows a four-pole motor with the field poles connected in series for alternate polarity. One pole of such a motor is shown separately in Fig. 36.34. The laminated pole has a slot cut across the laminations approximately one-third distance from one edge. Around the small part of the pole is placed a short-circuited Cu coil known as *shading coil*. This part of the pole is known as *shaded* part and the other as *unshaded* part. When an alternating current is passed through the exciting (or field) winding



Shaded pole single phase motor

surrounding the whole pole, the axis of the pole shifts from the unshaded part *a* to the shaded part *b*. This shifting of the magnetic axis is, in effect, equivalent to the actual physical movement of the pole. Hence, the rotor starts rotating in the direction of this shift *i.e.* from unshaded part to the shaded part.

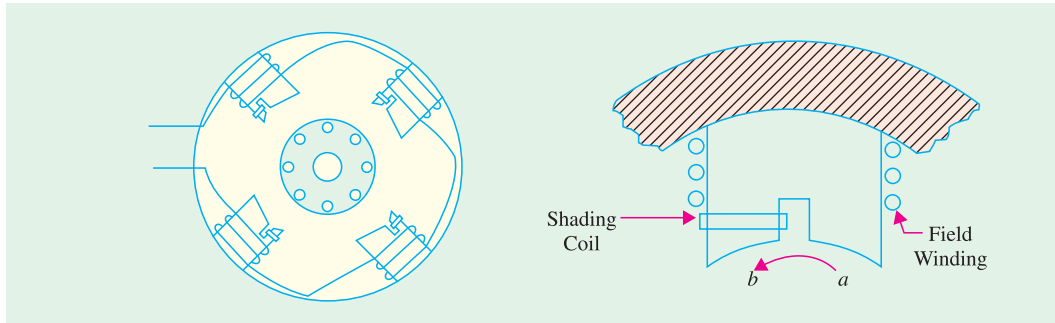


Fig. 36.33

Fig. 36.34

Let us now discuss why shifting of the magnetic axis takes place. It is helpful to remember that the shading coil is highly inductive. When the alternating current through exciting coil tends to increase, it induces a current in the shading coil by transformer action in such a direction as to oppose its growth. Hence, flux density decreases in the shaded part when exciting current increases. However, flux density increases in the shaded part when exciting current starts decreasing (it being assumed that exciting current is sinusoidal).

In Fig. 36.35 (a) exciting current is rapidly increasing along *OA* (shown by dots). This will produce an e.m.f. in the shading coil. As shading coil is of low resistance, a large current will be set up in such a direction (according to Lenz's law) as to oppose the rise of exciting current (which is responsible for its production). Hence, the flux mostly shifts to the unshaded part and the magnetic axis lies along the middle of this part *i.e.* along *NC*.

Next, consider the moment when exciting current is near its peak value *i.e.* from point *A* to *B* [Fig. 36.35 (b)]. Here, the change in exciting current is very slow. Hence, practically no voltage and, therefore, no current is induced in the shading coil. The flux produced by exciting current is at its maximum value and is uniformly distributed over the pole face. So the magnetic axis shifts to the centre of the pole *i.e.* along positions *ND*.

Fig. 36.35 (c) represents the condition when the exciting current is rapidly decreasing from *B* to *C*. This again sets up induced current in the shading coil by transformer action. This current will flow in such a direction as to oppose this decrease in exciting current, with the result that the flux is strengthened in the shaded part of the pole. Consequently, the magnetic axis shifts to the middle part of the shaded pole *i.e.* along *NE*.

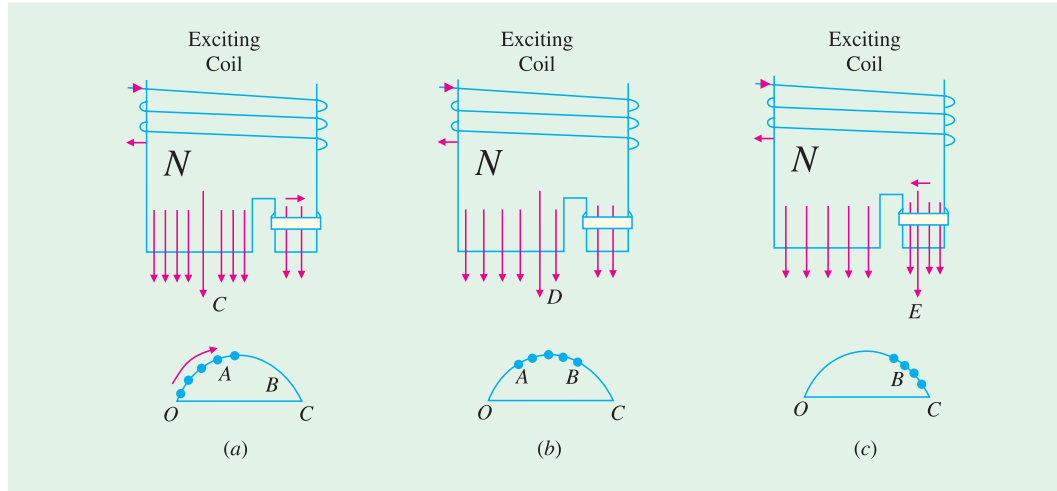


Fig. 36.35

From the above discussion we find that during the positive half-cycle of the exciting current, a *N*-pole shifts along the pole from the unshaded to the shaded part. During the next negative half-cycle of the exciting current, a *S*-pole trails along. The effect is as if a number of real poles were actually sweeping across the space from left to right.

Shaded pole motors are built commercially in very small sizes, varying approximately from 1/250 h.p. (3W) to 1/6 h.p. (125 W). Although such motors are simple in construction, extremely rugged, reliable and cheap, they suffer from the disadvantages of (i) low starting torque (ii) very little overload capacity and (iii) low efficiency. Efficiencies vary from 5% (for tiny sizes) to 35 (for higher ratings). Because of its low starting torque, the shaded-pole motor is generally used for small fans, toys, instruments, hair dryers, ventilators, circulators and electric clocks. It is also frequently used for such devices as churns, phonograph turntables and advertising displays etc. The direction of rotation of this motor cannot be changed, because it is fixed by the position of copper rings.

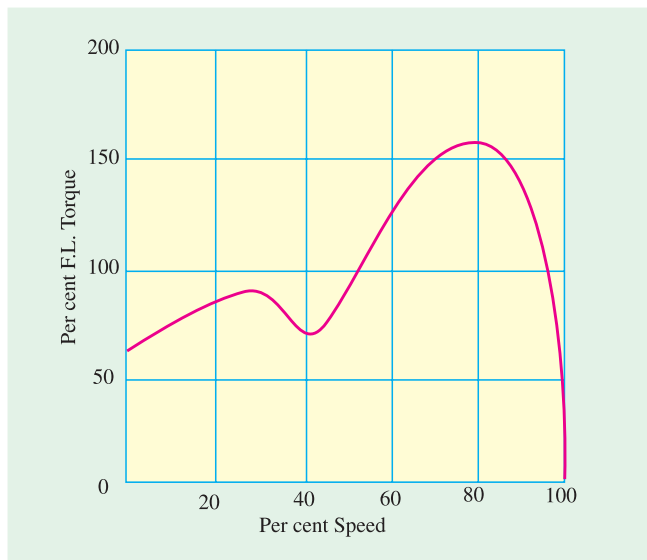


Fig. 36.36

A typical torque / speed curve for such a motor is shown in Fig. 36.36.

### 36.10. Repulsion Type Motors

These can be divided into the following four distinct categories :

- 1. Repulsion Motor.** It consists of (a) one stator winding (b) one rotor which is wound like a d.c. armature (c) commutator and (d) a set of brushes, which are short-circuited and remain in contact with the commutator at all times. It operates continuously on the 'repulsion' principle. No short-circuiting mechanism is required for this type.
- 2. Compensated Repulsion Motor.** It is identical with repulsion motor in all respects, except that (a) it carries an additional stator winding, called compensating winding (b) there is another set of two brushes which are placed midway between the usual short-circuited brush set. The compensating winding and this added set are connected in series.
- 3. Repulsion-start Induction-run Motor.** This motor starts as a repulsion motor, but normally runs as an induction motor, with constant speed characteristics. It consists of (a) one stator winding (b) one rotor which is similar to the wire-wound d.c. armature (c) a commutator and (d) a centrifugal mechanism which short-circuits the commutator bars all the way round (with the help of a short-circuiting necklace) when the motor has reached nearly 75 per cent of full speed.
- 4. Repulsion Induction Motor.** It works on the combined principle of repulsion and induction. It consists of (a) stator winding (b) two rotor windings : one squirrel cage and the other usual d.c. winding connected to the commutator and (c) a short-circuited set of two brushes.



Repulsion induction motor

It may be noted that repulsion motors have excellent characteristics, but are expensive and require more attention and maintenance than single-phase motors. Hence, they are being replaced by two-value capacitor motors for nearly all applications.

### 36.11. Repulsion Motor

Constructionally, it consists of the following :

- 1.** Stator winding of the distributed non-salient pole type housed in the slots of a smooth-cored stator (just as in the case of split-phase motors). The stator is generally wound for four, six or eight poles.
- 2.** A rotor (slotted core type) carrying a distributed winding (either lap or wave) which is connected to the commutator. The rotor is identical in construction to the d.c. armature.
- 3.** A commutator, which may be one of the two types : an axial commutator with bars parallel to the shaft or a radial or vertical commutator having radial bars on which brushes press horizontally.
- 4.** Carbon brushes (fitted in brush holders) which ride against the commutator and are used for conducting current through the armature (*i.e.* rotor) winding.

### 36.12. Repulsion Principle

To understand how torque is developed by the repulsion principle, consider Fig. 36.37 which shows a 2-pole salient pole motor with the magnetic axis vertical. For easy understanding, the stator winding has been shown with concentrated salient-pole construction (actually it is of distributed non-salient type). The basic functioning of the machine will be the same with either type of construction. As mentioned before, the armature is of standard d.c. construction with commutator and brushes (which are short-circuited with a low-resistance jumper).

Suppose that the direction of flow of the alternating current in the exciting or field (stator) winding is such that it creates a *N*-pole at the top and a *S*-pole at the bottom. The alternating flux produced by the stator winding will induce e.m.f. in the armature conductors by transformer action. The direction of the induced e.m.f. can be found by using Lenz's law and is as shown in Fig. 36.37 (a). However, the direction of the *induced* currents in the armature conductors will depend on the *positions of the short-circuited brushes*.

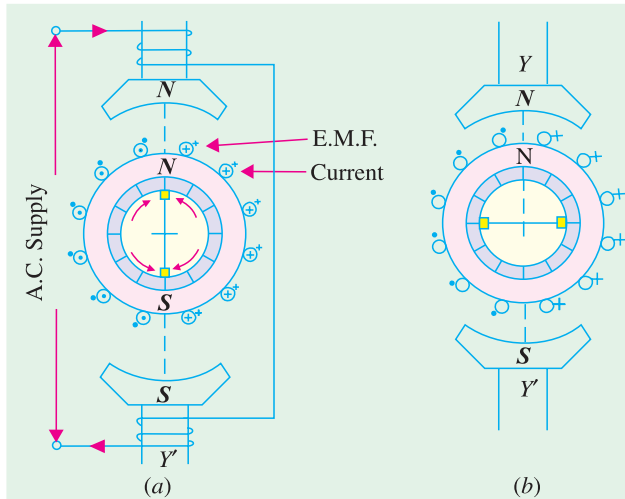


Fig. 36.37

If brush axis is colinear with magnetic axis of the main poles, the directions of the induced currents (shown by dots and arrows) will be as indicated in Fig. 36.37 (a)\*. As a result, the armature will become an electromagnet with a *N*-pole on its top, directly under the main *N*-pole and with a *S*-pole at the bottom, directly over the main *S*-pole. Because of this face-to-face positioning of the main and induced magnetic poles, no torque will be developed. The two forces of repulsion on top and bottom act along *Y Y'* in direct opposition to each other.\*\*

If brushes are shifted through 90° to the position shown in Fig. 36.37 (b) so that the brush axis is at right angles to

the magnetic axis of the main poles, the directions of the induced voltages at any time in the respective armature conductors are exactly the same as they were for the brush position of Fig. 36.37 (a). However, with brush positions of Fig. 36.37 (b), the voltages induced in the armature conductors in each path between the brush terminals will neutralize each other, hence there will be no net voltage across brushes to produce armature current. If there is no armature current, obviously, no torque will be developed.

If the brushes are set in position shown in Fig. 36.38 (a) so that the brush axis is neither in line with nor 90° from the magnetic axis *Y Y'* of the main poles, a net voltage\*\*\* will be induced between the brush terminals which will produce armature current. The armature will again act as an elec-

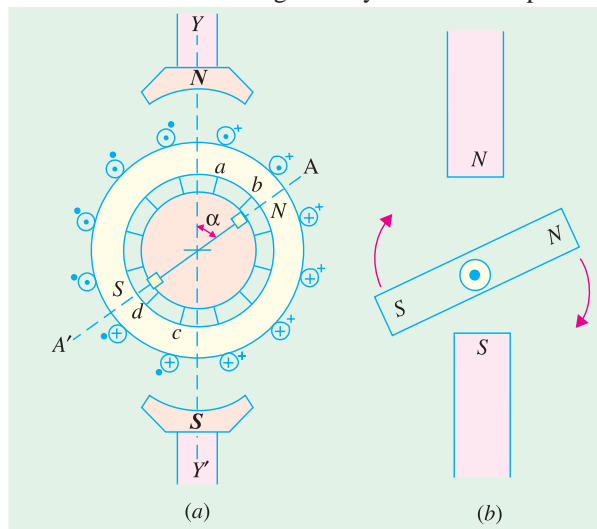


Fig. 36.38

\* It should be noted that during the next half-cycle of the supply current, the directions of the respective voltages will be in the opposite directions.

\*\* Alternatively, the absence of the torque may be explained by arguing that the torques developed in the four quadrants neutralize each other.

\*\*\* It will be seen from Fig. 36.38 (a) that the induced voltages in conductors *a* and *b* oppose the voltages in other conductors lying above brush-axis. Similarly, induced voltages in conductors *c* and *d* oppose the voltages in other conductors, lying below the brush-axis. Yet the net voltage across brush terminals will be sufficient to produce current which will make the armature a powerful magnet.

tromagnet and develop its own *N*- and *S*-poles which, in this case, will not directly face the respective main poles. As shown in Fig. 36.38 (a), the armature poles lie along *AA'* making an angle of  $\alpha$  with *YY*.

Hence, rotor *N*-pole will be repelled by the main *N*-pole and the rotor *S*-pole will, similarly, be repelled by the main *S*-pole. Consequently, the rotor will rotate in clockwise direction [Fig.36.38 (b)]. Since the forces are those of *repulsion*, it is appropriate to call the motor as repulsion motor.

It should be noted that if the brushes are shifted counter-clockwise from *YY'*, rotation will also be counter-clockwise. Obviously, direction of rotation of the motor is determined by the position of brushes with respect to the main magnetic axis.

It is worth noting that the value of starting torque developed by such a motor will depend on the *amount* of brush-shift whereas direction of rotation will depend on the *direction* of shift [Fig. 36.39 (a)]. Maximum starting torque is developed at some position where brush axis makes, an angle lying between  $0^\circ$  and  $45^\circ$  with the magnetic axis of main poles. Motor speed can also be controlled by means of brush shift. Variation of starting torque of a repulsion motor with brush-shift is shown in Fig. 36.39 (b).



Repulsion motor

A straight repulsion type motor has high starting torque (about 350 per cent) and moderate starting current (about 3 to 4 times full-load value).

Principal shortcomings of such a motor are :

1. speed varies with changing load, becoming dangerously high at no load.
2. low power factor, except at high speeds.
3. tendency to spark at brushes.

### 36.13. Compensated Repulsion Motor

It is a modified form of the straight repulsion motor discussed above. It has an additional stator winding, called compensating winding whose purpose is (i) to improve power-factor and (ii) to provide better speed regulation. This winding is much smaller than the stator winding and is usually wound in the inner slots of each main pole and is connected in series with the armature (Fig. 36.40) through an additional set of brushes placed mid-way between the usual short-circuited brushes.

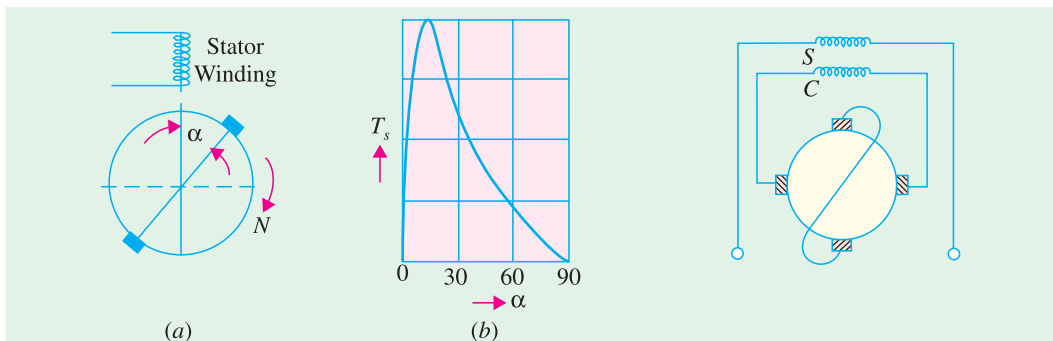


Fig. 36. 39

Fig. 36. 40

### 36.14. Repulsion-start Induction-Run Motor

As mentioned earlier, this motor starts as an ordinary repulsion motor, but after it reaches about 75 per cent of its full speed, centrifugal short-circuiting device short-circuits its commutator. From

then on, it runs as an induction motor, with a short-circuited squirrel-cage rotor. After the commutator is short-circuited, brushes do not carry any current, hence they may also be lifted from the commutator, in order to avoid unnecessary wear and tear and friction losses.

Repulsion-start motors are of two different designs :

1. Brush-lifting type in which the brushes are automatically lifted from the commutator when it is short-circuited. These motors generally employ radial form of commutator and are built both in small and large sizes.
2. Brush-riding type in which brushes ride on the commutator at all times. These motors use axial form of commutator and are always built in small sizes.

The starting torque of such a motor is in excess of 350 per cent with moderate starting current. It is particularly useful where starting period is of comparatively long duration, because of high inertia loads. Applications of such motors include machine tools, commercial refrigerators, compressors, pumps, hoists, floor-polishing and grinding devices etc.

### 36.15. Repulsion Induction Motor

In the field of repulsion motor, this type is becoming very popular, because of its good all-round characteristics which are comparable to those of a compound d.c. motor. It is particularly suitable for those applications where the load can be removed entirely by de-clutching or by a loose pulley.

This motor is a combination of the repulsion and induction types and is sometimes referred to as *squirrel-cage repulsion motor*. It possesses the desirable characteristics of a repulsion motor and the constant-speed characteristics of an induction motor.

It has the usual stator winding as in all repulsion motors. But there are two separate and independent windings in the rotor (Fig. 36.41).

- (i) a squirrel-cage winding and
- (ii) commutated winding similar to that of a d.c. armature.

Both these windings function during the *entire period* of operation of the motor. The commutated winding lies in the outer slots while squirrel-cage winding is located in the inner slots\*. At start, the commutated winding supplies most of the torque, the squirrel-cage winding being practically inactive because of its high reactance. When the rotor accelerates, the squirrel-cage winding takes up a larger portion of the load.

The brushes are short-circuited and ride on the commutator continuously. One of the advantages of this motor is that it requires no centrifugal short-circuiting mechanism. Sometimes such motors are also made with compensating winding for improving the power factor.

As shown in Fig. 36.42, its starting torque is high, being in excess of 300 per cent. Moreover, it has a fairly constant speed regulation. Its field of application includes house-hold refrigerators, garage air pumps, petrol pumps, compressors, machine tools, mixing machines, lifts and hoists etc.



Brushless d.c. seromotor

\* Hence, commutated winding has low resistance whereas the squirrel-cage winding has inherently a high reactance.



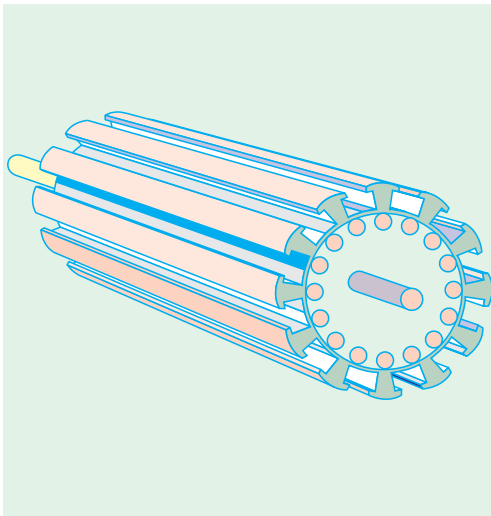


Fig. 36.41

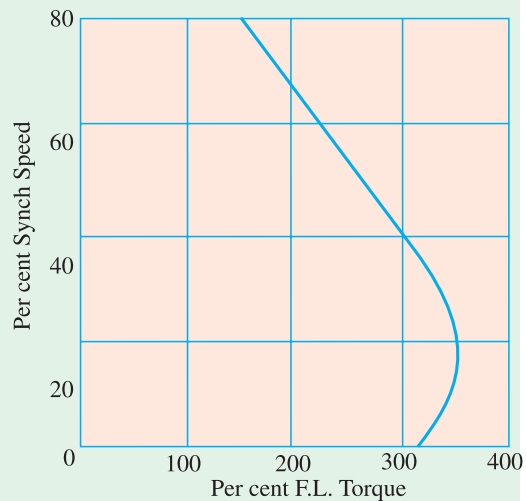


Fig. 36.42

These motors can be reversed by the usual brush-shifting arrangement.

### 36.16. A.C. Series Motors

If an ordinary d.c. series motor were connected to an a.c. supply, it will rotate and exert unidirectional torque\* because the current flowing both in the armature and field reverses at the same time. But the performance of such a motor will not be satisfactory for the following reasons :

1. the alternating flux would cause excessive eddy current loss in the yoke and field cores which will become extremely heated.
2. vicious sparking will occur at brushes because of the huge voltage and current induced in the short-circuited armature coils during their commutation period.
3. power factor is low because of high inductance of the field and armature circuits.



Induction type AC servo motor and control box

However, by proper modification of design and other refinements, a satisfactory single-phase motor has been produced.

The eddy current loss has been reduced by laminating the entire iron structure of the field cores and yoke.

Power factor improvement is possible only by reducing the magnitudes of the reactances of the field and armature windings. Field reactance is reduced by reducing the number of turns on the field windings. For a given current, it will reduce the field m.m.f. which will result in reduced air-gap flux. This will tend to increase the speed but reduce motor torque. To obtain the same torque, it will now be necessary to increase the number of armature turns proportionately. This will, however, result in increased inductive reactance of the armature, so that the overall reactance of the motor will not be

\* However, torque developed is not of constant magnitude (as in d.c. series motors) but pulsates between zero and maximum value each half-cycle.

significantly decreased. Increased armature m.m.f. can be neutralized effectively by using a compensating winding. In conductively-compensated motors, the compensating winding is connected in series with the armature [Fig. 36.43 (a)] whereas in inductively-compensated motors, the compensating winding is short-circuited and has no interconnection with the motor circuit [Fig. 36.43 (b)]. The compensating winding acts as a short-circuited secondary of a transformer, for which the armature winding acts as a primary. The current in the compensating winding will be proportional to the armature current and  $180^\circ$  out of phase with it.

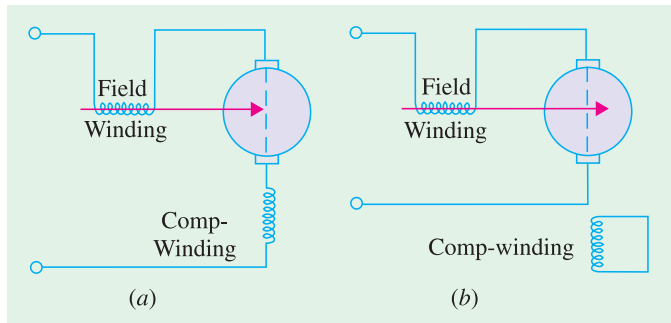


Fig. 36.43

Generally, all d.c. series motors are 'provided' with commutating poles for improving commutation (as in d.c. motors). But commutating poles alone will not produce satisfactory commutation, unless something is done to neutralize the huge voltage induced in the short-circuited armature coil by transformer action (this voltage is not there in d.c. series motor). It should be noted that in an a.c. series motor, the flux produced by the field winding is alternating and it induces voltage (by transformer action) in the short-circuited armature coil during its commutating period. The field winding, associated with the armature coil undergoing commutation, acts as primary and the armature coil during its commutating period acts as a short-circuited secondary. This transformer action produces heavy current in the armature coil as it passes through its commutating period and results in vicious sparking, unless the transformer voltage is neutralized. One method, which is often used for large motors, consists of shunting the winding of each commutating pole with a non-inductive resistance, as shown in Fig. 36.44 (a).

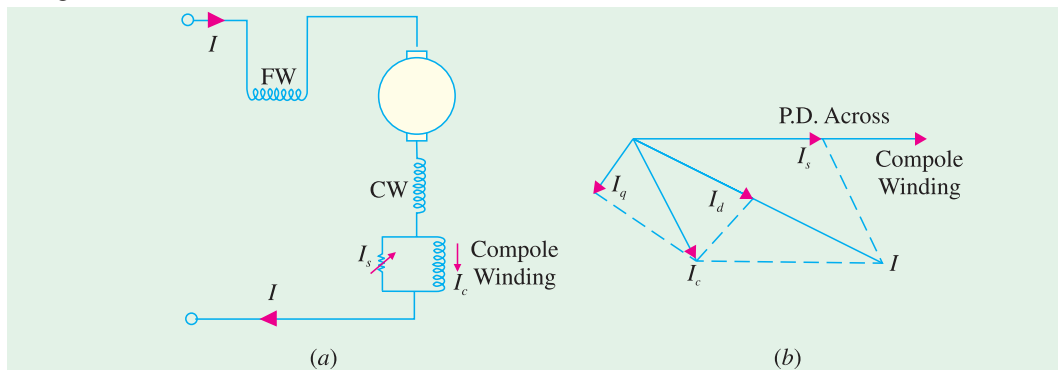


Fig. 36.44

Fig. 36.44. (b) shows the vector diagram of a shunted commutator pole. The current  $I_c$  through the commutating pole (which lags the total motor current) can be resolved into two rectangular components  $I_d$  and  $I_q$  as shown.  $I_d$  produces a flux which is in phase with total motor current  $I$  whereas flux produced by  $I_q$  lags  $I$  by  $90^\circ$ . By proper adjustment of shunt resistance (and hence  $I_s$ ), the speed voltage generated in a short-circuited coil by the cutting of the  $90^\circ$  lagging component of the commutating pole flux may be made to neutralize the voltage induced by transformer action.

### 36.17. Universal Motor

A universal motor is defined as a motor which may be operated either on direct or single-phase

a.c. supply at approximately the same speed and output.

In fact, it is a smaller version (5 to 150 W) of the a.c. series motor described in Art. 36.16. Being a series-wound motor, it has high starting torque and a variable speed characteristic. It runs at dangerously high speed on no-load. That is why such motors are usually built into the device they drive.

Generally, universal motors are manufactured in two types:

1. **concentrated-pole, non-compensated type** (low power rating)
2. **distributed-field compensated type** (high power rating)

The non-compensated motor has two salient poles and is just like a 2-pole series d.c. motor except that whole of its magnetic path is laminated (Fig. 36.45). The laminated stator is necessary because the flux is alternating when motor is operated from a.c. supply. The armature is of wound type and similar to that of a small d.c. motor. It consists essentially of a laminated core having either straight or skewed slots and a commutator to which the leads of the armature winding are connected.



Universal motor.

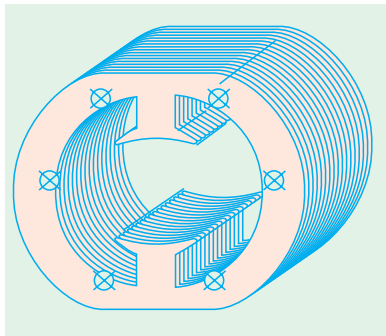


Fig. 36.45

The distributed-field compensated type motor has a stator core similar to that of a split-phase motor and a wound armature similar to that of a small d.c. motor. The compensating winding is used to reduce the reactance voltage present in the armature when motor runs on a.c. supply. This voltage is caused by the alternating flux by transformer action (Art. 36.16).

In a 2-pole non-compensated motor, the voltage induced by transformer action in a coil during its commutation period is not sufficient to cause any serious commutation trouble. Moreover, high-resistance brushes are used to aid commutation.

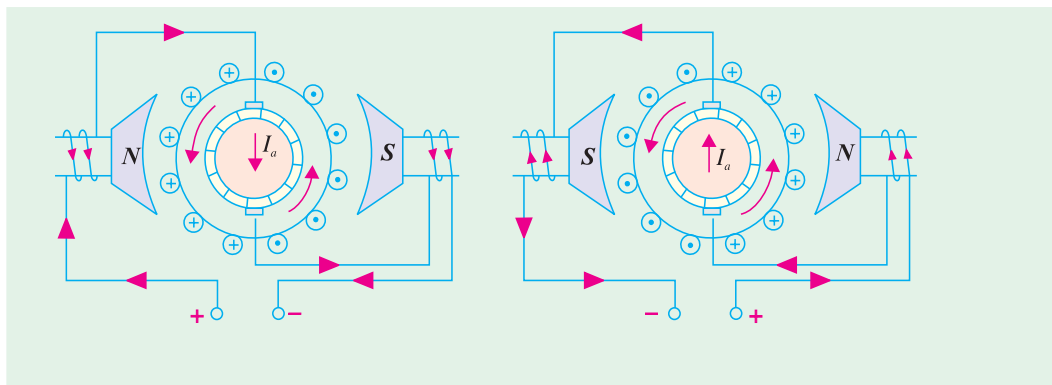


Fig. 36.46

Fig.36.47

**(a) Operation.** As explained in Art. 36.16, such motors develop unidirectional torque, regardless of whether they operate on d.c. or a.c. supply. The production of unidirectional torque, when the motor runs on a.c. supply can be easily understood from Fig. 36.46. The motor works on the same principle as a d.c. motor *i.e.* force between the main pole flux and the current-carrying armature conductors. This is true regardless of whether the current is alternating or direct (Fig. 36.47).

**(b) Speed/Load Characteristic.** The speed of a universal motor varies just like that of a d.c. series motor *i.e.* low at full-load and high on no-load (about 20,000 r.p.m. in some cases). In fact, on no-load the speed is limited only by its own friction and windage load. Fig. 36.48 shows typical torque characteristics of a universal motor both for d.c. and a.c. supply. Usually, gear trains are used to reduce the actual load speeds to proper values.

**(c) Applications.** Universal motors are used in vacuum cleaners where actual motor speed is the load speed. Other applications where motor speed is reduced by a gear train are : drink and food mixers, portable drills and domestic sewing machine etc.

**(d) Reversal of Rotation.** The concentrated-pole (or salient-pole) type universal motor may be reversed by reversing the flow of current through either the armature or field windings. The usual method is to interchange the leads on the brush holders (Fig.36.49).

The distributed-field compensated type universal motor may be reversed by interchanging either the armature or field leads and shifting the brushes against the direction in which the motor will rotate. The extent of brush shift usually amounts to several commutator bars.

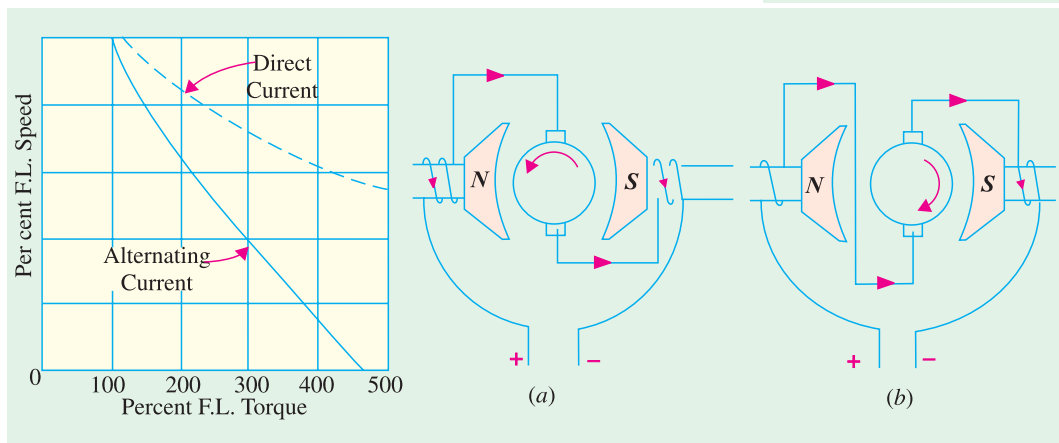
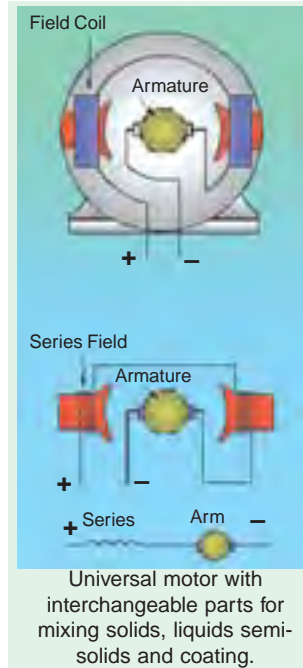


Fig. 36.48

Fig.36.49

### 36.18. Speed Control of Universal Motors

The following methods are usually employed for speed-control purposes :

**(i) Resistance Method.** As shown in Fig. 36.50, the motor speed is controlled by connecting a variable resistance  $R$  in series with the motor. This method is employed for motors used in sewing machines. The amount of resistance in the circuit is changed by means of a foot-pedal.

**(ii) Tapping-field Method.** In this method, a field pole is tapped at various points and speed is controlled by varying the field strength (Fig. 36.51). For this purpose, either of the following two arrangements may be used :

- (a)** The field pole is wound in various sections with different sizes of wire and taps are brought out from each section.
- (b)** Nichrome resistance wire is wound over one field pole and taps are brought out from this wire.

(iii) **Centrifugal Mechanism.** Universal motors, particularly those used for home food and drink mixers, have a number of speeds. Selection is made by a centrifugal device located inside the motor and connected, as shown in Fig. 36.52. The switch is adjustable by means of an external lever. If the motor speed rises above that set by the lever, the centrifugal device opens two contacts and inserts resistance  $R$  in the circuit, which causes the motor speed to decrease. When motor runs slow, the two contacts close and short-circuit the resistance, so that the motor speed rises. This process is repeated so rapidly that variations in speed are not noticeable.



Universal motor used for home food and drink mixers

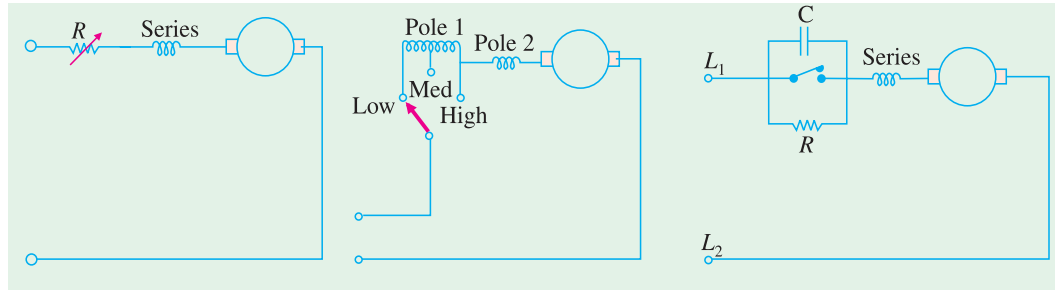


Fig. 36.50

Fig. 36.51

Fig. 36.52

The resistance  $R$  is connected across the governor points as shown in Fig. 36.52. A capacitor  $C$  is used across the contact points in order to reduce sparking produced due to the opening and closing of these points. Moreover, it prevents the pitting of contacts.

**Example 36.4.** A 250-W, single-phase, 50-Hz, 220-V universal motor runs at 2000 rpm and takes 1.0 A when supplied from a 220-V dc. supply. If the motor is connected to 220-V ac supply and takes 1.0 A (r.m.s), calculate the speed, torque and power factor. Assume  $R_a = 20 \Omega$  and  $L_a = 0.4$  H.

**Solution. DC Operation :**  $E_{b,dc} = V - I_a R_a$   
 $= 220 - 20 \times 1 = 200$  V

**AC Operation**

$X_a = 2 \pi \times 50 \times 0.4 = 125.7 \Omega$ .

As seen from Fig.36.53.

$V^2 = (E_{b,ac} + I_a R_a)^2 + (I_a \times X_a)^2$

$\therefore E_{b,ac} = -I_a R_a + \sqrt{V^2 - (I_a X_a)^2}$

$= -1 \times 20 + \sqrt{220^2 - (125.7 \times 1)^2} = 160.5$  V

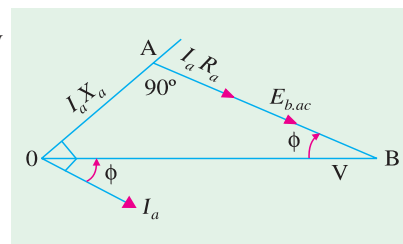


Fig. 36.53

Since armature current is the same for both dc and ac excitations, hence

$\frac{E_{b,dc}}{E_{b,ac}} = \frac{N_{dc}}{N_{ac}} ; \quad \therefore N_{ac} = 2000 \times \frac{160.5}{200} = 1605$  rpm

$\cos \phi = AB/OB = (E_{b,ac} + I_a R_a) / V = (160.5 + 20) / 220 = 0.82$  lag

$$P_{mech} = E_{b.ac} \cdot I_a = 160.5 \times 1 = 160.5 \text{ W}$$

$$T = 9.55 \times 160.5/1605 = \mathbf{0.955 \text{ N-m}}$$

**Example 36.5.** A universal series motor has resistance of  $30 \Omega$  and an inductance of  $0.5 \text{ H}$ . When connected to a  $250 \text{ V d.c.}$  supply and loaded to take  $0.8 \text{ A}$ , it runs at  $2000 \text{ r.p.m.}$  Estimate its speed and power factor, when connected to a  $250\text{-V, } 50\text{-Hz a.c.}$  supply and loaded to take the same current. (Elect. Machine, A.M.I.E. Sec. B, 1992)

**Solution. A.C. Operation**

$$X_a = 2 \pi \times 50 \times 0.5 = 157 \Omega,$$

$$R_a = 30 \Omega$$

$$I_a R_a = 0.8 \times 30 = 24 \Omega$$

$$I_a X_a = 0.8 \times 157 = 125.6 \text{ V}$$

The phasor diagram is shown in Fig. 36.54 (b)

$$V^2 = (E_{b.ac} + I_a R_a)^2 + (I_a X_a)^2$$

$$250^2 = (E_{b.ac} + 24)^2 + 125.6^2$$

$$\therefore E_{b.ac} = 192.6$$

**DC Operation**

$$E_{b.dc} = 250 - 0.8 \times 30 = 226 \text{ V}$$

Now,  $\frac{E_{b.ac}}{E_{b.dc}} = \frac{N_{ac}}{N_{dc}}$  or  $\frac{192.12}{226} = \frac{N_{ac}}{2000}$  ;

$$N_{ac} = 1700 \text{ rpm}$$

$$\cos \phi = (E_{b.ac} + I_a R_a)/V = 236.12/250 = \mathbf{0.864 \text{ lag}}$$

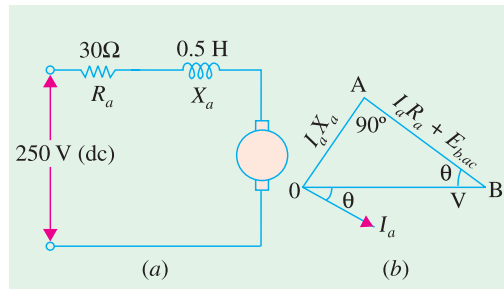


Fig. 36.54

**36.19. Unexcited Single-phase Synchronous Motors**

These motors

1. operate from a single-phase a.c. supply
2. run at a constant speed – the synchronous speed of the revolving flux
3. need no d.c. excitation for their rotors (that is why they are called *unexcited*)
4. are self-starting.

These are of two types (a) reluctance motor and (b) hysteresis motor.

**36.20. Reluctance Motor**

It has either the conventional split-phase stator and a centrifugal switch for cutting out the auxiliary winding (split-phase type reluctance motor) or a stator similar to that of a permanent-split capacitor-run motor (capacitor-type reluctance motor). The stator produces the revolving field.

The squirrel-cage rotor is of unsymmetrical magnetic construction. This type of unsymmetrical construction can be achieved by removing some of the teeth of a symmetrical squirrel-cage rotor punching. For example, in a 48-teeth, four-pole rotor following teeth may be cut away :

- 1, 2, 3, 4, 5, 6–13, 14, 15, 16, 17, 18–25, 26, 27, 28, 29, 30–37, 38, 39, 40, 41, 42.

This would leave four projecting or salient poles (Fig. 36.55) consisting of the following sets of teeth : 7–12; 19–24; 31–36 and 43–48. In this way, the rotor offers variable magnetic reluctance to the stator flux, the reluctance varying with the position of the rotor.

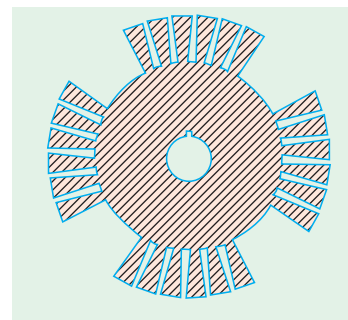


Fig. 36.55

### Working

For understanding the working of such a motor one basic fact must be kept in mind. And it is that when a piece of magnetic material is located in a magnetic field, a force acts on the material, tending to bring it into the most dense portion of the field. The force tends to align the specimen of material in such a way that the reluctance of the magnetic path that lies through the material will be minimum.

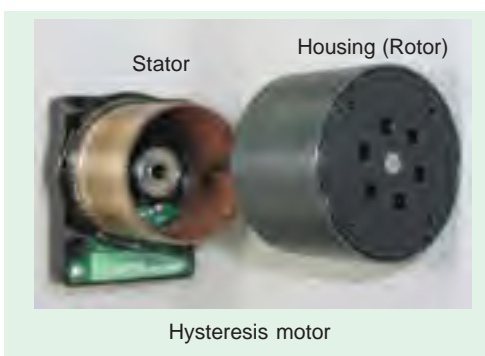
When the stator winding is energised, the revolving magnetic field exerts reluctance torque on the unsymmetrical rotor tending to align the salient pole axis of the rotor with the axis of the revolving magnetic field (because in this position, the reluctance of the magnetic path is minimum). If the reluctance torque is sufficient to start the motor and its load, the rotor will pull into step with the revolving field and continue to run at the speed of the revolving field.\*

However, even though the rotor revolves synchronously, its poles lag behind the stator poles by a certain angle known as torque angle, (something similar to that in a synchronous motor). The reluctance torque increases with increase in torque angle, attaining maximum value when  $\alpha = 45^\circ$ . If  $\alpha$  increases beyond  $45^\circ$ , the rotor falls out of synchronism. The average value of the reluctance torque is given by  $T = K (V/f)^2 \sin 2\delta$  where  $K$  is a motor constant.

It may be noted that the amount of load which a reluctance motor could carry at its *constant speed* would only be a fraction of the load that the motor could normally carry when functioning as an induction motor. If the load is increased beyond a value under which the reluctance torque cannot maintain synchronous speed, the rotor drops out of step with the field. The speed, then, drops to some value at which the slip is sufficient to develop necessary torque to drive the load by induction-motor action.

The constant-speed characteristic of a reluctance motor makes it very suitable for such applications as signalling devices, recording instruments, many kinds of timers and phonographs etc.

### 36.21. Hysteresis Motor



The operation of this motor depends on the presence of a *continuously-revolving* magnetic flux. Hence, for the split-phase operation, its stator has two windings which remain connected to the single-phase supply continuously both at starting as well as during the running of the motor. Usually, shaded-pole principle is employed for this purpose giving shaded-pole hysteresis motor. Alternatively, stator winding of the type used in capacitor-type motor may be used giving capacitor-type shaded-pole motor. Obviously, in either type, no centrifugal device is used.

The rotor is a smooth chrome-steel cylinder\*\* having

\* Actually, the motor starts as an induction motor and after it has reached its maximum speed as an induction motor, the reluctance torque pulls its rotor into step with the revolving field so that the motor now runs as a synchronous motor by virtue of its saliency.

\*\* Rotors of ceramic permanent magnet material are used whose resistivity approaches that of an insulator. Consequently, it is impossible to set up eddy currents in such a rotor. Hence, there is no eddy current loss but only hysteresis loss.

high retentivity so that the hysteresis loss is high. It has no winding. Because of high retentivity of the rotor material, it is very difficult to change the magnetic polarities once they are induced in the rotor by the revolving flux. The rotor revolves synchronously because the rotor poles magnetically lock up with the revolving stator poles of opposite polarity. However, the rotor poles always lag behind the stator poles by an angle  $\alpha$ . Mechanical power developed by rotor is given by  $P_m = P_h \left( \frac{1-s}{s} \right)$  where  $P_h$  is hysteresis loss in rotor. Also  $T_h = 9.55 P_m / N_s$ . It is seen that hysteresis torque depends solely on the area of rotor's hysteresis loop.

The fact that the rotor has no teeth or winding of any sort, results in making the motor extremely quiet in operation and free from mechanical and magnetic vibrations. This makes the motor particularly useful for driving tape-decks, tape-decks, turn-tables and other precision audio equipment. Since, commercial motors usually have two poles, they run at 3,000 r.p.m. at 50-Hz single-phase supply. In order to adopt such a motor for driving an electric clock and other indicating devices, gear train is connected to the motor shaft for reducing the load speed. The unit accelerates rapidly, changing from rest to full speed almost instantaneously. It must do so because it cannot accelerate gradually as an ordinary motor it is either operating at synchronous speed or not at all.

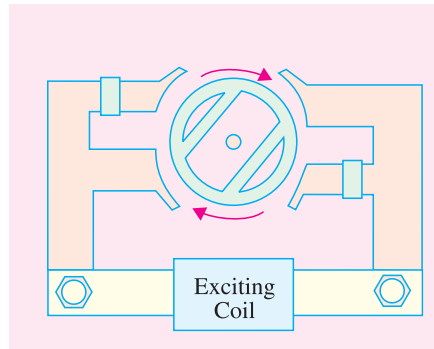


Fig. 36.56

Some unique features of a hysteresis motor are as under :

- (i) since its hysteresis torque remains practically constant from locked rotor to synchronous speed, a hysteresis motor is able to synchronise any load it can accelerate—something no other motor does.
- (ii) due to its smooth rotor, the motor operates quietly and does not suffer from magnetic pulsations caused by slots/salient-poles that are present in the rotors of other motors.

In Fig. 36.56. is shown a two-pole shaded-pole type hysteresis motor used for driving ordinary household electric clocks. The rotor is a thin metal cylinder and the shaft drives a gear train.

**Example 36.6.** A 8-kW, 4-pole, 220-V, 50-Hz reluctance motor has a torque angle of  $30^\circ$  when operating under rated load conditions. Calculate (i) load torque (ii) torque angle if the voltage drops to 205 V and (iii) will the rotor pulled out of synchronism ?

**Solution.** (i)  $N_s = 120 \times 50/4 = 1500$  rpm;  $T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 8000/1500 = 51N\text{-m}$

(ii) With the same load torque and constant frequency,

$$V_1 \sin 2\alpha_1 = V_2^2 \sin 2\alpha_2$$

$$\therefore 220^2 \times \sin (2 \times 30^\circ) = 205^2 \times \sin 2\alpha; \quad \therefore \alpha = 42.9^\circ$$

(iii) since the new load angle is less than  $45^\circ$ , the rotor will not pull out of synchronous.

### Tutorial Problems-36.2.

1. A 230-V, 50-Hz, 4-pole, class-A, single-phase induction motor has the following parameters at an operating temperature  $63^\circ\text{C}$  :

$$r_{1m} = 2.51 \text{ ohms}, r_2' = 7.81 \text{ ohm}, X_m = 150.88 \text{ ohm}, X_{im} = 4.62 \text{ ohm}, X_2' = 4.62 \text{ ohms}$$

Determine stator main winding current and power factor when the motor is running at a slip of 0.05 at the specified temperature of  $63^\circ\text{C}$ .

[3.74  $\angle 48.24^\circ$ , 0.666] (AMIE Sec. B Elect. Machines (E-B) Summer 1991)



2. A fractional horse-power universal motor has armature circuit resistance of 20 ohm and inductance of 0.4 H. On being connected to a 220-V d.c. supply, it draws 1.0 A from the mains and runs at 2000 r.p.m. Estimate the speed and power factor of the motor, when connected to a 230-V, 50-Hz supply drawing the same armature current. Draw relevant phasor diagram.  
 [1726 rpm, 0.84] (AMIE Sec. B Elect. Machines 1991)
3. A universal series motor, when operating on 220 V d.c. draws 10 A and runs at 1400 r.p.m. Find the new speed and power factor, when connected to 220 V, 25 Hz supply, the motor current remaining the same. The motor has total resistance of 1 ohm and total inductance of 0.1 H.  
 [961 rpm; 0.7] (AMIE Sec. B Elect. Machines 1990)

QUESTIONS AND ANSWERS ON SINGLE-PHASE MOTORS

**Q.1. How would you reverse the direction of rotation of a capacitor start-induction-run motor ?**

**Ans.** By reversing either the running or starting-winding leads where they are connected to the lines. Both must not be reversed.

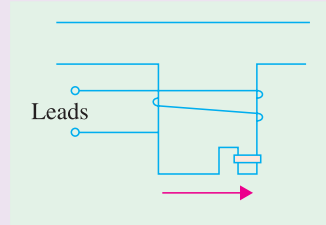


Fig. 36.57

**Q.2. In which direction does a shaded-pole motor run ?**

**Ans.** It runs from the unshaded to the shaded pole (Fig.36.57)

**Q.3. Can such a motor be reversed ?**

**Ans.** Normally, such motors are not reversible because that would involve mechanical dismantling and re-assembly. However, special motors are made having two rotors on a common shaft, each having one stator assembly for rotation in opposite direction.

**Q.4. What is a universal motor ?**

**Ans.** It is built like a series d.c. motor with the difference that both its stator and armature are laminated. They can be used either on d.c. or a.c. supply although the speed and power are greater on direct current. They cannot be satisfactorily made to run at less than about 2000 r.p.m.

**Q.5. How can a universal motor be reversed ?**

**Ans.** By reversing either the field leads or armature leads but not both.

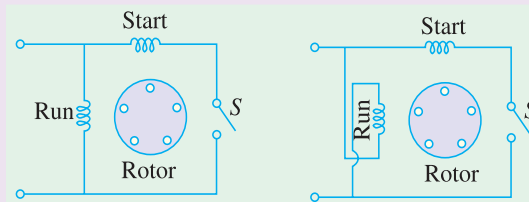


Fig. 36.58

**Q.6. How can we reverse the direction of rotation of repulsion, repulsion-induction and repulsion-induction and repulsion- start-induction-run motors ?**

**Ans.** By shifting the brush positions by about 15° electrical.

**Q.7. How can we reverse the rotation of a 1-phase, split-phase motor ?**

**Ans.** By reversing the leads to either the running or starter winding (Fig. 36.58) but not both.

**Q.8. What could be the reasons if a repulsion-induction motor fails to start ?**

**Ans.** Any one of the following :

1. no supply voltage
2. low voltage
3. excessive overload
4. the bearing lining may be stuck or 'frozen' to the shaft
5. armature may be rubbing
6. brush yoke may be incorrectly located
7. brush spacing may be wrong.

**Q.9. What could be the reasons if a split-phase motor fails to start and hums loudly ?**

**Ans.** It could be due to the starting winding being open or grounded or burnt out.

**Q.10. What could be the reasons if a split-phase motor runs too slow ?**

**Ans.** Any one of the following factors could be responsible :

1. wrong supply voltage and frequency
2. overload
3. grounded starting and running windings
4. short-circuited or open winding in field circuit.

### OBJECTIVE TESTS – 36

1. The starting winding of a single-phase motor is placed in the
  - (a) rotor
  - (b) stator
  - (c) armature
  - (d) field.
2. One of the characteristics of a single-phase motor is that it
  - (a) is self-starting
  - (b) is not self-starting
  - (c) requires only one winding
  - (d) can rotate in one direction only.
3. After the starting winding of a single-phase induction motor is disconnected from supply, it continues to run only on .....winding.
  - (a) rotor
  - (b) compensating
  - (c) field
  - (d) running
4. If starting winding of a single-phase induction motor is left in the circuit, it will
  - (a) draw excessive current and overheat
  - (b) run slower
  - (c) run faster
  - (d) spark at light loads.
5. The direction of rotation of a single-phase motor can be reversed by
  - (a) reversing connections of both windings
  - (b) reversing connections of starting winding
  - (c) using a reversing switch
  - (d) reversing supply connections.
6. If a single-phase induction motor runs slower than normal, the more likely defect is
  - (a) improper fuses
  - (b) shorted running winding
  - (c) open starting winding
  - (d) worn bearings.
7. The capacitor in a capacitor-start induction-run ac motor is connected in series with ..... winding.
  - (a) starting
  - (b) running
  - (c) squirrel-cage
  - (d) compensating
8. A permanent-split single-phase capacitor motor does not have
  - (a) centrifugal switch
  - (b) starting winding
  - (c) squirrel-cage rotor
  - (d) high power factor.
9. The starting torque of a capacitor-start induction-run motor is directly related to the angle  $\alpha$  between its two winding currents by the relation
  - (a)  $\cos \alpha$
  - (b)  $\sin \alpha$
  - (c)  $\tan \alpha$
  - (d)  $\sin \alpha/2$ .
10. In a two-value capacitor motor, the capacitor used for running purposes is a/an
  - (a) dry-type ac electrolytic capacitor
  - (b) paper-spaced oil-filled type
  - (c) air-capacitor
  - (d) ceramic type.
11. If the centrifugal switch of a two-value capacitor motor using two capacitors fails to open, then
  - (a) electrolytic capacitor will, in all probability, suffer breakdown
  - (b) motor will not carry the load
  - (c) motor will draw excessively high current
  - (d) motor will not come upto the rated speed.
12. Each of the following statements regarding a shaded-pole motor is true **except**
  - (a) its direction of rotation is from un-shaded to shaded portion of the poles
  - (b) it has very poor efficiency
  - (c) it has very poor p.f.
  - (d) it has high starting torque.
13. Compensating winding is employed in an ac series motor in order to

- (a) compensate for decrease in field flux  
 (b) increase the total torque  
 (c) reduce the sparking at brushes  
 (d) reduce effects of armature reaction.
14. A universal motor is one which  
 (a) is available universally  
 (b) can be marketed internationally  
 (c) can be operated either on dc or ac supply  
 (d) runs at dangerously high speed on no-load.
15. In a single-phase series motor the main purpose of inductively-wound compensating winding is to reduce the  
 (a) reactance emf of commutation  
 (b) rotational emf of commutation  
 (c) transformer emf of commutation  
 (d) none of the above.  
**(Power App.-II, Delhi Univ. Jan. 1987)**
16. A repulsion motor is equipped with  
 (a) a commutator (b) slip-rings  
 (c) a repeller  
 (d) neither (a) nor (b).
17. A repulsion-start induction-run single-phase motor runs as an induction motor only when  
 (a) brushes are shifted to neutral plane  
 (b) short-circuiter is disconnected  
 (c) commutator segments are short-circuited  
 (d) stator winding is reversed.
18. If a dc series motor is operated on ac supply, it will  
 (a) have poor efficiency  
 (b) have poor power factor  
 (c) spark excessively  
 (d) all of the above  
 (e) none of the above.
19. An outstanding feature of a universal motor is its  
 (a) best performance at 50 Hz supply  
 (b) slow speed at all loads  
 (c) excellent performance on dc. supply  
 (d) highest output kW/kg ratio.
20. The direction of rotation of a hysteresis motor is determined by the  
 (a) retentivity of the rotor material  
 (b) amount of hysteresis loss  
 (c) permeability of rotor material  
 (d) position of shaded pole with respect to the main pole.
21. Speed of the universal motor is  
 (a) dependent on frequency of supply  
 (b) proportional to frequency of supply  
 (c) independent of frequency of supply  
 (d) none of the above.  
**(Elect. Machines, A.M.I.E. Sec. B, 1993)**
22. In the shaded pole squirrel cage induction motor the flux in the shaded part always  
 (a) leads the flux in the unshaded pole segment  
 (b) is in phase with the flux in the unshaded pole segment  
 (c) lags the flux in the unshaded pole segment  
 (d) none of the above.  
**(Elect. Machines, A.M.I.E. Sec. B, 1993)**
23. Which of the following motor is an interesting example of beneficially utilizing a phenomenon that is often considered undesirable ?  
 (a) hysteresis motor  
 (b) reluctance motor  
 (c) stepper motor  
 (d) shaded-pole motor.
24. Usually, large motors are more efficient than small ones. The efficiency of the tiny motor used in a wrist watch is approximately..... per cent.  
 (a) 1  
 (b) 10  
 (c) 50  
 (d) 80

## ANSWERS

1. b 2. b 3. d 4. a 5. b 6. d 7. a 8. a 9. b 10. b 11. a 12. d 13. d 14. c 15. d 16. a  
 17. c 18. d 19. d 20. d 21. a 22. c 23. a 24. a

## ROUGH WORK

# CHAPTER 37

## Learning Objectives

- Basic Principle
- Stationary Armature
- Rotor
- Armature Windings
- Wye and Delta Connections
- Distribution or Breadth Factor or Winding Factor or Spread Factor
- Equation of Induced E.M.F.
- Factors Affecting Alternator Size
- Alternator on Load
- Synchronous Reactance
- Vector Diagrams of Loaded Alternator
- Voltage Regulation
- Rothert's M.M.F. or Ampere-turn Method
- Zero Power Factor Method or Potier Method
- Operation of Salient Pole Synchronous Machine
- Power Developed by a Synchronous Generator
- Parallel Operation of Alternators
- Synchronizing of Alternators
- Alternators Connected to Infinite Bus-bars
- Synchronizing Torque  $T_{sy}$
- Alternative Expression for Synchronizing Power
- Effect of Unequal Voltages
- Distribution of Load
- Maximum Power Output
- Questions and Answers on Alternators

## ALTERNATORS



 Alternator

### 37.1. Basic Principle

A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the **armature rotates** and the field system is **stationary**, the arrangement in alternators is just the reverse of it. In their case, standard construction consists of armature winding mounted on a stationary element called **stator** and field windings on a rotating element called rotor. The details of construction are shown in Fig. 37.1.

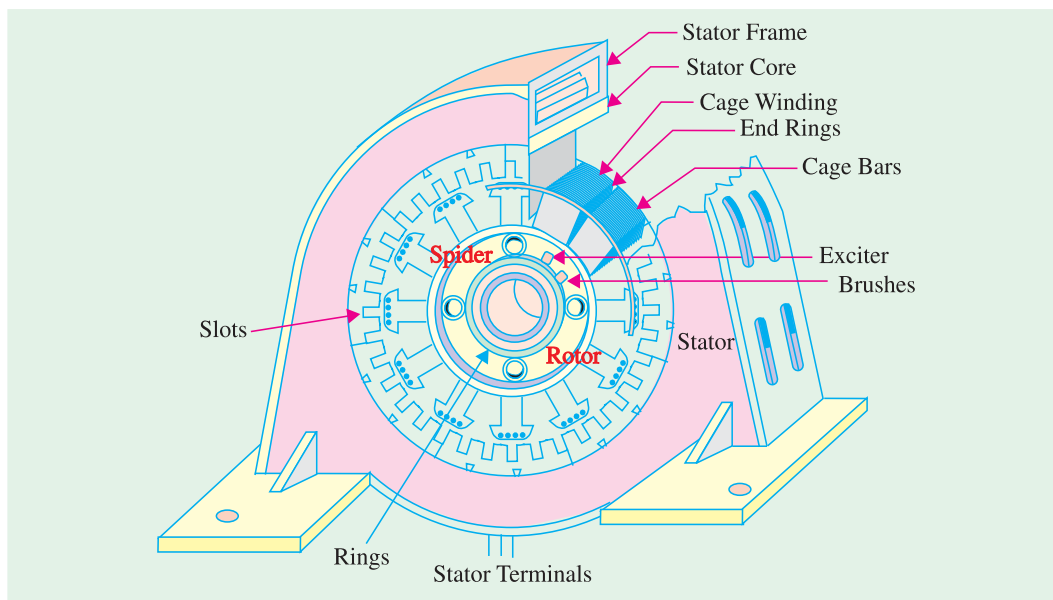


Fig. 37.1

The stator consists of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors. The rotor is like a flywheel having alternate *N* and *S* poles fixed to its outer rim. The magnetic poles are excited (or magnetised) from direct current supplied by a d.c. source at 125 to 600 volts. In most cases, necessary exciting (or magnetising) current is obtained from a small d.c. shunt generator which is belted or mounted on the shaft of the alternator itself. Because the field magnets are rotating, this current is supplied through two sliprings. As the exciting voltage is relatively small, the slip-rings and brush gear are of light construction. Recently, brushless excitation systems have been developed in which a 3-phase a.c. exciter and a group of rectifiers supply d.c. to the alternator. Hence, brushes, slip-rings and commutator are eliminated.

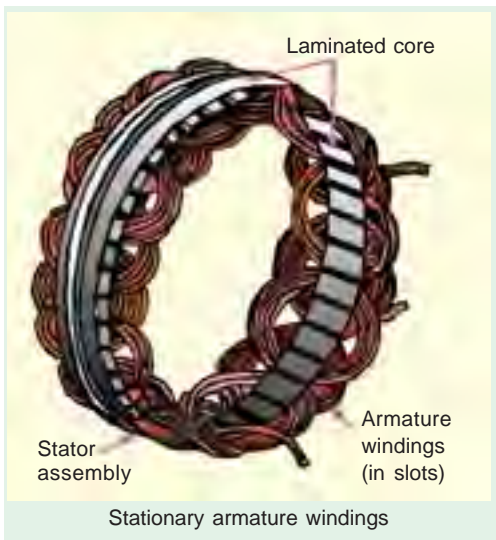
When the rotor rotates, the stator conductors (being stationary) are cut by the magnetic flux, hence they have induced e.m.f. produced in them. Because the magnetic poles are alternately *N* and *S*, they induce an e.m.f. and hence current in armature conductors, which first flows in one direction and then in the other. Hence, an alternating e.m.f. is produced in the stator conductors ( $i$ ) whose frequency depends on the

number of  $N$  and  $S$  poles moving past a conductor in one second and **(ii)** whose direction is given by Fleming's Right-hand rule.

### 37.2. Stationary Armature

Advantages of having stationary armature (and a rotating field system) are :

1. The output current can be led directly from fixed terminals on the stator (or armature windings) to the load circuit, without having to pass it through brush-contacts.



Alternators

2. It is easier to insulate stationary armature winding for high a.c. voltages, which may have as high a value as 30 kV or more.
3. The sliding contacts *i.e.* slip-rings are transferred to the low-voltage, low-power d.c. field circuit which can, therefore, be easily insulated.
4. The armature windings can be more easily braced to prevent any deformation, which could be produced by the mechanical stresses set up as a result of short-circuit current and the high centrifugal forces brought into play.

### 37.3. Details of Construction

#### 1. Stator Frame

In d.c. machines, the outer frame (or yoke) serves to carry the magnetic flux but in alternators, it is not meant for that purpose. Here, it is used for holding the armature

stampings and windings in position. Low-speed large-diameter alternators have frames which because of ease of manufacture, are cast in sections. Ventilation is maintained with the help of holes cast in the frame itself. The provision of radial ventilating spaces in the stampings assists in cooling the machine.

But, these days, instead of using castings, frames are generally fabricated from mild steel plates welded together in such a way as to form a frame having a box type section.

In Fig. 37.2 is shown the section through the top of a typical stator.

#### 2. Stator Core

The armature core is supported by the stator frame and is built up of laminations of

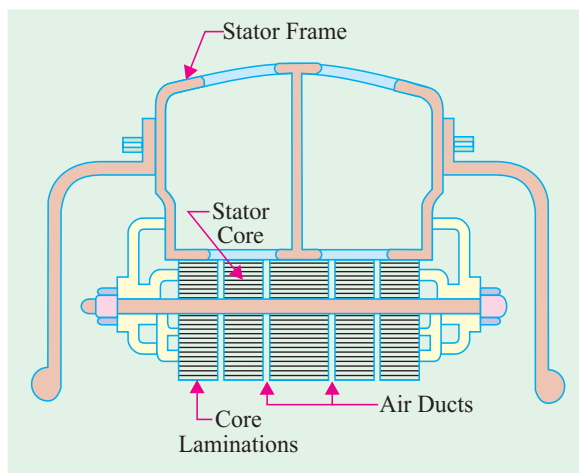


Fig. 37.2

special magnetic iron or steel alloy. The core is laminated to minimise loss due to eddy currents. The laminations are stamped out in complete rings (for smaller machine) or in segments (for larger machines). The laminations are insulated from each other and have spaces between them for allowing the cooling air to pass through. The slots for housing the armature conductors lie along the inner periphery of the core and are stamped out at the same time when laminations are formed. Different shapes of the armature slots are shown in Fig. 37.3.

The wide-open type slot (also used in d.c. machines) has the advantage of permitting easy installation of form-wound coils and their easy removal in case of repair. But it has the disadvantage of distributing the air-gap flux into bunches or tufts, that produce ripples in the wave of the generated e.m.f. The semi-closed type slots are better in this respect, but do not allow the use of form-wound coils. The wholly-closed type slots or tunnels do not disturb the air-gap flux but (i) they tend to increase the inductance of the windings (ii) the armature conductors have to be threaded through, thereby increasing initial labour and cost of winding and (iii) they present a complicated problem of end-connections. Hence, they are rarely used.



Streetbike stator

37.4. Rotor

Two types of rotors are used in alternators (i) salient-pole type and (ii) smooth-cylindrical type.

(i) Salient (or projecting) Pole Type

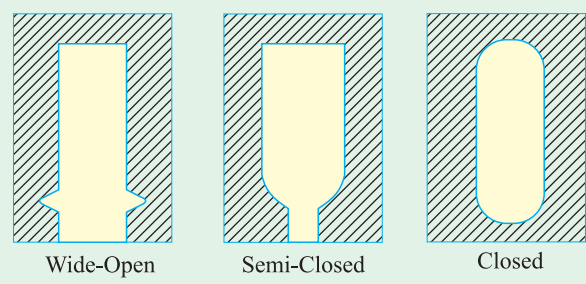
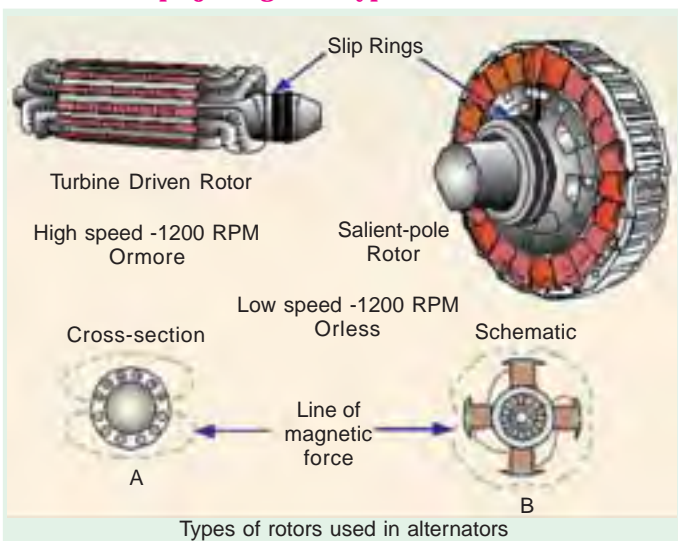


Fig. 37.3

It is used in low- and medium-speed (engine driven) alternators. It has a large number of projecting (salient) poles, having their cores bolted or dovetailed onto a heavy magnetic wheel of cast-iron, or steel of good magnetic quality (Fig. 37.4). Such generators are characterised by their large diameters and short axial lengths. The poles and pole-shoes (which cover 2/3 of pole-pitch) are laminated to minimize heating due to eddy currents. In large machines, field windings consist of rectangular copper strip wound on edge.



**(ii) Smooth Cylindrical Type**

It is used for steam turbine-driven alternators *i.e.* turbo-alternators, which run at very high speeds. The rotor consists of a smooth solid forged steel cylinder, having a number of slots milled out at intervals along the outer periphery (and parallel to the shaft) for accommodating field coils. Such rotors are designed mostly for 2-pole (or 4-pole) turbo-generators running at 3600 r.p.m. (or 1800 r.p.m.). Two (or four) regions corresponding to the central polar areas are left unslotted, as shown in Fig. 37.5 (a) and (b).

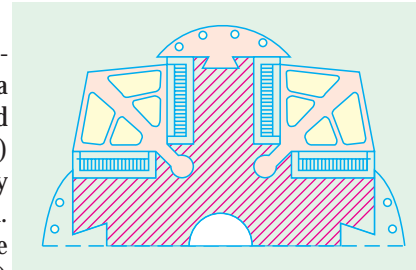


Fig. 37.4



Turbine alternator

The central polar areas are surrounded by the field windings placed in slots. The field coils are so arranged around these polar areas that flux density is maximum on the polar central line and gradually falls away on either side. It should be noted that in this case, poles are non-salient *i.e.* they do not project out from the surface of the rotor. To avoid excessive peripheral velocity, such rotors have very small diameters (about 1 metre or so). Hence, turbo-generators are characterised by small diameters and very long axial (or rotor) length. The cylindrical construction of the rotor gives better balance and

quieter-operation and also less windage losses.

**37.5. Damper Windings**

Most of the alternators have their pole-shoes slotted for receiving copper bars of a grid or damper winding (also known as squirrel-cage winding). The copper bars are short-circuited at both ends by heavy copper rings (Fig. 37.6). These dampers are useful in preventing the hunting (momentary speed fluctuations) in generators and are needed in synchronous motors to provide the starting torque. Turbo-generators usually do not have these damper windings (except in special case to assist in synchronizing) because the solid field-poles themselves act as efficient dampers. It should be clearly understood that under normal running conditions, damper winding does not carry any current because rotor runs at synchronous speed.

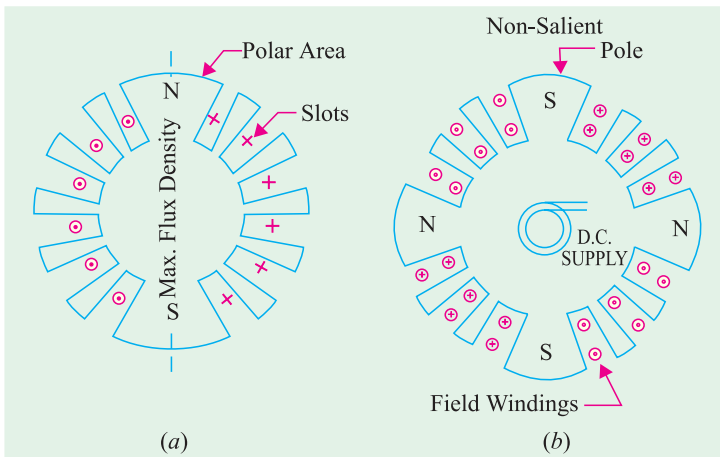


Fig. 37.5

The damper winding also tends to maintain balanced 3- $\phi$  voltage under unbalanced load conditions.

### 37.6. Speed and Frequency

In an alternator, there exists a definite relationship between the rotational speed ( $N$ ) of the rotor, the frequency ( $f$ ) of the generated e.m.f. and the number of poles  $P$ .

Consider the armature conductor marked  $X$  in Fig. 37.7 situated at the centre of a  $N$ -pole rotating in clockwise direction. The conductor being situated at the place of maximum flux density will have maximum e.m.f. induced in it.

The direction of the induced e.m.f. is given by Fleming's right hand rule. But while applying this rule, one should be careful to note that the thumb indicates the direction of the motion of the conductor relative to the field. To an observer stationed on the clockwise revolving poles, the conductor would seem to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced e.m.f. is downwards, in a direction at right angles to the plane of the paper.

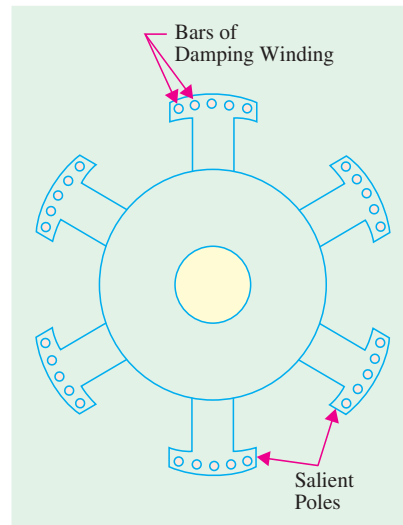


Fig. 37.6

When the conductor is in the interpolar gap, as at  $A$  in Fig. 37.7, it has minimum e.m.f. induced in it, because flux density is minimum there. Again, when it is at the centre of a  $S$ -pole, it has maximum e.m.f. induced in it, because flux density at  $B$  is maximum. But the direction of the e.m.f. when conductor is over a  $N$ -pole is opposite to that when it is over a  $S$ -pole.

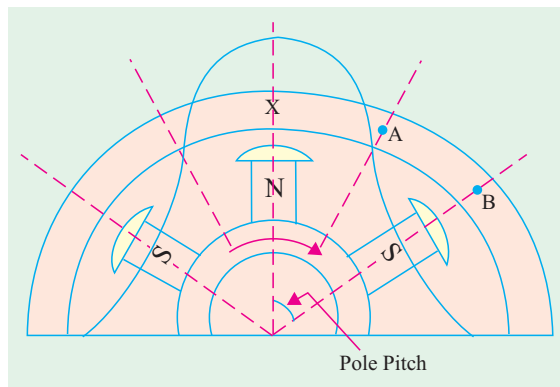


Fig. 37.7

Obviously, one cycle of e.m.f. is induced in a conductor when one pair of poles passes over it. In other words, the e.m.f. in an armature conductor goes through one cycle in angular distance equal to twice the pole-pitch, as shown in Fig. 37.7.

- Let  $P$  = total number of magnetic poles
- $N$  = rotative speed of the rotor in r.p.m.
- $f$  = frequency of generated e.m.f. in Hz.

Since one cycle of e.m.f. is produced when a pair of poles passes past a conductor, the number of cycles of e.m.f. produced in one revolution of the rotor is equal to the number of pair of poles.

$$\therefore \text{No. of cycles/revolution} = P/2 \text{ and No. of revolutions/second} = N/60$$

$$\therefore \text{frequency} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ Hz}$$

$$\text{or} \quad f = \frac{PN}{120} \text{ Hz}$$

$N$  is known as the synchronous speed, because it is the speed at which an alternator must run, in order to generate an e.m.f. of the required frequency. In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, the alternator will have to run at the following speeds :

No. of poles	2	4	6	12	24	36
Speed (r.p.m.)	3600	1800	1200	600	300	200

Referring to the above equation, we get  $P = 120/fN$

It is clear from the above that because of slow rotative speeds of engine-driven alternators, their number of poles is much greater as compared to that of the turbo-generators which run at very high speeds.

### 37.7. Armature Windings

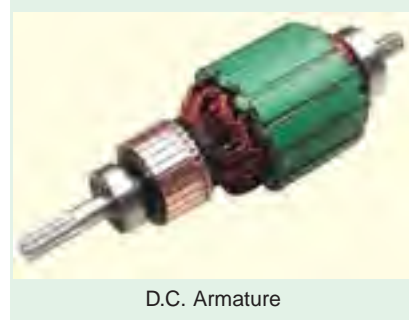
The armature windings in alternators are different from those used in d.c. machines. The d.c. machines have closed circuit windings but alternator windings are open, in the sense that there is no closed path for the armature currents in the winding itself. One end of the winding is joined to the neutral point and the other is brought out (for a star-connected armature).

The two types of armature windings most commonly used for 3-phase alternators are :

- (i) **single-layer winding**
  - (ii) **double-layer winding**
- Single-layer Winding**

It is variously referred to as concentric or chain winding. Sometimes, it is of simple bar type or wave winding.

The fundamental principle of such a winding is illustrated in Fig. 37.8 which shows a single-layer, one-turn, full-pitch winding for a four-pole generator. There are 12 slots in all, giving 3 slots per pole or 1 slot/phase/pole. The pole pitch is obviously 3. To get maximum e.m.f., two sides of a coil should be one pole-pitch apart *i.e.* coil span should be equal to one pole pitch. In other words, if one side of the coil is under the centre of a *N*-pole, then the other side of the same *coil* should be under the centre of a *S*-pole *i.e.* 180° (electrical) apart. In that case, the e.m.fs. induced in the two sides of the coil are added together.



D.C. Armature

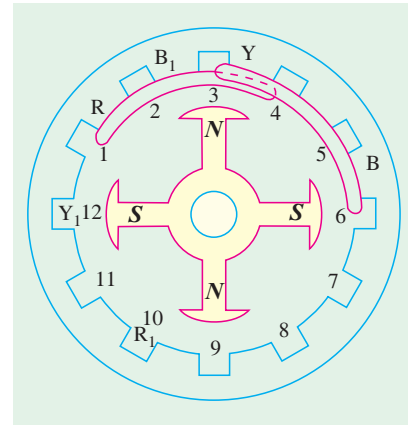
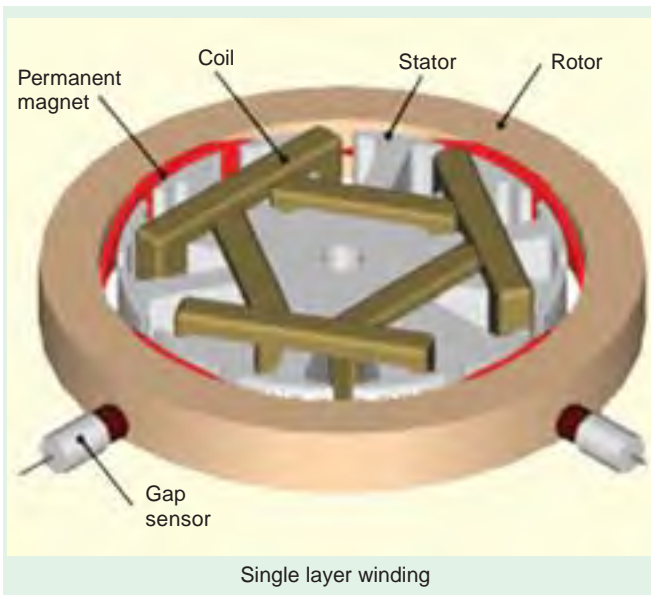


Fig. 37.8

It is seen from the above figure, that *R*-phase starts at slot No. 1, passes through slots 4, 7 and finishes at 10. The *Y*-phase starts 120° afterwards *i.e.* from slot No. 3 which is two slots away from the start of *R*-phase (because when 3 slots correspond to 180° electrical degrees, two slots correspond to an angular displacement of 120° electrical). It passes through slots 6, 9 and finishes at 12. Similarly, *B*-phase starts from slot No.5 *i.e.* two slots away from the start of *Y*-phase. It passes through slots 8, 11 and finishes at slot No. 2. The developed diagram is shown in Fig. 37.9. The ends of the windings are joined to form a star point for a *Y*-connection.



Single layer winding

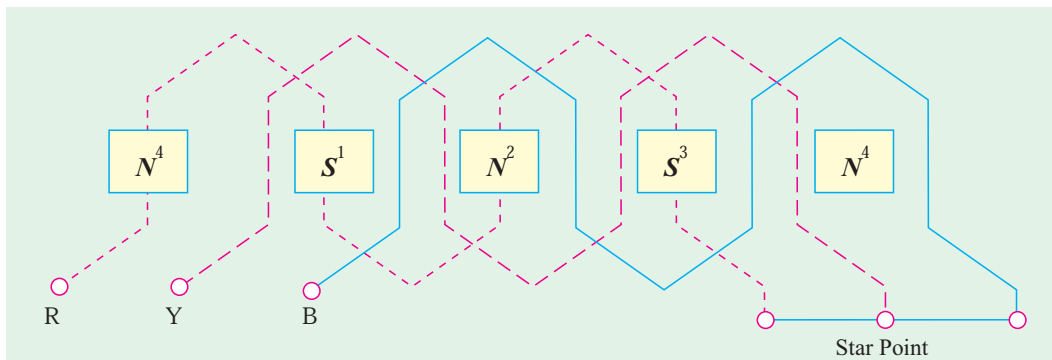


Fig. 37.9

### 37.8. Concentric or Chain Windings

For this type of winding, the number of slots is equal to twice the number of coils or equal to the number of coil sides. In Fig. 37.10 is shown a concentric winding for 3-phase alternator. It has one coil per pair of poles per phase.

It would be noted that the polar group of each phase is  $360^\circ$  (electrical) apart in this type of winding

1. It is necessary to use two different shapes of coils to avoid fouling of end connections.
2. Since polar groups of each phase are  $360$  electrical degrees apart, all such groups are connected in the same direction.
3. The disadvantage is that short-pitched coils cannot be used.

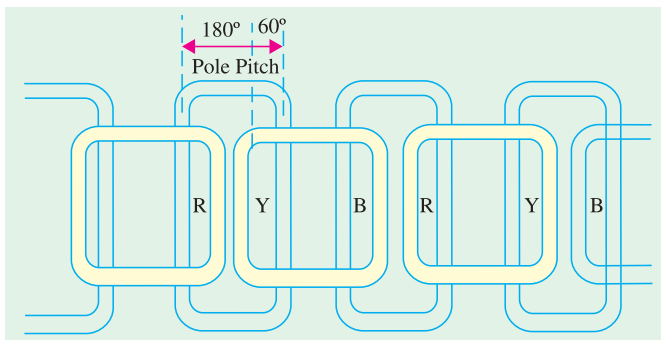


Fig. 37.10

In Fig. 37.11 is shown a concentric winding with two coils per group per pole. Different shapes of coils are required for this winding.

All coil groups of phase  $R$  are connected in the same direction. It is seen that in each group, one coil has a pitch of  $5/6$  and the other has a pitch of  $7/6$  so that pitch factor (explained later) is  $0.966$ . Such windings are used for large high-voltage machines.

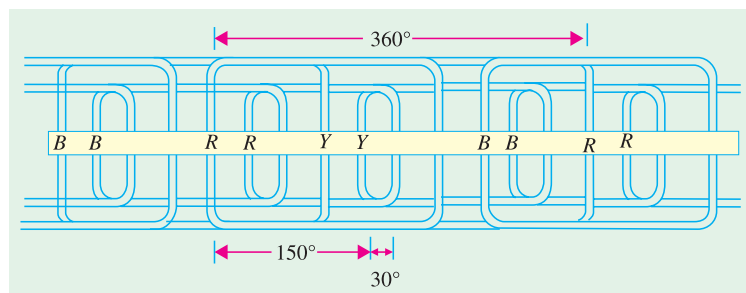


Fig. 37.11

### 37.9. Two-Layer Winding

This winding is either of wave-wound type or lap-wound type (this being much more common especially for high-speed turbo-generators). It is the simplest and, as said above, most commonly-used not only in synchronous machines but in induction motors as well.

Two important points regarding this winding should be noted :

- (a) Ordinarily, the number of slots in stator (armature) is a multiple of the number of poles and the number of phases. Thus, the stator of a 4-pole, 3-phase alternator may have 12, 24, 36, 48 etc. slots all of which are seen to be multiple of 12 (*i.e.*  $4 \times 3$ ).
- (b) The number of stator slots is equal to the number of coils (which are all of the same shape). In other words, each slot contains two coil sides, one at the bottom of the slot and the other at the top. The coils overlap each other, just like shingles on a roof top.

For the 4-pole, 24-slot stator machine shown in Fig. 37.12, the pole-pitch is  $24/4 = 6$ . For maximum voltage, the coils should be full-pitched. It means that if one side of the coil is in slot No.1, the other side should be in slot No.7, the two slots 1 and 7 being one pole-pitch or  $180^\circ$  (electrical) apart. To make matters simple, coils have been labelled as 1, 2, 3 and 4 etc. In the developed diagram of Fig. 37.14, the coil number is the number of the slot in which the left-hand side of the coil is placed.

Each of the three phases has  $24/3 = 8$  coils, these being so selected as to give maximum voltage when connected in series.

When connected properly, coils 1, 7, 13 and 19 will add directly in phase. Hence, we get 4 coils for this phase. To complete eight coils for this phase, the other four selected are 2, 8, 14 and 20 each of which is at an angular displacement of  $30^\circ$  (elect.) from the adjacent coils of the first. The coils 1 and 2 of this phase are said to constitute a polar group (which is defined as the group of coils/phase/pole). Other polar groups for this phase are 7 and 8, 13 and 14, 19 and 20 etc. After the coils are placed in slots, the polar groups are joined. These groups are connected together with alternate poles reversed (Fig. 37.13) which shows winding for one phase only.

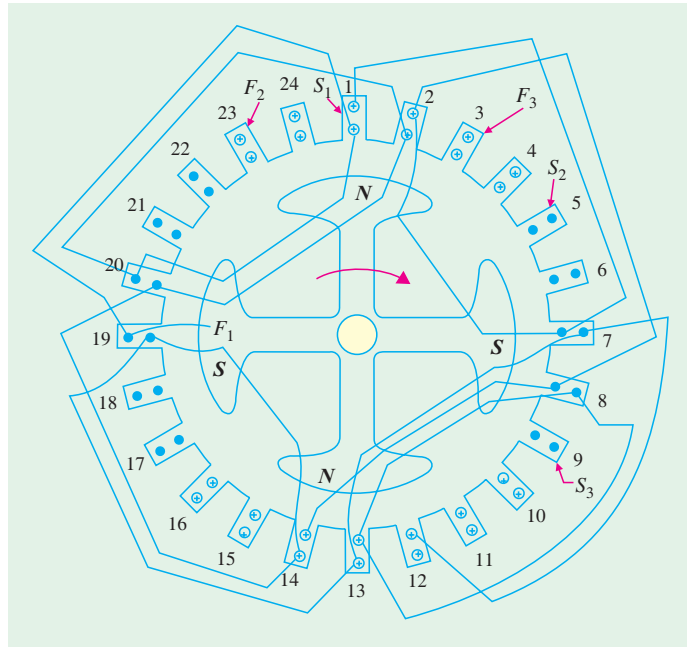


Fig. 37.12

Now, phase *Y* is to be so placed as to be  $120^\circ$  (elect.) away from phase *R*. Hence, it is started from slot 5 *i.e.* 4 slots away (Fig. 37.14). It should be noted that angular displacement between slot No. 1 and 5 is  $4 \times 30 = 120^\circ$  (elect). Starting from coil 5, each of the other eight coils of phase *Y* will be placed 4 slots to the right of corresponding coils for phase *R*. In the same way, *B* phase will start from coil 9. The complete wiring diagram for three phases is shown in Fig. 37.14. The terminals  $R_2$ ,  $Y_2$  and  $B_2$  may be connected together to form a neutral for *Y*-connection.

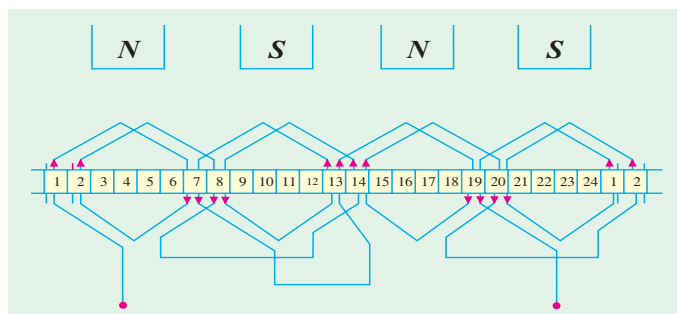


Fig. 37.13

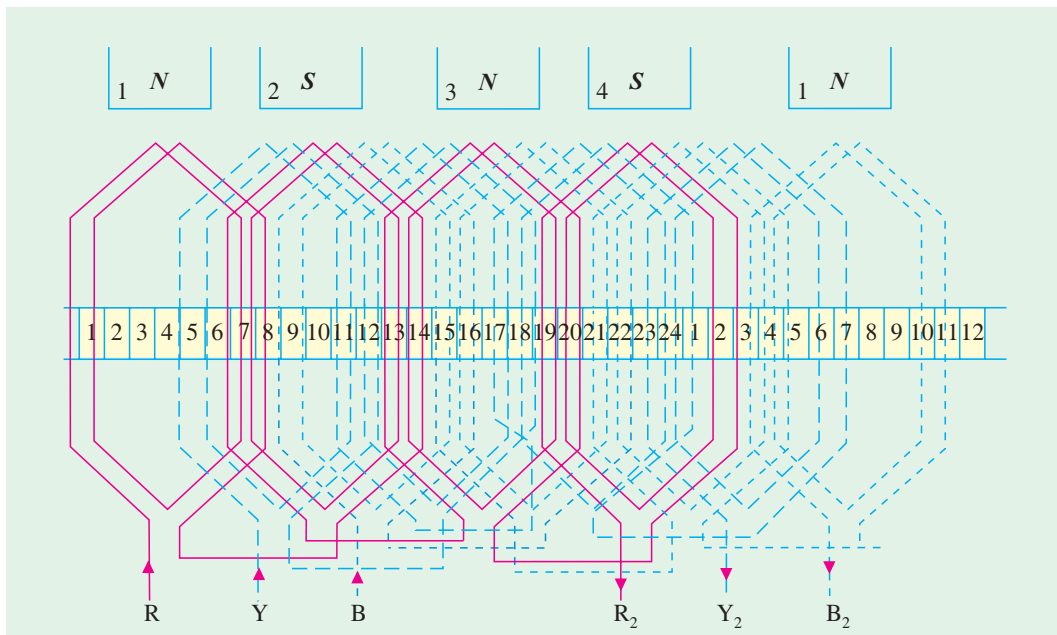


Fig. 37.14 (a)

A simplified diagram of the above winding is shown below Fig. 37.14. The method of construction for this can be understood by closely inspecting the developed diagram.

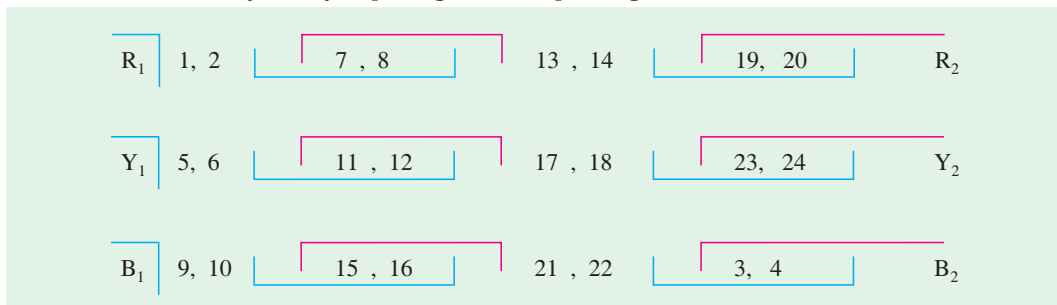


Fig. 37.14 (b)

### 37.10. Wye and Delta Connections

For *Y*-connection,  $R_1$ ,  $Y_1$  and  $B_1$  are joined together to form the star-point. Then, ends  $R_2$ ,  $Y_2$  and  $B_2$  are connected to the terminals. For delta connection,  $R_2$  and  $Y_1$ ,  $Y_2$  and  $B_1$ ,  $B_2$  and  $R_1$  are connected together and terminal leads are brought out from their junctions as shown in Fig. 37.15 (a) and (b).

### 37.11. Short-pitch Winding : Pitch factor/chording factor

So far we have discussed full-pitched coils *i.e.* coils having span which is equal to one pole-pitch *i.e.* spanning over  $180^\circ$  (electrical).

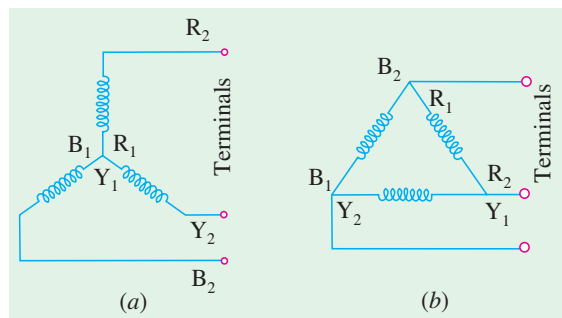


Fig. 37.15

As shown in Fig. 37.16, if the coil sides are placed in slots 1 and 7, then it is full-pitched. If the coil sides are placed in slots 1 and 6, then it is short-pitched or fractional-pitched because coil span is equal to 5/6 of a pole-pitch. It falls short by 1/6 pole-pitch or by  $180^\circ/6 = 30^\circ$ . Short-pitched coils are deliberately used because of the following advantages:

1. They save copper of end connections.
2. They improve the wave-form of the generated e.m.f. *i.e.* the generated e.m.f. can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated.
3. Due to elimination of high frequency harmonics, eddy current and hysteresis losses are reduced thereby increasing the efficiency.

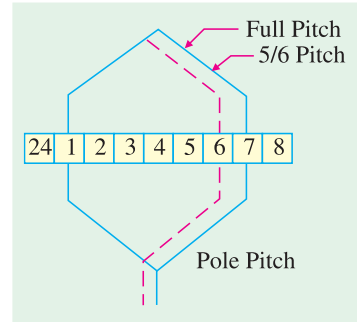


Fig. 37.16

But the disadvantage of using short-pitched coils is that the total voltage around the coils is somewhat reduced. Because the voltages induced in the two sides of the short-pitched coil are slightly out of phase, their resultant vectorial sum is less than their arithmetical sum.

The pitch factor or coil-span factor  $k_p$  or  $k_c$  is defined as

$$= \frac{\text{vector sum of the induced e.m.fs. per coil}}{\text{arithmetic sum of the induced e.m.fs. per coil}}$$

It is always less than unity.

Let  $E_s$  be the induced e.m.f. in each side of the coil. If the coil were full-pitched *i.e.* if its two sides were one pole-pitch apart, then total induced e.m.f. in the coil would have been  $= 2E_s$  [Fig. 37.17 (a)].

If it is short-pitched by  $30^\circ$  (elect.) then as shown in Fig. 37.17 (b), their resultant is  $E$  which is the vector sum of two voltage  $30^\circ$  (electrical) apart.

$$\therefore E = 2 E_s \cos 30^\circ/2 = 2E_s \cos 15^\circ$$

$$k_c = \frac{\text{vector sum}}{\text{arithmetic sum}} = \frac{E}{2 E_s} = \frac{2 E_s \cos 15^\circ}{2 E_s} = \cos 15^\circ = 0.966$$

Hence, pitch factor,  $k_c = 0.966$ .

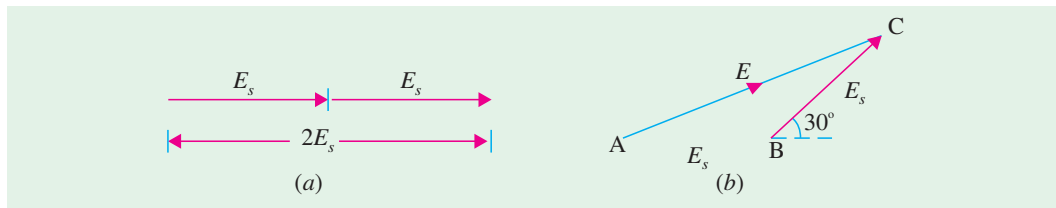


Fig. 37.17

In general, if the coil span falls short of full-pitch by an angle  $\alpha$  (electrical)\*,

then  $k_c = \cos \alpha/2$ .

Similarly, for a coil having a span of 2/3 pole-pitch,  $k_c = \cos 60^\circ/2 = \cos 30^\circ = 0.866$ .

It is lesser than the value in the first case.

**Note.** The value of  $\alpha$  will usually be given in the question, if not, then assume  $k_c = 1$ .

\* This angle is known as chording angle and the winding employing short-pitched coils is called chorded winding.

**Example 37.1.** Calculate the pitch factor for the under-given windings : (a) 36 stator slots, 4-poles, coil-span, 1 to 8 (b) 72 stator slots, 6 poles, coils span 1 to 10 and (c) 96 stator slots, 6 poles, coil span 1 to 12. Sketch the three coil spans.

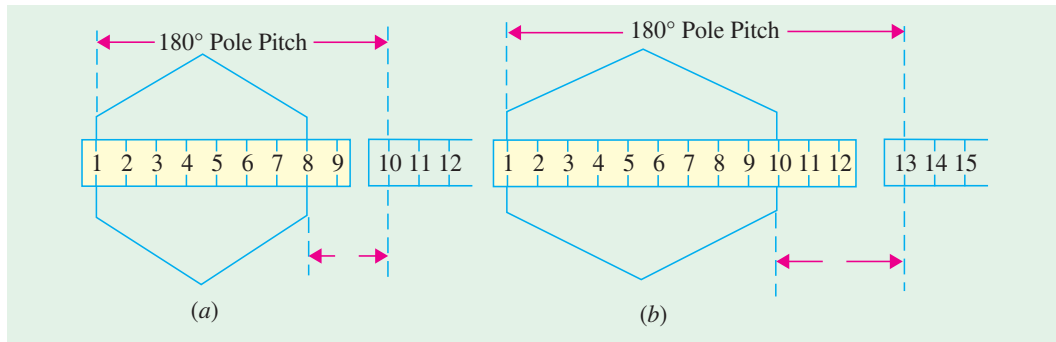


Fig. 37.18

**Solution. (a)** Here, the coil span falls short by  $(2/9) \times 180^\circ = 40^\circ$

$$\alpha = 40^\circ$$

$$\therefore k_c = \cos 40^\circ/2 = \cos 20^\circ = \mathbf{0.94}$$

**(b)** Here  $\alpha = (3/12) \times 180^\circ = 45^\circ \quad \therefore k_c = \cos 45^\circ/2 = \cos 22.5^\circ = \mathbf{0.924}$

**(c)** Here  $\alpha = (5/16) \times 180^\circ = 56^\circ 16' \quad \therefore k_c = \cos 28^\circ 8' = \mathbf{0.882}$

The coil spans have been shown in Fig. 37.18.

### 37.12. Distribution or Breadth Factor or Winding Factor or Spread Factor

It will be seen that in each phase, coils are not concentrated or bunched in one slot, but are distributed in a number of slots to form polar groups under each pole. These coils/phase are displaced from each other by a certain angle. The result is that the e.m.fs. induced in coil sides constituting a polar group are not in phase with each other but differ by an angle equal to angular displacement of the slots.

In Fig. 37.19 are shown the end connections of a 3-phase single-layer winding for a 4-pole alternator. It has a total of 36 slots *i.e.* 9 slots/pole. Obviously, there are 3 slots / phase / pole. For example, coils 1, 2 and 3 belong to R phase. Now, these three coils which constitute one polar group are not bunched in one slot but in three different slots. Angular displacement between any two adjacent slots =  $180^\circ/9 = 20^\circ$  (elect.)

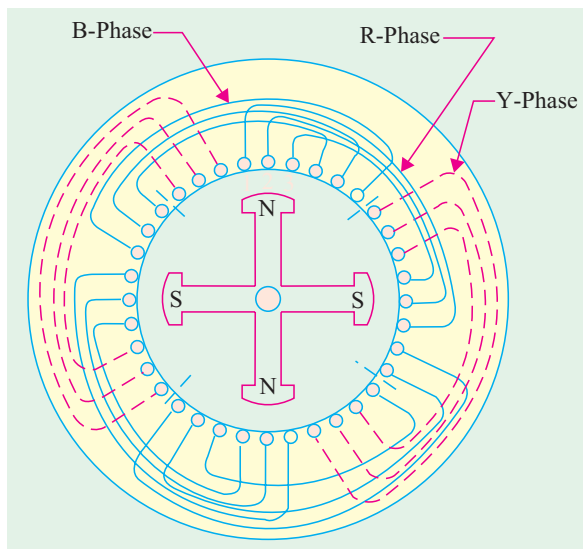


Fig. 37.19

If the three coils were bunched in one slot, then total e.m.f. induced in the three sides of the coil would be the arithmetic sum of the three e.m.f.s. *i.e.*  $= 3 E_s$ , where  $E_s$  is the e.m.f. induced in one coil *side* [Fig.37.20 (a)].

Since the coils are distributed, the individual e.m.fs. have a phase difference of  $20^\circ$  with each other. Their vector sum as seen from Fig. 35.20 (b) is



$$\begin{aligned}
 E &= E_S \cos 20^\circ + E_S + E_S \cos 20^\circ \\
 &= 2 E_S \cos 20^\circ + E_S \\
 &= 2 E_S \times 0.9397 + E_S = 2.88 E_S
 \end{aligned}$$

The distribution factor ( $k_d$ ) is defined as

$$= \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

In the present case

$$k_d = \frac{\text{e.m.f. with winding in 3 slots/pole/phase}}{\text{e.m.f. with winding in 1 slots/pole/phase}} = \frac{E}{3 E_S} = \frac{2.88 E_S}{3 E_S} = 0.96$$

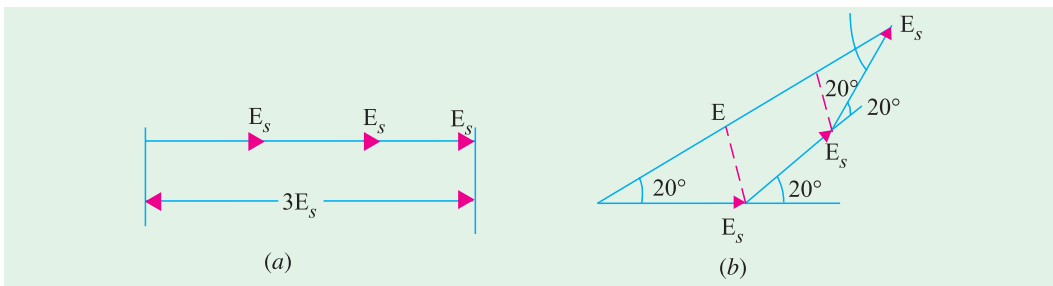


Fig. 37.20

**General Case**

Let  $\beta$  be the value of angular displacement between the slots. Its value is

$$\beta = \frac{180^\circ}{\text{No. of slots/pole}} = \frac{180^\circ}{n}$$

Let  $m =$  No. of slots/phase/pole

$m\beta =$  phase spread angle

Then, the resultant voltage induced in one polar group would be  $mE_S$

where  $E_S$  is the voltage induced in one coil side. Fig. 37.21 illustrates the method for finding the vector sum of  $m$  voltages each of value  $E_S$  and having a mutual phase difference of  $\beta$  (if  $m$  is large, then the curve  $ABCDE$  will become part of a circle of radius  $r$ ).

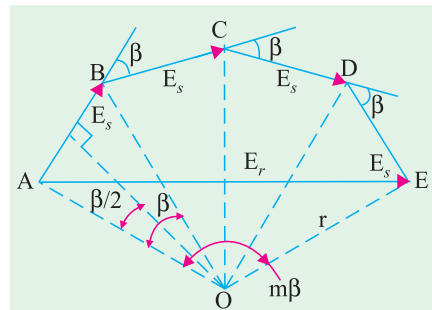


Fig. 37.21

$$AB = E_S = 2r \sin \beta/2$$

$$\text{Arithmetic sum is } = mE_S = m \times 2r \sin \beta/2$$

$$\text{Their vector sum} = AE = E_r = 2r \sin m\beta/2$$

$$\begin{aligned}
 k_d &= \frac{\text{vector sum of coils e.m.fs.}}{\text{arithmetic sum of coil e.m.fs.}} \\
 &= \frac{2r \sin m\beta/2}{m \times 2r \sin \beta/2} = \frac{\sin m\beta/2}{m \sin \beta/2}
 \end{aligned}$$

The value of distribution factor of a 3-phase alternator for different number of slots/pole/phase is given in table No. 37.1.

Table 37.1

Slots per pole	$m$	$\beta^\circ$	Distribution factor $k_d$
3	1	60	1.000
6	2	30	0.966
9	3	20	0.960
12	4	15	0.958
15	5	12	0.957
18	6	10	0.956
24	8	7.5	0.955

In general, when  $\beta$  is small, the above ratio approaches

$$= \frac{\text{chord}}{\text{arc}} = \frac{\sin m\beta/2}{m\beta/2} \quad \text{— angle } m\beta/2 \text{ in radians.}$$

**Example 37.2.** Calculate the distribution factor for a 36-slots, 4-pole, single-layer three-phase winding. (Elect. Machine-I Nagpur Univ. 1993)

**Solution.**

$$n = 36/4 = 9; \beta = 180^\circ/9 = 20^\circ; m = 36/4 \times 3 = 3$$

$$k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3 \times 20^\circ/2}{3 \sin 20^\circ/2} = \mathbf{0.96}$$

**Example 37.3.** A part of an alternator winding consists of six coils in series, each coil having an e.m.f. of 10 V r.m.s. induced in it. The coils are placed in successive slots and between each slot and the next, there is an electrical phase displacement of  $30^\circ$ . Find graphically or by calculation, the e.m.f. of the six coils in series.

**Solution.** By calculation

$$\text{Here } \beta = 30^\circ : m = 6 \therefore k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 90^\circ}{6 \times \sin 15^\circ} = \frac{1}{6 \times 0.2588}$$

Arithmetic sum of voltage induced in 6 coils =  $6 \times 10 = 60 \text{ V}$

$$\text{Vector sum} = k_d \times \text{arithmetic sum} = 60 \times 1/6 \times 0.2588 = \mathbf{38.64 \text{ V}}$$

**Example 37.4.** Find the value of  $k_d$  for an alternator with 9 slots per pole for the following cases :

(i) One winding in all the slots (ii) one winding using only the first 2/3 of the slots/pole (iii) three equal windings placed sequentially in  $60^\circ$  group.

**Solution.** Here,  $\beta = 180^\circ/9 = 20^\circ$  and values of  $m$  i.e. number of slots in a group are 9, 6 and 3 respectively.

$$\text{(i) } m = 9, \quad \beta = 20^\circ, \quad k_d = \frac{\sin 9 \times 20^\circ/2}{9 \sin 20^\circ/2} = \mathbf{0.64} \left[ \text{or } k_d = \frac{\sin \pi/2}{\pi/2} = 0.637 \right]$$

$$\text{(ii) } m = 6, \quad \beta = 20^\circ, \quad k_d = \frac{\sin 6 \times 20^\circ/2}{6 \sin 20^\circ/2} = \mathbf{0.83} \left[ \text{or } k_d = \frac{\sin \pi/3}{\pi/3} = 0.827 \right]$$

$$\text{(iii) } m = 3, \quad \beta = 20^\circ, \quad k_d = \frac{\sin 3 \times 20^\circ/2}{3 \sin 20^\circ/2} = \mathbf{0.96} \left[ \text{or } k_d = \frac{\sin \pi/6}{\pi/6} = 0.955 \right]$$

**37.13. Equation of Induced E.M.F.**

- Let  $Z$  = No. of conductors or coil sides in series/phase  
 =  $2T$  — where  $T$  is the No. of coils or turns per phase  
 (remember one turn or coil has two sides)
- $P$  = No. of poles  
 $f$  = frequency of induced e.m.f. in Hz  
 $\Phi$  = flux/pole in webers
- $k_d$  = distribution factor =  $\frac{\sin m\beta/2}{m \sin \beta/2}$
- $k_c$  or  $k_p$  = pitch or coil span factor =  $\cos \alpha/2$   
 $k_f$  = form factor = **1.11** —if e.m.f. is assumed sinusoidal  
 $N$  = rotor r.p.m.

In one revolution of the rotor (*i.e.* in  $60/N$  second) each stator conductor is cut by a flux of  $\Phi P$  webers.

$\therefore d\Phi = \Phi P$  and  $dt = 60/N$  second

$\therefore$  Average e.m.f. induced per conductor =  $\frac{d\Phi}{dt} = \frac{\Phi P}{60/N} = \frac{\Phi NP}{60}$

Now, we know that  $f = PN/120$  or  $N = 120 f/P$

Substituting this value of  $N$  above, we get

Average e.m.f. per conductor =  $\frac{\Phi P}{60} \times \frac{120 f}{P} = 2 f \Phi$  volt

If there are  $Z$  conductors in series/phase, then Average e.m.f./phase =  $2 f \Phi Z$  volt =  $4 f \Phi T$  volt

R.M.S. value of e.m.f./phase =  $1.11 \times 4 f \Phi T = 4.44 f \Phi T$  volt\*.

This would have been the actual value of the induced voltage if all the coils in a phase were (i) full-pitched and (ii) concentrated or bunched in one slot (instead of being distributed in several slots under poles). But this not being so, the actually available voltage is reduced in the ratio of these two factors.

$\therefore$  Actually available voltage/phase =  $4.44 k_c k_d f \Phi T = 4 k_f k_c k_d f \Phi T$  volt.

If the alternator is star-connected (as is usually the case) then the line voltage is  $\sqrt{3}$  times the phase voltage (as found from the above formula).

**37.14. Effect of Harmonics on Pitch and Distribution Factors**

(a) If the short-pitch angle or chording angle is  $\alpha$  degrees (electrical) for the fundamental flux wave, then its values for different harmonics are

for 3rd harmonic =  $3\alpha$ ; for 5th harmonic =  $5\alpha$  and so on.

$\therefore$  pitch-factor,  $k_c = \cos \alpha/2$  —for fundamental  
 $= \cos 3\alpha/2$  —for 3rd harmonic  
 $= \cos 5\alpha/2$  —for 5th harmonic etc.

(b) Similarly, the distribution factor is also different for different harmonics. Its value becomes

$k_d = \frac{\sin m\beta/2}{m \sin \beta/2}$  where  $n$  is the order of the harmonic

\* It is exactly the same equation as the e.m.f. equation of a transformer. (Art 32.6)

for fundamental,	$n = 1$	$k_{d1} = \frac{\sin m\beta/2}{m \sin \beta/2}$
for 3rd harmonic,	$n = 3$	$k_{d3} = \frac{\sin 3m\beta/2}{m \sin 3\beta/2}$
for 5th harmonic,	$n = 5$	$k_{d5} = \frac{\sin 5m\beta/2}{m \sin 5\beta/2}$

(c) Frequency is also changed. If fundamental frequency is 50 Hz i.e.  $f_1 = 50$  Hz then other frequencies are :

3rd harmonic,  $f_3 = 3 \times 50 = 150$  Hz, 5th harmonic,  $f_5 = 5 \times 50 = 250$  Hz etc.

**Example 37.5.** An alternator has 18 slots/pole and the first coil lies in slots 1 and 16. Calculate the pitch factor for (i) fundamental (ii) 3rd harmonic (iii) 5th harmonic and (iv) 7th harmonic.

**Solution.** Here, coil span is =  $(16 - 1) = 15$  slots, which falls short by 3 slots.

Hence,  $\alpha = 180^\circ \times 3/18 = 30^\circ$

(i)  $k_{c1} = \cos 30^\circ/2 = \cos 15^\circ = 0.966$  (ii)  $k_{c3} = \cos 3 \times 30^\circ/2 = 0.707$

(iii)  $k_{c5} = \cos 5 \times 30^\circ/2 = \cos 75^\circ = 0.259$  (iv)  $k_{c7} = \cos 7 \times 30^\circ/2 = \cos 105^\circ = \cos 75^\circ = 0.259$ .

**Example 37.6.** A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency rpm and the phase and line e.m.f. Assume full-pitched coil.

(Elect. Machines, AMIE Sec. B, 1991)

**Solution.**  $f = PN/120 = 16 \times 375/120 = 50$  Hz

Since  $k_c$  is not given, it would be taken as unity.

$n = 144/16 = 9; \beta = 180^\circ/9 = 20^\circ; m = 144/16 \times 3 = 3$

$k_d = \sin 3 \times (20^\circ/2)/3 \sin (20^\circ/2) = 0.96$

$Z = 144 \times 10 / 3 = 480; T = 480/2 = 240$  / phase

$E_{ph} = 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 240 = 15.34$  V

Line voltage,  $E_L = \sqrt{3} E_{ph} = \sqrt{3} \times 15.34 = 2658$  V

**Example 37.7.** Find the no-load phase and line voltage of a star-connected 3-phase, 6-pole alternator which runs at 1200 rpm, having flux per pole of 0.1 Wb sinusoidally distributed. Its stator has 54 slots having double layer winding. Each coil has 8 turns and the coil is chorded by 1 slot.

(Elect. Machines-I, Nagpur Univ. 1993)

**Solution.** Since winding is chorded by one slot, it is short-pitched by  $1/9$  or  $180^\circ/9 = 20^\circ$

$\therefore k_c = \cos 20^\circ/2 = 0.98; f = 6 \times 1200/120 = 60$  Hz

$n = 54/6 = 9; \beta = 180^\circ/9 = 20^\circ, m = 54/6 \times 3 = 3$

$k_d = \sin 3 \times (20^\circ/2)/3 \sin (20^\circ/2) = 0.96$

$Z = 54 \times 8/3 = 144; T = 144/2 = 72, f = 6 \times 1200/120 = 60$  Hz

$E_{ph} = 4.44 \times 0.98 \times 0.96 \times 60 \times 0.1 \times 72 = 1805$  V

Line voltage,  $E_L = \sqrt{3} \times 1805 = 3125$  V.

**Example 37.8.** The stator of a 3-phase, 16-pole alternator has 144 slots and there are 4 conductors per slot connected in two layers and the conductors of each phase are connected in series. If the speed of the alternator is 375 r.p.m., calculate the e.m.f. induced per phase. Resultant flux in the air-gap is  $5 \times 10^{-2}$  webers per pole sinusoidally distributed. Assume the coil span as  $150^\circ$  electrical.

(Elect. Machine, Nagpur Univ. 1993)

**Solution.** For sinusoidal flux distribution,  $k_f = 1.11$ ;  $\alpha = (180^\circ - 150^\circ) = 30^\circ$  (elect)

$$k_c = \cos 30^\circ/2 = 0.966^*$$

No. of slots / pole,

$$n = 144/16 = 9 ;$$

$$\beta = 180^\circ/9 = 20^\circ$$

$$m = \text{No. of slots/pole/phase} = 144/16 \times 3 = 3$$

$$\therefore k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3 \times 20^\circ/2}{3 \sin 20^\circ/2} = 0.96; f = 16 \times 375/120 = 50 \text{ Hz}$$

$$\text{No. of slots / phase} = 144/3 = 48; \text{No of conductors / slot} = 4$$

$$\therefore \text{No. of conductors in series/phase} = 48 \times 4 = 192$$

$$\therefore \text{turns / phase} = \text{conductors per phase}/2 = 192/2 = 96$$

$$E_{ph} = 4 k_f k_c k_d f \Phi T$$

$$= 4 \times 1.11 \times 0.966 \times 0.96 \times 50 \times 5 \times 10^{-2} \times 96 = 988 \text{ V}$$

**Example 37.9.** A 10-pole, 50-Hz, 600 r.p.m. alternator has flux density distribution given by the following expression

$$B = \sin \theta + 0.4 \sin 3\theta + 0.2 \sin 5\theta$$

The alternator has 180 slots wound with 2-layer 3-turn coils having a span of 15 slots. The coils are connected in  $60^\circ$  groups. If armature diameter is = 1.2 m and core length = 0.4 m, calculate

- (i) the expression for instantaneous e.m.f. / conductor
- (ii) the expression for instantaneous e.m.f./coil
- (iii) the r.m.s. phase and line voltages, if the machine is star-connected.

**Solution.** For finding voltage/conductor, we may either use the relation  $Blv$  or use the relation of Art. 35-13.

$$\text{Area of pole pitch} = (1.2 \pi/10) \times 0.4 = 0.1508 \text{ m}^2$$

$$\text{Fundamental flux/pole, } \phi_1 = \text{av. flux density} \times \text{area} = 0.637 \times 1 \times 0.1508 = 0.096 \text{ Wb}$$

(a) RMS value of fundamental voltage per conductor,

$$= 1.1 \times 2 \phi_1 = 1.1 \times 2 \times 50 \times 0.096 = 10.56 \text{ V}$$

$$\text{Peak value} = \sqrt{2} \times 10.56 = 14.93 \text{ V}$$

Since harmonic conductor voltages are in proportion to their flux densities,

$$\text{3rd harmonic voltage} = 0.4 \times 14.93 = 5.97 \text{ V}$$

$$\text{5th harmonic voltage} = 0.2 \times 14.93 = 2.98 \text{ V}$$

Hence, equation of the instantaneous e.m.f./conductor is

$$e = 14.93 \sin \theta + 5.97 \sin 3\theta + 2.98 \sin 5\theta$$

(b) Obviously, there are 6 conductors in a 3-turn coil. Using the values of  $k_c$  found in solved Ex. 37.5, we get

$$\text{fundamental coil voltage} = 6 \times 14.93 \times 0.966 = 86.5 \text{ V}$$

$$\text{3rd harmonic coil voltage} = 6 \times 5.97 \times 0.707 = 25.3 \text{ V}$$

$$\text{5th harmonic coil voltage} = 6 \times 2.98 \times 0.259 = 4.63 \text{ V}$$

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\* Since they are not of much interest, the relative phase angles of the voltages have not been included in the expression.

Hence, coil voltage expression is\*

$$e = 86.5 \sin \theta + 25.3 \sin 3\theta + 4.63 \sin 5\theta$$

$$(c) \text{ Here, } m = 6, \beta = 180^\circ/18 = 10^\circ; \quad k_{d1} = \frac{\sin 6 \times 10^\circ/2}{6 \sin 10^\circ/2} = 0.956$$

$$k_{d3} = \frac{\sin 3 \times 6 \times 10^\circ/2}{6 \sin 3 \times 10^\circ/2} = 0.644 \quad k_{d5} = \frac{\sin 5 \times 6 \times 10^\circ/2}{6 \sin 5 \times 10^\circ/2} = 0.197$$

It should be noted that number of coils per phase =  $180/3 = 60$

Fundamental phase e.m.f. =  $(86.5/\sqrt{2}) \times 60 \times 0.956 = 3510 \text{ V}$

3rd harmonic phase e.m.f. =  $(25.3/\sqrt{2}) \times 60 \times 0.644 = 691 \text{ V}$

5th harmonic phase e.m.f. =  $(4.63/\sqrt{2}) \times 60 \times 0.197 = 39 \text{ V}$

RMS value of phase voltage =  $(3510^2 + 691^2 + 39^2)^{1/2} = 3577 \text{ V}$

RMS value of line voltage =  $\sqrt{3} \times (3510^2 + 39^2)^{1/2} = 6080 \text{ V}$

**Example 37.10.** A 4-pole, 3-phase, 50-Hz, star-connected alternator has 60 slots, with 4 conductors per slot. Coils are short-pitched by 3 slots. If the phase spread is  $60^\circ$ , find the line voltage induced for a flux per pole of 0.943 Wb distributed sinusoidally in space. All the turns per phase are in series. (Electrical Machinery, Mysore Univ. 1987)

**Solution.** As explained in Art. 37.12, phase spread =  $m\beta = 60^\circ$  —given

Now,  $m = 60/4 \times 3 = 5 \quad \therefore 5\beta = 60^\circ, \beta = 12^\circ$

$$k_d = \frac{\sin 5 \times 12^\circ/2}{5 \sin 12^\circ/2} = 0.957; \alpha = (3/15) \times 180^\circ = 36^\circ; k_c = \cos 18^\circ = 0.95$$

$$Z = 60 \times 4/3 = 80; T = 80/2 = 40; \Phi = 0.943 \text{ Wb}; k_f = 1.11$$

$$\therefore E_{ph} = 4 \times 1.11 \times 0.95 \times 0.975 \times 50 \times 0.943 \times 40 = 7613 \text{ V}$$

$$E_L = \sqrt{3} \times 7613 = 13,185 \text{ V}$$

**Example 37.11.** A 4-pole, 50-Hz, star-connected alternator has 15 slots per pole and each slot has 10 conductors. All the conductors of each phase are connected in series' the winding factor being 0.95. When running on no-load for a certain flux per pole, the terminal e.m.f. was 1825 volt. If the windings are lap-connected as in a d.c. machine, what would be the e.m.f. between the brushes for the same speed and the same flux/pole. Assume sinusoidal distribution of flux.

**Solution.** Here  $k_f = 1.11, k_d = 0.95, k_c = 1$  (assumed)

$f = 50 \text{ Hz}; \text{ e.m.f./phase} = 1825/\sqrt{3} \text{ V}$

Total No. of slots =  $15 \times 4 = 60$

$\therefore$  No. of slots/phase =  $60/3 = 20$ ; No. of turns/phase =  $20 \times 10/2 = 100$

$$\therefore 1825/\sqrt{3} = 4 \times 1.11 \times 1 \times 0.95 \times \Phi \times 50 \times 100 \quad \therefore \Phi = 49.97 \text{ mWb}$$

**When connected as a d.c. generator**

$$E_g = (\Phi ZN/60) \times (P/A) \text{ volt}$$

$$Z = 60 \times 10 = 600, \quad N = 120 \text{ } \# P = 120 \times 50/4 = 1500 \text{ r.p.m.}$$

$$\therefore E_g = \frac{49.97 \times 10^{-3} \times 600 \times 1500}{60} \times \frac{4}{4} = 750 \text{ V}$$

\* Also  $k_d = \sin 150^\circ/2 = \sin 75^\circ = 0.966$

**Example 37.12.** An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field current is 2.4 A.

**Solution.** As seen from the e.m.f. equation of an alternator,

$$E \propto \Phi f \quad \therefore \frac{E_1}{E_2} = \frac{\Phi_1 f_1}{\Phi_2 f_2}$$

Since saturation is neglected,  $\Phi \propto I_f$  where  $I_f$  is the field current

$$\therefore \frac{E_1}{E_2} = \frac{I_{f1} \cdot f_1}{I_{f2} \cdot f_2} \quad \text{or} \quad \frac{360}{E_2} = \frac{3.6 \times 60}{2.4 \times 40}; E_2 = \mathbf{160 \text{ V}}$$

**Example 37.13.** Calculate the R.M.S. value of the induced e.m.f. per phase of a 10-pole, 3-phase, 50-Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is  $150^\circ$ . The flux per pole has a fundamental component of 0.12 Wb and a 20% third component. **(Elect. Machines-III, Punjab Univ. 1991)**

**Solution. Fundamental E.M.F.**

$$\alpha = (180^\circ - 150^\circ) = 30^\circ; k_{c1} = \cos \alpha/2 = \cos 15^\circ = 0.966$$

$$m = 2; \text{ No. of slots/pole} = 6; \beta = 180^\circ/6 = 30^\circ$$

$$\therefore k_{d1} = \frac{\sin m \beta / 2}{m \sin \beta / 2} = \frac{\sin 2 \times 30^\circ / 2}{2 \sin 30^\circ / 2} = 0.966$$

$$Z = 10 \times 2 \times 4 = 80; \text{ turn/phase, } T = 80/2 = 40$$

$$\therefore \text{ Fundamental E.M.F./phase} = 4.44 k_c k_d f \Phi T$$

$$\therefore E_1 = 4.44 \times 0.966 \times 0.966 \times 50 \times 0.12 \times 40 = 995 \text{ V}$$

Harmonic E.M.F.

$$K_{c3} = \cos 3 \alpha/2 = \cos 3 \times 30^\circ/2 = \cos 45^\circ = 0.707$$

$$k_{d3} = \frac{\sin mn \beta / 2}{m \sin n \beta / 2} \text{ where } n \text{ is the order of the harmonic i.e. } n = 3$$

$$\therefore k_{d3} = \frac{\sin 2 \times 3 \times 30^\circ / 2}{2 \sin 3 \times 30^\circ / 2} = \frac{\sin 90^\circ}{2 \sin 45^\circ} = 0.707, f_2 = 50 \times 3 = 150 \text{ Hz}$$

$$\Phi_3 = (1/3) \times 20\% \text{ of fundamental flux} = (1/3) \times 0.02 \times 0.12 = 0.008 \text{ Wb}$$

$$\therefore E_3 = 4.44 \times 0.707 \times 0.707 \times 150 \times 0.008 \times 40 = 106 \text{ V}$$

$$\therefore E \text{ per phase} = \sqrt{E_1^2 + E_3^2} = \sqrt{995^2 + 106^2} = \mathbf{1000 \text{ V}}$$

**Note.** Since phase e.m.fs. induced by the 3rd, 9th and 15th harmonics etc. are eliminated from the line voltages, the line voltage for a Y-connection would be  $= 995 \times \sqrt{3}$  volt.

**Example 37.14.** A 3-phase alternator has generated e.m.f. per phase of 230 V with 10 per cent third harmonic and 6 per cent fifth harmonic content. Calculate the r.m.s. line voltage for (a) star connection (b) delta-connection. Find also the circulating current in delta connection if the reactance per phase of the machine at 50-Hz is  $10 \Omega$ . **(Elect. Machines-III, Osmania Univ. 1988)**

**Solution.** It should be noted that in both star and delta-connections, the third harmonic components of the three phases cancel out at the line terminals because they are co-phased. Hence, the line e.m.f. is composed of the fundamental and the fifth harmonic only.

(a) Star-connection

$$E_1 = 230 \text{ V}; E_5 = 0.06 \times 230 = 13.8 \text{ V}$$

$$\text{E.M.F./phase} = \sqrt{E_1^2 + E_5^2} = \sqrt{230^2 + 13.8^2} = 230.2 \text{ V}$$

$$\text{R.M.S. value of line e.m.f.} = \sqrt{3} \times 230.2 = \mathbf{3.99 \text{ V}}$$

**(b) Delta-connection**

Since for delta-connection, line e.m.f. is the same as the phase e.m.f.

$$\text{R.M.S. value of line e.m.f.} = \mathbf{230.2 \text{ V}}$$

In delta-connection, third harmonic components are additive round the mesh, hence a circulating current is set up whose magnitude depends on the reactance per phase at the third harmonic frequency.

$$\text{R.M.S. value of third harmonic e.m.f. per phase} = 0.1 \times 230 = 23 \text{ V}$$

$$\text{Reactance at triple frequency} = 10 \times 3 = 30 \Omega$$

$$\text{Circulating current} = 23/30 = \mathbf{0.77 \text{ A}}$$

**Example 37.15 (a).** A motor generator set used for providing variable frequency a.c. supply consists of a three-phase, 10-pole synchronous motor and a 24-pole, three-phase synchronous generator. The motor-generator set is fed from a 25 Hz, three-phase a.c. supply. A 6-pole, three-phase induction motor is electrically connected to the terminals of the synchronous generator and runs at a slip of 5%. Determine :

(i) the frequency of the generated voltage of the synchronous generator.

(ii) the speed at which the induction motor is running. **(U.P. Technical University 2001)**

**Solution.** Speed of synchronous motor =  $(120 \times 25)/10 = 300 \text{ rpm}$ .

(i) At 300 rpm, frequency of the voltage generated by 24-pole synchronous generator

$$= \frac{24 \times 300}{120} = 60 \text{ Hz}$$

Synchronous speed of the 6-pole induction motor fed from a 60 Hz supply

$$= \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

(ii) With 5% slip, the speed of this induction motor =  $0.95 \times 1200 = 1140 \text{ rpm}$ .

Further, the frequency of the rotor-currents =  $s f = 0.05 \times 60 = 3 \text{ Hz}$ .

**Example 37.15 (b).** Find the no load line voltage of a star connected 4-pole alternator from the following :

Flux per pole = 0.12 Weber, Slots per pole per phase = 4

Conductors/slot = 4, Two layer winding, with coil span =  $150^\circ$

**[Bharthithasan University, April 1997]**

**Solution.** Total number of slots =  $4 \times 3 \times 4 = 48$ , Slot pitch =  $15^\circ$  electrical

Total number of conductors =  $48 \times 4 = 192$ , Total number of turns = 96

No. of turns in series per phase = 32

For a  $60^\circ$  phase spread,

$$k_b = \frac{\sin(60^\circ/2)}{4 \times \sin 7.5^\circ} = 0.958$$

For  $150^\circ$  coil-span, pitch factor  $k_p = \cos 15^\circ = 0.966$ , and for 50 Hz frequency,

$$E_{ph} = 4.44 \times 50 \times 0.12 \times 0.958 \times 0.966 \times 32 = 789 \text{ volts}$$

$$E_{line} = 789 \times 1.732 = 1366.6 \text{ volts}$$



## Tutorial Problems 37.1

1. Find the no-load phase and line voltage of a star-connected, 4-pole alternator having flux per pole of 0.1 Wb sinusoidally distributed; 4 slots per pole per phase, 4 conductors per slot, double-layer winding with a coil span of  $150^\circ$ .  
**[Assuming  $f = 50$  Hz; 789 V; 1366 V] (Elect. Technology-I, Bombay Univ. 1978)**
2. A 3- $\phi$ , 10-pole, Y-connected alternator runs at 600 r.p.m. It has 120 stator slots with 8 conductors per slot and the conductors of each phase are connected in series. Determine the phase and line e.m.fs. if the flux per pole is 56 mWb. Assume full-pitch coils.  
**[1910 V; 3300 V] (Electrical Technology-II, Madras Univ. April 1977)**
3. Calculate the speed and open-circuit line and phase voltages of a 4-pole, 3-phase, 50-Hz, star-connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.0496 Wb and is sinusoidally distributed.  
**[1500 r.p.m.; 3,300 V; 1,905 V] (Elect. Engg-II, Bombay Univ. 1979)**
4. A 4-pole, 3-phase, star-connected alternator armature has 12 slots with 24 conductors per slot and the flux per pole is 0.1 Wb sinusoidally distributed.  
 Calculate the line e.m.f. generated at 50 Hz. **[1850 V]**
5. A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 30 mWb sinusoidally distributed. Find the frequency, the phase and line voltage if the speed is 375 rpm. **[50 Hz; 1530 V; 2650 V] (Electrical Machines-I, Indore Univ. April 1977)**
6. A synchronous generator has 9 slots per pole. If each coil spans 8 slot pitches, what is the value of the pitch factor? **[0.985] (Elect. Machines, A.M.I.E. Sec. B. 1989)**
7. A 3-phase, Y-connected, 2-pole alternator runs at 3,600 r.p.m. If there are 500 conductors per phase in series on the armature winding and the sinusoidal flux per pole is 0.1 Wb, calculate the magnitude and frequency of the generated voltage from first principles. **[60 Hz; 11.5 kV]**
8. One phase of a 3-phase alternator consists of twelve coils in series. Each coil has an r.m.s. voltage of 10 V induced in it and the coils are arranged in slots so that there is a successive phase displacement of 10 electrical degrees between the e.m.f. in each coil and the next. Find graphically or by calculation, the r.m.s. value of the total phase voltage developed by the winding. If the alternator has six pole and is driven at 100 r.p.m., calculate the frequency of the e.m.f. generated. **[108 V; 50 Hz]**
9. A 4-pole, 50-Hz, 3-phase, Y-connected alternator has a single-layer, full-pitch winding with 21 slots per pole and two conductors per slot. The fundamental flux is 0.6 Wb and air-gap flux contains a third harmonic of 5% amplitude. Find the r.m.s. values of the phase e.m.f. due to the fundamental and the 3rd harmonic flux and the total induced e.m.f.  
**[3,550 V; 119.5 V; 3,553 V] (Elect. Machines-III, Osmania Univ. 1977)**
10. A 3-phase, 10-pole alternator has 90 slots, each containing 12 conductors. If the speed is 600 r.p.m. and the flux per pole is 0.1 Wb, calculate the line e.m.f. when the phases are (i) star connected (ii) delta connected. Assume the winding factor to be 0.96 and the flux sinusoidally distributed.  
**[(i) 6.93 kV (ii) 4 kV] (Elect. Engg-II, Kerala Univ. 1979)**
11. A star-connected 3-phase, 6-pole synchronous generator has a stator with 90 slots and 8 conductors per slot. The rotor revolves at 1000 r.p.m. The flux per pole is  $4 \times 10^{-2}$  weber. Calculate the e.m.f. generated, if all the conductors in each phase are in series. Assume sinusoidal flux distribution and full-pitched coils.  
**[Eph = 1,066 V] (Elect. Machines, A.M.I.E. Summer, 1979)**
12. A six-pole machine has an armature of 90 slots and 8 conductors per slot and revolves at 1000 r.p.m. the flux per pole being 50 milli weber. Calculate the e.m.f. generated as a three-phase star-connected machine if the winding factor is 0.96 and all the conductors in each phase are in series.  
**[1280 V] (Elect. Machines, AMIE, Sec. B, (E-3), Summer 1992)**
13. A 3-phase, 16 pole alternator has a star connected winding with 144 slots and 10 conductors per slot. The flux/pole is 0.04 wb (sinusoidal) and the speed is 375 rpm. Find the frequency and phase and line e.m.f. The total turns/phase may be assumed to series connected.  
**[50 Hz, 2035 Hz, 3525 V] (Rajiv Gandhi Technical University, Bhopal, 2000)**

### 37.15. Factors Affecting Alternator Size

The efficiency of an alternator always increases as its power increases. For example, if an alternator of 1 kW has an efficiency of 50%, then one of 10 MW will inevitably have an efficiency of about 90%. It is because of this improvement in efficiency with size that alternators of 1000 MW and above possess efficiencies of the order of 99%.

Another advantage of large machines is that power output per kilogram increases as the alternator power increases. If 1 kW alternator weighs 20 kg (*i.e.* 50 W/kg), then 10MW alternator weighing 20,000 kg yields 500 W/kg. In other words, larger alternators weigh relatively less than smaller ones and are, consequently, cheaper.

However, as alternator size increases, cooling problem becomes more serious. Since large machines inherently produce high power loss per unit surface area ( $\text{W/m}^2$ ), they tend to overheat.



Light weight alternator



Large weight alternator

To keep the temperature rise within acceptable limits, we have to design efficient cooling system which becomes ever more elaborate as the power increases. For cooling alternators of rating upto 50 MW, circulating cold-air system is adequate but for those of rating between 50 and 300 MW, we have to resort to hydrogen cooling. Very big machines in 1000 MW range have to be equipped with hollow water-cooled conductors. Ultimately, a point is reached where increased cost of cooling exceeds the saving made elsewhere and this fixes the upper limit of the alternator size.

So far as the speed is concerned, low-speed alternators are always bigger than high speed alternators of the same power. Bigness always simplifies the cooling problem. For example, the large 200-rpm, 500-MVA alternators installed in a typical

hydropower plant are air-cooled whereas much smaller 1800-r.p.m., 500-MVA alternators installed in a steam plant are hydrogen cooled.

### 37.16. Alternator on Load

As the load on an alternator is varied, its terminal voltage is also found to vary as in d.c. generators. This variation in terminal voltage  $V$  is due to the following reasons:

1. voltage drop due to armature resistance  $R_a$
2. voltage drop due to armature leakage reactance  $X_L$
3. voltage drop due to armature reaction

### (a) Armature Resistance

The armature resistance/phase  $R_a$  causes a voltage drop/phase of  $IR_a$  which is in phase with the armature current  $I$ . However, this voltage drop is practically negligible.

### (b) Armature Leakage Reactance

When current flows through the armature conductors, fluxes are set up which do not cross the air-gap, but take different paths. Such fluxes are known as *leakage fluxes*. Various types of leakage fluxes are shown in Fig. 37.22.

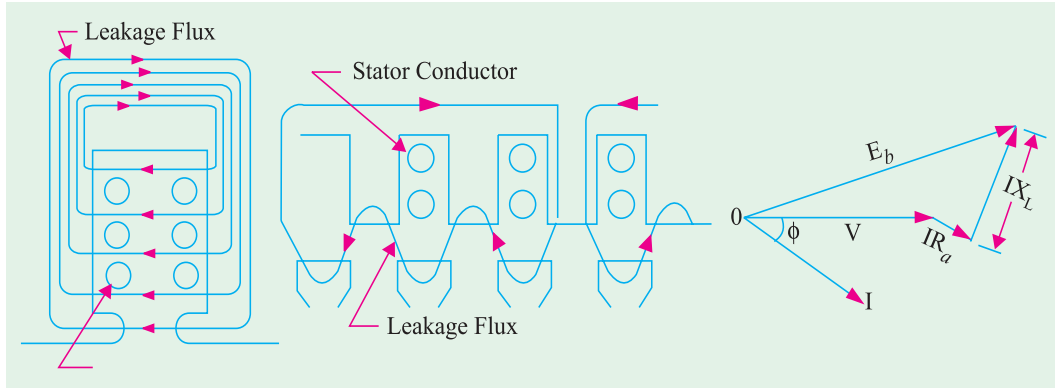


Fig. 37.22

Fig. 37.23

The leakage flux is practically independent of saturation, but is dependent on  $I$  and its phase angle with terminal voltage  $V$ . This leakage flux sets up an e.m.f. of self-inductance which is known as *reactance e.m.f.* and which is ahead of  $I$  by  $90^\circ$ . Hence, armature winding is assumed to possess leakage reactance  $X_L$  (also known as Potier reactance  $X_p$ ) such that voltage drop due to this equals  $IX_L$ . A part of the generated e.m.f. is used up in overcoming this reactance e.m.f.

$\therefore$

$$E = V + I(R + jX_L)$$

This fact is illustrated in the vector diagram of Fig. 37.23.

### (c) Armature Reaction

As in d.c. generators, armature reaction is the effect of armature flux on the main field flux. **In the case of alternators, the power factor of the load has a considerable effect on the armature reaction.** We will consider three cases : **(i)** when load of p.f. is unity **(ii)** when p.f. is zero lagging and **(iii)** when p.f. is zero leading.

Before discussing this, it should be noted that in a 3-phase machine the combined ampere-turn wave (or m.m.f. wave) is sinusoidal which moves synchronously. This amp-turn or m.m.f. wave is fixed relative to the poles, its amplitude is proportional to the load current, but its position depends on the p.f. of the load.

Consider a 3-phase, 2-pole alternator having a single-layer winding, as shown in Fig. 37.24 (a). For the sake of simplicity, assume that winding of each phase is concentrated (instead of being distributed) and that the number of turns per phase is  $N$ . Further suppose that the alternator is loaded with a resistive load of unity power factor, so that phase currents  $I_a$ ,  $I_b$  and  $I_c$  are in phase with their respective phase voltages. Maximum current  $I_a$  will flow when the poles are in position shown in Fig. 37.24 (a) or at a time  $t_1$  in Fig. 37.24 (c). When  $I_a$  has a maximum value,  $I_b$  and  $I_c$  have one-half their maximum values (the arrows attached to  $I_a$ ,  $I_b$  and  $I_c$  are only polarity marks and are not meant to give the instantaneous directions of these currents at time  $t_1$ ). The instantaneous directions of currents are shown in Fig. 37.24 (a). At the instant  $t_1$ ,  $I_a$  flows in conductor  $\alpha$  whereas  $I_b$  and  $I_c$  flow out.

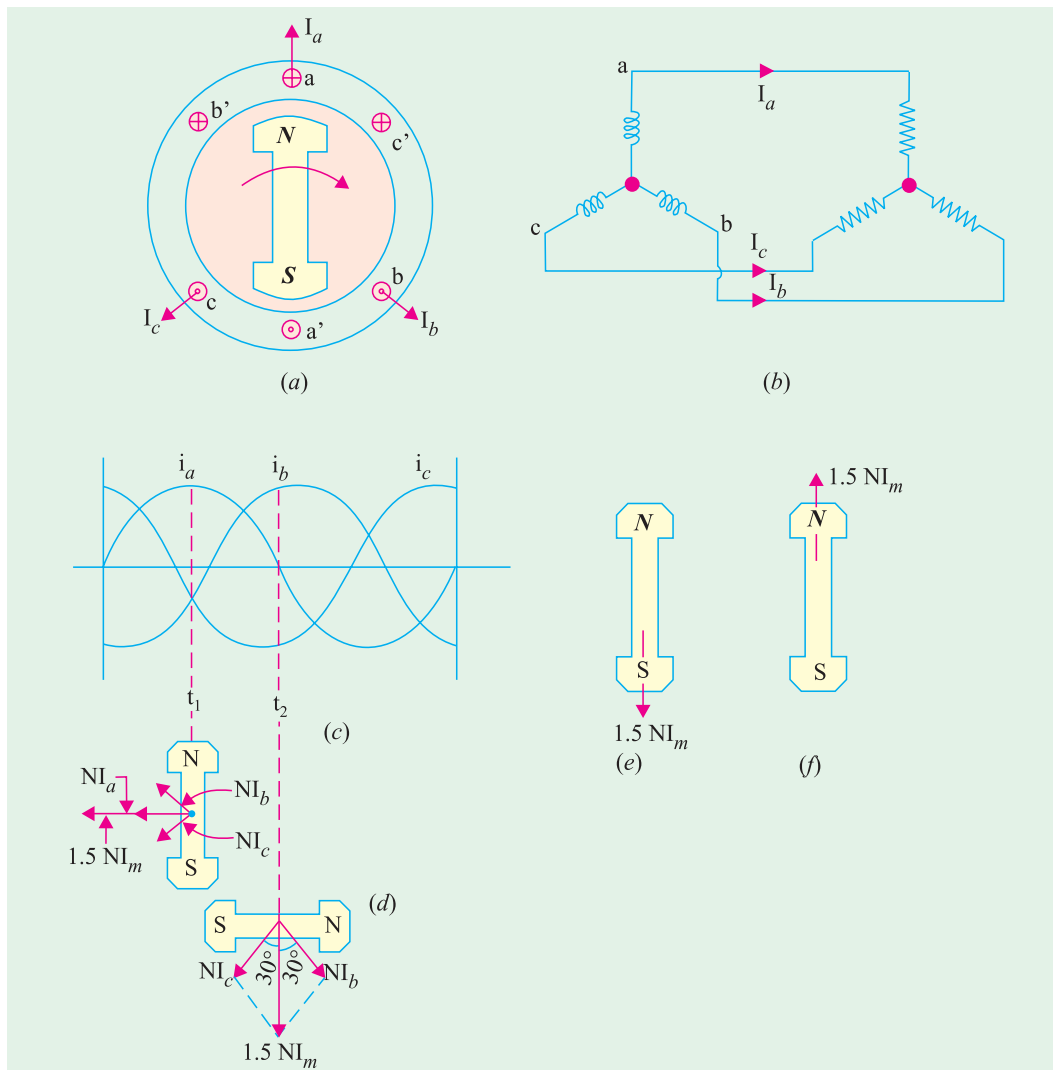


Fig. 37.24

As seen from Fig. 37.24 (d), the m.m.f. ( $= NI_m$ ) produced by phase  $a-a'$  is horizontal, whereas that produced by other two phases is  $(I_m/2) N$  each at  $60^\circ$  to the horizontal. The total armature m.m.f. is equal to the vector sum of these three m.m.fs.

$$\therefore \text{Armature m.m.f.} = NI_m + 2 \cdot (1/2 NI_m) \cos 60^\circ = 1.5 NI_m$$

As seen, at this instant  $t_1$ , the m.m.f. of the main field is upwards and the armature m.m.f. is behind it by  $90$  electrical degrees.

Next, let us investigate the armature m.m.f. at instant  $t_2$ . At this instant, the poles are in the horizontal position. Also  $I_a = 0$ , but  $I_b$  and  $I_c$  are each equal to  $0.866$  of their maximum values. Since  $I_c$  has not changed in direction during the interval  $t_1$  to  $t_2$ , the direction of its m.m.f. vector remains unchanged. But  $I_b$  has changed direction, hence, its m.m.f. vector will now be in the position shown in Fig. 37.24 (d). Total armature m.m.f. is again the vector sum of these two m.m.fs.

$$\therefore \text{Armature m.m.f.} = 2 \times (0.866 NI_m) \times \cos 30^\circ = 1.5 NI_m$$

If further investigations are made, it will be found that.

1. armature m.m.f. remains constant with time

2. it is 90 space degrees behind the main field m.m.f., so that it is only distortional in nature.
3. it rotates synchronously round the armature *i.e.* stator.

For a lagging load of zero power factor, all currents would be delayed in time  $90^\circ$  and armature m.m.f. would be shifted  $90^\circ$  with respect to the poles as shown in Fig. 37.24 (e). Obviously, armature m.m.f. would demagnetise the poles and cause a reduction in the induced e.m.f. and hence the terminal voltage.

For leading loads of zero power factor, the armature m.m.f. is advanced  $90^\circ$  with respect to the position shown in Fig. 37.24 (d). As shown in Fig. 37.24 (f), the armature m.m.f. strengthens the main m.m.f. In this case, armature reaction is wholly magnetising and causes an increase in the terminal voltage.

The above facts have been summarized briefly in the following paragraphs where the matter is discussed in terms of 'flux' rather than m.m.f. waves.

### 1. Unity Power Factor

In this case [Fig. 37.25 (a)] the armature flux is cross-magnetising. The result is that the flux at the leading tips of the poles is reduced while it is increased at the trailing tips. However, these two effects nearly offset each other leaving the average field strength constant. In other words, armature reaction for unity p.f. is distortional.

### 2. Zero P.F. lagging

As seen from Fig. 37.25 (b), here the armature flux (whose wave has moved backward by  $90^\circ$ ) is in direct opposition to the main flux.

Hence, the main flux is decreased. Therefore, it is found that armature reaction, in this case, is wholly **demagnetising**, with the result, that due to weakening of the main flux, less e.m.f. is generated. To keep the value of generated e.m.f. the same, field excitation will have to be increased to compensate for this weakening.

### 3. Zero P.F. leading

In this case, shown in Fig. 37.25 (c) armature flux wave has moved forward by  $90^\circ$  so that it is in phase with the main flux wave. This results in added main flux. Hence, in this case, armature reaction is wholly **magnetising**, which results in greater induced e.m.f. To keep the value of generated e.m.f. the same, field excitation will have to be reduced somewhat.

4. For intermediate power factor [Fig. 37.25 (d)], the effect is partly distortional and partly demagnetising (because p.f. is lagging).

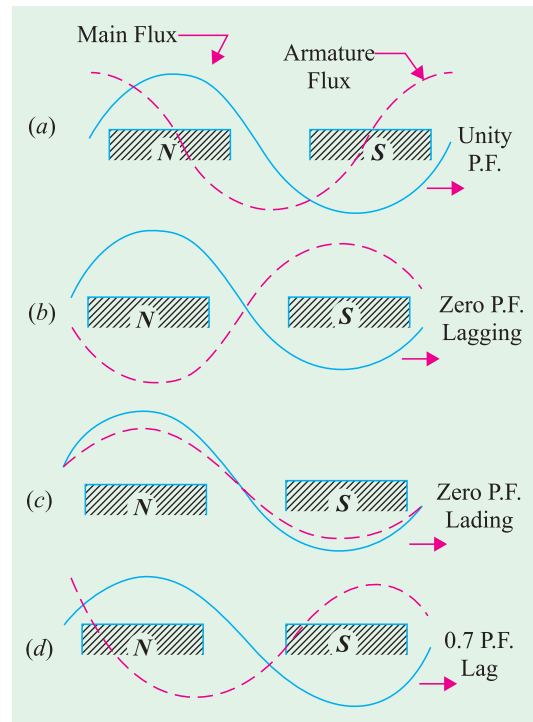


Fig. 37.25

## 37.17. Synchronous Reactance

From the above discussion, it is clear that for the same field excitation, terminal voltage is decreased from its no-load value  $E_0$  to  $V$  (for a lagging power factor). This is because of

1. drop due to armature resistance,  $IR_a$
2. drop due to leakage reactance,  $IX_L$
3. drop due to armature reaction.

The drop in voltage due to armature reaction may be accounted for by assuming the presence of a fictitious reactance  $X_a$  in the armature winding. The value of  $X_a$  is such that  $IX_a$  represents the voltage drop due to armature reaction.

The leakage reactance  $X_L$  (or  $X_p$ ) and the armature reactance  $X_a$  may be combined to give synchronous reactance  $X_S$ .

Hence  $X_S = X_L + X_a^*$

Therefore, total voltage drop in an alternator under load is  $= IR_a + jIX_S = I(R_a + jX_S) = IZ_S$  where  $Z_S$  is known as synchronous impedance of the armature, the word 'synchronous' being used merely as an indication that it refers to the working conditions.

Hence, we learn that the vector difference between no-load voltage  $E_0$  and terminal voltage  $V$  is equal to  $IZ_S$  as shown in Fig. 37.26.

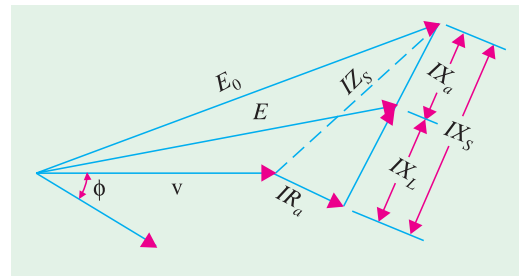


Fig. 37.26

### 37.18. Vector Diagrams of a Loaded Alternator

Before discussing the diagrams, following symbols should be clearly kept in mind.

$E_0$  = No-load e.m.f. This being the voltage induced in armature in the absence of three factors discussed in Art. 37.16. Hence, it represents the maximum value of the induced e.m.f.

$E$  = Load induced e.m.f. It is the induced e.m.f. after allowing for armature reaction.  $E$  is vectorially less than  $E_0$  by  $IX_a$ . Sometimes, it is written as  $E_a$  (Ex. 37.16).

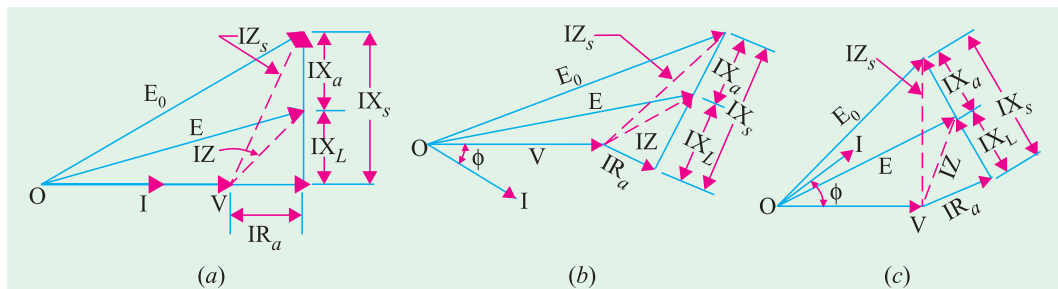


Fig. 37.27

$V$  = Terminal voltage, It is vectorially less than  $E_0$  by  $IZ_S$  or it is vectorially less than  $E$  by  $I Z$  where

$Z = \sqrt{(R_a^2 + X_L^2)}$ . It may also be written as  $Z_a$ .

$I$  = armature current/phase and  $\phi$  = load p.f. angle.

In Fig. 37.27 (a) is shown the case for unity p.f., in Fig. 37.27 (b) for lagging p.f. and in Fig. 37.27 (c) for leading p.f. All these diagrams apply to one phase of a 3-phase machine. Diagrams for the other phases can also be drawn similarly.

**Example 37.16.** A 3-phase, star-connected alternator supplies a load of 10 MW at p.f. 0.85 lagging and at 11 kV (terminal voltage). Its resistance is 0.1 ohm per phase and synchronous reactance 0.66 ohm per phase. Calculate the line value of e.m.f. generated.

(Electrical Technology, Aligarh Muslim Univ. 1988)

\* The ohmic value of  $X_a$  varies with the p.f. of the load because armature reaction depends on load p.f.

**Solution.** F.L. output current =  $\frac{10 \times 10^6}{\sqrt{3} \times 11,000 \times 0.85} = 618 \text{ A}$

$IR_a \text{ drop} = 618 \times 0.1 = 61.8 \text{ V}$

$IX_s \text{ drop} = 618 \times 0.66 = 408 \text{ V}$

Terminal voltage/phase =  $11,000 / \sqrt{3} = 6,350 \text{ V}$

$\phi = \cos^{-1}(0.85) = 31.8^\circ; \sin \phi = 0.527$

As seen from the vector diagram of Fig. 37.28 where  $I$  instead of  $V$  has been taken along reference vector,

$$E_0 = \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2}$$

$$= \sqrt{(6350 \times 0.85 + 61.8)^2 + (6350 \times 0.527 + 408)^2}$$

$$= 6,625 \text{ V}$$

Line e.m.f. =  $\sqrt{3} \times 6,625 = 11,486 \text{ volt}$

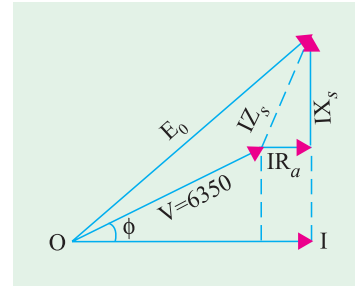


Fig. 37.28

### 37.19. Voltage Regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends **not only on the load but also on the load power factor.**

The voltage regulation of an alternator is defined as “the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage.”

$\therefore$  % regulation ‘up’ =  $\frac{E_0 - V}{V} \times 100$

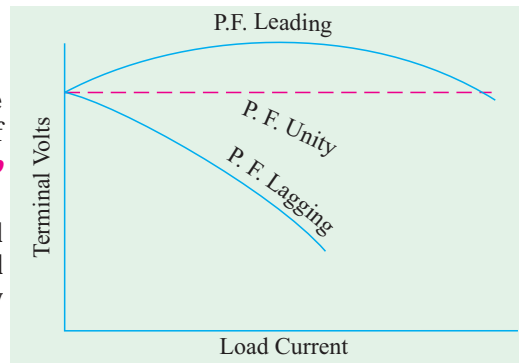


Fig. 37.29

**Note. (i)**  $E_0 - V$  is the arithmetical difference and not the vectorial one.

**(ii)** In the case of *leading* load p.f., terminal voltage will fall on removing the full-load. Hence, regulation is negative in that case.

**(iii)** The rise in voltage when full-load is thrown off is not the same as the fall in voltage when full-load is applied.

Voltage characteristics of an alternator are shown in Fig. 37.29.

### 37.20. Determination of Voltage Regulation

In the case of small machines, the regulation may be found by direct loading. The procedure is as follows :

The alternator is driven at synchronous speed and the terminal voltage is adjusted to its rated value  $V$ . The load is varied until the wattmeter and ammeter (connected for the purpose) indicate the rated values at desired p.f. Then the entire load is thrown off while the speed and field excitation are kept constant. The open-circuit or no-load voltage  $E_0$  is read. Hence, regulation can be found from

$$\% \text{ regn} = \frac{E_0 - V}{V} \times 100$$

In the case of large machines, the cost of finding the regulation by direct loading becomes prohibitive. Hence, other indirect methods are used as discussed below. It will be found that all these methods differ chiefly in the way the no-load voltage  $E_0$  is found in each case.

1. **Synchronous Impedance or E.M.F. Method.** It is due to Behn Eschenberg.
2. **The Ampere-turn or M.M.F. Method.** This method is due to Rothert.
3. **Zero Power Factor or Potier Method.** As the name indicates, it is due to Potier.

All these methods require—

1. Armature (or stator) resistance  $R_a$
2. Open-circuit/No-load characteristic.
3. Short-circuit characteristic (but zero power factor lagging characteristic for Potier method).

Now, let us take up each of these methods one by one.

**(i) Value of  $R_a$**

Armature resistance  $R_a$  per phase can be measured directly by voltmeter and ammeter method or by using Wheatstone bridge. However, under working conditions, the effective value of  $R_a$  is increased due to 'skin effect'\*. The value of  $R_a$  so obtained is increased by 60% or so to allow for this effect. Generally, a value 1.6 times the d.c. value is taken.

**(ii) O.C. Characteristic**

As in d.c. machines, this is plotted by running the machine on no-load and by noting the values of induced voltage and field excitation current. It is just like the  $B$ - $H$  curve.

**(iii) S.C. Characteristic**

It is obtained by short-circuiting the armature (*i.e.* stator) windings through a low-resistance ammeter. The excitation is so adjusted as to give 1.5 to 2 times the value of full-load current. During this test, the speed which is not necessarily synchronous, is kept constant.

**Example 37.17 (a).** *The effective resistance of a 2200V, 50Hz, 440 KVA, 1-phase, alternator is 0.5 ohm. On short circuit, a field current of 40 A gives the full load current of 200 A. The electromotive force on open-circuits with same field excitation is 1160 V. Calculate the synchronous impedance and reactance.* **(Madras University, 1997)**

**Solution.** For the 1-ph alternator, since the field current is same for O.C. and S.C. conditions

$$Z_S = \frac{1160}{200} = 5.8 \text{ ohms}$$

$$X_S = \sqrt{5.8^2 - 0.5^2} = 5.7784 \text{ ohms}$$

**Example 37.17 (b).** *A 60-KVA, 220 V, 50-Hz, 1- $\phi$  alternator has effective armature resistance of 0.016 ohm and an armature leakage reactance of 0.07 ohm. Compute the voltage induced in the armature when the alternator is delivering rated current at a load power factor of (a) unity (b) 0.7 lagging and (c) 0.7 leading.* **(Elect. Machines-I, Indore Univ. 1981)**

**Solution.** Full load rated current  $I = 60,000/220 = 272.2 \text{ A}$

$$IR_a = 272.2 \times 0.016 = 4.3 \text{ V ;}$$

$$IX_L = 272.2 \times 0.07 = 19 \text{ V}$$

**(a) Unity p.f.** — Fig. 37.30 (a)

$$E = \sqrt{(V + IR_a)^2 + (IX_L)^2} = \sqrt{(220 + 4.3)^2 + 19^2} = 225 \text{ V}$$

\* The 'skin effect' may sometimes increase the effective resistance of armature conductors as high as 6 times its d.c. value.



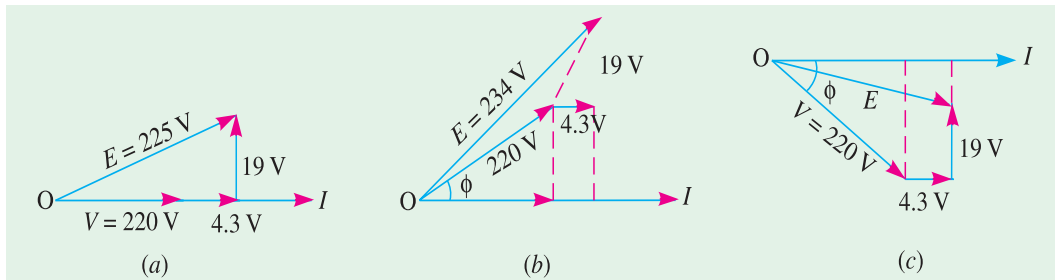


Fig. 37.30

(b) p.f. 0.7 (lag) —Fig. 37.30 (b)

$$E = [V \cos \phi + IR_a]^2 + (V \sin \phi + IX_L)^2]^{1/2}$$

$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 + 19)^2]^{1/2} = \mathbf{234 \text{ V}}$$

(c) p.f. = 0.7 (lead) —Fig. 37.30 (c)

$$E = [(V \cos \phi + IR_a)^2 + (V \sin \phi - IX_L)^2]^{1/2}$$

$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 - 19)^2]^{1/2} = \mathbf{208 \text{ V}}$$

**Example 37.18 (a).** In a 50-kVA, star-connected, 440-V, 3-phase, 50-Hz alternator, the effective armature resistance is 0.25 ohm per phase. The synchronous reactance is 3.2 ohm per phase and leakage reactance is 0.5 ohm per phase. Determine at rated load and unity power factor :

(a) Internal e.m.f.  $E_a$  (b) no-load e.m.f.  $E_0$  (c) percentage regulation on full-load (d) value of synchronous reactance which replaces armature reaction.

(Electrical Engg. Bombay Univ. 1987)

**Solution.** (a) The e.m.f.  $E_a$  is the vector sum of (i) terminal voltage  $V$  (ii)  $IR_a$  and (iii)  $IX_L$  as detailed in Art. 37.17. Here,

$$V = 440 / \sqrt{3} = 254 \text{ V}$$

F.L. output current at u.p.f. is

$$= 50,000 / \sqrt{3} \times 440 = 65.6 \text{ A}$$

Resistive drop =  $65.6 \times 0.25 = 16.4 \text{ V}$

Leakage reactance drop  $IX_L = 65.6 \times 0.5 = 32.8 \text{ V}$

$$\therefore E_a = \sqrt{(V + IR_a)^2 + (IX_L)^2}$$

$$= \sqrt{(254 + 16.4)^2 + 32.8^2} = 272 \text{ volt}$$

Line value =  $\sqrt{3} \times 272 = \mathbf{471 \text{ volt}}$ .

(b) The no-load e.m.f.  $E_0$  is the vector sum of (i)  $V$  (ii)  $IR_a$  and (iii)  $IX_S$  or is the vector sum of  $V$  and  $IZ_S$  (Fig. 37.31).

$$\therefore E_0 = \sqrt{(V + IR_a)^2 + (IX_S)^2} = \sqrt{(254 + 16.4)^2 + (65.6 \times 3.2)^2} = 342 \text{ volt}$$

Line value =  $\sqrt{3} \times 342 = \mathbf{592 \text{ volt}}$

(c) % age regulation 'up' =  $\frac{E_0 - V}{V} \times 100 = \frac{342 - 254}{254} \times 100 = \mathbf{34.65 \text{ per cent}}$

(d)  $X_a = X_S - X_L = 3.2 - 0.5 = \mathbf{2.7 \Omega}$  ...Art. 37.17

**Example 37.18 (b).** A 1000 kVA, 3300-V, 3-phase, star-connected alternator delivers full-load current at rated voltage at 0.80 p. f. Lagging. The resistance and synchronous reactance of the

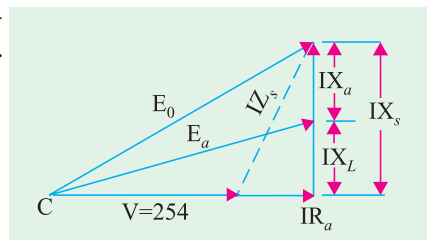


Fig. 37.31

machine per phase are 0.5 ohm and 5 ohms respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p. f. leading. (Amravati University, 1999)

**Solution.**  $V_{ph} = \frac{3300}{\sqrt{3}} = 1905$  volts

At rated load,  $I_{ph} = \frac{1000 \times 1000}{\sqrt{3} \times 3300} = 175$  amp

From phasor diagram for this case [Fig. 37.32 (a)]

Component of E along Ref =  $OD = OA + AB \cos \phi + BC \sin \phi$   
 =  $1905 + (87.5 \times 0.80) + (875 \times 0.60) = 2500$

Component of E along perpendicular direction  
 =  $CD = -AB \sin \phi + BC \cos \phi$   
 =  $87.5 \times 0.6 + 875 \times 0.80 = 647.5$  volts

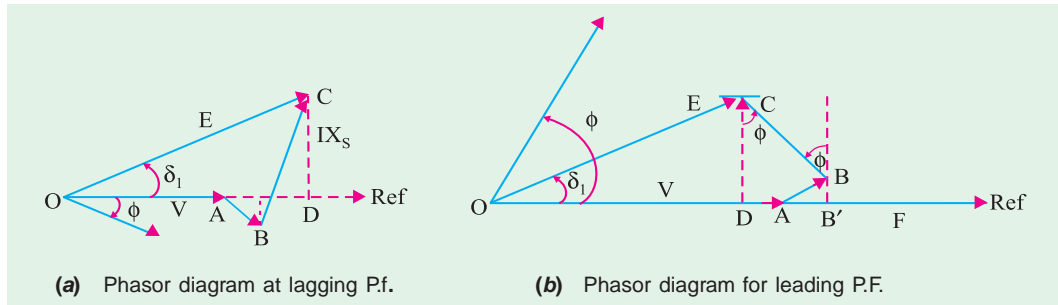


Fig. 37.32

$$OA = 1950, AB = I_r = 87.5, BC = IX_s = 875$$

$$OC = E = \sqrt{OD^2 + DC^2} = \sqrt{2500^2 + 647.5^2} = 2582.5 \text{ volts}$$

$$\delta_1 = \sin^{-1} \frac{CD}{OC} = \sin^{-1} (647.5 / 2582.5) = 14.52^\circ$$

Now, for E kept constant, and the alternator delivering rated current at 0.80 leading p.f., the phasor diagram is to be drawn to evaluate V.

Construction of the phasor diagram starts with marking the reference. Take a point A which is the terminating point of phasor V which starts from O. O is the point yet to be marked, for which the other phasors have to be drawn.

$$AB = 87.5, BC = 875$$

$$BAF = 36.8^\circ$$

BC perpendicular to AB. From C, draw an arc of length E, i.e. 2582.5 volts to locate O.

**Note.** Construction of Phasor diagram starts from known AE, V is to be found.

Along the direction of the current,  $AB = 87.5, \angle BAF = 36.8^\circ$ , since the current is leading.  $BC = 875$  which must be perpendicular to AB. Having located C, draw a line CD which is perpendicular to the reference, with point D, on it, as shown.

Either proceed graphically drawing to scale or calculate geometrically :

$$CD = AB \sin \phi + BC \cos \phi = (87.5 \times 0.60) + (875 \times 0.80) = 752.5 \text{ volts}$$

Since  $CD = E \sin \delta_2, \sin \delta_2 = 752.5 / 2582.5$  giving  $\delta_2 = 17^\circ$

$$OD = E \cos \delta = 2470 \text{ volts}$$

$$DA = DB - AB$$

$$= BC \sin \phi - AB \cos \phi - 875 \times 0.6 - 87.5 \times 0.8 = 455 \text{ volts}$$

Terminal voltage,  $V = OA = OD + DA = 2470 + 455 = 2925$  volts/phase

Since the alternator is star connected, line voltage  $\sqrt{3} \times 2925 = 5066$  volts

**Check :** While delivering lagging p.f. current,

Total power delivered =  $(100 \text{ kVA}) \times 0.80 = 800 \text{ kW}$

In terms of  $E$  and  $\delta_1$  referring to the impedance-triangle in Fig. 37.32 (c)

total power delivered

$$\begin{aligned}
 &= 3 \left[ \frac{VE}{Z_S} \cos(\theta - \delta_1) - \frac{V^2}{Z_S} \cos \theta \right] \\
 &= 3 \left[ \frac{1905 \times 2582.5}{5.025} \cos(84.3^\circ - 14.52^\circ) - 1905^2 \times \cos 84.3^\circ \right] \\
 &= 800 \text{ kW} \quad \dots \text{checked} \\
 OA &= 0.5, \quad AB = 5 \\
 OB &= \sqrt{0.5^2 + 5^2} = 5.025 \Omega \\
 \theta &= \angle BOA = \tan^{-1}(X_S/R) = \tan^{-1} 10 = 84.3^\circ
 \end{aligned}$$

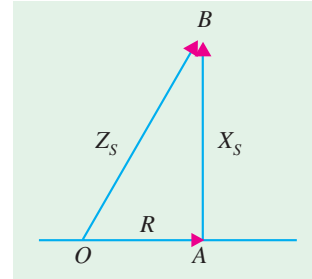


Fig. 37.32 (c) Impedance triangle for the Alternator

While delivering leading p.f. current the terminal voltage is 5.066 kV line to line.

Total power delivered in terms of  $V$  and  $I$

$$= \sqrt{3} \times 5.066 \times 175 \times 0.8 \text{ kW} = 1228.4 \text{ kW}$$

In terms of  $E$  and with voltages expressed in volts,

total power output

$$\begin{aligned}
 &= 3 \left[ \frac{VE}{Z_S} \cos(\theta - \delta_2) - \frac{V^2}{Z_S} \cos \theta \right] \times 10^{-3} \text{ kW} \\
 &= 3 \left[ \frac{2925 \times 2582.5}{5.025} \cos(84.3 - 17^\circ) - \frac{2925^2}{5.025} \times \cos 84.3^\circ \right] \times 10^{-3} \text{ kW} \\
 &= 3 \left[ \frac{2925 \times 2582.5}{5.025} \times \cos(67.3^\circ) - \frac{2925 \times 2925}{5.025} \cos 84.3^\circ \right] \text{ kW} \\
 &= 3(580.11 - 169.10) = 1233 \text{ kW, which agrees fairly closely to the previous figure}
 \end{aligned}$$

and hence checks our answer.

### 37.21. Synchronous Impedance Method

Following procedural steps are involved in this method:

1. O.C.C is plotted from the given data as shown in Fig. 37.33 (a).
2. Similarly, S.C.C. is drawn from the data given by the short-circuit test. It is a straight line passing through the origin. Both these curves are drawn on a common field-current base.

Consider a field current  $I_f$ . The O.C. voltage corresponding to this field current is  $E_1$ . When winding is short-circuited, the terminal voltage is zero. Hence, it may be assumed that the whole of this voltage  $E_1$  is being used to circulate the armature short-circuit current  $I_1$  against the synchronous impedance  $Z_S$ .

$$\therefore \quad E_1 = I_1 Z_S \quad \therefore Z_S = \frac{E_1 \text{ (open-circuit)}}{I_1 \text{ (short-circuit)}}$$

3. Since  $R_a$  can be found as discussed earlier,  $X_S = \sqrt{(Z_S^2 - R_a^2)}$

4. Knowing  $R_a$  and  $X_s$  vector diagram as shown in Fig. 37.33 (b) can be drawn for any load and any power factor.

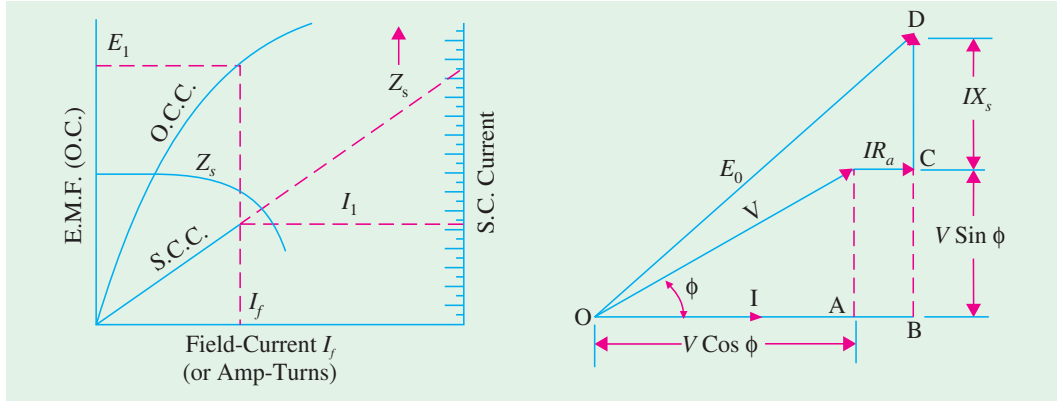


Fig. 37.33 (a)

Fig. 37.33 (b)

Here  $OD = E_0 \therefore E_0 = \sqrt{(OB^2 + BD^2)}$

or  $E_0 = \sqrt{[(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2]}$

$\therefore$  % regn. 'up' =  $\frac{E_0 - V}{V} \times 100$

**Note. (i)** Value of regulation for unity power factor or leading p.f. can also be found in a similar way.

**(ii)** This method is not accurate because the value of  $Z_s$  so found is always more than its value under normal voltage conditions and saturation. Hence, the value of regulation so obtained is always more than that found from an actual test. That is why it is called pessimistic method. The value of  $Z_s$  is not constant but varies with saturation. At low saturation, its value is larger because then the effect of a given armature ampere-turns is much more than at high saturation. Now, under short-circuit conditions, saturation is very low, because armature m.m.f. is directly demagnetising. Different values of  $Z_s$  corresponding to different values of field current are also plotted in Fig. 37.33 (a).

**(iii)** The value of  $Z_s$  usually taken is that obtained from full-load current in the short-circuit test.

**(iv)** Here, armature reactance  $X_a$  has not been treated separately but along with leakage reactance  $X_L$ .

**Example 37.19.** Find the synchronous impedance and reactance of an alternator in which a given field current produces an armature current of 200 A on short-circuit and a generated e.m.f. of 50 V on open-circuit. The armature resistance is 0.1 ohm. To what induced voltage must the alternator be excited if it is to deliver a load of 100 A at a p.f. of 0.8 lagging, with a terminal voltage of 200V. (Elect. Machinery, Banglore Univ. 1991)

**Solution.** It will be assumed that alternator is a single phase one. Now, for same field current,

$$Z_s = \frac{\text{O.C. volts}}{\text{S.C. current}} = \frac{50}{200} = 0.25 \Omega.$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{0.25^2 - 0.1^2} = 0.23 \Omega.$$

Now,  $IR_a = 100 \times 0.1 = 10 \text{ V}$ ,  $IX_s = 100 \times 0.23 = 23 \text{ V}$ ;  
 $\cos \phi = 0.8$ ,  $\sin \phi = 0.6$ . As seen from Fig. 37.34.

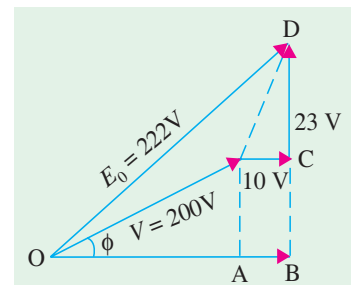


Fig. 37.34

$$\begin{aligned}
 E_0 &= \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2} \\
 &= [(200 \times 0.8 + 10)^2 + (200 \times 0.6 + 23)^2]^{1/2} = 222 \text{ V}
 \end{aligned}$$

**Example 37.20.** From the following test results, determine the voltage regulation of a 2000-V, 1-phase alternator delivering a current of 100 A at (i) unity p.f. (ii) 0.8 leading p.f. and (iii) 0.71 lagging p.f.

Test results : Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is  $0.8\Omega$ . (Elect. Engg.-II, M.S. Univ. 1987)

**Solution.**  $Z_s = \frac{\text{O.C. volts}}{\text{S.C. current}}$  —for same excitation  
for same excitation

$$= 500/100 = 5 \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.936 \Omega$$

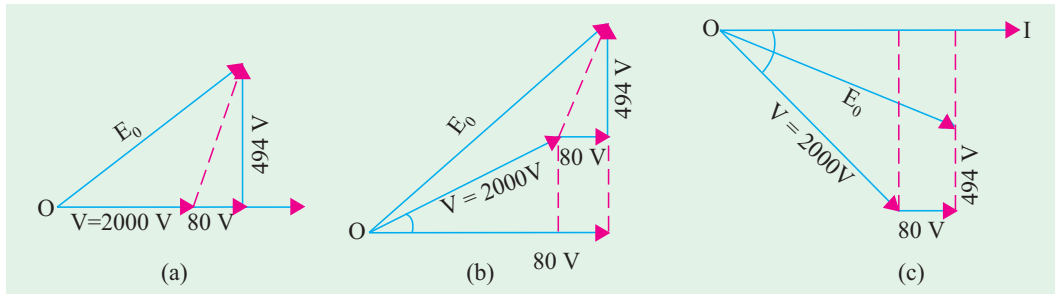


Fig. 37.35

(i) **Unity p.f.** [Fig. 37.35 (a)]

$$IR_a = 100 \times 0.8 = 80 \text{ V}; \quad IX_s = 100 \times 4.936 = 494 \text{ V}$$

$$\therefore E_0 = \sqrt{(2000 + 80)^2 + 494^2} = 2140 \text{ V}$$

$$\% \text{ regn} = \frac{2140 - 2000}{2000} \times 100 = 7\%$$

(ii) **p.f. = 0.8 (lead)** [Fig. 37.35 (c)]

$$E_0 = [(2000 \times 0.8 + 80)^2 + (2000 \times 0.6 - 494)^2]^{1/2} = 1820 \text{ V}$$

$$\% \text{ regn} = \frac{1820 - 2000}{2000} \times 100 = -9\%$$

(iii) **p.f. = 0.71 (lag)** [Fig. 37.35 (b)]

$$E_0 = [(2000 \times 0.71 + 80)^2 + (2000 \times 0.71 + 494)^2]^{1/2} = 2432 \text{ V}$$

$$\% \text{ regn} = \frac{2432 - 2000}{2000} \times 100 = 21.6\%$$

**Example 37.21.** A 100-kVA, 3000-V, 50-Hz 3-phase star-connected alternator has effective armature resistance of 0.2 ohm. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V (line value). Calculate the full-load voltage regulation at 0.8 p.f. lagging and 0.8 p.f. leading. Draw phasor diagrams.

(Basic Elect. Machines, Nagpur Univ. 1993)

**Solution.**

$$Z_S = \frac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}} \quad \text{— for same excitation}$$

$$= \frac{1040/\sqrt{3}}{200} = 3 \Omega$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{3^2 - 0.2^2}$$

$$= 2.99 \Omega$$

F.L. current,

$$I = 100,000/\sqrt{3} \times 3000$$

$$= 19.2 \text{ A}$$

$$IR_a = 19.2 \times 0.2 = 3.84 \text{ V}$$

$$IX_S = 19.2 \times 2.99 = 57.4 \text{ V}$$

Voltage/phase

$$= 3000/\sqrt{3} = 1730 \text{ V}$$

$$\cos \phi = 0.8 ; \sin \phi = 0.6$$

(i) **p.f. = 0.8 lagging**

—Fig. 37.36 (a)

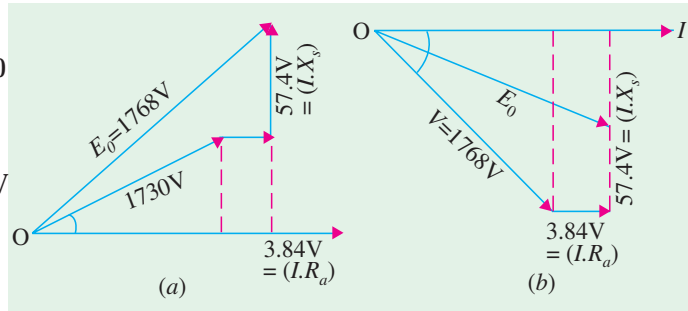


Fig. 37.36

$$E_0 = [(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$$

$$= [(1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 + 57.4)^2]^{1/2} = 1768 \text{ V}$$

$$\% \text{ regn. 'up'} = \frac{(1768 - 1730)}{1730} \times 100 = 2.2\%$$

(ii) **0.8 p.f. leading**—Fig. 37.36 (b)

$$E_0 = [(V \cos \phi + IR_a)^2 + (V \sin \phi - IX_S)^2]^{1/2}$$

$$= [(1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 - 57.4)^2]^{1/2}$$

$$= 1699 \text{ V}$$

$$\% \text{ regn.} = \frac{1699 - 1730}{1730} \times 100 = -1.8\%$$

**Example 37.22.** A 3-phase, star-connected alternator is rated at 1600 kVA, 13,500 V. The armature resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at 0.8 leading power factor.

(Advanced Elect. Machines AMIE Sec. B, 1991)

**Solution.**

$$1280,000 = \sqrt{3} \times 13,500 \times I \times 0.8;$$

$$\therefore I = 68.4 \text{ A}$$

$$IR_a = 68.4 \times 1.5 = 103 \text{ V} ; IX_S = 68.4 \times 30 = 2052$$

$$\text{Voltage/phase} = 13,500 / \sqrt{3} = 7795 \text{ V}$$

As seen from Fig. 37.37.

$$E_0 = [(7795 \times 0.8 + 103)^2 + (7795 \times 0.6 - 2052)^2]^{1/2} = 6663 \text{ V}$$

$$\% \text{ regn.} = \frac{(6663 - 7795)}{7795}$$

$$= -0.1411 \text{ or } -14.11\%$$

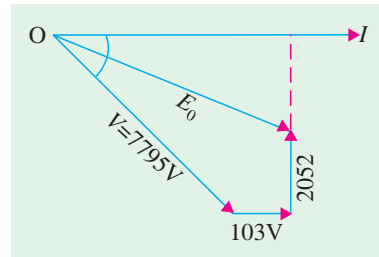


Fig. 37.37

**Example 37.23.** A 3-phase, 10-kVA, 400-V, 50-Hz, Y-connected alternator supplies the rated load at 0.8 p.f. lag. If arm. resistance is 0.5 ohm and syn. reactance is 10 ohms, find the power angle and voltage regulation. (Elect. Machines-I Nagpur Univ. 1993)

**Solution.** F.L. current,  $I = 10,000 / \sqrt{3} \times 400 = 14.4 \text{ A}$

$$IR_a = 14.4 \times 0.5 = 7.2 \text{ V}$$

$$IX_s = 14.4 \times 10 = 144 \text{ V}$$

$$\text{Voltage/phase} = 400 / \sqrt{3} = 231 \text{ V}$$

$$\phi = \cos^{-1} 0.8 = 36.87^\circ, \text{ as shown in Fig. 37.38.}$$

$$E_0 = [(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2]^{1/2}$$

$$= (231 \times 0.8 + 7.2)^2 + (231 \times 0.6 + 144)^2]^{1/2}$$

$$= 342 \text{ V}$$

$$\% \text{ regn.} = \frac{342 - 231}{231} \times 100 = 0.48 \text{ or } 48\%$$

The power angle of the machine is defined as the angle between  $V$  and  $E_0$  i.e. angle  $\delta$

$$\text{As seen from Fig. 37.38, } \tan(\phi + \delta) = \frac{BC}{OB} = \frac{231 \times 0.6 + 144}{231 \times 0.8 + 7.2} = \frac{282.6}{192} = 1.4419;$$

$$\therefore (\phi + \delta) = 55.26^\circ$$

$$\therefore \text{ power angle } \delta = 55.26^\circ - 36.87^\circ = 18.39^\circ$$

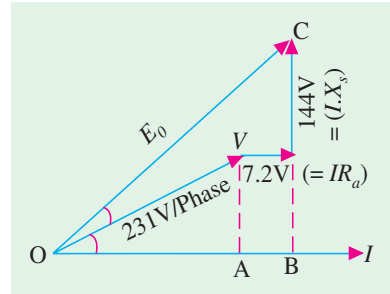


Fig. 37.38

**Example 37.24.** The following test results are obtained from a 3-phase, 6,000-kVA, 6,600 V, star-connected, 2-pole, 50-Hz turbo-alternator:

With a field current of 125 A, the open-circuit voltage is 8,000 V at the rated speed; with the same field current and rated speed, the short-circuit current is 800 A. At the rated full-load, the resistance drop is 3 per cent. Find the regulation of the alternator on full-load and at a power factor of 0.8 lagging. (Electrical Technology, Utkal Univ. 1987)

**Solution.**  $Z_s = \frac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}} = \frac{8000 / \sqrt{3}}{800} = 5.77 \Omega$

Voltage/phase =  $6,600 / \sqrt{3} = 3,810 \text{ V}$

Resistive drop =  $3\% \text{ of } 3,810 \text{ V} = 0.03 \times 3,810 = 114.3 \text{ V}$

Full-load current =  $6,000 \times 10^3 / \sqrt{3} \times 6,600 = 525 \text{ A}$

Now  $IR_a = 114.3 \text{ V}$

$\therefore R_a = 114.3 / 525 = 0.218 \Omega$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5.77^2 - 0.218^2} = 5.74 \Omega \text{ (approx.)}$$

As seen from the vector diagram of Fig. 37.33, (b)

$$E_0 = \sqrt{[3,810 \times 0.8 + 114.3]^2 + (3,810 \times 0.6 + 525 \times 5.74)^2} = 6,180 \text{ V}$$

$$\therefore \text{ regulation} = (6,180 - 3,810) \times 100 / 3,810 = 62.2\%$$

**Example 37.25.** A 3-phase 50-Hz star-connected 2000-kVA, 2300 V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the open circuit voltage was 900 V. The resistance between a pair of terminals was 0.12  $\Omega$ . Find full-load regulation at (i) UPF (ii) 0.8 p.f. lagging. (Elect. Machines, Nagpur Univ. 1993)

**Solution.**  $Z_s = \frac{\text{O.C. volts/phase}}{\text{S.C. current/phase}} = \frac{900 / \sqrt{3}}{600} = 0.866 \Omega$

Resistance between the terminals is  $0.12 \Omega$ . It is the resistance of two phases connected in series.

$\therefore$  Resistance/phase =  $0.12/2 = 0.06 \Omega$ ;  
 effective resistance/phase =  $0.06 \times 1.5 = 0.09 \Omega$ ;

$$X_S = \sqrt{0.866^2 - 0.09^2} = 0.86 \Omega$$

F.L.  $I = 2000,000 / \sqrt{3} \times 2300 = 500 \text{ A}$

$IR_a = 500 \times 0.06 = 30 \text{ V}$  ;

$IX_S = 500 \times 0.86 = 430 \text{ V}$

rated voltage/phase

=  $2300 / \sqrt{3} = 1328 \text{ V}$

(i) **U.P.F.** —Fig. 37.39 (a),

$$E_0 = [(V \cos \phi + IR_a)^2 + (IX_S)^2]^{1/2}$$

$$= \sqrt{(1328 + 30)^2 + 430^2} = 1425 \text{ V}$$

% regn. =  $(1425 - 1328)/1328 = 0.073$  or **7.3%**

(ii) **0.8 p.f. lagging** —Fig. 37.39 (b)

$$E_0 = [(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$$

$$= [(1328 \times 0.8 + 30)^2 + (1328 \times 0.6 + 430)^2]^{1/2} = 1643 \text{ V}$$

$\therefore$  % regn. =  $(1643 - 1328)/1328 = 0.237$  or **23.7%**.

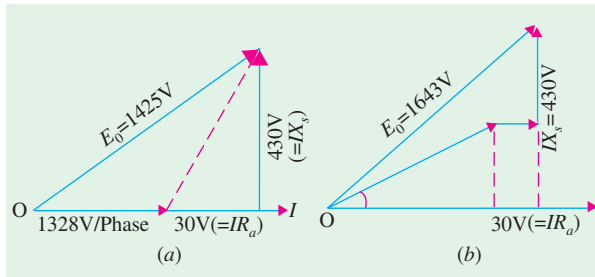


Fig. 37.39

**Example 37.26.** A 2000-kVA, 11-kV, 3-phase, star-connected alternator has a resistance of 0.3 ohm and reactance of 5 ohm per phase. It delivers full-load current at 0.8 lagging power factor at rated voltage. Compute the terminal voltage for the same excitation and load current at 0.8 power factor leading. (Elect. Machines, Nagpur Univ. 1993)

**Solution.** (i) **At 0.8 p.f. lagging**

F.L.  $I = 2000,000 / \sqrt{3} \times 11,000 = 105 \text{ A}$

Terminal voltage =  $11,000 / \sqrt{3} = 6350 \text{ V}$

$IR_a = 105 \times 0.3 = 31.5 \text{ V}$ ;

$IX_S = 105 \times 5 = 525 \text{ V}$

As seen from Fig. 37.40 (a)

$$E_0 = [6350 \times 0.8 + 31.5]^2 + (6350 \times 0.6 + 525)^2]^{1/2} = 6700 \text{ V}$$

As seen from Fig. 37.40 (b), now, we are given  $E_0 = 6700 \text{ V}$  and we are required to find the terminal voltage  $V$  at 0.8 p.f. leading.

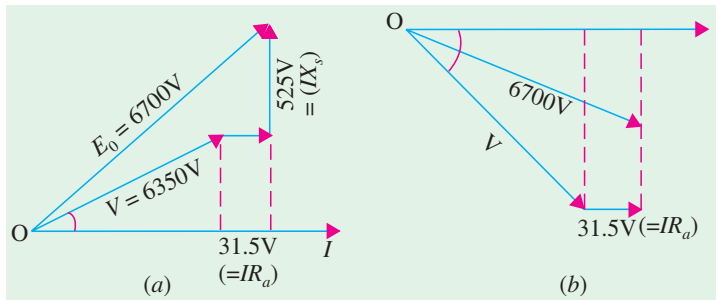


Fig. 37.40

$$6700^2 = (0.8V + 31.5)^2 + (0.6V - 525)^2; V = 6975 \text{ V}$$

**Example 37.27.** The effective resistance of a 1200-kVA, 3.3-kV, 50-Hz, 3-phase, Y-connected alternator is  $0.25 \Omega$ /phase. A field current of 35 A produces a current of 200 A on short-circuit and 1.1 kV (line to line) on open circuit. Calculate the power angle and p.u. change in magnitude of the terminal voltage when the full load of 1200 kVA at 0.8 p.f. (lag) is thrown off. Draw the corresponding phasor diagram. (Elect. Machines, A.M.I.E. Sec. B, 1993)



**Solution.**  $Z_s = \frac{\text{O.C. voltage}}{\text{S.C. voltage}}$

—same excitation

$$= \frac{1.1 \times 10^3 / \sqrt{3}}{200} = 3.175 \Omega$$

$$X_s = \sqrt{3.175^2 - 0.25^2} = 3.165 \Omega$$

$$V = 3.3 \times 10^3 / \sqrt{3} = 1905 \text{ V}$$

$$\tan \theta = X_s / R_a = 3.165 / 0.25, \theta = 85.48^\circ$$

$$\therefore Z_s = 3.175 \angle 85.48^\circ$$

$$\text{Rated } I_a = 1200 \times 10^3 / \sqrt{3} \times 3.3 \times 10^3 = 210 \text{ A}$$

$$\text{Let, } V = 1905 \angle 0^\circ, I_a = 210 \angle -36.87^\circ$$

As seen from Fig. 37.41,

$$E = V + I_a Z_s = 1905 + 210 \angle -36.87^\circ \times 3.175 \angle 85.48^\circ = 2400 \angle 12^\circ$$

$$\text{Power angle } = \delta = 12^\circ$$

Per unit change in terminal voltage is

$$= (2400 - 1905) / 1905 = 0.26$$

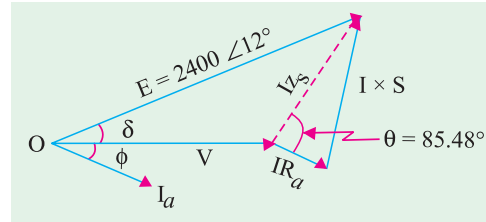


Fig. 37.41

**Example 37.28.** A given 3-MVA, 50-Hz, 11-kV, 3- $\phi$ , Y-connected alternator when supplying 100 A at zero p.f. leading has a line-to-line voltage of 12,370 V; when the load is removed, the terminal voltage falls down to 11,000 V. Predict the regulation of the alternator when supplying full-load at 0.8 p.f. lag. Assume an effective resistance of 0.4  $\Omega$  per phase.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** As seen from Fig. 37.42 (a), at zero p.f. leading

$$E_0^2 = (V \cos \phi + I R_a)^2 + (V \sin \phi - I X_s)^2$$

$$\text{Now, } E_0 = 11,000 / \sqrt{3} = 6350 \text{ V}$$

$$V = 12370 / \sqrt{3} = 7,142 \text{ V,}$$

$$\cos \phi = 0, \sin \phi = 1$$

$$\therefore 6350^2 = (0 + 100 \times 0.4)^2 + (7142 - 100 X_s)^2$$

$$\therefore 100 X_s = 790 \text{ or } X_s = 7.9 \Omega$$

F.L. current

$$I = \frac{3 \times 10^6}{\sqrt{3} \times 11,000} = 157 \text{ A}$$

$$I R_a = 0.4 \times 157 = 63 \text{ V; } I X_s = 157 \times 7.9 = 1240 \text{ V}$$

$$\therefore E_0 = [(6350 \times 0.8 + 63)^2 + (6350 \times 0.6 + 1240)^2]^{1/2} = 7210 \text{ V/phase}$$

$$\therefore \% \text{ regn} = \frac{7210 - 6350}{6350} \times 100 = 13.5\%$$

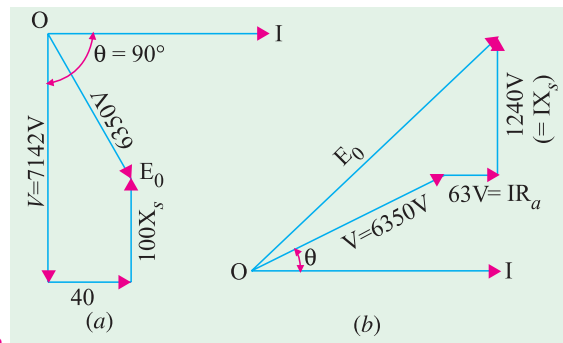


Fig. 37.42

**Example 37.29.** A straight line law connects terminal voltage and load of a 3-phase star-connected alternator delivering current at 0.8 power factor lagging. At no-load, the terminal voltage is 3,500 V and at full-load of 2,280 kW, it is 3,300 V. Calculate the terminal voltage when delivering current to a 3- $\phi$ , star-connected load having a resistance of 8  $\Omega$  and a reactance of 6  $\Omega$  per phase. Assume constant speed and field excitation.

(London Univ.)

**Solution.** No-load phase voltage =  $3,500/\sqrt{3} = 2,021 \text{ V}$

Phase voltage on full-load and 0.8 power factor =  $3,300/\sqrt{3} = 1,905 \text{ V}$

Full-load current is given by

$$\sqrt{3} V_L I_L \cos \phi = 2,280 \times 1000 \quad \therefore \quad I_L = \frac{2,280 \times 1000}{\sqrt{3} \times 3,300 \times 0.8} = 500 \text{ A}$$

drop in terminal voltage/phase for 500 A =  $2,021 - 1,905 = 116 \text{ V}$

Let us assume that alternator is supplying a current of  $x$  ampere.

Then, drop in terminal voltage per phase for  $x$  ampere is =  $116 x / 500 = 0.232 x$  volt

$\therefore$  terminal p.d./phase when supplying  $x$  amperes at a p.f. of 0.8 lagging is  
=  $2,021 - 0.232 x$  volt

Impedance of connected load/phase =  $\sqrt{(8^2 + 6^2)} = 10 \Omega$

load p.f. =  $\cos \phi = 8/10 = 0.8$

When current is  $x$ , the applied p.d. is =  $10 x$

$\therefore$   $10 x = 2021 - 0.232 x$  or  $x = 197.5 \text{ A}$

$\therefore$  terminal voltage/phase =  $2021 - (0.232 \times 197.5) = 1975.2 \text{ V}$

$\therefore$  terminal voltage of alternator =  $1975.2 \times \sqrt{3} = \mathbf{3,421 \text{ V}}$

### Tutorial Problem No. 37.2

1. If a field excitation of 10 A in a certain alternator gives a current of 150 A on short-circuit and a terminal voltage of 900 V on open-circuit, find the internal voltage drop with a load current of 60 A. **[360 V]**
2. A 500-V, 50-kVA, 1- $\phi$  alternator has an effective resistance of  $0.2 \Omega$ . A field current of 10 A produces an armature current of 200 A on short-circuit and an e.m.f. of 450 V on open-circuit. Calculate the full-load regulation at p.f. 0.8 lag. **[34.4%]**  
*(Electrical Technology, Bombay Univ. 1978)*
3. A 3- $\phi$  star-connected alternator is rated at 1600 kVA, 13,500 V. The armature effective resistance and synchronous reactance are  $1.5 \Omega$  and  $30 \Omega$  respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of (a) 0.8 leading and (b) 0.8 lagging. **[(a) -11.8% (b) 18.6%]** *(Elect. Engg.-II, Bombay Univ. 1977)*
4. Determine the voltage regulation of a 2,000-V, 1-phase alternator giving a current of 100 A at 0.8 p.f. leading from the test results. Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5 A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is  $0.8 \Omega$ . Draw the vector diagram.  
**[- 8.9%]** *(Electrical Machines-I, Gujarat Univ. Apr. 1976)*
5. In a single-phase alternator, a given field current produces an armature current of 250 A on short-circuit and a generated e.m.f. of 1500 V on open-circuit. Calculate the terminal p.d. when a load of 250 A at 6.6 kV and 0.8 p.f. lagging is switched off. Calculate also the regulation of the alternator at the given load.  
**[7,898 V; 19.7%]** *(Elect. Machines-II, Indore Univ. Dec. 1977)*
6. A 500-V, 50-kVA, single-phase alternator has an effective resistance of  $0.2 \Omega$ . A field current of 10 A produces an armature current of 200 A on short-circuit and e.m.f. of 450 V on open circuit. Calculate (a) the synchronous impedance and reactance and (b) the full-load regulation with 0.8 p.f. lagging.  
**[(a) 2.25  $\Omega$ , 2.24  $\Omega$ , (b) 34.4%]** *(Elect. Technology, Mysore Univ. 1979)*

7. A 100-kVA, 3,000-V, 50-Hz, 3-phase star-connected alternator has effective armature resistance of  $0.2 \Omega$ . A field current of 40 A produces short-circuit current of 200 A and an open-circuit e.m.f. of 1040 V (line value). Calculate the full-load percentage regulation at a power factor of 0.8 lagging. How will the regulation be affected if the alternator delivers its full-load output at a power factor of 0.8 leading?  
 [24.4% – 13.5%] (Elect. Machines-II, Indore Univ. July 1977)
8. A 3- $\phi$ , 50-Hz, star-connected, 2,000 kVA, 2,300-V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the O.C. voltage was 900 V. The resistance between a pair of terminals was  $0.12 \Omega$ . Find full-load regulation at (a) u.p.f. (b) 0.8 p.f. lagging (c) 0.8 p.f. leading.  
 [(a) 7.3% (b) 23.8% (c) -13.2%] (Elect. Machinery-III, Bangalore Univ. Aug. 1979)
9. A 3-phase star-connected alternator is excited to give 6600 V between lines on open circuit. It has a resistance of  $0.5 \Omega$  and synchronous reactance of  $5 \Omega$  per phase. Calculate the terminal voltage and regulation at full load current of 130 A when the P.F. is (i) 0.8 lagging, (ii) 0.6 leading.  
 [Rajive Gandhi Technical University, Bhopal, 2000]  
 [(i) 3318 Volts/Ph, + 14.83% (ii) 4265 Volts/Ph, - 10.65%]

37.22. Rothert's M.M.F. or Ampere-turn Method

This method also utilizes O.C. and S.C. data, but is the converse of the E.M.F. method in the sense that armature leakage reactance is treated as an additional armature reaction. In other words, it is assumed that the change in terminal p.d. on load is due entirely to armature reaction (and due to the ohmic resistance drop which, in most cases, is negligible). This fact is shown in Fig. 37.43.

Now, field A.T. required to produce a voltage of  $V$  on full-load is the vector sum of the following :

(i) Field A.T. required to produce  $V$  (or if  $R_a$  is to be taken into account, then  $V + IR_a \cos \phi$ ) on no-load. This can be found from O.C.C. and

(ii) Field A.T. required to overcome the demagnetising effect of armature reaction on full-load. This value is found from short-circuit test. The field A.T. required to produce full-load current on short-circuit balances the armature reaction and the impedance drop.

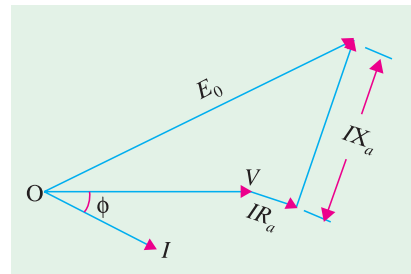


Fig. 37.43

The impedance drop can be neglected because  $R_a$  is usually very small and  $X_s$  is also small under short-circuit conditions. Hence, p.f. on short-circuit is almost zero lagging and the field A.T. are used entirely to overcome the armature reaction which is wholly demagnetising (Art. 37.15). In other words, the demagnetising armature A.T. on full-load are equal and opposite to the field A.T. required to produce full-load current on short-circuit.

Now, if the alternator, instead of being on short-circuit, is supplying full-load current at its normal voltage and zero p.f. lagging, then total field A.T. required are the vector sum of

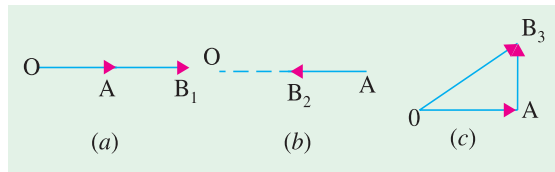


Fig. 37.44

(i) the field A.T. =  $OA$  necessary to produce normal voltage (as obtained from O.C.C.) and

(ii) the field A.T. necessary to neutralize the armature reaction  $AB_1$ . The total field A.T. are represented by  $OB_1$  in Fig. 37.44 (a) and equals the vector sum of  $OA$  and  $AB_1$

If the p.f. is zero leading, the armature reaction is *wholly magnetising*. Hence, in that case, the field A.T. required is  $OB_2$  which is less than  $OA$  by the field A.T. =  $AB_2$  required to produce full-load current on short-circuit [Fig. 37.44 (b)]

If p.f. is unity, the armature reaction is *cross-magnetising* i.e. its effect is distortional only. Hence, field A.T. required is  $OB_3$  i.e. vector sum of  $OA$  and  $AB_3$  which is drawn at right angles to  $OA$  as in Fig. 37.44 (c).

### 37.23. General Case

Let us consider the general case when the p.f. has any value between zero (lagging or leading) and unity. Field ampere-turns  $OA$  corresponding to  $V$  (or  $V + IR_a \cos \phi$ ) is laid off horizontally. Then  $AB_1$ , representing full-load short-circuit field A.T. is drawn at an angle of  $(90^\circ + \phi)$  for a lagging p.f. The total field A.T. are given by  $OB_1$  as in Fig. 37.45. (a). For a leading p.f., short-circuit A.T. =  $AB_2$  is drawn at an angle of  $(90^\circ - \phi)$  as shown in Fig. 37.45 (b) and for unity p.f.,  $AB_3$  is drawn at right angles as shown in Fig. 37.45 (c).

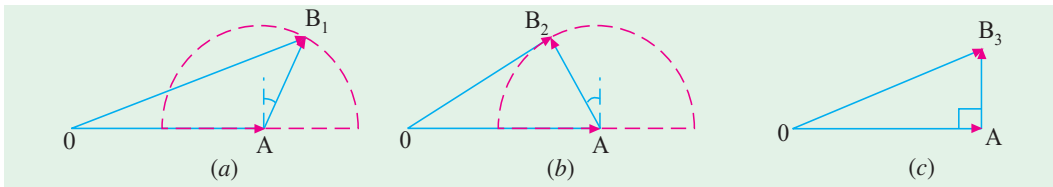


Fig. 37.45

In those cases where the number of turns on the field coils is not known, it is usual to work in terms of the field current as shown in Fig. 37.46.

In Fig. 37.47, is shown the complete diagram along with O.C. and S.C. characteristics.  $OA$  represents field current for normal voltage  $V$ .  $OC$  represents field current required for producing full-load current on *short-circuit*. Vector  $AB = OC$  is drawn at an angle of  $(90^\circ + \phi)$  to  $OA$  (if the p.f. is lagging). The total field current is  $OB$  for which the corresponding O.C. voltage is  $E_0$

$$\therefore \% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

It should be noted that this method gives results which are less than the actual results, that is why it is sometimes referred to as optimistic method.

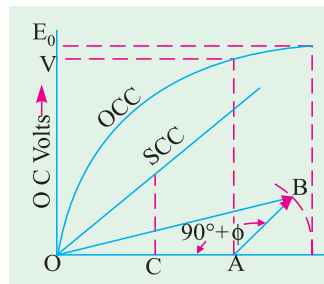


Fig. 37.46

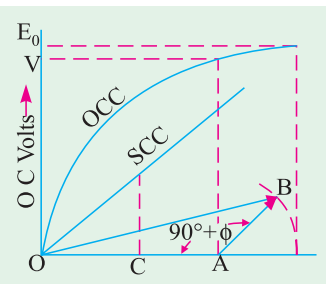


Fig. 37.47

**Example 37.30.** A 3.5-MVA, Y-connected alternator rated at 4160 volts at 50-Hz has the open-circuit characteristic given by the following data :

Field Current (Amps)	50	100	150	200	250	300	350	400	450
E.M.F. (Volts)	1620	3150	4160	4750	5130	5370	5550	5650	5750

A field current of 200 A is found necessary to circulate full-load current on short-circuit of the alternator. Calculate by (i) synchronous impedance method and (ii) ampere-turn method the full-load voltage regulation at 0.8 p.f. lagging. Neglect resistance. Comment on the results obtained.

(Electrical Machines-II, Indore Univ. 1984)

**Solution. (j)** As seen from the given data, a field current of 200 A produces O.C. voltage of 4750 (line value) and full-load current on short-circuit which is

$$= 3.5 \times 10^6 / \sqrt{3} \times 4160 = 486 \text{ A}$$

$$Z_S = \frac{\text{O.C. volt/phase}}{\text{S.C. current/phase}} = \frac{4750/\sqrt{3}}{486} = \frac{2740}{486} = 5.64 \Omega / \text{phase}$$

Since  $R_a = 0, X_S = Z_S \therefore IR_a = 0, IX_S = IZ_S = 486 \times 5.64 = 2740 \text{ V}$

$$\text{F.L. Voltage/phase} = 4160/\sqrt{3} = 2400 \text{ V, } \cos \phi = 0.8, \sin \phi = 0.6$$

$$E_0 = (V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$$

$$= [(2400 \times 0.8 + 0)^2 + (2400 \times 0.6 + 2740)^2]^{1/2} = 4600 \text{ V}$$

$$\% \text{ regn. up} = \frac{4600 - 2400}{2400} \times 100 = 92.5\%$$

**(ii)** It is seen from the given data that for normal voltage of 4160 V, field current needed is 150 A. Field current necessary to circulate F.L. current on short-circuit is 200 A.

In Fig. 37.48, *OA* represents 150 A. The vector *AB* which represents 200 A is vectorially added to *OA* at  $(90^\circ + \phi) = (90^\circ + 36^\circ 52') = 126^\circ 52'$ . Vector *OB* represents excitation necessary to produce a terminal p.d. of 4160 V at 0.8 p.f. lagging at full-load.

$$OB = [150^2 + 200^2 + 2 \times 150 \times 200 \times \cos(180^\circ - 126^\circ 52')]^{1/2}$$

$$= 313.8 \text{ A}$$

The generated phase e.m.f.  $E_0$ , corresponding to this excitation as found from OCC (if drawn) is 3140 V. Line value is  $3140 \times \sqrt{3} = 5440 \text{ V}$ .

$$\% \text{ regn.} = \frac{5440 - 4160}{4160} \times 100 = 30.7\%$$

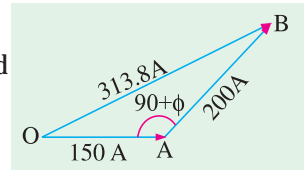


Fig. 37.48

**Example 37.31.** The following test results are obtained on a 6,600-V alternator:

Open-circuit voltage :	3,100	4,900	6,600	7,500	8,300
Field current (amps) :	16	25	37.5	50	70

A field current of 20 A is found necessary to circulate full-load current on short-circuit of the armature. Calculate by (i) the ampere-turn method and (ii) the synchronous impedance method the full-load regulation at 0.8 p.f. (lag). Neglect resistance and leakage reactance. State the drawbacks of each of these methods. **(Elect. Machinery-II, Bangalore Univ. 1992)**

**Solution. (i) Ampere-turn Method**

It is seen from the given data that for the normal voltage of 6,600 V, the field current needed is 37.5 A. Field-current for full-load current, on short-circuit, is given as 20 A.

In Fig. 37.49, *OA* represents 37.5 A. The vector *AB*, which represents 20 A, is vectorially added to *OA* at  $(90^\circ + 36^\circ 52') = 126^\circ 52'$ . Vector *OB* represents the excitation necessary to produce a terminal p.d. of 6,600 V at 0.8 p.f. lagging on full-load

$$OB = \sqrt{37.5^2 + 20^2 + 2 \times 37.5 \times 20 \times \cos 53^\circ 8'} = 52 \text{ A}$$

The generated e.m.f.  $E_0$  corresponding to this excitation, as found from O.C.C. of Fig. 37.49 is 7,600 V.

$$\text{Percentage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{7,600 - 6,600}{6,600} \times 100 = 15.16\%$$

**(ii) Synchronous Impedance Method**

Let the voltage of 6,600 V be taken as 100 per cent and also let 100 per cent excitation be that which is required to produce 6,600 V on open-circuit, that is, the excitation of 37.5 A.

Full-load or 100 per cent armature current is produced on short-circuit by a field current of 20 A. If 100 per cent field current were applied on short-circuit, then S.C. current would be  $100 \times 37.5/20 = 187.5$  per cent.

$$\therefore Z_s = \frac{\text{O.C. voltage}}{\text{S.C. current}} \Bigg|_{\text{same excitation}}$$

$$= 100/187.5 \text{ or } 0.533 \text{ or } 53.3\%$$

The impedance drop  $I Z_s$  is equal to 53.3% of the normal voltage. When the two are added vectorially (Fig. 37.50), the value of voltage is

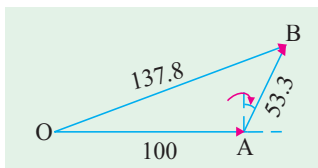


Fig. 37.50

$$E_0 = \sqrt{[100 + 53.3 \cos (90 - \phi)]^2 + [53.3 \sin (90 - \phi)]^2}$$

$$= \sqrt{(100 + 53.3 \times 0.6)^2 + (53.3 \times 0.8)^2} = 138.7\%$$

$$\% \text{ reg} = \frac{138.7 - 100}{100} \times 100 = 38.7\%$$

The two value of regulation, found by the two methods, are found to differ widely from each other. The first method gives somewhat lesser value, while the other method gives a little higher value as compared to the actual value. However, the first value is more likely to be nearer the actual value, because the second method employs  $Z_s$ , which does not have a constant value. Its value depends on the field excitation.

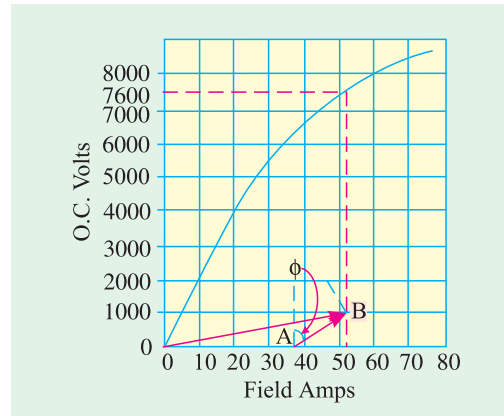


Fig. 37.49

**Example 37.32.** The open and short-circuit test readings for a 3- $\phi$ , star-connected, 1000-kVA, 2000 V, 50-Hz, synchronous generator are :

Field Amps ;	10	20	25	30	40	50
O.C. Terminal V	800	1500	1760	2000	2350	2600
S.C. armature current in A:	—	200	250	300	—	—

The armature effective resistance is 0.2  $\Omega$  per phase. Draw the characteristic curves and estimate the full-load percentage regulation at (a) 0.8 p.f. lagging (b) 0.8 p.f. leading.

**Solution.** The O.C.C. and S.C.C. are plotted in Fig. 37.51

The phase voltages are : 462, 866, 1016, 1155, 1357, 1502.

Full-load phase voltage =  $2000/\sqrt{3} = 1155$  V

Full-load current =  $1,000,000/2000 \times \sqrt{3} = 288.7$  A

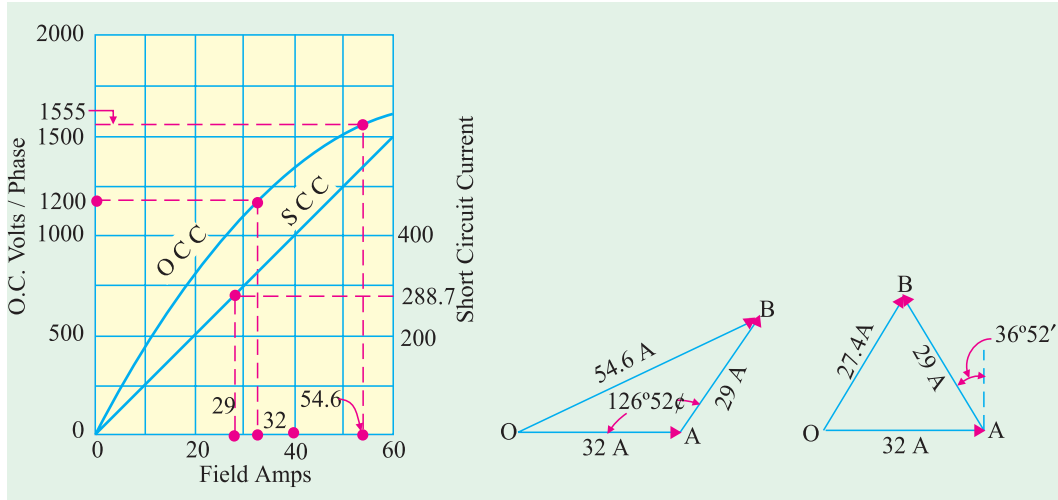


Fig. 37.51

Fig. 37.52

Fig. 37.53

Voltage/phase at full-load at 0.8 p.f. =  $V + IR_a \cos \phi = 1155 + (288.7 \times 0.2 \times 0.8) = 1200$  volt  
 From open-circuit curve, it is found that field current necessary to produce this voltage = 32 A.

From short-circuit characteristic, it is found that field current necessary to produce full-load current of 288.7 A is = 29 A.

(a)  $\cos \phi = 0.8, \phi = 36^\circ 52'$  (lagging)

In Fig. 37.52,  $OA = 32$  A,  $AB = 29$  A and is at an angle of  $(90^\circ + 36^\circ 52') = 126^\circ 52'$  with  $OA$ . The total field current at full-load 0.8 p.f. lagging is  $OB = 54.6$  A

O.C. volt corresponding to a field current of 54.6 A is = 1555 V

% regn. =  $(1555 - 1155) \times 100 / 1155 = 34.6\%$

(b) In this case, as p.f. is leading,  $AB$  is drawn with  $OA$  (Fig. 37.53) at an angle of  $90^\circ - 36^\circ 52' = 53^\circ 8'$ .  $OB = 27.4$  A.

O.C. voltage corresponding to 27.4 A of field excitation is 1080 V.

% regn. =  $\frac{1080 - 1155}{1155} \times 100 = -6.4\%$

**Example 37.33.** A 3-phase, 800-kVA, 3,300-V, 50-Hz alternator gave the following results:

Exciting current (A)	50	60	70	80	90	100
O.C. volt (line)	2560	3000	3300	3600	3800	3960
S.C. current	190	—	—	—	—	—

The armature leakage reactance drop is 10% and the resistance drop is 2% of the normal voltage. Determine the excitation at full-load 0.8 power factor lagging by the m.m.f. method.

**Solution.** The phase voltages are : 1478, 1732, 1905, 2080, 2195, 2287

The O.C.C. is drawn in Fig. 37.54.

Normal phase voltage =  $3300 / \sqrt{3} = 1905$  V;  $IR_a$  drop = 2% of 1905 = 38.1 volt

Leakage reactance drop = 10% of 1905 = 190.5 Volt

$\therefore E = \sqrt{[(1905 \times 0.8 + 38.1)^2 + (1905 \times 0.6 + 190.5)^2]} = 2,068$  V

The exciting current required to produce this voltage (as found from O.C.C.) is 82 A.

Full load current =  $800,000 / \sqrt{3} \times 3300 = 140$  A

As seen from S.C.C., the exciting current required to produce this full-load current of 140 A on short-circuit is 37 A.

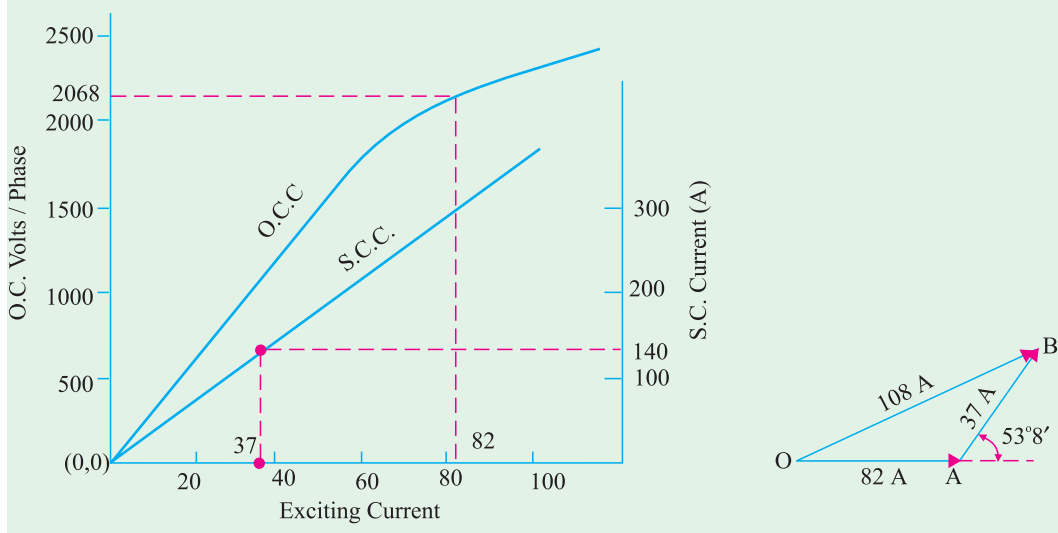


Fig. 37.54

Fig. 37.55

In Fig. 37.55, *OB* gives the excitation required on full-load to give a terminal phase voltage of 1905 V (or line voltage of 3300 V) at 0.8 p.f. lagging and its value is

$$= \sqrt{82^2 + 37^2 + 2 \times 82 \times 37 \times \cos 53^\circ 8'} = 108 \text{ A}$$

**Tutorial Problem No. 37.3**

1. A 30-kVA, 440-V, 50-Hz, 3- $\phi$ , star-connected synchronous generator gave the following test data :

Field current (A)	: 2	4	6	7	8	10	12	14
Terminal volts	: 155	287	395	440	475	530	570	592
S.C. current	: 11	22	34	40	46	57	69	80

Resistance between any two terminals is 0.3  $\Omega$

Find regulation at full-load 0.8 p.f. lagging by (a) synchronous impedance method and (b) Rother's ampere-turn method. Take  $Z_s$  corresponding to S.C. current of 80 A. **[(a) 51% (b) 29.9%]**

**37.24. Zero Power Factor Method or Potier Method**

This method is based on the separation of armature-leakage reactance drop and the armature reaction effects. Hence, it gives more accurate results. It makes use of the first two methods to some extent. The experimental data required is (i) no-load curve and (ii) full-load zero power factor curve (not the short-circuit characteristic) also called wattless load characteristic. It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero p.f.

The reduction in voltage due to armature reaction is found from above and (ii) voltage drop due to armature leakage reactance  $X_L$  (also called Potier reactance) is found from both. By combining these two,  $E_0$  can be calculated.

It should be noted that if we vectorially add to  $V$  the drop due to resistance and leakage reactance  $X_L$ , we get  $E$ . If to  $E$  is further added the drop due to armature reaction (assuming lagging p.f.), then we get  $E_0$  (Art. 37.18).

The zero p.f. lagging curve can be obtained.



- (a) if a similar machine is available which may be driven at no-load as a synchronous motor at practically zero p.f. or
- (b) by loading the alternator with pure reactors
- (c) by connecting the alternator to a 3- $\phi$  line with ammeters and wattmeters connected for measuring current and power and by so adjusting the field current that we get full-load armature current with zero wattmeter reading.

Point *B* (Fig. 37.56) was obtained in this manner when wattmeter was reading zero. Point *A* is obtained from a short-circuit test with full-load armature current. Hence, *OA* represents field current which is equal and opposite to the demagnetising armature reaction and for balancing leakage reactance drop at full-load (please refer to A.T. method). Knowing these two points, full-load zero p.f. curve *AB* can be drawn as under.

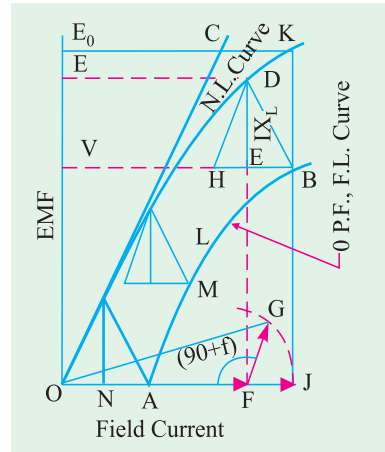


Fig. 37.56

From *B*, *BH* is drawn equal to and parallel to *OA*. From *H*, *HD* is drawn parallel to initial straight part of *N-L* curve i.e. parallel to *OC*, which is tangential to *N-L* curve. Hence, we get point *D* on no-load curve, which corresponds to point *B* on full-load zero p.f. curve. The triangle *BHD* is known as Potier triangle. This triangle is constant for a given armature current and hence can be transferred to give us other points like *M*, *L* etc. Draw *DE* perpendicular to *BH*. The length *DE* represents the drop in voltage due to armature leakage reactance  $X_L$  i.e.  $IX_L$ . *BE* gives field current necessary to overcome demagnetising effect of armature reaction at full-load and *EH* for balancing the armature leakage reactance drop *DE*.

Let *V* be the terminal voltage on full-load, then if we add to it vectorially the voltage drop due to armature leakage reactance alone (neglecting  $R_a$ ), then we get voltage  $E = DF$  (and not  $E_0$ ). Obviously, field excitation corresponding to *E* is given by *OF*. *NA* (= *BE*) represents the field current needed to overcome armature reaction. Hence, if we add *NA* vectorially to *OF* (as in Rothert's A.T. method) we get excitation for  $E_0$  whose value can be read from *N-L* curve.

In Fig. 37.56, *FG* (= *NA*) is drawn at an angle of  $(90^\circ + \phi)$  for a lagging p.f. (or it is drawn at an angle of  $90^\circ - \phi$  for a leading p.f.). The voltage corresponding to this excitation is  $JK = E_0$

$$\therefore \% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

The vector diagram is also shown separately in Fig. 37.57.

Assuming a lagging p.f. with angle  $\phi$ , vector for *I* is drawn at an angle of  $\phi$  to *V*.  $IR_a$  is drawn parallel to current vector and  $IX_L$  is drawn perpendicular to it. *OD* represents voltage *E*. The excitation corresponding to it i.e. *OF* is drawn at  $90^\circ$  ahead of it. *FG* (= *NA* = *BE* in Fig. 37.56) representing field current equivalent of full-load armature reaction, is drawn parallel to current vector *OI*. The closing side *OG* gives field excitation for  $E_0$ . Vector for  $E_0$  is  $90^\circ$  lagging behind *OG*. *DL* represents voltage drop due to armature reaction.

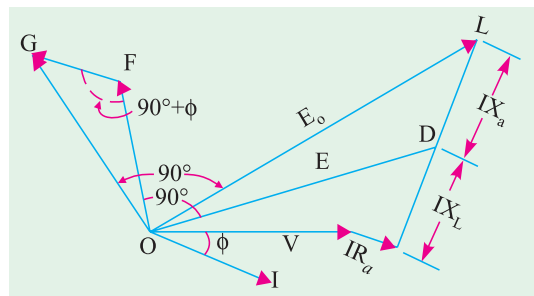


Fig. 37.57

### 37.25. Procedural Steps for Potier Method 1.

1. Suppose we are given *V*-the terminal voltage/phase.
2. We will be given or else we can calculate armature leakage reactance  $X_L$  and hence can calculate  $IX_L$ .

3. Adding  $IX_L$  (and  $IR_a$  if given) vectorially to  $V$ , we get voltage  $E$ .
4. We will next find from  $N-L$  curve, field excitation for voltage  $E$ . Let it be  $i_{f1}$ .
5. Further, field current  $i_{f2}$  necessary for balancing armature reaction is found from Potier triangle.
6. Combine  $i_{f1}$  and  $i_{f2}$  vectorially (as in  $A.T$  method) to get  $i_f$ .
7. Read from  $N-L$  curve, the e.m.f. corresponding to  $i_f$ . This gives us  $E_0$ . Hence, regulation can be found.

**Example 37.34.** A 3-phase, 6,00-V alternator has the following O.C.C. at normal speed :

Field amperes :	14	18	23	30	43
Terminal volts :	4000	5000	6000	7000	8000

With armature short-circuited and full-load current flowing the field current is 17 A and when the machine is supplying full-load of 2,000 kVA at zero power factor, the field current is 42.5 A and the terminal voltage is 6,000 V.

Determine the field current required when the machine is supplying the full-load at 0.8 p.f. lagging. (A.C. Machines-I, Jadavpur Univ. 1988)

**Solution.** The O.C.C. is drawn in Fig. 37.58 with phase voltages which are  
2310, 2828, 3465 4042 4620

The full-load zero p.f. characteristic can be drawn because two points are known i.e. (17, 0) and (42.5, 3465).

In the Potier  $\triangle BDH$ , line  $DE$  represents the leakage reactance drop ( $= IX_L$ ) and is (by measurement) equal to 450 V. As seen from Fig. 37.59.

$$E = \sqrt{(V \cos \phi)^2 + (V \sin \phi + IX_L)^2}$$

$$= \sqrt{(3465 \times 0.8)^2 + (3465 \times 0.6 + 450)^2}$$

$$= 3750 \text{ V}$$

From O.C.C. of Fig. 37.58, it is found that field amperes required for this voltage = 26.5 A.

Field amperes required for balancing armature reaction =  $BE = 14.5$  A (by measurement from Potier triangle  $BDH$ ).

As seen from Fig. 37.60, the field currents are added vectorially at an angle of  $(90^\circ + \phi) = 126^\circ 52'$ .

Resultant field current is  $OB = \sqrt{26.5^2 + 14.5^2 + 2 \times 26.5 \times 14.4 \cos 53^\circ 8'} = 37.2 \text{ A}$

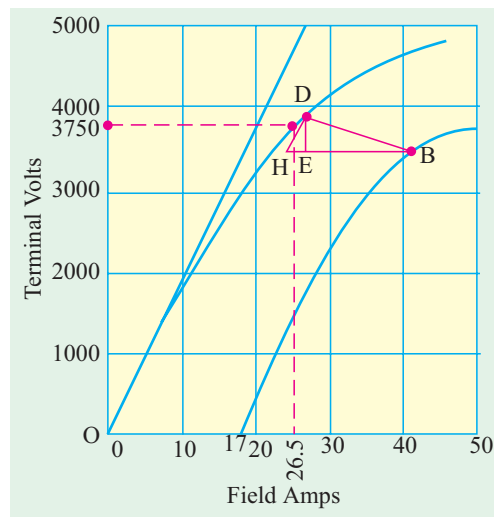


Fig. 37.58

**Example 37.35.** An 11-kV, 1000-kVA, 3-phase, Y-connected alternator has a resistance of  $2\Omega$  per phase. The open-circuit and full-load zero power factor characteristics are given below. Find the voltage regulation of the alternator for full load current at 0.8 p.f. lagging by Potier method.

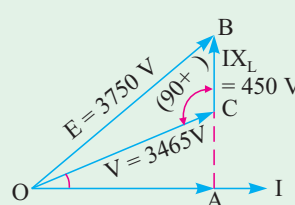


Fig. 37.59

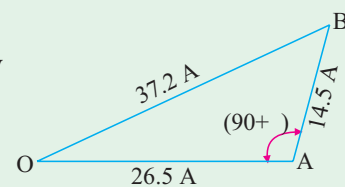


Fig. 37.60

Field current (A)	:	40	50	110	140	180
O.C.C. line voltage	:	5,800	7,000	12,500	13,750	15,000
Line volts zero p.f.	:	0	1500	8500	10,500	12,500

(Calcutta Univ. 1987 and S. Ramanandtirtha Univ. Nanded, 2001)

**Solution.** The O.C.C. and full-load zero p.f. curve for phase voltage are drawn in Fig. 37.61. The corresponding phase voltages are :

O.C.C. phase voltage	3350	4040	7220	7940	8660
Phase voltage zero p.f.	0	866	4900	6060	7220

Full-load current =  $1000 \times 1000 / \sqrt{3} \times 11,000 = 52.5 \text{ A}$

Phase voltage =  $11,000 / \sqrt{3} = 6,350 \text{ A}$

In the Potier  $\Delta ABC$ ,  $AC = 40 \text{ A}$ ,  $CB$  is parallel to the tangent to the initial portion of the O.C.C. and  $BD$  is  $\perp$  to  $AC$ .

$BD =$  leakage reactance drop  $IX_L = 1000 \text{ V}$  – by measurement

$AD = 30 \text{ A}$  — field current required to overcome demagnetising effect of armature reaction on full-load.

As shown in Fig. 37.62,

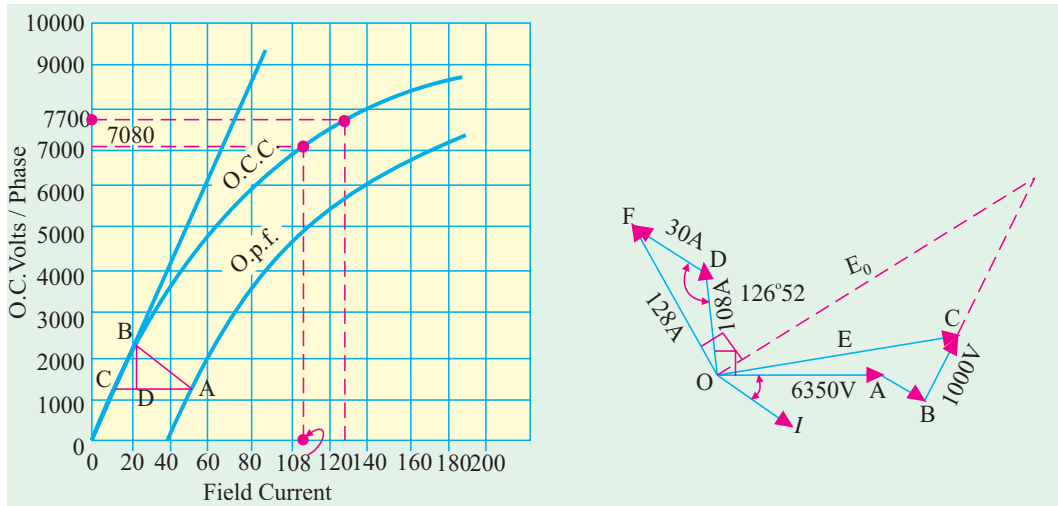


Fig. 37.61

Fig. 37.62

$OA = 6,350 \text{ V}; AB = IR_a = 52.5 \times 2 = 105 \text{ V}$

$IX_L = BC = 1000 \text{ V}$  —by measurement

$$OC = E = \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_L)^2}$$

$$= \sqrt{(6350 \times 0.8 + 105)^2 + (6350 \times 0.6 + 1000)^2}; E = 7,080 \text{ V}$$

As seen from O.C.C., field current required for 7,080 V is 108 A. Vector  $OD$  (Fig. 37.62) represents 108 A and is drawn  $\perp$  to  $OC$ .  $DF$  represents 30 A and is drawn parallel to  $OI$  or at  $(90^\circ + 36^\circ 52') = 126^\circ 52'$  with  $OD$ . Total field current is  $OF$ .

$$OF = \sqrt{108^2 + 30^2 + 2 \times 108 \times 30 \cos 53^\circ 8'} = 128 \text{ A}$$

From O.C.C., it is found that the e.m.f. corresponding to this field current is 7,700 V

$\therefore E_0 = 7,700 \text{ V}; \text{regulation} = \frac{7,700 - 6,350}{6,350} \times 100 = 21.3 \text{ per cent}$

**Example 37.36.** The following test results were obtained on a 275-kW, 3- $\phi$ , 6,600-V non-salient pole type generator:

Open-circuit characteristic :

Volts	:	5600	6600	7240	8100
Exciting amperes	:	46.5	58	67.5	96

Short-circuit characteristic : Stator current 35 A with an exciting current of 50 A. Leakage reactance on full-load = 8%. Neglect armature resistance. Calculate as accurately as possible the exciting current (for full-load) at power factor 0.8 lagging and at unity. (City & Guilds, London)

**Solution.** First convert the O.C. line volts into phase volts by dividing the given terminal values by  $\sqrt{3}$ .  
 $\therefore$  O.C. volts (phase) : 3233, 3810, 4180, 4677.

O.C.C. is plotted in Fig. 37.63. For plotting S.C.C., we need two points. One is (0, 0) and the other is (50 A, 35 A). In fact, we can do without plotting the S.C.C. because it being a straight line, values of field currents corresponding to any armature current can be found by direct ratio.

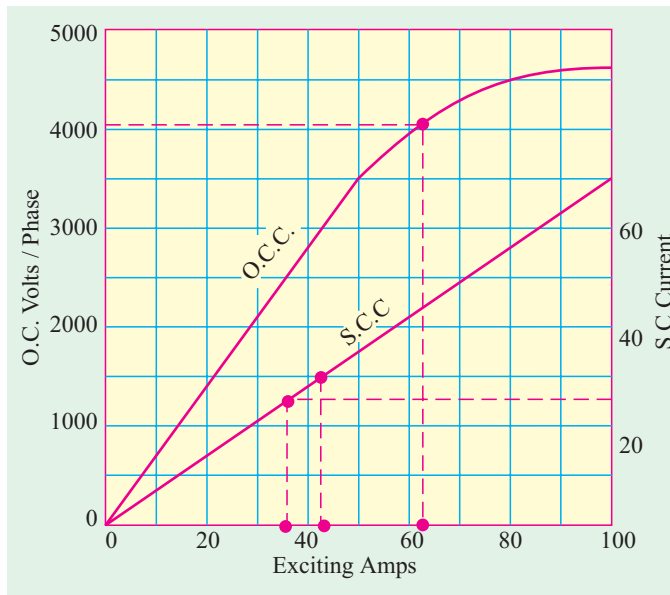


Fig. 37.63

Leakage reactance drop  
 $= \frac{3810 \times 8}{100} = 304.8 \text{ V}$

Normal phase voltage  
 $= 6,600 / \sqrt{3} = 3,810 \text{ V}$

In Fig. 37.64,  $OA = 3810 \text{ V}$  and at an angle  $\phi$  ahead of current vector  $OI$ .  
 $AB = 304.8 \text{ V}$  is drawn at right angles to  $OI$ . Resultant of the two is  $OB = 4010 \text{ V}$ .  
 From O.C.C., field current corresponding to 4,010 V is 62 A.

Full-load current at 0.8 p.f. =  $275,000 / \sqrt{3} \times 6600 \times 0.8 = 30 \text{ A}$

35 A of armature current need 50 A of field current, hence 30 A of armature current need  $30 \times 50 / 35 = 43 \text{ A}$ .

In Fig. 37.64,  $OC = 62 \text{ A}$  is drawn at right angles to  $OB$ . Vector  $CD = 43 \text{ A}$  is drawn parallel to  $OI$ . Then,  $OD = 94.3 \text{ A}$

**Note.** Here, 43 A pf field excitation is assumed as having all been used for balancing armature reaction. In fact, a part of it is used for balancing armature leakage drop of 304.8 V. This fact has been clarified in the next example.

**At Unity p.f.**

In Fig. 37.65,  $OA$  again represents  $V = 3810 \text{ V}$ ,  $AB = 304.8 \text{ V}$  and at right angles to  $OA$ .

The resultant  $OB = \sqrt{(3810^2 + 304.8^2)} = 3830 \text{ V}$

Field current from O.C.C. corresponding to this voltage = 59.8 A.

Hence,  $OC = 59.8 \text{ A}$  is drawn perpendicular to  $OB$  (as before)

Full-load current at u.p.f. =  $275,000 / \sqrt{3} \times 6600 \times 1 = 24 \text{ A}$

Now, 35 A armature current corresponds to a field current of 50 A, hence 24 A of armature current corresponds to  $50 \times 24/35 = 34.3$  A.

Hence,  $CD = 34.3$  A is drawn  $\parallel$  to  $OA$  (and  $\perp$  to  $OC$  approximately).\*

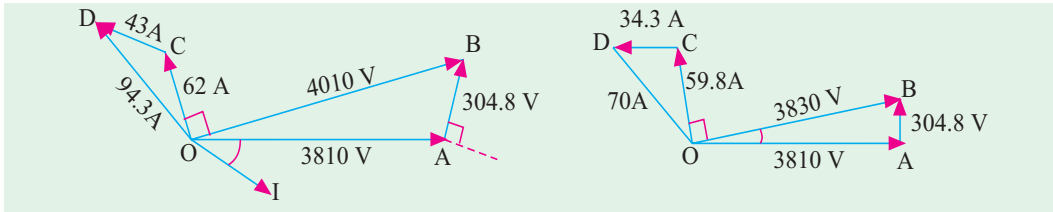


Fig. 37.64

Fig. 37.65

$$\therefore OD = \sqrt{(59.8^2 + 34.3^2)} = 70 \text{ A}$$

**Example 37.37.** A 600-kVA, 3,300-V, 8-pole, 3-phase, 50-Hz alternator has following characteristic :

Amp-turns/pole :	4000	5000	7000	10,000
Terminal E.M.F. :	2850	3400	3850	4400

There are 200 conductor in series per phase.

Find the short-circuit characteristic, the field ampere-turns for full-load 0.8 p.f. (lagging) and the voltage regulation, having given that the inductive drop at full-load is 7% and that the equivalent armature reaction in amp-turns per pole =  $1.06 \times$  ampere-conductors per phase per pole.

(London Univ.)

**Solution.** O.C. terminal voltages are first converted into phase voltages and plotted against field amp-turns, as shown in Fig. 37.66.

Full-load current

$$= \frac{600,000}{\sqrt{3} \times 3300} = 105 \text{ A}$$

Demagnetising amp-turns per pole per phase for full-load at zero p.f.

$$= 1.06 \times 105 \times 200/8 = 2,780$$

Normal phase voltage

$$= 3300 / \sqrt{3} = 1910 \text{ volt}$$

Leakage reactance drop

$$= \frac{3300 \times 7}{\sqrt{3} \times 100} = 133 \text{ V}$$

In Fig. 37.67,  $OA$  represents 1910 V.

$AB = 133$  V is drawn  $\perp$  to  $OI$ ,  $OB$  is the resultant voltage  $E$  (not  $E_0$ ).

$$\therefore OB = E = 1987 \text{ volt}$$

From O.C.C., we find that 1987 V correspond to 5100 field amp-turns. Hence,  $OC = 5100$  is drawn  $\perp$  to  $OB$ .  $CD = 2780$  is  $\parallel$  to  $OI$ . Hence,  $OD = 7240$  (approx). From O.C.C. it is found that this

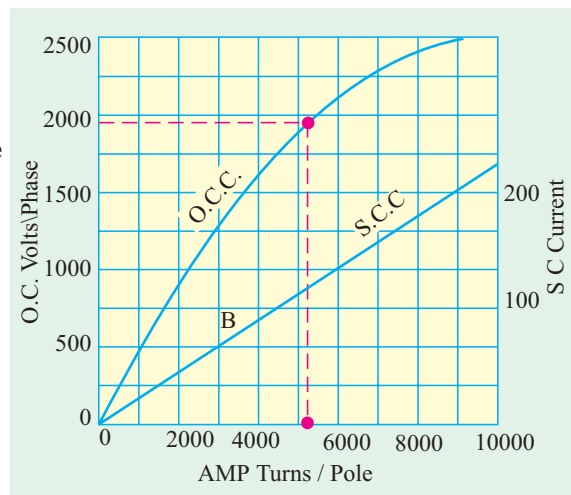


Fig. 37.66

\* It is so because angle between  $OA$  and  $OB$  is negligibly small. If not, then  $CD$  should be drawn at an angle of  $(90 + \alpha)$  where  $\alpha$  is the angle between  $OA$  and  $OB$ .

corresponds to an O.C. voltage of 2242 volt. Hence, when load is thrown off, the voltage will rise to 2242 V.

$$\begin{aligned} \therefore \quad \% \text{ regn.} &= \frac{2242 - 1910}{1910} \times 100 \\ &= 17.6\% \end{aligned}$$

**How to deduce S.C.C. ?**

We have found that field amp-turns for balancing armature reaction only are 2,780. To this should be added field amp-turns required for balancing the leakage reactance voltage drop of 133 V.

Field amp-turns corresponding to 133 volt on O.C. are 300 approximately. Hence, with reference to Fig. 37.56,  $NA = 2780$ ,  $ON = 300$

$$\begin{aligned} \therefore \text{Short-circuit field amp-turns} \\ &= OA = 2780 + 300 \\ &= 3080 \end{aligned}$$

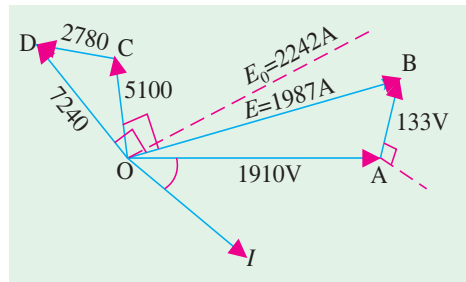


Fig. 37.67

Hence, we get a point B on S.C.C. i.e. (3080, 105) and the other point is the origin. So S.C.C. (which is a straight line) can be drawn as shown in Fig. 37.66.

**Example 37.38.** The following figures give the open-circuit and full-load zero p.f saturation curves for a 15,000-kVA, 11,000 V, 3- $\phi$ , 50-Hz, star-connected turbo-alternator:

Field AT in $10^3$	:	10	18	24	30	40	45	50
O.C. line kV	:	4.9	8.4	10.1	11.5	12.8	13.3	13.65
Zero p.f. full-load line kV	:	—	0	—	—	—	10.2	—

Find the armature reaction, the armature reactance and the synchronous reactance. Deduce the regulation for full-load at 0.8 power lagging.

**Solution.** First, O.C.C. is drawn between phase voltages and field amp-turns, as shown in Fig. 37.68.

Full-load, zero p.f. line can be drawn, because two points are known i.e. A (18, 0) and C (45, 5890). Other points on this curve can be found by transferring the Potier triangle. At point C, draw  $CD \parallel$  to and equal to  $OA$  and from D draw  $DE \parallel$  to  $ON$ . Join  $EC$ . Hence,  $CDE$  is the Potier triangle.

Line  $EF$  is  $\perp$  to  $DC$   
 $CF$  = field amp-turns for balancing armature-reaction only  
 = 15,700

$EF = GH = 640$  volt = leakage reactance drop/phase

Short-circuit A.T. required =  $OA = 18,000$

$$\text{Full-load current} = \frac{15,000 \times 1000}{\sqrt{3} \times 11,000} = 788 \text{ A}$$

$$\therefore 640 = I \times X_L \quad \therefore X_L = 640/788 = 0.812 \Omega$$

From O.C.C., we find that 18,000 A.T. correspond to an O.C. voltage of  $8,400/\sqrt{3} = 4,850$  V.

$$\begin{aligned} \therefore Z_s &= \frac{\text{O.C. volt}}{\text{S.C. current}} = \frac{4,850}{788} \\ &= 6.16 \Omega \end{aligned}$$

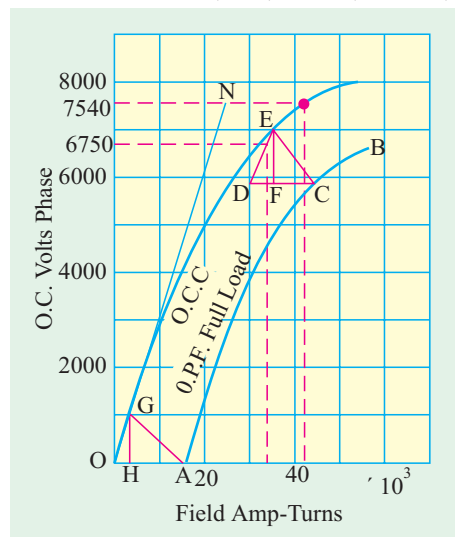


Fig. 37.68

(Art. 37.21)

As  $R_a$  is negligible, hence  $Z_s$  equals  $X_s$

**Regulation**

In Fig. 37.69,  $OA = \text{phase voltage} = 11,000/\sqrt{3} = 6,350 \text{ V}$

$AB = 640 \text{ V}$  and is drawn at right angles to  $OI$  or at  $(90^\circ + \phi)$  to  $OA$ .

Resultant is  $OB = 6,750 \text{ V}$

Field A.T. corresponding to O.C voltage of 6,750 V is  $= OC = 30,800$  and is drawn  $\perp$  to  $OB$ .

$CD = \text{armature reaction at F.L.} = 15,700$  and is drawn  $\parallel$  to  $OI$  or at  $(90^\circ + \phi)$  to  $OC$ .

Hence,  $OD = 42,800$ .

From O.C.C., e.m.f. corresponding to 42,800 A.T. of rotor = 7,540 V

$\therefore \% \text{ regn. up} = (7,540 - 6,350)/6,350 = 0.187$  or **18.7%**

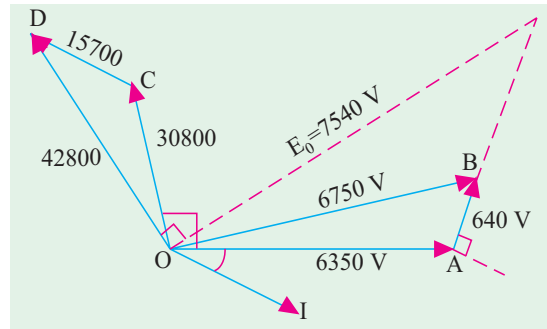


Fig. 37.69

**Tutorial Problem No. 37.4**

1. The following data relate to a 6,600-V, 10,000-kVA, 50-Hz, 3- $\phi$ , turbo- alternator:

O.C. kilovolt	4.25	5.45	6.6	7.3	8	9
Exciting A.T. in $10^3$	60	80	100	120	145	220

Excitation needed to circulate full-load current on short circuit : 117,000 A.T. Inductive drop in stator winding at full-load = 15%. Find the voltage regulation at full-load 0.8 power factor.

[34.4%] (City & Guilds, London)

2. Deduce the exciting current for a 3- $\phi$ , 3300-V generator when supplying 100 kW at 0.8 power factor lagging, given magnetisation curve on open-circuit :

Line voltage :	3300	3600	3900
Exciting current :	80	96	118

There are 16 poles, 144 slots, 5 conductors/slot, single-circuit, full-pitched winding, star- connected. The stator winding has a resistance per phase of 0.15  $\Omega$  and a leakage reactance of 1.2  $\Omega$ . The field coils have each 108 turns.

[124 A] (London Univ.)

3. Estimate the percentage regulation at full-load and power factor 0.8 lagging of a 1000-kVA, 6,600-V, 3- $\phi$ , 50-Hz, star-connected salient-pole synchronous generator. The open-circuit characteristic is as follows :

Terminal volt	4000	6000	6600	7200	8000
Field A.T.	5200	8500	10,000	12,500	17,500

Leakage reactance 10%, resistance 2%. Short-circuit characteristic : full-load current with a field excitation of 5000 A.T. Take the permeance to cross armature reaction as 35% of that to direct reaction.

[20% up]

4. A 1000-kVA, 11,000-V, 3- $\phi$ , 50-Hz, star-connected turbo-generator has an effective resistance of 2 $\Omega$ /phase. The O.C.C. and zero p.f. full-load data is as follows :

O.C. volt	5,805	7,000	12,550	13,755	15,000
Field current A	40	50	110	140	180
Terminal volt at F.L. zero p.f.	0	1500	8,500	10,500	12,400

Estimate the % regulation for F.L. at 0.8 p.f. lagging. [22 %]

5. A 5-MVA, 6.6 kV, 3- $\phi$ , star-connected alternator has a resistance of 0.075  $\Omega$  per phase. Estimate the regulation for a load of 500 A at p.f. (a) unity and (b) 0.9 leading (c) 0.71 lagging from the following open-circuit and full-load zero power factor curve.

Field current (A)	Open-circuit terminal voltage (V)	Saturation curve zero p.f.
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

[(a) 6.3% (b) -7.9% (c) 20.2%] (Electrical Machines-II, Indore Univ. Feb. 1978)

### 37.26. Operation of a Salient Pole Synchronous Machine

A multipolar machine with cylindrical rotor has a uniform air-gap, because of which its reactance remains the same, irrespective of the spatial position of the rotor. However, a synchronous machine with salient or projecting poles has non-uniform air-gap due to which its reactance varies with the rotor position. Consequently, a cylindrical rotor machine possesses one axis of symmetry (pole axis or direct axis) whereas salient-pole machine possesses two axes of geometric symmetry (i) field poles axis, called direct axis or  $d$ -axis and (ii) axis passing through the centre of the interpolar space, called the quadrature axis or  $q$ -axis, as shown in Fig. 37.70.

Obviously, two mmfs act on the  $d$ -axis of a salient-pole synchronous machine i.e. field m.m.f. and armature m.m.f. whereas only one m.m.f., i.e. armature mmf acts on the  $q$ -axis, because field mmf has no component in the  $q$ -axis. The magnetic reluctance is low along the poles and high between the poles. The above facts form the basis of the two-reaction theory proposed by Blondel, according to which

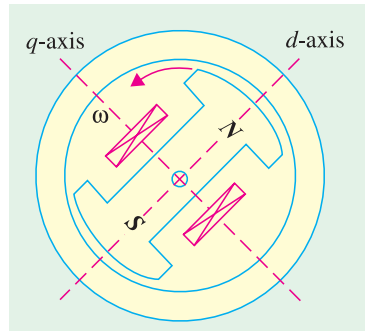


Fig. 37.70

- (i) armature current  $I_a$  can be resolved into two components i.e.  $I_d$  perpendicular to  $E_0$  and  $I_q$  along  $E_0$  as shown in Fig. 37.71 (b).
- (ii) armature reactance has two components i.e.  $q$ -axis armature reactance  $X_{aq}$  associated with  $I_d$  and  $d$ -axis armature reactance  $X_{ad}$  linked with  $I_q$ .

If we include the armature leakage reactance  $X_l$  which is the same on both axes, we get

$$X_d = X_{ad} + X_l \text{ and } X_q = X_{aq} + X_l$$

Since reluctance on the  $q$ -axis is higher, owing to the larger air-gap, hence,

$$X_{aq} < X_{ad} \text{ or } X_q < X_d \text{ or } X_d > X_q$$

### 37.27. Phasor Diagram for a Salient Pole Synchronous Machine

The equivalent circuit of a salient-pole synchronous generator is shown in Fig. 37.71 (a). The component currents  $I_d$  and  $I_q$  provide component voltage drops  $jI_d X_d$  and  $jI_q X_q$  as shown in Fig. 37.71 (b) for a lagging load power factor.

The armature current  $I_a$  has been resolved into its rectangular components with respect to the axis for excitation voltage  $E_0$ . The angle  $\psi$  between  $E_0$  and  $I_a$  is known as the internal power factor angle. The



vector for the armature resistance drop  $I_a R_a$  is drawn parallel to  $I_a$ . Vector for the drop  $I_d X_d$  is drawn perpendicular to  $I_d$  whereas that for  $I_q X_q$  is drawn perpendicular to  $I_q$ . The angle  $\delta$  between  $E_0$  and  $V$  is called the power angle. Following phasor relationships are obvious from Fig. 37.71 (b)

$$E_0 = V + I_a R_a + jI_d X_d + jI_q X_q \text{ and } I_a = I_d + I_q$$

If  $R_a$  is neglected the phasor diagram becomes as shown in Fig. 37.72 (a). In this case,

$$E_0 = V + jI_d X_d + jI_q X_q$$

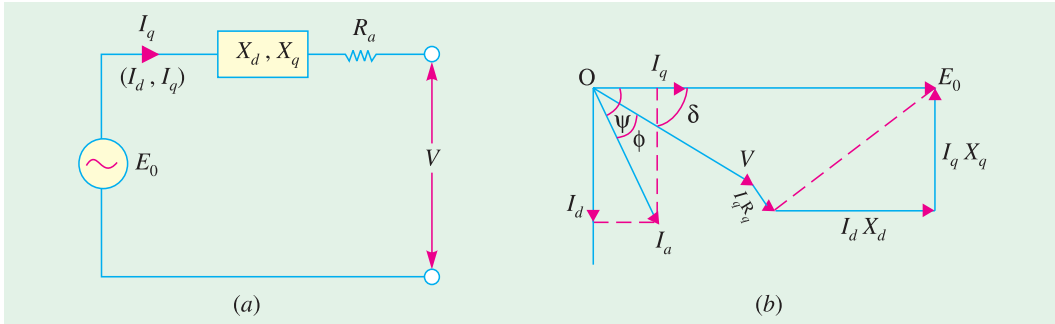


Fig. 37.71

Incidentally, we may also draw the phasor diagram with terminal voltage  $V$  lying in the horizontal direction as shown in Fig. 37.72 (b). Here, again drop  $I_a R_a$  is  $\parallel I_a$  and  $I_d X_d$  is  $\perp I_d$  and drop  $I_q X_q$  is  $\perp I_q$  as usual.

### 37.28. Calculations from Phasor Diagram

In Fig. 37.73, dotted line  $AC$  has been drawn perpendicular to  $I_a$  and  $CB$  is perpendicular to the phasor for  $E_0$ . The angle  $ACB = \psi$  because angle between two lines is the same as between their perpendiculars. It is also seen that

$$I_d = I_a \sin \psi ; I_q = I_a \cos \psi ; \text{ hence, } I_a = I_q / \cos \psi$$

In  $\triangle ABC$ ,

$$BC/AC = \cos \psi \text{ or } AC = BC / \cos \psi = I_q X_q / \cos \psi = I_a X_q$$

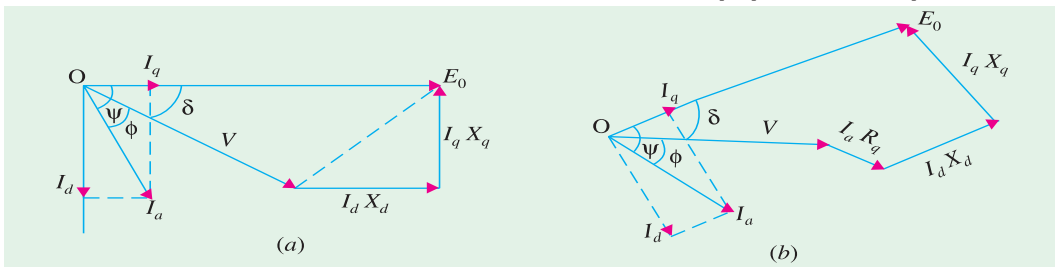


Fig. 37.72

From  $\triangle ODC$ , we get

$$\begin{aligned} \tan \psi &= \frac{AD + AC}{OE + ED} = \frac{V \sin \phi + I_a X_q}{V \cos \phi + I_a R_a} && \text{---generating} \\ &= \frac{V \sin \phi - I_a X_q}{V \sin \phi - I_a R_a} && \text{---motoring} \end{aligned}$$

The angle  $\psi$  can be found from the above equation. Then,  $\delta = \psi - \phi$  (generating) and  $\delta = \phi - \psi$  (motoring)

As seen from Fig. 37.73, the excitation voltage is given by

$$E_0 = V \cos \delta + I_q R_a + I_d X_d \quad \text{---generating}$$

$$= V \cos \delta - I_q R_a - I_d X_d \quad \text{---motoring}$$

**Note.** Since angle  $\phi$  is taken positive for lagging p.f., it will be taken negative for leading p.f.

If we neglect the armature resistance as shown in Fig. 37.72, then angle  $\delta$  can be found directly as under :

$$\psi = \phi + \delta \text{ (generating)}$$

$$\text{and } \psi = \phi - \delta \text{ (motoring).}$$

$$\text{In general, } \psi = (\phi \pm \delta).$$

$$I_d = I_a \sin \psi$$

$$= I_a \sin (\phi \pm \delta); I_q = I_a \cos \psi = I_a \cos (\phi \pm \delta)$$

As seen from Fig. 37.73,  $V \sin \delta = I_q X_q = I_a X_q \cos (\phi \pm \delta)$

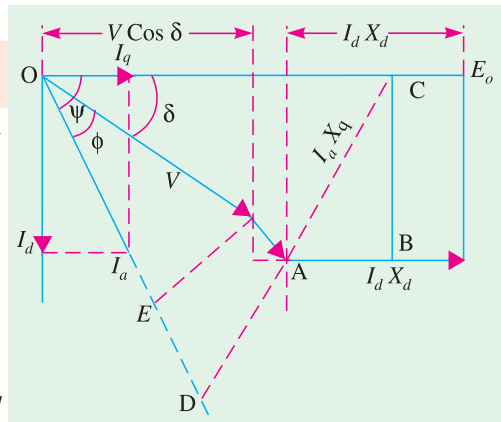


Fig. 37.73

$$\therefore V \sin \delta = I_a X_q (\cos \phi \cos \delta \pm \sin \phi \sin \delta)$$

$$\text{or } V = I_a X_q \cos \phi \cot \delta \pm I_a X_q \sin \phi$$

$$\therefore I_a X_q \cos \phi \cot \delta = V \pm I_a X_q \sin \phi$$

$$\therefore \tan \delta = \frac{I_a X_q \cos \phi}{V \pm I_a X_q \sin \phi}$$

In the above expression, plus sign is for synchronous generators and minus sign for synchronous motors.

Similarly, when  $R_a$  is neglected, then,

$$E_0 = V \cos \delta \pm I_d X_d$$

However, if  $R_a$  and hence  $I_a R_a$  drop is not negligible then,

$$\begin{aligned} E_0 &= V \cos \delta + I_q R_a + I_d X_d && \text{---generating} \\ &= V \cos \delta - I_q R_a - I_d X_d && \text{---motoring} \end{aligned}$$

### 37.29. Power Developed by a synchronous Generator

If we neglect  $R_a$  and hence Cu loss, then the power developed ( $P_d$ ) by an alternator is equal to the power output ( $P_{out}$ ). Hence, the per phase power output of an alternator is

$$P_{out} = VI_a \cos \phi = \text{power developed } (p_d) \quad \dots (i)$$

$$\text{Now, as seen from Fig., 37.72 (a), } I_q X_q = V \sin \delta; I_d X_d = E_0 - V \cos \delta \quad \dots (ii)$$

$$\text{Also, } I_d = I_a \sin (\phi + \delta); I_q = I_a \cos (\phi + \delta) \quad \dots (iii)$$

Substituting Eqn. (iii) in Eqn. (ii) and solving for  $I_a \cos \phi$ , we get

$$I_a \cos \phi = \frac{V}{X_d} \sin \delta + \frac{V}{2 X_q} \sin 2 \delta - \frac{V}{2 X_d} \sin 2 \delta$$

Finally, substituting the above in Eqn. (i), we get

$$P_d = \frac{E_0 V}{X_d} \sin \delta + \frac{1}{2} V^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2 \delta = \frac{E_0 V}{X_d} \sin \delta + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2 \delta$$

The total power developed would be three times the above power.

As seen from the above expression, the power developed consists of two components, the first term represents power due to field excitation and the second term gives the reluctance power *i.e.*

power due to saliency. If  $X_d = X_q$  i.e. the machine has a cylindrical rotor, then the second term becomes zero and the power is given by the first term only. If, on the other hand, there is no field excitation i.e.  $E_0 = 0$ , then the first term in the above expression becomes zero and the power developed is given by the second term. It may be noted that value of  $\delta$  is positive for a generator and negative for a motor.

**Example 37.39.** A 3-phase alternator has a direct-axis synchronous reactance of 0.7 p.u. and a quadrature axis synchronous reactance of 0.4 p.u. Draw the vector diagram for full-load 0.8 p.f. lagging and obtain therefrom (i) the load angle and (ii) the no-load per unit voltage.

(Advanced Elect. Machines, AMIE Sec. B 1991)

**Solution.**

$$V = 1 \text{ p.u.}; X_d = 0.7 \text{ p.u.}; X_q = 0.4 \text{ p.u.};$$

$$\cos \phi = 0.8; \sin \phi = 0.6; \phi = \cos^{-1} 0.8 = 36.9^\circ; I_a = 1 \text{ p.u.}$$

$$(i) \quad \tan \delta = \frac{I_a X_q \cos \phi}{V + I_a X_q \sin \phi} = \frac{1 \times 0.4 \times 0.8}{1 + 0.4 \times 0.6} = 0.258, \delta = 16.5^\circ$$

$$(ii) \quad I_d = I_a \sin(\phi + \delta) = 1 \sin(36.9^\circ + 14.9^\circ) = 0.78 \text{ A}$$

$$E_0 = V \cos \delta + I_d X_d = 1 \times 0.966 + 0.78 \times 0.75 = 1.553$$

**Example 37.40.** A 3-phase, star-connected, 50-Hz synchronous generator has direct-axis synchronous reactance of 0.6 p.u. and quadrature-axis synchronous reactance of 0.45 p.u. The generator delivers rated kVA at rated voltage. Draw the phasor diagram at full-load 0.8 p.f. lagging and hence calculate the open-circuit voltage and voltage regulation. Resistive drop at full-load is 0.015 p.u.

(Elect. Machines-II, Nagpur Univ. 1993)

**Solution.**

$$I_a = 1 \text{ p.u.}; V = 1 \text{ p.u.}; X_d = 0.6 \text{ p.u.}; X_q = 0.45 \text{ p.u.}; R_a = 0.015 \text{ p.u.}$$

$$\tan \psi = \frac{V \sin \phi + I_a X_q}{V \cos \phi + I_a R_a} = \frac{1 \times 0.6 + 1 \times 0.45}{1 \times 0.8 + 1 \times 0.015} = 1.288; \quad \psi = 52.2^\circ$$

$$\delta = \psi - \phi = 52.2^\circ - 36.9^\circ = 15.3^\circ$$

$$I_d = I_a \sin \psi = 1 \times 0.79 = 0.79 \text{ A}; I_q = I_a \cos \psi = 1 \times 0.61 = 0.61 \text{ A}$$

$$E_0 = V \cos \delta + I_q R_a + I_d X_d$$

$$= 1 \times 0.965 + 0.61 \times 0.015 + 0.79 \times 0.6 = 1.448$$

$$\therefore \% \text{ regn.} = \frac{1.448 - 1}{1} \times 100 = 44.8\%$$

**Example 37.41.** A 3-phase, Y-connected syn. generator supplies current of 10 A having phase angle of  $20^\circ$  lagging at 400 V. Find the load angle and the components of armature current  $I_d$  and  $I_q$  if  $X_d = 10 \text{ ohm}$  and  $X_q = 6.5 \text{ ohm}$ . Assume arm. resistance to be negligible.

(Elect. Machines-I, Nagpur Univ. 1993)

**Solution.**

$$\cos \phi = \cos 20^\circ = 0.94; \sin \phi = 0.342; I_a = 10 \text{ A}$$

$$\tan \delta = \frac{I_a X_q \cos \phi}{V + I_a X_q \sin \phi} = \frac{10 \times 6.5 \times 0.94}{400 + 10 \times 6.5 \times 0.342} = 0.1447$$

$$\delta = 8.23^\circ$$

$$I_d = I_a \sin(\phi + \delta) = 10 \sin(20^\circ + 8.23^\circ) = 4.73 \text{ A}$$

$$I_q = I_a \cos(\phi + \delta) = 10 \cos(20^\circ + 8.23^\circ) = 8.81 \text{ A}$$

Incidentally, if required, voltage regulation of the above generator can be found as under:

$$I_d X_d = 4.73 \times 10 = 47.3 \text{ V}$$

$$E_0 = V \cos \delta + I_d X_d = 400 \cos 8.23^\circ + 47.3 = 443 \text{ V}$$

$$\begin{aligned}\% \text{ regn.} &= \frac{E_0 - V}{V} \times 100 \\ &= \frac{443 - 400}{400} \times 100 = \mathbf{10.75\%}\end{aligned}$$

### Tutorial Problem No. 37.5.

1. A 20 MVA, 3-phase, star-connected, 50-Hz, salient-pole has  $X_d = 1$  p.u.;  $X_q = 0.65$  p.u. and  $R_a = 0.01$  p.u. The generator delivers 15 MW at 0.8 p.f. lagging to an 11-kV, 50-Hz system. What is the load angle and excitation e.m.f. under these conditions? [18°; 1.73 p.u.]
2. A salient-pole synchronous generator delivers rated kVA at 0.8 p.f. lagging at rated terminal voltage. It has  $X_d = 1.0$  p.u. and  $X_q = 0.6$  p.u. If its armature resistance is negligible, compute the excitation voltage under these conditions. [1.77 p.u.]
3. A 20-kVA, 220-V, 50-Hz, star-connected, 3-phase salient-pole synchronous generator supplies load at a lagging power factor angle of  $45^\circ$ . The phase constants of the generator are  $X_d = 4.0 \Omega$ ;  $X_q = 2 \Omega$  and  $R_a = 0.5 \Omega$ . Calculate (i) power angle and (ii) voltage regulation under the given load conditions. [(i) 20.6° (ii) 142%]
4. A 3-phase salient-pole synchronous generator has  $X_d = 0.8$  p.u.;  $X_q = 0.5$  p.u. and  $R_a = 0$ . Generator supplies full-load at 0.8 p.f. lagging at rated terminal voltage. Compute (i) power angle and (ii) no-load voltage if excitation remains constant. [(i) 17.1° (ii) 1.6 p.u.]

### 37.30. Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as **synchronizing**. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to **infinite** bus-bars.

It is never advisable to connect a stationary alternator to live bus-bars, because, stator induced e.m.f. being zero, a short-circuit will result. For proper synchronization of alternators, the following three conditions must be satisfied :

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.
2. The speed of the incoming machine must be such that its frequency ( $= PN/120$ ) equals bus-bar frequency.
3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage. It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

Condition (1) is indicated by a voltmeter, conditions (2) and (3) are indicated by synchronizing lamps or a synchroscope.

### 37.31. Synchronizing of Alternators

#### (a) Single-phase Alternators

Suppose machine 2 is to be synchronized with or 'put on' the bus-bars to which machine 1 is already connected. This is done with the help of two lamps  $L_1$  and  $L_2$  (known as synchronizing lamps) connected as shown in Fig. 37.74.

It should be noted that  $E_1$  and  $E_2$  are in-phase relative to the external circuit but are in direct phase opposition in the local circuit (shown dotted).

If the speed of the incoming machine 2 is not brought up to that of machine 1, then its frequency will also be different, hence there will be a phase-difference between their voltages (even when they are equal in magnitude, which is determined by field excitation). This phase-difference will be continuously changing with the changes in their frequencies. The result is that their resultant voltage will undergo changes similar to the frequency changes of beats produced, when two sound sources of nearly equal frequency are sounded together, as shown in Fig. 37.75.

Sometimes the resultant voltage is maximum and some other times minimum. Hence, the current is alternately maximum and minimum. Due to this changing current through the lamps, a flicker will be produced, the frequency of flicker being  $(f_2 - f_1)$ . Lamps will dark out and glow up alternately. Darkness indicates that the two voltages  $E_1$  and  $E_2$  are in exact phase opposition relative to the local circuit and hence

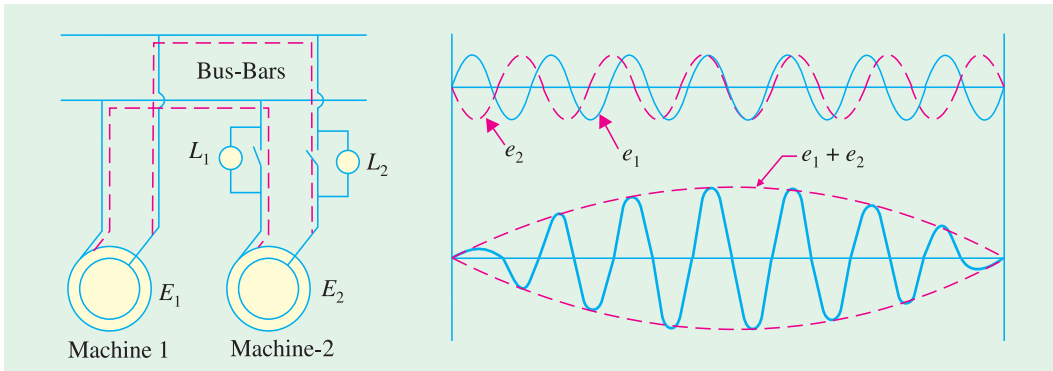


Fig. 37.74

Fig. 37.75

there is no resultant current through the lamps. Synchronizing is done at the middle of the dark period. That is why, sometimes, it is known as 'lamps dark' synchronizing. Some engineers prefer 'lamps bright' synchronization because of the fact the lamps are much more sensitive to changes in voltage at their maximum brightness than when they are dark. Hence, a sharper and more accurate synchronization is obtained. In that case, the lamps are connected as shown in Fig. 37.76. Now, the lamps will glow brightest when the two voltages are in-phase with the bus-bar voltage because then voltage across them is twice the voltage of each machine.

**(b) Three-phase Alternators**

In 3- $\phi$  alternators, it is necessary to synchronize one phase only, the other two phases will then be synchronized automatically. However, first it is necessary that the incoming alternator is correctly 'phased out' i.e. the phases are connected in the proper order of R, Y, B and not R, B, Y etc.

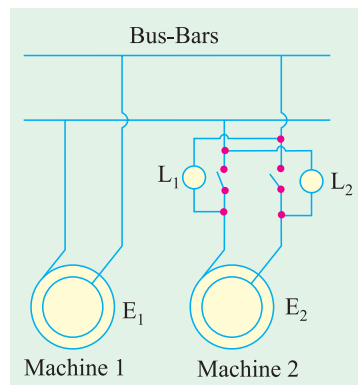


Fig. 37.76

In this case, three lamps are used. But they are deliberately connected asymmetrically, as shown in Fig. 37.77 and 37.78.

This transposition of two lamps, suggested by Siemens and Halske, helps to indicate whether the incoming machine is running too slow. If lamps were connected symmetrically, they would dark out or glow up simultaneously (if the phase rotation is the same as that of the bus-bars).

Lamp  $L_1$  is connected between R and R',  $L_2$  between Y and B' (not Y and Y) and  $L_3$  between B and Y (and not B and B'), as shown in Fig. 37.78.

Voltage stars of two machines are shown superimposed on each other in Fig. 37.79.

Two sets of star vectors will rotate at unequal speeds if the frequencies of the two machines are different. If the incoming alternator is running faster, then voltage star  $R'Y'B'$  will appear to rotate anticlockwise with respect to the bus-bar voltage star  $RYB$  at a speed corresponding to the difference between their frequencies. With reference to Fig. 37.79, it is seen that voltage across  $L_1$  is  $RR'$  and is seen to be increasing from zero, that across  $L_2$  is  $YB'$  which is decreasing, having just passed through its maximum, that across  $L_3$  is  $BY'$  which is increasing and approaching its maximum. Hence, the lamps will light up one after the other in the order 2, 3, 1 ; 2, 3, 1 or 1, 2, 3.



The rotor and stator of 3-phase generator

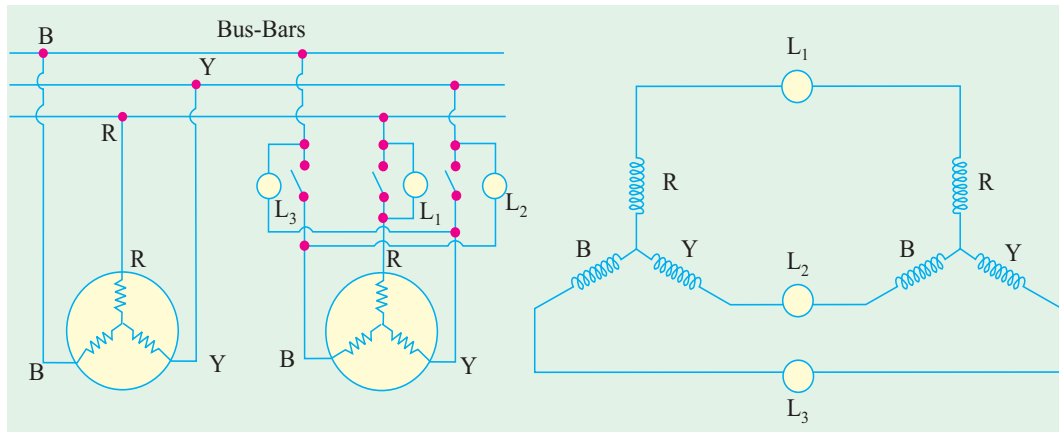


Fig. 37.77

Fig. 37.78

Now, suppose that the incoming machine is slightly slower. Then the star  $R'Y'B'$  will appear to be rotating clockwise relative to voltage star  $RYB$  (Fig. 37.80). Here, we find that voltage across  $L_3$  i.e.  $YB'$  is decreasing having just passed through its maximum, that across  $L_2$  i.e.  $YB'$  is increasing and approaching its maximum, that across  $L_1$  is decreasing having passed through its maximum earlier. Hence, the lamps will light up one after the other in the order 3, 2, 1 ; 3, 2, 1, etc. which is just the reverse of the first order. Usually, the three lamps are mounted at the three corners of a triangle and the apparent direction of rotation of light

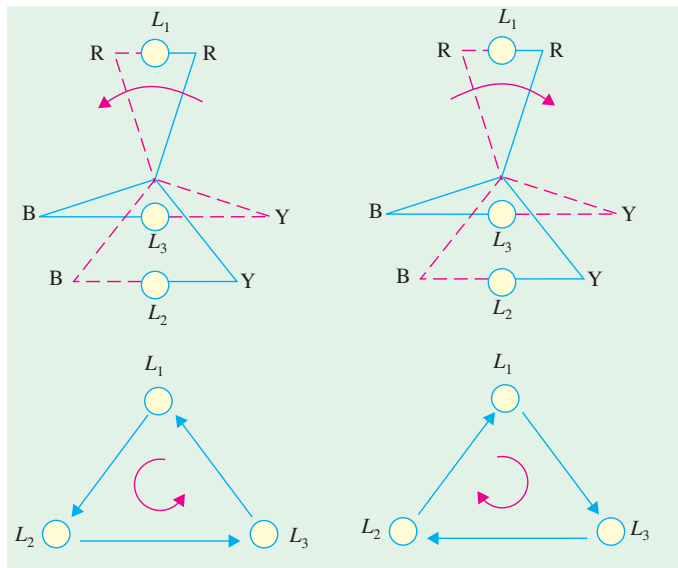


Fig. 37.79

Fig. 37.80

indicates whether the incoming alternator is running too fast or too slow (Fig. 37.81). Synchronization is done at the moment the uncrossed lamp  $L_1$  is in the middle of the dark period. When the alternator voltage is too high for the lamps to be used directly, then usually step-down transformers are used and the synchronizing lamps are connected to the secondaries.

It will be noted that when the uncrossed lamp  $L_1$  is dark, the other two 'crossed' lamps  $L_2$  and  $L_3$  are dimly but equally bright. Hence, this method of synchronizing is also sometimes known as 'two bright and one dark' method.

It should be noted that synchronization by lamps is not quite accurate, because to a large extent, it depends on the sense of correct judgement of the operator. Hence, to eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.

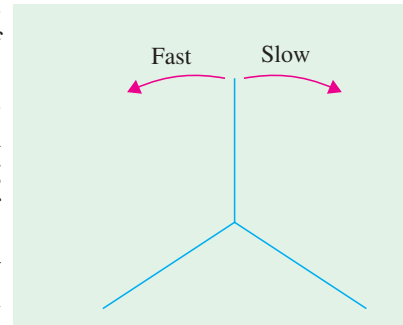


Fig. 37.81

**Example 37.42.** In Fig. 37.74,  $E_1 = 220 \text{ V}$  and  $f_1 = 60 \text{ Hz}$ , whereas  $E_2 = 222 \text{ V}$  and  $f_2 = 59 \text{ Hz}$ . With the switch open; calculate

- (i) maximum and minimum voltage across each lamp.
- (ii) frequency of voltage across the lamps.
- (iii) peak value of voltage across each lamp.
- (iv) phase relations at the instants maximum and minimum voltages occur.
- (v) the number of maximum light pulsations/minute.

**Solution. (i)**  $E_{\max}/\text{lamp} = (220 + 222)/2 = \mathbf{221 \text{ V}}$

$$E_{\min}/\text{lamp} = (222 - 220)/2 = \mathbf{1.0 \text{ V}}$$

(ii)  $f = (f_1 - f_2) = (60 - 59) = \mathbf{1.0 \text{ Hz}}$

(iii)  $E_{\text{peak}} = 221/0.707 = \mathbf{313 \text{ V}}$

(iv) in-phase and anti-phase respectively in the local circuit.

(v) No. of pulsation/min =  $(60 - 59) \times 60 = \mathbf{60}$ .

### 37.32. Synchronizing Current

Once synchronized properly, two alternators continue to run in synchronism. Any tendency on the part of one to drop out of synchronism is immediately counteracted by the production of a synchronizing torque, which brings it back to synchronism.

When in exact synchronism, the two alternators have equal terminal p.d.'s and are in exact phase opposition, so far as the local circuit (consisting of their armatures) is concerned. Hence, there is no current circulating round the local circuit. As shown in Fig. 37.82 (b) e.m.f.  $E_1$  of machine No. 1 is in exact phase opposition to the e.m.f. of machine No. 2 i.e.  $E_2$ . It should be clearly understood that the two e.m.f.s. are in opposition, so far as their local circuit is concerned but are in the same direction with respect to the external circuit. Hence, there is no resultant voltage (assuming  $E_1 = E_2$  in magnitude) round the local circuit.

But now suppose that due to change in the speed of the governor of second machine,  $E_2$  falls back\* by a phase angle of  $\alpha$  electrical degrees, as shown in Fig. 37.82 (c) (though still  $E_1 = E_2$ ). Now, they have a resultant voltage  $E_r$ , which when acting on the local circuit, circulates a current known as synchronizing current. The value of this current is given by  $I_{SY} = E_r / Z_S$  where  $Z_S$  is the synchronous impedance of the phase windings of both the machines (or of one machine only if it is connected to infinite bus-bars\*\*). The current  $I_{SY}$  lags behind  $E_r$  by an angle  $\theta$  given by  $\tan \theta = X_S / R_a$  where  $X_S$  is the combined synchronous reactance of the two machines and  $R_a$  their armature resistance. Since  $R_a$  is negligibly small,  $\theta$  is almost  $90^\circ$  degrees. So  $I_{SY}$  lags  $E_r$  by  $90^\circ$  and is almost in phase with  $E_1$ . It is seen that  $I_{SY}$  is generating current with respect to machine No. 1 and motoring current with respect to machine No. 2 (remember when the current flows in the same direction as e.m.f., then the alternator acts as a generator, and when it flows in the opposite direction, the machine acts as a motor). This current  $I_{SY}$  sets up a synchronising torque, which tends to retard the generating machine (i.e. No. 1) and accelerate the motoring machine (i.e. No. 2).

Similarly, if  $E_2$  tends to advance in phase [Fig. 37.82 (d)], then  $I_{SY}$ , being generating current for machine No. 2, tends to retard it and being motoring current for machine No. 1 tends to accelerate it. Hence, any departure from synchronism results in the production of a synchronizing current  $I_{SY}$  which sets up synchronizing torque. This re-establishes synchronism between the two machines by retarding the leading machine and by accelerating the lagging one. This current  $I_{SY}$ , it should be noted, is superimposed on the load currents in case the machines are loaded.

### 37.33. Synchronizing Power

Consider Fig. 37.82 (c) where machine No. 1 is generating and supplying the synchronizing power =  $E_1 I_{SY} \cos \phi_1$  which is approximately equal to  $E_1 I_{SY}$  ( $\because \phi_1$  is small). Since  $\phi_1 = (90^\circ - \theta)$ , synchronizing power =  $E_1 I_{SY} \cos \phi_1 = E_1 I_{SY} \cos (90^\circ - \theta) = E_1 I_{SY} \sin \theta \cong E_1 I_{SY}$  because  $\theta \cong 90^\circ$  so that

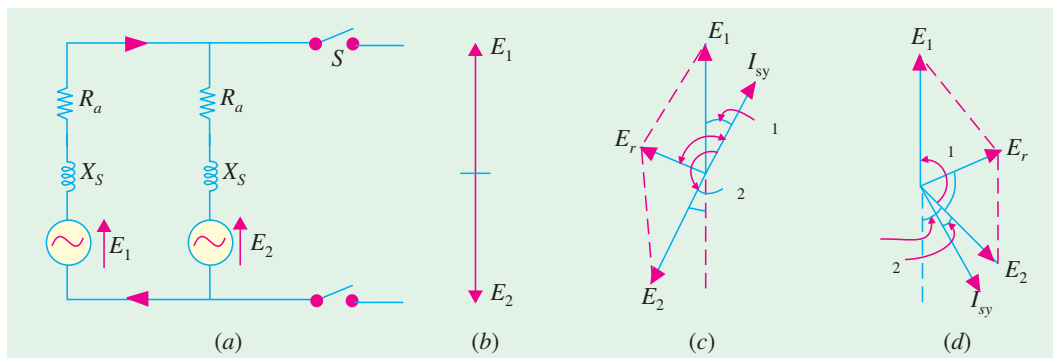


Fig. 37.82

$\sin \theta \cong 1$ . This power output from machine No. 1 goes to supply (a) power input to machine No. 2 (which is motoring) and (b) the Cu losses in the local armature circuit of the two machines. Power input to machine No. 2 is  $E_2 I_{SY} \cos \phi_2$  which is approximately equal to  $E_2 I_{SY}$

$$\therefore E_1 I_{SY} = E_2 I_{SY} + \text{Cu losses}$$

$$\text{Now, let } E_1 = E_2 = E \text{ (say)}$$

$$\text{Then, } E_r = 2 E \cos [(180^\circ - \alpha)/2]^{***} = 2 E \cos [90^\circ - (\alpha/2)]$$

\* Please remember that vectors are supposed to be rotating anticlockwise.

\*\* Infinite bus-bars are those whose frequency and the phase of p.d.'s are not affected by changes in the conditions of any one machine connected in parallel to it. In other words, they are constant-frequency, constant-voltage bus-bars.

\*\*\* Strictly speaking,  $E_r = 2 E \sin \theta$ .  $\sin \alpha/2 \cong 2 E \sin \alpha/2$ .



$$= 2 E \sin \alpha/2 = 2 E \times \alpha/2 = \alpha E \quad (\because \alpha \text{ is small})$$

Here, the angle  $\alpha$  is in electrical radians.

$$\text{Now, } I_{SY} = \frac{E_r}{\text{synch. impedance } Z_S} \cong \frac{E_r}{2 X_S} = \frac{\alpha E}{2 X_S}$$

—if  $R_a$  of both machines is negligible

Here,  $X_S$  represents synchronous reactance of one machine and not of both as in Art. 37.31 Synchronizing power (supplied by machine No. 1) is

$$P_{SY} = E_1 I_{SY} \cos \phi_1 = E I_{SY} \cos (90^\circ - \theta) = E I_{SY} \sin \theta \cong E I_{SY}$$

Substituting the value of  $I_{SY}$  from above,

$$P_{SY} = E \cdot \alpha E / 2 X_S = \alpha E^2 / 2 X_S \cong \alpha E^2 / 2 X_S \quad \text{—per phase}$$

(more accurately,  $P_{SY} = \alpha E^2 \sin \theta / 2 X_S$ )

Total synchronizing power for three phases

$$= 3 P_{SY} = 3 \alpha E^2 / 2 X_S \text{ (or } 3 \alpha E^2 \sin \theta / 2 X_S)$$

This is the value of the synchronizing power when two alternators are connected in parallel and are on no-load.

### 37.34. Alternators Connected to Infinite Bus-bars

Now, consider the case of an alternator which is connected to infinite bus-bars. The expression for  $P_{SY}$  given above is still applicable but with one important difference *i.e.* impedance (or reactance) of only that one alternator is considered (and not of two as done above). Hence, expression for synchronizing power in this case becomes

$$E_r = \alpha E \quad \text{—as before}$$

$$I_{SY} = E_r / Z_S \cong E_r / X_S = \alpha E / X_S \quad \text{—if } R_a \text{ is negligible}$$

$$\therefore \text{ Synchronizing power } P_{SY} = E I_{SY} = E \cdot \alpha E / X_S = \alpha E^2 / X_S \quad \text{— per phase}$$

Now,  $E / X_S \cong E / X_S = \text{S.C. current } I_{SC}$

$$\therefore P_{SY} = \alpha E^2 / X_S = \alpha E \cdot E / X_S = \alpha E \cdot I_{SC} \quad \text{—per phase}$$

(more accurately,  $P_{SY} = \alpha E^2 \sin \theta / X_S = \alpha E \cdot I_{SC} \sin \theta$ )

Total synchronizing power for three phases =  $3 P_{SY}$

### 37.35. Synchronizing Torque $T_{SY}$

Let  $T_{SY}$  be the synchronizing torque per phase in newton-metre (N-m)

**(a) When there are two alternators in parallel**

$$\therefore T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \therefore T_{SY} = \frac{P_{SY}}{2\pi N_S / 60} = \frac{\alpha E^2 / 2 X_S}{2\pi N_S / 60} \text{ N-m}$$

$$\text{Total torque due to three phases.} = \frac{3 P_{SY}}{2\pi N_S / 60} = \frac{3 \alpha E^2 / 2 X_S}{2\pi N_S / 60} \text{ N-m}$$

**(b) Alternator connected to infinite bus-bars**

$$T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \text{ or } T_{SY} = \frac{P_{SY}}{2\pi N_S / 60} = \frac{\alpha E^2 / X_S}{2\pi N_S / 60} \text{ N-m}$$

$$\text{Again, torque due to 3 phase} = \frac{3 P_{SY}}{2\pi N_S / 60} = \frac{3 \alpha E^2 / X_S}{2\pi N_S / 60} \text{ N-m}$$

where  $N_S = \text{synchronous speed in r.p.m.} = 120 / P$

37.36. Effect of Load on Synchronizing Power

In this case, instead of  $P_{SY} = \alpha E^2/X_S$  the approximate value of synchronizing power would be  $\cong \alpha EV/X_S$  where  $V$  is bus-bar voltage and  $E$  is the alternator induced e.m.f. per phase. The value of  $E = V + IZ_S$

As seen from Fig. 37.83, for a lagging p.f.,  
 $E = (V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2)^{1/2}$

**Example 37.43.** Find the power angle when a 1500-kVA, 6.6 kV, 3-phase, Y-connected alternator having a resistance of 0.4 ohm and a reactance of 6 ohm per phase delivers full-load current at normal rated voltage and 0.8 p.f. lag. Draw the phasor diagram.

(Electrical Machinery-II, Bangalore Univ. 1981)

**Solution.** It should be remembered that angle  $\alpha$  between  $V$  and  $E$  is known as power angle (Fig. 37.84)

Full-load  $I = 15 \times 10^3 / \sqrt{3} \times 6600 = 131 \text{ A}$   
 $IR_a = 131 \times 0.4 = 52.4 \text{ V}, IX_S = 131 \times 6 = 786 \text{ V}$

V/phase  $= 6600 / \sqrt{3} = 3810 \text{ V};$   
 $\phi = \cos^{-1}(0.8) = 36^\circ 50'$

As seen from Fig. 37.84

$$\tan(\phi + \alpha) = \frac{AB}{OA} = \frac{V \sin \phi + IX_S}{V \cos \phi + IR_a}$$

$$= \frac{3810 \times 0.6 + 786}{3810 \times 0.8 + 52.4} = 0.991$$

$\therefore (\phi + \alpha) = 44^\circ \therefore \alpha = 44^\circ - 36^\circ 50' = 7^\circ 10'$

The angle  $\alpha$  is also known as load angle or torque angle.

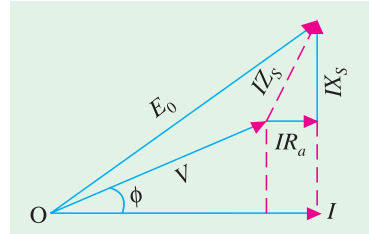


Fig. 37.83

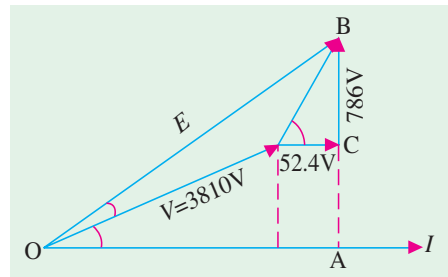


Fig. 37.84

37.37. Alternative Expression for Synchronizing Power

As shown in Fig. 37.85, let  $V$  and  $E$  (or  $E_0$ ) be the terminal voltage and induced e.m.f. per phase of the rotor. Then, taking  $V = V \angle 0^\circ$ , the load current supplied by the alternator is

$$I = \frac{E - V}{Z_S} = \frac{E \angle \alpha - V \angle 0^\circ}{Z_S \angle \theta}$$

$$= \frac{E}{Z_S} \angle \alpha - \theta - \frac{V}{Z_S} \angle -\theta$$

$$= \frac{E}{Z_S} [\cos(\theta - \alpha) - j \sin(\theta - \alpha)]$$

$$= -\frac{V}{Z_S} (\cos \theta - j \sin \theta)$$

$$= \left[ \frac{E}{Z_S} \cos(\theta - \alpha) - \frac{V}{Z_S} \cos \theta \right] - j \left[ \frac{E}{Z_S} \sin(\theta - \alpha) - \frac{V}{Z_S} \sin \theta \right]$$

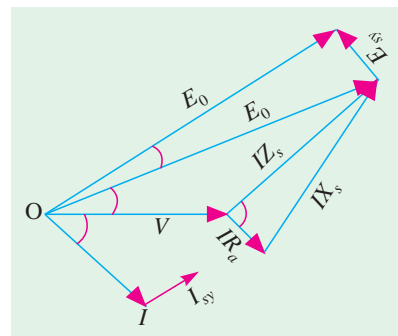


Fig. 37.85

These components represent the  $I \cos \phi$  and  $I \sin \phi$  respectively. The power  $P$  converted internally is given by the sum of the product of corresponding components of the current with  $E \cos \alpha$  and  $E \sin \alpha$ .

$$\begin{aligned} \therefore P &= E \cos \alpha \left[ \frac{E}{Z_S} \cos (\theta - \alpha) - \frac{V}{Z_S} \cos \theta \right] - E \sin \alpha \left[ \frac{E}{Z_S} \sin (\theta - \alpha) - \frac{V}{Z_S} \sin \theta \right] \\ &= E \left[ \frac{E}{Z_S} \cos \theta \right] - E \left[ \frac{V}{Z_S} \cos (\theta + \alpha) \right] = \frac{E}{Z_S} [E \cos \theta - V (\cos \theta + \alpha)] \quad \text{—per phase*} \end{aligned}$$

Now, let, for some reason, angle  $\alpha$  be changed to  $(\alpha \pm \delta)$ . Since  $V$  is held rigidly constant, due to displacement  $\pm \delta$ , an additional e.m.f. of divergence *i.e.*  $I_{SY} = 2E \sin \alpha/2$  will be produced, which will set up an additional current  $I_{SY}$  given by  $I_{SY} = E_{SY}/Z_S$ . The internal power will become

$$P' = \frac{E}{Z_S} [E \cos \theta - V \cos (\theta + \alpha \pm \delta)]$$

The difference between  $P'$  and  $P$  gives the synchronizing power.

$$\begin{aligned} \therefore P_{SY} &= P' - P = \frac{EV}{Z_S} [\cos (\theta + \alpha) - \cos (\theta + \alpha \pm \delta)] \\ &= \frac{EV}{Z_S} [\sin \delta \cdot \sin (\theta + \alpha) \pm 2 \cos (\theta + \alpha) \sin^2 \delta/2] \end{aligned}$$

If  $\delta$  is very small, then  $\sin^2 (\delta/2)$  is zero, hence  $P_{SY}$  per phase is

$$P_{SY} = \frac{EV}{Z_S} \cdot \sin (\theta + \alpha) \sin \delta \quad \dots(ii)$$

(i) In large alternators,  $R_a$  is negligible, hence  $\tan \theta = X_S/R_a = \infty$ , so that  $\theta \cong 90^\circ$ . Therefore,  $\sin (\theta + \alpha) = \cos \alpha$ .

$$\therefore P_{SY} = \frac{EV}{Z_S} \cdot \cos \alpha \sin \delta \quad \text{— per phase} \quad \dots(ii)$$

$$= \frac{EV}{X_S} \cos \alpha \sin \delta \quad \text{—per phase} \quad \dots(iii)$$

(ii) Consider the case of synchronizing an unloaded machine on to a constant-voltage bus-bars. For proper operation,  $\alpha = 0$  so that  $E$  coincides with  $V$ . In that case,  $\sin (\theta + \alpha) = \sin \theta$ .

$$\therefore P_{SY} = \frac{EV}{Z_S} \sin \theta \sin \delta \quad \text{—from (i) above.}$$

Since  $\delta$  is very small,  $\sin \delta = \delta$ ,

$$\therefore P_{SY} = \frac{EV}{Z_S} \delta \sin \theta = \frac{EV}{X_S} \delta \sin \theta \quad \text{Usually, } \sin \theta \cong 1, \text{ hence}$$

$$\therefore P_{SY} = \frac{EV}{Z_S} \cdot \delta^{**} = V \left( \frac{E}{Z_S} \right) \delta = V \left( \frac{E}{X_S} \right) \delta = VI_{SC} \cdot \delta \quad \text{—per phase}$$

### 37.38. Parallel Operation of Two Alternators

Consider two alternators with identical speed/load characteristics connected in parallel as shown in Fig. 37.86. The common terminal voltage  $V$  is given by

$$V = E_1 - I_1 Z_1 = E_2 - I_2 Z_2$$

$$\therefore E_1 - E_2 = I_1 Z_1 - I_2 Z_2$$

Also  $I = I_1 + I_2$  and  $V = IZ$

$$\therefore E_1 = I_1 Z_1 + IZ = I_1 (Z + Z_1) + I_2 Z$$

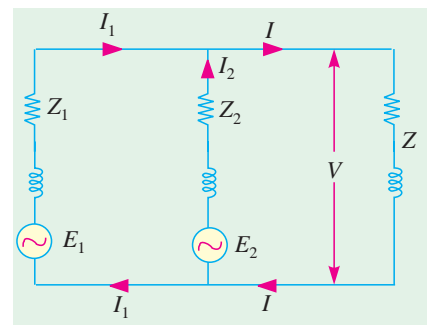


Fig. 37.86

\* In large machines,  $R_a$  is very small so that  $\theta = 90^\circ$ , hence  $P = \frac{E}{Z_S} V \cos(90^\circ \alpha) = \frac{E}{Z_S} V \sin \alpha = \alpha EV / Z_S$   
—if  $\alpha$  is so small that  $\sin \alpha = \alpha$

\*\* With  $E = V$ , the expression becomes  $P_{SY} = \frac{V^2}{Z_S} \delta = \frac{\delta V^2}{X_S}$  It is the same as in Art. 37.33 —per phase

$$\begin{aligned}
 E_2 &= I_2 Z_2 + IZ = I_2(Z_1 + Z_2) + I_1 Z \\
 \therefore I_1 &= \frac{(E_1 - E_2)Z + E_1 Z_2}{Z(Z_1 + Z_2) + Z_1 Z_2} \\
 I_2 &= \frac{(E_2 - E_1)Z + E_2 Z_1}{Z(Z_1 + Z_2) + Z_1 Z_2}; \\
 I &= \frac{E_1 Z_2 + E_2 Z_1}{Z(Z_1 + Z_2) + Z_1 Z_2} \\
 V &= IZ = \frac{E_1 Z_2 + E_2 Z_1}{Z_1 + Z_2 + (Z_1 Z_2/Z)}; I_1 = \frac{E_1 - V}{Z_1}; I_2 = \frac{E_2 - V}{Z_2}
 \end{aligned}$$

The circulating current under no-load condition is  $I_C = (E_1 - E_2)/(Z_1 + Z_2)$ .

#### Using Admittances

The terminal Voltage may also be expressed in terms of admittances as shown below:

$$V = IZ = (I_1 + I_2)Z \quad \therefore I_1 + I_2 = V/Z = VY \quad \dots (i)$$

$$\text{Also } I_1 = (E_1 - V)/Z_1 = (E_1 - V)Y_1; \quad I_2 = (E_2 - V)/Z_2 = (E_2 - V)Y_2$$

$$\therefore I_1 + I_2 = (E_1 - V)Y_1 + (E_2 - V)Y_2 \quad \dots (ii)$$

From Eq. (i) and (ii), we get

$$VY = (E_1 - V)Y_1 + (E_2 - V)Y_2 \quad \text{or} \quad V = \frac{E_1 Y_1 + E_2 Y_2}{Y_1 + Y_2 + Y}$$

#### Using Parallel Generator Theorem

$$\begin{aligned}
 V &= IZ = (I_1 + I_2)Z = \left( \frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2} \right) Z \\
 &= \left( \frac{E_1}{Z_1} + \frac{E_2}{Z_2} \right) Z - V \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) Z
 \end{aligned}$$

$$\therefore V \left( \frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2} \right) = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} = I_{SC1} + I_{SC2} = I_{SC}$$

where  $I_{SC1}$  and  $I_{SC2}$  are the short-circuit currents of the two alternators.

$$\text{If } \frac{1}{Z_0} = \left( \frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2} \right); \text{ then } V \times \frac{1}{Z_0} = I_{SC} \text{ or } V = Z_0 I_{SC}$$

**Example 37.44.** A 3,000-kVA, 6-pole alternator runs at 1000 r.p.m. in parallel with other machines on 3,300-V bus-bars. The synchronous reactance is 25%. Calculate the synchronizing power for one mechanical degree of displacement and the corresponding synchronizing torque.

(Elect. Machines-I, Gwalior Univ. 1984)

**Solution.** It may please be noted that here the alternator is working in parallel with many alternators. Hence, it may be considered to be connected to infinite bus-bars.

$$\text{Voltage/phase} = 3,300/\sqrt{3} = 1905 \text{ V}$$

$$\text{F.L. current} \quad I = 3 \times 10^6 / \sqrt{3} \times 3300 = 525 \text{ A}$$

$$\text{Now,} \quad IX_S = 25\% \text{ of } 1905 \quad \therefore X_S = 0.25 \times 1905/525 = 0.9075 \Omega$$

$$\text{Also,} \quad P_{SY} = 3 \times \alpha E^2 / X_S$$

$$\text{Here} \quad \alpha = 1^\circ \text{ (mech.); } \alpha \text{ (elect.)} = 1 \times (6/2) = 3^\circ$$

$$\therefore \quad \alpha = 3 \times \pi/180 = \pi/60 \text{ elect. radian.}$$

$$\therefore P_{SY} = \frac{3 \times \pi \times 1905^2}{60 \times 0.9075 \times 1000} = 628.4 \text{ kW}$$

$$T_{SY} = \frac{60 \cdot P_{SY}}{2\pi N_S} = 9.55 \frac{P_{SY}}{N_S} = 9.55 \frac{628.4 \times 10^3}{1000} = \mathbf{6,000 \text{ N-m}}$$

**Example 37.45.** A 3-MVA, 6-pole alternator runs at 1000 r.p.m on 3.3-kV bus-bars. The synchronous reactance is 25 percent. Calculate the synchronising power and torque per mechanical degree of displacement when the alternator is supplying full-load at 0.8 lag.

(Electrical Machines-1, Bombay Univ. 1987)

**Solution.**  $V = 3,300/\sqrt{3} = 1905 \text{ V/phase}$ , F.L.  $I = 3 \times 10^6/\sqrt{3} \times 3,300 = 525 \text{ A}$   
 $IX_S = 25\% \text{ of } 1905 = 476 \text{ V}$ ;  $X_S = 476/525 = 0.9075 \Omega$   
 Let,  $\mathbf{I} = 525 \angle 0^\circ$ , then,  $\mathbf{V} = 1905 (0.8 + j0.6) = 1524 + j1143$   
 $\mathbf{E}_0 = \mathbf{V} + \mathbf{IX}_S = (1524 + j1143) + (0 + j476) = (1524 + j1619) = 2220 \angle 46^\circ 44'$   
 Obviously,  $E_0$  leads  $I$  by  $46^\circ 44'$ . However,  $V$  leads  $I$  by  $\cos^{-1}(0.8) = 36^\circ 50'$ .  
 Hence,  $\alpha = 46^\circ 44' - 36^\circ 50' = 9^\circ 54'$   
 $\alpha = 1^\circ$  (mech.), No. of pair of poles =  $6/2 = 3 \therefore \alpha = 1 \times 3 = 3^\circ$  (elect.)

$$P_{SY} \text{ per phase} = \frac{EV}{X_S} \cos \alpha \sin \delta = \frac{2220 \times 1905}{0.9075} \times \cos 9^\circ 54' \sin 3^\circ = 218 \text{ kW}$$

$$P_{SY} \text{ for three phases} = 3 \times 218 = \mathbf{654 \text{ kW}}$$

$$T_{SY} = 9.55 \times P_{SY}/N_S = 9.55 \times 654 \times 10^2/1000 = 6245 \text{ N-m}$$

**Example 37.46.** A 750-kVA, 11-kV, 4-pole, 3- $\phi$ , star-connected alternator has percentage resistance and reactance of 1 and 15 respectively. Calculate the synchronising power per mechanical degree of displacement at (a) no-load (b) at full-load 0.8 p.f. lag. The terminal voltage in each case is 11 kV.  
 (Electrical Machines-II, Indore Univ. 1985)

**Solution.** F.L. Current  $I = 75 \times 10^3/\sqrt{3} \times 11 \times 10^3 = 40 \text{ A}$   
 $V_{ph} = 11,000/\sqrt{3} = 6,350 \text{ V}$ ,  $IR_a = 1\% \text{ of } 6,350 = 63.5$   
 or  $40 R_a = 63.5$ ,  $R_a = 1.6 \Omega$ ;  $40 \times X_S = 15\% \text{ of } 6,350 = 952.5 \text{ V}$   
 $\therefore X_S = 23.8 \Omega$ ;  $Z_S = \sqrt{1.6^2 + 23.8^2} \cong 23.8 \Omega$

(a) No-load

$$\alpha \text{ (mech)} = 1^\circ : \alpha \text{ (elect)} = 1 \times (4/2) = 2^\circ$$

$$= 2 \times \pi/180 = \pi/90 \text{ elect. radian.}$$

$$P_{SY} = \frac{\alpha E^2}{Z_S} \cong \frac{\alpha E^2}{X_S} = \frac{(\pi/90) \times 6350^2}{23.8}$$

$$= 59,140 \text{ W} = \mathbf{59.14 \text{ kW/phase.}}$$

On no-load,  $V$  has been taken to be equal to  $E$ .

(b) F.L. 0.8 p.f.

As indicated in Art. 37.35,  $P_{SY} = \alpha EV/X_S$ . The value of  $E$  (or  $E_0$ ) can be found from Fig. 37.87.

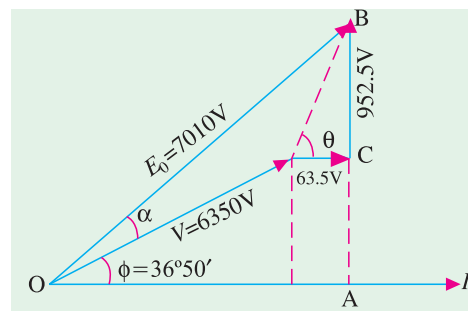


Fig. 37.87

$$E = [(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$$

$$= [(6350 \times 0.8 + 63.5)^2 + (6350 \times 0.6 + 952.5)^2]^{1/2} = 7010 \text{ V}$$

$$P_{SY} = \frac{\alpha EV}{X_S} = \frac{(\pi/90) \times 7010 \times 6350}{23.8} = 65,290 \text{ W}$$

$$= \mathbf{65.29 \text{ kW/phase}}$$

**More Accurate Method [Art. 37.35]**

$$P_{SY} = \frac{EV}{X_S} \cos \alpha \sin \delta$$

Now,  $E = 7010 \text{ V}, V = 6350 \text{ V}, \delta = 1^\circ \times (4/2) = 2^\circ$  (elect)

As seen from Fig. 37.87,  $\sin(\phi + \alpha) = AB/OB = (6350 \times 0.6 + 952.5)/7010 = 0.6794$

$$\therefore (\phi + \alpha) = 42^\circ 30'; \alpha = 42^\circ 30' - 36^\circ 50' = 5^\circ 40'$$

$$\begin{aligned} \therefore P_{SY} &= \frac{7010 \times 6350}{23.8} \times \cos 5^\circ 40' \times \sin 2^\circ \\ &= 7010 \times 6350 \times 0.9953 \times 0.0349/23.8 = 64,970 \text{ W} = \mathbf{64.97 \text{ kW/phase}} \end{aligned}$$

**Note.** It would be instructive to link this example with Ex. 38.1 since both are concerned with synchronous machines, one generating and the other motoring.

**Example 37.47.** A 2,000-kVA, 3-phase, 8-pole alternator runs at 750 r.p.m. in parallel with other machines on 6,000 V bus-bars. Find synchronizing power on full-load 0.8 p.f. lagging per mechanical degree of displacement and the corresponding synchronizing torque. The synchronous reactance is 6 ohm per phase. (Elect. Machines-II, Bombay Univ. 1987)

**Solution. Approximate Method**

As seen from Art. 37.37 and 38,  $P_{SY} = \alpha EV/X_S$  —per phase

Now  $\alpha = 1^\circ$  (mech); No. of pair of poles =  $8/2 = 4$ ;

$$\alpha = 1 \times 4 = 4^\circ \text{ (elect)}$$

$$= 4\pi/180 = \pi/45 \text{ elect. radian}$$

$$V = 6000/\sqrt{3} = 3,465 \text{ —assuming Y-connection}$$

$$\text{F.L. current } I = 2000 \times 10^3 / \sqrt{3} \times 6000 = 192.4 \text{ A}$$

As seen from Fig. 37.88,

$$\begin{aligned} E_0 &= [(V \cos \phi)^2 + (V \sin \phi + IX_S)^2]^{1/2} = 4295 \text{ V} \\ &= [(3465 \times 0.8)^2 + (3465 \times 0.6 + 192.4 \times 6)^2]^{1/2} \\ &= 4295 \text{ V} \end{aligned}$$

$$\begin{aligned} P_{SY} &= (\pi/45) \times 4295 \times 3465/6 = 173,160 \text{ W} \\ &= \mathbf{173.16 \text{ kW/phase}} \end{aligned}$$

$$P_{SY} \text{ for three phases} = 3 \times 173.16 = 519.5 \text{ kW}$$

If  $T_{SY}$  is the total synchronizing torque for three phases in N-m, then

$$T_{SY} = 9.55 P_{SY}/N_S = 9.55 \times 519,500/750 = \mathbf{6,614 \text{ N-m}}$$

**Exact Method**

As shown in the vector diagram of Fig. 37.89, I is full-load current lagging V by  $\phi = \cos^{-1}(0.8) = 36^\circ 50'$ . The reactance drop is  $IX_S$  and its vector is at right angles to (lag.)\*. The phase angle between  $E_0$  and V is  $\alpha$ .

$$\begin{aligned} \text{F.L. current } I &= 2,000,000 / \sqrt{3} \times 6,000 \\ &= 192.4 \text{ A} \end{aligned}$$

$$\text{Let, } I = 192.4 \angle 0^\circ$$

$$V = 3,465 (0.8 + j 0.6) = 2,772 + j 2,079$$

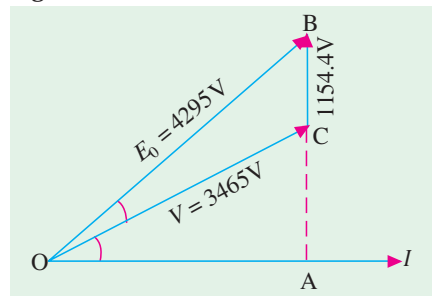


Fig. 37.88

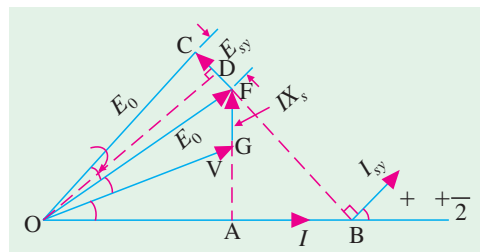


Fig. 37.89

\* Earlier, we had called this e.m.f. as E when discussing regulation.

$$IX_S = 192.4 \times 6 = 1154 \text{ V} = (0 + j1154) \text{ V}$$

$$\mathbf{E}_0 = \mathbf{V} + \mathbf{IX}_S$$

$$= (2,772 + j2,079) + (0 + j1154)$$

$$= 2,772 + j3,233 = 4,259 \angle 49^\circ 24'$$

$$\alpha = 49^\circ 24' - 36^\circ 50' = 12^\circ 34'$$

$$E_{SY} = 2E_0 \sin \delta/2 = 2E_0 \sin (4^\circ/2)$$

$$= 2 \times 4,259 \times 0.0349 = 297.3 \text{ V}$$

$$I_{SY} = 297.3/6 = 49.55 \text{ A}$$

As seen,  $V$  leads  $I$  by  $\phi$  and  $I_{SY}$  leads  $I$  by  $(\phi + \alpha + \delta/2)$ , hence  $I_{SY}$  leads  $V$  by  $(\alpha + \delta/2) = 12^\circ 34' + (4^\circ/2) = 14^\circ 34'$ .

$$\therefore P_{SY}/\text{phase} = VI_{SY} \cos 14^\circ 34' = 3465 \times 49.55 \times \cos 14^\circ 34' = 166,200 \text{ W} = 166.2 \text{ kW}$$

Synchronising power for three phases is  $= 3 \times 166.2 = \mathbf{498.6 \text{ kW}}$

If  $T_{SY}$  is the total synchronizing torque, then  $T_{SY} \times 2\pi \times 750/60 = 498,600$

$$\therefore T_{SY} = 9.55 \times 498,600/750 = \mathbf{6,348 \text{ N-m}}$$

#### Alternative Method

We may use Eq. (iii) of Art. 37.36 to find the total synchronizing power.

$$P_{SY} = \frac{EV}{X_S} \cos \alpha \sin \delta \quad \text{—per phase}$$

Here,  $E = 4,259 \text{ V}$ ;  $V = 3,465 \text{ V}$ ;  $\alpha = 12^\circ 34'$ ;  $\delta = 4^\circ$  (elect.)

$$\therefore P_{SY}/\text{phase} = 4,259 \times 3,465 \cos 12^\circ 34' \times \sin 4^\circ/6$$

$$= 4,259 \times 3,465 \times 0.976 \times 0.0698/6 = 167,500 \text{ W} = 167.5 \text{ kW}$$

$$P_{SY} \text{ for 3 phases} = 3 \times 167.5 = \mathbf{502.5 \text{ kW}}$$

Next,  $T_{SY}$  may be found as above.

**Example 37.48.** A 5,000-kV A, 10,000 V, 1500-r.p.m., 50-Hz alternator runs in parallel with other machines. Its synchronous reactance is 20%. Find for (a) no-load (b) full-load at power factor 0.8 lagging, synchronizing power per unit mechanical angle of phase displacement and calculate the synchronizing torque, if the mechanical displacement is  $0.5^\circ$ .

(Elect. Engg. V, M.S. Univ. Baroda, 1986)

**Solution.** Voltage / phase  $= 10,000 / \sqrt{3} = 5,775 \text{ V}$

Full-load current  $= 5,000,000 / \sqrt{3} \times 10,000 = 288.7 \text{ A}$

$$X_S = \frac{20}{100} \times \frac{5,775}{288.7} = 4 \Omega, P = \frac{120 f}{N_S} = \frac{120 \times 50}{1500} = 4$$

$$\alpha = 1^\circ \text{ (mech.)}; \text{ No. of pair of poles} = 2 \quad \therefore \alpha = 1 \times 2 = 2^\circ \text{ (elect.)} = 2\pi/180 = \pi/90 \text{ radian}$$

#### (a) At no-load

$$P_{SY} = \frac{3 \alpha E^2}{X_S} = 3 \times \frac{\pi}{90} \times \frac{5,775^2}{4 \times 1000} = \mathbf{873.4 \text{ kW}}$$

$$\therefore T_{SY} = 9.55 \times (873.4 \times 10^3) / 1500 = 5,564 \text{ N-m}$$

$$\therefore T_{SY} \text{ for } 0.5^\circ = 5564/2 = \mathbf{2,782 \text{ N-m}}$$

#### (b) At F.L. p.f. 0.8 lagging

Let  $\mathbf{I} = 288.7 \angle 0^\circ$ . Then  $\mathbf{V} = 5775 (0.8 + j0.6) = 4620 + j3465$

$$\mathbf{I.X}_S = 288.7 \angle 0^\circ \times 4 \angle 90^\circ = (0 + j1155)$$

$$\mathbf{E}_0 = \mathbf{V} + \mathbf{IX}_S = (4620 + j3465) + (0 + j1155)$$

$$= 4620 + j4620 = 6533 \angle 45^\circ$$

$$\cos \phi = 0.8, \phi = \cos^{-1}(0.8) = 36^\circ 50'$$

Now,  $E_0$  leads  $I$  by  $45^\circ$  and  $V$  leads  $I$  by  $36^\circ 50'$ . Hence,  $E_0$  leads  $V$  by  $(45^\circ - 36^\circ 50') = 8^\circ 10'$  i.e.  $\alpha = 8^\circ 10'$ . As before,  $\delta = 2^\circ$  (elect).

As seen from Art. 37.36,  $P_{SY} = \frac{EV}{X_s} \cos \alpha \sin \delta$  —per phase

$$= 6533 \times 5775 \times \cos 8^\circ 10' \times \sin 2^\circ / 4 = 326 \text{ kW}$$

$$P_{SY} \text{ for three phases} = 3 \times 326 = 978 \text{ kW}$$

$$T_{SY} / \text{unit displacement} = 9.55 \times 978 \times 10^3 / 1500 = 6,237 \text{ N-m}$$

$$T_{SY} \text{ for } 0.5^\circ \text{ displacement} = 6,237 / 2 = \mathbf{3118.5 \text{ N-m}}$$

(c) We could also use the approximate expression of Art. 37.36

$$P_{SY} \text{ per phase} = \alpha E V / X_s = (\pi / 90) \times 6533 \times 5775 / 4 = \mathbf{329.3 \text{ kW}}$$

**Example 37.49.** Two 3-phase, 6.6-kW, star-connected alternators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of machine A is  $(0.5 + j10) \Omega$  and of machine B is  $(0.4 + j12) \Omega$ . The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor and the governors are so set that load is shared equally between the machines.

Determine the current, power factor, induced e.m.f. and load angle of each machine.

(Electrical Machines-II, South Gujarat Univ. 1985)

**Solution.** It is given that each machine carries a load of 1500 kW. Also,  $V = 6600 / \sqrt{3} = 3810 \text{ V}$ . Let  $V = 3810 \angle 0^\circ = (3810 + j0)$ .

**For machine No. 1**

$$\sqrt{3} / 6600 \times 150 \times \cos \phi_1 = 1500 \times 10^3;$$

$$\cos \phi_1 = 0.874, \phi_1 = 29^\circ; \sin \phi_1 = 0.485$$

$$\text{Total current } I = 3000 / \sqrt{3} \times 6.6 \times 0.8 = 328 \text{ A}$$

$$\text{or } I = 828 (0.8 - j0.6) = 262 - j195$$

$$\text{Now, } \mathbf{I_1} = 150 (0.874 - j0.485) = 131 - j72.6$$

$$\therefore \mathbf{I_2} = (262 - j195) - (131 - j72.6)$$

$$= (131 - j124.4)$$

$$\text{or } I_2 = \mathbf{181 \text{ A}}, \cos \phi_2 = 131/181 = \mathbf{0.723 \text{ (lag)}}.$$

$$\mathbf{E_A} = \mathbf{V} + \mathbf{I_1 Z_1} = 3810 + (131 - j72.6) (0.5 + j10)$$

$$= 4600 + j1270$$

$$\text{Line value of e.m.f.} = \sqrt{3} \sqrt{(4600^2 + 1270^2)} = \mathbf{8,260 \text{ V}}$$

$$\text{Load angle } \alpha_1 = (1270/4600) = \mathbf{15.4^\circ}$$

$$\mathbf{E_B} = \mathbf{V} + \mathbf{I_2 Z_2} = 3810 + (131 - j124.4) (0.4 + j12)$$

$$= 5350 + j1520$$

$$\text{Line value of e.m.f.} = \sqrt{3} \sqrt{5350^2 + 1520^2} = \mathbf{9600 \text{ V}}$$

$$\text{Load angle } \alpha_2 = \tan^{-1} (1520/5350) = \mathbf{15.9^\circ}$$

**Example 37.50.** Two single-phase alternator operating in parallel have induced e.m.fs on open circuit of  $230 \angle 0^\circ$  and  $230 \angle 10^\circ$  volts and respective reactances of  $j2 \Omega$  and  $j3 \Omega$ . Calculate (i) terminal voltage (ii) currents and (iii) power delivered by each of the alternators to a load of impedance  $6 \Omega$  (resistive). (Electrical Machines-II, Indore Univ. 1987)

**Solution.** Here,  $\mathbf{Z_1} = j2, \mathbf{Z_2} = j3, \mathbf{Z} = 6; \mathbf{E_1} = 230 \angle 0^\circ$  and

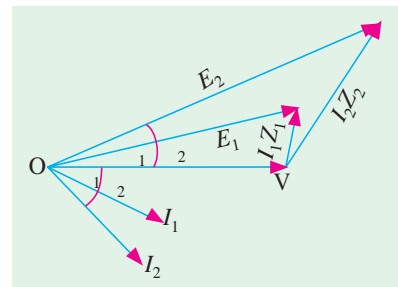


Fig. 37.90



$$E_2 = 230 \angle 10^\circ = 230 (0.985 + j0.174) = (226.5 + j39.9), \text{ as in Fig. 37.90}$$

$$(ii) \ I_1 = \frac{(E_1 - E_2) Z + E_1 Z_2}{Z(Z_1 + Z_2) + Z_1 Z_2} = \frac{[(230 + j0) - (226.5 + j39.9)] \times 6 + 230 \times j3}{6(j2 - j3) + j2 \times j3}$$

$$= 14.3 - j3.56 = 14.73 \angle -14^\circ \quad \text{---Art. 37.38}$$

$$I_2 = \frac{(E_2 - E_1) Z + E_2 Z_1}{Z(Z_1 + Z_2) + Z_1 Z_2} = \frac{(-3.5 + j39.9) + (222.5 + j39.9) \times j2}{6(j2 + j3) + j2 \times j3}$$

$$= 22.6 - j1.15 = 22.63 \angle -3.4^\circ$$

$$(i) \ I = I_1 + I_2 = 36.9 - j4.71 = 37.2 \angle -7.3^\circ$$

$$V = IZ = (36.9 - j4.71) \times 6 = 221.4 - j28.3 = 223.2 \angle -7.3^\circ$$

$$(iii) \ P_1 = VI_1 \cos \phi_1 = 223.2 \times 14.73 \times \cos 14^\circ = 3190 \text{ W}$$

$$P_2 = VI_2 \cos \phi_1 = 223.2 \times 22.63 \times \cos 3.4^\circ = 5040 \text{ W}$$

**Tutorial Problem No. 37.6.**

1. Calculate the synchronizing torque for unit mechanical angle of phase displacement for a 5,000-kVA, 3-φ alternator running at 1,500 r.p.m. when connected to 6,600-volt, 50-Hz bus-bars. The armature has a short-circuit reactance of 15%. **[43,370 kg-m] (City & Guilds, London)**
2. Calculate the synchronizing torque for one mechanical degree of phase displacement in a 6,000-kVA, 50-Hz, alternator when running at 1,500 r.p.m with a generated e.m.f. of 10,000 volt. The machine has a synchronous impedance of 25%. **[544 kg.m] (Electrical Engineering-III, Madras Univ. April 1978; Osmania Univ. May 1976)**
3. A 10,000-kVA, 6,600-V, 16-pole, 50-Hz, 3-phase alternator has a synchronous reactance of 15%. Calculate the synchronous power per mechanical degree of phase displacement from the full load position at power factor 0.8 lagging. **[10 MW] (Elect.Machines-I, Gwalior Univ. 1977)**
4. A 6.6 kV, 3-phase, star-connected turbo-alternator of synchronous reactance 0.5 ohm/phase is applying 40 MVA at 0.8 lagging p.f. to a large system. If the steam supply is suddenly cut off, explain what takes place and determine the current the machine will then carry. Neglect losses. **[2100 A] (Elect. Machines (E-3) AMIE Sec. B Summer 1990)**
5. A 3-phase 400 kVA, 6.6 kV, 1500 rpm., 50 Hz alternator is running in parallel with infinite bus bars. Its synchronous reactance is 25%. Calculate (i) for no load (ii) full load 0.8 p.f. lagging the synchronizing power and torque per unit mechanical angle of displacement. **[Rajive Gandhi Technical University, 2000] [(i) 55.82 kW, 355 Nw-m (ii) 64.2 kW, 409 Nw-m]**

**37.39. Effect of Unequal Voltages**

Let us consider two alternators, which are running exactly in-phase (relative to the external circuit) but which have slightly unequal voltages, as shown in Fig. 37.91. If  $E_1$  is greater than  $E_2$ , then their resultant is  $E_r = (E_1 - E_2)$  and is in-phase with  $E_1$ . This  $E_r$  or  $E_{SY}$  set up a local synchronizing current  $I_{SY}$  which (as discussed earlier) is almost  $90^\circ$  behind  $E_{SY}$  and hence behind  $E_1$  also. This lagging current produces demagnetising effect (Art. 37.16) on the first machine, hence  $E_1$  is reduced. The other machine runs as a synchronous motor, taking almost  $90^\circ$  leading current. Hence, its field is strengthened due to magnetising effect of armature reaction (Art. 37.16). This tends to increase  $E_2$ . These two effects act together and hence lessen the inequalities between the two voltages and tend to establish stable conditions.

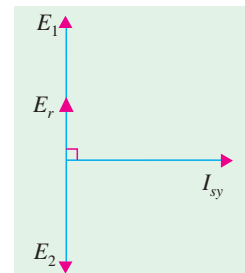


Fig. 37.91

**37.40. Distribution of Load**

It will, now be shown that the amount of load taken up by an alternator running, in parallel with other

machines, is solely determined by its driving torque *i.e.* by the power input to its prime mover (by giving it more or less steam, in the case of steam drive). Any alternation in its excitation merely changes its kVA output, but not its kW output. In other words, it merely changes the power factor at which the load is delivered.

**(a) Effect of Change in Excitation**

Suppose the initial operating conditions of the two parallel alternators are identical *i.e.* each alternator supplies one half of the active load (kW) and one-half of the reactive load (kVAR), the operating power factors thus being equal to the load p.f. In other words, both active and reactive powers are divided equally thereby giving equal apparent power triangles for the two machines as shown in Fig. 37.92 (b). As shown in Fig. 37.92 (a), each alternator supplies a load current  $I$  so that total output current is  $2I$ .

Now, let excitation of alternator No. 1 be increased, so that  $E_1$  becomes greater than  $E_2$ . The difference between the two e.m.fs. sets up a circulating current  $I_C = I_{SY} = (E_1 - E_2)/2Z_S$  which is confined to the local path through the armatures and round the bus-bars. This current is superimposed on the original current distribution. As seen,  $I_C$  is vectorially added to the load current of alternator No. 1 and subtracted from that of No. 2. The two machines now deliver load currents  $I_1$  and  $I_2$  at respective power factors of  $\cos \phi_1$  and  $\cos \phi_2$ . These changes in load currents lead to changes in power factors, such that  $\cos \phi_1$  is reduced, whereas  $\cos \phi_2$  is increased. However, effect on the

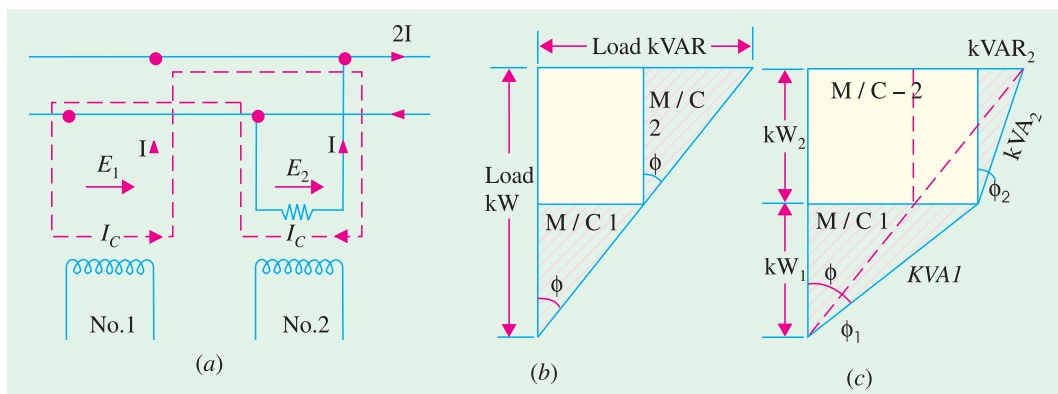


Fig. 37.92

kW loading of the two alternators is negligible, but  $kVAR_1$  supplied by alternator No. 1 is increased, whereas  $kVAR_2$  supplied by alternator No. 2 is correspondingly decreased, as shown by the kVA triangles of Fig. 37.92 (c).

**(b) Effect of Change in Steam Supply**

Now, suppose that excitations of the two alternators are kept the same but steam supply to alternator No. 1 is increased *i.e.* power input to its prime mover is increased. Since the speeds of the two machines are tied together by their synchronous bond, machine No. 1 cannot overrun machine No. 2. Alternatively, it utilizes its increased power input for carrying

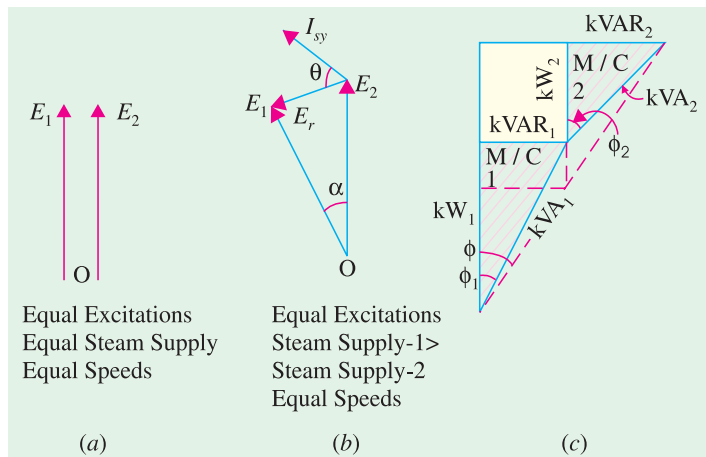


Fig. 37.93

more load than No. 2. This can be made possible only when rotor No. 1 advances its angular position with respect to No. 2 as shown in Fig. 37.93 (b) where  $E_1$  is shown advanced ahead of  $E_2$  by an angle  $\alpha$ . Consequently, resultant voltage  $E_r$  (or  $E_{sy}$ ) is produced which, acting on the local circuit, sets up a current  $I_{sy}$  which lags by almost  $90^\circ$  behind  $E_r$  but is almost in phase with  $E_1$  (so long as angle  $\alpha$  is small). Hence, power per phase of No. 1 is increased by an amount  $= E_1 I_{sy}$  whereas that of No. 2 is decreased by the same amount (assuming total load power demand to remain unchanged). Since  $I_{sy}$  has no appreciable reactive (or quadrature) component, the increase in steam supply does not disturb the division of reactive powers, but it increases the active power output of alternator No. 1 and decreases that of No. 2. Load division, when steam supply to alternator No. 1 is increased, is shown in Fig. 37.93 (c).

So, it is found that by increasing the input to its prime mover, an alternator can be made to take a greater share of the load, though at a different power factor.

The points worth remembering are :

1. The load taken up by an alternators directly depends upon its driving torque or in other words, upon the angular advance of its rotor.
2. The excitation merely changes the p.f. at which the load is delivered without affecting the load so long as steam supply remains unchanged.
3. If input to the prime mover of an alternator is kept constant, but its excitation is changed, then kVA component of its output is changed, not kW.

**Example 37.51.** Two identical 3-phase alternators work in parallel and supply a total load of 1,500 kW at 11 kV at a power factor of 0.867 lagging. Each machine supplies half the total power. The synchronous reactance of each is  $50 \Omega$  per phase and the resistance is  $4 \Omega$  per phase. The field excitation of the first machine is so adjusted that its armature current is 50 A lagging. Determine the armature current of the second alternator and the generated voltage of the first machine.

(Elect. Technology, Utkal Univ. 1983)

**Solution.** Load current at 0.867 p.f. lagging is

$$= \frac{1,500 \times 1,000}{\sqrt{3} \times 11,000 \times 0.867} = 90.4 \text{ A}; \cos \phi = 0.867; \sin \phi = 0.4985$$

Wattful component of the current =  $90.4 \times 0.867 = 78.5 \text{ A}$

Wattless component of the current =  $90.4 \times 0.4985 = 45.2 \text{ A}$

Each alternator supplies half of each of the above two component when conditions are identical (Fig. 37.94).

Current supplied by each machine =  $90.4/2 = 45.2 \text{ A}$

Since the steam supply of first machine is not changed, the working components of both machines would remain the same at  $78.5/2 = 39.25 \text{ A}$ . But the wattless or reactive components would be redivided due to change in excitation. The armature current of the first machine is changed from 45.2 A to 50 A.

$$\therefore \text{ Wattless component of the 1st machine} = \sqrt{50^2 - 39.25^2} = 31 \text{ A}$$

$$\text{ Wattless component of the 2nd machine} = 45.2 - 31 = 14.1 \text{ A}$$

The new current diagram is shown in Fig. 37.95 (a)

$$(i) \text{ Armature current of the 2nd alternator, } I_2 = \sqrt{39.25^2 + 14.1^2} = 41.75 \text{ A}$$

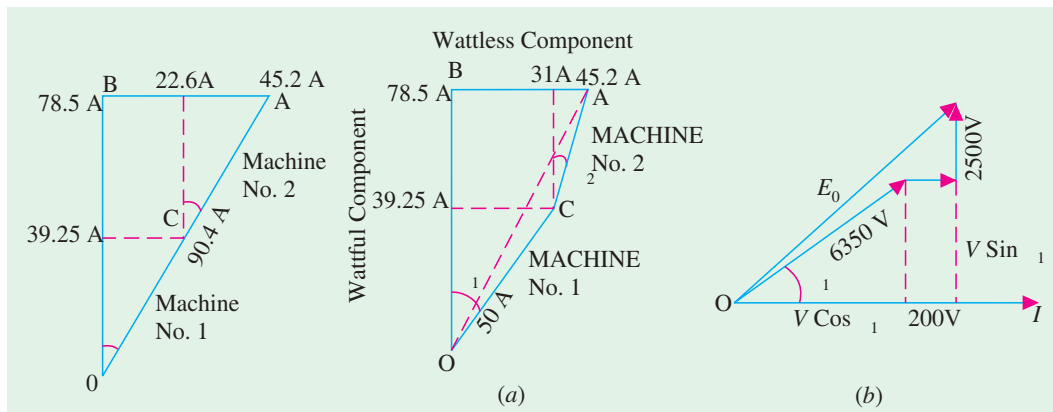


Fig. 37.94

Fig. 37.95

(ii) Terminal voltage / phase =  $11,000 / \sqrt{3} = 6350 \text{ V}$

Considering the first alternator,

$$IR \text{ drop} = 4 \times 50 = 200 \text{ V}; IX \text{ drop} = 50 \times 50 = 2,500 \text{ V}$$

$$\cos \phi_1 = 39.25/50 = 0.785; \sin \phi_1 = 0.62$$

Then, as seen from Fig. 37.95 (b)

$$\begin{aligned} E &= \sqrt{(V \cos \phi_1 + IR)^2 + (V \sin \phi_1 + IX)^2} \\ &= \sqrt{(6,350 \times 0.785 + 200)^2 + (6,350 \times 0.62 + 2,500)^2} = 8,350 \text{ V} \\ \text{Line voltage} &= 8,350 \times \sqrt{3} = \mathbf{14,450 \text{ V}} \end{aligned}$$

**Example 37.52.** Two alternators A and B operate in parallel and supply a load of 10 MW at 0.8 p.f. lagging (a) By adjusting steam supply of A, its power output is adjusted to 6,000 kW and by changing its excitation, its p.f. is adjusted to 0.92 lag. Find the p.f. of alternator B.

(b) If steam supply of both machines is left unchanged, but excitation of B is reduced so that its p.f. becomes 0.92 lead, find new p.f. of A.

**Solution.** (a)  $\cos \phi = 0.8, \phi = 36.9^\circ, \tan \phi = 0.7508; \cos \phi_A = 0.92, \phi_A = 23^\circ; \tan \phi_A = 0.4245$

$$\text{load kW} = 10,000, \text{ load kVAR} = 10,000 \times 0.7508 = 7508 \text{ (lag)}$$

$$\text{kW of A} = 6,000, \text{ kVAR of A} = 6,000 \times 0.4245 = 2547 \text{ (lag)}$$

Keeping in mind the convention that lagging kVAR is taken as negative we have,

$$\text{kW of B} = (10,000 - 6,000) = 4,000; \text{ kVAR of B} = (7508 - 2547) = 4961 \text{ (lag)}$$

$$\therefore \text{ kVA of B} = 4,000 - j4961 = 6373 \angle -51.1^\circ; \cos \phi_B = \cos 51.1^\circ = \mathbf{0.628}$$

(b) Since steam supply remains unchanged, load kW of each machine remains as before but due to change in excitation, kVARs of the two machines are changed.

$$\text{kW of B} = 4,000, \text{ new kVAR of B} = 4000 \times 0.4245 = 1698 \text{ (lead)}$$

$$\text{kW of A} = 6,000, \text{ new kVAR of A} = -7508 - (+1698) = -9206 \text{ (lag.)}$$

$$\therefore \text{ new kVA of A} = 6,000 - j9206 = 10,988 \angle -56.9^\circ; \cos \phi_A = \mathbf{0.546 \text{ (lag)}}$$

**Example 37.53.** A 6,000-V, 1,000-kVA, 3- $\phi$  alternator is delivering full-load at 0.8 p.f. lagging. Its reactance is 20% and resistance negligible. By changing the excitation, the e.m.f. is increased by 25% at this load. Calculate the new current and the power factor. The machine is connected to infinite bus-bars.

**Solution.** Full-load current  $I = \frac{1,000,000}{\sqrt{3} \times 6,000} = 87.5 \text{ A}$

$$\text{Voltage/phase} = 6,600 / \sqrt{3} = 3,810 \text{ V}$$

$$\text{Reactance} = \frac{3810 \times 20}{87.5 \times 100} = 8.7 \Omega$$

$$IX = 20\% \text{ of } 3810 = 762 \text{ V}$$

In Fig. 37.96, current vector is taken along X-axis. ON represents bus-bar or terminal voltage and is hence constant.

Current  $I$  has been split up into its active and reactive components  $I_R$  and  $I_X$  respectively.

$$NA_1 = I_X X = 52.5 \times 8.7 = 457 \text{ V}$$

$$A_1 C_1 = I_R X = 70 \times 8.7 = 609 \text{ V}$$

$$E_0 = OC_1 = \sqrt{[(V + I_X X)^2 + (I_R X)^2]}$$

$$= \sqrt{[(3,810 + 457)^2 + 609^2]} = 4,311 \text{ V}$$

When e.m.f. is increased by 25%, then  $E_0$  becomes equal to  $4,311 \times 1.25 = 5,389 \text{ V}$

The locus of the extremity of  $E_0$  lies on the line  $EF$  which is parallel to  $ON$ . Since the kW is unchanged,  $I_R$  and hence  $I_R X$  will remain the same. It is only the  $I_X X$  component which will be changed. Let  $OC_2$  be the new value of  $E_0$ . Then  $A_2 C_2 = A_1 C_1 = I_R X$  as before. But the  $I_X X$  component will change. Let  $I'$  be the new line current having active component  $I_R$  (the same as before) and the new reactive component  $I'_X$ . Then,  $I'_X X = NA_2$

From right-angled triangle  $OC_2 A_2$

$$OC_2^2 = OA_2^2 + A_2 C_2^2; 5,389^2 = (3810 + VA_2)^2 + 609^2$$

$$\therefore VA_2 = 1546 \text{ V or } I'_X X = 1546$$

$$\therefore I'_X = 1546 / 8.7 = 177.7 \text{ A}$$

$$\therefore \text{New line current } I = \sqrt{(70^2 + 177.7^2)} = \mathbf{191 \text{ A}}$$

$$\text{New angle of lag, } \phi' = \tan^{-1} (177.7/70) = 68^\circ 30'; \cos \phi' = \cos 68^\circ 30' = \mathbf{0.3665}$$

$$\text{As a check, the new power} = \sqrt{3} \times 6,600 \times 191 \times 0.3665 = 800 \text{ kW}$$

$$\text{It is the same as before} = 1000 \times 0.8 = 800 \text{ kW}$$

**Example 37.54.** A 6,600-V, 1000-kVA alternator has a reactance of 20% and is delivering full-load at 0.8 p.f. lagging. It is connected to constant-frequency bus-bars. If steam supply is gradually increased, calculate (i) at what output will the power factor become unity (ii) the maximum load which it can supply without dropping out of synchronism and the corresponding power factor.

**Solution.** We have found in Example 37.52 that

$$I = 87.5 \text{ A, } X = 8.7 \Omega, \text{ V/phase} = 3,810 \text{ V}$$

$$E_0 = 4,311 \text{ V, } I_R = 70 \text{ A, } I_X = 52.5 \text{ A and}$$

$$IX = 87.5 \times 8.7 = 762 \text{ V}$$

Using this data, vector diagram of Fig. 37.97 can be constructed.

Since excitation is constant,  $E_0$  remains constant, the extremity of  $E_0$  lies on the arc of a circle of radius  $E_0$  and centre  $O$ . Constant power lines have been shown dotted and they are all parallel to  $OV$ . Zero power output line coincides with  $OV$ . When p.f. is unity, the current vector lies along  $OV$ ,  $I_1 Z_1$  is  $\perp$  to  $OV$  and cuts the arc at  $B_1$ . Obviously

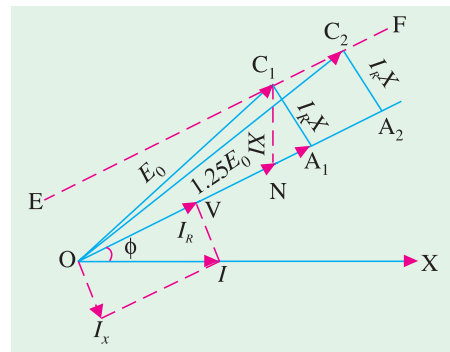


Fig. 37.96

$$VB_1 = \sqrt{(OB_1^2 - OV^2)}$$

$$= \sqrt{(4,311^2 - 3,810^2)} = 2018 \text{ V}$$

Now  $Z = X \therefore I_1 X = 2,018 \text{ V}$   
 $\therefore I_1 = 2018/8.7 = 232 \text{ A}$

(i)  $\therefore$  power output at u.p.f.  

$$= \frac{\sqrt{3} \times 6,600 \times 232}{1000} = 2,652 \text{ kW}$$

(ii) As vector  $OB$  moves upwards along the arc, output power goes on increasing *i.e.*, point  $B$  shifts on to a higher output power line. Maximum output power is reached when  $OB$  reaches the position  $OB_2$  where it is vertical to  $OV$ . The output power line passing through  $B_2$  represents the maximum output for that excitation. If  $OB$  is further rotated, the point  $B_2$  shifts down to a lower power line *i.e.* power is decreased. Hence,  $B_2V = I_2Z$  where  $I_2$  is the new current corresponding to maximum output.

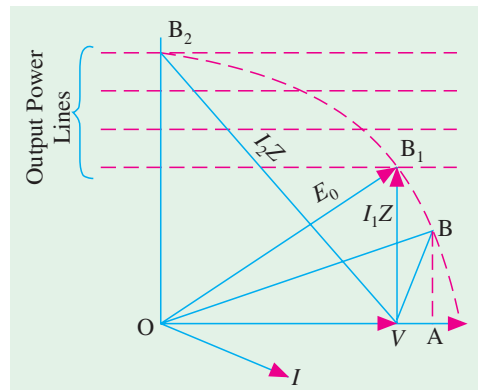


Fig. 37.97

From triangle  $OB_2V$ , it is seen that

$$B_2V = \sqrt{(OV^2 + OB_2^2)} = \sqrt{(3,810^2 + 4,311^2)} = 5,753 \text{ V}$$

$\therefore I_2Z = 5,753 \text{ V} \therefore I_2 = 5753/8.7 = 661 \text{ A}$

Let  $I_{2R}$  and  $I_{2X}$  be the power and wattless components of  $I_2$ , then

$$I_{2R} X = OB_2 = 4311 \text{ and } I_{2R} = 4311/8.7 = 495.6 \text{ A}$$

Similarly  $I_{2X} = 3810/8.7 = 438 \text{ A}; \tan \phi_2 = 438/495.6 = 0.884$

$\therefore \phi_2 = 41^\circ 28'; \cos \phi_2 = 0.749$

$\therefore$  Max. power output  $= \frac{\sqrt{3} \times 6,600 \times 661 \times 0.749}{1000} = 5,658 \text{ kW}$

**Example 37.55.** A 3-phase, star-connected turbo-alternator, having a synchronous reactance of  $10 \Omega$  per phase and negligible armature resistance, has an armature current of  $220 \text{ A}$  at unity p.f. The supply voltage is constant at  $11 \text{ kV}$  at constant frequency. If the steam admission is unchanged and the e.m.f. raised by  $25\%$ , determine the current and power factor.

If the higher value of excitation is maintained and the steam supply is slowly increased, at what power output will the alternator break away from synchronism?

Draw the vector diagram under maximum power condition.

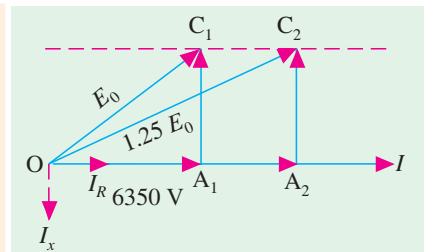


Fig. 37.98

(Elect.Machinery-III, Bangalore Univ. 1992)

**Solution.** The vector diagram for unity power factor is shown in Fig. 37.98. Here, the current is wholly active.

$$OA_1 = 11,000/\sqrt{3} = 6,350 \text{ V}$$

$$A_1C_1 = 220 \times 10 = 2,200 \text{ V}$$

$$E_0 = \sqrt{(6350^2 + 2,200^2)} = 6,810 \text{ V}$$

When e.m.f. is increased by  $25\%$ , the e.m.f. becomes  $1.25 \times 6,810 = 8,512 \text{ V}$  and is represented by  $OC_2$ . Since the kW remains unchanged,  $A_1C_1 = A_2C_2$ . If  $I$  is the new current, then its active component

$I_R$  would be the same as before and equal to 220 A. Let its reactive component be  $I_X$ . Then

$$A_1 A_2 = I_X X_S = 10 I_X$$

From right-angled  $\Delta OA_2 C_2$ , we have

$$8,512^2 = (6350 + A_1 A_2)^2 + 2,200^2$$

$$\therefore A_1 A_2 = 1870 \text{ V} \quad \therefore 10 I_X = 1870 \quad I_X = 187 \text{ A}$$

Hence, the new current has active component of 220 A and a reactive component of 187 A.

$$\text{New current} = \sqrt{220^2 + 187^2} = \mathbf{288.6 \text{ A}}$$

$$\text{New power factor} = \frac{\text{active component}}{\text{total current}} = \frac{220}{288.6} = \mathbf{0.762 \text{ (lag)}}$$

Since excitation remains constant,  $E_0$  is constant. But as the steam supply is increased, the extremity of  $E_0$  lies on a circle of radius  $E_0$  and centre  $O$  as shown in Fig. 37.99.

The constant-power lines (shown dotted) are drawn parallel to  $OV$  and each represents the locus of the e.m.f. vector for a constant power output at varying excitation. Maximum power output condition is reached when the vector  $E_0$  becomes perpendicular to  $OV$ . In other words, when the circular e.m.f. locus becomes tangential to the constant-power lines *i.e.* at point  $B$ . If the steam supply is increased further, the alternator will break away from synchronism.

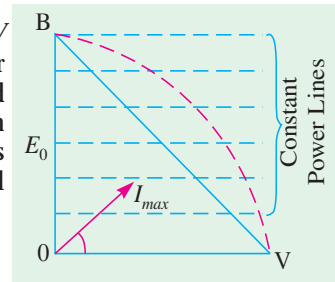


Fig. 37.99

$$B.V. = \sqrt{6350^2 + 8,512^2} = 10,620 \text{ V}$$

$$\therefore I_{\max} \times 10 = 10,620 \text{ or } I_{\max} = 1,062 \text{ A}$$

If  $I_R$  and  $I_X$  are the active and reactive components of  $I_{\max}$ , then

$$10 I_R = 8,512 \quad \therefore I_R = 851.2 \text{ A}; \quad 10 I_X = 6,350 \quad \therefore I_X = 635 \text{ A}$$

Power factor at maximum power output =  $851.2/1062 = 0.8$  (lead)

$$\text{Maximum power output} = \sqrt{3} \times 11,000 \times 1062 \times 0.8 \times 10^{-3} = \mathbf{16,200 \text{ kW}}$$

**Example 37.56.** Two 20-MVA, 3- $\phi$  alternators operate in parallel to supply a load of 35MVA at 0.8 p.f. lagging. If the output of one machine is 25 MVA at 0.9 lagging, what is the output and p.f. of the other machine? **(Elect. Machines, Punjab Univ. 1990)**

**Solution.** Load MW =  $35 \times 0.8 = 28$ ; load MVAR =  $35 \times 0.6 = 21$

**First Machine**  $\cos \phi_1 = 0.9, \sin \phi_1 = 0.436$ ;  $MVA_1 = 25, MW_1 = 25 \times 0.9 = 22.5$

$$MVAR_1 = 25 \times 0.436 = 10.9$$

**Second Machine**  $MW_2 = MW - MW_1 = 28 - 22.5 = 5.5$

$$MVAR_2 = MVAR - MVAR_1 = 21 - 10.9 = 10.1$$

$$\therefore MVA_2 = \sqrt{MW_2^2 + MVAR_2^2} = \sqrt{5.5^2 + 10.1^2} = \mathbf{11.5}$$

$$\cos \phi_2 = 5.5/11.5 = \mathbf{0.478 \text{ (lag)}}$$

**Example 37.57.** A lighting load of 600 kW and a motor load of 707 kW at 0.707 p.f. are supplied by two alternators running in parallel. One of the machines supplies 900 kW at 0.9 p.f. lagging. Find the load and p.f. of the second machine. **(Electrical Technology, Bombay Univ. 1988 & Bharatiar University, 1997)**

**Solution.**

	Active Power	kVA	Reactive Power
(a) Lighting Load (unity P.f.)	600 kW	600	—
(b) Motor, 0.707 P.f.	707 kW	1000	707 k VAR
Total Load :	1307	By Phasor addition	707 k VAR

One machine supplies an active power of 900 kW, and due to 0.9 lagging p.f., kVA = 1000 kVA and its kVAR =  $1000 \times \sqrt{1 - 0.9^2} = 436$  kVAR. Remaining share will be catered to by the second machine.

Active power shared by second machine =  $1307 - 900 = 407$  kW

Reactive power shared by second machine =  $707 - 436 = 271$  kVAR

**Example 37.58.** Two alternators, working in parallel, supply the following loads :

- (i) Lighting load of 500 kW
- (ii) 1000 kW at p.f. 0.9 lagging
- (iii) 800 kW at p.f. 0.8 lagging
- (iv) 500 kW at p.f. 0.9 leading

One alternator is supplying 1500 kW at 0.95 p.f. lagging. Calculate the kW output and p.f. of the other machine.

**Solution.** We will tabulate the kW and kVAR components of each load separately :

Load	kW	kVAR
(i)	500	
(ii)	1000	$\frac{1000}{0.9} \times 0.436 = 485$
(iii)	800	$\frac{800 \times 0.6}{0.8} = 600$
(iv)	500	$\frac{500 \times 0.436}{0.9} = -242$
Total	2800	+ 843

For 1st machine, it is given : kW = 1500, kVAR =  $(1500/0.95) \times 0.3123 = 493$

∴ kW supplied by other machine =  $2800 - 1500 = 1300$

kVAR supplied =  $843 - 493 = 350$  ∴  $\tan \phi = 350/1300 = 0.27$  ∴  $\cos \phi = 0.966$

**Example 37.59** Two 3-φ synchronous mechanically-coupled generators operate in parallel on the same load. Determine the kW output and p.f. of each machine under the following conditions: synchronous impedance of each generator :  $0.2 + j2$  ohm/phase. Equivalent impedance of the load :  $3 + j4$  ohm/phase. Induced e.m.f. per phase,  $2000 + j0$  volt for machine I and  $2,200 + j100$  for II.

[London Univ.]

**Solution.** Current of 1st machine =  $I_1 = \frac{E_1 - V}{0.2 + j2}$  or  $E_1 - V = I_1 (0.2 + j2)$

Similarly  $E_2 - V = I_2 (0.2 + j2)$

Also  $V = (I_1 + I_2) (3 + j4)$  where  $3 + j4 =$  load impedance

Now  $E_1 = 2,000 + j0$ ,  $E_2 = 2,200 + j100$

Solving from above, we get  $I_1 = 68.2 - j102.5$

Similarly  $I_2 = 127 - j196.4$ ;  $I = I_1 + I_2 = 195.2 - j299$

Now  $V = IZ = (192.2 - j299) (3 + j4) = 1781 - j115.9$

Using the method of conjugate for power calculating, we have for the first machine

$$P_{VA1} = (1781 - j115.9) (68.2 + j102.5) = 133,344 + j174,648$$

∴  $kW_1 = 133.344$  kW/phase =  $3 \times 133.344 = 400$  kW —for 3 phases

Now  $\tan^{-1} (102.5/68.2) = 56^\circ 24'$ ;  $\tan^{-1} (115.9/1781) = 3^\circ 43'$

∴ for 1st machine ;  $\cos(56^\circ 24' - 3^\circ 43') = 0.6062$



$$PVA_2 = (1781 - j115.9)(127 + j196.4) = 248,950 + j335,069$$

$$\therefore kW_2 = 248.95 \text{ kW/phase} = 746.85 \text{ kW} \quad \text{—for 3 phases}$$

$$\tan^{-1}(196.4/127) = 57^\circ 6'; \cos \phi = \cos(57^\circ 6' - 3^\circ 43') = \mathbf{0.596}$$

**Example 37.60.** The speed regulations of two 800-kW alternators A and B, running in parallel, are 100% to 104% and 100% to 105% from full-load to no-load respectively. How will the two alternators share a load of 1000 kW? Also, find the load at which one machine ceases to supply any portion of the load. **(Power Systems-I, A.M.I.E. 1989)**

**Solution.** The speed / load characteristics (assumed straight) for the alternators are shown in Fig. 37.100. Out of the combined load AB = 1000 kW, A's share is AM and B's share is BM. Hence, AM + BM = 1000 kW. PQ is the horizontal line drawn through point C, which is the point of intersection.

From similar  $\Delta s$  GDA and CDP, we have

$$\frac{CP}{GA} = \frac{PD}{AD} \text{ or}$$

$$CP = GA \cdot \frac{PD}{AD}$$

Since,

$$PD = (4 - h)$$

$$\therefore CP = 800(4 - h)/4$$

$$= 200(4 - h)$$

Similarly, from similar  $\Delta s$  BEF and QEC, we get

$$\frac{QC}{BF} = \frac{QE}{BE} \text{ or}$$

$$QC = 800(5 - h)/5$$

$$= 160(5 - h)$$

$$\therefore CP + QC = 1000 \text{ or } 200(4 - h) + 160(5 - h) = 1000 \text{ or } h = 5/3$$

$$\therefore CP = 200(4 - 5/3) = 467 \text{ kW, } QC = 160(5 - 5/3) = 533 \text{ kW}$$

Hence, alternator A supplies **467 kW** and B supplies **533 kW**.

Alternator A will cease supplying any load when line PQ is shifted to point D. Then, load supplied by alternator B (= BN) is such that the speed variation is from 105% to 104%.

Knowing that when its speed varies from 105% to 100%, alternator B supplies a load of 800 kW, hence load supplied for speed variation from 105% to 104% is (by proportion)

$$= 800 \times 1/5 = \mathbf{160 \text{ kW} (= BN)}$$

Hence, when load drops from 1000 kW to 160 kW, alternator A will cease supplying any portion of this load.

**Example 37.61.** Two 50-MVA, 3- $\phi$  alternators operate in parallel. The settings of the governors are such that the rise in speed from full-load to no-load is 2 per cent in one machine and 3 per cent in the other, the characteristics being straight lines in both cases. If each machine is fully loaded when the total load is 100 MW, what would be the load on each machine when the total load is 60 MW? **(Electrical Machines-II, Punjab Univ. 1991)**

**Solution.** Fig. 37.101 shows the speed/load characteristics of the two machines, NB is of the first machine and MA is that of the second. Base AB shows equal load division at full-load and speed. As the machines are running in parallel, their frequencies must be the same. Let CD be drawn through x% speed where total load is 60 MW.

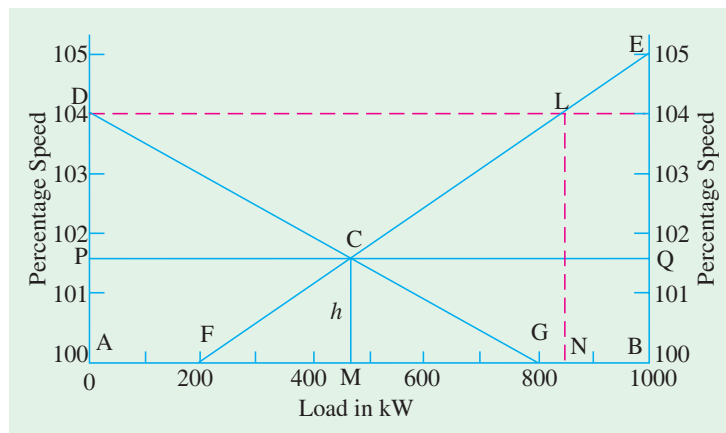


Fig. 37.100

$$CE = 50 - AP = 50 - \frac{50}{3} x$$

$$ED = 50 - QB = 50 - \frac{50}{2} x$$

$$\therefore CD = 50 - (50/3) x + 50 - 25 x$$

$$\therefore 60 = 50 - (50/3) x + 50 - 25 x$$

$$x = \frac{24}{25}; \therefore LE = 100 \frac{24}{25} \%$$

Load supplied by 1st machine

$$= ED = 50 - 25 \times \frac{24}{25} = \mathbf{26 \text{ MW}}$$

Load supplied by 2nd machine.

$$= CE = 50 - \left(\frac{50}{3}\right) \times \frac{24}{25} = \mathbf{34 \text{ MW}}$$

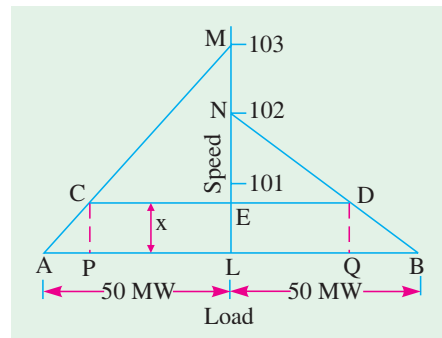


Fig. 37.101

**Example 37.62.** Two identical 2,000 -kVA alternators operate in parallel. The governor of the first machine is such that the frequency drops uniformly from 50-Hz on no-load to 48-Hz on full-load. The corresponding uniform speed drop of the second machines is 50 to 47.5 Hz (a) How will the two machines share a load of 3,000 kW? (b) What is the maximum load at unity p.f. that can be delivered without overloading either machine ? **(Electrical Machinery-II, Osmania Univ. 1989)**

**Solution.** In Fig. 37.102 are shown the frequency/load characteristics of the two machines, AB that of the second machine and AD that of the first. Remembering that the frequency of the two machines must be the same at any load, a line MN is drawn at a frequency  $x$  as measured from point A (common point).

Total load at that frequency is

$$NL + ML = 3000 \text{ kW}$$

From  $\Delta s ABC$  and  $ANL$ ,  $NL/2000 = x / 2.5$

$$\therefore NL = 2000 x / 2.5 = 800 x$$

Similarly,  $ML = 2000 x / 2 = 1000x$

$$\therefore 1800 x = 3000 \text{ or } x = 5/3$$

$$\text{Frequency} = 50 - 5/3 = 145/3 \text{ Hz.}$$

(a)  $NL = 800 \times 5/3 = \mathbf{1333 \text{ kW (assuming u.p.f.)}}$

$$ML = 1000 \times 5/3 = \mathbf{1667 \text{ kW (assuming u.p.f.)}}$$

(b) For getting maximum load, DE is extended to cut AB at F. Max. load = DF.

$$\text{Now, } EF = 2000 \times 2/3.5 = 1600 \text{ kW}$$

$$\therefore \text{Max. load} = DF = 2,000 + 1,600$$

$$= \mathbf{3,600 \text{ kW.}}$$

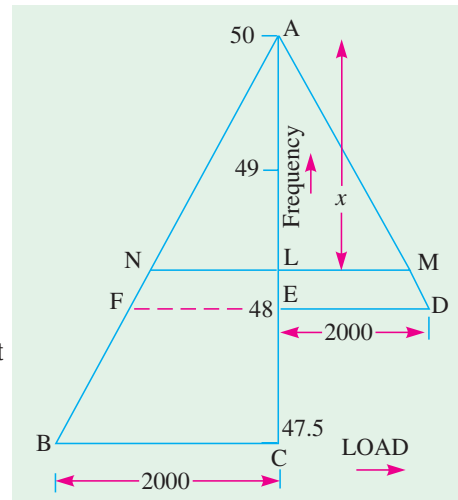


Fig. 37.102

### Tutorial Problem No. 37.7

- Two similar 6,600-V, 3- $\phi$ , generators are running in parallel on constant-voltage and frequency bus-bars. Each has an equivalent resistance and reactance of 0.05  $\Omega$  and 0.5  $\Omega$  respectively and supplies one half of a total load of 10,000 kW at a lagging p.f. of 0.8, the two machines being similarly excited. If the excitation of one machine be adjusted until the armature current is 438 A and the steam supply to the turbine remains unchanged, find the armature current, the e.m.f. and the p.f. of the other alternator. **[789 A, 7200 V, 0.556] (City & Guilds, London)**

2. A single-phase alternator connected to 6,600-V bus-bars has a synchronous impedance of  $10\Omega$  and a resistance of  $1\Omega$ . If its excitation is such that on open circuit the p.d. would be 5000 V, calculate the maximum load the machine can supply to the external circuit before dropping out of step and the corresponding armature current and p.f. **[2864 kW, 787 A, 0.551] (London Univ.)**
3. A turbo-alternator having a reactance of  $10\Omega$  has an armature current of 220 A at unity power factor when running on 11,000 V, constant-frequency bus-bars. If the steam admission is unchanged and the e.m.f. raised by 25%, determine graphically or otherwise the new value of the machine current and power factor. If this higher value of excitation were kept constant and the steam supply gradually increased, at what power output would the alternator break from synchronism? Find also the current and power factor to which this maximum load corresponds. State whether this p.f. is lagging or leading. **[360 A at 0.611 p.f. ; 15.427 kW ; 1785 A at 0.7865 leading] (City & Guilds, London)**
4. Two single-phase alternators are connected to a 50-Hz bus-bars having a constant voltage of  $10\angle 0^\circ$  kV. Generator A has an induced e.m.f. of  $13\angle 22.6^\circ$  kV and a reactance of  $2\Omega$ ; generator B has an e.m.f. of  $12.5\angle 36.9^\circ$  kV and a reactance of  $3\Omega$ . Find the current, kW and kVAR supplied by each generator. **(Electrical Machine-II, Indore Univ. July 1977)**
5. Two 15-kVA, 400-V, 3-ph alternators in parallel supply a total load of 25 kVA at 0.8 p.f. lagging. If one alternator shares half the power at unity p.f., determine the p.f. and kVA shared by the other alternator. **[0.5548; 18.03 kVA] (Electrical Technology-II, Madras Univ. Apr. 1977)**
6. Two 3- $\phi$ , 6,600-V, star-connected alternators working in parallel supply the following loads :  
 (i) Lighting load of 400 kW      (ii) 300 kW at p.f. 0.9 lagging  
 (iii) 400 kW at p.f. 0.8 lagging      (iv) 1000 kW at p.f. 0.71 lagging  
 Find the output, armature current and the p.f. of the other machine if the armature current of one machine is 110 A at 0.9 p.f. lagging. **[970 kW, 116 A, 0.73 lagging]**
7. A 3- $\phi$ , star-connected, 11,000-V turbo-generator has an equivalent resistance and reactance of  $0.5\Omega$  and  $8\Omega$  respectively. It is delivering 200 A at u.p.f. when running on a constant-voltage and constant-frequency bus-bars. Assuming constant steam supply and unchanged efficiency, find the current and p.f. if the induced e.m.f. is raised by 25%. **[296 A, 0.67 lagging]**
8. Two similar 13,000-V, 3-ph alternators are operated in parallel on infinite bus-bars. Each machine has an effective resistance and reactance of  $0.05\Omega$  and  $0.5\Omega$  respectively. When equally excited, they share equally a total load of 18 MW at 0.8 p.f. lagging. If the excitation of one generator is adjusted until the armature current is 400 A and the steam supply to its turbine remains unaltered, find the armature current, the e.m.f. and the p.f. of the other generator. **[774.6 A; 0.5165 ; 13,470 V] (Electric Machinery-II, Madras Univ. Nov. 1977)**

### 37.41. Time-period of Oscillation

Every synchronous machine has a natural time period of free oscillation. Many causes, including the variations in load, create phase-swinging of the machine. If the time period of these oscillations coincides with natural time period of the machine, then the amplitude of the oscillations may become so greatly developed as to swing the machine out of synchronism.

The expression for the natural time period of oscillations of a synchronous machine is derived below :

Let  $T =$  torque per mechanical radian (in N-m/mech. radian)  
 $J = \Sigma m r^2$  —moment of inertia in  $\text{kg-m}^2$ .

The period of undamped free oscillations is given by  $t = 2\pi \sqrt{\frac{J}{T}}$ .

We have seen in Art. 37.32 that when an alternator swings out of phase by an angle  $\alpha$  (electrical radian), then synchronizing power developed is

$$P_{SY} = \alpha E^2/Z \quad \text{— } \alpha \text{ in elect. radian}$$

$$= \frac{E^2}{Z} \text{ per electrical radian per phase.}$$

Now, 1 electrical radian =  $\frac{P}{2} \times$  mechanical radian—where  $P$  is the number of poles.

$$\therefore P_{SY} \text{ per mechanical radian displacement} = \frac{E^2 P}{2 Z}.$$

The synchronizing or restoring torque is given by

$$T_{SY} = \frac{P_{SY}}{2\pi N_S} = \frac{E^2 P}{4\pi Z N_S} \quad \text{---}N_S \text{ in r.p.s.}$$

Torque for three phases is  $T = 3T_{SY} = \frac{3 E^2 P}{4 \pi Z N_S}$  where  $E$  is e.m.f. per phase ... (i)

Now  $E/Z =$  short-circuit current =  $I_{SC}$   
 $f = PN_S/2$ ; hence  $P/N_S = 2 f/N_S^2$

Substituting these values in (i) above, we have

$$T_{SY} = \frac{3}{4\pi} \cdot \left(\frac{E}{Z}\right) \cdot E \cdot \frac{P}{N_S} = \frac{3}{4\pi} \cdot I_{SC} \cdot E \cdot \frac{2f}{N_S^2} = 0.477 \frac{E I_{SC} f}{N_S^2}$$

Now,  $t = 2\pi \sqrt{\frac{J}{0.477 E I_{SC} f / N_S^2}} = 9.1 N_S \sqrt{\frac{J}{E \cdot I_{SC} \cdot f}}$  second

$$= 9.1 N_S \sqrt{\frac{J}{\frac{1}{\sqrt{3}} \cdot E_L \cdot I \cdot (I_{SC}/D) \cdot f}}$$

$$= 9.1 N_S \sqrt{\frac{J}{\frac{1}{2} \cdot \sqrt{3} E_L \cdot I \cdot (I_{SC}/D) \cdot f}}$$

$$= 9.1 N_S \sqrt{\frac{J}{\frac{1000}{3} \cdot \frac{\sqrt{3} \cdot E_L \cdot I \left(\frac{I_{SC}}{I}\right) \cdot f}{1000}}}$$

$$= \frac{9.1 \times \sqrt{3}}{\sqrt{1000}} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC}/D) \cdot f}}$$

$$\therefore t = 0.4984 N_S \sqrt{\frac{J}{kVA \cdot (I_{SC}/D) \cdot f}}$$

where kVA = full-load kVA of the alternator;  $N_S =$  r.p.s. of the rotating system

If  $N_S$  represents the speed in r.p.m., then

$$t = \frac{0.4984}{60} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC}/D) \cdot f}} = 0.0083 N_S \sqrt{\frac{J}{kVA (I_{SC}/D) \cdot f}} \text{ second}$$

**Note.** It may be proved that  $I_{SC}/I = 100/\text{percentage reactance} = 100\% X_S$

**Proof.** Reactance drop =  $I X_S = \frac{V \times \% X_S}{100} \therefore X_S = \frac{\text{reactance drop}}{\text{full-load current}} = \frac{V \times \% X_S}{100 \times I}$

Now 
$$I_{SC} = \frac{V}{X_S} = \frac{V \times 100 \times I}{V \times \% X_S} = \frac{100}{\% X_S} \times I; \text{ or } \frac{I_{SC}}{I} = \frac{100}{\% X_S}$$

For example, if synchronous reactance is 25 per cent, then

$$I_{SC}/I = 100/25 = 4 \text{ (please see Ex. 37.64)}$$

**Example 37.63.** A 5,000-kVA, 3-phase, 10,000-V, 50-Hz alternator runs at 1500 r.p.m. connected to constant-frequency, constant-voltage bus-bars. If the moment of inertia of entire rotating system is  $1.5 \times 10^4 \text{ kg.m}^2$  and the steady short-circuit current is 5 times the normal full-load current, find the natural time period of oscillation. **(Elect. Engg. Grad. I.E.T.E. 1991)**

**Solution.** The time of oscillation is given by

$$t = 0.0083 N_S \sqrt{\frac{J}{kVA \cdot (I_{SC}/I) f}} \text{ . second}$$

Here,  $N_S = 1500 \text{ r.p.m.}; I_{SC}/I = 5^* ; J = 1.5 \times 10^4 \text{ kg.m}^2 ; f = 50 \text{ Hz}$

$$t = 0.0083 \times 1500 \sqrt{\frac{1.5 \times 10^4}{5000 \times 5 \times 50}} = \mathbf{1.364 \text{ s.}}$$

**Example 37.64.** A 10,000-kVA, 4-pole, 6,600-V, 50-Hz, 3-phase star-connected alternator has a synchronous reactance of 25% and operates on constant-voltage, constant frequency bus-bars. If the natural period of oscillation while operating at full-load and unity power factor is to be limited to 1.5 second, calculate the moment of inertia of the rotating system. **(Electric Machinery-II, Andhra Univ. 1990)**

**Solution.** 
$$t = 0.0083 N_S \sqrt{\frac{J}{kVA (I_{SC}/I) f}} \text{ second.}$$

Here  $I_{SC}/I = 100/25 = 4; N_S = 120 \times 50/4 = 1500 \text{ r.p.m.}$

$$\therefore 1.5 = 0.0083 \times 1500 \sqrt{\frac{J}{10,000 \times 4 \times 50}} = 12.45 \times \frac{\sqrt{J}}{10^3 \times \sqrt{2}}$$

$$\therefore J = (1.5 \times 10^3 \times \sqrt{2}/12.45)^2 = \mathbf{2.9 \times 10^4 \text{ kg.m}^2}$$

**Example 37.65.** A 10-MVA, 10-kV, 3-phase, 50-Hz, 1500 r.p.m. alternator is paralleled with others of much greater capacity. The moment of inertia of the rotor is  $2 \times 10^5 \text{ kg.m}^2$  and the synchronous reactance of the machine is 40%. Calculate the frequency of oscillation of the rotor. **(Elect. Machinery-III, Bangalore Univ. 1992)**

**Solution.** Here,  $I_{SC}/I = 100/40 = 2.5$

$$t = 0.0083 \times 1500 \sqrt{\frac{2 \times 10^5}{10^4 \times 2.5 \times 50}} = 5 \text{ second}$$

$$\text{Frequency} = 1/5 = \mathbf{0.2 \text{ Hz.}}$$

---

\* It means that synchronous reactance of the alternator is 20 %.

Tutorial Problem No. 37.8

1. Show that an alternator running in parallel on constant-voltage and frequency bus-bars has a natural time period of oscillation. Deduce a formula for the time of one complete oscillation and calculate its value for a 5000-kVA, 3-phase, 10,000 V machine running at 1500 r.p.m. on constant 50-Hz bus-bars.  
The moment of inertia of the whole moving system is 14112 kg-m<sup>2</sup> and the steady short-circuit current is five times the normal full-load value. **[1.33 second]**
2. A 10,000-kVA, 5-kV, 3-phase, 4-pole, 50-Hz alternator is connected to infinite bus-bars. The short-circuit current is 3.5 times the normal full-load current and the moment of inertia of the rotating system is 21,000 kg-m<sup>2</sup>. Calculate its normal period of oscillation. **[1.365 second]**
3. Calculate for full-load and unity p.f., the natural period of oscillation of a 50-Hz, 10,000-kVA, 11-kV alternator driven at 1500 r.p.m. and connected to an infinite bus-bar. The steady short-circuit current is four times the full-load current and the moment of the inertia of the rotating masses is 17,000 kg-m<sup>2</sup>. **[1.148 s.] (Electrical Machinery-II, Madras Univ. Apr. 1976)**
4. Calculate the rotational inertia in kg-m<sup>2</sup> units of the moving system of 10,000 kVA, 6,600-V, 4-pole, turbo-alternator driven at 1500 r.p.m. for the set to have a natural period of 1 second when running in parallel with a number of other machines. The steady short-circuit current of the alternator is five times the full-load value. **[16,828 kg-m<sup>2</sup>] (City & Guilds, London)**
5. A 3-φ, 4-pole, 6,000 kVA, 5,000-V, 50-Hz star-connected alternator is running on constant-voltage and constant-frequency bus-bars. It has a short-circuit reactance of 25% and its rotor has a moment of inertia of 16,800 kg-m<sup>2</sup>. Calculate its natural time period of oscillation. **[1.48 second]**

37.42. Maximum Power Output

For given values of terminal voltage, excitation and frequency, there is a maximum power that the alternator is capable of delivering. Fig. 37.103 (a) shows full-load conditions for a cylindrical rotor where  $IR_a$  drop has been neglected\*.

The power output per phase is

$$P = VI \cos \phi = \frac{VIX_S \cos \phi}{X_S}$$

Now, from  $\Delta OBC$ , we get

$$\frac{IX_S}{\sin \alpha} = \frac{E}{\sin (90 + \phi)} = \frac{E}{\cos \phi}$$

$$IX_S \cos \phi = E \sin \alpha$$

$$\therefore P = \frac{EV \sin \alpha}{X_S}$$

Power becomes maximum when  $\alpha = 90^\circ$ , if  $V, E$  and  $X_S$  are regarded as constant (of course,  $E$  is fixed by excitation).

$$\therefore P_{\max} = EV/X_S$$

It will be seen from Fig. 37.103 (b) that under maximum power output conditions,  $I$  leads  $V$  by  $\phi$  and since  $IX_S$  leads  $I$  by  $90^\circ$ , angle  $\phi$  and hence  $\cos \phi$  is fixed =  $E/IX_S$ .

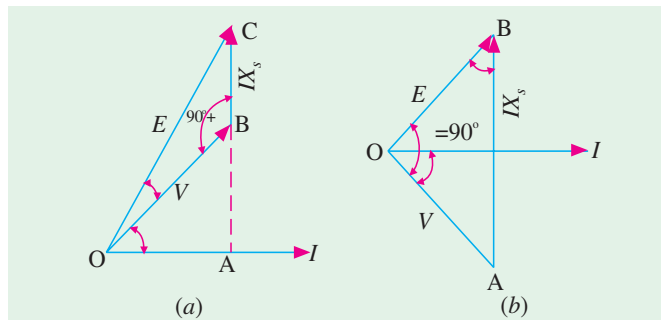


Fig. 37.103

\* In fact, this drop can generally be neglected without sacrificing much accuracy of results.

Now, from right-angled  $\triangle AOB$ , we have that  $IX_S = \sqrt{E^2 + V^2}$ . Hence, p.f. corresponding to maximum power output is

$$\cos \phi = \frac{E}{\sqrt{E^2 + V^2}}$$

The maximum power output per phase may also be written as

$$P_{\max} = VI_{\max} \cos \phi = VI_{\max} \frac{E}{\sqrt{E^2 + V^2}}$$

where  $I_{\max}$  represents the current/phase for maximum power output.

If  $I_f$  is the full-load current and  $\% X_S$  is the percentage synchronous reactance, then

$$\% X_S = \frac{I_f X_S}{V} \times 100 \quad \therefore \frac{V}{X_S} = \frac{I_f \times 100}{\% X_S}$$

Now,

$$P_{\max} = VI_{\max} \frac{E}{\sqrt{E^2 + V^2}} = \frac{EV}{X_S} = \frac{EI_f \times 100}{\% X_S}$$

Two things are obvious from the above equations.

(i) 
$$I_{\max} = \frac{100 I_f}{\% X_S} \times \frac{\sqrt{E^2 + V^2}}{V}$$

Substituting the value of  $\% X_S$  from above,

$$I_{\max} = \frac{100 I_f}{100 I_f X_S} \times V \times \frac{\sqrt{E^2 + V^2}}{V} = \frac{\sqrt{E^2 + V^2}}{X_S}$$

(ii) 
$$P_{\max} = \frac{100 EI_f}{\% X_S} = \frac{E}{V} \cdot \frac{100}{\% X_S} \times VI_f \text{ per phase}$$

$$= \frac{E}{V} \cdot \frac{100}{\% X_S} \times \text{F.L. power output at u.p.f.}$$

Total maximum power output of the alternator is

$$= \frac{E}{V} \cdot \frac{100}{\% X_S} \times \text{F.L. power output at u.p.f.}$$

**Example 37.66.** Derive the condition for the maximum output of a synchronous generator connected to infinite bus-bars and working at constant excitation.

A 3- $\phi$ , 11-kV, 5-MVA, Y-connected alternator has a synchronous impedance of  $(1 + j 10)$  ohm per phase. Its excitation is such that the generated line e.m.f. is 14 kV. If the alternator is connected to infinite bus-bars, determine the maximum output at the given excitation

(Electrical Machines-III, Gujarat Univ. 1984)

**Solution.** For the first part, please refer to Art. 37.41

$$P_{\max} \text{ per phase} = \frac{EV}{X_S} \text{ — if } R_a \text{ is neglected} = \frac{V}{Z_S} (E - V \cos \theta) \text{ — if } R_a \text{ is considered}$$

Now,

$$E = 14,000/\sqrt{3} = 8,083 \text{ V}; V = 11,000/\sqrt{3} = 6352 \text{ V}$$

$$\cos \theta = R_a/Z_S = 1/\sqrt{1^2 + 10^2} = 1/10.05$$

$$\therefore P_{\max} \text{ per phase} = \frac{8083 \times 6352}{10 \times 1000} = 5,135 \text{ kW}$$

Total 
$$P_{\max} = 3 \times 5,135 = 15,405 \text{ kW}$$

More accurately, 
$$P_{\max}/\text{phase} = \frac{6352}{10.05} \left( 8083 - \frac{6352}{10.05} \right) = \frac{6352}{10.05} \times \frac{7451}{1000} = 4,711 \text{ kW}$$

Total 
$$P_{\max} = 4,711 \times 3 = 14,133 \text{ kW.}$$

**Example 37.67.** A 3-phase, 11-kVA, 10-MW, Y-connected synchronous generator has synchronous impedance of  $(0.8 + j 8.0)$  ohm per phase. If the excitation is such that the open circuit voltage is 14 kV, determine (i) the maximum output of the generator (ii) the current and p.f. at the maximum output.

(Electrical Machines-III, Gujarat Univ. 1987)

**Solution.** (i) If we neglect  $R_a^*$ , the  $P_{\max}$  per phase =  $EV/X_S$

where  $V$  is the terminal voltage (or bus-bar voltage in general) and  $E$  the e.m.f. of the machine.

$$\therefore P_{\max} = \frac{(11,000/\sqrt{3}) \times (14,000/\sqrt{3})}{8} = \frac{154,000}{24} \text{ kW/phase}$$

$$\text{Total } P_{\max} = 3 \times 154,000/24 = 19,250 \text{ kW} = \mathbf{19.25 \text{ MW}}$$

Incidentally, this output is nearly twice the normal output.

$$(ii) I_{\max} = \frac{\sqrt{E^2 + V^2}}{X_S} = \frac{\sqrt{[(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2]}}{8} = \mathbf{1287 \text{ A}}$$

$$(iii) \text{ p.f.} = \frac{E}{\sqrt{E^2 + V^2}} = \frac{14000/\sqrt{3}}{\sqrt{(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2}} = \mathbf{0.786 \text{ (lead)}}.$$

## QUESTIONS AND ANSWERS ON ALTERNATORS

**Q. 1. What are the two types of turbo-alternators ?**

**Ans.** Vertical and horizontal.

**Q. 2. How do you compare the two ?**

**Ans.** Vertical type requires less floor space and while step bearing is necessary to carry the weight of the moving element, there is very little friction in the main bearings. The horizontal type requires no step bearing, but occupies more space.

**Q. 3. What is step bearing ?**

**Ans.** It consists of two cylindrical cast iron plates which bear upon each other and have a central recess between them. Suitable oil is pumped into this recess under considerable pressure.

**Q. 4. What is direct-connected alternator ?**

**Ans.** One in which the alternator and engine are directly connected. In other words, there is no intermediate gearing such as belt, chain etc. between the driving engine and alternator.

**Q. 5. What is the difference between direct-connected and direct-coupled units ?**

**Ans.** In the former, alternator and driving engine are directly and permanently connected. In the latter case, engine and alternator are each complete in itself and are connected by some device such as friction clutch, jaw clutch or shaft coupling.

**Q. 6. Can a d.c. generator be converted into an alternator ?**

**Ans.** Yes.

**Q. 7. How ?**

**Ans.** By providing two collector rings on one end of the armature and connecting these two rings to two points in the armature winding  $180^\circ$  apart.

**Q. 8. Would this arrangement result in a desirable alternator ?**

**Ans.** No.

\* If  $R_a$  is not neglected, then  $P_{\max} = \frac{V}{Z_S} (E - V \cos \theta)$  where  $\cos \theta = R_a/Z_S$  (Ex. 37.66)



**Q. 9. How is a direct-connected exciter arranged in an alternator ?**

**Ans.** The armature of the exciter is mounted on the shaft of the alternator close to the spider hub. In some cases, it is mounted at a distance sufficient to permit a pedestal and bearing to be placed between the exciter and the hub.

**Q. 10. Any advantage of a direct-connected exciter ?**

**Ans.** Yes, economy of space.

**Q. 11. Any disadvantage ?**

**Ans.** The exciter has to run at the same speed as the alternator which is slower than desirable. Hence, it must be larger for a given output than the gear-driven type, because it can be run at high speed and so made proportionately smaller.

**OBJECTIVE TESTS – 37**

1. The frequency of voltage generated by an alternator having 4-poles and rotating at 1800 r.p.m. is .....hertz.
 

(a) 60	(b) 7200
(c) 120	(d) 450.
2. A 50-Hz alternator will run at the greatest possible speed if it is wound for ..... poles.
 

(a) 8	(b) 6
(c) 4	(d) 2.
3. The main disadvantage of using short-pitch winding in alternators is that it
 

(a) reduces harmonics in the generated voltage	(b) reduces the total voltage around the armature coils
(c) produces asymmetry in the three phase windings	(d) increases Cu of end connections.
4. Three-phase alternators are invariably Y-connected because
 

(a) magnetic losses are minimised	(b) less turns of wire are required
(c) smaller conductors can be used	(d) higher terminal voltage is obtained.
5. The winding of a 4-pole alternator having 36 slots and a coil span of 1 to 8 is short-pitched by ..... degrees.
 

(a) 140	(b) 80
(c) 20	(d) 40.
6. If an alternator winding has a fractional pitch of 5/6, the coil span is ..... degrees.
 

(a) 300	(b) 150
(c) 30	(d) 60.
7. The harmonic which would be totally eliminated from the alternator e.m.f. using a fractional pitch of 4/5 is
 

(a) 3rd	(b) 7th
(c) 5th	(d) 9th.
8. For eliminating 7th harmonic from the e.m.f. wave of an alternator, the fractional-pitch must be
 

(a) 2/3	(b) 5/6
(c) 7/8	(d) 6/7.
9. If, in an alternator, chording angle for fundamental flux wave is  $\alpha$ , its value for 5th harmonic is
 

(a) $5\alpha$	(b) $\alpha/5$
(c) $25\alpha$	(d) $\alpha/25$ .
10. Regarding distribution factor of an armature winding of an alternator which statement is false?
 

(a) it decreases as the distribution of coils (slots/pole) increases	(b) higher its value, higher the induced e.m.f. per phase
(c) it is not affected by the type of winding either lap, or wave	(d) it is not affected by the number of turns per coil.
11. When speed of an alternator is changed from 3600 r.p.m. to 1800 r.p.m., the generated e.m.f./phases will become
 

(a) one-half	(b) twice
(c) four times	(d) one-fourth.
12. The magnitude of the three voltage drops in an alternator due to armature resistance, leakage reactance and armature reaction is solely determined by
 

(a) load current, $I_a$	
-------------------------	--

- (b) p.f. of the load  
(c) whether it is a lagging or leading p.f. load  
(d) field construction of the alternator.
13. Armature reaction in an alternator primarily affects  
(a) rotor speed  
(b) terminal voltage per phase  
(c) frequency of armature current  
(d) generated voltage per phase.
14. Under no-load condition, power drawn by the prime mover of an alternator goes to  
(a) produce induced e.m.f. in armature winding  
(b) meet no-load losses  
(c) produce power in the armature  
(d) meet Cu losses both in armature and rotor windings.
15. As load p.f. of an alternator becomes more leading, the value of generated voltage required to give rated terminal voltage  
(a) increases  
(b) remains unchanged  
(c) decreases  
(d) varies with rotor speed.
16. With a load p.f. of unity, the effect of armature reaction on the main-field flux of an alternator is  
(a) distortional (b) magnetising  
(c) demagnetising (d) nominal.
17. At lagging loads, armature reaction in an alternator is  
(a) cross-magnetising (b) demagnetising  
(c) non-effective (d) magnetising.
18. At leading p.f., the armature flux in an alternator ..... the rotor flux.  
(a) opposes (b) aids  
(c) distorts (d) does not affect.
19. The voltage regulation of an alternator having 0.75 leading p.f. load, no-load induced e.m.f. of 2400V and rated terminal voltage of 3000V is ..... percent.  
(a) 20 (b) -20  
(c) 150 (d) -26.7
20. If, in a 3- $\phi$  alternator, a field current of 50A produces a full-load armature current of 200 A on short-circuit and 1730 V on open circuit, then its synchronous impedance is ..... ohm.  
(a) 8.66 (b) 4  
(c) 5 (d) 34.6
21. The power factor of an alternator is determined by its  
(a) speed  
(b) load  
(c) excitation  
(d) prime mover.
22. For proper parallel operation, a.c. polyphase alternators must have the same  
(a) speed (b) voltage rating  
(c) kVA rating (d) excitation.
23. Of the following conditions, the one which does not have to be met by alternators working in parallel is  
(a) terminal voltage of each machine must be the same  
(b) the machines must have the same phase rotation  
(c) the machines must operate at the same frequency  
(d) the machines must have equal ratings.
24. After wiring up two 3- $\phi$  alternators, you checked their frequency and voltage and found them to be equal. Before connecting them in parallel, you would  
(a) check turbine speed  
(b) check phase rotation  
(c) lubricate everything  
(d) check steam pressure.
25. Zero power factor method of an alternator is used to find its  
(a) efficiency  
(b) voltage regulation  
(c) armature resistance  
(d) synchronous impedance.
26. Some engineers prefer 'lamps bright' synchronization to 'lamps dark' synchronization because  
(a) brightness of lamps can be judged easily  
(b) it gives sharper and more accurate synchronization  
(c) flicker is more pronounced  
(d) it can be performed quickly.
27. It is never advisable to connect a stationary alternator to live bus-bars because it  
(a) is likely to run as synchronous motor  
(b) will get short-circuited  
(c) will decrease bus-bar voltage though momentarily  
(d) will disturb generated e.m.fs. of other alternators connected in parallel.

28. Two identical alternators are running in parallel and carry equal loads. If excitation of one alternator is increased without changing its steam supply, then
- (a) it will keep supplying almost the same load
  - (b) kVAR supplied by it would decrease
  - (c) its p.f. will increase
  - (d) kVA supplied by it would decrease.
29. Keeping its excitation constant, if steam supply of an alternator running in parallel with another identical alternator is increased, then
- (a) it would over-run the other alternator
  - (b) its rotor will fall back in phase with respect to the other machine
  - (c) it will supply greater portion of the load
  - (d) its power factor would be decreased.
30. The load sharing between two steam-driven alternators operating in parallel may be adjusted by varying the
- (a) field strengths of the alternators
  - (b) power factors of the alternators
  - (c) steam supply to their prime movers
  - (d) speed of the alternators.
31. Squirrel-cage bars placed in the rotor pole faces of an alternator help reduce hunting
- (a) above synchronous speed only
  - (b) below synchronous speed only
  - (c) above and below synchronous speeds both
  - (d) none of the above.
- (Elect. Machines, A.M.I.E. Sec. B, 1993)**
32. For a machine on infinite bus active power can be varied by
- (a) changing field excitation
  - (b) changing of prime cover speed
  - (c) both (a) and (b) above
  - (d) none of the above .
- (Elect. Machines, A.M.I.E. Sec. B, 1993)**

**ANSWERS**

1. a    2. d    3. b    4. d    5. d    6. b    7. c    8. d    9. a    10. b    11. a  
 12. a    13. d    14. b    15. c    16. a    17. d    18. b    19. b    20. c    21. b    22. b  
 23. d    24. b    25. b    26. b    27. b    28. a    29. c    30. c    31. c    32. b

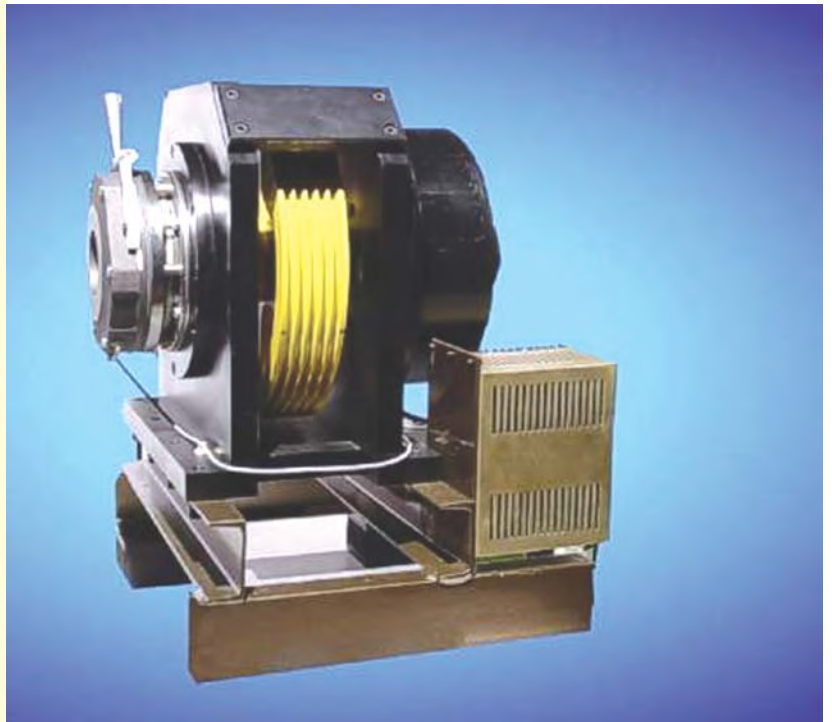
## ROUGHWORK

# CHAPTER 38

## Learning Objectives

- Synchronous Motor-General
- Principle of Operation
- Method of Starting
- Motor on Load with Constant Excitation
- Power Flow within a Synchronous Motor
- Equivalent Circuit of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Synchronous Motor with Different Excitations
- Effect of increased Load with Constant Excitation
- Effect of Changing Excitation of Constant Load
- Different Torques of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Alternative Expression for Power Developed
- Various Conditions of Maxima
- Salient Pole Synchronous Motor
- Power Developed by a Salient Pole Synchronous Motor
- Effects of Excitation on Armature Current and Power Factor
- Constant-Power Lines
- Construction of V-curves
- Hunting or Surging or Phase Swinging
- Methods of Starting
- Procedure for Starting a Synchronous Motor
- Comparison between Synchronous and Induction Motors
- Synchronous Motor Applications

## SYNCHRONOUS MOTOR



Rotary synchronous motor for lift applications

### 38.1. Synchronous Motor—General

A synchronous motor (Fig. 38.1) is electrically identical with an alternator or a.c. generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of d.c. machines. Most



Synchronous motor

synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m.

Some characteristic features of a synchronous motor are worth noting :

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency (because  $N_s = 120 f / P$ ).
2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.
3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

### 38.2. Principle of Operation

As shown in Art. 34.7, when a 3- $\phi$  winding is fed by a 3- $\phi$  supply, then a magnetic flux of constant magnitude but **rotating** at synchronous speed, is produced. Consider a two-pole stator of Fig. 38.2, in which are shown two stator poles (marked  $N_s$  and  $S_s$ ) rotating at synchronous speed, say, in clockwise direction. With the rotor position as shown, suppose the stator poles are at that instant situated at points  $A$  and  $B$ . The two similar poles,  $N$  (of rotor) and  $N_s$  (of stator) as well as  $S$  and  $S_s$  will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

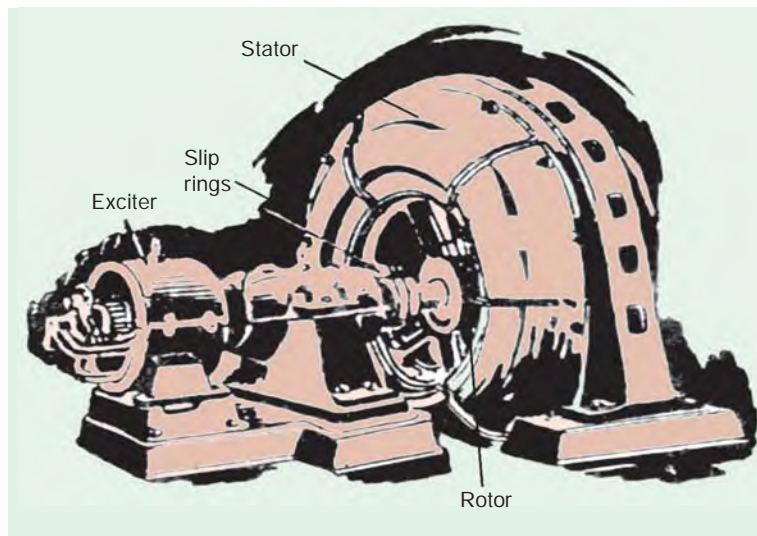


Fig. 38.1

But half a period later, stator poles, having rotated around, interchange their positions *i.e.*  $N_s$  is at point  $B$  and  $S_s$  at point  $A$ . Under these conditions,  $N_s$  attracts  $S$  and  $S_s$  attracts  $N$ . Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.

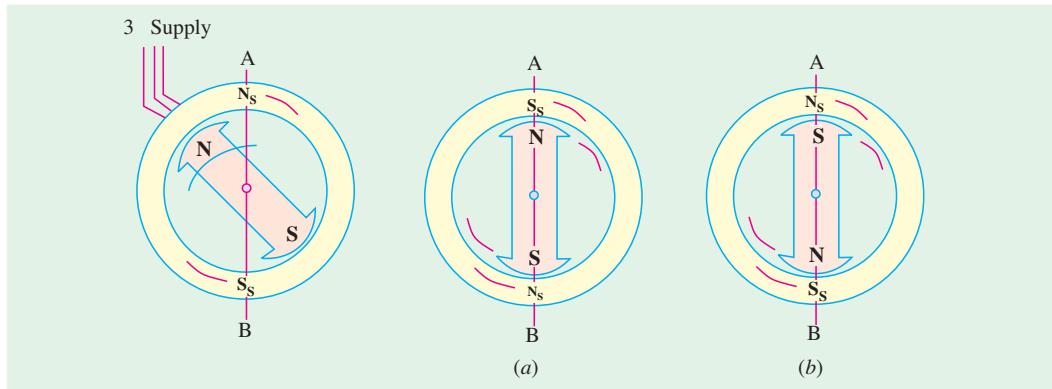


Fig. 38.2

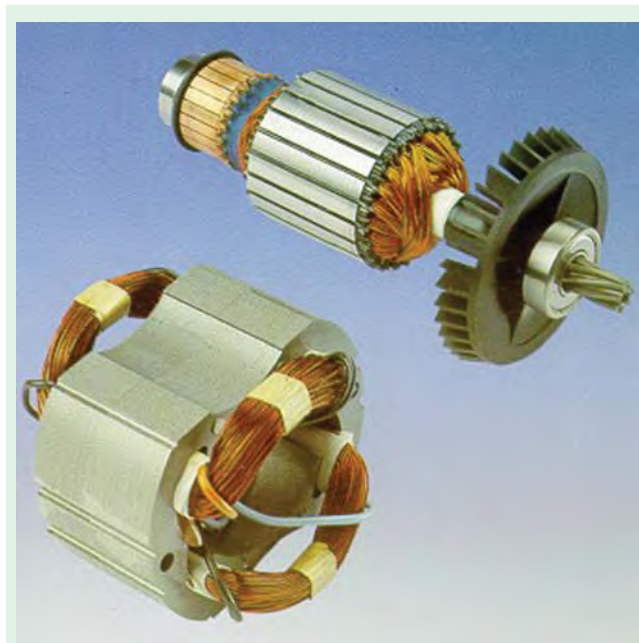
Fig. 38.3

Fig. 38.3

Now, consider the condition shown in Fig. 38.3 (a). The stator and rotor poles are attracting each other. Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig. 38.3 (b). Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e.*, clockwise torque, as shown in Fig. 38.3.

### 38.3. Method of Starting

The rotor (which is as yet unexcited) is speeded up to synchronous / near synchronous speed by some arrangement and then excited by the d.c. source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor



The rotor and the stator parts of motor.

poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. The synchronous speed is given by the usual relation  $N_s = 120 f / P$ .

However, it is important to understand that the arrangement between the stator and rotor poles is **not an absolutely rigid one**. As the load on the motor is increased, the rotor progressively tends to fall back **in phase** (but *not* in speed as in d.c. motors) by some angle (Fig. 38.4) **but it still continues to run synchronously**. The value of this load angle or coupling angle (as it is called) depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, say,  $\alpha$ .

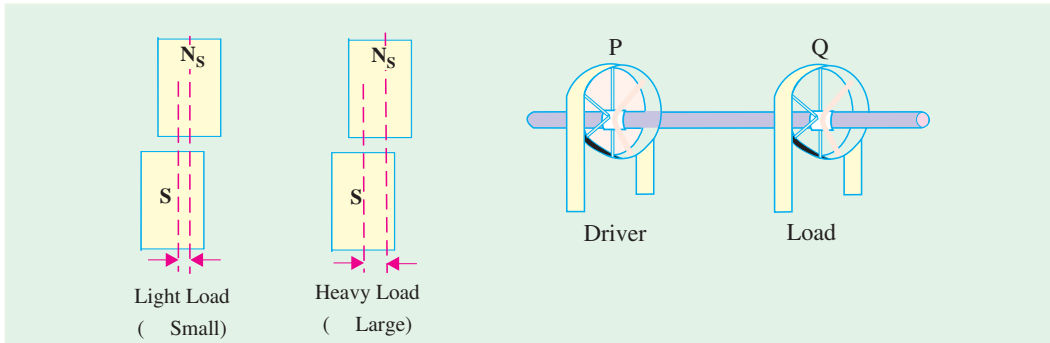


Fig. 38.4

Fig. 38.5

The working of a synchronous motor is, in many ways, similar to the transmission of mechanical power by a shaft. In Fig. 38.5 are shown two pulleys *P* and *Q* transmitting power from the driver to the load. The two pulleys are assumed to be keyed together (just as stator and rotor poles are interlocked) hence they run at exactly the same (average) speed. When *Q* is loaded, it slightly falls behind owing to the twist in the shaft (twist angle corresponds to  $\alpha$  in motor), the angle of twist, in fact, being a measure of the torque transmitted. It is clear that unless *Q* is so heavily loaded as to break the coupling, both **pulleys must run at exactly the same (average) speed**.

### 38.4. Motor on Load with Constant Excitation

Before considering as to what goes on inside a synchronous motor, it is worthwhile to refer briefly to the d.c. motors. We have seen (Art. 29.3) that when a d.c. motor is running on a supply of, say,  $V$  volts then, on rotating, a back e.m.f.  $E_b$  is set up in its armature conductors. The resultant voltage across the armature is  $(V - E_b)$  and it causes an armature current  $I_a = (V - E_b) / R_a$  to flow where  $R_a$  is armature circuit resistance. The value of  $E_b$  depends, among other factors, on the speed of the rotating armature. The mechanical power developed in armature depends on  $E_b I_a$  ( $E_b$  and  $I_a$  being in opposition to each other).

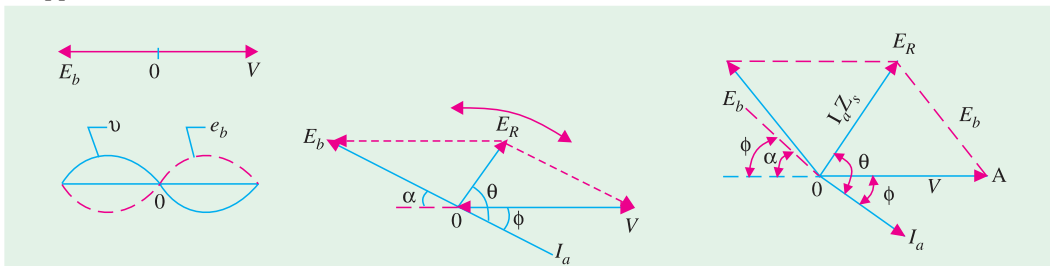


Fig. 38.6

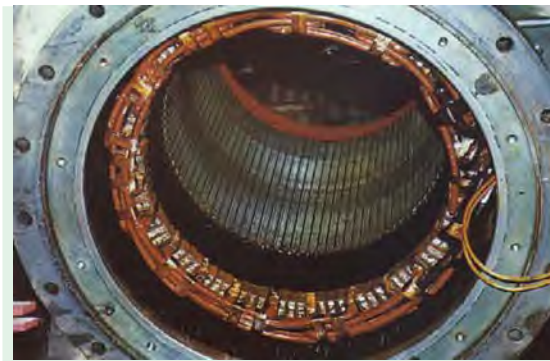
Fig. 38.7

Fig. 38.8

Similarly, in a synchronous machine, a back e.m.f.  $E_b$  is set up in the armature (stator) by the rotor flux which opposes the applied voltage  $V$ . This back e.m.f. depends on rotor excitation only (and not on speed, as in d.c. motors). The net voltage in armature (stator) is the **vector difference** (not arithmetical, as in d.c. motors) of  $V$  and  $E_b$ . Armature current is obtained by dividing this **vector difference** of voltages by armature impedance (not resistance as in d.c. machines).



Fig. 38.6 shows the condition when the motor (properly synchronized to the supply) is running on **no-load** and has **no losses**\* and is having field excitation which makes  $E_b = V$ . It is seen that vector difference of  $E_b$  and  $V$  is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be met by it. In other words, the motor just floats.



Stator of synchronous motor

If motor is on no-load, but it has losses, then the vector for  $E_b$  falls back (vectors are rotating anti-clockwise) by a certain small angle  $\alpha$  (Fig. 38.7), so that a resultant voltage  $E_R$  and hence current  $I_a$  is brought into existence, which supplies losses.\*\*

If, now, the motor is loaded, then its rotor will further fall back **in phase** by a greater value of angle  $\alpha$  – called the load angle or coupling angle (corresponding to the twist in the shaft of the pulleys). The resultant voltage  $E_R$  is increased and motor draws an increased armature current (Fig. 38.8), though at a slightly decreased power factor.

### 38.5. Power Flow within a Synchronous Motor

Let  $R_a$  = armature resistance / phase ;  $X_S$  = synchronous reactance / phase

then  $Z_S = R_a + jX_S$ ;  $I_a = \frac{E_R}{Z_S} = \frac{V - E_b}{Z_S}$ ; Obviously,  $V = E_b + I_a Z_S$

The angle  $\theta$  (known as internal angle) by which  $I_a$  lags behind  $E_R$  is given by  $\tan \theta = X_S / R_a$ .  
 If  $R_a$  is negligible, then  $\theta = 90^\circ$ . Motor input =  $V I_a \cos \phi$  —per phase  
 Here,  $V$  is applied voltage / phase.

Total input for a star-connected, 3-phase machine is,  $P = \sqrt{3} V_L \cdot I_L \cos \phi$ .

The mechanical power developed in the rotor is

$$P_m = \text{back e.m.f.} \times \text{armature current} \times \text{cosine of the angle between the two i.e., angle between } I_a \text{ and } E_b \text{ reversed.}$$

$$= E_b I_a \cos (\alpha - \phi) \text{ per phase} \quad \dots \text{Fig. 38.8}$$

Out of this power developed, some would go to meet iron and friction and excitation losses. Hence, the power available at the shaft would be less than the developed power by this amount.

Out of the input power / phase  $V I_a \cos \phi$ , and amount  $I_a^2 R_a$  is wasted in armature\*\*\*, the rest ( $V I_a \cos \phi - I_a^2 R_a$ ) appears as mechanical power in rotor; out of it, iron, friction and excitation losses are met and the rest is available at the shaft. If power input / phase of the motor is  $P$ , then

$$P = P_m + I_a^2 R_a$$

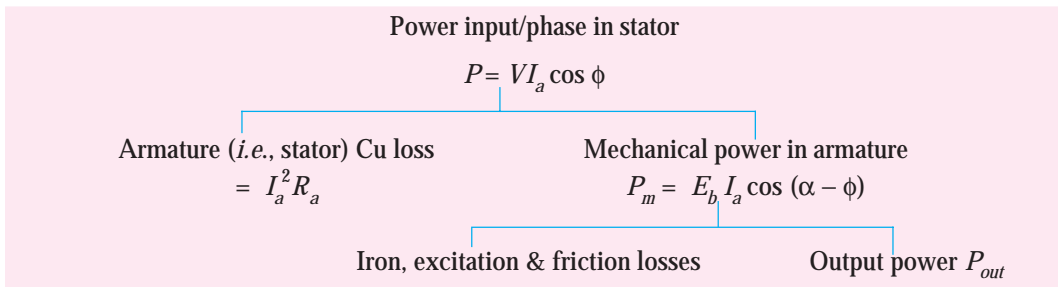
or mechanical power in rotor  $P_m = P - I_a^2 R_a$  —per phase

For three phases  $P_m = \sqrt{3} V_L I_L \cos \phi - 3 I_a^2 R_a$

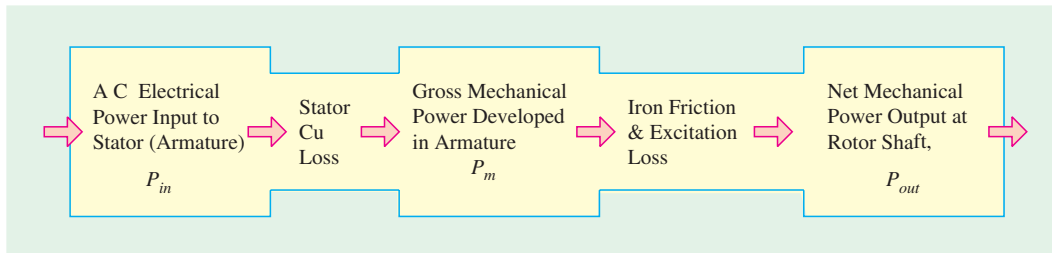
The per phase power development in a synchronous machine is as under :

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\* This figure is exactly like Fig. 37.74 for alternator except that it has been shown horizontally rather than vertically.  
 \*\* It is worth noting that magnitude of  $E_b$  does not change, only its phase changes. Its magnitude will change only when rotor dc excitation is changed i.e., when magnetic strength of rotor poles is changed.  
 \*\*\* The Cu loss in rotor is not met by motor ac input, but by the dc source used for rotor excitation.



Different power stages in a synchronous motor are as under :



### 38.6. Equivalent Circuit of a Synchronous Motor

Fig. 38.9 (a) shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. 38.9 (b) that the phase applied voltage  $V$  is the vector sum of reversed back e.m.f. i.e.,  $-E_b$  and the impedance drop  $I_a Z_s$ . In other words,  $V = (-E_b + I_a Z_s)$ . The angle  $\alpha^*$  between the phasor for  $V$  and  $E_b$  is called the load angle or power angle of the synchronous motor.

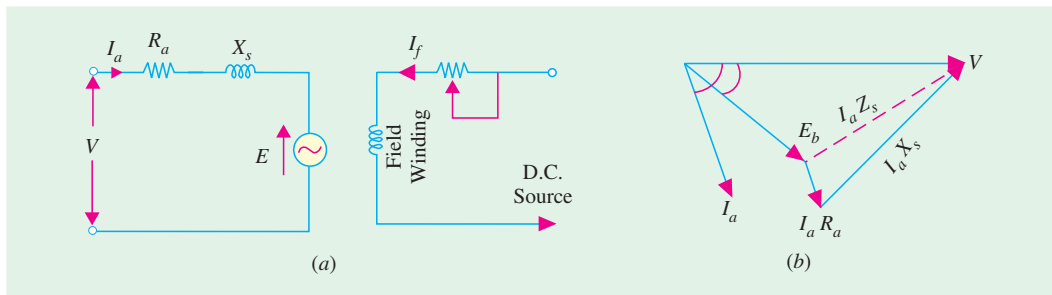


Fig. 38.9

### 38.7. Power Developed by a Synchronous Motor

Except for very small machines, the armature resistance of a synchronous motor is negligible as compared to its synchronous reactance. Hence, the equivalent circuit for the motor becomes as shown in Fig. 38.10 (a). From the phasor diagram of Fig. 38.10 (b), it is seen that

$$AB = E_b \sin \alpha = I_a X_s \cos \phi$$

$$\text{or } VI_a \cos \phi = \frac{E_b V}{X_s} \sin \alpha$$

Now,  $VI_a \cos \phi =$  motor power input/phase

\* This angle was designated as  $\delta$  when discussing synchronous generators.

$$\begin{aligned} \therefore P_{in} &= \frac{E_b V}{X_S} \sin \alpha && \dots \text{per phase}^* \\ &= 3 \frac{E_b V}{X_S} \sin \alpha && \dots \text{for three phases} \end{aligned}$$

Since stator Cu losses have been neglected,  $P_{in}$  also represents the gross mechanical power  $\{P_m\}$  developed by the motor.

$$\therefore P_m = \frac{3E_b V}{X_S} \sin \alpha$$

The gross torque developed by the motor is  $T_g = 9.55 P_m / N_s \text{ N-m}$  ...Ns in rpm.

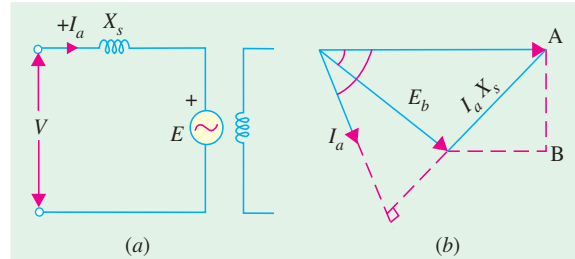


Fig. 38.10

**Example 38.1.** A 75-kW, 3- $\phi$ , Y-connected, 50-Hz, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency excluding field and stator losses, is 95% and  $X_S = 2.5 \Omega$ . Calculate (i) mechanical power developed (ii) armature current (iii) back e.m.f. (iv) power angle and (v) maximum or pull-out torque of the motor.

**Solution.**  $N_s = 120 \times 50/4 = 1500 \text{ rpm} = 25 \text{ rps}$

(i)  $P_m = P_{in} = P_{out} / \eta = 75 \times 10^3 / 0.95 = 78,950 \text{ W}$

(ii) Since power input is known

$$\therefore \sqrt{3} \times 440 \times I_a \times 0.8 = 78,950; \quad I_a = 129 \text{ A}$$

(iii) Applied voltage/phase =  $440/\sqrt{3} = 254 \text{ V}$ . Let  $V = 254 \angle 0^\circ$  as shown in Fig. 38.11.

Now,  $V = E_b + j I X_S$  or  $E_b = V - j I X_S = 254 \angle 0^\circ - 129 \angle 36.9^\circ \times 2.5 \angle 90^\circ = 250 \angle 0^\circ - 322 \angle 126.9^\circ = 254 - 322 (\cos 126.9^\circ + j \sin 126.9^\circ) = 254 - 322 (-0.6 + j 0.8) = 516 \angle -30^\circ$

(iv)  $\therefore \alpha = -30^\circ$

(v) pull-out torque occurs when  $\alpha = 90^\circ$

$$\text{maximum } P_m = 3 \frac{E_b V}{X_S} \sin \delta = 3 \frac{256 \times 516}{2.5} = \sin 90^\circ = 157,275 \text{ W}$$

$$\therefore \text{pull-out torque} = 9.55 \times 157,275 / 1500 = 1,000 \text{ N-m}$$

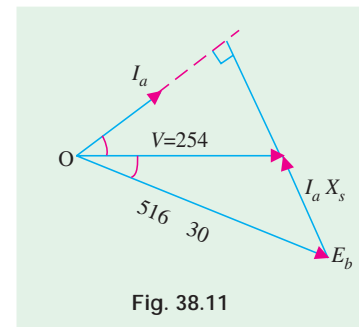


Fig. 38.11

### 38.8. Synchronous Motor with Different Excitations

A synchronous motor is said to have normal excitation when its  $E_b = V$ . If field excitation is such that  $E_b < V$ , the motor is said to be **under-excited**. In both these conditions, it has a lagging power factor as shown in Fig. 38.12.

On the other hand, if d.c. field excitation is such that  $E_b > V$ , then motor is said to be **over-excited** and draws a leading current, as shown in Fig. 38.13 (a). There will be some value of excitation for which armature current will be in phase with  $V$ , so that power factor will become unity, as shown in Fig. 38.13 (b).

\* Strictly speaking, it should be  $P_{in} = \frac{-E_b V}{X_S} \sin \alpha$

The value of  $\alpha$  and back e.m.f.  $E_b$  can be found with the help of vector diagrams for various power factors, shown in Fig. 38.14.

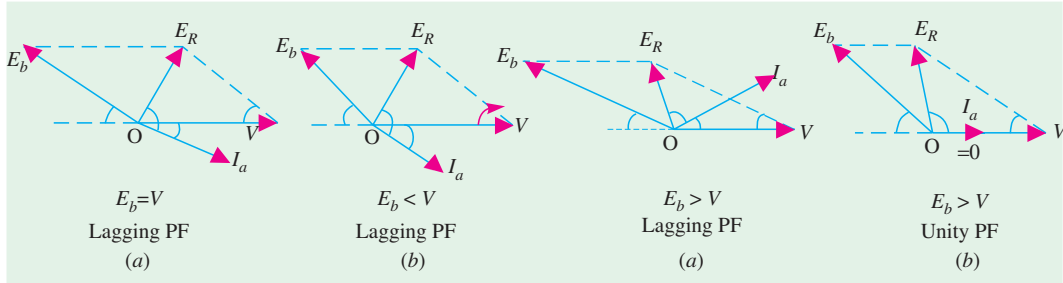


Fig. 38.12

Fig. 38.13

(i) **Lagging p.f.** As seen from Fig. 38.14 (a)

$$AC^2 = AB^2 + BC^2 = [V - E_R \cos(\theta - \phi)]^2 + [E_R \sin(\theta - \phi)]^2$$

$$\therefore E_b = \sqrt{[V - I_a Z_S \cos(\theta - \phi)]^2 + [I_a Z_S \sin(\theta - \phi)]^2}$$

$$\text{Load angle } \alpha = \tan^{-1} \left( \frac{BC}{AB} \right) = \tan^{-1} \left[ \frac{I_a Z_S \sin(\theta - \phi)}{V - I_a Z_S \cos(\theta - \phi)} \right]$$

(ii) **Leading p.f.** [38.14 (b)]

$$E_b = V + I_a Z_S \cos[180^\circ - (\theta + \phi)] + j I_a Z_S \sin[180^\circ - (\theta + \phi)]$$

$$\alpha = \tan^{-1}$$

(iii) **Unity p.f.** [Fig. 38.14 (c)]

Here,  $OB = I_a R_a$  and  $BC = I_a X_S$

$$\therefore E_b = (V - I_a R_a) + j I_a X_S; \alpha = \tan^{-1}$$

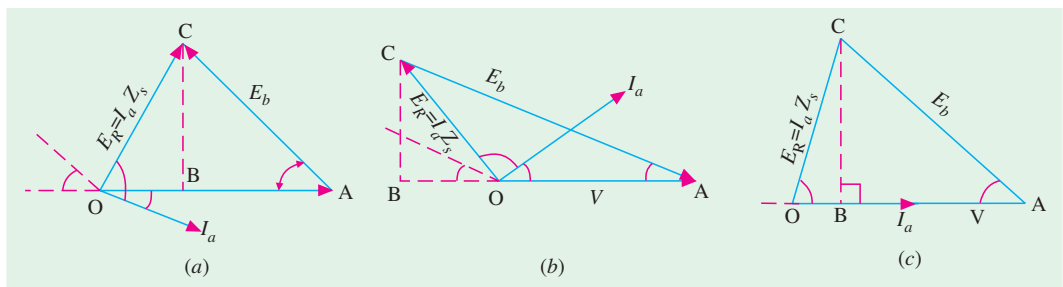


Fig. 38.14

### 38.9. Effect of Increased Load with Constant Excitation

We will study the effect of increased load on a synchronous motor under conditions of normal, under and over-excitation (ignoring the effects of armature reaction). With normal excitation,  $E_b = V$ , with under excitation,  $E_b < V$  and with over-excitation,  $E_b > V$ . Whatever the value of excitation, it would be kept **constant** during our discussion. It would also be assumed that  $R_a$  is negligible as compared to  $X_S$  so that phase angle between  $E_R$  and  $I_a$  i.e.,  $\theta = 90^\circ$ .

(i) **Normal Excitation**

Fig. 38.15. (a) shows the condition when motor is running with light load so that (i) torque angle

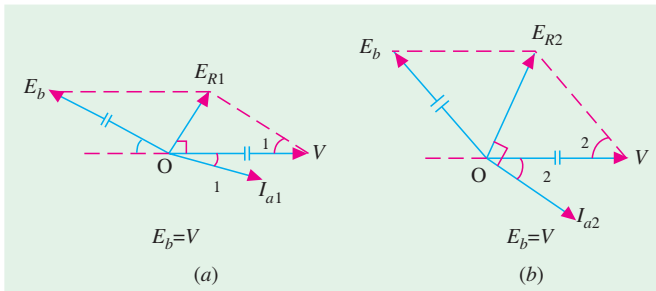


Fig. 38.15

$\alpha_1$  is small (ii) so  $E_{R1}$  is small (iii) hence  $I_{a1}$  is small and (iv)  $\phi_1$  is small so that  $\cos \phi_1$  is large.

Now, suppose that load on the motor is **increased** as shown in Fig. 38.15 (b). For meeting this extra load, motor must develop more torque by drawing more armature current. Unlike a d.c. motor, a synchronous motor cannot increase its  $I_a$  by

decreasing its speed and hence  $E_b$  because both are constant in its case. What actually happens is as under :

1. rotor falls back **in phase i.e.**, load angle increases to  $\alpha_2$  as shown in Fig. 38.15 (b),
2. the resultant voltage in armature is increased **considerably** to new value  $E_{R2}$ ,
3. as a result,  $I_{a1}$  increases to  $I_{a2}$ , thereby increasing the torque developed by the motor,
4.  $\phi_1$  increases to  $\phi_2$ , so that power factor **decreases** from  $\cos \phi_1$  to the new value  $\cos \phi_2$ .

Since increase in  $I_a$  is much greater than the **slight** decrease in power factor, the torque developed by the motor is **increased** (on the whole) to a new value sufficient to meet the extra load put on the motor. It will be seen that essentially it is by increasing its  $I_a$  that the motor is able to carry the extra load put on it.



Geared motor added to synchronous servo motor line offers a wide range of transmission ratios, and drive torques.

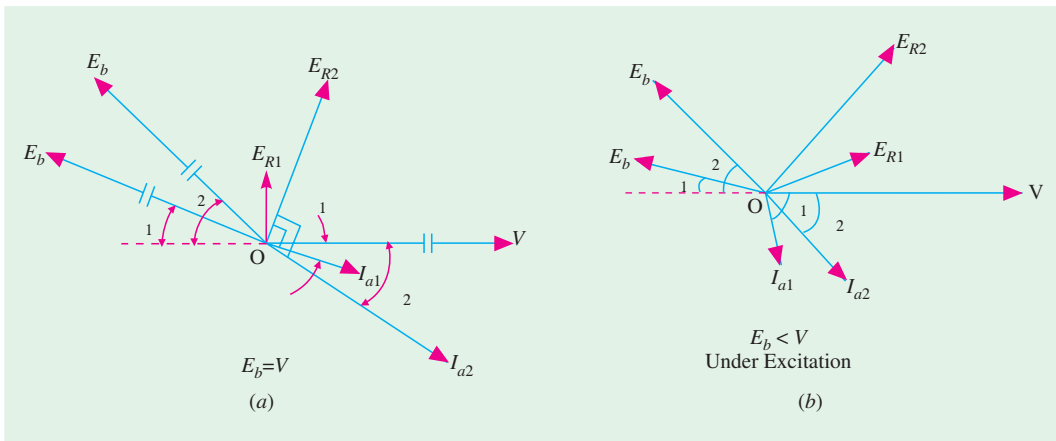


Fig. 38.16

A phase summary of the effect of increased load on a synchronous motor at **normal excitation** is shown in Fig. 38.16 (a) It is seen that there is a comparatively much greater **increase** in  $I_a$  than in  $\phi$ .

**(ii) Under-excitation**

As shown in Fig. 38.16 (b), with a small load and hence, small torque angle  $\alpha_1$ ,  $I_{a1}$  lags behind  $V$  by a **large** phase angle  $\phi_1$  which means poor power factor. Unlike normal excitation, a much larger armature current must flow for developing the same power because of poor power factor. That is why  $I_{a1}$  of Fig. 38.16 (b) is larger than  $I_{a1}$  of Fig. 38.15 (a).

As load increases,  $E_{R1}$  increases to  $E_{R2}$ , consequently  $I_{a1}$  increases to  $I_{a2}$  and p.f. angle **decreases** from  $\phi_1$  to  $\phi_2$  or p.f. **increases** from  $\cos \phi_1$  to  $\cos \phi_2$ . Due to increase both in  $I_a$  and p.f., power generated by the armature increases to meet the increased load. As seen, in this case, **change in power factor is more than the change in  $I_a$** .

**(iii) Over-excitation**

When running on light load,  $\alpha_1$  is small but  $I_{a1}$  is comparatively larger and **leads**  $V$  by a larger angle  $\phi_1$ . Like the under-excited motor, as more load is applied, the power factor improves and **approaches** unity. The armature current also increases thereby producing the necessary increased armature power to meet the increased applied load (Fig. 38.17). However, it should be noted that in this case, power factor angle  $\phi$  decreases (or p.f. increases) at a faster rate than the armature current thereby producing the necessary increased power to meet the increased load applied to the motor.

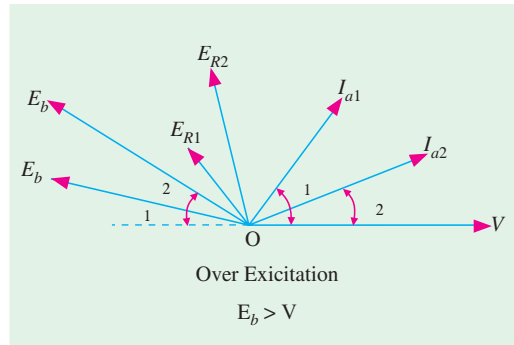


Fig. 38.17

**Summary**

The main points regarding the above three cases can be summarized as under :

1. As load on the motor increases,  $I_a$  **increases regardless of excitation.**
2. For under-and over-excited motors, p.f. tends to approach unity with increase in load.
3. Both with under-and over-excitation, change in p.f. is greater than in  $I_a$  with increase in load.
4. With normal excitation, when load is increased change in  $I_a$  is greater than in p.f. which tends to become increasingly lagging.

**Example 38.2.** A 20-pole, 693-V, 50-Hz, 3- $\phi$ ,  $\Delta$ -connected synchronous motor is operating at no-load with normal excitation. It has armature resistance per phase of zero and synchronous reactance of  $10\Omega$ . If rotor is retarded by  $0.5^\circ$  (mechanical) from its synchronous position, compute.

- |  |                                 |
|--|---------------------------------|
| (i) rotor displacement in electrical degrees | (iii) armature emf / phase      |
| (ii) armature emf / phase                    | (v) power developed by armature |
| (iv) power drawn by the motor                |                                 |

How will these quantities change when motor is loaded and the rotor displacement increases to  $5^\circ$  (mechanical) ?

(Elect. Machines, AMIE Sec. B, 1993)

**Solution.** (a)  $0.5^\circ$  (mech) Displacement [Fig 38.18 (a)]

$$\begin{aligned}
 \text{(i)} \quad \alpha \text{ (elect.)} &= \frac{P}{2} \times \alpha \text{ (mech)} \\
 \therefore \alpha \text{ (elect)} & \\
 &= \frac{20}{2} \times 0.5 = 5^\circ \text{ (elect)}
 \end{aligned}$$

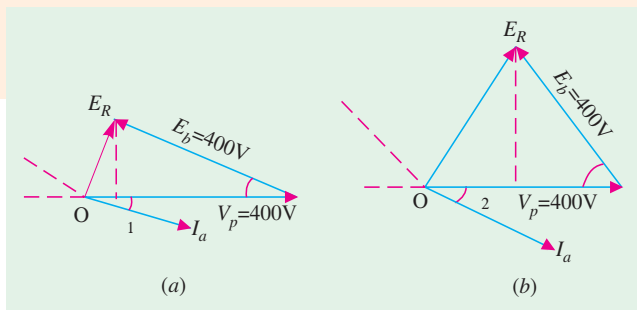


Fig. 38.18

- (ii)  $V_p = V_L / \sqrt{3} = 693 / \sqrt{3}$   
 $= 400 \text{ V}$   
 $E_b = V_p = 400 \text{ V}$   
 $\therefore E_R = (V_p - E_b \cos \alpha) + j E_b \sin \alpha = (400 - 400 \cos 5^\circ + j 400 \sin 5^\circ)$   
 $= 1.5 + j 35 = \mathbf{35 \angle 87.5^\circ \text{ V/phase}}$
- (iii)  $Z_S = 0 + j10 = 10 \angle 90^\circ$ ;  $I_a = E_R / Z_S = 35 \angle 87.5^\circ / 10 \angle 90^\circ = \mathbf{3.5 \angle -2.5^\circ \text{ A/phase}}$   
 Obviously,  $I_a$  lags behind  $V_p$  by  $2.5^\circ$
- (iv) Power input/phase  $V_p I_a \cos \phi = 400 \times 3.5 \times \cos 2.5^\circ = 1399 \text{ W}$   
 Total input power  $= 3 \times 1399 = \mathbf{4197 \text{ W}}$
- (v) Since  $R_a$  is negligible, armature Cu loss is also negligible. Hence 4197 W also represent power developed by armature.
- (b)  $5^\circ$  (mech) Displacement – Fig. 38.18 (b)
- (i)  $\alpha$  (elect)  $= \frac{20}{2} \times 5^\circ = \mathbf{50^\circ}$
- (ii)  $E_R = (400 - 400 \cos 50^\circ) + j400 \sin 50^\circ = 143 + j 306.4 = \mathbf{338.2 \angle 64.9^\circ}$
- (iii)  $I_a = 338.2 \times 64.9^\circ / 10 \angle 90^\circ = \mathbf{33.8 \angle -25.1^\circ \text{ A/phase}}$
- (iv) motor power/phase  $= V_p I_a \cos \phi = 400 \times 33.8 \cos 25.1^\circ = 12,244 \text{ W}$   
 Total power  $= 3 \times 12,244 = 36,732 \text{ W} = \mathbf{36.732 \text{ kW}}$

It is seen from above that as motor load is increased

1. rotor displacement increases from  $5^\circ$  (elect) to  $50^\circ$  (elect) *i.e.*  $E_b$  falls back in phase considerably.
2.  $E_R$  increases from 35 V to 338 V/phase
3.  $I_a$  increases from 3.5 A to 33.8 A
4. angle  $\phi$  increases from  $2.5^\circ$  to  $25.1^\circ$  so that p.f. *decreases* from 0.999 (lag) to 0.906 (lag)
5. increase in power is almost *directly proportional to increase in load angle*.

Obviously, increase in  $I_a$  is much more than decrease in power factor.

It is interesting to note that not only power but even  $I_a$ ,  $E_R$  and  $\phi$  also increase almost as many times as  $\alpha$ .

### Special Illustrative Example 38.3

#### Case of Cylindrical Rotor Machine :

A 3-Phase synchronous machine is worked as follows: Generator - mode : 400 V/Ph, 32 A/Ph, Unity p.f.  $X_S = 10$  ohms. Motoring - mode : 400 V/Ph, 32 A/Ph, Unity p.f. ,  $X_S = 10$  ohms. Calculate  $E$  and  $\delta$  in both the cases and comment.

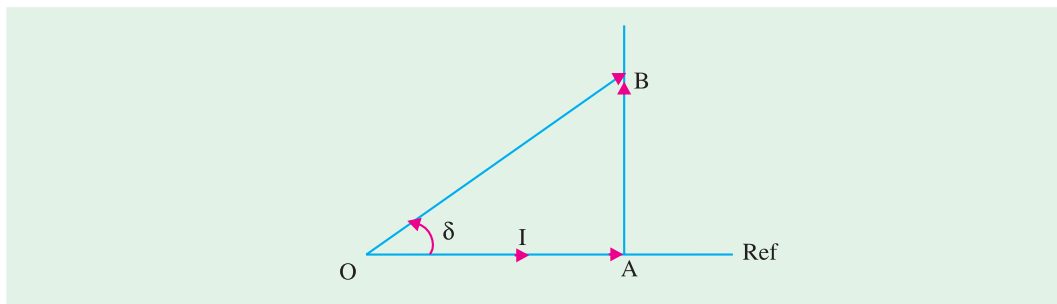


Fig. 38.19 (a) Generator-mode

**Solution.** In Fig. 38.19 (a),  $V = OA = 400$ ,  $IX_s = AB = 320$  V

$$E = OB = 512.25, \delta = \tan^{-1} \frac{320}{400} = 38.66^\circ$$

Total power in terms of parameters measurable at terminals (i.e.,  $V$ ,  $I$ , and  $\phi$ )

$$= 3 V_{ph} I_{ph} \cos \phi = 3 \times 400 \times 32 = 38.4 \text{ kW}$$

Total power using other parameters =  $3 \times \left[ \frac{VE}{X_s} \sin \delta \right] \times 10^{-3} \text{ kW}$

$$= 3 \times \frac{400 \times 512.25}{10} \times (\sin 38.66^\circ) \times 10^{-3} = 38.4 \text{ kW}$$

Since losses are neglected, this power is the electrical output of generator and also is the required mechanical input to the generator.

**For motoring mode :**  $V = OA = 400$ ,  $-IX_s = AB = 320$

$$E = OB = 512.25, \text{ as in Fig. 38.19 (b)}$$

Hence,  $|\delta| = 38.66^\circ$ , as before.

**Comments :** The change in the sign of  $\delta$  has to be noted in the two modes. It is +ve for generator and -ve for motor.  $E$  happens to be equal in both the cases due to unity p.f. At other p.f., this will be different.

As before, power can be calculated in two ways and it will be electrical power input to motor and also the mechanical output of the motor.

Naturally, Power = 38.4 kW

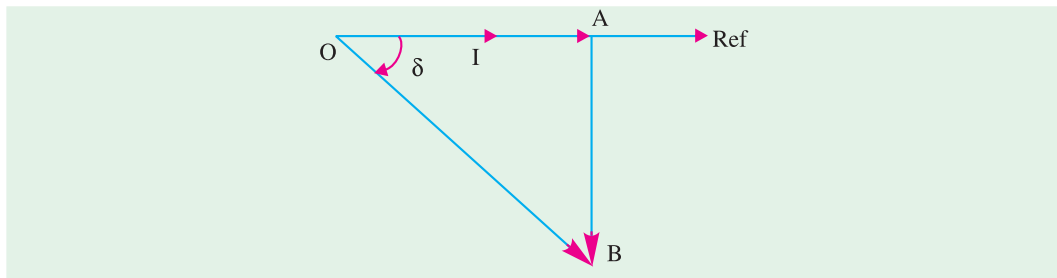


Fig. 38.19 (b) Motoring mode

### 38.10. Effect of Changing Excitation on Constant Load

As shown in Fig. 38.20 (a), suppose a synchronous motor is operating with normal excitation ( $E_b = V$ ) at unity p.f. with a given load. If  $R_a$  is negligible as compared to  $X_s$ , then  $I_a$  lags  $E_R$  by  $90^\circ$  and is in phase with  $V$  because p.f. is unity. The armature is drawing a power of  $V.I_a$  per phase which is enough to meet the mechanical load on the motor. Now, let us discuss the effect of decreasing or increasing the field excitation when the load applied to the motor **remains constant**.

#### (a) Excitation Decreased

As shown in Fig. 38.20 (b), suppose due to decrease in excitation, back e.m.f. is reduced to  $E_{b1}$  at the **same load angle**  $\alpha_1$ . The resultant voltage  $E_{R1}$  causes a lagging armature current  $I_{a1}$  to flow. Even though  $I_{a1}$  is larger than  $I_a$  in magnitude it is incapable of producing necessary power  $V.I_a$  for carrying the **constant** load because  $I_{a1} \cos \phi_1$  component is less than  $I_a$  so that  $V.I_{a1} \cos \phi_1 < V.I_a$

Hence, it becomes necessary for load angle to **increase** from  $\alpha_1$  to  $\alpha_2$ . It increases back e.m.f. from  $E_{b1}$  to  $E_{b2}$  which, in turn, increases resultant voltage from  $E_{R1}$  to  $E_{R2}$ . Consequently, armature current increases to  $I_{a2}$  whose in-phase component produces enough power ( $V.I_{a2} \cos \phi_2$ ) to meet the constant load on the motor.



**(b) Excitation Increased**

The effect of increasing field excitation is shown in Fig. 38.20 (c) where increased  $E_{b1}$  is shown at the original load angle  $\alpha_1$ . The resultant voltage  $E_{R1}$  causes a **leading** current  $I_{a1}$  whose in-phase component is larger than  $I_a$ . Hence, armature develops more power than the load on the motor. Accordingly, load angle **decreases** from  $\alpha_1$  to  $\alpha_2$  which decreases resultant voltage from  $E_{R1}$  to  $E_{R2}$ . Consequently, armature current decreases from  $I_{a1}$  to  $I_{a2}$  whose in-phase component  $I_{a2} \cos \phi_2 = I_a$ . In that case, armature develops power sufficient to carry the constant load on the motor.

Hence, we find that variations in the excitation of a synchronous motor running with a **given** load produce variations in its **load angle only**.

### 38.11. Different Torques of a Synchronous Motor

Various torques associated with a synchronous motor are as follows:

1. starting torque
2. running torque
3. pull-in torque and
4. pull-out torque

**(a) Starting Torque**

It is the torque (or turning effort) developed by the motor when full voltage is applied to its stator (armature) winding. It is also sometimes called **breakaway** torque. Its value may be as low as 10% as in the case of centrifugal pumps and as high as 200 to 250% of full-load torque as in the case of loaded reciprocating two-cylinder compressors.

**(b) Running Torque**

As its name indicates, it is the torque developed by the motor under running conditions. It is determined by the horse-power and speed of the **driven** machine. The peak horsepower determines the maximum torque that would be required by the driven machine. The motor must have a breakdown or a maximum running torque greater than this value in order to avoid stalling.

**(c) Pull-in Torque**

A synchronous motor is started as induction motor till it runs 2 to 5% below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously-rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

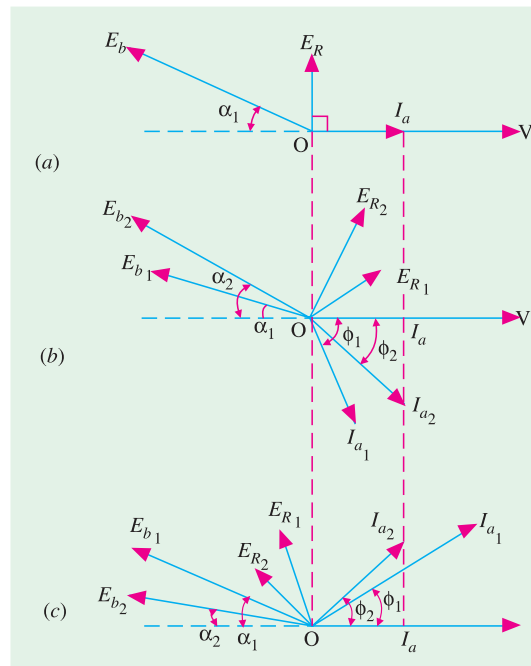


Fig. 38.20



Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions

**(d) Pull-out Torque**

The maximum torque which the motor can develop without pulling out of step or synchronism is called the pull-out torque.

Normally, when load on the motor is increased, its rotor progressively tends to fall back *in phase* by some angle (called load angle) behind the synchronously-revolving stator magnetic field though it keeps running synchronously. Motor develops maximum torque when its rotor is retarded by an angle of 90° (or in other words, it has shifted backward by a distance equal to half the distance between adjacent poles). Any further increase in load will cause the motor to pull out of step (or synchronism) and stop.

**38.12. Power Developed by a Synchronous Motor**

In Fig. 38.21,  $OA$  represents supply voltage/phase and  $I_a = I$  is the armature current,  $AB$  is back e.m.f. at a load angle of  $\alpha$ .  $OB$  gives the resultant voltage  $E_R = IZ_S$  (or  $IX_S$  if  $R_a$  is negligible).  $I$  leads  $V$  by  $\phi$  and lags behind  $E_R$  by an angle  $\theta = \tan^{-1}(X_S/R_a)$ . Line  $CD$  is drawn at an angle of  $\theta$  to  $AB$ .  $AC$  and  $ED$  are  $\perp$  to  $CD$  (and hence to  $AE$  also).

Mechanical power per phase developed in the rotor is

$$P_m = E_b I \cos \psi \quad \dots (i)$$

In  $\Delta OBD$ ,  $BD = I Z_S \cos \psi$

Now,  $BD = CD - BC = AE - BC$

$$I Z_S \cos \psi = V \cos (\theta - \alpha) - E_b \cos \theta$$

$$\therefore I \cos \psi = \frac{V}{Z_S} \cos (\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta$$

Substituting this value in (i), we get

$$P_m \text{ per phase} = E_b \left[ \frac{V}{Z_S} \cos (\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta \right] = \frac{E_b V}{Z_S} \cos (\theta - \alpha) - \frac{E_b^2}{Z_S} \cos \theta \quad \dots (ii)$$

This is the expression for the mechanical power developed in terms of the load angle ( $\alpha$ ) and the internal angle ( $\theta$ ) of the motor for a constant voltage  $V$  and  $E_b$  (or excitation because  $E_b$  depends on excitation only).

If  $T_g$  is the gross armature torque developed by the motor, then

$$T_g \times 2 \pi N_S = P_m \text{ or } T_g = P_m / \omega_s = P_m / 2 \pi N_S \quad - N_S \text{ in rps}$$

$$T_g = \frac{P_m}{2 \pi N_S / 60} = \frac{60}{2 \pi} \cdot \frac{P_m}{N_S} = 9.55 \frac{P_m}{N_S} \quad - N_S \text{ in rpm}$$

Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\therefore \frac{d P_m}{d \alpha} = - \frac{E_b V}{Z_S} \sin (\theta - \alpha) = 0 \quad \text{or} \quad \sin (\theta - \alpha) = 0 \quad \therefore \theta = \alpha$$

\* Since  $R_a$  is generally negligible,  $Z_S = X_S$  so that  $\theta \cong 90^\circ$ . Hence

$$P_m = \frac{E_b V}{X_S} \cos (90^\circ - \alpha) = \frac{E_b V}{X_S} \sin \alpha$$

This gives the value of mechanical power developed in terms of  $\alpha$  – the basic variable of a synchronous machine.

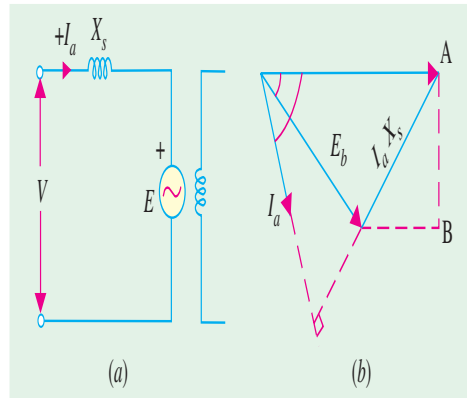


Fig. 38.21

$$\therefore \text{value of maximum power } (P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2}{Z_S} \cos \alpha \text{ or } (P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2}{Z_S} \cos \theta \dots (iii)$$

This shows that the maximum power and hence torque ( $\because$  speed is constant) depends on  $V$  and  $E_b$  i.e., excitation. Maximum value of  $\theta$  (and hence  $\alpha$ ) is  $90^\circ$ . For all values of  $V$  and  $E_b$ , this limiting value of  $\alpha$  is the same but maximum torque will be proportional to the maximum power developed as given in equation (iii). Equation (ii) is plotted in Fig. 38.22.

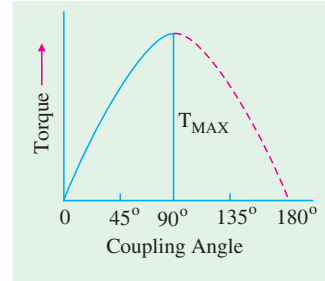


Fig. 38.22

If  $R_a$  is neglected, then  $Z_S \cong X_S$  and  $\theta = 90^\circ \therefore \cos \theta = 0$

$$P_m = \frac{E_b V}{X_S} \sin \alpha \dots (iv) \quad (P_m)_{max} = \frac{E_b V}{X_S} \dots \text{from equation}$$

(iii)\* The same value can be obtained by putting  $\alpha = 90^\circ$  in equation (iv). This corresponds to the 'pull-out' torque.

### 38.13. Alternative Expression for Power Developed

In Fig. 38.23, as usual,  $OA$  represents the supply voltage per phase i.e.,  $V$  and  $AB (= OC)$  is the induced or back e.m.f. per phase i.e.,  $E_b$  at an angle  $\alpha$  with  $OA$ . The armature current  $I$  (or  $I_a$ ) lags  $V$  by  $\phi$ .

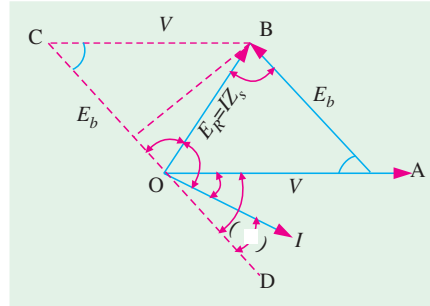


Fig. 38.23

Mechanical power developed is,

$$\begin{aligned} P_m &= E_b \cdot I \times \text{cosine of the angle between } E_b \text{ and } I \\ &= E_b I \cos \angle DOI \\ &= E_b I \cos (\pi - \angle COI) \\ &= -E_b I \cos (\theta + \gamma) \\ &= -E_b \left( \frac{E_R}{Z_S} \right) (\cos \theta \cos \gamma - \sin \theta \sin \gamma) \dots (i) \end{aligned}$$

Now,  $E_R$  and functions of angles  $\theta$  and  $\gamma$  will be eliminated as follows :

$$\text{From } \triangle OAB; V/\sin \gamma = E_R / \sin \alpha \quad \therefore \sin \gamma = V \sin \alpha / E_R$$

$$\text{From } \triangle OBC; E_R \cos \gamma + V \cos \alpha = E_b \quad \therefore \cos \gamma = (E_b - V \cos \alpha) / E_R$$

$$\text{Also} \quad \cos \theta = R_a / Z_S \text{ and } \sin \theta = X_S / Z_S$$

Substituting these values in Eq. (i) above, we get

$$\begin{aligned} P_m &= -\frac{E_b \cdot E_R}{Z_S} \left( \frac{R_a}{Z_S} \cdot \frac{E_b - V \cos \alpha}{E_R} - \frac{X_S}{Z_S} \cdot \frac{V \sin \alpha}{E_R} \right) \\ &= \frac{E_b V}{Z_S^2} (R_a \cos \alpha + X_S \sin \alpha) - \frac{E_b^2 R_a}{Z_S^2} \dots (ii) \end{aligned}$$

It is seen that  $P_m$  varies with  $E_b$  (which depends on excitation) and angle  $\alpha$  (which depends on the motor load).

**Note.** If we substitute  $R_a = Z_S \cos \theta$  and  $X_S = Z_S \sin \theta$  in Eq. (ii), we get

$$P_m = \frac{E_b V}{Z_S^2} (Z_S \cos \theta \cos \alpha + Z_S \sin \theta \sin \alpha) - \frac{E_b^2 Z_S \cos \theta}{Z_S^2} = \frac{E_b V}{Z_S} \cos (\theta - \alpha) - \frac{E_b^2}{Z_S} \cos \theta$$

It is the same expression as found in Art. 38.10.

\* It is the same expression as found for an alternator or synchronous generator in Art. 37.37.

38.14. Various Conditions of Maxima

The following two cases may be considered :

(i) **Fixed  $E_b$ ,  $V$ ,  $R_a$  and  $X_s$ .** Under these conditions,  $P_m$  will vary with load angle  $\alpha$  and will be maximum when  $dP_m / d\alpha = 0$ . Differentiating Eq. (ii) in Art. 38.11, we have

$$\frac{dP_m}{d\alpha} = \frac{E_b V}{Z_s^2} (X_s \cos \alpha - R_a \sin \alpha) = 0 \quad \text{or} \quad \tan \alpha = X_s / R_a = \tan \theta \quad \text{or} \quad \alpha = \theta$$

Putting  $\alpha = \theta$  in the same Eq. (ii), we get

$$\begin{aligned} (P_m)_{max} &= \frac{E_b V}{Z_s^2} (R_a \cos \theta + X_s \sin \theta) - \frac{E_b^2 R_a}{Z_s^2} = \\ &= \frac{E_b V}{Z_s^2} \left( \frac{R_a^2 + X_s^2}{Z_s} \right) - \frac{E_b^2 R_a}{Z_s^2} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} \quad \dots (i) \end{aligned}$$

This gives the value of power at which the motor falls out of step.

Solving for  $E_b$  from Eq. (i) above, we get

$$E_b = \frac{Z_s}{2R_a} \left[ V \pm \sqrt{V^2 - 4R_a \cdot (P_m)_{max}} \right]$$

The two values of  $E_b$  so obtained represent the excitation limits for any load.

(ii) **Fixed  $V$ ,  $R_a$  and  $X_s$ .** In this case,  $P_m$  varies with excitation or  $E_b$ . Let us find the value of the excitation or induced e.m.f.  $E_b$  which is necessary for maximum power possible. For this purpose, Eq. (i) above may be differentiated with respect to  $E_b$  and equated to zero.

$$\therefore \frac{d(P_m)_{max}}{dE_b} = \frac{V}{Z_s} - \frac{2R_a E_b}{Z_s^2} = 0; \quad E_b = \frac{V Z_s}{2R_a} \quad \dots (ii)$$

Putting this value of  $E_b$  in Eq. (i) above, maximum power developed becomes

$$(P_m)_{max} = \frac{V^2}{2R_a} - \frac{V^2}{4R_a} = \frac{V^2}{4R_a}$$

38.15. Salient Pole Synchronous Motor

Cylindrical-rotor synchronous motors are much easier to analyse than those having salient-pole rotors. It is due to the fact that cylindrical-rotor motors have a uniform air-gap, whereas in salient-pole motors, air-gap is much greater between the poles than along the poles. Fortunately, cylindrical rotor theory is reasonably accurate in predicting the steady-state performance of salient-pole motors. Hence, salient-pole theory is required only when very high degree of accuracy is needed or when problems concerning transients or power system stability are to be handled.

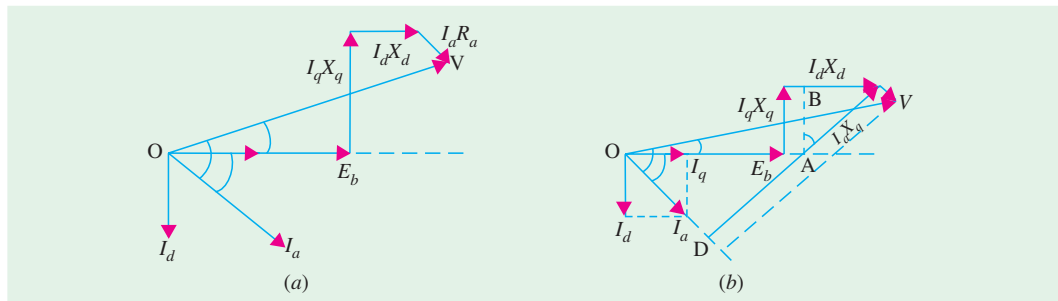


Fig. 38.24

\* This is the value of induced e.m.f. to give maximum power, but it is not the maximum possible value of the generated voltage, at which the motor will operate.

The  $d$ - $q$  currents and reactances for a salient-pole synchronous motor are exactly the same as discussed for salient-pole synchronous generator. The motor has  $d$ -axis reactance  $X_d$  and  $q$ -axis reactance  $X_q$ . Similarly, motor armature current  $I_a$  has two components:  $I_d$  and  $I_q$ . The complete phasor diagram of a salient-pole synchronous motor, for a lagging power factor is shown in Fig. 38.24 (a).

With the help of Fig. 38.24 (b), it can be proved that  $\tan \psi = \frac{V \sin \phi - I_q X_q}{V \cos \phi - I_a R_a}$

If  $R_a$  is negligible, then  $\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$

For an overexcited motor *i.e.*, when motor has leading power factor,

$$\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$$

The power angle  $\alpha$  is given by  $\alpha = \phi - \psi$

The magnitude of the excitation or the back e.m.f.  $E_b$  is given by

$$E_b = V \cos \alpha - I_q R_a - I_d X_d$$

Similarly, as proved earlier for a synchronous generator, it can also be proved from Fig. 38.24 (b) for a synchronous motor with  $R_a = 0$  that

$$\tan \alpha = \frac{I_a X_q \cos \phi}{V - I_a X_q \sin \phi}$$

In case  $R_a$  is not negligible, it can be proved that

$$\tan \alpha = \frac{I_a X_q \cos \phi - I_a R_a \sin \phi}{V - I_a X_q \sin \phi - I_a R_a \cos \alpha}$$

### 38.16. Power Developed by a Salient Pole Synchronous Motor

The expression for the power developed by a salient-pole synchronous generator derived in Chapter 35 also applies to a salient-pole synchronous motor.

$$\begin{aligned} \therefore P_m &= \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \quad \dots \text{per phase} \\ &= 3 \times \left[ \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \right] \dots \text{per three phases} \\ T_g &= 9.55 P_m / N_s \quad \dots N_s \text{ in rps.} \end{aligned}$$

As explained earlier, the power consists of two components, the first component is called excitation power or magnet power and the second is called reluctance power (because when excitation is removed, the motor runs as a reluctance motor).

**Example 38.4.** A 3- $\phi$ , 150-kW, 2300-V, 50-Hz, 1000-rpm salient-pole synchronous motor has  $X_d = 32 \Omega$  / phase and  $X_q = 20 \Omega$  / phase. Neglecting losses, calculate the torque developed by the motor if field excitation is so adjusted as to make the back e.m.f. twice the applied voltage and  $\alpha = 16^\circ$ .

**Solution.**

$$V = 2300 / \sqrt{3} = 1328 \text{ V}; E_b = 2 \times 1328 = 2656 \text{ V}$$

$$\text{Excitation power / phase} = \frac{E_b V}{X_d} \sin \alpha = \frac{2656 \times 1328}{32} \sin 16^\circ = 30,382 \text{ W}$$

$$\text{Reluctance power / phase} = \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha = \frac{1328^2 (32 - 20)}{2 \times 32 \times 20} \sin 32^\circ = 8760 \text{ W}$$

$$\text{Total power developed, } P_m = 3(30382 + 8760) = 117,425 \text{ W}$$

$$T_g = 9.55 \times 117,425 / 1000 = \mathbf{1120 \text{ N-m}}$$

**Example 38.5.** A 3300-V, 1.5-MW, 3- $\phi$ , Y-connected synchronous motor has  $X_d = 4\Omega$  / phase and  $X_q = 3\Omega$  / phase. Neglecting all losses, calculate the excitation e.m.f. when motor supplies rated load at unity p.f. Calculate the maximum mechanical power which the motor would develop for this field excitation. (Similar Example, Swami Ramanand Teertha Marathwada Univ. Nanded 2001)

**Solution.**

$$V = 3300 / \sqrt{3} = 1905 \text{ V}; \cos \phi = 1; \sin \phi = 0; \phi = 0^\circ$$

$$I_a = 1.5 \times 10^6 / \sqrt{3} \times 3300 \times 1 = 262 \text{ A}$$

$$\tan \psi = \frac{V \sin \phi - I_a X_q}{V \cos \phi} = \frac{1905 \times 0 - 262 \times 3}{1905} = -0.4125; \psi = -22.4^\circ$$

$$\alpha = \phi - \psi = 0 - (-22.4^\circ) = 22.4^\circ$$

$$I_d = 262 \times \sin(-22.4^\circ) = -100 \text{ A}; I_q = 262 \cos(-22.4^\circ) = 242 \text{ A}$$

$$E_b = V \cos \alpha - I_d X_d = 1905 \cos(-22.4^\circ) - (-100 \times 4) = 2160 \text{ V}$$

$$= 1029 \sin \alpha + 151 \sin 2\alpha$$

$$P_m = \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \quad \dots \text{ per phase}$$

$$= \frac{2160 \times 1905}{4 \times 1000} + \frac{1905^2 (4 - 3)}{2 \times 4 \times 3 \times 1000} \sin 2\alpha \quad \dots \text{ kW/phase}$$

$$= 1029 \sin \alpha + 151 \sin 2\alpha \quad \dots \text{ kW/phase}$$

If developed power has to achieve maximum value, then

$$\frac{dP_m}{d\alpha} = 1029 \cos \alpha + 2 \times 151 \cos 2\alpha = 0$$

$$\therefore 1029 \cos \alpha + 302 (2 \cos^2 \alpha - 1) = 0 \quad \text{or} \quad 604 \cos^2 \alpha + 1029 \cos \alpha - 302 = 0$$

$$\therefore \cos \alpha = \frac{-1029 \pm \sqrt{1029^2 + 4 \times 604 \times 302}}{2 \times 604} = 0.285; \alpha = 73.4^\circ$$

$$\therefore \text{maximum } P_m = 1029 \sin 73.4^\circ + 151 \sin 2 \times 73.4^\circ = 1070 \text{ kW/phase}$$

Hence, maximum power developed for three phases =  $3 \times 1070 = 3210 \text{ kW}$

**Example 38.6.** The input to an 11000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find (i) the power supplied to the motor (ii) mechanical power developed and (iii) induced emf for a power factor of 0.8 leading. (Elect. Engg. AMIETE (New Scheme) June 1990)

**Solution.** (i) Motor power input =  $\sqrt{3} \times 11000 \times 60 \times 0.8 = 915 \text{ kW}$

(ii) stator Cu loss/phase =  $60^2 \times 1 = 3600 \text{ W}$ ; Cu loss for three phases =  $3 \times 3600 = 10.8 \text{ kW}$

$$P_m = P_2 - \text{rotor Cu loss} = 915 - 10.8 = 904.2 \text{ kW}$$

$$V_p = 11000 / \sqrt{3} = 6350 \text{ V}; \phi = \cos^{-1} 0.8 = 36.9^\circ;$$

$$\theta = \tan^{-1} (30/1) = 88.1^\circ$$

$$Z_s \cong 30 \Omega; \text{stator impedance drop / phase} = I_a Z_s$$

$$= 60 \times 30 = 1800 \text{ V}$$

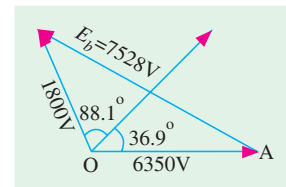


Fig. 38.25

As seen from Fig. 38.25,

$$E_b^2 = 6350^2 + 1800^2 - 2 \times 6350 \times 1800 \times \cos(88.1^\circ + 36.9^\circ)$$

$$= 6350^2 + 1800^2 - 2 \times 6350 \times 1800 \times -0.572$$

$$\therefore E_b = 7528 \text{ V}; \text{line value of } E_b = 7528 \times \sqrt{3} = 13042$$

**Special Example 38.7. Case of Salient - Pole Machines**

A synchronous machine is operated as below :

As a Generator : 3 -Phase,  $V_{ph} = 400$ ,  $I_{ph} = 32$ , unity p.f.

As a Motor : 3 -Phase,  $V_{ph} = 400$ ,  $I_{ph} = 32$ , unity p.f.

Machine parameters :  $X_d = 10\ \Omega$ ,  $X_q = 6.5\ \Omega$

Calculate excitation emf and  $\delta$  in the two modes and deal with the term power in these two cases.

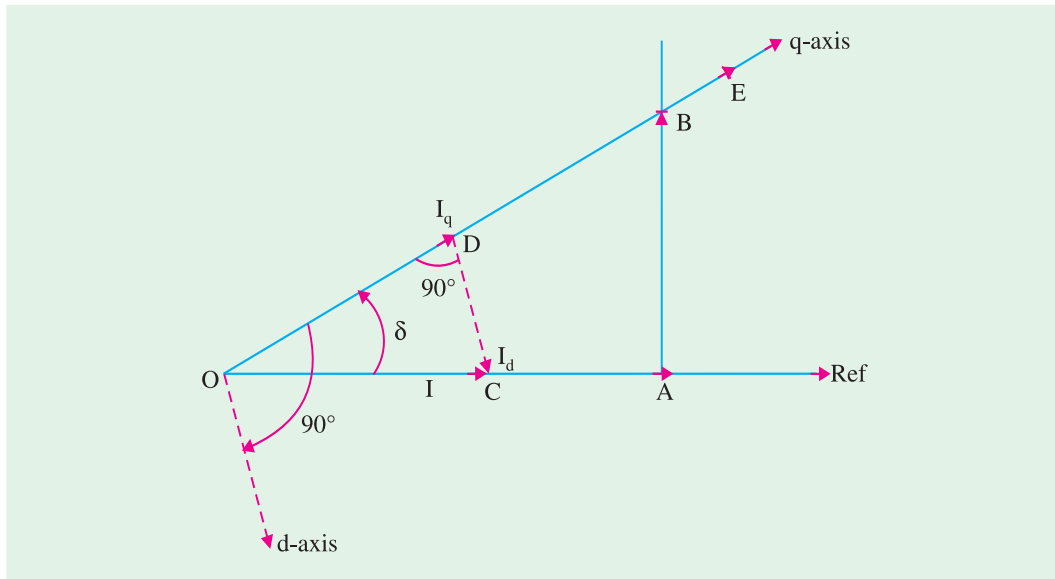


Fig. 38.26 (a) Generator-action

**Solution.**

**Generating Mode :**

**Voltagess :**

$$OA = 400 \text{ V}, AB = IX_q \\ = 32 \times 6.5 = 208 \text{ V}$$

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V},$$

$$\delta = \tan^{-1} \frac{AB}{OA} = \tan^{-1} \frac{208}{400} = 27.5^\circ$$

$$BE = I_d(X_d - X_q) \\ = 14.8 \times 3.5 = 51.8 \text{ V},$$

$$E = OE = OB + BE = 502.8 \text{ V}$$

**Currents :**  $I = OC = 32$ ,  $I_q = I \cos \delta = OD = 28.4$  amp.,  $I_d = DC = I \sin \delta = 14.8$  amp.  $E$  leads  $V$  in case of generator, as shown in Fig. 38.26 (a)

$$\text{Power (by one formula)} = 3 \times 400 \times 32 \times 10^{-3} = 38.4 \text{ kW}$$

$$\text{or Power (by another formula)} = 3 \times \left[ \frac{400 \times 502.8}{10} \sin 27.5^\circ + \frac{400^2}{2} \times \left( \frac{3.5}{65} \right) \times \sin 55^\circ \right] \\ = 38.44 \text{ kW}$$

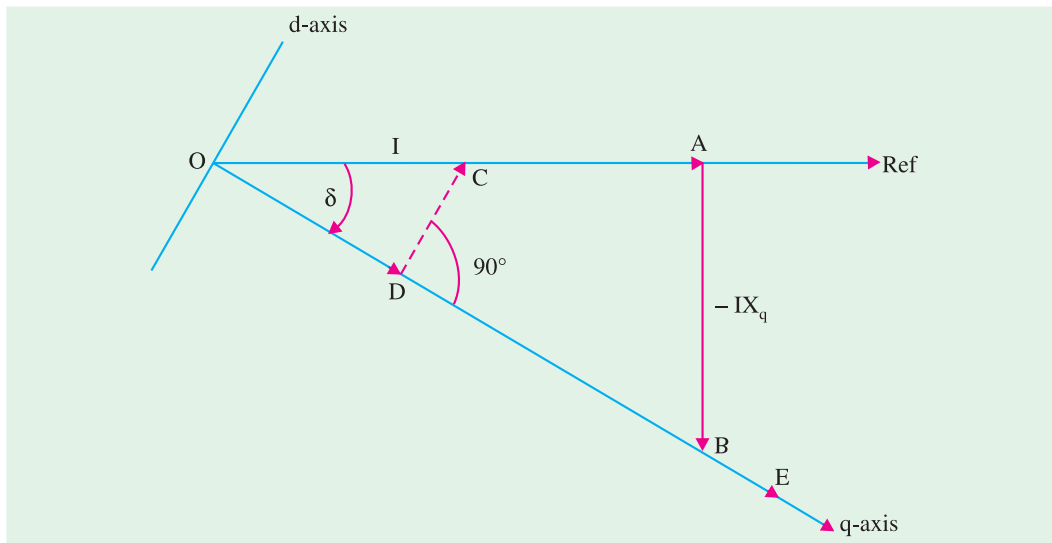


Fig. 38.26 (b) Phasor diagram : Motoring mode

**Motoring mode of a salient pole synchronous machine**

**Voltages :**  $OA = 400 \text{ V}, AB = -IX_q = 208 \text{ V}$

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V}$$

$$\delta = \tan^{-1} \frac{AB}{OA} = \tan^{-1} \frac{208}{400} = 27.5^\circ \text{ as before but now } E \text{ lags behind } V.$$

$$BE = I_d(X_d - X_q) = 51.8 \text{ V in the direction shown. } OE = 502.8 \text{ V as before}$$

**Currents :**  $OC = 32 \text{ amp. } OD = 28.4 \text{ amp. } DC = 14.8 \text{ amp.}$  Naturally,  $I_q = 28.4 \text{ amp.}$  and  $I_d = 14.8 \text{ amp}$

Power (by one formula) = 38.4 kW

Power (by another formula) = 38.44 kW

**Note.** Numerical values of  $E$  and  $\delta$  are same in cases of generator-mode and motor-mode, due to unity p.f.  $\delta$  has different signs in the two cases.

**Example 38.8.** A 500-V, 1-phase synchronous motor gives a net output mechanical power of 7.46 kW and operates at 0.9 p.f. lagging. Its effective resistance is  $0.8 \Omega$ . If the iron and friction losses are 500 W and excitation losses are 800 W, estimate the armature current. Calculate the commercial efficiency. **(Electrical Machines-I, Gujarat Univ. 1988)**

**Solution.** Motor input =  $VI_a \cos \phi$ ; Armature Cu loss =  $I_a^2 R_a$

Power developed in armature is  $P_m = VI_a \cos \phi - I_a^2 R_a$

$$\therefore I_a^2 R_a - VI_a \cos \phi + P_m = 0 \text{ or } I_a = \frac{V \cos \phi \pm \sqrt{V^2 \cos^2 \phi - 4 R_a P_m}}{2 R_a}$$

Now,

$$P_{out} = 7.46 \text{ kW} = 7,460 \text{ W}$$

$$P_m = P_{out} + \text{iron and friction losses} + \text{excitation losses}$$

$$= 7460 + 500 + 800 = 8760 \text{ W} \quad \dots \text{ Art. 38.5}$$

$$I_a = \frac{500 \times 0.9 \pm \sqrt{(500 \times 0.9)^2 - 4 \times 0.8 \times 3760}}{2 \times 0.8}$$

$$= \frac{450 \pm \sqrt{202,500 - 28,030}}{1.6} = \frac{450 \pm 417.7}{1.6} = \frac{32.3}{1.6} = 20.2 \text{ A}$$



Motor input =  $500 \times 20.2 \times 0.9 = 9090 \text{ W}$

$\eta_c = \text{net output} / \text{input} = 7460 / 9090 = 0.8206$  or **82.06%**.

**Example 38.9.** A 2,300-V, 3-phase, star-connected synchronous motor has a resistance of 0.2 ohm per phase and a synchronous reactance of 2.2 ohm per phase. The motor is operating at 0.5 power factor leading with a line current of 200 A. Determine the value of the generated e.m.f. per phase. (Elect. Engg.-I, Nagpur Univ. 1993)

**Solution.** Here,  $\phi = \cos^{-1}(0.5) = 60^\circ$  (lead)  
 $\theta = \tan^{-1}(2.2/0.2) = 84.8^\circ$   
 $\therefore (\theta + \phi) = 84.8^\circ + 60^\circ = 144.8^\circ$   
 $\cos 144.8^\circ = -\cos 35.2^\circ$   
 $V = 2300 / \sqrt{3} = 1328 \text{ volt}$   
 $Z_S = \sqrt{0.2^2 + 2.2^2} = 2.209 \Omega$   
 $I Z_S = 200 \times 2.209 = 442 \text{ V}$

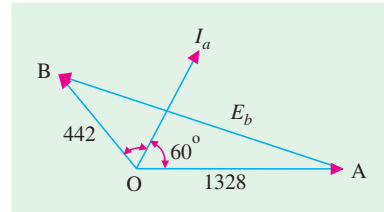


Fig. 38.27

The vector diagram is shown in Fig. 38.27.

$$E_b = \sqrt{V^2 + E_R^2 - 2 V \cdot E_R \cos(\theta + \phi)}$$

$$= \sqrt{1328^2 + 442^2 + 2 \times 1328 \times 442 \times \cos 35.2^\circ} = \mathbf{1708 \text{ Volt / Phase}}$$

**Example 38.10.** A 3-phase, 6,600-volts, 50-Hz, star-connected synchronous motor takes 50 A current. The resistance and synchronous reactance per phase are 1 ohm and 20 ohm respectively. Find the power supplied to the motor and induced emf for a power factor of (i) 0.8 lagging and (ii) 0.8 leading. (Eect. Engg. II pune Univ. 1988)

**Solution. (i) p.f. = 0.8 lag** (Fig. 38.28 (a)).

Power input =  $\sqrt{3} \times 6600 \times 50 \times 0.8 = 457,248 \text{ W}$

Supply voltage / phase =  $6600 / \sqrt{3} = 3810 \text{ V}$

$\phi = \cos^{-1}(0.8) = 36^\circ 52'$ ;  $\theta = \tan^{-1}(X_S / R_a) = (20/1) = 87.8'$

$Z_S = \sqrt{20^2 + 1^2} = 20 \Omega$  (approx.)

Impedance drop =  $I_a Z_S = 50 \times 20 = 1000 \text{ V/phase}$

$\therefore E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times \cos(87^\circ 8' - 36^\circ 52')$   $\therefore E_b = 3263 \text{ V / phase}$

Line induced e.m.f. =  $3263 \times \sqrt{3} = \mathbf{5651 \text{ V}}$

(ii) Power input would remain the same.

As shown in Fig. 38.28 (b), the current vector is drawn at a **leading** angle of

$\phi = 36^\circ 52'$

Now,  $(\theta + \phi) = 87^\circ 8' + 36^\circ 52' = 124^\circ$ ,  
 $\cos 124^\circ = -\cos 56^\circ$

$\therefore E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times -\cos 56^\circ$   $\therefore E_b = 4447 \text{ V / phase}$

Line induced e.m.f. =  $\sqrt{3} \times 4447$   
 = **7,700 V**

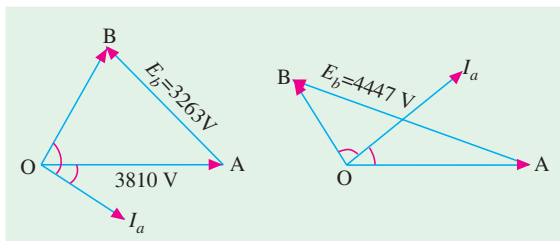


Fig. 38.28 (a)

Fig. 38.28 (b)

**Note.** It may be noted that if  $E_b > V$ , then motor has a leading power factor and if  $E_b < V$ .

**Example 38.11.** A synchronous motor having 40% reactance and a negligible resistance is to be operated at rated load at (i) u.p.f. (ii) 0.8 p.f. lag (iii) 0.8 p.f. lead. What are the values of induced e.m.f.? Indicate assumptions made, if any. (Electrical Machines-II, Indore Univ. 1990)

**Solution.** Let  $V = 100 \text{ V}$ , then reactance drop  $= I_a X_S = 40 \text{ V}$

(i) At unity p.f.

Here,  $\theta = 90^\circ$ ,  $E_b = \sqrt{100^2 + 40^2} = 108 \text{ V}$  ...Fig. 38.29 (a)

(ii) At p.f. 0.8 (lag.) Here  $\angle BOA = \theta - \phi = 90^\circ - 36^\circ 54' = 53^\circ 6'$

$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 53^\circ 6'$ ;  $E_b = 82.5 \text{ V}$ , as in Fig. 38.29 (b)

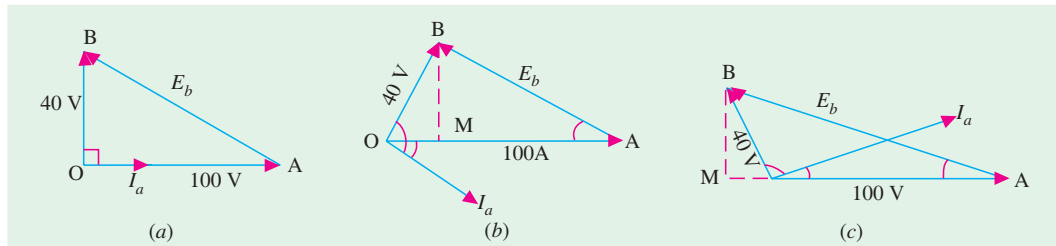


Fig. 38.29

Alternatively,  $E_b = AB = \sqrt{AM^2 + MB^2} = \sqrt{76^2 + 32^2} = 82.5 \text{ V}$

(iii) At p.f. 0.8 (lead.) Here,  $(\theta + \phi) = 90^\circ + 36.9^\circ = 126.9^\circ$

$E_b^2 = 100^2 + 40^2 - 2 \times 40 \times \cos 126.9^\circ = 128 \text{ V}$

Again from Fig. 38.29 (c),  $E_b^2 = (OM + OA)^2 + MB^2 = 124^2 + 32^2$ ;  $E_b = 128 \text{ V}$ .

**Example 38.12.** A 1,000-kVA, 11,000-V, 3- $\phi$ , star-connected synchronous motor has an armature resistance and reactance per phase of  $3.5 \Omega$  and  $40 \Omega$  respectively. Determine the induced e.m.f. and angular retardation of the rotor when fully loaded at (a) unity p.f. (b) 0.8 p.f. lagging (c) 0.8 p.f. leading. (Elect. Engineering-II, Bangalore Univ. 1992)

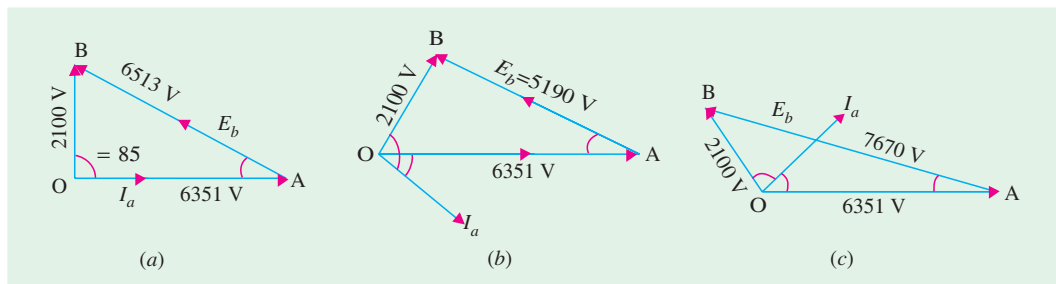


Fig. 38.30

**Solution.** Full-load armature current  $= 1,000 \times 1,000 / \sqrt{3} \times 11,000 = 52.5 \text{ A}$

Voltage / phase  $= 11,000 / \sqrt{3} = 6,351 \text{ V}$ ;  $\cos \phi = 0.8 \therefore \phi = 36^\circ 53'$

Armature resistance drop / phase  $= I_a R_a = 3.5 \times 52.5 = 184 \text{ V}$

reactance drop / phase  $= I_a X_S = 40 \times 52.5 = 2,100 \text{ V}$

$\therefore$  impedance drop / phase  $= I_a Z_S = \sqrt{(184^2 + 2100^2)} = 2,100 \text{ V (approx.)}$

$\tan \theta = X_S / R_a \therefore \theta = \tan^{-1} (40 / 3.5) = 85^\circ$

(a) At unity p.f. Vector diagram is shown in Fig. 38.30 (a)

$$E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \cos 85^\circ; E_b = 6,513 \text{ V per phase}$$

$$\text{Induced line voltage} = 6,513 \times \sqrt{3} = \mathbf{11,280 \text{ V}}$$

$$\text{From } \triangle OAB, \frac{2100}{\sin \alpha} = \frac{6153}{\sin 85^\circ} = \frac{6153}{0.9961}$$

$$\sin \alpha = 2,100 \times 0.9961 / 6,513 = 0.3212 \quad \therefore \alpha = \mathbf{18^\circ 44'}$$

(b) At p.f. 0.8 lagging – Fig. 38.30 (b)

$$\angle BOA = \theta - \phi = 85^\circ - 36^\circ 53' = 48^\circ 7'$$

$$E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \times \cos 48^\circ 7'$$

$$E_b = 5,190 \text{ V per phase}$$

$$\text{Induced line voltage} = 5,190 \times \sqrt{3} = \mathbf{8,989 \text{ V}}$$

Again from the  $\triangle OAB$  of Fig. 36.30 (b)

$$\frac{2100}{\sin \alpha} = \frac{5190}{\sin 48^\circ 7'} = \frac{5,190}{0.7443}$$

$$\therefore \sin \alpha = 2100 \times 0.7443 / 5190 = 0.3012 \quad \therefore \alpha = \mathbf{17^\circ 32'}$$

(c) At p.f. 0.8 leading [Fig. 38.30 (c)]

$$\angle BOA = \theta + \phi = 85^\circ + 36^\circ 53' = 121^\circ 53'$$

$$\therefore E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \times \cos 121^\circ 53'$$

$$\therefore E_b = 7,670 \text{ volt per phase.}$$

$$\text{Induced line e.m.f} = 7,670 \times \sqrt{3} = \mathbf{13,280 \text{ V}}$$

$$\text{Also, } \frac{2,100}{\sin \alpha} = \frac{7,670}{\sin 121^\circ 53'} = \frac{7,670}{0.8493}$$

$$\therefore \sin \alpha = 2,100 \times 0.8493 / 7,670 = 0.2325 \quad \therefore \alpha = \mathbf{13^\circ 27'}$$

**Special Example 38.13.** Both the modes of operation : Phase - angle = 20° Lag

Part (a) : A three phase star-connected synchronous generator supplies a current of 10 A having a phase angle of 20° lagging at 400 volts/phase. Find the load angle and components of armature current (namely  $I_d$  and  $I_q$ ) if  $X_d = 10$  ohms,  $X_q = 6.5$  ohms. Neglect  $r_a$ . Calculate voltage regulation.

**Solution.** The phasor diagram is drawn in Fig 38.31 (a)

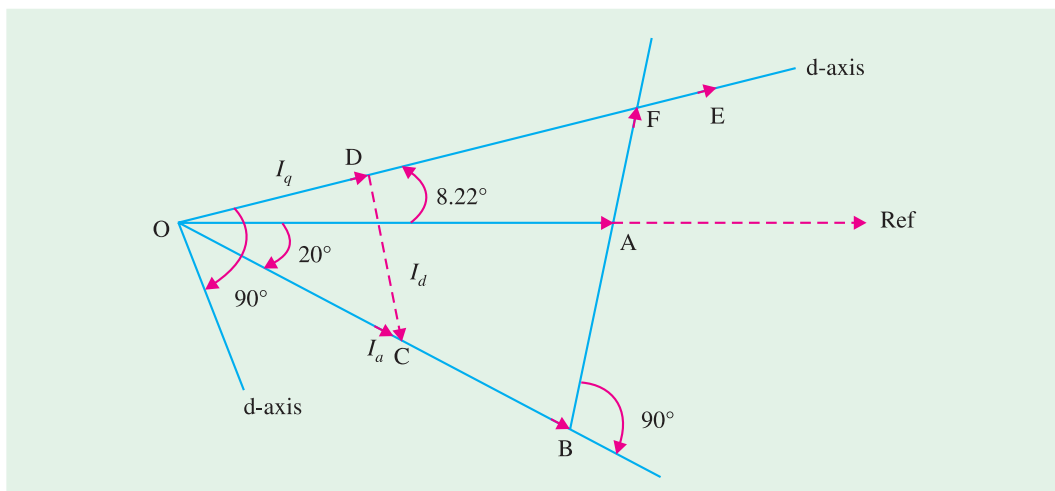


Fig. 38.31 (a) : Generator-mode

$$\begin{aligned}
 OA &= 400 \text{ V}, OB = 400 \cos 20^\circ = 376 \text{ V}, AB = 400 \sin 20^\circ = 136.8 \text{ V} \\
 AF &= IX_q = 10 \times 6.5 = 65 \text{ V}, BF = BA + AF = 201.8 \text{ V} \\
 OF &= \sqrt{376^2 + 201.8^2} = 426.7 \text{ V}, \delta = 8.22^\circ \\
 DC &= I_d = I_a \sin 28.22^\circ = 4.73 \text{ amp}, DC \text{ perpendicular to } OD, \\
 OD &= I_q = I_a \cos 28.22^\circ = 8.81 \text{ amp} \\
 FE &= I_d(X_d - X_q) = 4.73 \times 3.5 = 16.56 \text{ V. This is along the direction of '+q' -axis} \\
 E &= OE = OF + FE = 426.7 + 16.56 = 443.3 \text{ V}
 \end{aligned}$$

$$\% \text{ Regulation} = \frac{443 - 400}{400} \times 100\% = 10.75 \%$$

If the same machine is now worked as a synchronous motor with terminal voltage, supply-current and its power-factor kept unaltered, find the excitation emf and the load angle.

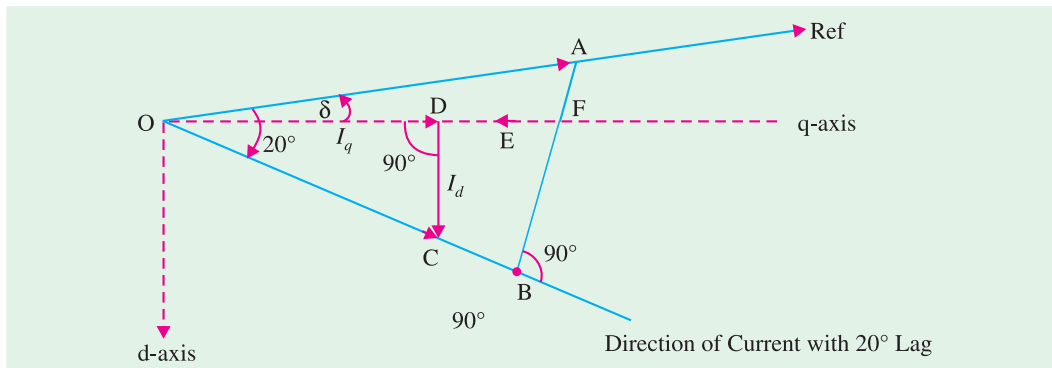


Fig. 38.31 (b) Motoring-mode

$$\begin{aligned}
 AF &= -I_a X_q = -65 \text{ V}, AB = 136.8 \text{ V}, FB = 71.8 \text{ V} \\
 OB &= 400 \cos 20^\circ = 376 \text{ V} \\
 OF &= \sqrt{376^2 + 71.8^2} = 382.8 \text{ V} \\
 20^\circ - \delta &= \tan^{-1} BF/OB = \tan^{-1} 71.8/376 = 10.8^\circ, \delta = 9.2^\circ \\
 FE &= -I_d(X_d - X_q) = -1.874 \times (3.5) = -6.56 \text{ volts, as shown in Fig.38.31 (b)} \\
 E &= OE = OF + FE = 382.8 - 6.56 = 376.24 \text{ volts}
 \end{aligned}$$

**Currents :**

$$\begin{aligned}
 I_a &= OC = 10 \text{ amp} \\
 I_q &= OD = 10 \cos 10.8^\circ = 9.823 \text{ amp}, I_d = DC = \sin 10.8^\circ = 1.874 \text{ amp}
 \end{aligned}$$

**Note.**  $I_d$  is in downward direction.

Hence,  $-I_d(X_d - X_q)$  will be from  $F$  towards  $O$  i.e., along '-q' direction.  
Thus, Excitation emf = 376.24 Volts, Load angle =  $9.2^\circ$

**(Note.** With respect to the generator mode,  $E$  has decreased, while  $\delta$  has increased.)

Power (by one formula) = 11276 watts, as before

Power (by another formula)

$$\begin{aligned}
 &= 3 [(V E/X_d) \sin \delta + (V^2/2) \{(1/X_q) \sin 2\delta\}] \\
 &= 3 [(400 \times 376.24/10) \sin 9.2^\circ + (400 \times 400/2) (3.5/65) \sin 18.4^\circ] \\
 &= 3 \times [2406 + 1360] = 11298 \text{ watts.}
 \end{aligned}$$

[This matches quite closely to the previous value calculated by other formula.]

**Example 38.14.** A 1- $\phi$  alternator has armature impedance of  $(0.5 + j0.866)$ . When running as a synchronous motor on 200-V supply, it provides a net output of 6 kW. The iron and friction losses amount to 500 W. If current drawn by the motor is 50 A, find the two possible phase angles of current and two possible induced e.m.fs. (Elec. Machines-I, Nagpur Univ. 1990)

**Solution.** Arm. Cu loss/phase =  $I_a^2 R_a = 50^2 \times 0.5 = 1250$  W  
 Motor intake =  $6000 + 500 + 1250 = 7750$  W  
 p.f. =  $\cos \phi = \text{Watts} / \text{VA} = 7750 / 200 \times 50 = 0.775 \therefore \phi = 39^\circ$  lag or lead.  
 $\theta = \tan^{-1} (X_S / R_a) = \tan^{-1} (0.866 / 0.5) = 60^\circ$  ;

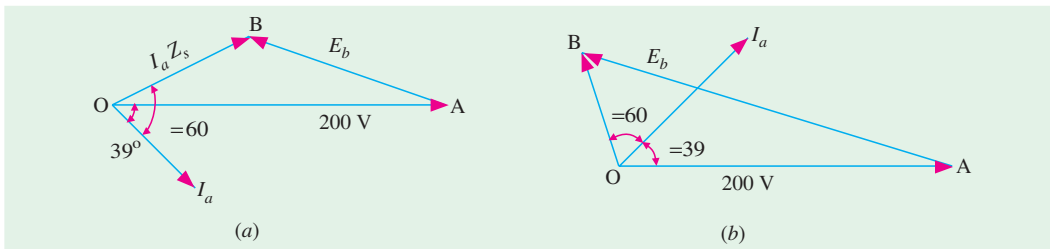


Fig. 38.32

$\angle BOA = 60^\circ - 39^\circ = 21^\circ$  - Fig. 38.32 (a)

$Z_S = \sqrt{0.5^2 + 0.866^2} = 1 \Omega ; I_a Z_S = 50 \times 1 = 50$  V

$AB = E_b = \sqrt{200^2 + 50^2 - 2 \times 200 \times 50 \cos 21^\circ} ; E_b = 154$  V.

In Fig. 38.32 (b),  $\angle BOA = 60^\circ + 39^\circ = 99^\circ$

$\therefore AB = E_b = \sqrt{(200^2 + 50^2) - 2 \times 200 \times 50 \cos 99^\circ} ; E_b = 214$  V.

**Example 38.15.** A 2200-V, 3- $\phi$ , Y-connected, 50-Hz, 8-pole synchronous motor has  $Z_S = (0.4 + j 6)$  ohm/phase. When the motor runs at no-load, the field excitation is adjusted so that E is made equal to V. When the motor is loaded, the rotor is retarded by  $3^\circ$  mechanical.

Draw the phasor diagram and calculate the armature current, power factor and power of the motor. What is the maximum power the motor can supply without falling out of step?

(Power Apparatus-II, Delhi Univ. 1988)

**Solution.** Per phase  $E_b = V = 2200 / \sqrt{3} = 1270$  V

$\alpha = 3^\circ$  (mech) =  $3^\circ \times (8/2) = 12^\circ$  (elect).

As seen from Fig 38.33 (a).

$E_R = (1270^2 + 1270^2 - 2 \times 1270 \times 1270 \times \cos 12^\circ)^{1/2}$   
 $= 266$  V;  $Z_S = \sqrt{0.4^2 + 6^2} = 6.013 \Omega$

$I_a = E_R / Z_S = 266 / 6.013 = 44.2$  A. From  $\Delta OAB$ ,

we get,  $\frac{1270}{\sin(\theta - \phi)} = \frac{266}{\sin 12^\circ}$

$\therefore \sin(\theta - \phi) = 1270 \times 0.2079 / 266 = 0.9926 \therefore (\theta - \phi) = 83^\circ$

Now,  $\theta = \tan^{-1} (X_S / R_a) = \tan^{-1} (6 / 0.4) = 86.18^\circ$

$\phi = 86.18^\circ - 83^\circ = 3.18^\circ \therefore \text{p.f.} = \cos 3.18^\circ = 0.998(\text{lag})$

Total motor power input =  $3 V I_a \cos \phi = 3 \times 1270 \times 44.2 \times 0.998 = 168$  kW

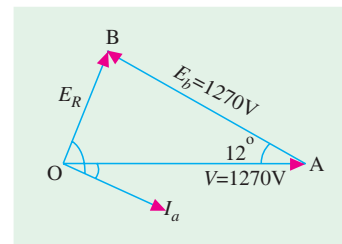


Fig. 38.33 (a)

$$\begin{aligned} \text{Total Cu loss} &= 3 I_a^2 R_a = 3 \times 44.2^2 \times 0.4 = 2.34 \text{ kW} \\ \text{Power developed by motor} &= 168 - 2.34 = \mathbf{165.66 \text{ kW}} \end{aligned}$$

$$P_{m(max)} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} = \frac{1270 \times 1270}{6.013} - \frac{1270^2 \times 0.4}{6.013^2} = \mathbf{250 \text{ kW}}$$

**Example 38.16.** A 1- $\phi$ , synchronous motor has a back e.m.f. of 250 V, leading by 150 electrical degrees over the applied voltage of 200 volts. The synchronous reactance of the armature is 2.5 times its resistance. Find the power factor at which the motor is operating and state whether the current drawn by the motor is leading or lagging.

**Solution.** As induced e.m.f. of 250 V is greater than the applied voltage of 200 V, it is clear that the motor is over-excited, hence it must be working with a leading power factor.

In the vector diagram of Fig. 38.33 (b), OA represents applied voltage, AB is back e.m.f. at an angle of 30° because  $\angle AOC = 150^\circ$  and  $\angle COD = \angle BAO = 30^\circ$ . OB represents resultant of voltage V and  $E_b$  i.e.  $E_R$

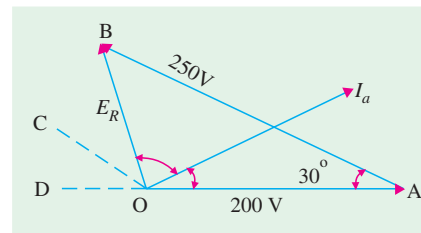


Fig. 38.33 (b)

In  $\triangle OBA$ ,

$$\begin{aligned} E_R &= \sqrt{(V^2 + E_b^2 - 2 V E_b \cos 30^\circ)} \\ &= \sqrt{(200^2 + 250^2 - 2 \times 200 \times 250 \times 0.866)} = 126 \text{ V} \end{aligned}$$

Now, 
$$\frac{E_R}{\sin 30^\circ} = \frac{E_b}{\sin (\theta + \phi)} \quad \text{or} \quad \frac{126}{0.5} = \frac{250}{\sin (\theta + \phi)}$$

$\therefore \sin (\theta + \phi) = 125/126$  (approx.)  $\therefore (\theta + \phi) = 90^\circ$

Now  $\tan \theta = 2.5 \quad \therefore \theta = 68^\circ 12'$   $\therefore \phi = 90^\circ - 68^\circ 12' = 21^\circ 48'$

$\therefore$  p.f. of motor =  $\cos 21^\circ 48' = \mathbf{0.9285}$  (leading)

**Example 38.17.** The synchronous reactance per phase of a 3-phase star-connected 6,600 V synchronous motor is 10  $\Omega$ . For a certain load, the input is 900 kW and the induced line e.m.f. is 8,900 V. (line value). Evaluate the line current. Neglect resistance.

(Basic Elect. Machines, Nagpur Univ. (1993))

**Solution.** Applied voltage / phase =  $6,600 / \sqrt{3} = 3,810 \text{ V}$

Back e.m.f. / phase =  $8,900 / \sqrt{3} = 5,140 \text{ V}$

Input =  $\sqrt{3} V_L \cdot I \cos \phi = 900,000$

$\therefore I \cos \phi = 9 \times 10^5 / \sqrt{3} \times 6,600 = 78.74 \text{ A}$

In  $\triangle ABC$  of vector diagram in Fig. 38.34, we have  $AB^2 = AC^2 + BC^2$

Now  $OB = I \cdot X_s = 10 I$

$$\begin{aligned} BC &= OB \cos \phi = 10 I \cos \phi \\ &= 10 \times 78.74 = 787.4 \text{ V} \end{aligned}$$

$\therefore 5,140^2 = 787.4^2 + AC^2 \quad \therefore AC = 5,079 \text{ V}$

$\therefore OC = 5,079 - 3,810 = 1,269 \text{ V}$

$\tan \phi = 1269/787.4 = 1.612; \phi = 58.2^\circ, \cos \phi = 0.527$

Now  $I \cos \phi = 78.74; I = 78.74/0.527 = \mathbf{149.4 \text{ A}}$

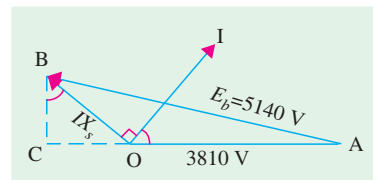


Fig. 38.34

**Example 38.18.** A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohms per phase and armature resistance negligible when the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.

(Elect. Machines, AMIE Sec. B 1991)

**Solution.** When Power Input is 1000 kW (Fig. 38.35 (a))

$$\begin{aligned} \sqrt{3} \times 6600 \times I_{a1} \times 0.8 &= 1000,000; I_{a1} = 109.3 \text{ A} \\ Z_S &= X_S = 20 \Omega; I_{a1} Z_S = 109.3 \times 20 = 2186 \text{ V}; \phi_1 = \cos^{-1} 0.8 = 36.9^\circ; \theta = 90^\circ \\ E_b^2 &= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times \cos(90^\circ + 36.9^\circ) \\ &= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times -\cos 53.1^\circ; \therefore E_b = 5410 \text{ V} \end{aligned}$$

Since excitation remains constant,  $E_b$  in the second case would remain the same i.e., 5410 V.

**When Power Input is 1500 kW :**

$$\begin{aligned} \sqrt{3} \times 6600 \times I_{a2} \cos \phi_2 &= 1500,000; I_{a2} \cos \phi_2 = 131.2 \text{ A} \end{aligned}$$

As seen from Fig. 38.35 (b),

$$\begin{aligned} OB &= I_{a2} Z_S = 20 I_{a2} \\ BC &= OB \cos \phi_2 = 20 I_{a2} \cos \phi_2 \\ \cos \phi_2 &= 20 \times 131.2 = 2624 \text{ V} \end{aligned}$$

In  $\triangle ABC$ , we have,  $AB^2 = AC^2 + BC^2$  or  $5410^2 = AC^2 + 2624^2$

$$\therefore AC = 4730 \text{ V}; OC = 4730 - 3810 = 920 \text{ V}$$

$$\tan \phi_2 = 920 / 2624; \phi_2 = 19.4^\circ; \text{p.f.} = \cos \phi_2 = \cos 19.4^\circ = 0.9432 \text{ (lead)}$$

$$\tan \alpha_2 = BC / AC = 2624 / 4730; \alpha_2 = 29^\circ$$

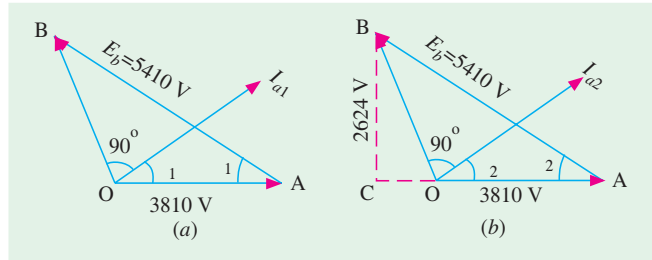


Fig. 38.35

**Example 38.19.** A 3-phase, star-connected 400-V synchronous motor takes a power input of 5472 watts at rated voltage. Its synchronous reactance is 10  $\Omega$  per phase and resistance is negligible. If its excitation voltage is adjusted equal to the rated voltage of 400 V, calculate the load angle, power factor and the armature current. (Elect. Machines AMIE Sec. B, 1990)

$$\begin{aligned} \text{Solution. } \sqrt{3} \times 400 \times I_a \cos \phi &= 5472; I_a \cos \phi = 7.9 \text{ A} \\ Z_S &= 10 \Omega; E_R = I_a Z_S = 10 I_a \end{aligned}$$

As seen from Fig. 38.36,  $BC = OB \cos \phi = 10, I_a \cos \phi = 7.9 \text{ V}$

$$AC = \sqrt{231^2 - 7.9^2} = 217 \text{ V}; OC = 231 - 217 = 14 \text{ V}$$

$$\tan \phi = 14 / 79; \phi = 10^\circ; \cos \phi = 0.985 \text{ (lag)}$$

$$I_a \cos \phi = 7.9; I_a = 7.9 / 0.985 = 8 \text{ A}; \tan \alpha = BC / AC = 7.9 / 217; \alpha = 20^\circ$$

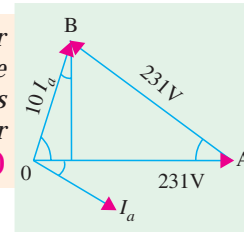


Fig. 38.36

**Example 38.20.** A 2,000-V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0.2  $\Omega$  and 2.2  $\Omega$  respectively. The input is 800 kW at normal voltage and the induced e.m.f. is 2,500 V. Calculate the line current and power factor. (Elect. Engg. A.M.I.E.T.E., June 1992)

**Solution.** Since the induced e.m.f. is greater than the applied voltage, the motor must be running with a leading p.f. If the motor current is  $I$ , then its in-phase or power component is  $I \cos \phi$  and reactive component is  $I \sin \phi$ .

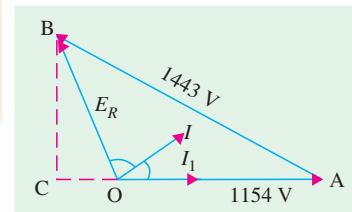


Fig. 38.37

Let  $I \cos \phi = I_1$  and  $I \sin \phi = I_2$  so that  $I = (I_1 + jI_2)$

$$I \cos \phi = I_1 = 800,00 / \sqrt{3} = 231 \text{ A}$$

Applied voltage / phase =  $2,000 / \sqrt{3} = 1,154 \text{ V}$

Induced e.m.f. / phase =  $2500 / \sqrt{3} = 1,443 \text{ V}$

In Fig. 38.37  $OA = 1154 \text{ V}$  and

$$AB = 1443 \text{ V, } OI \text{ leads } OA \text{ by } \phi$$

$$E_R = I Z_S \text{ and } \theta = \tan^{-1} (2.2 / 0.2) = 84.8^\circ$$

$BC$  is  $\perp AO$  produced.

Now,  $E_R = I Z_S = (I_1 + jI_2) (0.2 + j2.2)$

$$= (231 + jI_2) (0.2 + j2.2) = (46.2 - 2.2 I_2) + j (508.2 + 0.2 I_2)$$

Obviously,  $OC = (46.2 - 2.2 I_2)$ ;  $BC = j (508.2 + 0.2 I_2)$

From the right-angled  $\Delta ABC$ , we have

$$AB^2 = BC^2 + AC^2 = BC^2 + (AO + OC)^2$$

or  $1443^2 = (508.2 + 0.2 I_2)^2 + (1154 + 46.2 - 2.2 I_2)^2$

Solving the above quadratic equation, we get  $I_2 = 71 \text{ A}$

$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{231^2 + 71^2} = 242 \text{ A}$$

$$\text{p.f.} = I_1 / I = 231 / 242 = 0.95 \text{ (lead)}$$

**Example 38.21.** A 3 phase, 440-V, 50 Hz, star-connected synchronous motor takes 7.46 kW from the three phase mains. The resistance per phase of the armature winding is 0.5 ohm. The motor operates at a p. f. of 0.75 lag. Iron and mechanical losses amount to 500 watts. The excitation loss is 650 watts. Assume the source for excitation to be a separate one.

Calculate. (i) armature current, (ii) power supplied to the motor, (iii) efficiency of the motor  
(Amravati University 1999)

**Solution.** A 3-phase synchronous motor receives power from two sources :

- (a) 3-phase a. c. source feeding power to the armature.
- (b) D.C. source for the excitation, feeding electrical power only to the field winding.

Thus, power received from the d. c. source is utilized only to meet the copper-losses of the field winding.

3 Phase a.c. source feeds electrical power to the armature for following components of power:

- (i) Net mechanical power output from the shaft
- (ii) Copper-losses in armature winding
- (iii) Friction, and armature-core-losses.

In case of the given problem

$$\sqrt{3} \times I_a \times 440 \times 0.75 = 7460$$

$$I_a = 13.052 \text{ amp}$$

Total copper-loss in armature winding =  $3 \times 13.052^2 \times 0.50 = 255 \text{ watts}$

Power supplied to the motor =  $7460 + 650 = 8110 \text{ watts}$

$$\text{efficiency of the motor} = \frac{\text{Output}}{\text{Input}}$$

$$\text{Output from shaft} = (\text{Armature Input}) - (\text{Copper losses in armature winding})$$

$$- (\text{friction and iron losses})$$

$$= 7460 - 255 - 500 = 6705 \text{ watts}$$

$$\text{Efficiency of the motor} = \frac{6705}{8110} \times 100\% = 82.7\%$$



**Example 38.22.** Consider a 3300 V delta connected synchronous motor having a synchronous reactance per phase of 18 ohm. It operates at a leading pf of 0.707 when drawing 800 kW from mains. Calculate its excitation emf and the rotor angle (= delta), explaining the latter term. (Elect. Machines Nagpur Univ. 1993)

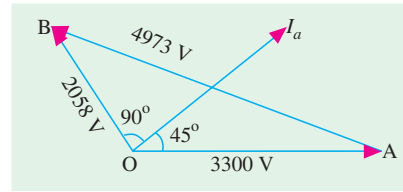


Fig. 38.38

**Solution.**  $\sqrt{3} \times 3300 \times I_a \times 0.707 = 800,000$

$\therefore$  Line current = 198 A, phase current,  $I_a = 198 / \sqrt{3} = 114.3$  A;

$$Z_s = 18 \Omega ; I_a Z_s = 114.3 \times 18 = 2058 \text{ V}$$

$$\phi = \cos^{-1} 0.707; \phi = 45^\circ; \theta = 90^\circ;$$

$$\cos(\theta + \phi) = \cos 135^\circ = -\cos 45^\circ = -0.707$$

From Fig. 38.38, we find

$$E_b^2 = 3300^2 + 2058^2 - 2 \times 3300 \times 2058 \times -0.707$$

$$\therefore E_b = 4973 \text{ V}$$

From  $\Delta OAB$ , we get  $2058/\sin \alpha = 4973/\sin 135^\circ$ . Hence,  $\alpha = 17^\circ$

**Example 38.23.** A 75-kW, 400-V, 4-pole, 3-phase star connected synchronous motor has a resistance and synchronous reactance per phase of 0.04 ohm and 0.4 ohm respectively. Compute for full-load 0.8 p.f. lead the open circuit e.m.f. per phase and mechanical power developed. Assume an efficiency of 92.5%. (Elect. Machines AMIE Sec. B 1991)

**Solution.** Motor input =  $75,000 / 0.925 = 81,080 \text{ W}$

$$I_a = 81,080 / (\sqrt{3} \times 400 \times 0.8) = 146.3 \text{ A}; Z_s = \sqrt{0.04^2 + 0.4^2} = 0.402 \Omega$$

$$I_a Z_s = 146.3 \times 0.402 = 58.8 \text{ V}; \tan \phi = 0.4 / 0.04 = 10 ;$$

$$\theta = 84.3^\circ; \phi = \cos^{-1} 0.8 ; \phi = 36.9^\circ; (\theta + \phi) = 121.2^\circ;$$

$$V_{ph} = 400 / \sqrt{3} = 231 \text{ V}$$

As seen from Fig. 36.39,

$$E_b^2 = 231^2 + 58.8^2 - 2 \times 231 \times 58.8 \times \cos 121.2^\circ; E_b / \text{phase} = 266 \text{ V}$$

Stator Cu loss for 3 phases =  $3 \times 146.3^2 \times 0.04 = 2570 \text{ W};$

$$N_s = 120 \times 50/40 = 1500 \text{ r.p.m.}$$

$$P_m = 81080 - 2570 = 78510 \text{ W}; T_g = 9.55 \times 78510/1500 = 490 \text{ N-m.}$$

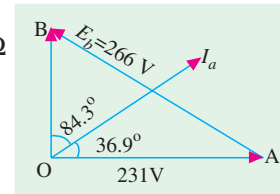


Fig. 38.39

**Example 38.24.** A 400-V, 3-phase, 50-Hz, Y-connected synchronous motor has a resistance and synchronous impedance of 0.5  $\Omega$  and 4  $\Omega$  per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is increased to 60 A, the field current remaining unchanged, calculate the gross torque developed and the new power factor. (Elect. Machines, AMIE Sec. B 1992)

**Solution.** The conditions corresponding to the first case are shown in Fig. 38.40.

$$\text{Voltage/phase} = 400/\sqrt{3} = 231 \text{ V}; I_a Z_s = OB = 15 \times 4 = 60 \text{ V}$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{4^2 - 0.5^2} = 3.968 \Omega$$

$$\theta = \tan^{-1} (3.968/0.5) = \tan^{-1} (7.936) = 81.48^\circ$$

$$E_b^2 = 231^2 + 60^2 - 2 \times 231 \times 60 \times \cos 81.48^\circ; E_b = 231 \text{ V}$$

It is obvious that motor is running with normal excitation because  $E_b = V$

When the motor load is increased, the phase angle between the applied voltage and the induced (or back) e.m.f. is increased. Art (38.7). The vector diagram is as shown in Fig. 38.41.

Let  $\phi$  be the new phase angle.

$$I_a Z_S = 60 \times 4 = 240 \text{ V}$$

$$\angle BOA = (81^\circ 48' - \phi).$$

Since the field current remains constant, the value of  $E_b$  remains the same.

$$\therefore 231^2 = 231^2 + 240^2 - 2 \times 231 \times 240 \cos (81^\circ 48' - \phi)$$

$$\therefore \cos (81^\circ 48' - \phi) = 0.4325 \text{ or } 81^\circ 48' - \phi = 64^\circ 24'$$

$$\therefore \phi = 81^\circ 48' - 64^\circ 24' = 17^\circ 24'. \text{ New p.f.} = \cos 17^\circ 24' = \mathbf{0.954 \text{ (lag)}}$$

$$\text{Motor input} = \sqrt{3} \times 400 \times 60 \times 0.954 = 39,660 \text{ W}$$

$$\text{Total armature Cu loss} = 3 \times 60^2 \times 0.5 = 5,400 \text{ W}$$

$$\text{Electrical power converted into mechanical power} = 39,660 - 5,400 = 34,260 \text{ W}$$

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m.} \quad T_g = 9.55 \times 34,260/1000 = \mathbf{327 \text{ N-m}}$$

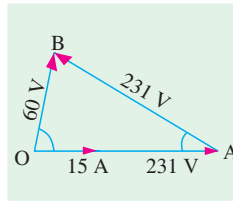


Fig. 38.40

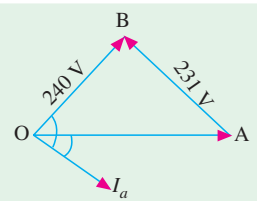


Fig. 38.41

**Example 38.25.** A 400-V, 10 h.p. (7.46 kW), 3-phase synchronous motor has negligible armature resistance and a synchronous reactance of 10  $\Omega$  / phase. Determine the minimum current and the corresponding induced e.m.f. for full-load conditions. Assume an efficiency of 85%.

(A.C. Machines-I, Jadavpur Univ. 1987)

**Solution.** The current is minimum when the power factor is unity i.e., when  $\cos \phi = 1$ . The vector diagram is as shown in Fig. 38.42.

$$\text{Motor input} = 7460 / 0.85 = 8,775 \text{ W}$$

$$\text{Motor line current} = 8,775 / \sqrt{3} \times 400 = \mathbf{12.67 \text{ A}}$$

$$\text{Impedance drop} = I_a X_S = 10 \times 12.67 = 126.7 \text{ V}$$

$$\text{Voltage / phase} = 400 / \sqrt{3} = 231 \text{ V}$$

$$E_b = \sqrt{231^2 + 126.7^2} = \mathbf{263.4 \text{ V}}$$

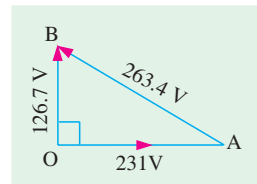


Fig. 38.42

**Example 38.26.** A 400-V, 50-Hz, 3-phase, 37.5 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is  $(0.2 + j 1.6)$  ohm per phase. If the excitation of the motor is adjusted to give a leading power factor of 0.9, calculate the following for full load :

(i) the excitation e.m.f.

(ii) the total mechanical power developed

(Elect.Machines, A.M.I.E. Sec. B, 1989)

**Solution.** Motor input =  $37.5/0.88 = 42.61 \text{ kW}$ ;  $I_a = 42,610 / \sqrt{3} \times 400 \times 0.9 = 68.3 \text{ A}$

$$V = 400 / \sqrt{3} = 231 \text{ V}; Z_S = 0.2 + j 1.6 = 1.612 \angle 82.87^\circ$$

$$E_R = I_a Z_S = 68.3 \times 1.612 = 110 \text{ V};$$

$$\phi = \cos^{-1} (0.9) = 25.84^\circ$$

$$\text{Now, } (\phi + \theta) = 25.84^\circ + 82.87^\circ = 108.71^\circ$$

$$\cos (\phi + \theta) = \cos 108.71^\circ = -0.32$$

$$\mathbf{(a) \therefore E_b^2 = V^2 + E_R^2 - 2 V E_R \cos 108.71^\circ \text{ or } E_b = 286 \text{ V}}$$

$$\text{Line value of excitation voltage} = \sqrt{3} \times 285 = \mathbf{495 \text{ V}}$$

**(b)** From  $\Delta OAB$ , (Fig. 38.43)  $E_R / \sin \alpha = E_b / \sin (\phi + \theta)$ ,  $\alpha = 21.4^\circ$

$$P_m = 3 \frac{E_b V}{Z_S} \sin \alpha = 3 \frac{286 \times 231}{1.612} \sin 21.4^\circ = \mathbf{14,954 \text{ W}}$$

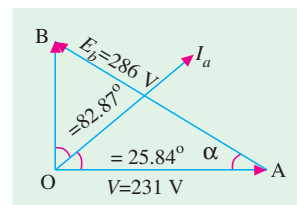


Fig. 38.43

**Example 38.27.** A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohm per phase and armature resistance negligible. When the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.

(Elect. Machines, A.M.I.E., Sec. B, 1991)

**Solution.**  $V = 6600 / \sqrt{3} = 3810 \text{ V}$ ,  $I_a = 1000 \times 10^3 / \sqrt{3} \times 6600 \times 0.8 = 109.3 \text{ A}$

The phasor diagram is shown in Fig. 38.44. Since  $R_a$  is negligible,  $\theta = 90^\circ$

$$E_R = I_a X_S = 109.3 \times 20 = 2186 \text{ V}$$

$$\cos \phi = 0.8, \phi = 36.87^\circ$$

$$E_b^2 = V^2 + E_R^2 - 2E_b V \cos (90^\circ + 36.87^\circ);$$

$$E_b = 5410 \text{ V}$$

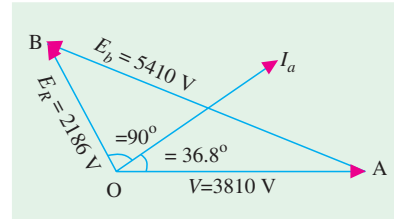


Fig. 38.44

Now, excitation has been kept constant but power has been increased to 1500 kW

$$\therefore 3 \frac{E_b V}{Z_S} \sin \alpha = P; 3 \times \frac{5410 \times 3810}{20} \sin \alpha = 1500 \times 10^3; \alpha = 29^\circ$$

Also, 
$$\frac{E_b}{(\sin 90^\circ + \phi)} = \frac{V}{\sin [180^\circ - (\alpha + 90^\circ + \phi)]} = \frac{V}{\cos (\alpha + \phi)}$$

or 
$$\frac{E_b}{\cos \phi} = \frac{V}{\cos (\alpha + \phi)} \text{ or } \frac{V}{E_b} = \frac{\cos (29^\circ + \phi)}{\cos \phi} = 0.3521$$

$$\therefore \phi = 19.39^\circ, \cos \phi = \cos 19.39^\circ = \mathbf{0.94 \text{ (lead)}}$$

**Example 38.28.** A 400-V, 50-Hz, 6-pole, 3-phase, Y-connected synchronous motor has a synchronous reactance of 4 ohm/phase and a resistance of 0.5 ohm/phase. On full-load, the excitation is adjusted so that machine takes an armature current of 60 ampere at 0.866 p.f. leading.

Keeping the excitation unchanged, find the maximum power output. Excitation, friction, windage and iron losses total 2 kW. (Electrical Machinery-III, Bangalore Univ. 1990)

**Solution.**  $V = 400 / \sqrt{3} = 231 \text{ V/phase}$ ;  $Z_S = 0.5 + j4 = 4.03 \angle 82.9^\circ$ ;  $\theta = 82.9^\circ$

$$I_a Z_S = 60 \times 4.03 = 242 \text{ V}; \cos \phi = 0.866$$

$$\phi = 30^\circ \text{ (lead)}$$

As seen from Fig. 36.45,

$$E_b^2 = 231^2 + 242^2 - 2 \times 231 \times 242 \cos 112.9^\circ$$

$$E_b = \mathbf{394 \text{ V}}$$

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{394 \times 231}{4.03} - \frac{394^2 \times 0.5}{4.03^2}$$

$$= 17,804 \text{ W/phase.} \quad \text{—Art. 38.12}$$

Maximum power developed in armature for 3 phases

$$= 3 \times 17,804 = 52,412 \text{ W}$$

$$\text{Net output} = 52,412 - 2,000 = 50,412 \text{ W} = \mathbf{50.4 \text{ kW}}$$

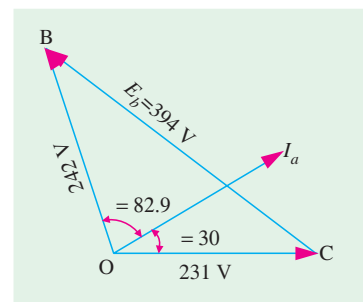


Fig. 38.45

**Example 38.29.** A 6-pole synchronous motor has an armature impedance of 10 ohm and a resistance of 0.5 ohm. When running on 2,000 volts, 25-Hz supply mains, its field excitation is such that the e.m.f. induced in the machine is 1600 V. Calculate the maximum total torque in N-m developed before the machine drops out of synchronism.

**Solution.** Assuming a three-phase motor,

$$V = 2000 \text{ V}, E_b = 1600 \text{ V}; R_a = 0.5 \Omega; Z_s = 10 \Omega; \cos \theta = 0.5/10 = 1/20$$

Using equation (iii) of Art. 37-10, the total max. power for 3 phases is

$$(P_m)_{max} = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \theta = \frac{2000 \times 1600}{10} - \frac{1600^2 \times 1}{10 \times 20} = 307,200 \text{ watt}$$

Now,  $N_s = 120 f/P = 120 \times 25/6 = 500 \text{ r.p.m.}$

Let  $T_{g,max}$  be the maximum gross torque, then

$$T_{g,max} = 9.55 \times \frac{307200}{500} = \mathbf{5,868 \text{ N-m}}$$

**Example 38.30.** A 2,000-V, 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m. The excitation is constant and corresponds to an open-circuit terminal voltage of 2,000 V. The resistance is negligible as compared with synchronous reactance of 3 Ω per phase. Determine the power input, power factor and torque developed for an armature current of 200 A.

(Elet. Engg.-I, Nagpur Univ. 1993)

**Solution.** Voltage/phase =  $2000/\sqrt{3} = 1150 \text{ V}$

Induced e.m.f. = 1150 V – given

Impedance drop =  $200 \times 3 = 600 \text{ V}$

As shown in Fig. 38.46, the armature current is assumed to lag behind  $V$  by an angle  $\phi$ . Since  $R_a$  is negligible,  $\theta = 90^\circ$ .

$$\angle BOA = (90^\circ - \phi)$$

Considering  $\Delta BOA$ , we have

$$1150^2 = 1150^2 + 600^2 - 2 \times 600 \times 1150 \cos (90 - \phi)$$

$$\sin \phi = 0.2605; \phi = 16.2^\circ; \text{p.f.} = \cos 16.2^\circ = \mathbf{0.965 \text{ (lag)}}$$

$$\text{Power input} = \sqrt{3} \times 2,000 \times 200 \times 0.965 = \mathbf{668.5 \text{ kW}}$$

$$N_s = 1500 \text{ r.p.m.} \quad \therefore T_g = 9.55 \times 66,850/1500 = \mathbf{4,255 \text{ N-m.}}$$

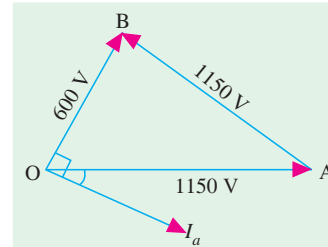


Fig. 38.46

**Example 38.31.** A 3-φ, 3300-V, Y-connected synchronous motor has an effective resistance and synchronous reactance of 2.0 Ω and 18.0 Ω per phase respectively. If the open-circuit generated e.m.f. is 3800 V between lines, calculate (i) the maximum total mechanical power that the motor can develop and (ii) the current and p.f. at the maximum mechanical power.

(Electrical Machines-III, Gujarat Univ. 1988)

**Solution.**  $\theta = \tan^{-1} (18/2) = 83.7^\circ$ ;  $V_{ph} = 3300/\sqrt{3} = 1905 \text{ V}$ ;  $E_b = 3800/\sqrt{3} = 2195 \text{ V}$

Remembering that  $\alpha = \theta$  for maximum power development (Ar. 38-10)

$$E_R = (1905^2 + 2195^2 - 2 \times 1905 \times 2195 \times \cos 83.7^\circ)^{1/2} = 2744 \text{ volt per phase}$$

$$\therefore I_a Z_s = 2,744; \text{ Now, } Z_s = \sqrt{2^2 + 18^2} = 18.11 \Omega$$

$$\therefore I_a = 2744/18.11 = 152 \text{ A/phase}; \text{ line current} = \mathbf{152 \text{ A}}$$

$$(P_m)_{max} \text{ per phase} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} = \frac{2195 \times 1905}{18.11} - \frac{2195^2 \times 2}{18.11^2}$$

$$= 230,900 - 29,380 = 201,520 \text{ W per phase}$$

Maximum power for three phases that the motor can develop *in its armature*

$$= 201,520 \times 3 = \mathbf{604,560 \text{ W}}$$

$$\text{Total Cu losses} = 3 \times 152^2 \times 2 = 138,700 \text{ W}$$

$$\text{Motor input} = 604,560 + 138,700 = 743,260 \text{ W}$$

$$\therefore \sqrt{3} \times 3300 \times 152 \times \cos \phi = 743,260 \quad \therefore \cos \phi = \mathbf{0.855 \text{ (lead).}}$$

**Example 38.32.** The excitation of a 415-V, 3-phase, mesh-connected synchronous motor is such that the induced e.m.f. is 520 V. The impedance per phase is  $(0.5 + j4.0)$  ohm. If the friction and iron losses are constant at 1000 W, calculate the power output, line current, power factor and efficiency for maximum power output. (Elect. Machines-I, Madras Univ. 1987)

**Solution.** As seen from Art. 38-12, for fixed  $E_b$ ,  $V$ ,  $R_a$  and  $X_s$ , maximum power is developed when  $\alpha = \theta$ .

$$\text{Now, } \theta = \tan^{-1} (4/0.5) = \tan^{-1} (8) = 82.90^\circ = \alpha$$

$$E_R = \sqrt{415^2 + 520^2 - 2 \times 415 \times 520 \times \cos 82.9^\circ} = 625 \text{ V per phase}$$

$$\text{Now, } IZ_S = 625; \quad Z_S = \sqrt{4^2 + 0.5^2} = 4.03 \Omega \quad \therefore I = 625/4.03 = 155 \text{ A}$$

$$\text{Line current} = 155 \times \sqrt{3} = 268.5 \text{ A}$$

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{520 \times 415}{4.03} - \frac{520^2 \times 0.5}{16.25} = 45,230 \text{ W}$$

$$\text{Max. power for 3 phases} = 3 \times 45,230 = 135,690 \text{ W}$$

$$\begin{aligned} \text{Power output} &= \text{power developed} - \text{iron and friction losses} \\ &= 135,690 - 1000 = 134,690 \text{ W} = \mathbf{134.69 \text{ kW}} \end{aligned}$$

$$\text{Total Cu loss} = 3 \times 155^2 \times 0.5 = 36,080 \text{ W}$$

$$\text{Total motor input} = 135,690 + 36,080 = 171,770 \text{ W}$$

$$\therefore \sqrt{3} \times 415 \times 268.5 \times \cos \phi = 171,770; \quad \cos \phi = \mathbf{0.89 \text{ (lead)}}$$

$$\text{Efficiency} = 134,690/171,770 = 0.7845 \text{ or } \mathbf{78.45\%}$$

### Tutorial Problems 38.1

- A 3-phase, 400-V, synchronous motor takes 52.5 A at a power factor of 0.8 leading. Calculate the power supplied and the induced e.m.f. The motor impedance per phase is  $(0.25 + j3.2)$  ohm. [29.1 kW; 670V]
- The input to a 11-kV, 3 $\phi$ , Y-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are  $1 \Omega$  and  $30 \Omega$  respectively. Find (a) power supplied to the motor and (b) the induced e.m.f. for a p.f. of 0.8 leading. [(a) 915 kW (b) 13kV] (Grad. I.E.T.E. Dec. 1978)
- A 2,200-V, 3-phase, star-connected synchronous motor has a resistance of  $0.6 \Omega$  and a synchronous reactance of  $6 \Omega$ . Find the generated e.m.f. and the angular retardation of the motor when the input is 200 kW at (a) power factor unity and (b) power factor 0.8 leading. [(a) 2.21 kV; 14.3° (b) 2.62 kV; 12.8°]
- A 3-phase, 220-V, 50-Hz, 1500 r.p.m., mesh-connected synchronous motor has a synchronous impedance of 4 ohm per phase. It receives an input line current of 30 A at a leading power factor of 0.8. Find the line value of the induced e.m.f. and the load angle expressed in mechanical degrees. If the mechanical load is thrown off without change of excitation, determine the magnitude of the current under the new conditions. Neglect losses. [268 V; 6°, 20.6 A]
- A 400-V, 3-phase, Y-connected synchronous motor takes 3.73 kW at normal voltage and has an impedance of  $(1 + j8)$  ohm per phase. Calculate the current and p.f. if the induced e.m.f. is 460 V. [6.28 A; 0.86 lead] (Electrical Engineering, Madras Univ. April 1979)
- The input to 6600-V, 3-phase, star-connected synchronous motor is 900 kW. The synchronous reactance per phase is  $20 \Omega$  and the effective resistance is negligible. If the generated voltage is 8,900 V

(line), calculate the motor current and its power factor.

[Hint. See solved Ex. 38.17] (*Electrotechnics, M.S. Univ. April 1979*)

7. A 3-phase synchronous motor connected to 6,600-V mains has a star-connected armature with an impedance of  $(2.5 + j15)$  ohm per phase. The excitation of machine gives 7000 V. The iron, friction and excitation losses are 12 kW. Find the maximum output of the motor. [153.68 kW]
8. A 3300-V, 3-phase, 50-Hz, star-connected synchronous motor has a synchronous impedance of  $(2 + j15)$  ohm. Operating with an excitation corresponding to an e.m.f. of 2,500 V between lines, it just falls out of step at full-load. To what open-circuit e.m.f. will it have to be excited to stand a 50% excess torque. [4 kV]
9. A 6.6 kV, star-connected, 3-phase, synchronous motor works at constant voltage and constant excitation. Its synchronous impedance is  $(2.0 + j20)$  per phase. When the input is 1000 kW, its power factor is 0.8 leading. Find the power factor when the input is increased to 1500 kW (solve graphically or otherwise).  
[0.925 lead] (*AMIE Sec. B Advanced Elect. Machines (E-9) Summer 1991*)
10. A 2200-V, 373 kW, 3-phase, star-connected synchronous motor has a resistance of  $0.3 \Omega$  and a synchronous reactance of  $3.0 \Omega$  per phase respectively. Determine the induced e.m.f. per phase if the motor works on full-load with an efficiency of 94 per cent and a p.f. of 0.8 leading.  
[1510 V] (*Electrical Machinery, Mysore Univ. 1992*)
11. The synchronous reactance per phase of a 3-phase star-connected 6600 V synchronous motor is  $20 \Omega$ . For a certain load, the input is 915 kW at normal voltage and the induced line e.m.f. is 8,942 V. Evaluate the line current and the p.f. Neglect resistance. [97 A; 0.8258 (lead)]
12. A synchronous motor has an equivalent armature reactance of  $3.3 \Omega$ . The exciting current is adjusted to such a value that the generated e.m.f. is 950 V. Find the power factor at which the motor would operate when taking 80 kW from a 800-V supply mains. [0.965 leading] (*City & Guilds, London*)
13. The input to an 11000 V, 3-phase star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohms. Find the power supplied to the motor and the induced electromotive force for a power factor of 0.8 leading.  
[914.5 kW, 13 kV] (*Elect. Machines, A.M.I.E. Sec. B, 1990*)
14. A 400-V, 6-pole, 3-phase, 50-Hz, star-connected synchronous motor has a resistance and synchronous reactance of 0.5 ohm per phase and 4-ohm per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 60 A, the field current remaining unchanged, find the gross torque developed, and the new power factor. [354 Nm; 0.93] (*Elect. Engg. AMIETE Dec. 1990*)
15. The input to a 11,000-V, 3-phase, star-connected synchronous motor is 60 amperes. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find the power supplied to the motor and the induced e.m.f. for power factor of 0.8 (a) leading and (b) lagging.  
[915 kW (a) 13 kV (b) 9.36 kV] (*Elect. Machines-II, South Gujarat Univ. 1981*)
16. Describe with the aid of a phasor diagram the behaviour of a synchronous motor starting from no-load to the pull-out point.  
What is the output corresponding to a maximum input to a 3- $\phi$  delta-connected 250-V, 14.92 kW synchronous motor when the generated e.m.f. is 320 V? The effective resistance and synchronous reactance per phase are  $0.3 \Omega$  and  $4.5 \Omega$  respectively. The friction, windage, iron and excitation losses total 800 watts and are assumed to remain constant. Give values for (i) output (ii) line current (iii) p.f.  
[(i) 47.52 kW (ii) 161 A (iii) 0.804] (*Elect. Machines, Indore Univ. Feb. 1982*)
17. A synchronous motor takes 25 kW from 400 V supply mains. The synchronous reactance of the motor is 4 ohms. Calculate the power factor at which the motor would operate when the field excitation is so adjusted that the generated EMF is 500 volts.  
[ 0.666 Leading] (*Rajiv Gandhi Technical University, Bhopal, 2000*)

### 38.17. Effect of Excitation on Armature Current and Power Factor

The value of excitation for which back e.m.f.  $E_b$  is equal (in magnitude) to applied voltage  $V$  is known as 100% excitation. We will now discuss what happens when motor is either over-excited or under-excited although we have already touched this point in Art. 38-8.

Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected).

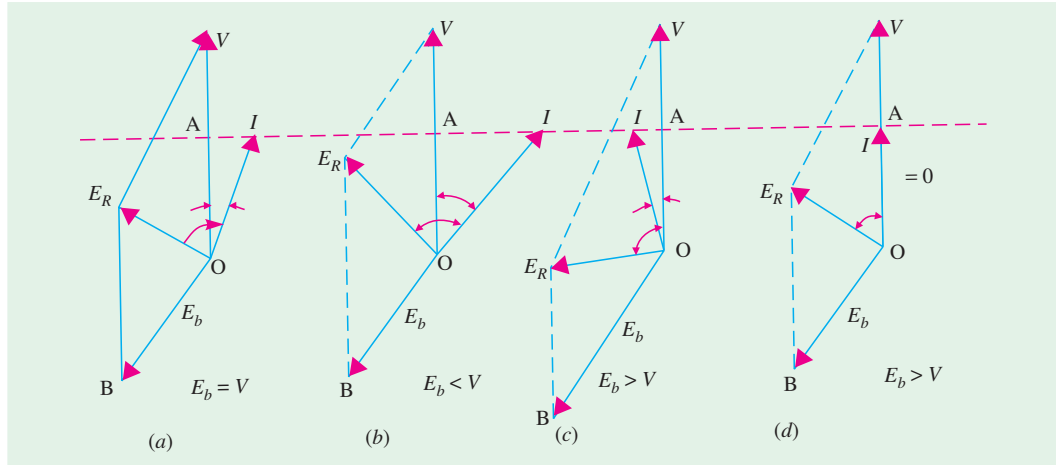


Fig. 38.47

Fig. 38.47 (a) shows the case for 100% excitation *i.e.*, when  $E_b = V$ . The armature current  $I$  lags behind  $V$  by a small angle  $\phi$ . Its angle  $\theta$  with  $E_R$  is fixed by stator constants *i.e.*  $\tan \theta = X_s / R_a$

In Fig. 38.47 (b)\* excitation is less than 100% *i.e.*,  $E_b < V$ . Here,  $E_R$  is advanced clockwise and so is armature current (because it lags behind  $E_R$  by fixed angle  $\theta$ ). We note that the magnitude of  $I$  is increased but its power factor is decreased ( $\phi$  has increased). Because input as well as  $V$  are constant, hence the power component of  $I$  *i.e.*,  $I \cos \phi$  remains the same as before, but wattless component  $I \sin \phi$  is increased. Hence, as excitation is decreased,  $I$  will increase but p.f. will decrease so that power component of  $I$  *i.e.*,  $I \cos \phi = OA$  will remain constant. In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.

Incidentally, it may be noted that when field current is reduced, the motor pull-out torque is also reduced in proportion.

Fig. 38.47 (c) represents the condition for overexcited motor *i.e.* when  $E_b > V$ . Here, the resultant voltage vector  $E_R$  is pulled anticlockwise and so is  $I$ . It is seen that now motor is drawing a leading current. It may also happen for some value of excitation, that  $I$  may be in phase with  $V$  *i.e.*, p.f. is unity [Fig. 38.47 (d)]. At that time, the current drawn by the motor would be **minimum**.

Two important points stand out clearly from the above discussion :

- (i) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of  $I$  with excitation are shown in Fig. 38.48 (a) which are known as 'V' curves because of their shape.
- (ii) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly. When over-excited, motor runs with leading p.f. and with lagging p.f. when under-excited. In between, the p.f. is unity. The variations of *p.f.* with excitation

\* These are the same diagrams as given in Fig. 38.7 and 8 expect that vector for  $V$  has been shown vertical.

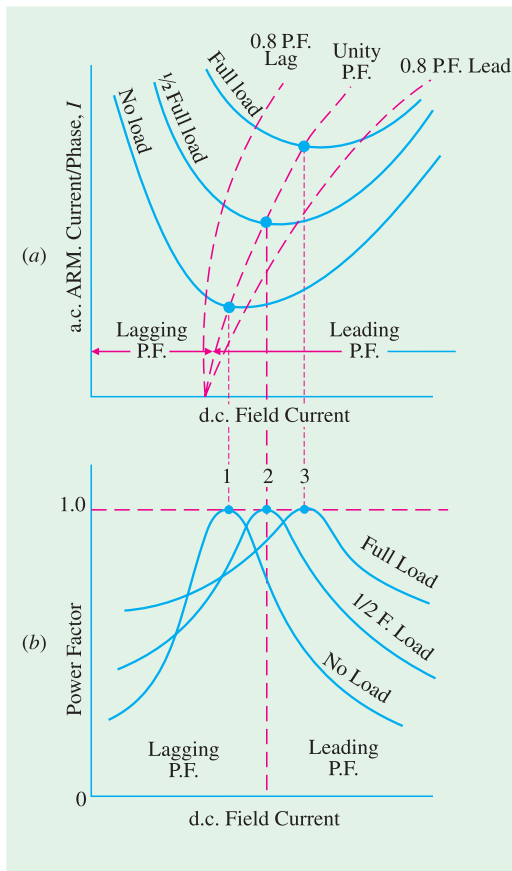


Fig. 38.48



Inductor motor

are shown in Fig. 38.48 (b). The curve for p.f. looks like inverted 'V' curve. It would be noted that **minimum armature current corresponds to unity power factor**.

It is seen (and it was pointed out in Art. 38.1) that an over-excited motor can be run with leading power factor. This property of the motor renders it extremely useful for phase advancing (and so power factor correcting) purposes in the case of industrial loads driven by induction motors (Fig. 38.49) and lighting and heating loads supplied through transformers. Both transformers and induction motors draw lagging currents from the line. Especially on light loads, the power drawn by them has a large reactive component and the power factor has a very low value. This reactive component, though essential for operating the electric machinery, entails appreciable loss in many ways. By using synchronous motors in conjunction with induction motors and transformers, the lagging reactive power required by the

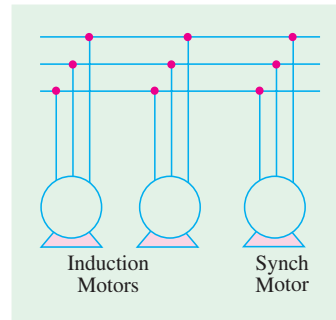


Fig. 38.49

latter is supplied locally by the leading reactive component taken by the former, thereby relieving the line and generators of much of the reactive component. Hence, they now supply only the active component of the load current. When used in this way, a synchronous motor is called a **synchronous capacitor**, because it draws, like a capacitor, leading current from the line. Most synchronous capacitors are rated between 20 MVAR and 200 MVAR and many are hydrogen-cooled.

**Example 38.33.** Describe briefly the effect of varying excitation upon the armature current and p.f. of a synchronous motor when input power to the motor is maintained constant.

A 400-V, 50-Hz, 3- $\phi$ , 37.3 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is  $(0.2 + j 1.6) \Omega$  per phase. If the excitation of the motor is adjusted to give a leading p.f. of 0.9, calculate for full-load (a) the induced e.m.f. (b) the total mechanical power developed.

**Solution.** Voltage / phase =  $400 / \sqrt{3} = 231 \text{ V}$ ;



$$Z_S = \sqrt{(1.6^2 + 0.2^2)} = 1.61 \Omega;$$

$$\text{Full-load current} = \frac{37,300}{\sqrt{3} \times 400 \times 0.88 \times 0.9} = 68 \text{ A}$$

$$\therefore IZ_S = 1.61 \times 68 = 109.5 \text{ V}$$

With reference to Fig. 38.50

$$\tan \theta = 1.6 / 0.2 = 8, \theta = 82^\circ 54'$$

$$\cos \phi = 0.9, \phi = 25^\circ 50'$$

$$\therefore (\theta + \phi) = 82^\circ 54' + 25^\circ 50' = 108^\circ 44'$$

Now  $\cos 108^\circ 44' = -0.3212$

(a) In  $\triangle OAB$ ,  $E_b^2 = 231^2 + 109.5^2 - 2 \times 231 \times 109.5 \times (-0.3212) = 285.6^2$ ;  $E_b = 285.6 \text{ V}$

Line value of  $E_b = \sqrt{3} \times 285.6 = 495 \text{ V}$

(b) Total motor input =  $37,300 / 0.88 = 42,380 \text{ W}$

Total Cu losses =  $3 \times I^2 R_a = 3 \times 68^2 \times 0.2 = 2,774 \text{ W}$

$\therefore$  Electric power converted into mechanical power =  $42,380 - 2,774 = 39.3 \text{ kW}$ .

**Example 38.34.** A 3- $\phi$ , star-connected synchronous motor takes 48 kW at 693 V, the power factor being 0.8 lagging. The induced e.m.f. is increased by 30%, the power taken remaining the same. Find the current and the p.f. The machine has a synchronous reactance of 2  $\Omega$  per phase and negligible resistance.

**Solution.** Full-load current

$$= 48,000 / \sqrt{3} \times 693 \times 0.8 = 50 \text{ A}$$

Voltage/phase =  $693 / \sqrt{3} = 400 \text{ V}$

$$Z_S = X_S = 2 \Omega \quad \therefore IZ_S = 50 \times 2 = 100 \text{ V}$$

$$\tan \theta = 2/0 = \infty \quad \therefore \theta = 90^\circ; \cos \phi = 0.8, \sin \phi = 0.6$$

The vector diagram is shown in Fig. 38.51. In  $\triangle OAB$ ,

$$E_b^2 = 400^2 + 100^2 - 2 \times 400 \times 100 \times \cos (90^\circ - \phi)$$

$$= 400^2 + 100^2 - 2 \times 400 \times 100 \times 0.6 = 349^2 \quad \therefore E_b = 349 \text{ V.}$$

The vector diagram for increased e.m.f. is shown in Fig. 38.52. Now,  $E_b = 1.3 \times 349 = 454 \text{ V}$ . It can be safely assumed that in the second case, current is leading  $V$  by some angle  $\phi'$ .

Let the new current and the leading angle of current by  $I$  and  $\phi'$  respectively. As power input remains the same and  $V$  is also constant,  $I \cos \phi$  should be the same for the same input.

$$\therefore I \cos \phi = 50 \times 0.8 = 40 = I' \cos \phi'$$

In  $\triangle ABC$ ,  $AB^2 = AC^2 + BC^2$

Now  $BC = I X_S \cos \phi' \quad (\because OB = I X_S)$

$$= 40 \times 2 = 80 \text{ V}$$

$$\therefore 454^2 = AC^2 + 80^2 \text{ or } AC = 447 \text{ V}$$

$$\therefore OC = 447 - 400 = 47 \text{ V}$$

$$\therefore \tan \phi' = 47/80, \phi' = 30^\circ 26'$$

$$\therefore \text{New p.f.} = \cos 30^\circ 26' = 0.8623 \text{ (leading)}$$

Also,  $I \cos \phi' = 40 \quad \therefore I = 40/0.8623 = 46.4 \text{ A.}$

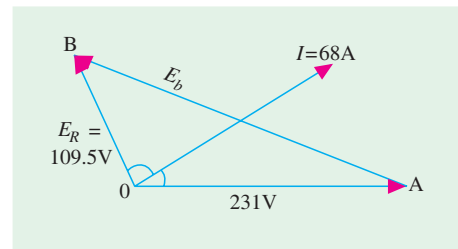


Fig. 38.50

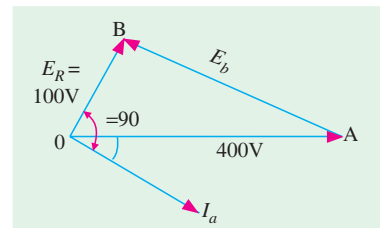


Fig. 38.51

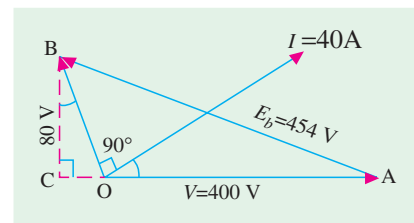


Fig. 38.52

**Example 38.35.** A synchronous motor absorbing 60 kW is connected in parallel with a factory load of 240 kW having a lagging p.f. of 0.8. If the combined load has a p.f. of 0.9, what is the value of the leading kVAR supplied by the motor and at what p.f. is it working ?

(Electrical Engineering-II, Bangalore Univ. 1990)

**Solution.** Load connections and phase relationships are shown in Fig. 38.53.

Total load = 240 + 60 = 300 kW; combined p.f. = 0.9 (lag)

$\phi = 25.8^\circ, \tan \phi = 0.4834, \text{ combined kVAR} = 300 \times 0.4834 = 145(\text{lag})$

**Factory Load**

$\cos \phi_L = 0.8, \phi_L = 36.9^\circ, \tan \phi_L = 0.75, \text{ load kVAR} = 240 \times 0.75 = 180 (\text{lag})$

or load kVA = 240/0.8 = 300, kVAR = 300 × sin  $\phi_L$  = 300 × 0.6 = 180

$\therefore$  leading kVAR supplied by synchronous motor = 180 – 145 = **35.**

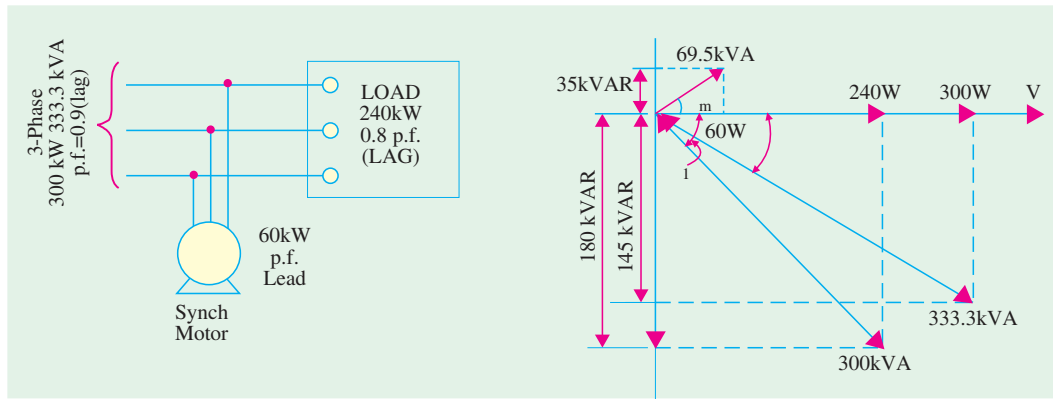


Fig. 38.53

**For Synchronous Motor**

kW = 60, leading kVAR = 35,  $\tan \phi_m = 35/60$  ;  $\phi_m = 30.3^\circ$  ;  $\cos 30.3^\circ = 0.863$

$\therefore$  motor p.f. = **0.863 (lead)**. Incidentally, motor kVA =  $\sqrt{60^2 + 35^2} = 69.5$ .

**38.18. Constant-power Lines**

In Fig. 38.54, OA represents applied voltage / phase of the motor and AB is the back e.m.f. / phase,  $E_b$ . OB is their resultant voltage  $E_R$ . The armature current is OI lagging behind  $E_R$  by an angle  $\theta = \tan^{-1} X_S / R_a$ . Value of  $I = E_R / Z_S$ . Since  $Z_S$  is constant,  $E_R$  or vector OB represents (to some suitable scale) the main current I. OY is drawn at an angle  $\phi$  with OB (or at an angle  $\theta$  with CA). BL is drawn perpendicular to OX which is at right angles to OY. Vector OB, when referred to OY, also represents, on a different scale, the current both in magnitude and phase.

Hence,  $OB \cos \phi = I \cos \phi = BL$

The power input / phase of the motor

=  $VI \cos \phi = V \times BL$ .

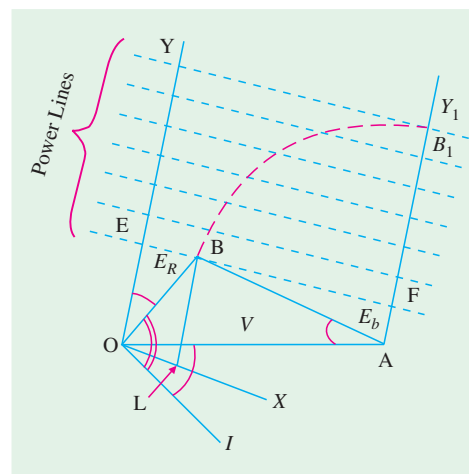


Fig. 38.54

As  $V$  is constant, power input is dependent on  $BL$ . If motor is working with a constant intake, then locus of  $B$  is a straight line  $\parallel$  to  $OX$  and  $\perp$  to  $OY$  i.e. line  $EF$  for which  $BL$  is constant. Hence,  $EF$ , represents a constant-power input line for a given voltage but varying excitation. Similarly, a series of such parallel lines can be drawn each representing a definite power intake of the motor. As regards these constant-power lines, it is to be noted that

1. for equal increase in intake, the power lines are parallel and equally-spaced
2. zero power line runs along  $OX$
3. the perpendicular distance from  $B$  to  $OX$  (or zero power line) represents the motor intake
4. If excitation is fixed i.e.  $AB$  is constant in length, then as the load on motor is increased, increases. In other words, locus of  $B$  is a circle with radius =  $AB$  and centre at  $A$ . With increasing load,  $B$  goes on to lines of higher power till point  $B_1$  is reached. Any further increase in load on the motor will bring point  $B$  down to a lower line. It means that as load increases beyond the value corresponding to point  $B_1$ , the motor intake decreases which is impossible. The area to the right of  $AY_1$  represents unstable conditions. For a given voltage and excitation, the maximum power the motor can develop, is determined by the location of point  $B_1$  beyond which the motor pulls out of synchronism.

### 38.19. Construction of V-curves

The  $V$ -curves of a synchronous motor show how armature current varies with its field current when motor **input is kept constant**. These are obtained by plotting a.c. armature current against d.c. field current while motor input is kept constant and are so called because of their shape (Fig. 38.55). There is a family of such curves, each corresponding to a definite power intake.

In order to draw these curves experimentally, the motor is run from constant voltage and constant-frequency bus-bars. Power input to motor is kept constant at a definite value. Next, field current is increased in small steps and corresponding armature currents are noted. When plotted, we get a  $V$ -curve for a particular constant motor input. Similar curves can be drawn by keeping motor input constant at different values. A family of such curves is shown in Fig. 38.55.

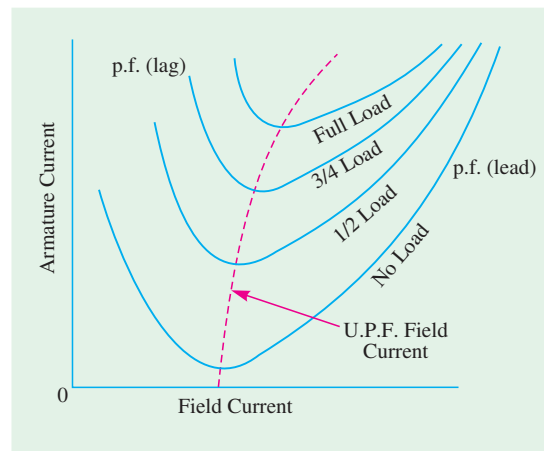


Fig. 38.55

Detailed procedure for graphic construction of  $V$ -curves is given below :

1. First, constant-power lines are drawn as discussed in Art. 38.14.
2. Then, with  $A$  as the centre, concentric circles of different radii  $AB, AB_1, AB_2$ , etc. are drawn where  $AB, AB_1, AB_2$ , etc., are the back e.m.fs corresponding to different excitations. The intersections of these circles with lines of constant power give positions of the working points for specific loads and excitations (hence back e.m.fs). The vectors  $OB, OB_1, OB_2$  etc., represent different values of  $E_R$  (and hence currents) for different excitations. Back e.m.f. vectors  $AB, AB_1$  etc., have not been drawn purposely in order to avoid confusion (Fig. 38.56).

3. The different values of back e.m.fs like  $A B$ ,  $A B_1$ ,  $A B_2$ , etc., are projected on the magnetisation and corresponding values of the field (or exciting) amperes are read from it.
4. The field amperes are plotted against the corresponding armature currents, giving us 'V' curves.

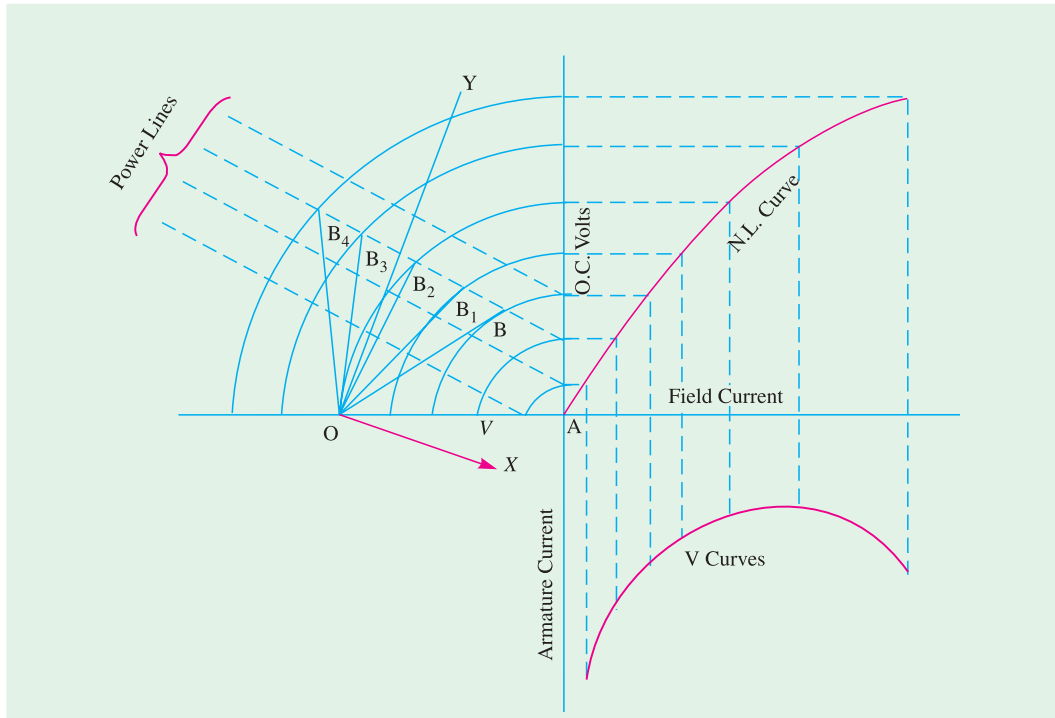


Fig. 38.56

### 38.20. Hunting or Surging or Phase Swinging

When a synchronous motor is used for driving a varying load, then a condition known as hunting is produced. Hunting may also be caused if supply frequency is pulsating (as in the case of generators driven by reciprocating internal combustion engines).

We know that when a synchronous motor is loaded (such as punch presses, shears, compressors and pumps etc.), its rotor falls back in phase by the coupling angle  $\alpha$ . As load is progressively increased, this angle also increases so as to produce more torque for coping with the increased load. If now, there is sudden decrease in the motor load, the motor is immediately pulled up or advanced to a new value of  $\alpha$  corresponding to the new load. But in this process, the rotor overshoots and hence is again pulled back. In this way, the rotor starts oscillating (like a pendulum) about its new position of

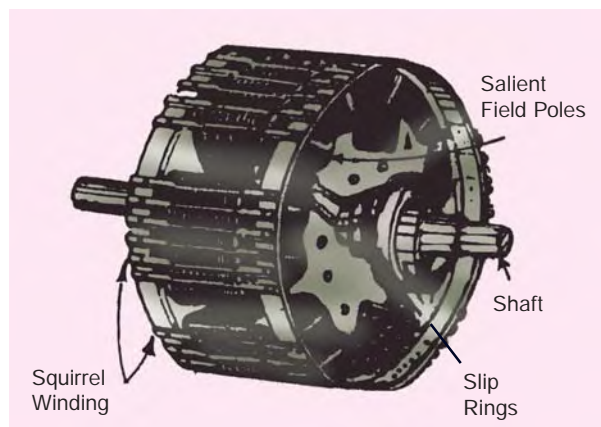
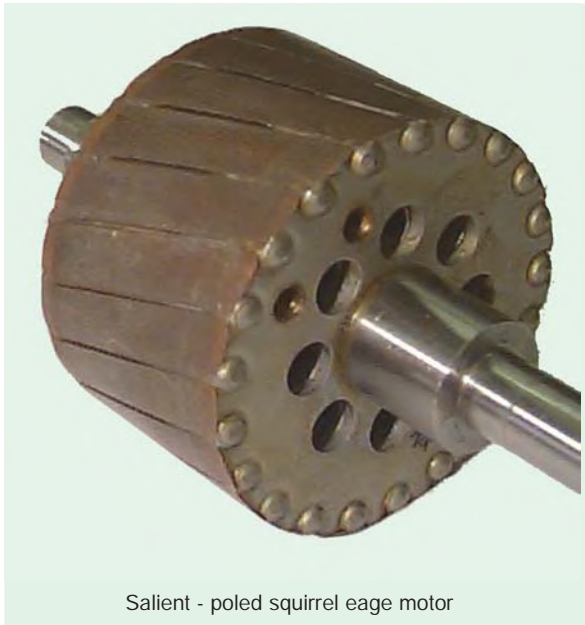


Fig. 38.57



Salient - poled squirrel cage motor

equilibrium corresponding to the new load. If the time period of these oscillations happens to be equal to the natural time period of the machine (refer Art. 37.36) then mechanical resonance is set up. The amplitude of these oscillations is built up to a large value and may eventually become so great as to throw the machine out of synchronism. To stop the build-up of these oscillations, dampers or damping grids (also known as squirrel-cage winding) are employed. These dampers consist of short-circuited Cu bars embedded in the faces of the field poles of the motor (Fig. 38.57). The oscillatory motion of the rotor sets up eddy currents in the dampers which flow in such a way as to suppress these oscillations.

But it should be clearly understood that dampers do not completely prevent hunt-

ing because their operation depends upon the presence of some oscillatory motion. However, they serve the additional purpose of making the synchronous motor self-starting.

### 38.21. Methods of Starting

As said above, almost all synchronous motors are equipped with dampers or squirrel cage windings consisting of Cu bars embedded in the pole-shoes and short-circuited at both ends. Such a motor starts readily, acting as an induction motor during the starting period. The procedure is as follows :

The line voltage is applied to the armature (stator) terminals and the field circuit is left *unexcited*. Motor starts as an induction motor and while it reaches nearly 95% of its synchronous speed, the d.c. field is excited. At that moment the stator and rotor poles get engaged or interlocked with each other and hence pull the motor into synchronism.

However, two points should be noted :

1. At the beginning, when voltage is applied, the rotor is stationary. The rotating field of the stator winding induces a very large e.m.f. in the rotor during the starting period, though the value of this e.m.f. goes on decreasing as the rotor gathers speed.

Normally, the field windings are meant for 110-V (or 250 V for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.

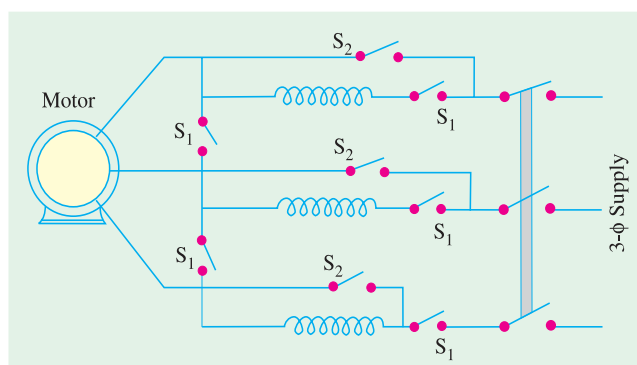


Fig. 38.58

2. When full line voltage is switched on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor. In some cases, this may not be objectionable but where it is, the applied voltage at starting, is reduced by using auto-transformers (Fig. 38.58). However, the voltage should not be reduced to a very low value because the starting torque of an induction motor varies approximately as the square of the applied voltage. Usually, a value of 50% to 80% of the full-line voltage is satisfactory. Auto-transformer connections are shown in Fig. 38.58. For reducing the supply voltage, the switches  $S_1$  are closed and  $S_2$  are kept open. When the motor has been speeded-up,  $S_2$  are closed and  $S_1$  opened to cut out the transformers.

### 38.22. Procedure for Starting a Synchronous Motor

While starting a modern synchronous motor provided with damper windings, following procedure is adopted.

1. First, main field winding is short-circuited.
2. Reduced voltage with the help of auto-transformers is applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak d.c. excitation is applied by removing the short-circuit on the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the auto-transformers.
5. The motor may be operated at any desired power factor by changing the d.c. excitation.

### 38.23. Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.
4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

### 38.24. Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation

#### (a) Power factor correction

Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors (Fig. 38.49) and other devices having lagging p.f. such as welders and fluorescent lights etc.

#### (b) Constant-speed applications

Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc.

Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

#### (c) Voltage regulation

The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor with a field regulator (for varying its excitation), this voltage rise can be controlled.

When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.

### QUESTIONS AND ANSWERS ON SYNCHRONOUS MOTORS

**Q. 1. Does change in excitation affect the synchronous motor speed ?**

**Ans.** No.

**Q. 2. The power factor ?**

**Ans.** Yes.

**Q. 3. How ?**

**Ans.** When over-excited, synchronous motor has leading power factor. However, when underexcited, it has lagging power factor.

**Q. 4. For what service are synchronous motors especially suited ?**

**Ans.** For high voltage service.

**Q. 5. Which has more efficiency; synchronous or induction motor ?**

**Ans.** Synchronous motor.

**Q. 6. Mention some specific applications of synchronous motor ?**

**Ans.** 1. constant speed load service      2. reciprocating compressor drives  
3. power factor correction                4. voltage regulation of transmission lines.

**Q. 7. What is a synchronous capacitor ?**

**Ans.** An overexcited synchronous motor is called synchronous capacitor, because, like a capacitor, it takes a leading current.

**Q. 8. What are the causes of faulty starting of a synchronous motor ?**

**Ans.** It could be due to the following causes :

1. voltage may be too low – at least half voltage is required for starting
2. there may be open-circuit in one phase – due to which motor may heat up
3. static friction may be large – either due to high belt tension or too tight bearings
4. stator windings may be incorrectly connected
5. field excitation may be too strong.

**Q. 9. What could be the reasons if a synchronous motor fails to start ?**

**Ans.** It is usually due to the following reasons :

1. voltage may be too low
2. some faulty connection in auxiliary apparatus
3. too much starting load
4. open-circuit in one phase or short-circuit
5. field excitation may be excessive.

**Q. 10. A synchronous motor starts as usual but fails to develop its full torque. What could it be due to ?**

**Ans.** 1. exciter voltage may be too low 2. field spool may be reversed 3. there may be either open-circuit or short-circuit in the field.

**Q. 11. Will the motor start with the field excited ?**

**Ans.** No.

**Q. 12. Under which conditions a synchronous motor will fail to pull into step ?**

**Ans.** 1. no field excitation 2. excessive load 3. excessive load inertia

## OBJECTIVE TESTS – 38

1. In a synchronous motor, damper winding is provided in order to
  - (a) stabilize rotor motion
  - (b) suppress rotor oscillations
  - (c) develop necessary starting torque
  - (d) both (b) and (c)
2. In a synchronous motor, the magnitude of stator back e.m.f.  $E_b$  depends on
  - (a) speed of the motor
  - (b) load on the motor
  - (c) both the speed and rotor flux
  - (d) d.c. excitation only
3. An electric motor in which both the rotor and stator fields rotates with the same speed is called a/an .....motor.
  - (a) d.c.
  - (b) chrage
  - (c) synchronous
  - (d) universal
4. While running, a synchronous motor is compelled to run at synchronous speed because of
  - (a) damper winding in its pole faces
  - (b) magnetic locking between stator and rotor poles
  - (c) induced e.m.f. in rotor field winding by stator flux
  - (d) compulsion due to Lenz's law
5. The direction of rotation of a synchronous motor can be reversed by reversing
  - (a) current to the field winding
  - (b) supply phase sequence
  - (c) polarity of rotor poles
  - (d) none of the above



6. When running under no-load condition and with normal excitation, armature current  $I_a$  drawn by a synchronous motor
- leads the back e.m.f.  $E_b$  by a small angle
  - is large
  - lags the applied voltage  $V$  by a small angle
  - lags the resultant voltage  $E_R$  by  $90^\circ$ .
7. The angle between the synchronously-rotating stator flux and rotor poles of a synchronous motor is called..... angle.
- synchronizing
  - torque
  - power factor
  - slip
8. If load angle of a 4-pole synchronous motor is  $8^\circ$  (elect), its value in mechanical degrees is .....
- 4
  - 2
  - 0.5
  - 0.25
9. The maximum value of torque angle  $\alpha$  in a synchronous motor is .....degrees electrical.
- 45
  - 90
  - between 45 and 90
  - below 60
10. A synchronous motor running with normal excitation adjusts to load increases *essentially* by increase in its
- power factor
  - torque angle
  - back e.m.f.
  - armature current.
11. When load on a synchronous motor running with normal excitation is increased, armature current drawn by it increases because
- back e.m.f.  $E_b$  becomes less than applied voltage  $V$
  - power factor is decreased
  - net resultant voltage  $E_R$  in armature is increased
  - motor speed is reduced
12. When load on a normally-excited synchronous motor is increased, its power factor tends to
- approach unity
  - become increasingly lagging
  - become increasingly leading
  - remain unchanged.
13. The effect of increasing load on a synchronous motor running with normal excitation is to
- increase both its  $I_a$  and p.f.
  - decrease  $I_a$  but increase p.f.
  - increase  $I_a$  but decrease p.f.
  - decrease both  $I_a$  and p.f.
14. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is increased, its torque angle must necessarily
- decrease
  - increase
  - remain constant
  - become twice the no-load value.
15. If the field of a synchronous motor is under-excited, the power factor will be
- lagging
  - leading
  - unity
  - more than unity
16. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is decreased from its normal value, it leads to
- increase in but decrease in  $E_b$
  - increase in  $E_b$  but decrease in  $I_a$
  - increase in both  $I_a$  and p.f. which is lagging
  - increase in both  $I_a$  and  $\phi$
17. A synchronous motor connected to infinite bus-bars has at *constant* full-load, 100% excitation and unity p.f. On changing the excitation only, the armature current will have
- leading p.f. with under-excitation
  - leading p.f. with over-excitation
  - lagging p.f. with over-excitation
  - no change of p.f.
- (Power App.-II, Delhi Univ. Jan 1987)**
18. The  $V$ -curves of a synchronous motor show relationship between
- excitation current and back e.m.f.
  - field current and p.f.
  - d.c. field current and a.c. armature current
  - armature current and supply voltage.

19. When load on a synchronous motor is increased, its armature currents is increased provided it is  
 (a) normally-excited  
 (b) over-excited  
 (c) under-excited  
 (d) all of the above
20. If main field current of a salient-pole synchronous motor fed from an infinite bus and running at no-load is reduced to zero, it would  
 (a) come to a stop  
 (b) continue running at synchronous speed  
 (c) run at sub-synchronous speed  
 (d) run at super-synchronous speed
21. In a synchronous machine when the rotor speed becomes more than the synchronous speed during hunting, the damping bars develop  
 (a) synchronous motor torque  
 (b) d.c. motor torque  
 (c) induction motor torque  
 (d) induction generator torque  
 (Power App.-II, Delhi Univ. Jan. 1987)
22. In a synchronous motor, the rotor Cu losses are met by  
 (a) motor input  
 (b) armature input  
 (c) supply lines  
 (d) d.c. source
23. A synchronous machine is called a doubly-excited machine because  
 (a) it can be overexcited  
 (b) it has two sets of rotor poles  
 (c) both its rotor and stator are excited  
 (d) it needs twice the normal exciting current.
24. Synchronous capacitor is  
 (a) an ordinary static capacitor bank  
 (b) an over-excited synchronous motor driving mechanical load  
 (c) an over-excited synchronous motor running without mechanical load  
 (d) none of the above 623  
 (Elect. Machines, A.M.I.E. Sec. B, 1993)

## ANSWERS

1. d    2. d    3. c    4. b    5. b    6. c    7. b    8. a    9. b    10. d    11. c  
 12. b    13. c    14. a    15. a    16. d    17. b    18. c    19. d    20. b    21. d    22. d  
 23. c    24. c

# CHAPTER 39

## Learning Objectives

- Introduction
- Stepper Motors
- Types of Stepper Motors
- Variable Reluctance Stepper Motors
- Multi-stack VR Stepper Motor
- Permanent-Magnet Stepping Motor
- Hybrid Stepper Motor
- Summary of Stepper Motors
- Permanent-Magnet DC Motor
- Low-inertia DC Motors
- Shell-type Low-inertia DC Motor
- Printed-circuit (Disc) DC Motor
- Permanent-Magnet Synchronous Motors
- Synchros
- Types of Synchros
- Applications of Synchros
- Control Differential Transmitter
- Control Differential Receiver
- Switched Reluctance Motor
- Comparison between VR Stepper Motor and SR Motor
- The Resolver
- Servomotors
- DC Servomotors
- AC Servomotors

## SPECIAL MACHINES



↑ Stepper motor

### 39.1. Introduction

This chapter provides a brief introduction to electrical machines which have special applications. It includes machines whose stator coils are energized by electronically switched currents. The examples are: various types of stepper motors, brushless d.c. motor and switched reluctance motor etc. There is also a brief description of d.c./a.c. servomotors, synchro motors and resolvers. These motors are designed and built primarily for use in feedback control systems.

### 39.2. Stepper Motors

These motors are also called stepping motors or step motors. The name stepper is used because this motor rotates through a fixed angular step in response to each input current pulse received by its controller. In recent years, there has been widespread demand of stepping motors because of the explosive growth of the computer industry. Their popularity is due to the fact that they can be controlled directly by computers, microprocessors and programmable controllers.

As we know, industrial motors are used to convert electric energy into mechanical energy but they cannot be used for precision positioning of an object or precision control of speed without using closed-loop feedback. Stepping motors are ideally suited for situations where either precise positioning or precise speed control or both are required in automation systems.

Apart from stepping motors, other devices used for the above purposes are synchros and resolvers as well as dc/ac servomotors (discussed later).

The unique feature of a stepper motor is that its output shaft rotates in a series of discrete angular intervals or steps, one step being taken each time a command pulse is received. When a definite number of pulses are supplied, the shaft turns through a definite known angle. This fact makes the motor well-suited for open-loop position control because no feedback need be taken from the output shaft.

Such motors develop torques ranging from 1  $\mu\text{N}\cdot\text{m}$  (in a tiny wrist watch motor of 3 mm diameter) upto 40 N·m in a motor of 15 cm diameter suitable for machine tool applications. Their power output ranges from about 1 W to a maximum of 2500 W. The only moving part in a stepping motor is its rotor which has no windings, commutator or brushes. This feature makes the motor quite robust and reliable.

#### Step Angle

The angle through which the motor shaft rotates for each command pulse is called the step angle  $\beta$ . Smaller the step angle, greater the number of steps per revolution and higher the resolution or accuracy of positioning obtained. The step angles can be as small as  $0.72^\circ$  or as large as  $90^\circ$ . But the most common step sizes are  $1.8^\circ$ ,  $2.5^\circ$ ,  $7.5^\circ$  and  $15^\circ$ .

The value of step angle can be expressed either in terms of the rotor and stator poles (teeth)  $N_r$  and  $N_s$  respectively or in terms of the number of stator phases ( $m$ ) and the number of rotor teeth.



Stepper Motor

$$\beta = \frac{(N_s - N_r)}{N_s \cdot N_r} \times 360^\circ$$

$$\text{or } \beta = \frac{360^\circ}{m N_r} = \frac{360^\circ}{\text{No. of stator phases} \times \text{No. of rotor teeth}}$$

For example, if  $N_s = 8$  and  $N_r = 6$ ,  $\beta = (8 - 6) \times 360 / 8 \times 6 = 15^\circ$

Resolution is given by the number of steps needed to complete one revolution of the rotor shaft. Higher the resolution, greater the accuracy of positioning of objects by the motor

$$\therefore \text{Resolution} = \text{No. of steps / revolution} = 360^\circ / \beta$$

A stepping motor has the extraordinary ability to operate at very high stepping rates (upto 20,000 steps per second in some motors) and yet to remain fully in synchronism with the command pulses. When the pulse rate is high, the shaft rotation seems continuous. Operation at high speeds is called 'slewing'. When in the slewing range, the motor generally emits an audible whine having a fundamental frequency equal to the stepping rate. If  $f$  is the stepping frequency (or pulse rate) in pulses per second (pps) and  $\beta$  is the step angle, then motor shaft speed is given by

$$n = \beta \times f / 360 \text{ rps} = \text{pulse frequency resolution}$$

If the stepping rate is increased too quickly, the motor loses synchronism and stops. Same thing happens if when the motor is slewing, command pulses are suddenly stopped instead of being progressively slowed.

Stepping motors are designed to operate for long periods with the rotor held in a fixed position and with rated current flowing in the stator windings. It means that stalling is no problem for such motors whereas for most of the other motors, stalling results in the collapse of back emf ( $E_b$ ) and a very high current which can lead to a quick burn-out.

#### Applications :

Such motors are used for operation control in computer peripherals, textile industry, IC fabrications and robotics etc. Applications requiring incremental motion are typewriters, line printers, tape drives, floppy disk drives, numerically-controlled machine tools, process control systems and X-Y plotters. Usually, position information can be obtained simply by keeping count of the pulses sent to the motor thereby eliminating the need for expensive position sensors and feedback controls. Stepper motors also perform countless tasks outside the computer industry. It includes commercial, military and medical applications where these motors perform such functions as mixing, cutting, striking, metering, blending and purging. They also take part in the manufacture of packed food stuffs, commercial end-products and even the production of science fiction movies.



Connecting a stepper motor to the interface

**Example 39.1.** A hybrid VR stepping motor has 8 main poles which have been castleated to have 5 teeth each. If rotor has 50 teeth, calculate the stepping angle.

**Solution.**

$$N_s = 8 \times 5 = 40; \quad N_r = 50$$

$$\therefore \beta = (50 - 40) \times 360 / 50 \times 40 = 1.8^\circ.$$

**Example 39.2.** A stepper motor has a step angle of  $2.5^\circ$ . Determine (a) resolution (b) number of steps required for the shaft to make 25 revolutions and (c) shaft speed, if the stepping frequency is 3600 pps.

**Solution.** (a) Resolution =  $360^\circ / \beta = 360^\circ / 2.5^\circ = 144$  steps / revolution.

(b) Now, steps / revolution = 144. Hence, steps required for making 25 revolutions =  $144 \times 25 = 3600$ .

(c)  $n = \beta \times f / 360^\circ = 2.5 \times 3600 / 360^\circ = 25$  rps

### 39.3. Types of Stepper Motors

There is a large variety of stepper motors which can be divided into the following three basic categories :

#### (i) Variable Reluctance Stepper Motor

It has wound stator poles but the rotor poles are made of a ferromagnetic material as shown in Fig.39.1 (a). It can be of the single stack type (Fig.39.2) or multi-stack type (Fig.39.5) which gives smaller step angles. Direction of motor rotation is independent of the polarity of the stator current. It is called variable reluctance motor because the reluctance of the magnetic circuit formed by the rotor and stator teeth varies with the angular position of the rotor.

#### (ii) Permanent Magnet Stepper Motor

It also has wound stator poles but its rotor poles are permanently magnetized. It has a cylindrical rotor as shown in Fig. 39.1 (b). Its direction of rotation depends on the polarity of the stator current.

#### (iii) Hybrid Stepper Motor

It has wound stator poles and permanently-magnetized rotor poles as shown in Fig.39.1(c). It is best suited when small step angles of  $1.8^\circ$ ,  $2.5^\circ$  etc. are required.



Permanent magnet stepper motor

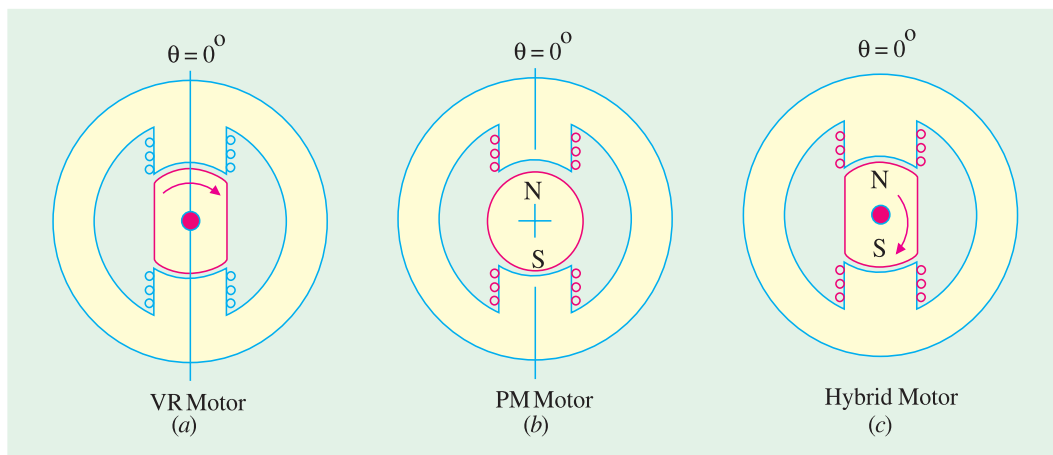


Fig. 39.1

As a variable speed machine, VR motor is sometime designed as a switched-reluctance motor. Similarly, PM stepper motor is also called variable speed brushless dc motor. The hybrid motor combines the features of VR stepper motor and PM stepper motor. Its stator construction is similar to the single-stack VR motor but the rotor is cylindrical and is composed of radially magnetized permanent magnets. A recent type uses a disc rotor which is magnetized axially to give a small stepping angle and low inertia.

### 39.4. Variable Reluctance Stepper Motors

**Construction :** A variable-reluctance motor is constructed from ferromagnetic material with salient poles as shown in Fig. 39.2. The stator is made from a stack of steel laminations and has six equally-spaced projecting poles (or teeth) each wound with an exciting coil. The rotor which may be solid or laminated has four projecting teeth of the same width as the stator teeth. As seen, there are three independent stator circuits or phases *A*, *B* and *C* and each one can be energised by a direct current pulse from the drive circuit (not shown in the figure).



Variable reluctance motor

A simple circuit arrangement for supplying current to the stator coils in proper sequence is shown in Fig. 39.2 (e). The six stator coils are connected in 2-coil groups to form three separate circuits called phases. Each phase has its own independent switch. Diametrically opposite pairs of stator coils are connected in series such that when one tooth becomes a *N*-pole, the other one becomes a *S*-pole. Although shown as mechanical switches in Fig. 39.2 (e), in actual practice, switching of phase currents is done with the help of solid-state control. When there is no current in the stator coils, the rotor is completely free to rotate. Energising one or more stator coils causes the rotor to step forward (or backward) to a position that forms a path of least reluctance with the magnetized stator teeth. The step angle of this three-phase, four rotor teeth motor is  $\beta = 360/4 \times 3 = 30^\circ$ .

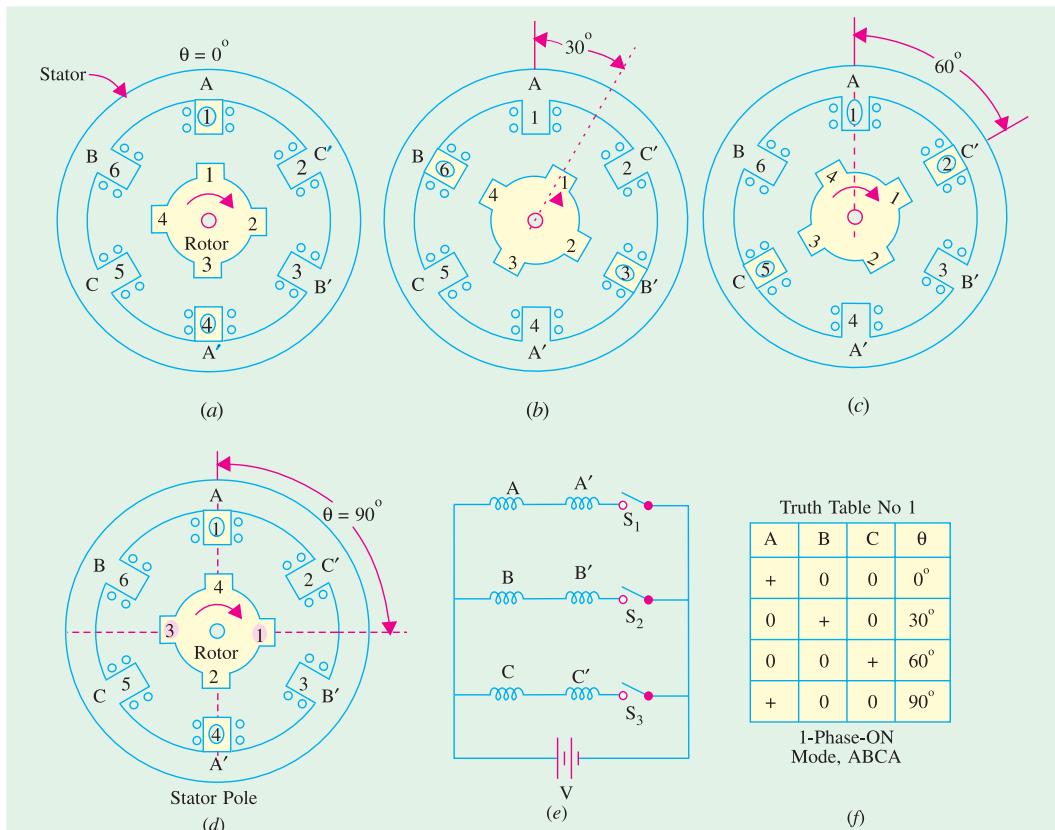


Fig. 39.2

**Working.** The motor has following modes of operation :

**(a) 1-phase-ON or Full-step Operation**

Fig. 39.2 (a) shows the position of the rotor when switch  $S_1$  has been closed for energising phase A. A magnetic field with its axis along the stator poles of phase A is created. The rotor is therefore, attracted into a position of minimum reluctance with diametrically opposite rotor teeth 1 and 3 lining up with stator teeth 1 and 4 respectively. Closing  $S_2$  and opening  $S_1$  energizes phase B causing rotor teeth 2 and 4 to align with stator teeth 3 and 6 respectively as shown in Fig. 39.2 (b). The rotor rotates through full-step of  $30^\circ$  in the clockwise (CW) direction. Similarly, when  $S_3$  is closed after opening  $S_2$ , phase C is energized which causes rotor teeth 1 and 3 to line up with stator teeth 2 and 5 respectively as shown in Fig. 39.2 (c). The rotor rotates through an additional angle of  $30^\circ$  in the clockwise (CW) direction. Next if  $S_3$  is opened and  $S_1$  is closed again, the rotor teeth 2 and 4 will align with stator teeth 4 and 1 respectively thereby making the rotor turn through a further angle of  $30^\circ$  as shown in Fig. 39.2 (d). By now the total angle turned is  $90^\circ$ . As each switch is closed and the preceding one opened, the rotor each time rotates through an angle of  $30^\circ$ . By repetitively closing the switches in the sequence 1-2-3-1 and thus energizing stator phases in sequence ABCA etc., the rotor will rotate clockwise in  $30^\circ$  steps. If the switch sequence is made 3-2-1-3 which makes phase sequence CBAC (or ACB), the rotor will rotate anticlockwise. This mode of operation is known as 1-phase-ON mode or full-step operation and is the simplest and widely-used way of making the motor step. The stator phase switching truth table is shown in Fig. 39.2 (f). It may be noted that the direction of the stator magnetizing current is not significant because a stator pole of either magnetic polarity will always attract the rotor pole by inducing opposite polarity.

**(b) 2-phase-ON Mode**

In this mode of operation, two stator phases are excited simultaneously. When phases A and B are energized together, the rotor experiences torques from both phases and comes to rest at a point mid-way between the two adjacent full-step positions. If the stator phases are switched in the sequence AB, BC, CA, AB etc., the motor will take full steps of  $30^\circ$  each (as in the 1-phase-ON mode) but its equilibrium positions will be interleaved between the full-step positions. The phase switching truth table for this mode is shown in Fig. 39.3 (a).

Truth Table No. 2				Truth Table No. 3			
A	B	C		A	B	C	
+	+	0	$15^\circ$	+	0	0	$0^\circ$
0	+	+	$45^\circ$	+	+	0	$15^\circ$
+	0	+	$75^\circ$	0	+	0	$30^\circ$
+	+	0	$105^\circ$	0	+	+	$45^\circ$
				0	0	+	$65^\circ$
				+	0	+	$75^\circ$
				+	0	0	$90^\circ$

2 Phase-ON Mode  
AB, BC, CA, AB

Half-Stepping Alternate  
1-Phase-On &  
2-Phase-on Mode  
A, AB, B, BC, C, CA, A

Fig. 39.3

The 2-phase-ON mode provides greater holding torque and a much better damped single-stack response than the 1-phase-ON mode of operation.



### (c) Half-step Operation

Half-step operation or 'half-stepping' can be obtained by exciting the three phases in the sequence  $A, AB, B, BC, C$  etc. *i.e.* alternately in the 1-phase-ON and 2-phase-ON modes. It is sometime known as 'wave' excitation and it causes the rotor to advance in steps of  $15^\circ$  *i.e.* half the full-step angle. The truth table for the phase pulsing sequence in half-stepping is shown in Fig. 39.3 (b).

Half-stepping can be illustrated with the help of Fig. 39.4 where only three successive pulses have been considered. Energizing only phase  $A$  causes the rotor position shown in Fig. 39.4 (a). Energizing phases  $A$  and  $B$  simultaneously moves the rotor to the position shown in Fig. 39.4 (b) where rotor has moved through half a step only. Energizing only phase  $B$  moves the rotor through another half-step as shown in Fig. 39.4 (c). With each pulse, the rotor moves  $30 / 2 = 15^\circ$  in the CCW direction.

It will be seen that in half-stepping mode, the step angle is halved thereby doubling the resolution. Moreover, continuous half-stepping produces a smoother shaft rotation.

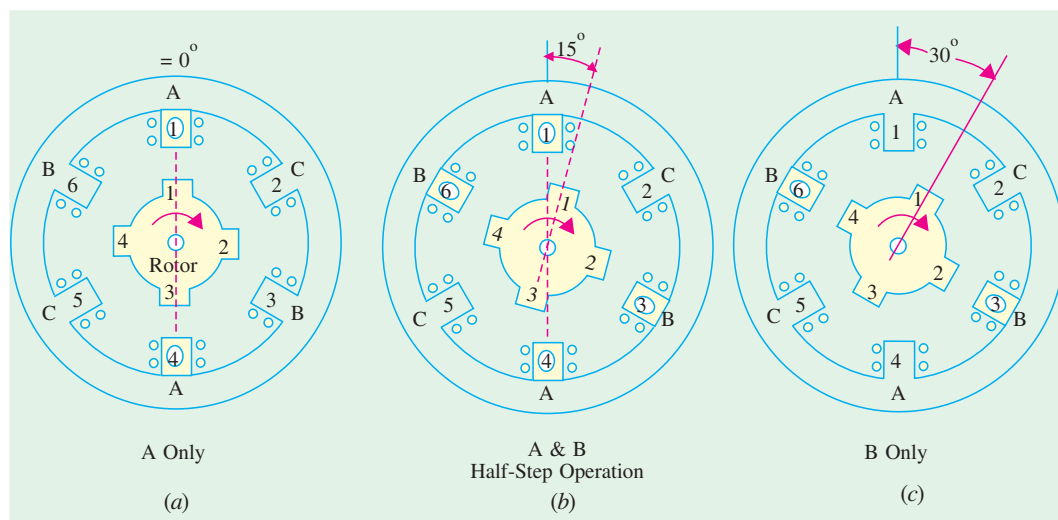


Fig. 39.4

### (d) Microstepping

It is also known as mini-stepping. It utilizes two phases simultaneously as in 2-phase-ON mode but with the two currents deliberately made unequal (unlike in half-stepping where the two phase currents have to be kept equal). The current in phase  $A$  is held constant while that in phase  $B$  is increased in very small increments until maximum current is reached. The current in phase  $A$  is then reduced to zero using the same very small increments. In this way, the resultant step becomes very small and is called a microstep. For example, a  $VR$  stepper motor with a resolution of 200 steps / rev ( $\beta = 1.8^\circ$ ) can with microstepping have a resolution of 20,000 steps / rev ( $\beta = 0.018^\circ$ ). Stepper motors employing microstepping technique are used in printing and phototypesetting where very fine resolution is called for. As seen, microstepping provides smooth low-speed operation and high resolution.

**Torque.** If  $I_a$  is the d.c. current pulse passing through phase  $A$ , the torque produced by it is given by  $T = (1/2) I_a^2 dL / d\theta$ .  $VR$  stepper motors have a high (torque / inertia) ratio giving high rates of acceleration and fast response. A possible disadvantage is the absence of detent torque which is necessary to retain the rotor at the step position in the event of a power failure.

### 39.5. Multi-stack VR Stepper Motor

So, far, we have discussed single-stack VR motors though multi-stack motors are also available which provide smaller step angles. The multi-stack motor is divided along its axial length into a number of magnetically-isolated sections or stacks which can be excited by a separate winding or phase. Both stator and rotor have the same number of poles. The stators have a common frame while rotors have a common shaft as shown in Fig. 39.5 (a) which represents a three-stack VR motor. The teeth of all the rotors are perfectly aligned with respect to themselves but the stator teeth of various stacks have a progressive angular displacement as shown in the developed diagram of Fig. 39.5 (b) for phase excitation.

Three-stack motors are most common although motors with upto seven stacks and phases are available. They have step angles in the range of  $2^\circ$  to  $15^\circ$ . For example, in a six-stack VR motor having 20 rotor teeth, the step angle  $\beta = 360^\circ / 6 \times 20 = 3^\circ$ .

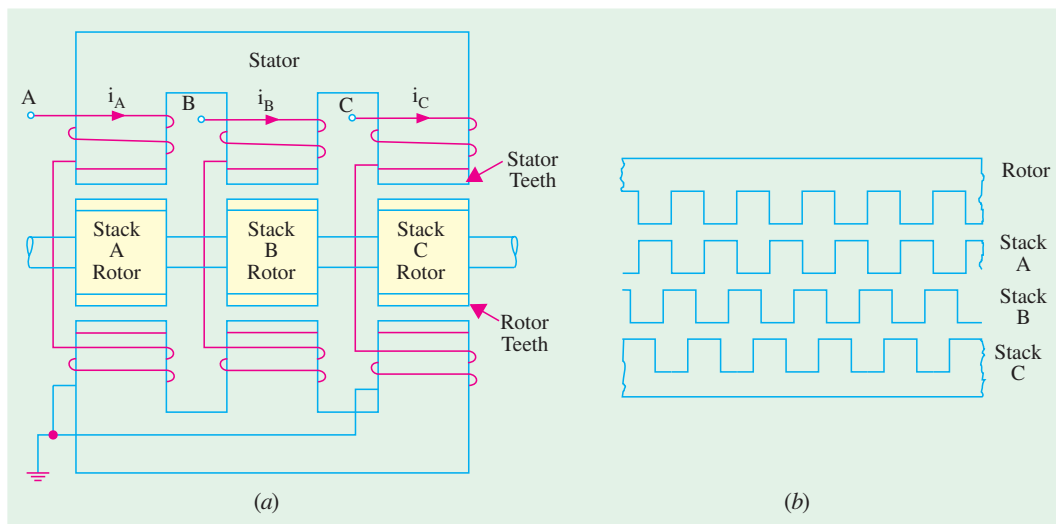
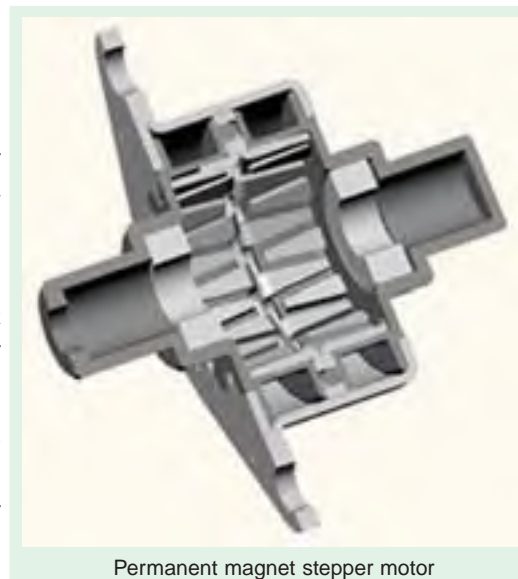


Fig. 39.5

### 39.6. Permanent-Magnet Stepping Motor

(a) **Construction.** Its stator construction is similar to that of the single-stack VR motor discussed above but the rotor is made of a permanent-magnet material like magnetically 'hard' ferrite. As shown in the Fig. 39.6 (a), the stator has projecting poles but the rotor is cylindrical and has radially magnetized permanent magnets. The operating principle of such a motor can be understood with the help of Fig. 39.6 (a) where the rotor has two poles and the stator has four poles. Since two stator poles are energized by one winding, the motor has two windings or phases marked A and B. The step angle of this motor  $\beta = 360^\circ / mN_r = 360^\circ / 2 \times 2 = 90^\circ$  or  $\beta = (4 - 2) \times 360^\circ / 2 \times 4 = 90^\circ$ .



Permanent magnet stepper motor

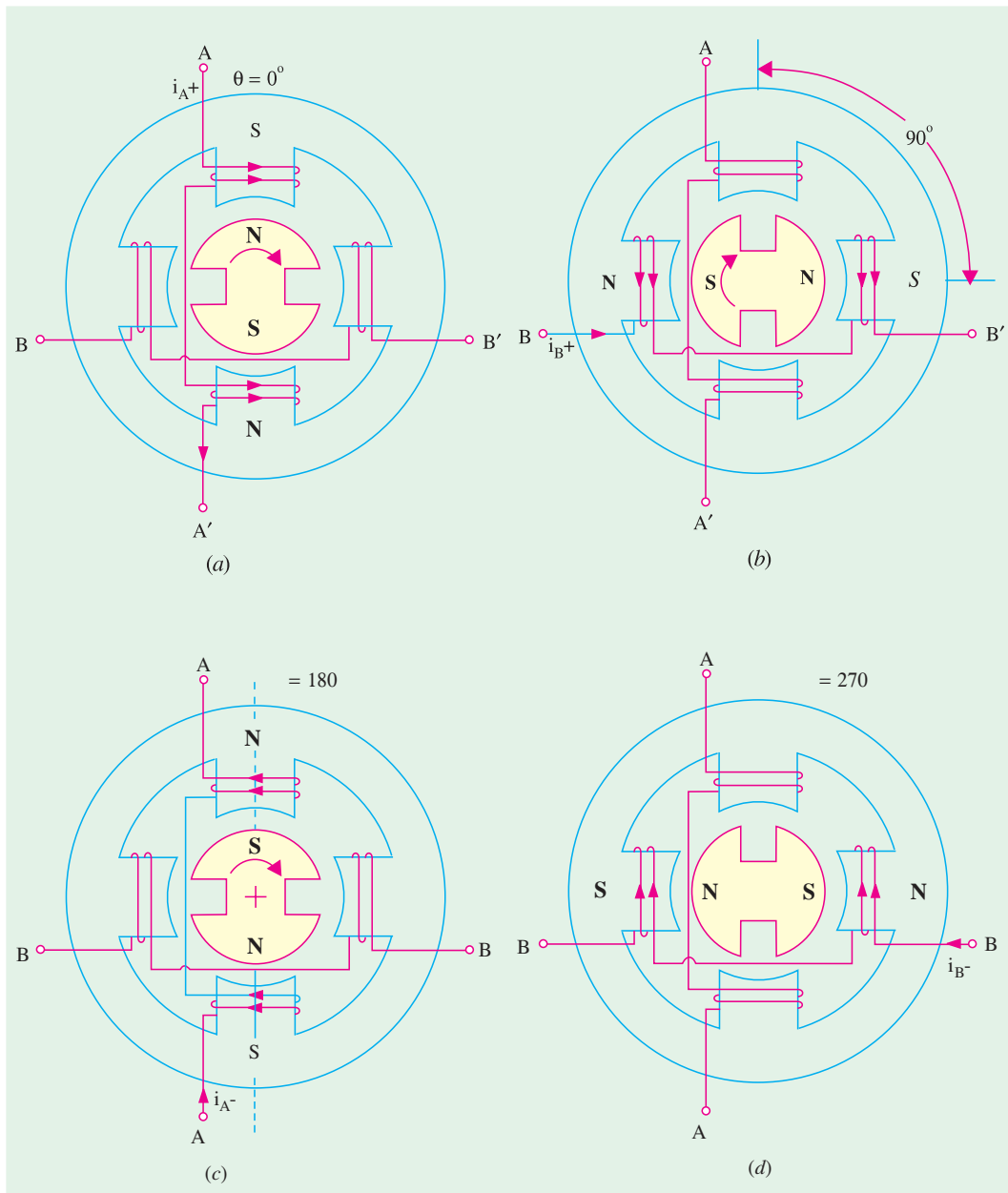


Fig. 39.6

**(b) Working.** When a particular stator phase is energized, the rotor magnetic poles move into alignment with the excited stator poles. The stator windings  $A$  and  $B$  can be excited with either polarity current ( $A^+$  refers to positive current  $i_{A^+}$  in the phase  $A$  and  $A^-$  to **negative current**  $i_{A^-}$ ). Fig. 39.6 (a) shows the condition when phase  $A$  is excited with positive current  $i_{A^+}$ . Here,  $\theta = 0^\circ$ . If excitation is now switched to phase  $B$  as in Fig. 39.6 (b), the rotor rotates by a full step of  $90^\circ$  in the clockwise direction. Next, when phase  $A$  is excited with negative current  $i_{A^-}$ , the rotor turns through another  $90^\circ$  in  $CW$  direction as shown in Fig. 39.6 (c). Similarly, excitation of phase  $B$  with  $i_{B^-}$  further turns the rotor through another  $90^\circ$  in the same direction as shown in Fig. 39.6 (d). After this, excitation of phase  $A$  with  $i_{A^+}$  makes the rotor turn through one complete revolution of  $360^\circ$ .

Truth Table No. 1			Truth Table No. 2			Truth Table No. 3		
A	B		A	B		A	B	
+	0	0°	+	+	45°	+	0	0°
0	+	90°		+	135°	+	+	45°
	0	180°			225°	0	+	90°
0		270°	+		315°			225°
+	0	0°	+	+	45°	0		270°
						+		315°
						+	0	0°

1-Phase-ON Mode
1-Phase-ON Mode
Alternate 1-Phase-On & 2-Phase-On Modes

Fig. 39.7

It will be noted that in a permanent-magnet stepper motor, the direction of rotation depends on the polarity of the phase currents as tabulated below :

$$\begin{aligned}
 & i_{A^+}; i_{B^+}; i_{A^-}; i_{B^-}; i_{A^+}, \dots \dots \dots \\
 & A^+; B^+; A^-; B^-; A^+, \dots \dots \dots \quad \text{for clockwise rotation} \\
 & i_{A^+}; i_{B^-}; i_{A^-}; i_{B^+}; i_{A^+}, \dots \dots \dots \\
 & A^+; B^-; A^-; B^+; A^+, \dots \dots \dots \quad \text{for CCW rotation}
 \end{aligned}$$

Truth tables for three possible current sequences for producing clockwise rotation are given in Fig. 39.7. Table No.1 applies when only one phase is energized at a time in 1-phase-ON mode giving step size of 90°. Table No.2 represents 2-phase-ON mode when two phases are energised simultaneously. The resulting steps are of the same size but the effective rotor pole positions are midway between the two adjacent full-step positions. Table No.3 represents half-stepping when 1-phase-ON and 2-phase-ON modes are used alternately. In this case, the step size becomes half of the normal step or one-fourth of the pole-pitch (i.e. 90° / 2 = 45° or 180° / 4 = 45°). Microstepping can also be employed which will give further reduced step sizes thereby increasing the resolution.

**(c) Advantages and Disadvantages.** Since the permanent magnets of the motor do not require external exciting current, it has a low power requirement but possesses a high detent torque as compared to a VR stepper motor. This motor has higher inertia and hence slower acceleration. However, it produces more torque per ampere stator current than a VR motor. Since it is difficult to manufacture a small permanent-magnet rotor with large number of poles, the step size in such motors is relatively large ranging from 30° to 90°. However, recently disc rotors have been manufactured which are magnetized axially to give a small step size and low inertia.

**Example 39.3.** A single-stack, 3-phase VR motor has a step angle of 15°. Find the number of its rotor and stator poles.

**Solution.** Now,  $\beta = 360^\circ / mN_r$  or  $15^\circ = 360^\circ / 3 \times N_r$ ;  $\therefore N_r = 8$ .

For finding the value of  $N_s$ , we will use the relation  $\beta = (N_s - N_r) \times 360^\circ / N_s \cdot N_r$ .

(i) When  $N_s > N_r$  Here,  $\beta = (N_s - N_r) \times 360^\circ / N_s \cdot N_r$

or  $15^\circ = (N_s - 8) \times 360^\circ / 8 N_s$ ;  $\therefore N_s = 12$

(ii) When  $N_s < N_r$  Here,  $15^\circ = (8 - N_s) \times 360^\circ / 8 N_s$ ;  $\therefore N_s = 6$ .

**Example 39.4.** A four-stack VR stepper motor has a step angle of  $1.8^\circ$ . Find the number of its rotor and stator teeth.

**Solution.** A four-stack motor has four phases. Hence,  $m = 4$ .

$\therefore 1.8^\circ = 360^\circ / 4 \times N_r$ ;  $\therefore N_r = 50$ .

Since in multi-stack motors, rotor teeth equal the stator teeth, hence  $N_s = 50$ .

### 39.7. Hybrid Stepper Motor

(a) **Construction.** It combines the features of the variable reluctance and permanent-magnet stepper motors. The rotor consists of a permanent-magnet that is magnetized axially to create a pair of poles marked  $N$  and  $S$  in Fig. 39.8 (b). Two end-caps are fitted at both ends of this axial magnet. These end-caps consist of equal number of teeth which are magnetized by the respective polarities of the axial magnet. The rotor teeth of one end-cap are offset by a half tooth pitch so that a tooth at one end-cap coincides with a slot at the other. The cross-sectional views perpendicular to the shaft along  $X-X'$  and  $Y-Y'$  axes are shown in Fig. 39.8 (a) and (c) respectively. As seen, the stator consists of four stator poles which are excited by two stator windings in pairs. The rotor has five  $N$ -poles at one end and five  $S$ -poles at the other end of the axial magnet. The step angle of such a motor is  $= (5 - 4) \times 360^\circ / 5 \times 4 = 18^\circ$ .



Hybrid stepper motor

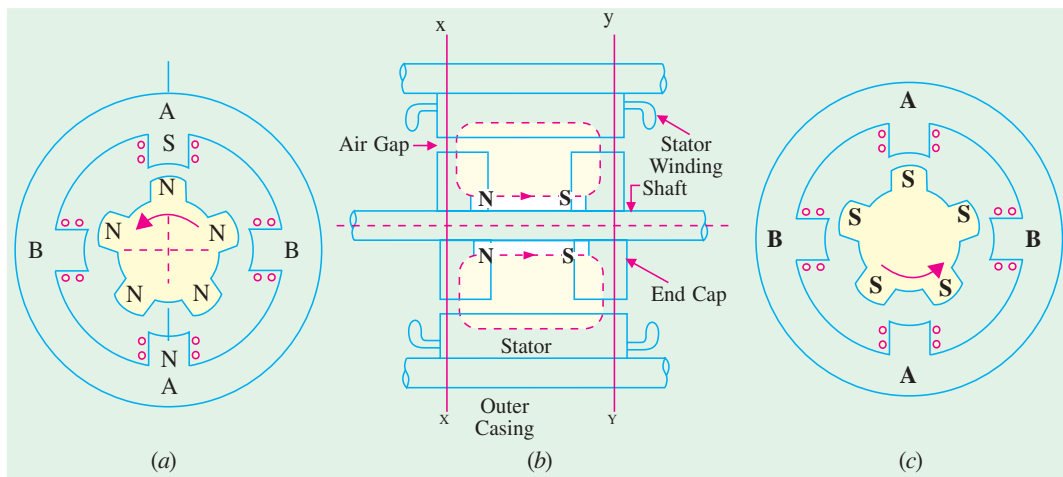


Fig. 39.8

(b) **Working.** In Fig.39.8 (a), phase A is shown excited such that the top stator pole is a  $S$ -pole so that it attracts the top  $N$ -pole of the rotor and brings it in line with the  $A-A'$  axis. To turn the rotor,

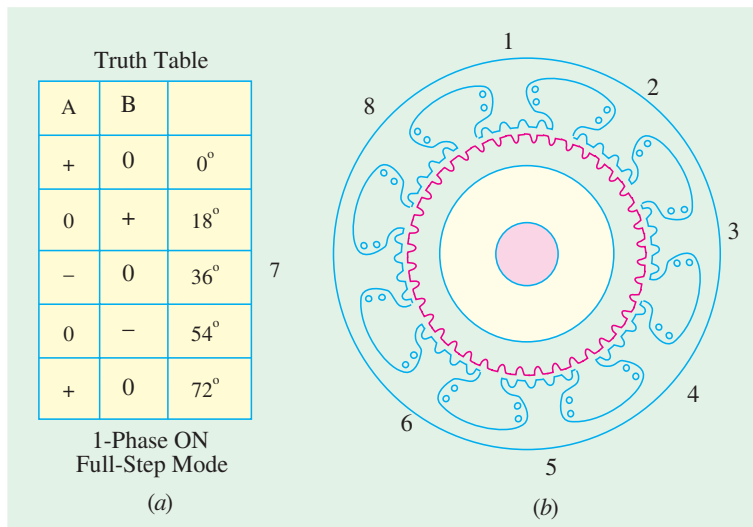


Fig. 39.9

Fig. 39.9 in order to give higher angular resolution. Hence, the stator poles are often slotted or castleated to increase the number of stator teeth. As shown in Fig. 39.9 (b), each of the eight stator poles has been allotted or castleated into five smaller poles making  $N_s = 8 \times 5 = 40$ . If rotor has 50 teeth, then step angle  $= (50 - 40) \times 360^\circ / 50 \times 40 = 1.8^\circ$ . Step angle can also be decreased (and hence resolution increased) by having more than two stacks on the rotor.

This motor achieves small step sizes easily and with a simpler magnet structure whereas a purely *PM* motor requires a multiple permanent-magnet. As compared to *VR* motor, hybrid motor requires less excitation to achieve a given torque. However, like a *PM* motor, this motor also develops good detent torque provided by the permanent-magnet flux. This torque holds the rotor stationary while the power is switched off. This fact is quite helpful because the motor can be left overnight without fear of its being accidentally moved to a new position.

### 39.8. Summary of Stepper Motors

1. A stepper motor can be looked upon as a digital electromagnetic device where each pulse input results in a discrete output *i.e.* a definite angle of shaft rotation. It is ideally-suited for open-loop operation because by keeping a count of the number of input pulses, it is possible to know the exact position of the rotor shaft.

2. In a *VR* motor, excitation of the stator phases gives rise to a torque in a direction which minimizes the magnetic circuit reluctance. The reluctance torque depends on the square of the phase current and its direction is independent of the polarity of the phase current. A *VR* motor can be a single-stack or multi-stack motor. The step angle  $\beta = 360^\circ / mN_r$ , where  $N_r$  is the number of rotor teeth and  $m$  is the number of phases in the single-stack motor or the number of stacks in the multi-stack motor.

3. A permanent-magnet stepper motor has a permanently-magnetized cylindrical rotor. The direction of the torque produced depends on the polarity of the stator current.

4. A hybrid motor combines the features of *VR* and *PM* stepper motors. The direction of its torque also depends on the polarity of the stator current. Its step angle  $\beta = 360^\circ / mN_r$ .

5. In the 1-phase *ON* mode of excitation, the rotor moves by one full-step for each change of excitation. In the 2-phase-*ON* mode, the rotor moves in full steps although it comes to rest at a point midway between the two adjacent full-step positions.

phase *A* is denergized and phase *B* is excited positively. The rotor will turn in the CCW direction by a full step of  $18^\circ$ .

Next, phase *A* and *B* are energized negatively one after the other to produce further rotations of  $18^\circ$  each in the same direction. The truth table is shown in Fig. 39.9 (a). For producing clockwise rotation, the phase sequence should be  $A^+; B^-; A^-; B^+; A^+$  etc.

Practical hybrid stepping motors are built with more rotor poles than shown in

6. Half-stepping can be achieved by alternating between the 1-phase-ON and 2-phase-ON modes. Step angle is reduced by half.
7. Microstepping is obtained by deliberately making two phase currents unequal in the 2-phase-ON mode.

### Tutorial Problems 39.1

1. A stepper motor has a step angle of  $1.8^\circ$ . What number should be loaded into the encoder of its drive system if it is desired to turn the shaft ten complete revolutions ? [2000]
2. Calculate the step angle of a single-stack, 4-phase, 8/6-pole VR stepper motor. What is its resolution ? [15°; 24 steps/rev]
3. A stepper motor has a step angle of  $1.8^\circ$  and is driven at 4000 pps. Determine (a) resolution (b) motor speed (c) number of pulses required to rotate the shaft through  $54^\circ$ . [(a) 200 steps/rev (b) 1200 rpm (c) 30]
4. Calculate the pulse rate required to obtain a rotor speed of 2400 rpm for a stepper motor having a resolution of 200 steps/rev. [4000 pps]
5. A stepper motor has a resolution of 500 steps/rev in the 1-phase-ON mode of operation. If it is operated in half-step mode, determine (a) resolution (b) number of steps required to turn the rotor through  $72^\circ$ . [(a) 1000 steps/rev (b) 200]
6. What is the required resolution for a stepper motor that is to operate at a pulse frequency of 6000 pps and a travel  $180^\circ$  in 0.025 s ? [300 steps/rev]

### 39.9. Permanent-Magnet DC Motor

A permanent-magnet d.c. (PMDC) motor is similar to an ordinary d.c. shunt motor except that its field is provided by permanent magnets instead of salient-pole wound-field structure. Fig. 39.10 (a) shows 2-pole PMDC motor whereas Fig. 39.10 (b) shows a 4-pole wound-field d.c. motor for comparison purposes.

#### (a) Construction

As shown in Fig. 39.10 (a), the permanent magnets of the PMDC motor are supported by a cylindrical steel stator which also serves as a return path for the magnetic flux. The rotor (*i.e.* armature) has winding slots, commutator segments and brushes as in conventional d.c. machines.



Permanent magnet DC - motor

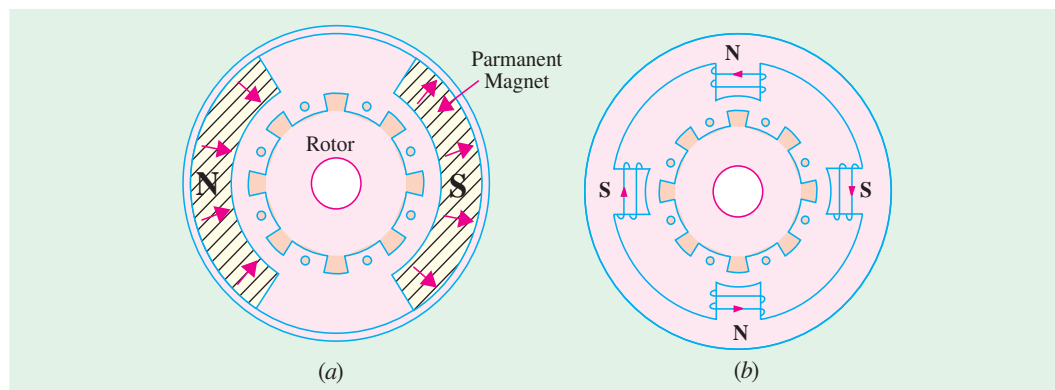


Fig. 39.10

There are three types of permanent magnets used for such motors. The materials used have residual flux density and high coercivity.

- (i) Alnico magnets – They are used in motors having ratings in the range of 1 kW to 150 kW.
- (ii) Ceramic (ferrite) magnets – They are much economical in fractional kilowatt motors.
- (iii) Rare-earth magnets – Made of samarium cobalt and neodymium iron cobalt which have the highest energy product. Such magnetic materials are costly but are best economic choice for small as well as large motors.

Another form of the stator construction is the one in which permanent-magnet material is cast in the form of a continuous ring instead of in two pieces as shown in Fig. 39.10 (a).

#### (b) Working

Most of these motors usually run on 6 V, 12 V or 24 V dc supply obtained either from batteries or rectified alternating current. In such motors, torque is produced by interaction between the axial current-carrying rotor conductors and the magnetic flux produced by the permanent magnets.

#### (c) Performance

Fig. 39.11 shows some typical performance curves for such a motor. Its speed-torque curve is a straight line which makes this motor ideal for a servomotor. Moreover, input current increases linearly with load torque. The efficiency of such motors is higher as compared to wound-field dc motors because, in their case, there is no field Cu loss.

#### (d) Speed Control

Since flux remains constant, speed of a PMDC motor cannot be controlled by using Flux Control Method (Art 33.2). The only way to control its speed is to vary the armature voltage with the help of an armature rheostat (Art 33.2) or electronically by using  $x$ -choppers. Consequently, such motors are found in systems where speed control below base speed only is required.

#### (e) Advantages

- (i) In very small ratings, use of permanent-magnet excitation results in lower manufacturing cost.
- (ii) In many cases a PMDC motor is smaller in size than a wound-field d.c. motor of equal power rating.
- (iii) Since field excitation current is not required, the efficiency of these motors is generally higher than that of the wound-field motors.
- (iv) Low-voltage PMDC motors produce less air noise.
- (v) When designed for low-voltage (12 V or less) these motors produced very little radio and TV interference.

#### (f) Disadvantages

- (i) Since their magnetic field is active at all times even when motor is not being used, these motors are made totally enclosed to prevent their magnets from collecting magnetic junk from neighbourhood. Hence, as compared to wound-field motors, their temperature

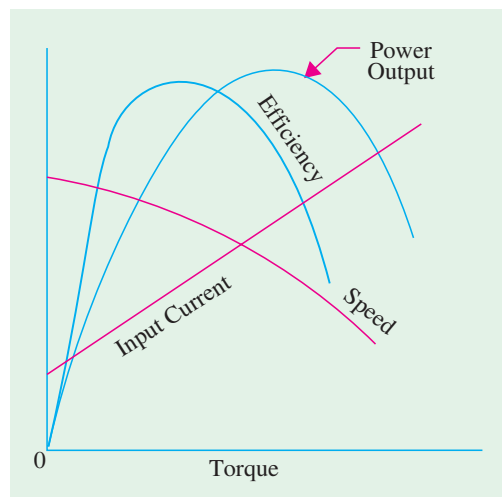


Fig. 39.11



tends to be higher. However, it may not be much of a disadvantage in situations where motor is used for short intervals.

- (ii) A more serious disadvantage is that the permanent magnets can be demagnetized by armature reaction mmf causing the motor to become inoperative. Demagnetization can result from (a) improper design (b) excessive armature current caused by a fault or transient or improper connection in the armature circuit (c) improper brush shift and (d) temperature effects.

**(g) Applications**

- (i) Small, 12-V PMDC motors are used for driving automobile heater and air conditioner blowers, windshield wipers, windows, fans and radio antennas etc. They are also used for electric fuel pumps, marine engine starters, wheelchairs and cordless power tools.
- (ii) Toy industry uses millions of such motors which are also used in other appliances such as the toothbrush, food mixer, ice crusher, portable vacuum cleaner and shoe polisher and also in portable electric tools such as drills, saber saws and hedge trimmers etc.

### 39.10. Low-inertia DC Motors

These motors are so designed as to make their armature mass very low. This permits them to start, stop and change direction and speed very quickly making them suitable for instrumentation applications. The two common types of low-inertia motors are (i) shell-type motor and (ii) printed-circuit (PC) motor.

### 39.11. Shell-type Low-inertia DC Motor

Its armature is made up of flat aluminium or copper coils bonded together to form a hollow cylinder as shown in Fig. 39.12. This hollow cylinder is not attached physically to its iron core which is stationary and is located inside the shell-type rotor. Since iron does not form part of the rotor, the rotor inertia is very small.

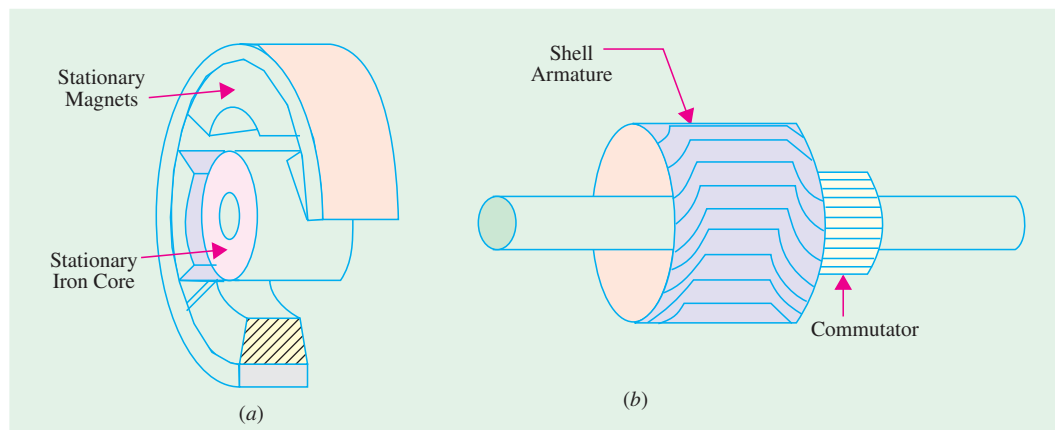


Fig. 39.12

### 39.12. Printed-circuit (Disc) DC Motor

**(a) Constructional Details**

It is a low-voltage dc motor which has its armature (rotor) winding and commutator printed on a thin disk of non-magnetic insulating material. This disk-shaped armature contains no iron and etched-copper conductors are printed on its both sides. It uses permanent magnets to produce the necessary

magnetic field. The magnetic circuit is completed through the flux-return plate which also supports the brushes. Fig. 39.13 shows an 8-pole motor having wave-wound armature. Brushes mounted in an axial direction bear directly on the inner parts of the armature conductors which thus serve as a commutator.



Low voltage DC motor

Since the number of armature conductors is very large, the torque produced is uniform even at low speeds. Typical sizes of these motors are in the fractional and subfractional horsepower ranges. In many applications, acceleration from zero to a few thousand rpm can be obtained within 10 ms.

#### (b) Speed Control

The speed can be controlled by varying either the applied armature voltage or current. Because of their high efficiency, fan cooling is not required in many applications.

The motor brushes require periodic inspection and replacement. The rotor disk which carries the conductors and commutator, being very thin, has a limited life. Hence, it requires replacing after some time.

#### (c) Main Features

The main features of this motor are (i) very low-inertia (ii) high overload current capability (iii) linear speed-torque characteristic (iv) smooth torque down to near-zero speed (v) very suitable for direct-drive control applications (vi) high torque/inertia ratio.

#### (d) Advantages

(i) High efficiency (ii) Simplified armature construction (iii) Being of low-voltage design, produces minimum of radio and TV interference.

#### (e) Disadvantages

(i) Restricted to low voltages only (ii) Short armature life (iii) Suited for intermittent duty cycle only because motor overheats in a very short time since there is no iron to absorb excess heat (v) liable to burn out if stalled or operated with the wrong supply voltage.

#### (f) Applications

These low-inertia motors have been developed specifically to provide high performance characteristics when used in direct-drive control applications. Examples are :

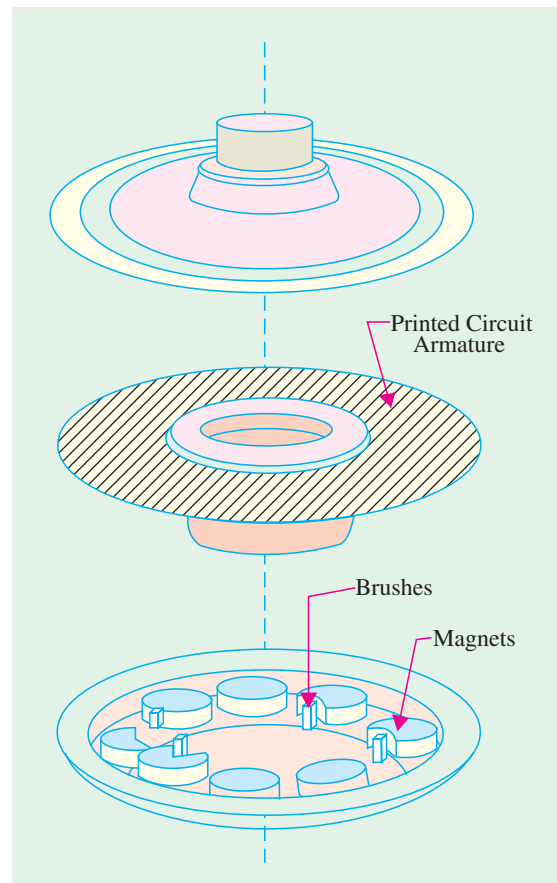


Fig. 39.13

(i) high speed paper tape readers (ii) oscillographs (iii) X-Y recorders (iv) layer winders (v) point-to-point tool positioners *i.e.* as positioning servomotors (vi) with in-built optical position encoder, it competes with stepping motor (vii) in high rating is being manufactured for heavy-duty drives such as lawn mowers and battery-driven vehicles etc.

### 39.13. Permanent-Magnet Synchronous Motors

#### (a) Construction and Performance

Such motors have a cage rotor having rare-earth permanent magnets instead of a wound field. Such a motor starts like an induction motor when fed from a fixed-frequency supply. A typical 2-pole and 4-pole surface-mounted versions of the rotor are shown in Fig. 39.14. Since no d.c. supply is needed for exciting the rotor, it can be made more robust and reliable. These motors have outputs ranging from about 100 W upto 100 kW. The maximum synchronous torque is designed to be around

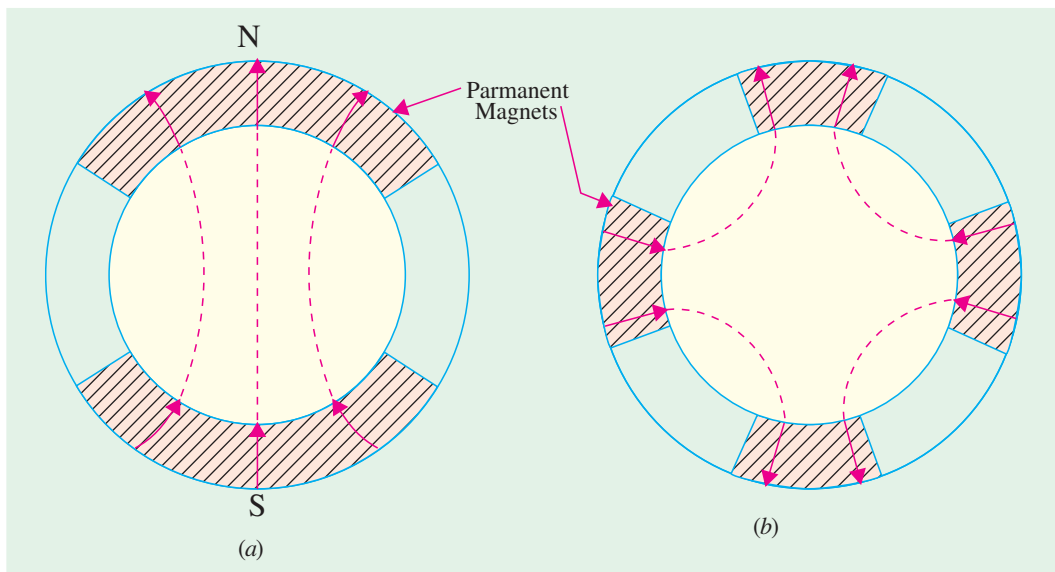


Fig. 39.14

150 per cent of the rated torque. If loaded beyond this point, the motor loses synchronism and will run either as an induction motor or stall.

These motors are usually designed for direct-on-line (DOL) starting. The efficiency and power factor of the permanent-magnet excited synchronous motors are each 5 to 10 points better than their reluctance motor counterparts.

#### (b) Advantages

Since there are no brushes or slip-rings, there is no sparking. Also, brush maintenance is eliminated. Such motors can pull into synchronism with inertia loads of many times their rotor inertia.

#### (c) Applications

These motors are used where precise speed must be maintained to ensure a consistent product. With a constant load, the motor maintains a constant speed.



Permanent magnetic synchronous motor

Hence, these motors are used for synthetic-fibre drawing where constant speeds are absolutely essential.

### 39.14. Synchros

It is a general name for self-synchronizing machines which, when electrically energized and electrically interconnected, exert torques which cause two mechanically independent shafts either to run in synchronism or to make the rotor of one unit follow the rotor position of the other. They are also known by the trade names of selsyns and autosyns. Synchros, in fact, are small cylindrical motors varying in diameter from 1.5 cm to 10 cm depending on their power output. They are low-torque devices and are widely used in control systems for transmitting shaft position information or for making two or more shafts to run in synchronism. If a large device like a robot arm is to be positioned, synchros will not work. Usually, a servomotor is needed for a higher torque.

### 39.15. Types of Synchros

There are many types of synchros but the four basic types used for position and error-voltage applications are as under :

(i) Control Transmitter (denoted by CX) – earlier called generator (ii) Control Receiver (CR) – earlier called motor (iii) Control-Transformer (CT) and (iv) Control Differential (CD). It may be further subdivided into control differential transmitter (CDX) and control differential receiver (CDR).

All of these synchros are single-phase units except the control differential which is of three-phase construction.

#### (a) Constructional Features

##### 1. Control Transmitter

Its constructional details are shown in Fig. 39.15 (a). It has a three-phase stator winding similar to that of a three-phase synchronous generator. The rotor is of the projecting-pole type using dumbbell construction and has a single-phase winding. When a single-phase ac voltage is applied to the rotor through a pair of slip rings, it produces an alternating flux field along the axes of the rotor. This alternating flux induces three unbalanced single phase/voltage in the three stator windings by transformer action. If the rotor is aligned with the axis of the stator winding 2, flux linkage of this stator winding is maximum and this rotor position is defined as the electrical zero. In Fig. 39.15 (b), the rotor axis is displaced from the electrical zero by an angle displaced  $120^\circ$  apart.

##### (b) Control Receiver (CR)

Its construction is essentially the same as that of the control transmitter shown in Fig. 39.15 (a). It has three stator windings and a single-phase salient-pole rotor. However, unlike a CX, a CR has a mechanical viscous damper on the shaft which permits CR rotor to respond without overshooting its mark. In normal use, both the rotor and stator windings are excited with single-phase currents. When the field of the rotor conductors interacts with the field of the stator conductors, a torque is developed which produces rotation.



Synchros

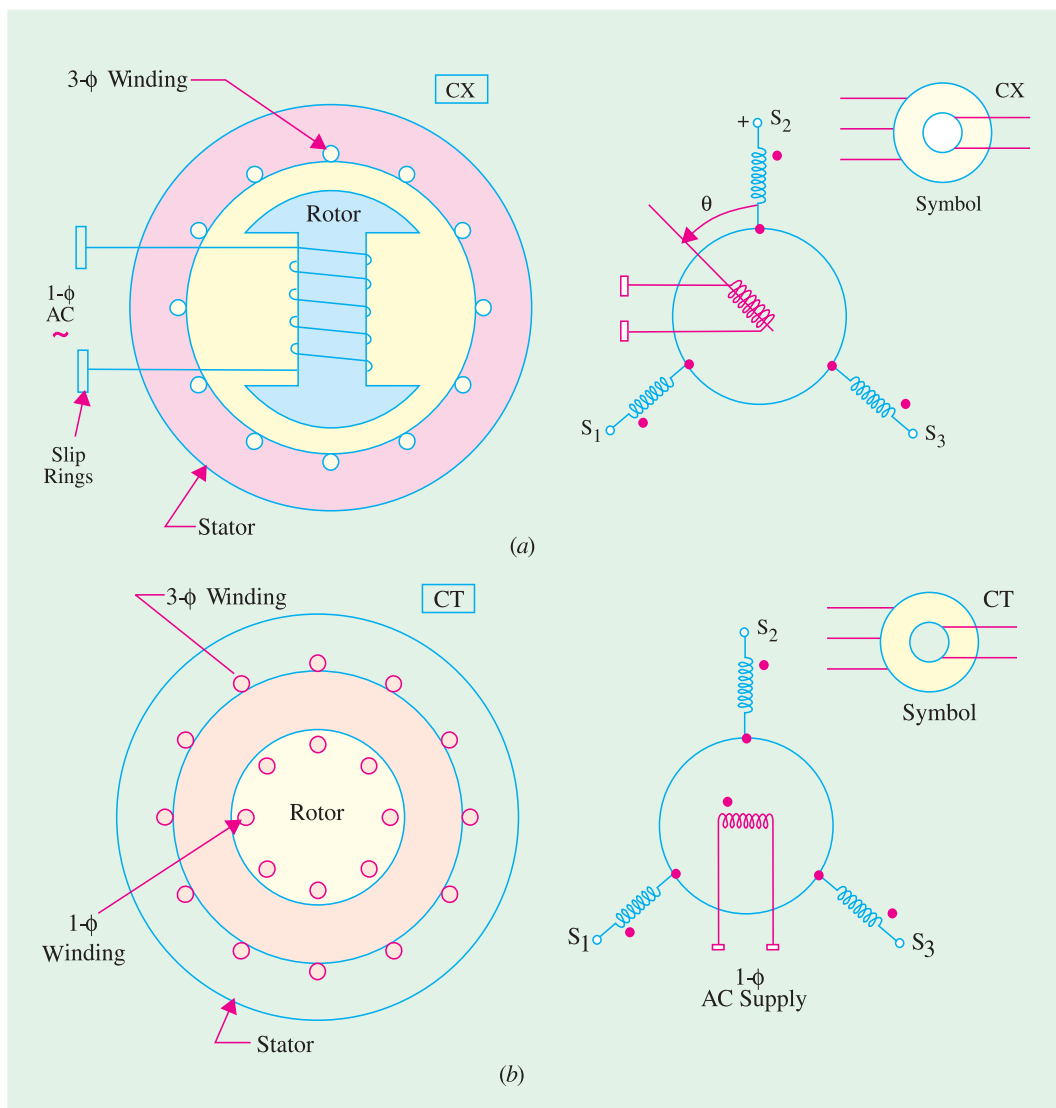


Fig. 39.15

**(c) Control Transformer (CT)**

As shown in Fig. 39.15 (b) its stator has a three-phase winding whereas the cylindrical rotor has a single-phase winding. In this case, the electrical zero is defined as that position of the rotor that makes the flux linkage with winding 2 of the stator zero. This rotor position has been shown in Fig. 39.15 (b) and is different from that of a control transmitter.

**(d) Control Differential (CD)**

The differential synchro has a balanced three-phase distributed winding in both the stator and the rotor. Moreover, it has a cylindrical rotor as shown in Fig. 39.16 (a). Although three-phase windings are involved, it must be kept in mind that these units deal solely with single-phase voltages. The three winding voltages are not polyphase voltages. Normally, the three-phase voltages are identical in magnitude but are separated in phase by  $120^\circ$ . In synchros, these voltages are in phase but differ in magnitude because of their physical orientation.

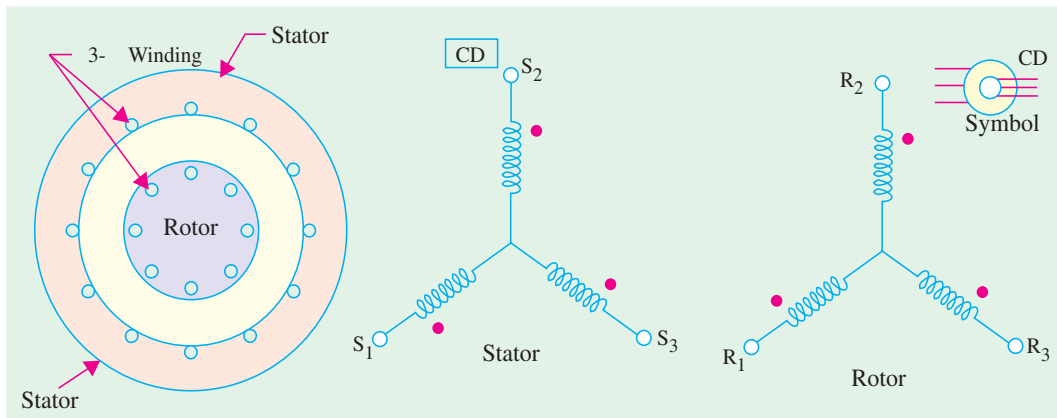


Fig. 39.16

(e) Voltage Relations

Consider the control transmitter shown in Fig. 39.17. Suppose that its rotor winding is excited by a single-phase sinusoidal ac voltage of rms value  $E_r$  and that rotor is held fast in its displaced position from the electrical zero. If  $K = \text{stator turns} / \text{rotor turns}$ , the rms voltage induced in the stator winding is  $E = KE_r$ . However, if we assume  $K = 1$ , then  $E = E_r$ .

The rms value of the induced emf in stator winding 2 when the rotor displacement is ' $\alpha$ ' is given by

$$E_{2s} = E_r \cos \alpha.$$

Since the axis of the stator winding 1 is located  $120^\circ$  ahead of the axis of winding 2, the rms value of the induced emf in this winding is

$$E_{1s} = E_r \cos (\alpha - 120^\circ).$$

In the same way since winding 3 is located behind the axis of winding 2 by  $120^\circ$ , the expression for the induced emf in winding 3 becomes

$$E_{3s} = E_r \cos (\alpha + 120^\circ).$$

We can also find the values of terminal induced voltages as

$$\begin{aligned} E_{12} &= E_{1s} + E_{s2} = E_{1s} - E_{2s} \\ &= E_r \cos \alpha \cos 120^\circ + E_r \sin \alpha \sin 120^\circ - E_r \cos \alpha \\ &= E_r \left( -\frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) \\ &= \sqrt{3} E_r \left( -\frac{1}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) \\ &= \sqrt{3} E_r \cos (\alpha - 150^\circ) \end{aligned}$$

$$\begin{aligned} E_{23} &= E_{2s} + E_{s3} = E_{2s} - E_{3s} \\ &= E_r \left( \frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) = \sqrt{3} E_r \left( \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) = \sqrt{3} E_r \cos (\alpha - 30^\circ) \end{aligned}$$

$$\begin{aligned} E_{31} &= E_{3s} + E_{s1} = E_{3s} - E_{1s} \\ &= E_r \cos (\alpha + 120^\circ) - E_r \cos (\alpha - 120^\circ) \\ &= -\sqrt{3} E_r \sin \alpha = \sqrt{3} E_r \cos (\alpha + 90^\circ) \end{aligned}$$

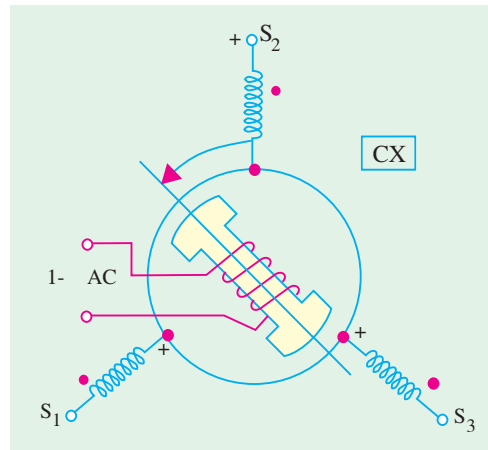


Fig. 39.17

**Example 39.5.** The rotor of a control transmitter (CX) is excited by a single-phase ac voltage of rms value 20 V. Find the value of  $E_{1s}$ ,  $E_{2s}$  and  $E_{3s}$  for rotor angle  $\alpha = +40^\circ$  and  $-40^\circ$ . Assume the stator/rotor turn ratio as unity. Also, find the values of terminal voltages when  $\alpha = +30^\circ$ .

**Solution.** Since  $K = 1$ , the voltage relations derived in will be used.

(a)  $\alpha = +40^\circ$

$$E_{2s} = E_r \cos \alpha = 20 \cos 40^\circ = 15.3 \text{ V}$$

$$E_{1s} = E_r \cos (\alpha - 120^\circ) = 20 \cos (40^\circ - 120^\circ) = \mathbf{3.5 \text{ V}}$$

$$E_{3s} = E_r \cos (\alpha + 120^\circ) = 20 \cos 160^\circ = \mathbf{-18.8 \text{ V}}$$

(b)  $\alpha = -40^\circ$

$$E_{2s} = 20 \cos (-40^\circ) = 15.3 \text{ V}$$

$$E_{1s} = 20 \cos (-40^\circ - 120^\circ) = 20 \cos (-160^\circ) = -18.8 \text{ V}$$

$$E_{3s} = 20 \cos (-40^\circ + 120^\circ) = 20 \cos 80^\circ = \mathbf{3.5 \text{ V}}$$

(c)  $E_{12} = \sqrt{3} \times 20 \times \cos (30^\circ - 150^\circ) = -17.3 \text{ V}$

$$E_{23} = \sqrt{3} E_r \cos (\alpha - 30^\circ) = \sqrt{3} E_r \cos (30^\circ - 30^\circ) = 34.6 \text{ V}$$

$$E_{31} = \sqrt{3} E_r \cos (\alpha + 90^\circ) = \sqrt{3} \times 20 \times \cos (30^\circ + 90^\circ) = \mathbf{-17.3 \text{ V}}$$

### 39.16. Applications of Synchronizers

The synchronizers are extensively used in servomechanism for torque transmission, error detection and for adding and subtracting rotary angles. We will consider these applications one by one.

#### (a) Torque Transmission

Synchronizers are used to transmit torque over a long distance without the use of a rigid mechanical connection. Fig. 39.18 represents an arrangement for maintaining alignment of two distantly-located shafts. The arrangement requires a control transmitter (CX) and a control receiver (CR) which acts as a torque receiver. As CX is rotated by an angle  $\alpha$ , CR also rotates through the same angle  $\alpha$ . As shown, the stator windings of the two synchronizers are connected together and their rotors are connected to the same single-phase ac supply.

**Working.** Let us suppose that CX rotor is displaced by an angle  $\alpha$  and switch  $SW_1$  is closed to energize the rotor winding. The rotor winding flux will induce an unbalanced set of three single-phase voltages (in time phase with the rotor voltage) in the CX stator phase windings which will circulate currents in the CR stator windings. These currents produce the CR stator flux field whose axis is fixed by the angle  $\alpha$ . If the CR rotor winding is now energized by closing switch  $SW_2$ , its flux field will interact with the flux field of the stator winding and thereby produce a torque. This torque will rotate the freely-moving CR rotor to a position which exactly corresponds with the CX rotor *i.e.* it will be displaced by the same angle  $\alpha$  as shown in Fig. 39.18. It should be noted that if the two rotors are in the same relative positions, the stator voltages in the two synchronizers will be exactly equal and opposite. Hence, there will be no current flow in the two stator windings and so no torque will be produced and the system will achieve equilibrium. If now, the transmitter rotor angle changes to a new value, then new set of voltages would be induced in the transmitter stator windings which will again drive currents through the receiver stator windings. Hence, necessary torque will be produced which will turn the CR rotor through an angle corresponding to that of the CX rotor. That is why the transmitter rotor is called the master and the receiver rotor as the slave, because it follows its master. It is worth noting that this master-slave relationship is reversible because when the receiver rotor is displaced through a certain angle, it causes the transmitter rotor to turn through the same angle.

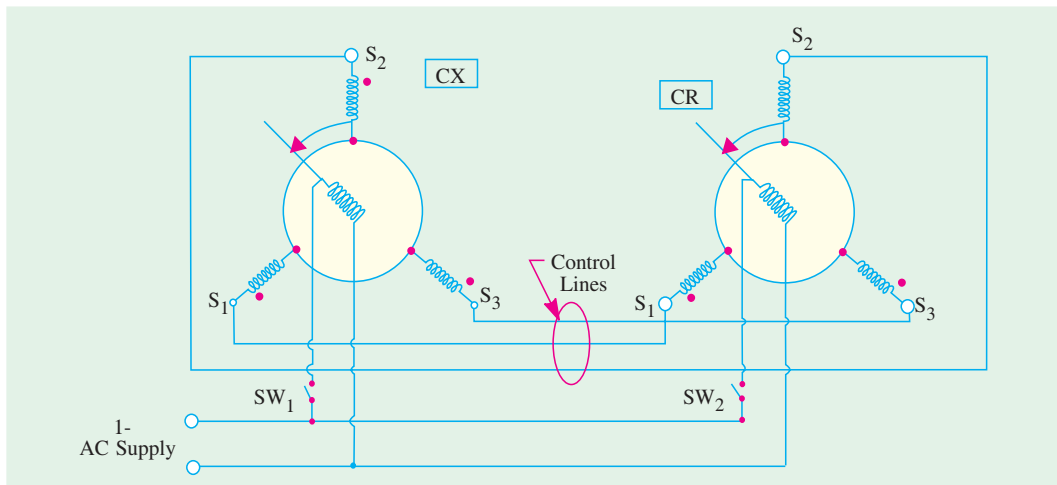


Fig. 39.18

**(b) Error Detection**

Synchros are also used for error detection in a servo control system. In this case, a command in the form of a mechanical displacement of the CX rotor is converted to an electrical voltage which appears at the CT rotor winding terminals which can be further amplified by an amplifier.

For this purpose, we require a CX synchro and a CT synchro as shown in Fig. 39.19. Only the CX rotor is energized from the single-phase ac voltage supply which produces an alternating air-gap flux field. This time-varying flux field induces voltages in the stator windings whose values for  $\alpha = 30^\circ$  are as indicated in the Fig. 39.19. The CX stator voltages supply magnetizing currents in the CT stator

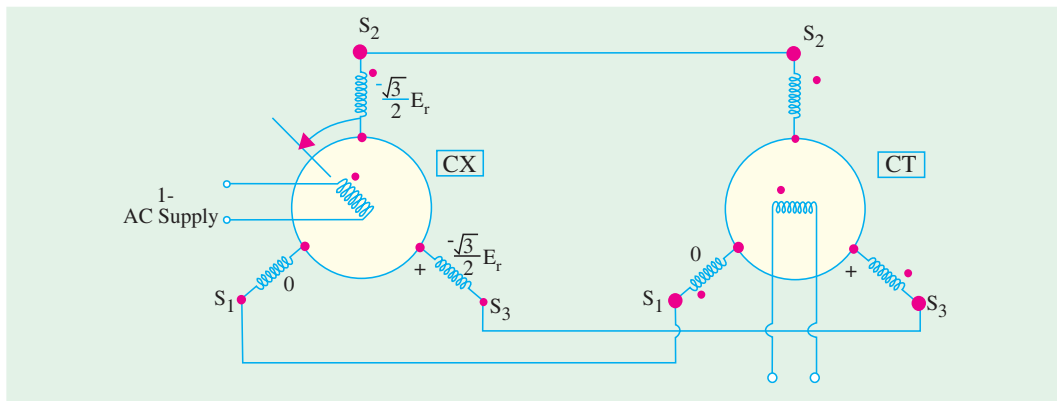


Fig. 39.19

windings which, in turn, create an alternating flux field in their own air-gap. The values of the CT stator phase currents are such that the air-gap flux produced by them induces voltages that are equal and opposite to those existing in the CX stator. Hence, the direction of the resultant flux produced by the CX stator phase currents is forced to take a position which is exactly identical to that of the rotor axis of the CT.

If the CT rotor is assumed to be held fast in its electrical zero position as shown in Fig.39.19, then the rms voltage induced in the rotor is given by  $E = E_{max} \sin \alpha$ , where  $E_{max}$  is the maximum voltage induced by the CT air-gap flux when coupling with the rotor windings is maximum and  $\alpha$  is the displacement angle of the CT rotor.



In general, the value of the rms voltage induced in the *CT* rotor winding when the displacement of the *CX* rotor is  $\alpha_x$  and that of the *CT* rotor is  $\alpha_T$  is given by

$$E = E_{max} \sin (\alpha_x - \alpha_T)$$

### 39.17. Control Differential Transmitter

It can be used to produce a rotation equal to the sum of difference of the rotations of two shafts. The arrangement for this purpose is shown in Fig. 39.20 (a). Here, a *CDX* is coupled to a control transmitter on one side and a control receiver on the other. The *CX* and *CR* rotor windings are energized from the same single-phase voltage supply.

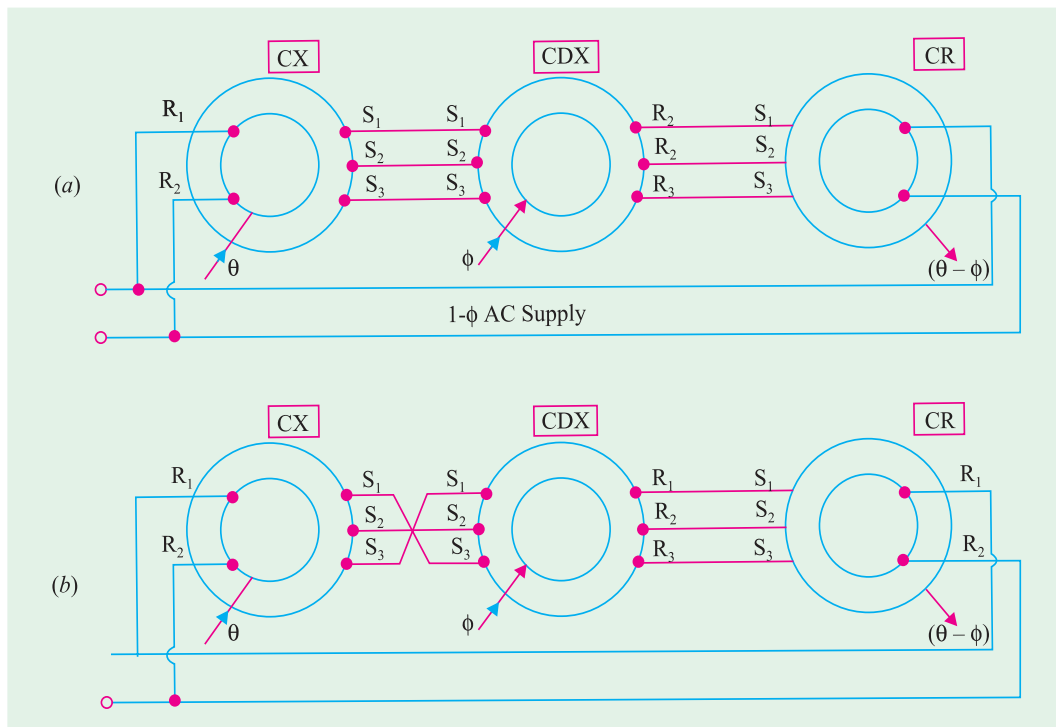


Fig. 39.20

It has two inputs : Mechanical  $\theta$  and Electrical  $\phi$  and the output is Mechanical  $(\theta - \phi)$ . The mechanical input ( $\theta$ ) to *CX* is converted and applied to the *CDX* stator. With a rotor input ( $\phi$ ), the electrical output of the *CDX* is applied to the *CR* stator which provides the mechanical output  $(\theta - \phi)$ .

As shown in Fig. 39.20 (b), if any two stator connections between *CX* and *CDX* are transposed, the electrical input from *CX* to *CDX* becomes  $-\theta$ , hence the output becomes  $(-\theta - \phi) = -(\theta + \phi)$ .

### 39.18. Control Differential Receiver

In construction, it is similar to a *CDX* but it accepts two electrical input angles and provide the difference angle as a mechanical output (Fig. 39.21).

The arrangement consists of two control transmitters coupled to a *CDR*. The two control transmitters provide inputs to the *CDX*, one ( $\theta$ ) to the stator and the other ( $\phi$ ) to the rotor. The *CDX* output is the difference of the two inputs *i.e.*  $(\theta - \phi)$ .

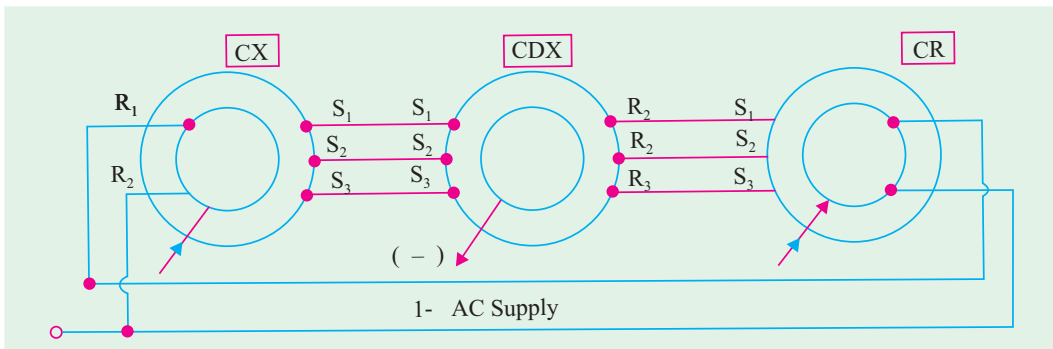


Fig. 39.21

### 39.19. Switched Reluctance Motor

The switched reluctance (SR) motor operates on the same basic principle as a variable reluctance stepper motor (Art. 39.4).

#### (a) Construction

Unlike a conventional synchronous motor, both the rotor and stator of a SR motor have salient poles as shown in Fig. 39.22. This doubly-salient arrangement is very effective for electromagnetic energy conversion.

The stator carries coils on each pole, the coils on opposite poles being connected in series. The eight stator coils shown in Figure are grouped to form four phases which are independently energized from a four-phase converter. The laminated rotor has no windings or magnets and is, therefore cheap to manufacture and extremely robust. The motor shown in Fig. 39.22 has eight stator poles and six rotor poles which is a widely-used arrangement although other pole combinations (like 6/4 poles) are used to suit different applications.

#### (b) Working

Usual arrangement is to energize stator coils sequentially with a single pulse of current at high speed. However, at starting and low speed, a



Switched reluctance motor

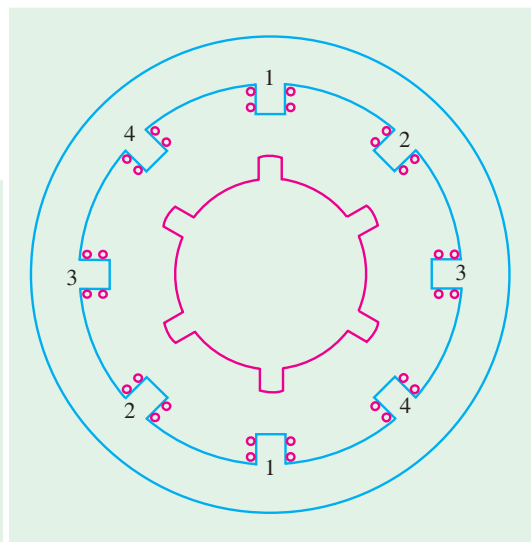


Fig. 39.22

current-chopper type control is used to limit the coil current.

The motor rotates in the anticlockwise direction when the stator phases are energized in the sequence 1, 2, 3, 4 and in clockwise direction when energized in the sequence 1, 4, 3, 2. When the stator coils are energized, the nearest pair of rotor poles is pulled into alignment with the appropriate stator poles by reluctance torque.

Closed-loop control is essential to optimize the switching angles of the applied coil voltages. The stator phases are switched by signals derived from a shaft-mounted rotor position detectors such as Hall-effect devices or optical sensors Fig. (39.23). This causes the behaviour of the SR motor to resemble that of a dc motor.

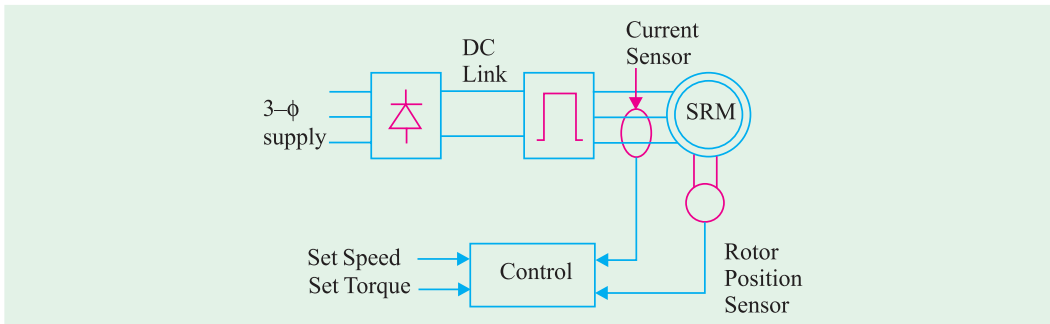


Fig. 39.23

**(c) Advantages and Disadvantages**

Although the newest arrival on the drives scene, the SR motor offers the following advantages:

- (i) higher efficiency
- (ii) more power per unit weight and volume
- (iii) very robust because rotor has no windings or slip rings
- (iv) can run at very high speed (upto 30,000 rpm) in hazardous atmospheres
- (v) has versatile and flexible drive features and
- (vi) four-quadrant operation is possible with appropriate drive circuitry.

However, the drawbacks are that it is (i) relatively unproven (ii) noisy and (iii) not well-suited for smooth torque production.

**(d) Applications**

Even though the SR technology is still in its infancy, it has been successfully applied to a wide range of applications such as (i) general purpose industrial drives (ii) traction (iii) domestic appliances like food processors, vacuum cleaners and washing machines etc., and (iv) office and business equipment.

**39.20. Comparison between VR Stepper Motor and SR Motor**

<i>VR Stepper Motor</i>	<i>SR Motor</i>
1. It rotates in steps.	It is meant for continuous rotation.
2. It is designed first and foremost for open-loop operation.	Closed-loop control is essential for its optimal working.
3. Its rotor poles are made of ferromagnetic material.	Its rotor poles are also made of ferromagnetic material.
4. It is capable of half-step operation and microstepping.	It is not designed for this purpose.
5. Has low power rating.	Has power ratings upto 75 kW (100 hp).
6. Has lower efficiency.	Has higher overall efficiency.

39.21. The Resolver

In many ways, it is similar to a synchro but differs from it in the following respects : (i) Electrical displacement between stator windings is  $90^\circ$  and not  $120^\circ$  (ii) It has two stator windings and two rotor windings (Fig. 39.24) (iii) Its input can be either to the stator or to the rotor (iv) They are usually not used as followers because their output voltage is put to further use.

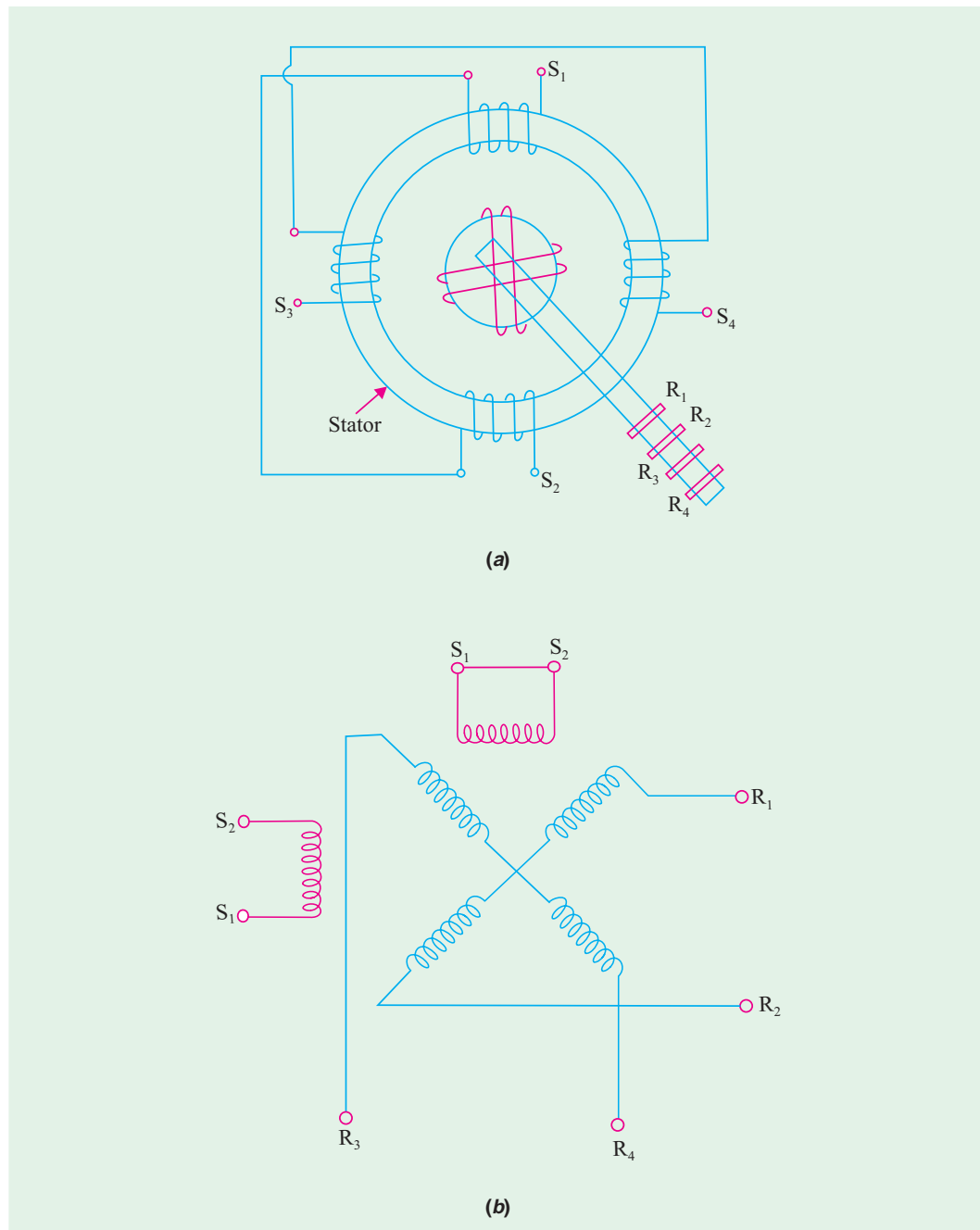


Fig. 39.24

**(a) Construction**

The main constructional features and the symbol for a resolver are shown in Fig. 39.24. There are two stator windings which are wound  $90^\circ$  apart. In most applications, only one stator winding is used, the other being short-circuited. The two rotor winding connections are brought out through slip rings and brushes.

**(b) Applications**

Resolvers find many applications in navigation and height determination as shown in Fig. 39.25 (a) and (c) where Fig. 39.25 (b) provides the key.

**(i) Navigation Application**

As shown in Fig. 39.25 (a), the purpose is to determine the distance  $D$  to the destination. Suppose the range  $R$  to a base station as found by a radar ranging device is 369 km. The angle  $\theta$  is also determined directly. If the amplifier scale is 4.5 V per 100 km, the range would be represented by  $369 \times (4.5 / 100) = 16.6$  V. Further suppose that angle  $\theta$  is found to be  $52.5^\circ$ . Now, set the resolver at  $52.5^\circ$  and apply 16.6 V to rotor terminals  $R_3 R_4$ . The voltage which appears at terminals  $S_1 S_2$  represents  $D$ . If we assume  $K = \text{stator turns} / \text{rotor turns} = 1$ , the voltage available at  $S_1 S_2$  will be  $= 16.6 / \cos 52.5^\circ = 16.6 / 0.6088 = 27.3$  V. Since 4.5 V represents 100 km, 27.3 V represents  $27.3 \times 100 / 4.5 = 607$  km.

**(ii) Height Determination**

Suppose the height  $H$  of a building is to be found. First of all, the oblique distance  $D$  to the top of the building is found by a range finder. Let  $D = 210$  m and the scale of the amplifier to the resolver stator be 9 V per 100 m. The equivalent voltage is  $9 \times 210 / 100 = 18.9$  V. This voltage is applied to stator terminals  $S_1 S_2$  of the resolver. Suppose the angle  $\theta$  read from the resolver scale is  $61.3^\circ$ . The height of the building is given in the form of voltage which appears across the rotor terminals  $R_1 R_2$ . Assuming stator / rotor turn ratio as unity and the same amplifier ratio for the rotor output, the voltage across  $R_1 R_2 = 18.9 \times \sin 61.3^\circ = 16.6$  V. Hence,  $H = 16.6 \times 100 / 9 = 184$  m. It would be seen that in using the resolver, there is no need to go through trigonometric calculations because the answers come out directly.

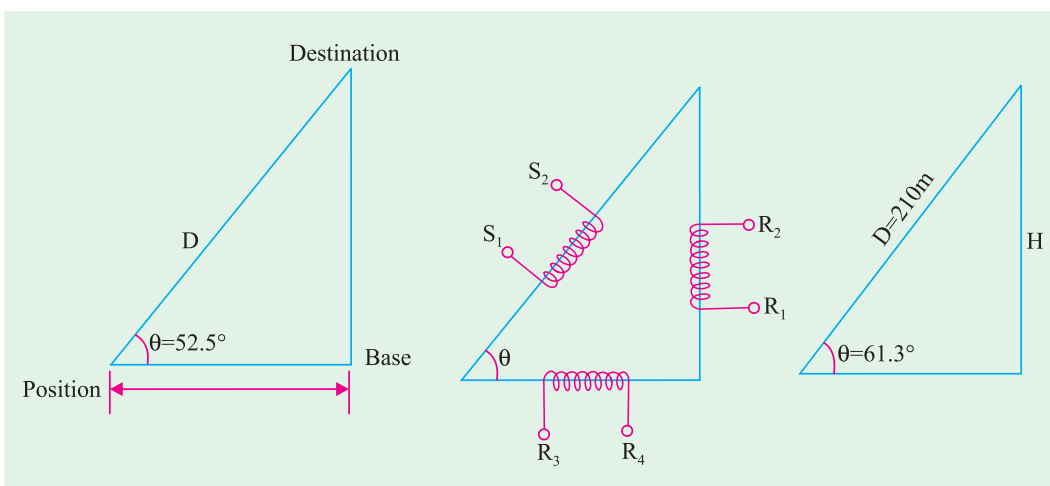


Fig. 39.25

### 39.22. Servomotors

They are also called control motors and have high-torque capabilities. Unlike large industrial motors, they are not used for continuous energy conversion but only for precise speed and precise position control at high torques. Of course, their basic principle of operation is the same as that of other electromagnetic motors. However, their construction, design and mode of operation are different. Their power ratings vary from a fraction of a watt upto a few 100 W. Due to their low-inertia, they have high speed of response. That is why they are smaller in diameter but longer in length. They generally operate at vary low speeds or sometimes zero speed. They find wide applications in radar, tracking and guidance systems, process controllers, computers and machine tools. Both dc and a.c. (2-phase and 3-phase) servomotors are used at present.

Servomotors differ in application capabilities from large industrial motors in the following respects :

1. They produce high torque at all speeds including zero speed.
2. They are capable of holding a static (*i.e.* no motion) position.
3. They do not overheat at standstill or lower speeds.
4. Due to low-inertia, they are able to reverse directions quickly.
5. They are able to accelerate and deaccelerate quickly.
6. They are able to return to a given position time after time without any drift.

These motors look like the usual electric motors. Their main difference from industrial motors is that more electric wires come out of them for power as well as for control. The servomotor wires go to a controller and not to the electrical line through contactors. Usually, a tachometer (speed indicating device) is mechanically connected to the motor shaft. Sometimes, blower or fans may also be attached for motor cooling at low speeds.



DC servo motor

### 39.23. DC Servomotors

These motors are either separately-excited dc motors or permanent-magnet dc motors. The schematic diagram of a separately-excited d.c. motor alongwith its armature and field MMFs and torque/speed characteristics is shown in Fig. 39.26. The speed of d.c. servomotors is normally controlled by varying the armature voltage. Their armature is deliberately designed to have large resistance so that torque-speed characteristics are linear and have a large negative slope as shown in Fig. 39.26 (c). The negative slope serves the purpose of providing the viscous damping for the servo drive system.

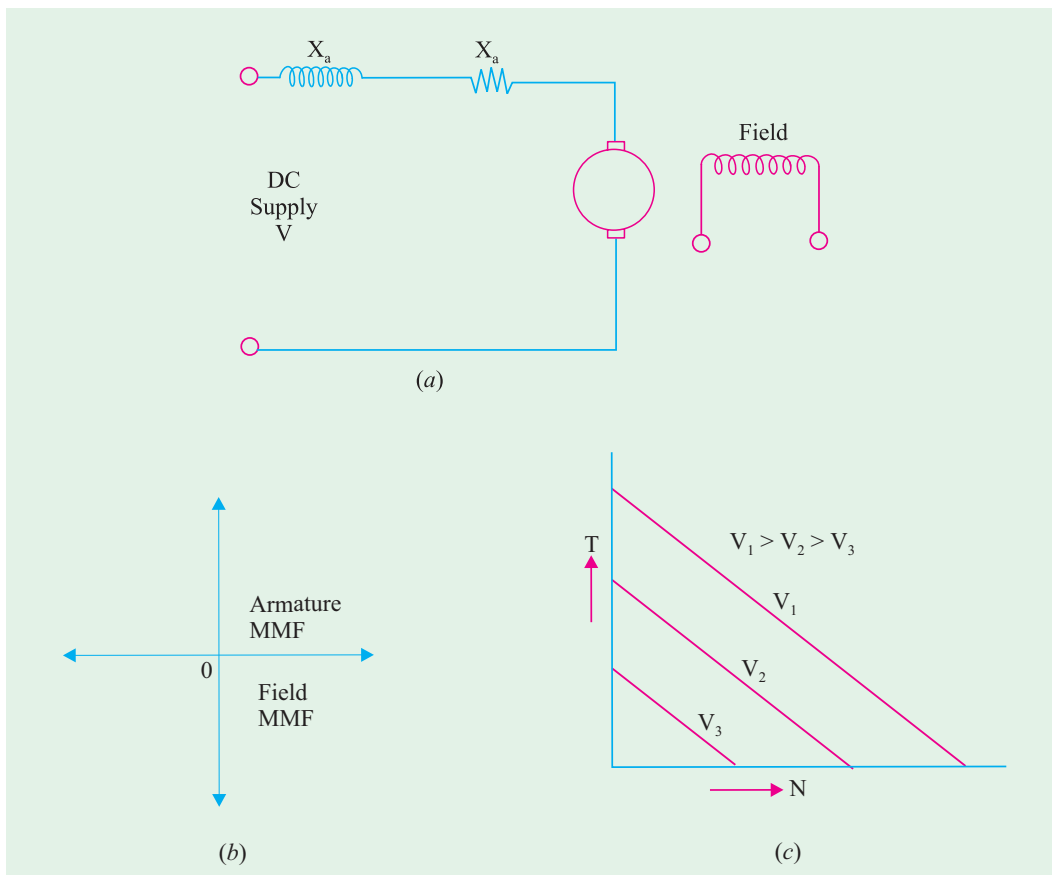


Fig. 39.26

As shown in Fig. 39.26 (b), the armature m.m.f. and excitation field mmf are in quadrature. This fact provides a fast torque response because torque and flux become decoupled. Accordingly, a step change in the armature voltage or current produces a quick change in the position or speed of the rotor.

### 39.24. AC Servomotors

Presently, most of the ac servomotors are of the two-phase squirrel-cage induction type and are used for low power applications. However, recently three-phase induction motors have been modified for high power servo systems which had so far been using high power d.c. servomotors.

#### (a) Two-phase AC Servomotor

Such motors normally run on a frequency of 60 Hz or 400 Hz (for airborne systems). The stator has two distributed windings which are displaced from each other by  $90^\circ$  (electrical). The main



Permanent magnet stepper motor

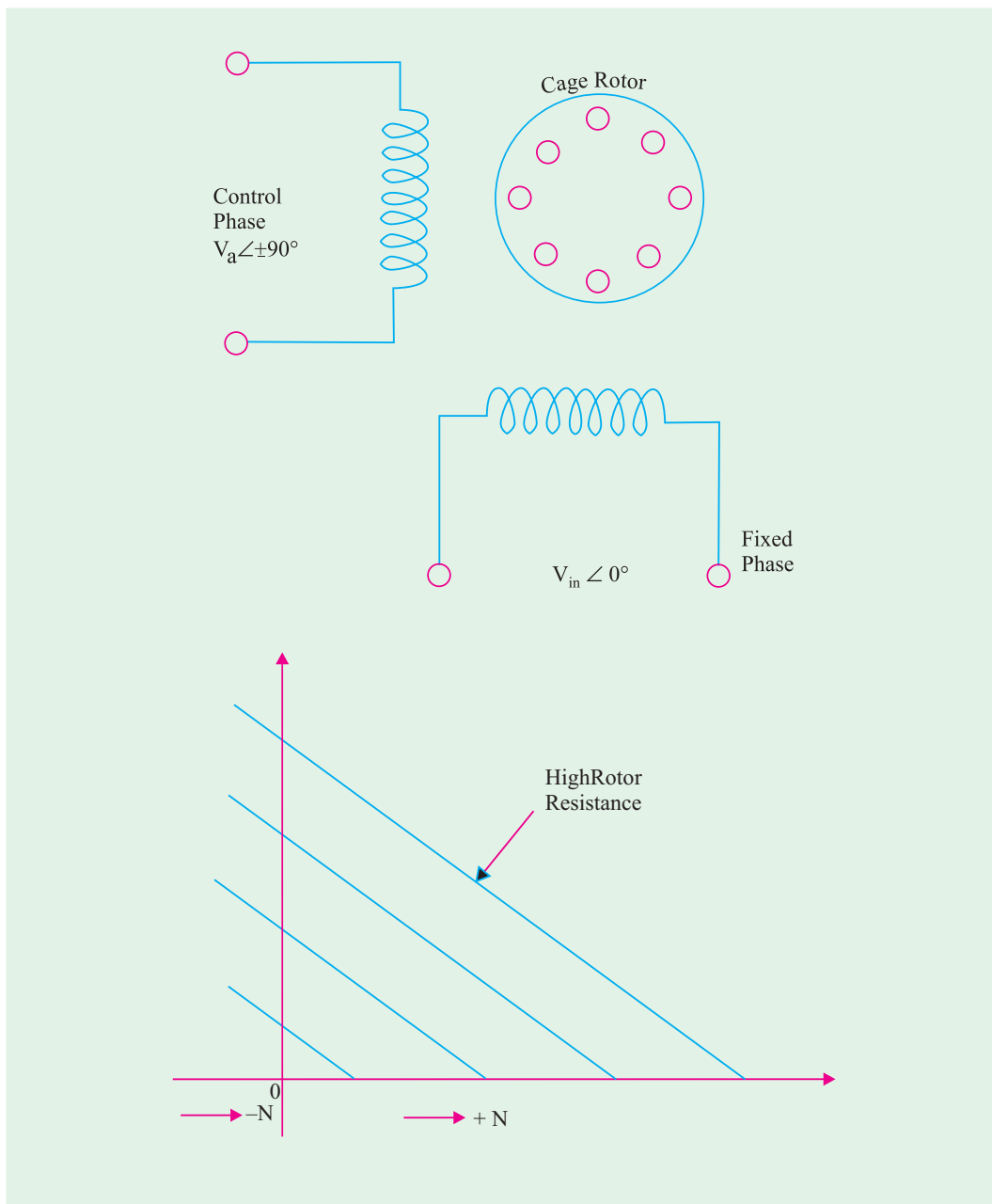


Fig. 39.27

winding (also called the reference or fixed phase) is supplied from a constant voltage source,  $V_m \angle 0^\circ$  (Fig. 39.27). The other winding (also called the control phase) is supplied with a variable voltage of the same frequency as the reference phase but is phase-displaced by  $90^\circ$  (electrical). The control-phase voltage is controlled by an electronic controller. The speed and torque of the rotor are controlled by the phase difference between the main and control windings. Reversing the phase difference from leading to lagging (or *vice-versa*) reverses the motor direction.



Since the rotor bars have high resistance, the torque-speed characteristics for various armature voltages are almost linear over a wide speed range particularly near the zero speed. The motor operation can be controlled by varying the voltage of the main phase while keeping that of the reference phase constant.

### (b) Three-phase AC Servomotors

A great deal of research has been to modify a three-phase squirrel-cage induction motor for use in high power servo systems. Normally, such a motor is a highly non-linear coupled-circuit device. Recently, this machine has been operated successfully as a linear decoupled machine (like a d.c. machine) by using a control method called vector control or field oriented control. In this method, the currents fed to the machine are controlled in such a way that its torque and flux become decoupled as in a dc machine. This results in a high speed and a high torque response.

## OBJECTIVE TESTS – 39

1. A single-stack, 4-phase, 6-pole VR stepper motor will have a step angle of
  - (a)  $15^\circ$
  - (b)  $30^\circ$
  - (c)  $45^\circ$
  - (d)  $90^\circ$
2. In a three-stack 12/8-pole VR motor, the rotor pole pitch is
  - (a)  $15^\circ$
  - (b)  $30^\circ$
  - (c)  $45^\circ$
  - (d)  $60^\circ$
3. A three-stack VR stepper motor has a step angle of  $10^\circ$ . What is the number of rotor teeth in each stack ?
  - (a) 36
  - (b) 24
  - (c) 18
  - (d) 12
4. If a hybrid stepper motor has a rotor pitch of  $36^\circ$  and a step angle of  $9^\circ$ , the number of its phases must be
  - (a) 4
  - (b) 2
  - (c) 3
  - (d) 6
5. What is the step angle of a permanent-magnet stepper motor having 8 stator poles and 4 rotor poles ?
  - (a)  $60^\circ$
  - (b)  $45^\circ$
  - (c)  $30^\circ$
  - (d)  $15^\circ$
6. A stepping motor is a ..... device.
  - (a) mechanical
  - (b) electrical
  - (c) analogue
  - (d) incremental
7. Operation of stepping motors at high speeds is referred to as
  - (a) fast forward
  - (b) slewing
  - (c) inching
  - (d) jogging
8. Which of the following phase switching sequence represents half-step operation of a VR stepper motor ?
  - (a) A, B, C, A .....
  - (b) A, C, B, A .....
  - (c) AB, BC, CA, AB .....
  - (d) A, AB, B, BC .....
9. The rotational speed of a given stepper motor is determined solely by the
  - (a) shaft load
  - (b) step pulse frequency
  - (c) polarity of stator current
  - (d) magnitude of stator current.
10. A stepper motor may be considered as a ..... converter.
  - (a) dc to dc
  - (b) ac to ac
  - (c) dc to ac
  - (d) digital-to-analogue
11. The rotor of a stepper motor has no

- (a) windings  
(b) commutator  
(c) brushes  
(d) all of the above.
12. Wave excitation of a stepper motor results in  
(a) microstepping  
(b) half-stepping  
(c) increased step angle  
(d) reduced resolution.
13. A stepper motor having a resolution of 300 steps/rev and running at 2400 rpm has a pulse rate of— pps.  
(a) 4000  
(b) 8000  
(c) 6000  
(d) 10,000
14. The torque exerted by the rotor magnetic field of a PM stepping motor with unexcited stator is called .....torque.  
(a) reluctance  
(b) detent  
(c) holding  
(d) either (b) or (c)
15. A variable reluctance stepper motor is constructed of ..... material with salient poles.  
(a) paramagnetic  
(b) ferromagnetic  
(c) diamagnetic  
(d) non-magnetic
16. Though structurally similar to a control transmitter, a control receiver differs from it in the following way :  
(a) it has three-phase stator winding  
(b) it has a rotor of dumbbell construction  
(c) it has a mechanical damper on its shaft  
(d) it has single-phase rotor excitation.
17. The control ..... synchro has three-phase winding both on its stator and rotor.  
(a) differential  
(b) transformer  
(c) receiver  
(d) transmitter
18. Regarding voltages induced in the three stator windings of a synchro, which statement is false ?  
(a) they depend on rotor position.  
(b) they are in phase.  
(c) they differ in magnitude.  
(d) they are polyphase voltages.
19. The low-torque synchros cannot be used for  
(a) torque transmission  
(b) error detection  
(c) instrument servos  
(d) robot arm positioning.
20. Which of the following synchros are used for error detection in a servo control system ?  
(a) control transmitter  
(b) control transformer  
(c) control receiver  
(d) both (a) and (b).
21. For torque transmission over a long distance with the help of electrical wires only, which of the following two synchros are used ?  
(a) *CX* and *CT*  
(b) *CX* and *CR*  
(c) *CX* and *CD*  
(d) *CT* and *CD*.
22. The arrangement required for producing a rotation equal to the sum or difference of the rotation of two shafts consists of the following coupled synchros.  
(a) control transmitter  
(b) control receiver  
(c) control differential transmitter  
(d) all of the above.
23. Which of the following motor would suit applications where constant speed is absolutely essential to ensure a consistent product ?  
(a) brushless dc motor  
(b) disk motor  
(c) permanent-magnet synchronous motor  
(d) stepper motor.
24. A switched reluctance motor differs from a *VR* stepper motor in the sense that it  
(a) has rotor poles of ferromagnetic material  
(b) rotates continuously  
(c) is designed for open-loop operation only  
(d) has lower efficiency.
25. The electrical displacement between the two stator windings of a resolver is  
(a) 120°  
(b) 90°  
(c) 60°  
(d) 45°.

26. Which of the following motor runs from a low dc supply and has permanently magnetized salient poles on its rotor ?
- permanent-magnet d.c. motor
  - disk d.c. motor
  - permanent-magnet synchronous motor
  - brushless d.c. motor.
27. A dc servomotor is similar to a regular d.c. motor except that its design is modified to cope with
- electronic switching
  - slow speeds
  - static conditions
  - both (b) and (c).
28. One of the basic requirements of a servomotor is that it must produce high torque at all
- loads
  - frequencies
  - speeds
  - voltages.
29. The most common two-phase ac servomotor differs from the standard ac induction motor because it has
- higher rotor resistance
  - higher power rating
  - motor stator windings
  - greater inertia.
30. Squirrel-cage induction motor is finding increasing application in high-power servo systems because new methods have been found to
- increase its rotor resistance
  - control its torque
  - decrease its inertia
  - decouple its torque and flux.

### ANSWERS

1. a 2. c 3. d 4. a 5. b 6. d 7. b 8. d 9. b 10. d 11. d 12. b 13. c 14. d 15. b 16. c  
17. a 18. d 19. d 20. d 21. b 22. d 23. c 24. b 25. b 26. a 27. d 28. c 29. a 30. d.

### QUESTIONS AND ANSWERS ON SPECIAL MACHINES

**Q.1. Do stepper motors have internal or external fans ?**

**Ans.** No. Because the heat generated in the stator winding is conducted through the stator iron to the case which is cooled by natural conduction, convection and radiation.

**Q.2. Why do hybrid stepping motors have many phases sometime more than six ?**

**Ans.** In order to obtain smaller step angles.

**Q.3. Any disadvantage(s) of having more phases?**

**Ans.** Minor ones are: more leads have to be brought out from the motor, more interconnections are required to the drive circuit and more switching devices are needed.

**Q.4. What is the main attraction of a multi-stack VR stepper motor ?**

**Ans.** It is well-suited to high stepping rates.

**Q.5. You are given a VR motor and a hybrid stepper motor which look exactly similar. How would you tell which is which ?**

**Ans.** Spin the rotor after short-circuiting the stator winding. If there is no mechanical resistance to rotation, it is a VR motor and if there is resistance, then it is a hybrid motor.

**Q.6. How do you explain it ?**

**Ans.** Since *VR* motor has magnetically neutral rotor, it will not induce any e.m.f. in the short-circuited winding *i.e.* the machine will not act as a generator and hence experience no drag on its rotation. However, the rotor of a hybrid motor has magnetic poles, hence it will act as a generator and so experience a drag.

**Q.7. Will there be any harm if the rotor of a hybrid stepper motor is pulled out of its stator ?**

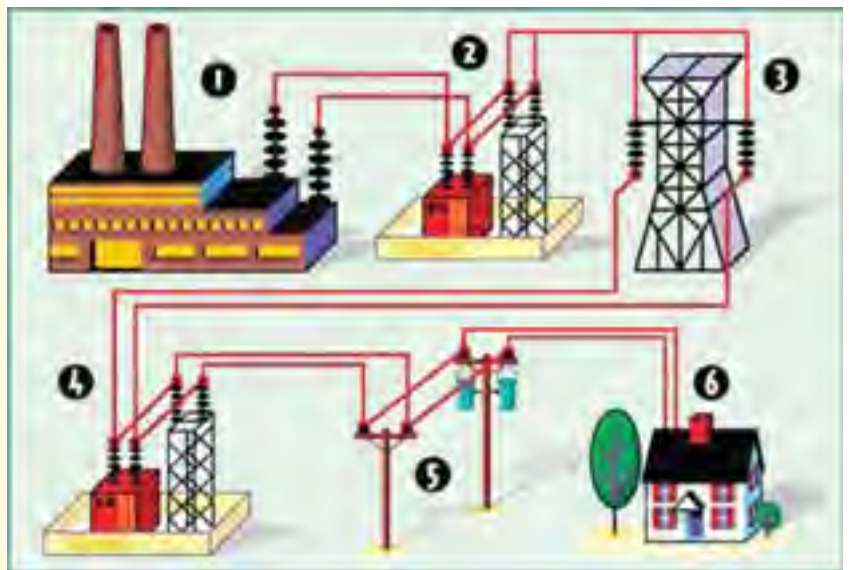
**Ans.** Yes. The rotor will probably become partially demagnetized and, on reassembling, will give less holding torque.

# C H A P T E R 40

## Learning Objectives

- Transmission and Distribution of D.C. Power
- Two-wire and Three-wire System
- Voltage Drop and Transmission Efficiency
- Methods of Feeding Distributor
- D.C. Distributor Fed at One End
- Uniformly Loaded Distributor
- Distributor Fed at Both Ends with Equal Voltage
- Distributor Fed at Both ends with Unequal Voltage
- Uniform Loading with Distributor Fed at Both Ends
- Concentrated and Uniform Loading with Distributor Fed at One End
- Ring Distributor
- Current Loading and Load-point Voltage in a 3-wire System
- Three-wire System
- Balancers
- Boosters
- Comparison of 2-wire and 3-wire Distribution System

## D.C. TRANSMISSION AND DISTRIBUTION



- ↑ (1) Electricity leaves the power plant, (2) Its voltage is increased at a step-up transformer, (3) The electricity travels along a transmission line to the area where power is needed, (4) There, in the substation, voltage is decreased with the help of step-down transformer, (5) Again the transmission lines carry the electricity, (6) Electricity reaches the final consumption points

### 40.1. Transmission and Distribution of D.C. Power

By transmission and distribution of electric power is meant its conveyance from the central station where it is generated to places, where it is demanded by the consumers like mills, factories, residential and commercial buildings, pumping stations etc. Electric power may be transmitted by two methods.

(i) By overhead system or (ii) By underground system—this being especially suited for densely-populated areas though it is somewhat costlier than the first method. In over-head system, power is conveyed by bare conductors of copper or aluminium which are strung between wooden or steel poles erected at convenient distances along a route. The bare copper or aluminium wire is fixed to an insulator which is itself fixed onto a cross-arm on the pole. The number of cross-arms carried by a pole depends on the number of wires it has to carry. Line supports consist of (i) pole structures and (ii) tower. Poles which are made of wood, reinforced concrete or steel are used up to 66 kV whereas steel towers are used for higher voltages.

The underground system employs insulated cables which may be single, double or triple-core etc.

A good system whether overhead or underground should fulfil the following requirements :

1. The voltage at the consumer's premises must be maintained within  $\pm 4$  or  $\pm 6\%$  of the declared voltage, the actual value depending on the type of load\*.
2. The loss of power in the system itself should be a small percentage (about 10%) of the power transmitted.
3. The transmission cost should not be unduly excessive.
4. The maximum current passing through the conductor should be limited to such a value as not to overheat the conductor or damage its insulation.
5. The insulation resistance of the whole system should be very high so that there is no undue leakage or danger to human life.

It may, however, be mentioned here that these days all production of power is as a.c. power and nearly all d.c. power is obtained from large a.c. power systems by using converting machinery like synchronous or rotary converters, solid-state converters and motor-generator sets etc. There are many sound reasons for producing power in the form of alternating current rather than direct current.

(i) It is possible, in practice, to construct large high-speed a.c. generators of capacities up to 500 MW. Such generators are economical both in the matter of cost per kWh of electric energy produced as well as in operation. Unfortunately, d.c. generators cannot be built of ratings higher than 5 MW because of commutation trouble. Moreover, since they must operate at low speeds, it necessitates large and heavy machines.

(ii) A.C. voltage can be efficiently and conveniently raised or lowered for economic transmission and distribution of electric power respectively. On the other hand, d.c. power has to be generated at comparatively low voltages by units of relatively low power ratings. As yet, there is no economical method of raising the d.c. voltage for transmission and lowering it for distribution.

Fig. 40.1 shows a typical power system for obtaining d.c. power from a.c. power. Other details such as instruments, switches and circuit breakers etc. have been omitted.

Two 13.8 kV alternators run in parallel and supply power to the station bus-bars. The voltage is stepped up by 3-phase transformers to 66 kV for transmission purposes\*\* and is again stepped down to 13.8 kV at the sub-station for distribution purposes. Fig. 40.1 shows only three methods commonly used for converting a.c. power to d.c. power at the sub-station.

\* According to Indian Electricity Rules, voltage fluctuations should not exceed  $\pm 5\%$  of normal voltage for L.T. supply and  $\pm 12\frac{1}{2}\%$  for H.T. supply.

\*\* Transmission voltages of upto 400 kV are also used.

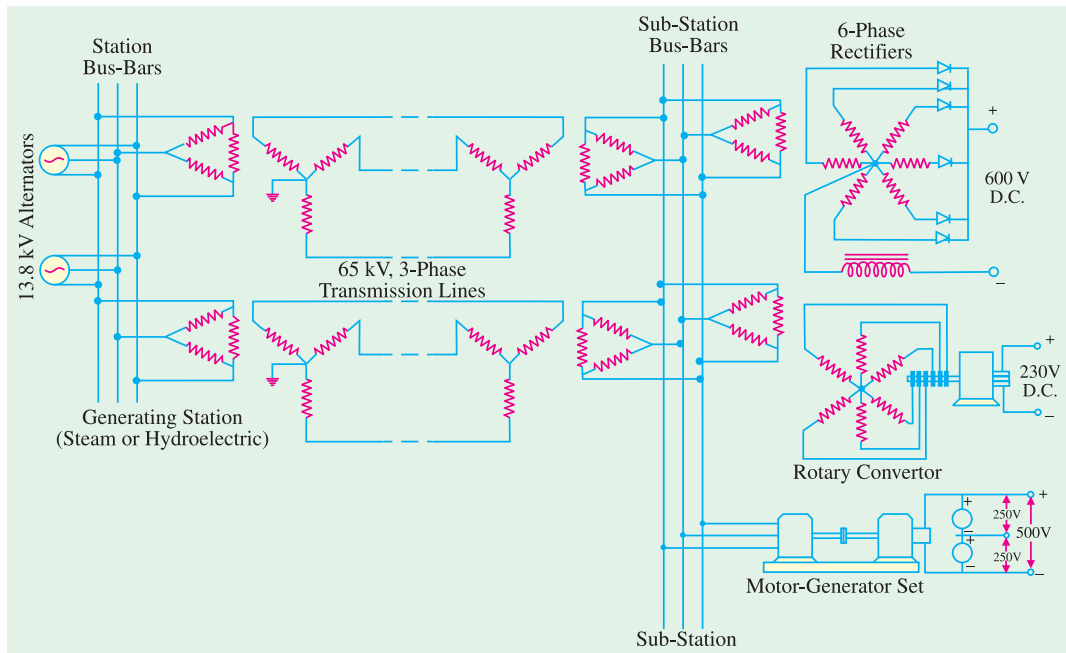


Fig. 40.1

- (a) a 6-phase mercury-arc rectifier gives 600 V d.c. power after the voltage has been stepped down to a proper value by the transformers. This 600-V d.c. power is generally used by electric railways and for electrolytic processes.
- (b) a rotary converter gives 230 V d.c. power.
- (c) a motor-generator set converts a.c. power to 500/250 d.c. power for 3-wire distribution systems.

In Fig. 40.2 is shown a schematic diagram of *low tension* distribution system for d.c. power. The whole system consists of a network of cables or conductors which convey power from central station to the consumer's premises. The station bus-bars are fed by a number of generators (only two shown in the figure) running in parallel. From the bus-bars, the power is carried by many *feeders* which radiate to various parts of a city or locality. These feeders deliver power at certain points to a distributor which runs along the various streets. The points *FF*, as shown in the figure, are known as *feeding points*. Power connections to the various consumers are given *from this distributor and not directly from the feeder*. The wires which convey power from the distributor to the consumer's premises are known as *service mains (S)*. Sometimes when there is only one distributor in a locality, several sub-distributors (*SD*) branching off from the distributor are employed and service mains are now connected to them instead of distributor as shown in the figure.

Obviously, a feeder is designed on the basis of its current-carrying capacity whereas the design of distributor is based on the voltage drop occurring in it.



The above figure shows a motor-generator set. Nowadays, we use solid-state devices, called rectifiers, to convert standard AC to DC current. Back in the olden days, they needed a "motor dynamo" set to make the conversion as shown above. An AC motor would turn a DC Generator, as pictured above.

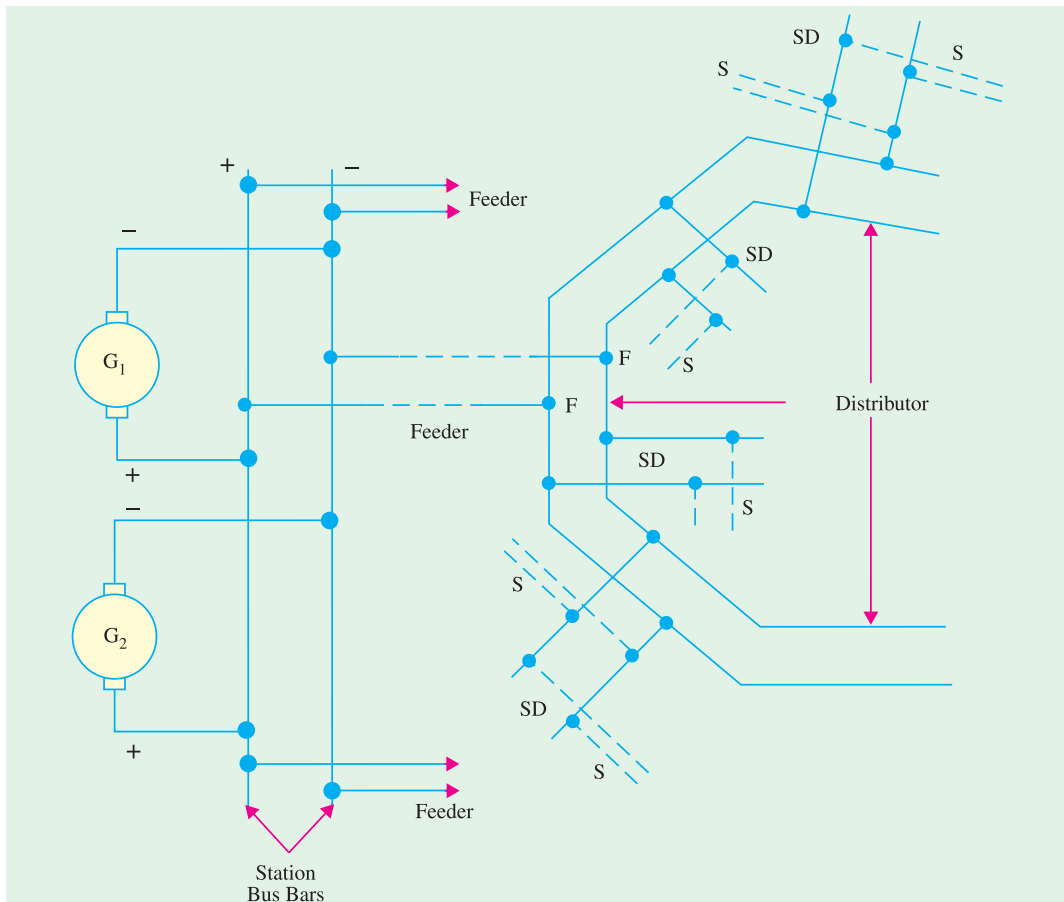


Fig. 40.2

### 40.2. Two-wire and Three-wire Systems

In d.c. systems, power may be fed and distributed either by (i) *2-wire system* or (ii) *3-wire system*.

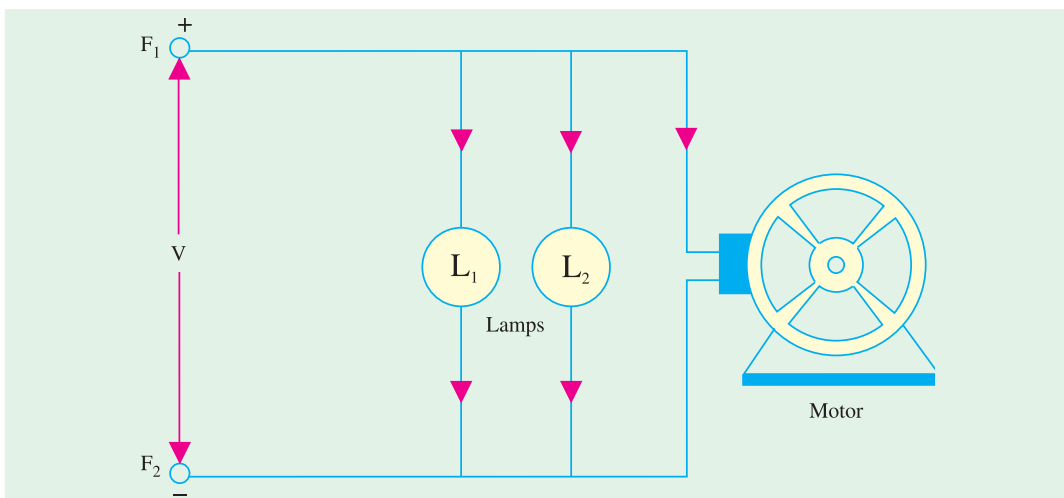


Fig. 40.3



In the 2-wire system, one is the outgoing or positive wire and the other is the return or negative wire. In the case of a 2-wire distributor, lamps, motors and other electrical apparatus are connected in parallel between the two wires as shown in Fig. 40.3. As seen, the potential difference and current have their maximum values at feeding points  $F_1$  and  $F_2$ . The standard voltage between the conductors is 220 V.

The 2-wire system when used for transmission purposes, has much lower efficiency and economy as compared to the 3-wire system as shown later.

A 3-wire has not only a higher efficiency of transmission (Fig. 40.4) but when used for distribution purposes, it makes available two voltages at the consumer's end (Fig. 40.5). This 3-wire system consists of two 'outers' and a middle or neutral wire which is earthed at the generator end. Its potential is midway between that of the outers *i.e.* if the p.d. between the outers is 460 V, then the p.d. of positive outer is 230 V above the neutral and that of negative outer is 230 V below the neutral. Motors requiring higher voltage are connected across the outers whereas lighting and heating circuits requiring less voltage are connected between any one of the outers and the neutral.

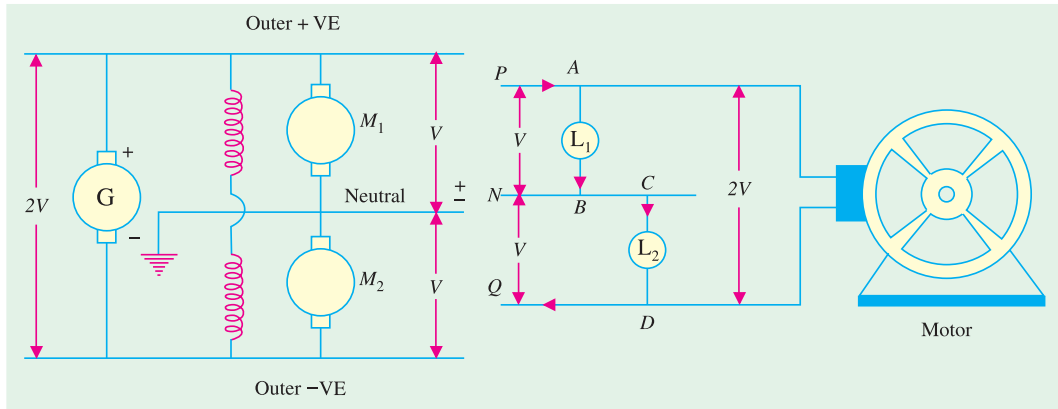


Fig. 40.4

Fig. 40.5

The addition of the middle wire is made possible by the connection diagram shown in Fig. 40.4. G is the main generator which supplies power to the whole system.  $M_1$  and  $M_2$  are two identical shunt machines coupled mechanically with their armatures and shunt field winding joined in series across the outers. The junction of their armatures is earthed and the neutral wire is taken out from there.

### 40.3 Voltage Drop and Transmission Efficiency

Consider the case of a 2-wire feeder (Fig. 40.6).  $AB$  is the sending end and  $CD$  the receiving end. Obviously, the p.d. at  $AB$  is higher than at  $CD$ . The difference in potential at the two ends is the potential drop or 'drop' in the cable. Suppose the transmitting voltage is 250 V, current in  $AC$  is 10 amperes, and resistance of each feeder conductor is  $0.5 \Omega$ , then drop in each feeder conductor is  $10 \times 0.5 = 5$  volt and drop in both feeder conductor is  $5 \times 2 = 10$  V.

$$\therefore \text{P.d. at Receiving end } CD \text{ is } = 250 - 10 = 240 \text{ V}$$

$$\text{Input power at } AB = 250 \times 10 = 2,500 \text{ W}$$

$$\text{Output power at } CD = 240 \times 10 = 2,400 \text{ W}$$

$$\therefore \text{ power lost in two feeders } = 2,500 - 2,400 = 100 \text{ W}$$

The above power loss could also be found by using the formula

$$\text{Power loss} = 2I^2R = 2 \times 10^2 \times 0.5 = 100 \text{ W}$$

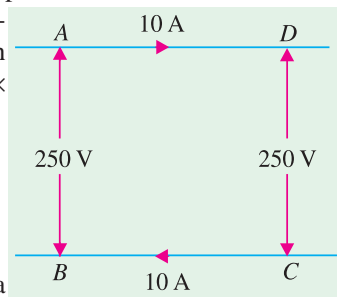


Fig. 40.6

The efficiency of transmission is defined, like any other efficiency, as the ratio of the output to input

$$\therefore \text{efficiency of transmission} = \frac{\text{power delivered by the line}}{\text{power received by the line}}$$

In the present case, power delivered by the *feeder* is = 2500 W and power received by it as 2400 W.

$$\therefore \eta = 2400 \times 100/2500 = 96\%$$

In general, if  $V_1$  is the voltage at the sending end and  $V_2$  at the receiving end and  $I$  the current delivered, then

$$\text{Input} = V_1 I, \quad \text{output} = V_2 I \quad \therefore \eta = \frac{V_2 I}{V_1 I} = \frac{V_2}{V_1}$$

$$\begin{aligned} \text{Now } V_2 &= V_1 - \text{drop in both conductors} \\ &= V_1 - IR, \quad \text{where } R \text{ is the resistance of both conductors} \end{aligned}$$

$$\therefore \eta = \frac{V_1 - IR}{V_1} = 1 - \frac{IR}{V_1} \quad \dots(i)$$

$$\text{or } \% \text{ efficiency} = 100 \times \left( 1 - \frac{IR}{V_1} \right) = 100 - \left( \frac{IR}{V_1} \times 100 \right)$$

Now,  $(IR/V_1) \times 100$  represents the voltage drop in both conductors expressed as a percentage of the voltage at the sending end. It is known as percentage drop.

$$\therefore \% \eta = 100 - \% \text{ drop}$$

In the present case total drop is 10 volt which expressed as percentage becomes  $(10/250) \times 100 = 4\%$ . Hence  $\% \eta = 100 - 4 = 96\%$  ...as before

It is seen from equation (i) above that for a given drop, transmission efficiency can be considerably increased by increasing the voltage at the transmitting end. *i.e.*,  $V_1$ . Moreover, the cross-section of copper in the cables is decreased in proportion to the increase in voltage which results in a proportionate reduction of the cost of copper in the cables.

The calculation of drop in a *feeder* is, as seen from above, quite easy because of the fact that current is constant throughout its length. But it is not so in the case of *distributors* which are tapped off at various places along their entire lengths. Hence, their different sections carry different currents over different lengths. For calculating the total voltage drop along the entire length of a distributor, following information is necessary.

- (i) value of current tapped at each load point.
- (ii) the resistance of each section of the distributor between tapped points.



A DC power distribution system consists of a network of cables or conductors which conveys power from central station to the consumer's premises.

**Example 40.1.** A DC 2-wire feeder supplies a constant load with a sending-end voltage of 220 V. Calculate the saving in copper if this voltage is doubled with power transmitted remaining the same.

**Solution.** Let

- $l$  = length of each conductor in metre
- $\sigma$  = current density in  $A/m^2$
- $P$  = power supplied in watts

(i) 220 V Supply

Current per feeder conductor  $I_1 = P/220$   
 Area of conductor required  $A_1 = I_1/\sigma = P/220 \sigma$   
 Volume of Cu required for both conductors is

$$V_1 = 2A_1 l = \frac{2Pl}{220\sigma} = \frac{Pl}{110\sigma}$$

(ii) 440 V Supply

$$V_2 = \frac{Pl}{220\sigma}$$

$$\% \text{ age saving in Cu} = \frac{V_1 - V_2}{V_1} \times 100 = \frac{\frac{Pl}{110\sigma} - \frac{Pl}{220\sigma}}{\frac{Pl}{110\sigma}} \times 100 = 50\%$$

40.4. Methods of Feeding a Distributor

Different methods of feeding a distributor are given below :

1. feeding at one end
2. feeding at both ends with equal voltages
3. feeding at both ends with unequal voltages
4. feeding at some intermediate point

In adding, the nature of loading also varies such as

- (a) concentrated loading      (b) uniform loading      (c) combination of (a) and (b).

Now, we will discuss some of the important cases separately.

40.5. D.C. Distributor Fed at One End

In Fig. 40.7 is shown one conductor AB of a distributor with concentrated loads and fed at one end.

Let  $i_1, i_2, i_3$  etc. be the currents tapped off at points C, D, E and F and  $I_1, I_2, I_3$  etc. the currents in the various sections of the distributor. Let  $r_1, r_2, r_3$  etc. be the ohmic resistances of these various sections and  $R_1, R_2, R_3$  etc. the total resistance from the feeding end A to the successive tapping points.

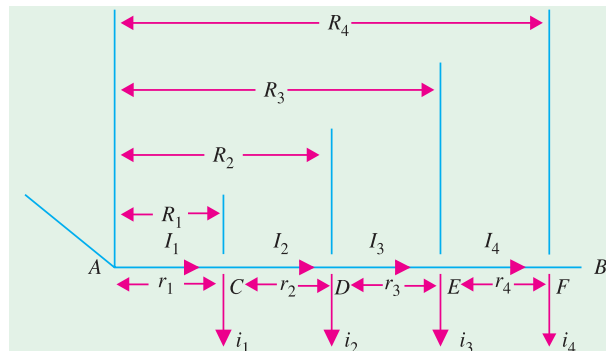


Fig. 40.7

Then, total drop in the distributor is

$$= r_1 I_1 + r_2 I_2 + r_3 I_3 + \dots$$

Now

$$I_1 = i_1 + i_2 + i_3 + i_4 + \dots; I_2 = i_2 + i_3 + i_4 + \dots; I_3 = i_3 + i_4 + \dots$$

∴

$$\begin{aligned} &= r_1(i_1 + i_2 + i_3 + \dots) + r_2(i_2 + i_3 + i_4 + \dots) + r_3(i_3 + i_4 + \dots) \\ &= i_1 r_1 + i_2(r_1 + r_2) + i_3(r_1 + r_2 + r_3) + \dots = i_1 R_1 + i_2 R_2 + i_3 R_3 + \dots \\ &= \text{sum of the moments of each load current about feeding point A.} \end{aligned}$$

1. Hence, the drop at the far end of a distributor fed at one end is given by the sum of the moments of various tapped currents about the feeding point i.e.  $v = \sum i R$ .

2. It follows from this that the total voltage drop is the same as that produced by a single load equal to the sum of the various concentrated loads, acting at the centre of gravity of the load system.

3. Let us find the drop at any intermediate point like E. The value of this drop is

$$= i_1 R_1 + i_2 R_2 + i_3 R_3 + R_3(i_4 + i_5 + i_6 + \dots)^*$$

\* The reader is advised to derive this expression himself.

$$= \left( \begin{array}{c} \text{sum of moments} \\ \text{upto } E \end{array} \right) + \left( \begin{array}{c} \text{moment of load beyond} \\ E \text{ assumed acting at } E \end{array} \right)$$

In general, the drop at any intermediate point is *equal to the sum of moments of various tapped currents upto that point plus the moment of all the load currents beyond that point assumed to be acting at that point.*

The total drop over both conductors would, obviously, be twice the value calculated above.

### 40.6. Uniformly Loaded Distributor

In Fig. 40.8 is shown one conductor  $AB$  of a distributor fed at one end  $A$  and uniformly loaded with  $i$  amperes per unit length. Any convenient unit of length may be chosen *i.e.* a metre or 10 metres but at every such unit length, the load tapped is the same. Hence, let

$$\begin{aligned} i &= \text{current tapped off per unit length} & l &= \text{total length of the distributor} \\ r &= \text{resistance per unit length of the distributor} \end{aligned}$$

Now, let us find the voltage drop at a point  $C$  (Fig. 40.9) which is at a distance of  $x$  units from feeding end  $A$ . The current at point  $C$  is  $(il - ix) = i(l - x)$ .

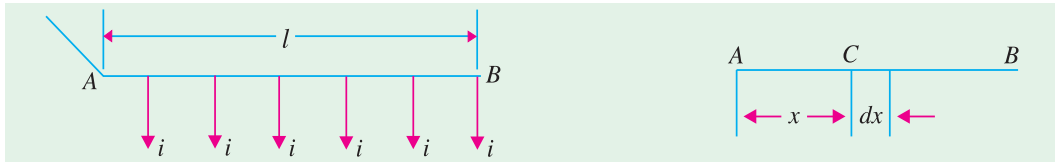


Fig. 40.8

Fig. 40.9

Consider a small section of length  $dx$  near point  $C$ . Its resistance is  $(rdx)$ . Hence, drop over length  $dx$  is

$$dv = i(l - x)(rdx) = (ilr - ixr)dx$$

The total drop up to point  $x$  is given by integrating the above quantity between proper limits.

$$\therefore \int_0^x dx = \int_0^x (ilx - ixr) dx \quad \therefore v = ilrx - \frac{1}{2} irx^2 = ir \left( lx - \frac{x^2}{2} \right)$$

The drop at point  $B$  can be obtained by putting  $x = l$  in the above expression.

$$\therefore \text{drop at point } B = ir \left( l^2 - \frac{l^2}{2} \right) = \frac{irl^2}{2} = \frac{(i \times l) \times (r \times l)}{2} = \frac{1}{2} IR = I \times \frac{1}{2} R$$

where  $i \times l = I$  – total current entering at point  $A$ ;  $r \times l = R$  – the total resistance of distributor  $AB$ .

$$\text{Total drop in the distributor } AB = \frac{1}{2} IR$$

(i) It follows that in a uniformly loaded distributor, total drop is equal to that produced by the whole of the load assumed concentrated at the middle point.

(ii) Suppose that such a distributor is fed at both ends  $A$  and  $B$  with *equal voltages*. In that case, the point of minimum potential is obviously the middle point. We can thus imagine as if the distributor were cut into two at the middle point, giving us two uniformly-loaded distributors each fed at one end with equal voltages. The resistance of each is  $R/2$  and total current fed into each distributor is  $I/2$ . Hence, drop at the middle point is

$$= \frac{1}{2} \times \left( \frac{I}{2} \right) \times \left( \frac{R}{2} \right) = \frac{1}{8} IR$$

It is 1/4th of that of a distributor fed at one end only. The advantage of feeding a distributor at both ends, instead of at one end, is obvious.

The equation of drop at point  $C$  distant  $x$  units from feeding point  $A = irlx - \frac{1}{2} irx^2$  shows that the diagram of drop of a uniformly-loaded distributor fed at one end is a parabola.

**Example 40.2.** A uniform 2-wire d.c. distributor 200 metres long is loaded with 2 amperes/metre. Resistance of single wire is 0.3 ohm/kilometre. Calculate the maximum voltage drop if the distributor is fed (a) from one end (b) from both ends with equal voltages.

(Elect. Technology ; Bombay Univ.)

**Solution. (a)** Total resistance of the distributor is  $R = 0.3 \times (200/1000) = 0.06 \text{ W}$

Total current entering the distributor is  $I = 2 \times 200 = 400 \text{ A}$

Total drop on the whole length of the distributor is (Art. 40.6)

$$= \frac{1}{2} IR = \frac{1}{2} \times 400 \times 0.06 = \mathbf{12 \text{ V}}$$

**(b)** Since distributor is fed from both ends, total voltage drop is

$$= \frac{1}{8} IR = \frac{1}{8} \times 400 \times 0.06 = \mathbf{3 \text{ V}}$$

As explained in Art. 40.6 the voltage drop in this case is one-fourth of that in case (a) above.

**Example 40.3.** A 2-wire d.c. distributor AB is 300 metres long. It is fed at point A. The various loads and their positions are given below :

At point	distance from A in metres	concentrated load in A
C	40	30
D	100	40
E	150	100
F	250	50

If the maximum permissible voltage drop is not to exceed 10 V, find the cross-sectional area of the distributor. Take  $\rho = (1.78 \times 10^{-8}) \Omega\text{-m}$ . (Electrical Power-I ; Bombay Univ.)

**Solution.** The distributor along with its tapped currents is shown in Fig. 40.10.

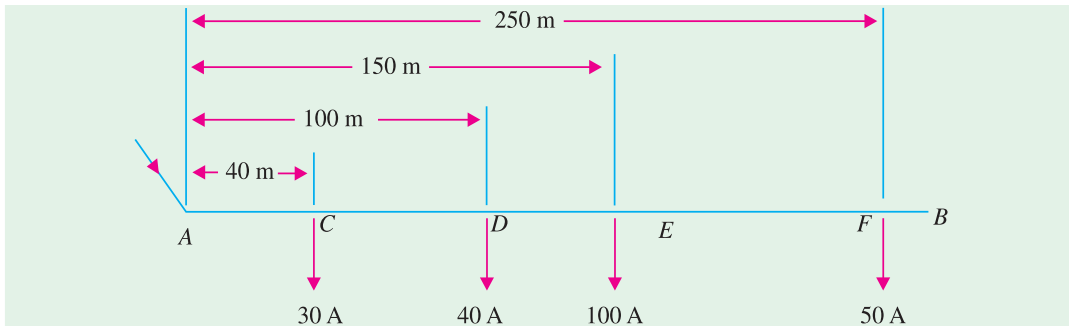


Fig. 40.10

Let 'A' be cross sectional area of distributor

As proved in Art. 40.5 total drop over the distributor is

$$v = i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4 \quad \text{—for one conductor}$$

$$= 2(i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4) \quad \text{—for two conductors}$$

where

$$R_1 = \text{resistance of AC} = 1.78 \times 10^{-8} \times 40/A$$

$$R_2 = \text{resistance of AD} = 1.78 \times 10^{-8} \times 100/A$$

$$R_3 = \text{resistance of AE} = 1.78 \times 10^{-8} \times 150/A$$

$$R_4 = \text{resistance of AF} = 1.78 \times 10^{-8} \times 250/A$$

Since

$$v = 10 \text{ V, we get}$$

$$10 = \frac{2 \times 1.78 \times 10^{-8}}{A} (30 \times 40 + 40 \times 100 + 100 \times 150 + 50 \times 250)$$

$$= \frac{2 \times 1.78 \times 10^{-8}}{A} \times 32,700$$

$$\therefore A = 3.56 \times 327 \times 10^{-7} \text{ m}^2 = \mathbf{1,163 \text{ cm}^2}$$

### 40.7. Distributor Fed at Both Ends with Equal Voltages

It should be noted that in such cases

(i) the maximum voltage drop must always occur at one of the load points and

(ii) if both feeding ends are at the same potential, then the voltage drop between each end and this point must be the same, which in other words, means that the sum of the moments about ends must be equal.

In Fig. 40.11 (a) is shown a distributor fed at two points  $F_1$  and  $F_2$  with equal voltages. The potential of the conductor will gradually fall from  $F_1$  onwards, reach a minimum value at one of the tapping, say,  $A$  and then rise again as the other feeding point  $F_2$  is approached. All the currents tapped off between points  $F_1$  and  $A$  will be supplied from  $F_1$  while those tapped off between  $F_2$  and  $A$  will be supplied from  $F_2$ . The current tapped at point  $A$  itself will, in general, be partly supplied by  $F_1$  and partly by  $F_2$ . Let the values of these currents be  $x$  and  $y$  respectively. If the distributor were actually cut off into two at  $A$ —the point of minimum voltage, with  $x$  amperes tapped off from the left and  $y$  amperes tapped off from the right, then potential distribution would remain unchanged, showing that we can regard the distributor as consisting of two separate distributors each fed from one end only, as shown in Fig. 40.11 (b). The drop can be calculated by locating point  $A$  and then values of  $x$  and  $y$  can be calculated.

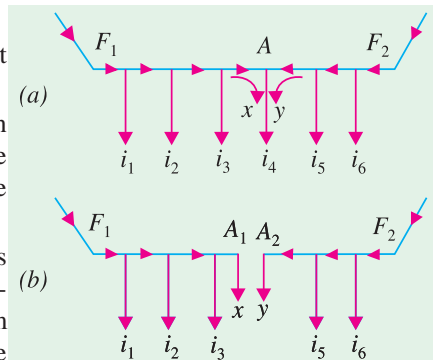


Fig. 40.11

This can be done with the help of the following pair of equations :

$$x + y = i_4 \quad \text{and} \quad \text{drop from } F_1 \text{ to } A_1 = \text{drop from } F_2 \text{ to } A_2.$$

**Example 40.4.** A 2-wire d.c. distributor  $F_1 F_2$  1000 metres long is loaded as under :

Distance from $F_1$ (in metres) :	100	250	500	600	700	800	850	920
Load in amperes :	20	80	50	70	40	30	10	15

The feeding points  $F_1$  and  $F_2$  are maintained at the same potential. Find which point will have the minimum potential and what will be the drop at this point ? Take the cross-section of the conductors as  $0.35 \text{ cm}^2$  and specific resistance of copper as  $(1.764 \times 10^{-6}) \Omega\text{-cm}$ .

**Solution.** The distributor along with its tapped currents is shown in Fig. 40.12. The numbers along the distributor indicate length in metres.

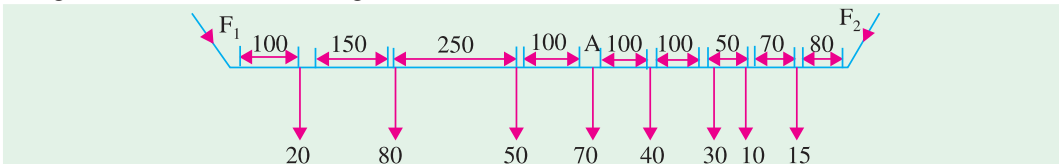


Fig. 40.12

The easiest method of locating the point of minimum potential is to take the moments about the two ends and then by comparing the two sums make a guess at the possible point. The way it is done is as follows :

It is found that 4th point from  $F_1$  is the required point *i.e.* point of minimum potential. Using the previous two equations, we have

$$x + y = 70 \quad \text{and} \quad 47,000 + 600x = 20,700 + 400y$$

Solving the two equations, we get  $x = 1.7 \text{ A}$ ,  $y = 70 - 1.7 = 68.3 \text{ A}$

Drop at A per conductor =  $47,000 + 600 \times 1.7 = 48,020$  ampere-metre.

$$\text{Resistance/metre} = \frac{\rho l}{A} = \frac{1.764 \times 10^{-8} \times 1}{0.35 \times 10^{-4}} = 50.4 \times 10^{-5} \Omega/\text{m}$$

Hence, drop per conductor =  $48,020 \times 50.4 \times 10^{-5} = 24.2 \text{ V}$

Reckoning both conductors, the drop at A is = **48.4 V**

Moments about $F_1$ in ampere-metres	Sum	Moments about $F_2$ in ampere-metres	Sum
$20 \times 100 = 2,000$	2,000	$15 \times 80 = 1,200$	1,200
$80 \times 250 = 20,000$	22,000	$10 \times 150 = 1,500$	2,700
$50 \times 500 = 25,000$	47,000	$30 \times 200 = 6,000$	8,700
		$40 \times 300 = 12,000$	20,700
		$70 \times 400 = 28,000$	48,700

**Alternative Solution**

The alternative method is to take the total current fed at one end, say,  $F_1$  as  $x$  and then to find the current distribution as shown in Fig. 40.13 (a). The drop over the whole distributor is equal to the sum of the products of currents in the various sections and their resistances. For a distributor fed at both ends with *equal voltages*, this drop equals zero. In this way, value of  $x$  can be found.

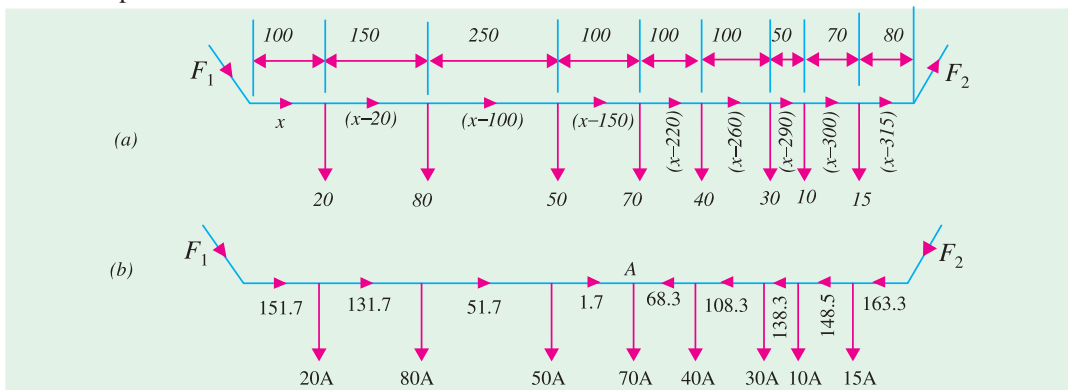
Resistance per metre single =  $5.04 \times 10^{-4} \Omega$

Resistance per metre double =  $10.08 \times 10^{-4} \Omega$

$$\therefore 10.08 \times 10^{-4} [100x + 150(x - 20) + 250(x - 100) + 100(x - 150) + 100(x - 200) + 100(x - 260) + 50(x - 290) + 70(x - 300) + 80(x - 315)] = 0 \quad \text{or} \quad 1000x = 151,700$$

$$\therefore x = 151.7 \text{ A}$$

This gives a current distribution as shown in Fig. 40.13 (b). Obviously, point A is the point of minimum potential.



**Fig. 40.13**

Drop at A (considering both conductors) is

$$= 10.08 \times 10^{-4} (100 \times 151.7 + 150 \times 131.7 + 250 \times 51.7 + 100 \times 1.7)$$

$$= 10.08 \times 10^{-4} \times 48,020 = \mathbf{48.4 \text{ V}} \quad \text{—as before.}$$

**Example 40.5.** The resistance of a cable is  $0.1 \Omega$  per 1000 metre for both conductors. It is loaded as shown in Fig. 40.14 (a). Find the current supplied at A and at B. If both the ends are supplied at 200 V. **(Electrical Technology-II, Gwalior Univ.)**

**Solution.** Let the current distribution be as shown in Fig. 40.14 (b). Resistance for both conductors =  $0.1/1000=10^{-4} \Omega/m$ . The total drop over the whole cable is zero because it is fed at both ends by equal voltages.

$$\therefore 10^{-4} [500 i + 700 (i - 50) + 300 (i - 150) + 250 (i - 300)] = 0$$

$$i = \mathbf{88.6 \text{ A}}$$

This gives the current distribution as shown in Fig. 40.14 (c).

- Current in AC = 88.6 A
- Current in CD =  $(88.6 - 50) = 38.6 \text{ A}$
- Current in DE =  $(38.6 - 100) = -61.4 \text{ A}$
- Current in EB =  $(-61.4 - 150) = -211.4 \text{ A}$

Hence, current entering the cable at point B is **211.4 A**.

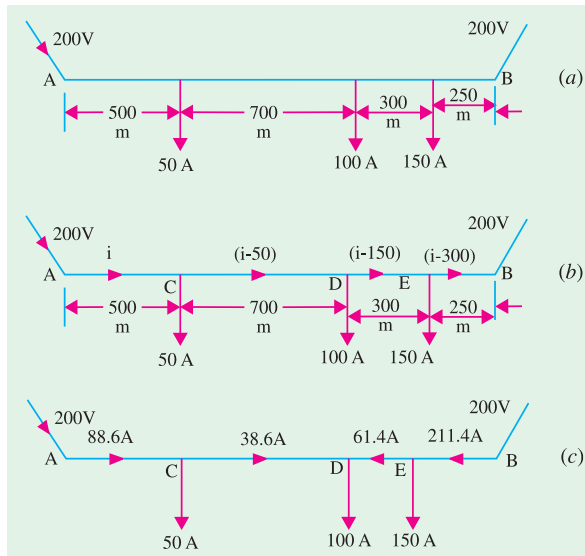


Fig. 40.14

**Example 40.6.** The resistance of two conductors of a 2-conductor distributor shown in Fig. 39.15 is  $0.1 \Omega$  per 1000 m for both conductors. Find (a) the current supplied at A (b) the current supplied at B (c) the current in each section (d) the voltages at C, D and E. Both A and B are maintained at 200 V. **(Electrical Engg. Grad. I.E.T.E.)**

**Solution.** The distributor along with its tapped currents is shown in Fig. 40.15. Let the current distribution be as shown. Resistance per metre double is  $= 0.1/1000 = 10^{-4} \Omega$ .

The total drop over the whole distributor is zero because it is fed at both ends by equal voltages.

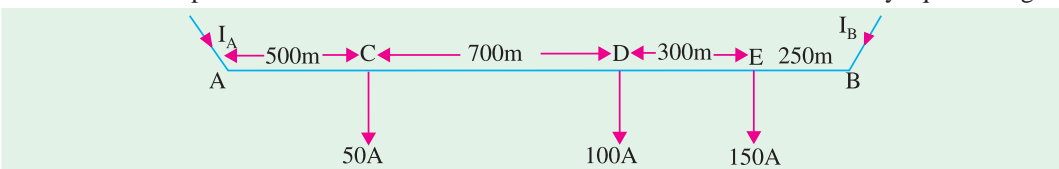


Fig. 40.15

$$\therefore 10^{-4} [500i + 700 (i - 50) + 300 (i - 150) + 250 (i - 300)] = 0$$

or  $1750i = 155,000$  or  $i = 88.6 \text{ A}$

This gives the current distribution shown in Fig. 40.16.

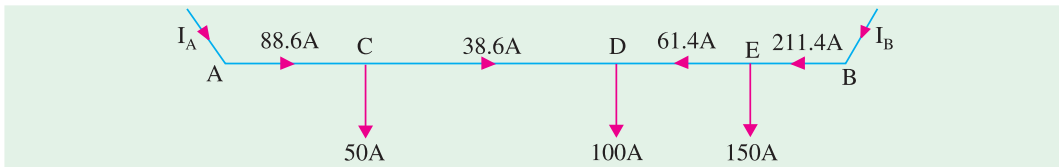


Fig. 40.16

- (a)  $I_A = \mathbf{88.6 \text{ A}}$
- (b)  $I_B = \mathbf{-211.4 \text{ A}}$
- (c) Current in section AC = **88.6 A**; Current in CD =  $(88.6 - 50) = \mathbf{38.6 \text{ A}}$   
Current in section DE =  $(31.6 - 100) = \mathbf{-61.4 \text{ A}}$   
Current in EB =  $(-61.4 - 150) = \mathbf{-211.4 \text{ A}}$
- (d) Drop over AC =  $10^{-4} \times 500 \times 88.6 = 4.43 \text{ V}$ ; Drop over CD =  $10^{-4} \times 700 \times 38.6 = \mathbf{2.7 \text{ V}}$



Drop over  $DE = 10^{-4} \times 300 \times -61.4 = -1.84 \text{ V} \therefore$  Voltage at  $C = 200 - 4.43 = \mathbf{195.57 \text{ V}}$   
 Voltage at  $D = (195.57 - 2.7) = \mathbf{192.87 \text{ V}}$ , Voltage at  $E = 192.87 - (-1.84) = \mathbf{194.71 \text{ V}}$

**Example 40.7.** A 200 m long distributor is fed from both ends A and B at the same voltage of 250 V. The concentrated loads of 50, 40, 30 and 25 A are coming on the distributor at distances of 50, 75, 100 and 150 m respectively from end A. Determine the minimum potential and locate its position. Also, determine the current in each section of the distributor. It is given that the resistance of the distributor is 0.08 Ω per 100 metres for go and return.

(Electric Power-I (Trans & Dist) Punjab Univ. 1993)

**Solution.** As shown in Fig. 40.17, let current fed at point A be  $i$ . The currents in various sections are as shown. Resistance per metre of the distributor (go and return) is  $0.08/100 = 0.0008 \text{ Ω}$ .

Since the distributor is fed at both ends with equal voltages, total drop over it is zero.

Hence,  $0.0008 [50i + 25(i - 50) + 25(i - 90) + 50(i - 120) + 50(i - 145)] = 0$

or  $200i = 16,750$  or  $i = 83.75 \text{ A}$

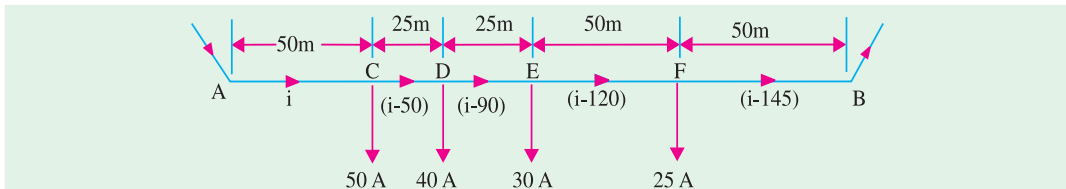


Fig. 40.17

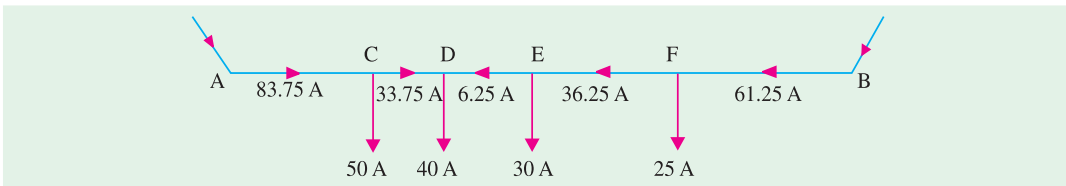


Fig 40.18

The actual current distribution is shown in Fig. 40.18. Obviously, point  $D$  is the point of minimum potential.

Drop at point  $D$  is  $= 0.0008(50 \times 83.75 + 25 \times 33.75) = 4.025 \text{ V}$

$\therefore$  minimum potential  $= 250 - 4.025 = \mathbf{245.98 \text{ V}}$

**40.8. Distributor Fed at Both Ends with Unequal Voltages**

This case can be dealt with either by taking moments (in amp-m) about the two ends and then making a guess about the point of minimum potential or by assuming a current  $x$  fed at one end and then finding the actual current distribution.

Consider the case already discussed in Ex. 40.4. Suppose, there is a difference of  $v$  volts between ends  $F_1$  and  $F_2$  with  $F_1$  being at higher potential. Convert  $v$  volts into ampere-metres with the help of known value of resistance/metre. Since  $F_2$  is at a lower potential, these ampere-metres appear in the column for  $F_2$  as initial drop.

If, for example,  $v$  is 4 volts, then since resistance/metre is  $5.04 \times 10^{-4} \text{ Ω}$ , initial ampere-metres for  $F_2$  are

$$= \frac{4 \times 10^4}{5.04} = 7,938 \text{ amp-metres}$$

The table of respective moments will be as follows :

Moments about $F_1$	Sum	Moments about $F_2$	Sum
$20 \times 100 = 2000$	2,000	initial = 7,938	
$80 \times 250 = 20,000$	22,000	$15 \times 80 = 1,200$	9,138
$50 \times 500 = 25,000$	47,000	$10 \times 150 = 1,500$	10,638
		$30 \times 200 = 6,000$	16,638
		$40 \times 300 = 12,000$	28,638
		$70 \times 400 = 28,000$	56,638

As seen, the dividing point is the same as before.

$$\therefore \quad x + y = 70 \quad \text{and} \quad 47,000 + 600x = 28,638 + 400y$$

Solving for  $x$  and  $y$ , we get  $x = 9.64 \text{ A}$  and  $y = 60.36 \text{ A}$

After knowing the value of  $x$ , the drop at  $A$  can be calculated as before.

The alternative method of solution is illustrated in Ex. 40.12.

### 40.9. Uniform Loading with Distributor Fed at Both Ends

Consider a distributor  $PQ$  of length  $l$  units of length, having resistance per unit length of  $r$  ohms and with loading per unit length of  $i$  amperes. Let the difference in potentials of the two feeding points be  $v$  volts with  $Q$  at the lower potential. The procedure for finding the point of minimum potential is as follows:

Let us assume that point of minimum potential  $M$  is situated at a distance of  $x$  units from  $P$ . Then drop from  $P$  to  $M$  is  $= irx^2/2$  volts (Art. 40.6)

Since the distance of  $M$  from  $Q$  is  $(l - x)$ , the drop from  $Q$  to  $M$  is  $= \frac{ir(l-x)^2}{2}$  volt.

Since potential of  $P$  is greater than that of  $Q$  by  $v$  volts,

$$\therefore \frac{irx^2}{2} = \frac{ir(l-x)^2}{2} + v \quad \text{or} \quad x = \frac{l}{2} + \frac{v}{irl}$$



A circuit-board as shown above uses DC current

### 40.10. Concentrated and Uniform Loading with Distributor Fed at One End

Such cases are solved in two stages. First, the drop at any point due to concentrated loading is found. To this add the voltage drop due to uniform loading as calculated from the relation.

$$ir \left( lx - \frac{x^2}{2} \right)$$

As an illustration of this method, please look up Ex. 40.9 and 40.13.

**Example 40.8.** Each conductor of a 2-core distributor, 500 metres long has a cross-sectional area of  $2 \text{ cm}^2$ . The feeding point  $A$  is supplied at 255 V and the feeding point  $B$  at 250 V and load currents of 120 A and 160 A are taken at points  $C$  and  $D$  which are 150 metres and 350 metres respectively from the feeding point  $A$ . Calculate the voltage at each load. Specific resistance of copper is  $1.7 \times 10^{-6} \Omega\text{-cm}$ . (Elect. Technology-I, Bombay Univ.)

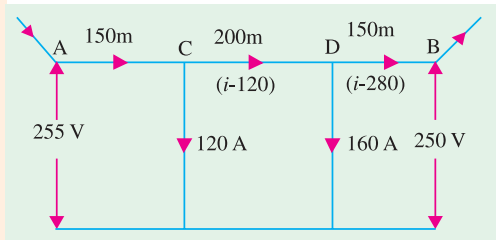


Fig. 40.19

**Solution.**

$$\begin{aligned}\rho &= 1.7 \times 10^{-6} \Omega\text{-cm} \\ &= 1.7 \times 10^{-8} \Omega\text{.m}\end{aligned}$$

$$\text{Resistance per metre single} = \rho \frac{l}{A} = 1.7 \times 10^{-8} / 2 \times 10^{-4} = 85 \times 10^{-6} \text{ m}$$

$$\text{Resistance per metre double} = 2 \times 85 \times 10^{-6} = 17 \times 10^{-5} \Omega$$

$$\text{The drop over the whole distributor} = 255 - 250 = 5 \text{ V}$$

$$\therefore 17 \times 10^{-5} [150i + 200(i - 120) + 150(i - 280)] = 5$$

$$\therefore$$

$$i = 190.8 \text{ A}$$

— (Fig. 40.19)

$$\text{Current in section } AC = 190.8 \text{ A; Current in section } CD = (190.8 - 120) = 70.8 \text{ A}$$

$$\text{Current in section } DB = (190.8 - 280) = -89.2 \text{ A}$$

(from B to D)

Voltage at point

$$C = 255 - \text{drop over } AC$$

$$= 255 - (17 \times 10^{-5} \times 150 \times 190.8) = \mathbf{250.13 \text{ V}}$$

Voltage at point

$$D = 250 - \text{drop over } BD$$

$$= 250 - (17 \times 10^{-5} \times 150 \times 89.2) = \mathbf{247.72 \text{ V}}$$

**Example 40.9.** A 2-wire distributor 500 metres long is fed at P at 250 V and loads of 40A, 20A, 60A, 30A are tapped off from points A, B, C and D which are at distances of 100 metres, 150 metres, 300 metres and 400 metres from P respectively. The distributor is also uniformly loaded at the rate of 0.1 A/m. If the resistance of the distributor per metre (go and return) is 0.0005  $\Omega$ , calculate the voltage at (i) point Q and (ii) point B.

**Solution.** First, consider drop due to concentrated load only.

$$\text{Drop in } PA = 150 \times (100 \times 0.0005) = 7.5 \text{ V}$$

$$\text{Drop in } AB = 110 \times (50 \times 0.0005) = 2.75 \text{ V}$$

$$\text{Drop in } BC = 90 \times (150 \times 0.0005) = 6.75 \text{ V}$$

$$\text{Drop in } CD = 30 \times (100 \times 0.0005) = 1.5 \text{ V}$$

$$\therefore \text{ total drop due to this load} = 18.5 \text{ V}$$

Now, let us consider drop due to uniform load only.

$$\text{Drop over length } l = ir \frac{l^2}{2} = 0.1 \times 0.0005 \times 500^2 / 2 = 6.25 \text{ V}$$

$$(i) \therefore \text{ potential of point } Q = 250 - (18.5 + 6.25) = \mathbf{225.25 \text{ V}}$$

(ii) Consider point B at the distance of 150 metres from P.

$$\text{Drop due to concentrated loading} = 7.5 + 2.75 = 10.25 \text{ V.}$$

$$\text{Drop due to uniform loading} = ir (lx - x^2/2); \quad \text{Here } l = 500 \text{ m ; } x = 150 \text{ m}$$

$$\therefore \text{ drop} = 0.1 \times 0.0005 \left( 500 \times 150 - \frac{150^2}{2} \right) = 3.1875 \text{ V}$$

$$\therefore \text{ potential of point } P = 250 - (10.25 + 3.1875) = \mathbf{13.44 \text{ V}}$$

**Example 40.10.** A distributor AB is fed from both ends. At feeding point A, the voltage is maintained at 236 V and at B at 237 V. The total length of the distributor is 200 metres and loads are tapped off as under :

(i) 20 A at 50 metres from A

(ii) 40 A at 75 metres from A

(iii) 25 A at 100 metres from A

(iv) 30 A at 150 metres from A

The resistance per kilometre of one conductor is 0.4  $\Omega$ . Calculate the currents in the various sections of the distributor, the minimum voltage and the point at which it occurs.

(Electrical Technology, Calcutta Univ.)

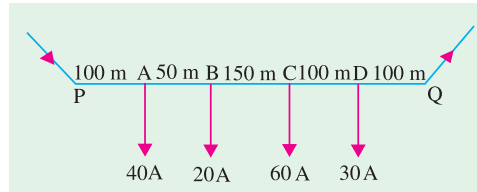


Fig. 40.20

**Solution.** The distributor along with currents in its various sections is shown in Fig. 40.21.

$$\text{Resistance/metre single} = 0.4/1000 = 4 \times 10^{-4} \Omega$$

$$\text{Resistance/metre double} = 8 \times 10^{-4} \Omega$$

Voltage drop on both conductors of 200-metre long distributor is

$$= 8 \times 10^{-4} [50i + 26(i - 20) + 25(i - 60) + 50(i - 85) + 50(i - 115)]$$

$$= 8 \times 10^{-4} (200i - 12,000) \text{ volt}$$

This drop must be equal to the potential difference between A and B.

$$\therefore 8 \times 10^{-4} (200i - 12,000) = 236 - 237 = -1$$

$$\therefore i = 53.75 \text{ A}$$

$$\text{Current in section AC} = 53.75 \text{ A}$$

$$\text{Current in section CD} = 53.75 - 20 = 33.75 \text{ A}$$

$$\text{Current in section DE} = 53.75 - 60 = -6.25 \text{ A}$$

$$\text{Current in section EF} = 53.75 - 85 = -31.25 \text{ A}$$

$$\text{Current in section FB} = 53.75 - 115 = -61.25 \text{ A}$$

The actual current distribution is as shown in Fig. 40.22.

Obviously, minimum voltage occurs at point D i.e. 75 metre from point A (or 125 m from B)

Voltage drop across both conductors of the distributor over the length AD is

$$= 8 \times 10^{-4} (50 \times 53.75 + 25 \times 33.75) = 2.82 \text{ V}$$

$\therefore$  potential of point

$$D = 236 - 2.82 = 233.18 \text{ V}$$

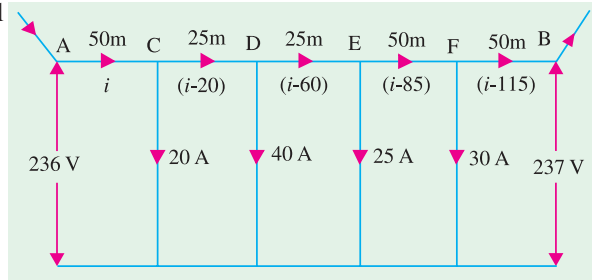


Fig. 40.21

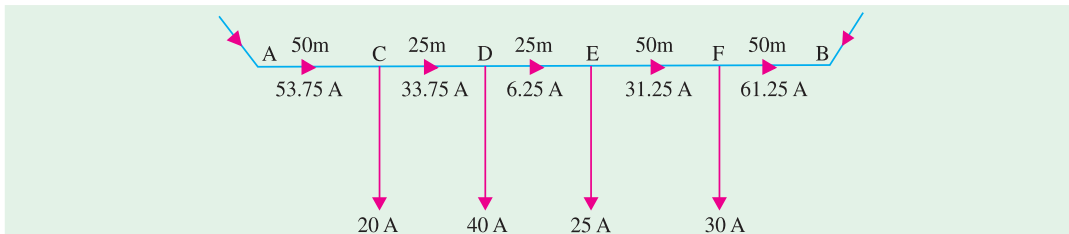


Fig. 40.22

**Example 40.11.** A distributor cable AB is fed at its ends A and B. Loads of 12, 24, 72 and 48 A are taken from the cable at points C, D, E and F. The resistances of sections AC, CD, DE, EF and FB of the cable are 8, 6, 4, 10 and 5 milliohm respectively (for the go and return conductors together).

The p.d. at point A is 240 V, the p.d. at the load F is also to be 240 V. Calculate the voltage at the feeding point B, the current supplied by each feeder and the p.d.s. at the loads C, D and E.

(Electrical Technology ; Utkal Univ.)

**Solution.** Let the current fed at the feeding point A be  $i$ . The current distribution in various sections becomes as shown in Fig. 40.23.

Voltage drop on both sides of the distributor over the section AF is

$$= [8i + 6(i - 12) + 4(i - 36) + 10(i - 108)] \times 10^{-3} = (28i - 1296) \times 10^{-3} \text{ V}$$

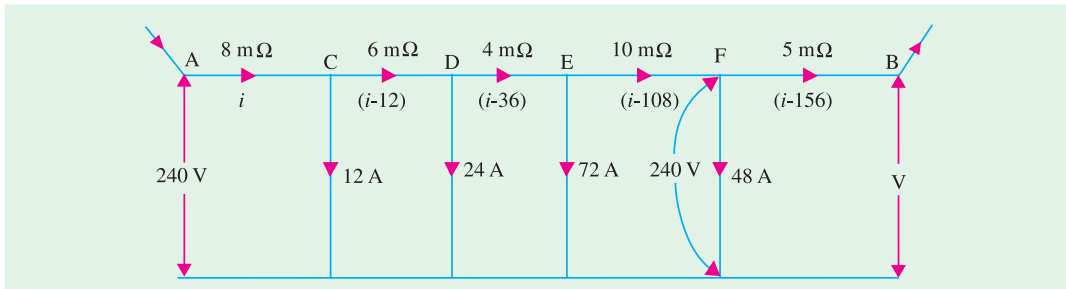


Fig. 40.23

Since points A and F are at the same potential, the p.d. between A and F is zero.

$$\therefore (28i - 1296) \times 10^{-3} = 0 \quad \text{or} \quad i = 46.29 \text{ A}$$

Current in section AC = 46.29 A

Current in section CD = 46.29 - 12 = 34.29 A

Current in section DE = 46.29 - 36 = 10.29 A

Current in section EF = 46.29 - 108 = -61.71 A

Current in section FB = 46.29 - 156 = -109.71 A

The actual current distribution is as shown in Fig. 40.24.

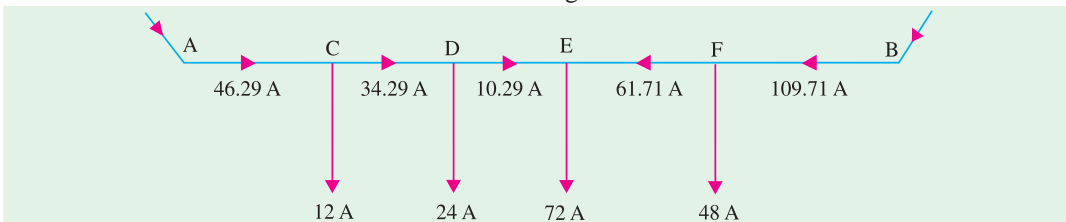


Fig. 40.24

Current applied by feeder at point A is 46.29 A and that supplied at point B is 109.71 A.

Voltage at feeding point B = 240 - drop over FB = 240 - (-5 × 10<sup>-3</sup> × 109.71) = **240.55 V**.

Voltage at point C = 240 - drop over AC = 240 - (8 × 10<sup>-3</sup> × 46.29) = **239.63 V**

Voltage at point D = 239.63 - drop over CD = 239.63 - (6 × 10<sup>-3</sup> × 34.29) = **239.42 V**

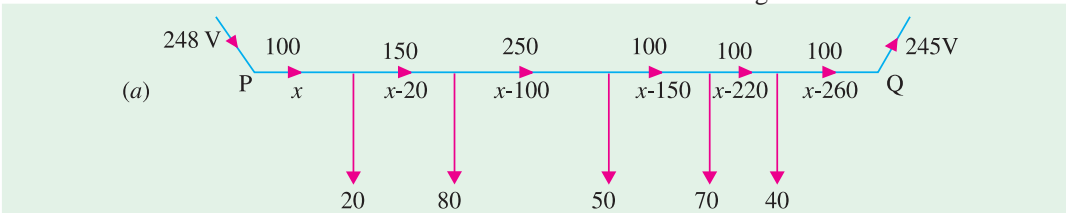
Voltage at point E = 239.42 - drop over DE = 239.42 - (4 × 10<sup>-3</sup> × 10.29) = **239.38 V**

**Example 40.12.** A two-wire, d.c. distributor PQ, 800 metre long is loaded as under :

Distance from P (metres) :	100	250	500	600	700
Loads in amperes :	20	80	50	70	40

The feeding point at P is maintained at 248 V and that at Q at 245 V. The total resistance of the distributor (lead and return) is 0.1 Ω. Find (a) the current supplied at P and Q and (b) the power dissipated in the distributor.

**Solution.** As shown in Fig. 40.25 (a), let x be the current supplied from end P. The other currents in the various sections of the distributor are as shown in the figure.



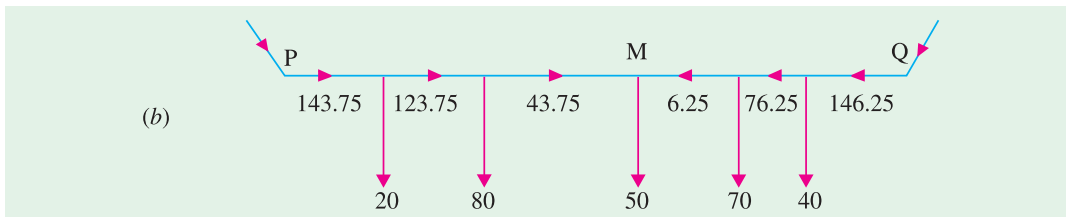


Fig. 40.25

Now, the total drop over  $PQ$  should equal the potential difference between ends  $P$  and  $Q$  i.e.,  $= 248 - 245 = 3$ .

Resistance/metre of both conductors  $= 0.1/800 = 1/8000 \Omega$

$$\therefore \frac{1}{8000} [100x + 150(x - 20) + 250(x - 100) + 100(x - 150) + 100(x - 220) + 100(x - 246)] = 3$$

or  $800x = 115,000 \quad \therefore x = 143.75 \text{ A}$

The actual distribution is shown in Fig. 40.25 (b) from where it is seen that point  $M$  has the minimum potential.

$$I_p = 143.75 \text{ A.} \quad I_Q = 116.25 \text{ A}$$

$$\text{Power loss} = \Sigma I^2 R = \frac{1}{8000} [143.75^2 \times 100 + 123.75^2 \times 150 + 43.75^2 \times 250 + 6.25^2 \times 100 + 76.25^2 \times 100 + 116.25^2 \times 100] = 847.3 \text{ W}$$

**Example 40.13.** The two conductors of a d.c. distributor cable 1000 m long have a total resistance of  $0.1 \Omega$ . The ends  $A$  and  $B$  are fed at 240 V. The cable is uniformly loaded at  $0.5 \text{ A per metre}$  length and has concentrated loads of 120 A, 60 A, 100 A and 40 A at points distant 200, 400, 700 and 900 m respectively from the end  $A$ . Calculate (i) the point of minimum potential on the distributor (ii) the value of minimum potential and (iii) currents fed at the ends  $A$  and  $B$ .

(Power System-I, AMIE, Sec. B, 1993)

**Solution.** Concentrated loads are shown in Fig. 40.26. Let us find out the point of minimum potential and currents at points  $A$  and  $B$ .

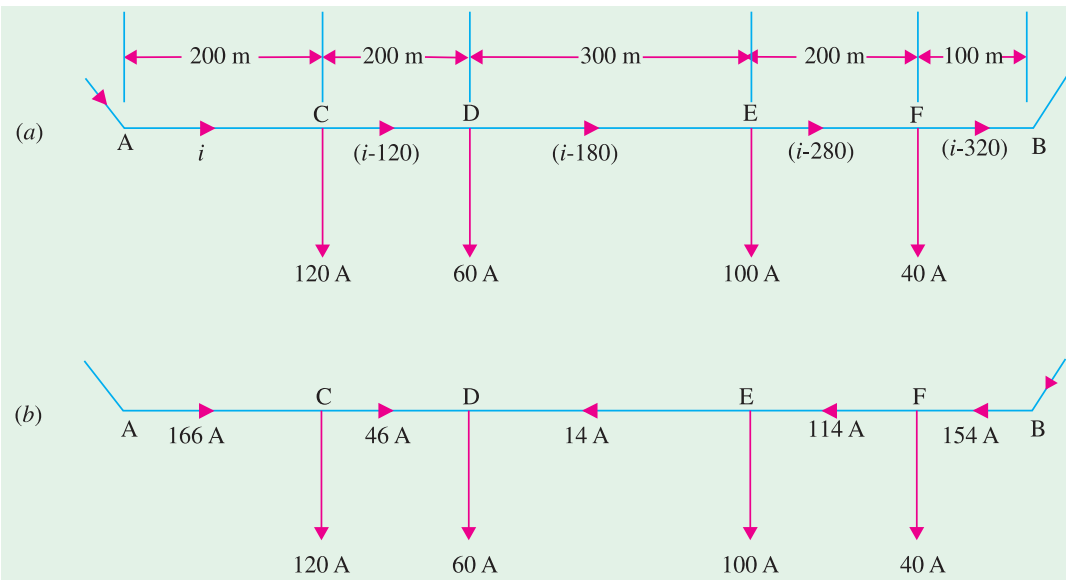


Fig. 40.26

It should be noted that location of point of minimum potential is not affected by the uniformly-spread load of 0.5 A/m. In fact, let us, for the time being, imagine that it is just not there. Then, assuming  $i$  to be the input current at A, the different currents in various sections are as shown. Since points A and B are fed at equal voltages, total drop over the distributor is zero. Distributor resistance per metre length (go and return) is  $0.1/1000 = 10^{-4} \Omega$ .

$$\therefore 10^{-4} [200i + 200(i - 120) + 300(i - 180) + 200(i - 280) + 100(i - 320)] = 0$$

or  $1000i = 166,000 \quad \therefore i = 166 \text{ A}$

Actual current distribution is shown in Fig. 40.26 (b) from where it is seen that point D is the point of minimum potential. The uniform load of 0.5 A/m upto point D will be supplied from A and the rest from point B.

Uniform load from A to D =  $400 \times 0.5 = 200 \text{ A}$ . Hence,  $I_A = 166 + 200 = 366 \text{ A}$ .

Similarly,  $I_B = 154 + (600 \times 0.5) = 454 \text{ A}$ .

Drop at D due to concentrated load is  $= 10^{-4} (166 \times 200 + 46 \times 200) = 4.24 \text{ V}$ .

Drop due to uniform load can be found by imagining that the distributor is cut into two at point D so that AD can be looked upon as a distributor fed at one end and loaded uniformly. In that case, D becomes the other end of the distributor.

$$\therefore \text{drop at D due to uniform load (Art. 40.6)}$$

$$= \frac{ir^2}{2} = 0.5 \times 10^{-4} \times 400^2/2 = 4 \text{ V}$$

$$\therefore \text{total drop at D} = 4.24 + 4 = 8.24 \text{ V} \quad \therefore \text{potential of D} = 240 - 8.24 = 231.76 \text{ V}$$

**Example 40.14.** It is proposed to lay out a d.c. distribution system comprising three sections—the first section consists of a cable from the sub-station to a point distant 800 metres from which two cables are taken, one 350 metres long supplying a load of 22 kW and the other 1.5 kilometre long and supplying a load of 44 kW. Calculate the cross-sectional area of each cable so that the total weight of copper required shall be minimum if the maximum drop of voltage along the cable is not to exceed 5% of the normal voltage of 440 V at the consumer's premises. Take specific resistance of copper at working temperature equal to  $2 \mu \Omega\text{-cm}$ .

**Solution.** Current taken from 350-m section is  $I_1 = 22,000/440 = 50 \text{ A}$

Current taken from 1.5 km section,  $I_2 = 44,000/440 = 100 \text{ A}$

$\therefore$  Current in first section  $I = 100 + 50 = 150 \text{ A}$

Let  $V =$  voltage drop across first section ;  $R =$  resistance of the first section

$A =$  cross-sectional area of the first section

Then  $R = V/I = V/150 \Omega$

Now,  $A = \frac{\rho l}{R} = \rho l \frac{I}{V} = 80,000 \times 2 \times 10^{-6} \times 150/V = 24/V \text{ cm}^2$

Now, maximum allowable drop = 5% of 440 = 22 V

$\therefore$  voltage drop along other sections =  $(22 - V)$  volt

Hence, cross-sectional area of 350-m section is

$$A_1 = 35,000 \times 2 \times 10^{-6} \times 50/(22 - V) = 3.5/(22 - V) \text{ cm}^2$$

Also, cross-sectional area of 1500-m section is

$$A_2 = 150,000 \times 2 \times 10^{-6} \times 100/(22 - V) = 30/(22 - V) \text{ cm}^2$$

Now, total weight of copper required is proportional to the total volume.

$$\therefore W = K[(800 \times 24/V) + 350 \times 3.5/(22 - V) + 1500 \times 30/(22 - V)]$$

$$= K[1.92/V + 4.62/(22 - V)] \times 10^4$$

Weight of copper required would be minimum when  $dW/dV = 0$

$$\therefore \frac{dW}{dV} = K \left[ \frac{-1.92}{V^2} + \frac{4.62}{(22-V)^2} \right] \times 10^4 = 0$$

or  $\frac{1.92}{V^2} = \frac{4.62}{(22-V)^2}$  or  $(22-V)^2 = 2.4 V^2$

or  $V = 22/2.55 = 8.63 \text{ volt}$   $\therefore A = 24/8.63 = 2.781 \text{ cm}^2$   
 $A_1 = 3.5/(22 - 8.63) = 0.2618 \text{ cm}^2$   $A_2 = 30/(22 - 8.63) = 2.246 \text{ cm}^2$

**Example 40.15.** A d.c. two-wire distributor AB is 450 m long and is fed at both ends at 250 volts. It is loaded as follows : 20A at 60 m from A, 40A at 100m from A and a uniform loading of 1.5 A/m from 200 to 450 m from A. The resistance of each conductor is 0.05 Ω/km. Find the point of minimum potential and its potential. **(Electrical Power-II, Bangalore Univ. 1993)**

**Solution.** In Fig. 40.27, let D be the point of minimum potential and let *i* be the current in distributor section CD. Then, current supplied to load D from end B is (40 - *i*). If *r* is the resistance of the distributor/metre (both go and return), then

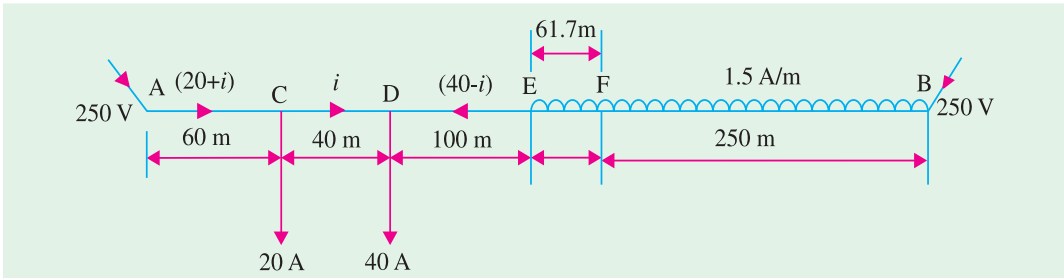


Fig. 40.27

$$\begin{aligned} \text{Drop over AD} &= (20 + i) \times 60r + i \times 40r \\ \text{Drop over BD} &= \text{drop due to concentrated load} + \text{drop due to distributed load} \\ &= (40 - i) \times 350r + 1.5 \times r \times 250^2/2 \quad \text{—Art. 40.6} \end{aligned}$$

Since the two feeding points A and B are at the same potential of 250 V, the two drops must be equal.

$$\therefore (20 + i) \times 60r + i \times 40r = (40 - i) \times 350r + 1.5 \times r \times 250^2/2 \quad \therefore i = 132.6 \text{ A}$$

Since (40 - *i*) comes out to be negative, it means that D is not the point of minimum potential. The required point is somewhat nearer the other end B. Let it be F. Obviously, current in section DF = 132.6 - 40 = 92.6 A. Hence, distance of minimum potential point F from end A is

$$= 60 + 40 + 100 + 92.6/1.5 = 261.7 \text{ m}$$

$$\begin{aligned} \text{Total voltage drop over section AF is} \\ &= 2 \times 0.05 \times 10^{-3} (152.6 \times 60 + 132.6 \times 40 + 92.6 \times 100 + 92.6 + 61.7/4) = 2.65 \text{ V} \\ \therefore \text{potential of point F} &= 250 - 2.65 = 247.35 \text{ V.} \end{aligned}$$

**Example 40.16.** A two-wire d.c. distributor AB, 1000 metres long, is supplied from both ends, 240 V at A and 242 V at B. There is a concentrated load of 200 A at a distance of 400 metre from A and a uniformly distributed load of 1.0 A/m between the mid-point and end B. Determine (i) the currents fed at A and B (ii) the point of minimum potential and (iii) voltage at this point. Take cable resistance as 0.005 Ω per 100 metre each core.

**Solution.** The resistance per 100 metres of both cores = 0.005 × 2 = 0.01 Ω.

Let us take 100 m as the unit of length. Let current fed at end B be *I<sub>B</sub>* as shown in Fig. 40.28.



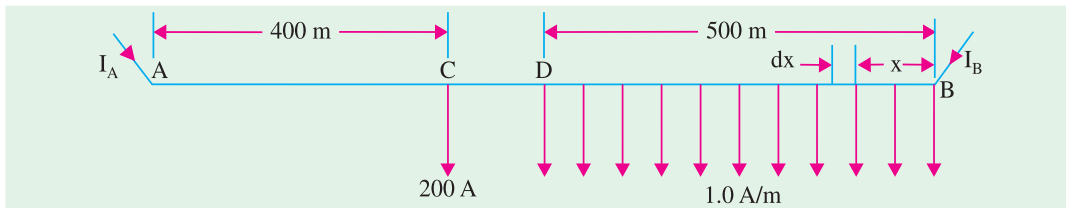


Fig. 40.28

Consider any element  $dx$  anywhere within distance  $BD$  i.e. distance over which uniformly distributed load is applied. Suppose it is at a distance of  $x$  units (of 100 metre each). The current through  $dx$  is  $I = (I_B - 100 \times 1.0 \times x) = (I_B - 100x)$

$$\therefore \text{voltage drop over } dx = (I_B - 100x) \times 0.01 \times dx$$

Voltage drop over distance  $BD$

$$\begin{aligned} &= \int_0^5 (I_B - 100x) \times 0.01 \times dx = 0.01 \int_0^5 (I_B - 100x) dx \\ &= 0.01 \left[ I_B x - 500x^2 \right]_0^5 = (0.05 I_B - 12.5) \text{ V} \end{aligned}$$

Voltage drop over  $DC = (I_B - 500) \times 0.01$ ; Voltage drop over  $CA = (I_B - 700) \times 0.01 \times 4$

Since total drop over  $AB = 242 - 240 = 2$  volt

$$\therefore 0.05 I_B - 12.5 + 0.01 I_B - 5 + 0.04 I_B - 28 = 2 \quad \therefore I_B = 455 \text{ A}$$

Now, total current is  $= 500 + 200 = 700$  A

$$\therefore I_A = 700 - 455 = 245 \text{ A}$$

It is obvious that  $(245 - 200) = 45$  A is fed into the distributed load at  $D$ . Hence, point of minimum potential  $M$  will be  $45/1.0 = 45$  metres from  $D$ . Its distance from  $B = 500 - 45 = 455$  metres or 4.55 units of length.

$$\text{Voltage drop upto this point from end } B = \int_0^{4.55} (455 - 100x) \times 0.01 \times dx = 10.35 \text{ V}$$

$$\therefore \text{potential of } M = 242 - 10.35 = 231.65 \text{ V}$$

### Tutorial Problem No. 40.1

1. A 2-wire direct-current distributor  $PQ$  is 500 metres long. It is supplied by three feeders entering at  $P$ ,  $Q$  and  $R$  where  $R$  is midway between  $P$  and  $Q$ . The resistance of the distributor, go and return, is  $0.05 \Omega$  per 100 metres. The distributor is loaded as follows :

Point	A	B
Distance from $P$ (metres)	100	350
Load (ampere)	30	40

In addition, a distributed load of  $0.5$  A/metre exists from  $P$  to  $Q$ . Calculate (a) the current supplied by each feeder if  $P$  and  $R$  are maintained at  $220$  V while  $Q$  is at  $215$  V (b) the voltage at a point between  $P$  and  $Q$  and  $50$  metres from  $P$ .

**[(a) current at P = 18 A ; at Q = 28.5 A ; at R = 98.5 A (b) voltage at stated point = 214.8 V]**

2. A section of a 2-wire distributor network is 1,200 metres long and carries a uniformly distributed load of  $0.5$  A/ metre. The section is supplied at each end by a feeder from a distribution centre at which the voltage is maintained constant. One feeder is 900 and the other 600 metres long and each has a cross-sectional area 50% greater than that of the distributor. Find the current in each feeder cable and the distance from one end of the distributor at which p.d. is a minimum. (*London Univ.*)

**[Current in 900 metres: 272.7 A; current in 600 metres: 327.3 A; 545.5 metres from the 900 metre feeder point]**

3. A pair of distributing mains of uniform cross-section 1,000 metres in length having a resistance of  $0.15 \Omega$  each, are loaded with currents of 50, 100, 57.5, 10 and 75 A at distances measured from one end where the voltage between mains is 211.6, of 100, 300, 540, 740 and 850 metres respectively. If the voltage of the other end is maintained at 210, calculate the total current entering the system at each end of the mains and the position of the point of minimum potential. **(I.E.E. London)**  
**[160.63 A and 131.87 A ; 540 metre load]**
4. A 2-core distribution cable  $AB$ , 400 metre long, supplies a uniformly distributed lighting load of 1 A/m. There are concentrated loads of 120, 72, 48 and 120 A at 40, 120, 200 and 320 metres respectively from end  $A$ . This cable has a resistance of  $0.15 \Omega$  per 1,000 m run. Calculate the voltage and the position of the lowest run lamp when the cable is fed at 250 Volts (a) from both ends  $A$  and  $B$  (b) from end  $A$  only. **[(a) 239.1 volt at 200 m (b) 207.6 volt at B]**
5. A 2-wire d.c. distributor  $AB$ , 300 m long, is fed at both ends  $A$  and  $B$ . The distributor supplies a uniformly distributed load of 0.25 A per m together with concentrated loads of 40 A at  $C$  and 60 A at  $D$ ,  $AC$  and  $BD$  being 120 m each.  $A$  and  $B$  are maintained at 300 V, the loop resistance of the distributor is  $0.1 \text{ ohm}/100 \text{ m}$ . Determine the current fed at  $A$  and  $B$  and also the potential of points  $C$  and  $D$ . **[85.5 A, 89.5 A, 291.54 V, 291.06 V]**

### 40.11. Ring Distributor

A ring distributor is a distributor which is arranged to form a closed circuit and which is fed at one or more than one points. For the purpose of calculating voltage distribution, it can be looked upon as consisting of a series of open distributors fed at both ends. By using a ring distributor fed properly, great economy in copper can be affected.

If the ring distributor is fed at one point then, for the purposes of calculation, it is equivalent to a straight distributor fed at both ends with equal voltages (Ex. 40.17).

**Example 40.17.** A 400-metre ring distributor has loads as shown in Fig. 40.29 (a) where distances are in metres. The resistance of each conductor is  $0.2 \Omega$  per 1,000 metres and the loads tapped off at points  $B$ ,  $C$  and  $D$  are as shown. If the distributor is fed at  $A$ , find voltages at  $B$ ,  $C$  and  $D$ .

**Solution.** Let us assume a current of  $I$  in section  $AD$  [Fig. 40.29 (a)] and then find the total drop which should be equated to zero.

$$\therefore 70 I + 90 (I - 50) + 80 (I - 120) + 60 (I - 220) = 0 \quad \therefore 300 I = 27,300 \text{ or } I = 91 \text{ A}$$

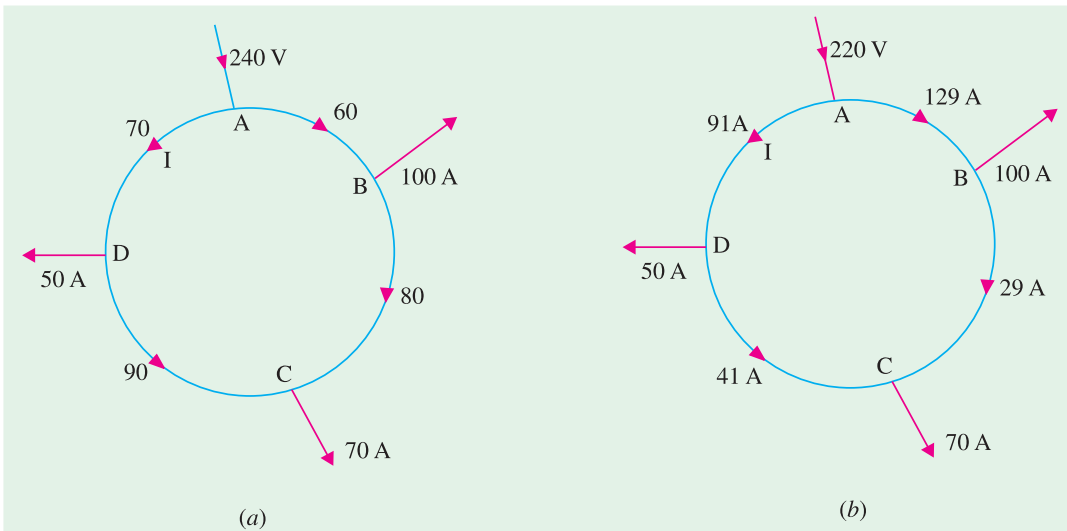


Fig. 40.29

The current distribution becomes as shown in Fig. 40.29 (b) from where it is seen that C is the point of minimum potential.

Drop in AD =  $2(91 \times 70 \times 0.2/1,000) = 2.55 \text{ V}$  ; Drop in DC =  $2(41 \times 90 \times 0.2/1,000) = 1.48 \text{ V}$   
 Drop in CB =  $2(29 \times 80 \times 0.2/1,000) = 0.93 \text{ V}$  ; Drop in BA =  $2(129 \times 60 \times 0.2/1,000) = 3.1 \text{ V}$   
 Voltage at D =  $240 - 2.55 = 237.45 \text{ V}$  ; Voltage at C =  $237.45 - 1.48 = 235.97 \text{ V}$   
 Voltage at B =  $240 - 3.1 = 236.9 \text{ V}$

**Example 40.18.** In a direct current ring main, a voltage of 400 V is maintained at A. At B, 500 metres away from A, a load of 150 A is taken and at C, 300 metres from B, a load of 200 A is taken. The distance between A and C is 700 metres. The resistance of each conductor of the mains is 0.03 Ω per 1,000 metres. Find the voltage at B and C and also find the current in the section BC.

(Elect. Technology, Kerala Univ.)

**Solution.** Let us assume a current of I in section AB, then find the total drop round the ring main and equate it to zero. As seen from Fig. 40.30 (a).

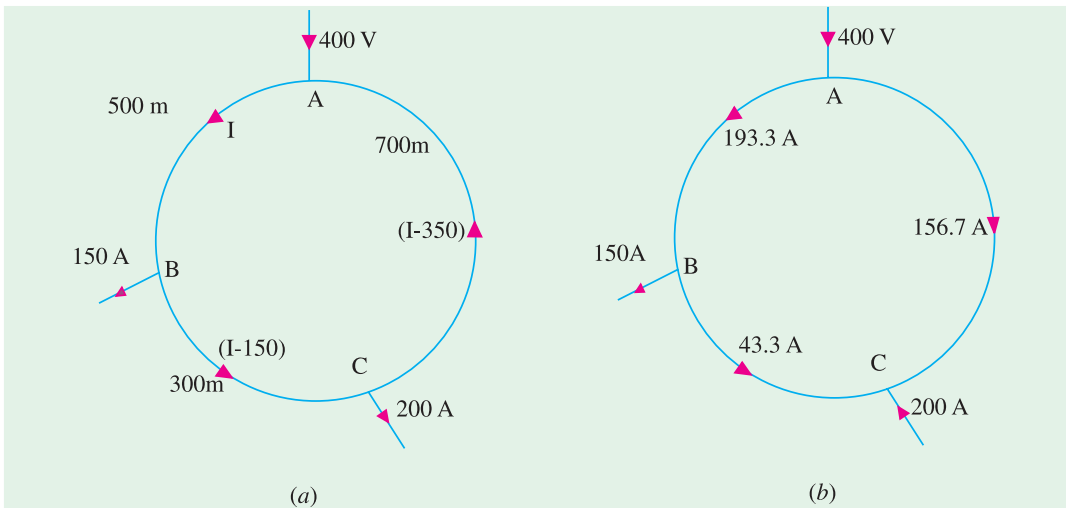


Fig. 40.30

$$500 I + 300 (I - 150) + 700 (I - 350) = 0 \quad \therefore I = 193.3 \text{ A}$$

The current distribution becomes as shown in Fig. 40.30 (b) from where it is seen that C is the point of minimum potential.

Drop over AB =  $2(193.3 \times 50 \times 0.03/1,000) = 5.8 \text{ V}$   
 Drop over BC =  $2(43.3 \times 300 \times 0.03/1,000) = 0.78 \text{ V}$   
 Voltage at B =  $400 - 5.8 = 394.2 \text{ V}$  ; Voltage at C =  $394.2 - 0.78 = 393.42 \text{ V}$

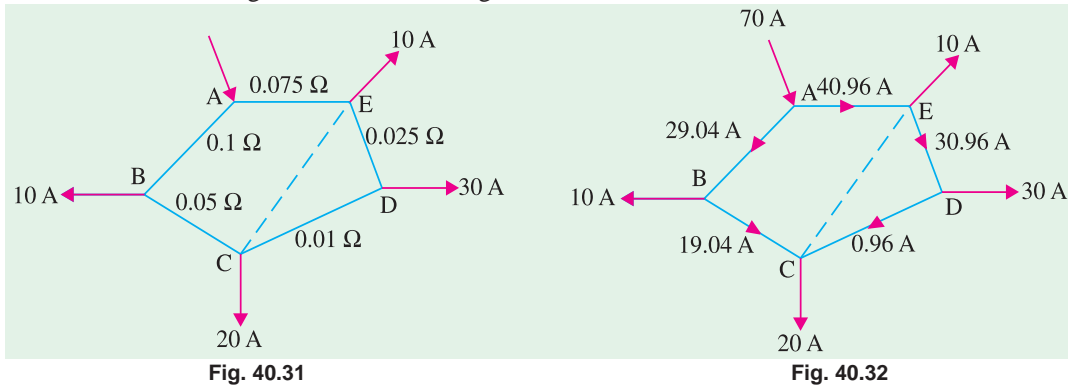
The current in section BC is as shown in Fig. 40.30 (b).

**Example 40.19.** A d.c. ring main ABCDE is fed at point A from a 220-V supply and the resistances (including both lead and return) of the various sections are as follows (in ohms) : AB = 0.1 ; BC = 0.05 ; CD = 0.01 ; DE = 0.025 and EA = 0.075. The main supplies loads of 10 A at B ; 20 A at C ; 30 A at D and 10 A at E. Find the magnitude and direction of the current flowing in each section and the voltage at each load point.

If the points C and E are further linked together by a conductor of 0.05 Ω resistance and the output currents from the mains remain unchanged, find the new distribution of the current and voltage in the network.

(London Univ.)

**Solution.** The ring main is shown in Fig. 40.31.



Let us assume a current of  $I$  amperes in section  $AB$  and put the total drop round the ring equal to zero.

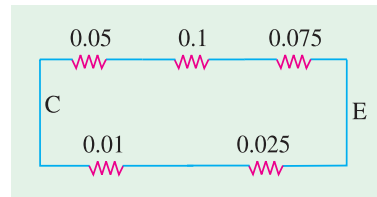
$$\therefore 0.1 I + 0.05(I - 10) + 0.01(I - 30) + 0.025(I - 60) + 0.075(I - 70) = 2 \text{ or } I = 29.04 \text{ A}$$

Current distribution now becomes as shown in Fig. 40.32.

$$\begin{aligned} \text{Drop in } AB &= 29.04 \times 0.1 = 2.9 \text{ V}; & \text{Drop in } BC &= 19.04 \times 0.05 = 0.95 \text{ V} \\ \text{Drop in } ED &= 30.96 \times 0.025 = 3.77 \text{ V}; & \text{Drop in } AE &= 40.96 \times 0.075 = 3.07 \text{ V} \end{aligned}$$

$$\therefore \begin{aligned} \text{Potential of } B &= 217.1 \text{ V}, & \text{Potential of } C &= \mathbf{216.15 \text{ V}} \\ \text{Potential of } E &= 216.93 \text{ V}, & \text{Potential of } D &= \mathbf{216.16 \text{ V}} \end{aligned}$$

The interconnector between points  $C$  and  $E$  is shown in Fig. 40.31. It may be noted here that the function of the interconnector is to reduce the drop of voltage in various sections. For finding current in the interconnector, first p.d. across its ends is calculated. Then we calculate the resistance, viewed from points  $E$  and  $C$  of the network composed resistances of the distribution lines only, ignoring the load (Art, 2-22). Then current through the interconnector is given by



$$I = \frac{\text{p.d. between points } E \text{ and } C}{\text{resistance of distribution network} + \text{interconnector}}$$

$$\text{P.D. between points } E \text{ and } C = 216.93 - 216.15 = 0.78 \text{ V}$$

To determine the resistance viewed from  $C$  and  $E$ , the network is drawn as shown in Fig. 40.33. Since the two branches are in parallel, the combined resistance is

$$= 0.225 \times 0.035 / (0.225 + 0.035) = 0.03 \Omega$$

$$\therefore \text{current in interconnector} = 0.78 / (0.03 + 0.05) = 9.75 \text{ A from } E \text{ to } C.$$

The currents in the other sections must be calculated again.

Let us assume a current  $I_1$  and  $ED$ , then since the voltage round the closed mesh  $EDC$  is zero, hence

$$-0.025 I_1 - 0.01 (I_1 - 30) + 0.05 \times 9.75 = 0 \text{ or } 0.035 I_1 = 0.7875 \therefore I_1 = \mathbf{22.5 \text{ A}}$$

$$\text{Current in } AE = 10 + 22.5 + 9.75 = \mathbf{42.25 \text{ A}}; \text{ Current in } AB = 70 - 42.25 = \mathbf{27.75 \text{ A}}$$

$$\text{Drop in } AB = 27.75 \times 0.1 = \mathbf{2.775 \text{ V}}; \text{ Drop in } BC = 17.75 \times 0.05 = \mathbf{0.888 \text{ V}}$$

$$\text{Drop in } ED = 32.25 \times 0.025 = \mathbf{0.806 \text{ V}}; \text{ Drop in } AE = 42.25 \times 0.075 = \mathbf{3.169 \text{ V}}$$

$$\text{Potential of } B = 220 - 2.775 = \mathbf{217.225 \text{ V}}; \text{ Potential of } C = 217.225 - 0.888 = \mathbf{216.337 \text{ V}}$$

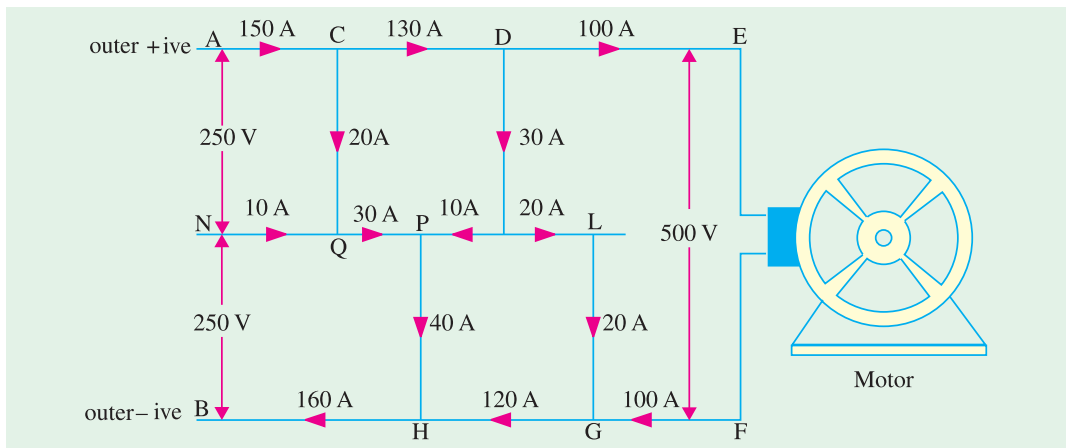
$$\text{Potential of } E = 220 - 3.169 = \mathbf{216.83 \text{ V}}; \text{ Potential of } D = 216.83 - 0.806 = \mathbf{216.024 \text{ V}}$$

**Tutorial Problem No. 40.2**

- Four power loads  $B, C, D$  and  $E$  are connected in this order to a 2-core distributor cable, arranged as a ring main, and take currents of 20, 30, 25 and 30 A respectively. The ring is supplied from a substation at the point  $A$  between  $B$  and  $E$ . An interconnector cable joins the points  $C$  and  $E$  and from a point  $F$  on this inter-connector cable a current of 20 A is taken. The total resistance of the cable between the load points is :  $AB = 0.04 \Omega$  ;  $BC = 0.03 \Omega$  ;  $CD = 0.02 \Omega$  ;  $DE = 0.03 \Omega$  ;  $EA = 0.04 \Omega$  ;  $CF = 0.02 \Omega$  and  $EF = 0.01 \Omega$ . Calculate the current in each section of the ring and the interconnector. (London Univ.)  
[AB = 53.94 A ; BC = 33.94 A ; CD = 8.35 A ; ED = 16.65 A ;  
AE = 71.06 A ; FC = 4.42 A ; EF = 24.42 A]
- A 2-core ring feeder cable  $ABCDEA$  is connected to a sub-station at  $A$  and supplies feeding points to a distribution network at  $B, C, D$  and  $E$ . The points  $C$  and  $E$  are connected by an inter-connector  $CFE$  and a load is taken at  $F$ . The total resistance in ohms of both conductors in the several sections is  $AB = 0.05$  ;  $BC = 0.4$  ;  $CD = 0.03$  ;  $DE = 0.04$  ;  $EA = 0.05$  ;  $CF = 0.02$  ;  $FE = 0.1$ . The currents taken at the load points are  $B = 12$  A ;  $C = 15$  A ;  $D = 12$  A ;  $E = 15$  A and  $F = 10$  A. Calculate the current in each section of the cable and the p.d. at each load point, if the p.d. at  $A$  is maintained constant at 250 V. (City & Guilds, London)  
[Currents : AB = 27.7 A ; FC = 3.3 A ; P.D. s at B = 248.6 V ; C = 248 V ;  
D = 247.87 V ; E = 248.18 V ; F = 248 V]
- A distributor cable in the form of a ring main  $ABCDEA$ , supplies loads of 20, 60, 30, and 40 A taken at the points  $B, C, D$  and  $E$  respectively, the feeding point being at  $A$ . The resistances of the sections are  $AB = 0.1 \Omega$ ,  $BC = 0.15 \Omega$ ,  $CD = 0.1 \Omega$ ,  $DE = 0.05 \Omega$  and  $EA = 0.1 \Omega$ . The points  $E$  and  $C$  are connected by a cable having a resistance of  $0.3 \Omega$ . Calculate the current in each section of the network. (City & Guilds, London)  
[A to B : 60 A ; B to C : 40 A ; E to C : 10 A ; D to C : 10 A ; E to D : 40 A ; A to E : 90 A]

**40.12. Current Loading and Load-point Voltages in a 3-wire System**

Consider a 3-wire, 500/250-V distributor shown in Fig. 40.34. The motor requiring 500 V is connected across the outers whereas other loads requiring lower voltage of 250 V are connected on both sides of the neutral.



**Fig. 40.34**

The current in positive outer  $AE$  flows from the left to right and in negative outer  $FB$  from right to left. The current through the various sections of the neutral wire  $NL$  may flow in either direction depending on the load currents on the positive side and negative side but is independent of the loads connected between outers.

Since 150 A enters the +ve outer but 160 A comes out of the network, it means that a current of 10 A must flow into the neutral at point *N*. Once the direction and magnitude of current in *NQ* is known, the directions and magnitudes of currents in other sections of the neutral can be found very easily. Since *PH* takes 40 A, currents meeting at *P* should add up to 40 A. As seen, 20 A of *CQ* and 10A of *NQ* flow towards *P*, the balance of 10 A flows from point *M*. Then, 30 A current of *DM* is divided into two parts, 10 A flowing along *MP* and the other 20 A flowing along *ML* to feed the load *LG*.

Knowing the values of currents in the various conductors, voltage drops can be calculated provided resistances are known. After that, voltages at different load points can be calculated as illustrated in Ex. 40.20.

**Example 40.20.** In a 3-wire distribution system, the supply voltage is 250 V on each side. The load on one side is a 3 ohm resistance and on the other, a 4 ohm resistance. The resistance of each of the 3 conductors is 0.05 Ω. Find the load voltages.

(Elements of Elect. Engg-I, Bangalore Univ.)

**Solution.** Let the assumed directions of unknown currents in the three conductors be as shown in Fig. 40.35. Applying KVL (Art. 2.2) to closed circuit *ABCD*A, we have

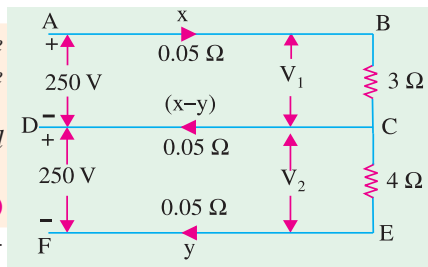


Fig. 40.35

$$-3.05x - 0.05(x - y) + 250 = 0$$

$$\text{or } 310x - 5y = -25,000 \dots (i)$$

Similarly, circuit *DCEFD* yields

$$0.05(x - y) - 4.05y + 250 = 0$$

$$\text{or } 5x - 410y = -25,000 \dots (ii)$$

From (i) and (ii), we get  $x = 81.64 \text{ A}$ ,  $y = 61.97 \text{ A}$ . Since both currents come out to be positive, it means that their assumed directions of flow are correct.

$$\therefore V_1 = 250 - 0.05 \times 81.64 - 0.05(81.64 - 61.97) = 244.9 \text{ V}$$

$$V_2 = 250 + 0.05(81.64 - 61.97) - 0.05 \times 61.97 = 247.9 \text{ V}$$

**Example 40.21.** A 3-wire d.c. distributor *PQ*, 250 metres long, is supplied at end *P* at 500/250 V and is loaded as under :

Positive side: 20 A 150 metres from *P* ; 30 A 250 metres from *P*.

Negative side: 24 A 100 metres from *P* ; 36 A 220 metres from *P*.

The resistance of each outer wire is 0.02 Ω per 100 metres and the cross-section of the middle wire is one-half that of the outer. Find the voltage across each load point.

**Solution.** The current loading is shown in Fig. 40.36.

The current flowing into the positive side is  $30 + 20 = 50 \text{ A}$ . Since current flowing out of the negative side is  $36 + 24 = 60 \text{ A}$ , it means that 10 A must be flowing in the section *NC* of the neutral. To make up 24 A in load *CD*. 10 A are contributed by *NC* and 14 A by *BC*. The balance of 6 A flows through *BF* and adds up with 30 A of *KF* to make up 36A through load *FE*.

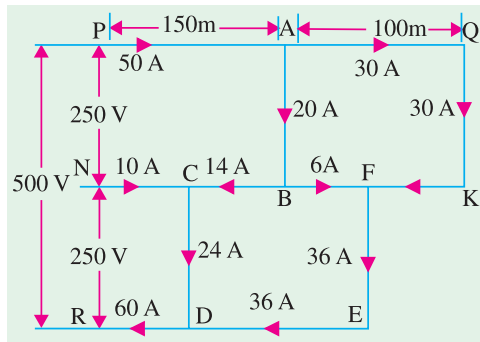


Fig. 40.36

It is given that the resistance of outers is 0.02 Ω per 100 m. Since neutral is of half the cross-section, its resistance is 0.04 Ω per 100 m. Knowing them, voltage at load points can be determined as under. Let us see how we will calculate voltage across load *AB*.

Voltage at  $AB = 250 - \text{drop in } PA - \text{drop in } BC + \text{drop in } CN$ .

It should be particularly noted that drop in  $CN$  has been added instead of being *subtracted*. The reason is this : as we start from  $P$  and go round  $PABCN$ , we go along the current over section  $PA$  *i.e.* we go 'downstream' hence drop is taken negative, but along  $CN$  we go 'upstream', hence the drop is taken as positive\*. Proceeding in this way, we tabulate the currents, resistances and voltage drops in various sections as given below :

Section	Resistance ( $\Omega$ )	Current (A)	Drop (V)
$PA$	0.03	50	1.5
$AQ$	0.02	30	0.6
$KF$	0.012	30	0.36
$BC$	0.02	14	0.28
$BF$	0.028	6	0.168
$NC$	0.04	10	0.4
$ED$	0.024	36	0.864
$DR$	0.02	60	1.2

$$\text{P.D. across } AB = 250 - 1.5 - 0.28 + 0.4 = \mathbf{248.62 \text{ V}}$$

$$\text{P.D. across } QK = 248.62 - 0.6 - 0.36 + 0.168 = \mathbf{247.83 \text{ V}}$$

$$\text{P.D. across } CD = 250 - 0.4 - 1.2 = \mathbf{248.4 \text{ V}}$$

$$\text{P.D. across } FE = 248.4 + 0.28 - 0.168 - 0.864 = \mathbf{247.65 \text{ V}}$$

### Tutorial Problem No. 40.3

1. A 3-wire system supplies three loads (a) 10 (b) 20 and (c) 30 amperes situated at distances of 100, 150 and 300 metres respectively from the supply point on the positive side of the neutral wire. Connected between negative and neutral wires are two loads (d) 30 A and (e) 20A, situated 120 and 200 metres respectively from the supply point. Give a diagram showing the values and directions of the currents in various parts of the neutral wire.

If the resistance of the outers is  $0.05 \Omega$  per 1000 metres and that of the neutral  $0.1 \Omega$  per 1000 metres, calculate the potential difference at the load points (b) and (e); the pressure at the supply point being 100 V between outers and neutral. **[10A and 5A; Volts at (b) = 99.385 V ; at (e) = 99.86 V]**

2. A 3-wire d.c. distributor 400 metres long is fed at both ends at 235 volts between each outer and neutral. Two loads  $P$  and  $Q$  are connected between the positive and neutral and two loads  $R$  and  $S$  are connected between the negative outer and the neutral. The loads and their distances from one end ( $X$ ) of the distributor are as follows : Load  $P$ , 50 A, 100 metres from  $X$  ; Load  $Q$ , 70 A, 300 metres from  $X$  ; Load  $R$ , 60 A, 150 metres from  $X$  ; Load  $S$ , 60 A, 350 metres from  $X$ . Determine the p.d. at each load point and the current at each feeding point. The resistance of each outer is  $0.25 \Omega$  per 1000 metres and that of the neutral is  $0.5 \Omega$  per 1000 metres.

**[Current into the +ve outer at  $X = 55 \text{ A}$  ; other end  $Y = 65 \text{ A}$  ; Out from -ve outer at  $X = 45 \text{ A}$  ; other end  $Y = 75 \text{ A}$  ; P.D. s at  $P = 233.125 \text{ V}$  ;  $Q = 232.375 \text{ V}$  ;  $R = 232.812 \text{ V}$  ;  $S = 233.812 \text{ V}$ ]**

### 40.13. Three-wire System

As already mentioned briefly in Art. 40.2, it consists of the 'outer' conductors (between which the voltage is twice the normal value for lighting) and the third wire which is called the middle or neutral wire. It is of half the cross-section as compared to any one of the two outers and is earthed at the generator end. The voltage of the neutral is thus approximately half way between that of the outers.

\* Hence, sign convention in this : While going 'upstream' take the drop as positive and while going 'downstream' take the drop as negative. (Art. 2-3).

If the total voltage between outers is 460 V, then the positive outer is 230 V *above* the neutral and the negative outer 230 V *below* the neutral. Motor loads requiring higher voltage are connected across the outers directly whereas lighting and heating loads requiring lesser voltage of 230 V are connected between any one of the outers and the neutral. If the loads on both sides of the neutral are equal *i.e.* balanced as shown in Fig. 40.37, then there is no current in the middle wire and the effect is as if the different loads were connected in series across the outers. However, in practice, although effort is made to distribute the various loads equally on the two sides of the neutral, yet it is difficult to achieve exact balance, with the result that some out-of-balance current does flow in the neutral as shown in Fig. 40.38. In the present case as viewed from generator end, the load is equivalent to a resistance of  $25/3$  ohms in series with a resistance of  $25/2$  ohms (Fig. 40.39). Obviously, the voltages across the two will become unequal. The voltage across the positive side will fall to  $\frac{500 \times 25/2}{(25/2 + 25/3)} = 200$  V and on the negative side, it will rise to  $\frac{500 \times 25/3}{(25/2 + 25/3)} = 300$  V. This difficulty of unequal voltages with unbalanced loads is removed by using balancers as discussed below in Art. 40.14.

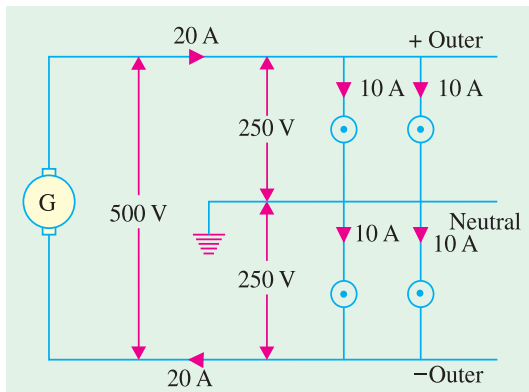


Fig. 40.37

neutral, yet it is difficult to achieve exact balance, with the result that some out-of-balance current does flow in the neutral as shown in Fig. 40.38. In the present case as viewed from generator end, the load is equivalent to a resistance of  $25/3$  ohms in series with a resistance of  $25/2$  ohms (Fig. 40.39). Obviously, the voltages across the two will become unequal. The voltage across the positive side will fall to  $\frac{500 \times 25/2}{(25/2 + 25/3)} = 200$  V and on the negative side, it will rise to  $\frac{500 \times 25/3}{(25/2 + 25/3)} = 300$  V. This difficulty of unequal voltages with unbalanced loads is removed by using balancers as discussed below in Art. 40.14.

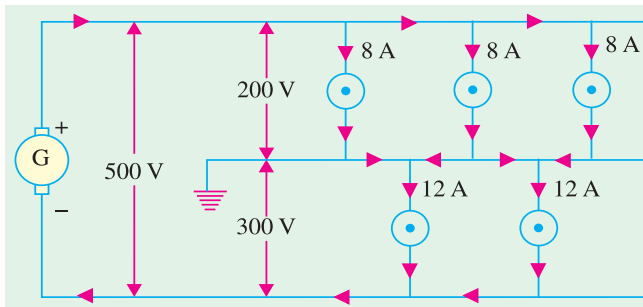


Fig. 40.38

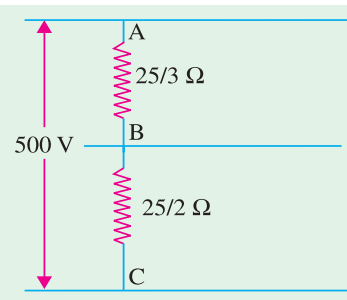


Fig. 40.39

### 40.14. Balancers

In order to maintain p.d.s on the two sides of neutral equal to each other, a balancer set is used. The commonest form of balancer consists of two identical shunt-wound machines which are not only coupled mechanically but have their armatures and field circuits joined in series across the outers. The neutral is connected to the junction of the armatures as shown. When the system is unloaded or when the loads on the two sides are balanced, then

1. both machines run as unloaded motors and
2. since their speeds and field currents are equal, their back e.m.f.s. are the same.

When the two sides are unbalanced *i.e.* when the load supplied by + ve outer is different from that

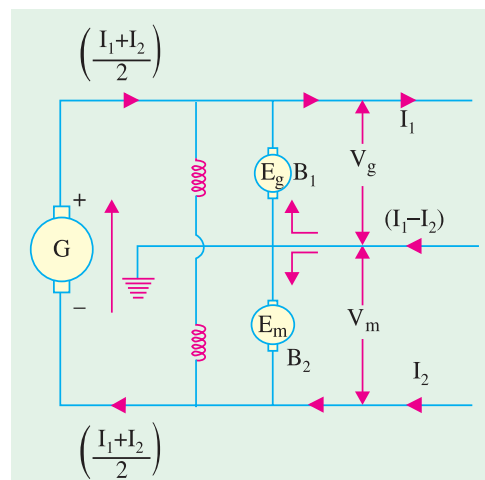


Fig. 40.40



supplied by negative outer, then out-of-balance current  $(I_1 - I_2)$  flows through the mid-wire to the balancers where it divides into two halves each equal to  $(I_1 - I_2)/2$  as shown in Fig. 40.40.

If  $I_1$  is greater than  $I_2$ , then +ve side is more heavily loaded (than -ve side) hence the p.d. on this side tends to fall below the e.m.f. of the balancer on this side so that  $B_1$  runs as a generator. However, the p.d. on the lightly-loaded side rises above the e.m.f. of the balancers on that side, hence  $B_2$  runs as a motor. In this way, energy is transferred through balancers from the lightly-loaded side to the heavily-loaded side. In Fig. 40.40, machine  $B_2$  is running as a motor and driving machine  $B_1$  as a generator.

Let

- $R_a$  = armature resistance of each machine
- $V_g$  = terminal p.d. of machine running as a generator *i.e.*  $B_1$
- $E_g$  = induced e.m.f. of  $B_1$
- $V_m$  = terminal p.d. of motoring machine *i.e.*  $B_2$
- $E_m$  = induced e.m.f. of  $B_2$

then

$$V_g = E_g - \frac{(I_1 - I_2)}{2} R_a \quad \text{and} \quad V_m = E_m + \frac{(I_1 - I_2)}{2} R_a$$

$$\therefore V_m - V_g = (E_m - E_g) + (I_1 - I_2) R_a \quad \dots(i)$$

Since the speed and excitation of the two machines are equal,

$$\therefore E_g = E_m \quad \therefore V_m - V_g = (I_1 - I_2) R_a \quad \dots(ii)$$

Hence, we find that the difference of voltages between the two sides of the system is proportional to —

- (i) out of balance current  $(I_1 - I_2)$  and (ii) armature resistance of the balancer.

For this reason,  $R_a$  is kept very small and effort is made to arrange the loads on the two sides such that out-of-balance current is as small as possible.

The value  $(V_m - V_g)$  can be still further reduced *i.e.* the voltages on the two sides can be more closely balanced by cross-connecting the balancer fields as shown in Fig. 40.41. In this way, generator draws its excitation from the lightly-loaded side which is at a higher voltage, hence  $E_g$  is increased. The motoring machine draws its excitation from the heavily loaded side which is at a little lower voltage, hence  $E_m$  is decreased. In this way, the difference  $(E_m - E_g)$  is decreased and so is  $(V_m - V_g)$ . Further, regulation of the voltage can be accomplished by connecting an adjustable regulator in series with the two balancer fields as shown in Fig. 40.42.

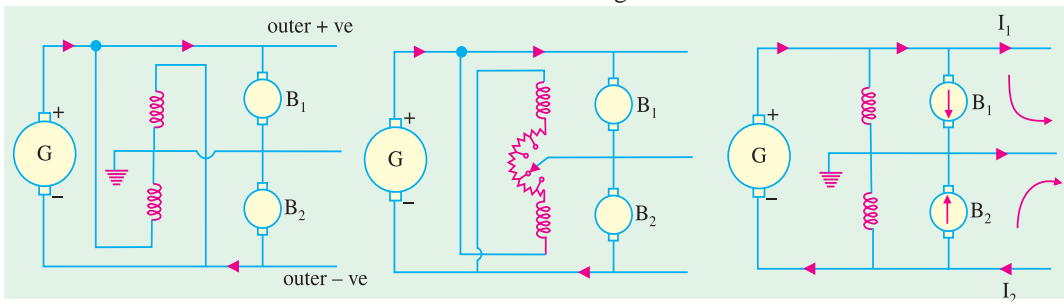


Fig. 40.41

Fig. 40.42

Fig. 40.43

It should be noted that since machine  $B_1$  is running as a generator and  $B_2$  as a motor, the directions of currents in  $B_1$ ,  $B_2$  and the neutral are as shown in Fig. 40.40. If, however,  $B_2$  runs as a generator and  $B_1$  as a motor *i.e.* if negative side is more heavily loaded than the +ve side, then directions of currents through  $B_1$ ,  $B_2$  and the neutral are as shown in Fig. 40.43. In particular, the change in the direction of the current through midwire should be noted.

**Example 40.22.** A d.c. 3-wire system with 500-V between outers has lighting load of 100 kW on the positive and 50 kW on the negative side. If, at this loading, the balancer machines have each a loss of 2.5 kW, calculate the kW loading of each balancer machine and the total load on the system.

**Solution.** The connections are shown in Fig. 40.44.

Total load on main generator =  $100 + 50 + (2 \times 2.5)$   
 = **155 kW**  
 Output current of main generator =  $155 \times 1000/500$   
 = 310 A  
 Load current on +ve side,  $I_1 = 100 \times 1000/250$   
 = 400 A  
 Load current on -ve side,  $I_2 = 50 \times 1000/250$   
 = 200 A

Out-of-balance current =  $400 - 200 = 200$  A  
 Since +ve side is more heavily loaded,  $B_1$  is working as a generator and  $B_2$  as motor.

$\therefore$  current of  $B_1 = 400 - 310 = 90$  A  
 current of  $B_2 = 200 - 90 = 110$  A  
 Loading of  $B_1 = 250 \times 90/1000 = 22.5$  kW  
 Loading of  $B_2 = 250 \times 110/1000 = 27.5$  kW

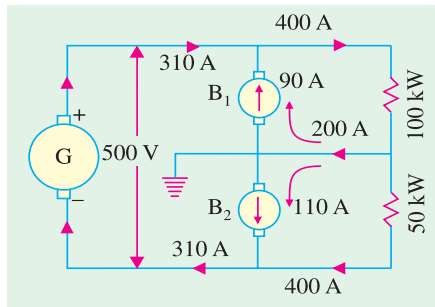


Fig. 40.44

**Example 40.23.** In a 500/250-V d.c. 3-wire system, there is a current of 2000 A on the +ve side, 1600 A on the negative side and a load of 300 kW across the outers. The loss in each balancer set is 8 kW. Calculate the current in each armature of the balancer set and total load on the main generator.

**Solution.** Connections are shown in Fig. 40.45. It should be noted that loading across 'outers' directly in no way determines the current in the neutral.

+ve loading =  $2000 \times 250/1000$   
 = 500 kW  
 -ve loading =  $1600 \times 250/1000$   
 = 400 kW

Total loading on main generator is  
 =  $500 + 400 + 300 + (2 \times 8) = 1216$  kW

$\therefore$  Current of main generator =  $1216 \times 1000/500 = 2,432$  A

Out-of-balance current =  $2000 - 1600 = 400$  A

Current through  $B_1 = 2,600 - 2,432 = 168$  A

Current through  $B_2 = 400 - 168 = 232$  A

**Example 40.24.** On a 3-wire d.c. distribution system with 500 V between outers, there is a load of 1500 kW on the positive side and 2,000 kW on the negative side. Calculate the current in the neutral and in each of the balancer armatures and the total current supplied by the generator. Neglect losses.

(Electrical Engineering ; Madras Univ.)

**Solution.** Since negative side is more heavily loaded than the positive side, machine  $B_2$  runs as a generator and  $B_1$  as a motor. The directions of current through  $B_1$  and  $B_2$  are as shown in Fig. 40.46. Total loading on the main generator

=  $2,000 + 1,500 = 3,500$  kW

Current supplied by main generator =  $3,500 \times 1000/500 = 7,000$  A

Current on +ve side =  $1500 \times 1000/250 = 6,000$  A

Current on -ve side =  $2000 \times 1000/250 = 8,000$  A

Out-of-balance current =  $8,000 - 6000 = 2,000$  A

Current through the armature of each machine = **1000 A**

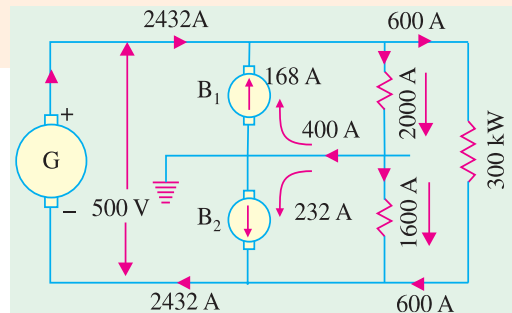


Fig. 40.45

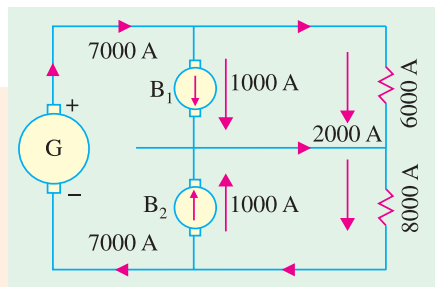


Fig. 40.46

**Example 40.25.** A 125/250 V, 3-wire distributor has an out-of-balance current of 50 A and larger load of 500 A. The balancer set has a loss of 375 W in each machine. Calculate the current in each of the balancer machines and output of main generator.

(Electrical Technology-II, Gwalior Univ.)

**Solution.** As shown in Fig. 40.47, let larger load current be  $I_1 = 500$  A. Since  $(I_1 - I_2) = 50$   
 $\therefore I_2 = 450$  A  
 Larger load =  $500 \times 125/1000 = 62.5$  kW  
 Smaller load =  $450 \times 125/1000 = 56.25$  kW  
 Balancer loss =  $2 \times 375 = 0.75$  kW  
 Output of main generator  
 =  $62.5 + 56.25 + 0.75 = 119.5$  kW  
 Current of main generator  
 =  $119.5 \times 1000/250 = 478$  A

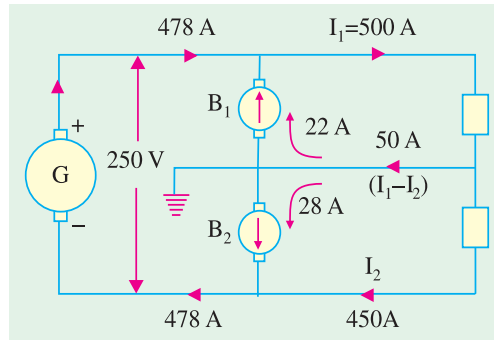


Fig. 40.47

As seen from Fig. 40.47, current of

$$B_1 = (500 - 478) = 22 \text{ A}$$

and current of  $B_2 = (50 - 22) = 28 \text{ A}$

**Example 40.26.** The load on a d.c. 3-wire system with 500 V between outers consists of lighting current of 1500 A on the positive side and 1300 A on the negative side while motors connected across the outers absorb 500 kW. Assuming that at this loading, the balancer machines have each a loss of 5 kW, calculate the load on the main generator and on each of the balancer machines.

(Electrical Engineering ; Madras Univ.)

**Solution.** Connections are shown in Fig. 40.48.  
 Positive loading =  $1500 \times 250/1000 = 375$  kW  
 Negative loading =  $1300 \times 250/1000 = 325$  kW  
 Total load on the main generator is  
 =  $375 + 325 + 500 + (2 \times 5) = 1210$  kW  
 Current supplied by the main generator is  
 =  $1210 \times 1000/500 = 2,420$  A

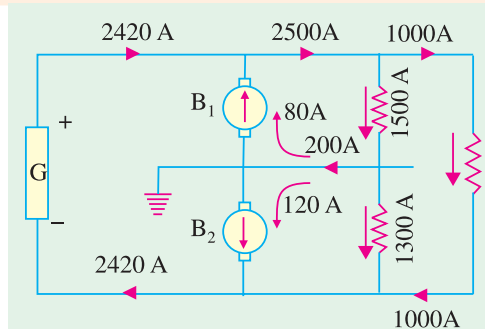


Fig. 40.48

Out-of-balance current =  $1500 - 1300 = 200$  A  
 Current through  $B_1 = 2500 - 2420 = 80$  A  
 Current through  $B_2 = 200 - 80 = 120$  A

Loading of  $B_1 = 80 \times 250/1000 = 20$  kW

Loading of  $B_2 = 120 \times 250/1000 = 30$  kW

**Example 40.27.** A d.c. 3-wire system with 480 V across outers supplies 1200 A on the positive and 1000 A on the negative side. The balancer machines have each an armature resistance of 0.1W and take 10 A on no-load. Find

- (a) the voltage across each balancer and
- (b) the total load on the main generator and the current loading of each balancer machine.

The balancer field windings are in series across the outers.

**Solution.** As shown in Fig. 40.49,  $B_1$  is generating and  $B_2$  is motoring.

The out-of-balance current is  $(1200 - 1000) = 200$  A. Let current through the motoring machine be  $I_m$ , then that through the generating machine is  $(200 - I_m)$ . Let  $V_g$  and  $V_m$  be p.d.s. of the two machines.

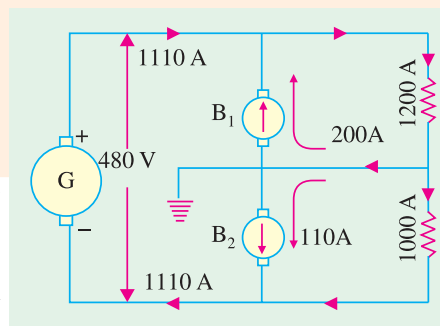


Fig. 40.49

Since  $B_2$  is driving  $B_1$ , output of the motor supplies the losses in the set plus the output of the generator. Total losses in the set

$$\begin{aligned}
 &= \text{no-load losses} + \text{Cu losses in two machines} \\
 &= 480 \times 10 + 0.1 I_m^2 + (200 - I_m)^2 \times 0.1 \\
 V_m I_m &= V_g (200 - I_m) + \text{total losses} \\
 &= V_g (200 - I_m) + 4800 + 0.1 I_m^2 + (200 - I_m)^2 \times 0.1 \\
 \text{Now } V_m &= E_b + I_m R_a \quad \text{and} \quad V_g = E_b - I_g R_a \\
 \text{Now, back e.m.f. } E_b &= (240 - 0.1 \times 10) = 239 \text{ V} \\
 \therefore V_m &= (239 + 0.1 I_m) \quad \text{and} \quad V_g = 239 - (200 - I_m) \times 0.1 \\
 \therefore (239 + 0.1 I_m) I_m &= [239 - (200 - I_m) \times 0.1] (200 - I_m)^2 + 4800 + 0.1 I_m^2 \\
 &\quad + (200 - I_m)^2 \times 0.1 \\
 \therefore I_m &= 110 \text{ A} \quad \text{and} \quad I_g = 200 - 110 = 90 \text{ A} \\
 (a) \therefore V_m &= 239 + 0.1 \times 110 = \mathbf{250 \text{ V}} \quad \text{and} \quad V_g = 239 - (0.1 \times 90) = \mathbf{230 \text{ V}} \\
 (b) \text{ Load on main generator} &= 1200 - 90 = \mathbf{1110 \text{ A}}
 \end{aligned}$$

**Example 40.28.** A d.c. 3-wire system with 460 V between outers supplies 250 kW on the positive and 400 kW on the negative side, the voltages being balanced. Calculate the voltage on the positive and negative sides respectively, if the neutral wire becomes disconnected from the balancer set.

(Electrical Power-III, Bangalore Univ.)

**Solution.** Before the disconnection of the mid-wire, voltages on both sides of the system are equal *i.e.* 230 V. The loads are 250 kW on the +ve side and 400 kW on the -ve side. If  $R_1$  and  $R_2$  are the resistances of the two loads, then

$$230^2/R_1 = 250,000 ; \quad R_1 = 0.2116 \Omega$$

Similarly,  $R_2 = 230^2/400,000 = 0.1322 \Omega$

When the mid-wire is disconnected from the balancer set *i.e.* from the generator side, then the two resistances  $R_1$  and  $R_2$  are put in series across 460 V as shown in Fig. 40.50 (b).

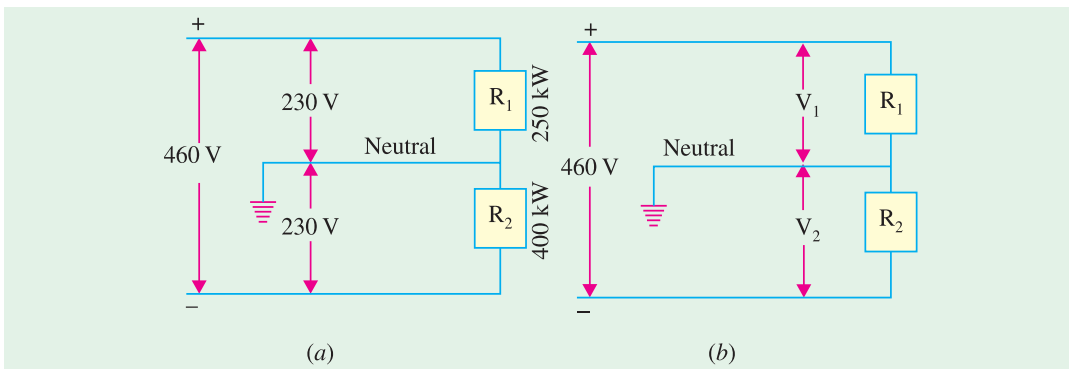


Fig. 40.50

$$\begin{aligned}
 \therefore V_1 &= \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.02116 \times 460}{(0.2116 + 0.1322)} = \mathbf{283 \text{ V}} \\
 V_2 &= \frac{R_2}{R_1 + R_2} \times 460 = \frac{0.1322 \times 460}{0.3438} = \mathbf{177 \text{ V}} \quad (\text{or } V_2 = 460 - V_1)
 \end{aligned}$$

**Tutorial Problem No. 40.4**

1. In a 500/250-V d.c. 3-wire system there is an out-of-balance load of 125 kW on the positive side. The loss in each balancer machine is 7.5 kW and the current in the negative main is 1500 A. Calculate the total load on the generator and the current in each armature of the balancer set.

[890 kW ; 220 A ; 280 A]

2. A 460-V d.c. 2-wire supply is converted into 3-wire supply with the help of rotary balancer set, each machine having a no-load loss of 2.3 kW. If the load on the positive side of 69 kW and on the negative side 57.5 kW, calculate the currents flowing in each of the balancer machines. [15 A ; 35 A]
3. In a 500/250 volt 3-wire d.c. system there is an out-of-balance load of 200 kW on the positive side. The loss in each balancer is 10 kW and the current in the negative main is 2800 A. Calculate the current in each armature of the balancer set and the total load on the generators. (I.E.E. London)

[Motoring machine = 440 A ; Generating machine = 360 A ; Total load = 1620 kW ]

### 40.15. Boosters

A booster is a generator whose function is to add to or inject into a circuit, a certain voltage that is sufficient to compensate for the  $I_R$  drop in the feeders etc.

In a d.c. system, it may sometimes happen that a certain feeder is much longer as compared to others and the power supplied by it is also larger. In that case, the voltage drop in this particular feeder will exceed the allowable drop of 6% from the declared voltage. This can be remedied in two ways (i) by increasing the cross-section of the feeder, so that its resistance and hence  $I_R$  drop is decreased (ii) or by increasing the voltage of the station bus-bars.

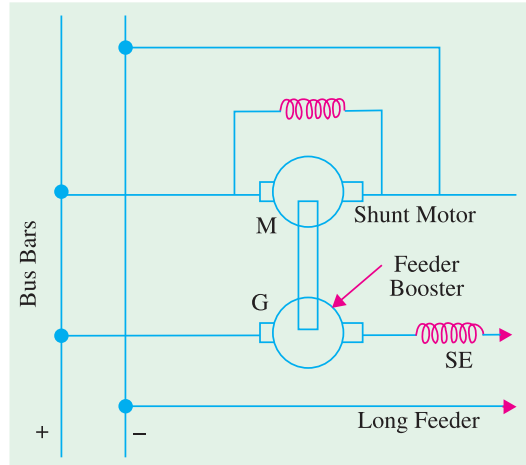


Fig. 40.51

The second method is not practicable because it will disturb the voltage of other feeders, whereas the first method will involve a large initial investment towards the cost of increased conductor material.

To avoid all these difficulties, the usual practice is to install a booster in series with this longer feeder as shown in Fig. 40.51. Since it is used for compensating drop in a feeder, it is known as feeder booster. It is a (series) generator connected in series with the feeder and driven at a constant speed by a shunt-motor working from the bus-bars. The drop in a feeder is proportional to the load current, hence the voltage injected into the feeder by the booster must also be proportional to the load current, if exact compensation is required. In other words, the booster must work on the straight or linear portion of its voltage characteristic.

**Example 40.29.** A 2-wire system has the voltage at the supply end maintained at 500. The line is 3 km long. If the full-load current is 120 A, what must be the booster voltage and output in order that the far end voltage may also be 500 V.

Take the resistance of the cable at the working temperature as 0.5 ohm/kilometre.

(Elect. Machinery-I, Calcutta Univ.)

**Solution.** Total resistance of the line is =  $0.5 \times 3 = 1.5 \Omega$

Full-load drop in the line is =  $1.5 \times 120 = 180 \text{ V}$

Hence, the terminal potential difference of the booster is **180 V** (i.e.  $180/120 = 1.5$  volt per ampere of line current).

$$\text{Booster-output} = 120 \times 180/1000 = \mathbf{21.6 \text{ kW}}$$

### 40.16. Comparison of 2-wire and 3-wire Distribution Systems

We will now compare the 2-wire and 3-wire systems from the point of view of economy of conductor material. For this comparison, it will be assumed that—

1. the amount of power transmitted is the same in both cases.
2. the distance of transmission is the same.

3. the efficiency of transmission (and hence losses) is the same.
4. voltage at consumer's terminals is the same.
5. the 3-wire system is balanced and
6. in the 3-wire system, the mid-wire is of half the cross-section of each outer.

Let  $W$  be the transmitted power in watts and  $V$  the voltage at the consumer's terminals. Also, let

$R_2$  = resistance in ohms of each wire of 2-wire system.

$R_3$  = resistance in ohms of each outer in 3-wire system.

The current in 2-wire system is  $W/V$  and the losses are  $2(W/V)^2 R_2$ .

In the case of 3-wire system, voltage between outers is  $2V$ , so that current through outers is  $(W/2V)$ , because there is no current in the neutral according to our assumption (5) above. Total losses in the two outers are  $2(W/2V)^2 R_3$ .

Since efficiencies are the same, it means the losses are also the same.

$$\therefore 2(W/V)^2 R_2 = 2(W/2V)^2 \times R_3 \quad \text{or} \quad \frac{R_3}{R_2} = \frac{4}{1}$$

Since the cross-section and hence the volume of a conductor of given length, is inversely proportional to its resistance,

$$\therefore \frac{\text{volume of each 3-wire conductor}}{\text{volume of each 2-wire conductor}} = \frac{1}{4}$$

Let us represent the volume of copper in the 2-wire system by 100 so that volume of each conductor is 50.

Then, volume of each outer in 3-wire system =  $50/4 = 12.5$

volume of neutral wire ,, ,, =  $12.5/2 = 6.25$

$\therefore$  total volume of copper in 3-wire system =  $12.5 + 6.25 + 12.5 = 31.25$

$$\therefore \frac{\text{total copper vol. in 3-wire feeder}}{\text{total copper vol. in 2-wire feeder}} = \frac{31.25}{100} = \frac{5}{6}$$

Hence, a 3-wire system requires only 5/16th (or 31.25%) as much copper as a 2-wire system.

### OBJECTIVE TESTS – 40

1. If in a d.c. 2-wire feeder, drop per feeder conductor is 2%, transmission efficiency of the feeder is ..... percent.
  - (a) 98
  - (b) 94
  - (c) 96
  - (d) 99
2. Transmitted power remaining the same, if supply voltage of a dc 2-wire feeder is increased by 100 percent, saving in copper is ..... percent.
  - (a) 50
  - (b) 25
  - (c) 100
  - (d) 75
3. A uniformly-loaded d.c. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at middle point is
  - (a) one-half
  - (b) one-fourth
  - (c) one-third
  - (d) twice.
4. In a d.c. 3-wire distributor using balancers and having unequal loads on the two sides
  - (a) both balancers run as motors
  - (b) both balancers run as generators
  - (c) balancer connected to heavily-loaded side runs as a motor
  - (d) balancer connected to lightly-loaded side runs as a motor.
5. As compared to a dc 2-wire distributor, a 3-wire distributor with same maximum voltage to earth uses only..... percent of copper.
  - (a) 66.7
  - (b) 33.3
  - (c) 31.25
  - (d) 150
6. In a d.c. 3-wire distribution system, balancer fields are cross-connected in order to
  - (a) make both machines run as unloaded motors.
  - (b) equalize voltages on the positive and negative outers
  - (c) boost the generated voltage
  - (d) balance loads on both sides of the neutral.

### ANSWERS

1. (c)    2. (a)    3. (b)    4. (d)    5. (c)    6. (a)

# CHAPTER 41

## Learning Objectives

- General Layout of the System
- Power System and System Networks
- Systems of A.C. Distribution
- Effect of Voltage on Transmission Efficiency
- Comparison of Conductor Materials Required for Various Overhead Systems
- Reactance of an Isolated Single-Phase Transmission Line
- A.C. Distribution Calculations
- Load Division Between Parallel Lines
- Suspension Insulators
- Calculation of Voltage Distribution along Different Units
- Interconnectors
- Voltage Drop Over the Interconnectors
- Sag and Tension with Support at Unequal Levels
- Effect of Wind and Ice

## **A.C. TRANSMISSION AND DISTRIBUTION**



The above figure shows a mini AC power plant. AC has distinct advantages over DC in generation as well as transmission.

### 41.1. General Layout of the System

The conductor system by means of which electric power is conveyed from a generating station to the consumer's premises may, in general, be divided into two distinct parts *i.e.* transmission system and distribution system. Each part can again be sub-divided into two—primary transmission and secondary transmission and similarly, primary distribution and secondary distribution and then finally the system of supply to individual consumers. A typical layout of a generating, transmission and distribution network of a large system would be made up of elements as shown by a single-line diagram of Fig. 41.1 although it has to be realized that one or more of these elements may be missing in any particular system. For example, in a certain system, there may be no secondary transmission and in another case, when the generating station is nearby, there may be no transmission and the distribution system proper may begin at the generator bus-bars.

Now-a-days, generation and transmission is almost exclusively three-phase. The secondary transmission is also 3-phase whereas the distribution to the ultimate customer may be 3-phase or single-phase depending upon the requirements of the customers.

In Fig. 41.1, *C.S.* represents the central station where power is generated by 3-phase alternators at 6.6 or 11 or 13.2 or even 32 kV. The voltage is then stepped up by suitable 3-phase transformers for transmission purposes. Taking the generated voltage as 11 kV, the 3-phase transformers step it up to 132 kV as shown. Primary or high-voltage transmission is carried out at 132 kV\*. The transmission voltage is, to a very large extent, determined by economic considerations. High voltage transmission requires conductors of smaller cross-section which results in economy of copper or aluminium. But at the same time cost of insulating the line and other expenses are increased. Hence, the economical voltage of transmission is that for which the saving in copper or aluminium is not offset (i) by the increased cost of insulating the line (ii) by the increased size of transmission-line structures and (iii) by the increased size of generating stations and sub-stations. A rough basis of determining the most economical transmission voltage is to use 650 volt per km of transmission line. For example, if transmission line is 200 km, then the most economical transmission voltage will be  $200 \times 650 \cong 132,000 \text{ V}$  or 132 kV.

The 3-phase, 3-wire overhead high-voltage transmission line next terminates in step-down transformers in a sub-station known as Receiving Station (*R.S.*) which usually lies at the outskirts of a city because it is not safe to bring high-voltage overhead transmission lines into thickly-populated areas. Here, the voltage is stepped down to 33 kV. It may be noted here that for ensuring continuity of service **transmission is always by duplicate lines**.

From the Receiving Station, power is next transmitted at 33 kV by underground cables (and occasionally by overhead lines) to various sub-stations (*SS*) located at various strategic points in the city. This is known as secondary or low-voltage transmission. From now onwards starts the primary and secondary distribution.

At the sub-station (*SS*) voltage is reduced from 33kV to 3.3kV 3-wire for primary distribution. Consumers whose demands exceeds 50 kVA are usually supplied from *SS* by special 3.3 kV feeders.

The secondary distribution is done at 400/230 V for which purpose voltage is reduced from 3.3 kV to 400 V at the distribution sub-stations. Feeders radiating from distribution sub-station supply power to distribution networks in their respective areas. If the distribution network happens to be at a great distance from sub-station, then they are supplied from the secondaries of distribution transformers which are either pole-mounted or else housed in kiosks at suitable points of the distribution networks. The most common system for secondary distribution is 400/230-V, 3-phase 4-wire system. The single-phase residential lighting load is connected between any one line and the neutral

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\* High voltages like 750 kV are in use in USSR (Konakovo-Moscow line) and 735 kV in Canada (Montreal-Manicogan Scheme).



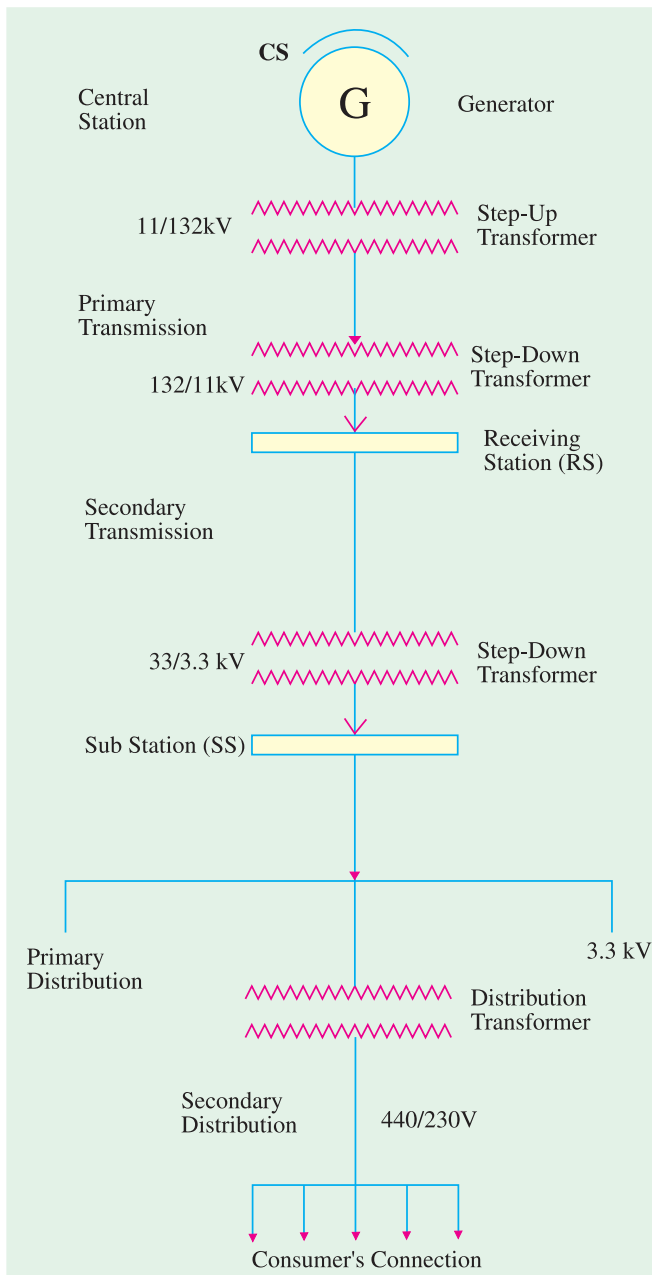


Fig. 41.1

This combination of generating stations forms what is known as **power system**. The various elements of such a system like generating stations, transmission lines, the substations, feeders and distributors etc. become tied into a whole by the integrated process of continuous generation and consumption of electric energy.

**A system network (or grid)** is the name given to that part of power system which consists of the sub-stations and transmission lines of various voltage rating.

Fig. 41.2 shows single-line diagram representing the main connections of a power system consisting of a heating and power central station (HCS), a large-capacity hydro-electric station (HS) and

whereas 3-phase, 400-V motor load is connected across 3-phase lines directly.

It should be noted that low-voltage distribution system is sub-divided into feeders, distributors and service mains. No consumer is given direct connection from the feeders, instead consumers are connected to distribution network through their service mains. The a.c. distributors are, in many ways, similar to the d.c. distributors as regards their constructional details and restrictions on drops in voltage.

Summarizing the above, we have

1. Generating voltage :  
6.6, 11, 13.2 or 33 kV.
2. High voltage transmission :  
220 kV, 132 kV, 66kV.
3. High voltage or primary distribution : 3.3, 6.6 kV.
4. Low-voltage distribution :  
A.C. 400/230, 3-phase, 4-wire  
D.C. 400/230 ; 3-wire system

The standard frequency for a.c. working is 50 Hz or 60 Hz (as in U.S.A.). For single-phase traction work, frequencies as low as 25 or 16 2/3Hz are also used.

### 41.2. Power Systems and System Networks

It is a common practice now-a-days to interconnect many types of generating stations (thermal and hydroelectric etc.) by means of a common electrical network and operate them all in parallel.

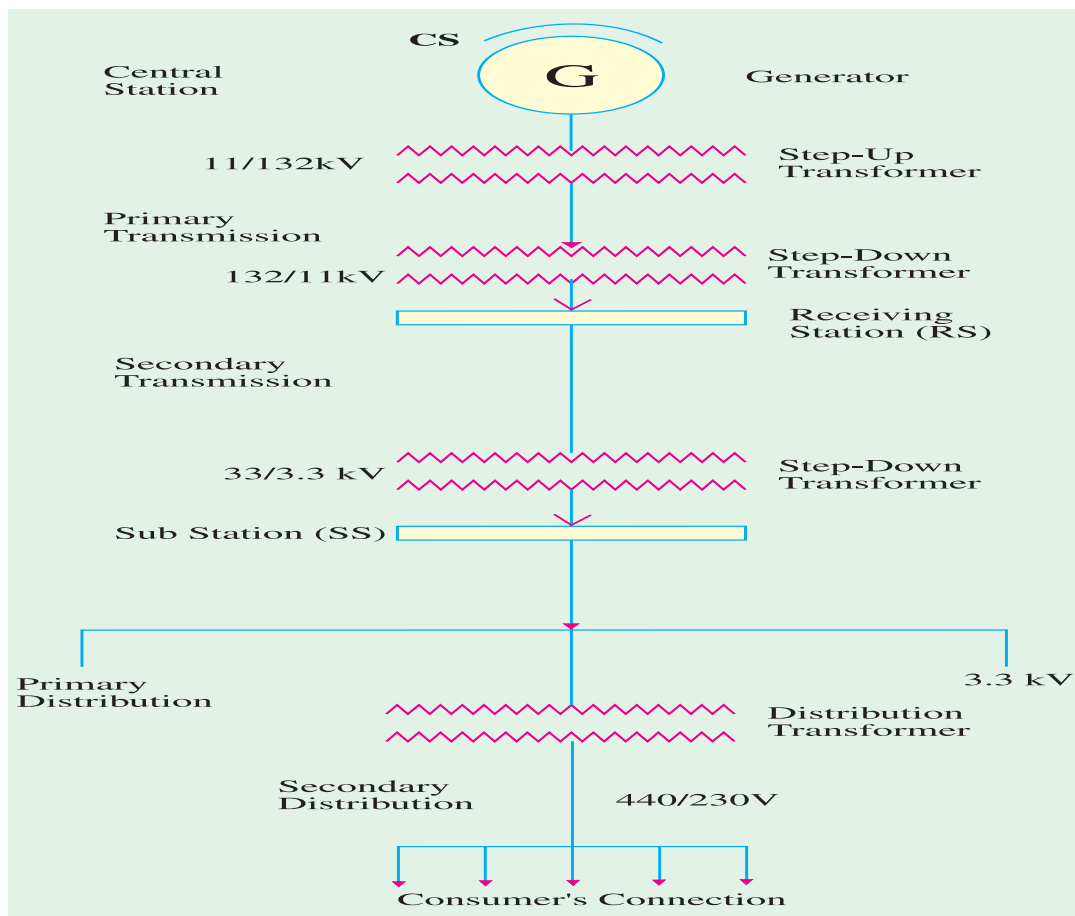


Fig. 41.2

two regional thermal power stations (*RTS-1* and *RTS-2*). The stations *HS*, *RTS-1* and *RTS-2* are situated at large distances from the consumers, hence voltage of the electric power generated by them has to be stepped up by suitable transformers before it is fed into the system transmission network.

As shown in Fig. 41.2, *HS* is connected with the main 110-kV network of the system with the help of (i) two-220 kV transmission lines *L-1* and (ii) main (regional) sub-station *A* which houses two 220/110 kV, 2-winding transformers for interconnecting the two transmission lines.

Transmission lines *L-2*, *L-3* and *L-4* constitute a high-voltage loop or ring mains. As seen, disconnection of any one of the lines will not interrupt the connections between the elements of the system. Station *RTS-1* feeds directly into the 110-kV line loop whereas *RTS-2* is connected to the main network of the system by lines *L-5* and *L-6* through the buses of substations *B* and *C*. *HCS* is interconnected with 110-kV system through the buses of substation *A* by means of 10/110-kV transformers and line *L-7*.

It may be pointed out here that the main substations of the system are *A* and *B*. Substation *B* houses 3-winding transformers. The 35-kV supply is provided over quite large areas whereas 6-10 kV supply is supplied to consumers situated with a limited radius from the substations. Substations *C* and *D* are equipped with 2-winding transformers which operate at voltages indicated in the diagram. Substation *C* is known as a *through substation* whereas *D* is known as a *spur* or *terminal* substation.

Obviously, Fig. 41.2 shows only part of 220-kV and 110-kV lines and leaves out the 35, 10 and 6-kV circuits originating from the buses of the substations. Also, left out are low-voltage circuits for transmission and distribution (see Fig. 41.9).

### 41.3. Systems of A.C. Distribution

As mentioned earlier, a.c. power transmission is always at high voltage and mostly by 3-phase system. The use of single-phase system is limited to single-phase electric railways. Single-phase power transmission is used only for short distances and for relatively low voltages. As will be shown later, 3-phase power transmission requires less copper than either single-phase or 2-phase power transmission.

The distribution system begins either at the sub-station where power is delivered by overhead transmission lines and stepped down by transformers or in some cases at the generating station itself. Where a large area is involved, primary and secondary distributions may be used.

With respect to phases, the following systems are available for the distribution of a.c. power.

1. Single-phase, 2-wire system.
2. Single-phase, 3-wire system.
3. Two-phase, 3-wire system.
4. Two-phase, 4-wire system.
5. Three-phase, 3-wire system.
6. Three-phase, 4-wire system.

#### 41.4. Single-phase, 2-wire System

It is shown in Fig. 41.3 (a) and (b). In Fig. 41.3 (a), one of the two wires is earthed whereas in Fig. 41.3 (b) mid-point of the phase winding is earthed.

#### 41.5. Single-phase, 3-wire System

The 1-phase, 3-wire system is identical in principle with the 3-wire d.c. system. As shown in Fig. 41.4, the third wire or neutral is connected to the centre of the transformer secondary and earthed for protecting personnel from electric shock should the transformer insulation break down or the secondary main contact high voltage wire.



The above figure shows an electrical substation. A substation is a high-voltage electric system facility. It is used to switch generators, equipment, and circuits or lines in and out of a system. It is also used to change AC voltages from one level to another, and/or change alternating current to direct current or direct current to alternating current. Some substations are small with little more than a transformer and associated switches.

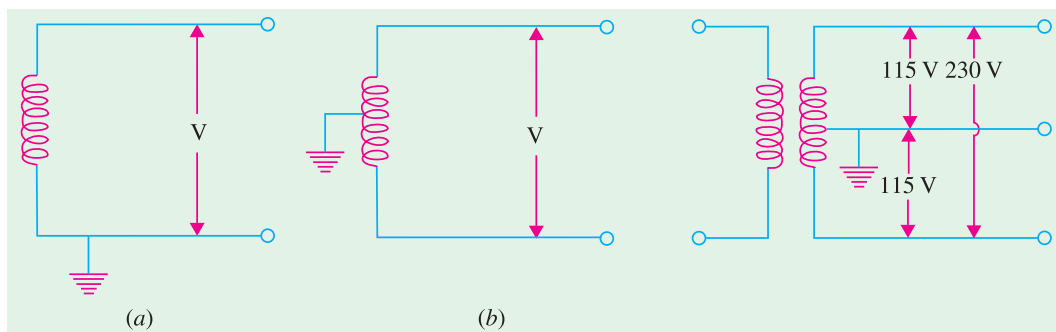


Fig. 41.3

Fig. 41.4

### 41.6. Two-phase, 3-wire System

This system is still used at some places. The third wire is taken from the junction of the two-phase windings I and II, whose voltages are in quadrature with each other as shown in Fig. 41.5. If the voltage between the third or neutral wire and either of the two wires is  $V$ , then the voltage between the outer wires is  $V$  as shown. As compared to 2-phase, 4-wire system, the 3-wire system suffers from the defect that it produces voltage unbalance because of the unsymmetrical voltage drop in the neutral.

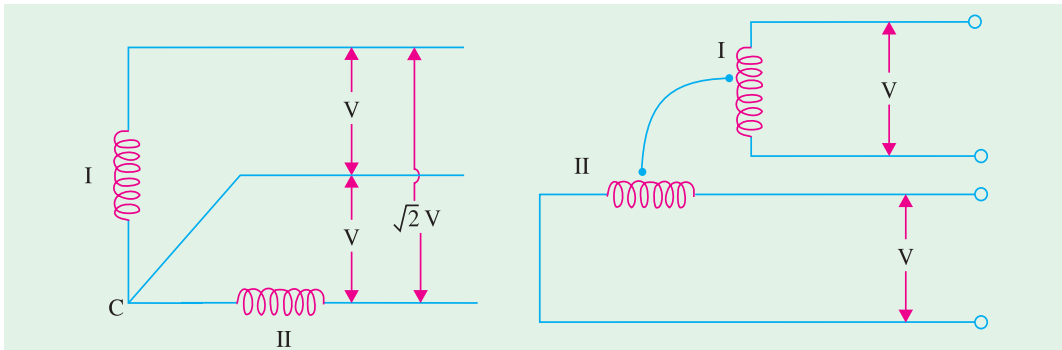


Fig. 41.5

Fig. 41.6

### 41.7. Two-phase, 4-wire System

As shown in Fig. 41.6, the four wires are taken from the ends of the two-phase windings and the mid-points of the windings are connected together. As before, the voltage of the two windings are in quadrature with each other and the junction point may or may not be earthed. If voltage between the two wires of a phase winding be  $V$ , then the voltage between one wire of phase I and one wire of phase II is  $0.707 V$ .

### 41.8. Three-phase, 3-wire System

Three-phase systems are used extensively. The 3-wire system may be delta-connected or star-connected whose star point is usually earthed. The voltage between lines is  $V$  in delta-connection and  $\sqrt{3} V$  in case of star connection where  $V$  is the voltage of each phase as shown in Fig. 41.7 (a) and (b) respectively.

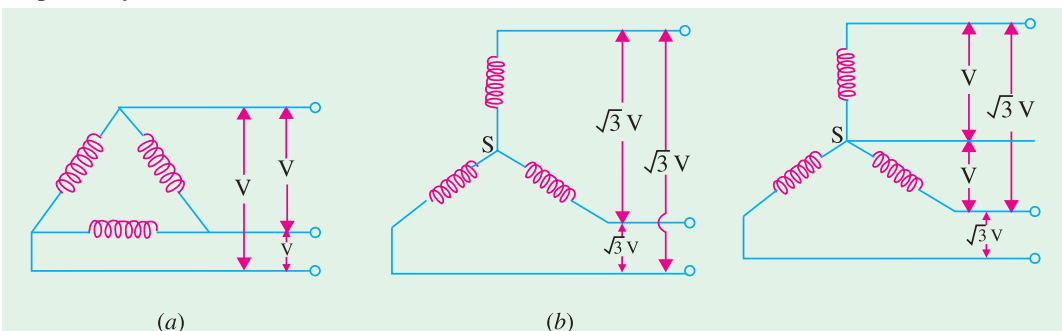


Fig. 41.7

Fig. 41.8

### 41.9. Three-phase, 4-wire System

The 4th or neutral wire is taken from the star-point of the star-connection as shown in Fig. 41.8 and is of half the cross-section of the outers or line conductors. If  $V$  is the voltage of each winding,

then line voltage is  $\sqrt{3}$  V. Usually, phase voltage *i.e.* voltage between any outer and the neutral for a symmetrical system is 230 V so that the voltage between any two lines or outers is  $\sqrt{3} \times 230 = 400$  V.

Single-phase residential lighting loads or single-phase motors which run on 230 V are connected between the neutral and any one of the line wires. These loads are connected symmetrically so that line wires are loaded equally. Hence, the resultant current in the neutral wire is zero or at least minimum. The three phase induction motors requiring higher voltages of 400 V or so are put across the lines directly.

### 41.10. Distribution

The distribution system may be divided into feeders, distributors, sub-distributors and service mains. As already explained in Art. 41.1, feeders are the conductors which connect the sub-station (in some cases the generating station) to the distributors serving a certain allotted area. From distributors various tappings are taken. The connecting link between the distributors and the consumers' terminals are the service mains. The essential difference between a feeder and a distributor is that whereas the current loading of a feeder is the same throughout its length, the distributor has a distributed loading which results in variations of current along its entire length. In other words, no direct tappings are taken from a feeder to a consumer's premises.

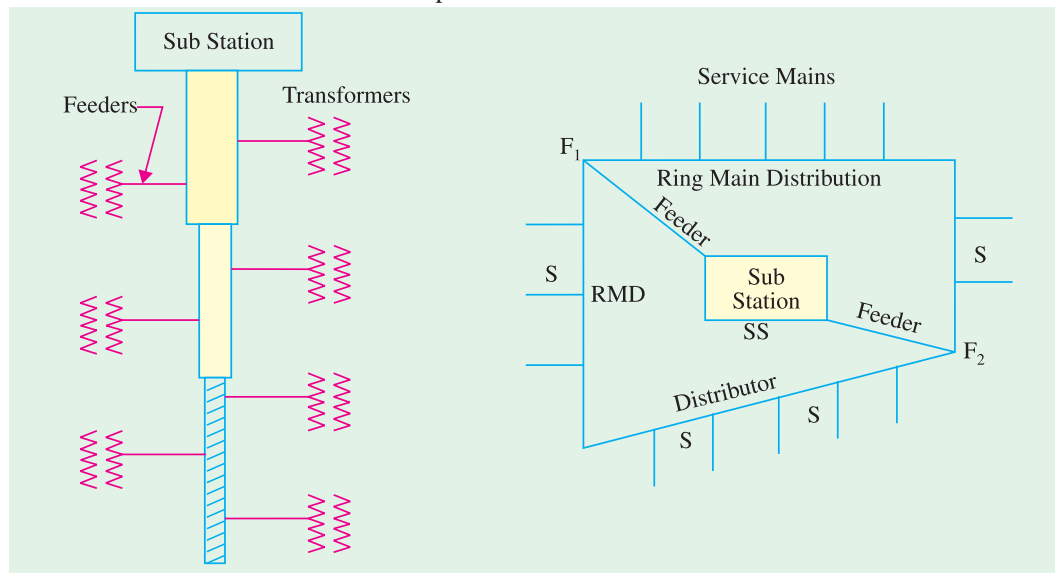


Fig. 41.9

Fig. 41.10

In early days, *radial* distribution of tree-system type, as shown in Fig. 41.9, was used. In this system, a number of independent feeders branch out radially from a common source of supply *i.e.* a sub-station or generating station. The distribution transformers were connected to the taps along the length of the feeders. One of the main disadvantages of this system was that the consumer had to depend on one feeder only so that if a fault or breakdown occurred in his feeder, his supply of power was completely cut off till the fault was repaired. Hence, there was no absolute guarantee of continuous power supply.

For maintaining continuity of service, ring-main distributor (*R.M.D.*) system as shown in Fig. 41.10, is employed almost universally. *SS* represents the sub-station from which two feeders supply power to the ring-main distributor at feeding points  $F_1$  and  $F_2$ . The ring-main forms a complete loop and has *isolating* switches provided at the poles at strategic points for isolating a particular section in case of fault. In this way, continuity of service can be maintained to other consumers on

healthy sections of the ring-main. The number of feeders of the ring-main depends on (i) the nature of loading—heavy or light (ii) the total length of the *R.M.D.* and (iii) on the permissible/allowable drop of voltage. Service mains (*S*) are taken off at various points of the *R.M.D.* Sometimes sub-distributors are also used. Since a loop or closed ring-main can be assumed to be equivalent to a number of straight distributors fed at both ends, the voltage drop is small which results in economy of conductor material. The service mains are the connecting link between the consumer's terminals and the *R.M.D.* or sub-distributor.

#### 41.11. Effect of Voltage on Transmission Efficiency

Let us suppose that a power of  $W$  watt is to be delivered by a 3-phase transmission line at a line voltage of  $E$  and power factor  $\cos \phi$ .

$$\text{The line current} \quad I = \frac{W}{\sqrt{3} E \cos \phi}$$

$$\text{Let} \quad l = \text{length of the line conductor}; \quad \sigma = \text{current density}$$

$$\rho = \text{specific resistance of conductor material, } A = \text{cross-section of conductor}$$

$$\text{then} \quad A = \frac{I}{\sigma} = \frac{W}{\sqrt{3} E \sigma \cos \phi}$$

$$\text{Now} \quad R = \frac{\rho l}{A} = \frac{\sqrt{3} \sigma \rho l E \cos \phi}{W}$$

$$\text{Line loss} \quad = 3 \times \text{loss per conductor} = 3 I^2 R$$

$$= 3 \frac{W^2}{3 E^2 \cos^2 \phi} \times \frac{\sqrt{3} \sigma \rho l \cos \phi}{W} = \frac{\sqrt{3} \sigma \rho l W}{E \cos \phi} \quad \dots (1)$$

$$\text{Line intake or input} = \text{output} + \text{losses} = W + \frac{\sqrt{3} \sigma \rho l W}{E \cos \phi} = W \left( 1 + \frac{\sqrt{3} \sigma \rho l}{E \cos \phi} \right)$$

$$\therefore \text{efficiency of transmission} = \frac{\text{output}}{\text{input}} = \frac{W}{W \left( 1 + \frac{\sqrt{3} \sigma \rho l}{E \cos \phi} \right)} = \left( 1 - \frac{\sqrt{3} \sigma \rho l}{E \cos \phi} \right) \text{approx} \quad \dots (2)$$

$$\text{Voltage drop per line} = IR = \frac{\sqrt{3} \sigma \rho l E \cos \phi}{W} \times \frac{W}{\sqrt{3} E \cos \phi} = \sigma \rho l \quad \dots (3)$$

$$\text{Volume of copper} = 3 l A = \frac{3 W l}{\sqrt{3} \sigma E \cos \phi} = \frac{\sqrt{3} W l}{\sigma E \cos \phi} \quad \dots (4)$$

For a given value of transmitted power  $W$ , line length  $l$ , current density  $\sigma$  and specific resistance  $\rho$  of the given conductor material, the effect of supply voltage on transmission can be seen as follows :

1. From equation (1), line loss is inversely proportional to  $E$ . It is also inversely proportional to power factor,  $\cos \phi$ .
2. Transmission efficiency increases with voltage of transmission and power factor as seen from equation (2).
3. As seen from equation (3) for a given current density, the resistance drop per line is constant (since  $\rho$  and  $l$  have been assumed fixed in the present case). Hence, percentage drop is decreased as  $E$  is increased.
4. The volume of copper required for a transmission line is inversely proportional to the voltage and the power factor as seen from equation (4).

It is clear from the above that for long distance transmission of a.c. power, high voltage and high power factors are essential. But as pointed out earlier in Art. 41.1, economical upper limit of voltage is reached when the saving in cost of copper or aluminium is offset by the increased cost of insulation and increased cost of transformers and high-voltage switches etc. Usually, 650 volt per route km is taken as a rough guide.

### 41.12. Comparison of Conductor Materials Required for Various Overhead Systems

We will now calculate the amounts of conductor material required for various systems of a.c. power transmission. To do it without prejudice to any system, we will make the following assumptions :

1. Amount of power transmitted by each system is the same.
2. Distance of transmission is the same in each case.
3. Transmission efficiency is the same *i.e.* the losses are the same in each case.
4. Loads are balanced in the case of 3-wire systems.
5. Cross-section of the neutral wire is half that of any outer.
6. Maximum voltage to earth is the same in all cases.

By way of illustration a few cases will be compared, although the reader is advised to attempt others as well.

#### (i) Two-wire d.c. System and Three-phase, 3-wire System

Let in both cases,  $E$  be the maximum voltage between conductors and  $W$  the power transmitted.

In d.c. system, let  $R_1$  be the resistance of each conductor and  $I_1$  the current, then  $I_1 = (W/E)$ , hence losses in the two conductors are  $= 2I_1^2 R_1 = 2 (W/E)^2 R_1$  ...**(i)**

For a.c. system, let resistance per conductor be  $R_2$  and power factor  $\cos \phi$ , then since  $E$  is the maximum voltage, the *R.M.S.* voltage is  $E/\sqrt{2}$ .

$$\text{Current in each line, } I_2 = \frac{W}{\sqrt{3} (E/\sqrt{2}) \cos \phi}$$

Total losses in three conductors are,

$$3I_2^2 R_2 = \left[ \frac{W}{\sqrt{3} (E/\sqrt{2}) \cos \phi} \right]^2 R_2 = \frac{2W^2}{E^2} \times \frac{1}{\cos^2 \phi} \cdot R_2 \quad \dots\text{(ii)}$$

Since transmission efficiency and hence losses are the same, therefore equating **(i)** and **(ii)**, we get

$$\frac{2W^2 R_1}{E^2} = \frac{2W^2}{E^2} \cdot \frac{1}{\cos^2 \phi} R_2 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{1}{\cos^2 \phi}$$

Since area of cross-section is inversely proportional to the resistance.

$$\therefore \frac{\text{area of one a.c. conductor}}{\text{area of one d.c. conductor}} = \frac{1}{\cos^2 \phi}$$

Now, for a given length, volumes are directly proportional to the areas of cross-section.

$$\therefore \frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{1}{\cos^2 \phi}$$

Keeping in mind the fact, that there are two conductors in d.c. system and three in a.c. system under consideration, we have

$$\frac{\text{total volume in 3-wire a.c. system}}{\text{total volume in 2-wire d.c. system}} = \frac{1}{\cos^2 \phi} \cdot \frac{3}{2} = \frac{1.5}{\cos^2 \phi}$$

#### (ii) Three-phase, 4-wire and 3-wire d.c. Systems

The neutral conductor of each system is earthed. Let  $E$  be the maximum voltage to earth and  $W$  the power to be transmitted in both cases.

For d.c. 3-wire system, voltage between outers is  $2E$ . If  $I_1$  and  $R_1$  are the current in and resistance of each conductor, then  $I_1 = W/2E$ . Assuming a balanced load (in which case there would be no current flowing in the neutral conductor), the value of loss in two conductors is

$$2I_1^2R_1 = 2(W/2E)^2R_1 = W^2R_1/2E^2$$

In the case of a.c. system, if  $E$  is the maximum voltage between any wire and neutral, then its R.M.S. value is  $E/\sqrt{2}$ . Hence, the line voltage =  $\sqrt{3} \cdot E/\sqrt{2}$ . The line current is

$$I_2 = \frac{W}{\sqrt{3} \times (\sqrt{3} E/\sqrt{2}) \cos \phi}$$

where  $\cos \phi$  is the power factor. If  $R_2$  is the resistance of each line, then total loss in 3 lines is

$$= 3I_2^2R_2 = 3 \times \left[ \frac{W}{\sqrt{3} (\sqrt{3} E/\sqrt{2}) \cos \phi} \right]^2 \times R_2 = \frac{2W^2R_2}{3E^2 \cos^2 \phi}$$

For equal transmission efficiencies, the two losses should be the same.

$$\therefore \frac{W^2R_1}{2E^2} = \frac{2W^2R_2}{3E^2 \cos^2 \phi} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{4}{3 \cos^2 \phi}$$

Since volume is directly proportional to the area of cross-section which is itself inversely proportional to the resistance, hence

$$\frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{4}{3 \cos^2 \phi}$$

Assuming the neutral wires of each system to have half the cross-section of the outers, there is 3.5 times the quantity of one conductor in a.c. and 2.5 times in the d.c. system. Hence

$$\frac{\text{total volume in the a.c. system}}{\text{total volume in the d.c. system}} = \frac{3.5}{2.5} \times \frac{4}{3 \cos^2 \phi} = \frac{1.867}{\cos^2 \phi}$$

In Table 41.1 are given the ratios of copper in any system as compared with that in the corresponding d.c. 2-wire system which has been allotted a basic number of 100.

<b>Table No. 41.1</b>		
<i>System</i>	<i>With same maximum voltage to earth</i>	<i>With same maximum voltage between conductors</i>
D.C. 2-wire	100	100
D.C. 2-wire Mid-point earthed	25	100
D.C. 3-wire Neutral = 1/2 outer	31.25	125
D.C. 3-wire Neutral = 1/2 outer	37.5	150
Single-phase, 2-wire	$200/\cos^2 \phi$	$200/\cos^2 \phi$
Single-phase, 2-wire Mid-point earthed	$50/\cos^2 \phi$	$200/\cos^2 \phi$
Single-phase, 3-wire Neutral = 1/2 outer	$62.5/\cos^2 \phi$	$250/\cos^2 \phi$
Two-phase, 4-wire	$50/\cos^2 \phi$	$200/\cos^2 \phi$



Two-phase, 3-wire	$146/\cos^2 \phi$	$291/\cos^2 \phi$
Three-phase, 3-wire	$50/\cos^2 \phi$	$150/\cos^2 \phi$
Three-phase, 4-wire		
Neutral = outer	$67/\cos^2 \phi$	$200/\cos^2 \phi$

A comparison of costs for underground systems employing multicore cables can also be worked out on similar lines. In these case, however, maximum voltage between the conductors is taken as the criterion.

It would be clear from the above discussion, that for a.c. systems, the expression for volume contains  $\cos^2 \phi$  in denominator. Obviously, reasonable economy in copper can be achieved only when  $\cos \phi$  approaches unity as closely as possible. For equal line loss, the volume of copper required for  $\cos \phi = 0.8$  will be  $1/0.8^2 = 1.6$  times that required for  $\cos \phi = 1$ .

Since the cost of distribution system represents a large percentage of the total capital investment of power supply companies, it is quite reasonable that a consumer should be penalized for low power factor of his load. This point is fully dealt with in Chapt. 47.

**Example 41.1.** A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire D.C. system with same line voltage. Assume the same losses and balanced load. The neutral is one half the cross-section of one of the respective outers.

**Solution.** (a) **Two-wire DC** Let  $V$  = voltage between conductors  
 $P$  = power delivered,  $R_1$  = resistance/conductor  
 Current  $I_1 = P/V$   
 power loss =  $2I_1^2R_1 = 2P^2R_1/V^2$

(b) **Three-phase, 4-wire**

Let  $V$  be the line-to-neutral voltage and  $R_2$  resistance of each phase wire.

$$P = 3VI_2 \cos \phi = 3VI_2 \quad \text{—if } \cos \phi = 1$$

$$\text{Power loss} = 3I_2^2R_2 = 3(P/3V)^2R_2 = P^2R_2/3V^2$$

Since power losses in both systems are equal

$$\therefore \quad 2P^2R_1/V^2 = P^2R_2/3V^2 \quad \text{or} \quad R_1/R_2 = 1/6$$

If  $A_1$  and  $A_2$  are the cross-sectional areas of conductors in the two systems, then  $A_1/A_2 = 6$ . Because  $R \propto l/A$

$$\therefore \quad \text{Cu reqd. for 2-wire system} = 2A_1l$$

$$\text{Cu reqd. for 3-}\phi, \text{ 4-wire system} = (3A_2l + A_2l/2)$$

$$\therefore \quad \frac{\text{Cu for 3-}\phi \text{ system}}{\text{Cu for d.c. system}} = \frac{3.5A_2l}{2A_1l} = \frac{3.5}{2} \times \frac{1}{6} = 0.292$$

$$\text{Cu for 3-}\phi \text{ system} = \mathbf{0.292 \times \text{Cu for d.c. system.}}$$

**Example 41.2.** Estimate the weight of copper required to supply a load of 100 MW at upf by a 3-phase, 380-kV system over a distance of 100 km. The neutral point is earthed. The resistance of the conductor is 0.045 ohm/cm<sup>2</sup>/km. The weight of copper is 0.01 kg/cm<sup>3</sup>. The efficiency of transmission can be assumed to be 90 percent. **(Power Systems, AMIE, Sec. B, 1994)**

**Solution.** Power loss in the line =  $(1 - 0.9) \times 100 = 10$  MW

$$\text{Line current} = 100 \times 10^6 / \sqrt{3} \times 380 \times 10^3 \times 1 = 152 \text{ A}$$

Since  $I^2R$  loss in 3-conductors is  $10 \times 10^6$  W, loss per conductor is  $= 10 \times 10^6 / 3 = 3.333 \times 10^6$  W

$$\text{Resistance per conductor} = 10 \times 10^6 / 3 \times 152^2 = 144.3 \Omega$$

$$\text{Resistance per conductor per km} = 144.3 / 100 = 1.443 \Omega$$

$$\therefore \text{Conductor cross-section} = 0.045 / 1.443 = 0.03 \text{ m}^3$$

$$\text{Volume of copper per meter run} = 0.03 \times 100 = 3 \text{ cm}^3$$

$$\text{Weight of copper for 3-conductor for 100 km length} = 3 \times (3 \times 0.01) \times 100 \times 1000 = \mathbf{9000 \text{ kg}}$$

**Example 41.3.** A d.c. 2-wire distribution system is converted into a.c. 3-phase, 3-wire system by adding a third conductor of the same size as the two existing conductors. If voltage between conductors and percentage power loss remain the same, calculate the percentage additional balanced load which can now be carried by the conductors at 0.95 p.f.

**Solution.** (a) DC 2-wire system [Fig. 40.11 (a)]

If  $R$  is the resistance per conductor, then power transmitted is  $P = VI_1$  and power loss =  $2I_1^2R$ . Percentage power loss

$$= \frac{2I_1^2R \times 100}{VI_1}$$

$$= \frac{2I_1R \times 100}{V}$$

(b) as 3-phase, 3-wire system [Fig. 40.11 (b)]

$$P_2 = \sqrt{3} VI_2 \cos \phi, \text{ power loss} = 3I_2^2R$$

$$\% \text{ power loss} = \frac{3I_2^2R}{P_2} = \frac{\sqrt{3}I_2R \times 100}{V \cos \phi}$$

Since losses in the two cases are the same

$$\therefore \frac{2I_1R \times 100}{V} = \frac{\sqrt{3}I_2R \times 100}{V \cos \phi} \text{ or } I_2 = \frac{2 \cos \phi}{\sqrt{3}} I_1$$

$$\therefore P_2 = \sqrt{3} V \cdot \frac{2 \cos \phi}{\sqrt{3}} I_1 \cos \phi = 2VI_1 \cos^2 \phi = 2VI_1 (0.95)^2 = 1.8 VI_1$$

Percentage additional power transmitted in a 3-phase, 3-wire system

$$= \frac{P_2 - P_1}{P_1} \times 100 = \frac{1.8 VI_1 - VI_1}{VI_1} \times 100 = \mathbf{80\%}$$

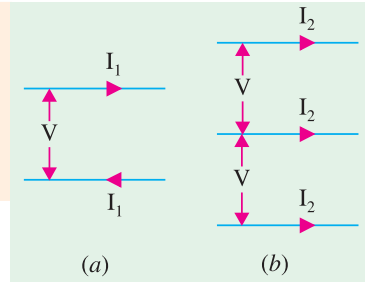


Fig. 41.11

**Example 41.4.** A 2-phase, 3-wire a.c. system has a middle conductor of same cross-sectional area as the outer and supplies a load of 20 MW. The system is converted into 3-phase, 4-wire system by running a neutral wire. Calculate the new power which can be supplied if voltage across consumer terminal and percentage line losses remain the same. Assume balanced load.

**Solution.** The two systems are shown in Fig. 41.12. Let  $R$  be the resistance per conductor.

$$P_1 = 2VI_1 \cos \phi; \text{ Cu loss, } W_1 = 2I_1^2R$$

$$\text{Percentage Cu loss} = \frac{W_1}{P_1} \times 100$$

$$= \frac{2I_1^2R}{2VI_1 \cos \phi} \times 100$$

$$= \frac{I_1R \times 100}{V \cos \phi}$$

$$P_2 = 3VI_2 \cos \phi$$

$$W_2 = 3I_2^2R$$

$$\% \text{ line loss} = \frac{W_2}{P_2} \times 100$$

$$= \frac{3I_2^2R}{3VI_2 \cos \phi} \times 100$$

$$= \frac{I_2R \times 100}{V \cos \phi}$$

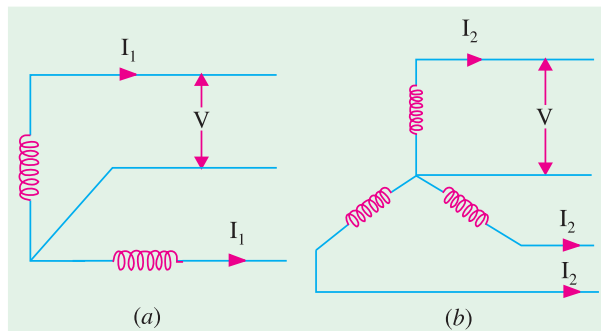


Fig. 41.12

Since percentage line losses are the same in both cases

$$\begin{aligned} \therefore I_1 R \times 100/V \cos \phi &= I_2 R \times 100/V \cos \phi & \therefore I_1 = I_2 \\ \text{Now, } P_1 &= 2VI_1 \cos \phi = \mathbf{20 \text{ MW}} ; & \therefore VI_1 \cos \phi = 10 \text{ MW} \\ P_2 &= 3VI_2 \cos \phi = 3VI_1 \cos \phi = 3 \times 10 = \mathbf{30 \text{ MW}} \end{aligned}$$

### 41.13. Constants of a Transmission Line

A transmission line not only has an ohmic resistance but also inductance and capacitance between its conductors. These are known as the constants of a transmission line. While calculating the drop in a.c. transmission and distribution circuits, we will have to consider (i) resistive or ohmic drop—in phase with the current (ii) inductive drop—leading the current by 90° and (iii) the capacitive drop and charging current taken by the capacitance of the line. The capacitance and hence the charging current is usually negligible for short transmission lines.

### 41.14. Reactance of an Isolated Single-Phase Transmission Line

In Fig. 41.13 (a) are shown the cross-sections of two conductors of a single-phase line spaced  $D$  from centre to centre.

Since currents through the two conductors will, at any time, always be flowing in opposite directions, the fields surrounding them always reinforce each other in the space between them as shown in Fig. 41.13.

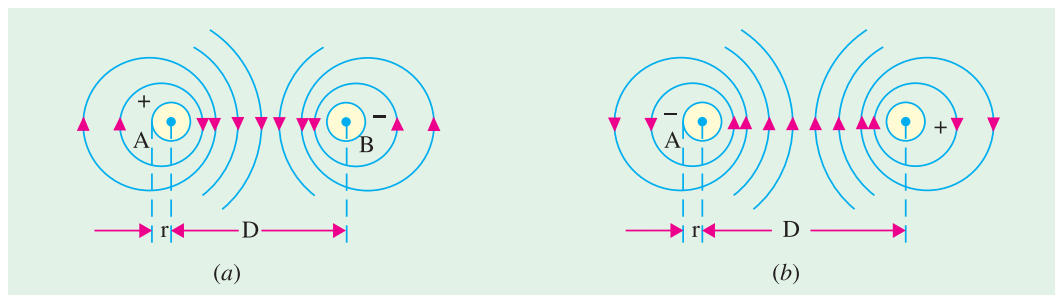


Fig. 41.13

The two parallel conductors form a rectangular loop of one turn through which flux is produced by currents in the two conductors. Since this flux links the loop, the loop possesses inductance. It might be thought that this inductance is negligible because the loop has only one turn and the entire flux-path lies through air of high reluctance. But as the cross-sectional area of the loop is large, from 1 to 10 metre wide and several km long, even for a small flux density, the total flux linking the loop is large and so inductance is appreciable.

It can be proved that inductance per loop metre (when  $r \leq D$ ) is ...(i)

$$L = \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \text{ henry/metre}$$

where

$\mu$  = absolute permeability of the surrounding medium

$\mu_i$  = absolute permeability of the conductor material

Now,  $\mu = \mu_0 \mu_r$  and  $\mu_i = \mu_0 \mu_r'$ , where  $\mu_r$  and  $\mu_r'$  are the relative permeabilities of the surrounding medium and the conductor material. If surrounding medium is air, then  $\mu_r = 1$ . Also, if conductor is made of copper, then  $\mu_r' = 1$ . Hence, the above expression becomes.

$$L = \left( \frac{\mu_0 \mu_r}{\pi} \log_e D/r + \frac{\mu_0 \mu_r'}{4\pi} \right) \text{ H/m}$$

$$= \frac{\mu_0}{4\pi} (\log h D/r + 1) H/m = 10^{-7} (1 + \log h D/r) H/m$$

Loop reactance  $X = 2\pi fL$  ohm/metre

Obviously, the inductance of each single conductor is half the above value\*.

$$\begin{aligned} \text{inductance/conductor} &= \frac{1}{2} (1 + \log h D/r) \times 10^{-7} \text{ H/m} && \dots(ii) \\ \text{reactance/conductor} &= 2 \pi f \times \frac{1}{2} (1 + \log h D/r) \times 10^{-7} \Omega/\text{m} \end{aligned}$$

### 41.15. Reactance of 3-phase Transmission Line

In 3-phase transmission, it is more convenient to consider the reactance of each conductor instead of the looped line or of the entire circuit. Two cases will be considered for 3-phase lines.

#### (i) Symmetrical Spacing

In Fig. 41.14 (a) are shown the three conductors spaced symmetrically *i.e.* each conductor is at the apex of the same equilateral triangle *ABC* of side *D*. The inductance per conductor per metre is found by using equation (ii) in Art 41.14 because it can be shown that inductance per km of one conductor of 3-phase circuit is the same as the inductance per conductor per km of single-phase circuit with equivalent spacing.

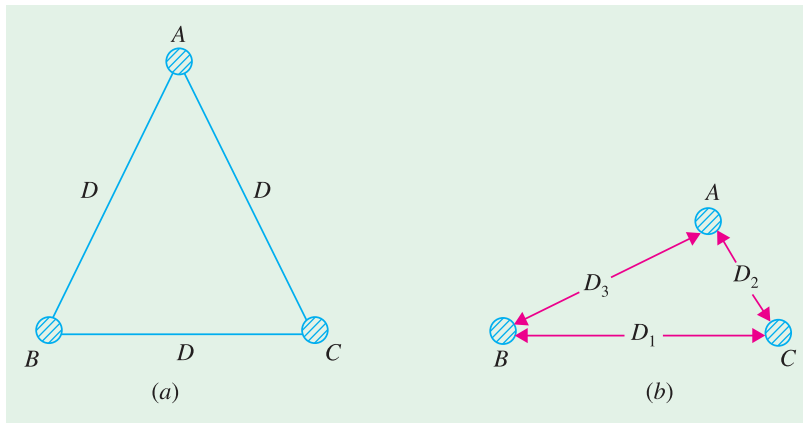


Fig. 41.14

#### (ii) Unsymmetrical Spacing

In Fig. 41.14 (b) is shown a 3-phase circuit with conductors placed unsymmetrically. In this case also, the inductance is given by equation (ii) of Art. 41.14 with the only modification that *D* is put equal to  $\sqrt[3]{(D_1 D_2 D_3)}$ .

### 41.16. Capacitance of a Single-phase Transmission Line

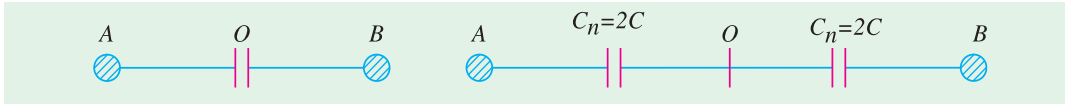
We know that any two conductors which are separated by an insulating medium constitute a capacitor. When a potential difference is established across two such conductors, the current flows in at one conductor and out at the other so long as that p.d. is maintained. The conductors of an overhead transmission line fulfil these conditions, hence when an alternating potential difference is applied across a transmission line, it draws a leading current even when it is unloaded. This leading current is in quadrature with the applied voltage and is known as the **charging current**. Its value depends upon voltage, the capacitance of the line and the frequency of alternating current.

As shown in Art 5.10, the capacitance between conductors of a single-phase line is approximately given by

\* It may be noted that standard conductors have a slightly higher inductance.

$$C = \frac{\pi \epsilon}{\log_e h D/r} = \frac{\pi \epsilon_0 \epsilon_r}{2.3 \log_{10} D/r} \quad F/m = \frac{0.0121 \epsilon_r}{\log_{10} D/r} \mu F/km$$

Here,  $D$  is the distance between conductor centres and  $r$  the radius of each conductor, both expressed in the same units (Fig. 41.15).



As shown in Fig. 41.16, the capacitance to neutral  $C_n = 2C$  where point  $O$  is the neutral of the system. Obviously, the total capacitance between conductors  $A$  and  $B$  is given by finding the resultant of two capacitances each of value  $C_n$  joined in series, the resultant, obviously, being equal to  $C$ .

It is important to remember that if capacitance to neutral is used for calculating the charging current, then voltage to neutral must also be used.

$$\therefore C_n = 2C = \frac{2 \pi \epsilon}{\log_e D/r} \text{ F/m} \quad \dots (ii)$$

$$\therefore \text{line charging current} = \frac{V}{X_C} = \frac{V}{1/(2\pi f C_n)} = 2\pi f C_n V \text{ A/m}$$

where  $V$  is the **voltage to neutral**.

However, it may be noted that ground effect has been neglected while deriving the above expression. This amounts to the tacit assumption that height  $h$  of the conductors is very large as compared to their spacing ' $d$ '. In case ground effect is to be taken into account, the expression for capacitance becomes.

$$C_n = \frac{2 \pi \epsilon}{\log_e h \frac{d}{r \sqrt{1 + d^2/4h^2}}} \text{ F/m} \quad (\text{Ex. 41.7})$$

**Example 41.5.** What is the inductance per loop metre of two parallel conductors of a single phase system if each has a diameter of 1 cm and their axes are 5 cm apart when conductors have a relative permeability of (a) unity and (b) 100. The relative permeability of the surrounding medium is unity in both cases. End effects may be neglected and the current may be assumed uniformly distributed over cross-section of the wires.

**Solution.** (a) Here,  $\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}; \mu_i = 1$

$$\therefore L = \frac{4\pi \times 10^{-7}}{\pi} \left( \log h^5/0.5 + \frac{1}{4} \right) = 1.02 \mu\text{H/m}$$

(b) Here,  $\mu = \mu_0 = 4\pi \times 10^{-7}; \mu_i = 100 \mu_0$

$$L = \frac{4\pi \times 10^{-7}}{\pi} \left( \log h^5/0.5 + \frac{100}{4} \right) = 10.9 \mu\text{H/m}$$

**Example 41.6.** A 20-km single-phase transmission line having 0.823 cm diameter has two line conductors separated by 1.5 metre. The conductor has a resistance of 0.311 ohm per kilometre. Find the loop impedance of this line at 50 Hz. (Gen. Trans. & Dist. Bombay Univ. 1992)

**Solution.** Loop length = 20 km =  $2 \times 10^4$  m

Total loop inductance is  $L = 2 \times 10^4 \left( \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \right)$  henry

Here,  $D = 1.5 \text{ m}; r = 0.823/2 = 0.412 \text{ cm} = 4.12 \times 10^{-3} \text{ m}$

Assuming  $\mu_r = 1$  for copper conductors since they are non-magnetic and also taking  $\mu_r = 1$  for air, we have

$$L = 2 \times 10^4 \left( \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \right) = \frac{2 \times 10^4 \times 4\pi \times 10^{-7}}{\pi} \left( \log_e^{1.5/4.12 \times 10^{-3}} + \frac{1}{4} \right) \text{H}$$

$$= 8 \times 10^{-3} (5.89 + 0.25) = 49.12 \times 10^{-3} \text{H}$$

Reactance  $X = 2\pi \times 50 \times 49.12 \times 10^{-3} = 15.44 \Omega$  ; Loop resistance =  $2 \times 20 \times 0.311 = 12.44 \Omega$

$$\text{Loop impedance} = \sqrt{12.44^2 + 15.44^2} = \mathbf{19.86 \Omega}$$

**Example 41.7.** The conductors in a single-phase transmission line are 6 m above ground. Each conductor has a diameter of 1.5 cm and the two conductors are spaced 3 m apart. Calculate the capacitance per km of the line (i) excluding ground effect and (ii) including the ground effect.

**Solution.** The line conductors are shown in Fig. 41.17.

(i) 
$$C_n = \frac{2\pi\epsilon_0}{\log h d/r} = \frac{2\pi \times 8.854 \times 10^{-12}}{\log h 3/0.75 \times 10^{-2}} \text{ F/m}$$

$$= 9.27 \times 10^{-12} \text{ F/m} = \mathbf{9.27 \times 10^{-3} \mu\text{F/km}}$$

(ii) In this case,

$$C_n = \frac{2\pi\epsilon_0}{\log h \frac{d}{r\sqrt{1+d^2/4h^2}}} \text{ F/m}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\log h \frac{3}{0.75 \times 10^{-2} \sqrt{1+3^2/4 \times 6^2}}}$$

$$= 9.33 \times 10^{-12} \text{ F/m} = \mathbf{9.33 \times 10^{-3} \mu\text{F/km}}$$

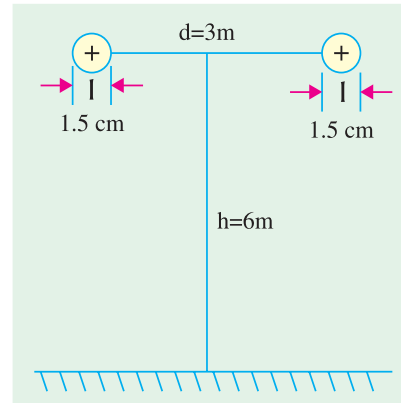


Fig. 41.17

It is seen that line capacitance when considering ground effect is more (by about 0.64% in the present case).

### 41.17. Capacitance of a Three-phase Transmission Line

In Fig. 41.18 (a) are shown three conductors spaced symmetrically and having a capacitance of  $C$  between each pair. The value of  $C$  i.e. **line-to-line** capacitance is the same as given by equation (i) in Art. 41.16.

Considering the two conductors  $A$  and  $B$ , their line capacitance  $C$  can be regarded as consisting of two line-to-neutral capacitances connected in series, so that for the line-to-neutral capacitance of  $A$  with respect to neutral plane 1, as in Art. 41.16, we have

$$C_{n1} = \frac{2\pi\epsilon}{\log_e D/r} \text{ F/m}$$

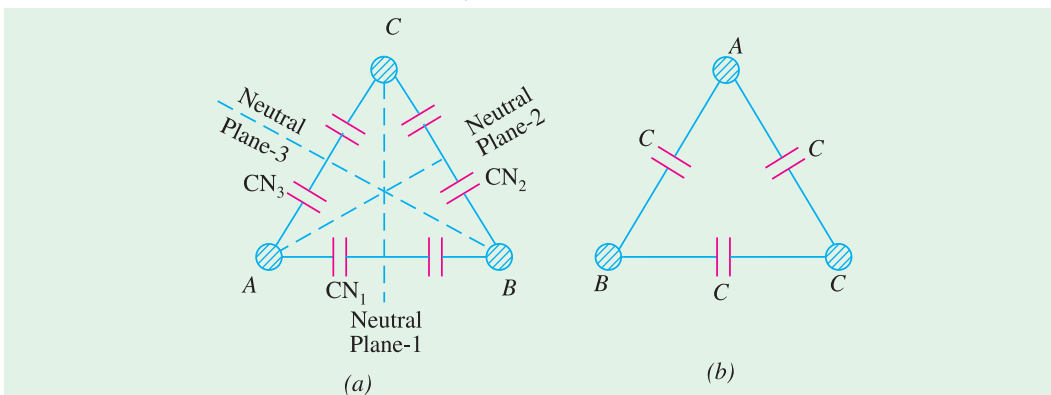


Fig. 41.18

Similarly, line-to-neutral capacitance of conductor A with respect to neutral plane 3 is

$$C_{n3} = \frac{2\pi\epsilon}{\log_e D/r} \text{ F/m}$$

The vector sum of these capacitances is equal to either because their phase angle is  $120^\circ$  i.e. the phase angle of the voltages which cut across them.

Hence, the capacitance to neutral per metre of one conductor of a 3-phase transmission line is

$$C_n = \frac{2\pi\epsilon}{\log_e D/r} \text{ F/m}$$

In case the conductors are placed asymmetrically, the distance used is  $D = \sqrt[3]{(D_1 D_2 D_3)}$ .

### 41.18. Short Single-phase Line Calculations

Single-phase circuits are always short and work at relatively low voltages. In such cases, the line constants can be regarded as 'lumped' instead of being distributed and capacitance can be neglected.

- Let  $E_S$  = voltage at sending end ;  $E_R$  = voltage at receiving end
- $I$  = line current  $\cos \phi_R$  = power factor at receiving end
- $R$  = resistance of both conductors;  $X$  = reactance of both conductors =  $\omega L$

Then, the equivalent circuit of a short single-phase line is as shown in Fig. 41.19.

- Resistive drop =  $IR$  —in phase with  $I$
- Reactive drop =  $IX$  —in quadrature with  $I$

The vector diagram is shown in Fig. 41.20. From the right-angled triangle  $OMA$ , we get

$$OM^2 = OA^2 + AM^2 = (OK + KA)^2 + (AC + CM)^2$$

$$E_S = \sqrt{[(E_R \cos \phi_R + IR)^2 + (E_R \sin \phi_R + IX)^2]}$$

An approximate expression for the voltage drop is as follows :

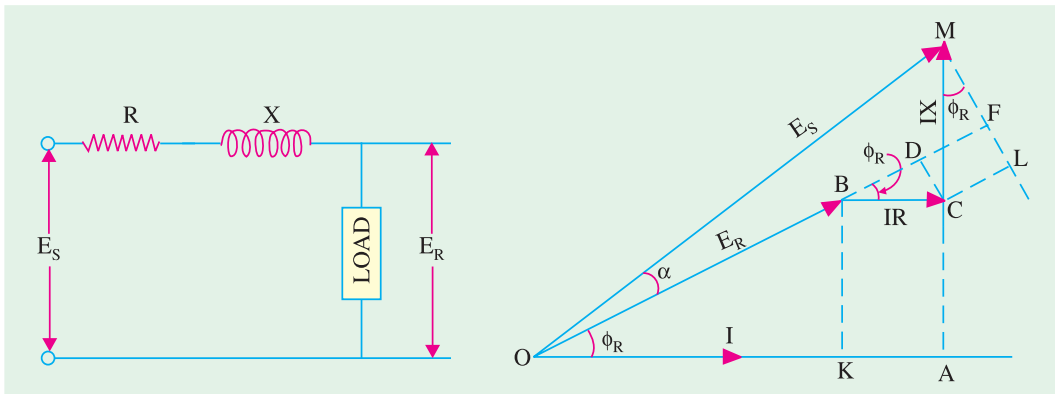


Fig. 41.19

Fig. 41.20

Draw the various dotted lines as in Fig. 41.20. Here,  $ML$  is  $\perp OF$ ,  $CL$  is  $\parallel OF$  and  $CD$  is  $\perp OF$ . As seen

$$OM = OF \text{ (approximately)}$$

$$= OD + DF = OB + BD + DF$$

or  $E_S = E_R + BD + DF = E_R + BD + CL = E_R + IR \cos \phi_R + IX \sin \phi_R$

or  $E_S - E_R = IR \cos \phi_R + IX \sin \phi_R \therefore \text{drop} = I(R \cos \phi_R + X \sin \phi_R) \text{ (approx.)}$

Voltage regn. of line =  $[(E_S - E_R)/E_R] \times 100$

**Solution in Complex Notation**

Let us take  $E_R$  as reference vector as shown in Fig. 41.21 so that  $E_R = (E_R + j0)$

As seen from  $\Delta OAB$ ,  $E_S$  is equal to the vector sum of  $E_R$  and  $IZ$  or  $E_S = E_R + IZ$

Now,  $I = I \angle -\phi_R = I (\cos \phi_R - j \sin \phi_R)$

Similarly,  $Z = Z \angle \theta = (R + jX)$

$\therefore E_S = E_R + IZ \angle \theta - \phi_R$

or  $E_S = E_R + I (\cos \phi_R - j \sin \phi_R) (R + jX)$

$= E_R + (IR \cos \phi_R + IX \sin \phi_R) + j (IX \cos \phi_R - IR \sin \phi_R)^2$

$\therefore E_S = \sqrt{(E_R + IR \cos \phi_R + IX \sin \phi_R)^2 + (IX \sin \phi_R - IR \sin \phi_R)^2}$

If the p.f. is leading, then

$I = I \angle \phi_R = I (\cos \phi_R + j \sin \phi_R)$

$\therefore E_S = E_R + IZ \angle \theta + \phi_R = E_R + I (\cos \phi_R + j \sin \phi_R) (R + jX)$

It may be noted that the sending end power factor angle is  $\phi_s = (\phi_R + \alpha)$ .

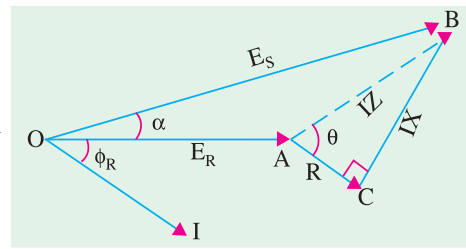


Fig. 41.21

**Example 41.8.** A single-phase line has an impedance of  $5 \angle 60^\circ$  and supplies a load of 120 A, 3,300 V at 0.8 p.f. lagging. Calculate the sending-end voltage and draw a vector diagram.

(City & Guides, London)

**Solution.** The vector diagram is similar to that shown in Fig. 41.21. Here

$E_R = 3,300 \angle 0^\circ, Z = 5 \angle 60^\circ$

Since  $\phi_R = \cos^{-1}(0.8) = 36^\circ 52', \therefore I = 120 \angle -36^\circ 52'$

Voltage drop  $= IZ = 120 \times 5 \angle 60^\circ - 36^\circ 52'$   
 $= 600 \angle 23^\circ 8' = 600 (0.9196 + j0.3928) = 551.8 + j 235.7$

$\therefore E_S = (3,300 + j0) + (551.8 + j 235.7) = 3,851.8 + j 235.7$

$E_S = \sqrt{3851.8^2 + 235.7^2} = 3,860 \text{ V}$

**Example 40.9.** An overhead, single-phase transmission line delivers 1100 kW at 33 kV at 0.8 p.f. lagging. The total resistance of the line is 10  $\Omega$  and total inductive reactance is 15  $\Omega$ . Determine (i) sending-end voltage (ii) sending-end p.f. and (iii) transmission efficiency.

(Electrical Technology-I, Bombay Univ.)

**Solution.** Full-load line current is  $I = 1100/33 \times 0.8 = 41.7 \text{ A}$

Line loss  $= I^2 R = 41.7^2 \times 10 = 17,390 \text{ W} = 17.39 \text{ kW}$

(iii) Transmission efficiency  $= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{1100 \times 100}{1100 + 17.39} = 98.5\%$

(i) Line voltage drop  $= IZ = 41.7(0.8 - j 0.6) (10 + j 15) = 709 + j 250$

Sending-end voltage is

$E_S = E_R + IZ = (33,000 + j0) + (709 + j 250)$   
 $= 33,709 + j 250 = 33,710 \angle 25^\circ$

Hence, sending-end voltage is 33.71 kV

(ii) As seen from Fig. 41.22,  $\alpha = 0^\circ 25'$

Sending-end p.f. angle is

$\theta + \alpha = 36^\circ 52' + 0^\circ 25' = 37^\circ 17'$

$\therefore \text{p.f.} = \cos 37^\circ 17' = 0.795 \text{ (lag).}$

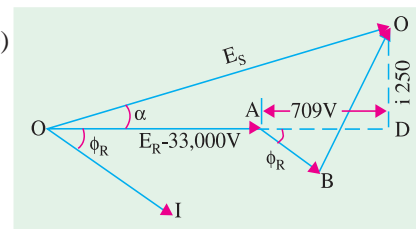


Fig. 41.22



**Note.** As seen from Fig. 41.22, approximate line drop is

$$\begin{aligned} &= I(R \cos \phi_R + X \sin \phi_R) = 41.7 (10 \times 0.8 + 15 \times 0.6) = 709 \text{ V} \\ \therefore E_S &= 33,000 + 709 = 33,709 \text{ V} \text{ —as above} \end{aligned}$$

**Example 41.10.** What is the maximum length in km for a 1-phase transmission line having copper conductors of  $0.775 \text{ cm}^2$  cross-section over which 200 kW at unity power factor and at 3300 V can be delivered? The efficiency of transmission is 90 per cent. Take specific resistance as  $(1.725 \times 10^{-8}) \Omega\text{-m}$ . **(Electrical Technology, Bombay Univ.)**

**Solution.** Since transmission efficiency is 90 per cent, sending-end power is  $200/0.9 \text{ kW}$ .

$$\text{Line loss} = (200/0.9 - 200) = 22.22 \text{ kW} = 22,220 \text{ W}$$

$$\text{Line current} = 200,000/3300 \times 1 = 60.6 \text{ A}$$

If  $R$  is the resistance of one conductor, then

$$2I^2R = \text{line loss} \quad \text{or} \quad 2 \times 60.62 \times R = 22,220 \quad \text{or} \quad R = 3.03 \text{ W}$$

$$\text{Now, } R = \rho \frac{l}{A} \quad \therefore 3.03 = 1.725 \times 10^{-8} \times l / 0.775 \times 10^{-4} \quad \therefore l = 13,600, m = \mathbf{13.6 \text{ km}}$$

**Example 41.11.** An industrial load consisting of a group of induction motors which aggregate 500 kW at 0.6 power factor lagging is supplied by a distribution feeder having an equivalent impedance of  $(0.15 + j0.6) \text{ ohm}$ . The voltage at the load end of the feeder is 2300 volts.

(a) Determine the load current.

(b) Find the power, reactive power and voltampere supplied to the sending end of the feeder.

(c) Find the voltage at the sending end of the feeder.

**(Electrical Technology, Vikram Univ., Ujjain)**

**Solution. (a)**  $I = 500 \times 10^3 / \sqrt{3} = 209 \text{ A}$

(c) Let  $V_R = (2300 + j0)$ ;  $I = 209 (0.6 - j 0.8)$

Voltage drop  $= \mathbf{IZ} = 209 (0.6 - j 0.8) (0.15 + j 0.6) = 119 + j50$

$$\mathbf{V_S} = \mathbf{V_R} + \mathbf{IZ} = 2300 + 119 + j 50 = 2419 + j 50 = 2420 \angle 1.2'$$

(b) Sending power  $= \sqrt{3} \times 2420 \times 209 \times 0.5835 = \mathbf{511.17 \text{ kW}}$

Sending-end reactive power  $= \sqrt{3} \times 2420 \times 209 \times 0.8121 = \mathbf{711.42 \text{ kVAR}}$

Sending-end volt ampere kVA  $= \sqrt{3} \times 2420 \times 209 = \mathbf{876 \text{ kVA}}$

or  $\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2} = \sqrt{511.17^2 + 711.42^2} = \mathbf{876}$

### 41.19. Short Three-phase Transmission Line Constants

Three-phase calculations are made in the same way as single-phase calculations because a 3-phase unit can be regarded as consisting of 3 single-phase units each transmitting one-third of the total power. In Fig. 41.23 is shown a 3-phase system in which each conductor has a resistance of  $R\Omega$  and inductance of  $X\Omega$ . In the case of a 3-phase system, it is advantageous to work in phase instead of in line-to-line values. In Fig. 41.24, one phase is shown separately. Since in a balanced load, there is no current in the neutral wire hence no current flows through the ground.

The resistive and reactive drops are calculated as before. By adding these drops **vectorially** to the receiving-end voltage, the sending-end voltage can be calculated. The line voltage at the sending-end can be found by multiplying this value by  $\sqrt{3}$ .

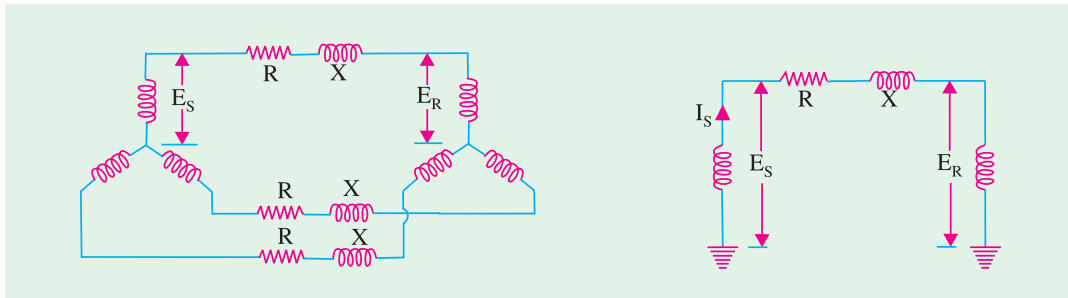


Fig. 41.23

Fig. 41.24

In the figure, a star-connected load is assumed. But if it is a delta-connected load then it can be replaced by an equivalent star-connected load.

**Example 41.12.** A 33-kV, 3-phase generating station is to supply 10 MW load at 31 kV and 0.9 power factor lagging over a 3-phase transmission line 3 km long. For the efficiency of the line to be 96%, what must be the resistance and reactance of the line?

(Electrical Power-III, Bangalore Univ.)

**Solution.** Power output = 10 MW ;  $\eta = 0.96$  ; Power input =  $10/0.96 = 10.417$  MW  
 Total loss = 0.417 MW

Now,  $I = 10 \times 10^6 / \sqrt{3} \times 31 \times 10^3 \times 0.9 = 207$  A

If R is the resistance per phase, then  $3 \times 207^2 \times R = 0.417 \times 10^6 \therefore R = 3.24 \Omega$

Now,  $V_S$  per phase =  $33/\sqrt{3} = 19.052$  kV and  $V_R$  per phase =  $31/\sqrt{3} = 17.898$  kV

Using the approximate relation of Art. 36-18, we get

$$V_S = V_R + I(R \cos \phi_R + X \sin \phi_R)$$

$$19,052 = 17,898 + 207(3.24 \times 0.9 + X \times 0.4368) \therefore X = 6.1 \Omega/\text{phase.}$$

**Example 41.13.** A balanced Y-connected load of  $(300 + j100) \Omega$  is supplied by a 3-phase line 40 km long with an impedance of  $(0.6 + j0.7) \Omega$  per km (line-to-neutral). Find the voltage at the receiving end when the voltage at the sending end is 66 kV. What is the phase angle between these voltages? Also, find the transmission efficiency of the line.

(Elect. Power Systems, Gujarat Univ.)

**Solution.** The circuit connections are shown in Fig. 41.25.

Resistance for 40 km conductor length =  $40 \times 0.6 = 24 \Omega$

Reactance for 40 km conductor length =  $40 \times 0.7 = 28 \Omega$

Total resistance/phase =  $300 + 24 = 324 \Omega$

Total reactance/phase =  $100 + 28 = 128 \Omega$

Total impedance/phase =  $\sqrt{324^2 + 128^2} = 348 \Omega$

Line current =  $\frac{66,000/\sqrt{3}}{348} = \frac{38,100}{348} = 110$  A

Now,  $\tan \phi_R = 100/300$  ;  $\phi_R = 18.4'$  ;  $\cos \phi_R = 0.949$ ,  $\sin \phi_R = 0.316$

Voltage drop in conductor resistance =  $110 \times 24 = 2640$  V

Voltage drop in conductor reactance =  $110 \times 28 = 3080$  V

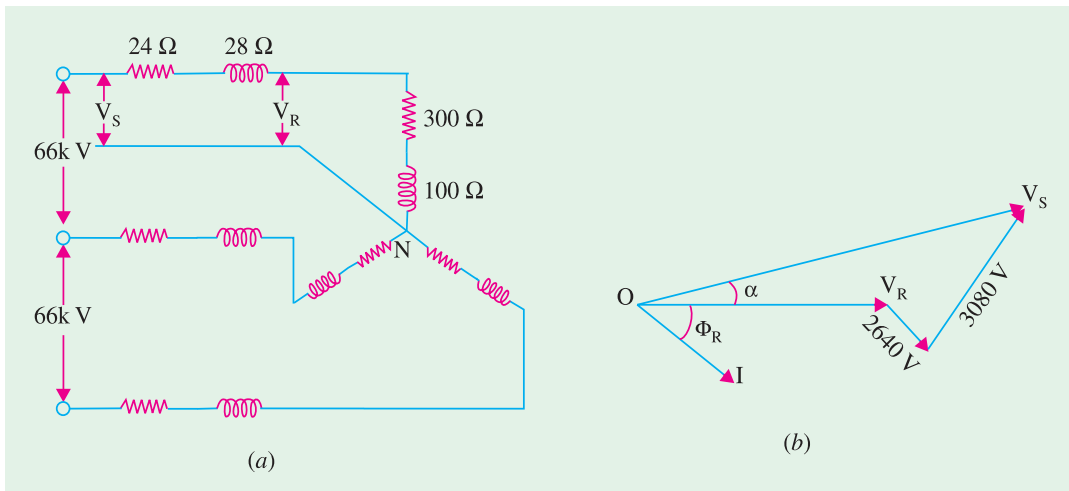


Fig. 41.25

It is seen from Art. 41.18 that

$$\begin{aligned} \mathbf{V}_S &= (V_R + IR \cos \phi_R + IX \sin \phi_R) + j(IX \cos \phi_R - IR \sin \phi_R) \\ &= (V_R + 2640 \times 0.949 + 3080 \times 0.316) + j(3080 \times 0.949 - 2640 \times 0.316) \\ &= (V_R + 3475) + j 2075 \end{aligned}$$

Now,  $\mathbf{V}_S = 66000/\sqrt{3} = 38,100 \text{ V}$

$\therefore 38,100^2 = (V_R + 3475)^2 + 2075^2, \quad V_R = 34,585 \text{ V}$

Line voltage across load =  $34,585 \times \sqrt{3} = 59.88 \text{ kV}$

$\therefore \mathbf{V}_S = (34,585 + 3475) + j 2075 = 38,060 + j 2075 = 38,100 \angle 3.1^\circ$

Obviously,  $V_S$  leads  $V_R$  by  $3.1^\circ$  as shown in Fig. 41.25 (b).

Power delivered to the load/phase =  $110^2 \times 300 \text{ W}$

Power transmitted/phase =  $110^2 \times 324 \text{ W}$

$\therefore$  transmission efficiency  $\eta = (300/324) \times 100 = 92.6\%$

**Example 41.14.** Define 'regulation' and 'efficiency' of a short transmission line.

A 3-phase, 50-Hz, transmission line having resistance of  $5\Omega$  per phase and inductance of  $30 \text{ mH}$  per phase supplies a load of  $1000 \text{ kW}$  at  $0.8$  lagging and  $11 \text{ kV}$  at the receiving end. Find.

(a) sending end voltage and power factor (b) transmission efficiency (c) regulation.

(Electrical Engineering-III, Poona Univ. 1990)

**Solution.** VR per phase =  $11,000/\sqrt{3} = 6,350 \text{ V}$

Line current, =  $1000 \times 10^3/\sqrt{3} \times 11,000 \times 0.8 = 65.6 \text{ A}$

$X_L = 2\pi \times 50 \times 30 \times 10^{-3} = 9.4 \Omega, R = 5\Omega; Z_{ph} = (5 + j 9.4) \Omega$

$\therefore$  drop per conductor =  $65.6 (0.8 - j 0.6) (5 + j 9.4) = 632 + j 296$

(a)  $V_S = 6350 + (632 + j 296) = 6,982 + j 296 = 6,988 \angle 2.4^\circ$

Sending-end line voltage =  $6,988 \times \sqrt{3} = 12,100 \text{ V} = 12.1 \text{ kV}$

As seen from Fig. 41.25 (b),  $\alpha = 2.4^\circ$ , Now,  $\cos \phi_R = 0.8, \phi_R = 36.9^\circ$

$\phi_S = 36.9^\circ + 2.4^\circ = 39.3^\circ, \cos \phi_S = 0.774 \text{ (lag)}$ .

(b) Total power loss =  $3 I^2 R = 3 \times 65.6^2 \times 5 = 64,550 \text{ W} = 64.55 \text{ kW}$

Input power =  $1000 + 64.55 = 1064.55 \text{ kW}$

∴ power transmission  $\eta = 1000/1064.55 = 0.9394$  or **93.94%**

(c) Now, line regulation is defined as the rise in voltage when full-load is thrown off the line divided by voltage at the load end.

∴ % regn. =  $\frac{(12.1 - 11)}{11} \times 100 = 10\%$

**Example 41.15.** A short 3- $\phi$  line with an impedance of  $(6 + j8) \Omega$  per line has sending and receiving end line voltages of 120 and 110 kV respectively for some receiving-end load at a p.f. of 0.9. Find the active power and the reactive power at the receiving end.

(Transmission and Distribution, Madras Univ.)

**Solution.** As seen from Art. 41.18, considering phase values, we have

$$V_S = V_R + I(R \cos \phi R + X \sin \phi R)$$

Now,  $V_S$  per phase =  $120/\sqrt{3} = 69,280$  V

Similarly,  $V_R$  per phase =  $110/\sqrt{3} = 63,507$  V

∴  $69,280 = 63,507 + I(6 \times 0.9 + 8 \times 0.435)$

∴ Line current  $I = 5,773/8.88 = 650$  A

Active power at receiving end =  $\sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 110 \times 650 \times 0.9 = 111,400$  kW

Reactive power at receiving end =  $\sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 110 \times 650 \times 0.435 = 53,870$  kVAR

**Example 41.16.** A 3-phase, 20 km line delivers a load of 10 MW at 11 kV having a lagging p.f. of 0.707 at the receiving end. The line has a resistance of  $0.02 \Omega/\text{km}$  phase and an inductive reactance of  $0.07 \Omega/\text{km}/\text{phase}$ . Calculate the regulation and efficiency of the line. If, now, the receiving-end p.f. is raised to 0.9 by using static capacitors, calculate the new value of regulation and efficiency.

(Electrical Engg. ; Bombay Univ.)

**Solution. (i) When p.f. = 0.707 (lag)**

Line current =  $10 \times 10^6/\sqrt{3} \times 11,000 \times 0.707 = 743$  A

$V_R$  per phase =  $11,000/\sqrt{3} = 6,352$  V

Total resistance/phase for 20 km =  $20 \times 0.02 = 0.4$  W

Total reactance/phase for 20 km =  $20 \times 0.07 = 1.4$  W

∴ Total impedance/phase =  $(0.4 + j 1.4) \Omega$

If  $V_R$  is taken as the reference vector, then drop per phase

$$= 743 (0.707 - j 0.707) (0.4 + j 1.4) = (945 + j 525)$$

∴  $V_S = 6,352 + 945 + j 525 = 7,297 + j 525$

or  $V_S = \sqrt{7297^2 + 525^2} = 7,315$  V

∴ % regulation =  $\frac{7,315 - 6,352}{6,352} \times 100 = 15.1 \%$

Total line loss =  $3I^2R = 3 \times 743^2 \times 0.4 = 662$  kW

Total output =  $10 + 0.662 = 10.662$  MW ∴  $\eta = 10 \times 100/10.662 = 94\%$

**(ii) When p.f. = 0.9 (lag)**

Line current =  $10^7/\sqrt{3} \times 11,000 \times 0.9 = 583$  A

Drop/phase =  $583 (0.9 - j 0.435) (0.4 + j 1.4) = 565 + j 633$

$$V_S = 6,352 + (565 + j 633) = 6,917 + j 633$$

∴  $V_S = \sqrt{6,917^2 + 633^2} = 6,947$  V

$$\begin{aligned} \therefore \% \text{ regulation} &= \frac{6,947 - 6,352}{6,352} \times 100 = \mathbf{9.37\%} \\ \text{Total line loss} &= 3 \times 5832 \times 0.4 = 408 \text{ kW} ; \text{ Total output} = 10.408 \text{ MW} \\ \therefore \eta &= 10 \times 100/10.408 = \mathbf{96.1\%} \end{aligned}$$

**Example 41.17.** A load of 1,000 kW at 0.8 p.f. lagging is received at the end of a 3-phase line 10 km long. The resistance and inductance of each conductor per km are 0.531 W and 1.76 mH respectively. The voltage at the receiving end is 11 kV at 50 Hz. Find the sending-end voltage and the power loss in the line. What would be the reduction in the line loss if the p.f. of the load were improved to unity? (Elect. Power Systems, Gujarat Univ.)

**Solution.**

$$\begin{aligned} \text{Line current} &= 1,000 \times 1,000/\sqrt{3} \times 11 \times 1,000 \times 0.8 = 65.6 \text{ A} \\ \text{Voltage/phase} &= 11,000/\sqrt{3} = 6,352 \text{ V} \\ \mathbf{X} &= 2\pi \times 50 \times 1.76 \times 10 \times \sqrt{3} \times 10 = 5.53 \Omega ; \mathbf{R} = 0.531 \times 10 = 5.31 \Omega \quad \therefore \mathbf{Z} = (5.31 + j 5.53) \\ \text{Voltage drop/phase} &= 65.6 (0.8 - j 0.6) (5.31 + j 5.53) = 496.4 + j 81.2 \\ \therefore \mathbf{V}_S &= 6,352 + 496.4 + j 81.2 = 6,848 + j 81 = 6,848 \text{ V numerically (approx.)} \\ \therefore \text{line-to-line sending-end voltage} &= 6,848 \times \sqrt{3} = 11,860 \text{ V} = \mathbf{11.86 \text{ kV}} \\ \text{Total loss} &= 3 \times 5.31 \times 65.62 = 68,550 \text{ W} = \mathbf{68.55 \text{ kW}} \\ \text{Line current for unity p.f.} &= 1,000/11 \times \sqrt{3} = 52.49 \text{ A} \\ \therefore \text{New losses} &= 3 \times 5.31 \times 52.492 = 43.89 \text{ kW} \\ \therefore \text{reduction} &= 68.55 - 43.89 = \mathbf{24.66 \text{ kW}} \end{aligned}$$

**Example 41.18.** Estimate the distance over which a load of 15,000 kW at 0.85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.905 W per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7.5% of the load. (Transmission and Distribution, Madras Univ.)

**Solution.**

$$\begin{aligned} \text{Line current} &= 15,000/132 \times \sqrt{3} \times 0.85 = 77.2 \text{ A} \\ \text{Total loss} &= 7.5\% \text{ of } 15,000 = 1,125 \text{ kW} \\ \text{If } R \text{ is the resistance of one conductor, then} & \\ 3 I^2 R &= 1,125,000 \quad \text{or} \quad 3 \times (77.2)^2 \times R = 1,125,000 ; R = 62.94 \Omega \\ \text{Length of the line} &= 62.94/0.905 = \mathbf{69.55 \text{ km.}} \end{aligned}$$

**Example 41.19.** A 3- $\phi$  line has a resistance of 5.31  $\Omega$  and inductance of 0.0176 H. Power is transmitted at 33 kV, 50-Hz from one end and the load at the receiving end is 3,600 kW at 0.8 p.f. lagging. Find the line current, receiving-end voltage, sending-end p.f. and efficiency of transmission. (Transmission and Distribution-I, Madras Univ.)

**Solution.**

$$\begin{aligned} V_S &= V_R + I_R \cos \phi_R + IX \sin \phi_R \text{ approximately} \\ \text{Now, power delivered/phase} &= V_R I \cos \phi_R \\ \therefore 1,200 \times 1,000 &= V_R \cdot I \cdot 0.8 \quad \therefore I = 15 \times 10^5/V_R \\ \text{Also, } V_S \text{ per phase} &= 33,000/\sqrt{3} = 19,050 \text{ V} \\ R &= 5.31 \Omega ; X = 0.0176 \times 314 = 5.54 \Omega \\ \therefore 19,050 &= V_R + \frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8 + \frac{15 \times 10^5}{V_R} \times 5.54 \times 0.6 \\ \therefore V_R^2 - 19,050 V_R + 11,358,000 &= 0 \text{ or } V_R = \frac{19,050 \pm 17,810}{2} = \frac{36,860}{2} = 18,430 \text{ V} \\ \text{Line voltage at the receiving end} &= 18,430 \times \sqrt{3} = \mathbf{32 \text{ kV}} \\ I &= 15 \times 10^5/18,430 = \mathbf{81.5 \text{ A}} \end{aligned}$$

$$\begin{aligned}
 \text{Now,} \quad V_S &= V_R + I(\cos \phi_R - j \sin \phi_R)(R + jX) \\
 &= 18,430 + 81.5(0.8 - j0.6)(5.31 + j5.54) \\
 &= 18,430 + 615 + j100 = 19,050 \angle 0.3^\circ \\
 \phi_R &= \cos^{-1}(0.8) = 36^\circ 52'; \\
 \phi_S &= 36^\circ 52' + 18' = 37^\circ 10' \\
 \therefore \text{ sending-end p.f.} &= \cos \phi_S = \cos 37^\circ 10' = \mathbf{0.797 A} \\
 \text{Power lost in line} &= 3 I^2 R = 3 \times 81.52 \times 5.31 = 106 \text{ kW} \\
 \text{Power at sending end} &= 3,600 + 106 = 3,706 \text{ kW} \\
 \text{transmission} \quad \eta &= 3,600 \times 100/3,706 = \mathbf{97.2\%}
 \end{aligned}$$

**Example 41.20.** A 3-phase short transmission line has resistance and reactance per phase of  $15\Omega$  and  $20\Omega$  respectively. If the sending-end voltage is  $33\text{ kV}$  and the regulation of the line is not to exceed  $10\%$ , find the maximum power in kW which can be transmitted over the line. Find also the kVAR supplied by the line when delivering the maximum power.

**Solution.** As seen from Art. 41.18

$$\begin{aligned}
 V_S^2 &= (V_R + I_R \cos \phi_R + IX \sin \phi_R)^2 + (IX \cos \phi_R - I_R \phi_R)^2 \\
 &= V_R^2 + 2IV_R(R \cos \phi_R + X \sin \phi_R) + I^2(R^2 + X^2) \\
 \text{real power/phase,} \quad P &= V_R I \cos \phi_R; \text{ reactive power/ phase, } Q = V_R I \sin \phi_R \\
 \therefore P^2 + Q^2 &= V_R^2 I^2 \quad \text{or} \quad I^2 = (P^2 + Q^2)/V_R^2 \\
 \text{Substituting this value above, we get}
 \end{aligned}$$

$$\begin{aligned}
 V_S^2 &= V_R^2 + 2PR + 2QX + \frac{P^2 + Q^2}{V_R^2} (R^2 + X^2) \\
 \text{or} \quad V_S^2 &= V_R^2 - 2PR - 2QX - \frac{P^2 + Q^2}{V_R^2} (R^2 + X^2) = 0 \quad \dots(i)
 \end{aligned}$$

To find the maximum power transmitted by the line, differentiate the above equation w.r.t.  $Q$  and put  $dP/dQ = 0$  (treating  $V_S$  and  $V_R$  as constants).

$$\therefore -2R \frac{dP}{dQ} - 2X - 2P \cdot \frac{(R^2 + X^2)}{V_R} \cdot \frac{dP}{dP} - 2Q \frac{(R^2 + X^2)}{V_R^2} = 0$$

$$\text{Since} \quad dP/dQ = 0 \quad \therefore 2X + 2Q \frac{(R^2 + X^2)}{V_R^2} = 0 \quad \text{or} \quad Q = \frac{-V_R^2 X}{R^2 + X^2} = \frac{-V_R^2 X}{Z^2}$$

Putting this value of  $Q$  in Eq. (i) above we have

$$\begin{aligned}
 \frac{P^2 Z^2}{V_R^2} + 2PR - V_S^2 + V_R^2 - \frac{V_R^2 X^2}{Z^2} &= 0 \\
 \text{or} \quad \frac{P^2 Z^2}{V_R^2} + 2PR - V_S^2 + V_R^2 \left(1 - \frac{X^2}{Z^2}\right) &= 0 \\
 \text{or} \quad \frac{P^2 Z^2}{V_R^2} + 2PR + \frac{V_S^2 + V_R^2}{Z^2} - V_S &= 0
 \end{aligned}$$

$$\text{Solving for } P, \text{ we get} \quad P_{max} = \frac{V_R^2}{Z^2} \left( Z \cdot \frac{V_S}{V_R} - R \right) \text{ watts/phase}$$

In the present case,  $V_S = 33,000/\sqrt{3} = 19,050\text{ V}$ ; Since regulation is limited to  $10\%$ ,

$$\therefore V_R + 10\% V_R = V_S \quad \text{or} \quad 1.1 V_R = 19,050; V_R = 17,320\text{ V}; Z = \sqrt{15^2 + 20^2} = 25\Omega$$

$$\begin{aligned} \therefore P_{\max} \text{ per phase} &= \left( \frac{17,320}{25} \right)^2 \left( 25 \times \frac{19,050}{17,320} - 15 \right) = 6 \times 10^6 \text{ W} = 6 \text{ MW} \\ \text{Total maximum power} &= 3 \times 6 = \mathbf{18 \text{ MW}} \\ \text{kVAR supplied/phase } Q &= \frac{V_R^2 X}{Z^2} = - \frac{17,320^2 \times 20}{25^2} \times 10^{-3} = 9,598 \\ \text{Total kVAR supplied} &= 3 \times 9,598 = \mathbf{28,794.} \end{aligned}$$

**Example 41.21.** A 3- $\phi$ , 50-Hz generating station supplies a load of 9,900 kW at 0.866 p.f. (lag) through a short overhead transmission line. Determine the sending-end voltage if the receiving-end voltage is 66 kV and also the efficiency of transmission. The resistance per km is  $4\Omega$  and inductance 40 mH. What is the maximum power in kVA that can be transmitted through the line if both the sending and receiving-end voltages are kept at 66 kV and resistance of the line is negligible.

**Solution.**

$$R = 4\Omega; X = 40 \times 10^{-3} \times 314 = 12.56\Omega$$

Line current  $I = 9,900/\sqrt{3} \times 66 \times 0.866 = 100 \text{ A}$

$$V_R = 66,000/\sqrt{3} = 38,100 \text{ V}; \cos \phi_R = 0.866; \sin \phi_R = 0.5$$

$$V_S = V_R + I(R \cos \phi_R + X \sin \phi_R) = 38,100 + 100(4 \times 0.866 + 12.56 \times 0.5) = 39,075 \text{ V}$$

Line value of sending-end voltage  $= 39,075 \times \sqrt{3} = \mathbf{67.5 \text{ kV}}$

$$\text{Total line loss} = 3I^2R = 3 \times 100^2 \times 4 = 120 \text{ kW}$$

$$\therefore \eta = 9,900/(9,900 + 120) = 0.988 \text{ or } 98.8\%$$

$$\text{Max. value of } Q \text{ for 3-phases} = \frac{3 \cdot V_R^2}{Z^2} \cdot X$$

Now,  $V_S = V_R = 38,100 \text{ V}$  and resistance is negligible.

$$\therefore \text{Max. value of } Q = \frac{3 \cdot V_R^2}{Z^2} \cdot X \cdot 10^{-3} = 3 \times 3,810 \times 10^{-3} / 12.56 = \mathbf{3,48,000 \text{ kVA}}$$

**Example 41.22.** A 3-phase load of 2,000 kVA, 0.8 p.f. is supplied at 6.6 kV, 50-Hz by means of a 33 kV transmission line 20 km long and a 5 : 1 transformer. The resistance per km of each conductor is  $0.4\Omega$  and reactance  $0.5\Omega$ . The resistance and reactance of the transformer primary are  $7.5\Omega$  and  $13.2\Omega$ , whilst the resistance of the secondary is  $0.35\Omega$  and reactance  $0.65\Omega$ . Find the voltage necessary at the sending end of transmission line when 6.6 kV is maintained at the load-end and find the sending-end power factor. Determine also the efficiency of transmission.

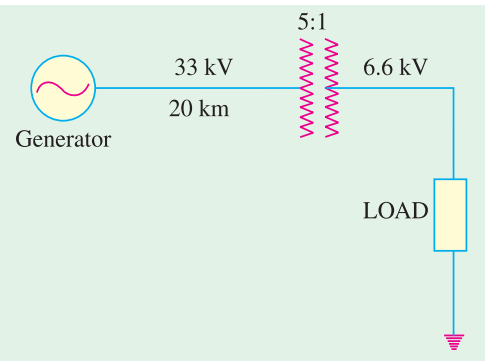


Fig. 41.26

**Solution.** One phase of the system is shown in Fig. 41.26. Impedance per phase of high voltage line  $= (8 + j 10)$

$$\begin{aligned} \text{Impedance of the transformer primary} \\ &= (7.5 + j 13.2) \text{ ohm} \end{aligned}$$

$$\begin{aligned} \text{Total impedance on the high-tension side} \\ &= (8 + j 10) + (7.5 + j 13.2) = 15.5 + j 23.2 \end{aligned}$$

This impedance can be transferred to secondary side by using the relation given in Art. 30.12.

$$\begin{aligned} \text{Hence, impedance as referred to secondary side is} \\ &= (15.5 + j 23.2)/5^2 = 0.62 + j 0.928 \end{aligned}$$

Adding to them the impedance of the transformer secondary, we get the total impedance as referred to low-voltage side

$$= (0.62 + j 0.928) + (0.35 + j 0.65) = 0.97 + j 1.578$$

Now, kVA load per phase =  $2,000/3.0 = 667$

Receiving-end voltage/phase =  $6.6/\sqrt{3} = 3.81$  kV

$\therefore$  current in the line =  $667/3.81 = 175$  A

Drop per conductor =  $I(R \cos \phi + X \sin \phi) = 175(0.97 \times 0.8 + 1.578 \times 0.6) = 302$  V

Now,  $E_S = E_R + I(R \cos \phi + X \sin \phi)$

Hence, sending-end voltage (phase to neutral as referred to the lower voltage side) is  $3,810 + 302 = 4,112$  V. As referred to high-voltage side, its value =  $4,112 \times 5 = 20,560$  V

Line voltage =  $20,560 \times \sqrt{3}/1000 = 35.6$  kV

If  $\phi_S$  is the power factor angle at the sending-end, then

$$\tan \phi_S = \frac{\sin \phi + (IX/E_B)}{\cos \phi + (IR/E_R)} = \frac{0.6 + (175 \times 1.578/3810)}{0.8 + (175 \times 0.97/3.10)} = 0.796$$

$\therefore \phi_S = \tan^{-1}(0.796) = 38^\circ 31'$   $\therefore \cos \phi_S = \cos 38^\circ 31' = 0.782$

Power loss/phase =  $175^2 \times 0.97/1000 = 29.7$  kW

Power at the receiving end/phase =  $2000 \times 0.8/3 = 533.3$  kW

$\therefore$  transmission efficiency =  $\frac{533.3 \times 100}{533.3 + 29.7} = 94.7\%$

### Tutorial Problem No. 41.1

- 500 kW at 11kV are received from 3-phase transmission line each wire of which has a resistance of  $1.2 \Omega$  and a reactance of  $1 \Omega$ . Calculate the supply pressure when the power factor of the load is (i) unity and (ii) 0.5 leading. **[(i) 11,055 V (ii) 10,988 V]**
- What load can be delivered by a 3-phase overhead line 5 km long with a pressure drop of 10%. Given that the station voltage is 11 kV, resistance per km of each line  $0.09 \Omega$ , reactance per km  $0.08 \Omega$  and the power factor of the load 0.8 lagging. **[14,520 kW]**
- Estimate the distance over which a load of 15,000 kW at 0.85 power factor can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance  $0.56 \Omega$  per km. The p.d. at the receiving end is to be 132 kV and the loss in transmission is not to exceed 7.5% **[121.9 km] (I.E.E. London)**
- A d.c. 2-wire system is to be converted into 3-phase, 3-wire a.c. system by adding a third conductor of the same size as the two existing conductors. Calculate the percentage additional balanced load that can now be carried by the conductors at 0.96 p.f. lagging. Assume the same voltage between the conductors and the same percentage power loss. **[84%]**
- A 3-phase short transmission line of resistance  $8 \Omega$  and reactance  $11 \Omega$  per phase is supplied with a voltage of 11 kV. At the end of the line is a balanced load of  $P$  kW per phase at a p.f. of 0.8 leading. For what value of  $P$  is the voltage regulation of the line zero? **[210 kW] (Electrical Technology, M.S. Univ. Baroda)**
- A 3-ph, 50-Hz transmission line 10 km long delivers 2,500 kVA at 10 kV. The p.f. of the load is 0.8 (lag). The resistance of each conductor is  $0.3 \text{ W/km}$  and the inductance  $1.82 \text{ mH/km}$ . Find (a) the voltage and p.f. at the sending end (b) the efficiency of transmission and (c) the percentage regulation of the line. **[(a) 11.48 kV ; 0.763 (b) 91.4% (c) 14.8%]**
- The conductors in a single-phase transmission line are 6 m above the ground. Each conductor has 1.5 cm diameter and the conductors are spaced 3 m apart. Starting from the fundamentals, determine the capacitance per kilometre of the line first including and then excluding the effect of ground. What do you conclude ? **[ $4.60 \times 10^{-3} \mu\text{F/km}$  ;  $4.63 \times 10^{-3} \mu\text{F/km}$ ] (Ranchi Univ.)**



41.20. Effect of Capacitance

So far we have neglected the effect of capacitance on the line regulation because the capacitances of short lines transmitting at relatively low voltages (up to 20 kV) are negligible. But as the voltage and length of the transmission line increase, the capacitance gradually becomes of greater importance. Similarly, the leakage across insulators also assumes greater importance. Hence, exact calculations of regulation for long lines take into consideration the capacitance and leakage reactance of the lines and are quite elaborate, the amount of elaboration depending on the transmitting voltage.

(i) In the case of short lines, ordinarily, the capacitance is negligible. But if in a problem, the line capacitance is given and if the line is less than 80 km, then the line capacitance can be lumped at the receiving or load end as shown in Fig. 41.27 (a) although this method of localizing the line capacitance at the load end over-estimates the effect of capacitance. In that case, the line current  $I_S$  is the vector sum of the load current  $I_R$  and the charging current  $I_C$  of the capacitance. Hence,  $I_S = I_R + I_C$

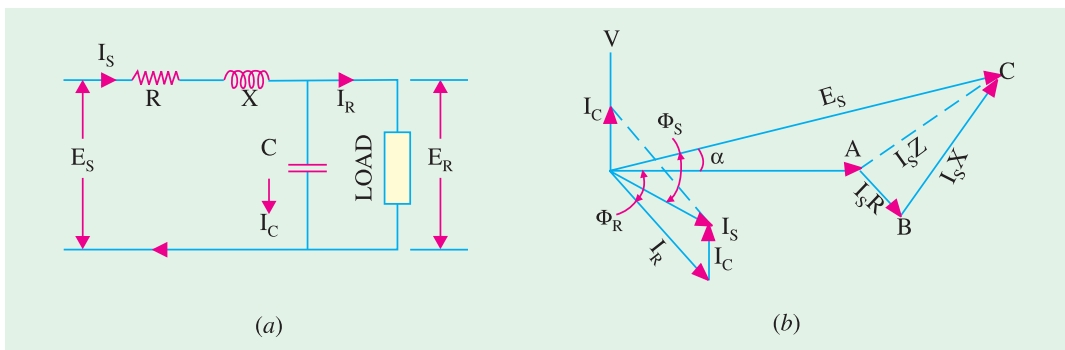


Fig. 41.27

Now, charging current  $I_C = j\omega CE_R \therefore I_R = I_R (\cos \phi_R - j \sin \phi_R)$   
 $\therefore I_S = I_R \cos \phi_R - jI_R \sin \phi_R + j CE_R = I_R \cos \phi_R + j(-I_R \sin \phi_R + \omega CE_R)$   
 Line drop  $= I_S(R + jX) \therefore E_S = E_R + I_S(R + jX)$

The vector diagram is shown in Fig. 41.27 (b).

(ii) In the case of lines with voltages up to 100 kV and 150 km in length, satisfactory solutions can be obtained by the so-called T-method and pi or pi method as described below.

**Example 41.23.** A (medium) single-phase transmission line 50 km long has the following constants :

resistance/km = 0.5 Ω; reactance/km = 1.6 Ω  
 susceptance/km = 28 × 10<sup>-6</sup> S; receiving-end line voltage = 66,000 V

Assuming that total capacitance of the line is located at receiving end alone, determine the sending-end voltage, the sending-end current and regulation. The line is delivering 15,000 kW at 0.8 p.f. lagging. Draw a vector diagram to illustrate your answer.

**Solution.** Let  $E_S$  and  $E_R$  be sending-end and receiving-end voltages respectively as shown in Fig. 41.28.

Load current at the receiving-end is  $I_R = 15 \times 10^4 / 66 \times 10^3 \times 0.8 = 284$  A

Total resistance = 0.5 × 50 = 25 Ω ; Total reactance = 1.6 × 50 = 80 Ω

Susceptance B = 28 × 10<sup>-6</sup> × 50 = 14 × 10<sup>-4</sup> Siemens

Capacitive admittance Y = B = 14 × 10<sup>-4</sup> Siemens

As seen from vector diagram of Fig. 41.29, sending-end current  $I_S$  is the vector sum of load current  $I_R$  and capacitive current  $I_C$ .

Now,  $I_C = E_R Y = 66,000 \times 14 \times 10^{-4} = 92 \text{ A}$

Let  $E_R = (66,000 + j 0)$

$I_R = 284 (0.8 - j 0.6) = 227 - j 190$ ;  $I_C = j 92$

$I_S = I_R + I_C = (227 - j 190 + j 92) = 240 \angle -18^\circ 57'$ ;  $Z = 25 + j 80 = 84 \angle 72^\circ 36'$

Line drop =  $I_S Z = 240 \angle -18^\circ 57' \times 84 \angle 72^\circ 36' = 20,160 \angle 53^\circ 39' = 11,950 + j 16,240$

$E_S = E_R + I_S Z = 66,000 + (11,950 + j 16,240) = 79,500 \angle 11^\circ 44'$

Regulation =  $\frac{E_S - E_R}{E_S} \times 100 = \frac{79,500 - 66,000}{66,000} \times 100 = 20.5\%$

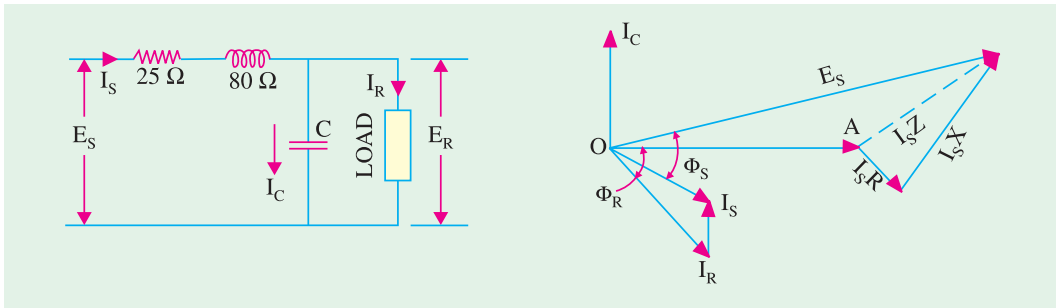


Fig. 41.28

Fig. 41.29

41.21. "Nominal" T-method

In this T-method, also known as mid-capacitor method, the whole of the line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 41.30.

It is seen that  $E_1 = E_R + I_R Z_{BC}$ . Knowing  $E_1$ , we can find  $I_C$  as under.

$I_C = j\omega C E_1 \therefore I_S = I_C + I_R$

Obviously, current through portion AB is  $I_S$ ; hence voltage drop =  $I_S Z_{AB}$

$\therefore E_S = E_1 + I_S Z_{AB}$

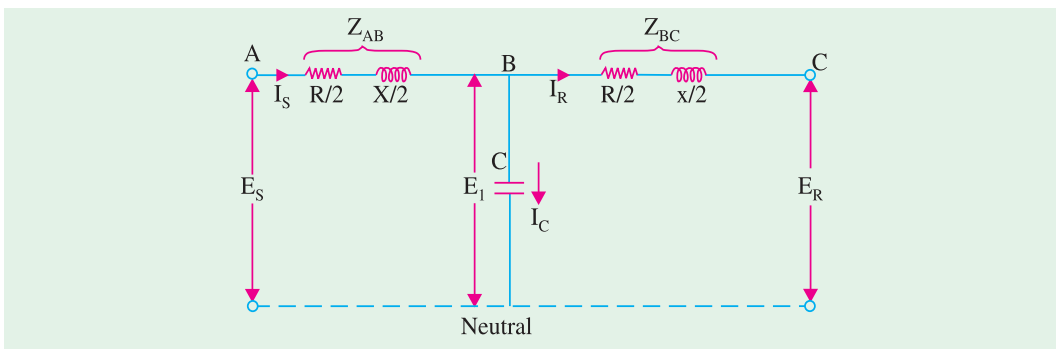


Fig. 41.30

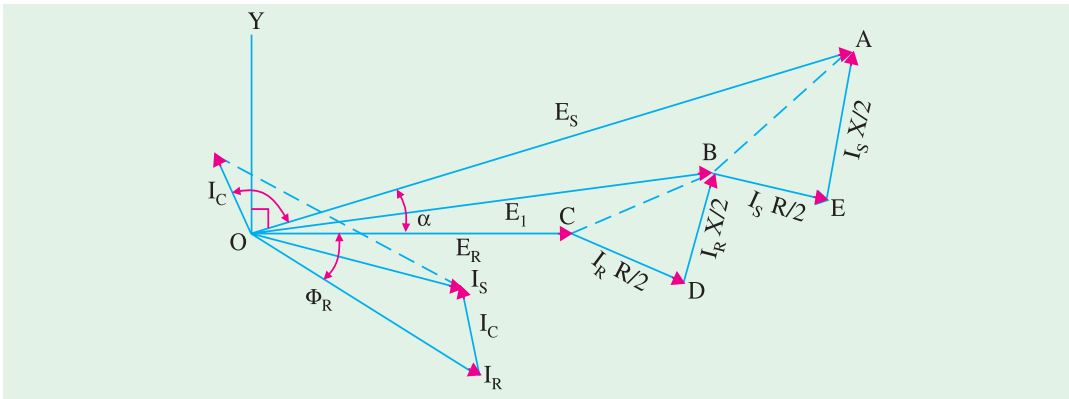


Fig. 41.31

The vector diagram is shown in Fig. 41.31. Receiving-end voltage  $E_R$  is taken as the reference vector. It may be pointed out here that all values are phase values *i.e.* line to neutral values.  $I_R$  is the load current lagging  $E_R$  by  $\phi_R$ ,  $CD = I_R \cdot R/2$  and parallel to  $I_R$ .  $BD = I_R \cdot X/2$  and perpendicular to  $I_R$ .  $OB$  represents  $E_1$  *i.e.* the voltage across the middle capacitor.  $BE$  represents  $I_S \cdot R/2$  and is parallel to  $I_S$ . Similarly,  $EA = I_S \cdot X/2$  and perpendicular to  $I_S$ .  $OA$  represents the voltage at the sending end.

It may be noted that if leakage is appreciable, then leakage conductance  $G$  can be assumed to be concentrated at the middle point of the line and can be represented by non-inductive conductance  $G$  shunting the middle capacitor as shown in Fig. 41.32.

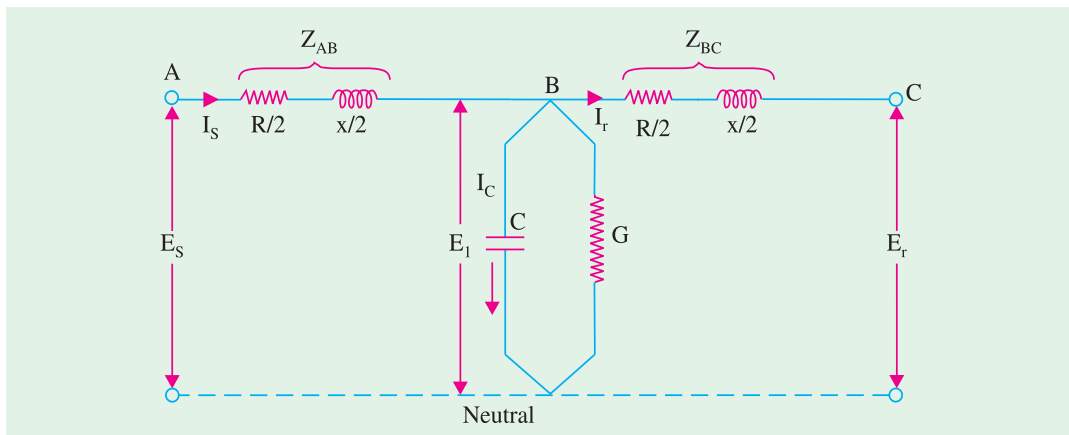


Fig. 41.32

**Example 41.24.** A 3-phase, 50-Hz overhead transmission line 100 km long with 132 kV between lines at the receiving end has the following constants :

resistance/km/phase = 0.15  $\Omega$  ;      inductance/km/phase = 1.20 mH  
 capacitance/km/phase = 0.01 mF

Determine, using an approximate method of allowing for capacitance, the voltage, current and p.f. at the sending end when the load at the receiving end is 72 MW at 0.8 p.f. lagging. Draw vector diagram for the circuit assumed. (Electrical Power System ; Gujarat Univ.)

**Solution.** For a 100-km length of the line,

$$R = 0.15 \times 100 = 15 \Omega ; X_L = 314 \times 1.2 \times 10^{-3} \times 100 = 37.7 \Omega$$

$$X_C = 106/314 \times 0.01 \times 100 = 3187 \Omega$$

Using the nominal  $T$ -method, the equivalent circuit is shown in Fig. 41.33 (a)

$$V_R = 132/\sqrt{3} = 76.23 \text{ kV} = 76,230 \text{ V. Hence, } \mathbf{V}_R = 76,230 + j 0$$

Load current,  $\mathbf{I}_R = 72 \times 102/\sqrt{3} \times 132 \times 103 \times 0.8 = 394 \text{ A}$

$$\therefore \mathbf{I}_R = 394 (0.8 - j 0.6) = 315 - j 236 \text{ A ; } \mathbf{Z}_{BC} = (7.5 + j 18.85) \text{ W}$$

Drop/phase over  $BC = \mathbf{I}_R \mathbf{Z}_{BC} = (315 - j 236) (7.5 + j 18.85) = 6802 + j 4180$

$$\mathbf{V}_1 = \mathbf{V}_R + \mathbf{I}_R \mathbf{Z}_{BC} = (76,230 + j 0) + (6802 + j 4180) = 83,030 + j 4180$$

$$\mathbf{I}_C = \frac{\mathbf{V}_1}{\mathbf{X}_C} = \frac{83,030 + j 4180}{-j 3187} = -1.31 + j 26$$

$$\mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R = (-1.31 + j 26) + (315 - j 236) = (313.7 - j 210) = 377.3 \angle -33.9^\circ$$

Drop/phase over  $AB = \mathbf{I}_S \mathbf{Z}_{AB} = (313.7 - j 210) (7.5 + j 18.85) = 6320 + j 4345$

$$\therefore \mathbf{V}_S = \mathbf{V}_1 + \mathbf{I}_S \mathbf{Z}_{AB} = (83,030 + j 4180) + (6320 + j 4345) = 89,350 + j 8525 = 89,750 \angle 5.4^\circ$$

Line value of sending-end voltage =  $\sqrt{3} \times 89,750 \times 10^{-3} = 155.7 \text{ kV}$

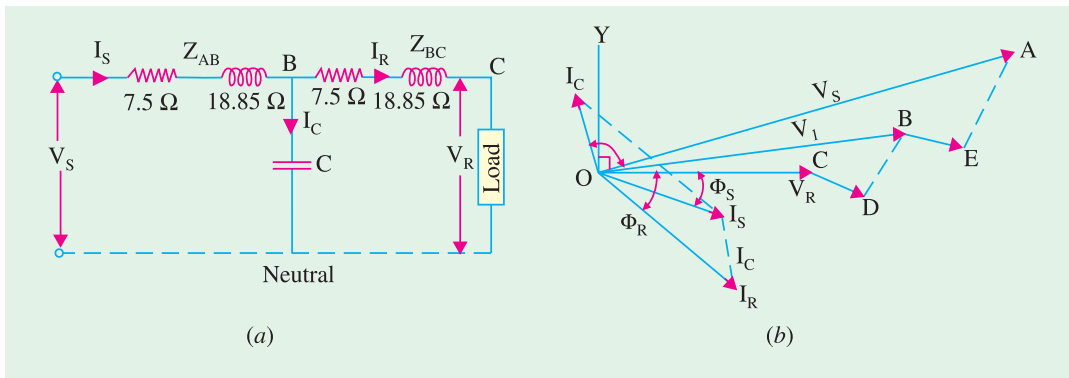


Fig. 41.33

Phase difference between  $V_S$  and  $I_S = 33.9^\circ + 5.4^\circ = 39.3^\circ$  with current lagging as shown in Fig. 41.33 (b)  $\cos \phi_S = \cos 39.3^\circ = 0.774$  (lag)

**Example 41.25.** A 3-phase, 50-Hz transmission line, 100 km long delivers 20 MW at 0.9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0.2 Ω and 0.4 Ω respectively while the capacitive admittance is  $2.5 \times 10^{-6} \text{ S}$  per km. Calculate (a) the voltage and current at the sending end and (b) the efficiency of transmission. Use the nominal  $T$ -method.

(Electrical Power-I, M.S. Univ. Baroda)

**Solution.** Resistance for 100 km =  $0.2 \times 100 = 20 \text{ } \Omega$

Reactance for 100 km =  $0.4 \times 100 = 40 \text{ } \Omega$

Capacitive admittance for 100 km =  $2.5 \times 10^{-6} \times 100 = 2.5 \times 10^{-4} \text{ S}$

Let us take the receiving-end voltage  $E_R$  as reference vector.

$$E_R = 110/\sqrt{3} = 63.5 \text{ kV ; } I_R = 20 \times 106/\sqrt{3} \times 110 \times 103 \times 0.9 = 116.6 \text{ A}$$

$$\cos \phi_R = 0.9 ; \sin \phi_R = 0.435 \text{ (from tables)}$$

With reference to Fig. 41.34, we have

$$\mathbf{E}_R = (63.5 + j 0) \text{ kV ; } \mathbf{I}_R = 116.6(0.9 - j 0.435) = 105 - j 50.7 ; \mathbf{Z}_{BC} = (10 + j 20)$$

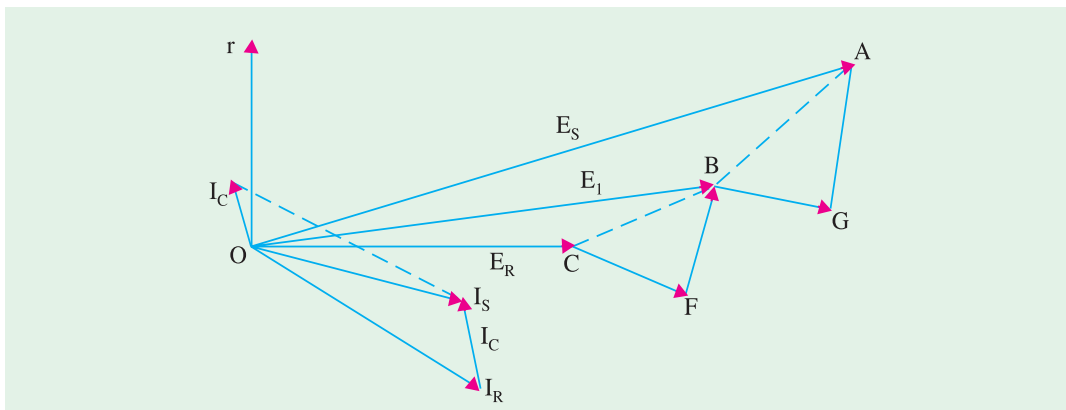


Fig. 41.34

Voltage drop between points  $B$  and  $C$  is

$$\mathbf{V}_{BC} = \mathbf{I}_R \mathbf{Z}_{BC} = (105 - j 50.7) (10 + j 20) = (2,064 + j 1,593) \text{ V}$$

$$\mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R \mathbf{Z}_{BC} = (63,500 + 2,064 + j 1,593) = 65,564 + j 1,593$$

$$\mathbf{I}_C = \mathbf{E}_1 \mathbf{Y} = (65,564 + j 1,593) \times j 2.5 \times 10^{-4} = (-0.4 + j 16.4) \text{ A}$$

$$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (105 - j 50.7) + (-0.4 + j 16.4) = (104.6 - j 34.3) = 110.1 \angle 18^\circ 9'$$

Drop between points  $A$  and  $B$  is

$$\mathbf{V}_{AB} = \mathbf{I}_S \mathbf{Z}_{AB} = (104.6 - j 34.3) (10 + j 20) = 1,732 + j 1,749$$

$$\mathbf{E}_S = \mathbf{E}_1 + \mathbf{V}_{AB} = (65,564 + j 1,593) + (1,732 + j 1,749) = 67,296 + j 3,342 = 67,380 \angle 2^\circ 51'$$

(a) Sending-end voltage (line value) is  $67,380 \times \sqrt{3} = 116,700 \text{ V} = \mathbf{116.7 kV}$

Sending-end current =  $\mathbf{110.1 A}$

**Note.** Phase difference between  $E_S$  and  $I_S$  is  $(-18^\circ 9' - 2^\circ 51') = -21^\circ$  with current lagging. Hence, p.f. at sending-end is  $\cos \phi_s = \cos 21^\circ = 0.934$  (lag).

(b) Copper loss for three phases between points  $B$  and  $C$  (Fig. 41.32) is

$$= 3 \times 116.6^2 \times 10 = 0.408 \text{ MW}$$

Copper loss for three phases between points  $A$  and  $B$  is  $3 \times 110.1^2 \times 10 = 0.363 \text{ MW}$

Total Cu loss for 100 km of line length =  $0.408 + 0.363 = 0.771 \text{ MW}$

Transmission  $\eta = 20 \times 100 / 20.771 = \mathbf{96.27 \%}$

**Note.** Line losses could also be computed by finding the line input which =  $\sqrt{3} E_S I_S \cos \phi_s$ .

#### 41.22. "Nominal" $\pi$ -method

In this method, the line-to-neutral capacitance is divided into two halves ; one half being concentrated or localized at the sending-end and the other half at the receiving-end as shown in Fig. 41.35 (a). The capacitance at the sending or generating end has no effect on line drop or line regulation but its charging current must be added to the line current in order to obtain the total sending-end current  $I_S$ .

It is obvious that  $I_S$  is the vector sum of  $I_{C2}$  and  $I_L$  where  $I_L$  is the vector sum of  $I_{C1}$  and  $I_R$ . The vector diagram is shown in Fig. 41.35 (b).  $E_R$  is taken as the reference vector. Current  $I_L$  is vector sum of  $I_R$  and  $I_{C1}$  (which is ahead of  $E_R$  by  $90^\circ$ ). The drop  $AB = I_L R$  is in phase with vector for  $I_L$  and reactive drop  $BC = I_L X$  is in quadrature with  $I_L$ .  $OC$  represents the sending-end voltage. The sending-end current  $I_S$  is the vector sum of  $I_L$  and  $I_{C2}$  (which itself is ahead of  $E_S$  by  $90^\circ$ ). It may be noted that

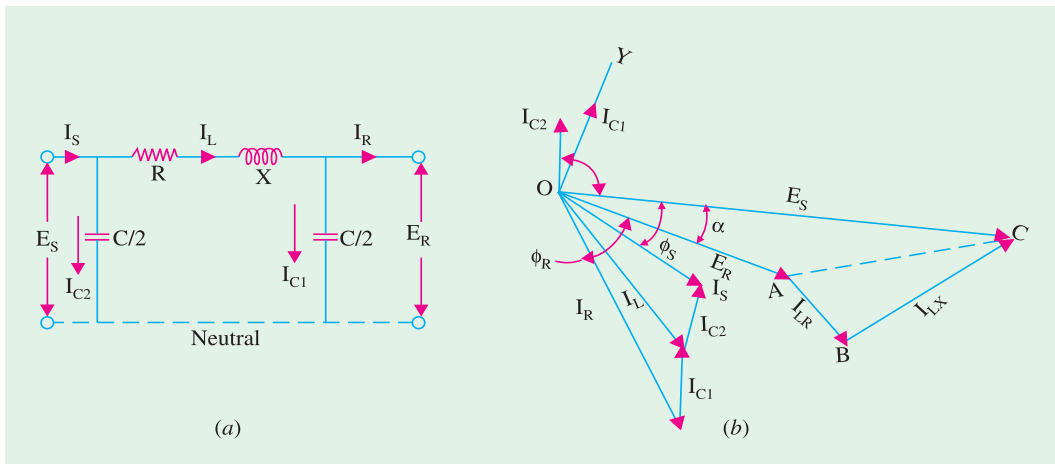


Fig. 41.35

if leakage reactance is not negligible, then leakage conductance  $G$  can also be divided into two equal halves and put at both ends in parallel with the capacitors as shown in Fig. 41.36.

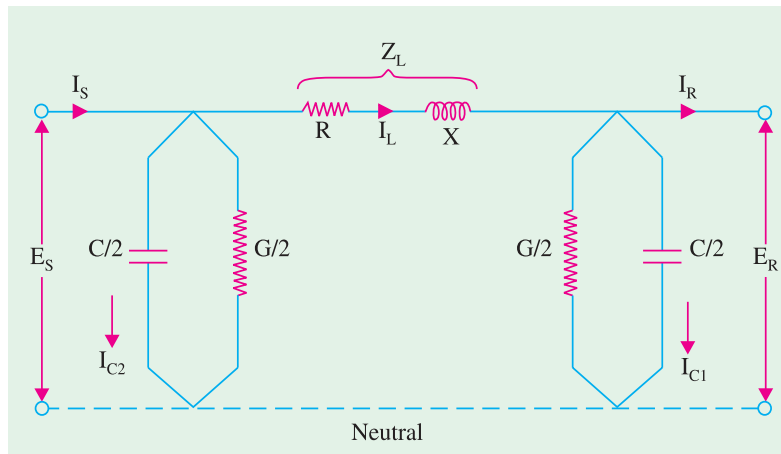


Fig. 41.36

### 41.23. Ferranti Effect

A long or medium transmission line has considerable capacitance and so draws leading charging current from the generating-end *even when unloaded*. Moreover, receiving-end voltage  $V_R$  under no-load condition is found to be *greater* than sending-end voltage  $V_S$ . This phenomenon is known as *Ferranti effect*.

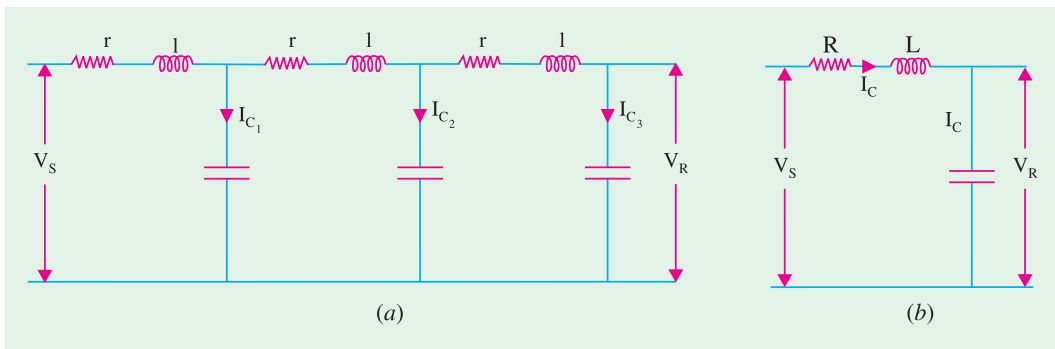


Fig. 41.37

Fig. 41.37 (a) shows the distributed parameters of such a line. It may be replaced by the circuit of Fig. 41.37 (b) where these distributed parameters have been lumped. As shown in the phasor

diagram of Fig. 41.38, the charging current  $I_C$  leads  $V_R$  by  $90^\circ$  and produces a phase voltage drop  $= I_C Z = I_C (R + j X_L)$ .

Obviously,  $V_S < V_R$  Now,  $I_C = V_R \omega C$

As seen from Fig. 41.38

$$V_S = \sqrt{(V_R - I_C X_L)^2 + (I_C R)^2}$$

If  $R$  is negligible, then

$$V_S = (V_R - I_C X_L) \text{ or } V_R = V_S + I_C X_L$$

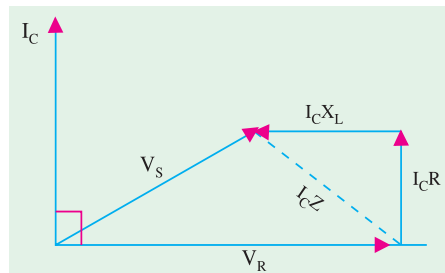


Fig. 41.38

### 41.24. Charging Current and Line Loss of an Unloaded Transmission Line

Fig. 41.39 shows the distribution of capacitance in a long transmission line of length  $l$ . Obviously, charging current  $I_C$  has maximum value at the sending end and linearly falls to zero at the receiving end. Accordingly, value of charging current at distance  $x$  from the sending end is proportionally equal to  $I_C (l - x)/l$ . The RMS value  $I$  of this current is given by

$$\begin{aligned} I^2 &= \frac{1}{l} \int_0^l \frac{I_C^2 (l - x)^2}{l^2} \cdot dx = \frac{I_C^2}{l^3} \int_0^l (l^2 + x^2 - 2lx) dx \\ &= \frac{I_C^2}{l^3} \left[ l^2 x + \frac{x^3}{3} - lx^2 \right]_0^l = \frac{I_C^2}{3} \end{aligned}$$

$$\therefore I = I_C / \sqrt{3}$$

If  $R$  is the resistance of the line per phase, then total power loss in the line is

$$= 3 I^2 R = 3 \left( \frac{I_C}{\sqrt{3}} \right)^2 R = I_C^2 R.$$

**Example 41.26.** A 3-phase transmission line, 100 km long has following constants: resistance per km per phase =  $0.28 \Omega$ ; inductive reactance per km per phase =  $0.63 \Omega$ . Capacitive susceptance per km per phase =  $4 \times 10^{-6}$  siemens. If the load at the receiving end is 75 MVA at 0.8 p.f. lagging with 132 kV between lines calculate sending-end voltage, current and p.f. Use nominal- $\pi$ -method.

(Power System-I, AMIE, Sec. B. 1994)

**Solution.** For a line of length 100 km,  
 resistance/phase =  $0.28 \times 100 = 28 \Omega$  ;  
 inductive reactance/phase =  $0.63 \times 100 = 63 \Omega$   
 Capacitive susceptance/phase =  $4 \times 10^{-6} \times 100 = 4 \times 10^{-4}$  S

Capacitive susceptance at each end =  $2 \times 10^{-4}$  S

$V_R = 132 \times 10^3 / \sqrt{3} = 76,230$  V ;  $V_R = (76,230 + j 0)$  ;  $I_R = 75 \times 10^6 / \sqrt{3} \times 132 \times 10^3 \times 0.8 = 410$

$I_R = 410 (0.8 - j 0.6) = 328 - j 246$

$Y_{C1} = j 2 \times 10^{-4}$  S ;  $I_{C1} = V_R \cdot Y_{C1}$   
 $= 76,230 \times j 2 \times 10^{-4} = j 15.25$  A

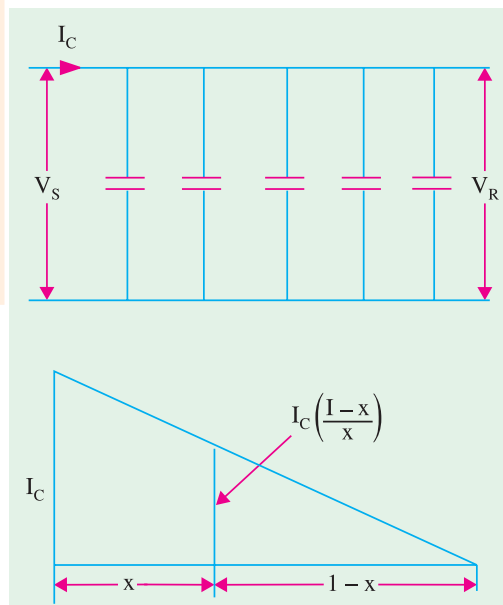


Fig. 41.39

$$I_L = I_R + I_{C1} = 328 - j 231 \text{ A}$$

$$\text{Drop per conductor} = I_L \cdot Z_L = (328 - j 231) (28 + j 63) = 23,737 + j 14,196$$

$$V_S = V_R + I_L Z_L = 76,230 + 23,737 + j 14,196 = 99,967 + j 14,196 = 100,970 \angle 8.1^\circ$$

$$\text{Line value of sending-end voltage} = 100,970 \times 10^{-3} \times \sqrt{3} = \mathbf{174.9 \text{ kW}}$$

$$I_C = Y_S Y_{C2} = 100,970 \angle 8.1^\circ \times 2 \times 10^{-4} \angle 90^\circ = 20.19 \angle 98.1^\circ = -2.84 + j 20$$

$$I_S = I_C + I_L = (-2.84 + j 20) + (328 - j 231) = 325.2 - j 211 = 388 \angle -33^\circ$$

$$\text{Angle between } V_S \text{ and } I_S = 8.1^\circ + 33^\circ = 41.4^\circ ; \cos \phi = \mathbf{0.877 \text{ (lag)}}$$

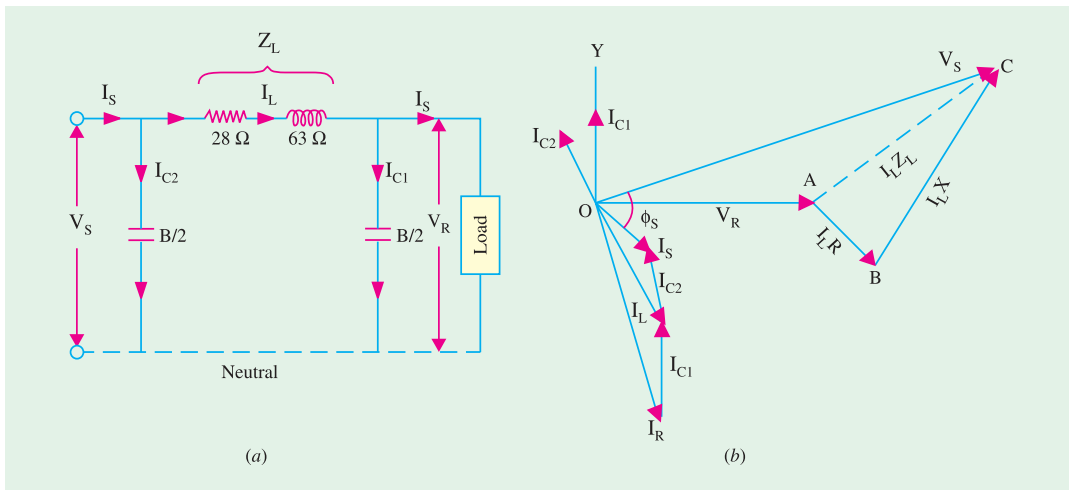


Fig. 41.40

**Example 41.27.** A 100-km long, three-phase, 50-Hz transmission line has resistance/phase/km = 0.1 Ω ; reactance/phase/km = 0.5 Ω ; susceptance/phase/km = 10 × 10<sup>-6</sup> siemens.

If the line supplies a load of 20 MW at 0.9 p.f. lagging at 66 kV at the receiving end, calculate by nominal 'p' method, the regulation and efficiency of the line. Neglect leakage.

(Electrical Power-I, Bombay Univ. 1991)

**Solution.** For a 100 km line

$$\text{resistance/phase} = 0.1 \times 100 = 10 \Omega ; \text{ inductive reactance/phase} = 0.5 \times 100 = 50 \Omega$$

$$\text{Capacitive susceptance/phase} = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4} \text{ siemens}$$

$$\text{Susceptance at each end} = 5 \times 10^{-4} \text{ siemens}$$

$$V_R = 66 \times 10^3 / \sqrt{3} = 38,100 \text{ V} \quad \therefore V_R = (38,100 + j 0)$$

$$I_R = 20 \times 10^6 / \sqrt{3} \times 66 \times 10^3 \times 0.9 = 195 \text{ A} ; I_R = 195(0.9 - j 0.435) = 176 - j 85$$

$$I_{C1} = 38100 \times j 5 \times 10^{-4} = j 19.1 \text{ A} ; I_L = I_R + I_{C1} = 176 - j 66$$

$$\text{drop/conductor} = I_L Z_L = (176 - j 66) (10 + j 50) = 5060 + j 8140$$

$$V_S = V_R + I_L Z_L = 38,100 + 5060 + j 8140 = 43,160 + j 8140 = 43,920 \angle 10.7^\circ$$

$$\text{Line value} = 43,920 \times 10^{-3} \times \sqrt{3} = 76 \text{ kV}$$

$$I_{C2} = V_S \cdot Y_{C2} = 43,920 \angle 10.7^\circ \times 5 \times 10^{-4} \angle 90^\circ = 21.96 \angle 100.7^\circ = -4.1 + j 21.6$$

$$I_S = I_{C2} + I_L = 172 - j 44.4 = 177.6 \angle -14.5^\circ ; \phi_S = 14.5^\circ + 10.7^\circ = 25.2^\circ$$

$$\cos \phi_S = 0.905 \text{ (lag)}$$

$$(i) \quad \% \text{ regulation} = \frac{76 - 66}{66} \times 100 = \mathbf{15.15\%}$$



- (ii) line input =  $\sqrt{3} \times 76 \times 103 \times 177.6 \times 0.905 = 21.5 \times 10^6 \text{ W} = 21.15 \text{ MW}$   
 $\therefore$  transmission  $\eta = 20 \times 100/21.15 = 94.56\%$

**Example 41.28. (a)** A 50-Hz, 3-phase, 100-km long line delivers a load of 40 MVA at 110 kV and 0.7 p.f. lag. The line constants (line to neutral) are : resistance of 11 ohms, inductive reactance of 38 ohms and capacitive susceptance of  $3 \times 10^{-4}$  siemens. Find the sending-end voltage, current, power factor and efficiency of power transmission.

(b) draw the vector diagram.

(c) If the sending-end voltage is held constant and load is removed, calculate the receiving-end voltage and current. (Electrical Power-II, Bangalore Univ.)

**Solution. (a)** Nominal  $\pi$ -method will be used to solve this problem. Capacitive susceptance (or admittance) at each end =  $Y/2 = 3 \times 10^{-4}/2 = 1.5 \times 10^{-4}$  siemens as shown in Fig. 41.41.

$$V_R = 110/\sqrt{3} = 63.5 \text{ kV/phase}; \quad I_R = 40 \times 10^6/\sqrt{3} \times 110 \times 10^3 = 210 \text{ A}$$

Let  $V_R = (63,500 + j 0)$  ;  $I_R = 210 (0.7 - j 0.714) = (147 - j 150)$

$$Y_{C1} = 1.5 \times 10^{-4} \text{ siemens}; \quad I_{C1} = V_R Y_{C1} = 63,500 \times j 1.5 \times 10^{-4} = j 9.5 \text{ A}$$

As seen from Fig. 41.36

$$I_L = I_R + I_{C1} = (147 - j 140.5) \text{ A}$$

$$\text{Voltage drop/phase} = I_L Z_L = (147 - j 140.5) (11 + j 38) = 6956 + j 4041$$

$$V_S = V_R + I_L Z_L = 63,500 + 6,956 + j 4041 = 70,456 + j 4041 = 70,570 \angle 3.2^\circ$$

Line value of sending-end voltage

$$= \sqrt{3} \times 70,570 = 122.2 \text{ kV}$$

$$I_{C1} = V_S Y_{C2} = (70,570 + j 4041) \times j 1.5 \times 10^{-4} = -0.6 + j 10.6 = 10.6 \angle 93.2^\circ$$

$$I_S = I_{C2} + I_L = (-0.6 + j 10.6) + (147 - j 140.5) = 146.4 - j 129.9 = 195.7 \angle -41.6^\circ$$

Angle between  $V_S$  and  $I_S = 41.6^\circ + 3.2^\circ = 44.8^\circ$ ;

$$\cos \phi_S = \cos 44.8^\circ = 0.71 \text{ (lag)}$$

$$\text{Power input} = \sqrt{3} \times 122.2 \times 10^3 \times 195.7 \times 0.7 = 29.41 \text{ MW}$$

$$\text{Power output} = 40 \times 0.7 = 28 \text{ MW}$$

- $\therefore$  power transmission  $\eta = 28/29.41 = 0.952$  or **95.2%**

(b) vector diagram is similar to that shown in Fig. 41.35 (b).

(c) As seen from Fig. 41.41 under no-load condition, current in the conductor is

$$= I_{C1} = j 9.5 \text{ A}$$

$$\text{Drop/phase} = I_{C1} Z_L = j 9.5(11 + j 38) = -361 + j 105$$

$$V_S = 63,500 + (-361 + j 105) = 63,140 + j 105 = 63,140 \angle 0.1^\circ$$

$$I_{C2} = (63,140 + j 105) \times j 1.5 \times 10^{-4} = (-0.016 + j 9.47) \text{ A}$$

$$I_S = I_{C1} + I_{C2} = j 9.5 - 0.016 + j 9.47 = (-0.016 + j 18.97) \text{ A}$$

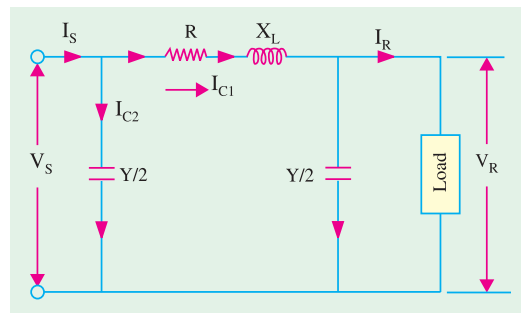


Fig. 41.41

### Tutorial Problem No. 41.2

1. A 50-Hz, 3-phase line 100 km long delivers a load of 40,000 kVA at 110 kV and a lagging power factor of 0.7. The line constants (line-to-neutral) are : resistance  $11 \text{ } \Omega$  ; inductive reactance  $38 \text{ } \Omega$ ; capacitive susceptance  $3 \times 10^{-4} \text{ S}$  (half at each end), leakage negligible. Find the sending-end voltage, current, power factor and power input. [122 kV ; 195.8 A ; 0.715 ; 29.675 kW]

2. A 50-Hz, 3-phase transmission line has the following constants (line-to-neutral). Resistance  $11 \Omega$ , reactance  $38 \Omega$ ; susceptance  $3 \times 10^{-4} \text{ S}$ , leakage negligible. The capacitance can be assumed located half at each end of the line. Calculate the sending-end voltage, the line current and the efficiency of transmission when the load at the end of the line is 40,000 kVA at 110 kV power factor 0.7 lagging.  
[122 kV ; 203.35 A ; 95.5%]
3. A 3-phase transmission line has the following constants (line-to-neutral) ;  $R = 10 \Omega$  ; inductive reactance =  $20 \Omega$ , capacitive reactance =  $2.5 \text{ k}\Omega$ . Using the nominal  $T$ -method, calculate the voltage, line current and power factor at sending-end and the efficiency of transmission when the transmission line supplies a balanced load of 10 MW at 66 kV and power factor 0.8 lagging.  
[69.5 kV ; 100 A ; 0.85 lag ; 96.8 %] (Electrical Technology ; M.S. Univ. Baroda)
4. A 3-phase line has a resistance of 5.31 ohms and inductance of 0.0176 henry. Power is transmitted at 33 kV, 50 Hz from one end and the load at the receiving end is 3600 kW at 80 per cent power factor. Find the line current, receiving-end voltage, sending-end power factor and the efficiency of transmission.  
[81.5 A, 31.9 kV, 0.796, 97.2%] (U.P.S.C.)
5. Calculate the % regulation of 6.6 kV single-phase A.C. transmission line delivering 40 amps current at 0.8 lagging power factor. The total resistance and reactance of the line are 4.0 ohm and 5.0 ohm per phase respectively.  
[3.76%]
6. A-3 phase load of 2,000 kVA at 0.8 power factor is supplied at 6.6 kV, 50 Hz by means of a 33 kV transmission line, 32 km long in association with a 33/6.6 kV step-down transformer. The resistance and reactance of each conductor per km are 0.4 ohm and 0.5 ohm respectively. The resistance and reactance of the transformer primary are 7.5 ohm and 13.2 ohm while those of secondary are 0.35 ohm and  $0.65 \Omega$  respectively. Find the voltage necessary at the sending end of the transmission line when the voltage is maintained at 6.6 kV at the receiving end. Determine also the sending-end power and efficiency of transmission.  
[35.4 kV, 1,689.5 kW, 94.7%]
7. The generalised  $A$  and  $B$  constants of a transmission line are  $0.96 \angle 1^\circ$  and  $120 \angle 80^\circ$  respectively. If the line to line voltages at the sending and receiving ends are both 110 kV and the phase angle between them is  $30^\circ$ , find the receiving-end power factor and the current.  
With the sending-end voltage maintained at 110 kV, if the load is suddenly thrown off, find the corresponding receiving-end voltage.  
[0.868 (lead), 326 A, 114.5 kV]
8. A 3-phase, 50-Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of  $0.1 \Omega$ ,  $0.5 \Omega$  and  $3 \times 10^{-6}$  siemens/km/phase. If the line delivers 50 MW at 110 kV and 0.8 p.f. lag, determine the sending-end voltage and current. Assume a nominal  $\pi$ -circuit for the line.  
[143.7 kV, 306.3 A]

#### 41.25. Generalised Circuit Constants of a Transmission Line

For any 4-terminal network *i.e.* one having two input and two output terminals (like a transmission line) the sending-end voltage per phase and the currents at the receiving and sending-end can be expressed by the following two equations :

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R \quad \dots(i)$$

$$\mathbf{I}_S = \mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R \quad \dots(ii)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$  and  $\mathbf{D}$  are the constants known as 'generalized circuit constants' of the transmission line. Their values depend on the particular method adopted for solving the transmission network. Let us consider the following cases.

(i) **Short Line.** In the case of lines up to 50 km, the effect of capacitance on the line performance is negligible. Hence, such a line can be represented as shown in Fig. 41.42 (a).

Here,  $\mathbf{I}_S = \mathbf{I}_R$  and  $\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_R \mathbf{Z}$

Comparing these with Eq. (i) and (ii) above, we get that

$$\mathbf{A} = 1 ; \quad \mathbf{B} = \mathbf{Z}, \quad \mathbf{C} = 0 \quad \text{and} \quad \mathbf{D} = 1$$

Incidentally, it may be noted that  $AD - BC = 1$

**(ii) Medium Line-Nominal-T Method**

The circuit is shown in Fig. 41.42 (b).

It is seen that,  $V_S = V_1 + I_S Z/2$  ...**(iii)**

Also  $V_1 = V_R + I_R Z/2$  ...**(iv)**

Now,  $I_C = I_S - I_R = V_1 Y = Y (V_R + I_R Z/2)$

$\therefore I_S = V_R Y + I_R \left(1 + \frac{YZ}{2}\right)$  ...**(v)**

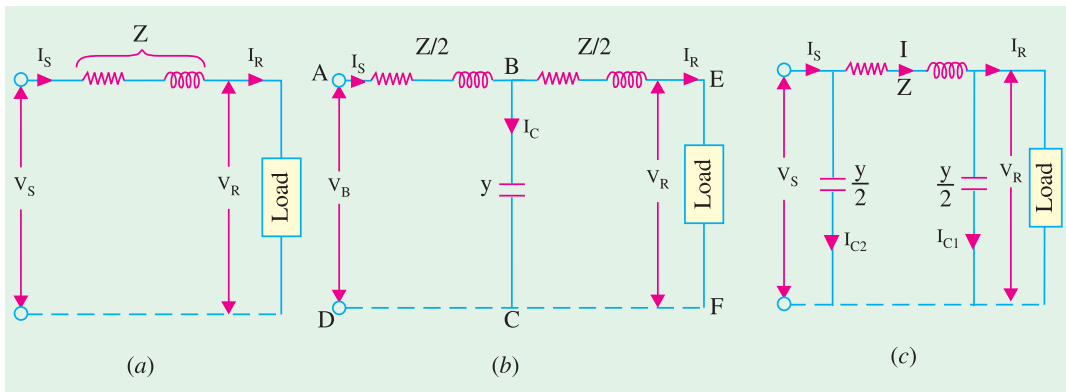


Fig. 41.42

Eliminating  $V_1$  from Eq. (iii) and (iv) we get

$$V_S = V_R + \frac{I_S Z}{2} + \frac{I_R Z}{2} \quad \dots\text{(vi)}$$

Substituting the value of  $I_S$ , we get

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + \left(Z + \frac{YZ^2}{4}\right) I_R \quad \dots\text{(vii)}$$

Comparing Eq. (vii) and (v) with Eq. (i) and (ii) respectively, it is found that

$$A = D = 1 + \frac{YZ}{2}; B = \left(1 + \frac{YZ}{4}\right) \text{ and } C = Y$$

It can again be proved that  $AD - BC = 1$ .

**(iii) Medium Line-Nominal-  $\pi$  Method**

The circuit is shown in Fig. 41.42 (c). Here, series impedance per phase =  $(R + jX)$  and admittance is  $Y = j\omega C$ .

As seen,  $I_S = I + I_{C2} = I + V_S Y/2$  ...**(viii)**

Also  $I = I_{C1} + I_R = I_R + V_R Y/2$  ...**(ix)**

Now,  $V_S = V_R + IZ = V_R + Z \left( I_R + \frac{V_R Y}{2} \right)$   
 $= \left(1 + \frac{YZ}{2}\right) V_R + Z I_R$  ...**(x)**

Eliminating  $I$  from Eq. (viii) and (ix), we get

$$I_S = I_R + \frac{V_R Y}{2} + \frac{V_S Y}{2}$$

Now, substituting the value of  $V_S$ , we have

$$I_S = I_R + \frac{V_R Y}{2} + \frac{Y}{2} \left[ \left( 1 + \frac{YZ}{2} \right) V_R + Z I_R \right]$$

or 
$$I_S = Y \left( 1 + \frac{YZ}{4} \right) V_R + \left( 1 + \frac{YZ}{2} \right) I_R \quad \dots(xi)$$

Comparing Eq. (x) and (ii) with Eq. (i) and (xi) above, we get,

$$A = D = \left( 1 + \frac{YZ}{2} \right); B = Z, C = Y \left( 1 + \frac{YZ}{4} \right)$$

Again, it can be shown that  $AD - BC = 1$

**Example 41.29.** Find the following for a single-circuit transmission line delivering a load of 50 MVA at 110 kV and p.f. 0.8 lagging :

(i) sending-end voltage, (ii) sending-end current, (iii) sending-end power, (iv) efficiency of transmission. (Given  $A = D = 0.98 \angle 3^\circ$ ,  $B = 110 \angle 75^\circ$  ohm,  $C = 0.0005 \angle 80^\circ$  ohm)

(Power Systems, AMIE, Sec. B, 1993)

**Solution.** Receiving-end voltage,  $V_R = 110/\sqrt{3} = 63.5$  kV

Taking this voltage as reference voltage, we have

$$\begin{aligned} V_R &= (63,500 + j 0) \\ I_R &= 50 \times 10^6 / \sqrt{3} \times 110 \times 10^3 = 262.4 \text{ A} \\ I_R &= 262.4 \angle \cos^{-1} 0.8 = 262.4 (0.8 - j 0.6) = (210 - j 157.5) \text{ A} \\ AV_R &= 0.98 \angle 3^\circ \times 63,500 \angle 0^\circ = 62,230 \angle 3^\circ = (62145 + j 3260) \text{ V} \\ BI_R &= 110 \angle 75^\circ \times 262.4 \angle 36.86^\circ = 28,865 \angle 111.8^\circ = (-10,720 + j 26,800) \text{ V} \\ (i) \quad V_S &= AV_R + BI_R = 51,425 + j 30,060 = 59,565, \angle 30.3^\circ \text{ V} \\ CV_R &= 0.0005 \angle 80^\circ \times 63,500 \angle 0^\circ = 31.75 \angle 80^\circ = (5.5 + j 31.3) \text{ A} \end{aligned}$$

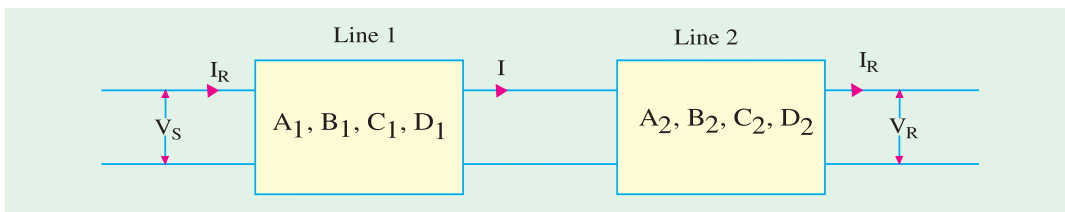


Fig. 41.43

$$DI_R = 0.98 \angle 3^\circ \times 262.4 \angle 36.86^\circ = 257.75 \angle 39.86^\circ = 197.4 + j 164.8$$

(ii)  $I_S = CV_R + DI_R = 203 + j 196 = 282 \angle 44^\circ \text{ A}$

(iii) Sending-end power =  $3V_S I_S \cos \phi_S = 3 \times 59.565 \times 282 \times \cos \angle 44.30^\circ - 30.3^\circ = 48.96 \text{ MW}$

Receiving end power =  $50 \times 0.8 = 40 \text{ MW}$

(iv) Transmission  $\eta = 40 \times 100/48.96 = 81.7\%$

**Example 41.30.** A 150 km, 3- $\phi$ , 110-V, 50-Hz transmission line transmits a load of 40,000 kW at 0.8 p.f. lag at receiving end.

resistance/km/phase = 0.15  $\Omega$ , reactance/km/phase = 0.6  $\Omega$ ; susceptance/km/phase =  $10^{-5}$  S

(a) determine the A, B, C and D constants of the line (b) find regulation of the line.

(Power System-I, AMIE, 1993)

**Solution.** We will use the nominal- $\pi$  method to solve the problem. For a length of 150 km :

$$R = 0.15 \times 150 = 22.5 \Omega; X = 0.6 \times 150 = 90 \Omega; Y = 150 \times 10^{-5} \text{ S} = 15 \times 10^{-4} \text{ S}$$

$$Z = (R + j X) = (22.5 + j 90) = 92.8 \angle 75^\circ \text{ ohm}; Y = 15 \times 10^{-4} \angle 90^\circ \text{ S}$$

$$(a) \quad \mathbf{A} = \mathbf{D} = \left(1 + \frac{\mathbf{YZ}}{2}\right) = 1 + \frac{j 15 \times 10^{-4}}{2} (22.5 + j 90) = (0.9675 + j 0.01688) \\ = 0.968 \angle 1.0^\circ$$

$$\mathbf{B} = \mathbf{Z} = 92.8 \angle 7.5^\circ; \quad \mathbf{C} = \mathbf{Y} \left(1 + \frac{\mathbf{YZ}}{4}\right) = j 15 \times 10^{-4} \left[1 + \frac{1}{4} \cdot j 15 \times 10^{-4} (22.5 + j 90)\right] \\ = -0.00001266 + j 0.00145 = 0.00145 \angle 90.5^\circ$$

(b) Now, the regulation at the receiving-end is defined as the change in voltage when full-load is thrown off, the sending-end voltage being held constant.

Now, when load is thrown off,  $\mathbf{I}_R = 0$ . Hence, putting this value in  $\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R$ , we get

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_{RO} \text{ or } \mathbf{V}_{RO} = \frac{\mathbf{V}_S}{\mathbf{A}} \quad \therefore \quad \% \text{ regn.} = \frac{V_{RO} - V_R}{V_R} \times 100 = \frac{(V_S/A - V_R)}{V_R} \times 100$$

Now,  $V_R = 100/\sqrt{3}$  kV = 63,520 V—at load

$$\mathbf{I}_R = 40 \times 10^6/\sqrt{3} \times 110 \times 10^3 \times 0.8 = 263 \text{ A}; \quad \mathbf{I}_R = 263 (0.8 - j 0.6) = (210 - j 158)$$

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R = 63,520 (0.9675 + j 0.01688) + (22.5 + j 90) (210 - j 158) \\ = 80,450 + j 16,410 = 82,110 \angle 11.5^\circ$$

$$V_S/A = 82,110/0.968 = 84,800; \quad \text{regn.} = \frac{84,800 - 63,520}{63,520} \times 100 = 33.5\%$$

**Example 41.31.** A 132-kV, 50-Hz, 3-phase transmission line delivers a load of 50 MW at 0.8 p.f. lagging at receiving-end.

The generalised constants of the transmission line are

$$\mathbf{A} = \mathbf{D} = 0.95 \angle 1.4^\circ; \quad \mathbf{B} = 96 \angle 7.8^\circ; \quad \mathbf{C} = 0.0015 \angle 90^\circ$$

Find the regulation of the line and the charging current. Use nominal T-method.

(Electrical Power-I, Bombay Univ.)

**Solution.**

$$\mathbf{I}_R = 50 \times 10^6/\sqrt{3} \times 132 \times 10^3 \times 0.8 = 273 \text{ A}$$

$$\mathbf{I}_R = 273 (0.8 - j 0.6) = 218 - j 164 = 273 \angle -36.9^\circ$$

$$\mathbf{V}_R = 132,000/\sqrt{3} = 76,230; \quad \mathbf{V}_R = 76,230 + j 0$$

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R = 76,230 \angle 0^\circ \times 0.95 \angle 1.4^\circ + 96 \angle 7.8^\circ \cdot 273 \angle -36.9^\circ \\ = 72,418 \angle 1.4^\circ + 26,208 \angle 41.1^\circ = 89,000 + j 21,510 = 92,150 \angle 13^\circ$$

$$\mathbf{I}_S = \mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R = 76,230 \times 0.0015 \angle 90^\circ + 0.95 \angle 1.4^\circ \cdot 273 \angle -36.9^\circ \\ = j 114 + 259.3 \angle -35.5^\circ = 211 - j 36$$

$$V_{RO} = V_S/A = 92,150/0.95 = 97,000; \quad \% \text{ regn.} = \frac{97,000 - 76,230}{76,230} = 27.1$$

$$\mathbf{I}_C = \mathbf{I}_S - \mathbf{I}_R = (211 - j 36) - (218 - j 164) = -7 + j 128 = 128.2 \angle 93.1^\circ$$

**Example 41.32.** A 3-phase transmission line consists of two lines 1 and 2 connected in series, line 1 being at the sending end and 2 at the receiving end. The respective auxiliary constants of the two lines are :  $A_1, B_1, C_1, D_1$  and  $A_2, B_2, C_2, D_2$ . Find the A, B, C, D constants of the whole line which is equivalent to two series-connected lines.

**Solution.** The two series-connected lines along with their constants as shown in Fig. 41.43.

For line No. 1

$$\mathbf{V}_S = \mathbf{A}_1\mathbf{V} + \mathbf{B}_1\mathbf{I} \quad ; \quad \mathbf{I}_S = \mathbf{C}_1\mathbf{V} + \mathbf{D}_1\mathbf{I} \quad \dots(i)$$

For line No. 2

$$\mathbf{V} = \mathbf{A}_2\mathbf{V}_R + \mathbf{B}_2\mathbf{I}_R \quad ; \quad \mathbf{I} = \mathbf{C}_2\mathbf{V}_R + \mathbf{D}_2\mathbf{I}_R \quad \dots(ii)$$

Substituting the values of  $\mathbf{V}$  and  $\mathbf{I}$  from Eq. (ii) into Eq. (i), we get

$$V_S = A_1(A_2V_R + B_2I_R) + B_1(C_2V_R + D_2I_R) = (A_1A_2 + B_1C_2)V_R + (A_1B_2 + B_1D_2)I_R$$

and  $I_S = C_1(A_2V_R + B_2I_R) + D_1(C_2V_R + D_2I_R) = (C_1A_2 + D_1C_2)V_R + (C_1B_2 + D_1D_2)I_R$

Hence, the two lines connected in series have equivalent auxiliary constants of

$$\begin{aligned} \mathbf{A} &= \mathbf{A_1A_2 + B_1C_2}, & \mathbf{B} &= \mathbf{A_1B_2 + B_1D_2} \\ \mathbf{C} &= \mathbf{C_1A_2 + D_1C_2} \quad \text{and} & \mathbf{D} &= \mathbf{C_1B_2 + D_1D_2} \end{aligned}$$

### 41.26. Corona

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameter, there is no apparent change in the condition of the atmospheric air surrounding the wires if voltage is low. However, when the p.d. is increased, then a point is reached when a faint luminous glow of bluish colour appears along the lengths of conductors and at the same time a hissing sound is heard. This bluish discharge is known as **corona**. Corona is always accompanied by the production of ozone which is readily detected because of its characteristic odour. If the p.d. is further increased, then the glow and hissing both increase in intensity till a spark-over between the conductors takes place due to the break-down of air insulation. If the conductors are smooth and polished, the corona glow is uniform along their length but if there is any roughness, they will be picked up by relatively brighter illumination. In the case of conductors with a spacing shorter as compared to their diameters, sparking may take place without any visible glow. If the p.d. between wires is direct instead of alternating, there is a difference in the appearance of the two wires. The positive wire has a smooth glow about it whereas the glow about the negative wire is spotty.

Corona occurs when the electrostatic stress in the air around the conductors exceeds 30 kVA (maximum)/cm or 21.1 kV (r.m.s.)/cm. The **effective disruptive critical voltage** to neutral is given by the relation.

$$V_C = m_0g_0 \delta r \log D/r \text{ kV/phase} = 2.3 m_0g_0 \delta r \log_{10} D/r \text{ kV/phase}$$

where  $m_0$  = irregularity factor which takes into account the surface conditions of the conductor.

$D$  = distance between conductors in cm.

$r$  = radius of the conductor in cm or the radius of the circumscribing circle in a stranded cable.

$g_0$  = breakdown strength or disruptive gradient of air at 76 cm of mercury and 25°C

$$= 30 \text{ kV (max.)}/\text{cm} = 21.1 \text{ kV (r.m.s.)}/\text{cm}$$

$\delta$  = air density factor

Substituting the value of  $g_0$ , we have

$$\begin{aligned} V_C &= 21.1 m_0 \delta r \log D/r \text{ kV/phase} = 21.1 m_0 \delta r \times 2.3 \log_{10} D/r \text{ kV/phase} \\ &= 48.8 m_0 \delta r \log_{10} D/r \text{ kV/phase} \end{aligned}$$

The value of  $\delta$  is given by  $\delta = \frac{3.92 b}{273 + t}$

where  $b$  = barometric pressure in cm of Hg and  $t$  = temp. in degrees centigrade.

when  $b = 76 \text{ cm}$  and  $t = 25^\circ \text{C}$ ,  $\delta = 1$

The irregularity factor  $m_0$  depends on the shape of cross-section of the wire and the state of its surface. Its value is unity for an absolutely smooth wire of one strand of circular section and less than unity for wires roughened due to weathering as shown below :

<b>Irregularity Factor</b>		
Polished wires	...	1.0
Weathered wires	...	0.93 to 0.98
7-strand cables, concentric lay	...	0.83 to 0.87
Cables with more than 7-strands	...	0.80 to 0.85

### 41.27. Visual Critical Voltage

This voltage is higher than the disruptive critical voltage  $V_c$ . It is the voltage at which corona appears all along the line. The *effective* or r.m.s. value of visual critical voltage is given by the following empirical relation.

$$\begin{aligned} V_v &= m_v g_0 \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \log_e D/r \text{ kV/phase} \\ &= 21.1 m_v \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \times 2.3 \log_{10} D/r \text{ kV/phase} \\ &= 48.8 m_v \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \log_{10} D/r \text{ kV/phase} \end{aligned}$$

where  $m_v$  is another irregularity factor having a value of 1.0 for smooth conductors and 0.72 to 0.82 for rough conductors.

### 41.28. Corona Power

Formation of corona is always accompanied by dissipation of energy. This loss will have some effect on efficiency of the line but will not have any appreciable effect on the line regulation. This loss is affected both by atmospheric and line conditions. Soon after the critical voltage is reached, the corona loss increases as the square of the excess voltage. The loss for voltage of  $V$  kilovolt to neutral is given by the following empirical relation.

$$P = 241 \frac{(f + 25)}{\delta} \sqrt{\left(\frac{r}{D}\right)} (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where  $f$  is the frequency of the a.c. in Hz.

Obviously, total loss equals 3 times the above.

### 41.29. Disadvantages of Corona

1. There is a definite dissipation of power although it is not so important except under abnormal weather conditions like storms etc.
2. Corrosion due to ozone formation.
3. The current drawn by the line due to corona losses is non-sinusoidal in character, hence it causes non-sinusoidal drop in the line which may cause some interference with neighbouring communication circuits due to electromagnetic and electrostatic induction. Such a shape of corona current tends to introduce a large third harmonic component.  
However, it has been found that corona works as a safety valve for surges.
4. Particularly intense corona effects are observed at a working voltage of 35 kV or higher. Hence, designs have to be made to avoid any corona on the bus-bars of substations rated for 35 kV and higher voltages during their normal operation. Corona discharge round bus-bars is extremely undesirable because the intense ionization of the air reduces its dielectric strength, makes it easier for the flashover to occur in the insulators and between phases particularly when the surfaces concerned are dirty or soiled with other deposits. The ozone produced due to corona discharge aggressively attacks the metallic components in the substations and switchgear, covering them with oxides. Moreover, the crackling sound of the corona discharge in a substation masks other sounds like light crackling noise due to arcing in a loose contact, the sound of an impending breakdown or creepage discharge in the equipment, the rattling noise due to the loosening of steel in a transformer core etc. The timely detection of such sounds is very important if any serious breakdown is to be avoided.

**Example 41.33.** Find the disruptive critical voltage for a transmission line having :  
 conductor spacing = 1 m ; conductor (stranded) radius = 1 cm  
 barometric pressure = 76 cm of Hg ; temperature = 40°C  
 Air break-down potential gradient (at 76 cm of Hg and at 25°C) = 21.1 kV (r.m.s.)/cm.

(Electrical Power Systems-I, Gujarat Univ.)

**Solution.**  $V_c = 2.3 m_0 g_0 r \log_{10} D/r$  kV/phase  
 Here,  $g_0 = 21.1$  kV (r.m.s.)/cm ;  $m_0 = 0.85$  (assumed)  
 $\delta = 3.92 \times 76 / (273 + 40) = 0.952$  ;  $\log_{10} D/r = \log_{10} 100/1 = 2$   
 $\therefore V_c = 2.3 \times 0.85 \times 21.1 \times 0.952 \times 1 \times 2 = 78.54$  kV (r.m.s.)/phase  
 Line value =  $78.54 \times \sqrt{3} = 136$  kV (r.m.s.)/phase

**Example 41.34.** Find the disruptive critical and visual corona voltage of a grid-line operating at 132 kV.

conductor dia = 1.9 cm ; conductor spacing = 3.81 m  
 temperature = 44°C ; barometric pressure = 73.7 cm  
 conductor surface factor :  
 fine weather = 0.8 ; rough weather = 0.66.

(Electrical Power Systems-III, Gujarat Univ.)

**Solution.**  $V_c = 48.8 m_0 \delta r \log_{10} D/r$  kV/phase  
 Here,  $m_0 = 0.8$  ;  $\delta = 3.92 \times 73.7 / (273 + 44) = 0.91$   
 $\log_{10} 381/1.9 = \log_{10} 200.4 = 2.302$   
 $\therefore V_c = 48.8 \times 0.8 \times 0.91 \times 1.9 \times 2.302 = 155.3$  kV/phase  
 $V_v = 48.8 m_v \delta r \left( 1 + \frac{0.3}{\sqrt{\delta r}} \right) \log_{10} D/r$  kV/phase  
 Here,  $m_v = 0.66$  ;  $\delta = 0.91$  ;  $\sqrt{\delta r} = \sqrt{0.91 \times 1.9} = 1.314$   
 $\therefore V_v = 48.8 \times 0.66 \times 0.91 \times 1.9 \left( 1 + \frac{0.3}{1.314} \right) 2.302 = 157.5$  kV/phase

**Example 41.35.** A certain 3-phase equilateral transmission line has a total corona loss of 53 kW at 106 kV and a loss of 98 kW at 110.9 kV. What is the disruptive critical voltage between lines? What is the corona loss at 113 kV? (Electrical Power Systems-II, Gujarat Univ.)

**Solution.** As seen from Art. 41.28, the total corona loss for three phases is given by

$$P = 3 \times \frac{241(f+25)}{\delta} \cdot \sqrt{\frac{r}{D}} (V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

Other things being equal,  $P \propto (V - V_c)^2$

$$\therefore 53 \propto \left( \frac{106}{\sqrt{3}} - V_c \right)^2 \propto (61.2 - V_c)^2 \quad \text{--- 1st case}$$

$$98 \propto \left( \frac{110.9}{\sqrt{3}} - V_c \right)^2 \propto (64 - V_c)^2 \quad \text{--- 2nd case}$$

$$\therefore \frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2} \quad \text{or } V_c = 54.2 \text{ kV/km}$$

$$\text{Similarly, } W \propto \left( \frac{113}{\sqrt{3}} - V_c \right)^2 \propto (65.2 - V_c)^2$$



$$\frac{W}{98} = \frac{(65.2 - V_c)^2}{(64 - V_c)^2} = \frac{(65.2 - 54.2)^2}{(64 - 54.2)^2} \quad \therefore W = 123.4 \text{ kW}$$

**Example 41.36.** A 3-phase, 50-Hz, 220-kV transmission line consists of conductors of 1.2 cm radius spaced 2 metres at the corners of an equilateral triangle. Calculate the corona power loss per km of the line at a temperature of 20°C and barometric pressure of 72.2 cm. Take the surface factors of the conductor as 0.96. (Electrical Power-II, Bangalore Univ.)

**Solution.** As seen from Art. 41.28, corona loss per phase is

$$P = 241 \frac{(f + 25)}{\delta} \sqrt{(r/D)} \cdot (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

Here, 
$$\delta = \frac{3.92 b}{273 + t} = \frac{3.92 \times 72.2}{273 + 20} = 0.966$$

$$V_c = 48.8 m_0 \delta r \log_{10} D/r = 48.8 \times 0.96 \times 0.966 \times 1.2 \times \log_{10} \frac{200}{1.2}$$

$$= 120.66 \text{ kV/phase}$$

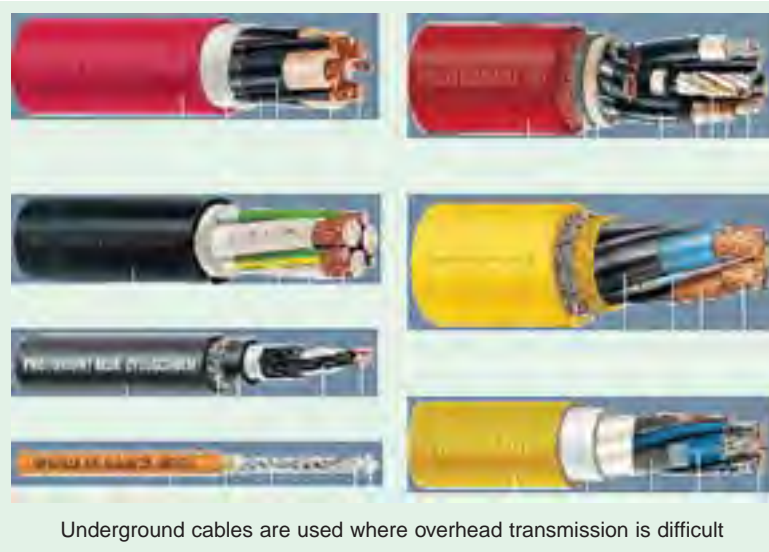
$$V = 220/\sqrt{3} = 127 \text{ kV/phase}$$

$$\therefore P = 241 \times \frac{75}{0.966} \times \sqrt{\left(\frac{1.2}{200}\right)} \times (127 - 120.66)^2 \times 10^{-5} = 0.579 \text{ kW/km/phase}$$

$$\text{Total loss for 3 phase} = 3 \times 0.579 = 1.737 \text{ kW/km}$$

### 41.30. Underground Cables

Underground cables are used where overhead lines are not possible as in large cities despite the fact that in their case, cost per kW per km is much more as compared to overhead transmission lines. Another advantage of overhead system for distributors is that tapping can be made at any time without any disturbance, which is of great importance in rapidly developing areas. However, underground cables are more advanta-



geous for feeders which are not likely to be disturbed for tapping purposes because, being less liable to damage through storms or lightning or even wilful damage, they offer a safer guarantee of supply. But this advantage may be offset by the cost of trenching and expensive jointing necessary in case of repairs.

However, cables score over overhead lines in cases where voltage regulation is more important, because, due to very small spacing of their conductors, they have a very low inductance and hence low inductive drops.

Cables may be classified in two ways according to (i) the type of insulating material used in their manufacture or (ii) the voltage at which they transmit power. The latter method of classification is, however, more generally used according to which cables are divided into three groups :

1. Low-tension cables—up to 1000 V
2. High-tension cables—up to 23,000 V
3. Super-tension cables—from 66 kV to 132 kV

For all cables, the conductor is tinned stranded copper of high conductivity. Stranding is done to secure flexibility and the number of conductors in a core is generally 3, 7, 19 and 37 etc. Except for 3-strand, all numbers have a centrally-disposed conductor with all others surrounding it. A cable may have one or more than one core depending on the type of service for which it is intended. It may be (i) single-core (ii) two-core (iii) three-core and (iv) four-core etc.

In Fig. 41.44 is shown a section through a twin-cored, high-tension lead-covered underground cable sheathed with a continuous tube of pure lead whereas Fig. 41.45 shows a section of a typical concentric type 2-core cable used for single-phase distribution. The cores are arranged concentrically. The outer core is arranged in the form of hollow tubing. Both cores are of stranded copper and paper-insulated and are protected by a lead-sheath.

In Fig. 41.46 is shown a section through a 3-core extra-high-tension paper-insulated lead-covered and steel-wire armoured cable.

The cores are surrounded by insulation or impregnated paper, varnished cambric or vulcanised bitumen.

The insulation is, in turn, surrounded by a metal sheath made of lead or a lead alloy and prevents the entry of moisture into the inner parts. On the sheath is applied the bedding which consists of two compounded paper tapes along with suitable compounded fibrous materials.

Next comes 'armouring' which is placed over the bedding and consists of either galvanized steel wires or two layers of steel tape. Armouring may not be done in the case of some cables.

Next comes 'serving' which consists of compounded fibrous material (jute etc.) placed over the armouring in the case of armoured cables or over the metal sheath in the case of unarmoured cables—in which case serving consists of two layers of compounded paper tapes and a final covering of compounded fibrous material.

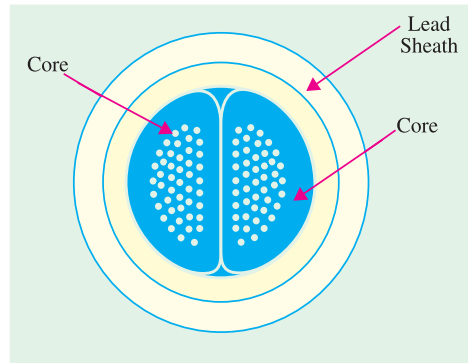


Fig. 41.44

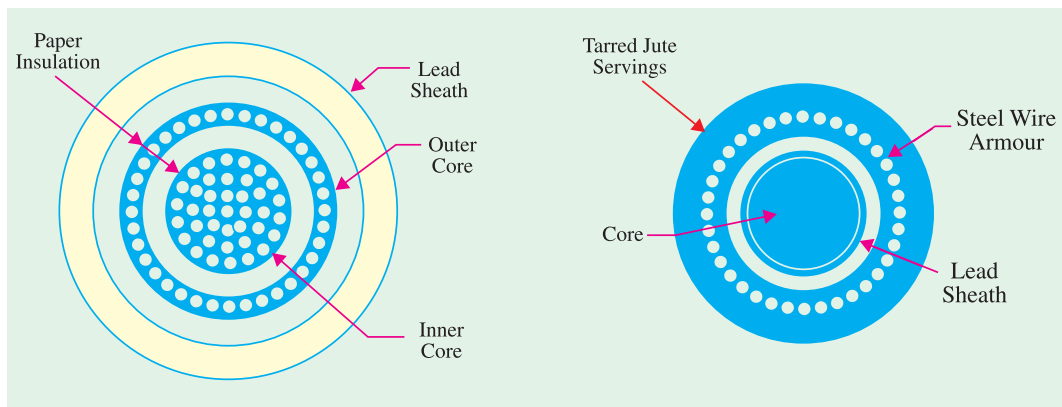


Fig. 41.45

Fig. 41.46

**41.31. Insulation Resistance of a Single-core Cable**

The expression for the insulation resistance of such a cable, as derived in Art. 5.14.

$$= \frac{2.3 \rho}{2 \pi l} \log_{10} \frac{r_2}{r_1} \text{ ohm}$$

The value of specific resistance for paper is approximately 500 Ω-m.

**41.32. Capacitance and Dielectric Stress**

This has already been dealt with in Art. 5.9 and 5.13.

**41.33. Capacitance of 3-core Belted Cables**

The capacitance of a cable is much more important than of the overhead wires of the same length because in cables (i) conductors are nearer to each other and the sheath and (ii) they are separated by a dielectric medium of higher permittivity as compared to air. Fig. 41.47 shows a system of capacitances

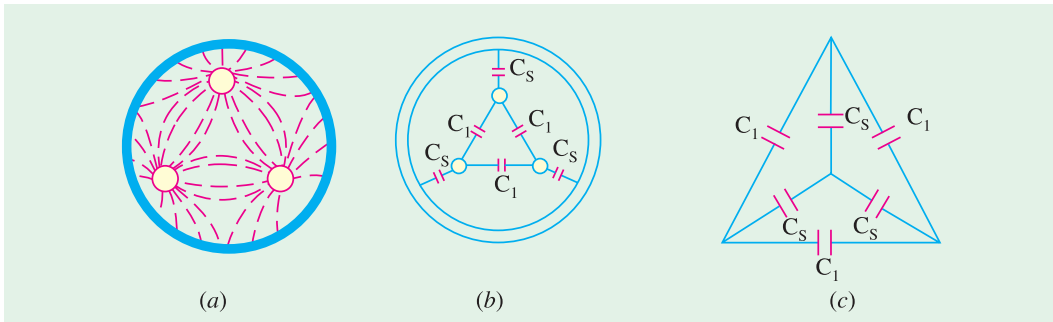


Fig. 41.47

in a belted 3-core cable used for 3-phase system. It can be regarded as equivalent to three-phase cables having a common sheath. Since there is a p.d. between pairs of conductors and between each conductor and sheath, there exist electrostatic fields as shown in Fig. 41.47 (a) which shows average distribution of electrostatic flux, though, actually, the distribution would be continually changing because of changing potential difference between conductors themselves and between conductors and sheath. Because of the existence of this electrostatic coupling, there exist six capacitances as shown in Fig. 41.47 (b). The three capacitances between three cores are delta-connected whereas the other three between each core and the sheath are star-connected, the sheath forming the star-point [Fig. 41.47 (c)].

The three delta-connected capacitances each of value  $C_1$  can be converted into equivalent star-capacitance  $C_2$  which will be three times the delta-capacitances  $C_1$  as shown in Fig. 41.48.

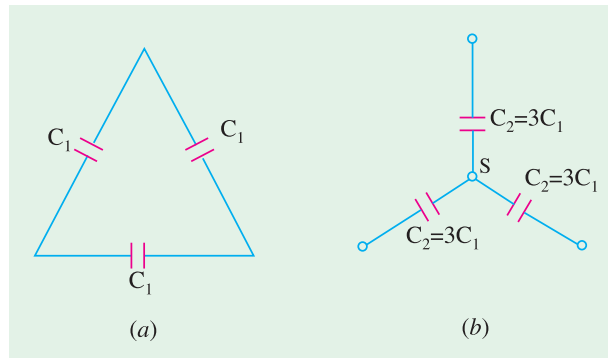


Fig. 41.48

The two star capacitances can now be combined as shown in Fig. 41.49 (a). In this way, the whole cable is equivalent to three star-connected capacitors each of capacitance  $C_n = 3C_1 + C_s$  as shown in Fig. 41.49 (b).

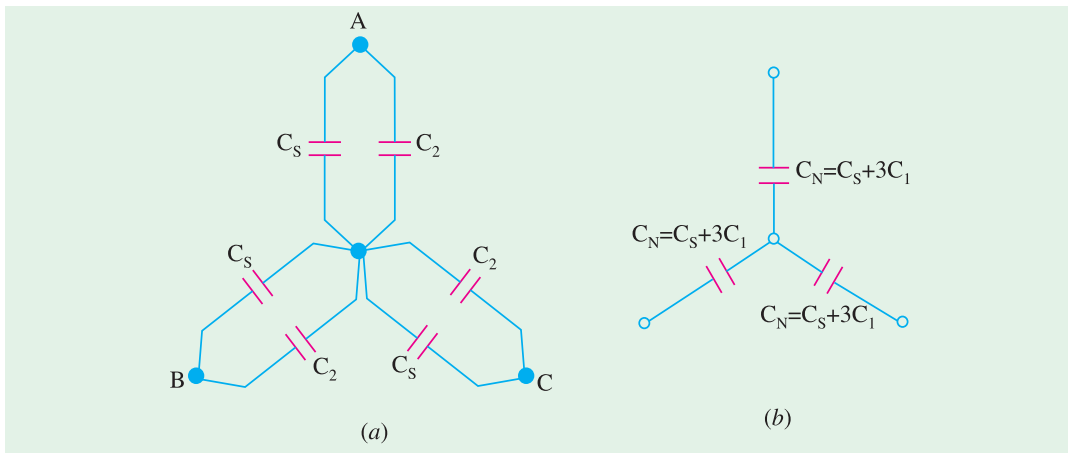


Fig. 41.49

If  $V_P$  is the phase voltage, then the charging current is given by

$$I_C = V_P (C_S + 3C_1)\omega = V_P \omega C_n$$

### 41.34. Tests for Three-phase Cable Capacitance

The following three tests may be used for the measurement of  $C_1$  and  $C_S$  :

- (i) In the first test, all the three cores are bunched together and then capacitance is measured by usual methods between these bunched cores and the earthed sheath. This gives  $3 C_S$  because the three capacitances are in parallel.
- (ii) In the second method, two cores are bunched with the sheath and capacitance is measured between these and the third core. It gives  $2C_1 + C_S$ . From this value,  $C_1$  and  $C_S$  can be found.
- (iii) In the third method, the capacitance  $C_L$  between two cores or lines is measured with the third core free or shorted to earth. This gives

$$\left( C + \frac{C_1}{2} + \frac{C_S}{2} \right) = \frac{1}{2} (3 C_1 + C_S) = \frac{C_n}{2}$$

Hence,  $C_n$  is twice the measured value i.e.  $C_n = 2C_L$

Therefore, charging current  $I_C = \omega V_P C_n = \frac{2}{\sqrt{3}} \cdot \omega V_L C_L$

where  $V_L$  is the line voltage (not phase voltage).

**Example 41.37.** A single-core lead-covered cable is to be designed for 66-kV to earth. Its conductor radius is 1.0 cm and its three insulating materials A, B, C have permittivities of 5.4 and 3 respectively with corresponding maximum safe working stress of 38 kV per cm (r.m.s. value), 26-kV per cm and 20-kV per cm respectively. Find the minimum diameter of the lead sheath.

**Solution.**

$$g_{1\max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r1} r} \text{ or } 38 = \frac{Q}{2\pi\epsilon_0 \times 5 \times 1} \quad \dots(i)$$

$$g_{2\max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r2} r_1} \text{ or } 26 = \frac{Q}{2\pi\epsilon_0 \times 4 \times r_1} \quad \dots(ii)$$

$$g_{3\max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r3} r_2} \text{ or } 20 = \frac{Q}{2\pi\epsilon_0 \times 3 \times r_2} \quad \dots(iii)$$

From (i) and (ii), we get,  $38/26 = 4r_1/5$ ,  $r_1 = 1.83$  cm

Similarly, from (i) and (iii),

$$38/20 = 3r_2/5 ; r_2 = 3.17 \text{ cm}$$

$$V_1 = g_{1\max} \times r \times 2.3 \log_{10} r_1/r$$

$$= 38 \times 1 \times 2.3 \log_{10} \frac{1.83}{1} = 22.9 \text{ kV}$$

$$V_2 = g_{2\max} \times r_1 \times 2.3 \log_{10} \frac{r_2}{r_1}$$

$$= 26 \times 1.83 \times 2.3 \log_{10} \frac{3.17}{1.83}$$

$$= 26.1 \text{ kV}$$

$$V_3 = g_{3\max} \times r_2 \times 2.3 \log_{10} \frac{r_3}{r_2}$$

$$= 20 \times 3.17 \times 2.3 \log_{10} \frac{r_3}{3.17}$$

$$= 145.82 \log_{10} \frac{r_3}{3.17}$$

Now,  $V = V_1 + V_2 + V_3$

$$\therefore 66 = 22.9 + 26.1 + 145.82 \log_{10} \frac{r_3}{3.17}$$

$$\therefore r_3 = 4.15 \text{ cm}$$

$$\therefore \text{diameter of the sheath} = 2r_3 = 2 \times 4.15 = \mathbf{8.3 \text{ cm}}$$

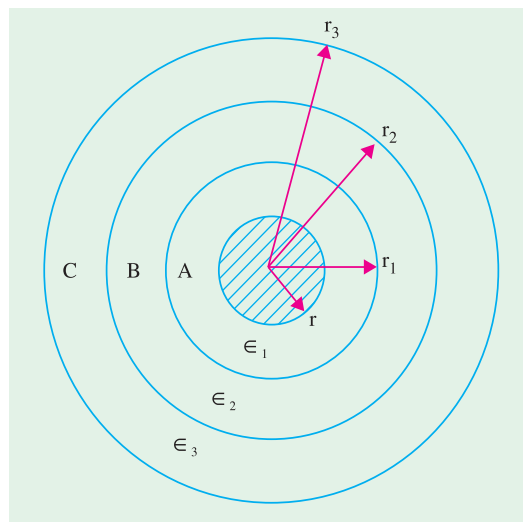


Fig. 41.50

**Example 41.38.** The capacitances per kilometer of a 3-phase cable are  $0.63 \mu\text{F}$  between the three cores bunched and the sheath and  $0.37 \mu\text{F}$  between one core and the other two connected to sheath. Calculate the charging current taken by eight kilometres of this cable when connected to a 3-phase, 50-Hz, 6,600-V supply.

**Solution.** As shown in Art. 40.33,  $0.63 = 3C_s \therefore C_s = 0.21 \mu\text{F/km}$   
 From the second test,

$$0.37 = 2C_1 + C_2 = 2C_1 + 0.21 \therefore C_1 = 0.08 \mu\text{F/km}$$

$$\therefore C_s \text{ for } 8 \text{ km} = 0.21 \times 8 = 1.68 \mu\text{F} ; C_1 \text{ for } 8 \text{ km} = 0.08 \times 8 = 0.64 \mu\text{F}$$

$$C_n = C_s + 3C_1 = 1.68 + (3 \times 0.64) = 3.6 \mu\text{F}$$

Now,

$$V_p = (6,600/\sqrt{3}) ; \omega = 314 \text{ rad/s}$$

$$I_C = \omega V_p C_n = (6,600/\sqrt{3}) \times 3.6 \times 10^{-6} \times 314 = \mathbf{4.31 \text{ ampere}}$$

**Example 41.39.** A 3-core, 3-phase belted cable tested for capacitance between a pair of cores on single phase with the third core earthed, gave a capacitance of  $0.4 \text{ mF}$  per km. Calculate the charging current for 1.5 km length of this cable when connected to 22 kV, 3-phase, 50-Hz supply.

**Solution.**

$$C_L = 0.4 \mu\text{F}, V_L = 22,000 \text{ V}, \omega = 314 \text{ rad/s}$$

$$I_C = \frac{2}{\sqrt{3}} \omega V_L C_L = \frac{2}{\sqrt{3}} \times 22,000 \times 0.4 \times 10^{-6} \times 314 = 3.2 \text{ A}$$

Charging current for 15 km =  $3.2 \times 15 = \mathbf{48 \text{ A}}$

**Example 41.40.** A 3-core, 3-phase metal-sheathed cable has (i) capacitance of  $1 \mu\text{F}$  between shorted conductors and sheath and (ii) capacitance between two conductors shorted with the sheath and the third conductor  $0.6 \mu\text{F}$ . Find the capacitance (a) between any two conductors (b) between any two shorted conductors and the third conductor. (Power Systems-I, AMIE, Sec. B, 1993)

**Solution.** (a) The capacitance between two cores or lines when the third core is free or shorted to earth is given by  $1/2 (3C_1 + C_s)$   
 Now, (i) we have  $3C_s = 1 \mu\text{F}$  or  $C_s = 1/3 \mu\text{F} = 0.333 \mu\text{F}$

From (ii) we get,  $2C_1 + C_s = 0.6 \mu F$ ,  $2C_1 = 0.6 - 0.333 = 0.267 \mu F$ ,

$$\therefore C_1 = 0.133 \mu F$$

(b) The capacitance between two shorted conductors and the other is given by

$$2C_1 + \frac{2C_s \times C_s}{3C_s} = 2C_1 + \frac{2}{3} C_s = 2 \times 0.133 + \frac{2}{3} \times 0.333 = \mathbf{0.488 \text{ mF}}$$

### 41.35. A.C. Distributor Calculations

These calculations are similar to those for d.c. distributor but with the following differences :

1. The loads tapped off will be at different power factors. Each power factor is taken with respect to the voltage at the feeding point which is regarded as a reference vector.
2. The currents in the sections of the distributor will be given by the vector sum of load currents and not by their arithmetic sum as in a d.c. distributor. Currents can be added algebraically only when they are expressed in the complex notation.
3. The voltage drop, in the case of a.c. circuits, is not only due to ohmic resistance but due to inductive reactance as well (neglecting capacitive reactance if any).

It has already been shown that voltage drop in an inductive circuit is given by

$$I (R \cos \phi + X \sin \phi)$$

The total drop will be given by  $\Sigma I (R \cos \phi + X \sin \phi)$ .

Questions on a.c. distributors may be solved in the following three ways :

1. Express voltages, currents and impedances in complex notation and then proceed exactly as in d.c. distributors.
2. Split the various currents into their active and reactive components. Now, the drop in the case of active components will be due to resistance only and in the case of reactive components due to reactance only. Find out these two drops and then add the two to find the total drop.
3. In cases where approximate solutions are sufficient, quick results can be obtained by finding the “distribution centre” or centre of gravity of the load.

All these three methods are illustrated by Ex. 41.42 given on the next page :

**Example 41.41.** A 2-wire a.c. feeder 1 km long supplies a load of 100 A at 0.8 p.f. lag 200 volts at its far end and a load of 60 A at 0.9 p.f. lag at its mid-point. The resistance and reactance per km (lead and return) are 0.06 ohm and 0.08 ohm respectively. Calculate the voltage drop along the distributor from sending end to mid-point and from mid-point to far end.

(Power Systems-I, AMIE, Sec. B, 1993)

**Solution.** Fig. 41.51 shows the feeder AC 1 km long having B as its mid-point and A as its sending-end point.

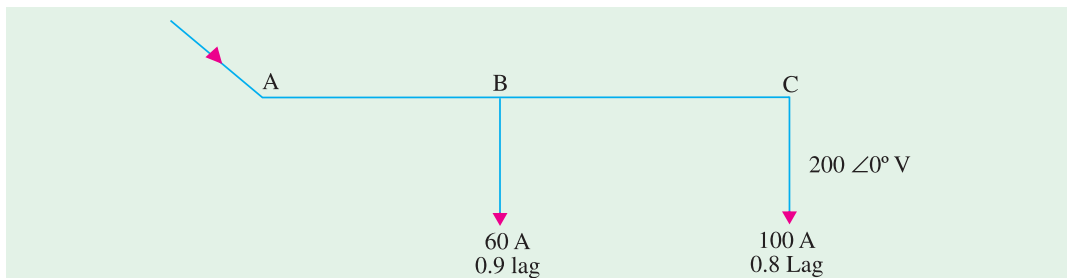


Fig. 41.51

Let the voltage of point C be taken as reference voltage.

$$V_C = 200 + j_0, I_C = 100(0.8 - j 0.6) = (80 - j 60) \text{ A}$$

Loop impedance of feeder  $BC$  (lead and return) =  $(0.06 + j 0.08)/2 = (0.03 + j 0.04) \text{ ohm}$

Voltage drop in  $BC = (80 - j 60) (0.03 + j 0.04) = (4.8 + j 1.4) \text{ V}$

$\therefore V_B = 200 + 4.8 + j 1.4 = (204.8 + j 1.4) \text{ V}$

Current  $I_B = 60 (0.9 - j 0.4357) = (54 - j 26.14) \text{ A}$

Current in feeder  $AB$  i.e.  $I_{AB} = I_C + I_B = (80 - j 60) + (54 - j 26.14) = (134 - j 86.14) \text{ A}$

Drop in section  $AB = (134 - j 86.14) (0.03 + j 0.04) = (7.46 + j 2.78) \text{ V}$

Voltage drop from point  $A$  to point  $B = (7.46 + j 2.78) \text{ V}$

**Example 41.42.** A single-phase a.c. distributor 500 m long has a total impedance of  $(0.02 + j 0.04) \Omega$  and is fed from one end at 250V. It is loaded as under :

- (i) 50 A at unity power factor 200 m from feeding point.
- (ii) 100 A at 0.8 p.f. lagging 300 m from feeding point.
- (iii) 50 A at 0.6 p.f. lagging at the far end.

Calculate the total voltage drop and voltage at the far end. (Power System-I, AMIE, 1994)

**Solution. First Method**

Current in section  $AD$  (Fig. 41.52) is the vector sum of the three load currents.

$$\begin{aligned} \therefore \text{current in } AD &= 50 + 100(0.8 - j 0.6) + 50(0.6 - j 0.8) \\ &= 160 - j 100 \end{aligned}$$

$$\begin{aligned} \text{Impedance of section } AD &= (200/500) (0.02 + j 0.04) \\ &= (0.008 + j 0.016) \text{ W} \end{aligned}$$

$$\text{Voltage drop in section } AD = (160 - j 100) \times (0.008 + j 0.016) = (2.88 + j 1.76) \text{ V}$$

$$\text{Current in section } DC = (160 - j 100) - 50 = (110 - j 100) \text{ A}$$

$$\text{Impedance of } DC = (0.004 + j 0.008) \Omega$$

$$\text{Drop in } CD = (100 - j 100) (0.004 + j 0.008) = (1.24 + j 0.48) \text{ V}$$

$$\text{Current in } CB = 50 (0.6 - j 0.8) = (30 - j 40) \text{ A}$$

$$\text{Impedance of } CB = (0.008 + j 0.016) \Omega$$

$$\therefore \text{ drop in } CB = (30 - j 40) (0.008 + j 0.016) = (0.88 + j 0.16) \text{ V}$$

$$\text{Total drop} = (2.88 + j 1.76) + (1.24 + j 0.48) + (0.88 + j 0.16) = (5 + j 2.4) \text{ V}$$

$$\text{Voltage at far end} = (250 + j 0) - (5 + j 2.4) = 245 - j 2.4 \text{ volt}$$

$$\text{Its magnitude is} = \sqrt{245^2 + 2.4^2} = 245 \text{ V (approx)}$$

**Second Method**

We will split the currents into their active and reactive components as under :

$$50 \times 1 = 50 \text{ A}; \quad 100 \times 0.8 = 80 \text{ A}; \quad 50 \times 0.6 = 30 \text{ A}$$

These are shown in Fig. 41.51 (a). The reactive or wattless components are

$$50 \times 0 = 0; \quad 100 \times 0.6 = 60 \text{ A}; \quad 50 \times 0.8 = 40 \text{ A}$$

These are shown in Fig. 41.53 (b). The resistances and reactances are shown in their respective figures.

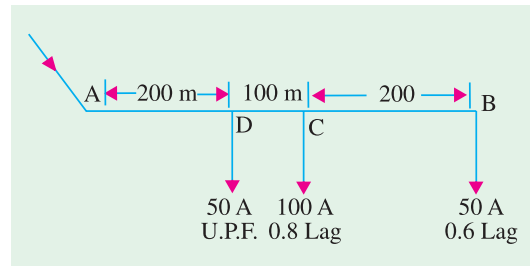


Fig. 41.52

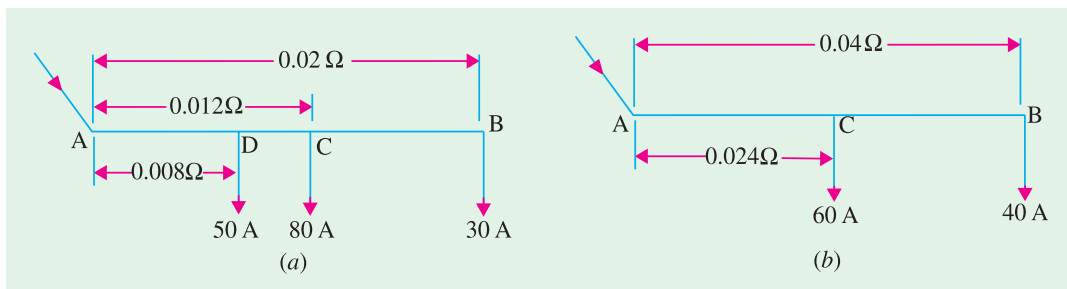


Fig. 41.53

Drops due to active components of currents are given by taking moments

$$= 50 \times 0.008 + 80 \times 0.012 + 30 \times 0.02 = 1.96 \text{ V}$$

Drops due to reactive components =  $60 \times 0.024 + 40 \times 0.04 = 3.04$

Total drop =  $1.96 + 3.04 = 5 \text{ V}$

This is approximately the same as before.

### Third Method

The centre of gravity (C.G.) of the load is at the following distance from the feeding end

$$\frac{50 \times 200 + 100 \times 300 + 50 \times 500}{200} = 325 \text{ m}$$

Value of resistance upto C.G. =  $325 \times 0.02/500 = 0.013 \text{ } \Omega$ .

Value of reactance upto C.G. =  $325 \times 0.04/500 = 0.026 \text{ } \Omega$ .

Average p.f. =  $\frac{50 \times 1 + 100 \times 0.8 + 50 \times 0.6}{200} = 0.8$

$$\cos \phi_{av} = 0.8 ; \sin \phi_{av} = 0.6$$

Drop =  $200(0.013 \times 0.8 + 0.026 \times 0.6) = 5.2 \text{ V}$

This is approximately the same as before.

**Example 41.43.** A single-phase distributor, one km long has resistance and reactance per conductor of  $0.2 \text{ } \Omega$  and  $0.3 \text{ } \Omega$  respectively. At the far end, the voltage  $V_B = 240 \text{ V}$  and the current is  $100 \text{ A}$  at a power factor of  $0.8$  lag. At the mid-point A of the distributor current of  $100 \text{ A}$  is tapped at a power factor of  $0.6$  lag with reference to the voltage  $V_A$  at the mid-point. Calculate the supply voltage  $V_S$  for the distributor and the phase angle between  $V_S$  and  $V_B$ .

**Solution.** As shown in Fig. 41.54 (a), let  $SB$  be the distributor with A as the mid point. Total impedance of the distributor is  $(0.4 + j0.6) \text{ } \Omega$ .

Let the voltage  $V_B$  at point B be taken as the reference voltage.

$$\therefore \mathbf{V}_B = (240 + j0) \text{ V} ; \mathbf{I}_B = 100 (0.8 - j0.6) = 80 - j60 \text{ A}$$

$$\text{Drop in section } AB = (80 - j60) (0.2 + j0.3) = (34 + j12) \text{ V}$$

$$\mathbf{V}_A = \mathbf{V}_B + \text{drop over } AB = (240 + j0) + (34 + j12) = (274 + j12) \text{ V}$$

The phase difference between  $V_A$  and  $V_B$  is  $\tan^{-1} (12/274) = 2^\circ 30'$

The load current  $I_A$  has lagging power factor of  $0.6$  with respect to  $V_A$ . It lags  $V_A$  by an angle  $\phi = \cos^{-1}(0.6) = 53^\circ 8'$ .

Hence, it lags behind  $V_B$  by an angle of  $(53^\circ 8' - 2^\circ 30') = 50^\circ 38'$  as shown in the vector diagram of Fig. 41.54 (b).



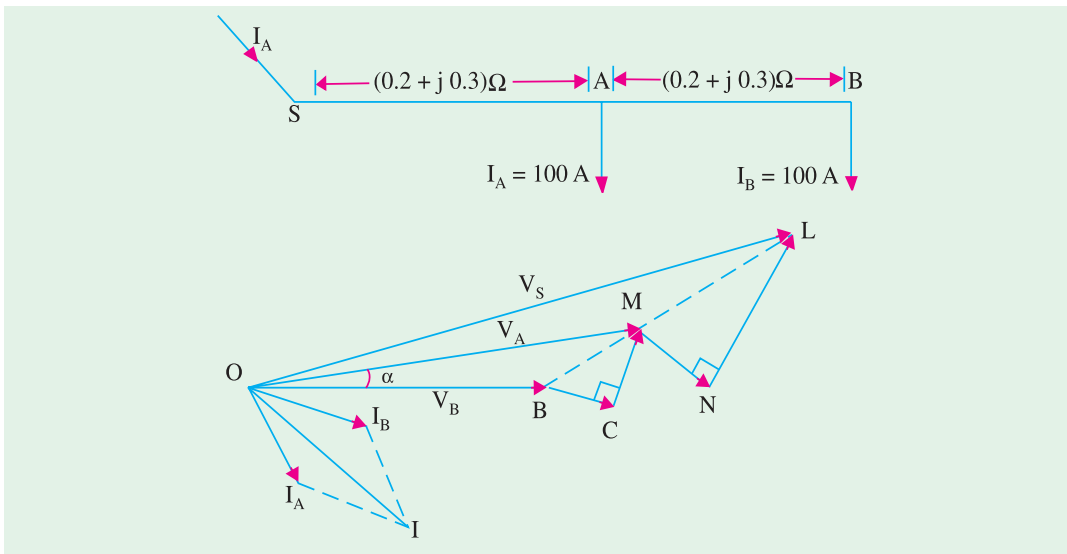


Fig. 41.54

$$\begin{aligned}
 \mathbf{I}_A &= 100 (\cos 50^\circ 38' - j \sin 50^\circ 38') = (63.4 - j77.3) \text{ A} \\
 \mathbf{I} &= \mathbf{I}_A + \mathbf{I}_B = (80 - j60) + (63.4 - j77.3) = (143.4 - j137.3) \text{ A} \\
 \text{Drop in section } S_A &= (143.4 - j137.3)(0.2 + j0.3) = (69.87 + j15.56) \\
 \mathbf{V}_S &= \mathbf{V}_A + \text{drop in section } S_A \\
 &= (274 + j12) + (69.87 + j15.56) = 343.9 + j27.6 = 345 \angle 5^\circ 28' \text{ V}
 \end{aligned}$$

Hence, supply voltage is 345 V and lead  $V_B$  by  $5^\circ 28'$

**Example 41.44.** A 1-phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at A. The total impedances of the sections AB, BC and CA are  $(1 + j1)$ ,  $(1 + j2)$  and  $(1 + j3)$  ohm respectively. Find the total current fed at A and the current in each section.

(Transmission and Distribution-II, Madras Univ.)

**Solution.** Thevenin's theorem will be used to solve this problem. The ring distributor is shown in Fig. 41.55 (a). Imagine feeder BC to be removed [Fig. 41.55 (b)].

$$\begin{aligned}
 \text{Current in } AB &= 20(0.8 - j0.6) = (16 - j12) \text{ A} & \text{Current in section } AC &= 15(0.6 - j0.8) = (9 - j12) \text{ A} \\
 \text{Drop over } AB &= (16 - j12)(1 + j1) = (28 + j4) \text{ V} & \text{Drop over } AC &= (9 - j12)(1 + j3) = (45 + j15) \text{ V}
 \end{aligned}$$

Obviously, point C is at a lower potential as compared to point B.

$$\text{p.d. between } B \text{ and } C = (45 + j15) - (28 + j4) = (17 + j11) \text{ V}$$

Impedance of the network as looked into from points B and C is  $(1 + j1) + (1 + j3) = (2 + j4)\Omega$ . The equivalent Thevenin's source is shown in Fig. 41.55 (c) with feeder BC connected across it.

$$\text{Current in } BC = \frac{17 + j11}{(2 + j4) + (1 + j2)} = (2.6 - j1.53) \text{ A}$$

$$\text{Current in } AB = (16 - j12) + (2.6 - j1.53) = 18.6 - j13.53 = 23 \angle -36^\circ \text{ A}$$

$$\text{Current in } BC = (9 - j12) - (2.6 - j1.53) = 6.4 - j11.5 = 13.2 \angle -60.9^\circ \text{ A}$$

$$\text{Total current fed at point A} = (16 - j12) + (9 - j12) = 25 - j24 = 34.6 \angle -43.8^\circ \text{ A}$$

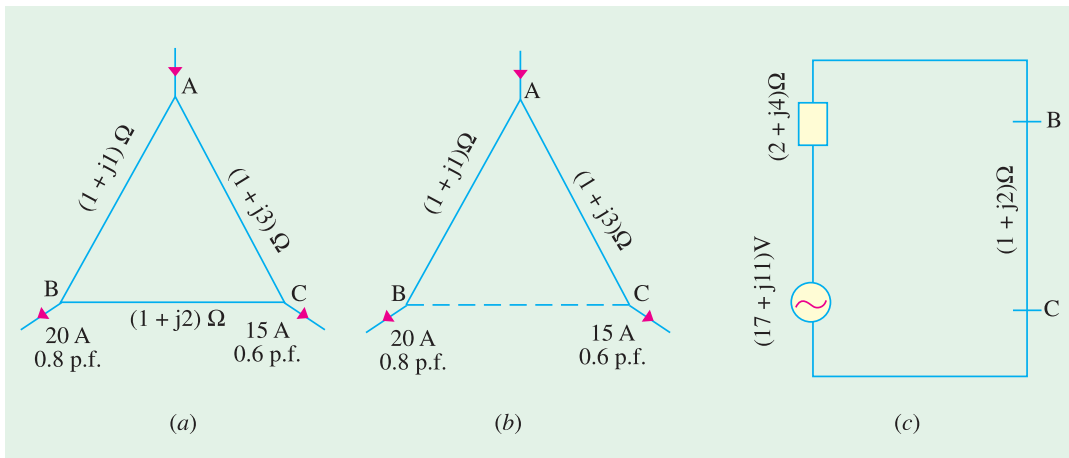


Fig. 41.55

**Example 41.45.** A 2-wire ring distributor ABC is supplied at A at 400 V. Point loads of 20 A at a p.f. of 0.8 lagging and 30 A at a p.f. 0.6 lagging are tapped off at B and C respectively. Both the power factors refer to the voltage at A. The respective go-and-return impedances of sections AB, BC and CA are  $(1 + j2)$  ohm,  $(2 + j3)$  ohm and  $(1 + j3)$  ohm. Calculate the current flowing through each section and the potentials at B and C. Use Superposition theorem.

(Power Systems-I, M.S. Univ. Baroda 1992)

**Solution.** The distributor circuit is shown in Fig. 41.56 (a). Currents in various sections are as shown. First, consider the load at point B acting alone as in Fig. 41.56 (b). The input current at point A divides in the inverse ratio of the impedances of the two paths A B and A C B.

Let the currents be  $I_1'$  and  $I_2'$

$$I_1' = (16 - j12) \times \frac{(1 + j3) + (2 + j3)}{(1 + j3)(2 + j3) + (1 + j3)} = (16 - j12) \times \frac{(3 + j6)}{4 + j8} = (12 - j9) \text{ A}$$

$$I_2' = (16 - j12) - (12 - j9) = (4 - j3) \text{ A}$$

Now, consider the load at point C to act alone as shown in Fig. 41.56 (c). Let the currents now be  $I_1''$  and  $I_2''$ .

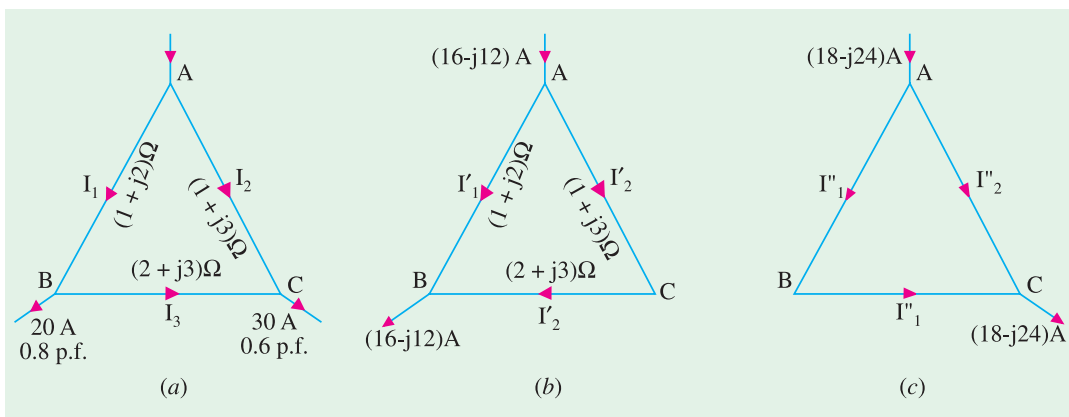


Fig. 41.56

$$I_1'' = (18 - j24) \times \frac{1 + j3}{4 + j8} = 7.5 - j7.5; \quad I_2'' = (18 - j24) - (7.5 - j7.5) = (10.5 - j16.5) \text{ A}$$

As per Superposition theorem, the current  $I_1$  in section  $AB$  is the vector sum of  $I_1'$  and  $I_1''$ .

$$\therefore I_1 = I_1' + I_1'' = (12 - j9) + (7.5 - j7.5) = (19.5 - j16.5) \text{ A}$$

$$I_2 = I_2' + I_2'' = (4 - j3) + (10.5 - j16.5) = (14.5 - j19.5) \text{ A}$$

$$I_3 = I_1'' - I_2' = (7.5 - j7.5) - (4 - j3) = (3.5 - j4.5) \text{ A}$$

$$\text{Potential of } B = 400 - \text{drop over } AB \\ = 400 - (19.5 - j16.5)(1 + j2) = 347.5 - j22.5 = 348 \angle -3.7^\circ \text{ V}$$

$$\text{Potential of } C = 400 - (14.5 - j19.5)(1 + j3) = 327 - j24 = 328 \angle -4.2^\circ \text{ V}$$

**Example 41.46.** A 3-phase ring main  $ABCD$ , fed from end  $A$ , supplies balanced loads of 50 A at 0.8 p.f. lagging at  $B$ , 120 A at u.p.f. at  $C$  and 70 A at 0.866 p.f. lagging at  $D$ , the load currents being referred to the voltage at point  $A$ .

The impedance per phase of the various line sections are :

$$\text{section } AB = (1 + j0.6) \Omega ; \text{ section } BC = (1.2 + j0.9) \Omega$$

$$\text{section } CD = (0.8 + j0.5) \Omega ; \text{ section } DA = (3 + j2) \Omega$$

Determine the currents in the various sections.

(Electrical Power-I, Bombay Univ.)

**Solution.** One phase of the ring main is shown in Fig. 41.57. Let the current in section  $AB = (x + jy)$  A.

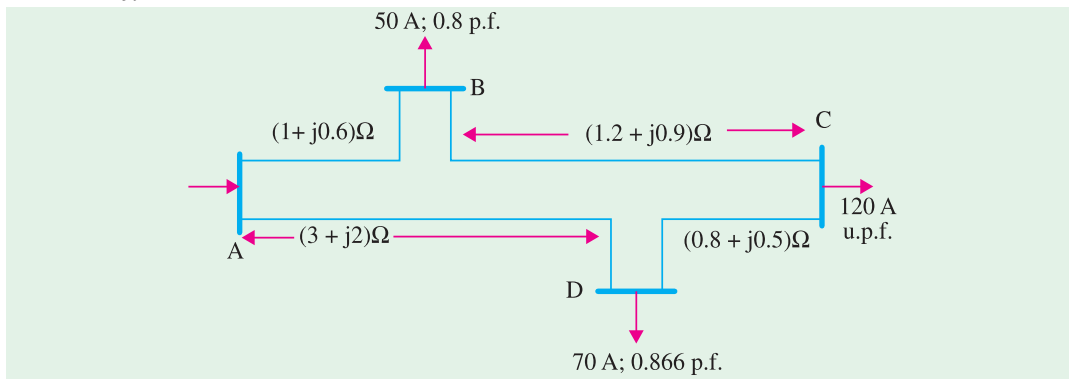


Fig. 41.57

$$\text{Current in } BC = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)$$

$$\text{Current in } CD = (x - 40) + j(y + 30) - (120 + j0) = (x - 160) + j(y + 30)$$

$$\text{Current in } DA = (x - 160) + j(y + 30) - 70(0.866 - j0.5) = (x - 220.6) + j(y + 65)$$

Applying Kirchhoff's voltage law to the closed loop  $ABCD$ , we have

$$(1 + j0.6)(x + jy) + (1.2 + j0.9) \\ [(x - 40) + j(y + 30)] + (0.8 + j0.5) \\ [(x - 160) + j(y + 30)] + (3 + j2) \\ [(x - 220.6) + j(y + 65)] = 0$$

$$\text{or } (6x - 4y + 1009.8) + j(4x + 6y - 302.2) = 0$$

Since the real (or active) and imaginary (or reactive) parts have to be separately zero.

$$\therefore 6x - 4y + 1009.8 = 0 \quad \text{and} \quad 4x + 6y - 302.2 = 0$$

Solving for  $x$  and  $y$ , we get

$$x = 139.7 \text{ and } y = -42.8$$

$$\therefore \text{Current in section } AB = (139.7 - j42.8) \text{ A}$$

$$\text{Current in section } BC = (139.7 - 40) + j(-42.8 + 30) = (99.7 - j12.8) \text{ A}$$

$$\text{Current in section } CD = (139.7 - 160) + j(-42.8 + 30) = (-20.3 - j12.8) \text{ A}$$

$$\text{Current in section } DA = (139.7 - 220.6) + j(-42.8 + 65) = (-80.9 + j22.2) \text{ A}$$

### 41.36. Load Division Between Parallel Lines

It is common practice to work two or more cables or overhead lines in parallel when continuity of supply is essential. In the case of a fault developing in one line of cable, the other lines or cables carry the total load till the fault is rectified."

Let us take the case of two lines in parallel and having impedances of  $Z_1$  and  $Z_2$ . Their combined impedance is

$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

If  $I$  is the current delivered to both lines, then total drop =  $I Z = I \times \frac{Z_1 Z_2}{Z_1 + Z_2}$

If  $I_1$  and  $I_2$  are the respective currents flowing in the two lines, then

$$I_1 = \frac{\text{voltage drop}}{Z_1} = \frac{I Z_2}{Z_1 + Z_2}; \quad \text{Similarly, } I_2 = \frac{I Z_1}{Z_1 + Z_2}$$

It may be noted that in the case of two impedances in parallel, it is convenient to take voltage vector as the reference vector.

**Example 41.47.** A total load of 12,000 kW at a power factor of 0.8 lagging is transmitted to a substation by two overhead three-phase lines connected in parallel. One line has a conductor resistance of  $2 \Omega$  per conductor and reactance (line to neutral) of  $1.5 \Omega$ , the corresponding values for the other line being  $1.5$  and  $1.2 \Omega$  respectively. Calculate the power transmitted by each overhead line.

(London Univ.)

**Solution.** Let us assume a line voltage of 1000 kV for convenience.

$$Z_1 = (2 + j1.5); \quad Z_2 = (1.5 + j1.2) \Omega$$

$$\text{Total load current } I = 12,000/\sqrt{3} \times 1000 \times 0.8 = 8.66 \text{ A}$$

Taking voltage along reference vector, we have  $I = 8.66 (0.8 - j0.6)$

$$I_1 = \frac{8.66 (0.8 - j0.6) (1.5 + j1.2)}{(2 + j1.5) (1.5 + j1.2)} = 4.437(6.882 - j5.39) \text{ A}$$

$\therefore$  power transmitted by 1st line is

$$W_1 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 6.882 = \mathbf{5,280 \text{ kW}}$$

$$\text{Similarly } I_2 = \frac{8.66 (0.8 - j0.6) (2 + j1.5)}{(2 + j1.5) (1.5 + j1.2)} = 4.437(8.75 - j6.75) \text{ A}$$

$$W_2 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 8.75 = \mathbf{6,720 \text{ kW}}$$

$$\text{As a check, total power} = 5,280 + 6,720 = \mathbf{12,000 \text{ kW}}$$

### 41.37. Suspension Insulators

Suspension insulators are used when transmission voltage is high. A number of them are connected in series to form a chain and the line conductor is carried by the bottom most insulator.

As shown in Fig. 41.58 a 'cap' type suspension insulator consists of a single disc-shaped piece of porcelain grooved on the under surface to increase the surface leakage path. A galvanized cast iron cap is cemented at the top of the insulator. In the hollow cavity of the insulator is cemented a galvanized forged steel pin, the lower enlarged end of which fits into the cavity of the steel cap of the lower suspension insulator and forms a ball and socket connection.

A string of suspension insulators consists of many units, the number of units depending on the value of the transmission voltage. There exists mutual capacitance between different units and, in addition, there is capacitance to ground of each unit because of the nearness of the tower, the cross-arm and the line. Due to this capacitance to ground, the total system voltage is not equally distributed over the different units of the string. The unit nearest to the line conductor carries the maximum percentage of voltage, the figure progressively decreasing as the unit nearest to the tower is approached. The inequality of voltage distribution between individual units becomes more pronounced as the number of insulators increases and it also depends on the ratio (capacitance of insulator/capacitance of earth).

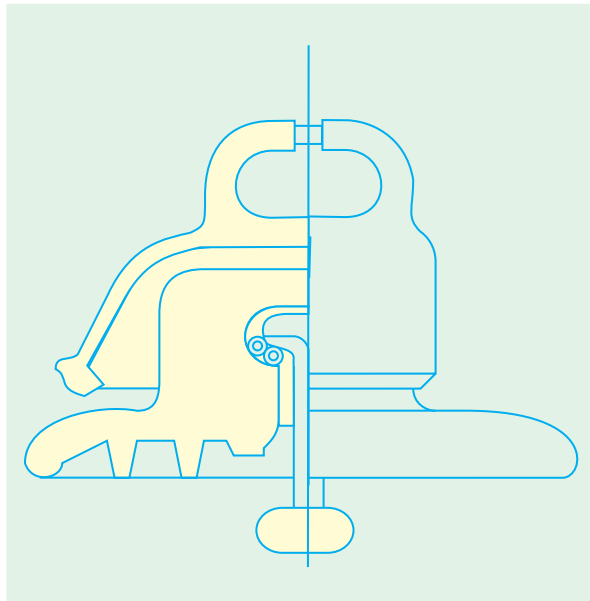


Fig. 41.58

### String Efficiency

If there are  $n$  units in the string, then its efficiency is given by

$$\% \text{ string } \eta = \frac{\text{total voltage across the string}}{n \times \text{voltage across unit adjacent the line}} \times 100$$

### 41.38. Calculation of Voltage Distribution along Different Units

Let,  $C$  = capacitance to ground

$kC$  = mutual capacitance between units

The current and voltage distribution is as shown in Fig. 41.59. It is seen that

$$I_1 = \frac{V_1}{1/\omega kC} = \omega kC V_1; \text{ Similarly } i_1 = \frac{V_1}{1/\omega C} = \omega C V_1$$

$$\text{Now, } I_2 = I_1 + i_1 = \omega C V_1 (1 + k) \text{ and } V_2 = \frac{I_2}{\omega kC} = \frac{V_1 (1 + k)}{k}$$

The current  $i_2$  is produced by the voltage combination of  $(V_1 + V_2)$

$$\text{Now, } V_1 + V_2 = V_1 \left(1 + \frac{1+k}{k}\right) = V_1 \left(1 + \frac{1+2k}{k}\right); \therefore i_2 = \omega C V_1 \left(1 + \frac{1+2k}{k}\right)$$

At junction  $B$ , we have

$$\begin{aligned} I_3 &= I_2 + i_2 = \omega C V_1 \left\{1 + k \frac{(1+2k)}{k}\right\} \\ &= \omega C V_1 \frac{(1+3k+k^2)}{k} \end{aligned}$$

$$\text{However, } I_3 = \omega kC V_3; \therefore V_3 = \frac{V_1 (1+3k+k^2)}{k^2} = V_1 \left(1 + \frac{3}{k} + \frac{1}{k^2}\right)$$

The current  $i_3$  is produced by the voltage combination of  $(V_1 + V_2 + V_3)$

Now,  $V_1 + V_2 + V_3$   
 $= V_1 \left[ 1 + \frac{(1+k)}{k} + \frac{(1+3k+k^2)}{k^2} \right]$

$\therefore i_3 = \omega C V_1 \left( \frac{1+4k+3k^2}{k^2} \right)$

At junction C, we have

$I_4 = I_3 + i_3 = \omega C V_1 \left[ \frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2} \right]$

Now,  $I_4 = \omega k C V_4$ ,

$\therefore V_4 = V_1 \left[ \frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2} \right]$

$= V_1 \left( 1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^2} \right)$

For the fifth insulator from the top, we have

$V_5 = \left( 1 + \frac{10}{k} + \frac{15}{k^2} + \frac{7}{k^3} + \frac{1}{k^4} \right)$

and so on.

If the string has  $n$  units, the total voltage is given by

$V = (V_1 + V_2 + V_3 + V_4 + \dots + V_n)$

**Example 41.48.** For a string insulator with four discs, the capacitance of the disc is ten times the capacitance between the pin and earth. Calculate the voltage across each disc when used on a 66-kV line. Also, calculate the string efficiency. (Power Systems-I, AMIE Sec. B, 1994)

**Solution.** Let  $C$  be the self-capacitance of each disc and  $kC$  the capacitance between each link pin and earth. We are given that  $k = 10$

As seen from Art. 41.38,

$V_2 = \frac{(1+k)}{k} V_1 = \frac{11}{10} V_1$

$V_3 = V_1 \left( 1 + \frac{3}{k} + \frac{1}{k^2} \right) = \frac{131}{100} V_1$

$V_4 = V_1 \left( 1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^3} \right) = \frac{1561}{100} V_1$

$V = V_1 + V_2 + V_3 + V_4 = V_1 + \frac{11}{10} V_1 + \frac{131}{100} V_1 + \frac{1561}{1000} V_1 = \frac{4971}{1000} V_1$

$\therefore V_1 = V \frac{1000}{4971} = \frac{66}{\sqrt{3}} \times \frac{1000}{4971} = 7.66 \text{ kV}$

$V_2 = \frac{11}{10} V_1 = \frac{11}{10} \times 7.66 = 8.426 \text{ kV}$

$V_3 = \frac{131}{100} V_1 = \frac{131}{100} \times 7.66 = 10.03 \text{ kV}$

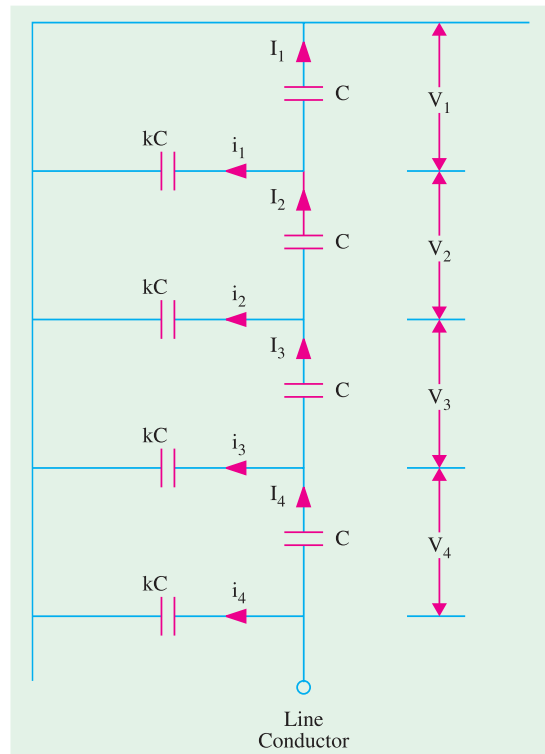


Fig. 41.59

$$V_4 = \frac{1561}{1000} V_1 = \frac{1561}{1000} \times 7.66 = 11.96 \text{ kV}$$

$$\text{String } \eta = \frac{V}{4 \times V_4} \times 100 = \frac{66 \times 100}{\sqrt{3} \times 4 \times 11.96} = 79.65 \%$$

**Example 41.49.** Explain what is meant by string efficiency and how it can be improved. Each line of a 3-phase, 33-kV system is suspended by a string of 3 identical insulator discs. The capacitance of each disc is 9 times the capacitance to ground. Find voltage distribution across each insulator and the string efficiency. Suggest a method for improving the string efficiency.

(Power Systems-I, AMIE, Sec. B, 1993)

**Solution.** Let  $C$  be the self-capacitance of each disc and  $kC$  the mutual capacitance between the units. We are given that  $k = 9$ .

$$V_2 = V_1 \frac{(1+k)}{k} = \frac{10}{9} V_1;$$

$$V_3 = V_1 \frac{(1+3k+k^2)}{k^2} = \frac{109}{81} V_1$$

Total voltage,

$$V = V_1 + V_2 + V_3 = V_1 + \frac{10}{9} V_1 + \frac{109}{81} V_1 = \frac{280}{81} V_1$$

$\therefore$

$$\frac{33}{\sqrt{3}} = \frac{280}{81} V_1; \therefore V_1 = \frac{33 \times 81}{\sqrt{3} \times 280} = 5.51 \text{ kV}$$

$$V_2 = (10/9) V_1 = 6.12 \text{ kV}; V_3 = (109/81) V_1 = 7.41 \text{ kV}$$

String efficiency

$$= \frac{V}{3 \times V_3} \times 100 = \frac{33/\sqrt{3}}{3 \times 7.41} \times 100 = 85\%$$

The string efficiency can be improved by providing a guard ring surrounding the lower-most unit which is connected to the metal work at the bottom. This ring increases the capacitance between the metal work and the line and helps in equalising the voltage distribution along the different units. The efficiency can also be improved by grading the insulators and by making the ratio (capacitance to earth/capacitance per insulator) as small as possible.

### 41.39. Interconnectors

An interconnector is a tieline which enables two generating stations to operate in parallel. It facilitates the flow of electric power in either direction between the two stations.

### 41.40. Voltage Drop Over the Interconnector

Let station 1 supply a current of 1 to station 2 along an interconnector having a resistance of  $R$  and reactance of  $X$  per phase as shown in Fig. 41.60 (a). If the receiving end p.f. is  $\cos \phi$  lagging, then the vector diagram will be as shown in Fig. 41.60 (b).

Voltage drop over the interconnector

$$= I (\cos \phi - j \sin \phi) (R + jX)$$

$$= I (R \cos \phi + X \sin \phi) + jI (X \cos \phi - R \sin \phi)$$

% voltage drop

$$= \left\{ \frac{I (R \cos \phi + X \sin \phi) + jI (X \cos \phi - R \sin \phi)}{E_2} \right\} \times 100$$

Let  $I \cos \phi = I_d$  and  $I \sin \phi = I_q$ , where  $I_d$  and  $I_q$  are the in-phase and quadrature components.

$$\therefore \text{ voltage drop} = \left\{ \frac{(I_d R + I_q X) + j (I_d X - I_q R)}{E_2} \right\} \times 100$$

$$\therefore \text{in-phase voltage drop} = \frac{(I_d R + I_q X)}{E_2} \times 100$$

$$\text{and quadrature voltage drop} = \frac{(I_d X^2 - I_q R)}{E_2} \times 100$$

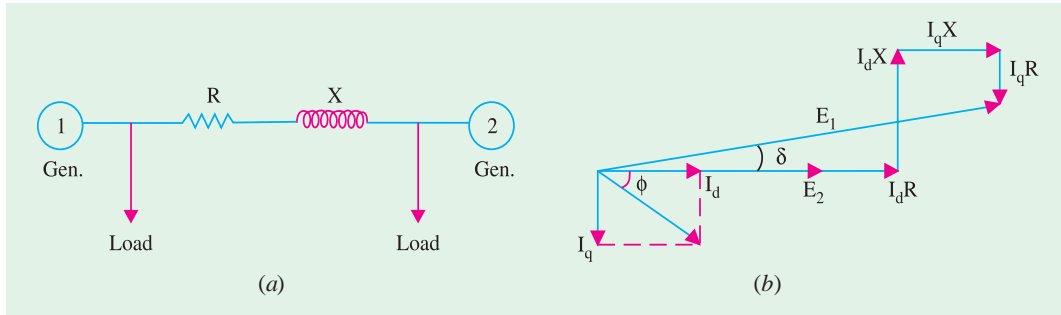


Fig. 41.60

**Example 41.50.** The bus-bar voltages of two stations A and B are 33 kV and are in phase. If station A sends 8.5 MW power at u.p.f. to station B through an interconnector having an impedance of  $(3 + j4)\Omega$ , determine the bus-bar voltage of station A and the phase angle shift between the bus-bar voltages.

**Solution.** With reference to Fig. 41.61,

Voltage of station B,  $V_B = 33,000/\sqrt{3} = 19,050$  V/phase

Since power transferred to station B is 8.5 MW, current in the interconnector is

$$I = 8.5 \times 10^3 / \sqrt{3} = 148.7 \text{ A}$$

Taking  $V_B$  as the reference vector,

$$V_A = V_B + I(\cos \phi - j \sin \phi)(R + jX)$$

$$= 19,050 + 148.7(1 - j0)(3 + j4)$$

$$= 19,496 + j595 = 19,502 \angle 1.75^\circ$$

The line-to-line bus-bar voltage of station

A is

$$= 19,502 \times \sqrt{3} = 33.78 \text{ kV}$$

The phase-shift angle between bus-bar voltages is  $1.75^\circ$

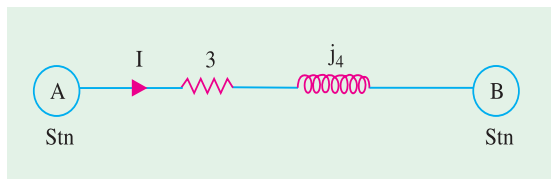


Fig. 41.61

### 41.41. Sag and Stress Analysis

The conductors of a transmission line are attached to suitable insulators carried on supports of wood, iron, steel or reinforced concrete. Obviously, the supports must be strong enough to withstand not only the dead weight of the conductors themselves but also the loads due to ice and sleet that may adhere to them and to wind pressure. Moreover, the minimum factor of safety for the conductors should be 2.0 based on ultimate strength.

Sag and stresses vary with temperature on account of thermal expansion and contraction of the line conductors. The value of sag as well as tension of a conductor would now be calculated when (i) supports are at equal levels and (ii) supports are at unequal levels.

### 41.42. Sag and Tension with Supports at Equal Levels

Fig. 41.62 shows a span of a wire with the two supports at the same elevation and separated by a horizontal distance  $2l$ . It can be proved that the conductor AB forms a catenary with the lowest point O forming the mid-point (where the curve is straight).



Let  $W$  be the weight of the wire per unit length and let point  $O$  be chosen as the reference point for measuring the co-ordinates of different points on the wire. Consider a point  $P$  having co-ordinates of  $x$  and  $y$ . The tension  $T$  at point  $P$  can be resolved into two rectangular components,  $T_x$  and  $T_y$ , so that  $T = \sqrt{T_x^2 + T_y^2}$ . If  $S$  is the length of the arc  $OP$ , then its weight is  $WS$  which acts vertically downward through the centre of gravity of  $OP$ . There are four forces acting on  $OP$ —two vertical and two horizontal. Since  $OP$  is in equilibrium, the net force is zero. Equating the horizontal and vertical components, we have,

$$T_0 = T_x \text{ and } T_y = WS.$$

It may be noted that the horizontal component of tension is constant throughout the length of the wire :

Since line  $PT$  is tangential to the curve  $OB$  at point  $P$ ,  $\tan \theta = \frac{T_y}{T_x}$

It is also seen from the elementary piece  $PP'$  of the line that  $\tan \theta = dy/dx$

$$\therefore \quad dy/dx = \tan \theta = \frac{T_y}{T_x} \text{ or } dy/dx = WS/T_0 \quad \dots (i)$$

$$\text{If } PP' = dS, \text{ then } dS = \sqrt{(dx)^2 + (dy)^2} = dx \sqrt{1 + (dy/dx)^2}$$

$$\text{or } \frac{dS}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{WS}{T_0}\right)^2} \text{ or } dx = \frac{dS}{\sqrt{1 + \left(\frac{W^2 S^2}{T_0^2}\right)}}$$

$$\text{Integrating both sides, we have } x = \left(\frac{T_0}{W}\right) \sinh^{-1} \left(\frac{WS}{T_0}\right) + C$$

where  $C$  is the constant of integration.

Now, when  $x = 0, S = 0$ . Putting these values above, we find that  $C = 0$ .

$$\therefore \quad x = \left(\frac{T_0}{W}\right) \sinh^{-1} \left(\frac{WS}{T_0}\right) \text{ or } S = \frac{T_0}{W} \sinh \left(\frac{Wx}{T_0}\right) \quad \dots (ii)$$

Substituting this value of  $S$  in Eq. (i), we get

$$\frac{dy}{dx} = \sinh \left(\frac{Wx}{T_0}\right) \text{ or } dy = \sinh \left(\frac{Wx}{T_0}\right) dx$$

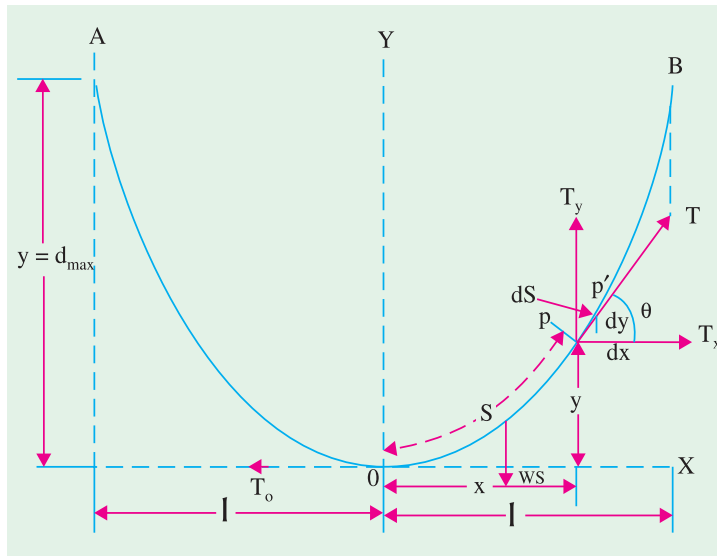


Fig. 41.62

$$\therefore y = \int \sinh\left(\frac{Ex}{T_0}\right) dx = \left(\frac{T_0}{W}\right) \cosh\left(\frac{Wx}{T_0}\right) + D$$

where  $D$  is also the constant of integration. At the origin point  $O$ ,  $x = 0$  and  $y = 0$ . Hence, the above equation becomes

$$0 = \left(\frac{T_0}{W}\right) \cosh 0 + D + \left(\frac{T_0}{W}\right) + D \quad \therefore D = -\frac{T_0}{W}$$

Substituting this value of  $D$  in the above equation, we get

$$y = \left(\frac{T_0}{W}\right) \cosh\left(\frac{Wx}{T_0}\right) - \frac{T_0}{W} = \frac{T_0}{W} \left[ \cosh\left(\frac{Wx}{T_0}\right) - 1 \right] \quad \dots(iii)$$

This is the equation of the curve known as catenary. Hence, when a wire is hung between two supports, it forms a catenary

(a) The tension at point  $P(x, y)$  is given by

$$\begin{aligned} T^2 &= T_{x2} + T_{y2} = T_{02} + W^2 S^2 = T_{02} + T_{02} \sinh^2\left(\frac{Wx}{T_0}\right) \quad \text{---from Eq. (iii)} \\ &= T_{02} \left[ 1 + \sinh^2\left(\frac{Wx}{T_0}\right) \right] = T_{02} \cosh^2\left(\frac{Wx}{T_0}\right) \quad \therefore T = T_0 \cosh\left(\frac{Wx}{T_0}\right) \quad \dots(iv) \end{aligned}$$

(b) Tension at points  $A$  and  $B$  where  $x = \pm l$  is given by  $T = T_0 \cosh(Wl/T_0)$  ... (v)

(c) The maximum sag is represented by the value of  $y$  at either of the two points  $A$  and  $B$  for which  $x = +l$  and  $x = -l$  respectively. Writing  $y = d_{\max}$  and putting  $x = \pm l$  in Eq. (iii), we get,

$$d_{\max} = \frac{T_0}{W} \left[ \cosh\left(\frac{Wl}{T_0}\right) - 1 \right] \quad \dots(vi)$$

(d) The length of the wire or conductor in a half span is as seen from Eq. (ii) above,

$$S = \frac{T_0}{W} \sinh\left(\frac{Wl}{T_0}\right)$$

### Approximate Formulae

The hyperbolic sine and cosine functions can be expanded into the following series

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \dots \quad \text{and} \quad \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \dots$$

Using the above, the approximate values of  $T$ ,  $d$  and  $S$  points at  $A$  and  $B$  may be found as follows :

$$\begin{aligned} (i) \quad T &= T_0 \cosh\left(\frac{Wl}{T_0}\right) = T_0 \left( 1 + \frac{W^2 l^2}{2T_0^2} + \dots \right) \text{---neglecting higher powers} \\ &= T_0 + \frac{W^2 l^2}{2T_0} \cong T_0 \quad \text{---if } \frac{W^2 l^2}{2T_0} \text{ is negligible} \end{aligned}$$

$\therefore T = T_0$  — *i.e.* tension at the supports is very approximately equal to the horizontal tension acting at any point on the wire.

$$\begin{aligned} (ii) \quad d &= \frac{T_0}{W} \left[ \cosh\left(\frac{Wl}{T_0}\right) - 1 \right] = \frac{T_0}{W} \left[ \left( 1 + \frac{W^2 l^2}{2T_0^2} + \dots \right) - 1 \right] \\ &= \frac{Wl^2}{2T_0} = \frac{Wl^2}{2T} \text{ (approx)} \quad \therefore d = \frac{Wl^2}{2T} \end{aligned}$$

It should be noted that  $W$  and  $T$  should be in the same units *i.e.* kg-wt or newton.

$$(iii) S = \frac{T_0}{W} \sinh + \left( \frac{WL}{T_0} \right) = \frac{T_0}{W} \left( \frac{WL}{T_0} + \frac{W^3 l^3}{6T_0^3} + \dots \right) = l + \frac{W^2 l^3}{6T_0^2} \quad \text{---neglecting higher terms.}$$

The total length of the wire along the curve is  $L = 2S = 2l + \frac{W^2 l^2}{3T^2}$

This length consists of the unstretched length  $L_u$  and the stretch or extension.

∴ unstretched length  $L_u = L$  —extension  $\Delta L$

Now,  $E = \frac{T/A}{\Delta l/2l} \quad \text{or} \quad \Delta l = \frac{2lT}{EA}$

Substituting the value of  $T$  from the relation  $d = Wl^2/2T$ , we have

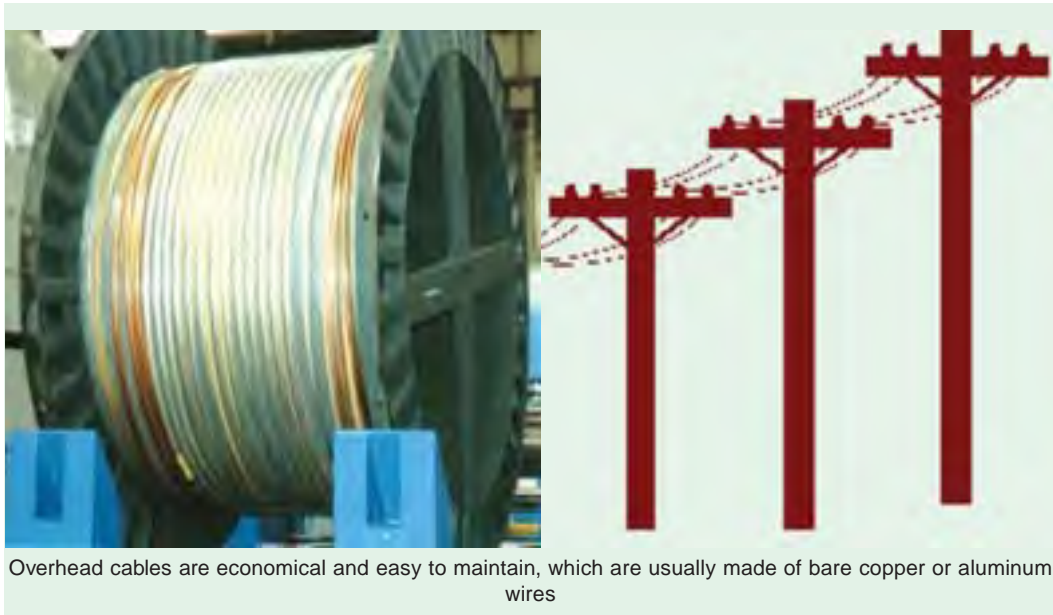
$$\Delta l = \frac{2l}{EA} \cdot \frac{Wl^2}{2d} = \frac{Wl^3}{EAd} \quad \therefore L_u = L - \frac{Wl^3}{EAd}$$

$$(iv) y = \frac{T_0}{W} \left[ \cosh \left( \frac{Wx}{T_0} \right) - 1 \right] = \frac{T_0}{W} \left[ \left( 1 + \frac{W^2 x^2}{2T_0^2} + \dots \right) - 1 \right]$$

or  $y = \frac{Wx^2}{2T}$  —the equation of a parabola

It shows that the catenary curve formed by the sagging is very approximately like a parabola in shape.

The above formulae are sufficiently accurate for all practical purposes provided sag is less than 10% of the span.



### 41.43. Sag and Tension with Supports at Unequal Levels

Fig. 41.63 shows a span between two supports  $A$  and  $B$  whose elevations differ by  $\eta$ , their horizontal spacing being  $2l$  as before. Such spans are generally met with in a hilly country. Let  $O$  be the lowest point of the catenary  $AOB$ . Obviously,  $OA$  is a catenary of half-span  $x_1$  and  $OB$  of half-span  $x_2$ .

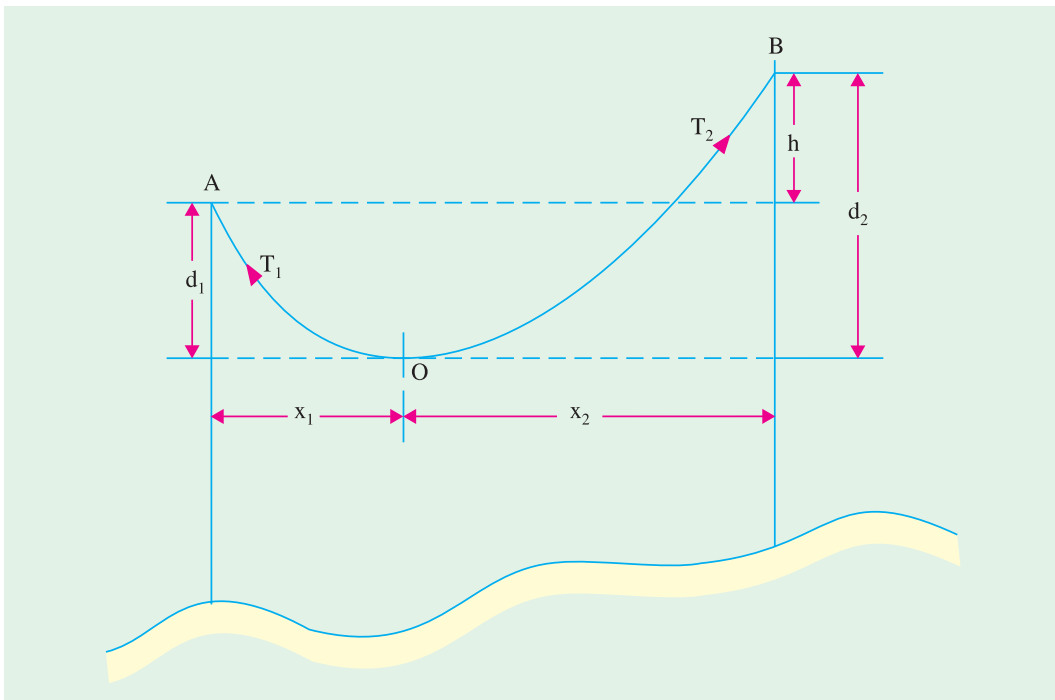


Fig. 41.63

The equations derived in Art. 41.42 also apply to this case.

(i) As seen from Eq. (vi) of Art. 41.42.

$$d_1 = \frac{T_0}{W} \left[ \cosh \left( \frac{Wx_1}{T_0} \right) - 1 \right] \quad \text{--- putting } l = x_1$$

$$d_2 = \frac{T_0}{W} \left[ \cosh \left( \frac{Wx_2}{T_0} \right) - 1 \right] \quad \text{--- putting } l = x_2$$

(ii)  $T_1 = T_0 \cosh \left( \frac{Wx_1}{T_0} \right)$  and  $T_2 = T_0 \cosh \left( \frac{Wx_2}{T_0} \right)$

It is obvious that maximum tension occurs at the higher support B.

$$\therefore \text{max. permissible tension} = T_0 \cosh \left( \frac{Wx_2}{T_0} \right)$$

**Approximate Formulae**

It is obvious that

$$d_2 - d_1 = \frac{T_0}{W} \left[ \cosh \left( \frac{Wx_2}{T_0} \right) - \cosh \left( \frac{Wx_1}{T_0} \right) \right] = h \text{ and } x_1 + x_2 = 2l$$

Using approximations similar to those in Art. 41.42, we get

$$d_1 = \frac{Wx_1^2}{2T_0} = \frac{Wx_1^2}{2T} \quad \text{and} \quad d_2 = \frac{Wx_2^2}{2T_0} = \frac{Wx_2^2}{2T}$$

(it has been assumed, as before, that  $T = T_0$ )

$$\therefore d_2 - d_1 = h = \frac{W}{2T} (x_2^2 - x_1^2) = \frac{W}{2T} (x_2 + x_1)(x_2 - x_1) = \frac{Wl}{T} (x_2 - x_1)$$

$$\therefore x_2 - x_1 = \frac{hT}{Wl}; \text{ Now, } x_2 - x_1 = hT/Wl \text{ and } x_2 + x_1 = 2l$$

$$\therefore x_1 = 1 - \frac{hT}{2Wl} \text{ and } x_2 = 1 + \frac{hT}{2Wl} *$$

Having found  $x_1$  and  $x_2$ , values of  $d_1$  and  $d_2$  can be easily calculated. It is worth noting that in some cases,  $x_1$  may be negative which means that there may be no horizontal point (like point  $O$ ) in the span. Such a thing is very likely to happen when the line runs up a steep mountain side.

### 41.44. Effect of Wind and Ice

In the formulae derived so far, effect of ice and wind loading has not been taken into account. It is found that under favourable atmospheric conditions, quite an appreciable thickness of ice is formed on transmission lines. The weight of ice acts vertically downwards *i.e.* in the same direction as the weight of the conductor itself as shown in Fig. 41.64. In addition, there may be high wind which exerts considerable force on the conductor. This force is supposed to act in a horizontal direction. If  $W_i$  is the weight of ice per unit length of the conductor and  $W_u$  the force per unit length exerted by the wind, then total weight of the conductor per unit length is



Effects of wind, ice and snow are important considerations while designing electric cables

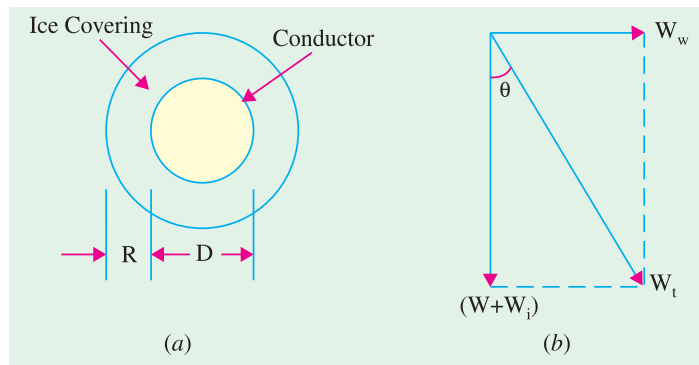


Fig. 41.64

$$W_t = \sqrt{(W + W_i)^2 + W_w^2}$$

It is obvious that in all equations derived in Art. 41.42 and 41.43,  $W$  should be replaced by  $W_t$ . As seen from Fig. 41.64 (b), the conductor sets itself in a plane at an angle of  $\theta = \tan^{-1} [W_w / (W + W_i)]$  to the vertical but keeps the shape of a catenary (or a parabola approximately).

**Note.** (i) If  $P$  is the wind pressure per unit projected area of the conductor, then wind load or force per unit length of the ice-covered conductor is

$$W_w = P \times (D + 2R) \times l = P \times D \quad \text{—if conductor is without ice}$$

\* If we put  $h = 0$  *i.e.* assume supports at the same level, then as expected  $x_1 = x_2 = l$

(ii) Ice is in the form of a hollow cylinder of inner diameter =  $D$  and outer diameter =  $(D + 2R)$ . Hence, volume per unit length of such a cylinder is

$$= \frac{\pi}{4} [(D + 2R)^2 - D^2] \times 1 = \frac{\pi}{4} (4DR + 4R^2) = \pi R(D + R)$$

If  $\rho$  is the ice density, then weight of ice per unit length of the conductor is

$$W_i = \rho \times \text{volume} = \pi \rho R(D + R)$$

When ice and wind loads are taken into account, then the approximate formulae derived in Art. 41.42 become

$$d = \frac{W_t l^2}{2T}; \quad S = 1 + \frac{W_t^2 l^3}{6T^2} \quad \text{and} \quad y = \frac{W_t x^2}{2T}$$

Here 'd' represents the slant sag in a direction making an angle of  $\theta$  with the vertical. The vertical sag would be =  $d \cos \theta$ .

Similarly, formulae given in Art. 41.43 become

$$d_1 = \frac{W_t \cdot x_1^2}{2T}; \quad d_2 = \frac{W_t \cdot x_2^2}{2T}; \quad x_1 = 1 - \frac{hT}{2W_t \cdot l} \quad \text{and} \quad x_2 = 1 + \frac{hT}{2W_t \cdot l}$$

**Example 41.51.** A transmission line conductor at a river crossing is supported from two towers at heights of 70 m above water level. The horizontal distance between towers is 300 m. If the tension in conductor is 1,500 kg, find the clearance at a point midway between the towers. The size of conductor is  $0.9 \text{ cm}^2$ . Density of conductor material is  $8.9 \text{ g/cm}^3$  and suspension length of the string is 2 metres.

**Solution.** Sag,  $d = Wl^2/2T$

$$l = 150 \text{ m}, \quad W = lAp = 2 \times (0.9 \times 10^{-4}) \times (8.9 \times 10^3) = 0.8 \text{ kg wt}; \quad T = 1500 \text{ kg wt}$$

$$d = Wl^2/2T = 0.8 \times 150^2/2 \times 1500 = 6 \text{ m}$$

Clearance between conductor and water at mid-way between the towers  
 =  $70 - 6 - 2 = 62 \text{ m}$

**Example 41.52.** The effective diameter of a line is 1.96 cm and it weighs 90 kg per 100 metre length. What would be the additional loading due to ice of radial thickness 1.25 cm and a horizontal wind pressure of  $30 \text{ kg/m}^2$  of projected area? Also, find the total weight per metre run of the line. Density of ice is  $920 \text{ kg/m}^3$ .

**Solution.** It should be noted that weights of the conductor and ice act vertically downwards whereas wind pressure is supposed to act horizontally. Hence, the total force on one metre length of the conductor is found by adding the horizontal and vertical forces vectorially.

$$\therefore \text{ total weight} \quad W_t = \sqrt{(W + W_i)^2 + W_w^2}, \quad W_i = \pi \rho R(D + R)$$

$$\text{Here} \quad \rho = 920 \text{ kg/m}^3, \quad R = 0.0125 \text{ m}; \quad D = 0.0196 \text{ m}$$

$$W_w = P(D + 2R) = 30(0.0196 + 2 \times 0.0125) = 1.34 \text{ kg wt/m}; \quad W = 90/100 = 0.9 \text{ kg wt/m}$$

$$\therefore \quad W_i = \pi \times 920 \times 0.0125(0.0196 + 0.0125) = 1.16 \text{ kg wt/m}$$

$$\therefore \quad W_t = \sqrt{(W + W_i)^2 + W_w^2} = \sqrt{(0.9 + 1.16)^2 + 1.34^2} = 2.46 \text{ kg wt/m}$$

**Example 41.53.** A transmission line has a span of 150 metres between supports, the supports being at the same level. The conductor has a cross-sectional area of  $2 \text{ cm}^2$ . The ultimate strength is  $5,000 \text{ kg/cm}^2$ . The specific gravity of the material is 8.9. If the wind pressure is  $1.5 \text{ kg/m}$  length of the conductor, calculate the sag at the centre of the conductor if factor of safety is 5.

(Electrical Technology-I, Bombay Univ.)

**Solution.** Safety factor =  $\frac{\text{breaking or ultimate stress}}{\text{working stress}}$

$\therefore$  working stress =  $5,000/5 = 10^3 \text{ kg/cm}^2$ ; Working tension,  $T = 10^2 \times 2 = 2,000 \text{ kg wt.}$

Vol. of one metre length of conductor =  $2 \times 100 = 200 \text{ cm}^3$

Wt. of 1 m of material =  $8.9 \times 200 \text{ g}$  or  $W = 8.9 \times 200/1,000 = 1.98 \text{ kg-wt.}$

Total Wt. per metre =  $\sqrt{W^2 + W_w^2} = \sqrt{1.98^2 + 1.5^2} = 2.48 \text{ kg-wt}$

Now  $d = Wl^2/2T = 2.48 \times (150/2)^2/2 \times 2,000 = \mathbf{3.5 \text{ metre}}$

**Example 41.54.** A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of  $1.29 \text{ cm}^2$ , weighs  $1,170 \text{ kg/km}$  and has a breaking stress of  $4,218 \text{ kg/cm}^2$ . Calculate the sag for a factor of safety of 5 allowing a wind pressure of  $122 \text{ kg per m}^2$  of projected area. What is the vertical sag?

(Transmission and Distribution-II, Kerala Univ.)

**Solution.** Safety factor =  $\frac{\text{breaking or ultimate stress}}{\text{working stress}}$

working stress =  $4,218/5 = 843.6 \text{ kg/cm}^2$

Working tension  $T = 843.6 \times 1.29 = 1,088 \text{ kg.wt}$

$\therefore W = 1,170 \text{ kg/km} = 1.17 \text{ kg.wt/metre}$

Let us now find diameter of the conductor from the equation

$\pi d^2/4 = 1.29$  or  $d = 1.28 \text{ cm}$

$\therefore$  Projected area of the conductor per metre length is =  $1.28 \times 10^{-2} \times 1 = 1.28 \times 10^{-2} \text{ m}^2$

$W_w = 122 \times 1.28 \times 10^{-2} = 1.56 \text{ kg-wt/m}$

$\therefore W_t = \sqrt{W^2 + W_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95 \text{ kg-wt/m}$

Slant sag  $d = W_t l^2/2T = 1.95 \times 100^2/2 \times 1088 = \mathbf{8.96 \text{ m}}$

Now,  $\tan \theta = W_w/W = 1.56/1.17 = 1.333$ ;  $\theta = 53.2^\circ$

$\therefore$  vertical sag =  $d \cos 53.2^\circ = 8.96 \times 0.599 = \mathbf{5.3 \text{ m}}$

**Example 41.55.** A transmission line has a span of 214 metres. The line conductor has a cross-section of  $3.225 \text{ cm}^2$  and has an ultimate breaking strength of  $2,540 \text{ kg/cm}^2$ . Assuming that the line is covered with ice and provides a combined copper and ice load of  $1.125 \text{ kg/m}$  while the wind pressure is  $1.5 \text{ kg/m run}$  (i) calculate the maximum sag produced. Take a factor of safety of 3 (ii) also determine the vertical sag. (Electrical Power Systems, Gujarat Univ.)

**Solution.** (i) Maximum sag in a direction making an angle of  $\theta$  (Fig. 41.64) is  $d = W_t l^2/2T$

Here,  $W + W_i = 1.125 \text{ kg-wt/m}$ ;  $W_w = 1.5 \text{ kg-wt/m}$

$W_t = \sqrt{1.125^2 + 1.5^2} = 1.874 \text{ kg-wt/m}$ ;  $l = 214/2 = 107 \text{ m}$

Safety factor =  $\frac{\text{ultimate breaking stress}}{\text{working stress}}$

$\therefore$  working stress =  $2,540/3 = 847 \text{ kg-wt/cm}^2$

Permissible tension  $T = 847 \times 3.225 = 2,720 \text{ kg-wt}$

$\therefore d = 1.874 \times 107^2/2 \times 2,720 = \mathbf{3.95 \text{ m}}$

(ii) Now  $\tan \theta = W_w/(W_i + W) = 1.5/1.125 = 0.126$ ;  $\theta = 53.2^\circ$ ;  $\cos \theta = 0.599$

$\therefore$  vertical sag =  $d \cos \theta = 3.95 \times 0.599 = \mathbf{2.35 \text{ m}}$

**Example 41.56.** Two towers of height 30 and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 500 m. If the tension in the conductor is 1,600 kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5 kg/m. Bases of the towers can be considered to be at the water level. (Electrical Power-I, Bombay Univ.)

**Solution.**  $x_1 = l - \frac{hT}{2Wl}$

Here,  $l = 500/2 = 250$  m  
 $h = 90 - 30 = 60$  m  
 $T = 1,600$  kg-wt ;  
 $W = 1.5$  kg-wt/m

$$\therefore x = 250 - \frac{60 \times 1,600}{2 \times 1.5 \times 250}$$

$$= 250 - 128 = 122 \text{ m}$$

$$x_1 = 250 + 128 = 378 \text{ m}$$

$$d_1 = \frac{Wx_1^2}{2T} = \frac{1.5 \times 122^2}{2 \times 1600} = 7 \text{ m}$$

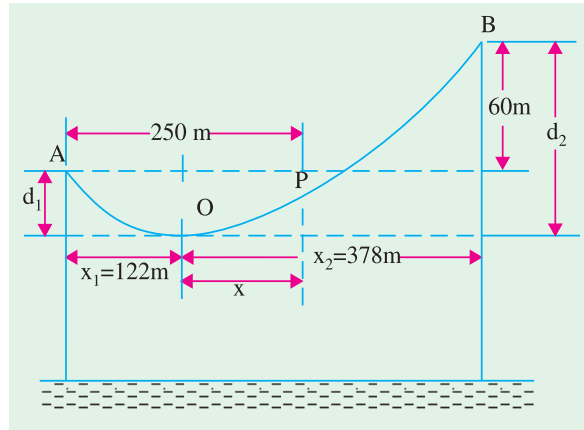


Fig. 41.65

As seen from Fig. 41.65, clearance of the lowest point O from the water level

$$= 30 - 7 = 23 \text{ m}$$

The horizontal distance of mid-point P from the reference point O is  $x = (250 - 122) = 128\text{m}^*$ . The height of the point P above O is

$$d_{mid} = \frac{Wx_1^2}{2T} = \frac{1.5 \times 128^2}{2 \times 1,600} = 7.68 \text{ m}$$

Hence, clearance of mid-point above water level is  $23 + 7.68 = 30.68 \text{ m}$

**Example 41.57.** An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 1,800 kg and the conductor weighs 1 kg/m, find the clearance between the conductor and water at a point mid-way between the towers. (Power System-I, AMIE, Sec. B, 1994)

**Solution.** Here,  $h = 100 - 50 = 50$  m ;  
 $l = 400/2 = 200$  m  
 $T = 1800$  kg-wt  
 $W = 1$  kg-wt/m

$$x_1 = l - \frac{hT}{2Wl}$$

$$= 200 - \frac{50 \times 1800}{2 \times 1 \times 200}$$

$$= -25 \text{ m}$$

$$x_2 = l + \frac{hT}{2Wl} = 200 + 225$$

$$= 425 \text{ m}$$

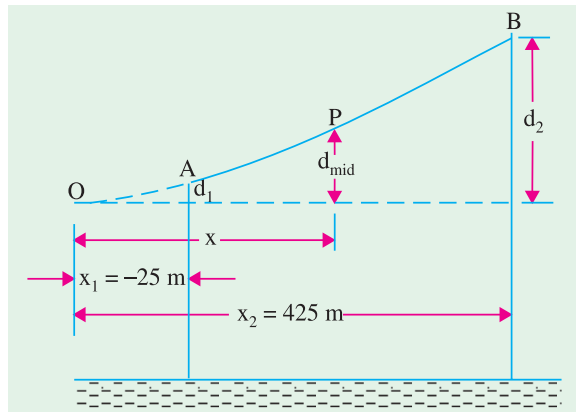


Fig. 41.66

\* Or  $x = \frac{1}{2} (x_2 - x_1)$ . In this case,  $x = \frac{1}{2} (425 - (-25)) = 225$  m



Since  $x_1$  turns out to be negative, point  $A$  lies on the same side of  $O$  as  $B$  (Fig. 41.66). Distance of mid-point  $P$  from  $O$  is  $x = (425 + 25) = 225$  m. Hence, height of  $P$  above  $O$  is

$$d_{mid} = \frac{Wx^2}{T} = \frac{1 \times 225^2}{2 \times 1,800} = 14 \text{ m}$$

Now, 
$$d_2 = \frac{Wx_2^2}{2T} = \frac{1 \times 425^2}{2 \times 1,800} = 50.2 \text{ m}$$

Hence,  $P$  is  $(50.2 - 14) = 36.2$  m below point  $B$ . It means that mid-point  $P$  is  $(100 - 36.2) = 63.8$  m above water level.

**Example 41.58.** A conductor is strung across a river, being supported at the two ends at heights of 20 m and 16 m respectively, from the bed of the river. The distance between the supports is 375 m and the weight of the conductor = 1.2 kg/m.

If the clearance of the conductor from the river bed be 9 m, find the horizontal tension in the conductor. Assume a parabolic configuration and that there is no wind or ice loading.

(Electrical Power-I, Bombay Univ.)

**Solution.** Here,  $l = 375/2 = 187.5$  m;  $h = 20 - 16 = 4$  m;  $W = 1.2$  kg-wt/m;  $T = ?$

$$x_1 = l - \frac{hT}{2Wl} = 187.5 - \frac{4 \times T}{2 \times 1.2 \times 187.5} = 187.5 - \frac{T}{112.5}$$

or  $x_1 = (187.5 - m)$  where  $m = T/112.5$

Also,  $d_1 + 9 = 16$  or  $d_1 = 7 = \frac{Wx_1^2}{2T}$

$$\therefore 7 = \frac{1.2 (187.5 - m)^2}{225 m} \quad (\because 2T = 2m \times 112.5 = 225 \text{ m})$$

or  $1.2 m^2 - 2,025 m + 42,180 = 0$  or  $m = 1,677$  or  $10.83$

Rejecting the bigger value which is absurd, we have

$$m = 10.83 \text{ or } T/112.5 = 10.83 \text{ or } T = \mathbf{1,215 \text{ kg-wt/m}}$$

### Tutorial Problem No. 41.3

- Show diagrammatically the distribution of electrostatic capacitance in a 3-core, 3-phase lead-sheathed cable.

The capacitance of such a cable measured between any two of the conductors, the sheathing being earthed, is  $0.3 \mu\text{F}$  per km. Find the equivalent star-connected capacitance and the kVA required to keep 10 km of the cable charged when connected to 20,000-V, 50 Hz bus-bars. **[0.6  $\mu\text{F}$  ; 754 kVA]**

- The 3-phase output from a hydro-electric station is transmitted to a distributing centre by two overhead lines connected in parallel but following different routes. Find how a total load of 5,000 kW at a p.f. of 0.8 lagging would divide between the two routes if the respective line resistances are 1.5 and  $1.0 \Omega$  and their reactances at 25 Hz are 1.25 and  $1.2 \Omega$ .

**[2,612 kW ; 2,388 kW](City & Guilds, London)**

- Two 3-phase cables connected in parallel supply a 6,600-V, 1,000-kW load at a lagging power factor of 0.8. The current in one of the cables is 70 A and it delivers 600 kW. Calculate its reactance and resistance, given that the other cable has reactance of  $2.6 \Omega$  and a resistance of  $2 \Omega$ .

**[R = .795  $\Omega$  ; X = 1.7  $\Omega$ ] (London Univ.)**

- A concentric cable has a conductor diameter of 1 cm and an insulation thickness of 1.5 cm. Find the maximum field strength when the cable is subjected to a test pressure of 33 kV.

**[47.6 kV/cm (r.m.s.) or 67.2 kV/cm (peak)] (London Univ.)**

5. A single-phase ring distributor  $XYZ$  is fed at  $X$ . The loads at  $Y$  and  $Z$  are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at  $X$ . The total impedances of the three sections  $XY$ ,  $YZ$  and  $ZX$  are  $(1 + j1)$ ,  $(1 + j2)$  and  $(1 + j3)$  ohms respectively. Find the total current fed at  $X$  and the current in each section with respect to supply voltage at  $X$ .  
(Allahabad Univ.)  
[34.6  $\angle -43.8^\circ$  A ;  $AXY = 23.1 \angle -32^\circ$  A,  $AXZ = 13.1 \angle -60.8^\circ$ .  $YZ = 3.01 \angle -30.5^\circ$  A]
6. A single-phase distributor has a resistance of 0.2 ohm and reactance 0.3 ohm. At the far end, the voltage  $V_B = 240$  volts, the current is 100 A and the power factor is 0.8. At the mid point A, current of 100 A is supplied at a power factor 0.6 with reference to voltage  $V_A$  at point A. Find supply voltage  $V_S$  and the phase angle between  $V_S$  and  $V_B$ . [292 V, 2.6°]
7. Estimate the corona loss for a 3-phase 110-kV, 50-Hz, 150-km long transmission line consisting of three conductors, each of 10 mm diameter and spaced 2.5 metre apart in an equilateral triangle formation. The temperature of air is 30°C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. [385 kW](AMIE)
8. A transmission line conductor at a river crossing is supported from two towers at heights of 20 m and 60 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductor is 1800 kg and the conductor weighs 1.0 kg per metre, find the clearance between the conductor and the water level at a point mid-way between the towers. Use approximate method. (18.75 m) (AMIE)
9. Show that the positive and negative sequence impedances of transmission lines are same where as its zero sequence impedance is higher than positive sequence impedance.  
(Nagpur University, Summer 2004)
10. A 132 kV, 3 phase, 50 Hz overhead line of 100 km length has a capacitance to earth of each line of 0.01  $\mu\text{F}/\text{km}$ . Determine inductance and kVA rating of the arc suppression coil suitable for this system.  
(Nagpur University, Summer 2004)

### OBJECTIVE TESTS – 41

- With same maximum voltage between conductors, the ratio of copper volumes in 3-phase, 3-wire system and 1-phase, 2-wire system is  
(a) 4/3 (b) 3/4  
(c) 5/3 (d) 3/5
- The volume of copper required for an a.c. transmission line is inversely proportional to  
(a) current (b) voltage  
(c) power factor (d) both (b) and (c)  
(e) both (a) and (c).
- For a.c. transmission lines less than 80 km in length, it is usual to lump the line capacitance at  
(a) the receiving end  
(b) the sending end  
(c) the mid-point  
(d) any convenient point.
- Corona occurs between two transmission wires when they  
(a) are closely-spaced  
(b) are widely-spaced  
(c) have high potential difference  
(d) carry d.c. power.
- The only advantage of corona is that it  
(a) makes line current non-sinusoidal  
(b) works as a safety-valve for surges  
(c) betrays its presence by hissing sound  
(d) produces a pleasing luminous glow.
- The sag produced in the conductor of a transmission wire depends on  
(a) weight of the conductor per unit length  
(b) tension in the conductor  
(c) length of the conductor  
(d) all of the above  
(e) none of the above.
- Suspension insulators are used when transmission voltage is  
(a) high (b) low  
(c) fluctuating (d) steady
- The string efficiency of suspension insulators can be increased by  
(a) providing a guard ring  
(b) grading the insulators

- (c) using identical insulator disc  
(d) both (a) & (b).
9. An interconnector between two generating stations facilitates to  
(a) keep their voltage constant  
(b) run them in parallel  
(c) transfer power in either direction  
(d) both (b) & (c)
10. The effective disruptive critical voltage of a transmission line does NOT depend on  
(a) irregularity factor  
(b) conductor radius  
(c) distance between conductors  
(d) material of the conductors.
11. By which of the following systems electric power may be transmitted?  
(a) Overhead system  
(b) Underground system  
(c) Both (a) and (b)  
(d) None of the above
12. .... are the conductors, which connect the consumer's terminals to the distribution  
(a) Distributors  
(b) Service mains  
(c) Feeders  
(d) None of the above
13. The underground system cannot be operated above  
(a) 440 V (b) 11 kV  
(c) 33 kV (d) 66 kV
14. Overhead system can be designed for operation upto  
(a) 11 kV (b) 33 kV  
(c) 66 kV (d) 400 kV
15. If variable part of annual cost on account of interest and depreciation on the capital outlay is equal to the annual cost of electrical energy wasted in the conductors, the total annual cost will be minimum and the corresponding size of conductor will be most economical. This statement is known as  
(a) Kelvin's law (b) Ohm's law  
(c) Kirchhoff's law (d) Faraday's law  
(e) none of the above
16. The wooden poles well impregnated with creosote oil or any preservative compound have life  
(a) from 2 to 5 years  
(b) 10 to 15 years  
(c) 25 to 30 years  
(d) 60 to 70 years
17. Which of the following materials is not used for transmission and distribution of electrical power?  
(a) Copper  
(b) Aluminium  
(c) Steel  
(d) Tungsten
18. Galvanised steel wire is generally used as  
(a) stay wire  
(b) earth wire  
(c) structural components  
(d) all of the above
19. The usual spans with R.C.C. poles are  
(a) 40–50 metres  
(b) 60–100 metres  
(c) 80–100 metres  
(d) 300–500 metres
20. The corona is considerably affected by which of the following?  
(a) Size of the conductor  
(b) Shape of the conductor  
(c) Surface condition of the conductor  
(d) All of the above
21. Which of the following are the constants of the transmission lines?  
(a) Resistance  
(b) Inductance  
(c) Capacitance  
(d) All of the above
22. %age regulation of a transmission line is given by  
(a)  $\frac{V_R - V_S}{V_R^2} \times 100$   
(b)  $\frac{V_S - V_R}{V_R} \times 100$   
(c)  $\frac{V_S - V_R}{V_S} \times 100$   
(d)  $\frac{V_S - V_R}{V_R^2} \times 100$
- where  $V_S$  and  $V_R$  are the voltages at the sending end and receiving end respectively.
23. The phenomenon of rise in voltage at the receiving end of the open-circuited or lightly loaded line is called the  
(a) Seeback effect  
(b) Ferranti effect  
(c) Raman effect  
(d) none of the above

24. The square root of the ratio of line impedance and shunt admittance is called the
- surge impedance of the line
  - conductance of the line
  - regulation of the line
  - none of the above
25. Which of the following is the demerit of a 'constant voltage transmission system'?
- Increase of short-circuit current of the system
  - Availability of steady voltage at all loads at the line terminals
  - Possibility of better protection for the line due to possible use of higher terminal reactances
  - Improvement of power factor at times of moderate and heavy loads
  - Possibility of carrying increased power for a given conductor size in case of long-distance heavy power transmission
26. Low voltage cables are meant for use up to
- 1.1 kV
  - 3.3 kV
  - 6.6 kV
  - 11 kV
27. The operating voltage of high voltage cables is upto
- 1.1 kV
  - 3.3 kV
  - 6.6 kV
  - 11 kV
28. The operating voltage of supertension cables is upto
- 3.3 kV
  - 6.6 kV
  - 11 kV
  - 33 kV
29. The operating voltage of extra high tension cables is upto
- 6.6 kV
  - 11 kV
  - 33 kV
  - 66 kV
  - 132 kV
30. Which of the following methods is used for laying of underground cables?
- Direct laying
  - Draw-in-system
  - Solid system
  - All of the above
31. Which of the following is the source of heat generation in the cables?
- Dielectric losses in cable insulation
  - $I^2R$  losses in the conductor
  - Losses in the metallic sheathings and armourings
  - All of the above
32. Due to which of the following reasons the cables should not be operated too hot?
- The oil may lose its viscosity and it may start drawing off from higher levels
  - Expansion of the oil may cause the sheath to burst
  - Unequal expansion may create voids in the insulation which will lead to ionization
  - The thermal instability may rise due to the rapid increase of dielectric losses with temperature
33. Which of the following D.C. distribution system is the simplest and lowest in first cost?
- Radial system
  - Ring system
  - Inter-connected system
  - Non of the above
34. A booster is a
- series wound generator
  - shunt wound generator
  - synchronous generator
  - none of the above
35. Besides a method of trial and error, which of the following methods is employed for solution of network problems in interconnected system?
- Circulating current method
  - Thevenin's theorem
  - Superposition of currents
  - direct application of Kirchhoff's laws
  - All of the above
36. Which of the following faults is most likely to occur in cables?
- Cross or short-circuit fault
  - Open circuit fault
  - Breakdown of cable insulation
  - all of the above
37. The cause of damage to the lead sheath of a cable is
- crystallisation of the lead through vibration
  - chemical action on the lead when
  - mechanical damage
  - all of the above
38. The voltage of the single phase supply to residential consumers is
- 110 V
  - 210 V
  - 230 V
  - 400 V
39. Most of the high voltage transmission lines in India are
- underground
  - overhead

- (c) either of the above  
(d) none of the above
40. The distributors for residential areas are  
(a) single phase  
(b) three-phase three wire  
(c) three-phase four wire  
(d) non of the above
41. The conductors of the overhead lines are  
(a) solid  
(b) stranded  
(c) both solid and stranded  
(d) none of the above
42. High voltage transmission lines use  
(a) suspension insulators  
(b) pin insulators  
(c) both (a) and (b)  
(d) none of the above
43. Multicore cables generally use  
(a) square conductors  
(b) circular conductors  
(c) rectangular conductors  
(d) sector-shaped conductors  
(e) none of the above
44. Distributio lines in India generally use  
(a) wooden poles  
(b) R.C.C. poles  
(c) steel towers  
(d) none of the above
45. The material commonly used for insulation in high voltage cables is  
(a) lead  
(b) paper  
(c) rubber  
(d) none of the above
46. The loads on distributors systems are generally  
(a) balanced  
(b) unbalanced  
(c) either of the above  
(d) none of the above
47. The power factor of industrial loads is generally  
(a) unity  
(b) lagging  
(c) leading  
(d) zero
48. Overhead lines generally use  
(a) copper conductors  
(b) all aluminium conductors  
(c) A.C.S.R. conductors  
(d) none of these
49. In transmission lines the cross-arms are made of  
(a) copper  
(b) wood  
(c) R.C.C.  
(d) steel
50. The material generally used for armour of high voltage cables is  
(a) aluminium  
(b) steel  
(c) brass  
(d) copper
51. Transmission line insulators are made of  
(a) glass  
(b) porcelain  
(c) iron  
(d) P.V.C.
52. The material commonly used for sheaths of underground cables is  
(a) lead  
(b) rubber  
(c) copper  
(d) iron
53. The minimum clearance between the ground and a 220 kV line is about  
(a) 4.3 m  
(b) 5.5 m  
(c) 7.0 m  
(d) 10.5 m
54. The spacing between phase conductors of a 220 kV line is approximately equal to  
(a) 2 m  
(b) 3.5 m  
(c) 6 m  
(d) 8.5 m
55. Large industrial consumers are supplied electrical energy at  
(a) 400 V  
(b) 11 kV  
(c) 66 kV  
(d) 400 kV
56. In a D.C. 3-wire distribution system, balancer fields are cross-connected in order to  
(a) boost the generated voltage  
(b) balance loads on both sides of the neutral  
(c) make both machines run as unloaded motors  
(d) equalize voltages on the positive and negative outers
57. In a D.C. 3-wire distributor using balancers and having unequal loads on the two sides  
(a) both balancers run as generators  
(b) both balancers run as motors  
(c) balancer connected to lightly-loaded side runs as a motor  
(d) balancer connected to heavily-loaded side runs as a motor
58. Transmitted power remaining the same, if supply voltae of a D.C. 2-wire feeder is increased 100 percent, saving in copper is

- (a) 25 percent  
(b) 50 percent  
(c) 75 percent  
(d) 100 percent
59. A uniformly-loaded D.C. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at a middle point is  
(a) one-fourth  
(b) one-third  
(c) one-half  
(d) twice  
(e) none of the above
60. As compared to a 2-wire D.C. distributor, a 3-wire distributor with same maximum voltage to earth uses only  
(a) 31.25 percent of copper  
(b) 33.3 percent of copper  
(c) 66.7 percent of copper  
(d) 125 percent of copper
61. Which of the following is usually not the generating voltage?  
(a) 6.6 kV  
(b) 8.8 kV  
(c) 11 kV  
(d) 13.2 kV
62. For an overhead line, the surge impedance is taken as  
(a) 20–40 ohms  
(b) 70–80 ohms  
(c) 100–200 ohms  
(d) 500–1000 ohms  
(e) none of the above
63. The presence of ozone due to corona is harmful because it  
(a) reduces power factor  
(b) corrodes the material  
(c) gives odour  
(d) transfer energy to the ground  
(e) none of the above
64. A feeder, in a transmission system, feeds power to  
(a) distributors  
(b) generating stations  
(c) service mains  
(d) all of the above
65. The power transmitted will be maximum when  
(a) corona losses are minimum  
(b) reactance is high  
(c) sending end voltage is more  
(d) receiving end voltage is more
66. A 3-phase 4 wire system is commonly used on  
(a) primary transmission  
(b) secondary transmission  
(c) primary distribution  
(d) secondary distribution
67. Which of the following materials is used for overhead transmission lines?  
(a) Steel cored aluminium  
(b) Galvanised steel  
(c) Cadmium copper  
(d) Any of the above
68. Which of the following is not a constituent for making porcelain insulators?  
(a) Quartz  
(b) Kaolin  
(c) Felspar  
(d) Silica
69. There is a greater possibility of occurrence of corona during  
(a) dry weather  
(b) winter  
(c) summer heat  
(d) humid weather  
(e) none of the above
70. Which of the following relays is used on long transmission lines?  
(a) Impedance relay  
(b) Mho's relay  
(c) Reactance relay  
(d) None of the above
71. The steel used in steel cored conductors is usually  
(a) alloy steel  
(b) stainless steel  
(c) mild steel  
(d) high speed steel  
(e) all of the above
72. Which of the following distribution system is more reliable?  
(a) Radial system  
(b) Tree system  
(c) Ring main system  
(d) All are equally reliable
73. Which of the following characteristics should the line supports for transmission lines possess?  
(a) Low cost  
(b) High mechanical strength  
(c) Longer life  
(d) All of the above
74. Transmission voltage of 11 kV is normally used for distance upto  
(a) 20–25 km  
(b) 40–50 km  
(c) 60–70 km  
(d) 80–100 km
75. Which of the following regulations is considered best?  
(a) 50%  
(b) 20%

- (c) 10%  
(d) 2%
76. Skin effect is proportional to  
(a) (conductor diameter)<sup>4</sup>  
(b) (conductor diameter)<sup>3</sup>  
(c) (conductor diameter)<sup>2</sup>  
(d) (conductor diameter)<sup>1/2</sup>  
(e) none of the above
77. A conductor, due to sag between two supports, takes the form of  
(a) semi-circle  
(b) triangle  
(c) ellipse  
(d) catenary
78. In A.C.S.R. conductors, the insulation between aluminium and steel conductors is  
(a) insulin  
(b) bitumen  
(c) varnish  
(d) no insulation is required
79. Which of the following bus-bar schemes has the lowest cost?  
(a) Ring bus-bar scheme  
(b) Single bus-bar scheme  
(c) Breaker and a half scheme  
(d) Main and transfer scheme
80. Owing to skin effect  
(a) current flows through the half cross-section of the conductor  
(b) portion of the conductor near the surface carries more current and core of the conductor carries less current  
(c) portion of the conductor near the surface carries less current and core of the conductor carries more current  
(d) none of the above
81. By which of the following methods string efficiency can be improved?  
(a) Using a guard ring  
(b) Grading the insulator  
(c) Using long cross arm  
(d) Any of the above  
(e) None of the above
82. In aluminium conductors, steel core is provided to  
(a) compensate for skin effect  
(b) neutralise proximity effect  
(c) reduce line inductance  
(d) increase the tensile strength
83. By which of the following a bus-bar is rated?  
(a) Current only  
(b) Current and voltage  
(c) Current, voltage and frequency  
(d) Current, voltage, frequency and short time current
84. A circuit is disconnected by isolators when  
(a) line is energized  
(b) there is no current in the line  
(c) line is on full load  
(d) circuit breaker is not open
85. For which of the following equipment current rating is not necessary?  
(a) Circuit breakers  
(b) Isolators  
(c) Load break switch  
(d) Circuit breakers and load break switches
86. In a substation the following equipment is not installed  
(a) exciters  
(b) series capacitors  
(c) shunt reactors  
(d) voltage transformers
87. Corona usually occurs when the electrostatic stress in air around the conductor exceeds  
(a) 6.6 kV (r.m.s. value)/cm  
(b) 11 kV (r.m.s. value)/cm  
(c) 22 kV (maximum value)/cm  
(d) 30 kV (maximum value)/cm
88. The voltage drop, for constant voltage transmission is compensated by installing  
(a) inductors  
(b) capacitors  
(c) synchronous motors  
(d) all of above  
(e) none of the above
89. The use of strain type insulators is made where the conductors are  
(a) dead ended  
(b) at intermediate anchor towers  
(c) any of the above  
(d) none of the above
90. The current drawn by the line due to corona losses is  
(a) non-sinusoidal  
(b) sinusoidal  
(c) triangular  
(d) square
91. Pin type insulators are generally not used for voltages beyond  
(a) 1 kV  
(b) 11 kV  
(c) 22 kV  
(d) 33 kV
92. Aluminium has a specific gravity of  
(a) 1.5  
(b) 2.7  
(c) 4.2  
(d) 7.8
93. For transmission of power over a distance of 200 km, the transmission voltage should be

- (a) 132 kV  
(b) 66 kV  
(c) 33 kV  
(d) 11 kV
94. For aluminium, as compared to copper, all the following factors have higher values except  
(a) specific volume  
(b) electrical conductivity  
(c) co-efficient of linear expansion  
(d) resistance per unit length for same cross-section
95. Which of the following equipment, for regulating the voltage in distribution feeder, will be most economical?  
(a) Static condenser  
(b) Synchronous condenser  
(c) The changing transformer  
(d) Booster transformer
96. In a tap changing transformer, the tappings are provided on  
(a) primary winding  
(b) secondary winding  
(c) high voltage winding  
(d) any of the above
97. Constant voltage transmission entails the following disadvantage  
(a) large conductor area is required for same power transmission  
(b) short-circuit current of the system is increased  
(c) either of the above  
(d) none of the above
98. On which of the following factors skin effect depends?  
(a) Frequency of the current  
(b) Size of the conductor  
(c) Resistivity of the conductor material  
(d) All of the above
99. The effect of corona can be detected by  
(a) presence of zone detected by odour  
(b) hissing sound  
(c) faint luminous glow of bluish colour  
(d) all of the above
100. for transmission of power over a distance of 500 km, the transmission voltage should be in the range  
(a) 150 to 220 kV  
(b) 100 to 120 kV  
(c) 60 to 100 kV  
(d) 20 to 50 kV
101. In the analysis of which of the following lines shunt capacitance is neglected?  
(a) Short transmission lines  
(b) Medium transmission lines  
(c) Long transmission lines  
(d) Medium as well as long transmission lines
102. When the interconnector between two stations has large reactance  
(a) the transfer of power will take place with voltage fluctuation and noise  
(b) the transfer of power will take place with least loss  
(c) the stations will fall out of step because of large angular displacement between the stations  
(d) none of the above
103. The frequency of voltage generated, in case of generators, can be increased by  
(a) using reactors  
(b) increasing the load  
(c) adjusting the governor  
(d) reducing the terminal voltage  
(e) none of the above
104. When an alternator connected to the bus-bar is shut down the bus-bar voltage will  
(a) fall  
(b) rise  
(c) remain uncharged  
(d) none of the above
105. The angular displacement between two interconnected stations is mainly due to  
(a) armature reactance of both alternators  
(b) reactance of the interconnector  
(c) synchronous reactance of both the alternators  
(d) all of the above
106. Electro-mechanical voltage regulators are generally used in  
(a) reactors  
(b) generators  
(c) transformers  
(d) all of the above
107. Series capacitors on transmission lines are of little use when the load VAR requirement is  
(a) large  
(b) small  
(c) fluctuating  
(d) any of the above
108. The voltage regulation in magnetic amplifier type voltage regulator is effected by  
(a) electromagnetic induction  
(b) varying the resistance  
(c) varying the reactance  
(d) variable transformer
109. when a conductor carries more current on the surface as compared to core, it is due to  
(a) permeability variation  
(b) corona  
(c) skin effect  
(d) unsymmetrical fault



- (e) none of the above
- 110.** The following system is not generally used
- 1-phase 3 wire
  - 1-phase 4 wire
  - 3-phase 3 wire
  - 3-phase 4 wire
- 111.** The skin effect of a conductor will reduce as the
- resistivity of conductor material increases
  - permeability of conductor material increases
  - diameter increases
  - frequency increases
- 112.** When a live conductor of public electric supply breaks down and touches the earth which of the following will happen?
- Current will flow to earth
  - Supply voltage will drop
  - Supply voltage will increase
  - No current will flow in the conductor
- 113.** 310 km line is considered as
- a long line
  - a medium line
  - a short line
  - any of the above
- 114.** The conductors are bundled primarily to
- increase reactance
  - reduce reactance
  - reduce ratio interference
  - none of the above
- 115.** The surge impedance in a transmission line having negligible resistance is given as
- $\sqrt{LC}$
  - $\sqrt{L/C}$
  - $\sqrt{1/LC}$
  - $\sqrt{L + C}$
  - none of the above
- 116.** The top most conductor in a high transmission line is
- earth conductor
  - R-phase conductor
  - Y-phase conductor
  - B-phase conductor
- 117.** In A.C.S.R. conductor the function of steel is to
- provide additional mechanical strength
  - prevent corona
  - take care of surges
  - reduce inductance and subsequently improve power factor
- 118.** In transmission and distribution system the permissible voltage variation is
- $\pm 1$  percent
  - $\pm 10$  percent
  - $\pm 20$  percent
  - $\pm 30$  percent
  - none of the above
- 119.** By which of the following methods voltage of transmission can be regulated?
- use of series capacitors to neutralise the effect of series reactance
  - Switching in shunt capacitors at the receiving end during heavy loads
  - Use of tap changing transformers
  - Any of the above methods
- 120.** Which of the following distribution systems is the most economical?
- A.C. 1-phase system
  - A.C. 3-phase 3 wire system
  - A.C. 3-phase 4 wire system
  - Direct current system
- 121.** Which of the following is the main advantage of A.C. transmission system over D.C. transmission system?
- Less instability problem
  - Less insulation problems
  - Easy transformation
  - Less losses in transmission over long distances
- 122.** A tap changing transformer is used to
- supply low voltage current for instruments
  - step up the voltage
  - step down the voltage
  - step up as well as step down the voltage
- 123.** Which of the following bar schemes is the most expensive?
- Double bus-bar double breaker
  - Ringbus-bar scheme
  - Single bus-bar scheme
  - Main and transfer scheme
- 124.** By which of the following methods the protection against direct lightning strokes and high voltage surge waves is provided?
- Lightning arresters
  - Ground wire
  - Lightning arresters and ground wires
  - Earthing of neutral
  - None of the above
- 125.** In which of the following voltage regulators the effect of dead zone is found?
- Electromagnetic type
  - Magnetic amplifier
  - Electronic type using integrated circuits
  - all of the above
- 126.** Corona results in

- (a) radio interference  
 (b) power factor improvement  
 (c) better regulation  
 (d) none of the above
- 127.** Which of the following has least effect on corona?  
 (a) Atmospheric temperature  
 (b) Number of ions  
 (c) Size and charge per ion  
 (d) Mean free path
- 128.** In context of corona, if the conductors are polished and smooth, which of the following statements is correct?  
 (a) Hissing sound will be more intense  
 (b) Power loss will be least  
 (c) Corona glow will be uniform along the length of the conductor  
 (d) Corona glow will not occur
- 129.** Power loss due to corona is not directly proportional to  
 (a) spacing between conductors  
 (b) supply voltage frequency  
 (c) phase-neutral voltage  
 (d) all of the above
- 130.** Poles which carry transformers are usually  
 (a) circular  
 (b) I-type  
 (c) A-type  
 (d) H-type  
 (e) none of the above
- 131.** Out of the following which type of poles are bulky?  
 (a) Transmission towers  
 (b) Concrete poles  
 (c) Tubular steel poles  
 (d) Wooden poles
- 132.** The effect of ice on transmission line conductors is to increase the  
 (a) transmission losses  
 (b) weight of the conductor  
 (c) tendency for corona  
 (d) resistance to flow of current
- 133.** If the height of transmission tower is increased  
 (a) the line capacitance will decrease but line inductance will remain unchanged  
 (b) the line capacitance and inductance will not change  
 (c) the line capacitance will increase but line inductance will decrease  
 (d) the line capacitance will decrease and line inductance will increase
- 134.** If string efficiency is 100 percent it means that  
 (a) potential across each disc is zero  
 (b) potential across each disc is same  
 (c) one of the insulator discs is shorted  
 (d) none of the above
- 135.** In a 70/6 A.C.S.R. conductor there are  
 (a) 35 aluminium conductors and 3 steel conductors  
 (b) 70 aluminium conductors and 6 steel conductors  
 (c) 70 steel conductors and 6 aluminium conductors  
 (d) none of the above
- 136.** On which of the following does the size of a feeder depend?  
 (a) Voltage drop  
 (b) Voltage  
 (c) Frequency  
 (d) Current carrying capacity
- 137.** Which of the following are connected by the service mains?  
 (a) Transformer and earth  
 (b) Distributor and relay system  
 (c) Distributor and consumer terminals  
 (d) Distributor and transformer
- 138.** In the design of a distributor which of the following is the major consideration?  
 (a) Voltage drop  
 (b) Current carrying capacity  
 (c) Frequency  
 (d) kVA of system  
 (e) None of the above
- 139.** In a distribution system major cost is that of  
 (a) earthing system  
 (b) distribution transformer  
 (c) conductors  
 (d) meters
- 140.** A booster is connected in  
 (a) parallel with earth connection  
 (b) parallel with the feeder  
 (c) series with the feeder  
 (d) series with earth connection
- 141.** With which of the following are step-up substations associated?  
 (a) Concentrated load  
 (b) Consumer location  
 (c) Distributors  
 (d) Generating stations  
 (e) None of the above
- 142.** Which of the following equipment should be installed by the consumers having low power factor?  
 (a) Synchronous condensers  
 (b) Capacitor bank  
 (c) Tap changing transformer  
 (d) Any of the above  
 (e) None of the above
- 143.** Which of the following equipment is used to

- limit short-circuit current level in a substation?
- (a) Isolator
  - (b) Lightning switch
  - (c) Coupling capacitor
  - (d) Series reactor
144. Steepness of the travelling waves is alternated by ..... of the line
- (a) capacitance
  - (b) inductance
  - (c) resistance
  - (d) all of the above
145. The limit of distance of transmission line may be increased by the use of
- (a) series resistances
  - (b) shunt capacitors and series reactors
  - (c) series capacitors and shunt reactors
  - (d) synchronous condensers
  - (e) none of the above
146. By which of the following factors is the sag of a transmission line least affected?
- (a) Current through the conductor
  - (b) Ice deposited on the conductor
  - (c) Self weight of conductor
  - (d) Temperature of surrounding air
  - (e) None of the above
147. Which of the following cause transient disturbances?
- (a) Faults
  - (b) Load variations
  - (c) Switching operations
  - (d) Any of the above
148. A guy wire
- (a) protects conductors against shortcircuiting
  - (b) provides emergency earth route
  - (c) provides protection against surges
  - (d) supports the pole
149. Which of the following is neglected in the analysis of short transmission lines?
- (a) Series impedance
  - (b) Shunt admittance
  - (c)  $I^2R$  loss
  - (d) None of the above
  - (e) All of the above
150. Basically the boosters are
- (a) synchronous motors
  - (b) capacitors
  - (c) inductors
  - (d) transformers
151. Which of the following is a static exciter?
- (a) Rectifier
  - (b) Rotorol
  - (c) Amplidyne
  - (d) D.C. separately excited generator
152. For exact compensation of voltage drop in the feeder the booster
- (a) must be earthed
  - (b) must work on line voltage
  - (c) must work on linear portion of its V-I characteristics
  - (d) must work on non-linear portion of its V-I characteristics
153. The purpose of using a booster is to
- (a) increase current
  - (b) reduce current
  - (c) reduce voltage drop
  - (d) compensate for voltage drop
  - (e) none of the above
154. Induction regulators are used for voltage control in
- (a) alternators
  - (b) primary distribution
  - (c) secondary distribution
  - (d) none of the above
155. A synchronous condenser is generally installed at the ..... of the transmission line
- (a) receiving end
  - (b) sending end
  - (c) middle
  - (d) none of the above
156. The area of cross-section of the neutral in a 3-wire D.C. system is generally ..... the area of cross-section of main conductor
- (a) same as
  - (b) one-fourth
  - (c) one half
  - (d) double
157. For which of the following, the excitation control method is satisfactory?
- (a) Low voltage lines
  - (b) High voltage lines
  - (c) Short lines
  - (d) Long lines
158. In which of the following cases shunt capacitance is negligible?
- (a) Short transmission lines
  - (b) Medium transmission lines
  - (c) Long transmission lines
  - (d) All transmission lines
159. A lightning arrester is usually located nearer to
- (a) transformer
  - (b) isolator
  - (c) busbar
  - (d) circuit breaker
  - (e) none of the above
160. The material used for the manufacture of grounding wires is
- (a) cast iron
  - (b) aluminium

- (c) stainlesssteel  
(d) galvanised steel
- 161.** Surge absorbers protect against ..... oscillations  
(a) high voltage high frequency  
(b) high voltage low frequency  
(c) low voltage high frequency  
(d) low voltage low frequency
- 162.** Skin effect is noticeable only at ..... frequencies  
(a) audio  
(b) low  
(c) high  
(d) all
- 163.** Per system stability is least affected by  
(a) reactance of generator  
(b) input torque  
(c) losses  
(d) reactance of transmission line
- 164.** When the load at the receiving end of a long transmission line is removed, the sending end voltage is less than the receiving end voltage. This effect is known as  
(a) Ferranti effect  
(b) Proximity effect  
(c) Kelvin effect  
(d) Faraday effect  
(e) Skin effect
- 165.** In medium transmission lines the shunt capacitance is taken into account in  
(a) T-method  
(b)  $\pi$ -method  
(c) Steinmetz method  
(d) all of the above
- 166.** System grounding is done so that  
(a) inductive interference between power and communication circuits can be controlled  
(b) the floating potential on the lower voltage winding for a transformer is brought down to an insignificant value  
(c) the arcing faults to earth would not set up dangerously high voltage on healthy phases  
(d) for all above reasons
- 167.** Which of the following can be used for bus-bars?  
(a) Tubes  
(b) Rods  
(c) Bars  
(d) Any of the above
- 168.** If the height of transmission tower is increased, which of the following parameters is likely to change?  
(a) Capacitance  
(b) Inductance  
(c) Resistance  
(d) All of the above  
(e) None of the above
- 169.** A.C.S.R. conductor having 7 steel strands surrounded by 25 aluminium conductors will be specified as  
(a) 25/7  
(b) 50/15  
(c) 7/25  
(d) 15/50
- 170.** Impedance relay is used on ..... transmission lines  
(a) short  
(b) medium  
(c) long  
(d) all
- 171.** Corona is likely to occur maximum in  
(a) transmission lines  
(b) distribution lines  
(c) domestic wiring  
(d) all of the above
- 172.** The effect of wind pressure is more predominant on  
(a) supporting towers  
(b) neutral wires  
(c) transmission lines  
(d) insulators
- 173.** As compared to cables, the disadvantages of transmission lines is  
(a) inductive interference between power and communication circuits  
(b) exposure to lightning  
(c) exposure to atmospheric hazards like smoke, ice, etc.  
(d) all of the above
- 174.** In overhead transmission lines the effect of capacitance can be neglected when the length of line is less than  
(a) 80 km  
(b) 110km  
(c) 150 km  
(d) 210 km
- 175.** The effective resistance of a conductor will be the same as 'ohmic resistance' when  
(a) power factor is unity  
(b) current is uniformly distributed in the conductor cross-section  
(c) voltage is low  
(d) current is in true sine wave form
- 176.** Conductors for high voltage transmission lines are suspended from towers to  
(a) increase clearance from ground  
(b) reduce clearance from ground  
(c) take care of extension in length during summer  
(d) reduce wind and snow loads

- (e) none of the above
177. To increase the capacity of a transmission line for transmitting power which of the following must be decreased?
- (a) Capacitance
  - (b) Line inductance
  - (c) Voltage
  - (d) All of the above
178. By using bundled conductors which of the following is reduced?
- (a) Power loss due to corona
  - (b) Capacitance of the circuit
  - (c) Inductance of the circuit
  - (d) None of the above
  - (e) All of the above
179. Which of the following short-circuits is most dangerous?
- (a) Dead short-circuit
  - (b) Line to ground short-circuit
  - (c) Line to line short-circuit
  - (d) Line to line and ground short-circuit
  - (e) all of the above
180. Due to which of the following reasons aluminium is being favoured as busbar material?
- (a) Low density
  - (b) Low cost
  - (c) Ease of fabrication
  - (d) None of the above
181. In case of transmission line conductors with the increase in atmospheric temperature
- (a) length decreases but stress increases
  - (b) length increases but stress decreases
  - (c) both the length and stress increases
  - (d) both the length and stress decrease
182. Skin effect exists only in
- (a) a.c. transmission
  - (b) high voltage d.c. overhead transmission
  - (c) low voltage d.c. overhead transmission
  - (d) cables carrying d.c. current
183. Floating neutral, in 3-phase supply, is undesirable because it causes
- (a) low voltage across the load
  - (b) high voltage across the load
  - (c) unequal line voltages across the load
  - (d) none of the above
184. The surge resistance of cables is
- (a) 20 ohms
  - (b) 50 ohms
  - (c) 200 ohms
  - (d) 300 ohms
185. The electrostatic stress in underground cables is
- (a) zero at the conductor as well as on the sheath
  - (b) same at the conductor and sheath
  - (c) minimum at the conductor and minimum at the sheath
  - (d) maximum at the conductor and minimum at the sheath
186. The ground ring transmission lines are used to
- (a) reduce the transmission losses
  - (b) reduce the earth capacitance of the lowest unit
  - (c) increase the earth capacitance of the lowest unit
  - (d) none of the above
187. The string efficiency of an insulator can be increased by
- (a) correct grading of insulators of various capacitances
  - (b) reducing the number of strings
  - (c) increasing the number of strings in the insulator
  - (d) none of the above
188. High voltages for transmitting power is economically available from
- (a) d.c. currents
  - (b) a.c. currents
  - (c) carrier currents
  - (d) none of the above
189. High voltage is primarily used, for long distance power transmission, to
- (a) reduce the time of transmission
  - (b) reduce the transmission losses
  - (c) make the system reliable
  - (d) none of the above
190. By using bundle conductors, the critical voltage for the formation of corona will
- (a) remain same
  - (b) decrease
  - (c) increase
  - (d) not occur
191. If the voltage is increased  $x$  times, the size of the conductor would be
- (a) reduced to  $1/x^2$  times
  - (b) reduced to  $1/x$  times
  - (c) increased  $x$  times
  - (d) increased to  $x^2$  times
  - (e) none of the above
192. The colour of the neutral of three-core flexible cable is
- (a) blue
  - (b) brown
  - (c) red
  - (d) black
193. In the cables sheaths are used to
- (a) prevent the moisture from entering the cable

- (b) provide strength to the cable  
(c) provide proper insulation  
(d) none of the above
- 194.** The charging current in the cables  
(a) leads the voltage by  $180^\circ$   
(b) leads the voltage by  $90^\circ$   
(c) lags the voltage by  $90^\circ$   
(d) lags the voltage by  $180^\circ$
- 195.** Ground wire is used to  
(a) avoid overloading  
(b) give the support to the tower  
(c) give good regulation  
(d) connect a circuit conductor or other device to an earth-plate
- 196.** Earthing is necessary to give protection against  
(a) danger of electric shock  
(b) voltage fluctuation  
(c) overloading  
(d) high temperature of the conductors
- 197.** Resistance grounding is used for voltage between  
(a) 3.3kV to 11 kV  
(b) 11 kV to 33 kV  
(c) 33 kV to 66 kV  
(d) none of the above
- 198.** Solid grounding is adopted for voltages below  
(a) 100 V  
(b) 200 V  
(c) 400 V  
(d) 660 V
- 199.** The size of the earth wire is determined by  
(a) the atmospheric conditions  
(b) the voltage of the service wires  
(c) the ampere capacity of the service wires  
(d) none of the above
- 200.** Transmission lines link  
(a) generating station to receiving and station  
(b) receiving and station to distribution transformer  
(c) distribution transformer to consumer premises  
(d) service points to consumer premises  
(e) none of the above

### ANSWERS

- |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| <b>1.</b> (b)   | <b>2.</b> (d)   | <b>3.</b> (a)   | <b>4.</b> (c)   | <b>5.</b> (b)   | <b>6.</b> (d)   | <b>7.</b> (a)   | <b>8.</b> (d)   | <b>9.</b> (d)   | <b>10.</b> (d)  |
| <b>11.</b> (c)  | <b>12.</b> (b)  | <b>13.</b> (d)  | <b>14.</b> (d)  | <b>15.</b> (a)  | <b>16.</b> (c)  | <b>17.</b> (d)  | <b>18.</b> (d)  | <b>19.</b> (c)  | <b>20.</b> (d)  |
| <b>21.</b> (d)  | <b>22.</b> (b)  | <b>23.</b> (b)  | <b>24.</b> (a)  | <b>25.</b> (a)  | <b>26.</b> (c)  | <b>27.</b> (d)  | <b>28.</b> (d)  | <b>29.</b> (d)  | <b>30.</b> (d)  |
| <b>31.</b> (d)  | <b>32.</b> (e)  | <b>33.</b> (a)  | <b>34.</b> (a)  | <b>35.</b> (e)  | <b>36.</b> (d)  | <b>37.</b> (d)  | <b>38.</b> (c)  | <b>39.</b> (b)  | <b>40.</b> (c)  |
| <b>41.</b> (b)  | <b>42.</b> (a)  | <b>43.</b> (d)  | <b>44.</b> (b)  | <b>45.</b> (b)  | <b>46.</b> (b)  | <b>47.</b> (b)  | <b>48.</b> (c)  | <b>49.</b> (d)  | <b>50.</b> (b)  |
| <b>51.</b> (b)  | <b>52.</b> (a)  | <b>53.</b> (c)  | <b>54.</b> (c)  | <b>55.</b> (c)  | <b>56.</b> (d)  | <b>57.</b> (c)  | <b>58.</b> (b)  | <b>59.</b> (a)  | <b>60.</b> (a)  |
| <b>61.</b> (b)  | <b>62.</b> (c)  | <b>63.</b> (b)  | <b>64.</b> (a)  | <b>65.</b> (c)  | <b>66.</b> (d)  | <b>67.</b> (d)  | <b>68.</b> (d)  | <b>69.</b> (d)  | <b>70.</b> (b)  |
| <b>71.</b> (c)  | <b>72.</b> (c)  | <b>73.</b> (d)  | <b>74.</b> (a)  | <b>75.</b> (b)  | <b>76.</b> (c)  | <b>77.</b> (d)  | <b>78.</b> (d)  | <b>79.</b> (b)  | <b>80.</b> (b)  |
| <b>81.</b> (d)  | <b>82.</b> (d)  | <b>83.</b> (d)  | <b>84.</b> (b)  | <b>85.</b> (b)  | <b>86.</b> (a)  | <b>87.</b> (d)  | <b>88.</b> (c)  | <b>89.</b> (c)  | <b>90.</b> (a)  |
| <b>91.</b> (d)  | <b>92.</b> (b)  | <b>93.</b> (a)  | <b>94.</b> (b)  | <b>95.</b> (d)  | <b>96.</b> (b)  | <b>97.</b> (b)  | <b>98.</b> (d)  | <b>99.</b> (d)  | <b>100.</b> (a) |
| <b>101.</b> (a) | <b>102.</b> (c) | <b>103.</b> (c) | <b>104.</b> (c) | <b>105.</b> (b) | <b>106.</b> (b) | <b>107.</b> (b) | <b>108.</b> (c) | <b>109.</b> (c) | <b>110.</b> (b) |
| <b>111.</b> (a) | <b>112.</b> (a) | <b>113.</b> (a) | <b>114.</b> (b) | <b>115.</b> (b) | <b>116.</b> (a) | <b>117.</b> (a) | <b>118.</b> (b) | <b>119.</b> (d) | <b>120.</b> (d) |
| <b>121.</b> (d) | <b>122.</b> (d) | <b>123.</b> (a) | <b>124.</b> (c) | <b>125.</b> (a) | <b>126.</b> (a) | <b>127.</b> (a) | <b>128.</b> (c) | <b>129.</b> (a) | <b>130.</b> (d) |
| <b>131.</b> (b) | <b>132.</b> (b) | <b>133.</b> (a) | <b>134.</b> (b) | <b>135.</b> (b) | <b>136.</b> (d) | <b>137.</b> (c) | <b>138.</b> (a) | <b>139.</b> (b) | <b>140.</b> (c) |
| <b>141.</b> (d) | <b>142.</b> (b) | <b>143.</b> (d) | <b>144.</b> (c) | <b>145.</b> (c) | <b>146.</b> (a) | <b>147.</b> (d) | <b>148.</b> (d) | <b>149.</b> (b) | <b>150.</b> (d) |
| <b>151.</b> (a) | <b>152.</b> (c) | <b>153.</b> (d) | <b>154.</b> (b) | <b>155.</b> (a) | <b>156.</b> (c) | <b>157.</b> (c) | <b>158.</b> (a) | <b>159.</b> (a) | <b>160.</b> (d) |
| <b>161.</b> (c) | <b>162.</b> (c) | <b>163.</b> (c) | <b>164.</b> (a) | <b>165.</b> (d) | <b>166.</b> (d) | <b>167.</b> (d) | <b>168.</b> (a) | <b>169.</b> (a) | <b>170.</b> (b) |
| <b>171.</b> (a) | <b>172.</b> (a) | <b>173.</b> (d) | <b>174.</b> (a) | <b>175.</b> (b) | <b>176.</b> (a) | <b>177.</b> (b) | <b>178.</b> (a) | <b>179.</b> (a) | <b>180.</b> (b) |
| <b>181.</b> (b) | <b>182.</b> (a) | <b>183.</b> (c) | <b>184.</b> (b) | <b>185.</b> (d) | <b>186.</b> (b) | <b>187.</b> (a) | <b>188.</b> (b) | <b>189.</b> (b) | <b>190.</b> (c) |
| <b>191.</b> (a) | <b>192.</b> (a) | <b>193.</b> (a) | <b>194.</b> (b) | <b>195.</b> (d) | <b>196.</b> (a) | <b>197.</b> (a) | <b>198.</b> (d) | <b>199.</b> (c) | <b>200.</b> (a) |

# CHAPTER 42

## Learning Objectives

- Introduction
- Need Based Energy Management (NBEM)
- Advantages of NBEM
- Automated System
- Sectionalizing Switches
- Remote Terminal Units (RTU's)
- Data Acquisition System (DAS)
- Communication Interface
- Radio Communication
- Machine Interface
- A Typical SCADA System
- Load Management in DMS Automated Distribution System
- Substation Automation
- Control System
- Protective System
- Feeder Automation
- Distribution Equipment
- Automation Equipment
- Consumer Side Automation
- Energy Auditing
- Reduced Line Loss
- Power Quality
- Deferred Capital Expenses
- Energy Cost Reduction
- Optimal Energy Use

## DISTRIBUTION AUTOMATION



➤ An automated electric distribution system ensures that the power is transmitted efficiently and without interruptions. Faults in transmission are identified and rectified with minimum human intervention

### 42.1. Introduction

The power industry may seem to lack competition. This thought arises because each power company operates in a geographic region not served by other companies. Favorable electric rates are a compelling factor in the location of an industry, although this factor is much less important in times when costs are rising rapidly and rates charged for power are uncertain than in periods of stable economic conditions. Regulation of rates by State Electricity Boards, however, places constant pressure on companies to achieve maximum economy and earn a reasonable profit in the face of advancing costs of production.

Power shortage in India is endemic. Despite heavy investments in the power sector and additional power generation every year, there appears to be no chance of the shortage syndrome easing-up in the near future. Barring one or two small States with hydro-potential, all other States and Union Territories have perpetual power deficits. There seems to be no respite from these crippling deficits as seen from the figures that are being published from time to time.

In power sector, it has almost become a ritual to insistently clamor for more generation. This school of thought which is predominant majority, believes in classical concept of load management which emphasizes continuous and copious supply of energy to all sectors of consumers at all times. This ideal proposition could be achieved, only if unlimited resources at our command are available. But in actual practice, these resources are not only limited but scarce. These scarce resources are to be distributed throughout the country uniformly and called for immediate attention. A demand based energy management would therefore only result in shortage syndrome repeating itself endlessly since the ever-growing 'demand' could never be met fully and satisfactorily. A better alternative would be to replace the Demand Based Energy Management with a Need Based one.

### 42.2. Need Based Energy Management (NBEM)

In power sector, there is a distinct difference between 'demand' and 'need'. Consumers of electric power could be classified into five broad categories: the industrial users, agricultural sector, commercial organizations, domestic consumers and essential services. Industrial users could be further sub-divided as shift based industries and continuous process industries. Agricultural sector would include irrigation tubewells and rural industries. Out of these several groups and sub-groups, only three - viz - continuous process industries, domestic consumers and essential services need power round the clock, others may demand power for 24 hours of the day, but they don't need it.

A Need Based Energy Management would :

- (i) Identify the needs of various consumers
- (ii) Forecast the generation requirement based on the need
- (iii) Plan power generation as per forecast
- (iv) Laydown a suitable transmission and distribution network
- (v) Regulate distribution as per need
- (vi) Monitor matching of need with supply

The greatest bugbear of NBEM is distribution network. Consumers cannot be supplied power as per their need since the distribution network is not designed for that purpose. Present day power distribution network is full of constraints and is clumsy to the core. The ills of the system are many – poor reliability, high line losses, low voltage profiles, overloading of transformers, poor maintenance, absence of conservation measures, stealing of power, haphazard layouts, whimsical load connections, inadequate clearances etc. With a single feeder connected to all types of consumers, there is no load discipline and the distribution network is exposed to several malpractices and distortions. Generation of power also suffers and as a result, engineers incharge of power generation in state electricity boards are not enthusiastic about optimizing power generation because they feel that their efforts are wasted due to the chaotic and sub-optimal distribution system. So if power problem is to be solved, distribution holds the key.



Figure 42.1 shows utility system with SCADA (Supervisory Control and Data Acquisition) and Distribution Automation.

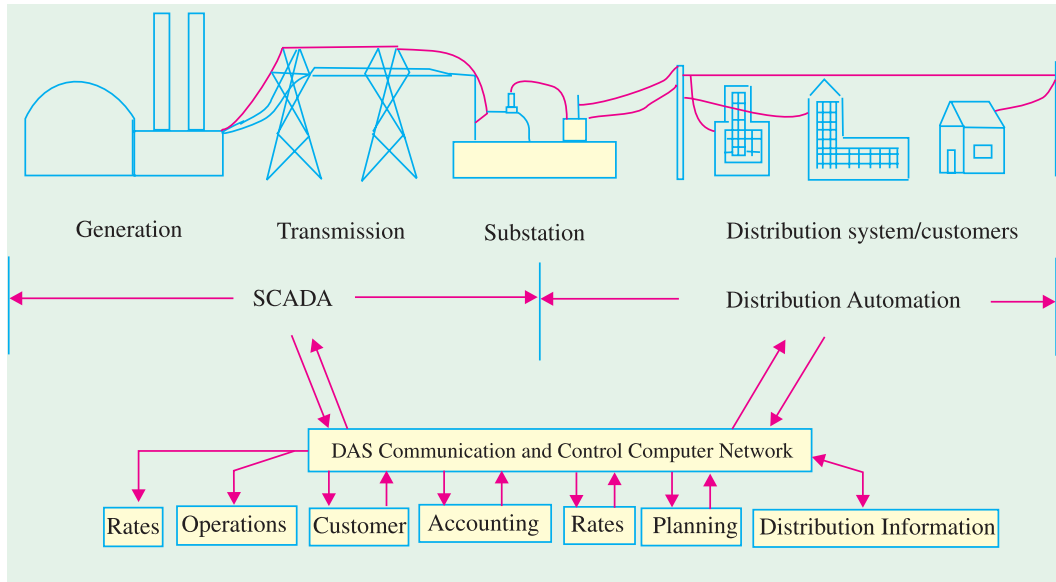


Fig. 42.1 Utility systems with SCADA and Distribution Automation

#### 42.2.1. Advantages of NBEM

The distinct advantages of NBEM are:

- (1) It ensures high reliability of supply to consumers meeting the specific demand effectively for periods of actual requirements.
- (2) The system losses can be substantially reduced since line and equipment does not get overloaded at any point of time.
- (3) The voltage profile at all levels is improved thus safeguarding the customer's equipment from losing their efficiency and performance at low voltage.
- (4) The scheme facilitates the adoption of energy conservation schemes and energy audit policy.
- (5) Power cuts are reduced and quality of power improves leading to better industrial and agricultural health and productivity.

#### 42.3. Conventional Distribution Network

The power system network, which generally concerns (or which is in close proximity of) the common man, is the distribution network of 11 KV lines or feeders downstream of the 33 KV substation. Each 11 KV feeder, which emanates from the 33 KV substation branches further into several subsidiary 11 KV feeders to carry power close to the load points, where it is further step-down to either 230 V or 415 V.

The present structure of the distribution feeders doesn't support quick fault detection, isolation of faulty region and restoration of supply to the maximum outage area, which is healthy. In the absence of switches at different points in the distribution network, it is not possible to isolate certain loads for load shedding as and when required. The only option available in the present distribution network is the circuit breaker (one each for every main 11 KV feeder) at the 33 KV substation. However, these circuit breakers are actually provided as a means of protection to completely isolate the downstream network in the event of a fault. Using this as a tool for load management is not desirable, as it disconnects the power supply to a very large segment of consumers. Clearly, there is a need to put in place a system that can achieve a finer resolution in load management.

In the event of a fault on any feeder section downstream, the circuit breaker at the 33 KV substation trips (opens). As a result, there is a blackout over a large section of the distribution network. If the faulty segment could be precisely identified, it would be possible to substantially reduce the blackout area, by re-routing the power to the healthy feeder segments through the operation of sectionalizing switches, placed at strategic locations in various feeder segments.

Thus, lack of information at the base station (33 KV sub-station) of the loading and health status of the 11 KV/ 415 V distribution transformers and associated feeders is one primary cause of inefficient power distribution. Also, due to absence of monitoring, overloading occurs, which results in low voltage at the customer end and increases the risk of frequent breakdowns of transformers and feeders.

#### 42.4. Automated System

The inefficient operation of the conventional distribution system can be mainly attributed to the frequent occurrence of faults and the uncertainty in detecting them. To enhance the electrical power distribution reliability, sectionalizing switches are provided along the way of primary feeders. Thus, by adding fault detecting relays to the sectionalizing switches along with circuit breaker and protective relays at the distribution substations, the system is capable to determine fault sections. To reduce the service disruption area in case of power failure, normally open (NO) sectionalizing switches called as route (tie) switches are used for supply restoration process. The operation of these switches is controlled from the control center through the Remote Terminal Units (RTU'S).

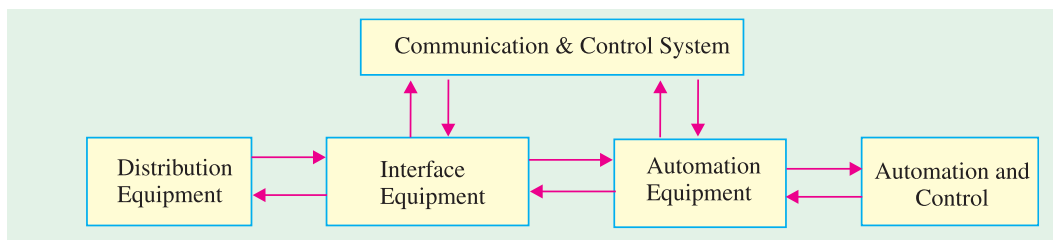


Fig. 42.2. Interconnection of distribution, control and communication system.

In distribution automation (DA) system, the various quantities (*e.g.*, voltage, current, switch status, temperature and oil level, etc.) are recorded in the field at the distribution transformers and feeders, using a data acquisition device called Remote Terminal Unit. These quantities are transmitted on-line to the base station through a communication media. The acquired data is processed at the base station for display at multiple computers through a Graphic user interface (GUI). In the event of a system quantity crossing a pre-defined threshold, an alarm is generated for operator intervention. Any control action, for opening or closing of the switch or circuit breaker, is initiated by the operator and transmitted from the 33 KV base station through the communication channel to the remote terminal unit associated with the corresponding switch or CB. The desired switching takes place and the action is acknowledged back to the operator.

Interconnection of distribution, control and communication system is shown in Figure 42.2.

All the above mentioned functions of data collection, data transmission, data monitoring, data processing, man-machine interface, etc. are realized using an integrated distribution SCADA (Supervisory Control And Data Acquisition) system.

The implementation of SCADA system in the electric utility involves the installation of following units :

1. Sectionalizing Switches
2. Remote Terminal Unit
3. Data Acquisition System
4. Communication Interface
5. Control Computer

### 42.5. Sectionalizing Switches

These sectionalizing switches are basically either air-brake switches or Load Break Switches (LBS) or Moulded Case Circuit Breaker (MCCB). These are remotely operable switches designed specifically for 11 KV and 415 V feeders. However, switches of appropriate rating corresponding to the rated feeder current can also be chosen.

Generally, 11 KV Vacuum break line sectionalizers are installed away from the substation and on the pole. A fault-indicating device is provided for location of fault at any section. To avoid opening of switches due to transient voltage drop or any other mal-operation, generally, 1.5 - 3 seconds delay is provided. Benefits obtained from installing these sectionalizing switches include :

1. Immediate isolation with indication of faulty section
2. Immediate restoration of supply to the healthy section
3. No down time for transient faults
4. Improvement of revenue due to lesser outage of section and low down time
5. Requires less man-power in the system and
6. Better Reliability of power supply

### 42.6. Remote Terminal Units (RTU's)

A typical SCADA system consists of remote terminal units, to record and check, measured values and meter readings, before transmitting them to control station and in the opposite direction, to transmit commands, set point values and other signals to the switchgear and actuator.

The functions of RTU's can be given as following :

- (a) Acquisition of information such as measured values, signals, meter readings, etc.
- (b) Transmit commands or instructions (binary plus type or continuous), set points, control variables, etc. including their monitoring as a function of time.
- (c) Recognition of changes in signal input states plus time data allocation and sequential recording of events by the master control station.
- (d) Processing of information transmitted to and from the telecommunication equipment such as data compression, coding and protection.
- (e) Communication with master control station.

A typical architecture RTU interfacing is shown in Fig 42.3.

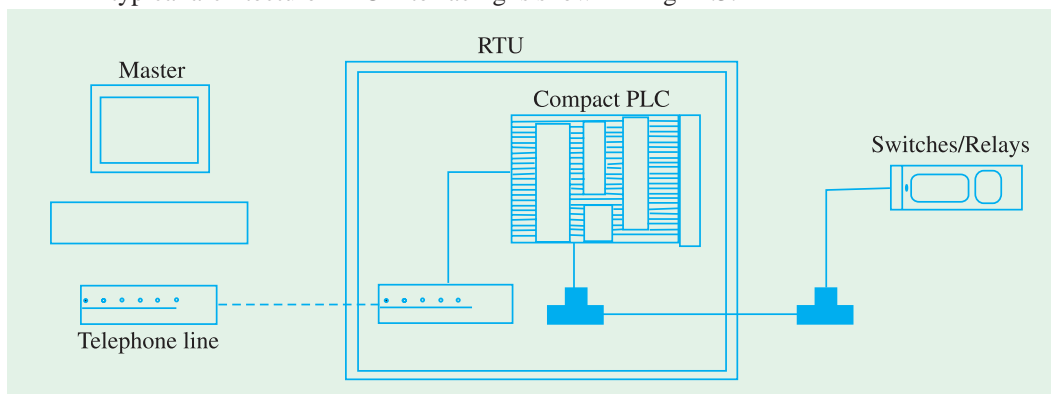


Fig. 42.3. Typical Architecture of RTU Interfacing

### 42.7. Data Acquisition System (DAS)

The data regarding the complete network consists of electrical and mechanical variables, on / off states, analog quantities, digital quantities, changes of state, sequence of events, time of occurrence

and several other data, which the control room operators will like to know.

Data is acquired by means of current transformers (CTs), potential transformers (PTs), transducers and other forms of collecting information. Transducers convert the data into electrical form to enable easy measurement and transmission. Data may be collected at low level or high level. Then it is amplified in signal amplifier and conditioned in data signal conditioner. The data is transmitted from the process location to the control room and from the control room to the control center.

The large number of electrical, mechanical and other data are scanned at required interval, recorded and displayed as per the requirement. Some of the data is converted from analog to digital form through A/D (Analogue to Digital) converters.

The data loggers perform the following functions :

1. Input Scanning
2. Signal Amplification and A/D Conversion
3. Display, Recording and Processing

The input scanner is generally a multi-way device, which selects input signals at regular periodic intervals in a sequence decided by the rate of change of input data. Slow varying quantities are scanned with a lower period of time-intervals. Output of scanner is given to A/D converter. Digital signals are obtained through DSP by micro-controllers or the control computer. This acquired signal can be displayed, recorded and processed for appropriate actions to be performed later.

#### 42.8. Communication Interface

A good data communication system to transmit the control commands and data between Distribution Control Centre (DCC) and a large number of device remotely located on the distribution network is a pre-requisite for the good performance of Distribution Automation System (DAS). The communication requirements of each DAS is unique, depending upon the Distribution Automation functions selected for the implementation. A wide range of communication technologies are available to perform the tasks of DAS. The choice of communication technology also has a big impact on the cost of DAS.

RTU's communicate with the control room through a communication interface, which could be any of the following :

##### (a) Power line carrier communication (PLCC) :

Each end of the transmission line is provided with identical carrier equipment in the frequency range of 30 to 500 kHz. The high frequency signals are transmitted through power lines. The carrier current equipment comprises the coupling capacitor and the tuning circuit.

##### (b) Fibre optics data communication :

Application of fibre optic communication is presently in infant stage and has a vast scope due to freedom from electromagnetic interference and enormous data handling capacity of a single pair of optical fibre. The information is exchanged in the form of digitised light signals transmitted through optical fibres.

Fibre optics, with its explicit downward cost trend in terms of product as well as installation and maintenance costs has become a widely accepted choice, as it offers both technical and commercial advantages over conventional systems that use metallic cables and radio links. The communication basically consists of a transmitter and a receiver for information signals coming from the user's device which is connected through copper wire to the switching center or exchange, where it is changed into a digital signal like 1s or 0 s for easy handling. The signal is then transferred to the transmitter. In the transmitter, the information signal which is electrical, drives an optical source : Laser or a light emitting diode (LED), which in turn, optically modulates the information signal,

which gets coupled into the optical fibre. The receiver located at the other side of the link detects the original signal and demodulates or converts back the optical signal to the original information signal (Electrical). The signal is then connected through copper wires to the switching device or exchange for selection and connection to the proper user or user device. This has advantage of high data rate (9600 bits and much more) and immunity from noise.

**(c) Radio communication :**

Radio communication utilizes frequency bands between 85 MHz to 13 GHz, Point to point radio links, Multi terminal radio communication facility, Limited area radio scheme, Mobile radio sets, Emergency radio communication etc. are the types of radio communication facilities used in electrical power systems.

**(d) Public telephone communication :**

Dial-up and dedicated leased telephone lines are often used for Distribution Automation. The dial-up lines are suitable for infrequent data transmission. The leased line are suitable for continuous communication but are expensive. The reliability of communication varies greatly and is dependent upon the telephone company. Telephone communication service through packet switching network, cellular radio are viable and may have the advantages of providing services in other-wise inaccessible places.

**(e) Satellite communication :**

A satellite communication system using very small aperture terminal (VSAT) is suitable for DAS. VSAT is a point to multi-point star network like TDMA. It consists of one single Hub and number of remote Personal Earth Stations (PES). The communication between transponder and Hub is TDM access and between Hub and PES is TDMA access. The communication system between Hub remote PES is through two separate radio links. The link from remote PES to Hub is called inroute and from Hub to the remote PES is called as outroute. No end user transmission either originates or terminates at the satellite. In India extended 'C' band with up link in the 6.15/ 6.815 and down link in the 4.09/ 4.59 KHz range is used. Fig. 42.4 shows schematic diagram of VSAT network.



Nowadays even telephone connections are via wireless systems. The figure shows the transmission tower from where radio waves can be transmitted to other stations and satellites

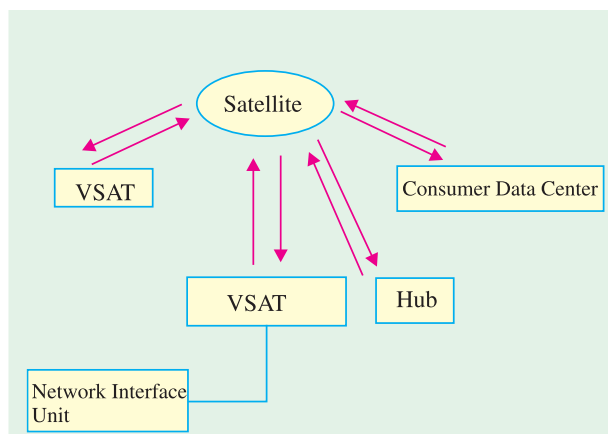
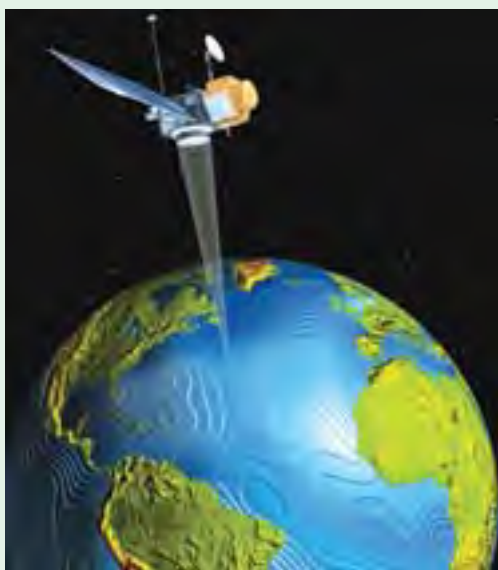


Fig. 42.4 Schematic diagram of VSAT network

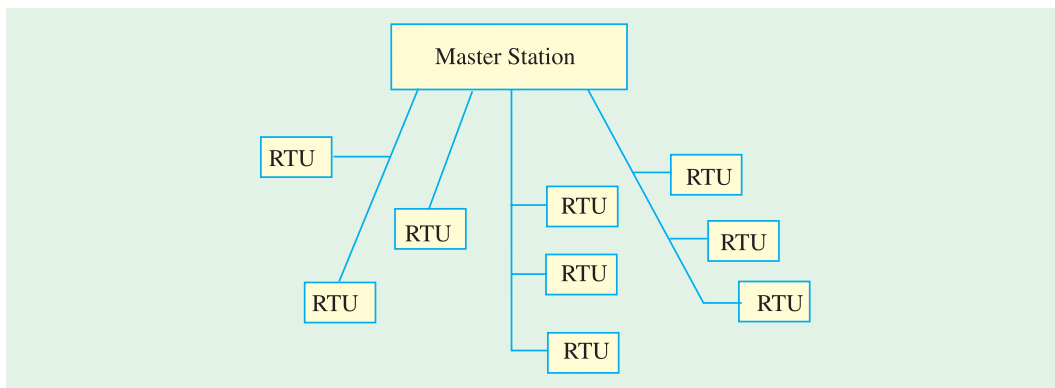


A satellite helps to develop instant communication links with the stations spread across the globe

**(f) Polling scheme :**

SCADA systems intended for electric system operations almost universally use a polling scheme between the central master and individual RTUs.

The master station controls all activities and RTUs respond only to polling requests. Fig. 42.5 illustrates the most common communication arrangement. Multiple two or four wire telephone grade circuits radiate from the master. The media for these circuits may be leased telephone circuits from a common carrier, private microwave, fibre optic cable systems, two-way cable TV, power line carrier, or even satellite. Polling and command requests and RTU responses are time multiplexed on each circuit. Each circuit terminating at the master station is independently serviced on an asynchronous basis by the master station. The most commonly used information rates is 1200 bits/sec. using asynchronous byte-oriented message formats.



**Fig. 42.5.** Typical multidrop communication system

### 42.9. Distribution SCADA

Distribution SCADA involves collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying whether the desired results are achieved. Fig 42.6 gives flow diagram of the SCADA functions.

Data acquisition in an electric utility SCADA system concentrates on the power system performance quantities like bus line volts, transformer currents, real and reactive power flow, C.B. status (circuit breaker status), isolator status and secondary quantities such as transformer temperature, insulating gas pressure, tank oil levels, flow levels etc. Often transformer tap positions, usual positions or other multiple position quantities are also transmitted in analog format.

The usual reason for installing supervisory control is to provide the system with sufficient information and control to operate the power system or some part of it in a safe, secure and economic manner.

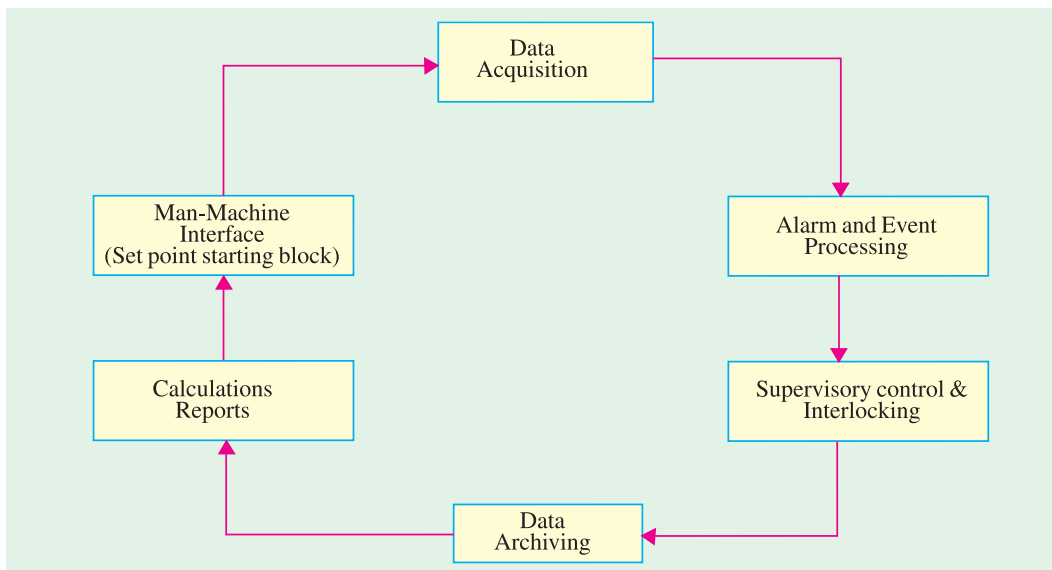


Fig. 42.6. SCADA functions

**42.10. Man – Machine Interface**

The implementation of SCADA system in an electric utility requires the installation of remote terminal units. RTU’s are designed to acquire data and transfer the same to the master station through a communication link. They collect data from transducers, transmitters, connect input from equipment/ instruments, meter readings etc., performs analogue/ digital conversions, check data scaling and corrections (typically at I/O levels) performs preprocessing tasks and send/ receive messages from/ to master stations via interfaces. Fig 42.7 shows the flow diagram for man-machine interface.

Man-Machine Interface (MMI) is the interface between man and technology for control of the technical process. The computer system at master control centre or central control room integrates with RTU over the communication link with its transmission protocol, acquires the remote substation or distribution transformer/ feeder data and transfers the same to the computer system for man-machine interface.

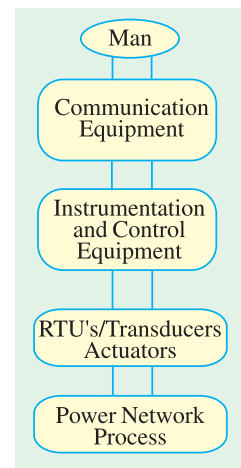


Fig. 42.7. Man - Machine Interface

**42.11. A Typical SCADA System**

A typical SCADA system may therefore comprises hardware and the software.

The hardware may include :

1. User friendly man-machine interface
2. Work station
3. Service having a particular function
4. Communication sub system
5. RTU’s

All the above components communicate with each other through a local area network (LAN) with internationally standardised protocols. A flexible redundancy is provided, assigning hot standby server to any server fulfilling time critical functions.

A software must offer the following features :

1. Use of high level programming language
2. Modular structure with clear and standardised interfaces between software modules and the database
3. Inter module communication only via a 'soft bus' independent of their computer residency
4. Easy addition of further application programs
5. Comfortable on-line diagnostics, development tools and file editors.

Artificial Intelligence (AI) plays a pivotal role in the development of software, rule based experts systems, logic based systems like fuzzy logic, neural networks etc. fall under this category. Few applications of the recent times are stated on the next page :

1. Application of expert system to power system restoration using flow control rules, energisation rules, line switching rules, load shedding rules, voltage correction rules etc.
2. Knowledge-based expert systems for fault location and diagnosis on electric power distribution feeders.
3. Fuzzy based logic for reconfiguration of distribution networks.

The application of model based expert system results in an intelligent power distribution system with learning self organizing and diagnostic capabilities. In addition, it advises the control center operations in cases of emergency. Distribution automated systems can therefore be operated in two operational modes: Online and Off-line. The Online operation permits faulty equipment identification, restoration planning, network maintenance scheduling and emergency operations. The Off-line simulator mode allows the user to verify the validity of acquired knowledge by setting an imaginary fault on the system and provides a convenient way of training the inexperienced operator.

#### 42.12. Distribution Automation

Distribution Automation functions provide a means to more effectively manage minute by minute continuous operation of a distribution system. Distribution Automation provides a tool to achieve a maximum utilization of the utility's physical plant and to provide the highest quality of service to its customers. Obviously, both the utility and its customers are beneficiaries of successful Distribution Automation.

Distribution Automation systems are modular, hence they may be implemented in stages, commencing from a modest degree of capability and complexity and growing as necessary to achieve tangible and intangible economic benefits. For example, a utility may start with a limited capability SCADA System for sub station monitoring and control, extend this to the feeders and finally implement a complete integration of automation functions. Systems implemented in this fashion must be designed to accommodate future expansion.

Distribution Automation System offers an integrated 'Distributed Management System' (DMS). The functions of DMS are shown in Fig 42.8. As in any other SCADA system, Distribution SCADA involves

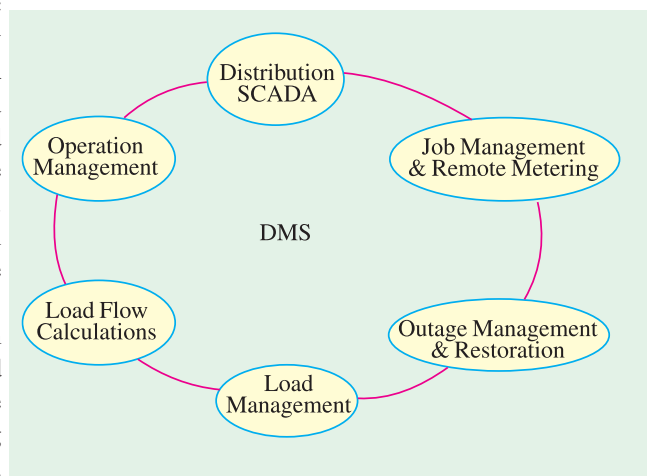


Fig. 42.8. Functions of distributed management system



collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying that the desired results are achieved. Operation management supports the analysis of distribution network. It models the load profile to present state of the network. Load flow calculations estimate voltage levels and power flows at each feeder. Job management makes switching order handling easier and work protection tagging ensures the safety of repair crew on duty. Outage management and service restoration facilitates to reduce outage time thereby increasing the reliability of the supply. Remote metering provides for the appropriate selection of energy registers where time-of-use rates are in effect, thus improving energy metering services to be more accurate and more frequent.

### 42.13. Load Management in DMS

This involves controlling system loads by remote control of individual customer loads. Control includes suppressing or biasing automatic control of cyclic loads, as well as load switching. Load Management can also be effected by inducing customers to suppress loads during utility selected daily periods by means of time-of-day rate incentives. Distribution Automation provides the control and monitoring ability required for both the load management scenarios -viz - direct control of customers loads and the monitoring necessary to verify that programmed levels are achieved. Execution of load management provides several possible benefits to the utility and its customers. Maximum utilization of the existing distribution system can lead to deferrals of capital expenditure. This is achieved by :

- (i) Shaping the daily (or monthly, annual) load characteristic by suppressing loads at peak times and encouraging energy consumption at off-peak times.
- (ii) Minimizing the requirement for more costly generation or power purchases by suppressing loads.
- (iii) Relieving the consequences of significant loss of generation or similar emergency situations by suppressing load.
- (iv) Reducing cold load pick-up during re-energization of circuits using devices with cold load pick-up features.

The effectiveness of direct control of customer loads is obviously enhanced by selecting the larger and more significant customer loads. These include electric space and water heating, air conditioning, washing machines, dryers and others of comparable magnitude.

More sophisticated customer's activated load management strategies are under study, taking advantage of the capabilities of Distribution Automation and of customers installed load control PC. With such an arrangement, the utility could vary rates throughout the day, reflecting actual generation costs and any system supply capability constraints and broadcast this information to all customer's PC. Each PC could then control it's loads to conform to a customer selected cost bias. This is sometimes called as 'spot pricing'.

### 42.14. Automated Distribution System

In the conventional distribution system the abnormal conditions are detected manually which costs lots of time and money to both consumers and power industry. In order to maintain high service quality and reliability and minimize loss in revenues, automation is required. Automation may be applied to the power distribution system so that problems on the distribution network may be detected and operated upon so as to minimize the outage time.

The equipments (either fixed wired or /and programmable), which are used for distribution automation, include :

1. Data collection equipment
2. Data transmission (telemetry) equipment

3. Data monitoring equipment
4. Data processing equipment
5. Man-Machine interface.

All the above equipments are integrated through distribution SCADA system.

Distribution SCADA involves collecting and analyzing information to take decisions and then implementing them and finally verifying whether desired results are achieved .

The implementation of SCADA system in the electric utility involves the installation of following units :

**(a) Data acquisition unit :** The basic variables (data) required for control, monitoring and protection include current, voltage, frequency, time, power factor, reactive power and real power. The data may be tapped in analog or digital form as required. For data collection purpose CT's and PT's are used. Transducers may be needed to convert the data into electrical form to enable easy measurement and transmission .The data is amplified in signal amplifier and conditioned in data signal conditioner. The data is then transmitted from the process location to the transmission room. In the control room data processing and data logging are performed which includes input scanning at required intervals, recording, programming and display by microprocessor, PLC, PC etc.

**(b) Remote terminal unit (RTU) :** RTU is used to record and check signals, measured values and meter readings before transmitting them to control station and in the opposite direction to transmit commands, set point values and other signals to the switchgear and actuator.

**(c) Communication unit :** A good data communication system to transmit the control command and data between distribution control centre and large number of remotely located devices is a prerequisite for a good performance of DA system. Wide range of communication technologies are available to perform the task of DA system which include public telephone communication (leased or dedicated line), power line carrier communication (PLCC), UHF MARS (ultra high frequency multi address radio system) and VHF Radio, as discussed earlier.

**Distribution Automation can be broadly classified as :**

1. Substation Automation.
2. Feeder Automation.
3. Consumer side Automation

#### 42.15. Substation Automation

Substation automation is the cutting edge technology in electrical engineering. It means having an intelligent, interactive power distribution network including :

1. Increased performance and reliability of electrical protection.
2. Advanced disturbance and event recording capabilities, aiding in detailed electrical fault analysis.
3. Display of real time substation information in a control center.
4. Remote switching and advanced supervisory control.
5. Increased integrity and safety of the electrical power network including advanced interlocking functions.
6. Advanced automation functions like intelligent load-shedding.

##### 42.15.1 Requirements

The general requirements for selecting an automation system while designing a new substation are :

1. The system should be adaptable to any vendor's hardware.
2. It should incorporate distributed architecture to minimize wiring.
3. It should be flexible and easily set up by the user.

4. The substation unit should include a computer to store data and pre-process information.

#### 42.15.2. Functioning

Bus voltages and frequencies, line loading, transformer loading, power factor, real and reactive power flow, temperature, etc. are the basic variables related with substation control and instrumentation. The various supervision, control and protection functions are performed in the substation control room. The relays, protection and control panels are installed in the controlled room. These panels along with PC aids in automatic operation of various circuit breakers, tap changers, autoreclosers, sectionalizing switches and other devices during faults and abnormal conditions. Thus, primary control in substation is of two categories.

1. Normal routine operation by operator's command with the aid of analog and digital control system.
2. Automatic operation by action of protective relays, control systems and PC.

The automated substation functioning can be treated as integration of two subsystems, as discussed below :

**(a) Control System :** The task of control system in a substation includes data collection, scanning, event reporting and recording; voltage control, power control, frequency control, other automatic and semiautomatic controls etc. The various switching actions like auto reclosing of line circuit breakers, operation of sectionalizing switches, on-load tap changers are performed by remote command from control room. The other sequential operations like load transfer from one bus to another, load shedding etc. are also taken care by control center.

**(b) Protective System :** The task of protective system includes sensing abnormal condition, annunciation of abnormal condition, alarm, automatic tripping, back-up protection, protective signaling.

The above two systems work in close co-operation with each other. Most of the above functions *i.e.* automatic switching sequences, sequential event recording, compiling of energy and other reports, etc. are integrated in software in the substation computer. This software is of modular design, which facilitates addition of new functions.

The communication between circuit breakers, autoreclosers and sectionalizing switches in the primary and secondary distribution circuits located in the field and the PC in distribution substation control room is through radio telecontrol or fibre optic channel or power line carrier channel as is feasible.

#### 42.16. Feeder Automation

Automating the fault diagnosis and supply restoration process significantly reduces the duration of service interruptions. The key objective behind automating the service restoration process is to restore supply to maximum loads in out-of-service zones. This is achieved by reconfiguring the network such that the constraints of the system are not violated. Providing timely restoration of supply to outage areas of the feeder enhances the value of service to customers and retains the revenue for the power industry.

The system data consisting of the status signals and electrical analog quantities are obtained using a suitable Data Acquisition System and processed by the control computer for typical functions of fault detection, isolation and network reconfiguration for supply restoration. The equipments normally required in Feeder Automation are discussed below :

**(a) Distribution equipment :**

This includes transformers, breakers, load break switches and motor operators, power reclosures, voltage regulators, capacitor banks etc.

**(b) Interface equipment :**

Interface equipment is required for the purpose of data acquisition and control. Potential transformers, current transformer, watt, var meter and voltage transducers, relays are some examples.

**(c) Automation equipment :**

Automation Equipment includes a DAS, communication equipment, substation remote terminal units (RTU) and distribution feeder RTU, current-to-voltage converters, etc.

**42.17. Consumer Side Automation**

Consumer side automation is very important for a distribution company as almost 80% of all the losses are taking place on distribution side alone. It is needed to evaluate the performance of a specific area in the distribution system and judge the overall losses.

**42.17.1. Energy Auditing**

Energy audit has a very wide range of applications in the electrical systems. It means overall accounting of energy generated, transmitted and distributed. As far as distribution side is concerned energy audit would mean overall accounting of energy supplied to and utilized by the consumers. Energy audit can also be used for rethinking about billing strategy, usefulness of an individual subscriber, loading of a given feeder etc.

Remote metering is used in energy auditing in which the energy used by a consumer is billed from a remote (distant) location without actually going to the place. In remote metering, the concept of TOD (time of day) metering can be introduced wherein the electronic meters at consumer's service entrance point are programmed to read the following meter readings on monthly basis.

1. KW hrs. consumed during calendar month by the consumer during low tariff and high tariff hours.
2. KVA maximum demand by the consumer during the calendar month (based on maximum demand lasting for 30 minutes duration).
3. Low tariff for off peak hour consumption.

These readings are telemetered to the control room for the purpose of monthly billing and cash collection through the various modes of communication available (- viz - Telephonic and wireless communication) depending on the load condition of the consumer. Tampering of energy - meters (if done) is also telemetered for taking action/penalty and disconnection of service.

Thus energy audit, though a very cumbersome and tedious job, can make a non-profitable business of distribution into a highly profitable one.

**42.18. Advantages of Distribution Automation**

More and more electric utilities are looking to distribution automation as an answer to the three major economic challenges facing the industry : the rising cost of adding generating capacity, increased saturation of existing distribution networks and greater sensitivity to customer service. Therefore, utilities that employ distribution automation expect both cost and service benefits. These benefits accumulate in areas that are related to investments, interruptions and customer service, as well as in areas related to operational cost savings, as given below :

**(a) Reduced line loss :**

The distribution substation is the electrical hub for the distribution network. A close coordination between the substation equipment, distribution feeders and associated equipment is necessary to increase system reliability. Volt/VAR control is addressed through expert algorithms which monitors and controls substation voltage devices in coordination with down-line voltage devices to reduce line loss and increase line throughput.

**(b) Power quality :**

Mitigation equipment is essential to maintain power quality over distribution feeders. The substation RTU in conjunction with power monitoring equipment on the feeders monitors, detects,

and corrects power-related problems before they occur, providing a greater level of customer satisfaction.

**(c) Deferred capital expenses :**

A preventive maintenance algorithm may be integrated into the system. The resulting ability to schedule maintenance, reduces labour costs, optimizes equipment use and extends equipment life.

**(d) Energy cost reduction :**

Real-time monitoring of power usage throughout the distribution feeder provides data allowing the end user to track his energy consumption patterns, allocate usage and assign accountability to first line supervisors and daily operating personnel to reduce overall costs.

**(e) Optimal energy use :**

Real-time control, as part of a fully-integrated, automated power management system, provides the ability to perform calculations to reduce demand charges. It also offers a load-shedding/preservation algorithm to optimize utility and multiple power sources, integrating cost of power into the algorithm.

**(f) Economic benefits :**

Investment related benefits of distribution automation came from a more effective use of the system. Utilities are able to operate closer to the edge to the physical limits of their systems. Distribution automation makes this possible by providing increased availability of better data for planning, engineering and maintenance. Investment related benefits can be achieved by deferring addition of generation capacity, releasing transmission capacity and deferring the addition, replacement of distribution substation equipment. Features such as voltage/VAR control, data monitoring and logging and load management contribute to capital deferred benefits.

Distribution automation can provide a balance of both quantitative and qualitative benefits in the areas of interruption and customer service by automatically locating feeder faults, decreasing the time required to restore service to unfaulted feeder sections, and reducing costs associated with customer complaints.

**(g) Improved reliability :**

On the qualitative side, improved reliability adds perceived value for customer and reduce the number of complaints. Distribution automation features that provide interruption and customer service related benefits include load shedding and other automatic control functions.

Lower operating costs are another major benefits of distribution automation. Operating cost reduction are achieved through improved voltage profiles, controlled VAR flow, repairs and maintenance savings, generation fuel savings from reduced substation transformer load losses, reduced feeder primary and distribution transformer losses, load management and reduced spinning reserve requirements. In addition, data acquisition and processing and remote metering functions play a large role in reducing operating costs and should be considered an integral part of any distribution automation system.

Through real time operation, the control computer can locate the faults much faster and control the switches and reclosures to quickly reroute power and minimize the total time-out, thus increasing the system reliability.

**(h) Compatibility :**

Distribution automation spans many functional and product areas including computer systems, application software, RTUs, communication systems and metering products. No single vendor provides all the pieces. Therefore, in order to be able to supply a utility with a complete and integrated system, it is important for the supplier to have alliances and agreements with other vendors.

An effective distribution automation system combines complementary function and capabilities and require an architecture that is flexible or “open” so that it can accommodate products from different vendors. In addition, a distribution automation system often requires interfaces with existing system in order to allow migration and integration, still monitoring network security.

## Tutorial Problem No. 42.1

1. Why we need Transmission System Interconnection? *(Nagpur University, Summer 2004)*
2. Describe different distribution automation methods and their advantages and disadvantages.

## OBJECTIVE TESTS – 42

1. Need based energy management is better than demand based energy management  
(a) true (b) false
2. Round the clock power is required to  
(a) agricultural uses  
(b) shift based industries  
(c) commercial organisations  
(d) essential services
3. Need based energy management would include  
(a) plan power generation as per forecast  
(b) monitor matching of need with supply  
(c) identify the needs of various consumers  
(d) all of the above
4. Existing Distribution systems are  
(a) properly designed  
(b) chaotic  
(c) optimally operated  
(d) automated
5. Distribution systems are not radial  
(a) true (b) false
6. The major aim of distribution automation is to keep minimum area in dark for a minimum time  
(a) true (b) false
7. Distribution automation is used for  
(a) electrical quantities only  
(b) physical quantities only  
(c) both of the above
8. Sectionalizing switches are circuit breakers  
(a) true (b) false
9. RTU is a -  
(a) transmitter (b) receiver  
(c) trans-receiver
10. The main problem in implanting Distribution Automation in metros is -  
(a) RTUs (b) communication  
(c) D.A.S. (d) control computers
11. Distribution SCADA means -  
(a) data acquisition  
(b) man – machine interface  
(c) data archiving  
(d) all of the above
12. Now - a - days the trend is towards -  
(a) manned substation  
(b) unmanned substation
13. Artificial Intelligence plays a pivotal data in distribution automation  
(a) true (b) false
14. Distribution Automation means -  
(a) substation Automation  
(b) feeder Automation  
(c) consumer side Automation  
(d) all of the above
15. Distribution Automotive system are modular  
(a) true (b) false
16. Existing distribution systems are controlled  
(a) automatically  
(b) manually
17. Distribution Automotive systems operate in real time  
(a) true (b) false
18. Until recent years, much of the investment and technical exploitation was done in -  
(a) generation  
(b) transmission  
(c) generation and transmission  
(d) distribution
19. Demand supply gap in power sector is expected to grow by - - - - percent every year.  
(a) 5 (b) 10  
(c) 15 (d) 20
20. Major revenue losses in distribution system occur in -  
(a) domestic sector  
(b) industrial sector  
(c) agricultural sector  
(d) commercial sector

## ANSWERS

1. (a) 2. (d) 3. (d) 4. (b) 5. (b) 6. (a) 7. (c) 8. (b) 9. (c) 10. (b)  
11. (d) 12. (b) 13. (a) 14. (d) 15. (a) 16. (b) 17. (a) 18. (c) 19. (d) 20. (c)

# CHAPTER 43

## Learning Objectives

- General
- Traction System
- Direct Steam Engine Drive
- Advantages of Electric Traction
- Saving in High Grade Coal
- Disadvantages of Electric Traction
- System of Railway Electrification
- Three Phase Low-Frequency A.C. System
- Block Diagram of an AC Locomotive
- The Tramways
- Collector Gear for OHE
- Confusion Regarding Weight and Mass of Train
- Tractive Efforts for Propulsion of a Train
- Power Output from Driving Axles
- Energy Output from Driving Axles
- Specific Energy Output
- Evaluation of Specific Energy Output
- Energy Consumption
- Specific Energy Consumption
- Adhesive Weight
- Coefficient of Adhesion

## ELECTRIC TRACTION



In electric traction, driving force (or tractive force) is generated by electricity, using electric motors. Electric trains, trams, trolley, buses and battery run cars are some examples where electric traction is employed.

**43.1. General**

By electric traction is meant locomotion in which the driving (or tractive) force is obtained from electric motors. It is used in electric trains, tramcars, trolley buses and diesel-electric vehicles etc. Electric traction has many advantages as compared to other non-electrical systems of traction including steam traction.

**43.2. Traction Systems**

Broadly speaking, all traction systems may be classified into two categories :

**(a) non-electric traction systems**

They do not involve the use of electrical energy at *any stage*. Examples are : steam engine drive used in railways and internal-combustion-engine drive used for road transport.



The above picture shows a diesel train engine. These engines are now being rapidly replaced by electric engines

**(b) electric traction systems**

They involve the use of electric energy at some stage or the other. They may be further subdivided into two groups :

1. First group consists of self-contained vehicles or locomotives. Examples are : battery-electric drive and diesel-electric drive etc.
2. Second group consists of vehicles which receive electric power from a distribution network fed at suitable points from either central power stations or suitably-spaced sub-stations. Examples are : railway electric locomotive fed from overhead ac supply and tramways and trolley buses supplied with dc supply.

**43.3. Direct Steam Engine Drive**

Though losing ground gradually due to various reasons, steam locomotive is still the most widely-adopted means of propulsion for railway work. Invariably, the reciprocating engine is employed because

1. it is inherently simple.
2. connection between its cylinders and the driving wheels is simple.
3. its speed can be controlled very easily.



However, the steam locomotive suffers from the following disadvantages :

1. since it is difficult to install a condenser on a locomotive, the steam engine runs non-condensing and, therefore, has a very low thermal efficiency of about 6-8 percent.
2. it has strictly limited overload capacity.
3. it is available for hauling work for about 60% of its working days, the remaining 40% being spent in preparing for service, in maintenance and overhaul.

#### 43.4. Diesel-electric Drive

It is a self-contained motive power unit which employs a diesel engine for direct drive of a dc generator. This generator supplies current to traction motors which are geared to the driving axles.

In India, diesel locomotives were introduced in 1945 for shunting service on broad-gauge (BG) sections and in 1956 for high-speed main-line operations on metre-gauge (MG) sections. It was only in 1958 that Indian Railways went in for extensive main-line dieselisation.\*

Diesel-electric traction has the following advantages :

1. no modification of existing tracks is required while converting from steam to diesel-electric traction.
2. it provides greater tractive effort as compared to steam engine which results in higher starting acceleration.
3. it is available for hauling for about 90% of its working days.
4. diesel-electric locomotive is more efficient than a steam locomotive (though less efficient than an electric locomotive).

#### Disadvantages

1. for same power, diesel-electric locomotive is costlier than either the steam or electric locomotive.
2. overload capacity is limited because diesel engine is a constant-kW output prime mover.
3. life of a diesel engine is comparatively shorter.
4. diesel-electric locomotive is heavier than plain electric locomotive because it carries the main engine, generator and traction motors etc.
5. regenerative braking cannot be employed though rheostatic braking can be.

#### 43.5. Battery-electric Drive

In this case, the vehicle carries secondary batteries which supply current to dc motors used for driving the vehicle. Such a drive is well-suited for shunting in railway yards, for traction in mines, for local delivery of goods in large towns and large industrial plants. They have low maintenance cost and are free from smoke. However, the scope of such vehicles is limited because of the small capacity of the batteries and the necessity of charging them frequently.



The above picture shows a battery run car. Battery run vehicles are seen as alternatives for future transport due to their pollution-free locomotion

#### 43.6. Advantages of Electric Traction

As compared to steam traction, electric trac-

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\* The Diesel Locomotive Works at Varanasi turns out 140 locomotives of 2700 hp (2015 kW) annually. Soon it will be producing new generation diesel engines of 4000 hp (2985 kW).

tion has the following advantages :

**1. Cleanliness.** Since it does not produce any smoke or corrosive fumes, electric traction is most suited for underground and tube railways. Also, it causes no damage to the buildings and other apparatus due to the absence of smoke and flue gases.

**2. Maintenance Cost.** The maintenance cost of an electric locomotive is nearly 50% of that for a steam locomotive. Moreover, the maintenance time is also much less.

**3. Starting Time.** An electric locomotive can be started at a moment's notice whereas a steam locomotive requires about two hours to heat up.

**4. High Starting Torque.** The motors used in electric traction have a very high starting torque. Hence, it is possible to achieve higher accelerations of 1.5 to 2.5 km/h/s as against 0.6 to 0.8 km/h/s in steam traction. As a result, we are able to get the following additional advantages:

- (i) high schedule speed
- (ii) increased traffic handling capacity
- (iii) because of (i) and (ii) above, less terminal space is required—a factor of great importance in urban areas.

**5. Braking.** It is possible to use regenerative braking in electric traction system. It leads to the following advantages :

- (i) about 80% of the energy taken from the supply during ascent is returned to it during descent.
- (ii) goods traffic on gradients becomes safer and speedier.
- (iii) since mechanical brakes are used to a very small extent, maintenance of brake shoes, wheels, tyres and track rails is considerably reduced because of less wear and tear.

**6. Saving in High Grade Coal.** Steam locomotives use costly high-grade coal which is not so abundant. But electric locomotives can be fed either from hydroelectric stations or pit-head thermal power stations which use cheap low-grade coal. In this way, high-grade coal can be saved for metallurgical purposes.

**7. Lower Centre of Gravity.** Since height of an electric locomotive is much less than that of a steam locomotive, its centre of gravity is comparatively low. This fact enables an electric locomotive to negotiate curves at higher speeds quite safely.

**8. Absence of Unbalanced Forces.** Electric traction has higher coefficient of adhesion since there are no unbalanced forces produced by reciprocating masses as is the case in steam traction. It not only reduces the weight/kW ratio of an electric locomotive but also improves its riding quality in addition to reducing the wear and tear of the track rails.

### 43.7. Disadvantages of Electric Traction

**1.** The most vital factor against electric traction is the initial high cost of laying out overhead electric supply system. Unless the traffic to be handled is heavy, electric traction becomes uneconomical.

**2.** Power failure for few minutes can cause traffic dislocation for hours.

**3.** Communication lines which usually run parallel to the power supply lines suffer from electrical interference. Hence, these communication lines have either to be removed away from the rail track or else underground cables have to be used for the purpose which makes the entire system still more expensive.

**4.** Electric traction can be used only on those routes which have been electrified. Obviously, this restriction does not apply to steam traction.

**5.** Provision of a negative booster is essential in the case of electric traction. By avoiding the

flow of return currents through earth, it curtails corrosion of underground pipe work and interference with telegraph and telephone circuits.

### 43.8. Systems of Railway Electrification

Presently, following four types of track electrification systems are available :

1. Direct current system—600 V, 750 V, 1500 V, 3000 V
2. Single-phase ac system—15-25 kV,  $16\frac{2}{3}$ , 25 and 50 Hz
3. Three-phase ac system—3000-3500 V at  $16\frac{2}{3}$  Hz
4. Composite system—involving conversion of single-phase ac into 3-phase ac or dc.

### 43.9. Direct Current System

Direct current at 600-750 V is universally employed for tramways in urban areas and for many suburban railways while 1500-3000 V dc is used for main line railways. The current collection is from third rail (or conductor rail) up to 750 V, where large currents are involved and from overhead wire for 1500 V and 3000 V, where small currents are involved. Since in majority of cases, track (or running) rails are used as the return conductor, only one conductor rail is required. Both of these contact systems are fed from substations which are spaced 3 to 5 km for heavy suburban traffic and 40-50 km for main lines operating at higher voltages of 1500 V to 3000 V. These sub-stations themselves receive power from 110/132 kV, 3-phase network (or grid). At these substations, this high-voltage 3-phase supply is converted into low-voltage 1-phase supply with the help of Scott-connected or V-connected 3-phase transformers (Art. 31.9). Next, this low ac voltage is converted into the required dc voltage by using suitable rectifiers or converters (like rotary converter, mercury-arc, metal or semiconductor rectifiers). These substations are usually automatic and are remote-controlled.

The dc supply so obtained is fed via suitable contact system to the traction motors which are either dc series motors for electric locomotive or compound motors for tramway and trolley buses where regenerative braking is desired.

It may be noted that for *heavy suburban service*, low voltage dc system is undoubtedly superior to 1-phase ac system due to the following reasons :

1. dc motors are better suited for frequent and rapid acceleration of heavy trains than ac motors.
2. dc train equipment is lighter, less costly and more efficient than similar ac equipment.
3. when operating under similar service conditions, dc train consumes less energy than a 1-phase ac train.
4. the conductor rail for dc distribution system is less costly, both initially and in maintenance than the high-voltage overhead ac distribution system.
5. dc system causes no electrical interference with overhead communication lines.

The only disadvantage of dc system is the necessity of locating ac/dc conversion sub-stations at relatively short distances apart.

### 43.10. Single-Phase Low-frequency AC System

In this system, ac voltages from 11 to 15 kV at  $16\frac{2}{3}$  or 25 Hz are used. If supply is from a generating station exclusively meant for the traction system, there is no difficulty in getting the electric supply of  $16\frac{2}{3}$  or 25 Hz. If, however, electric supply is taken from the high voltage transmission lines at 50 Hz, then in addition to step-down transformer, the substation is provided with a frequency

converter. The frequency converter equipment consists of a 3-phase synchronous motor which drives a 1-phase alternator having 16  $\frac{2}{3}$  or 25 Hz frequency.

The 15 kV 16  $\frac{2}{3}$  or 25 Hz supply is fed to the electric locomotor via a single over-head wire (running rail providing the return path).

A step-down transformer carried by the locomotive reduces the 15-kV voltage to 300-400 V for feeding the ac series motors. Speed regulation of ac series motors is achieved by applying variable voltage from the tapped secondary of the above transformer.

Low-frequency ac supply is used because apart from improving the commutation properties of ac motors, it increases their efficiency and power factor. Moreover, at low frequency, line reactance is less so that line impedance drop and hence line voltage drop is reduced. Because of this reduced line drop, it is feasible to space the substations 50 to 80 km apart. Another advantage of employing low frequency is that it reduces telephonic interference.

### 41.11. Three-phase Low-frequency AC System

It uses 3-phase induction motors which work on a 3.3 kV, 16  $\frac{2}{3}$  Hz supply. Sub-stations receive power at a very high voltage from 3-phase transmission lines at the usual industrial frequency of 50 Hz. This high voltage is stepped down to 3.3 kV by transformers whereas frequency is reduced from 50 Hz to 16  $\frac{2}{3}$  Hz by frequency converters installed at the sub-stations. Obviously, this system employs *two* overhead contact wires, the track rail forming the third phase (of course, this leads to insulation difficulties at the junctions).

Induction motors used in the system are quite simple and robust and give trouble-free operation. They possess the merits of high efficiency and of operating as a generator when driven at speeds above the synchronous speed. Hence, they have the property of automatic regenerative braking during the descent on gradients. However, it may be noted that despite all its advantages, this system has not found much favour and has, in fact, become obsolete because of its certain inherent limitations given below :

1. the overhead contact wire system becomes complicated at crossings and junctions.
2. constant-speed characteristics of induction motors are not suitable for traction work.
3. induction motors have speed/torque characteristics similar to dc shunt motors. Hence, they are not suitable for parallel operation because, even with little difference in rotational speeds caused by unequal diameters of the wheels, motors will become loaded very unevenly.

### 43.12. Composite System

Such a system incorporates good points of two systems while ignoring their bad points. Two such composite systems presently in use are :

1. 1-phase to 3-phase system also called Kando system
2. 1-phase to dc system.

### 43.13. Kando System

In this system, single-phase 16-kV, 50 Hz supply from the sub-station is picked up by the locomotive through the single overhead contact wire. It is then converted into 3-phase ac supply at the same frequency by means of phase converter equipment carried on the locomotives. This 3-phase supply is then fed to the 3-phase induction motors.

As seen, the complicated overhead two contact wire arrangement of ordinary 3-phase system is replaced by a single wire system. By using silicon controlled rectifier as inverter, it is possible to get variable-frequency 3-phase supply at 1/2 to 9 Hz frequency. At this low frequency, 3-phase motors develop high starting torque without taking excessive current. In view of the above, Kando system is likely to be developed further.

#### 43.14. Single-phase AC to DC System

This system combines the advantages of high-voltage ac distribution at industrial frequency with the dc series motors traction. It employs overhead 25-kV, 50-Hz supply which is stepped down by the transformer installed in the locomotive itself. The low-voltage ac supply is then converted into dc supply by the rectifier which is also carried on the locomotive. This dc supply is finally fed to dc series traction motor fitted between the wheels. The system of traction employing 25-kV, 50-Hz, 1-phase ac supply has been adopted for all future track electrification in India.

#### 43.15. Advantages of 25-kV, 50-Hz AC System

Advantages of this system of track electrification over other systems particularly the dc system are as under :

##### 1. Light Overhead Catenary

Since voltage is high (25 kV), line current for a given traction demand is less. Hence, cross-section of the overhead conductors is reduced. Since these small-sized conductors are light, supporting structures and foundations are also light and simple. Of course, high voltage needs higher insulation which increases the cost of overhead equipment (OHE) but the reduction in the size of conductors has an overriding effect.

##### 2. Less Number of Substations

Since in the 25-kV system, line current is less, line voltage drop which is mainly due to the resistance of the line is correspondingly less. It improves the voltage regulation of the line which fact makes larger spacing of 50-80 km between sub-stations possible as against 5-15 km with 1500 V dc system and 15-30 km with 3000 V dc system. Since the required number of substations along the track is considerably reduced, it leads to substantial saving in the capital expenditure on track electrification.

##### 3. Flexibility in the Location of Substations

Larger spacing of substations leads to greater flexibility in the selection of site for their proper location. These substations can be located near the national high-voltage grid which, in our country, fortunately runs close to the main railway routes. The substations are fed from this grid thereby saving the railway administration lot of expenditure for erecting special transmission lines for their substations. On the other hand, in view of closer spacing of dc substations and their far away location, railway administration has to erect its own transmission lines for taking feed from the national grid to the substations which consequently increases the initial cost of electrification.

##### 4. Simplicity of Substation Design

In ac systems, the substations are simple in design and layout because they do not have to install and maintain rotary converters or rectifiers as in dc systems. They only consist of static transformers alongwith their associated switchgear and take their power directly from the high-voltage national grid running over the length and breadth of our country. Since such sub-stations are remotely controlled, they have few attending personnel or even may be unattended.

**5. Lower Cost of Fixed Installations**

The cost of fixed installations is much less for 25 kV ac system as compared to dc system. In fact, cost is in ascending order for 25 kV ac, 3000 V dc and 1500 V dc systems. Consequently, traffic densities for which these systems are economical are also in the ascending order.

**6. Higher Coefficient of Adhesion**

The straight dc locomotive has a coefficient of adhesion of about 27% whereas its value for ac rectifier locomotive is nearly 45%. For this reason, a lighter ac locomotive can haul the same load as a heavier straight dc locomotive. Consequently, ac locomotives are capable of achieving higher speeds in coping with heavier traffic.

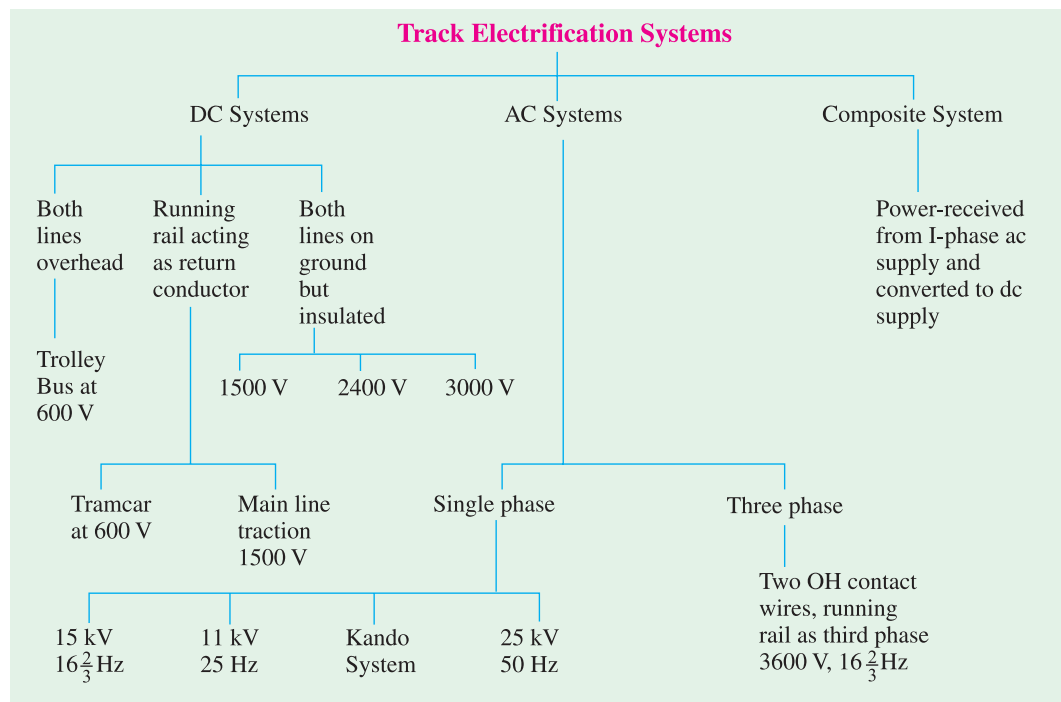
**7. Higher Starting Efficiency**

An ac locomotive has higher starting efficiency than a straight dc locomotive. In dc locomotive supply voltage at starting is reduced by means of ohmic resistors but by on-load primary or secondary tap-changer in ac locomotives.

**43.16. Disadvantages of 25-kV AC System**

1. Single-phase ac system produces both current and voltage unbalancing effect on the supply.
2. It produces interference in telecommunication circuits. Fortunately, it is possible at least to minimize both these undesirable effects.

Different track electrification systems are summarised below :



**43.17. Block Diagram of an AC Locomotive**

The various components of an ac locomotive running on single-phase 25-kV, 50-Hz ac supply are numbered in Fig. 43.1.

1. OH contact wire

- 2. pantograph
- 3. circuit breakers
- 4. on-load tap-changers
- 5. transformer
- 6. rectifier
- 7. smoothing choke
- 8. dc traction motors.

As seen, power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low ac voltage so obtained is converted into pulsating dc voltage by means of the rectifier. The pulsations in the dc voltage are then removed by the smoothing choke before it is fed to dc series traction motors which are mounted between the wheels.

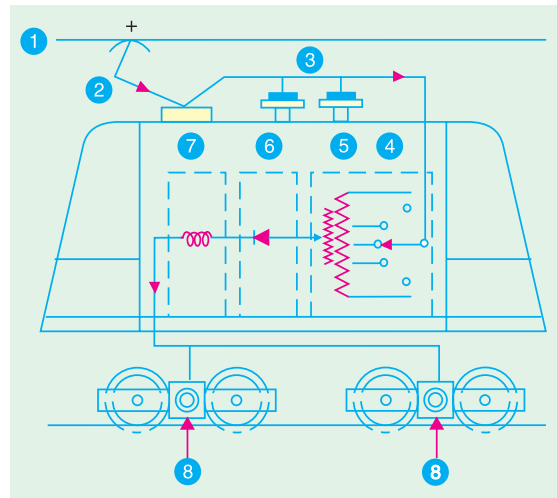


Fig. 43.1

The function of circuit breakers is to immediately disconnect the locomotive from the overhead supply in case of any fault in its electrical system. The on-load tap-changer is used to change the voltage across the motors and hence regulate their speed.

### 43.18. The Tramways

It is the most economical means of transport for very dense traffic in the congested streets of large cities. It receives power through a bow collector or a grooved wheel from an overhead conductor at about 600 V dc, the running rail forming the return conductor. It is provided with at least two driving axles in order to (i) secure necessary adhesion (ii) start it from either end and (iii) use two motors with series-parallel control. Two drum-type controllers, one at each end, are used for controlling the tramcar. Though these controllers are connected in parallel, they have suitable interlocking arrangement meant to prevent their being used simultaneously.

Tramcars are being replaced by trolley-buses and internal-combustion-engined omnibuses because of the following reasons :

- 1. tramcars lack flexibility of operation in congested areas.
- 2. the track constitutes a source of danger to other road users.

### 43.19. The Trolleybus

It is an electrically-operated pneumatic-tyred vehicle which needs *no track in the roadway*. It receives its power at 600 V dc from two overhead contact wires. Since adhesion between a rubber-tyred wheel and ground is sufficiently high, only a single driving axle and, hence, a single motor is used. The trolleybus can manoeuvre through traffic a metre or two on each side of the centre line of the trolley wires.



Trolley Bus

### 43.20. Overhead Equipment (OHE)

Broadly speaking, there are two systems of current collection by a traction unit :

(i) third rail system and (ii) overhead wire system.

It has been found that current collection from overhead wire is far superior to that from the third rail. Moreover, insulation of third rail at high voltage becomes an impracticable proposition and endangers the safety of the working personnel.

The simplest type of OHE consists of a single contact wire of hard drawn copper or silico-bronze supported either by bracket or an overhead span. To facilitate connection to the supports, the wire is grooved as shown in Fig. 43.2. Because there is appreciable sag of the wire between supports, it limits the speed of the traction unit to about 30 km/h. Hence, single contact wire system is suitable for tramways and in complicated yards and terminal stations where speeds are low and simplicity of layout is desirable.

For collection of current by high-speed trains, the contact (or trolley) wire has to be kept level without any abrupt changes in its height between the supporting structures. It can be done by using the single catenary system which consists of one catenary or messenger wire of steel with high sag and the trolley (or contact) wire supported from messenger wire by means of droppers clipped to both wires as shown in Fig. 43.3.

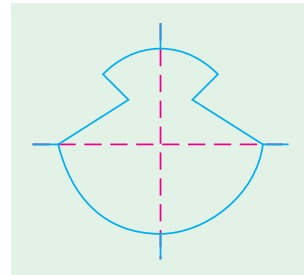


Fig. 43.2

#### 43.21. Collector Gear for OHE

The most essential requirement of a collector is that it should keep continuous contact with trolley wire at all speeds. Three types of gear are in common use :

1. trolley collector 2. bow collector and 3. pantograph collector.

To ensure even pressure on OHE, the gear equipment must, be flexible in order to follow variations in the sag of the contact wire. Also, reasonable precautions must be taken to prevent the collector from leaving the overhead wire at points and crossings.

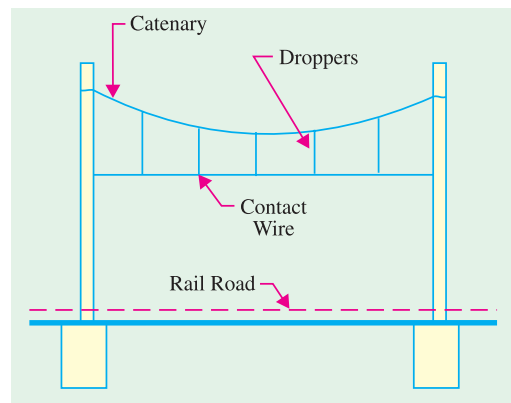


Fig. 43.3

#### 43.22. The Trolley Collector

This collector is employed on tramways and trolley buses and is mounted on the roof of the vehicle. Contact with the OH wire is made by means of either a grooved wheel or a sliding shoe carried at the end of a light trolley pole attached to the top of the vehicle and held in contact with OH wire by means of a spring. The pole is hinged to a swivelling base so that it may be reversed for reverse running thereby making it unnecessary for the trolley wire to be accurately maintained above the centre of the track. Trolley collectors always operate in the trailing position.

The trolley collector is suitable for low speeds upto 32 km/h beyond which there is a risk of its jumping off the OH contact wire particularly at points and crossing.

#### 43.23. The Bow Collector

It can be used for higher speeds. As shown in Fig. 43.4, it consists of two roof-mounted trolley poles at the ends of which is placed a light metal strip (or bow) about one metre long for current collection. The collection strip is purposely made of soft material (copper, aluminium or carbon) in order that most of the wear may occur on it rather than on the trolley wire. The bow collector also



operates in the trailing position. Hence, it requires provision of either duplicate bows or an arrangement for reversing the bow for running in the reverse direction. Bow collector is not suitable for railway work where speeds up to 120 km/h and currents up to 3000 A are encountered. It is so because the inertia of the bow collector is too high to ensure satisfactory current collection.

#### 43.24. The Pantograph Collector

Its function is to maintain link between overhead contact wire and power circuit of the electric locomotive at different speeds under all wind conditions and stiffness of OHE. It means that positive pressure has to be maintained at all times to avoid loss of contact and sparking but the pressure must be as low as possible in order to minimize wear of OH contact wire.

A 'diamond' type single-pan pantograph is shown in Fig. 43.5. It consists of a pentagonal framework of high-tensile alloy-steel tubing. The contact portion consists of a pressed steel pan fitted with renewable copper wearing strips which are forced against the OH contact wire by the upward action of pantograph springs. The pantograph can be raised or lowered from cabin by air cylinders.

#### 43.25. Conductor Rail Equipment

The conductor rails may be divided into three classes depending on the position of the contact surface which may be located at the top, bottom or side of the rail. The top contact rail is adopted universally for 600 V dc electrification. The side contact rail is used for 1200 V dc supply. The under contact rail has the advantage of being protected from snow, sleet and ice.

Fig. 43.6 shows the case when electric supply is collected from the top of an insulated conductor rail  $C$  (of special high-conductivity steel) running parallel to the track at a distance of 0.3 to 0.4 m from the running rail ( $R$ ) which forms the return path.  $L$  is the insulator and  $W$  is the wooden protection used at stations and crossings.

The current is collected from top surface of the rail by flat steel shoes (200 mm  $\times$  75 mm), the necessary contact pressure being obtained by gravity. Since it is not always possible to provide conductor rail on the same side of the track, shoes are provided on both sides of the locomotive or train. Moreover two shoes are provided on each side in order to avoid current interruption at points and crossings where there are gaps in the running rail.

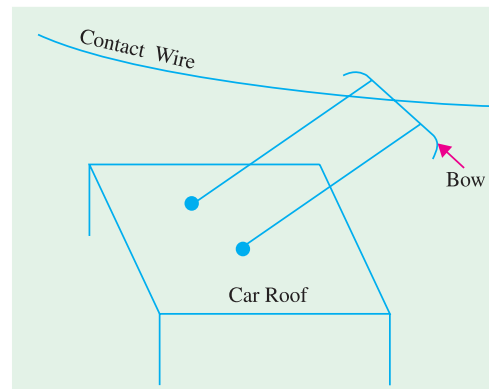


Fig. 43.4

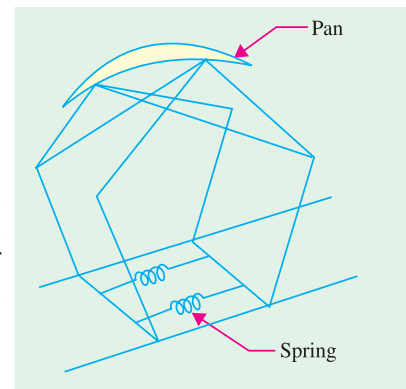


Fig. 43.5



The pantograph mechanism helps to maintain a link between the overhead contact wire and power circuit of the electric locomotive

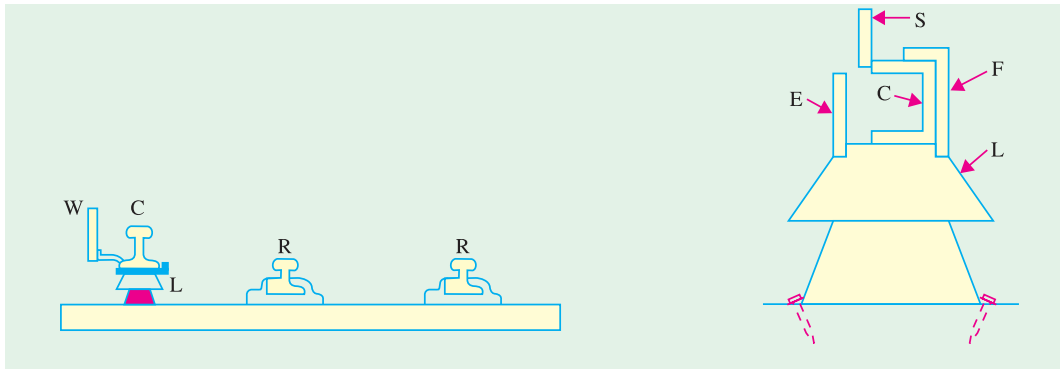


Fig. 43.6

Fig. 43.7

Fig. 43.7 shows the side contact rail and the method of the mounting. The conductor rail (C) rests upon a wooden block recessed into the top of the procelain insulator L. Current is collected by steel shoes (S) which are kept pressed on the contact rail by springs. E and F are the guards which rest upon ledges on the insulator.

### 43.26. Types of Railway Services

There are three types of passenger services offered by the railways :

**1. City or Urban Service.** In this case, there are frequent stops, the distance between stops being nearly 1 km or less. Hence, high acceleration and retardation are essential to achieve moderately high schedule speed between the stations.

**2. Suburban Service.** In this case, the distance between stops averages from 3 to 5 km over a distance of 25 to 30 km from the city terminus. Here, also, high rates of acceleration and retardation are necessary.

**3. Main Line Service.** It involves operation over long routes where stops are infrequent. Here, operating speed is high and accelerating and braking periods are relatively unimportant.

On goods traffic side also, there are three types of services (i) main-line freight service (ii) local or pick-up freight service and (iii) shunting service.

### 43.27. Train Movement

The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various *times* after the start of the run and the later gives speed at various *distances* from the starting point. Out of the two, speed/time curve is more important because

1. its slope gives acceleration or retardation as the case may be.
2. area between it and the horizontal (*i.e.* time) axis represents the distance travelled.
3. energy required for propulsion can be calculated if resistance to the motion of train is known.

### 43.28. Typical Speed/Time Curve

Typical speed/time curve for electric trains operating on passenger services is shown in Fig. 43.8. It may be divided into the following **five** parts :

#### 1. Constant Acceleration Period (0 to $t_1$ )

It is also called notching-up or starting period because during this period, starting resistance of the motors is gradually cut out so that the motor current (and hence, tractive effort) is maintained nearly constant which produces constant acceleration alternatively called 'rheostatic acceleration' or 'acceleration while notching'.

**2. Acceleration on Speed Curve ( $t_1$  to  $t_2$ )**

This acceleration commences after the starting resistance has been all cut-out at point  $t_1$  and full supply voltage has been applied to the motors. During this period, the motor current and torque decrease as train speed increases. Hence, acceleration gradually *decreases* till torque developed by motors exactly balances that due to resistance to the train motion. The shape of the portion *AB* of the speed/time curve depends primarily on the torque/speed characteristics of the traction motors.

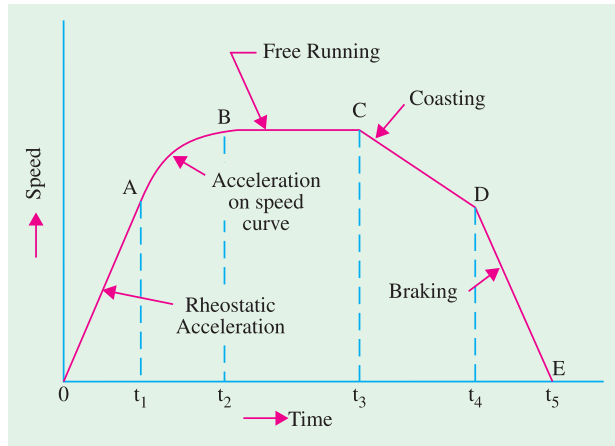


Fig. 43.8

**3. Free-running Period ( $t_2$  to  $t_3$ )**

The train continues to run at the speed reached at point  $t_2$ . It is represented by portion *BC* in Fig. 43.8 and is a constant-speed period which occurs on level tracks.

**4. Coasting ( $t_3$  to  $t_4$ )**

Power to the motors is cut off at point  $t_3$  so that the train runs under its momentum, the speed gradually falling due to friction, windage etc. (portion *CD*). During this period, retardation remains practically constant. Coasting is desirable because it utilizes some of the kinetic energy of the train which would, otherwise, be wasted during braking. Hence, it helps to reduce the energy consumption of the train.

**5. Braking ( $t_4$  to  $t_5$ )**

At point  $t_4$ , brakes are applied and the train is brought to rest at point  $t_5$ .

It may be noted that coasting and braking are governed by train resistance and allowable retardation respectively.

**43.29. Speed/Time Curves for Different Services**

Fig. 43.9 (a) is representative of city service where relative values of acceleration and retardation are high in order to achieve moderately high average speed between stops. Due to short

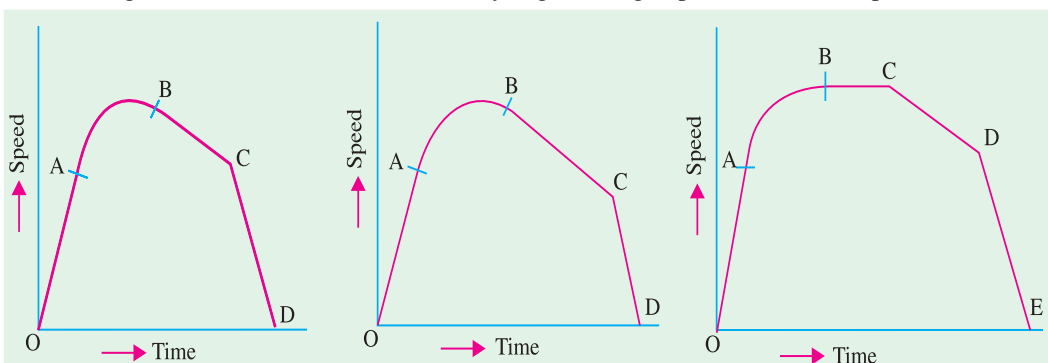


Fig. 43.9

distances between stops, there is no possibility of free-running period though a short coasting period is included to save on energy consumption.

In suburban services [Fig. 43.9 (b)], again there is no free-running period but there is comparatively *longer* coasting period because of longer distances between stops. In this case also, relatively

high values of acceleration and retardation are required in order to make the service as attractive as possible.

For main-line service [Fig. 43.9 (c)], there are long periods of free-running at high speeds. The accelerating and retardation periods are relatively unimportant.

### 43.30. Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 43.8 is replaced by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retardation, average speed and distance etc. The simple curve would be fairly accurate provided it *(i) retains the same acceleration and retardation and (ii) has the same area as the actual speed/time curve*. The simplified speed/time curve can have either of the two shapes :

(i) trapezoidal shape  $OA_1B_1C$  of Fig. 43.10 where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant-speed period.

(ii) quadrilateral shape  $OA_2B_2C$  where the same two periods are replaced by the extensions of initial constant acceleration and coasting periods.

It is found that trapezoidal diagram  $OA_1B_1C$  gives simpler relationships between the principal quantities involved in train movement and also gives closer approximation of actual energy consumed during *main-line service on level track*. On the other hand, quadrilateral diagram approximates more closely to the actual conditions in *city and suburban services*.

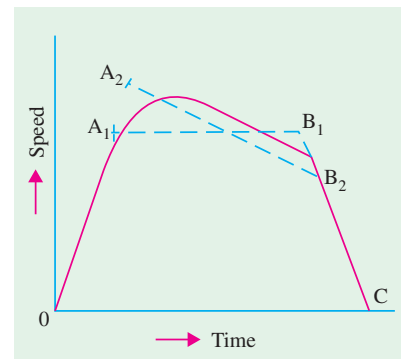


Fig. 43.10

### 43.31. Average and Schedule Speed

While considering train movement, the following three speeds are of importance :

1. **Crest Speed.** It is the maximum speed ( $V_m$ ) attained by a train during the run.

2. **Average Speed** =  $\frac{\text{distance between stops}}{\text{actual time of run}}$

In this case, only running time is considered but *not the stop time*.

3. **Schedule Speed** =  $\frac{\text{distance between stops}}{\text{actual time of run} + \text{stop time}}$

Obviously, schedule speed can be obtained from average speed by including the duration of stops. For a given distance between stations, higher values of acceleration and retardation will mean lesser running time and, consequently, higher schedule speed. Similarly, for a given distance between stations and for fixed values of acceleration and retardation, higher crest speed will result in higher schedule speed. For the same value of average speed, increase in duration of stops decreases the schedule speed.

### 43.32. SI Units in Traction Mechanics

In describing various quantities involved in the mechanics of train movement, only the latest SI system will be used. Since SI system is an 'absolute system', only absolute units will be used while gravitational units (used hitherto) will be discarded.

1. **Force.** It is measured in newton (N)
2. **Mass.** Its unit is kilogram (kg). Commonly used bigger units is tonne ( $t$ ), 1 tonne = 1000 kg

- 3. Energy.** Its basic unit is joule (J). Other units often employed are watt-hour (Wh) and kilowatt-hour (kWh).
- $$1 \text{ Wh} = 1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600 \text{ J} = 3.6 \text{ kJ}$$
- $$1 \text{ kWh} = 1000 \times 1 \frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J} = 3.6 \text{ MJ}$$
- 4. Work.** Its unit is the same as that of energy.
- 5. Power.** Its unit is watt (W) which equals 1 J/s. Other units are kilowatt (kW) and megawatt (MW).
- 6. Distance.** Its unit is metre. Other unit often used is kilometre (km).
- 7. Velocity.** Its absolute unit is metre per second (m/s). If velocity is given in km/h (or km.ph), it can be easily converted into the SI unit of m/s by multiplying it with a factor of  $(1000/3600) = 5/18 = 0.2778$ . For example,  $72 \text{ km.ph} = 72 \times 5/18 = 72 \times 0.2778 = 20 \text{ m/s}$ .
- 8. Acceleration.** Its unit is metre/second<sup>2</sup> (m/s<sup>2</sup>). If acceleration is given in km/h/s (or km-ph.ps), then it can be converted into m/s<sup>2</sup> by simply multiplying it by the factor  $(1000/3600) = 5/18 = 0.2778$  *i.e.* the same factor as for velocity. For example,  $1.8 \text{ km.ph.ps} = 1.8 \times 5/18 = 1.8 \times 0.2778 = 0.5 \text{ m/s}^2$

### 43.33. Confusion Regarding Weight and Mass of a Train

Many students often get confused regarding the correct meaning of the terms ‘weight’ and ‘mass’ and their units while solving numericals on train movement particularly when they are not expressed clearly and consistently in their absolute units. It is primarily due to the mixing up of absolute units with gravitational units. There would be no confusion at all if ***we are consistent in using only absolute units*** as required by the SI system of units which disallows the use of gravitational units.

Though this topic was briefly discussed earlier, it is worth repeating here.

- 1. Mass (M).** It is the quantity of matter contained in a body.

Its absolute unit is kilogram (kg). Other multiple in common use is tonne.

- 2. Weight (W).** It is the **force** with which earth pulls a body downwards.

The weight of a body can be expressed in (i) the **absolute** unit of newton (N) or (ii) the **gravitational** unit of kilogram-weight (kg. wt) which is often writing as ‘kgf’ in engineering literature.

Another still bigger **gravitational** unit commonly used in traction work is tonne-weight (t-wt)

$$1 \text{ t-wt} = 1000 \text{ kg-wt} = 1000 \times 9.8 \text{ N} = 9800 \text{ N}$$

- (i) Absolute Unit of Weight**

It is called newton (N) whose definition may be obtained from Newton’s Second Law of Motion.

Commonly used multiple is kilo-newton (kN). Obviously,  $1 \text{ kN} = 1000 \text{ N} = 10^3 \text{ N}$ .

For example, if a mass of 200 kg has to be given an acceleration of  $2.5 \text{ m/s}^2$ , force required is  $F = 200 \times 2.5 = 500 \text{ N}$ .

If a train of mass 500 tonne has to be given an acceleration of  $0.6 \text{ m/s}^2$ , force required is

$$F = ma = (500 \times 1000) \times 0.6 = 300,000 \text{ N} = 300 \text{ kN}$$

- (ii) Gravitational Unit of Weight**

It is ‘g’ times bigger than newton. It is called kilogram-weight (kg.wt.)

$$1 \text{ kg.wt} = g \text{ newton} = 9.81 \text{ N} \cong 9.8 \text{ N}$$

Unfortunately, the word ‘wt’ is usually omitted from kg-wt when expressing the weight of the body on the assumption that it can be understood or inferred from the language used.

Take the statement “a body has a *weight* of 100 kg”. It looks as if the weight of the body has been

expressed in terms of the mass unit 'kg'. To avoid this confusion, statement should be 'a body has a weight of 100 kg. wt.' But the first statement is justified by the writers on the ground that since the word 'weight' has already been used in the statement, it should be automatically understood by the readers that 'kg' is not the 'kg' of mass but is kg-wt. It would be mass kg if the statement is 'a body has a mass of 100 kg'. Often kg-wt is written as 'kgf' where 'f' is the first letter of the word force and is added to distinguish it from kg of mass.

Now, consider the statement "a body *weighing* 500 kg travels with a speed of 36 km/h....."

Now, weight of the body  $W = 500 \text{ kg.wt.} = 500 \times 9.8 \text{ N}$

Since we know the weight of the body, we can find its mass from the relation  $W = mg$ . But while using this equation, it is essential that we must **consistently use the absolute** units only. In this equation,  $W$  must be in newton (not in kg. wt),  $m$  in kg and  $g$  in  $\text{m/s}^2$ .

$$\therefore 500 \times 9.8 = m \times 9.8 ; \quad \therefore m = 500 \text{ kg}$$

It means that a body which *weighs* 500 kg (wt) has a *mass* of 500 kg.

As a practical rule, weight of a body in **gravitational** units is numerically equal to its mass in **absolute** units. This simple fact must be clearly understood to avoid any confusion between weight and mass of a body.

A train which weighs 500 tonne has a mass of 500 tonne as proved below :

$$\text{train weight, } W = 500 \text{ tonne-wt} = 500 \times 1000 \text{ kg-wt} = 500 \times 1000 \times 9.8 \text{ N}$$

$$\text{Now, } W = mg ; \quad \therefore 500 \times 1000 \times 9.8 = m \times 9.8$$

$$\therefore m = 500 \times 1000 \text{ kg} = 500 \times 1000/1000 = 500 \text{ tonne}$$

To avoid this unfortunate confusion, it would be helpful to change our terminology. For example, instead of saying "a train weighing 500 tonne is....." it is better to say "a 500-t train is ....." or "a train having a mass of 500 t is ....."

In order to remove this confusion, SI system of units has disallowed the use of gravitational units. There will be no confusion if **we consistently use only absolute units**.

### 43.34. Quantities Involved in Traction Mechanics

Following principal quantities are involved in train movement :

$D$ = distance between stops	$M$ = dead mass of the train
$M_e$ = effective mass of the train	$W$ = dead weight of the train
$W_e$ = effective weight of the train	$\alpha$ = acceleration during starting period
$\beta_c$ = retardation during coasting	$\beta$ = retardation during braking
$V_a$ = average speed	$V_m$ = maximum (or crest) speed.
$t$ = total time for the run	$t_1$ = time of acceleration
$t_2$ = time of free running = $t - (t_1 + t_3)$	$t_3$ = time of braking
$F_t$ = tractive effort	$T$ = torque

### 43.35. Relationship Between Principal Quantities in Trapezoidal Diagram

As seen from Fig. 43.11.

$$\alpha = V_m/t_1 \quad \text{or} \quad t_1 = V_m/\alpha$$

$$\beta = V_m/t_3 \quad \text{or} \quad t_3 = V_m/\beta$$

As we know, total distance  $D$  between the two stops is given by the area of trapezium  $OABC$ .

$$\begin{aligned} \therefore D &= \text{area } OABC \\ &= \text{area } OAD + \text{area } ABED + \text{area } BCE \\ &= \frac{1}{2} V_m t_1 + V_m t_2 + \frac{1}{2} V_m t_3 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} V_m t_1 + V_m [t - (t_1 + t_3)] + \frac{1}{2} V_m t_3 \\
 &= V_m \left[ \frac{t_1}{2} + t - t_1 - t_3 + \frac{t_3}{2} \right] \\
 &= V_m \left[ t - \frac{1}{2} (t_1 + t_3) \right] \\
 &= V_m \left[ t - \frac{V_m}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \right]
 \end{aligned}$$

Let,  $K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)$ . Substituting this value

of  $K$  in the above equation, we get

$$\begin{aligned}
 D &= V_m (t - KV_m) \\
 \text{or } KV_m^2 - V_m t + D &= 0 \quad \dots(i)
 \end{aligned}$$

$$\therefore V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

Rejecting the positive sign which gives impracticable value, we get

$$V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

From Eq. (i) above, we get

$$KV_m^2 = V_m t - D \quad \text{or} \quad K = \frac{t}{V_m} - \frac{D}{V_m^2} = \frac{D}{V_m^2} \left( V_m \cdot \frac{t}{D} - 1 \right)$$

$$\text{Now, } V_a = \frac{D}{t} \quad \therefore K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right)$$

Obviously, if  $V_m$ ,  $V_a$  and  $D$  are given, then value of  $K$  and hence of  $\alpha$  and  $\beta$  can be found (Ex. 43.2).

### 43.36. Relationship Between Principal Quantities in Quadrilateral Diagram

The diagram is shown in Fig. 43.12. Let  $\beta_c$  represent the retardation during coasting period. As before,

$$\begin{aligned}
 t_1 &= V_1/\alpha, \quad t_2 = (V_2 - V_1)/\beta_c \quad \text{and} \quad t_3 = V_2/\beta \\
 D &= \text{area } OABC \\
 &= \text{area } OAD + \text{area } ABED + \text{area } BCE \\
 &= \frac{1}{2} V_1 t_1 + t_2 \left( \frac{V_1 + V_2}{2} \right) + \frac{1}{2} V_2 t_3 \\
 &= \frac{1}{2} V_1 (t_1 + t_2) + \frac{1}{2} V_2 (t_2 + t_3) \\
 &= \frac{1}{2} V_1 (t - t_3) + \frac{1}{2} V_2 (t - t_1) \\
 &= \frac{1}{2} t (V_1 + V_2) - \frac{V_1 t_1}{2} - \frac{V_2 t_3}{2} \\
 &= \frac{1}{2} t (V_1 + V_2) - \frac{1}{2} V_1 V_2 \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \\
 &= \frac{1}{2} t (V_1 + V_2) - KV_1 V_2
 \end{aligned}$$

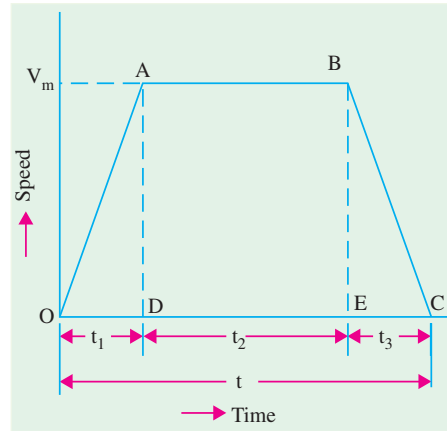


Fig. 43.11

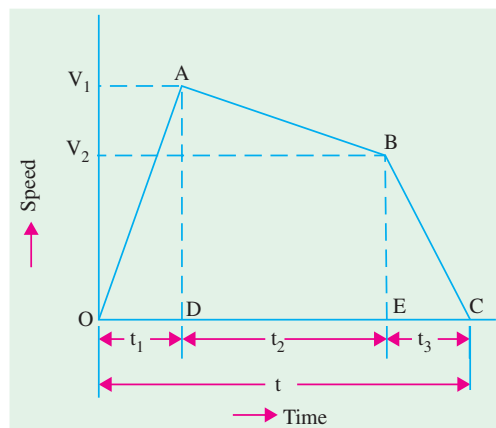


Fig. 43.12

where  $K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) = \frac{\alpha + \beta}{2\alpha\beta}$  Also,  $\beta_c = \frac{(V_1 - V_2)}{t_2}$

$$\therefore V_2 = V_1 - \beta_c t_2 = V_1 - \beta_c (t - t_1 - t_3)$$

$$= V_1 - \beta_c \left( t - \frac{V_1}{\alpha} - \frac{V_2}{\beta} \right) = V_1 \beta_c \left( t - \frac{V_1}{\alpha} \right) + \beta_c \frac{V_2}{\beta}$$

or  $V_2 \left( 1 - \frac{\beta_c}{\beta} \right) = V_1 - \beta_c \left( t - \frac{V_1}{\alpha} \right) \quad \therefore V_2 = \frac{V_1 - \beta_c (t - V_1/\alpha)}{(1 - \beta_c/\beta)}$

**Example 43.1.** A suburban train runs with an average speed of 36 km/h between two stations 2 km apart. Values of acceleration and retardation are 1.8 km/h/s and 3.6 km/h/s.

Compute the maximum speed of the train assuming trapezoidal speed/time curve.

(Electric Traction, Punjab Univ. 1994)

**Solution.** Now,  $V_a = 36 \text{ km/h} = 36 \times 5/18 = 10 \text{ m/s}$   
 $\alpha = 1.8 \text{ km/h/s} = 1.8 \times 5/18 = 0.5 \text{ m/s}^2$ ,  $\beta = 3.6 \text{ km/h/s} = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$   
 $t = D/V_a = 2000/10 = 200 \text{ s}$ ;  $K = (\alpha + \beta)/2\alpha\beta = (0.5 + 1.0)/2 \times 0.5 \times 1 = 1.5$

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{200 - \sqrt{200^2 - 4 \times 1.5 \times 2000}}{2 \times 1.5}$$

$$= 11 \text{ m/s} = 11 \times 18/5$$

$$= \mathbf{39.6 \text{ km/h}}$$

**Example 43.2.** A train is required to run between two stations 1.5 km apart at a schedule speed of 36 km/h, the duration of stops being 25 seconds. The braking retardation is 3 km/h/s. Assuming a trapezoidal speed/time curve, calculate the acceleration if the ratio of maximum speed to average speed is to be 1.25

(Elect. Power, Bombay Univ. 1980)

**Solution.** Here,  $D = 1500 \text{ m}$  ;  
 schedule speed = 36 km/h = 36 × 5/18 = 10 m/s

$$\beta = 3 \text{ km/h/s} = 3 \times 5/18 = 5/6 \text{ m/s}^2$$

Schedule time of run = 1500/10 = 150 s ; Actual time of run = 150 - 25 = 125 s

$$\therefore V_a = 1500/125 = 12 \text{ m/s} ; V_m = 1.25 \times 12 = 15 \text{ m/s}$$

Now,  $K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right) = \frac{1500}{15^2} (1.25 - 1) = \frac{5}{3}$

Also,  $K = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)$  or  $\frac{5}{3} = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{6}{5} \right)$

$$\therefore \alpha = 0.47 \text{ m/s}^2 = 0.47 \times 18/5 = \mathbf{1.7 \text{ km/h/s}}$$

**Example 43.3.** Find the schedule speed of an electric train for a run of 1.5 km if the ratio of its maximum to average speed is 1.25. It has a braking retardation of 3.6 km/h/s, acceleration of 1.8 km/h/s and stop time of 21 second. Assume trapezoidal speed/time curve.



Electric traction provides high starting torque and low maintenance costs making it the best choice for trains



**Solution.**  $\alpha = 1.8 \times 5/18 = 0.5 \text{ m/s}^2$ ,  $\beta = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$   
 $D = 1.5 \text{ km} = 1500 \text{ m}$   
 $K = \frac{1}{2} \left( \frac{1}{0.5} + \frac{1}{1} \right) = \frac{3}{2}$       Now,  $K = \frac{D}{V_m^2} \left( \frac{V_m}{V_a} - 1 \right)$   
 or  $V_m^2 = \frac{D}{K} \left( \frac{V_m}{V_a} - 1 \right)$        $\therefore V_m^2 = \frac{1500}{3/2} (1.25 - 1) = 250$ ;  $V_m = 15.8 \text{ m/s}$   
 $V_a = V_m / 1.25 = 15.8/1.25 = 12.6 \text{ m/s}$   
 Actual time of run =  $1500/12.6 = 119 \text{ seconds}$   
 Schedule time =  $119 + 21 = 140 \text{ second}$   
 $\therefore$  Schedule speed =  $1500/140 = 10.7 \text{ m/s} = 38.5 \text{ km/h}$

**Example 43.4.** A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.

Assume a simplified speed/time curve.

**Solution.** Given :  $D = 1.6 \text{ km} = 1600 \text{ m}$ ,  $V_a = 36 \text{ km/h} = 10 \text{ m/s}$   
 $V_1 = 72 \text{ km/h} = 20 \text{ m/s}$ ;  $\alpha = 2.7 \text{ km/h/s} = 0.75 \text{ m/s}^2$   
 $\beta_c = 0.18 \text{ km/h/s} = 0.05 \text{ m/s}^2$ ;  $\beta = 3.6 \text{ km/h/s} = 1.0 \text{ m/s}^2$

With reference to Fig. 43.12, we have

Duration of acceleration,  $t_1 = V_1/\alpha = 20/0.75 = 27 \text{ s}$   
 Actual time of run,  $t = 1600/10 = 160 \text{ s}$   
 Duration of braking,  $t_3 = V_2/1.0 = V_2 \text{ second}$   
 Duration of coasting,  $t_2 = (V_1 - V_2)/\beta_c = (20 - V_2)/0.05 = (400 - 20 V_2) \text{ second}$   
 Now,  $t = t_1 + t_2 + t_3$  or  $160 = 27 + (400 - 20 V_2) + V_2$        $\therefore V_2 = 14 \text{ m/s}$   
 $\therefore t_2 = (20 - 14)/0.05 = 120 \text{ s}$ ;  $t_3 = 14/1.0 = 14 \text{ s}$

### 43.37. Tractive Effort for Propulsion of a Train

The tractive effort ( $F_t$ ) is the force developed by the traction unit at the rim of the driving wheels for moving the unit itself and its train (trailing load). The tractive effort required for train propulsion on a level track is

$$F_t = F_a + F_r$$

If gradients are involved, the above expression becomes

$$F_t = F_a + F_g + F_r \quad \text{— for ascending gradient}$$

$$F_t = F_a - F_g + F_r \quad \text{— for descending gradient}$$

where

$F_a$  = force required for giving linear acceleration to the train  
 $F_g$  = force required to overcome the effect of gravity  
 $F_r$  = force required to overcome resistance to train motion.

#### (a) Value of $F_a$

If  $M$  is the dead (or stationary) mass of the train and  $a$  its linear acceleration, then

$$F_a = Ma$$

Since a train has rotating parts like wheels, axles, motor armatures and gearing etc., its effective (or accelerating) mass  $M_e$  is more (about 8 – 15%) than its stationary mass. These parts have to be given angular acceleration at the same time as the whole train is accelerated in the linear direction. Hence,  $F_e = M_e a$

- (i) If  $M_e$  is in kg and  $\alpha$  in  $m/s^2$ , then  $F_a = M_e a$  newton
- (ii) If  $M_e$  is in tonne and  $\alpha$  in  $km/h/s$ , then converting them into absolute units, we have

$$F_a = (1000 M_e) \times (1000/3600) a = 277.8 M_e a \text{ newton}$$

**(b) Value of  $F_g$**

As seen from Fig. 43.13,  $F_g = W \sin \theta = Mg \sin \theta$

In railway practice, gradient is expressed as the rise (in metres) a track distance of 100 m and is called percentage gradient.

$$\therefore \% G = \frac{BC}{AC/100} = 100 \frac{BC}{AC} = 100 \sin \theta$$

Substituting the value of  $\sin \theta$  in the above equation, we get

$$F_g = Mg G/100 = 9.8 \times 10^{-2} MG$$

- (i) When  $M$  is in kg,  $F_g = 9.8 \times 10^{-2} MG$  newton
- (ii) When  $M$  is given in tonne, then

$$F_g = 9.8 \times 10^{-2} (1000 M) G = 98 MG \text{ newton}$$

**(c) Value of  $F_r$**

Train resistance comprises all those forces which oppose its motion. It consists of mechanical resistance and wind resistance. Mechanical resistance itself is made up of internal and external resistances. The internal resistance comprises friction at journals, axles, guides and buffers etc. The external resistance consists of friction between wheels and rails and flange friction etc. Mechanical resistance is almost independent of train speed but depends on its weight. The wind friction varies directly as the square of the train speed.

If  $r$  is specific resistance of the train *i.e.* resistance offered per unit mass of the train, then  $F_r = M.r$ .

- (i) If  $r$  is in newton per kg of train mass and  $M$  is the train mass in kg, then

$$F_r = M.r \text{ newton}$$

- (ii) If  $r$  is in newton per tonne train mass (N/t) and  $M$  is in tonne ( $t$ ), then

$$F_r = M \text{ tonne} \times r = M_r \text{ newton}^*$$

Hence, expression for total tractive effort becomes

$$F_t = F_a \pm F_g + F_r = (277.8 \alpha M_e \pm 98 MG + Mr) \text{ newton}$$

Please remember that here  $M$  is in tonne,  $\alpha$  in  $km/h/s$ ,  $G$  is in metres per 100 m of track length (*i.e.* %  $G$ ) and  $r$  is in newton/tonne (N/t) of train mass.

The positive sign for  $F_g$  is taken when motion is along an ascending gradient and negative sign when motion is along a descending gradient.

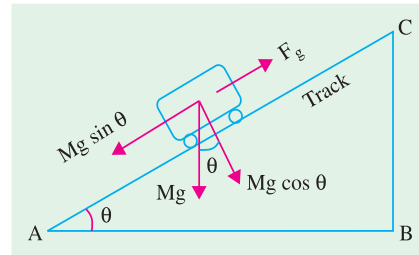


Fig. 43.13

**43.38. Power Output from Driving Axles**

If  $F_t$  is the tractive effort and  $v$  is the train velocity, then

$$\text{output power} = F_t \times v$$

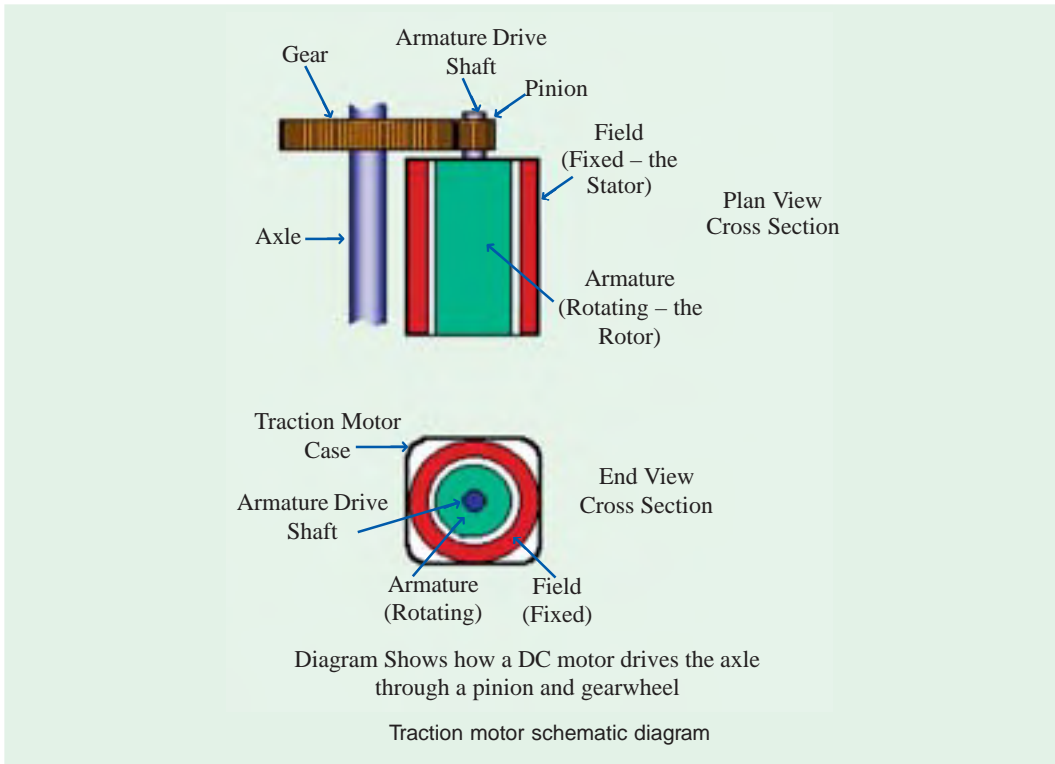
- (i) If  $F_t$  is in newton and  $v$  in  $m/s$ , then

$$\text{output power} = F_t \times v \text{ watt}$$

- (ii) If  $F_t$  is in newton and  $v$  is in  $km/h$ , then converting  $v$  into  $m/s$ , we have

---

\* If  $r$  is in kg (wt) per tonne train mass and  $M$  is in tonne, then  $F_r = M \text{ tonne} \times (r \times 9.8) \text{ newton/tonne} = 9.8 Mr \text{ newton}$ .



$$\text{output power} = F_t \times \left(\frac{1000}{3600}\right) v \text{ watt} = \frac{F_t v}{3600} \text{ kW}$$

If  $\eta$  is the efficiency of transmission gear, then power output of motors is

$$= F_t \cdot v / \eta \text{ watt} \quad \text{--- } v \text{ in m/s}$$

$$= \frac{F_t v}{3600 \eta} \text{ kW} \quad \text{--- } v \text{ in km/h}$$

### 43.39. Energy Output from Driving Axles

Energy (like work) is given by the product of power and time.

$$E = (F_t \times v) \times t = F_t \times (v \times t) = F_t \times D$$

where  $D$  is the distance travelled in the direction of tractive effort.

Total energy output from driving axles for the run is

$$E = \text{energy during acceleration} + \text{energy during free run}$$

As seen from Fig. 43.11

$$E = F_t \times \text{area } OAD + F_t' \times \text{area } ABED = F_t \times \frac{1}{2} V_m t_1 + F_t' \times \frac{1}{2} V_m t_2$$

where  $F_t$  is the tractive effort during accelerating period and  $F_t'$  that during free-running period. Incidentally,  $F_t$  will consist of all the three components given in Art. 43.37 whereas  $F_t'$  will consist of  $(98 MG + Mr)$  provided there is an ascending gradient.

### 43.40. Specific Energy Output

It is the energy output of the driving wheel expressed in watt-hour (Wh) per tonne-km ( $t$ -km) of

the train. It can be found by first converting the energy output into Wh and then dividing it by the mass of the train in tonne and route distance in km.

Hence, unit of specific energy output generally used in railway work is : Wh/tonne-km (Wh/t-km).

### 43.41. Evaluation of Specific Energy Output

We will first calculate the total energy output of the driving axles and then divide it by train mass in tonne and route length in km to find the specific energy output. It will be presumed that :

- (i) there is a gradient of G throughout the run and
- (ii) power remains ON upto the end of free run in the case of trapezoidal curve (Fig. 43.11) and upto the accelerating period in the case of quadrilateral curve (Fig. 43.12).

Now, output of the driving axles is used for the following purposes :

1. for accelerating the train
2. for overcoming the gradient
3. for overcoming train resistance.

#### (a) Energy required for train acceleration ( $E_a$ )

As seen from trapezoidal diagram of Fig. 43.11,

$$\begin{aligned}
 E_a &= F_a \times \text{distance } OAD = 277.8 \alpha M_e \times \frac{1}{2} V_m t_1 \text{ joules} \\
 &= 277.8 \alpha M_e \times \frac{1}{2} V_m \times \frac{V_m}{\alpha} \text{ joules} \quad \left( \text{as } t_1 = \frac{V_m}{\alpha} \right) \\
 &= 277.8 \alpha M_e \times \left[ \frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha} \right] \text{ joules}
 \end{aligned}$$

It will be seen that since  $V_m$  is in km/h, it has been converted into m/s by multiplying it with the conversion factor of (1000/3600). In the case of ( $V_m/t$ ), conversion factors for  $V_m$  and  $\alpha$  being the same, they cancel out. Since 1 Wh = 3600 J.

$$\therefore E_a = 277.8 \alpha M_e \left[ \frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha} \right] \text{ Wh} = 0.01072 \frac{V_m^2}{M_e} \text{ Wh}$$

#### (b) Energy required for over coming gradient ( $E_g$ )

$$E_g = F_g \times D'$$

where 'D'' is the **total distance over which power remains ON**. Its maximum value equals the distance represented by the area *OABE* in Fig. 43.11 *i.e.* from the start to the end of free-running period in the case of trapezoidal curve [as per assumption (i) above].

Substituting the value of  $F_g$  from Art. 43.37, we get

$$E_g = 98 MG. (1000 D') \text{ joules} = 98,000 MGD' \text{ joules}$$

It has been assumed that  $D'$  is in km.

When expressed in Wh, it becomes

$$E_g = 98,000 MGD' \frac{1}{3600} \text{ Wh} = 27.25 MGD' \text{ Wh}$$

#### (c) Energy required for overcoming resistance ( $E_r$ )

$$\begin{aligned}
 E_r &= F_r \times D' = M \cdot r \times (1000 D') \text{ joules} && \text{--- } D' \text{ in km} \\
 &= \frac{1000 Mr D'}{3600} \text{ Wh} = 0.2778 Mr D' \text{ Wh} && \text{--- } D' \text{ in km}
 \end{aligned}$$

$\therefore$  total energy output of the driving axles is

$$E = E_a + E_g + E_r$$

$$= (0.01072 V_m^2 / M_e + 27.25 MGD' + 0.2778 Mr D' \text{ Wh})$$

Specific energy output

$$E_{spo} = \frac{E}{M \times D} \quad \text{--- } D \text{ is the total run length}$$

$$= \left( 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 27.25 G \frac{D'}{D} + 0.2778 r \frac{D'}{D} \right) \text{ Wh/t-km}$$

It may be noted that if there is no gradient, then

$$E_{spo} = \left( 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778 r \frac{D'}{D} \right) \text{ Wh/t-km}$$

**Alternative Method**

As before, we will consider the trapezoidal speed/time curve. Now, we will calculate energy output not **force-wise** but **period-wise**.

**(i) Energy output during accelerating period**

$$E_a = F_t \times \text{distance travelled during accelerating period}$$

$$= F_t \times \text{area } OAD \quad \text{---Fig. 43.11}$$

$$= F_t \times \frac{1}{2} V_m t_1 = \frac{1}{2} F_t \cdot V_m \cdot \frac{V_m}{\alpha}$$

$$= \frac{1}{2} \cdot F_t \left( \frac{1000}{3600} \cdot V_m \right) \cdot \frac{V_m}{\alpha} \text{ joules}$$

$$= \frac{1}{2} \cdot F_t \left( \frac{1000}{3600} \cdot V_m \right) \cdot \frac{V_m}{\alpha} \cdot \frac{1}{3600} \text{ Wh}$$

Substituting the value of  $F_t$ , we get

$$E_a = \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2 \alpha} (277.8 \alpha M_e + 98 MG + Mr) \text{ Wh}$$

It must be remembered that during this period, **all the three forces are at work** (Art. 43.37)

**(ii) Energy output during free-running period**

Here, work is required only against two forces *i.e.* gravity and resistance (as mentioned earlier).

$$\text{Energy } E_{fr} = F'_t \times \text{area } ABED \quad \text{---Fig. 43.11}$$

$$= F'_t \times (V_m \times t_2) = F'_t \times \left( \frac{1000}{3600} V_m \right) \cdot t_2 \text{ joules}$$

$$= F'_t \times \left( \frac{1000}{3600} V_m \right) \times t_2 \times \frac{1}{3600} \text{ Wh} = \left( \frac{1000}{3600} \right) F'_t \times V_m t_2 \cdot \frac{1}{3600} \text{ Wh}$$

$$= \left( \frac{1000}{3600} \right) \cdot F'_t \times D_{fr} \text{ Wh} = \left( \frac{1000}{3600} \right) (98 MG + Mr) D_{fr} \text{ Wh}$$

where  $D_{fr}$  is the distance in km travelled during the free-running period\*

Total energy required is the sum of the above two energies.

$$\therefore E = E_a + E_{fr}$$

$$= \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2 \alpha} (277.8 \alpha \cdot M_e + 98 MG + Mr) + \frac{1000}{3600} (98 MG + Mr) D_{fr} \text{ Wh}$$

---

\*  $D_{fr} = \text{velocity in km/h} \times \text{time in hours}$   
 $= V_m \times (t_2 / 3600)$  because times are always taken in seconds.

$$= \frac{1000}{(3600)^2} \frac{V_m^2}{2\alpha} 277.8 \alpha M_e + \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (98 MG + Mr) + \frac{1000}{3600} (98 MG + Mr) \cdot D_{fr} \text{ Wh}$$

$$= 0.01072 V_m^2 \cdot M_e + \frac{1000}{3600} (98 MG + Mr) \left( \frac{V_m^2}{2\alpha \times 3600} + D_{fr} \right) \text{ Wh}$$

Now, 
$$\frac{V_m^2}{2\alpha \times 3600} = \frac{1}{2} \left( \frac{V_m}{3600} \right) \cdot \frac{V_m}{\alpha} = \frac{1}{2} \left( \frac{V_m}{3600} \right) \cdot t_1$$

= distance travelled during accelerating period *i.e.*  $D_a$

$$\therefore E = 0.01072 V_m^2 \cdot M_e + \frac{1000}{3600} (98 MG + Mr) (D_a + D_{fr}) \text{ Wh}$$

$$= 0.01072 V_m^2 \cdot M_e + (27.25 MG + 0.2778 Mr) D' \text{ Wh}$$

It is the same expression as found above.

### 43.42. Energy Consumption

It equals the total energy input to the traction motors from the supply. It is usually expressed in Wh which equals 3600 J. It can be found by dividing the energy output of the driving wheels with the combined efficiency of transmission gear and motor.

$$\therefore \text{energy consumption} = \frac{\text{output of driving axles}}{\eta_{motor} \times \eta_{gear}}$$

### 43.43. Specific Energy Consumption

It is the energy consumed (in Wh) per tonne mass of the train per km length of the run. Specific energy consumption,

$$E_{spc} = \frac{\text{total energy consumed in Wh}}{\text{train mass in tonne} \times \text{run length in km}} = \frac{\text{specific energy output}}{\eta}$$

where  $\eta$  = overall efficiency of transmission gear and motor =  $\eta_{gear} \times \eta_{motor}$

As seen from Art. 43.41, specific energy consumption is

$$E_{spc} = \left( 0.01072 \cdot \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km}$$

If no gradient is involved, then specific energy consumption is

$$E_{spc} = \left( 0.01072 \cdot \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km}$$

The specific energy consumption of a train running at a given schedule speed is influenced by

**1.** Distance between stops **2.** Acceleration **3.** Retardation **4.** Maximum speed **5.** Type of train and equipment **6.** Track configuration.

### 43.44. Adhesive Weight

It is given by the total weight carried on the driving wheels. Its value is  $W_a = x W$ , where  $W$  is dead weight and  $x$  is a fraction varying from 0.6 to 0.8.

### 43.45. Coefficient of Adhesion

Adhesion between two bodies is due to interlocking of the irregularities of their surfaces in contact. The adhesive weight of a train is *equal to the total weight to be carried on the driving*

**wheels.** It is less than the dead weight by about 20 to 40%.

If  $x = \frac{\text{adhesive weight, } W_a}{\text{dead weight } W}$ , then,  $W_a = x W$

Let,  $F_t = \text{tractive effort to slip the wheels}$   
 or  
 $= \text{maximum tractive effort possible without wheel slip}$

Coefficient of adhesion,  $\mu_a = F_t/W_a$

$\therefore F_t = \mu_a W_a = \mu_a x W = \mu_a x M g$

If  $M$  is in tonne, then

$$F_t = 1000 \times 9.8 \times \mu_a M = 9800 \mu_a x M \text{ newton}$$

It has been found that tractive effort can be increased by increasing the motor torque but only upto a certain point. Beyond this point, any increase in motor torque does not increase the tractive effort but merely causes the driving wheels to slip. It is seen from the above relation that for increasing  $F_t$ , it is not enough to increase the kW rating of the traction motors alone but the weight on the driving wheels has also to be increased.

Adhesion also plays an important role in braking. If braking effort exceeds the adhesive weight of the vehicle, skidding takes place.

### 43.46. Mechanism of Train Movement

The essentials of driving mechanism in an electric vehicle are illustrated in Fig. 43.14. The armature of the driving motor has a pinion which meshes with the gear wheel keyed to the axle of the driving wheel. In this way, motor torque is transferred to the wheel through the gear.

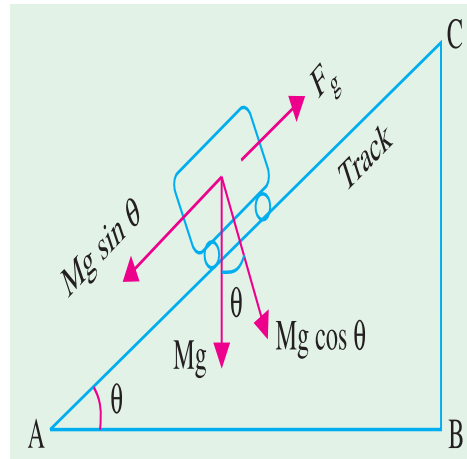


Fig. 43.14

Let,  $T = \text{torque exerted by the motor}$

$F_1 = \text{tractive effort at the pinion}$

$F_t = \text{tractive effort at the wheel}$

$\gamma = \text{gear ratio}$

Here,  $d_1, d_2 = \text{diameters of the pinion and gear wheel respectively}$

$D = \text{diameter of the driving wheel}$

$\eta = \text{efficiency of power transmission from the motor to driving axle}$

Now,  $T = F_1 \times d_1/2$  or  $F_1 = 2T/d_1$

Tractive effort transferred to the driving wheel is

$$F_t = \eta F_1 \left( \frac{d_2}{D} \right) = \eta \cdot \frac{2T}{d_1} \left( \frac{d_2}{D} \right) = \eta T \left( \frac{2}{D} \right) \left( \frac{d_2}{d_1} \right) = 2 \gamma \eta \frac{T}{D}$$

For obtaining motion of the train without slipping,  $F_t \leq \mu_a W_a$  where  $\mu_a$  is the coefficient of adhesion (Art. 43.45) and  $W_a$  is the adhesive weight.

**Example 43.5.** The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.

**Solution.** Maximum number of revolutions per second made by armature

$$= \frac{\text{armature velocity}}{\text{armature circumference}} = \frac{44}{0.42 \pi} = \frac{100}{3} \text{ rps.}$$

Maximum number of revolutions per second made by the driving wheel

$$= \frac{100}{3} \times \frac{18}{75} = 8 \text{ rps.}$$

Maximum distance travelled by the driving wheel in one second

$$= 8 \times 0.91 \pi \text{ m/s} = 22.88 \text{ m/s}$$

Hence, limiting value of train speed

$$= 22.88 \text{ m/s} = 22.88 \times 18/5 = \mathbf{82 \text{ km/h}}$$

**Example 43.6.** A 250-tonne motor coach driven by four motors takes 20 seconds to attain a speed of 42 km/h, starting from rest on an ascending gradient of 1 in 80. The gear ratio is 3.5, gear efficiency 92%, wheel diameter 92 cm train resistance 40 N/t and rotational inertia 10 percent of the dead weight. Find the torque developed by each motor.

**Solution.**  $F_t = (277.8 \times M_e a + 98 \text{ MG} + Mr)$  newton

Now,  $\alpha = V_m / t_1 = 42/20 = 2.1 \text{ km/h/s}$  Since gradient is 1 in 80, it becomes 1.25 in 100. Hence, percentage gradient  $G = 1.25$ . Also,  $M_e = 1.1 M$ . The tractive effort at the driving wheel is

$$\begin{aligned} F_t &= 277.8 \times (1.1 \times 250) \times 2.1 + 98 \times 250 \times 1.25 + 250 \times 40 \\ &= 160,430 + 30,625 + 10,000 = 201,055 \text{ N} \end{aligned}$$

Now,  $F_t = 2\gamma\eta T/D$  or  $201,055 = 2 \times 3.5 \times 0.92 \times T/0.92 \therefore T = 28,744 \text{ N-m}$

Torque developed by each motor =  $28,744/4 = \mathbf{7,186 \text{ N-m}}$

**Example 43.7.** A 250-tonne motor coach having 4 motors, each developing a torque of 8000 N-m during acceleration, starts from rest. If up-gradient is 30 in 1000, gear ratio 3.5, gear transmission efficiency 90%, wheel diameter 90 cm, train resistance 50 N/t, rotational inertia effect 10%, compute the time taken by the coach to attain a speed of 80 km/h.

If supply voltage is 3000 V and motor efficiency 85%, calculate the current taken during the acceleration period.

**Solution.** Tractive effort (Art. 43.46) at the wheel

$$= 2\gamma\eta T/D = 2 \times 3.5 \times 0.9 \times (8000 \times 4)/0.9 = 224,000 \text{ N}$$

Also,  $F_t = (277.8 a M_e + 98 \text{ MG} + Mr)$  newton

$$\begin{aligned} &= (277.8 \times (1.1 \times 250) \times a + 98 \times 250 \times 3 + 250 \times 50 \text{ N}) \\ &= (76,395 a + 86,000) \text{ N} \end{aligned}$$

Equating the two expression for tractive effort, we get

$$224,000 = 76,395 a + 86,000 ; a = 1.8 \text{ km/h/s}$$

Time taken to achieve a speed of 80 km/h is

$$t_1 = V_m / a = 80/1.8 = \mathbf{44.4 \text{ second}}$$

Power taken by motors (Art. 41.36) is

$$\begin{aligned} &= \frac{F_t \times v}{\eta} = \frac{F_t \times V_m}{\eta} = F_t \cdot \left(\frac{1000}{3600}\right) \cdot \frac{V_m}{\eta} \text{ watt} \\ &= 22,000 \times 0.2778 \times 80/0.85 = 58.56 \times 10^5 \text{ W} \end{aligned}$$

$$\text{Total current drawn} = 58.56 \times 10^5 / 3000 = 1952 \text{ A}$$

$$\text{Current drawn/motor} = 1952/4 = \mathbf{488 \text{ A.}}$$



**Example 43.8.** A goods train weighing 500 tonne is to be hauled by a locomotive up an ascending gradient of 2% with an acceleration of 1 km/h/s. If coefficient of adhesion is 0.25, train resistance 40 N/t and effect of rotational inertia 10%, find the weight of locomotive and number of axles if load is not to increase beyond 21 tonne/axle.

**Solution.** It should be clearly understood that a train weighing 500 tonne has a mass of 500 (Art. 43.33).

Tractive effort required is

$$F_t = (277.8 a M_e + 98 MG + Mr) \text{ newton} = M \left( 277.8 a \cdot \frac{M_e}{M} + 98 G + r \right) \text{ newton}$$

$$= M (277.8 \times 1 \times 1.1 + 98 \times 2 + 40) = 541.6 M \text{ newton}$$

If  $M_L$  is the mass of the locomotive, then

$$F_t = 541.6 (M + M_L) = 541.6 (500 + M_L) \text{ newton}$$

Maximum tractive effort (Art. 43.45) is given by

$$F_t = 1000 \mu_a M_L \cdot g = 1000 \times 0.25 M_L \times 9.8 \quad \text{--- } x = 1$$

$$\therefore 541.6 (500 + M_L) = 1000 \times 0.25 M_L \times 9.8 \quad \therefore M_L = \mathbf{142 \text{ tonne}}$$

Hence, weight of the locomotive is 142 tonne. Since, weight per axle is not to exceed 21 tonne, the number of axles required is = 142/21 = 7.

**Example 43.9.** An electric locomotive weighing 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an up-gradient of 0.1%. Train resistance is 45 N/t and rotational inertia is 10%. If this locomotive is helped by another locomotive of weight 120 tonne, find :

- (i) the trailing weight that can now be hauled up the same gradient under the same conditions.
- (ii) the maximum gradient, if the trailing hauled load remains unchanged.

Assume adhesive weight expressed as percentage of total dead weight as 0.8 for both locomotives. **(Utilization of Elect. Power ; AMIE, Summer)**

**Solution.** Dead weight of the train and locomotive combined = (100 + 500) = 600 tonne. Same is the value of the dead mass.

$$\therefore F_t = (277.8a M_e + 98 MG + Mr) \text{ newton}$$

$$= 277.8 \times 1 \times (1.1 \times 600) + 98 \times 600 \times 0.1 + 600 \times 45 = 216,228 \text{ N}$$

Maximum tractive effort (Art. 43.45) of the first locomotive

$$= 9800x \mu_a M_L = 9800 \times 0.8 \times \mu_a \times 1000 = 784,000 \mu_a$$

$$\therefore 784,000 \mu_a = 216,288 ; \quad \mu_a = 0.276$$

With two locomotive,  $M_L' = (100 + 120) = 220$  tonne

$$\therefore F_t = 9800 x \mu_a M_L' = 9800 \times 0.8 \times 0.276 \times 220 = 476,045 \text{ N}$$

(i) Let trailing load which the two combined locomotives can haul be  $M$  tonne. In that case, total dead mass becomes  $M = (100 + 120 + M) = (220 + M)$  tonne. Tractive effort required is

$$= (277.8 M_e' + 98 M'G + M'r) \text{ newton}$$

$$= M' (277.8 \times 1 \times 1.1 + 98 \times 0.1 + 45) = 360.4 M' \text{ newton}$$

$$\therefore 360.4 M' = 476,045; M' = 1321 \text{ tonne} \quad \therefore \text{trailing load, } M = 1321 - 220 = \mathbf{1101 \text{ tonne}}$$

(ii) Total hauled load = 500 + 100 + 120 = 720 tonne

Let  $G$  be the value of maximum percentage gradient. Then

$$F_t = (277.8 a M_e + 98MG + Mr) \text{ newton} = M \left( 277.8 a \frac{M_e}{M} + 98G + r \right) \text{ newton}$$

$$= 720 (277.8 \times 1 \times 1.1 + 98G + 45) \text{ newton} = (252,418 + 70,560 G) \text{ newton}$$

Equating it with the combined tractive effort of the two locomotive as calculated above, we have,

$$476,045 = 252,418 + 70,560 G \quad \therefore G = \mathbf{3.17 \text{ percent}}$$

**Example 43.10.** The average distance between stops on a level section of a railway is 1.25 km. Motor-coach train weighing 200 tonne has a schedule speed of 30 km/h, the duration of stops being 30 seconds. The acceleration is 1.9 km/h/s and the braking retardation is 3.2 km/h/s. Train resistance to traction is 45 N/t. Allowance for rotational inertia is 10%. Calculate the specific energy output in Wh/t-km. Assume a trapezoidal speed/time curve. **(Elect. Power, Bombay Univ.)**

**Solution.**  $\alpha = 1.9 \times 5/18 = 9.5/18 \text{ m/s}^2$ ;  $\beta = 3.2 \times 5/18 = 8/9 \text{ m/s}^2$

$$K = (\alpha + \beta)/2\alpha\beta = 1.5; \quad D = 1.25 \text{ km} = 1250 \text{ m}$$

$$\text{Schedule time} = 1.25 \times 3600/30 = 150 \text{ s. Running time} = 150 - 30 = 120 \text{ s}$$

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{120 - \sqrt{120^2 - 4 \times 1.5 \times 1250}}{2 \times 1.5} = 10.4 \text{ m/s} = \mathbf{37.4 \text{ km/h}}$$

$$\text{Braking distance } D = V_m^2/2\beta = 10.42^2/2 \times (8/9) = 0.06 \text{ km}$$

$$\therefore D = D - \text{braking distance} = 1.25 - 0.06 = 1.19 \text{ km}$$

$$\begin{aligned} \text{Specific energy output} &= 0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778 r \frac{D'}{D} \\ &= 0.01072 \times \frac{37.4^2}{1.25} \times 1.1 + 0.2778 \times 50 \times \frac{1.19}{1.25} \text{ Wh/t-km} \\ &= 16.5 + 13.2 = \mathbf{29.7 \text{ Wh/t-km}} \end{aligned}$$

**Example 43.11.** A 300-tonne EMU is started with a uniform acceleration and reaches a speed of 40 km/h in 24 seconds on a level track. Assuming trapezoidal speed/time curve, find specific energy consumption if rotational inertia is 8%, retardation is 3 km/h/s, distance between stops is 3 km, motor efficiency is 0.9 and train resistance is 40 N/tonne.

**(Elect. Traction, AMIE Summer)**

**Solution.** First of all, let us find  $D'$  – the distance upto which energy is consumed from the supply. It is the distance travelled upto the end of free-running period. It is equal to the total distance minus the distance travelled during braking.

$$\text{Braking time, } t_2 = V_m/\beta = 40/3 = 13.33 \text{ second}$$

Distance travelled during braking period

$$= \frac{1}{2} V_m t_2 = \frac{1}{2} \times 40 \times \left(\frac{13.33}{3600}\right) = 0.074 \text{ km}$$

$$\therefore D' = D - \text{braking distance} = 3 - 0.074 = 2.926 \text{ km}$$

Since,  $M_e/M = 1.08$ , using the relation derived in Art. 43.43, we get the value of specific energy consumption as

$$\begin{aligned} &= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778 \frac{r}{\eta} \frac{D'}{D} \right) \text{ Wh/t-km} \\ &= \left( 0.01072 \times \frac{40^2}{0.9 \times 3} \times 1.08 + 0.2778 \times \frac{49}{0.9} \times \frac{2.926}{3} \right) = \mathbf{21.6 \text{ Wh/t-km.}} \end{aligned}$$

**Example 43.12.** An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 70 seconds against a constant resistance of 60 N/t and is then braked to rest with uniform retardation of 3.0 km/h/s in 12 seconds. Compute

(i) uniform acceleration

(ii) coasting retardation

(iii) schedule speed if station stops are of 20-second duration

Allow 10% for rotational inertia. How will the schedule speed be affected if duration of stops is reduced to 15 seconds, other factors remaining the same ?

**Solution. (i)** As seen from Fig. 43.15,  $\alpha = V_1/t_1 = 50/25 = 2 \text{ km/h/s}$

(ii) The speeds at points B and C are connected by the relation

$$0 = V_2 + \beta t_3 \quad \text{or} \quad 0 = V_2 + (-3) \times 12 \quad \therefore V_2 = 36 \text{ km/h}$$

Coasting retardation,  $\beta_c = (V_2 - V_1)/t_2 = (36 - 50)/70 = -0.2 \text{ km/h/s}$

(iii) Distance travelled during acceleration

$$\begin{aligned} &= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 50 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h} \\ &= 0.174 \text{ km} \end{aligned}$$

Distance travelled during coasting can be found from the relation

$$\begin{aligned} V_{22} - V_{12} &= 2 \beta_c D \quad \text{or} \\ D &= (362 - 502)/2 \times -0.2 \times 3600 \\ &= 0.836 \text{ km} \end{aligned}$$

Distance covered during braking

$$\begin{aligned} &= \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 36 \frac{\text{km}}{\text{h}} \times \frac{12}{3600} \text{ h} \\ &= 0.06 \text{ km} \end{aligned}$$

Total distance travelled from start to stop

$$= 0.174 + 0.836 + 0.06 = 1.07 \text{ km}$$

Total time taken including stop time

$$= 25 + 70 + 12 + 20 = 127 \text{ second}$$

Schedule speed =  $1.07 \times 3600/127 = 30.3 \text{ km/s}$

Schedule speed with a stop of 15 s is  $1.07 \times 3600/122 = 31.6 \text{ km/h}$

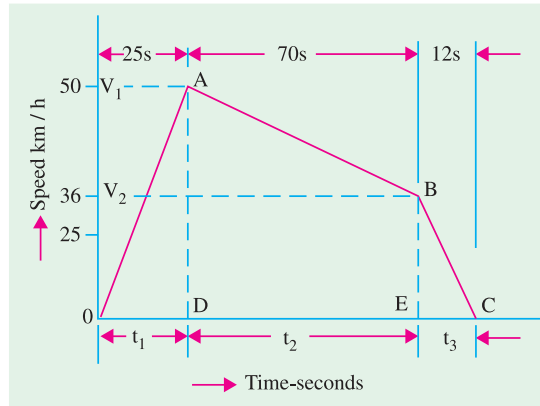


Fig. 43.15

**Example 43.13.** A 350-tonne electric train runs up an ascending gradient of 1% with the following speed/time curves :

1. uniform acceleration of 1.6 km/h/s for 25 seconds
2. constant speed for 50 seconds
3. coasting for 30 seconds
4. braking at 2.56 km/h/s to rest.

Compute the specific energy consumption if train resistance is 50 N/t, effect of rotational inertia 10%, overall efficiency of transmission gear and motor, 75%.

**Solution.** As seen from Fig. 43.16,  $V_1 = \alpha \cdot t_1 = 1.6 \times 25 = 40 \text{ km/h}$

Tractive force during coasting is

$$\begin{aligned} F_t &= (98 MG + M.r) \\ &= M (98 \times 1 + 50) \\ &= 148 M \text{ newton} \end{aligned}$$

Also,  $F_t = 277.8 M_e \beta_c$  during coasting.

Equating the two expressions, we get  $277.8 M_e$

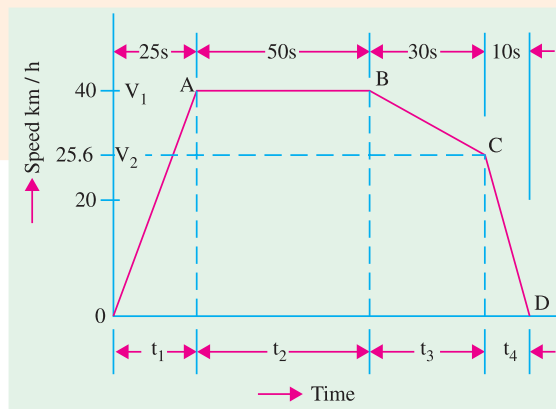


Fig. 43.16

$$\beta_c = 148 M$$

$$\therefore \beta_c = \frac{148}{277.8} \times \frac{M}{M_e} = \frac{148}{277.8} \times \frac{1}{1.1};$$

$$\beta_c = 0.48 \text{ km/h/s}$$

$$\begin{aligned} \text{Now, } V_2 &= V_1 + \beta_c t_3 \\ &= 40 + (-0.48) \times 30 \\ &= 25.6 \text{ km/h} \end{aligned}$$

$$t_4 = V_2 / \beta = 25.6 / 2.56 = 10 \text{ second}$$

$$\text{Distance travelled during acceleration period} = \frac{1}{2} \times 40 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h} = 0.139 \text{ km}$$

$$\text{Distance travelled during constant speed period is} = V_1 \times t_2 = 40 \times 50/3600 = 0.555 \text{ km}$$

$$\text{Distance travelled during coasting} = \left( \frac{V_1 + V_2}{2} \right) \times t_3 = \frac{40 + 25.6}{2} \times \frac{30}{3600} = 0.273 \text{ km}$$

$$\text{Distance travelled during braking} = \frac{1}{2} V_2 t_4 = \frac{1}{2} \times 25.6 \times \frac{10}{3600} = 0.035 \text{ km}$$

$$\text{Total distance between stops} = 0.139 + 0.555 + 0.273 + 0.035 = 1.002 \text{ km}$$

Distance travelled during acceleration and free-running period is

$$D' = 0.139 + 0.555 = 0.694 \text{ km}$$

Specific energy consumption (Art. 43.43) is

$$\begin{aligned} &= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km} \\ &= \left( 0.01072 \times \frac{40^2}{0.75 \times 1.002} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.694}{1.002} + 0.2778 \times \frac{50}{0.75} \times \frac{0.694}{1.002} \right) \\ &= 25.1 + 25.2 + 12.8 = \mathbf{63.1 \text{ Wh/t-km}} \end{aligned}$$

**Example 43.14.** An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 : 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.29 km/h/s. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting = 29.4 N/t, Train resistance at 30 km/h = 49 N/t

Coefficient of adhesion = 0.3, Rotational inertia = 10%

(Utilization of Elect. Power, AMIE)

**Solution.** Downward force due to gravity

$$= Mg \sin \theta = (5000 \times 1000) \times 9.8 \times 1/50 = 980,000 \text{ N}$$

$$\text{Train resistance} = 49 \times 5000 = 245,000 \text{ N}$$

$$\text{Braking force to be supplied by brakes} = 980,000 - 245,000 = 735,000 \text{ N}$$

Max. braking force which one locomotive can provide

$$= 1000 \mu_a Mg \text{ newton} \quad \text{--- } M \text{ in tonne}$$

$$= 1000 \times 0.3 \times 75 \times 9.8 = 220,500 \text{ N}$$

$$\text{No. of locomotives required for braking} = 735,000/220,500 = 3.33$$

Since fraction is meaningless, it means that 4 locomotives are needed.

Tractive effort required to haul the train on level track

$$= (277.8 \alpha M_e + Mr) \text{ newton}$$

$$= 277.8 \times (5000 \times 1.1) \times 0.29 + 5000 \times 29.4 = 590,090 \text{ N}$$

No. of locomotives required =  $590,090/220,500 = 2.68 \cong 3$

It means that **4 locomotives** are enough to look after braking as well as starting.

**Example 43.15.** A 200-tonne electric train runs according to the following quadrilateral speed/time curve:

1. uniform acceleration from rest at 2 km/h/s for 30 seconds
2. coasting for 50 seconds
3. duration of braking : 15 seconds

If up-gradient is = 1%, train resistance = 40 N/t, rotational inertia effect = 10%, duration of stops = 15 s and overall efficiency of gear and motor = 75%, find

(i) schedule speed (ii) specific energy consumption (iii) how will the value of specific energy consumption change if there is a down-gradient of 1% rather than the up-gradient ?

(Electric Traction Punjab Univ. 1993)

**Solution.**  $V_1 = \alpha \cdot t_1 = 2 \times 30 = 60 \text{ km/h/s}$

During coasting, gravity component and train resistance will cause coasting retardation  $\beta_c$ .

Retarding force =  $(98 MG + Mr)$  newton =  $(98 \times 200 \times 1.0 + 200 \times 40) = 27,600 \text{ N}$

As per Art, 43.37,  $277.8 M_e \beta_c = 27,600$  or  $277.8 \times (200 \times 1.1) \times \beta_c = 27,600$

$\therefore \beta_c = 0.45 \text{ km/h/s}$

Now,  $V_2 = V_1 + \beta_c t_2$  or  $V_2 = 60 + (-0.45) \times 50 = 37.5 \text{ km/h}$

Braking retardation  $\beta = V_2/t_3 = 37.5/15 = 2.5 \text{ km/h/s}$

Distance travelled during acceleration (area  $OAD$  in Fig. 43.17)

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 60 \times \left( \frac{30}{3600} \right) = 0.25 \text{ km}$$

Distance travelled during coasting

$$= \text{area } ABED = \left( \frac{V_1 + V_2}{2} \right) \times t_2 = \left( \frac{60 + 37.5}{2} \right) \times \frac{50}{3600} = 0.677 \text{ km}$$

Distance travelled during braking

$$= \text{area } BCE = \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 37.5 \times \frac{15}{3600} = 0.078 \text{ km}$$

Total distance travelled,  $D = 0.25 + 0.677 + 0.078 = 1.005 \text{ km}$

Total schedule time =  $30 + 50 + 15 + 15 = 110 \text{ s}$

(i)  $\therefore$  Schedule speed =  $\frac{1.005}{110/3600} = 32.9 \text{ km/h}$

(ii) As per Art. 43.43, specific energy consumption

$$= \left( 0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \cdot \frac{r}{\eta} \cdot \frac{D'}{D} \right) \text{ Wh/t-km}$$

$$= \left( 0.01072 \times \frac{60^2}{0.75 \times 1.005} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.005} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.005} \right) \text{ Wh/t-km}$$

(iii) the speed/time curve for this case is shown in Fig. 43.18. As before,  $V_1 = 60 \text{ km/h}$ . Here, we will take  $G = -1\%$

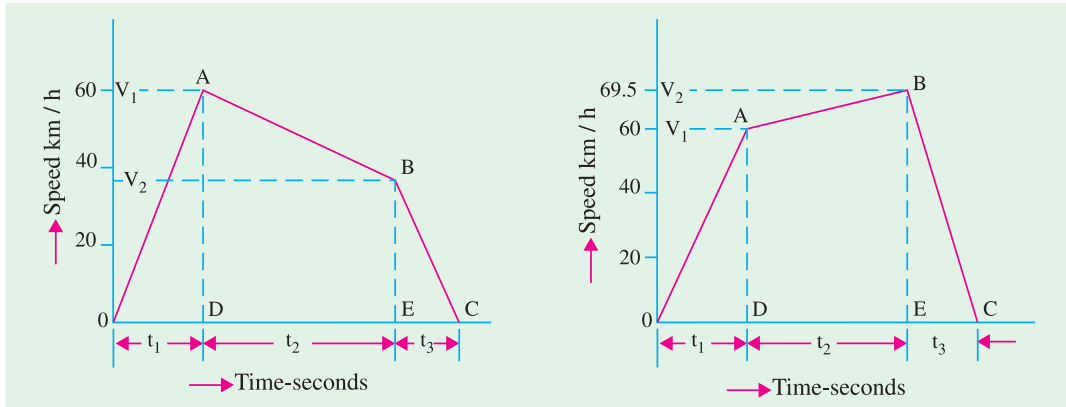


Fig. 43.17

Fig. 43.18

$$\therefore \text{Retarding force} = (98 MG + Mr) \text{ newton} = 98 \times 200 \times (-1.0) + 200 \times 40 = -11,600 \text{ N}$$

The negative sign indicates that instead of being a retarding force, it is, in fact, an accelerating force. If  $\alpha_c$  is the acceleration produced, then

$$11,600 = 277.8 \times (200 \times 1.1) \times \alpha_c$$

$$\alpha_c = 0.19 \text{ km/h/s}$$

Also,  $V_2 = V_1 + \alpha_c t_2 = 60 + 0.19 \times 50 = 69.5 \text{ km/h}$

$$\beta = V_2 / t_3 = 69.5 / 15 = 4.63 \text{ km/h/s}$$

Distance travelled during acceleration = 0.25 km —as before

$$\text{Distance travelled during coasting} = \frac{60 + 69.5}{2} \times \frac{50}{3600} = 0.9 \text{ km}$$

$$\text{Distance travelled during braking} = \frac{1}{2} \times 69.5 \times \frac{15}{3600} = 0.145 \text{ km}$$

$$\therefore D = 0.25 + 0.9 + 0.145 = 1.295 \text{ km}$$

Hence, specific energy consumption is

$$= \left( 0.01072 \times \frac{60^2}{0.75 \times 1.295} \times 1.1 - 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.295} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.295} \right) \text{Wh/t-km}$$

$$= 43.7 - 7.01 + 2.86 = 39.55 \text{ Wh/t-km}$$

As seen, energy consumption has decreased from 69 to 39.55 Wh/t-km.

**Example 43.16.** An electric train has an average speed of 45 kmph on a level track between stops 1500 m apart. It is accelerated at 1.8 kmph/s and is braked at 3 kmph/s. Draw the speed - time curve for the run.

**Solution.**

Acceleration  $\alpha = 1.8 \text{ kmph/s}$

Retardation  $\beta = 3.0 \text{ kmph/s}$

Distance of run  $S = 1.5 \text{ km}$

Average speed  $V_a = 45 \text{ kmph}$

Time of run,  $T = \frac{S}{V_a} \times 3600 = \frac{1.5}{45} \times 3600 = 120 \text{ seconds.}$

Using equation  $V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3600 S}{K}}$

Where

$$K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2 \times 1.8} + \frac{1}{2 \times 3.0} = 0.4444$$

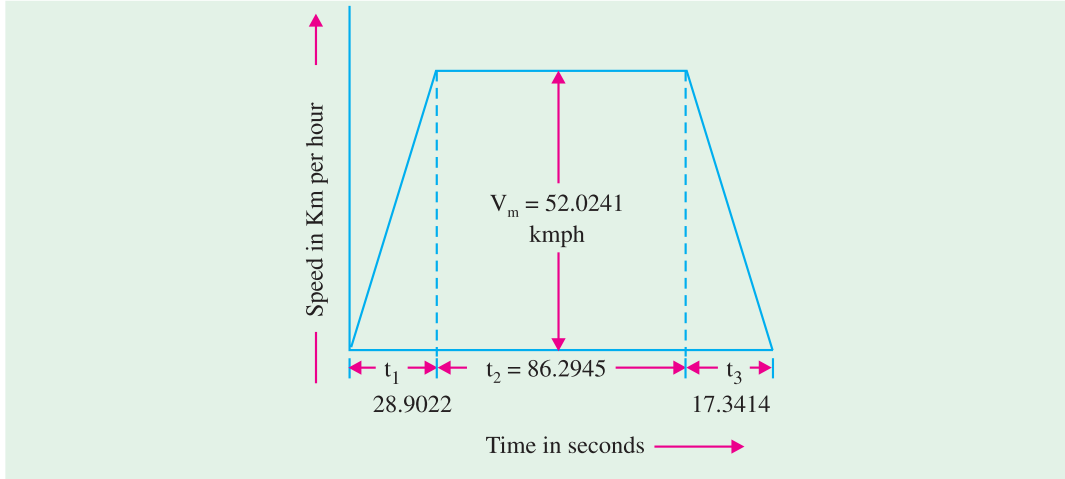


Fig. 43.19

$$\therefore \text{Maximum speed, } V_m = \frac{120}{2 \times 0.4444} - \sqrt{\frac{(120)^2}{(2 \times 0.4444)^2} - \frac{3600 \times 1.5}{0.4444}}$$

$$\therefore V_m = 52.0241 \text{ kmph}$$

$$\text{Acceleration period, } t_1 = \frac{V_m}{\alpha} = \frac{52.0241}{1.8} = 28.9022 \text{ seconds}$$

$$\text{Braking period, } t_3 = \frac{V_m}{\beta} = \frac{52.0241}{3.0} = 17.3414 \text{ seconds}$$

$$\text{Free running period, } t_2 = T - (t_1 + t_3) = 120 - (28.9622 + 17.3414) = 86.2945 \text{ seconds}$$

**Example 43.17.** A train has schedule speed of 60 km per hour between the stops which are 9 km apart. Determine the crest speed over the run, assuming trapezoidal speed – time curve. The train accelerates at 3 kmphs and retards at 4.5 kmphs. Duration of stops is 75 seconds.

**Solution.**

Acceleration  $\alpha = 3$  kmphs

Retardation  $\beta = 4.5$  kmphs

Distance of run,  $S = 9$  km

Schedule speed,  $V_s = 60$  kmph

Schedule time,  $T_s = \frac{S}{V_s} \times 3,600 \text{ seconds} = \frac{9}{60} \times 3,600 = 540 \text{ seconds}$

Actual time of run,  $T = T_s - \text{Time of stop} = 540 - 75 = 465 \text{ seconds}$

Using the equation

$$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3,600 S}{K}}$$

where  $K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{6} + \frac{1}{9} = 0.2777$

$$\therefore \text{Maximum speed, } V_m = \frac{465}{2 \times 0.2777} - \sqrt{\frac{(465)^2}{4 \times (0.2777)^2} - \frac{3,600 \times 9}{0.2777}}$$

$$V_m = 837 - \sqrt{700569 - 116640}$$

$$\therefore V_m = 72.8475 \text{ kmph}$$

**Example 43.18.** An electric train is to have acceleration and braking retardation of 1.2 km/hour/sec and 4.8 km/hour/sec respectively. If the ratio of maximum to average speed is 1.6 and time for stops 35 seconds, find schedule speed for a run of 3 km. Assume simplified trapezoidal speed-time curve.

**Solution.**

Acceleration  $\alpha = 1.2$  kmphps

Retardation  $\beta = 4.8$  kmphps

Distance of run,  $S = 3$  km

Let the actual time of run be  $T$  seconds

Average speed,  $V_a = \frac{3,600 S}{T} = \frac{3,600 \times 3}{T} = \frac{10800}{T}$  kmph

Maximum speed,  $V_m = 1.6 \times \frac{10800}{T} = \frac{17280}{T}$  kmph

Since  $V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$

$$\therefore V_m^2 = \frac{V_m T - 3,600 S}{\frac{1}{2\alpha} + \frac{1}{2\beta}} = \frac{\frac{17280}{T} T - 3600 \times 3}{\frac{1}{2 \times 1.2} + \frac{1}{2 \times 4.8}}$$

or  $V_m = 111.5419$  kmph

and  $V_a = \frac{V_m}{1.5} = \frac{111.5419}{1.5} = 74.3613$  kmph

Actual time of run  $T = \frac{3,600 S}{V_a} = \frac{3600 \times 3}{74.3613} = 145.2369$  seconds

Schedule time,  $T_s = \text{Actual time of run} + \text{Time of stop} = 145.2369 + 35 = 180$  seconds

$\therefore$  Schedule speed,  $V_s = \frac{S \times 3,600}{T_s} = \frac{3 \times 3600}{180} = 60$  kmph

**Example 43.19.** An electric train has a schedule speed of 25 kmph between stations 800 m apart. The duration of stop is 20 seconds, the maximum speed is 20 percent higher than the average running speed and the braking retardation is 3 kmphps. Calculate the rate of acceleration required to operate this service.

**Solution.**

Schedule speed,  $V_s = 25$  kmph

Distance of run,  $S = 800$  metres = 0.8 km

Retardation,  $\beta = 3$  kmphps

Schedule time of run,  $T_s = \frac{3,600 \times S}{V_s} = \frac{3600 \times 0.8}{25} = 115.2$  seconds

Actual time of run,  $T = T_s - \text{duration of stop} = 115.2 - 20 = 95.2$  seconds

Average speed,  $V_a = \frac{3,600 \times S}{T} = \frac{3600 \times 0.8}{95.2} = 30.25$  kmphs

Maximum speed,  $V_m = 1.2 V_a = 1.2 \times 30.25 = 36.3$  kmph



$$\text{Since } V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$$

$$\therefore \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600 S}{V_m^2}$$

$$\text{or } \frac{1}{2\alpha} + \frac{1}{2 \times 3} = \frac{36.3 \times 95.2 - 3,600 \times 0.8}{(36.3)^2} = 0.4369$$

$$\text{or } \alpha = 1.85 \text{ kmphs}$$

**Example 43.20.** A suburban electric train has a maximum speed of 80 kmph. The schedule speed including a station stop of 35 seconds is 50 kmph. If the acceleration is 1.5 kmphs, find the value of retardation when the average distance between stops is 5 km.

**Solution.**

Schedule speed,	$V_s = 50 \text{ kmph}$
Distance of run,	$S = 5 \text{ km}$
Acceleration,	$\alpha = 1.5 \text{ kmphs}$
Maximum speed,	$V_m = 80 \text{ kmph}$
Duration of stop	$= 35 \text{ seconds}$

$$\text{Schedule time of run, } T_s = \frac{3,600 \times S}{V_s} = \frac{3,600 \times 5}{50} = 360 \text{ seconds}$$

$$\text{Actual time of run, } T = \frac{T_s - \text{duration of stop}}{V_m^2} = 360 - 30 = 330 \text{ seconds}$$

$$\text{Since } V_m^2 \left( \frac{1}{2\alpha} + \frac{1}{2\beta} \right) - V_m T + 3,600 S = 0$$

$$\text{or } \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{V_m T - 3,600 S}{V_m^2} = \frac{80 \times 330 - 3,600 \times 5}{(80)^2} = 1.3125$$

$$\text{or } \frac{1}{2\beta} = 1.3125 - \frac{1}{2\alpha} = 1.3125 - \frac{1}{2 \times 1.5} = 0.9792$$

$$\text{or } \beta = \frac{1}{2 \times 0.9792} = 0.51064 \text{ kmphs}$$

**Example 43.21.** A train is required to run between two stations 1.6 km apart at the average speed of 40 kmph. The run is to be made to a simplified quadrilateral speed-time curve. If the maximum speed is to be limited to 64 kmph, acceleration to 2.0 kmphs and coasting and braking retardation to 0.16 kmphs and 3.2 kmphs respectively, determine the duration of acceleration, coasting and braking periods.

**Solution.**

Distance of run,	$S = 1.6 \text{ km}$
Average speed,	$V_a = 40 \text{ kmph}$
Maximum speed,	$V_m = 64 \text{ kmph}$
Acceleration,	$\alpha = 2.0 \text{ kmphs}$
Coasting retardation,	$\beta_C = 0.16 \text{ kmphs}$
Braking retardation,	$\beta = 3.2 \text{ kmphs}$

$$\text{Duration of acceleration, } t_1 = \frac{V_m}{\alpha} = \frac{64}{2.0} = 32 \text{ seconds}$$

$$\text{Actual time of run, } T = \frac{3,600 S}{V_a} = \frac{3,600 \times 1.6}{40} = 144 \text{ seconds}$$

Let the speed before applying brakes be  $V_2$

$$\text{then duration of coasting, } t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - V_2}{0.16} \text{ seconds}$$

$$\text{Duration of braking, } t_3 = \frac{V_2}{\beta} = \frac{V_2}{3.2} \text{ seconds}$$

$$\text{Since actual time of run, } T = t_1 + t_2 + t_3$$

$$\therefore 144 = 32 + \frac{64 - V_2}{0.16} + \frac{V_2}{3.2}$$

$$\text{or } V_2 \left( \frac{1}{0.16} - \frac{1}{3.2} \right) = 32 + 400 - 144$$

$$\text{or } V_2 = \frac{288}{6.25 - 0.3125} = 48.5 \text{ kmph}$$

$$\text{Duration of coasting, } t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - 48.5}{0.16} = \mathbf{96.85 \text{ seconds}}$$

$$\text{Duration of braking, } t_3 = \frac{V_2}{\beta} = \frac{48.5}{3.2} = \mathbf{15.15 \text{ seconds}}$$

#### 43.47. General Features of Traction Motor

Electric Features

- High starting torque
- Series Speed - Torque characteristic
- Simple speed control
- Possibility of dynamic/regenerative braking
- Good commutation under rapid fluctuations of supply voltage.

Mechanical Features

- Robustness and ability to withstand continuous vibrations.
- Minimum weight and overall dimensions
- Protection against dirt and dust

No type of motor completely fulfills all these requirements. Motors, which have been found satisfactory are D.C. series for D.C. systems and A.C. series for A.C. systems. While using A.C. three phase motors are used. With the advent of Power Electronics it is very easy to convert single phase A.C. supply drawn from pantograph to three phase A.C.

#### 43.48. Speed - Torque Characteristic of D.C. Motor

$$V = E_b + I_a R_a$$

$$V \cdot I_a = E_b \cdot I_a + I_a^2 R_a$$

where  $E_b I_a$  = Power input to armature = Electrical power converted into mechanical power at the shaft of motor.

$$\text{Mechanical Power} = T \cdot \omega = T \times \frac{2\pi N}{60}$$

$$\therefore \frac{2\pi NT}{60} = E_b \cdot I_a \therefore T = \frac{60 E_b I_a}{2\pi N} = 9.55 \frac{E_b I_a}{N}$$

$$\text{But } E_b = \frac{\phi ZNP}{60 A}$$

$$\begin{aligned} \therefore T &= 9.55 \frac{\phi ZNP}{60 A} \frac{I_a}{N} = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A} \\ &= 0.1592 \times \phi \times \left[ Z \frac{I_a}{A} \right] P \text{ Nw-m} \end{aligned}$$

$\therefore$  Torque  $T = 0.1592 \times \text{flux per pole} \times \text{armature amp. conductors} \times \text{Number of poles}$   
 Also speed ' $N$ ' can be calculated as:

$$\begin{aligned} E_b &= \frac{\phi ZNP}{60 A} \quad \therefore N = \frac{(E_b)}{\phi ZP} 60 A \\ N &= \frac{(V - I_a R_a) 60 A}{\phi ZP} \quad \therefore N \propto \frac{V - I_a R_a}{\phi} \end{aligned}$$

But  $T = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A}$  from the equation of torque

$$\therefore \frac{T}{I_a} = \frac{9.55 \phi ZP}{60 A} \Rightarrow \frac{9.55 I_a}{T} = \frac{60 A}{\phi ZP} \text{ Put this value in the above equation of } N$$

$$\therefore N = \frac{(V - I_a R_a) \times 9.55 I_a}{T}$$

$$\text{Speed } N = \frac{9.55 (V - I_a R_a)}{T / I_a}$$

The torque - current and speed - torque curves for D.C. motors are shown in Fig. 43.20 (a) and (b) respectively.

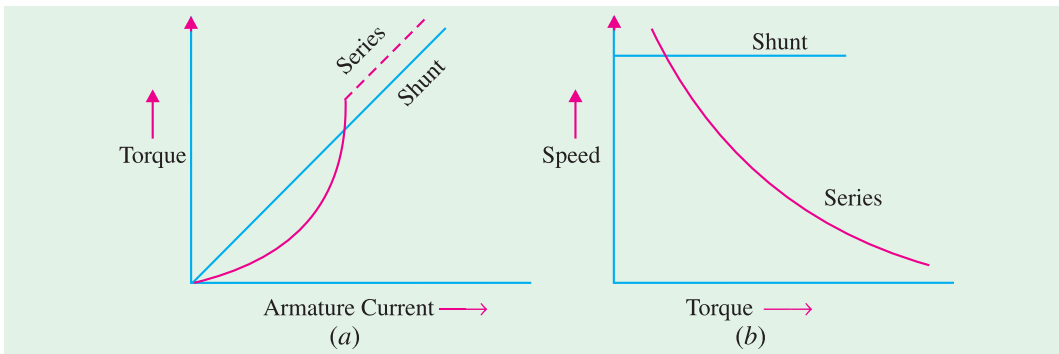


Fig. 43.20

### 43.49. Parallel Operation of Series Motors with Unequal Wheel Diameter

An electric locomotive uses more than one motor. Each motor drives different set of axles and wheels. Due to wear and tear the diameter of wheels become different, after a long service. But the linear speed of locomotive and wheels will be the same. Therefore, motor speeds will be different due to difference in diameter of wheels driven by them as shown in Fig. 43.21. Therefore, when the motors are connected in parallel they will not share the torques equally, as the current shared by them will be different.

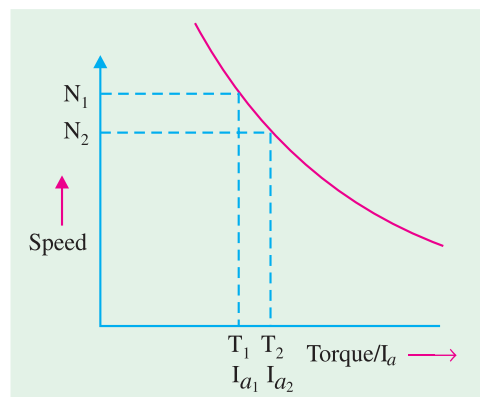


Fig. 43.21

Let the motor wheels ratio is 1.04 : 1 *i.e.* speed of rotation of motor-1 is 1.04 times that of motor-2, as shown in Fig. 43.21.

Let motor 1 drives wheel with 100 c.m. dia. and motor 2 drives wheel with 104 c.m. dia. Then speed of rotation of motor -1 will be  $\frac{104}{100} = 1.04$  times that of 2 *i.e.*  $N_2 = 1.04 N_1$ , for a given speed of locomotive.

**43.50. Series operation of Series Motor with unequal wheel diameter**

Let the motors ‘A’ and ‘B’ be identical having armature resistance  $R$  in series, as shown in Fig. 43.22.

Since they are in series, the same current ‘ $I$ ’ will flow through both. But due to unequal wheel diameter; they deliver different loads *i.e.* voltage across each will be different

$$V = V_A + V_B \quad \text{and} \quad N \propto V - IR$$

$$\therefore \frac{N_A}{N_B} = \frac{V_A - IR}{V_B - IR}$$

$$\therefore V_A - IR = \frac{N_A}{N_B} (V_B - IR)$$

$$V_A = \frac{N_A}{N_B} (V_B - IR) + IR$$

$$V_A = \frac{N_A}{N_B} (V - V_A - IR) + IR$$

$$V_A = \frac{N_A}{N_B} (V - IR) + IR - \frac{N_A}{N_B} V_A$$

$$V_A + \frac{N_A}{N_B} V_A = \frac{N_A}{N_B} (V - IR) + IR$$

$$V_A \left( 1 + \frac{N_A}{N_B} \right) = \frac{N_A}{N_B} (V - IR) + IR$$

$$V_A = \frac{\frac{N_A}{N_B} (V - IR) + IR}{1 + \frac{N_A}{N_B}}$$

Similarly,

$$V_B = \frac{\frac{N_B}{N_A} (V - IR) + IR}{1 + \frac{N_B}{N_A}}$$

**43.51. Series Operation of Shunt Motors with Unequal Wheel Diameter**

It is similar to the case of series operation of series motors, and hence the same equation holds good.

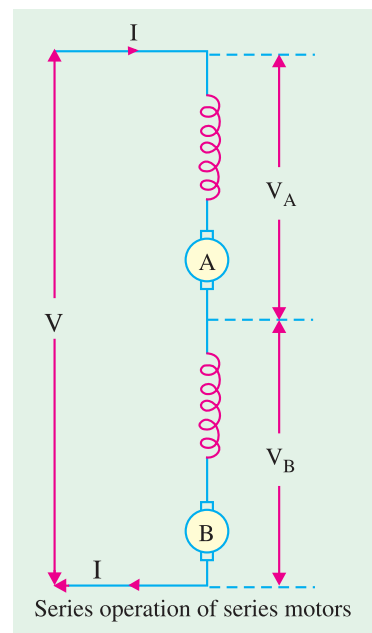


Fig. 43.22

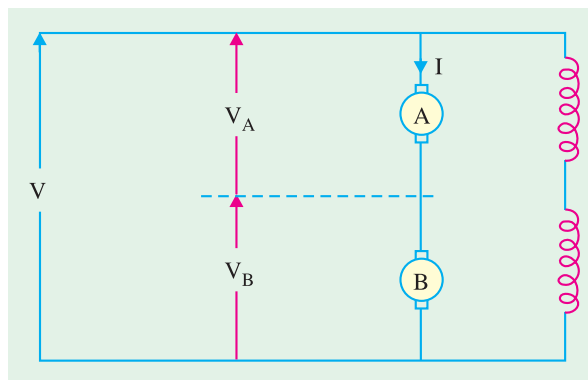


Fig. 43.23

**43.52. Parallel Operation of Shunt Motors with Unequal Wheel Diameter**

As seen from the Fig. 43.24, a small difference in speeds of two motors, causes motors to be loaded very unequally due to flat speed current curve of D.C. shunt motor.

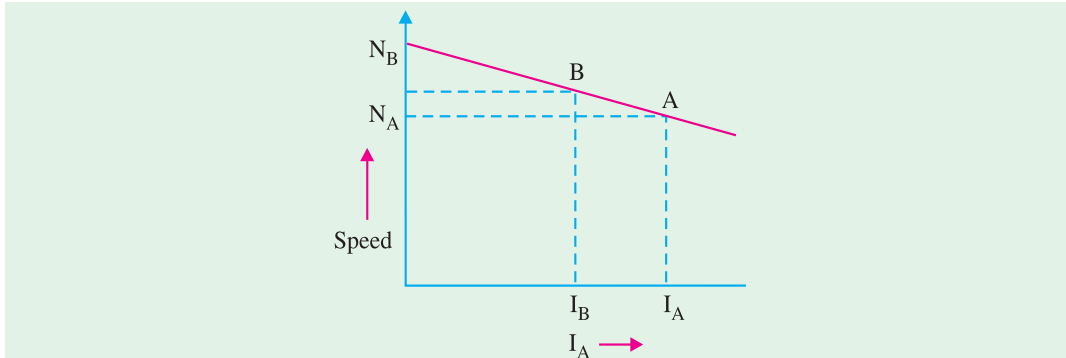


Fig. 43.24

**Example 43.22.** The torque-armature current characteristics of a series traction motor are given as:

Armature Current (amp) :	5	10	15	20	25	30	35	40
Torque (N-m) :	20	50	100	155	215	290	360	430

The motor resistance is  $0.3\Omega$ . If this motor is connected across 230 V, deduce the speed armature current characteristics.

**Solution.**

Supply voltage,  $V = 230$  V.

Total Resistance of series motor  $R_m = R_a + R_{se} = 0.3 \Omega$ .

Armature current, $I_a$ in amperes	5	10	15	20	25	30	35	40
Torque, T in N-m	20	50	100	155	215	290	360	430
Back e.m.f., $E_b = (V - I_a R_m)$ in volts	228.5	227.0	225.5	224.0	222.5	221.0	219.5	218.0
Speed, $N_a$ $\frac{60 E_b}{2\pi \tau}$ in R.P.M.	545	434	323	276	247	218	204	194

The deduced speed-armature current characteristic is shown in Fig. 43.25.

**Example 43.23.** The following figures give the magnetization curve of d.c. series motor when working as a separately excited generator at 600 rpm.:

Field Current (amperes) :	20	40	60	80
E.M.F. (volts) :	215	381	485	550

The total resistance of the motor is 0.8 ohm. Deduce the speed – torque curve for this motor when operating at a constant voltage of 600 volts.

**Solution.**

Voltage applied across the motor,  $V = 600$  volts

Resistance of the motor,  $R_m = (R_a + R_{se}) = 0.8 \Omega$

Speed,  $N_1 = 600$  r.p.m.

Field Current (amperes)	20	40	60	80
Back e.m.f., $E_1$ (Volts) at speed $N_1$ (600 r.p.m.) = e.m.f. generated by the armature (given)	215	381	485	550
Back e.m.f., $E$ (Volts) at speed $N$ (to be determined) = $V - I_a R_m$	584	568	552	536
Speed, $N$ (to be determined in r.p.m.) = $N_1/E_1 \times (V - I_a R_m)$	1,630	895	683	585
Torque, $T = \frac{9.55 (V - I_a R_m) I_a}{N}$ N-m	68.4	240	462	700

The deduced speed-torque curve for the motor is shown in Fig. 43.26.

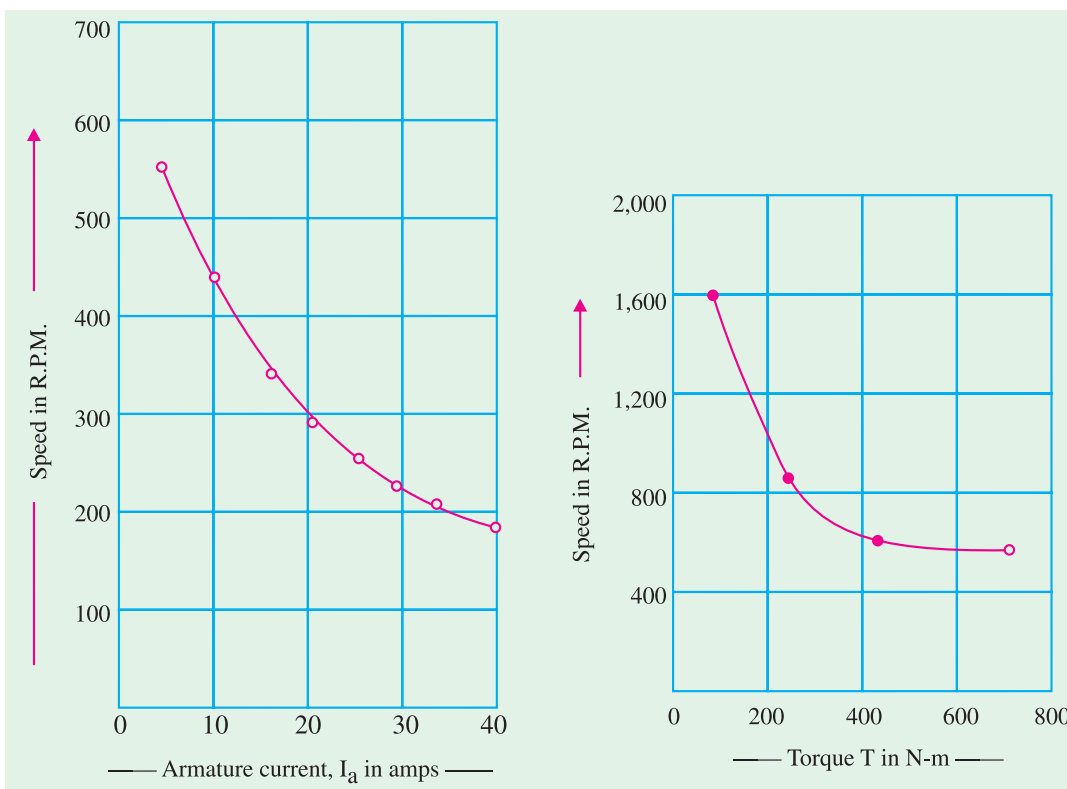


Fig. 43.25

Fig. 43.26

**Example 43.24.** Two d.c. traction motors run at a speed of 900 r.p.m. and 950 r.p.m. respectively when each takes a current of 50 A from 500 V mains. Each motor has an effective resistance of 0.3  $\Omega$ . Calculate the speed and voltage across each machine when mechanically coupled and electrically connected in series and taking a current of 50 A from 500 V mains, the resistance of each motor being unchanged.

**Solution.**

Let the two motors be A and B of speed  $N_A = 900$  rpm. And  $N_B = 950$  r.p.m. respectively. Resistance of each motor  $R_m = 0.3 \Omega$

Applied voltage across each motor,  $V = 500$  V.

Back e.m.f. of motor  $A$  when taking a current of 50 A

$$E_{b_A} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

Back e.m.f. of motor  $B$  when taking a current of 50 A

$$E_{b_B} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

When the machines are mechanically coupled and connected in series, the speed of each motor will be same, say  $N$ , current will be same and equal to 50 A (given) and the sum of voltage across the two motors will be equal to 500 V.

Let the voltage across motors  $A$  and  $B$  be  $V_A$  and  $V_B$  respectively

$$\text{Now } V_A + V_B = 500 \quad \dots(i)$$

$$\text{Back e.m.f. of motor } A, E_{b'_A} = E_{b_A} \times \frac{N}{N_A} = \frac{485}{900} \times N$$

$$\text{Voltage across motor } A, V_A = E_{b'_A} + I R_m = \frac{485}{900} \times N + 50 \times 0.3 = \frac{485}{900} \times N + 15$$

$$\text{Back e.m.f. of motor } B, E_{b'_B} = E_{b_B} \times \frac{N}{N_B} = \frac{485}{950} N$$

$$\text{Voltage across motor } B, V_B = E_{b'_B} + I R_m = \frac{485}{950} N + 15$$

Substituting  $V_A = \frac{485}{900} N + 15$  and  $V_B = \frac{485}{950} N + 15$  in expression (i) we get

$$\frac{485}{900} N + 15 + \frac{485}{950} N + 15 = 500$$

$$\text{or } \left( \frac{485}{900} + \frac{485}{950} \right) N = 470$$

$$\text{or } N \left( \frac{1}{900} + \frac{1}{950} \right) = \frac{470}{485}$$

$$\therefore N = 447.87 \text{ r.p.m.}$$

$$\text{P.D. across machine } A, V_A = \frac{485 N}{900} + 15 = 256.35 \text{ V}$$

$$\text{P.D. across machine } B, V_B = \frac{485 N}{950} + 15 = 243.65 \text{ V}$$

**Example 43.25.** A tram car is equipped with two motors which are operating in parallel. Calculate the current drawn from the supply main at 500 volts when the car is running at a steady speed of 50 kmph and each motor is developing a tractive effort of 2100 N. The resistance of each motor is 0.4 ohm. The friction, windage and other losses may be assumed as 3500 watts per motor.

**Solution.**

Voltage across each motor,  $V = 500$  volts

Maximum speed,  $V_m = 50$  kmph

Tractive effort,  $F_t = 2100$  Newtons

Motor resistance,  $R_m = 0.4$  W

Losses per motor = 3500 watts

$$\text{Power output of each motor} = \frac{F_t \times V_m \times 1000}{3600} \text{ watts}$$

$$= \frac{2100 \times 50 \times 1000}{3600} \text{ watts} = 29166.67 \text{ watts.}$$

Constant losses = 3500 watts

$$\text{Copper losses} = I^2 R_m = 0.4 I^2$$

Where  $I$  is the current drawn from supply mains

$$\text{Input to motor} = \text{Motor output} + \text{constant losses} + \text{copper losses}$$

$$VI = 29166.67 + 3500 + 0.4 I^2$$

$$0.4 I^2 - 500 I + 32666.67 = 0$$

$$I = \frac{500 \pm \sqrt{(500)^2 - 4 \times 0.4 \times 32666.67}}{2 \times 0.4}$$

$$I = 69.16 \text{ A} \quad \text{or} \quad 1180.84 \text{ A}$$

Current drawn by each motor = 69.16A

∴ 1180.84 A being unreasonably high can not be accepted

Total current drawn from supply mains =  $69.16 \times 2 = 138.32 \text{ A}$

**Example 43.26.** A motor coach is being driven by two identical d.c. series motors. First motor is geared to driving wheel having diameter of 90 cm and other motor to driving wheel having diameter of 86 cm. The speed of the first motor is 500 r.p.m. when connected in parallel with the other across 600 V supply. Find the motor speeds when connected in series across the same supply. Assume armature current to remain same and armature voltage drop of 10% at this current.

**Solution.**

Speed of first motor,  $N_1 = 500 \text{ r.p.m.}$

$$\begin{aligned} \text{Back e.m.f.,} \quad E_{b_1} &= 600 - \frac{10}{100} \times 600 \\ &= 540 \text{ volts.} \end{aligned}$$

When the motors are connected in series across 600 V supply, as shown in Fig. 43.27.

Let the supply voltage across motors I and II be  $V_1$  and  $V_2$  volts and speed  $N'_1$  and  $N'_2$  respectively.

$$\text{Since speed,} \quad N \propto \frac{V - IR}{\phi}$$

Current through the motors remains the same, therefore flux produced by it also remains the same and  $N \propto (V - IR)$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{V_1 - IR}{V_2 - IR} = \frac{V_1 - \frac{10}{100} \times 600}{V_2 - \frac{10}{100} \times 600} = \frac{V_1 - 60}{V_2 - 60} \quad \dots(i)$$

$$\text{And also} \quad N'_1 D_1 = N'_2 D_1 = N'_2 D_2$$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{D_2}{D_1} = \frac{86}{90}$$

$$\frac{V_1 - 60}{V_2 - 60} = \frac{86}{90} \quad (ii) \text{ Since peripheral speed is equal}$$

$$\text{or} \quad 90 V_1 - 5,400 = 86 V_2 - 5,160$$

$$\text{or} \quad 90 V_1 - 86 V_2 = 5,400 - 5,160 = 240 \quad \dots(iii)$$

$$\text{and also} \quad V_1 + V_2 = 600 \text{ V} \quad \dots(iv)$$

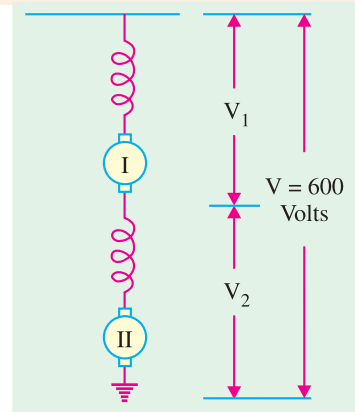


Fig. 43.27



Solving expressions (iii) and (iv)

$$V_1 = 294.55 \text{ V}$$

And  $V_2 = 305.45 \text{ V}$

Now the speeds of the motors can be calculated as follows :

$$\frac{N'_1}{N'_2} = \frac{r_{\phi_1}}{r_{\phi_2}}$$

or  $N'_1 = N_1 \times \frac{E_{b1}}{E_b} = 500 \times \frac{294.55 - 60}{600 - 60} = 217 \text{ r.p.m}$

and  $N'_2 = N'_1 \times \frac{D_1}{D_2} = 217 \times \frac{90}{86} = 277 \text{ r.p.m}$

**Example 43.27.** Two similar series type motors are used to drive a locomotive. The supply fed to their parallel connection is 650 V. If the first motor 'A' is geared to drive wheels of radius 45 cms. and other motor 'B' to 43 cms. And if the speed of first motor 'A' when connected in parallel to 2<sup>nd</sup> motor 'B' across the main supply lines is 400 rpm., find voltages and speeds of motors when connected in series. Assume  $I_a$  to be constant and armature voltage drop of 10% at this current.

**Solution.**

$N \propto V - IR$  as flux  $\phi$  is constant, since  $I_a$  is constant

$$N_A = V_A - IR \quad N_B = V_B - IR \quad \text{Also } V = V_A + V_B$$

Assume  $\frac{N_A}{N_B} = \rho$

$$V_A = \frac{\rho(V - IR) + IR}{1 + \rho}$$

Armature voltage drop = 10% of 650 V  $\therefore IR = 65$

But  $\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{43}{45} = \rho$

$$V_A = \frac{43/45(650 - 65) + 65}{1 + 43/45} = 320 \text{ V}$$

$$V_B = V - V_A = 650 - 320 = 330 \text{ V}$$

Speed  $N_A$  of motor A is 400 rpm with a supply of 650 V.

$\therefore$  Speed  $N'_A$  of motor A with supply voltage of 320 V will be

$$\frac{N'_A}{N_A} = \frac{320 - IR}{650 - IR} = \frac{320 - 65}{650 - 65} = \frac{255}{585}$$

$\therefore N'_A = \frac{255}{585} N_A = \frac{255}{585} \times 400 = 175 \text{ r.p.m.}$

$$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{N'_A}{N'_B} = \frac{43}{45}$$

$$N'_B = \frac{45}{43} N'_A = \frac{45}{43} \times 175 = 184 \text{ r.p.m.}$$

### 43.53. Control of D.C. Motors

The starting current of motor is limited to its normal rated current by starter during starting. At the instant of switching on the motor, back e.m.f.  $E_b = 0$

$\therefore$  Supply voltage =  $V = IR + \text{Voltage drop across } R_s$ .

At any other instant during starting

$$V = IR + \text{Voltage across } R_s + E_b$$

At the end of accelerating period, when total  $R_s$  is cut-off

$$V = E_b + IR$$

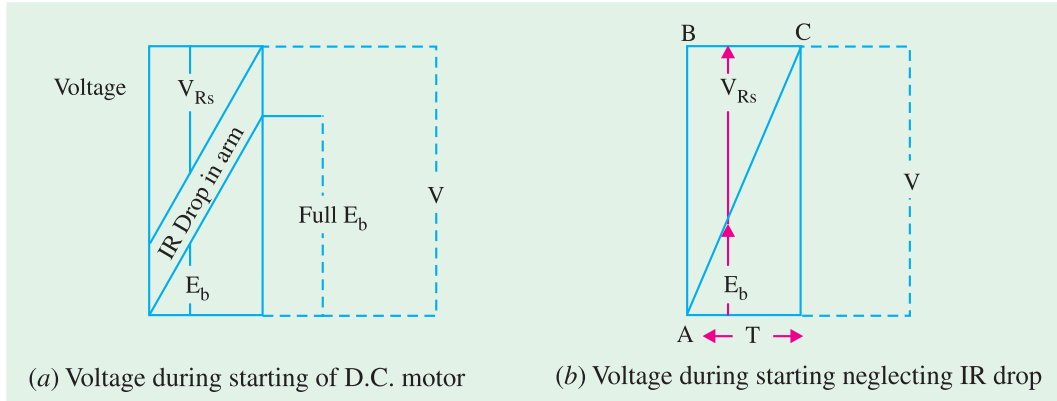


Fig. 43.28

If  $T$  is the time in sec. for starting and neglecting  $IR$  drop, total energy supplied =  $V.I.T$  watt-sec

From Fig. 43.28 (b) Energy wasted in  $R_s = \text{Area of triangle } ABC \times I = \frac{1}{2} \cdot T.V.I.$  watt - sec. =  $\frac{1}{2} VIT$  watt - sec. But total energy supplied =  $V.I.T$  watt - sec.

$\therefore$  Half the energy is wasted in starting

$\therefore \eta_{\text{starting}} = 50\%$

### 43.54. Series - Parallel Starting

With a 2 motor equipment  $\frac{1}{2}$  the normal voltage will be applied to each motor at starting as shown in Fig. 43.29 (a) (Series connection) and they will run upto approximate  $\frac{1}{2}$  speed, at which instant they are switched on to parallel and full voltage is applied to each motor.  $R_s$  is gradually cut-out, with motors in series connection and then reinserted when the motors are connected in parallel, and again gradually cut-out.

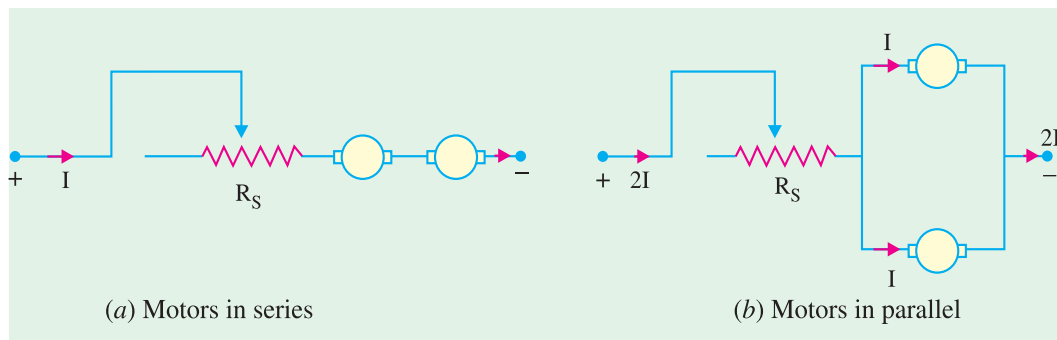


Fig. 43.29

In traction work, 2 or more similar motors are employed. Consider 2 series motors started by series parallel method, which results in saving of energy.

(a) **Series operation.** The 2 motors, are started in series with the help of  $R_s$ . The current during starting is limited to normal rated current ' $I$ ' per motor. During series operation, current ' $I$ ' is drawn

from supply. At the instant of starting  $OA = AB = IR$  drop in each motor.  $OK =$  Supply voltage ' $V$ '. The back e.m.fs. of 2 motors jointly develop along  $OM$  as shown in Fig. 43.30 (a). At point  $E$ , supply voltage  $V =$  Back e.m.fs of 2 motors  $+ IR$  drops of 2 motor. Any point on the line  $BC$  represents the sum of Back e.m.fs. of 2 motors  $+ IR$  drops of 2 motors  $+ Voltage$  across resistance  $R_s$  of 2 motors

$$OE = \text{time taken for series running.}$$

At pt ' $E$ ' at the end of series running period, each motor has developed a back e.m.f.

$$= \frac{V}{2} - IR$$

$$EL = ED - LD$$

**(b) Parallel operation.** The motors are switched on in parallel at the instant ' $E$ ', with  $R_s$  reinserted as shown in Fig. 43.29 (b). Current drawn is  $2I$  from supply. Back e.m.f. across each motor  $= EL$ . So the back e.m.f. now develops along  $LG$ . At point ' $H$ ' when the motors are in full parallel, ( $R_s = 0$  and both the motors are running at rated speed)

$$\text{Supply voltage} = V = HF = HG + GF$$

$$= \text{Normal Back e.m.f. of each motor} + IR \text{ drop in each motor.}$$

### 43.55. To find $t_s$ , $t_p$ and $\eta$ of starting

The values of time  $t_s$  during which the motors remain in series and  $t_p$  during which they are in parallel can be determined from Fig. 43.30 (a), (c). From Fig. 43.30 (a), triangles  $OLE$  and  $OGH$  are similar

$$\therefore \frac{OE}{OH} = \frac{LE}{GH} \therefore \frac{t_s}{T} = \frac{DE - DL}{FH - FG} = \frac{V/2 - IR}{V - IR}$$

$$\therefore t_s = \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T$$

$$t_p = T - t_s = T - \left\{ \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T \right\}$$

$$t_p = T \left\{ 1 - \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) \right\}$$

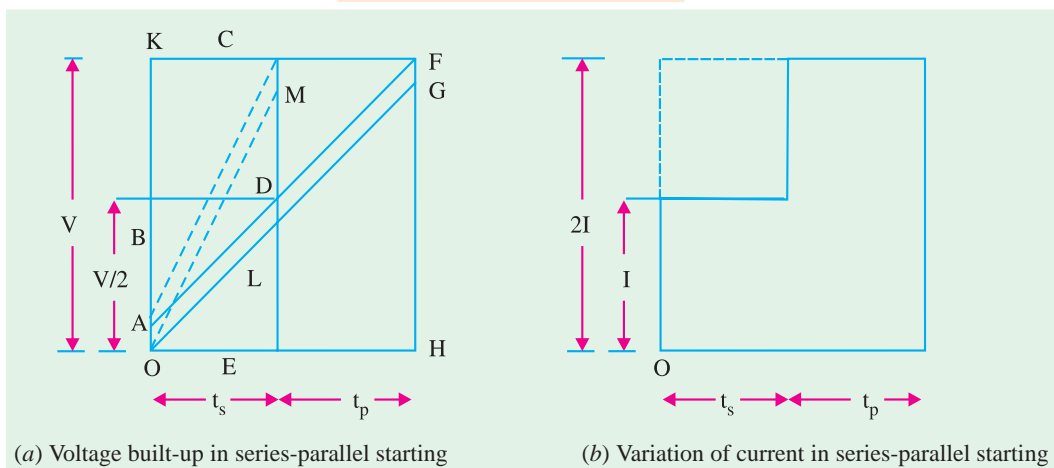


Fig. 43.30

To calculate  $\eta$  of starting, neglect  $IR$  drop in armature circuit.

This modifies Fig. 43.30 (a) to Fig. 43.30 (c). 'D' is midpoint of CE and back e.m.f. develops along DF in parallel combination.  $KC = CF$  i.e. time for series combination = time for parallel combination

i.e.  $t_s = t_p = t$  and average starting current =  $I$  per motor.

Energy lost in  $R_s =$  Area under triangle OKC + Area under triangle CDF

$$= \left(\frac{1}{2} VI\right) \times t + \left(\frac{1}{2} \frac{V}{2} 2I\right) \times t = VI t$$

But total energy supplied

$$= IVt + 2IVt$$

(Series) (Parallel)

$$= 3VI t$$

$$\therefore \eta \text{ of starting} = \frac{3VI t - VI t}{3VI t}$$

$$= \frac{2}{3} = 66.6\%$$

$\therefore \eta$  is increased by 16.66% as compared to previous case. If there are 4 motors then  $\eta_{\text{starting}} = 73\%$ . So there is saving of energy lost in  $R_s$ , during starting period as compared with starting by both motors in parallel.

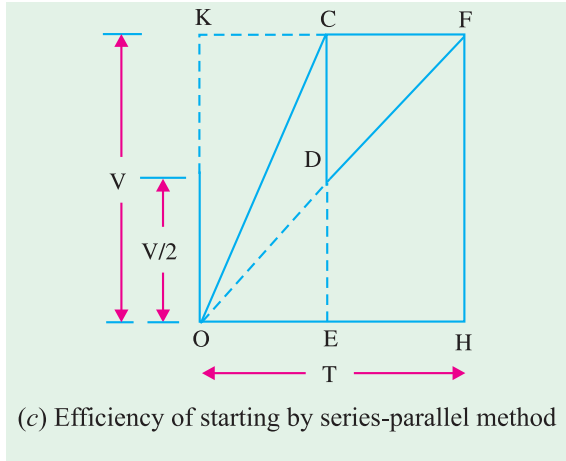


Fig. 43.30

**Example 43.28.** Two motors of a motor coach are started on series - parallel system, the current per motor being 350 A (Considered as being maintained constant) during the starting period which is 18 sec. If the acceleration during starting period is uniform, the line voltage is 600 V and resistance of each motor is 0.1 W. Find (a) the time during which the motors are operated in series. (b) the energy loss in the rheostat during starting period. [Nagpur University, Summer 2002]

**Solution.**

Time during which motors are in series is given by

$$t_s = \frac{1}{2} \left( \frac{V - 2IR}{V - IR} \right) T = \frac{1}{2} \left( \frac{600 - 2 \times 350 \times 0.1}{600 - 350 \times 0.1} \right) 18$$

$$t_s = 8.44 \text{ sec.}$$

Time during which motors are in parallel.

$$t_p = T - t_s = 18 - 8.44 = 9.56 \text{ sec.}$$

Back e.m.f.  $E_b$  of each motor, in series operation (from Fig. 43.30a)

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 350(0.1) = 265 \text{ V.}$$

When 2 motors are in series,

Total  $E_b = 265 + 265 = 530 \text{ V}$

$$E_{b_p} = V - IR = 600 - 350(0.1) = 565 \text{ V}$$

Energy lost when motors are connected in series

$$= \frac{1}{2} E_b I t_s = \frac{1}{2} \times 530 \times 350 \times \frac{8.44}{3600} = 217 \text{ watt - hours}$$

Energy lost when motors are connected in parallel

$$\frac{1}{2} \frac{E_b}{2} 2I t_p = \frac{1}{2} \times \frac{565}{2} \times 2 \times 350 \times \frac{9.56}{3600} = 262.5 \text{ watt - hour}$$

j 263 watt - hours

∴ Total energy lost = (217 + 263) watt - hours = **480 watt - hours**

**43.56. Series Parallel Control by Shunt Transition Method**

The various stages involved in this method of series – parallel control are shown in Fig. 43.31

In steps 1, 2, 3, 4 the motors are in series and are accelerated by cutting out the  $R_s$  in steps. In step 4, motors are in full series. During transition from series to parallel,  $R_s$  is reinserted in circuit– step 5. One of the motors is bypassed -step 6 and disconnected from main circuit – step 7. It is then connected in parallel with other motor -step 8, giving 1<sup>st</sup> parallel position.  $R_s$  is again cut-out in steps completely and the motors are placed in full parallel.

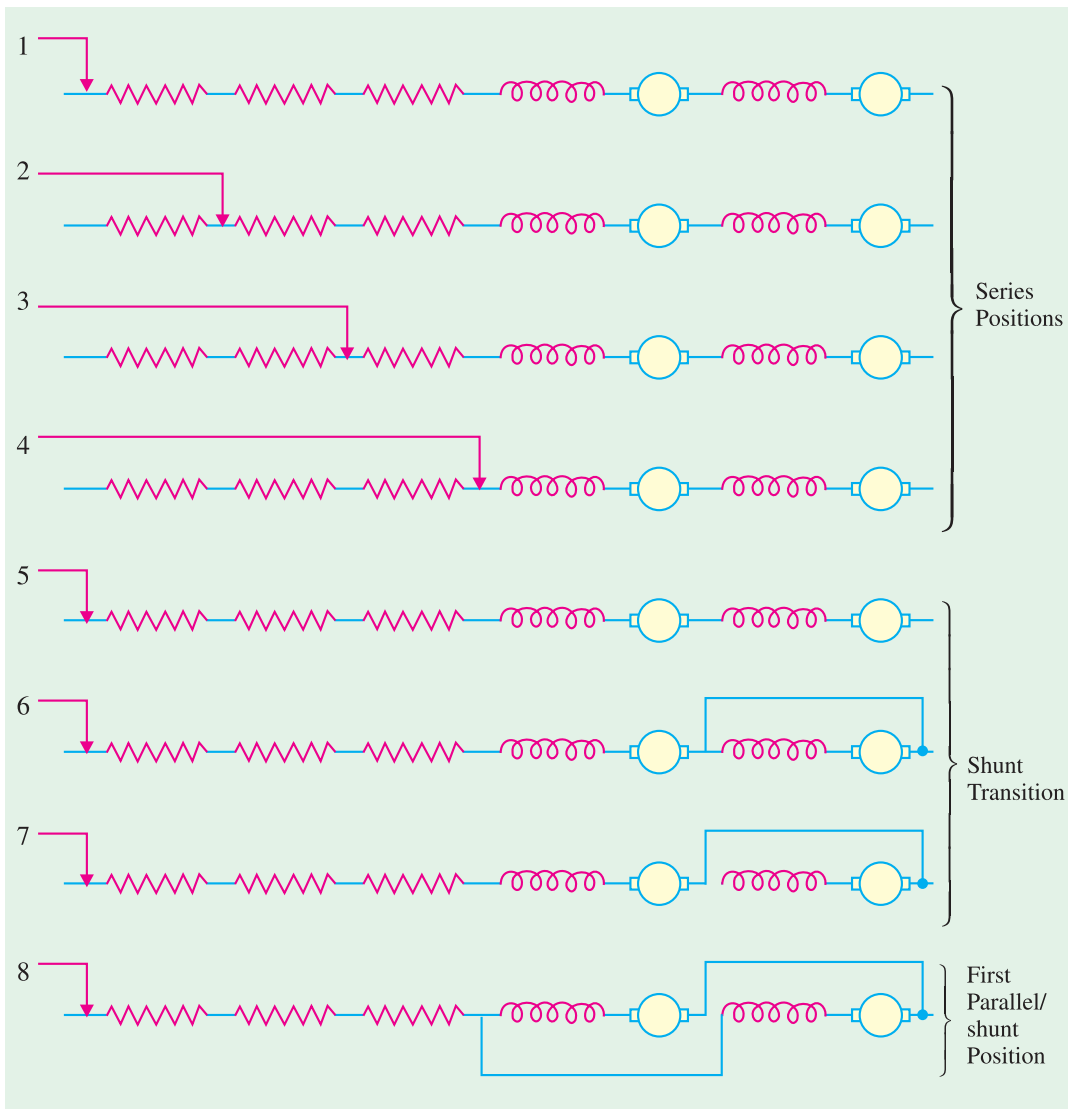


Fig. 43.31

The main difficulty with series parallel control is to obtain a satisfactory method of transition from series to parallel without interrupting the torque or allowing any heavy rushes of current.

In shunt transition method, one motor is short circuited and the total torque is reduced by about 50% during transition period, causing a noticeable jerk in the motion of vehicle.

The Bridge transition is more complicated, but the resistances which are connected in parallel with or 'bridged' across the motors are of such a value that current through the motors is not altered in magnitude and the total torque is therefore held constant and hence it is normally used for railways. So in this method it is seen that, both motors remain in circuit through-out the transition. Thus the jerks will not be experienced if this method is employed.

**43.57. Series Parallel Control by Bridge Transition**

- (a) At starting, motors are in series with  $R_s$  i.e. link P in position = AA'
- (b) Motors in full series with link P in position = BB' (No  $R_s$  in the circuit)

The motor and  $R_s$  are connected in the form of Wheatstone Bridge. Initially motors are in series with full  $R_s$  as shown in Fig. 43.32 (a). A and A' are moved in direction of arrow heads. In position BB' motors are in full series, as shown in Fig. 43.32 (b), with no  $R_s$  present in the circuit.

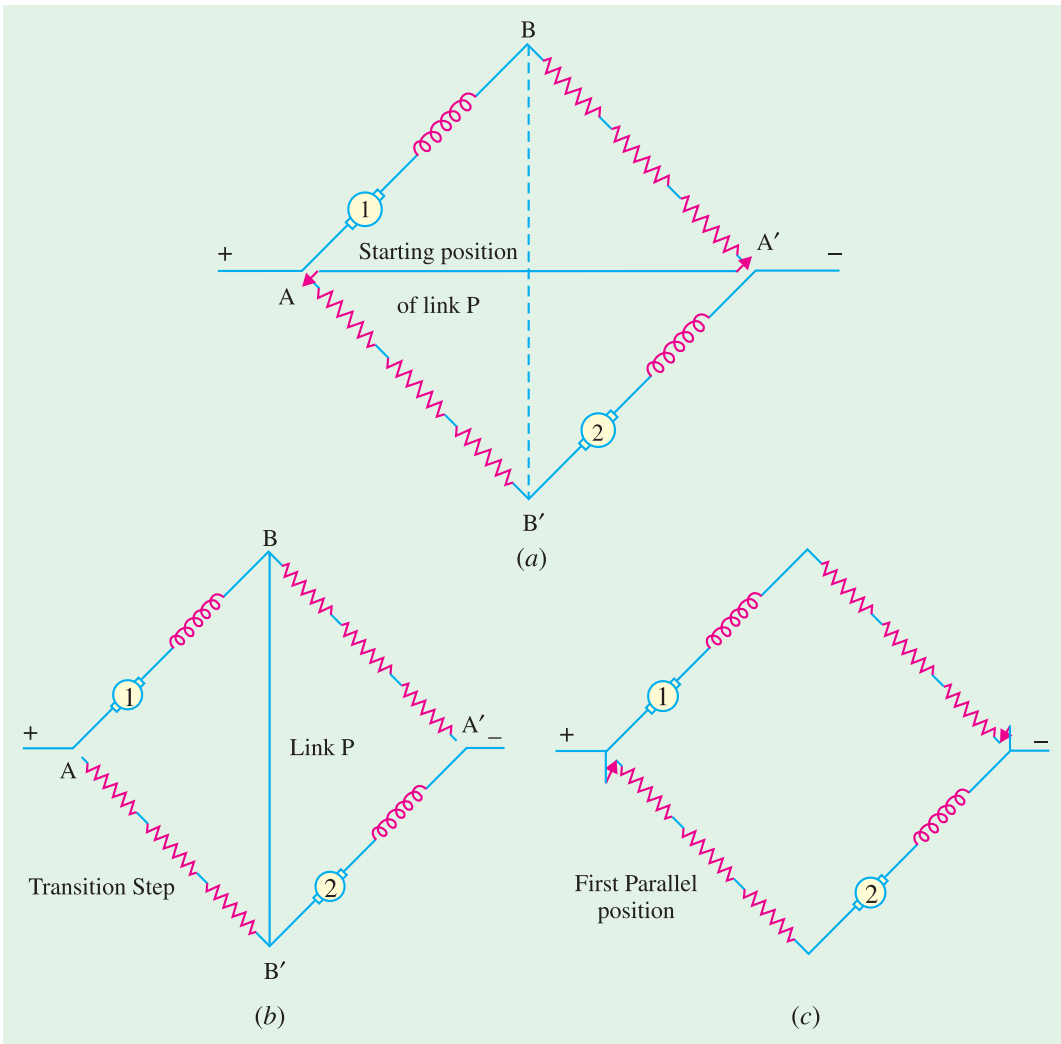


Fig. 43.32

In transition step the  $R_s$  is reinserted.

In I<sup>st</sup> parallel step, link  $P$  is removed and motors are connected in parallel with full  $R_s$  as shown in Fig. 43.32 (c). Advantage of this method is that the normal acceleration torque is available from both the motors, through - out starting period. Therefore acceleration is smoother, without any jerks, which is very much desirable for traction motors.

**43.58. Braking in Traction**

Both electrical and mechanical braking is used. Mechanical braking provides holding torque. Electric Braking reduces wear on mechanical brakes, provides higher retardation, thus bringing a vehicle quickly to rest. Different types of electrical braking used in traction are discussed.

**43.59. Rheostatic Braking**

(a) Equalizer Connection

(b) Cross Connection

**(a) Equalizer Connection**

For traction work, where 2 or more motors are employed, these are connected in parallel for braking, because series connection would produce too high voltage. K.E. of the vehicle is utilized in driving the machines as generators, which is dissipated in braking resistance in the form of heat.

To ensure that the 2 machines share the load equally, an equalizer connection is used as shown in Fig. 43.33 (a). If it is not used, the machine whose acceleration builds-up first would send a current through the 2<sup>nd</sup> machine in opposite direction, causing it to excite with reverse voltage. So that the 2 machines would be short circuited on themselves. The current would be dangerously high. Equalizer prevents such conditions. Hence Equalizer connection is important during braking in traction.

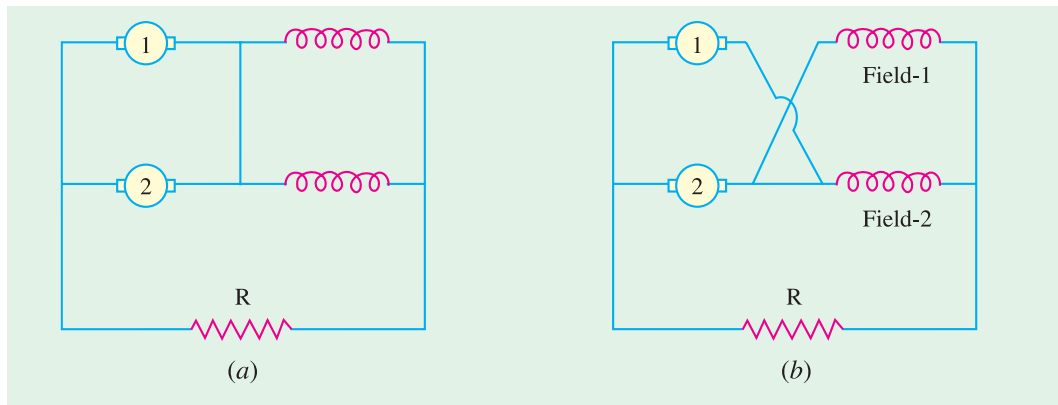


Fig. 43.33

**(b) Cross Connection**

In cross connection the field of machine 2 is connected in series with armature of machine 1 and the field of machine 1 is connected in series with armature of machine 2 as shown in Fig. 43.33 (b). Suppose the voltage of machine 1 is greater than that of 2. So it will send greater current through field of machine 2, causing it to excite to higher voltage. At the same time machine 1 excitation is low, because of lower voltage of machine 2. Hence machine 2 will produce more voltage and machine 1 voltage will be reduced. Thus automatic compensation is provided and the 2 machines operate satisfactorily.

Because of cross - connection during braking of traction motors, current in any of the motor will not go to a very high value.

### 43.60. Regenerative Braking with D.C. Motors

In order to achieve the regenerative braking, it is essential that (i) the voltage generated by the machine should exceed the supply voltage and (ii) the voltage should be kept at this value, irrespective of machine speed. Fig. 43.34 (a) shows the case of 4 series motors connected in parallel during normal running *i.e.* motoring.

One method of connection during regenerative braking, is to arrange the machines as shunt machines, with series fields of 3 machines connected across the supply in series with suitable resistance. One of the field winding is still kept in series across the 4 parallel armatures as shown in figure 43.34 (b).

The machine acts as a compound generator. (with slight differential compounding) Such an arrangement is quite stable; any change in line voltage produces a change in excitation which produces corresponding change in e.m.f. of motors, so that inherent compensation is provided *e.g.* let the line voltage tends to increase beyond the e.m.f. of generators. The increased voltage across the shunt circuit increases the excitation thereby increasing the generated voltage. Vice-versa is also true. The arrangement is therefore self compensating.

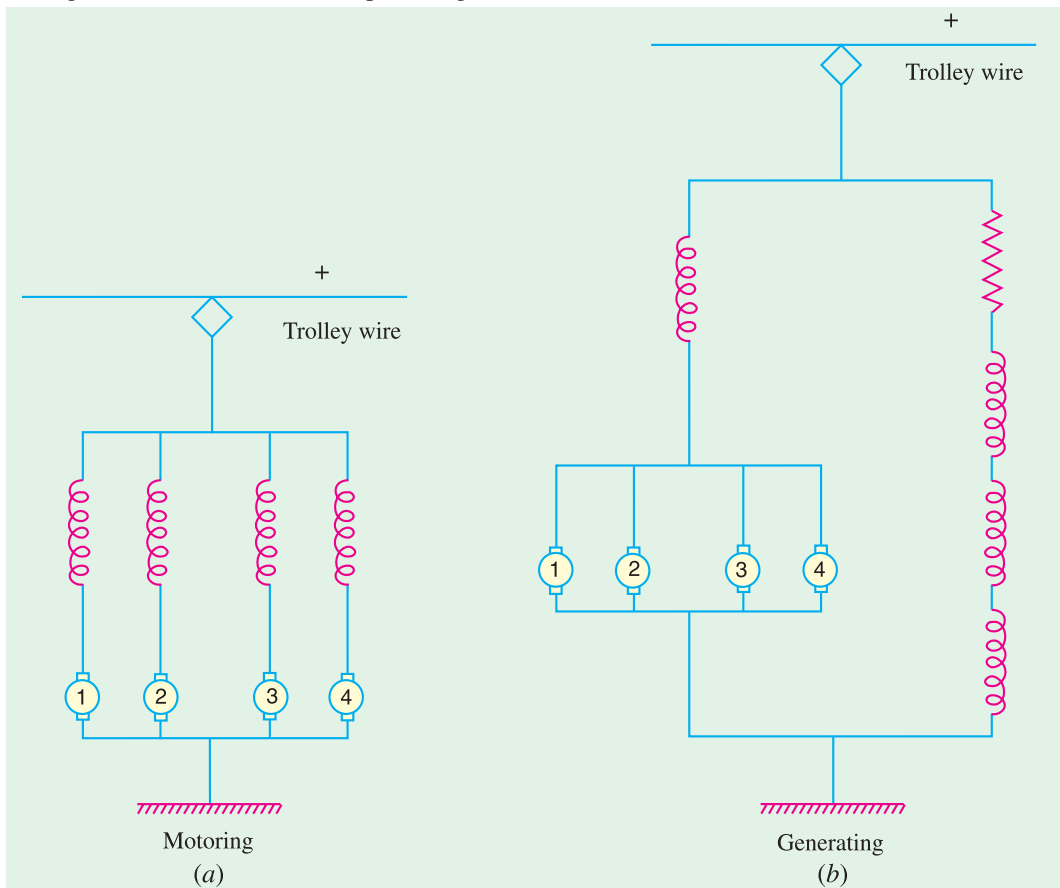


Fig. 43.34

D.C. series motor can't be used for regenerative braking without modification for obvious reasons. During regeneration current through armature reverses; and excitation has to be maintained. Hence field connection must be reversed.



**Example 43.29.** Two 750 V D.C. series motors each having a resistance of 0.1  $\Omega$  are started on series - parallel system. Mean current through - out the starting period is 300 A. Starting period is 15 sec. and train speed at the end of this period is 25 km/hr. Calculate

- (i) Rheostatic losses during series and parallel combination of motors
- (ii) Energy lost in motor
- (iii) Motor output
- (iv) Starting  $\eta$
- (v) Train speed at which transition from series to parallel must be made.

[Nagpur University, Summer 2000]

**Solution.**

$$(i) \quad t_s = \frac{1}{2} \left[ \frac{V - 2IR}{V - IR} \right] T$$

$$t_s = \frac{1}{2} \left[ \frac{750 - 2(300)0.1}{750 - (300)0.1} \right] 15 = 7.1875 \text{ sec.}$$

$$\therefore t_p = T - t_s = 7.8125 \text{ sec.}$$

$$\begin{aligned} \text{Energy lost in Rheostat} &= \frac{1}{2} E_{b_s} I t_s + \frac{1}{2} \frac{E_{b_p}}{2} 2 I t_p \\ &= \frac{1}{2} \left[ 2 \times \left[ \frac{V}{2} - IR \right] \right] I \cdot t_s + \frac{1}{2} [[V - IR]/2] 2I \cdot t_p \\ &= \frac{1}{2} \left[ 2 \times \left[ \frac{750}{2} - 300(0.1) \right] \right] 300 \times 7.1875 + \frac{1}{2} \left[ \frac{750 - 300(0.1)}{2} \right] \times 2(300) \times 7.1825 \\ &= 743906.25 + 843750 \\ &= 1587656.25 \text{ watt - sec.} \\ &= \frac{1587656.25}{3600} = \mathbf{441.00 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (ii) \text{ Total Energy supplied} &= V I t_s + 2 I \cdot V \cdot t_p \\ &= 750 \times 300 (7.1875) + 2 (300) 750 (7.8125) \\ &= 1617187.5 + 3515625 \\ &= 5132812.5 \text{ watt-sec} = 1425.7812 \text{ watt - hrs.} \end{aligned}$$

$$\begin{aligned} \text{Energy lost in 2 Motors} &= (I_a^2 \times R_a) \times 2 \times 15 \\ &= (300^2 \times 0.1) \times 2 \times 15 = 270000 \text{ watt - sec.} = \mathbf{75 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (iii) \text{ Motor O/P} &= \text{Total Energy supplied} - \text{Energy lost in Rheostat} - \text{Energy lost in armature} \\ &= 1425.7812 - 441 - 75 \\ &= \mathbf{909.7812 \text{ watt - hrs.}} \end{aligned}$$

$$\begin{aligned} (iv) \quad \eta \text{ starting} &= \frac{\text{Total Energy Supplied} - \text{Energy lost in Rheostat}}{\text{Total Energy Supplied}} \\ &= \frac{1425.7812 - 441.00}{1425.7812} \times 100 \\ &= \mathbf{69.0605\%} \end{aligned}$$

(v) Acceleration is uniform during starting period of 15 sec. Therefore speed after which series to parallel transition must be made is given as –

$$= \frac{\text{Speed after starting period}}{\text{Total starting period}} \times t_s$$

$$= \frac{25}{15} \times 7.1875$$

$$= \mathbf{11.9791 \text{ km/hr.}}$$

**Example 43.30.** Two 600-V motors each having a resistance of  $0.1\Omega$  are started on the series-parallel system, the mean current per motor throughout the starting period being 300A. The starting period is 20 seconds and the train speed at the end of this period is 30 km per hour. Calculate (i) the rheostatic losses (in kwh) during (a) the series and (b) the parallel combinations of motors (ii) the train speed at which transition from series to parallel must be made.

**Solution.**

Number of motors operating = 2  
 Line voltage,  $V = 600$  volts  
 Current per motor,  $I = 300$  amperes  
 Starting period,  $T_s = 20$  seconds  
 Motor resistance,  $R = 0.1 \Omega$   
 Maximum speed,  $V_m = 30$  kmph.  
 Back e.m.f. of each motor in full series position,

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 300 \times 0.1 = 270 \text{ volts.}$$

Back e.m.f. of each motor in full parallel position,

$$E_{b_p} = V - IR = 600 - 300 \times 0.1 = 570 \text{ volts}$$

Assuming smooth acceleration, back e.m.f. will be built up at constant rate.

Since motors take 20 seconds to build up 570 volts, therefore time taken to build up 270 volts e.m.f. will be :

$$T_{\text{series}} = 20 \times \frac{270}{570} = 9.4737 \text{ seconds}$$

$$T_{\text{parallel}} = 20 - 9.4737 = 10.5263 \text{ seconds}$$

(i) (a) Voltage drop in the starting rheostat in series combination at the starting instant  
 $= V - 2IR = 600 - 2 \times 300 \times 0.1 = 540$  volts,

which reduces to zero in full series position

Energy dissipated in starting resistance during series combination

$$= \frac{(V - 2IR) + 0}{2} \times I \times \frac{T_{\text{series}}}{3600} = \frac{540 + 0}{2} \times 300 \times \frac{9.4737}{3600}$$

$$= \mathbf{213.1579 \text{ watt - hours}}$$

(b) Voltage drop across the starting resistance in first parallel position is equal to  $V/2$  i.e. 300 volts which gradually reduces to zero.

Energy dissipated in starting resistance during parallel combination

$$= \frac{\frac{V}{2} + 0}{2} \times 2I \times \frac{T_{\text{parallel}}}{3600} = \frac{\frac{600}{2} + 0}{2} \times 2 \times 300 \times \frac{10.5263}{3600}$$

$$= \mathbf{263.1579 \text{ watt - hours}}$$

(ii) Acceleration,  $\alpha = \frac{\text{Maximum speed}}{\text{Starting period}} = \frac{V_m}{T_s} = \frac{30}{20} = 1.5 \text{ kmphps.}$

Speed at the end of series period =  $\alpha T_{\text{series}} = 1.5 \times 9.4737 = \mathbf{14.21 \text{ km/hour}}$

**Example 43.31.** Two d.c. series motors of a motor coach have resistance of  $0.1 \Omega$  each. These motors draw a current of 500 A from 600 V mains during series – parallel starting period of 25 seconds. If the acceleration during starting period remains uniform, determine:

- (i) time during which the motors operate in (a) series (b) parallel.
- (ii) the speed at which the series connections are to be changed if the speed just after starting period is 80 kmph.

**Solution.**

Number of motors operating = 2  
 Line voltage,  $V = 600 \text{ V}$   
 Current per motor,  $I = 500 \text{ A}$   
 Motor resistance,  $R = 0.1 \Omega$   
 Maximum speed,  $V_m = 80 \text{ kmph.}$

Back e.m.f. of each motor in full series position.

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 500 \times 0.1 = 250 \text{ V}$$

Back e.m.f. each motor in full parallel operation,

$$E_{b_p} = V - IR = 600 - 500 \times 0.1 = 550 \text{ V}$$

Since motors take 25 seconds to build up 550 V, therefore, time taken to build up 250 V, will be: (assuming smooth acceleration and building up of e.m.f. at constant rate.)

(i) Period of series operation,  $T_{\text{series}} = 25 \times \frac{255}{550} = \mathbf{11.3636 \text{ seconds}}$

Period of parallel operation,  $T_{\text{parallel}} = T - T_{\text{se}} = 25 - 11.3636 = \mathbf{13.6363 \text{ seconds}}$

(iii) Speed at which the series connections are to be changed

$$= \alpha T_{\text{series}} = \frac{V_m}{T} \cdot T_{\text{series}} = \frac{80}{25} \times 11.3636 = \mathbf{36.3636 \text{ kmph}}$$

**Example 43.32.** The following figures refer to the speed-current and torque – current characteristics of a 600 V d.c. series traction motor.

Current, amperes :	50	100	150	200	250
Speed, kmph :	73.6	48	41.1	37.3	35.2
Torque, N-m :	150	525	930	1,335	1,750

Determine the braking torque at a speed of 48 kmph when operating as self excited d.c. generator. Assume resistance of motor and braking rheostat to be  $0.6\Omega$  and  $3.0 \Omega$  respectively.

**Solution.**

**As motor :**

Terminal voltage,  $V = 600 \text{ volts.}$   
 The motor current at a speed of 48 kmph (from speed-current characteristic curve),  
 $I = 100 \text{ A}$

Back e.m.f. developed by the motor,  $E_b = V - IR_m = 600 - 100 \times 0.6 = \mathbf{540 \text{ V}}$

**As Generator:**

At the instant of applying rheostatic braking at speed of 48 kmph, the terminal voltage of machine will be equal to e.m.f. developed by the machine i.e. 540 volts.

Total resistance in the circuit =  $R_m + R_{\text{rheostat}} = 0.6 + 3 = 3.6 \Omega$

Current delivered by the machine,  $I = \frac{540}{3.6} = \mathbf{150 \text{ amps}}$

The braking torque (the torque corresponding to 150 amperes from torque-current curve)  
 $= \mathbf{930 \text{ N-m}}$

## Tutorial Problem No. 43.1

1. A train weighs 500 tonnes. What is its mass in (i) tonnes and (ii) kilograms.  
[*(i) 500 t (ii) 500,000 kg*]
2. A train has a mass of 200 tonnes. What is its weight in (i) newtons and (ii) kg-wt (iii) tonnes-wt.  
[*(i)  $19.6 \times 10^5$  N (ii) 200,000 kg. wt (iii) 200 t-wt )*]
3. A train has a speed of 100 km/h. What is its value in m/s ? [27.78 m/s]
4. A certain express train has an acceleration of 3.6 km/h/s. What is its value in  $\text{m/s}^2$  ? [1.0  $\text{m/s}^2$ ]
5. If there is an ascending gradient of 15 m in a track length of 1 km, what is the value of percentage gradient ? [1.5%]
6. A train runs at an average speed of 45 km per hour between stations 2.5 km apart. The train accelerates at 2 km/h/s and retards at 3 km/h/s. Find its maximum speed assuming a trapezoidal speed/time curve. Calculate also the distance travelled by it before the brakes are applied.  
[50.263 km/h, 2.383 km] (*Elect. Traction and Utilization B.H.U. )*
7. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20 percent higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with free running at the maximum speed.  
[2.73 km/h/s] (*Traction and Utilization of Elect. Power, Agra Univ.*)
8. A suburban electric train has a maximum speed of 65 km/h.. The schedule speed including a station stop of 30 seconds is 43.5 km/h. If the acceleration is 1.3 km/h/s, find the value of retardation when the average distance between stops is 3 km.  
[ $\beta = 1.21$  km/h/s] (*Utilization of Elect. Power and Traction, Gorakhpur Univ.,*)
9. An electric train is accelerated uniformly from rest to a speed of 40 km/h, the period of acceleration being 20 seconds. If it coasts for 60 seconds against a constant resistance of 50 N/t and is brought to rest in a further period of 10 seconds by braking, determine :  
(i) the acceleration (ii) the coasting retardation (iii) the braking retardation (iv) distance travelled and (v) schedule speed with station stops of 10 seconds duration.  
Allow 10 percent for rotational inertia. (*Elect. Traction, Punjab Univ.*)  
[ $\alpha = 2$  km/h/s,  $\beta_c = 0.1636$  km/h/s,  $\beta = 3$  km/h/s.,  $D = 0.736$  km.,  $V = 27.5$  km/h]
10. The speed-time curve of an electric train on a uniform rising gradient of 1 in 100 comprises :  
(i) uniform acceleration from rest at 2 km/h/s/ for 30 seconds.  
(ii) coasting with power off for 70 seconds.  
(iii) braking at 3 km/h/s to a standstill.  
The weight of the train is 250 tonnes, the train resistance on level track being 49 N/tonne and allowance for rotary inertia 1%.  
Calculate the maximum power developed by traction motors and total distance travelled by the train. Assume transmission efficiency as 97%.  
[3,3258 kW, 1.12 km] (*Traction and Utilization of Elect. Power, Agra Univ.*)
11. A 400-tonne goods train is to be hauled by a locomotive up a gradient of 2% with an acceleration of 1 km/h/s. Co-efficient of adhesion is 20%, track resistance 40 N/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and number of axles if the axle load is not increased beyond 22 tonnes. [152.6 tonnes, 7] (*Traction and Utilization of Elect. Power, Agra Univ.*)
12. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 20% with an acceleration of 1.2 km/h/s. Co-efficient of adhesion is 25%, track resistance 40 N/ tonne and effective rotating masses 10% of dead weight. Find the weight of the locomotive and number of axles if axle load is not to exceed 20 tonnes. [160 tonnes, 8] (*Utilization of Elect. Power, A.M.I.E. Winter*)

13. Determine the maximum adhesive weight of a loco required to start a 2340 tonne weight (inclusive of loco) on 1 : 150 gradient and accelerate it at 0.1 km/h/s. Assume co-efficient of adhesion as 0.25, train resistance 39.2 N/tonne and rotary inertia as 8%.  
**[128.5 tonnes]**  
*(Elect. Traction, A.M.I.E., May)*
14. Ore carrying trains weighing 5000 tonne each are to be hauled down a gradient of 1 in 60 at a maximum speed of 40 km/h and started on a level track at an acceleration of  $0.1 \text{ m/s}^2$ . How many locomotives, each weighing 75 tonne, will have to be employed ?  
 Train resistance during starting = 29.4 N/tonne  
 Train resistance at 40 km/h = 56.1 kg/tonne  
 Co-efficient of adhesion =  $1/3$  ; Rotational inertia =  $1/10$   
**[3 Loco]** *(Engg. Service Examination U.P.S.C.)*
15. A locomotive accelerates a 400-tonne train up a gradient of 1 in 100 at 0.8 km/h/s. Assuming the coefficient of adhesion to be 0.25, determine the minimum adhesive weight of the locomotive. Assume train resistance of 60 N/tonne and allow 10% for the effect of rotational inertia.  
**[65.7 t]** *(Elect. Traction and Utilization, Nagpur Univ.)*
16. Calculate the specific energy consumption if a maximum speed of 12.20 metres/sec and for a given run of 1525 m an acceleration of  $0.366 \text{ m/s}^2$  are desired. Train resistance during acceleration is 52.6 N/1000 kg and during coasting is 6.12 N/1000 kg, 10% being allowable for rotational inertia. The efficiency of the equipment during the acceleration period is 50%. Assume a quadrilateral speed-time curve.  
**[3.38 Wh/kg-m]** *(Util. of Elect. Power, A.M.I.E. Sec. B)*
17. An electric locomotive of 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an upgradient of 1/1000. Tractive resistance of the track is 45 N per tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find, (i) the trailing weight that can be hauled up the same gradient under the same conditions and (ii) the maximum gradient, the trailing weight hauled remaining unchanged.  
 Assume adhesive weight expressed as percentage of total dead weight to be the same for both the locomotive.  
**[(i) 1120 t (ii) 3.15%]** *(Util. of Elect. Power, A.M.I.E. Sec. B.)*
18. An electric train has quadrilateral speed-time curve as follows :  
 (i) uniform acceleration from rest at 2 km/h/s for 30 sec,  
 (ii) coasting for 50 sec.  
 (iii) uniform braking to rest for 20 seconds.  
 If the train is moving uniform upgradient of a 10/1000, train resistance is 40 N/tonne, rotational inertia effect 10% of dead weight and duration of stop 30 seconds, find the schedule speed.  
**[28.4 km/h]** *(Util. of Elect. Power, A.M.I.E. Sec. B.)*
19. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20% higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with the free running at the maximum speed.  
**[2.73 km/h/s]** *(Traction & Util. of Elect. Power, Agra Univ.)*
20. An electric train has an average speed of 42 km/h on a level track between stops 1,400 metre apart. It is accelerated at 1.7 km/h/s and is braked at 3.3 km/h/s. Draw the speed-time curve for the run. Estimate the sp. energy consumption. Assume tractive resistance as 50 N/t and allow 10% for rotational inertia.  
**[39.48 Wh/t-km]** *(Util. of Elect. Power, A.M.I.E. Sec. B.)*
21. An electric train weighing 200 tonne has eight motors geared to driving wheels, each wheel is 90 cm diameter. Determine the torque developed by each motor to accelerate the train to a speed of 48 km/h in 30 seconds up a gradient of 1 in 200. The tractive resistance is 50 N/t, the effect of rotational inertia is 10% of the train weight, the gear ratio is 4 to 1 and gearing efficiency is 80%.  
**[2,067 N-m]** *(Traction & Util. of Elect. Power, Agra Univ.)*

22. An electric train accelerates uniformly from rest to a speed of 48 km/h in 24 seconds. It then coasts for 69 seconds against a constant resistance of 58 N/t and is braked to rest at 3.3 km/h/s in 11 seconds.  
Calculate (i) the acceleration (ii) coasting retardation and (iii) the schedule speed, if the station stops are of 20 seconds duration. What would be the effect on schedule speed of reducing the station stops to 15 second duration, other conditions remaining the same? Allow 10% for the rotational inertia.  
**[(i) 2 km/h/s (ii) 0.19 km/h/s (iii) 30.25 km/h]**  
**(Util. of Elect. Power, A.M.I.E. Sec. B.)**
23. An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 1 minute 10 seconds against a constant resistance of 70 N/t and is braked to rest at 4 km/h/s in 10 seconds. Calculate the schedule speed, if the station stops are of 15 second duration.  
**[31.125 km/h] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
24. An electric train has a quadrilateral speed-time curve as follows :  
(i) uniform acceleration from rest at 2.5 km/h/s for 25 second  
(ii) coasting for 50 second (iii) duration of braking 25 second.  
If the train is moving along a uniform upgradient of 1 in 100 with a tractive resistance of 45 N/t, rotational inertia 10% of dead weight, duration of stops at stations 20 second and overall efficiency of transmission gear and motor 80%, calculate the schedule speed and specific energy consumption of run.  
**[69 km/h, 26.61 Wh/t-km] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
25. An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 in 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of  $0.08 \text{ m/s}^2$ . How many locomotives, each weighing 75 tonne, will have to be employed ?  
Train resistance during starting = 3 kg/t  
Train resistance at 30 km/h = 5 kg/t  
Co-efficient of adhesion = 0.3, Rotational inertia = 10%.  
**[4 loco] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
26. A train with an electric locomotive weighing 300 tonne is to be accelerated up a gradient of 1 in 33 at an acceleration of 1 km/h/s. If the train resistance, co-efficient of adhesion and effect of rotational inertia are 80 N/t, 0.25 and 12.5% of the dead weight respectively, determine the minimum adhesive weight of the locomotive.  
**[88 t] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
27. A train weighing 400 tonne has speed reduced by regenerative braking from 40 to 20 km/h over a distance of 2 km at a down gradient of 20%. Calculate the electrical energy and average power returned to the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion 75%.  
**[324 kW/h, 4860 kW] (Util. & Traction Power, Agra Univ.)**
28. A 250-tonne motor coach having 4 motors, each developing 5,000 N-m torque acceleration, starts from rest. If upgradient is 25 in 1000, gear ratio 5, gear transmission efficiency 88%, wheel radius 44 cm, train resistance 50 N/t addition of rotational inertia 10%, calculate the time taken to reach a speed of 45 km/h.  
If the supply voltage were 1500 V d.c. and efficiency of motor is 83.4%, determine the current drawn per motor during notching period.  
**[27.25 s, 500 A] (Util. of Elect. Power, A.M.I.E. Sec. B.)**
29. An electric train weighing 100 tonne has a rotational inertia of 10%. This train while running between two stations which are 2.5 km apart has an average speed of 50 km/h. The acceleration and retardation during braking are respectively 1 km/h/s and 2 km/h/s. The percentage gradient between these two stations is 1% and the train is to move up the incline. The track resistance is 40 N/t. If the combined efficiency of the electric train is 60%, determine (i) maximum power at driving axle (ii) total energy consumption and (iii) specific energy consumption. Assume that journey estimation is being made in simplified trapezoidal speed-time curve.  
**[(i) 875 kW (ii) 23.65 kW/h (iii) 94.6 Wh/t-km] (Util. of Elect. Power, A.M.I.E. Sec. B.)**

30. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 1 in 40 with an acceleration of 1.5 km/h/s. Determine the weight of the locomotive and number of axles, if axle load is not to exceed 24 tonne. Co-efficient of adhesion is 0.31, track resistance 45 N/t and effective rotating masses 10% of dead weight. *[7] (Util. of Elect. Power, A.M.I.E. Sec. B.)*
31. Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600V mains during series-parallel starting period of 20 seconds. If the acceleration during starting period remains uniform, determine :
- (i) time during which motor operates in (a) series, (b) parallel
- (ii) the speed at which the series connections are to be changed if the speed just after starting period is 70 km/h.
- [(i) 9.098, 10.971 Sec. (ii) 31.82 km/h] (Utili. of Elect. Power and Traction, Agra Univ.)*
32. Explain how series motors are ideally suited for traction service. *(Nagpur University, Summer 2004)*
33. Explain any one method for regenerative braking of D.C. motor for traction. *(Nagpur University, Summer 2004)*
34. Discuss the effect of unequal wheel diameters on the parallel operation of traction motors. *(Nagpur University, Summer 2004)*
35. Explain the various modes of operation in traction services with neat speed-time curve. *(Nagpur University, Summer 2004)*
36. A 100 tonne motor coach is driven by 4 motors, each developing a torque of 5000 N-m during acceleration. If up-gradient is 50 in 1000, gear ratio  $a = 0.25$ , gear transmission efficiency 98%, wheel radius 0.54 M, train resistance 25 N/tonne, effective mass on account of rotational inertia is 10% higher, calculate the time taken to attain a speed of 100 kmph. *(Nagpur University, Summer 2004)*
37. What are the requirements of an ideal traction system? *(J.N. University, Hyderabad, November 2003)*
38. What are the advantages and disadvantages of electric traction? *(J.N. University, Hyderabad, November 2003)*
39. Write a brief note on the single phase a.c. series motor and comment upon its suitability for traction services. How does it compare in performance with the d.c. Services motor. *(J.N. University, Hyderabad, November 2003)*
40. Draw the speed-time curve of a main line service and explain. *(J.N. University, Hyderabad, November 2003)*
41. A train has a scheduled speed of 40 km/hr between two stops, which are 4 kms apart. Determine the crest speed over the run, if the duration of stops is 60 sec and acceleration and retardation both are 2 km/hr/sec each. Assume simplified trapezoidal speed-time curve. *(J.N. University, Hyderabad, November 2003)*
42. What are the various electric traction systems in India? Compare them. *(J.N. University, Hyderabad, November 2003)*
43. Give the features of various motors used in electric traction. *(J.N. University, Hyderabad, November 2003)*
44. Draw the speed-time curve of a suburban service train and explain. *(J.N. University, Hyderabad, November 2003)*
45. A train accelerates to a speed of 48 km/hr in 24 sec. then it coasts for 69 sec under a constant resistance of 58 newton/tonne and brakes are applied at 3.3 km/hr/sec in 11 sec. Calculate (i) the acceleration (ii) the coasting retardation (iii) the scheduled speed if station stoppage is 20 secs. What is the effect of scheduled speed if station stoppage is reduced to 15 sec duration, other conditions remaining same. Allow 10% for rotational inertia. *(J.N. University, Hyderabad, November 2003)*

46. Derive an expression for specific energy output on level track using a simplified speed-time curve. What purpose is achieved by this quantity? *(J.N. University, Hyderabad, November 2003)*
47. A 400 tonne goods train is to be hauled by a locomotive up a gradient of 2% with acceleration of 1 km/hr/sec, coefficient of adhesion 20%, track resistance 40 newtons/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and the number of axles if the axle load is not to increase beyond 22 tonnes. *(J.N. University, Hyderabad, November 2003)*
48. A motor has the following load cycle :  
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.  
Full speed period 15-85 sec Load constant at 600 h.p.  
Decelerating period 85-100 sec h.p. returned to line falls uniformly from 200 to zero  
Decking period 100-120 sec Motor stationary. Estimate the size of the motor. *(J.N. University, Hyderabad, November 2003)*
49. Explain the characteristics of series motors and also explain how they are suitable for electric traction work? *(J.N. University, Hyderabad, November 2003)*
50. For a trapezoidal speed-time curve of a electric train, derive expression for maximum speed and distance between stops. *(J.N. University, Hyderabad, November 2003)*
51. A mail is to be run between two stations 5 kms apart at an average speed of 50 km/hr. If the maximum speed is to be limited to 70 km/hr, acceleration to 2 km/hr/sec, braking retardation to 4 km/hr/sec and coasting retardation to 0.1 km/hr/sec, determine the speed at the end of coasting, duration of coasting period and braking period. *(J.N. University, Hyderabad, November 2003)*
52. Discuss the merits and demerits of the D.C. and 1- $\phi$  A.C. systems for the main and suburban line electrification of the railways. *(J.N. University, Hyderabad, April 2003)*
53. Which system do you consider to be the best for the suburban railways in the vicinity of large cities? Given reasons for your answer. *(J.N. University, Hyderabad, April 2003)*
54. Derive expression for the tractive effort for a train on a level track. *(J.N. University, Hyderabad, April 2003)*
55. The maximum speed of a suburbanelectric train is 60 km/hr. Its scheduled speed is 40 km/hr and duration of stops is 30 sec. If the acceleration is 2 km/hr/sec and distance between stops is 2 kms, determine the retardation. *(J.N. University, Hyderabad, April 2003)*
56. What are various types of traction motors? *(J.N. University, Hyderabad, April 2003)*
57. What are the advantages of series parallel control of D.C. motors? *(J.N. University, Hyderabad, April 2003)*
58. Describe about duplication of railway transmission lines. *(J.N. University, Hyderabad, April 2003)*
59. Write a note on feeding and distributing system on A.C. Traction and for d.c. tram ways. *(J.N. University, Hyderabad, April 2003)*
60. For a quadrilateral speed-time curve of a electric train, derive expression for the distance between stops and speed at the end of the coasting period. *(J.N. University, Hyderabad, April 2003)*
61. A train is required to run between stations 1.6 kms apart at an average speed of 40 km/hr. The run is to be made from a quadrilateral speed-time curve. The acceleration is 2 km/hr/sec. The coasting and braking retardations are 0.16 km/hr/sec and 3.2 km/hr/sec respectively. Determine the duration of acceleration, coasting and braking and the distance covered in each period. *(J.N. University, Hyderabad, April 2003)*
62. Explain the characteristics of D.C. compound motors and explain its advantage over the series motor. *(J.N. University, Hyderabad, April 2003)*
63. What are the requirements to be satisfied by an ideal traction system? *(J.N. University, Hyderabad, April 2003)*
64. What are the advantages and disadvantages of electrification of track? *(J.N. University, Hyderabad, April 2003)*



65. Discuss why a D.C. series motor is ideally suited for traction services.  
(J.N. University, Hyderabad, April 2003)
66. An electric locomotive of 100 tonnes can just accelerate a train of 500 tonnes (trailing weight) with an acceleration of 1 km/hr/sec on an up gradient 1 in 1000. Tractive resistance of the track is 45 newtons/tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find (i) the trailing weight that can be hauled up the same gradient, under the same condition (ii) the maximum gradient, the trailing hauled load remaining unchanged. Assume adhesive weight expressed as percentage of total dead weight to be same for both the locomotives.  
(J.N. University, Hyderabad, April 2003)
67. Explain how electric regeneration braking is obtained with a D.C. locomotive. How is the braking torque varied?  
(J.N. University, Hyderabad, April 2003)
68. Explain why a series motor is preferred for the electric traction.  
(J.N. University, Hyderabad, April 2003)
69. The characteristics of a series motor at 525 – V are as follows :
- |             |      |     |     |     |
|-------------|------|-----|-----|-----|
| Current (A) | 50   | 100 | 150 | 200 |
| Speed (RPM) | 1200 | 952 | 840 | 745 |
- Determine the current when working as a generator at 1000 R.P.M. and loaded with a resistance of 3 ohms. The resistance of the motor is 0.5 ohms.  
(J.N. University, Hyderabad, April 2003)
70. Briefly explain the a.c. motors used in traction.  
(J.N. University, Hyderabad, April 2003)
71. The scheduled speed of a trolley service is to be 53 km/hr. The distance between stops is 2.8 km. The track is level and each stop is of 30 sec duration. Using simplified speed-time curve, calculate the maximum speed, assuming the acceleration to be 2 km/hr/sec, retardation 3.2 km/hr/sec, the dead weight of the car as 16 tonnes, rotational inertia as 10% of the dead weight and track resistance as 40 newtons/tonne. If the overall efficiency is 80%, calculate (i) the maximum power output from the driving axles (ii) the specific energy consumption in watt-hr/tonne-km.  
(J.N. University, Hyderabad, April 2003)
72. Discuss various traction systems you know of?  
(J.N. University, Hyderabad, December 2002/January 2003)
73. Explain the requirements for ideal traction and show which drive satisfies almost all the requirements.  
(J.N. University, Hyderabad, December 2002/January 2003)
74. Define the adhesive weight of a locomotive which accelerates up a gradient of 1 in 100 at 0.8 kmphs. The self weight of locomotive is 350 Tonnes. Coefficient of adhesion is 0.25. Assume a train resistance of 45 N–m/Tonne and allow 10% for the effect of rotational inertia.  
(J.N. University, Hyderabad, December 2002/January 2003)
75. State Factors affecting specific energy consumption.  
(J.N. University, Hyderabad, December 2002/January 2003)
76. Explain with the help of a diagram, the four quadrant speed-torque characteristic of an induction motor when running in (i) forward direction (ii) reverse direction.  
(J.N. University, Hyderabad, December 2002/January 2003)
77. Explain the general features of traction motors.  
(J.N. University, Hyderabad, December 2002/January 2003)
78. A 250 tonne electric train maintains a scheduled speed of 30 kmph between stations situated 5 km apart, with station stops of 30 sec. The acceleration is 1.8 kmph ps and the braking retardation is 3 kmph ps. Assuming a trapezoidal speed-time curve, calculate (i) maximum speed of the train (ii) energy output of the motors if the tractive resistance is 40 NW per tonne.  
(J.N. University, Hyderabad, December 2002/January 2003)
79. Discuss the relative merits of electric traction and the factors on which the choice of traction system depends.  
(J.N. University, Hyderabad, December 2002/January 2003)
80. Explain the terms (i) tractive effort (ii) coefficient of adhesion (iii) specific energy consumption of train (iv) tractive resistance.  
(J.N. University, Hyderabad, December 2002/January 2003)

81. Existing traction systems in India. *(J.N. University, Hyderabad, December 2002/January 2003)*
82. Explain the terms tractive effort, coefficient of adhesion, train resistance and specific energy consumption of train. *(J.N. University, Hyderabad, December 2002/January 2003)*
83. An electric train maintains a scheduled speed of 40 kmph between stations situated at 1.5 km apart. It is accelerated at 1.7 kmph.ps and is braked at 3.2 kmph.ps. Draw the speed-time curve for the run. Estimate the energy consumption at the axle of the train. Assume tractive resistance constants at 50 NW per tonne and allow 10% for the effect of rotation inertia. *(J.N. University, Hyderabad, December 2002/January 2003)*
84. Explain the advantages of series parallel control of starting as compared to the rheostatic starting for a pair of dc traction motors. *(J.N. University, Hyderabad, December 2002/January 2003)*
85. Discuss the main features of various train services. What type of services correspond to trapezoidal and quadrilateral speed-time curves. *(J.N. University, Hyderabad, December 2002/January 2003)*
86. Existing electric traction system in India. *(J.N. University, Hyderabad, December 2002/January 2003)*
87. Briefly explain the controlling of D.C. Motor. *(Anna Univ., Chennai 2003)*

### OBJECTIVE TESTS – 43

- Diesel electric traction has comparatively limited overload capacity because
  - diesel electric locomotive is heavier than a plain electric locomotive
  - diesel engine has shorter life span
  - diesel engine is a constant-kW output prime mover
  - regenerative braking cannot be employed.
- The most vital factor against electric traction is the
  - necessity of providing a negative booster
  - possibility of electric supply failure
  - high cost of its maintenance
  - high initial cost of laying out overhead electric supply system.
- The direct current system used for tramways has a voltage of about .....volt.
  - 750
  - 1500
  - 3000
  - 2400
- In electric traction if contact voltage exceeds 1500 V, current collection is invariably via a
  - contact rail
  - overhead wire
  - third rail
  - conductor rail.
- For the single-phase ac system of track electrification, low frequency is desirable because of the following advantages
  - it improves commutation properties of ac motors
  - it increases ac motor efficiency
  - it increases ac motor power factor
  - all of the above.
- In Kando system of track electrification, .....is converted into .....
  - 1-phase ac, dc
  - 3-phase ac, 1-phase ac
  - 1-phase ac, 3-phase ac
  - 3-phase ac, dc.
- The main reason for choosing the composite 1-phase ac-to-dc system for all future track electrification in India is that it
  - needs less number of sub-stations
  - combines the advantages of high-voltage ac distribution at 50 Hz with dc series traction motors
  - provides flexibility in the location of sub-stations
  - requires light overhead catenary.
- Ordinary, tramway is the most economical means of transport for
  - very dense traffic of large city
  - medium traffic densities
  - rural services
  - suburban services.
- Unlike a tramway, a trolleybus requires no
  - overhead contact wire
  - driving axles
  - hand brakes
  - running rail.

10. The current collector which can be used at different speeds under all wind conditions and stiffness of OHE is called ..... collector.
- trolley
  - bow
  - pantograph
  - messenger.
11. The speed/time curve for city service has no..... period.
- coasting
  - free-running
  - acceleration
  - braking.
12. For the same value of average speed, increase in the duration of stops..... speed.
- increases the schedule
  - increases the crest
  - decreases the crest
  - decreases the schedule.
13. A train weighing 490 tonne and running at 90 km/h has a mass of ..... kg and a speed of ..... m/s.
- 50,000, 25
  - 490,000, 25
  - 490, 25
  - 50, 324.
14. A train has a mass of 500 tonne. Its weight is
- 500 t.wt
  - 500,000 kg-wt
  - 4,900,000 newton
  - all of the above
  - none of the above.
15. The free-running speed of a train does NOT depend on the
- duration of stops
  - distance between stops
  - running time
  - acceleration.
16. A motor coach weighing 100 tonnes is to be given an acceleration of 1.0 km/h/s on an ascending gradient of 1 percent. Neglecting rotational inertia and train resistance, the tractive force required is ..... newton.
- 109,800
  - 37,580
  - 28,760
  - 125,780.
17. In a train, the energy output of the driving axles is used for
- accelerating the train
  - overcoming the gradient
  - overcoming train resistance
  - all of the above.
18. Longer coasting period for a train results in
- higher acceleration
  - higher retardation
  - lower specific energy consumption
  - higher schedule speed.
19. Tractive effort of an electric locomotive can be increased by
- increasing the supply voltage
  - using high kW motors
  - increasing dead weight over the driving axles
  - both (b) and (c) (e)both (a) and (b).
20. Skidding of a vehicle always occurs when
- braking effort exceeds its adhesive weight
  - it negotiates a curve
  - it passes over points and crossings
  - brake is applied suddenly.
21. Which of the following is an advantage of electric traction over other methods of traction?
- Faster acceleration
  - No pollution problems
  - Better braking action
  - All of the above
22. Which of the following is the voltage for single phase A.C. system?
- 22 V
  - 440 V
  - 5 kV
  - 15 kV
  - None of the above
23. Long distance railways use which of the following?
- 200 V D.C.
  - 25 kV single phase A.C.
  - 25 kV two phase A.C.
  - 25 kV three phase A.C.
24. The speed of a locomotive is controlled by
- flywheel
  - gear box
  - applying brakes
  - regulating steam flow to engine

25. Main traction system used in India are, those using
- electric locomotives
  - diesel engine locomotives
  - steam engine locomotives
  - diesel electric locomotives
  - all of the above
26. In India diesel locomotives are manufactured at
- Ajmer
  - Varanasi
  - Bangalore
  - Jamalpur
27. For diesel locomotives the range of horsepower is
- 50 to 200
  - 500 to 1000
  - 1500 to 2500
  - 3000 to 5000
28. .... locomotive has the highest operational availability.
- Electric
  - Diesel
  - Steam
29. The horsepower of steam locomotives is
- upto 1500
  - 1500 to 2000
  - 2000 to 3000
  - 3000 to 4000
30. The overall efficiency of steam locomotive is around
- 5 to 10 percent
  - 15 to 20 percent
  - 25 to 35 percent
  - 35 to 45 percent
31. In tramways which of the following motors is used?
- D.C. shunt motor
  - D.C. series motor
  - A.C. three phase motor
  - A.C. single phase capacitor start motor
32. In a steam locomotive electric power is provided through
- overhead wire
  - battery system
  - small turbo-generator
  - diesel engine generator
33. Which of the following drives is suitable for mines where explosive gas exists?
- Steam engine
  - Diesel engine
  - Battery locomotive
  - Any of the above
34. In case of locomotives the tractive power is provided by
- single cylinder double acting steam engine
  - double cylinder, single acting steam engine
  - double cylinder, double acting steam engine
  - single stage steam turbine
35. Overload capacity of diesel engines is usually restricted to
- 2 percent
  - 10 percent
  - 20 percent
  - 40 percent
36. In case of steam engines the steam pressure is
- 1 to 4 kgf/cm<sup>2</sup>
  - 5 to 8 kgf/cm<sup>2</sup>
  - 10 to 15 kgf/cm<sup>2</sup>
  - 25 to 35 kgf/cm<sup>2</sup>
37. The steam engine provided on steam locomotives is
- single acting condensing type
  - single acting non-condensing type
  - double acting condensing type
  - double acting non-condensing type
38. Electric locomotives in India are manufactured at
- Jamalpur
  - Bangalore
  - Chitranjan
  - Gorakhpur
39. The wheels of a train, engine as well as bogies, are slightly tapered to
- reduce friction
  - increase friction
  - facilitate braking
  - facilitate in taking turns
40. Automatic signalling is used for which of the following trains?

- (a) Mail and express trains
  - (b) Superfast trains
  - (c) Suburban and Urban electric trains
  - (c) All trains
41. The efficiency of diesel locomotives is nearly
- (a) 20 to 25 percent
  - (b) 30 to 40 percent
  - (c) 45 to 55 percent
  - (d) 60 to 70 percent
42. The speed of a superfast train is
- (a) 60 kmph
  - (b) 75 kmph
  - (c) 100 kmph
  - (d) more than 100 kmph
43. The number of passenger coaches that can be attached to a diesel engine locomotive on broad gauge is usually restricted to
- (a) 5
  - (b) 10
  - (c) 14
  - (d) 17
44. Which of the following state capitals is not on broad gauge track?
- (a) lucknow
  - (b) Bhopal
  - (c) Jaipur
  - (d) Chandigarh
45. Which of the following is the advantage of electric braking?
- (a) It avoids wear of track
  - (b) Motor continues to remain loaded during braking
  - (c) It is instantaneous
  - (d) More heat is generated during braking
46. Which of the following braking systems on the locomotives is costly?
- (a) Regenerative braking on electric locomotives
  - (b) Vacuum braking on diesel locomotives
  - (c) Vacuum braking on steam locomotives
  - (d) All braking systems are equally costly
47. Tractive effort is required to
- (a) overcome the gravity component of train mass
  - (b) overcome friction, windage and curve resistance
  - (c) accelerate the train mass
  - (d) do all of the above
48. For given maximum axle load tractive efforts of A.C. locomotive will be
- (a) less than that of D.C. locomotive
  - (b) more than that of D.C. locomotive
  - (c) equal to that of D.C. locomotive
  - (d) none of the above
49. Co-efficient of adhesion reduces due to the presence of which of the following?
- (a) Sand on rails
  - (b) Dew on rails
  - (c) Oil on the rails
  - (d) both (b) and (c)
50. Due to which of the following co-efficient of adhesion improves?
- (a) Rust on the rails
  - (b) Dust on the rails
  - (c) Sand on the rails
  - (d) All of the above
51. Quadrilateral speed-time curve pertains to which of the following services?
- (a) Main line service
  - (b) Urban service
  - (c) Sub-urban service
  - (d) Urban and sub-urban service
52. Which of the following is the disadvantage of electric traction over other systems of traction?
- (a) Corrosion problems in the underground pip work
  - (b) Short time power failure interrupts traffic for hours
  - (c) High capital outlay in fixed installations beside route limitation
  - (d) Interference with communication lines
  - (e) All of the above
53. Co-efficient of adhesion is
- (a) high in case of D.C. traction than in the case of A.C. traction
  - (b) low in case of D.C. traction than in the case of A.C. traction
  - (c) equal in both A.C. and D.C. traction
  - (d) any of the above
54. Speed-time curve of main line service differs from those of urban and suburban services on following account
- (a) it has longer free running period
  - (b) it has longer coasting period

- (c) accelerating and braking periods are comparatively smaller  
(d) all of the above
55. The rate of acceleration on suburban or urban services is restricted by the consideration of  
(a) engine power  
(b) track curves  
(c) passenger discomfort  
(d) track size
56. The specific energy consumption of a train depends on which of the following?  
(a) Acceleration and retardation  
(b) Gradient  
(c) Distance covered  
(d) all of the above
57. The friction at the track is proportional to  
(a)  $1/\text{speed}$   
(b)  $1/(\text{speed})^2$   
(c) speed  
(d) none of the above
58. The air resistance to the movement of the train is proportional to  
(a) speed  
(b)  $(\text{speed})^2$   
(c)  $(\text{speed})^3$   
(d)  $1/\text{speed}$
59. The normal value of adhesion friction is  
(a) 0.12  
(b) 0.25  
(c) 0.40  
(d) 0.75
60. The pulsating torque exerted by steam locomotives causes which of the following?  
(a) Jolting and skidding  
(b) Hammer blow  
(c) Pitching  
(d) All of the above
61. Which of the following braking systems is used on steam locomotives?  
(a) Hydraulic system  
(b) Pneumatic system  
(c) Vacuum system  
(d) None of the above
62. Vacuum is created by which of the following?  
(a) Vacuum pump  
(b) Ejector  
(c) Any of the above  
(d) None of the above
63. The resistance encountered by a train in motion is on account of  
(a) resistance offered by air  
(b) friction at the track  
(c) friction at various parts of the rolling stock  
(d) all of the above
64. Battery operated trucks are used in  
(a) steel mills  
(b) power stations  
(c) narrow gauge traction  
(d) factories for material transportation
65. .... method can bring the locomotive to dead stop.  
(a) Plugging braking  
(b) Rheostatic braking  
(c) Regenerative braking  
(d) None of the above
66. The value of co-efficient of adhesion will be high when rails are  
(a) greased  
(b) wet  
(c) sprayed with oil  
(d) cleaned with sand
67. The voltage used for suburban trains in D.C. system is usually  
(a) 12 V  
(b) 24 V  
(c) 220 V  
(d) 600 to 750 V
68. For three-phase induction motors which of the following is the least efficient method of speed control?  
(a) Cascade control  
(b) Pole changing  
(c) Rheostatic control  
(d) Combination of cascade and pole changing
69. Specific energy consumption becomes  
(a) more on steeper gradient  
(b) more with high train resistance  
(c) less if distance between stops is more  
(d) all of the above
70. In main line service as compared to urban and suburban service

- (a) distance between the stops is more  
 (b) maximum speed reached is high  
 (c) acceleration and retardation rates are low  
 (d) all of the above
71. Locomotive having monomotor bogies  
 (a) has better co-efficient of adhesion  
 (b) are suited both for passenger as well as freight service  
 (c) has better riding qualities due to the reduction of lateral forces  
 (d) has all above qualities
72. Series motor is not suited for traction duty due to which of the following account?  
 (a) Less current drain on the heavy load torque  
 (b) Current surges after temporary switching off supply  
 (c) self relieving property  
 (d) Commutating property at heavy load
73. When a bogie negotiates a curve, reduction in adhesion occurs resulting in sliding. Thus sliding is acute when  
 (a) wheel base of axles is more  
 (b) degree of curvature is more  
 (c) both (a) and (b)  
 (d) none of the above
74. Energy consumption in propelling the train is required for which of the following?  
 (a) Work against the resistance to motion  
 (b) Work against gravity while moving up the gradient  
 (c) Acceleration  
 (d) All of the above
75. An ideal traction system should have .....  
 (a) easy speed control  
 (b) high starting tractive effort  
 (c) equipment capable of with standing large temporary loads  
 (d) all of the above
76. .... have maximum unbalanced forces  
 (a) Diesel shunters  
 (b) Steam locomotives  
 (c) Electric locomotives  
 (d) Diesel locomotives
77. Specific energy consumption is affected by which of the following factors?  
 (a) Retardation and acceleration values  
 (b) Gradient  
 (c) Distance between stops  
 (d) All of the above
78. In case of ..... free running and coasting periods are generally long.  
 (a) main-line service  
 (b) urban service  
 (c) sub-urban service  
 (d) all of the above
79. Overhead lines for power supply to tramcars are at a minimum height of  
 (a) 3 m  
 (b) 6 m  
 (c) 10 m  
 (d) 20 m
80. The return circuit for tram cars is through .....  
 (a) neutral wire  
 (b) rails  
 (c) cables  
 (d) common earthing
81. Specific energy consumption is least in ..... service.  
 (a) main line  
 (b) urban  
 (c) suburban
82. Locomotives with monomotor bogies have  
 (a) uneven distribution of tractive effort  
 (b) suitability for passenger as well as freight service  
 (c) lot of skidding  
 (d) low co-efficient of adhesion
83. .... was the first city in India to adopt electric traction.  
 (a) Delhi  
 (b) Madras  
 (c) Calcutta  
 (d) Bombay
84. .... frequency is not common in low frequency traction system  
 (a) 40 Hz  
 (b) 25 Hz  
 (c) 16 Hz
85. For 25 kV single phase system power supply frequency is .....  
 (a) 60 Hz  
 (b) 50 Hz  
 (c) 25 Hz  
 (d)  $16\frac{2}{3}$  Hz

86. Power for lighting in passenger coach, in a long distance electric train, is provided
- (a) directly through overhead electric
  - (b) through individual generator of bogie and batteries
  - (c) through rails
  - (d) through locomotive
87. In India, electrification of railway track was done for the first time in which of the following years?
- (a) 1820–1825
  - (b) 1880–1885
  - (c) 1925–1932
  - (d) 1947–1954
88. Suri transmission is .....
- (a) electrical-pneumatic
  - (b) mechanical-electrical
  - (c) hydro-mechanical
  - (d) hydro-pneumatic
89. In case of a steam engine an average coal consumption per km is nearly
- (a) 150 to 175 kg
  - (b) 100 to 120 kg
  - (c) 60 to 80 kg
  - (d) 28 to 30 kg
90. Which of the following happens in Kando system?
- (a) Three phase A.C. is converted into D.C.
  - (b) Single phase A.C. is converted into D.C.
  - (c) Single phase supply is converted into three phase system
  - (d) None of the above
91. For which of the following locomotives the maintenance requirements are the least?
- (a) Steam locomotives
  - (b) Diesel locomotives
  - (c) Electric locomotives
  - (d) Equal in all of the above
92. Which of the following methods is used to control speed of 25 kV, 50 Hz single phase traction?
- (a) Reduced current method
  - (b) Tapchanging control of transformer
  - (c) Series parallel operation of motors
  - (d) All of the above
93. If the coefficient of adhesion on dry rails is 0.26, which of the following could be the value for wet rails?
- (a) 0.3
  - (b) 0.26
  - (c) 0.225
  - (d) 0.16
94. .... watt-hours per tonne km is usually the specific energy consumption for suburban services.
- (a) 15–20
  - (b) 50–75
  - (c) 120–150
  - (d) 160–200
95. The braking retardation is usually in the range
- (a) 0.15 to 0.30 km phps
  - (b) 0.30 to 0.6 km phps
  - (c) 0.6 to 2.4 km phps
  - (d) 3 to 5 km phps
  - (e) 10 to 15 km phps
96. The rate of acceleration on suburban or urban service is in the range
- (a) 0.2 to 0.5 km phps
  - (b) 1.6 to 4.0 km phps
  - (c) 5 to 10 km phps
  - (d) 15 to 25 km phps
97. The coasting retardation is around
- (a) 0.16 km phps
  - (b) 1.6 km phps
  - (c) 16 km phps
  - (d) 40 km phps
98. which of the following track is electrified
- (a) Delhi–Bombay
  - (b) Delhi–Madras
  - (c) Delhi–Howrah
  - (d) Delhi–Ahmedabad
99. .... is the method of braking in which motor armature remains connected to the supply and draws power from it producing torque opposite to the direction of motion.
- (a) Rheostatic braking
  - (b) Regenerative braking
  - (c) Plugging
100. For 600 V D.C. line for tramcars, track is connected to .....
- (a) positive of the supply
  - (b) negative of the supply
  - (c) mid voltage of 300 V
  - (d) none of the above



## ANSWERS

1. (c) 2. (d) 3. (a) 4. (b) 5. (d) 6. (c) 7. (b) 8. (a) 9. (d) 10. (c)  
11. (b) 12. (d) 13. (b) 14. (d) 15. (a) 16. (b) 17. (d) 18. (c) 19. (d) 20. (a)  
21. (d) 22. (d) 23. (b) 24. (d) 25. (e) 26. (b) 27. (c) 28. (a) 29. (a) 30. (a)  
31. (b) 32. (c) 33. (c) 34. (c) 35. (b) 36. (c) 37. (d) 38. (c) 39. (d) 40. (c)  
41. (a) 42. (d) 43. (d) 44. (c) 45. (a) 46. (a) 47. (d) 48. (b) 49. (d) 50. (d)  
51. (d) 52. (e) 53. (b) 54. (d) 55. (c) 56. (d) 57. (c) 58. (b) 59. (b) 60. (a)  
61. (c) 62. (c) 63. (d) 64. (d) 65. (a) 66. (d) 67. (d) 68. (c) 69. (d) 70. (d)  
71. (d) 72. (b) 73. (c) 74. (d) 75. (d) 76. (b) 77. (d) 78. (a) 79. (c) 80. (b)  
81. (a) 82. (b) 83. (d) 84. (a) 85. (b) 86. (b) 87. (c) 88. (c) 89. (d) 90. (c)  
91. (c) 92. (b) 93. (d) 94. (b) 95. (d) 96. (b) 97. (a) 98. (c) 99. (c) 100. (b)

## ROUGH WORK

# CHAPTER 44

## Learning Objectives

- Advantages of Electric Drive
- Classification of Electric Drives
- Advantages of Individual Drives
- Selection Drive
- Electric Characteristics
- Types of Enclosures
- Bearings Transmission of Power
- Noise
- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Motors for Different Industrial Drives
- Types of Electric Braking
- Plugging Applied to DC Motors
- Plugging of Induction Motors
- Rheostatic Braking
- Rheostatic Braking of DC Motors
- Rheostatic Braking Torque
- Rheostatic Braking of Induction Motors
- Regenerative Braking
- Energy Saving in Regenerative Braking

## INDUSTRIAL APPLICATIONS OF ELECTRIC MOTORS



The above figure shows a squirrel cage motor. In industries electric drive is preferred over mechanical drive, because electric drive has the advantages of quick start, high torques and comparatively hassle free operation

### 44.1. Advantages of Electric Drive

Almost all modern industrial and commercial undertakings employ electric drive in preference to mechanical drive because it possesses the following advantages :

1. It is simple in construction and has less maintenance cost
2. Its speed control is easy and smooth
3. It is neat, clean and free from any smoke or flue gases
4. It can be installed at any desired convenient place thus affording more flexibility in the layout
5. It can be remotely controlled
6. Being compact, it requires less space
7. It can be started immediately without any loss of time
8. It has comparatively longer life.

However, electric drive system has two inherent disadvantages :

1. It comes to stop as soon as there is failure of electric supply and
2. It cannot be used at far off places which are not served by electric supply.

However, the above two disadvantages can be overcome by installing diesel-driven dc generators and turbine-driven 3-phase alternators which can be used either in the absence of or on the failure of normal electric supply.

### 44.2. Classification of Electric Drives

Electric drives may be grouped into three categories : group drive, individual drive and multimotor drive.

In group drive, a single motor drives a number of machines through belts from a common shaft. It is also called line shaft drive. In the case of an individual drive, each machine is driven by its own separate motor with the help of gears, pulley etc. In multi-motor drives separate motors are provided for actuating different parts of the driven mechanism. For example, in travelling cranes, three motors are used : one for hoisting, another for long travel motion and the third for cross travel motion. Multimotor drives are commonly used in paper mills, rolling mills, rotary printing presses and metal working machines etc.

Each type of electric drive has its own advantages and disadvantages. The group drive has following advantages :

1. It leads to saving in initial cost because one 150-kW motor costs much less than ten 15-kW motors needed for driving 10 separate machines.
2. Since all ten motors will seldomly be required to work simultaneously, a single motor of even 100-kW will be sufficient to drive the main shaft. This diversity in load reduces the initial cost still further.
3. Since a single large motor will always run at full-load, it will have higher efficiency and power factor in case it is an induction motor.
4. Group drive can be used with advantage in those industrial processes where there is a sequence of continuity in the operation and where it is desirable to stop these processes simultaneously as in a flour mill.

However, group drive is seldom used these days due to the following disadvantages :

1. Any fault in the driving motor renders all the driven equipment idle. Hence, this system is unreliable.
2. If all the machines driven by the line shaft do not work together, the main motor runs at reduced load. Consequently, it runs with low efficiency and with poor power factor.

3. Considerable amount of power is lost in the energy transmitting mechanism.
4. Flexibility of layout of different machines is lost since they have to be so located as to suit the position of the line shaft.
5. The use of line shaft, pulleys and belts etc. makes the drive look quite untidy and less safe to operate.
6. It cannot be used where constant speed is required as in paper and textile industry.
7. Noise level at the worksite is quite high.

### 44.3. Advantages of Individual Drive

It has the following advantages :

1. Since each machine is driven by a separate motor, it can be run and stopped as desired.
2. Machines not required can be shut down and also replaced with a minimum of dislocation.
3. There is flexibility in the installation of different machines.
4. In the case of motor fault, only its connected machine will stop whereas others will continue working undisturbed.
5. The absence of belts and line shafts greatly reduces the risk of accidents to the operating personnel.
6. Each operator has full control of the machine which can be quickly stopped if an accident occurs.
7. Maintenance of line shafts, bearings, pulleys and belts etc. is eliminated. Similarly there is no danger of oil falling on articles being manufactured—something very important in textile industry.

The only disadvantage of individual drive is its initial high cost (Ex 44.1). However, the use of individual drives and multimotor drives has led to the introduction of automation in production processes which, apart from increasing the productivity of various undertakings, has increased the reliability and safety of operation.

**Example 44.1.** A motor costing Rs. 10,000/- is used for group drive in a certain installation. How will its total annual cost compare with the case where four individual motors each costing Rs. 4000/- were used? With group drive, the energy consumption is 50 MWh whereas it is 45 MWh for individual drive. The cost of electric energy is 20 paise/kWh. Assume depreciation, maintenance and other fixed charges at 10% in the case of group drive and 15 per cent in the case of individual drive.

**Solution.** Group Drive

Capital cost	= Rs. 10,000/-
Annual depreciation, maintenance and other fixed charges	= 10% of Rs. 10,000 = Rs. 1,000/-
Annual cost of energy	= Rs. $50 \times 10^3 \times (20/100)$ = Rs. 10,000/-
Total annual cost	= Rs. 1,000 + Rs. 10,000 = <b>Rs. 11,000/-</b>

**Individual Drive**

Capital cost	= 4 × Rs. 4000 = Rs. 16,000/-
Annual depreciation, maintenance and other fixed charges	= 15% of Rs. 16,000 = Rs. 2400/-
Annual cost of energy	= Rs. $45 \times 10^3 \times (20/100)$ = Rs. 9000/-
Total annual cost	= Rs. 9000 + Rs. 2400 = <b>Rs. 11,400/-</b>

It is seen from the above example that individual drive is costlier than the group drive.

#### 44.4. Selection of a Motor

The selection of a driving motor depends primarily on the conditions under which it has to operate and the type of load it has to handle. Main guiding factors for such a selection are as follows :

**(a) Electrical characteristics**

- |                             |                            |
|-----------------------------|----------------------------|
| 1. Starting characteristics | 2. Running characteristics |
| 3. Speed control            | 4. Braking                 |

**(b) Mechanical considerations**

- |                                 |                     |
|---------------------------------|---------------------|
| 1. Type of enclosure            | 2. Type of bearings |
| 3. Method of power transmission | 4. Type of cooling  |
| 5. Noise level                  |                     |

**(c) Size and rating of motors**

1. Requirement for continuous, intermittent or variable load cycle
2. Overload capacity

**(d) Cost**

- |                 |                 |
|-----------------|-----------------|
| 1. Capital cost | 2. Running cost |
|-----------------|-----------------|

In addition to the above factors, one has to take into consideration the type of current available whether alternating or direct. However, the basic problem is one of matching the mechanical output of the motor with the load requirement *i.e.* to select a motor with the correct speed/torque characteristics as demanded by the load. In fact, the complete selection process requires the analysis and synthesis of not only the load and the proposed motor but the complete drive assembly and the control equipment which may include rectification or frequency changing.

#### 44.5. Electrical Characteristics

Electrical characteristics of different electric drives have been discussed in Vol. II of this book entitled "A.C. and D.C. Machines".

#### 44.6. Types of Enclosures

The main function of an enclosure is to provide protection not only to the working personnel but also to the motor itself against the harmful ingress of dirt, abrasive dust, vapours and liquids and solid foreign bodies such as a spanner or screw driver etc. At the same time, it should not adversely affect the proper cooling of the motor. Hence, different types of enclosures are used for different motors depending upon the environmental conditions. Some of the commonly used motor enclosures are as under :

**1. Open Type.** In this case, the machine is open at both ends with its rotor being supported on pedestal bearings or end brackets. There is free ventilation since the stator and rotor ends are in free contact with the surrounding air. Such, machines are housed in a separate neat and clean room. This type of enclosure is used for large machines such as d.c. motors and generators.

**2. Screen Protected Type.** In this case, the enclosure has large openings for free ventilation. However, these openings are fitted with screen covers which safeguard against accidental contacts and rats entering the machine but afford no protection from dirt, dust and falling water. Screen-protected type motors are installed where dry and neat conditions prevail without any gases or fumes.

**3. Drip Proof Type.** This enclosure is used in very damp conditions. *i.e.* for pumping sets. Since motor openings are protected by over-hanging cowls, vertically falling water and dust are not able to enter the machine.

**4. Splash-proof Type.** In such machines, the ventilating openings are so designed that liquid or dust particles at an angle between vertical and  $100^\circ$  from it cannot enter the machine. Such type of motors can be safely used in rain.

**5. Totally Enclosed (TE) Type.** In this case, the motor is completely enclosed and no openings are left for ventilation. All the heat generated due to losses is dissipated from the outer surface which is finned to increase the cooling area. Such motors are used for dusty atmosphere *i.e.* sawmills, coal-handling plants and stone-crushing quarries etc.

**6. Totally-enclosed Fan-cooled (TEFC) Type.** In this case, a fan is mounted on the shaft external to the totally enclosed casing and air is blown over the ribbed outer surfaces of the stator and endshields (Fig. 44.1). Such motors are commonly used in flour mills, cement works and sawmills etc. They require little maintenance apart from lubrication and are capable of giving years of useful service without any interruption of production.



Fig. 44.1. A three-phase motor

**7. Pipe-ventilated Type.** Such an enclosure is used for very dusty surroundings. The motor is totally enclosed but is cooled by neat and clean air brought through a separate pipe from outside the dust-laden area. The extra cost of the piping is offset by the use of a smaller size motor on account of better cooling.

**8. Flame-proof (FLP) Type.** Such motors are employed in atmospheres which contain inflammable gases and vapours *i.e.* in coal mines and chemical plants. They are totally enclosed but their enclosures are so constructed that any explosion within the motor due to any spark does not ignite the gases outside. The maximum operating temperature at the surface of the motor is much less than the ignition temperature of the surrounding gases.

#### 44.7. Bearings

These are used for supporting the rotating parts of the machines and are of two types :

1. Ball or roller bearings
  2. Sleeve or bush bearings
- (a) **Ball Bearings**

Upto about 75kW motors, ball bearings are preferred to other bearings because of their following advantages :

1. They have low friction loss
2. They occupy less space
3. They require less maintenance
4. Their use allows much smaller air-gap between the stator and rotor of an induction motor
5. Their life is long.

Their main disadvantages are with regard to cost and noise particularly at high motor speeds.

(b) **Sleeve Bearings**

These are in the form of self-aligning porous bronze bushes for fractional kW motors and in the



A ball-bearing

form of journal bearings for larger motors. Since they run very silently, they are fitted on super-silent motors used for driving fans and lifts in offices or other applications where noise must be reduced to the absolute minimum.

#### 44.8. Transmission of Power

There are many ways of transmitting mechanical power developed by a motor to the driven machine.

**1. Direct Drive.** In this case, motor is coupled directly to the driven machine with the help of solid or flexible coupling. Flexible coupling helps in protecting the motor from sudden jerks. Direct drive is nearly 100% efficient and requires minimum space but is used only when speed of the driven machine equals the motor speed.

**2. Belt Drive.** Flat belts are extensively used for line-shaft drives and can transmit a maximum power of about 250 kW. Where possible, the minimum distance between the pulley centres should be 4 times the diameter of the larger pulley with a maximum ratio between pulley diameters of 6 : 1. The power transmitted by a flat belt increases in proportion to its width and varies greatly with its quality and thickness. There is a slip of 3 to 4 per cent in the belt drive.

**3. Rope Drive.** In this drive, a number of ropes are run in V-grooves over the pulleys. It has negligible slip and is used when the power to be transmitted is beyond the scope of belt drive.



Fig. 44.2. Geared Motor Unit

consisting of a flange motor bolted to a high-efficiency gear box which is usually equipped with feet, the motor being overhung.

**4. Chain Drive.** Though somewhat more expensive, it is more efficient and is capable of transmitting larger amounts of power. It is noiseless, slipless and smooth in operation.

**5. Gear Drive.** It is used when a high-speed motor is to drive a low-speed machine. The coupling between the two is through a suitable ratio gear box. In fact motors for low-speed drives are manufactured with the reduction gear incorporated in the unit itself. Fig. 44.2 shows such a unit



A sleeve bearing

#### 44.9. Noise

The noise produced by a motor could be magnetic noise, windage noise and mechanical noise. Noise level must be kept to the minimum in order to avoid fatigue to the workers in a workshop. Similarly, motors used for domestic and hospital appliances and in offices and theatres must be almost noiseless. Transmission of noise from the building where the motor is installed to another building can be reduced if motor foundation is flexible *i.e.* has rubber pads and springs.

#### 44.10. Motors for Different Industrial Drives

**1. D.C. Series Motor.** Since it has high starting torque and variable speed, it is used for heavy duty applications such as electric locomotives, steel rolling mills, hoists, lifts and cranes.

**2. D.C. Shunt Motor.** It has medium starting torque and a nearly constant speed. Hence, it is used for driving constant-speed line shafts, lathes, vacuum cleaners, wood-working machines, laundry washing machines, elevators, conveyors, grinders and small printing presses etc.

**3. Cumulative Compound Motor.** It is a varying-speed motor with high starting torque and



is used for driving compressors, variable-head centrifugal pumps, rotary presses, circular saws, shearing machines, elevators and continuous conveyors etc.

**4. Three-phase Synchronous Motor.** Because its speed remains constant under varying loads, it is used for driving continuously-operating equipment at constant speed such as ammonia and air compressors, motor-generator sets, continuous rolling mills, paper and cement industries.

**5. Squirrel Cage Induction Motor.** This motor is quite simple but rugged and possesses high over-load capacity. It has a nearly constant speed and poor starting torque. Hence, it is used for low and medium power drives where speed control is not required as for water pumps, tube wells, lathes, drills, grinders, polishers, wood planers, fans, blowers, laundry washing machines and compressors etc.

**6. Double Squirrel Cage Motor.** It has high starting torque, large overload capacity and a nearly constant speed. Hence, it is used for driving loads which require high starting torque such as compressor pumps, reciprocating pumps, large refrigerators, crushers, boring mills, textile machinery, cranes, punches and lathes etc.

**7. Slip-ring Induction Motor.** It has high starting torque and large overload capacity. Its speed can be changed upto 50% of its normal speed. Hence, it is used for those industrial drives which require high starting torque and speed control such as lifts, pumps, winding machines, printing presses, line shafts, elevators and compressors etc.

**8. Single-phase Synchronous Motor.** Because of its constant speed, it is used in teleprinters, clocks, all kinds of timing devices, recording instruments, sound recording and reproducing systems.

**9. Single-phase Series Motor.** It possesses high starting torque and its speed can be controlled over a wide range. It is used for driving small domestic appliances like refrigerators and vacuum cleaners etc.

**10. Repulsion Motor.** It has high starting torque and is capable of wide speed control. Moreover, it has high speed at high loads. Hence, it is used for drives which require large starting torque and adjustable but constant speed as in coil winding machines.

**11. Capacitor-start Induction-run Motor.** It has fairly constant speed and moderately high starting torque. Speed control is not possible. It is used for compressors, refrigerators and small portable hoists.

**12. Capacitor-start-and-run Motor.** Its operating characteristics are similar to the above motor except that it has better power factor and higher efficiency. Hence, it is used for drives requiring quiet operations.



#### 44.11. Advantages of Electrical Braking Over Mechanical Braking

1. In mechanical braking; due to excessive wear on brake drum, liner etc. it needs frequent and costly replacement. This is not needed in electrical braking and so electrical braking is more economical than mechanical braking.
2. Due to wear and tear of brake liner frequent adjustments are needed thereby making the maintenance costly.
3. Mechanical braking produces metal dust, which can damage bearings. Electrical braking has no such problems.
4. If mechanical brakes are not correctly adjusted it may result in shock loading of machine or machine parts in case of lift, trains which may result in discomfort to the occupants.
5. Electrical braking is smooth.

6. In mechanical braking the heat is produced at brake liner or brake drum, which may be a source of failure of the brake. In electric braking the heat is produced at convenient place, which in no way is harmful to a braking system.
7. In regenerative braking electrical energy can be returned back to the supply which is not possible in mechanical braking.
8. Noise produced is very high in mechanical braking.

Only disadvantage in electrical braking is that it is ineffective in applying holding torque.

#### 44.12. Types of Electric Braking

There are three types of electric braking as applicable to electric motors in addition to eddy-current braking. These have already been discussed briefly in Art. 44.7.

1. Plugging or reverse-current braking.
2. Rheostatic or dynamic braking.
3. Regenerative braking.

In many cases, provision of an arrangement for stopping a motor and its driven load is as important as starting it. For example, a planing machine must be quickly stopped at the end of its stroke in order to achieve a high rate of production. In other cases, rapid stops are essential for preventing any danger to operator or damage to the product being manufactured. Similarly, in the case of lifts and hoists, effective braking must be provided for their proper functioning.

#### 44.13. Plugging Applied to D.C. Motors

As discussed earlier in Art. 42.7, in this case, armature connections are reversed whereas *field winding connections remains unchanged*. With reversed armature connections, the motor develops a torque in the *opposite* direction. When speed reduces to zero, motor will accelerate in the opposite direction. Hence, the arrangement is made to disconnect the motor from the supply as soon as it comes to rest. Fig. 44.3 shows running and reversed connections for shunt motors whereas Fig. 44.4 shows similar conditions for series motors.

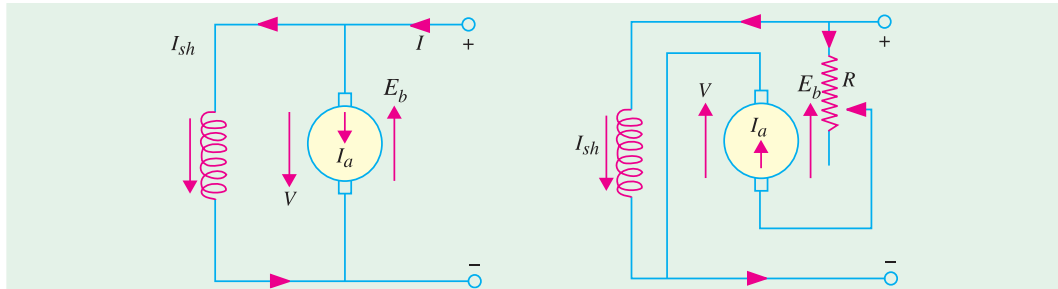


Fig. 44.3

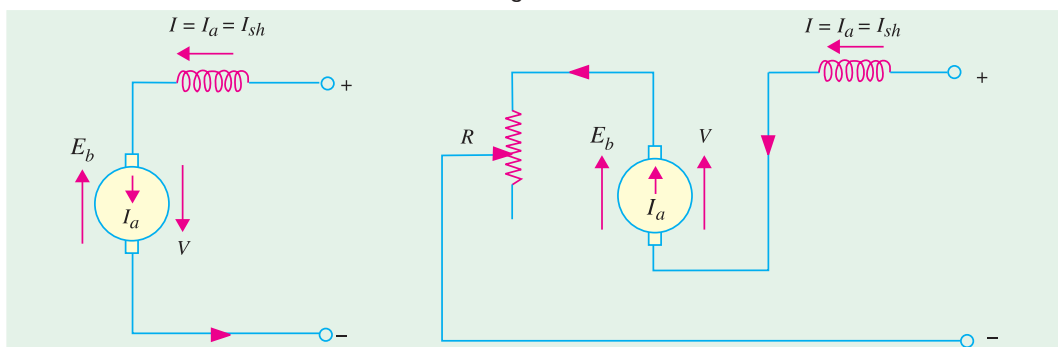


Fig. 44.4

Since with reversed connection,  $V$  and  $E_b$  are in the same direction, voltage across the armature is almost double of its normal value. In order to avoid excessive current through the armature, additional resistance  $R$  is connected in series with armature.

This method of braking is wasteful because in addition to wasting kinetic energy of the moving parts, it draws additional energy from the supply during braking.

**Braking Torque.** The electric braking torque is given by

$$T_B \propto \Phi I_a = k_1 \Phi I_a; \text{ Now, } I_a = (V + E_b)/R$$

$$\therefore T_B = K_1 \Phi \cdot \frac{V + E_b}{R} = k_1 \Phi \frac{V + k_2 \Phi N}{R} \quad (\because E_b \propto \Phi N)$$

$$= \frac{K_1 \Phi V}{R} + \frac{k_1 k_2 \Phi^2 N}{R} = k_3 \Phi + k_4 \Phi^2 N$$

**Shunt Motor**

Since in the case,  $\Phi$  is practically constant,  $T_B = k_5 + k_6 N$ .

**Series Motor**

$$T_B = k_3 \Phi + k_4 \Phi^2 N = k_5 I_a + k_6 N I_a^2 \quad (\because \phi \propto I_a)$$

The value of braking torque can be found with the help of magnetisation curve of a series motor.

**Example 44.2.** A 40-kW, 440-V, d.c. shunt motor is braked by plugging. Calculate (i) the value of resistance that must be placed in series with the armature circuit to limit the initial braking current to 150 A (ii) the braking torque and (iii) the torque when motor speed falls to 360 rpm.

Armature resistance  $R_a = 0.1 \Omega$ , full-load  $I_a = 100$  A, full-load speed = 600 rpm.

(Electric Drives & Util. Punjab Univ. : 1994)

**Solution.** Full-load  $E_b = 440 - 100 \times 0.1 = 430$  V

Voltage across the armature at the start of braking =  $V + E_b = 440 + 430 = 870$  V

(i) Since initial braking current is limited to 150 A, total armature circuit resistance required is

$$R_t = 870 / 150 = 5.8 \Omega \quad \therefore R = R_t - R_a = 5.8 - 0.1 = 5.7 \Omega$$

(ii) For a shunt motor,  $T_B \propto \Phi I_a \propto I_a$   $\therefore \Phi$  is constant

$$\text{Now, } \frac{\text{initial braking torque}}{\text{full-load torque}} = \frac{\text{initial braking current}}{\text{full-load current}}$$

$$\text{Full-load torque} = 40 \times 103 / 2\pi (600/60) = 636.6 \text{ N-m}$$

$$\therefore \text{initial braking torque} = 636.6 \times 150 / 100 = 955 \text{ N-m}$$

(iii) The decrease in  $E_b$  is directly proportional to the decrease in motor speed.

$$\therefore E_b \text{ at 360 rpm} = 430 \times 360/600 = 258 \text{ V}$$

$$I_a \text{ at 360 rpm} = (440 + 258) / 5.8 = 120 \text{ A}$$

$$T_B \text{ at 360 rpm} = 636.6 \times 120/100 = 764 \text{ N-m}$$

**44.14. Plugging of Induction Motors**

This method of braking is applied to an induction motor by transposing any of its two line leads as shown in Fig. 44.5. It reverses the direction of rotation of the synchronously-rotating magnetic field which produces a torque in the reverse direction, thus applying braking on the motor. Hence, at the **first instant** after plugging, the rotor is running in a direction opposite to that of the stator field. It means that speed of the rotor relative to the magnetic field is  $(N_s + N) \cong 2N_s$  as shown in Fig. 44.6.

In Fig 44.6. ordinate  $BC$  represents the braking torque at the instant of plugging. As seen, this torque gradually increases as motor approaches standstill condition after which motor is disconnected from the supply (otherwise it will start up again in the reverse direction).

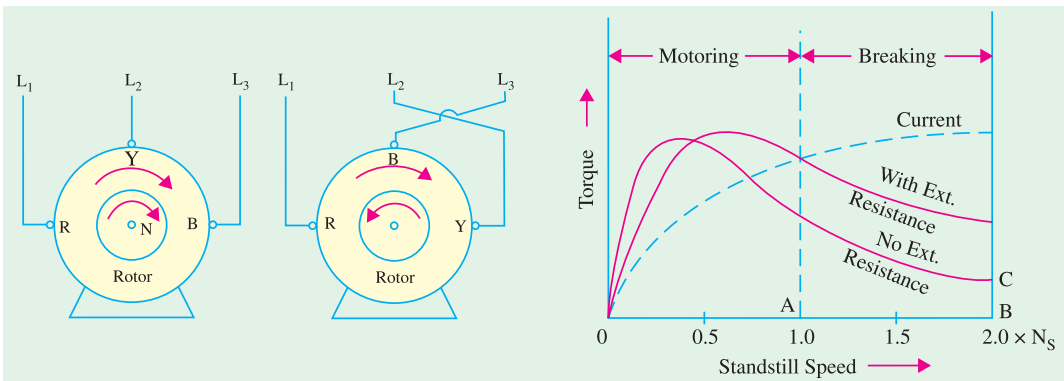


Fig. 44.5

Fig. 44.6

As compared to squirrel cage motors, slip-ring motors are more suitable for plugging because, in their case, external resistance can be added to get the desired braking torque.

**Example 44.3.** A 30-kW, 400-V, 3-phase, 4-pole, 50-Hz induction motor has full-load slip of 5%. If the ratio of standstill reactance to resistance per motor phase is 4, estimate the plugging torque at full speed. (Utilisation of Elect. Energy, Punjab Univ.)

**Solution.**

$$N_s = 120 f / P = 120 \times 50 / 4 = 1500 \text{ rpm}$$

$$\text{Full-load speed, } N_f = N_s (1-s) = 1500 (1 - 0.05) = 1425 \text{ rpm}$$

$$\text{Full-load torque, } T_f = \frac{30 \times 10^3}{2\pi \times 1425 / 60} = 200 \text{ N-m}$$

Since,

$$T \propto \frac{s R_2 E_2^2}{R_2^2 + s^2 X_2^2} \therefore \frac{T_2}{T_1} = \frac{s_2 R_2 E_2^2 / (R_2^2 + s_2^2 X_2^2)}{s_1 R_2 E_2^2 / (R_2^2 + s_1^2 X_2^2)}$$

$$= \frac{s_2 (R_2^2 + s_1^2 X_2^2)}{s_1 (R_2^2 + s_2^2 X_2^2)} = \frac{s_2}{s_1} \cdot \frac{1 + s_1^2 (X_2/R_2)^2}{1 + s_2^2 (X_2/R_2)^2}$$

$$= \frac{s_1}{s_2} \cdot \frac{1 + 16s_1^2}{1 + 16s_2^2} \left( \frac{X_2}{R_2} = 4 \right)$$

Slip,

$$S_p = 2 - 0.05 = 1.95$$

$$\therefore \text{plugging torque, } T_p = \frac{1.95}{0.05} \cdot \frac{1 + 16 \times (0.05)^2}{1 + 16 (1.95)^2} \times T_f$$

$$= 39 \times \frac{1.04}{61.84} \times 200 = 131 \text{ N-m.}$$

### 44.15. Rheostatic Braking

In this method of electric braking, motor is disconnected from the supply though its field continues to be energised in *the same direction*. The motor starts working as a generator and all the kinetic energy of the equipment to be braked is converted into electrical energy and is further dissipated in the variable external resistance *R* connected across the motor during the braking period. This external resistance must be less than the critical resistance otherwise there will not be enough current for generator excitation (Art. 44.3).



Rheostat

D.C. and synchronous motors can be braked this way but induction motors require separate d.c. source for field excitation.

This method has advantage over plugging because, in this case, no power is drawn from the supply during braking.

### 44.16. Rheostatic Braking of D.C. Motors

Fig. 44.7 shows connections for a d.c. shunt motor. For applying rheostatic braking armature is disconnected from the supply and connected to a variable external resistance  $R$  while the field remains on the supply. The motor starts working as a generator whose induced emf  $E_b$  depends upon its speed. At the start of braking, when speed is high,  $E_b$  is large, hence  $I_a$  is large. As speed decreases,  $E_b$  decreases, hence  $I_a$  decreases. Since  $T_b \propto \Phi I_a$ , it will be high at high speeds but low at low speeds. By gradually cutting out  $R$ ,  $I_a$  and, hence,  $T_B$  can be kept constant throughout. Value of  $I_a = E_b / (R + R_a)$ .

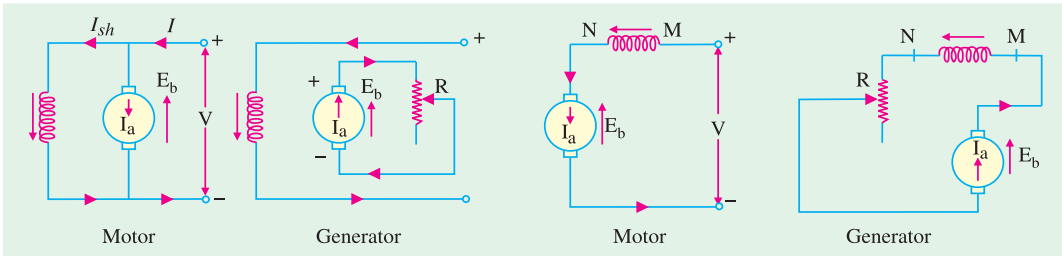


Fig. 44.7

Fig. 44.8

Fig. 44.8 shows running and braking conditions for a d.c. series motor. In this case also, for rheostatic braking, the armature is disconnected from the supply and, at the same time, is connected across  $R$ . However, connections are so made that current keeps flowing through the series field *in the same direction* otherwise no braking torque would be produced. The motor starts working as a series generator provided  $R$  is less than the critical resistance.

### 44.17. Rheostatic Braking Torque

$$T_B \propto \Phi I_a. \quad \text{Now, } I_a = E_b / (R + R_a) = E_b / R_t$$

$$\text{Since } E_b \propto \Phi N, I_a \propto \Phi N / R_t \therefore T_B \propto \Phi^2 N / R_t = k_1 \Phi^2 N$$

1. For D.C. shunt motors and synchronous motors,  $\Phi$  is constant. Hence

$$T_B = k_1 N$$

2. In the case of series motors, flux depends on current. Hence, braking torque can be found from its magnetisation curve.

When rheostatic braking is to be applied to the series motors used for traction work, they are connected in parallel (Fig. 44.9) rather than in series because series connection produces excessive voltage across the loading rheostats.

However, it is essential to achieve electrical stability in parallel operation of two series generators. It can be achieved either by equalizing the exciting currents *i.e.* by connecting the two fields in parallel [Fig. 44.9 (a)] or by cross-connection [Fig. 44.9 (b)] where field of one machine is excited by the armature current of the other. If equalizer is not used, then the machine which happens to build up first will send current through the other *in the opposite direction* thereby exciting it with reverse voltage. Consequently, the two machines would be short-circuited upon themselves and may burn out on account of excessive voltage and, hence, current.

In the cross-connection of Fig. 44.9 (b), suppose the voltage of machine No. 1 is greater than that of No. 2. It would send a larger current through  $F_2$ , thereby exciting it to a higher voltage. This results in stability of their parallel operation because stronger machine always helps the weaker one.

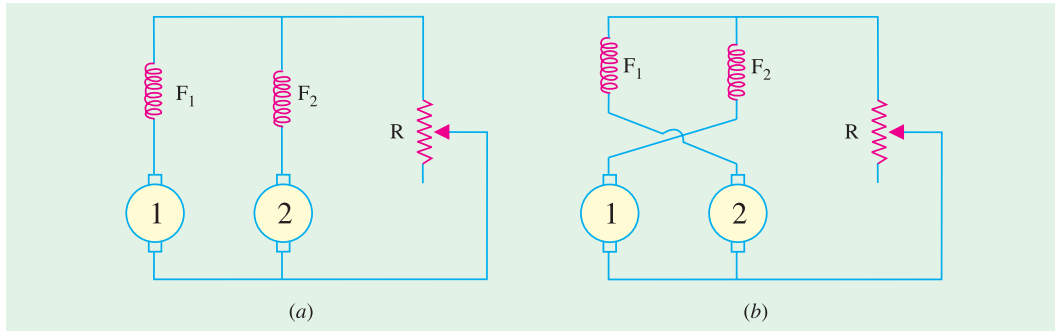


Fig. 44.9

The cross-connection method has one special advantage over equalizer-connection method. If due to any reason (say, a run-back on a gradient) direction of rotation of the generators is reversed, no braking effect would be produced with connections of Fig. 44.9 (a) since the machines will fail to excite. However, with cross-excited fields, the machines will build up in series and being short-circuited upon themselves, will provide an emergency braking and would not allow the coach/car to run back on a gradient.

#### 44.18. Rheostatic Braking of Induction Motors

If an induction motor is disconnected from the supply for rheostatic braking, there would be no magnetic flux and, hence, no generated emf in the rotor and no braking torque. However, if after disconnection, direct current is passed through the stator, steady flux would be set up in the air-gap which will induce current, in the short-circuited rotor. This current which is proportional to the rotor speed, will produce the required braking torque whose value can be regulated by either controlling d.c. excitation or varying the rotor resistance.

#### 44.19. Regenerative Braking

In this method of braking, motor is not disconnected from the supply but is made to run as a generator by utilizing the kinetic energy of the moving train. Electrical energy is fed back to the supply. The magnetic drag produced on account of generator action offers the braking torque. It is the most efficient method of braking. Take the case of a shunt motor. It will run as a generator whenever its  $E_b$  becomes greater than  $V$ . Now,  $E_b$  can exceed  $V$  in two ways :

1. by increasing field excitation
2. by increasing motor speed beyond its normal value, field current remaining the same. It happens when load on the motor has overhauling characteristics as in the lowering of the cage or a hoist or the down-gradient movement of an electric train.

Regenerative braking can be easily applied to d.c. shunt motors though not down to very low speeds because it is not possible to increase field current sufficiently.

In the case of d.c. series motors, reversal of current necessary to produce regeneration would



DC Shunt Motor

cause reversal of the field and hence of  $E_b$ . Consequently, modifications are necessary if regenerative braking is to be employed with d.c. series motors used in electric traction.

It may, however, be clearly understood that regenerative braking cannot be used for stopping a motor. Its main advantages are (i) reduced energy consumption particularly on main-line railways having long gradients and mountain railways (ii) reduced wear of brake shoes and wheel tyres and (iii) lower maintenance cost for these items.

#### 44.20. Energy Saving in Regenerative Braking

We will now compute the amount of energy recuperated between any two points on a level track during which regenerative braking is employed. The amount of energy thus recovered and then returned to the supply lines depends on :

- (i) initial and final velocities of the train during braking
- (ii) efficiency of the system and
- (iii) train resistance.

Suppose regenerative braking is applied when train velocity is  $V_1$  km/h and ceases when it is  $V_2$  km/h. If  $M_e$  tonne is the effective mass of the train, then

$$\begin{aligned} \text{K.E. of the train at } V_1 &= \frac{1}{2} M_e V_1^2 = \frac{1}{2} (1000 M_e) \times \left( \frac{1000 V_1}{3600} \right)^2 \text{ joules} \\ &= \frac{1}{2} (1000 M_e) \left( \frac{1000 V_1}{3600} \right)^2 \times \frac{1}{3600} \text{ Wh} \\ &= 0.01072 M_e V_1^2 \text{ Wh} = 0.01072 \frac{M_e}{M} V_1^2 \text{ Wh/tonne} \end{aligned}$$

$$\text{K.E. at } V_2 = 0.01072 \frac{M_e}{M} V_2^2 \text{ Wh/tonne}$$

Hence, energy available for recovery is  $= 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) \text{ Wh/tonne}$

If  $r$  N/t is the specific resistance of the train, then total resistance  $= rM$  newton.

If  $d$  km is the distance travelled during braking, then

$$\text{energy spent} = rM \times (1000 d) \text{ joules} = rMd \times \frac{1000}{3600} \text{ Wh} = 0.2778 rd \text{ Wh/tonne}$$

Hence, net energy recuperated during regenerative braking is

$$= 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) - 0.2778 rd \text{ Wh/tonne}$$

**Gradient.** If there is a *descending* gradient of  $G$  per cent over the same distance of  $d$  km, then downward force is  $= 98 MG$  newton

Energy provided during braking

$$= 98 MG \times (1000 d) \text{ joules} = 98 MG d (1000 / 3600) \text{ Wh} = 27.25 Gd \text{ Wh/tonne}$$

Hence, net energy recuperated in this case is

$$\begin{aligned} &= \left[ 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) - 0.2778 rd + 27.25 Gd \right] \text{ Wh/tonne} \\ &= 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) + d (27.25 G - 0.2778 r) \text{ Wh/tonne} \end{aligned}$$

If  $\eta$  is the system efficiency, net energy returned to the line is

(i) **level track**

$$= \eta \left[ 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) - 0.2778 rd \right] \text{ Wh/tonne}$$

(ii) descending gradient

$$= \eta \left[ 0.01072 \frac{M_e}{M} (V_1^2 - V_2^2) + d (27.25 G - 0.2778 r) \right] \text{ Wh/tonne}$$

**Example 44.4.** A 500-t electric train travels down a descending gradient of 1 in 80 for 90 seconds during which period its speed is reduced from 100 km/h to 60 km/h by regenerative braking. Compute the energy returned to the lines in kWh if tractive resistance = 50 N/t; allowance for rotational inertia = 10%; overall efficiency of the system = 75%.

**Solution.** Here  $G = 1 \times 100 / 80 = 1.25\%$        $M_e/M = 1.1$

$$d = \left( \frac{V_1 + V_2}{2} \right) \times t = \left( \frac{100 + 60}{2} \right) \times \frac{90}{3600} = 2 \text{ km}$$

Hence, energy returned to the supply line

$$\begin{aligned} &= 0.75 [(0.01072 \times 1.1 (100^2 - 60^2) + 2 (27.25 \times 1.25 - 0.2778 \times 50))] \text{ Wh/t} \\ &= 0.75 [75.5 + 2 (34 - 13.9)] = 86.77 \text{ Wh/t} \\ &= 86.77 \times 500 \text{ Wh} = 86.77 \times 500 \times 10^{-3} \text{ kWh} = \mathbf{43.4 \text{ kWh}} \end{aligned}$$

**Example 44.5.** A 350-t electric train has its speed reduced by regenerative braking from 60 to 40 km/h over a distance of 2 km along down gradient of 1.5%. Calculate (i) electrical energy and (ii) average power returned to the line. Assume specific train resistance = 50 N/t; rotational inertia effect = 10%; conversion efficiency of the system = 75%. **(Elect. Power, Bombay Univ.)**

**Solution .** (i) Energy returned to the line is

$$\begin{aligned} &= 0.75 [0.01072 \times 1.1 (60^2 - 40^2) + 2 (27.25 \times 1.5 - 0.2778 \times 50)] \text{ Wh/t} \\ &= 58.2 \text{ Wh/t} = 58.2 \times 350 \times 10^{-3} = \mathbf{20.4 \text{ kWh}} \end{aligned}$$

(ii) Average speed =  $(60 + 40)/2 = 50 \text{ km/h}$ ; time taken =  $2/50 \text{ h} = 1/25 \text{ h}$

$$\therefore \text{ power returned} = \frac{20.4 \text{ kWh}}{1/25 \text{ h}} = \mathbf{510 \text{ kW}}$$

**Example 44.6.** If in Example 42.4, regenerative braking is applied in such a way that train speed on down gradient remains constant at 60 km/h, what would be the power fed into the line?

**Solution :** Since no acceleration is involved, the down-gradient tractive effort which drives the motors as generators is

$$F_t = (98 MG - Mr) \text{ newton} = (98 \times 350 \times 1.5 - 350 \times 50) = 33,950 \text{ N}$$

Power that can be recuperated is

$$= F_t \times \left( \frac{1000}{3600} \right) V = 0.2778 F_t V \text{ watt} = 0.2778 \times 33,950 \times 60 = 565,878 \text{ W}$$

Since  $\eta = 0.75$ , the power that is actually returned to the line is

$$= 0.75 \times 565,878 \times 10^{-3} = \mathbf{424.4 \text{ kW}}$$

**Example 44.7.** A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion of 75%. **(Util. of Elect. Power, A.M.I.E. Sec. B.)**

**Solution.** Down-gradient tractive effort which drives the motors as generators is

$$F_t = (98 MG - Mr) = (98,000 \times 500 \times 2 - 500 \times 40) = 78,000 \text{ N}$$

Power that can be recuperated is  $= 0.2778 F_t V = 0.2778 \times 78,000 \times 40 = 866,736 \text{ W}$

Since  $\eta = 0.75$ , the power that is actually fed into the lines is

$$= 0.75 \times 866,736 \times 10^{-3} = \mathbf{650 \text{ kW}}$$



**Example 44.8.** A 250-V d.c. shunt motor, taking an armature current of 150 A and running at 550 r.p.m. is braked by reversing the connections to the armature and inserting additional resistance in series with it. Calculate :

- (a) the value of series resistance required to limit the initial current to 240 A.  
 (b) the initial value of braking torque.  
 (c) the value of braking torque when the speed has fallen to 200 r.p.m.

The armature resistance is  $0.09 \Omega$ . Neglect winding friction and iron losses.

(Traction and Util. of Elect. Power, Agra Univ.)

**Solution.** Induced emf at full-load,  $E_b = 250 - 150 \times 0.09 = 236.5 \text{ V}$

Voltage across the armature at braking instant  $= V + E_b = 250 + 236.5 = 486.5 \text{ V}$

- (a) Resistance required in the armature circuit to limit the initial current to 240 A

$$= \frac{486.5}{240} = 2.027 \Omega$$

Resistance to be added in the armature circuit  $= 2.027 - 0.09 = 1.937 \Omega$

- (b) F.L. Torque,  $T_f = VI/2\pi \text{ (N/60)} = 250 \times (550/60) = 650 \text{ N-m}$

$$\text{Initial braking torque} = T_f \frac{\text{initial braking current}}{\text{full-load current}} = \frac{650 \times 240}{150} = 1040 \text{ N-m}$$

- (c) When speed falls to 200 r.p.m., back emf also falls in the same proportion as the speed.

$$\therefore E_b = E_b \times 200/550 = 236.5 \times 200/550 = 94.6 \text{ V}$$

$$\therefore \text{current drawn} = (250 + 94.6)/2.027 = 170 \text{ A}$$

$$\therefore \text{braking torque} = 650 \times 170/150 = 737 \text{ N-m}$$

**Example 44.9.** A 400 V 3-ph squirrel cage induction motor has a full load slip of 4%. A standstill impedance of  $1.54 \Omega$  and the full load current  $= 30 \text{ A}$ . The maximum starting current which may be taken from line is 75A. What tapping must be provided on an auto-transformer starter to limit the current to this value and what would be the starting torque available in terms of full load torque ?

[Nagpur University, Winter 1994]

**Solution.**

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = X$$

or

$$V_1 I_1 = V_2 I_2 = X$$

$$I_1 = 75 \text{ A}$$

$$V_1 = \frac{400}{\sqrt{3}} = 231 \text{ V and } I_1 = I_2 X$$

$$I_1 = \frac{V_2}{Z} X$$

$$I_1 = \frac{X V_1}{1.54} X$$

$$I_1 = \frac{X^2 \times 231}{1.54}$$

$$\Rightarrow 75 = \frac{X^2 \times 231}{1.54}$$

$$\Rightarrow X^2 = \frac{75 \times 1.54}{231}$$

$$X = 0.7071$$

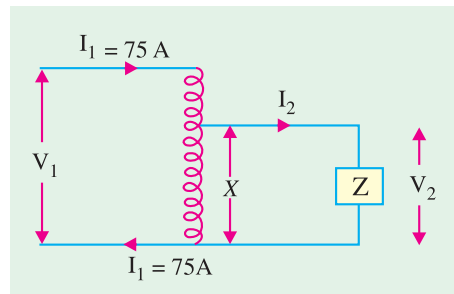


Fig. 44.10

$$\frac{T_s}{T_{FL}} = X^2 \left( \frac{I_s}{I_{FL}} \right)^2 \times \text{Slip}_{(FL)}$$

Now  $I_2 = I_s = \frac{I_1}{X} = \frac{75}{0.708} = 106 \text{ A}$

$$s_{FL} = 0.04 \quad I_s = 106 \text{ A} \quad I_{FL} = 30 \text{ A.}$$

$$\therefore \frac{T_s}{T_{HL}} = (0.701)^2 \left( \frac{106}{30} \right)^2 \times 0.04$$

$$\therefore T_s = 0.25 T_{FL}$$

**Example 44.10.** A 220V, 10 H.P. shunt motor has field and armature resistances of 122Ω and 0.3Ω respectively. Calculate the resistance to be inserted in the armature circuit to reduce the speed to 80% assuming motor η at full load to be 80%.

- (a) When torque is to remain constant.  
 (b) When torque is proportional to square of the speed. [Nagpur University, Winter 1994]

**Solution.**  $I_f = \frac{220}{112} = 1.8 \text{ Amp.}$

Motor O/P = 10 × 746 = 7460 W

$$\therefore \text{Motor I/P} = \frac{7460}{0.8} = 9300 \text{ W}$$

Line current  $I_L = \frac{9300}{220} = 42.2 \text{ Amp.}$

$$\therefore I_a = 42.2 - 1.8 = 40.4 \text{ A}$$

$$\therefore E_{b_1} = 220 - 40.4 \times 0.3 = 208 \text{ V}$$

Now  $\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \quad \because \phi \text{ is constant}$

$$0.8 = \frac{E_{b_2}}{208} \quad \therefore 0.8 = \frac{E_{b_2}}{208} \Rightarrow E_{b_2} = 166.4 \text{ V}$$

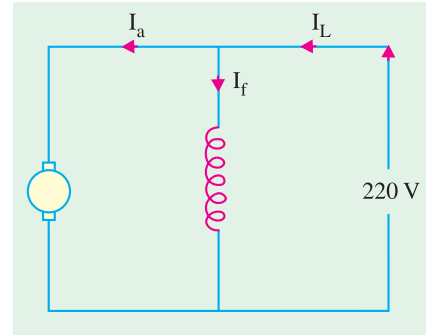


Fig. 44.11

- (a)  $\because$  Torque remains constant and  $\phi$  is constant  
 $\therefore I_a$  at reduced speed will also remain same
- $$\therefore E_{b_2} = V - I_{a_2} R \text{ where } R \text{ is total resistance}$$
- $$166.4 = 220 - 40.4 \times R \quad \therefore R = \frac{220 - 166.4}{40.0} = 1.34 \Omega$$
- $\therefore$  Additional resistance in armature circuit = 1.34 - 0.3 = **1.04 Ω**

- (b)  $\frac{T_2}{T_1} = \left( \frac{N_2}{N_1} \right)^2$
- $T \propto I_a$  also  $T \propto \phi I_a$
- $\therefore T \propto I_a \quad (\because \phi \text{ is constant})$
- $$\frac{T_2}{T_1} = (0.8)^2 = 0.64$$
- $$\therefore \frac{T_2}{T_1} = \frac{I_{a_2}}{I_{a_1}} \quad \therefore 0.64 = \frac{I_{a_2}}{40.4} \Rightarrow I_{a_2} = 25.3 \text{ A}$$
- $$E_{b_2} = 220 - 25.3 \times R$$

$$166.4 = 220 - 25.5936 \times R$$

$$\therefore R = 2.0943 \Omega$$

$$\therefore \text{Additional resistance} = 2.0943 - 0.3 = 1.7943 \Omega$$

**Example 44.11.** A 37.5 H.P., 220 V D.C. shunt motor with a full load speed of 535 r.p.m. is to be braked by plugging. Estimate the value of resistance which should be placed in series with it to limit the initial braking current to 200 amps. What would be the initial value of the electric braking torque and the value when the speed had fallen to half its full load value? Armature resistance of motor is 0.086  $\Omega$  and full load armature current is 140 amps.

**Solution.**

$$E = V - I_a R_a$$

Back e.m.f. of motor =  $E = 220 - 140 \times 0.086 = 220 - 12 = 208$  Volts.

Total voltage during braking =  $E + V$

$$= 220 + 208 = 428 \text{ V}$$

$$R = \frac{V}{I}$$

Resistance required =  $\frac{428}{200} = 2.14 \Omega$

There is already 0.086  $\Omega$  present in armature.

$$\therefore \text{Resistance to be added} = 2.14 - 0.086 = 2.054 \Omega$$

Torque  $\propto \phi I$  or Torque  $\propto I$  ( $\because \phi$  is constant for shunt motor)

$$\frac{\text{Initial braking torque}}{\text{Initial braking current}} = \frac{\text{F.L. torque}}{\text{F.L. current}}$$

Power = Torque  $\times \omega$

$$\omega = \frac{2\pi N}{60} \text{ rad/sec.}$$

$$37.5 \times 746 = T \times \frac{2\pi \times 535}{60}$$

Full load torque = 499.33 Nw-m.

Initial braking torque =  $499.33 \times \frac{200}{140} = 713.328$  Nw-m.

→ At half – speed back e.m.f. falls to half its original value =  $208/2 = 104$  V

$$\text{Current} = \frac{220 + 104}{2.14} = 151 \text{ Amps.}$$

Electric braking torque at  $\frac{1}{2}$  speed =  $499.33 \times \frac{151}{140} = 538.56$  Nw-m.

**Example 44.12.** A 500 V series motor having armature and field resistances of 0.2 and 0.3  $\Omega$ , runs at 500 r.p.m. when taking 70 Amps. Assuming unsaturated field find out its speed when field diverter of 0.684  $\Omega$  is used for following load whose torque

- (a) remains constant  
(b) varies as square of speed.

**Solution.** When no diverter connected,  $E_{b_1} = 500 - 70(0.2 + 0.3) = 465$  V

(a) If  $I_{a_2}$  be the armature current when diverter is used, then current flowing through

$$\text{field} = I_{f_2} = I_{a_2} \times \frac{0.684}{0.3 + 0.684} = 0.695 I_{a_2}$$

$\therefore$  Load torque is constant

$$\therefore I_{a_1} \phi_1 = I_{a_2} \phi_2 \quad (\because \phi \propto I_a)$$

$$\therefore I_{a_1} \phi_1 = I_{a_2} (0.695) I_{a_2} \Rightarrow I_{a_2} = \frac{I_{a_1}}{\sqrt{0.695}} = \frac{70}{\sqrt{0.695}} = 84 \text{ A}$$

$$\therefore \text{Field current} = I_{f_2} = 0.695 I_{a_2} = 0.695 \times 84 = 58.4 \text{ A}$$

$$\text{Resistance of field with diverter} = \frac{0.3 \times 0.684}{0.3 + 0.684} = 0.208 \Omega$$

$$\text{Total field and armature resistance} = 0.2 + 0.208 = 0.408 \Omega$$

$$E_{b_2} = 500 - 84 (0.408) = 465.8 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{500}{N_2} = \frac{465}{465.8} \times \frac{58.4}{70} \Rightarrow N_2 = \mathbf{600 \text{ r.p.m}}$$

$$(b) \quad \frac{T_1}{T_2} = \left( \frac{N_1}{N_2} \right)^2 \therefore \frac{T_1}{T_2} = \frac{I_{a_1} \phi_1}{I_{a_2} \phi_2} = \frac{I_{a_1} \cdot I_{a_1}}{I_{a_2} \times 0.695 I_{a_2}}$$

$$\therefore \left( \frac{N_1}{N_2} \right)^2 = \frac{I_{a_1} \cdot I_{a_1}}{I_{a_2}^2 \times 0.695} \Rightarrow \frac{N_1}{N_2} = \frac{I_{a_1}}{I_{a_2} \sqrt{0.695}} = \frac{70}{I_{a_2} \sqrt{0.695}}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{70}{I_{a_2} \sqrt{0.695}} = \frac{465}{500 - I_{a_2} (0.2 + 0.208)} \times \frac{0.695 I_{a_2}}{70}$$

$$I_{a_2}^2 + 7.42 I_{a_2} - 9093 = 0$$

$$\Rightarrow I_{a_2} = 91.7 \text{ A} \quad \therefore \text{negative value is absurd.}$$

$$\frac{N_1}{N_2} = \frac{70}{I_{a_2} \sqrt{0.695}} \Rightarrow \frac{500}{N_2} = \frac{70}{91.7 \sqrt{0.695}}$$

$$\therefore N_2 = \mathbf{546 \text{ r.p.m.}}$$

**Example 44.13.** A 200 V series motor runs at 1000 r.p.m. and takes 20 Amps. Armature and field resistance is 0.4 W. Calculate the resistance to be inserted in series so as to reduce the speed to 800 r.p.m., assuming torque to vary as cube of the speed and unsaturated field.

$$\text{Solution.} \quad \frac{T_1}{T_2} = \left( \frac{N_1}{N_2} \right)^3 = \left( \frac{1000}{800} \right)^3 = \frac{125}{64}$$

$$\therefore \frac{T_1}{T_2} = \frac{I_{a_1} \phi_1}{I_{a_2} \phi_2} = \frac{20 \times 20}{I_{a_2} \times I_{a_2}} \quad \therefore \phi \propto I_a \text{ for series motor.}$$

$$\frac{125}{64} = \frac{20^2}{I_{a_2}^2} \quad I_{a_2} I_{a_2} = 14.3 \text{ Amp}$$

$$E_{b_1} = 200 - 20 \times 0.4 = 192 \text{ V.}$$

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2}$$

$$\frac{E_{b_2}}{E_{b_1}} = \frac{1000}{800} \times \frac{20}{14.3}$$

$$E_{b_2} = 110 \text{ V}; \quad E_{b_2} = V - IR$$

$$110 = 200 - 14.3 \times R; \quad R = \frac{90}{14.3} = 6.3 \Omega$$

Additional resistance required =  $6.3 - 0.4 = 5.9 \Omega$

**Example 44.14.** A 220V, 500 r.p.m. D.C. shunt motor with an armature resistance of  $0.08 \Omega$  and full load armature current of 150 Amp. is to be braked by plugging. Estimate the value of resistance which is to be placed in series with the armature to limit initial braking current to 200 Amps. What would be the speed at which the electric braking torque is 75% of its initial value.

**Solution.** Back e.m.f. of motor =  $E_{b_1} = V - I_a R_a$   
 $= 220 - 150 \times 0.08 = 208 \text{ V}$

Voltage across armature when braking starts  
 $= 220 + 208 = 428 \text{ V}$

Initial braking current to be limited to 200 A.

$$\therefore \text{Resistance in armature circuit} = \frac{428}{200} = 2.14 \Omega$$

$$\therefore \text{External resistance required} = 2.14 - 0.08 = 2.06 \Omega$$

Since field Flux  $\phi$  is constant therefore 75% torque will be produced when armature current is 75% of 200 Amp. i.e. 150 Amp.

Let  $N_2$  be the speed in r.p.m. at which 75% braking torque is produced. At this speed generated e.m.f. in armature

$$\therefore \frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2}; \quad \frac{208}{E_{b_2}} = \frac{500}{N_2} \quad \therefore E_{b_2} = \frac{208}{500} N_2$$

Voltage across armature when braking starts  
 $150 \times 2.14 = \left(220 + \frac{208}{500} N_2\right) \text{ Volts}$

$$\therefore N_2 = 243 \text{ r.p.m.}$$

**Example 44.15.** A D.C. series motor operating at 250 V D.C. mains and draws 25 A and runs at 1200 r.p.m.  $R_a = 0.1 \Omega$  and  $R_{se} = 0.3 \Omega$ .

A resistance of  $25 \Omega$  is placed in parallel with the armature of motor. Determine:

(i) The speed of motor with the shunted armature connection, if the magnetic circuit remains unsaturated and the load torque remains constant.

(ii) No load speed of motor.

[Nagpur University Winter 1995]

**Solution.**  $\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$

Voltage across diverter =  $250 - 0.3 I_2$

$$I_{\text{div}} = \frac{250 - 0.3 I_2}{25}$$

$$I_{a_2} = I_2 - \frac{250 - 0.3 I_2}{25}$$

$$= 1.012 I_2 - 10$$

As  $T$  is constant  $\therefore \phi_1 I_{a_1} = \phi_2 I_{a_2}$

$$\therefore I_{a_1}^2 = I_2 (I_{a_2})$$

$$(25)^2 = I_2 (1.012 I_2 - 10)$$

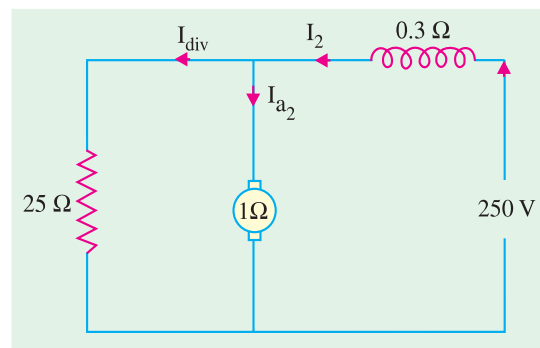


Fig. 44.12

$$1.012 I_2^2 - 10 \times I_2 - 625 = 0 \Rightarrow I_2 = 30.27 \text{ A (neglecting negative value)}$$

$$(25)^2 = (30.27) I_{a_2} \Rightarrow I_{a_2} = 20.65 \text{ A}$$

$$E_{b_1} = V - I_{a_1} (R_a + R_{se}) = 250 - 25 (0.4) = 240 \text{ Volts}$$

$$E_{b_2} = V - I_2 (R_{se}) - I_{a_2} (R_a) = 250 - 30.27 (0.3) - 0.1 (20.65) = 238.85 \text{ V.}$$

$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{1200} = \frac{238.85}{240} \times \frac{2.5}{20.27} \therefore N_2 = 986 \text{ r.p.m.}$$

(ii) Series motor on no load.

Series motor can't be started on no load. When flux is zero, motor tries to run at infinite speed, which is not possible. So in the process, it tries to draw very high current from supply and fuse blows-out.

**Example 44.16.** A 4 pole, 50Hz, slip ring Induction Motor has rotor resistance and stand still reactance referred to stator of  $0.2 \Omega$  and  $1 \Omega$  per phase respectively. At full load, it runs at 1440 r.p.m. Determine the value of resistance to be inserted in rotor in ohm/ph to operate at a speed of 1200 r.p.m., if:

- (i) Load torque remains constant. (ii) Load torque varies as square of the speed.  
Neglect rotor resistance and leakage reactance.

**Solution.** (i) Load torque constant

$$\Rightarrow T \propto \frac{s}{R_2} \quad N_s = 1500 \text{ rpm}$$

$$\therefore T_1 \propto \frac{s_1}{R_2}$$

$$T_2 \propto \frac{s_2}{(R_1 + r)}$$

$$s_1 = \frac{1500 - 1400}{1500} = 0.04 \quad s_2 = \frac{1500 - 1240}{1500} = 0.2$$

As  $T_1 = T_2 \therefore \frac{s_1}{R_2} = \frac{s_2}{R_1 + r}$

$$\frac{0.04}{0.2} = \frac{0.2}{0.2 + r}$$

$\therefore r = 0.8 \Omega$

(ii) Load torque varies as square of the speed.

$$\frac{T_1}{T_2} = \left[ \frac{N_1}{N_2} \right]^2 = \left[ \frac{1440}{1200} \right]^2 = 1.44$$

$$\frac{T_1}{T_2} = 1.44 = \frac{\frac{R_2 s_1}{R_2^2 + (s_1 X_2)^2}}{\frac{(R_2 + r) s_2}{(R_2 + r)^2 + (s_2 X_2)^2}} = \frac{0.2 \times 0.04}{(0.2^2 + (0.04 \times 1)^2)} \div \frac{(0.02 + r) (0.2)}{(0.2 + r)^2 + (0.2 \times 1)^2}$$

Substituting

$$R_2 + r = R$$

$$1.44 = \frac{0.1923}{0.2 R} \div \frac{R^2 + 0.04}{R^2 + 0.04}$$

$$\therefore 0.1923 R^2 - 0.288 R + 0.007652 = 0$$

$$\Rightarrow R = 1.47 \text{ and } 0.0272, \text{ But } R > 0.2 \therefore R = 1.47 \Omega$$

$$\therefore R = 0.2 + r; 1.47 = 0.2 + r \Rightarrow r = 1.27 \Omega$$

## Tutorial Problem No. 44.1

- The characteristics of a series traction motor at 525 V are as follows :
 

current	:	50	70	80	90	A
speed	:	33.8	26.9	25.1	23.7	km/h
Gross torque	:	217	352	423	502	N-m

 Determine the gross braking torque at a speed of 25.7 km/h when operating as self-excited series generator and loaded with an external resistance of 6  $\Omega$ . Resistance of motor = 0.5  $\Omega$ .  
**[382.4 N-m] (London Univ.)**
- The characteristics of a series motor at 525 V are as follows :
 

current	:	75	125	175	225	A
speed	:	1200	950	840	745	r.p.m.

 Calculate the current when operating as a generator at 1000 r.p.m. and loaded on a rheostat having a resistance of 3.25  $\Omega$ . The resistance of motor is 3.5  $\Omega$ .  
**[150 A] (London Univ.)**
- A train weighing 400 tonne travels a distance of 10 km down a gradient of 2%, getting its speed reduced from 40 to 20 km/h, the train resistance is = 50 N/t, allowance for rotational inertia = 10% and overall efficiency = 72%. Estimate (i) power and (ii) energy returned to the line.  
**[(i) 363 kW (ii) 121 kWh] (Elect. Power, Bombay Univ.)**
- A 400-tonne train travels down a gradient of 1 in 100 for 20 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if the tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%.  
**[28.2 kWh] (A.M.I.E.)**
- A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the line if tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%.  
**[30.12 kWh] (A.M.I.E.)**
- A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion of 75%.  
**[650 kW] (Utilization of Elect. Power, A.M.I.E.)**
- A 18.65 kW, 220-V D.C. shunt motor with a full-load speed of 600 r.p.m. is to be braked by plugging. Estimate the value of the resistance which should be placed in series with it to limit the current to 130A. What would be the initial value of the electric braking torque and value when speed has fallen to half of its full-load value? Armature resistance of motor is 0.1  $\Omega$ . Full-load armature current is 95 A.  
**[3.211  $\Omega$ , 400.5 N-m, 302.57 N-m] (Util of Elect. Power, A.M.I.E. Sec. B.)**
- A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if tractive resistance is 5 kg / tonne and allowance for rotational inertia is 7.5%. Overall efficiency of motor is 75%.  
**[30.64%]**
- What are the advantages of Electrical Drive over other Drives? What are the main features of Group Drive and an Individual Drive?  
**(Nagpur University, Summer 2004)**
- What are the essential requirements of starting of any motor? With the help of neat diagram explain 'open circuit transition' and 'closed circuit transition' in Auto transformer starting of Induction Motor.  
**(Nagpur University, Summer 2004)**
- What is the principle of speed control of D.C. motors for, below the base speed and above the base speed. Explain with neat N-T characteristics.  
**(Nagpur University, Summer 2004)**
- A 400 V, 25 h.p., 450 rpm, D.C. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2  $\Omega$ .  
**(Nagpur University, Summer 2004)**
- Mention the Advantage of PLC over conventional motor control.  
**(Nagpur University, Summer 2004)**

14. Suggest the motors required for following Drives :-  
(i) Rolling mills (ii) Marine drive (iii) Home appliances (iv) Pump  
(v) Refrigeration and air-conditioning (vi) Lifts. *(Nagpur University, Summer 2004)*
15. Explain with neat block diagram the digital control of Electrical Drives.  
*(Nagpur University, Summer 2004)*
16. Explain Series parallel control of traction motor. *(Nagpur University, Summer 2004)*
17. Write short Notes on Speed reversal by contactor and relay. *(Nagpur University, Summer 2004)*
18. Write short Notes on Ratings of contactors. *(Nagpur University, Summer 2004)*
19. Write short Notes on Magnetic time-delay relay. *(Nagpur University, Summer 2004)*
20. Discuss the advantages and disadvantages of electric drive over other drives.  
*(J.N. University, Hyderabad, November 2003)*
21. Though a.c. is superior to d.c. for electric drives, sometimes d.c. is preferred. Give the reasons and mention some of the applications. *(J.N. University, Hyderabad, November 2003)*
22. A d.c. series motor drives a load, the torque of which varies as the square of the speed. The motor takes current of 30 amps, when the speed is 600 r.p.m. Determine the speed and current when the field winding is shunted by a diverter, the resistance of which is 1.5 times that of the field winding. The losses may be neglected. *(J.N. University, Hyderabad, November 2003)*
23. State the condition under which regenerative braking with d.c. services motor is possible and with the aid of diagrams of connection, explain the various methods of providing regeneration.  
*(J.N. University, Hyderabad, November 2003)*
24. Explain what you mean by “Individual drive” and “Group drive”. Discuss their relative merits and demerits. *(J.N. University, Hyderabad, November 2003)*
25. A 500 V d.c. series motor runs at 500 r.p.m. and takes 60 amps. The resistances of the field and the armature are 0.3 and 0.2 Ohms, respectively. Calculate the value of the resistance to be shunted with the series field winding in order that the speed may be increased to 600 r.p.m., if the torque were to remain constant. Saturation may be neglected.  
*(J.N. University, Hyderabad, November 2003)*
26. A motor has the following duty cycle :  
Load rising from 200 to 400 h.p. – 4 minutes  
Uniform load 300 h.p. – 2 minutes  
Regenerative braking h.p. Returned to supply from 50 to zero – 1 minute.  
Remains idle for 1 minute.  
Estimate the h.p. of the motor. *(J.N. University, Hyderabad, November 2003)*
27. What are various types of electric braking used?*(J.N. University, Hyderabad, November 2003)*
28. Explain how rheostatic braking is done in D.C. shunt motors and series motors.  
*(J.N. University, Hyderabad, November 2003)*
29. Describe how plugging, rheostatic braking and regenerative braking are employed with D.C. series motor. *(J.N. University, Hyderabad, November 2003)*
30. Where is the use of Individual drive recommended and why?  
*(J.N. University, Hyderabad, November 2003)*
31. The speed of a 15 h.p. (Metric) 400 V d.c. shunt motor is to be reduced by 25% by the use of a controller. The field current is 2.5 amps and the armature resistance is 0.5 Ohm. Calculate the resistance of the controller, if the torque remains constant and the efficiency is 82%.  
*(J.N. University, Hyderabad, November 2003)*
32. Explain regenerative braking of electric motors. *(J.N. University, Hyderabad, November 2003)*
33. “If a high degree of speed control is required, d.c. is preferable to a.c. for an electric drive”. Justify.  
*(J.N. University, Hyderabad, April 2003)*
34. A 200 V shunt motor has an armature resistance of 0.5 ohm. It takes a current of 16 amps on full load and runs at 600 r.p.m. If a resistance of 0.5 ohm is placed in the armature circuit, find the



- ratio of the stalling torque to the full load torque. (J.N. University, Hyderabad, April 2003)
35. What are the requirements of good electric braking? (J.N. University, Hyderabad, April 2003)
36. Explain the method of rheostatic braking. (J.N. University, Hyderabad, April 2003; Anna University, Chennai 2003)
37. Mean horizontal Candlepower (J.N. University, Hyderabad, April 2003)
38. Mean hemispherical Candlepower (J.N. University, Hyderabad, April 2003)
39. Luminous flux. (J.N. University, Hyderabad, April 2003)
40. Define : (i) Luminous intensity (ii) Point source (iii) Lumen and (iv) Uniform point source. (J.N. University, Hyderabad, April 2003)
41. Prove that Luminous intensity of a point source is equal to the luminous flux per unit solid angle. (J.N. University, Hyderabad, April 2003)
42. Discuss the various factors that govern the choice of a motor for a given service. (J.N. University, Hyderabad, April 2003)
43. A 6 pole, 50 Hz slip ring induction motor with a rotor resistance per phase of 0.2 ohm and a stand still reactance of 1.0 ohm per phase runs at 960 r.p.m. at full load. Calculate the resistance to be inserted in the rotor circuit to reduce the speed to 800 r.p.m., if the torque remains unaltered. (J.N. University, Hyderabad, April 2003)
44. Compare the features of individual and group drives. (J.N. University, Hyderabad, April 2003)
45. What is an electric drive? Classify various types of electric drives and discuss their merits and demerits. (J.N. University, Hyderabad, December 2002/January 2003)
46. Suggest, with reasons the electric drive used for the following applications. (i) Rolling mills (ii) Textile mills (iii) Cement mills (iv) Paper mills (v) Coal mining (vi) Lift, Cranes, Lathes and pumps. (J.N. University, Hyderabad, December 2002/January 2003)
47. A 100 hp, 500 rpm d.c. shunt motor is driving a grinding mill through gears. The moment of inertia of the mill is  $1265 \text{ kgm}^2$ . If the current taken by the motor must not exceed twice full load current during starting, estimate the minimum time taken to run the mill upto full speed. (J.N. University, Hyderabad, December 2002/January 2003)
48. Explain the different methods of electric braking of a 3 phase induction motor. (J.N. University, Hyderabad, December 2002/January 2003)
49. A 50 hp, 400V, 750 rpm synchronous motor has a moment of inertia  $20 \text{ kgm}^2$  and employs rheostatic braking for obtaining rapid stopping in case of emergency when the motor is running at full load, star connected braking resistor of 2 ohm per phase is switched on. Determine the time taken and the number of revolutions made before the motor is stopped. Assume as efficiency of 90% and a full load power factor of 0.95. (J.N. University, Hyderabad, December 2002/January 2003)
50. Explain regenerative braking of induction motor. (J.N. University, Hyderabad, December 2002/January 2003)
51. What is dynamic braking? (Anna University, Chennai, Summer 2003)
52. What is regenerative braking? (Anna University, Chennai, Summer 2003)
53. What are braking systems applicable to a DC shunt motor? (Anna University, Chennai, Summer 2003)
54. What for Series motor Regenerative Braking is not suited? (Anna University, Chennai, Summer 2003)
55. What are the important stages in controlling an electrical drive. (Anna University, Chennai, Summer 2003)
56. Explain rheostatic braking of D.C. motors. (Anna University, Chennai 2003)

### OBJECTIVE TESTS – 44

- A steel mill requires a motor having high starting torque, wide speed range and precise speed control. Which one of the following motors will you choose ?
  - d.c. shunt motor
  - synchronous motor
  - d.c. series motor
  - slip-ring induction motor.
- Heavy-duty steel-works cranes which have wide load variations are equipped with ..... motor.
  - double squirrel-cage

- (b) d.c. series  
(c) slip-ring induction  
(d) cumulative compound.
3. A reciprocating pump which is required to start under load will need .....motor.  
(a) repulsion  
(b) squirrel-cage induction  
(c) synchronous  
(d) double squirrel-cage induction.
4. Motors used in wood-working industry have ..... enclosure.  
(a) screen protected (b) drip proof  
(c) TEFC (d) TE
5. Single-phase synchronous motors are used in teleprinters, clocks and all kinds of timing devices because of their  
(a) low starting torque  
(b) high power factor  
(c) constant speed  
(d) over-load capacity.
6. Which motor is generally used in rolling mills, paper and cement industries ?  
(a) d.c. shunt motor  
(b) double squirrel-cage motor  
(c) slip-ring induction motor  
(d) three-phase synchronous motor
7. Direct drive is used for power transmission only when  
(a) negligible slip is required  
(b) large amount of power is involved  
(c) speed of the driven machine equals the motor speed  
(d) high-speed motor is to drive a low-speed machine.
8. Which type of enclosure will be most suitable for motors employed in atmospheres containing inflammable gases and vapours ?  
(a) pipe-ventilated  
(b) totally enclosed, fan-cool  
(c) flame proof  
(d) screen-protected.
9. While plugging d.c. motors, ..... connections are reversed  
(a) supply  
(b) armature  
(c) field  
(d) both armature and field
10. During rheostatic braking of a d.c., motor,  
(a) its field is disconnected from the supply  
(b) its armature is reverse-connected  
(c) it works as a d.c. generator  
(d) direction of its field current is reversed.
11. Rheostatic braking may be applied to an induction motor provided  
(a) separate d.c. source for field excitation is available  
(b) it is a squirrel cage type  
(c) it is slip-ring type  
(d) variable external resistance is available
12. During regenerative braking of electric motors, they are  
(a) disconnected from the supply  
(b) reverse-connected to the supply  
(c) made to run as generators  
(d) made to stop.
13. Regenerative braking  
(a) can be used for stopping a motor  
(b) cannot be easily applied to d.c. series motors  
(c) can be easily applied to d.c. shunt motors  
(d) cannot be used when motor load has overhauling characteristics
14. Net energy saved during regenerative braking of an electric train  
(a) increases with increase in specific resistance  
(b) is high with high down gradient  
(c) decreases with reduction in train speed due to braking  
(d) is independent of the train weight.
15. The selection of an electric motor for any application depends on which of the following factors?  
(a) Electrical characteristics  
(b) Mechanical characteristics  
(c) Size and rating of motors  
(d) cost  
(e) All of the above
16. For a particular application the type of electric and control gear are determined by which of the following considerations?  
(a) Starting torque  
(b) Conditions of environment  
(c) Limitation on starting current  
(d) Speed control range and its nature  
(e) all of the above
17. Which of the following motors is preferred for traction work?  
(a) Universal motor  
(b) D.C. series motor  
(c) Synchronous motor  
(d) three-phase induction motor
18. Which of the following motors always starts

- on load?  
(a) Conveyor motor (b) Floor mill motor  
(c) Fan motor (d) All of the above
19. .... is preferred for automatic drives.  
(a) Squirrel cage induction motor  
(b) Synchronous motors  
(c) Ward-Leonard controlled D.C. motors  
(d) Any of the above
20. When the load is above ..... a synchronous motor is found to be more economical.  
(a) 2 kW (b) 20 kW  
(c) 50 kW (d) 100 kW
21. The load cycle for a motor driving a power press will be .....  
(a) variable load  
(b) continuous  
(c) continuous but periodical  
(d) intermittent and variable load
22. Light duty cranes are used in which of the following?  
(a) Power houses  
(b) Pumping station  
(c) Automobile workshops  
(d) all of the above
23. While selecting an electric motor for a floor mill, which electrical characteristics will be of least significance?  
(a) Running characteristics  
(b) Starting characteristics  
(c) Efficiency  
(d) Braking
24. Which of the following motors are preferred for overhead travelling cranes?  
(a) Slow speed motors  
(b) Continuous duty motors  
(c) Short time rated motors  
(d) None of the above
25. .... is preferred for synthetic fibre mills.  
(a) Synchronous motor  
(b) Reluctance motor  
(c) Series motor  
(d) Shunt motor
26. Ward-Leonard controlled D.C. drives are generally used for ..... excavators.  
(a) Light duty (b) Medium duty  
(c) Heavy duty (d) All of the above
27. Which of the following motors is used for elevators?  
(a) Induction motor  
(b) Synchronous motor  
(c) Capacitor start single phase motor  
(d) Any of the above
28. Which part of a motor needs maximum attention for maintenance?  
(a) Frame (b) Bearing  
(c) Stator winding (d) Rotor winding
29. .... need frequent starting and stopping of electric motors.  
(a) Paper mills  
(b) Grinding mills  
(c) Air-conditioners  
(d) Lifts and hoists
30. Which feature, while selecting a motor for centrifugal pump, will be of least significance?  
(a) Starting characteristics  
(b) Operating speed  
(c) Horse power  
(d) Speed control
31. .... motor is a constant speed motor.  
(a) Synchronous motor  
(b) Schrage motor  
(c) Induction motor  
(d) Universal motor
32. The starting torque is case of centrifugal pumps is generally  
(a) less than running torque  
(b) same as running torque  
(c) slightly more than running torque  
(d) double the running torque
33. Which of the following motors are best for the rolling mills?  
(a) Single phase motors  
(b) Squirrel cage induction motors  
(c) Slip ring induction motors  
(d) D.C. motors
34. .... is not a part of ball bearing?  
(a) Inner race (b) Outer race  
(c) Cage (d) Bush
35. The starting torque of a D.C. motor is independent of which of the following?  
(a) Flux  
(b) Armature current  
(c) Flux and armature current  
(d) Speed
36. Rotor of a motor is usually supported on ..... bearings.  
(a) ball or roller (b) needle  
(c) bush (d) thrust
37. For which of the following applications D.C. motors are still preferred?  
(a) High efficiency operation  
(b) Reversibility

- (c) Variable speed drive  
(d) High starting torque
38. In a paper mill where constant speed is required  
(a) synchronous motors are preferred  
(b) A.C. motors are preferred  
(c) individual drive is preferred  
(d) group drive is preferred
39. A reluctance motor .....  
(a) is provided with slip rings  
(b) requires starting gear  
(c) has high cost  
(d) is compact
40. The size of an excavator is usually expressed in terms of  
(a) 'crowd' motion (b) angle of swing  
(c) cubic metres (d) travel in metres
41. For blowers which of the following motors is preferred?  
(a) d.C. series motor  
(b) D.C. shunt motor  
(c) Squirrel cage induction motor  
(d) Wound rotor induction motor
42. Belted slip ring induction motor is almost invariably used for  
(a) water pumps  
(b) jaw crushers  
(c) centrifugal blowers  
(d) none of the above
43. Which of the following is essentially needed while selecting a motor?  
(a) Pulley (b) Starter  
(c) Foundation pedal (d) Bearings
44. Reluctance motor is a .....  
(a) variable torque motor  
(b) low torque variable speed motor  
(c) self starting type synchronous motor  
(d) low noise, slow speed motor
45. .... method of starting a three phase induction motor needs six terminals.  
(a) Star-delta  
(b) Resistance starting  
(c) Auto-transformer  
(d) None of the above
46. In .... method of starting three phase induction motors the starting voltage is not reduced.  
(a) auto-transformer  
(b) star-delta  
(c) slip ring  
(d) any of the above
47. In jaw crushers a motor has to often start against ..... load.  
(a) heavy (b) medium  
(c) normal (d) low
48. For a motor-generator set which of the following motors will be preferred?  
(a) Synchronous motor  
(b) Slip ring induction motor  
(c) Pole changing induction motor  
(d) Squirrel cage induction motor
49. Which of the following motors is usually preferred for kiln drives?  
(a) Cascade controlled A.C. motor  
(b) slip ring induction motor  
(c) three phase shunt wound commutator motor  
(d) Any of the above
50. Heat control switches are used in .....  
(a) transformers  
(b) cooling ranges  
(c) three phase induction motors  
(d) single phase
51. .... has relatively wider range of speed control  
(a) Synchronous motor  
(b) Slip ring induction motor  
(c) Squirrel cage induction motor  
(d) D.C. shunt motor
52. In squirrel cage induction motors which of the following methods of starting cannot be used?  
(a) Resistance in rotor circuit  
(b) Resistance in stator circuit  
(c) Auto-transformer starting  
(d) Star-delta starting
53. In which of the following applications the load on motor changes in cyclic order?  
(a) Electric shovels  
(b) Cranes  
(c) Rolling mills  
(d) All of the above
54. Flame proof motors are used in  
(a) paper mills  
(b) steel mills  
(c) moist atmospheres  
(d) explosive atmospheres
55. Which of the following machines has heavy fluctuation of load?  
(a) Printing machine  
(b) Punching machine  
(c) Planer  
(d) Lathe
56. For derricks and winches which of the following drives can be used?  
(a) Pole changing squirrel cage motors  
(b) D.C. motors with Ward-leonard control

- (c) A.C. slip ring motors with variable resistance  
(d) Any of the above
57. Battery operated scooter for braking uses  
(a) plugging  
(b) mechanical braking  
(c) regenerative braking  
(d) rheostatic braking
58. .... has least range of speed control.  
(a) Slip ring induction motor  
(b) Synchronous motor  
(c) D.C. shunt motor  
(d) Schrage motor
59. .... has the least value of starting torque to full load torque ratio.  
(a) D.C. shunt motor  
(b) D.C. series motor  
(c) Squirrel cage induction motor  
(d) Slip ring induction motor
60. In case of ..... speed control by injecting e.m.f. in the rotor circuit is possible.  
(a) d.c. shunt motor  
(b) schrage motor  
(c) synchronous motor  
(d) slip ring induction motor
61. A pony motor is used for the starting which of the following motors?  
(a) Squirrel cage induction motor  
(b) Schrage motor  
(c) Synchronous motor  
(d) None of the above
62. In ..... the speed can be varied by changing the position of brushes.  
(a) slip ring motor  
(b) schrage motor  
(c) induction motor  
(d) repulsion motor
63. In which of the following applications variable speed operation is preferred?  
(a) Exhaust fan  
(b) Ceiling fan  
(c) Refrigerator  
(d) Water pump
64. Heavy duty cranes are used in  
(a) ore handling plants  
(b) steel plants  
(c) heavy engineering workshops  
(d) all of the above
65. the travelling speed of cranes varies from  
(a) 20 to 30 m/s  
(b) 10 to 15 m/s  
(c) 5 to 10 m/s  
(d) 1 to 2.5 m/s
66. Besides a constant speed a synchronous rotor possesses which of the following advantages?  
(a) Lower cost  
(b) Better efficiency  
(c) High power factor  
(d) All of the above
67. By the use of which of the following D.C. can be obtained from A.C.?  
(a) Silicon diodes  
(b) Mercury arc rectifier  
(c) Motor generator set  
(d) any of the above
68. Which of the following motors is preferred when quick speed reversal is the main consideration?  
(a) Squirrel cage induction motor  
(b) Wound rotor induction motor  
(c) Synchronous motor  
(d) D.C. motor
69. Which of the following motors is preferred when smooth and precise speed control over a wide range is desired?  
(a) D.C. motor  
(b) Squirrel cage induction motor  
(c) Wound rotor induction motor  
(d) Synchronous motor
70. For crane travel which of the following motors is normally used?  
(a) Synchronous motor  
(b) D.C. differentially compound motor  
(c) Ward-Leonard controlled D.C. shunt motor  
(d) A.C. slip ring motor
71. The capacity of a crane is expressed in terms of  
(a) type of drive  
(b) span  
(c) tonnes  
(d) any of the above
72. the characteristics of drive for crane hoisting and lowering are which of the following?  
(a) Precise control  
(b) Smooth movement  
(c) Fast speed control  
(d) All of the above
73. Which of the following motors is preferred for boom hoist of a travelling crane?  
(a) Single phase motor  
(b) Synchronous motor  
(c) A.C. slip ring motor

- (d) Ward-Leonard controlled D.C. shunt motor
74. A wound rotor induction motor is preferred, as compared to squirrel cage induction motor, when major consideration is
- slop speed operation
  - high starting torque
  - low windage losses
  - all of the above
75. Which of the following motors has series characteristics?
- Shadel pole motor
  - Repulsion motor
  - Capacitor start motor
  - None of the above
76. Which of the following happens when star-delta starter is used?
- Starting voltage is reduced
  - Starting current is reduced
  - Both (a) and (b)
  - None of the above
77. For a D.C. shunt motor which of the following is incorrect?
- Unsuitable for heavy duty starting
  - Torque varies as armature current
  - Torque-armature current is a straight line
  - Torque is zero for zero armature current
78. For which of the following applications motor has to start with high acceleration?
- Oil expeller
  - Floor mill
  - Lifts and hoists
  - centrifugal pump
79. Which of the following types of motor enclosure is safest?
- totally enclosed
  - Totally enclosed fan cooled
  - Open type
  - Semi closed
80. While selecting motor for an air conditioner which of the following characteristics is of great importance?
- Type of bearings
  - Type of enclosure
  - Noise
  - Arrangement for power transmission
  - None of the above
81. The diameter of the rotor shaft for an electric motor depends on which of the following?
- r.p.m. only
  - Horse power only
  - Horse power and r.p.m.
  - Horse power, r.p.m. and power factor
82. Which of the following alternatives will be cheaper?
- A 100 H.P. A.C. three phase motor
  - Four motors of 25 H.P. each
  - Five motors of 20 H.P. each
  - Ten motors of 10 H.P. each
83. The cost of an induction motor will increase as
- horsepower rating increases but r.p.m. decreases
  - horsepower rating decreases but r.p.m. increases
  - horsepower rating and operating speed increases
  - horsepower rating and operating speed decreases
84. in series motor which of the following methods can be used for changing the flux per pole?
- Tapped field control
  - Diverter field control
  - Series-parallel control
  - Any of the above

### ANSWERS

1. (c) 2. (b) 3. (d) 4. (c) 5. (c) 6. (d) 7. (c) 8. (c) 9. (b) 10. (c)  
 11. (a) 12. (c) 13. (c) 14. (b) 15. (e) 16. (e) 17. (b) 18. (d) 19. (c) 20. (d)  
 21. (d) 22. (d) 23. (d) 24. (c) 25. (b) 26. (c) 27. (a) 28. (b) 29. (d) 30. (d)  
 31. (a) 32. (a) 33. (d) 34. (d) 35. (d) 36. (a) 37. (c) 38. (c) 39. (d) 40. (c)  
 41. (c) 42. (b) 43. (b) 44. (c) 45. (a) 46. (c) 47. (a) 48. (a) 49. (d) 50. (b)  
 51. (d) 52. (a) 53. (d) 54. (d) 55. (b) 56. (d) 57. (b) 58. (b) 59. (c) 60. (d)  
 61. (c) 62. (b) 63. (b) 64. (d) 65. (d) 66. (c) 67. (d) 68. (c) 69. (a) 70. (d)  
 71. (c) 72. (d) 73. (c) 74. (b) 75. (b) 76. (c) 77. (a) 78. (c) 79. (b) 80. (c)  
 81. (c) 82. (a) 83. (a) 84. (d)

# CHAPTER 45

## Learning Objectives

- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Heating of Motor or Temperature Rise
- Equation for Heating of Motor
- Heating Time Constant
- Equation for Cooling of Motor or Temperature Fall
- Cooling Time Constant
- Heating and Cooling Curves
- Load Equalization
- Use of Flywheels
- Flywheel Calculations
- Load Removed (Flywheel Accelerating)
- Choice of Flywheel

## RATING AND SERVICE CAPACITY



Generator converts mechanical energy into electrical energy using electromagnetic induction

### 45.1. Size and Rating

The factors which govern the size and rating of motor for any particular service are its maximum temperature rise under given load conditions and the maximum torque required. It is found that a motor which is satisfactory from the point of view of maximum temperature rise usually satisfies the requirement of maximum torque as well. For class-A insulation, maximum permissible temperature rise is 40°C whereas for class – B insulation, it is 50°C. This temperature rise depends on whether the motor has to run continuously, intermittently or on variable load.

Different ratings for electrical motors are as under:

**1. Continuous Rating.** It is based on the maximum load which a motor can deliver for an indefinite period without its temperature exceeding the specified limits and also possessing the ability to take 25% overload for a period of time not exceeding two hours under the same conditions.

For example, if a motor is rated continuous 10 KW, it means that it is capable of giving an output of 10 KW continuously for an indefinite period of time and 12.5 KW for a period of two hours without its temperature exceeding the specified limits.

**2. Continuous Maximum Rating.** It is the load capacity as given above but without overload capacity. Hence, these motors are a little bit inferior to the continuous-rated motors.

**3. Intermittent Rating.** It is based on the output which a motor can deliver for a specified period, say one hour or ½ hour or ¼ hour without exceeding the temperature rise.

This rating indicates the maximum load of the motor for the specified time followed by a no-load period during which the machine cools down to its original temperature.

### 45.2. Estimation of Motor Rating

Since primary limitation for the operation of an electric motor is its temperature rise, hence motor rating is calculated on the basis of its average temperature rise. The average temperature rise depends on the average heating which itself is proportional to the square of the current and the time for which the load persists.

For example, if a motor carries a load  $L_1$  for time  $t_1$  and load  $L_2$  for time  $t_2$  and so on, then

$$\text{Average heating} \propto L_1^2 t_1 + L_2^2 t_2 + \dots + L_n^2 t_n$$

In fact, heating is proportional to square of the current but since load can be expressed in terms of the current drawn, the proportionality can be taken for load instead of the current.

$$\therefore \text{size of the motor} = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + \dots + L_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$

Generally, load on a motor is expressed by its load cycle. Usually, there are periods of no-load in the cycle. When motor runs on no-load, heat generated is small although heat dissipation continues at the same rate as long as the machine is running. Hence, there is a difference in the heating of a motor running at no-load and when at rest. It is commonly followed practice in America to consider the period at rest as one – third while calculating the size of motor. It results in giving a higher motor rating which is advantageous and safe.

**Example 45.1** An electric motor operates at full-load of 100 KW for 10 minutes, at ¾ full load for the next 10 minutes and at ½ load for next 20 minutes, no-load for the next 20 minutes and this cycle repeats continuously. Find the continuous rating of the suitable motor.

**Solution.**

$$\begin{aligned} \text{Size of the motor required} &= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + 20}} \\ &= 61 \text{ KW} \end{aligned}$$



According to American practice, we will consider the period of rest as (20/3) minutes. In that case, the motors size is

$$= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + (20/3)}} = 66 \text{ KW}$$

**Example 45.2.** An electric motor has to be selected for a load which rises uniformly from zero to 200 KW in 10 minutes after which it remains constant at 200 KW for the next 10 minutes, followed by a no-load period of 15 minutes before the cycle repeats itself. Estimate a suitable size of continuously rated motor.

**Solution.**

$$\text{Motor size} = \sqrt{\frac{(200/2)^2 \times 10 + (200)^2 \times 10 + 0 \times 15}{10 + 10 + (15 \times 1/3)}} = 140 \text{ KW}$$

According to American practice, no-load has been taken as one third.

**Example 45.3.** A certain motor has to perform the following duty cycle:

100 KW for 10 minutes                      No-load for 5 minutes  
50 KW for 8 minutes                        No-load for 4 minutes

The duty cycle is repeated indefinitely. Draw the curve for the load cycle. Assuming that the heating is proportional to the square of the load, determine suitable size of a continuously-rated motor. [Utilisation of Electric Power A.M.I.E.]

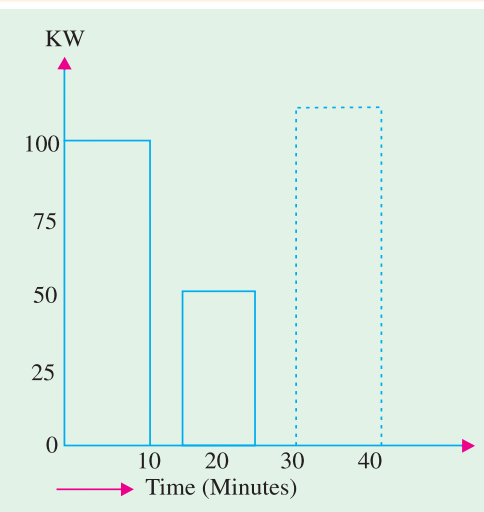


Fig. 45.1

**Solution.**

As explained above, heating is proportional to the square of the current and hence, to the square of the load.

∴ size of the continuously-rated motor

$$= \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}} = 66.67 \text{ kW}$$

Hence, the motor of **70 KW** would be adequate. The curve of the load cycle is shown in Fig. 45.1

The ultimate usefulness of the above factors is to select a motor of as small a size as possible compatible with temperature rise and to ensure that the motor has ample overload torque to cater for maximum-load conditions. Obviously, over-motoring

of any industrial drive will result in a waste of electrical energy, a low power factor and unnecessarily high capital cost for the motor and control gear.

### 45.3. Different Types of Industrial Loads

The three different types of industrial loads under which electric motors are required to work are as under:

(i) continuous load (ii) intermittent load and (iii) variable or fluctuating load.

The size of the motor depends on two factors. Firstly, on the temperature rise which, in turn,

will depend on whether the motor is to operate on continuous, intermittent or variable load. Secondly, it will depend on the maximum torque to be developed by the motor. Keeping in mind the load torque requirements, the rating of the motor will be decided by the load conditions as described below.

**(i) Continuous Load.** In such cases, the calculation of motor size is simpler because the loads like pumps and fans require a constant power input to keep them operating. However, it is essential to calculate the KW rating of the motor correctly. If the KW rating of the motor is less than what is required, the motor will overheat and consequently burn out. If, on the other hand, KW rating is more than what is needed by the load, the motor will remain cool but will operate at lower efficiency and power.

**(ii) Intermittent Loads.** Such loads can be of the following two types:

**(a)** In this type of load, motor is loaded for a short time and then shut of for a sufficient by long time, allowing the motor to cool down to room temperature as shown in Fig. 45.2. In such cases, a motor with a short time rating is used as in a kitchen mixie.



Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions. Their applications are in servo and positioning systems, tension reels, automatic door openers, and filament winding equipment.

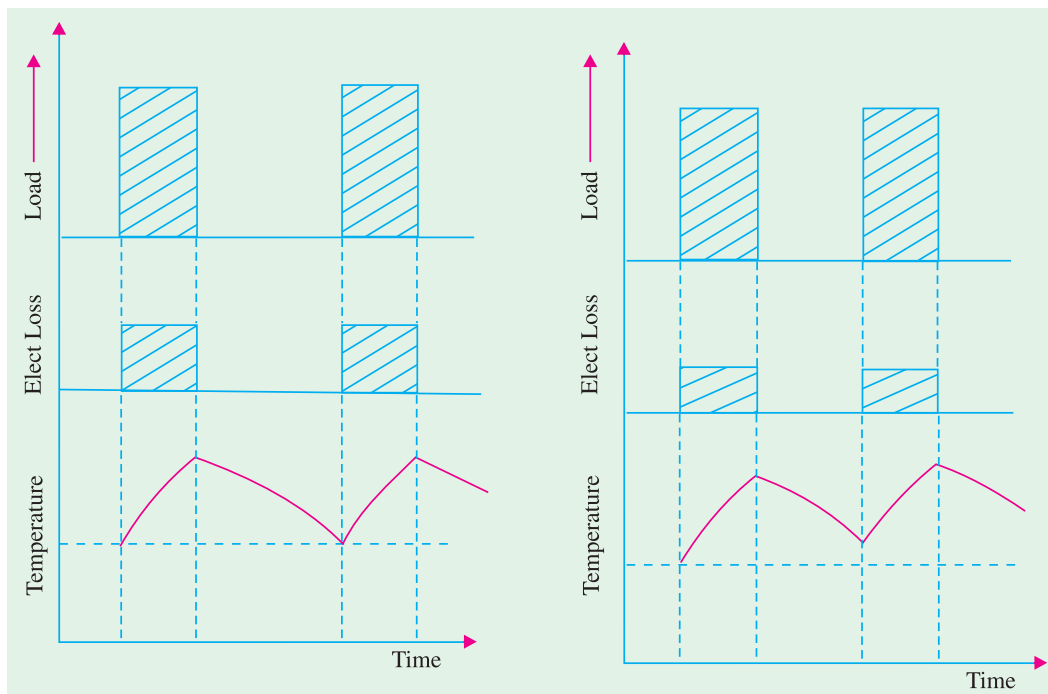


Fig. 45.2

Fig. 45.3

(b) In this type of load, motor is loaded for a short time and then it is shut off for a short time. The shut off time is so short that the motor cannot cool down to the room temperature as shown in Fig.45.3. In such cases, a suitable continuous or short-time rated motor is chosen which, when operating on a given load cycle, will not exceed the specified temperature limit.

(iii) **Variable Loads.** In the case of such loads, the most accurate method of selecting a suitable motor is to draw the heating and cooling curves as per the load fluctuations for a number of motors. The smallest size motor which does not exceed the permitted temperature rise when operating on the particular load cycle should be chosen for the purpose.

However, a simpler but sufficiently accurate method of selection of a suitable rating of a motor is to assume that heating is proportional to the square of the current and hence the square of the load. The suitable continuous rating of the motor would equal the r.m.s. value of the load current.

**Example 45.4.** A motor has to perform the following duty cycle

100 H.P.	For	10 min
No Load	"	5 min
60 H.P.	"	8 min
No Load	"	4 min

which is repeated infinitely. Determine the suitable size of continuously rated motor.

**Solution.**

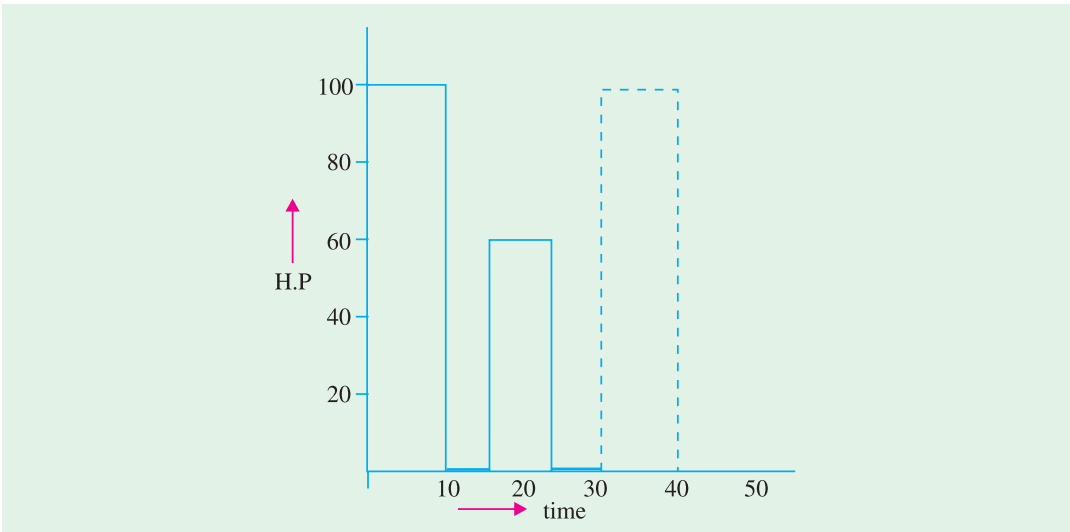


Fig. 45.4

$$\text{R.M.S. H.P.} = \sqrt{\left\{ \frac{1}{\text{Time for one cycle}} \right\} \int \text{HP}^2 dt}$$

$$\text{R.M.S. H.P.} = \left[ \frac{\sum \text{HP}^2 \times \text{time}}{\text{Time for one cycle}} \right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}} = 69.07 \text{ H.P.}$$

≈ 75 H.P. motor can be used.

**Example 45.5.** A motor working in a coal mine has to exert power starting from zero and rising uniformly to 100 H.P. in 5 min after which it works at a constant rate of 50 H.P. for 10 min. Then, a no load period of 3 min. The cycle is repeated indefinitely, estimate suitable size of motor.

[Nagpur University Summer 2000]

**Solution.**

(a) For time period : 0 - 5 min

$$\Rightarrow y = mx + c$$

$$\text{Slope} = \frac{(100-0)}{5}$$

$$m = 20 \text{ HP/min}$$

$$\therefore y = 20x + 0$$

$$y = 20x$$

(b) For total time period : 0 - 18 min

R.M.S. H P<sup>2</sup>

$$= \left\{ \int_0^5 y^2 dx + 50^2 \times 10 + 0^2 \times 3 \right\} / 18$$

$$\Rightarrow \text{H.P.}^2 \times 18 = \left[ \int_0^5 (20x)^2 dx \right] + 25000 = \left[ \frac{400x^3}{3} \right]_0^5 + 25000$$

$$\therefore \text{H.P.}^2 \times 18 = \frac{400 \times 125}{3} + 25000$$

$$\Rightarrow \text{H.P.} = \sqrt{\frac{41666.67}{18}} = 48.11 \text{ H.P.} \approx 50 \text{ H.P. motor can used}$$

or Same problem can be solved by Simpson's 1/3<sup>rd</sup> Rule of Integration

$$\text{H. P.} = \sqrt{\frac{\frac{1}{3} \times 100^2 \times 5 + 50^2 \times 10}{18}}$$

$$\text{H. P.} = 48.11 \text{ H.P. ;}$$

$$\therefore \text{H. P.} \approx 50 \text{ H.P.}$$

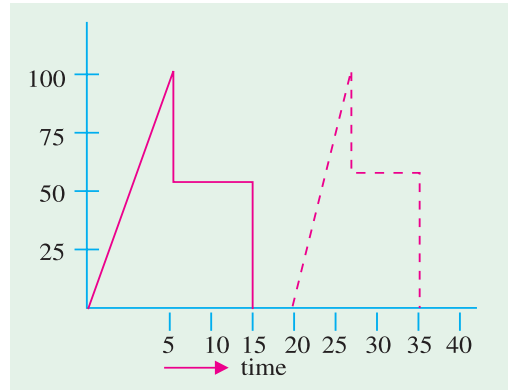


Fig. 45.5

**Example 45.6.** A motor has following duty cycle

Load rising from 200 to 400 H.P. - 4 min.

Uniform load 300 H. P. - 2 min.

Regenerative braking - H.P. returned to supply from 50 to zero - 1 min.

Remaining idle for - 1 min.

Estimate suitable H. P. rating of the motor. motor can be used.

[Nagpur University Winter 1994]

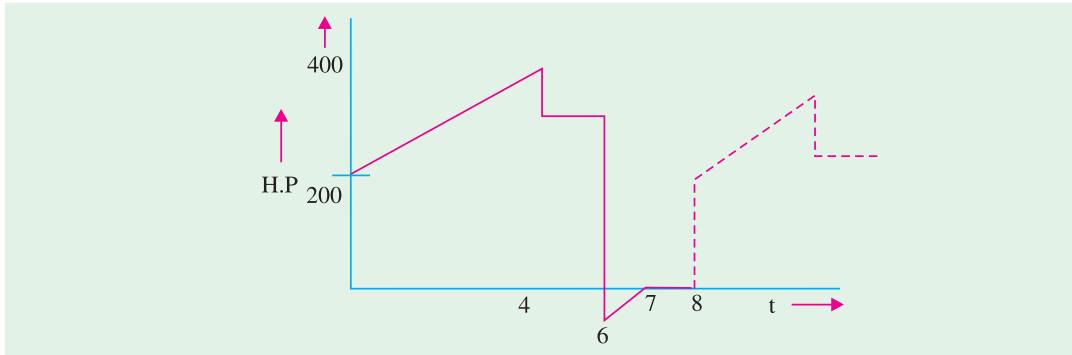


Fig. 45.6

**Solution.**

$$\begin{aligned}
 \text{H. P.} &= \sqrt{\frac{\frac{1}{3}(H_1^2 + H_1H_2 + H_2^2)t_1 + H_3^2t_2 + \frac{1}{3}H_4^2t_3}{8}} \\
 &= \sqrt{\frac{\frac{1}{3}(200^2 + 200 \times 400 + 400^2) \times 4 + 300^2 \times 2 + \frac{1}{3}50^2 \times 1}{8}} \\
 &= \sqrt{\frac{1662500}{24}} = \mathbf{263 \text{ H. P.}}
 \end{aligned}$$

**Note.** During regenerative braking, even though H.P. is returned to line, machine will be carrying current. So far heating is concerned, it is immaterial whether machine is taking current from or giving current to line.

This problem can be solved by another method as follows:-

(a) For time period : 0 - 4 min

$$\begin{aligned}
 \rightarrow \int_0^4 (50x + 200)^2 dx &= \int_0^4 (250x^2 + 20000x + 40000) dx \\
 &= 2500 \left[ \frac{x^3}{3} \right]_0^4 + 20000 \left[ \frac{x^2}{2} \right]_0^4 + 40000x \\
 &= 2500 \left[ \frac{4^3}{3} \right] + 20000 \left[ \frac{4^2}{2} \right] + 40000 \times 4 = \mathbf{373333.3 \text{ H.P.}}
 \end{aligned}$$

(b) For time period : 4 - 6 min

$$\rightarrow (300)^2 \times 2 = \mathbf{180000 \text{ H.P.}}$$

(c) For time period : 6 - 7 min

$$\rightarrow \int_0^1 (50x)^2 dx = 2500 \left[ \frac{x^3}{3} \right]_0^1 = \frac{2500}{3} = 833.33 \text{ H.P.}$$

$$\begin{aligned}
 \therefore \text{R.M.S. H.P.} &= \sqrt{\frac{373333.33 + 180000 + 833.33}{8}} = 263.1 \text{ H.P.} \\
 &\approx \mathbf{300 \text{ H.P. motor will be suitable}}
 \end{aligned}$$

**Example 45.7.** The load cycle of a motor for 15 min. in driving some equipment is as follows :

0 - 5 min	-	30 H. P.
5 - 9 min	-	No Load
9 - 12 min	-	45 H. P.
12 - 15 min	-	No Load

The load cycle is repeated indefinitely. Suggest a suitable size of continuously rated motor.

**Solution.**

$$\begin{aligned} \text{R.M.S. H.P} &= \left[ \frac{30^2 \times 5 + 45^2 \times 3}{15} \right]^{\frac{1}{2}} \\ &= 26.55 \text{ H. P.} \end{aligned}$$

∴ R.M.S. H.P  $\simeq$  **30 H. P. motor will be suitable.**

**Example 45.8.** A motor driving a colliery winder has the following acceleration period load cycle

0 - 15 sec.	:	Load rising uniformly from 0 - 1000 H.P.
Full speed period	:	15 - 85 sec. Load const. at 600 H.P.
Deceleration period	:	85 to 95 sec. regenerative braking the H. P. returned uniformly from 200 to 0 H. P.
95 - 120 sec.	:	Motor stationary.

Estimate the size of continuously rated motor.

**Solution.**

$$\begin{aligned} \text{R.M.S. H. P.} &= \left[ \frac{\frac{1}{3}(1000)^2 \times 15 + 600^2 \times 70 + \frac{1}{3}(200)^2 \times 10}{120} \right]^{\frac{1}{2}} \\ &= 502 \text{ H. P.} \\ &\equiv \text{505 H. P. motor can be used.} \end{aligned}$$

#### 45.4. Heating of Motor or Temperature Rise

The rise in temperature of a motor results from the heat generated by the losses and an expression for this temperature rise is obtained by equating the rate at which heat is being generated by these losses to the rate at which heat is being absorbed by the motor for raising the temperature of motor and in dissipation from the surfaces exposed to cooling media.

So long as the temperature of machine rises, the generated heat will be stored in body and the rest will be dissipated to cooling medium depending upon the temperature difference. This is called as unstable or transient situation.

If the temperature of body rises, it has to store heat. The amount of heat *i.e.* stored depends upon the heat capacity of the body. If the temperature of the machine remains constant *i.e.* it doesn't rise, then no further storage of heat takes place and all the heat *i.e.* generated must be dissipated. So rate of heat generation in motor equals rate of heat dissipation from the cooling surface. This is called a stable situation.

#### 45.5. Equation for Heating of Motor

Let,

$W \rightarrow$  Heat generated in motor due to powerloss in watts.

$G \rightarrow$  Weight of motor (kg)

- $S \rightarrow$  Average specific heat in (Watt - Sec.) to raise the temperature of unit weight through  $1^\circ\text{C}$ .
- $G \times S \rightarrow$  Heat required to raise the temperature of motor through  $1^\circ\text{C}$  (Watt - Sec.)
- $\theta \rightarrow$  Temperature rise above cooling medium in  $^\circ\text{C}$ .
- $\theta_f \rightarrow$  Final temperature rise in  $^\circ\text{C}$ .
- $A \rightarrow$  Cooling surface area of motor.
- $\lambda \rightarrow$  Rate of heat dissipation from the cooling surface.  
[(Watts/Unit area/ $^\circ\text{C}$  rise in temperature.) above cooling medium]
- $A\lambda \rightarrow$  Rate of heat dissipation in Watts / $^\circ\text{C}$  rise in temperature for a motor.

**Assumptions**

1. Loss 'W' remains constant during temperature rise.
2. Heat dissipation is proportional to the temperature difference between motor and cooling medium.
3. Temperature of cooling medium remains constant.

{Rate of heat generation in motor}  
= {Rate of heat absorption by the motor} + {Rate of heat dissipation from cooling surface}

$$\rightarrow W = GS \frac{d\theta}{dt} + A\lambda\theta$$

$$\text{or } W - A\lambda\theta = GS \frac{d\theta}{dt}$$

$$\rightarrow \frac{W}{A\lambda} - \theta = \frac{GS}{A\lambda} \frac{d\theta}{dt}$$

$$\rightarrow \frac{d\theta}{\left(\frac{W}{A\lambda} - \theta\right)} = \frac{dt}{\frac{GS}{A\lambda}}$$

By integrating,

$$\log_e \left( \frac{W}{A\lambda} - \theta \right) = -\frac{A\lambda}{GS} t + C \tag{2}$$

At  $t = 0$ ,  $\theta = \theta_1$  [ Initial temperature rise *i.e.* difference between the temperature of cooling medium and temperature of motor, during starting]

If starting from cold position,  $\theta_1 = 0$

Substituting the values of  $t$  and  $\theta$  in above equation.

$$C = \log_e \left( \frac{W}{A\lambda} - \theta_1 \right)$$

$$\therefore (2) \Rightarrow \log_e \left[ \frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)} \right] = -\frac{A\lambda}{GS} t$$

by taking antilog, 
$$\frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)} = e^{-\frac{A\lambda}{GS} t}$$

$$\therefore \theta = \frac{W}{A\lambda} - \left( \frac{W}{A\lambda} - \theta_1 \right) e^{-\frac{A\lambda}{GS}t} \quad \dots (3)$$

When, the final temperature rise of  $\theta_f$  is reached, all the heat generated is dissipated from the cooling surface so that,

equation (1) becomes  $W = A\lambda \theta_f$  or  $\theta_f = \frac{W}{A\lambda}$

And  $\frac{GS}{A\lambda} = \text{Heating time constant}$

$$\therefore \frac{A\lambda}{GS} = \frac{1}{T}$$

Then equation (3) becomes;

$$\theta = \theta_f - (\theta_f - \theta_1) e^{-\frac{t}{T}}$$

If starting from cold, then  $\theta_1 = 0$

$$\therefore \theta = \theta_f (1 - e^{-\frac{t}{T}})$$

### 45.6. Heating Time Constant

Heating time constant of motor is defined as the time required to heat up the motor upto 0.633 times its final temperature rise.

$$\theta = (1 - e^{-t/T}) \theta_f$$

At  $t = T$ ,  $\theta = 0.633 \theta_f$

After time	$t = T$	$\theta$ reaches to 63.3 % of $\theta_f$
	$t = 2T$	$\theta$ reaches to 86.5 % of $\theta_f$
	$t = 3T$	$\theta$ reaches to 95 % of $\theta_f$
	$t = 4T$	$\theta$ reaches to 98.2 % of $\theta_f$
	$t = 5T$	$\theta$ reaches to 99.3 % of $\theta_f$

- $T = \text{Heating time constant.}$
- $= 90 \text{ min for motors upto 20 H.P.}$
- $= 300 \text{ min for larger motors.}$

### 45.7. Equation for Cooling of Motor or Temperature Fall

If rate of heat generation is less than rate of heat dissipation, cooling will take place.

$\therefore \{ \text{Rate of heat generation in motor} \} + \{ \text{Rate of heat absorption by motor} \} = \{ \text{Rate of heat dissipation from cooling surface} \}$

$$W + GS \frac{d\theta}{dt} = A\lambda'\theta \quad \text{where } \lambda' = \text{Rate of heat dissipation during cooling surface}$$

$$W - A\lambda'\theta = -GS \frac{d\theta}{dt} \Rightarrow \frac{W}{A\lambda'} - \theta = -\frac{GS}{A\lambda'} \frac{d\theta}{dt}$$

$$\theta - \frac{W}{A\lambda'} = \frac{GS}{A\lambda'} \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \frac{dt}{\frac{GS}{A\lambda'}}$$



$$\therefore \int \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \int \frac{dt}{\frac{GS}{A\lambda'}}$$

$$\log_e \left( \theta - \frac{W}{A\lambda'} \right) = - \frac{A\lambda'}{GS} t + C$$

At  $t = 0$  Let  $\theta = \theta_0$  Difference of temperature between cooling medium and motor (Temperature rise at which cooling starts.)

$$\therefore C = \log_e \left( \theta_0 - \frac{W}{A\lambda'} \right) \text{ Put this value of } C \text{ in the above equation.}$$

$$\therefore \log_e \frac{\theta - \frac{W}{A\lambda'}}{\theta_0 - \frac{W}{A\lambda'}} = - \frac{A\lambda'}{GS} t$$

If  $\theta_f'$  is final temperature drop (above that of cooling medium), then at this temperature whatever heat is generated will be dissipated.

$$\therefore W = A\lambda' \theta_f' \Rightarrow \theta_f' = \frac{W}{A\lambda'}$$

$$\therefore \log_e \left\{ \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} \right\} = - \frac{t}{T'} \quad \text{Where } T' \text{ is cooling time constant} = \frac{GS}{A\lambda'}$$

$$\therefore \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} = e^{-t/T'} \Rightarrow (\theta - \theta_f') = (\theta_0 - \theta_f') e^{-t/T'}$$

$$\theta = \theta_f' + (\theta_0 - \theta_f') e^{-t/T'}$$

If motor is disconnected from supply, there will be no losses taking place and so final temperature reached will be ambient temperature. Hence  $\theta_f' = 0$  ( $\because W = 0$ )

$$\therefore \theta = \theta_0 \cdot e^{-t/T'}$$

$$\text{If } t = T', \text{ then } \theta = \theta_0 \cdot e^{-1} \Rightarrow \theta = \frac{\theta_0}{e} = 0.368 \theta_0 \quad ; \quad \therefore \theta = 0.368 \theta_0$$

### 45.8. Cooling Time Constant

Cooling time constant is defined as the time required to cool machine down to 0.368 times the initial temperature rise above ambient temperature.

By putting different values of  $T'$  in  $\theta = \theta_0 \cdot e^{-t/T'}$

$\therefore$ After time $t$	$=$	$T'$	$\theta$ has fallen to	36.8% of $\theta_0$	
	$t$	$=$	$2T'$	$\theta$ has fallen to	13.5% of $\theta_0$
	$t$	$=$	$3T'$	$\theta$ has fallen to	5% of $\theta_0$

$$\begin{aligned}
 t &= 4T' & \theta &\text{ has fallen to } 1.8\% \text{ of } \theta_0 \\
 t &= 5T' & \theta &\text{ has fallen to } 0.7\% \text{ of } \theta_0
 \end{aligned}$$

### 45.9. Heating and Cooling Curves

(a) Motor continuously worked on Full Load.

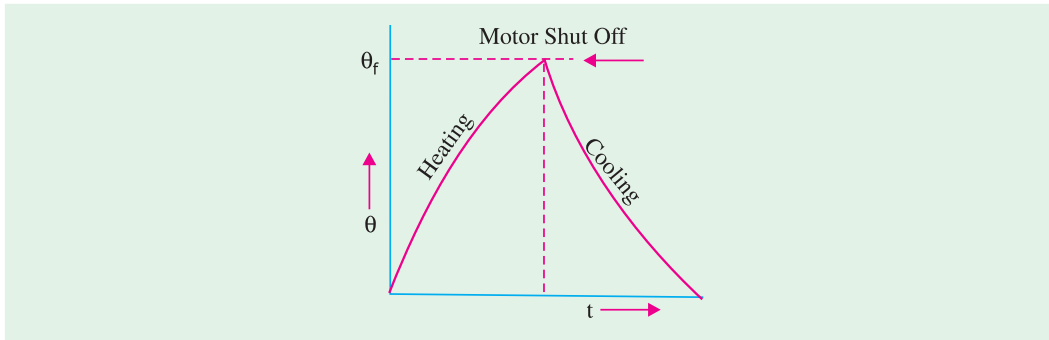


Fig. 45.7

Maximum permissible temperature rise.  
 Motor reaches final temperature rise and then cooling is carried out to ambient temperature.

(b) Motor Run for short time

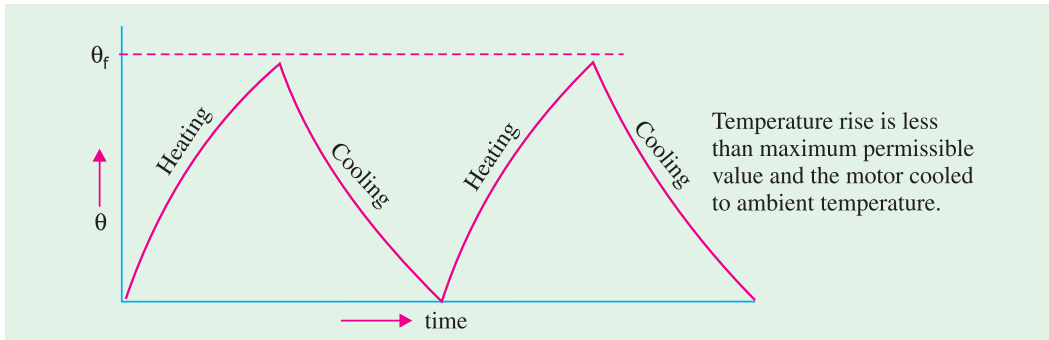


Fig. 45.8

(c) Cooling period not sufficient to cool down the motor to its ambient temperature.

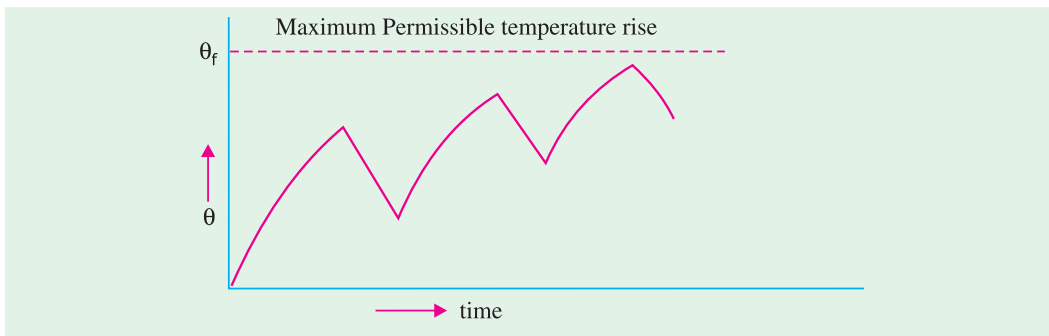


Fig. 45.9

\* For intermittent loads, a motor of smaller rating can be used without exceeding maximum permissible temperature rise.

**Example 45.9.** A 40 KW motor when run continuously on full load, attains a temperature of  $35^{\circ}\text{C}$ , above the surrounding air. Its heating time constant is 90 min. What would be the 1/2 hour rating of the motor for this temperature rise? Assume that the machine cools down completely between each load period and that the losses are proportional to square of the load,

**Solution.**

Let 'P' KW be the  $\frac{1}{2}$  hour rating of the motor

$\theta_f$  – Final temperature rise at P K W

$\theta_f'$  – Final temperature rise at 40 KW

$\therefore$  Losses at P KW  $\propto P^2$

Losses at 40 K W  $\propto 40^2$

$$\Rightarrow \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at P KW}}{\text{Losses at 40 KW}} = \left(\frac{P}{40}\right)^2 \quad \therefore \theta_f = \left(\frac{P}{40}\right)^2 \theta_f'$$

As the machine cools down completely, for 'P' KW the equation will be

$$\theta = \theta_f \left(1 - e^{-\frac{t}{T}}\right) \quad \text{where} \quad \theta_f = \left(\frac{P}{40}\right)^2 \times 35$$

$$\Rightarrow 35 = \left(\frac{P}{40}\right)^2 \times 35 \left(1 - e^{-\frac{0.5}{1.5}}\right); \quad \therefore P = 75.13 \text{ KW}$$

**Example 45.10.** Determine the one - hour rating of a 15 H.P. motor having heting time constant of 2 hours. The motor attains the temperature rise of  $40^{\circ}\text{C}$  on continuous run at full load. Assume that the losses are proportional to square of the load and the motor is allowed to cool down to the ambient temperature before being loaded again. [Nagpur University Summer 2001]

**Solution.**

Let 'P' H. P be one - hour rating of the motor

Losses at this load = Original losses  $\times \left(\frac{P}{15}\right)^2$ .

Let  $\theta_f$  be the final temperature rise at P H.P. and  $\theta_f'$  at 15 H P

$$\therefore \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at P H.P.}}{\text{Original Losses}} = \left(\frac{P}{15}\right)^2$$

$$\theta_f = \theta_f' \left(\frac{P}{15}\right)^2 = 40 \left(\frac{P}{15}\right)^2$$

$$\theta = \theta_f \left(1 - e^{-\frac{t}{T}}\right)$$

$$40 = 40 \left[\frac{P}{15}\right]^2 \left(1 - e^{-\frac{1}{2}}\right)$$

$$P = 23.96 \text{ H.P.}$$

$$P \simeq 24 \text{ H.P.}$$

**Example 45.11.** The heating and cooling time constants of a motor are 1 hour and 2 hours respectively. Final temperature rise attained is  $100^{\circ}\text{C}$ . This motor runs at full load for 30 minutes and then kept idle for 12 min. and the cycle is repeated indefinitely. Determine the temperature rise of motor after one cycle. [Nagpur University Winter 1997]

**Solution.**

$$\theta = \theta_f \left( 1 - e^{-\frac{t}{T}} \right)$$

$$= 100 \left( 1 - e^{-\frac{30}{60}} \right) = 39.34^{\circ}$$

$$\theta = \theta_0 e^{-\frac{t}{T}} = 39.34 e^{-\frac{12}{120}} = 35.6^{\circ}\text{C}$$

= Temperature rise of motor after 1 cycle.

**Example 45.12.** Calculate the maximum overload that can be carried by a 20 KW output motor, if the temperature rise is not to exceed  $50^{\circ}\text{C}$  after one hour on overload. The temperature rise on full load, after 1 hour is  $30^{\circ}\text{C}$  and after 2 hours is  $40^{\circ}\text{C}$ . Assume losses proportional to square of load.

**Solution.**

$$\theta = \theta_f \left( 1 - e^{-\frac{t}{T}} \right)$$

$$30 = \theta_f \left( 1 - e^{-\frac{1}{T}} \right) \quad \text{and} \quad 40 = \theta_f \left( 1 - e^{-\frac{2}{T}} \right)$$

$$\frac{1 - e^{-\frac{2}{T}}}{1 - e^{-\frac{1}{T}}} = \frac{40}{30}$$

Put  $x = e^{-\frac{1}{T}}$

$$\therefore \frac{1 - x^2}{1 - x} = \frac{4}{3} \quad \therefore \frac{(1 - x)(1 + x)}{(1 - x)} = \frac{4}{3}$$

$$\Rightarrow 1 + x = \frac{4}{3}$$

$$\therefore x = \frac{1}{3} = e^{-\frac{1}{T}} \Rightarrow T = 0.91 \text{ hrs.}$$

To find  $\theta_f \Rightarrow 30 = \theta_f \left( 1 - e^{-\frac{1}{T}} \right) \Rightarrow 30 = \theta_f \left( 1 - \frac{1}{3} \right) \Rightarrow \theta_f = 45^{\circ}\text{C}$

After 1 hr.  $50 = \theta_f \left( 1 - \frac{1}{e^T} \right)$

$$50 = \theta_f \left( 1 - \frac{1}{3} \right) \quad \therefore \theta_f = 75^{\circ}\text{C}$$

Let the maximum overload capacity of 20 KW motor is  $P$  KW

$$\therefore \frac{\theta_f}{\theta'_f} = \frac{\text{Losses at PK W}}{\text{Original Losses}} = \left( \frac{P}{20} \right)^2$$

$$\therefore \frac{75}{45} = \left(\frac{P}{20}\right)^2 \quad \therefore P = 25.8 \text{ kW}$$

**Example 45.13.** In a transformer the temperature rise is 25°C after 1 hour and 37.5°C after 2 hours, starting from cold conditions. Calculate its final steady temperature rise and the heating time constant. If the transformer temperature falls from the final steady value to 40°C in 1.5 hours when disconnected, calculate its cooling time constant. Ambient temperature is 30°C.

**Solution.**  $\theta = \theta_f \left(1 - e^{-\frac{t}{T}}\right)$

$$25 = \theta_f \left(1 - e^{-\frac{1}{T}}\right) \quad \text{and} \quad 37.5 = \theta_f \left(1 - e^{-\frac{2}{T}}\right)$$

$$\therefore \frac{37.5}{25} = \frac{1 - e^{-\frac{2}{T}}}{1 - e^{-\frac{1}{T}}}, \quad \text{Put } x = e^{-\frac{1}{T}}$$

$$1.5 = \frac{1 - x^2}{1 - x}, \quad 1.5 = \frac{(1 - x)(1 + x)}{(1 - x)}$$

$$1.5 = 1 + X \quad \therefore X = 0.5$$

$$\therefore e^{-\frac{1}{T}} = 0.5 \quad \therefore T = 1.44 \text{ hrs}$$

$$25 = \theta_f \left(1 - e^{-\frac{1}{1.44}}\right) \Rightarrow 25 = \theta_f \left(1 - e^{-0.694}\right) \Rightarrow \theta_f = 50^\circ\text{C}$$

**Cooling :** Temperature rise after 1.5 hours above ambient temperature = 40 – 30 = 10°C.

$\therefore$  The transformer is disconnected  $\theta = \theta_0 e^{-\frac{t}{T'}}$

$$10 = 50 e^{-\frac{1.5}{T'}} \quad \therefore T' = 0.932 \text{ hrs}$$

**Example 45.14.** The initial temperature of machine is 45°C. Calculate the temperature of machine after 1.2 hours, if its final steady temperature rise is 85°C and the heating time constant is 2.4 hours. Ambient temperature is 25°C

**Solution.**

$$\theta = \theta_f - (\theta_f - \theta_1) e^{-\frac{t}{T}} \quad \theta = 85 - (85 - 20) e^{-\frac{1.2}{2.4}}$$

$$\theta = 45.54^\circ\text{C} \quad \text{- Temperature rise above cooling medium}$$

$$\therefore \text{Temperature of machine after 1.2 hours is} = 45.54 + 25 = 70.54^\circ\text{C}$$

**Example 45.15.** The following rises were observed in a temperature rise test on a D.C. machine at full loads.

After	1 hour	—	15°C
After	2 hours	—	25°C

**Find out (i)** Final steady temperature rise and time constant.

**(ii)** The steady temperature rise after 1 hour at 50% overload, from cold.

Assume that the final temperature rise on 50% overload is 90°C.

[Nagpur University Summer 1998]

**Solution.**  $\theta = \theta_f (1 - e^{-\frac{t}{T}})$ , as motor is starting from cold.

$$15 = \theta_f (1 - e^{-\frac{1}{T}}) \quad \text{and} \quad 25 = \theta_f (1 - e^{-\frac{2}{T}})$$

$$\therefore \frac{25}{15} = \frac{\theta_f \left(1 - e^{-\frac{2}{T}}\right)}{\theta_f \left(1 - e^{-\frac{1}{T}}\right)}$$

$$\therefore e^{-\frac{1}{T}} = \frac{25}{15} - 1, \quad e^{-\frac{1}{T}} = \frac{2}{3}$$

$$\therefore T = 2.466 \text{ hours}, \quad 15 = \theta_f \left(1 - e^{-\frac{1}{T}}\right)$$

by putting value of  $T$ ,

$$15 = \theta_f \left(1 - e^{-\frac{1}{2.466}}\right)$$

$$\Rightarrow \theta_f = 45^\circ\text{C}$$

(ii) On 50% overload  $\theta_f = 90^\circ\text{C}$

$\therefore$  Final temperature rise after 1 hour at 50% overload is  $\theta = \theta_f (1 - e^{-\frac{t}{T}})$

$$\theta = 90 \left(1 - e^{-\frac{1}{2.466}}\right) = 30^\circ\text{C}$$

### 45.10. Load Equalization

If the load fluctuates between wide limits in space of few seconds, then large peak demands of current will be taken from supply and produce heavy voltage drops in the system. Large size of conductor is also required for this.

Process of smoothing out these fluctuating loads is commonly referred to as load equalization and involves storage of energy during light load periods which can be given out during the peak load period, so that demand from supply is approximately constant. Tariff is also affected as it is based on M.D. (Maximum Demand)

For example, in steel rolling mill, when the billet is in between the rolls it is a peak load period and when it comes out it is a light load period, when the motor has to supply only the friction and internal losses, as shown in figure 45.10.

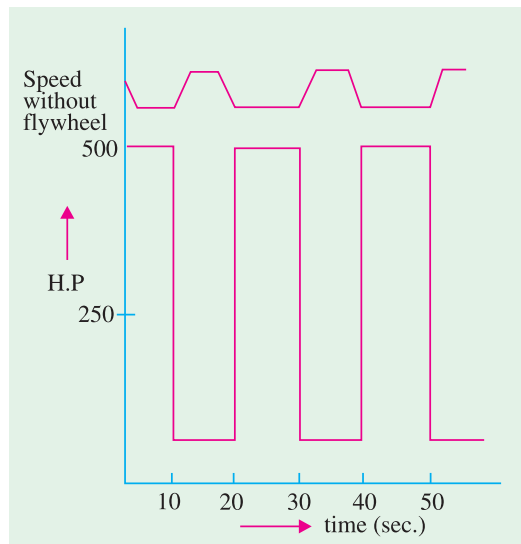


Fig. 45.10

### 45.11. Use Of Flywheels

The method of Load Equalization most commonly employed is by means of a flywheel. During peak load period, the flywheel decelerates and gives up its stored kinetic energy, thus reducing the load demanded from the supply. During light load periods, energy is taken from supply to accelerate flywheel, and replenish its stored energy ready for the next peak. Flywheel is mounted on the motor shaft near the motor. The motor must have drooping speed characteristics, that is, there should be a drop in speed as the load comes to enable flywheel to give up its stored energy. When the Ward - Leonard system is used with a flywheel, then it is called as Ward - Leonard Ilgner control.

### 45.12. Flywheel Calculations

The behaviour of flywheel may be determined as follows.

#### Fly wheel Decelerating :- (or Load increasing)

Let	$T_L$	→	Load torque assumed constant during the time for which load is applied in kg-m
	$T_f$	→	Torque supplied by flywheel in kg-m
	$T_o$	→	Torque required on no load to overcome friction internal losses etc., in kg-m
	$T_m$	→	Torque supplied by the motor at any instant, in kg-m
	$\omega_o$	→	No Load speed of motor in rad/sec.
	$\omega$	→	Speed of motor at any instant in rad/sec.
	$s$	→	motor slip speed ( $\omega_o - \omega$ ) in rad/sec.
	$I$	→	Moment of inertia of flywheel in kg-m <sup>2</sup>
	$g$	→	Acceleration due to gravity in m/sec <sup>2</sup>
	$t$	→	time in sec.

When the flywheel decelerates, it gives up its stored energy.

$$\rightarrow T_m = T_L - T_f \quad \text{or} \quad T_L = T_m + T_f \quad \dots (1)$$

Energy stored by flywheel when running at speed 'ω' is 1/2 Iω<sup>2</sup>/g.

If speed is reduced from ω<sub>0</sub> to ω.

The energy given up by flywheel is

$$= \frac{1}{2} \frac{I}{g} (\omega_o^2 - \omega^2)$$

$$= \frac{1}{2} \frac{I}{g} (\omega_o + \omega) (\omega_o - \omega) \quad \dots (2)$$

$\left(\frac{\omega_o + \omega}{2}\right)$  = mean speed. Assuming speed drop of not more than 10%., this may be assumed equal to ω.

$$\therefore \left(\frac{\omega_o + \omega}{2}\right) \approx \omega \quad \text{Also} \quad (\omega_o - \omega) = s$$

$$\therefore \text{From equation (2), Energy given up} = \frac{I}{g} \omega \cdot s$$

$$\text{Power given up} = \frac{I}{g} \omega \frac{ds}{dt}$$

$$\text{but Torque} = \frac{\text{Power}}{\omega}$$

∴ Torque supplied by flywheel.

$$T_f = \frac{I}{g} \frac{ds}{dt}$$

∴ From equation (1),  $T_m = T_L - \frac{I}{g} \frac{ds}{dt}$

For values of slip speed upto 10% of No - load speed, slip is proportional to Torque

or  $s = K T_m$

∴  $T_m = T_L - \frac{I}{g} K \frac{dT_m}{dt}$

This equation is similar to the equation for heating of the motor  $W - A\lambda\theta = G.S. \frac{d\theta}{dt}$

i.e.  $(T_L - T_m) = \frac{I}{g} K \frac{dT_m}{dt} \Rightarrow g \frac{dt}{IK} = \frac{dT_m}{(T_L - T_m)}$

By integrating both sides.

$$-\ln(T_L - T_m) = \frac{tg}{IK} + C_1 \quad \dots (3)$$

At  $t = 0$ , when load starts increasing from no load i.e.  $T_m = T_0$

Hence, at  $t = 0$   $T_m = T_0$

∴  $C_1 = -\ln(T_L - T_0)$

By substituting the value of  $C_1$  above, in equation (3)  $-\ln(T_L - T_m) = \frac{tg}{IK} - \ln(T_L - T_0)$

$$\therefore \ln\left(\frac{T_L - T_m}{T_L - T_0}\right) = -\frac{tg}{IK} \Rightarrow \left(\frac{T_L - T_m}{T_L - T_0}\right) = e^{-\frac{tg}{IK}}$$

$$\Rightarrow (T_L - T_m) = (T_L - T_0) e^{-\frac{tg}{IK}} \quad \therefore T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}}$$

If the Load torque falls to zero between each rolling period, then  $T_m = T_L - \left(1 - e^{-\frac{tg}{IK}}\right)$  ( $\therefore T_0 = 0$ )

### 45.13. Load Removed (Flywheel Accelerating)

Slip speed is decreasing and therefore  $\frac{ds}{dt}$  is negative

$$T_m = T_0 + T_f = T_0 - \frac{I}{g} \frac{ds}{dt} \Rightarrow T_0 - T_m = \frac{I}{g} K \frac{dT_m}{dt}$$

$$\frac{g dt}{IK} = \frac{dT_m}{T_0 - T_m}$$

After integrating both sides,

$$-\ln(T_0 - T_m) = \frac{tg}{IK} + C \quad \text{At } t = 0, T_m = T_m' \text{ motor torque at the instant, when load is removed}$$

∴  $C = -\ln(T_0 - T_m')$  Putting this value of C in the above equation

$$-\ln(T_0 - T_m) = \frac{tg}{IK} - \ln(T_0 - T_m')$$



$$\therefore \ln \left( \frac{T_0 - T_m}{T_0 - T_m'} \right) = \frac{-tg}{IK} \qquad \therefore T_0 - T_m = (T_0 - T_m') e^{\frac{-tg}{IK}}$$

$$\therefore T_m = T_0 + (T_m' - T_0) e^{\frac{-tg}{IK}}$$

Where  $T_m'$  = the motor torque, at the instant the load is removed.

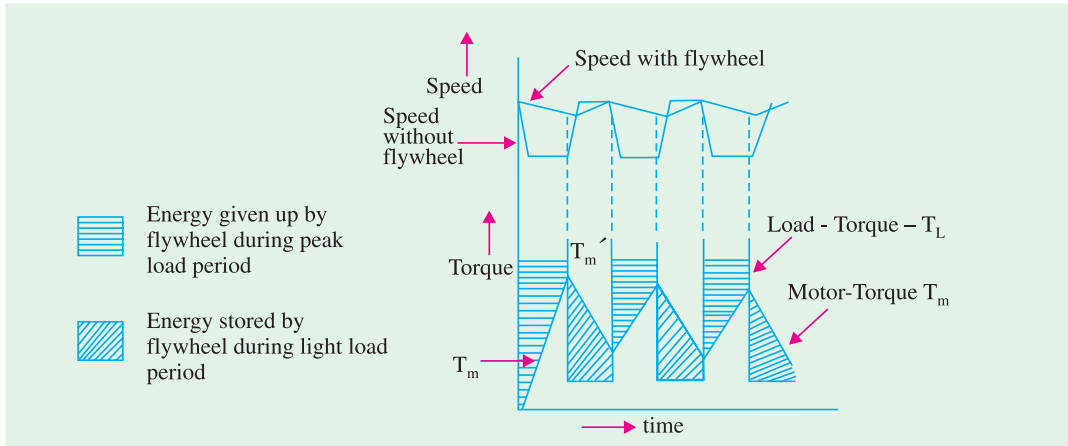


Fig. 45.11. Rolling mill drive with Flywheel

#### 45.14. Choice of Flywheel

There are two choices left for selecting a flywheel to give up its maximum stored energy:

1. Large drop in speed and small flywheel (But with this the quality of production will suffer, since a speed drop of 10 to 15% for maximum load is usually employed).
2. Small drop in speed and large flywheel. (This is expensive and creates additional friction losses. Also design of shaft and bearing of motor is to be modified.) So compromise is made between the two and a proper flywheel is chosen.



The above figure shows the flywheel of a motor as a separate part

**Example 45.16.** The following data refers to a 500 H.P. rolling mill, induction motor equipped with a flywheel.

No load speed	→	40 r.p.m.
Slip at full load (torque)	→	12%
Load torque during actual rolling	→	41500 kg - m
Duration of each rolling period	→	10 sec.

Determine inertia of flywheel required in the above case to limit motor torque to twice its full load value. Neglect no - load losses and assume that the rolling mill torque falls to zero between each rolling period. Assume motor slip proportional to full load torque.

[Nagpur University Summer 1996]

**Solution.**  $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189 \text{ rad/sec}$

given,  $T_0 = 0$   $T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}}$

$t = 10 \text{ sec.}$

$T_L = 41500 \text{ kg - m}$

$T_m = 2 \times T_{\text{Full Load}}$

Now  $T_{\text{Full Load}} = \frac{500 \times 735.5}{0.88 \times 4.189} \text{ N-m.} \quad \therefore s = 12\%$

$(1-s) = 0.88$

$= 99765 \text{ N-m}$

$= 10169.7 \text{ kg-m}$

$\therefore T_m = 2 \times 10169.7$

$= 20339.5 \text{ kg}$

$s = \omega_0 - \omega$

$= \frac{2\pi}{60} (N_0 - N)$

$= \frac{2\pi}{60} (40 - 0.88(40))$

$= \frac{2\pi}{60} (4.8) = 0.503 \text{ rad/sec.}$

$s = K T_{\text{FL}}$

$0.503 = K (10169.7) \Rightarrow K = 4.91 \times 10^{-5}$

$T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}}$

$\therefore T_0 = 0 \quad \therefore T_m = T_L \left( 1 - e^{-\frac{tg}{IK}} \right)$

$20339.5 = 41500 \left( 1 - \frac{-10 \times 981}{e^{1 \times 4.91 \times 10^{-5}}} \right)$

$\therefore I = 2.9663 \times 10^6 \text{ kg - m}^2$

**Example 45.17.** A 6 pole, 50 Hz Induction Motor has a flywheel of 1200 kg-m<sup>2</sup> as moment of inertia. Load torque is 100 kg - m. for 10 sec. No load period is long enough for the flywheel, to regain its full speed. Motor has a slip of 6% at a torque of 50 kg-m. Calculate

(i) Maximum torque exerted by motor.

(ii) Speed at the end of deceleration period.

[Nagpur University Winter 1996]

**Solution.** (i)  $T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}}$

Assume  $T_0 = 0, \rightarrow T_m = T_L (1 - e^{-\frac{tg}{IK}})$

$T_L = 100 \text{ kg - m,} \quad t = 10 \text{ sec,} \quad g = 9.81 \text{ m/sec}^2, \quad I = 1200 \text{ kg - m}^2 \quad s = \omega_0 - \omega$

$$s = KT \quad \therefore \text{slip speed} = s = \frac{2\pi}{60} (N_0 - N)$$

$$N_s = \frac{120 f}{p} = \frac{120 \times 50}{6} = 1000 \text{ rpm} = N_0$$

$$N = 0.94 \times 1000 = 940 \text{ rpm.}$$

$$\therefore S = \frac{2\pi}{60} (1000 - 940) = 2\pi \text{ rad/sec} \quad \text{or} \quad s = \frac{2\pi N}{60} = \frac{2\pi \times 1000 \times 0.06}{60}$$

$$s = 2\pi = 6.283 \text{ rad/sec}$$

$$\therefore K = \frac{S}{T} = \frac{2\pi}{50} = 0.04\pi = 0.125$$

$$\therefore T_m = T_L (1 - e^{-\frac{tg}{IK}}) = \left( 1 - e^{-\frac{-10 \times 9.81}{1200 \times 0.04\pi}} \right)$$

$$T_m = 47.83 \text{ kg - m}$$

(ii) Slip speed =  $0.04 \pi \times 47.8 \text{ rad/sec}$

$$s = KT_m$$

$$s = 0.125 (47.83)$$

$$s = 5.98 \text{ rad/sec}$$

$$s = 5.98 \frac{60}{2\pi} \text{ rpm} = 57.5 \text{ rpm} = \text{slip speed}$$

$$\therefore \text{Actual speed} = 1000 - 57.5 = 942.5 \text{ rpm}$$

**Example 45.18.** An Induction Motor equipped with a flywheel is driving a rolling mill which requires a Load Torque of 1900 N - m for 10 sec. followed by 250 N - m for 30 sec. This cycle being repeated indefinitely. The synchronous speed of motor is 750 r.p.m and it has a slip of 10% when delivering 1400 N-m Torque. The total Moment of Inertia of the flywheel and other rotating parts is 2100 kg-m<sup>2</sup>. Draw the curves showing the torque exerted by the motor and the speed for five complete cycles, assuming that initial torque is zero. [Nagpur University Summer 1998]

**Solution.**

$$T_L = 1900 \text{ N- m for 10 sec.}$$

$$T_L = 250 \text{ N- m for 30 sec.}$$

$$T_0 = 0 \text{ (assumed)}$$

$$\text{Slip} = 10\% \text{ at } 1400 \text{ N - m torque}$$

$$\text{Slip} = 750 \times 0.1 = 75 \text{ r.p.m.}$$

$$N_s = 750 \text{ r.p.m.}$$

$$s = 10\%$$

$$I = 2100 \text{ Kg- m}^2, T_m = 1400 \text{ N - m}$$

$$= \frac{75 \times 2\pi}{60} = 7.85 \text{ rad/sec.}$$

$$s = K T_m; K = \frac{S}{T_m} = \frac{7.85}{1400} = 0.0056$$

(i) During 1st cycle :

(a) Flywheel de-accelerating :

$$T_m = T_L - (T_L - T_0) e^{-\frac{t}{IK}} \quad [\text{When torque is taken in N - m.}]$$

→ After 10 sec

$$T_m = 1900 - (1900 - 0)e^{-0.085 \times 10} \quad \therefore \frac{1}{JK} = 0.085$$

$$T_m = 1088 \text{ N-m}$$

$$\text{Slip} = 0.0056 \times 1088 = 6.08 \text{ rad/sec}$$

$$\text{Slip} = 58 \text{ r.p.m.}$$

$$\text{Speed} = 750 - 58 = \mathbf{692 \text{ r.p.m.}}$$

(b) Flywheel acclerating (Off Load Period)

$$T_m = T_0 + (T_m' - T_0)e^{-\frac{t}{JK}}$$

$$T_0 = \text{No load torque} = 280 \text{ N-m}$$

$$T_m' = 1088 \text{ N-m (} T_m \text{ at the beginning of the period i.e. the motor torque at the instant when load is removed)}$$

After 30 sec.,

$$T_m = 280 + (1088 - 280)e^{-0.085 \times 30}$$

$$T_m = 343 \text{ N-m}$$

$$\therefore \text{Slip at this } T_m = 0.0056 \times 343 = 1.92 \text{ rad/sec} = 18.34 \text{ r.p.m.}$$

$$\therefore \text{Speed} = (750 - 18.34) \text{ r.p.m.} = \mathbf{731.6 \text{ r.p.m.}}$$



Flywheel

(ii) During 2nd cycle :

(a) Flywheel deaccelerating  $T_0 = 343 \text{ N-m}$ .

$$T_m = 1900 - (1900 - 343)e^{-0.085 \times 30}$$

$$= 1235 \text{ N-m.}$$

$$\therefore \text{Slip at this } T_m = 0.0056 \times 1235 = 6.92 \text{ rad/sec}$$

$$= 66 \text{ r.p.m.}$$

$$\rightarrow \text{speed} = 750 - 66 = \mathbf{684 \text{ r.p.m.}}$$

(b) Off Load Period:

$$T_m = 280 + (1235 - 280)e^{-0.085 \times 30}$$

$$= 354.6 \text{ N-m.}$$

$$\therefore \text{Slip at this } T_m = 0.0056 \times 354.6 = 1.99 \text{ rad/sec}$$

$$= 19 \text{ r.p.m.}$$

$$\text{speed} = 750 - 19 = \mathbf{731.0 \text{ r.p.m.}}$$

(iii) During 3rd Cycle :

(a) On Load period :  $T_m$  can be found as above.

$$T_m = 1263 \text{ N-m}$$

$$\text{Speed} = \mathbf{683.6 \text{ r.p.m.}}$$

(b) Off Load period

$$T_m = 354.6 \text{ N-m}$$

$$\text{speed} = \mathbf{731.0 \text{ r.p.m.}}$$

Initial condition at the beginning of the 3<sup>rd</sup> peak load are thus practically the same as that at the beginning of 2<sup>nd</sup>. Therefore Motor Torque in this and all succeeding load cycles will follow a similar curve to that in second period.

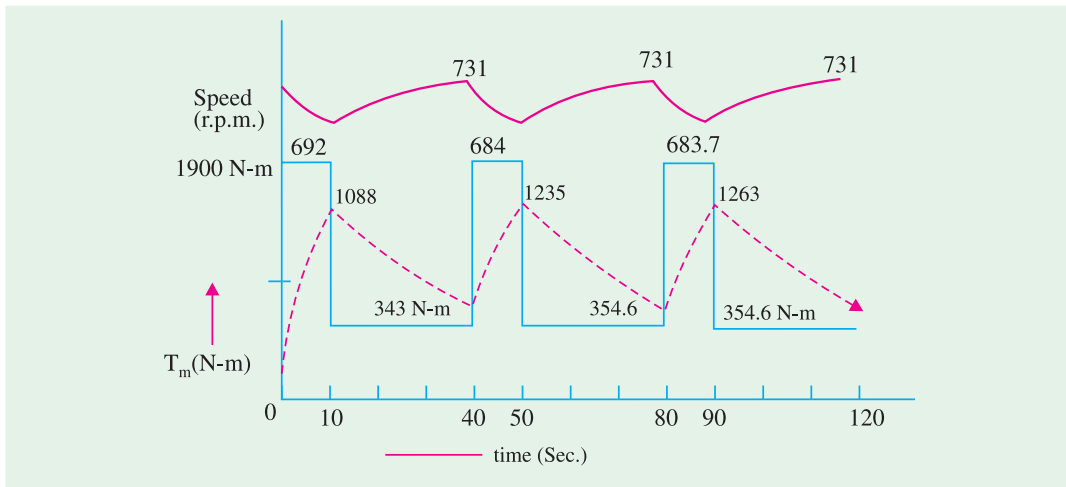


Fig. 45.12

**Example 45.19.** A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no-load period, the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine moment of inertia of flywheel. The no-load speed of motor is 500 r.p.m. and it has a slip of 10% on full load.

**Solution.**

$$T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}}$$

$$T_m = T_L \left( 1 - e^{-\frac{tg}{IK}} \right), \quad \because T_0 = 0 \text{ kg-m}$$

$$T_m = 85 \text{ kg-m}, \quad T_L = 150 \text{ kg-m}, \quad T_0 = 0 \text{ kg-m}, \quad t = 15 \text{ sec.}, \quad I = ?, \quad g = 9.81 \text{ m/sec}^2$$

$$s = K T_{F.L.}, \quad \text{where } s = \omega_0 - \omega$$

$$\frac{2\pi (500) \times 0.1}{60} = K \times 85 \Rightarrow K = 0.0617$$

$$\therefore 85 = 150 \left( 1 - e^{-\frac{15 \times 9.81}{I \times 0.0617}} \right); \quad \therefore I = 2884 \text{ kg-m}^2$$

**Example 45.20.** A 3- $\phi$ , 50 KW, 6 pole, 960 r.p.m. induction motor has a constant load torque of 300 N-m and at wide intervals additional torque of 1500 N-m for 10 sec. Calculate

- (a) The moment of inertia of the flywheel used for load equalization, if the motor torque is not to exceed twice the rated torque.
- (b) Time taken after removal of additional load, before the motor torque becomes 700 N-m.

**Solution.**

(a)  $P = T \times \omega \quad \therefore T = P / \omega$

$$\therefore T_{F.L.} = \frac{50 \times 10^3}{\frac{2\pi \times 960}{60}} = 497.36 \text{ N-m.}$$

$$\therefore T_m = 2 \times T_{FL} = 2 \times 497.36 = 994.72 \text{ N-m}$$

$$T_L = 1500 + 300 = 1800 \text{ N-m}$$

$$N_S = \frac{120 f}{P} = 1000 \text{ r.p.m.}$$

$$\therefore \text{F.L. slip} = 1000 - 960 = 40 \text{ rpm} = 40 \text{ r.p.m.} = 4\%$$

$$\therefore s = K T_{FL}$$

$$\frac{2\pi(40)}{60} = K \times 497.36 \quad \therefore K = 8.42 \times 10^{-3}$$

$$T_m = T_L - (T_L - T_0) e^{\frac{-t}{K}} \text{ as torque is in N - m}$$

$$994.72 = 1800 - (1800 - 300) \times e^{\frac{-10}{I \times 8.42 \times 10^{-3}}}$$

$$\therefore I = 1909 \text{ kg- m}^2$$

$$(b) \quad T_m = T_0 + (T_m' - T_0) e^{\frac{-t}{K}}$$

$$700 = 300 + (994.72 - 300) e^{\frac{-t}{1909 \times 8.42 \times 10^{-3}}}$$

$$\therefore t = 8.87 \text{ sec.}$$

**Example 45.21.** A 3-phase, 8 pole, 50 c.p.s. Induction Motor equipped with a flywheel supplies a constant load torque of 100 N-m and at wide intervals an additional load torque of 300 N-m for 6 sec. The motor runs at 735 r.p.m., at 100 N-m torque. Find moment of inertia of the flywheel, if the motor torque is not to exceed 250 N-m.

**Solution.**

$$T_0 = 100 \text{ N-m} \quad \therefore T_L = 100 + 300 = 400 \text{ N-m}$$

$$N_S = \frac{120 f}{P} = \frac{120 f}{P} = 750 \text{ rpm.}$$

$$\text{Slip at 100 N-m torque} = 750 - 735 = 15 \text{ r.p.m.}$$

$$s = K T_m$$

$$\frac{2\pi}{60}(15) = K(100) \quad \therefore K = 0.0157 \quad ; \quad T_m = T_L - (T_L - T_0) e^{\frac{-t}{K}}$$

$$250 = 400 - (400 - 100) e^{\frac{-6}{I(0.0157)}}$$

$$\therefore I = 552 \text{ kg - m}^2$$

**Example 45.22.** A 6 pole, 50 Hz, 3 -  $\phi$  wound rotor Induction Motor has a flywheel coupled to its shaft. The total moment of inertia is 1000 kg-m<sup>2</sup>. Load torque is 1000 N-m for 10 sec. followed by a no load period which is long enough for the motor to reach its no - load speed. Motor has a slip of 5% at a torque of 500 N-m. Find

(a) Maximum torque developed by motor

(b) Speed at the end of deceleration period.

[Nagpur University Winter 1996]

**Solution.**

$$(a) \quad T_m = T_L \left( 1 - e^{\frac{-t}{K}} \right)$$

$$s = KT_m \quad \text{But } N_s = \frac{120f}{P} = 1000 \text{ r.p.m.}$$

$$\frac{2\pi}{60} (1000 \times 0.05) = K (500) ; \quad K = 6.2 \times 10^{-3}$$

$$T_m = 1000 \left( 1 - e^{-\frac{10}{1000 \times 6.2 \times 10^{-3}}} \right) ; \quad T_m = 796.39 \text{ N-m}$$

$$(b) \quad s = K T_{F.L.}$$

$$\frac{2\pi}{60} (100 - N) = 6.2 \times 10^{-3} \times 790.39$$

$$\therefore N = 952.2 \text{ r.p.m.}$$

**Example 45.23.** A motor fitted with a flywheel supplies a load torque of 1000 N-m. for 2 sec. During no load period, the flywheel regains its original speed. The motor torque is to be limited to 500 N-m. Find moment of inertia of the flywheel. No load speed of the motor is 500 r.p.m. and its full load slip is 10%.

**Solution.**

$$s = K T_{F.L.}$$

$$\frac{2\pi}{60} (500 \times 0.1) = K 500, \quad K = 0.0104 ; \quad T_m = T_L \left( 1 - e^{-\frac{t}{K}} \right)$$

$$500 = 1000 \left\{ 1 - e^{-\frac{2}{1(0.0104)}} \right\} ; \quad I = 277.44 \text{ kg-m}^2$$

### Tutorial Problem No. 45.1

1. A motor driving a colliery winding equipment has to deliver a load rising uniformly from zero to a maximum of 1500 KW in 20 sec. during the accelerating period, 750 KW for 40 sec. during the full speed period and during the deceleration period of 10 sec., when regenerative braking is taking place from an initial value of 250 KW to zero and then a no load period of 20 sec. Estimate remittable KW rating of the motor. [648 KW]
2. A constant speed drive has the following duty cycle:
 

Load rising from 0 to 400 KW	–	5 minutes
Uniform load of 400 KW	–	5 minutes
Regenerative power of 400 KW returned to supply	–	4 minutes
Remains idle for	–	2 minutes

 Estimate power rating of motor. [380 H. P.]  
[Nagpur University Winter 96]
3. Determine the rated current of a transformer for the following duty cycle:
  - 500 A for 3 minutes
  - Sharp increase to 1000 A and constant at this value for 1 minute
  - Gradually decreasing to 200 A. for 2 minutes
  - Constant at this value for 2 minutes
  - Gradually increasing to 500 A during 2 minutes repeated indefinitely.[540 A]
4. An induction motor has to perform the following duty cycle:
 

75 KW for 10 minutes,	No load for 5 minutes
45 KW for 8 minutes,	No load for 4 minutes

which is repeated indefinitely.

Determine suitable capacity of a continuously rated motor.

[70 H. P.]

5. A 25 H.P. motor has heating time constant of 90 min. and when run continuously on full load attains a temperature of 45°C. above the surrounding air. What would be the half hour rating of the motor for this temperature rise, assuming that it cools down completely between each load period and that the losses are proportional to square of the load. [47 H.P.]
6. At full load of 10 H.P., temperature rise of a motor is 25°C. after 1 hr. and 40°C after 2 hrs. Find (a) Heating time constant of motor, (b) Final temperature rise on full load. [T = 1.96 hrs,  $\theta_r = 62.5^\circ\text{C}$ ]
7. A totally enclosed motor has a temperature rise of 20°C after half an hour and 35°C after one hour on full load. Determine temperature rise after 2 hours on full load. [54.68°C]
8. A 25 H.P., 3- $\phi$ , 10 pole, 50 c.p.s. induction motor provided with a flywheel has to supply a load torque of 800 N-m for 10 sec, followed by a no load period, during which the flywheel regains its full speed. Full load slip of motor is 4% and torque-speed curve may be assumed linear over the working range. Find moment of inertia of flywheel, if the motor torque is not to exceed twice the full load torque. Assume efficiency = 90%. [718 kg-m<sup>2</sup>]
9. A motor fitted with a flywheel has to supply a load torque of 200 kg-m for 10 sec, followed by a no load period. During the no load period, the motor regains its speed. It is desired to limit the motor torque to 100 kg-m. What should be the moment of inertia of flywheel. No load speed of motor is 500 r.p.m. and has a slip of 10% at a torque of 100 kg-m. [I = 2703 kg-m<sup>2</sup>]
10. A 50 Hz., 3- $\phi$ , 10 pole, 25H.P., induction motor has a constant load torque of 20 kg-m and at wide intervals additional torque of 100 kg-m for 10 sec. Full load slip of the motor is 4% and its efficiency is 88%. Find -  
 (a) Moment of inertial of flywheel , if motor torque not to exceed twice full load torque.  
 (b) Time taken after removal of additional load, before motor torque is 45 kg- m. [ I = 1926 kg-m<sup>2</sup>, t = 9.99 sec.]
11. Define the following terms regarding the ratings of motor :-  
 (i) Continuous rating (ii) short time rating (iii) Intermittent rating.  
 (Nagpur University, Summer 2004)
12. With the help of heating and cooling curves define and explain the terms :  
 (i) Heating time constant (ii) Cooling time constant. (Nagpur University, Summer 2004)
13. What do you mean by 'load-equilisation' it is possible to apply this scheme for reversible drive? Why? (Nagpur University, Summer 2004)
14. A motor is equipped with the flywheel has to supply a load torque of 600 N-m for 10 seconds, followed by no load period long enough for flywheel to regain its full speed. It is desired to limit the motor torque of 450 N-m. What should be moment of inertia of the flywheel? the no load speed of the motor is 600 rpm and has 8% slip at a torque of 450 N-m. The speed-torque characteristics of the motor can be assumed to be a straight line in the region of interest.  
 (Nagpur University, Summer 2004)
15. A motor has the following load cycle :  
 Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.  
 Full speed period 15-85 sec Load constant at 600 h.p.  
 Decelerating period 85-100 sec h.p. returned to line falls uniformly 200 to zero  
 Decking period 100-120 sec Motor stationary. Estimate the size the motor.  
 (J.N. University, Hyderabad, November 2003)
16. A motor driving a load has to deliver a load rising uniformly from zero to maximum of 2000 h.p. in 20 sec during the acceleration period, 1000 h.p. for 40 sec during the full speed period and during the deceleration period of 10 sec when regenerating braking taking place the h.p. returned to the supply falls from 330 to zero. The interval for decking before the next load cycle starts is 20 sec. Estimate the h.p. Rating of the motor. (J.N. University, Hyderabad, November 2003)
17. Draw and explain the output vs. time characteristics of any three types of loads.  
 (J.N. University, Hyderabad, November 2003)



18. Discuss series and parallel operation of series and shunt motors with unequal wheel diameters. Comment on the load sharing in each case. *(J.N. University, Hyderabad, November 2003)*
19. Discuss the various factors that govern the size and the rating of a motor for a particular service. *(J.N. University, Hyderabad, April 2003)*
20. A motor has to deliver a load rising uniformly from zero to a maximum of 1500 Kw in 20 sec during the acceleration period, 1,000 Kw for 50 sec during the full load period and during the deceleration period of 10 sec when regenerative braking takes place the Kw returned to the supply falls from an initial value of 500 to zero uniformly. The interval for decking before the next load cycle starts is 20 sec. Estimate the rating of the motor. *(J.N. University, Hyderabad, April 2003)*
21. Derive an expression for the temperature rise of an equipment in terms of the heating time constant. *(J.N. University, Hyderabad, April 2003)*
22. At full load of 10 h.p., the temperature rise of a motor is 25 degree C after one hour, and 40 degree C after 2 hours. Find the final temperature rise on full load. Assume that the iron losses are 80% of full load copper losses. *(J.N. University, Hyderabad, April 2003)*
23. Explain what you mean by Load Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*
24. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*
25. Discuss the various losses that occur in magnetic conductors which cause the temperature rise in any electrical apparatus and suggest how they can be reduced. *(J.N. University, Hyderabad, April 2003)*
26. The outside of a 12 h.p. (metric) motor is equivalent to a cylinder of 65 cms diameter and 1 meter length. The motor weighs 400 Kg and has a specific heat of 700 Joules per kg per degree C. The outer surface is capable of heat dissipation of 12 W per meter square per degree C. Find the final temperature rise and thermal constant of the motor when operating at full load with an efficiency of 90%. *(J.N. University, Hyderabad, April 2003)*
27. "A flywheel is not used with a synchronous motor for load equalization". Discuss. *(J.N. University, Hyderabad, April 2003)*
28. A 25 h.p. 3-phase 10 pole, 50 Hz induction motor fitted with flywheel has to supply a load torque of 750 Nm for 12 sec followed by a no load period during which the flywheel regains its original speed. Full load slip of the motor is 4% and the torque-speed curve is linear. Find the moment of inertia of the flywheel if the motor torque is not to exceed 2 times the full load torque. *(J.N. University, Hyderabad, April 2003)*
29. Explain what do you mean by Load Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*
30. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*
31. A 100 hp motor has a temperature rise of 50°C when running continuously on full load. It has a time constant of 90 minutes. Determine 1/2 hr rating of the motor for same temperature rise. Assume that the losses are proportional to the square of the load and motor cools completely between each load period. *(J.N. University, Hyderabad, December 2002/January 2003)*
32. Explain 'load equalisation'. How this can be achieved in industrial drives. *(J.N. University, Hyderabad, December 2002/January 2003)*

33. Obtain the expression for temperature rise of a electrical machine. State the assumptions made if any. *(J.N. University, Hyderabad, December 2002/January 2003)*
34. A 75 kW, 500 rpm dc shunt motor is used to drive machinery for which the stored energy per kW is 5400 Joules. Estimate the time taken to start the motor, if the load torque is equal to full load torque during the starting period and the current is limited to 1 1/2 times the full load current. *(J.N. University, Hyderabad, December 2002/January 2003)*

### OBJECTIVE TESTS – 45

1. Heat dissipation is assumed proportional to
  - (a) Temperature difference
  - (b) Temperature difference between motor and cooling medium
  - (c) Temperature of cooling medium
2. Temperature of cooling medium is assumed
  - (a) constant
  - (b) variable
3. When the motor reaches final temperature rise its temperature remains
  - (a) constant
  - (b) falls
  - (c) rises.
4. For intermittent load, a motor of smaller rating can be used
  - (a) true
  - (b) false
5. If motor is disconnected from supply, final temperature reached will be the ambient temperature
  - (a) true
  - (b) false
6. Final temperature rise is theoretically attained only after
  - (a) fixed time
  - (b) variable time
  - (c) infinite time
7. Motor is derated when taken at altitude
  - (a) Yes
  - (b) No
8. The rolling mill load
  - (a) is constant
  - (b) fluctuates widely within long intervals of time
  - (c) fluctuates widely within short intervals of time
  - (d) varies
9. Size of motor is decided by
  - (a) load
  - (b) current
  - (c) heat produced in motor
  - (d) torque
10. Tariff is affected by sudden load drawn by motor
  - (a) true
  - (b) false
11. Flywheel helps in smoothing only
  - (a) speed fluctuations
  - (b) current fluctuations
  - (c) both of the above
12. To use flywheel, motor should have
  - (a) constant speed characteristics
  - (b) drooping speed characteristics
  - (c) variable speed characteristics
13. During light load period
  - (a) flywheel absorbs energy
  - (b) flywheel gives up energy
  - (c) flywheel does nothing
14. During peak load periods
  - (a) flywheel absorbs energy
  - (b) flywheel gives up energy
  - (c) flywheel does nothing
15. Large size of flywheel
  - (a) can be used practically
  - (b) can't be used practically

### ANSWERS

1. (b)    2. (a)    3. (a)    4. (a)    5. (a)    6. (c)    7. (a)    8. (c)    9. (c)    10. (a)  
 11. (c)    12. (b)    13. (a)    14. (b)    15. (b)

# CHAPTER 46

## Learning Objectives

- Classes of Electronic AC Drives
- Variable Frequency Speed Control of a SCIM
- Variable Voltage Speed Control of a SCIM
- Chopper Speed Control of a WRIM
- Electronic Speed Control of Synchronous Motors
- Speed Control by Current-fed D.C. Link
- Synchronous Motor and Cycloconverter

## **ELECTRONIC CONTROL OF A.C. MOTORS**



Efficient control of motors becomes critical where high precision, accuracy, flexibility, reliability and faster response are of paramount importance. Electronic and digital controls are employed in such conditions

### 46.1. Classes of Electronic A.C. Drives

AC motors, particularly, the squirrel-cage and wound-rotor induction motors as well as synchronous motors lend themselves well to electronic control of their speed and torque. Such a control is usually exercised by varying voltage and frequency. Majority of the electronic a.c. drives can be grouped under the following broad classes :

1. **static frequency changers** like cyclo-converters which convert incoming high line frequency directly into the desired low load frequency. Cyclo-converters are used both for synchronous and squirrel-cage induction motors.
2. **variable-voltage controllers** which control the speed and torque by varying the a.c. voltage with the help of SCRs and gate turn-off thyristors (GTOs).
3. **rectifier-inverter systems** with natural commutation.
4. **rectifier-inverter systems** with self-commutation.

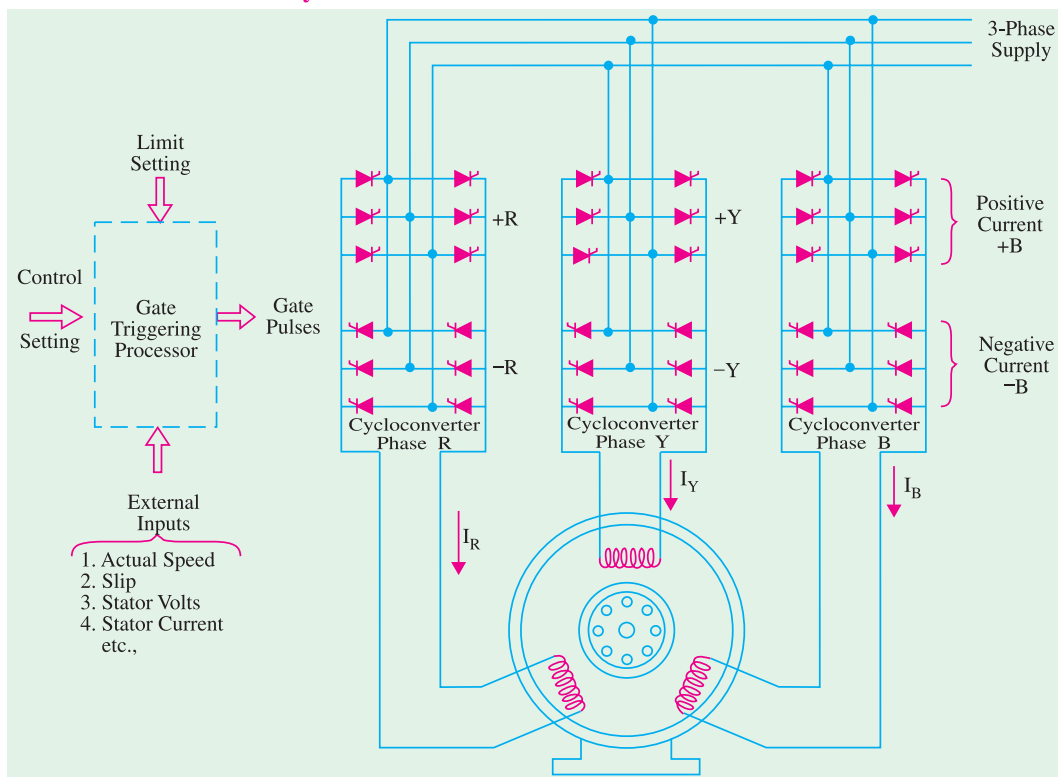


Fig. 46.1

### 46.2. Variable-frequency Speed Control of a SCIM

Fig. 46.1 shows a 3-phase SCIM connected to the outputs of three 3-phase cycloconverters. As seen, each cyclo-converter consists of two 3-phase thyristor bridges, each fed by the same 3-phase, 50-Hz line. The + $R$  bridge generates the positive half-cycle for  $R$ -phase whereas - $R$  generates the negative half. The frequency of the cycloconverter output can be reduced to any value (even upto zero) by controlling the application of firing pulses to the thyristor gates. This low frequency permits excellent speed control. For example, the speed of a 4-pole induction motor can be varied from zero to 1200 rpm on a 50-Hz line by varying the output frequency of the cycloconverter from zero to 40 Hz. The stator voltage is adjusted in proportion to the frequency in order to maintain a constant flux in the motor.

This arrangement provides excellent torque/speed characteristics in all 4-quadrants including regenerative braking. However, such cycloconverter-fed motors run about 10°C hotter than normal and hence require adequate cooling. A small part of the reactive power required by SCIM is provided by the cycloconverter, the rest being supplied by the 3-phase line. Consequently, power factor is poor which makes cycloconverter drives feasible only on small and medium power induction motors.

### 46.3. Variable Voltage Speed Control of a SCIM

In this method, the speed of a SCIM is varied by varying the stator voltage with the help of three sets of SCRs connected back-to-back (Fig. 46.2). The stator voltage is reduced by delaying the firing (or triggering) of the thyristors. If we delay the firing pulses by 100°, the voltage obtained is about 50% of the rated voltage which decreases the motor speed considerably.

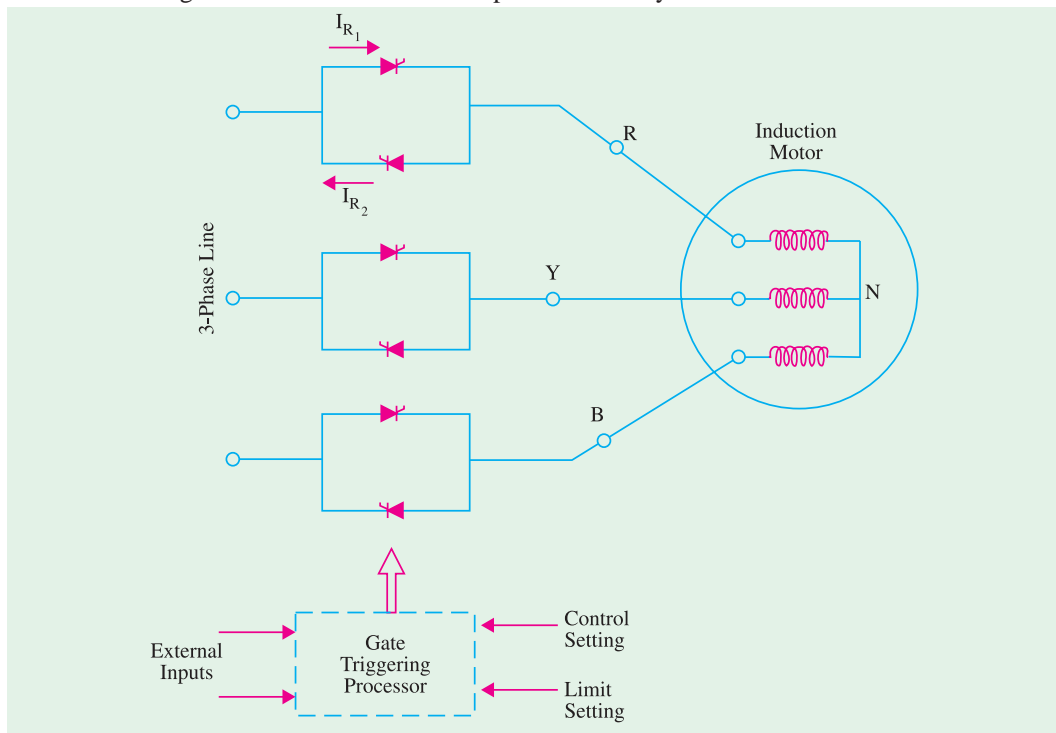


Fig. 46.2

Unfortunately,  $I^2R$  losses are considerable due to distortion in voltage. Moreover, p.f. is also low due to large lag between the current and voltage. Hence, this electronic speed control method is feasible for motors rated below 15 kW but is quite suitable for small hoists which get enough time to cool off because of intermittent working. Of course, p.f. can be improved by using special thyristors called gate turn-off thyristors (GTOs) which force the current to flow almost in phase with the voltage (or even lead it).

### 46.4. Speed Control of a SCIM with Rectifier-Inverter System

A rectifier-inverter system with a d.c. link is used to control the speed of a SCIM. The inverter used is a self-commutated type (different from a naturally commutated type) which converts d.c. power into a.c. power at a frequency determined by the frequency of the



A commonly used electronic power inverter

pulses applied to the thyristor gates. The rectifier is connected to the 3-phase supply line whereas the inverter is connected to the stator of the SCIM.

Two types of links are used :

1. constant-current d.c. link —for speed control of *individual* motors.
2. constant-voltage d.c. link —for speed control of several motors.

As shown in Fig. 46.3, the constant-current link supplies constant current to the inverter which then feeds it sequentially (through proper switching sequence) to the three phases of the motor. Similarly, the constant-voltage dc link (Fig. 46.4) provides a constant voltage to the inverter which is switched from one phase of the motor to the next in a proper sequence.

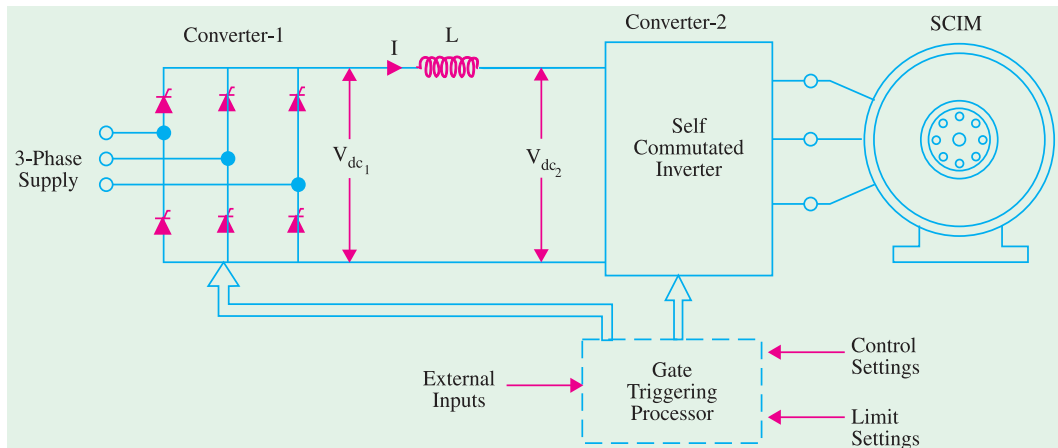


Fig. 46.3

The arrangement of Fig. 46.3 gives speed control with high efficiency in all 4 quadrants in addition to the facility of regenerative braking. Heavy inertia loads can be quickly accelerated because motor develops full break-down torque right from the start. The output frequency of the inverter varies over a range of 20 : 1 with a top frequency of about 1 kHz. The a.c. voltage supplied by the inverter is changed in proportion to the frequency so as to maintain the stator flux constant.

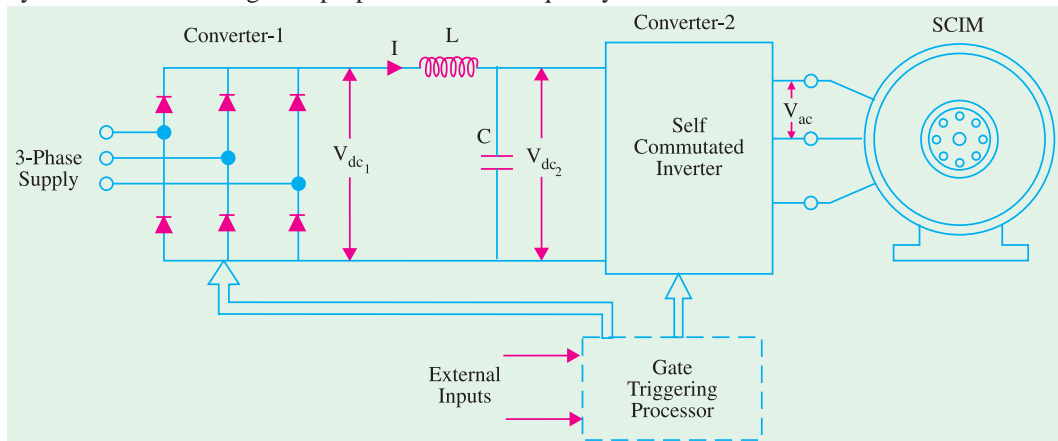


Fig. 46.4

Consequently, d.c. link voltage  $V_{dc1}$  has to be reduced as the motor speeds up. This is accomplished by increasing the firing angle of the thyristors in converter 1. Unfortunately, this leads to increase in the reactive power drawn from the 3-phase line which results in poor power factor. To improve the p.f., use of capacitors is necessary. The direction of rotation can be changed by altering the phase sequence of the pulses that trigger the gates of converter 2.

The voltage-fed frequency converter of Fig. 46.4 consists of a rectifier and a self-commutated inverter connected by a d.c. link and is often used for group drives in textile mills. The 3-phase bridge rectifier produces d.c. voltage  $V_{dc1}$  which is smoothed up by the LC filter before being applied to the inverter. The inverter successively switches its output ac voltage  $V_{ac}$  to the three phases of the motor. This voltage is varied in proportion to the frequency so as to maintain constant flux in the motor. Since,  $V_{ac}$  depends on  $V_{dc2}$  which itself depends on  $V_{dc1}$ , it is  $V_{dc1}$  which is changed as frequency varies. In this system, motor speed can be controlled from zero to maximum while developing full breakdown torque.



### 46.5. Chopper Speed Control of a WRIM

As discussed in Art. 30.18 (d), the speed of a WRIM can be controlled by inserting three variable resistors in the rotor circuit. The all-electronic control of speed can be achieved by connecting a 3-phase bridge rectifier across the rotor terminals and then feed the rotor output to a single fixed resistor or  $R_0$  via a chopper (Fig. 46.5).

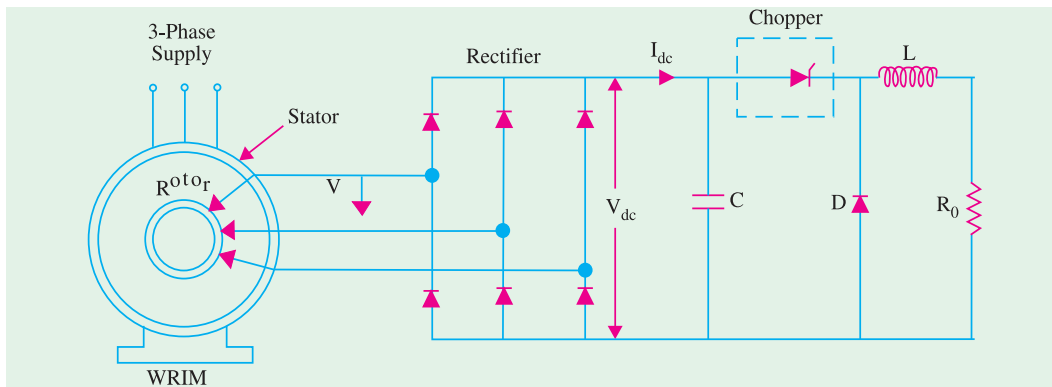


Fig. 46.5

The capacitor  $C$  supplies the high current pulses drawn by the chopper. The inductor  $L$  and free-wheeling diode  $D$  serve the same purpose as discussed in Art. 28.14. By varying the chopper on-time  $T_{ON}$ , the apparent resistance  $R_a$  across the bridge rectifier can be made either high or low. The value of apparent resistance is given by  $R_a = R_0/f^2 T_{ON}^2$ , where  $f$  is the OFF/ON switching frequency of the chopper. The resulting torque/speed characteristic is similar to the one obtained with a 3-phase rheostat.

**Example 46.1.** The wound-rotor induction motor of Fig.43.5 is rated at 30-kW, 975 rpm, 440-V, 50 Hz. The open-circuit line voltage is 400 V and the load resistance is 0.5  $\Omega$ . If chopper frequency is 200 Hz, calculate  $T_{ON}$  so that the motor develops a gross torque of 200 N-m at 750 rpm. Also, calculate the magnitude of the current pulses drawn from the capacitor.

**Solution.** Obviously,  $N_s = 1000$  rpm. Hence, slip at 750 rpm is  $= (1000 - 750)/1000 = 0.25$ . The rotor line voltage at 750 rpm is  $= sE_2 = 0.25 \times 400 = 100$  V.

The d.c. voltage of 3-phase bridge rectifier is  $V_{dc} = 1.35 V = 1.35 \times 100 = 135$  V.

Now,  $T_g = P^2/2\pi N_s; P^2 = T_g \times 2\pi N_s = 200 \times 2\pi \times (1000/60) = 20,950$  W

Part of  $P^2$  dissipated as heat  $= sP^2 = 0.25 \times 20,950 = 5,238$  W

The power is actually dissipated in  $R_0$  and is, obviously, equal to the rectifier output  $V_{dc} \cdot I_{dc}$ .

$$\therefore V_{dc} \cdot I_{dc} = 5238 \quad \text{or} \quad I_{dc} = 5238/135 = 38.8 \text{ A}$$

The apparent resistance at the input to the chopper is

$$R_a = V_{dc} / I_{dc} = 135 / 38.8 = 3.5 \Omega$$

Now,  $R_a = R_0 / f^2 \frac{1}{\alpha_{eff}^2}$  or  $T_{ON} = \sqrt{R_0 / f^2 R_a} = \sqrt{0.5 / 200^2 \times 3.5} = 1.9 \text{ ms}$

Current in  $R_0$  can be found from the relation

$$I_0^2 R_0 = 5238 \text{ or } I_0 = \sqrt{5238 / 0.5} = 102 \text{ A}$$

As seen, capacitor delivers current pulses of magnitude  $\frac{1}{\alpha_{eff}^2}$  A and a pulse width of 1.9 ms at the rate of 200 pulses/second. However, the rectifier continuously charges C with a current of 38.8 A.

### 46.6. Electronic Speed Control of Synchronous Motors

The speed of such motors may be controlled efficiently by using (i) current-fed delink and (ii) cycloconverter as discussed below :

### 46.7. Speed Control by Current-fed DC Link

As shown in Fig. 46.6, the typical circuit consists of three converters two of which are connected between the three-phase source and the synchronous motor whereas the third converter (bridge rectifier) supplies dc field excitation for the rotor. Converter-1 (C-1) acts as a controlled rectifier and feeds d.c. power to converter-2 (C-2). The converter-2 behaves as a naturally commutated inverter whose a.c. voltage and frequency are established by the motor. The function of the smoothing inductor  $L$  is to maintain a ripple-free current in the d.c. link between the two converters. Converter-1 acts as a current source and controls  $I$ .

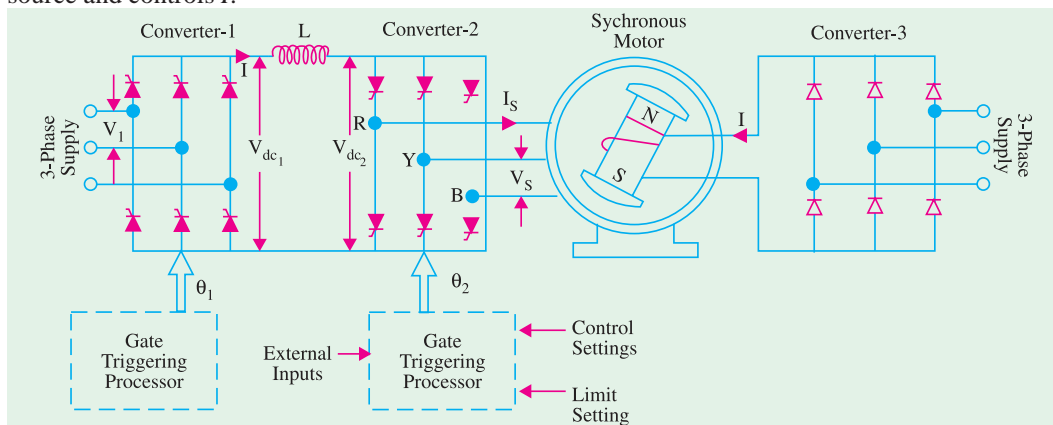


Fig. 46.6

The converter-2 is naturally commutated by voltage  $V_s$  induced across motor terminals by its revolving magnetic flux. The revolving flux which depends on the stator currents and the d.c. field exciting current  $I_{f1}$  is usually kept constant. Consequently,  $V_s$  is proportional to motor speed.

As regards various controls, information picked off from various points is processed in the gate-triggering processors which then send out appropriate gate firing pulses to converters 1 and 2. The processors receive information about the desired rotor speed, its actual speed, instantaneous rotor position, field current, stator voltage and current etc. The processors check whether these inputs represent normal or abnormal conditions and send appropriate gate firing pulses either to correct the situation or meet a specific demand.

Gate triggering of C-1 is done at line frequency (50 Hz) whereas that of C-2 is done at motor frequency. In fact, gate pulses of C-2 are controlled by rotor position which is sensed by position transducers mounted at the end of the shaft. The motor speed can be increased by increasing either d.c. link current  $I$  or exciting current  $I_f$ .

Now,  $V_{dc2} = 1.35 V_s \cos \alpha_1$  and  $V_{dc1} = 1.35 V_s \cos \alpha_1$

where  $V_{dc2}$  = d.c. voltage generated by C-2,  $V_{dc1}$  = d.c. voltage supplied by C-1





Special features of A C Synchronous motors: 1. Bi-directional, 2. Instantaneous Start, Stop and Reverse, 3. Identical Starting and Running Currents, 4. Residual Torque always present, 5. No damage due to stalling, 6. Low speed of 60 rpm. Applications of AC Synchronous Motors are found in: 1. Actuators, 2. Remote control of switches 3. Winding machines, 4. Machine tool applications, 6. Valve controls, 6. Printing machines, 7. Automatic welding machines, 8. Medical equipment, 9. Conveyor systems, 10. Paper feeders

$$\alpha_2 = \text{firing angle of } C-2 ; \alpha_1 = \text{firing angle of } C-1$$

The firing angle  $\alpha_1$  is automatically controlled and supplies  $I$  which is sufficient to develop the required torque. This method of speed control is applied to motors ranging from 1 kW to several MW. Permanent-magnet synchronous motors used in textile industry and brushless synchronous motors for nuclear reactor pumps are controlled by this method.

### 46.8. Synchronous Motor and Cycloconverter

As shown in Fig. 46.7, the arrangement consists of three cycloconverters connected to the three

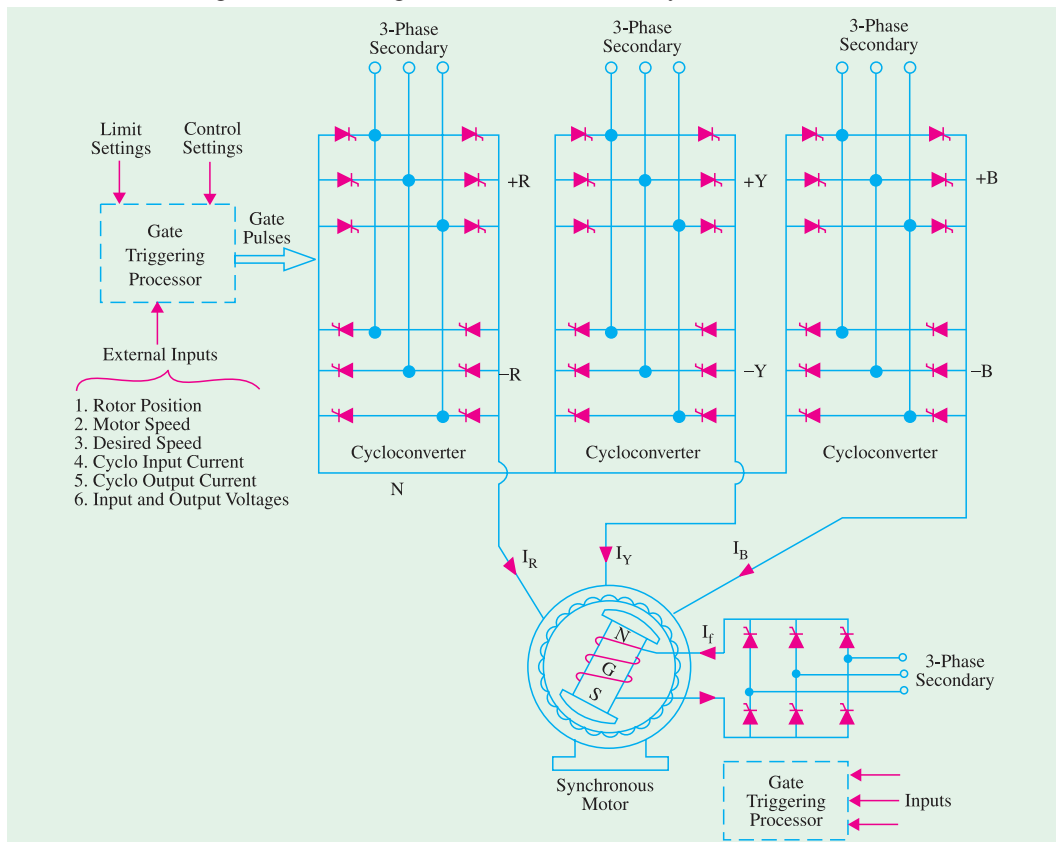


Fig. 46.7

one phases of the synchronous motor and one controlled rectifier for supplying field exciting current  $I_f$  to the rotor. Each cycloconverter is composed of two three-phase bridges and supplies a single-phase output. As is well known, a cycloconverter can directly convert a.c. power at higher frequency to one at a lower frequency. With a line frequency of 50 Hz, the cycloconverter output frequency can be varied from zero to 10 Hz.

The cycloconverters and the controlled rectifier function as current sources. The air-gap flux is kept constant by controlling the magnitude of the stator currents and exciting current  $I_f$ . By proper timing of gate pulses, motor can be made to operate at unity power factor.

The speed of cycloconverter-driven large slow-speed synchronous motors can be continuously varied from zero to 15 rpm. Such low speeds permit direct drive of the ball mill without using a gear reducer. Such high-power low-speed systems are also being introduced as propeller drives on board the ships.

## 46.9. Digital Control of Electric Motors

### Advantages of Digital Control

1. High precision and accuracy
2. Better speed regulation
3. Faster response
4. Flexibility
5. Better time response
6. Improved performance
7. Economical
8. Easy software control
9. Reliability
10. The greatest advantage of the digital control is that by changing the program, desired control technique can be implemented without any change in the hardware.

The speed information can be fed into microcomputer using a D.C. Tacho (Speed encoder) and A/D converter (Speed I/P module). The motor current data is usually fed into the computer through a fast A/D converter. A synchronizing circuit interface (Line synchronizing circuit) is required so that the micro-computer can synchronize the generation of the firing pulse data with the supply line frequency. The gate pulse generator is shown as receiving a firing signal from microcomputer.

A set of instruction (Program) is stored in memory and those are executed by computer for proper functioning of a drive. A typical program flow - chart for this drive system is shown in figure (46.9). The sequence of instructions allows the computer to process data for speed regulation, current regulation and reversal operation.

### 46.10. Application of Digital Control

The above operations can be clearly understood by considering one of the applications of Digital Control system, such as Digital Control System for Speed Control of D.C. drives using a Micro-computer :

Various components and their operations shown in Fig. 46.8 are discussed below :

#### (i) Thyristor Converter

PC based control systems can be built where a phase-controlled rectifier supplies a D.C. motor. The main control to be handled is to turn on & off SCRs. Thyristor power converter in this case is a dual converter – one for forward and other for reverse direction.

#### (ii) Gate Pulse Generator and Amplifier

PC is used for firing angle control of dual converter. It can be programmed using suitable software to perform the function of firing range selection, firing pulse generation, etc. The firing pulses so obtained are amplified, if needed to turn ON the SCR reliably. Changeover signal decides whether to

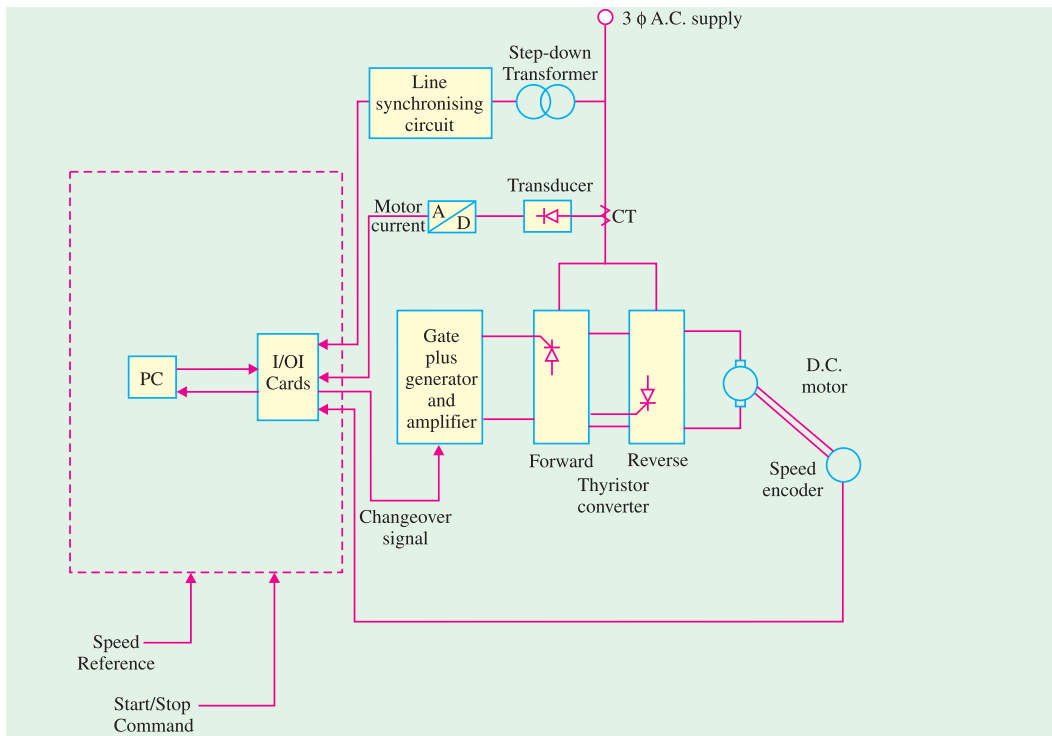


Fig. 46.8

switch ON forward or reverse group of SCRs. The gate pulse generator is shown as receiving a firing signal from PC.

**(iii) Speed Encoder and Input Module**

The speed information can be fed to PC through speed input module. The speed measurement is done digitally by means of speed/shaft encoder. It consists of a disc with definite number of holes drilled on it. This disc is fixed on to the shaft. Using a light source and a phototransistor; a series of pulses is obtained, as the shaft rotates. This pulse train is processed and shaped. These optically coded pulses are counted to get actual speed of motor.

**(iv) A/D Converter and Transducer**

The motor current drawn from supply is stepped down with the help of current transformer. It is converted to D.C. voltage output with the help of current transducer. As PC can't process analog signals, this analog current signal is fed to A/D converter to obtain digital signal which is fed to PC.

**(v) Line Synchronizing Circuit**

This is required so that PC can synchronize the generation of firing pulse data, with supply line frequency.

**(vi) I/O Cards**

Input/ Output cards are required to interface PC with the outside world.

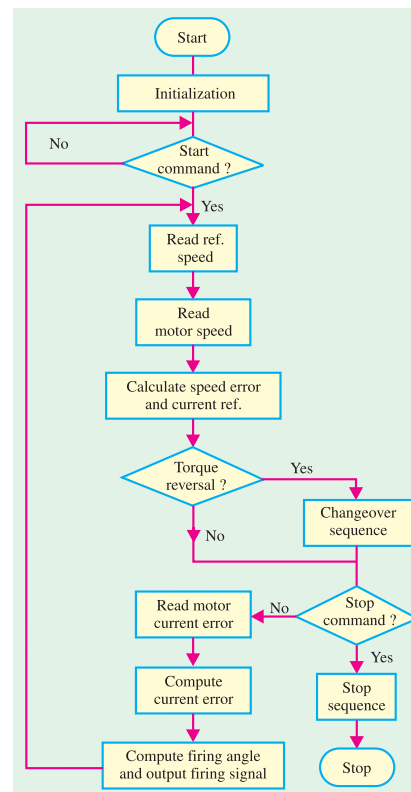


Fig. 46.9. Program flow chart for digital control of motor.

## OBJECTIVE TESTS – 46

1. The function of a cycloconverter is to convert
  - (a) ac power into d.c. power
  - (b) direct current into alternating current
  - (c) high ac frequency directly to low ac frequency
  - (d) a sine wave into a rectangular wave.
2. Major disadvantage of using three sets of SCRs for variable-voltage speed control of a SCIM is the
  - (a) considerable  $I^2R$  loss
  - (b) poor power factor
  - (c) long delay of thyristor firing pulses
  - (d) necessity of using a processor.
3. In the current-fed frequency converter arrangement for controlling the speed of an individual SCIM, the direction of rotation can be reversed by
  - (a) changing the output frequency of the inverter
  - (b) altering the phase sequence of pulses that trigger converter-2
  - (c) interchanging any two line leads
  - (d) reversing the d.c. link current.
4. In the chopper speed control method for a WRIM, the motor speed inversely depends on
  - (a) fixed resistor across the rectifier
  - (b) chopper switching frequency
  - (c) chopper ON time TON
  - (d) both (b) and (c).
5. In the synchronous motor drive using current-fed dc link
  - (a) converter-2 functions as a self-commutated inverter
  - (b) converter-1 works as an uncontrolled rectifier
  - (c) converter-3 is a controlled rectifier
  - (d) gate triggering of converter-2 is done at motor frequency.
6. In the three cycloconverter drive of a synchronous motor
  - (a) each cycloconverter produces a 3-phase output
  - (b) all cycloconverters act as voltage sources
  - (c) a 3-phase controlled rectifier provides field exciting current.
  - (d) air-gap flux is kept constant by controlling stator currents only.

## ANSWERS

1. (c) 2. (a) 3. (b) 4. (d) 5. (d) 6. (c)

# CHAPTER 47

## Learning Objectives

- Introduction
- Advantages of Electric Heating
- Different Methods of Heat Transfer
- Methods of Electric Heating
- Resistance Heating
- Requirement of a Good Heating Element
- Resistance Furnaces or Ovens
- Temperature Control of Resistance Furnaces
- Design of Heating Element
- Arc Furnaces
- Direct Arc Furnace
- Indirect Arc Furnace
- Induction Heating
- Core Type Induction Furnace
- Indirect Core Type Induction Furnace
- Coreless Induction Furnace
- High Frequency Eddy Current Heating
- Dielectric Heating
- Dielectric Loss
- Advantages of Dielectric Heating
- Applications of Dielectric Heating
- Choice of Frequency
- Infrared Heating

## ELECTRIC HEATING



The above figure shows an electric arc furnace, producing steel. Electric heating is widely used in furnaces in metallurgical and chemical industries

### 47.1. Introduction

Electric heating is extensively used both for domestic and industrial applications. Domestic applications include (i) room heaters (ii) immersion heaters for water heating (iii) hot plates for cooking (iv) electric kettles (v) electric irons (vi) pop-corn plants (vii) electric ovens for bakeries and (viii) electric toasters etc.

Industrial applications of electric heating include (i) melting of metals (ii) heat treatment of metals like annealing, tempering, soldering and brazing etc. (iii) moulding of glass (iv) baking of insulators (v) enamelling of copper wires etc.

### 47.2. Advantages of Electric Heating

As compared to other methods of heating using gas, coal and fire etc., electric heating is far superior for the following reasons :

- (i) **Cleanliness.** Since neither dust nor ash is produced in electric heating, it is a clean system of heating requiring minimum cost of cleaning. Moreover, the material to be heated does not get contaminated.
- (ii) **No Pollution.** Since no flue gases are produced in electric heating, no provision has to be made for their exit.
- (iii) **Economical.** Electric heating is economical because electric furnaces are cheaper in their initial cost as well as maintenance cost since they do not require big space for installation or for storage of coal and wood. Moreover, there is no need to construct any chimney or to provide extra heat installation.
- (iv) **Ease of Control.** It is easy to control and regulate the temperature of an electric furnace with the help of manual or automatic devices. Temperature can be controlled within  $\pm 5^\circ\text{C}$  which is not possible in any other form of heating.
- (v) **Special Heating Requirement.** Special heating requirements such as uniform heating of a material or heating one particular portion of the job without affecting its other parts or heating with no oxidation can be met only by electric heating.
- (vi) **Higher Efficiency.** Heat produced electrically does not go away waste through the chimney and other by products. Consequently, most of the heat produced is utilised for heating the material itself. Hence, electric heating has higher efficiency as compared to other types of heating.
- (vii) **Better Working Conditions.** Since electric heating produces no irritating noises and also the radiation losses are low, it results in low ambient temperature. Hence, working with electric furnaces is convenient and cool.
- (viii) **Heating of Bad Conductors.** Bad conductors of heat and electricity like wood, plastic and bakery items can be uniformly and suitably heated with dielectric heating process.
- (ix) **Safety.** Electric heating is quite safe because it responds quickly to the controlled signals.
- (x) **Lower Attention and Maintenance Cost.** Electric heating equipment generally will not require much attention and supervision and their maintenance cost is almost negligible. Hence, labour charges are negligibly small as compared to other forms of heating.

### 47.3. Different Methods of Heat Transfer

The different methods by which heat is transferred from a hot body to a cold body are as under:

#### 1. Conduction

In this mode of heat transfer, one molecule of the body gets heated and transfers some of the



heat to the adjacent molecule and so on. There is a temperature gradient between the two ends of the body being heated.

Consider a solid material of cross-section  $A$  sq.m. and thickness  $x$  metre as shown in Fig. 47.1. If  $T_1$  and  $T_2$  are the temperatures of the two sides of the slab in  $^{\circ}K$ , then heat conducted between the two opposite faces in time  $t$  seconds is given by:

$$H = \frac{KA(T_1 - T_2)t}{x}$$

where  $K$  is thermal conductivity of the material.

### 2. Convection

In this process, heat is transferred by the flow of hot and cold air currents. This process is applied in the heating of water by immersion heater or heating of buildings. The quantity of heat absorbed by the body by convection process depends mainly on the temperature of the heating element above the surroundings and upon the size of the surface of the heater. It also depends, to some extent, on the position of the heater. The amount of heat dissipated is given by

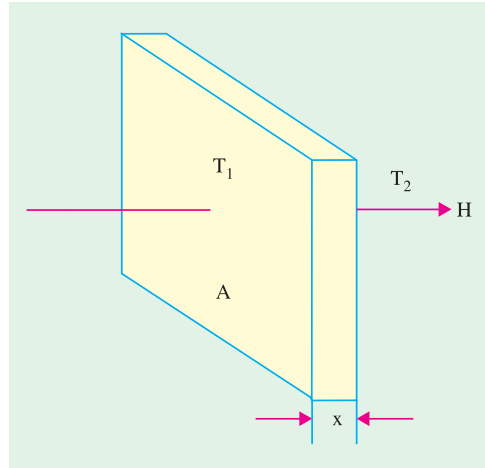


Fig. 47.1

$H = a(T_1 - T_2)$ , where  $a$  and  $b$  are constants and  $T_1$  and  $T_2$  are the temperatures of the heating surface and the fluid in  $^{\circ}K$  respectively.

In electric furnaces, heat transferred by convection is negligible.

### 3. Radiation

It is the transfer of heat from a hot body to a cold body in a straight line without affecting the intervening medium. The rate of heat emission is given by Stefan's law according to which

$$\text{Heat dissipated, } H = 5.72 eK \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \text{ W/m}^2$$

where  $K$  is radiating efficiency and  $e$  is known as emissivity of the heating element.

If  $d$  is the diameter of the heating wire and  $l$  its total length, then its surface area from which heat is radiated =  $\pi \times d \times l$ . If  $H$  is the power radiated per  $\text{m}^2$  of the heating surface, then total power radiated as heat =  $H \times \pi d l$ . If  $P$  is the electrical power input to the heating element, then  $P = \pi d l \times H$ .

#### 47.4. Methods of Electric Heating

Basically, heat is produced due to the circulation of current through a resistance. The current may circulate directly due to the application of potential difference or it may be due to induced eddy currents. Similarly, in magnetic materials, hysteresis losses are used to create heat. In dielectric heating, molecular friction is employed for heating the substance. An arc established between an electrode and the material to be heated can be made a source of heat. Bombarding the surface of material by high energy particles can be used to heat the body.



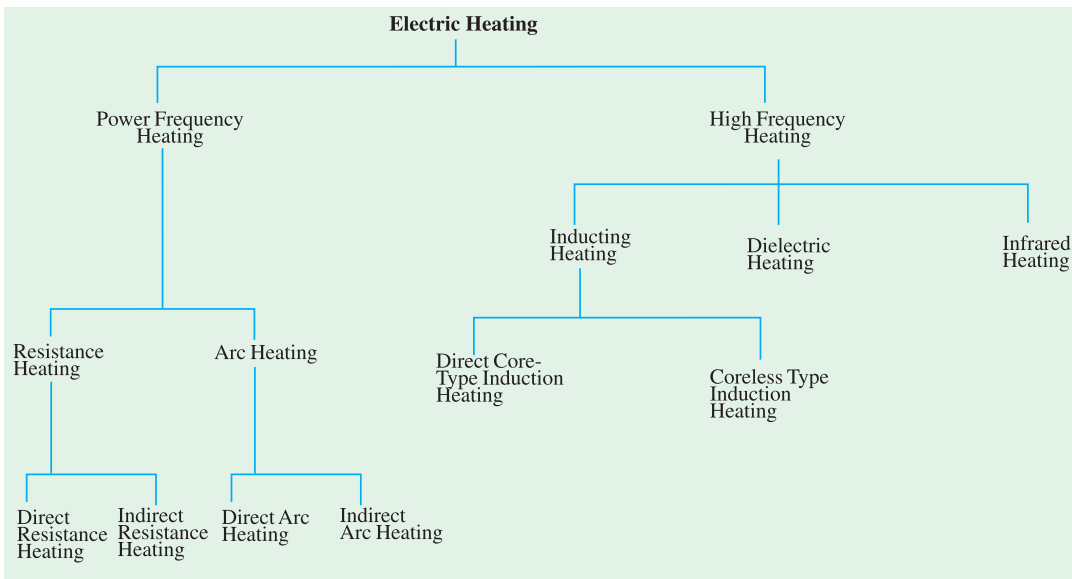
Room heater is a familiar appliance where electric heating is employed

Different methods of producing heat for general industrial and domestic purposes may be classified below :

### 47.5. Resistance Heating

It is based on the  $I^2R$  effect. When current is passed through a resistance element  $I^2R$  loss takes place which produces heat. There are two methods of resistance heating.

(a) **Direct Resistance Heating.** In this method the material (or charge) to be heated is treated as a resistance and current is passed through it. The charge may be in the form of powder, small solid pieces or liquid. The two electrodes are inserted in the charge and connected to either a.c. or d.c.



supply (Fig. 47.2). Obviously, two electrodes will be required in the case of d.c. or single-phase a.c. supply but there would be three electrodes in the case of 3-phase supply. When the charge is in the form of small pieces, a powder of high resistivity material is sprinkled over the surface of the charge to avoid direct short circuit. Heat is produced when current passes through it. This method of heating has high efficiency because the heat is produced in the charge itself.

(b) **Indirect Resistance Heating.** In this method of heating, electric current is passed through a resistance element which is placed in an electric oven. Heat produced is proportional to  $I^2R$  losses in the heating element. The heat so

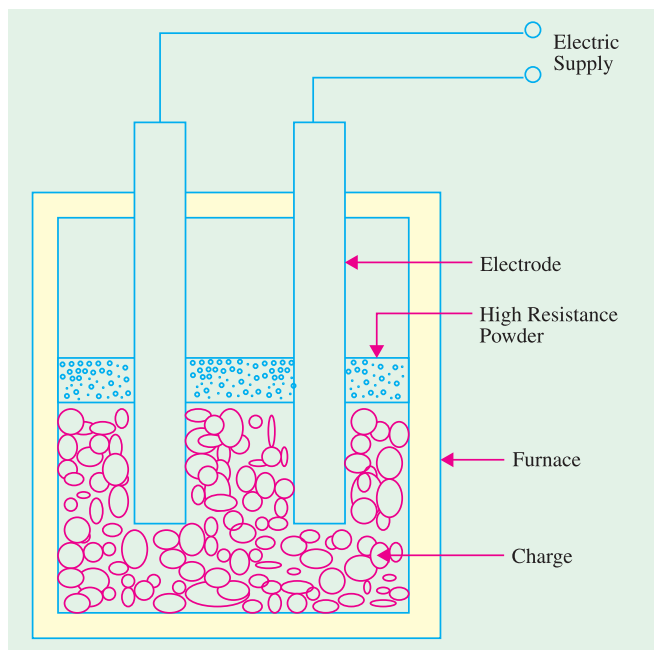


Fig. 47.2



produced is delivered to the charge either by radiation or convection or by a combination of the two.

Sometimes, resistance is placed in a cylinder which is surrounded by the charge placed in the jacket as shown in the Fig. 47.3. This arrangement provides uniform temperature. Moreover, automatic temperature control can also be provided.

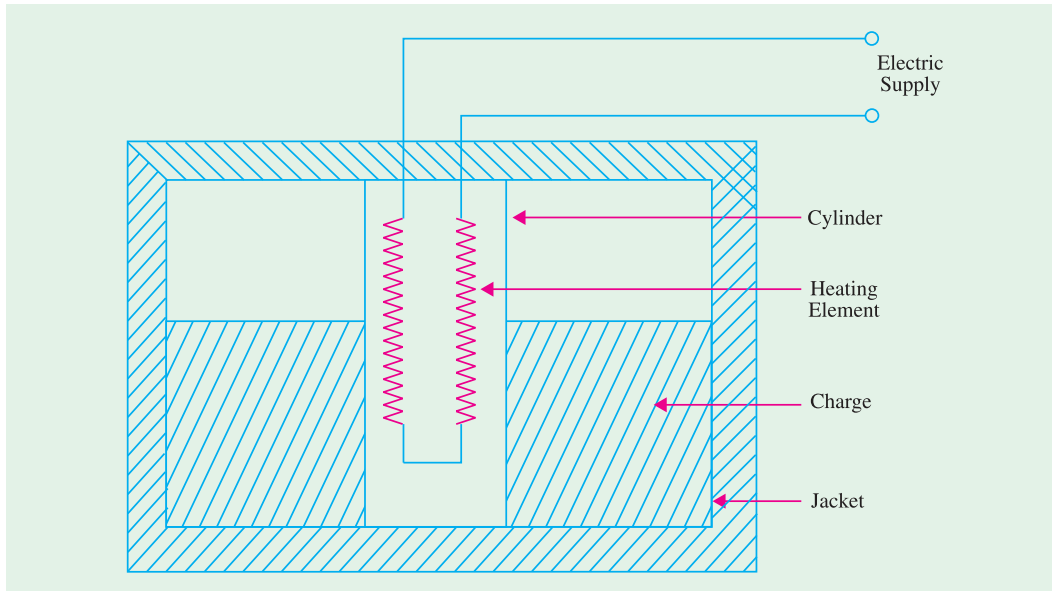


Fig. 47.3

#### 47.6. Requirement of a Good Heating Element

Indirect resistance furnaces use many different types of heating elements for producing heat. A good heating element should have the following properties :

(1) **High Specific Resistance.** When specific resistance of the material of the wire is high, only short length of it will be required for a particular resistance (and hence heat) or for the same length of the wire and the current, heat produced will be more.

(2) **High Melting Temperature.** If the melting temperature of the heating element is high, it would be possible to obtain higher operating temperatures.

(3) **Low Temperature Coefficient of Resistance.** In case the material has low temperature coefficient of resistance, there would be only small variations in its resistance over its normal range of temperature. Hence, the current drawn by the heating element when cold (*i.e.*, at start) would be practically the same when it is hot.

(4) **High Oxidising Temperature.** Oxidisation temperature of the heating element should be high in order to ensure longer life.

(5) **Positive Temperature Coefficient of Resistance.** If the temperature coefficient of the resistance of heating element is negative, its resistance will decrease with rise in temperature and it will draw more current which will produce more wattage and hence heat. With more heat, the resistance will decrease further resulting in instability of operation.

(6) **Ductile.** Since the material of the heating elements has to have convenient shapes and sizes, it should have high ductility and flexibility.

(7) **Mechanical Strength.** The material of the heating element should possess high mechanical strength of its own. Usually, different types of alloys are used to get different operating temperatures. For example maximum working temperature of *constantan* (45% Ni, 55% Cu) is 400°C, that of

*nichrome* (50% Ni, 20% Cr) is 1150°C, that of *Kantha* (70% Fe, 25% Cr, 5% Al) is 1200°C and that of *silicon carbide* is 1450°C.

With the passage of time, every heating element breaks open and becomes unserviceable. Some of the factors responsible for its failure are :

- (1) Formation of hot spots which shine brighter during operation,
- (2) Oxidation
- (3) Corrosion
- (4) Mechanical failure.

### 47.7. Resistance Furnaces or Ovens

These are suitably-insulated closed chambers with a provision for ventilation and are used for a wide variety of purposes including heat treatment of metals like annealing and hardening etc., stoving of enamelled wares, drying and baking of potteries, vulcanizing and hardening of synthetic materials and for commercial and domestic heating. Temperatures upto 1000°C can be obtained by using heating elements made of nickel, chromium and iron. Ovens using heating elements made of graphite can produce temperatures upto 3000°C. Heating elements may consist of circular wires or rectangular ribbons. The ovens are usually made of a metal framework having an internal lining of fire bricks. The heating element may be located on the top, bottom or sides of the oven. The nature of the insulating material is determined by the maximum temperature required in the oven.

An enclosure for charge which is heated by radiation or convection or both is called a *heating chamber*.

### 47.8. Temperature Control of Resistance Furnaces

The temperature of a resistance furnace can be changed by controlling the  $I^2R$  or  $V^2/R$  losses.

Following different methods are used for the above purpose :

**(1) Intermittent Switching.** In this case, the furnace voltage is switched ON and OFF intermittently. When the voltage supply is switched off, heat production within the surface is stalled and hence its temperature is reduced. When the supply is restored, heat production starts and the furnace temperature begins to increase. Hence, by this simple method, the furnace temperature can be limited between two limits.

**(2) By Changing the Number of Heating Elements.** In this case, the number of heating elements is changed without cutting off the supply to the entire furnace. Smaller the number of heating elements, lesser the heat produced.

In the case of a 3-phase circuit, equal number of heating elements is switched off from each phase in order to maintain a balanced load condition.

**(3) Variation in Circuit Configuration.** In the case of 3-phase secondary load, the heating elements give less heat when connected in a star than when connected in delta because in the two cases, voltages across the elements is different (Fig. 47.4). In single-phase circuits, series and parallel grouping of the heating elements causes change in power dissipation resulting in change of furnace temperature.



Electric resistance furnace

As shown in Fig. 47.5 heat produced is more when all these elements are connected in parallel than when they are connected in series or series-parallel.

**(4) Change of Applied Voltage. (a)** Obviously, lesser the magnitude of the voltage applied to the load, lesser the power dissipated and hence, lesser the temperature produced. In the case of a furnace transformer having high voltage primary, the tapping control is kept in the primary winding because the magnitude of the primary current is less. Consider the multi-tap step-down transformer shown in Fig. 47.6.



Resistance heating tube furnace

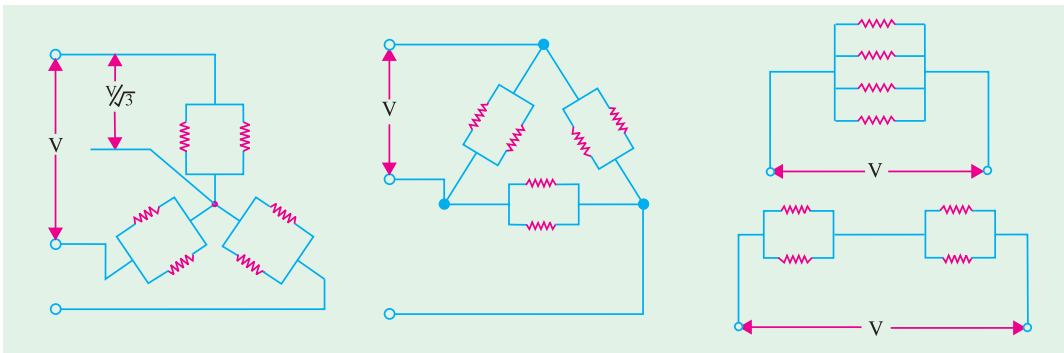


Fig. 47.4

Fig. 47.5

Let the four tappings on the primary winding have 100%, 80%, 60% and 50%. When 100% primary turns are used, secondary voltage is given by  $V_2 = (N_2/N_1)V_i$  where  $V_i$  is the input voltage. When 50% tapping is used, the number of primary turns involved is  $N_1/2$ . Hence, available secondary voltage  $V_2 = (2N_2/N_1)V_i$ . By selecting a suitable primary tapping, secondary voltage can be increased or decreased causing a change of temperature in the furnace.

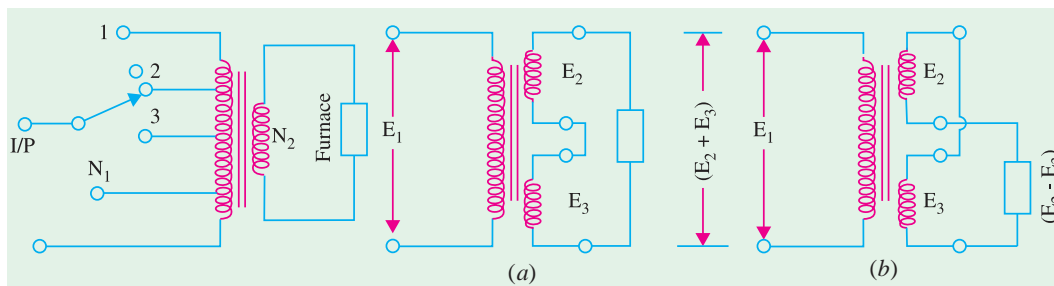


Fig. 47.6

Fig. 47.7

**(b) Bucking-Boosting the Secondary Voltage.** In this method, the transformer secondary is wound in two sections having unequal number of turns. If the two sections are connected in series-aiding, the secondary voltage is boosted *i.e.*, increased to  $(E_2 + E_3)$  as shown in Fig. 47.7 (a).

When the two sections are connected in series-opposing [Fig. 47.7 (b)] the secondary voltage is reduced *i.e.*, there is bucking effect. Consequently, furnace voltage becomes  $(E_2 - E_3)$  and, hence, furnace temperature is reduced.

**(c) Autotransformer Control.** Fig. 47.8 shows the use of tapped autotransformer used for decreasing the furnace voltage and, hence, temperature of small electric furnaces. The required voltage can be selected with the help of a voltage selector.

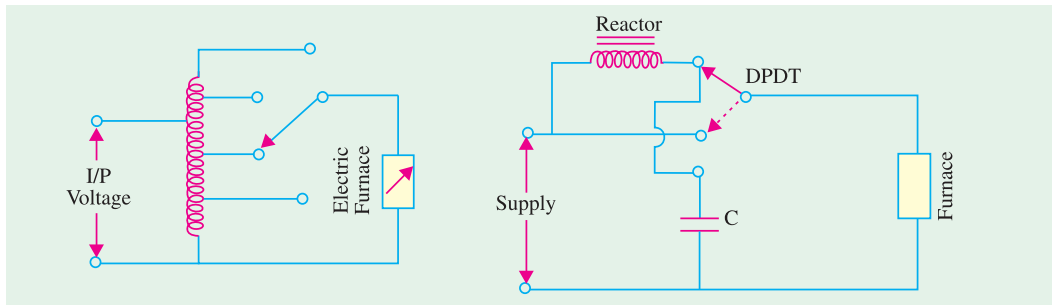


Fig. 47.8

Fig. 47.9

**(d) Series Reactor Voltage.** In this case, a heavy-duty core-wound coil is placed in series with the furnace as and when desired. Due to drop in voltage across the impedance of the coil, the voltage available across the furnace is reduced. With the help of D.P.D.T. switch, high/low, two-mode temperature control can be obtained as shown in the Fig. 47.9. Since the addition of series coil reduces the power factor, a power capacitor is simultaneously introduced in the circuit for keeping the p.f. nearly unity. As seen, the inductor is connected in series, whereas the capacitor is in parallel with the furnace.

### 47.9. Design of Heating Element

Normally, wires of circular cross-section or rectangular conducting ribbons are used as heating elements. Under steady-state conditions, a heating element dissipates as much heat from its surface as it receives the power from the electric supply. If  $P$  is the power input and  $H$  is the heat dissipated by radiation, then  $P = H$  under steady-state conditions.

As per Stefan's law of radiation, heat radiated by a hot body is given by

$$H = 5.72 eK \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \text{W/m}^2$$

where  $T_1$  is the temperature of hot body in  $^{\circ}\text{K}$  and  $T_2$  that of the cold body (or cold surroundings) in  $^{\circ}\text{K}$

$$\text{Now, } P = \frac{V^2}{R}, \text{ and } R = \rho \frac{l}{A} = \rho \frac{l}{\pi d^2 / 4} = \frac{4\rho l}{\pi d^2}$$

$$\therefore P = \frac{V^2}{4\rho l / \pi d^2} = \frac{\pi d^2 V^2}{4\rho l} \text{ or } \frac{l}{d^2} = \frac{\pi V^2}{4\rho l} \quad \dots (i)$$

Total surface area of the wire of the element =  $(\pi d) \times l$

If  $H$  is the heat dissipated by radiation per second per unit surface area of the wire, then heat radiated per second

$$= (\pi d) \times l \times H \quad \dots (ii)$$

Equating (i) and (ii), we have

$$P = (\pi d) \times l \times H \text{ or } \frac{\pi d^2 V^2}{4\rho l} = (\pi d) \times H \text{ or } \frac{d}{l^2} = \frac{4\rho H}{V^2} \quad \dots (iii)$$

We can find the values of  $l$  and  $d$  from Eq. (i) and (iii) given above.

### Ribbon Type Element

If  $w$  is the width of the ribbon and  $t$  its thickness, then

$$P = \frac{V^2}{R} = \frac{V^2}{\rho l / A} = \frac{V^2}{\rho l / Twt} = \frac{wtV^2}{\rho l} \text{ or } \frac{t}{wt} = \frac{V^2}{\rho P} \quad \dots (iv)$$

Heat lost from ribbon surface =  $2wlH$  (neglecting the side area  $2tl$ )

$$\therefore \frac{wtV^2}{\rho l} = 2wlH \quad \text{or} \quad \frac{t}{l^2} = \frac{2\rho H}{V^2} \quad \dots (v)$$

Values of  $l$  and  $w$  for a given ribbon of thickness  $t$  can be found from Eqn. (iv) and (v) given above.

**Example 47.1.** A resistance oven employing nichrome wire is to be operated from 220 V single-phase supply and is to be rated at 16 kW. If the temperature of the element is to be limited to  $1,170^\circ\text{C}$  and average temperature of the charge is  $500^\circ\text{C}$ , find the diameter and length of the element wire.

Radiating efficiency = 0.57, Emmissivity = 0.9, Specific resistance of nichrome =  $(109 \times 10^{-8})$  ohm-m. **(Utili. of Elect. Energy, Punjab Univ.)**

**Solution.**  $P = 16 \text{ kW} = 16,000 \text{ W}$

$$\text{From Article 47.9, } \frac{l}{d^2} = \frac{\pi V^2}{4\rho P} = \frac{\pi \times (220)^2}{4 \times 109 \times 10^{-8} \times 16,000} = 2,179,660 \quad \dots (i)$$

$$\begin{aligned} \text{Now, } H &= 5.72eK \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \text{ W/m}^2 = 5.72 \times 0.9 \times 0.57 \left[ \left( \frac{1443}{100} \right)^4 - \left( \frac{773}{100} \right)^4 \right] \\ &= 116,752 \text{ W/m}^2 \end{aligned}$$

Now, total heat dissipated/s = electrical power input

$$\therefore (\pi d) \times l \times 116,752 = 16,000; \quad \therefore dl = 0.0436$$

$$\text{or } d^2 l^2 = 0.0019 \quad \dots (ii)$$

$$\text{From Eqn. (i) and (ii), } l^3 = 2,179,660 \times 0.0019 = 4141$$

$$\therefore l = 16.05 \text{ m}$$

$$d = 0.0436/16.05 = 2.716 \times 10^{-3} \text{ m} = 2.716 \text{ mm}$$

**Example 47.2.** A 30-kW, 3- $\phi$ , 400-V resistance oven is to employ nickel-chrome strip 0.254 mm thick for the three star-connected heating elements. If the wire temperature is to be  $1,100^\circ\text{C}$  and that of the charge to be  $700^\circ\text{C}$ , estimate a suitable width for the strip. Assume emissivity = 0.9 and radiating efficiency to be 0.5 and resistivity of the strip material is  $101.6 \times 10^{-8} \Omega \cdot \text{m}$ . What would be the temperature of the wire if the charge were cold ?

**(Utili. of Elect. Power A.M.I.E. Sec. B)**

**Solution.** Power/phase =  $30 \times 1000/3 = 10,000 \text{ W}$ ,  $V_{\text{ph}} = 400/\sqrt{3} = 231 \text{ V}$

If  $R$  is the resistance of the strip,  $R = V_{\text{ph}}^2/P = 231^2/10,000 = 5.34 \Omega$

$$\text{Resistance of the strip, } R = \frac{\rho l}{wt} \quad \text{or} \quad \frac{l}{w} = \frac{5.34 \times 0.245 \times 10^{-3}}{101.6 \times 10^{-8}} = 1335 \quad \dots (i)$$

Heat dissipated from surface of the strip,

$$H = 5.72 \times 0.9 \times 0.5 \left[ \left( \frac{1373}{100} \right)^4 - \left( \frac{973}{100} \right)^4 \right] = 68,400 \text{ W/m}^2$$

Surface area of the strip =  $2wl$ ; Total heat dissipated =  $2wl \times H$

$$\therefore 68,400 \times 2 \times wl = 10,000 \quad \text{or} \quad wl = 0.0731 \quad \dots (ii)$$

From Eqn. (i) and (ii), we get  $w = 0.0731/1335$  or  $w = 7.4 \text{ mm}$

**Example 47.3.** A cubic water tank has surface area of  $6.0 \text{ m}^2$  and is filled to 90% capacity six times daily. The water is heated from  $20^\circ\text{C}$  to  $65^\circ\text{C}$ . The losses per square metre of tank surface per  $1^\circ\text{C}$  temperature difference are  $6.3 \text{ W}$ . Find the loading in kW and the efficiency of the tank. Assume specific heat of water =  $4,200 \text{ J/kg}^\circ\text{C}$  and one kWh =  $3.6 \text{ MJ}$ . (A.M.I.E. Sec. B)

**Solution.** If  $l$  is the side of the tank, then total surface area of the tank =  $6l^2$

$$\therefore 6l^2 = 6 \quad \text{or} \quad l = 6/6 = 1 \text{ m}^2$$

$$\text{Volume of the tank} = l^3 = 1 \text{ m}^3$$

$$\text{Volume of water to be heated daily} = 6 \times (1 \times 0.9) = 5.4 \text{ m}^3$$

$$\text{Since } 1 \text{ m}^3 \text{ of water weighs } 1000 \text{ kg, mass of water to be heated daily} = 5.4 \times 1000 = 5400 \text{ kg}$$

$$\text{Heat required to raise the temperature of water} = 5400 \times 4200 (65 - 20) = 1020 \text{ MJ} = 1020/3.6 = 283.3 \text{ kWh}$$

$$\text{Daily loss from the surface of the tank} = 6.3 \times 6 \times (65 - 20) \times 24/1000 = 40.8 \text{ kWh}$$

$$\text{Energy supplied per day} = 283.3 + 40.8 = 324.1 \text{ kWh}$$

$$\text{Loading in kW} = 324.1/24 = \mathbf{3.5 \text{ kW}}$$

$$\text{Efficiency of the tank} = 283.3 \times 100/324.1 = 87.4\%$$

#### 47.10. Arc Furnaces

If a sufficiently high voltage is applied across an air-gap, the air becomes ionized and starts conducting in the form of a continuous spark or arc thereby producing intense heat. When electrodes are made of carbon/graphite, the temperature obtained is in the range of  $3000^\circ\text{C}$ - $3500^\circ\text{C}$ . The high voltage required for striking the arc can be obtained by using a step-up transformer fed from a variable a.c. supply as shown in Fig. 47.10 (a).

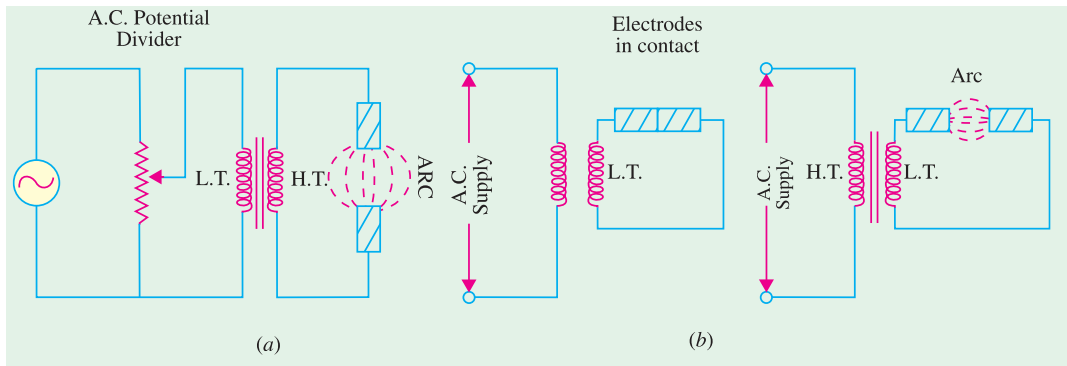


Fig. 47.10

An arc can also be obtained by using low voltage across two electrodes initially in contact with each other as shown in Fig. 47.10 (b). The low voltage required for this purpose can be obtained by using a step-down transformer. Initially, the low voltage is applied, when the two electrodes are in contact with each other. Next, when the two electrodes are gradually separated from each other, an arc is established between the two.

Arc furnaces can be of the following two types :

##### 1. Direct Arc Furnace

In this case, arc is formed between the two electrodes and the charge in such a way that electric current passes through the body of the charge as shown in Fig. 47.11 (a). Such furnaces produce very high temperatures.

## 2. Indirect Arc Furnace

In this case, arc is formed between the two electrodes and the heat thus produced is passed on to the charge by radiation as shown in Fig. 47.11 (b).

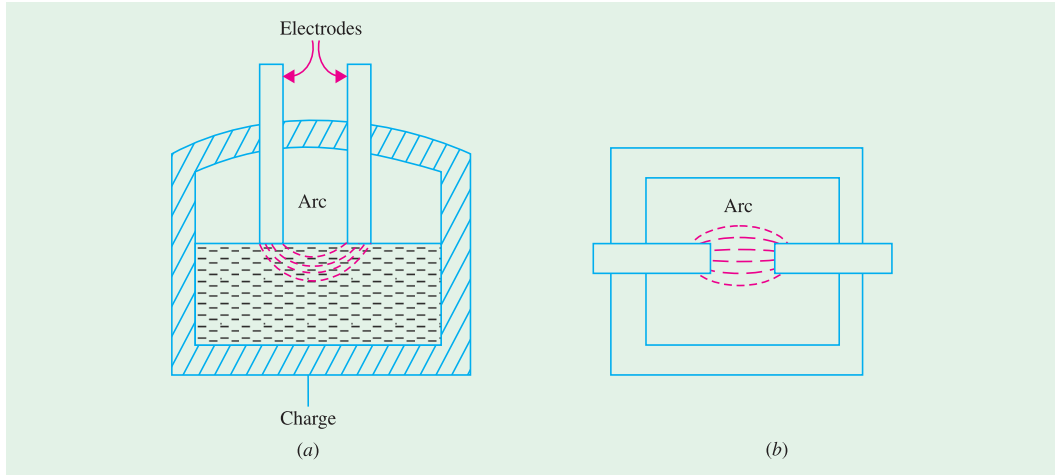


Fig. 47.11

## 47.11. Direct Arc Furnace

It could be either of conducting-bottom type [Fig. 47.12 (a)] or non-conducting bottom type [Fig. 47.12 (b)].

As seen from Fig. 47.12 (a), bottom of the furnace forms part of the electric circuit so that current passes through the body of the charge which offers very low resistance. Hence, it is possible

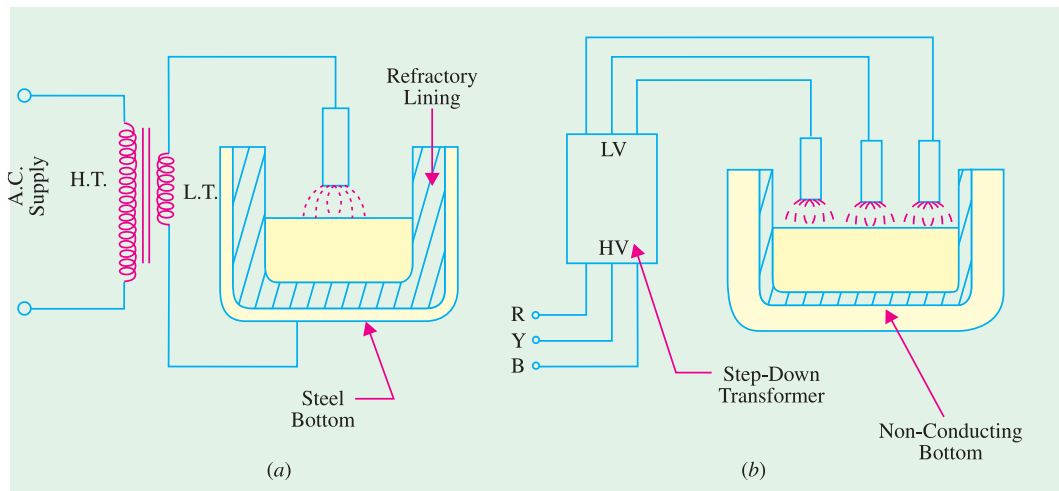


Fig. 47.12

to obtain high temperatures in such furnaces. Moreover, it produces uniform heating of charge without stirring it mechanically. In Fig. 47.12 (b), no current passes through the body of the furnace.

Most common application of these furnaces is in the production of steel because of the ease with which the composition of the final product can be controlled during refining.

Most of the furnaces in general use are of non-conducting bottom type due to insulation problem faced in case of conducting bottom.

### 47.12. Indirect Arc Furnace

Fig. 47.13 shows a single-phase indirect arc furnace which is cylindrical in shape. The arc is struck by short-circuiting the electrodes manually or automatically for a moment and then, withdrawing them apart. The heat from the arc and the hot refractory lining is transferred to the top layer of the charge by radiation. The heat from the hot top layer of the charge is further transferred to other parts of the charge by conduction.

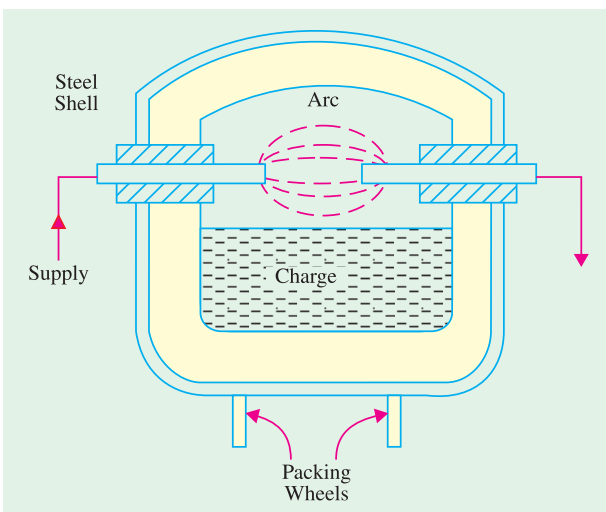
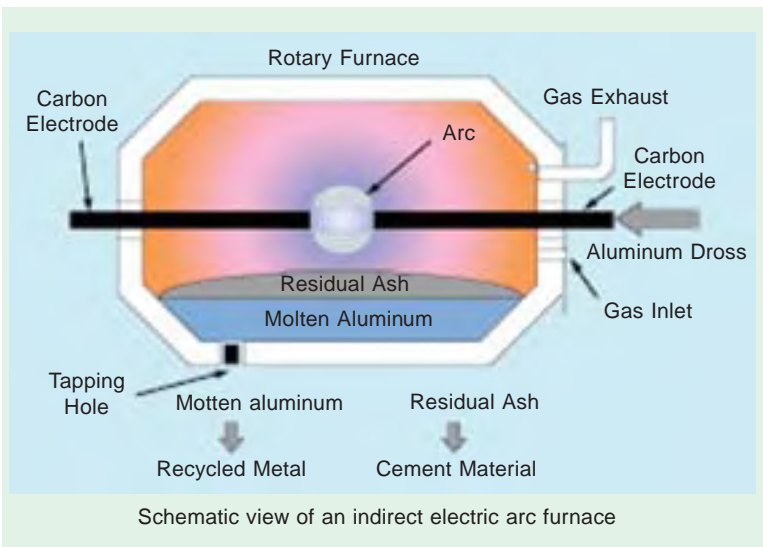


Fig. 47.13

**Example 47.4.** A 4-phase electric arc furnace has the following data :

Current drawn = 5000 A ;	Arc voltage = 50 V
Resistance of transformer referred to secondary = 0.002 $\Omega$	
Resistance of transformer referred to secondary = 0.004 $\Omega$	

- Calculate the power factor and kW drawn from the supply.
- If the overall efficiency of the furnace is 65%, find the time required to melt 2 tonnes of steel if latent heat of steel = 8.89 kcal/kg, specific heat of steel = 0.12, melting point of steel = 1370°C and initial temperature of steel = 20°C.

(Utilisation of Elect. Power, A.M.I.E. Sec. B)

**Solution.** Voltage drop due to transformer resistance =  $5000 \times 0.002 = 10$  V  
 Voltage drop due to transformer reactance =  $5000 \times 0.004 = 20$  V.



Since arc voltage drop is resistive in nature, it is vectorially added to the transformer resistance drop.

$$\text{Open circuit transformer secondary voltage/phase} = \sqrt{(50+10)^2 + 20^2} = 63.25 \text{ V}$$

$$(i) \text{ Supply p.f.} = (50 + 10)/63.25 = \mathbf{0.9487}$$

$$\text{Power drawn/phase by the secondary} = 5000 \times 63.25 \times 0.9487 = 300,000 \text{ W} = 300 \text{ kW}$$

$$\text{Total power drawn from the supply} = 3 \times 300 = \mathbf{900 \text{ kW}}$$

$$(ii) \text{ Energy required to melt 2 tonnes of steel} = ms(t_2 - t_1) + mL = 2000 \times 0.12 \times (1370 - 20) + 2000 \times 8.89 = 341,780 \text{ kcal.} = 341,780/860 = 397.4 \text{ kWh}$$

$$\text{Power actually utilised} = 900 \times 0.65 = 585 \text{ kW}$$

$$\text{Time required for melting steel} = 397.4/585 = 0.679 \text{ hours} = \mathbf{40 \text{ minutes } 46 \text{ seconds.}}$$

**Example. 47.5.** If a 3-phase arc furnace is to melt 10 tonne steel in 2 hours, estimate the average input to the furnace if overall efficiency is 50%. If the current input is 9,000 A with the above kW input and the resistance and reactance of furnace leads (including transformer) are 0.003 Ω and 0.005 Ω respectively, estimate the arc voltage and total kVA taken from the supply  
Specific heat of steel = 444 J kg<sup>-1</sup>°C<sup>-1</sup>

$$\text{Latent heat of fusion of steel} = 37.25 \text{ kJ/kg}$$

$$\text{Melting point of steel} = 1,370 \text{ °C} \quad (\text{Utilisation of Elect. Energy, Punjab Univ. 1989.})$$

**Solution.** Energy required to melt 10 tonnes of steel = 10,000 × 444 (1370 – 20) + 10,000 × 37,250 = 6366.5 × 10<sup>6</sup> J = 1768.5 kWh

It has been assumed in the above calculation that the initial melting temperature of steel is 20°C.

$$\text{Since time taken is two hours, average output power} = 1768.5/2 = 884.25 \text{ kW}$$

$$\text{Average input power} = 884.25/0.5 = 1768.5 \text{ kW}$$

$$\text{Voltage drop due to the resistance of the furnace leads (including transformer)} = 9000 \times 0.003 = 27 \text{ V}$$

Voltage drop due to reactance of the furnace leads (including transformer) = 9000 × 0.005 = 45 V  
If VA is the arc drop (which is assumed resistive in nature) then

$$\text{O.D. secondary voltage/phase} = \sqrt{(V_A + 27)^2 + 45^2}$$

$$\text{P.F.} = \frac{V_A + 27}{\sqrt{(V_A + 27)^2 + (45)^2}}$$

$$\text{Total power input} = 3 \times \text{power drawn/phase}$$

$$\therefore 1768.5 \times 10^3 = 3 \times 9000 \sqrt{(V_A + 27)^2 + (35)^2} \times (V_A + 27) / \sqrt{(V_A + 27)^2 + (45)^2}$$

$$\therefore V_A + 27 = 65.5 \quad \text{or} \quad V_A = 65.5 - 27 = \mathbf{38.5 \text{ V}}$$

$$\text{O.C. secondary voltage/phase} = \sqrt{(V_A + 27)^2 + (45)^2} = 79.5 \text{ V}$$

$$\text{Total kVA taken from supply line} = 3 \times 9000 \times 79.5 \times 10^{-3} = \mathbf{2145 \text{ kVA.}}$$

### 47.13. Induction Heating

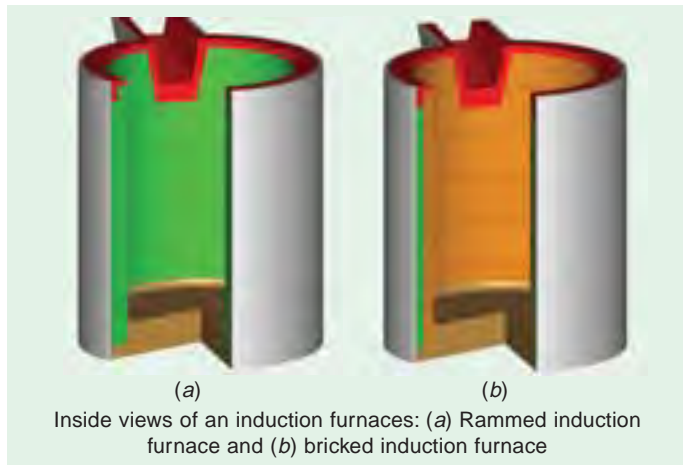
This heating process makes use of the currents induced by the electro-magnetic action in the charge to be heated. In fact, induction heating is based on the principle of transformer working. The primary winding which is supplied from an a.c. source is magnetically coupled to the charge which acts as a short circuited secondary of single turn. When an a.c. voltage is applied to the primary, it induces voltage in the secondary *i.e.* charge. The secondary current heats up the charge in the same way as any electric current does while passing through a resistance. If *V* is the voltage induced in the charge and *R* is the charge resistance, then heat produced = *V*<sup>2</sup>/*R*. The value of current induced in the charge depends on (i) magnitude of the primary current (ii) turn ratio of the transformer

(iii) co-efficient of magnetic coupling.

Low-frequency induction furnaces are used for melting and refining of different metals. However, for other processes like case hardening and soldering etc., high-frequency eddy-current heating is employed. Low-frequency induction furnaces employed for the melting of metals are of the following two types :

(a) **Core-type Furnaces** — which operate just like a two winding transformer. These can be further sub-divided into (i) Direct core-type furnaces (ii) Vertical core-type furnaces and (iii) Indirect core-type furnaces.

(b) **Coreless-type Furnaces** — in which an inductively-heated element is made to transfer heat to the charge by radiation.



#### 47.14. Core Type Induction Furnace

It is shown in Fig. 47.14 and is essentially a transformer in which the charge to be heated forms a single-turn short-circuited secondary and is magnetically coupled to the primary by an iron core. The furnace consists of a circular hearth which contains the charge to be melted in the form of an annular ring. When there is no molten metal in the ring, the secondary becomes open-circuited there-by cutting off the secondary current. Hence, to start the furnace, molten metal has to be poured in the annular hearth. Since, magnetic coupling between the primary and secondary is very poor, it results in high leakage and low power factor. In order to nullify the effect of increased leakage reactance, low primary frequency of the order of 10 Hz is used. If the transformer secondary current density exceeds  $500 \text{ A/cm}^2$  then, due to the interaction of secondary current with the alternating magnetic field, the molten metal is squeezed to the extent that secondary circuit is interrupted. This effect is known as “pinch effect”.

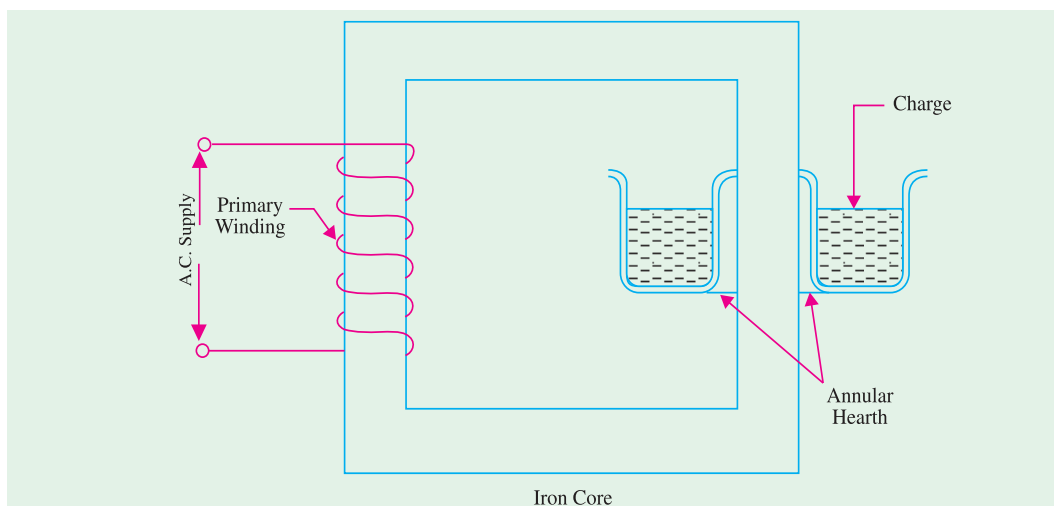


Fig. 47.14

This furnace suffers from the following drawbacks :

1. It has to be run on low-frequency supply which entails extra expenditure on motor-generator set or frequency convertor.
2. It suffers from pinching effect.
3. The crucible for charge is of odd shape and is very inconvenient for tapping the molten charge.
4. It does not function if there is no molten metal in the hearth *i.e.* when the secondary is open. Every time molten metal has to be poured to start the furnace.
5. It is not suitable for intermittent service.

However, in this furnace, melting is rapid and clean and temperature can be controlled easily. Moreover, inherent stirring action of the charge by electro-magnetic forces ensures greater uniformity of the end product.

### 47.15. Vertical Core-Type Induction Furnace

It is also known as Ajax-Wyatt furnace and represents an improvement over the core-type furnace discussed above. As shown in Fig. 47.15, it has vertical channel (instead of a horizontal one) for the charge, so that the crucible used is also vertical which is convenient from metallurgical point of view. In this furnace, magnetic coupling is comparatively better and power factor is high. Hence, it can be operated from normal frequency supply. The circulation of the molten metal is kept up round the Vee portion by convection currents as shown in Fig. 47.15.

As Vee channel is narrow, even a small quantity of charge is sufficient to keep the secondary circuit closed. However, Vee channel must be kept full of charge in order to maintain continuity of secondary circuit. This fact makes this furnace suitable for continuous operation. The tendency of the secondary circuit to rupture due to pinch-effect is counteracted by the weight of the charge in the crucible.

The choice of material for inner lining of the furnace depends on the type of charge used. Clay lining is used for yellow brass. For red brass and bronze, an alloy of magnetia and alumina or corundum is used. The top of the furnace is covered with an insulated cover which can be removed for charging. The furnace can be tilted by the suitable hydraulic arrangement for taking out the molten metal.

This furnace is widely used for melting and refining of brass and other non-ferrous metals. As said earlier, it is suitable for continuous operation. It has a p.f. of 0.8-0.85. With normal supply frequency, its efficiency is about 75% and its standard size varies from 60-300 kW, all single-phase.

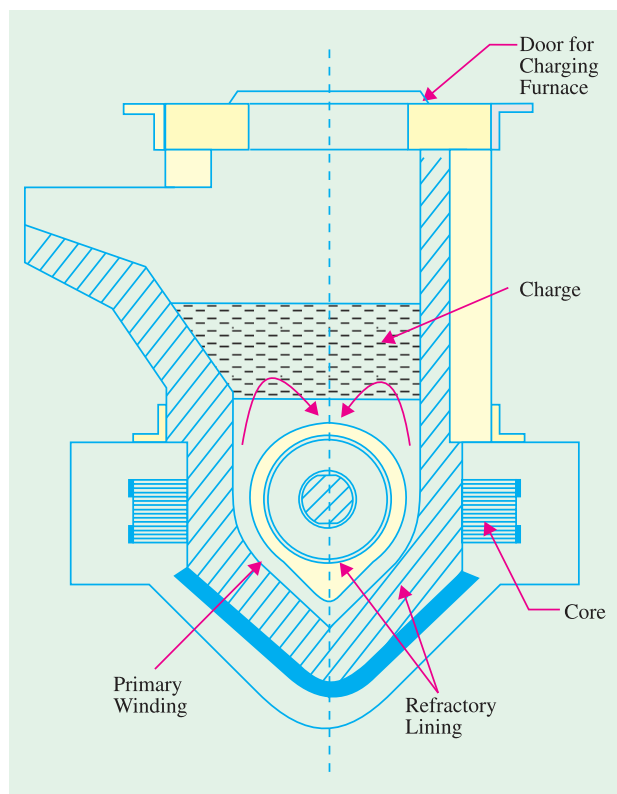


Fig. 47.15

### 47.16. Indirect Core-Type Induction Furnace

In this furnace, a suitable element is heated by induction which, in turn, transfers the heat to the charge by radiation. So far as the charge is concerned, the conditions are similar to those in a resistance oven.

As shown in Fig. 47.16, the secondary consists of a metal container which forms the walls of the furnace proper. The primary winding is magnetically coupled to this secondary by an iron core. When primary winding is connected to a.c. supply, secondary current is induced in the metal container by transformer action which heats up the container. The metal container transfers this heat to the charge. A special advantage of this furnace is that its temperature can be automatically controlled without the use of an external equipment. The part AB of the magnetic circuit situated inside the oven chamber consists of a special alloy which loses its magnetic properties at a particular temperature but regains them when cooled back to the same temperature. As soon as the chamber attains the critical temperature, reluctance of the magnetic circuit increases manifold thereby cutting off the

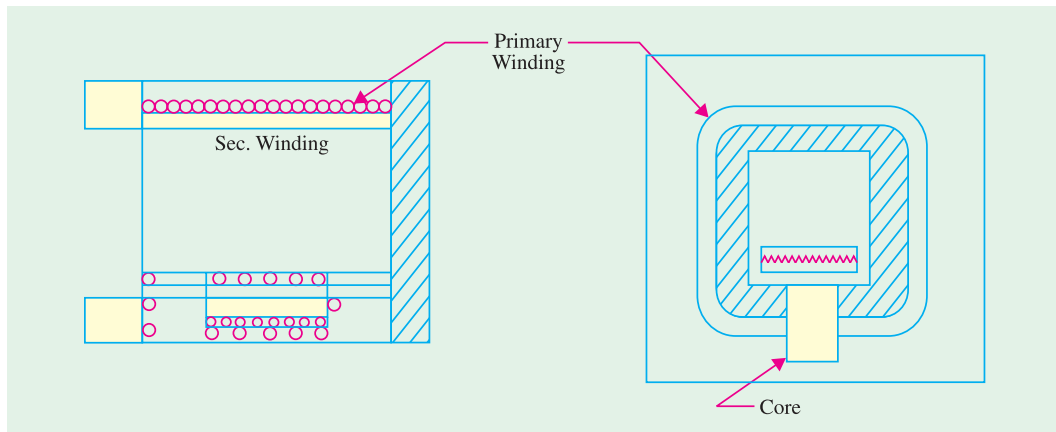


Fig. 47.16

heat supply. The bar *AB* is detachable and can be replaced by other bars having different critical temperatures.

### 47.17. Coreless Induction Furnace

As shown in Fig. 47.17, the three main parts of the furnace are (i) primary coil (ii) a ceramic crucible containing charge which forms the secondary and (iii) the frame which includes supports and tilting mechanism. The distinctive feature of this furnace is that it contains no heavy iron core with the result that there is no continuous path for the magnetic flux. The crucible and the coil are relatively light in construction and can be conveniently tilted for pouring.

The charge is put into the crucible and primary winding is connected to a high-frequency a.c. supply. The flux produce by the primary sets up eddy-currents in the charge and heats it up to the melting point. The charge need not be in the molten state at the start as was required by core-type furnaces. The eddy-currents also set up electromotive forces which produce stirring action which is essential for obtaining uniform quality of metal. Since flux density is low (due to the absence of the magnetic core) high frequency supply has to be used because eddy-current loss  $W_e \propto B^2 f^2$ . However, this high frequency increases the resistance of the primary winding due to skin effect, thereby increasing primary Cu losses. Hence, the primary winding is not made of Cu wire but consists of hollow Cu tubes which are cooled by water circulating through them.

Since magnetic coupling between the primary and secondary windings is low, the furnace p.f. lies between 0.1 and 0.3. Hence, static capacitors are invariably used in parallel with the furnace to improve its p.f.

Such furnaces are commonly used for steel production and for melting of non-ferrous metals like brass, bronze, copper and aluminum etc., along with various alloys of these elements. Special application of these furnaces include vacuum melting, melting in a controlled atmosphere and melting for precision casting where high frequency induction heating is used. It also finds wide use in electronic industry and in other industrial activities like soldering, brazing hardening and annealing and sterilizing surgical instruments etc.

Some of the advantages of coreless induction furnaces are as follows :

- (1) They are fast in operation.
- (2) They produce most uniform quality of product.
- (3) They can be operated intermittently.
- (4) Their operation is free from smoke, dirt, dust and noises.
- (5) They can be used for all industrial applications requiring heating and melting.
- (6) They have low erection and operating costs.
- (7) Their charging and pouring is simple.

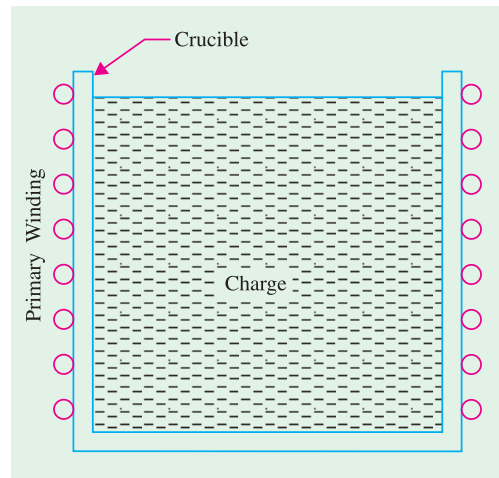


Fig. 47.17

### 47.18. High Frequency Eddy-current Heating

For heating an article by eddy-currents, it is placed inside a high frequency a.c. current-carrying coil (Fig. 47.18). The alternating magnetic field produced by the coil sets up eddy-currents in the article which, consequently, gets heated up. Such a coil is known as heater coil or work coil and the material to be heated is known as *charge or load*. Primarily, it is the eddy-current loss which is responsible for the production of heat although hysteresis loss also contributes to some extent in the case of non-magnetic materials.

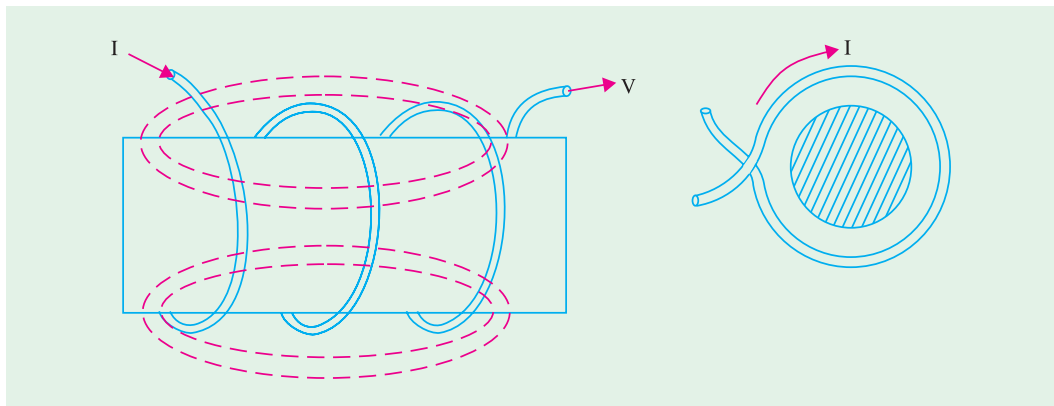


Fig. 47.18

The eddy-current loss  $W_e \propto B^2 f^2$ . Hence, this loss can be controlled by controlling flux density  $B$  and the supply frequency  $f$ . This loss is greatest on the surface of the material but decreases as we go deep inside. The depth of the material upto which the eddy-current loss penetrates is given by

$$d = \frac{1}{2\pi} \sqrt{\frac{\rho \times 10^9}{\mu_r \cdot f}}$$

where  $\rho$  = resistivity of the molten metal  
 $f$  = supply frequency  
 $\mu_r$  = relative permeability of the charge

### Advantages of Eddy-current Heating

- (1) There is negligible wastage of heat because the heat is produced in the body to be heated.
- (2) It can take place in vacuum or other special environs where other types of heating are not possible.
- (3) Heat can be made to penetrate any depth of the body by selecting proper supply frequently.



Eddy Current Heater

### Applications of Eddy-current Heating

(1) **Surface Hardening.** The bar whose surface is to be hardened by heat treatment is placed within the working coil which is connected to an a.c. supply of high frequency. The depth upto which the surface is to be hardened can be obtained by the proper selection of frequency of the coil current. After a few seconds, when surface has reached the proper temperature, a.c. supply is cut off and the bar is at once dipped in water.

(2) **Annealing.** Normally, annealing process takes long time resulting in scaling of the metal which is undesirable. However, in eddy-current heating, time taken is much less so that no scale formation takes place.

(3) **Soldering.** Eddy-current heating is economical for precise high-temperature soldering where silver, copper and their alloys are used as solders.

**Example. 47.6.** Determine the efficiency of a high-frequency induction furnace which takes 10 minutes to melt 2 kg of a aluminium initially at a temperature of 20°C. The power drawn by the furnace is 5 kW, specific heat of aluminium = 0.212, melting point of aluminium = 660° C and latent heat of fusion of aluminium. = 77 kcal/kg.

**Solution.** Heat required to melt aluminium = 2 × 77 = 154 kcal

Heat required to raise the temperature of aluminium from 20°C to 660°C

$$= 2 \times 0.212 \times (660 - 20) = 2 \times 0.212 \times 640 = 271.4 \text{ kcal}$$

$$\text{Total heat required} = 154 + 271.4 = 425.4 \text{ kcal}$$

$$\text{Heat required per hour} = 425.4 \times 60/10 = 2552.4 \text{ kcal}$$

$$\text{Power delivered to aluminium} = 2552.4/860 = 2.96 \text{ kW}$$

$$\therefore \text{efficiency} = \text{output/input} = 2.97/5 = \mathbf{0.594 \text{ or } 59.4\%}$$

**Example 47.7.** A low-frequency induction furnace has a secondary voltage of 20V and takes 600 kW at 0.6 p.f. when the hearth is full. If the secondary voltage is kept constant, determine the power absorbed and the p.f. when the hearth is half-full. Assume that the resistance of the secondary circuit is doubled but the reactance remains the same.

**Solution.** Secondary current =  $600 \times 10^3 / 20 \times 0.6 = 5 \times 10^4$  A

If this current is taken as the reference quantity, then secondary voltage is

$$V_2 = 20(0.6 + j 0.8) = (12 + j16)V$$

Hence, secondary impedance,  $Z_2 = (12 + j 16)/5 \times 10^4 = (2.4 + j 3.2) \times 10^{-4}$  ohm

Now, if the secondary resistance is double, then total impedance when the hearth is half-full is

$$= Z_2 = (4.8 + j3.2) \times 10^{-4} \text{ ohm}$$

Now, secondary current  $I_2 = 20 / (4.8 + j3.2) \times 10^{-4}$

$$= 20 / 5.77 \angle 33.7^\circ \times 10^4 = 3.466 \angle -33.7^\circ \times 10^4 \text{ A}$$

Now p.f. =  $\cos 33.7^\circ = 0.832$

Hence, power absorbed =  $20 \times 3.466 \times 10^4 \times 0.832 \times 10^{-3} = 580 \text{ kW}$

**Example 47.8.** Estimate the energy required to melt 0.5 tonne of brass in a single-phase induction furnace. If the melt is to be carried out in 0.5 hour, what must be the average power input to the furnace?

Specific heat of brass = 0.094

Latent heat of fusion of brass = 30 kilocal/kg

Melting point of brass =  $920^\circ\text{C}$

Furnace efficiency = 60.2%

The temperature of the cold charge may be taken as  $20^\circ\text{C}$

**Solution.** Total amount of heat required to melt 0.5 kg of brass.

$$= (0.5 \times 1000) \times 39 + 500 \times 0.094 \times (920 - 20)$$

$$= 61,800 \text{ kcal} = 61,800 / 860 = 71.86 \text{ kWh}$$

Total furnace input =  $71.86 / 0.602 = 119.4 \text{ kWh}$

**Example 47.9.** A low-frequency induction furnace whose secondary voltage is maintained constant at 10 V, takes 400 kW at 0.6 p.f. when the hearth is full. Assuming the resistance of the secondary circuit to vary inversely as the height of the charge and reactance to remain constant, find the height upto which the hearth should be filled to obtain maximum heat.

(Utili. of Elect. Power and Traction, Gorakhpur Univ.)

**Solution.** Secondary current  $I_2 = P / V_2 \cos \phi$

$$= 400 \times 10^3 / 10 \times 0.6 = 6.667 \times 10^4 \text{ A}$$

Impedance of the secondary circuit when hearth is full

$$Z_2 = V_2 / I_2 = 10 / 6.667 \times 10^4 = 1.5 \times 10^{-4} \Omega$$

Secondary resistance when hearth is full,  $R_2 = Z_2 \cos \phi$

$$= 1.5 \times 10^{-4} \times 0.6 = 0.9 \times 10^{-4} \Omega$$

Reactance of the secondary circuit,  $X_2 = Z_2 \sin \phi$

$$= 1.5 \times 10^{-4} \times 0.8 = 1.2 \times 10^{-4} \Omega$$

In the second, let the height of the charge be  $x$  times of the full hearth i.e.  $h = xH$

Since resistance varies inversely as the height of the charge

$$= R_2 = R_2 / x = 0.9 \times 10^{-4} / x \Omega$$

Power drawn and hence heat produced will be maximum where secondary resistance equals its reactance.

$$\therefore 0.9 \times 10^{-4} / x = 1.2 \times 10^{-4} \text{ or } x = 3/4$$

Hence, maximum heat would be produced in the charge when its height is three-fourth the height of the hearth.

### 47.19. Dielectric Heating

It is also called high-frequency capacitive heating and is used for heating insulators like wood, plastics and ceramics etc. which cannot be heated easily and uniformly by other methods. The supply frequency required for dielectric heating is between 10-50 MHz and the applied voltage is upto 20 kV. The overall efficiency of dielectric heating is about 50%.

### 47.20. Dielectric Loss

When a practical capacitor is connected across an a.c. supply, it draws a current which leads the voltage by an angle  $\phi$ , which is a little less than  $90^\circ$  or falls short of  $90^\circ$  by an angle  $\delta$ . It means that there is a certain component of the current which is in phase with the voltage and hence produces some loss called dielectric loss. At the normal supply frequency of 50 Hz, this loss is negligibly small but at higher frequencies of 50 MHz or so, this loss becomes so large that it is sufficient to heat the dielectric in which it takes place. The insulating material to be heated is placed between two conducting plates in order to form a parallel-plate capacitor as shown in Fig. 47.19 (a).

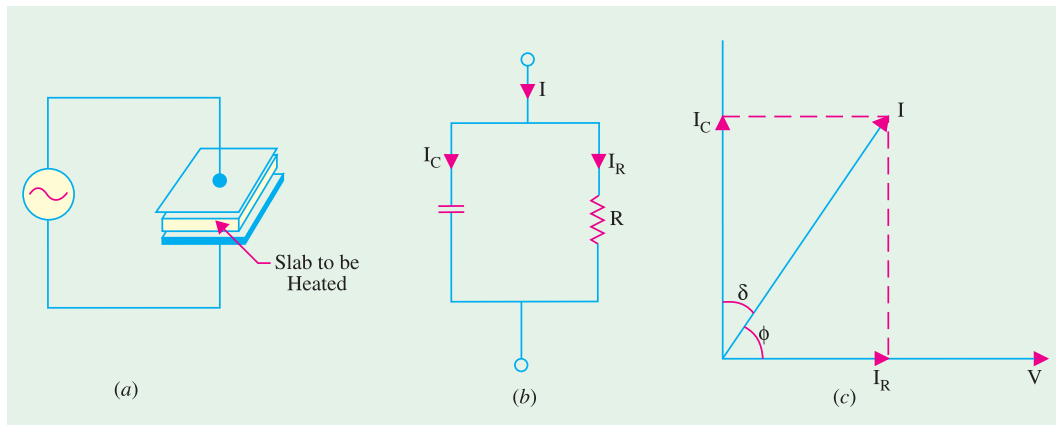


Fig. 47.19

Fig. 47.19 (b) shows the equivalent circuit of the capacitor and Fig. 47.19 (c) gives its vector diagram.

$$\text{Power drawn from supply} = VI \cos \phi$$

$$\text{Now, } I_C = I = V/X_C = 2\pi f CV$$

$$\therefore P = V(2\pi f CV) \cos \phi = 2\pi f CV^2 \cos \phi$$

$$\text{Now, } \phi = (90 - \delta), \cos \phi = \cos (90 - \delta) = \sin \delta = \tan \delta = \delta$$

where  $\delta$  is very small and is expressed in radians.

$$P = 2\pi f CV^2 \delta \text{ watts}$$

Here,  $C = \epsilon_0 \epsilon_r \frac{A}{d}$  where  $d$  is the thickness and  $A$  is the surface area of the dielectric slab.

This power is converted into heat. Since for a given insulator material,  $C$  and  $\delta$  are constant, the dielectric loss is directly proportional to  $V^2 f$ . That is why high-frequency voltage is used in dielectric heating. Generally, a.c. voltage of about 20 kV at a frequency of 10-30 MHz is used.

### 47.21. Advantages of Dielectric Heating

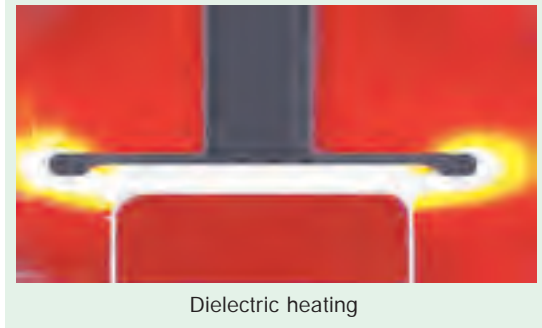
1. Since heat is generated within the dielectric medium itself, it results in uniform heating.
2. Heating becomes faster with increasing frequency.
3. It is the only method for heating bad conductors of heat.
4. Heating is fastest in this method of heating.
5. Since no naked flame appears in the process, inflammable articles like plastics and wooden products etc., can be heated safely.
6. Heating can be stopped immediately as and when desired.



### 47.22. Applications of Dielectric Heating

Since cost of dielectric heating is very high, it is employed where other methods are not possible or are too slow. Some of the applications of dielectric heating are as under :

1. For gluing of multilayer plywood boards.
2. For baking of sand cores which are used in the moulding process.
3. For preheating of plastic compounds before sending them to the moulding section.
4. For drying of tobacco after glycerine has been mixed with it for making cigarettes.
5. For baking of biscuits and cakes etc. in bakeries with the help of automatic machines.
6. For electronic sewing of plastic garments like raincoats etc. with the help of cold rollers fed with high-frequency supply.
7. For dehydration of food which is then sealed in air-tight containers.
8. For removal of moistures from oil emulsions.
9. In diathermy for relieving pain in different parts of the human body.
10. For quick drying of glue used for book binding purposes.



Dielectric heating

**Example. 47.10.** A slab of insulating material  $150 \text{ cm}^2$  in area and  $1 \text{ cm}$  thick is to be heated by dielectric heating. The power required is  $400 \text{ W}$  at  $30 \text{ MHz}$ . Material has relative permittivity of  $5$  and p.f. of  $0.05$ . Determine the necessary voltage. Absolute permittivity =  $8.854 \times 10^{-12} \text{ F/m}$ .

(Utilisation of Elect. Energy, Punjab Univ.)

**Solution.**  $P = 400 \text{ W}$ , p.f. =  $0.05$ ,  $f = 30 \times 10^6 \text{ Hz}$

$$C = \epsilon_0 \epsilon_r A/d = 8.854 \times 10^{-12} \times 5 \times 150 \times 10^{-4}/1 \times 10^{-2} = 66.4 \times 10^{-12} \text{ F}$$

$$\text{Now, } P = 2\pi f C V^2 \cos\phi \text{ or } 400 = 2\pi \times 30 \times 10^6 \times 66.4 \times 10^{-12} \times V^2 \times 0.05 \text{ or } V = \mathbf{800 \text{ V}}$$

**Example. 47.11.** An insulating material  $2 \text{ cm}$  thick and  $200 \text{ cm}^2$  in area is to be heated by dielectric heating. The material has relative permittivity of  $5$  and power factor of  $0.05$ . Power required is  $400 \text{ W}$  and frequency of  $40 \text{ MHz}$  is to be used. Determine the necessary voltage and the current that will flow through the material. If the voltage were to be limited to  $700 \text{ V}$ , what will be the frequency to get the same loss? (A.M.I.E. Sec. B)

$$\text{Solution. } C = 8.854 \times 10^{-12} \times 5 \times 200 \times 10^{-4}/2 \times 10^{-2} = 44.27 \times 10^{-12} \text{ F}$$

$$P = 2\pi f C V^2 \cos\phi \text{ or } V = \sqrt{400/2\pi \times 40 \times 10^6 \times 44.27 \times 10^{-12} \times 0.05} = \mathbf{848 \text{ V}}$$

Current flowing through the material,

$$I = P/V \cos\phi = 400/848 \times 0.05 = \mathbf{9.48 \text{ A}}$$

Heat produced  $\propto V^2 f$

$$\therefore V_2^2 f_2 = V_1^2 f_1 \text{ or } f_2 = f_1 (V_1/V_2)^2 = 40 \times 10^6 (848/700)^2 = \mathbf{58.7 \text{ MHz}}$$

**Example 47.12.** A plywood board of  $0.5 \times 0.25 \times 0.02$  metre is to be heated from  $25$  to  $125^\circ\text{C}$  in  $10$  minutes by dielectric heating employing a frequency of  $30 \text{ MHz}$ . Determine the power required in this heating process. Assume specific heat of wood  $1500 \text{ J/kg}^\circ\text{C}$ ; weight of wood  $600 \text{ kg/m}^3$  and efficiency of process  $50\%$ . (Utilisation of Traction, B.H.U.)

**Solution.** Volume of plywood to be heated =  $0.5 \times 0.25 \times 0.02 = 0.0025 \text{ m}^3$

$$\text{Weight of plywood} = 0.0025 \times 600 = 1.5 \text{ kg}$$

Heat required to raise the temperature of plywood board from 25° to 125°

$$= 1.5 \times 1500 (125 - 25) = 2,25,000 \text{ J or W-s}$$

$$\text{or } H = 2,25,000/60 \times 60 = 62.5 \text{ Wh}$$

Since heating is to be done in 10 minutes, power required =  $62.5/(10/60) = 375 \text{ W}$

Since efficiency is 50%, power input =  $375/0.5 = 700 \text{ W}$

### 47.23. Choice of Frequency

The selection of frequency for heating is important because it has a great bearing on the work to be heated and the method of its heating whether by induction heating or dielectric heating. Furnaces running on power frequency of 50 Hz can be of 1 MW capacity whereas those running on medium frequencies (500 Hz to 1000 Hz) have a capacity of 50 kW and those running on high frequency (1 MHz to 2 MHz) have capacities ranging from 200 kW to 500 kW.

**1. Induction Heating.** While choosing frequency for induction heating, the following factors are considered :

- (a) Thickness of the surface to be heated. Higher the frequency, thinner the surface that will get heated.
- (b) The time of continuous heating. Longer the duration of heating, deeper the penetration of heat in the work due to conduction.
- (c) The temperature to be obtained. Higher the temperature, higher the capacity of the generator required.

**2. Dielectric Heating.** The power consumed during dielectric heating,  $P = 2\pi f C V^2 \cos \phi$ . As seen,  $P \propto f \times C \times V^2 \times \cos \phi$ . Hence, rate of heat production can be increased by increasing voltage or voltage across any specimen is limited by its thickness or because of the consideration of potential gradient, breakdown voltage and safety etc., Voltages ranging from 600 V to 3000 V are used for dielectric heating, although voltages of 20 kV or so are also used sometimes.

Rate of heat production can also be increased by applying high potential but it is also limited because of the following considerations :

- (a) Possibility of formation of standing waves between the surface of two electrodes having wavelength nearly equal to or more than one quarter of the wavelength of the particular frequency used.
- (b) Necessity of employing special matching circuit at higher frequencies due to the fact that maximum power transfer takes place when the oscillator impedance equals the load impedance.
- (c) At higher frequencies it is difficult for tuning inductance to resonate with the charge capacitance.
- (d) At higher frequencies, it is almost impossible to get uniform voltage distribution.
- (e) Since higher frequencies disturb near-by radio station services, special arrangement has to be made to stop radiations from the high-frequency generator used for the purpose.

### 47.24. Infrared Heating

When tungsten filament lamps are operated at about 2300°C (instead of 3000°C), they produce plenty of heat radiations called *infrared radiations*. With the help of suitable reflectors, these infrared radiations are focused on the surface to be heated. The lamps so employed have ratings varying from 250 W to 1000 W operating at 115 V. Lower voltage results in robust filaments. With this arrangement, the charge temperature obtain is between 200°C and 300°C. The heat emission intensity obtained is about 7000 W/m<sup>2</sup> as compared to 1500 W/m<sup>2</sup> obtained with ordinary resistance furnaces. In this type of heating, heat absorption remains practically constant whatever the charge temperature whereas

it falls rapidly as the temperature of charge rises in the ordinary resistance furnace.

Infrared heating is used for paint drying and for drying foundry moulds, for low temperature heating of plastics and for various dehydration and other processes.

### Tutorial Problem No. 47.1

- A slab of insulating material 150 cm<sup>2</sup> in area and 1 cm. thick is to be heated by dielectric heating. The power required is 400W at 30 MHz. Material has permittivity of 5 and p.f. 0.05. Determine the voltage necessary. Absolute permittivity =  $8.854 \times 10^{-12}$  F/m.  
**[800 V] (Utilisation of Elect. Energy, Punjab Univ.)**
- A 20-kW, 1-phase, 220-V resistance oven employs circular nickel chrome wire for its heating elements. If the wire temperature is not to exceed 1107° C and the temperature of the charge is to be 500°C, calculate the length and size of wire required. Assume a radiating efficiency of 0.6 and the specific resistance of nickel chrome as  $100 \times 10^{-6}$  Ω cm and emissivity 0.9.  
**[l =17.1 m, d = 0.3 cm] (Utilisation of Elect. Power, A.M.I.E.)**
- An electric toaster consists of two resistance elements each of 190 ohm. Calculate the power drawn from 250 V a.c. single-phase supply, when the elements are connected in (i) parallel and (ii) series.  
**[(i) 658 W (ii) 164.5 W]**
- A 15-kW, 220-V, single-phase resistance oven employs nickel-chrome wire for its heating elements. If the wire temperature is not to exceed 1,000°C and the temperature of the charge is to be 600°C, calculate the diameter and length of the wire. Assume radiating efficiency to be 0.6 and emissivity as 0.9. For nickel chrome resistivity is  $1.016 \times 10^{-6}$  Ω-m. **[3.11 mm, 24.24 m]**  
**(Utili. of Elect. Power, A.M.I.E. Sec. B)**
- A 30-kW, 3-phase, 400-V resistance oven is to employ nickel-chrome strip 0.025 cm thick for the 3-phase star-connected heating elements. If the wire temperature is to be 1100°C and that of charge is to be 700°C, estimate a suitable width for the strip. Assume radiating efficiency as 0.6 and emissivity as 0.90. The specific resistance of the nichromealloy is  $1.03 \times 10^{-6}$  Ω-m. State any assumptions made. **[6.86 mm] (Utili. of Elect. Power, A.M.I.E. Sec. B.)**
- Estimate the efficiency of a high-frequency induction furnace which takes 10 minutes to melt 1.8 kg of a aluminium, the input to the furnace being 5 kW and initial temperature 15°C. Given :  
 Specific heat of aluminium = 880 J/kg/°C  
 Melting point of aluminium = 660°C  
 Latent heat of fusion of aluminium = 32 kJ/kg and 1J =  $2.78 \times 10^{-7}$  kWh.  
**[36 %] (Elect. Drives and Util. of Elect. Energy, Punjab Univ.)**
- A 3-phase electric arc furnace has the following data :  
 Current drawn = 5,000 A ; Arc voltage = 50 V  
 Resistance of transformer referred to primary =0.002 Ω  
 Resistance of transformer referred to secondary=0.004 Ω  
 (i) Calculate the power factor and kW drawn from the supply.  
 (ii) If the overall efficiency of furnace is 65%, find the time required to melt 2 tonnes of steel when latent heat of steel = 8.89 kcal/kg. Specific heat of steel = 0.12, melting point of steel = 1370°C and initial temperature of steel = 20°C.  
**[(i) 0.9847; 900 kW, (ii) 40 min. 46 s.] (Utili. of Elect. Power, A.M.I.E. Sec. B.)**
- Dielectric heating is to be employed to heat a slab of insulating material 20 mm thick and 1530 mm<sup>2</sup> in area. Power required is 200 W and a frequency of 3 MHz is to be used. The material has a permittivity of 5 and p.f. of 0.05. Determine the voltage necessary and the current which will flow through the material. **[8000 V ; 0.5 A] (Elect. Drives and Utili of Elect. Energy, Punjab Univ.)**
- What are the advantages of electrically produced heat? What are the properties to be possessed by the element used in resistance oven? **(J.N. University, Hyderabad, November 2003)**
- A 20 kW single-phase, 220 V resistance oven employs circular nichrome wire for its heating element, if the wire temperature is not to exceed 1227° and the temperature of the charge is to be 427°C, calculate the size and length of the wire required. Assume emissivity = 0.9, radiating efficiency = 0.6 and specific resistance of wire =  $1.09 \times 10^{-6}$ Ω-m.  
**(J.N. University, Hyderabad, November 2003)**

11. Discuss the various modes of heat dissipation. *(J.N. University, Hyderabad, November 2003)*
12. Explain in brief how heating is done in the following cases?  
(i) Resistance heating, (ii) Induction heating, (iii) Dielectric heating.  
*(J.N. University, Hyderabad, November 2003)*
13. 90 Kg of tin is to smelt during an hour in smelting furnace. Determine the suitable rating of the furnace, if melting temperature = 230°C, specific heat = 0.055, latent heat of liquidification is 13.3 Kcal/kg. Take the initial temperature of the metal as 35°C.  
*(J.N. University, Hyderabad, November 2003)*
14. Explain the principle of Induction heating, What are the applications of induction heating.  
*(J.N. University, Hyderabad, November 2003)*
15. With a neat sketch explain the working principle of coreless type induction furnace.  
*(J.N. University, Hyderabad, November 2003)*
16. Explain with a neat sketch the principle of coreless type induction furnace.  
*(J.N. University, Hyderabad, November 2003)*
17. 100 kg of tin is to smelt in one hour in a smelting furnace. Determine the suitable rating of furnace if smelting temperature of tin is 235°C; specific heat is 0.055, latent heat of liquidification 13.3 Kcal/kg. Take initial temperature of metal as 35°C.  
*(J.N. University, Hyderabad, November 2003)*
18. A low frequency Induction Furnace whose secondary voltage is maintained constant at 12 Volts takes 300 Kw at 0.65 p.f. when the head of the charge and reactance to remain constant, find the height upto which the hearth should be filled to obtain maximum heat.  
*(J.N. University, Hyderabad, November 2003)*
19. Give relative advantages and disadvantages of direct and indirect electric arc furnaces.  
*(J.N. University, Hyderabad, April 2003)*
20. An electric arc furnace consuming 5 KW takes 15 minutes to just melt 1.5 Kgs of aluminium, the initial temperature being 15°C. Find the efficiency of the furnace. Specific heat of aluminium is 0.212, melting point 658°C and latent heat of fusion is 76.8 Cal per gram.  
*(J.N. University, Hyderabad, April 2003)*
21. What are the causes of failure of heating elements? *(J.N. University, Hyderabad, April 2003)*
22. Six resistances each of 40ohms are used as heating elements in furnace. Find the power of the furnace for various connections to a three phase 230 V supply.  
*(J.N. University, Hyderabad, April 2003)*
23. What are different methods of heat transfer Explain in brief ?  
*(J.N. University, Hyderabad, April 2003)*
24. What are the advantages of radiant heating ? *(J.N. University, Hyderabad, April 2003)*
25. Describe various types of electric heating equipment. *(J.N. University, Hyderabad, April 2003)*
26. What are the causes of failure of heating elements? *(J.N. University, Hyderabad, April 2003)*
27. Six resistances each of 40 ohms are used as heating elements in furnace. Find the power of the furnace for various connections to a three phase 230 V supply.  
*(J.N. University, Hyderabad, April 2003)*
28. Explain why very high frequencies should not be used for dielectric heating.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
29. A wooden board 30 cms × 15 cms × 2 cms is to be heated from 20°C to 180°C in 10 minutes by dielectric heating using 40 MHz supply. Specific heat of wood 0.35 and density 0.55 gm/cc.  $\epsilon_r = 5$  and p.f. 0.05. Estimate the voltage across the specimen and current during heating. Assume loss of energy by conduction, convection and radiation as 10%.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
30. Write short Notes on The Ajax-yatt furnace.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
31. Discuss the different methods of electric heating and their relative merits  
*(J.N. University, Hyderabad, December 2002/January 2003)*
32. Dielectric heating is to be employed to heat a slab of insulating material of 20 mm thick and 1500 mm<sup>2</sup> in area. The power required is 200 watts at a frequency of 30 MHz. The material has a permittivity of 5 and a power factor of 0.05. Determine the voltage necessary and the current which flows through the material. *(J.N. University, Hyderabad, December 2002/January 2003)*

33. State the advantages of electric heating.  
(J.N. University, Hyderabad, December 2002/January 2003)
34. Briefly explain the different methods of electric heating.  
(J.N. University, Hyderabad, December 2002/January 2003)
35. Estimate the energy required to melt 500 kg of brass in a single phase Ajax-wyatt furnace. If the melting is to be carried out in 3/4 hour, what must be the average power input to the furnace.  
(J.N. University, Hyderabad, December 2002/January 2003)

### OBJECTIVE TESTS – 47

- As compared to other methods of heating using gas and coal etc, electric heating is far superior because of its.
  - cleanliness
  - ease of control
  - higher efficiency
  - all of the above.
- Magnetic materials are heated with the help of
  - hysteresis loss
  - electric arc
  - electric current
  - radiation.
- In the indirect resistance heating method, heat is delivered to the charge
  - directly
  - by radiation
  - by convection
  - both (b) and (c).
- The main requirements of a good heating element used in a resistance furnaces are
  - high resistivity
  - high melting-temperature
  - positive resistance-temperature coefficient
  - all of the above.
- Electric ovens using heating elements of ..... can produce temperatures up to 3000°C
  - nickel
  - graphite
  - chromium
  - iron.
- The temperature of resistance furnaces can be controlled by changing the
  - applied voltage
  - number of heating elements
  - circuit configuration
  - all of the above.
- Which of the following heating method is based on the transformer principle ?
  - resistance heating
  - eddy-current heating
  - induction heating
  - dielectric heating.
- When graphite electrodes are used in arc furnaces, the temperature obtained is in the range of .....degree centi-  
grade.
  - 3000-3500
  - 2500-3000
  - 2000-2500
  - 1500-2000
- Which of the following furnace suffers from pinch effect?
  - resistance furnace
  - core type induction furnace
  - coreless induction furnace
  - vertical core type induction furnace.
- Which of the following induction furnace has the lowest power factor?
  - vertical core type
  - indirect core type
  - coreless type
  - core type.
- The coreless induction furnace uses high-frequency electric supply in order to obtain high
  - flux density
  - eddy-current loss
  - primary resistance
  - power factor.
- Inflammable articles like plastic and wooden products etc, can be safely heated by using..... heating.
  - eddy-current
  - dielectric
  - induction
  - resistance
- Which of the following is an advantages of heating by electricity?
  - Quicker operation
  - Higher efficiency
  - Absence of flue gases
  - All of the above
- ..... has the highest value of thermal conductivity.
  - Copper
  - Aluminium
  - Brass
  - Steel
- Which of the following heating methods has maximum power factor?
  - Arc heating
  - Dielectric heating
  - Induction heating
  - Resistance heating

16. .... method has leading power factor  
(a) Resistance heating  
(b) Dielectric heating  
(c) Arc heating  
(d) Induction heating
17. .... is used for heating non-conducting materials.  
(a) Eddy current heating  
(b) Arc heating  
(c) Induction heating  
(d) Dielectric heating
18. Which of the following methods of heating is not dependent on the frequency of supply?  
(a) Induction heating  
(b) Dielectric heating  
(c) Electric resistance heating  
(d) All of the above
19. When a body reflects entire radiation incident on it, then it is known as  
(a) white body (b) grey body  
(c) black body (d) transparent body
20. For the transmission of heat from one body to another  
(a) temperature of the two bodies must be different  
(b) both bodies must be solids  
(c) both bodies must be in contact  
(d) at least one of the bodies must have some source of heating
21. Heat transfer by condition will not occur when  
(a) bodies are kept in vacuum  
(b) bodies are immersed in water  
(c) bodies are exposed to thermal radiations  
(d) temperatures of the two bodies are identical
22. A perfect black body is one that  
(a) transmits all incident radiations  
(b) absorbs all incident radions  
(c) reflects all incident radiations  
(d) absorbs, reflects and transmits all incident radiations
23. Heat is transferred simultaneously by condition, convection and radiation  
(a) inside boiler furnaces  
(b) during melting of ice  
(c) through the surface of the insulated pipe carrying steam  
(d) from refrigerator coils to freezer of a refrigerator
24. The process of heat transfer during the re-entry of satellites and missiles, at very high speeds, into earth's atmosphere is known as  
(a) ablation  
(b) radiation  
(c) viscous dissipation  
(d) irradiation
25. Which of the following has the highest value of thermal conductivity?  
(a) Water (b) Steam  
(c) Solid ice (d) Melting ice
26. Induction heating process is based on which of the following principles?  
(a) Thermal ion release principle  
(b) Nucleate heating principle  
(c) Resistance heating principle  
(d) Electro-magnetic induction principle
27. Which of the following insulating materials is suitable for low temperature applications?  
(a) Asbestos paper  
(b) Diatomaceous earth  
(c) 80 percent magnesia  
(d) Cork
28. A non-dimensional number generally associated with natural convection heat transfer is  
(a) Prandtl number  
(b) Grashoff number  
(c) Peclet number  
(d) Nusselt number
29. The temperature inside a furnace is usually measured by which of the following?  
(a) Optical pyrometer  
(b) Mercury thermometer  
(c) Alcohol thermometer  
(d) Any of the above
30. Which of the following will happen if the thickness of refractory wall of furnace is increased?  
(a) Heat loss through furnace wall will increase  
(b) Temperature inside the furnace will fall  
(c) Temperature on the outer surface of furnace walls will drop  
(d) Energy consumption will increase
31. The material of the heating element for a furnace should have  
(a) lower melting point  
(b) higher temperature co-efficient  
(c) high specific resistance  
(d) all of the above
32. In a resistance furnace the atmosphere is .....  
(a) oxidising (b) deoxidising  
(c) reducing (d) neutral
33. By which of the following methods the temperature inside a resistance furnace can be varied?  
(a) By disconnecting some of the heating elements  
(b) By varying the operating voltage  
(c) By varying the current through heating elements  
(d) By any of the above method

34. In induction heating ..... is abnormally high.  
(a) phase angle (b) frequency  
(c) current (d) voltage
35. By the use of which of the following high frequency power supply for induction furnaces can be obtained?  
(a) Coreless transformers  
(b) Current transformers  
(c) Motor-generator set  
(d) Multi-phase transformer
36. Induction furnaces are employed for which of the following?  
(a) Heat treatment of castings  
(b) Heating of insulators  
(c) Melting aluminium  
(d) None of the above
37. In an electric room heat convactor the method of heating used is  
(a) arc heating  
(b) resistance heating  
(c) induction heating  
(d) dielectric heating
38. In a domestic cake baking oven the temperature is controlled by  
(a) voltage variation  
(b) thermostat  
(c) auto-transformer  
(d) series-parallel operation
39. In an electric press mica is used  
(a) as an insulator  
(b) as a device for power factor improvement  
(c) for dielectric heating  
(d) for induction heating
40. Induction heating takes place in which of the following?  
(a) Insulating materials  
(b) Conducting materials which are magnetic  
(c) Conducting materials which are non-magnetic  
(d) Conducting materials which may or may not be magnetic
41. For heating element high resistivity material is chosen to  
(a) reduce the length of heating element  
(b) increase the life of the heating element  
(c) reduce the effect of oxidation  
(d) produce large amount of heat
42. In resistance heating highest working temperature is obtained from heating elements made of .....  
(a) nickel copper (b) nichrome  
(c) silicon carbide (d) silver
43. For intermittent work which of the following furnaces is suitable?  
(a) Indirect arc furnace  
(b) Core less furnace  
(c) Either of the above  
(d) None of the above
44. Due to which of the following reasons it is desirable to have short arc length?  
(a) To achieve better heating  
(b) To increase the life of roof refractory  
(c) To have better stirring action  
(d) To reduce problem of oxidation  
(e) All of the above
45. In the indirect resistance heating method, maximum heat-transfer takes place by  
(a) radiation (b) convection  
(c) conduction (d) any of the above
46. Property of low temperature co-efficient of heating element is desired due to which of the following reasons?  
(a) To avoid initial rush of current  
(b) To avoid change in kW rating with temperature  
(c) Both (a) and (b)  
(d) Either (a) or (b)
47. Which of the following methods is used to control temperature in resistance furnaces?  
(a) Variation of resistance  
(b) Variation of voltage  
(c) Periodical switching on and off of the supply  
(d) All of the above methods
48. It is desirable to operate the arc furnace at power factor of  
(a) zero (b) 0.707 lagging  
(c) unity (d) 0.707 leading
49. Radiations from a black body are proportional to  
(a) T (b)  $T^2$   
(c)  $T^3$  (d)  $T^4$
50. In arc furnace the function of choke is  
(a) to stabilize the arc  
(b) to improve power factor  
(c) to reduce severity of the surge  
(d) none of the above
51. Ajax Wyatt furnace is started when  
(a) it is filled below core level  
(b) it is filled above core level  
(c) it is fully empty  
(d) none of the above
52. In electric press, mica is used because it is ..... conductor of heat but/and ..... conductor of electricity.  
(a) bad, good (b) bad, bad  
(c) good, bad (d) good, good
53. Resistance variation method of temperature control is done by connecting resistance elements in

- (a) series  
 (b) parallel  
 (c) series-parallel connections  
 (d) star-delta connections  
 (e) all of the above ways
- 54.** Hysteresis loss and eddy current loss are used in  
 (a) induction heating of steel  
 (b) dielectric heating  
 (c) induction heating of brass  
 (d) resistance heating
- 55.** In heating the ferromagnetic material by induction heating, heat is produced due to  
 (a) induced current flow through the charge  
 (b) hysteresis loss taking place below curie temperature  
 (c) due to hysteresis loss as well as eddy current loss taking place in the charge  
 (d) one of the above factors
- 56.** Radiant heating is used for which of the following?  
 (a) Annealing of metals  
 (b) Melting of ferrous metals  
 (c) Heating of liquids in electric kettle  
 (d) Drying of paints and varnishes
- 57.** Which of the following devices is necessarily required for automatic temperature control in a furnace?  
 (a) Thermostat  
 (b) Thermocouple  
 (c) auto-transformer  
 (d) Heating elements of variable resistance material
- 58.** For radiant heating around 2250°C, the heating elements are made of  
 (a) copper alloy      (b) carbon  
 (c) tungsten alloy      (d) stainless steel alloy
- 59.** Which of the following is an advantage of eddy current heating?  
 (a) The amount of heat generated can be controlled accurately  
 (b) Heat at very high rate can be generated  
 (c) The area of the surface over which heat is produced can be accurately controlled  
 (d) All of the above
- 60.** The electrode of a direct arc furnace is made of  
 (a) tungsten  
 (b) graphite  
 (c) silver  
 (d) copper
- 61.** Direct arc furnaces have which of the following power factors?  
 (a) Unity  
 (b) Low, lagging  
 (c) Low, leading  
 (d) Any of the above
- 62.** In direct arc furnace, which of the following has high value?  
 (a) Current  
 (b) Voltage  
 (c) Power factor  
 (d) All of the above

**ANSWERS**

- |                |                |                |                |                |                |                |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| <b>1.</b> (d)  | <b>2.</b> (a)  | <b>3.</b> (d)  | <b>4.</b> (d)  | <b>5.</b> (b)  | <b>6.</b> (d)  | <b>7.</b> (c)  |
| <b>8.</b> (a)  | <b>9.</b> (b)  | <b>10.</b> (d) | <b>11.</b> (b) | <b>12.</b> (b) | <b>13.</b> (d) | <b>14.</b> (a) |
| <b>15.</b> (d) | <b>16.</b> (b) | <b>17.</b> (d) | <b>18.</b> (c) | <b>19.</b> (a) | <b>20.</b> (a) | <b>21.</b> (d) |
| <b>22.</b> (b) | <b>23.</b> (a) | <b>24.</b> (a) | <b>25.</b> (c) | <b>26.</b> (d) | <b>27.</b> (b) | <b>28.</b> (b) |
| <b>29.</b> (a) | <b>30.</b> (c) | <b>31.</b> (c) | <b>32.</b> (a) | <b>33.</b> (d) | <b>34.</b> (b) | <b>35.</b> (c) |
| <b>36.</b> (a) | <b>37.</b> (b) | <b>38.</b> (b) | <b>39.</b> (a) | <b>40.</b> (d) | <b>41.</b> (a) | <b>42.</b> (c) |
| <b>43.</b> (a) | <b>44.</b> (e) | <b>45.</b> (a) | <b>46.</b> (c) | <b>47.</b> (d) | <b>48.</b> (b) | <b>49.</b> (d) |
| <b>50.</b> (a) | <b>51.</b> (d) | <b>52.</b> (c) | <b>53.</b> (e) | <b>54.</b> (a) | <b>55.</b> (c) | <b>56.</b> (d) |
| <b>57.</b> (b) | <b>58.</b> (c) | <b>59.</b> (d) | <b>60.</b> (b) | <b>61.</b> (b) | <b>62.</b> (a) |                |



# CHAPTER 48

## Learning Objectives

- Definition of Welding
- Welding Processes
- Four Positions of Arc Welding
- Electrodes for Metal Arc Welding
- Advantages of Coated Electrodes
- Arc Welding Machines
- V-I Characteristics of Arc Welding D.C. Machines
- D.C. Welding Machines with Motor Generator Set
- AC Rectified Welding Unit
- AC Welding Machines
- Carbon Arc Welding
- Submerged Arc Welding
- Gas Shield Arc Welding
- TIG Welding
- MIG Welding
- MAG Welding
- Resistance Welding
- Spot Welding
- Seam Welding
- Projection Welding
- Butt Welding
- Flash Butt Welding
- Upset Welding
- Stud Welding
- Electrogas Welding
- Electron Beam Welding

## ELECTRIC WELDING



Electricity is used to generate heat necessary to melt the metal to form the necessary joints

### 48.1. Definition of Welding

It is the process of joining two pieces of metal or non-metal at faces rendered plastic or liquid by the application of heat or pressure or both. Filler material may be used to effect the union.

### 48.2. Welding Processes

All welding processes fall into two distinct categories :

**1. Fusion Welding**—It involves melting of the parent metal. Examples are:

- (i) Carbon arc welding, metal arc welding, electron beam welding, electroslag welding and electrogas welding which utilize electric energy and
- (ii) Gas welding and thermit welding which utilize chemical energy for the melting purpose.

**2. Non-fusion Welding**—It does not involve melting of the parent metal. Examples are:

- (i) Forge welding and gas non-fusion welding which use chemical energy.
- (ii) Explosive welding, friction welding and ultrasonic welding etc., which use mechanical energy.
- (iii) Resistance welding which uses electrical energy.

Proper selection of the welding process depends on the (a) kind of metals to be joined (b) cost involved (c) nature of products to be fabricated and (d) production techniques adopted. The principal welding processes have been tabulated in Fig. 48.1

### 48.3. Use of Electricity in Welding

Electricity is used in welding for generating heat at the point of welding in order to melt the material which will subsequently fuse and form the actual weld joint. There are many ways of producing this localised heat but the two most common methods are as follows :

**1. resistance welding**—here current is passed through the inherent resistance of the joint to be welded thereby generating the heat as per the equation  $I^2 R t / J$  kilocalories.

**2. arc welding**—here electricity is conducted in the form of an arc which is established between the two metallic surfaces

### 48.4. Formation and Characteristics of Electric Arc

An electric arc is formed whenever electric current is passed between two metallic electrodes which are separated by a short distance from each other. The arc is started by momentarily touching the positive electrode (anode) to the negative metal (or plate) and then withdrawing it to about 3 to 6 mm from the plate. When electrode first touches the plate, a large short-circuit current flows and as it is later withdrawn from the plate, current continues to flow in the form of a spark across the air gap so formed. Due to this spark (or discharge), the air in the gap becomes ionized *i.e.* is split into negative electrons and positive ions. Consequently, air becomes conducting and current is able to flow across the gap in the form of an arc.

As shown in Fig. 48.2, the arc consists of *lighter* electrons which flow from cathode to anode and *heavier* positive ions which flow from anode to cathode. Intense heat is generated when high-velocity electrons strike the anode. Heat generated at the cathode is much less because of the low velocity of the impinging ions. It is found that nearly **two-third** of the heat is developed at the anode which burns into the form of a crater where temperature rises to a value of 3500-4000°C. The remaining one-third of the heat is developed near the cathode. The above statement is true in all d.c. systems of welding where positive side of the circuit is the hottest side. As a result, an electrode connected to the positive end of the d.c. supply circuit will burn 50% faster than if connected to the negative end. This fact can be used for obtaining desired penetration of the base metal during welding.

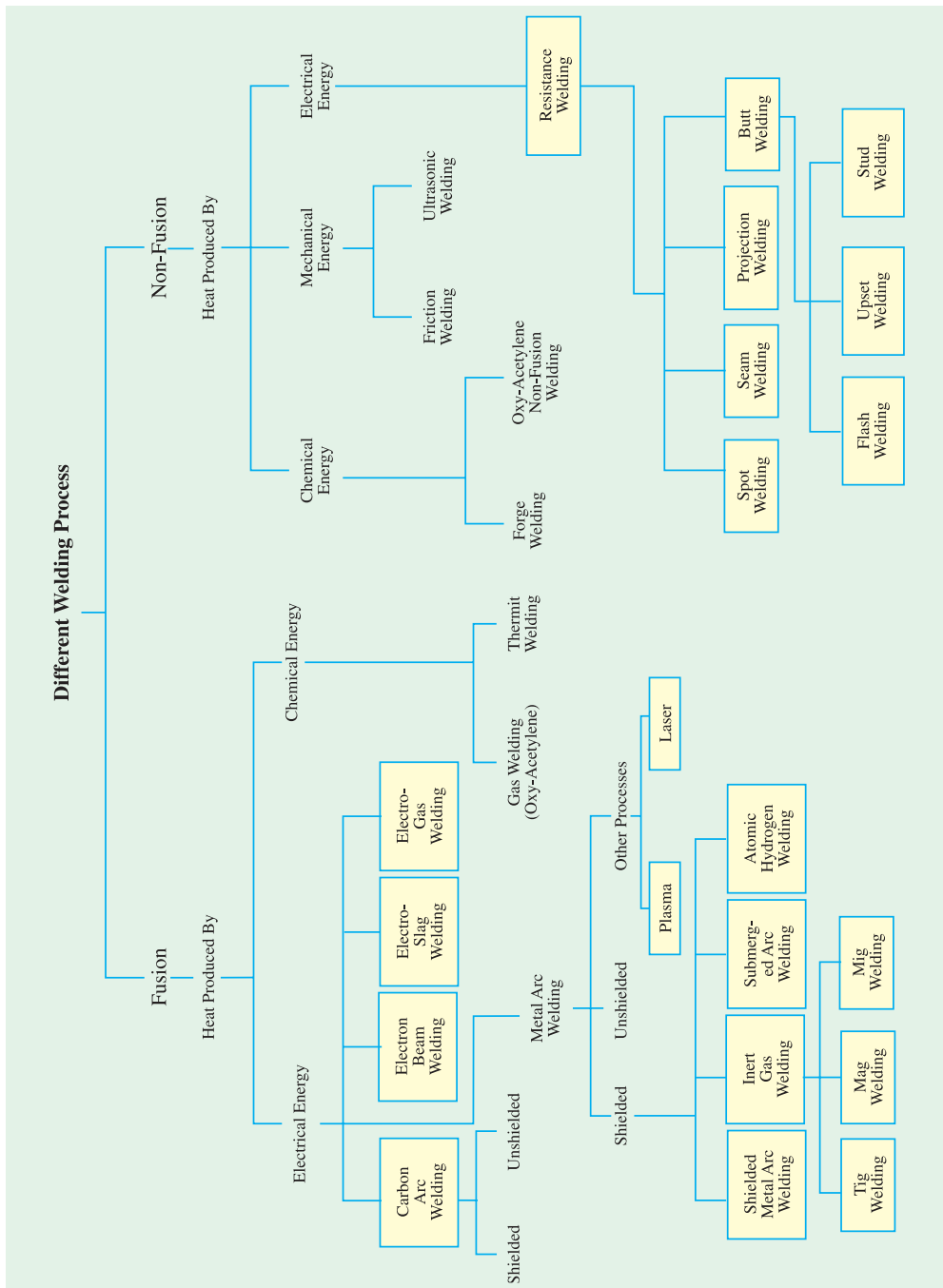


Fig. 48.1

If positive supply end is connected to the base metal (which is normally grounded), penetration will be greater due to more heat and, at the same time, the electrode will burn away slowly [Fig. 48.3 (a)] since it is connected to the negative end of the supply. If supply connections are reversed, the penetration of heat zone in the base metal will be comparatively shallow and, at the same time, electrode will burn fast [Fig. 48.3 (b)]. AC supply produces a penetration depth that is nearly halfway between that achieved by the d.c. positive ground and negative ground as shown in Fig. 48.3 (c).

It may be noted that with a.c. supply, heat is developed equally at the anode and cathode due to rapid reversal of their polarity. The arc utilized for arc welding is a low-voltage high-current discharge. The voltage required for striking the arc is higher than needed for maintaining it. Moreover, amperage increases as voltage decreases after the arc has been established. Fig 48.4 shows V/I characteristics of an electric arc for increasing air-gap lengths. The voltage required to strike a d.c. arc is about 50-55 V and that for a.c. arc is 80-90 V. The voltage drop across the arc is nearly 15-20 V. It is difficult to maintain the arc with a voltage less than 14 V or more than 40 V.

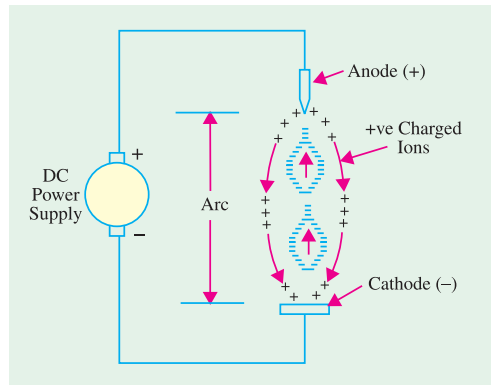


Fig. 48.2

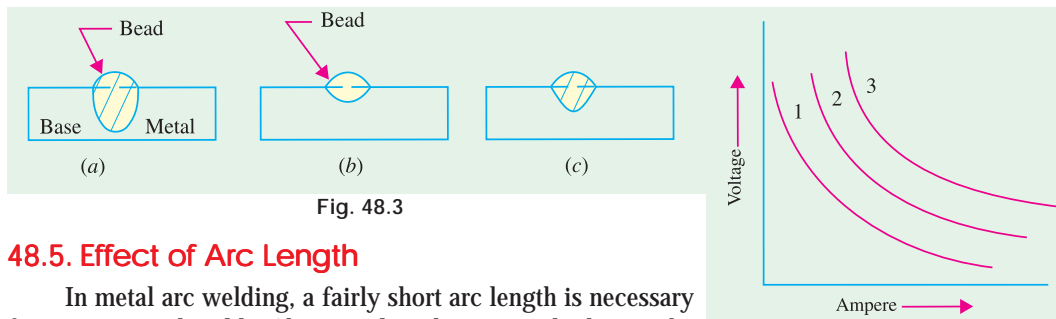


Fig. 48.3

Fig. 48.4

### 48.5. Effect of Arc Length

In metal arc welding, a fairly short arc length is necessary for getting good welds. Short arc length permits the heat to be concentrated on the workpiece, is more stable because effect of magnetic blow is reduced and the vapours from the arc surround the electrode metal and the molten pool thereby preventing air from destroying the weld metal. When arc length is long

1. large amount of heat is lost into the surrounding area thus preventing good penetration and fusion;
2. arc flame is very unstable since effect of magnetic blow is increased. Hence, arc flame will have a tendency to blow out;
3. air is able to reach the molten globule of metal as it passes from the electrode to the weld and weld pool. It leads to the contamination of the weld due to absorption of oxygen and nitrogen;
4. weld deposits have low strength, poor ductility, high porosity, poor fusion and excessive spatter.

The length of arc required for welding will depend on the kind of electrode used, its coating, its diameter, position of welding and the amount of current used. Usually, shorter arc length are necessary for vertical, horizontal and over-head welding than for flat welding.

### 48.6. Arc Blow

An arc column can be considered as a flexible current-carrying conductor which can be easily deflected by the magnetic field set up in its neighbourhood by the positive and negative leads from the d.c. welding set. The



An electric arc is produced when electricity is passed between two electrodes

two leads carry currents in the opposite directions and hence, set up a repulsive magnetic force which pulls the arc away from the weld point particularly when welding corners where field concentration is maximum. The deflection of the arc is called **arc blow**. This condition is encountered only with d.c. welding sets and is especially noticeable when welding with bare electrodes. It is experienced most when using currents above 200 A or below 40 A.

Due to arc blow, heat penetration in the required area is low which leads to incomplete fusion and bead porosity apart from excessive weld spatter.

Arc blow can be avoided by using a.c. rather than d.c. welding machines because reversing currents in the welding leads produce magnetic fields which cancel each other out thereby eliminating the arc blow. However, with d.c. welding machines, arc blow effects can be minimized by (i) welding away from the earth ground connection, (ii) changing the position of the earth connection on the work, (iii) wrapping the welding electrode cable a few turns around the work, (iv) reducing the welding current or electrode size, (v) reducing the rate of travel of the electrode and (vi) shortening the arc column length etc.

### 48.7. Polarity in DC Welding

Arc welding with the electrode connected to the positive end of the d.c. supply is called reverse polarity.\* Obviously, the workpiece is connected to the negative end.

A better name for d.c. reverse polarity (DCRP) is **electrode-positive** as shown in Fig. 48.5 (a). As stated earlier in Art. 48.4, two-third of the arc heat is developed at the anode. Hence, in DCRP welding, electrode is the hottest whereas workpiece is comparatively cooler. Consequently, electrode burns much faster but weld bead is relatively shallow and wide. That is why thick and heavily-coated electrodes are used in DCRP welding because they require more heat for melting.

Arc welding with the electrode connected to the negative end of the d.c. supply is called **straight polarity**\*\* Obviously, the workpiece is connected to the positive end as shown in Fig. 48.5 (b). A better name for d.c. straight polarity (DCSP) is **electrode-negative**.

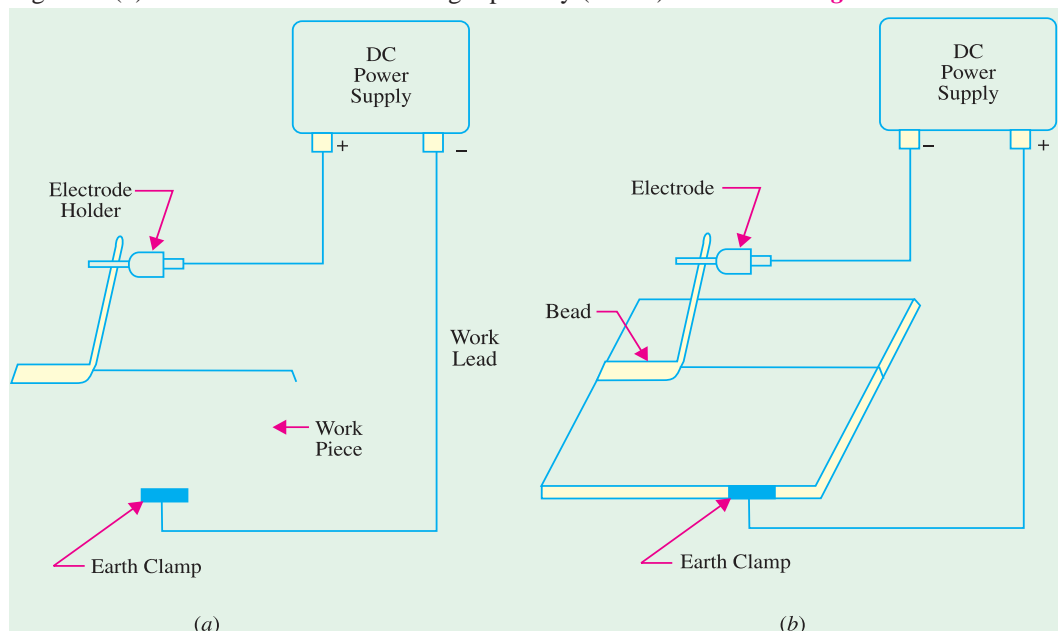


Fig. 48.5

\* In British literature, it is called straight polarity.

\*\* In British literature, it is called reverse polarity.

In DCSP welding, workpiece is the hottest, hence base metal penetration is narrow and deep. Moreover, bare and medium-coated electrodes can be used in this welding as they require less amount of heat for melting.

It is seen from the above discussion that polarity necessary for the welding operation is determined by the type of electrode used.

It is also worth noting that in a.c. welding, there is no choice of polarity because the circuit becomes alternately positive, first on one side and then on the other. In fact, it is a combination of D C S P and D C R P.

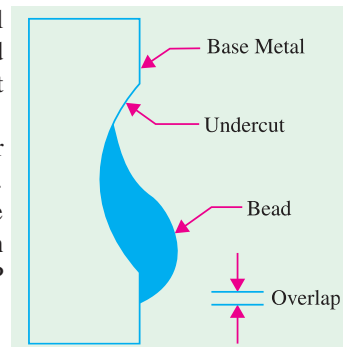


Fig. 48.6

### 48.8. Four Positions of Arc Welding

There are four basic positions in which manual arc welding is done.

**1. Flat position.** It is shown in Fig. 48.7 (a). Of all the positions, flat position is the easiest, most economical and the most used for all shielded arc welding. It provides the strongest weld joints. Weld beads are exceedingly smooth and free of slag spots. This position is most adaptable for welding of both ferrous and non-ferrous metals particularly for cast iron.

**2. Horizontal Position.** It is the second most popular position and is shown in Fig. 48.7 (b). It also requires a short arc length because it helps in preventing the molten puddle of the metal from sagging. However, major errors that occur while welding in horizontal position are under-cutting and over-lapping of the weld zone (Fig. 48.6).

**3. Vertical Position.** It is shown in Fig. 48.7 (c). In this case, the welder can deposit the bead either in the uphill or downhill direction. Downhill welding is preferred for thin metals because it is faster than the uphill welding. Uphill welding is suited for thick metals because it produces stronger welds.

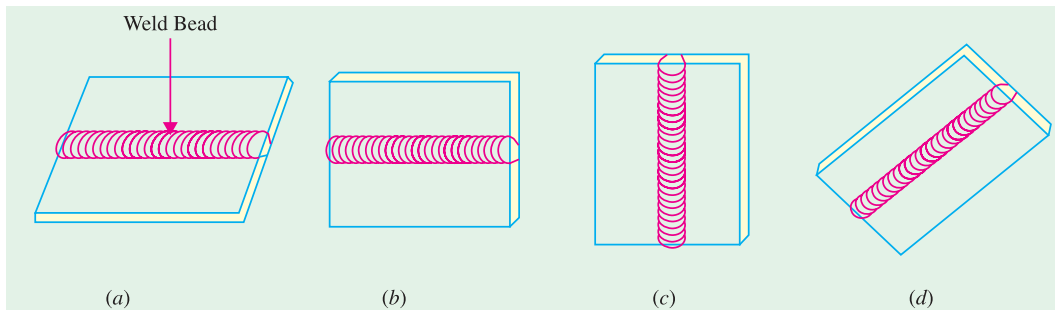


Fig. 48.7.

**4. Overhead Position.** It is shown in Fig. 48.7 (d). Here, the welder has to be very cautious otherwise he may get burnt by drops of falling metal. This position is thought to be the most hazardous but not the most difficult one.

### 48.9. Electrodes for Metal Arc Welding

An electrode is a filler metal in the form of a wire or rod which is either bare or coated uniformly with flux. As per IS : 814-1970, the contact end of the electrode is left bare and clean to a length of 20-30 mm. for inserting it into electrode holder (Fig. 48.8).

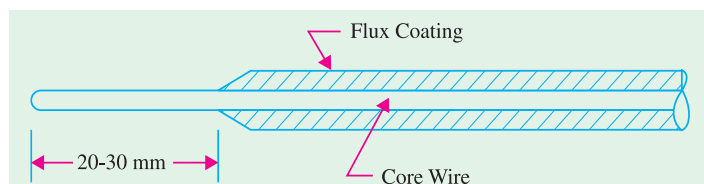


Fig. 48.8

Metal arc welding was originally done with bare electrodes which consisted of a piece of wire or rod of the same metal as the base metal. However, due to atmospheric contamination, they produced brittle and poor quality welds.

Hence, bare wire is no longer used except for automatic welding in which case arrangement is made to protect the weld area from the atmosphere by either powdered flux or an inert gas. Since 1929, coated electrodes are being extensively used for shielded arc welding. They consist of a metal core wire surrounded by a thick flux coating applied by

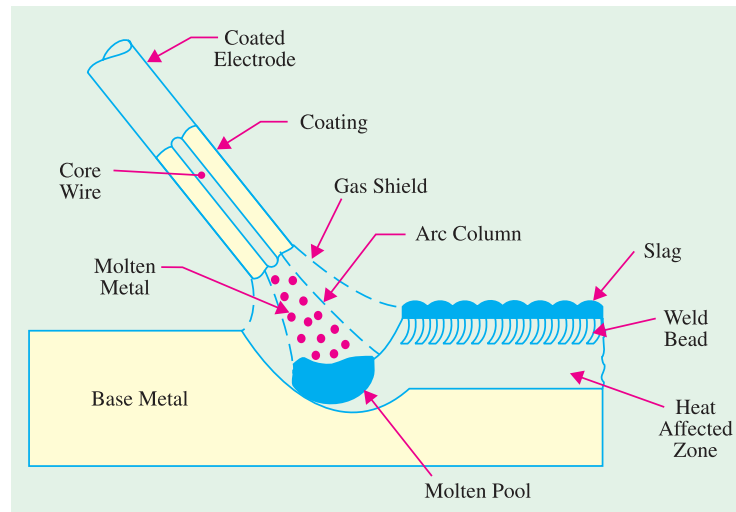


Fig. 48.9

extrusion, winding or other processes. Depending on the thickness of the flux coating, coated electrodes may be classified into (i) lightly-dusted (or dipped) electrodes and (ii) semi-coated (or heavy-coated) electrodes. Materials commonly used for coating are (i) titanium oxide (ii) ferromanganese (iii) silica flour (iv) asbestos clay (v) calcium carbonate and (vi) cellulose with sodium silicate often used to hold ingredients together.

Electrode coating contributes a lot towards improving the quality of the weld. Part of the coating burns in the intense heat of the arc and provides a gaseous shield around the arc which prevents oxygen, nitrogen and other impurities in the atmosphere from combining with the molten metal to cause a poor quality brittle and weak weld. Another portion of the coating flux melts and mixes with the impurities in the molten pool causing them to float to the top of the weld where they cool in the form of slag (Fig. 48.9). This slag improves the bead quality by protecting it from the contaminating effects of the atmosphere and causing it to cool down more uniformly. It also helps in controlling the basic shape of the weld bead.

The type of electrode used depends on the type of metal to be welded, the welding position, the type of electric supply whether a.c. or d.c. and the polarity of the welding machine.

#### 48.10. Advantages of Coated Electrodes

The principal advantages of using electrode coating are as under :

1. It stabilizes the arc because it contains ionizing agents such as compounds of sodium and potassium.
2. It fluxes away impurities present on the surface being welded.
3. It forms slag over the weld which (i) protects it from atmospheric contamination (ii) makes it cool uniformly thereby reducing the changes of brittleness and (iii) provides a smoother surface by reducing 'ripples' caused by the welding operation.
4. It adds certain materials to the weld metal to compensate for the loss of any volatile alloying elements or constituents lost by oxidation.
5. It speeds up the welding operation by increasing the rate of melting.
6. It prevents the sputtering of metal during welding.

- 7. it makes it possible for the electrode to be used on a.c. supply. In a.c. welding, arc tends to cool and interrupt at zero-current positions. But the shielding gases produced by the flux keep the arc space ionized thus enabling the coated electrodes to be used on a.c. supply.

It is worth noting that efficiency of all coated (or covered) electrodes is impaired by dampness. Hence, they must always be stored in a dry space. If dampness is suspected, the electrodes should be dried in a warm cabinet for a few hours.

### 48.11. Types of Joints and Types of Applicable Welds

Bureau of Indian Standards (B.I.S.) has recommended the following types of joints and the welds applicable to each one of them (Fig. 48.10).

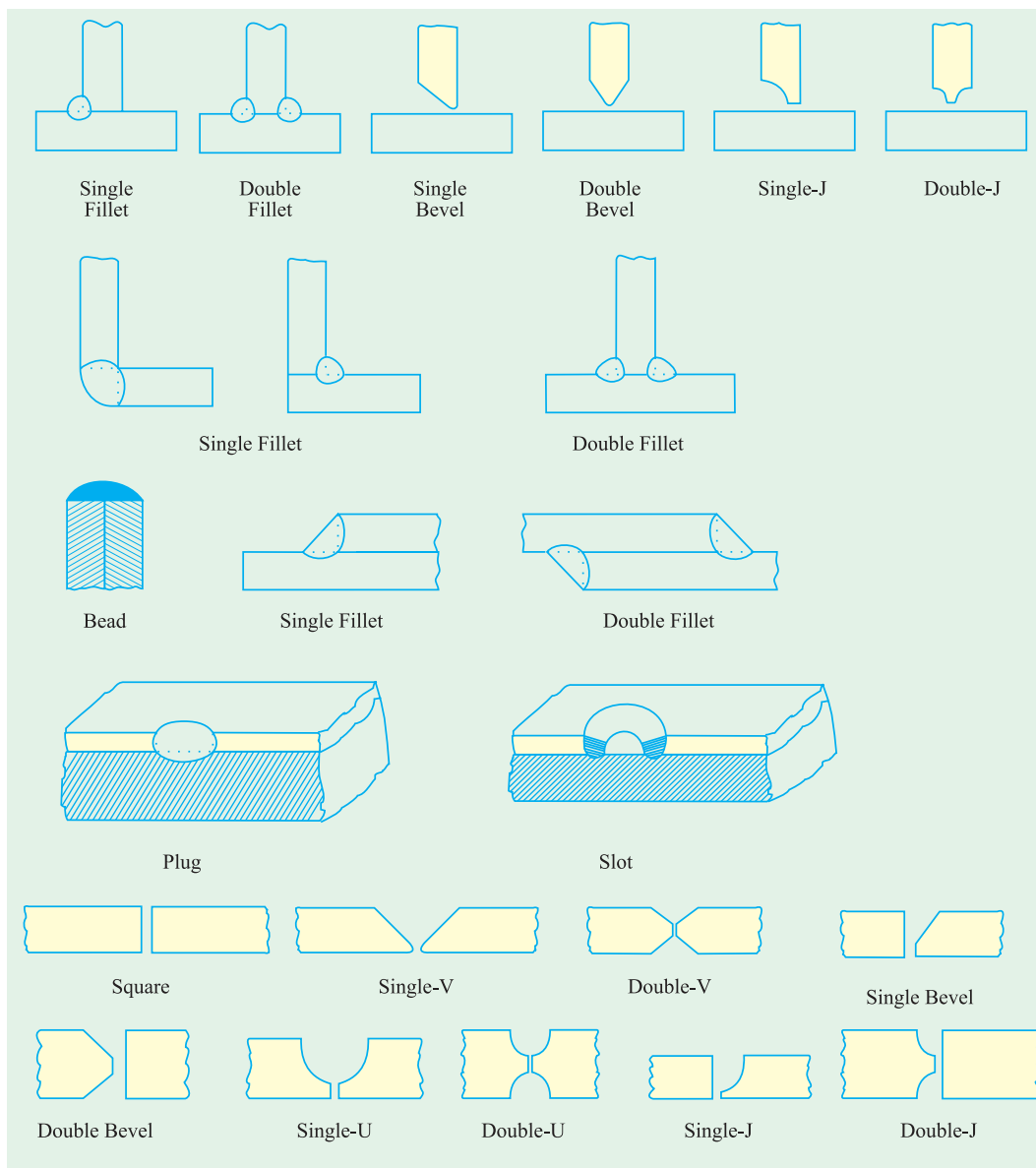


Fig. 48.10



- |                 |   |                           |
|-----------------|---|---------------------------|
| 1. Tee joint    | — | with six types of welds.  |
| 2. Corner joint | — | with two types of welds.  |
| 3. Edge joint   | — | with one type of weld.    |
| 4. Lap joint    | — | with four types of welds. |
| 5. Butt joint   | — | with nine types of welds. |

### 48.12. Arc Welding Machines

Welding is never done directly from the supply mains. Instead, special welding machines are used which provided currents of various characteristics. Use of such machines is essential for the following reasons :

1. To convert a.c. supply into d.c. supply when d.c. welding is desired.
2. To reduce the high supply voltage to a safer and suitable voltage for welding purposes.
3. To provide high current necessary for arc welding without drawing a corresponding high current from the supply mains.
4. To provide suitable voltage/current relationships necessary for arc welding at minimum cost.

There are two general types of arc welding machines :

**(a) d.c. welding machines**

- (i) motor-generator set
- (ii) a.c. transformers with rectifiers

**(b) a.c. welding machines**

### 48.13. V-I Characteristics of Arc Welding DC Machines

It is found that during welding operation, large fluctuations in current and arc voltage result from the mechanism of metal transfer and other factors. The welding machine must compensate for such changes in arc voltage in order to maintain an even arc column. There are three major voltage/current characteristics used in modern d.c. welding machines which help in controlling these current fluctuations :

1. drooping arc voltage (DAV).
2. constant arc voltage (CAV).
3. rising arc voltage (RAV).

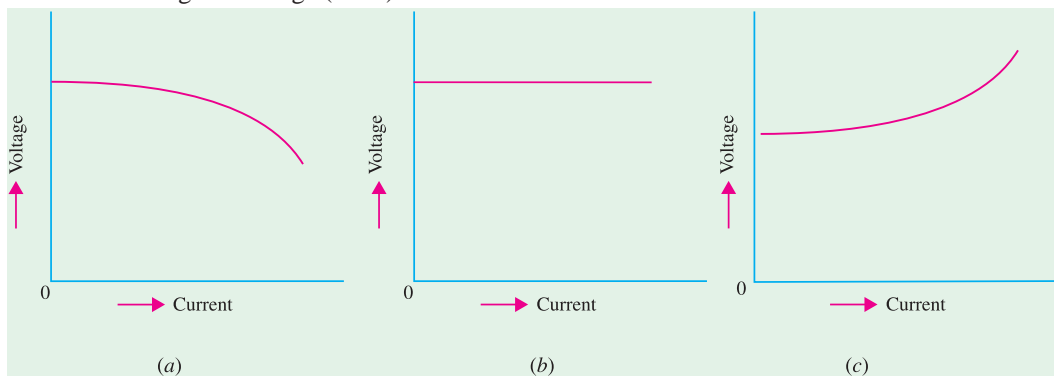


Fig. 48.11

The machines with DAV characteristics have high open-circuit voltage which drops to a minimum when arc column is started. The value of current rises rapidly as shown in Fig. 48.11 (a). This type of characteristic is preferred for manual shield metal arc welding.

The CAV characteristic shown in Fig. 48.11 (b) is suitable for semi-automatic or automatic welding processes because voltage remains constant irrespective of the amount of current drawn.

Because of its rising voltage characteristic, RAV has an advantage over CAV because it maintains a constant arc gap even if short circuit occurs due to metal transfer by the arc. Moreover, it is well-adapted to fully automatic process.

DC welding machines can be controlled by a simple rheostat in the exciter circuit or by a combination of exciter regulator and series of field taps. Some arc welding are equipped with remote-controlled current units enabling the operator to vary voltage-ampere requirement without leaving the machines.



DC and AC welding machines

#### 48.14. DC Welding Machines with Motor Generator Set

Such a welding plant is a self-contained single-operator motor-generator set consisting of a reverse series winding d.c. generator driven by either a d.c. or an a.c. motor (usually 3-phase). The series winding produces a magnetic field which opposes that of the shunt winding. On open-circuit, only shunt field is operative and provides maximum voltage for striking the arc. After the arc has been established, current flows through the series winding and sets up a flux which opposes the flux produced by shunt winding. Due to decreases in the net flux, generator voltage is decreased (Art. 48.13).

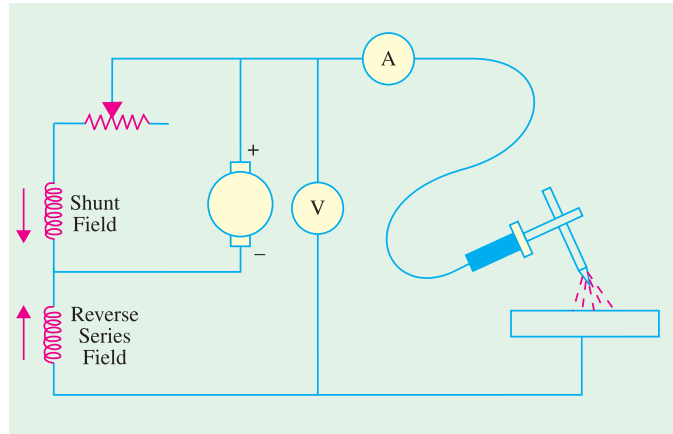


Fig. 48.12

With the help of shunt regulator, generator voltage and current values can be adjusted to the desired level. Matters are so arranged that despite changes in arc voltage due to variations in arc length, current remains practically constant. Fig. 48.12 shows the circuit of a d.c. motor-generator type of welding machine.

**Advantages.** Such a d.c. welder has the following advantages :

1. It permits portable operation.
2. It can be used with either straight or reverse polarity.
3. It can be employed on nearly all ferrous and non-ferrous materials.
4. It can use a large variety of stick electrodes.
5. It can be used for all positions of welding.

**Disadvantages**

1. It has high initial cost.
2. Its maintenance cost is higher.
3. Machine is quite noisy in operation.
4. It suffers from arc blow.

**48.15. AC Rectified Welding Unit**

It consists of a transformer (single-or three-phase) and a rectifier unit as shown in Fig. 48.13. Such a unit has no moving parts, hence it has long life. The only moving part is the fan for cooling the transformer. But this fan is not the basic part of the electrical system. Fig. 48.13 shows a single-phase full-wave rectified circuit of the welder. Silicon diodes are used for converting a.c. into d.c. These diodes are hermetically sealed and are almost ageless because they maintain rectifying characteristics indefinitely.

Such a transformer-rectifier welder is most adaptable for shield arc welding because it provides both

d.c. and a.c. polarities. It is very efficient and quiet in operation. These welders are particularly suitable for the welding of (i) pipes in all positions (ii) non-ferrous metals (iii) low-alloy and corrosion-heat and creep-resisting steel (iv) mild steels in thin gauges.

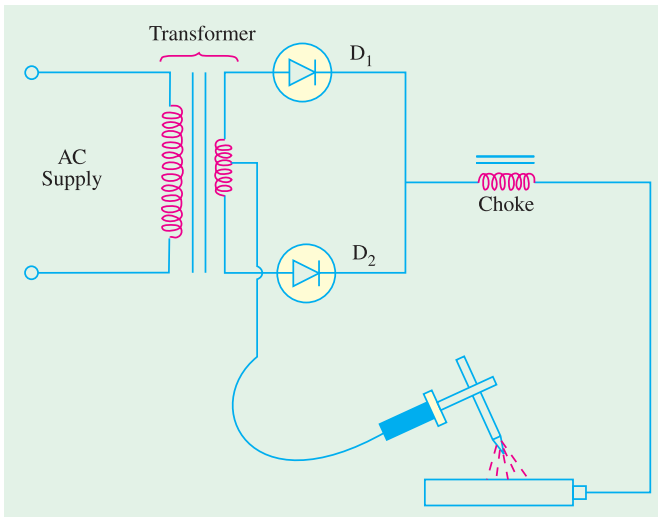


Fig. 48.13

**48.16. AC Welding Machines**

As shown in Fig. 48.14, it consists of a step-down transformer with a tapped secondary having an adjustable reactor in series with it for obtaining drooping V/I characteristics. The secondary is tapped to give different voltage/current settings.

**Advantages.** This a.c. welder which can be operated from either a single-phase or 3-phase supply has the following advantages :

- |                      |   |
|----------------------|---|
| (i) Low initial cost | (ii) Low operation and maintenance cost |
| (iii) Low wear       | (iv) No arc blow                        |

**Disadvantages.** (i) its polarity cannot be changed (ii) it is not suitable for welding of cast iron and non-ferrous metals.

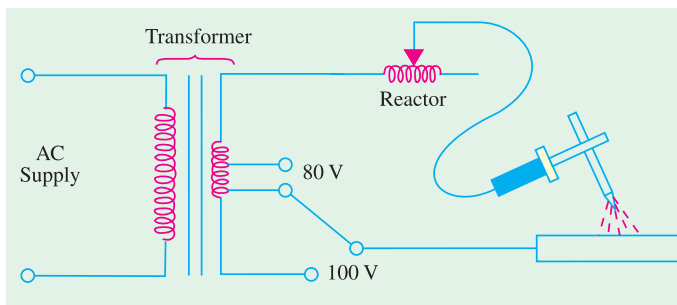


Fig. 48.14

**48.17. Duty Cycle of a Welder**

The duty cycle of an arc welder is based on a working period of 10 minutes. For example, if a welder is operated for 2 minutes in a period of 10 minutes, then its percentage duty cycle is  $(2/10) \times 100 = 20$  percent. Conversely, a 10 percent duty cycle would mean that the welder would

be operated for 10 percent of 10 minutes *i.e.* for one minute only in a period of 10 minutes.

Usually, values of maximum amperage and voltage are indicated along with the duty cycle. It is advisable to adhere to these values. Suppose a welding machine has maximum amperage of 300A and voltage of 50 V for a duty cycle of 60 percent. If this machine is operated at higher settings and for periods longer than 6 minutes, then its internal insulation will deteriorate and cause its early failure.

### 48.18. Carbon Arc Welding

#### (a) General

Carbon arc welding was the first electric welding process developed by a French inventor Auguste de Meritens in 1881. In this process, fusion of metal is accomplished by the heat of an electric arc. No pressure is used and generally, no shielding atmosphere is utilized. Filler rod is used only when necessary. Although not used extensively these days, it has, nevertheless, certain useful fields of application.

Carbon arc welding differs from the more common shield metal arc welding in that *it uses non-consumable carbon or graphic electrodes* instead of the consumable flux-coated electrodes.

#### (b) Welding Circuit

The basic circuit is shown in Fig. 48.15 and can be used with d.c. as well as a.c. supply. When direct current is used, the electrode is mostly negative (DCSP). The process is started by adjusting the amperage on the d.c. welder, turning welder ON and bringing the electrode into contact with the workpiece. After the arc column starts, electrode is withdrawn 25 – 40 mm away and the arc is maintained at this distance. The arc can be extinguished by simply removing the electrode from the workpiece completely. The only function of the carbon arc is to supply heat to the base metal. This heat is used to melt the base metal or filler rod for obtaining fusion weld. Depending on the type and size of electrodes, maximum current values range from 15 A to 600 A for single-electrode carbon arc welding.

#### (c) Electrodes

These are made of either carbon or graphite, are usually 300 mm long and 2.5 – 12 mm in diameter. Graphite electrodes are harder, more brittle and last longer than carbon electrodes. They can withstand higher current densities but their arc column is harder to control. Though considered non-consumable, they do disintegrate gradually due to vaporisation and oxidation.

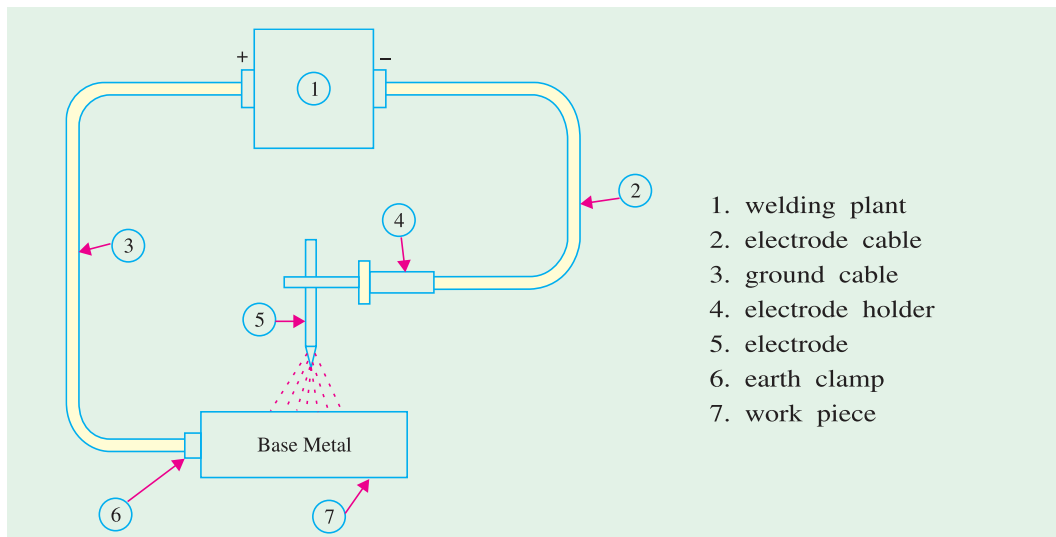


Fig. 48.15

**(d) Applications**

1. The joint designs that can be used with carbon arc welding are butt joints, bevel joints, flange joints, lap joints and fillet joints.
2. This process is easily adaptable for automation particularly where amount of weld deposit is large and materials to be fabricated are of simple geometrical shapes such as water tanks.
3. It is suitable for welding galvanised sheets using copper-silicon-manganese alloy filler metal.
4. It is useful for welding thin high-nickel alloys.
5. Monel metal can be easily welded with this process by using a suitable coated filler rod.
6. Stainless steel of thinner gauges is often welded by the carbon-arc process with excellent results.

**(e) Advantages and Disadvantages**

1. The main advantage of this process is that the temperature of the molten pool can be easily controlled by simply varying the arc length.
2. It is easily adaptable to automation.
3. It can be easily adapted to inert gas shielding of the weld and
4. It can be used as an excellent heat source for brazing, braze welding and soldering etc.

Its disadvantages are as under :

1. A separate filler rod has to be used if any filler material is required.
2. Since arc serves only as a heat source, it does not transfer any metal to help reinforce the weld joint.
3. The major disadvantage of the carbon-arc process is that blow holes occur due to magnetic arc blow especially when welding near edges of the workpiece.

**48.19. Submerged Arc Welding**

In this *fusion* process, welding is done under a blanket of granulated flux which shields the weld from all bad effects of atmospheric gases while a consumable electrode is continuously and mechanically fed into the arc. The arc, the end of the bare metal electrode and the molten weld pool are all submerged under a thick mound of finely-divided granulated powder that contains deoxidisers, cleansers and other fluxing agents. The fluxing powder is fed from a hopper that is carried on the welding head itself (Fig. 48.16). This hopper spread the powder in a continuous mound ahead of the

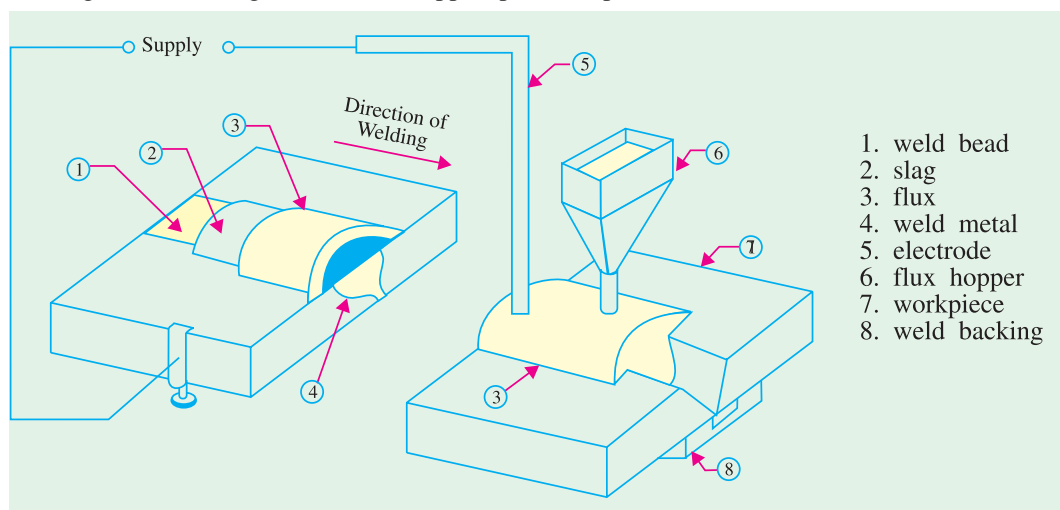


Fig. 48.16

electrode in the direction of welding. Since arc column is completely submerged under the powder, there is no splatter or smoke and, at the same time, weld is completely protected from atmospheric contamination. Because of this protection, weld beads are extremely smooth. The flux adjacent to the arc column melts and floats to the top of the molten pool where it solidifies to form slag. This slag is easy to remove. Often it cracks off by itself as it cools. The unused flux is removed and is reused again and again.

The electrode is either a bare wire or has a slight mist of copper coated over it to prevent oxidation. In automatic or semi-automatic submerged arc welding, wire electrode is fed mechanically through an electrically contacting collet. Though a.c. power supply may be used, yet d.c. supply is more popular because it assures a simplified and positive control of the welding process. This process requires high current densities about 5 to 6 times of those used in ordinary manual stick electrode welding. As a result, melting rate of the electrode as well as welding speed become much higher. Faster welding speed minimizes distortion and warpage.

The submerged arc process is suitable for

1. Welding low-alloy, high-tensile steels.
2. Welding mild, low-carbon steels.
3. Joining medium-carbon steel, heat-resistant steels and corrosion-resistant steels etc.
4. Welding nickel, Monel and other non-ferrous metals like copper.

This process has many industrial applications such as fabrication of pipes, boiler pressure vessels, railroad tank cars, structural shapes etc. which demand welding in a straight line. Welds made by this process have high strength and ductility. A major advantage of this process is that fairly thick sections can be welded in a single pass without edge preparation.

Submerged arc welding can be done manually where automatic process is not possible such as on curved lines and irregular joints. Such a welding gun is shown in Fig. 48.17. Both manual and automatic submerged arc processes are most suited for flat and slightly downhill welding positions.

#### 48.20. Twin Submerged Arc Welding

As shown in Fig. 48.18, in this case, two electrodes are used simultaneously instead of one. Hence, weld deposit size is increased considerably. Moreover, due to increase in welding current (upto 1500 A), much deeper penetration of base metal is achieved.

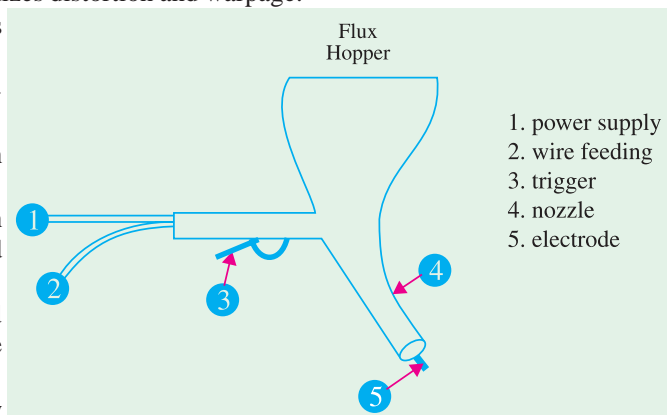


Fig. 48.17

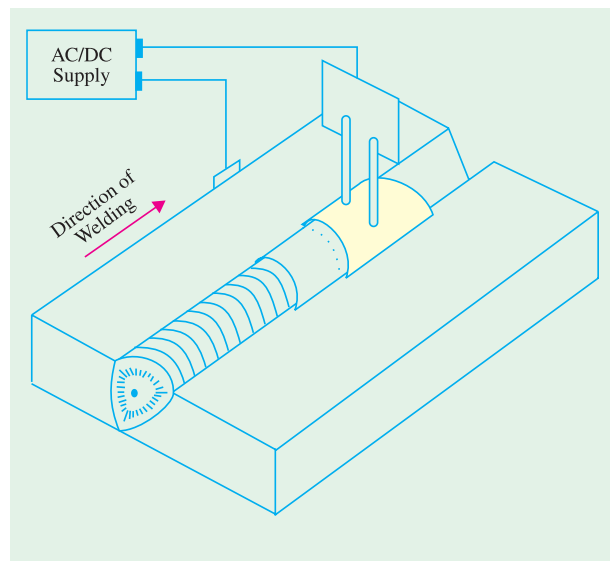


Fig. 48.18

### 48.21. Gas Shield Arc Welding

In this fusion process, welding is done with bare electrodes but weld zone is shielded from the atmosphere by a gas which is piped to the arc column. Shielding gases used are carbon dioxide, argon, helium, hydrogen and oxygen. No flux is required. Different processes using shielding gas are as follows.

#### (a) Tungsten inert-gas (TIG) Process

In this process, non-consumable tungsten electrode is used and filler wire is fed separately. The weld zone is shielded from the atmosphere by the inert gas (argon or helium) which is ducted directly to the weld zone where it surrounds the tungsten and the arc column.

#### (b) Metal inert-gas (MIG) Process

It is a refinement of the TIG process. It uses a bare consumable (*i.e.* fusible) wire electrode which acts as the source for the arc column as well as the supply for the filler material. The weld zone is shielded by argon gas which is ducted directly to the electrode point.

### 48.22. TIG Welding

#### (a) Basic Principle

It is an electric process which uses a bare non-consumable tungsten electrode for striking the arc only (Fig. 48.19). Filler material is added separately. It uses an inert gas to shield the weld puddle from atmospheric contamination. This gas is ducted directly to the weld zone from a gas cylinder.

#### (b) Welding Equipment

The usual TIG welding system consists of the following (Fig. 48.20).

1. A standard shield arc welding machine complete with cables etc.
2. A supply of inert gas complete with hose, regulators etc.
3. A source of water supply (in the case of water-cooled torches).
4. A TIG torch with a control switch to which all the above are connected.

#### (c) Electrodes

The electrodes are made of either pure tungsten or zirconiated or thoriated tungsten. Addition of zirconium or thorium (0.001 to 2%) improves electron emission tremendously.

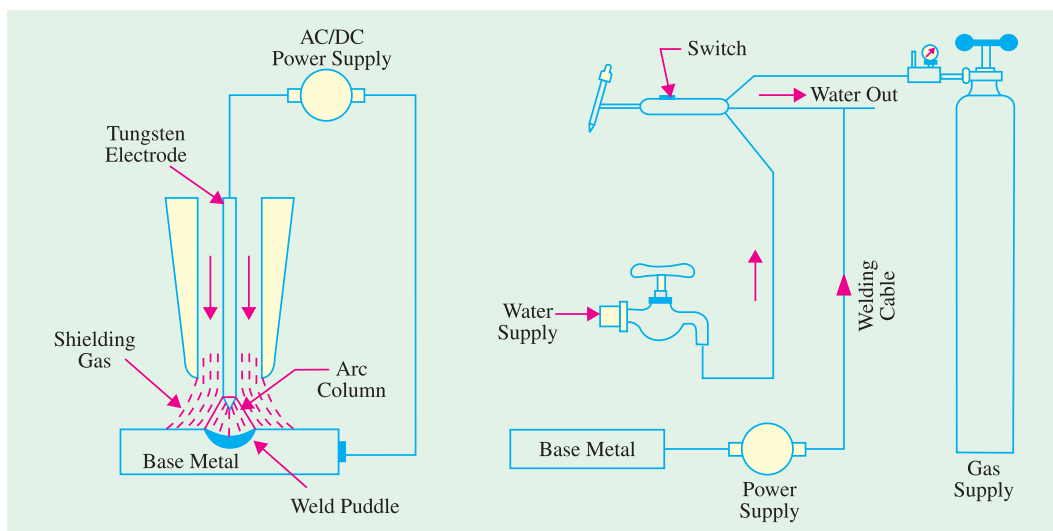


Fig. 48.19

Fig. 48.20

**(d) Power Supply**

The three basic power supplies used in TIG operation are :

1. DCSP power supply—here electrode is negative, runs cooler and, hence, can be thin.
2. DCRP power supply—here electrode is positive and hot. Hence, it has to be large.
3. A.C. high frequency (ACHF) power supply—it is a combination of standard a.c. supply of 50 Hz and high-voltage high-frequency d.c. supply. The function of this d.c. supply is to sustain the arc when a.c. supply is at zero current positions.

**(e) Advantages of TIG Welding**

1. It provides maximum protection to weld bead from atmospheric contamination.
2. TIG welds are stronger, more ductile and more corrosion-resistant than those of shield metal arc welding.
3. Since no flux is used, there is no flux entrapment in the bead.
4. Since no flux is required, a wider variety of joint designs can be used.
5. No post-weld cleansing is necessary.
6. There is no weld splatter or sparks that could damage the surface of the base metal.
7. It gives relatively fast welding speeds.
8. It is suitable for welding food or medical containers where entrapment of any decaying organic matter could be extremely harmful.
9. It is suitable for all welding positions—the flat, horizontal, vertical and overhead positions. The joints suitable for TIG welding process are (i) butt joint (ii) lap joint (iii) T-joint, (iv) corner joint and (v) edge joint.

**(f) Applications**

- |   |   |         |
|---|---|---------|
| 1. aluminium and its alloys   | — | AC/DCRP |
| 2. magnesium and its alloys   | — | ACHF    |
| 3. stainless steel  | — | DCSP    |
| 4. mild steel, low-alloy steel, medium<br>-carbon steel and cast iron | — | DCSP    |
| 5. copper and alloys  | — | DCSP    |
| 6. nickel and alloys  | — | DCSP    |

TIG welding is also used for dissimilar metals, hardfacing and surfacing of metals. Special industrial applications include manufacture of metal furniture and air-conditioning equipment.

Fig. 48.21 shows Phillips 400-D compact fan-cooled DC TIG welding set which has an open-circuit voltage of 80 V and a welding current of 400 A with 60% duty cycle and 310 A with 100% duty cycle.

**48.23. MIG Welding****(a) Basic Principle**

It is also called inert-gas consumable- electrode process. The fusible wire electrode is driven by the drive wheels. Its function is two-fold: to produce arc column and to provide filler material. This process uses inert gas for shielding the weld zone from atmospheric contamination. Argon is used to weld non-ferrous metals though helium gives better control of porosity and arc stability. This



Fig. 48.21 (TIG welding set)



process can deposit large quantities of weld metal at a fast welding speed. The process is easily adaptable to semi-automatic or fully automatic operations.

**(b) Welding Equipment**

The basic MIG welding system (Fig. 48.23) consists of the following :

1. Welding power supply
2. Inert gas supply with a regulator and flow meter
3. Wire feed unit containing controls for wire feed, gas flow and the ON/OFF switch for MIG torch
4. MIG torch
5. Depending on amperage, a water cooling unit.

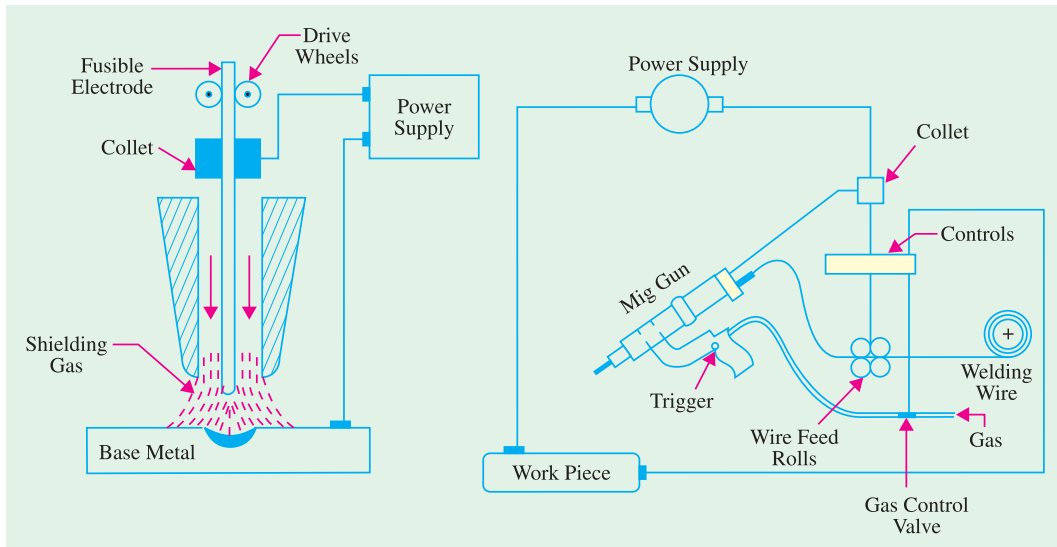


Fig. 48.22

Fig. 48.23

**(c) Electrode**

It is a bare wire fed to the MIG gun by a suitable wire-feed mechanism.

**(d) Power Supply**

The major power supply used for MIG welding is DCRP and the machines which provide this supply are motor-generator sets or a.c. transformers with rectifiers (Art. 48.14). They have either CAV or RAV characteristics (Art. 48.12). The CAV supply gives the operator great latitude in arc length and is helpful in preventing the wire electrode from stubbing. A DCRP current produces deeper penetration and a cleaner weld surface than other types of current.

The RAV machines are more suitable for automatic operation. They are capable of handling large diameter wires than CAV machines.

Fig. 48.24 shows semi-automatic forced-air cooled arc welding set MIG-400. It consists of

- (i) Indarc 400 MMR rectifier which is basically a 3-phase transformer rectifier with silicon



Fig. 48.24.

MIG-400 Welding Set. (Courtesy : Indian Oxygen Ltd. Calcutta)

diodes and a constant potential output. It provides maximum current of 400 A at 40 V for 75% duty cycle and 350 A at 42 V for 100% duty cycle.

- (ii) Indarc Wire Feeder which has a twin roll drive system, designed to feed 0.8 to 2.4 mm diameter welding wires to a hand-operated MIG welding torch.
- (iii) MIG Torches which are available in both air-cooled and water-cooled varieties. Fig. 48.25 (a) and (b) show light-weight swan-necked torches which are designed to operate upto 360 A and 400 A with CO<sub>2</sub> as shielding gas. Fig. 48.25 (c) shows a heavy-duty water-cooled torch designed to operate upto 550 A with CO<sub>2</sub>/mixed shielding gases at 100% duty cycle.
- (iv) CO<sub>2</sub> Kit for hard wire applications and Argon Kit for soft wire applications.

**(e) Advantages of MIG Welding**

1. Gives high metal deposit rates varying from 2 to 8 kg/h.
2. Requires no flux.
3. Requires no post-welding cleaning.
4. Gives complete protection to weld bead from atmospheric contamination.
5. Is adaptable for manual and automatic operations.
6. Can be used for a wide range of metals both ferrous and non-ferrous.
7. Is easy to operate requiring comparatively much less operating skill.
8. Is especially suited for horizontal, vertical and overhead welding positions.

**(f) Applications**

With inert gas shielding, this process is suitable for fusion welding of (i) aluminium and its alloys (ii) nickel and its alloys (iii) copper alloys (iv) carbon steels (v) low-alloy steels (vi) high strength steels and (vii) titanium.

### 48.24. MAG Welding

As discussed earlier, in MIG welding process, the shielding gas used is monoatomic (argon or helium) and is inert *i.e.* chemically inactive and metal transfer takes place by axial pulverization. In MAG (metal-active-gas) process, shielding gas used is chemically active *i.e.* carbon dioxide or its mixture with other gases. Transfer of metal takes place in big drops.

### 48.25. Atomic Hydrogen Welding

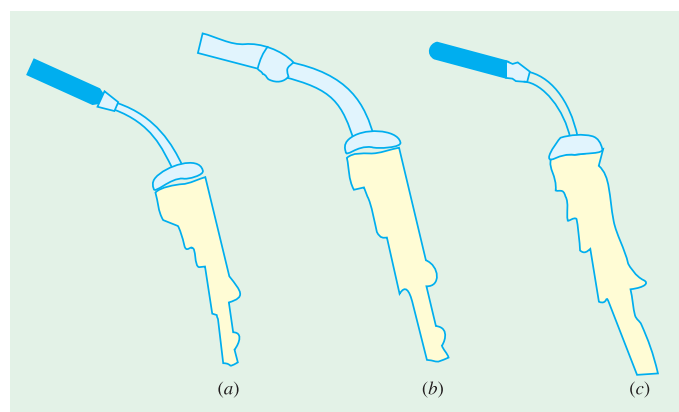
**(a) General**

It is a non-pressure fusion welding process and the welder set is used only as heat supply for the base metal. If additional metal is required, a filler rod can be melted into the joint. It uses two tungsten electrodes between which an arc column (actually, an arc fan) is maintained by an a.c. supply.

**(b) Basic Principle**

As shown in Fig. 48.26, an arc column is struck between two tungsten electrodes with an a.c.

power supply. Soon after, normal molecular hydrogen (H<sub>2</sub>) is forced through this arc column. Due to intense heat of the arc column, this diatomic hydrogen is dissociated into atomic hydrogen (H).



**Fig. 48.25**

However, atomic hydrogen being unstable, recombines to form stable molecular hydrogen. In so doing, it releases intense heat at about  $3750^{\circ}\text{C}$  which is used to fuse the metals.

### (c) Welding Equipment

The welding equipment essentially consists of the following :

1. Standard welding machine consisting of a step-down transformer with tapped secondary (not shown in Fig. 48.27) energised from normal a.c. supply. Amperage requirement ranges from 15 A to 150 A
2. Hydrogen gas supply with an appropriate regulator
3. Atomic hydrogen welding torch having an ON-OFF switch and a trigger for moving the two tungsten electrodes close together for striking and maintaining the arc column.

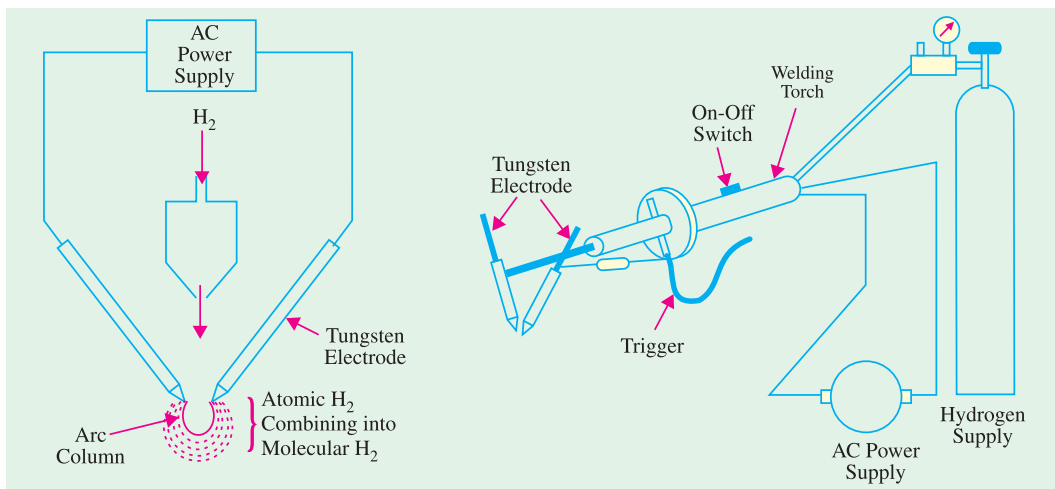


Fig. 48.26

Fig. 48.27

### (d) Method of Welding

The torch is held in the right hand with first finger resting lightly on the trigger. The arc is struck either by allowing the two tungsten electrodes to touch and separate or by drawing the separated electrodes over a carbon block. At the same time, a stream of hydrogen is allowed to flow through the arc. As soon as the arc strikes, an intensely hot flame extends fanwise between the electrodes. When this fan touches the workpiece, it melts it down quickly. If filler material is required, it can be added from the rod held in the left hand as in gas welding.

### (e) Advantages

1. Arc and weld zone are shrouded by burning hydrogen which, being an active reducing agent, protects them from atmospheric contamination.
2. Can be used for materials too thin for gas welding.
3. Can weld quite thick sections.
4. Gives strong, ductile and sound welds.
5. Can be used for welding of mild steel, alloy steels and stainless steels and aluminium alloys.
6. Can also be used for welding of most non-ferrous metals such as nickel, monel, brass, bronze, tungsten and molybdenum etc.

## 48.26. Resistance Welding

It is fundamentally a heat and squeeze process. The term '*resistance welding*' denotes a group

of processes in which welding heat is produced by the resistance offered to the passage of electric current through the two metal pieces being welded. These processes differ from the fusion processes in the sense that no extra metal is added to the joint by means of a filler wire or electrode. According to Joule's law, heat produced electrically is given by  $H = I^2Rt/J$ . Obviously, amount of heat produced depends on.

(i) square of the current (ii) the time of current and (iii) the resistance offered.

As seen, in simple resistance welding, high-amperage current is necessary for adequate weld. Usually, R is the contact resistance between the two metals being welded together. The current is passed for a suitable length of time controlled by a timer. The various types of resistance welding processes may be divided into the following four main groups :

(i) spot welding (ii) seam welding (iii) projection welding and (iv) butt welding which could be further subdivided into flash welding, upset welding and stud welding etc.

#### Advantages

Some of the advantages of resistance welding are as under :

1. Heat is localized where required
2. Welding action is rapid
3. No filler material is needed
4. Requires comparatively lesser skill
5. Is suitable for large quantity production
6. Both similar and dissimilar metals can be welded
7. Parent metal is not harmed
8. Difficult shapes and sections can be welded.

Only disadvantages are with regard to high initial as well as maintenance cost.

It is a form of resistance welding in which the two surfaces are joined by spots of fused metal caused by fused metal between suitable electrodes under pressure.

### 48.27. Spot Welding

The process depends on two factors :

1. Resistance heating of small portions of the two workpieces to plastic state and

2. Application of forging pressure for welding the two workpieces.

Heat produced is  $H = I^2Rt/J$ . The resistance R is made up of (i) resistance of the electrodes and metals themselves (ii) contact resistance between electrodes and workpieces and (iii) contact resistance between the two workpieces. Generally, contact resistance between the two workpieces is the greatest.

As shown in 48.28 (b), mechanical pressure is applied by the tips of the two electrodes. In fact, these electrodes not only provide the forging pressure but also carry the welding current and concentrate the welding heat on the weld spot directly below them.



Spot welding machine

Fig. 48.28 (a) shows diagrammatically the basic parts of a modern spot welding. It consists of a step-down transformer which can supply huge currents (upto 5,000 A) for short duration of time.

The lower arm is fixed whereas the upper one is movable. The electrodes are made of low-resistance, hard-copper alloy and are either air cooled or butt-cooled by water circulating through the rifled drillings in the electrode. Pointed electrodes [Fig. 48.29 (a)] are used for ferrous materials

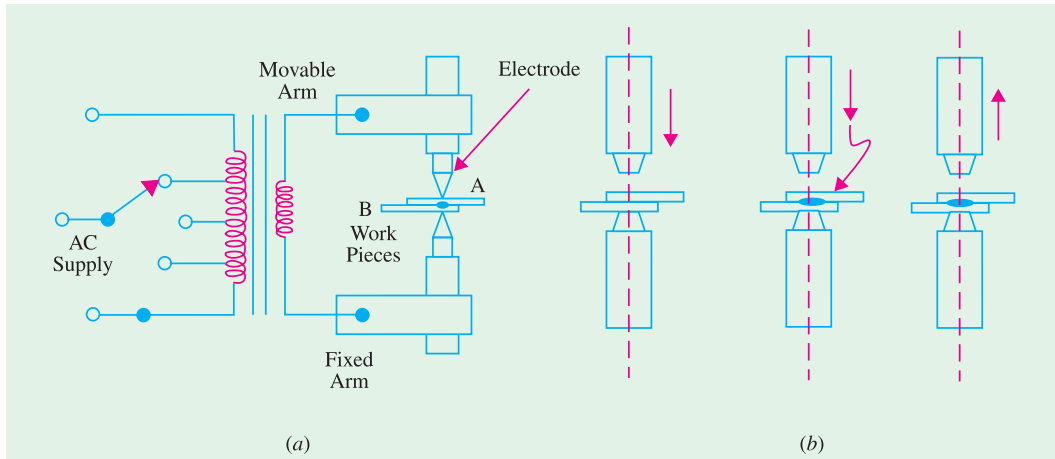


Fig. 48.28

whereas domed electrodes [Fig. 48.25 (b)] are used for non-ferrous materials. Flat domes are used when spot-welding deformation is not desired. The weld size is determined by the diameter of the electrode.

The welding machine is cycled in order to produce the required heat timed to coincide with the pressure exerted by the electrodes as shown in Fig. 48.28 (a). As the movable electrode comes down and presses the two workpieces A and B together, current is passed through the assembly. The metals under the pressure zone get heated upto about 950°C and fuse together. As they fuse, their resistance is reduced to zero, hence there is a surge of current. This surge is made to switch off the welding current automatically. In motor-driven machines, speeds of 300 strokes/minute are common.

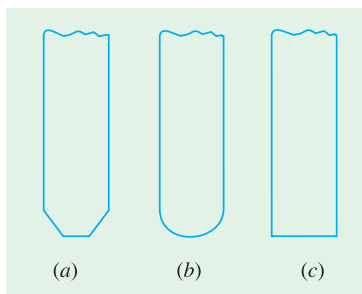


Fig. 48.29

Spot welders are of two different types. One is a stationary welder which is available in different sizes. The other has a stationary transformer but the electrodes are in a gun form.

Electric resistance spot welding is probably the best known and most widely-used because of its low cost, speed and dependability. It can be easily performed by even a semi-skilled operator. This process has a fast welding rate and quick set-up time apart from having low unit cost per weld.

Spot welding is used for galvanized, tinned and lead-coated sheets and mild steel sheet work. This technique is also applied to non-ferrous materials such as brass, aluminium, nickel and bronze etc.

### 48.28. Seam Welding

The seam welder differs from ordinary spot welder only

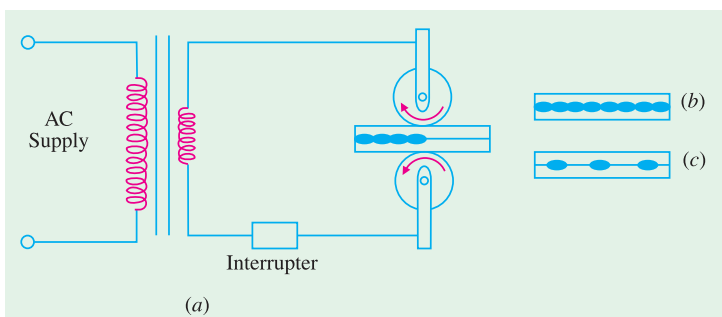


Fig. 48.30

in respect of its electrodes which are of disc or roller shape as shown in Fig. 48.30 (a). These copper wheels are power driven and rotate whilst gripping the work. The current is so applied through the wheels that the weld spots either overlap as in Fig. 48.30 (b) or are made at regular intervals as in Fig. 48.30 (c). The continuous or overlapped seam weld is also called *stitch weld* whereas the other is called roll weld.

Seam welding is confined to welding of thin materials ranging in thickness from 2 mm to 5 mm. It is also restricted to metals having low hardenability rating such as hot-rolled grades of low-alloy steels. Stitch welding is commonly used for long water-tight and gas-tight joints. Roll welding is used for simple joints which are not water-tight or gas-tight. Seam welds are usually tested by pillow test.

### 48.29. Projection Welding

It can be regarded as a mass-production form of spot welding. Technically, it is a cross between spot welding and butt welding. It uses the same equipment as spot welding. However, in this process, large-diameter flat electrodes (also called platens) are used. This welding process derives its name

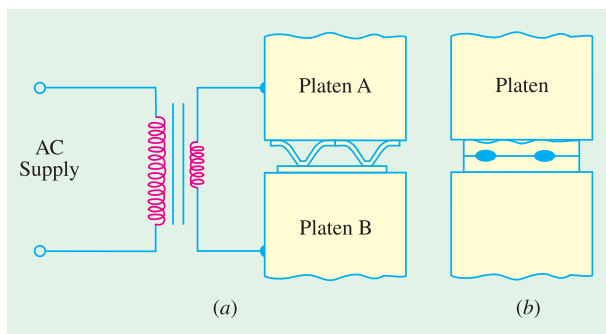


Fig. 48.31

from the fact that, prior to welding, projections are raised on the surfaces to be welded [Fig. 48.31 (a)]. As seen, the upper and lower platens are connected across the secondary of a step-down transformer and are large enough to cover all the projections to be welded at one stroke of the machine. When platen A touches the workpiece, welding current flows *through each projection*.

The welding process is started by first lowering the upper platen A on to the work-piece and then applying mechanical pressure to ensure correctly-forged welds. Soon after, welding current is switched on as in spot welding. As projection areas heat up, they collapse and union takes place at all projections simultaneously [Fig. 48.31 (b)].

It is seen that projections serve many purposes :

1. They increase the welding resistance of the material locally.
2. They accurately locate the positions of the welds.
3. They speed up the welding process by making it possible to perform several small welds simultaneously.
4. They reduce the amount of current and pressure needed to form a good bond between two surfaces.
5. They prolong the life of the electrode considerably because the metal itself controls the heat produced.

Projection welding is used extensively by auto manufactures for joining nuts, bolts and studs to steel plates in car bodies. This process is especially suitable for metals like brass, aluminium and copper etc. mainly due to their high thermal conductivity.

A variation of projection welding is the metal fibre welding which uses a metal fibre rather than a projection point (Fig. 48.32). This metal fibre is generally a felt material. Instead of projections, tiny elements of this felt material are placed between the two metals which are then projection-welded in the usual way.

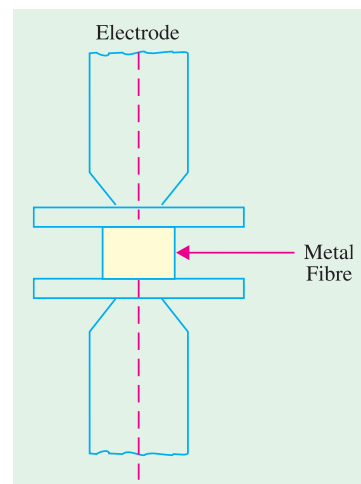


Fig. 48.32

### 48.30. Butt Welding

In this case, the two workpieces are brought into contact end-to-end and the butted ends are heated by passing a heavy current through the joint. As in other forms of resistance welding, the weld heat is produced mainly by the electrical resistance of the joint faces. In this case, however, the electrodes are in the form of powerful vice clamps which hold the work-pieces and also convey the forging pressure to the joint [Fig. 48.33].

This process is useful where parts have to be joined end-to-end or edge-to-edge. *i.e.* for welding pipes, wires and rods. It is also employed for making continuous lengths of chain.

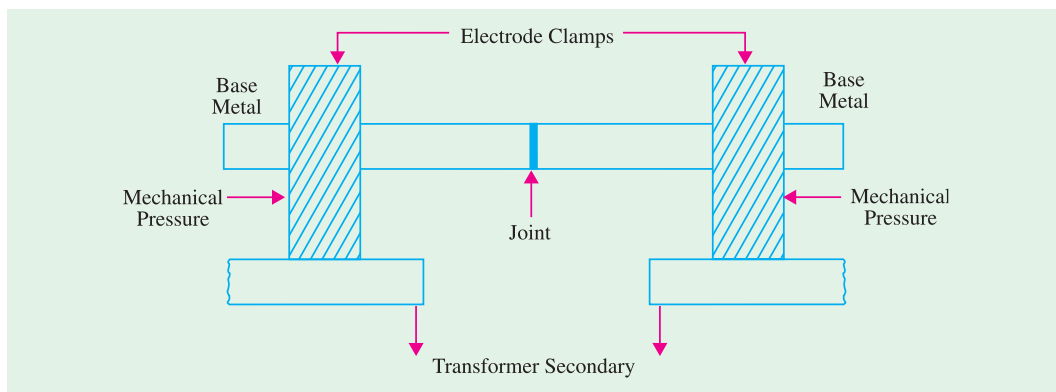


Fig. 48.33

### 48.31. Flash Butt Welding

It is also called by the simple name of *flash welding*. It is similar to butt welding but with the difference that here current is applied when ends of the two metal pieces are quite close to each other **but do not touch intimately**. Hence, an arc or flash is set up between them which supplies the necessary welding heat. As seen, in the process heat is applied **before** the two parts are pressed together.

As shown in Fig. 48.34 (a), the workpieces to be welded are clamped into specially designed electrodes one of which is fixed whereas the other is movable. After the flash has melted their faces, current is cut off and the movable platen applies the forging pressure to form a fusion weld. As shown in Fig. 48.34 (b), there is increase in the size of the weld zone because of the pressure which forces the soft ends together.

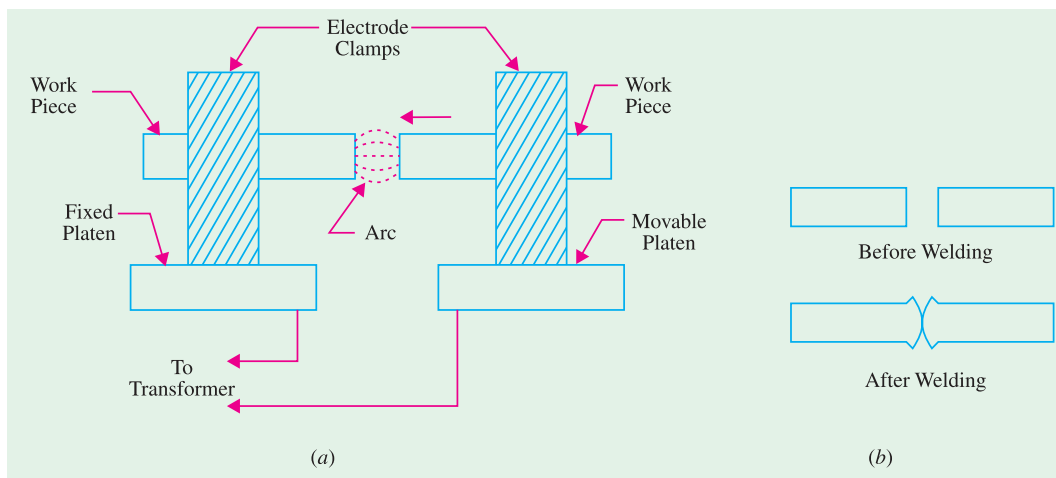


Fig. 48.34

**Advantages**

1. Even rough or irregular ends can be flash-welded. There is no need to level them by machining and grinding because all irregularities are burnt away during flashing period.
2. It is much quicker than butt welding.
3. It uses considerably less current than butt welding.
4. One of its major advantages is that dissimilar metals with different welding temperatures can be flash-welded.

**Applications**

1. To assemble rods, bars, tubings, sheets and most ferrous metals.
2. In the production of wheel rims for automobiles and bicycles.
3. For welding tubular parts such as automobile break cross-shafts.
4. For welding tube coils for refrigeration plants etc.

**48.32. Upset Welding**

In this process, *no flash is allowed to occur* between the two pieces of the metals to be welded. When the two base metals are brought together to a single interface, heavy current is passed between them which heats them up. After their temperature reaches a value of about 950°C, the two pieces of base metal are pressed together more firmly. This pressing together is called *upsetting*. This upsetting takes place *while current is flowing and continues even after current is switched off*. This upsetting action mixes the two metals homogeneously while pushing out many atmospheric impurities.

**48.33. Stud Welding****(a) Basic Principle**

It is similar to flash welding because it incorporates a method of drawing an arc between the stud (a rod) and the surface of the base metal. Then, the two molten surfaces are brought together under pressure to form a weld. Stud welding eliminates the need for drilling holes in the main structure.

**(b) Welding Equipment**

The stud welding equipment consists of a stud welding gun, a d.c. power supply capable of giving currents upto 400 A, a device to control current and studs and ferrules which are used not only as arc shields but also as containing walls for the molten metal.

**(c) Applications**

It is a low-cost method of fastening extensions (studs) to a metal surface. Most of the ferrous and non-ferrous metals can be stud-welded successfully. Ferrous metals include stainless steel, carbon steel and low-alloy steel. Non-ferrous metals include aluminium, lead-free brass, bronze and chrome-plated metals.

Stud welding finds application in the installations of conduit pipe hangers, planking and corrugated roofings.

This process is also used extensively in shipbuilding, railroad and automotive industries.

**48.34. Plasma Arc Welding****(a) Basic Principle**

It consists of a high-current electronic arc which is forced through a small hole in a water-cooled metallic nozzle [Fig. 48.35 (a)]. The plasma gas itself is used to protect the nozzle from the extreme heat of the arc. The plasma arc is shielded by inert gases like argon and helium which are pumped through an extra passageway within the nozzle of the plasma torch. As seen, plasma arc consists of electronic arc, plasma gas and gases used to shield the jet column. The idea of using the nozzle is to



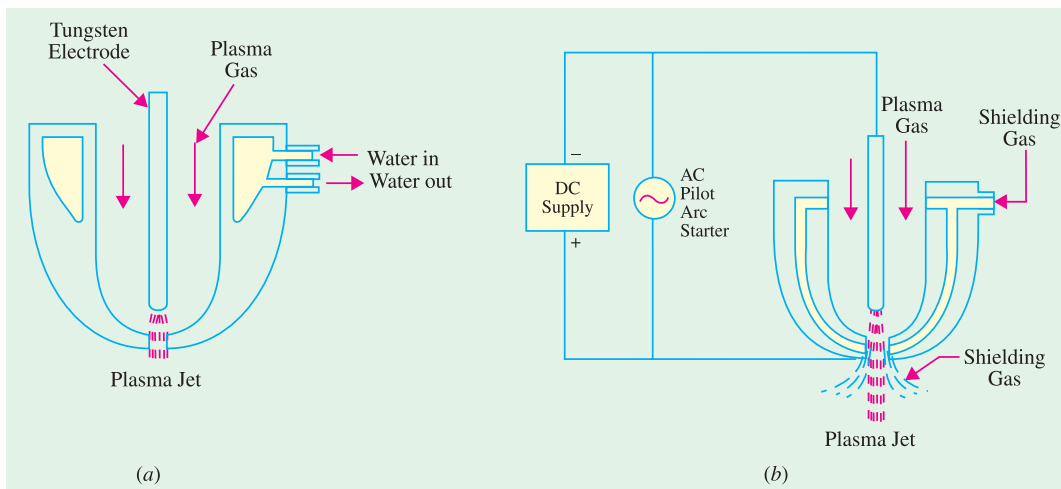


Fig. 48.35

constrict the arc thereby increasing its pressure. Collision of high-energy electrons with gas molecules produces the plasma which is swept through the nozzle and forms the current path between the electrode and the workpiece. Plasma jet torches have temperature capability of about  $35,000^{\circ}\text{C}$ .

#### (b) Electrodes

For stainless steel welding and most other metals, straight polarity tungsten electrodes are used. But for aluminium welding, reverse polarity water-cooled copper electrodes are used.

#### (c) Power Supply

Plasma arc welding requires d.c. power supply which could be provided either by a motor-generator set or transformer-rectifier combination. The latter is preferred because it produces better arc stability. The d.c. supply should have an open-circuit voltage of about 70V and drooping voltage-ampere characteristics. A high-frequency pilot arc circuit is employed to start the arc [Fig. 48.35 (b)].

#### (d) Method of Welding

Welding with plasma arc jet is done by a process called 'keyhole' method. As the plasma jet strikes the surface of the workpiece, it burns a hole through it. As the torch progresses along the work-piece, this hole also progresses along with but is filled up by the molten metal as it moves along. obviously, 100 percent penetration is achieved in this method of welding. Since plasma jet melts a large surface area of the base metal, it produces a weld bead of wineglass design as shown in Fig. 48.36. The shape of the bead can be changed by changing the tip of the nozzle of the torch. Practically, all welding is done mechanically.



Plasma arc welding



Fig. 48.36

#### (e) Applications

1. Plasma arc welding process has many aerospace applications.
2. It is used for welding of reactive metals and thin materials.
3. It is capable of welding high-carbon steel, stainless steel, maraging steel, copper and copper alloys, brass alloys, aluminium and titanium.
4. It is also used for metal spraying.

- It can be modified for metal cutting purposes. It has been used for cutting aluminium, carbon steel, stainless steel and other hard-to-cut steels. It can produce high-quality cross-free aluminium cuts 15 cm deep.

(f) **Disadvantages**

- Since it uses more electrical equipment, it has higher electrical hazards.
- It produces *ultra-violet and infra-red* radiations necessitating the use of tinted lenses.
- It produces high-pitched noise (100 dB) which makes it necessary for the operator to use ear plugs.

### 48.35. Electroslag Welding

(a) **General**

It is a metal-arc welding process and may be considered as a further development of submerged-arc welding.

This process is used for welding joints of thick sections of ferrous metals in a single pass and without any special joint preparation. Theoretically, there is no upper limit to the thickness of the weld bead. It is usually a vertical uphill process.

It is called *electroslag* process because heat is generated by passing current through the molten slag which floats over the top of the metal.

(b) **Welding Equipment**

As shown in Fig. 48.37, two water-cooled copper shoes (or dams) are placed on either side of the joint to be welded for the purpose of confining the molten metal in the joint area. The electrode is fed into the weld joint almost vertically from special wire guides. There is a mechanical device which raises the shoes and wire-feed mechanism as the weld continues upwards till it is completed. An a.c. welding machine has 100 percent duty cycle and which can supply currents upto 1000 A if needed.

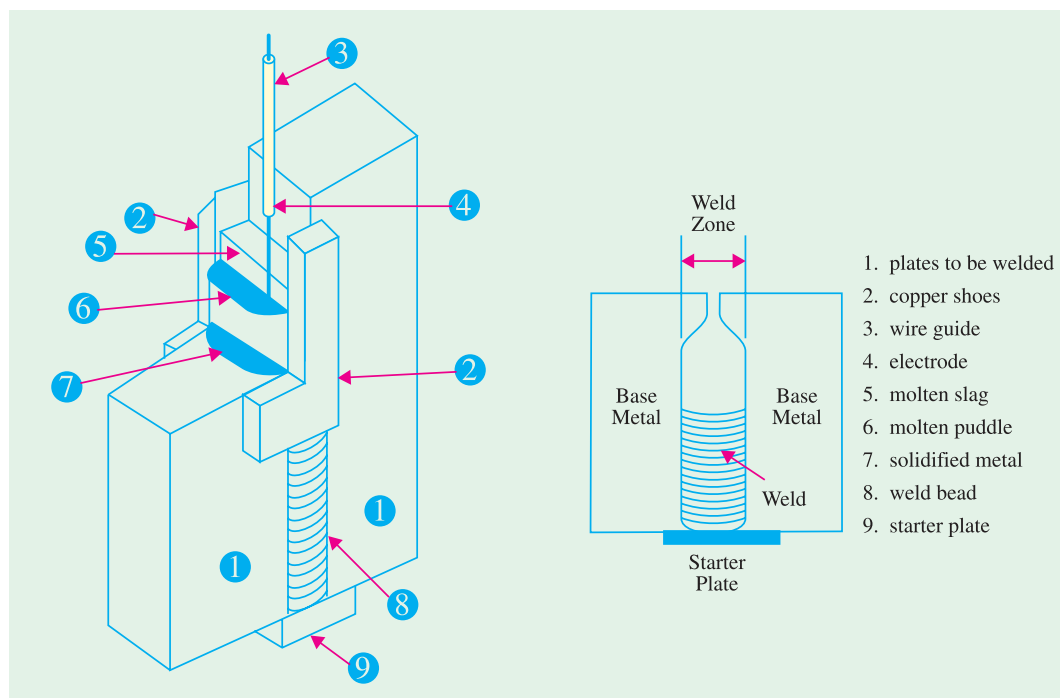


Fig. 48.37

### (c) Welding Process

The electroslag process is initiated just like submerged arc process by starting an electric arc beneath a layer of granular welding flux. When a sufficient thick layer of hot flux or molten slag is formed, arc action stops and from then onwards, current passes from the electrode to workpiece through the molten slag. At this point, the process becomes truly electroslag welding. A starting plate is used in order to build up proper depth of conductive slag before molten pool comes in contact with the workpieces.

The heat generated by the resistance to the flow of current through the molten slag is sufficient to melt the edges of the workpiece and the filler electrode. The molten base metal and filler metal collect at the bottom of the slag pool forming the weld pool. When weld pool solidifies, weld bead is formed which joins the faces of the base metal as shown in Fig. 48.33 (b).

As welding is continued upwards, flux flows to the top in the form of molten slag and cleanses the impurities from the molten metal. A mechanism raises the equipment as the weld is completed in the uphill vertical position.

### (d) Advantages

1. It needs no special joint preparation.
2. It does welding in a single pass rather than in costly multiple passes.
3. There is theoretically no maximum thickness of the plate it can weld.
4. There is also no theoretical upper limit to the thickness of the weld bead. Weld beads upto 400 mm thick have been performed with the presently-available equipment.
5. This process requires less electrical power per kg of deposited metal than either the submerged arc welding process or the shield arc process.
6. It has high deposit rate of upto 20 kg of weld metal per hour.
7. It has lower flux consumption.
8. Due to uniform heating of the weld area, distortion and residual stresses are reduced to the minimal amounts.

However, for electroslag welding, it is necessary to have only a square butt joint or a square edge on the plates to be welded.

### (e) Applications

It is commonly used in the fabrication of large vessels and tanks. Low-carbon steels produce excellent welding properties with this process.

## 48.36. Electro gas Welding

This process works on the same basic principle as the electroslag process but has certain additional features of submerged arc welding. Unlike electroslag process, the electro gas process uses an inert gas for shielding the weld from oxidation and there is a continuous arc (as in submerged arc process) to heat the weld pool.

## 48.37. Electron Beam Welding

In this process, welding operation is performed in a vacuum chamber with the help of a sharply-focussed beam of high-velocity electrons. The electrons after being emitted from a suitable electrode are accelerated by the high anode voltage and are then focussed into a fine beam which is finally directed to the workpiece. Obviously, this process needs no electrodes. The electron beam produces intense local heat which can melt not only the metal but can even boil it. A properly-focussed electron beam can completely penetrate through the base metal thereby creating a small hole whose walls are molten. As the beam moves along the joint, it melt the material coming in contact with it. The molten metal flows back to the previously-melted hole where it fuses to make a perfect weld for the entire depth of penetration.

Electron-beam welding has following advantages :

1. It produces deep penetration with little distortion.
2. Its input power is small as compared to other electrical welding devices.
3. Electron-beam weld is much narrower than the fusion weld.
4. It is especially suitable for reactive metals which become contaminated when exposed to air because this process is carried out in vacuum.
5. It completely eliminates the contamination of the weld zone and the weld bead because operation is performed in a vacuum chamber.
6. It is especially suited to the welding of beryllium which is being widely used in the fabrication of industrial and aerospace components.
7. Its high deposition rate produces welds of excellent quality with only a single pass.
8. It is the only process which can join high temperature metals such as columbium.

At present, its only serious limitations are that it is extremely expensive and is not available in portable form. However, recently a non-vacuum electron-beam welder has been developed.

### 48.38. Laser Welding

It uses an extremely concentrated beam of coherent monochromatic light *i.e.* light of only one colour (or wavelength). It concentrates tremendous amount of energy on a very small area of the workpiece to produce fusion. It uses solid laser (ruby, sapphire), gas laser ( $\text{CO}_2$ ) and semiconductor laser. Both the gas laser and solid laser need capacitor storage to store energy for later injection into the flash tube which produces the required laser beam.

The gas laser welding equipment consists of (i) capacitor bank for energy storage (ii) a triggering device (iii) a flash tube that is wrapped with wire (iv) lasing material (v) focussing lens and (vi) a worktable that can rotate in the three X, Y and Z directions.

When triggered, the capacitor bank supplies electrical energy to the flash tube through the wire. This energy is then converted into short-duration beam of laser light which is pin-pointed on the workpiece as shown in Fig. 48.38. Fusion takes place immediately and weld is completed fast.



Electron beam welding facility

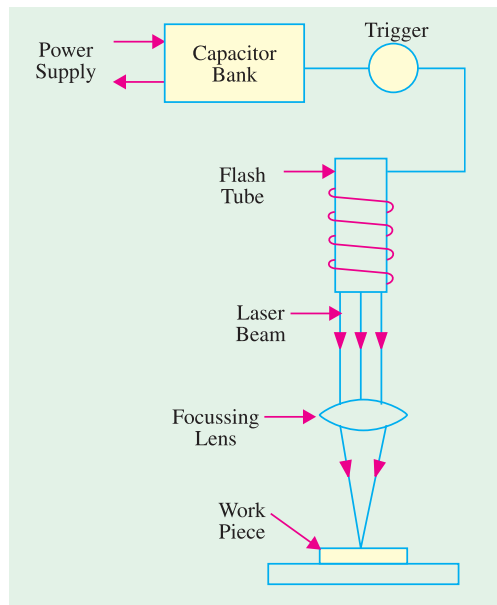


Fig. 48.38

Since duration of laser weld beam is very short (2 ms or so), two basic welding methods have been adopted. In the first method, the workpiece is moved so fast that the entire joint is welded in a single burst of the light. The other method uses a number of pulses one after the other to form the weld joint similar to that formed in electric resistance seam welding (Art 48.31).

Laser welding is used in the aircraft and electronic industries for lighter gauge metals.

Some of the advantages of laser welding process are as follows :

1. It does not require any electrode.
2. It can make welds with high degree of precision and on materials as thin as 0.025 mm.
3. It does not heat the workpiece except at one point. In fact, heat-affected zone is virtually non-existent.
4. Liquidus is reached only at the point of fusion.
5. It can produce glass-to-metal seals as in the construction of klystron tubes.
6. Since laser beam is small in size and quick in action, it keeps the weld zone uncontaminated.
7. It can weld dissimilar metals with widely varying physical properties.
8. It produces minimal thermal distortion and shrinkage because area of heat-affected zone is the minimum possible.
9. It can easily bond refractory materials like molybdenum, titanium and tantalum etc.

However, the major disadvantage of this process is its slow welding speed. Moreover, it is limited to welding with thin metals only.

#### Tutorial Problem No. 48.1

1. Describe various types of electric arc welding processes.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
2. Compare resistance welding and arc welding.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
3. Briefly explain the different methods of electric welding and state their relative merits.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
4. Give the comparison between A.C. and D.C. welding.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
5. Explain the different methods of electric welding and their relative advantages.  
*(J.N. University, Hyderabad, December 2002/January 2003)*

#### OBJECTIVE TESTS – 48

1. The basic *electrical* requirement in arc welding is that there should be
  - (a) coated electrodes
  - (b) high open-circuit voltage
  - (c) no arc blow
  - (d) d.c. power supply.
2. Welding is not done directly from the supply mains because
  - (a) it is customary to use welding machines
  - (b) its voltage is too high
  - (c) its voltage keeps fluctuating
  - (d) it is impracticable to draw heavy currents.
3. A.C. welding machine cannot be used for welding
  - (a) MIG
  - (b) atomic hydrogen
  - (c) resistance
  - (d) submerged arc.
4. In electric welding, arc blow can be avoided by
  - (a) using bare electrodes
  - (b) welding away from earth ground connection
  - (c) using a.c. welding machines
  - (d) increasing arc length.

5. In DCSP welding
  - (a) electrode is the hottest
  - (b) workpiece is relatively cool
  - (c) base metal penetration is deep
  - (d) heavily-coated electrodes are used.
6. Overhead welding position is thought to be the most
  - (a) hazardous
  - (b) difficult
  - (c) economical
  - (d) useful.
7. The *ultimate* aim of using electrode coating is to
  - (a) provide shielding to weld pool
  - (b) prevent atmospheric contamination
  - (c) improve bead quality
  - (d) cleanse the base metal.
8. In electrode-positive welding ..... of the total heat is produced at the electrode.
  - (a) one-third
  - (b) two-third
  - (c) one-half
  - (d) one-fourth.
9. Submerged arc process is characterised by
  - (a) deep penetration
  - (b) high welding current
  - (c) exceptionally smooth beads
  - (d) all of the above.
10. The major disadvantage of carbon arc welding is that
  - (a) there is occurrence of blow holes
  - (b) electrodes are consumed fast
  - (c) separate filler rod is needed
  - (d) bare electrodes are necessary.
11. In atomic hydrogen welding, electrodes are long-lived because
  - (a) two are used at a time
  - (b) arc is in the shape of a fan
  - (c) of a.c. supply
  - (d) it is a non-pressure process.
12. Unlike TIG welding, MIG welding
  - (a) requires no flux
  - (b) uses consumable electrodes
  - (c) provides complete protection from atmospheric contamination
  - (d) requires no post-weld cleansing.
13. The major power supply used in MIG welding is
  - (a) a.c. supply
  - (b) DCSP
  - (c) electrode-negative
  - (d) DCRP.
14. MIG welding process is becoming increasingly popular in welding industry mainly because of
  - (a) its easy operation
  - (b) its high metal deposit rate
  - (c) its use in both ferrous and non-ferrous metals.
  - (d) both (a) and (b).
15. A weld bead of wineglass design is produced in ..... welding.
  - (a) plasma arc
  - (b) electron beam
  - (c) laser
  - (d) MAG.
16. Spot welding process basically depends on
  - (a) Ohmic resistance
  - (b) generation of heat
  - (c) application of forging pressure
  - (d) both (b) and (c).
17. Electric resistance seam welding uses ..... electrodes.
  - (a) pointed
  - (b) disc
  - (c) flat
  - (d) domed.
18. Projection welding can be regarded as a mass production form of ..... welding.
  - (a) seam
  - (b) butt
  - (c) spot
  - (d) upset.
19. In the process of electroslag welding, theoretically there is no upper limit to the
  - (a) thickness of weld bead
  - (b) rate of metal deposit
  - (c) slag bath temperature
  - (d) rate of slag consumption.
20. High temperature metals like columbium can be easily welded by ..... welding.
  - (a) flash
  - (b) MIG
  - (c) TIG
  - (d) electron beam.
21. During resistance welding heat produced at the joint is proportional to
  - (a)  $I^2R$
  - (b) kVA
  - (c) current
  - (d) voltage
22. Grey iron is usually welded by ..... welding
  - (a) gas
  - (b) arc

- (c) resistance  
(d) MIG
23. The metal surfaces, for electrical resistance welding must be.....  
(a) lubricated  
(b) cleaned  
(c) moistened  
(d) rough
24. In a welded joint poor fusion is due to which of the following?  
(a) Improper current  
(b) High welding speed  
(c) Uncleaned metal surface  
(d) Lack of flux
25. For arc welding, D.C. is produced by which of the following?  
(a) motor-generator set  
(b) regulator  
(c) transformer  
(d) none of the above
26. .... welding process uses consumable electrodes.  
(a) TIG  
(b) MIG  
(c) Laser  
(d) All of the above
27. Which of the following equipment is generally used for arc welding?  
(a) single phase alternator  
(b) two phase alternator  
(c) three phase alternator  
(d) transformer
28. Which of the following is not an inert gas?  
(a) argon  
(b) carbon dioxide  
(c) helium  
(d) all of the above
29. Electronic components are joined by which of the following methods?  
(a) brazing  
(b) soldering  
(c) seam welding  
(d) spot welding  
(e) none of the above
30. Resistance welding cannot be used for  
(a) dielectrics  
(b) ferrous materials  
(c) non-ferrous metals  
(d) any of the above
31. Electric arc welding process produces temperature upto  
(a) 1000°C  
(b) 1500°C  
(c) 3500°C  
(d) 5550°C
32. Increased heat due to shorter arc is harmful on account of  
(a) under-cutting of base material  
(b) burn through  
(c) excessive porosity  
(d) all of the above
33. Arc blow results in which of the following?  
(a) Non-uniform weld beads  
(b) Shallow weld puddle given rise to weak weld  
(c) Splashing out of metal from weld puddle  
(d) All of the above defects
34. Inseam welding  
(a) the work piece is fixed and disc electrodes move  
(b) the work piece moves but rotating electrodes are fixed  
(c) any of the above  
(d) none of the above
35. In arc welding major personal hazards are  
(a) flying sperks  
(b) weld spatter  
(c) harmful infrared and ultra-violet rays from the arc  
(d) all of the above
36. In spot welding composition and thickness of the base metal decides  
(a) the amount of squeeze pressure  
(b) hold time  
(c) the amount of weld current  
(d) all above
37. Helium produces which of the following?  
(a) deeper penetration  
(b) faster welding speeds  
(c) narrower heat affected zone in base metal  
(d) all of the above
38. Due to which of the following reasons aluminium is difficult to weld?  
(a) it has an oxide coating  
(b) it conducts away heat very rapidly  
(c) both (a) and (b)  
(d) none of the above
39. Welding leads have  
(a) high flexibility  
(b) high current handling capacity  
(c) both (a) and (b)  
(d) none of the above
40. Air craft body is  
(a) spot welded

- (b) gas welded  
(c) seam welded  
(d) riveted
41. For arc welding current range is usually  
(a) 10 to 15 A  
(b) 30 to 40 A  
(c) 50 to 100 A  
(d) 100 to 350 A
42. Spot welding is used for  
(a) thin metal sheets  
(b) rough and irregular surfaces  
(c) costings only  
(d) thick sections
43. Galvanising is a process of applying a layer of  
(a) aluminium  
(b) lead  
(c) copper  
(d) zinc
44. A seamless pipe has  
(a) steam welded joint  
(b) spot welded joint  
(c) arc welded joint  
(d) no joint
45. Motor-generator set for D.C. arc welding has generator of  
(a) series type  
(b) shunt type  
(c) differentially compound type  
(d) level compound type
46. Plain and butt welds may be used on materials upto thickness of nearly  
(a) 5 mm  
(b) 10 mm  
(c) 25 mm  
(d) 50 mm
47. In argon arc welding argon is used as a  
(a) flux  
(b) source of heat  
(c) agent for heat transfer  
(d) shield to protect the work from oxidation
48. During arc welding as the thickness of the metal to be welded increases  
(a) current should decrease, voltage should increase  
(b) current should increase, voltage remaining the same  
(c) current should increase, voltage should decrease  
(d) voltage should increase, current remaining the same
49. In D.C. arc welding  
(a) electrode is made positive and workpiece negative  
(b) electrode is made negative and workpiece positive  
(c) both electrode as well as workpiece are made positive  
(d) both electrode as well as workpiece are made negative
50. The purpose of coating on arc welding electrodes is to  
(a) stabilise the arc  
(b) provide a protecting atmosphere  
(c) provide slag to protect the molten metal  
(d) all of the above
51. 50 percent duty cycle of a welding machine means  
(a) machine input is 50 percent of rated input  
(b) machine efficiency is 50 percent  
(c) machine work on 50 percent output  
(d) machine works for 5 minutes in a duration of 10 minutes

### ANSWERS

1. (b)    2. (d)    3. (a)    4. (c)    5. (c)    6. (a)    7. (c)    8. (b)    9. (d)  
 10. (a)    11. (c)    12. (b)    13. (d)    14. (d)    15. (a)    16. (d)    17. (b)    18. (c)  
 19. (a)    20. (d)    21. (a)    22. (a)    23. (b)    24. (a)    25. (a)    26. (b)    27. (d)  
 28. (b)    29. (b)    30. (a)    31. (d)    32. (d)    33. (d)    34. (c)    35. (d)    36. (d)  
 37. (d)    38. (c)    39. (d)    40. (c)    41. (d)    42. (a)    43. (d)    44. (d)    45. (d)  
 46. (c)    47. (d)    48. (b)    49. (b)    50. (d)    51. (d)



# CHAPTER 49

## Learning Objectives

- Radiations from a Hot Body
- Solid Angle
- Definitions
- Calculation of Luminance (L)
- Laws of Illumination or Illuminance
- Polar Curves of C.P. Distribution
- Uses of Polar Curves
- Determination of M.S.C.P and M.H.C.P. from Polar Diagrams
- Integrating Sphere or Photometer
- Diffusing and Reflecting Surfaces
- Lighting Schemes
- Illumination Required for Different Purposes
- Space / Height Ratio
- Design of Lighting Schemes and Layouts
- Utilisation Factor ([h])
- Depreciation Factor (P)
- Floodlighting
- Artificial Source of Light
- Incandescent Lamp
- Filament Dimensions
- Incandescent Lamp Characteristics
- Clear and Inside
- Frosted Gas-filled Lamps
- Discharge Lamps
- Sodium Vapour Lamp

## ILLUMINATION



When some materials are heated above certain temperatures, they start radiating energy in the form of light. This phenomenon is called luminance. Electric lamps are made based on this phenomenon.

### 49.1. Radiations From a Hot Body

The usual method of producing artificial light consists in raising a solid body or vapour to incandescence by applying heat to it. It is found that as the body is gradually heated above room temperature, it begins to radiate energy in the surrounding medium in the form of electromagnetic waves of various wavelengths. The *nature* of this radiant energy depends on the temperature of the hot body. Thus, when the temperature is low, radiated energy is in the form of heat waves only but when a certain temperature is reached, light waves are also radiated out in addition to heat waves and the body becomes luminous. Further increase in temperature produces an increase in the amount of both kinds of radiations but the colour of light or visible radiation changes from bright red to orange, to yellow and then finally, if the temperature is high enough, to white. As temperature is increased, the wavelength of the visible radiation goes on becoming shorter. It should be noted that heat waves are identical to light waves except that they are of longer wavelength and hence produce no impression on the retina. Obviously, from the point of view of light emission, heat energy represents so much wasted energy.

The ratio  $\frac{\text{energy radiated out in the form of light}}{\text{total energy radiated out by the hot body}}$  is called the *radiant* efficiency of the luminous source and, obviously, depends on the temperature of the source. As the temperature is increased beyond that at which light waves were first given off, the radiant efficiency increases, because light energy will increase in greater proportion than the total radiated energy. When emitted light becomes white *i.e.*, it includes all the visible wavelengths, from extreme red to extreme violet, then a further increase in temperature produces radiations which are of wavelength smaller than that of violet radiations. Such radiations are invisible and are known as ultra-violet radiations. It is found that maximum radiant efficiency would occur at about  $6200^{\circ}\text{C}$  and even then the value of this maximum efficiency would be 20%. Since this temperature is far above the highest that has yet been obtained in practice, it is obvious that the actual efficiency of all artificial sources of light *i.e.* those depending on *temperature incandescence*, is low.

As discussed above, light is radiant energy which is assumed to be propagated in the form of transverse waves through an invisible medium known as ether. These light waves travel with a velocity of  $2.99776 \times 10^8$  m/s or  $3 \times 10^8$  m/s approximately but their wavelengths are different. The wavelength of red light is nearly 0.000078 cm and that of violet light 0.000039 cm. Since these wavelengths are very small, instead of using 1 cm as the unit for their measurement, a submultiple  $10^{-8}$  cm is used. This submultiple is known as Angstrom Unit (A.U.)

$$1 \text{ A.U.} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

Hence, the wave-length of red light becomes  $\lambda_r = 7800 \times 10^{-10}$  m or 7800 A.U. and  $\lambda_v = 3900 \times 10^{-10}$  m or 3900 A.U. The sensation of colour is due to the difference in the wavelengths and hence frequencies of the light radiations.

### 49.2. Solid Angle

Consider an area  $A$  which is part of a sphere of radius  $r$  (Fig. 49.1). Let us find the solid angle  $\omega$  subtended by this area at the centre  $C$  of the sphere. For this purpose, let point  $C$  be joined to every point on the edges of the area  $A$ . Then, the angle enclosed by the cone at point  $C$  gives the solid angle. Its value is

$$\omega = \frac{A}{r^2} \text{ steradian}$$

The unit of solid angle is *steradian* (sr). If, in the above equation,  $A = r^2$ , then  $\omega = 1$  steradian. Hence, steradian is defined as the angle subtended at the centre of a sphere by a part of its surface having an area equal to (radius)<sup>2</sup>.

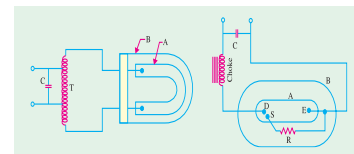


Fig. 49.1

Obviously, the solid angle subtended at the centre by whole of the spherical surface =  $4\pi r^2/r^2 = 4\pi$  steradian (sr).

**49.3. Definitions**

Before proceeding further, definitions of a few principal terms employed in connection with illumination, are given below :

**1. Candela.** It is the unit of luminous intensity of a source. It is defined as 1/60th of the luminous intensity per  $\text{cm}^2$  of a black body radiator at the temperature of solidification of platinum (2045°K).

A source of one candela (*cd*) emits one lumen per steradian. Hence, total flux emitted by it allround is  $4\pi \times 1 = 4\pi$  lumen.

**2. Luminous Flux (F or Φ).** It is the light energy radiated out per second from the body in the form of luminous light waves.

Since, it is a rate of flow of energy, it is a sort of *power* unit. Unit of luminous flux is *lumen* (lm). It is defined as the **flux contained per unit solid angle of a source of one candela or standard candle** (Fig. 49.2).

Approximate relation between lumen and electric unit of power *i.e.* watt is given as

$$1 \text{ lumen} = 0.0016 \text{ watt (approx.)}$$

**3. Lumen-hour.** It is the quantity of light delivered in one hour by a flux of one lumen.\*

**4. Luminous Intensity (I) or Candle-power** of a point source in any particular direction is given by the **luminous flux radiated out per unit solid angle in that direction**. In other words, it is solid angular flux density of a source in a specified direction.

If  $d\Phi$  is the luminous flux radiated out by a source within a solid angle of  $d\omega$  steradian in any particular direction, then  $I = d\Phi/d\omega$ .

If flux is measured in lumens and solid angle in steradian, then its unit is lumen/steradian (lm/sr) or candela (*cd*).

If a source has an average luminous intensity of  $I$  lm/sr (or  $I$  candela), then total flux radiated by it all around is  $\Phi = \omega I = 4\pi I$  lumen.

Generally, the luminous intensity or candle power of a source is different in different directions. The average candle-power of a source is the average value of its candle power in all directions. Obviously, it is given by total flux (in lm) emitted in all directions in all planes divided by  $4\pi$ . This average candle-power is also known as **mean spherical candle-power** (M.S.C.P.).

$$\therefore \text{M.S.C.P.} = \frac{\text{total flux in lumens}}{4\pi}$$

If the average is taken over a hemisphere (instead of sphere), then this average candle power is known as **mean hemispherical candle-power** (M.H.S.C.P.).

It is given by the total flux emitted in a hemisphere (usually the lower one) divided by the solid angle subtended at the point source by the hemisphere.

$$\therefore \text{M.H.S.C.P.} = \frac{\text{flux emitted in a hemisphere}}{2\pi}$$



Fireworks radiate light energy of different frequencies, which appear in different colours

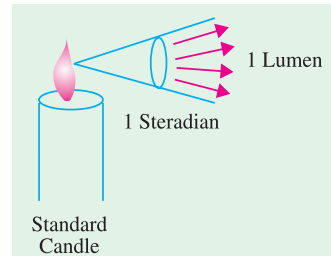


Fig. 49.2

\* It is similar to watt-hour (Wh)

5. **Reduction Factor** of a source is given by the ratio,  $f = \text{M.S.C.P.}/\text{M.H.C.P.}$  where M.H.C.P. is the mean horizontal candle power.

It is also referred to as spherical reduction factor.

6. **Illuminance or Illumination (E).** When the luminous flux falls on a surface, it is said to be illuminated. The illumination of a surface is measured by the normal luminous flux per unit area received by it.

If  $d\Phi$  is the luminous flux incident normally on an area  $dA$ , then  $E = d\Phi/dA$  or  $E = \Phi/A$ .

**Unit.** Since flux  $\Phi$  is measured in lumens and area in  $\text{m}^2$ , unit of  $E$  is  $\text{lm}/\text{m}^2$  or lux. The alternative name is metre-candle (m-cd). Let us see why? Imagine a sphere of radius of one metre around a point source of one candela. Flux radiated out by this source is  $4\pi$  lumen. This flux falls normally on the curved surface of the sphere which is  $4\pi\text{m}^2$ . Obviously, illumination at every point on the inner surface of this sphere is  $4\pi \text{ lm}/4\pi \text{ m}^2 = 1 \text{ lm}/\text{m}^2$ . However, the term  $\text{lm}/\text{m}^2$  is to be preferred to metre-candle.

7. **Luminance (L) of an Extended Source.** Suppose  $\Delta A$  is an element of area of an *extended* source and  $\Delta I$  its luminous intensity when viewed in a direction making an angle  $\phi$  with the perpendicular to the surface of the source (Fig. 49.3), then luminance of the source element is given by

$$L = \frac{\Delta I}{\Delta A \cos \phi} = \frac{\Delta I}{\Delta A'} \text{ cd}/\text{m}^2 \dots(i)$$

where  $\Delta A' = \Delta A \cos \phi$   
 = area of the source element projected onto a plane perpendicular to the specified direction.

As will be seen from Art. 49.5.

$$E = \frac{I \cos \theta}{d^2} \text{ or } \Delta E = \frac{\Delta I}{d^2} \cos \theta$$

Substituting the value of  $\Delta I$  from Eq. (i) above, we get

$$\Delta E = \frac{L \cdot \Delta A'}{d^2} \cos \theta = L \cos \theta \cdot d\omega$$

where  $d\omega = \Delta A'/d^2$  steradian

$$E = \int L \cos \theta \cdot d\omega = L \int \cos \theta \cdot d\omega$$

—if  $L$  is constant.

8. **Luminous Exitance (M) of a Surface.** The luminous exitance ( $M$ ) at a point on a surface is defined as luminous flux emitted per unit area in all directions. If an element of an illuminated area  $\Delta A$  emits a total flux of  $\Delta\Phi$  in all directions (over a solid angle of  $2\pi$  steradian) then

$$M = \Delta\Phi/\Delta A \text{ lm}/\text{m}^2$$

It can be proved that  $M = \pi L$  in the case of a uniform diffuse source.

9. **Transmittance (T) of an Illuminated Diffuse Reflecting Surface.** It is defined as the ratio of the total luminous flux transmitted by it to the total flux incident on it.

The relation between luminous exitance ( $M$ ) of a surface transmitting light and illuminance ( $E$ ) on the other side of it is

$$M = TE \text{ or } T = M/E$$

Since light falling on a surface is either transmitted, reflected or absorbed the following relation holds good

$$T + \rho + \alpha = 1 \text{ where } \alpha \text{ is the absorptance of the surface.}$$

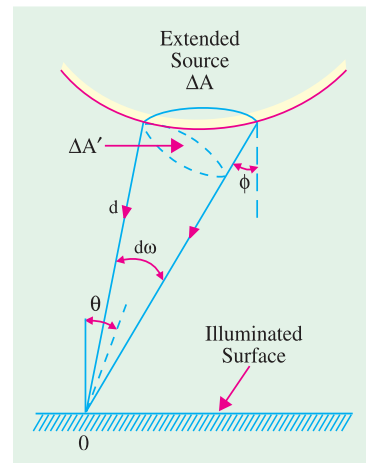


Fig. 49.3

**10. Reflection Ratio or Coefficient of Reflection or Reflectance ( $\rho$ ).** It is given by the luminous flux reflected from a small area of the surface to the total flux incident upon it

$$\rho = M/E \text{ i.e. ratio of luminous exitance and illuminance.}$$

It is always less than unity. Its value is zero for ideal ‘black body’ and unity for a perfect reflector.

**11. Specific Output or Efficiency** of a lamp is the ratio of luminous flux to the power intake. Its unit is lumen/watt (lm/W). Following relations should be taken note of :

(a) 
$$\frac{\text{lumen}}{\text{watt}} = \frac{4\pi \times \text{M.S.C.P.}}{\text{watt}}$$

or 
$$\frac{\text{lm}}{\text{W}} = \frac{4\pi}{\text{watt/M.S.C.P.}}$$

(b) since  $f = \text{M.S.C.P./M.H.C.P.} \therefore \text{lm/W} = \frac{4\pi f}{\text{watt/M.S.C.P.}}$

(c) Obviously,  $\text{watts/M.S.C.P.} = \frac{4\pi}{\text{lm/W}} = \frac{\text{watt/M.H.C.P.}}{f}$

(d) Also  $\text{watts/M.H.C.P.} = \frac{4\pi f}{\text{lm/W}} = f \times \text{watts/M.S.C.P.}$

**12. Specific Consumption.** It is defined as the ratio of the power input to the average candle-power. It is expressed in terms of watts per average candle or watts/M.S.C.P.

The summary of the above quantities along with their units and symbol is given in Table 49.1.

Name of Qty	Unit	Symbols
Luminous Flux	Lumen	$F$ or $\Phi$
Luminous Intensity (candle-power)	Candela	$I$
Illumination or Illuminance	$\text{lm/m}^2$ or lux	$E$
Luminance or Brightness	$\text{cd/m}^2$	$L$ or $B$
Luminous Exitance	$\text{lm/m}^2$	$M$

**49.4. Calculation of Luminance (L) of a Diffuse Reflecting Surface**

The luminance (or brightness) of a surface largely depends on the character of the surface, if it is itself not the emitter. In the case of a polished surface, the luminance depends on the angle of viewing. But if the surface is matt and diffusion is good, then the luminance or brightness is practically independent of the angle of viewing. However, the reflectance of the surface reduces the brightness proportionately. In Fig. 49.4 is shown a perfectly diffusing surface of small area  $A$ . Suppose that at point  $M$  on a hemisphere with centre  $O$  and radius  $R$ , the illuminance is  $L \text{ cd/m}^2$ . Obviously, **luminous intensity** at point  $M$  is  $L \times A \cos \theta$  candela (or lumen/steradian). Now, the hemisphere can be divided into a number of zones as shown. Consider one such zone  $MN$  between  $\theta$  and  $(\theta + d\theta)$ . The width of this zone is  $R.d\theta$  and length  $2\pi R \sin \theta$  so that its area (shown shaded) is  $= 2\pi R^2 \sin \theta . d\theta$ . Hence, it subtends a solid angle  $= 2\pi R^2 \sin \theta . d\theta / R^2 = 2\pi \sin \theta . d\theta$  steradian at point  $O$ . The luminous flux passing through this zone is

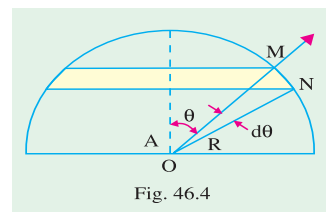


Fig. 46.4

Fig. 49.4

$d\Phi = L A \cos \theta \times 2\pi \sin \theta . d\theta = \pi L A \times 2 \sin \theta \cos \theta d\theta = \pi L A \sin 2\theta d\theta$  lumen

Total flux passing through the whole hemisphere is

$$\Phi = \int_0^{\pi/2} \pi L A \sin 2\theta . d\theta = \pi L A \text{ lumen}$$

If the *illumination* of the surface (produced by a light source) is  $E \text{ lm/m}^2$  and  $\rho$  is its reflection coefficient, then  $\Phi = \rho A E$  lumen.

Equating the two values of flux, we have  $\pi L A = \rho A E$   
 or  $L = \rho E / \pi \text{ cd/m}^2 = \rho E \text{ lm/m}^2$

For example, consider a perfectly diffusing surface having  $\rho = 0.8$  and held at a distance of 2 metres from a source of luminous intensity 100 candela at right angles to the direction of flux. Then

$$E = 100/2^2 = 25 \text{ lm/m}^2$$

$$L = \rho E / \pi = 25 \times 0.8 / \pi = 6.36 \text{ cd/m}^2 = 636 \times \pi = 20 \text{ lm/m}^2$$

### 49.5. Laws of Illumination or Illuminance

The illumination ( $E$ ) of a surface depends upon the following factors. The source is assumed to be a point source or is otherwise sufficiently away from the surface to be regarded as such.

- (i)  $E$  is directly proportional to the luminous intensity ( $I$ ) of the source or  $E \propto I$
- (ii) *Inverse Square Law*. The illumination of a surface is inversely proportional to the square of the distance of the surface from the source.

In other words,  $E \propto 1/r^2$

**Proof**

In Fig. 49.5 are shown portions of the surfaces of three spheres whose radii are in the ratio 1 : 2 : 3. All these portions, obviously, subtend the same solid angle at the source and hence receive the same amount of total flux. However, since their areas are in the ratio of 1 : 4 : 9, their illuminations are in the ratio  $1 : \frac{1}{4} : \frac{1}{9}$ .

(iii) *Lambert's Cosine Law*. According to this law,  $E$  is directly proportional to the cosine of the angle made by the normal to the illuminated surface with the direction of the incident flux.

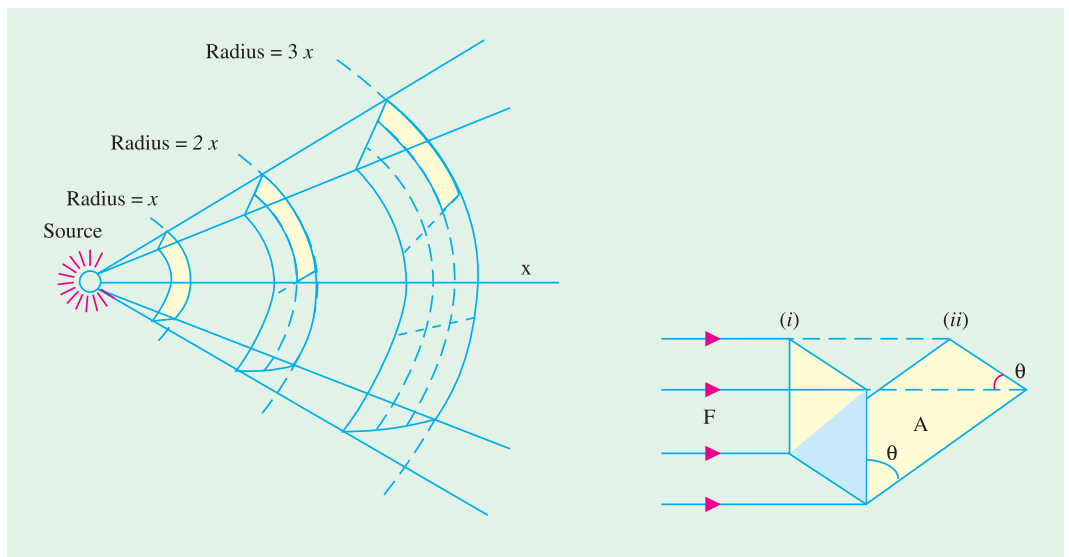


Fig. 49.5

Fig. 49.6

**Proof**

As shown in Fig. 49.6, let  $\Phi$  be the flux incident on the surface of area  $A$  when in position 1. When this surface is turned back through an angle  $\theta$ , then the flux incident on it is  $\Phi \cos \theta$ . Hence, illumination of the surface when in position 1 is  $E_1 = \Phi / A$ . But when in position 2,

$$E_2 = \frac{\Phi \cos \theta}{A} \quad \therefore \quad E_2 = E_1 \cos \theta$$

Combining all these factors together, we get  $E = I \cos \theta / r^2$ . The unit is  $\text{lm}/\text{m}^2$ .

The above expression makes the determination of illumination possible at a given point provided the position and the luminous intensity or candle power (in the given direction) of the source (or sources) by which it is illuminated, are known as illustrated by the following examples.

Consider a lamp of uniform luminous intensity suspended at a height  $h$  above the working plane as shown in Fig. 49.7. Let us consider the value of illumination at point  $A$  immediately below the lamp and at other points  $B, C, D$  etc., lying in the working plane at different distances from  $A$ .

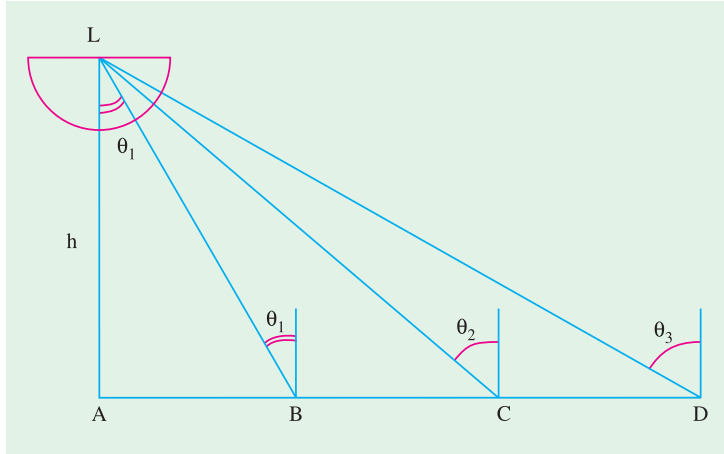


Fig. 49.7

$$E_A = \frac{I}{h^2} \text{ —since } \theta = 0 \text{ and } \cos \theta = 1$$

$$E_B = \frac{I}{LB^2} \times \cos \theta_1. \quad \text{Since, } \cos \theta_1 = h / LB$$

$$\therefore E_B = \frac{I}{LB^2} \times \frac{h}{LB} = I \times \frac{h}{LB^3} = \frac{1}{h^2} \cdot \frac{h^3}{LB^3} = \frac{1}{h^2} \left( \frac{h}{LB} \right)^3$$

Now  $\frac{1}{h^2} = E_A$  and  $\left( \frac{h}{LB} \right)^3 = \cos^3 \theta_1$

$$\therefore E_B = E_A \cos^3 \theta_1$$

Similarly,  $E_C = E_A \cdot \cos^3 \theta_2$  and  $E_D = E_A \cos^3 \theta_3$  and so on.

**Example 49.1.** A lamp giving out 1200 lm in all directions is suspended 8 m above the working plane. Calculate the illumination at a point on the working plane 6 m away from the foot of the lamp. (Electrical Technology, Aligarh Muslim Univ.)

**Solution.** Luminous intensity of the lamp is

$$I = 1200/4\pi = 95.5 \text{ cd}$$

As seen from Fig. 49.8.

$$L_B = \sqrt{8^2 + 6^2} = 10 \text{ m}; \quad \cos \theta = 8/10 = 0.8$$

Now,  $E = I \cos \theta / r^2$

$$\therefore E_B = 95.5 \times 0.8 / 10^2 = \mathbf{0.764 \text{ lm}/\text{m}^2}$$

**Example 49.2.** A small light source with intensity uniform in all directions is mounted at a height of 10 metres above a horizontal surface. Two points A and B both lie on the surface with point

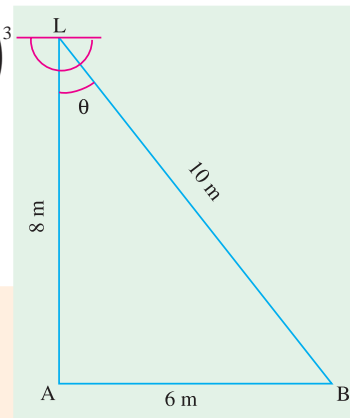


Fig. 49.8

A directly beneath the source. How far is B from A if the illumination at B is only 1/10 as great as at A? (A.M.I.E.)

**Solution.** Let the intensity of the lamp be  $I$  and the distance between A and B be  $x$  metres as shown in Fig. 49.9.

Illumination at point A,  $E_A = I/10^2 = I/100$  lux

Illumination at point B,

Illumination at point B,

$$E_B = \frac{I}{10^2} \times \left[ \frac{10}{\sqrt{10^2 + x^2}} \right]^2 = \frac{10I}{(100 + x^2)^{3/2}}$$

Since  $E_B = \frac{E_A}{10}$ ,

$$\therefore \frac{10I}{(100 + x^2)^{3/2}} = \frac{1}{10} \times \frac{I}{100}, \quad \therefore x = 19.1 \text{ m}$$

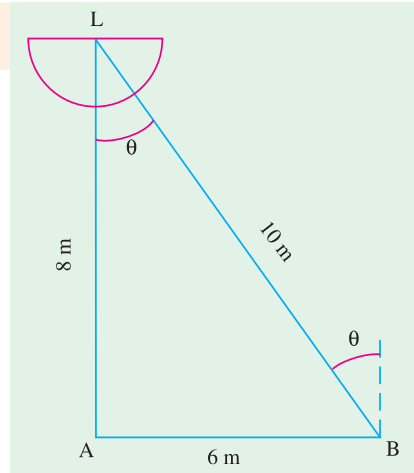


Fig. 49.9

**Example 49.3.** A corridor is lighted by 4 lamps spaced 10 m apart and suspended at a height of 5 m above the centre line of the floor. If each lamp gives 200 C.P. in all directions below the horizontal, find the illumination at the point on the floor mid-way between the second and third lamps.

(Electrical Engineering, Bombay Univ.)

**Solution.** As seen from 49.10, illumination at point C is due to all the four lamps. Since point C is symmetrically situated between the lamps, illumination at this point is twice that due to  $L_1$  and  $L_2$ .

(i) illumination due to  $L_1 = I \cos \theta_1 / L_1 C^2$   $L_1 C = \sqrt{5^2 + 15^2} = 15.8$   
 $\cos \theta = 5/15.8$

illumination due to  $L_1 = \frac{220 \times (5/15.8)}{250} = 0.253 \text{ lm/m}^2$

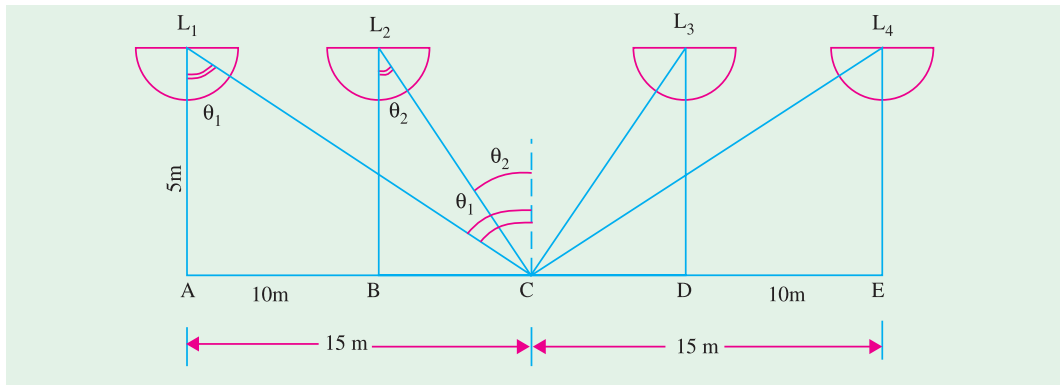


Fig. 49.10

(ii)  $L_2 C = 5/\sqrt{2} \text{ m}$ ;  $\theta_2 = 45^\circ$ ;  $\cos \theta_2 = 1/\sqrt{2}$

Illumination due to  $L_2 = \frac{200 \times 1/\sqrt{2}}{50} = 2.83 \text{ lm/m}^2$

$\therefore$  illumination at C due to  $L_1$  and  $L_2 = 3.08 \text{ lm/m}^2$

Illumination at C due to all the four lamps,  $E_C = 2 \times 3.08 = 6.16 \text{ lm/m}^2$



**Example 49.4.** Two lamps A and B of 200 candela and 400 candela respectively are situated 100 m apart. The height of A above the ground level is 10 m and that of B is 20 m. If a photometer is placed at the centre of the line joining the two lamp posts, calculate its reading.

(Electrical Technology, Gujarat Univ.)

**Solution.** When the illumination photometer is placed at the centre point, it will read the value of combined illumination produced by the two lamps (Fig. 49.11).

$$\begin{aligned} \text{Now, } L_1C &= \sqrt{10^2 + 50^2} \\ &= 51 \text{ m} \\ L_2C &= \sqrt{20^2 + 50^2} \\ &= 53.9 \text{ m} \\ \cos \theta_1 &= 10/51 ; \\ \cos \theta_2 &= 20/53.9 \\ \text{Illumination at point C due to lamp } L_1 &= \frac{200 \times 10}{51 \times 2600} \\ &= \mathbf{0.015 \text{ lm/m}^2} \end{aligned}$$

Similarly, illumination due to lamp  $L_2$

$$\begin{aligned} &= \frac{400 \times 20 / 53.9}{2900} = \mathbf{0.051 \text{ lm/m}^2} \\ \therefore E_C &= 0.015 + 0.051 \\ &= \mathbf{0.066 \text{ lm/m}^2 \text{ or lux}} \end{aligned}$$

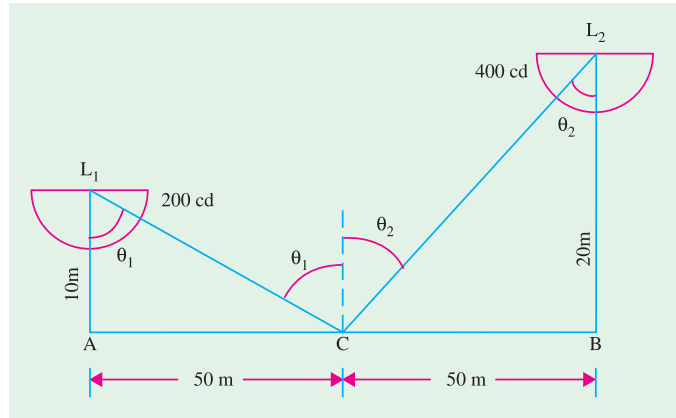


Fig. 49.11

**Example 49.5.** The average luminous output of an 80-W fluorescent lamp 1.5 metre in length and 3.5 cm diameter is 3300 lumens. Calculate its average brightness. If the auxiliary gear associated with the lamp consumes a load equivalent to 25 percent of the lamp, calculate the cost of running a twin unit for 2500 hours at 30 paise per kWh.

**Solution.** Surface area of the lamp =  $\pi \times 0.035 \times 1.5 = 0.165 \text{ m}^2$

$$\text{Flux emitted per unit area} = 3300/0.165 = 2 \times 10^4 \text{ lm/m}^2$$

$$\therefore B = \frac{\text{flux emitted per unit area}}{\pi} \text{ cd/m}^2 = 2 \times \frac{10^4}{\pi} = \mathbf{6,382 \text{ cd/m}^2}$$

$$\text{Total load of twin fitting} = 2[80 + 0.25 \times 80] = 200 \text{ W}$$

$$\text{Energy consumed for 2500 hr} = 2500 \times 200 \times 10^{-3} = 500 \text{ kWh}$$

$$\text{cost} = \text{Rs. } 500 \times 0.3 = \mathbf{\text{Rs. } 150.}$$

**Example 49.6.** A small area 7.5 m in diameter is to be illuminated by a lamp suspended at a height of 4.5 m over the centre of the area. The lamp having an efficiency of 20 lm/w is fitted with a reflector which directs the light output only over the surface to be illuminated, giving uniform candle power over this angle. Utilisation coefficient = 0.40. Find out the wattage of the lamp. Assume 800 lux of illumination level from the lamp. (A.M.I.E.)

$$\text{Solution. } A = \pi d^2/4 = 44.18 \text{ m}^2, \quad E = 800 \text{ lux}$$

$$\text{Luminous flux reaching the surface} = 800 \times 44.18 = 35,344 \text{ lm}$$

Total flux emitted by the lamp =  $35,344/0.4 = 88,360$  lm  
 Lamp wattage =  $88,360/20 = 4420$  W

**Example 49.7.** A lamp of 100 candela is placed 1 m below a plane mirror which reflects 90% of light falling on it. The lamp is hung 4 m above ground. Find the illumination at a point on the ground 3 m away from the point vertically below the lamp.

**Solution.** The lamp and mirror arrangement is shown in Fig. 49.12. The lamp  $L$  produces an image  $L'$  as far behind the mirror as it is in front. Height of the image from the ground is  $(5 + 1) = 6$  m.  $L'$  acts as the secondary source of light and its candle power is  $= 0.9 \times 100 = 90$  candela.

Illumination at point  $B$  equals the sum of illumination due to  $L$  and that due to  $L'$ .

$$\begin{aligned} \therefore E_B &= \frac{100}{(LB)^2} \times \cos \theta + \frac{90}{(L'B)^2} \cos \theta_1 \\ &= \frac{100}{5^2} \times \frac{4}{5} \times \frac{90}{45} \times \frac{6}{\sqrt{45}} = 5 \text{ lux} \end{aligned}$$

**Example 49.8.** A light source having an intensity of 500 candle in all directions is fitted with a reflector so that it directs 80% of its light along a beam having a divergence of  $15^\circ$ . What is the total light flux emitted along the beam? What will be the average illumination produced on a surface normal to the beam direction at a distance of 10 m? (A.M.I.E.)

**Solution.** Total flux emitted along the beam =  $0.8 (4\pi \times 500) = 5,227$  lm

Beam angle,  $\theta = 15^\circ, l = 10$  m

Radius of the circle to be illuminated,  $r = l \tan \theta/2$   
 $= 10 \tan 15^\circ/2 = 1.316$  m

Area of the surface to be illuminated,  $A = \pi r^2 = \pi \times 1.316^2$   
 $= 5.44$  m<sup>2</sup>

$\therefore$  Average illumination =  $5227/5.44 = 961$  lux

**Example 49.9.** A lamp has a uniform candle power of 300 in all directions and is fitted with a reflector which directs 50% of the total emitted light uniformly on to a flat circular disc of 20 m diameter placed 20 m vertically below the lamp. Calculate the illumination (a) at the centre and (b) at the edge of the surface without the reflector. Repeat these two calculations with the reflector provided. (Electrical Engg., Grad I.E.T.E.)

**Solution.** It should be noted that the formula  $E = I \cos \theta/r^2$  will not be applicable when the reflector is used. Moreover, with reflector, illumination would be uniform.

**Without Reflector**

(a)  $E = 300 \times 1/20^2 = 0.75$  lm/m<sup>2</sup>

(b) Here,  $\theta = \tan^{-1} (10/20) = 26.6^\circ, \cos \theta = 0.89; x^2 = 10^2 + 20^2 = 500$

$\therefore E = 300 \times 0.89/500 = 0.534$  lm/m<sup>2</sup>

**With Reflector**

Luminous output of the lamp =  $300 \times 4\pi$  lumen

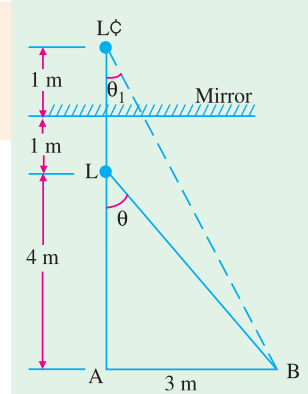


Fig. 49.12

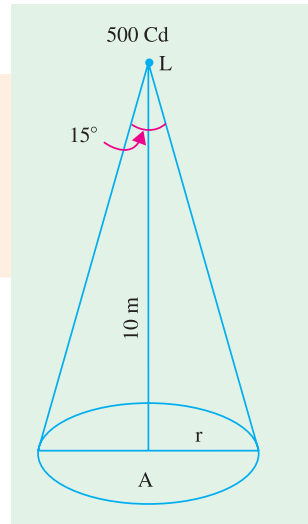


Fig. 49.13

Flux directed by the reflector =  $0.5 \times 1200 \pi = 600 \pi \text{ lm}$

Illumination produced on the disc =  $600 \pi / 100 \pi = 6 \text{ lm/m}^2$

It is the same at every point of the disc.

**Example 49.10.** A light is placed 3 m above the ground and its candle power is  $100 \cos \theta$  in any downward direction making an angle  $\theta$  with the vertical. If P and Q are two points on the ground, P being vertically under the light and the distance PQ being 3 m, calculate.

(a) the illumination of the ground at P and also at Q.

(b) the total radiations sent down by the lamp.

**Solution.** With reference to Fig. 49.14

(a) C.P. along LP =  $100 \times \cos 0^\circ = 100 \text{ cd}$   $\therefore E_P = 100/3^2 = 11.1 \text{ lm/m}^2$

C.P. along LQ =  $100 \times \cos 45^\circ = 70.7$   $\therefore E_Q = 70.7 \times \cos 45^\circ / 18 = 1.39 \text{ lm/m}^2$

(b) Consider an imaginary hemisphere of radius  $r$  metre at whose centre lies the given lamp (Fig. 49.15).

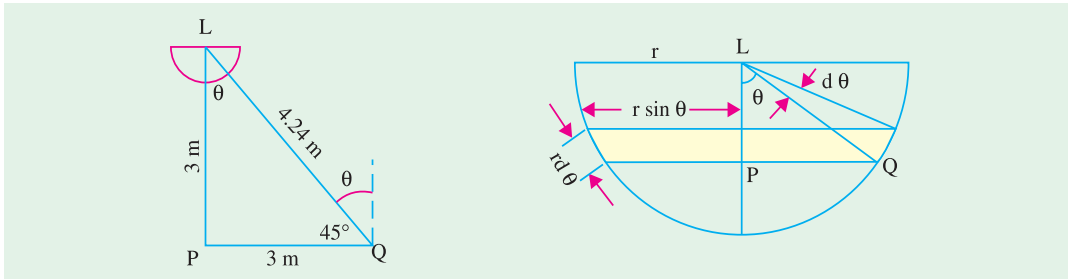


Fig. 49.14

Fig. 49.15

C.P. along LQ =  $100 \cos \theta$   $\therefore E_Q = 100 \cos \theta / r^2$

The area of the elementary strip at an angular distance  $\theta$  from the vertical and of width  $PQ = r \cdot d\theta$  is  $= (2\pi r \sin \theta) \times r \cdot d\theta = 2\pi r^2 \sin \theta \cdot d\theta$ .

Flux incident on the shaded area

$$= \frac{100 \cos \theta}{r^2} 2\pi r^2 \sin \theta \cdot d\theta = 100 \pi \cdot 2 \sin \theta \cdot \cos \theta \cdot d\theta = 100 \pi \sin 2\theta \cdot d\theta$$

Total flux over the hemisphere can be obtained by integrating the above expression between proper limits.

$$\therefore \text{total flux} = \int_0^{\pi/2} 100 \pi \sin 2\theta \cdot d\theta = 100 \pi \left[ -\frac{\cos 2\theta}{2} \right]_0^{\pi/2} = \frac{100 \pi}{2} = 100 \pi = 314 \text{ lm}$$

**Example 49.11.** A drawing office containing a number of boards and having a total effective area of  $70 \text{ m}^2$  is lit by a number of 40 W incandescent lamps giving  $11 \text{ lm/W}$ . An illumination of  $80 \text{ lux}$  is required on the drawing boards. Assuming that  $60\%$  of the total light emitted by the lamps is available for illuminating the drawing boards, estimate the number of lamps required.

**Solution.** Let  $N$  be the number of 40 W lamps required.

Output/lamp =  $40 \times 11 = 440 \text{ lm}$  ; Total flux =  $440 N \text{ lm}$

Flux actually utilized =  $0.6 \times 440 N = 264 N \text{ lm}$

Illumination required =  $80 \text{ lux} = 80 \text{ lm/m}^2$

Total flux required at the rate of  $80 \text{ lm/m}^2 = 80 \times 70 \text{ lm} = 5600 \text{ lm}$

$264 N = 5600 \therefore N = 21$

**Example 49.12.** A perfectly diffusing surface has a luminous intensity of 10 candelas at an angle of  $60^\circ$  to the normal. If the area of the surface is  $100 \text{ cm}^2$ , determine the brightness and total flux radiated.

**Solution.** Brightness  $B$  is defined as the luminous intensity divided by the projected area (Fig. 49.16).

$$\begin{aligned} \text{Luminous intensity} &= 10 \text{ candela} \\ \text{Projected area} &= A \cos \theta \\ &= 100 \times \cos 60^\circ \\ &= 50 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore B &= 10/50 \times 10^{-4} \\ &= 2 \times 10^3 \text{ cd/m}^2 \\ &= \mathbf{2\pi \times 10^3 \text{ lm/m}^2} \text{—Art. 46.4} \\ \text{Total flux radiated} &= 2\pi \times 10^3 \times 100 \times 10^{-4} \\ &= \mathbf{68.2 \text{ lm}} \end{aligned}$$

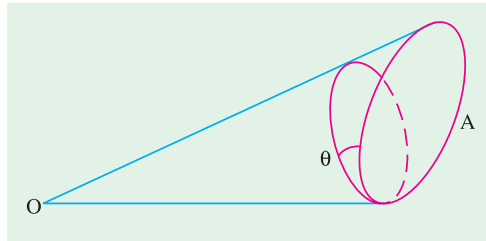


Fig. 49.16

**Example 49.13.** Calculate the brightness (or luminance) of snow under an illumination of (a) 44,000 lux and (b) 0.22 lux. Assume that snow behaves like a perfect diffusor having a reflection factor of 85 per cent.

**Solution.**

$$\begin{aligned} \text{(a) } L &= \rho E/\pi = 44,000 \times 0.85/\pi \text{ cd/m}^2 = \mathbf{1.19 \times 10^4 \text{ cd/m}^2} \\ \text{(b) } L &= \frac{0.22 \times 0.85}{\pi} = \mathbf{5.9 \times 10^{-2} \text{ cd/m}^2} \end{aligned}$$

**Example 49.14.** A 21 cm diameter globe of dense opal glass encloses a lamp emitting 1000 lumens and has uniform brightness of  $4 \times 10^3$  lumen/m<sup>2</sup> when viewed in any direction. What would be the luminous intensity of the globe in any direction? Find what percentage of the flux emitted by the lamp is absorbed by the globe.

**Solution.**

$$\begin{aligned} \text{Surface area of the globe} &= \pi d^2 = \pi \times 21^2 = 1,386 \text{ cm}^2 = 0.1386 \text{ m}^2 \\ \text{Flux emitted by the globe is} &= 0.1386 \times 4 \times 10^3 = 554.4 \text{ lumen} \\ \text{Now,} & \qquad \qquad \qquad 1 \text{ candela} = 4\pi \text{ lumens} \\ \text{Hence, luminous intensity of globe is} &= 554.4/4\pi = \mathbf{44 \text{ cd}} \\ \text{Flux absorbed by the globe is} &= 1000 - 554.4 = 445.6 \text{ lm.} \\ \therefore \text{percentage absorption} &= 445.6 \times 100/1000 = \mathbf{44.56\%} \end{aligned}$$

**Example 49.15.** A 2.5 cm diameter disc source of luminance  $1000 \text{ cd/cm}^2$  is placed at the focus of a specular parabolic reflector normal to the axis. The focal length of the reflector is 10 cm, diameter 40 cm and reflectance 0.8. Calculate the axial intensity and beam-spread. Also show diagrammatically what will happen if the source were moved away from the reflector along the axis in either direction. (A.M.I.E. Sec. B, Winter 1991)

**Solution.** The axial or beam intensity  $I$  depends upon

- (i) luminance of the disc source i.e.  $L$
- (ii) aperture of the reflector i.e.  $A$
- (iii) reflectivity of the reflector i.e.  $r$

$$\begin{aligned} \therefore I &= \rho A L \text{ candela} \\ \text{Now, } L &= 1000 \text{ cd/cm}^2 = 10^7 \text{ cd/m}^2 \\ A &= \pi d^2/4 = \pi \times 0.4^2/4 = 125.7 \times 10^{-3} \text{ m}^2 \\ \therefore I &= 0.8 \times 125.7 \times 10^{-3} \times 10^7 = \mathbf{1,005,600 \text{ cd}} \end{aligned}$$

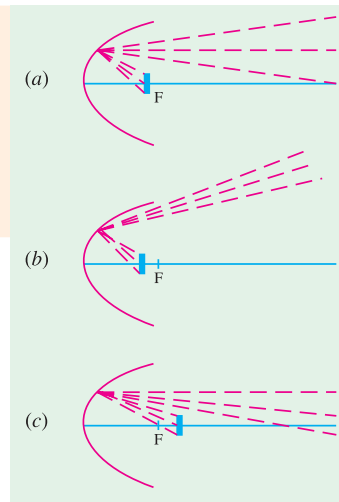


Fig. 49.17

To a first approximation, the beam spread for disc source is determined by reflector focal length and the size of the disc source. If  $\theta$  is the total beam spread when the source is at the focus of the reflector [Fig. 49.17] (a)] then

$$\theta = 2 \tan^{-1} (r/f)$$

Here,

$$r = 2.5/2 = 1.25 \text{ cm ; } f = 10 \text{ cm}$$

$\therefore$

$$\theta = 2 \tan^{-1} (1.25/10) = 2 \times 7^\circ 7' = \mathbf{14^\circ 14'}$$

The effect of axial movement of the source is shown in Fig. 49.17 (b) and (c).

**Example 49.16.** A 22 diameter globe of opal glass encloses a lamp of uniform luminous intensity 120 C.P. Thirty per cent of light emitted by the lamp is absorbed by globe. Determine (a) luminance of globe (b) C.P. of globe in any direction.

**Solution.**

(a) Flux emitted by source =  $120 \times 4\pi \text{ lm}$  ; flux emitted by globe =  $0.7 \times 480 \pi \text{ lm}$

$$\therefore L = \frac{0.7 \times 480 \pi}{\pi \times 0.22^2} = \mathbf{6,940 \text{ lm/m}^2}$$

(b) Since 1 candela =  $4 \pi \text{ lm}$

$\therefore$  candle-power or luminous intensity of the globe is

$$= \frac{\text{flux in lumens}}{4 \pi} = \frac{0.7 \times 480 \pi}{\pi \times 4} = \mathbf{84 \text{ cd}}$$

**Example 49.17.** A 0.4 m diameter diffusing sphere of opal glass (20 percent absorption) encloses an incandescent lamp with a luminous flux of 4850 lumens. Calculate the average luminance of the sphere. **(A.M.I.E. Sec. B, Summer 1993)**

**Solution.** Flux emitted by the globe 80% or  $4850 = 3880 \text{ lm}$

Surface area of the globe =  $4\pi r^2 = \pi d^2 \text{ m}^2$

$$B = \frac{\text{flux emitted}}{\text{surfacer area}} = \frac{3880}{\pi \times 0.4^2} = \mathbf{7,720 \text{ lm/m}^2}$$

### Tutorial Problem No. 49.1

1. A high-pressure mercury-vapour lamp is mounted at a height of 6 m in the middle of a large road crossing. A special reflector directs 100 C.P. maximum in a cone of  $70^\circ$  to the vertical line. Calculate the intensity of illumination on the road surface due to this beam of 100 C.P.  
**(Electrical Engineering, Bombay Univ.)**
2. A room  $6\text{ m} \times 4 \text{ m}$  is illuminated by a single lamp of 100 C.P. in all directions suspended at the centre 3 m above the floor level. Calculate the illumination (i) below the lamp and (ii) at the corner of the room.  
**(Mech. & Elect. Engg. : Gujarat Univ.)**
3. A lamp of 100 candle-power is placed at the centre of a room  $10 \text{ m} \times 6 \text{ m} \times 4 \text{ m}$  high. Calculate the illumination in each corner of the floor and at a point in the middle of a 6 m wall at a height of 2 m from the floor.  
**(Utilization of Elect. Power A.M.I.E.)**
4. A source of 5000 lumen is suspended 6.1 m. above ground. Find out the illumination (i) at a point just below the lamp and (ii) at a point 12.2 m away from the first, assuming uniform distribution of light from the source.  
**[(i) 10.7 lux (ii) 0.96 lux] (A.M.I.E. Sec. B)**
5. Determine the average illumination of a room measuring 9.15 m by 12.2 m illuminated by a dozen 150 W lamps. The luminous efficiency of lamps may be taken as 14 lm/W and the co-efficient of utilisation as 0.35.  
**[79 lux] (A.M.I.E. Sec. B)**
6. Two lamps are hung at a height of 9 m from the floor level. The distance between the lamps is one metre. Lamp one is of 500 candela. If the illumination on the floor vertically below this lamp is 20 lux, find the candle power of the lamp number two. **[1140 candela] (Utili. of Elect. Power A.M.I.E.)**

7. Two powerful street lamps of 1,000 candela and 800 candela (assumed uniform in all directions) are mounted 12.5 m above the road level and are spaced 25 metres apart. Find the intensity of horizontal illumination produced at a point on the ground in-between the lamp posts and just below the lamp posts. [4.07 lux, 6.86 lux, 5.69 lux] (A.M.I.E.)
8. It is required to provide an illumination of 100 lux in a factory hall 30 m by 15 m. Assume that the depreciation factor is 0.8, coefficient of utilisation is 0.4 and efficiency of lamp is 14 lm/W. Suggest the number of lamps and their ratings. The sizes of the lamps available are 100, 250, 400 and 500 W. [40 lamps of 250 W in 5 rows]
9. It is required to provide an illumination of 100 lm/m<sup>2</sup> in a workshop hall 40m × 10m and efficiency of lamp is 14 lm/W. Calculate the number and rating of lamps and their positions when trusses are provided at mutual distance of 5m. Take coefficient of utilisation as 0.4 and depreciation factor as 0.8. [14 lamps of 750 W each]
10. A drawing hall 30 m by 15 m with a ceiling height of 5 m is to be provided with a general illumination of 120 lux. Taking a coefficient of utilization of 0.5 and depreciation factor of 1.4, determine the number of fluorescent tubes required, their spacing, mounting height and total wattage. Taking luminous efficiency of fluorescent tube as 40 lm/W for 80 W tubes. [48, 24 twin-tube units each tube of 80 W; row spacing of 5 m and unit spacing of 3.75m, 3840 W] (Utilisation of Elect. Power, A.M.I.E.)

### 49.6. Laws Governing Illumination of Different Sources

The laws applicable to the illumination produced by the following three types of sources will be considered.

(i) Point Source

As discussed in Art. 49.5, the law governing changes in illumination due to point source of light is  $E = I \cos \theta / d^2$ .

(ii) Line Source

Provided the line source is of infinite length and of uniform intensity, the illumination at a point lying on a surface parallel to and facing the line source is given by

$$E = \frac{\pi I}{2d} \text{ lm/m}^2$$

where

$I$  = luminous intensity normal to the line source (in candles per-meter length of the sources)

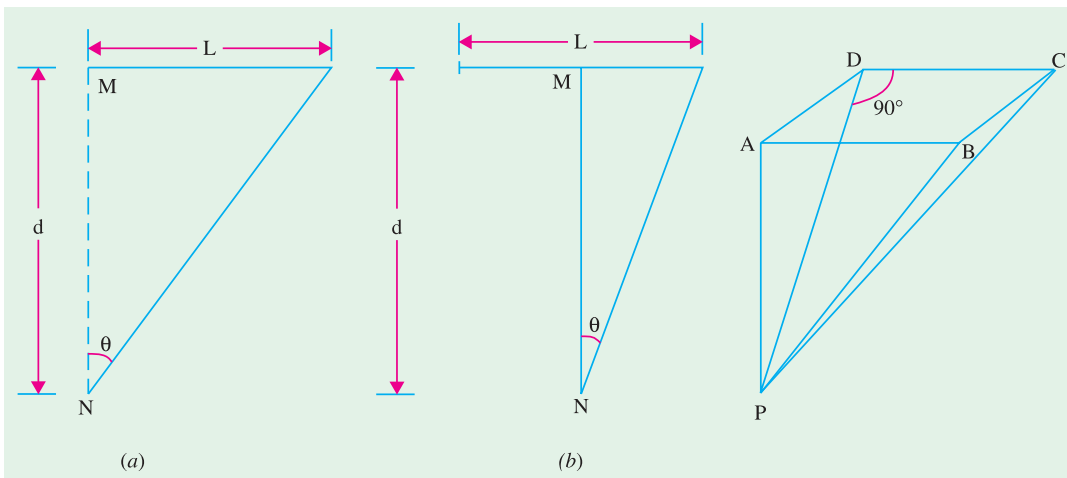


Fig. 49.18

Fig. 49.19

However, in practice, the line sources are of finite length, so that the following law applies

$$E = \frac{I}{4d} (\sin 2\theta + 2\theta) \text{ lm/m}^2 \quad \text{---Fig. 49.18 (a)}$$

$$= \frac{I}{2d} (\sin 2\theta + 2\theta) \text{ lm/m}^2 \quad \text{---Fig. 49.18 (b)}$$

where

$I$  = candle power per metre length in a direction normal to the line source

$$= \frac{\Phi}{\pi^2 L} \text{ cd/m}$$

where  $\Phi$  is the total flux of the source in lumens and  $L$  is the length of the line source in metres.

**(iii) Surface Source**

Provided the surface source is of infinite area and of uniform brightness, illumination at any point facing the source is independent of the distance between the point and the surface source. Mathematically, its value is  $E = \pi L \text{ lm/m}^2$  where  $L$  is the luminance of the surface source in  $\text{cd/m}^2$ .

In case the surface source is limited and rectangular in shape (Fig. 49.19), the law governing the illumination at a point  $P$  is

$$E = \frac{L}{2} (\alpha' \sin \beta + \beta' \sin \alpha)$$

where

$$\alpha = \angle APD ; \alpha' = \angle BPC ; \beta = \angle APB ; \beta' = \angle DPC$$

**Note. (i)** In case, distance  $d$  is more than 5 times the greatest dimension of the source, then irrespective of its shape, the illumination is found to obey inverse square law. This would be the case for illumination at points 5 metres or more away from a fluorescent tube of length one metre.

**(ii)** In the case of surface sources of large area, such as luminous daylights covering the whole ceiling of a large room, illumination is found to be practically constant irrespective of the height of the working place.

It may be noted that a point source produces deep shadows which may, however, be cancelled by installing a large number of fittings. Usually, glare is present. However, point sources are of great practical importance where accurate light control is required as in search-lights.

Line sources give more diffusion but cast shadows of objects lying parallel to them thus hindering vision.

Large-area surface sources though generally of low brightness, produce minimum glare but no shadows. However, the final effect is not liveliness but one of deadness.

**Example 49.18.** A show case is lighted by 4 metre of architectural tubular lamps arranged in a continuous line and placed along the top of the case. Determine the illumination produced on a horizontal surface 2 metres below the lamps in a position directly underneath the centre of the 4 m length of the lamps on the assumption that in tubular lamps emit 1,880 lm per metre run. Neglect the effect of any reflectors which may be used.

**Solution.** As seen in Art 49.10

$$E = \frac{I}{2d} (\sin 2\theta + 2\theta) \text{ lm/m}^2 \text{ and } I = \frac{\Phi}{\pi^2 L} \text{ cd/m}$$

As seen from Fig. 49.15

$$\begin{aligned} \theta &= \tan^{-1} (L/2d) \\ &= \tan^{-1} (4/2) = 45^\circ \\ I &= 4 \times 1,880 / \pi^2 \times 4 \\ &= \mathbf{188 \text{ cd/m}} \end{aligned}$$

$$\therefore E = \frac{188}{2 \times 2} \left( \sin 90^\circ + \frac{\pi}{2} \right) = \mathbf{121 \text{ lm/m}^2}$$

49.7. Polar Curves of C.P. Distribution

All our calculations so far were based on the tacit assumption that the light source was of equal luminous intensity or candle-power in all directions. However, lamps and other sources of light, as a rule, do not give uniform distribution in the space surrounding them.

If the actual luminous intensity of a source in various directions be plotted to scale along lines radiating from the centre of the source at corresponding angles, we obtain the polar curve of the candle power.

Suppose we construct a figure consisting of large number of spokes radiating out from a point—the length of each spoke representing to some scale the candle power or luminous intensity of the source in that particular direction. If now we join the

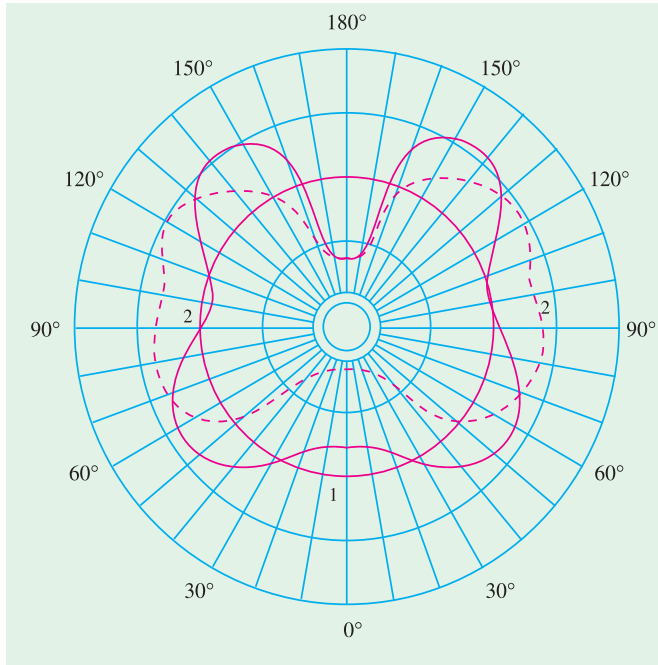


Fig. 49.20

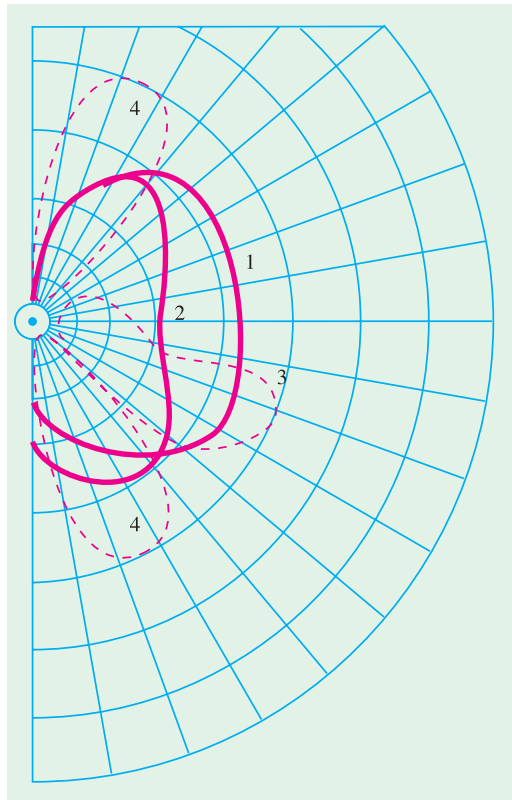


Fig. 49.21

ends of these spokes by some suitable material, say, by linen cloth, then we get a surface whose shape will represent to scale the three dimensional candle power distribution of the source placed at the centre. In the ideal case of a point source having equal distribution in all directions, the surface would be spherical.

It would be realized that it is difficult to give a graphic representation of such a 3-dimensional model in a plane surface. Therefore, as with engineering drawings, it is usual to draw only one or more elevations and a plan of sections through the centre of the source. Elevations represent c.p. distribution in the *vertical* plane and the plans represent c.p. distribution in *horizontal* plane. The number of elevations required to give a complete idea of the c.p. distribution of the source in all directions depends upon the shape of the plan *i.e.* on the horizontal distribution. If the distribution is uniform in every horizontal plane *i.e.* if the polar curve of horizontal distribution is a circle, then only one vertical curve is sufficient to give full idea of the space distribution.

In Fig. 49.20 are shown two polar curves of c.p. distribution in a vertical plane. Curve 1 is for



vacuum type tungsten lamp with zig-zag filament whereas curve 2 is for gas filled tungsten lamp with filament arranged as a horizontal ring.

If the polar curve is symmetrical about the vertical axis as in the figures given below, then it is sufficient to give only the polar curve within one semicircle in order to completely define the distribution of c.p. as shown in Fig. 49.21.

The curves 1 and 2 are as in Fig. 49.20, curves 3 is for d.c. open arc with plain carbons and curve 4 is for a.c. arc with plain carbons. However, if the source and/or reflector are not symmetrical about vertical axis, it is impossible to represent the space distribution of c.p. by a single polar diagram and even polar diagrams for two planes at right angles to each other give no definite idea as to the distribution in the intermediate planes.

Consider a filament lamp with a helmet-type reflector whose axis is inclined and cross-section elliptical—such reflectors are widely used for lighting shop windows. Fig. 49.22 represents the distribution of luminous intensity of such source and its reflector in two planes at right angles to each other.

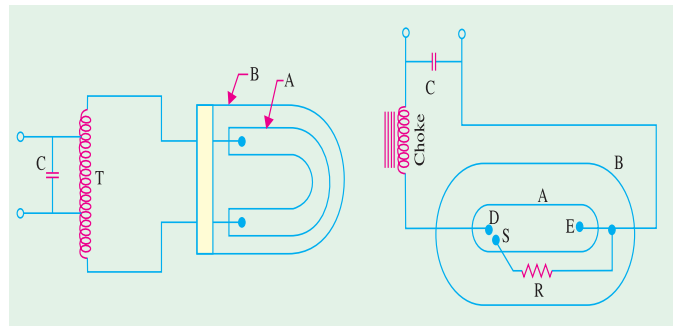


Fig. 49.22

The importance of considering the polar curves in different planes when the c.p. distribution in asymmetrical is even more strikingly depicted by the polar curves in  $YY$  plane and  $XX$  plane of a lamp with a special type of reflector designed for street lighting purposes (Fig. 49.23).

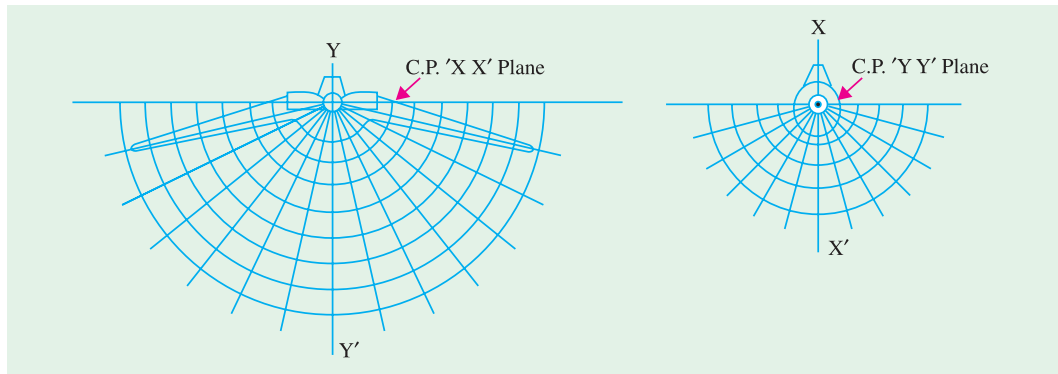


Fig. 49.23

It would be realized from above that the polar distribution of light from any source can be given any desired form by using reflectors and/or refractors of appropriate shape.

In Fig. 49.24 is shown the polar curve of c.p. distribution of a straight type of lamp in a horizontal plane.

### 49.8. Uses of Polar Curves

Polar curves are made use of in determining the M.S.C.P. etc. of a source. They are also used in determining the actual illumination of a surface *i.e.* while calculating the illumination in a particular direction, the c.p. in that particular direction as read from the vertical polar curve, should be employed.

**49.9. Determination of M.S.C.P. and M.H.C.P. from Polar Diagrams**

In Fig. 49.25 (a) is shown the polar distribution curve of a filament lamp in a horizontal plane and the polar curve in Fig. 49.25 (b) represents the c.p. distribution in a vertical plane. It will be seen that the horizontal candle-power is almost uniform in all directions in that plane. However, in the vertical plane, there is a large variation in the candle power which falls to zero behind the cap of the lamp. The curve in Fig. 49.25 (a) has been drawn with the help of a photometer while the lamp is rotated about a vertical axis, say,  $10^\circ$  at a time. But the curve in Fig. 49.25 (b) was drawn while the lamp was rotated in a vertical plane about a horizontal axis passing through the centre of the filament.

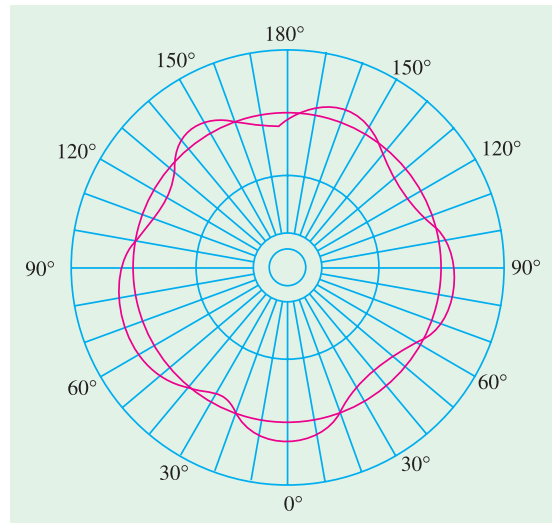


Fig. 49.24

The M.H.C.P. is taken as the mean of the readings in Fig. 49.25 (a). However, a more accurate result can be obtained by plotting candle power on an angular base along the rectangular axes and by determining the mean height of the curve by the mid-ordinate or by Simpson's rule.

The M.S.C.P. of the lamp can be obtained from the vertical polar curve of Fig. 49.25 (b) by Rousseau's construction as explained below :

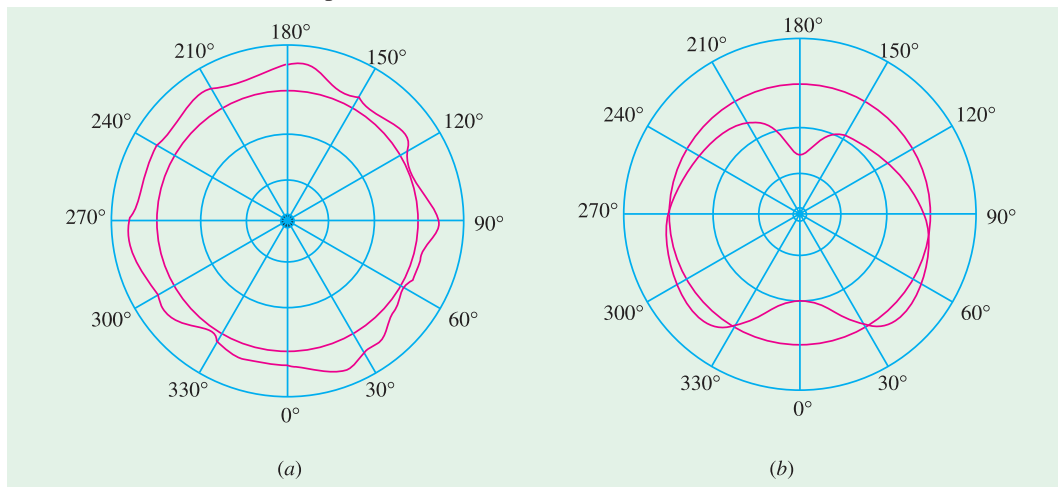


Fig. 49.25

Only half of the vertical polar curve is shown in the figure (Fig. 49.26) since it is symmetrical about the vertical axis. With  $O$  is the centre and radius  $OR$  equal to the maximum radius of the polar curve, a semi-circle  $LRM$  is drawn. A convenient number of points on this semi-circle (say  $10^\circ$  points) are projected onto any vertical plane as shown. For example, points  $a, b, c$  etc. are projected to  $d, e, f$  and so on. From point  $d$ , the horizontal line  $dg$  is drawn equal to the intercept  $OA$  of the polar diagram on the radius  $oa$ . Similarly,  $eh = OB$ ,  $fk = OC$  and so on. The points  $g, h, k$  etc., define the Rousseau figure. The average width  $w$  of this figure represents the M.S.C.P. to the same scale as that of the candle powers in the polar curve. The average width is obtained by dividing the Rousseau area by the base of the Rousseau figure *i.e.* length  $lm$  which is the projection of the semi-circle  $LM$  on the vertical axis. The area may be determined by Simpson's rule or by using a planimeter.

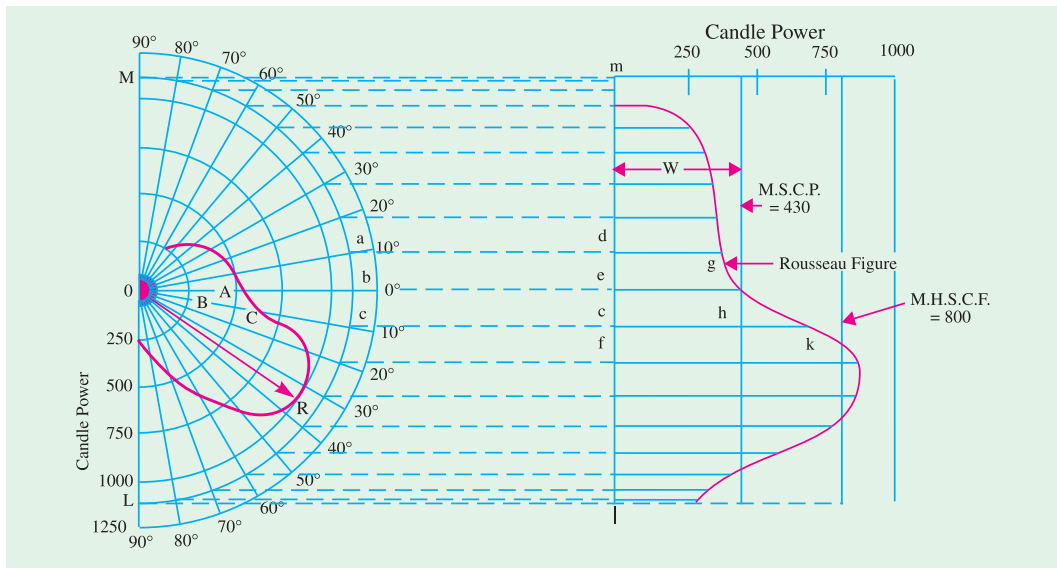


Fig. 49.26

∴ 
$$\text{M.S.C.P.} = \frac{\text{area of Rousseau figure}}{\text{length of the base}}$$

As explained earlier, the M.H.C.P. of an incandescent lamp can be easily obtained by mounting the lamp with its axis vertical and taking photometer readings in the horizontal plane while the lamp is rotated about its axis in steps of 10° or so. A definite ratio exists between the M.H.C.P. and M.S.C.P. of each particular type of filament. M.S.C.P. of a lamp can be found by multiplying M.H.C.P. by a factor known as spherical reduction factor which, as defined earlier, is

Spherical reduction factor  $f = \frac{\text{M.S.C.P.}}{\text{M.H.C.P.}}$  ∴  $\text{M.S.C.P.} = f \times \text{M.H.C.P.}$

For the particular lamp considered,  $f = 430/80 = 0.54$  (approx.)

Typical values of this factor are :

Ordinary vacuum-type tungsten lamp having zig-zag filament	0.76 – 0.78
Gas-filled tungsten lamp with filament in the form of broad shallow V's	0.85 – 0.94
Gas-filled tungsten lamp with filament in the shape of a horizontal ring	1.0 – 1.2

The total lumen output is given by the relation ; lumen output =  $4\pi \times \text{M.S.C.P.}$

In the present case, lumen output =  $4\pi \times 430$   
= 5,405 lm

### 49.10. Integrating Sphere or Photometer

The M.S.C.P. is usually measured by means of an integrating photometer, the most accurate form of which consists of a hollow sphere (as originally proposed by Ulbricht) whose diameter is large (at least 6 times) as compared to that of the lamp under test. The interior surface of the hollow sphere is whitened by means of a special matt white paint. When the lamp is placed inside the sphere (not

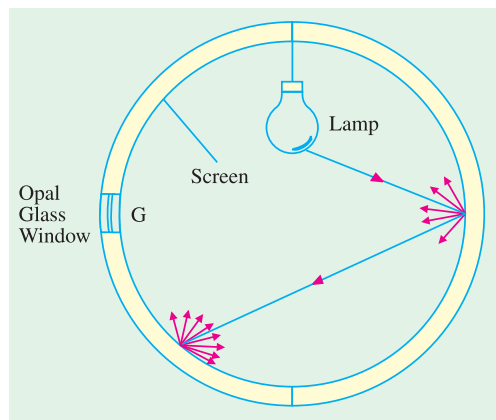


Fig. 49.27

necessarily at its centre) then due to successive reflections, its light is so diffused as to produce a uniform illumination over the whole surface. At some point, a small matt opal-glass window, shaded from the direct rays of the source, is made in the hollow sphere.

The brightness of the matt opal glass is proportional to that of the interior surface of the sphere. By using a suitable illumination photometer, the illumination of the window can be measured which can be used to find out the total flux emitted by the source.

$$\text{Total flux} = \text{illumination (lm/m}^2) \times \text{surface area of the sphere (m}^2)$$

$$\therefore \text{M.S.C.P.} = \frac{\text{total flux}}{4\pi} \text{ candela}$$

**Theory.** In Fig. 49.28 is shown a light source  $L$  of luminous intensity  $I$  candela and having a total flux output of  $F_L$  placed at the centre of an integrating sphere of radius  $r$  and reflection factor  $\rho$ . Let  $E_A$  and  $E_B$  represent the illuminations at two points  $A$  and  $B$ , each of infinitely small area  $da$  and  $db$  respectively and distance  $m$  apart. We will now consider total illumination (both direct and reflected) at point  $A$ .

$$\text{Obviously, } E_A \text{ directly due to } L = I/r^2$$

$$E_B \text{ directly due to } L = I/r^2$$

Luminous intensity of  $B$  in the direction of  $A$  is

$$I_B = \frac{\rho \cdot E_B \cdot A_B}{\pi} \text{ candela} \quad \text{—Art. 45.4}$$

where  $A_B =$  projected area of  $B$  at right angles to the line  $BA = db \cdot \cos \theta$

$$\therefore I_B = \frac{\rho \cdot I \cdot db \cos \theta}{\pi r^2} \text{ candela}$$

$$\text{Hence, illumination of } A \text{ due to } B \text{ is} = \frac{I_B \cos \theta}{m^2} = \frac{\rho I \cdot db \cos^2 \theta}{\pi r^2 \times m^2}$$

$$\text{Now, as seen from Fig. 49.28, } m = 2r \cos \theta$$

$\therefore$  illumination of  $A$  due to  $B$  becomes

$$= \frac{\rho \cdot I \cdot db \cos^2 \theta}{\pi r^2 \times 4r^2 \cos^2 \theta} = \frac{\rho}{4\pi r^2} \times \frac{I}{r^2} \times db = \frac{\rho}{S} \cdot E_B \cdot db = \frac{\rho F_B}{D}$$

where

$$F_B = \text{flux incident on } B \text{ and } A = \text{surface area of the sphere}$$

$$\text{Hence, total illumination due to first reflection} = \sum \frac{\rho F_B}{S} = \frac{\rho F_B}{S}$$

Now, consider any other point  $C$ . Illumination on  $B$  due to point  $C = \rho F_L/S$ . The illumination on  $A$  due to  $C$  as reflected from  $B$ .

$$\begin{aligned} &= \left[ \rho \cdot \left( \frac{\rho F_L}{S} \right) \times \frac{db \cos \theta}{\pi} \right] \times \frac{\cos \theta}{m^2} = \frac{\rho F_L}{S} \times \frac{\rho \cdot db \cos \theta}{\pi} \times \frac{\cos \theta}{4r^2 \cos^2 \theta} \\ &= \frac{\rho F_L}{S} \times \frac{\rho \cdot db}{S} \end{aligned}$$

$$\text{Total illumination due to two reflections} = \sum \frac{\rho F_L}{S} \times \frac{\rho \cdot db}{S} = \frac{\rho^2 F_L}{S} \quad (\because \sum ab = S)$$

Continuing this way, it can be proved that total illumination at point  $A$  from all reflections from all points

$$= \frac{\rho F_L}{S} (1 + \rho^2 + \rho^3 + \dots + \rho^{n-1}) = \frac{\rho F_L}{S} \left( \frac{1}{1 - \rho} \right)$$

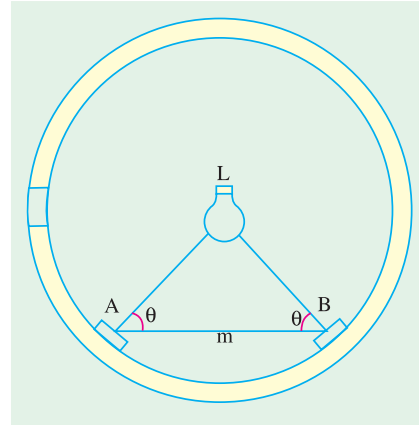


Fig. 49.28

Hence, total illumination at A from direct and reflected lights is

$$= E_A + \frac{\rho F_L}{S} \left( \frac{1}{1-\rho} \right)$$

If A is shielded from lamp L, then its illumination is proportional to  $F_L$  because  $\frac{\rho}{S} \left( \frac{1}{1-\rho} \right)$  is a constant factor. Obviously, if either brightness or illumination at one point in the sphere is measured, it would be proportional to the light output of the source. This fact is made use of while using this sphere as a globe photometer.

**Example 49.19.** If an integrating sphere 0.6 m in diameter whose inner surface has a reflection coefficient of 0.8 contains a lamp producing on the portion of the sphere, screened from direct radiation, a luminance of 1000 cd/m<sup>2</sup>, what is the luminous flux yield of the source ?

(A.M.I.E. Sec. B. Summer 1986)

**Solution.** Obviously, the screened portion of the sphere receives light by reflection only. Reflection illumination of the screened portion is

$$E = \frac{\rho F_L}{S} \left( \frac{1}{1-\rho} \right) = \frac{0.8 F_L}{\pi \times 0.6^2} \left( \frac{1}{1-0.8} \right) = \frac{100 F_L}{9\pi} \text{ lm/m}^2$$

Also  $L = \rho E / \pi$  — Art. 49.4

$$\therefore 1000 = \frac{100 F_L}{9\pi} \times \frac{0.8}{\pi} \quad \text{or} \quad F_L = 1/25 \text{ lm}$$

### 49.11. Diffusing and Reflecting Surfaces : Globes and Reflectors

When light falls on polished metallic surfaces or silvered surfaces, then most of it is reflected back according to the laws of reflection *i.e.* the angle of incidence is equal to the angle of reflection. Only a small portion of the incident light is absorbed and there is always the image of the source. Such reflection is known as *specular* reflection.

However, as shown in Fig. 49.29 (b), if light is incident on coarse surfaces like paper, frosted glass, painted ceiling etc., then it is scattered or diffused in all directions, hence no image of the source is formed. Such reflection of light is called *diffuse reflection*. A perfect diffuser is one that scatters light uniformly in all directions and hence appears equally bright from whatever direction it is viewed. A white blotting paper is the nearest approach to a diffuser.

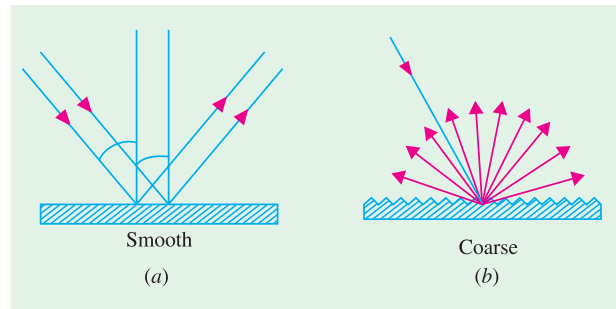


Fig. 49.29

By reflecting factor of a surface is meant the ratio =  $\frac{\text{reflected light}}{\text{incident light}}$

It is also known as reflection ratio or coefficient of reflection of a surface.

If the light is incident on a transparent surface, then some of it is absorbed and greater percentage of it passes through and emerges on the other side.

To avoid direct glare from electric arcs and incandescent filament lamps, they are surrounded more or less completely by diffusing shades or globes. In addition, a reflector may also be embodied to prevent the escape of light in directions where it serves no useful purpose. In that case, so far as the surroundings are concerned, the diffusing globe is the source of light. Its average brilliancy is lower

the more its diffusing area. Depending on the optical density, these globes absorb 15 to 40% of light emitted by the encircled bulb. The bulbs may also be frosted externally by etching or sand-blasting but internal frosting is better because there is no sharp scratching or cracks to weaken the glass.

In domestic fittings, a variety of shades are used whose main purpose is to avoid glare. Properly designed and installed prismatic glass shades and holophane type reflectors have high efficiency and are capable of giving accurate predetermined distribution of light.

Regular metallic reflection is used in search-light mirrors and for general lighting purposes. But where it is used for general lighting, the silvered reflectors are usually fluted to make the illumination as uniform as possible.

Regular cleaning of all shades, globes, and reflectors is very important otherwise the loss of light by absorption by dust etc., collected on them becomes very serious.

Various types of reflectors are illustrated in Figs. 49.30 to 49.34. Fig. 49.30 shows a holophane stiletto reflector used where extensive, intensive or focussing light distribution is required.

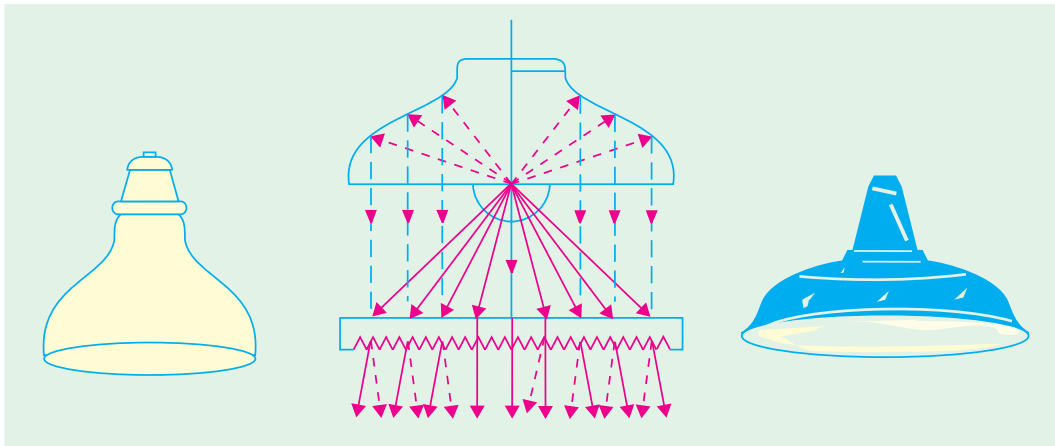


Fig. 49.30

Fig. 49.31

Fig. 49.32

The optical combination of a lamp, reflector and a lens plate, as shown in Fig. 49.31, provides a high degree of light control. Multiple panels can be conveniently incorporated in fittings suited to different architectural schemes.

The dispersive reflector of Fig. 49.32 is suitable for practically all classes of industrial installations. The reflector is a combination of concave and cylindrical reflecting surfaces in the form of a deep bowl of wide dispersive power. It gives maximum intensity between  $0^\circ$  and  $45^\circ$  from the vertical.

The concentrating reflector of parabolic form shown in Fig. 49.33 is suitable for situations requiring lofty installations and strongly-concentrated illumination as in public halls, foundries and power stations etc. They give maximum intensity in zones from  $0^\circ$  to  $25^\circ$  from the vertical.

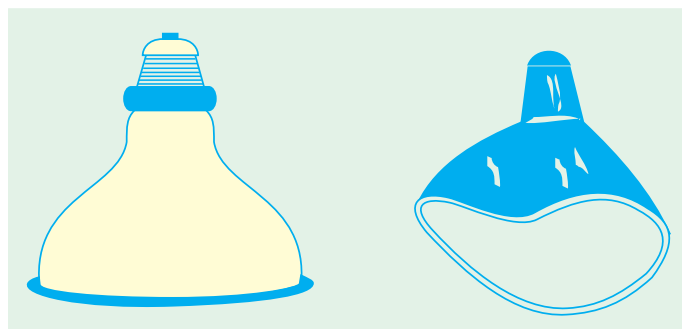


Fig. 49.33

Fig. 49.34

The elliptical angle reflector shown in Fig. 49.34 is suitable for

the side lighting of switchboards, show windows etc., because they give a forward projection of light in the vertical plane and spread the light in the horizontal plane.

### 49.12. Lighting Schemes

Different lighting schemes may be classified as (i) direct lighting (ii) indirect lighting and (iii) semi-direct lighting (iv) semi-indirect lighting and (v) general diffusing systems.

#### (i) Direct Lighting

As the name indicates, in the form of lighting, the light from the source falls directly on the object or the surface to be illuminated (Fig. 49.35). With the help of shades and globes and reflectors of various types as discussed in Art. 49.11, most of the light is directed in the lower hemisphere and also the brilliant source of light is kept out of the direct line of vision. Direct illumination by lamps in suitable reflectors can be supplemented by standard or bracket lamps on desk or by additional pendant fittings over counters.

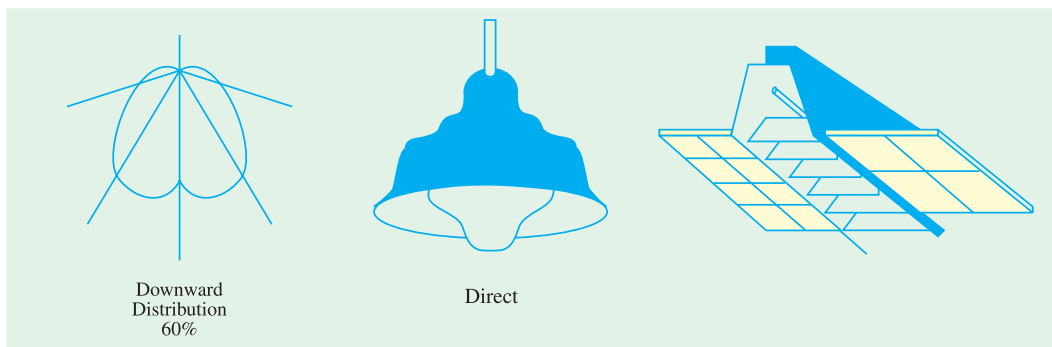


Fig. 49.35

The fundamental point worth remembering is planning any lighting installation is that sufficient and sufficiently uniform lighting is to be provided at the working or reading plane. For this purpose, lamps of suitable size have to be so located and furnished with such fittings as to give correct degree and distribution of illumination at the required place. Moreover, it is important to keep the lamps and fittings clean otherwise the decrease in effective illumination due to dirty bulbs or reflectors may amount to 15 to 25% in offices and domestic lighting and more in industrial areas as a result of a few weeks neglect.

Direct lighting, though most efficient, is liable to cause glare and hard shadows.

#### (ii) Indirect Lighting

In this form of lighting, light does not reach the surface directly from the source but indirectly by diffuse reflection (Fig. 49.36). The lamps are either placed behind a cornice or in suspended *opaque* bowls. In both cases, a silvered reflector which is corrugated for eliminating striations is placed beneath the lamp.

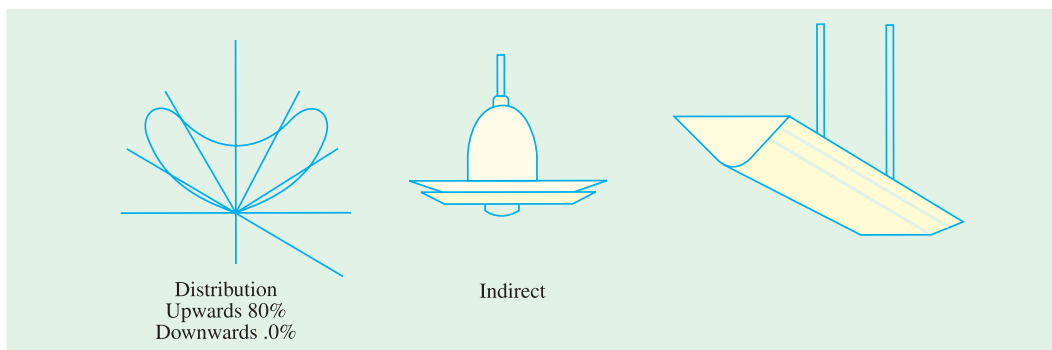


Fig. 49.36

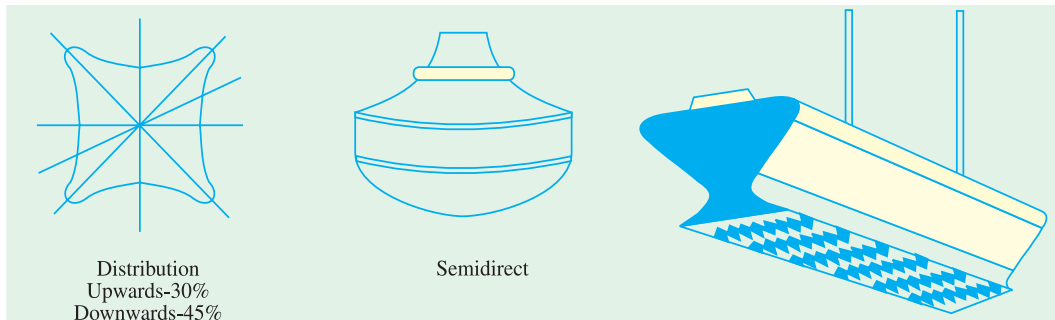
In this way, maximum light is thrown upwards on the ceiling from which it is distributed all over the room by diffuse reflection. Even gradation of light on the ceiling is secured by careful adjustment of the position and the number of lamps. In the cornice and bowl system of lighting, bowl fittings are generally suspended about three-fourths the height of the room and in the case of cornice lighting, a frieze of curved profile aids in throwing the light out into the room to be illuminated. Since in indirect lighting whole of the light on the working plane is received by diffuse reflection, it is important to keep the fittings clean.

One of the main characteristics of indirect lighting is that it provides shadowless illumination which is very useful for drawing offices, composing rooms and in workshops especially where large machines and other obstructions would cast troublesome shadows if direct lighting were used.

However, many people find purely indirect lighting flat and monotonous and even depressive. Most of the users demand 50 to 100% more light at their working plane by indirect lighting than with direct lighting. However, for appreciating relief, a certain proportion of direct lighting is essential.

### (iii) Semi-direct System

This system utilizes luminaires which send most of the light downwards directly on the working plane but a considerable amount reaches the ceilings and walls also (Fig. 49.37).

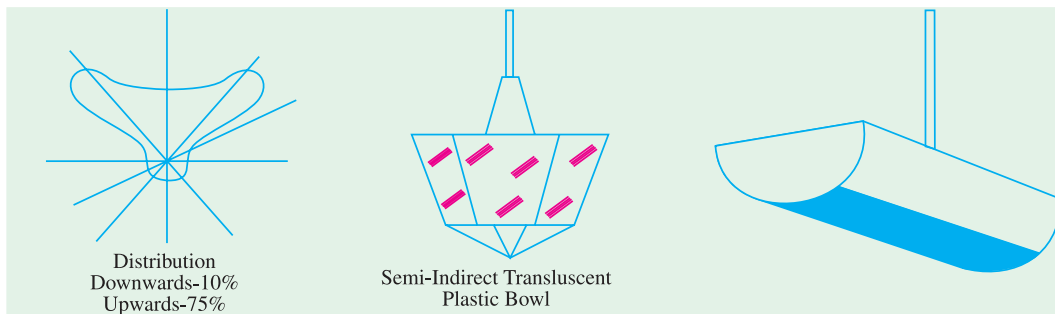


**Fig. 49.37**

The division is usually 30% upwards and 45% downwards. Such a system is best suited to rooms with high ceilings where a high level of uniformly-distributed illumination is desirable. Glare in such units is avoided by using diffusing globes which not only improve the brightness towards the eye level but improve the efficiency of the system with reference to the working plane.

### (iv) Semi-indirect Lighting

In this system which is, in fact, a compromise between the first two systems, the light is partly received by diffuse reflection and partly direct from the source (Fig. 49.38). Such a system, therefore, eliminates the objections of indirect lighting mentioned above. Instead of using opaque bowls with reflectors, translucent bowls without reflector are used. Most of the light is, as before, directed upwards to the ceiling for diffuse reflection and the rest reaches the working plane directly except for some absorption by the bowl.



**Fig. 49.38**



**(v) General Diffusing System**

In this system, luminaires are employed which have almost equal light distribution downwards and upwards as shown in Fig. 49.39.

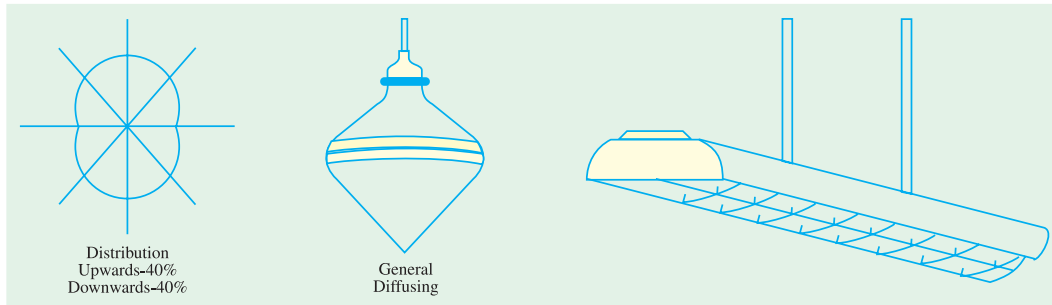


Fig. 49.39

**49.13. Illumination Required for Different Purposes**

There has been a steady movement towards higher intensities for artificial illumination during the last few decades. The movement is likely to continue because the highest intensities in average installations are much less than those of the diffused daylight. The human eye poses a tremendous power of accommodation and it can work comfortably within an enormous range of illuminations.

For example, at full noon, sun provides about 120,000 lm/m<sup>2</sup>, diffuse day-light near a window is of the order of 600 lm/m<sup>2</sup> (value varying widely) and full moon-light gives 0.1 to 0.3 lm/m<sup>2</sup>. For reading, usually 20 to 30 lm/m<sup>2</sup> is generally considered sufficient, though daylight illumination is much higher.

Some persons can read without much strain even when illumination is as low as 3 lm/m<sup>2</sup>. Because of this, it is difficult to lay down definite values of illumination for various purposes but the following summary will be found useful :

<i>Purpose and Places</i>	<i>lm/m<sup>2</sup></i>
Precision work, displays, tasks requiring rapid discrimination	above 500
Extra fine machine work, around needles of sewing machines, fine engraving, inspection of fine details having low contrast	200-500
Proof-reading, drawing, sustained reading, fine assembling, skilled bench-work	100-200
Drawing offices, art exhibition, usual reading	60-100
In museums, drill halls, for work of simple nature not involving close attention to fine details	40-60
Usual observation as in bed-rooms, waiting rooms, auditoriums and general lighting in factories	20-40
Hospital wards, yards, railway platforms and corridors	5-10

**49.14. Space/Height Ratio**

It is given by the ratio :  $\frac{\text{horizontal distance between two lamps}}{\text{mounting height of lamps}}$

This ratio depends on the nature of the polar curve of a lamp *when used along with its reflector*. A reflector has tremendous influence on the shape of the polar curve of the lamp, hence the value of space/height ratio, in fact, depends entirely on the type of reflector used. For obtaining uniform illumination on the working plane, it is essential to choose a correct value for this ratio.

In other words, it means that a reflector gives uniform illumination for a definite value of this ratio only. The ratio may be found easily if the polar curve of the type of fixture used is known. For reflectors normally used in indoor lighting, the value of this ratio lies between 1 and 2.

#### 49.15. Design of Lighting Schemes and Lay-outs

A well-designed lighting scheme is one which

(i) provides adequate illumination (ii) avoids glare and hard shadows (iii) provides sufficiently uniform distribution of light all over the working plane.

Before explaining the method of determining the number, size and proper arrangement of lamps in order to produce a given uniform illumination over a certain area, let us first consider the following two factors which are of importance in such calculations.

#### 49.16. Utilization Factor or Coefficient of Utilization ( $\eta$ )

It is the ratio of the lumens actually received by a particular surface to the total lumens emitted by a luminous source.

$$\therefore \eta = \frac{\text{lumens actually received on working plane}}{\text{lumens emitted by the light source}}$$

The value of this factor varies widely and depends on the following factors :

1. the type of lighting system, whether direct or indirect etc.
2. the type and mounting height of the fittings
3. the colour and surface of walls and ceilings and
4. to some extent on the shape and dimensions of the room.

For example, for direct lighting, the value of  $\eta$  varies between 0.4 and 0.6 and mainly depends on the shape of the room and the type and mounting height of fittings but very little on the colour of walls and ceiling. For indirect lighting, its value lies between 0.1 and 0.35 and the effect of walls and ceiling, from which light is reflected on the working plane, is much greater. Exact determination of the value of utilization factor is complicated especially in small rooms where light undergoes multiple reflections.

Since the light leaving the lamp in different directions is subjected to different degrees of absorption, the initial polar curve of distribution has also to be taken into account. Even though manufacturers of lighting fittings supply tables giving utilization factors for each type of fitting under specified conditions yet, since such tables apply only to the fittings for which they have been compiled, a good deal of judgment is necessary while using them.

#### 49.17. Depreciation Factor ( $p$ )

This factor allows for the fact that effective candle power of all lamps or luminous sources deteriorates owing to blackening and/or accumulation of dust or dirt on the globes and reflectors etc. Similarly, walls and ceilings etc., also do not reflect as much light as when they are clean. The value of this factor may be taken as 1/1.3 if the lamp fittings are likely to be cleaned regularly or 1/1.5 if there is much dust etc.

$$p = \frac{\text{illumination under actual conditions}}{\text{illumination when everything is perfectly clean}}$$

Since illumination is specified in  $\text{lm/m}^2$ , the area in square metre multiplied by the illumination required in  $\text{lm/m}^2$  gives the total useful luminous flux that must reach the working plane. Taking into consideration the utilization and depreciation or maintenance factors, the expression for the gross lumens required is

$$\text{Total lumens, } \Phi = \frac{E \times A}{\eta \times p}$$

where  $E$  = desired illumination in  $\text{lm/m}^2$ ;  $A$  = area of working plane to be illuminated in  $\text{m}^2$   
 $p$  = depreciation or maintenance factor;  $\eta$  = utilization factor.

The size of the lamp depends on the number of fittings which, if uniform distribution is required, should not be far apart. The actual spacing and arrangement is governed by space/height values and by the layout of ceiling beams or columns. Greater the height, wider the spacing that may be used, although the larger will be the unit required. Having settled the number of units required, the lumens per unit may be found from (total lumens/number of units) from which the size of lamp can be calculated.

**Example 49.20.** A room  $8\text{ m} \times 12\text{ m}$  is lighted by 15 lamps to a fairly uniform illumination of  $100\text{ lm/m}^2$ . Calculate the utilization coefficient of the room given that the output of each lamp is 1600 lumens.

**Solution.** Lumens emitted by the lamps =  $15 \times 1600 = 24,000\text{ lm}$   
 Lumens received by the working plane of the room =  $8 \times 12 \times 100 = 9600\text{ lm}$   
 Utilization coefficient =  $9600/24,000 = 0.4$  or **40%**.

**Example 49.21.** The illumination in a drawing office  $30\text{ m} \times 10\text{ m}$  is to have a value of 250 lux and is to be provided by a number of 300-W filament lamps. If the coefficient of utilization is 0.4 and the depreciation factor 0.9, determine the number of lamps required. The luminous efficiency of each lamp is 14  $\text{lm/W}$ . **(Elect. Drives & Utilization, Punjab Univ. Dec. 1994)**

**Solution.**  $\Phi = EA/\eta p$ ;  $E = 250\text{ lm/m}^2$ ,  $A = 30 \times 10 = 300\text{ m}^2$ ;  $\eta = 0.4$ ,  $p = 0.9$   
 $\therefore \Phi = 250 \times 300 / 0.4 \times 0.9 = 208,333\text{ lm}$   
 Flux emitted/lamp =  $300 \times 14 = 4200\text{ lm}$ ; No. of lamps reqd. =  $208,333/4200 = 50$ .

**Example 49.22.** Find the total saving in electrical load and percentage increase in illumination if instead of using twelve 150 W tungsten-filament lamps, we use twelve 80 W fluorescent tubes. It may be assumed that (i) there is a choke loss of 25 per cent of rated lamp wattage (ii) average luminous efficiency throughout life for each lamp is 15  $\text{lm/W}$  and for each tube 40  $\text{lm/W}$  and (iii) coefficient of utilization remains the same in both cases.

**Solution.** Total load of filament lamps =  $12 \times 150 = 1800\text{ W}$   
 Total load of tubes =  $12(80 + 0.25 \times 80) = 1200\text{ W}$   
 Net saving in load =  $1800 - 1200 = 600\text{ W}$   
 If  $A$  is the room area and  $\eta$  the coefficient of utilization, then

$$\text{illumination with lamps, } E_1 = \frac{12 \times 150 \times 15\eta}{A} = 27,000 \eta/A \text{ lm/m}^2$$

$$\text{illumination with tubes, } E_2 = \frac{12 \times 80 \times 40\eta}{A} = 38,400 \eta/A \text{ lm/m}^2$$

$$\text{increase in illumination} = \frac{38,400 - 27,000}{27,000} = 0.42 \text{ or } 42\%$$

**Example 49.23.** A football pitch  $120\text{ m} \times 60\text{ m}$  is to be illuminated for night play by similar banks of equal 1000 W lamps supported on twelve towers which are distributed around the ground to provide approximately uniform illumination of the pitch. Assuming that 40% of the total light emitted reaches the playing pitch and that an illumination of  $1000\text{ lm/m}^2$  is necessary for television purposes, calculate the number of lamps on each tower. The overall efficiency of the lamp is to be taken as 30  $\text{lm/W}$ . **(Elect. Technology-I, Bombay Univ.)**

**Solution.** Area to be illuminated =  $120 \times 60 = 7,200\text{ m}^2$

Flux required =  $7,200 \times 1,000 = 7.2 \times 10^6$  lm

Since only 40% of the flux emitted reaches the ground, the total luminous flux required to be produced is =  $7.2 \times 10^6 / 0.4 = 18 \times 10^6$  lm

Flux contributed by each tower bank =  $18 \times 10^6 / 12 = 1.5 \times 10^6$  lm

Output of each 1000-W lamp =  $30 \times 1000 = 3 \times 10^4$  lm

Hence, number of such lamps on each tower is =  $1.5 \times 10^6 / 3 \times 10^4 = 50$

**Example 49.24.** Design a suitable lighting scheme for a factory 120 m × 40 m with a height of 7 m. Illumination required is 60 lux. State the number, location and mounting height of 40 W fluorescent tubes giving 45 lm/W.

Depreciation factor = 1.2 ; utilization factor = 0.5

(Electric Drives & Util. Punjab Univ. 1993)

**Solution.**  $\Phi = \frac{60 \times 120 \times 40}{0.5 \times (1/1.2)} = 691,200$  lm; Flux per tube =  $45 \times 40 = 1800$  lm.

No. of fluorescent tubes reqd. =  $691,200 / 1800 = 384$ . If twin-tube fittings are employed, then number of such fittings required. =  $384 / 2 = 192$ .

These can be arranged in 8 rows of 24 fittings each. Assuming that the working plane is 1 metre above the floor level and the fittings are fixed 1 metre below the ceiling, we get a space/height factor of unity both along the length as well as width of the factory bay.

**Example 49.25.** A drawing hall in an engineering college is to be provided with a lighting installation. The hall is 30 m × 20 m × 8 m (high). The mounting height is 5 m and the required level of illumination is 144 lm/m<sup>2</sup>. Using metal filament lamps, estimate the size and number of single lamp luminaries and also draw their spacing layout. Assume :

Utilization coefficient = 0.6; maintenance factor = 0.75; space/height ratio=1

lumens/watt for 300-W lamp = 13, lumens/watt for 500-W lamp = 16.

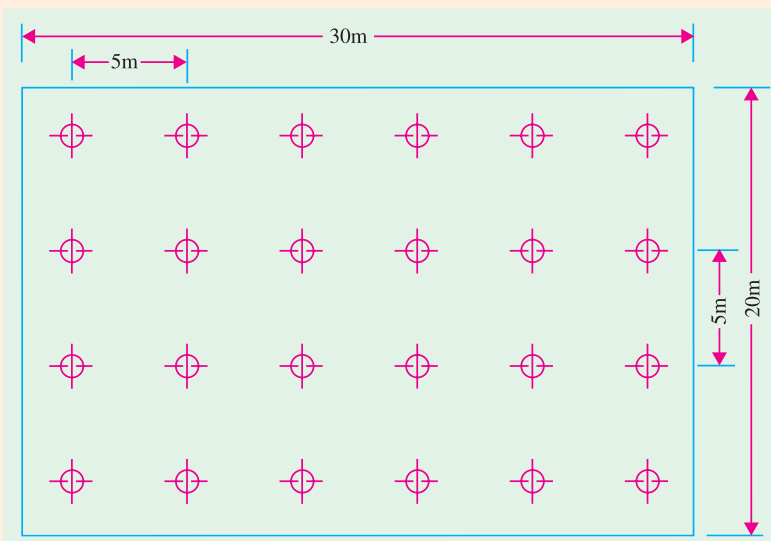


Fig. 49.40

**Solution.** Flux is given by  $\Phi = EA/\eta_p = 30 \times 20 \times 144 / 0.6 \times 0.75 = 192,000$  lm

Lumen output per 500-W lamp =  $500 \times 16 = 8,000$

∴ No. of 500-W lamps required =  $192,000 / 8000 = 24$

Similarly, No. of 300-W lamps required =  $192,000 / 3900 = 49$

The 300-W lamps cannot be used because their number cannot be arranged in a hall of 30 m × 20 m with a space/height ratio of unity. However, 500-W lamps can be arranged in 4 rows of 6 lamps each with a spacing of 5 m both in the width and the length of the hall as shown in Fig. 49.40.

**Example 49.26.** Estimate the number and wattage of lamps which would be required to illuminate a workshop space  $60 \times 15$  metres by means of lamps mounted 5 metres above the working plane. The average illumination required is about 100 lux.

Coefficient of utilization = 0.4 ; Luminous efficiency = 16 lm/W.

Assume a spacing/height ratio of unity and a candle power depreciation of 20%.

(Utilization of Electrical Energy, Madras Univ.)

**Solution.** Luminous flux is given by  $\Phi = \frac{EA}{\eta p} = \frac{100 \times (60 \times 15)}{0.4 \times 1/1.2} = 27 \times 10^4$  lm

Total wattage reqd. =  $27 \times 10^4 / 16 = 17,000$  W

For a space/height ratio of unity, only three lamps can be mounted along the width of the room. Similarly, 12 lamps can be arranged along the length of the room. Total number of lamps required is  $12 \times 3 = 36$ .

Wattage of each lamp

is =  $17,000 / 36 = 472$  W. We will take the nearest standard lamp of 500 W. These thirty-six lamps will be arranged as shown in Fig. 49.41.

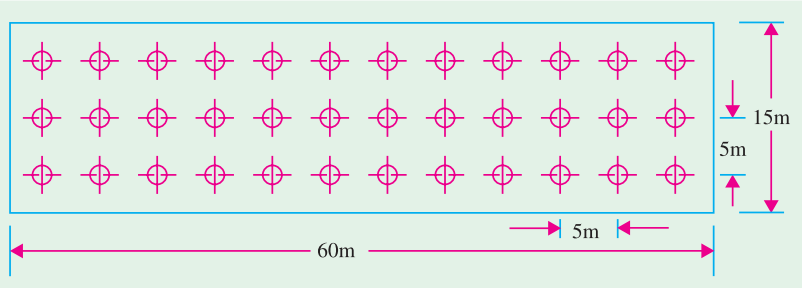


Fig. 49.41

**Example 49.27.** A drawing hall  $40 \text{ m} \times 25 \text{ m} \times 6$  high is to be illuminated with metal-filament gas-filled lamps to an average illumination of  $90 \text{ lm/m}^2$  on a working plane 1 metre above the floor. Estimate suitable number, size and mounting height of lamps. Sketch the spacing layout.

Assume coefficient of utilization of 0.5, depreciation factor of 1.2 and spacing/height ratio of 1.2

Size of lamps	200 W	300 W	500 W
Luminous efficiency (in lm/W)	16	18	20

(Elect. Technology, Bombay Univ.)

**Solution.** Total flux required is  $\Phi = \frac{40 \times 25 \times 90}{0.5 \times 1/1.2} = 216,000$  lm

Lumen output of each 200-W lamp is 3200 lm, of 300-W lamp is 5,400 lm and of 500-W lamp is 10,000 lm.

No. of 200-W lamps reqd. =  $\frac{216,000}{3,200} = 67$ ; No. of 300-W lamps reqd. =  $\frac{216,000}{5,400} = 40$

No. of 500-W lamps reqd. =  $216,000 / 10,000 = 22$

With a spacing/height ratio of 1.2, it is impossible to arrange both 200-W and 300-W lamps. Hence, the choice falls on 500-W lamp. If instead of the calculated 22, we take 24 lamps of 500 wattage, they can be arranged in four rows each having six lamps as shown in Fig. 49.42. Spacing along the length of the hall is  $40/6 = 6.67$  m and that along the width is  $25/4 = 6.25$  m.

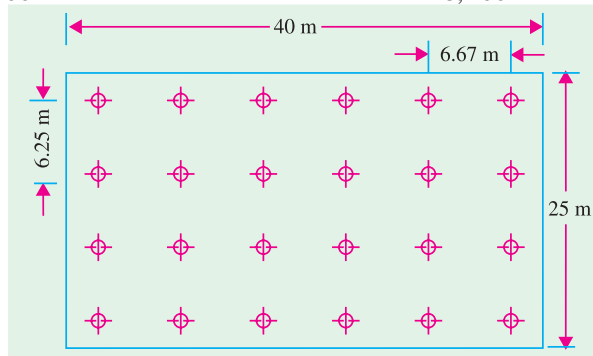


Fig. 49.42

Since mounting height of the lamps is 5 m above the working plane, it gives a space/height ratio of  $6.7/5 = 1.5$  along the length and  $6.25/5 = 1.4$  along the width of the hall.

**Example 49.28.** A school classroom,  $7\text{ m} \times 10\text{ m} \times 4\text{ m}$  high is to be illuminated to  $135\text{ lm/m}^2$  on the working plane. If the coefficient of utilization is 0.45 and the sources give 13 lumens per watt, work out the total wattage required, assuming a depreciation factor of 0.8. Sketch roughly the plan of the room, showing suitable positions for fittings, giving reasons for the positions chosen.

**Solution.** Total flux required is  $\Phi = EA/\eta p$   
 Now  $E = 135\text{ lm/m}^2$ ;  
 $A = 7 \times 10 = 70\text{ m}^2$ ;  
 $\eta = 0.45$ ;  $p = 0.8$   
 $F = 135 \times 7 \times 10 / 0.45 \times 0.8 = 26,250\text{ lm}$

Total wattage reqd.  $= 26,250/13 = 2020\text{ W}$

Taking into consideration the dimensions of the room, light fitting of 200 W would be utilized.

No. of fittings required  $= 2020/200 = 10$

As shown in Fig. 49.43, the back row of fittings has been located  $2/3\text{ m}$  from the rear wall so as to (i) provide adequate illumination on the rear desk and (ii) to minimise glare from paper because light would be incident practically over the shoulders of the students. The two side fittings help eliminate shadows while writing. One fitting has been provided at the chalk board end of the classroom for the benefit of the teacher. The

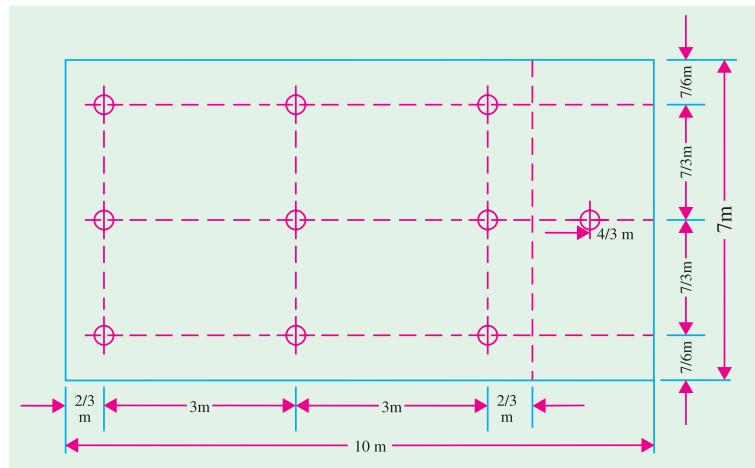


Fig. 49.43

The fittings should be of general diffusing pendant type at a height of 3 m from the floor.

**Example 49.29.** A hall  $30\text{ m}$  long and  $12\text{ m}$  wide is to be illuminated and the illumination required is  $50\text{ lm/m}^2$ . Calculate the number, the wattage of each unit and the location and mounting height of the units, taking a depreciation factor of 1.3 and utilization factor of 0.5, given that the outputs of the different types of lamp are as under :

Watts :	100	200	300	500	1000
Lumens :	1615	3650	4700	9950	21500

(Utili. of Elect. Power, A.M.I.E.)

**Solution.** Area to be illuminated,  $A = 30 \times 12 = 360\text{ m}^2$   
 Illumination required,  $E = 50\text{ lm/m}^2$ ,  $p = 1/1.3$ ,  $\eta = 0.5$   
 Now,  $\Phi = EA/\eta p = 50 \times 360 / 0.5 \times (1/1.3) = 46,800\text{ lm}$   
 If 100-W lamps are used, No. reqd.  $= 46,800/1615 = 29$   
 If 200-W lamps are used, No. reqd.  $= 46,800/3650 = 13$   
 If 300-W lamps are used, No. reqd.  $= 46,800/4700 = 10$   
 If 500-W lamps are used, No. reqd.  $= 46,800/9950 = 5$   
 If 1000-W lamps are used, No. reqd.  $= 46,800/21500 = 2$

If we take the mounting height of 5 m, then 300 W lamps would be suitable. The No. of lamps required would be 10, arranged in two rows, each row having 5 lamps thus giving a spacing of 6 m in lengths as well as width and space/height ratio of  $6/5 = 1.2$ .

If we use lamps of low power, their number would be large thereby increasing the number of fittings and hence cost. Lamps of higher voltage would be few in number but will not give a desirable space/height ratio.

### Tutorial Problem No. 49.2

1. A room  $30\text{ m} \times 15\text{ m}$  is to be illuminated by 15 lamps to give an average illumination of  $40\text{ lm/m}^2$ . The utilization factor is 4.2 and the depreciation factor is 1.4. Find the M.S.C.P. of each lamp.  
**[561 cd] (Elect. Technology-I, Bombay Univ.)**
2. A factory space of  $33\text{ m} \times 13\text{ m}$  is to be illuminated with an average illumination of  $72\text{ lm/m}^2$  by 200 W lamps. The coefficient of utilization is 0.4 and the depreciation factor is 1.4. Calculate the number of lamps required. The lumen output of a 200-W lamp is 2,730 lm.  
**[40] (Elect. Technology-I, Bombay Univ.)**
3. A drawing hall  $30\text{ m} \times 15\text{ m}$  with a ceiling height of 5 m is to be provided with an illumination of 120 lux. Taking the coefficient of utilization of 0.5, depreciation factor of 1.4, determine the No. of fluorescent tubes required and their spacing, mounting height and total wattage. Take luminous efficiency of fluorescent tube as 40 lm/W for 80-W tube. **(A.M.I.E. Sec. B, Summer)**
4. A room  $40\text{ m} \times 15\text{ m}$  is to be illuminated by 1.5 m 80-W fluorescent tubes mounted 3.5 m above the working plane on which an average illumination of  $180\text{ lm/m}^2$  is required. Using maintenance factor of 0.8 and the utilization factor of 0.5, design and sketch a suitable layout. The 80-W fluorescent tube has an output of 4,500 lm. **(Electrical Technology, Bombay Univ.)**
5. A hall is to be provided with a lighting installation. The hall is  $30\text{ m} \times 20\text{ m} \times 8\text{ m}$  (high). The mounting height is 5 m and the required level of illumination is 110 lux. Using metal filament lamps, estimate the size and number of single lamp luminaries and draw their spacing layout. Assume depreciation factor = 0.8, utilization coefficient = 0.6 and space/height ratio = 1.  

Watt :	200	300	500
Lumen/watt :	10	12	12.3

**[24 lamps, 500 W] (Services & Equipment-II, Calcutta Univ.)**
6. It is required to provide an illumination of 100 lux in a factory hall  $30\text{ m} \times 12\text{ m}$ . Assume that the depreciation factor is 0.8, the coefficient of utilization 0.4 and the efficiency of proposed lamps 14 lm/W. Calculate the number of lamps and their disposition. **(Utilization of Elect. Energy, Madras Univ.)**
7. Define the terms : (i) Lux (ii) Luminous Flux (iii) Candle Power.  
A workshop  $100\text{ m} \times 50\text{ m}$  is to be illuminated with intensity of illumination being 50 lux. Design a suitable scheme of lighting if coefficient of utilization = 0.9 ; Depreciation factor = 0.7 and efficiency of lamps = 80 lm/W. Use 100-W lamps. **(Electrical Engineering-III, Poona Univ.)**

### 49.18. Floodlighting

It means ‘flooding’ of large surfaces with the help of light from powerful projectors. Flooding is employed for the following purposes :

1. For aesthetic purposes as for enhancing the beauty of a building by night *i.e.* flood lighting of ancient monuments, religious buildings on important festive occasions etc.
2. For advertising purposes *i.e.* flood lighting, huge hoardings and commercial buildings.
3. For industrial and commercial purposes as in the case of railway yards, sports stadiums and quarries etc.

Usually, floodlight projectors having suitable reflectors fitted with standard 250-, 500-, or 1,000-watt gas-filled tungsten lamps, are employed. One of the two typical floodlight installations often used is as shown in Fig. 49.44 (a). The projector is kept 15 m to 30 m away from the surface to be floodlighted and provides approximately parallel beam having beam spread of 25° to 30°. Fig. 49.44 (b) shows the case when the projector cannot be located away from the building. In that case, an asymmetric reflector is used which directs more intense light towards the top of the building.

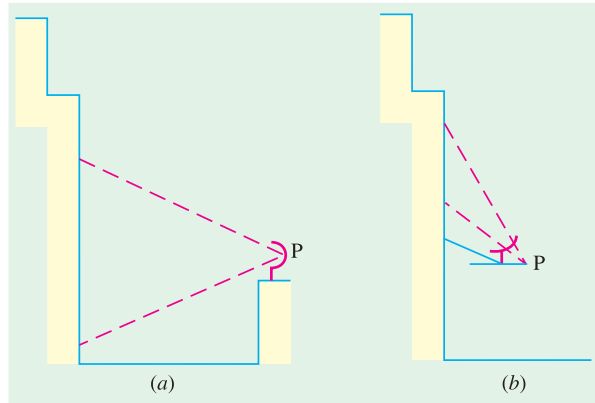


Fig. 49.44

The total luminous flux required to floodlight a building can be found from the relation,  $\Phi = EA/\eta \times p$ .

However, in the case of flood-lighting, one more factor has to be taken into account. That factor is known as waste-light factor ( $W$ ). It is so because when several projectors are used, there is bound to be a certain amount of overlap and also because some light would fall beyond the edges of the area to be illuminated. These two factors are taken into account by multiplying the theoretical value of the flux required by a waste-light factor which has a value of nearly 1.2 for regular surfaces and about 1.5 for irregular objects like statues etc. Hence, the formula for calculation of total flux required for floodlighting purposes is

$$\Phi = \frac{EAW}{\eta p}$$

**Example 49.30.** It is desired to floodlight the front of a building 42 m wide and 16 m high. Projectors of 30° beam spread and 1000-W lamps giving 20 lumen/watt are available. If the desired level of illumination is 75 lm/m<sup>2</sup> and if the projectors are to be located at ground level 17 m away, design and show a suitable scheme. Assume the following :

Coefficient of utilization = 0.4 ; Depreciation factor = 1.3; Waste-light factor = 1.2.

(Electrical Power-II ; M.S. Univ. Baroda)

**Solution.**  $\Phi = \frac{EAW}{\eta p}$

Here  $E = 75 \text{ lm/m}^2$  ;  $A = 42 \times 16 = 672 \text{ m}^2$  ;  $W = 1.2$  ;  $\eta = 0.4$  ;  $p = 1/1.3$

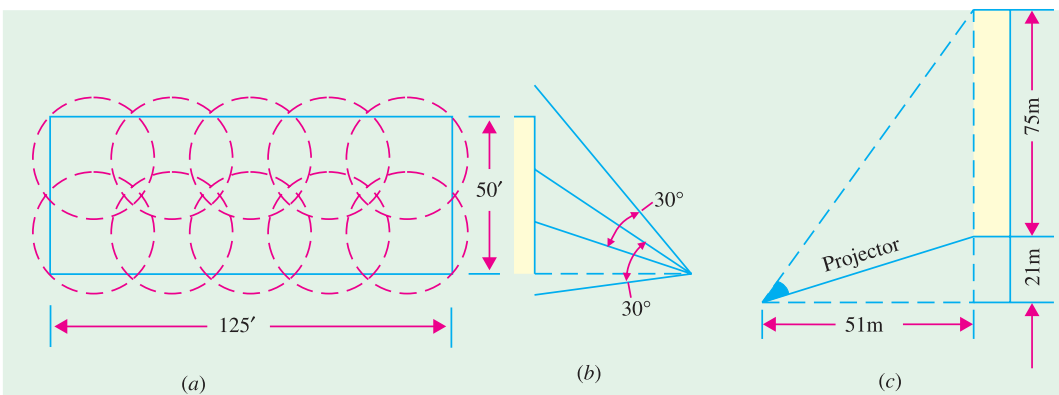


Fig. 49.45



$$\therefore \Phi = \frac{7.5 \times 672 \times 1.2}{0.4 \times 1/1.3} = 196,500 \text{ lm}$$

Lumen output of each 1,000-W lamp =  $1,000 \times 20 = 20,000 \text{ lm}$

No. of lamps required =  $196,500/20,000 = 10$ .

With a beam spread\* of  $30^\circ$ , it is possible to cover the whole length and width of the building by arranging the 10 projectors in two rows as shown in Fig. 49.45 (a).

**Example 49.31.** Estimate the number of 1000-W floodlight projectors required to illuminate the up per 75 m of one face of a 96 m tower of width 13 m if approximate initial average luminance is to be  $6.85 \text{ cd/m}^2$ . The projectors are mounted at ground level 51m from base of the tower. Utilization factor is = 0.2; reflection factor of wall = 25% and efficiency of each lamp = 18 lm/W.

(A.M.I.E. Sec. B., Winter 1992)

**Solution.**

$$B = 6.85 \text{ cd/m}^2 \quad \text{Now, } B = \rho E / \pi \text{ cd/m}^2 \quad \text{---Art 49.4}$$

$$\therefore E = \pi B / \rho = 6.85 \pi / 0.25 = 27.4 \pi \text{ lm/m}^2$$

$$\text{Area to be floodlighted} = 13 \times 75 = 975 \text{ m}^2$$

$$\therefore \text{flux required} = 27.4 \pi \times 975 \text{ lm}$$

Taking utilization factor into account, the flux to be emitted by all the lamps

$$= 27.4 \pi \times 975 / 0.2 \text{ lm}$$

Flux emitted by each lamp =  $18 \times 1000 = 18,000 \text{ lm}$

$$\therefore \text{No. of lamps reqd.} = \frac{27.4 \pi \times 975}{0.2 \times 18,000} = 24 \text{ (approx.)}$$

#### 49.19. Artificial Sources of Light

The different methods of producing light by electricity may, in a broad sense, be divided into three groups.

**1.** By *temperature incandescence*. In this method, an electric current is passed through a filament of thin wire placed in vacuum or an inert gas. The current generates enough heat to raise the temperature of the filament to luminosity.

Incandescent tungsten filament lamps are examples of this type and since their output depends on the temperature of their filaments, they are known as *temperature radiators*.

**2.** By establishing an arc between two carbon electrodes. The source of light, in their case, is the incandescent electrode.

**3. Discharge Lamps.** In these lamps, gas or vapour is made luminous by an electric discharge through them. The colour and intensity of light *i.e.* candle-power emitted depends on the nature of the gas or vapour only. It should be particularly noted that these discharge lamps are luminous-light lamps and do not depend on temperature for higher efficiencies. In this respect, they differ radically from incandescent lamps whose efficiency is dependent on temperature. Mercury vapour lamp, sodium-vapour lamp, neon-gas lamp and fluorescent lamps are examples of light sources based on discharge through gases and vapours.

#### 49.20. Incandescent Lamp

An incandescent lamp essentially consists of a fine wire of a high-resistance metal placed in an evacuated glass bulb and heated to luminosity by the passage of current through it. Such lamps were

\* It indicates the divergence of a beam and may be defined as the angle within which the minimum illumination on a surface normal to the axis of the beam is 1/10th of the maximum.

produced commercially for the first time by Edison in 1879. His early lamps had filaments of carbonized paper which were, later on, replaced by carbonized bamboo. They had the disadvantage of negative temperature coefficient of resistivity. In 1905, the metallized carbon-filament lamps were put in the market whose filaments had a positive temperature coefficient of resistivity (like metals). Such lamps gave 4 lm/W.

At approximately the same time, osmium lamps were manufactured which had filaments made of osmium which is very rare and expensive metal. Such lamps had a very fair maintenance of candle-power during their useful life and an average efficiency of 5 lm/W. However, osmium filaments were found to be very fragile.

In 1906 tantalum lamps having filaments of tantalum were produced which had an initial efficiency of 5 lm/watt.

All these lamps were superseded by tungsten lamps which were commercially produced in about 1937 or so. The superiority of tungsten lies mainly in its ability to withstand a high operating temperature without undue vaporisation of the filament. The necessity of high working temperature is due to the fact that the amount of visible radiation increases with temperature and so does the radiant efficiency of the luminous source. The melting temperature of tungsten is 3655°K whereas that of osmium is 2972°K and that of tantalum is 3172°K. Actually, carbon has a higher melting point than tungsten but its operating temperature is limited to about 2073°K because of rapid vaporization beyond this temperature.

In fact, the ideal material for the filament of incandescent lamps is one which has the following properties :

1. A high melting and hence operating temperature
2. A low vapour pressure
3. A high specific resistance and a low temperature coefficient
4. Ductility and
5. Sufficient mechanical strength to withstand vibrations.

Since tungsten possesses practically all the above mentioned qualities, it is used in almost all modern incandescent lamps. The earlier lamps had a square-cage type filament supported from a central glass stem enclosed in an evacuated glass bulb. The object of vacuum was two fold :

- (a) to prevent oxidation and
- (b) to minimize loss of heat by convection and the consequent lowering of filament temperature. However, vacuum favoured the evaporation of the filament with the resulting blackening of the lamp so that the operating temperature had to be kept as low as 2670° K with serious loss in luminous efficiency.



An incandescent lamp

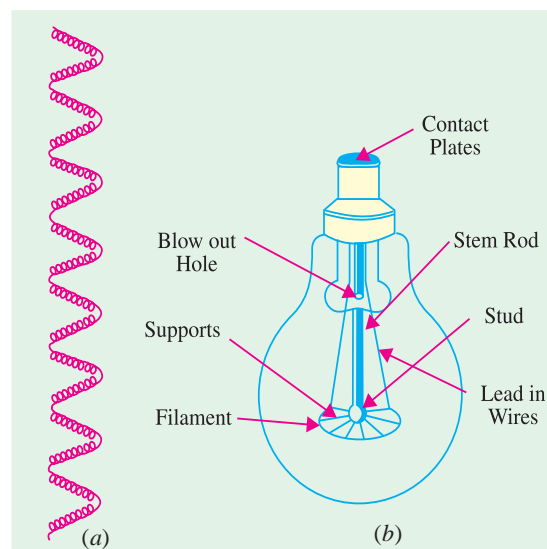


Fig. 49.46

It was, later on, found that this difficulty could be solved to a great extent by inserting a chemically inert gas like nitrogen or argon. The presence of these gases within the glass bulb decreased the evaporation of the filament and so lengthened its life. The filament could now be run at a relatively higher temperature and hence higher luminous efficiency could be realized. In practice, it was found that an admixture of 85% argon and about 15 percent nitrogen gave the best results.

However, introduction of gas led to another difficulty *i.e.* loss of heat due to convection which offsets the additional increase in efficiency. However, it was found that for securing greater efficiency, a concentrated filament having a tightly-wound helical construction was necessary. Such a coiled filament was less exposed to circulating gases, its turns supplying heat to each other and further the filament was mechanically stronger. The latest improvement is that the coiled filament is itself 'coiled' resulting in 'coiled-coil' filament Fig. 49.46 (a) which leads to further concentrating the heat, reducing the effective exposure to gases and allows higher temperature operation, thus giving greater efficiency. The construction of a modern coiled-coil gas-filled filament lamp is shown in Fig. 49.46 (b). The lamp has a 'wreath' filament *i.e.* a coiled filament arranged in the form of a wreath on radial supports.



A electric filament is a metallic wire, usually made of tungsten, when heated to luminance by passing electricity, radiates light

**49.21. Filament Dimensions**

There is found to be a definite relation between the diameter of a given filament and the current. Consider a filament operating at a fixed temperature and efficiency. Then since no heat is being utilized for further raising the temperature, all the heat produced in a given time is mostly lost by radiation (if vacuum is good). In other words, Heat produced per second = heat lost per second by radiation.

$$\text{Now, power intake} = I^2 R = I^2 \times \frac{\rho l}{A} = \frac{I^2 \rho l}{\pi d^2 / 4} = I^2 \left( \frac{4\rho l}{\pi d^2} \right)$$

where,  $I$  = filament current in amperes,  $l$  = filament length  
 $A$  = filament cross-section  $d$  = filament diameter  
 $\rho$  = resistivity of filament material at the working temperature.

Heat radiated per second from the surface is proportional to the area of the surface and emissivity of the material

$$\begin{aligned} \therefore \text{heat lost/second} &\propto \text{surface area} \times \text{emissivity } \sigma \\ \therefore I^2 (4\rho l / \pi d^2) &\propto l \times \pi d \times \sigma \quad \text{or} \quad I^2 \propto d^3 \quad \dots(i) \\ \therefore I &\propto d^{1.5} \quad \text{or} \quad d \propto I^{2/3} \end{aligned}$$

In general, for two filaments of the same material working at the same temperature and efficiency, the relation as seen from (i) above is

$$\left( \frac{I_1}{I_2} \right)^2 = \left( \frac{d_1}{d_2} \right)^3$$

It would be noticed that the above expressions are similar to those concerning fusing current of a given material under stated conditions (Preece's Rule).

Moreover, for two filaments working at the same temperature, the flux per unit area is the same. Denoting their lengths by  $l_1$  and  $l_2$  and their diameters by  $d_1$  and  $d_2$  respectively, we have, Lumen output  $\propto l_1 d_1 \propto l_2 d_2$  or  $l_1 d_1 = l_2 d_2 = \text{constant}$ .

**Example 49.32.** *If the filament of a 32 candela, 100-V lamp has a length  $l$  and diameter  $d$ , calculate the length and diameter of the filament of a 16 candela 200-V lamp, assuming that the two lamps run at the same intrinsic brilliance.*

**Solution.** Using the above relation  $32 \propto l_1 d_1$  and  $16 \propto l_2 d_2$

$$\therefore l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$32 \propto 100 I_1 \text{ and } 16 \propto 200 I_2 \quad \therefore I_2 = I_1 \times (16/200) \times (100/32) = \frac{1}{4} I_1$$

$$\text{Also } I_1 \propto d_1^{3/2} \text{ and } I_2 \propto d_2^{3/2} \quad \therefore (d_2/d_1)^{3/2} = I_2/I_1 = \frac{1}{4}$$

$$\therefore d_2 = 0.4 d_1 \text{ (approx.)}$$

$$\therefore l_2 = \frac{1}{2} l_1 \times (d_1/d_2) = \frac{1}{2} \times (1/0.3968) \times l_1 = 1/26 l_1.$$

Actually, this comparison is not correct because a thicker filament can be worked at a somewhat higher temperature than a thinner one.

**Example 49.33.** *An incandescent lamp has a filament of 0.005 cm diameter and one metre length. It is required to construct another lamp of similar type to work at double the supply voltage and give half the candle power. Assuming that the new lamp operates at the same brilliancy, determine suitable dimensions for its filament. (Elect. Technology, Utkal Univ.)*

**Solution.** Let  $I_1$  and  $I_2$  be the luminous intensities of the two lamps. Then

$$I_1 \propto l_1 d_1 \text{ and } I_2 \propto l_2 d_2 \quad \therefore \frac{l_2 d_2}{l_1 d_1} = \frac{I_2}{I_1} = \frac{1}{2} \text{ or } l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$I_1 \propto V_1 i_1 \text{ and } I_2 \propto V_2 i_2 \quad \therefore \frac{V_2 i_2}{V_1 i_1} = \frac{I_2}{I_1}$$

$$\therefore i_2 = i_1 (V_1/V_2) (I_2/I_1) = i_1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} i_1 \quad \text{Now, } i_1 \propto d_1^{3/2} \text{ and } i_2 \propto d_2^{3/2}$$

$$\therefore (d_2/d_1)^{3/2} = (i_2/i_1) = \frac{1}{4} \quad \therefore d_2 = 0.3968 d_1$$

$$\therefore l_2 = \frac{1}{2} l_1 \frac{d_1}{d_2} = \frac{1}{2} l_1 \times \left( \frac{1}{0.3968} \right) = 1.26 l_1$$

$$\text{Now, } d_1 = \mathbf{0.005 \text{ cm}} ; l_1 = \mathbf{100 \text{ cm}}$$

$$\therefore d_2 = 0.3968 \times 0.005 = \mathbf{0.001984 \text{ cm.}} \quad l_2 = 1.26 \times 100 = \mathbf{126 \text{ cm.}}$$

**Example 49.34.** *A 60 candle power, 250-V metal filament lamp has a measured candle power of 71.5 candela at 260 V and 50 candela at 240 V.*

(a) *Find the constant for the lamp in the expression  $C = aV^b$  where  $C$  = candle power and  $V$  = voltage.*

(b) Calculate the change of candle power per volt at 250 V. Determine the percentage variation of candle power due to a voltage variation of  $\pm 4\%$  from the normal value. (A.M.I.E. Sec. B)

**Solution.** (a)  $C = aV^b \therefore 71.5 = a \times 260^b$  and  $50 = a \times 240^b$   
 $\therefore 71.5/50 = (260/240)^b, b = 4.5$

Substituting this value of  $b$  in the above equation, we get

$a = 50/240^{4.5}, a = 0.98 \times 10^{-9}$

Hence, the expression for the candle power of the lamp becomes  $C = 0.98 \times 10^{-9} V^{4.5}$  candela

(b) Differentiating the above expression and putting  $V = 250$  V, we get

$\frac{dC}{dV} = 0.98 \times 10^{-9} \times 4.5 \times 250^{3.5} = 4.4$  candela per volt

When voltage increases by 4%,  $C_2/C_1 = 1.04^{4.5}$

% change in candle power  $= \frac{C_2 - C_1}{C_1} \times 100 = (1.04^{4.5} - 1) \times 100 = 19.3$

When voltage falls by 4%,  $C_2/C_1 = 0.96^{4.5}$

$\therefore$  % change in candle power  $= (0.96^{4.5} - 1) \times 100 = -16.8$

### 49.22. Incandescent Lamp Characteristics

The operating characteristics of an incandescent lamp are materially affected by departure from its normal working voltage. Initially, there is a rapid heating up of the lamp due to its low thermal capacity, but then soon its power intake becomes steady. If the filament resistance were not dependent on its temperature, the rate of generation of heat would have been directly proportional to the square of voltage applied across the lamp. However, because of (i) positive temperature coefficient of resistance and (ii) complex mechanism of heat transfer from filament to gas, the relations between the lamp characteristics and its voltage are mostly experimental. Some of the characteristics of gas-filled lamps are given below.

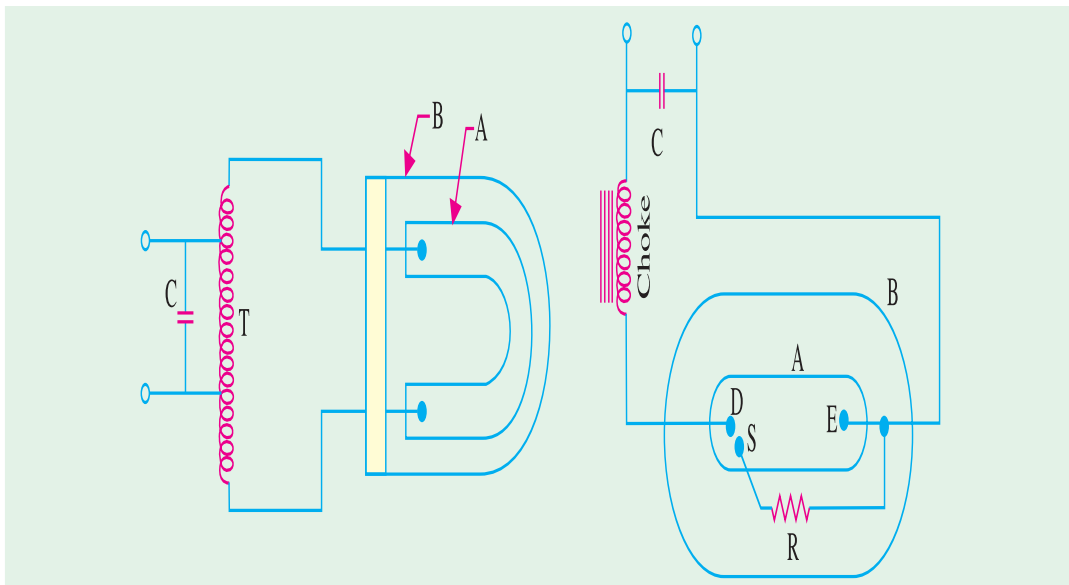


Fig. 49.47

(i) It is found that candle power or lumen output of the lamp varies with the voltage as lumen output  $\propto V^{3.3}$ .

- (ii) Variation of lumen output in terms of current is given by : lumen output  $\propto I^5$
- (iii) Life of the lamp is given by : life  $\propto 1/V^{13}$
- (iv) Wattage is given by  $W \propto V^{1.43}$
- (v) Its lumen/watt is given by : lm/watt  $\propto V^2$

The characteristic curves are plotted in Fig. 49.47. The life characteristic is very revealing. Even a small undervoltage considerably increases its life whereas overvoltage of as small a value as 5% shortens its life by 50%.

#### 49.23. Clear and Inside-frosted Gas-filled Lamps

The advantages of clear glass-filled lamps are that they facilitate light control and are necessary for use in lighting units where accurate distribution is required such as in flood-lights for buildings, projectors and motor-car headlights. However, they produce hard shadows and glare from filaments. Inside-frosted gas-filled lamps have luminous output nearly 2 per cent less than clear glass lamps of the same rating, but they produce softer shadows and practically eliminate glare from filaments. Such lamps are ideal for use in industrial open fittings located in the line of sight at low mounting heights and in diffuse fittings of opal glass type in order to avoid the presence of filament striations on the surface of the glassware etc.

Another new type of incandescent lamp is the inside-silica coated lamp which, due to the fine coating of silica on the inside of its bulb, has high diffusion of light output. Hence, the light from the filament is evenly distributed over the entire bulb surface thus eliminating the noticeably-bright spot around the filament area of an inside-frosted lamp. Such lamps are less glaring, soften shadows and minimize the brightness of reflections from specular (shiny) surfaces.

#### 49.24. Discharge Lamps

In all discharge lamps, an electric current is passed through a gas or vapour which renders it luminous. The elements most commonly used in this process of producing light by gaseous conduction are neon, mercury and sodium vapours. The colours (*i.e.* wavelength) of light produced depends on the nature of gas or vapour. For example, the neon discharge yields orange-red light of nearly 6,500 A.U. which is very popular for advertising signs and other spectacular effects. The pressure used in neon tubes is usually from 3 to 20 mm of Hg. Mercury-vapour light is always bluish green and deficient in red rays, whereas sodium vapour light is orange-yellow.

Discharge lamps are of two types. The first type consists of those lamps in which the colour of light *is the same as produced by the discharge through the gas or vapour*. To this group belong the neon gas lamps, mercury vapour (M.V.) and sodium vapour lamps. The other type consists of vapour lamps which use the phenomenon of fluorescence. In their case, the discharge through the vapour produces ultra-violet waves which cause fluorescence in certain materials known as phosphors. The radiations from the mercury discharge (especially 2537 A° line) impinge on these phosphors which absorb them and then re-radiate them at longer wave-lengths of visible spectrum. The inside of the fluorescent lamp is coated with these phosphors for this purpose. Different phosphors have different exciting ranges of frequency and give lights of different colours as shown in table 49.2.

Table 49.2

Phosphor	Lamp Colour	Exciting range A°	Emitted wavelenght A°
Calcium Tangstate	Blue	2200 - 3000	4400
Zinc Silicate	Green	2200 - 2960	5250
Cadmium Borate	Pink	2200 - 3600	6150
Cadmium Silicate	Yellow-pink	2200 - 3200	5950

### 49.25. Sodium Vapour Lamp

One type of low-pressure sodium-vapour lamp along with its circuit connection is shown in Fig. 49.48. It consists of an inner U-tube *A* made of a special sodium-vapour-resisting glass. It houses the two electrodes and contains sodium together with the small amount of neon-gas at a pressure of about 10 mm of mercury and one per cent of argon whose main function is to reduce the initial ionizing potential. The discharge is first started in the neon gas (which gives out redish colour). After a few minutes, the heat of discharge through the neon gas becomes sufficient to vaporise sodium and then discharge passes through the sodium vapour. In this way, the lamp starts its normal operation emitting its characteristic yellow light.



Sodium vapour lamp

The tungsten-coated electrodes are connected across auto-transformer *T* having a relatively high leakage reactance. The open-circuit voltage of this transformer is about 450 V which is sufficient to initiate a discharge through the neon gas. The leakage reactance is used not only for starting the current but also for limiting its value to safe limit. The electric discharge or arc strikes immediately after the supply is switched on whether the lamp is hot or cold. The normal burning position of the lamp is horizontal although two smaller sizes of lamp may be burnt vertically. The lamp is surrounded by an outer glass envelope *B* which serves to reduce the loss of heat from the inner discharge tube *A*. In this way, *B* helps to maintain the necessary high temperature needed for the operation of a sodium vapour lamp irrespective of draughts. The capacitor *C* is meant for improving the power factor of the circuit.

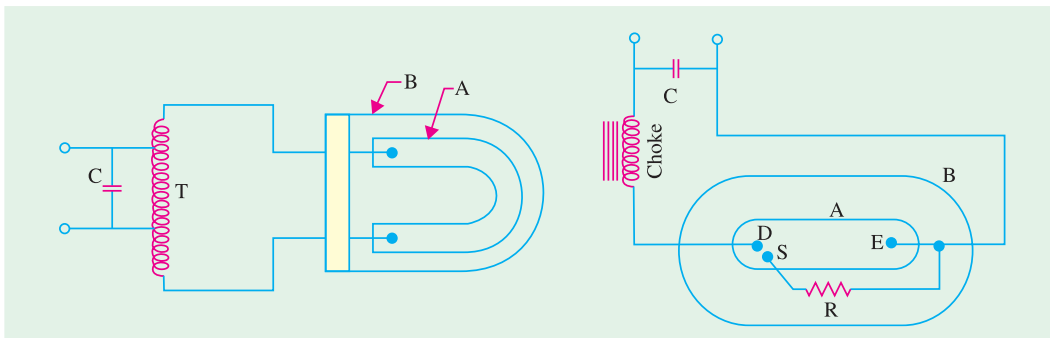


Fig. 49.48

Fig. 49.49

The light emitted by such lamps consists entirely of yellow colour. Solid objects illuminated by sodium-vapour lamp, therefore, present a picture in monochrome appearing as various shades of yellow or black.

### 49.26. High-pressure Mercury Vapour Lamp

Like sodium-vapour lamp, this lamp is also classified as electric discharge lamp in which light is produced by gaseous conduction. Such a lamp usually consists of two bulbs — an arc-tube containing the electric discharge and an outer bulb which protects the arc-tube from changes in temperature. The

inner tube or arc tube *A* is made of quartz (or hard glass) the outer bulb *B* of hard glass. As shown in Fig. 49.49, the arc tube contains a small amount of mercury and argon gas and houses three electrodes *D*, *E* and *S*. The main electrodes are *D* and *E* whereas *S* is the auxiliary starting electrode. *S* is connected through a high resistance *R* (about 50 k $\Omega$ ) to the main electrode situated at the outer end of the tube. The main electrodes consist of tungsten coils with electron-emitting coating or elements of thorium metal.

When the supply is switched on, initial discharge for the few seconds is established in the argon gas between *D* and *S* and then in the argon between *D* and *E*. The heat produced due to this discharge through the gas is sufficient to vaporise mercury. Consequently, pressure inside *A* increases to about one or two atmospheres and the p.d. across *D* and *E* grows from about 20 to 150 V, the operation taking about 5-7 minutes. During this time, discharge is established through the mercury vapours which emit greenish-blue light.

The choke serves to limit the current drawn by the discharge tube *A* to a safe limit and capacitor *C* helps to improve the power factor of the circuit.

True colour rendition is not possible with mercury vapour lamps since there is complete absence of red-light from their radiations. Consequently, red objects appear black, all blues appear mercury-spectrum blue and all greens the mercury-spectrum green with the result that colour values are distorted.

Correction for colour distortion can be achieved by

1. Using incandescent lamps (which are rich in red light) in combination with the mercury lamps.
2. Using colour-corrected mercury lamps which have an inside phosphor coat to add red colour to the mercury spectrum.

Stroboscopic (Flickering) effect in mercury vapour lamps is caused by the 100 on and off arc strikes when the lamps are used on the 50-Hz supply. The effect may be minimized by

1. Using two lamps on lead-lag transformer
2. Using three lamps on separated phases of a 3-phase supply and
3. Using incandescent lamps in combination with mercury lamps.

In the last few years, there has been tremendous improvement in the construction and operation of mercury-vapour lamps, which has increased their usefulness and boosted their application for all types of industrial lighting, floodlighting and street lighting etc. As compared to an incandescent lamp, a mercury-vapour lamp is (a) smaller in size (b) has 5 to 10 times longer operating life and (c) has 3 times higher efficiency *i.e.* 3 times more light output for given electrical wattage input.

Typical mercury-vapour lamp applications are :

1. High-bay industrial lighting — where high level illumination is required and colour rendition is not important.
2. Flood-lighting and street-lighting
3. Photochemical applications — where ultra-violet output is useful as in chlorination, water sterilization and photocopying etc.
4. For a wide range of inspection techniques by ultra-violet activation of fluorescent and phosphorescent dyes and pigments.
5. Sun-tan lamps — for utilizing the spectrum lines in the erythema region of ultra-violet energy for producing sun-tan.

### 49.27. Fluorescent Mercury-vapour Lamps

Basically, a fluorescent lamp consists of a long glass tube internally coated with a suitable fluorescent powder. The tube contains a small amount of mercury along with argon whose function is to facilitate the starting of the arc. There are two sealed-in electrodes at each end of the tube. Two basic types of electrodes are used in fluorescent lamps :



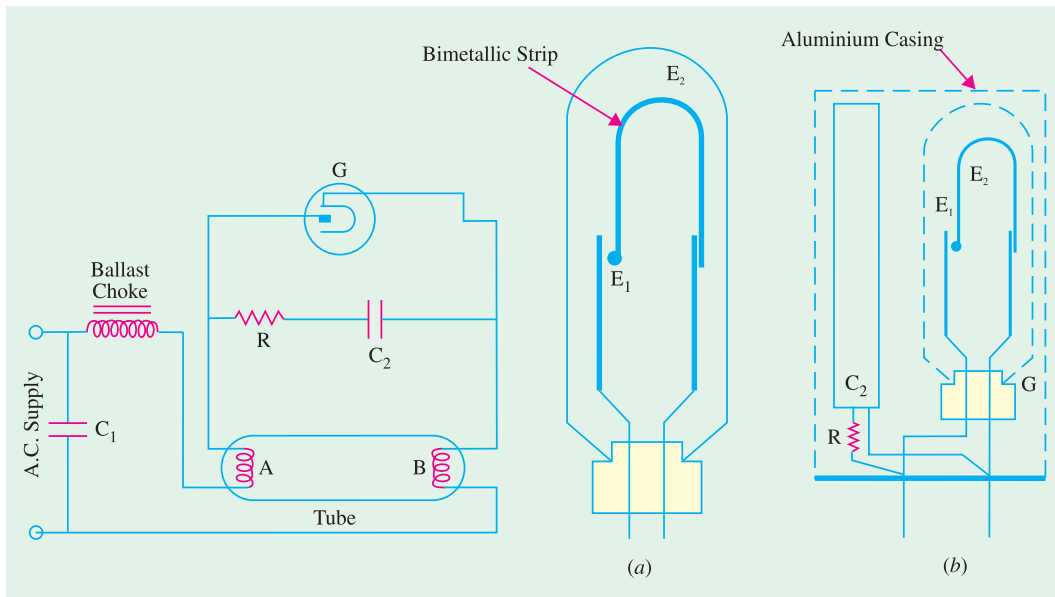


Fig. 49.50

Fig. 49.51

1. The coated-coil tungsten wire type. This type is used in standard pre-heat, rapid-start, instant-start lamps etc.

2. The inside-coated metal cylinder type which operates at a lower and more even temperature than the tungsten type and is called 'cold cathode'.\*

Circuits employed for the control of fluorescent lamps can be divided into two main groups (i) switch-start circuits and (ii) startless circuits requiring no starter. There are two types of starters (a) glow type — which is a voltage-operated device and (b) thermal switch — which is a current-operated device. Fig. 49.50 shows a fluorescent tube fitted with a glow starter  $G$ . As shown separately in Fig. 49.51 (a), the glow switch consists of two electrodes enclosed in a glass bulb filled with a mixture of helium and hydrogen or argon or neon at low pressure. One electrode  $E_1$  is fixed whereas the other  $E_2$  is movable and is made of a U-shaped bimetallic strip. To reduce radio interference, a small capacitor  $C_2$  is connected across the switch. The resistor  $R$  checks capacitor surges and prevents the starter electrodes or contacts from welding together. The complete starter switch along with the capacitor and resistor is contained in an aluminium casing is shown in Fig. 49.51 (b). Normally, the contacts are open and when supply is switched on, the glow switch receives almost full mains voltage\*\*. The voltage is sufficient to start a glow discharge between the two electrodes  $E_1$  and  $E_2$  and the heat generated is sufficient to bend the bimetallic strip  $E_2$  till it makes contact with the fixed electrode  $E_1$ , thus closing the contacts. This action completes the main circuit through the choke and the lamp electrodes  $A$  and  $B$  (Fig. 49.50). At the same time, since the glow between  $E_1$  and  $E_2$  has been shorted out, the bimetallic strip cools and the contacts  $E_1$  and  $E_2$  open. By this time, lamp electrodes  $A$  and  $B$  become heated to incandescence and the argon gas in their immediate vicinity is ionized. Due to opening of the glow switch contacts, a high inductive e.m.f. of about 1000 volts is induced in the choke. This voltage surge is sufficient to initiate a discharge in the argon gas lying between electrodes  $A$  and  $B$ . The heat thus produced is sufficient to vaporize mercury and the p.d. across the fluorescent tube falls to about 100 or 110 V which is not sufficient to restart the glow in

\* A cold cathode fluorescent lamp requires higher operating voltage than the other type. Although cold cathode lamps have less efficiency, they have much longer life than other lamps.

\*\* It is so because only the small discharge current flows and voltage drop across the choke is negligible.

G. Finally, the discharge is established through the mercury vapour which emits ultra-violet radiations. These radiations impinge on the fluorescent powder and make it emit visible light.

The function of the capacitor  $C_1$  is to improve the power factor of the circuit. It may be noted that the function of the highly-inductive choke (also called ballast) is (i) to supply large potential for starting the arc or discharge and (ii) to limit the arc current to a safe value.

#### 49.28. Fluorescent Lamp Circuit with Thermal Switch

The circuit arrangement is shown in Fig. 49.52. The switch has a bimetallic strip close to a resistance  $R$  which produces heat. The switch is generally enclosed in hydrogen-filled glass bulb  $G$ . The two switch electrodes  $E_1$  and  $E_2$  are normally closed when the lamp is not in operation. When normal supply is switched on, the lamp filament electrodes  $A$  and  $B$  are connected together through the thermal switch and a large current passes through them. Consequently, they are heated to incandescence. Meanwhile heat produced in resistance  $R$  causes the bimetallic strip  $E_2$  to break contact. The inductive surge of about 1000 V produced by the choke is sufficient to start discharge through mercury vapours as explained in Art. 49.27. The heat produced in  $R$  keeps the switch contacts  $E_1$  and  $E_2$  open during the time lamp is in operation.

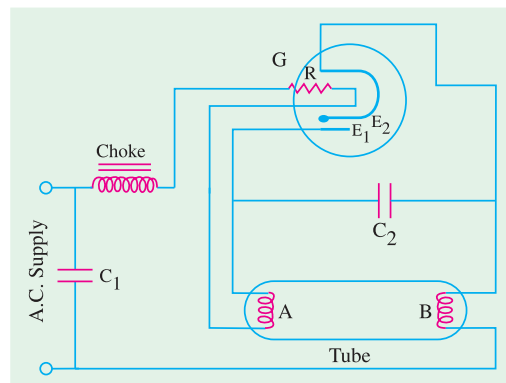


Fig. 49.52

#### 49.29. Startless Fluorescent Lamp Circuit

Such a circuit (Fig. (49.53) which does not require the use of a starter switch is commercially known as 'instant-start' or 'quick-start'. In this case, the normal starter is replaced by a filament heating transformer whose secondaries  $SS$  heat up the lamp electrodes  $A$  and  $B$  to incandescence in a fraction of a second. This combination of pre-heating and application of full supply voltage across lamp electrodes  $A$  and  $B$  is sufficient to start ionization in the neighbourhood of the electrodes which further spreads to the whole tube. An earthed strip  $E$  is used to ensure satisfactory starting.

The advantages of startless method are

1. It is almost instantaneous starting.
2. There is no flickering and no false starts.
3. It can start and operate at low voltage of 160-180 V.
4. Its maintenance cost is lower due to the elimination of any starter-switch replacements.
5. It lengthens the life of the lamp.

### 49.30. Stroboscopic Effect of Fluorescent Lamps

Stroboscopic or flickering effect produced by fluorescent lamps is due to the periodic fluctuations in the light output of the lamp caused by cyclic variations of the current on a.c. circuits. This phenomenon creates multiple-image appearance of moving objects and makes the movement appear jerky. In this connection, it is worth noting that

1. This flicker effect is more pronounced at lower frequencies.
2. Frequency of such flickers is twice the supply frequency.
3. The fluorescent powder used in the tube is slightly phosphorescent, hence stroboscopic effect is reduced to some extent due to after-glow.

Stroboscopic effect is very troublesome in the following cases :

1. When an operator has to move objects very quickly particularly those having polished finish. These objects would appear to move with jerky motion which over a long period would produce visual fatigue.
2. In the case of rotating machines whose frequency of rotation happens to be a multiple of flicker frequency, the machines appear to decrease in speed of rotation or be stationary. Sometimes the machines may even seem to rotate in the opposite direction.

Some of the methods employed for minimizing stroboscopic effect are given below :

1. By using three lamps on the separate phases of a 3-phase supply. In this case, the three light waves reaching the working plane would overlap by  $120^\circ$  so that the resultant fluctuation will be very much less than from a single fluorescent lamp.
2. By using a 'twin lamp' circuit on single-phase supply as shown in Fig. 49.54, one of the chokes has a capacitor in series with it and the lamp. In this way, a phase displacement of nearly  $120^\circ$  is introduced between the branch currents and also between the two light waves thereby reducing the resultant fluctuation.
3. By operating the lamps from a high frequency supply. Obviously, stroboscopic effect will entirely disappear on d.c. supply.

### 49.31. Comparison of Different Light Sources

**1. Incandescent Lamps.** They have instantaneous start and become momentarily off when the supply goes off. The colour of their light is very near the natural light. Their initial cost of installation is minimum but their running cost is maximum. They work equally well both on d.c. and a.c. supply and frequent switching does not affect their life of operation. Change of supply voltage affects their efficiency, output and life in a very significant

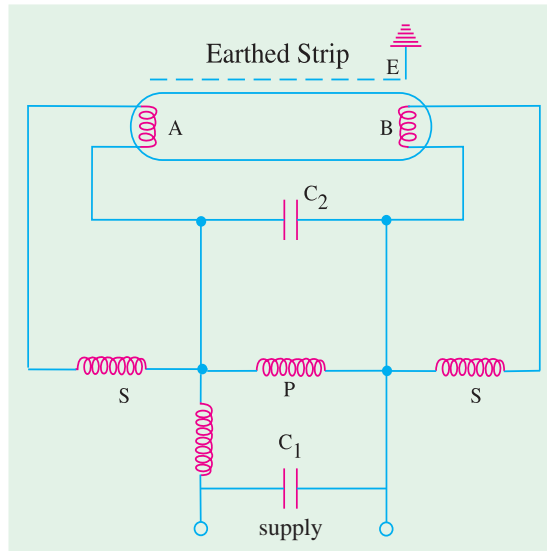


Fig. 49.53

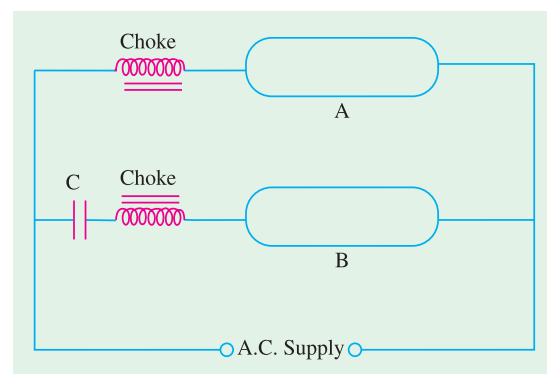


Fig. 49.54

way. They have an average working life of 1000 hours and luminous efficiency of 12 lm/W. Since their light has no stroboscopic effects, the incandescent lamps are suitable for domestic, industrial, street lighting and floodlights etc. They are available in a wide range of voltage ratings and, hence are used in automobiles, trains, emergency lights, aeroplanes and signals for railways etc.

**2. Fluorescent Lamps.** They have a reaction time of one second or a little more at the start. They go off and restart when the supply is restored. The colour of their light varies with the phosphor coating. Their initial cost of installation is maximum but running cost is minimum. Since stroboscopic effect is present, they are suitable for semi-direct lighting, domestic, industrial, commercial, roads and halls etc. Change in voltage affects their starting although light output does not change as remarkably as in the case of incandescent lamps. Colour of their light changes with the different phosphor coating on the inner side of the tube. Frequent switching affects their life period. They have quite high utility but their voltage rating is limited. Hence, their use is confined to mains voltage or complicated inverter circuits which convert 12 V d.c. into high volt d.c.. They have an average working life of 4000 hours and a luminous efficiency of 40 lm/W.

**3. Mercury Vapour Lamps.** They take 5 to 6 minutes for starting. They go off and cannot be restarted after the recovery of the voltage till the pressure falls to normal. They suffer from high colour distortion. Their initial cost of installation is high but lesser than that of fluorescent lamps. Their running cost is much less than incandescent lamps but higher than fluorescent tubes for the same levels of illumination. Stroboscopic effect is present in their light. They are suitable for open space like yards, parks and highway lighting etc. Change in voltage effects their starting time and colour of radiations emitted by them. Switching does not affect their life period. They have very limited utility that too on mains voltage. They are suitable for vertical position of working. They have an average working life of 3000 hours and an efficiency of 40 lm/W.

**4. Sodium Vapour Lamps.** They have a starting time of 5 to 6 minutes. They go off and cannot be restarted after the recovery of the voltage till its value falls to the normal value. The colour of their light is yellowish and produces colour distortion. Their initial cost of installation is maximum although their running cost is less than for filament lamps but more than for fluorescent lamps. They have stroboscopic effect and are suitable for use in open spaces, highways and street lighting etc. Change in voltage affects their starting time and colour of their radiations. They work on a.c. voltage and frequent switching affects their life. They are not suitable for local lighting. The colour of their light cannot be changed. They are very suitable for street lighting purposes. Their position of working is horizontal. They have a working life of about 3000 hours and efficiency of 60-70 lm/W.

### Tutorial Problem No. 49.3

1. Explain  $\cos^3 \phi$  law. *(J.N. University, Hyderabad, November 2003)*
2. A lamp of 500 candle power is placed at the centre of a room, (20 m × 10 m × 5 m.) Calculate the illumination in each corner of the floor and a point in the middle of a 10 m wall at a height of 2 m from floor. *(J.N. University, Hyderabad, November 2003)*
3. Enumerate the various factors, which have to be considered while designing any lighting scheme. *(J.N. University, Hyderabad, November 2003)*
4. Prove that 1 candle/sq. feet =  $(\pi f l - L.)$  *(J.N. University, Hyderabad, November 2003)*
5. A lamp giving 300 c.p. in all directions below the horizontal is suspended 2 metres above the centre of a square table of 1 metre side. Calculate the maximum and minimum illumination on the surface of the table. *(J.N. University, Hyderabad, November 2003)*
6. Explain the various types of lighting schemes with relevant diagrams. *(J.N. University, Hyderabad, November 2003)*
7. Discuss inverse square law? Corire law of Illustration. *(J.N. University, Hyderabad, November 2003)*

8. A lamp fitted with 120 degrees angled cone reflector illuminates circular area of 200 meters in diameter. The illumination of the disc increases uniformly from 0.5 metre-candle at the edge to 2 meter-candle at the centre.  
Determine :  
(i) the total light received (ii) Average illumination of the disc (iii) Average c.p. of the source.  
*(J.N. University, Hyderabad, November 2003)*
9. Discuss the flood lighting with suitable diagrams. *(J.N. University, Hyderabad, November 2003)*
10. Explain the measurement techniques used for luminous intensity.  
*(J.N. University, Hyderabad, November 2003)*
11. Write short notes on :  
(i) Bunsen photometer head (ii) Lummer-Brodtherm photometer head (iii) Flicker photometer head.  
*(J.N. University, Hyderabad, November 2003)*
12. What do you understand by polar curves as applicable to light source? Explain.  
*(J.N. University, Hyderabad, November 2003)*
13. Mean spherical Candlepower. *(J.N. University, Hyderabad, April 2003)*
14. Explain how you will measure the candlepower of a source of light.  
*(J.N. University, Hyderabad, April 2003)*
15. Explain the Rosseau's construction for calculating M.S.C.P. of a lamp.  
*(J.N. University, Hyderabad, April 2003)*
16. What do you mean by International Luminosity curve? Explain.  
*(J.N. University, Hyderabad, April 2003)*
17. Explain in detail the primary standard of luminous intensity with relevant diagram.  
*(J.N. University, Hyderabad, April 2003)*
18. Explain with sketches the constructional features of a filament lamp.  
*(J.N. University, Hyderabad, April 2003)*
19. Explain how the standard lamps can be calibrated w.r.t. primary and secondary standards.  
*(J.N. University, Hyderabad, April 2003)*
20. Briefly explain the various laboratory standards used in Illumination.  
*(J.N. University, Hyderabad, April 2003)*
21. Write short notes on :  
(a) High pressure mercury vapour lamp (i) M.A. Type (ii) M.T. Type  
(b) Mercury fluorescent lamp.  
*(J.N. University, Hyderabad, April 2003)*
22. Explain with connection diagram the operation of the low pressure fluorescent lamp and state its advantage.  
*(J.N. University, Hyderabad, April 2003)*
23. Explain clearly the following :  
Illumination, Luminous efficiency, MSCP, MHCP and solid angle.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
24. A small light source with uniform intensity is mounted at a height of 10 meters above a horizontal surface. Two points A and B both lie on the surface with point A directly beneath the source. How far is B from A if the illumination at B is only 1/15th of that at A?  
*(J.N. University, Hyderabad, December 2002/January 2003)*
25. Discuss the : (i) Specular reflection principle (ii) Diffusion principle of street lighting.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
26. Determine the height at which a light source having uniform spherical distribution should be placed over a floor in order that the intensity of horizontal illumination at a given distance from its vertical line may be greatest.  
*(J.N. University, Hyderabad, December 2002/January 2003)*
27. What is a Glare? *(J.N. University, Hyderabad, December 2002/January 2003)*
28. With the help of a neat diagram, explain the principle of operation of fluorescent lamp.  
*(J.N. University, Hyderabad, December 2002/January 2003)*

29. A machine shop 30 m long and 15 m wide is to have a general illumination of 150 lux on the work plane provided by lamps mounted 5 m above it. Assuming a coefficient of utilization of 0.55, determine suitable number and position of light. Assume any data if required.  
(J.N. University, Hyderabad, December 2002/January 2003)
30. Laws of illumination. (J.N. University, Hyderabad, December 2002/January 2003)
31. Working principle of sodium vapour lamp.  
(J.N. University, Hyderabad, December 2002/January 2003)
32. Explain the principle of operation of sodium vapour lamp and its advantages.  
(J.N. University, Hyderabad, December 2002/January 2003)
33. A corridor is lighted by lamps spaced 9.15 m apart and suspended at a height of 4.75 m above the centre line of the floor. If each lamp gives 100 candle power in all directions, find the maximum and minimum illumination on the floor along the centre line. Assume and data if required.  
(J.N. University, Hyderabad, December 2002/January 2003)
34. Discuss the laws of illumination and its limitations in practice.  
(J.N. University, Hyderabad, December 2002/January 2003)
35. State the functions of starter and choke in a fluorescent lamp.  
(J.N. University, Hyderabad, December 2002/January 2003)
36. Mercury Vapour Lamp. (J.N. University, Hyderabad, December 2002/January 2003)
37. Describe briefly (i) conduit system (ii) C.T.S. system of wiring.  
(Bangalore University, January/February 2003)
38. With a neat circuit diagram, explain the two way control of a filament lamp.  
(Belgaum Karnataka University, February 2002)
39. With a neat sketch explain the working of a sodium vapour lamp.  
(Belgaum Karnataka University, February 2002)
40. Mention the different types of wiring. With relevant circuit diagrams and switching tables, explain two-way and three-way control of lamps.  
(Belgaum Karnataka University, January/February 2003)
41. With a neat sketch explain the working of a fluorescent lamp.  
(Belgaum Karnataka University, January/February 2003)

### OBJECTIVE TESTS – 49

- Candela is the unit of ..... candela.
  - flux
  - luminous intensity
  - illumination
  - luminance.
- The unit of illuminance is
  - lumen
  - $\text{cd/m}^2$
  - lux
  - steradian.
- The illumination at various points on a horizontal surface illuminated by the same source varies as
  - $\cos^3\theta$
  - $\cos\theta$
  - $1/r^2$
  - $\cos^2\theta$ .
- The M.S.C.P. of a lamp which gives out a total luminous flux of  $400\pi$  lumen is
  - 200
  - 100
  - 50
  - 400.
- A perfect diffuser surface is one that
  - diffuses all the incident light
  - absorbs all the incident light
  - transmits all the incident light
  - scatters light uniformly in all directions.
- The direct lighting scheme is most efficient but is liable to cause
  - monotony
  - glare
  - hard shadows
  - both (b) and (c).
- Total flux required in any lighting scheme depends inversely on
  - illumination

- (b) surface area  
(c) utilization factor  
(d) space/height ratio.
8. Floodlighting is NOT used for ..... purposes.  
(a) reading  
(b) aesthetic  
(c) advertising  
(d) industrial.
9. Which of the following lamp has minimum initial cost of installation but maximum running cost ?  
(a) incandescent  
(b) fluorescent  
(c) mercury vapour  
(d) sodium vapour.
10. An incandescent lamp can be used  
(a) in any position  
(b) on both ac and dc supply  
(c) for street lighting  
(d) all of the above.
11. The average working life of a fluorescent lamp is about..... hours.  
(a) 1000  
(b) 4000  
(c) 3000  
(d) 5000.
12. The luminous efficiency of a sodium vapour lamp is about ..... lumen/watt.  
(a) 10  
(b) 30  
(c) 50  
(d) 70
13. Which of the following statements is correct?  
(a) Light is a form of heat energy  
(b) Light is a form of electrical energy  
(c) Light consists of shooting particles  
(d) Light consists of electromagnetic waves
14. Luminous efficiency of a fluorescent tube is  
(a) 10 lumens/watt  
(b) 20 lumens/watt  
(c) 40 lumens/watt  
(d) 60 lumens/watt
15. Candela is the unit of which of the following?  
(a) Wavelength  
(b) Luminous intensity  
(c) Luminous flux  
(d) Frequency
16. Colour of light depends upon  
(a) frequency  
(b) wave length  
(c) both (a) and (b)  
(d) speed of light
17. Illumination of one lumen per sq. metre is called .....  
(a) lumen metre  
(b) lux  
(c) foot candle  
(d) candela
18. A solid angle is expressed in terms of .....  
(a) radians/metre  
(b) radians  
(c) steradians  
(d) degrees
19. The unit of luminous flux is .....  
(a) watt/m<sup>2</sup>  
(b) lumen  
(c) lumen/m<sup>2</sup>  
(d) watt
20. Filament lamps operate normally at a power factor of  
(a) 0.5 lagging  
(b) 0.8 lagging  
(c) unity  
(d) 0.8 leading
21. The filament of a GLS lamp is made of  
(a) tungsten  
(b) copper  
(c) carbon  
(d) aluminium
22. Find diameter tungsten wires are made by  
(a) turning  
(b) swaging  
(c) compressing  
(d) wire drawing
23. What percentage of the input energy is radiated by filament lamps?  
(a) 2 to 5 percent  
(b) 10 to 15 percent  
(c) 25 to 30 percent  
(d) 40 to 50 percent
24. Which of the following lamps is the cheapest for the same wattage?  
(a) Fluorescent tube  
(b) Mercury vapour lamp  
(c) GLS lamp  
(d) Sodium vapour lamp
25. Which of the following is not the standard rating of GLS lamps?  
(a) 100 W  
(b) 75 W  
(c) 40 W  
(d) 15 W
26. In houses the illumination is in the range of  
(a) 2–5 lumens/watt  
(b) 10–20 lumens/watt

- (c) 35–45 lumens/watt  
(d) 60–65 lumens/watt
27. “The illumination is directly proportional to the cosine of the angle made by the normal to the illuminated surface with the direction of the incident flux”. Above statement is associated with  
(a) Lambert's cosine law  
(b) Planck's law  
(c) Bunsen's law of the illumination  
(d) Macbeth's law of illumination
28. The colour of sodium vapour discharge lamp is  
(a) red  
(b) pink  
(c) yellow  
(d) bluish green
29. Carbon arc lamps are commonly used in  
(a) photography  
(b) cinema projectors  
(c) domestic lighting  
(d) street lighting
30. Desired illumination level on the working plane depends upon  
(a) age group of observers  
(b) whether the object is stationary or moving  
(c) Size of the object to be seen and its distance from the observer  
(d) whether the object is to be seen for longer duration or shorter duration of time  
(e) all above factors
31. On which of the following factors does the depreciation or maintenance factor depend?  
(a) Lamp cleaning schedule  
(b) Ageing of the lamp  
(c) Type of work carried out at the premises  
(d) All of the above factors
32. In lighting installing using filament lamps 1% voltage drop results into  
(a) no loss of light  
(b) 1.5 percent loss in the light output  
(c) 3.5 percent loss in the light output  
(d) 15 percent loss in the light output
33. For the same lumen output, the running cost of the fluorescent lamp is  
(a) equal to that of filament lamp  
(b) less than that of filament lamp  
(c) more than that of filament lamp  
(d) any of the above
34. For the same power output  
(a) high voltage rated lamps will be more sturdy  
(b) low voltage rated lamps will be more sturdy  
(c) both low and high voltage rated lamps will be equally sturdy
35. The cost of a fluorescent lamp is more than that of incandescent lamp because of which of the following factors?  
(a) More labour is required in its manufacturing  
(b) Number of components used is more  
(c) Quantity of glass used is more  
(d) All of the above factors
36. Filament lamp at starting will take current  
(a) less than its full running current  
(b) equal to its full running current  
(c) more than its full running current
37. A reflector is provided to  
(a) protect the lamp  
(b) provide better illumination  
(c) avoid glare  
(d) do all of the above
38. The purpose of coating the fluorescent tube from inside with white powder is  
(a) to improve its life  
(b) to improve the appearance  
(c) to change the colour of light emitted to white  
(d) to increase the light radiations due to secondary emissions
39. .... will need lowest level of illumination.  
(a) Auditoriums  
(b) Railway platform  
(c) Displays  
(d) Fine engravings
40. Due to moonlight, illumination is nearly  
(a) 3000 lumens/m<sup>2</sup>  
(b) 300 lumens/m<sup>2</sup>  
(c) 30 lumens/m<sup>2</sup>  
(d) 0.3 lumen/m<sup>2</sup>
41. Which of the following instruments is used for the comparison of candle powers of different sources?  
(a) Radiometer  
(b) Bunsen meter  
(c) Photometer  
(d) Candle meter
42. .... photometer is used for comparing the lights of different colours?  
(a) Grease spot  
(b) Bunsen  
(c) Lummer brodhum  
(d) Guilds flicker
43. In the fluorescent tube circuit the function of choke is primarily to  
(a) reduce the flicker



- (b) minimise the starting surge  
(c) initiate the arc and stabilize it  
(d) reduce the starting current
44. .... cannot sustain much voltage fluctuations.  
(a) Sodium vapour lamp  
(b) Mercury vapour lamp  
(c) Incandescent lamp  
(d) Fluorescent lamp
45. The function of capacitor across the supply to the fluorescent tube is primarily to  
(a) stabilize the arc  
(b) reduce the starting current  
(c) improve the supply power factor  
(d) reduce the noise
46. .... does not have separate choke  
(a) Sodium vapour lamp  
(b) Fluorescent lamp  
(c) Mercury vapour lamp  
(d) All of the above
47. In sodium vapour lamp the function of the leak transformer is  
(a) to stabilize the arc  
(b) to reduce the supply voltage  
(c) both (a) and (b)  
(d) none of the above
48. Most affected parameter of a filament lamp due to voltage change is  
(a) wattage  
(b) life  
(c) luminous efficiency  
(d) light output
49. In electric discharge lamps for stabilizing the arc  
(a) a reactive choke is connected in series with the supply  
(b) a condenser is connected in series to the supply  
(c) a condenser is connected in parallel to the supply  
(d) a variable resistor is connected in the circuit
50. For precision work the illumination level required is of the order of  
(a) 500-1000 lumens/m<sup>2</sup>  
(b) 200-2000 lumens/m<sup>2</sup>  
(c) 50-100 lumens/m<sup>2</sup>  
(d) 10-25 lumens/m<sup>2</sup>
51. .... is a cold cathode lamp.  
(a) Fluorescent lamp  
(b) Neon lamp  
(c) Mercury vapour lamp  
(d) Sodium vapour lamp
52. In case of .... least illumination level is required.  
(a) skilled bench work  
(b) drawing offices  
(c) hospital wards  
(d) find machine work
53. For normal reading the illumination level required is around  
(a) 20-40 lumens/m<sup>2</sup>  
(b) 60-100 lumens/m<sup>2</sup>  
(c) 200-300 lumens/m<sup>2</sup>  
(d) 400-500 lumens/m<sup>2</sup>
54. In electric discharge lamps light is produced by  
(a) cathode ray emission  
(b) ionisation in a gas or vapour  
(c) heating effect of current  
(d) magnetic effect of current
55. A substance which change its electrical resistance when illuminated by light is called .....  
(a) photoelectric  
(b) photovoltaic  
(c) photoconductive  
(d) none of the above
56. In case of .... power factor is the highest.  
(a) GLS lamps  
(b) mercury arc lamps  
(c) tube lights  
(d) sodium vapour lamps
57. A mercury vapour lamp gives .... light.  
(a) white  
(b) pink  
(c) yellow  
(d) greenish blue
58. Sometimes the wheels of rotating machinery, under the influence of fluorescent lamps appear to be stationary. This is due to the  
(a) low power factor  
(b) stroboscopic effect  
(c) fluctuations  
(d) luminescence effect
59. Which of the following bulbs operates on least power?  
(a) GLS bulb  
(b) Torch bulb  
(c) Neon bulb  
(d) Night bulb
60. The flicker effect of fluorescent lamps is more pronounced at  
(a) lower frequencies  
(b) higher frequencies  
(c) lower voltages  
(d) higher voltages

- 61.** Which of the following application does not need ultraviolet lamps?  
(a) Car lighting  
(b) Medical purposes  
(c) Blue print machine  
(d) Aircraft cockpit dashboard lighting
- 62.** Which gas can be filled in GLS lamps?  
(a) Oxygen  
(b) Carbon dioxide  
(c) Xenon  
(d) Any inert gas

**ANSWERS**

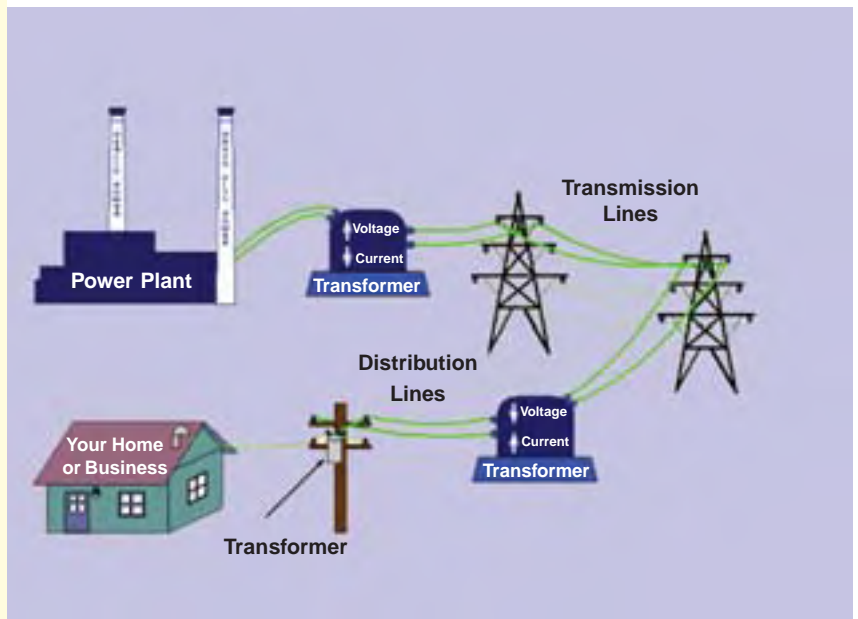
- 1.** (b)   **2.** (c)   **3.** (a)   **4.** (b)   **5.** (d)   **6.** (d)   **7.** (c)   **8.** (a)   **9.** (a)   **10.** (d)  
**11.** (b)   **12.** (d)   **13.** (d)   **14.** (d)   **15.** (b)   **16.** (c)   **17.** (b)   **18.** (c)   **19.** (b)   **20.** (c)  
**21.** (a)   **22.** (d)   **23.** (b)   **24.** (c)   **25.** (b)   **26.** (d)   **27.** (a)   **28.** (c)   **29.** (b)   **30.** (e)  
**31.** (d)   **32.** (c)   **33.** (b)   **34.** (b)   **35.** (d)   **36.** (c)   **37.** (d)   **38.** (d)   **39.** (b)   **40.** (d)  
**41.** (c)   **42.** (d)   **43.** (c)   **44.** (c)   **45.** (c)   **46.** (a)   **47.** (c)   **48.** (b)   **49.** (a)   **50.** (a)  
**51.** (b)   **52.** (c)   **53.** (b)   **54.** (b)   **55.** (c)   **56.** (a)   **57.** (d)   **58.** (b)   **59.** (b)   **60.** (a)  
**61.** (a)   **62.** (d)

# CHAPTER 50

## Learning Objectives

- Economic Motive
- Depreciation
- Indian Currency
- Factors Influencing Cost and Tariffs of Electric Supply
- Demand
- Average Demand
- Maximum Demand
- Demand Factor
- Diversity of Demand
- Diversity Factor
- Load Factor
- Plant Factor or Capacity Factor
- Utilization Factor (or Plant use Factor)
- Connected Load Factor
- Tariffs
- Flat Rate
- Sliding Scale
- Two-part Tariff
- Kelvin's Law
- Effect of Cable Insulation
- Note on Power Factor
- Disadvantages of Low Power Factor
- Economics of Power Factor
- Economical Limit of Power Factor Correction

## TARIFFS AND ECONOMIC CONSIDERATIONS



↑ For the successful running of an electricity production, transmission and distribution system, it is necessary to properly account for the various direct and indirect costs involved, before fixing the final kWh charges for the consumers

### 50.1. Economic Motive

In all engineering projects with the exception of the construction of works of art or memorial buildings, the question of cost is of first importance. In fact, in most cases the cost decides whether a project will be undertaken or not although political and other considerations may intervene sometimes. However, the design and construction of an electric power system is undertaken for the purpose of producing electric power to be sold at a profit. Hence, every effort is made to produce the power as cheaply as possible. The problem of calculating the cost of any scheme is often difficult because the cost varies considerably with time, tariffs and even with convention. In general, the cost of producing electric power can be roughly divided into the following two portions :

**(a) Fixed Cost.** These do not vary with the operation of the plant *i.e.* these are independent of the number of units of electric energy produced and mainly consist of :

1. Interest on capital investment,
2. Allowance for depreciation (*i.e.* wearing out of the depreciable parts of the plant augmented by obsolescence, buildings, the transmission and distribution system etc.),
3. Taxes and insurance, 4. most of the salaries and wages, 5. small portion of the fuel cost.

**(b) Running or Operating Costs.** These vary with the operation of the plant *i.e.* these are proportional to the number of units of electric energy generated and are mostly made up of :

1. most of the fuel cost, 2. small portion of salaries and wages, 3. repair and maintenance.

### 50.2. Depreciation

It is obvious that from the very day the construction of a generating plant is completed, deterioration starts and due to wear and tear from use and the age and physical decay from lapse of time, there results a reduction in the value of the plant — a loss of some part of the capital investment in the perishable property. The rate of wear and disintegration is dependent upon **(i)** conditions under which the plant or apparatus is working, **(ii)** how it is protected from elements and **(iii)** how promptly the required repairs are carried out.

Hence, as the property decreases from its original cost when installed, to its final scrap or salvage value at the end of its useful life, it is essential that the owner will have in hand at any given time as much money as represents the shrinkage in value and at the time of actual retirement of the plant, he must certainly have in hand the full sum of the depreciable part of the property. By adding this amount to the net salvage value of the plant, the owner can rebuild the same type of property as he did in the first instance or he can build some other property of an equivalent earning capacity.

The useful life of the apparatus ends when its repair becomes so frequent and expensive that it is found cheaper to retire the equipment and replace it by a new one.

It may be pointed out here that in addition to depreciation from wear and tear mentioned above, there can also be depreciation of the apparatus due to the inadequacy from obsolescence, both sentimental and economic, from the requirements of the regulating authorities and from accidental damages and if any of these factors become operative, they may force the actual retirement of the apparatus much before the end of its normal useful life and so shorten the period during which its depreciation expenses can be collected. These factors will necessitate increased depreciation rate and the consequent build up of the depreciation reserve as to be adequate for the actual retirement.

Some of the important methods of providing for depreciation are :

1. Straight-line method,
2. Diminishing-value method,
3. Retirement-expense method,
4. Sinking-fund method.

In the straight-line method, provision is made for setting aside each year an equal proportional part of the depreciable cost based on the useful life of the property. Suppose a machine costs

Rs. 45,000 and its useful life is estimated as ten years with a scrap value of Rs. 5,000, then the annual depreciation value will be 1/10 of Rs. 40,000 *i.e.* Rs. 4000. This method is extremely simple and easy to apply when the only causes for retirement of the machine are the wear and tear or the slow action of elements. But it is extremely difficult to estimate when obsolescence or accidental damage may occur to the machine. This method ignores the amount of interest earned on the amount set aside yearly.

In the diminishing value method, provision is made for setting aside each year a fixed rate, first applied to the original cost and then to the diminishing value; such rate being based upon the estimated useful life of the apparatus. This method leads to heaviest charges for depreciation in early years when maintenance charges are lowest and so evens out the total expense on the apparatus for depreciation plus the maintenance over its total useful life. This method has the serious disadvantage of imposing an extremely heavy burden on the early years of a new plant which has as yet to develop its load and build up its income as it goes along.

The retirement expense method which is not based on the estimated life of the property, aims at creating an adequate reserve to take care of retirement before such retirements actually occurs. Because of many objections raised against this method, it is no longer used now.

In the sinking-fund method, provision is made for setting aside each year such a sum as, invested at certain interest rate compounded annually, will equal the amount of depreciable property at the end of its useful life. As compared to straight-line method, it requires smaller annual amounts and also the amounts for annuity are uniform. This method would be discussed in detail in this chapter.

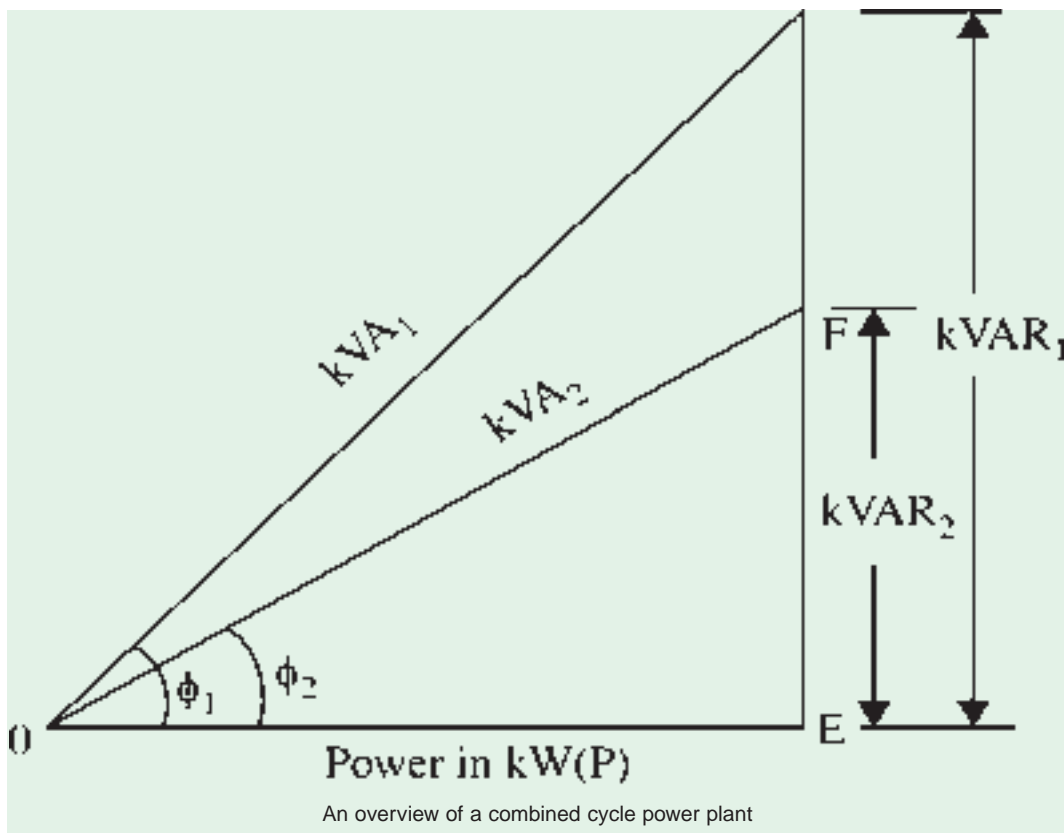
Suppose  $P$  is the capital outlay required for an installation and  $r$  p.a. is the interest per unit (6% is equivalent to  $r = 0.06$ ). The installation should obviously provide  $rP$  as annual interest which is added to its annual running cost. Were the installation to last forever, then this would have been the only charge to be made. But as the useful life of the installation has a definite value, it is necessary to provide a sinking fund to produce sufficient amount at the end of the estimated useful life to replace the installation by a new one. Let the cost of replacement be denoted by  $Q$ . This  $Q$  will be equal to  $P$  if the used installation has zero scrap value, less than  $P$  if it has positive scrap value and greater than  $P$  if it has a negative scrap value. If the useful life is  $n$  years, then the problem is to find the annual charge  $q$  to provide a sinking fund which will make available an amount  $Q$  at the end of  $n$  years. Since amount  $q$  will earn an annual interest  $rq$ , hence its value after one year becomes  $q + rq = q(1 + r)$ . This sum will earn an interest of  $r \times q(1 + r)$  and hence its value at the end of two years will become  $q(1 + r) + qr(1 + r)$  or  $q(1 + r)^2$ . Similarly, its value at the end of three years is  $q(1 + r)^3$ . *i.e.* its value is multiplied by  $(1 + r)$  every year so that the first payment becomes worth  $q(1 + r)^n$  at the end of  $n$  years. The second payment to the sinking-fund is made at the beginning of the second year, hence its value at the end of the useful life of the installation becomes  $q(1 + r)^{n-1}$  because this amount earns interest only for  $(n - 1)$  years. The total sum available at the end of  $n$  years is therefore

$$\begin{aligned}
 &= q(1 + r)^n + q(1 + r)^{n-1} + \dots\dots\dots q(1 + r)^2 + q(1 + r) \\
 &= q \frac{(1 + r)^{n+1} - (1 + r)}{(1 + r) - 1} = q \frac{1 + r}{r} [(1 + r)^n - 1]
 \end{aligned}$$

This sum must, obviously, be equal to the cost of renewal  $Q$ .

$\therefore$   $Q = q [(1 + r)^n - 1]$  or  $q = Q \frac{r}{1 + r} \div [(1 + r)^n - 1]$

Hence, the total annual charge on the installation is  $(rP + q)$  *i.e.* the plant should bring in so much money every year.



### 50.3. Indian Currency

The basic unit of Indian currency is rupee (Re). Its plural form is rupees (Rs.) One rupee contain 100 paise. Higher multiples of rupees in common use are :

1 lakh (or lac) = Rs. 100,000 = Rs.  $10^5$  = Rs. 0.1 million

1 crore = 100 lakh = Rs.  $10^7$  = Rs. 10 million

**Example 50.1.** Find the total annual charge on an installation costing Rs. 500,000 to buy and install, the estimated life being 30 years and negligible scrap value. Interest is 4% compounded annually.

**Solution.** Since scrap value is negligible,  $Q = P$ . Now  $Q = \text{Rs. } 500,000$ ;  $r = 0.04$ ,  $n = 30$  years.

$$\therefore q = 500,000 \times \frac{0.04}{1.04} \div [1.04^{30} - 1] = \frac{500,000 \times 0.04}{1.04 \times 2.236} = 8,600$$

Hence, the total annual charge on the installation is

$$= rP + q = (0.04 \times 500,000) + 8,600 = \text{Rs. } 28,600$$

**Example 50.2.** A power plant having initial cost of Rs. 2.5 lakhs has an estimated salvage value of Rs. 30,000 at the end of its useful life of 20 years. What will be the annual deposit necessary if it is calculated by :

(i) straight-line depreciation method. (ii) sinking-fund method with compound interest at 7%.

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**Solution.** Here,  $Q = P - \text{scrap value} = \text{Rs. } 250,000 - \text{Rs. } 30,000 = \text{Rs. } 220,000$

$$r = 0.07, n = 20$$

(i) Total depreciation of 20 years = Rs. 220,000

$$\therefore \text{annual depreciation} = \text{Rs. } 220,000/20 = \text{Rs. } 11,300.$$

$$\therefore \text{annual deposit} = rP + q = 0.07 \times 250,000 + 11,000 = \text{Rs. } 28,500$$

(ii) 
$$q = Q \frac{r}{r+1} \div [(1+r)^n - 1]$$

$$= \text{Rs. } 220,000 [1.0720 - 1] = \text{Rs. } 5015.$$

$$\text{Annual deposit} = 0.07 \times 250,000 + 5015 = \text{Rs. } 22,515$$

**Example 50.3.** A plant initially costing Rs. 5 lakhs has an estimated salvage value of Rs. 1 lakh at the end of its useful life of 20 years. What will be its valuation half-way through its life (a) on the basis of straight-line depreciation and (b) on the sinking-fund basis at 8% compounded annually?

**Solution.** (a) In this method, depreciation is directly proportional to time.

$$\text{Total depreciation in 20 years} = \text{Rs. } (5 - 1) = \text{Rs. } 4 \text{ lakhs}$$

$$\therefore \text{depreciation in 10 years} = \text{Rs. } 4/2 = \text{Rs. } 2 \text{ lakhs}$$

$$\therefore \text{its value after 10 years} = (5 - 2) = \text{Rs. } 3 \text{ lakhs.}$$

(b) Now,  $Q = 5 - 1 = \text{Rs. } 4 \text{ lakhs ; } r = 0.08, n = 20$

$$\text{The annual charge is } q = Q \frac{r}{r+1} \div [(1+r)^n - 1]$$

$$\therefore q = 4 \times 10^5 \times \frac{0.08}{1.08} [1.08^{20} - 1] = \text{Rs. } 8095$$

At the end of 10 years, the amount deposited in the sinking fund would become

$$= q \frac{1+r}{r} [(1+r)^n - 1] = 8095 \times \frac{1.08}{0.08} \times (1.08^{10} - 1) = \text{Rs. } 126,647$$

$$\therefore \text{value at the end of 10 years} = \text{Rs. } 500,000 - \text{Rs. } 126,647$$

$$= \text{Rs. } 373,353 = \text{Rs. } 3.73353 \text{ lakhs.}$$

#### 50.4. Factors Influencing Costs and Tariffs of Electric Supply

In the succeeding paragraphs we will discuss some of the factors which determine the cost of



Automation of electricity production, transmission and distribution helps in the effective cost management

generating electric energy and hence the rates or tariffs of charging for this energy. The cost is composed of (i) *standing charges which are independent of the output* and (ii) *running or operating charges which are proportional to the output*. The size or capacity of the generating plant and hence the necessary capital investment is determined by the maximum demand imposed on the generating plant.

### 50.5. Demand

By '*demand*' of a system is meant its load requirement (usually in kW or kVA) averaged over a suitable and specified *interval* of time of short duration.

It should be noted that since '*demand*' means the load averaged over an *interval* of time, there is no such thing as *instantaneous* demand.

### 50.6 Average Demand

By *average* demand of an installation is meant its average power requirement during some specified period of time of considerable duration such as a day or month or year giving us daily or monthly or yearly average power respectively.

Obviously, the average power demand of an installation during a specific period can be obtained by dividing the energy consumption of the installation in kWh by the number of hours in the period.

In this way, we get the arithmetical average.

$$\text{Average power} = \frac{\text{kWh consumed in the period}}{\text{hours in the period}}$$

### 50.7. Maximum Demand

The maximum demand of an installation is defined as the greatest of all the demands which have occurred during a given period.

It is measured, according to specifications, over a prescribed time interval during a certain period such as a day, a month or a year.

It should be clearly understood that it is not the greatest instantaneous demand but the greatest average power demand occurring during any of the relatively short intervals of 1-minute, 15-minute or 30 minute duration within that period.

In Fig. 50.1 is shown the graph of an imaginary load extending over a period of 5 hours. The maximum demand on 30 min. interval basis occurs during the interval *AB* *i.e.* from 8-30 p.m to 9-00 p.m. Its value as calculated in Fig. 50.1 is 288 kW. A close inspection of the figure shows that average load is greater during the 30 min. interval *AB* than it is during any other 30-min interval during this period of 5 hours. The average load over 30-min. interval *AB* is obtained first by scaling kW instantaneous demands at five equidistant points between ordinates *AC* and *BD* and then by taking arithmetic average of these values as shown. Hence, 30 min. maximum demand from the above load graph is 288 kW.

It may be noted that the above method of averaging can be made to yield more accurate results by (i) considering a large number of ordinates and (ii) by scaling the ordinates more precisely.

It may also be noted that if the maximum demand were to be based on a 15 min. interval, then it will occur during the 15-min. interval *MN* and its value will be 342 kW as shown in Fig. 50.1. It is seen that not only has the position of maximum demand changed but its value has also changed. The 30-min. maximum demand has lesser value than 15-min. max. demand. In the present case, 1-min. max. demand will have still greater value and will occur somewhere near point *M*.

From the above discussion, it should be clear that the unqualified term "maximum demand" is indefinite and has no specific meaning. For example, a statement that "maximum demand is 150 kW" carries no specific meaning. To render any statement of maximum demand meaningful, it is necessary



(i) to indicate the *period* of load duration under consideration and (ii) to specify the *time interval* used *i.e.* 15-min. or 30-min. etc. and also (iii) the method used for averaging the demand during that interval.

Now, let us see why it is the *average maximum demand* over a definite *interval* of time that is of interest rather than the *instantaneous maximum demand*.

Maximum demand determinations are mostly used for estimating the capacity (and hence cost) of the generator and other electrical apparatus required for serving a certain specific load.

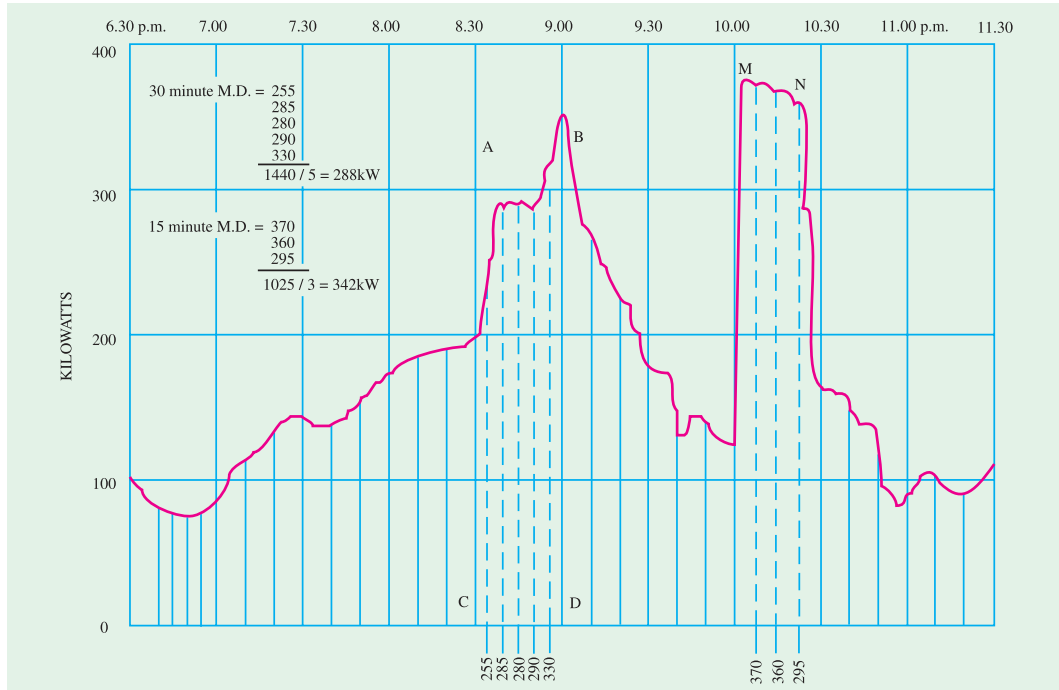


Fig. 50.1

The one main reason why maximum demand values are important is because of the direct bearing they have in establishing the capacity of the generating equipment or indirectly, the initial investment required for serving the consumers. The amount of this investment will have a further effect on fixing of rates for electric service. Since all electric machines have ample overload capacity *i.e.* they are capable of taking 100% or more overloads for short periods without any permanent adverse effects, it is not logical or economically desirable to base the continuous capacity requirements of generators on instantaneous maximum loads which will be imposed on them only momentarily or for very short periods.

Consider the graph of the power load (Fig. 50.2) to be impressed on a certain generator. Let it be required to find the rating of a generator capable of supplying this load. It is seen that there are peak loads of short durations at point A, B, C and D of values 250, 330, 230 and 260 kW. However, during the interval EF a demand of 210 kW persists for more than half an hour. Hence, in this particular case, the capacity of the generator required, as based on 30-min. maximum demand, should be 210 kW, it being of course, assumed that 4-hour load conditions graphed in Fig. 50.2 are typical of the conditions which exist during any similar period of generator's operation.

In the end, it may be remarked that the exact time interval for maximum demand determinations, over which the greatest demand is averaged varies not only with the characteristics of the load but with the policy of the firm measuring the load. However, 15-min. interval is now most generally

used, peak load of shorter durations being considered as temporary overloads.

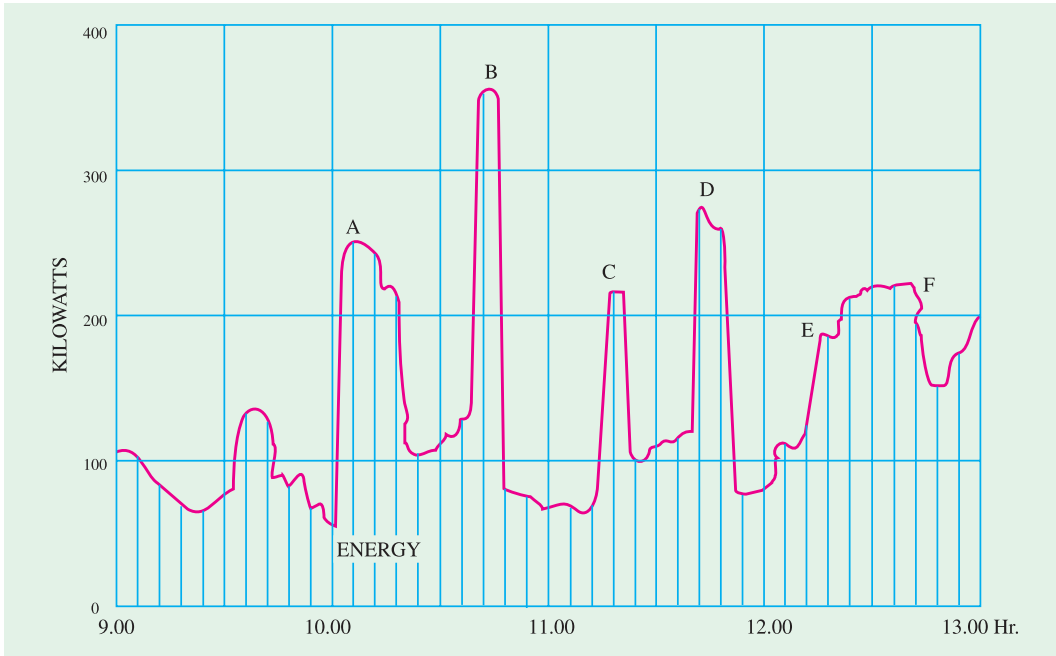


Fig. 50.2

### 50.8. Demand Factor

Demand factors are used for estimating the proportion of the total connected load which will come on the power plant at one time. It is defined as the *ratio of actual maximum demand made by the load to the rating of the connected load*.

$$\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}}$$

The idea of a demand factor was introduced because of the fact that normally the kW or kVA maximum demand of a group of electrical devices or 'receivers' is always less than the sum of the kW or kVA ratings or capacities of these receivers. There are two reasons for the existence of this condition (i) the electrical apparatus is usually selected of capacity somewhat greater than that actually required in order to provide some reserve or overload capacity and (ii) in a group of electrical devices it very rarely happens that all devices will, at the same time, impose their maximum demands which each can impose *i.e.* rarely will all 'receivers' be running full-load simultaneously.

The demand factor of an installation can be determined if (i) maximum demand and (ii) connected load are known.

Maximum demand can be determined as discussed in Art. 50.7 whereas connected load can be calculated by adding together the name-plate ratings of all the electrical devices in the installation. The value of demand factor is always less than unity.

Demand factors are generally used for determining the capacity and hence cost of the power equipment required to serve a given load. And because of their influence on the required investment, they become important factors in computing rate schedules.

As an example, suppose a residence has the following connected load : three 60-W lamps; ten 40-W lamps; four 100-W lamps and five 10-W lamps. Let us assume that the demand meter indicates a 30-min. maximum demand of 650 W. The demand factor can be found as follows :

$$\begin{aligned}\text{Connected load} &= (3 \times 60) + (10 \times 40) + (4 \times 100) + (5 \times 10) = 1,030 \text{ W} \\ \text{30-min. max.demand} &= 650 \text{ W}\end{aligned}$$

Hence, the demand factor of this lighting installation is given as

$$= \frac{\text{max. demand}}{\text{connected load}} = \frac{650}{1,030} = \mathbf{0.631 \text{ or } 63.1\%}$$

Demand factors of lighting installations are usually fairly constant because lighting loads are not subject to such sudden and pronounced variations as like power loads.

### 50.9. Diversity of Demand

In central-station parlance, diversity of demand implies that maximum demands of various consumers belonging to different classes and the various circuit elements in a distribution system are not coincident. In other words, the maximum demands of various consumers occur at different times during the day and not simultaneously. It will be shown later that from the economic angle, it is extremely fortunate that there exists a diversity or non-simultaneity of maximum demand of various consumers which results in lower costs of electric energy.

For example, residence lighting load is maximum in the evening whereas manufacturing establishments require their maximum power during daytime hours. Similarly, certain commercial establishments like department stores usually use more power in day-time than in the evening whereas some other stores like drug stores etc. use more power in the evening.

The economic significance of the concept of diversity of demand can only be appreciated if one considers the increase in the capacity of the generating and distributing plant (and hence the corresponding increase in investment) that would be necessary, if the maximum demands of all the consumers occurred simultaneously. It is of great concern to the engineer because he has to take it into consideration while planning his generating and distributing plant. Also diversity is an important element in fixing the rates of electric service. If it were not for the fact that the coincident maximum demand imposed on a certain station is much less than the sum of maximum demands of all the consumers fed by that station, the investment required for providing the electric service would have been far in excess of that required at present. Because of the necessity of increase in investment, that cost of electric supply would also have been increased accordingly.

### 50.10. Diversity Factor

The non-coincidence of the maximum demands of various consumers is taken into consideration in the so-called diversity factor which is defined as the ratio of the sum of the individual maximum demands of the different elements of a load during a specified period to the simultaneous (or coincident) maximum demand of all these elements of load during the same period.

$$\text{Diversity factor}^* = \frac{\text{maximum demand}}{\text{connected load}}$$

Its value is usually much greater than unity. It is clear that if all the loads in a group impose their maximum demands simultaneously, then diversity factor is equal to unity. High value of diversity factor means that more consumers can be supplied for a given station maximum demand and so lower prices can be offered to consumers. Usually domestic load gives higher value of diversity factor than industrial load. As shown in Fig. 50.3, suppose that the maximum demands of six elements of a load as observed from their maximum demand meters  $M_1$  and  $M_2$  etc. are 620 W, 504 W, 435 W, 380 W, 160 W and 595 W respectively.

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\* Sometimes, the diversity factor is given by certain authors as the reciprocal of the value so obtained.

Also, suppose that the (coincident) maximum demand of the whole group as observed by the maximum demand meter  $M_T$  is only 900 W. It is so because the maximum demands of all load elements did not occur simultaneously.

Sum of individual maximum demands

$$= 620 + 504 + 435 + 380 + 160 + 595 = 2694 \text{ W} = 2.694 \text{ kW}$$

$$\therefore \text{diversity factor} = 2.694/0.9 = 2.99$$

It may be noted here that because of the diversity of demand, the maximum demand on a transformer is less than the sum of the maximum demands of the consumers supplied by that transformer. Further, the maximum demand imposed on a feeder is less than the sum of the maximum demands of transformers connected to that feeder. Similarly, the maximum demand imposed on the generating station is less than the sum of the maximum demands of all the feeders supplied station is less than the sum of the maximum demands of all the feeders supplied from the station. The effective demand of a consumer on a generator is given as follows :

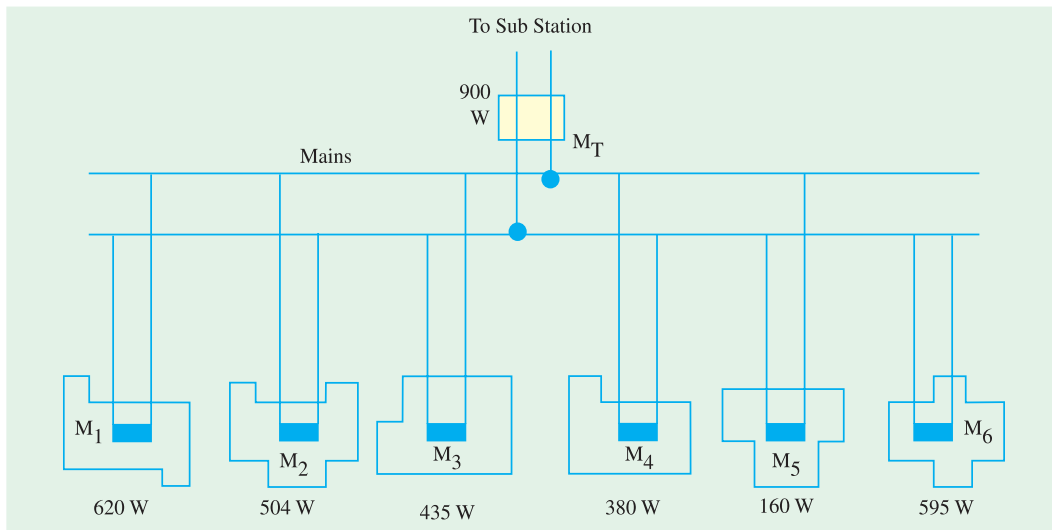


Fig. 50.3

Multiply this connected load by demand factor and then divide the product by diversity factor for consumer to generator.

As an example, let us find the total diversity factor for a residence lighting system whose component diversity factors are : between consumers 2.6 ; between transformers 1.32 ; between feeders 1.13 and between sub-stations 1.1. The total diversity factor between the consumers and the generating equipment would be the product of these component factors *i.e.*

$$= 2.6 \times 1.32 \times 1.13 \times 1.1 = 4.266$$

The factor may now be used in determining the effective demand of consumers on the generator.

It may be proved that generating equipment can be economized by grouping on one supply source different elements of load having high diversity factor. In fact, the percentage of the generating equipment which can be eliminated is equal to 100 percent minus the reciprocal of diversity factor expressed as a percentage. Suppose four loads of maximum demand 120, 360, 200 and 520 kVA respectively are to be supplied. If each of these loads were supplied by a separate transformer, then aggregate transformer capacity required would be =  $120 + 360 + 200 + 520 = 1200$  kVA. Suppose these loads had a diversity factor of 2.5 among themselves, then (coincident) maximum demand of the whole group would be  $1200/2.5 = 480$  kVA.

In other words, a single 480 kVA transformer can serve the combined load. The saving =  $1200 - 480 = 720$  kVA which expressed as a percentage is  $720 \times 100/1200 = 60\%$ . Now, reciprocal of diversity factor =  $1/2.5 = 0.4$  or 40 %. The percentage saving in the required apparatus is also =  $100 - 40 = 60\%$  which proves the statement made above.

### 50.11. Load Factor

It is defined *as the ratio of the average power to the maximum demand.*

It is necessary that in each case the time interval over which the maximum demand is based and the *period\** over which the power is averaged must be definitely specified.

If, for example, the maximum demand is based on a 30-min. interval and the power is averaged over a month, then it is known as 'half-hour monthly' load factor.

Load factors are usually expressed as percentages. The average power may be either generated or consumed depending on whether the load factor is required for generating equipment or receiving equipment.

When applied to a generating station, annual load factor is

$$= \frac{\text{No. of units actually supplied/year}}{\text{Max. possible No. of units that can be supplied}}$$

It may be noted that *maximum* in this definition means the value of the maximum peak load and *not the maximum kW installed capacity of the plant equipment of the station.*

$$\therefore \text{ annual load factor} = \frac{\text{No. of units actually supplied/year}}{\text{Max. possible demand} \times 8760}$$

$$\text{Monthly load factor} = \frac{\text{No. of units actually supplied/month}}{\text{Max. possible demand} \times 24 \times 30}$$

When applied to a consuming equipment

$$\text{annual load factor} = \frac{\text{No. of units consumed/year}}{\text{Max. demand} \times 8760}$$

$$\text{monthly load factor} = \frac{\text{No. of units consumed/month}}{\text{Max. demand} \times 24 \times 30}$$

$$\text{Daily load factor} = \frac{\text{No. of units consumed/day}}{\text{Max. demand} \times 24}$$

$$\text{In general, load factor} = \frac{\text{Average power}}{\text{Max. demand}} \text{ per year or per month or per day}$$

The value of maximum demand can be found by using a maximum demand meter set for 30-min. or 15-min. interval as already explained in Art. 50.7. The average power can also be found either by graphic method explained below or by using a planimeter.

In the graphic method, momentary powers are scaled or read from the load-graph at the end of a number of suitable and equal time intervals over the entire time comprehended by the graph. Then these are added up. Average power is obtained by dividing this sum by the number of periods into which the total time was apportioned.

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\* If not specified, it is assumed to be one year of  $24 \times 365 = 8,760$  hours.

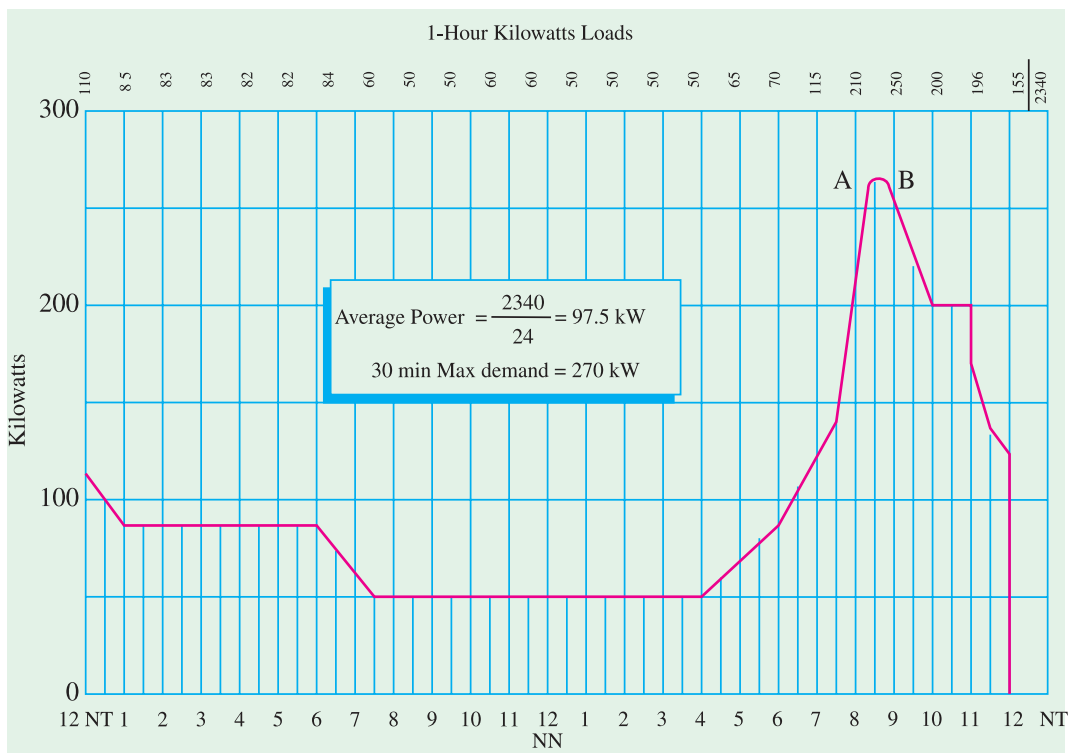


Fig. 50.4

The number of time intervals into which the entire time is apportioned is determined by the contour of the graph and the degree of accuracy required.

The general, the accuracy will increase with the increase in the number of the intervals. For a graph having a smooth contour and comprehending 24 hours, sufficient accuracy can be obtained by taking 1 hour intervals. However, in the case of a graph which has extremely irregular contours and comprehends short time intervals, reasonable accuracy can only be obtained if 15-min. or even 1 min. intervals are used.

As an example, let us find the load factor of a generating equipment whose load graph (imaginary one) is shown in Fig. 50.4. For calculating average power over a period of 24 hours, let us take in view of the regularity of the curve, a time interval of 1 hour as shown. The average power is 97.5 kW. Now, the 30-min. maximum demand is 270 kW and occurs during 30-min. interval of *A B*. Obviously load factor =  $97.5 \times 100/270 = 36.1\%$ .

### 50.12. Significance of Load Factor

Load factor is, in fact, an index to the proportion of the whole time a generator plant or system is being worked to its full capacity. The generating equipment has to be selected on the basis of maximum power demand that is likely to be imposed on it. However, because of general nature of things, it seldom happens that a generating equipment has imposed on it during all the 8,760 hrs of a year the maximum load which it can handle. But whether the equipment is being worked to its full capacity or not, there are certain fixed charges (like interest, depreciation, taxes, insurance, part of staff salaries etc.) which are adding up continuously. In other words, the equipment is costing money to its owner whether working or idle. The equipment earns a net profit only during those hours when it is fully loaded and the more it is fully loaded, the more is the profit to the owner. Hence, from the standpoint

of economics, it is desirable to keep the equipment loaded for as much time as possible i.e. it is economical to obtain high load factors.

If the load factor is poor i.e. kWh of electric energy produced is small, then charge per kWh would obviously be high. But if load factor is high i.e. the number of kWh generated is large, then cost of production and hence charge per kWh are reduced because now the standing charges are distributed over a larger number of units of energy.

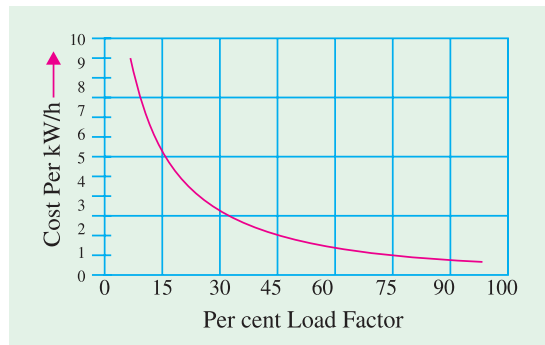


Fig. 50.5

The fact that fixed charges per kWh increase with decreasing load factor and vice versa is brought out in Ex. 50.15 and is graphically shown in Fig. 50.5.

It may be remarked here that increase of diversity in demand increases the load factor almost in direct proportion.

Load factor of a generating plant may be improved by seeking and accepting off-peak loads at reduced rates and by combining lighting, industrial and inter-urban railway loads.

**Example 50.4.** A consumer has the following connected load : 10 lamps of 60 W each and two heaters of 1000 W each. His maximum demand is 1500 W. On the average, he uses 8 lamps for 5 hours a day and each heater for 3 hours a day. Find his total load, monthly energy consumption and load factor. (Power Systems-I, AMIE, Sec. B, 1993)

**Solution.** Total connected load =  $10 \times 60 + 2 \times 1000 = 2600 \text{ W}$ .

Daily energy consumption is =  $(8 \times 60 \times 5) + (2 \times 1000 \times 3) = 8400 \text{ Wh} = 8.4 \text{ kWh}$

Monthly energy consumption =  $8.4 \times 30 = 252 \text{ kWh}$

$$\text{Monthly load factor} = \frac{252}{1500 \times 10^{-3} \times 24 \times 30} = 0.233 \text{ or } 23.3 \%$$

**Example 50.5.** The load survey of a small town gives the following categories of expected loads.

	Type	Load in kW	% D.F.	Group D.F.
1.	Residential lighting	1000	60	3
2.	Commercial lighting	300	75	1.5
3.	Street lighting	50	100	1.0
4.	Domestic power	300	50	1.5
5.	Industrial power	1800	55	1.2

What should be the kVA capacity of the S/S assuming a station p.f. of 0.8 lagging ?

**Solution. (i) Residential lighting.** Total max. demand =  $1000 \times 0.6 = 600 \text{ kW}$

Max. demand of the group =  $600/3 = 200 \text{ kW}$

**(ii) Commercial lighting** Total max. demand =  $300 \times 0.75 = 225 \text{ kW}$

Max. demand of the group =  $225/1.5 = 150 \text{ kW}$

**(iii) Street lighting** Total max. demand =  $50 \text{ kW}$

Max. demand of the group =  $50/1 = 50 \text{ kW}$

**(iv) Domestic power** Total max. demand =  $300 \times 0.5 = 150 \text{ kW}$

Max. demand of the group =  $150/1.5 = 100 \text{ kW}$

(v) **Industrial power**      Total max. demand =  $1800 \times 0.55 = 990 \text{ kW}$   
 Max. demand of the group =  $990/1.2 = 825 \text{ kW}$

Total max. demand at the station =  $200 + 150 + 50 + 100 + 825 = 1325 \text{ kW}$ .

Capacity of the sub-station required at a p.f. of 0.8 lagging =  $1325 / 0.8 = 1656 \text{ kVA}$

**Example 50.6.** A consumer has the following load-schedule for a day :

From midnight (12 p.m.) to 6 a.m. = 200 W ; From 6 a.m. to 12 noon = 3000 W

From 12 noon to 1 p.m. = 100 W ; From 1 p.m. to 4 p.m. = 4000 W

From 4 p.m. to 9 p.m. = 2000 W ; From 9 p.m. to mid-night (12 p.m.) = 1000 W

Find the load factor.

If the tariff is 50 paise per kW of max. demand plus 35 paise per kWh, find the daily bill the consumer has to pay. (Electrical Engineering-III, Poona Univ.)

**Solution.** Energy consumed per day i.e. in 24 hours

$$= (200 \times 6) + (3000 \times 6) + (100 \times 1) + (4000 \times 3) + (2000 \times 5) + (1000 \times 3) = 44,300 \text{ Wh}$$

$$\text{Average power} = 44,300/24 = 1846 \text{ W} = 1.846 \text{ kW}$$

$$\text{Daily load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{1846}{4000} = 0.461 \text{ or } 46.1\%$$

Since max. demand = 4 kW M.D. charge =  $4 \times 1/2 = \text{Rs. } 2/-$

Energy consumed = 44.3 kWh      Energy charge =  $\text{Rs. } 44.3 \times 35/100 = \text{Rs. } 15.5/-$

∴ daily bill of the consumer =  $\text{Rs. } 2 + \text{Rs. } 15.5 = \text{Rs. } 17.5$

**Example 50.7.** A generating station has a connected load of 43,000 kW and a maximum demand of 20,000 kW, the units generated being 61,500,000 for the year. Calculate the load factor and demand factor for this case.

**Solution.** Demand factor =  $\frac{\text{maximum demand}}{\text{connected load}} = 0.465 \text{ or } 46.5\%$

$$\text{Average power} = 61,500,000/8,760 = 7020 \text{ W} \quad (\because 1 \text{ year} = 8760 \text{ hr})$$

$$\therefore \text{Load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{7020}{20,000} = 0.351 \text{ or } 35.1\%$$

**Example 50.8.** A 100 MW power station delivers 100 MW for 2 hours, 50 MW for 6 hours and is shut down for the rest of each day. It is also shut down for maintenance for 45 days each year. Calculate its annual load factor. (Generation and Utilization, Kerala Univ.)

**Solution.** The station operates for  $(365 - 45) = 320$  days in a year. Hence, number of MWh supplied in one year =  $(100 \times 2 \times 320) + (50 \times 6 \times 320) = 160,000 \text{ MWh}$

Max. No. of MWh which can be supplied per year with a max. demand of 100 MW is

$$= 100 \times (320 \times 24) = 768,000 \text{ MWh}$$

$$\therefore \text{load factor} = \frac{160,000}{768,000} \times 100 = 20.8\%$$

**Example 50.9.** Differentiate between fixed and running charges in the operation of a power company.

Calculate the cost per kWh delivered from the generating station whose

(i) capital cost =  $\text{Rs. } 10^6$ ,

(ii) annual cost of fuel =  $\text{Rs. } 10^5$ ,

(iii) wages and taxes =  $\text{Rs. } 5 \times 10^5$ ,

(iv) maximum demand load = 10,000 kW,

(v) rate of interest and depreciation = 10%      (vi) annual load factor = 50%.

Total number of hours in a year is 8,760.

(Electrical Technology-I, Bombay Univ.)



**Solution.** Average power demand = max. load  $\times$  load factor =  $10,000 \times 0.5 = 5,000$  kW  
 Units supplied/year =  $5,000 \times 8,760 = 438 \times 10^5$  kWh  
 Annual cost of the fuel plus wages and taxes = Rs.  $6 \times 10^5$   
 Interest and depreciation charges/year = 10% of Rs.  $106 = \text{Rs. } 10^5$   
 Total annual charges = Rs.  $7 \times 10^5$  ; Cost / kWh =  $\text{Rs. } 7 \times 10^5 / 438 \times 10^5 = 1.6$  paisa

**Example 50.10.** A new colony of 200 houses is being established, with each house having an average connected load of 20 kW. The business centre of the colony will have a total connected load of 200 kW. Find the peak demand of the city sub-station given the following data.

	Demand factor	Group D.F.	Peak D.F.
Residential load	50%	3.2	1.5
Business load	60%	1.4	1.2

**Solution.** The three demand factors are defined as under :

$$\text{Demand factor} = \frac{\text{max. demand}}{\text{connected load}}$$

$$\text{group D.F.} = \frac{\text{sum of individual max. demands}}{\text{actual max. demand of the group}}$$

$$\text{Peak D.F.} = \frac{\text{max. demand of consumer group}}{\text{demand of consumer group at the time of system peak demand}}$$

Max. demand of each house =  $2 \times 0.5 = 1.0$  kW

Max. demand of residential consumer =  $1 \times 200 / 3.2 = 62.5$  kW

Demand of the residential consumer at the time of the system peak =  $62.5 / 1.5 = 41.7$  kW

Max. demand of commercial consumer =  $200 \times 0.6 = 120$  kW

Max. demand of commercial group =  $120 / 1.4 = 85.7$  kW

Commercial demand at the time of system peak =  $85.7 / 1.2 = 71.4$  kW

Total demand of the residential and commercial consumers at the time of system peak  
 =  $41.7 + 71.4 = 113$  kW

**Example 50.11.** In Fig. 50.6 is shown the distribution network from main sub-station. There are four feeders connected to each load centre sub-station. The connected loads of different feeders and their maximum demands are as follows :

Feeder No.	Connected load, kW	Maximum Demand, kW
1.	150	125
2.	150	125
3.	500	350
4.	750	600

If the actual demand on each load centre is 1000 kW, what is the diversity factor on the feeders? If load centres B, C and D are similar to A and the diversity factor between different load centres is 1.1, calculate the maximum demand of the main sub-station. What would be the kVA capacity of the transformer required at the main sub-station if the overall p.f. at the main sub-station is 0.8 ?

**Solution.** Diversity factor of the feeders

$$= \frac{\text{total of max. demand of different feeders}}{\text{simultaneous max. demand}} = \frac{125 + 125 + 350 + 600}{1000} = 1.2$$

Total max. demand of all the 4 load centre sub-stations =  $4 \times 1000 = 4000 \text{ kW}$

Diversity factor of load centres = 1.1

Simultaneous max. demand on the main sub-station =  $4000/1.1 = 3636 \text{ kW}$

The kVA capacity of the transformer to be used at the sub-station =  $3636/0.8 = 4545$

**Example 50.12.** If a generating station had a maximum load for the year of 18,000 kW and a load factor of 30.5% and the maximum loads on the sub-stations were 7,500, 5,000, 3,400, 4,600 and 2,800 kW, calculate the units generated for the year and the diversity factor.

**Solution.** Load factor =  $\frac{\text{average power}}{\text{maximum power demand}} \therefore 0.305 = \frac{\text{average power}}{18,000}$

$\therefore$  average power =  $18,000 \times 0.305 \text{ kW}$

kWh generated per year =  $18,000 \times 0.305 \times 8760 = 48.09 \times 10^6 \text{ kWh}$

Sum of individual maximum demands =  $7,500 + 5,000 + 3,400 + 4,600 + 2,800 = 23,300$

$\therefore$  diversity factor =  $23,300/18,000 = 1.3$  (approx.)

**Example 50.13.** A power station is supplying four regions of load whose peak loads are 10 MW, 5 MW, 8 MW and 7 MW. the diversity factor of the load at the station is 1.5 and the average annual load factor is 60%. Calculate the maximum demand on the station and the annual energy supplied from the station. Suggest the installed capacity and the number of units taking all aspects into account. (A.M.I.E. Sec. B, Winter 1990)

**Solution.** Diversity factor =  $\frac{\text{sum of individual max. demands}}{\text{max. demands of the whole load}}$

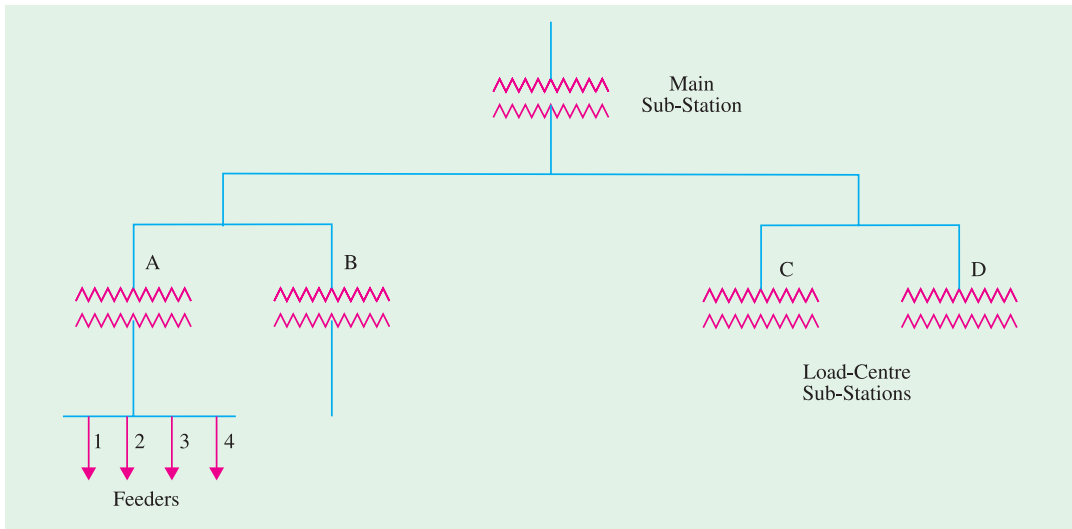


Fig. 50.6

$\therefore$  Max. demand of the whole load imposed on the station =  $(10 + 5 + 8 + 7)/1.5 = 20 \text{ MW}$

Now, annual load factor =  $\frac{\text{No. of units supplied/year}}{\text{Max. demand} \times 8760}$

$\therefore$  No. of units supplied / year =  $0.6 \times 20 \times 10^3 \times 8760 = 105.12 \times 10^6 \text{ kWh}$

Provision for future growth in load may be made by making installed capacity 50% more than the maximum demand of the whole load. Hence, installed capacity is  $20 \times 1.5 = 30 \text{ MW}$ . Four generators, two of 10 MW each and other two of 5 MW each may be installed.

**Example 50.14.** The capital cost of 30 MW generating station is Rs.  $15 \times 10^6$ . The annual expenses incurred on account of fuel, taxes, salaries and maintenance amount to Rs.  $1.25 \times 10^6$ . The station operates at an annual load factor of 35%. Determine the generating cost per unit delivered, assuming rate of interest 5% and rate of depreciation 6%. (Electrical Power-I, Bombay Univ.)

**Solution.** Average power = max. power  $\times$  load factor =  $30 \times 10^6 \times 0.35 = 10,500$  kW  
 Units produced/year =  $10,500 \times 8,760 = 91.98 \times 10^6$  kWh; Annual expenses = Rs.  $1.25 \times 10^6$   
 Depreciation plus interest = 11% of capital cost = 11% of Rs.  $15 \times 10^6 =$  Rs.  $1.65 \times 10^6$   
 Total expenses/year = Rs.  $1.25 \times 10^6 +$  Rs.  $1.65 \times 10^6 =$  Rs.  $2.9 \times 10^6$   
 $\therefore$  cost/kWh = Rs.  $2.9 \times 10^6 / 91.98 \times 10^6 =$  **3.15 paisa / kWh.**

**Example 50.15.** A generating plant has a maximum capacity of 100 kW and costs Rs. 300,000. The fixed charges are 12% consisting of 5% interest, 5% depreciation and 2% taxes etc. Find the fixed charges per kWh generated if load factor is (i) 100% and (ii) 25%.

**Solution.** Annual fixed charges = Rs.  $300,000 \times 12/100 =$  Rs. 36,000

With a load factor of 100%, number of kWh generated per year =  $100 \times 1 \times 8,760 =$  876,000 kWh.

Similarly, units generated with a load factor of 25% =  $100 \times 0.25 \times 8,760 =$  219,000 kWh.

(i) Fixed charge / kWh =  $36,000 \times 100/876,000 =$  **4.1 paisa**

(ii) Fixed charge / kWh =  $36,000 \times 100/219,000 =$  **16.4 paisa**

As seen, the charge has increased four-fold. In fact, charge varies inversely as the load factor.

**Example 50.16.** The annual working cost of a thermal station is represented by the formula Rs.  $(a + b \text{ kW} + c \text{ kWh})$  where  $a$ ,  $b$  and  $c$  are constants for that particular station, kW is the total installed capacity and kWh is the energy produced per annum.

Determine the values of  $a$ ,  $b$  and  $c$  for a 100 MW station having annual load factor of 55% and for which (i) capital cost of buildings and equipment is Rs. 90 million, (ii) the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs. 1,20,000, (iii) interest and depreciation on buildings and equipment are 10% p.a., (iv) annual cost of organisation, interest on cost of site etc. is Rs. 80,000.

**Solution.** In the given formula,  $a$  represents the fixed cost,  $b$  semi-fixed cost and  $c$  the running cost. Here,  $a =$  **Rs. 80,000**



An overview of a thermal power plant

Now,  $b \times \text{kW}$  minimum demand = semi-fixed cost

or,  $b \times 100 \times 10^3 = 90 \times 10^6 \therefore b = \mathbf{900}$

Total units generated per annum = kW max. demand  $\times$  load factor  $\times$  8760  
 $= 100 \times 10^3 \times 0.55 \times 8760 = 438 \times 10^6 \text{ kWh}$

Since running cost is Rs. 1,20,000

$\therefore c \times 438 \times 10^6 = 1,20,000$  or  $c = \mathbf{0.00027}$

**Example 50.17.** In a steam generating station, the relation between the water evaporated  $W$  kg and coal consumed  $C$  kg and power in kW generated per 8-hour shift is as follows :

$$W = 28,000 + 5.4 \text{ kWh}; C = 6000 + 0.9 \text{ kWh}$$

What would be the limiting value of the water evaporated per kg of coal consumed as the station output increases ? Also, calculate the amount of coal required per hour to keep the station running at no-load.

**Solution.** For an 8-hour shift, Wt. of water evaporated per kg of coal consumed is

$$\frac{W}{C} = \frac{28,000 + 5.4 \text{ kWh}}{6,000 + 0.9 \text{ kWh}}$$

As the station output increases, the ratio  $W/C$  approaches the value  $5.4 / 0.9 = 6$

Hence, weight of water evaporated per kg of coal approaches a limiting value of **6 kg** as the station output increases.

Since at no-load, there is no generation of output power, kWh = 0. Substituting this value of kWh in the above ratio we get,

Coal consumption per 8-hour shift = 6000 kg

$\therefore$  coal consumption per hour on no-load =  $\frac{6000}{8} = \mathbf{725 \text{ kg.}}$

**Example 50.18.** Estimate the generating cost per kWh delivered from a generating station from the following data :

Plant capacity = 50 MW ; annual load factor = 40%; capital cost = Rs. 3.60 crores; annual cost of wages, taxation etc. = Rs. 4 lakhs; cost of fuel, lubrication, maintenance etc. = 2.0 paise per kWh generated, interest 5% per annum, depreciation 5% per annum of initial value.

(Electrical Technology, M.S. Univ. Baroda)

**Solution.** Average power over a year = maximum power  $\times$  load factor  
 $= 50 \times 10^6 \times 0.4 = 2 \times 10^7 \text{ W} = 2 \times 10^4 \text{ kW}$   
 Units produced/year =  $20,000 \times 8,760 = 1,752 \times 10^5 \text{ kWh}$   
 Depreciation plus interest = 10% of initial investment =  $0.1 \times 3.6 \times 10^7$   
 $= \mathbf{Rs. 3.6 \times 10^6}$   
 Annual wages and taxation etc. = Rs. 4 lakhs =  $\text{Rs. } 0.4 \times 10^6$   
 Total cost/year =  $\text{Rs. } (3.6 + 0.4) \times 10^6 = \text{Rs. } 4 \times 10^6$   
 Cost / kWh =  $4 \times 10^6 \times 100 / 1,752 \times 10^5 = \mathbf{2.28 \text{ paisa}}$

Adding the cost of fuel, lubrication and maintenance etc., we get

Cost per kWh delivered =  $2.0 + 2.28 = \mathbf{4.28 \text{ paisa.}}$

**Example 50.19.** The following data relate to a 1000 kW thermal station :

Cost of Plant	= Rs. 1,200 per kW
Interest, insurance and taxes	= 5% p.a.
Depreciation	= 5% p.a.

Cost of primary distribution system	=	Rs. 4,00,000
Interest, insurance, taxes and depreciaton	=	5% p.a.
Cost of coal including transportation	=	Rs. 40 per tonne
Operating cost	=	Rs. 4,00,000 p.a.
Plant maintenance cost :		
fixed	=	Rs. 20,000 p.a.
variable	=	Rs. 30,000 p.a.
Installed plant capacity	=	10,000 kW
Maximum demand	=	9,000 kW
Annual load factor	=	60%
Consumption of coal	=	25,300 tonne

Find the cost of power generation per kilowatt per year, the cost per kilowatt-hour generated and the total cost of generation per kilowatt-hour. Transmission/primary distribution is chargeable to generation. **(Power Systems-I, AMIE, Sec. B, 1993)**

**Solution.** Cost of the plant = Rs. 1200 per kW

Fixed cost per annum is as under :

$$(i) \text{ on account of capital cost} = (1200 \times 10,000) \times 0.1 + 400 \times 103 \times 0.05 \\ = \mathbf{Rs. 1.22 \times 10^6}$$

$$(ii) \text{ part of maintenance cost} = Rs. 20,000 = Rs. 0.02 \times 10^6 \\ \therefore \text{ total fixed cost} = 1.22 \times 10^6 + 0.02 \times 10^6 = \mathbf{Rs. 1.24 \times 10^6}$$

Running or variable cost per annum is as under : (i) operation cost = Rs. 4,00,000, (ii) part of maintenance cost = Rs. 30,000,

$$(iii) \text{ fuel cost} = Rs. 25,300 \times 40 = Rs. 10,12,000$$

$$\text{Total cost} = 4,00,000 + 30,000 + 10,12,000 = Rs. 1.442 \times 10^6$$

$$\text{Load factor} = \frac{\text{average demand}}{\text{maximum demand}} \text{ or } 0.6 = \frac{\text{average demand}}{9,000 \times 8,760}$$

$$\therefore \text{ average demand} = \mathbf{47,305 \text{ MWh}}$$

$$\text{Total cost per annum} = 1.24 \times 10^6 + 1.442 \times 10^6 = Rs. 2.682 \times 10^6$$

$$\text{Cost per kWh generated} = 2.682 \times 10^6 / 47,305 \times 10^3 = Rs. 0.0567 = \mathbf{5.7 \text{ paisa.}}$$

$$\text{Since total installed capacity is 10,000 kW, the cost per kW per year} \\ = 2.682 \times 10^6 / 10,000 = \mathbf{Rs. 268.2}$$

**Example 50.20.** A consumer has an annual consumption of 176,400 kWh. The charge is Rs. 120 per kW of maximum demand plus 4 paisa per kWh.

(i) Find the annual bill and the overall cost per kWh if the load factor is 36%.

(ii) What is the overall cost per kWh, if the consumption were reduced 25% with the same load factor ?

(iii) What is the overall cost per kWh, if the load factor is 27% with the same consumption as in (i) **(Utili. of Elect. Power, AMIE Sec. B)**

**Solution.** (i) Since load factor is 0.36 and there are 8760 hrs in a year,

$$\text{Annual max. demand} = 176,400 / 0.36 \times 8760 = \mathbf{55.94 \text{ kW}}$$

The annual bill will be based on maximum annual demand charges plus the annual energy consumption charge.

$$\therefore \text{ annual bill} = Rs. (55.94 \times 120 + 176,400 \times 0.04) = \mathbf{Rs. 13,768}$$

$$\text{Overall cost/kWh} = Rs. 13,768 / 176,400 = \mathbf{7.8 \text{ paisa.}}$$

(ii) In this case, the annual consumption is reduced to  $176,400 \times 0.75 = 132,300$  kWh but the load factor remains the same.

$$\begin{aligned} \text{Annual max. demand} &= 132,300/0.36 \times 8760 = 41.95 \text{ kW} \\ \therefore \text{annual bill} &= \text{Rs. } (41.95 \times 120 + 132,300 \times 0.04) = \text{Rs. } 10,326 \\ \text{Overall cost/kWh} &= 10,326/132,300 = 7.8 \text{ paisa} \end{aligned}$$

It will be seen that the annual max. demand charge is reduced but the overall cost per kWh remains the same.

(iii) Since, load factor has decreased to 0.27,

$$\begin{aligned} \text{Annual max. demand} &= 176,400/0.27 \times 8760 = 74.58 \text{ kW} \\ \text{Annual bill} &= \text{Rs. } (74.58 \times 120 + 176,400 \times 0.04) = \text{Rs. } 16,006 \\ \text{Overall cost/kWh} &= \text{Rs. } 16,006/176,400 = 9.1 \text{ paisa} \end{aligned}$$

Here, it will be seen that due to decrease in load factor, the annual bill as well as cost per kWh have increased.

### 50.13. Plant Factor or Capacity Factor

This factor relates specifically to a generating plant unlike load factor which may relate either to generating or receiving equipment for the whole station.

It is defined as the *ratio of the average load to the rated capacity of the power plant i.e. the aggregate rating of the generators*. It is preferable to use continuous rating while calculating the aggregate.

$$\therefore \text{plant factor} = \frac{\text{average load}}{\text{rated capacity of plant}} = \frac{\text{average demand on station}}{\text{max. installed capacity of the station}}$$

It may be of interest to note that if the maximum load corresponds exactly to the plant ratings, then load factor and plant factor will be identical.

### 50.14. Utilization Factor (or Plant Use Factor)

It is given by the ratio of the kWh generated to the product of the capacity of the plant and the number of hours the plant has been actually used.

$$\text{Utilization factor} = \frac{\text{station output in kWh}}{\text{plant capacity} \times \text{hours of use}}$$

If there are three units in a plant of ratings  $\text{kW}_1$ ,  $\text{kW}_2$  and  $\text{kW}_3$  and their operation hours are  $h_1$ ,  $h_2$  and  $h_3$  respectively, then

$$\text{Utilization factor} = \frac{\text{station output in kWh}}{(\text{kW}_1 \times h_1) + (\text{kW}_2 \times h_2) + (\text{kW}_3 \times h_3)}$$

### 50.15. Connected Load Factor

The factor relates only to the receiving equipment and is defined as the ratio of the average power input to the connected load.

To render the above value specific, it is essential\*

- (i) to define the period during which average is taken and
- (ii) to state the basis on which the connected load is computed.

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\* Wherever feasible, it should be stated on continuous-rating basis. Lighting connected load is taken equal to the sum of the wattages of all lamps in the installation whereas motor connected load is equal to the sum of the name-plate outputs of all motors (and not their input ratings).

$$\text{Connected-load factor} = \frac{\text{average power input}}{\text{connected load}}$$

It can be proved that the connected-load factor of a *receiving* equipment is equal to the product of its demand factor and its load factor.

$$\begin{aligned} \text{Connected-load factor} &= \frac{\text{average power input}}{\text{connected load}} \\ &= \frac{\text{Average power}}{\text{Max. demand}} \times \frac{\text{max demand}}{\text{connected load}} = \text{load factor} \times \text{demand factor} \end{aligned}$$

### 50.16. Load Curves of a Generating Station

The total power requirement of a generating station can be estimated provided variation of load with time is known. Following curves help to acquire this knowledge.

#### (i) Load Curve (or Chronological Curve)

It represents the load in its proper time sequence. As shown in Fig. 50.7 (a), this curve is obtained by plotting the station load (in kW) along Y-axis and the time when it occurs along X-axis. Usually, such curves are plotted for one day *i.e.* for 24 hours by taking average load (kW) on hourly basis. The area under the curve represents the total energy consumed by the load in one day. Following information can be obtained from the load curve :

(a) maximum load imposed on the station, (b) size of the generating unit required and (c) daily operating schedule of the station.

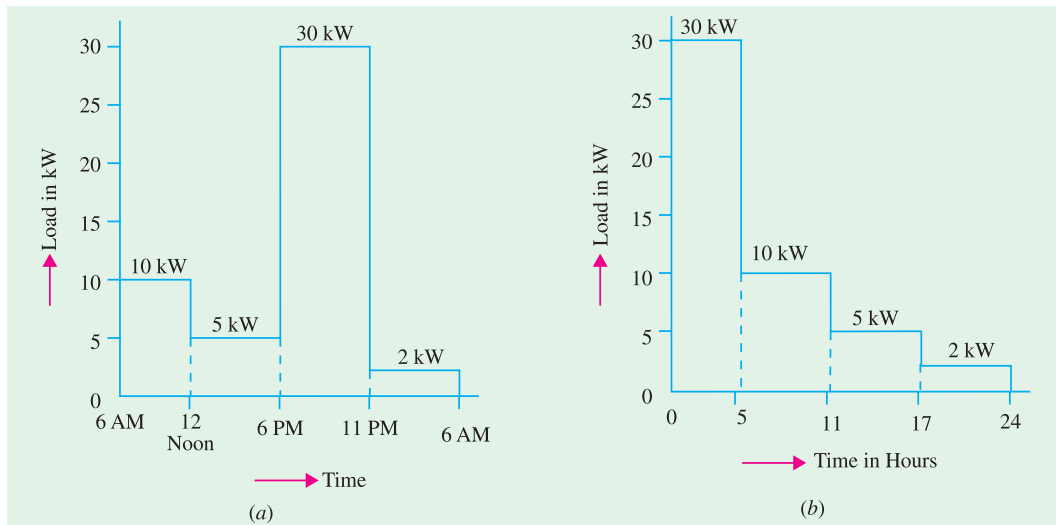


Fig. 50.7

#### (ii) Load Duration Curve

It represents the same data (*i.e.* load vs time) but the ordinates are rearranged in magnitude sequence (not time sequence). Here, greatest load is plotted on the left, lesser load towards the right and the least load on the extreme right. In other words, loads are plotted in descending order. As seen from Fig. 50.7 (a) maximum load on the station is 30 kW which lasts for 5 hours from 6 p.m. to 11 p.m. It is plotted first in Fig. 50.7 (b). The next lower load is 10 kW from 6 a.m. to 12 noon *i.e.* for 6 hours. It has been plotted next to the highest load. The other lesser loads are plotted afterwards. The areas under the load curve and load duration curve are equal and each represents the total units consumed during a day of 24 hours.

(iii) Load Energy Curve (or Integrated Load Duration Curve)

It represents the relation between a particular load on the station and the total number of kWhs produced *at or below this load*. The load in kW is taken along the ordinate (Y-axis) and kWh generated *upto this load* along the abscissa (X-axis) as shown in Fig. 50.8. This curve is derived from the load duration curve. For example, for a load of 2 kW, the number of units generated is  $2 \times 24 = 48$  kWh. It corresponds to point A on the curve. For a load of 5 kW, the units generated are  $5 \times 17 + 2 \times 7 = 99$  kWh. It corresponds to point B. For a load of 10 kW, the units generated are  $10 \times 11 + 5 \times 6 + 2 \times 7 = 154$  kWh (point C). Finally, for a load of 30 kW, the number of units generated is  $30 \times 5 + 10 \times 6 + 5 \times 6 + 2 \times 7 = 254$  kWh (point D).

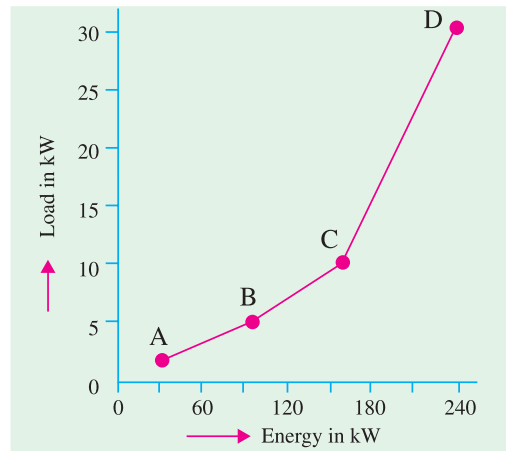


Fig. 50.8

**Example 50.21.** A power station has a load cycle as under :

60 MW for 6 hr; 200 MW for 8 hr; 160 MW for 4 hr; 100 MW for 6 hr

If the power station is equipped with 4 sets of 75 MW each, calculate the load factor and the capacity factor from the above data. Calculate the daily fuel requirement if the calorific value of the oil used were 10,000 kcal/kg and the average heat rate of the station were 2,860 kcal/kWh.

(Electric Power Systems-III, Gujarat Univ.)

**Solution.** Daily load factor =  $\frac{\text{units actually supplied in a day}}{\text{max. demand} \times 24}$

Now, MWh supplied per day =  $(260 \times 6) + (200 \times 8) + (160 \times 4) + (100 \times 6) = 4,400$

$\therefore$  station daily load factor =  $\frac{4,440}{260 \times 24} = 0.704$  or **70.4%**

Capacity factor =  $\frac{\text{average demand on station}}{\text{installed capacity of the station}}$

No. of MWh supplied/day = 4,400  $\therefore$  average power/day =  $4,400/24$  MW

Total installed capacity of the station =  $75 \times 4 = 300$  MW

$\therefore$  capacity factor =  $\frac{4,400/24}{300} = 0.611$  or **61.1%**

Energy supplied/day = 4,400 MWh =  $44 \times 10^5$  kWh

Heat required/day =  $44 \times 10^5 \times 2,860$  kcal

Amount of fuel required/day =  $44 \times 2,860 \times 10^5 / 10^5$  kg = **125 tonne.**

**Example 50.22.** A generating station has two 50 MW units each running for 8,500 hours in a year and one 30 MW unit running for 1,250 hours in one year. The station output is  $650 \times 10^6$  kWh per year. Calculate (i) station load factor, (ii) the utilization factor.

**Solution.** (i)  $\text{kW}_1 \times h_1 = 50 \times 10^3 \times 8,500 = 425 \times 10^6$  kWh

(ii)  $\text{kW}_2 \times h_2 = 50 \times 10^3 \times 8,500 = 425 \times 10^6$  kWh

(iii)  $\text{kW}_3 \times h_3 = 30 \times 10^3 \times 1,250 = 37.5 \times 10^6$  kWh

$\therefore \Sigma (\text{kW}) \times h = (2 \times 425 + 37.5) \times 10^6 = 887.5 \times 10^6$  kWh

Total installed capacity of the station =  $2 \times 50 + 30 = 130 \times 10^3$  kW

(i) Assuming that maximum demand equals installed capacity of the station,



$$\text{annual load factor} = \frac{\text{units generated/year}}{\text{max. demand} \times 8,760} = \frac{650 \times 10^6}{130 \times 10^3 \times 8760} = \mathbf{0.636 \text{ or } 63.6\%}$$

**Note.** In view of the above assumption, this also represents the plant or capacity factor.

$$(ii) \quad \text{utilization factor} = \frac{\text{station output in kWh}}{\Sigma(\text{kW}) \times h} = \frac{650 \times 10^6}{887.5 \times 10^6} = \mathbf{0.732 \text{ or } 73.2\%}$$

**Example 50.23.** The yearly duration curve of a certain plant may be considered as a straight line from 40,000 kW to 8,000 kW. To meet this load, three turbo-generators, two rated at 20,000 kW each and one at 10,000 kW are installed. Determine (a) the installed capacity, (b) plant factor, (c) maximum demand, (d) load factor and (e) utilization factor. **(Ranchi Univ.)**

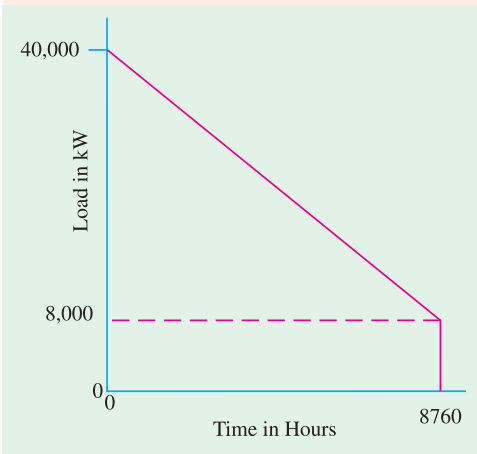


Fig. 50.9

- Solution.** (a) installed capacity  
 $= 20 + 20 + 10 = \mathbf{50 \text{ MW}}$
- (b) Average demand on plant  $= (40,000 + 8,000)/2$   
 $= 24,000 \text{ kW}$   
 plant factor  $= \text{av. demand} / \text{installed capacity}$   
 $= 24,000/50,000 = \mathbf{0.48\% \text{ or } 48\%}$
- (c) Max. demand, obviously, is  $\mathbf{40,000 \text{ kW}}$
- (d) From load duration curve, total energy generated/year  
 $= 24,000 \times 8760 \text{ kWh} = 21 \times 10^7 \text{ kWh.}$   
 Load factor  $= 21 \times 10^7 / 40,000 \times 8,760 = \mathbf{0.6 \text{ or } 60\%}$
- (e) u.f.  $= \frac{\text{max. demand}}{\text{plant capacity}} \times 100 = \frac{40,000}{50,000} \times 100 = \mathbf{80\%}$

**Example 50.24.** The load duration curve of a system is as shown in Fig. 50.10. The system is supplied by three stations; a steam station, a run-of-river station and a reservoir hydro-electric station. The ratios of number of units supplied by the three stations are as below :

Steam	:	Run of river	:	Reservoir
7	:	4	:	1

The run-of-river station is capable of generating power continuously and works as a peak load station. Estimate the maximum demand on each station and also the load factor of each station.

**(Ranchi Univ.)**

**Solution.** Here 100% time will be taken as 8760 hours.

$$\begin{aligned} \text{Total units generated} &= \text{area under the curve} \\ &= \frac{1}{2} (160 + 80) \times 10^3 \times 8760 = 1051.2 \times 10^6 \text{ kWh} \end{aligned}$$

From the given ratio, the number of units supplied by each station can be calculated

Units Generated

$$\text{Run-of-river-station} = 1051.2 \times 10^6 \times 4/12 = 350.4 \times 10^6 \text{ kWh}$$

$$\text{Steam station} = 1051.2 \times 10^6 \times 7/12 = 613.2 \times 10^6 \text{ kWh}$$

$$\text{Reservoir HE station} = 1051.2 \times 10^6 \times 1/12 = 87.6 \times 10^6 \text{ kWh}$$

$$\text{Max. demand of ROR station} = 350.4 \times 10^6 / 8760 = \mathbf{40 \text{ MW}}$$

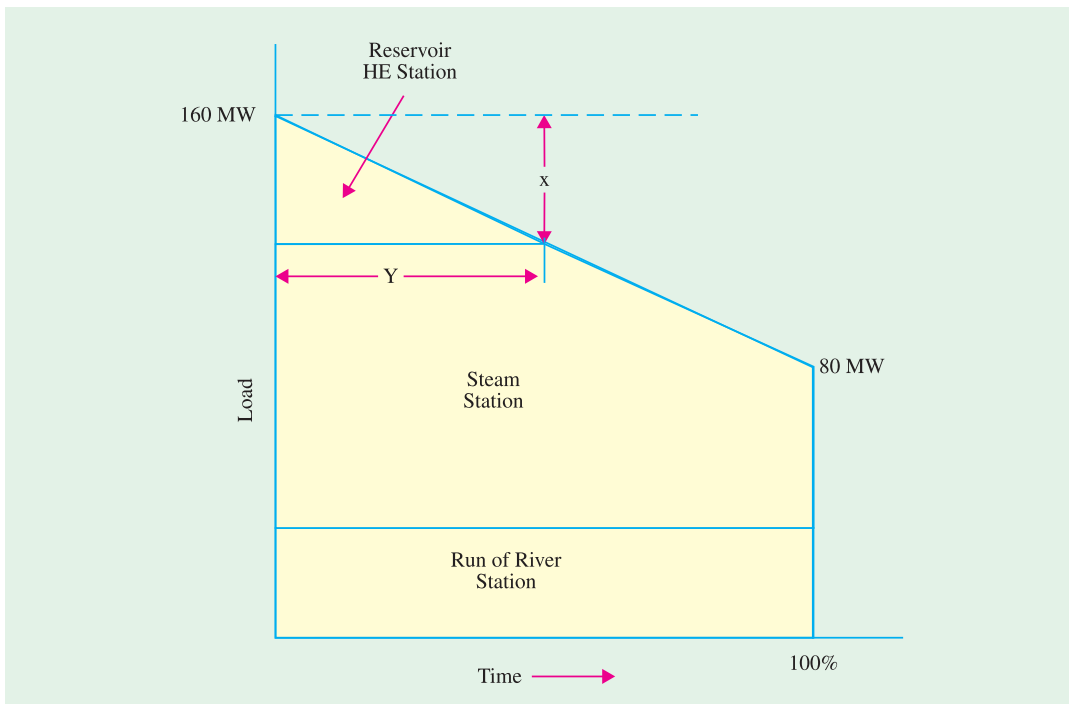


Fig. 50.10

Let  $x$  MW be the maximum demand of the reservoir plant. As shown in Fig. 50.10, let it operate for  $y$  hours.

Obviously, 
$$y = \frac{x}{80} \times 8760$$

Area under the curve for reservoir station =  $\frac{1}{2}xy \times 10^3$  kWh  

$$= \frac{1}{2}x \times \frac{x}{80} \times 8760 \times 10^3 = 54,750 x^2$$

$\therefore 54,750 x^2 = 87.6 \times 10^6 ; x = 40$  MW  
 $\therefore$  max. demand of steam station =  $160 - (40 + 40) = 80$  MW.

**Load factor**

Since ROR station works continuously as a base load station, its load factor is **100%**.

Reservoir station =  $87.6 \times 10^6 / 40 \times 10^3 \times 8760 = 0.25$  or **25%**  
 Steam station =  $613.2 \times 10^6 / 80 \times 10^3 \times 8760 = 0.875$  or **87.5%**.

**Example 47.25.** A load having a maximum value of 150 MW can be supplied by either a hydro-electric plant or a steam power plant. The costs are as follows :

- Capital cost of steam plant = Rs. 700 per kW installed
- Capital cost of hydro-electric plant = Rs. 1,600 per kW installed
- Operating cost of steam plant = Rs. 0.03 per kWh
- Operating cost of hydro-electric plant = Rs. 0.006 per kWh

Interest on capital cost 8 per cent. Calculate the minimum load factor above which the hydro-electric plant will be more economical.

**Solution.** Let  $x$  be the total number of units generated per annum.

**Steam Plant**

$$\begin{aligned} \text{Capital cost} &= \text{Rs. } 700 \times 150 \times 10^3 = \text{Rs. } 10.5 \times 10^7 \\ \text{Interest charges} &= 8\% \text{ of Rs. } 10.5 \times 10^7 = \text{Rs. } 8.4 \times 10^6 \\ \therefore \text{ fixed cost/unit} &= \text{Rs. } 8.4 \times 10^6 / x; \text{ operating cost / unit} = \text{Re. } 0.03 \\ \therefore \text{ total cost/unit generated} &= \text{Rs. } (8.4 \times 10^4 / x + 0.03) \end{aligned}$$

**Hydro Plant**

$$\begin{aligned} \text{Capital cost} &= \text{Rs. } 1600 \times 150 \times 10^3 = \text{Rs. } 24 \times 10^7 \\ \text{Interest charges} &= 8\% \text{ of Rs. } 24 \times 10^7 = \text{Rs. } 19.2 \times 10^6 \\ \text{Total cost/unit} &= \text{Rs. } (19.2 \times 10^6 / x + 0.006) \end{aligned}$$

The two overall costs will be equal when

$$(8.4 \times 10^6 / x) + 0.03 = (19.2 \times 10^6 / x) + 0.006; x = 45 \times 10^7 \text{ kWh}$$

Obviously, if units generated are more than  $45 \times 10^7$  kWh, hydro-electric station will be cheaper.

$$\text{Load factor} = 45 \times 10^7 / 150 \times 10^3 \times 8760 = \mathbf{0.342 \text{ or } 34.2\%}$$

This represents the minimum load factor beyond which hydro-electric station would be economical.

**Example 50.26.** A power system having maximum demand of 100 MW has a load 30% and is to be supplied by either of the following schemes :

(a) a steam station in conjunction with a hydro-electric station, the latter supplying  $100 \times 10^6$  units per annum with a max. output of 40 MW,

(b) a steam station capable of supply the whole load,

(c) a hydro station capable of supplying the whole load,

Compare the overall cost per unit generated assuming the following data :

	Steam	Hydro
Capital cost / kW	Rs. 1,250	Rs. 2,500
Interest and depreciation on the capital cost	12%	10%
Operating cost/kWh	5 paise	1.5 paise
Transmission cost/kWh	Negligible	0.2 paise

Show how overall cost would be affected in case (ii) and (iii) above if the system load factor were improved to 90 per cent. **(Elect. Power System-III, Gujarat Univ.)**

**Solution.** Average power =  $100 \times 0.3 = 30 \text{ MW} = 3 \times 10^4 \text{ kW}$

Units generated in one year =  $3 \times 10^4 \times 8,760 = 262.8 \times 10^6 \text{ kWh}$

**(a) Steam Station in Conjunction with Hydro Station**

Units supplied by hydro-station =  $100 \times 10^6 \text{ kWh}$

Units supplied by steam station =  $(262.8 - 100) \times 10^6 = 162.8 \times 10^6 \text{ kWh}$

Since, maximum output of hydro-station is 40 MW, the balance  $(100 - 40) = 60 \text{ MW}$  is supplied by the steam station.

**(i) Steam Station**

Capital cost =  $\text{Rs. } 60 \times 10^3 \times 1,250 = \text{Rs. } 75 \times 10^6$

Annual interest and depreciation -  $\text{Rs. } 0.12 \times 75 \times 10^6 = \text{Rs. } 9 \times 10^6$

Operating cost =  $\text{Rs. } 0.5 \times 162.8 \times 10^6 / 100 = \text{Rs. } 8.14 \times 10^6$

Transmission cost = negligible

Total annual cost =  $\text{Rs. } (9 + 8.14) \times 10^6 = \text{Rs. } 17.14 \times 10^6$

**(ii) Hydro Station**

$$\begin{aligned} \text{Capital cost} &= \text{Rs. } 40 \times 10^3 \times 2,500 = \text{Rs. } 100 \times 10^6 \\ \text{Annual interest and depreciation} &= \text{Rs. } 0.1 \times 100 \times 10^6 = \text{Rs. } 10 \times 10^6 \\ \text{Operating cost} &= \text{Rs. } 0.15 \times 100 \times 10^6 / 100 = \text{Rs. } 1.5 \times 10^6 \\ \text{Transmission cost} &= \text{Rs. } .02 \times 100 \times 10^6 / 100 = \text{Rs. } 0.2 \times 10^6 \\ \text{Total annual cost} &= \text{Rs. } (10 + 1.5 + 1.2) \times 10^6 = \text{Rs. } 11.7 \times 10^6 \\ \text{Combined annual charge for steam and hydro stations} &= \text{Rs. } (17.14 + 11.7) \times 10^6 = \text{Rs. } 28.84 \times 10^6 \end{aligned}$$

$$\therefore \text{Overall cost/kWh} = \text{Rs. } \frac{28.84 \times 10^6}{262.8 \times 10^6} = \mathbf{10.97 \text{ paise}}$$

**(b) Steam Station Alone**

$$\begin{aligned} \text{Capital cost} &= \text{Rs. } 1,250 \times 100 \times 10^3 = \text{Rs. } 125 \times 10^6 \\ \text{Annual interest and depreciation} &= \text{Rs. } 0.12 \times 125 \times 10^6 = \text{Rs. } 15 \times 10^6 \\ \therefore \text{fixed charge / unit} &= \text{Rs. } 15 \times 10^6 / 262.8 \times 10^6 = 5.71 \text{ paise} \\ \text{Operating cost/unit} &= 5 \text{ paise ; Transmission cost / unit} = 0 \\ \therefore \text{overall cost per unit} &= (5.71 + 5) = \mathbf{10.71 \text{ paise}} \end{aligned}$$

**(c) Hydro Station Alone**

$$\begin{aligned} \text{Annual interest and depreciation on capital cost} &= \text{Rs. } 0.1 (2,500 \times 100 \times 10^3) = \text{Rs. } 25 \times 10^6 \\ \therefore \text{fixed charge / unit} &= \text{Rs. } 25 \times 10^6 / 262.8 \times 10^6 = 9.51 \text{ paise} \\ \text{Operating cost/unit} &= 1.5 \text{ paise ; Transmission cost/unit} = 0.2 \text{ paise} \\ \therefore \text{overall cost/unit} &= (9.51 + 1.5 + 0.2) = \mathbf{11.21 \text{ paise}} \end{aligned}$$

**(d) (i) Steam Station**

Since number of units generated will increase three-fold, fixed charge per unit will decrease to one-third of its previous value *i.e.* to  $5.71/3 = 1.9$  paise. Since other charges are unaffected by change in load factor.

$$\therefore \text{Overall cost/unit} = (1.9 + 5) = \mathbf{6.9 \text{ paise}}$$

**(ii) Hydro Station**

For same reasons, fixed cost per unit becomes  $9.51/3 = 3.17$  paise Overall cost/unit =  $(3.17 + 1.5 + 0.2) = \mathbf{4.87 \text{ paise}}$



An overview of a hydroelectric plant

**Example 50.27.** The capital cost of a hydro-power station of 50,000 kW capacity is Rs. 1,200 per kW. The annual charge on investment including depreciation etc. is 10%. A royalty of Rs. 1 per kW per year and Rs. 0.01 per kWh generated is to be paid for using the river water for generation of power. The maximum demand is 40,000 kW and the yearly load factor is 80%. Salaries, maintenance charges and supplies etc. total Rs. 6,50,000. If 20% of this expense is also chargeable as fixed charges, determine the generation cost in the form of A per kW plus B per kWh.

(A.M.I.E. Sec. B.)

**Solution.** Capital cost of station = Rs.  $1200 \times 50,000 = \text{Rs. } 6 \times 10^7$   
 Annual charge on investment including depreciation  
     = 10% of Rs.  $6 \times 10^7 = \text{Rs. } 6 \times 10^6$   
 Total running charges = 80% of Rs. 6,50,000 = Rs. 5,20,000  
 Fixed charges = 20% of Rs. 6,50,000 = Rs. 1,30,000  
 Total annual fixed charges = Rs.  $6 \times 10^6 + \text{Rs. } 0.13 \times 10^6 = \text{Rs. } 6.13 \times 10^6$   
 Cost per M.D. kW due to fixed charges = Rs.  $6.13 \times 10^6 / 40,000 = \text{Rs. } 153.25$   
 Cost per M.D. kW due to royalty = Rs. 1  
 Total cost per M.D. kW = Rs. 154.25  
 Total No. of units generated per annum =  $40,000 \times 0.8 \times 8760 = 28 \times 10^7$  kWh  
 Cost per unit due to running charges = Rs.  $5,20,000 / 28 \times 10^7 = 0.18$  p  
 Royalty cost/unit = 1 p  
 $\therefore$  total cost/unit = 1.18 p  
 $\therefore$  generation cost = Rs. 154.25 kWh + 1.18 p kWh  
     = **Rs. (154.25 kWh + 1.18  $\times 10^{-2}$  kWh)**

**Example 50.28.** The capital costs of steam and water power stations are Rs. 1,200 and Rs. 2,100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.

The reserve capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor would the overall cost per kWh be the same in both cases? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydro-electric station. What would be the cost of generating 500 million kWh at this load factor?

**Solution.** Let  $x$  be the maximum demand in kWh and  $y$  the load factor.

Total No. of units produced =  $xy \times 8760$  kWh

#### Steam Station

installed capacity = 1.25  $x$  (including reserve capacity)  
 capital cost = Rs.  $1200 \times 1.25 x = \text{Rs. } 1500 x$   
 annual interest and depreciation = 9% of Rs.  $1500 x = \text{Rs. } 135 x$   
 annual running cost = Rs.  $8760 xy \times 5/100 = \text{Rs. } 438 xy$   
 Total annual cost = Rs.  $(135 x + 438 xy)$   
 $\therefore$  total cost / unit = **Rs. (135  $x$  + 438  $xy$  / 8760  $xy$**

#### Hydro-electric Station

installed capacity = 1.33  $x$  (including reserve capacity)  
 capital cost = Rs.  $1.33 x \times 2100 = \text{Rs. } 2800 x$   
 annual interest and depreciation = 7.5% of Rs.  $2800 x = \text{Rs. } 210 x$   
 annual running cost = Rs.  $8760 xy \times 3.2/100 = \text{Rs. } 280 xy$

$$\begin{aligned} \therefore \text{total annual cost} &= \text{Rs. } (210x + 280xy) \\ \therefore \text{total cost/unit} &= \text{Rs. } (210x + 280xy)/8760xy \end{aligned}$$

For the two costs to be the same, we have

$$\frac{135x + 438xy}{8760xy} = \frac{210x + 280xy}{8760xy}, \quad y = \mathbf{0.475 \text{ or } 47.5\%}$$

**Cost of  $500 \times 10^6$  Units**

$$\begin{aligned} \text{Now, maximum demand} &= 500 \times 10^6 / 8760 \times 0.475 = 12 \times 10^4 \text{ kW} \\ \therefore x &= 12 \times 10^4, y = 0.475 \\ \therefore \text{generating cost} &= \text{Rs. } (136 \times 12 \times 10^4 + 438 \times 12 \times 10^4 \times 0.475) = \mathbf{41,166,000} \end{aligned}$$

**Example 50.29.** In a particular area, both steam and hydro-stations are equally possible. It has been estimated that capital cost and the running costs of these two types will be as follows:

Capital cost/kW	Running cost/kWh	Interest
Hydro : Rs. 2,200	1 Paise	5%
Steam : Rs. 1,200	5 Paise	5%

If expected average load factor is only 10%, which is economical to operate : steam or hydro? If the load factor is 50%, would there be any change in the choice ? If so, indicate with calculation.

(Electric Power-II Punjab Univ. 1991)

**Solution.** Let  $x$  be the capacity of power station in kW.

**Case I. Load factor = 10%**

$$\text{Total units generated/annum} = x \times 0.1 \times 8760 = 876x \text{ kWh}$$

**(a) Hydro Station**

$$\begin{aligned} \text{capital cost} &= \text{Rs. } 2200x \\ \text{annual fixed charges} &= 5\% \text{ of Rs. } 2200x = \text{Rs. } 110x \\ \text{annual running charges} &= \text{Rs. } 876x \times 1/100 = \text{Rs. } 8.76x \\ \text{total annual charges} &= \text{Rs. } (110 + 8.76)x \\ \text{total cost/unit} &= \text{Rs. } (110 + 8.76)x / 876x = \mathbf{13.5 \text{ p}} \end{aligned}$$

**(b) Steam Station**

$$\begin{aligned} \text{capital cost} &= \text{Rs. } 1200x \\ \text{annual fixed charges} &= 5\% \text{ of Rs. } 1200x = \text{Rs. } 60x \\ \text{annual running charges} &= \text{Rs. } 876x \times 5/100 = \text{Rs. } 43.8x \\ \text{total annual charges} &= \text{Rs. } (60 + 43.8)x \\ \text{overall cost/unit} &= \text{Rs. } 103.8x / 876x = \mathbf{11.85 \text{ p}} \end{aligned}$$

Obviously, steam station is more economical to operate.

**Case II. Load factor = 50%**

$$\text{Total units generated/annum} = x \times 8760 \times 0.5 = 4380x$$

**(a) Hydro Station**

$$\text{If we proceed as above, we find that total cost/unit} = \mathbf{3.5 \text{ p}}$$

**(b) Steam Station**

$$\text{total cost/unit} = \mathbf{6.35 \text{ p}}$$

Obviously, in this case, hydro-station is more economical.

**Example 50.30.** The annual working cost of a thermal station can be represented by a formula Rs.  $(a + b \text{ kW} + c \text{ kWh})$  where  $a$ ,  $b$  and  $c$  are constants for a particular station, kW is the total installed capacity and kWh the energy produced per annum. Explain the significance of the constants  $a$ ,  $b$  and  $c$  and the factors on which their values depend.

Determine the values of  $a$ ,  $b$  and  $c$  for a 60 MW station operating with annual load factor of 40% for which :

- (i) capital cost of buildings and equipment is Rs.  $5 \times 10^5$
- (ii) the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs. 90,000
- (iii) the interest and depreciation on buildings and equipment are 10% per annum
- (iv) annual cost of organisation and interest on cost of site etc. is Rs. 50,000.

**Solution.** Here,  $a$  represents fixed charge due to the annual cost of the organisation, interest on the capital investment on land or site etc.

The constant  $b$  represents semi-fixed cost. The constant  $b$  is such that when multiplied by the maximum kW demand on the station, it equals the annual interest and depreciation on the capital cost of the buildings equipment and the salary of the charge engineer.

Constant  $c$  represents running cost and its value is such that when multiplied by the annual total kWh output of the station, it equals the annual cost of the fuel, oil, taxation, wages and salaries of the operating staff.



An overview of a thermal power plant near Tokyo, Japan

- (i) Here,  $a = 50,000$
- (ii)  $b \times \text{max. kW demand} = \text{annual interest on the capital cost of the buildings and equipment etc.} = 0.1 \times 5 \times 10^5$   
 $\therefore b \times 60 \times 10^3 = 0.5 \times 10^5$  or  $b = 0.834$
- (iii) annual average power =  $0.5 \times 60 \times 10^3 = 30 \times 10^3$  kW  
 Units produced annually =  $30 \times 10^3 \times 8,760 = 262.8 \times 10^6$  kWh  
 $\therefore c \times 262.8 \times 10^6 = 90,000$  ;  $c = 0.0034$

### 50.17. Tariffs

The size and cost of installations in a generating station is determined by the maximum demand by the different consumers on the station. Each consumer expects his maximum demand to be met at any time of the day or night. For example, he may close down his workshop or house for a month or so but on his return he expects to be able to switch on his light, motor and other equipment without any previous warning to the supply company. Since electric energy, unlike gas or water cannot be stored, but must be produced as and when required, hence the generating equipment has to be held in 'readiness' to meet every consumer's full requirement at all hours of the day.

This virtually amounts to allocating a certain portion of the generating plant and the associated distribution system to each consumer for his individual use. Hence, it is only fair that a consumer should pay the fixed charges on that portion of the plant that can be assumed to have been exclusively allocated to him plus the charges proportional to the units actually used by him.

Hence, any method of charging or tariff, in the fairness to the supply company, should take into account the two costs of producing the electric energy (i) fixed or standing cost proportional to the maximum demand and (ii) running cost proportional to the energy used. Such two-part tariffs are in common use. Some of the different ways of rate making are described below :

### 50.18. Flat Rate

This was the earliest type of tariff though it is not much used these days because, strictly speaking, it is not based on the considerations discussed above. In this system, charge is made at a simple flat rate per unit. But the lighting loads and power loads are metered separately and charged at different rates. Since the lighting load has a poor load factor *i.e.* the number of units sold is small in relation to the installed capacity of the generating plant, the fixed cost per kWh generated is high and this is taken into account by making the price per unit comparatively high. But since the power load is more predictable and has a high load factor, the cost per kWh generated is much lower which results in low rate per unit.

### 50.19. Sliding Scale

In this type of tariff, the fixed costs are collected by charging the first block of units at a higher rate and then reducing the rates, usually in many steps, for units in excess of this quantity.

### 50.20. Two-part Tariff

This tariff is based on the principles laid down in Art. 50.16. It consists of two parts (i) a fixed charge proportional to the maximum demand (but independent of the units used) and (ii) a low running charge proportional to the actual number of units used.

The maximum demand during a specified period, usually a quarter, is measured by a maximum demand indicator. The maximum demand indicator is usually a watt-hour meter which returns to zero automatically at the end of every half hour but is fitted with a tell-tale pointer which is left behind at the maximum reducing reached during the quarter under consideration.

This type of tariff is expressed by a first degree equation like Rs.  $A \times \text{kW} + B \times \text{kWh}$  where Rs.  $A$  is the charge per annum per kW of maximum demand and  $B$  is the price per kWh.

Sometimes, the customer is penalized for his poor load power factor by basing the fixed charges on kVA instead of per kW of maximum demand.

**Example 50.31.** Compute the cost of electrical energy and average cost for consuming 375 kWh under 'block rate tariff' as under :

First 50 kWh at 60 paisa per kWh ; next 50 kWh at 50 paisa per kWh; next 50 kWh at 40 paisa per kWh; next 50 kWh at 30 paisa per kWh.

Excess over 200 kWh at 25 paisa per kWh. **(Utilisation of Elect. Power, AMIE Sec. B)**

**Solution.** Energy charge for the first 50 kWh is = Rs.  $0.6 \times 50 = \text{Rs. } 30$

Energy charge for the next 50 kWh at 50 paisa / kWh = Rs.  $0.5 \times 50 = \text{Rs. } 25$

Energy charge for the next 50 kWh at 40 paisa / kWh = Rs.  $0.4 \times 50 = \text{Rs. } 20$

Energy charge for the next 50 kWh at 30 paisa / kWh = Rs.  $0.3 \times 50 = \text{Rs. } 15$

Energy charge for the rest (375 – 200) *i.e.* 175 kWh =Rs.  $0.25 \times 175 = \text{Rs. } 43.75$

Total cost of energy for 375 kWh = Rs.  $(30 + 25 + 20 + 15 + 43.75) = \text{Rs. } 133.75$

Average cost of electrical energy/kWh = Rs.  $133.75 / 375 = \text{Rs. } 36 \text{ paise.}$

**Example 50.32.** The output of a generating station is  $390 \times 10^6$  units per annum and installed capacity is 80,000 kW. If the annual fixed charges are Rs. 18 per kW of installed plant and running charges are 5 paisa per kWh, what is the cost per unit at the generating station ?

**(Electrical Technology, Bombay Univ.)**



**Solution.** Annual fixed charges = Rs.  $18 \times 80,000 = \text{Rs. } 1.44 \times 10^6$   
 Fixed charges/kWh = Rs.  $1.44 \times 10^6 / 390 \times 10^6 = 0.37$  paise  
 Running charges/kWh = 5 paise  
 Hence, cost at the generating station is =  $5 + 0.37 = \mathbf{5.37 \text{ paise/kWh}}$ .

**Example 50.33.** A power station has an installed capacity of 20 MW. The capital cost of station is Rs. 800 per kW. The fixed costs are 30% of the cost of investment. On full-load at 100% load factor, the variable costs of the station per year are 1.5 times the fixed cost. Assume no reserve capacity and variable cost to be proportional to the energy produced, find the cost of generation per kWh at load factors of 100% and 20%. Comment on the results. **(Ranchi University)**

**Solution.** Capital cost of the station = Rs.  $800 \times 20,000 = \text{Rs. } 16 \times 10^6$

**(a) At 100% load factor**

Fixed cost = Rs.  $16 \times 10^6 \times 13/100 = \text{Rs. } 2.08 \times 10^6$   
 Variable cost = Rs.  $1.5 \times 2.08 \times 10^6 = \text{Rs. } 3.12 \times 10^6$   
 Total operating cost per annum = Rs.  $(2.08 + 3.12) \times 10^6 = \text{Rs. } 5.2 \times 10^6$   
 Total No. of units generated = kW max. demand  $\times$  LF  $\times$  8760 =  $20,000 \times 1 \times 8760$   
 =  $175.2 \times 10^6$  kWh  
 Cost of generation per kWh =  $5.2 \times 10^6 \times 100 / 175.2 \times 10^6 = \mathbf{3.25 \text{ paise}}$ .

**(b) At 20% load factor :**

Fixed cost = Rs.  $2.08 \times 10^6$  as before  
 Total units generated per annum =  $20,000 \times 0.2 \times 8760 = 35.04 \times 10^6$  kWh  
 Since variable cost is proportional = Rs.  $3.12 \times 10^6 \times 35.04 \times 10^6 / 175.2 \times 10^6$  to kWh generated, the variable cost at load of 20% is = Rs.  $0.624 \times 10^6$   
 Total operating cost = Rs.  $(2.08 + 0.624) \times 10^6 = 2.704 \times 10^6$   
 Cost of generation per kWh =  $2.704 \times 10^6 \times 100 / 35.04 \times 10^6 = \mathbf{7.7 \text{ paise}}$

It is obvious from the above calculations that as the station load factor is reduced, the cost of electric generation is increased.

**Example 50.34.** The annual output of a generating sub-station is  $525.6 \times 10^6$  kWh and the average load factor is 60%. If annual fixed charges are Rs. 20 per kW installed plant and the annual running charges are 1 paise per kWh, what would be the cost per kWh at the bus bars ?

**Solution.** Average power supplied per annum =  $525.6 \times 10^6 / 8760 = 60,000$  kW

Max. demand =  $\frac{\text{average power}}{\text{load factor}} = \frac{60,000}{0.6} = 100,000$  kW\*

Annual fixed charges = Rs.  $20 \times 100,000 = \text{Rs. } 2 \times 10^6$

Fixed charges / kWh =  $\frac{2 \times 10^6}{525.6 \times 10^6} = \frac{2 \times 100}{525.6} = 0.38$  paise

Annual running charges per kWh = 1 paise. Hence, cost per kWh at the bus-bars = **1.38 paise**.

\* It has been assumed that the installed capacity is equal to the maximum demand of 100,000 kW.

**Example 50.35.** A certain factory working 24 hours a day is charged at Rs. 10 per kVA of max. demand plus 5 paise per kVARh. The meters record for a month of 30 days; 135,200 kWh, 180,020 kVARh and maximum demand 310 kW. Calculate

- (i) M.D. charges, (ii) monthly bill, (iii) load factor, (iv) average power factor.

(Electric Engineering-II, Bangalore Univ.)

**Solution.** Average demand =  $135,200/24 \times 30 = 188 \text{ kW}$   
 Average reactive power =  $180,020/24 \times 30 = 250 \text{ kVAR}$   
 Now,  $\tan \phi = \text{kVAR/kW} = 250/188$ ;  $\phi = 53^\circ$ ;  $\cos \phi = 0.6$ ;  
 M.D. kVA =  $310/0.6 = 517$

- (i) M.D. charge = Rs.  $10 \times 517 = \text{Rs. } 5170$   
 (ii) reactive energy charges = Rs.  $5 \times 180,020/100 = \text{Rs. } 9001$   
 $\therefore$  monthly bill = **Rs. 14,171**  
 (iii) monthly load factor =  $188/310 = 0.606$  or 60.6% (iv) average p.f. = **0.6**

**Example 50.36.** The cost data of a power supply company is as follows :

Station maximum demand = 50 MW ; station load factor = 60% ; Reserve capacity = 20% ; capital cost = Rs. 2,000 per kW; interest and depreciation = 12% ; salaries (annual) = Rs.  $5 \times 10^5$ ; fuel cost (annual) = Rs.  $5 \times 10^6$  ; maintenance and repairs (annual) Rs.  $2 \times 10^5$  ; losses in distribution = 8% ; load diversity factor = 1.7.

Calculate the average cost per unit and the two-part tariff, assuming 80 per cent of salaries and repair and maintenance cost to be fixed. (Electrical Power-I, Bombay Univ.)

**Solution.** Station capacity =  $50 + 20\% \text{ of } 50 = 60 \text{ MW}$   
 Average power =  $60 \times 0.6 = 36 \text{ MW} = 36 \times 10^3 \text{ kW}$   
 Capital investment = Rs.  $60 \times 10^3 \times 2000 = \text{Rs. } 12 \times 10^7$   
 Interest + depreciation = Rs.  $0.12 \times 12 \times 10^7 = \text{Rs. } 14.4 \times 10^6$   
 Total cost both fixed and running  
 = Rs.  $14.4 \times 10^6 + \text{Rs. } 5 \times 10^5 + \text{Rs. } 5 \times 10^6 + \text{Rs. } 2 \times 10^5 = \text{Rs. } 201 \times 10^5$   
 No. of units generated annually =  $8760 \times 36 \times 10^3$   
 $\therefore$  overall cost/unit = Rs.  $\frac{201 \times 10^5}{8760 \times 36 \times 10^3} = \text{6.37 paise}$

#### Fixed charges

Annual interest and depreciation = Rs.  $14.4 \times 10^6$   
 80% of salaries = Rs.  $0.8 \times 5 \times 10^5 = \text{Rs. } 4 \times 10^5$   
 80% of repair and maintenance cost = Rs.  $0.8 \times 2 \times 10^5 = \text{Rs. } 1.6 \times 10^5$   
 Total fixed charges = Rs.  $149.6 \times 10^5$

Aggregate maximum demand of all consumers

= Max. demand on generating station  $\times$  diversity factor  
 =  $60 \times 10^3 \times 1.7 = 102 \times 10^3 \text{ kW}$

$\therefore$  annual cost/kW of maximum demand = Rs.  $149.6 \times 10^5 / 102 \times 10^3 = \text{Rs. } 149.6$

#### Running Charges

Cost of fuel = Rs.  $50 \times 10^5$  ; 20% of salaries = Rs.  $1 \times 10^5$   
 20% of maintenance =  $0.4 \times 10^5$  ;  
 Total = Rs.  $51.4 \times 10^5$

Considering distribution loss of 8%, cost per unit delivered to the consumer is  
 $= \text{Rs. } 51.4 \times 105/0.92 \times 8760 \times 36 \times 10^3 = \mathbf{1.77 \text{ paise}}$

Hence, two-part tariff is : **Rs. 149.6 per kW max.** demand plus **1.77 paise per kWh** consumed.

**Example 50.37.** A certain electric supply undertaking having a maximum demand of 110 MW generates  $400 \times 10^6$  kWh per year. The supply undertaking supplies power to consumers having an aggregate demand of 170 MW. The annual expenses including capital charges are :

$\text{Fuel} = \text{Rs. } 5 \times 10^6$   
 $\text{Fixed expenses connected with generation} = \text{Rs. } 7 \times 10^6$   
 $\text{Transmission and distribution expenses} = \text{Rs. } 8 \times 10^6$   
 Determine a two-part tariff for the consumers on the basis of actual cost.

Assume 90% of the fuel cost as variable charges and transmission and distribution losses as 15% of energy generated. **(Electrical Power-I. Bombay Univ.)**

**Solution.** Total fixed charges per annum are as under :

Fixed charges for generation =  $\text{Rs. } 7 \times 10^6$   
 Transmission and distribution expenses =  $\text{Rs. } 8 \times 10^6$   
 10% of annual fuel cost =  $\text{Rs. } 0.5 \times 10^6$  Total =  $\text{Rs. } 15.5 \times 10^6$

This cost has to be spread over the aggregate maximum demand of all consumers i.e. 170 MW.

$\therefore$  cost per kW of maximum demand =  $\text{Rs. } 15.5 \times 10^6 / 170 \times 10^3 = \mathbf{\text{Rs. } 91.2}$

Running charges = 90% of fuel cost =  $\text{Rs. } 4.5 \times 10^6$

These charges have to be spread over the number of kWh actually delivered.

No. of units delivered = 85% of No. of units generated  
 $= 0.85 \times 400 \times 10^6 = 340 \times 10^6$

$\therefore$  running cost / kWh =  $\text{Rs. } 4.5 \times 10^6 / 340 \times 10^6 = \mathbf{1.32 \text{ paise}}$

Hence, the two-part tariff for the consumer would be **Rs. 91.2 per kW** of maximum demand and **1.32 paise per kWh** consumed.

**Example 50.38.** A customer is offered power at Rs. 80 per annum per kVA of maximum demand plus 8 paise per unit metered. He proposes to install a motor to carry his estimated maximum demand of 300 b.h.p. (223.8 kW). The motor available has a power factor of 0.85 at full-load. How many units will he require at 20% load factor and what will be his annual bill ?

**(Electric Power II Punjab Univ. 1992)**

**Solution.** Assuming a motor efficiency of 90%, the full-load power intake of motor =  $223.8/0.9 = 746/3$  kW. This represents the max. demand.

Now, load factor =  $\frac{\text{average}}{\text{max. demand}}$

$\therefore$  average power = max. demand  $\times$  load factor =  $(746/3) \times 0.2 = \mathbf{149.2/3 \text{ kW}}$

$\therefore$  annual consumption =  $149.2 \times 8,760/3 = 435,700 \text{ kWh}$

Max. kVA of demand =  $300 \times 746/0.9 \times 0.85 \times 1000 = 292.5$

$\therefore$  cost per kVA of maximum demand =  $292.5 \times 80 = \text{Rs. } 23,400$

Cost of units consumed/annum =  $435,700 \times 8/100 = \text{Rs. } 34,856$

$\therefore$  annual bill =  $\text{Rs. } 23,400 + \text{Rs. } 34,856 = \mathbf{\text{Rs. } 58,256}$

**Example 50.39.** How two-part tariff is modified for penalising low p.f. consumers ?  
 An industry consumes 4 million kWh/year with a maximum demand of 1000 kW at 0.8 p.f. What is its load factor ?

- (a) Calculate the annual energy charges if tariff in force is as under :  
Max. demand charge = Rs. 5 per kVA per month. Energy charges = Rs. 0.35 per kWh
- (b) Also calculate reduction in this bill if the maximum demand is reduced to 900 kW at 0.9 p.f. lagging. **(Electrotechnics, Gujarat Univ.)**

**Solution.** Load factor =  $4 \times 10^6 / 1000 \times 8760 = 0.4566$  or 45.66%

(a) Max. kVA demand =  $1000 / 0.8 = 1250$

Annual M.D. charge/kVA =  $5 \times 12 = \text{Rs. } 60$

M.D. charge for 1250 kVA =  $\text{Rs. } 60 \times 1250 = \text{Rs. } 75 \times 10^3$

Annual energy charge =  $\text{Rs. } 4 \times 10^6 \times 0.35 = \text{Rs. } 1400 \times 10^3$

Annual energy bill = **Rs.  $1475 \times 10^3$**

(b) Since load factor remains the same, annual average power =  $900 \times 0.4566 \text{ kW}$

Units consumed annually =  $8760 \times 900 \times 0.4566 = 3.6 \times 10^6$

Max. kVA demand =  $900 / 0.9 = 1000$ , M.D. charge for 1000 kVA =  $\text{Rs. } 60 \times 1000 = \text{Rs. } 60 \times 10^3$

Energy charge =  $\text{Rs. } 3.6 \times 10^6 \times 0.35 = \text{Rs. } 1260 \times 10^3$ ; Annual bill =  $\text{Rs. } 1,320 \times 10^3$

Annual saving =  $\text{Rs. } (1475 - 1320) \times 10^3 = \text{Rs. } 155 \times 10^3$

**Example 50.40.** A supply is offered on the basis of fixed charges of Rs. 30 per annum plus 3 paise per unit or alternatively, at the rate of 6 paise per unit for the first 400 units per annum and 5 paise per unit for all the additional units. Find the number of units taken per annum for which the cost under these two tariffs becomes the same. **(Electrical Technology-I, Bombay Univ.)**

**Solution.** Let  $x$  kWh be the annual consumption of the consumer for which the two tariffs are equally advantageous.

Cost according to the first tariff =  $\text{Rs. } 30 + 3x / 100$

Charges according to the alternative tariff are

=  $\text{Rs. } 6 \times 400 / 100 + \text{Rs. } (x - 400) \times 5 / 100 = \text{Rs. } [24 + (x - 400) / 20]$

Since charges in both cases are to be equal

$$\therefore 30 + \frac{3x}{100} = 24 + \frac{(x - 400)}{20}$$

or  $x = \text{1300 kWh.}$

**Example 50.41.** If power is charged for at the rate of Rs. 75 per kVA of maximum demand and 4 paise per unit, what is the cost per unit at 25% yearly load factor (a) for unity power factor demand and (b) for 0.7 power factor demand.

**Solution. (a) At 25% load factor and unity power factor**

Maximum demand charge per unit =  $\frac{75 \times 100}{8760 \times 0.25} = 3.43 \text{ paise}$

Energy charge per unit = 4 paise ; Cost per unit =  $4 + 3.43 = \text{7.43 paise}$

**(b) At 25% load factor and 0.7 power factor**

Maximum demand charge per unit =  $\frac{75 \times 100}{0.7 \times 0.25 \times 8760} = \text{4.9 paise}$

Energy charge per unit = 4 paise ; Cost per unit =  $4 + 4.9 = \text{8.9 paise}$

**Example 50.42.** Explain different methods of tariff. A tariff in force is Rs. 50 per kVA of max. demand per year plus 10 p per kWh. A consumer has a max. demand of 10 kW with a load factor of 60% and p.f. 0.8 lag.

- (i) Calculate saving in his annual bill if he improves p.f. to 0.9 lag.  
 (ii) Show the effect of improving load factor to 80% with the same max. demand and p.f. 0.8 lag on the total cost per kWh. **(Electrical Engineering-III, Poona Univ.)**

**Solution.** (i) Max. kVA demand at 0.8 p.f. =  $10/0.8 = 12.5$

$$\text{M.D. charges} = \text{Rs. } 50 \times 12.5 = \text{Rs. } 625$$

$$\text{Max. kVA demand at 0.9 p.f.} = 10/0.9 ; \text{M.D. charge} = \text{Rs. } 50 \times 10/0.9 = \text{Rs. } 555.55$$

Since energy consumed remains constant, saving is due only to reduction in M.D. charges.

$$\therefore \text{ saving} = \text{Rs. } (625 - 555.55) = \text{Rs. } 69.45$$

(ii) With 60% load factor, average power =  $10 \times 0.6 = 6$  kW. The number of units consumed annually =  $6 \times 8760 = 52,560$  kWh. Also, annual M.D. charge =  $\text{Rs. } 50 \times 12.5 = \text{Rs. } 625$

$$\text{M.D. charge/unit consumed} = 625 \times 100/52,560 = 1.19 \text{ paise}$$

$$\text{Total cost per unit} = 10 + 1.19 = \text{11.19 paise}$$

$$\text{With a load p.f. of 80\% average power} = 10 \times 0.8 = 8 \text{ kW}$$

Number of units consumed annually =  $8 \times 8,760 = 70,080$  kWh. The M.D. charge as before = Rs. 625.

$$\therefore \text{ M.D. charge/unit consumed} = 625 \times 100/70,080 = 0.89 \text{ paise}$$

$$\therefore \text{ total cost/unit consumed} = 10 + 0.89 = \text{10.89 paise}$$

Obviously, with improvement in load factor, total cost per unit is reduced.

**Example 50.43.** A consumer has a maximum demand (M.D.) of 20 kW at 0.8 p.f. lagging and an annual load factor of 60%. There are two alternative tariffs (i) Rs. 200 per kVA of M.D. plus 3p per kWh consumed and (ii) Rs. 50 per kVA of M.D. plus 7p per kWh consumed. Determine which of the tariffs will be economical for him. **(Electrical Engineering, Banaras Hindu Univ.)**

**Solution.** An M.D. of 20 kW at 0.8 p.f. =  $20/0.8 = 25$  kVA

$$\text{Average power} = \text{M.D.} \times \text{load factor} = 20 \times 0.6 = 12 \text{ kW}$$

$$\text{Energy consumed/year} = 12 \times 8760 = 105120 \text{ kWh}$$

$$(a) \text{ M.D. charge} = \text{Rs. } (200 \times 25) = \text{Rs. } 5000$$

$$\text{Energy charge} = \text{Rs. } 3 \times 105,120/100 = \text{Rs. } 3153.60$$

$$\text{Annual charges} = \text{Rs. } 5000 + \text{Rs. } 3153.60 = \text{Rs. } 8153.60$$

$$(b) \text{ M.D. charge} = \text{Rs. } (50 \times 25) = \text{Rs. } 1250$$

$$\text{Energy charge} = \text{Rs. } \frac{7 \times 105120}{100} = \text{Rs. } 7358.40$$

$$\text{Annual charge} = \text{Rs. } 1250 + \text{Rs. } 7358.40 = \text{Rs. } 8608.40$$

Obviously, tariff (a) is economical.

**Example 50.44.** Determine the load factor at which the cost of supplying a unit of electricity from a Diesel station and from a steam station is the same if the respective annual fixed and running charges are as follows.

Station	Fixed charges	Running charges
Diesel	Rs. 300 per kW	25 paise/kWh
Steam	Rs. 1200 per kW	6.25 paise/kWh

**(Electrical Power-I, Gujarat Univ.)**

**Solution.** (i) Diesel Station

Suppose that the energy supplied in one year is one unit i.e. one kWh.

$$\therefore \text{ annual average power} = 1 \text{ kWh}/8760 \text{ h} = 1/8760 \text{ kW}$$

$$\begin{aligned} \text{Now, annual load factor (L)} &= \frac{\text{annual average power}}{\text{annual max. demand}} \\ \therefore \text{max. demand} &= \frac{\text{average power}}{L} = \frac{1}{8760 L} \text{ kW} \\ \therefore \text{fixed charge} &= \text{Rs. } 300 \times 1/8760 L = 30,000 / 8760 L \text{ paise} \\ \text{Running charge} &= 1 \times 25 = 25 \text{ paise} \end{aligned}$$

Hence, fixed and running charges per kWh supplied by Diesel station =  $\left(\frac{30,000}{8760 L}\right)$  paise

**(ii) Steam Station**

$$\begin{aligned} \text{In the same way, fixed charge} &= \text{Rs. } 1200/8760 L = 120,000/8760 L \text{ paise} \\ \text{Running charge} &= 6.25 \text{ paise} \\ \therefore \text{fixed plus running charges for supplying one kWh of energy are} &= (120,000/8760 L) + 6.25 \text{ paise.} \end{aligned}$$

Since the two charges have to be the same

$$\therefore \frac{120,000}{8760 L} + 6.25 = \frac{30,000}{8760 L} + 25 ; L = \mathbf{0.55 \text{ or } 55\%}$$

**Example 50.45.** A factory has a maximum load of 300 kW at 0.72 p.f. with an annual consumption of 40,000 units, the tariff is Rs. 4.50 per kVA of maximum demand plus 2 paise/unit. Find out the average price per unit. What will be the annual saving if the power factor be improved to unity ?

(Electrical Technology-I, Bombay Univ.)

**Solution.** kVA load at 0.72 p.f. =  $300/0.72 = 1250/3$

$$\begin{aligned} \therefore \text{max. kVA demand charge} &= \text{Rs. } 4.5 \times 1250/3 = \text{Rs. } 1875 \\ \therefore \text{M.D. charge per unit} &= \text{Rs. } 1875 / 40,000 = 4.69 \text{ paise} \\ \therefore \text{total charge per unit} &= 2 + 4.69 = \mathbf{6.69 \text{ paise}} \\ \text{Max. kVA demand at unity p.f.} &= 300 \\ \text{M.D. charge per unit} &= \text{Rs. } 300 \times 4.5 = \text{Rs. } 1350 \\ \text{Annual saving} &= \text{Rs. } 1875 - \text{Rs. } 1350 = \mathbf{\text{Rs. } 525.} \end{aligned}$$

**Example 50.46.** There is a choice of two lamps, one costs Rs. 1.2 and takes 100 W and the other costs Rs. 5.0 and takes 30 W ; each gives the same candle power and has the same useful life of 1000 hours. Which will prove more economical with electrical energy at Rs. 60 per annum per kW of maximum demand plus 3 paise per unit ? At what load factor would they be equally advantageous?

**Solution. (i) First Lamp**

$$\begin{aligned} \text{Initial cost of the first lamp per hour} &= \text{Rs. } 120/1000 = 0.12 \text{ paise} \\ \text{Max. demand / hr.} &= 100/1000 = 0.1 \text{ kW} \\ \text{Max. demand charge/hr.} &= (60 \times 100) \times 0.1/8760 = 0.069 \text{ paise} \\ \text{Energy charge/hr.} &= 3 \times 0.1 = 0.3 \text{ paise} \\ \therefore \text{Total cost/hr.} &= 0.12 + 0.069 + 0.3 = \mathbf{0.489 \text{ paise}} \end{aligned}$$

**(ii) Second Lamp**

$$\begin{aligned} \text{Initial cost/hr.} &= 500/1000 = 0.5 \text{ paise ; Max. demand /hr ;} = 30/1000 = 0.03 \text{ kW} \\ \text{Max. demand charge/hr.} &= (60 \times 100) \times 0.03 / 8760 = 0.02 \text{ paise} \\ \text{Energy charge/hr.} &= 3 \times 0.03 = 0.09 \text{ paise} \therefore \text{total cost/hr.} = 0.5 + 0.02 + 0.09 = \mathbf{0.61 \text{ paise}} \\ \text{Hence, the first lamp would be more economical.} & \end{aligned}$$

It would be seen that the only charge which will vary with load factor is the max. demand charge. Moreover the maximum demand charge varies inversely as the load factor. Let  $x$  be the load factor at which both lamps are equally advantageous. Then

$$0.12 + \frac{0.069}{x} + 0.3 = 0.05 + \frac{0.02}{x} + 0.09 \quad \therefore x = 0.29$$

Hence, both lamps would be equally advantageous at **29%** load factor.

**Example 50.47.** The following data refers to a public undertaking which supplies electric energy to its consumers at a fixed tariff of 11.37 paise per unit.

Total installed capacity = 344 MVA ; Total capital investment = Rs. 22.4 crores;

Annual recurring expenses = Rs. 9.4 crores ; Interest charge = 6% ; depreciation charge = 5%

Estimate the annual load factor at which the system should operate so that there is neither profit nor loss to the undertaking. Assume distribution losses at 7.84% and the average system p.f. at 0.86.

(Electrical Power-I, Bombay Univ.)

**Solution.** Yearly load factor ( $L$ ) =  $\frac{\text{No. of kWh supplied in a year}}{\text{Max. No. of kWh which can be supplied}^*}$

$$= \frac{\text{kWh supplied/year}}{\text{Max. output in kW} \times 8760}$$

$$\therefore \text{kWh supplied/year} = \text{Max. output in kW} \times 8760 \times L$$

$$= (344,000 \times 0.86) \times 8760 \times L = L \times 25.92 \times 10^8$$

Considering distribution losses of 7.84%, the units actually supplied

$$= 92.16 \text{ per cent of } (L \times 25.92 \times 10^8) = L \times 23.89 \times 10^8$$

Amount collected @ of 11.37 paise / kWh = Rs.  $L \times 23.89 \times 10^8 \times 11.37 / 100$

If there is to be no profit or gain, then this amount must just equal the fixed and running charges.

Annual interest and depreciation on capital investment

$$= 11\% \text{ of Rs. } 22.4 \times 10^7 = \text{Rs. } 2.46 \times 10^7$$

Total annual expenses = Rs.  $2.46 \times 10^7 + \text{Rs. } 9.4 \times 10^7 = \text{Rs. } 11.86 \times 10^7$

$$\therefore L \times 23.89 \times 10^8 \times 11.37 / 100 = 11.86 \times 10^7 ; L = 0.437 \text{ or } 4.37\%$$

**Example 50.48.** An area has a M.D. of 250 MW and a load factor of 45%. Calculate the overall cost per unit generated by (i) steam power station with 30 per cent reserve generating capacity and (ii) nuclear station with no reserve capacity.

Steam station : Capital cost per kW = Rs. 1000 ; interest and depreciation on capital costs = 15% ; operating cost per unit = 5 paise.

Nuclear station : capital cost per kW = Rs. 2000 ; interest and depreciation on capital cost = 12% ; operating cost per unit = 2 paise.

For which load factor will the overall cost in the two cases become equal ?

(Electrical Power-I, Bombay Univ.)

**Solution. (i) Steam Station**

Taking into consideration the reserve generating capacity, the installed capacity of the steam station

$$= 250 + (30\% \text{ of } 250) = 325 \text{ MW} = 325 \times 10^3 \text{ kW}$$

average power (annual) = M.D.  $\times$  load factor =  $325 \times 10^3 \times 0.45 = 146,250 \text{ kW}$

$$\therefore \text{No. of units produced/year} = 8760 \times 146,250 = 128 \times 10^7$$

\* In this case max. demand has been taken as equal to the installed capacity.

$$\begin{aligned} \text{Capital investment} &= \text{Rs. } 325 \times 10^3 \times 1000 = \text{Rs. } 325 \times 10^6 \\ \text{Annual interest and depreciation} &= \text{Rs. } 325 \times 10^6 \times 0.15 = \text{Rs. } 48.75 \times 10^6 \\ \text{These fixed charges have to be spread over the total number of units produced by the station.} \\ \therefore \text{ fixed charges / unit} &= \text{Rs. } 48.75 \times 10^6 / 128 \times 10^7 = 3.8 \text{ paise} \\ \therefore \text{ overall cost per unit generated} &= 3.8 + 5 = \mathbf{8.8 \text{ paise}} \end{aligned}$$

**(ii) Nuclear Station**

Since there is no reserve capacity, installed capacity of the station equals the maximum demand of 250 MW =  $25 \times 10^4$  kW.

$$\begin{aligned} \text{Average power} &= 0.45 \times 25 \times 10^4 = 112,500 \text{ kW} \\ \text{Units produced annually} &= 8760 \times 112,500 = 98.3 \times 10^7 \\ \text{Capital investment} &= \text{Rs. } 2000 \times 250 \times 10^3 = \text{Rs. } 5 \times 10^8 \\ \text{Annual interest and depreciation} &= \text{Rs. } 0.12 \times 5 \times 10^8 = \text{Rs. } 6 \times 10^7 \\ \therefore \text{ fixed charges / unit} &= \text{Rs. } 6 \times 10^7 / 98.3 \times 10^7 = 6.1 \text{ paise} \\ \text{Overall cost per unit generated} &= 6.1 + 2.2 = \mathbf{8.3 \text{ paise}} \end{aligned}$$

**Load Factor**

Suppose  $L$  is the load factor at which the overall cost per unit generated is the same.

$$\text{Cost / unit for steam station} = \left( \frac{48.75 \times 10^8}{325 \times 10^3 \times 8760 L} + 5 \right) = \left( \frac{1.712}{L} + 5 \right) \text{ paise.}$$

Similarly, overall cost/unit for nuclear station is

$$= \left( \frac{6 \times 10^7 \times 100}{250 \times 10^3 \times 8760 L} + 2 \right) = \left( \frac{2.74}{L} + 2 \right) \text{ paise}$$

$$\text{Equating the two, we get } [(1.712/L) + 5] = [2.74/L + 2] \quad \therefore L = \mathbf{0.34 \text{ or } 34\%}$$

**Example 50.49.** The maximum demand of a customer is 25 amperes at 220 volt and his total energy consumption is 9750 kWh. If the energy is charged at the rate of 20 paise per kWh for 500 hours' use of the maximum demand plus 5 paise power unit for all additional units, estimate his annual bill and the equivalent flat rate.

**Solution. Charge at the max. demand rate.**

$$\begin{aligned} \text{Max. demand} &= 25 \times 220/1,000 = 5.5 \text{ kW} \\ \text{Units consumed at maximum demand rate} &= 5.5 \times 500 = 2,750 \text{ kWh} \\ \text{Max. demand charge} &= 2,750 \times 20/100 = \mathbf{Rs. 550} \end{aligned}$$

**Energy Charge**

$$\text{Units to be charged at lower rate} = 9,750 - 2,750 = 7,000$$

$$\text{Charge} = \frac{7,000 \times 5}{100} = \text{Rs. } 350$$

$$\therefore \text{ annual bill} = \text{Rs. } 550 + 350 = \mathbf{Rs. 900}$$

$$\text{Equivalent flat rate} = 900 \times 100/9,750 = \mathbf{9.2 \text{ paise}}$$

**Example 50.50.** A workshop having a number of induction motors has a maximum demand of 750 kW with a power factor of 0.75 and a load factor of 35%. If the tariff is Rs. 75 per annum per kVA of maximum demand plus 3 paise per unit, estimate what expenditure would it pay to incur to raise the power factor of 0.9.

**Solution.** Max. kVA demand at 0.75 p.f. =  $750/0.75 = 1000$

$$\text{M.D. charge} = 75 \times 1000 = \text{Rs. } 75,000 ; \text{ Max. kVA demand at 0.9 p.f.} = 750/0.9$$



$$\text{M.D. charge} = (7,500/9) \times 75 = \text{Rs. } 62,500$$

$$\text{Difference in one year} = 75,000 - 62,500 = \text{Rs. } 12,500$$

If the annual interest on money borrowed for purchasing the p.f. improvement plant is assumed 10%, then an expenditure of Rs. 125,000 is justified.

**Example 50.51.** *The owner of a new factory is comparing a private oil-engine generating station with public supply. Calculate the average price per unit his supply would cost him in each case, using the following data :*

*Max. demand, 600 kW; load factor, 30%; supply tariff, Rs. 70 per kW of maximum demand plus 3 paise per unit; capital cost of plant required for public supply, Rs.  $10^5$ ; capital cost of plant required for private generating station, Rs.  $4 \times 10^5$ ; cost of fuel, Rs. 80 per tonne; consumption of fuel oil; 0.3 kg per unit generated. Other work costs for private plant are as follows : lubricating oil, stores and water = 0.35 paise per unit generated; wages 1.1 paise; repairs and maintenance 0.3 paise per unit.*

**Solution. Public Supply Charges**

$$\text{Running charge/unit} = 3.0 \text{ paise}; \text{ Average power consumption} = 600 \times 0.3 = 180 \text{ kW}$$

$$\text{Annual energy consumption} = 180 \times 8,760 \text{ kWh}$$

$$\text{Fixed annual charges} = \text{Rs. } 70 \times 600 = \text{Rs. } 42,000$$

Let us assume a capital charge rate of 10%. Hence, further annual amount to be charged from the customer is  $0.1 \times 10^5 = \text{Rs. } 10,000$

$$\therefore \text{ total fixed charges/annum} = \text{Rs. } 42,000 + \text{Rs. } 10,000 = \text{Rs. } 52,000$$

$$\therefore \text{ fixed charge per unit generated} = \frac{52,000 \times 100}{180 \times 8760} = 3.3 \text{ paise}$$

$$\therefore \text{ fixed plus running charges per unit generated} = 3.0 + 3.3 = \text{6.3 paise.}$$

**Private Supply Charges**

$$\text{Annual capital charges} = \text{Rs. } 4 \times 10^5 \times 0.1 = \text{Rs. } 40,000$$

$$\text{No. of units generated annually} = 180 \times 8,760 \text{ kWh}$$

$$\therefore \text{ fixed charges per unit generated} = \frac{4,000 \times 100}{180 \times 8760} = 2.54 \text{ paise}$$

$$\text{Cost of oil per unit generated} = \frac{80 \times 100 \times 0.3}{1,000} = 2.4 \text{ paise}$$

Running charges per unit generated are :

$$\text{Lubricating oil, stores, water} = 0.35 \text{ paise}; \text{ Wages} = 1.1 \text{ paise}$$

$$\text{Repairs and maintenance} = 0.3 \text{ paise}; \text{ Total} = 0.35 + 1.1 + 0.3 = 1.75 \text{ paise}$$

$$\text{Total running charges} = 2.4 + 1.75 = \text{4.15 paise}$$

$$\text{Fixed plus running charges per unit generated} = 2.54 + 4.15 = \text{6.69 paise}$$

**Example 50.52.** *Calculate the minimum two-part tariff to be charged to the consumers of a supply undertaking from the following data :*

*Generating cost per kWh; 3.6 paise; Generating cost per kW of maximum demand, Rs. 50*

*Total energy generated per year;  $4,380 \times 10^4$  kWh*

*Load factor at the generating station, 50%*

*Annual charges for distribution Rs. 125,000*

*Diversity factor for the distribution network, 1.25*

*Total loss between station and consumer, 10%.*

**Solution.** Average generating power =  $4,380 \times 10^4 / 8760 = 5,000$  kW  
 $\therefore$  maximum load on generator =  $5,000 / \text{load factor} = 5,000 / 0.5 = 10,000$  kW  
 $\therefore$  annual fixed charges = Rs.  $10,000 \times 50 =$  Rs. 500,000  
Fixed charges = Rs. 125,000 Total fixed charges = Rs. 625,000

Since diversity factor is known, consumer's max. demand =  $10,000 \times 1.25 = 12,500$  kW  
Hence, Rs. 625,000 have to be equally distributed over the 12,500 kW maximum demand.

$\therefore$  cost per kW max. demand =  $625,000 / 12,500 =$  Rs. 50  
 $\therefore$  monthly kW maximum charges is = Rs.  $50 / 12 =$  Rs. 4.17

Since losses are 10%, consumer gets 0.9 kWh for every kWh generated at the station. Hence cost per kWh to the consumer is  $3.6 / 0.9 = 4$  paise.

Therefore, minimum charges are **Rs. 4.17 per kW** of max. demand per month and **4 paise per kWh** consumed.

**Example 50.53.** Two systems of tariffs are available for a factory working 8 hours a day for 300 working days in a year.

(a) High-voltage supply at 5 paise per unit plus Rs. 4.50 per month per kVA of maximum demand.

(b) Low-voltage supply at Rs. 5 per month per kVA of maximum demand plus 5.5 paise per unit. The factory has an average load of 200 kW at 0.8 power factor and a maximum demand of 250 kW at the same p.f.

The high-voltage equipment costs Rs. 50 per kVA and losses can be taken as 4 per cent. Interest and depreciation charges are 12 per cent. Calculate the difference in the annual cost between the two systems.

**Solution.** First, let us find the annual cost according to rate (a).

(a) Capacity of h.v. switchgear =  $(250 / 0.8) \times (100 / 96)$  kVA  
Annual interest on capital investment and depreciation  
=  $\left( \frac{250}{0.8} \times \frac{100}{96} \right) \times 50 \times 0.12 =$  Rs. 1953

Annual charge due to kVA max. demand is  
=  $\frac{250}{0.8} \times \frac{100}{96} \times 12 \times 4.5 =$  Rs. 17,580

Annual charge due to kWh consumption  
=  $200 \times \frac{100}{96} \times \frac{5}{100} \times (8 \times 300) =$  Rs. 25,000

$\therefore$  total charges = Rs.  $(1953 + 17,580 + 25,000) =$  **Rs. 44,533**

(b) The total annual cost due to rate (b) would be as under.  
Annual charge due to kVA max. demand is =  $250 \times 5 \times 12 / 0.8 =$  Rs. 18,750

Annual charge due to kWh consumption is =  $200 \times \frac{5.5}{100} \times 2,400 =$  Rs. 26,400

$\therefore$  total charges = Rs.  $18,750 +$  Rs. 26,400 = Rs. 45,150

Hence, high-voltage supply is cheaper by  $45,150 - 44,533 =$  **Rs. 617.**

**Example 50.54.** Estimate what the consumption must be in order to justify the following maximum demand tariff in preference to the flat rate if the maximum demand is 6 kW.

On Maximum Demand Tariff. A max. demand rate of 37 paise per unit for the first 200 hr. at the maximum demand rate plus 3 paise for all units in excess.

Flat-rate tariff, 20 paise per unit.

**Solution.** Let  $x$  = the number of units to be consumed (within a specific period)

**On Max. Demand Tariff**

$$\begin{aligned} \text{Units consumed at max. demand rate} &= 6 \times 200 = 1200 \text{ kWh} \\ \text{Units in excess of the max. demand units} &= (x - 1200) \\ \text{Cost of max. demand units} &= 1200 \times 37 = 44,400 \text{ paise} \\ \text{Cost of excess units} &= 3(x - 1200) = (3x - 3600) \text{ paise} \\ \therefore \text{total cost on this tariff} &= 44,400 + 3x - 3600 = 3x + 40,800 \text{ paise} \end{aligned}$$

**Flat-rate Tariff**

$$\begin{aligned} \text{Total cost of units on this rate} &= 20x \text{ paise} \\ \therefore 20x &= 3x + 40,800 \\ \text{or } 17x &= 40,800 \quad \therefore x = \mathbf{2,400 \text{ units}} \end{aligned}$$

**Tutorial Problem No. 50.1**

1. A plant costing Rs. 650,000 has a useful life of 15 years. Find the amount which should be saved annually to replace the equipment at the end of that time.  
(i) by the straight line method and (ii) by the sinking fund method if the annual rate of compound interest is 15%.

Assume that the salvage value of equipment is Rs. 5000.

**[(i) Rs. 4,000 (ii) Rs. 1,261] (Elect. Generation, Punjab Univ.)**

2. A plant costs Rs.  $7.56 \times 10^5$  and it is estimated that after 25 years, it will have to be replaced by a new one. At that instant, its salvage value will be Rs.  $1.56 \times 10^5$ . Calculate.  
(i) the annual deposit to be made in order to replace the plant after 25 years and  
(ii) value of the plant after 10 years on the 'reducing balance depreciation method'.

**[(i) 0.0612 (ii) Rs.  $4.02 \times 10^5$ ] (Util. of Elect. Power, AMIE Sec. B.)**

3. From the following data, estimated the generating cost per unit delivered at the station.  
Capacity of the generating plant = 10 MW; annual load factor = 0.4 ; capital cost = Rs. 5 million;  
annual cost of fuel, oil, wages, taxes and salaries = Rs.  $2 \times 10^5$ ; Rate of interest = 5%; rate of depreciation = 5% of initial value.

**[2 paisa/kWh]**

4. From the following data, find the cost of generation per unit delivered from the station.  
Capacity of the plant installed = 100 MW; annual load factor = 35% ; capital cost of power plant = Rs. 1.25 crores ; Annual cost of fuel, oil, salaries and taxation = Rs. 0.15 crore; Interest and depreciation on capital = 12%.

If the annual load factor of the station is raised to 40%, find the percentage saving in cost per unit.

**[1 paisa/kWh ; 12.2%]**

5. The capital costs of steam and water power station are Rs. 1200 and Rs. 2100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.  
The reserve-capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor will the overall cost per kWh be the same in both cases ? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydroelectric station. What would be cost of generating 500 million kWh at this load factor ? **[47.5% ; Rs. 4.12 crores]**

6. In a particular area, both steam and hydro stations are equally possible. It has been estimated that capital costs and the running costs of these two types will be as follows :

	Capital cost /kW	running cost/kW	interest
Hydro	Rs. 2,200	1 paise	5%
Steam	Rs. 1,200	5 paise	5%

If expected average load factor is only 10% which is economical to operate, steam or hydro ? If the load factor is 50%, would there be any changes in the choice ? If so, indicate with calculations.

**[10% load factor ; hydro ; 13.5 paise / kWh ; Steam = 11.85 paise/ kWh 50% load factor ; hydro ; 3.5 paise/kWh ; Steam = 6.35 paise/kWh]**

7. A consumer takes a steady load of 200 kW at a power factor of 0.8 lagging for 10 hours per day and 300 days per annum. Estimate his annual payment under each of the following tariffs :
- (i) 10 paise per kWh plus Rs. 100 per annum per kVA  
(ii) 10 paise per kWh plus Rs. 100 per annum per kW plus 2 paise per kVARh.  
**[(i) Rs. 85,000 (ii) Rs. 170,000.] (Util. of Elect. Power, Punjab Univ.)**
8. A consumer requires one million units per year and his annual load factor is 50%. The tariff in force is Rs. 120 per kW per annum plus 5 paise per unit. Estimate the saving in his energy charges which would result if he improved his load factor to 100%.  
**[Rs. 13,698.60] (Util. of Elect. Power, AMIE)**
9. A factory has a maximum demand of 1000 kW and a load factor of 40%. It buys power from a company at the rate of Rs. 45 per kW plus Rs. 0.025/kWh consumed. Calculate the annual electricity bill of the factory. Also work out the overall cost of one unit of electricity.  
**[Rs. 1,32,600, 3.78 paise] (Util. of Elect. Power, AMIE)**
10. A power station having a maximum demand of 100MW has a load factor of 30%. It is to be supplied by any of the following schemes :
- (a) a steam power station in conjunction with a hydro station, the latter supplying 108 kWh/annum with a maximum output of 40 MW.  
(b) a steam station capable of supplying whole load,  
(c) a hydro station capable of supplying whole load.
- The following data may be assumed :
- |  | steam   | hydro    |
|--|---------|----------|
| (i) Capital cost/kW installed capacity         | Rs. 600 | Rs. 1500 |
| (ii) Interest and depreciation on capital cost | 12%     | 10%      |
| (iii) Operating cost/unit                      | 5 p     | 1 p      |
| (iv) Transmission cost/unit negligible         | 0.25 p  |          |
- Neglect spares. Calculate the overall cost per unit generated in each case.  
**[(a) 7.5 P/ kWh (b) 7.74 P/kWh (c) 6.95 P/kWh]**
11. An electricity undertaking having a maximum load of 100 MW generates 375 million kWh per annum and supplies consumers having an aggregate maximum demand of 165 MW.  
The annual expenses including capital charges are : for fuel Rs. 30 lakhs, fixed charges concerning generation Rs. 40 lakhs. Fixed charges connected with transmission and distribution: Rs. 50 lakhs. Assume that 90% of the fuel cost is essential to running charges and with the losses in transmission and distribution as 15% of the kWh generated, deduce a two-part tariff to find the actual cost of supply to consumers.  
**[Rs. 56.3 per kW Max. demand ; 0.84 P/kWh]**
12. Obtain a tariff for the consumers of supply undertaking which generates  $39 \times 10^7$  kWh/year and has maximum demand of 130 MW connected to it. The cost is distributed as follows :  
Fuel : Rs. 37.5 lakhs ; Generation : Rs. 18 lakhs ; Transmission : Rs. 37.5 lakhs ; Distribution : Rs.25.5 lakhs.  
Of these items, 90%, 10% 5% and 7% respectively are allocated to running charges, the remainder being fixed charges. The total loss between the station and the consumers is 10% of the energy generated.  
Find also the load factor and overall cost per unit.  
**[Rs. 61 per kW max. demand and 1.1 P/kWh ; 34.2% ; 3.38 P/kWh]**
13. A generating station has got maximum demand of 80 kW. Calculate the cost per kWh delivered from the following data :

Capital cost = Rs.  $16 \times 10^6$   
 Annual cost of fuel and oil = Rs.  $14 \times 10^5$   
 Taxes, wages and salaries = Rs.  $9 \times 10^5$

The rate of interest and depreciation is 12%; Annual load factor is 60%.

[10.036 paise] (Utili. of Elect. Power. AMIE)

14. A S/S supplies power by four feeders to its consumers. Feeder No. 1 supplies six consumers whose individual daily maximum demands are 70 kW, 90 kW, 20 kW, 50 kW, 10 kW and 20 kW, while the maximum demand on feeder is 200 kW. Feeder No. 2 supplies four consumers, whose daily max. demands are 60 kW, 40 kW, 70 kW, and 30 kW, while the maximum demand on feeder No. 2 is 160 kW. Feeders Nos. 3 and 4 have a daily maximum demand of 150 kW and 200 kW respectively, while the maximum demand on the station is 600 kW.

Determine the diversity factor for the consumers of feeder No. 1, feeder No. 2 and for the four feeders.

[1.3, 1.25, 1.18] (AMIE)

15. A generating station has a M.D. of 75 MW and a yearly load factor of 40%. Generating costs inclusive of station capital cost are Rs. 60 per annum per kW demand plus 1 paise/kWh transmitted. The annual capital charge for the transmission system is Rs. 1.5 million, the respective diversity factors being 1.2 and 1.25 respectively. The efficiency of transmission system is 90% and that of the distribution system inclusive of substation losses 85%. Find the yearly cost per kW demand and the cost per kWh supplied (a) at the substation and (b) at the consumer's premises.

[(a) Rs. 72.2 per kW M.D. and 1.1. P/kWh (b) Rs. 71.1 per kW M.D. and 1.31 P/kWh]

### 50.21. Kelvin's Law

We will now consider the application of Kelvin's economy law to power transmission through feeder cables. Since, in feeders, voltage drop is not of vital importance, they can be designed on the basis of their current-carrying capacity and where feasible, of minimum financial losses.

The financial loss occurring in a current-carrying conductor is made up of two parts :

- (i) interest on the capital cost of the conductor plus allowance for depreciation and
- (ii) the cost of energy loss due to (a) ohmic resistance *i.e.*,  $I^2R$  loss, (b) losses in the metallic sheaths of sheathed cables and (c) in the case of high tension insulated cables, loss in the insulating material used.

For a given length of the cable, the weight and hence cost of copper required is directly proportional to the cable cross-section. The annual combined interest on capital cost and depreciation is also directly proportional to the cross-section of the cable and can be written as Rs.  $PA$  where  $P = a$  constant and  $A =$  cross-section of the cable.

Now, we will consider the cost of  $I^2R$  loss, neglecting at the moment losses other than ohmic. The ohmic resistance of the conductor is proportional to  $1/A$ . For a given annual load curve, the energy loss is proportional to resistance and so proportional to  $1/A$ . It can be written as  $Q/A$  where  $Q$  is another constant. If the load is variable, then current used in calculating  $I^2R$  loss will be the r.m.s. value of current reckoned over a period of one year.

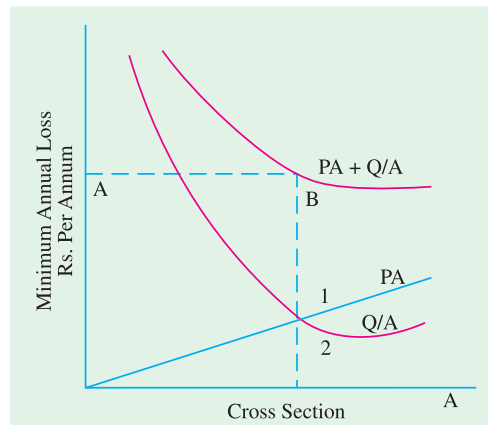


Fig. 50.11

$$\therefore \text{ total annual financial loss is } L = \text{Rs.} \left( PA + \frac{Q}{A} \right)$$

Now, it can be proved that this annual financial loss will be minimum when  $PA = Q/A$  i.e., **when annual charge on the capital outlay is equal to the annual value of the energy loss.\*** The most economical cross-section which will make the above cost equal is given by :

$$PA = \frac{Q}{A} \text{ or } A^2 = Q/P \therefore A = \sqrt{Q/P}$$

The most economical cross section can be also found graphically as shown in Fig. 50.11. It is seen that graph of  $PA$  is a straight line (curve 1) and that of  $Q/A$  is a rectangular hyperbola (curve 2). The graph for total annual loss is obtained by combining curves 1 and 2 and is found to exhibit a minimum for that value of 'A' which corresponds to the intersection of the two curves.

Since we have neglected the dielectric and sheath losses while deriving the above condition of minimum financial loss, it is obvious that it applies only to **bare conductors.**

In practice, Kelvin's law may have to be modified because the most economical size so calculated may not always be the practical one because it may be too small to carry the current.

**Example 50.55.** If the cost of an overhead line is Rs. 2000 A (where A is the cross-sectional area in  $\text{cm}^2$ ) and if the interest and depreciation charges on the line are 8%, estimate the most economical current density to use for a transmission requiring full-load current for 60% of the year. The cost of generating electric energy is 5 paise/kWh. The resistance of a conductor one kilometre long and  $1 \text{ cm}^2$  cross-section is  $0.18 \Omega$ .

**Solution.** Take one kilometre of the overhead line and let I be the full-load current, then if R is the resistance of each line, the full-load power loss in the line is

$$= 2I^2R = 2I^2 \times 0.18 \times \frac{1}{A} \text{ watt} = \frac{0.36 I^2}{A} \times 10^{-3} \text{ kW}$$

Since the line works for 60% of the year i.e. for  $(0.6 \times 365 \times 24)$  hours, hence total annual loss is  $= \frac{0.35 I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24 \text{ kWh}$

$$\therefore \text{annual cost of this loss} = \text{Rs.} \left( \frac{0.35 I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24 \times 0.05 \right) = 0.0946 I^2/A$$

The annual value of interest on capital outlay and depreciation = 8% of  $2,000 A = \text{Rs. } 160 A$

For minimum total financial loss :  $160 A = 0.0946 I^2/A \quad I/A = \sqrt{(160/0.0946)} = 41.12 \text{ A/cm}^2$

**Example 50.56.** A 500-V, 2-core feeder cable 4 km long supplies a maximum current of 200 A and the demand is such that the copper loss per annum is such as would be produced by the full-load current flowing for six months. The resistance of the conductor 1 km long and  $1 \text{ sq cm}$ . cross-sectional area is  $0.17 \Omega$ . The cost of cable including installation is Rs.  $(120 A + 24)$  per metre where A is the area of cross-section in  $\text{sq. cm}$  and interest and depreciation charges are 10%. The cost of energy is 4 paise per kWh. Find the most economical cross-section.

(Electrical Technology, M.S. Univ. Baroda)

**Solution.** Let us consider one kilometer length of the feeder cable.\*\*

\* For this expression to have minimum value  $\frac{dL}{dA} = 0$

$$\frac{d}{dA}(P + Q/A) = 0 \quad \text{or} \quad P - \frac{Q}{A^2} = 0 \quad \therefore \quad P = \frac{Q}{A^2} \quad \text{or} \quad PA = \frac{Q}{A} \quad \text{or} \quad A = \sqrt{Q/P}$$

\*\* Even though we are given 4 km length of the cable, so far as our calculations are concerned, any convenient unit of length can be taken.

Cable cost/metre	= Rs. (120 A + 24)
Cost of 1 km long cable	= Rs. 120 A × 1000 = Rs. 12 × 10 <sup>4</sup> A
Interest and depreciation per annum is	= 10% of Rs. 12 × 10 <sup>4</sup> A = Rs. 12 × 10 <sup>3</sup> A
Resistance of one km long cable is	R = 0.17/A ohm. —where A is in cm <sup>2</sup>
Cu loss in the cable = 2I <sup>2</sup> R = 2 × 200 <sup>2</sup> × 0.17/A	W = 13.6 × 10 <sup>3</sup> /AW = 13.6/A kW
Energy loss over six months	= (13.6/A) × (8760/2) kWh = 59,568/A kWh
Cost of this energy loss	= Rs. 59,568 × 0.04/A = Rs. 2383/A

For most economical cross-section ;  $12 \times 10^3 A = 2,383/A, A = \sqrt{0.1986} = 0.446 \text{ cm}^2$

**Example 50.57.** A 2-core, 11-kV cable is to supply 1 MW at 0.8 p.f. lag for 3000 hours in a year. Capital cost of the cable is Rs. (20 + 400a) per metre where a is the cross-sectional area of core in cm<sup>2</sup>. Interest and depreciation total 10% and cost per unit of energy is 15 paise. If the length of the cable is 1 km, calculate the most economical cross-section of the conductor. The specific resistance of copper is  $1.75 \times 10^{-6}$  ohm-cm. **(Power Systems-I, AMIE, Sec. B, 1993)**

**Solution.** Cost of 1 km length of the cable = Rs. (20 + 400a) × 1000 = (20,000 + 400 × 10<sup>3</sup>a)  
 If a is the cross-sectional area of each core of the cable, then resistance of 1 km cable length is  
 =  $1.75 \times 10^{-6} \times 1000 \times 100/a = 0.175/a \Omega$ ; F.L. current =  $1 \times 10^6/11 \times 10^3 \times 0.8 = 113.6 \text{ A}$   
 Power loss in the cable =  $2I^2R = 2 \times 113.6^2 \times (0.175/a) = 4516.7/a \text{ W}$   
 Annual cost of energy loss = Rs. (4516.7/a) × 3000 × 10<sup>-3</sup> × (15/100) = Rs. 2032.5/a.  
 Interest and depreciation per annum =  $0.1 \times 400 \times 10^3 a = 40,000 a$   
 As per Kelvin’s law, the most economical cross-section would be given by

$$40,000 a = 2032.5/a \quad a = 0.2254 \text{ cm}^2 \text{ and}$$

$$d = \sqrt{0.2254 \times 4/\pi} = 0.536 \text{ cm.}$$

### 50.22. Effect of Cable Insulation

Let us now consider the effect of insulation in the case of an insulated cable. The cost of insulation would now be added to the annual value of interest and depreciation. Since for a given type of cable, a given type of armoring and for a given voltage of transmission, the insulation cost does not vary much with the cross-section of the cable, it is taken care of by merely adding a constant R to the term Rs. PA. Hence, the annual interest and depreciation, and insulation cost is represented by the term Rs. (PA + R) where R is a constant representing insulation cost. The addition of term Rs. R results in merely raising the graph of Rs. PA vertically through a distance representing Rs. R, as shown in Fig. 50.12.

The total annual financial loss of insulated cable is thus represented by Rs. (PA + R + Q/A). It should be noted that curve for Q/A is not disturbed at all. However, the graph for total loss i.e. (PA + Q/A + R) is only shifted vertically through a distance equal to Rs.

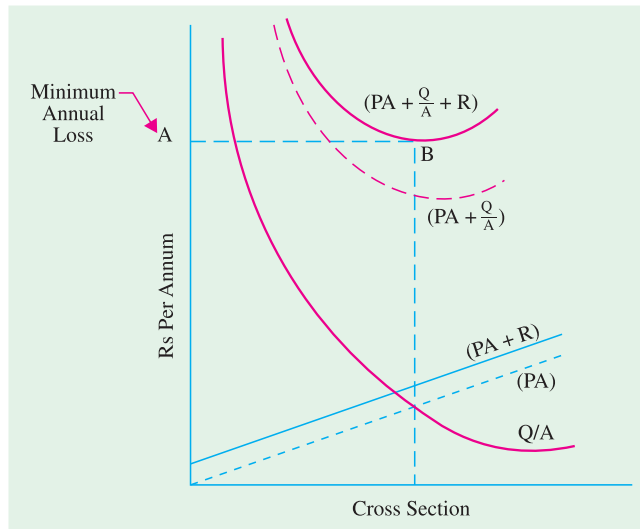


Fig. 50.12

$R$  without any horizontal displacement of the point of minimum financial loss. In other words, the insulation cost does not affect the value of most economical cross-section. Hence, for an insulated cable, Kelvin's law is : *the most economical cross-section is that which makes the interest on the capital outlay plus depreciation due to the conductor in the cable equal to the annual cost of energy lost.*

**Example 50.58.** *The cost of a two-core feeder cable including insulation is Rs.  $(130 A + 24)$  per metre and the interest and depreciation charges 10% per annum. The cable is two km in length and the cost of energy is 4 paisa per unit. The maximum current in the feeder is 250 amperes and the demand is such that the copper loss is equal to that which would be produced by the full current flowing for six months. If the resistance of a conductor of 1 sq. cm cross-sectional area and one km in length be  $0.18 \Omega$ , find the most economical section of the same.*

**Solution.** Cable cost/m = Rs.  $(130 A + 24)$  where  $A$  is the cable cross-section in  $\text{cm}^2$ .

Cost of 1 km long conductor = Rs.  $(130 A \times 1000) = \text{Rs. } (13 \times 10^4 A)$

Interest and depreciation per annum =  $0.1 \times 13 \times 10^4 A = \text{Rs. } (13 \times 10^3 A)$

Resistance of one km long conductor =  $0.18/A$  ohm

Cu loss in the cable =  $2I^2R = 2 \times 250^2 \times \frac{0.18}{A} \text{ W} = 2 \times 250^2 \times \frac{0.18}{A} \times 10^{-3} \text{ kW}$

Annual value of this energy loss =  $\text{Rs. } \left( 2 \times 250^2 \times \frac{0.18}{A} \times 10^{-3} \times \frac{4}{100} \right) \times \frac{8760}{2}$   
 = Rs.  $3941/A$

For most economical cross-section  $13 \times 10^3 A = 3,942/A \therefore A^2 = 3942/13 \times 10^3$

$\therefore A = 0.505 \text{ cm}^2$

**Example 50.59.** *An 11-kV, 3-core cable is to supply a works with 500-kW at 0.9 p.f. lagging for 2,000 hours p.a. Capital cost of the cable per core when laid is Rs.  $(10,000 + 32,00 A)$  per km where  $A$  is the cross-sectional area of the core in sq. cm. The resistance per km of conductor of  $1 \text{ cm}^2$  cross-section is  $0.16 \Omega$ .*

*If the energy losses cost 5 paise per unit and the interest and sinking fund is recovered by a charge of 8% p.a., calculate the most economical current density and state the conductor diameter.*

(Power Systems-I, AMIE-1994)

**Solution.** The annual charge on cost of conductors per km is =  $0.08 \times 32,000 A = \text{Rs. } 2,560 A$

Current per conductor is =  $500,000/\sqrt{3} \times 11,000 \times 0.9 = 29.2 \text{ A}$

Losses in all the three cores =  $3I^2R = 3 \times 29.2^2 \times \frac{0.16}{A} = \frac{409.3}{A} \times 10^{-3} \text{ kW}$

Annual cost of this loss =  $\text{Rs. } \frac{409.3 \times 10^{-3}}{A} \times \frac{5}{100} \times 200 = \text{Rs. } \frac{40.93}{A}$

Obviously, the values of the two constants per km are  $P = 2,560$  and  $Q = 40.93$

The most economical cross-section is given by  $A = \sqrt{Q/P} = \sqrt{(40.93/2,560)} = 0.1265 \text{ cm}^2$

The current density is  $29.2/0.1265 = 231 \text{ A/cm}^2$  and the conductor diameter is

$$d = \sqrt{4 \times 0.1265 / \pi} = 0.4 \text{ cm.}$$

**Example 50.60.** *Discuss limitations of the application of Kelvin's law.*

*An industrial load is supplied by a 3-phase cable from a sub-station at a distance of 6 km. The voltage at the load is 11 kV. The daily load cycle for six days in a week for the entire year is as given below :*



- (i) 700 kW at 0.8 p.f. for 7 hours,                      (ii) 400 kW at 0.9 p.f. for 3 hours,  
 (iii) 88 kW at unity p.f. for 14 hours.

Compute the most economical cross-section of conductors for a cable whose cost is Rs.  $(5000 A + 1500)$  per km (including the cost of laying etc.). The tariff for the energy consumed at the load is Rs. 150 per annum per kVA of M.D. plus 5 paise per unit. Assume the rate of interest and depreciation as 15%. The resistance per km of the conductor is  $(0.173/A) \Omega$ .

(Electrical Power-I ; Bombay Univ.)

**Solution.** Capital cost of the cable =  $6 \times \text{Rs. } (5000 A + 1500) = \text{Rs. } (30,000 A + 9000)$ .

Annual cost of interest and depreciation =  $\text{Rs. } 0.15 (30,000 A + 9000) = \text{Rs. } (4500 A + 1350)$ .

Resistance of each conductor =  $6 \times 0.173/A = 1038/A \text{ ohm}$

Line currents due to different loads are :

- (i) At 700 kW, 0.8 p.f.;  $I = 700 \times 10^3 / \sqrt{3} \times 11 \times 10^3 \times 0.8 = 45.9 \text{ A}$   
 (ii) At 400 kW, 0.9 p.f.;  $I = 400 \times 10^3 / \sqrt{3} \times 11 \times 10^3 \times 0.9 = 23.3 \text{ A}$   
 (iii) At 88 kW, u.p.f.  $I = 88 / 11 = 4.62 \text{ A}$

The corresponding energy losses per week in the cable are :

- (i) loss =  $3 \times 45.92 \times (1.038/A) \times (6 \times 7) / 1000 = (276/A) \text{ kWh}$   
 (ii) loss =  $3 \times 23.32 \times (1.038/A) \times (6 \times 3) / 1000 = (30.4/A) \text{ kWh}$   
 (iii) loss =  $3 \times 4.622 \times (1.038/A) \times (6 \times 14) \times 10^{-3} = (5.6/A) \text{ kWh}$

Total weekly loss =  $(276/A) + (30.4/A) + (5.6/A) = (312/A) \text{ kWh}$

Taking 52 weeks in one year, we have

$$\text{Annual Cu loss} = \text{Rs. } \frac{5}{100} \cdot \frac{312}{A} \times 52 = \text{Rs. } \frac{811}{A}$$

Max. voltage drop in each conductor =  $45.9 \times 1.038/A = (47.64/A) \text{ volt}$

Max. kVA demand charge due to this drop for three conductors is

$$= \text{Rs. } 3 \times (47.64/A) \times 45.9 \times 10^{-3} \times 150 = \text{Rs. } 984/A$$

Total annual charges due to cable loss =  $\text{Rs. } (811/A) + \text{Rs. } (984/A) = \text{Rs. } (1795/A)$

For the most economical size of the cable ;  $4500 A = 1795/A$  ;  $A = 0.63 \text{ cm}^2$ .

**Note.** Though not given, it has been assumed that A appearing in Rs.  $(5000 A + 1500)$  is in  $\text{cm}^2$ .

### Tutorial Problem No. 50.2

- The daily load cycle of a 3-phase, 110-kV transmission line is as follows :  
 (a) 6 hours–20 MW, (b) 12 hours–5 MW, (c) 10 hours – 6 MW. The load p.f. is 0.8 lag for all the three loads. Determine the most economical cross-section if the cost of the line including erection is Rs.  $(9,000 + 600 A)$  per km where A is the cross-section of each conductor in  $\text{cm}^2$ . The rate of interest and depreciation is 10% and energy cost is 6 paise/unit. The line is in use all the year. The resistance per km of each conductor is  $0.176/A \text{ ohm}$ . [1.64  $\text{cm}^2$ ]
- Find the most economical cross-section of conductor for a system to transmit 120 A at 250 V all the year round with a cost of 10 paise per kWh for energy wasted. The two-core cable costs Rs. 20 A per metre where A is the cross-section of each core in  $\text{cm}^2$ . The length of feeder is one km. The interest and depreciation charges total 8% of total cost. One km of Cu wire  $1 \text{ cm}^2$  in cross-section area has a resistance of 0.15 W. [1.52  $\text{cm}^2$ ]

50.23. Note on Power Factor

Irrespective of the nature of voltage and current, the power factor may be defined as the ratio of true power consumed to the apparent power (product of voltage and current).

$$\therefore \text{p.f.} = \frac{W}{VA} = \frac{kW}{kVA}$$

If voltage and current are both sinusoidal, then p.f. = kW/kVA = cos φ where φ is the phase difference between voltage and current.

The power factor of all a.c. motors (except over-excited synchronous motors and certain types of commutator motors) and transformers is less than unity (lagging). Majority of industrial motors are induction motors which have a reasonable high power factor (0.9 or so) at full-load but very low power factor at light loads and at no-load as low a value as 0.1. Induction motor is, in fact, a simple resistance-inductance circuit. The current taken by such a circuit has two components (i) active or wattful component  $I_w$  which is in phase with the voltage. Its value is  $I \cos \phi$  where  $I$  is total circuit current. It is this component which represents the useful power in the circuit and (ii) reactive or wattless or idle component  $I_\mu$  which lags behind the applied voltage by 90°. Its value is  $I \sin \phi$  and its purpose is to produce alternating magnetic flux. Hence, it is also known as wattless or magnetising current.

Obviously  $I^2 = I_w^2 + I_\mu^2$

Multiplying both sides by  $V^2$ , we get

$$V^2 I^2 = V^2 I_w^2 + V^2 I_\mu^2$$

or

$$(VI)^2 = (VI_w)^2 + (VI_\mu)^2$$

Now

$$VI = \text{volt-amperes written as } VA ; VI_w = \text{wattage } W$$

$$VI_\mu = \text{reactive volt-ampere written as } VA R.$$

Hence, in any inductive circuit as shown in Fig. 50.13

$$kVA^2 = kW^2 + kVAR^2$$

For example, suppose an R-L circuit draws a current of 100 A from a supply of 250 V at a power factor of 0.8, then total kilo-voltampere of the circuit is  $250 \times 100/1000 = 25$  kVA. The true power is  $250 \times 100 \times 0.8/1000 = 20$  kW and reactive voltampere is  $250 \times 100 \times 0.6/1000 = 15$  kVAR.

Obviously,  $kVA = \sqrt{20^2 + 15^2} = 25$

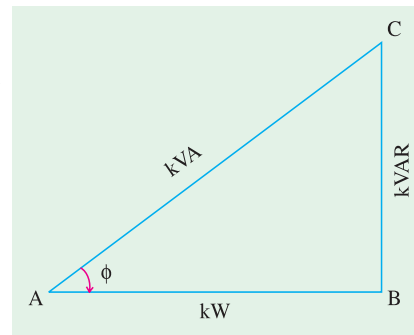


Fig. 50.13

50.24. Disadvantages of Low Power Factor

Suppose a 3-phase balanced system is supplying a load  $W$  at a voltage of  $V$  and p.f.  $\cos \phi$ , then current flowing through the conductor is  $I = W/\sqrt{3} \cdot V \cdot \cos \phi$ .

A lower power factor, obviously, means a higher current and this affects in the following three ways :

1. Line losses are proportional to  $I^2$  which means proportional to  $1/\cos^2 \phi$ . Thus losses at  $\cos \phi = 0.8$  are  $1/0.8^2 = 1.57$  times those at unity power factor.
2. Ratings of generators and transformers etc. are proportional to current, hence to  $1/\cos \phi$ , therefore, larger generators and transformers are required.
3. Low lagging power factor causes a large voltage drop, hence extra regulation equipment is required to keep voltage drop within prescribed limits.

**Explanation**

Low power factor means higher wattless current which presents a serious problem from supply

viewpoint. Consider the following numerical example. Suppose an alternator (single-phase) is to have an output of 1500 kW at 10,000 V.

(a) Load at unity power factor  $P = VI \times 1 = VI$  watt and  $I = \frac{P}{V} = \frac{1,500,000}{10,000} = 150$  A

In this case kVA = kW = 1500

(b) If the load has a lagging p.f. of 0.6, then

$$I = \frac{P}{V \cos \phi} = \frac{1,500,000}{10,000 \times 0.6} = 250 \text{ A and kVA} = \mathbf{2500}$$

It is obvious that in case (b), the current-carrying capacity of alternator winding will have to be (250-150)/150 = 0.67 or 67% greater. Now, the size and hence cost of a given type of alternator is decided by kVA output and not by kW.

In case (a), kVA = kW 1500. In case (b) kVA = 1500/0.6 = 2500.

It is seen that with a load p.f. of 0.6, the cost of an alternator to give the same power output and hence the earning capacity, is about 67% greater than when p.f. is unity. Moreover, the switchgear will have to carry 250 A instead of 150 A and so, will be correspondingly larger and more costly. Since transformers are almost always used in the line, they too will be more expensive.

The cables connecting the alternator with the load will also have to carry more current. If current density is the same, then cable cross-section and hence the weight of copper required will be increased.

Therefore, it is seen that a low power factor leads to a high capital cost for the alternator, switchgear, transformers and cables etc. Since the value of power factor is decided by the consuming devices like motors etc., the electric supply undertakings encourage the consumers to make their power factor as high as possible. As an inducement, they offer cheaper tariffs in different ways discussed in Art. 50.25.

The disadvantages of low power factor are summarized below :

In a transmission line (supplying an inductive load) it is only the in-phase component of line current which is active in the transmission of power. When power factor is low, then in-phase (or active) component is small, but reactive component is large hence unnecessarily large current is required to transmit a given amount of power. Large reactive component means large voltage drop and hence greater Cu losses with the result that the regulation is increased and efficiency is decreased.

Lighting and heating loads, being mainly resistive, have unity p.f. but electromagnetic machinery has a low p.f. especially on no-load or light loads.

A low power factor (lagging) system has the following disadvantages as compared to a system carrying the same power but at higher p.f.

1. As current is large and at a large lagging angle, it cause greater losses and requires higher excitation in the generators, thereby reducing the efficiency of generation.
2. Increased Cu losses are incurred in the cables and machinery because of large current. Both these factors raise the running cost of the system.
3. The ratings of the generators, transformers, switchgears and cables etc., have to be increased, which means additional capital charges with increased depreciation and interest.
4. Due to voltage drop in generators, transformers and cables etc. the regulation becomes poor. As supply authorities are usually bound to maintain the voltage at consumer's terminals within prescribed limits, they have to incur additional capital cost of tap-changing gear on transformers to compensate for the voltage drop. Hence, the supply authorities penalise industrial consumers for their low power factor by charging increased tariff for kVA maximum demand in addition to usual kWh charge. Obviously, it is advantageous for the consumer to improve his load p.f. with the help of phase-advancing equipment (*i.e.* synchronous capacitors) or static capacitors.

### 50.25. Economics of Power Factor

In order to induce the customers to keep their load p.f. as high as possible, tariff rates for a.c. power are such that the overall charge per kWh of the energy consumed depends, in some way or the other, on the load p.f. of the consumer. Some of the ways in which it is done are given below :

(i) The total bill for consumption is so adjusted as to make it depend on the deviation of the load p.f. from a standard value, say 0.9. The bill is increased by a constant percentage for each unit p.f. deviation from 0.9, say, for every decrease of 0.01 from 0.9. Similarly, there would be bonus for each increase of unit p.f. value.

(ii) The total power bill is adjusted in some way on the total reactive kilo-volt ampere-hour *i.e.* kVARh instead of kWh. Special meters are installed for measuring kVARh.

(iii) But the most commonly-used tariff is the two-part tariff (Art. 50.20). The fixed charges instead of being based on kW maximum demand are based on kVA maximum demand so that they become inversely proportional to the power factor (because  $kVA = kW/\cos \phi$ )

#### Explanation

Consider a single-phase, 1500-kW, 10,000-V alternator already discussed in Art. 50.24. Suppose whole of its output is taken up by one consumer and the two-part tariff is Rs. 60 per annum per kVA max. demand plus 2 paise per kWh. Let the load factor be 30%. Then,

$$\begin{aligned} \text{Average power throughout the year} &= 0.3 \times 1500 = 450 \text{ kW} \\ \therefore \text{annual consumption} &= 450 \times 8,760 \text{ kWh} \\ \text{Annual cost at 2 paise per kWh} &= \text{Rs. } 450 \times 8,760 \times 0.02 = \text{Rs. } 78,840 \end{aligned}$$

For the fixed kVA charge, we have

$$\begin{aligned} \text{Annual charge at unit p.f.} &= \text{Rs. } 60 \times 1500 = \text{Rs. } 90,000 \\ \text{Annual charge at of 0.6 p.f.} &= \text{Rs. } 60 \times (1500/0.6) = \text{Rs. } 150,000 \\ \text{Total charges at unity p.f.} &= \text{Rs. } 90,000 + 78,840 = \text{Rs. } 168,840 \\ \text{Total charges of 0.6 p.f.} &= \text{Rs. } 150,000 + 78,840 = \text{Rs. } 228,840 \end{aligned}$$

It is seen that annual saving for unity p.f. is Rs. 60,000. Of course, this may not represent the net total saving when p.f. correction has been done either by phase-advancing equipment or static capacitors because the cost and maintenance of p.f. correcting apparatus has still to be taken into account. The cost will be by way of interest on capital required to install the p.f. improvement apparatus plus depreciation, maintenance expenses etc.

Consider the undergiven case. To illustrate how two-part tariff operates for different cases, suppose that rate of charging power to a large factory is Rs. 90 per annum per kVA maximum demand and 2 paise per kWh consumed. Also, suppose that the factory has 450 kW maximum load demand and an annual consumption of 10,000,000 kWh.

(i) If load p.f. were 0.9, then 450 kW max. demand is equal to  $450/0.9 = 500$  kVA max. demand.

(ii) If load p.f. were 0.7, then 450 kW max. demand is equal to  $450/0.7 = 643$  kVA max. demand.

Cost on kWh consumed will be the same for both power factors, it is only the maximum demand charge which will vary with the power factor.

$$\text{At 0.7, the annual fixed charge for 643 kVA at Rs. 90 is } \text{Rs. } 90 \times 643 = \text{Rs. } 57,870$$

$$\text{At 0.9, the annual fixed charge for 500 kVA} = \text{Rs. } 90 \times 500 = \text{Rs. } 45,000$$

$$\therefore \text{extra charge} = \text{Rs. } 12,870$$

Now, let us assume that phase advancing equipment required to improve the power factor from 0.7 to 0.9 costs Rs. 50,000. Then, taking combined interest and depreciation at 12%, the annual charge will be  $\text{Rs. } 50,000 \times 0.12 = \text{Rs. } 6,000$ .

Hence, the net annual saving comes to Rs. 12,870 – Rs. 6,000 = Rs. 6,870.

If, suppose, the p.f. correcting plant had cost Rs. 107,250, then interest and depreciation allowance on it would be = Rs. 107,250 × 0.12 = Rs. 12,870. In that case, there would be no point in installing such an apparatus because there would be no net saving.

It can be seen from the above example that higher the desired power factor improvement *i.e.* greater the kVAR reduction, the more costly the p.f. improvement plant and hence greater the charge on interest on capital outlay and depreciation. A point is reached, in practice, when any further improvement in power factor costs more than the saving in the power bill. Hence, it is necessary for the consumer to find out the value of power factor at which his net saving will be maximum. The value can be found if (i) annual charge per kVA maximum demand and (ii) the cost per kVA rating of phase advancing equipment are known.

### 50.26. Economical Limit of Power Factor Correction

Suppose a consumer is charged at Rs.  $A$  per kVA maximum demand plus a flat rate per kWh. Further, suppose that he is taking power of  $P$  kW at a power factor of  $\cos \phi_1$ . As shown in Fig. 50.14, his  $kVA_1$  is  $P/\cos \phi_1$  and his  $kVAR_1$  is  $P \tan \phi_1$ . Suppose by installing static capacitors or synchronous capacitors, he improves his power factor to  $\cos \phi_2$  (his power consumption  $P$  remaining the same). In that case, his  $kVA_2$  is  $P/\cos \phi_2$  and  $kVAR_2$  is  $P \tan \phi_2$ .

Reduction in his kVA maximum demand is =  $(kVA_1 - kVA_2) = (P/\cos \phi_1 - P/\cos \phi_2)$ . Since charge is Rs.  $A$  per kVA maximum demand, his annual saving on this account is

$$= A (P/\cos \phi_1 - P/\cos \phi_2).$$

His kVAR is reduced from  $kVAR_1$  to  $kVAR_2$ , the difference  $(kVAR_1 - kVAR_2) = (P \tan \phi_1 - P \tan \phi_2)$  being neutralized by the leading kVAR supplied by the phase advancer.

$\therefore$  Leading kVAR supplied by phase advancer is  $= (P \tan \phi_1 - P \tan \phi_2)$ .

If the cost per kVAR of advancing plant is Rs.  $B$  and the rate of interest and depreciation is  $p$  percent per year, then its cost per annum is

$$\begin{aligned} &= \frac{BP}{100} (P \tan \phi_1 - P \tan \phi_2) \\ &= C (P \tan \phi_1 - P \tan \phi_2) \text{ where } C = BP/100 \end{aligned}$$

$\therefore$  Net annual saving  $S = A (P/\cos \phi_1 - P/\cos \phi_2) - C(P \tan \phi_1 - P \tan \phi_2)$

This net saving is maximum when  $dS/d\phi_2 = 0$

$$\begin{aligned} \therefore \frac{dS}{d\phi_2} &= \frac{d}{d\phi_2} [A (P/\cos \phi_1 - P/\cos \phi_2) - C (P \tan \phi_1 - P \tan \phi_2)] \\ &= -AP \sec \phi_2 \tan \phi_2 + CP \sec^2 \phi_2 \end{aligned}$$

For maximum saving,  $dS/d\phi_2 = 0$

$$\therefore -AP \sec \phi_2 \tan \phi_2 + CP \sec^2 \phi_2 = 0 \quad \text{or} \quad \sin \phi_2 = C/A = BP/A$$

From this expression,  $\phi_2$  and hence  $\cos \phi_2$  can be found. It is interesting to note that the most economical angle of lag  $\phi_2$  is independent of the original value  $\phi_1$ .

Obviously, most economical p.f. is

$$\cos \phi_2 = \sqrt{1 - \sin^2 \phi_2} = \sqrt{1 - (C/A)^2} = \sqrt{1 - (BP/100 A)^2}$$

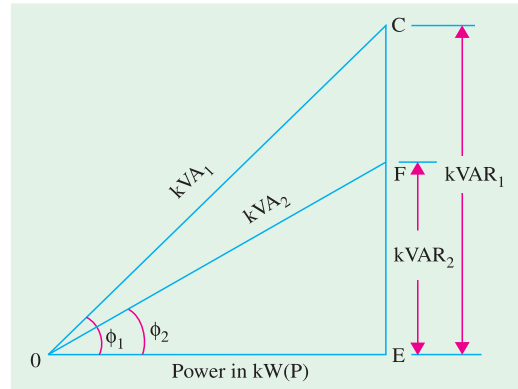


Fig. 50.14

**Example 50.61.** A 3-phase, 50-Hz, 3,000-V motor develops 600 h.p. (447.6 kW), the power factor being 0.75 lagging and the efficiency 0.93. A bank of capacitors is connected in delta across the supply terminals and power factor raised to 0.95 lagging. Each of the capacitance units is built of five similar 600-V capacitors. Determine capacitance of each capacitor. (London Univ.)

**Solution.** Power input  $P = 447,600/0.93 = 480,000 \text{ W}$   
 $\cos \phi_1 = 0.75 ; \phi_1 = \cos^{-1}(0.75) = 41^\circ 24'$ ,  $\cos \phi_2 = \cos^{-1}(0.95) = 18^\circ 12'$   
 $\therefore \tan \phi_1 = \tan 41^\circ 24' = 0.8816$ ,  $\tan \phi_2 = \tan 18^\circ 12' = 0.3288$

As shown in Fig. 50.14, leading VAR supplied by capacitor bank is  
 $= P(\tan \phi_1 - \tan \phi_2) = 480,000(0.8816 - 0.3288) = 48 \times 5,528$   
 Leading VAR supplied by each of the three sets  $= 48 \times 5,528/3 = 16 \times 5,528 \dots(i)$

Phase current of capacitor is  $I_{CP} = \frac{V}{X_C} = \frac{3,000}{X_C} = \frac{3,000}{1/\omega C} = 3,000 \times 314 C$

where C is the total capacitance in each phase.

$\therefore$  VAR of each phase  $= VI_{cap} = 3,000 \times 3,000 \times 314 C \dots(ii)$

Equating (i) and (ii) we get,  $3,000 \times 3,000 \times 314 C = 16 \times 5,528 \therefore C = 31.22 \mu\text{F}$

Since it is the combined capacitance of five equal capacitors joined in series, the capacitance of each  $= 5 \times 31.22 = 156 \mu\text{F}$ .

**Example 50.62.** A synchronous motor having a power consumption of 50 kW is connected in parallel with a load of 200 kW having a lagging power factor of 0.8. If the combined load has a p.f. of 0.9, what is the value of leading reactive kVA supplied by the motor and at what p.f. is it working? (I.E.E. London)

**Solution.** Let  $\phi_1 =$  p.f. angle of motor ;  $\phi_2 =$  p.f. angle of load  
 $\phi_t =$  combined p.f. angle ;  $\phi_2 = \cos^{-1}(0.8) = 36^\circ 52'$   
 $\tan \phi_2 = \tan 36^\circ 52' = 0.75$  ;  $\phi_t = \cos^{-1}(0.9) = 25^\circ 51'$   
 $\tan \phi_t = \tan 25^\circ 51' = 0.4854$

Combined power  $P = 200 + 50 = 250 \text{ kW}$   
 Total kVAR  $= P \tan \phi_t = 250 \times 0.4854 = 121.1$   
 Load kVAR  $= 200 \times \tan \phi_2 = 200 \times 0.75 = 150$

$\therefore$  leading kVAR supplied by synchronous motor  $= 150 - 121.1 = 28.9$   
 $\tan \phi_1 = 28.9/50 ; \phi_1 = 30.1^\circ$ ,  $\cos \phi_1 = 0.86$  (lead)

**Example 50.63.** A generating station supplies power to the following, lighting load 100-kW ; an induction motor 400 h.p. (298.4 kW), power factor 0.8, efficiency, 0.92 ; a rotary converter giving 100 A at 500 V at an efficiency of 0.94. What must be the power factor of the rotary converter in order that the power factor of the supply station may be unity.

**Solution.** Since, lighting load has no kVAR (unity p.f. assumed), the lagging kVAR of induction motor are neutralized by the leading kVAR of rotary converter only,

(i) Motor input  $= 298.4/0.92 = 324.4 \text{ kW}$   
 motor p.f. angle  $\phi_m = \cos^{-1}(0.8) = 36^\circ 52' \therefore \tan \phi_m = \tan 36^\circ 52' = 0.75$   
 $\therefore$  lagging motor kVAR  $= 324.4 \times \tan \phi_m = 324.4 \times 0.75 = 243.3$

$\therefore$  leading kVAR to be supplied by rotary converter  $= 243.3$   
 Rotary converter intake  $= 500 \times 100/0.94 \times 1000 = 53.2 \text{ kW}$

$\tan \phi = \frac{\text{kVAR}}{\text{kW}} = \frac{243.3}{53.2} = 4.573$   
 $\therefore \phi = 77^\circ 40'$  or  $\cos \phi = 0.214$  (leading)

**Example 50.64.** A factory has an average annual demand of 50 kW and an annual load factor of 0.5. The power factor is 0.75 lagging. The tariff is Rs. 100 per kVA maximum demand per annum plus five paise per kWh. If loss-free capacitors costing Rs. 600 per kVAR are to be utilized, find the value of the power factor at which maximum saving will result. The interest and depreciation together amount to ten per cent. Also, determine the saving affected by improving the power factor to this value. **(Electrical Technology ; M.S. Univ. Baroda)**

**Solution.** The most economical power factor angle is given by  $\sin \phi = C/A$

$$C = 10\% \text{ of Rs. } 600 = \text{Rs. } 60, A = \text{Rs. } 100 \quad \therefore \sin \phi = 60/100 = 0.6$$

$$\phi = \sin^{-1}(0.6) = 36^\circ 52' ; \text{ New p.f.} = \cos 36^\circ 52' = \mathbf{0.8}$$

The net annual saving due to improving in power factor can be found as follows :

$$\text{Max. demand} = 50/0.5 = 100 \text{ kW}$$

At load p.f. of 0.75, maximum demand of 100 kW represents  $100/0.75 = 400/3$  kVA maximum demand.

At load p.f. of 0.8, 100 kW represent  $100/0.8 = 125$  kVA maximum demand.

Max. kVA demand charge

$$\text{At } 0.75 \text{ p.f.} = \text{Rs. } 100 \times 400/3 = \text{Rs. } 13,333.3$$

$$\text{At } 0.8 \text{ p.f.} = \text{Rs. } 100 \times 125 = \text{Rs. } 12,500$$

$$\text{Annual saving} = \text{Rs. } (13,333.3 - 12,500) = \mathbf{\text{Rs. } 838.3}$$

**Example 50.65.** For increasing the kW capacity of a plant working at 0.7 lag p.f. the necessary increase of power can be obtained by raising the p.f. to 0.85 or by installing additional plant. What is the maximum cost per kVA of p.f. correction apparatus to make its use more economical than additional plant at Rs. 500 kVA ? **(Gen. Protect. and Switchgear, Madras Univ.)**

**Solution.** Let  $kVA_1$  be the initial capacity of the plant and  $kVA_2$  its increased capacity with extra or additional plant. As seen from Fig. 50.15.

$$kVA_1 \cos \phi_2 = kVA_2 \cos \phi_1$$

$$\therefore kVA_2 = kVA_1 \times \cos \phi_2 / \cos \phi_1 \\ = kVA_1 \times 0.85 / 0.7 = (17/14) kVA_1$$

$$kVA \text{ of the additional plant} = kVA_2 - kVA_1 = (17/14) kVA_1 - kVA_1 = (3/14) kVA_1$$

$$\text{Capital cost of additional plant} = \text{Rs. } (500 \times 3/14) kVA_1 = \text{Rs. } (750/7) kVA_1$$

$$\text{Now, } \cos \phi_1 = 0.7, \phi_1 = 45^\circ 34' ; \sin \phi_1 = 0.714$$

$$\cos \phi_2 = 0.85, \phi_2 = 31.8^\circ ; \sin \phi_2 = 0.527$$

$\therefore$  kVAR supplied by p.f. correction apparatus

$$= (kVA_2 \sin \phi_1 - kVA_1 \sin \phi_2) = \left( \frac{7}{14} \times 0.714 kVA_1 - kVA_1 \times 0.527 \right) = 0.34 kVA_1$$

If the capital cost of the p.f. correction apparatus be Rs.  $x$  per kVAR, the total cost is  $= 0.34x kVA_1$ .

If the annual cost of the additional plant is to be the same as that of the p.f. correction apparatus (assuming same annual interest and depreciation), then

$$(750/7) kVA_1 = 0.34 x kVA_1 \quad x = \text{Rs. } 315.2$$

If p.f. correction apparatus is loss-free, then its kVAR = kVA. Hence, the maximum cost per kVA of p.f. correction apparatus that can be paid is **Rs. 315.2**.

**Example 50.66.** A consumer taking a steady load of 160 kW at a p.f. of 0.8 lag is charged at Rs. 80 per annum per kVA of maximum demand plus 5 paise per kWh consumed. Calculate the value to which he should improve the p.f. in order to affect the maximum saving if the leading kVA cost Rs.100 per kVA and interest and depreciation be at 12% per annum. Calculate also the saving.

(Electrical Technology ; Gujarat Univ.)

**Solution.** With reference to Art. 50.26,  $C = BP/100 = 12 \times 100/100 = 12$ ,  $A = 80$

$$\begin{aligned} \text{Most economical p.f. } \cos \phi_2 &= \sqrt{1 - (BP/100A)^2} \\ &= \sqrt{1 - (12/80)^2} \\ &= \mathbf{0.9887 \text{ (lag)}} \end{aligned}$$

Since load remains steady, maximum load is also 160 kW.

At load p.f. of 0.8, maximum load of 160 kW represents  $160/0.8 = 200$  kVA.

Similarly, at 0.9887 p.f., it represents  $160/0.9887 = 162$  kVA.

Max. kVA demand charge at 0.8 p.f. = Rs.  $80 \times 200 =$  Rs. 16000

Max. kVA demand charge at 0.9887 p.f. = Rs.  $80 \times 162 =$  Rs. 12,960

$\therefore$  annual saving = Rs.  $(16,000 - 12,960) =$  **Rs. 3,040.**

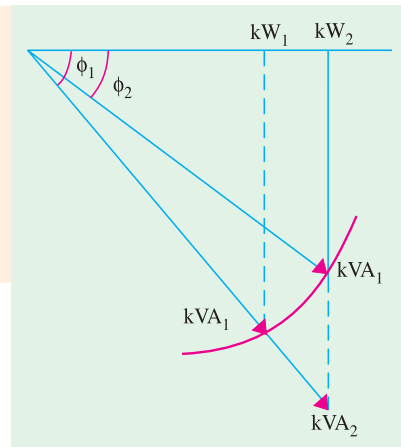


Fig. 50.15

**Example 50.67.** A consumer takes a steady load of 1500 kW at a p.f. of 0.71 lagging and pays Rs. 50 per annum per kVA of maximum demand. Phase advancing plant costs Rs. 80 per kVA. Determine the capacity of the phase advancing plant required for minimum overall annual expenditure. Interest and depreciation total 10%. What will be the value of the new power factor of the supply?

(Electrical Power-III, Bangalore Univ.)

**Solution.** Minimum overall annual expenditure corresponds to most economical power factor given by  $\cos \phi_2 = \sqrt{1 - (BP/100A)^2}$  as shown in Art. 50.26.

Now,  $B = 80, P = 10, A = 50 \cos \phi_2 = \sqrt{1 - (80 \times 10 / 100 \times 50)^2} = \mathbf{0.987}$

Now,  $\cos \phi_1 = 0.71, \phi_1 = 44.7^\circ$

$\tan \phi_1 = 0.9896; \cos \phi_2 = 0.987, \phi_2 = 9.2^\circ, \tan \phi_2 = 0.162$

kVA supplied by phase advancing plant =  $P (\tan \phi_1 - \tan \phi_2) = 1500 (0.9896 - 0.162) = \mathbf{1240.}$

**Example 50.68.** A factory takes a load of 200 kW at 0.85 p.f. (lagging) for 2,500 hours per annum and buys energy on tariff of Rs. 150 per kVA plus 6 paise per kWh consumed. If the power factor is improved to 0.9 lagging by means of capacitors costing Rs. 525 per kVA and having a power loss of 100 W per kVA, calculate the annual saving affected by their use. Allow 8% per annum for interest and depreciation on the capacitors.

**Solution.** Factory load = 200 kW

$\cos \phi_1 = 0.85 \text{ (lagging)}$

$\therefore \phi_1 = \cos^{-1}(0.85) = 31^\circ 48' \tan \phi_1 = 0.62$

$\therefore$  lagging kVAR of factory load =  $0.62 \times 200 = 124$

(or first find total kVA =  $200/0.85$  and then multiply it by  $\sin \phi_1$  to get kVAR)

Suppose,  $x =$  capacitor's kVAR (leading), Total kVAR =  $124 - x$

Because loss per kVA is 100 W i.e., 1/10 kW per kVA



$$\begin{aligned} \therefore \text{capacitor's loss} &= x/10 \text{ kW} \\ \therefore \text{Total kW} &= 200 + (x/10) \\ \text{Now, overall p.f.} &= \cos \phi = 0.9 \quad \therefore \phi = 25^\circ 51' \quad \therefore \tan 25^\circ 51' = 0.4845 \\ \text{Now} \quad \tan \phi &= \frac{\text{total kVAR}}{\text{total kW}} = \frac{124 - x}{200 + (x/10)} \quad \therefore x = 25.9 \text{ kVAR} \end{aligned}$$

**Cost per annum before improvement**

$$\begin{aligned} \text{kVA} &= 200/0.85 = 235.3 \text{ Units consumed per annum} = 200 \times 2500 = 5 \times 10^5 \text{ kWh} \\ \therefore \text{annual cost} &= \text{Rs. } (235.3 \times 150 + 5 \times 10^5 \times 6/100) = \text{Rs. } 65,295 \end{aligned}$$

**Cost per annum after improvement**

$$\begin{aligned} \text{kVA} &= 200/0.9 = 222.2, \text{ Cost} = \text{Rs. } 222.2 \times 150 = \text{Rs. } 33,330 \\ \text{Cost of energy} &= \text{Rs. } 5 \times 10^5 \times 6/100 = \text{Rs. } 30,000 \text{ (as before)} \\ \text{Cost of losses occurring in capacitors} &= \text{Rs. } \frac{25.9 \times 100 \times 2500 \times 6}{1000 \times 100} = \text{Rs. } 389 \\ \text{Annual interest and depreciation cost on capacitors} &= \text{Rs. } 525 \times 25.9 \times 8/100 = \text{Rs. } 1088 \\ \therefore \text{total annual cost is} &= \text{Rs. } (33,330 + 30,000 + 389 + 1088) = \text{Rs. } 64,807 \\ \therefore \text{annual saving} &= \text{Rs. } (65,295 - 64,807) = \text{Rs. } 488. \end{aligned}$$

**Example 50.69.** A 30 h.p. (22.38 kW) induction motor is supplied with energy on a two-part tariff of Rs. 60 per kVA of maximum demand per annum plus 5 paise per unit. Motor (A) has an efficiency of 89% and a power factor of 0.83. Motor (B) with an efficiency of 90% and a p.f. of 0.91 costs Rs. 160 more. With motor (A) the p.f. would be raised to 0.91 (lagging) by installing capacitors at a cost Rs. 50 per kVA.

If the service required from the motor is equivalent to 2,280 hr. per annum at full load, compare the annual charges in the two cases. Assume interest and depreciation charges to be % per annum for the motor and 8% per annum for the capacitors.

**Solution. For Motor A**

$$\begin{aligned} \text{Full-load motor input} &= 22.38 \times 10^3 / 0.89 = 25.14 \text{ kW} ; \text{ kVA} = 25.14 / 0.83 = 30.29 \\ \text{Now, when p.f. is raised to 0.91, then kVA demand of motor} &= 25.14 / 0.91 = 27.6 \\ \text{Annual cost of energy supplied to motor} &= 27.6 \times 60 + (25.14 \times 2,280 \times 5/100) \\ &= 1,656 + 2,866 = \text{Rs. } 4,522 \end{aligned}$$

By using capacitors, the phase angle of load is decreased from  $\phi_1$  ( $\cos \phi_1 = 0.83$ ) to  $\phi_2$  ( $\cos \phi_2 = 0.91$ ).

$$\begin{aligned} \text{kVAR necessary for this improvement} &= \text{load kW} (\tan \phi_1 - \tan \phi_2) \\ &= 25.14 (0.672 - 0.4557) = 5.439 \end{aligned}$$

$$\text{Annual charges on capacitors} = \text{Rs. } 50 \times 5.439 \times 8/100 = \text{Rs. } 21.8$$

$$\begin{aligned} \text{Total charges per annum including interest and depreciation on motor} \\ &= \text{Rs. } (4,522 + 21.8) = \text{Rs. } 4,544. \end{aligned}$$

**For Motor B**

$$\text{F.L. motor input} = 22.38 / 0.9 = 24.86 \text{ kW} ; \text{ kVA} = 24.86 / 0.91 = 27.32$$

$$\text{Annual cost of energy supplied to motor} = 27.32 \times 60 + (24.86 \times 2280 \times 5/100) = \text{Rs. } 4,473$$

If we do not include the interest and depreciation on the motor itself, then B is cheaper than A by Rs. (4544 - 4473) i.e., by Rs. 71. But B cost Rs. 160 more than A. Hence, additional annual charges on B are % of Rs. 160 i.e., Rs. 20. Hence, B is cheaper than A by Rs. (71 - 20) = Rs. 51 per annum.

**Example 50.70.** The motor of a 22.5 kW condensate pump has been burnt beyond economical repairs. Two alternatives have been proposed to replace it by

Motor A. Cost = Rs. 6000 ;  $\eta$  at full-load = 90% ; at half-load = 86%.

Motor B. Cost = Rs. 4000 ;  $\eta$  at full-load = 85% ; at half-load = 82%.

The life of each motor is 20 years and its salvage value is 10% of the initial cost. The rate of interest is 5% annually. The motor operates at full-load for 25% of the time and at half-load for the remaining period. The annual maintenance cost of motor A is Rs. 420 and that of motor B is Rs. 240. The energy rate is 10 paise per kWh.

Which motor will you recommend ?

(Power Plant Engg., AMIE)

**Solution. Motor A**

$$\text{Annual interest on capital cost} = \text{Rs. } 6000 \times 5/100 = \text{Rs. } 300$$

$$\begin{aligned} \text{Annual depreciation charges} &= \frac{\text{original cost-salvage value}}{\text{life of motor in years}} \\ &= \text{Rs. } \frac{6000 - (6000 \times 10/100)}{20} = \text{Rs. } 270 \end{aligned}$$

$$\text{Annual maintenance cost} = \text{Rs. } 420$$

$$\text{Energy input per annum} = \frac{22.5 \times 0.25 \times 8760}{0.9} + \frac{11.25 \times 0.75 \times 8760}{0.86} = 1,40,695 \text{ kWh}$$

$$\text{Annual energy cost} = \text{Rs. } 0.1 \times 1,40,695 = \text{Rs. } 14,069$$

$$\text{Total annual cost} = \text{Rs. } (300 + 270 + 420 + 14,069) = \text{Rs. } 15,059$$

**Motor B**

$$\text{Annual interest on capital cost} = \text{Rs. } 4000 \times 5/100 = \text{Rs. } 200$$

$$\text{Annual depreciation charges} = \text{Rs. } \frac{4000 - (4000 \times 10/100)}{20} = \text{Rs. } 180$$

$$\text{Annual maintenance charges} = \text{Rs. } 240$$

$$\text{Energy input per annum} = \frac{22.5 \times 0.25 \times 8760}{0.85} + \frac{11.25 \times 0.75 \times 8760}{0.82} = 1,48,108 \text{ kWh}$$

$$\text{Annual energy cost} = \text{Rs. } 0.1 \times 1,48,108 = \text{Rs. } 14,811$$

$$\text{Total annual cost} = \text{Rs. } (200 + 180 + 240 + 14,811) = \text{Rs. } 15,431$$

Since the annual cost of Motor B is Rs. (15431 – 15059) = Rs. 372 more than that of Motor A, hence **motor A would be recommended.**

**Example 50.71.** An industrial load takes  $10^6$  kWh a year, the power factor being 0.707 lagging. The maximum demand is 500 kVA. The tariff is Rs. 75 per annum per kVA maximum demand plus 3 paise per unit. Calculate the yearly cost of supply and find the annual saving in cost by installing phase advancing plant costing Rs. 45 per kVA which raises the plant power factor from 0.707 to 0.9 lagging. Allow 10% per annum on the cost of the phase advancing plant to cover all additional costs.

**Solution.** Max. demand charge per annum = Rs.  $500 \times 75 = \text{Rs. } 37,500$

$$\text{Annual energy charges} = \text{Rs. } 10^6 \times 3/100 = \text{Rs. } 30,000$$

$$\therefore \text{ yearly cost of supply} = \text{Rs. } (37,500 + 30,000) = \text{Rs. } 67,500$$

Now, when p.f. is increased from 0.707 to 0.9, then maximum kVA demand is reduced to  $500 \times 0.707/0.9 = 392.8$ .

$$\begin{aligned} \text{Now, annual cost of the supply} &= \text{Rs. } 75 \times 392.8 + \text{Rs. } 30,000 \\ &= 29,460 + 30,000 = \text{Rs. } 59,460 \end{aligned}$$

kVAR to be supplied by the phase-advancing plant for improving the p.f. from 0.707 to 0.9 = kW demand  $(\tan \phi_1 - \tan \phi_2)$  where  $\phi_1 = \cos^{-1}(0.707)$  and  $\phi_2 = \cos^{-1}(0.9)$ .

$$\therefore \text{ kVAR} = 500 \times 0.707 \times 0.516 = 182.4$$

$$\begin{aligned} \text{Annual cost by way of interest and depreciation etc. on the phase advancing plant} \\ &= \text{Rs. } 182.4 \times 45 \times 0.1 = \text{Rs. } 821 \end{aligned}$$

The annual saving would be due to reduction in the maximum demand charge only because units consumed remain the same throughout.

$\therefore$  yearly saving = initial charges for kVA – (new charges for kVA + annual cost of phase advancer) = 67,500 – (59,460 + 821) = **Rs. 7,219.**

**Example 50.72.** It is necessary to choose a transformer to supply a load which varies over 24 hour period in the manner given below :

500 kVA for 4 hours, 1000 kVA for 6 hours, 1500 kVA for 12 hours and 2000 kVA for the rest of the period.

Two transformers each rated at 1500 kVA have been quoted. Transformer I has iron loss of 2.7 kW and full-load copper loss of 8.1 kW while transformer II has an iron loss and full-load copper loss of 5.4 kW each.

- (i) Calculate the annual cost of supplying losses for each transformer if electrical energy costs 10 paise per kWh.
- (ii) Determine which transformer should be chosen if the capital cost of the transformer I is Rs. 1000 more than that of the transformer II and annual charges of interest and depreciation are 10%.
- (iii) What difference in capital cost will reverse the decision made in (ii) above ?

(Utilisation of Elect. Power AMIE)

**Solution. (i) Transformer No. 1**

$$\text{Iron loss/day} = 2.7 \times 24 = 64.8 \text{ kW}$$

$$\begin{aligned} \text{Cu loss/day} &= 8.1 (500/1500)^2 \times 4 + 8.1 \times (1000/1500)^2 \times 6 + 8.1 \times (1500/1500)^2 \\ &\quad \times 12 + 8.1 \times (2000/1500)^2 \times 2 = 151.2 \text{ kWh} \end{aligned}$$

$$\text{Annual energy loss} = 365 (64.8 + 151.2) = 78,840 \text{ kWh}$$

$$\text{Annual cost of both losses} = \text{Rs. } 0.1 \times 78,840 = \text{Rs. } 7884$$

**Transformer No. 2**

$$\text{Iron loss/day} = 5.4 \times 24 = 129.6 \text{ kWh}$$

$$\begin{aligned} \text{Cu loss/day} &= 5.4 (500/1500)^2 \times 4 + 5.4 (1000/1500)^2 \times 6 + 5.4 (1500/1500)^2 \times 12 \\ &\quad + 5.4 (2000/1500)^2 \times 2 = 100.8 \text{ kWh} \end{aligned}$$

$$\text{Annual energy loss} = 365 (129.6 + 100.8) = 84,096 \text{ kWh}$$

$$\text{Annual cost of both losses} = \text{Rs. } 0.1 \times 84,096 = \text{Rs. } 8410$$

(ii) Since the cost of transformer No. 1 is Rs. 1000 more than that of transformer No. 2, extra annual charge in case transformer No. 1 is selected = Rs. 1000  $\times$  10/100 = Rs. 100

$$\text{Annual saving in energy cost due to losses} = \text{Rs. } (8410 - 7884) = \text{Rs. } 526$$

Since the annual saving in the energy cost of supplying losses is much than the extra annual charges due to the higher cost of the transformer No. 1, therefore transformer No. 1 will be selected.

(iii) Transformer No. 2 would be chosen in case annual charges due to extra capital cost of transformer No. 1 exceed the annual saving on the energy cost of supplying losses.

If  $x$  is the extra capital cost of transformer No. 1, then transformer No. 2 could be selected if

$$x \times 10/100 > 526 \text{ or } x > \text{Rs. } 5260$$

**Example 50.73.** Three-phase 50-Hz power is supplied to a mill, the voltage being stepped down to 460-V before use.

The monthly power rate is 7.50 per kVA. It is found that the average power factor is 0.745 while the monthly demand is 611 kVA.

To improve power factor, 210 kVA capacitors are installed in which there is negligible power loss. The installed cost of the equipment is Rs. 11,600 and fixed charges are estimated at 15% per year. What is the yearly saving introduced by the capacitors.

**Solution.** Monthly demand = 611 kVA ; p.f. = 0.745  
 $\phi = \cos^{-1}(0.745) = 41^{\circ}5'$   $\therefore \sin \phi = \sin 41^{\circ}5' = 0.667$   
 $\therefore$  kW component = kVA  $\cos \phi = 611 \times 0.745 = 455$   
kVAR component = kVA  $\sin \phi = 611 \times 0.667 = 408$  (lagging)  
leading kVAR supplied by capacitor = 210  
Hence, kVAR after p.f. improvement =  $408 - 210 = 198$   
 $\therefore$  kVA after p.f. improvement =  $\sqrt{455^2 + 198^2} = 491$   
Reduction in kVA =  $611 - 491 = 120$   
Hence, monthly saving on kVA charge = Rs.  $120 \times 7.5 =$  Rs. 900  
Yearly saving on kVA charge = Rs.  $12 \times 900 =$  Rs. 10,800  
Capital cost on capacitors = Rs. 11,600  
Fixed charge per annum = Rs.  $0.15 \times 11,600 =$  Rs. 1,740  
Hence, net saving per annum = Rs.  $(10,800 - 1,740) =$  **Rs. 9,060**

**Example 50.74.** A supply undertaking is offering the following two tariffs to prospective customers :

Tariff A : Lighting : 20 paise per unit; domestic power: 5 paise per unit, meter rent: 30 paise per meter per month.

Tariff B : 12 per cent on the rateable value of the customer's premises plus 3 paise per unit for all purposes.

If the annual rateable value of the customer's premises is Rs. 2,500 and his normal consumption for lighting per month is 40 units, determine what amount of domestic power consumption will make both the tariffs equally advantageous.

**Solution.** Let  $x$  = minimum number of power units consumed per month.

**Tariff A**

$$\begin{aligned} \text{Total cost per month} &= \text{meter rent} + \text{lighting charges} + \text{power charges} \\ &= (2 \times 30) + (40 \times 20) + 5x = 860 + 5x \text{ paise} \end{aligned}$$

**Tariff B**

$$\begin{aligned} \text{Total cost per month} &= 12\% \text{ of } 2500 + \text{energy charges for all purposes} \\ &= 0.12 \times 2500 \times 100/12 + 3(40 + x) \end{aligned}$$

$$\therefore 860 + 5x = 2500 + 120 + 3x \quad \therefore 2x = 1760 \quad \therefore x = \mathbf{880 \text{ units}}$$

**Example 50.75.** Transformers and low-tension motors of a certain size can be purchased at Rs. 12 per kVA of full output and Rs. 24 per kW output respectively. If their respective efficiencies are 98% and 90%, what price per kW output could be paid for high-tension motors of the same size but of average efficiency only 89%? Assume an annual load factor of 30%, the cost of energy per unit as 7 paise and interest and depreciation at the rate of 8% for low-tension motors.

**Solution.** Let  $S$  = the price of h.t. motors per output kW in rupees.

**Low-tension Motors with Transformers**

(a) Interest and depreciation on the motor input kW = Rs.  $\frac{24 \times 0.08}{0.9} =$  Rs. 2.13

(b) Let  $\infty$  a p.f. or the motors and calculate their standing charges on the basis of their input expressed in kW.

∴ interest and depreciation on transformers per input kW = Rs.  $\frac{12 \times 0.9}{0.99} \times 0.08 = \text{Rs. } 0.88$

$$(c) \quad \text{Running charges} = \frac{(8,760 \times 0.3) \times 7}{(0.98 \times 0.9) \times 100} = \text{Rs. } 208.6$$

Hence, total cost of transformers and low-tension motors per input kW  
 $= 2.13 + 0.88 + 208.6 = \text{Rs. } 211.6$

**High Tension Motors**

$$\text{Standing charges/output kW} = \text{Rs. } \frac{S}{89} \times .12 = \text{Rs. } \frac{12S}{89}$$

$$\text{Running cost} = \frac{(1 \times 8,760 \times 0.3) \times 7}{0.89 \times 100} = \text{Rs. } 206.7$$

$$\text{Total cost of h.t. motors} = (12 S/89) + 206.7.$$

$$\therefore 12 S/89 + 206.7 = 211.6 \quad \therefore S = \text{Rs. } 36.34 \text{ kW output.}$$

**Example 50.76.** An industrial load can be supplied on the following alternative tariffs (a) high-voltage supply at Rs. 45 per kVA per annum plus 1.5 paise per kWh or (b) low-voltage supply at Rs. 50 per annum plus 1.8 paise per kWh. Transformers and switchgear etc. for the H.V. supply cost Rs. 35 per kVA, the full-load transformer losses being 2%. The fixed charges on the capital cost of the high-voltage plant are 25% and the installation works at full-load. If there are 50 working weeks in a year, find the number of working hours per week above which the H.V. supply is cheaper.

**Solution.** Let  $x$  be the number of working hours per week above which H.V. supply is cheaper than the L.V. supply. Also, suppose that the load = 100 kW.

Rating of transformers and switchgear =  $100/0.98 = 10,000/98$  kW, because transformer losses are 2%.

$$\text{Cost of switchgear and transformer} = (10,000/98) \times 35 = \text{Rs. } 3,571.5$$

$$\text{Annual fixed charges} = 3,571.5 \times 0.25 = \text{Rs. } 892.9$$

$$\text{Annual energy consumption} = 100 \times x \times 50 = 5,000 x \text{ kWh}$$

On the H.V., the total kWh which will have to be paid for is  $5,000 x/0.98$ .

**(a) H.V. Supply**

$$\begin{aligned} \text{Total annual cost} &= 45 \times \text{kVA} + \text{energy charges} + \text{charge on H.V. plant} \\ &= 45 \times \frac{100}{0.98} + \frac{1.5 \times 5,000 x}{100 \times 0.98} + \text{Rs. } 892.9 \\ &= \text{Rs. } 4,592 + \text{Rs. } 76.54 x + \text{Rs. } 892.9 = \text{Rs. } (5,485 + 76.54 x) \end{aligned}$$

**(b) L.V. Supply**

$$\begin{aligned} \text{Total annual cost} &= \text{Rs. } 50 \times \text{kVA} + \text{energy charges} = 50 \times 100 + \frac{1.8 \times 5,000 x}{100} \\ &= \text{Rs. } (5,000 + 90 x) \end{aligned}$$

If the two annual costs are to be equal, then

$$5,485 + 76.54 x = 5,000 + 90 x$$

$$\therefore 13.46 x = 485 \quad \text{or} \quad x = 36 \text{ hr.}$$

Hence, above 36 hours per week, the H.V. supply would be cheaper.

**Example 50.77.** For a particular drive in a factory requiring 10 h.p. (7.46 kW) motors, following tenders have been received. Which one will you select ?

	cost	efficiency
Motor X	Rs. 1,150	86%
Motor Y	Rs. 1,000	85%

Electrical tariff is Rs. 50 per kW + 5 paise per kWh. Assume interest and depreciation as 10% and that the factory works for 10 hours a day for 300 days a year.

(Electrical Engineering, Bombay Univ.)

**Solution. (i) Motor X**

F.L. power input	= 7.46/0.86	= 8.674 kW
Units consumed/year	= 8760 × 8.674	= 75,980 kWh
kW charges	= Rs. 50 × 8.674	= Rs. 433.7
kWh charges	= Rs. 5 × 75,980/100	= Rs. 3,799
Fixed charges	= Rs. 0.1 × 1150	= Rs. 115
Total annual charges	= (3,799 + 433.7 + 115)	= <b>Rs. 4,347.7</b>

**(ii) Motor Y**

F.L. power input	= 7.46/0.85	= 8.776 kW
units consumed/year	= 8760 × 8.766	= 76,880 kWh
kW charges	= Rs. 50 × 8.766	= Rs. 438.8
kWh charges	= Rs. 5 × 76,880/100	= Rs. 3,844
Fixed charges	= Rs. 0.1 × 1000	= Rs. 100
Total annual charges	= (438.8 + 3,844 + 100)	= Rs. 4,382.8
Obviously, motor X is cheaper by Rs. (4,382.8 × 4,347.7)		= <b>Rs. 35.1</b>

**Example 50.78.** A 200-h p. (149.2 kW) motor is required to operate at full-load for 1500 hr, at half-load for 3000 hr per year and to be shut down for the remainder of the time. Two motors are available.

Motor A : efficiency at full load = 90% ; at half-load = 88%

Motor B : efficiency at full load = 90% ; at half-load = 89%

The unit of energy is 5 paise/kWh and interest and depreciation may be taken as 12 per cent per year. If motor A cost Rs. 9,000 ; what is the maximum price which could economically be paid for motor B ?

(Electrical Power-II, Bombay Univ.)

**Solution. (a) Motor A**

F.L. power input	= 149.2/0.9	= 165.8 kW
Half-load power input	= (149.2/2)/0.88	= 84.7 kW
Total energy consumed per year	= (1500 × 165.8) + (3000 × 84.7)	= 502,800 kWh
Cost of energy	= Rs. 5 × 502,800/100	= Rs. 25,140
Interest and depreciation on motor	= Rs. 0.12 × 9000	= Rs. 1,080
Total charge	= Rs. 25,140 + Rs. 1,080	= <b>Rs. 26,220</b>

**(b) Motor B**

F.L. input = 149.2/0.9 = 165.8 kW	Half-load input = (149.2/2)/0.89 = 83.8 kW
Total energy consumed per year	= (1500 × 165.8) + (3000 × 83.8) = 497,100 kWh
Cost of energy consumed	= Rs. 5 × 497,100/100 = Rs. 24,855

Let  $x$  be the maximum price which could be paid for motor  $B$ .

$$\text{Annual interest and depreciation} = \text{Rs. } 0.12x$$

$$\text{annual charges} = \text{Rs. } (24,855 + 0.12x)$$

$$\text{For the two charges to be equal} = 24,855 + 0.12x = 26,220 ; x = \text{Rs. } 11,375$$

**Example 50.79.** Two tenders  $A$  and  $B$  for a 1000-kVA, 0.8 power factor transformer are :  $A$ , full-load efficiency = 98.5% and iron loss = 6 kW at rated voltage ;  $B$ , 98.8% and iron loss 4 kW but costs Rs. 1,500 more than  $A$ . The load cycle is 2000 hours per annum at full-load, 600 hours at half-load and 400 hours at 25 kVA. Annual charges for interest and depreciation are 12.5% of capital cost and energy costs 3 paise per kWh. Which tender is better and what would be the annual saving.

**Solution. Tender A** Transformer full-load output =  $1000 \times 0.8 = 800$  kW

$$\text{Efficiency} = 98.5\% \qquad \text{Input} = 800/0.985$$

$$\text{Total losses} = (800/0.985) - 800 = 12.2 \text{ kW} ; \text{F.L. Cu losses} = 12.2 - 6 = 6.2 \text{ kW}$$

Total losses per year for a running period of 3000 hr. are—

$$(i) \text{ Iron loss} = 3,000 \times 6 = 18,000 \text{ kWh}$$

$$(ii) \text{ F.L. Cu losses for 2000 hours} = 2,000 \times 6.2 = 12,400 \text{ kWh}$$

$$(iii) \text{ Cu loss at half-load for 600 hours} = \frac{1}{4} \times 6.2 \times 600 = 930 \text{ kWh}$$

$$(iv) \text{ Cu loss at 25 kVA load for 400 hours} = \left(\frac{25}{1000}\right)^2 \times 6.2 \times 400 = 1.5 \text{ kWh}$$

$$\text{Total energy loss per year} = 18,000 + 12,400 + 930 + 1.5 = 31,332 \text{ kWh}$$

$$\text{Cost} = \text{Rs. } 31,332 \times 3/100 = \text{Rs. } 939.96$$

**Tender B.**

$$\text{Total loss of transformer} = (800/0.988) - 800 = 9.7 \text{ kW}$$

$$\text{Iron loss} = 4 \text{ kW} ; \text{Full-load Cu loss} = 9.7 - 4 = 5.7 \text{ kW}$$

Total losses for 3,000 hours per year are as follows :

$$(i) \text{ Iron loss} = 3,000 \times 4 = 12,000 \text{ kWh}$$

$$(ii) \text{ F.L. Cu loss for 2000 hours} = 5.7 \times 2000 = 11,400 \text{ kWh}$$

$$(iii) \text{ Cu loss at half-load for 600 hours} = (5.7/4) \times 600 = 855 \text{ kWh}$$

$$(iv) \text{ Cu loss at 25 kVA for 400 hours} = (25/1000)^2 \times 5.7 \times 400 = 1.4 \text{ kWh}$$

$$\text{Total yearly loss} = 12,000 + 11,400 + 855 + 1.4 = 24,256 \text{ kWh}$$

$$\text{Cost} = \text{Rs. } 24,256 \times 3/100 = \text{Rs. } 727.68 ; \text{Extra cost of } B = \text{Rs. } 1500$$

$$\text{Annual interest and depreciation etc.} = 12.5\% \text{ or } 1500 = \text{Rs. } 187.50$$

$$\therefore \text{ yearly cost of } B = \text{Rs. } 727.68 + \text{Rs. } 187.50 = \text{Rs. } 915.18$$

Obviously, tender  $B$  is cheaper by Rs.  $(939.96 - 915.18) = \text{Rs. } 24.78$  annually.

**Example 50.80.** Transformer  $A$  has iron loss of 150 kWh and load loss of 140 kWh daily while the corresponding losses of transformer  $B$  are 75 kWh and 235 kWh. If annual charges are 12.5% of the capital costs and energy costs 5 paise per kWh, what should be the difference in the cost of the two transformers so as to make them equally economical ?

**Solution. Transformer A**

$$\text{Yearly loss} = 365 \times (150 + 140) = 365 \times 290 \text{ kWh}$$

**Transformer B**

$$\text{Yearly loss} = 365 \times (75 + 235) = 365 \times 310 \text{ kWh}$$

Difference in yearly loss = 365 (310 – 290) = 7,300 kWh  
 Value of this loss = Rs. 7,300 × 5/100 = Rs. 365

Hence, transformer B has greater losses per year and so is costlier by Rs. 365. If it is to be equally advantageous, then let it cost Rs.  $x$  less than A. Annual interest and depreciation on it is Rs.  $0.125x$ .

∴  $0.125x = 365$   
 or  $x = \text{Rs. } 2,920$

Hence, transformer B should cost Rs. 2,920 less than A.

**Example 50.81.** Quotations received from three sources for transformers are :

	Price	No-load loss	Full-load loss
A	Rs. 41,000	16 kW	50 kW
B	Rs. 45,000	14 kW	45 kW
C	Rs. 38,000	19 kW	60 kW

If the transformers are kept energized for the whole of day (24 hours), but will be on load for 12 hours per day, the remaining period on no-load, the electricity cost being 5 paise per kWh and fixed charges Rs. 125 per kW of loss per annum and if depreciation is 10% of the initial cost, which of the transformers would be most economical to purchase ?

**Solution. Transformer A**

F.L. loss = 50 kW  
 No-load losses = 16 kW (this represents iron loss)  
 ∴ F.L. Cu loss = 50 – 16 = 34 kW  
 F.L. Cu loss for 12 hours\* = 12 × 34 = 408 kWh  
 Iron losses for 24 hours = 24 × 16 = 384 kWh  
 Total loss per day = 408 + 384 = 792 kWh  
 Annual loss = 792 × 365 kWh  
 Cost of this loss = Rs. 792 × 365 × 5/100 = Rs. 14,454  
 Annual fixed charges = Rs. 125 × 50 = Rs. 6,250  
 Annual depreciation = Rs. 0.1 × 41,000 = Rs. 4,100  
 Total annual charges for transformer  
 A = Rs. 14,454 + Rs. 6,250 + Rs. 4,100 = Rs. 24,804

**Transformer B**

F.L. loss = 45 kW ; Iron or no-load loss = 14 kW  
 F.L. Cu loss = 45 – 14 = 31 kW; Iron loss/day = 24 × 14 = 336 kWh  
 F.L. Cu losses for 12 hours\* = 12 × 31 = 372 kWh  
 Total loss/day = 708 kWh ; Annual loss = 708 × 365 kWh  
 Cost of this loss = 708 × 365 × 5/100 = Rs. 12,921  
 Fixed charges = Rs. 125 × 45 = Rs. 5,625  
 Annual depreciation = Rs. 0.1 × 45,000 = Rs. 4,500  
 Total annual charges for transformer B  
 = Rs. 12,921 + Rs. 5,625 + Rs. 4,500 = Rs. 23,046

**Transformer C**

F.L. loss = 60 kW ; Iron or N.L. loss = 19 kW  
 F.L. Cu loss = 60 – 19 = 41 kW

\* Cu loss is proportional to (kVA)<sup>2</sup>



Iron loss for 24 hr.	= $24 \times 19 = 456$ kWh
F.L. Cu loss for 12 hr	= $12 \times 41 = 492$ kWh
Total losses per day	= 948 kWh ; Annual losses = $948 \times 365$ kWh
Cost of these losses	= Rs. $948 \times 365 \times 5/100 =$ Rs. 17,301
Fixed charges	= Rs. $125 \times 60 =$ Rs. 7,500 ;
Annual depreciation	= Rs. $0.1 \times 38,000 =$ Rs. 3,800
Total annual charges on transformer C	= Rs. 17,301 + Rs. 7,500 + Rs. 3,800 = Rs. 28,601

Obviously, transformer B is cheaper and should be purchased.

**Example. 50.82.** A 37.3 kW induction motor has power factor 0.9 and efficiency 0.9 at full-load, power factor 0.6 and efficiency 0.7 at half-load. At no-load, the current is 25% of the full-load current and power factor 0.1. Capacitors are supplied to make the line power factor 0.8 at half-load. With these capacitors in circuit, find the line power factor at (i) full-load and (ii) no-load.

(Utilisation of Elect. Power, AMIE)

**Solution.** Full-load motor input,  $P_1 = 37.3/0.9 = 40.86$  kW

Lagging kVAR drawn by the motor at full-load,  $kVAR_1 = P_1 \tan \phi_1 = 40.86 \tan (\cos^{-1} 0.9) = 19.79$ .

Half-load motor input,  $P_2 = (0.5 \times 37.3)/0.7 = 26.27$  kW

Lagging kVAR drawn by the motor at half-load,

$$kVAR_2 = P_2 \tan \phi_2 = 26.27 \tan (\cos^{-1} 0.6) = 35.02$$

Full-load current,  $I_1 = 37.3 \times 10^3 / \sqrt{3} V_L \times 0.9 \times 0.9 = 26,212 / V_L$

Current at no-load,  $I_0 = 0.25 I_1 = 0.25 \times 26,212 / V_L = 6553 / V_L$

Motor input at no-load,  $P_0 = \sqrt{3} V_L I_0 \cos \phi_0 = \sqrt{3} \times 6553 \times V_L \times 0.1 / V_L = 1135$  W = 1.135 kW

Lagging kVAR drawn by the motor at no-load,  $kVAR_0 = 1.135 \tan (\cos^{-1} 0.1) = 11.293$

Lagging kVAR drawn from the mains at half-load with capacitors,

$$kVAR_{2C} = 26.27 \tan (\cos^{-1} 0.8) = 19.7$$

kVAR supplied by capacitors,  $kVAR_C = kVAR_2 - kVAR_{2C} = 35.02 - 19.7 = 15.32$

kVAR drawn from the main at full-load with capacitors  $kVAR_{1C} = kVAR_K - kVAR_C = 19.79 - 15.32 = 4.47$ .

(i) Line power factor at full-load =  $\cos (\tan^{-1} kVAR_{1C} / P_1) = \cos (\tan^{-1} 4.47 / 40.86) = \mathbf{0.994}$  lagging

(ii) kVAR drawn from mains at no-load with capacitors =  $11.293 - 15.32 = -4.027$

Line power factor at no-load =  $\cos (\tan^{-1} -4.027 / 1.135) = \cos (-74.26^\circ) = \mathbf{0.271}$  leading.

### Tutorial Problem No. 50.3

- A power plant is working at its maximum kVA capacity with a lagging p.f. of 0.7. It is now required to increase its kW capacity to meet the demand of additional load. This can be done by raising the p.f. to 0.85 by correction apparatus or by installing extra generating plant which costs Rs. 800 per kVA. Find the minimum cost per kVA of p.f. apparatus to make its use more economical than the additional generating plant. **[Rs. 502 per kVAR]**
- An industrial load of 4 MW is supplied at 11 kV, the p.f. being 0.8 lag. A synchronous motor is required to meet additional load of 1500 h.p. (1,119 kW) and at the same time to raise the resultant p.f. to 0.95 lag. Find the kVA capacity of the motor and the p.f. at which it must operate. The efficiency of the motor is 80%. **[1875 kVA ; 0.7466 (lead)]**

3. A factory takes a load of 300 kW at 0.8 p.f. lag for 4800 hours per annum and buys energy at a tariff of Rs. 100 per kVA plus 3 p/kWh consumed. If p.f. is improved to 0.9 (lag) by means of capacitors costing Rs. 200 per kVA and having a power loss of 50 W per kVA, calculate the annual saving affected by their use. Allow 10% per annum for interest and depreciation of the capacitors.  
[Rs. 693 per year]
4. The following two tenders are received in connection with the purchase of 1000-kVA transformer. Transformer A : full-load efficiency = 98% ; core loss = 8 kW  
Transformer B : full-load efficiency = 91.5% ; core loss = 6 kW  
The service needed per year is 200 hours as 1000 kVA, 1000 hours at 500 kVA and 1000 hours at 200 kVA. If the transformer A is costing Rs. 25,000, estimate the maximum price that could be paid for transformer B. Take interest and depreciation at 10 per cent per annum and cost of energy 5 paise per kWh. Assume working at unity power factor. [Max. Price for B = Rs. 21,425]
5. Tenders A and B for a 1000-kVA, 0.8 p.f. transformer are :  
A : full-load  $\eta$  = 98.3% ; core loss = 7 kW at rated voltage  
B : full-load  $\eta$  = 98.7% ; core loss = 4 kW but costs Rs. 5000 more than A.  
The service needed is 1800 hours per annum at 1000 kVA, 600 hours at 600 kVA, 400 hours at 25 kVA and remaining period shut down. Take the annual charges for interest and depreciation at 12.5 per cent of capital cost and energy costs at 8 paise/kWh ; which is the better tender and what will be annual saving ? [Tender B is cheaper by Rs. 211]
6. A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three D-connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor.  
[11.45 kVA ; 2.7  $\mu$ F] (I.E.E. London)
7. A generator station supplies power to the following : lighting 100 kW; an induction motor 400 h.p. (298.4 kW) power factor 0.8, efficiency 0.92, a rotary converter giving 100 A at 800 V at an efficiency of 0.94. What must be the p.f. of the rotary converter in order that the p.f. of the supply station may be unity. [0.214 leading] (C. & G. London)
8. A substation which is fed by a single feeder cable supplies the following loads : 1,000 kW at p.f. 0.85 lagging, 1,500 kW at 0.8 lagging, 2,000 kW at 0.75 lagging ; 500 kW at 0.9 leading. Find the p.f. of the supply to the substation and the load the feeder cable could carry at unity p.f. with the same cable heating. [0.84 ; 5,960 kW] (I.E.E. London)
9. A 3-phase synchronous motor is connected in parallel with a load of 200 kW at 0.8 p.f. (lagging) and its excitation is adjusted until it raises the total p.f. to 0.9 (lagging). If the mechanical load on the motor, including losses, takes 50 kW, calculate the kVA input to the motor.  
[57.8 kVA] (I.E.E. London)
10. A 3-phase, 3,000-V, 50-Hz motor develops 375 h.p. (223.8 kW), the p.f. being 0.75 lagging and the efficiency 92%. A bank of capacitors is star-connected in parallel with the motor and the total p.f. raised to 0.9 lagging. Each phase of the capacitor bank is made up of 3 capacitors joined in series. Determine the capacitance of each. [118  $\mu$ F] (London Univ.)
11. For increasing the kW capacity of a power plant working at 0.7 lagging power factor, the necessary increase in power can be obtained by raising the power factor to 0.9 or by installing additional plant. What is the maximum cost per kVA of power factor correction apparatus to make its use more economical than additional plant at Rs. 800 per kVA?  
[Rs. 525] (Utili. of Elect. Power, AMIE)
12. A 340-kW, 3300-V, 50-Hz, 3 $\phi$  star-connected induction motor has full-load efficiency of 85% and power factor of 0.8 lagging. It is desired to improve power factor to 0.96 lagging by using a bank of three capacitors. Calculate

- (i) the kVA rating of the capacitor bank,
- (ii) the capacitance of each limb of the capacitor bank connected in delta,
- (iii) the capacitance of each capacitor, if each one of the limbs of the delta-connected capacitor bank is formed by using 6 similar 3300-V capacitors.

**[(i) 183.33 kVAR (ii) 17,86 μF (iii) 2.977 μF] (Util. of Elect. Power, AMIE)**

- 13.** A system is working at its maximum kVA capacity with a lagging power factor of 0.71. An anticipated increase of load could be met by

- (i) raising the power factor of the system to 0.87 by the installation of phase advancers,
- (ii) installing extra generating plant, cables etc. to meet the increased power demand.

Estimate the limiting cost per kVA of phase advancing plant which would justify its use if the cost for generating plant is Rs. 60 per kVA. Interest and depreciation charges may be assumed to be 1% in each case.

**[Rs. 36.5] (Util. of Elect. Power, AMIE)**

- 14.** A consumer requires an induction motor of 36.775 kW. He is offered two motors of the following specifications :

Motor A : efficiency 88% and p.f. = 0.9 Motor B : efficiency 90% and p.f. = 0.81

The consumer is being charged on a two-part tariff of Rs. 70 per kVA of the maximum demand plus 5 paise per unit. The motor (B) power factor is raised to 0.89 by installing capacitors. The motor B costs Rs. 150 less than A.

The cost of capacitor is Rs.60 per kVAR. Determine which motor is more economical and by how much. Assume rate of interest and depreciation as 10% and working hours of motors as 2400 hours in a year.

**[Motor B, Rs. 111.10] (Power Systems-II, AMIE)**

- 15.** A 250-V, 7.46 kW motor is to be selected for a workshop from two motors A and B. The cost of each motor is same but the losses at full load are different as given below :

	Motor A	Motor B
Stray loss	1000 W	900 W
Shunt field loss	250 W	200 W
Armature copper loss	300 W	450 W

The motor has to work on full-load for 8 hours, half-load for 4 hours and quarter load for 4 hours each day. Which motor should be selected ?

**[Motor B] (Util. of Elect. Power AMIE)**

- 16.** What is load duration curve? **(Anna University, April 2002)**
- 17.** How will you classify loads? **(Anna University, April 2002)**
- 18.** Why should there be diversity? **(Anna University, April 2002)**
- 19.** Define load factor. **(Anna University, April 2002)**
- 20.** What is M.D.? **(Anna University, April 2002)**
- 21.** Distinguish between a base load plant and a peak load plant. **(Anna University, April 2002)**

### OBJECTIVE TESTS – 50

- 1.** While calculating the cost of electric power generation, which of the following is NOT considered a fixed cost ?
  - (a) interest on capital investment
  - (b) taxes and insurance
  - (c) most of the salaries and wages
  - (d) repair and maintenance.
- 2.** Maximum demand of an installation is given by its
  - (a) instantaneous maximum demand
  - (b) greatest average power demand
  - (c) average maximum demand over a definite interval of time during a certain period.
  - (d) average power demand during an interval of 1-minute.
- 3.** A diversity factor of 2.5 gives a saving of ..... percent in the generating equipment.
  - (a) 60
  - (b) 50
  - (c) 40
  - (d) 25.

4. Mark the WRONG statement.  
High load factor of a generating equipment
  - (a) leads to lesser charges per kWh
  - (b) implies lower diversity in demand
  - (c) gives more profit to the owner
  - (d) can be obtained by accepting off-peak loads.
5. In a generating station, fixed, charges at 100% load factor are 6 paise/kWh. With 25% load factor, the charges would become ..... paise/kWh.
  - (a) 1.5
  - (b) 10
  - (c) 24
  - (d) 3
6. When considering the economics of power transmission, Kelvin's law is used for finding the
  - (a) cost of energy loss in bare conductors
  - (b) most economical cross-section of the conductors
  - (c) interest on capital cost of the conductor
  - (d) the maximum voltage drop in feeders.
7. A 3-phase balanced system working at 0.9 lagging power factor has a line loss of 3600 kW. If p.f. is reduced to 0.6, the line loss would become ..... kW.
  - (a) 8100
  - (b) 1600
  - (c) 5400
  - (d) 2400.
8. Mark the WRONG statement.  
While considering power factor improvement, the most economical angle of lag depends on the
  - (a) cost/kVA rating of phase advancer
  - (b) rate of interest on capital outlay
  - (c) rate of depreciation
  - (d) value of original lagging p.f. angle.
9. Load factor of a power station is defined as
  - (a) maximum demand / average load
  - (b) average load  $\times$  maximum demand
  - (c) average load / maximum demand
  - (d)  $(\text{average load} \times \text{maximum demand})^{1/2}$
10. Load factor of a power station is generally
  - (a) equal to unity
  - (b) less than unity
  - (c) more than unity
  - (d) equal to zero
11. Diversity factor is always
  - (a) equal to unity
  - (b) less than unity
  - (c) more than unity
  - (e) more than twenty
12. Load factor for heavy industries may be taken as
  - (a) 10 to 20%
  - (b) 25 to 40%
  - (c) 50 to 70%
  - (d) 70 to 80%
13. The load factor of domestic load is usually
  - (a) 10 to 15%
  - (b) 30 to 40%
  - (c) 50 to 60%
  - (d) 60 to 70%
14. Annual depreciation cost is calculated by
  - (a) sinking fund method
  - (b) straight line method
  - (c) both (a) and (b)
  - (d) none of the above
15. Depreciation charges are high in case of
  - (a) thermal plant
  - (b) diesel plant
  - (c) hydroelectric plant
16. Demand factor is defined as
  - (a) average load/maximum load
  - (b) maximum demand/connected load
  - (c) connected load/maximum demand
  - (d) average load  $\times$  maximum load
17. High load factor indicates
  - (a) cost of generation per unit power is increased
  - (b) total plant capacity is utilised for most of the time
  - (c) total plant capacity is not properly utilised for most of the time
  - (d) none of the above
18. A load curve indicates
  - (a) average power used during the period
  - (b) average kWh (kW) energy consumption during the period
  - (c) either of the above
  - (d) none of the above
19. Approximate estimation of power demand can be made by
  - (a) load survey method
  - (b) statistical methods
  - (c) mathematical method
  - (d) economic parameters
  - (e) all of the above

20. Annual depreciation as per straight line method, is calculated by  
(a) the capital cost divided by number of year of life  
(b) the capital cost minus the salvage value, is divided by the number of years of life  
(c) increasing a uniform sum of money per annum at stipulated rate of interest  
(d) none of the above
21. A consumer has to pay lesser fixed charges in  
(a) flat rate tariff  
(b) two part tariff  
(c) maximum demand tariff  
(d) any of the above
22. In two part tariff, variation in load factor will affect  
(a) fixed charges  
(b) operating or running charges  
(c) both (a) and (b)  
(d) either (a) & (b)
23. In Hopkinson demand rate or two part tariff the demand rate for fixed charges are  
(a) dependent upon the energy consumed  
(b) dependent upon the maximum demand of the consumer  
(c) both (a) and (b)  
(d) neither (a) and (b)
24. Which plant can never have 100 percent load factor?  
(a) Peak load plant  
(b) Base load plant  
(c) Nuclear power plant  
(d) Hydro electric plant
25. The area under a load curve gives  
(a) average demand  
(b) energy consumed  
(c) maximum demand  
(d) none of the above
26. Different generating stations use following prime movers  
(a) diesel engine  
(b) hydraulic turbine  
(c) gas turbine  
(d) steam turbine  
(e) any of the above
27. Diversity factor has direct effect on the  
(a) fixed cost of unit generated  
(b) running cost of unit generated  
(c) both (a) and (b)  
(d) neither (a) nor (b)
28. Following power plant has instant starting  
(a) nuclear power plant  
(b) hydro power plant  
(c) both (a) and (b)  
(d) none of the above
29. Which of the following generating station has minimum running cost?  
(a) Nuclear  
(b) Hydro  
(c) Thermal  
(d) Diesel
30. Power plant having maximum demand more than the installed rated capacity will have utilisation factor  
(a) equal to unity  
(b) less than unity  
(c) more than unity  
(d) none of the above
31. Load curve is useful in deciding the  
(a) operating schedule of generating units  
(b) sizes of generating units  
(c) total installed capacity of the plant  
(d) all of the above
32. Load curve of a power plant has always  
(a) zero slope  
(b) positive slope  
(c) negative slope  
(d) any combination of (a), (b) and (c)
33. Annual operating expenditure of a power plant consists of  
(a) fixed charges  
(b) semi-fixed charges  
(c) running charges  
(d) all of the above
34. Maximum demand on a power plant is  
(a) the greatest of all "short time interval averaged" demand during a period  
(b) instantaneous maximum value of kVA supplied during a period  
(c) both (a) or (b)  
(d) none of the above
35. Annual instalment towards depreciation reduces as rate of interest increases with  
(a) sinking fund depreciation  
(b) straight line depreciation  
(c) reducing balances depreciation  
(d) none of the above
36. Annual depreciation of the plant is proportional to the earning capacity of the plant vide

- (a) sinking fund depreciation  
(b) straight line depreciation  
(c) reducing balances depreciation  
(d) none of the above
37. For high value of diversity factor, a power station of given installed capacity will be in a position to supply  
(a) less number of consumers  
(b) more number of consumers  
(c) neither (a) nor (b)  
(d) either (a) or (b)
38. Salvage value of the plant is always  
(a) positive  
(b) negative  
(c) zero  
(d) any of the above
39. Load curve helps in deciding  
(a) total installed capacity of the plant  
(b) size of the generating units  
(c) operating schedule of generating units  
(d) all of the above
40. .... can generate power at unpredictable or uncontrolled times.  
(a) Solar power plant  
(b) Tidal power plant  
(c) Wind power plant  
(d) Any of the above
41. Direct conversion of heat into electric power is possible through  
(a) fuel cell  
(b) batteries  
(c) thermionic converter  
(d) all of the above
42. A low utilization factor for a plant indicates that  
(a) plant is used for stand by purpose only  
(b) plant is under maintenance  
(c) plant is used for base load only  
(d) plant is used for peak load as well as base load
43. Which of the following is not a source of power?  
(a) Thermocouple  
(b) Photovoltaic cell  
(c) Solar cell  
(d) Photoelectric cell
44. Which of the following should be used for extinguishing electrical fires?  
(a) Water  
(b) Carbon tetrachloride fire extinguisher  
(c) Foam type fire extinguisher  
(d) CO<sub>2</sub> fire extinguisher
45. Low power factor is usually not due to  
(a) arc lamps  
(b) induction motors  
(c) fluorescent tubes  
(d) incandescent lamp
46. Ships are generally powered by  
(a) unclear power plants  
(b) hydraulic turbines  
(c) diesel engines  
(d) steam accumulators  
(e) none of the above
47. Direct conversion of heat into electrical energy is possible through  
(a) fuel cells  
(b) solar cells  
(c) MHD generators  
(d) none of the above
48. Which of the following place is not associated with nuclear power plants in India?  
(a) Narora  
(b) Tarapur  
(c) Kota  
(d) Benglore
49. During load shedding  
(a) system power factor is changed  
(b) some loads are switched off  
(c) system voltage is reduced  
(d) system frequency is reduced
50. Efficiency is the secondary consideration in which of the following plants?  
(a) Base load plants  
(b) Peak load plants  
(c) Both (a) and (b)  
(d) none of the above
51. Air will not be the working substance in which of the following?  
(a) Closed cycle gas turbine  
(b) Open cycle gas turbine  
(c) Diesel engine  
(d) Petrol engine
52. A nuclear power plant is invariably used as a  
(a) peak load plant  
(b) base load plant  
(c) stand-by plant  
(d) spinning reserve plant  
(e) any of the above
53. ....power plant is expected to have the longest life.

- (a) Steam  
(b) Diesel  
(c) Hydroelectric  
(d) Any of the above
54. .... power plant cannot have single unit of 100 MW.  
(a) Hydroelectric  
(b) Nuclear  
(c) Steam  
(d) Diesel  
(e) Any of the above
55. Which of the following, in a thermal power plant, is not a fixed cost?  
(a) Fuel cost  
(b) Interest on capital  
(c) Depreciation  
(d) Insurance charges
56. .... will offer the least load.  
(a) Vacuum cleaner  
(b) Television  
(c) Hair dryer  
(d) Electric shaver
57. In ..... fuel transportation cost is least.  
(a) nuclear power plants  
(b) diesel generating plants  
(c) steam power stations
58. Which of the following equipment provides fluctuating load?  
(a) Exhaust fan  
(b) Lathe machine  
(c) Welding transformer  
(d) All of the above
59. The increased load during summer months is due to  
(a) increased business activity  
(b) increased water supply  
(c) increased use of fans and air conditioners  
(d) none of the above
60. .... is the reserved generating capacity available for service under emergency conditions which is not kept in operation but in working order.  
(a) Hot reserve  
(b) Cold reserve  
(c) Spinning reserve  
(d) Firm power
61. Generating capacity connected to the bus bars and ready to take load when switched on is known as .....  
(a) firm power  
(b) cold reserve  
(c) hot reserve  
(d) spinning reserve
62. .... offers the highest electric load.  
(a) Television set  
(b) Toaster  
(c) Vacuum cleaner  
(d) Washing machine
63. .... industry has the least power consumption per tonne of product.  
(a) Soap  
(b) Sugar  
(c) Vegetable oil  
(d) Caustic soda
64. With reference to a power station which of the following is not a fixed cost?  
(a) Fuel cost  
(b) Interest on capital  
(c) Insurance charges  
(d) Depreciation
65. .... is invariably used as base load plant.  
(a) Diesel engine plant  
(b) Nuclear power plant  
(c) Gas turbine plant  
(d) Pumped storage plant
66. In a power plant if the maximum demand on the plant is equal to the plant capacity, then  
(a) plant reserve capacity will be zero  
(b) diversity factor will be unity  
(c) load factor will be unity  
(d) load factor will be nearly 60%
67. In case of ..... fuel transportation is the major problem.  
(a) diesel power plants  
(b) nuclear power plants  
(c) hydro-electric power plants  
(d) thermal power plants
68. Which of the following power plants need the least period for installation?  
(a) Thermal power plant  
(b) Diesel power plant  
(c) Nuclear power plant  
(d) Hydro-electric power plant
69. For which of the following power plants highly skilled engineers are required for running the plants?  
(a) Nuclear power plants  
(b) Gas turbine power plants  
(c) Solar power plants  
(d) Hydro-electric power plants

70. In which of the following power plants the maintenance cost is usually high?  
(a) Nuclear power plant  
(b) Hydro-electric power plants  
(c) Thermal power plants  
(d) Diesel engine power plants
71. .... is invariably used for peak load  
(a) Nuclear power plant  
(b) Steam turbine plant  
(c) Pumped storage plant  
(d) None of the above
72. Which of the following is not an operating cost?  
(a) Maintenance cost  
(b) Fuel cost  
(c) Salaries of high officials  
(d) Salaries of operating staff
73. Which of the following is the essential requirement of peak load plant?  
(a) It should run at high speed  
(b) It should produce high voltage  
(c) It should be small in size  
(d) It should be capable of starting quickly
74. Large capacity generators are invariably  
(a) water cooled  
(b) natural air cooled  
(c) forced air cooled  
(d) hydrogen cooled
75. By the use of which of the following power factor can be improved?  
(a) Phase advancers  
(b) Synchronous compensators  
(c) Static capacitors  
(d) Any of the above
76. An induction motor has relatively high power factor at  
(a) rated r.p.m.  
(b) no load  
(c) 20 percent load  
(d) near full load  
(e) none of the above
77. Which of the following is the disadvantage due to low power factor?  
(a) Poor voltage regulation  
(b) Increased transmission losses  
(c) High cost of equipment for a given load  
(d) All of the above
78. In a distribution system, in order to improve power factor, the synchronous capacitors are installed  
(a) at the receiving end  
(b) at the sending end  
(c) either (a) or (b)  
(d) none of the above
79. Satic capacitors are rated in terms of  
(a) kW  
(b) kWh  
(c) kVAR  
(d) none of the above
80. Base load plants usually have ..... capital cost, ..... operating cost and ..... load factor.  
(a) high, high, high  
(b) high, low, high  
(c) low, low, low  
(d) low, high, low
81. Which of the following is the disadvantage of a synchronous condenser?  
(a) High maintenance cost  
(b) Continuous losses in motor  
(c) Noise  
(d) All of the above
82. For a consumer the most economical power factor is generally  
(a) 0.5 lagging  
(b) 0.5 leading  
(c) 0.95 lagging  
(d) 0.95 leading
83. A synchronous condenser is virtually which of the following?  
(a) Induction motor  
(b) Underexcited synchronous motor  
(c) Over excited synchronous motor  
(d) D.C. generator  
(e) None of the above
84. For a power plant which of the following constitutes running cost?  
(a) Cost of wages  
(b) Cost of fuel  
(c) Cost of lubricants  
(d) All of the above



85. In an interconnected system, the diversity factor of the whole system
- (a) remains unchanged
  - (b) decreases
  - (c) increases
  - (d) none of the above
86. Generators for peak load plants are usually designed for maximum efficiency at
- (a) 25 to 50 percent full load
  - (b) 50 to 75 percent full load
  - (c) full load
  - (d) 25 percent overload
87. .... will be least affected due to change in supply voltage frequency.
- (a) Electric clock
  - (b) Mixer grinder
  - (c) Ceiling fan
  - (d) Room heater
88. For the same maximum demand, if load factor is decreased, the cost generation will
- (a) remain unchanged
  - (b) decrease
  - (c) increase
89. The connected load of a domestic consumer is around
- (a) 5 kW
  - (b) 40 kW
  - (c) 80 kW
  - (d) 120 kW
90. Which of the following is not necessarily an advantage of interconnecting various power stations?
- (a) Improved frequency of power supplied
  - (b) Reduction in total installed capacity
  - (c) Increased reliability
  - (d) Economy in operation of plants
91. A power transformer is usually rated in
- (a) kW
  - (b) kVAR
  - (c) kWh
  - (d) kVA
92. .... public sector undertaking is associated with erection and sometimes running of thermal power plants
- (a) NTPC
  - (b) SAIL
  - (c) BEL
  - (d) BHEL
93. Most efficient plants are normally used as
- (a) peak load plants
  - (b) base load plants
  - (c) either (a) or (b)
  - (d) none of the above
94. For a diesel generating station the useful life is expected to be around
- (a) 15 to 20 years
  - (b) 20 to 50 years
  - (c) 50 to 75 years
  - (d) 75 to 100 years
95. Which of the following is not a method for estimating depreciation charges?
- (a) Sinking fund method
  - (b) Straight line method
  - (c) Diminishin value method
  - (d) Halsey's 50–50 formula
96. The expected useful life of an hydroelectric power station is around
- (a) 15 years
  - (b) 30 years
  - (c) 60 years
  - (d) 100 years
97. In a load curve the highest point represents
- (a) peak demand
  - (b) average demand
  - (c) diversified demand
  - (d) none of the above
98. Which of the following source of power is least reliable?
- (a) Solar energy
  - (b) Geothermal power
  - (c) Wind power
  - (d) MHD
99. In India production and distribution of electrical energy is confined to
- (a) private sector
  - (b) public sector
  - (c) government sectors
  - (d) joint sector
  - (e) none of the above
100. A pilot exciter is provided on generators for which of the following reasons?
- (a) To excite the poles of main exciter
  - (b) To provide requisite starting torque to main exciter

- (c) To provide requisite starting torque to generator  
 (d) None of the above
- 101.** The primary reason for low power factor in supply system is due to installation of  
 (a) induction motors  
 (b) synchronous motors  
 (c) single phase motors  
 (d) d.c. motors
- 102.** An over excited synchronous motor on no-load is known as  
 (a) synchronous condenser  
 (b) generator  
 (c) induction motor  
 (d) alternator
- 103.** Which of the following is an advantage of static capacitor for power factor improvement?  
 (a) Little maintenance cost  
 (b) Ease in installation  
 (c) Low losses  
 (d) All of the above
- 104.** For any type of consumer the ideal tariff is  
 (a) two part tariff  
 (b) three part tariff  
 (c) block rate tariff  
 (d) any of the above
- 105.** The efficiency of a plant is of least concern when it is selected as  
 (a) peak load plant  
 (b) casual run plant  
 (c) either (a) or (b)  
 (d) base load plant
- 106.** Power generation cost reduces as  
 (a) diversity factor increases and load factor decreases  
 (b) diversity factor decreases and load factor increases  
 (c) both diversity factor as well as load factor decrease  
 (d) both diversity factor as well as load factor increase
- 107.** The depreciation charges in diminishing value method are  
 (a) light in early years  
 (b) heavy in early years  
 (c) heavy in later years  
 (d) same in all years
- 108.** The area under daily load curve divided by 24 hours gives  
 (a) average load  
 (b) least load  
 (c) peak demand  
 (d) total kWh generated
- 109.** Maximum demand tariff is generally not applied to domestic consumers because  
 (a) they consume less power  
 (b) their load factor is low  
 (c) their maximum demand is low  
 (d) none of the above
- 110.** A 130 MW generator is usually ..... cooled  
 (a) air  
 (b) oxygen  
 (c) nitrogen  
 (d) hydrogen
- 111.** For cooling of large size generators hydrogen is used because  
 (a) it is light  
 (b) it offers reduced fire risk  
 (c) it has high thermal conductivity  
 (d) all of the above
- 112.** Major share of power produced in India is through  
 (a) diesel power plants  
 (b) hydroelectric power plants  
 (c) thermal power plants  
 (d) nuclear power plants
- 113.** Which of the following may not be the effect of low plant operating power factor?  
 (a) Improved illumination from lighting  
 (b) Reduced voltage level  
 (c) Over loaded transformers  
 (d) Overloaded cables
- 114.** Which of the following plants is almost inevitably used as base load plant?  
 (a) Diesel engine plant  
 (b) Gas turbine plant  
 (c) Nuclear power plant  
 (d) Pumped storage plant
- 115.** Which of the following component, in a steam power plant, needs maximum maintenance attention?  
 (a) Steam turbine  
 (b) Condenser  
 (c) Water treatment plant  
 (d) Boiler
- 116.** For the same cylinder dimensions and speed, which of the following engine will produce least power?  
 (a) Supercharged engine  
 (b) Diesel engine

- (c) Petrol engine  
(d) All of the above engines will equal power
- 117.** The least share of power is provided in India, by which of the following power plants?  
(a) Diesel power plants  
(b) Thermal power plants  
(c) Hydro-electric power plants  
(d) Nuclear power plants
- 118.** Submarines for under water movement, are powered by which of the following?  
(a) Steam accumulators  
(b) Air motors  
(c) Diesel engines  
(d) Batteries
- 119.** An alternator coupled to a ..... runs at slow speed, as compared to as compared to others.  
(a) diesel engine  
(b) hydraulic turbine  
(c) steam turbine  
(d) gas turbine
- 120.** The effect of electric shock on human body depends on which of the following  
(a) current  
(b) voltage  
(c) duration of contact  
(d) all of the above
- 121.** Which lightning stroke is most dangerous?  
(a) Direct stroke on line conductor  
(b) Indirect stroke on conductor  
(c) Direct stroke on tower top  
(d) Direct stroke on ground wire
- 122.** Which of the following devices may be used to provide protection against lightning over voltages?  
(a) Horn gaps  
(b) Rod gaps  
(c) Surge absorbers  
(d) All of the above
- 123.** When the demand of consumers is not met by a power plant, it will resort to which of the following?  
(a) Load shedding  
(b) Power factor improvement at the generators  
(c) Penalising high load consumers by increasing the charges for electricity  
(d) Efficient plant operation.
- 124.** Load shedding is possible through which of the following?  
(a) Switching of the loads  
(b) Frequency reduction  
(c) Voltage reduction  
(d) Any of the above
- 125.** In power plants insurance cover is provided for which of the following?  
(a) Unskilled workers only  
(b) Skilled workers only  
(c) Equipment only  
(d) All of the above
- 126.** A company can raise funds through  
(a) fixed deposits  
(b) shares  
(c) bonds  
(d) any of the above
- 127.** Which of the following are not repayable after a stipulated period?  
(a) Shares  
(b) Fixed deposits  
(c) Cash certificates  
(d) Bonds
- 128.** The knowledge of diversity factor helps in determining  
(a) plant capacity  
(b) average load  
(c) peak load  
(d) kWh generated  
(e) none of the above
- 129.** Load shedding is done to  
(a) improve power factor  
(b) run the equipment efficiently  
(c) repair the machine  
(d) reduce peak demand
- 130.** when a plant resorts to load shedding it can be concluded that  
(a) peak demand is more than the installed capacity  
(b) daily load factor is unity  
(c) diversity factor is zero  
(d) plant is under repairs
- 131.** Which of the following is the disadvantage of static capacitor for power factor improvement?  
(a) Easily damaged by high voltage  
(b) Cannot be repaired  
(c) Short service life  
(d) All of the above
- 132.** If the tariff for electrical energy charges provides incentive by way of reduced charges for higher consumption, then it can be concluded that  
(a) Load factor is unity

- (b) power is generated through hydroelectric plant  
 (c) plant has sufficient reserve capacity  
 (d) station has more than two generators
- 133.** Anything having some heat value can be used as fuel in case of  
 (a) open cycle gas turbines  
 (b) closed cycle gas turbines  
 (c) petrol engines  
 (d) diesel engines

## ANSWERS

- |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|
| 1. (d)   | 2. (c)   | 3. (a)   | 4. (b)   | 5. (c)   | 6. (b)   | 7. (a)   |
| 8. (d)   | 9. (c)   | 10. (b)  | 11. (c)  | 12. (d)  | 13. (a)  | 14. (c)  |
| 15. (a)  | 16. (b)  | 17. (b)  | 18. (b)  | 19. (e)  | 20. (b)  | 21. (c)  |
| 22. (b)  | 23. (b)  | 24. (a)  | 25. (b)  | 26. (e)  | 27. (a)  | 28. (d)  |
| 29. (b)  | 30. (c)  | 31. (d)  | 32. (d)  | 33. (d)  | 34. (a)  | 35. (a)  |
| 36. (c)  | 37. (b)  | 38. (d)  | 39. (d)  | 40. (d)  | 41. (c)  | 42. (a)  |
| 43. (a)  | 44. (b)  | 45. (d)  | 46. (c)  | 47. (c)  | 48. (d)  | 49. (b)  |
| 50. (b)  | 51. (a)  | 52. (b)  | 53. (c)  | 54. (d)  | 55. (a)  | 56. (d)  |
| 57. (a)  | 58. (c)  | 59. (c)  | 60. (b)  | 61. (d)  | 62. (b)  | 63. (c)  |
| 64. (a)  | 65. (b)  | 66. (a)  | 67. (d)  | 68. (b)  | 69. (a)  | 70. (c)  |
| 71. (c)  | 72. (c)  | 73. (d)  | 74. (d)  | 75. (d)  | 76. (d)  | 77. (d)  |
| 78. (a)  | 79. (c)  | 80. (b)  | 81. (d)  | 82. (c)  | 83. (c)  | 84. (d)  |
| 85. (c)  | 86. (b)  | 87. (f)  | 88. (c)  | 89. (a)  | 90. (a)  | 91. (d)  |
| 92. (a)  | 93. (b)  | 94. (a)  | 95. (d)  | 96. (d)  | 97. (a)  | 98. (c)  |
| 99. (b)  | 100. (a) | 101. (a) | 102. (a) | 103. (d) | 104. (b) | 105. (c) |
| 106. (d) | 107. (b) | 108. (a) | 109. (c) | 110. (d) | 111. (d) | 112. (c) |
| 113. (a) | 114. (c) | 115. (d) | 116. (d) | 117. (a) | 118. (d) | 119. (b) |
| 120. (d) | 121. (a) | 122. (d) | 123. (a) | 124. (d) | 125. (c) | 126. (d) |
| 127. (d) | 128. (a) | 129. (d) | 130. (a) | 131. (d) | 132. (c) | 133. (b) |

# CHAPTER 51

## Learning Objectives

- The Atom
- Bohr's Atomic Model
- Electron Energy Levels in Hydrogen Atom
- Orbital (or Azimuthal) Quantum Number
- Orbital Magnetic Quantum Number ( $m_l$ )
- Magnetic Spin Quantum Number ( $m_s$ )
- Pauli's Exclusion Principle
- Energy Bands in Lithium and Their Occupancy
- Valence and Conduction Bands
- Insulators, Conductors and Semiconductors
- Crystal Structure
- Types of Semiconductors
- Mobile Charge Carriers and Immobile Ions
- Electron Conductivity of a Metal
- Combined Drift and Diffusion Currents
- Relation Between  $D$  and  $\mu$
- Carrier Life Time
- P-N Junction
- Formation of Depletion Layer
- Junction or Barrier Voltage
- Forward Biased P-N Junction
- Reverse Biased P-N Junction

## SEMI- CONDUCTOR PHYSICS



- ↑ A worker at a hydro-electric power station is carrying out routine maintenance on a high-voltage insulator made of a ceramic material.

### 51.1. The Atom

According to the model of atom proposed by Bohr in 1913, an atom is composed of a number of electrons moving in circular or elliptical orbits around a relatively heavy nucleus of protons and neutrons as shown in Fig. 51.1. Although, this simple model of an atom has been replaced by later models, yet it affords a convenient method of understanding the working of semi-conductor devices. Electron has a mass of nearly  $9.1 \times 10^{-31}$  kg and a charge of  $1.6 \times 10^{-19}$  C. The diameter of an atom is approximately  $10^{-10}$  m and that of the nucleus about  $10^{-15}$  m. The number of protons in the atom of an element gives its atomic number while the atomic mass number is determined by the number of protons and neutrons present in the nucleus.

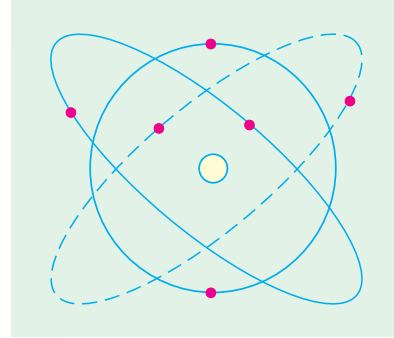


Fig. 51.1

### 51.2. Bohr's Atomic Model

The nuclear atomic model proposed by Rutherford in 1911 was found to suffer from two serious drawbacks concerning distribution of extra-nuclear electrons and stability of the atom as a whole. It was later on superseded by atomic model proposed by Bohr in 1913. Using Planck's Quantum Theory, Bohr made the following postulates :

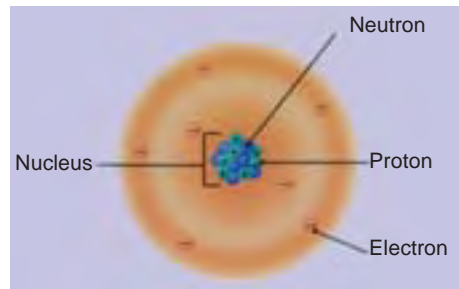
1. The atom has a massive positively-charged nucleus;
2. The electrons revolve round their nucleus in circular orbits, the centrifugal force being balanced by the electrostatic pull between the nucleus and electrons;
3. An electron cannot revolve round the nucleus in any arbitrary orbit but in just certain definite discrete orbits. *Only those orbits are possible (or permitted) for which the orbital angular momentum (i.e. moment of momentum) of the electron is equal to an integral multiple of  $h/2\pi$  i.e. orbital angular momentum =  $nh/2\pi$  where  $n$  is an integer and  $h$  is Planck's constant.* Such orbits are also known as *stationary orbits*;
4. While revolving in these permitted stationary (or stable) orbits, the electron does not radiate out any electromagnetic energy. In other words, *the permissible orbits are non-radiating paths of the electron*;
5. The atom radiates out energy only when an electron *jumps* from one orbit to another. If  $E_2$  and  $E_1$  are the energies corresponding to two orbits before and *after* the jump, the frequency of the emitted photon is given by the relation

$$E_2 - E_1 = hf \quad \text{or} \quad \Delta E = hf$$

where  $f$  is the frequency of the emitted radiations.

**Explanation.** If  $I$  is the moment of inertia of an electron and  $\omega$  its angular velocity, then as per assumption (3) given above  
 $\omega I = nh/2\pi$  or  $(mr^2)\omega = nh/2\pi$  or  $(mr^2)v/r = nh/2\pi$   
 or  $mvr = n.h/2\pi$

Alternatively, since the momentum of the revolving electron is  $mv$ , its moment about the nucleus is  $mvr$  (Fig. 51.2).



Neils Bohr  
(1885–1962)

Hence,

$$mvr = nh/2\pi$$

when  $n = 1, 2, 3$  etc. for the first, second and third orbits respectively. It is called the **principal quantum number** and because it can take whole number values only, **it fixes the size of the allowed orbits** (also called Bohr's circular orbits).

Let the different permitted orbits have energies of  $E_1, E_2, E_3$  etc. as shown in Fig. 51.3 (a). The electron can be raised from  $n = 1$  orbit to any other higher orbit if it is given proper amount of energy.

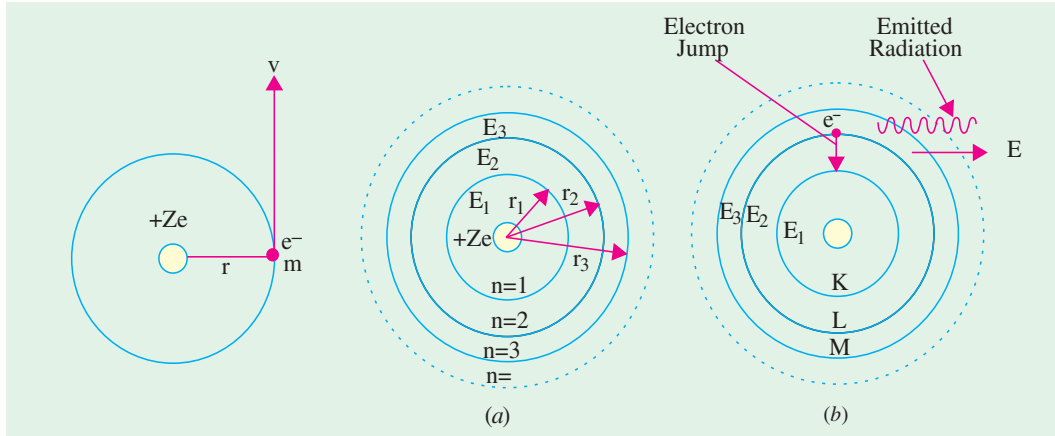


Fig. 51.2

Fig. 51.3

When it drops back to  $n = 1$  orbit after a short interval of time, it gives out the energy difference  $\Delta E$  in the form of a radiation as shown in Fig. 51.3 (b). The relation between the energy released and frequency of the emitted radiation is

$$E_2 - E_1 = hf \quad \text{or} \quad \Delta E = hf$$

### 51.3. Calculations Concerning Bohr's Atomic Model

The above postulates concerning Bohr's atomic model can be utilized to calculate not only the radii of different electron orbits but also the velocity and orbital frequency possessed by different electrons.

Now, the stability of the atom requires that the centrifugal force acting on the revolving electron be balanced by the electrostatic pull exerted by the positively charged nucleus on the electron.

$$\therefore \frac{mv^2}{r} = \frac{Ze \cdot e}{4\pi\epsilon_0 r^2} \quad \text{or} \quad mv^2 r = \frac{Ze^2}{4\pi\epsilon_0}$$

Also, according to Bohr's postulates,  $mvr = nh/2\pi$ .

The above two equations may be used to find the radii of different Bohr's circular orbits.

**(a) Radii of Orbits.** Eliminating  $v$  from the above two equations, we get

$$r = \frac{\epsilon_0 n^2 h^2}{\pi mZe^2} \quad \text{or} \quad r_n = \frac{\epsilon_0 n^2 h^2}{\pi mZe^2}$$

It is seen that the radii of the permitted orbits vary as the square of the principal quantum number  $n$ . Also,  $r_n = n^2 r_1$ .

For hydrogen atom,  $r_1 = 0.53 \times 10^{-10}$  m,  $r_2 = 2^2 r_1 = 2.12 \times 10^{-10}$  m. All values between  $r_1$  and  $r_2$  are forbidden.

**(b) Velocity of Revolving Electrons.** The velocity of a revolving electron as found from the above equations is

$$v = Ze^2/2\epsilon_0 nh = 9 \times 10^9 \cdot 2\pi Ze^2/nh.$$

It shows that velocity is inversely proportional to  $n$  i.e.  $v \propto 1/n$ . Hence, the electron in the innermost orbit has the highest velocity (nearly  $1/37$  of the velocity of light).

(c) **Orbital Frequency.** The orbital *rotational* frequency\* of an electron is

$$f = \frac{v}{2\pi r} = \frac{mZ^2 e^4}{4\epsilon_0^2 n^3 h^3} \cdot \quad \text{It is seen that } f \propto 1/n^3$$

(d) **Electron Energy.** The orbital energy of a revolving electron is of two types :

(i) Kinetic energy due to the motion of the electron. Its value is  $(1/2)mv^2$ .

As seen from above,  $mv^2 = Ze^2/4\pi\epsilon_0 r$ .

Hence, K.E.  $= (1/2)mv^2 = Ze^2/8\pi\epsilon_0 r$ . —as represented by curve 2 in Fig. 51.4.

Eliminating  $r$  from above,

$$\text{K.E.} = mZ^2 e^4 / 8\epsilon_0^2 n^2 h^2.$$

(ii) Potential energy—because the electron lies in the electric field of the positive nucleus.

Now, the potential at a point distant  $r$  from the nucleus is

$$V = Q/4\pi\epsilon_0 r = Ze/4\pi\epsilon_0 r$$

The potential energy of an electron (of charge  $-e$ ) is

$$\text{P.E.} = V \times (-e) = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

or  $= -\frac{mZ^2 e^4}{4\epsilon_0^2 n^2 h^2}$  — curve 1 in Fig. 51.4

$\therefore$  Orbital energy = K.E. + P.E.

$$= \frac{Ze^2}{8\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\epsilon_0 r} \quad \text{— curve 3 in Fig. 51.4}$$

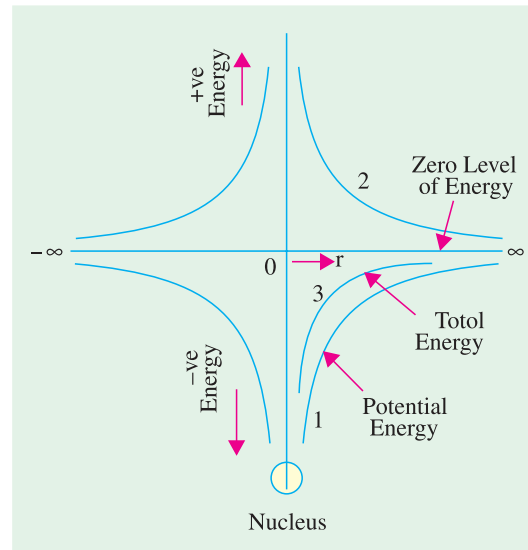


Fig. 51.4

Variations of potential energy and total energy are shown in Fig. 51.4.

If  $r$  is eliminated from the above equation, the orbital energy\*\* is

$$E_n = -mZ^2 e^4 / 8\epsilon_0^2 n^2 h^2$$

where  $n = 1, 2, 3, \dots$  etc. for the energy states that it is possible for the electron to have. It will be seen that  $E_n = E_1/n^2$ .

### 51.4. Normal, Excited and Ionized Atom

Consider the case of the simplest atom i.e. hydrogen atom. When its only electron is in its innermost orbit ( $n = 1$ ), then the atom is said to be in its normal (or unexcited) state. Generally, it is this condition in which most of the free hydrogen atoms in a gas are found to exist at normal room temperature and pressure. However, if spark is passed through hydrogen gas contained in a vessel, then high-speed electrons produced by the spark collide with hydrogen atoms and may either completely remove the  $n = 1$  electron from them or raise it to higher permitted orbits having  $n = 2, 3, 4$  etc.

When the electron is completely removed from the atom, the atom is said to be ionized. If, however, the electron is forced into an outer or higher  $n$ -value orbit, then the atom, is said to be

\* It is the mechanical frequency of rotation and should not be confused with the frequency of emitted radiations.

\*\* The negative sign only indicates that this much energy is required to remove the electron from the atom.



**excited** (or in an excited state). The atom does not remain in the excited state longer than  $10^{-8}$  second because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. In doing so, the electron loses the energy it had earlier gained during collision. However, the electron may return by several jumps, thereby emitting many different radiations of different frequencies.

### 51.5. Electron Energy Levels in Hydrogen Atom

As seen from Art. 51.3, orbital energy of an electron revolving in  $n_{th}$  orbit or shell is

$$E_n = -m Z^2 e^4 / 8\epsilon_0^2 n^2 h^2$$

In the case of hydrogen atom,  $Z = 1$ , hence

$$\begin{aligned} E_n &= -\frac{me^4}{8\epsilon_0^2 n^2 h^2} = -\frac{me^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n^2} = -\frac{21.7 \times 10^{-19}}{n^2} \text{ joules} \\ &= \frac{-21.7 \times 10^{-19}}{1.6 \times 10^{-19}} \cdot \frac{1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}) \end{aligned}$$

This expression gives the total energy of an electron when it occupies any one of the different orbits (or shells) of the hydrogen atom.

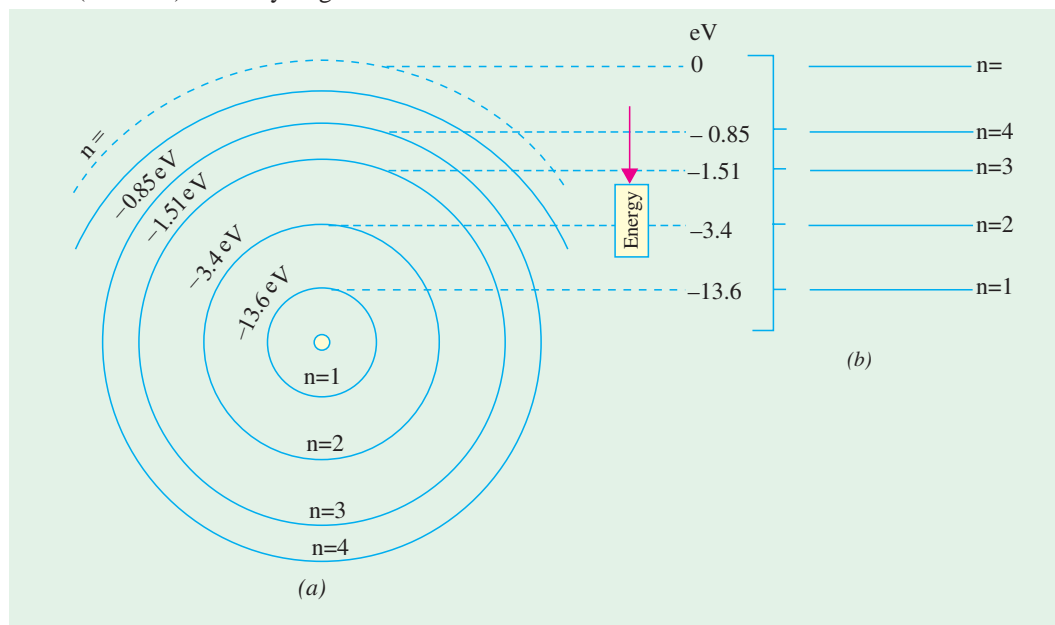


Fig. 51.5

For *K*-shell,  $n = 1$ ,

$$E_1 = -13.6 \text{ eV}$$

For *L*-shell,  $n = 2$ ,

$$E_2 = -13.6/2^2 = -3.4 \text{ eV}$$

For *M*-shell,  $n = 3$ ,

$$E_3 = -13.6/3^2 = -1.51 \text{ eV etc.}$$

Instead of drawing various electron orbits to the scale of their radii as in Fig. 51.5 (a), it is customary to draw horizontal lines to an energy scale as shown in Fig. 51.5 (b) and such a diagram is called energy level diagram (ELD) of an atom. In this array of energies, the higher (*i.e.* less negative) energies are at the top while the lower (*i.e.* more negative) energies are at the bottom. The various electron jumps between allowed orbits now become vertical arrows between energy levels. Greater the length of the arrow, greater is the energy ' $hf$ ' of the radiated photon.

**Example 51.1.** Calculate the value of the kinetic, potential and total energy of an electron revolving in Bohr's first orbit in a hydrogen atom.

**Solution.** (i) 
$$K.E. = \frac{me^4}{8\epsilon_0^2 n^2 h^2} = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.854 \times 10^{-12})^2 \times 1^2 \times (6.625 \times 10^{-34})^2}$$

$$= 21.7 \times 10^{-19} \text{ joules} = \mathbf{13.6 \text{ eV}}$$
(ii) 
$$P.E. = \frac{me^4}{4\epsilon_0^2 n^2 h^2} = -43.4 \times 10^{-19} \text{ joules} = \mathbf{-27.2 \text{ eV.}}$$
(iii) Total energy =  $K.E. + P.E. = 13.6 + (-27.2) = \mathbf{-13.6 \text{ eV}}$

### 51.6. Orbital (or Azimuthal) Quantum Number

According to Bohr's postulates (Art. 51.2), there is only one orbit (and hence one energy level) corresponding to each value of the principal quantum number  $n$ . However, subsequent experimental evidence revealed that all orbits (except  $n = 1$  orbit) consist of more than one orbit called sub-orbits. This group of sub-orbits is collectively known as **shell**. For example,  $n = 1$  shell or *K*-shell consists of only one orbit which may also be called its own sub-orbit or sub-shell. The  $n = 2$  shell or *L*-shell consists of two sub-shells. Similarly,  $n = 3$  shell or *M*-shell consists of three sub-shells. In other words, **the number of sub-shells is equal to the  $n$ -value of the shell**. The  $2n^2$  electrons of the shell now get divided between these sub-shells.

In order to distinguish between different sub-shells belonging to a given shell, a new quantum number called orbital (or azimuthal) quantum number  $l$  has been introduced. This quantum number can have integral values lying between zero and  $(n - 1)$  i.e.  $0 \leq l \leq (n - 1)$ . It is helpful in the following ways :

- (a) It gives the number of sub-shells which are contained in one shell. The **number** of sub-shells is equal to the number of values which  $l$  can have subject to the restriction  $0 \leq l \leq (n - 1)$  as shown in Fig. 51.6 (a).

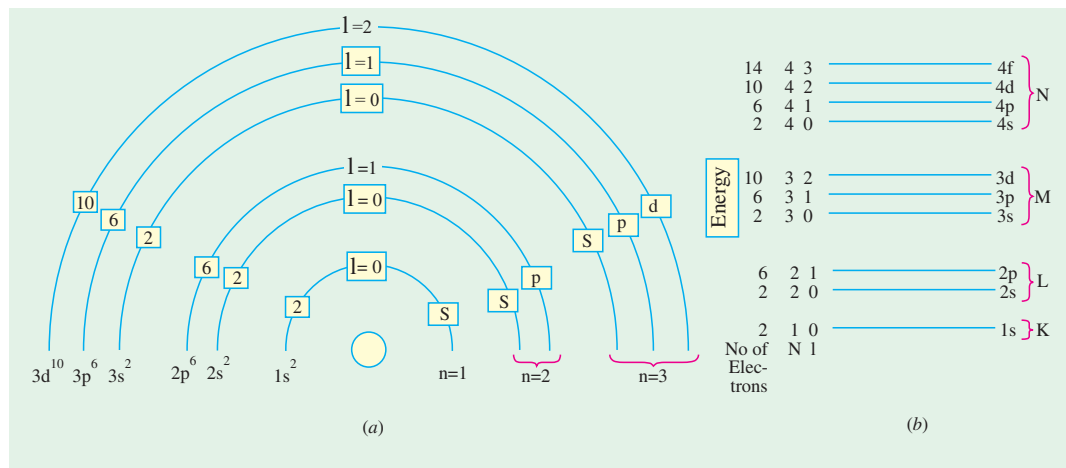


Fig. 51.6

- (b) It helps to distinguish between different sub-shells of a shell by its different values for each sub-shell. Consider the following cases :

- n = 1 shell.** Here,  $l$  can have only one value i.e. zero. Hence, *K*-shell has only one sub-shell with two quantum numbers of  $n = 1$  and  $l = 0$ .

2.  **$n = 2$  shell.** Here,  $l = 0, 1$ . Hence, this shell has two sub-shells\* which are distinguishable from each other by their different quantum numbers of  $n = 2, l = 0$  and  $n = 2, l = 1$ . (Fig. 51.6).
3.  **$n = 3$  shell.** Here  $l = 0, 1, 2$ , showing that  $M$ -shell has **three** sub-shells which differ in their  $l$ -values. The two quantum numbers for the three sub-shells are :  
 $n = 3, l = 0; n = 3, l = 1; n = 3, l = 2$ .
- (c) It helps to determine the **shape** of different sub-shells of a given shell. Some sub-shells are circular whereas others are elliptical in shape. If  $a$  and  $b$  are semi-major and semi-minor axes respectively of an ellipse,  $n$  and  $l$  are its principal quantum number and orbital quantum number respectively, then the relation between them is

$$\frac{b}{a} = \frac{l+1}{n}$$

Consider the following cases :

1. **K-shell or  $n = 1$  shell**

Here,  $l = 0$ , hence  $b/a = (0 + 1)/1$  or  $b/a = 1$  or  $b = a$ . In other words, the first shell (also called sub-shell or  $K$ -shell) around the nucleus of an atom is circular in shape as shown in Fig. 51.7 (a).

2. **L-shell or  $n = 2$  shell**

As stated earlier and shown in Fig. 51.6, it has two more sub-shells.

First sub-shell with  $l = 0$ . Here,  $\frac{b}{a} = \frac{0+1}{2} = \frac{1}{2}$  or  $b = \frac{a}{2}$ . It is elliptical in shape as shown in Fig. 51.7 (b).

Second sub-shell with  $l = 1$ . Here  $\frac{b}{a} = \frac{1+1}{2} = 1$  or  $b = a$ .

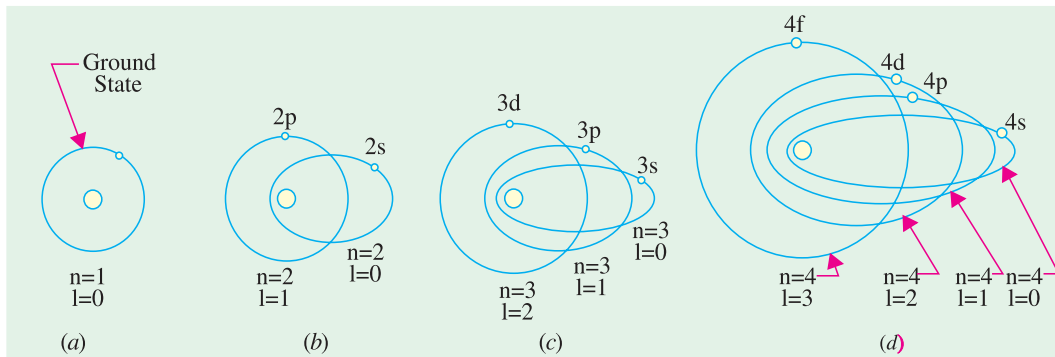


Fig. 51.7

This sub-shell has circular shape as shown in Fig. 51.7 (b).

3. **M-shell or  $n = 3$  shell**

It has three sub-shells with  $l = 0, 1, 2$ .

First sub-shell with  $l = 0$ . Here,  $b/a = (0 + 1)/3 = 1/3$ . It is elliptical in shape.

Second sub-shell with  $l = 1$ . Here,  $b/a = (1 + 1)/3 = 2/3$ . It is also elliptical in shape.

Third sub-shell with  $l = 2$ . Here,  $b/a = (2 + 1)/3 = 1$  or  $b = a$ . Obviously, this sub-shell is circular in shape.

Fig. 51.7 shows to shapes of different sub-shells per values of  $n$  from 1 to 4.

- (d) It determines the distribution of electrons in various sub-shells of a shell. The maximum

\* Different sub-shells have been shown circular for the sake of simplicity although, in practice, most of them are elliptical.

number of electrons which a sub-shell of any  $n$ -value can accommodate is  $= 2(2l + 1)$ . Consider the following :

1. **K-shell.** Here  $l = 0$ , hence it can have  $2(2 \times 0 + 1) = 2$  electrons. They are designated as  $1s^2$  electrons.
2. **L-shell.** First sub-shell with  $l = 0$  can have  $2(2 \times 0 + 1) = 2$  electrons which are designated as  $2s^2$  electrons.  
Second sub-shell with  $l = 1$  can have a maximum of  $2(2 \times 1 + 1) = 6$  electrons which are designated as  $2p^6$  electrons.  
Total number of electrons in the two sub-shells is  $2 + 6 = 8$  which tallies with  $2n^2 = 2 \times 2^2 = 8$ .
3. **M-shell.** First sub-shell with  $l = 0$  has, as explained above, 2 electrons which are designated as  $3s^2$  electrons.  
Second sub-shell  $l = 1$  can have 6 electrons written as  $3p^6$  electrons.  
Third sub-shell with  $l = 2$  can accommodate a maximum of  $2(2 \times 2 + 1) = 10$  electrons which are known as  $3d^{10}$  electrons.  
Again, it will be seen that total number of electrons in the three sub-shells of  $n = 3$  shell is  $= 2 + 6 + 10 = 18$  which equals  $2n^2 = 2 \times 3^2 = 18$ .
4. **N-shell.** First sub-shell with  $l = 0$  can have maximum of two electrons designated as  $4s^2$  electrons. Second sub-shell can contain 6 electrons known as  $4p^6$  electrons. Similarly, third sub-shell can have 10 electrons designated as  $4d^{10}$  electrons. The fourth sub-shell can have a maximum of  $2(2 \times 3 + 1) = 14$  electrons whose designation is  $4f^{14}$  electrons.

The total number of electrons is  $= 2 + 6 + 10 + 14 = 32$  which is in accordance with the number  $2n^2 = 2 \times 4^2 = 32$ .

Incidentally, it should be noted that different  $l$  values of 0, 1, 2, 3, 4..... etc. are identified as  $s, p, d, f, g$ ..... etc.

The different sub-shells and maximum number of electrons they can accommodate are tabulated below.

Shell	K		L			M			N			
$n$	1		2		3			4				
$l$	0	0	1	1	0	1	2	0	1	2	3	
sub-shell	$s$	$s$	$p$	$p$	$s$	$p$	$d$	$s$	$p$	$d$	$f$	
Maximum No. of electrons $= 2(2l + 1)$	2	2	6	6	2	6	10	2	6	10	14	
Total No. of electrons $= 2n^2$	2		8			18			32			

- (e) The orbital quantum number  $l$  also quantizes the orbital angular momentum  $p_l$  associated with each sub-shell

$$p_l = l.h/2\pi \quad \text{where } l = 0, 1, 2, 3, \dots \text{etc.}^*$$

However, quantum mechanical considerations indicate that  $p_l \neq l.h/2\pi$  as stated above but instead  $p_l = l^*.h/2\pi$  where  $l^* = [l(l + 1)]^{1/2}$ .

---

\* According to Bohr's theory,  $p_l = n . h/2\pi$ .

### 51.7. Electron Configuration of Atoms

By electron configuration of an atom is meant the distribution of its electrons in its various sub-shells around the nucleus. Following three rules govern the electron distribution :

1. **Maximum number** of electrons a shell can have is  $= 2n^2$ .
2. In the  $n$ th shell, there are  $n$  sub-shells having different values of  $l$  such as 0, 1, 2.....  $(n - 1)$ .
3. Each sub-shell can accommodate a maximum of  $2(2l + 1)$  electrons.

Consider the following atoms :

- (i) **Sodium atom,  $Z = 11$ .** It has 11 electrons. Hence, its electronic configuration is  $1s^2, 2s^2, 2p^6, 3s^1$ . Obviously, *Na* has a single electron in its outermost sub-shell and hence is said to be monovalent. Same property is possessed by other alkali metals like Li, K, Rb and Cs. They have similar chemical properties and are, therefore, included in the same group in the periodic table.

The electron in the inner sub-shells are very tightly bound to the nucleus and cannot be easily removed. In other words, they have high **binding energy**.

- (ii) **Copper atom,  $Z = 29$ .** It has 29 electrons. Its electronic configuration is  $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^1$ .

### 51.8. Orbital Magnetic Quantum Number ( $m_l$ )

It determines the spatial **orientation** of elliptical electron orbits with respect to an applied magnetic field. There are restrictions on the orientations of the electron orbits because of which they are said to be **space quantized**. Whereas  $l$  determines the orbital angular momentum,  $m_l$  represents the magnitude of the component of angular momentum along the direction of the magnetic field.

Looked from a different angle,  $m_l$  determines the number of sub-subshells in a given shell.

This quantum number can have any one of the  $(2l + 1)$  values ranging from  $+l$  to  $-l$  including zero *i.e.*  $l, (l - 1), (l - 2), \dots, 2, 1, 0, -1, -2, \dots, -(l - 2), -(l - 1), -l$ .

Each sub-subshell can accommodate a maximum of 2 electrons, so that maximum number of electrons in a shell becomes  $2(2l + 1)$  as stated earlier in Art. 51.6 (d).

### 51.9. Magnetic Spin Quantum Number ( $m_s$ )

It has been found that an electron spins around its own mechanical axis as it rotates in orbit around the nucleus. When the electron is subjected to a magnetic field, its spin axis orientates itself either parallel to or antiparallel to the direction of the field.

This quantum number arises out of quantization of the electron spin angular momentum. It determines the spin orientation up or down and has correspondingly two values of  $+1/2$  and  $-1/2$ .

### 51.10. Pauli's Exclusion Principle

This principle which was enunciated by Pauli in 1925 states that in an atom, **no two electrons can have the same set of values for its four quantum number  $n, l, m_l$  and  $m_s$** . In other words, no two electrons can be described by an identical set of four quantum numbers. They may have at the most three numbers alike but at least one must be different. Consequently, it restricts the number of electrons an atom can have.

Consider the case of He atom which has two electrons. These electrons occupy *K*-shell ( $n = 1$ ) and are designated as  $1s^2$  electrons. Their four quantum numbers are as follows :

	$n$	$l$	$m_l$	$m_s$
1st electron :	1	0	0	+ 1/2
2nd electron :	1	0	0	- 1/2

As seen, the two electrons have different sets of four quantum numbers  $(1, 0, 0, +1/2)$  and  $(1, 0, 0, -1/2)$  as required by Pauli's Exclusion Principle.

This principle affords a very rational theoretical justification for the electronic shell structure of the atoms deduced experimentally.

Pauli's Exclusion Principle may be used to find the total number of electrons an atom can have in its various shells. Consider the following examples :

**n = 1, K-shell**

$l = 0, 1s \text{ sub-shell}, m_l = 0; m_s = \pm 1/2$	2 electrons
Total	2 electrons

**n = 2, L-shell**

(i) $l = 0, 2s\text{-shell}, m_l = 0; m_s = \pm 1/2$	2 electrons
(ii) $l = 1, 2p \text{ sub-shell}, m_l = -1, 0, +1; m_s = \pm 1/2$	6 electrons
Total	8 electrons

**n = 3, M-shell**

(i) $l = 0, 3s \text{ sub-shell}, m_l = 0; m_s = \pm 1/2$	2 electrons
(ii) $l = 1, 3p \text{ sub-shell}, m_l = -1, 0, +1; m_s = \pm 1/2$	6 electrons
(iii) $l = 2, 3d \text{ sub-shell}, m_l = -2, -1, 0, +1, +2; m_s = \pm 1/2$	10 electrons
Total	18 electrons

**n = 4, N-shell**

(i) $l = 0, 4s \text{ sub-shell}, m_l = 0, m_s = \pm 1/2$	2 electrons
(ii) $l = 1, 4p \text{ sub-shell}, m_l = -1, 0, 1; m_s = \pm 1/2$	6 electrons
(iii) $l = 2, 4d \text{ sub-shell}, m_l = -2, -1, 0, +1, +2, m_s = \pm 1/2$	10 electrons
(iv) $l = 3, 4f \text{ sub-shell}, m_l = -3, -2, -1, 0, +1, +2, +3, m_s = \pm 1/2$	14 electrons
Total	32 electrons

### 51.11. Energy Bands in Solids

In the case of an *isolated single* atom, there are *single* energy levels as shown for a hydrogen atom in Fig. 51.5. But there are significant changes in the energy levels when atoms are brought close together as in solids.

It is found that each of the energy levels of an atom splits into  $N$  levels of energy where  $N$  is the number of atoms in the crystal. Each original energy level becomes a *band* of very closely-spaced levels of slightly different energy. The individual energies within the band are so close together that, for many purposes, the energy band may be considered to be a continuous one.

Fig. 51.8 (a) shows the splitting of  $K, L$  and  $M$  levels as the distance between different atoms is reduced. At first, only valence level or  $M$ -level is affected as shown by dotted vertical line marked  $A$ , then as separation is reduced, inner shells also become affected as indicated by dotted vertical line  $B$ .

Consider the case of  $Na$  crystal which consists of an ordered array of many closely-packed sodium atoms usually referred to as crystal lattice. Each  $Na$  atom has 11 electrons arranged in different shells and sub-shells as shown in Fig. 51.9 (a). As seen, the  $1s, 2s$  and  $2p$  sub-shells are filled but  $3s$  sub-shell is incomplete and could hold one electron more. The electrons in each sub-shell occurs specific energy levels as shown in Fig. 51.9 (b). For a small sodium crystal containing  $10^{20}$  atoms, the band formed by splitting of  $s$ -subshell has 2 electronic levels (one with spin up and the others with spin down). Similarly in a  $p$ -band there will be  $6 \times 10^{20}$  closely packed levels because there are 6 electrons in a filled  $p$  subshell.

In general, in an assembly of  $N$  atoms, the number of possible energy states is  $N$ . Since only two electrons of opposite spin can occupy the same state (as per Pauli's Exclusion Principle discussed earlier), the maximum number of electrons which these  $N$  states can occupy is  $2N$ .

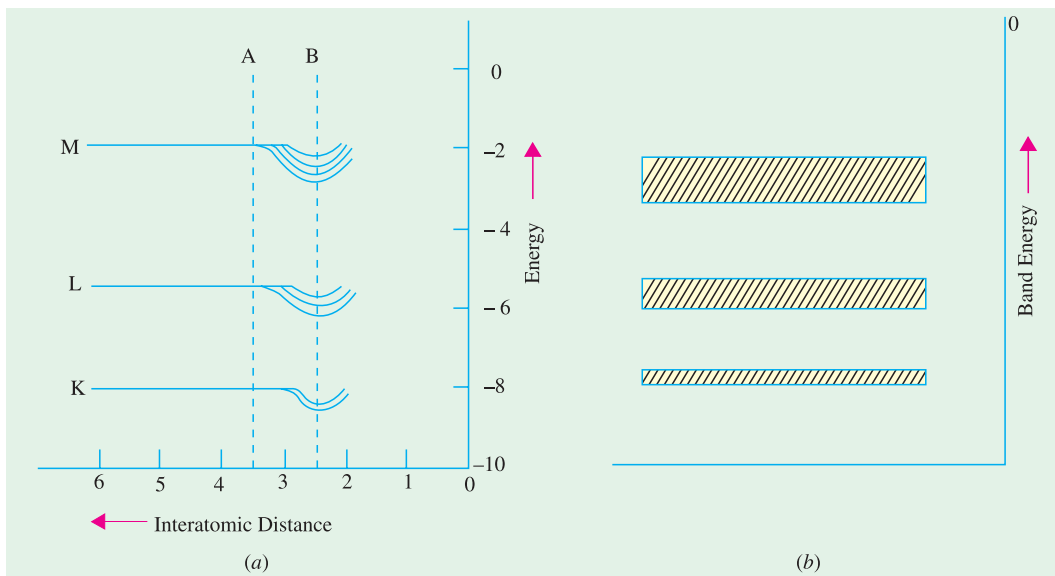


Fig. 51.8

### 51.12. Spacing Between Energy Levels of a Band

It will be quite interesting as well as instructive to calculate the spacing between different energy levels in an energy band. A crystal weighing one milligram contains about  $10^{19}$  atoms. If we assume the valency band to be an s-band, it will contain  $2 \times 10^{19}$  levels. Suppose the width of the energy band is 2 eV. Then, it is obvious that  $2 \times 10^{19}$  levels per milligram are spread over an energy band width of 2 eV. Hence, spacing between different levels =  $2/(2 \times 10^{19}) = 10^{-19}$  eV. It will be appreciated that even though energy levels are discrete, the picture of a band as a continuum of energy levels is a very good approximation. This splitting of the single energy level of an *isolated* atom into a band of

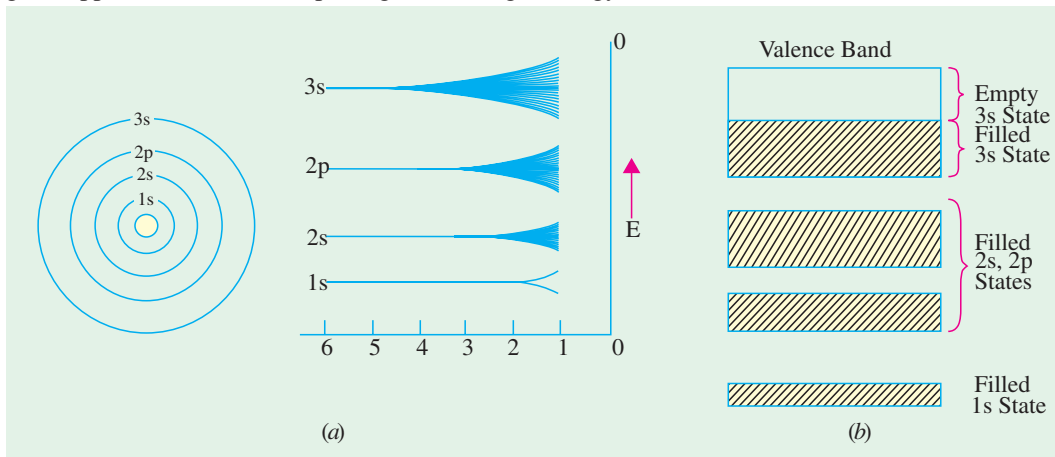


Fig. 51.9

energy in the case of a *solid* is responsible for most of the electrical, magnetic and optical properties of that solid. It is worth pointing out here that in gases under normal conditions of temperature and pressure, the atomic spacing is so great that there is no splitting of energy levels and hence no band formation.

### 51.13. Energy Bands in Lithium and their Occupancy

Consider the case of lithium metal—the simplest atom which forms a solid at ordinary temperature. Its atom has three electrons, two of which have the same energy and the third one has higher value of energy. In an isolated single atom, two electrons move round the electron orbit with  $n = 1$  whereas the third occupies the orbit with  $n = 2$  as shown in Fig. 51.10 (a). Now, consider a piece of lithium metal containing 100 atoms. It will be found that the lower level (with  $n = 1$ ) forms a band of 200 electrons occupying 100 different energy states. The higher level (with  $n = 2$ ) forms a wider band of 100 energy states which could, as before, accommodate 200 electrons. But as there are only 100 electrons available (one from each atom), this energy band remains half-filled [Fig. 51.10 (b)].

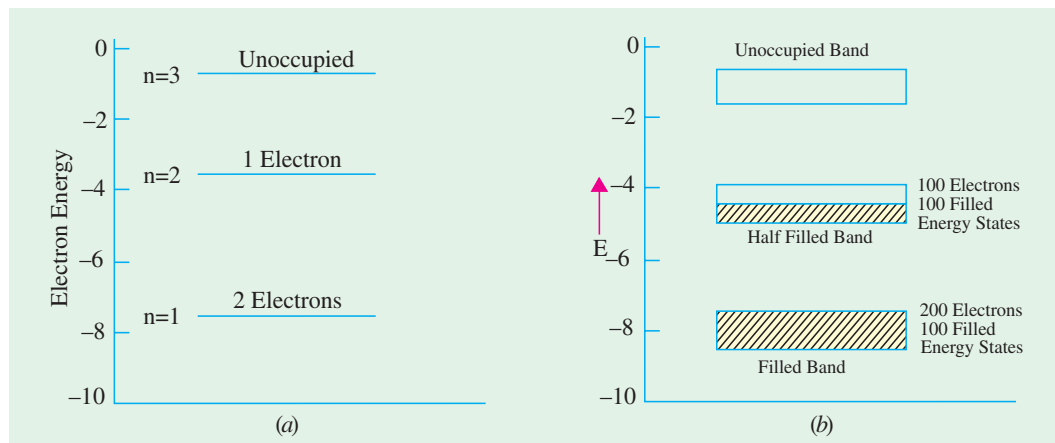


Fig. 51.10

### 51.14. Valence and Conduction Bands

The outermost electrons of an atom *i.e.* those in the shell furthest from the nucleus are called **valence** electrons and have the **highest** energy\* or least binding energy. It is these electrons which are most affected when a number of atoms are brought very close together as during the formation of a solid. The states of lower-energy electrons orbiting in shells nearer to the nucleus are little, if at all, affected by this atomic proximity.

The band of energy occupied by the valence electrons is called the **valence band** and is, obviously, the **highest occupied band**. It may be completely filled or partially filled with electrons but never empty.

The next higher permitted energy band is called the **conduction** band and may either be **empty** or **partially filled** with electrons. In fact, it may be defined as the lowest unfilled energy band.

In conduction band, electrons can move freely and hence are known as **conduction** electrons. The gap between these two bands is known as the **forbidden energy gap**.

It may be noted that the covalent force of the crystal lattice have their source in the valence band. If a valence electron happens to absorb enough energy, it jumps across the forbidden energy gap and enters the conduction band (Fig. 51.12). An electron in the conduction band can jump to an adjacent conduction band more readily than it can jump back to the valence band from where it had come earlier. However, if a conduction electron happens to radiate too much energy, it will suddenly reappear in the valence band once again.

When an electron is ejected from the valence band, a covalent bond is broken and a positively-charged hole is left behind. This hole can travel to an adjacent atom by acquiring an electron from

\* In the algebraic sense only. Their binding energy is the least.



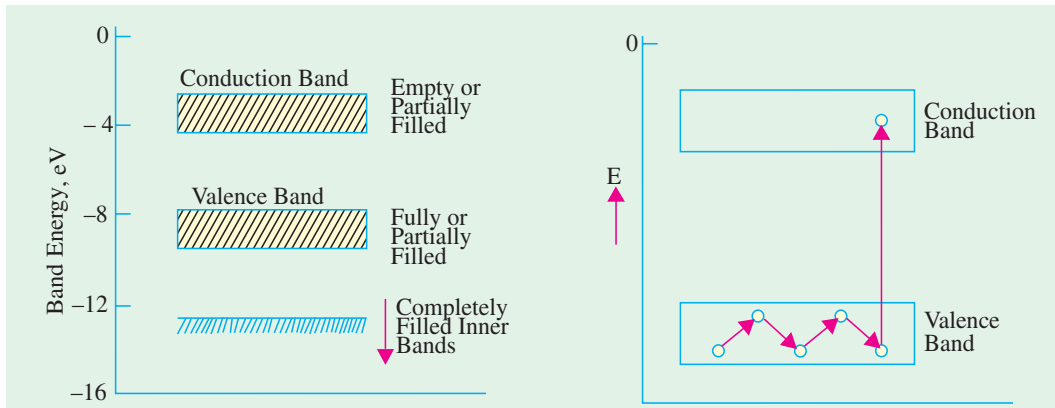


Fig. 51.11

Fig. 51.12

that atom which involves breaking an existing covalent bond and then re-establishing a covalent bond by filling up the hole. It is to be noted carefully that holes are filled by electrons which move from adjacent atoms without passing through the forbidden energy gap.

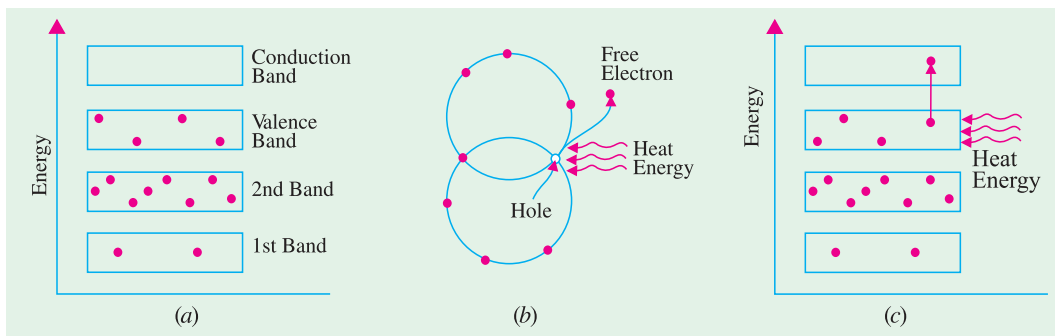


Fig. 51.13

It is simply another way of saying that conditions in the conduction band have nothing to do with the hole flow. It points to a very important distinction between the hole current and electron current—although holes flow with ease, they *experience more opposition than electron flow in the conduction band*.

To summarize the above, it may be repeated that :

1. conduction electrons are found in and freely flow in the *conduction band* ;
2. holes exist in and flow in the valence band ;
3. conduction electrons move almost twice as fast as the holes.

Fig. 51.13 (a) shows the energy band diagram of an unexcited silicon atom ( $Z = 14$ ) with its electronic distribution. When silicon crystal is given thermal or light energy from outside [Fig. 51.13 (b)], some electrons gain sufficient energy to jump the gap from the valence band into the conduction band thereby becoming free electrons [Fig. 51.13 (c)]. For every electron which jumps to conduction band, a hole is created in the valence band. In this way, an electron-hole pair is created.

### 51.15. Insulators, Conductors and Semiconductors

The electrical conduction properties of different elements and compounds can be explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.

(i) **Insulators.** Stated simply, insulators are those materials in which valence electrons are

bound very tightly to their parent atoms, thus requiring very large electric field to remove them from the attraction of their nuclei. In other words, insulators have no free charge carriers available with them under normal conditions.

In terms of energy bands, it means that insulators (a) have a full valence band,

1. have an empty conduction band,
2. have a large energy gap (of several eV) between them and
3. at ordinary temperatures, the probability of electrons from full valence band gaining sufficient energy so as to surmount energy gap and thus become available for conduction in the conduction band, is slight.

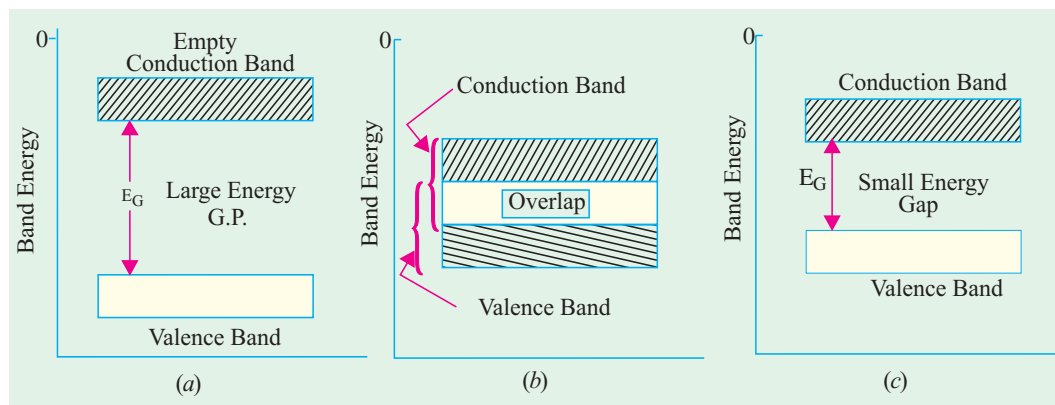


Fig. 51.14

This is shown in Fig. 51.14 (a). For conduction to take place, electrons must be given sufficient energy to jump from the valence band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which fact accounts for the negative resistance-temperature coefficient of insulators.

**(ii) Conductors.** Put in a simple way, conducting materials are those in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Fig. 51.14 (b).

In fact, there is no physical distinction *between the two bands*. Hence, the availability of a large number of conduction electrons.

Another point worth noting is that in the absence of forbidden energy gap in good conductors, there is no structure to establish holes. The *total current in such conductors is simply a flow of electrons*. It is exactly for this reason that the existence of holes was not discovered until semi-conductors were studied thoroughly.

**(iii) Semiconductors.** A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. Examples are : germanium and silicon.

In terms of energy bands, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV) separating the two.

At 0°K, there are no electrons in the conduction band and the valence band is completely filled. However, with increase in temperature, width of the forbidden energy bands is decreased so that



Insulators

some of the electrons are liberated into the conduction band. In other words, conductivity of semiconductors increases with temperature. Moreover, such departing electrons leave behind positive holes in the valence band (Fig. 51.12). Hence, semiconductor current is the sum of electron and hole currents flowing in opposite directions.

### 51.16. Crystal Structure

Semiconductors like germanium and silicon, have crystalline structure. That is the atoms are arranged in three-dimensional periodic fashion. The periodic arrangement of atoms in a crystal is called **lattice**. In a crystal, an atom strays far from a single, fixed position. The thermal vibrations associated with the atom are centred about this position. For a given semiconductor (silicon or germanium), there is

a **unit cell** that is representative of the entire lattice. By repeating the unit cell throughout the crystal, we can generate the entire lattice.

There are several different types of crystal lattice depending upon the symmetry and internal structure. One of them is the cubic crystal lattice. There are three basic types of unit cells in a cubic crystal lattice. These are simple cubic (SC), base centred cubic (BCC) and face centred cubic (FCC). Fig. 51.15 (a) shows a SC crystal. In this unit cell, each corner of the cubic lattice is occupied by an atom (indicated by a small sphere) that has six equidistant nearest neighbouring atoms. The dimension “ $a$ ” is called the **lattice constant**. Only **potassium** is crystallised into the simple cubic lattice.

Fig. 51.15 (b) shows a BCC unit cell. In this unit cell, each atom has eight nearest neighbouring atoms. Crystals exhibiting the BCC lattices include those of **sodium** and **tungsten**. Fig. 51.15 (c) shows a FCC unit cell. This unit cell has one atom at each of the six cubic faces in addition to the eight corner atoms. In an FCC lattice, each atom has 12 nearest neighbouring atoms. A large number of elements exhibit the FCC lattice form. These include **aluminium, copper, gold** and **platinum**.

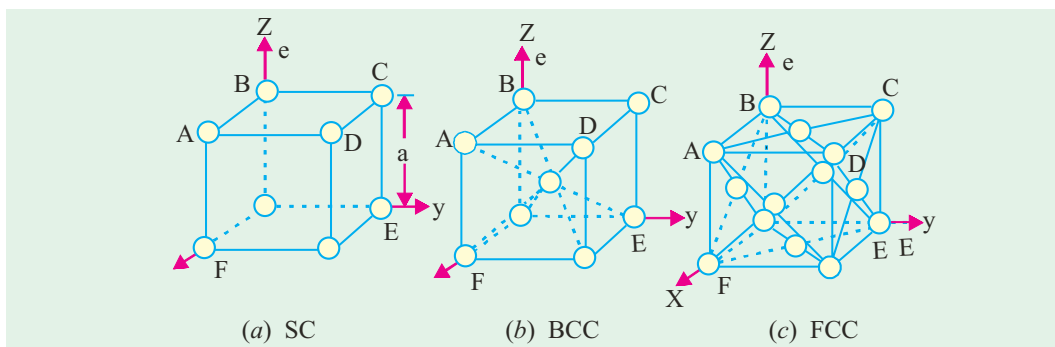
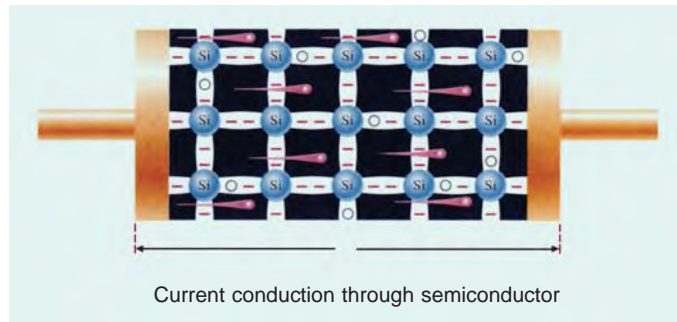


Fig. 51.15

Fig. 51.16 shows the crystal structure of silicon and germanium—the element semiconductors. This type of structure is called diamond crystal structure and it belongs to cubic crystal family. All atoms are identical in a diamond lattice. Note that each atom in diamond lattice is surrounded by four equidistant neighbours that lie at the corners of a tetrahedron.



Current conduction through semiconductor

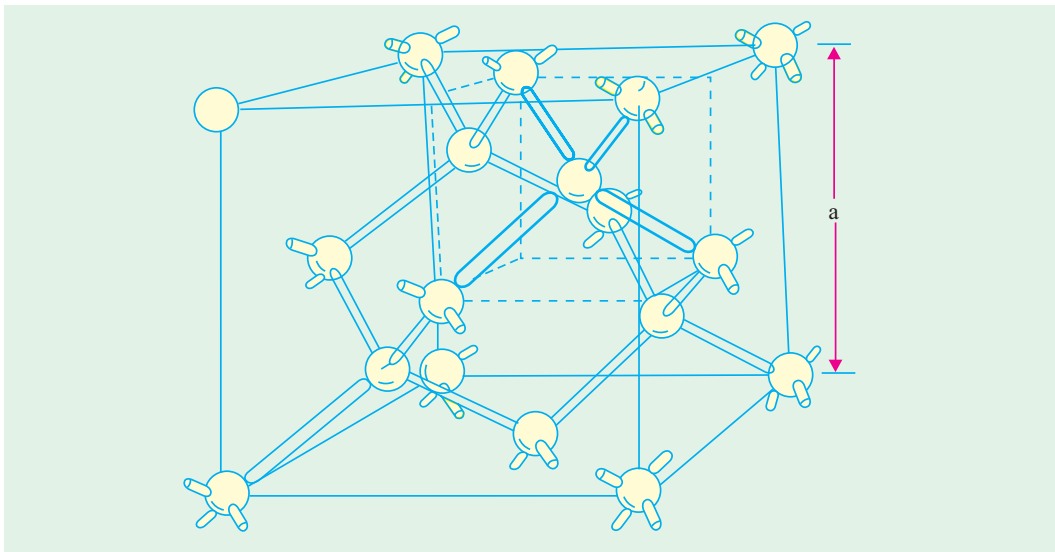


Fig. 51.16

### 51.17. Representation of Crystal Planes and Directions

Referring to Fig. 51.15 (b) again, we note that there are four atoms in the ABCD plane and four atoms in the ACEF plane (four atoms from the corners and one from the centre). Moreover, the atomic spacings are different for the two planes. Therefore, the crystal properties along different planes are different. The electrical and other device characteristics are dependent on the crystal orientation. A convenient method of defining or representing the various planes in a crystal is to use *Miller indices*. These indices are obtained using the following steps :

1. Find the intercepts of the plane on the three coordinate axes in terms of lattice constant.
2. Take the reciprocals of these numbers and reduce them to the smallest three integers having the same ratio.
3. Enclose the result in parentheses ( $hk\ l$ ) as the Miller indices for a single plane.

For example, consider the plane ABC having the intercept at  $a$ ,  $2a$  and  $2a$  along the three rectangular coordinate axes as shown in Fig. 51.17. Taking the reciprocal of these intercepts, we get  $1/a$ ,  $1/2a$ ,  $1/2a$ . Multiplying each fraction by the least common multiplier  $2a$ , we find that the smallest three integers having the same ratio are 2, 1 and 1. Thus the Miller indices from the plane are (211). The plane ABC can also be referred to as (211) plane.

Fig. 51.18 shows the Miller indices of important planes in cubic crystal. It may be noted that for a plane that intercepts the X-axis on the negative side of the origin, the Miller indices are represented by  $(\bar{h}k\ l)$ . Similarly for a plane that intercepts the Y-axis on the negative side of the origin, the Miller indices are represented by  $(h\bar{k}\ l)$  and so on.

Sometimes it is convenient to represent the Miller indices of the planes of equivalent symmetry by  $(hk\ l)$ . For example, in cubic crystal, the planes (100), (010), (001),  $(\bar{1}00)$ ,  $(0\bar{1}0)$  and  $(00\bar{1})$  have the same symmetry. Therefore, these planes can be represented by Miller indices as  $\{100\}$ . Note that we have used the curly brackets to represent a set of planes with equivalent symmetry.

The line that originates from the origin and passes through the plane at right angles to it is called *crystal direction*. The crystal direction is indicated by enclosing the Miller indices in square brackets.

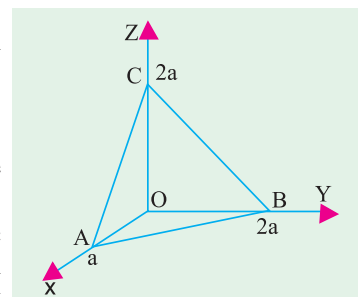


Fig. 51.17

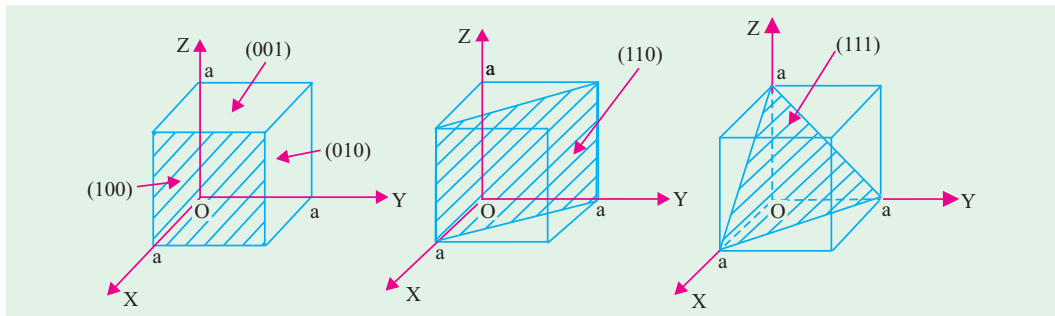


Fig. 51.18

*i.e.*,  $[hk1]$ . For example, the direction indicated by  $[100]$  is a direction for the  $X$ -axis and it is perpendicular to  $(100)$  plane. Similarly  $[111]$  direction is perpendicular to  $(111)$  plane. It is possible to represent a set of equivalent directions by  $\langle hk1 \rangle$ . Notice the use of carat signs. Thus  $\langle 100 \rangle$  represents a set of directions for  $[100]$ ,  $[010]$ ,  $[10\bar{0}]$  and  $[00\bar{1}]$ .

### 51.18. Atomic Binding in Semiconductors

Semiconductors like germanium\* and silicon, have crystalline structure. Their atoms are arranged in an ordered array known as crystal lattice. Both these materials are tetravalent *i.e.* each has four valence electrons in its outermost shell. The neighbouring atoms form *covalent* bonds by sharing four electrons with each other so as to achieve inert gas structure (*i.e.* 8 electrons in the outermost orbit). A two-dimensional view of the germanium crystal lattice is shown in Fig. 51.19 (b) in which circles represent atom cores consisting of the nuclei and inner 28 electrons. Each *pair* of lines represents a covalent bond. The dots represent the valence electrons. It is seen that each atom has 8 electrons under its influence.

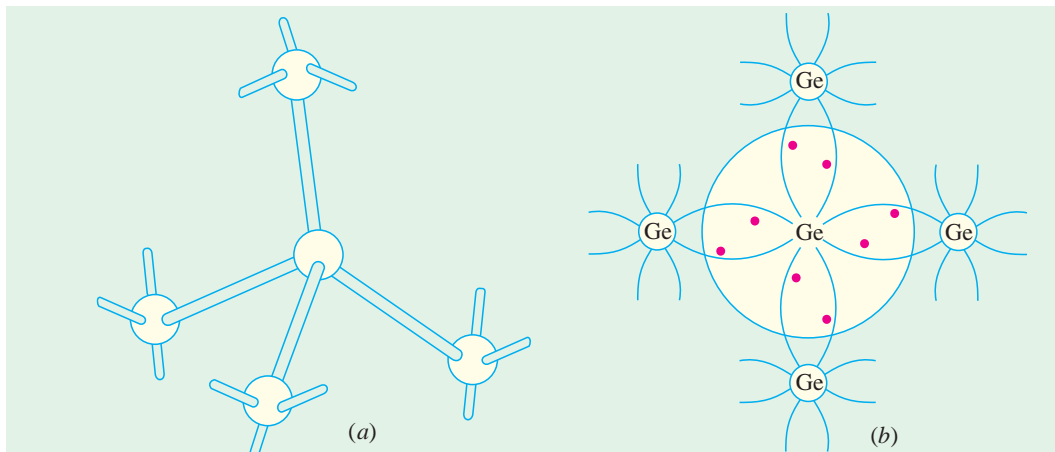
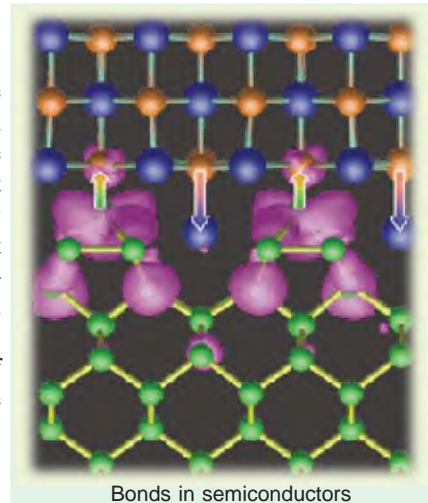


Fig. 51.19

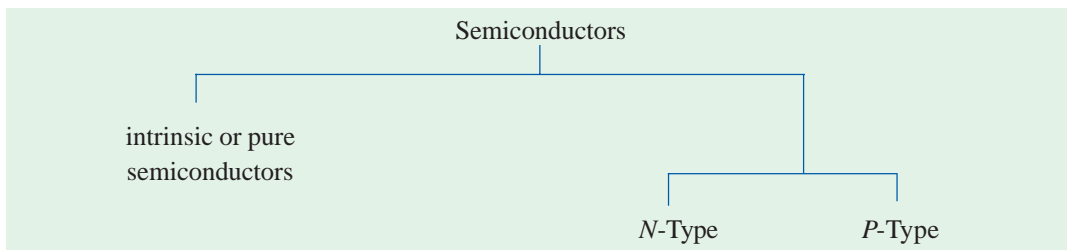
\* A single germanium atom has 32 electrons out of which only four electrons take part in electrical properties of germanium, the remaining 28 electrons being tightly bound to the nucleus. The four electrons revolve in the outermost shell and are called valence electrons.

A 3-dimensional view of germanium crystal lattice is shown in Fig. 51.19 (a) where each atom is surrounded symmetrically by four other atoms forming a tetrahedral crystal. Each atom shares a valence electron with each of its four neighbours, thereby forming a stable structure.

In the case of pure (*i.e.* intrinsic) germanium, the covalent bonds have to be broken to provide electrons for conduction. There are many ways of rupturing the covalent bond and thereby setting the electrons free. One way is to increase the crystal temperature above 0°K.

It may be noted that covalent crystals are characterised by their hardness and brittleness. Their brittleness is due to the fact that in such crystals, adjacent atoms must remain in accurate alignment since the bond is strongly directional and formed along a line joining the atoms. The hardness is due to the great strength of the covalent bond itself.

### 51.19. Types of Semiconductors



Semiconductors can be classified as shown below :

### 51.20. Intrinsic Semiconductors

An intrinsic semiconductor is one *which is made of the semiconductor material in its extremely pure form.*

Common examples of such semiconductors are : pure germanium and silicon which have forbidden energy gaps of 0.72 eV and 1.1 eV respectively. The energy gap is so small that even at ordinary room temperature, there are many electrons which possess sufficient energy to jump across the small energy gap between the valence and the conduction bands. However, it is worth noting that for each electron liberated into the conduction band, a positively charged hole is created in the valence band (Fig. 1.20). When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0°K, conduction electrons move to the anode and the holes in the valence band move to the cathode. Hence semiconductor current consists of movement of electrons and holes in opposite directions. Electron current is due to movement of electrons in the conduction band whereas hole current is within the valence band as a result of the holes 'jumping' from one atom to another.

As stated above, in pure semiconductors, electric conduction is due to the thermally-generated electron hole pairs. Hence in pure semiconductors kept in the dark, thermally-generated charge carriers are the only means of conduction. The number of such charge carriers per unit volume (*i.e.* intrinsic carrier density) is given by

$$n_i = N \exp(-E_g / 2kT)$$

where  $N$  is constant for a given semiconductor,  $E_g$  is the band gap energy in joules,  $k$  is Boltzmann's constant and  $T$  is the temperature in °K.

**Example 51.2.** Find the intrinsic carrier concentration in silicon at 300° K for which  $N = 3 \times 10^{25} \text{ m}^{-3}$ ,  $E_g = 1.1 \text{ eV}$ ,  $\mu_e = 0.14 \text{ m}^2/\text{V-s}$  and  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$ . Also, find the conductivity of silicon. **(Electronics-II, Madras Univ. 1993)**

**Solution.** The intrinsic carrier concentration in pure silicon is given by

$$n_i = N \exp(-E_g/2kT)$$

Now,  $N = 3 \times 10^{25} \text{ m}^{-3}$ ,  $E_g = 1.1 \text{ eV} = 1.1 \times 1.6 \times 10^{-19} = 1.76 \times 10^{-19} \text{ J}$   
 $k = 1.38 \times 10^{-23} \text{ J/K}$ ,  $T = 300^\circ\text{K}$

$$\therefore n_i = 3 \times 10^{25} (-1.76 \times 10^{-19}) / (2 \times 1.38 \times 10^{-23} \times 300) = 2 \times 10^{16} \text{ m}^{-3}$$

$$\sigma = n_i e (\mu_e + \mu_h) = 2 \times 10^{16} \times 1.6 \times 10^{-19} (0.14 + 0.05) = 0.61 \times 10^{-3} \text{ S/m}$$

### 51.21. Hole Formation in Semiconductors

The formation of a hole which is a positive charge carrier is explained below :

As shown in Fig. 51.21, suppose the covalent bond is broken at A and the electron has moved through the crystal lattice leaving behind a hole in the covalent bond. An electron at B may jump into the vacant hole at A and later, an electron at C may jump into the hole at B and so on. In this way, by a succession of electron movements, a hole will appear at G and a negative charge would have moved from G to A. It would, however, be more convenient to regard positive charge to have moved from A to G and this conception gives rise to a hole as a positive charge carrier as if it were an electron with a positive charge. It should be clearly understood that these holes are due to the movement of electrons in the valence band and that each electron movement corresponds to a collision. The drift velocity of holes is, obviously, much less than the drift velocity of electrons.

Alternatively, an intrinsic semiconductor may be defined as one *in which the number of conduction electrons is equal to the number of holes*.

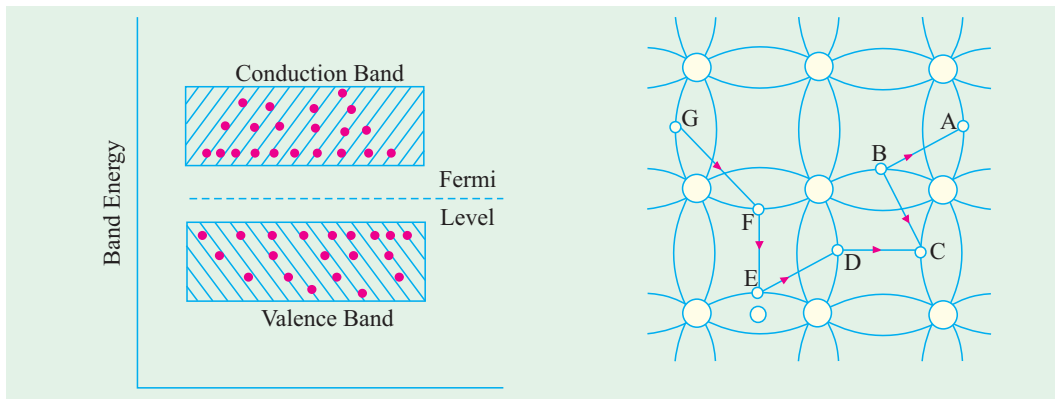


Fig. 51.20

Fig. 51.21

Schematic energy band diagram of an intrinsic semiconductor at room temperature is shown in Fig. 51.20. Only two bands *i.e.* valence and conduction bands have been shown since lower filled bands are not of any consequence. Here, Fermi level\* lies exactly in the middle of the forbidden energy gap.

### 51.22. Fermi Level in an Intrinsic Semiconductor

It can be proved that in an intrinsic semiconductor, Fermi energy level\*  $E_F$  lies in the middle of the energy gap *i.e.* midway between the conduction and valence bands.

Let, at any temperature  $T^\circ\text{K}$

$$n_c = \text{No. of electrons in the conduction band}$$

$$n_v = \text{No. of electrons in the valence band}$$

\* For the present discussion, Fermi level may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes weighted according to their energies.

$$N = n_c + n_v$$

= No. of electrons in both bands.

Further, let us make the following simplifying assumptions :

1. width of energy bands are small as compared to forbidden energy gap between them;
2. since band widths are small, all levels in a band have the same energy;
3. energies of all levels in valence band are zero as shown in Fig. 51.22;
4. energies of all levels in the conduction band are equal to  $E_g$ .

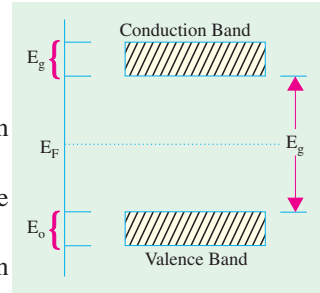


Fig. 51.22

In Fig. 51.22, the zero-energy reference level has been arbitrarily taken at the top of valence band.

Now, number of electrons in the conduction band is

$$n_c = N.P(E_g)$$

where  $P(E_g)$  represents the probability of an electron having energy  $E_g$ . Its value can be found from Fermi-Dirac probability distribution function given by

$$P(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

where  $P(E)$  is the probability of finding an electron having any particular value of energy  $E$  and  $E_F$  is Fermi level.

$$\therefore P(E_g) = \frac{1}{1 + e^{(E_g - E_F)/kT}} \quad \therefore n_c = \frac{N}{1 + e^{(E_g - E_F)/kT}}$$

Now, number of electrons in the valence band is  $n_v = N.P(0)$

The probability  $P(0)$  of an electron being found in the valence band with zero energy can again be calculated by putting  $E = 0$  in the Femi-Dirac probability distribution function.

$$P(0) = \frac{1}{1 + e^{(0 - E_F)/kT}} = \frac{1}{1 + e^{-E_F/kT}} \quad \text{Hence, } n_v = \frac{N}{1 + e^{-E_F/kT}}$$

$$\text{Now } N = n_c + n_v \quad \text{or } N = \frac{N}{1 + e^{(E_g - E_F)/kT}} + \frac{N}{1 + e^{-E_F/kT}}$$

$$\therefore 1 - \frac{1}{1 + e^{-E_F/kT}} = \frac{1}{1 + e^{(E_g - E_F)/kT}}$$

which on simplification gives  $E_F = E_g/2$ .

Hence, it shows that in an intrinsic semiconductor, the Fermi level lies midway between the conduction and valence bands. That this conclusion is physically possible can be best seen with the help of Fig. 51.23.

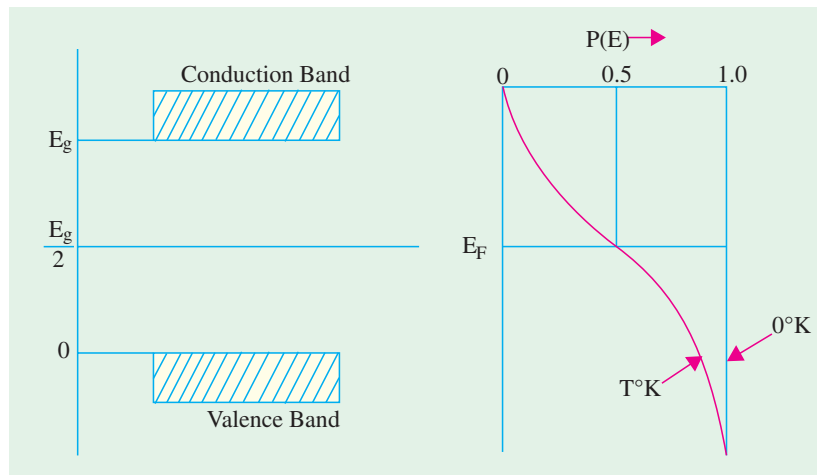


Fig. 51.23



### 51.23. Extrinsic Semiconductors

Those intrinsic semiconductors to which some suitable impurity or doping agent or dopant has been added in extremely small amounts (about 1 part in  $10^8$ ) are called *extrinsic* or impurity semiconductors.

The usual doping agents are :

1. *pentavalent* atoms having five valence electrons (arsenic, antimony, phosphorus) or
2. *trivalent* atoms having three valence electrons (gallium, indium, aluminium, boron).

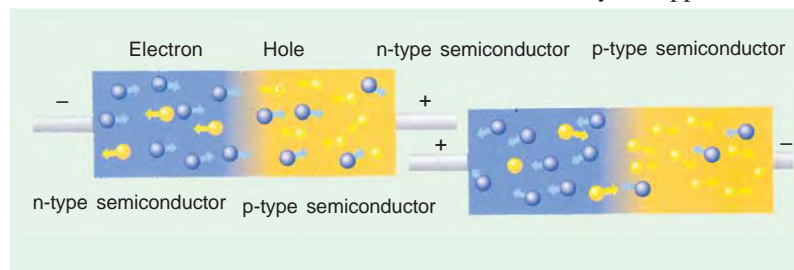
Pentavalent doping atom is known as *donor* atom because it donates or contributes one electron to the conduction band of pure germanium. The trivalent atom, on the other hand, is called *acceptor* atom because it accepts one electron from the germanium atom.

Depending on the type of doping material used, extrinsic semiconductors can be sub-divided into two classes :

- (i) *N*-type semiconductors and
- (ii) *P*-type semiconductors.

(a) ***N*-type Extrinsic Semiconductor.** This type of semiconductor is obtained when a pentavalent material like antimony (Sb) is added to pure germanium crystal. As shown in Fig. 51.24 (a), each antimony atom forms covalent bonds with the surrounding four germanium atoms with the help of four of its five electrons. The fifth electron is superfluous and is loosely bound to the antimony atom. Hence, it can be easily excited from the valence band to the conduction band by the application of electric field or increase

in thermal energy. Thus, practically every antimony atom introduced into the germanium lattice, contributes one conduction electron into the germanium lattice *without creating*



*a positive hole*. Antimony is called *donor* impurity and makes the pure germanium an *N*-type (*N* for negative) extrinsic semiconductor. As an aid to memory, the student should associate the *N* in doNOR with *N* in the *N*-type material and *N* in Negative charge carrier.

It may be noted that by giving away its one valence electron, the donor atom becomes a positively-charged *ion*. But it cannot take part in conduction because it is firmly fixed or tied into the crystal lattice. It will be seen that apart from electrons and holes *intrinsically available in germanium*, the addition of antimony greatly increases the number of conduction electrons. Hence, concentration of electrons in the conduction band is increased and exceeds the concentration of holes in the valence band. Because of this, Fermi level shifts upwards towards the bottom of the conduction band as shown in Fig. 51.24 (b),\* because the number of charge carriers has become more in conduction band than in valence band.

In terms of energy levels, the fifth antimony electron has an energy level (called donor level) just below the conduction band. Usually, the donor level is 0.01 eV below conduction band for germanium and 0.054 eV for silicon.

It is seen from the above description that in *N*-type semiconductors, electrons *are the majority carriers* while holes constitute the minority carriers. Hence, *N*-type semiconductor conducts principally by electrons in the nearly empty conduction band and the process is called '*excess*' conduction.

Another point worth noting is that even though *N*-type semiconductor has excess of electrons,

\* Since the number of electrons as compared to the number of holes increases with temperature, the position of Fermi level also changes considerably with temperature.

still it is electrically neutral. It is so because by the addition of donor impurity, number of electrons available for conduction purposes becomes more than the number of holes available intrinsically. But the total charge of the semiconductor does not change because the donor impurity brings in as much negative charge (by way of electrons) as positive charge (by way of protons in its nucleus).

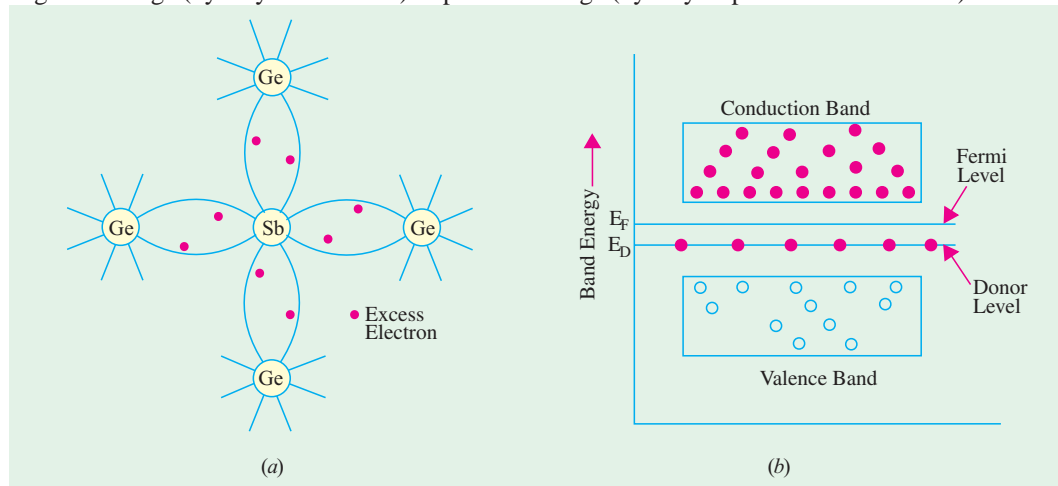


Fig. 51.24

**(b) P-type Extrinsic Semiconductor.** This type of semiconductor is obtained when traces of a trivalent like boron (*B*) are added to a pure germanium crystal.

In this case, the three valence electrons of boron atom form covalent bonds with four surrounding germanium atoms but one bond is left incomplete and gives rise to a hole as shown in Fig. 51.25 (a).

Thus, boron which is called an *acceptor* impurity causes as many positive holes in a germanium crystal as there are boron atoms thereby producing a *P*-type (*P* for positive) extrinsic semiconductor. As an aid to memory, the student should associate the **P** in acce**P**tor with **P** in *P*-type material and **P** with **P**ositive charge carrier.

In this type of semiconductor, conduction is by the movement of holes in the valence band. Accordingly, holes form the majority carriers *whereas electrons constitute minority carriers*. The process of conduction is called 'deficit' conduction.

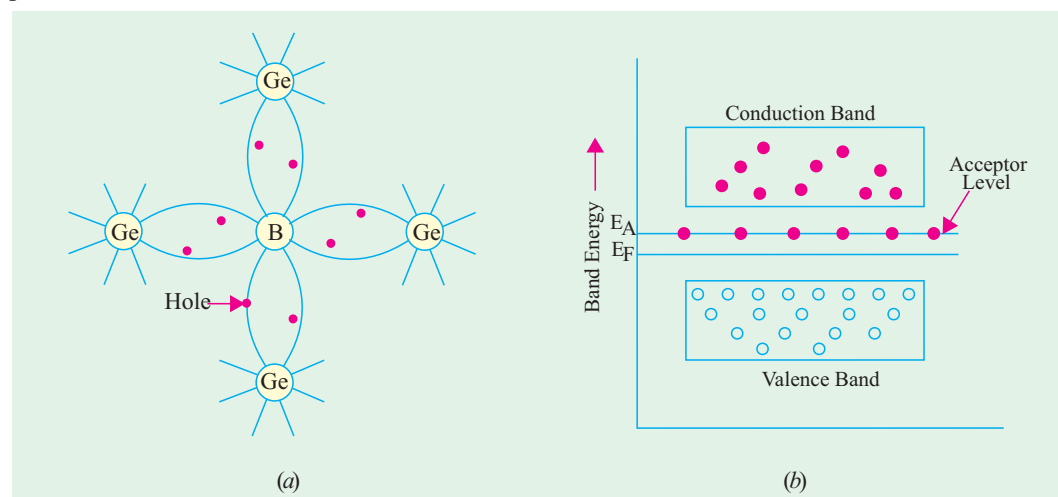


Fig. 51.25

Since concentration of holes in the valence band is more than the concentration of electrons in the conduction band, Fermi level shifts nearer to the valence band [Fig. 51.25 (b)]. The acceptor level lies immediately above the Fermi level. Conduction is by means of hole movement at the top of valence band, acceptor level readily accepting electrons from the valence band.

Again, it may be noted that even though *P*-type semiconductor has excess of holes for conduction purposes, on the whole it is electrically neutral for the same reasons as given above.

### 51.24. Majority and Minority Carriers

In a piece of *pure* germanium or silicon, no free charge carriers are available at 0°K. However, as its temperature is raised to room temperature, some of the covalent bonds are broken by heat energy and as a result, electron-hole pairs are produced. These are called thermally-generated charge carriers. They are also known as intrinsically-available charge carriers. Ordinarily, their number is quite small.

An intrinsic of pure germanium can be converted into a *P*-type semiconductor by the addition of an acceptor impurity which adds a large number of holes to it. Hence, a *P*-type material contains following charge carriers :

- (a) large number of positive holes—most of them being the added impurity holes with only a very small number of thermally generated ones;
- (b) a very small number of thermally-generated electrons (the companions of the thermally generated holes mentioned above).

Obviously, in a *P*-type material, the number of holes (both added and thermally-generated) is much more than that of electrons. Hence, in such a material, holes constitute *majority* carriers and electrons form *minority* carriers as shown in Fig. 51.26 (a).

Similarly, in an *N*-type material, the number of electrons (both added and thermally-generated) is much larger than the number of thermally-generated holes. Hence, in such a material, electrons are majority carriers whereas holes are minority carriers as shown in Fig. 51.26 (b).

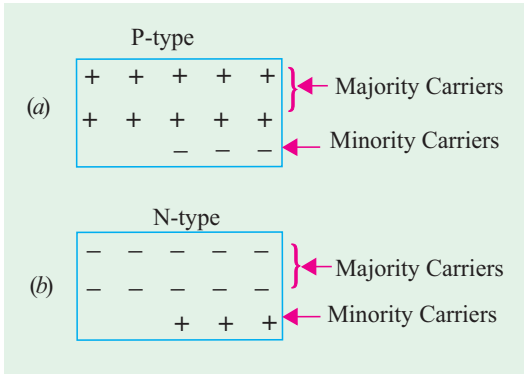


Fig. 51.26

### 51.25. Mobile Charge Carriers and Immobile Ions

As discussed in Art. 51.23, *P*-type material is formed by the addition of acceptor impurity atoms like boron to the pure *Ge* or *Si* crystals. The number of holes added is equal to the number of boron atoms because each such atom contributes one hole. Now, when a hole moves *away* from its parent atom, the remaining atom becomes a *negative ion*. Unlike the mobile and free-moving hole, this ion

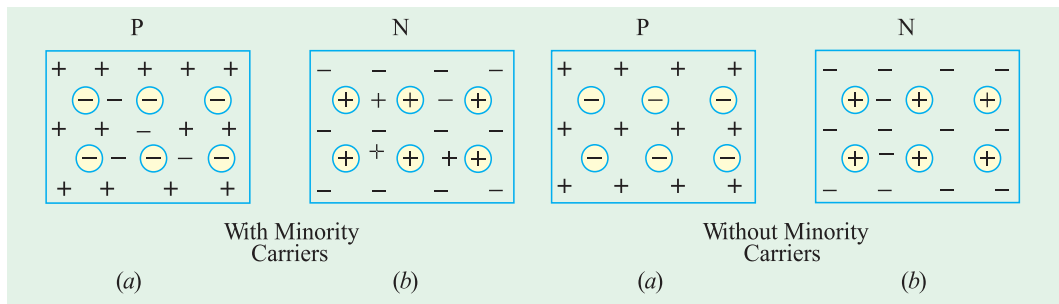


Fig. 51.27

Fig. 51.28

cannot take part in conduction because it is fixed in the crystal lattice. In Fig. 51.27 (a), these immobile ions are shown by **circled** minus signs whereas free and mobile holes are shown by **uncircled** plus signs. Thermally-generated electrons (which form minority carriers) are shown by **uncircled** minus signs.

Similarly, addition of penta-valent atoms like antimony to pure *Ge* or *Si* crystal produces an *N*-type material. The number of free and mobile electrons which are added equals the number of donor *Sb* atoms. Again, when an electron moves **away** from its parent atom, it leaves behind a positive ion. This ion, being fixed in crystal structure, cannot take part in conduction. As shown in Fig. 51.27 (b); these immobile ions are represented by **circled** plus signs whereas free and mobile electrons are represented by **uncircled** minus signs. The thermally-generated holes (which form minority carriers in this case) are shown by **uncircled** plus signs. In Fig. 51.28, minority carriers of both types have been neglected. Hence, the figure does not show the small number of free electrons in the *P*-type material or the small number of holes in the *N*-type material.

### 51.26. Electron Conductivity of a Metal

According to free electron model of an atom, the valence electrons are not attached to individual atoms but are free to move about in all directions among the atoms. These electrons are called **conduction** electrons and are said to form 'free electron cloud' or free electron 'gas' or the Fermi gas. For example in copper there is one such free electron per atom, the other 28 electrons remaining bound to the copper nuclei to form positive ion cores.



When no external field is applied to the metal, the free electrons move randomly in all directions as shown in Fig. 51.29 (a). However, when an external electric field is applied to the metal, the free electron motion becomes directed as shown in Fig. 51.29 (b).

This directed flow of electrons results in a net charge displacement in a definite direction. This type of motion is known as **drift** and the phenomenon is referred to as process of conduction by drift charge. The drift velocity ( $v$ ) of the electrons is dependent upon the electron mobility ( $\mu_e$ ) and the applied electric field  $E$ . The actual relation is  $v = \mu_e E$

- Let,
- $e$  = electron charge (coloumb)
  - $v$  = electron drift velocity (m/s)
  - $A$  = conductor cross-section ( $\text{m}^2$ )
  - $n$  = number of free electrons per unit volume of the conductor  
i.e. electron density (per  $\text{m}^3$ )
  - $l$  = length of the conductor (m)
  - $E = V/l$  — applied electric field (V/m)

Now, electric current flowing in any conductor is given by the amount of charge which flows in one second across any plane of the conductor (Fig. 51.30). The **total** number of electrons which cross the plane  $P$  of cross-section  $A$  in one second =  $n \times (v \times A)$ .

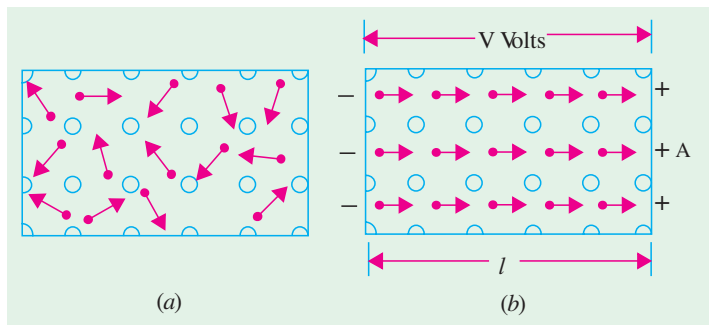


Fig. 51.29

Charge carried by them per second is  $envA$ . Hence,  $I = venA$ .  
 Substituting the value of  $v$ , we get

$$I = neA\mu_e E = nAe\mu_e V/l$$

$$\therefore R = \frac{V}{I} = \frac{l}{A} \left( \frac{1}{ne\mu_e} \right) = \rho \frac{l}{A}$$

$\therefore$  resistivity  $\rho = 1/ne \mu_e$  ohm-m and conductivity  $\sigma = ne\mu_e$  Siemens/m

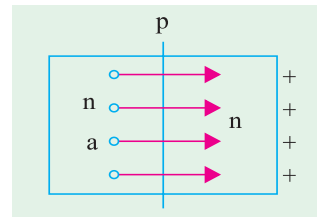


Fig. 51.30

Incidentally, it may be noted that conductivity of a semiconductor differs from that of a metal in one important respect *i.e.* in a semiconductor, charge carriers are both holes as well as electrons whereas in metals, electrons are the only charge carriers (Art. 51.27).

**Example 51.3.** A copper wire of 2mm diameter with conductivity of  $5.8 \times 10^7$  Siemens/m and electron mobility of  $0.0032 \text{ m}^2/\text{V-s}$  is subjected to an electric field of 20 V/m. Find (a) the charge density of free electrons, (b) the current density, (c) current flowing in the wire, (d) the electron drift velocity. (U.P.S.C. Engg. Services 2002)

**Solution.**  $d = 2\text{mm} = 2 \times 10^{-3} \text{ m}$ ,  $\sigma = 5.8 \times 10^7 \text{ S/m}$ ,  
 $\mu_e = 0.0032 \text{ m}^2/\text{V-s}$ ,  $E = 20 \text{ V/m}$   
 $\sigma = qn\mu$

(a)  $n = \frac{\sigma}{q\mu_e} = \frac{5.8 \times 10^7}{1.6 \times 10^{-19} \times 0.0032} = 1.132 \times 10^{29} / \text{m}^3$

(b)  $J = \sigma E = (5.8 \times 10^7) \times (20 \times 10^{-3}) = 1.16 \times 10^6 \text{ A/m}^2$

(c) Let  $I =$  current flowing through the wire.

Area of cross-section of a wire,

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (2 \times 10^{-3})^2}{4} = 3.16 \times 10^{-6} \text{ m}^2$$

$$J = \frac{I}{A} \text{ or } I = JA = (1.16 \times 10^6) \times (3.16 \times 10^{-6}) = 3.67 \text{ A}$$

(d)  $v = \mu E = 0.0032 \times (20 \times 10^{-3}) = 64 \text{ m/s}$

### 51.27. Conductivity of Intrinsic Semiconductors

In their case, current flow is due to the movement of electrons and holes in opposite directions. However, since their charges are of opposite sign, the current due to each is in the same direction. Even though the number of electrons equals the number of holes, hole mobility  $\mu_h$  is practically half of electron mobility  $\mu_e$ .

As shown in Fig. 51.31, the total current flow which is due to the sum of electron flow and hole flow, is given by

Let  $I = I_e + I_h$   
 $v_e =$  drift velocity of electrons (m/s)  
 $v_h =$  drift velocity of holes (m/s)  
 $n_i =$  density of free electrons in an intrinsic semiconductor (per  $\text{m}^3$ )

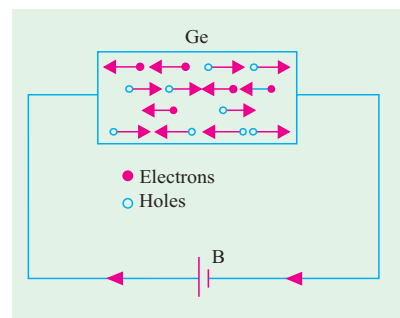


Fig. 51.31

- $p_i$  = density of holes in an intrinsic semiconductor (per  $m^3$ )
- $e$  = electron charge (coulomb)
- $A$  = cross-section of the semiconductor ( $m^2$ )

Since in an intrinsic semiconductor  $n_i = p_i$

$$\therefore I = n_i e (v_e + v_h) A = n_i e (\mu_h + \mu_e) EA$$

where  $\mu_e$  = electron mobility =  $v_e/E$

$\mu_h$  = hole mobility =  $v_h/E$

Since  $E = V/l$  where  $l$  is the length of the intrinsic semiconductor,

$$\therefore I = n_i e (\mu_e + \mu_h) AV/l$$

$$\therefore \frac{V}{I} = \frac{l}{A} \cdot \frac{1}{n_i e (\mu_e + \mu_h)} = \rho_i \frac{l}{A}$$

where  $\rho$  is the resistivity of the semiconductor. It is given by

$$\rho_i = \frac{1}{n_i e (\mu_e + \mu_h)} \text{ ohm-m}$$

The electrical conductivity which is the reciprocal of resistivity is given by

$$\sigma_i = n_i e (\mu_e + \mu_h) \text{ S/m}$$

Now, current density,  $J = I/A \quad \therefore J = n_i e (\mu_e + \mu_h) E = \sigma_i E \quad \therefore \sigma_i = J/E$

Obviously, conductivity of semiconductors depends on two factors (i) number of current carriers present per unit volume and (ii) the mobility of the current carriers. It is found that with increase in temperature,  $n_i$  as well  $p_i$  increase and correspondingly, the conductivity of intrinsic semiconductors increases *i.e.* resistivity decreases.

### 51.28. Conductivity of Extrinsic Semiconductors

The general expression for current density (derived above) in the case of an extrinsic semiconductor when an electric field is employed is

$$J = (ne\mu_e + pe\mu_h)E \quad \dots(i)$$

(a) If it is an *N*-type semiconductor, then the above expression becomes

$$J_n = e(n_n\mu_e + p_n\mu_h)E$$

where  $n_n$  and  $p_n$  represent the electron and hole densities in the *N*-type semiconductor *after doping*.

(b) If it is a *P*-type semiconductor, then

$$J_p = e(n_p\mu_e + p_p\mu_h)E$$

where  $n_p$  and  $p_p$  represent similar quantities in a *P*-type semiconductor after doping.

The conductivity is given by  $\sigma = J/E$

$$\therefore \sigma = ne\mu_e + pe\mu_h$$

or  $\sigma_n = e(n_n\mu_e + p_n\mu_h)$  — for *N*-type

and  $\sigma_p = e(n_p\mu_e + p_p\mu_h)$  — for *P*-type

(i) In *N*-type semiconductors, electrons form the majority carriers although holes are also available as minority carriers.

The current density in such a semiconductor is given by Eq. (i) above. However, since electron density in such extrinsic semiconductors is much more than hole density *i.e.*  $n_n \gg p_n$ , the above expressions are simplified to

$$J_n = n_n e \mu_e E \quad \text{and} \quad \sigma_n = n_n e \mu_e \quad \dots(ii)$$

(ii) In *P*-type semiconductors, conduction is by means of holes in the valence band which form majority carriers in this case although electrons are available as minority carriers.

Since in such extrinsic semiconductors,  $n_p \ll p_p$ , the above expressions become

$$J_p = p_p e \mu_h E \quad \text{and} \quad \sigma_p = p_p e \mu_h \quad \dots(iii)$$

### 51.29. Conductivity when Intrinsic Charge Carrier Densities are Neglected

In case density of charge carriers available intrinsically is negligible as compared to the added impurity atoms (whether of donor or a acceptor type), then the formulae for conductivity given by Eq. (ii) and (iii) above will be changed as follows :

#### (a) For *N*-type semiconductor

As seen from Eq. (ii) above, the conductivity is given by  $\sigma_n = n_n e \mu_e$  where  $n_n$  is electron density after doping.

In this relation, intrinsic hole density has already been neglected. The remaining electron density is also made up of the following two components :

1. intrinsic hole density due to holes available in a *pure* semiconductor ;
2. electron density  $N_d$  contributed by added donor impurity.

However, if we further neglect the intrinsic electron density, then  $\sigma_n = N_d e \mu_e$

#### (b) For *P*-type semiconductor

As seen from Eq. (iii) above, the conductivity is given by  $\sigma_p = p_p e \mu_h$  where  $p_p$  represents hole density. Again, in this relation, intrinsic electron density has been already neglected. This hole density further consists of the following two components :

1. intrinsic hole density due to holes available in a pure semiconductor ;
2. hole density ( $N_a$ ) contributed by added acceptor impurity.

However, if we further neglect the intrinsic hole density, then  $\sigma_p = N_a e \mu_h$

### 51.30. Conductivity of Pure and P-type Germanium

As shown in Art. 51.25, the conductivity of pure germanium is given by

$$\sigma = n_i e (\mu_e + \mu_h) = p_i e (\mu_e + \mu_h)$$

When germanium is doped by a trivalent impurity like indium, it becomes a *P*-type semiconductor. After doping, its conductivity depends on the number of charge carriers available in it. The law of Mass Action can be used for finding this number. For acceptor impurity, the law may be stated as follows :

$$n_p p_p = n_i p_i = n_i^2 = p_i^2$$

where  $n_p$  and  $p_p$  represent the 'free' electron and hole densities respectively in the semi-conductor after doping and  $n_i$  and  $p_i$ , the electron and hole densities *before* doping *i.e.* in an intrinsic semiconductor. Put in another way, it simply means that, at constant temperature, the product of the number of electron carriers and the number of hole carriers is independent of the density of acceptor atoms. In physical terms, it means that the introduction of *P*-type impurity fills some of the electron levels produced by thermal action.

By calculating  $n_p$  and  $p_p$  from above and knowing  $\mu_e$  and  $\mu_h$ , conductivity after doping can be found out as illustrated by the following example.

#### (a) Pure Germanium

$$\sigma = n_i e (\mu_e + \mu_h) = p_i e (\mu_e + \mu_h)$$

Let,

$$\begin{aligned} n_i = p_i &= 2 \times 10^{19} \text{ per m}^3, \quad \mu_e = 0.36 \text{ m}^2/\text{V-s} \\ \mu_h &= 0.17 \text{ m}^2/\text{V-s} \quad \text{and} \quad e = 1.6 \times 10^{-19} \text{ C} \\ \sigma &= 2 \times 10^{19} \times 1.6 \times 10^{-19} (0.36 + 0.17) = 1.69 \text{ S/m.} \end{aligned}$$

#### (b) *P*-type Germanium

here,

$$\sigma_p = e (n_p \mu_e + p_p \mu_h)$$

Suppose, we add  $10^{22}$  atoms/ $\text{m}^3$  of indium and that  $n_i = 2 \times 10^{19}$  charge carriers (either electrons or holes) per  $\text{m}^3$ .

$$\begin{aligned} \text{Then,} \quad n_p p_p &= n_i^2 = (2 \times 10^{19})^2 = 4 \times 10^{38} \quad \text{and} \quad p_p - n_p = 10^{20} \\ \therefore \quad p_p - 4 \times 10^{38}/p_p &= 10^{20} \quad \text{or} \quad p_p^2 - 10^{20} p_p - (4 \times 10^{38}) = 0 \end{aligned}$$

Solving the above quadratic equation and taking positive value only.

$$p_p = 1.04 \times 10^{20} \quad \text{and} \quad n_p = 0.04 \times 10^{20}$$

$$\sigma_p = 1.6 \times 10^{-19} (0.04 \times 10^{20} \times 0.36 + 1.04 \times 10^{20} \times 0.17) = 3.1 \text{ S/m}^*$$

It is seen that conductivity is almost doubled.

**Example 51.4.** What length of a round copper wire of diameter 1 mm will have a resistance of 1 k  $\Omega$  if copper conductivity is 60 MS/m. A cylindrical piece of silicon having a diameter of 1 mm is doped with  $10^{20} \text{ m}^{-3}$  atoms of phosphorous which are fully ionized. What length of this silicon would be required to give a resistance of 1 k  $\Omega$  if electronic mobility in silicon is  $0.1 \text{ m}^2/\text{V-s}$  ?

(Electronic Devices & Circuits, Pune Univ. 1994)

**Solution.**  $R = 1 \text{ k } \Omega = 1000 \text{ } \Omega$ ,  $\sigma = 60 \times 10^6 \text{ S/m}$ ,  $A = \pi d^2/4 = \pi \times (1 \times 10^{-3})^2/4 \text{ m}^2$   
 $R = l/\sigma A$   
 $l = \sigma AR = 60 \times 10^6 \times (\pi \times 10^{-6}/4) \times 1000 = 47,100 \text{ m} = \mathbf{47.1 \text{ km}}$

**For Silicon Wire**

$$\sigma = n_i e \mu_e = 10^{20} \times 1.6 \times 10^{-19} \times 0.1 = 1.6 \text{ S/m}$$

$$l = \sigma AR = 1.6 \times (\pi \times 10^{-6}/4) \times 1000$$

$$= 1.26 \times 10^{-3} = \mathbf{1.26 \text{ mm}}$$

**Example 51.5.** Calculate the intrinsic conductivity of silicon at room temperature if  $n = 1.41 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_e = 0.145 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ . What are the individual contributions made by electrons and holes ?

(Electronic Engg., Nagpur Univ. 1991)

**Solution.** As seen from Art. 1.27, the conductivity of an intrinsic semiconductor is given by

$$\sigma_i = n_i e \mu_e + n_i e \mu_h$$

$$= 1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.145 + 1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.05$$

$$= 0.325 \times 10^{-3} + 0.112 \times 10^{-3} \text{ S/m} = \mathbf{0.437 \times 10^{-3} \text{ S/m}}$$

Contribution by electrons =  $\mathbf{0.325 \times 10^{-3} \text{ S/m}}$

Contribution by holes =  $\mathbf{0.112 \times 10^{-3} \text{ S/m}}$

**Example 51.6.** Calculate the donor concentration in N-type germanium having resistivity of 100  $\Omega\text{-m}$ . Derive the formula you use. Take  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $\mu_e = 0.36 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ .

(Electronics ; Nagpur Univ. 1990)

**Solution.** As seen from Art. 1.29,  $\rho_n = 1/N_d e \mu_e$   
 $\therefore 100 = 1/N_d \times 1.6 \times 10^{-19} \times 0.36$ ;  $N_d = \mathbf{1.74 \times 10^{17} \text{ atoms/m}^3}$

**Example 51.7.** An N-type silicon has a resistivity of 1500  $\Omega\text{-m}$  at a certain temperature. Compute the electron-hole concentration given that  $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_e = 0.14 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ .

**Solution.** Being N-type silicon, it is assumed that  $n \gg p$

$$\therefore \sigma = e (n \mu_e + p \mu_h) = n_e \mu_e$$

$$\therefore \rho = 1/n_e \mu_e \quad \text{or} \quad 15 = 1/n \times 1.6 \times 10^{-19} \times 0.14 \quad \text{or} \quad n = \mathbf{3.1 \times 10^{20} \text{ m}^{-3}}$$

Now,  $np = n_i^2$  or  $p = n_i^2/n = (1.5 \times 10^{16})^2/(3.1 \times 10^{20}) = \mathbf{2 \times 10^{12} \text{ m}^{-3}}$

**Example 51.8.** A specimen of pure germanium at  $300^\circ \text{ K}$  has a density of charge carriers of  $2.5 \times 10^{19}/\text{m}^3$ . It is doped with donor impurity atoms at the rate of one impurity atom for every  $10^6$  atoms of germanium. All impurity atoms may be supposed to be ionized. The density of germanium atom is  $4.2 \times 10^{28} \text{ atoms/m}^3$ .

Find the resistivity of the doped germanium if electron mobility is  $0.36 \text{ m}^2/\text{V-s}$ .

**Solution.** Density of added impurity atoms is

$$N_d = 4.2 \times 10^{28}/10^6 = 4.2 \times 10^{22} \text{ atoms/m}^3$$

\* It is the new name for the old unit of mho/m.



As seen, it is very large as compared to the intrinsic charge carrier density of  $2.5 \times 10^{19}/\text{m}^3$  which will, therefore, be neglected. Now, as seen from Art. 1.29.

$$\begin{aligned}\sigma_n &= N_d e \mu_e = 4.2 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.36 = 2.42 \times 10^3 \text{ S/m} \\ \rho_n &= 1/\sigma_n = 1/2.42 \times 10^3 = \mathbf{0.413 \times 10^{-3} \Omega\text{-m.}}\end{aligned}$$

**Example 51.9.** Compute the relative concentration of silicon atoms and electron-hole pairs at  $300^\circ\text{K}$ . Also, calculate intrinsic resistivity of silicon. Given Avogadro's number =  $6.02 \times 10^{23}$  atoms/g-atom, density =  $2.33 \times 10^6 \text{ g/m}^3$ , atomic Wt. = 28.1, intrinsic carrier density =  $1.5 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_e = 0.14 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$ .

**Solution.**

$$\begin{aligned}n_A &= \text{Avogadro's No.} \times \text{density/atomic Wt.} \\ &= 6.02 \times 10^{23} \times 2.33 \times 10^6 / 28.1 \cong 5 \times 10^{28} \text{ atoms/m}^3\end{aligned}$$

Since intrinsic concentration  $n_i$  i.e., electron-hole pairs/ $\text{m}^3$  is  $1.5 \times 10^{16}$

$$\therefore \frac{n_A}{n_i} = \frac{5 \times 10^{28}}{1.5 \times 10^{16}} \cong \mathbf{3.3 \times 10^{12}}$$

It means that there are  $3.3 \times 10^{12}$  Si atoms for each electron-hole pair.

Since in a pure semiconductor,  $n = p = n_i$ , intrinsic conductivity is given by

$$\sigma_i = n_i e (\mu_e + \mu_h) = 1.5 \times 10^{16} \times 1.6 \times 10^{-19} (0.14 + 0.05) = 0.456 \times 10^{-3} \text{ S/m}$$

Hence, intrinsic resistivity is given by

$$\rho_i = 1/\sigma_i = 1/0.456 \times 10^{-3} = \mathbf{2193 \Omega\text{-m}}$$

**Example 51.10.** Silicon is doped with acceptor atoms to a density of  $10^{22} \text{ m}^{-3}$ . If it is assumed that all acceptor centres are ionized, calculate the conductivity of the extrinsic silicon. Given that intrinsic density is  $1.4 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_e = 0.145 \text{ m}^2/\text{V-s}$  and  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$ .

(Electronic Devices & Circuits, Pune Univ. 1992)

**Solution.** The minority carrier density can be found from the equation given in Art. 1.30, i.e.  $np = n_i^2$ .

$$\text{Now, } p = 10^{22} \quad \therefore n \times 10^{22} = (1.4 \times 10^{16})^2; \quad n = 1.96 \times 10^{10}/\text{m}^3$$

$$\begin{aligned}\text{Now, } \sigma &= ne \mu_e + pe \mu_h \\ &= 1.96 \times 10^{10} \times 0.145 \times 1.6 \times 10^{-19} + 10^{22} \times 0.05 \times 1.6 \times 10^{-19} = \mathbf{80 \text{ S/m}}\end{aligned}$$

### 51.31. Drift

Directed motion of charge carriers in semiconductors occurs through two mechanisms :

- (i) charge drift under the influence of applied electric field and
- (ii) diffusion of charge from a region of high charge density to one of low charge density.

Consider the drift phenomenon first. When no electric field is applied to the semiconductor which is above  $0^\circ\text{K}$ , the conduction electrons (as well as holes) move within the crystal with random motion and repeatedly collide with each other and the fixed ions. Due to randomness of their motion, the net average velocity of these charge carriers in any given direction is zero. Hence, no current exists in the crystal under this condition of no field.

Now, consider the case when an electric field is applied to the crystal. Under the influence of this field, the charge carriers attain a *directed* motion which is superimposed on their random thermal motion. This results in a net average velocity called *drift* velocity in the direction of the applied electric field. Of course, electrons and holes move in opposite directions but because of their opposite charges, both produce current in the same direction. In extrinsic semiconductors, this current is essentially a majority carrier flow.

The drift velocity is proportional to electric field strength  $E$ , the constant of proportionality being called mobility  $\mu$ . The exact relation between the two is  $v = \mu E$ .

Let us find the value of drift current in a semiconductor.

(i) current density due to electron drift is  $J_e = e\mu_e nE$  where  $\mu_e$  is electron mobility,  $n$  is electron density and  $E$  is the electric field strength.

(ii) current density due to hole drift is  $J_h = e\mu_h pE$  where  $p$  is hole density.

Total current density due to electron and hole drift is

$$\begin{aligned} J &= J_e + J_h = e\mu_e nE + e\mu_h pE = e(n\mu_e + p\mu_h) E \\ &= e\mu_h (p + bn)E \quad \text{where } b = \mu_e/\mu_h \end{aligned}$$

### 51.32. Diffusion

It is gradual flow of charge from a region of high density to a region of low density. It is a force-free process based on non-uniform distribution of charge carriers in a semiconductor crystal. It leads to an electric current without the benefit of an applied field. This flow or diffusion of carriers is proportional to the carrier density gradient, the constant of proportionality being called diffusion constant or diffusion coefficient  $D$  which has a unit of  $\text{m}^2/\text{s}$ .

Current density due to hole diffusion is  $J_h = -eD_h dp/dx$ . Similarly, current density due to electron diffusion is  $J_e = eD_e dn/dx$ .

where  $D_e, D_h$  = electron and hole diffusion constants respectively  
 $dn/dx$  = density gradient of electrons  
 $dp/dx$  = density gradient of holes.

It is obvious that diffusion depends on charge in homogeneity or on the presence of a space gradient of charge density. It can occur in regions free of electric field. On the other hand, drift current is a function of both electric field and charge density.

Incidentally, it may be noted that, generally, diffusion leads to redistribution of charges which further results in the development of potential difference between different parts of the semiconductor. The electric field due to this potential difference sets up drift current in opposition to diffusion current. Final equilibrium is achieved when the potential difference developed becomes sufficiently large so as to create a drift current equal and opposite to the diffusion current thus resulting in zero net flow of current.

### 51.33. Combined Drift and Diffusion Currents

In semiconductors, drift and diffusion processes may be present simultaneously. The expressions for total electron and hole densities become.

$$J_e = e\mu_e nE + eD_e dn/dx \text{ A/m}^2 \quad \text{and} \quad J_h = e\mu_h pE - eD_h dp/dx \text{ A/m}^2$$

### 51.34. Relation Between $D$ and $\mu$

Both diffusion constant and mobility are statistical thermodynamic phenomena and are related to each other by the following equation.

$$\mu_e = \frac{e}{kT} D_e \quad \text{and} \quad \mu_h = \frac{e}{kT} D_h \quad \text{or} \quad \frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{kT}{e} = \frac{T}{11,600}$$

The relationship is known as Einstein's equation

At  $t = 23^\circ\text{C}$ ,  $T = 300^\circ\text{K}$ , hence  $D/\mu = 300/11,600 = 1/39$  or  $\mu = 39 D$

**Example 51.11.** Calculate diffusion constants for electrons and holes at  $300^\circ\text{K}$  in silicon if  $\mu_e = 0.15 \text{ m}^2/\text{V}\cdot\text{s}$  and  $\mu_h = 0.05 \text{ m}^2/\text{V}\cdot\text{s}$ .

**Solution.** According to Einstein's equation,

$$\begin{aligned} D &= \mu kT/e \quad \text{or} \quad D = \mu/39 \text{ m}^2/\text{s} \quad \text{--- at } 300^\circ\text{K} \\ D_e &= \mu_e/39 = 0.15/39 = 3.85 \times 10^{-3} \text{ m}^2/\text{s} \\ D_h &= \mu_h/39 = 0.05/39 = 6.4 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

**Example 51.12.** Find the diffusion coefficients of holes and electrons for germanium at 300 K. The carrier mobilities in  $\text{cm}^2/\text{volt-sec}$  at 300 K for electrons and holes are respectively 3600 and 1700. Density of carriers is  $2.5 \times 10^{13}$ . Boltzmann constant,  $k = 1.38 \times 10^{23}$ .

(U.P.S.C. Engg. Services, 1996)

**Solution.** According to the Einstein's equation,

$$D = \mu k T / e \quad \text{or} \quad D = \mu / 39 \text{ m}^2/\text{s} - \text{at } 300^\circ \text{K}$$

$$D_e = \mu_e / 39 = 3600 (\text{cm}^2/\text{V-s}) / 39 = 0.36 (\text{m}^2/\text{V-s}) / 39 = 9.2 \times 10^{-3} \text{ m}^2/\text{s}$$

$$D_h = \mu_h / 39 = 1700 (\text{cm}^2/\text{V-s}) / 39 = 0.17 (\text{m}^2/\text{V-s}) / 39 = 4.36 \times 10^{-3} \text{ m}^2/\text{s}$$

### 51.35. Recombination

Apart from drift and diffusion, a third phenomenon which occurs in semiconductors is called **recombination** that results from the collision of an electron with a hole.

The process is essentially the return of a free conduction electron to the valence band and is accompanied by the emission of energy. Obviously, the recombination rate is directly proportional to the carrier concentration for the simple reason that larger the number of carriers, the more likely is the occurrence of electron-hole recombination. This phenomenon is important in describing minority carrier flow.

As is well-known, in a semiconductor, thermal generation of electron-hole pairs also takes place continuously. Hence, there is **net recombination** rate given by the difference between the recombination and generation rates.

### 51.36. Carrier Life Time

It is defined as the time for which, on an average, a charge carrier will exist before recombination with a carrier of opposite charge. Its value varies from nanoseconds ( $10^{-9}$ ) to hundreds of microseconds ( $\mu\text{s}$ ) and depends on temperature and impurity concentration in the semiconductor material.

### 51.37. Total Carrier Flow

The total carrier flow in a semiconductor is the sum of the three flow phenomena discussed above. Each type of carrier has to be treated separately and the number of electrons or holes leaving the sample being accounted for by drift or diffusion or net recombination. The current in the semiconductor is then the sum of the electron and hole currents.

### 51.38. P-N Junction

It is possible to manufacture a single piece of a semiconductor material half of which is doped by P-type impurity and the other half by N-type impurity as shown in Fig. 51.32. The plane dividing the two zones is called **junction**. Theoretically, junction plane is assumed to lie where the density of donors and acceptors is equal. The P-N junction is fundamental to the operation of diodes, transistors and other solid-state devices.

Let us see if anything unusual happens at the junction. It is found that following three phenomena take place :

1. A thin **depletion layer** or region (also called space-charge region or transition region) is established on both sides of the junction and is so called because it is depleted of **free charge carriers**. Its thickness is about  $10^{-6}$  m.
2. A barrier potential or junction potential is developed across the junction.
3. The presence of depletion layer gives rise to junction and diffusion capacitances (Art. 51.5).

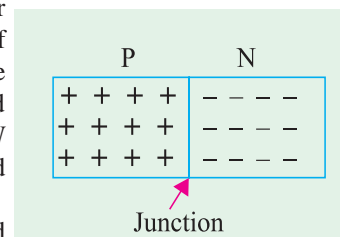
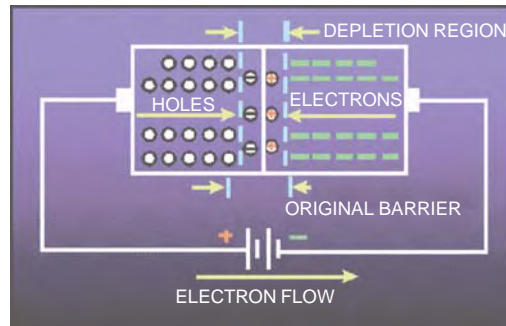


Fig. 51.32

### 51.39. Formation of Depletion Layer

Suppose that a junction has just been formed. At that instant, holes are still in the *P*-region and electrons in the *N*-region. However, there is greater concentration of holes in *P*-region than in *N*-region (where they exist as minority carriers). Similarly, concentration of electrons is greater in *N*-region than in *P*-region (where they exist as minority carriers). This concentration differences establishes density gradient across the junction resulting in carrier diffusion. Holes diffuse from *P* to *N*-region and electrons from *N*-to *P*-region and terminate their existence by recombination



[Fig. 51.33 (a)]. This recombination of free and mobile electrons and holes produces the narrow region at the junction called depletion layer. It is so named because this region is devoid of (or depleted of) *free and mobile charge carriers like electrons and holes*—there being present only positive ions which are not free to move.

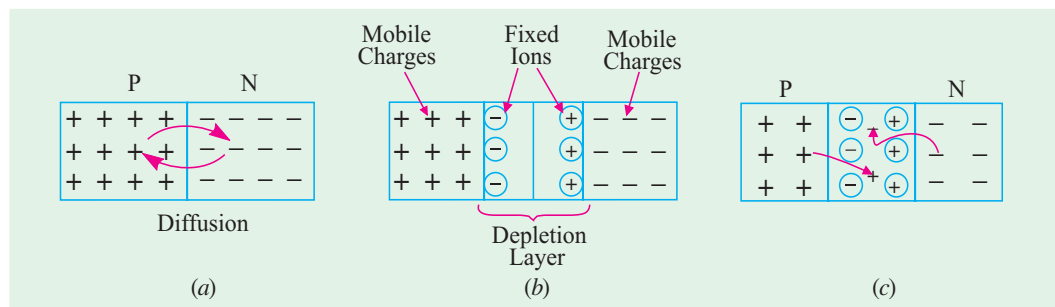
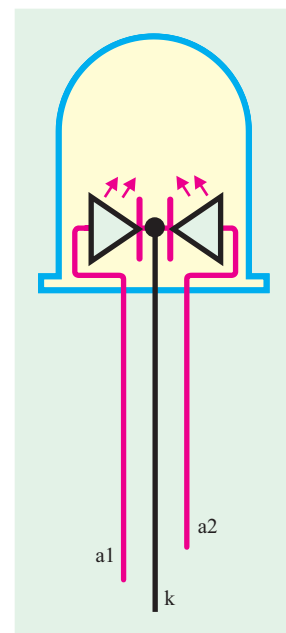


Fig. 51.33

It might seem from above that eventually all the holes from the *P*-side would diffuse to the *N*-side and all the electrons from the *N*-side would diffuse to the *P*-side but this does not occur due to the formation of ions on the two sides of the junction. The impurity atoms which provide these migratory electrons and holes are left behind in an ionized state bearing a charge which is opposite to that of the departed carrier. Also, these impurity ions, just like germanium atoms, are fixed in their positions in the crystal lattice in the *P*- and *N*- regions of the diode. Hence, as shown in Fig. 51.33 (b), they form parallel rows or ‘plates’ of opposite charges facing each other across the depletion layer. Obviously, row of *fixed* positive ions in the *N*-region is produced by the migration of electrons from the *N*- to *P*- region. Similarly, the row of *fixed* negative ions in the *P*-region is produced by the migration of holes from the *P*- to *N*-region.

If a majority carrier (either an electron or a hole) tries to cross into depletion layer, it can meet either of the following two facts :

- (i) either it can be trapped or captured by the row of fixed impurity ions of opposite sign which guard its own region. For example, a hole trying to approach the depletion layer may be neutralized by the row of fixed negative ions situated in the *P*-region itself



at the edge of the depletion layer. So will be the case with the electron trying to approach the depletion layer from *N*-region [Fig. 51.33 (c)]; or

- (ii) it may succeed in entering the depletion layer where it will be repelled by the row of similarly-charged impurity ions guarding the other region. But its life will be cut short by recombination with a majority carrier of opposite sign which has similarly entered the depletion layer from the other half of the diode.

Ultimately, an equilibrium condition is reached when depletion layer has widened to such an extent that no electrons or holes can cross the *P-N* junction.

### 51.40. Junction or Barrier Voltage

Even though depletion layer is cleared of charge carriers, it has oppositely-charged fixed rows of ions on its two sides. Because of this charge separation, an electric potential difference  $V_B$  is established across the junction even when the *junction is externally isolated* (Fig. 51.34). It is known as *junction or barrier potential*. It stops further flow of carriers across the junction unless supplied by energy from an external source. At room temperature of 300°K,  $V_B$  is about 0.3 V for Ge and 0.7 V for Si.

The value of barrier voltage is given by  $V_B = V_T \log_e N_a N_d / n_i^2$  where  $N_a, N_d, n_i^2$  and  $V_T$  have the meanings explained in Art. 51.29 and 34. The value of  $V_T$  at room temperature of 300°K is given by

$$V_T = V_{300} = \frac{kT}{e} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 26 \text{ mV}$$

$$\therefore V_B = 26 \log_e (N_a N_d / n_i^2) \text{ mV}$$

Barrier voltage depends on doping density, electronic charge and temperature. For a given junction, the first two factors are constant, thus making  $V_B$  dependent on temperature. With increase in temperature, more minority charge carriers are produced, leading to their increased drift across the junction. As a result, equilibrium occurs at a lower barrier potential. It is found that both for Ge and Si,  $V_B$  decreases by about 2 mV / °C.

$$\therefore \Delta V_B = -0.002 \Delta t \quad \text{where } \Delta t \text{ is the rise in temperature in } ^\circ\text{C}.$$

The strong field set up by  $V_B$  causes drift of carriers through depletion layer. As seen from [Fig. 51.35 (b)], under the influence of this field, *holes drift from N-to P-region and electrons from P-to -N region*. This drift current must be equal and opposite to the diffusion current [Fig. 51.35 (a)] because under condition of equilibrium and with no external supply, net current through the crystal is zero.

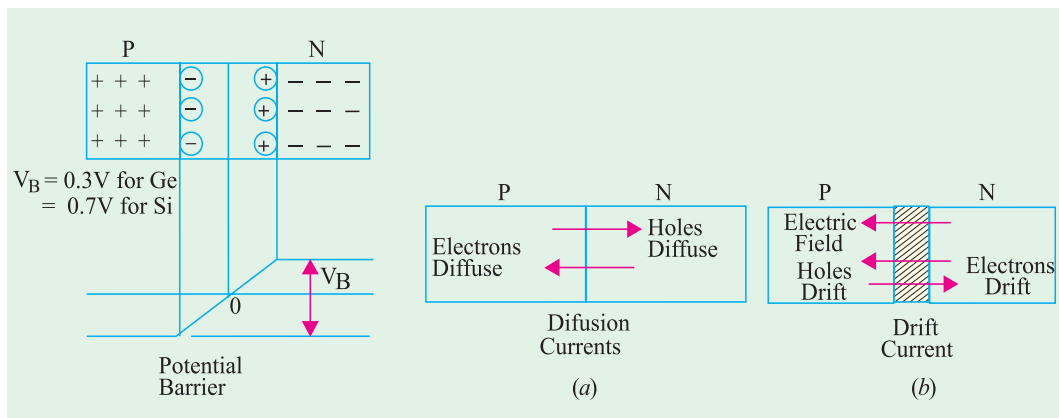


Fig. 51.34

Fig. 51.35

To summarize the main points of Art. 51.39 and 51.40, we have :

1. As soon as *P-N* junction is formed, free electrons and holes start diffusing across the junction and recombining.

2. Their recombination leads to the appearance of a depletion layer across the junction which contains no *mobile* carriers but only immobile ions.
3. These immobile ions set up a barrier potential and hence an electric field which sets up drift current that is equal and opposite to the diffusion current when final equilibrium is reached.

**Example 51.13.** Calculate the barrier potential at room temperature for P-N junction in silicon which is doped to a carrier density of  $10^{21} \text{ m}^{-3}$  on the P-side and  $10^{22} \text{ m}^{-3}$  on the N-side. The intrinsic carrier density for silicon is  $1.4 \times 10^{16} \text{ m}^{-3}$ . **(Electronics-I, Bangalore Univ. 1992)**

**Solution.** Using the relation given in Art 1.40, we have

$$V_B = 26 \log_e (N_a N_d / n_i^2) \text{ mV}$$

$$= 26 \log_e 10^{21} \times 10^{22} / (1.4 \times 10^{16})^2 = 641 \text{ mV} = \mathbf{0.641 \text{ V}}$$

**Example 51.14.** Calculate the change in barrier potential of a P-N junction at  $300^\circ\text{K}$  if doping on the N-side is increased 1000 times while keeping doping on P-side unchanged.

**Solution.** As seen from Art. 51.40, at  $300^\circ\text{K}$

$$V_B = 26 \log_e (N_a N_d / n_i^2) \text{ mV}$$

$$\therefore V_{B1} = 26 \ln (N_{a1} N_{d1} / n_i^2); \quad V_{B2} = 26 \ln (N_{a2} N_{d2} / n_i^2)$$

$$\therefore V_{B2} - V_{B1} = 26 \log_e (N_{a2} N_{d2}) / (N_{a1} N_{d1})$$

$$= 26 \log_e (N_{d2} / N_{d1}) = 26 \ln 1000 = \mathbf{179 \text{ mV}}$$

### 51.41. Energy Band Diagram of a P-N Junction

Now, let us consider the operation of a P-N junction in terms of its energy bands. At the instant of junction formation, energy bands of the trivalent impurity atoms in the P-region are at a slightly higher level than those of the pentavalent impurity atoms in the N-region as shown in Fig. 51.36. It is so because core attraction for valence electrons (+3) in a trivalent atom is less than the core attraction for valence electrons (+5) in a pentavalent atom. Consequently, trivalent valence electrons are in slightly higher orbit and, hence, at a higher energy level. However, there is some over-lap between respective bands of the two regions. Due to this reason, some high-energy electrons near the top of N-region conduction band diffuse into the lower part of the P-region conduction band. Soon after, they recombine with the holes in the valence band as shown in Fig. 51.36 (a). As diffusion continues, depletion layer begins to form. Another side-effect of this electron diffusion is that energy bands in

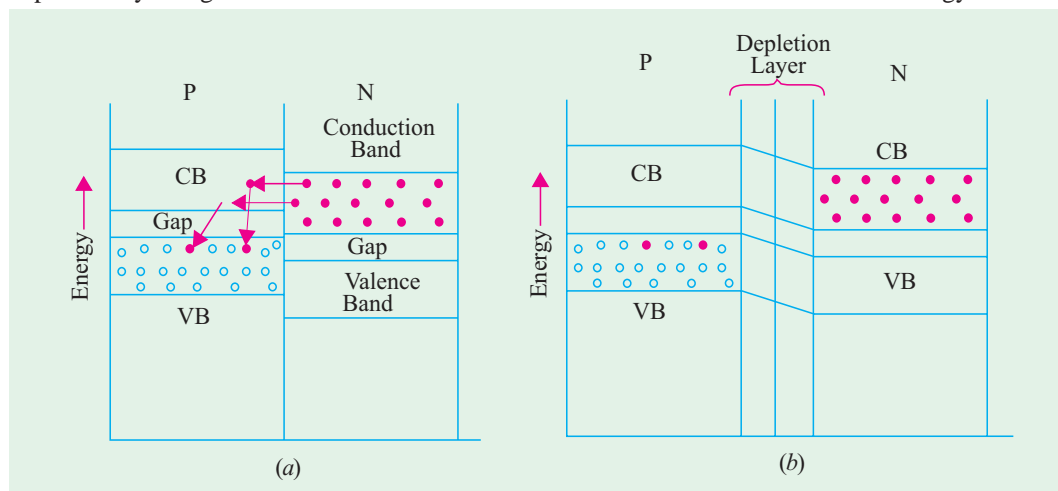


Fig. 51.36

the  $N$ -region shift downward due to loss of high-energy electrons. When the top of the conduction band in  $N$ -region reaches the same level as the bottom of the conduction band in  $P$ -region, further diffusion ceases and equilibrium condition is reached as shown in Fig. 51.36 (b). To an electron which might still try to diffuse across the junction, the path looks like a steep energy hill. It cannot climb this energy hill unless it receives energy from an external source.

### 51.42. Forward Biased P-N Junction

Suppose, positive battery terminal is connected to  $P$ -region of a semiconductor and the negative battery terminal to the  $N$ -region as shown in Fig. 51.37 (a). In that case the junction is said to be biased in the **forward direction** because it permits easy flow of current across the junction. This current flow may be explained in the following two ways :

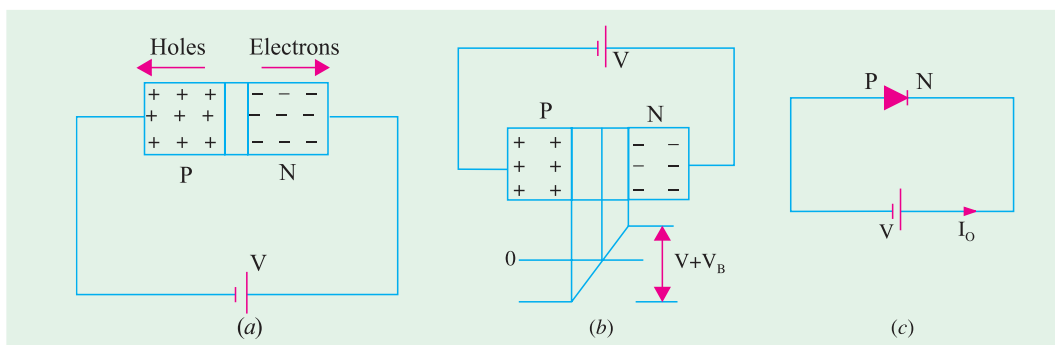


Fig. 51.37

- (i) As soon as battery connection is made, holes are **repelled** by the positive battery terminal and electrons by the negative battery terminal with the result that both the electrons and the holes are driven **towards** the junction where they recombine. This **en masse** movement of electrons to the left and that of holes to the right of the junction constitutes a large current flow through the semiconductor. Obviously, the junction offers **low resistance** in the forward direction.

A more detailed picture of carrier flow is as under :

As free electrons move to the left, new **free** electrons are injected by the negative battery terminal into the  $N$ -region of the semiconductor. Thus, a flow of electrons is set up in the wire connected to the negative battery terminal. As holes are driven towards the junction, more holes are created in the  $P$ -region by the breakage of covalent bonds (Art. 51.21). These newly-created holes are driven towards the junction to keep up a continuous supply. But the electrons so produced are attracted to the left by the positive battery terminal from where they go to the negative terminal and finally to the  $N$ -region of the crystal.

Incidentally, it may be noted that though there is movement of both electrons and holes **inside** the crystal, **only free electrons move in the external circuit i.e.** in the battery-connecting wires.

**Note.** There is also present an extremely small amount of current  $I_0$  due to minority carriers on either side of the junction. But it is negligible as compared to forward current which is due to majority carriers. This current  $I_0$  depends on temperature but is almost independent of applied voltage.

- (ii) Another way to explain current flow in forward direction is to say that forward bias of  $V$  volts lowers the barrier potential to  $(V - V_B)$  which now allows more current to flow across the junction [Fig. 51.37 (b)].

Incidentally, it may be noted that forward bias reduces the thickness of the depletion layer as shown in Fig. 51.38.

Energy band diagram for forward bias is shown in Fig. 51.39. By comparing this figure with Fig.

51.36 it is seen that energy hill has been reduced. Because of this reduction, conduction electrons in *N*-region are able to cross over to *P*-region. After reaching there, each electron falls into a hole (path A) and becomes a valence electron. In this way, it is able to continue its journey towards the left end of the crystal.

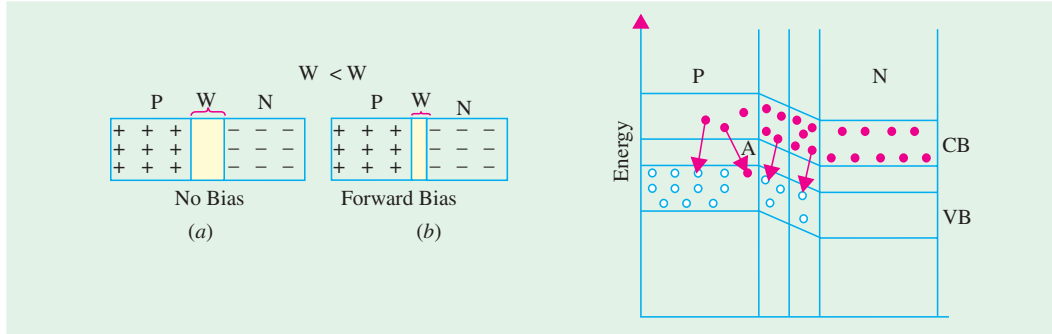


Fig. 51.38

Fig. 51.39

### 51.43. Forward V/I Characteristic

A typical *V/I* characteristic for a forward-biased *P-N* junction is shown in Fig. 51.40. It is seen that forward current rises exponentially with the applied forward voltage. However, at ordinary room temperature, a p.d. of about 0.3 V is required before a reasonable amount of forward current starts flowing in a germanium junction. This voltage is known as **threshold voltage ( $V_{th}$ )** or **cut-in voltage** or **knee voltage  $V_K$** . It is practically the same as barrier voltage  $V_B$ . Its value for silicon junction is about 0.7 volt. For  $V < V_{th}$ , current flow is negligible. But as applied voltage increases beyond the threshold value, the forward current increases sharply. If forward voltage is increased beyond a certain safe value, it will produce an extremely large current which may destroy the junction due to overheating.

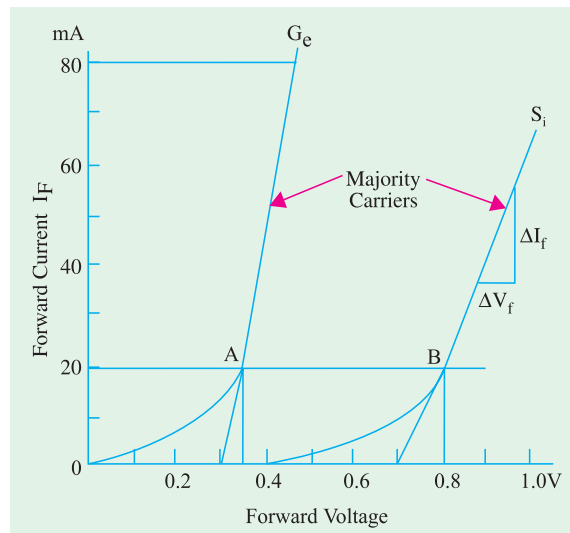


Fig. 51.40

*Ge* devices can stand junction temperatures around 100°C whereas *Si* units can function upto 175°C.

Obviously, the **forward-biased junction has a low resistance**. For point B in Fig. 51.40, the forward resistance for Si is

$$R_F = 0.8V/20 \text{ mA} = 40 \Omega$$

Similarly, for point A on the *Ge* curve,  $R_F = 0.36 \text{ V}/20 \text{ mA} = 18 \Omega$

In practice, this static forward resistance is not used. Instead, the **dynamic** resistance or **incremental** resistance or **ac resistance** of the junction is used. It is given by the reciprocal of the slope of the forward characteristic.

$$r_{ac} = \frac{1}{\Delta I_F / \Delta V_F} = \frac{\Delta V_F}{\Delta I_F}$$

Here,

$$\Delta V_F = 0.19 \text{ V} \quad \text{and} \quad \Delta I_F = 37.6 \text{ mA}$$



$$\therefore r_{ac} = \frac{0.19}{37.6 \times 10^{-3}} \cong 5 \Omega$$

### 51.44. Reverse Biased P-N Junction

When battery connections to the semiconductor are made as shown in Fig. 51.41 (a), the junction is said to **reverse-biased**. In this case, holes are attracted by the negative battery terminal and electrons by the positive terminal so that both holes and electrons move **away** from the junction and **away** from each other. Since there is no electron-hole combination, no current flows and the junction offers high resistance.

Another way of looking at the process is that in this case, the applied voltage increases the barrier potential to  $(V + V_B)$ , thereby blocking the flow of majority carriers.

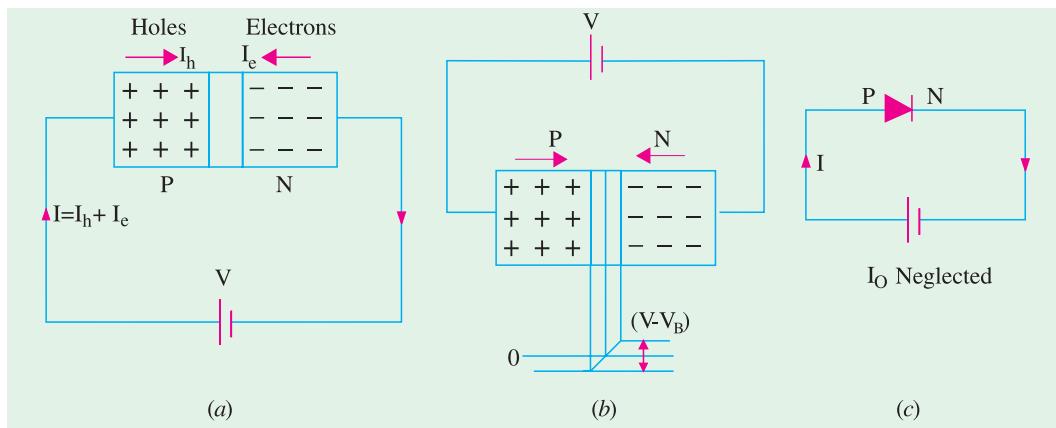


Fig. 51.41

Incidentally, it may be noted that under reverse bias condition, width of depletion layer is increased because of increased barrier potential as shown in Fig. 51.42.

Although, in this case, there is practically no current due to **majority** carriers, yet there is a small amount of current (a few  $\mu A$  only) due to the flow of **minority** carriers across the junction. As explained earlier in Art. 51.24, due to thermal energy, there are always generated some holes in the *N*-type region and some electrons in the *P*-type region of the semiconductor as shown in Fig. 51.28. The battery drives these minority carriers across the junction thereby producing a small current called **reverse current or reverse saturation current  $I_0$  or  $I_S$** . Since minority carriers are thermally-generated,  $I_0$  is extremely **temperature dependent**. For the same reason, forward current is also temperature dependent but to a much less degree because minority current forms a very small percentage of the majority current. The name saturation has been used because we cannot get minority current more than what is produced by thermal energy. In other words,  **$I_S$  does not increase with increase in reverse bias**.

$I_S$  is found to increase approximately 7 percent per  $0^\circ C$  rise in temperature both for Ge and Si. Since,  $(1.07)^{10} = 2$ , it means that reverse current **doubles for every  $10^\circ C$  rise in temperature**. It is worth noting that reverse saturation current is also referred to as leakage current of the *P-N* junction diode.

With reverse bias, energy hill becomes too steep for majority carriers to go up the hill and cross over.

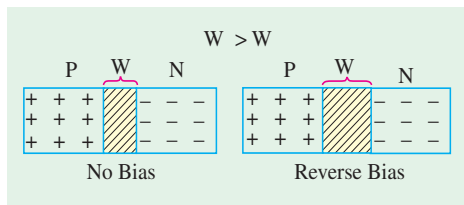


Fig. 51.42

### 51.45. Reverse V/I Characteristic

As said earlier, the reverse saturation current is also referred to as **leakage current** of the *P-N* junction. Fig. 51.43 shows *V/I* characteristics of a reverse-biased *P-N* junction. It is seen that as reverse voltage is increased from zero, the reverse current quickly rises to its maximum or saturation value. Keeping temperature constant as the reverse voltage is increased,  $I_o$  is found to increase only slightly. This slight increase is due to the impurities on the surface of the semiconductor which behaves as a resistor and hence obeys Ohm's law. This gives rise to a very small current called **surface leakage current**.

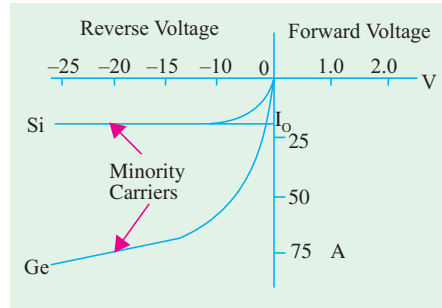


Fig. 51.43

Unlike the main leakage (or saturation) current, this surface leakage current is independent of temperature but depends on the magnitude of the reverse voltage.

A reverse-biased junction can be represented by a very large resistance. As seen from Fig. 51.43, in the case of Si, for a reverse voltage of about 15 V,  $I_o = 10 \mu\text{A}$ . Hence, reverse resistance is  $R_R = 15 \text{ V} / 10 \mu\text{A} = 1.5 \text{ M}\Omega$

### 51.46. Hall Effect

If a specimen (whether of a metal or a semiconductor) carrying a current *I* is placed in a **transverse** magnetic field of flux density *B*, an electric field is developed along a direction perpendicular to both *B* and *I*. This phenomenon is known as **Hall effect** and is used for the following purposes :

1. to determine whether a semiconductor is of *N*-type or *P*-type;
2. to find carrier concentration;
3. to measure the conductivity of the material;
4. to find carrier mobility;
5. to detect and measure magnetic fields one million times smaller than that of earth with the help of Hall-effect magnetometers.

As shown in Fig. 51.44 (a), a current *I* is flowing through the semiconductor in the direction *MN* under the influence of an external applied electric field  $E_L$ . Obviously, electrons comprising this current move along *NM* with a velocity of  $v_e$ . Consider one such electron shown in the figure. The direction of the force exerted on it by the magnetic field *B* can be found by using Fleming's left-hand rule and is as shown\*. The magnitude of force is  $Be v_e$ .

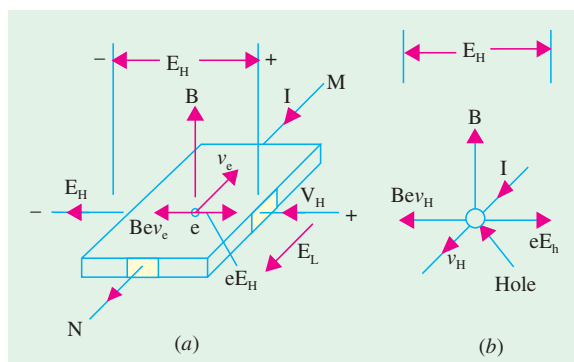


Fig. 51.44

Under the influence of this force, electrons will tend to crowd towards the left side of semiconductor. This collection of electrons to one side gives rise to electric potential difference  $V_H$  (called Hall voltage) and hence to an electric field  $E_H$ . This field begins to prevent additional electrons from arriving there. Force exerted by this field on the electrons is  $eE_H$ . Equilibrium is established when the two oppositely-directed magnetic and electric forces acting on the electron become equal in magnitude.

\* While using the rule, the middle finger must point in the direction of conventional current *i.e.* along *MN* and not along the direction of electronic current *i.e.* along *NM*.

$$\begin{aligned} \text{i.e.} \quad & B e v_e = e E_H \\ \text{or} \quad & B v_e = E_H \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{Also} \quad & v_e = \mu_e E_L \\ \therefore \quad & B \mu_e E_L = E_H \quad \text{or} \quad \mu_e B = E_H / E_L \end{aligned}$$

Hence, knowing  $E_L$  and  $B$  and measuring  $E_H$ , we can find electron mobility.

$$\text{Now, current density} \quad J = I/A = n e v_e A / A = n e v_e$$

Substituting the value of  $v_e$  from (i) above, we get

$$J = n e E_H / B \quad \text{or} \quad n = J B / e E_H$$

This equation may be used to find electron density.

$$\text{Now, Hall coefficient,} \quad R_H = 1/\text{charge density} = 1/n e \quad \text{or} \quad R_H = E_H / J B$$

$$\text{As shown in Art. 51.26.} \quad \mu_e = 1/n e \rho = R_H / \rho$$

where  $\rho$  is the resistivity of the semiconductor.

It may be noted that the above treatment is equally applicable to  $P$ -type semiconductors where current flow is made up of hole movement. If holes were to move in the same direction as that of the electrons in Fig. 51.44 (a), then polarity of Hall voltage  $E_H$  would be reversed as shown in Fig. 51.44 (b). In fact, it was the observation of the polarity of Hall voltage associated with  $P$ -type material which led to the concept of a positive hole.

**Example 51.15.** The Hall-coefficient of a specimen of doped semiconductor is  $3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$  and the resistivity of the specimen is  $8.93 \times 10^{-3} \Omega\text{-m}$ . Determine the carrier mobility in  $\text{m}^2 \text{ v}^{-1} \text{ s}^{-1}$ .

**Solution.** Using equation  $\mu_e = R_H / \rho$

$$\mu_e = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}} = 0.041 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

**Example 51.16.** A sample of  $N$ -type semiconductor has a hall coefficient of  $160 \text{ cm}^3/\text{Coulomb}$ . If its resistivity is  $0.16 \Omega\text{-cm}$ , estimate the electron mobility in the sample,

**Solution.** Using the relation,  $\mu_e = R_H / \rho$ , we have

$$\mu_e = \frac{160}{0.16} = 1000 \text{ cm}^2/\text{Volt-Sec.}$$

**Example 51.17.** A current of  $50 \text{ A}$  is passed through a metal strip, which is subjected to a magnetic flux of density  $1.2 \text{ Wb/m}^2$ . The magnetic field is directed at right angles to the current direction and the thickness of the strip in the direction of magnetic field is  $0.5 \text{ mm}$ . The Hall voltage is found to be  $100 \text{ V}$ . Calculate the number of conduction electrons per cubic metre in the metal.

**Solution.** We know that  $n = \frac{J B}{e E_H}$

Since  $J = I/A = I/d.t$  where  $d$  is the distance between the two surfaces between the ends across which the electric field is measured. The above equation can be rewritten as,

$$n = \frac{(I/A) B}{e E_H} = \frac{(I/d.t) B}{e V_H / d} = \frac{B I}{e V_H t} \quad (\because E_H = V_H / d)$$

Substituting the values of  $V_H$ ,  $t$ ,  $B$ ,  $I$  and  $e$ , we get

$$n = \frac{1.2 \times 50}{(1.6 \times 10^{-19}) \times 100 \times (0.5 \times 10^{-3})} = 7.51 \times 10^{21} / \text{m}^3$$

**Example 51.18.** An  $N$ -type semiconductor has a resistivity of  $20 \times 10^{-2} \text{ ohm-m}$ . The mobility of electrons through a separate experiment was found to be  $100 \times 10^{-4} \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$ . Find the number of electron carriers per  $\text{m}^3$ .

**Solution.** Using the equation,  $\mu_e = 1/n e \rho$ , we have

$$n = \frac{1}{e\mu_e\rho} = \frac{1}{1.6 \times 10^{-19} \times (100 \times 10^{-4}) \times (20 \times 10^{-2})}$$

$$= 3.1 \times 10^{21}/\text{m}^3$$

**Example 51.19.** The Hall-coefficient of a specimen of doped silicon is found to be  $3.66 \times 10^{-4} \text{ m}^3\text{C}$ ; the resistivity of the specimen is  $8.93 \times 10^{-3} \Omega\text{-m}$ . Find the mobility and density of charge carriers assuming single carrier conduction. (U.P.S.C. Engg. Services 2002)

**Solution.**  $R_H = 3.66 \times 10^{-4} \text{ m}^3\text{C}$ ;  $\rho = 8.93 \times 10^{-3} \Omega\text{-m}$ .

$$\mu = \sigma R_H = \frac{1}{\rho} R_H = \frac{1}{8.93 \times 10^{-3}} \times 3.66 \times 10^{-4} = 0.041 \text{ m}^2/\text{V.s}$$

$$R_H = \frac{1}{ne} \text{ or } n = \frac{1}{R_H e} = \frac{1}{(3.66 \times 10^{-4}) \times (1.6 \times 10^{-19})} = 1.7 \times 10^{22}/\text{m}^3$$

**Example 51.20.** Resistivity of a sample semiconductor is  $9 \text{ m} \Omega\text{-m}$ . Its holes have mobility of  $0.03 \text{ m}^2/\text{V.s}$ . Calculate the value of Hall-coefficient of the sample. (U.P.S.C. Engg. Services 2002)

**Solution.**  $\rho = 9 \text{ m} \Omega\text{-m} = 9 \times 10^{-3} \Omega\text{-m}$ ,  $\mu_p = 0.03 \text{ m}^2/\text{V.s}$

$$\sigma = \frac{1}{\rho} = \frac{1}{9 \times 10^{-3}} = 111.1 \text{ S/m}$$

$$\mu_p = \sigma R_H$$

$$R_H = \frac{\mu_p}{\sigma} = \frac{0.03}{111.1} = 2.7 \times 10^{-4} \text{ m}^3\text{C}$$

### Tutorial Problem No. 51.1

1. Compute the intrinsic conductivity of a specimen of pure silicon at room temperature given that  $n_i = 1.4 \times 10^6 \text{ m}^{-3}$ ,  $\mu_e = 0.145 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ . Also, calculate the individual contributions from electrons and holes.  
[0.437  $\times 10^{-3} \text{ S/m}$  ; 0.325  $\times 10^{-3} \text{ S/m}$  ; 0.112  $\times 10^{-3} \text{ S/m}$ ]
2. Find (i) conductivity and (ii) resistance of a bar of pure silicon of length 1 cm and cross-sectional area  $1 \text{ mm}^2$  at  $300^\circ\text{K}$ . Given :  $n_i = 1.5 \times 10^{16} \text{ per m}^3$ ,  $\mu_e = 0.13 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ .  
[(i) 4.32  $\times 10^{-4} \text{ S/m}$  (ii) 23.15 M $\Omega$ ]
3. A specimen of silicon is doped with acceptor impurity to a density of  $10^{22}$  atoms per  $\text{m}^3$ . Given that  $n_i = 1.4 \times 10^{16} \text{ per m}^3$ ,  $\mu_e = 0.145 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ . All impurity atoms may be assumed to be ionized.  
[nearly 80 S/m]
4. Calculate the conductivity of a specimen of pure Si at room temperature of  $300^\circ\text{K}$  for which  $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$ ,  $\mu_e = 0.13 \text{ m}^2/\text{V-s}$ ,  $\mu_h = 0.05 \text{ m}^2/\text{V-s}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ . The Si specimen is now doped 2 parts per  $10^8$  of a donor impurity. If there are  $5 \times 10^{28} \text{ Si atoms/m}^3$ , calculate its conductivity. By what factor has the conductivity increased ?  
[4.32  $\times 10^{-4} \text{ S/m}$  ; 20.8 S/m ;  $\approx 48,000$ ]
5. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are  $0.36 \text{ m}^2/\text{V-s}$  and  $0.17 \text{ m}^2/\text{V-s}$  respectively. If the electron and hole densities are each equal to  $2.5 \times 10^{19} \text{ per m}^3$ , calculate the conductivity.  
[2.12 S/m](Electronic-I, Bangalore Univ.)
6. A p.d. of 10 V is applied longitudinally to a rectangular specimen of intrinsic germanium of length 2.5 cm, width 0.4 cm and thickness 0.15 cm. Calculate at room temperature  
(i) electron and hole drift velocities;  
(ii) the conductivity of intrinsic Ge if intrinsic carrier density is  $= 2.5 \times 10^{19}/\text{m}^3$ ;

(iii) the total current.

Given,  $\mu_e = 0.38 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ ,  $\mu_h = 0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ ,  $k = 1.38 \times 10^{-23} \text{ J-deg}^{-1}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$

**[(i) 152 m/s, 72 m/s (ii) 2.24 S/m (iii) 5.38 mA]**

**(Applied Electronics & Circuits, Grad. I.E.T.E. 1987)**

5. The resistivity of a doped silicon material is  $9 \times 10^{-3}$ . The Hall coefficient is  $3.6 \times 10^{-4} \text{ Coulomb}^{-1}$ . Assuming single carrier conduction, find the mobility and density of charge carriers.  $e = 1.6 \times 10^{-19} \text{ Coulomb}$ .  
**(U.P.S.C. Engg. Services 1994)**
6. Consider an intrinsic Ge bar with material time constant of  $100 \mu\text{sec}$  across a cross-section of  $1 \text{ mm}^2$  and length  $1 \text{ cm}$ . One side of the bar is illuminated with  $10^{15}$  photons/sec. Assume that each incident photon generate one electron hole pair and these are uniformly distributed throughout the bar. Find the bar resistance under constant excitation at room temperature.  
**(Electronic Devices and Circuits, Nagpur Univ. Summer, 2004)**
7. Find the concentration of holes and electrons in a  $p$ -type germanium at  $300\text{K}$ , if the conductivity is  $100 \text{ per ohm-cm}$ . Also find these values for  $n$ -type silicon, if the conductivity is  $0.1 \text{ per ohm-cm}$ . Given that  
for germanium  $n_i = 2.5 \times 10^{13}/\text{cm}^3$   
 $M_n = 3800 \text{ cm}^2/\text{V-s}$ ,  $M_p = 1800 \text{ cm}^2/\text{V-s}$   
for silicon,  $n_i = 1.5 \times 10^{10} \text{ per cm}^3$   
 $M_n = 1300 \text{ cm}^2/\text{V-s}$  and  $M_p = 500 \text{ cm}^2/\text{V-s}$   
**(Electronics Engg; Bangalore Univ. 2004)**
8. A germanium P-N junction at  $300 \text{ K}$  has the following parameters;  $N_D = 5 \times 10^{18}/\text{cm}^3$ ,  $N_A = 6 \times 10^{16}/\text{cm}^3$ ,  $n_i = 1.5 \times 10^{10}/\text{cm}^3$ ; Calculate the minority electron density in the P-region and the minority hole density in the N-region.  
**(Electronics Engg. Bangalore Univ. 2003)**

### OBJECTIVE TESTS – 51

- The total energy of a revolving electron in an atom can
  - have any value above zero
  - never be positive
  - never be negative
  - not be calculated.
- An atom is said to be ionised when any one of its orbiting electron
  - Jumps from one orbit to another
  - is raised to a higher orbit
  - comes to the ground state
  - is completely removed.
- The maximum number of electrons which the M-shell of an atom can contain is
  - 32
  - 8
  - 18
  - 50.
- Electronic distribution of an Si atom is
  - 2, 10, 2
  - 2, 8, 4
  - 2, 7, 5
  - 2, 4, 8.
- Semiconductor materials have ..... bonds.
  - ionic
  - covalent
  - mutual
  - metallic.
- The maximum number of electrons which the valence shell of an atom can have is
  - 6
  - 8
  - 18
  - 2
- Silicon has  $Z = 14$ . Its outermost orbit is
  - partially filled
  - half filled
  - completely occupied
  - empty
- Major part of the current in an intrinsic semiconductor is due to
  - conduction-band electrons
  - valence-band electrons
  - holes in the valence band
  - thermally-generated electron.
- Conduction electrons have more mobility than holes because they
  - are lighter
  - experience collisions less frequently
  - have negative charge
  - need less energy to move them.
- Doping materials are called impurities because they

- (a) decrease the number of charge carriers  
 (b) change the chemical properties of semiconductors  
 (c) make semiconductors less than 100 percent pure  
 (d) alter the crystal structures of the pure semiconductors.
11. Current flow in a semiconductor depends on the phenomenon of  
 (a) drift (b) diffusion  
 (c) recombination (d) all of the above.
12. The process of adding impurities to a pure semiconductor is called  
 (a) mixing (b) doping  
 (c) diffusing (d) refining.
13. The most widely used semiconducting material in electronic devices is  
 (a) germanium (b) silicon  
 (c) copper (d) carbon
14. Electron-hole pairs are produced by  
 (a) recombination (b) thermal energy  
 (c) ionization (d) doping
15. Recombination takes place when  
 (a) an electron falls into a hole  
 (b) a positive and a negative ion bond together  
 (c) a valence electron becomes a conduction  
 (d) a crystal is formed
16. When a *P-N* junction is formed, diffusion current causes  
 (a) mixing of current carriers  
 (b) forward bias  
 (c) reverse bias  
 (d) barrier potential.
17. The leakage current of a *P-N* diode is caused by  
 (a) heat energy (b) chemical energy  
 (c) barrier potential (d) majority carriers.
18. Electronic components which are made of a semiconductor material are often called ..... devices.  
 (a) solid-state (b) silicon  
 (c) germanium (d) intrinsic.
19. Any voltage that is connected across a *P-N* junction is called ..... voltage.  
 (a) breakdown (b) barrier  
 (c) bias (d) reverse.
20. The area within a semiconductor diode where no mobile current carriers exist when it is formed is called ..... region.  
 (a) depletion (b) saturation  
 (c) potential barrier (d) space charge.
21. The depletion region of a semiconductor diode is due to  
 (a) reverse biasing  
 (b) forward biasing  
 (c) crystal doping  
 (d) migration of mobile charge carriers.
22. If an intrinsic semiconductor is doped with a very small amount of boron, then in the extrinsic semiconductor so formed, the number of electrons and holes will,  
 (a) decrease  
 (b) increase and decrease respectively  
 (c) increase  
 (d) decrease and increase respectively.
23. Two initially identical samples A and B of pure germanium are doped with donors to concentrations of  $1 \times 10^{20}$  and  $3 \times 10^{20}$  respectively. If the hole concentration in A is  $9 \times 10^{12}$ , then the hole concentration in B at the same temperature will be  
 (a)  $3 \times 10^{12} \text{ m}^{-3}$  (b)  $7 \times 10^{12} \text{ m}^{-3}$   
 (c)  $11 \times 10^{12} \text{ m}^{-3}$  (d)  $27 \times 10^{12} \text{ m}^{-3}$
24. Hall effect is observed in a specimen when it (metal or a semiconductor) is carrying current and is placed in a magnetic field. The resultant electric field inside the specimen will be in  
 (a) a direction normal to both current and magnetic field  
 (b) the direction of current  
 (c) a direction antiparallel to the magnetic field  
 (d) an arbitrary direction depending upon the conductivity of the specimen.

ANSWERS

- |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (d)  | 3. (c)  | 4. (b)  | 5. (b)  | 6. (b)  | 7. (b)  |
| 8. (a)  | 9. (d)  | 10. (d) | 11. (d) | 12. (b) | 13. (b) | 14. (b) |
| 15. (a) | 16. (d) | 17. (a) | 18. (a) | 19. (c) | 20. (a) | 21. (d) |
| 22. (d) | 23. (a) | 24. (a) |         |         |         |         |

# CHAPTER 52

## Learning Objectives

- P-N Junction Diode
- Derivation of Junction Resistance
- Junction Breakdown
- Junction Capacitance
- Equivalent Circuit of P-N Junction
- Diode Fabrication
- Grown Junction
- Alloy Junction
- Diffused Junction
- Epitaxial Junction
- Point Contact Junction
- The Ideal Diode
- The Real Diode
- Diode Circuits with D.C. and A.C. Voltage Sources
- Diode Clipper and Clamper Circuits
- Clippers
- Some Clipping Circuits
- Clampers
- Summary of Clamping Circuits

## P-N JUNCTION DIODE



Chemist, led the research for the molecular diode (In the semiconductor industry, called p-n junctions)

52.1. P-N Junction Diode

(a) Construction

It is a two-terminal device consisting of a P-N junction formed either in Ge or Si crystal. Its circuit symbol is shown in Fig. 52.1 (a). The P and N-type regions are referred to as anode and cathode respectively. In Fig. 52.1 (b), arrow-head indicates the conventional direction of current flow when forward-biased. It is the same direction in which hole flow takes place.

Commercially available diodes usually have some means to indicate which lead is P and which lead is N. Standard notation consists of type numbers preceded by 'IN' such as IN 240 and IN 1250. Here, 240 and 1250 correspond to colour bands. Fig. 52.2 (a) shows typical diodes having a variety of physical structures whereas Fig. 52.2 (b) illustrates terminal identifications. Also refer to the picture of two commercial diodes shown in Fig 52.1(c).

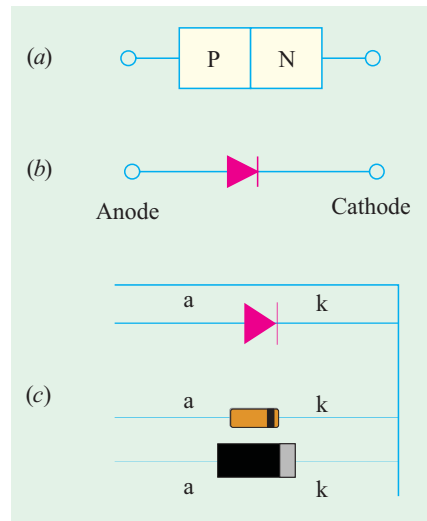


Fig. 52.1

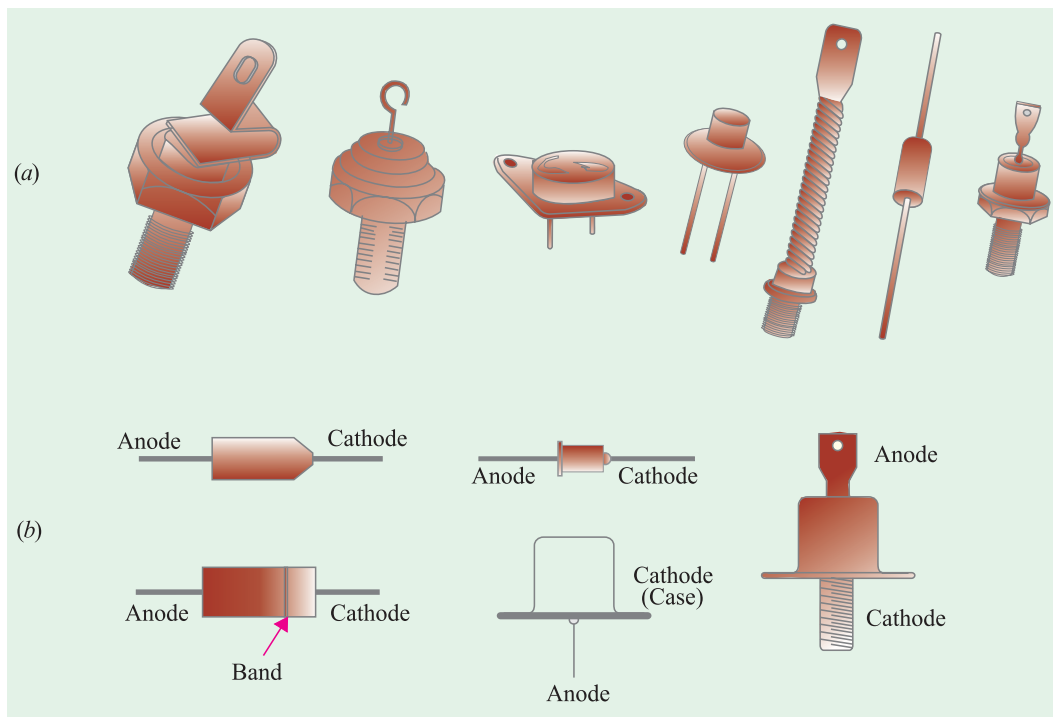


Fig. 52.2

The low-current diodes whose body is about 3 mm long can carry a forward current of about 100 mA, have saturation current of 5  $\mu$ A at room temperature (25°C) and can withstand a reverse voltage of 75 V without breaking down. The medium-current diodes can pass a forward current of about 500 mA and can withstand a reverse voltage of 250 V. The high-current diodes or power diodes can pass a forward current of many amperes and can survive several hundred volts of reverse voltage.

(b) Diode Mounting

Low and medium-current diodes are usually mounted by soldering their leads to the connecting



terminals. The heat generated by these diodes (when operating) is small enough to be carried away by air convection and conduction along the connecting leads. However, high-current stud-mounted diodes generate large amounts of heat for which air convection is totally inadequate. For cooling, they need heat sinks made of metals such as copper or aluminium which are good conductors of heat. The sink absorbs heat from the device and then transfers it to the surrounding air by convection and radiation since it has large surface area.

**(c) Working**

A P-N junction diode is one-way device offering low resistance when forward-biased [Fig. 52.3 (a)] and behaving almost as an insulator when reverse-biased [Fig. 52.3 (b)]. Hence, such diodes are mostly used as rectifiers *i.e.* for converting alternating current into direct current.

**(d) V/I Characteristic**

Fig. 52.4 shows the static voltage-current characteristics for a low-power P-N junction diode.

**1. Forward Characteristic**

When the diode is forward-biased and the applied voltage is increased from zero, hardly any current flows through the device in the beginning. It is so because the external voltage is being opposed by the internal barrier voltage  $V_B$  whose value is 0.7 V for Si and 0.3 V for Ge. As soon as  $V_B$  is neutralized, current through the diode increases rapidly with increasing applied battery voltage. It is found that as little a voltage as 1.0 V produces a forward current of about 50 mA. A burnout is likely to occur if forward voltage is increased beyond a certain safe limit.

**2. Reverse Characteristic**

When the diode is reverse-biased, majority carriers are blocked and only a small current (due to minority carriers) flows through the diode. As the reverse voltage is increased from zero, the reverse current very quickly reaches its maximum or saturation value  $I_0$  which is also known as **leakage current**. It is of the order of nanoamperes (nA) for Si and microamperes ( $\mu A$ ) for Ge. The value of  $I_0$  (or  $I_s$ ) is independent of the applied reverse voltage but depends on (a) temperature, (b) degree of doping and (c) physical size of the junction.

As seen from Fig. 52.4, when reverse voltage exceeds a certain value called break-down voltage  $V_{BR}$  (or Zener voltage  $V_z$ ), the leakage current suddenly and sharply increases, the curve indicating zero resistance at this point. Any further increase in voltage is likely to produce burnout unless protected by a current-limiting resistor.

When P-N junction diodes are employed primarily because of this breakdown property as voltage regulators, they are called Zener diodes (Art. 54.1).

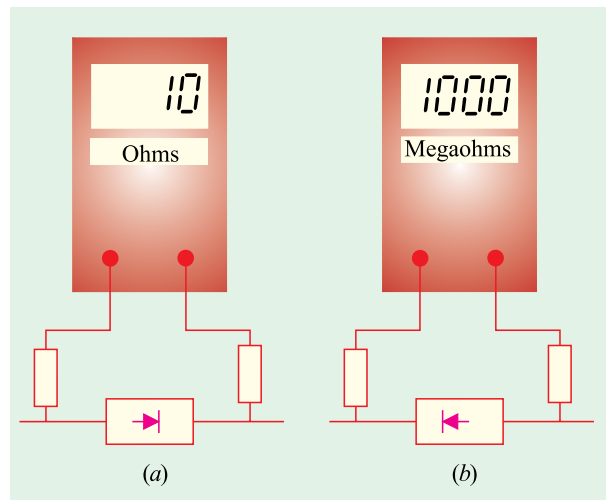
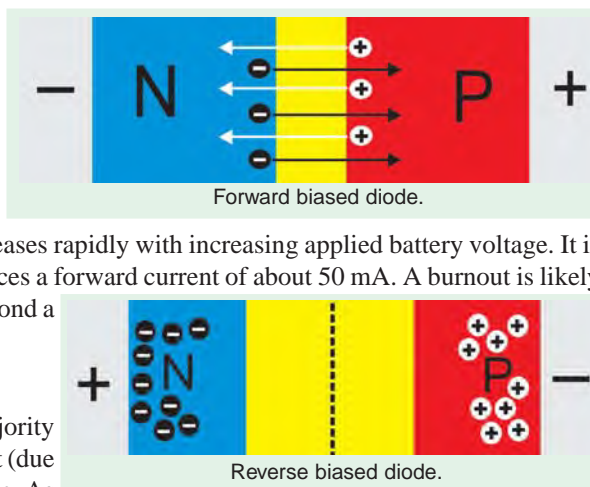


Fig. 52.3



### 3. Equation of the Static Characteristic

The volt-ampere characteristics described above are called *static* characteristics because they describe the d.c. behaviour of the diode. The forward and reverse characteristics have been combined into a single diagram of Fig. 52.4.

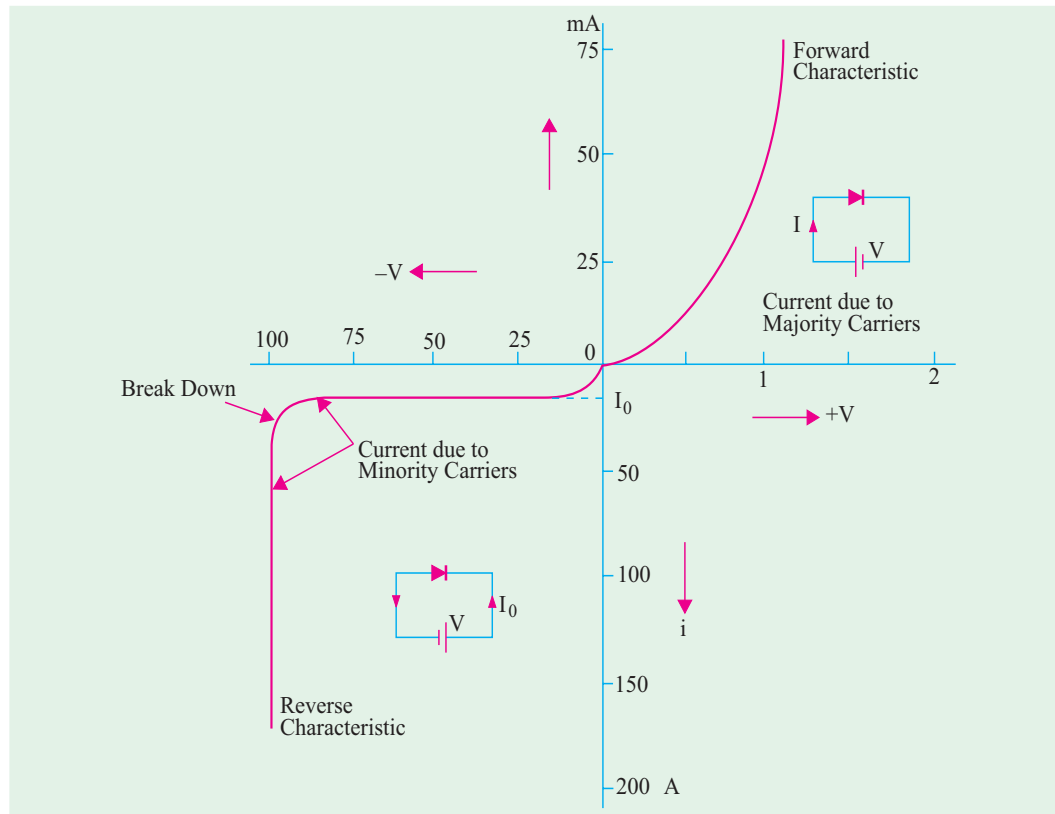


Fig. 52.4

These characteristics can be described by the analytical equation called *Boltzmann diode equation* given below :

$$I = I_0 \left( e^{\frac{eV}{kT}} - 1 \right) \text{ ampere}$$

- where
- $I_0$  = diode reverse saturation current
  - $V$  = voltage across junction – positive for forward bias and negative for reverse bias.
  - $k$  = Boltzmann constant =  $1.38 \times 10^{-23}$  J/°K
  - $T$  = crystal temperature in °K
  - $\eta = 1$  – for germanium
  - $\eta = 2$  – for silicon

Hence, the above diode equation becomes

$$I = I_0 \left( e^{\frac{eV}{kT}} - 1 \right) \text{ – for germanium}$$

$$I = I_0 \left( e^{\frac{eV}{2kT}} - 1 \right) \text{ – for silicon}$$

Now,  $e/k = 11,600$  and putting  $T/11,600 = V_T$ , the above equation may be written as

$$I = I_0 \left( e^{\frac{11,600V}{\eta T}} - 1 \right) = I_0 \left( e^{\frac{V}{\eta V_T}} - 1 \right) \text{ ampere}$$

Now, at room temperature of  $(273 + 20) = 293^\circ\text{K}$ ,  $V_T = 293/11,600 = 0.025 \text{ V} = 25 \text{ mV}$ . Substituting

the value of  $\eta$ , we have

$$\begin{aligned}
 I &= I_0 (e^{40V-1}) && \text{– for Ge} \\
 &\cong I_0 e^{40V} && \text{– if } V > 1 \text{ volt} \\
 I &= I_0 (e^{20V-1}) && \text{– for Si} \\
 &\cong I_0 e^{20V} && \text{– if } V > 1 \text{ volt}
 \end{aligned}$$

We may also write the above diode equation as under

$$\begin{aligned}
 I &= I_0 (e^{V_f/\eta V_T-1}) && \text{– forward bias} \\
 &= I_0 (e^{V_R/\eta V_T-1}) && \text{– reverse bias}
 \end{aligned}$$

**(e) Diode Parameters**

The diode parameters of greatest interest are as under :

**1. Bulk resistance ( $r_B$ )**

It is the sum of the resistance values of the P-and N-type semiconductor materials of which the diode is made of.

$$\therefore r_B = r_P + r_N \quad \text{– Fig. 52.5}$$

Usually, it is very small. It is given by

$$r_B = (V_F - V_B)/I_F$$

It is the resistance offered by the diode well above the barrier voltage *i.e.* when current is large. Obviously, this resistance is offered in the forward direction.

**2. Junction resistance ( $r_j$ )**

Its value for forward-biased junction depends on the magnitude of forward *dc* current.

$$\begin{aligned}
 r_j &= 25 \text{ mV}/I_F \text{ (mA)} && \text{– for Ge} \\
 &= 50 \text{ mV}/I_F \text{ (mA)} && \text{– for Si}
 \end{aligned}$$

Obviously, it is a *variable* resistance.

**3. Dynamic or ac resistance**

$$r_{ac} \text{ or } r_d = r_B + r_j$$

For large values of forward current,  $r_j$  is negligible. Hence,  $r_{ac} = r_B$ . For small values of  $I_F$ ,  $r_B$  is negligible as compared to  $r_j$ . Hence  $r_{ac} = r_j$ .

**4. Forward voltage drop**

It is given by the relation

$$\text{forward voltage drop} = \frac{\text{power dissipated}}{\text{forward dc current}}$$

**5. Reverse saturation current ( $I_0$ ).** It has already been discussed in Art. 52.42.

**6. Reverse breakdown voltage ( $V_{BR}$ ).** It is discussed in Art. 51.3.

**7. Reverse dc resistance  $R_R = \frac{\text{reverse voltage}}{\text{reverse current}}$**

**(f) Applications**

The main applications of semiconductor diodes in modern electronic circuitry are as under :

1. As power or rectifier diodes. They convert ac current into dc current for dc power supplies of electronic circuits.
2. As signal diodes in communication circuits for modulation and demodulation of small signals.
3. As Zener diodes in voltage stabilizing circuits.
4. As varactor diodes—for use in voltage-controlled tuning circuits as may be found in radio and TV receivers. For this purpose, the diode is deliberately made to have a certain range of junction capacitance. The capacitance of the reverse-biased diode is given by  $C = K/\sqrt{V_R}$

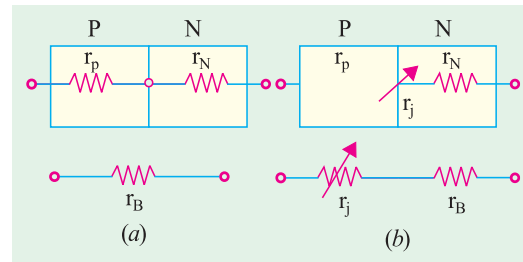


Fig. 52.5

where  $V_R$  is the reverse voltage.

5. In logic circuits used in computers.

**Example 52.1.** Using approximate Boltzmann's diode equation, find the change in forward bias for doubling the forward current of a germanium semiconductor at 290°K.

(Basic Electronics, Osmania Univ. 1993)

**Solution.** The approximate Boltzmann's diode equation is given by  $I = I_0 \exp(eV/kT)$

$$\therefore I_1 = I_0 \exp(eV_1/kT) \text{ and } I_2 = I_0 \exp(eV_2/kT)$$

$$\therefore \frac{I_2}{I_1} = \exp\left[\frac{e}{kT}(V_2 - V_1)\right]$$

$$\text{or } (V_2 - V_1) = \frac{kT}{e} \ln\left(\frac{I_2}{I_1}\right) = 25 \ln\left(\frac{I_2}{I_1}\right) \text{ mV}$$

$$\text{Since } I_2 = 2I_1 \text{ or } I_2/I_1 = 2$$

$$\therefore (V_2 - V_1) = 25 \ln 2 = 25 \times 0.693 = \mathbf{17.3 \text{ mV}}$$

**Example 52.2.** A certain P-N junction diode has a leakage current of  $10^{-14}$  A at room temperature of 27°C and  $10^{-9}$  A at 125°C. The diode is forward-biased with a constant-current source of 1 mA at room temperature. If current is assumed to remain constant, calculate the junction barrier voltage at room temperature and at 125°C.

**Solution.** As given in Art. 52.1,

$$I = I_0 \exp(eV/kT - 1) \text{ or } (I/I_0) + 1 = \exp(eV/kT)$$

Taking logarithm of both sides and solving for V, we get

$$V = \frac{kT}{e} \ln\left(\frac{I}{I_0} + 1\right)$$

Now, at 27°C or  $(273 + 27) = 300^\circ\text{K}$

$$\therefore kT/e = 1.38 \times 10^{-23} \times 300 / 1.6 \times 10^{-19} = 26 \text{ mV}$$

At  $(273 + 125) = 398^\circ\text{K}$ ,

$$kT/e = 1.38 \times 10^{-23} \times 398 \times (1.6 \times 10^{-19}) = 36 \text{ mV}$$

$$\text{Hence, at } 27^\circ\text{C, } V_B = 26 \ln\left(\frac{10^{-3}}{10^{-14}} + 1\right) = 660 \text{ mV} = \mathbf{0.66 \text{ V}}$$

$$\text{At } 125^\circ\text{C, } V_B = 36 \ln\left(\frac{10^{-3}}{10^{-9}} + 1\right) = 500 \text{ mV} = \mathbf{0.5 \text{ V}}$$

## 52.2. Derivation of Junction Resistance

Junction resistance is also known as *incremental or dynamic resistance* and is an important parameter particularly in connection with small-signal operations of the diode.

$$r_j = dV/dI \text{ or } g_j = dI/dV$$

$$\text{Now, } I = I_0(e^{V/\eta V_T} - 1) = I_0 e^{V/\eta V_T} - I_0$$

$$\therefore g_j = \frac{dI}{dV} = \frac{I_0 e^{V/\eta V_T}}{\eta V_T} = \frac{I + I_0}{\eta V_T}$$

### (a) Reverse bias

When reverse bias is greater than a few tenths of a volt *i.e.* when  $|V/\eta V_T| \gg 1$ , then  $g_j$  is extremely small so that  $r_j$  is very large. That high value is also represented by  $R_R$ .

### (b) Forward bias

Again, for a forward bias greater than a few tenths of a volt,  $I \gg I_0$ , hence,  $g_j = I/\eta V_T$  and  $r_j = \eta V_T/I$ .

Now, at room temperature of 293°K,  $V_T = T/11,600 = 293/11,600 = 25 \text{ mV}$ . Also  $\eta = 1$  for Ge and 2 for Si.

$$\therefore \begin{aligned} r_j &= 25 \text{ mV/I mA} && \text{– for germanium} \\ &= 50 \text{ mV/I mA} && \text{– for silicon} \end{aligned}$$

**Example 52.3.** A silicon diode has a forward voltage drop of 1.2 V for a forward dc current of 100 mA. It has a reverse current of 1 μA for a reverse voltage of 10 V. Calculate

- (a) bulk and reverse resistance of the diode  
 (b) ac resistance at forward dc current of (i) 2.5 mA and (ii) 25 mA.

**Solution. (a)**  $r_B = \frac{V_F - V_B}{I_F} = \frac{1.2 \text{ V} - 0.7 \text{ V}}{100 \text{ mA}} = 5 \Omega$

$$R_R = V_R / I_R = 10 \text{ V} / 1 \mu\text{A} = 10 \text{ M}$$

(b) (i)  $r_j = 25 \text{ mV} / 2.5 \text{ mA} = 10 \Omega$   $r_{ac} = r_B + r_j = 5 + 10 = 15 \Omega$

(ii)  $r_j = 25 \text{ mV} / 25 \text{ mA} = 1 \Omega$   $\therefore r_{ac} = 5 + 1 = 6 \Omega$

**Example 52.4.** Using analytical expression for diode current, calculate the dynamic slope resistance of a germanium diode at 290 K when forward biased at current of (i) 10 μA and (ii) 5 mA.

(Electronics-I, Mysore Univ. 1992)

**Solution.**  $I = I_0 (e^{eV/kT} - 1) \approx I_0 e^{eV/kT}$

$$\therefore dI = \frac{e}{kT} I_0 [e^{eV/kT}] dV = \frac{e}{kT} I dV$$

$$\therefore r_d = \frac{dV}{dI} = \frac{kT}{eI} = \frac{25 \times 10^{-3}}{I} \text{ – } I \text{ in ampere}$$

(i) Now,  $I = 10 \mu\text{A} = 10 \times 10^{-6} = 10^{-5} \text{ A}$

$$r_d = 25 \times 10^{-3} / 10^{-5} = 2500 \Omega$$

(ii) Now,  $I = 5 \text{ mA} = 5 \times 10^{-3} \text{ A}$

$$\therefore r_d = 25 \times 10^{-3} / 5 \times 10^{-3} = 5 \Omega$$

**Example 52.5.** Find the current through the 20 Ω resistor shown in Fig. 52.6 (a). Each silicon diode has a barrier potential of 0.7 V and a dynamic resistance of 2 Ω. Use the diode equivalent circuit technique.

(Semiconductor Devices, Gujarat BTE, 1993)

**Solution.** In Fig. 52.6 (b) each diode has been replaced by its equivalent circuit. It is seen that diodes  $D_1$  and  $D_3$  are forward-biased by 5 V battery whereas  $D_2$  and  $D_4$  are reverse-biased. Hence, the current will flow from point A to B, then to C via 20 Ω resistance and then back to the negative terminal of the 5 V battery.

The net voltage in the equivalent circuit is

$$V_{net} = 5 - 0.7 - 0.7 = 3.6 \text{ V}$$

Total resistance seen by this net voltage is

$$R_T = 2 + 20 + 2 = 24 \Omega$$

The circuit current  $I = V_{net} / R_T = 3.6 / 24 = 0.15 \text{ A}$

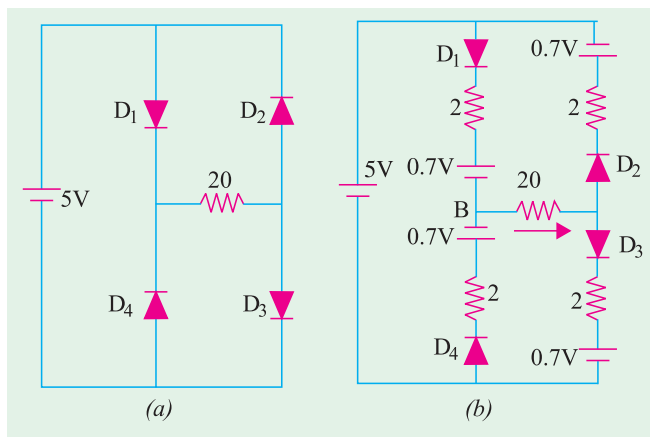


Fig. 52.6

### 52.3. Junction Breakdown

If the reverse bias applied to a P-N junction is increased, a point is reached when the junction breaks down and reverse current rises sharply to a value limited only by the external resistance

connected in series with the junction (Fig. 52.7). This critical value of the voltage is known as **breakdown voltage** ( $V_{BR}$ ). It is found that once breakdown has occurred, very little further increase in voltage is required to increase the current to relatively high values. The junction itself offers almost zero resistance at this point.

The breakdown voltage depends on the width of the depletion region which, in turn, depends on the doping level.

The following two mechanism are responsible for breakdown under increasing reverse voltage:

### 1. Zener Breakdown

This form of breakdown occurs in junctions which, being heavily doped, have narrow depletion layers. The breakdown voltage sets up a very strong electric field (about  $10^8$  V/m) across this narrow layer. This field is strong enough to **break or rupture the covalent bonds** thereby generating electron-hole pairs. Even a small further increase in reverse voltage is capable of producing large number of current carriers. That is why the junction has very low resistance in the break-down region.

### 2. Avalanche Breakdown

This form of breakdown occurs in junctions which being lightly-doped, have wide depletion layers where the electric field is not strong enough to produce Zener breakdown. Instead, the minority carriers (accelerated by this field) collide with the semiconductor atoms in the depletion region. Upon collision with valence electrons, covalent bonds are broken and electron-hole pairs are generated. These newly-generated charge carriers are also accelerated by the electric field resulting in more collisions and hence further production of charge carriers. This leads to an avalanche (or flood) of charge carriers and, consequently, to a very low reverse resistance. The two breakdown phenomena are shown in Fig. 52.7

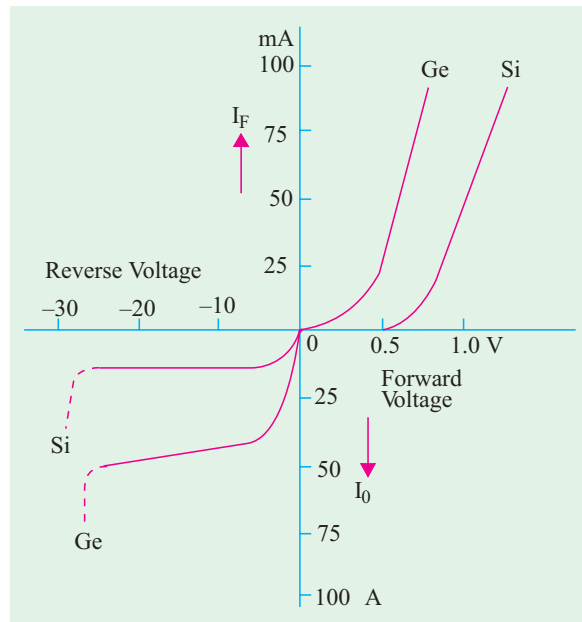
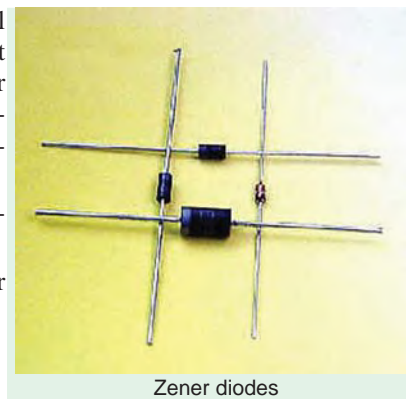


Fig. 52.7

## 52.4. Junction Capacitance

Capacitive effects are exhibited by  $P-N$  junctions when they are either forward-biased or reverse-biased.

### (a) Transition Capacitance ( $C_T$ ) or Space-charge Capacitance

When a  $P-N$  junction is reverse-biased, the depletion region acts like an insulator or as a dielectric material essential for making a capacitor. The  $P$ - and  $N$ -type regions on either side have low resistance and act as the plates. We, therefore, have all the components necessary for making a parallel-plate capacitor. This junction capacitance is called **transition or space charge** capacitance ( $C_{pm}$  or  $C_T$ ). It may be calculated by the usual formula  $C = \epsilon A/d$ . Its typical value is 40 pF. Since thickness of depletion (or transition) layer depends on the amount of reverse bias, capacitance  $C_T$  can be

controlled with the help of applied bias. This property of variable capacitance possessed by a reverse-biased P-N junction is used in the construction of a device known as **varicap or varactor**.

This capacitance is voltage dependent as given by the relation

$$C_T = \frac{K}{(V_K + V_R)^n}$$

where  $V_K$  = knee voltage ;  $V_R$  = applied reverse voltage  
 $K$  = constant depending on semiconductor material

$n = \frac{1}{2}$  – for alloy junction and  $= \frac{1}{3}$  –for diffused junction

The voltage-variance capacitance of a reverse-biased P-N junction is used in many circuits one of which is automatic frequency control (AFC) in an FM tuner. Other applications include self-balancing bridge circuits, special type of amplifiers known as **parametric** amplifiers and electronic tuners in TV.

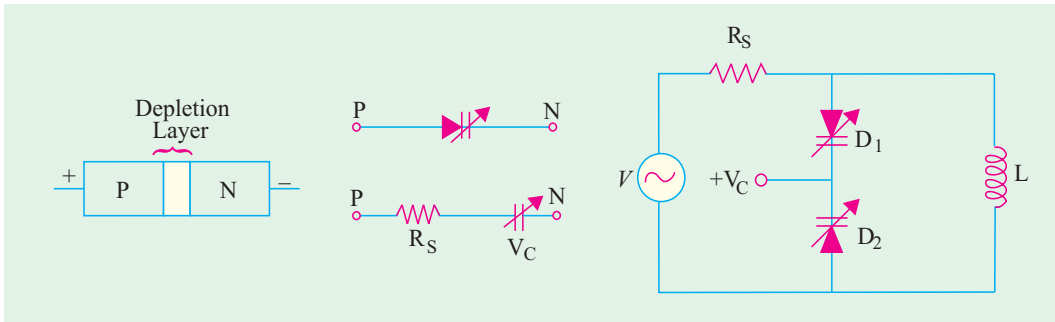


Fig. 52.8

Fig. 52.9

When used in such a role, the diodes are called **varactors, varicaps or voltacaps**. The symbol of these diodes when used in this role is shown in Fig. 52.8 along with its equivalent circuit.

When used in a resonant circuit, varactor acts as a variable capacitor and allows the resonant frequency to be adjusted by a variable voltage level. In Fig. 52.8, two varactors have been used to provide total variable capacitance in a parallel resonant circuit. Here  $V_C$  is a variable dc voltage that controls the reverse bias and hence the capacitance of the diodes.

**(b) Diffusion or Storage Capacitance ( $C_D$ )**

This capacitive effect is present when the junction is **forward-biased**. It is called diffusion capacitance to account for the time delay in moving charges across the junction by diffusion process.\* Due to this fact, this capacitance cannot be identified in terms of a dielectric and plates. It varies directly with the magnitude of forward current as explained below in more details.

Consider a forward-biased junction which is carrying a forward current  $I_F$ . Suppose the applied voltage is suddenly reversed, then  $I_F$  ceases suddenly but leaves lot of majority charge carriers in the depletion region. These charge carriers must get out of the region which, to their bad luck, becomes wider under the reverse bias. Hence, it is seen that when a forward-biased P-N junction is suddenly reverse-biased, a reverse current flows which is large initially but gradually decreases to the level of saturation current  $I_0$ . This effect can be likened to the discharging current of a capacitor and is, therefore, rightly represented by a capacitance called **diffusion capacitance  $C_D$** . Since the number of charge carriers left in depletion layer is proportional to forward current,  $C_D$  is directly proportional to  $I_F$ . Its typical value is  $0.02 \mu\text{F}$  which is 5000 times  $C_T$ .

The capacitance assumes great significance in the operation of devices which are required to switch rapidly from forward to reverse bias. If  $C_D$  is large, this switchover cannot be rapid. It will delay both the switch-on and the switch-off. This effect of  $C_D$  is variously known as **recovery time** or carrier storage.

\* In the case of forward bias, the diode current is almost entirely due to diffusion (drift current being negligible).

If  $\tau$  is mean lifetime of charge carriers, then a flow of charge  $Q$  yields a diode current of

$$I = \frac{Q}{\tau} = I_0 (e^{V/\eta V_T} - 1) \cong I_0 e^{V/\eta V_T}, Q = I\tau = \tau I_0 e^{V/\eta V_T}$$

$$\therefore C_D = \frac{dQ}{dV} = \frac{\tau I_0}{\eta V_T} e^{V/\eta V_T} \cong \frac{\tau I}{\eta V_T}$$

### 52.5. Equivalent Circuit of a P-N Junction

We have seen from above that a forward-biased junction offers ac resistance  $r_{ac}$  and possesses diffusion capacitance  $C_D$  (which comes into the picture only when frequency of the applied voltage is very high). Hence, it can be represented by the equivalent circuit of Fig. 52.10 (a). An opposing battery has been connected in series with  $r_{ac}$  to account for the junction barrier potential.

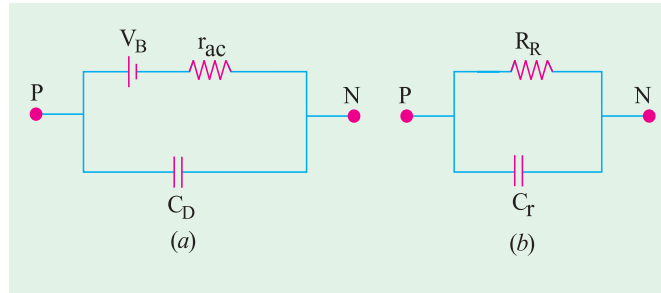


Fig. 52.10

As seen from Fig. 52.10 (b), a reverse-biased junction can be simply represented by a reverse resistance  $R_R$  connected in parallel with a capacitance  $C_T$  or  $C_{pn}$ .

**Example 52.6.** Calculate the barrier potential for Si junction at (a)  $100^\circ\text{C}$  and (b)  $0^\circ\text{C}$  if its value at  $25^\circ\text{C}$  is  $0.7\text{ V}$ .

**Solution.** As seen from Art. 1.38.

$$\begin{aligned} \Delta V &= -0.002. \quad \Delta t = -0.002 (t_2 - t_1) \\ (a) \quad \Delta t &= (100 - 25) = 75^\circ\text{C} & \therefore \Delta V &= -0.002 \times 75 = -0.15\text{ V} \\ \therefore V_B \text{ at } 100^\circ\text{C} &= 0.7 + (-0.15) = \mathbf{0.55\text{ V}} \\ (b) \quad \Delta t &= (0 - 25) = -25^\circ\text{C} & \therefore \Delta V &= -0.002 \times (-25) = 0.05\text{ V} \\ \therefore V_B \text{ at } 0^\circ\text{C} &= 0.7 + 0.05 = \mathbf{0.75\text{ V}} \end{aligned}$$

**Example 52.7.** A germanium diode draws  $40\text{ mA}$  with a forward bias of  $0.25\text{ V}$ . The junction is at room temperature of  $293^\circ\text{K}$ . Calculate the reverse saturation current of the diode.

**Solution.**  $I = I_0 (e^{40V} - 1)$  or  $40 \times 10^{-3} = I_0 (e^{40 \times 0.25} - 1)$   
 $\therefore I_0 = 40 \times 10^3 / (22,027 - 1) = \mathbf{1.82\ \mu\text{A}}$

**Example 52.8.** Calculate forward current in Ge diode at  $20^\circ\text{C}$  when forward voltage is  $0.3\text{ V}$ . Compare this value with that after a temperature rise of  $50^\circ\text{C}$ . Assume that reverse saturation current doubles for every  $10^\circ\text{C}$  rise in temperature. (Electronics-I, Mysore Univ. 1991)

**Solution.** According to Boltzmann's diode equation

$$I_{20} = I_0 (e^{40V} - 1) = I_0 (e^{40 \times 0.3} - 1) = I_0 (e^{12} - 1) = 162,755 I_0$$

At,  $t = (20 + 50) = 70^\circ\text{C}$  or  $T = 70 + 273 = 343^\circ\text{K}$

$$V_T = 343 / 11,600 = 0.0296\text{ V} \cong 0.03\text{ V, hence, } V/\eta V_T = 0.3/1 \times 0.03 = 10$$

The value of  $I_0$  at  $70^\circ\text{C}$  is  $I_0' = 2^5 \times I_0 = 32 I_0$  because it doubles for every  $10^\circ\text{C}$  rise in temperature.

$$\therefore I_{70} = I_0' (e^{10} - 1) = 32 I_0 (22,026 - 1) = 704,800 I_0$$

It means that  $I_{70} = (704,800, 162,755) I_{20} = 4.3 I_{20}$  i.e. forward current has increased only 4.3 times whereas  $I_0$  has increased 32 times for the same rise in temperature.



**Example 52.9.** The capacitance of a varactor varies from 5 to 50 pF. Two such varactor diodes are used in the tuning circuit of Fig. 52.11. If  $L = 10\text{ mH}$ , determine the tuning range of the circuit.

**Solution.** It should be noted that the two varactor capacitances are connected in series (Fig. 52.11).

$$C_{T(min)} = 5/2 = 2.5\text{ pF} ; C_{T(max)} = 50/2 = 25\text{ pF}$$

$$f_{r(max)} = 1/2\pi \sqrt{LC} = 1/2\pi \sqrt{10 \times 10^{-3} \times 2.5 \times 10^{-12}}$$

$$= \mathbf{1\text{ MHz}}$$

$$f_{r(min)} = 1/2\pi \sqrt{10 \times 10^{-3} \times 25 \times 10^{-12}} = \mathbf{318\text{ KHz}}$$

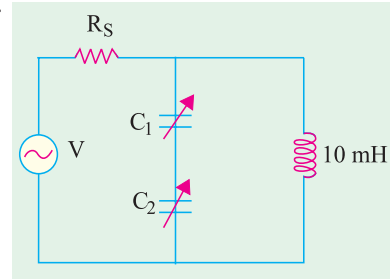


Fig. 52.11

### 52.6. Diode Fabrication

The electrical characteristics of a semiconductor diode depend on two factors (i) the material employed and (ii) the type of P-N junction used.

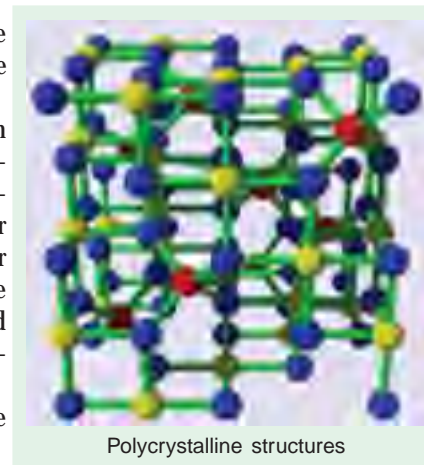
The two most commonly-used materials are germanium (Ge) and silicon (Si). Since Ge has higher electrical conduction than Si, it is often used in low- and medium-power diodes. On the other hand, Si has been found more suitable for high-power applications because it can be operated at higher temperatures than Ge. A new material called gallium-arsenide (GaAs) is found to combine desirable features of both Ge and Si and is finding ever-increasing use in many new applications.

The P-N junction may be produced by any one of the following methods :

1. grown junction
2. alloy junction
3. diffused junction
4. epitaxial growth
5. point contact junction.

The first step in the manufacture of any semiconductor device is to obtain the semiconductor material in an extremely pure form. The accepted impurity level is less than one part of impurity in one billion ( $10^9$ ) parts of the semiconductor material. To begin with, the raw material is subjected to a series of chemical reactions and then to a zone refining process which employs induction heating to reduce the impurity level of the polycrystalline structure. Next, the Czochralski or floating zone technique is used to form single crystals of Ge or Si for fabrication of diodes. These crystals are then cut into wafers as thin as 0.025 mm (nearly one fourth the thickness of this paper).

Now, we will briefly discuss the four basic processes commonly used in the manufacture of semiconductor diodes.



### 52.7. Grown Junction

Such junctions are produced by employing either the Czochralski or floating zone technique. The apparatus used for Czochralski technique is shown in Fig. 52.12. A single crystal seed of the desired impurity level is immersed in the molten semiconductor material contained in a crucible. Then, it is gradually withdrawn while the shaft holding the seed is slowly turning. When crystal is being pulled out, impurities of P- and N-type are alternately added to produce a P-N junction. This large area crystal is then cut into a large number of smaller-area diodes.

### 52.8. Alloy Junction

The alloy process produces junction diodes that have high PIV and current ratings but which

have large junction capacitance due to their large junction area.

In this process, a tiny dot (or pellet) of indium (or any other *P*-type impurity) is placed on the

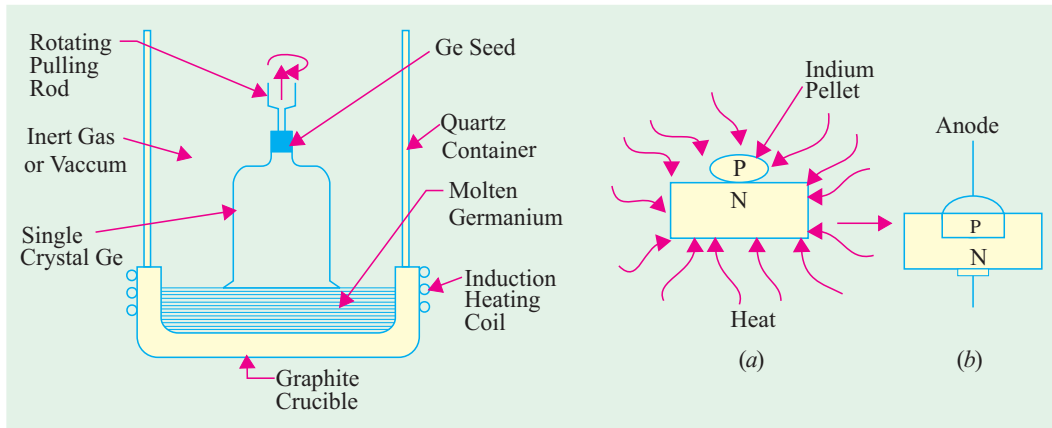


Fig. 52.12

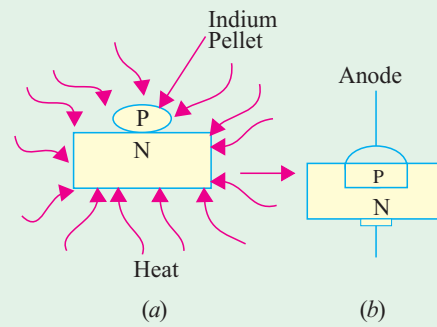


Fig. 52.13

surface of an *N*-type silicon wafer and the two are heated well above the melting temperature of indium (about 150°C) as shown in Fig. 52.13 (a). Consequently, indium melts and dissolves some of the silicon. The temperature is then lowered and silicon refreezes to form a single crystal having a *P-N* junction as shown in Fig. 52.13 (b).

### 52.9. Diffused Junction

The diffusion process employs either solid or gaseous diffusion. This process takes more time than alloy process but is relatively cheaper and more accurately controllable. In this process, particles from an area of high concentration drift to surrounding region of lesser concentration.

#### (i) Solid Diffusion

The solid diffusion process starts with the ‘painting’ of a *P*-type impurity (say, indium) on an *N*-type substrate and heating the two until the impurity (say, indium) on an *N*-type substrate and heating the two until the impurity diffuses a short distance into the substrate to form *P*-type layer (Fig. 52.14).

#### (ii) Gaseous Diffusion

In the gaseous diffusion process, an *N*-type material is heated in a chamber containing a high concentration of an acceptor impurity in vapour form (Fig. 52.15). Some of the acceptor atoms are diffused (or absorbed) into the *N*-type substrate to form the *P*-type layer thus creating a *P-N* junction. By exposing only part of the *N*-type material during the diffusion process (the remainder being covered by a thin coating of  $\text{SiO}_2$ ), the size of the *P*-region can be controlled. Finally, metal contacts are electroplated on the surface of each region for connecting the leads.

The diffusion technique enables simultaneous fabrication of many hundreds of diodes on one small disc of a semiconductor material. That is why it is the most frequently-used technique not only for the manufacture of semi-conductor diodes but also for the production of transistors and integrated circuits etc.

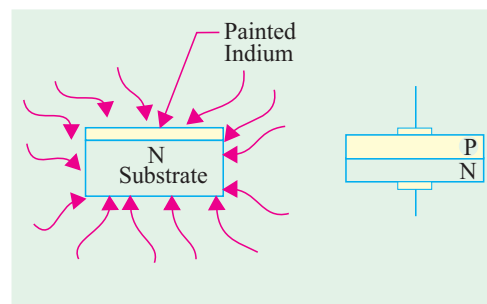


Fig. 52.14

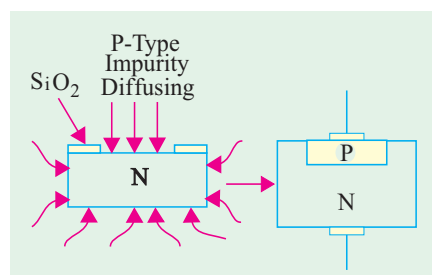


Fig. 52.15

### 52.10. Epitaxial Junction

This junction differs from the diffusion junction only in the manner in which the junction is fabricated. Such junctions are grown on top of an *N*-type wafer in a high temperature chamber. The growth proceeds atom by atom and hence is exactly similar to the crystal lattice of the wafer on which it is grown. Such junctions have the advantage of low resistance.

### 52.11. Point Contact Junction

It consists of an *N*-type germanium or silicon wafer about 1.25 mm square by 0.5 mm thick, one face of which is soldered to a metal base by radio-frequency heating as shown in Fig. 52.16 (a). The other face has a phosphor bronze (or tungsten) spring (called a cat's whisker) pressed against it. The *P-N* junction is formed by passing a large current for a second or two through the wire while the crystal face with wire point is kept positive. The heat so produced drives away some of the electrons from the atoms in the small region around the point of contact thereby leaving holes behind. This

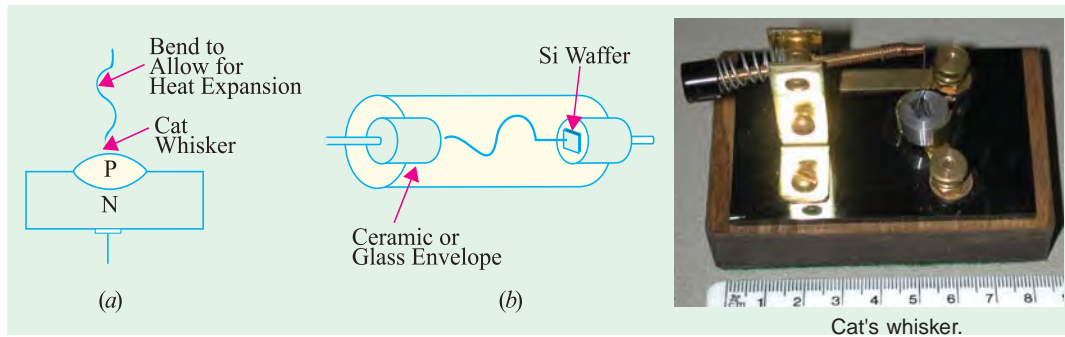


Fig. 52.16

small region of the *N*-type material is, consequently, converted into *P*-type material as shown in Fig. 52.16 (a). The small area of the *P-N* junction results in very low junction capacitance as mentioned earlier.

### 52.12. The Ideal Diode

There is no such thing as an *ideal* diode or perfect diode. The existence of such a diode is visualized simply as an aid in analysing the diode circuits.

An ideal diode may be defined as a two-terminal device which



- (a) conducts with zero resistance when forward-biased, and
- (b) appears as an infinite resistance when reverse-biased.

In other words, such a device acts as a short-circuit in the forward direction and as an open-circuit in the reverse direction as shown in Fig. 52.17.

Also, in the forward direction, there is no voltage drop (even though current is there) since a short has zero resistance. On the other hand, there is no reverse current because reverse resistance is infinite.

It is helpful to think of an ideal diode as a bistable switch which is closed in the forward direction and open in the reverse direction. Hence, it has two stable states : ON or OFF.

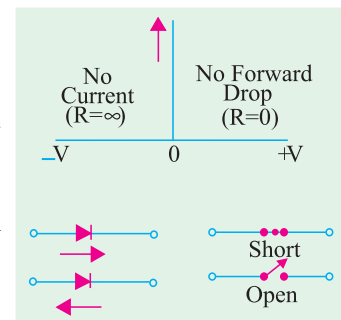


Fig. 52.17

**Example 52.10.** Calculate the circuit current and power dissipated in the (a) ideal diode (b)  $6\ \Omega$  resistor of the circuit shown in Fig. 52.18 (a).

**Solution.** The diode is an ideal one and is forward-biased. Hence, it can be replaced by a short (closed switch) as shown in Fig. 52.18 (b). The circuit current, as given by Ohm's law, is

$$I = 12/6 = 2\ \text{A}$$

(a) Since there is no voltage drop across the diode, power consumed by it is **zero**.

As we know, there is no power when either the voltage or current is zero. In the forward direction, there is current but no voltage drop, hence power dissipated by the ideal diode is zero. In the reverse direction, there is voltage but no current. Hence, power dissipated by the diode is again zero. In fact, an ideal diode never dissipates any power.

(b) power consumed by  $6\ \Omega$  resistor =  $2^2 \times 6 = 24\ \text{W}$ .

**Example 52.11.** Calculate the current in the circuit of Fig. 52.19 (a).

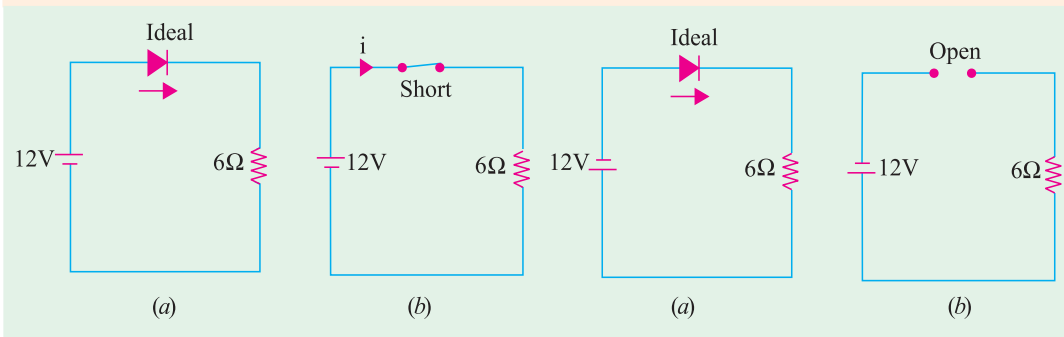


Fig. 52.18

Fig. 52.19

**Solution.** As seen from the polarity of the battery terminals, the diode is reverse-biased. Hence, it acts like an open switch as shown in Fig. 52.18 (b). Obviously, current in such a circuit is zero.

**Example 52.12.** Find the current, if any flowing in the circuit of Fig. 52.20 (a) which uses two oppositely-connected ideal diodes in parallel.

**Solution.** The diode  $D_1$  is reverse-biased and acts as an open switch as shown in Fig. 52.20 (b). So, there is no current through  $D_1$  and the  $8\ \Omega$  resistor.

However,  $D_2$  is forward-biased and acts like a short-current or closed switch. The current drawn is  $I = 12/(2 + 4) = 2\ \text{A}$ .

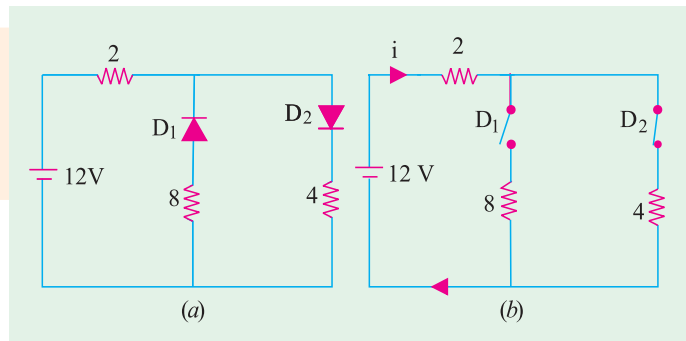


Fig. 52.20

**Example 52.13.** A sinusoidal voltage of peak value  $50\ \text{V}$  is applied to a diode as shown in Fig. 52.21. Sketch the waveform of voltage  $V_O$  treating the diode as an ideal one.

**Solution.** First, consider the positive half-cycle of the input signal. The diode acts as a short in the forward direction and the moment  $V_{in}$  exceeds battery voltage of  $10\ \text{V}$ , current will start flowing through the circuit. The value of  $V_O$  will remain steady at  $10\ \text{V}$ , the balance of  $40\ \text{V}$  dropping across  $5\ \text{K}$  resistance. It is seen that value of  $V_O$  is set by the battery voltage.

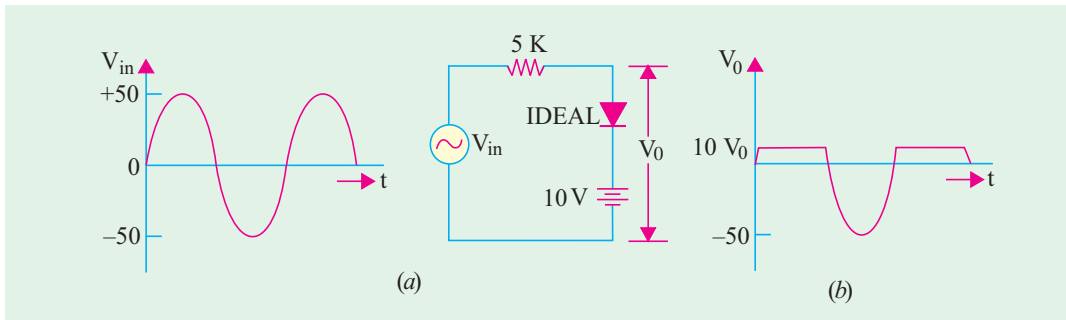


Fig. 52.21

Next, consider the negative input half-cycle. Now, the diode acts like an open switch. Consequently, there is no circuit current and thus no voltage drop across 5 K resistor. Hence,  $V_0$  equals source voltage of peak value 50 V.

It is interesting to note that the above circuit acts as a **positive** clipper with a clipping level of 10 V *i.e.* equal to battery voltage. All positive parts of  $V_{in}$  above 10 V have been clipped off [Fig. 52.21 (b)].

**Example 52.14.** Sketch the waveform of the output voltage  $V_0$  of the circuit shown in Fig. 52.22 (a).

**Solution.** It is obvious that diode  $D_1$  and its 10 V battery act as positive clipper with positive clipping level at +10 V. Similarly,  $D_2$  and its 10 V battery act as negative clipper with a clipping level at -10 V.

It is clear that output voltage  $V_0$  would be clipped during both the positive and negative half-cycles as shown in Fig. 52.22 (b).

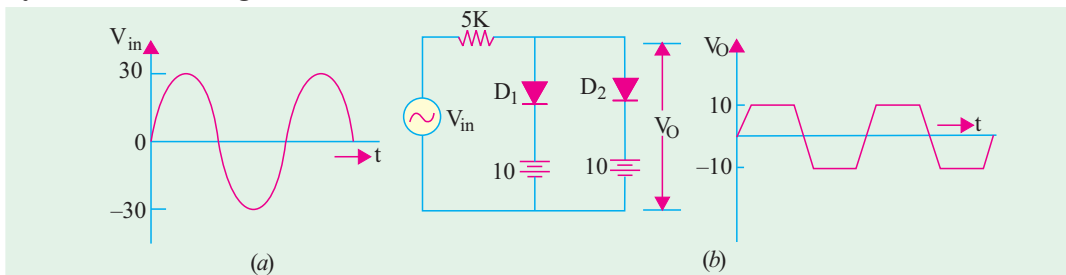


Fig. 52.22

Incidentally, the above example represents one way of obtaining an approximate square waveform from a sine wave. In fact, clipping takes place at +10 V and -10 V regardless of the shape of the input wave *i.e.* whether it is triangular or sawtooth etc.

**Example 52.15.** In Fig 52.23 draw input and output waveforms to scale. Consider diode forward resistance of 50 ohms. (Nagpur Univ. Winter 2003)

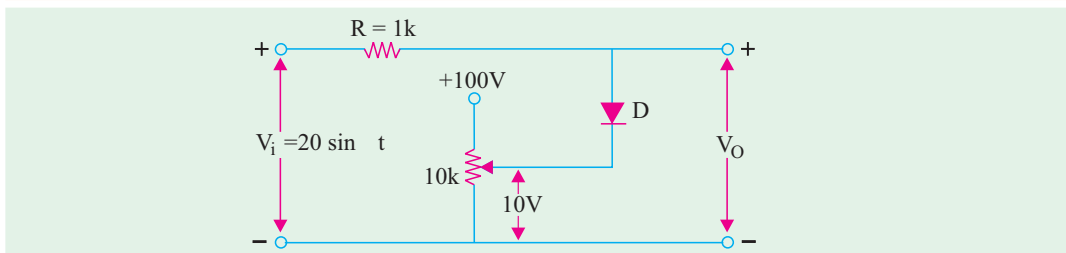


Fig. 52.23

**Solution.** It is obvious that the diode D and its 10V voltage drop across the 10k variable resistor act as positive clipper. Since the value of  $R \gg R_f$  (i.e. forward resistance of a diode), therefore, the output voltage is clipped at 10V as shown in Fig 52.24.

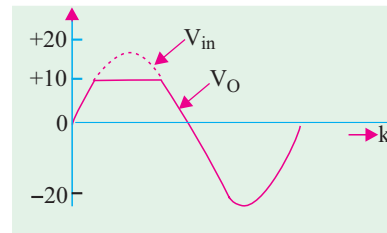


Fig. 52.24

### 52.13. The Real Diode

A real diode neither conducts in the forward direction with zero resistance nor it offers infinite resistance in the reverse direction.

#### (a) Forward Direction

In this case, we have to take two factors into account. One is that forward current does not start flowing until the voltage applied to the diode exceeds its threshold or knee voltage  $V_K$  (0.3 V for Ge and 0.7 V for Si). Hence, a real diode is shown as equivalent to an ideal diode in series with a small **oppositely-connected** battery of e.m.f.  $V_K$  as shown in Fig. 52.25 (a).

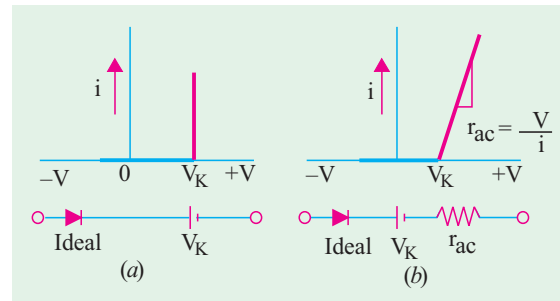


Fig. 52.25

The second factor to be considered is the forward dynamic or ac resistance ( $r_{ac}$ ) offered by the circuit. So far, we considered this resistance to be zero implying that forward characteristic is a straight vertical line [Fig. 52.25 (a)].

If we take  $r_{ac}$  into account, the forward characteristic becomes as shown in Fig. 52.25 (b). Here, the reciprocal of the slope of this characteristic represents  $r_{ac}$ .

#### (i) Large Signal Operation

Large signal sources are those whose voltage is much greater than the diode knee voltage  $V_K$  (nearly equal to barrier potential  $V_B$ ). Under such conditions, forward current would be large, so that  $r_j$  (Art. 52.1) would be negligible.

$$\therefore r_{ac} = r_j + r_B \cong r_B$$

#### (ii) Small Signal Operation

In this case, the signal voltage is much smaller than  $V_K$  (0.3 V for Ge and 0.7 V for Si). Since  $I_F$  would be small,  $r_j$  would be very large as compared to  $r_B$ .

$$\therefore r_{ac} = r_j + r_B \cong r_j$$

#### (b) Reverse Direction

An actual or real diode does not have infinite resistance in the reverse direction because it will always have some reverse saturation current prior to breakdown. For example, if with a  $V_R = 50$  V,  $I_R$  is  $10 \mu\text{A}$ , then  $R_R = 5 \times 10^6 \Omega = 5$  M. Silicon diodes have reverse resistance of many thousands of megohms. Hence, an actual diode in the reverse direction can be thought of as equivalent to a high resistor. This would be true only in the case of signals of low frequencies. For high-frequency signals, we will have to take into account the capacitive effects (Art 52.4).

**Example 52.16.** Sketch the waveform for the output voltage  $V_O$  in Fig. 52.26. Take  $V_K = 0.3$  V for germanium diode which has a forward current of 28 mA at 1 V.

**Solution.**  $r_B = (1 - 0.3)/28 \text{ mA} = 25 \Omega$ ;  $r_j = 25/28 = 0.9 \Omega$  – negligible

$$\therefore r_{ac} = r_j + r_B \cong r_B = 25 \Omega.$$

Hence, the equivalent circuit becomes as shown in Fig. 52.27. Whenever  $V_{in}$  exceeds 0.3 V in the forward direction, diode is shorted and the circuit offers a total resistance =  $(25 + 75) = 100 \Omega$ .

The peak voltage =  $20 - 0.3 = 19.7$  V. Hence, peak value of forward current is  $= 19.7/100 = 197$  mA.  
 $\therefore$  peak value of output voltage =  $197 \times 10^{-3} \times 75 = 14.77$  V

Hence, the peak value of the half-wave rectified voltage is 14.77 V as shown in Fig. 52.27 (b). During the negative half-cycle of the applied voltage, diode acts as an open switch so that  $V_o = 0$ . The waveform sketch of  $V_o$  is shown in Fig. 52.27 (b).

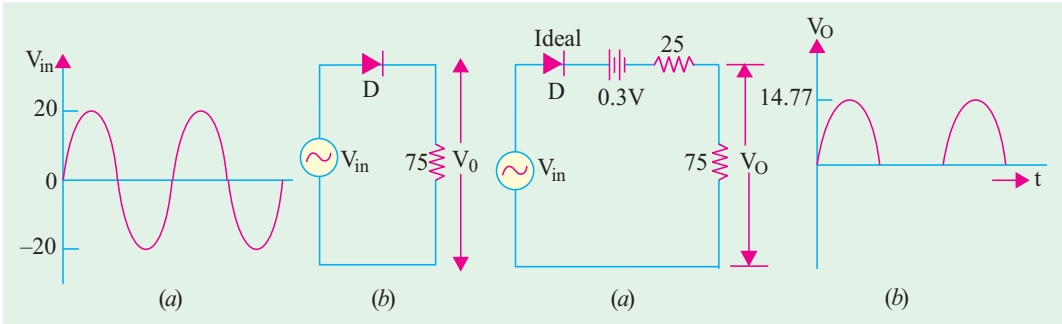


Fig. 52.26

Fig. 52.27

### 52.14. Diode Circuits with DC and AC Voltage Sources

We will often come across diode and transistor circuits which will contain both dc and ac voltage sources. Such circuits can be easily analysed by using Superposition Theorem (Art 52.15). We will first draw the dc equivalent circuit while neglecting ac-sources and find the required current and voltage values. Next, we will draw the ac equivalent circuit while neglecting the dc sources and again find the voltage and current values. Finally, we will superimpose the two sets of values to get the final result.

While drawing the equivalent circuits, following points must be remembered :

1. Direct current cannot flow ‘through’ a capacitor. Hence, all capacitors look *like an open switch to a dc source*.
2. Usually, capacitors used in most circuits are large enough to *look like a short to an ac source* particularly one of very high frequency. Similarly, dc batteries would also act as short circuits unless they have very high internal resistances.

**Example 52.17.** The silicon diode shown in Fig. 52.28 has a bulk resistance of  $1 \Omega$ . The frequency of the  $10$  mV (peak) signal is so high that the reactance of the coupling, capacitor may be taken as zero. Sketch the approximate waveform of the total voltage ‘V’ across the diode.

(Basic Electronics, Punjab Univ.)

**Solution.** We will apply Superposition theorem to find V. First, voltage drop due to dc source would be found and then it would be added to the drop due to the ac source.

#### (i) DC Equivalent Circuit

It is seen from Fig. 52.26 that the circuit to the left of point A is ‘open’ to the dc source of 20 V because of capacitor C. Hence, the equivalent dc circuit is as shown in Fig. 52.29 (a). As seen, diode is forward-biased by the battery. Hence, only 0.7 V dc appear across the diode.

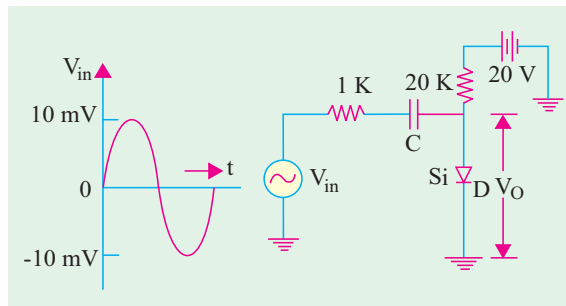


Fig. 52.28

The dc current  $I = (20 - 0.7)/20 \text{ K} \cong 1 \text{ mA}$ .

(ii) AC Equivalent Circuit

Here, the capacitor  $C$  and the 20-V battery would be treated as shorts thereby giving us the ac equivalent circuit of Fig. 52.29 (b).

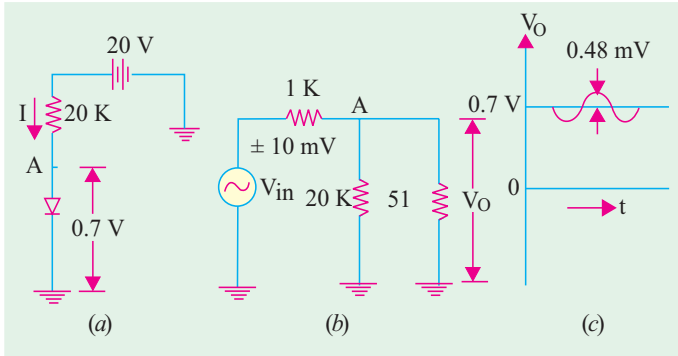


Fig. 52.29

Since, it is a silicon diode  
 $r_j = 50 \text{ mV}/1 \text{ mA} = 50 \Omega$   
 $r_{ac} = r_j + r_B = 50 + 1 = 51 \Omega$   
 As shown in Fig. 52.29 (b), so far as the signal source is concerned, 20 K resistor and ac resistance of the diode are connected in parallel at point A. Now,  $20 \text{ K} \parallel 51 \Omega = 51 \Omega$ . Hence, 1 K and 51  $\Omega$  are put in series across the signal source of peak value 10 mV. The peak value of the ac voltage drop over 51  $\Omega$  resistance is

$$= 10 \times \frac{51}{51 + 1000} = 0.48 \text{ mV}$$

The total drop across the diode is the sum of the ac and dc drops. The combined voltage waveform is shown in Fig. 52.29 (c). It consists of a dc voltage of 0.7 V over which rides an ac voltage of peak value  $\pm 0.48 \text{ mV}$ .

**Example 52.18.** The Ge diode shown in Fig. 52.30 has a bulk resistance of 2  $\Omega$ . The two capacitors offer negligible reactance to the a.c. signal. Sketch the waveform of the output voltage.

**Solution.** As seen from Fig. 52.30 (b), no dc current can pass through 100 K resistor because it is blocked by  $C_2$ . Hence, no dc voltage appears across 100-K resistor. The battery current passes through 10 K resistor, through diode, through 20 K resistor and finally to ground.

$$I = (30 - 0.3)/(10 + 20) \times 10^3 = 0.99 \text{ mA} \cong 1 \text{ mA}$$

$$r_{ac} = r_j + r_B = (25 \text{ mV}/1 \text{ mA}) + 2 = 27 \Omega$$

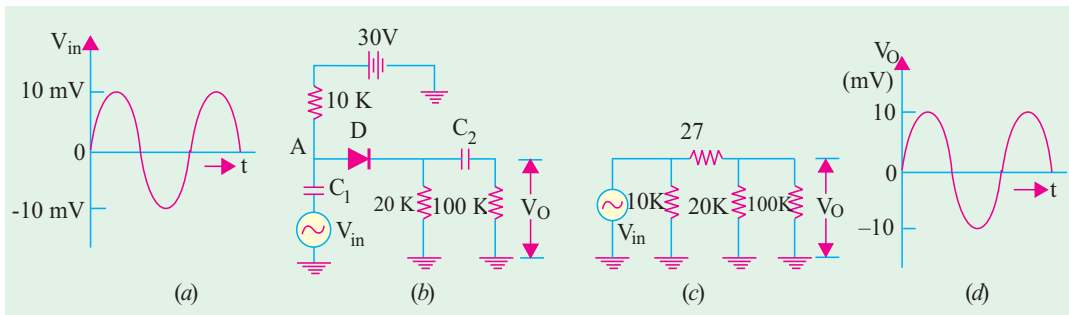


Fig. 52.30

AC equivalent circuit is shown in Fig. 52.30 (c) where capacitors and battery have been shorted. As seen, diode resistance of 27  $\Omega$  is in series with  $20 \text{ K} \parallel 100 \text{ K}$ . It means that only a very small part of ac. Source voltage drops across the diode. Hence, almost all a.c.source voltage appears across 100 K resistor. Obviously,  $V$  is practically the same as ac source voltage as shown in Fig. 52.30 (d).

52.15. Diode Clipper and Clamper Circuits

These are diode waveshaping circuits *i.e.* circuits meant to control the shape of the voltage and current waveforms to suit various purposes. Each performs the waveshaping function indicated by its



name. The output of the clipping circuit appears as if a portion of the input signal were *clipped off*. But clamper circuits simply clamp (*i.e.* lift up or down) the input signal to a different dc level.

### 52.16. Clippers

A clipping circuit requires a minimum of two components *i.e.* a diode and a resistor. Often, dc battery is also used to fix the clipping level. The input waveform can be clipped at different levels by simply changing the battery voltage and by interchanging the position of various elements. We will use an ideal diode which acts like a closed switch when forward-biased and as an open switch when reverse-biased.

Such circuit are used in radars and digital computers etc. when it is desired to remove signal voltages above or below a specified voltage level. Another application is in radio receivers for communication circuits where noise pulses that rise well above the signal amplitude are clipped down to the desired level.

**Example 52.19.** For the simple parallel clipper of Fig. 52.31, find the shape of the output voltage  $V_o$  across the diode if the input sine wave signal is as shown in Fig. 52.31 (a). What will happen when diode and resistor are inter-changed ?

**Solution.** When positive half-cycle of the signal voltage is applied to the clipper *i.e.* when A is positive with respect to B, the diode D is reverse-biased. Hence, it acts as an open switch. Consequently, the entire input voltage appears across it.

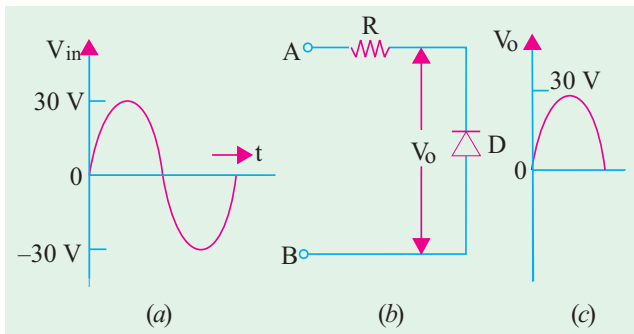


Fig. 52.31

During the negative half-cycle of the signal voltage when circuit terminal B becomes positive with respect to A, the diode is forward-biased. Hence, it acts like a closed switch (or short) across which no voltage is dropped. Hence, the waveshape of  $V_o$  is as shown in Fig. 52.31 (c). It is seen that the negative portion of the signal voltage has been removed. Hence, such a circuit is called a **negative clipper**.

### When Diode and Resistor are Interchanged

In this case, the circuit becomes as shown in Fig. 52.32 (b). Now, the output voltage  $V_o$  is that which is dropped across R. During the positive half-cycle of the signal voltage, D acts as an open switch. Hence, all applied voltage drops across D and none across R. So, there is no output signal voltage.

During the negative input half-cycle, terminal B is positive and so it is forward-biases B which acts as a short. Hence, there is no voltage drop across D. Consequently, all the applied signal voltage drops across R and none across D. As a result, the negative half-cycle of the input signal is allowed to pass through the clipper circuit. Obviously, now the circuit acts as a **positive clipper**.

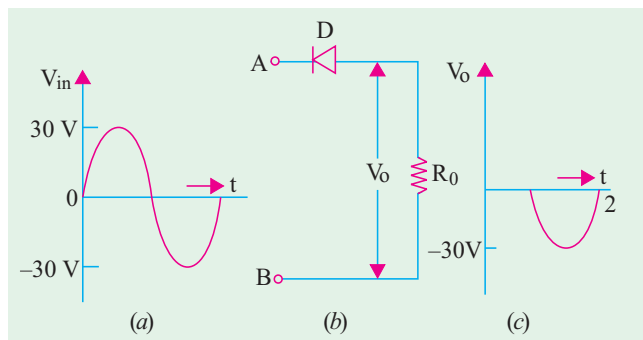


Fig. 52.32

**Example 52.20.** What would be the output waveform displayed by the oscilloscope in Fig. 52.33? The silicon diode has a barrier voltage of 0.7 V.

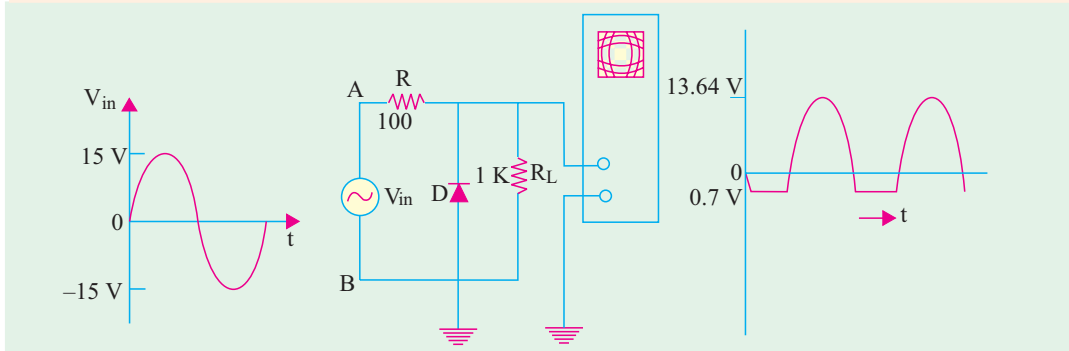


Fig. 52.33

**Solution.** Consider the negative input half-cycle first *i.e.* when point B is positive with respect to point A. The diode starts conducting when applied voltage exceeds 0.7 V. Since D and  $R_L$  are in parallel, voltage across them cannot exceed 0.7 V. Obviously, negative half-cycle beyond 0.7 V gets clipped. Hence, circuit behaves like a negative clipper.

During the positive input half-cycle when point A is positive, diode becomes reverse-biased and hence, becomes open-circuited. The applied voltage drops across the resistors R and  $R_L$  connected in series. The peak value of the output voltage is

$$= 15 \left( \frac{R_L}{R + R_L} \right) = 15 \times \frac{1}{1.1} = 13.64 \text{ V}$$

Hence, the output voltage as displayed by the oscilloscope would be as shown in Fig. 52.33.

**Example 52.21.** With the sine wave signal input of Fig. 52.34 (a), find the shape of the output signal  $V_o$  in the biased series clipper of Fig. 52.34 (b). What would happen if battery connections are reversed?

**Solution.** Let us consider the positive half-cycle of the signal *i.e.* when terminal A of the circuit becomes positive with respect to B. The diode appears as a short since it is forward-biased.

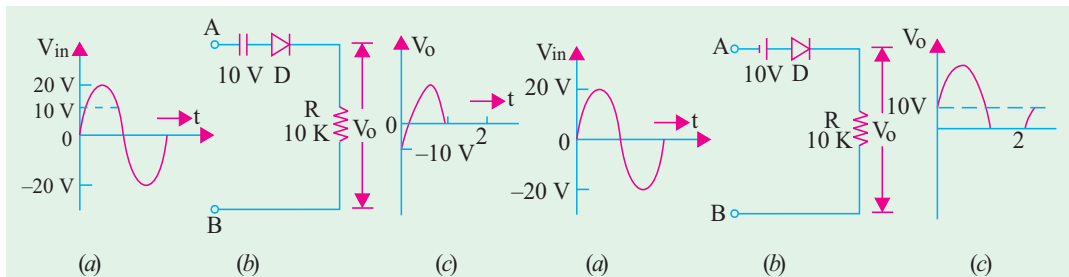


Fig. 52.34

Fig. 52.35

But no current flows till  $V_{in}$  exceeds the opposing battery voltage of 10 V. Hence, only upper part of the positive signal voltage passes through the clipper circuit and appears as  $V_o$  across R. Its shape is shown in Fig. 52.34 (c). The negative half-cycle of the signal voltage is clipped off.

In fact, in this circuit, the entire input is clipped off except positive peak portions.

### Reversed Battery Connections

The battery connections have been reversed in Fig. 52.35. In this case, during the positive half-cycle of the signal, the voltage across R would be the sum of the signal voltage and the battery voltage *i.e.* signal voltage would be lifted up by 10 V as shown in Fig. 52.35 (c).

During the negative input half-cycle, the lower peak portions of the signals would be clipped off because of the 10 V battery.

**Example. 52.22.** The triangular voltage of Fig. 52.36 (a) is applied to the biased parallel clipper circuit of Fig 52.36 (b). Find the wave-shape of the output voltage.

**Solution.** During the positive half-cycle,  $D_1$  would conduct but  $D_2$  will act as an open-circuit. However, value of  $V_0$  cannot exceed 10V because points C and D are electrically connected across the 10 V battery since  $D_1$  is shorted. Hence, signal voltage above 10 V level would be clipped off as shown in Fig. 52.36 (c).

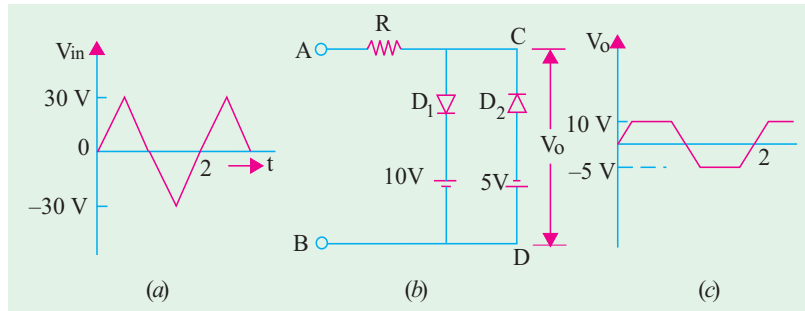


Fig. 52.36

During the negative half-cycle,  $D_1$  is open but  $D_2$  conducts. Again,  $V_0$  cannot exceed 5 V since it is the voltage across points C and D whose value is fixed by the battery connected in that branch. Hence, signal voltage beyond 5 V is clipped off. The wave-shape of  $V_0$  is as shown in Fig. 52.36 (c).

### 52.17. Some Clipping Circuits

For the following circuits, a sinusoidal input signal as shown in Fig. 52.37 would be assumed.

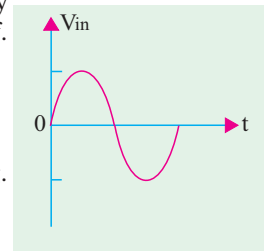


Fig. 52.37

#### (a) Biased Series Clippers

The output voltage has the waveform as shown in Fig. 52.38.

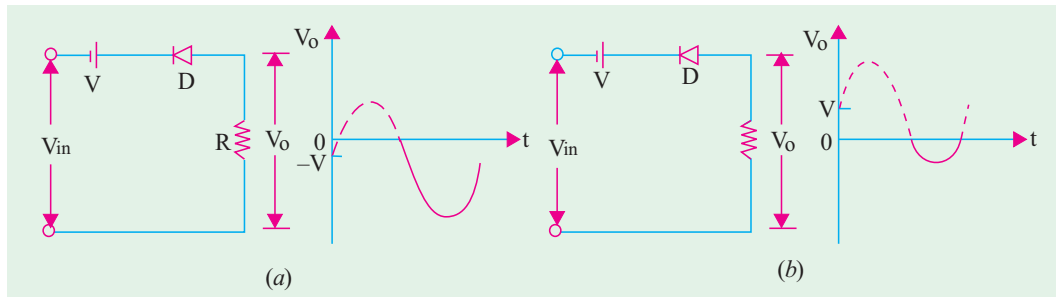


Fig. 52.38

#### (b) Biased Parallel Clippers

The waveforms of the output voltage are as shown in Fig. 52.39.

Clipping has been changed by changing the battery and diode connections.

### 52.18. Clampers

To put it simply, clamping is the process of introducing a dc level into an ac signal. Clampers are also sometimes known as dc restorers.

By way of illustration, consider the signal shown in Fig. 52.40 (a). It is a sine wave with equal positive and negative swings of  $\pm 5$  V about 0 V. Hence, its average value over one cycle is zero (it has no dc level).

In Fig. 52.40 (b), the signal waveform has been lifted up so as to just touch the horizontal axis. It is now said to have acquired a dc level of 5 V. This output wave-form is said to be **positively** clamped at 0 V. Fig. 52.40 (c) shows an output waveform which is negatively clamped at 0 V.

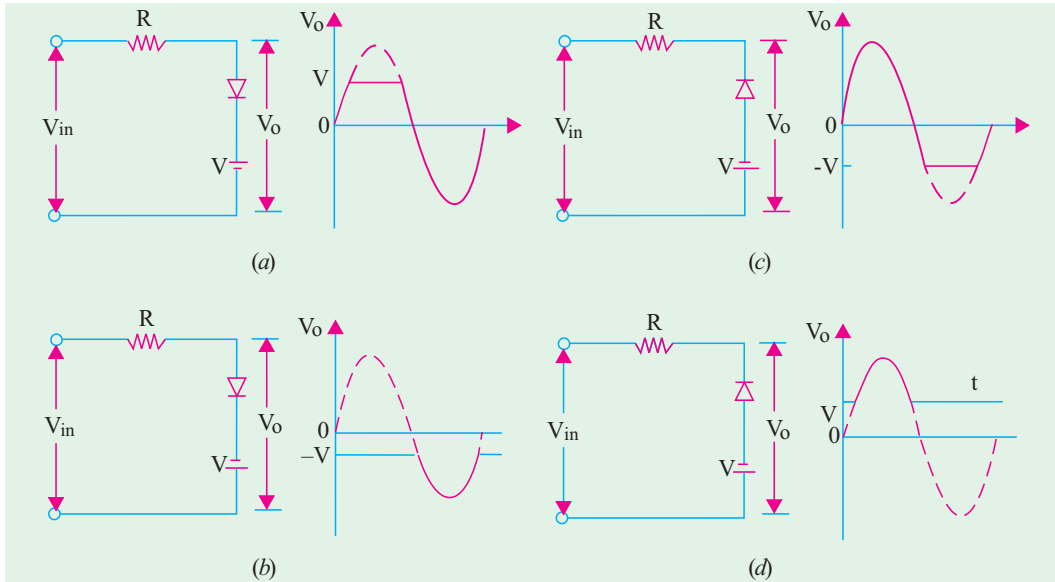


Fig. 52.39

A circuit capable of accepting the input signal shown in Fig. 52.40 (a) and delivering the output shown in Fig. 52.40 (b) or (c) is called a **clammer**. Such a circuit has a minimum requirement of **three** elements.

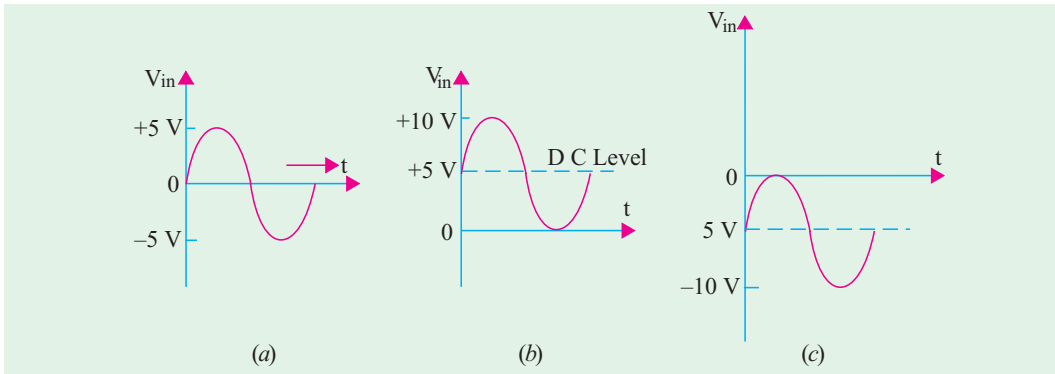


Fig. 52.40

1. a diode
2. a capacitor and
3. a resistor.

It will generally need a dc battery also. Following additional points regarding clamper circuits are worth keeping in mind.

1. both  $R$  and  $C$  affect the waveform.
2. values of  $R$  and  $C$  should produce a time constant ( $\lambda = CR$ ) which is large enough to ensure that capacitor remains almost fully charged during the time-period of the signal. In other words, time constant  $\lambda \gg T/2$  where  $T$  is the time-period of the input signal. For good clamping action, the  $RC$  time constant should be at least ten times the time-period of the input signal voltage.

3. it is advantageous to first consider the condition under which the diode becomes forward biased.
4. for all clamping circuits, voltage swing of the input and output waveforms is the same.
5. such circuits are often used in TV receivers as dc restorers. The incoming composite video signal is normally processed through capacitively-coupled amplifiers which eliminate the dc component thereby losing the black and white reference levels and the blanking level. These reference levels have to be restored before applying the video signal to the picture tube.

**Example 52.23.** The input signal of Fig. 52.41 (a) is applied to the clamper circuit shown in Fig. 52.41 (b). Draw the waveform of the output voltage  $V_o$ . How will it change if  $R$  is made  $100 \Omega$ ? (Electronic circuits, Bangalore Univ.)

**Solution.** As seen, time-period of the input signal is  $T = 1/1000$  second = 1 ms

$$\therefore 0 \rightarrow t_1 = t_1 \rightarrow t_2 = t_2 \rightarrow t_2 = T/2 \dots\dots = 0.5 \text{ ms.}$$

$$\lambda = C_R = 1 \times 10^{-6} \times 10 \times 10^3 = 10 \text{ ms}$$

As seen,  $\lambda \gg T/2$ . Hence, once charged, the capacitor will have hardly any time to discharge by the time signal polarity reverses.

**(a) Positive Input Half-cycle**

When positive half-cycle of the input signal voltage is applied to the clamper circuit, its terminal  $A$  becomes positive with respect to terminal  $B$ . Hence,  $D$  acts like a short as shown in Fig. 52.41 (c). A steady positive voltage of 5 V remains applied to  $A$  for 0.5 ms. At the same time,  $R$  is also shorted out [Fig. 52.41 (c)] because it is in parallel with  $D$ . Hence,  $C$  will rapidly charge to 5 V. Being across a short,  $V_o = 0$  during positive half-cycle as shown in Fig. 52.41 (d).

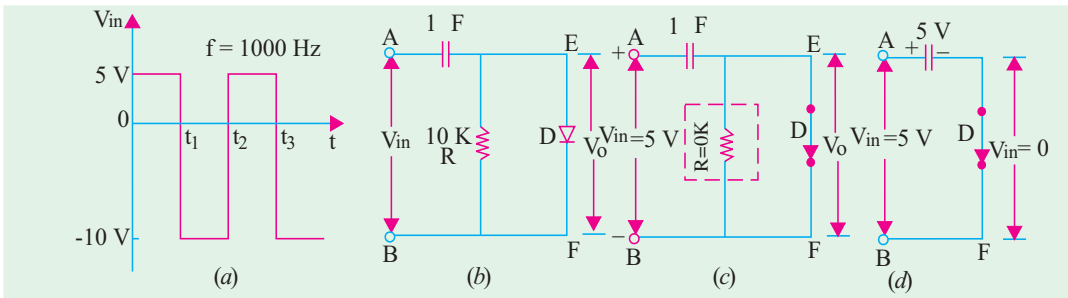


Fig. 52.41

**(b) Negative Input Half-cycle**

In this case, terminal  $B$  becomes positive and so reverse-biases  $D$  by 10 V. Hence,  $D$  acts like an open switch as shown in Fig. 52.42 (a). Now,  $R$  and  $C$  get connected in series so that their  $\lambda = RC = 10$  ms.

As stated earlier, capacitor will take a time of  $5\lambda = 50$  ms to get fully discharged. But the input signal will allow it just 0.5 ms during which to discharge. Obviously,  $C$  would hardly get discharged in this extremely short time interval of 0.5 ms. Hence, it can be assumed to be still fully charged with the original polarity during this negative half-cycle.

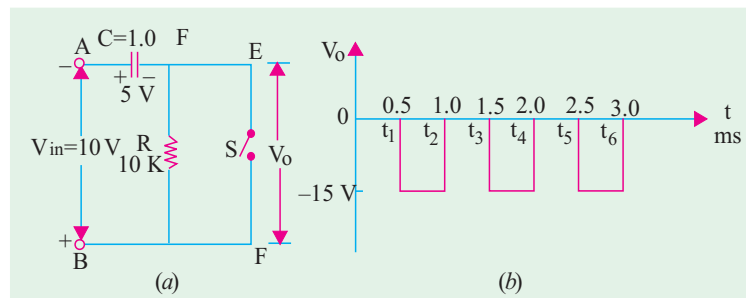


Fig. 52.42

The output voltage  $V_0$  across the 'open' will be  
 = voltage from  $E \rightarrow A \rightarrow B \rightarrow F$   
 =  $5 + 10 = 15 \text{ V}$

– Fig. 52.40 (a)  
 – with  $E$  negative

The waveform of the output voltage is shown in Fig. 52.42 (b). It has same frequency as that of the input signal. However, it has been clamped down in the negative region. It is seen that voltage swing of both input and output circuits is the same *i.e.* 15 V. It is never the case in clipping circuits.

**When  $R = 100$**

Now,  $\lambda = 100 \times 1 \times 10^{-6} \text{ ms} = 0.1 \text{ ms}$ . Hence, the capacitor which is almost instantaneously charged to +5 V during the positive input half-cycle, will be almost completely discharged during the negative half-cycle because, now,  $5 \lambda$  (full discharge time) equals the half time-period (0.5 s) of the signal.

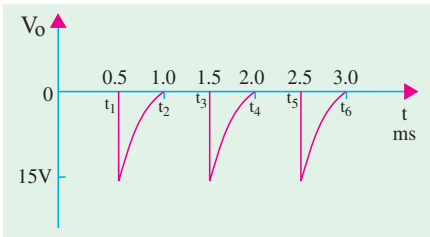


Fig. 52.43

Hence, in this case,  $V_0$  would be momentarily equal to  $-15 \text{ V}$  at the beginning of the negative half-cycle but will fall off to almost 0 V before the signal reverses its polarity (Fig. 52.43). As seen,  $v_0$  consists of voltage spikes of amplitude  $-15 \text{ V}$ .

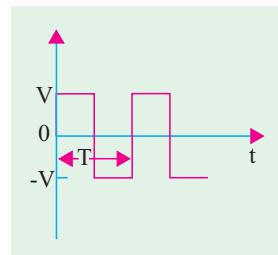


Fig. 52.44

### 52.19. Summary of Clamping Circuits

In the following clamping circuits, it would be assumed that the amount of the time  $5\lambda = 5RC \gg T/2$  where  $T$  is the time-period of the input signal. For all circuits, we will take the same input signal shown in Fig. 52.44 with a peak value of  $V$ . We will also take note of the change in the output waveform when diode connections are reversed.

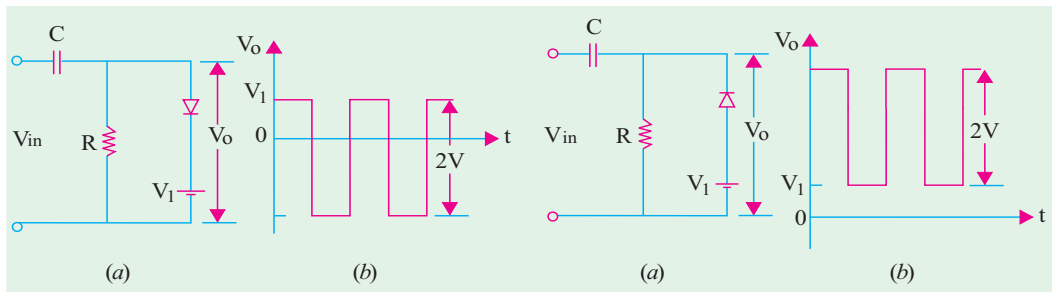


Fig. 52.45

Fig. 52.46

It is seen from Fig. 52.44 and 52.45 that negative clamping has changed to positive clamping when the diode connections are reversed.

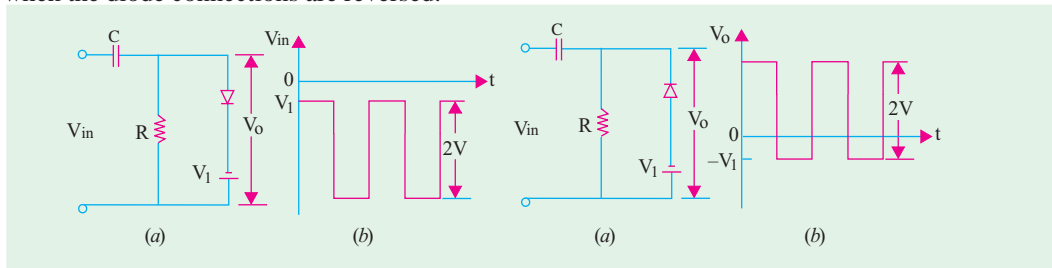


Fig. 52.47

Fig. 52.48

The same thing happens in the case of clamping circuits shown in Fig. 52.47 and 52.48.

**Tutorial Problems No. 52.1**

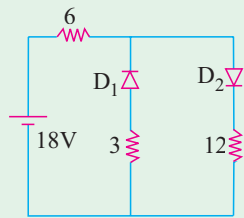


Fig. 52.49

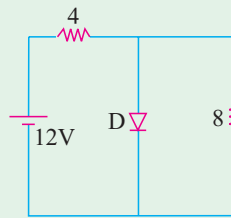


Fig. 52.50

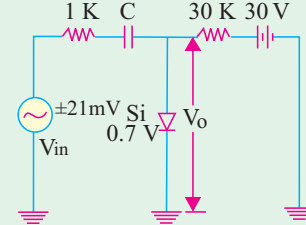


Fig. 52.51

1. Find the current supplied, if any, by the battery in the circuit of Fig. 52.49 which uses two oppositely-connected ideal diodes in parallel. [1 A]
2. What is the current supplied by the battery in Fig. 52.50 if *D* is an ideal diode. [3 A]
3. In Fig. 52.51, signal voltage has a peak value of  $\neq 21$  mV and its frequency is so high that reactance of the coupling capacitor can be neglected. If bulk resistance of the silicon diode is neglected, what

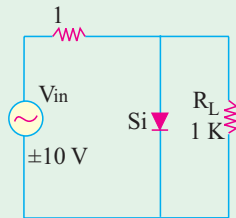


Fig. 52.52

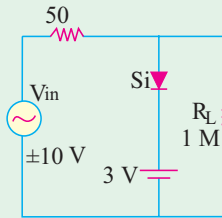


Fig. 52.53

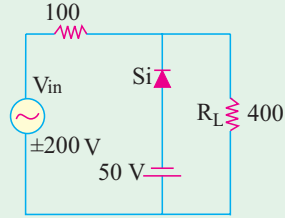


Fig. 52.54

would be the approximate waveform of the total voltage ' $V_o$ ' across the diode. [0.7 V dc on which rides an ac voltage of peak value = 1 mV]

4. Sketch the voltage across  $R_L$  in the clipper circuit of Fig. 52.52.
5. Sketch the voltage across  $R_L$  in the clipper circuit of Fig. 52.53.
6. What is the waveform of the voltage across  $R_L$  in Fig. 52.54 ?
7. What is the approximate dc component in the output of a positive diode clamper with a peak input of 20 V ? [20 V]

**OBJECTIVE TESTS – 52**

1. For a silicon diode, the value of the forward-bias voltage typically
  - (a) must be greater than 0.3 V
  - (b) must be greater than 0.7 V
  - (c) depends on the width of the depletion region
  - (d) depends on the concentration of majority carriers.
2. When forward biased, a diode
  - (a) blocks current
  - (b) conducts current
  - (c) has a high resistance
  - (d) drops a large voltage
3. The term bias means
  - (a) the ratio of majority carriers to minority carriers
  - (b) the amount of current across the P-N junction
  - (c) a dc voltage applied across the P-N junction to control its operation
  - (d) none of the above
4. To forward-bias a P-N junction, diode,
  - (a) an external voltage is applied that is positive at the anode and negative at the cathode
  - (b) an external voltage is applied that is negative at the anode and positive at the cathode.

- (c) an external voltage is applied that is positive at the P-region and negative at the N-region  
 (d) a and c above
5. When a P-N junction is forward-biased  
 (a) the only current is the hole current  
 (b) the only current is the electron current  
 (c) the only current is produced by majority carriers  
 (d) the current is produced by both holes and electrons.
6. A P-N junction diode's dynamic conductance is directly proportional to  
 (a) the applied voltage  
 (b) the temperature  
 (c) the current  
 (d) the thermal voltage  
 (Hint : Conductance is reciprocal of the resistance)
7. The junction capacitance of a linearly graded junction varies with the applied reverse bias,  $V_R$  as  
 (a)  $V_R^{-1}$  (b)  $V_R^{-1/2}$   
 (c)  $V_R^{-1/3}$  (d)  $V_R^{1/2}$   
 (UPSC Engg. Services 2002)
8. The diffusion capacitance of a forward biased  $P^+N$  ( $P^+$  indicates heavily doped P-region) junction diode with a steady current  $I$  depends on  
 (a) width of the depletion region  
 (b) mean life-time of holes  
 (c) mean life-time of electrons  
 (d) junction-area  
 (UPSC Engg. Services 2002)
9. The width of depletion layer of a P-N junction  
 (a) decreases with light doping  
 (b) increases with heavy doping  
 (c) is independent of applied voltage  
 (d) is increased under reverse bias.
10. At room temperature of  $25^\circ\text{C}$ , the barrier potential for silicon is 0.7 V. Its value at  $125^\circ\text{C}$  is ..... volt.  
 (a) 0.5 (b) 0.3  
 (c) 0.9 (d) 0.7.
11. Junction breakdown occurs  
 (a) under high temperature condition  
 (b) with forward bias  
 (c) under reverse bias  
 (d) because of manufacturing defect.
12. Avalanche breakdown is primarily dependent on the phenomenon of  
 (a) collision (b) doping  
 (c) ionization (d) recombination.

13. Reverse current in a silicon junction nearly doubles for every.....  $^\circ\text{C}$  rise in temperature.  
 (a) 10 (b) 2  
 (c) 6 (d) 5.
14. In the forward region of its characteristic, a diode appears as a/an

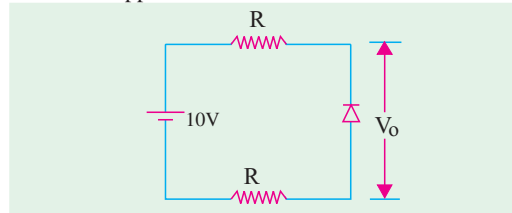


Fig. 52.55

- (a) OFF switch (b) high resistance  
 (c) capacitor (d) ON switch.
15. The approximate value of  $V_o$  across the diode in Fig. 52.53 is

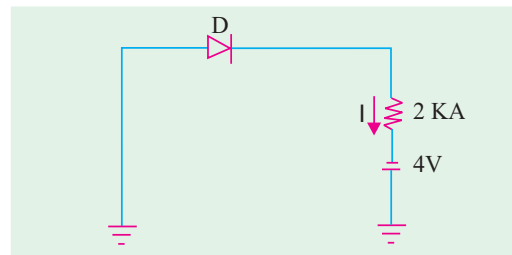


Fig. 52.56

- (a) zero (b) 10 V  
 (c) 5 V (d) dependent on value of R.
16. The diode 'D' is ideal in the network shown in Fig. 52.56. The current 'I' will be.  
 (a)  $\dot{\text{A}} 2 \text{ n A}$  (b) zero  
 (c) 2 m A (d) 4 mA
17. The voltages at  $V_1$  and  $V_2$  of the arrangement shown in Fig. 52.57 will be respectively.

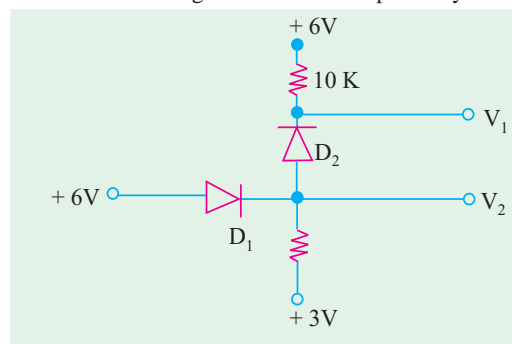


Fig. 52.57

- (a) 6 V and 5.4 V (b) 5.4 V and 6 V  
 (c) 3 V and 5.4 V (d) 6 V and 5 V



18. Without a dc source, a clipper acts like a  
 (a) rectifier (b) clamper  
 (c) demodulator (d) chopper
19. The primary function of a clamper circuit is to  
 (a) suppress variations in signal voltage  
 (b) raise positive half-cycle of the signal  
 (c) lower negative half-cycle of the signal  
 (d) introduce a dc level into an ac signal
20. For an input  $V_s = 5 \sin t$  (assuming ideal diode), the circuit shown in Fig. 52.58 will behave as a  
 (a) clipper, sine wave clipped at  $-2 \text{ V}$

- (b) clamper, sine wave clamped at  $-2 \text{ V}$   
 (c) clamper, sine wave clamped at zero volt  
 (d) clipped, sine wave clipped at  $2 \text{ V}$

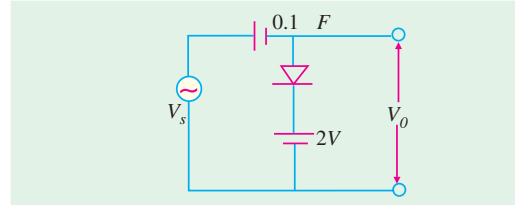


Fig. 52.58

21. A clipping circuit is shown in Fig. 52.59. Its transfer characteristic will be

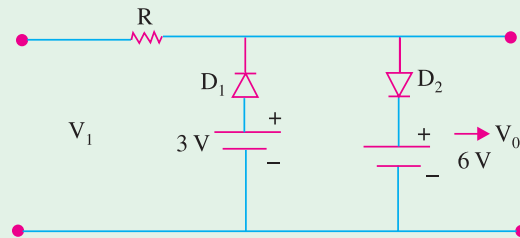
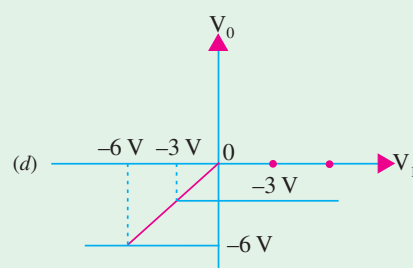
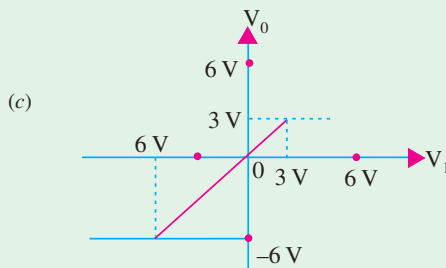
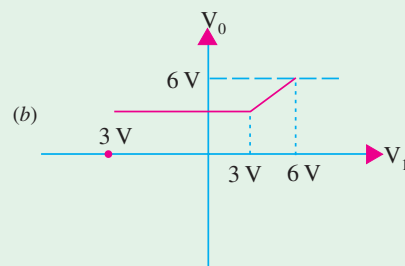
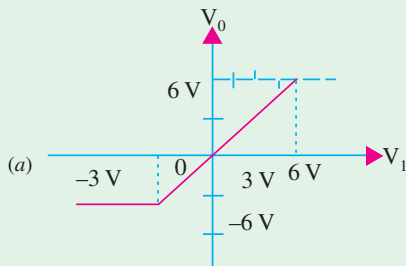


Fig. 52.59



ANSWERS

- |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (b)  | 3. (c)  | 4. (d)  | 5. (d)  | 6. (c)  | 7. (c)  |
| 8. (c)  | 9. (d)  | 10. (c) | 11. (c) | 12. (a) | 13. (c) | 14. (d) |
| 15. (b) | 16. (c) | 17. (a) | 18. (a) | 19. (d) | 20. (b) | 21. (b) |

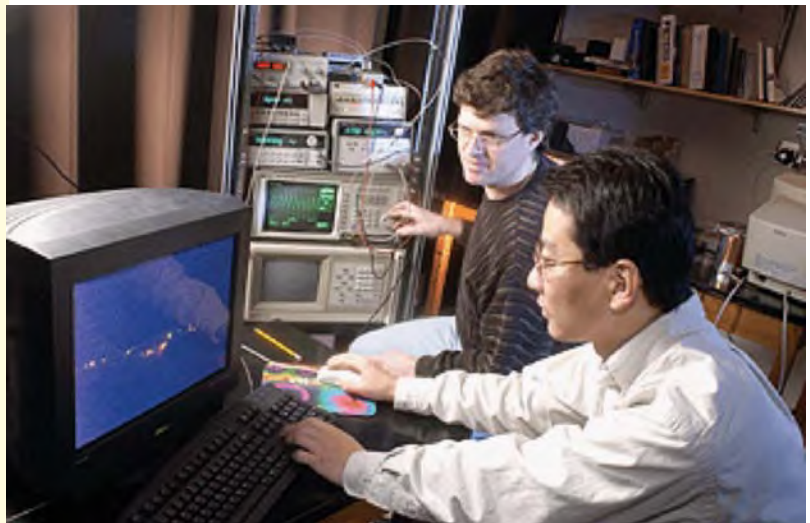
## ROUGH WORK

# C H A P T E R 53

## Learning Objectives

- Fundamentals of Light
- Light Emitting Diode (LED)
- Use of LEDs in Facsimile Machines
- Liquid Crystal Displays
- P-N Junction Photodiode
- Dust Sensor
- Photoconductive Cell
- Phototransistor
- Photodarlington
- Photo voltaic or Solar Cell
- Laser Diode
- Optical Disks
- Read-only Optical Disk Equipment
- Printers Using Laser Diodes
- Hologram Scanners
- Laser Range Finder
- Light-activated SCR(LASCR)
- Optical Isolators
- Optical Modulators
- Optical Fibre Communication Systems
- Optical Fibre Data Links

## OPTO-ELECTRONIC DEVICES



Researchers have demonstrated a new type of nanometer scale optoelectronic device that performs addition and other complex logic operations

### 53.1. Fundamentals of Light

According to the Quantum Theory, light consists of discrete packets of energy called photons. The energy contained in a photon depends on the **frequency** of the light and is given by the relation  $E = hf$  where  $h$  is Planck's constant ( $6.625 \times 10^{-34}$  Joule-second). In this equation, energy  $E$  is in Joules and frequency  $f$  is in hertz (Hz). As seen, photon energy is directly proportional to frequency: higher the frequency, greater the energy. Now, velocity of light is given by  $c = f\lambda$  where  $c$  is the velocity of the light ( $3 \times 10^8$  m/s) and  $\lambda$  is the wavelength of light in metres. The wavelength of light determines its colour in the visible range and whether it is ultraviolet or infrared outside the visible range.

Now,  $E = hf = hc/\lambda$  or  $\lambda = hc/E$  metres

$$\therefore \lambda = (6.625 \times 10^{-34}) \times (3 \times 10^8) / E = (19.875 \times 10^{-26}) / E \quad \text{--- } E \text{ in Joules}$$

If  $E$  is in electron-volt (eV), then since  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$\therefore \lambda = (19.875 \times 10^{-26}) / (E \times 1.6 \times 10^{-19}) = (12.42 \times 10^{-7}) / E \text{ metre}$$

or  $\lambda = 1.242 \mu\text{m}$

In a forward-biased  $P-N$  junction, electrons and holes both cross the junction. In the process, some electrons and holes recombine with the result that electrons lose energy. The amount of energy lost is equal to the difference in energy between the conduction and valence bands, this being known as the semiconductor energy band gap  $E_g$ . The value of  $E_g$  for silicon is 1.1 eV, for GaAs is 1.43 eV and for InAs is 0.36 eV. For example, the wavelength of light emitted by silicon  $P-N$  junction is  $\lambda = 1.242/E_g = 1.242/1.1 = 1.13 \mu\text{m}$ .

### 53.2. Light Emitting Diode (LED)

#### (a) Theory

As the name indicates, it is a forward-biased  $P-N$  junction which emits visible light when energised. As discussed earlier (Art. 53.40), charge carrier recombination takes place when electrons from the  $N$ -side cross the junction and recombine with the holes on the  $P$ -side.

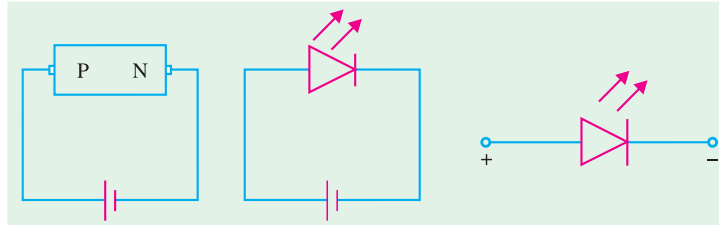
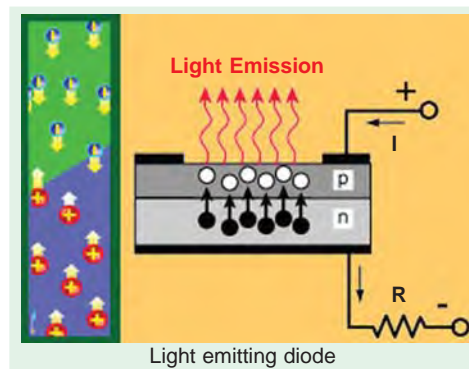


Fig. 53.1

Now, electrons are in the higher conduction band on the  $N$ -side whereas holes are in the lower valence band on the  $P$ -side. During recombination, some of the energy difference is given up in the form of heat and light (*i.e.* photons). For Si and Ge junctions, greater percentage of this energy is given up in the form of heat so that the amount emitted as light is insignificant. But in the case of other semiconductor materials like gallium arsenide (GaAs), gallium phosphide (GaP) and gallium-arsenide-phosphide (GaAsP), a greater percentage of energy released during recombination is given out in the form of light. If the semiconductor material is translucent, light is emitted and the junction becomes a light source *i.e.* a light-emitting diode (LED) as shown schematically in Fig. 53.1. The colour of the emitted light depends on the type of material used as given on the next page.

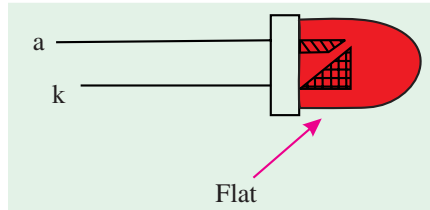


Light emitting diode

The colour of the emitted light depends on the type of material used as given on the next page.

1. GaAs — infrared radiation (invisible).
2. GaP — red or green light.
3. GaAsP — red or yellow (amber) light.

LEDs that emit *blue* light are also available but red is the most common. LEDs emit no light when reverse-biased. In fact, operating LEDs in reverse direction will quickly destroy them. Fig. 53.1 shows a picture of LEDs that emits different colours of light.



**(b) Construction**

Broadly speaking, the LED structures can be divided into two categories :

1. **Surface-emitting LEDs :** These LEDs emit light in a direction **perpendicular** to the *PN* junction plane.
2. **Edge-emitting LEDs :** These LEDs emit light in a direction **parallel** to the *PN* junction plane.

Fig. 53.2 shows the construction of a surface-emitting LED. As seen from this figure, an *N*-type layer is grown on a substrate and a *P*-type layer is deposited on it by diffusion. Since carrier recombination takes place in the *P*-layer, it is kept upper most. The metal anode connections are made at the outer edges of the *P*-layer so as to allow more central surface area for the light to escape. LEDs are manufactured with domed lenses in order to lessen the reabsorption problem.

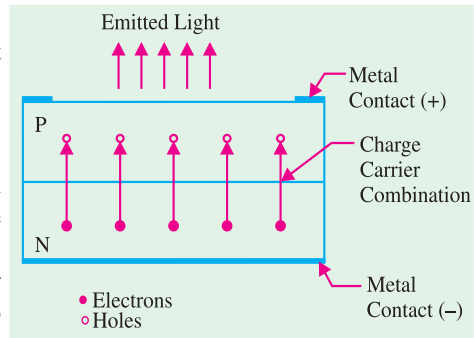


Fig. 53.2

A metal (gold) film is applied to the bottom of the substrate for reflecting as much light as possible to the surface of the device and also to provide cathode connection. LEDs are always encased in order to protect their delicate wires.

Being made of semiconductor material, it is rugged and has a life of more than 10,000 hours.

**(c) Working**

The forward voltage across an LED is considerably greater than for a silicon *PN* junction diode. Typically the maximum forward voltage for LED is between 1.2 V and 3.2 V depending on the device. Reverse breakdown voltage for an LED is of the order of 3 V to 10 V. Fig. 53.3 (a) shows a simple circuit to illustrate the working of an LED. The LED emits light in response to a sufficient forward current. The amount of power output translated into light is directly proportional to the forward current as shown in Fig. 53.3 (b). It is evident from this figure that greater the forward current, the greater the light output.

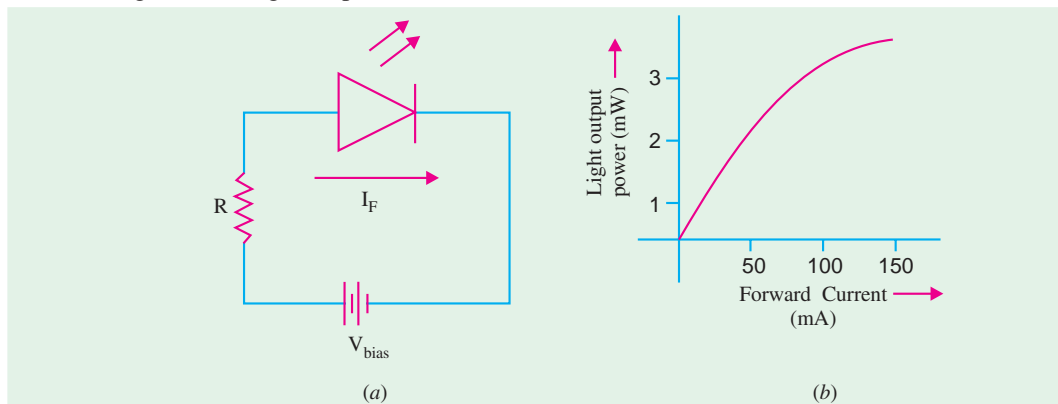


Fig. 53.3

**(d) Applications**

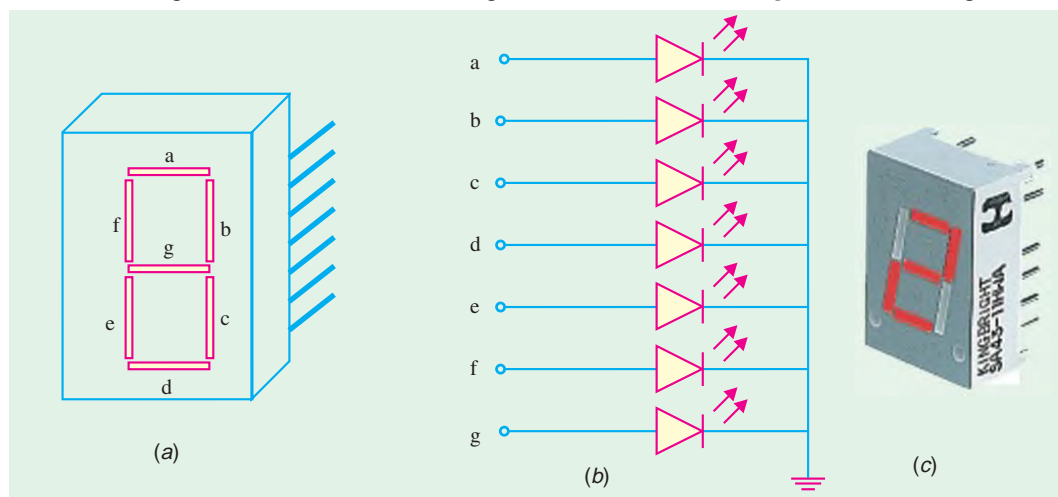
To choose emitting diodes for a particular application, one or more of the following points have to be considered : wavelength of light emitted, input power required, output power, efficiency, turn-on and turn-off time, mounting arrangement, light intensity and brightness etc.

Since LEDs operate at voltage levels from 1.5 V to 3.3 V, they are highly compatible with solid-state circuitry.

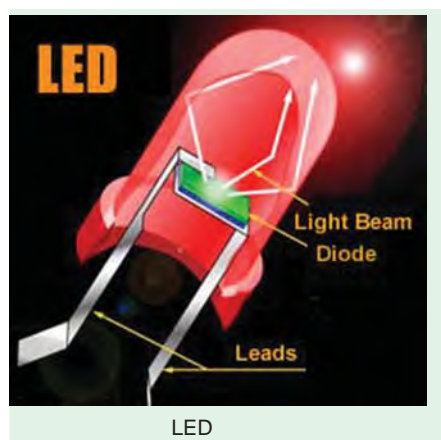
Their uses include the following :

1. LEDs are used in burglar-alarm systems;
2. for solid-state video displays which are rapidly replacing cathode-ray tubes (CRT);
3. in image sensing circuits used for 'picturephone';
4. in the field of optical fibre communication systems where high-radiance GaAs diodes are matched into the silica-fibre optical cable;
5. in data links and remote controllers;
6. in arrays of different types for displaying alphanumeric (letters and numbers) or supplying input power to lasers or for entering information into optical computer memories;
7. for numeric displays in hand-held or pocket calculators.

As shown in Fig. 53.4 (a) a seven-segment display consists of seven rectangular LEDs which can form the digits 0 to 9. The seven LED segments are labelled 'a' to 'g'. Each of this segments is



**Fig. 53.4**

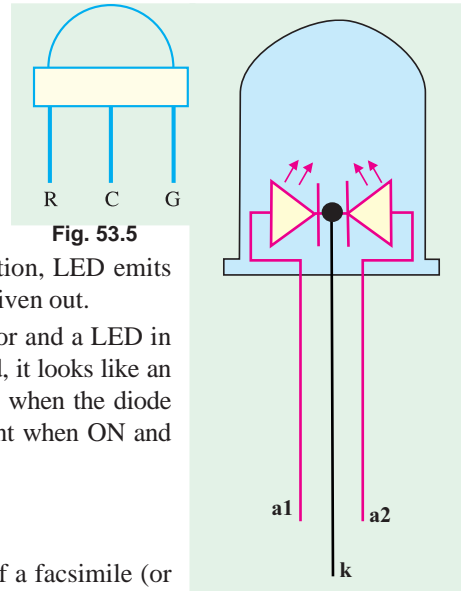


controlled through one of the display LEDs. Seven-segment displays come in two types, common-cathode and common-anode type. In the common-cathode type, all the cathodes of the diodes are tied together as shown in Fig. 53.4 (b). This makes it possible to light any segment by forward-biasing that particular LED. For example, to light number 5, segments *a*, *f*, *g*, *c* and *d* must be forward-biased. Since the cathodes are tied to ground, only 5 volt is to be applied to the anode of these segments to light them.

The common-anode seven-segment display has all its anodes tied together to +5 volt and ground is used to light the individual segments. Fig. 53.4(c) shows a picture of a seven-segment display.

**(e) Multicoloured LEDs**

LEDs are available which gives out light in either two or three colours. There are also blinking LEDs. A two-colour LED is a three-terminal device as shown in Fig. 3.5. The longest lead is the cathode and the remaining two leads are the anodes. When leads R and C are forward-biased, the LED emits red light and when leads G and C are forward-biased, LED emits green light. The tricolour LED looks similar to the ordinary LED but emits, red, green or yellow light depending on operating conditions. It has two leads and each of these acts as both anode and cathode. When dc current flows through it in one direction, LED emits red light but when current flows in the opposite direction, LED emits green light. However, with ac current, yellow light is given out.

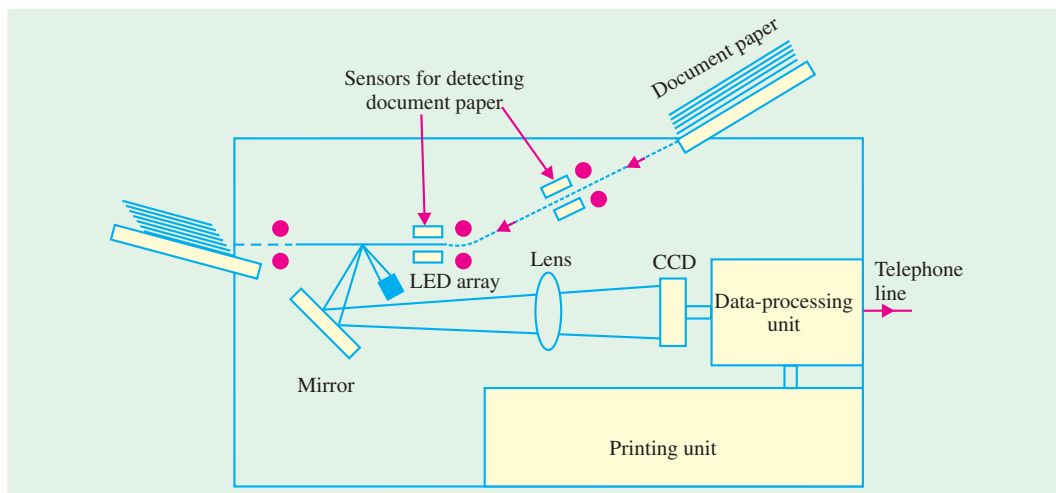


**Fig. 53.5**

The blinking LED is a combination of an oscillator and a LED in one package. Since it has an anode and a cathode lead, it looks like an ordinary LED. The blinking frequency is usually 3 Hz when the diode forward bias is 5 V. It conducts about 20 mA of current when ON and 0.9 mA when OFF.

**53.3. Use of LEDs in Facsimile Machines**

Fig. 53.6 shows a simplified schematic diagram of a facsimile (or fax) machine. As seen, the light from the LED array is focussed on the document paper. The light reflected at the paper is focussed on a charge-coupled device (CCD) by a combination of mirror and a lens. This causes the optical information to be converted into electrical information. The electrical information is then sent through the data-processing unit to its destination via telephone line.



**Fig. 53.6**

**53.4. Liquid Crystals Displays**

**(a) General**

A liquid crystal is a material (usually, an organic compound) which flows like a liquid at room temperature but whose molecular structure has some properties normally associated with solids (examples of such compounds are : cholesteryl nonanoate and p-azoxyanisole). As is well-known,

the molecules in ordinary liquids have random orientation but in a liquid crystal they are oriented in a definite crystal pattern. Normally, a thin layer of liquid crystal is transparent to incident light but when an electric field is applied across it, its molecular arrangement is disturbed causing changes in its optical properties. When light falls on an activated layer of a liquid crystal, it is either absorbed or else is scattered by the disoriented molecules.

#### (b) Construction

As shown in Fig. 53.7 (a), a liquid crystal 'cell' consists of a thin layer (about 10  $\mu\text{m}$ ) of a liquid crystal sandwiched between two glass sheets with transparent electrodes deposited on their inside faces. With both glass sheets transparent, the cell is known as **transmittive** type cell. When one glass is transparent and the other has a reflective coating, the cell is called **reflective** type. The LCD does not produce any illumination of its own. It, in fact, depends entirely on illumination falling on it from an external source for its visual effect.

#### (c) Working

The two types of display available are known as (i) **field-effect display** and (ii) **dynamic scattering display**. When field-effect display is energized, the energized areas of the LCD absorb the incident light and, hence give localized black display. When dynamic scattering display is energized, the molecules of energized area of the display become turbulent and scatter light in all directions. Consequently, the activated areas take on a frosted glass appearance resulting in a silver display. Of course, the un-energized areas remain translucent.

As shown in Fig. 53.7 (b), a digit on an LCD has a segment appearance. For example, if number 5 is required, the terminals 8, 2, 3, 6 and 5 would be energized so that only these regions would be activated while the other areas would remain clear.

#### (d) Advantages

An LCD has the distinct advantage of extremely low power requirement (about 10-15  $\mu\text{W}$  per 7-segment display as compared to a few mW for a LED). It is due to the fact that it does not itself generate any illumination but depends on external illumination for its visual effect (colour depending on the incident light). They have a life-time of about 50,000 hours.

#### (e) Uses

1. Field-effect LCDs are normally used in watches and portable instruments where source of energy is a prime consideration.
2. Thousands of tiny LCDs are used to form the picture elements (pixels) of the screen in one type of B & W pocket TV receiver.
3. Recent desk top LCD monitors.
4. Note book computer display
5. Cellular phone display, to display data on personal digital assistant (PDAs) such as Palm Vx etc.

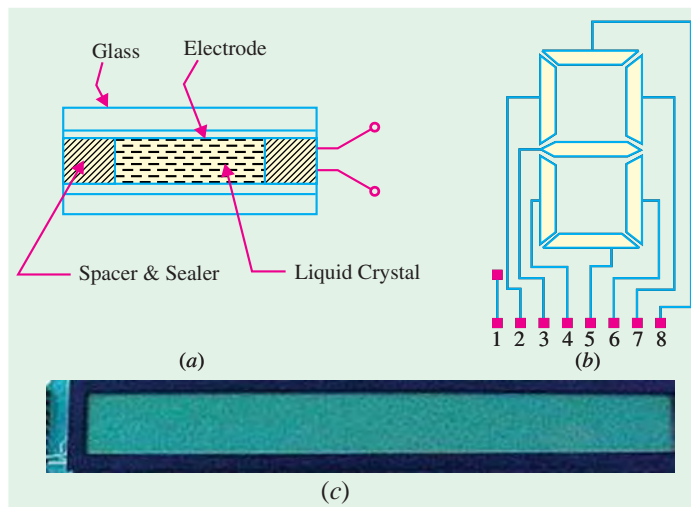


Fig. 53.7



The liquid crystal display (LCDs) commonly used on notebook computers and handheld PDAs are also appearing on desktop. These flat panel displays promise great clarity at increasingly high resolutions and are available in screen sizes upto 15 inches. The LCD monitor offers benefits and drawbacks. The first benefit is size. Because of the need to house the tube itself, cathode-ray tube (CRT) monitors are big and heavy. LCD monitors are only a few inches deep and they are much lighter in weight. However LCD monitors are expensive than CRTs at present. Another problem is the viewing angle. The optimal viewing angle of an LCD is from straight in front and as you move further to the side the screen becomes harder to read, much more so than with a CRT. Moreover screen resolutions generally reach only as high as  $1,024 \times 768$ , which is insufficient for some applications. Fig. 53.7(c) shows the picture of an LCD used in portable instrument.

**53.5. P-N Junction Photodiode**

It is a two-terminal junction device which is operated by first reverse-biasing the junction and then illuminating it. A reverse-biased P-N junction has a small amount of reverse saturation current  $I_s$  (or  $I_0$ ) due to thermally-generated electron-hole pairs. In silicon,  $I_s$  is the range of nanoamperes. The number of these minority carriers depends on the intensity of light incident on the junction. When the diode is in glass package, light can reach the junction and thus change the reverse current.

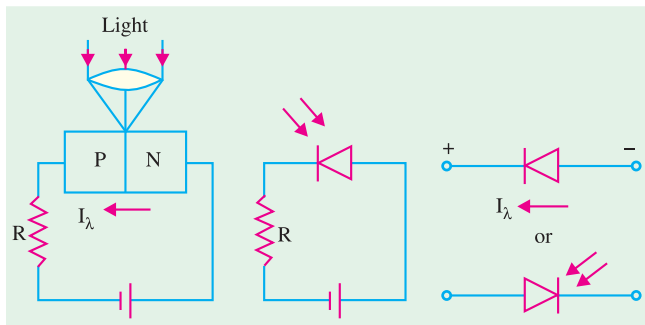
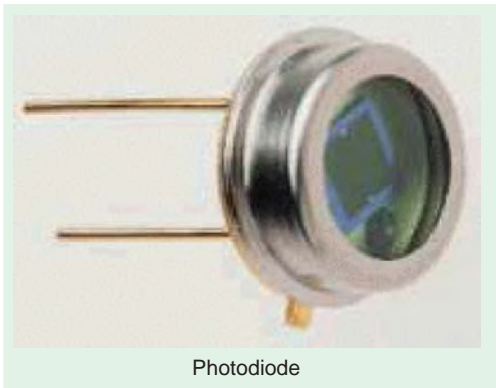


Fig. 53.8



Photodiode

The basic biasing arrangement, construction and symbols of a photodiode are shown in Fig. 53.8. As seen, a lens has been used in the cap of the unit to focus maximum light on the reverse-biased junction. The active diameter of these devices is about 2.5 mm but they are mounted in standard TO-5 packages with a window to allow maximum incident light.

The characteristics of Fig. 53.9 show that for a given reverse voltage,  $I_\lambda$  (or  $I_s$ ) increases with increase in the level of illumination. The dark current refers

to the current that flows when no light is incident. By changing the illumination level, reverse current can be changed. In this way, reverse resistance of the diode can be changed by a factor of nearly 20.

A photodiode can turn its current ON and OFF in nanoseconds. Hence, it is one of the fastest photodetectors. It is used where it is required to switch light ON and OFF at a maximum rate. Applications of a photodiode include

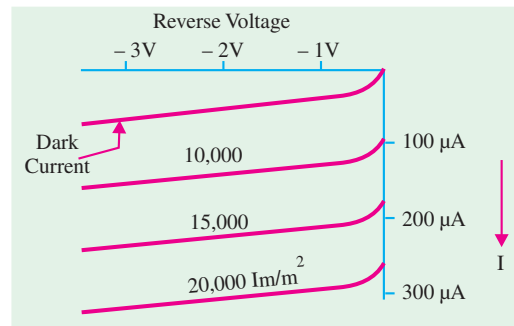


Fig. 53.9

1. detection, both visible and invisible ;
2. demodulation ;
3. switching ;
4. logic circuit that require stability and high speed ;
5. character recognition ;
6. optical communication equipment ;
7. encoders etc.

### 53.6. Dust Sensor

Fig. 53.10 shows a combination of an LED and a photodiode used as a dust sensor. As seen, the light emitted from the LED gets reflected by the dust particles. The reflected light is collected by the photodiode and is converted into an electrical signal. The dust sensor is employed in cleaners.

The combination of an LED and a photodiode is also used as : (1) a paper sensor in facsimile machines, (2) as a tape-end sensor in videotape recorders/players, and (3) as a dirt detector for rinsing in washing machines.

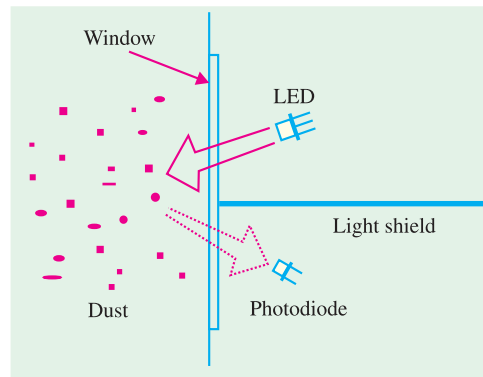


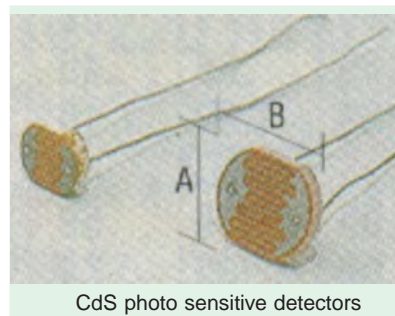
Fig. 53.10

### 53.7. Photoconductive Cell

It is a semiconductor device whose resistance varies inversely with the intensity of light that falls upon it. It is also known as *photoresistive* cell or *photoresistor* because it operates on the principle of photoresistivity.

#### (a) Theory

The resistivity (and, hence, resistance) of a semiconductor depends on the number of free charge carriers available in it. When the semiconductor is not illuminated, the number of charge carriers is small and, hence, resistivity is high. But when light in the form of photons strikes the semiconductor, each photon delivers energy to it. If the photon energy is greater than the energy band gap of the semiconductor, free mobile charge carriers are liberated and, as a result, resistivity of the semiconductor is decreased.



CdS photo sensitive detectors

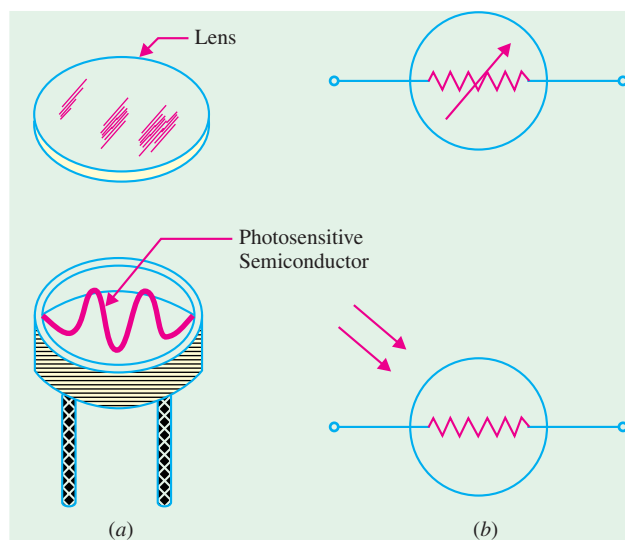


Fig. 53.11

#### (b) Construction and Working

Photoconductive cells are generally made of cadmium compounds such as cadmium sulphide (CdS) and cadmium selenide (CdSe). Spectral response of CdS cell is similar to the human eye, hence such cells are often used to simulate the human eye. That is why they find

use in light metering circuits in photographic cameras.

The construction of a typical photo conductive cell and its two alternative circuit symbols are shown in Fig. 53.11 (a) and (b) respectively. As seen, a thin layer of photosensitive semiconductor material is deposited in the form of a long strip zig-zagged across a disc-shaped ceramic base with protective sides. For added protection, a glass lens or plastic cover is used. The two ends of the strip are brought out to connecting pins below the base.

The terminal characteristic of a photoconductive cell is shown in Fig. 53.12. It depicts how the resistance of the cell varies with light intensity. Typically, the dark resistance of the cell is 1 MΩ or larger. Under illumination, the cell resistance drops to a value between 1 and 100 kΩ depending on surface illumination.

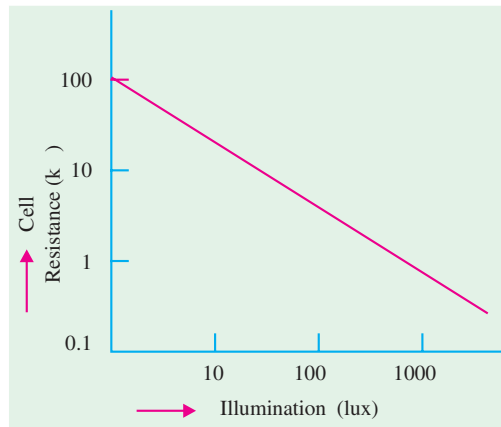


Fig. 53.12

**(c) Applications**

A photoconductive cell is an inexpensive and simple detector which is widely used in OFF/ON circuits, light-measurement and light-detecting circuits.

**Example 53.1.** A relay is controlled by a photoconductive cell which has resistance of 100 kΩ when illuminated and 1 kΩ when in the dark. The relay is supplied with 10 mA from a 30-V supply when cell is illuminated and is required to be de-energized when the cell is in the dark. Sketch a suitable circuit and calculate the required series resistance and value of dark current.

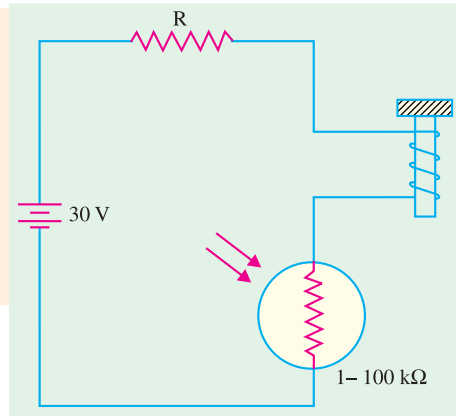


Fig. 53.13

**(Optoelectronic Devices, Gujarat Univ. 1993)**

**Solution.** The circuit is as shown in Fig. 30.13 where R is a current-limiting resistor.

$$I = 30/(R + r)$$

—where r is cell resistance

$$\therefore R = (30/I) - r$$

**When illuminated**

$$R = (30/10 \times 10^{-3}) - 1 \times 10^3 = 2 \times 10^3 = 2 \text{ k}\Omega$$

Dark current is given by

$$I_d = 30/(2 + 100) \times 10^3 = 0.3 \times 10^{-3} \text{ A} = 0.3 \text{ mA}$$

**53.8. Phototransistor**

It is light-sensitive transistor and is similar to an ordinary bipolar junction transistor (BJT) except that it has no connection to the base terminal. Its operation is based on the photodiode that exists at the CB junction. Instead of the base current, the input to the transistor is provided in the form of light as shown in the schematic symbol of Fig. 53.14 (a).

Silicon NPNs are mostly used as photo transistors. The device is usually packed in a TO-type can with a lens on top although it is



Phototransistor

sometimes encapsulated in clear plastic. When there is no incident light on the  $CB$  junction, there is a small thermally-generated collector-to-emitter leakage current  $I_{CEO}$  which, in this case, is called dark current and is in the  $nA$  range.

When light is incident on the  $CB$  junction, a base current  $I_\lambda$  is produced which is directly proportional to the light intensity. Hence, collector current  $I_C = \beta I_\lambda$

Typical collector characteristic curves of a phototransistor are shown in Fig. 53.14 (b). Each individual curve corresponds to a certain value of light intensity expressed in  $mW/cm^2$ . As seen,  $I_C$  increases with light intensity.

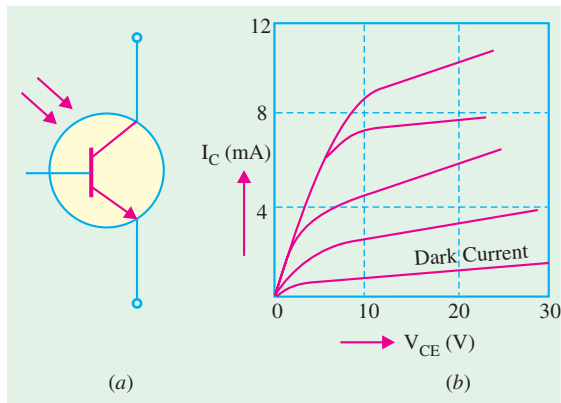


Fig. 53.14

The phototransistor has applications similar to those of a photodiode. Their main differences are in the current and response time. The photo-transistor has the advantages of greater sensitivity and current capacity than photodiodes. However, photodiodes are faster of the two, switching in less than a nanosecond.

### 53.9. Photodarlington

As shown in Fig. 53.15 a photodarlington consists of a phototransistor in a Darlington arrangement with a common transistor. It has a much greater sensitivity to incident radiant energy than a phototransistor because of higher current gain. However, its switching time of  $50 \mu s$  is much longer than the phototransistor ( $2 \mu s$ ) or the photodiode ( $1 ns$ ). Its circuit diagram is shown in Fig. 53.15.

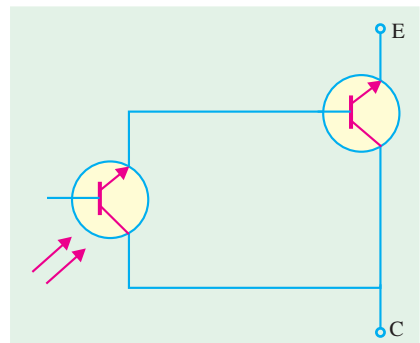


Fig. 53.15

#### Applications

Photodarlington are used in a variety of applications some of which are given below.

A light-operated relay is shown in Fig. 53.16 (a). The phototransistor  $T_1$  drives the bipolar transistor  $T_2$ . When sufficient light falls on  $T_2$ , it is driven into saturation so that  $I_C$  is increased manifold. This collector current while passing through the relay coil energizes the relay.

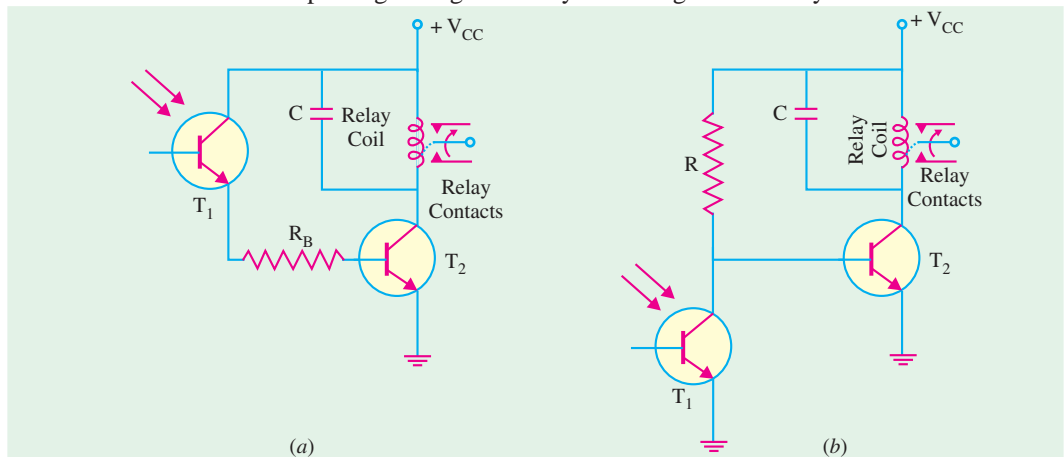


Fig. 53.16

Fig. 53.16 (b) shows a dark-operated relay circuit *i.e.* one in which relay is deenergized when light falls on the phototransistor. Here,  $T_1$  and  $R$  form a potential divider across  $V_{CC}$ . With insufficient light incident on  $T_1$ , transistor  $T_2$  is biased ON thereby keeping the relay energized. However, when there is sufficient light,  $T_1$  turns ON and pulls the base of  $T_2$  low thereby turning  $T_2$  OFF and hence, deenergizing the relay.

Such relays are used in many applications such as (i) automatic door activators, (ii) process counters and (iii) various alarm systems for smoke or intrusion detection.

**53.10. Photo voltaic or Solar Cell**

Such cells operate on the principle of photovoltaic action *i.e.* conversion of light energy into electrical energy. This action occurs in all semiconductors which are constructed to absorb energy.

**(a) Construction**

As shown in Fig. 53.17 (a), a basic solar cell consists of *P*-type and *N*-type semiconductor material (usually, silicon or selenium) forming a *P-N* junction. The bottom surface of the cell (which is always away from light) covered with a continuous conductive contact to which a wire lead is attached. The upper surface has a maximum area exposed to light with a small contact either along the edge or around the perimeter. The surface layer of *P*-type material is extremely thin (0.5 mm) so that light can penetrate to the junction.

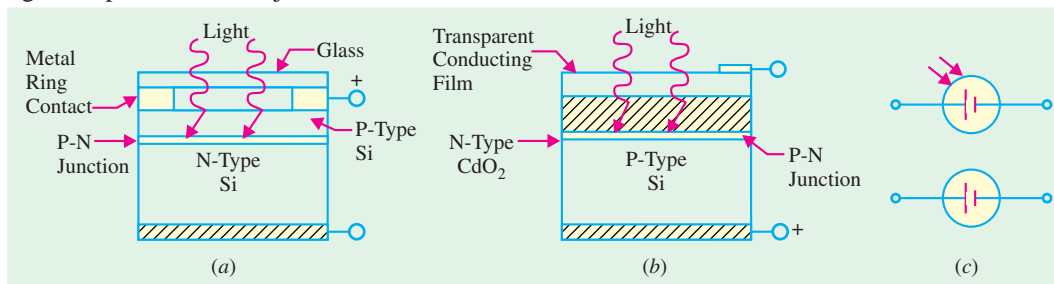
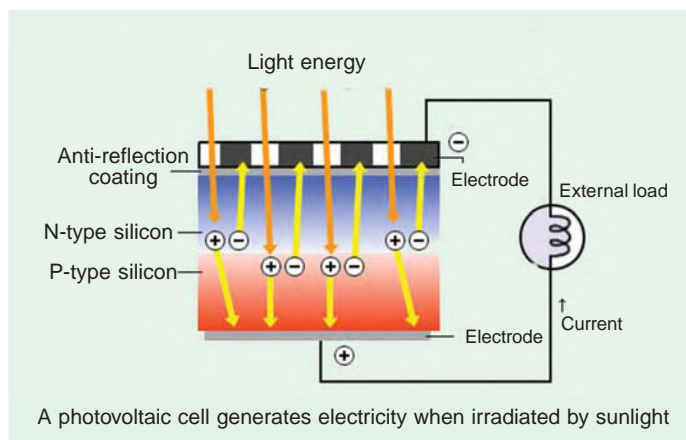


Fig. 53.17

Although silicon is commonly used for fabricating solar cells, another construction consists of *P*-type selenium covered with a layer of *N*-type cadmium oxide to form *P-N* junction as shown in Fig. 53.17 (b). Two alternative circuit symbols are shown in Fig. 53.17 (c). Power solar cells are also fabricated in flat strips to form efficient coverage of available surface area. Incidentally, the maximum efficiency of a solar cell in converting sunlight into electrical energy is nearly 15 per cent at the present.

**(b) Theory**

When the *P-N* junction of a solar cell is illuminated, electron-hole pairs are generated in much the same way, as in photovoltaic cell. An electric field is established near the *P-N* junction by the positive and negative ions created due to the production of electron-hole pairs which leads to the development of potential across the junction. Since the number of electron-hole pairs far exceeds the number needed for thermal equilibrium, many of the



electrons are pulled across the junction by the force of the electric field. Those that cross the junction contribute to the current in the cell and through the external load. The terminal voltage of the cell is directly proportional to the intensity of the incident light. The voltage may be as high as 0.6 V depending on the external load. Usually a large number of cells are arranged in an array in order to obtain higher voltages and currents as shown in Fig. 53.18.

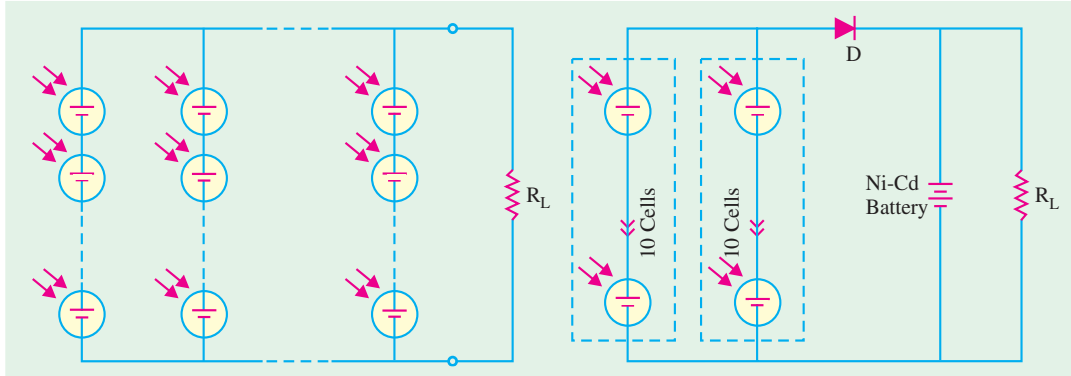


Fig. 53.18

Fig. 53.19

Solar cells act like a battery when connected in series or parallel. Fig. 53.19 show two groups of 10 series cells connected in parallel with each other. If each cell provides 0.5 V at 150 mA, the overall value of the solar bank is 5 V at 150 mA. The two parallel solar banks provide 5 V at 300 mA. This solar power source supplies the load and also charges the Ni-Cd battery. The battery provides power in the absence of light. A blocking diode  $D$  is used to isolate the solar cells from the Ni-Cd battery otherwise in the absence of light, the battery will discharge through the cells thereby damaging them.

A solar cell operates with fair efficiency, has unlimited life, can be easily mass-produced and has a high power capacity per weight. It is because of these qualities that it has become an important source of power for earth satellites.

**Example 53.2.** An earth satellite has on board 12-V battery which supplies a continuous current of 0.5 A. Solar cells are used to keep the battery charged. The solar cells are illuminated by the sun for 12 hours in every 24 hours. If during exposure, each cell gives 0.5 V at 50 mA, determine the number of cells required. (Optoelectronics Devices, Gujarat Univ. 1994)

**Solution.** The solar cell battery-charging circuit is shown in Fig. 53.20. The cells must be connected in series to provide the necessary voltage and such groups must be connected in parallel to provide the necessary current. The charging voltage has to be greater than the battery voltage of 12 V. Allowing for different drops, let the solar bank voltage be 13.5 V.

Number of series connected solar cells =  $13.5/0.5 = 27$

The charge given out by batteries during a 24 hour period =  $12 \times 0.5 = 6$  Ah. Hence, solar cells must supply this much charge over the same period. However, solar cells deliver current only when they illuminated *i.e.* for 12 hours in every 24 hours. Necessary charging current

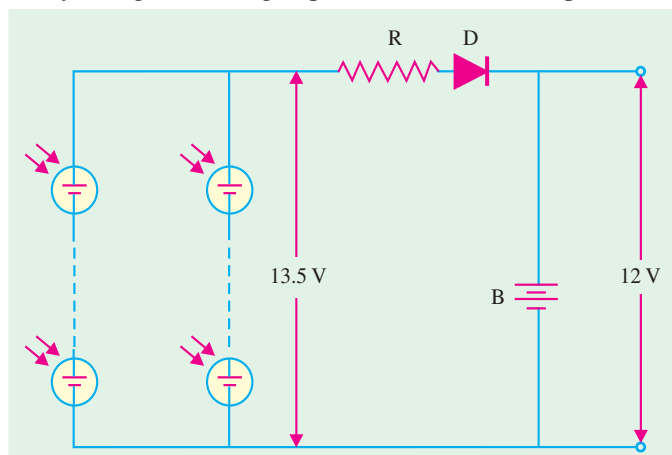


Fig. 53.20

required from the solar cells = 6 Ah/12 h = 0.5 A.

Total number of groups of solar cells required to be connected in parallel is

$$= \text{output current} / \text{cell current} = 0.5 / 50 \times 10^3 = 10$$

∴ total number of solar cells required for the earth satellite = 27 × 10 = 270

### 53.11. Laser Diode

Like LEDs, laser diodes are typical *PN* junction devices used under a forward-bias. The word LASER is an acronym for *Light Amplification by Stimulated Emission of Radiation*. The use of laser is (becoming increasing common) in medical equipment used in surgery and in consumer products like compact disk (CD) players, laser printers, hologram scanners etc.

#### (a) Construction

Broadly speaking, the laser diode structure can be divided into two categories :

1. **Surface-emitting laser diodes** : These laser diodes emit light in a direction **perpendicular** to the *PN* junction plane.
2. **Edge-emitting laser diodes** : These laser diodes emit light in a direction **parallel** to the *PN* junction plane.

Fig. 53.21 (a) shows the structure of an edge-emitting laser diode. This type of structure is called Fabry-Perot type laser. As seen from the figure, a *P-N* junction is formed by two layers of doped gallium arsenide (GaAs). The length of the *PN* junction bears a precise relationship with the wavelength of the light to be emitted. As seen, there is a highly reflective surface at one end of the junction and a partially reflective surface at the other end. External leads provide the anode and cathode connections.

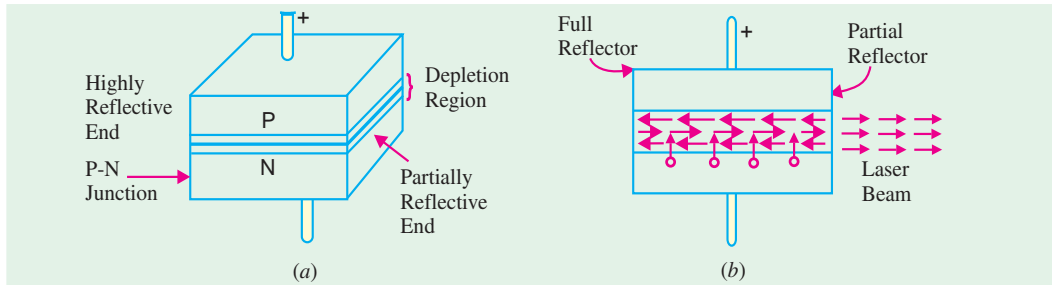
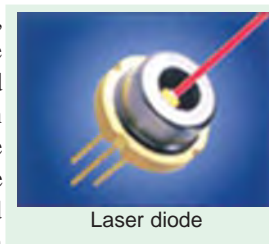


Fig. 53.21

#### (b) Theory

When the *P-N* junction is forward-biased by an external voltage source, electrons move across the junction and usual recombination occurs in the depletion region which results in the production of photons. As forward current is increased, more photons are produced which drift at random in the depletion region. Some of these photons strike the reflective surface perpendicularly. These reflected photons enter the depletion region, strike other atoms and release more photons. All these photons move back and forth between the two reflective surfaces. [Fig. 53.21 (b)] The photon activity becomes so intense that at some point, a strong beam of laser light comes out of the partially reflective surface of the diode.



Laser diode

#### (c) Unique Characteristics of Laser Light

The beam of laser light produced by the diode has the following unique characteristics :

1. It is coherent *i.e.* there is no path difference between the waves comprising the beam;
2. It is monochromatic *i.e.* it consists of one wavelength and hence one colour only;

3. It is collimated *i.e.* emitted light waves travel parallel to each other.

Laser diodes have a threshold level of current above which the laser action occurs but below which the laser diode behaves like a LED emitting incoherent light. The schematic symbol of a laser diode is similar to that of LED. Incidentally, a filter or lens is necessary to view the laser beam.

#### (d) Applications

Laser diodes are used in variety of applications ranging from medical equipment used in surgery to consumer products like optical disk equipment, laser printers, hologram scanners etc. Laser diodes emitting visible light are used as pointers. Those emitting visible and infrared light are used to measure range (or distance). The laser diodes are also widely used in parallel processing of information and in parallel interconnections between computers. Some of these applications are discussed in the following articles.

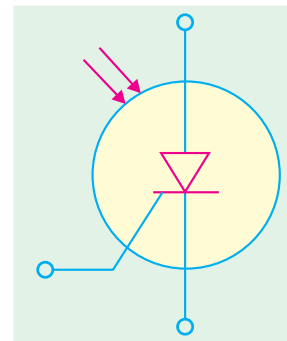


Fig. 53.22

### 53.12. Optical Disks

The major application field for laser diodes is in optical disk equipment. This equipment is used for reading or recording information and can be broadly divided into two groups :

1. Reading-only and
2. Recording-and-reading type.

The optical disk equipment of either type make use of a laser diode, lenses and photodiodes. During recording, it changes electrical information into optical information and then records the information on the optical disk. During reading (or playback), the head optically reads the recorded information and changes the optical information into electrical information. Fig. 53.22 shows the different types of optical disks used in practice. The commercial systems make use of disks that are 90, 120, 130 and 300 mm in diameter. A mini disk, 64 mm in diameter is also used for digital audio.

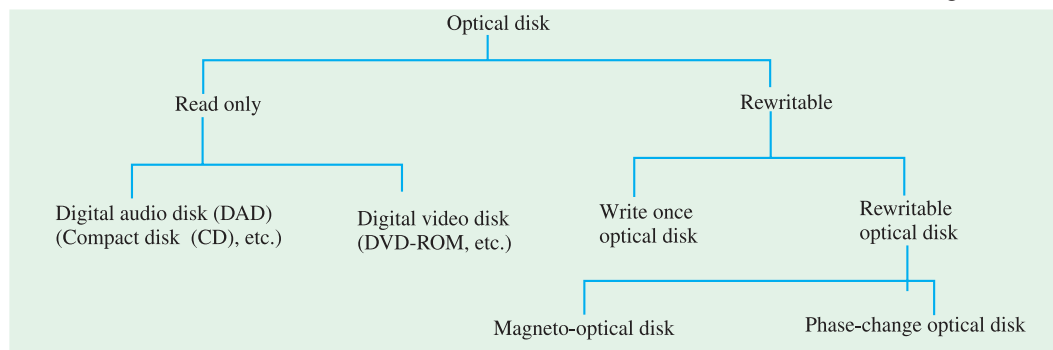


Fig. 53.23

The optical disks have several advantages over semiconductor memories. Some of these include their larger data storage capacity, shorter access time and smaller size. Therefore they are used in terminal equipment of computers as well as in audio visual equipment.

### 53.13. Read-only Optical Disks Equipment

Fig. 53.24 shows an optical equipment for reading data from digital audio (compact) disks. Compact disks (CDs) which are 120 mm in diameter are typical digital audio disks. Compact disks usually means digital audio compact disk, but it also includes the read-only memory (CD-ROM) for data memory and interactive compact disk (CD-I) for multimedia use.



Audio information (*i.e.* sound) is digitally recorded in stereo on the surface of a CD in the form of microscopic “pits” and “flats”. As seen from Fig. 53.24, the light emitted from the laser diode passes through the lens and is focussed to a diameter of about  $1\ \mu\text{m}$  on the surface of a disk. As the CD rotates, the lens and beam follow the track under control of a servo motor. The laser light which is altered by the pits and flats along the recorded track is reflected back from the track through the lens and optical system to infrared photodiodes. The signal from the photodiodes is then used to reproduce the digitally recorded sound.

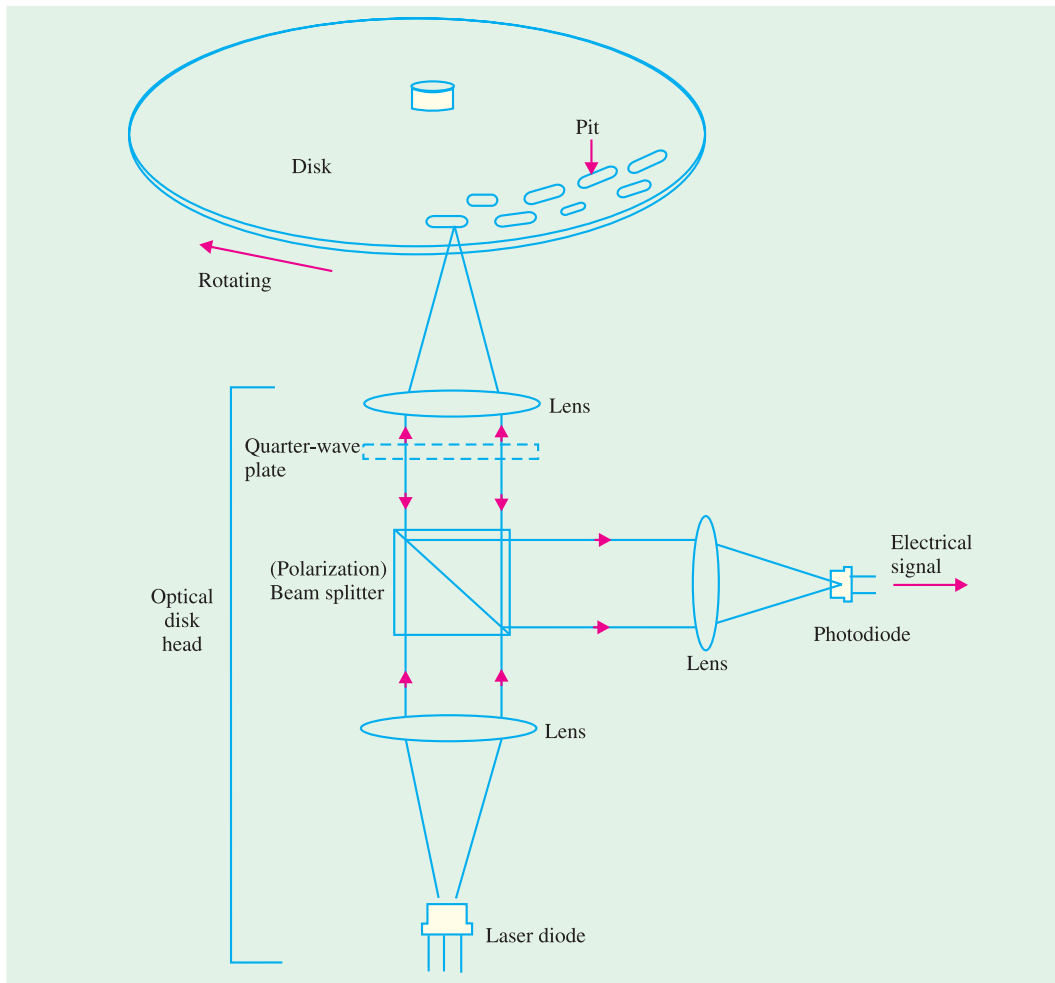


Fig. 53.24

### 53.14. Printers Using Laser Diodes

There are two types of optical sources usually used in printers ; (1) laser diodes and (2) LED arrays. The printers using laser diodes are called laser beam printers (or simply laser printers). These are one of the most attractive type of equipment in office automation in today’s world. Words and figures can be printed rapidly and clearly more easily by a laser printer than by other types of printers.

(Courtesy optical semiconductor devices by M.Fukuda published by John Weliy & Sons Inc.)

Fig. 53.25 shows a simplified diagram of a laser printer. As seen the laser diode is driven by modulated signals from the computer. The optical beam after passing through the lens is reflected by the rotating polygon mirror and scanned on the photosensitive drum. The drum is homogeneously charged when it passes through the charging unit consisting of an LED array. The homogeneous electrification is partially erased in accordance with the scanned optical beam. This is because of the fact that the electrical resistance at the light-irradiated part decreases and the electric charge is released. This causes the signals (*i.e.* data) from the computer to be written on to the drum. At the developing unit, an electrically charged powder (called toner) is electrostatically attached to the written parts. At the transcribing unit, the powder is transferred to the paper. Next, the transferred pattern is fixed by heating and pressing at the fixing unit. The data from the computer is thus printed on the paper.

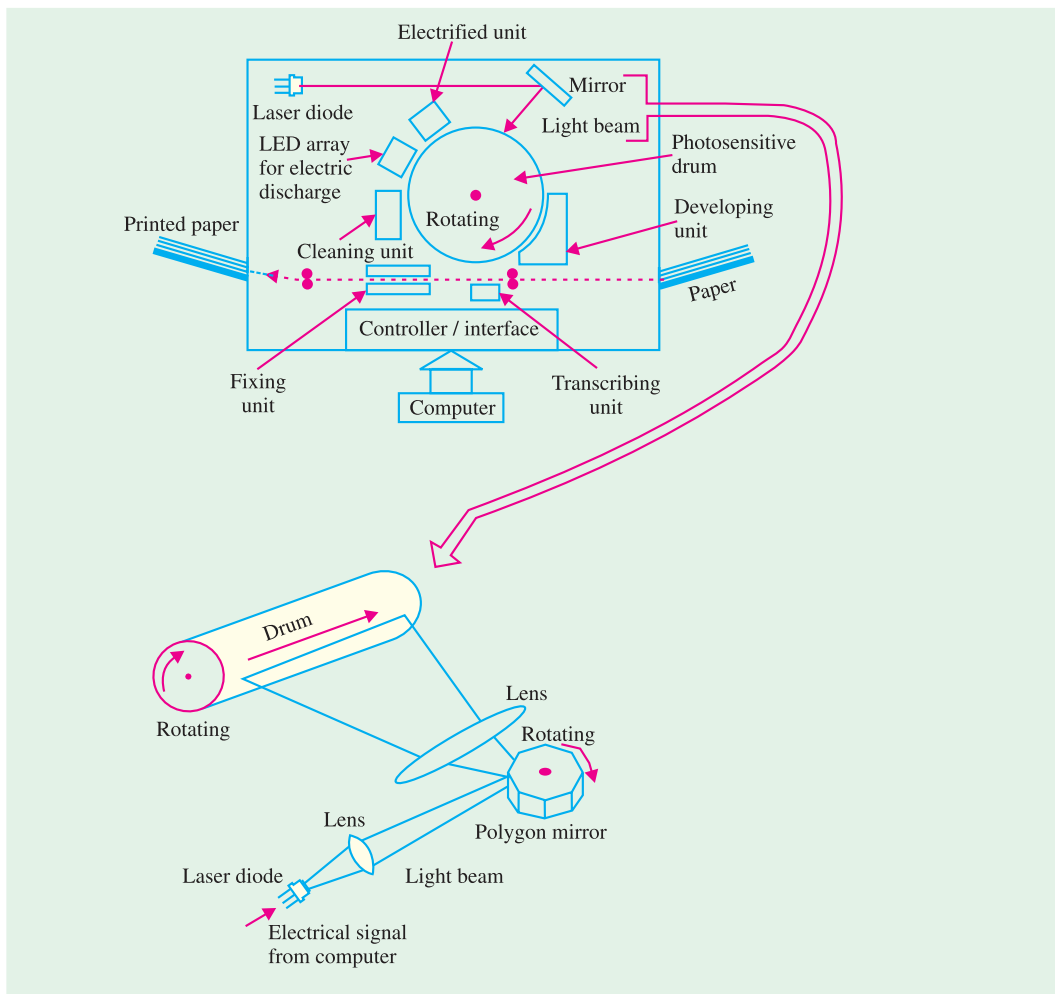


Fig. 53.25

### 53.15. Hologram Scanners

The hologram scanner is widely used in various equipment and is ordinarily used in bar-code readers in point-of-sale systems (such as super marked checkout counters). It is also used in laser printers for scanning the laser beam on the drum precisely.

Fig. 53.26 shows a simplified schematic of a hologram scanner. As seen, the optical beam for reading the bar-code is focussed by a lens through the hologram disk and scanned on the bar-code by rotating the hologram disk. Gratings with coaxial circles are formed on the hologram disk. This

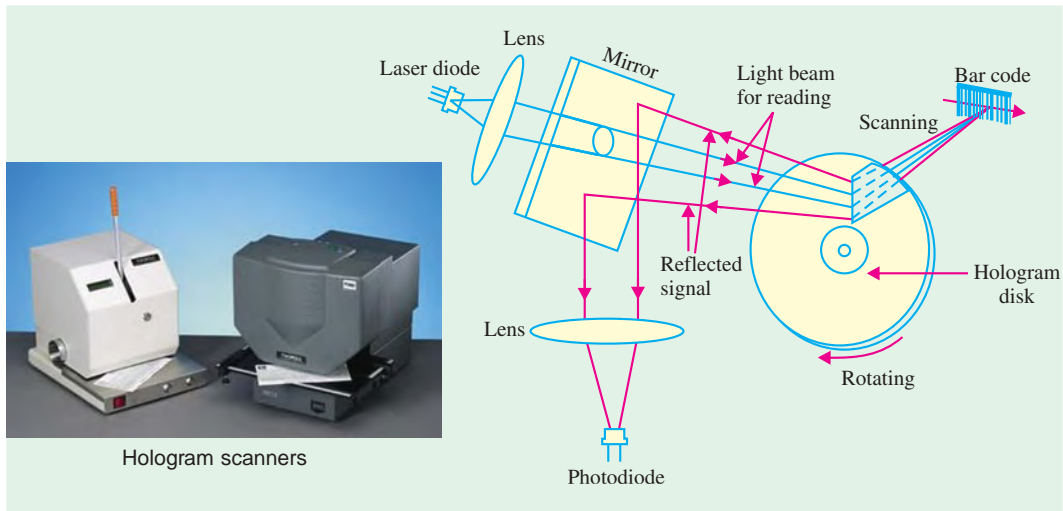


Fig. 53.26

causes the incident laser beam to bend at the grating by an amount determined by the grating pitch. The reflected light modulated according to the bar-code is reflected by the mirror and monitored by the photodiode. The monitored optical signal is then translated into an electrical signal.

### 53.16. Laser Range Finder

The laser diodes along with photodiodes can be used to measure the range (*i.e.* a distance) of an object. Fig. 53.27 shows a simple schematic of a laser range finder. As seen, the laser diode is modulated with high current pulses. The pulsed high-power beam is emitted in the direction of an object. The beam is reflected from the object. The reflected beam is detected with a photo detector (or photodiode). The range is calculated as the difference between the time the light was emitted from the laser diode and the time it was detected by the photodiode.

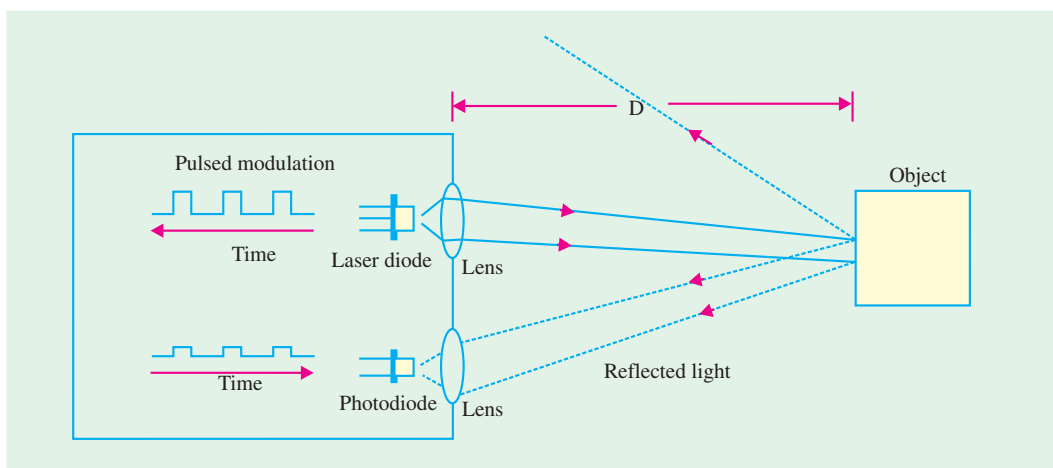


Fig. 53.27

Let  $D$  = distance between the laser range finder and the object.  
 $\Delta T$  = Time difference between the instance when the light was emitted from the laser diode and the instance when it was detected by the photodiode

Then  $D = \frac{1}{2} \times \text{speed of light} \times \Delta T$

A 2-dimensional array of laser diodes and photodetectors can be constructed. Such a system is used to obtain 3-D images of an object.

### 53.17. Light-activated SCR (LASCR)

The operation of a LASCR is essentially similar to that of a conventional SCR except that it is light-triggered (Fig.53.28). Moreover, it has a window and lens which focuses light on the gate junction area. The LASCR operates like a latch. It can be triggered ON by a light input on the gate area but does not turn OFF when light source is removed. It can be turned OFF only by reducing the current through it below its holding current. Depending on its size, a LASCR is capable of handling larger amount of current that can be handled by a photodiode or a photo-transistor.

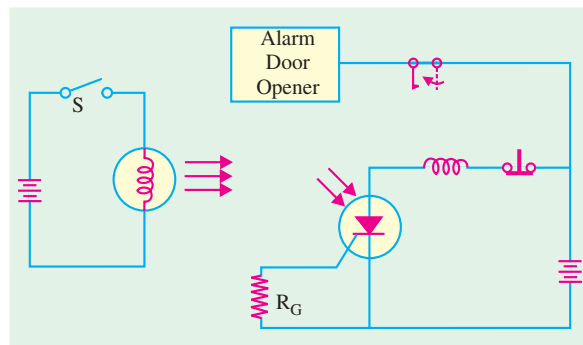


Fig. 53.28

Fig. 53.28 shows how a LASCR can be used for energizing a latching relay. The input dc source turns on the electric lamp and the resulting incident light triggers the LASCR into conduction. The anode current energizes the relay and closes the contact. It is seen that the input dc source is electrically isolated from the rest of the circuit.

### 53.18. Optical Isolators

Optical isolators are designed to electrically isolate one circuit from another while allowing one circuit to control the other. The usual purpose of isolation is to provide protection from high-voltage transients, surge voltages and low-level electrical noise that could possibly result in an erroneous output or damage to the device. Such isolators allow interfacing of circuits with different voltage levels and different grounds etc.

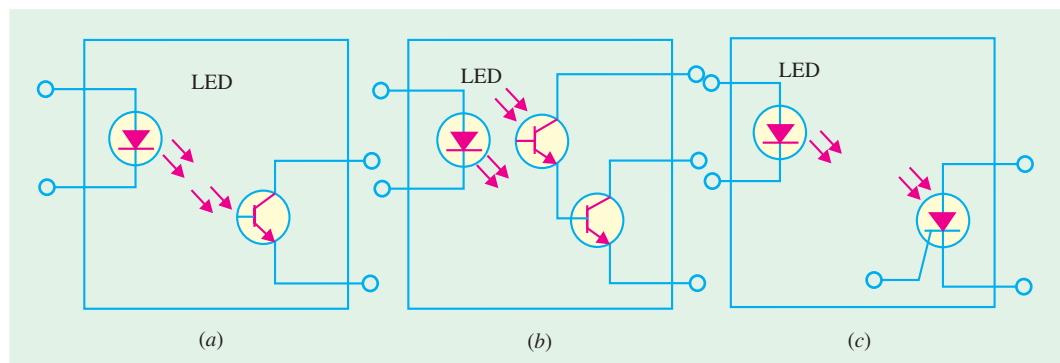


Fig. 53.29

An optical isolator (or coupler) consists of a light source such as LED and a photodetector such as a photo transistor as shown in Fig. 53.29 (a) and is available in a standard IC package.

When LED is forward-biased, the light produced by it is transferred to the phototransistor which is turned ON thereby producing current through the external load.

Fig. 53.29 (b) shows a Darlington transistor coupler which is used when increased output current capability is needed beyond that provided by the phototransistor output. The LASCR output coupler of Fig. 53.29 (c) can be used in applications where a low-level input voltage is required to latch a high voltage relay for activating some kind of electro-mechanical device.

### 53.19. Optical Modulators

Light emitting *PN* junction devices such as LEDs and laser diodes are easily modulated by superimposing signals on to the injected current. This is **direct modulation**. Laser diodes in high-bit rate and long-span optical communication systems are frequently used under direct modulation.

However direct modulation results in **chirping** which limits transmission quality because of **dispersion** in optical fibres. An optical modulator can modulate the light output from laser diodes with little or no chirping. There are two types of optical modulators :

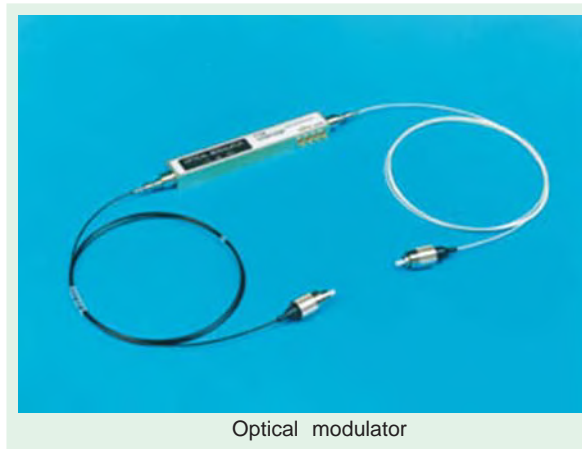
1. The semiconductor optical modulators
2. Optical modulators composed of dielectric materials such as **lithium nitrate** ( $\text{LiNO}_3$ )

The semiconductor optical modulators are *PN* junction diodes and can further be subdivided into two types :

1. Devices used under forward bias (as LEDs and laser diodes are used). The optical modulation in these devices is carried out by changing gain or loss within the modulators.

2. Devices used under reverse bias (*i.e.*, as photodiodes are used). Most high-performance semiconductor optical modulators are used under reverse bias. The reverse bias is needed to generate strong electric field. Optical modulation is basically performed by modulating the **refractive index or optical absorption coefficient** of the modulators. The devices which make use of refractive index phenomenon for modulation are called phase modulation type devices while those that use optical absorption coefficient phenomenon are called intensity modulation type devices.

There are several different types of optical modulators available today. But the waveguide type optical modulator is more common in use. Further there are several different waveguide type optical modulator structures possible. Fig. 53.30 shows a mesa type optical modulator structure.



Optical modulator

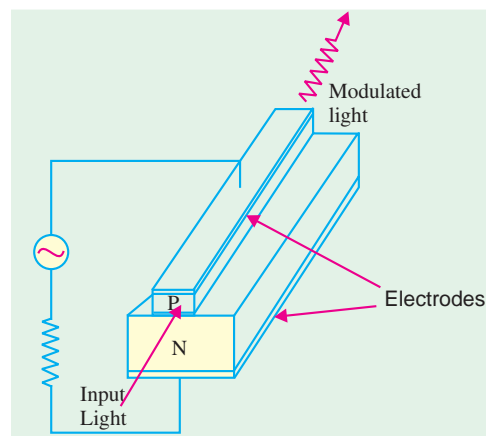


Fig. 53.30

It may be noted that although we have shown the structure making use of a simple  $N$ - and  $P$ -layer but in reality each layer ( $N$ -type or  $P$ -type) is made up of several different semiconductors.

### 53.20. Optical Fibre Communication Systems

The optical fibre communication systems (such as public communication networks and data links) are the basic infrastructure of the information hungry society. There are several advantages of the optical fibre system over metallic transmission systems as listed below :

1. Data can be transmitted at a very high-frequency over longer distances without much loss.
2. Electromagnetic induction (EMI) noise is never induced during transmission through optical fibre cables.
3. Optical fibre cable is light, flexible and economical.

Fig. 53.31 shows the public optical fibre communication system broadly divided into two groups: (1) Submarine systems, and (2) Land systems. Submarine systems have already been used to connect countries all over the world. The submarine systems help people to talk overseas without any time delay.

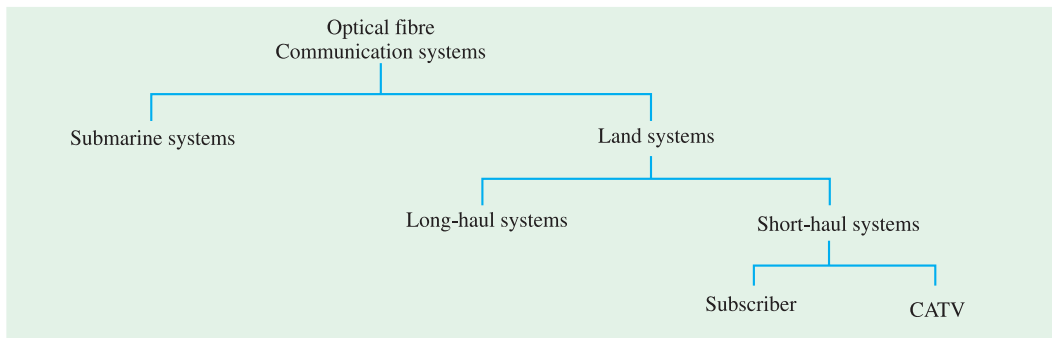


Fig. 53.31

In land systems, long-haul systems have been connected between large cities. The land systems also include systems such as subscriber systems and CATV (*i.e.* community or common antenna television, cable and telecommunication television system, or cable television system).

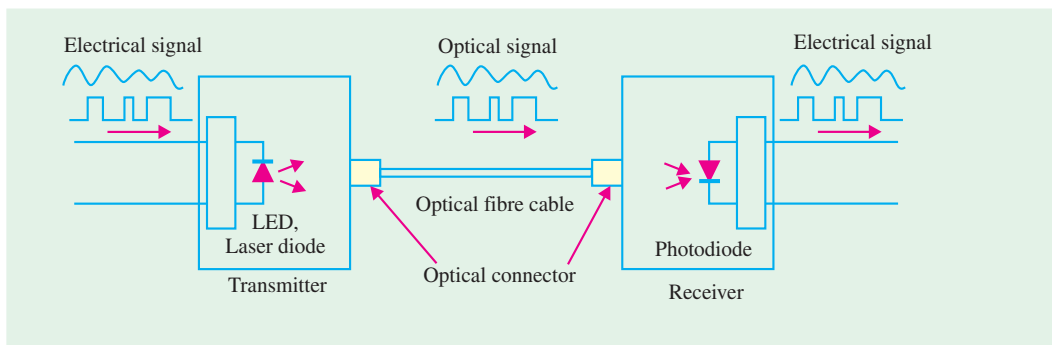


Fig. 53.32

Fig. 53.32 shows an application of LEDs, laser diodes and photodiodes in a simplified optical fibre communication systems. The LEDs and laser diodes emit light modulated with a signal. The optical signal is then transmitted through the optical fibre and is received with photodiodes on the destination side. In this type of a system LEDs or laser diodes emit the light directly through the optical fibre and therefore is referred to as direct modulation type systems. But in more recent

systems, the **optical modulators** modulate the light emitted from the laser diodes and then the modulated light is transmitted through the optical fibre [refer to Fig. 53.33].

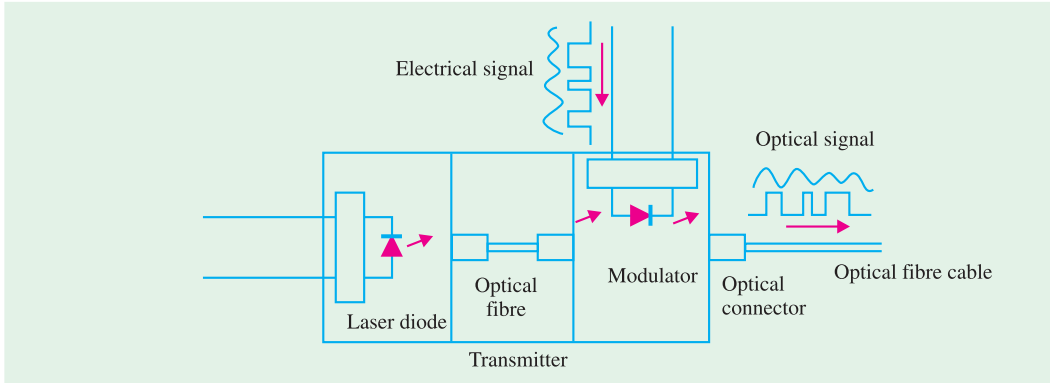


Fig. 53.33

In long-haul systems, **repeaters** (which include photodiodes and laser diodes and electronic circuits) are inserted. In the repeater, the weak optical signal being transmitted through the optical fibre is detected by the photodiode. The detected signal is reformed and amplified by the electronic circuits. The amplified signal is converted again into an optical signal by a laser diode and transmitted again through the optical fibre cable. Fig. 53.34 shows a simple schematic of a repeater.

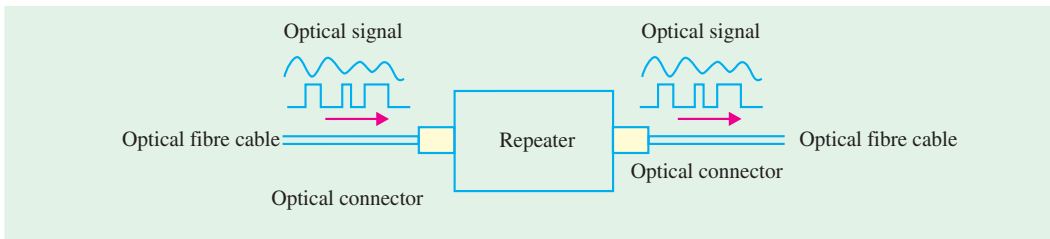


Fig. 53.34

From the modulation point of view, the optical fibre communication systems can be divided into **digital systems** and **analog systems**. Most long-haul and large capacity optical fibre communication systems are digital systems. The analog systems are used for transmitting information over a short distance.

### 53.21. Optical Fibre Data Links

The use of optical fibre data links has wide spread in the past few decades. Its application ranges from local area networks (LANs) to the computer, digital audio and mobile fields. Several different types of LEDs and laser diodes emitting light at wavelengths ranging from visible to the infrared are used as optical sources. The transmission data rate is a function of transmission distance and varies from application to application. For computer links where the distance varies from 1 m to 100 m, the data transmission rate varies from 1 M bits/s to 100 M bits/s. For local-area-networks used in factory, office and building automation, the data transmission rate varies from 10 K bits/s to 5 M bits/s. In digital audio field, where the distance is below 1 m the data transmission rate varies from 500 bits/s to over 10 M bits/s. Similarly in mobile fields (such as ship, aircraft, train and automotive applications) where the distance could vary from 1 m to 100 m, the data transmission rate varies from 1 K bits/s to 1 M bits/s.

**1. Optical fibre local area networks.** The optical fibre local area networks (LANs) are similar to the public communication systems. Some of their advantages over the systems using metallic cables are : (1) high transmission capacity and bit rate and (2) longer transmission distance. However, the range of LANs is restricted. They are more commonly used within factories, offices, buildings etc. Computers, printers, facsimile machines and other office equipment are connected with each other by optical fibre cables as shown in Fig. 53.35.

The LEDs and laser diodes are used to transmit data through the optical fibre cable and photodiodes are used to receive data. The different types of equipment connected in the LAN could be one of the following two types :

(1) An optical ethernet having a radial-shape network as shown in Fig. 53.36 (a) or (2) a fibre-distributed data interface (FDDI) having a ring-shape network as shown in Fig. 53.36 (b).

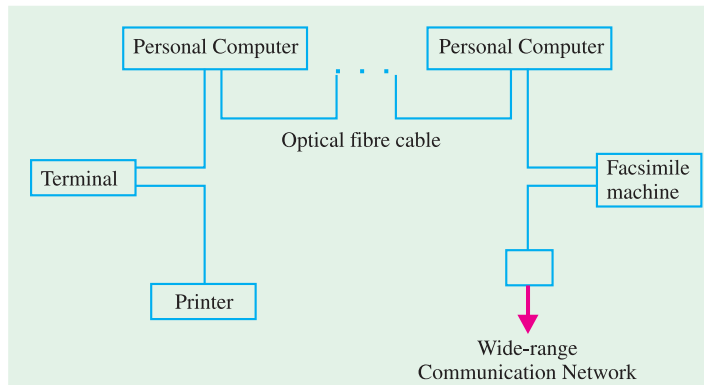


Fig. 53.35

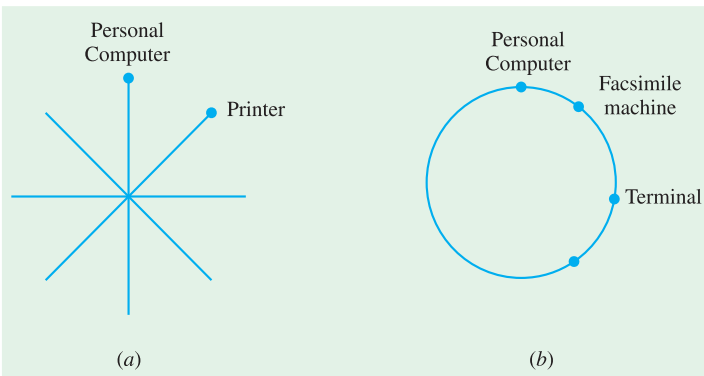


Fig. 53.36

**2. Digital audio field.** Fig. 53.37 shows an example of a data link in digital audio field. As seen, the optical fibre cable is used to connect compact disk (CD) player, laser disk (LD) player, digital audio tape (DAT) and tuner with the amplifier and speaker. The connection between the amplifier and everything except DAT is unidirectional. The audio digital signals from CD, LD player, DAT and tuner are converted into optical signals by LEDs or laser diodes at one end of the fibre optic cable and then transmitted through the cable to the opposite end. At the opposite end, the signals are received by photodiodes and converted into an electrical signal for amplification and finally speaker for reproduction to a sound.

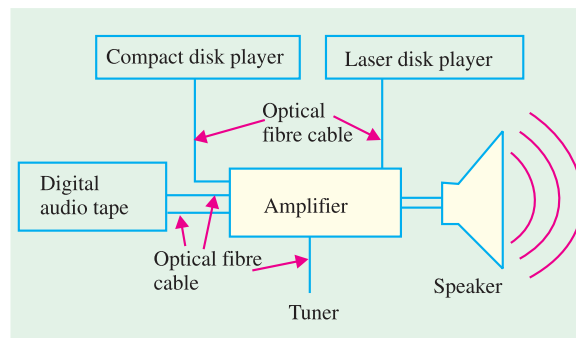


Fig. 53.37

**3. Mobile fields.** The optical data links are very suitable in mobile fields such as ship, aircraft, train, automotive etc. The reason is that optical data links are very compact, and light in weight than metallic data links. In addition to this, the optical data links are not subjected to noise induced by electromagnetic induction.



## OBJECTIVE TESTS – 53

1. LEDs are commonly fabricated from gallium compounds like gallium arsenide and gallium phosphide because they
  - (a) are cheap
  - (b) are easily available
  - (c) emit more heat
  - (d) emit more light.
2. A LED is basically a ..... P-N junction.
  - (a) forward-biased
  - (b) reverse-biased
  - (c) lightly-doped
  - (d) heavily-doped.
3. As compared to a LED display, the distinct advantage of an LCD display is that it requires
  - (a) no illumination
  - (b) extremely low power
  - (c) no forward-bias
  - (d) a solid crystal
4. Before illuminating a P-N junction photodiode, it has to be
  - (a) reverse-biased
  - (b) forward-biased
  - (c) switched ON
  - (d) switched OFF.
5. In a photoconductive cell, the resistance of the semiconductor material varies ..... with the intensity of incident light.
  - (a) directly
  - (b) inversely
  - (c) exponentially
  - (d) logarithmically.
6. A photoconductive cell is known as ..... cell.
  - (a) phototransistor
  - (b) photoresistor
  - (c) photovoltaic
  - (d) both (a) and (b).
7. A phototransistor excels a photodiode in the matter of
  - (a) faster switching
  - (b) greater sensitivity
  - (c) higher current capacity
  - (d) both (a) and (b)
- (e) both (b) and (c).
8. A photodarlington comprises of
  - (a) a phototransistor
  - (b) a transistor
  - (c) a photodiode
  - (d) both (a) and (b).
9. A solar cell operates on the principle of
  - (a) diffusion
  - (b) recombination
  - (c) photo voltaic action
  - (d) carrier flow.
10. Solar cells are used as source of power in earth satellites because they have
  - (a) very high efficiency
  - (b) unlimited life
  - (c) higher power capacity per weight
  - (d) both (b) and (c)
  - (e) both (a) and (b).
11. The device possessing the highest sensitivity is a
  - (a) photo conductive cell
  - (b) photovoltaic cell
  - (c) photodiode
  - (d) phototransistor
12. The unique characteristics of LASER light are that it is
  - (a) coherent
  - (b) monochromatic
  - (c) collimated
  - (d) all of the above
13. The LASCR operates like a
  - (a) latch
  - (b) LED
  - (c) photodiode
  - (d) phototransistor.
14. Optical couplers are designed to ..... one circuit from another.
  - (a) control
  - (b) isolate
  - (c) disconnect
  - (d) protect.
15. The main purpose of using optical isolators is to provide protection to devices from
  - (a) high-voltage transients
  - (b) surge voltages
  - (c) low-level noise
  - (d) all of the above.

16. A LED emits visible light when its .....
- (a) P-N junction is reverse-biased
  - (b) depletion region widens
  - (c) holes and electrons recombine
  - (d) P-N junction becomes hot.
17. In LED, light is emitted because
- (a) recombination of charge carriers takes place
  - (b) diode gets heated up
  - (c) light falling on the diode gets amplified
  - (d) light gets reflected due to lens action.
18. GaAs, LEDs emit radiation in the
- (a) ultraviolet region
  - (b) violet-blue green range of the visible region
  - (c) visible region
  - (d) infra-red region
19. Phototransistors respond much like a conventional transistor except that, in their case, light energy is used to .....
- (a) alter leakage current
  - (b) change base voltage
  - (c) switch it ON
  - (d) alter emitter current.

**ANSWERS**

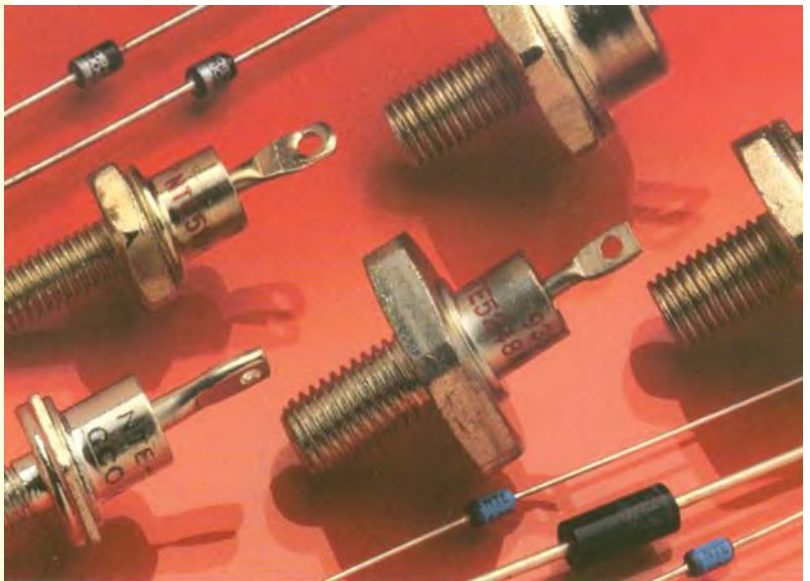
1. (d) 2. (a) 3. b 4. (a) 5. (b) 6. (d) 7. (e) 8. (d) 9. (c) 10. (d) 11. (d) 12. (d)  
13. (a) 14. (b) 15. (d) 16. (c) 17. (a) 18. (d) 19. (c)

# CHAPTER 54

## Learning Objectives

- Zener Diode
- Voltage Regulation
- Zener Diode as Peak Clipper
- Meter Protection
- Zener Diode as a Reference Element
- Tunneling Effect
- Tunnel Diode
- Tunnel Diode Oscillator
- Varactor Diode
- PIN Diode
- Schottky Diode
- Step Recovery Diode
- Gunn Diode
- IMPATT Diode

## SPECIAL DIODES



↑ A major application for zener diodes is voltage regulation in dc power supplies. Zener diode maintains a nearly constant dc voltage under the proper operating conditions.

### 54.1. Zener Diode

It is a reverse-biased heavily-doped silicon (or germanium)  $P-N$  junction diode which is operated in the **breakdown region** where current is limited by both external resistance and power dissipation of the diode. Silicon is preferred to Ge because of its higher temperature and current capability. As seen from Art. 52.3, when a diode breaks down, both Zener and avalanche effects are present although usually one or the other predominates depending on the value of reverse voltage. At reverse voltages less than 6 V, Zener effect predominates whereas above 6 V, avalanche effect is predominant. Strictly speaking, the first one should be called Zener diode and the second one as avalanche diode but the general practice is to call both types as Zener diodes.

Zener breakdown occurs due to **breaking of covalent bonds by the strong electric field set up in the depletion region by the reverse voltage**. It produces an extremely large number of electrons and holes which constitute the reverse saturation current (now called Zener current,  $I_z$ ) whose value is limited only by the external resistance in the circuit. **It is independent of the applied voltage**. Avalanche breakdown occurs at higher reverse voltages when thermally-generated electrons acquire sufficient energy to produce more carriers by collision.

#### (a) V/I Characteristic

A typical characteristic is shown by Fig. 54.1 in the negative quadrant. The forward characteristic is simply that of an ordinary forward-biased junction diode. The important points on the reverse characteristic are :

- $V_z$  = Zener breakdown voltage
- $I_{z\ min}$  = minimum current to sustain breakdown
- $I_{z\ max}$  = maximum Zener current limited by maximum power dissipation.

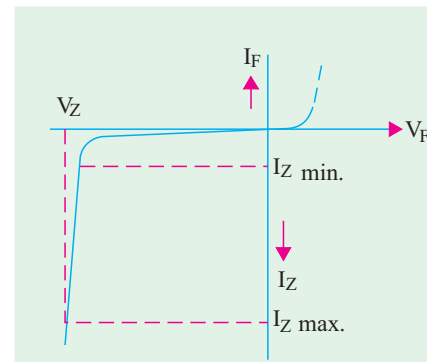


Fig. 54.1

Since its reverse characteristic is not exactly vertical, the diode possesses some resistance called **Zener dynamic impedance\***. However, we will neglect it assuming that the characteristic is truly vertical. In other words, we will assume an ideal Zener diode for which voltage **does not change once it goes into breakdown**. It means that  $V_z$  remains constant even when  $I_z$  increases considerably.

The schematic symbol of a Zener diode and its equivalent circuit are shown in Fig. 54.2 (a). The complete equivalent circuit is shown in Fig. 54.2 (b) and the approximate one in Fig. 54.2 (c) where it looks like a battery of  $V_z$  volts.

The schematic symbol of Fig. 54.2 (a) is similar to that of a normal diode except that the line representing the cathode is bent at both ends. With a little mental effort, the cathode symbol can be imagined to look like the letter Z for Zener.

#### (b) Zener Voltages

Zener diodes are available having Zener voltages of 2.4 V to 200 V. This voltage is temperature dependent. Their power dissipation is given by the product  $V_z I_z$ ... maximum ratings vary from 150 mW to 50 W.

#### (c) Zener Biasing

For proper working of a Zener diode in any circuit, it is essential that it must

1. be reverse-biased;
2. have voltage across it greater than  $V_z$ ;

\* Its value is given by  $Z_z = \Delta V_z / \Delta I_z$ . It is negligible as compared to large external resistance connected in the circuit.



3. be in a circuit where current is less than  $I_{z,max}$ ;

**(d) Diode Identification**

Physically, a Zener diode looks like any other diode and is recognized by its IN number such as IN 750 (10 W power) or IN 4000 (high power). Fig 54.2(d) shows a picture of a zener diode with  $V_z = 4.7V$ .

**(e) Uses**

Zener diodes find numerous applications in transistor circuitry. Some of their common uses are :

1. as voltage regulators;
2. as a fixed reference voltage in a network for biasing and comparison purposes and for calibrating voltmeters;
3. as peak clippers or voltage limiters;
4. for metre protection against damage from accidental application of excessive voltage;
5. for reshaping a waveform.

**Example 54.1.** Determine whether the ideal Zener diode of Fig. 54.3 is properly biased. Explain why ?

**Solution.** Since positive battery terminal is connected to its cathode, the diode is reverse-biased.

Since applied voltage is less than  $V_z$ , the diode is not properly voltage-biased.

**Example 54.2.** Find out if the Zener diode of Fig. 54.4 is properly-biased. If so, find diode current assuming it to be an ideal one.

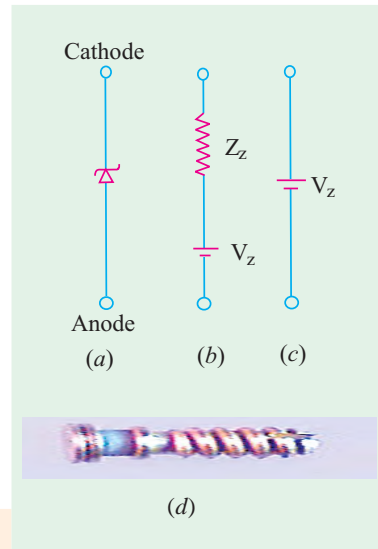


Fig. 54.2

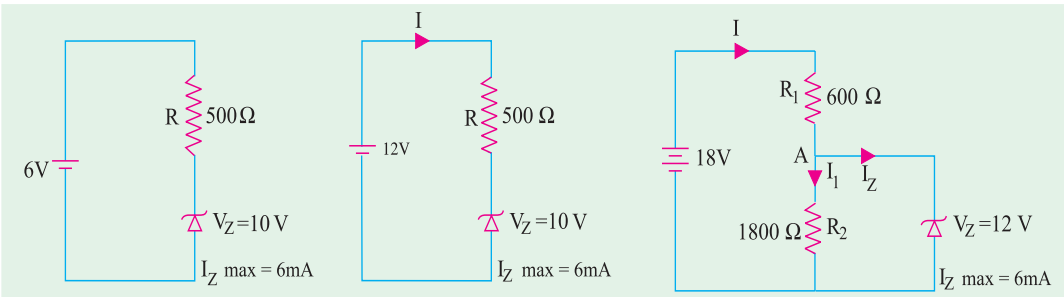


Fig. 54.3

Fig. 54.4

Fig. 54.5

**Solution.** Polarity-wise, the diode is properly-biased. Since applied voltage is greater than  $V_z$ , the diode is properly voltage-biased.

Drop across  $R = 12 - 10 = 2\text{ V} \therefore I = 2/500 = 4\text{ mA}$

Since this current is less than the maximum diode current, the diode is properly-biased according to the criteria laid down in Art. 54.1 (c).

**Example 54.3.** Determine if the Zener diode of Fig. 54.5 is biased properly. If so, find  $I_z$  and the power dissipated by the diode.

**Solution.** Since its anode is connected to the negative battery terminal, the Zener diode is correctly reverse-biased.

Now,  $V_{AB} = V_z = 12\text{ V}$ . Hence, drop across  $R_1 = 18 - 12 = 6\text{ V}$   
 $\therefore I = 6/600 = 0.01\text{ A} = 10\text{ mA}$   
 $I_1 = 12/1800 = 6.7 \times 10^{-3}\text{ A} = 6.7\text{ mA}$   
 $I_z = I - I_1 = 10 - 6.7 = 3.3\text{ mA}$

Since  $I_z$  is less than  $I_{z,max}$ , the diode is properly-biased in every respect as per Art 4.1. Power dissipated  $= V_z I_z = 12 \times 3.3 = 39.6\text{ mW}$ .



**Example. 54.4.** Calculate the value of  $E_0$  in the given circuit of Fig. 54.6. Given  $E_{in} = 6\text{ V}$  and  $20\text{ V}$ . (Electrical Engg. II, Indore Univ.)

**Solution.** When  $E_{in}$  is  $6\text{ V}$ , the diode acts like an open circuit. It is so because  $6\text{ V}$  is not enough to cause Zener break-down which will take place only when  $E_{in}$  exceeds  $10\text{ V}$ . Hence, in this case,  $E_0 = 0$ .

When  $E_{in} = 20\text{ V}$ , breakdown occurs but voltage across diode remains constant at  $10\text{ V}$ . The balance  $(20 - 10) = 10\text{ V}$  appears across  $100\ \Omega$  resistor. Hence,  $E_0 = \text{drop across } R = 10\text{ V}$ .

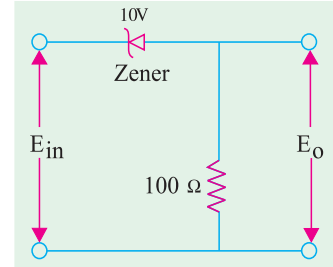


Fig. 54.6

### 54.2. Voltage Regulation

It is a measure of a circuit's ability to maintain a constant output voltage even when either input voltage or load current varies. A Zener diode, when working in the breakdown region, can serve as a voltage regulator. In Fig. 54.7,  $V_{in}$  is the input dc voltage whose variations are to be regulated. The Zener diode is reverse-connected across  $V_{in}$ . When p.d. across the diode is greater than  $V_z$ , it conducts and draws relatively large current through the series resistance  $R$ .

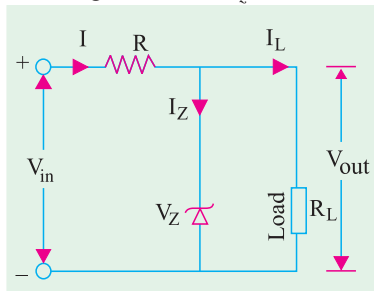


Fig. 54.7

The load resistance  $R_L$  across which a constant voltage  $V_{out}$  is required, is connected in parallel with the diode. The total current  $I$  passing through  $R$  equals the sum of diode current and load current i.e.  $I = I_z + I_L$ .

It will be seen that under all conditions  $V_{out} = V_z$ . Hence,  $V_{in} = IR + V_{out} = IR + V_z$ .

**Case 1.** Suppose  $R_L$  is kept fixed but supply voltage  $V_{in}$  is increased slightly. It will increase  $I$ . This increase in  $I$  will be absorbed by the Zener diode without affecting  $I_L$ . The increase in  $V_{in}$  will be dropped across  $R$  thereby keeping  $V_{out}$  constant.

Conversely if supply voltage  $V_{in}$  falls, the diode takes a smaller current and voltage drop across  $R$  is reduced, thus again keeping  $V_{out}$  constant. Hence, when  $V_{in}$  changes,  $I$  and  $IR$  drop change in such a way as to keep  $V_{out} (= V_z)$  constant.

**Case 2.** In this case,  $V_{in}$  is fixed but  $I_z$  is changed. When  $I_L$  increases, diode current  $I_z$  decreases thereby keeping  $I$  and hence  $IR$  drop constant. In this way,  $V_{out}$  remains unaffected.

Should  $I_L$  decrease,  $I_z$  would increase in order to keep  $I$  and hence  $IR$  drop constant. Again,  $V_{out}$  would remain unchanged because

$$V_{out} = V_{in} - IR = V_{in} - (I_z + I_L)R$$

Incidentally, it may be noted that  $R = (V_{in} - V_{out}) / (I_z + I_L)$

It may also be noted that when diode current reaches its maximum value,  $I_L$  becomes zero. In that case

$$R = \frac{V_{in} - V_{out}}{I_{Zmax}}$$

In Fig. 54.7, only one reference voltage level is available. Fig. 54.8 shows the circuits for establishing two reference levels. Here, two diodes having different Zener voltages have been connected in series.

**Example 54.5.** Calculate the battery current  $I$ ,  $I_z$  and  $I_L$  in the circuit of Fig. 54.9. How will these values be affected if source voltage increases to  $70\text{ V}$ ? Neglect Zener resistance. (Industrial Electronics, Pune Univ.)

**Solution.** When  $V_{in} = 40\text{ V}$   
 Now,  $V_{AB} = V_z = 10\text{ V}$   
 $\therefore I = 30/3\text{ K} = 10\text{ mA}$   
 $\therefore$  drop across  $3\text{K}$  series (or line) resistor is  $= 40 - 10 = 30\text{ V}$   
 $I_L = V_z/R_L = V_{AB}/R_L = 10/2\text{ K} = 5\text{ mA}$



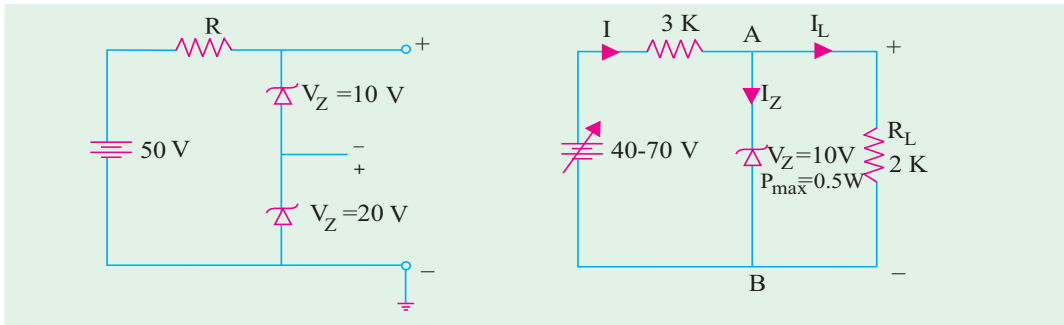


Fig. 54.8

Fig. 54.9

∴  $I_z = I - I_L = 10 - 5 = 5 \text{ mA}$   
 Now,  $P_{max} = V_z \cdot I_{z(max)}$  or  $0.5 = 10 \times I_{z(max)}$   
 or  $I_{z(max)} = 0.5/10 = 0.05 \text{ A} = 50 \text{ mA}$   
 Obviously, diode current of 5 mA is very much within the current range of the diode.

(b) When,  $V_{in} = 70 \text{ V}$   
 Drop across  $R = 70 - 10 = 60 \text{ V}$  ∴  $I = 60/3 \text{ K} = 20 \text{ mA}$   
 $I_L = 5 \text{ mA}$  (as before);  $I_z = I - I_L = 20 - 5 = 15 \text{ mA}$

**Example 54.6.** Using the ideal Zener approximations, find current through the diode of Fig. 54.10 when load resistance  $R_L$  is (i) 30 K (ii) 5 K (iii) 3 K. (Electronics, Madurai Kamraj Univ. )

**Solution.** (i)  $R_L = 30 \text{ K}$

$V_{AB} = V_z = 30 \text{ V}$ ; drop across  $R = 60 - 30 = 30 \text{ V}$   
 ∴  $I = 30/3 \text{ K} = 10 \text{ mA}$ ;  $I_L = V_{AB}/R_L = 30/30 \text{ K} = 1 \text{ mA}$   
 ∴  $I_z = I - I_L = 10 - 1 = 9 \text{ mA}$

(ii) When  $R_L = 5 \text{ K}$   
 $I = 10 \text{ mA}$  — as before;  $I_L = 30/5 \text{ K} = 6 \text{ mA}$   $I_z = 10 - 6 = 4 \text{ mA}$

(iii) When  $R_L = 3 \text{ K}$   
 $I = 10 \text{ mA}$  — as before;  $I_L = 30/3 \text{ K} = 10 \text{ mA}$   $I_z = I - I_L = 10 - 10 = 0$

In this case, it is obvious that the diode is just on the **verge of coming out of breakdown region**. If  $R_L$  is reduced further, the diode will come out of breakdown region and would no longer act as a voltage regulator.



Fig. 54.10

Fig. 54.11

**Example 54.7.** A 24-V, 600-mW Zener diode is to be used for providing a 24-V stabilized supply to a variable load (Fig. 54.11). If input voltage is 32 V, calculate the (i) series resistance R required (ii) diode current when  $R_L = 1200 \Omega$ . (Applied Electronics, Punjab Univ. 1991)

**Solution.** (i)  $V_z I_{z(max)} = 600 \text{ mW}$ ;  $I_{z(max)} = 600/24 = 25 \text{ mA}$

∴  $R = \frac{V_{in} - V_{out}}{I_{z(max)}} = \frac{32 - 24}{25 \times 10^{-3}} = 320 \Omega$

(ii) When  $R_L = 1200 \Omega$ ,  $I_L = V_z/R_L = 24/1200 = 20 \text{ mA}$ ;  $I_z = 25 - 20 = 5 \text{ mA}$



### 54.3. Zener Diode as Peak Clipper

Use of Zener diodes in wave-shaping circuits is illustrated in Fig. 54.12. The two similar diodes  $D_1$  and  $D_2$  have been joined back-to-back across the input sine wave voltage of peak value  $\pm 25$  V. Both have  $V_z = 20$  V. As seen, the output is a semi-square wave with a peak value of  $\pm 20$  V.

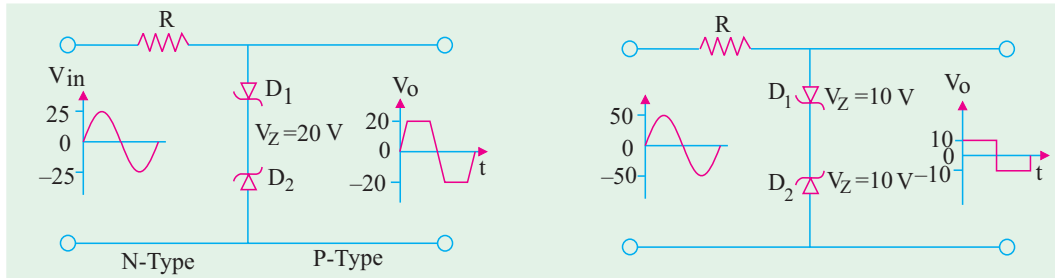


Fig. 54.12

Fig. 54.13

It is well-known that a Zener diode acts like a ‘short’ (or very low resistance) in the forward direction and an ‘open’ in the reverse direction till it goes into breakdown at  $V_z$ . During positive input half-cycle,  $D_1$  is shorted (being forward-biased) but  $D_2$  acts like an open upto 20 V. Thereafter, it goes into breakdown and holds the output voltage constant till input voltage falls below 20 V in the later part of the half-cycle. At that point,  $D_2$  comes out of the breakdown and again acts like an open across which the entire input voltage is dropped.

During the negative input half-cycle, roles of  $D_1$  and  $D_2$  are reversed. As a result, the output wave is clipped on both peaks as shown in Fig. 54.12.

If we increase the peak value of the input signal voltage and use Zener diodes of lesser  $V_z$  value, we can get an almost square output voltage wave from a sinusoidal input wave as shown in Fig. 54.13.

### 54.4. Meter Protection

Zener diodes are frequently used in volt-ohm-milliammeters (VOM) for protecting meter movement against burn-out from accidental overloads. If VOM is set to its 2.5 V range and the test leads are accidentally connected to a 25 V circuit, an unprotected meter will be burned out or at least get severely damaged.

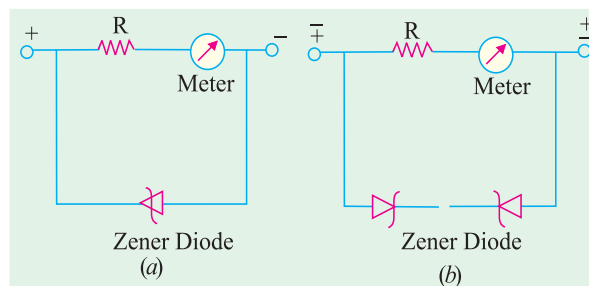


Fig. 54.14

This hazard can be avoided by connecting a Zener diode in parallel with the meter as shown in Fig. 54.14 (a). In the event of an accidental overload, most of the current will pass through the diode. Two Zener diodes connected as shown in Fig. 54.14 (b) can provide overload protection regardless of the applied polarity.

### 54.5. Zener Diode as a Reference Element

In many electronic circuits, it is desirable to maintain a constant voltage between two points and use it as a reference voltage for comparing other voltages against it. The difference between the two voltages is amplified and then used for performing some control function. This type of arrangement is used for power supply voltage regulator circuits, measurement circuits and servomechanism circuits. The constant-voltage characteristic in its breakdown region makes a Zener diode desirable for this application. Fig. 54.15 shows a circuit in which Zener diode is used as a reference element. The reference voltage equals the Zener breakdown voltage. The value of  $R$  is so chosen that the diode operates well within its breakdown region. The difference  $(E_{in} - E_{ref})$  gives the control output.





### 54.6. Tunneling Effect

In a normally-doped P-N junction, the depletion layer is relatively wide and a potential barrier exists across the junction. The charge carriers on either side of the junction cannot cross over unless they possess sufficient energy to overcome this barrier (0.3 V for Ge and 0.7 V for Si). As is well-known, width of the depletion region depends directly on the doping density of the semiconductor. If a P-N junction is doped very heavily (1000 times or more)\*, its depletion layer

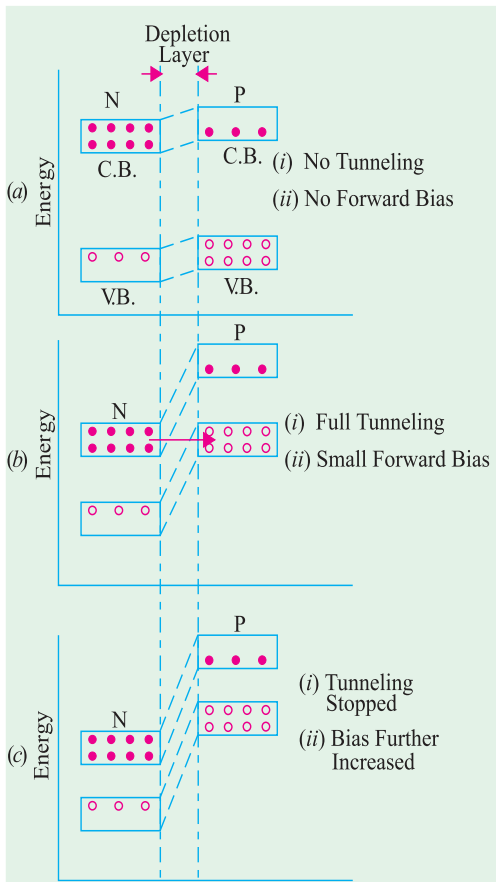


Fig. 54.17

becomes extremely thin (about 0.00001 mm). It is found that under such conditions, many carriers can ‘punch through’ the junction with the speed of light even when they do not possess enough energy to overcome the potential barrier. Consequently, large forward current is produced even when the applied bias is much less than 0.3 V.

#### (a) No Forward Bias

This conduction mechanism in which charge carriers (possessing very little energy) bore through a barrier directly instead of climbing over it is called **tunneling**.  
**Explanation**  
 Energy band diagrams (EBD) of *N*-type and *P*-type semiconductor materials can be used to explain this tunneling phenomenon. Fig. 54.16 shows the energy band diagram of the two types of silicon separately. As explained earlier (Art. 51.21), in the *N*-type semiconductor, there is increased concentration of electrons in the conduction band. It would be further increased under heavy doping. Similarly, in a *P*-type material, there is increased concentration of holes in the valence band for similar reasons.

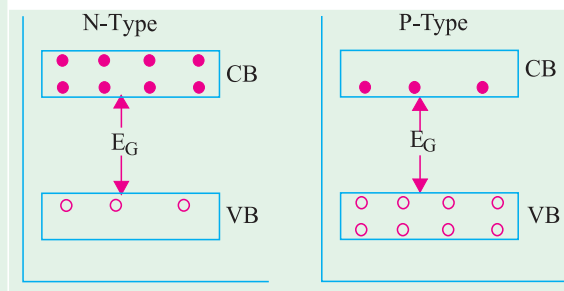


Fig. 54.16

When the *N*-type and *P*-type materials are joined, the EBD under no-bias condition becomes as shown in Fig. 54.17 (a). The junction barrier produces only a rough alignment of the two materials and their respective valence and conduction bands. As seen, the depletion region between the two is extremely narrow due to very heavy doping on both sides of the junction. The potential hill is also increased as shown.

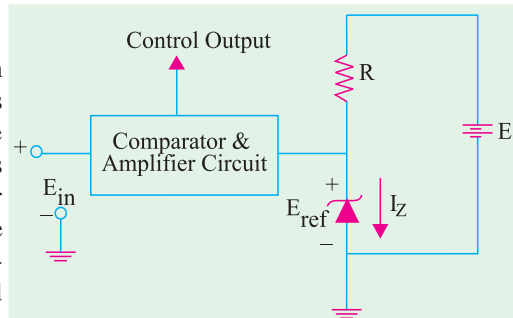


Fig. 54.15

\* Much more than even for a Zener diode.



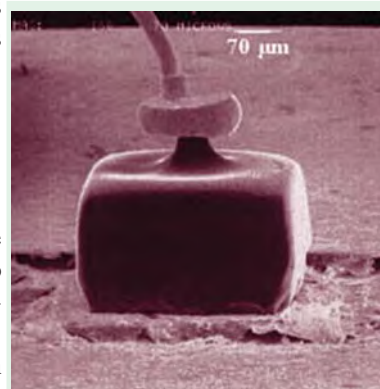
**(b) Small Forward Bias**

When a very small forward voltage ( $\cong 0.1\text{ V}$ ) is applied, the EBDs become as shown in Fig. 54.17 (b). Due to the downward movement of the *N*-region, the *P*-region valence band becomes exactly aligned with the *N*-region conduction band. At this stage, electrons tunnel through the thin depletion layer with the velocity of light thereby giving rise to a large current called peak current  $I_p$ .

**(c) Large Forward Bias**

When the forward bias is increased further, the two bands get out of alignment as shown in Fig. 54.17 (c). Hence, tunneling of electrons stops thereby decreasing the current. Since current decreases with increase in applied voltage (*i.e.*  $dV/dI$  is negative), the junction is said to possess negative resistance at this stage. This resistance increases throughout the negative region.

However, it is found that when applied forward voltage is increased still further, the current starts increasing once again as in a normal junction diode.



Discrete commercial Si tunnel diode

**54.7. Tunnel Diode**

This diode was first introduced by Dr. Leo Esaki in 1958.

**(a) Construction**

It is a high-conductivity two-terminal *P-N* junction diode having doping density about 1000 times higher as compared to an ordinary junction diode. This heavy doping produces following three unusual effects :

1. Firstly, it reduces the width of the depletion layer to an extremely small value (about 0.00001 mm).
2. Secondly, it reduces the reverse breakdown voltage to a very small value (approaching zero) with the result that the diode appears to be broken down for any reverse voltage.
3. Thirdly, it produces a negative resistance section on the  $V/I$  characteristic of the diode.

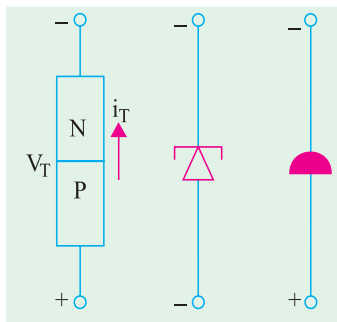


Fig. 54.18

It is called a **tunnel diode** because due to its extremely thin depletion layer, electrons are able to **tunnel through** the potential barrier at relatively low forward bias voltage (less than 0.05 V). Such diodes are usually fabricated from germanium, gallium-arsenide (GaAs) and gallium antimonide (GaSb).

The commonly-used schematic symbols for the diode are shown in Fig. 54.18. It should be handled with caution because being a low-power device, it can be easily damaged by heat and static electricity.

**(b)  $V/I$  Characteristic**

It is shown in Fig. 54.19. As seen, forward bias produces immediate conduction *i.e.* as soon as forward bias is applied, significant current is produced. The current quickly rises to its peak value  $I_p$  when the applied forward voltage reaches a value  $V_p$  (point A). When forward voltage is increased further, diode current starts decreasing till it achieves its minimum value called valley current  $I_v$  corresponding to valley voltage  $V_v$  (point B). For voltages greater than  $V_v$ , current starts increasing again as in any ordinary junction diode.

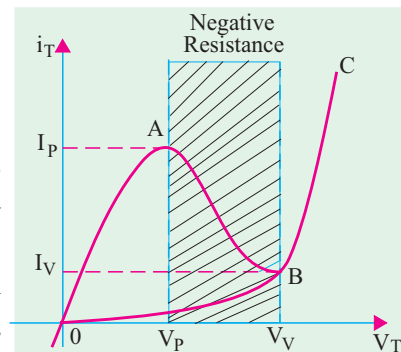


Fig. 54.19



As seen from Fig. 54.19, between the peak point  $A$  and valley point  $B$ , current decreases with increase in the applied voltage. In other words, tunnel diode possesses negative resistance ( $-R_N$ ) in this region. In fact, this constitutes the most useful property of the diode. Instead of absorbing power, a negative resistance produces power. By offsetting losses in  $L$  and  $C$  components of a tank circuit, such a negative resistance permits oscillations. Hence, a tunnel diode is used as a very high frequency oscillator.

Another point worth noting is that this resistance increases as we go from point  $A$  to  $B$  because as voltage is increased, current keeps decreasing which means that diode negative resistance keeps increasing.

### (c) Tunneling Theory

At zero forward bias, the energy levels of conduction electrons in  $N$ -region of the junction are slightly out of alignment with the energy levels of holes in the  $P$ -region. As the forward voltage is slightly increased, electron levels start getting aligned with the hole levels on the other side of junction thus permitting some electrons to cross over. This kind of junction crossing is called tunneling.

As voltage is increased to peak voltage ( $V_p$ ), all conduction band electrons in the  $N$ -region are able to cross over to the valence band in the  $P$ -region because the two bands are exactly aligned. Hence, maximum current (called peak current  $I_p$ ) flows in the circuit.

After  $V_p$ , as the applied voltage is increased, current starts decreasing because the two bands start gradually getting out of alignment. It reaches minimum value (called valley current) when the two are totally out of alignment at a forward bias of  $V_v$  (valley voltage).

For voltages greater than  $V_v$ , current starts increasing again exactly as it does in the case of an ordinary P-N junction diode.

Tunneling is much faster than normal crossing which enables a tunnel diode to switch ON and OFF much faster than an ordinary diode. That is why a tunnel diode is extensively used in special applications requiring very fast switching speeds like high-speed computer memories and high frequency oscillators etc.

### (d) Diode Parameters

(i) **Negative Resistance ( $-R_N$ )**. It is the resistance offered by the diode within the negative-resistance section of its characteristic (shown shaded in Fig. 54.19). It equals the reciprocal of the slope of the characteristic in this region.

It may also be found from the following relation  $R_N = -dV/dI$ .

Its value depends on the semiconductor material used (varying from  $-10\ \Omega$  to  $-200\ \Omega$ ).

### (ii) $I_p/I_v$ Ratio

It is almost as important a factor (particularly for computer applications) as the negative resistance of the diode.

Silicon diodes have a low  $I_p/I_v$  ratio of 3 : 1 and their negative resistance can be approximated from  $R_N = -200/I_p$ . Such diodes are used mainly for switches operating in high ambient temperatures.

Germanium diodes have an  $I_p/I_v$  ratio of 6 : 1 and negative resistance formula  $R_N = -120/I_p$ . GaAs diodes (used exclusively in oscillators) have an  $I_p/I_v$  ratio of about 10 : 1 and a negative resistance nearly equal to that of silicon diodes.

The minimum  $I_p/I_v$  ratio for GaSb diode is about 12 : 1 and has the lowest resistance of all given by  $R_N = -60/I_p$ . Hence, such diodes have the lowest noise.

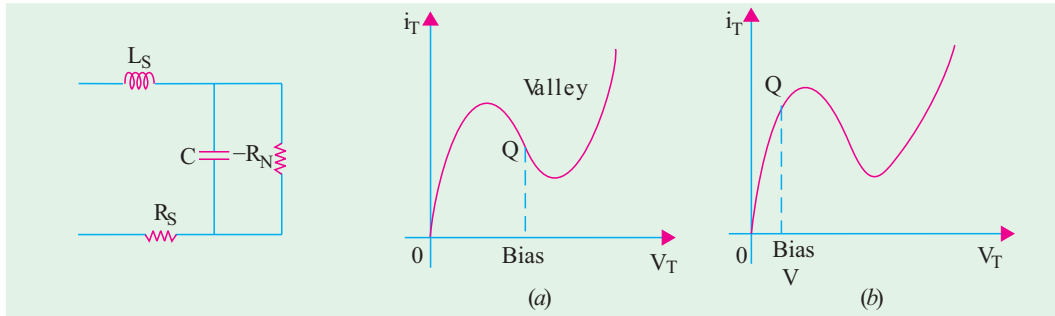
### (e) Equivalent Circuit

The equivalent circuit of a tunnel diode is shown in Fig. 54.20. The capacitance  $C$  is the junction diffusion capacitance (1 to 10 pF) and ( $-R_N$ ) is the negative resistance. The inductor  $L_S$  is due mainly to the terminal leads (0.1 to 4 nH). The resistance  $R_S$  is due to the leads, ohmic contact and semiconductor materials (1 – 5  $\Omega$ ). These factors limit the frequency at which the diode may be used. They are also important in determining the switching-speed limit.



**(f) Biasing the Diode**

The tunnel diode has to be biased from some dc source for fixing its  $Q$ -point on its characteristic when used as an amplifier or as anoscillator and modulator.



**Fig. 54.20**

**Fig. 54.21**

The diode is usually biased in the negative region [Fig. 54.21 (a)]. In mixer and relaxation oscillator applications, it is biased in the positive-resistance region nearest zero [Fig. 54.21 (b)].

**(g) Applications**

Tunnel diode is commonly used for the following purposes :

1. as an ultrahigh-speed switch-due to tunneling mechanism which essentially takes place at the speed of light. It has a switching time of the order of nanoseconds or even picoseconds;
2. as logic memory storage device – due to triple-valued feature of its curve for current.
3. as microwave oscillator at a frequency of about 10 GHz – due to its extremely small capacitance and inductance and negative resistance.
4. in relaxation oscillator circuits – due to its negative resistance. In this respect, it is very similar to the unijunction transistor.

**(h) Advantages and Disadvantages**

The advantages of a tunnel diode are :

1. low noise,
2. ease of operation,
3. high speed,
4. low power,
5. Insensitivity to nuclear radiations

The disadvantages are :

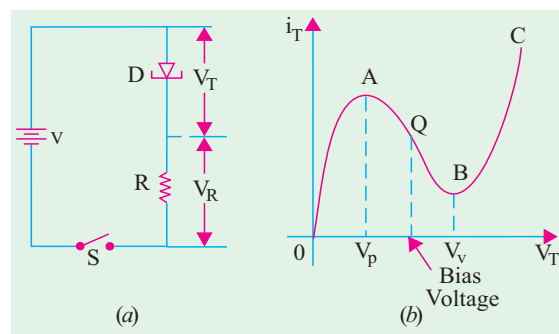
1. the voltage range over which it can be operated properly is 1 V or less;
2. being a two-terminal device, it provides no isolation between the input and output circuits.

**54.8. Tunnel Diode Oscillator**

The basic job of an oscillator is to convert dc power into ac power. Ordinarily, we do not expect an ac signal from a circuit which has no input ac source. But the circuit shown in Fig. 54.22 does exactly that as explained below.

The value of  $R$  is so selected as to bias the diode  $D$  in the negative-resistance region  $A - B$ . The working or quiescent point  $Q$  is almost at the centre of the curve  $A - B$ . When  $S$  is closed, the current immediately rises to a value determined by  $R$  and the diode resistance which are in series. The applied voltage  $V$  divides across  $D$  and  $R$  according to the ratio of their resistances.

However, as  $V_T$  exceeds  $V_p$ , diode is driven into the negative area and its resistance starts to increase (Art 4.7). Hence,  $V_T$  increases further till it becomes equal to valley voltage  $V_v$  (point  $B$ ). At this point, further increase in  $V_T$  drives the diode into the positive- resistance region  $BC$  [Fig. 54.22



**Fig. 54.22**



(b)]. The resulting increase in current now increases  $V_R$  but correspondingly decreases  $V_T$ , thereby bringing the diode back into the negative-resistance region. This decrease in  $V_T$  increases the circuit current till point A is reached when  $V_T$  equals  $V_p$ .

It describes one cycle of operation. In this way, the circuit will continue to oscillate back and forth through the negative-resistance region *i.e.* between points A and B on its characteristic. Its output across R is like a sine wave.

Fig. 54.23 shows a practical circuit drawn in two slightly different ways. Here,  $R_2$  sets the proper bias level for the diode whereas  $R_1$  (in parallel with the LC tank circuit) sets proper current level for it. The capacitor  $C_c$  is the coupling capacitor. As the switch S is closed, the diode is set into oscillations whose frequency equals the resonant frequency of the tank.

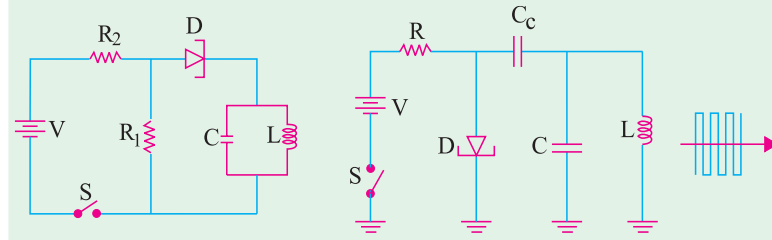


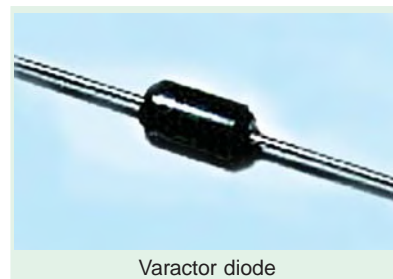
Fig. 54.23

### 54.9. Varactor Diode

The varactor diode is a semiconductor, voltage-dependent variable capacitor alternatively known as varicap or voltacap or voltage-variable capacitor (VVC) diode. Basically, it is just a reverse-biased junction diode whose mode of operation depends on its transition capacitance ( $C_T$ ). As explained earlier in Art. 52.4, reverse-biased junctions behave like capacitors whose capacitance is  $\propto 1/V_R^n$  where  $n$  varies from 1/3 to 1/2. As reverse voltage  $V_R$  is increased, depletion layer widens thereby decreasing the junction capacitance. Hence, we can change diode capacitance by simply changing  $V_R$ . Silicon diodes which are optimised for this variable capacitance effect are called varactors.

The picture, schematic symbol and a simple equivalent circuit for a varactor are shown in Fig. 54.24.

Varactors may be of two types as shown in Fig. 54.25. The doping profile of the abrupt-junction diode is shown in Fig. 54.25 (a) and that of the hyperabrupt-junction diode in Fig. 54.25 (b). The abrupt-junction diode has uniform doping and a capacitive tuning ratio (TR) of 4 : 1. For example, if its maximum transition capacitance is 100 pF and minimum 25 pF, then its TR is 4 : 1 which is not enough to tune a broadcast receiver over its entire frequency range of 550 to 1050 kHz.



Varactor diode

The hyperabrupt-junction diode has highest impurity concentration near the junction. It results in narrower depletion layer and larger capacitance. Also, changes in  $V_R$  produce larger capacitance changes. Such a diode has a tuning range of 10 : 1 enough to tune a broadcast receiver through its frequency range of nearly 3 : 1.

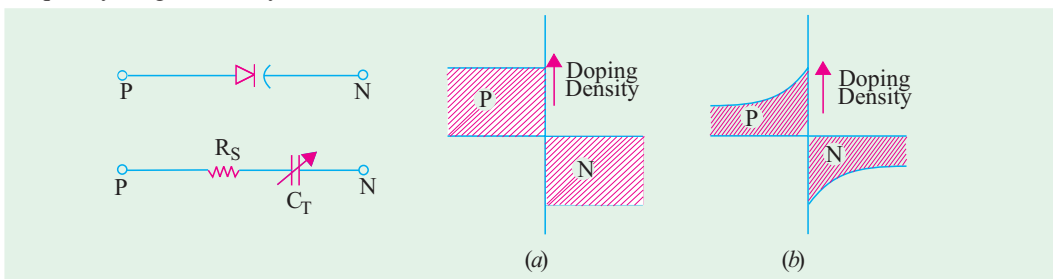


Fig. 54.24

Fig. 54.25



**Applications**

Since the junction capacitance of a varactor is in the pF range, it is suitable for use in high-frequency circuits. Its main applications are as

1. automatic frequency control device,
2. FM modulator,
3. adjustable band-pass filter,
4. Parametric amplifier.

**54.10. PIN Diode**

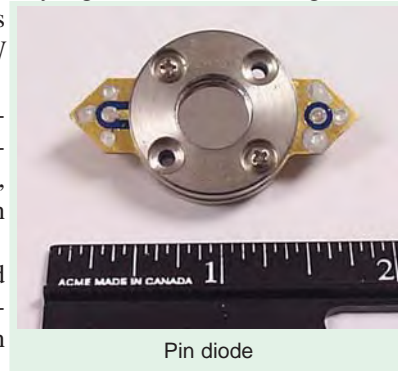
**(a) Construction**

It is composed of three sections. These are the usual *P* and *N*-regions but sandwiched between them is an intrinsic layer or *I*-layer of pure silicon (Fig. 54.26). Being intrinsic (or undoped) layer, it offers relatively high resistance. This high-resistance region gives it two advantages as compared to an ordinary *P-N* diode. The advantages are :

1. decrease in capacitance  $C_{pn}$  because capacitance is inversely proportional to the separation of *P*- and *N*-regions. It allows the diode a faster response time. Hence, PIN diodes are used at high frequencies (more than 300 MHz);
2. possibility of greater electric field between the *P*- and *N*-junctions. It enhances the electron-hole pair generation thereby enabling PIN diode to process even very weak input signals.



Fig. 54.26



Pin diode

**(b) Diode Resistance**

1. When forward-biased, it offers a variable resistance  $r_{ac} \cong 50/I$  where *I* is the dc current in mA (Art. 52.2). For large dc currents, it would look like a *short*.
2. When reverse-biased, it looks like an ‘open’ i.e. it offers infinite resistance in the reverse direction.

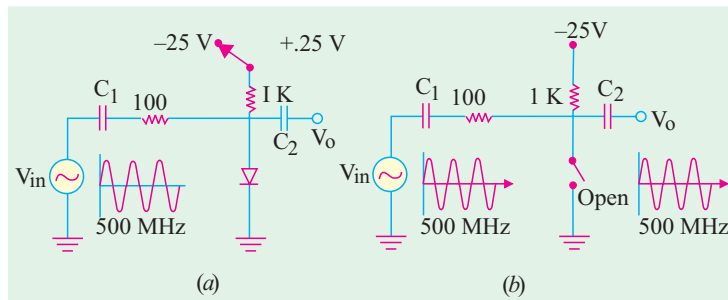


Fig. 54.27

**(c) Operation**

**(i) High Frequency Switching.** Its use in electronic high frequency switching is illustrated in Fig. 54.27.

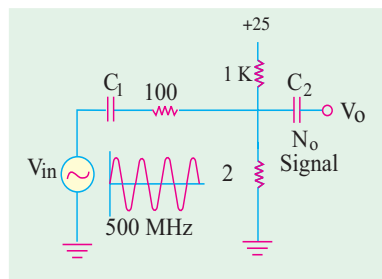


Fig. 54.28

When the diode is reverse-biased, it looks like an ‘open’ as shown in Fig. 54.27 (b). The 500-MHz input signal voltage divides across the series-connected 100 Ω resistance and the diode in proportion to their resistances. Since the diode has infinite resistance (being open), the entire input signal appears across it. Hence, the whole input signal passes out *via* coupling capacitor  $C_2$  without any attenuation (or loss). When the diode is forward-biased by the + 25 V dc source,  $I = 25/1 K = 25$  mA. Hence, diode resistance  $r_{ac} = 50/25 = 2 \Omega$  as shown by its equivalent circuit in Fig. 54.28. Now, almost all the input signal voltage drops across 100 Ω resistance and practically none



across the  $2\ \Omega$  resistance. Hence, there is hardly any signal output.

In practice instead of mechanically switching the diode-biasing supply from  $-25\ \text{V}$  to  $+25\ \text{V}$ , a transistor is used to do this switching operation. In this way, we can turn a very high frequency signal (MHz range) OFF and ON with the speed of a transistor switching circuit.

(ii) **Use as AM Modulator.** The way in which the 500-MHz signal is modulated at 1 kHz rate is illustrated in Fig. 54.29.

A 1 kHz signal is fed into a PNP transistor where it varies its dc output current at the same rate. This varying dc current is applied as biasing current to the PIN diode as shown in Fig. 54.29. It varies the diode ac resistance as seen by the 500 MHz signal. Hence, the signal is modulated at 1 kHz rate as shown.

**(d) Applications**

- (i) as a switching diode for signal frequencies upto GHz range;
- (ii) as an AM modulator of very high frequency signals.

### 54.11. Schottky Diode

It is also called Schottky barrier diode or *hot-carrier* diode. It is mainly used as a rectifier at signal frequencies exceeding 300 MHz. It has more uniform junction region and is more rugged than PIN diode – its main rival.

**(a) Construction**

It is a metal-semiconductor junction diode with *no depletion layer*. It uses a metal (like gold, silver, platinum, tungsten etc.) on the side of the junction and usually an *N*-type doped silicon semiconductor on the other side. The diode and its schematic symbol are shown in Fig. 54.30.

**(b) Operation**

When the diode is unbiased, electrons on the *N*-side have lower energy levels than electrons in the metal. Hence, they cannot surmount the junction barrier (called Schottky barrier) for going over to the metal.

When the diode is forward-biased, conduction electrons on *N*-side gain enough energy to cross the junction and enter the metal. Since these electrons plunge into the metal with very large energy, they are commonly called '*hot-carriers*'. That is why this diode is often referred to as hot-carrier diode.

**(c) Applications**

This diode possesses two unique features as compared to an ordinary *P-N* junction diode :

1. it is a unipolar device because it has electrons as majority carriers on both sides of the junction. An ordinary *P-N* junction diode is a bipolar device because it has both electrons and holes as majority carriers;
2. since no holes are available in metal, there is *no depletion layer or stored charges* to worry about. Hence, Schottky diode can switch OFF faster than a bipolar diode.

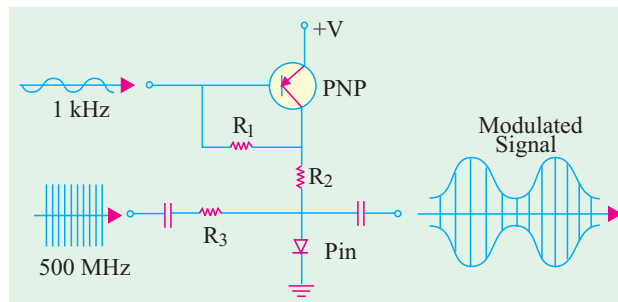


Fig. 54.29



Schottky barrier diode family expands to meet needs of power related application



Because of these qualities, Schottky diode can easily rectify signals of frequencies exceeding 300 MHz. As shown in Fig. 54.31, it can produce an almost perfect half-wave rectified output.

The present maximum current rating of the device is about 100 A. It is commonly used in switching power supplies that operate at frequencies of 20 GHz. Another big advantage of this diode is its low noise figure which is extremely important in communication receivers and radar units etc.

It is also used in clipping and clamping circuits, computer gating, mixing and detecting networks used in communication systems.

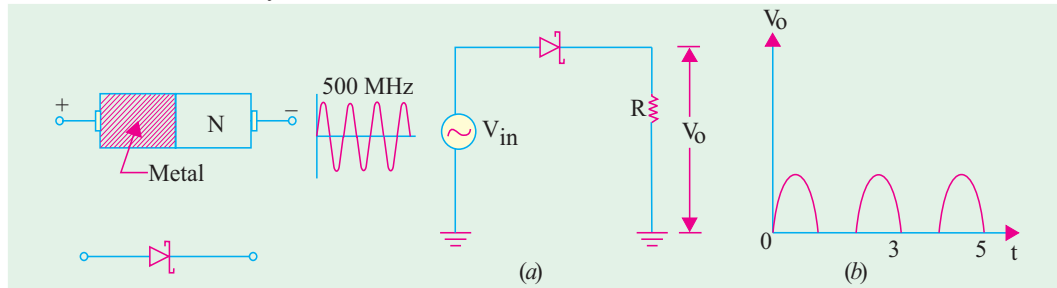


Fig. 54.30

Fig. 54.31

### 54.12. Step Recovery Diode

It is another type of VVC diode having a *graded doping profile* where doping density decreases near the junction as shown in Fig. 54.32. This results in the production of strong electric fields on both sides of the junction.

#### (a) Theory

At low frequencies, an ordinary diode acts as a rectifier. It conducts in the forward direction but not in the reverse direction *i.e.* it recovers immediately from ON state to the OFF state. However, it is found that when driven forward-to-reverse by a high-frequency signal (above a few MHz), the diode does not recover immediately. Even during the negative half-cycle of the input signal when the diode is reverse-biased, it keeps conducting for a while after which the reverse current ceases abruptly in one step. This reverse conduction is due to the fact that charges stored in the depletion layer during the period of forward bias take time to drain away from the junction.

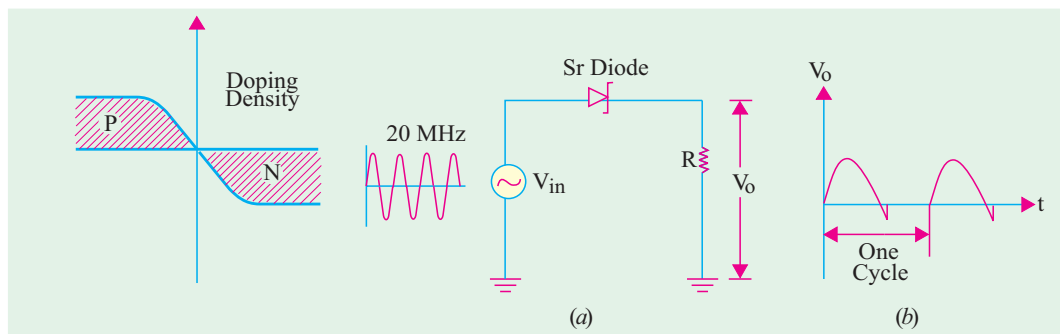


Fig. 54.32

Fig. 54.33

Fig. 54.33 (a) shows a step-recovery diode being driven by a 20-MHz signal source. As seen from Fig. 54.33 (b), it conducts in the forward direction like any diode. During the reverse half-cycle, we get reverse current due to the draining of the stored charge after which current suddenly drops to zero. It looks as though diode has suddenly *snapped open* during the early part of the reverse cycle. That is why it is sometimes called a *snap diode*.

The step or sudden recovery from reverse current ON to reverse current OFF gives the diode its name.





**(b) Applications**

Its main use is in high-frequency harmonic generator circuits as a frequency multiplier as explained below.

It is found that whenever a waveform has sudden step or transition, it contains all the harmonics of the input signal (*i.e.* multiples of its fundamental frequency). For example, the output waveform of Fig. 54.33 (b) contains waves of frequencies 40 MHz, 60 MHz, 80 MHz and so on.

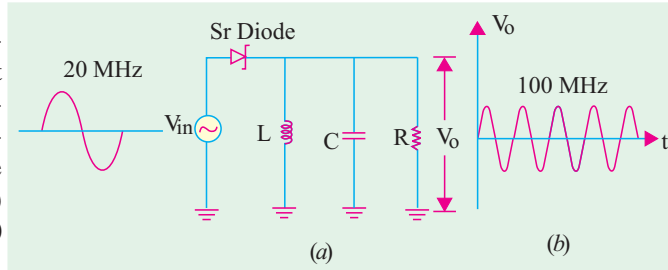


Fig. 54.34

Fig. 54.34 shows how the output of a step recovery diode can be used to drive a tuning circuit which can be made to tune out all harmonics except one *i.e.* fifth in this case (100 MHz).

With an input signal of 20 MHz, the step recovery diode generates harmonics of different multiple frequencies listed above. However, the resonant *L-C* circuit is tuned to 5th harmonic of  $f = 100$  MHz. Hence, all except this harmonic are filtered out of the circuit. The signal appearing across *R* is almost a pure sine wave with  $f = 100$  MHz as shown separately in Fig. 54.34 (b).

Step-recovery diodes are also used in pulse and digital circuits for generating very fast pulses with rise time of less than 1 nanosecond.

**54.13. Gunn Diode**

It is a negative-resistance microwave device for oscillator applications.

As shown in Fig. 54.35, it consists of a thin slice of *N*-type gallium arsenide sandwiched between two metal conductors. The central section is *N*-gallium arsenide whereas the two outer sections are epitaxially grown from GaAs with increased doping and higher conductivity. As an oscillator, its frequencies range from 5 GHz and 100 mW output upto 35 GHz and 1 mW output.

Efficiencies of 3 to 5 per cent are possible at present. Fig. 54.35(b) shows the picture of a Gunn diode.

**54.14. IMPATT Diode**

IMPATT stands for *impact avalanche and transit time* diode. As the name indicates, it is a microwave diode that utilizes the delay time required for attaining an avalanche condition plus transit time to produce a negative-resistance characteristic. It is used as a microwave oscillator within a frequency range of 10-100 GHz.

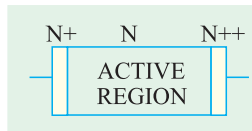


Fig. 54.35(a)

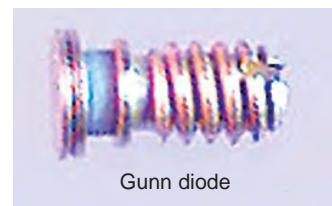


Fig. 54.35(b)

**Tutorial Problems 54.1**

1. Is the ideal Zener diode shown in Fig. 54.36 properly biased ? If not, explain why ? [No]
2. Check up if the diode in Fig. 54.37 is biased properly for normal operation. What is the current taken by the diode ? [Yes : 4 mA]
3. Check up if Zener diode of Fig. 54.38 is reverse-biased as well as properly voltage-biased. Calculate diode current and power dissipation. [6 mA, 96 mW]
4. Using ideal Zener diode approximations, find the minimum and maximum currents through the diode in Fig. 54.39. [4 mA, 8 mA]



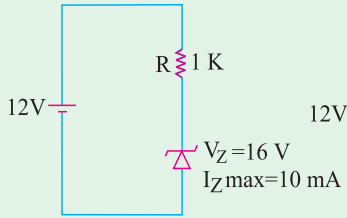


Fig. 54.36

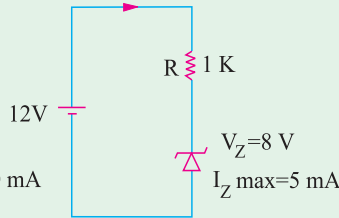


Fig. 54.37

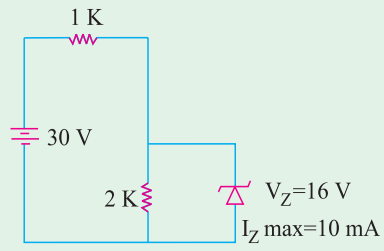


Fig. 54.38

5. A 9 V stabilized voltage supply is required to run a car stereo system from car's 12 V battery. A Zener diode with  $V_z = 9\text{ V}$  and  $P_{max} = 0.25\text{ W}$  is used as a voltage regulator as shown in Fig. 54.40.

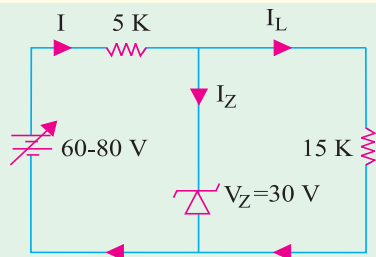


Fig. 54.39

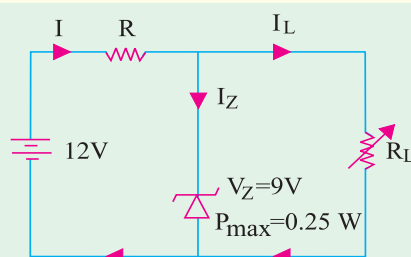


Fig. 54.40

Find the value of the series resistor  $R$ .

[120  $\Omega$ ]

6. A load of 1kW is connected across a 10 V Zener regulator as shown in Fig. 54. 41. The zener current can vary between 5mA to 55mA while maintaining the voltage constant. Find the minimum and maximum voltage level at input.

*(Electronic Devices and Circuits, Nagpur Univ. Summer, 2004)*

7. A 24V, 600 mW zener diode is used for providing a 24V stabilized supply to a variable load. If the input voltage is 32V, calculate
- the value of series resistance required.
  - diode current when the load is 1200  $\Omega$

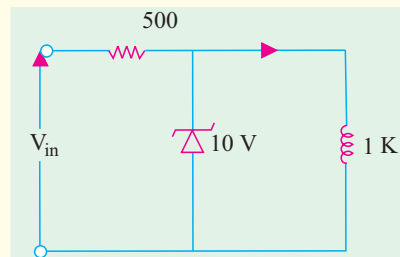


Fig. 54.41

*(Electronics Engg., Bangalore Univ. 2003)*

### OBJECTIVE TESTS – 54

- Silicon is preferred for manufacturing Zener diodes because it
  - is relatively cheap
  - needs lower doping level
  - has higher temperature and current capacity
  - has lower break-down voltage.
- When used in a circuit, a Zener diode is always
  - forward-biased
  - connected in series
  - troubled by overheating
  - reverse-biased.
- The main reason why electrons can tunnel through a  $P-N$  junction is that
  - they have high energy
  - barrier potential is very low
  - depletion layer is extremely thin
  - impurity level is low.
- The  $I_P/I_V$  ratio of a tunnel diode is of primary importance in



- (a) determining tunneling speed of electrons  
(b) the design of an oscillator  
(c) amplifier designing  
(d) computer applications.
5. Mark the INCORRECT statement. A varactor diode
- (a) has variable capacitance  
(b) utilizes transition capacitance of a junction  
(c) has always a uniform doping profile  
(d) is often used as an automatic frequency control device.
6. The microwave device used as an oscillator within the frequency range 10-1000 GHz is ..... diode.
- (a) Schottky  
(b) IMPATT  
(c) Gunn  
(d) Step Recovery.
7. A PIN diode is frequently used as a
- (a) peak clipper  
(b) voltage regulator  
(c) harmonic generator  
(d) switching diode for frequencies upto GHz range.
8. Mark the WRONG statement. A Schottky diode
- (a) has no depletion layer  
(b) has metal-semiconductor junction  
(c) has fast recovery time  
(d) is a bipolar device  
(e) is also called hot-carrier diode  
(f) can easily rectify high-frequency signals.
9. A special purpose diode which uses metals like gold, silver or platinum on one side of the junction, *n*-type doped silicon on another side and has almost no charge storage in the junction, is a
- (a) Schottky diode  
(b) tunnel diode  
(c) varactor diode  
(d) zener diode
10. A step-recovery diode
- (a) has an extremely short recovery time  
(b) conducts equally well in both directions  
(c) is mainly used as a harmonic generator  
(d) is an ideal rectifier of high-frequency signals.
11. A semiconductor device that resembles a voltage variable capacitor is called diode.
- (a) tunnel  
(b) PIN  
(c) Schottky  
(d) varactor
12. A diode that has no depletion layers and operates with hot carriers is called ..... diode.
- (a) Schottky  
(b) Gunn  
(c) step recovery  
(d) PIN
13. In switching devices, gold doping is used to
- (a) improve bonding  
(b) reduce storage time  
(c) increase the mobility of the carrier  
(d) protect the terminals against corrosion
14. When the reverse bias voltage of a varactor diode increases, its
- (a) capacitance decreases  
(b) leakage current decreases  
(c) negative resistance increases  
(d) depletion zone decreases.
15. Which of the following are negative-resistance microwave diodes oscillator applications ?
- (a) Gunn  
(b) IMPATT  
(c) step recovery  
(d) both (a) and (b)  
(e) both (b) and (c).
16. A negative-resistance microwave diode having a thin slice of a semiconductor material sandwiched between two metal conductors is called ..... diode.
- (a) Schottky  
(b) PIN  
(c) Gunn  
(d) varactor.
17. Zener diodes are used primarily as
- (a) rectifiers  
(b) voltage regulators  
(c) oscillators  
(d) amplifiers.



18. The diode which is often used for voltage regulation in electronic circuits is called ..... diode  
 (a) zener (b) varactor  
 (c) silicon (d) germanium.
19. Avalanche photodiodes are preferred over PIN diodes in optical communication systems because of  
 (a) speed of operation  
 (b) higher sensitivity  
 (c) larger bandwidth  
 (d) larger power handling capacity
20. The current through the Zener diode in Fig. 54. 42.

- (a) 33 mA  
 (b) 3.3 mA  
 (c) 2 mA  
 (d) 0 mA

(GATE ; 2004)

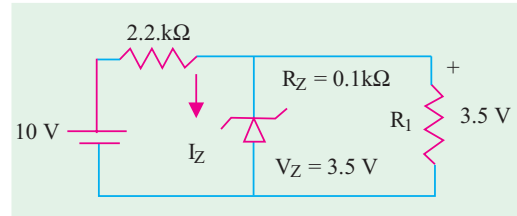


Fig. 54.42

ANSWERS

1. (c) 2. (d) 3. (c) 4. (d) 5. (c) 6. (b) 7. (d) 8. (d) 9. (a) 10. (c)  
 11. (d) 12. (a) 13. (b) 14. (a) 15. (d) 16. (b) 17. (b) 18. (a) 19. (a) 20. (c)



# C H A P T E R 55

## Learning Objectives

- Unregulated Power Supply
- Regulated Power Supply
- Rectifiers
- Single-phase Half-wave Rectifier
- Six-phase Half-wave Rectifier
- Filters
- Shunt Capacitor Filter
- Effect of increasing Filter Capacitance
- Series Inductor Filter
- The Choke Input of L-C Filter
- The R-C Filter
- The C-L-C or Pi Filter
- Bleeder Resistor
- Voltage Dividers
- Complete Power Supply
- Voltage Multipliers
- Half-wave Voltage Doubler
- Full-wave Voltage Doubler
- Voltage Tripler and Quadrupler Circuits
- Troubleshooting Power Supplies
- Controlled Rectification
- Output Voltage and Current Values in Controlled Rectifiers
- Average Values for FW Controlled Rectifier
- Silicon Controlled Rectifier
- $180^\circ$  Phase Control of SCR
- SCR Controlled Load Circuit
- UJT Controlled Load Circuit
- Chopper
- Inverters
- Single Phase Inverter
- Push-pull Inverter

## DC POWER SUPPLIES



↑ Low cost DC Power Supply





**5. Voltage Divider:** Its function is to provide different dc-voltages needed by different electronic circuits. It consists of a number of resistors connected in series across the output terminals of the voltage regulator. Obviously, it eliminates the necessity of providing separate dc power supplies to different electronic circuits working on different dc levels.

**Comments.** Strictly speaking, all that is really required for conversion from ac to dc is a transformer and a rectifier (in fact, even the transformer could be eliminated if no voltage transformation is required). The filter, voltage regulator and voltage divider are mere refinements of a dc power supply though they are essential for most applications except for battery charging and running small dc motors etc.

**55.4. Rectifiers**

We will consider the following circuits :

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 1. single-phase half-wave rectifier, | 2. single-phase full-wave rectifier, |
| 3. full-wave bridge circuit,         | 4. three-phase half-wave rectifier,  |
| 5. three-phase full-wave rectifier,  | 6. six-phase half-wave rectifier,    |
| 7. three-phase bridge circuit,       | 8. voltage multiplier circuits.      |

Many semiconductor devices or systems (like car stereo systems) require a negative dc source or both a negative and a positive dc source. For the sake of simplicity, we will analyse only the positive dc power supplies. However, a positive dc supply can be converted into a negative one by simply reversing the two leads in the same way as we reverse the polarity of a dry cell.

Quite a number of integrated circuits (ICs) require both positive and negative source with common ground. In that case, the polarised components in the negative portion of the supply will have to be reversed. For example, its rectifier, filter capacitor and voltage/current regulation devices will have to be reversed as compared to the positive supply.

**55.5. Single-phase Half-Wave Rectifier**

The basic circuit of a half-wave rectifier with a resistive load (but no filter circuit) is shown in Fig. 55.2 (a). The alternating secondary voltage is applied to a diode connected in series with a load resistor  $R_L$ . Let the equation of the alternating secondary voltage be  $V_s = V_{sm} \sin \omega t$ .

**(a) Working**

During the positive half-cycle of the input ac voltage, the diode  $D$  is forward-biased (ON) and conducts. While conducting, the diode acts as a short-circuit so that circuit current flows and hence, positive half-cycle of the input ac voltage is dropped across  $R_L$ . It constitutes the output voltage  $V_L$  as shown in Fig. 55.2 (b). Waveform of the load voltage is also shown in Fig. 55.2 (b). It consists of half-wave rectified sinusoids of peak value  $V_{LM}$

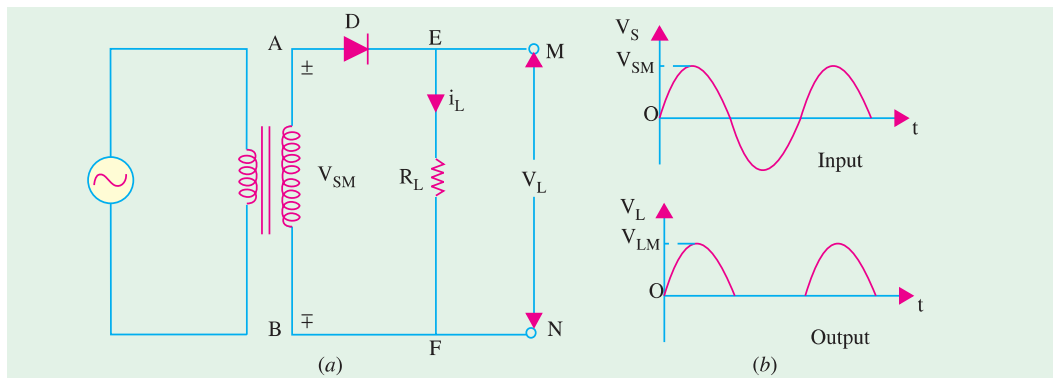


Fig. 55.2

During the negative input half-cycle, the diode is reverse-biased (OFF) and so, does not conduct *i.e.* there is no current flow. Hence, there is no voltage drop across  $R_L$ . In other words  $i_L = 0$  and



$V_L = 0$ . Obviously, the negative input half-cycle is suppressed *i.e.* it is not utilized for delivering power to the load. As seen, the output is not a steady dc but **only a pulsating dc wave** having a ripple frequency equal to that of the input voltage frequency. This wave can be observed by an oscilloscope connected across  $R_L$ . When measured by a dc meter, it will show some **average** positive value both for voltage and current. Since only one half-cycle of the input wave is used, it is called a half-wave rectifier. It should be noted that forward voltage drop across the diode has been neglected in the above discussion. We have, in fact, assumed an ideal diode (having zero forward resistance and infinite reverse resistance).

**(b) Average and RMS Values**

- Let
- $V_{sm}$  = maximum value of transformer secondary voltage
  - $V_s$  = rms value of secondary voltage
  - $V_{LM}$  = maximum value of load voltage  
=  $V_{sm}$  – diode drop – secondary resistance drop
  - $V_L$  = rms value of load voltage
  - $I_L$  = rms value of load current
  - $V_{L(dc)}$  = average value of load voltage
  - $I_{L(dc)}$  = average value of load current
  - $I_{LM}$  = maximum value of load current
  - $R_L$  = load resistance
  - $R_S$  = transformer secondary resistance
  - $r_d$  = diode forward resistance
  - $R_0 = R_S + r_d$
  - $I_{LM} = \frac{V_{sm} - V_B}{(R_S + r_d) + R_L} = \frac{V_{sm} - V_B}{R_0 + R_L}$ ,  $V_{LM} = I_{LM} \cdot R_L$
  - $V_{L(dc)} = \frac{V_{LM}}{\pi} = 0.318 V_{LM}$ ,  $I_{L(dc)} = \frac{I_{LM}}{\pi} = 0.318 I_{LM}$
  - $I_L = I_{LM}/\sqrt{2} = 0.5 I_{LM} = 0.5 V_{LM}/R_L$

**(c) Efficiency**

The efficiency of rectification is given by the ratio of the output dc power to the total amount of input power supplied to the circuit. It is also called the conversion efficiency.

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{\text{power in the load}}{\text{input power}}$$

Now,

$$P_{dc} = I_{L(dc)}^2 R_L = \left(\frac{I_{LM}}{\pi}\right)^2 R_L = \frac{I_{LM}^2}{\pi^2} \cdot R_L$$

$$P_{in} = I_L^2 (R_L + R_0) = \left(\frac{I_{LM}}{2}\right)^2 (R_L + R_0) = \frac{I_{LM}^2}{4} (R_L + R_0)$$

$$\therefore \eta = \frac{P_{dc}}{P_{in}} = \left(\frac{4}{\pi^2}\right) \frac{R_L}{(R_L + R_0)} = \frac{0.406}{1 + R_0/R_L} = \frac{40.6\%}{(1 + R_0/R_L)}$$

If  $R_0$  is neglected  $\eta = 40.6\%$ . Obviously, it is the maximum possible efficiency of a half-wave rectifier.

**(d) Frequency Components of H.W. Rectified Voltage and Current**

As shown in Fig. 55.3, the load current  $I_L$  consist of a dc component  $I_{L(dc)}$  and an ac component  $I_{L(ac)}$ . The Fourier series of the half-wave rectified current flowing through the load is found to be

$$i_L = I_{LM} \left( \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t - \frac{2}{15\pi} \cos 4\omega t + \dots \right)$$





As seen, the half-wave rectified current consists of a large number of ac components (which constitute the ripple) in addition to the dc component. The first term is  $I_{LM}/\pi$  which represents the dc component  $I_{L(dc)}$ . The second term  $(I_{LM}/2) \sin \omega t$  has peak value of  $(I_{LM}/2)$ . It is called the fundamental or first harmonic component and its rms value is  $I_{L1} = I_{LM}/2 \sqrt{2}$ .

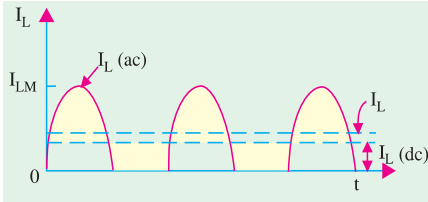


Fig. 55.3

The third term represents the second harmonic component whose frequency is double that of the supply frequency. The rms value is  $I_{L2} = \text{peak value}/\sqrt{2} = 2 I_{LM}/3 \pi \sqrt{2} = I_{LM}/3\pi$ .

The fourth term represents the third harmonic component whose frequency is four times the supply frequency. Its rms value is  $2 I_{LM}/15 \pi \times \sqrt{2} = \sqrt{2} \cdot I_{LM}/15 \pi$ .

The rms values of other components can be similarly calculated. However, they are found to be of continuously diminishing value.

As discussed above, the rectified output (or load) current consists of

- (i) dc component,  $I_{L(dc)} = I_{LM}/\pi$  and
- (ii) ac components of rms values  $I_{L1}, I_{L2}$  and  $I_{L3}$  etc. Their combined rms value is given by

$$I_{L(ac)} = \sqrt{I_{L1}^2 + I_{L2}^2 + I_{L3}^2 + \dots}$$

The rms (or effective) value of the total load current is given by

$$I_L = \sqrt{I_{L(dc)}^2 + I_{L(ac)}^2} = \sqrt{I_{L(dc)}^2 + (I_{L1}^2 + I_{L2}^2 + I_{L3}^2 + \dots)}$$

Similarly, the Fourier series of the load voltage is given by

$$V_L = V_{LM} \left( \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2 \omega t - \frac{2}{15\pi} \cos 4 \omega t \dots \right)$$

It also consists of

- (i) a dc component,  $V_{L(dc)} = V_{LM}/\pi$
- (ii) ac components of rms values  $V_{L1}, V_{L2}$  and  $V_{L3}$  etc. which are given by  $V_{L1} = V_{LM}/\sqrt{2}, V_{L2} = \sqrt{2} \cdot V_{LM}/3 \pi; V_{L3} = \sqrt{2} V_{LM}/15 \pi$  etc.

Again,  $V_{L(ac)} = \sqrt{V_{L1}^2 + V_{L2}^2 + V_{L3}^2 + \dots}$

The rms value of the entire load voltage is given by

$$V = \sqrt{V_{L(dc)}^2 + V_{L(ac)}^2} = \sqrt{V_{L(dc)}^2 + V_{L1}^2 + V_{L2}^2 + V_{L3}^2 + \dots}$$

**(e) Ripple Factor**

When defined in terms of voltage, it is given by

$$\gamma = \frac{\text{rms value of ac components}}{\text{dc value of load voltage}} = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(ms)}}{V_{L(ms)}}$$

In terms of current, we have  $\gamma = I_{L(ac)}/I_{L(dc)}$

As seen from above,  $I_{L(ac)} = \sqrt{I_L^2 - I_{L(dc)}^2}$

$$\gamma = \frac{I_{L(ac)}}{I_{L(dc)}} = \frac{\sqrt{I_L^2 - I_{L(dc)}^2}}{I_{L(dc)}} = \sqrt{\left(\frac{I_L}{I_{dc}}\right)^2 - 1}$$

Now,  $I_L/I_{L(dc)} = \text{form factor } K_f \text{ (Art 12.18)} \therefore \gamma = \sqrt{K_f^2 - 1}$

In the case of a half-wave rectifies with resistive load but no filter  $K_f = \pi/2 = 1.57$

$$\therefore \gamma = \sqrt{1.57^2 - 1} = 1.21$$



Alternatively, the value of  $\gamma$  could be found as under :

If we neglect fourth and higher harmonics in the load current, then as seen from above

$$I_{L(ac)} = \sqrt{I_{L1}^2 + I_{L2}^2 + I_{L3}^2 + \dots}$$

$$= \sqrt{(I_{LM} / 2\sqrt{2})^2 + (\sqrt{2}I_{LM} / 3\pi)^2 + (\sqrt{2} \cdot I_{LM} / 15\pi)^2 + \dots} = 0.385 I_{LM}$$

$$\therefore \gamma = \frac{I_{L(ac)}}{I_{L(dc)}} = \frac{0.385 I_{LM}}{I_{LM} / \pi} = \frac{0.385 I_{LM}}{0.318 I_{LM}} = 1.21$$

**(f) Peak Inverse Voltage (PIV)**

It is the maximum voltage that occurs across the rectifying diode *in the reverse direction*. As seen from Fig. 55.2, the diode is reverse-biased during the negative half-cycle and the maximum voltage applied across it equals the maximum secondary voltage *i.e.*  $V_{sm}$ .

**(g) Transformer Utilization Factor (TUF)**

While designing any power supply, it is necessary to determine the rating of the transformer. It can be done provided *TUF* is known. The value of *TUF* depends on the amount of power to be delivered to the load and the type of rectifier circuit to be used.

$$TUF = \frac{\text{dc power delivered to the load}}{\text{ac rating of transformer secondary}}$$

$$= \frac{P_{dc}}{P_{ac.rated}} = \frac{P_{dc}}{P_{in.rated}}$$

At first sight it might appear as if the above ratio is the same as the conversion efficiency. Actually, it is not so because the rating of the transformer secondary is different from the actual power delivered by the secondary.

$$P_{dc} = V_{L(dc)} \cdot I_{L(dc)} = \frac{V_{LM}}{\pi} \cdot \frac{V_{LM}}{R_L} = \frac{V_{LM}^2}{\pi R_L}$$

$$= \frac{V_{sm}^2}{\pi R_L} \quad \text{—if drop over } R_0 \text{ is neglected}$$

Now, the rated voltage of transformer secondary is  $V_{sm}/\sqrt{2}$  but the actual current flowing through the secondary is  $I_L = I_{LM}/2$  (and not  $I_{LM}/\sqrt{2}$ ) since it is a half-wave rectified current.

$$P_{ac.rated} = \frac{V_{sm}}{\sqrt{2}} \cdot \frac{I_{LM}}{2} = \frac{V_{sm}}{\sqrt{2}} \cdot \frac{V_{LM}}{2 R_L} = \frac{V_{sm}^2}{2\sqrt{2} R_L}$$

$$TUF = \frac{V_{sm}^2 / \pi R_L}{V_{sm}^2 / 2\sqrt{2} R_L} = \frac{2\sqrt{2}}{\pi} = 0.287$$

However, due to saturation effects produced by the flow of direct current through the transformer secondary, the value of *TUF* is further reduced to 0.2.

Obviously, dc power delivered to the load = ac transformer rating  $\times$  *TUF*

If, we have a 1-kVA transformer, then the power which it would be able to deliver to a resistive load in a half-wave rectifier without over-heating would be =  $0.2 \times 1000 = 200$  W.

**Example 55.1.** In the half-wave rectifier circuit of Fig. 55.4, determine

- (i) maximum and rms values of load voltage,
- (ii) peak and rms values of load current,
- (iii) power absorbed by the load,
- (iv) PIV of the diode,
- (v) rms value of ripple voltage.

Neglect resistance of transformer secondary and that of the diode.



**Solution.** Here,  $K = N_2/N_1 = 1/10$ . Peak primary voltage is

$$V_{pm} = 220\sqrt{2} = 310 \text{ V.}$$

Hence,  $V_{sm} = KV_{pm} = 310/10 = 31 \text{ V}$

(i)  $V_{LM} = V_{sm} = 31 \text{ V}$

$$V_L = V_{LM}/2 = 31/2 = 15.5 \text{ V}$$

(ii)  $I_{LM} = V_{LM}/R_L = 31/100 = 0.31 \text{ A}$

$$I_L = I_{LM}/2 = 0.31/2 = 0.155 \text{ A}$$

(iii)  $P_L = V_L I_L = 15.5 \times 0.155 = 2.4 \text{ W}$

(iv)  $P_{IV} = 2 V_{sm} = 2 \times 31 = 62 \text{ W}$

(v)  $V_{L(ac)} = \sqrt{V_L^2 - V_{L(dc)}^2}$

Now,  $V_L = V_{LM}/2$  and  $V_{L(dc)} = V_{LM}/\pi$

$$\therefore V_{L(ac)} = \sqrt{(V_{LM}/2)^2 - (V_{LM}/\pi)^2} = 0.385 V_{LM}$$

$$\therefore V_{r(rms)} = V_{L(ac)} = 0.385 \times 31 = 11.9 \text{ V}$$

It represents the rms value of the ripple voltage.

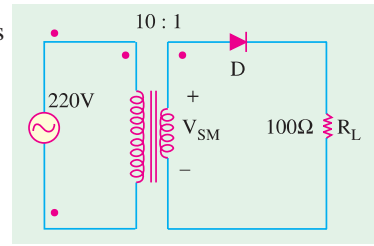


Fig. 55.4

### 55.6. Equivalent Circuit of a HW Rectifier

Such a circuit is shown in Fig. 55.5. Here, the diode has been replaced by its equivalent circuit (Art. 55.13). The transformer secondary of Fig. 55.2 has been replaced by an ac sinusoidal generator having a peak value of  $V_{sm}$ . Resistance  $R_s$  represents transformer secondary resistance. Obviously,

$$I_{LM} = \frac{V_{sm} - V_B}{(R_s + r_d) + R_L} = \frac{V_{sm} - V_B}{R_0 + R_L}$$

$$V_{LM} = I_{LM} \cdot R_L$$

$$V_{L(dc)} = V_{LM}/\pi, I_{L(dc)} = I_{LM}/\pi$$

$$V_L = V_{LM}/2 \text{ and } I_L = I_{LM}/2$$

(i)  $\eta = \left( \frac{4}{\pi^2} \right) \frac{R_L}{(R_s + r_d) + R_L}$

$$= \left( \frac{4}{\pi^2} \right) \frac{R_L}{R_0 + R_L} = \frac{40.6\%}{1 + R_0/R_L}$$

(ii) Voltage regulation is given by

$$V_R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

(iii) Under no-load condition *i.e.* when no output current flows, the voltage has maximum value. When rectifier is fully loaded *i.e.* when output current flows, there is drop over  $R_0$ . Hence, output voltage is decreased by this much amount.

$$V_{FL} = V_{NL} \frac{R_L}{R_0 + R_L}$$

Substituting this value in the above equation, we get  $V_R = R_0/R_L$

**Example 55.2.** A half-wave rectifier using silicon diode has a secondary emf of 14.14 V (rms) with a resistance of 0.2 Ω. The diode has a forward resistance of 0.05 Ω and a threshold voltage of 0.7 V. If load resistance is 10 Ω, determine



(i) dc load current (ii) dc load voltage (iii) voltage regulation and (iv) efficiency.  
 (Applied Electronics-I, Punjab University, 1992)

**Solution.**

$$V_{sm} = \sqrt{2} \times 14.14 = 20 \text{ V}, R_0 = 0.2 + 0.05 = 0.25 \Omega$$

(i) 
$$I_{LM} = \frac{V_{sm} - V_B}{R_0 + R_L} = \frac{20 - 0.7}{10.25} = 1.88 \text{ A}; I_{L(dc)} = \frac{I_{LM}}{\pi} = \frac{1.88}{\pi} = 0.6 \text{ A}$$

(ii) 
$$V_{L(dc)} = I_{L(dc)} \cdot R_L = 0.6 \times 10 = 6 \text{ V}$$

(iii) 
$$V_R = R_0/R_L = 0.25/10 = 0.025 \text{ or } 2.5\%$$

(iv) 
$$\eta = \frac{40.6}{1 + 0.25/10} = 39.6\%$$

### 55.7. Single-phase Full-wave Rectifier

In this case, both half-cycles of the input are utilized with the help of two diodes working alternately. For full-wave rectification, use of a transformer is essential (though it is optional for half-wave rectification).

The full-wave rectifier circuit using two diodes and a centre-tapped transformer shown in 55.6 (a). The centre-tap is usually taken as the ground or zero voltage reference point.

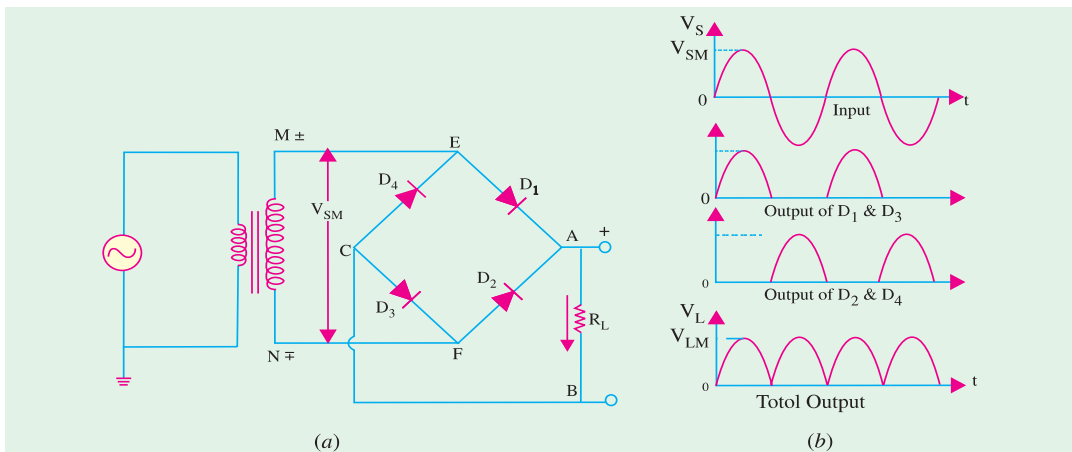


Fig. 55.6

Fig. 55.7 shows two different ways of drawing the circuit. In Fig. 55.7 (a),  $R_L$  becomes connected to point G via the earth whereas in Fig. 55.7 (b). It is connected directly to G.

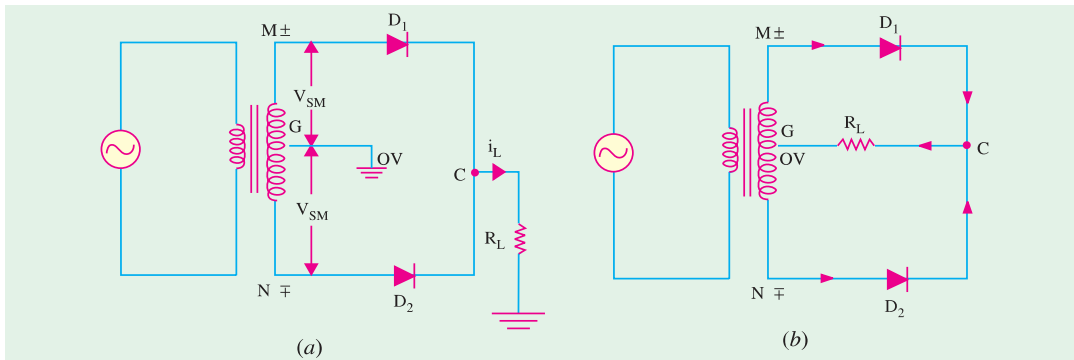


Fig. 55.7



**(a) Working**

When input ac supply is switched on, the ends  $M$  and  $N$  of the transformer secondary become +ve and -ve alternately. During the positive half-cycle of the ac input, terminal  $M$  is +ve,  $G$  is at zero potential and  $N$  is at -ve potential. Hence, being forward-biased, diode  $D_1$  conducts (but not  $D_2$  which is reverse-biased) and current flows along  $MD_1CABG$ . As a result, positive half-cycle of the voltage appears across  $R_L$ .

During the negative half-cycle, when terminal  $N$  becomes +ve, then  $D_2$  conducts (but not  $D_1$ ) and current flows along  $ND_2CABG$ . So, we find that current keeps on flowing through  $R_L$  in the same direction (i.e. from  $A$  to  $B$ ) in both half-cycles of ac input. It means that both half-cycles of the input ac supply are utilized as shown in Fig. 55.6 (b). Also, the frequency of the rectified output voltage is twice the supply frequency. Of course, this rectified output consists of a dc component and many ac components of diminishing amplitudes.

**(b) Average and RMS Values**

As proved earlier in and now shown in Fig. 5.8

$$V_L = V_{LM}/\sqrt{2} = 0.707 V_{LM}; V_{L(dc)} = 2 V_{LM}/\pi = 0.636 V_{LM}$$

$$V_{L(ac)} = \text{rms value of ac components in the output voltage}$$

$$= \sqrt{V_L^2 - V_{L(dc)}^2}$$

Similarly,

$$I_{LM} = \frac{V_{LM}}{R_L}; I_L = \frac{I_{LM}}{\sqrt{2}} = 0.707 I_{LM}$$

$$I_{L(dc)} = \frac{2 I_{LM}}{\pi} = 0.636 I_{LM}; I_{L(ac)} = \sqrt{I_L^2 - I_{L(dc)}^2}$$

Incidentally,  $I_{L(ac)}$  is the same thing as  $I_{r(rms)}$ .

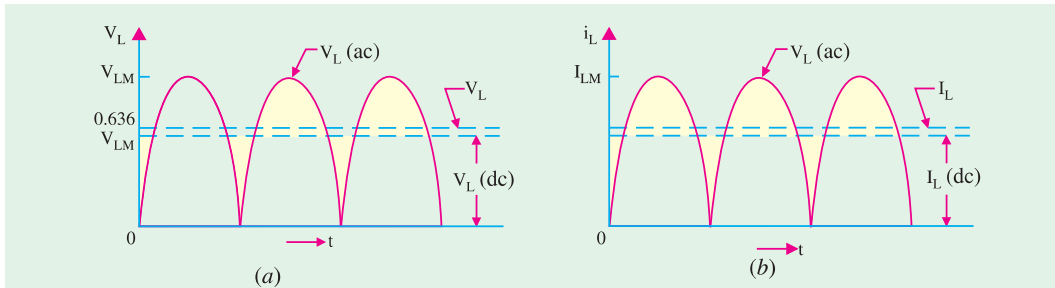


Fig. 55.8

**(c) Efficiency**

$$P_{in} = I_L^2 (R_0 + R_L) = \left( \frac{I_{LM}}{\sqrt{2}} \right)^2 (R_0 + R_L) = \frac{1}{2} I_{LM}^2 (R_0 + R_L)$$

$$P_{dc} = I_{L(dc)}^2 (R_0 + R_L) = \left( \frac{2 I_{LM}}{\pi} \right)^2 (R_0 + R_L) = \frac{4 I_{LM}^2}{\pi^2} (R_0 + R_L)$$

$$\therefore \eta = \frac{P_{dc}}{P_{in}} = \left( \frac{8}{\pi^2} \right) \left( \frac{R_L}{R_0 + R_L} \right) = \frac{0.812}{(1 + R_0/R_L)} = \frac{81.2\%}{(1 + R_0/R_L)}$$

It is twice the value for the half-wave rectifier for the simple reason that a full-wave rectifier utilizes both half-cycles of the input ac supply.

**(d) Frequency Components**

As in the case of a HW rectifier, the output of a full-wave rectifier also consists of (i) a dc



component and (ii) a number of ac components which form the ripple. The Fourier series for rectifier output voltage is

$$V_L = V_{LM} \left( \frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t - \frac{4}{35} \cos 6\omega t - \dots \right)$$

As seen,  $V_{L(dc)} = \frac{2V_{LM}}{\pi}$ ;  $V_{L1} = \frac{4V_{LM}}{\sqrt{2} \cdot 3\pi}$ ,  $V_{L2} = \frac{4V_{LM}}{\sqrt{2} \cdot 15\pi}$  etc.

$$V_{L(ac)} = \sqrt{V_{L1}^2 + V_{L2}^2} = \sqrt{\left(\frac{4V_{LM}}{\sqrt{2} \cdot 3\pi}\right)^2 + \left(\frac{4V_{LM}}{\sqrt{2} \cdot 15\pi}\right)^2} = 0.305 V_{LM}$$

Similarly,  $I_{L(ac)} = I_{r(rms)} = \sqrt{I_{L1}^2 + I_{L2}^2} = 0.305 I_{LM}$

**(e) Ripple Factor**

$$\gamma = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(rms)}}{V_{L(rms)}} = \frac{0.305 V_{LM}}{0.636 V_{LM}} = 0.482$$

It is much less as compared to 1.21 for half-wave rectifier.

**(f) PIV**

Its value is  $= 2 V_{sm}$

**(g) TUF**

Its value is found by considering the primary and secondary windings of the transformer separately. Its value is 0.693 (as compared to 0.287 for a half-wave rectifier). In such a rectifier, there is no problem due to dc saturation of flux in the core because the dc currents in the two halves of the secondary flow in opposite directions.

**Example 55.3.** With reference to the full-wave rectifier of Fig. 55.9, determine

- (i) peak, dc component, rms and ac component of load voltage,
  - (ii) peak, dc component, rms and ac component of load current,
  - (iii) ripple factor,
  - (iv) peak and average diode currents,
  - (v) total power supplied to the load.
- Neglect diode and secondary winding resistances.

**Solution.** Here,

$$K = 1/2$$

$$V_{pm} = 220 = 312 \text{ V. Hence,}$$

$$V_{MN} = 312/2 = 156 \text{ V so that}$$

$$V_{MG} = V_{GN} = 156/2 = 78 \text{ V}$$

$$V_{sm} = V_{LM} = 78 \text{ V}$$

$$V_{L(dc)} = 0.636 V_{LM} = 0.636 \times 78$$

$$= 49.6 \text{ V}$$

$$V_L = 0.707 V_{LM} = 0.707 \times 78$$

$$= 55 \text{ V}$$

$$V_{L(ac)} = \sqrt{55^2 - 49.6^2} = 23.8 \text{ V}$$

It also represents  $V_{r(rms)}$ .

(ii)  $I_{LM} = V_{LM}/R_L = 78/100 = 0.78 \text{ A}$

$$I_{L(dc)} = 0.636 \times 0.78 = 0.496 \text{ A}$$

$$I_L = 0.707 \times 0.78 = 0.55 \text{ A}$$

$$I_{L(ac)} = \sqrt{0.55^2 - 0.496^2} = 0.238 \text{ A}$$

It also represents  $I_{r(rms)}$ .

(iii)  $\gamma = \frac{I_{L(ac)}}{I_{L(dc)}} = \frac{I_{r(max)}}{I_{L(max)}} = \frac{0.238}{0.496} = 0.48$

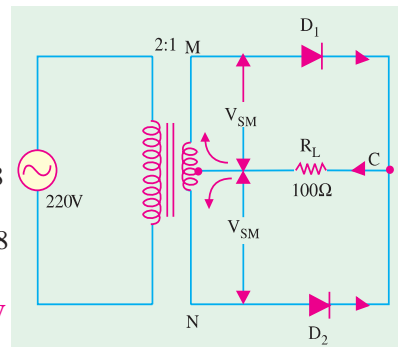


Fig. 55.9



(iv) peak diode current = peak load current = **0.78 A**

For finding the average current of a diode, it must be remembered that each diode carries current for one half-cycle only.

$$I_{D(av)} = I_{D(max)}/\pi = 0.318 \times 0.78 = \mathbf{0.25 \text{ A}}$$

(v)  $PL = V_L I_L = 55 \times 0.55 = \mathbf{30.25 \text{ W}}$

**Example 55.4.** A 1- $\phi$ , full-wave rectifier supplies power to a 1 k W load. The ac voltage applied to the diode is 300-0-300 V (rms). If diode resistance is 25  $\Omega$  and that of the transformer secondary negligible, determine“

- (i) average load current, (ii) average value of load voltage,  
 (iii) rms value of ripple, (iv) efficiency.

(Applied Electronics, Bombay Univ.)

**Solution.** It may be noted that rms value of ac voltage across each secondary half is 300 V.

(i)  $V_{sm} = 300 = 424 \text{ V}, I_{LM} = V_{sm}/(r_d + R_L) = 424/1025 = 0.414 \text{ A}$

$$I_{L(dc)} = I_{LM}/\pi = 2 \times 0.414/\pi = \mathbf{0.263 \text{ A}}$$

(ii)  $V_{L(dc)} = I_{L(dc)} \cdot R_L = 0.263 \times 1000 = \mathbf{263 \text{ V}}$

(iii) 
$$\gamma = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(rms)}}{V_{L(dc)}}$$

$\therefore V_{r(rms)} = \gamma \cdot V_{L(dc)} = 0.482 \times 263 = \mathbf{126.8 \text{ V}}$

(iv) 
$$\eta = \frac{81.2\%}{1 + r_d / R_L} = \frac{81.2\%}{1 + 25/1000} = \mathbf{79.2\%}$$

**Example 55.5.** A full-wave rectifier is built up by using the same components as in Ex. 55.2. Determine :

- (i) dc load current, (ii) dc load voltage,  
 (iii) voltage regulation, (iv) circuit efficiency and  
 (v) diode PIV and current rating.

**Solution.**

(i)  $I_{L(dc)} = 0.636 \times 1.88 = \mathbf{1.2 \text{ A}}$

(ii)  $V_{L(dc)} = I_{L(dc)} \cdot R_L = 1.2 \times 10 = \mathbf{12 \text{ V}}$

(iii)  $VR = 0.25/10 = 0.025$  or **2.5%**

(iv) 
$$\eta = \frac{81.2\%}{1 + R_0 / R_L} = \frac{81.2\%}{1 + 0.25/10} = \mathbf{79.2 \%$$

(v)  $PIV = 2 V_{sm} = 2 \times 20 = \mathbf{40 \text{ V}}$

With a safety factor of 1.5,  $PIV = 40 \times 1.5 = 60 \text{ V}$ . A dc current rating of about 2 A would be satisfactory.

**Example 55.6.** Silicon diodes are used in a two-diode full-wave rectifier circuit to supply a load with 12 volts D.C. Assuming ideal diodes and that the load resistance is 12 ohms, compute (i) the transformer secondary voltage, (ii) the load ripple voltage, (iii) the efficiency of the rectifier. Derive equations used. (Electronics-I, Bangalore Univ.)

**Solution.**

(i)  $V_{L(dc)} = 2 V_{sm}/\pi; \quad V_{sm} = \pi \times 12/2 = \mathbf{18.8 \text{ V}}$

It is the maximum value of the voltage across one half of the secondary.

(ii) RMS value of ripple voltage =  $\gamma V_{L(dc)} = 0.482 \times 12 = \mathbf{5.78 \text{ V}}$

(iii) Since diodes are ideal ones, their forward resistance is negligible. Hence,



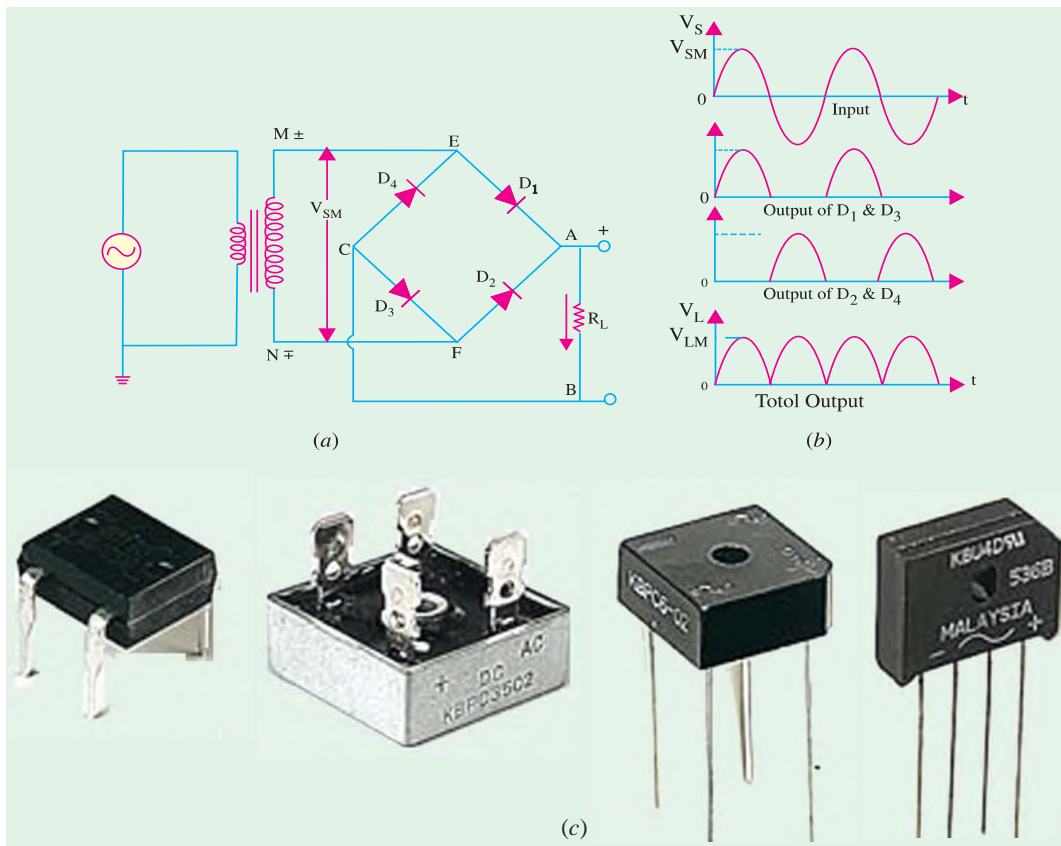


Fig. 55.10

$$\eta = \frac{81.2\%}{(1 + r_d / R_L)} = \frac{81.2\%}{1 + 0/12} = 81.2\%$$

### 55.8. Full-Wave Bridge Rectifier

It is the most frequently-used circuit for electronic dc power supplies. It requires four diodes but the transformer used is not centre-tapped and has a maximum voltage of  $V_{sm}$ . The full-wave bridge-rectifier is available in three distinct physics forms.

1. four discrete diodes,
2. one device inside a four-terminal case,
3. as part of an array of diodes in an IC.

The circuit using four discrete diodes is shown in Fig. 55.10 (a) and 55.10 (c) shows some pictures of the bridge rectifier available as one device in a four terminal case.

#### (a) Working

During the positive input half-cycle, terminal  $M$  of the secondary is positive and  $N$  is negative as shown separately in Fig. 55.11 (a). Diodes  $D_1$  and  $D_3$  become forward-biased (ON) whereas  $D_2$  and  $D_4$  are reverse-biased (OFF). Hence, current flows along  $MEABCFN$  producing a drop across  $R_L$ .

During the negative input half-cycle, secondary terminal  $N$  becomes positive and  $M$  negative. Now,  $D_2$  and  $D_4$  are forward-biased. Circuit current flows along  $NFABCEM$  as shown in Fig. 55.11 (b). Hence, we find that current keeps flowing through load resistance  $R_L$  in the same direction  $AB$  during both half-cycles of the ac input supply. Consequently, point  $A$  of the bridge rectifier always





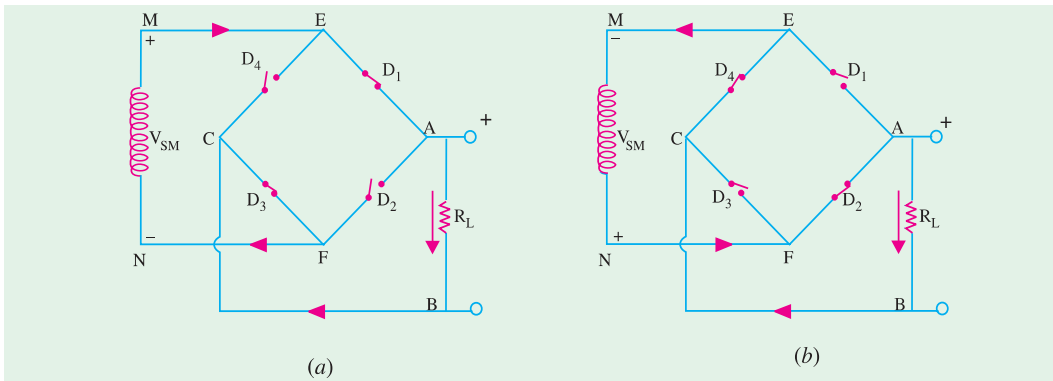


Fig. 55.11

acts as an anode and point C as cathode. The output voltage across  $R_L$  is as shown in Fig. 55.10 (b). Its frequency is twice that of the supply frequency.

**(b) Average and RMS Values**

These are the same as for the centre-tapped full-wave rectifier discussed in Art 55.7.

**(c) Efficiency**

$$\% \eta = \frac{81.2}{1 + 2 r_d / R_L}$$

**(d) Ripple Factor**

It is the same as for a full-wave rectifier *i.e.*  $\gamma = 0.482$

**(e) PIV**

The *PIV* rating of each of the four diodes is equal to  $V_{SM}$  —the entire voltage across the secondary.

**When Secondary and Diode Resistances are considered**

(i)  $I_{LM} = \frac{V_{sm} - 2V_B}{(R_S + 2r_d) + R_L} = \frac{V_{sm} - 2V_B}{R_0 + R_L}$       (ii)  $V_{LM} = I_{LM} \cdot R_L$

(iii)  $\eta = \left( \frac{8}{\pi^2} \right) \left( \frac{R_L}{(R_S + 2r_d + R_L)} \right) = \left( \frac{8}{\pi^2} \right) \frac{R_L}{R_0 + R_L}$

(iv)  $V_R = \frac{R_S + 2r_d}{R_L} = \frac{R_0}{R_L}$

**(f) Advantages**

After the advent of low-cost, highly-reliable and small-sized silicon diodes, bridge circuit has become much more popular than the centre-tapped transformer *FW* rectifier. The main reason for this is that for a bridge rectifier, a much smaller transformer is required for the same output because it utilizes the transformer secondary continuously unlike the 2-diode *FW* rectifier which uses the two halves of the secondary alternately.

So, the advantages of the bridge rectifier are :

1. no centre-tap is required on the transformer;
2. much smaller transformers are required;
3. it is suitable for high-voltage applications ;
4. it has less *PIV* rating per diode.

The obvious disadvantage is the need for twice as many diodes as for the centre-tapped transformer version. But ready availability of low-cost silicon diodes has made it more economical despite its requirement of four diodes.



### 55.9. Three-phase Half-wave Rectifier

Rectification of a 3-phase supply with the help of diodes is shown in Fig. 55.12 along with a smoothing circuit. The three diodes are connected to the three phases of star-connected secondary of a 3-phase transformer. Neutral point  $N$  of the secondary is the negative terminal for the rectified output and is earthed as shown.

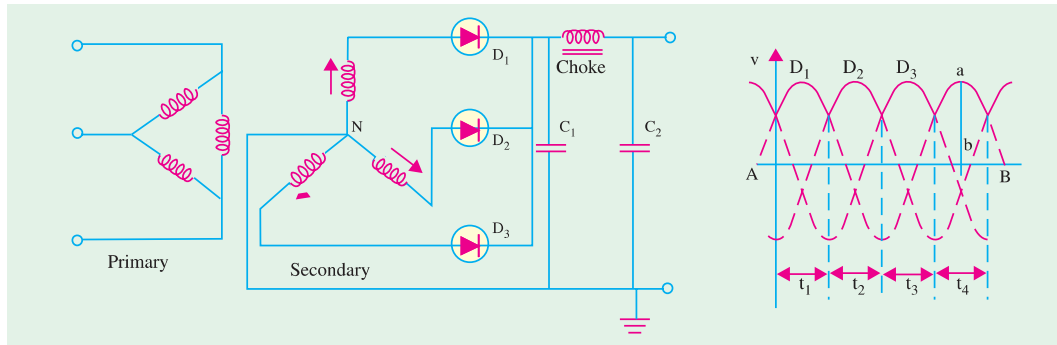


Fig. 55.12

Fig. 55.13

The shape of the output is shown in Fig. 55.13. The horizontal line  $AB$  represents the potential of the negative d.c. terminal output and the sine waves 1, 2 and 3 each represent the anode potentials of the three diodes.

During one-third of the cycle *i.e.*, during time  $t_1$ , only diode  $D_1$  will conduct. It will cease conducting at  $t_2$  and then  $D_2$  will conduct current upto  $t_2$  after which  $D_2$  will take over and will supply anode current till  $t_3$ . When one diode conducts, the other two remain inactive because then their anodes are more  $-ve$  than their cathodes. This process repeats itself during each ensuing cycle, with the conducting period of each diode being as indicated. The output is given by the vertical distance 'ab' between the upper envelope and the line  $AB$ . Obviously, the output fluctuates between the maximum and minimum values thrice in each cycle. The variations of output lie between  $V_{sm}$  and  $0.5 V_{sm}$  (neglecting voltage drop in diodes) and has a mean value of  $V_{dc} = 0.83 V_{sm}$  or  $1.17 V_S$  where  $V_S$  is the r.m.s. value of the secondary phase voltage. Its maximum conversion  $\eta = 96.5\%$  and  $\gamma = 0.17$ . But it should be noted that the magnitude of these fluctuations or pulsations is lesser than for a 1-phase, full-wave rectified output since the current never touches zero. It is further smoothed up by a  $C-L-C$  filter circuit as shown in Fig. 55.12.

It is seen that direct current of each diode appears in the secondary phase winding and so causes transformer saturation resulting in large primary current. It can be avoided by using zig-zag secondary.

### 55.10. Full-wave Rectification of 3-phase Currents

As shown in Fig. 55.14, a three-phase full-wave rectifier requires a transformer with six secondary windings connected to give two separate three-phase supplies  $180^\circ$  out of phase with each other. The centre taps are connected by an interphase transformer which enables the two rectifier units to operate independently of each

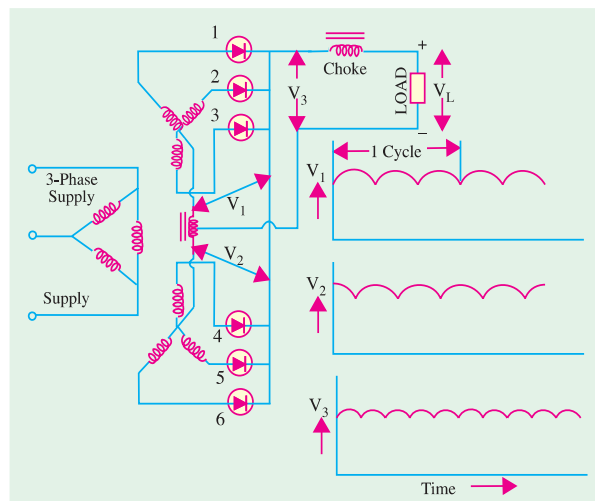


Fig. 55.14



other. Obviously, each diode conducts for one-third cycle. However, the addition of two outputs cancels the lowest frequency component of the ripple.

The output voltage has a mean value of  $0.83 V_{sm}$  (less the diode voltage drop) and a ripple having a fundamental frequency six times the supply frequency.

Three-phase full-wave rectifier circuit is preferred for high powers because

- (i) each secondary carries current for one-third of a cycle;
- (ii) each primary carries current for two-thirds of a cycle;
- (iii) Cu loss in the transformer windings is comparatively lower.

### 55.11. Six-phase Half-wave Rectifier

Such a rectifier can be operated from a 3-phase supply using a transformer with three centre-tapped secondary windings with all the centre taps connected together as shown in Fig. 55.15. Obviously, each diode conducts for one-sixth of a cycle. The output voltage has a mean value of  $0.955 V_{sm}$  (less the voltage drop in the diode). Ripple has a very small value and a fundamental frequency six times the supply frequency.

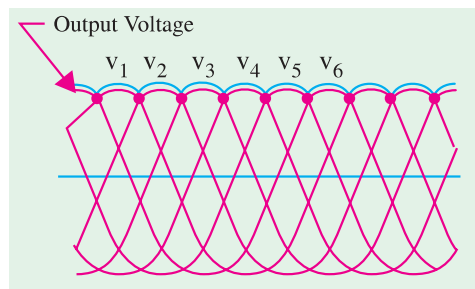


Fig. 55.15

### 55.12. Three-phase Bridge Circuit

This circuit is very frequently used because apart from being simple, it does not require centre-tap transformer. Only that diode supplies the load whose phase voltage is more positive than the others. For example, when  $D_1$  is forward-biased,  $D_2$  and  $D_3$  are reverse-biased. The current returns to the supply via  $D_5$  and  $D_6$ .

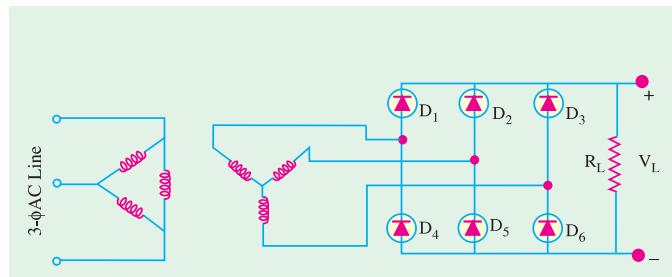


Fig. 55.16

In this circuit,  $I_{dc}$  is 0.955 times the peak current through each diode and only one-third of it flows through each diode (rather than one-sixth as in Fig. 5.15). Similarly,  $V_{dc}$  is twice of that in 3- $\phi$ , half-wave rectifier or  $2 \times 1.17 = 2.34$  times the r.m.s. a.c. voltage across each secondary leg. Accordingly, r.m.s. voltage across each secondary leg need be only  $1/2.34 = 0.428$  times the desired d.c. output voltage.

**Example 55.7.** The r.m.s. value of transformer secondary voltage per leg ( $V_s$ ) in a full-wave,  $D/Y$ , 6-f rectifier is 150 V. If average value of load current is 2 A, find (i)  $V_{dc}$  (ii) peak and average current through each diode and (iii) average power delivered to the load i.e.  $P_{dc}$ .

- Solution.** (i)  $V_{dc} = 2.34 \times V_s = 2.34 \times 150 = 351 \text{ V}$   
 (ii) peak current/diode =  $(1/0.955) \times I_{dc} = 1.05 \times 2 = 2.1 \text{ A}$   
 average d.c. current/diode =  $2/3 = 0.667 \text{ A}$   
 (In  $D/Y$  half-wave rectifier, its value is 1/6th rather than 1/3rd)  
 (iii)  $P_{dc} = V_{dc} I_{dc} = 351 \times 2 = 702 \text{ W}$

### 55.13. Calculations with Resistive Load

Let us suppose that no filter is connected across the rectifier (Fig. 55.12) but only a load of



resistance  $R$ . Further, we will neglect transformer resistance and leakage and internal diode drops. In that case,

$$\begin{aligned}
 V_{dc} &= 0.827 V_{sm}; I_{dc} = 0.827 V_{sm}/R \\
 P_{dc} &= V_{dc} I_{dc} = (0.827 V_{sm})^2/R; & P_{ac} = P_{in} = 0.706 V_{sm}^2/R \\
 \therefore \text{rectifier } \eta &= \frac{P_{dc}}{P_{ac}} = \frac{(0.827 V_{sm})^2/R}{0.706 V_{sm}^2/R} = 0.965 = 96.5\% \\
 \gamma &= \sqrt{(1.014)^2 - 1} = 0.17 \text{ or } 17\%.
 \end{aligned}$$

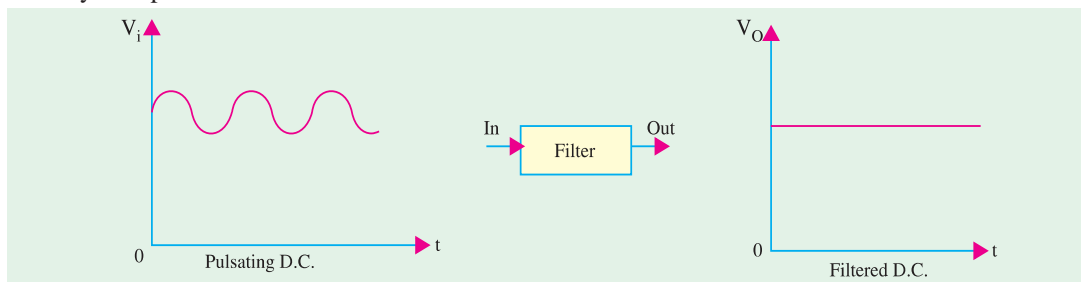
**Conclusions**

Here, average d.c. current is 0.827 times the peak current as compared to 0.318 times for 1- $\phi$ , half-wave circuit and 0.636 times for 1- $\phi$ , full-wave circuit. Moreover,  $V_{dc}$  is also correspondingly high, it is 51.17 times the r.m.s. voltage of each secondary leg ( $\because 1.414 \times 0.827 = 1.17$ ). Conversely, r.m.s. voltage ( $V_s$ ) across each leg of the secondary need only be  $1/1.17 = 0.855$  times the average desired d.c. output voltage across the load.

**55.14. Filters**

The main function of a filter circuit (Fig. 55.17) is to minimize the ripple content in the rectifier output.

As seen, output of various rectifier circuits is pulsating. It has a dc value and some ac components called ripples. This type of output is not useful for driving sophisticated electronic circuits/devices. In fact, these circuits require a very steady dc output that approaches the smoothness of a battery's output.



**Fig. 55.17**

A circuit that converts a pulsating output from a rectifier into a very steady dc level is known as filter because it filters out or smoothens out the pulsations in the output.

We will consider the following popular filter circuits :

1. Series capacitor filter,
2. series inductor filter,
3.  $L$ - $C$  filter (or  $L$ -type),
4.  $R$ - $C$  filter,
5.  $R$ - $L$ - $C$  filter.

**55.15. Shunt Capacitor Filter**

In this circuit, a suitable single capacitor  $C$  is connected across the rectifier and in parallel with the load  $R_L$  to achieve filtering action. This type of filter is known as **capacitor input filter**.

This filter circuit depends for its operation on the property of a capacitor to charge up (*i.e.* store energy) during conducting half-cycle and to discharge (*i.e.* deliver energy) during the non-conducting half-cycle. In simple words, a capacitor opposes any change in voltage. When connected across a pulsating d.c. voltage, it tends to smoothen out or filter out the voltage pulsations (or ripples). The filtering action of the simple capacitor filter when used in a half-wave rectifier can be understood with the help of Fig. 55.18.



**(a) Circuit Analysis**

When positive half-cycle of the ac input is applied, the diode is forward-biased and hence is turned ON. This allows  $C$  to quickly charge up to peak value of input voltage  $V_{ip}$  [point  $b$  in Fig. 55.18 (b)] because charging time constant is almost zero. It is so because there is no resistance in the charging path except diode forward resistance which is negligible. Hence, capacitor follows the charging voltage as shown. After being fully charged, the capacitor holds the charge till input ac supply to the rectifier goes negative. During the negative half-cycle, the capacitor attempts to discharge. However, it cannot discharge through diode which, being now reverse-biased, is OFF. Hence, it discharges through  $R_L$  from point  $b$  to  $c$  in Fig. 55.18 (c) and its voltage decreases somewhat. The discharging time constant ( $= CR_L$ ) is usually 100 times more than the charging time. Hence,  $C$  does not have sufficient time to discharge appreciably. It is seen that even during negative half-cycle of the input supply, the capacitor maintains a sufficiently large voltage across  $R_L$ .

During the next positive half-cycle, when rectifier voltage exceeds the capacitor voltage represented by point  $c$  in Fig. 55.18 (c),  $C$  is again charged quickly to  $V_{ip}$  as represented by point  $d$ . Once more, input voltage goes negative, opening the diode and forcing  $C$  to discharge through  $R_L$  during the interval  $de$ . In

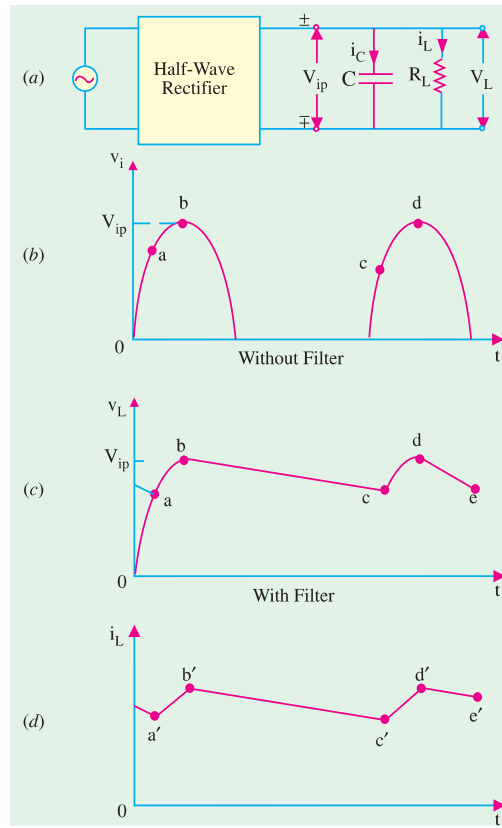


Fig. 55.18

this way,  $R_L$  sees a nearly constant dc voltage across it at all times.

The filtering action of this simple capacitor filter on a full-wave rectifier is shown in Fig. 55.19. It is seen that as compared to a HW rectifier.

- (i) dc load voltage increases slightly towards  $V_{ip}$ ,
- (ii) ripple voltage has been reduced by half.

The decreased ripple is because of shorter discharge time before the capacitor is reenergised by another pulse of current.

**(b) Load Current**

The load current has the same wave-shape as  $v_L$  because load is purely resistive. It is shown in Fig. 55.18 (d). During periods  $d'b'$  and  $c'd'$  etc., current is supplied by the diode and during periods  $b'c'$  and  $d'e'$  etc. by the capacitor.

**(c) Diode Current**

Diode current flows during short intervals of time like  $ab$  and  $cd$  etc. in Fig. 55.18 (c) which is reproduced in Fig. 55.20. During these intervals, diode

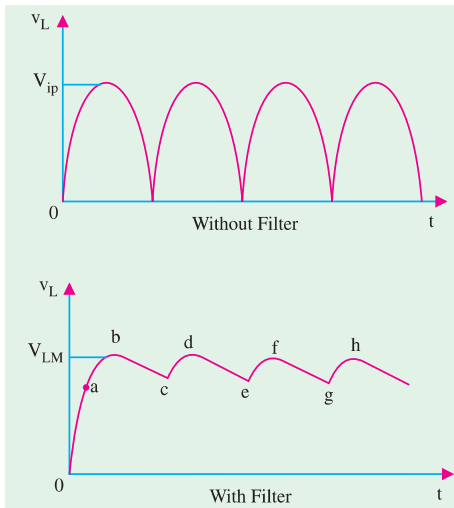


Fig. 55.19

During these intervals, diode



output voltage is greater than the capacitor voltage which is also the load voltage. Hence, diode current is a surging current *i.e.* it takes the form of short-duration pulses as shown in Fig. 55.20. A small resistor is always connected in series with the diode to limit this surge current. It is known as surge limiting resistor.

*The sole function of the diode is to recharge C and the sole function of C is to supply load current by discharge.*

**55.16. Effect of Increasing Filter Capacitance**

A capacitor has the basic property of opposing changes in voltage. Hence, a bigger capacitor would tend to reduce the ripple magnitude. It has been found that increasing the capacitor size.

1. increases  $V_{dc}$  towards the limiting value  $V_{ip}$  ;
2. reduces the magnitude of ripple voltage ;
3. reduces the time of flow of current pulse through the diode ;
4. increases the peak current in the diode.

**55.17. Calculations of Shunt Capacitor Filter**

Consider the rectifier and filter circuit of Fig. 55.21 where a capacitor has been connected across  $R_L$ . The output voltage waveforms of a half-wave rectifier with a shunt capacitor filter are shown in Fig. 55.22 (a) whereas Fig. 55.22 (b) shows those for a full-wave rectifier. The ripple voltage which occurs under light load conditions can be approximated by a triangular wave which has a peak-to-peak value of  $V_{r(p-p)}$  and a time period of  $T_r^*$  centred around the dc level.

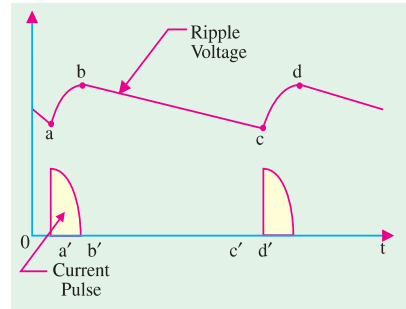


Fig. 55.20

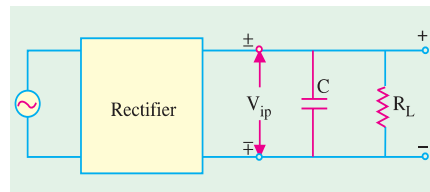


Fig. 55.21

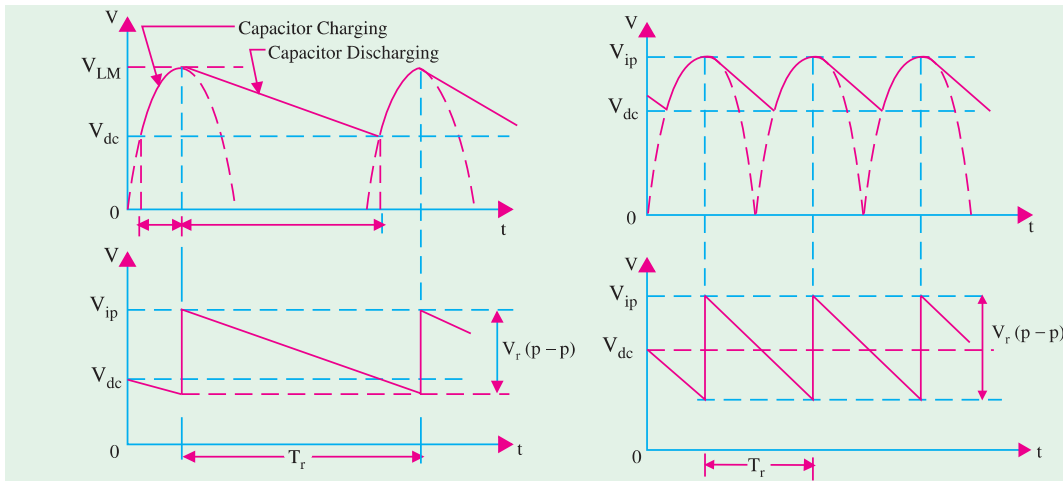


Fig. 55.22

In fact,  $V_{r(p-p)}$  is the amount by which capacitor voltage falls during discharge period  $T_r$ . This discharge is actually exponential\*\* but can be approximated by a straight line discharge if we assume

\* Since charging time is negligibly small, the approximate discharging time represents the full time-period.

\*\* given by  $V_c = V_{ip} e^{-t/CR_L}$



the discharge rate to remain constant at the dc level  $I_{dc}$ . In that case, charge lost  $dQ$  in time  $T_r$  is  $I_{dc} T_r$

$$\therefore V_{r(p-p)} = \frac{dQ}{C} = \frac{I_{dc} T_r}{C} = \frac{V_{dc}}{f_r C R_L} \quad \left( \because I_{dc} = \frac{V_{dc}}{R_L} \right)$$

The triangular ripple has an rms value given by  $V_{r(rms)} = V_{r(p-p)} / 2\sqrt{3}$

$$\therefore V_{r(rms)} = \frac{V_{r(p-p)}}{2\sqrt{3}} = \frac{V_{dc}}{2\sqrt{3} f_r C R_L} \quad \therefore \gamma = \frac{V_{r(rms)}}{V_{dc}} = \frac{1}{2\sqrt{3} f_r C R_L}$$

Now,  $f_r$  is the frequency of the ripple voltage. For a half-wave rectifier,  $f_r$  equals the rectifier line input frequency whereas it is double the line input frequency for a full-wave rectifier. If  $f$  is the line frequency, then

$$\begin{aligned} \gamma &\cong \frac{1}{2\sqrt{3} f C R_L} && \text{— for HW rectifier} \\ &\cong \frac{1}{4\sqrt{3} f C R_L} && \text{— for FW rectifier} \end{aligned}$$

It can be further proved that

$$\begin{aligned} \gamma &= \frac{1}{2\sqrt{3} f C R_L} = \frac{I_{dc}}{4\sqrt{3} f C} \left( \frac{1}{V_{ip}} - \frac{1}{V_{dc}} \right) && \text{— half-wave rectifier} \\ &= \frac{1}{4\sqrt{3} f C R_L} = \frac{I_{dc}}{4\sqrt{3} f C V_{ip}} && \text{— for full-wave rectifier} \end{aligned}$$

It is seen from above that ripple increases with increase in load (*i.e.* output) current.

Incidentally, 
$$V_{dc} = V_{ip} - \frac{V_{r(p-p)}}{2}$$

where  $V_{ip}$  = peak rectifier output voltage.

Substituting the value of  $V_{r(p-p)} = V_{dc} / f_r C R_L$ , we get

$$V_{dc} = V_{ip} - \frac{V_{dc}}{2 f_r C R_L} \quad \text{or} \quad V_{dc} = \frac{V_{ip}}{1 + \frac{1}{2 f_r C R_L}}$$

$$\begin{aligned} \therefore V_{dc} &= V_{ip} = \left( \frac{2 f C R_L}{1 + 2 f C R_L} \right) = \frac{V_{ip} - I_{dc} / 4 f C}{1 + I_{dc} / 4 f C V_{ip}} && \text{— half-wave rectifier} \\ &= V_{ip} \left( \frac{4 f C R_L}{1 + 4 f C R_L} \right) = \frac{V_{ip}}{1 + I_{dc} / 4 f C V_{ip}} && \text{— full-wave rectifier} \end{aligned}$$

**Example 55.8.** A half-wave rectifier has a peak output voltage of 12.2 V at 50 Hz and feeds a resistive load of 100 Ω. Determine (i) the value of the shunt capacitor to give 1 percent ripple factor and (ii) the resulting dc voltage across the load resistor.

**Solution. (i)** 
$$\gamma = 1 / 2\sqrt{3} f C R_L \quad \text{or} \quad 0.01 = 1 / 2\sqrt{3} \quad \therefore C = 5770 \mu\text{F}$$

**(ii)** 
$$V_{dc} = \frac{V_{ip}}{1 + 1 / 2 f C R_L} = \frac{12.2}{1 + 1 / 2 \times 5770 \times 10^{-6} \times 100} = 12 \text{ V}$$

**Example 55.9.** Find the ripple factor and dc output voltage for the filtered bridge rectifier shown in Fig. 55.23. Each silicon diode has a threshold voltage of 0.7 V.

**Solution.** Peak primary voltage =  $230 \times \sqrt{3}$   
= 325 V



$$\begin{aligned} \text{Peak secondary voltage} &= 325 \times 1/10 \\ &= 32.5 \text{ V} \end{aligned}$$

Peak full-wave rectified voltage at the filter input

$$V_{ip} = 32.5 - 2 \times 0.7 = 31.1 \text{ V}$$

$$\begin{aligned} \gamma &= \frac{1}{4\sqrt{3}fCR_L} \\ &= \frac{1}{4\sqrt{3} \times 50 \times 5 \times 10^{-6} \times 20 \times 10^3} \\ &= 0.028 \text{ or } \mathbf{2.8\%} \end{aligned}$$

$$V_{dc} = \frac{V_{ip}}{1 + \frac{1}{4fCR_L}} = \frac{31.1}{1 + \frac{1}{4 \times 50 \times 5 \times 10^{-6} \times 20 \times 10^3}} = \mathbf{29.6 \text{ V}}$$

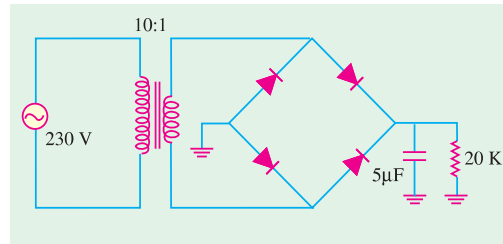


Fig. 55.23

**Example 55.10.** Derive an expression for the ripple in the output of a full-wave rectifier circuit, with a simple capacitor element as the filter.

The load current from the above circuit operating from 200-V, 50-Hz supply is 12 mA. Calculate minimum value of filter capacitor which is required to keep the ripple voltage below 2%.

(App. Electronics and Circuits ; Grad. I.E.T.E.)

**Solution.** As seen from Art. 5.17  $\gamma = I_{dc} / 4\sqrt{3}fCV_{ip}$

$$\therefore C = I_{dc} / 4\sqrt{3}f\gamma V_{ip} = 12 \times 10^{-3} / 4\sqrt{3} \times 50 \times 0.02 \times 200 = 6 \mu\text{F}$$

### 55.18. Series Inductor Filter

The filter consists of a choke in series with the load resistor  $R_L$  as shown in Fig. 52.24. The operation of such a filter depends on the fundamental property of an inductor to oppose any sudden changes in the current flowing through it. Since this inductor presents high impedance to the ac components in the filter output, it reduces their amplitude with respect to the dc component thereby producing only a small ripple as shown in Fig. 55.24 (b).

The Fourier series for the rectifier output voltage is

$$v_i = V_{ip} \left( \frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t - \dots \right)$$

For finding the ripple factor, we will calculate the dc as well as ac drop over  $R_L$ . If we neglect choke resistance ( $R_C$ ), then the entire dc component of filter output is available across  $R_L$  and its value is  $V_{dc} = 2V_{ip}/\pi$  \*.

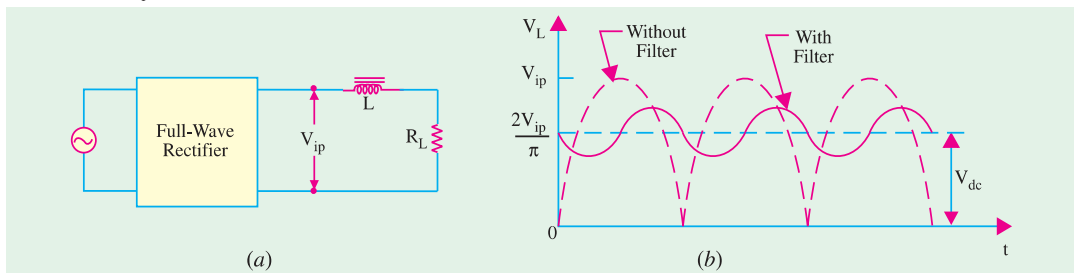


Fig. 55.24

\* If  $R_C$  is taken into account, then filter output dc voltage drops partly on  $R_C$  and partly on  $R_L$ . The drop over  $R_L$  would be

$$= \frac{2V_{ip}}{\pi} \cdot \frac{R_L}{R_L + R_C}$$





We will consider only the second harmonic voltage  $(4V_{ip}/3\pi) \cos 2\omega t$  of frequency  $2\omega$  and neglect higher harmonic voltages. This ac voltage partly drops over  $X_L$  and partly over  $R_L$ . Since choke and  $R_L$  are connected in series, the maximum value of drop over  $R_L$  is

$$= \frac{4V_{ip}}{3\pi} \cdot \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

The rms value of this ac voltage drop across  $R_L$  is

$$V_{\alpha} = \frac{4V_{ip}}{\sqrt{2.3}\pi} \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

$$\therefore \gamma = \frac{V_{\alpha}}{V_{dc}} = \frac{4V_{ip}}{\sqrt{2.3}\pi} \frac{R_L}{\sqrt{R_L^2 + X_L^2}} \times \frac{\pi}{2V_{ip}}$$

$$= \frac{\sqrt{2} R_L}{3\sqrt{(R_L^2 + X_L^2)}} = \frac{\sqrt{2}}{3\sqrt{1 + X_L^2 + R_L^2}^{1/2}}$$

Since

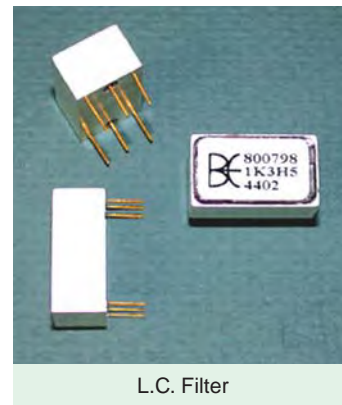
$$X_L = 2\omega L, \text{ hence } \gamma = \frac{\sqrt{2}}{3(1 + 4\omega^2 L^2 / R_L^2)^{1/2}}$$

$$\text{If } 4\omega^2 L^2 / R_L^2 \gg 1, \text{ then } \gamma = \frac{\sqrt{2}R_L}{3 \times 2\omega L} = \frac{R_L}{\sqrt{2.3}\omega L}$$

It is seen that ripple decreases as  $R_L$  decreases or load current increases (just the opposite of what happens in the case of shunt capacitor filter).

### 55.19. The Choke Input or L-C Filter

It is a combination of two filters considered in Art. 55.15 and 55.18 and provides a lower ripple than is possible with either  $L$  or  $C$  alone. As is known, in an inductor filter, ripple increases with  $R_L$  but decreases in a capacitor filter. The combination of  $L$  and  $C$  (*i.e.*  $L$ -section) filter makes the ripple independent of  $R_L$ . Fig. 55.25 (a) shows the filter and (b) the voltage variations.



L.C. Filter

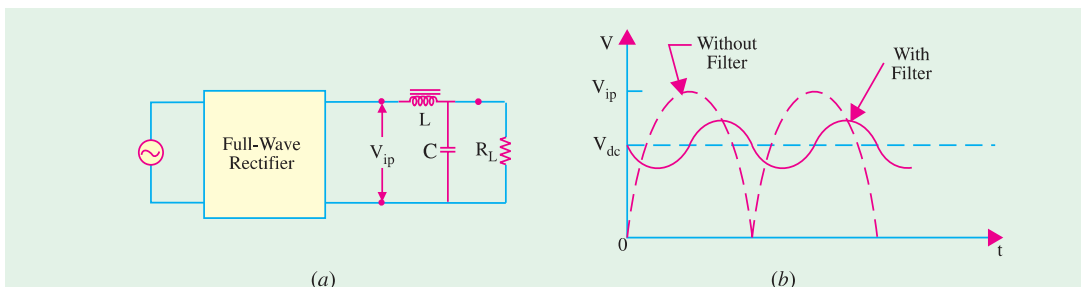


Fig. 55.25

#### Ripple Factor

If choke resistance  $R_C$  is neglected, then dc voltage available across  $R_L = 2V_{ip}/\pi$ . The ac drop over  $R_L$  is the same as across  $C$ . Since  $X_C \ll R_L$ , the parallel combination of  $R_L$  and  $X_C$  has impedance  $\cong X_C$ . The second harmonic voltage  $(4V_{ip}/3\pi) \cos 2\omega t$  can be assumed to drop over the  $L$ - $C$  series combination because  $R_L$  is effectively not there.

Maximum value of ac drop over  $C$  is



$$\begin{aligned}
 &= \frac{4 V_{ip}}{3 \pi} \cdot \frac{X_C}{(X_L + X_C)} \\
 \text{RMS value} &= \frac{4 V_{ip}}{\sqrt{2} \cdot 3 \pi} \cdot \frac{X_C}{(X_L + X_C)} \\
 \therefore \gamma &= \frac{V_{ac}}{V_{dc}} \frac{4 V_{ip}}{\sqrt{2} \cdot 3 \pi} \cdot \frac{X_C}{X_L + X_C} \times \frac{\pi}{2 V_{ip}} = \frac{\sqrt{2} X_C}{3 X_L} \text{---if } X_C \ll X_L \\
 &= \frac{\sqrt{2}}{3(2\omega C)(2\omega L)} = \frac{\sqrt{2}}{12\omega^2 LC} = \frac{1.19}{LC} \text{---if } C \text{ is in } \mu\text{F and } L \text{ in henrys}
 \end{aligned}$$

Now,  $I_{dc} = 2V_{ip} / \pi RL$ ; maximum value of second harmonic current  $I_{2h} = 4V_{ip} / 3\pi \times 2\omega L$ . The critical (or minimum) value of choke inductance essential for proper working of the filter is reached when  $I_{dc} = I_{2h}$  or  $2V_{ip} / \pi RL = 4V_{ip} / 3\pi \cdot 2\omega L$  or  $L = R_L / 3\omega$ . For  $f = 50$  Hz,  $L = R_L / 940$ .

**Example 55.11.** A single-phase full-wave rectifier uses 300-0-300 V, 50-Hz transformer. For a load current of 60 mA, design an L-filter using 10 H coil and a suitable capacitor to ensure a ripple factor of not more than 1%. (Electronics-II, Bangalore Univ.)

**Solution.**

$$\begin{aligned}
 \gamma &= 1.19/L \\
 0.01 &= 1.19/LC; \quad \therefore L_C = 119 \\
 \text{Hence, } 10 \times C &= 119 \\
 C &= 11.9 \mu\text{F} \approx 12 \mu\text{F}.
 \end{aligned}$$

### 55.20. The R-C Filter

Such a filter is shown in Fig. 55.26. Suppose that it is connected to a full-wave rectifier having a filtered output voltage of  $V_{ip}$ . The dc component voltage which drops over  $R_L$  is

$$= \frac{2 V_{ip}}{\pi} \cdot \frac{R_L}{R + R_L} \quad \dots (i)$$

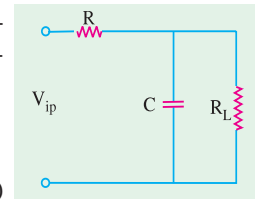


Fig. 55.26

Again, we would consider only the second harmonic voltage ( $4 V_{ip} / 3\pi$ ) cos  $2\omega t$ . As before, it will be assumed that  $X_C \ll R_L$  so that  $R_L \parallel X_C \approx X_C$ . In that case, ac voltage would be assumed to drop across R-C combination.

$$V_a = \frac{1}{\sqrt{2}} \cdot \frac{4 V_{ip}}{3\pi} \cdot \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{4 V_{ip}}{\sqrt{2} \cdot 3\pi} \cdot \frac{1}{\sqrt{1 + R^2 / X_C^2}}$$

Same is the drop across  $R_L$ .

$$\begin{aligned}
 \therefore \gamma &= \frac{V_a}{V_{dc}} = \frac{4 V_{ip}}{\sqrt{2} \cdot 3\pi} \cdot \frac{1}{\sqrt{1 + R^2 / X_C^2}} \times \frac{\pi(R + R_L)}{2 V_{ip} \cdot R_L} \\
 &= \frac{\sqrt{2}}{3} \cdot \frac{1 + R/R_L}{\sqrt{1 + R^2 / X_C^2}} = \frac{(1 + R/R_L)}{3\sqrt{2} \omega CR} \quad \text{---if } R^2 / X_C^2 \gg 1
 \end{aligned}$$

### 55.21. The C-L-C or Pi Filter

As shown in Fig. 55.27, it consists of one inductor and two capacitors connected across its each end. The three components are arranged in the shape of the Greek letter  $\pi$ . It is also called capacitor input  $\pi$ -filter. The input capacitor  $C_1$  is selected to offer very low reactance to the ripple frequency. Hence, major part of filtering is done by  $C_1$ . Most of the remaining ripple is removed by the combined action of  $L$  and  $C_2$ .

The charging and discharging action of  $C_1$  is exactly the same as described in Art. 55.15. The output voltage waveform is also like that shown in Fig. 55.25 (b).



This circuit gives much better filtering than  $LC$  filter circuit. However,  $C_1$  is still directly connected across the supply and would need high pulses of current if load current is large. Since these high peak current pulses are likely to damage the rectifier diode, this filter is used *with low-current equipment*.

Though this filter-gives somewhat higher output voltage, its voltage regulation is inferior to that of the  $LC$  filter.

The ripple factor of this filter is given by

$$\gamma = \sqrt{2} \frac{X_{C1} X_{C2}}{R_L X_L} = \frac{\sqrt{2}}{8\omega^3 C_1 C_2 L R_L} = \frac{5700}{L C_1 C_2 R_L} \quad \text{---when } f = 50 \text{ Hz}$$

Here,  $C_1, C_2$  are in  $\mu\text{F}$ ,  $L$  in henrys and  $R_L$  in ohms.

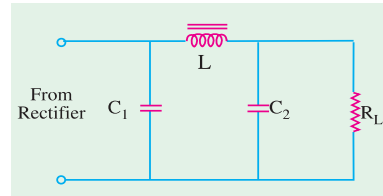


Fig. 55.27

### 55.22. Bleeder Resistor

Very often, a resistor (called bleeder resistor) is placed across the filter output (Fig. 55.28) because it provides the following advantages :

1. It improves voltage regulation of the supply. By acting as a pre-load on the supply, it causes an initial voltage drop. When the real load is connected, there is only a small amount of additional drop. In this way, difference between no-load and full-load voltage is reduced thereby improving the regulation.
2. It provides safety to the technicians handling the equipment. When power supply is switched off, it provides a path for the filter capacitor to discharge through. That is why it is called bleeder resistor. Without it, the capacitor will retain its charge for quite sometime even when the power supply is switched off. This high voltage can be dangerous for people working with the equipment.
3. By maintaining a minimum current through the choke, it improves its filtering action. Value of  $R_B$  should be such as to conduct 10 per cent of the total load current.

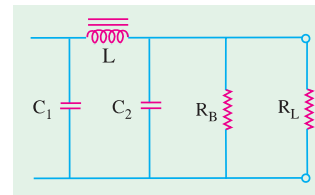


Fig. 55.28

### 55.23. Voltage Dividers

Often more than one dc voltage is needed for the operation of electronic circuits. A single power supply can provide as many voltages as are needed by using a voltage divider. As shown in Fig. 55.29 a voltage divider is a single tapped resistor connected across the output terminals of the supply. The tapped resistor may consist of two or three resistors connected in series across the supply. In fact, bleeder resistor may also be used as a voltage divider.

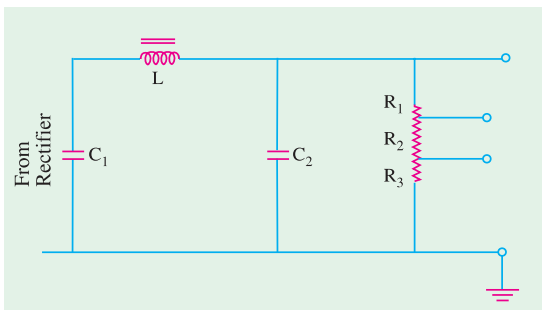
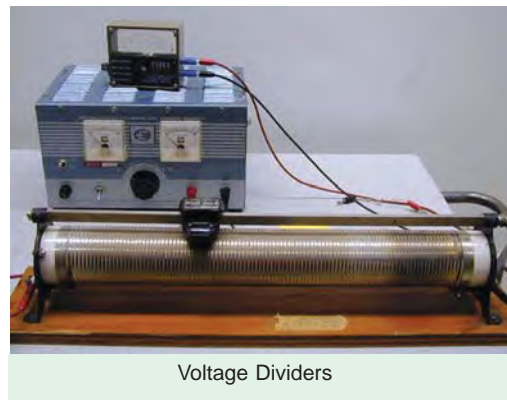


Fig. 55.29



Voltage Dividers



### 55.24. Complete Power Supply

Fig. 55.30 shows a complete solid-state power supply. From left to right, it consists of a transformer with a current-limiting resistor  $R_1$ , rectifier diodes for full-wave rectification, a  $\pi$ -type filter, a transistor series voltage regulator and a voltage divider.

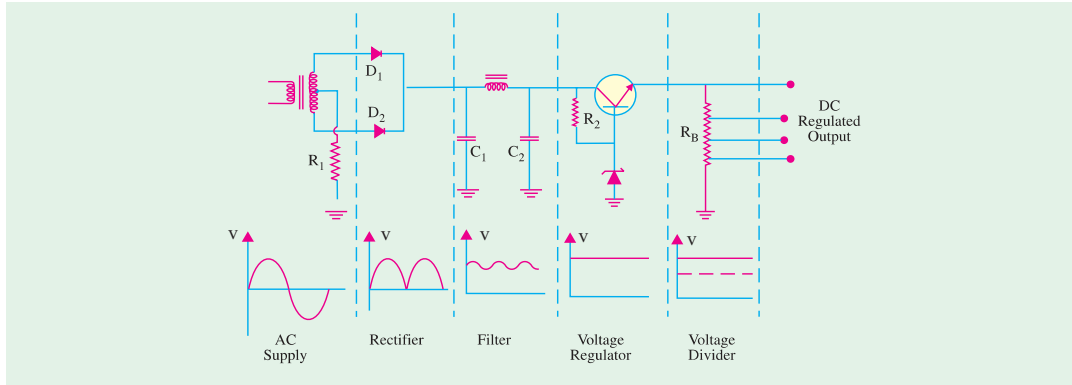


Fig. 55.30

As seen, unregulated ac voltage is fed from the transformer through a full-wave rectifier. It is then filtered by the CLC filter and finally regulated by a transistor regulator. The regulated dc supply becomes available across voltage divider resistance  $R_B$ . The output is practically ripple-free.

### 55.25. Voltage Multipliers

A voltage multiplier is a circuit which produces a greater dc output voltage than ac input voltage to the rectifiers. Multipliers are required in many circuit applications where it is necessary to have high voltages with low currents as for electron accelerating purposes in a cathode-ray tube (CRT).

We will consider the following circuit :

1. half-wave voltage doubler,
2. full-wave voltage doubler,
3. voltage tripler,
4. voltage quadrupler.

### 55.26. Half-wave Voltage Doubler

It is also known as cascade voltage doubler. The circuit is shown in Fig. 55.31.

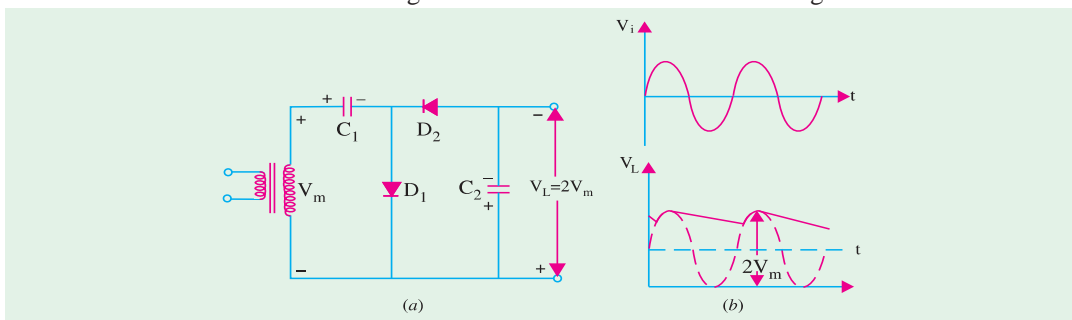


Fig. 55.31

#### Circuit Analysis

During the positive half-cycle of the input voltage,  $D_1$  conducts (not  $D_2$ ) and charges  $C_1$  to peak value of secondary voltage ( $V_m$ ) with the polarity as shown in Fig. 55.31 (a). During the negative half-cycle,  $D_2$  conducts (not  $D_1$ ) and charges  $C_2$ . The voltage across  $C_2$  is the sum of peak supply



voltage and the voltage across  $C_1$  (Fig. 55.31). It can be proved by applying *KVL* to the outer loop. Starting from the bottom of the transformer secondary in Fig. 55.32 and going clockwise, we get

$$-V_m - V_m + V_{C_2} = 0$$

or  $V_{C_2} = 2V_m = 2 \times \text{peak input voltage}$

During the next positive half-cycle,  $D_2$  is open and  $C_2$  will discharge through the load if it is connected. If no load is connected across  $C_2$ , then both capacitors stay charged *i.e.*  $C_1$  to  $V_m$  and  $C_2$  to  $2V_m$ . If there is a load connected across  $C_2$ , it will discharge a little bit and, as a result of it, voltage across it will drop slightly. But it will get recharged in the next half-cycle.

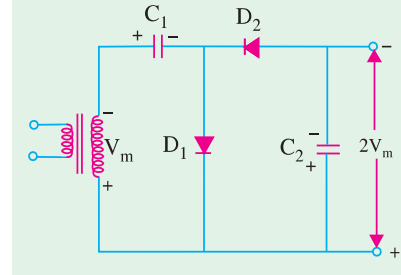


Fig. 55.32

The output waveform shown in Fig. 55.31 (b) is that of a half-wave rectifier filtered by a shunt capacitor. *PIV* across each diode is  $2V_m$ . Ripple frequency is equal to the supply frequency.

This circuit has very poor regulation and its ripple content is also high. This circuit has a common connection between the line and load (which a full-wave doubler does not have).

### 55.27. Full-wave Voltage Doubler

This circuit is shown in Fig. 55.33 (a). During the positive half-cycle of the input voltage,  $D_1$  conducts (but not  $D_2$ ) and charges capacitor  $C_1$  to the peak voltage  $V_m$  with the polarity as shown.

During the negative half-cycle,  $D_2$  conducts (but not  $D_1$ ) charging  $C_2$  to  $V_m$ . As far as the load is concerned, voltages across  $C_1$  and  $C_2$  are in series-aiding. If there is no load connected across the output, then load voltage  $V_L = 2V_m$  as shown in Fig. 55.33 (a). For example, if 220-V, 50-Hz is the supply, then  $V_{dc} = 2V_m = 2 \times \sqrt{2} = 620$  V. Of course, if a load is connected across the output terminals, then  $V_L$  would be less than  $2V_m$ .

The waveform of the output voltage is shown in Fig. 55.33 (b).

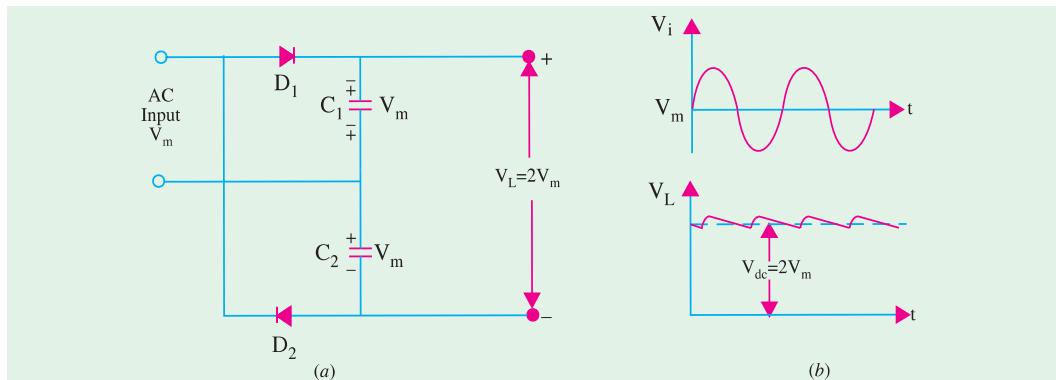


Fig. 55.33

The *PIV* rating of each diode is  $2V_m$ . Ripple frequency is twice the supply frequency. As seen, there is no common connection between the supply line and the load.

It is worth noting that where the expense of a line transformer is justified, it is preferable to use the superior conventional full-wave rectifier (Art. 55.7).

### 55.28. Voltage Tripler and Quadrupler Circuits

#### (a) General

The half-wave voltage doubler circuit (Fig. 55.31) can be extended to obtain any multiple of the peak input voltage ( $V_m$ ) *i.e.*  $3V_m, 4V_m, 5V_m$  etc. Theoretically speaking, there is no upper limit to the



amount of voltage multiplication that can be obtained. Though voltage triplers and quadruplers are commonly used, practical considerations limit additional multiplications. The main handicap is that total amount of capacitance becomes unduly large to maintain the desired dc output voltage for any thing except extremely light loads.

### (b) Circuit

The circuit for different multipliers is shown in Fig. 55.34. It should be obvious from the repetitive pattern of the circuit connections how additional diodes and capacitors may be connected to the doubler circuit for obtaining higher multiplications of the peak output voltage  $V_m$ .

### (c) Analysis

During the first positive half-cycle,  $C_1$  charges to  $V_m$  as diode  $D_1$  conducts. During negative half-cycle,  $C_2$  is charged through  $D_2$  to  $2V_m$  i.e. to the sum of voltage across  $C_1$  and peak input voltage  $V_m$  (Art 55.26).

During the second positive half-cycle,  $D_3$  conduct and voltage across  $C_2^*$  charges  $C_3$  to same voltage  $2V_m$ . During the negative half-cycle, diodes  $D_2$  and  $D_4$  conduct allowing  $C_3$  to charge  $C_4$  to the same peak voltage  $2V_m$ .

If it is seen from Fig. 55.34 that voltage across  $C_2$  is  $2V_m$ , across  $C_1$  and  $C_3$  is  $3V_m$  and across  $C_2$  and  $C_4$  is  $4V_m$ .

If additional diodes and capacitors are used, each capacitor would be charged to a peak voltage of  $2V_m$ .

The *PIV* rating of each diode is  $2V_m$  and ripple frequency is twice the line frequency. Generally, these circuits are used where both the supply voltage and load are maintained constant.

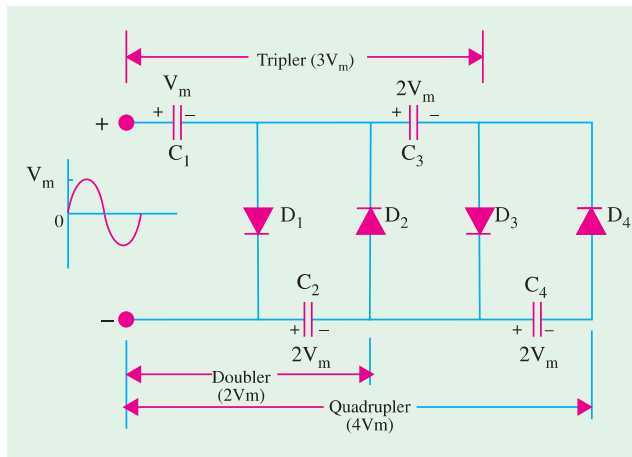


Fig. 55.34

## 55.29. Troubleshooting Power Supplies

There are usually two types of problems with power supplies i.e. either no dc output or low dc output.

The situation of no dc output can occur due to any one of the following reasons :

1. when there is no output from the rectifiers,
2. when there is no ac input to power supply,
3. when filter choke is open,
4. when the first input capacitor shorts.

A low dc output can occur in the following situations :

1. decreased input ac voltage,
2. open input capacitor of the filter circuit,
3. partial short across the load.

## 55.30. Controlled Rectification

It is that rectification in which the output of a rectifier circuit can be varied by controlling the point in the ac cycle at which the circuit is turned ON. A thyristor or *SCR* can be used for the purpose

\*  $C_1$  cannot charge  $C_3$  because it is shorted by  $D_1$ .



because under proper firing conditions, it can control the conduction angle of the rectifier circuit.

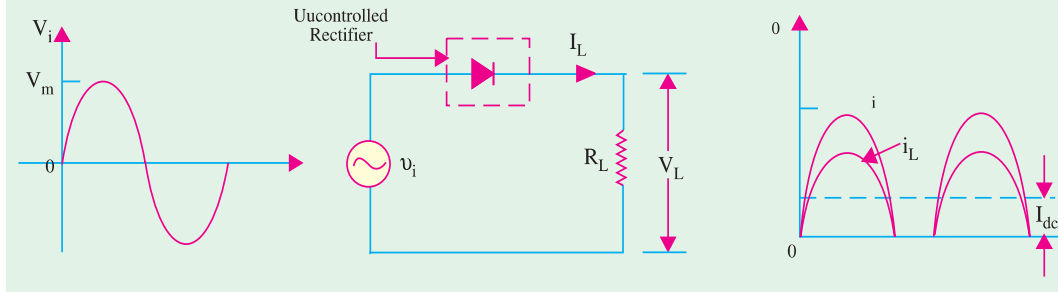


Fig. 55.35

In an ordinary diode rectifier circuit, current flows through the diode whenever instantaneous value of the ac supply voltage is greater than the voltage across the load at that instant.

For a resistive load shown in Fig. 55.35, load current flows at all times during the positive half-cycles of the supply.

In a controlled rectifier, on the other hand, load current flows only when a control signal is applied to turn on the rectifier at a specific point [like A in Fig. 55.36 (b)] in the ac cycle. Point A corresponds to the angle  $\theta_1$  so that conduction is delayed by this much period.

Once the HW rectifier is turned ON, it remains in conduction for the rest of the positive half-cycle *i.e.* upto  $180^\circ$ . Obviously, the firing point A is determined by the angle of delay in a plying the firing signal by the control circuit. As  $\theta_1$  increases, conduction occurs later in the cycle thereby decreasing the load current further.

The curve for voltage drop across rectifier diode is shown in Fig. 55.36 (c). During the positive half-cycle when the rectifier is fired into conduction, it acts like a short and voltage across it drops to zero (neglecting forward voltage of the diode). During negative half-cycle when diode is reverse-biased (and hence open) the full supply voltage appears across it as shown.

The curve for load voltage is shown in Fig. 55.36 (d). Since current obeys Ohm's law, it follows the load voltage.

**Average Value of Load Voltage**

Let the equation of the supply voltage be  $V_i = V_m \sin \theta$

For a half-wave rectifier, average value is found by taking average over the whole cycle ( $2\pi$ ) even though conduction takes

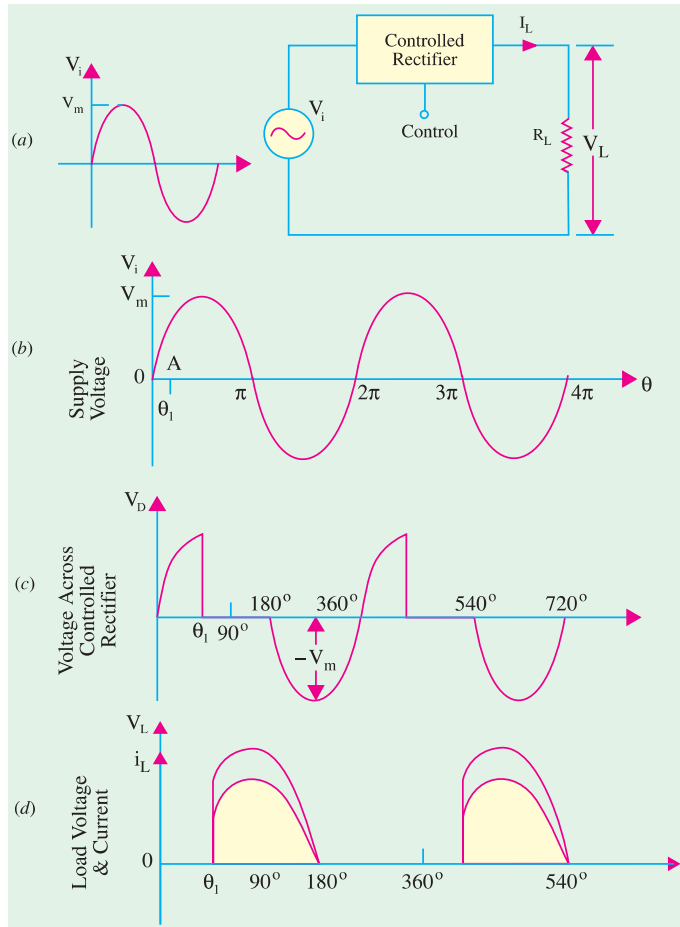


Fig. 55.36



place only from  $\theta_1 - \pi$  in the entire cycle.

$$\begin{aligned} \therefore V_L &= \frac{1}{2\pi} \int_{\theta_1}^{\pi} V_i d\theta \\ &= \frac{1}{2\pi} \int_{\theta_1}^{\pi} V_m \sin \theta d\theta \\ &= \frac{V_m}{2\pi} \int_{\theta_1}^{\pi} \sin \theta d\theta \\ &= \frac{V_m}{2\pi} [-\cos \theta]_{\theta_1}^{\pi} \\ &= \frac{V_m}{2\pi} |(-\cos \pi) - (-\cos \theta_1)| \\ &= \frac{V_m}{2\pi} (1 + \cos \theta_1) \\ \therefore V_{dc} = V_L &= \frac{V_m}{2\pi} (1 + \cos \theta_1) \end{aligned}$$

For a resistive load,

$$I_{dc} = I_L = \frac{V_m}{2\pi R_L} (1 + \cos \theta_1)$$

**Note :** If

$$\begin{aligned} \theta_1 &= 0, \text{ then } V_{dc} \\ &= \frac{V_m}{\pi} \text{ and } I_{dc} = \frac{V_m}{\pi R_L} \end{aligned}$$

These are the same values as for an uncontrolled or ordinary HW rectifier (Art 55.6).

### 55.31. Output Waveforms for Different Firing Angles

In all waveforms given in Fig. 55.37 it has been assumed that the ac supply voltage is sinusoidal given by the equation  $V_i = V_m \sin \theta$ . Also, a resistive load has been assumed.

Following points must always be kept in mind while drawing these diagrams. When the diode is forward-biased, it conducts and behaves like a short. Hence, drop across it is almost zero. Instead, whole of the applied voltage drops across load resistance. When during the negative input half-cycle, diode is reverse-biased, it does not conduct and behaves like an 'open'. Hence, all the applied voltage appears across the rectifier diode and none across  $R_L$ . In other words, diode anode voltage and load voltage are mutually exclusive *i.e. when one is there, the other is not.*

It is seen from Fig. 55.38 that as delay angle  $\theta_1$  is increased, the output keeps decreasing till for  $\theta_1 = 180^\circ$ , *the output is zero.*

### 55.32. Output Voltage and Current Values in Controlled Rectifiers

We will use the equation derived in Art 55.30 for a half-wave rectifier *i.e.*

$$\begin{aligned} V_{dc} &= \frac{V_m}{2\pi} (1 + \cos \theta_1) \\ I_{dc} &= \frac{V_m}{2\pi R_L} (1 + \cos \theta_1) = \frac{V_{dc}}{R_L} \end{aligned}$$





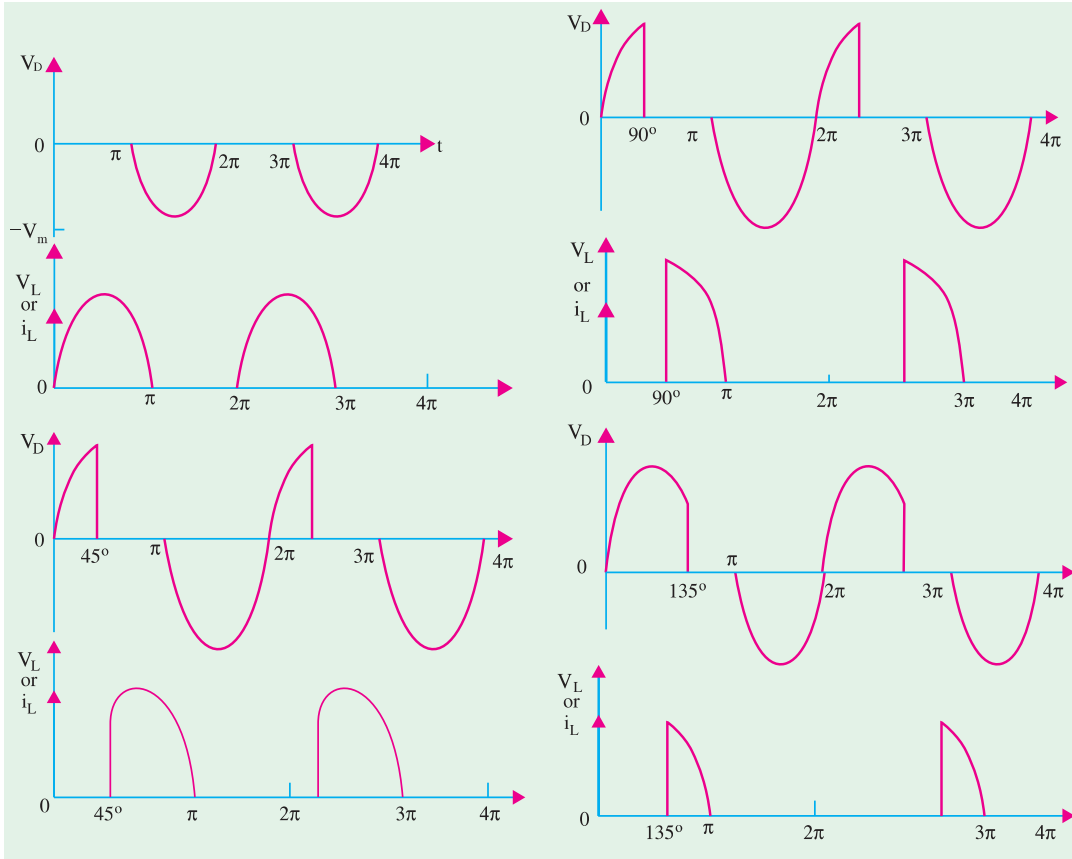


Fig. 55.37

Fig. 55.38

(a) when  $\theta_1 = 0$

$$\begin{aligned} \cos \theta_1 &= \cos 0^\circ = 1 \\ V_{dc} &= \frac{V_m}{2\pi} (1 + 1) = \frac{V_m}{\pi} = 0.318 V_m \\ I_{dc} &= \frac{V_{dc}}{R_L} = \frac{V_m}{\pi R_L} = 0.318 \frac{V_m}{R_L} \end{aligned}$$

(b) when  $\theta_1 = 30^\circ$

$$\begin{aligned} \cos \theta_1 &= \cos 30^\circ = 0.866 \\ \therefore V_{dc} &= \frac{V_m}{2\pi} (1 + 0.866) = 0.297 V_m ; I_{dc} = \frac{V_{dc}}{R_L} = 0.297 \frac{V_m}{R_L} \end{aligned}$$

(c) when  $\theta_1 = 45^\circ$

$$\begin{aligned} \cos \theta_1 &= \cos 45^\circ = 0.707 \\ \therefore V_{dc} &= \frac{V_m}{2\pi} (1 + 0.707) = 0.27 V_m ; I_{dc} = \frac{V_{dc}}{R_L} = 0.27 \frac{V_m}{R_L} \end{aligned}$$

(d) when  $\theta_1 = 60^\circ$

$$\begin{aligned} \cos \theta_1 &= \cos 60^\circ = 0.5 \\ \therefore V_{dc} &= \frac{V_m}{2\pi} (1 + 0.5) = 0.239 V_m ; I_{dc} = \frac{V_{dc}}{R_L} = 0.239 \frac{V_m}{R_L} \end{aligned}$$



(e) when  $\theta_1 = 90^\circ$

$$\begin{aligned}\cos \theta_1 &= \cos 90^\circ = 0 \\ V_{dc} &= \frac{V_m}{2\pi} (1 + 0) = \frac{V_m}{2\pi} = 0.159 V_m \\ I_{dc} &= \frac{V_{dc}}{R_L} = \frac{V_m}{2\pi R_L} = 0.159 \frac{V_m}{R_L}\end{aligned}$$

(f) when  $\theta_1 = 120^\circ$

$$\begin{aligned}\cos \theta_1 &= \cos 120^\circ = -0.5 \\ \therefore V_{dc} &= \frac{V_m}{2\pi} (1 - 10.5) = 0.08 V_m ; I_{dc} = \frac{V_{dc}}{R_L} = 0.08 \frac{V_m}{R_L}\end{aligned}$$

(g) when  $\theta_1 = 135^\circ$

$$\begin{aligned}\cos \theta_1 &= \cos 135^\circ = -0.707 \\ V_{dc} &= \frac{V_m}{2\pi} (1 - 0.797) = 0.0466 V_m ; I_{dc} = 0.0466 \frac{V_m}{R_L}\end{aligned}$$

**Example 55.12.** A  $100\text{-}\Omega$  load is connected to a peak supply of  $300\text{ V}$  through a controlled half-wave diode rectifier. The load power is to be varied from  $25\text{ W}$  to  $80\text{ W}$ . What is the angular firing control required? Neglect forward drop of the diode. (Basic Electronics, Pune Univ. 1990)

**Solution.**

$$P = V_{dc} \cdot I_{dc}$$

Now, 
$$V_{dc} = \frac{V_m}{2\pi} (1 + \cos \theta), \quad I_{dc} = \frac{V_m}{2\pi R_L} (1 + \cos \theta)$$

$$\begin{aligned}\therefore P &= \left(\frac{V_m}{2\pi}\right)^2 \cdot \frac{1}{R_L} (1 + \cos \theta)^2 = \left(\frac{300}{2\pi}\right)^2 \cdot \frac{1}{100} (1 + \cos \theta)^2 \\ &= 22.5 (1 + \cos \theta)^2\end{aligned}$$

(i) when  $P = 80\text{ W}$

$$\begin{aligned}\therefore 80 &= 22.5 (1 + \cos \theta)^2 \quad \text{or} \quad (1 + \cos \theta) = 1.8856 \\ \therefore \cos \theta &= 0.8856 \quad \text{or} \quad \theta = 27.7^\circ\end{aligned}$$

(ii) when  $P = 25\text{ W}$

$$\therefore 25 = 22.5 (1 + \cos \theta)^2 \quad \therefore \cos \theta = 0.1111 \quad \text{or} \quad \theta = 83.6^\circ$$

**Example 55.13.** In a controlled half-wave rectifier, peak supply voltage is  $200\text{ V}$  and the value of load resistor is  $1\text{ k}\Omega$ . Calculate the power delivered to the load circuit for firing angles of (i)  $0^\circ$  (ii)  $45^\circ$  (iii)  $90^\circ$  and (iv)  $135^\circ$ . (Solid State Devices and Ckts, BHU)

**Solution.** (i)  $V_{dc} = 0.318 \times 200 = 63.6\text{ V}$  ;  $I_{dc} = 63.6/1000 = 63.6\text{ mA}$   
 $P = V_{dc} \cdot I_{dc} = 63.6 \times 63.6 = 4045\text{ mW} = 4.045\text{ W}$

(ii)  $V_{dc} = 0.27 \times 200 = 54\text{ V}$  ;  $I_{dc} = 54/1000 = 54\text{ mA}$   
 $P = 54 \times 54 = 2916\text{ mW} = 2.916\text{ W}$

(iii)  $V_{dc} = 0.159 \times 200 = 31.8\text{ V}$  ;  $I_{dc} = 31.8\text{ mA}$   
 $P = 31.8 \times 31.8 = 1011\text{ mW} = 1.011\text{ W}$

(iv)  $V_{dc} = 0.0466 \times 200 = 9.3\text{ V}$  ;  $I_{dc} = 9.3\text{ mA}$   
 $P = 9.3 \times 9.3 = 86.5\text{ mW} = 0.0865\text{ W}$

### 55.33. Average Values for FW Controlled Rectifier

In this case, the average values would be doubled because there are two half sinusoids to be averaged as shown in Fig. 55.39 for  $\theta_1 = 60^\circ$ .



$$V_{dc} = 2 \times \frac{V_m}{2\pi} (1 + \cos \theta_1) = \frac{V_m}{\pi} (1 + \cos \theta_1)$$

$$\left[ I_{dc} = \frac{V_{dc}}{R_L} = \frac{V_m}{\pi R_L} (1 + \cos \theta_1) \right]$$

In the present case,  $\theta_1 = 60^\circ$ ,  $\cos \theta_1 = 0.5$

$$\therefore V_{dc} = \frac{V_m}{\pi} (1 + 0.5) = 0.477 V_m$$

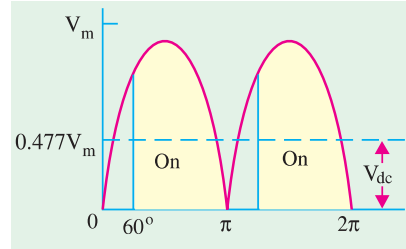


Fig. 55.39

### 55.34. Silicon Controlled Rectifier (SCR)

It is a trijunction PNPN device having three external connections : anode (A), cathode (C) and gate (G) as shown in Fig. 55.40.

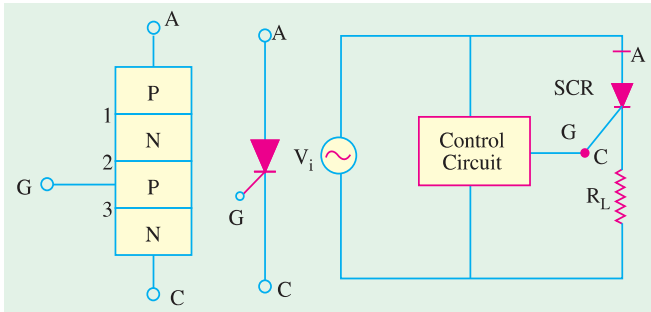


Fig. 55.40

When A is made positive with respect to C, the junctions marked 1 and 3 are forward-biased but the centremost junction marked 2 is reverse-biased. Hence, the SCR does not conduct. However, if a sufficiently positive voltage pulse is applied to this junction via the gate G, the SCR starts conducting like an ordinary diode. Gate has no control over SCR once it has been triggered into conduction. Conduction can only be terminated by removing positive voltage from its anode.

As shown in Fig. 55.40 (b), SCR needs a control circuit which triggers it into conduction by a gate pulse. Two popular ways of turning ON an SCR are:

1. Amplitude firing—in which gate current flows for the whole period of conduction.
2. UJT oscillator firing—in this method, gate current is supplied only momentarily. This method allows control over the power delivered to SCR's load from zero watt to fully ON.

Some of the simple ways of controlling the conduction of an SCR are discussed below.

### 55.35. Pulse Control of SCR

The simplest SCR control circuit of this type is shown in Fig. 55.41. If SCR were an ordinary rectifier, it would produce half-wave rectified ac voltage across  $R_L$ . However, SCR will not conduct even during positive half-cycle of the input ac voltage unless G is given positive voltage to forward-bias its centremost junction. By applying a trigger pulse at any time during the positive input half-cycle, it can be fired into conduction.

The resultant load waveform across  $R_L$  will consist of a portion of the positive half-cycle commencing at the instant at which the SCR is triggered [Fig. 55.41 (c)].



SCR and diode tester



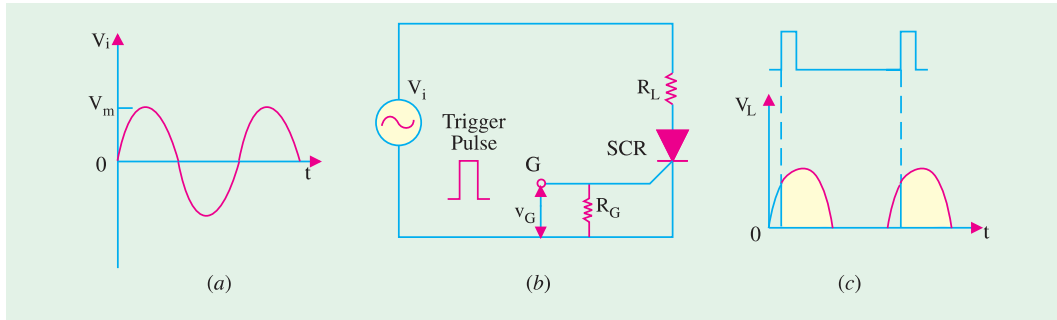


Fig. 55.41

**55.36. 90° Phase Control of SCR**

Fig. 55.42 shows a circuit which can trigger (or switch on) SCR any where from the commencement of the ac cycle to the peak of its positive half-cycle *i.e.* between 0° and 90°. As shown, gate current is derived from the ac supply (often it is from rectifier output as in Fig. 5.44) *via* R. If R is set at low value, the SCR will trigger almost at the commencement of the positive half-cycle of the ac input. On the other hand, if R is set at high resistance, the SCR may not switch ON until the peak positive half-cycle when sufficiently large gate current would become available.

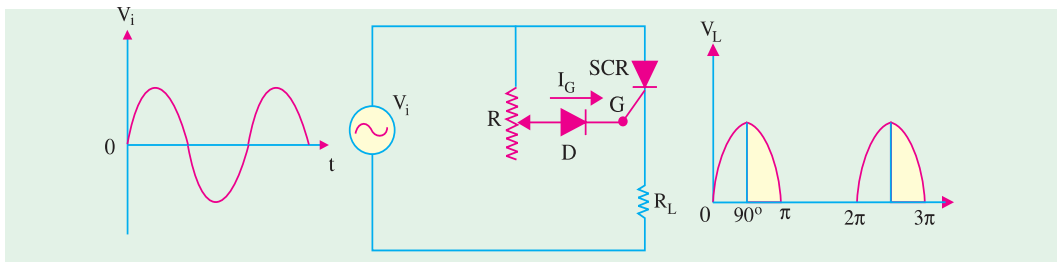


Fig. 55.42

If  $I_G$  is not large enough even at peak positive half-cycle, then SCR will not trigger at all because  $I_G$  has the greatest value at the peak input and then falls off as the voltage falls. The purpose of diode D is to protect the SCR gate from negative voltage during the negative input half-cycle. This method is also known as the amplitude firing of an SCR.

**55.37. 180° Phase Control of SCR**

The circuit shown in Fig. 55.43 can trigger the SCR from 0° to 180° of the input waveform.

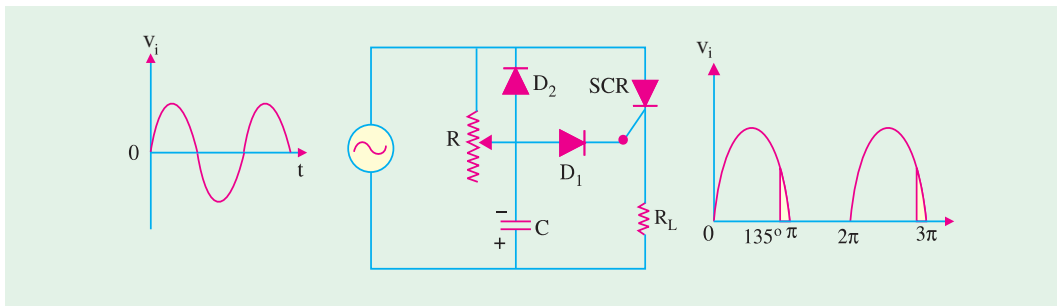


Fig. 55.43

The circuit is identical to that of Fig. 55.42 except for the addition of the diode  $D_2$  and capacitor C.



We will start the analysis with the negative half-cycle. During the negative half-cycle of the input,  $C$  is charged immediately (with the polarity as shown) to the peak of the input voltage because  $D_2$  is forward-biased. When peak of the negative half-cycle passes over,  $D_2$  becomes reverse-biased because its anode (connected to  $C$ ) becomes more negative than its cathode (connected to the supply). Hence,  $C$  starts to discharge through  $R$ . Depending on the time constant ( $= CR$ ),  $C$  may almost be completely discharged at the commencement of the oncoming positive half-cycle or may retain partial charge until almost  $180^\circ$  of positive half-cycle has passed. So long as  $C$  remains negatively-charged,  $D_1$  is reverse-biased and the gate cannot become positive to trigger the  $SCR$  into conduction. Hence,  $R$  and/or  $C$  can be adjusted to trigger the  $SCR$  anywhere from  $0^\circ$  to  $180^\circ$  of the input ac cycle.

**55.38. SCR Controlled Load Circuit**

We will now consider a circuit where an amplitude-fired  $SCR$  is used to control the power in a full-wave rectifier circuit.

In Fig. 55.44,  $R_L$  is the load resistance which may be a lamp load, heater or a small dc motor whose power we wish to control. The full-wave bridge rectifier furnishes the rectified output shown in Fig 55.45 (a). Obviously, control over both half-cycles of the ac input is possible.

The rectifier output voltage is dropped across the potentiometer  $AA$ . When potentiometer is set at point 1, drop across  $R_3$  is not enough to provide sufficient gate current to switch ON the  $SCR$ . Hence, there is no load current and, consequently, no drop across the load. When potentiometer is set at point 2 and then 3, the  $SCR$  gets fired yielding load voltage waveforms shown in Fig. 55.45 (b) and (c). Hence, changing the conduction angle and hence the amount of power delivered to the load.

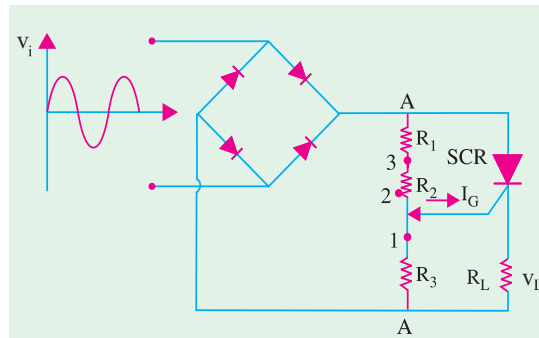


Fig. 55.44

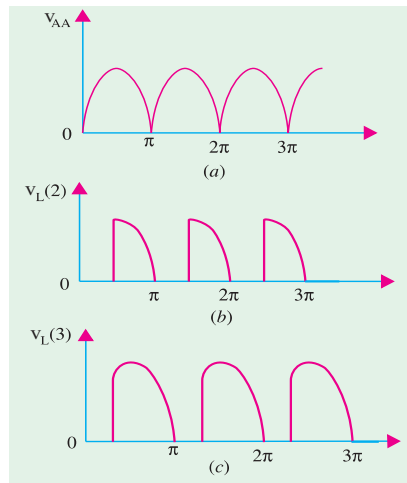


Fig. 55.45

**55.39. UJT Controlled Load Circuit**

This circuit employs a  $UJT$  oscillator to control the firing of the  $SCR$  as shown in Fig. 55.46. The full-wave rectified output voltage  $V_{AF}$  is available across two parallel paths  $AF$  and  $BF$ . When  $SCR$  is not conducting (*i.e.* it acts as an open), then whole output voltage drops across it and none across  $R_L$ . When it conducts (*i.e.* acts as a short), whole of  $V_{AF}$  drops across  $R_L$  and none across it.

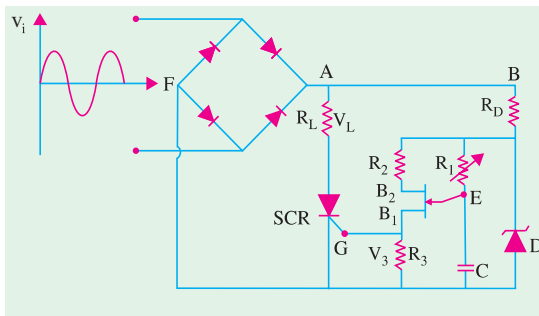


Fig. 55.46

Now, consider the other parallel circuit  $BF$ .  $R_D$  is the voltage dropping resistor so that a suitable voltage  $V_D$  is applied to the  $UJT$  oscillator circuit for the  $UJT$  to work without damage. The full-wave rectified voltage is clipped at a convenient level by Zener diode  $D$  for proper operation of  $UJT$ . Varying  $R_1$  varies the time it takes for  $C$  to charge to the  $UJT$ 's firing voltage. Charging of  $C$  always starts when  $V_U$  is at 0 V dc or at  $0^\circ$  of  $V_{AF}$ .

If  $R_1$  is very small,  $C$  charges very quickly ( $\because$  time constant  $CR_1$  is very short) and reaches the



*UJT* firing voltage soon after  $V_{AF}$  starts to go positive. Hence, *UJT* fires early on every cycle of  $V_{AF}$ . When *UJT* fires, it becomes almost a short between its emitter ( $E$ ) and  $B_1$ . Thus, it discharges  $C$  very quickly through  $R_3$  which is in parallel with the *SCR*'s gate. This triggers the *SCR* early in the cycle of  $V_{AF}$  and the *SCR* latches itself ON until  $V_{AF}$  falls to zero. When  $V_{AF}$  goes to zero, the *SCR* is shut OFF. The cycle repeats itself with the *SCR* being turned ON early in the cycle thereby delivering nearly full-power to the load.

When  $R_1$  is increased,  $V_C$  rises slowly and reaches the *UJT* firing voltage later during each cycle of  $V_{AF}$ . Therefore, *SCR* is turned ON much later thereby considerably cutting down the power delivered to the load as shown in Fig. 55.47. Obviously, the *SCR* is being fired during the last few degrees of the cycle.

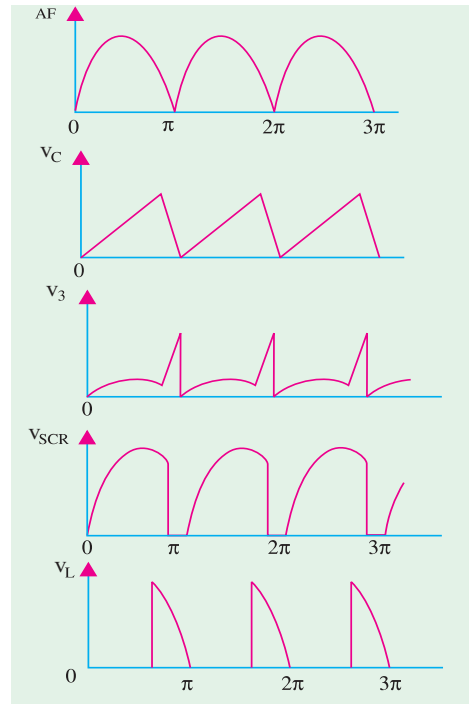


Fig. 55.47

### 55.40 Chopper

#### (a) Definition

To put it simply, a chopper is a **dc-to-dc converter**. It converts a given constant dc voltage into a variable average dc voltage across a load by placing a high-speed static switch between the dc source and the load. This high-speed static switch is called a chopper because it chops off the dc supply into ON and OFF periods of flow.

#### (b) Basic Circuit

A basic chopper circuit is shown in Fig. 55.48. When switch  $S$  is closed, the dc supply voltage  $V_{dc}$  is applied across the load and when it is open, the load is disconnected from the supply. By varying the ratio of the switch-closed time ( $T_{ON}$ ) to the switch-open time ( $T_{OFF}$ ) at a fixed frequency, the value of the average output dc voltage can be controlled.

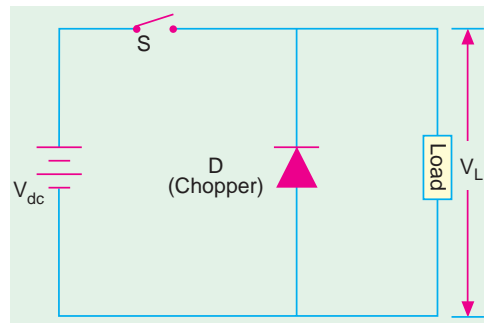


Fig. 55.48

The switch  $S$  in Fig. 55.48 could be either a transistor or an *SCR* depending on the amount of

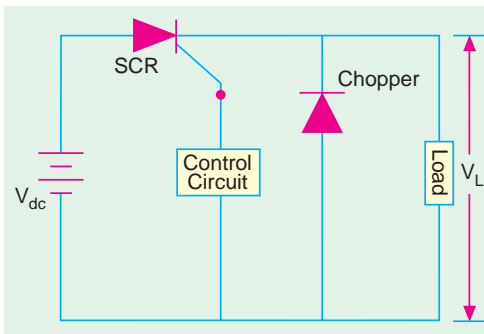


Fig. 55.49

power involved. An *SCR* is used in high-power applications whereas transistors are used when power involved is low. A dc chopper circuit using an *SCR* is shown in Fig. 55.49. The *SCR* acts like a static switch and has two states of ON and OFF. The duration of ON and OFF states can be varied with the help of triggering and commutating circuits (not shown) respectively. By changing the values of ON and OFF periods, average dc load voltage can be changed.

#### (c) Working

When *SCR* is triggered into conduction with the help of control circuitry, full dc voltage  $V_{dc}$  is applied across



the load for a period of  $T_{ON}$ . When the SCR is switched OFF by the control circuitry, there is no voltage across the load. The output load voltage is in the form of a square wave as shown in Fig. 55.50. The waveshapes of the load current and chopper current for an inductive load are as shown. The freewheeling diode  $D$  provides path for the stored inductive energy to flow.

**(d) Calculations**

If  $T_{ON}$  is the ON time and  $T_{OFF}$  is the OFF time of the chopper, the duty cycle of the chopper is given by

$$\text{duty cycle} = \frac{T_{ON}}{T} = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

The load voltage  $V_L$  is given by

$$V_L = V_{dc} \left( \frac{T_{ON}}{T} \right) = V_{dc} \times \text{duty cycle} = f V_{dc} T_{ON}$$

where  $f (= 1/T)$  is the switching frequency of the chopper.

It is seen from the above that  $V_L$  depends on the duty cycle since  $V_{dc}$  is constant. Hence, following different methods of controlling  $V_L$  are available.

1. keeping  $T_{OFF}$  constant, varying  $T_{ON}$  ;
2. keeping  $T_{ON}$  constant, varying  $T_{OFF}$  ;
3. varying the ratio  $T_{ON}/T_{OFF}$  ;
4. any combination of the above.

**Example 55.14.** A dc chopper has ON time of  $30 \mu\text{s}$  and OFF time of  $10 \mu\text{s}$ . Calculate (i) chopper duty cycle, (ii) chopping frequency.

**Solution.** (i) duty cycle =  $T_{ON}/T = 30/(30 + 10) = 0.75$   
 (ii) chopping frequency,  $f = 1/T = 1/(30 + 10) \times 10^{-6}$   
 $= 25,000 \text{ Hz} = 25 \text{ kHz}$

**Example 55.15.** A chopper supplied by a 200 V dc has ON time of 30 ms and OFF time of 10 ms. Determine the value of the average dc output voltage.

(Industrial Electronics, Mysore Univ. 1993)

**Solution.**  $T_{ON} = 30 \times 10^{-3} \text{ s}$ ,  $T_{OFF} = 10 \times 10^{-3} \text{ s}$ ,  
 $T = 40 \times 10^{-3} \text{ s}$   
 Duty cycle of the chopper =  $30 \times 10^{-3} / 40 \times 10^{-3} = 0.75$   
 $V_L = V_{dc} \times \text{duty cycle} = 200 \times 0.75 = 150 \text{ V}$

**55.41. Inverters**

An inverter is a device that **changes dc power into ac power** (just the opposite of converters). The inversion process can be achieved with the help of transistors, SCRs and tunnel diodes etc. For low and medium outputs, transistorised inverters are suitable but for high power outputs, SCR inverters are essential. For very low voltage and high current requirements, tunnel diode inverters are used.

For inverter applications, transistors have definite advantages over SCRs regarding the switching speed, simplicity of control circuitry, high efficiency and greater reliability. It is mainly due to this fact the SCR inverters require

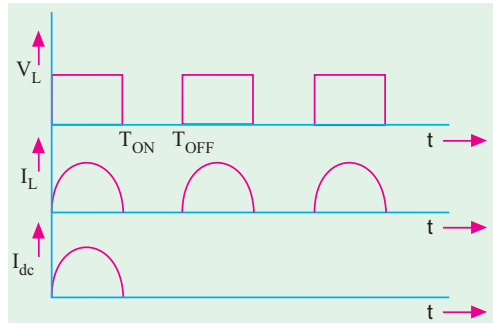


Fig. 55.50



Power Inverter



complicated circuitry for triggering and commutation.

The basic working principle of an inverter may be explained with the help of circuit shown in Fig. 55.51. It is called voltage-driven inverter because a dc voltage source is connected through semiconductor switches directly to the primary of a transformer.

In Fig. 55.51,  $S_1$  and  $S_2$  are switching devices (transistors or SCRs) which open and close alternately at regular intervals of time. The two switching devices are generally driven by an astable multi-vibrator operating at the desired frequency.

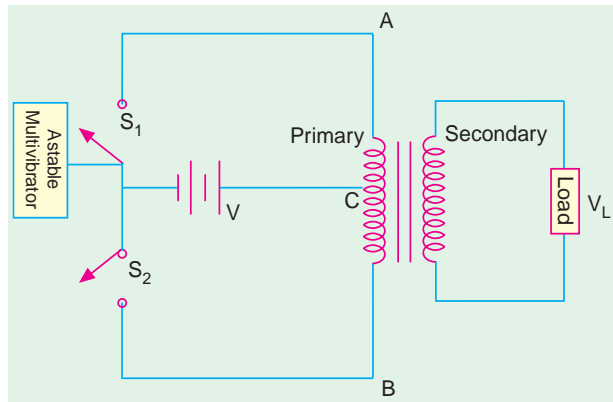


Fig. 55.51

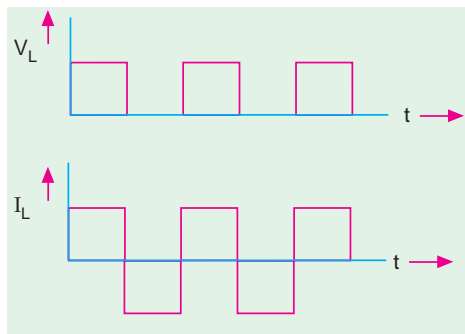


Fig. 55.52

When  $S_1$  is closed, the entire dc source voltage  $V$  is applied across points  $A$  and  $B$  of the transformer primary.  $S_1$  remains closed for a certain period of time after which it is cut off and  $S_2$  closes. It also remains closed for the same period of time during which the source voltage  $V$  is impressed across points  $B$  and  $C$  of the primary.  $S_2$  then opens out and  $S_1$  closes. In this way, an alternating voltage is applied across the primary which induces an ac voltage in the secondary. Since dc supply voltage is connected directly across the primary, the output waveform of the secondary voltage is a square wave (Fig. 55.52) irrespective of the type of load and load power factor. However, the waveforms of both the primary and secondary currents depend on the type of load whether resistive, inductive or capacitive.

### 55.42. Single-phase Inverter

Fig. 55.53 (a) shows a single-phase inverter with a load resistor using 4 SCRs working in pairs. The triggering and commutating circuitry of the SCRs has not been shown in the figure. The two thyristors  $SCR_1$  and  $SCR_4$  are triggered simultaneously so that load current passes through  $R_L$  from left to right. Exactly when these two SCRs are switched off by the commutating circuitry, thyristors  $SCR_2$  and  $SCR_3$  are triggered into conduction

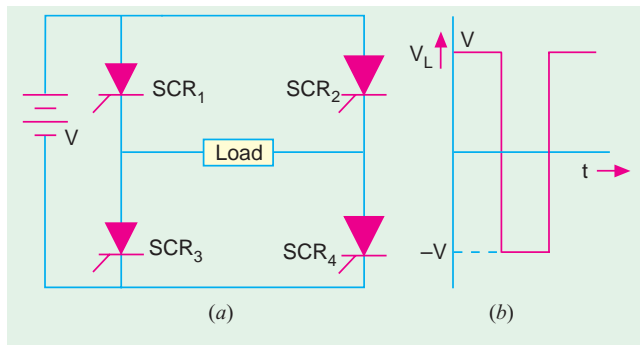


Fig. 55.53



Single-phase inverter

thereby sending current through  $R_L$  from right to left. Hence, an ac voltage is developed across the load whose waveform is as shown in Fig. 55.53 (b).

### 55.43. Push pull Inverter

Fig. 55.54 shows an inverter which





employs two *SCRs* and one transformer. These two *SCRs* are triggered into conduction alternately for the same period of time. As a result, current through the primary becomes alternating which induces an ac voltage across the secondary and hence the load. As explained earlier in Art 55.42, the secondary ac voltage has a square waveform. The capacitor  $C$  is connected across the anodes of the two *SCRs* and provides commutation *i.e.* switching off of the *SCRs*. The capacitor charges to double the supply voltage as a result of transformer action between the two halves of the primary winding. This large voltage is sufficient to reverse-bias the *SCRs* and drive the holding current below its rated value.

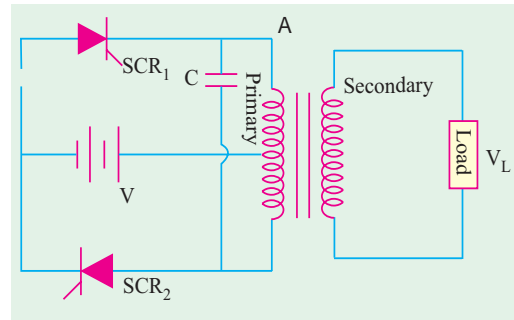


Fig. 55.54

### Tutorial Problems No. 55.1

- A 1- $\phi$  half-wave rectifier using a 10 : 1 transformer supplies power to a  $9\ \Omega$  load. If the primary input voltage has an r.m.s. value of 200 V and forward diode resistance is  $0.2\ \Omega$  and transformer secondary resistance is  $0.8\ \Omega$ , determine  
(i)  $I_L(dc)$ , (ii) r.m.s. ripple voltage and (iii) efficiency [ (i) 0.9 A (ii) 9.8 V (iii) 36.54% ]
- A single-phase full-wave rectifier using a power transformer has secondary voltage of 100-0-100V (r.m.s.). It supplies a load of 1 K. Neglecting transformer losses and forward voltage drop of the diode, determine :  
(a) dc output voltage, (b) dc output current, (c) ripple voltage and PIV rating of the diodes. [ (a) 90 V (b) 90 mA (c) 43.4 V (rms.) ; 282 V ]
- A single-phase half-wave rectifier supplies power to a 1 K load. The rectifier voltage is 200 V (r.m.s.). Neglecting diode resistance, calculate (i) dc load voltage, (ii) dc load current and (iii) r.m.s. ripple voltage. [ (i) 90 V (ii) 90 mA (iii) 1.09 V ]
- A single-phase half-wave diode rectifier supplies power to a 2-k $\Omega$  resistive load. The input ac supply voltage has a peak value of 300 V. Neglecting forward drop of the diode, calculate  
(a)  $V_{dc}$ , (b)  $I_{dc}$ , (c) power delivered to the load, (d) ripple voltage (rms value) [ (a) 95.4 V (b) 47.7 mA (c) 4550 mW (d) 115.4 V ]
- A full-wave diode rectifier supplies a load of 10 k $\Omega$ . The ac voltage applied to the diode is 300-0-300 V rms. Its diode resistance is neglected, calculate: (a)  $V_{dc}$ , (b)  $I_{dc}$ , (c)  $I_{rms}$ , (d) form factor, (e) ripple voltage. [ (a) 270 V (b) 27 mA (c) 30 mA (d) 1.11 (e) 130.1 V ]
- A dc and an ac voltmeter are used to measure the output voltage of a filter circuit. The readings of the two voltmeters are 50 V and 5 V respectively. Calculate the ripple factor of the filter. [ 10% ]
- In a controlled full-wave rectifier, peak supply voltage is 200 V and load resistance 1 kW. Calculate the power delivered to the load for firing angles of (a)  $60^\circ$  and (b)  $120^\circ$ . [ (a) 9.1 W (b) 1.01 W ]

### OBJECTIVE TESTS – 55

- The ripple factor of a power supply is a measure of  
(a) its filter efficiency  
(b) its voltage regulation  
(c) diode rating  
(d) purity of power output.
- The basic reason why a FW rectifier has twice the efficiency of a HW rectifier is that  
(a) it makes use of a transformer  
(b) its ripple factor is much less  
(c) it utilizes both half-cycle of the input  
(d) its output frequency is double the line frequency.
- The output of a half-wave rectifier is suitable only for  
(a) running car radios  
(b) running ac motors  
(c) charging batteries  
(d) running tape-recorders.
- The ripple factor of a bridge rectifier is  
(a) 0.406 (b) 0.812  
(c) 1.21 (d) 1.11



5. The ripple factor of a power supply is given by (symbols have the usual meaning).
- (a)  $\frac{P_{dc}}{P_{ac}}$  (b)  $\sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2} - 1$   
 (c)  $\sqrt{\left(\frac{I_{dc}}{I_{rms}}\right)^2} - 1$  (d)  $\frac{I_{dc}}{I_{rms}}$
6. The PIV of a half-wave rectifier circuit with a shunt capacitor filter is  
 (a)  $2 V_{sm}$  (b)  $V_{sm}$   
 (c)  $V_{sm}/2$  (d)  $3 V_{sm}$ .
7. The primary function of a rectifier filter is to  
 (a) minimise ac input variations  
 (b) suppress odd harmonics in the rectifier output  
 (c) stabilise dc level of the output voltage  
 (d) remove ripples from the rectified output
8. In a rectifier, larger the value of shunt capacitor filter  
 (a) larger the  $p-p$  value of ripple voltage  
 (b) larger the peak current in the rectifying diode  
 (c) longer the time that current pulse flows through the diode  
 (d) smaller the dc voltage across the load.
9. In a LC filter, the ripple factor,  
 (a) increases with the load current  
 (b) increases with the load resistance  
 (c) remains constant with the load current  
 (d) has the lowest value.
10. The main reason why a bleeder resistor is used in a dc power supply is that it  
 (a) keeps the supply ON  
 (b) improves voltage regulation  
 (c) improves filtering action  
 (d) both (b) and (c).
11. Which stage of a dc power supply uses a Zener as the main component?  
 (a) rectifier (b) voltage divider  
 (c) regulator (d) filter.
12. Which rectifier requires four diodes ?  
 (a) half-wave voltage doubler  
 (b) full-wave voltage doubler  
 (c) full-wave bridge circuit  
 (d) voltage quadrupler.
13. For a half-wave controlled rectifier, the average value of output dc voltage is given by

- (a)  $V_{dc} = \frac{V_m}{2\pi} (1 - \cos \theta_1)$   
 (b)  $V_{dc} = \frac{2V_m}{\pi} (1 + \cos \theta_1)$   
 (c)  $V_{dc} = \frac{V_m}{\pi} (\cos \theta_1 - 1)$   
 (d)  $V_{dc} = \frac{V_m}{2\pi} (\cos \theta_1 + 1)$ .

14. If, by mistake, ac source in a bridge rectifier is connected across the dc terminals, it will burn out and hence short ..... diodes.  
 (a) one (b) two  
 (c) three (d) four.
15. The circuit in Fig. 55.55 shows a full-wave rectifier. The input voltage is (rms) single-phase ac. The peak reverse voltage across the diodes D1 and D2 .

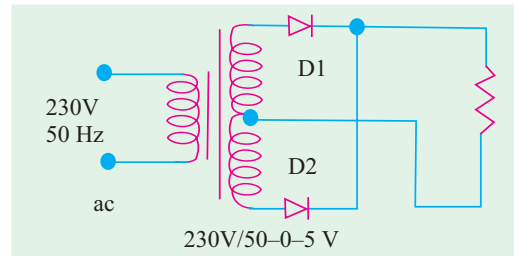


Fig. 55.55

- (a)  $100\sqrt{2}$  V (b) 100 V  
 (c)  $50\sqrt{2}$  V (d) 50V

(GATE ; 2004)

16. The circuit in Fig. 55.56 shows a 3-phase half-wave rectifier. The source is a symmetrical, 3-phase four-wire system. The line-to-line voltage of the source is 100 V. The supply frequency is 400 Hz. The ripple frequency at the output is

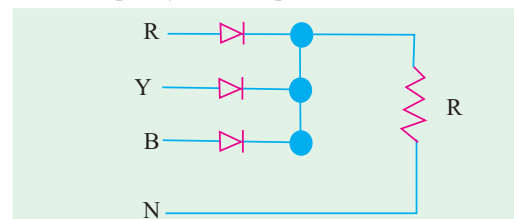


Fig. 55.56

- (a) 400 Hz (b) 800 Hz  
 (c) 1200 Hz (d) 2400 Hz

(GATE ; 2004)

ANSWERS

1. (a) 2. (c) 3. (b) 4. (c) 5. (b) 6. (a) 7. (d) 8. (b) 9. (c) 10. (d) 11. (c)  
 12. (b) 13. (c) 14. (d) 15. (a) 16. (c)

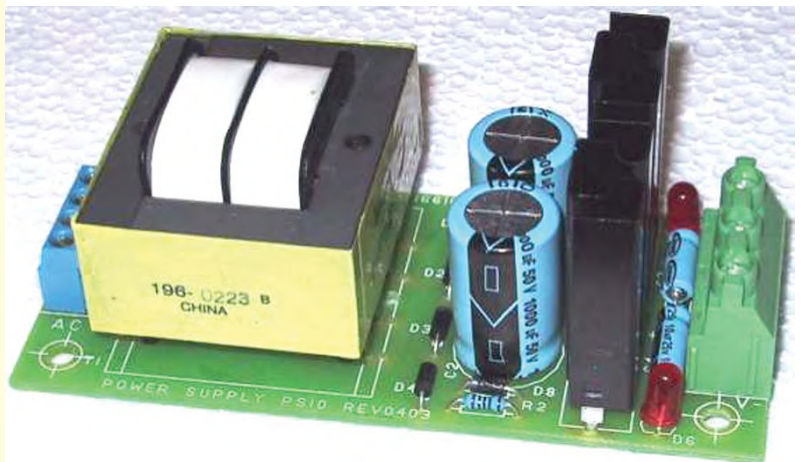


# CHAPTER 56

## Learning Objectives

- General
- Voltage Regulation
- Zener Diode Shunt Regulator
- Transistor Series Voltage Regulator
- Controlled Transistor Series Regulator
- Transistor Shunt Voltage Regulator
- Transistor Current Regulator
- Variable Feedback Regulator
- Basic OP-AMP Series Regulator
- Basic OP-AMP Shunt Regulator
- Switching Regulators
- Step-down Switching Regulator
- Step-up Switching Regulator
- Inverting Switching Regulator
- IC Voltage Regulators
- Fixed Positive Linear Voltage Regulators
- Fixed Negative Linear Voltage Regulators
- Adjustable Positive Output Linear Voltage Regulators
- Adjustable Negative Output Linear Voltage Regulators
- Use of External Pass Transistor with Linear Voltage Regulators
- Use of Linear Voltage Regulator as a Current Regulator
- Switching Voltage IC Regulators

## REGULATED POWER SUPPLY



➤ Voltage regulator provides a constant dc output voltage that is essentially independent of the input voltage, output load current and temperature.

56.1. General

The various power-supply circuits considered in Chapter 5 suffer from the drawback that their dc output voltage changes with changes in load or input voltage. Such a dc power supply is called unregulated power supply. Regulated power supply can be obtained by using a voltage regulator circuit. A regulator is an electronic control circuit which is capable of providing a nearly constant dc output voltage even when there are variations in load or input voltage. A source of regulated dc power is essential for all communication, instrumentation, computers or any other electronic system.

We will consider both *linear* regulators and *switching* regulators which are also available in integrated circuit form. In linear regulators, the transistor operates somewhere between saturation and cut-off. It is always ON and dissipates power. Hence, its efficiency (output power/input power) is 50 per cent or less. In switching regulators, the transistor operates like a switch *i.e.* it is either saturated or cut-off. Hence, its power efficiency is 90 per cent or more.

The linear regulators are of two basic type *i.e.* series regulators and shunt regulators. Likewise switching regulators can be of three basic types (i) step-down type, (ii) step-up type and (iii) inverting type.

56.2. Voltage Regulation

As stated above, in an unregulated power supply, output voltage changes whenever input supply voltage or load resistance changes. It is never constant. The change in voltage from no-load to full-load condition is called *voltage regulation*. The aim of a voltage regulator circuit is to reduce these variations to zero or, at least, to the minimum possible value.

The percentage regulation or, simply, regulation of a power supply is given by

$$\% \text{ regulation} = \frac{V_{max} - V_{min}}{V_{max}} \times 100$$

where  $V_{max}$  = maximum dc output voltage and  $V_{min}$  = minimum dc output voltage.

When we say that 10 V regulated dc power supply has a regulation of 0.005 per cent, it means that dc output voltage will vary within an envelope 0.005 per cent of 10 V.

$$\begin{aligned} \text{Now, } 0.005\% \text{ of } 10 \text{ V} &= \frac{V_{max} - V_{min}}{V_{max}} \times 100 \\ &= 0.0005 \text{ V} = 0.5 \text{ mV} \end{aligned}$$

Hence, output voltage will vary by  $\pm 0.25 \text{ mV}$ . So, we see that instead of expressing voltage regulation by unwieldy expression 0.005 per cent, we can express it by a simple figure of  $\pm 0.25 \text{ mV}$ .

$$\text{In general, } \% \text{ regn.} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

where,  $V_{NL}$  = no-load or open-circuit terminal voltage of the supply (Fig. 56.1).  
 $V_{FL}$  = full-load terminal voltage of the supply

In an ideal or perfectly regulated dc power supply, the percentage voltage regulation is zero. This voltage regulation is also called *load regulation*.

56.3. Zener Diode Shunt Regulator

A simple shunt voltage regulating system using a zener diode is shown in Fig. 56.2. The

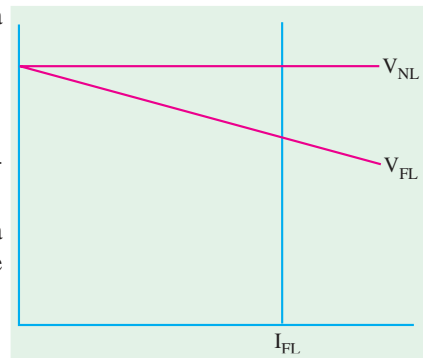


Fig. 56.1



input voltage  $V_{in}$ , in fact, is the unregulated output of a rectifier. This simple regulator restricts output voltage variations within reasonable limits around  $V_z$  in the face of changing load current or changing input voltage. Obviously, the Zener diode will regulate so long as it is kept in reverse conduction.

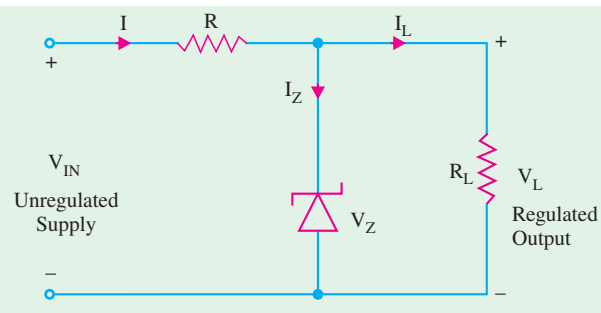


Fig. 56.2

**Example 56.1.** The Zener diode of Fig. 56.2 has the following ratings :

$$\begin{array}{ll}
 V_z = 6.8 \text{ V} & \text{at } I_z = 50 \text{ mA} \\
 r_z = 2 \Omega & \text{at } I_z = 50 \text{ mA} \\
 I_{z(\min)} = 5 \text{ mA} & I_{z(\max)} = 150 \text{ mA}
 \end{array}$$

What would be the load voltage when load current  $I_L$  varies from 10 mA to 120 mA ? Also, calculate voltage regulation of the regulator. (Power Electronics-I, Punjab Univ. 1992)

**Solution.** We will call  $V_z = 6.8 \text{ V}$  and  $I_z = 50 \text{ mA}$  as reference values and calculate changes in voltage with respect to these values.

(i)  $I_L = 120 \text{ mA}$

$$\begin{aligned}
 \text{Obviously,} \quad I_z &= 150 - 120 = 30 \text{ mA} \\
 \text{Deviation from 50 mA} &= 30 - 50 = -20 \text{ mA} \\
 \text{Drops across diode} &= I_z r_z = -20 \times 2 = -40 \text{ mV} \\
 V_L &= V_z + I_z r_z = 6.8 + (-40 \times 10^{-3}) = \mathbf{6.76 \text{ V}}
 \end{aligned}$$

(ii)  $I_L = 10 \text{ mA}$

$$\begin{aligned}
 \text{Now,} \quad I_z &= 150 - 10 = 140 \text{ mA} \\
 \text{Deviation from reference value} &= 140 - 50 = 90 \text{ mA} \\
 \text{Diode drop} &= I_z r_z = 90 \times 10^{-3} \times 2 = 0.18 \text{ V} \\
 \therefore V_L &= V_z + I_z r_z = 6.8 + 0.18 = \mathbf{6.98 \text{ V}}
 \end{aligned}$$

$$\% \text{ regn.} = \frac{6.98 - 6.76}{6.98} \times 100 = 3.15 \%$$

For many applications, a change in load voltage of 3.15% is acceptable but, in some, it may be intolerable. This regulation can be reduced to 1% or less with the help of circuits discussed below.

### 56.4. Transistor Series Voltage Regulator

The circuit is shown in Fig. 56.3. It is also called *emitter-follower* regulator because the voltage at the emitter follows the base voltage. In this set-up, the transistor behaves like a variable resistor whose resistance is determined by the base current. It is called *pass* transistor because total current to be regulated passes through it.

Keeping in mind the polarities of different voltages, they are related by the equation derived from *KVL*

$$V_L + V_{BE} - V_z = 0$$

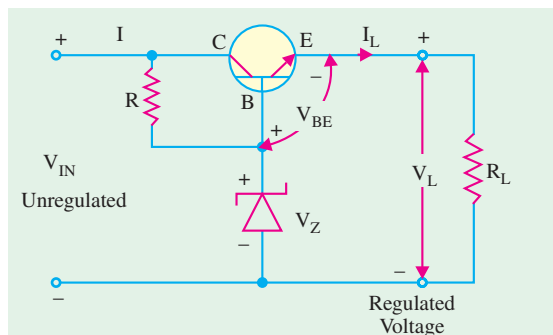


Fig. 56.3



$$\therefore V_{BE} = V_z - V_L \text{ (fixed)}$$

When current demand is increased by decreasing  $R_L$ ,  $V_L$  tends to decrease. As seen from the above equation, it will increase  $V_{BE}$  because  $V_z$  is fixed. This will increase forward bias of the transistor thereby increasing its level of conduction. This, in turn, will lead to decrease in the collector-emitter resistance of the transistor which will slightly increase the input current in order to compensate for decrease in  $R_L$  so that  $V_L = (I_L R_R)$  will remain at a constant value. Incidentally,  $R$  is used for limiting current passing through the Zener diode.

### 56.5. Controlled Transistor Series Regulator

The circuit employing a second transistor  $T_2$  as a sensing element is shown in Fig. 56.4. It has the additional feature of control with the help of potentiometer  $R_1 - R_2$ . In the discussion to follow, it will be assumed that  $I$  is much greater than  $I_{B2}$ . Now, there is a drop of  $V_L$  on  $(R_1 + R_2)$  and a drop of  $(V_z + V_{BE2})$  across  $R_2$ .

$$\frac{V_L}{V_z + V_{BE2}} = \frac{R_1 + R_2}{R_2} \text{ or } V_L = \frac{R_1 + R_2}{R_2} (V_z + V_{BE2})$$

Now,  $(R_1 + R_2)$  and  $(V_z + V_{BE2})$  both have constant values so that  $V_L \propto 1/R_2$ . If the potentiometer is adjusted so that  $R_2$  decreases, then  $V_L$  increases and *vice versa*.

Suppose  $R_L$  is decreased. Then,  $I_L$  increases but  $V_L$  decreases. Decreases in  $V_L$  decreases  $I_{B2}$  and  $I_{C2}$ . Assuming  $I_3$  to be relatively constant (or decreasing only slightly),  $I_{B1}$  is increased thereby decreasing the terminal (collector-emitter) resistance of  $T_1$ . This leads to decrease in  $V_{CE1}$  thereby offsetting the decrease in  $V_L$  which is, therefore, returned to its original value.

In sequential logic, we have

$$V_L \downarrow \quad I_{B2} \downarrow \quad I_{C2} \downarrow \quad I_{B1} \uparrow \quad V_{CE1} \downarrow \quad V_2 \uparrow$$

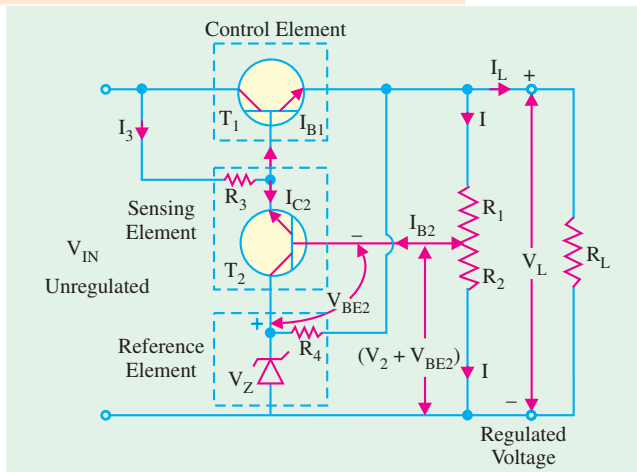


Fig. 56.4

### 56.6. Transistor Shunt Voltage Regulator

It employs the transistor in shunt configuration as shown in Fig. 56.5.

Since path  $AB$  is in parallel across  $V_L$ , we have from Kirchhoff's Voltage Law

$$V_L - V_z - V_{BE} = 0 \text{ or } V_{BE} = V_L - V_z \text{ (fixed)}$$

Since  $V_z$  is fixed, any decrease or increase in  $V_L$  will have a corresponding effect on  $V_{BE}$ . Suppose,  $V_L$  decreases, then as seen from the above relation,  $V_{BE}$  also decreases. As a result,  $I_B$  decreases, hence,

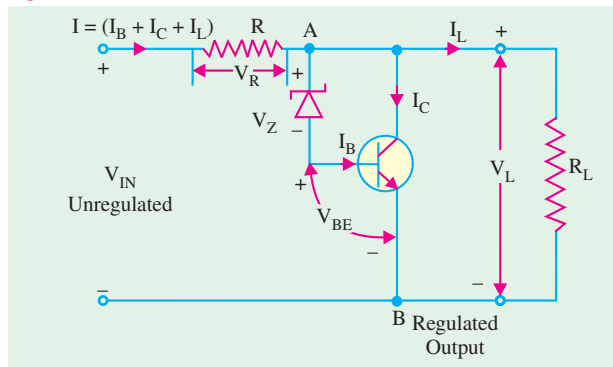


Fig. 56.5



$I_C (= \beta I_B)$  decreases, thereby decreasing  $I$  and hence  $V_R (= IR)$ . Consequently,  $V_L$  increases because at all times

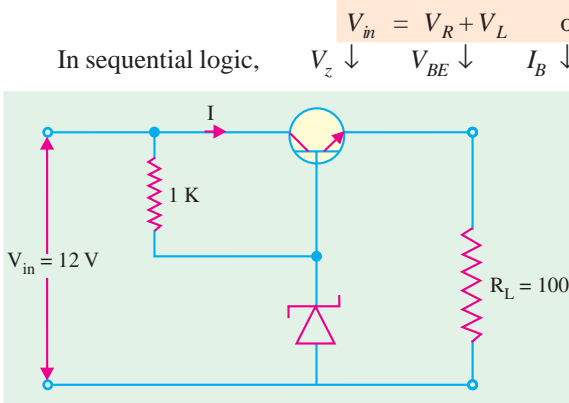


Fig. 56.6

Same line of logic applies in case  $V_L$  tries to increase.

**Example 56.2.** In the NPN emitter-follower regulator circuit of Fig. 56.6, calculate (i)  $V_L$ , (ii)  $V_{CE}$ , (iii)  $I_E$  and (iv) power dissipated. Take  $V_{BE} = 0.7$  V.

**Solution.**

- (i)  $V_L = V_{out} = V_Z - V_{BE} = 9 - 0.7 = 8.3$  V
- (ii)  $V_{CE} = V_{in} - V_{out} = 12 - 8.3 = 3.7$  V
- (iii)  $I_E = I_L = V_L / R_L$

$= 8.3/100 = 83$  mA

- (iv) Power dissipated  $= V_{CE} I_E = 3.7$  V  $\times$  83 mA = 310 mW

**Example 56.3.** Compute the output voltage  $V_{out}$  for the op-amp series regulator shown in Fig. 56.7. (Industrial Electronics, Bombay Univ. 1992)

**Solution.** We are given that  $V_{REF} = 6$  V and  $R_2 = R_3 = 1$  K  
 $V_{out} = V_{REF} (1 + R_z/R_3) = 6(1 + 10/10) = 6 \times 2 = 12$  V

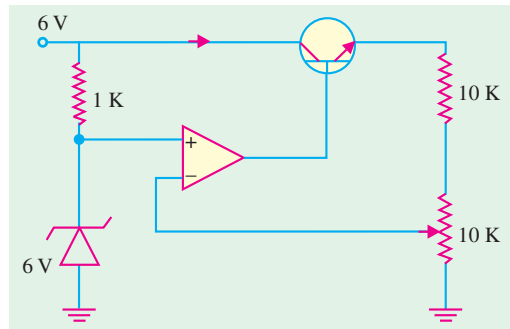


Fig. 56.7

### 56.7. Transistor Current Regulator

The main function of a current regulator is to maintain a fixed current through the load despite variations in the terminal voltage. Such a circuit employing a Zener diode and a PNP transistor is shown in Fig. 56.8. Suppose, due to drop in  $V_L$ , current  $I_L (= I_C)$  is decreased. This will decrease  $I_E (\cong I_C)$ . Hence, drop across  $R_E$  i.e.  $V_{RE}$  will decrease. As per Kirchoff's Voltage Law

$-V_{RE} - V_{BE} + V_z = 0$ , or  $V_{BE} = V_z - V_{RE}$  (fixed)

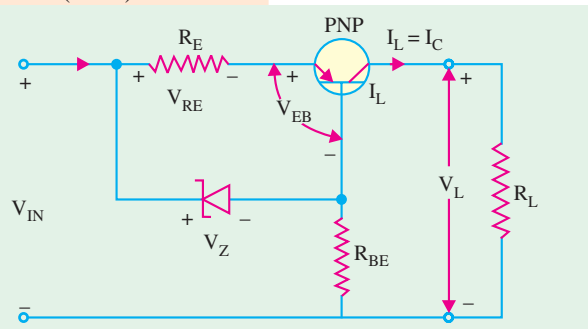


Fig. 56.8



Hence, a decrease in  $V_{RE}$  will increase  $V_{BE}$  and, hence, the conductivity of the transistor thereby keeping  $I_L$  at a fixed level.

A similar logic applies when there is increase in  $V_L$ .

### 56.8. Variable Feedback Regulator

The regulators considered so far provide a non-adjustable output voltage. This would be fine if only single value of regulated voltage is required. Fig. 56.9 shows a feedback regulator which provides different values of regulated dc voltage.

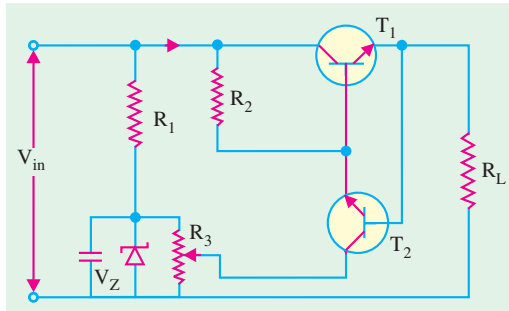


Fig. 56.9

In Fig. 56.9,  $T_1$  is the pass transistor and  $T_2$  is the feedback transistor whose job is to provide and sample output (*i.e.* load) voltage. It offsets any change in the output voltage. Since potentiometer  $R_3$  is connected in parallel with Zener diode  $D$ , it has Zener voltage  $V_z$  applied across it. Voltage across the wiper varies from 0 to  $V_z$ . Capacitor  $C$  ensures that voltage across  $D$  and  $R_3$  does not change suddenly.

Voltage at the base of  $T_2$  is 0.7 V more positive than the voltage at its emitter. Its emitter voltage and hence the base voltage can be changed with the help of  $R_3$ . Since base of  $T_2$  is tied to the output, it is responsible for providing output or load voltage. The voltage  $V_{CE1}$  across the pass transistor is given by the difference of input voltage and output voltage. The current through  $T_1$  is equal to the load current.  $R_2$  prevents saturation of transistors whereas  $R_1$  limits the current flowing through  $D$ .

The working of feedback transistor can be explained as follows :

Since base voltage of  $T_2$  is directly related to  $V_{out}$  it will change if  $V_{out}$  changes. The base and collector of  $T_2$  are 180° out of phase with each other. If base voltage increases due to increase in  $V_{out}$  collector voltage would decrease. Now, collector of  $T_2$  controls base of  $T_1$ . As the base voltage of  $T_1$  decreases, its collector-emitter resistance increases which lowers the load current. This, in turn, lowers the output voltage thereby offsetting the attempted increases in  $V_{out}$ . The opposite of these steps provides the action of an attempted decrease in output voltage.

**Example 56.4.** In the variable feedback regulator circuit of Fig. 56.8,  $V_{in} = 25\text{ V}$ ,  $V_z = 15\text{ V}$  and  $R_L = 1\text{ K}$ . If the wiper of  $R_3$  is adjusted half-way and assuming silicon transistor, compute (i)  $V_{out}$  (ii)  $I_L$  (iii)  $I_{E1}$  (iv)  $P_1$ .

- Solution.** (i)  $V_{out} = \text{voltage at wiper} + V_{BE2} = (15/2) + 0.7 = 8.2\text{ V}$   
 (ii)  $I_L = V_{out}/R_L = 8.2\text{ V}/1\text{ K} = 8.2\text{ mA}$   
 (iii)  $I_{E1} = I_L = 8.2\text{ mA}$   
 (iv)  $V_{CE1} = V_{in} - V_{out} = 25 - 8.2 = 16.8\text{ V}$   
 $\therefore P_1 = 16.8\text{ V} \times 8.2\text{ mA} = 140\text{ mW}$

### 56.9. Basic Op-amp Series Regulator

Its circuit is shown in Fig. 56.10 and its operation is as follows :

The potentiometer  $R_2$ - $R_3$  senses any change in out-

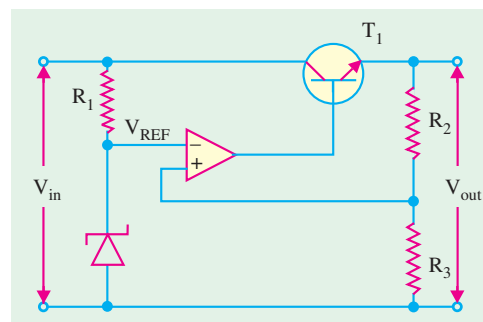


Fig. 56.10





put voltage  $V_{out}$ . When  $V_{out}$  attempts to decrease because of decrease in  $V_{in}$  or because of the increase in  $I_L$ , a proportional voltage decrease is applied to the inverting output of the op-amp by the potentiometer. Since, the other op-amp input is held by the Zener voltage at a fixed reference voltage  $V_{REF}$ , a small difference voltage (called error voltage) is developed across the two inputs of the op-amp. This difference voltage is amplified and op-amp's output voltage increases. This increase in voltage is applied to the base of  $T_1$  causing the emitter voltage ( $= V_{out}$ ) to increase till the voltage to the inverting input again equals the reference (Zener) voltage. This action offsets attempted decrease in the output voltage thus keeping it almost constant. The opposite action occurs if the output voltage tries to increase.

**Calculations**

It will be seen that the op-amp of Fig. 56.10 is actually connected as a non-inverting amplifier where  $V_{REF}$  is the input at the noninverting terminal and the  $R_2/R_3$  voltage divider forms the negative feed-back network. The closed-loop voltage gain is given by  $A = 1 + (R_2/R_3)$ . Neglecting base-emitter voltage of  $T_1$ , we get

$$V_{out} = V_{REF}(1 + R_2/R_3)$$

It is seen that  $V_{out}$  depends on Zener voltage and potential divider resistors  $R_2$  and  $R_3$  but is independent of input voltage  $V_{in}$ .

**56.10. Basic Op-amp Shunt Regulator**

Such a shunt type linear regulator is shown in Fig. 56.11. Here, the control element is a series resistor  $R_1$  and a transistor  $T_1$  in parallel with the load. In such a regulator, regulation is achieved by controlling the current through  $T_1$ .

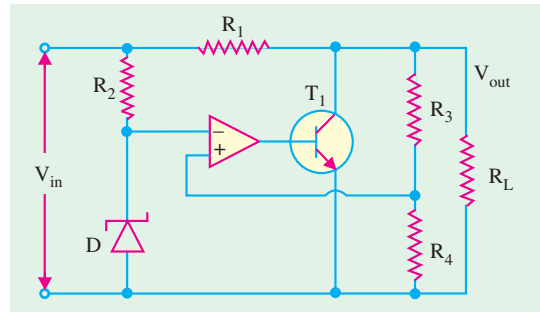


Fig. 56.11

**Working**

When output voltage tries to decrease due to change in either the input voltage or load current or temperature, the attempted decrease is sensed by  $R_3$  and  $R_4$  and applied to the non-inverting input of the op-amp. The resulting difference in voltage reduces the op-amp's output, driving  $T_1$  less thus reducing its collector current (shunt current), and increasing its collector-to-emitter resistance. Since collector-to-emitter resistance acts as a voltage divider with  $R_1$ , this action offsets the attempted decrease in output voltage and hence, maintains it at a constant value. The opposite action occurs when output voltage tries to increase. The shunt regulator is less efficient than the series type but offers inherent short-circuit protection.

**56.11. Switching Regulators**

In the linear regulators considered so far, the control element *i.e.* the transistor conducts all the time, the amount of conduction varying with changes in output voltage or current. Due to continuous power loss, the efficiency of such a regulator is reduced to 50 per cent or less.

A switching regulator is different because its control element operates like a switch *i.e.* either it is saturated (closed) or cut-off (open). Hence, there is no unnecessary wastage of power which results in higher efficiency of 90% or more.

Switching regulators are of three basic types : (i) step-down regulator, (ii) step-up regulator and (iii) inverting regulator.



### 56.12. Step-down Switching Regulator

In this regulator (Fig. 56.12),  $V_{out}$  is always less than  $V_{in}$ . An unregulated positive dc voltage is applied to the collector of the NPN transistor. A series of pulses from an oscillator is sent to the base of transistor  $T$  which gets saturated (closed) on each of the positive pulses. It is so because an NPN transistor needs a positive voltage pulse on its base in order to turn ON. A saturated transistor acts as a closed switch, hence it allows  $V_{in}$  to send current through  $L$  and charge  $C$  to the value of output voltage during the on-time ( $T_{ON}$ ) of the pulse. The diode  $D_1$  is reverse-biased at this point and hence, does not conduct.

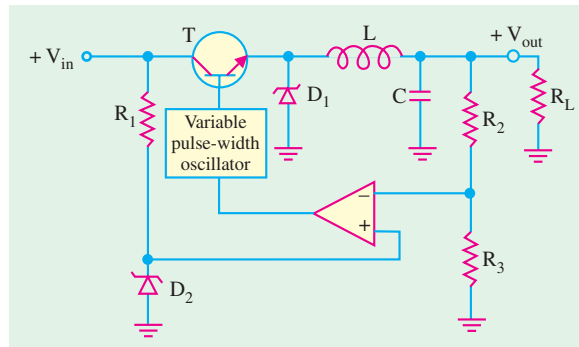


Fig. 56.12

Eventually when positive pulse turns to zero,  $T$  is cut-off and acts like an open switch during the off period ( $T_{OFF}$ ) of the pulse. The collapsing magnetic field of the coil produces self-induced voltage and keeps the current flowing by returning energy to the circuit.

The value of output voltage depends on input voltage and pulse width *i.e.* on-time

of the transistor. When on-time is increased relative to off-time,  $C$  charges more thus increasing  $V_{out}$ . When  $T_{ON}$  is decreased,  $C$  discharges more thus decreasing  $V_{out}$ . By adjusting the duty cycle ( $T_{ON}/T$ ) of the transistor,  $V_{out}$  can be varied.

$$\therefore V_{out} = V_{in} (T_{ON}/T)$$

where  $T$  is the period of the ON-OFF cycle of the transistor and is related to frequency by  $T = 1/f$ . Also,  $T = T_{ON} + T_{OFF}$  and the ratio ( $T_{ON}/T$ ) is called the **duty cycle**.

The regulating action of the circuit is as follows :

When  $V_{out}$  tries to decrease, on-time of the transistor is increased causing an additional charge on the capacitor  $C$  to offset the attempted decrease. When  $V_{out}$  tries to increase,  $T_{ON}$  of the transistor is decreased causing  $C$  to discharge enough to offset the attempted increase.

### 56.13. Step-up Switching Regulator

The circuit is shown in Fig. 56.13. When transistor  $T$  turns ON on the arrival of the positive pulse at its base, voltage across  $L$  increases quickly to  $V_{in} - V_{CE(sat)}$  and magnetic field of  $L$  expands quickly. During on-time of the transistor,  $V_L$  keeps decreasing from its initial maximum value. The longer transistor is ON, the smaller  $V_L$  becomes.

When transistor turns OFF, magnetic field of  $L$  collapses and its polarity reverses so that

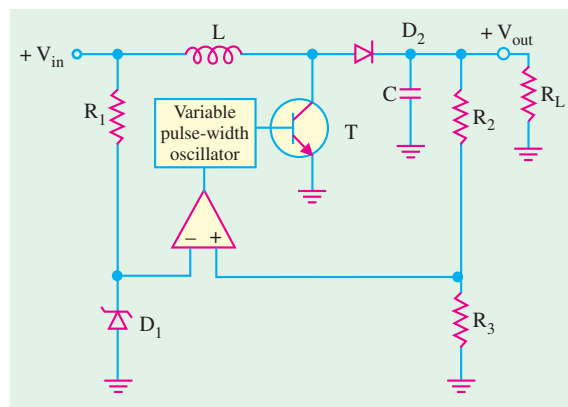


Fig. 56.13



its voltage adds to the input voltage thus producing an output voltage greater than the input voltage. During off-time of the transistor,  $D_2$  is forward-biased and allows  $C$  to charge. The variations in  $V_{out}$  due to charging and discharging action are sufficiently smoothed by filtering action of  $L$  and  $C$ .

It may be noted that shorter the on-time of the transistor, greater the inductor voltage and hence greater the output voltage (because greater  $V_L$  adds to  $V_{in}$ ). On the other hand, the longer the on-time, the smaller the inductor voltage and hence, lesser the output voltage (because smaller  $V_L$  adds to  $V_{in}$ ).

The regulating action can be understood as follows :

When  $V_{out}$  tries to decrease (because of either increasing load or decreasing  $V_{in}$ ), transistor on time decreases thereby offsetting attempted decrease in  $V_{out}$ . When  $V_{out}$  tries to increase, on-time increases and attempted increase in  $V_{out}$  is offset.

As seen, the output voltage is inversely related to the duty cycle.

$$\therefore V_{out} = V_{in} (T/T_{ON})$$

### 56.14. Inverting Switching Regulator

The basic diagram of such a regulator is shown in Fig. 56.14. This regulator provides an output voltage that is opposite in polarity to the input voltage.

When transistor  $T$  turns ON by the positive pulse, the inductor voltage  $V_L$  jumps to  $V_{in} - V_{CE(sat)}$  and the magnetic field of the inductor expands rapidly. When

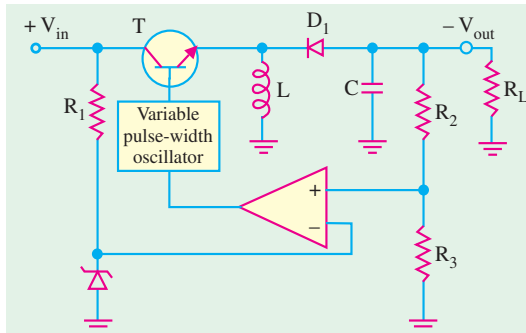


Fig. 56.14

transistor is ON, the diode  $D_2$  is reverse-biased and  $V_L$  decreases from its initial maximum value. When transistor turns OFF, the magnetic field collapses and inductor's polarity reverses. This forward-biases  $D_2$ , charges  $C$  and produces a negative output voltage. This repetitive ON-OFF action of the transistor produces a repetitive charging and discharging that is smoothed by  $LC$  filter action. As in the case of a step-up regulator, lesser the time for which transistor is ON, greater the output voltage and *vice versa*.

### 56.15. IC Voltage Regulators

Due to low-cost fabrication technique, many commercial integrated-circuit ( $IC$ ) regulators are available since the past two decades. These include fairly simple, fixed-voltage types of high-quality precision regulators. These  $IC$  regulators have much improved performance as compared to those made from discrete components. They have a number of unique build-in features such as current limiting, self-protection against overtemperature, remote control operation over a wide range of input voltages and foldback current limiting.

Now we will study the following types of  $IC$  voltage regulators : (1) fixed positive linear voltage regulators, (2) fixed negative linear voltage regulators, (3) adjustable positive linear voltage regulators, and (4) adjustable negative linear voltage regulators.

### 56.16. Fixed Positive Linear Voltage Regulators

There are many  $IC$  regulators available in the market that produces a fixed positive output voltage. But 7800 series of  $IC$  regulators is representative of three terminal devices that are available



with several fixed positive output voltages making them useful in a wide range of applications. Fig. 56.15 (a) shows a standard configuration of a fixed positive voltage IC regulator of 7800 series. Notice that it has three terminals labelled as input, output and ground. The last two digits (marked xx) in the part number designate the output voltage. For example, IC 7805 is a +5 V regulator. Similarly IC 7812 is a +12 V regulator and IC 7815 is a +15 V regulator. The capacitor  $C_1$  (typically  $0.33 \mu\text{F}$ ) is required only if the power supply filter is located more than 3 inches from the IC regulator. The capacitor  $C_2$  (typically  $0.01 \mu\text{F}$ ) acts basically as a line filter to improve transient response.

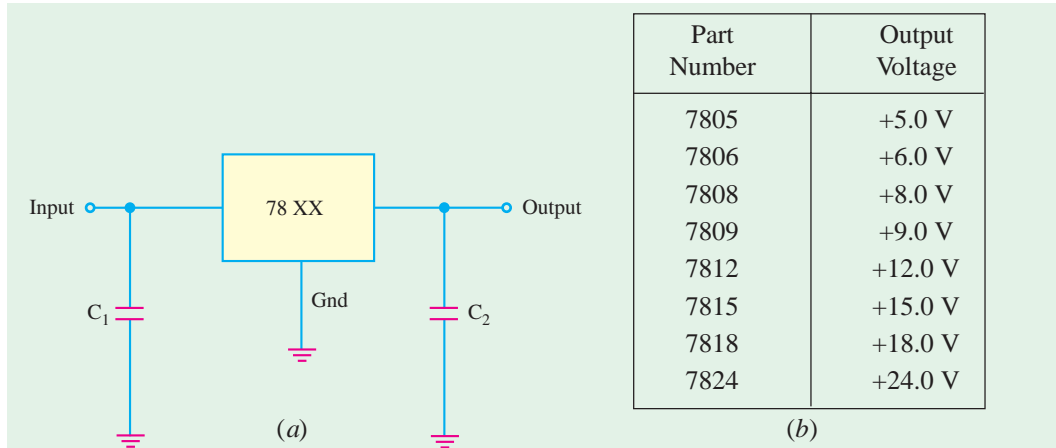


Fig. 56.15

Fig. 56.15 (b) shows the part number and the output voltage of 7800 series IC voltage regulators. As seen from this figure, the 7800 series has IC regulators that can produce output voltages ranging from +5.0 to +24.0 volt. It may be carefully noted that although these regulators are designed primarily to produce fixed output voltage but they can be used with external components to obtain adjustable output voltage and current.

Fig. 56.16 shows the circuit indicating the use of 78XX as an adjustable voltage regulator. The output voltage is given by the equation,

$$V_{out} = V_{fixed} + \left( \frac{V_{fixed}}{R_1} + I_Q \right) R_2$$

For example, for a 7805 IC regulator,  $V_{fixed} = 5 \text{ V}$ . Let  $R_1 = R_2 = 1 \text{ k}\Omega$  and  $I_Q = 5 \text{ mA}$ , then its output voltage is,

$$V_{out} = 5 + \left( \frac{5}{1 \text{ k}\Omega} + 5 \text{ mA} \right) \times 1 \text{ k}\Omega = 15 \text{ V}$$

Thus output voltage of IC 7805 regulator can be adjusted anywhere between 5 V to 15 V.

This example indicates that the output of IC 7805 regulator is adjusted to 15 V using external resistances  $R_1$  and  $R_2$ .

The standard 7800 series can produce output current in excess of 1 A when used with adequate

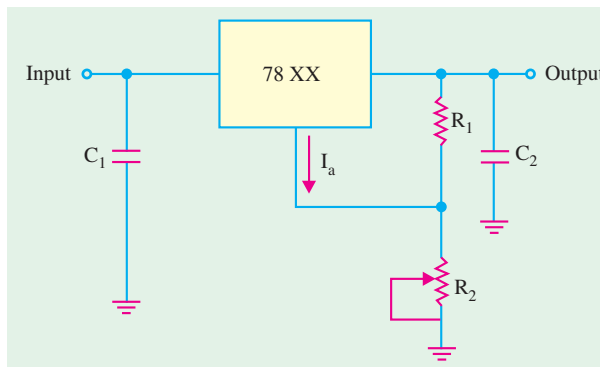


Fig. 56.16



heat sink. It is available in aluminium can package TO-3 (indicated by *K* in the part number) and plastic package TO-220 (indicated by *T* in the part number), The 78L00 series can provide up to 100 mA and is available in TO-92 and metal TO-39 low profile packages. The 78M00 series can provide upto 0.5 A and is available in plastic TO-202 package. Fig. 6.17 shows the typical metal and plastic packages for the *IC* voltage regulators.

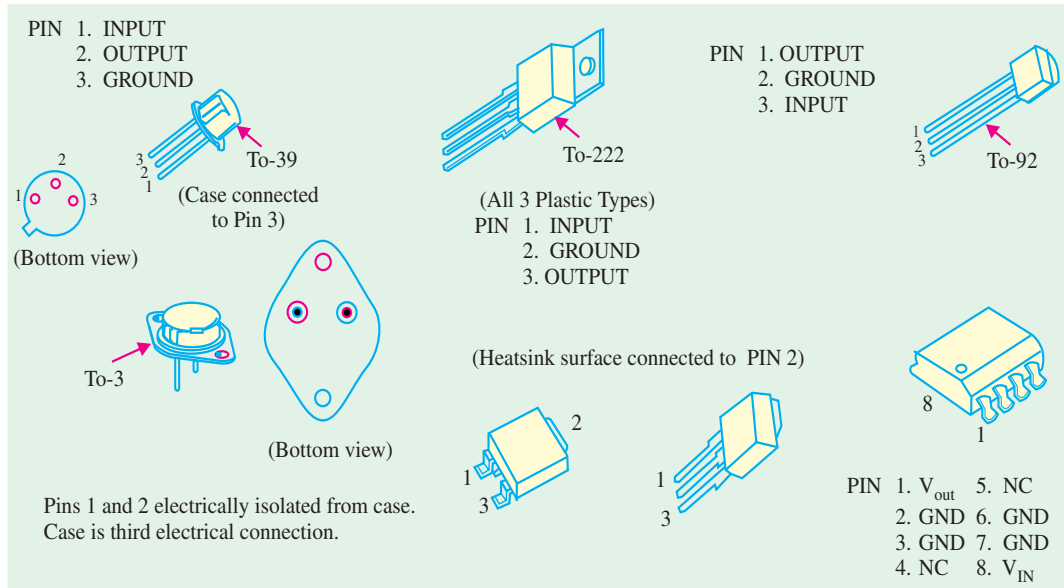


Fig. 56.17

It may be noted that the input voltage for the *IC* regulator must be at least 2 V above the output voltage. This is required in order to maintain regulation. The input voltage should not be more than 35 or 40 volts depending upon the part number. The circuit inside all the *IC* regulators have internal thermal overload protection and short-circuit current-limiting features. Thermal overload in a *IC* regulator occurs whenever the internal power dissipation becomes excessive and the temperature of the device exceeds a certain value.

In India, Bharat Electronic Limited, Bangalore manufactures the *IC* voltage regulators with output voltages of 5 and 12 V. These are available in the market with part numbers BEL 7805 and BEL 7812 respectively.

Fig 56.18 shows a picture of a transformer bridge rectifier and voltage regulator in a dc power supply.

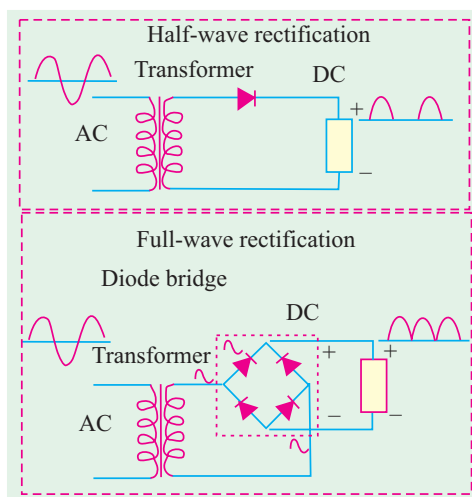


Fig. 56.18

### 56.17. Fixed Negative Linear Voltage Regulators

The 7900 series is typical of three-terminal *IC* regulators that provide a fixed negative output voltage. This series is a negative-voltage counterpart of the 7800 series and shares most of the same features, characteristics and package types. Fig. 56.19 (a) indicates the standard configuration and Fig. 56.19 (b), the part numbers with corresponding output voltages that are available in 7900 series.



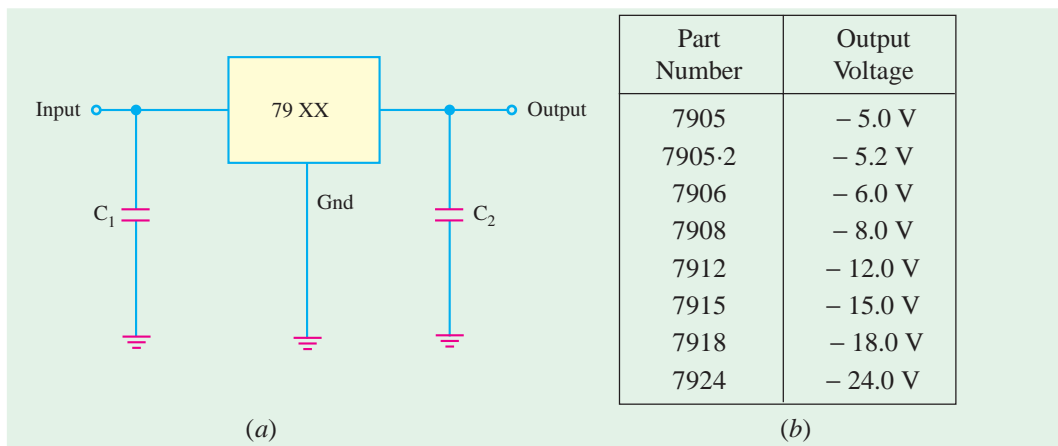


Fig. 56.19

The capacitor  $C_1$  (typically  $0.22 \mu\text{F}$ ) is required only if the power supply filter is located more than 3 inches away from the IC regulator. The capacitor  $C_2$  (typically  $1 \mu\text{F}$ ) is required for stability of the output voltage. Both capacitors  $C_1$  and  $C_2$  must be solid tantalum capacitors.

Fig. 56.20 shows the use of 79XX to produce an adjustable output voltage. The capacitor  $C_3$  (typically  $25 \mu\text{F}$ ) improves the transient response of the output voltage. The output voltage is given by the equation,

$$V_{out} = V_{fixed} \left( \frac{R_1 + R_2}{R_2} \right)$$

The recommended value of  $R_2$ , for 7905 is  $300 \Omega$ , for 7915, its value is  $750 \Omega$  and for 7915 is  $1 \text{ k}\Omega$ .

In India, BEL manufactures IC regulators with output voltage of  $-5 \text{ V}$  and  $-12 \text{ V}$ . These are available in the market with part numbers BEL 7905 and BEL 7915 respectively.

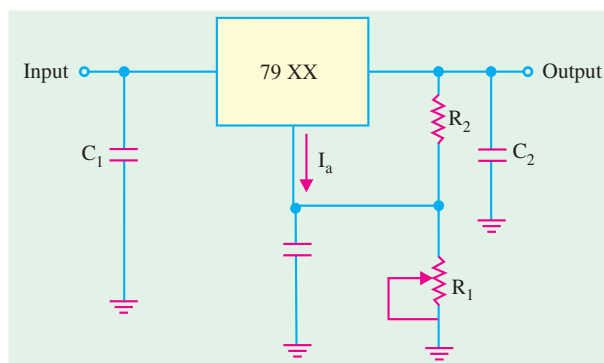


Fig. 56.20

### 56.18. Adjustable Positive Output Linear Voltage Regulators

We have already seen in Art. 56.16 that by adding external resistors, we can adjust the output voltage of 7800 series IC regulators higher than their fixed (or set) voltages. For example, the output voltage of 7805 can be adjusted higher than  $5 \text{ V}$ . But the performance and reliability of 7800 series to produce voltage higher than its fixed value is not considered to be good.

The LM 317 and LM 723 are IC regulators whose output voltage can be adjusted over a wide range. The output voltage of LM 317 can be adjusted from  $1.2 \text{ V}$  to  $37 \text{ V}$ , it can supply output current of  $100 \text{ mA}$  and is available in TO-92 package *i.e.*, it is also a 3 terminal IC regulator. On the other hand, the output voltage of LM 723 can be adjusted from  $2 \text{ V}$  to  $37 \text{ V}$ , it can supply output current of  $150 \text{ mA}$  without external transistor. But with the addition of external transistor, the output current capability can be increased in excess of  $10 \text{ A}$ . The LM 723 is available in dual-in-line package and in a metal can package.

Fig. 56.21 (a) shows the LM 317 connected to the external resistors  $R_1$  and  $R_2$  to produce an adjustable output voltage.



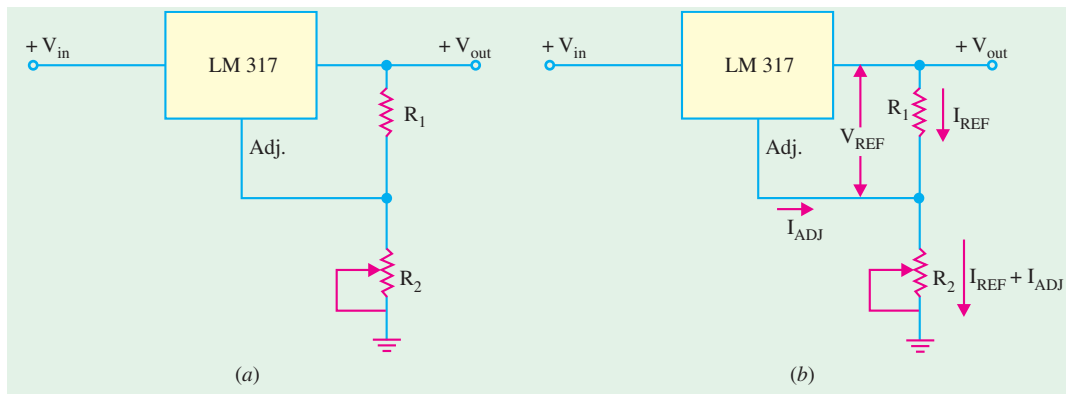


Fig. 56.21

In operation, the LM 317 develops a constant 1.25 V reference voltage ( $V_{REF}$ ) between the output and adjustment terminal. This constant reference voltage produces a constant current, ( $I_{REF}$ ) through  $R_1$ , regardless of the value of  $R_2$  Fig. 56.20 (b). Notice that the value of current through  $R_2$  is the sum of  $I_{REF}$  and  $I_{ADJ}$ , where  $I_{ADJ}$  is a very small current at the adjustment terminal. The value of  $I_{ADJ}$  is typically around 100  $\mu$ A. It can be shown that the output voltage.

$$V_{out} = V_{REF} \left( 1 + \frac{R_2}{R_1} \right) + I_{ADJ} \cdot R_2$$

It is evident from the above equation that the output voltage is a function of  $R_1$  and  $R_2$ . Usually the value of  $R_1$  is recommended to be around 220  $\Omega$ . Once the value of  $R_1$  is set, the output voltage is adjusted by varying  $R_2$ .

**Example 56.5.** Calculate the minimum and maximum output voltages for the IC voltage regulator shown in Fig. 56.22. Assume  $I_{ADJ} = 100 \mu$ A,  $V_{in} = +35$  V.

**Solution.** The equation for output voltage of the IC voltage regulator is given by,

$$V_{out} = V_{REF} \left( 1 + \frac{R_2}{R_1} \right) + I_{ADJ} \cdot R_2$$

When  $R_2$  is set at its minimum value (i.e. 0  $\Omega$ ), the output voltage,

$$\begin{aligned} V_{out(min)} &= 1.25 \left( 1 + \frac{0}{220} \right) + (100 \times 10^{-6}) \times 0 \\ &= \mathbf{1.25 \text{ V}} \end{aligned}$$

When  $R_2$  is set at its maximum values (i.e. 5 k $\Omega$ ), the output voltage,

$$\begin{aligned} V_{out(max)} &= 1.25 \left( 1 + \frac{5000}{220} \right) + (100 \times 10^{-6}) \times 5000 \\ &= 29.66 + 0.5 = \mathbf{30.16 \text{ V}}. \end{aligned}$$

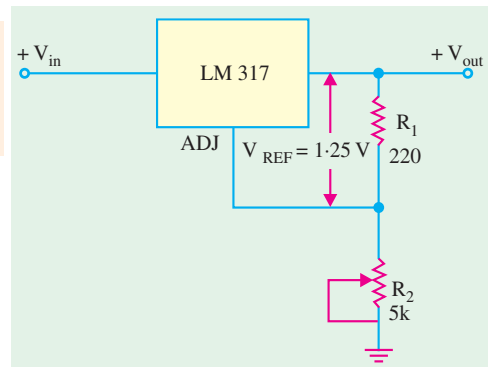


Fig. 56.22

### 56.19. Adjustable Negative Output Linear Voltage Regulators

A good example of this type of regulators is LM 337. The regulator is a negative output counterpart of LM 317. The LM 337 (like LM 317) requires two external resistors for adjustment of output



voltage as shown in Fig. 56.23. The output voltage can be adjusted anywhere from  $-1.2\text{ V}$  to  $-37\text{ V}$  depending upon the external resistor values.

The LM 723 can also be used as an adjustable negative output voltage regulator. The output voltage of this IC regulator can be adjusted anywhere from  $-2.0\text{ V}$  to  $-37\text{ V}$  depending upon the external resistor values.

### 56.20. Use of External Pass Transistor with Linear Voltage Regulators

We have already mentioned in the last two articles that a linear voltage regulator (7800 and 7900 series) is capable of delivering only a certain amount of output current to a load. For example, the 7800 series regulators can handle a maximum output current of at least  $1.3\text{ A}$  and typical  $2.5\text{ A}$ . If the load current exceeds the maximum allowable value, there will be a **thermal overload** and the regulator will **shut down**. *A thermal overload condition means that there is excessive power dissipation inside the regulator.*

If an application requires a larger value of load current than the maximum current that the regulator can deliver, we will have to use an external pass transistor as shown in Fig. 56.24.

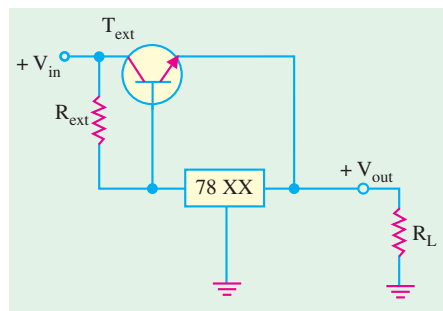


Fig. 56.24

the value of current that the voltage regulator is to handle **internally**.

When the current is sufficient to produce at least a  $0.7\text{ V}$  drop across  $R_{ext}$ , the transistor  $T_{ext}$  turns on and conducts any current in excess of  $I_{max}$ . The transistor  $T_{ext}$  will conduct current depending on the load requirement. For example, if the total load current is  $5\text{ A}$  and  $I_{max}$  was selected to be  $1\text{ A}$ , then the external pass Transistor ( $T_{ext}$ ) will conduct  $4\text{ A}$  of current through it.

It may be noted that the external pass transistor is a power transistor with heat sink that must be capable of handling a maximum power given by the equation,

$$P_{ext} = I_{ext} (V_{in} - V_{out})$$

One major drawback of the circuit shown in Fig. 56.24 is that the external pass transistor is not protected from excessive current, such as would result from a shorted output. This drawback can be overcome by using an additional current limiting circuit as shown in Fig. 56.25.

The operation of this circuit may be explained as follows. The current sensing resistor,  $R_{lim}$ , sets the base-to-emitter voltage of  $T_{lim}$ . The base-to-emitter voltage of  $T_{ext}$  is now determined by  $(V_{R_{ext}} - V_{R_{lim}})$  because they have opposite polarities. So for normal operation, the drop across  $R_{ext}$  must be sufficient to overcome the opposing drop across  $R_{lim}$ .

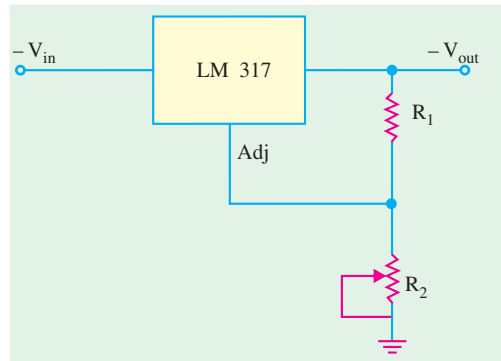


Fig. 56.23





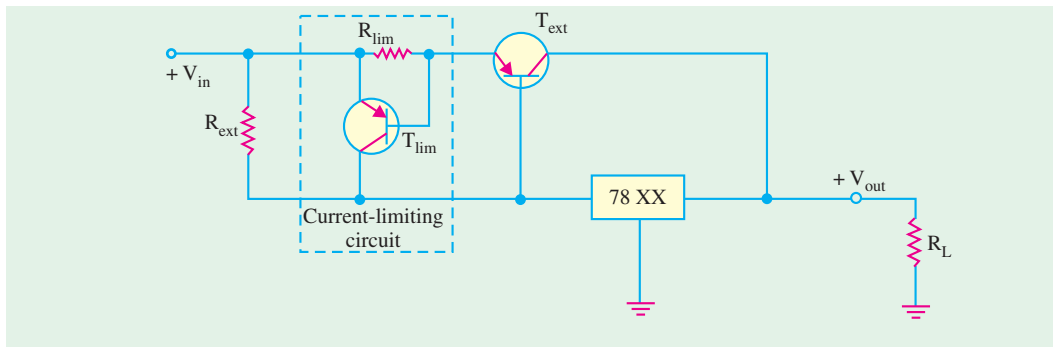


Fig. 56.25

If the current through  $T_{ext}$  exceeds a certain maximum value, ( $I_{ext(max)}$ ) because of a shorted output or a faulty load, the voltage across  $R_{lim}$  reaches 0.7 V and turns  $T_{lim}$  on. As a result,  $T_{lim}$  now conducts current away from  $T_{ext}$  and through the regulator. This forces a thermal overload to occur and shut down the regulator. Remember, the IC voltage regulator circuitry is internally protected from thermal overload as part of its design. This way the external pass transistor is protected from excessive current.

### 56.21. Use of Linear Voltage Regulators as a Current Regulator

The 3-terminal linear voltage regulator can be used as a current source when an application requires that a constant current be supplied to a variable load. The basic circuit is shown in Fig. 56.26. Here  $R_1$  is the current-setting resistor. The regulator provides a fixed output voltage,  $V_{out}$  between the ground terminal and the output terminal. However, it may be noted that the ground pin of the regulator is not connected to the circuit ground. The constant current supplied to the load, is given by the equation,

$$I_L = \frac{V_{out}}{R_L} + I_Q$$

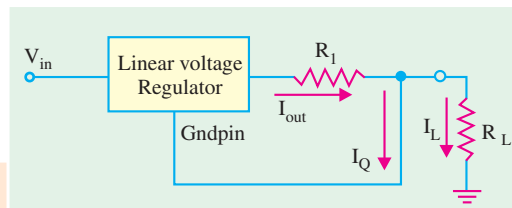


Fig. 56.26

Usually the current,  $I_Q$  is very small as compared to the output current and hence can be neglected, therefore

$$I_L \approx \frac{V_{Out}}{R_L}$$

For example, if we use 7805 regulator to provide a constant current of 1 A to a variable load, then

$$R_1 = \frac{5}{1} = 5 \Omega$$

Please note that input voltage must be at least 2 V greater than the output voltage. Thus for 5 V regulator,  $V_{in}$  must be greater than 7 V.

### 56.22. Switching Voltage IC Regulators

There are several switching voltage IC regulators available in the market. The choice depends upon the desired application and the cost. However, we will illustrate it with the IC 78S40. This device is a universal device that can be used with external components to provide step-up, step down and inverting operation.

Fig. 56.27 shows the internal circuitry of the IC 78C40. The circuitry can be compared to the basic switching regulators discussed in Art. 56.12, 56.13, and 56.14. As seen from this diagram, the



oscillator and comparator functions are directly comparable. The logic gate and flip-flop in the 78S40 were not included in the basic circuit of Fig. 56.12, but they provide additional regulation. Transistor  $T_1$  and  $T_2$  perform the same function as  $T$  in the basic circuit. The 1.25 V reference block in the 78S40 has the same purpose as that of the zener diode in the basic circuit and diode  $D_1$  in the 78S40 corresponds to  $D_1$  in the basic circuit.

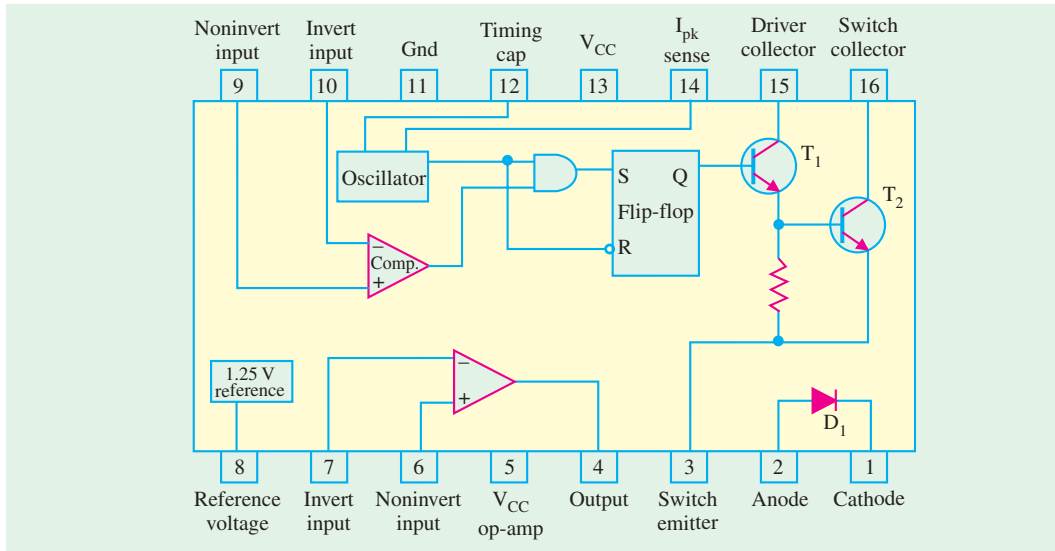


Fig. 56.27

The circuitry of 78S40 has also an uncommitted or unused OP-AMP. We require external circuitry to operate this device as a regulator.

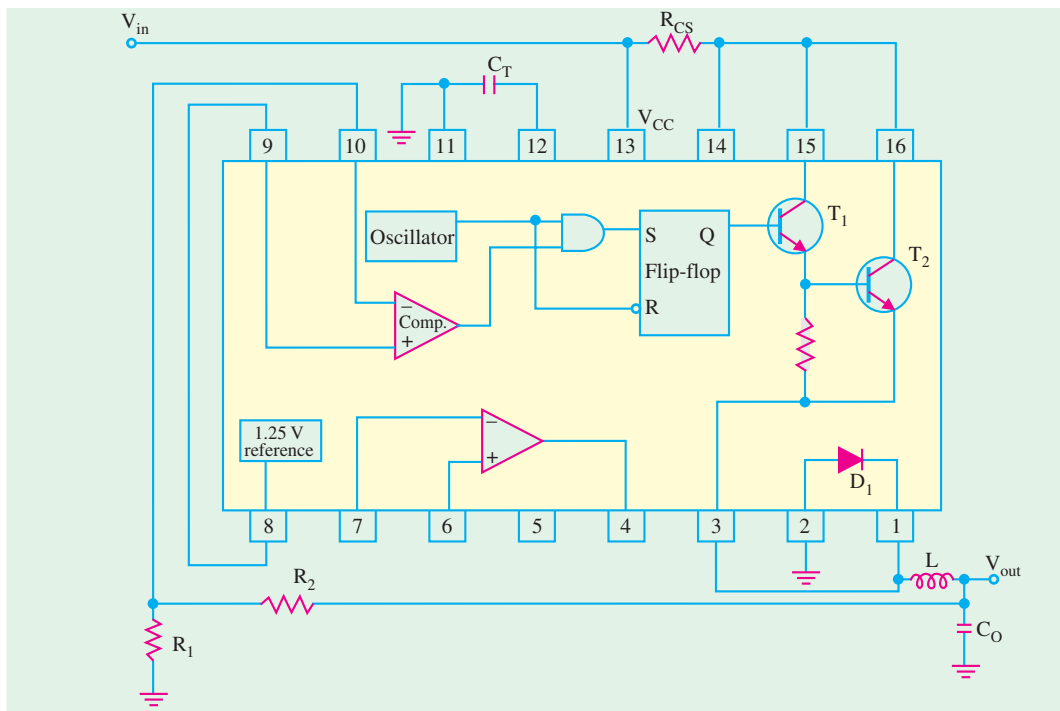


Fig. 56.28



Fig. 56.28 shows the external connections of the IC 78S40 for a step-down switching regulator configuration. Note that in this configuration the circuit produces an output voltage which is less than input voltage.

Fig. 56.28 shows the IC 78S40 connected to the external components for a step-up switching regulator configuration. In this case the output voltage is greater than the input voltage. An inverting configuration is also possible but is not shown here.

In both the circuits of Fig. 56.27 and Fig. 56.28, the capacitor  $C_T$  (called timing capacitor) controls the pulse width and frequency of the oscillator and thus establishes the on-time of transistor  $T_1$ . The voltage across the resistor  $R_{CS}$  (called current-sensing resistor) is used internally by the oscillator to vary the duty cycle based on the desired peak current. The voltage divider made up of  $R_1$  and  $R_2$  reduces the output voltage to a value equal to the reference voltage. If the output voltage ( $V_{out}$ ) exceeds its set value, the output of the comparator switches to the low state, disabling the gate to turn  $T_2$  off until the output decreases. This regulating action is in addition to that produced by the duty cycle variation of the oscillator.

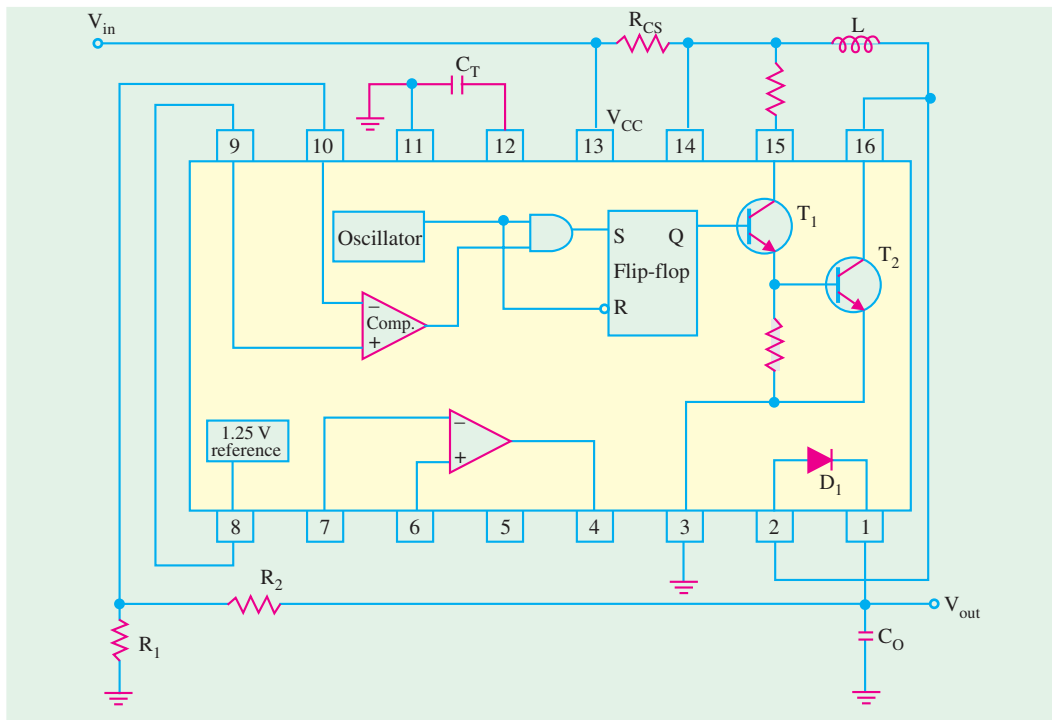


Fig. 56.29

### Tutorial Problems No. 56.1

- The Zener diode of Fig. 56.30 has the following ratings :

$$\begin{array}{lll}
 V_z = 15 \text{ V} & \text{at} & I_z = 17 \text{ mA} \\
 r_z = 14 \ \Omega & \text{at} & I_z = 17 \text{ mA} \\
 I_{z(\min)} = 0.25 \text{ mA} & & I_{z(\max)} = 66 \text{ mA}
 \end{array}$$

What would be the load voltage when load current  $I_L$  varies from 1 mA to 60 mA. Also calculate the voltage regulation of the regulator.

(14.846 V, 15.672 V, 5.27%)



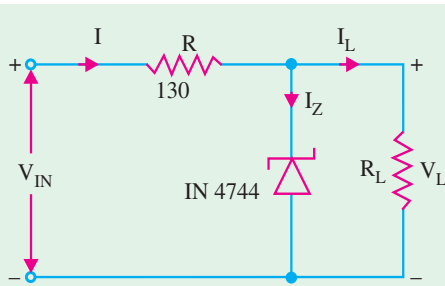


Fig. 56.30

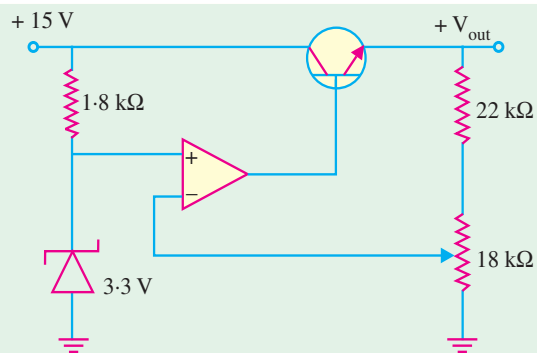


Fig. 56.31

2. Determine the output voltage,  $V_{out}$  for the op-amp series regulator shown in Fig. 56.31. (7.3 V)
3. Calculate the minimum and maximum output voltages for the IC voltage regulator shown in Fig. 53.32. Assume  $I_{ADJ} = 50 \mu\text{A}$ ,  $V_{in} = +35 \text{ V}$ ,  $V_{REF} = 1.2 \text{ V}$  (1.25 V, 12.71 V)

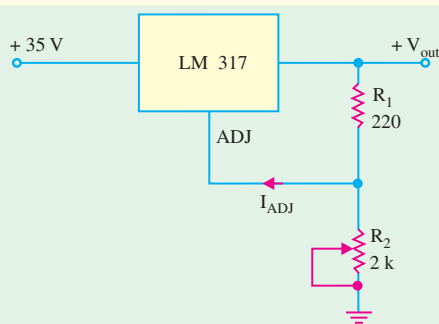


Fig. 56.32

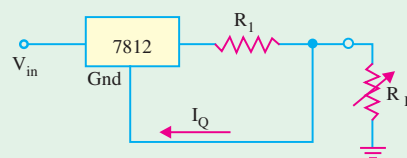


Fig. 56.33

4. Fig. 56.33 shows the circuit of a current regulator. What value of  $R_1$  is necessary to provide a constant current of 1 A. (12 Ω)

### OBJECTIVE TESTS – 56

1. The main job of a voltage regulator is to provide a nearly—output voltage.
  - (a) sinusoidal
  - (b) constant
  - (c) smooth
  - (d) fluctuating.
2. A 10-V dc regulator power supply has a regulation of 0.005 per cent. Its output voltage will vary within an envelope of ..... millivolt.
  - (a)  $\pm 2.5$
  - (b)  $\pm 0.5$
  - (c)  $\pm 5$
  - (d)  $\pm 0.05$
3. An ideal voltage regulator has a voltage regulation of
  - (a) 1
  - (b) 100
  - (c) 50
  - (d) 0.
4. In a Zener diode shunt voltage regulator, the diode regulates so long as it is kept in ..... condition.
  - (a) forward
  - (b) reverse
  - (c) loaded
  - (d) unloaded
5. The power efficiency of a switching voltage regulator is much higher than that of a linear regulator because it operates.
  - (a) in saturation
  - (b) in cut-off
  - (c) like a switch
  - (d) on high duty cycle.
6. A transistor series voltage regulator is called emitter-follower regulator because the emitter of the pass transistor follows the ..... voltage.
  - (a) output
  - (b) input
  - (c) base
  - (d) collector



- 7. In an op-amp series voltage regulator, output voltage depends on
  - (a) Zener voltage
  - (b) voltage divider resistors
  - (c) output voltage
  - (d) both (a) and (b)
- 8. In a feedback series regulator circuit, the output voltage is regulated by controlling the
  - (a) magnitude of input voltage
  - (b) gain of the feedback transistor
  - (c) reference voltage
  - (d) voltage drop across the series pass transistor
- 9. An op-amp shunt regulator differs from the series regulator in the sense that its control element is connected in
  - (a) series with line resistor
  - (b) parallel with line resistor
  - (c) parallel with load resistor
  - (d) parallel with input voltage.
- 10. A switching voltage regulator can be of the following type :
  - (a) step-down      (b) step-up
  - (c) inverting      (d) all of the above
- 11. In an inverting type switching regulator, output voltage is ..... input voltage.
  - (a) lesser than      (b) greater than
  - (c) equal to      (d) opposite to.
- 12. The output voltage of a step-down type switching voltage regulator depends on
  - (a) input voltage      (b) duty cycle
  - (c) transistor on-time      (d) all of the above.

- 13. As compared to voltage regulators made up of discrete components, IC regulators have the inherent advantage/s of
  - (a) self protection against over-temperature
  - (b) remote control
  - (c) current limiting
  - (d) all of the above
- 14. A 12 V monolithic regulator is adjusted to obtain a higher output voltage as shown in Fig. 56.34. The  $V_0$  will be

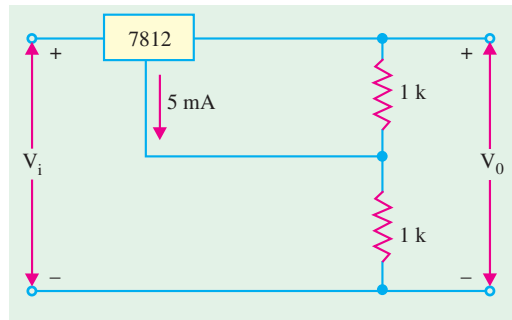


Fig. 56.34

- (a) 12 V      (b) 17 V
- (c) 24 V      (d) 29 V
- 15. A three terminal monolithic IC regulator can be used as
  - (a) an adjustable output voltage regulator alone
  - (b) an adjustable output voltage regulator and a current regulator
  - (c) a current regulator and a power switch
  - (d) a current regulator alone

ANSWERS

1. (b) 2. (a) 3. (d) 4. (b) 5. (c) 6. (c) 7. (d) 8. (d) 9. (a) 10. (d) 11. (d) 12. (d)  
 13. (d) 14. (d) 15. (d).



## ROUGH WORK

### Learning Objectives

- Bipolar Junction Transistor
- Transistor Biasing
- Transistor Currents
- Transistor Circuit Configurations
- CB Configuration
- CE Configuration
- Relation between  $\alpha$  and  $\beta$
- CC Configuration
- Relation between Transistor Currents
- Leakage Currents in a Transistor
- Thermal Runaway
- Transistor Static Characteristics
- Common Base Test Circuit
- Common Base Static Characteristics
- Common Emitter Static Characteristics
- Common Collector Static Characteristic
- Different Ways of Drawing Transistor Circuits
- The Beta Rule
- Importance of  $V_{CE}$
- Cut-off and Saturation Points
- BJT Operating Regions
- Active Region DC Model of BJT
- BJT Switches
- Normal DC Voltage Transistor Indications
- Transistor Fault Location
- Increase/Decrease Notation

# BIPOLAR JUNCTION TRANSISTOR



Bipolar junction transistor is used in two broad areas—as a linear amplifier to boost or amplify an electrical signal and as an electronic switch



### 57.1. Bipolar Junction Transistor

The transistor was invented by a team of three scientists at Bell Laboratories, USA in 1947. Although the first transistor was not a bipolar junction device, yet it was the beginning of a technological revolution that is still continuing in the twenty first century. All of the complex electronic devices and systems developed or in use today, are an outgrowth of early developments in semiconductor transistors.

There are two basic types of transistors : (1) the bipolar junction transistor (BJT) which we will study in this chapter and the field-effect transistor (FET) which is covered in chapter 13. The bipolar junction transistor is used in two broad areas of electronics : (1) as a linear amplifier to boost an electrical signal and (2) as an electronic switch.

Basically, the bipolar junction transistor consists of two back-to-back  $P-N$  junctions manufactured in a single piece of a semiconductor crystal. These two junctions give rise to three regions called **emitter**, **base** and **collector**. As shown in Fig. 57.1 (a) junction transistor is simply a sandwich of one type of semiconductor material between two layers of the other type. Fig. 57.1 (a) shows a layer of  $N$ -type material sandwiched between two layers of  $P$ -type material. It is described as a  $PNP$  transistor. Fig. 57.1 (b) shown an  $NPN$  transistor consisting of a layer of  $P$ -type material sandwiched between two layers of  $N$ -type material.

The emitter, base and collector are provided with terminals which are labelled as  $E$ ,  $B$  and  $C$ . The two junctions are : emitter-base ( $E/B$ ) junction and collector-base ( $C/B$ ) junction.

The symbols employed for  $PNP$  and  $NPN$  transistors are also shown in Fig. 57.1. The arrowhead is always at the emitter (not at the collector) and in each case, its direction indicates the **conventional** direction of current flow. For a  $PNP$  transistor, arrowhead points from emitter to base meaning that emitter is positive with respect to base (and also with respect to collector)\* For  $NPN$  transistor, it points from base to emitter meaning that base (and collector as well)\* is positive with respect to the emitter.

#### 1. Emitter

It is more heavily doped than any of the other regions because its main function is to supply majority charge carriers (either electrons or holes) to the base.

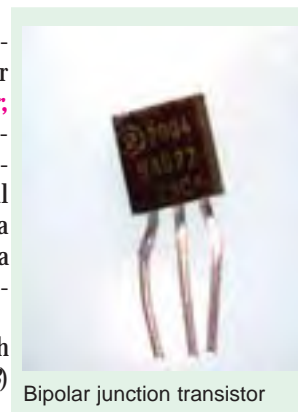
#### 2. Base

It forms the middle section of the transistor. It is very thin ( $10^{-6}$  m) as compared to either the emitter or collector and is very **lightly-doped**.

#### 3. Collector

Its main function (as indicated by its name) is to collect majority charge carriers coming from the emitter and passing through the base.

\* In a transistor, for normal operation, collector and base have the same polarity with respect to the emitter (Art. 57.3)



Bipolar junction transistor

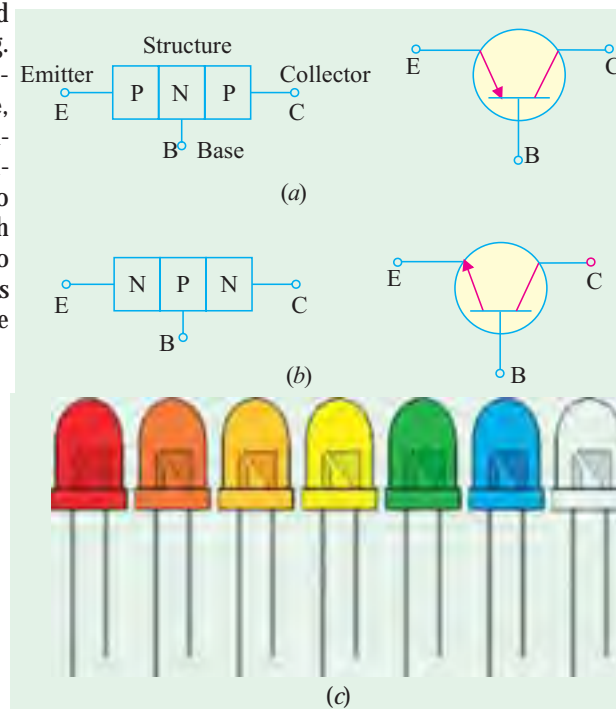


Fig. 57.1





In most transistors, collector region is made physically larger than the emitter region because it has to dissipate much greater power. Because of this difference, there is no possibility of inverting the transistor *i.e.* making its collector the emitter and its emitter the collector. Fig 57.1 (c), shows the picture of C1815 (front and the back view) transistor.

### 57.2. Transistor Biasing

For proper working of a transistor, it is essential to apply voltages of correct polarity across its two junctions. It is worthwhile to remember that for normal operation;

1. emitter-base junction is always forward-biased and
2. collector-base junction is always reverse-biased.

This type of biasing is known as *FR* biasing.

In Fig. 57.2, two batteries respectively provide the dc emitter supply voltage  $V_{EE}$  and collector supply voltage  $V_{CC}$  for properly biasing the two junctions of the transistor. In Fig. 57.2 (a), Positive terminal of  $V_{EE}$  is connected to P-type emitter in order to repel or Push holes into the base.

The negative terminal of  $V_{CC}$  is connected to the collector so that it may attract or pull holes through the base. Similar considerations apply to the NPN transistor of Fig. 57.2 (b). It must be remembered that a transistor will never conduct any current if its emitter-base junction is not forward-biased.\* Also refer to the picture shown in Fig. 57.2 (c).

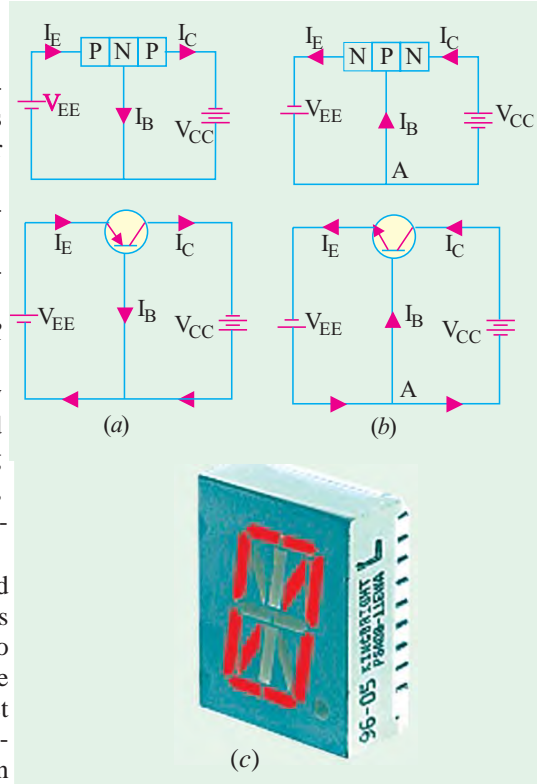


Fig. 57.2

### 57.3. Important Biasing Rule

For a PNP transistor, both collector and base are negative with respect to the emitter (the letter N of Negative being the same as the middle letter of PNP). Of course, collector is *more negative* than base [Fig. 57.3 (a)]. Similarly, for NPN transistor, both collector and base are positive with respect to the emitter (the letter P of Positive being the same as the middle letter of NPN). Again, collector is *more positive* than the base as shown in Fig. 57.3 (b).

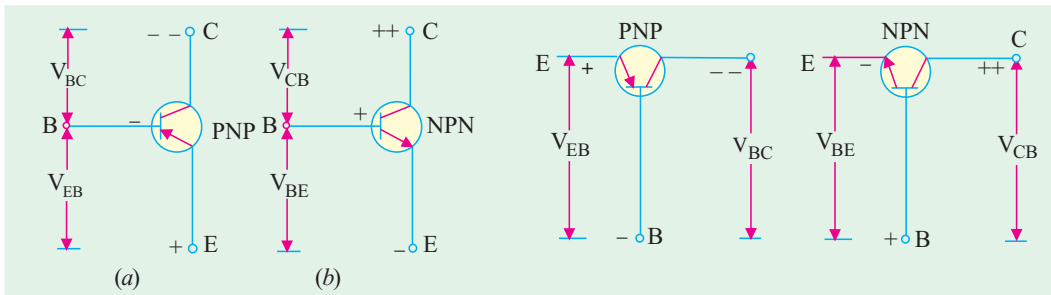


Fig. 57.3

Fig. 57.4

\* There would be no current due to majority charge carriers. However, there would be an extremely small current due to minority charge carriers which is called leakage current of the transistor (Art. 57.12).



It may be noted that different potentials have been designated by double subscripts. The first subscript always represents the point or terminal which is more positive (or less negative) than the point or terminal represented by the second subscript. For example, in Fig. 57.3 (a), the potential difference between emitter and base is written as  $V_{EB}$  (and not  $V_{BE}$ ) because **emitter is positive with respect to base**. Now, between the base and collector themselves, collector is more negative than base. Hence, their potential difference is written as  $V_{BC}$  and not as  $V_{CB}$ . Same is the case with voltages marked in Fig. 57.4.

### 57.4. Transistor Currents

The three primary currents which flow in a properly-biased transistor are  $I_E$ ,  $I_B$  and  $I_C$ . In Fig. 57.5 (a) are shown the directions of flow as well as relative magnitudes of these currents for a PNP transistor connected in the common-base mode. It is seen that again,

$$I_E = I_B + I_C$$

It means that a small part (about 1—2%) of emitter current goes to supply base current and the remaining major part (98—99%) goes to supply collector current.

Moreover,  $I_E$  flows into the transistor whereas both  $I_B$  and  $I_C$  flow out of it.

Fig. 57.5 (b) shows the flow of currents in the same transistor when connected in the common-emitter mode. It is seen that again,

$$I_E = I_B + I_C$$

By normal convention, currents flowing **into** a transistor are taken as positive whereas those flowing **out** of it are taken as negative. Hence,  $I_E$  is positive whereas both  $I_B$  and  $I_C$  are negative. Applying Kirchoff's Current Law, we have

$$I_E + (-I_B) + (-I_C) = 0 \quad \text{or} \quad I_E - I_B - I_C = 0 \quad \text{or} \quad I_E = I_B + I_C$$

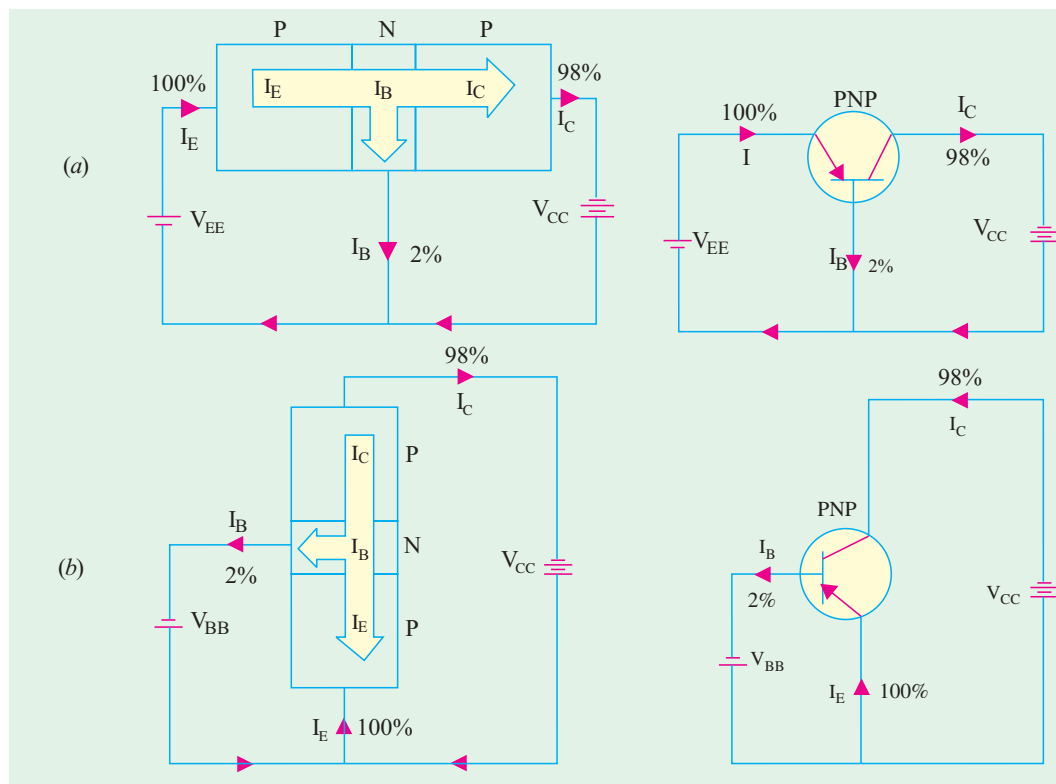


Fig. 57.5

This statement is true **regardless of transistor type or transistor configuration**.



**Note.** For the time being, we have not taken into account the leakage currents which exist in a transistor (Art. 57.12).

### 57.5. Summing Up

- The four basic guideposts about all transistor circuits are :
1. conventional current flows along the arrow whereas electrons flow against it;
  2. E/B junction is always forward-biased;
  3. C/B junction is always reverse-biased;
  4.  $I_E = I_B + I_C$

### 57.6. Transistor Circuit Configurations

Basically, there are three types of circuit connections (called configurations) for operating a transistor.

1. common-base (CB),
2. common-emitter (CE),
3. common-collector (CC).

The term ‘common’ is used to denote the electrode that is common to the input and output circuits. Because the common electrode is generally grounded, these modes of operation are frequently referred to as grounded-base, grounded-emitter and grounded-collector configurations as shown in Fig. 57.6 for a PNP – transistor.

Since a transistor is a 3-terminal (and not a 4-terminal) device, one of its terminals has to be common to the input and output circuits.

### 57.7. CB Configuration

In this configuration, emitter current  $I_E$  is the input current and collector current  $I_C$  is the output current. The input signal is applied between the emitter and base whereas output is taken out from the collector and base as shown in Fig. 57.6 (a).

The ratio of the collector current to the emitter current is called dc alpha ( $\alpha_{dc}$ ) of a transistor.

$$\therefore \alpha_{dc}^* = \frac{-I_C}{I_E}$$

The negative sign is due to the fact that current  $I_E$  flows into the transistor whereas  $I_C$  flows out of it. Hence,  $I_E$  is taken as positive and  $I_C$  as negative.

$$\therefore I_C = -\alpha_{dc} \cdot I_E$$

If we write  $\alpha_{dc}$  simply as  $\alpha^{**}$ , then  $\alpha = I_E/I_C$

It is also called forward current transfer ratio ( $-h_{FB}$ ). In  $h_{FB}$ , subscript  $F$  stands for forward and  $B$  for common-base. The subscript d.c. on  $\alpha$  signifies that this ratio is defined from dc values of  $I_C$  and  $I_E$ .

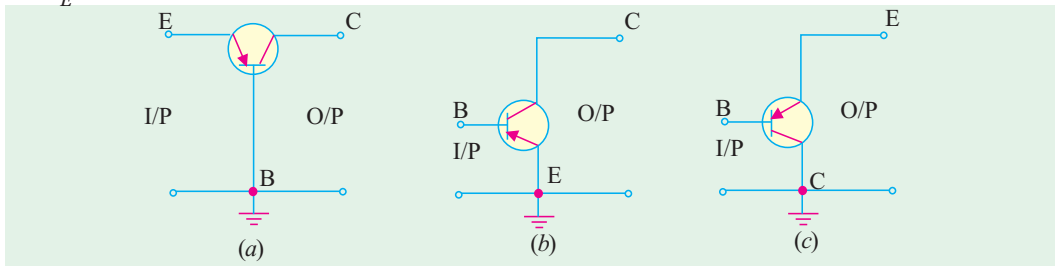


Fig. 57.6

The  $\alpha$  of a transistor is a measure of the quality of a transistor ; higher the value of  $\alpha$ , better the transistor in the sense that collector current more closely equals the emitter current. Its value ranges

\* More accurately,  $\alpha_{dc} = \frac{I_C - I_{CBO}}{I_E}$  ...Art.57.12

\*\* Negative sign has been omitted, since we are here concerned with only magnitudes of the currents involved.



from 0.95 to 0.999. Obviously, it applies only to *CB* configuration of a transistor. As seen from above and Fig. 57.7.

$$I_C = \alpha I_E$$

Now,  $I_B = I_E - \alpha I_E = (1 - \alpha) I_E$

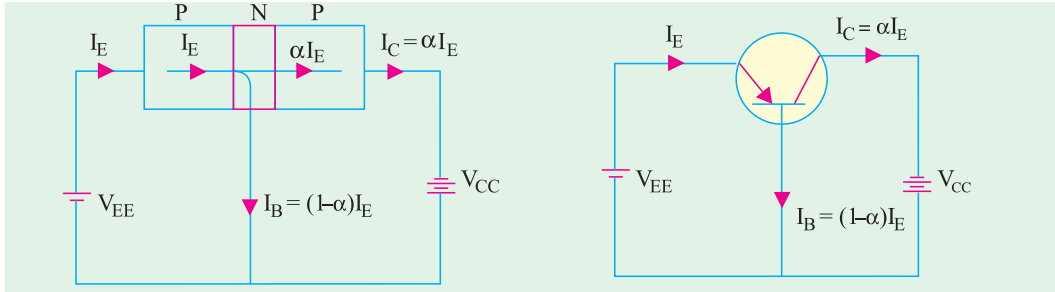


Fig. 57.7

Incidentally, there is also an a.c.  $\alpha$  for a transistor. It refers to the ratio of **change** in collector current to the **change** in emitter current.

$$\therefore \alpha_{ac} = \frac{-\Delta I_C}{\Delta I_E}$$

It is also, known as short-circuit gain of a transistor and is written as  $-h_{fb}$ . It may be noted that upper case subscript '*FB*' indicates dc value whereas lower case subscript '*fb*' indicates ac value. For all practical purposes,  $\alpha_{dc} = \alpha_{ac} = \alpha$ .

**Example 57.1.** Following current readings are obtained in a transistor connected in *CB* configuration :  $I_E = 2 \text{ mA}$  and  $I_B = 20 \text{ mA}$ . Compute the values of  $\alpha$  and  $I_C$

(Electronics-II, Punjab Univ. 1992)

**Solution.**  $I_C = I_E - I_B = 2 \times 10^{-3} - 20 \times 10^{-6} = 1.98 \text{ mA}$   
 $\alpha = I_C / I_E = 1.98 / 2 = 0.99$

### 57.8. CE Configuration

Here, input signal is applied between the base and emitter and output signal is taken out from the collector and emitter circuit. As seen from Fig. 57.6 (b),  $I_B$  is the input current and  $I_C$  is the output current.

The ratio of the d.c. collector current to dc base current is called dc beta ( $\beta_{dc}$ ) or just  $\beta$  of the transistor.

$$\therefore \beta = -I_C / -I_B = I_C / I_B \quad \text{or} \quad I_C = \beta I_B \quad \text{--- Fig. 57.8 (a)}$$

It is also called common-emitter d.c. **forward transfer ratio** and is written as  $h_{FE}$ . It is possible for  $\beta$  to have as high a value as 500.

While analysing ac operation of a transistor, we use ac  $\beta$  which is given by  $\beta_{ac} = \Delta I_C / \Delta I_B$

It is also written as  $h_{fe}$ .

The flow of various currents in a *CE* configuration both for *PNP* and *NPN* transistor is shown in Fig. 57.8. As seen

$$I_E = I_B + I_C = I_B + \beta I_B = (1 + \beta) I_B$$

### 57.9. Relation Between $\alpha$ and $\beta$

$$\beta = \frac{I_C}{I_B} \quad \text{and} \quad \alpha = \frac{I_C}{I_E} \quad \therefore \quad \frac{\beta}{\alpha} = \frac{I_E}{I_B} \quad \text{--- only numerical value of } \alpha$$



Now,  $I_B = I_E - I_C \quad \therefore \quad \beta = \frac{I_C}{I_E - I_C} = \frac{I_C / I_E}{I_E / I_E - I_C / I_E} \quad \text{or} \quad \beta = \frac{\alpha}{1 - \alpha}$

Cross-multiplying the above equation and simplifying it, we get  
 $\beta(1 - \alpha) = \alpha$  or  $\beta = \alpha(1 + \beta)$  or  $\alpha = \beta / (1 + \beta)$   
 It is seen from the about 2 equations that  $1 - \alpha = 1 / (1 + \beta)$

**57.10. CC Configuration**

In this case, input signal is applied between base and collector and output signal is taken out from emitter-collector circuit [Fig. 57.6 (c)]. Conventionally speaking, here  $I_B$  is the input current and  $I_E$  is the output current as shown in Fig. 57.9. The current gain of the circuit is

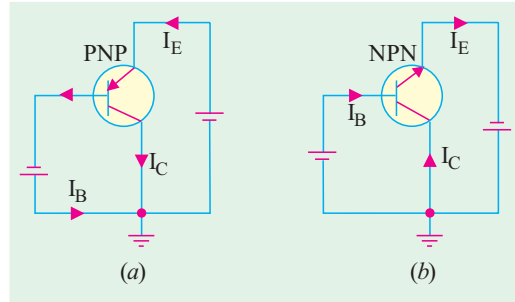


Fig. 57.9

$$\frac{I_E}{I_B} = \frac{I_E}{I_C} \cdot \frac{I_C}{I_B} = \frac{\beta}{\alpha} = \frac{\beta}{\beta / (1 + \beta)} = (1 + \beta)$$

The flow paths of various currents in a CC configuration are shown in Fig. 57.9. It is seen that  $I_E = I_B + I_C = I_B + \beta I_B = (1 + \beta) I_B$   
 $\therefore$  output current =  $(1 + \beta) \times$  input current.

**57.11. Relations Between Transistor Currents**

While deriving various equations, following definitions should be kept in mind.

$$\alpha = \frac{I_C}{I_E}, \quad \beta = \frac{I_C}{I_B}, \quad \alpha = \frac{\beta}{(1 + \beta)} \quad \text{and} \quad \beta = \frac{\alpha}{(1 - \alpha)}$$

(i)  $I_C = \beta I_B = \alpha I_E = \frac{\beta}{1 + \beta} I_E$

(ii)  $I_B = \frac{I_C}{\beta} = \frac{I_E}{1 + \beta} = (1 - \alpha) I_E$

(iii)  $I_E = \frac{I_C}{\alpha} = \frac{1 + \beta}{\beta} I_C = (1 + \beta) I_B = \frac{I_B}{(1 - \alpha)}$

(iv) The three transistor d.c. currents always bear the following ratio\*

$I_E : I_B : I_C \quad \therefore \quad 1 : (1 - \alpha) : \alpha$

Incidentally, it may be noted that for ac currents, small letters  $i_e$ ,  $i_b$  and  $i_c$  are used.

**57.12. Leakage Currents in a Transistor**

**(a) CB Circuit**

Consider the CB transistor circuit shown in Fig. 57.11. The emitter current (due to majority carriers) initiated by the forward-biased emitter base junction is split into two parts :

(i)  $(1 - \alpha) I_E$  which becomes base current  $I_B$  in the external circuit and

(ii)  $\alpha I_E$  which becomes collector current  $I_C$  in the external circuit.

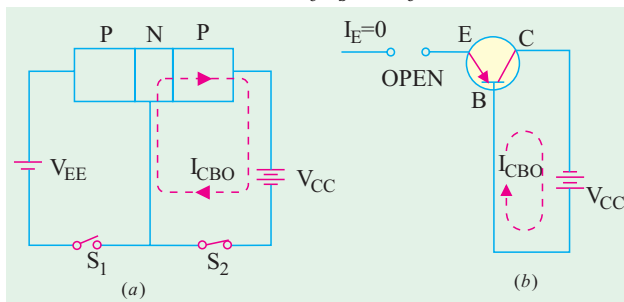


Fig. 57.10

\* It reminds us of the power distribution relationship in an induction motor.

As mentioned earlier (Art. 57.2), though *C/B* junction is reverse-biased for majority charge carriers (*i.e.* holes in this case), it is forward-biased so far as thermally-generated minority charge carriers (*i.e.* electrons in this case) are concerned. This current flows even when emitter is disconnected from its dc supply as shown in Fig. 57.10 (a) where switch,  $S_1$  is open. It flows in the **same direction\*** as the collector current of majority carriers. It is called leakage current  $I_{CBO}$ . The subscripts *CBO* stand for ‘Collector to Base with emitter Open.’ Very often, it is simply written as  $I_{CO}$ .

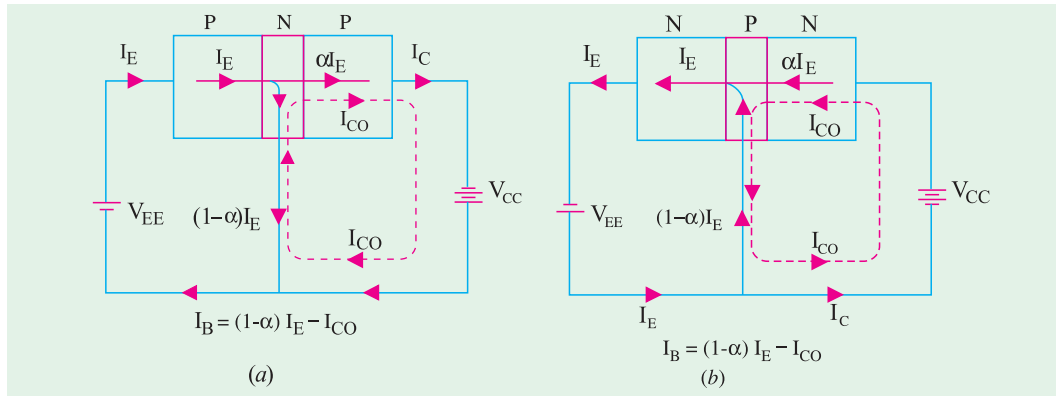


Fig. 57.11

It should be noted that

(i)  $I_{CBO}$  is exactly like the reverse saturation current  $I_S$  or  $I_0$  of a reverse-biased diode discussed in Art. 57.1.

(ii)  $I_{CBO}$  is extremely temperature-dependent because it is made up of thermally-generated minority carriers. As mentioned earlier,  $I_{CBO}$  doubles for every 10°C rise in temperature for *Ge* and 6°C for *Si*.

If we take into account the leakage current, the current distribution in a *CB* transistor circuit becomes as shown in Fig. 57.11 both for *PNP* and *NPN* type transistors.

It is seen that total collector current is actually the sum of two components :

(i) current produced by normal transistor action *i.e.* component controlled by emitter current. Its value is  $\alpha I_E$  and is due to majority carriers.

(ii) temperature-dependent leakage current  $I_{CO}$  due to minority carriers.

$$\therefore I_C = \alpha I_E + I_{CO} \quad \dots(i) \quad \therefore \alpha = \frac{I_C - I_{CO}}{I_E}$$

Since  $I_{CO} \ll I_C$ , hence  $\alpha \cong I_C / I_E$

(iii) Substituting the value of  $I_E = (I_C + I_B)$  in Eq. (i) above, we get

$$I_C = (I_C + I_B) + I_{CO} \quad \text{or} \quad I_C(1 - \alpha) = \alpha I_B + I_{CO}$$

$$\therefore I_C = \frac{\alpha I_B}{1 - \alpha} + \frac{I_{CO}}{1 - \alpha}$$

(iv) Eliminating  $I_C$  from Eq. (i) above, we get

$$(I_E - I_B) = \alpha I_E + I_{CO} \quad \text{or} \quad I_B = (1 - \alpha) I_E - I_{CO}$$

**(b) CE Circuit**

In Fig. 57.12 (a) is shown a common-emitter circuit of an *NPN* transistor whose base lead is

\* Actually, electrons (which form minority charge carriers in collector) flow from negative terminal of collector battery, to collector, then to base through *C/B* junction and finally, to positive terminal of  $V_{CC}$ . However, conventional current flows in the opposite direction as shown by dotted line in Fig. 57.10 (a)



open. It is found that despite  $I_B = 0$ , there is a leakage current from collector to emitter. It is called  $I_{CEO}$ , the subscripts *CEO* standing for 'Collector to Emitter with base Open'.

Taking this leakage current into account, the current distribution through a *CE* circuit becomes as shown in Fig. 57.12 (c).

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1 + \beta) I_{CO} = \beta I_B + I_{CO} / (1 - \alpha)$$

(i) 
$$\therefore I_C = \frac{\alpha I_B}{1 - \alpha} + \frac{I_{CO}}{1 - \alpha}$$

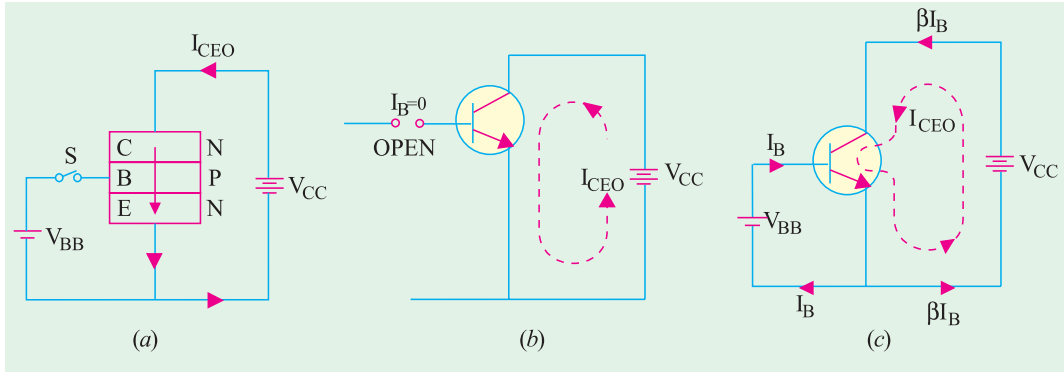


Fig. 57.12

Now,  $\beta I_B = \alpha I_E$  Substituting this value above, we get,

$$I_C = \alpha I_E + I_{CEO} \quad \text{Also, } I_B = I_E - I_C$$

Substituting the value of  $I_C$  from above, we have

(ii) 
$$I_B = I_E - \alpha I_E - I_{CEO} = (1 - \alpha) I_E - I_{CEO}$$

### 57.13. Thermal Runaway

As seen from Art. 57.12, for a *CE* circuit

$$I_C = \beta I_B + (1 + \beta) I_{CO}$$

The leakage current is extremely temperature-dependent. It almost doubles for every 6°C rise in temperature in *Ge* and for every 10°C rise in *Si*. Any increase in  $I_{CO}$  is magnified  $(1 + \beta)$  times i.e. 300 to 500 times. Even a slight increase in  $I_{CO}$  will affect  $I_C$  considerably. As  $I_C$  increases, collector power dissipation increases which raises the operating temperature that leads to further increase in  $I_C$ . If this succession of increases is allowed to continue, soon  $I_C$  will increase beyond safe operating value thereby damaging the transistor itself—a condition known as **thermal runaway**. Hence, some form of stabilization is necessary to prevent this thermal runaway.

**Example 57.2.** The reverse saturation current of an NPN transistor in common-base circuit is 12.5 μA. For an emitter current of 2 mA, collector current is 1.97 mA. Determine the current gain and base current. **(Electronics-1, Gwalior Univ. 1988)**

**Solution.** Given :  $I_{CBO} = 12.5 \mu\text{A}$ ;  $I_E = 2 \text{ mA}$ ,  $I_C = 1.97 \text{ mA}$  ;  $\alpha = ?$ ,  $I_B = ?$

$$I_C = \alpha I_E + I_{CBO} \quad \therefore \alpha = \frac{I_C - I_{CBO}}{I_E} = \frac{1.97 - 12.5 \times 10^{-3}}{2} = 0.978$$

$$I_B = I_E - I_C = 2 - 1.97 = 0.03 \text{ mA.}$$

**Example 57.3.** Derive an expression for forward current gain and leakage current of common-emitter configuration in terms of current gain and leakage current of common-base configuration. If  $\alpha = 0.98$ ,  $I_{CBO} = 5 \text{ mA}$ , calculate  $\beta$  and  $I_{CEO}$ . **(Electronics-I, Mysore Univ. 1990)**

**Solution.**  $\beta = \alpha / (1 - \alpha) = 0.98 / (1 - 0.98) = 49$



$$I_{CEO} = (1 + \beta) I_{CO} = (1 + 49) \times 5 = 250 \mu\text{A} = \mathbf{0.25 \text{ mA}}$$

**Example 57.4.** For a transistor,  $I_B = 100 \mu\text{A}$ ,  $\alpha_{dc} = 0.98$  and  $I_{CO} = 5 \mu\text{A}$ . Find the values of  $I_C$  and  $I_E$ .

**Solution.** As seen from Art. 57.12,  $I_C = \frac{\alpha I_B}{1 - \alpha} + \frac{I_{CO}}{1 - \alpha} = \frac{0.98 \times 100}{1 - 0.98} + \frac{5}{1 - 0.98} = \mathbf{5.15 \text{ mA}}$

$$I_E = I_C + I_B = 5.15 + 100 \times 10^{-3} = \mathbf{5.25 \text{ mA}}$$

**Example 57.5.** A transistor operating in CB configuration has  $I_C = 2.98 \text{ mA}$ ,  $I_E = 3.00 \text{ mA}$  and  $I_{CO} = 0.01 \text{ mA}$ . What current will flow in the collector circuit of this transistor when connected in CE configuration with a base current of  $30 \mu\text{A}$ . (Electronics-II, M.S. Univ. Vadodra 1990)

**Solution.** For CE configuration,  $I_C = \beta I_B + (1 + \beta) I_{CO}$

Let us find the value of  $\beta$  from data given for CB configuration. For such a circuit  $I_C = \alpha I_E + I_{CO}$  or  $2.98 = \alpha \times 3 + 0.01$ ;  $\alpha = 0.99$ ;  $\beta = \alpha / (1 - \alpha) = 0.99 / (1 - 0.09) = 99$ .

$\therefore$  For CE circuit,  $I_C = 99 \times 0.03 + (1 + 99) \times 0.01 = \mathbf{3.97 \text{ mA}}$ .

**Example 57.6.** For a certain transistor,  $I_C = 5.505 \text{ mA}$ ,  $I_B = 50 \text{ mA}$ ,  $I_{CO} = 5 \text{ mA}$ . Determine (i) values of  $\alpha$ ,  $\beta$  and  $I_E$  (ii) the new level of  $I_B$  required to make  $I_C = 10 \text{ mA}$ .

**Solution.** (i)  $I_C = \beta I_B + (1 + \beta) I_{CO}$  or  $5.505 \times 10^3 = \beta \times 50 + (1 + \beta) \times 5 \therefore \beta = 100$

Now,  $I_E = I_C + I_B = 5.505 + 50 \times 10^{-3} = \mathbf{5.555 \text{ mA}}$ .

Also,  $I_C = \alpha I_E + I_{CO}$ ;  $5.505 = \alpha \times 5.555 + 5 \times 10^{-3} \therefore \alpha = 5.500 / 5.555 = \mathbf{0.99}$

(ii) As seen from Art. 7.12,  $I_C = \beta I_B + (1 + \beta) I_{CO}$

$\therefore 10 = 100 I_B + 101 \times 5 \times 10^{-3}$ ;  $I_B = 0.09495 \text{ mA} = \mathbf{94.95 \mu\text{A}}$ .

**Example. 57.7.** Discuss the operation of a PNP transistor.

The reverse saturation current in a PNP germanium transistor type OC 71 is  $8 \mu\text{A}$ . If the transistor common base current gain is 0.979, calculate the collector and emitter current for  $40 \mu\text{A}$  base current. What is the collector current when base current is zero ?

(Electronics-1, Gwalior Univ. 1986)

**Solution.** Given :  $I_{CO} = 8 \mu\text{A} = 0.008 \text{ mA}$ ,  $\alpha = 0.979$ ;  $I_B = 40 \mu\text{A} = 0.04 \text{ mA}$

In a CE circuit:  $I_C = \beta I_B + I_{CEO} = \beta I_B + I_{CO} / (1 - \alpha)$ .

Now,  $\beta = \alpha / (1 - \alpha) = 0.979 / (1 - 0.979) = 46.6$

$\therefore I_C = 46.6 \times 0.04 + (1 + 46.6) \times 0.008 = \mathbf{1.9 \text{ mA}}$ ;  $I_E = I_C + I_B = 1.9 + 0.04 = \mathbf{1.94 \text{ mA}}$

### 57.14. Transistor Static Characteristics

There are the curves which represents relationship between different d.c. currents and voltages of a transistor. These are helpful in studying the operation of a transistor when connected in a circuit. The three important characteristics of a transistor are :

1. Input characteristic,
2. Output characteristic,
3. Constant-current transfer characteristic.

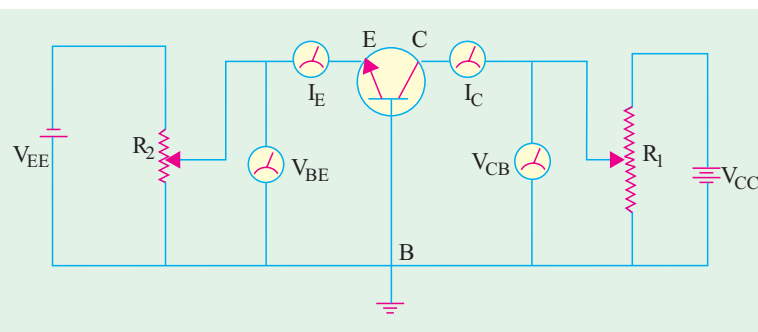


Fig. 57.13





57.15 Common Base Test Circuit

The static characteristics of an *NPN* transistor connected in common-base configuration can be determined by the use of test circuit shown in Fig. 57.13. Milliammeters are included in series with the emitter and collector circuits to measure  $I_E$  and  $I_C$ . Similarly, voltmeters are connected across  $E$  and  $B$  to measure voltage  $V_{BE}$  and across  $C$  and  $B$  to measure  $V_{CB}$ . The two potentiometer resistors  $R_1$  and  $R_2$  supply variable voltages from the collector and emitter dc supplies respectively.

57.16. Common Base Static Characteristics

(a) Input Characteristic

It shows how  $I_E$  varies with  $V_{BE}$  when voltage  $V_{CB}$  is held constant. The method of determining this characteristic is as follows :

First, voltage  $V_{CB}$  is adjusted to a suitable value with the help of  $R_1$  (Fig. 57.13). Next, voltage  $V_{BE}$  is increased in a number of discrete steps and corresponding values of  $I_E$  are noted from the milliammeter connected for the purpose. When plotted, we get the input characteristic shown in Fig. 57.14, one for *Ge* and the other for *Si*. Both curves are exactly similar to the forward characteristic of a *P-N* diode which, in essence, is what the emitter-base junction is.

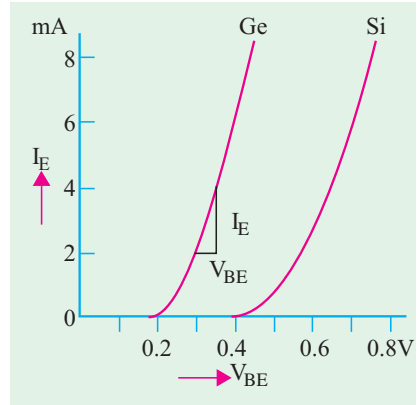


Fig. 57.14

This characteristic may be used to find the input resistance of the transistor. Its value is given by the reciprocal of its slope.

$$R_{in} = \Delta V_{BE} / \Delta I_E \quad \text{--- } V_{CB} \text{ constant.}$$

Since the characteristic is initially nonlinear,  $R_{in}$  will vary with the point of measurement. Its value over linear part of the characteristic is about  $50 \Omega$  but for low values of  $V_{BE}$ , it is considerably greater. This change in  $R_{in}$  with change in  $V_{BE}$  gives rise to distortion of signals handled by the transistor.

This characteristic is hardly affected by changes either in  $V_{CB}$  or temperature.

(b) Output Characteristic

It shows the way  $I_C$  varies with  $V_{CB}$  when  $I_E$  is held constant. The method of obtaining this characteristic is as follows:

First, movable contact, on  $R_2$  (Fig. 57.13) is changed to get a suitable value of  $V_{BE}$  and hence that of  $I_E$ . While keeping  $I_E$  constant at this value,  $V_{CB}$  is increased from zero in a number of steps and the corresponding collector current  $I_C$  that flows is noted.

Next,  $V_{CB}$  is reduced back to zero,  $I_E$  is increased to a value a little higher than before and the whole procedure is repeated. In this way, whole family of curves is obtained, a typical family being shown in Fig. 57.15.

1. The reciprocal of the near horizontal part of the characteristic gives the output resistance  $R_{out}$  of the transistor which it would offer to an input signal. Since the characteristic is linear over most of its length (meaning that  $I_C$  is virtually independent of  $V_{CB}$ ).  $R_{out}$  is very high, a typical value being  $500 \text{ k}\Omega$ .

$$R_{out} = \frac{1}{\Delta I_C / \Delta V_{CB}} = \frac{\Delta V_{CB}}{\Delta I_C}$$

- It is seen that  $I_C$  flows even when  $V_{CB} = 0$ . For example, it has a value =  $1.8 \text{ mA}$  corresponding to  $V_{CB} = 0$  for  $I_E = 2 \text{ mA}$  as shown in Fig. 57.15. It is due to the fact that electrons are being injected into the base under the action of forward-biased  $E/B$  junction and are being collected by the collector due to the action of the internal junction voltage at the  $C/B$  junction (Art. 57.2). For reducing  $I_C$  to zero, it is essential to neutralize this potential barrier by applying a small forward bias across  $C/B$  junction.



3. Another important feature of the characteristic is that a small amount of collector current flows even when emitter current  $I_E = 0$ . As we know (Art. 57.12), it is collector leakage current  $I_{CBO}$ .
4. This characteristic may be used to find  $\alpha_{ac}$  of the transistor as shown in Fig. 57.15.

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E} = \frac{DE}{BC}$$

$$= \frac{6.2 - 4.3}{2} = 0.95$$

5. Another point worth noting is that although  $I_C$  is practically independent of  $V_{CB}$  over the working range of the transistor, yet if  $V_{CB}$  is permitted to increase beyond a certain value,  $I_C$  eventually increases rapidly due to avalanche breakdown as shown in Fig. 57.15.

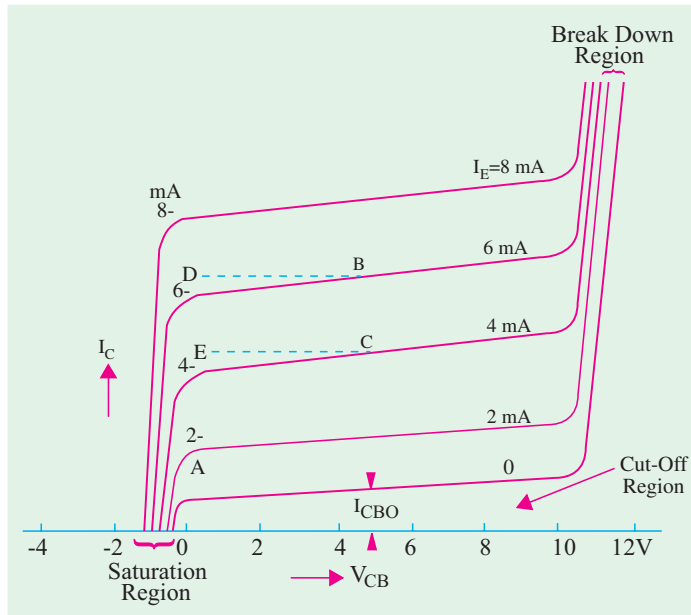


Fig. 57.15

**(c) Current Transfer Characteristic**

It shows how  $I_C$  varies with changes in  $I_E$  when  $V_{CB}$  is held constant. For drawing this characteristic, first  $V_{CB}$  is set to a convenient value and then  $I_E$  is increased in steps and corresponding values of  $I_C$  noted. A typical transfer characteristic is shown in Fig. 57.16 (a). Fig. 57.16 (b) shows a more detailed view of the portion near the origin.

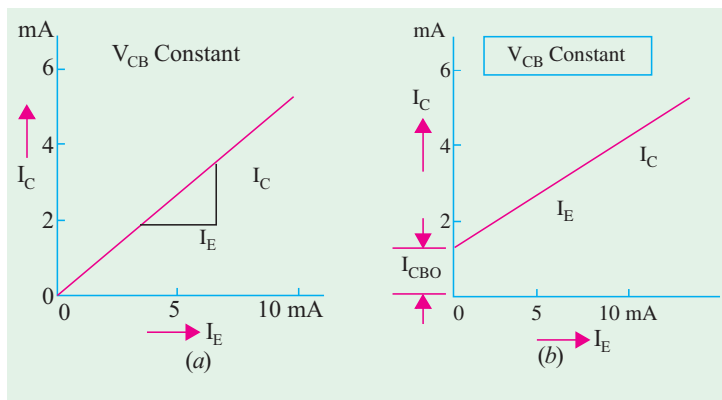


Fig. 57.16

As seen,  $\alpha_{ac}$  may be found from the equation

$$\alpha_{ac} = \Delta I_C / \Delta I_E$$

Usually,  $\alpha_{ac}$  is found from output characteristic than from this characteristic.

It may be noted in the end that *CB* connection is rarely employed for audio-frequency circuits because (i) its current gain is less than unity and (ii) its input and output resistances are so different.

**57.17. Common Emitter Test Circuit**

The static characteristics of an *NPN* transistor connected in *CE* configuration may be determined by the use of circuit diagram shown in Fig. 57.17. A milliammeter (or a microammeter in the case of a low-power transistor) is connected in series with the base to measure  $I_B$ . Similarly, a milliammeter is included in the collector circuit to measure  $I_C$ . A voltmeter with a typical range of 0–1 V is connected across base and emitter terminals for measuring  $V_{BE}$ .



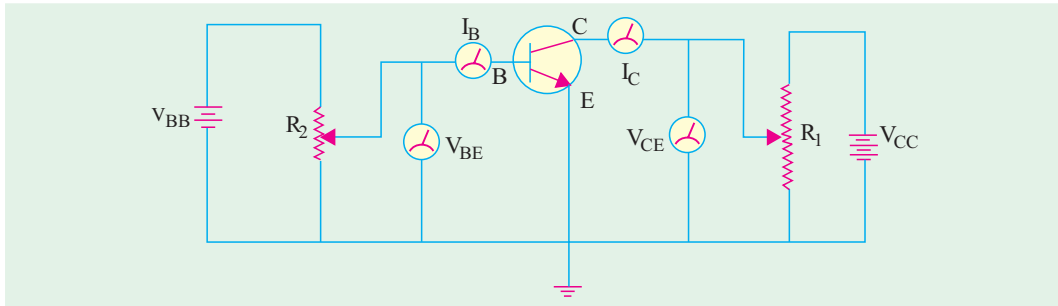


Fig. 57.17

Potentiometer  $R_2$  connected across dc supply  $V_{BB}$  is used to vary  $I_B$  and  $V_{BE}$ . A second voltmeter with a typical range of 0–20 V is connected across collector-emitter terminals to measure the output collector-emitter voltage  $V_{CE}$ .

### 57.18. Common Emitter Static Characteristics

#### (a) Input Characteristic

It shows how  $I_B$  varies with changes in  $V_{BE}$  when  $V_{CE}$  is held constant at a particular value. To begin with, voltage  $V_{CE}$  is maintained constant at a convenient value and then  $V_{BE}$  is increased in steps. Corresponding values of  $I_B$  are noted at each step. The procedure is then repeated for a different but constant value of  $V_{CE}$ . A typical input characteristic is shown in Fig. 57.18. Like  $CB$  connection, the overall shape resembles the forward characteristic of a  $P-N$  diode. The reciprocal of the slope gives the input resistance  $R_{in}$  of the transistor.

$$R_{in} = \frac{1}{\Delta I_B / \Delta V_{BE}} = \frac{\Delta V_{BE}}{\Delta I_B}$$

Due to initial non-linearity of the curve,  $R_{in}$  varies considerably from a value of 4 k $\Omega$  near the origin to a value of 600  $\Omega$  over the more linear part of the curve.

#### (b) Output or Collector Characteristic

It indicates the way in which  $I_C$  varies with changes in  $V_{CE}$  when  $I_B$  is held constant.

For obtaining this characteristic, first  $I_B$  is set to a convenient value and maintained constant and then  $V_{CE}$  is increased from zero in steps,  $I_C$  being noted at each step. Next,  $V_{CE}$  is reduced to zero and  $I_B$  increased to another convenient value and the whole procedure repeated. In this way, a family of curves (Fig. 57.19) is obtained.

It is seen that as  $V_{CE}$  increases from zero,  $I_C$  rapidly increases to a near saturation level for a fixed value of  $I_B$ . As shown, a small amount of collector current flows even when  $I_B = 0$ . It is called  $I_{CEO}$  (Art. 57.12). Since main collector current is zero, the transistor is said to be **cut-off**.

It may be noted that if  $V_{CE}$  is allowed to increase too far,  $C/B$  junction completely breaks down and due to this avalanche breakdown,  $I_C$  increases rapidly and may cause damage to the transistor.

When  $V_{CE}$  has very low value (ideally zero), the transistor is said to be saturated and it operates in the saturation region of the characteristic. Here, change in  $I_B$  does not produce a corresponding change in  $I_C$ .

This characteristic can be used to find  $\beta_{ac}$  at a specific value of  $I_B$  and  $V_{CE}$ . It is given by  $\beta_{ac} = \Delta I_C / \Delta I_B$

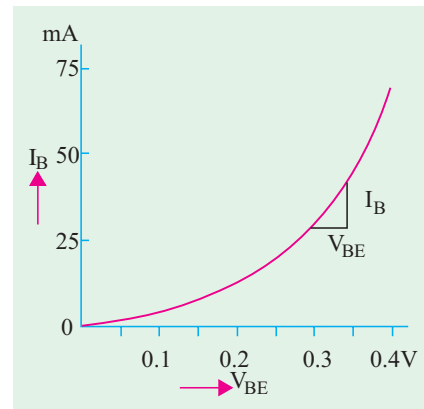


Fig. 57.18





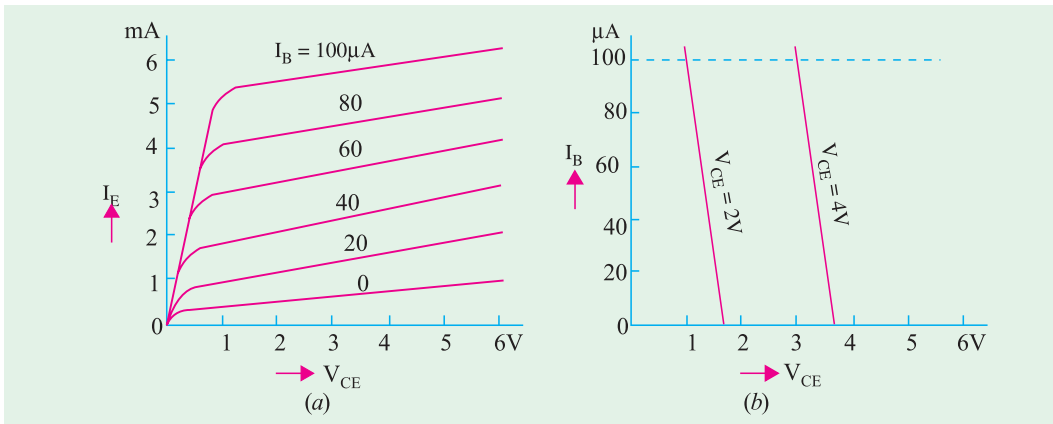


Fig. 57.22

57.20. Different Ways of Drawing Transistor Circuits

In Fig. 57.23 (a) is shown a *CB* transistor circuit which derives its voltage and current requirements from two independent power sources *i.e.* two different batteries. Correct battery connections can be done by remembering the transistor polarity rule (Art. 57.2) that in an *NPN* transistor, both collector and base have to be Positive with respect to the emitter. Of course, collector is a *little bit more* positive than base which means that between themselves, collector is at a *slightly higher positive* potential with the respect to the base. Conversely, base is at a little lower potential with respect to the collector.

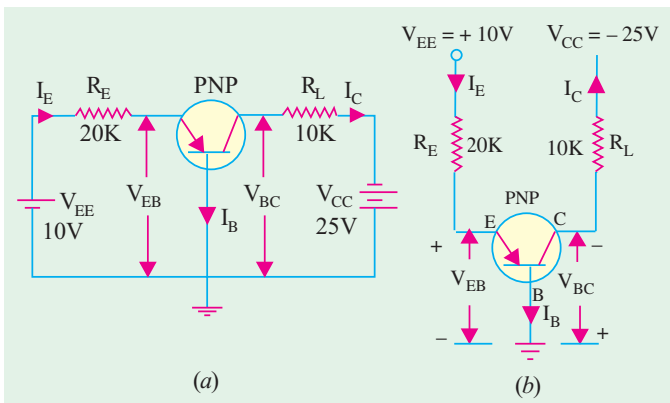


Fig. 57.23

Putting it in a slightly different way, we can say that collector is positive w.r.t. base and conversely, base is negative w.r.t. collector. That is why, potential difference between collector and base is written as  $V_{CB}$  (and not  $V_{BC}$ ) because terminal at higher potential is mentioned first. Same reasoning applies to  $V_{BE}$ . Fig. 57.23 (b) shows another and more popular way of indicating power supply voltage. Only one terminal of the

battery is shown, the other terminal is *understood to be grounded so as to provide a complete path for the current.*

For example, negative terminal of  $V_{CC}$  and positive terminal of  $V_{EE}$  are supposed to be grounded (as is the base) even though not shown as such in the diagram.

Fig. 57.24 (a) shows an *NPN* transistor connected in *CE* configuration with volt-

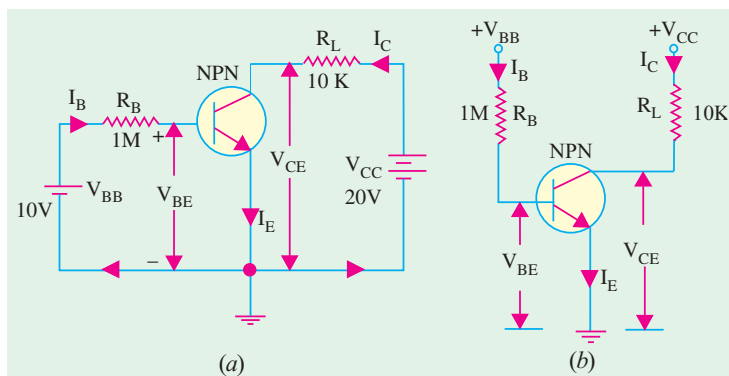


Fig. 57.24



ages and currents drawn from two independent power sources. As seen, battery connections and voltage markings are as per the rule given in Art. 57.2. Fig. 57.24 (b) shows the more popular way of indicating power supply voltages.

As seen, both collector and base are positive with respect to the common electrode *i.e.* emitter. Hence, a single battery can be used to get proper voltages across the two as shown in Fig. 57.25.

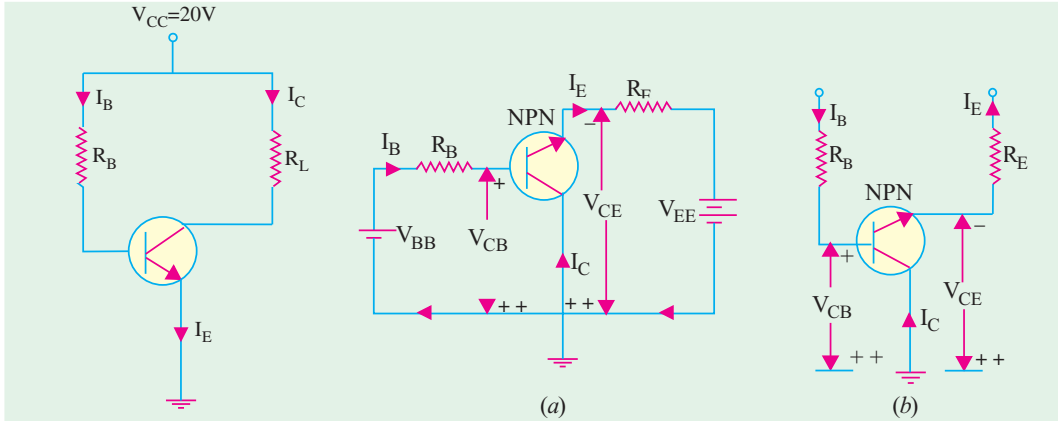


Fig. 57.25

Fig. 57.26

Fig. 57.26 (a) shows the *CC* configuration of an *NPN* transistor and Fig. 57.26 (b) shows the same circuit drawn differently.

### 57.21. Common Base Formulas

Let us find the values of different voltages and currents for the circuit in Fig. 57.23 (b). Consider the circuit *MEBM*. Applying Kirchhoff's voltage law and starting from point *B* (or ground) upwards, we get

$$(a) \quad -V_{BE} - I_E R_E + V_{EE}^* = 0 \quad \text{or} \quad I_E = \frac{V_{EE} - V_{BE}}{R_E}$$

where  $V_{BE} = 0.3 \text{ V}$  (for *Ge*) and  $0.7 \text{ V}$  (for *Si*)

Since, generally,  $V_{EE} \gg V_{BE}$ , we can simplify the above to  $I_E \cong V_{EE}/R_E = 10 \text{ V}/20 \text{ K} = 0.5 \text{ mA}$  (Fig. 57.23).

Taking  $V_{BE}$  into account and assuming silicon transistor

$$I_E = (10 - 0.7) \text{ V}/20 \text{ K} = 0.465 \text{ mA}$$

(b)  $I_C = \alpha I_E \cong I_E = 0.5 \text{ mA}$  neglecting leakage current.

(c) From circuit *NCBN*, we get

$$V_{CB} = V_{CC} - I_C \cong V_{CC} - I_E R_L = 25 - 0.5 \times 10 = 20 \text{ V} \quad (\because I_C \cong I_E)$$

**Example 57.8.** In the circuit of Fig. 57.27 (a), what value of  $R_L$  causes  $V_{CB} = 5 \text{ V}$ ?

**Solution.**  $I_E \cong V_{EE}/R_E = 10 \text{ V}/10 \text{ K} = 1 \text{ mA}$

$$I_C = \alpha I_E \cong I_E = 1 \text{ mA}$$

Now,  $V_{CC} = I_C R_L + V_{CB}$

$$\therefore R_L = \frac{V_{CC} - V_{CB}}{I_C} = \frac{20 - 5}{1 \text{ mA}} = 15 \text{ K}$$

**Example 57.9.** For the circuit shown in Fig. 57.27 (b), find the value of  $R_E$  which causes  $V_{BC} = 10 \text{ V}$ .

\* It is taken positive because we are going from the negative to the positive terminal of the emitter battery.



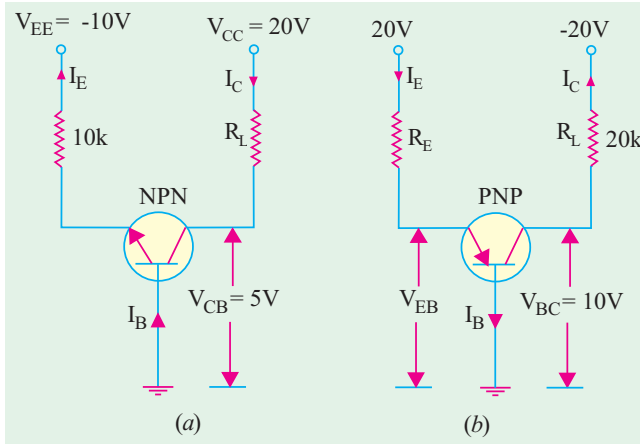


Fig. 57.27

**Solution.** 
$$I_C = \frac{V_{CC} - V_{BC}}{R_L}$$

$$= \frac{20 - 10}{20 \text{ K}} = 0.5 \text{ mA}$$

Now,  $I_E = I_C / \alpha \cong I_C = 0.5 \text{ mA}$

If we neglect  $V_{BE}$ , then entire  $V_{EE} = 20 \text{ V}$  has to be dropped across  $R_E$   
 $\therefore 0.5 R_E = 20$

or  $R_E = 20 / 0.5 \text{ mA} = 40 \text{ K}$

**57.22. Common Emitter Formulas**

Consider the CE circuit of Fig. 57.28. Taking the emitter-base circuit, we have

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} \cong \frac{V_{BB}}{R_B}$$

$$I_C = \beta I_B \quad \text{--- neglecting leakage current } I_{CEO}$$

$$V_{CE} = V_{CC} - I_C R_L$$

**Example 57.10.** For the circuit of Fig. 57.28, find (i)  $I_B$  (ii)  $I_C$  (iii)  $I_E$  and (iv)  $V_{CE}$ . Neglect  $V_{BE}$

**Sol. (i)**  $I_B \cong \frac{V_{BB}}{R_B} = \frac{10}{1M} = 10 \mu\text{A}$

**(ii)**  $I_C = \beta I_B = 100 \times 10 \mu\text{A} = 1 \text{ mA}$

**(iii)**  $I_E = I_B + I_C = 1 \text{ mA} + 10 \mu\text{A} = 1.01 \text{ mA}$

**(iv)**  $V_{CE} = V_{CC} - I_C R_L = 15 - 1 \times 10 = 5 \text{ V}$

**Example 57.11.** Find the exact value of emitter current  $I_E$  in the two-supply emitter bias circuit of Fig. 57.29.

(Electronics-1, Bangalore Univ. 1989)

**Solution.** Let us apply Kirchoff's voltage law to the loop containing  $R_B$ ,  $R_E$  and  $V_{EE}$ . Starting from emitter and going clock-wise, we get

$$-I_E R_E + V_{EE} - I_B R_B - V_{BE} = 0$$

or  $I_E R_E + I_B R_B = V_{EE} - V_{BE}$  ... (i)

Now  $\beta = I_C / I_B \cong I_E / I_B \therefore I_B \cong I_E / \beta$

Substituting this value in Eq. (i) above, we get

$$I_E R_E + \frac{I_E R_B}{\beta} = V_{EE} - V_{BE} \quad \text{or}$$

$$I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B / \beta}$$

Since, in most cases,  $(R_B / \beta) \ll R_E$

$$\therefore I_E = (V_{EE} - V_{BE}) / R_E \cong V_{EE} / R_E$$

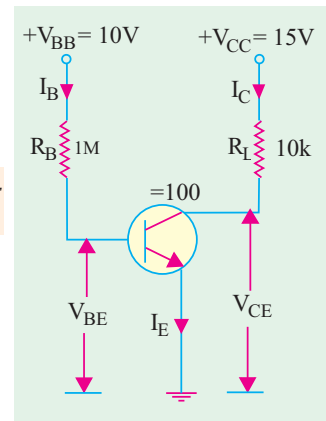


Fig. 57.28

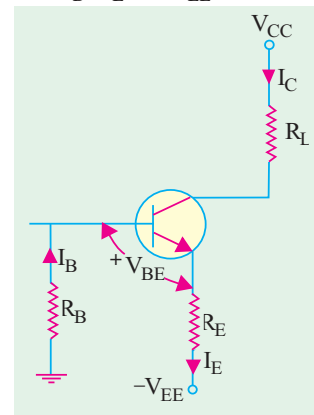


Fig. 57.29



Also,  $I_B = I_E / (1 + \beta) \cong I_E / \beta$

**Example 57.12.** In the circuit of Fig. 57.30, find (i)  $I_E$  (ii)  $I_B$  (iii)  $I_C$  and (iv)  $V_{CE}$ . Neglect  $V_{BE}$  and take  $\beta = 100$ .

- Sol. (i)**  $I_E = \frac{V_{EE}}{R_E + R_B / \beta} = \frac{30}{30 + 20 / 100} \cong 1 \text{ mA}$   
**(ii)**  $I_B \cong I_E / \beta = 1 / 100 = 0.01 \text{ mA}$   
**(iii)**  $I_C = I_E - I_B = 1 - 0.01 = 0.99 \text{ mA}$   
**(iv)**  $V_{CE} = V_{CC} - I_C R_L = 30 - 10 \times 0.99 = 20.1 \text{ V}$ .

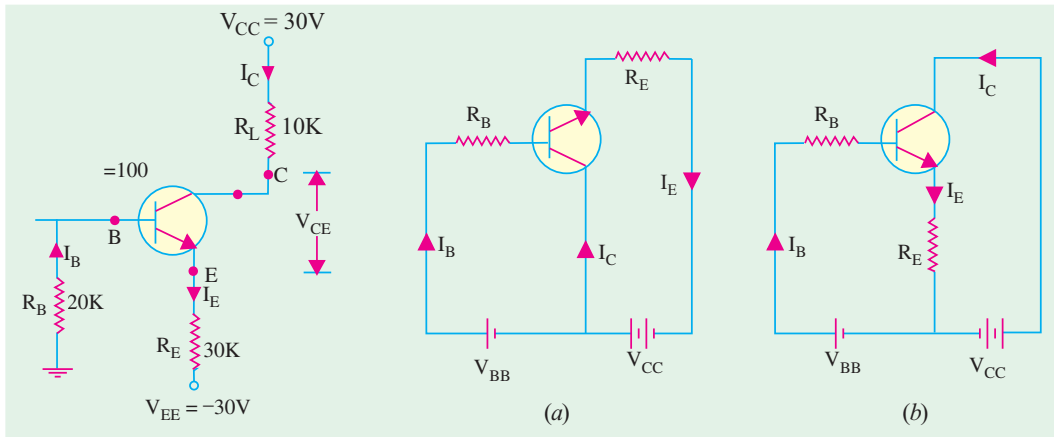


Fig. 57.30

Fig. 57.31

**57.23. Common Collector Formulas**

The CC circuit with its proper d.c. biasing voltage sources is shown in Fig. 57.31 (a). The two circuits given in Fig. 57.31 represent the same thing.

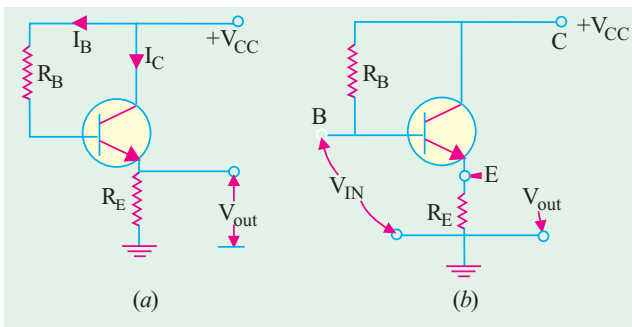


Fig. 57.32

Another way of drawing the same circuit is shown in Fig. 57.32 (a) where only one battery has been used. It should be noted that load resistor is not in the collector lead but in the emitter lead as shown.

Fig. 57.32 (b) makes the circuit connection quite clear. Input is between base and collector terminals whereas output is between emitter and collector terminals.

It is seen that

$$I_E = \frac{V_{CC} - V_{BE}}{R_E + R_B / \beta}; \quad V_{CC} = V_{CE} + I_E R_E; \quad I_E = \frac{V_{CC} - V_{BE}}{R_E + \beta R_B}; \quad I_C = \beta I_B$$

**Example 57.13.** In the CC circuit of Fig. 57.33, find (a)  $I_B$ , (b)  $I_E$ , (c)  $V_{CE}$ , (d)  $V_E$  and (e)  $V_B$

Take  $\beta = 49$  and  $V_{BE} = 0.7 \text{ V}$ .





**Solution.** (a) 
$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (1 + \beta) R_E}$$

$$= \frac{9.0 - 0.7}{100 + 50 \times 2} = 41.5 \mu\text{A}$$

(b)  $I_E = (1 + \beta) I_B = 50 \times 41.5 = 2.075 \text{ mA}$

(c)  $V_{CE} = V_{CC} - I_E R_E = 9 - 2.075 \times 2 = 5.85 \text{ V}$

(d)  $V_E = I_E R_E = 2.075 \times 2 = 4.15 \text{ V}$

(e)  $V_B = V_{BE} + I_E R_E = 0.7 + 4.15 = 4.85 \text{ V}$

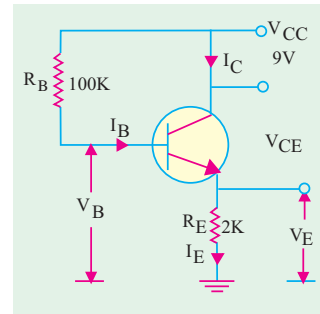


Fig. 57.33

**57.24. The Beta Rule**

According to this rule, resistance from one part of a transistor circuit can be referred to another of its parts (as we do with the primary and secondary winding impedances of a transformer). For example, resistance  $R_L$  in the collector circuit can be referred to the base circuit and *vice versa*. Similarly,  $R_E$  can be referred to the base circuit and, reciprocally,  $R_B$  can be referred to the emitter circuit. Since current through  $R_L$  is  $I_C (= \beta I_B)$ , hence  $\beta$ -factor comes into the picture. Similarly, current through  $R_E$  is  $I_E$  which is  $(1 + \beta)$  times  $I_B$ , hence  $(1 + \beta)$  or approximately  $\beta$ -factor comes into the picture again. Use of this ‘ $\beta$ -rule’ makes transistor circuit calculations quite quick and easy. It makes the calculation of  $I_B$  quite simple.

The ‘ $\beta$ -rule’ may be stated as under :

1. When referring  $R_L$  or  $R_C$  to the base circuit, **multiply** it by  $\beta$ . When referring  $R_B$  to the collector circuit, **divide** it by  $\beta$ .
2. When referring  $R_E$  to base circuit, **multiply** it by  $(1 + \beta)$  or just  $\beta$  (as a close approximation).
3. Similarly, when referring  $R_B$  to emitter circuit, **divide** it by  $(1 + \beta)$  or  $\beta$ .

Before you apply this rule to any circuit, you must remember one very important point otherwise you are likely to get wrong answers. The point is that **only those resistances are transferred which lie in the path of the current being calculated**. Not otherwise. The utility of this rule will be demonstrated by solving the following problems.

**Example 57.14.** Calculate the value of  $V_{CE}$  in the collector stabilisation circuit of Fig. 57.34.

**Solution.** We will use  $\beta$ -rule to find  $I_C$  in the following two ways.

**(i) First Method**

Here, we will transfer  $R_L$  to the base circuit.

$$I_B = \frac{V_{CC}}{R_B + \beta R_L} = \frac{20}{1000 + 100(10)} = 10 \text{ mA}$$

$$I_C = \beta I_B = 100 \times 10 = 1000 \text{ mA} = 1 \text{ A}$$

$$V_{CE} \cong V_{CC} - I_C R_L = 20 - 1 \times 10 = 10 \text{ V}$$

**(ii) Second Method**

Now, we will refer  $R_B$  to collector circuit.

$$I_C \cong \frac{V_{CC}}{R_L + R_B / \beta} = \frac{20}{10 + 1000 / 100} = 1 \text{ mA}$$

$$V_{CE} = V_{CC} - I_C R_L = 10 \text{ V} \quad \text{— as above}$$

It was a simple circuit because  $R_E = 0$  and  $R_B$  was connected to  $V_{CC}$  through  $R_L$  and not directly (in which case,  $R_L$  would not lie in the path of  $I_B$ ). Now, we will consider the case when  $R_E$  is present and  $R_L$  does not lie in the path of  $I_B$ .

**Example 57.15.** Calculate the three transistor currents in the circuit of Fig. 57.35.

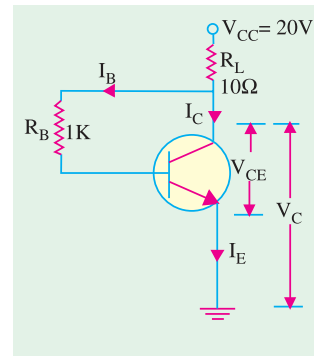


Fig. 57.34



**Solution. (j) First Method**

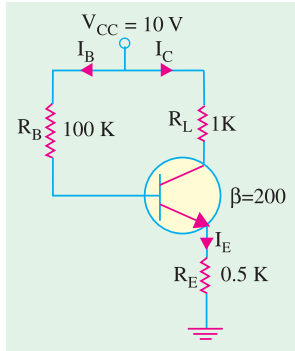


Fig. 57.35

Since  $R_E$  lies in the path of  $I_B$

$$\therefore I_B = \frac{V_{CC}}{R_L + \beta R_E} \quad \text{--- neglecting } V_{BE}$$

$$= \frac{10}{100 + 200(0.5)} = \mathbf{0.05 \text{ mA}}$$

$$I_C = \beta I_B = 200 \times 0.05 = \mathbf{10 \text{ mA}}, \quad I_E = I_B + I_C = \mathbf{10.05 \text{ mA}}$$

**(ij) Second Method**

Now, we will transfer  $R_B$  to emitter circuit and find  $I_E$  directly.

$$I_E = \frac{V_{CC}}{R_E + R_B/\beta} = \frac{10}{0.5 + 100/200} = \mathbf{10 \text{ mA}} \quad \text{--- as before}$$

$$I_B = I_C/\beta \cong 10/200 = \mathbf{0.05 \text{ mA}}$$

**Example 57.16.** Calculate  $I_E$  in the circuit of Fig. 57.36.

(Electronic & Commu., Ranchi Univ. 1990)

**Solution.** If we neglect  $V_{BE}$  then as seen from the circuit of Fig. 57.36.

$$I_E = \frac{V_{EE}}{R_E + R_B/\beta} = \frac{10}{10 + 10/100} = \mathbf{0.99 \text{ mA}}$$

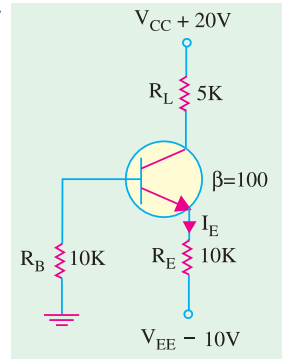


Fig. 57.36

**57.25. Importance of  $V_{CE}$**

The voltage  $V_{CE}$  is very important in checking whether the transistor is

- (a) defective,
- (b) working in cut-off,
- (c) in saturation or well into saturation (Example 57.17 and 57.18)

When  $V_{CE} = V_{CC}$  the transistor is in cut-off *i.e.* it is turned OFF.

When  $V_{CE} = 0$ , the transistor is in saturation *i.e.* it is turned fully ON.

When  $V_{CE}$  is less than zero *i.e.* negative, the transistor is said to be well into saturation. In practice, both these conditions are avoided. For am-

pplier operation,  $V_{CE} = \frac{1}{2} V_{CC}$  *i.e.* transistor is operated at approximately  $\frac{1}{2}$  ON. In this way, varia-

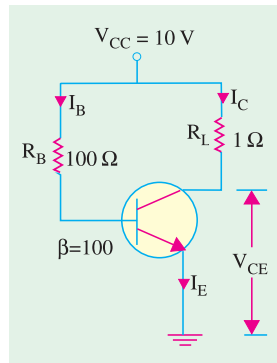


Fig. 57.37

tions in  $I_B$  in either direction will control  $I_C$  in both directions. In other words, when  $I_B$  increases or decreases,  $I_C$  also increases or decreases. However, if  $I_B$  is OFF,  $I_C$  is also OFF. On the other hand, if collector has been turned fully ON, maximum  $I_C$  flows. Hence, no further increase in  $I_B$  can be reflected in  $I_C$ .

**Example 57.17.** For the CE circuit of Fig. 57.37, find the value of  $V_{CE}$ . Take  $\beta = 100$  and neglect  $V_{BE}$ . Is the transistor working in cut-off or saturation?

**Solution.**

$$I_B = 10/100 = 0.1 \text{ A}$$

$$I_C = \beta I_B = 100 \times 0.1 = 10 \text{ A}$$

$$V_{CE} = V_{CC} - I_C R_L = 10 - 10 \times 1 = 0$$

Obviously, the transistor is operating just at saturation and not well into saturation.

**Example 57.18.** Find out whether the transistor of Fig. 57.38 is working in saturation or well into saturation. Neglect  $V_{BE}$ .

(Basic Electronics, Bombay Univ.)

**Solution.**

$$I_B = 10/10 = 1 \text{ A}$$

$$I_C = 100 \times 1 = 100 \text{ A}$$



Obviously,  $I_C$  cannot be that large because its maximum value is given by  $V_{CC}/R_L = 10/1 = 10$  A. However, let us assume that  $I_C$  takes this value temporarily. Then,

$$V_{CE} = V_{CC} - I_C R_L = 10 - 100 \times 1 = -90 \text{ V}$$

It means that the transistor is **working well into saturation**.

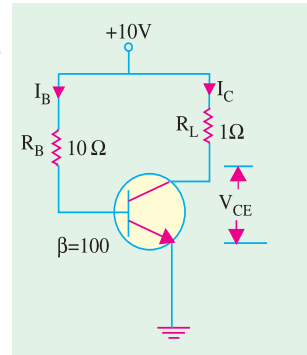


Fig. 57.38

### 57.26. Cut-Off And Saturation Points

Consider the circuit of Fig. 57.39 (a). As seen from Art 57.22,

$$V_{CE} = V_{CC} - I_C R_L$$

Since,  $I_B = 0, \therefore I_C = 0.$

Hence,  $V_{CE} = V_{CC}$

Under these conditions, the transistor is said to be cut-off for the

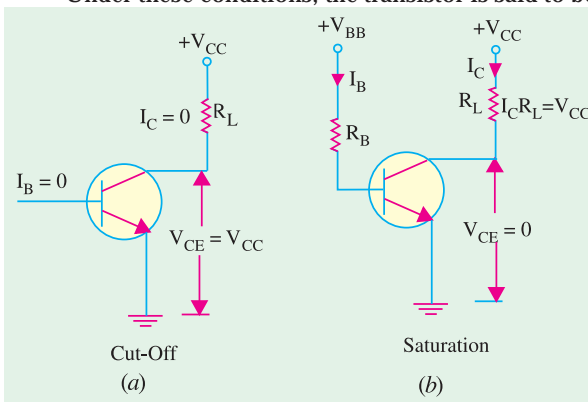


Fig. 57.39

simple reason that it does not **conduct any current**. This value of  $V_{CE}$  is written as  $V_{CE(\text{cut-off})}$ . Incidentally, a transistor when cut-off acts like an open switch.

If, in Fig. 57.39 (b), values of  $R_B$  and  $R_L$  are such that  $V_{CE}$  comes out to be zero, then transistor is said to be saturated. Putting  $V_{CE} = 0$  in the above equation, we get

$$0 = V_{CC} - I_C R_L$$

or  $I_C = V_{CC} / R_L$

It should be noted that a transistor, when saturated, acts as a closed switch of negligible resistance.

It is obvious that under saturation

condition,

- (i) whole of  $V_{CC}$  drops across  $R_L$ .
- (ii) collector current has maximum possible value called  $I_C(\text{sat})$ .

Normal operation of a transistor lies between the above two extreme conditions of cut-off and saturation.

**Example. 57.19.** In a simple amplifier circuit (Fig. 57.40) with base resistance,  $R_B = 50 \text{ K}$ ,  $R_E = 2 \text{ K}$ ,  $R_C = 3 \text{ K}$ ,  $V_{CC} = 10 \text{ V}$ ,  $h_{FE} = 100$ , determine whether or not the silicon transistor is in the saturation and find  $I_B$  and  $I_C$ . Explain the saturation region in common-emitter characteristics.

(Electronics, MS. Univ. Baroda,)

**Solution.** Whether the transistor is in saturation or not will depend on the value of  $V_{CE}$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B / \beta} \cong \frac{V_{BB}}{R_E + R_B / \beta}$$

$$= \frac{5}{2 + 50/100} = 2 \text{ mA}$$

$$I_C \cong I_E = 2 \text{ mA}; I_B = I_C / \beta = 2/100 = 0.02 \text{ mA}$$

Now,  $V_{CC} = I_C R_C + V_{CE} + I_E R_E$   
or  $V_{CE} = 10 - (2 \times 3) - (2 \times 2) = 0$

Obviously, the transistor has entered saturation.

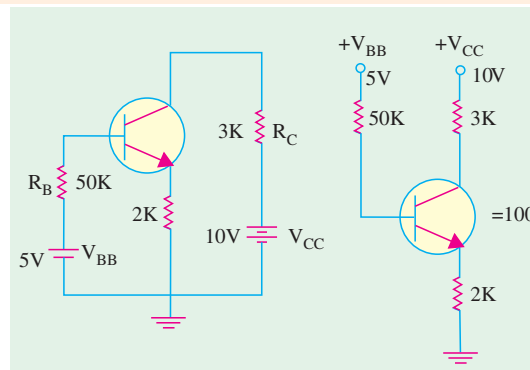


Fig. 57.40



57.27. BJT Operating Regions

A BJT has two junctions *i.e.* base-emitter and base-collector junctions either of which could be forward-biased or reverse-biased. With two junctions, there are four possible combinations of bias condition.

- (i) both junctions reverse-biased,
- (ii) both junctions forward-biased,
- (iii) BE junction forward-biased, BC junction reverse-biased.
- (iv) BE junction reverse-biased, BC junction forward-biased.

Since condition (iv) is generally not used, we will tabulate the remaining three conditions below.

Table No. 57.1: Transistor Operation Regions		
BE Jn	BC Jn	Region
RB*	RB	cut-off
FB**	FB	saturation
FB	RB	active

\* Reverse-biased,      \*\* Forward-biased

(a) Cut-off

This condition corresponds to reverse-bias for both base-emitter and base-collector junctions. In fact, both diodes act like open circuits under these conditions as shown in Fig. 57.41, which is true for an ideal transistor. The reverse leakage current (Art 57.12) has been neglected. As seen, the three transistor terminals are uncoupled from each other. In cut-off,  $V_{CE} = V_{CC}$ .

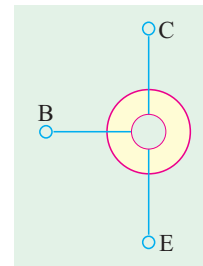


Fig. 57.41

(b) Saturation

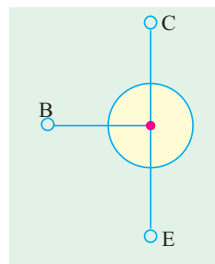


Fig. 57.42

This condition corresponds to forward-bias for both base-emitter and base-collector junctions. The transistor becomes saturated *i.e.* there is perfect short-circuit for both base-emitter and base-collector diodes. The ideal case is shown in Fig. 57.42, where the three transistor terminals have been connected together thereby acquiring equal potentials. In this case,  $V_{CE} = 0$ .

(c) Active Region

This condition corresponds to forward-bias for base-emitter junction and reverse bias for base-collector junction. In this,  $V_{CE} > 0$ .

57.28. Active Region DC Model of a BJT

Such a model is used for predicting transistor operation in the active region. This condition is shown in Fig. 57.43 both for a PNP and an NPN transistor. A base-emitter junction voltage of 0.7 V has been assumed for silicon transistor. The BE junction is represented by a constant voltage source since it is forward-biased. As seen, in an NPN transistor, base is 0.7 V higher than the emitter terminal. However, in a PNP transistor, base is 0.7 V lower than the emitter terminal.

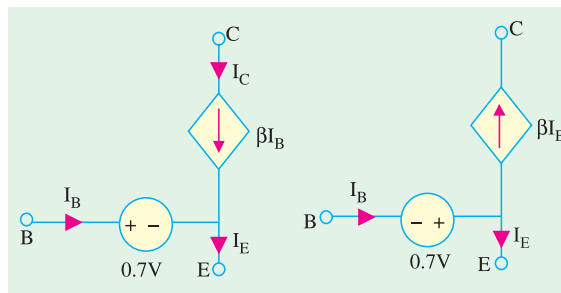


Fig. 57.43

To account for the effect of base control, a current source of  $\beta I_B$  is placed between collector and base terminals. It is called a dependent or controlled source because it is a function of a variable in another circuit. It may be noted that  $I_E = (I_B + I_C)$  in both cases.



**57.29. BJT Switches**

Very often, bipolar junction transistors are used as electronic switches. With the help of such a switch, a given load can be turned ON or OFF by a small control signal. This control signal might be the one appearing at the output of a digital logic or a microprocessor. The power level of the control signal is usually very small and, hence, it is incapable of switching the load directly. However, such a control signal is certainly capable of providing enough base drive to switch a transistor ON or OFF and, hence, the transistor is made to switch the load.

When using BJT as a switch, usually two levels of control signal are employed. With one level, the transistor operates in the cut-off region (open) whereas with the other level, it operates in the saturation region and acts as a short-circuit. Fig. 57.44 (b) shows the condition when control signal  $v_i = 0$ . In this case, the  $BE$  junction is reverse-biased and the transistor is open and, hence acts as an open switch. However, as shown in Fig. 57.44 (c) if  $v_i$  equals a positive voltage of sufficient magnitude to produce saturation *i.e.* if  $v_i = v_i$  the transistor acts as a closed switch.

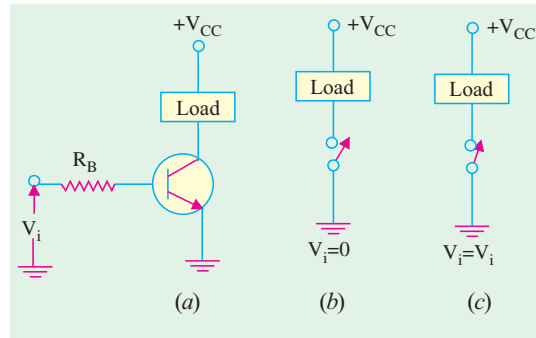


Fig. 57.44

Fig. 57.45 shows a form of series switching circuit utilizing an *NPN* transistor with a negative dc supply and a control signal voltage having levels of zero and  $-v_i$ .

**Example 57.20.** The circuit of Fig. 57.46 is designed to produce nearly constant current through the variable collector load resistance. An ideal 6V source is used to establish the current. Determine (a) value of  $I_C$  and  $V_E$ , (b) range of  $R_C$  over which the circuit will function properly. Assume silicon transistor and a  $\beta$  large enough to justify the assumptions used.

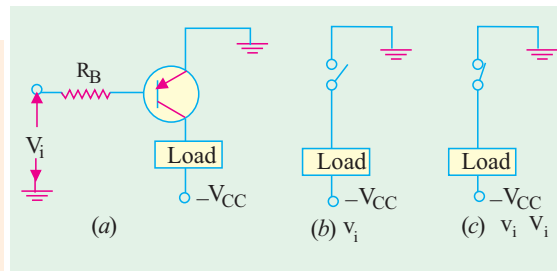


Fig. 57.45

(Applied Electronics-II, Punjab Univ. 1993)

**Solution. (a)**  $I_C \cong I_E = (6 - 0.7)/530 = 10 \text{ mA}$   
 $V_E = 6 - 530 \times (10 \times 10) = 5.3 \text{ V}.$

This voltage will remain constant so long as transistor operation is confined to active region.

**(b) When  $R_C = 0$**

$$V_{CE} = 12 - 5.3 = 6.7 \text{ V}$$

It is certainly well within the active region. As  $R_C$  increases, its drop increases and hence,  $V_{CE}$  decreases. There will be some value of  $R_C$  at which active region operation ceases.

Now,  $V_{CE} = 12 - 5.3 - I_C R_C = 6.7 - I_C R_C$

Value of  $R_{C(max)}$  can be found by putting  $V_{CE} = 0$

$$\therefore 0 = 6.7 - I_C R_{C(max)}$$

or  $R_{C(max)} = 6.7/I_C = 6.7/0.01 = 670 \Omega$

Hence, circuit will function as a constant current source so long as  $R_C$  is in the range  $0 < R_C < 670 \Omega$ . When  $R_C$  exceeds  $670 \Omega$ , the *BJT* becomes saturated.

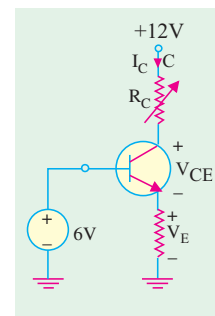


Fig. 57.46



57.30. Normal DC Voltage Transistor Indications

For a transistor to operate as an amplifier, it is desirable that  $V_{CE} = \frac{1}{2} V_{CC}$ . However, in actual practice, wide tolerances are allowed. Generally,  $V_{CE}$  varies between 25% to 75% of  $V_{CC}$ . Any transistor amplifier with  $V_{CE} = V_{CC}$  is either open or is operating in cut-off. When operating with  $V_{CE}$  near cut-off, the amplifier causes lot of distortion. Same is the case when  $V_{CE}$  is nearly zero. Hence, any transistor amplifier with  $V_{CE}$  more than 75%  $V_{CC}$  or less than 25%  $V_{CC}$  should be suspected of having a problem and further investigated.

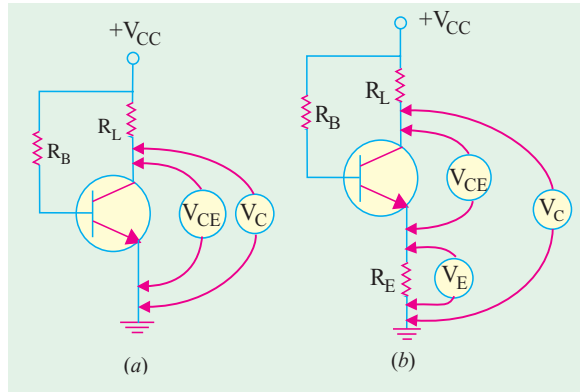


Fig. 57.47

In the circuit shown in Fig. 57.47 (a),  $V_{CE}$  should be in the range 25 – 75% of  $V_{CC}$ . In the circuit of Fig. 57.47 (b),  $V_{CE}$  may be normal but either  $R_L$  or  $R_E$  could be shorted. Hence,  $V_C$  and  $V_E$  should be measured separately. Moreover,  $V_{CE}$  could be found by subtracting  $V_E$  from  $V_C$ .

For the circuit of Fig. 57.47 (b), the normal voltmeter readings are

$$V_{CE} = \frac{1}{2} V_{CC} ; \quad V_E = \frac{1}{4} V_{CC} ; \quad V_C = \frac{3}{4} V_{CC}$$

If instead of  $R_L$ , there is a low-resistance coil in the circuit, then

$$V_{CE} = \frac{1}{2} V_{CC} ; \quad V_E = \frac{1}{2} V_{CC} ; \quad V_C = V_{CC}$$

57.31. Transistor Fault Location

Voltage measurements are employed in the vast majority of trouble situations because current measurements are comparatively difficult to make. Magnitude of  $V_{CE}$  is of great diagnostic value in finding and locating faults in a transistor circuit. Following possibilities are considered :

(a)  $V_{CE} = 0$

Possibilities are that the transistor is

1. shorted out,
2. operating in saturation,
3. disconnected from  $V_{CC}$

(b)  $V_{CE} = \frac{1}{2} V_{CC}$

It shows that the circuit is operating normally and is well-designed.

(c)  $V_{CE} = V_{CC}$

Possibilities are that the transistor is

1. open-circuited,
2. operating in out-off
3. having all resistors in series with  $V_{CE}$  shorted.

**Example 57.21.** Compute the value of  $V_{CE}$  for the CE circuit shown in Fig. 57.48.

**Solution.** Since the collector is disconnected from the supply due to ‘open’ in the

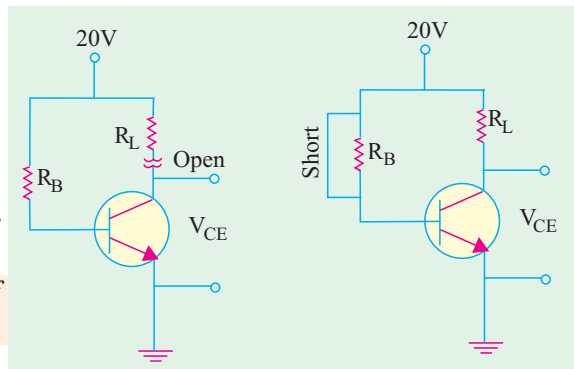


Fig. 57.48

Fig. 57.49



circuit,  $V_{CE} = 0$ . It represents fault condition No. (a) 3 in Art. 57.31.

**Example 57.22.** What is the value of  $V_{CE}$  in the CE circuit of Fig. 57.49.  
**Solution.** Since  $R_B$  is shorted out,  $I_B$  would increase and probably burn out the E/B junction. But this burn out is not indicated in the question. Hence, with high base current, the transistor is operating in saturation so that  $V_{CE} = 0$ .

**Example 57.23.** What is the value of  $V_{CE}$  in the circuit of Fig. 57.50.  
**Solution.** Since  $R_L$  is the only resistor in series with the transistor and is shorted out, it means that there is no voltage drop anywhere. Hence,  $V_{CE} = V_{CC}$ . It represents fault No. (c) 3 stated in Art. 7.31 above.

**Example 57.24.** Find the possible value of  $V_{CE}$ ,  $V_C$  and  $V_E$  for the circuit shown in Fig. 57.51.

**Solution.** In the circuit of Fig. 57.51, there is neither a short nor an open and the voltage polarities are correct for an NPN transistor. It looks like a well-designed circuit operating normally. Hence, according to Art. 57.30.

$$V_{CE} = \frac{1}{2} V_{CC} = 10 \text{ V}; \quad V_E = \frac{1}{4} V_{CC} = 5 \text{ V}$$

$$V_C = \frac{3}{4} V_{CC} = 15 \text{ V}$$

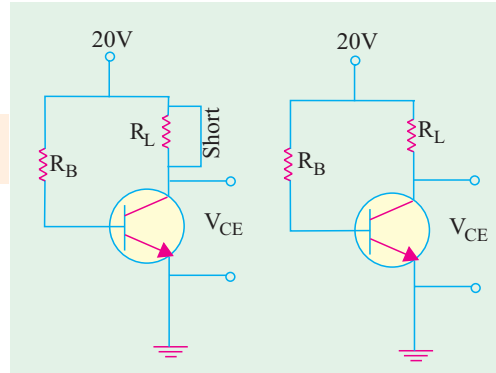


Fig. 57.50

Fig. 57.51

**Example 57.25.** Find the values of  $V_C$ ,  $V_E$  and  $V_{CE}$  in the circuit of Fig. 57.52.

**Solution.** Since  $I_B = 0$ , transistor is cut off.  
 Hence,  $V_E = 0$   
 Also  $V_C = -20 \text{ V}$   
 and  $V_{CE} = -20 \text{ V}$

**Example 57.26.** What would be the values of  $V_C$ ,  $V_E$  and  $V_{CE}$  for the circuit shown in Fig. 57.53.

**Solution.** Since emitter is open, no current flows in any part of the circuit. The transistor is essentially cut off. Without IR drops, all points above the emitter are at 30 V.

$$\therefore \begin{aligned} V_E &= 30 \text{ V}; \\ V_C &= 30 \text{ V} \text{ and} \\ V_{CE} &= 0 \text{ V} \end{aligned}$$

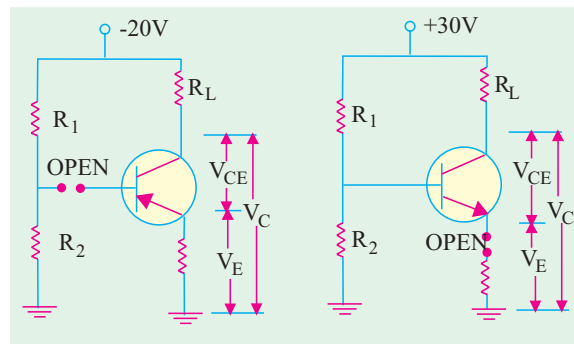


Fig. 57.52

Fig. 57.53

### 57.32. Solving Universal

#### Stabilization Circuit

Such a circuit is shown in Fig. 57.54 in which  $R_E$  appears to be in parallel with  $R_2$ . But according to the  $\beta$ -rule (Art 57.24),  $R_2$  is actually in parallel with  $\beta R_E$ . In a well-designed circuit, the resistance  $\beta R_E$  is much larger than  $R_2$ . Hence, their combined resistance =  $R_2 \parallel \beta R_E \cong R_2$ . On this assumption as well as another that  $I_B$  is practically zero, we can find voltage drop across  $R_2$  by the Proportional Voltage Formula. Since  $V_{CC}$  is applied across  $R_1 - R_2$  potential divider circuit, drop across  $R_2$ .

$$= V_{CC} \cdot R_2 / (R_1 + R_2)$$

If we neglect  $V_{BE}$  then this drop equals  $V_E$



$$\therefore V_E \cong V_{CC} \frac{R_2}{R_1 + R_2}$$

and  $I_E \cong \frac{V_E}{R_E}$

Having found  $I_E$ , other currents and voltage drops can be easily found.

$$V_{EE} = V_{CC} - I_C R_L - I_E R_E$$

Since,  $I_E \cong I_C$

$$\therefore V_{CE} = V_{CC} - I_E R_L - I_E R_E = V_{CC} - I_E (R_L + R_E)$$

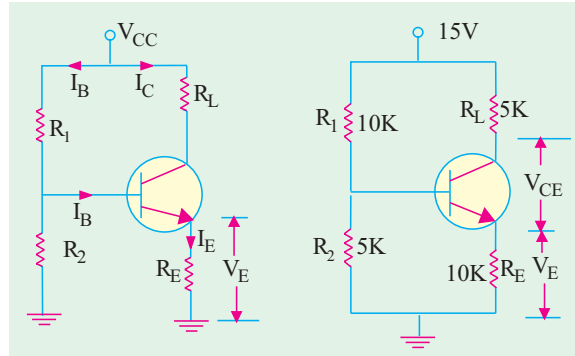


Fig. 57.54

Fig. 57.55

**Example 57.27.** Find  $V_{CE}$  and  $V_E$  for the circuit shown in Fig. 57.55. Neglect  $V_{BE}$

**Solution.** As explained above

$$V_E = V_2 = V_{CC} \frac{R_E}{R_1 + R_2} = 15 \times \frac{5}{15} = 5 \text{ V}$$

$$I_E = V_E / R_E = 5 \text{ V} / 10 \text{ K} = 0.5 \text{ mA}$$

$$I_C \cong I_E = 0.5 \text{ mA}, V_{CE} = V_{CC} - I_E (R_L + R_E) = 15 - 0.5 \times 15 = 7.5 \text{ V}$$

### 57.33. Notation for Voltages and Currents

In order to avoid confusion while dealing with dc and ac voltages and currents, following notation will be employed :

**1. For d.c. or non-time-varying quantities**

We will use capital letters with capital subscripts such as

$I_E, I_B, I_C$  — for dc currents

$V_E, V_B, V_C$  — for dc voltages to ground

$V_{BE}, V_{CB}, V_{CE}$  — for dc potential differences

$V_{EE}, V_{CC}, V_{BB}$  — for dc source or supply voltages

**2. For ac quantities**

We will use the following symbols :

$i_e, i_b, i_c$  — for instantaneous values of ac currents

$I_e, I_b, I_c$  — for r.m.s values of a.c. currents

$v_e, v_b, v_c$  — for instantaneous values of a.c. voltages to ground

$v_{be}, v_{eb}, v_{ce}$  — for a.c. voltage differences

**3. Total ac and dc voltages and currents**

In this case, we will use a hybrid notation. For example,  $i_E$  will be used to represent the total emitter current, i.e. sum of dc and ac currents in the emitter.

Fig. 57.56 illustrates the notation discussed above.

### 57.34. Increase/Decrease Notation

This notation is very helpful in analysing transistor operation when ac signal is applied to it. It is simply this:

↑ means increases and ↓ means decrease.

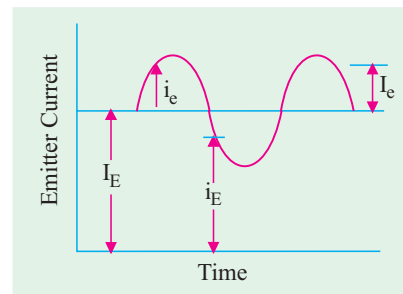


Fig. 57.56





As an illustration, consider the transistor circuit of Fig. 57.57. If  $V_{BB}$  were increased ( $\uparrow$ ),  $I_B$  would increase ( $\uparrow$ ). This would increase  $I_C$  ( $\uparrow$ ) because it equals  $\beta I_B$ . The drop  $I_C R_L$  would increase ( $\downarrow$ ) and, hence,  $V_{CE}$  will decrease ( $\downarrow$ ) because  $V_{CE} = V_C - I_C R_L$ .

Using increase/decrease notation, the above sequence of changes can be written as

$$V_{BB} \uparrow, I_B \uparrow, I_C \uparrow, I_C R_L \uparrow, V_{CE} \downarrow$$

At one look, we can straight away say that as input voltage is increased, output voltage is decreased.

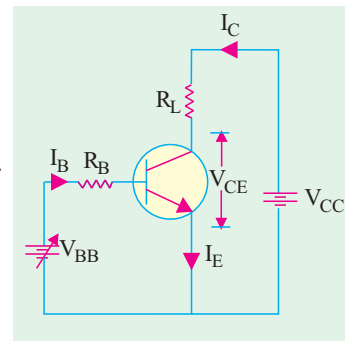


Fig. 57.57

### 57.35. Applying AC to a DC Biased Transistor

Suppose we want to apply an ac signal to the input emitter-base circuit of a properly-biased transistor shown in Fig. 57.58. If we apply the a.c. source directly across the *EBJ* as shown in Fig. 57.58 (a), it will upset the d.c. bias. It should be kept in mind that most **ac signal sources are nearly a short to dc**. Hence, nearly whole of  $I_B$  would pass through a.c. source rather than the base thereby spoiling the transistor bias.

In order to connect the ac source and at the same time not upset the d.c. bias, the ac source is connected via a coupling capacitor  $C$  as shown in Fig. 57.58 (b). This capacitor acts as an 'open' for dc but almost a short for ac source provided it is of sufficiently large capacitance.

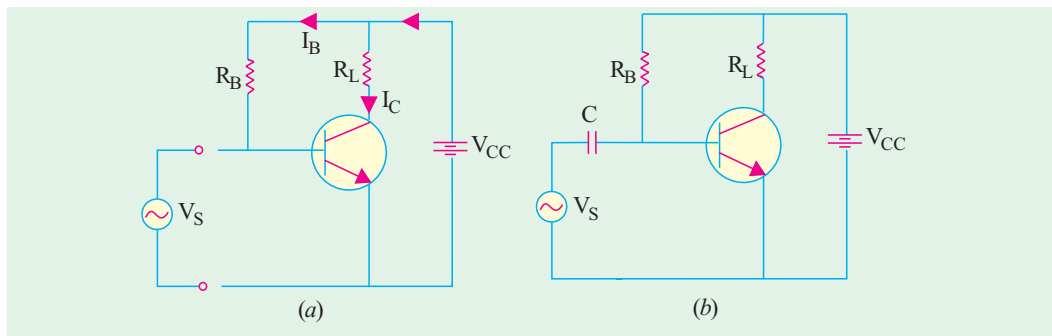


Fig. 57.58

### 57.36. Transistor AC/DC Analysis

In Fig. 57.59 is shown a *CE* amplifier circuit having an ac signal voltage  $v_{be}^*$  applied across its *E/B* junction. This voltage will be added to the dc voltage  $V_{BE}$  as if the two were connected in series. The resultant voltage is shown in Fig. 57.59 (b) which shows ac voltage riding the d.c. level. The variations in the resultant output voltage  $V_{CE}$  [Fig. 57.59 (b)] can be expressed in terms of the increase/decrease notation. It will be assumed that  $V_{BE}$  is such as to bias  $V_{CE}$  at  $V_{CC}$  when no a.c. signal is applied.

#### (i) First Quarter Cycle

In the first quarter-cycle of the input signal, both  $V_{BE}$  and  $V_{BE}$  increase thereby giving rise to the following sequence of changes :

$$V_{BE} \uparrow, i_B \uparrow, i_C \uparrow, i_C R_L \uparrow, V_{CE} \downarrow$$

Hence, output voltage decreases as shown in Fig. 57.59 (c)

#### (ii) Second Quarter Cycle

Here,  $V_{bc}$  as well as  $V_{BE}$  decrease. Hence,

$$V_{BE} \downarrow, i_B \downarrow, i_C \downarrow, i_C R_L \downarrow, V_{CE} \downarrow$$

\* Normally, we will use the notation  $v_i$  or  $c_{in}$  or  $c_i$  while discussing amplifiers.



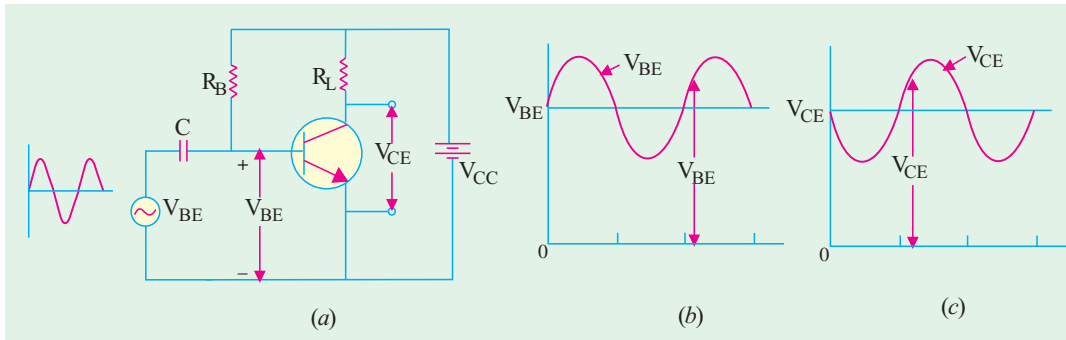


Fig. 57.59

Again,  $V_{CE}$  does the opposite of  $V_{BE}$

Same changes will happen in third quarter cycle as happened in the first quarter-cycle and so on. It is seen from Fig. 57.60 (c) that output ac voltage is  $180^\circ$  out of phase with the input voltage.

**Example 57.28.** Calculate the value of  $V_{CE}$  in the circuit of Fig. 57.60 (a) if a.c. signal voltage is sinusoidal with a peak value of 0.01 V. Take voltage gain  $A_v$  of the circuit as 100 and  $\beta = 100$ . Depict the waveform of the output voltage separately.

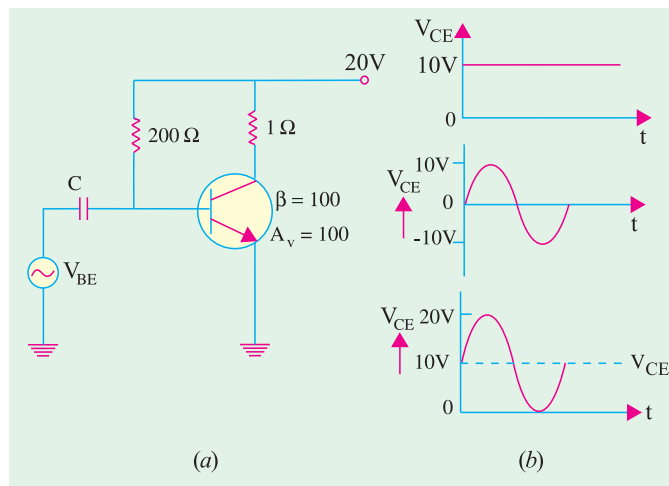


Fig. 57.60

**Solution.**

$$I_B = \frac{V_{CC}}{R_B} = \frac{20}{200} = 0.1A$$

$$I_C = \beta I_B = 100 \times 0.1 = 10 A$$

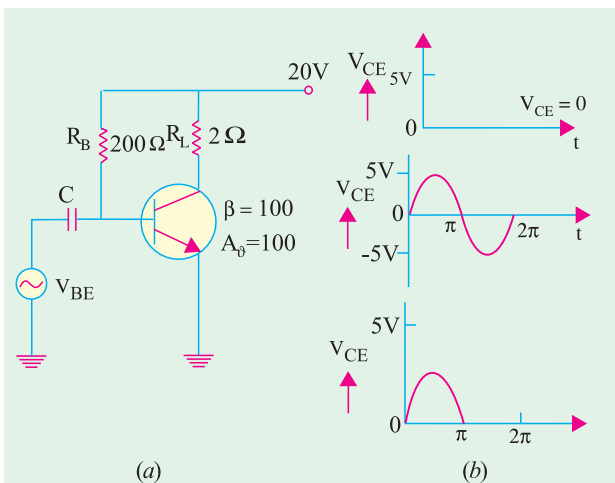


Fig. 57.61

$$V_{CE} = V_{CC} - I_C R_L = 20 - 10 \times 1 = 10 V;$$

$$v_{CE} = A_v v_{BE} = 100 \times 0.01 = 10 V$$

The combined output voltage  $v_{CE}$  is the sum of  $v_{CE}$  and  $v_{CE}$  and is shown graphically in Fig. 57.60 (b). It is seen that 100 times amplified ac signal rides the dc voltage.

**Example 57.29.** Find  $v_{CE}$  in the circuit of Fig. 57.61 (a) and sketch its waveform. Take  $A_u = 100$  and  $\beta = 100$  and peak input signal voltage as 0.05 V.

**Solution.**  $I_B = 20/200 = 0.1 A$  ;  $I_C = 100 \times 0.1 = 10 A$  ;  $V_{CE} = 20 - (10 \times 2) = 0.$



Obviously, the transistor has been biased at saturation as shown in Fig. 57.61 (b).  
 The addition of  $v_{CE}$  and  $v_{CE}$ , is shown graphically in Fig. 57.61 (b). During the positive half-cycle of the signal, the transistor comes out of saturation and lets pass the half-cycle. However, during the negative half-cycle of input signal, transistor is further driven into saturation. Since it is already biased at  $V_{CC}$ 's most negative limit (0 volt), it cannot further go negative. Hence, the negative half-cycle of the signal is lost in saturation.

**Tutorial Problems No. 57.1**

1. A CB-connected transistor has  $\alpha = 0.96$  and  $I_E = 2$  mA. Find its  $I_C$  and  $I_B$ . [1.92 mA, 80  $\mu$ A]
2. A CB-connected transistor has  $I_B = 20$   $\mu$ A and  $I_E = 2$  mA. Compute the value of  $\alpha$  and  $I_C$ . [0.99, 1.98 mA]
3. A CE-connected transistor has  $\alpha = 100$  and  $I_B = 50$   $\mu$ A. Compute the values of  $\alpha$ ,  $I_C$  and  $I_E$ . [0.99 ; 5 mA ; 5.05 mA]
4. The following quantities are measured in a CE transistor :  $I_C = 5$  mA ;  $I_B = 100$   $\mu$ A. Determine  $\beta$  and  $I_E$ . [0.98 ; 50 ; 51 mA]
5. A transistor has  $\alpha = 0.98$ ,  $I_{CBO} = 5$   $\mu$ A and  $I_B = 100$   $\mu$ A. Find the values of  $I_C$  and  $I_E$ . [5.15 mA ; 5.25 mA]
6. Following measurements are made in a transistor ;  $I_C = 5.202$  mA,  $I_B = 50$  mA,  $I_{CBO} = 2$  mA. Compute the values of  $\alpha$ ,  $\beta$  and  $I_E$ . [0.99 ; 100 ; 5.252 mA]
7. Following measurements were made in a certain transistor :  
 $I_C = 5.202$  mA ;  $I_B = 50$  mA ;  $I_{CBO} = 2$  mA.  
 Determine (i)  $\alpha$ ,  $\beta$  and  $I_E$  (ii) new value of  $I_B$  required to make  $I_C = 10$  mA.  
 [(i) 0.99 ; 100 ; 5.252 mA (ii) 97.98 A]
8. For the CB circuit of Fig. 57.62, find the value of  $V_{CB}$ . Neglect junction voltage  $V_{BE}$ . [5 V]
9. In the CB circuit of Fig. 57.63, what value of  $R_E$  causes  $V_{BC} = 10$  V? Neglect  $V_{EB}$ . [5 K]
10. For the CE circuit of Fig. 57.64, calculate the values of  $I_B$ ,  $I_C$ ,  $I_E$  and  $V_{CE}$ . Take  $\beta = 50$  and neglect  $V_{BE}$ . [100  $\mu$ A, 5 mA, 5.1 mA, 7.5 V]
11. In the circuit of Fig. 57.65, calculate  $I_B$ ,  $I_C$ ,  $I_E$  and  $V_{CE}$ . Take transistor  $\beta = 50$  and neglect  $V_{BE}$ . [100  $\mu$ A, 5 mA, 5.1 mA, 5V]

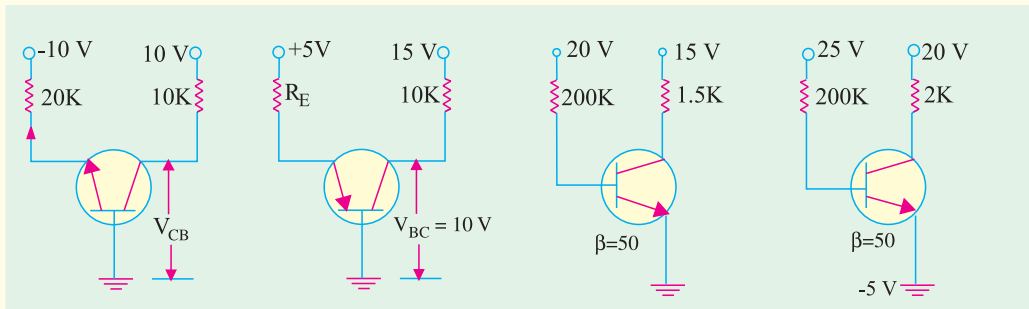


Fig. 57.62

Fig. 57.63

Fig. 57.64

Fig. 57.65

12. In the CC circuit of Fig. 57.66, compute the values of  $I_E$ ,  $I_B$ ,  $I_C$  and  $V_{CE}$ . Neglect  $V_{BE}$ . [0.25 mA, 2.48  $\mu$ A, 0.248 mA, 5 V]
13. In the CC circuit of Fig. 57.67, find  $I_E$ ,  $I_B$ ,  $I_C$  and  $V_{CE}$ . Neglect  $V_{BE}$ . [0.25 mA, 2.48  $\mu$ A, 0.248 mA, 5 V]
14. In the circuit of Fig. 57.68, find the drop across  $R_L$ . The transistor  $\beta = 100$ . [5 V]



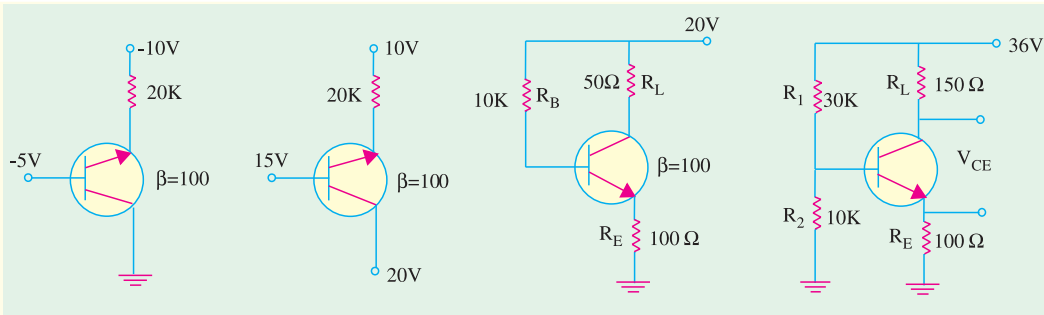


Fig. 57.66

Fig. 57.67

Fig. 57.68

Fig. 57.69

15. Find the value of  $V_{CE}$  for the universal stabilization circuit of Fig. 57.69.

[13.5 V]

16. (i) The reverse saturation current for the Ge transistor in Fig. 57.70 is  $2\mu\text{ A}$  at room temperature ( $25^\circ\text{C}$ ) and increases by a factor of 2 for each temperature increase of  $10^\circ\text{C}$ . Bias voltage  $V_{BB} = 5\text{ V}$ . Find the maximum allowable value for  $R_B$  if the transistor is to remain cutoff at a temperature of  $75^\circ\text{C}$ .

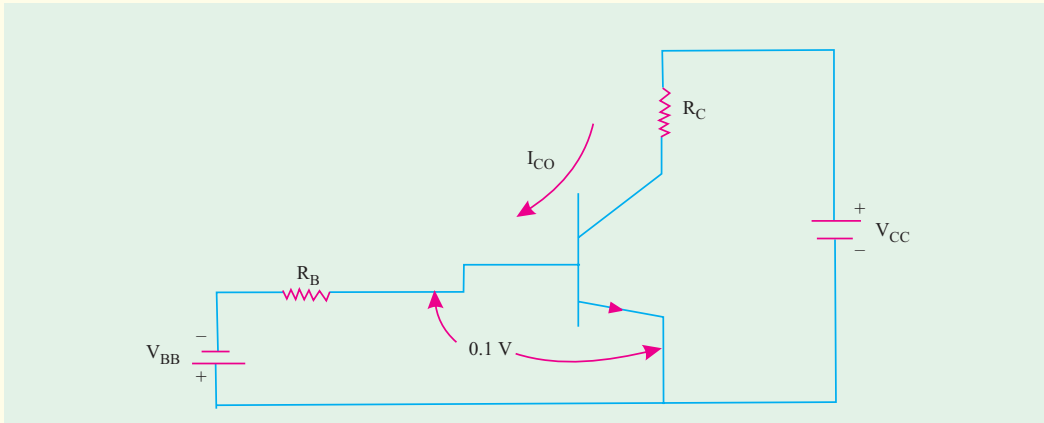


Fig. 57.70

(ii) If  $V_{BB} = 1.0\text{ V}$  and  $R_B = 50\text{ k}\Omega$ , how high may the temperature increase before the transistor comes out of cut-off *(Electronic Devices and Circuits Nagpur University Summer, 2004)*

17. Calculate the exact value of emitter current and  $V_{CE}$  in the circuit shown in Fig. 57.71 Assume transistor to be silicon and  $\beta = 100$ .

*(Electronic Devices of Circuits, Nagpur University Summer, 2004)*

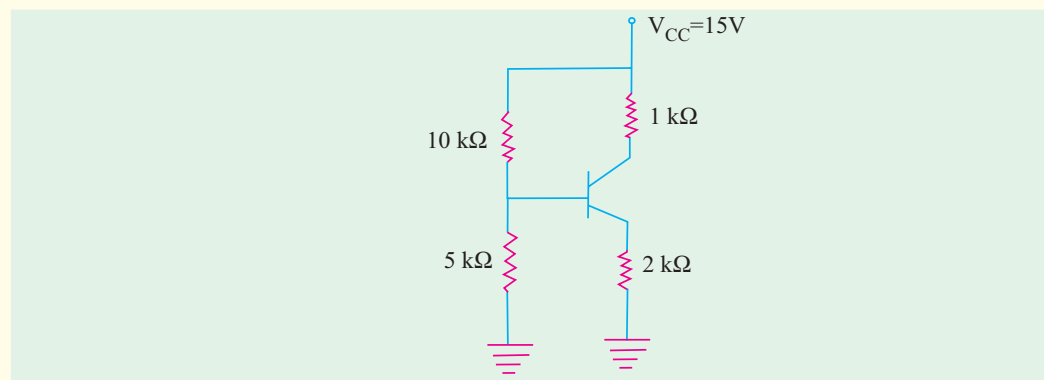


Fig. 57.71



## OBJECTIVE TESTS – 57

- The emitter of a transistor is generally doped the heaviest because it
  - has to dissipate maximum power
  - has to supply the charge carriers
  - is the first region of the transistor
  - must possess low resistance.
- For current working of an *NPN* bipolar junction transistor, the different electrodes should have the following polarities with respect to emitter.
  - collector +ve, base –ve
  - collector –ve, base +ve
  - collector –ve, base –ve
  - collector +ve, base +ve
- Select the CORRECT alternative.  
In a bipolar transistor
  - emitter region is of low/high resistivity material which is lightly/ heavily-doped.
  - collector region is of lower/higher conductivity than emitter region
  - base region is of high/low resistivity material which is only lightly/ heavily doped.
- In a properly-biased *NPN* transistor, most of the electrons from the emitter
  - recombine with holes in the base
  - recombine in the emitter itself
  - pass through the base to the collector
  - are stopped by the junction barrier.
- The following relationships between  $\alpha$  and  $\beta$  are correct EXCEPT
  - $\beta = \frac{\alpha}{1-\alpha}$
  - $\alpha = \frac{\beta}{1-\beta}$
  - $\alpha = \frac{\beta}{1+\beta}$
  - $1-\alpha = \frac{1}{1+\beta}$
- The value of total collector current in a *CB* circuit is
  - $I_C = \alpha I_E$
  - $I_C = \alpha I_E + I_{CO}$
  - $I_C = \alpha I_E - I_{CO}$
  - $I_C = \alpha I_E$ .
- In a junction transistor, the collector cut off current  $I_{CBO}$  reduces considerably by doping the
  - emitter with high level of impurity
  - emitter with low level of impurity
  - collector with high level of impurity
  - collector with low level of impurity
- In a transistor amplifier, the reverse saturation current  $I_{CO}$ 
  - doubles for every  $10^\circ\text{C}$  rise in temperature
  - doubles for every  $1^\circ\text{C}$  rise in temperature
  - increases linearly with the temperature
  - doubles for every  $5^\circ\text{C}$  rise in temperature
- In the case of a bipolar transistor,  $\alpha$  is
  - positive and  $> 1$
  - positive and  $< 1$
  - negative and  $> 1$
  - negative and  $< 1$ .
- The *EBJ* of a given transistor is forward-biased and its *CBJ* reverse-biased. If the base current is increased, then its
  - $I_C$  will decrease
  - $V_{CE}$  will increase
  - $I_C$  will increase
  - $V_{CC}$  will increase.
- The collector characteristics of a *CE*-connected transistor may be used to find its
  - input resistance
  - base current
  - output resistance
  - voltage gain.
- Which of the following approximations is often used in electronic circuits ?
  - $I_C \cong I_E$
  - $I_B \cong I_C$
  - $I_B \cong I_E$
  - $I_E \cong I_B + I_C$
- When a transistor is fully switched ON, it is said to be
  - shorted
  - saturated
  - open
  - cut-off
- If a change in base current does not change the collector current, the transistor amplifier is said to be
  - saturated
  - cut-off
  - critical
  - complemented.
- When an *NPN* transistor is saturated, its  $V_{CE}$ 
  - is zero and  $I_C$  is zero
  - is low and  $I_C$  is high
  - equals  $V_{CC}$  and  $I_C$  is zero
  - equals  $V_{CC}$  and  $I_C$  is high.
- When an *NPN* transistor is cut-off, its  $V_{CC}$ 
  - equals  $V_{CC}$  and  $I_C$  is high
  - equals  $V_{CC}$  and  $I_C$  is zero
  - is low and  $I_C$  is high
  - is high and  $I_C$  is low.
- If, in a bipolar junction transistor,  $I_B = 100 \mu\text{A}$  and  $I_C = 10 \text{ mA}$ , in what range does the value of its beta lie ?
  - 0.1 to 1.0
  - 1.01 to 10
  - 10.1 to 100
  - 100.1 to 1000.
- In a *BJT*, largest current flow occurs
  - in the emitter
  - in the collector
  - in the base
  - through *CB* junction.
- In a properly-connected *BJT*, an increase in base current causes increase in
  - $I_C$  only
  - $I_E$  only
  - both  $I_C$  and  $I_E$
  - leakage current.



20. When a BJT operates in cut-off  
 (a)  $V_{CE} = 0$   
 (b)  $V_{CE} = V_{cc}$   
 (c)  $V_{CE}$  has negative value  
 (d)  $I_C$  is maximum.
21. When a BJT is in saturation  
 (a)  $I_C = 0$   
 (b)  $I_B$  controls  $I_C$   
 (c)  $V_{CE} = 0$   
 (d)  $V_{CE}$  has positive value.
22. The best approximation for  $V_C$  in the circuit shown in Fig. 54.72 will be (assume  $\beta$  to be high)  
 (a) 4 V (b) 6.8 V  
 (c) 8.7 V (d) 10.7 V

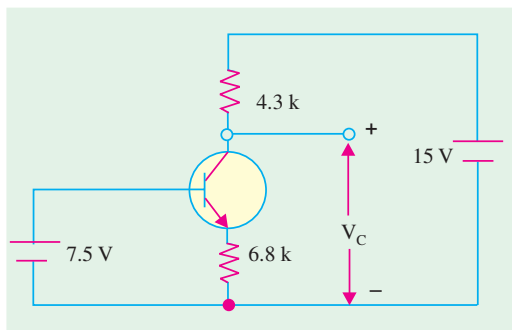


Fig. 57.72

23. Assume  $V_{BE} = 0.7$  V and  $\beta = 50$  for the transistor in the circuit shown in Fig. 57.73. For  $V_{CE} = 2$  V, the value of  $R_B$  is

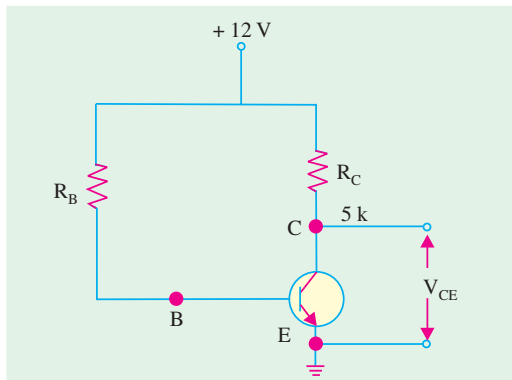


Fig. 57.73

- (a) 200  $\Omega$  (b) 242  $\Omega$   
 (c) 283  $\Omega$  (d) 300  $\Omega$

24. In the circuit shown in Fig. 57.74, if  $R_L = R_C = K\Omega$ , then the value of  $V_O$  will be

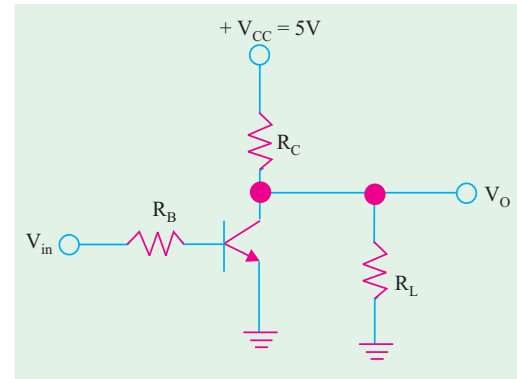


Fig. 57.74

- (a) 4.55 V (b) 2.5 V  
 (c) 1 V (d) zero
25. A transistor is operated as a non-saturated switch to eliminate  
 (a) storage time  
 (b) turn-off time  
 (c) turn-on time  
 (d) delay time
26. Early-effect in BJT refers to  
 (a) avalanche break down  
 (b) thermal break down  
 (c) base narrowing  
 (d) zener break-down
- (UPSC Engg. Services 2002)*
- (Hint. Early effect also called base-width modulation) is the variation of effective base width by the collector voltage)*
27. A bipolar junction transistor (BJT) is used as power control switch by biasing it in the cut-off region (OFF state) or in the saturation region (ON state). In the ON state, for the BJT.  
 (a) both the base-emitter junction and base-collector junctions are reverse biased  
 (b) the base-emitter is reverse biased, and the base-collector junction is forward biased  
 (c) the base-emitter junction is forward biased, and the base-collector junction is reverse biased  
 (d) both the base-emitter and base-collector junctions are forward biased.

ANSWERS

1. (a) 2. (d) 3. (a) low, heavily (b) lower (c) high, lightly 4. (c) 5. (b) 6. (b) 7. (d)  
 8. (a) 9. (d) 10. (c) 11. (c) 12. (a) 13. (b) 14. (a) 15. (b) 16. (b) 17. (b) 18. (a)  
 19. (c) 20. (b) 21. (c) 22. (d) 23. (d) 24. (b) 25. (a) 26. (c) 27. (d)

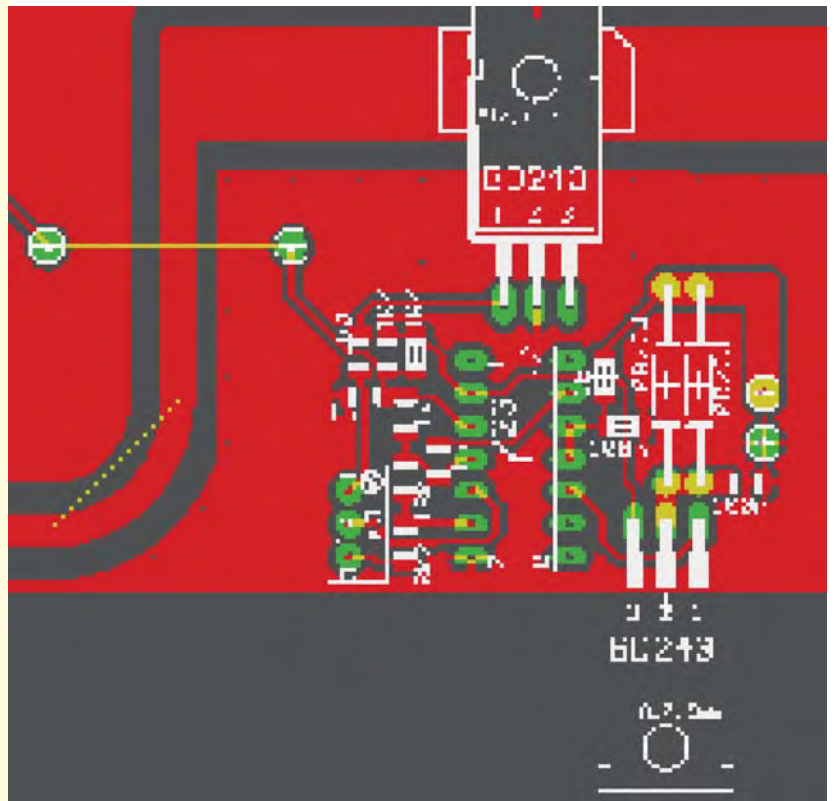


# CHAPTER 58

## Learning Objectives

- DC Load Line
- Q-Point and Maximum Undistorted Output
- Need for Biasing a Transistor
- Factor Affecting Bias Variations
- Stability Factor
- Beta Sensitivity
- Stability Factor for CB and CE Circuits
- Different Methods for Transistor Biasing
- Base Bias
- Base Bias with Emitter Feedback
- Base Bias with Collector Feedback
- Base Bias with Collector and Emitter Feedbacks
- Emitter Bias with two Supplies
- Voltage Divider Bias
- Load Line and Output Characteristics
- AC Load Line

## LOAD LINES AND DC BIAS CIRCUITS



↑ DC biasing is used to establish a steady level of transistor current and voltage called the dc operating point or quiescent point (Q-Point)

58.1. D.C. Load Line

For drawing the dc load line of a transistor, one need to know only its cut-off and saturation points. It is a straight line joining these two points. For the CE circuit of Fig. 58.28, the load line is drawn in Fig. 58.1. A is the cut-off point and B is the saturation point.

The voltage equation of the collector-emitter is

$$V_{CC} = I_C R_L + V_{CE} \quad \therefore I_C = \frac{V_{CC}}{R_L} - \frac{V_{CE}}{R_L}$$

Consider the following two particular cases :

- (i) when  $I_C = 0$ ,  $V_{CE} = V_{CC}$  — cut-off point A
- (ii) when  $V_{CE} = 0$ ,  $I_C = V_{CC}/R_L$  — saturation point B

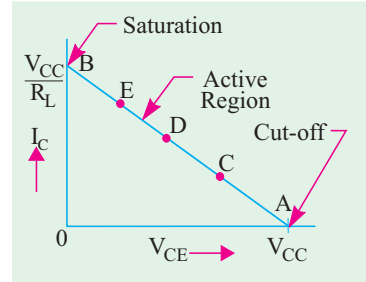


Fig. 58.1

Obviously, load line can be drawn if only  $V_{CC}$  and  $R_L$  are known. Incidentally slope of the load line  $AB = -1/R_L$

**Note.** The above given equation can be written as

$$I_C = \frac{V_{CE}}{R_L} + \frac{V_{CC}}{R_L}$$

It is a linear equation similar to  $y = -mx + c$

The graph of this equation is a straight line whose slope is  $m = -1/R_L$

Active Region

All operating points (like C, D, E etc. in Fig. 58.1) lying between cut-off and saturation points form the **active region** of the transistor. In this region, E/B junction is forward-biased and C/B junction is reverse-biased—conditions necessary for the proper operation of a transistor.

Quiescent Point

It is a point on the dc load line, which represents the values of  $I_C$  and  $V_{CE}$  that exist in a transistor circuit when **no input signal is applied**.

It is also known as the **dc operating point or working point**. The best position for this point is midway between cut-off and saturation points where  $V_{CE} = \frac{1}{2} V_{CC}$  (like point D in Fig. 58.1).

**Example 58.1.** For the circuit shown in Fig. 58.2 (a), draw the dc load line and locate its quiescent or dc working point.

**Solution.** The cut-off point is easily found because it lies along X-axis where  $V_{CE} = V_{CC} = 20\text{ V}$  i.e. point A in Fig. 58.2 (b). At saturation point B, saturation value of collector current is  $I_{C(sat)} = V_{CC}/R_L = 20\text{ V}/5\text{ K} = 4\text{ mA}$ .

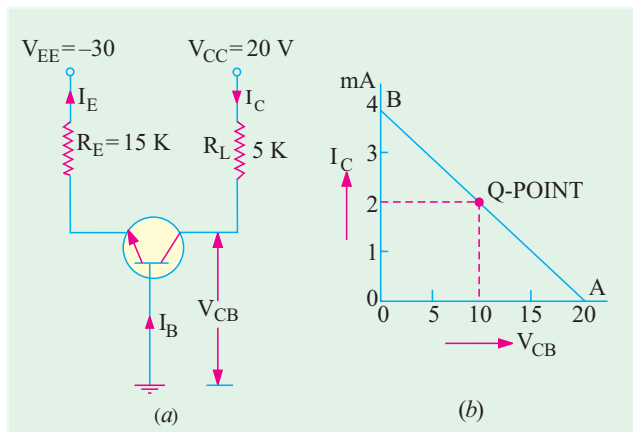


Fig. 58.2





The line  $AB$  represents the load line for the given circuit. We will now find the actual operating point.

$$I_E = V_{EE} / R_E = 30 \text{ V} / 15 \text{ K} = 2 \text{ mA} \quad \text{---neglecting } V_{BE}$$

$$I_C = \alpha I_E \cong I_E = 2 \text{ mA}; \quad \therefore V_{CB} = V_{CC} - I_C R_L = 20 - 2 \times 5 = 10 \text{ V}$$

Hence,  $Q$ -point is located at **(10 V ; 2 mA)** as shown in Fig. 58.2 (b)

**Example 58.2.** In the CB circuit of Fig. 8.3 (a), find

- (a) dc operating point and dc load line
- (b) maximum peak-to-peak unclipped signal
- (c) the approximate value of ac source voltage that will cause clipping.

(Electronics and Telecom Engg. Jadavpur Univ.)

**Solution.**

$$(a) I_{C(sat)} = \frac{V_{CC}}{R_L} = 2 \text{ mA}$$

--- point B

$$V_{CB} \text{ at cut-off} = V_{CC} = 20 \text{ V}$$

---point A

Hence,  $AB$  is the dc load line and is shown in Fig. 58.3 (b).

$$\text{Now, } I_E = 10 / 20 = 0.5 \text{ mA}$$

$$I_C \cong I_E = 0.5 \text{ mA}$$

$$\therefore V_{CB} = V_{CC} - I_C R_L = 20 - 0.5 \times 10 = 15 \text{ V}$$

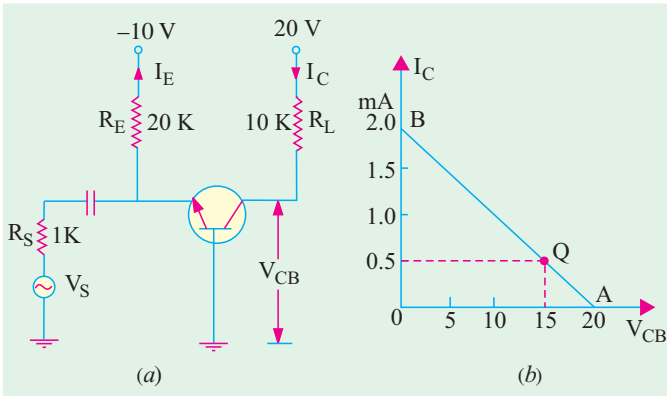


Fig. 58.3

The  $Q$ -point is located at **(15 V, 0.5 mA)**

(b) It is obvious from Fig. 58.3 (b) that maximum positive swing can be from 15 V to 20 V i.e. 5 V only. Of course, on the negative swing, the output swing can go from 15 V down to zero volt. The limiting factor being **cut-off on positive half-cycle**, hence maximum unclipped peak-to-peak voltage that we can get from this circuit is  $2 \times 5 = 10 \text{ V}$ .

(c) The approximate voltage gain of the above circuit is

$$A_v = \frac{V_0}{V_S} = \frac{R_L}{R_S} = \frac{10 \text{ K}}{1 \text{ K}} = 10$$

It means that signal voltage will be amplified 10 times. Hence, maximum value of source voltage for obtaining unclipped or undistorted output is

$$V_s = \frac{V_0}{10} = \frac{10 \text{ V}_{p-p}}{10} = 1 \text{ V}_{p-p}$$

**Example 58.3.** For the CE circuit shown in Fig. 58.4 (a), draw the dc load line and mark the dc working point on it. Assume  $b=100$  and neglect  $V$ .

(Applied Electronics, Punjab Univ.)

**Solution.** Cut-off point A is located where,  $I_C = 0$  and  $V_{CE} = V_{CC} = 30 \text{ V}$ . Saturation point B is given where  $V_{CE} = 0$  and

$$I_{C(sat)} = 30 \text{ V} / 5 \text{ K} = 6 \text{ mA}.$$

Line  $AB$  represents the load line in Fig. 58.4 (b).

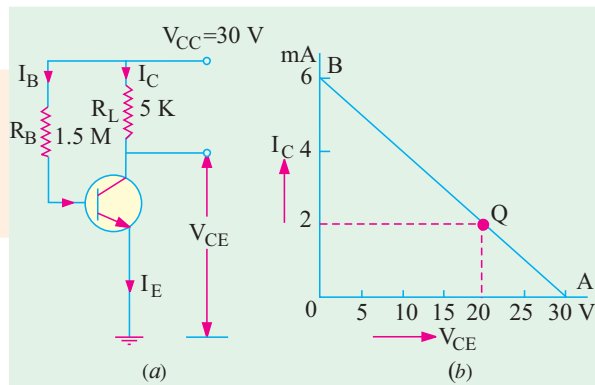


Fig. 58.4

Let us find the dc working point from the given values of resistances and supply voltage.

$$I_B = 30 \text{ V} / 1.5 \text{ M} = 20 \text{ } \mu\text{A}; \quad I_C = \beta I_B = 100 \times 20 = 2000 \text{ } \mu\text{A} = 2 \text{ mA};$$

$$V_C = V_{CC} - I_C R_L = 30 - 2 \times 5 = 20 \text{ V}$$

Hence,  $Q$ -point is (20 V ; 2 mA) as shown in Fig. 58.4.

### 58.2. Q-Point and Maximum Undistorted Output

Position of the  $Q$ -point on the dc load line determines the maximum signal that we can get from the circuit before clipping occurs. Consider the cases shown in Fig. 58.5.

In Fig. 58.5 (a), when  $Q$  is located near cut-off point, signal first starts to clip at A. It is called **cut-off clipping** because the positive swing of the signal drives the transistor to cut-off. In fact, as seen from Fig. 58.5 (a), maximum positive swing is  $I_{CQ} R_{ac}$ .

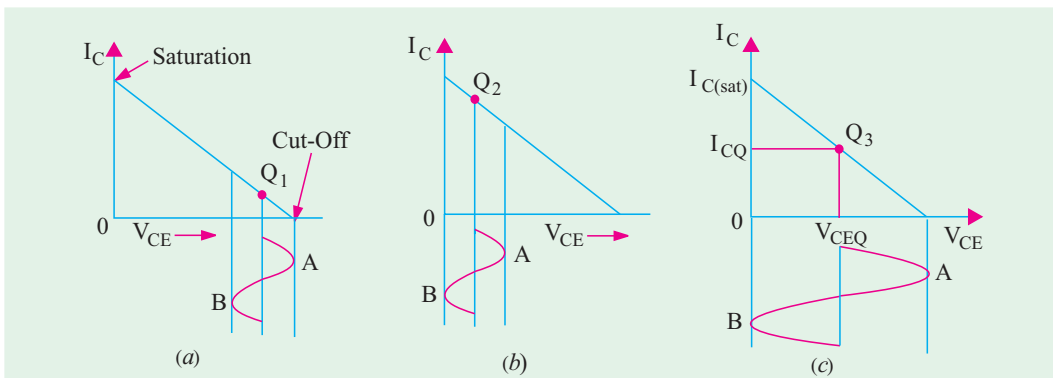


Fig. 58.5

If the  $Q$ -point  $Q_2$  is located near saturation point, then clipping first starts at point B as shown in Fig. 58.5 (b). It is caused by saturation. The maximum negative swing  $= V_{CEQ}$ .

In Fig. 58.5 (c), the  $Q$ -point  $Q_3$  is located at the centre of the load line. In this condition, we get the **maximum possible output signal**. The point  $Q_3$  gives the optimum  $Q$ -point. The maximum undistorted signal  $= 2V_{CEQ}$ .

In general, consider the case shown in Fig. 58.6. Since  $A < B$ , maximum possible peak-to-peak output signal  $= 2A$ .

If the operating point were so located that  $A > B$ , then maximum possible peak-to-peak output signal  $= 2B$ .

When operating point is located at the centre of the load line, then maximum undistorted peak-to-peak signal is  $= 2A = 2B = V_{CC} = 2V_{CEQ}$ .

Under optimum working conditions corresponding to Fig. 58.5 (c),  $I_{CQ}$  is half the saturation value given by  $V_{CC}/R_L$  (Art. 8.1).

$$\therefore I_{CQ} = \frac{1}{2} \cdot \frac{V_{CC}}{R_L} = \frac{V_{CC}}{2R}$$

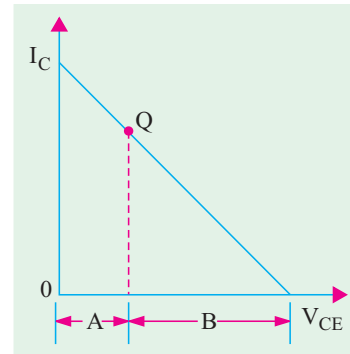


Fig. 58.6

**Example 58.4.** Determine the value of  $R_B$  required to adjust the circuit of Fig. 58.7 to optimum operating point. Take  $\beta = 50$  and  $V_{BE} = 0.7 \text{ V}$ .

**Solution.** As seen from above

$$I_{CQ} = \frac{V_{CC}}{2R_L} = \frac{20}{2 \times 10} = 1 \text{ mA}$$

The corresponding base current is

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1}{50} = 2 \text{ } \mu\text{A}$$

$$\begin{aligned} \text{Now, } V_{CC} &= I_B R_B + V_{BE} \\ \therefore R_B &= \frac{V_{CC} - V_{BE}}{I_B} = \frac{20 - 0.7}{20 \times 10^{-6}} = \mathbf{965 \text{ K}} \end{aligned}$$

### 58.3. Need For Biasing a Transistor

For normal operation of a transistor amplifier circuit, it is essential that there should be a

- (a) forward bias on the emitter-base junction and
- (b) reverse bias on the collector-base junction.

In addition, amount of bias required is important for establishing the  $Q$ -point which is dictated by the mode of operation desired.

If the transistor is not biased correctly, it would

1. work inefficiently and
2. produce distortion in the output signal.

It is desirable that once selected, the  $Q$ -point should remain stable *i.e.* should not shift its position due to temperature rise etc. Unfortunately, this does not happen in practice unless special efforts are made for the purpose.

### 58.4. Factors Affecting Bias Variations

In practice, it is found that even after careful selection,  $Q$ -point tends to shift its position. This bias instability is the direct result of thermal instability which itself is produced by cumulative increase in  $I_C$  that may, if unchecked, lead to thermal runaway (Art. 58.13).

The collector current for  $C_E$  circuit is given by

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1 + \beta) I_{CO}$$

This equation has three variables:  $\beta$ ,  $I_B$  and  $I_{CO}$  all of which are found to increase with temperature. In particular, increase in  $I_{CO}$  produces significant increase in collector current  $I_C$ . This leads to increased power dissipation with further increase in temperature and hence  $I_C$ . Being a cumulative process, it can lead to thermal runaway which will destroy the transistor itself!

However, if by some circuit modification,  $I_C$  is made to decrease with temperature automatically, then decrease in the term  $\beta I_B$  can be made to neutralize the increase in the term  $(1 + \beta) I_{CO}$ , thereby keeping  $I_C$  constant. This will achieve thermal stability resulting in bias stability.

### 58.5. Stability Factor

The degree of success achieved in stabilizing  $I_C$  in the face of variations in  $I_{CO}$  is expressed in terms of current stability factor  $S$ . It is defined as the rate of change of  $I_C$  with respect to  $I_{CO}$  when both  $\beta$  and  $I_B$  ( $V_{BE}$ ) are held constant.

$$\therefore S = \frac{dI_C}{dI_{CO}} \quad \text{--- } \beta \text{ and } I_B \text{ constant}$$

Larger the value of  $S$ , greater the thermal instability and *vice versa* (in view of the above, this factor should, more appropriately, be called instability factor!).

The stability factor may be alternatively expressed by using the well-known equation  $I_C = \beta I_B + (1 + \beta) I_{CO}$  which, on differentiation with respect to  $I_C$ , yields.

$$I = \beta \frac{dI_B}{dI_C} + (1 + \beta) \frac{dI_{CO}}{dI_C} = \beta \frac{dI_B}{dI_C} + (1 + \beta) \frac{1}{S} \quad \therefore S = \frac{(1 + \beta)}{1 - \beta(dI_B/dI_C)}$$

The stability factor of any circuit can be found by using the general formula

$$S = \frac{1 + R_B/R_E}{1 + (1 - \alpha)(R_B/R_E)}$$

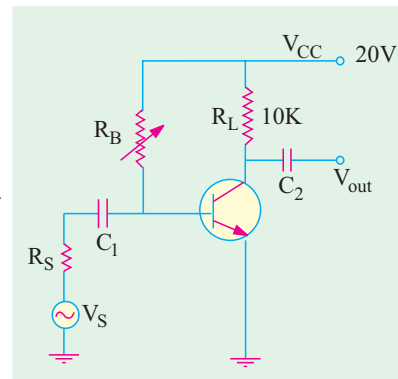


Fig. 58.7

where  $R_B$  = **total** series parallel resistance in the base  
 $R_E$  = **total** series dc resistance in the emitter  
 $\alpha$  = dc alpha of the transistor

### 58.6. Beta Sensitivity

By  $\beta$ -sensitivity of a circuit is meant the influence that the  $\beta$ -value has on its dc operating point. Variations in  $\beta$ -value are caused by variations in the circuit operating conditions or by the substitution of one transistor with another. Beta sensitivity  $K_\beta$  is given by

$$\frac{dI_C}{I_C} = K_\beta \frac{dI_B}{I_B} \quad \therefore \quad K_\beta = \frac{\beta}{I_C} \cdot \frac{dI_C}{dI_B}$$

Obviously,  $K_\beta$  is dimensionless ratio and can have values ranging from zero to unity.

### 58.7. Stability Factor for CB and CE Circuits

#### (i) CB Circuit

Here, collector current is given by

$$I_C = \alpha I_E + I_{CO} \quad \therefore \quad \frac{dI_C}{dI_{CO}} = 0 + 1 \quad \text{or} \quad S = 1$$

#### (ii) CE Circuit

$$I_C = \beta I_B + (1 + \beta) I_{CO} \quad \therefore \quad \frac{dI_C}{dI_{CO}} = (1 + \beta) \quad \text{—treating } I_B \text{ as a constant}$$

$$\therefore \quad S = (1 + \beta).$$

If  $\beta = 100$ , then  $S = 101$  which means that  $I_C$  changes 101 times as much as  $I_{CO}$ .

### 58.8. Different Methods for Transistor Biasing

Some of the methods used for providing bias for a transistor are :

#### 1. base bias or fixed current bias (Fig. 58.9)

It is not a very satisfactory method because bias voltages and currents do not remain constant during transistor operation.

#### 2. base bias with emitter feedback (Fig. 58.10)

This circuit achieves good stability of dc operating point against changes in  $\beta$  with the help of emitter resistor which causes degeneration to take place.

#### 3. base bias with collector feedback (Fig. 58.11)

It is also known as collector-to-base bias or collector feedback bias. It provides better bias stability.

#### 4. base bias with collector and emitter feedbacks

It is a combination of (2) and (3) above.

#### 5. emitter bias with two supplies (Fig. 58.13)

This circuit uses both a positive and a negative supply voltage. Here, base is at approximately 0 volt *i.e.*  $V_B \cong 0$ .

#### 6. voltage divider bias (Fig. 58.15)

It is most widely used in linear discrete circuits because it provides good bias stability. It is also called universal bias circuit or base bias with one supply. Each of the above circuits will now be discussed separately.

### 58.9. Base Bias

It has already been discussed in Art. 58.20 and is shown in Fig. 58.25. For such a circuit,  $S = (1 + \beta) \cong \beta$  and  $K_\beta = 1$ .

**58.10. Base Bias with Emitter Feedback**

This circuit is obtained by simply adding an emitter resistor to the base bias circuit as shown in Fig. 58.8.

1. At saturation,  $V_{CE}$  is essentially zero, hence  $V_{CC}$  is distributed over  $R_L$  and  $R_E$ .

$$\therefore I_{C(sat)} = \frac{V_{CC}}{R_E + R_L}$$

2.  $I_C$  can be found as follows :

Consider the supply, base, emitter and ground route. Applying Kirchhoff's Voltage Law, we have

$$-I_B R_B - V_{BE} - I_E R_E + V_{CC} = 0$$

or  $V_{CC} = I_B R_B + V_{BE} + I_E R_E$  ... (i)

Now  $I_B = I_C / \beta$  and  $I_E \cong I_C$

Substituting these values in the above equation, we have

$$V_{CC} \cong \frac{I_C R_B}{\beta} + V_{BE} + I_C R_E$$

$$\therefore I_C \cong \frac{V_{CC} - V_{BE}}{R_E + R_B / \beta} \cong \frac{V_{CC}}{R_E + R_B / \beta}$$

—neglecting  $V_{BE}$

(we could have applied the  $\beta$ -rule given in Art. 57.24)

3. collector-to-ground voltage  $V_C = V_{CC} - I_C R_L$

4. emitter-to-ground voltage  $V_E = I_E R_E \cong I_C R_E$

5. 
$$S = \frac{1 + R_B / R_E}{1 + R_B / (1 + \beta) R_E} = \frac{1 + R_B / R_E}{1 + R_B / \beta R_E}$$

6. The  $\beta$ -sensitivity of this circuit is  $K_\beta = \frac{1}{1 + \beta R_E / R_B}$

**Example 58.5.** For the circuit shown in Fig. 58.9, find (i)  $I_{C(sat)}$ , (ii)  $I_C$ , (iii)  $V_C$ , (iv)  $V_E$ , (v)  $V_{CE}$  and (vi)  $K_\beta$ .

**Solution.** (i) 
$$I_{C(sat)} = \frac{V_{CC}}{R_E + R_L} = \frac{30}{1 + 2} = 10 \text{ mA}$$

(ii) actual 
$$I_C \cong \frac{V_{CC}}{R_E + R_B / \beta} = \frac{30}{1 + 300 / 100} = 7.5 \text{ mA}$$

(iii) 
$$V_C = V_{CC} - I_C R_L = 30 - 2 \times 7.5 = 15 \text{ V}$$

(iv) 
$$V_E \cong I_E R_E \cong I_C R_E = 7.5 \times 1 = 7.5 \text{ V}$$

(v) 
$$V_{CE} = V_C - V_E = 15 - 7.5 = 7.5 \text{ V}$$

(vi) 
$$K_\beta = \frac{1}{1 + 100 \times 1 / 300} = 0.75$$

**Example 58.6.** The base-biased transistor circuit of Fig. 58.10 is subjected to increase in junction temperature from 25°C to 75°C. If  $\beta$  increases from 100 to 150 with rising temperature, calculate the percentage change in Q-point values ( $I_C$ ,  $V_{CE}$ ) over the temperature range. Assume that  $V_{BE}$  remains constant at 0.7 V.

**Solution.** At 25°C 
$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{100 \times 10^3} = 0.113 \text{ mA}$$

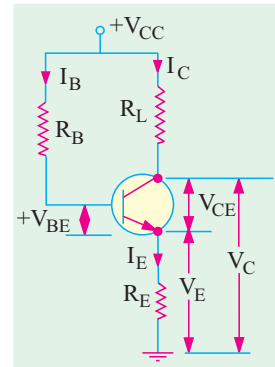


Fig. 58.8

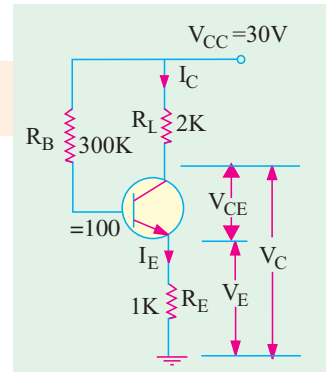


Fig. 58.9

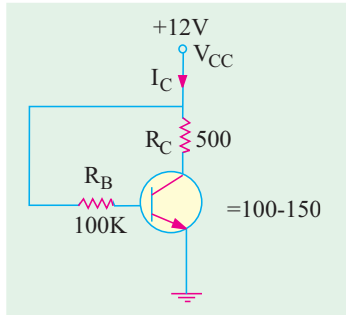


Fig. 58.10

$$I_C = \beta I_B = 100 \times 0.113 = 11.3 \text{ mA}$$

$$V_{CE} = I_C R_C = 12 - (11.3 \times 10^{-3} \times 500) = 6.35 \text{ V}$$

$$V_{CE} = V_{CC} - I_C R_C = 12 - (11.3 \times 10^{-3} \times 500) = 6.35 \text{ V}$$

At 25°

$$I_B = 0.113 \text{ mA}$$

$$I_C = \beta I_B = 150 \times 0.113 = 16.95 \text{ mA}$$

$$V_{CE} = 12 - (16.95 \times 10^{-3} \times 500) = 3.52 \text{ V}$$

$$\% \Delta I_C = \frac{16.95 - 11.3}{11.3} \times 100 = 50 \text{ (increase)}$$

$$\% \Delta V_{CE} = \frac{3.52 - 6.35}{6.35} \times 100 = -44.57 \text{ (decrease)}$$

It is seen that  $Q$ -point is very much dependent on temperature and makes the base-bias arrangement very unstable.

### 58.11. Base Bias with Collector Feedback

This circuit (Fig. 58.11) is like the base bias circuit except that base resistor is returned to collector rather than to the  $V_{CC}$  supply. It derives its name from the fact that since voltage for  $R_B$  is derived from collector, there exists a negative feed back effect which tends to stabilise  $I_C$  against changes in  $\beta$ . To understand this action, suppose that somehow  $\beta$  increases. It will increase  $I_C$  as well as  $I_C R_L$  but decrease  $V_C$  which is applied across  $R_B$ . Consequently,  $I_B$  will be decreased which will partially compensate for the original increase in  $\beta$ .

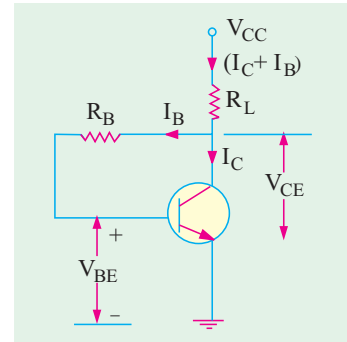


Fig. 58.11

- (i)  $I_{C(sat)} = V_{CC} / R_L$  —since  $V_{CE} = 0$
  - (ii)  $V_C = V_{CC} - (I_B + I_C) R_L \cong V_{CC} - I_C R_L$
- Also,  $V_C = I_B R_B + V_{BE}$

Equating the two expressions for  $V_C$ , we have

$$I_B R_B + V_{BE} \cong V_{CC} - I_C R_L$$

Since  $I_B = I_C / \beta$ , we get

$$\frac{I_C}{\beta} \cdot R_B + V_{BE} \cong V_{CC} - I_C R_L$$

$$\therefore I_C = \frac{V_{CC} - V_{BE}}{R_L + R_B / \beta} \cong \frac{V_{CC}}{R_L + R_B / \beta}$$

This is also the approximate value of  $I_E$  (again, we could take the help of  $\beta$ -rule). The  $\beta$ -sensitivity factor is given by

$$K_\beta = \frac{1}{\beta R_L / R_B} = 1 - \frac{I_C}{I_{C(sat)}}$$

$$S = \frac{1 + R_B / R_L}{1 + R_B / (1 + \beta) R_L} \cong \frac{V_{CC}}{R_L + R_B / \beta}$$

**Example 58.7.** In Fig. 58.11,  $V_{CC} = 12 \text{ V}$ ,  $V_{BE} = 0.7 \text{ V}$ ,  $R_L = 1\text{K}$ ,  $R_B = 100\text{K}$ ,  $\beta = 100$ . Find (i)  $I_C$ , (ii)  $V_{CE}$ , (iii)  $I_B$ , (iv)  $K_\beta$  and (v)  $S$ .

**Solution.** (i)  $I_C \cong I_E = \frac{12 - 0.7}{1 + 100/100} = 5.6 \text{ mA}$

(ii)  $V_{CE} \cong 12 - (5.65 \times 1) = 6.35 \text{ V}$  (iii)  $I_B = I_C / \beta = 5.65/100 = 56.5 \mu\text{A}$

(iv)  $K_B = \frac{1}{1 + 100 \times 1/100} = 0.5$  (v)  $S = \frac{1 + 100/1}{1 + 100 \times 1/101} = 50.5$

### 58.12. Base Bias with Collector and Emitter Feedbacks

In the circuit of Fig. 58.12, both collector and emitter feedbacks have been used in an attempt to reduce circuit sensitivity to changes in  $\beta$ . If  $\beta$  increases, emitter voltage increases but collector voltage decreases. It means that voltage across  $R_B$  is reduced causing  $I_B$  to decrease thereby partially off-setting the increase in  $\beta$ .

Under saturation conditions,  $V_{CC}$  is distributed over  $R_L$  and  $R_E$ . Assuming  $I_B$  to be negligible as compared to  $I_C$ , we get,  $I_{C(sat)} = V_{CC} / (R_E + R_L)$ .

$$\text{Actual value of } I_C \text{ is } = \frac{V_{CC} - V_{BE}}{R_E + R_L + R_B / \beta}$$

—going via  $R_B$  because  $V_{CE}$  is unknown.

$$V_C = V_{CC} - (I_C + I_B)R_L; \quad R_L \cong V_{CC} - I_C R_L$$

$$V_E = I_E R_E \cong I_C R_E; \quad \bar{V}_{CE} = V_C - V_E$$

$$V_{CE} \cong V_{CC} - I_C (R_L + R_E)$$

$$S = \frac{1 + R_B / (R_E + R_L)}{1 + R_B / \beta (R_E + R_L)}$$

$$\text{It can be proved that } K_\beta = \frac{1}{1 + \beta(R_E + R_L) / R_B} = 1 - \frac{I_C}{I_{C(sat)}}$$

Obviously,  $K_\beta$  will be degraded with increase in  $R_B$ .

**Example 58.8.** For the circuit shown in Fig. 58.13, find (a)  $I_{C(sat)}$  (b)  $V_{CE}$  and (c)  $K_\beta$ . Neglect  $V_{BE}$  and take  $\beta = 100$ .

**Solution.** (a)  $I_{C(sat)} = 15 / (10 + 10) = 0.75 \text{ mA}$

(b)  $I_C = \frac{V_{CC}}{R_E + R_L + R_B / \beta} = \frac{15}{10 + 10 + 500/100} = 0.6 \text{ mA}$

$$V_{CE} = 15 - 0.6 (10 + 10) = 3 \text{ V}$$

(c)  $K_\beta = \frac{1}{1 + 100(10 + 10) / 50} = 0.2$

or  $K_\beta = 1 - I_C / I_{C(sat)} = 1 - 0.6 / 0.75 = 0.2$

Obviously,  $K_\beta$  will be degraded with increase in  $R_B$ .

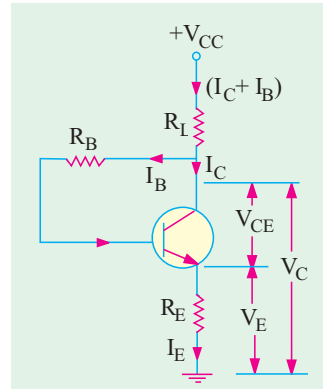


Fig. 58.12

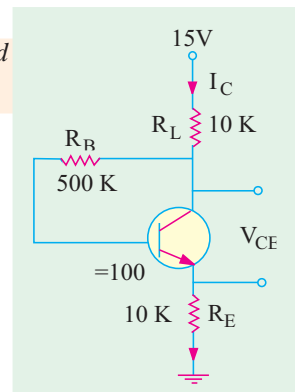


Fig. 58.13

### 58.13. Emitter Bias with Two Supplies

This circuit gives a reasonably stable  $Q$ -point and is widely used whenever two supplies (positive and negative) are available. Its popularity is due to the fact that  $I_C$  is essentially independent of  $\beta$ .

It can be shown that  $V_B \cong 0$  and  $V_E = -V_{BE}$

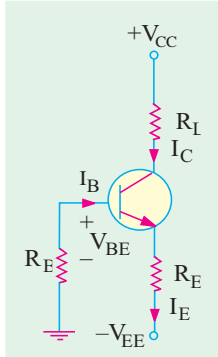


Fig. 58.14

Starting from ground and going clockwise round the base-emitter circuit, we get according to KVL.

$$-I_B R_B - V_{BE} - I_E R_E + V_{EE} = 0$$

$$\text{or } I_B R_B + I_E R_E = V_{EE} - V_{BE} \quad \dots(i)$$

Now,  $I_B = I_C / \beta \cong I_E / \beta$ . Substituting this in (i) above we have

$$\frac{I_E R_B}{\beta} + I_E R_E = V_{EE} - V_{BE} \quad \text{or} \quad I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B / \beta}$$

If  $V_{EE} \gg V_{BE}$  and  $R_E \gg R_B / \beta$ ,  $I_E = V_{EE} / R_E$ .

If  $V_E$  is the emitter to ground voltage, then

$$-I_B R_B - V_{BE} - V_E = 0$$

$$\text{or } V_E = -(I_B R_B + V_{BE}) = -(V_{BE} + I_C R_B / \beta) \cong -V_{BE}$$

$$\text{For this circuit, } S = \frac{1 + R_B / R_E}{1 + R_B / \beta R_E} \quad \text{and} \quad K_\beta = \frac{1}{1 + \beta R_E / R_B}$$

**Example 58.9.** For the circuit of Fig. 58.15, find (i)  $I_E$ , (ii)  $I_C$ , (iii)  $V_C$ , (iv)  $V_E$ , (v)  $V_{CE}$ , (vi) stability factor and (vii)  $K_\beta$  for a  $\beta$  of 50. Take  $V_{BE} = 0.7$  V. (Electronics-II, Bangalore Univ. 1995)

**Solution.** (i)  $I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B / \beta} = \frac{10 - 0.7}{20 + 10 / 50} = 0.46 \text{ mA}$

(ii)  $I_C \cong I_E = 0.46 \text{ mA}$

(iii)  $V_C = V_{CC} - I_C R_L = 20 - 0.46 \times 10 = 15.4 \text{ V}$

(iv)  $V_E = -(V_{BE} + I_C R_B / \beta) = -(0.7 + 0.46 \times 10 / 50) = -0.8 \text{ V}$

(v)  $V_{CE} = V_C - V_E = 15.4 - (-0.8) = 16.2 \text{ V}$

(vi)  $S = \frac{1 + R_B / R_E}{1 + R_B / \beta R_E} = \frac{1 + 10 / 20}{1 + 10 / 50 \times 20} = 1.485$

(vii)  $K_\beta = \frac{1}{1 + R_E / R_\beta} = \frac{1}{1 + 50 \times 20 / 10} \cong 0.01$

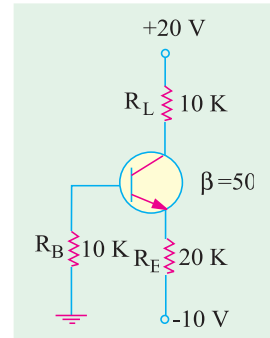


Fig. 58.15

### 58.14. Voltage Divider Bias

The arrangement is commonly used for transistors incorporated in integrated circuits (ICs).

The name ‘voltage divider’ is derived from the fact that resistors  $R_1$  and  $R_2$  form a potential divider across  $V_{CC}$  (Fig. 58.16)\*. The voltage drop  $V_2$  across  $R_2$  forward-biases the emitter whereas  $V_{CC}$  supply reverse-biases the collector.

As per voltage divider theorem.

$$V_2 = V_{CC} \cdot R_2 / (R_1 + R_2)$$

As seen,  $V_E = V_2 - V_{BE}$

$$\therefore I_E = \frac{V_E}{R_E} = \frac{V_2 - V_{BE}}{R_E} \cong \frac{V_2}{R_E}$$

Also,  $V_C = V_{CC} - I_C R_L$

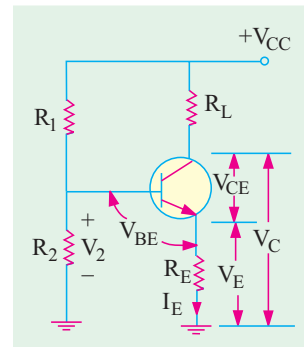


Fig. 58.16

\* It is also known as Universal Bias Stabilization Circuit.



$$V_{CE} = V_C - V_E = V_{CC} - I_C R_L - I_E R_E$$

$$\cong V_{CC} - I_C (R_L + R_E) \quad \therefore I_C \cong I_E$$

As before, 
$$I_{C(sat)} \cong \frac{V_{CC}}{R_L + R_E}$$

It is seen from above calculations that value of  $\beta$  was never used anywhere. The base voltage is set by  $V_{CC}$  and  $R_1$  and  $R_2$ . The *dc* bias circuit is independent of transistor  $\beta$ . That is why it is such a very popular bias circuit.

$$K_\beta = \frac{1}{1 + \beta R_E / (R_1 \parallel R_2)}$$

$$S = \frac{1 + (R_1 \parallel R_2) / R_E}{1 + (R_1 \parallel R_2) / (1 + \beta) R_E}$$

$$\cong \frac{1 + (R_1 \parallel R_2) / R_E}{1 + (R_1 \parallel R_2) / \beta R_E}$$

### Using Thevenin's Theorem

More accurate results can be obtained by Thevenizing the voltage divider circuit as shown in Fig. 58.17. The first step is to open the base lead at point A and remove the transistor along with  $R_L$  and  $R_E$  thereby leaving the voltage divider circuit behind as in Fig. 58.17 (a) and (b).

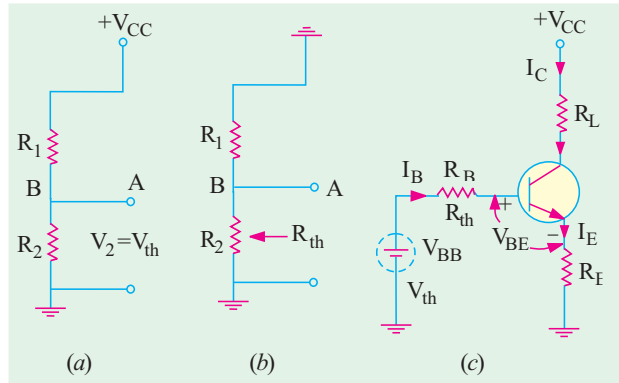


Fig. 58.17

$$V_{th} = V_2 = V_{CC} \cdot \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_{th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

The original circuit is reduced to that shown in Fig. 58.17 (c) where  $V_{th} = V_{BB'}$ ;  $R_{th} = R_{B'}$ . Now, applying *KVL* to the base-emitter loop, we get

$$V_{BB'} - I_B R_{B'} - V_{BE} - I_E R_E = 0$$

Substituting the value of  $I_E = (1 + \beta) I_B$  in (i) above, we get

$$I_B = \frac{V_{BB'} - V_{BE}}{R_{B'} + (1 + \beta) R_E}$$

However, if we substitute the value of  $I_B = I_E / (1 + \beta)$ , we get

$$I_E = \frac{V_{BB'} - V_{BE}}{R_E + R_{B'} / (1 + \beta)}$$

$$V_{CE} = V_{CC} - I_C R_L - I_E R_E \cong V_{CC} - I_C (R_L + R_E)$$

The above results could also be obtained directly by applying  $\beta$ -rule (Art. 58.12)

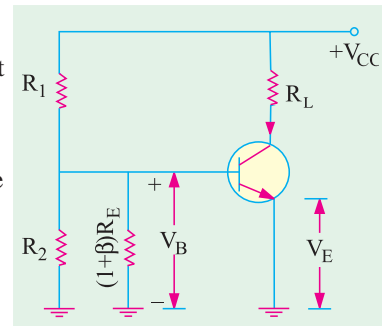


Fig. 58.18

### Using $\beta$ -rule

As per  $\beta$ -rule (Art. 58.12) when  $R_E$  is transferred to the base circuit, it becomes  $(1 + \beta) R_E$  and is in parallel with  $R_2$  as shown in Fig. 8.18. Now,  $V_{CC}$  drops over  $R_1$  and  $R_2 \parallel (1 + \beta) R_E$

$$\therefore V_B = V_{CC} \frac{R_2 \parallel (1+\beta)R_E}{R_1 + R_2 \parallel (1+\beta)R_E}$$

$$V_E = V_B - V_{BE}; I_E = V_E/R_E \quad \text{and so on.}$$

**Example. 58.10.** For the circuit of Fig. 58.19, find (a)  $I_{C(sat)}$ , (b)  $I_C$ , (c)  $V_{CE}$ , (d)  $K_\beta$ . Neglect  $V_{BE}$  and take  $\beta = 50$ .  
(Electronics, Gorakhpur Univ.)

**Solution.**

- (a)  $I_{C(sat)} = \frac{V_{CC}}{R_L + R_E} = \frac{20}{2+6} = 2.5 \text{ mA}$
- (b)  $I_C \cong I_E \cong V_2/R_E = 6/6 = 1 \text{ mA}$
- (c)  $V_{CE} = V_{CC} - I_C(R_L + R_E) = 20 - 1(2 + 6) = 12 \text{ V}$
- (d)  $R_1 \parallel R_2 = 84/20 = 4.2 \text{ K}$

$$K_\beta = \frac{1}{1 + 50 \times 6 / 4.2} = 0.0138$$

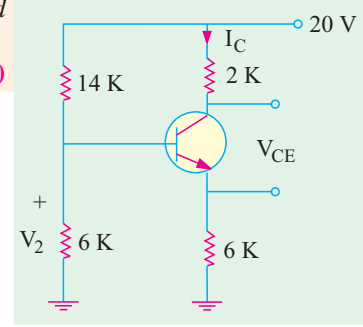


Fig. 58.19

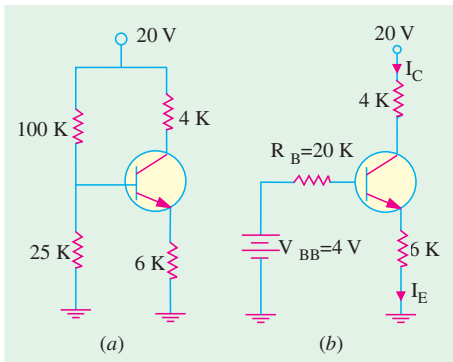


Fig. 58.20

**Example 58.11.** For the circuit shown in Fig. 58.20 (a), draw the dc load line and mark the  $Q$ -point of the circuit. Assume germanium material with  $V_{BE} = 0.3 \text{ V}$  and  $\beta = 50$ .  
(Electronics-I, M.S. Univ. 1991)

**Solution.**  $V_{CC(\text{cut-off})} = V_{CC} = 20 \text{ V}$

$$I_{C(sat)} = \frac{V_{CC}}{R_L + R_E} = \frac{20}{4+6} = 2 \text{ mA}$$

(a) **Approximate Method**

$$V_2 = V_{CC} \frac{R_2}{R_1 + R_2} = 20 \times \frac{25}{125} = 4 \text{ V}$$

$$I_E = \frac{V_2 - V_{BE}}{R_E} = \frac{4 - 0.3}{6} = 0.62 \text{ mA}$$

$\therefore V_{CE} = V_{CC} - I_C(R_L + R_E) = 20 - 0.62 \times 10 = 13.8 \text{ V}$   
This  $Q$ -point (13.8 V, 0.62 mA) is shown in Fig. 58.21 (a).

As seen from Fig. 58.20 (b)

$$V_{BB} = 20 \times 25/125 = 4 \text{ V}$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B + (1+\beta)R_E} = \frac{4 - 0.3}{20 + 51 \times 6} = 11.3 \mu\text{A}$$

$$I_C = \beta I_B = 50 \times 11.3 = 0.565 \text{ mA}$$

$$I_E = (1 + \beta) I_B = 576 \mu\text{A} = 0.576 \text{ mA}$$

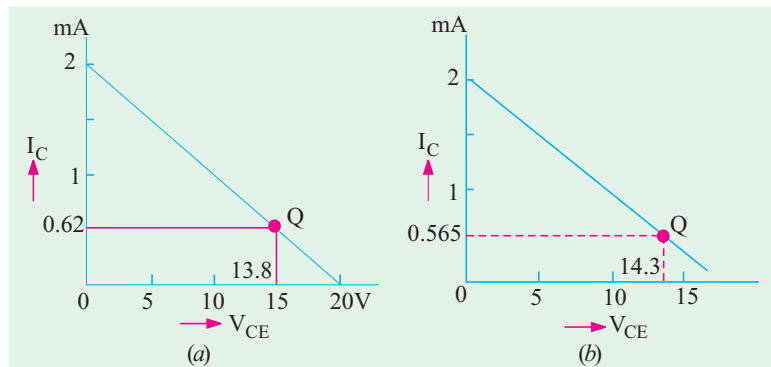


Fig. 58.21

$$\therefore V_{CE} = V_{CC} - I_C R_C - I_E R_E = 20 - 0.565 \times 4 - 0.576 \times 6 = 14.3 \text{ V}$$

The new and more accurate  $Q$ -point (14.3 V, 0.565 mA) is shown in Fig. 58.21 (b).

### 58.15. Load Line and Output Characteristics

In order to study the effect of bias conditions on the performance of a  $CE$  circuit, it is necessary to superimpose the dc load line on the transistor output ( $V_{CE}/I_C$ ) characteristics. Consider a silicon  $NPN$  transistor which is connected in  $CE$  configuration (Fig. 58.22) and whose output characteristics are given in Fig. 58.23. Let its  $\beta = 100$ .

First, let us find the cut-off and saturation points for drawing the dc load line and then mark in the  $Q$ -point.

$$I_{C(sat)} = 10/2 = 5 \text{ mA} \quad \text{---point B in Fig. 58.23}$$

$$V_{CE(cut-off)} = V_{CC} = 10 \text{ V} \quad \text{---point A in Fig. 58.23}$$

The load line is drawn in Fig. 58.23 below.

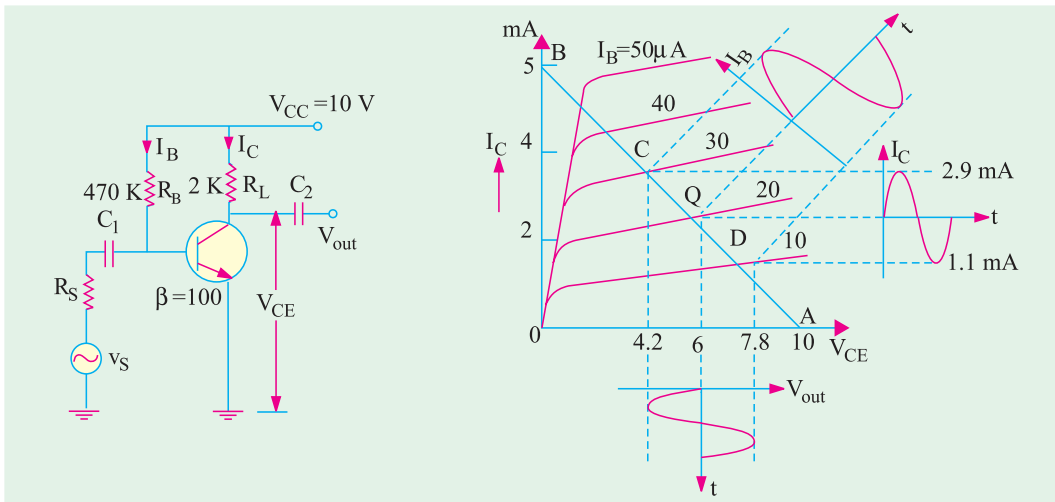


Fig. 58.22

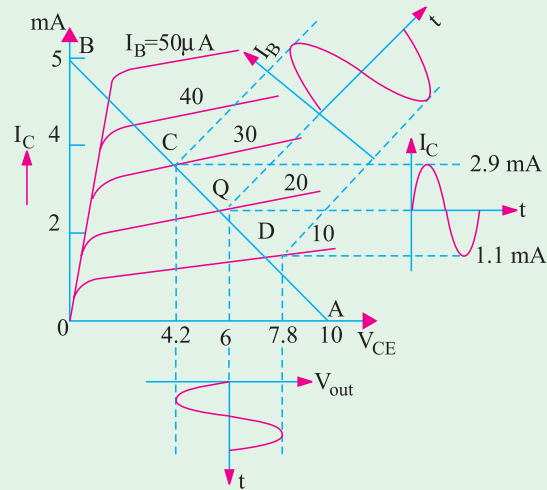


Fig. 58.23

Actual 
$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0.7}{470} = 20 \mu\text{A}$$

$$I_C = \beta I_B = 100 \times 20 = 2000 \mu\text{A} = 2 \text{ mA}$$

$$V_{CE} = V_{CC} - I_C R_L = 10 - 2 \times 2 = 6 \text{ V}$$

This locates the  $Q$ -point in Fig. 58.23.

Suppose an ac input signal voltage injects a sinusoidal base current of peak value  $10 \mu\text{A}$  into the circuit of Fig. 58.22. Obviously, it will swing the operating or  $Q$ -point up and down along the load line.

When positive half-cycle of  $I_B$  is applied, the  $Q$ -point shifts to point  $C$  which lies on the  $(20 + 10) = 30 \text{ mA}$  line.

Similarly, during negative half-cycle of input base current,  $Q$ -point shifts to point  $D$  which lies on the  $(20 - 10) = 10 \mu\text{A}$  line.

By measurement, at point  $C$ ,  $I_C = 2.9 \text{ mA}$ . Hence,  $V_{CE} = 10 - 2 \times 2.9 = 4.2 \text{ V}$

Similarly, at point  $D$ ,  $I_C$  measures  $1.1 \mu\text{A}$ . Hence,  $V_{CE} = 10 - 2 \times 1.1 = 7.8 \text{ V}$

It is seen that  $V_{CE}$  decreases from  $6 \text{ V}$  to  $4.2 \text{ V}$  i.e. by a peak value of  $(6 - 4.2) = 1.8 \text{ V}$  when base current goes positive. On the other hand,  $V_{CE}$  increases from  $6 \text{ V}$  to  $7.8 \text{ V}$  i.e. by a peak value of  $(7.8 - 6) = 1.8 \text{ V}$  when input base current signal goes negative. Since changes in  $V_{CE}$  represent changes in output voltage, it means that when input signal is applied,  $I_b$  varies according to the signal

amplitude and causes  $I_C$  to vary, thereby producing voltage variations.

Incidentally, it may be noted that variations in voltage drop across  $R_L$  are exactly the same as in  $V_{CE}$ .

Steady drop across  $R_L$  when no signal is applied =  $2 \times 2 = 4$  V. When base signal goes positive, drop across  $R_L = 2 \times 2.9 = 5.8$  V. When base signal goes negative,  $I_C = 1.1$  mA and drop across  $R_L = 2 \times 1.1 = 2.2$  V.

Hence, voltage variation is =  $5.8 - 4 = 1.8$  V during positive input half-cycle and  $4 - 2.2 = 1.8$  V during negative input half-cycle. Obviously, rms voltage variation =  $1.8\sqrt{2} = 1.27$  V

Now, power dissipated in  $R_L$  by ac component of output voltage is

$$P_{ac} = 1.27^2/2 = 0.81 \text{ mW and } P_{dc} = I_C^2 R = 2^2 \times 2 = 8 \text{ mW}$$

Total power dissipated in  $R_L = 8.81$  mW.

### 58.16. AC Load Line

It is the line along which  $Q$ -point shifts up and down when changes in output voltage and current of an amplifier are caused by an ac signal.

This line is steeper than the dc line but the two intersect at the  $Q$ -point determined by biasing dc voltage and currents.

AC load line takes into account the ac load resistance whereas dc load line considers only the dc load resistance.

#### (i) DC Load Line

The cut-off point for this line is where  $V_{CE} = V_{CC}$ . It is also written as  $V_{CE(\text{cut-off})}$ .

Saturation point is given by

$$I_C = V_{CC}/R_L. \text{ It is also written as } I_{C(\text{sat})}.$$

It is represented by straight line  $AQB$  in Fig.

58.24.

#### (ii) AC Load Line

The cut-off point is given by  $V_{CE(\text{out-off})} = V_{CEQ} + I_{CQ} R_{ac}$  where  $R_{ac}$  is the ac load resistance\*.

Saturation point is given by

$$I_{C(\text{sat})} = I_{CQ} + V_{CEQ}/R_{ac}.$$

It is represented by straight line  $CQD$  in Fig.

58.24.

The slope of the ac load line is given by  $y = \times 1/R_{ac}$ .

It is seen from Fig. 58.24 that maximum possible positive signal swing is =  $I_{CQ} R_{ac}$ . Similarly, maximum possible negative signal swing is  $V_{CEQ}$ . In other words, peak-signal handling capacity is limited to  $I_{CQ} R_{ac}$  or  $V_{CEQ}$  whichever is smaller.

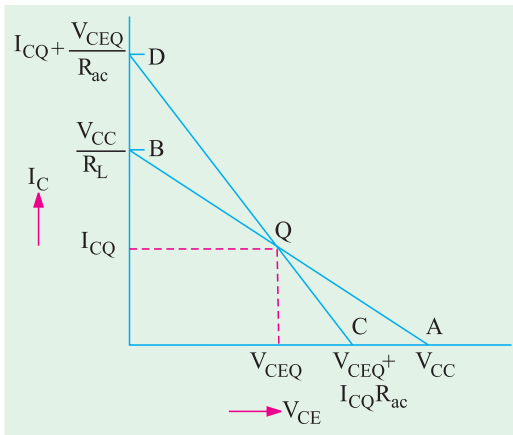


Fig. 58.24

**Example 58.12.** Draw the dc and ac load lines for the CE circuit shown in Fig. 58.25 (a). What is the maximum peak-to-peak signal that can be obtained ?

(Applied Electronics, Kerala Univ. 1991)

#### Solution. DC Load Line [Fig. 58.25 (b)]

$V_{CE(\text{cut-off})} = V_{CC} = 20$  V (point A) and  $I_{C(\text{sat})} = V_{CC}/(R_1 + R_E) = 20/5 = 4$  mA (point B). Hence,  $AQB$  represents dc load line for the given circuit.

Approximate bias conditions can be quickly found by assuming that  $I_b$  is too small to affect the base bias in Fig. 58.25 (a).

\* Written as  $r_L$  in Art. 59.4

$V_2 = 20 \times 4 / (4 + 16) = 4 \text{ V}$   
 If we neglect  $V_{BE}$ ,  $V_2 = V_E$ ;  $I_E$ ,  

$$I_E = \frac{V_E}{R_E} = \frac{V_2}{R_E} = \frac{4}{2} = 2 \text{ mA}$$
  
 Also,  $I_C \cong I_E = 2 \text{ mA}$ .  
 Hence,  $I_{CQ} = 2 \text{ mA}$   
 The corresponding value of  $V_{CEQ}$  can be found by drawing dotted line in Fig. 58.25 (b) or may be calculated as under :

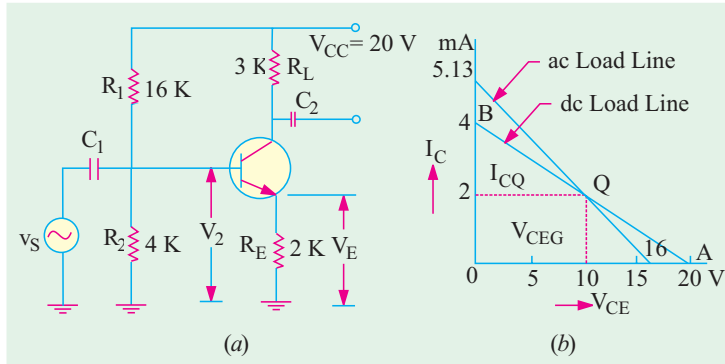


Fig. 58.25

**AC Load Line**

Cut-off point,

$$V_{CE(cut-off)} = V_{CEQ} + I_{CQ} R_{ac}$$

Now, for the given circuit, ac load resistance is  $R_{ac} = R_C = 3\text{K}$

Cut-off point =  $10 + 2 \times 3 = 16 \text{ V}$  [Fig. 58.25 (b)].

Saturation point, 
$$I_{C(sat)} = I_{CQ} + \frac{V_{CEQ}}{R_{ac}} = 2 + \frac{10}{3} = 5.13 \text{ mA}$$

Hence, line joining 16-V point and 5.13 mA point gives ac load line as shown in Fig. 58.25 (b). As expected, this line passes through the Q-point.

Now,  $I_{CQ} \cdot R_{ac} = 2 \times 3 = 6 \text{ V}$  and  $V_{CEQ} = 10 \text{ V}$ . Taking the smaller quantity, maximum peak output signal = 6 V. Hence, peak-to-peak value =  $2 \times 6 = 12 \text{ V}$ .

**Example 58.13.** Find the dc and ac load lines for CE circuit shown in Fig. 58.26 (a).

**Solution.** The given circuit is identical to that shown in Fig. 58.25 (a) except for the addition of 6-K resistor. This makes  $R_{AC} = 3 \text{ K} \parallel 6 \text{ K} = 2 \text{ K}$  because collector feeds these two resistors in parallel. The dc loadline would remain unaffected. Change would occur only in the ac load line.

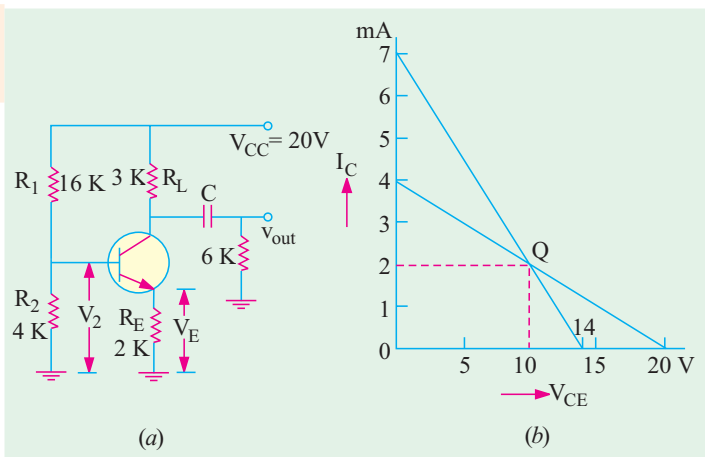


Fig. 58.26

**AC Load Line**

$$V_{CE(cut-off)} = V_{CEQ} + I_{CQ} R_{ac} = 10 + 2 \times 2 = 14 \text{ V}$$

$$I_{C(sat)} = I_{CO} + V_{CEQ} / R_{ac} = 2 + 10 / 2 = 7 \text{ mA}$$

**Example 58.14.** Draw the dc and ac load lines for the CB circuit shown in Fig. 58.27 (a). Which swing starts clipping first ?

**Solution.** The dc load line passes through cut-off point of  $30 \text{ V}$  and saturation point of  $V_{CC} / R_L = 1 \text{ mA}$ .

Now,  $I_E \cong 20 / 40 = 0.5 \text{ mA}$ ;  $I_C \cong I_E = 0.5 \text{ mA}$ ;  $I_{CQ} = 0.5 \text{ mA}$

$$V_{CB} = V_{CC} - I_C R_C = 30 - 0.5 \times 30 = 15 \text{ V}; V_{CBQ} = 15 \text{ V}$$

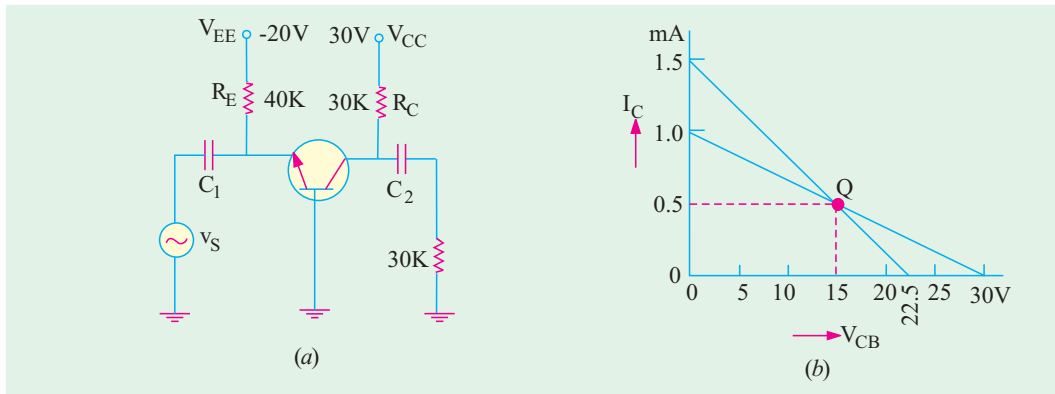


Fig. 58.27

Hence, *dc* operating point or *Q*-point is (15 V, 0.5 mA) as shown in Fig. 58.27 (b).

The cut-off point for ac load line is  $= V_{CBQ} + I_{CQ} R_{ac}$

Since collector sees an ac load of two 30 K resistors in parallel, hence  $R_{ac} = 30\text{ K} \parallel 30\text{ K} = 15\text{ K}$ .

$$\therefore V_{CBQ} + I_{CQ} R_{ac} = 15 + 0.5 \times 15 = 22.5\text{ V}$$

Saturation current for ac line is  $= I_{CQ} + V_{CBQ}/R_{ac} = 0.5 + 15/15 = 1.5\text{ mA}$

The line joining these two points (and also passing through *Q*) gives the ac load line as shown in Fig. 58.27 (b).

**Note.** Knowing ac cut-off point and *Q*-point, we can draw the ac load line. Hence, we need not find the value of saturation current for this purpose.

As seen, positive swing starts clipping first because  $I_{CQ} R_{ac}$  is less than  $V_{CBQ}$ . Obviously, maximum peak-to-peak signal that can be obtained from this circuit  $= 2 \times 7.5 = 15\text{ V}$ .

**Example. 58.15.** For the circuit shown in Fig. 58.27 (a), find the approximate value of source voltage *us* that will cause clipping. The voltage source has an internal resistance of 1 K.

**Solution.** As found out in Ex. 58.14, the maximum swing of the unclipped output  $= 2 \times 7.5 = 15\text{ V}$ .

$$\text{Now, } A_v = \frac{V_{out}}{V_s} = \frac{R_{ac}}{R_s} = \frac{15}{1} = 15$$

$$\therefore V_s = \frac{V_{out}}{A_v} = \frac{15V_{p-p}}{15} = 1V_{p-p}$$

### Tutorial Problems No. 58.1

- For the CB circuit shown in Fig. 58.28, find the approximate location of *Q*-point. [10 V, 2 mA]
- For the circuit of Fig. 58.29, find
  - dc operating-point,
  - maximum peak-to-peak unclipped signal. [(15 V, 0.5 mA) ; 10 V<sub>p-p</sub>]
- What is the maximum peak-to-peak signal that can be obtained from the circuit of Fig. 58.30 ? [20 V<sub>p-p</sub>]
- Find the value of maximum peak-to-peak output of signal that can be obtained from the circuit of Fig. 58.31. [10 V<sub>p-p</sub>]

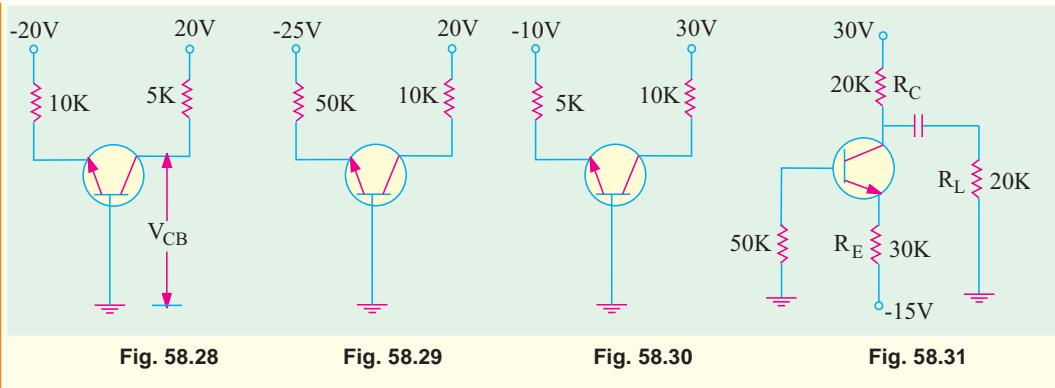


Fig. 58.28

Fig. 58.29

Fig. 58.30

Fig. 58.31

## OBJECTIVE TESTS — 58

- The dc load line of a transistor circuit
  - has a negative slope
  - is a curved line
  - gives graphic relation between  $I_C$  and  $I_B$
  - does not contain the  $Q$ -point.
- The positive swing of the output signal in a transistor circuit starts clipping first when  $Q$ -point of the circuit moves
  - to the centre of the load line
  - two-third way up the load line
  - towards the saturation point
  - towards the cut-off point.
- To avoid thermal run away in the design of analog circuit, the operating point of the  $BJT$  should be such that it satisfies the condition
  - $V_{CE} = 1/2 V_{CC}$
  - $V_{CE} < 1/2 V_{CC}$
  - $V_{CE} > 1/2 V_{CC}$
  - $V_{CE} < 0.78 V_{CC}$
- In the case of a  $BJT$  amplifier, bias stability is achieved by
  - keeping the base current constant
  - changing the base current in order to keep the  $I_C$  and  $V_{CB}$  constant
  - keeping the temperature constant
  - keeping the temperature and the base current constant
- For a transistor amplifier with self-biasing network, the following components are used :  
 $R_1 = 4 \text{ k } \Omega$ ,  $R_2 = 4 \text{ k } \Omega$  and  $R_E = 1 \text{ k } \Omega$   
 the approximate value of the stability factor 'S' will be
  - 4
  - 3
  - 2
  - 1.5
- A transistor circuit employing base bias with collector feedback has greater stability than the one without feedback because
  - $I_C$  decrease in magnitude
  - $V_{BE}$  is decreased
  - of negative feedback effect
  - $I_C$  becomes independent of  $\beta$ .
- The universal bias stabilization circuit is most popular because
  - $I_C$  does not depend on transistor characteristics
  - its  $\beta$ -sensitivity is high
  - voltage divider is heavily loaded by transistor base
  - $I_C$  equals  $I_E$ .
- Improper biasing of a transistor circuit leads to
  - excessive heat production at collector terminal
  - distortion in output signal
  - faulty location of load line
  - heavy loading of emitter terminal.
- The negative output swing in a transistor circuit starts clipping first when  $Q$ -point
  - has optimum value
  - is near saturation point
  - is near cut-off point
  - is in the active region of the load line.
- When a  $BJT$  is employed as an amplifier, it operates

- (a) in cut-off  
 (b) in saturation  
 (c) well into saturation  
 (d) over the active region.
11. Which of the following method used for biasing a *BJT* in integrated circuits is considered independent of transistor beta?  
 (a) fixed biasing  
 (b) voltage divider bias  
 (c) collector feedback bias  
 (d) base bias with collector feedback.
12. The voltage  $V_0$  of the circuit shown in Fig. 58.32 is,

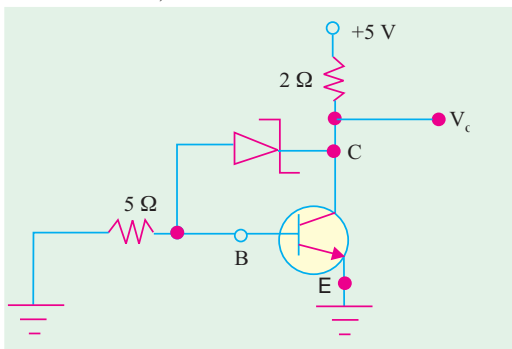


Fig. 58.32

- (a) 5.1 V  
 (b) 3.1 V  
 (c) 2.5 V  
 (d) zero
13. The collector voltage  $V_c$  of the circuit shown in Fig. 58.33, is approximately  
 (a) 2 V  
 (b) 4.6 V  
 (c) 9.6 V  
 (d) 8.6 V

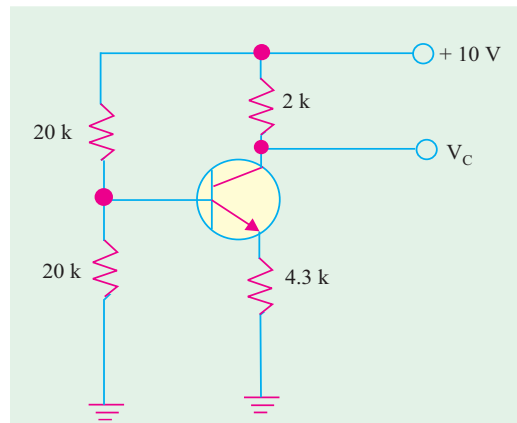


Fig. 58.33

ANSWERS

- |         |        |        |         |         |         |
|---------|--------|--------|---------|---------|---------|
| 1. (a)  | 2. (d) | 3. (c) | 4. (a)  | 5. (b)  | 6. (b)  |
| 7. (a)  | 8. (b) | 9. (b) | 10. (d) | 11. (b) | 12. (d) |
| 13. (a) |        |        |         |         |         |



Learning Objectives

- General
- DC Equivalent Circuit
- AC Equivalent Circuit
- Equivalent Circuit of a CB Amplifier
- Effect of Source Resistance  $R_s$  on Voltage Gain
- Equivalent Circuit of a CE Amplifier
- Equivalent Circuit of a CC Amplifier
- Small-signal Low-frequency Model or Representation
- T-Model
- Formulas for T-Equivalent of a CC Circuit
- What are h-parameters ?
- Input Impedance of a Two Port Network
- Voltage Gain of a Two Port Network
- The h-parameters of an Ideal CB Transistor
- The h-parameters of an ideal CE Transistor
- Approximate Hybrid Equivalent Circuits
- Transistor Amplifier Formulae Using h-parameters
- Typical Values of Transistor h-parameters
- Approximate Hybrid Formulas
- Common Emitter h-parameter Analysis
- Common Collector h-parameter Analysis
- Conversion of h-parameters

# TRANSISTOR EQUIVALENT CIRCUITS AND MODELS



Generalised hybrid parameter equivalent circuit

### 59.1. General

We will begin by idealizing a transistor with the help of simple approximations that will retain its essential features while discarding its less important qualities. These approximations will help us to **analyse transistor circuits easily and rapidly**.

We will discuss only the **small signal** equivalent circuits in this chapter. Small signal operation is that in which the ac input signal voltages and currents are in the order of  $\pm 10$  per cent of  $Q$ -point voltages and currents.

There are two prominent schools of thought today regarding the equivalent circuit to be substituted for the transistor. The two approaches make use of

- (a) four  $h$ -parameters of the transistor and the values of circuit components,
- (b) the beta ( $\beta$ ) of the transistor and the values of the circuit components.

Since long, industrial and educational institutions have heavily relied on the hybrid parameters because they produce more accurate results in the analysis of amplifier circuits. In fact, hybrid-parameter equivalent circuit continues to be popular even to-day. But their use is beset with the following difficulties :

1. The values of  $h$ -parameters are not so readily or easily available.
2. Their values vary considerably with individual transistors **even of the same type number**.
3. Their values are limited to a particular set of operating conditions for reasonably accurate results.

The second method which employs transistor beta and resistance values is gaining more popularity of late. It has the following advantages :

1. The required values are easily available;
2. The procedure followed is simple and easy to understand;
3. The results obtained are quite accurate for the study of amplifier circuit characteristics.

To begin with, we will consider the second method first.

### 59.2. DC Equivalent Circuit

#### (a) CB Circuit

In an ideal transistor,  $\alpha = 1$  which means that  $I_C = I_E$ .

The emitter diode acts like any **forward-biased ideal diode**. However, due to transistor action, collector diode acts as a **current source**. In other words, for the purpose of drawing dc equivalent circuit, we can view an ideal transistor as nothing more than a rectifier diode in emitter and a current source in collector.

In the dc equivalent circuit of Fig. 59.1 (b), current arrow always points in the direction of conventional current.

As per the polarities of transistor terminals (Art. 59.3) shown in Fig. 59.1 (a), emitter current flow from  $E$  to  $B$  and collector current from  $B$  to  $C$ .

The dc equivalent circuit shown in Fig. 59.2 for an  $NPN$  transistor is exactly similar except that direction of current flow is opposite.

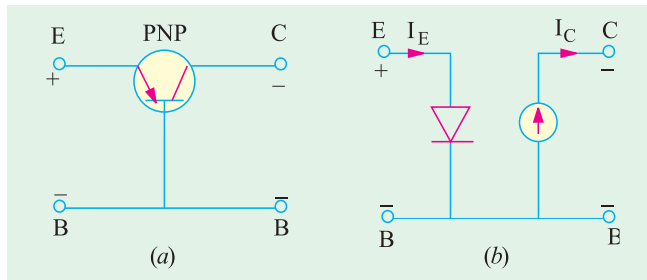


Fig. 59.1

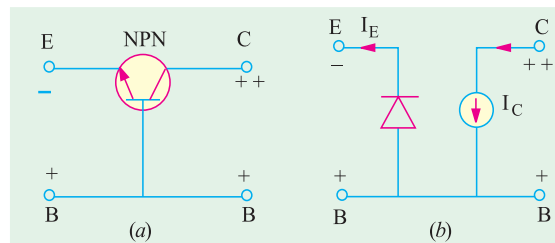


Fig. 59.2



**(b) CE Circuit**

Fig. 59.3 shows the dc equivalent circuit of an *NPN* transistor when connected in the *CE* configuration. Direction of current flow can be easily found by remembering the transistor polarity rule given in Art 59.3. In an ideal *CE* transistor, we disregard leakage current and take a.c beta equal to dc beta.

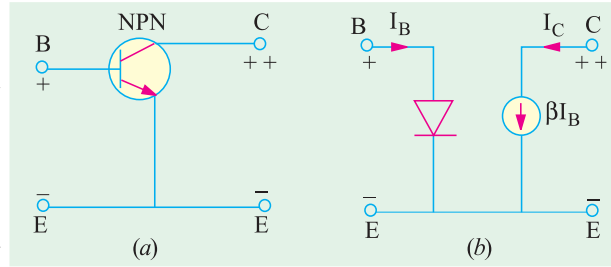


Fig. 59.3

**59.3. AC Equivalent Circuit**

**(a) CB Circuit**

In the case of *small input* ac signals, the emitter *diode does not rectify*, instead it offers resistance called ac *resistance*. As usual, collector diode acts as a current source.

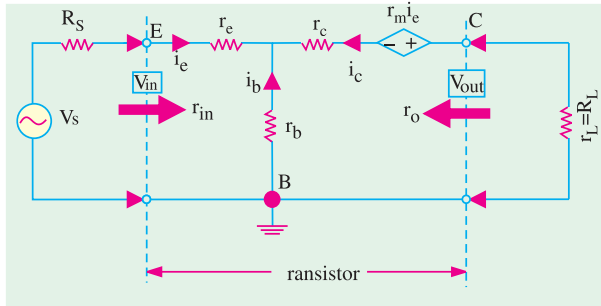


Fig. 59.4

Fig. 59.4 shows the ac equivalent circuit of a transistor connected in the *CB* configuration. \* Here, ac resistance offered by the emitter diode is

$r_{ac} = \text{junction resistance } (r_j) + \text{base-spreading resistance } (r_B)**$

$$= r_j \quad r_B \text{ is negligible}$$

$$= \frac{25 \text{ mV}}{I_E}$$

— where  $I_E$  is dc emitter current in mA

It is written as  $r_e'$  signifying junction resistance of the emitter *i.e.* a.c resistance looking into the emitter.

$$r_e' = \frac{25 \text{ mV}}{I_E}$$

Hence, the a.c equivalent circuit of a *CB* circuit becomes as shown in Fig. 59.5. Since changes in collector current are almost equal to changes in emitter current,  $\Delta i_c = \Delta i_e$ .

**(b) CE Circuit**

Fig. 59.6 (a) shows the equivalent circuit when an *NPN* transistor has been connected in the *CE* configuration.

The a.c resistance *looking into the base is*

$$r_{ac} = \frac{25 \text{ mV}}{I_B} \quad \text{— d.c current is } I_B \text{ not } I_E$$

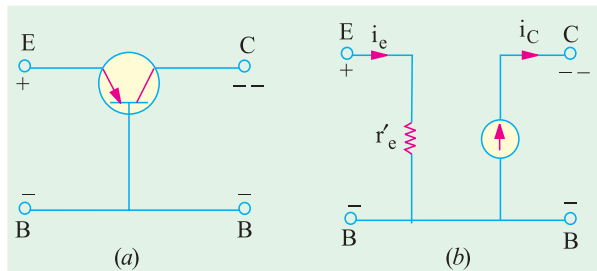


Fig. 59.5

\* This circuit is valid both for *PNP* as well as *NPN* transistors because difference in the direction of ac current does not matter.

\*\* Also called bulk-resistance.



$$\frac{25\text{mV}}{I_C'} , \frac{25\text{mV}}{I_C}$$

$$, \frac{25\text{mV}}{I_E} \quad r_e'$$

Strictly speaking,

$$r_{ac} = (1 + \beta) r_e' \cong \beta r_e' \text{ — Art 7.24}$$

The a.c collector current is  $\beta$  times the base current *i.e.*,  $i_c = \beta i_b$ .

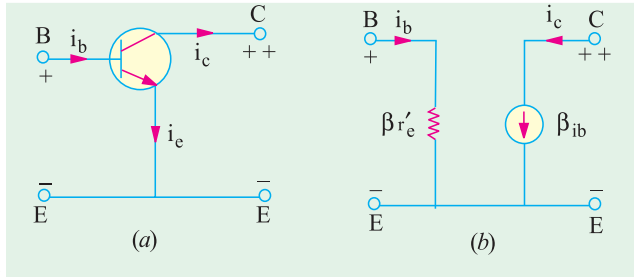


Fig. 59.6

### 59.4. Equivalent Circuit of a CB Amplifier

In Fig. 59.7 (a) is shown the circuit of a common-base amplifier. As seen, emitter is forward-biased by  $-V_{EE}$  and collector is reverse-biased by  $+V_{CC}$ . The a.c signal source voltage  $v_s$  drives the

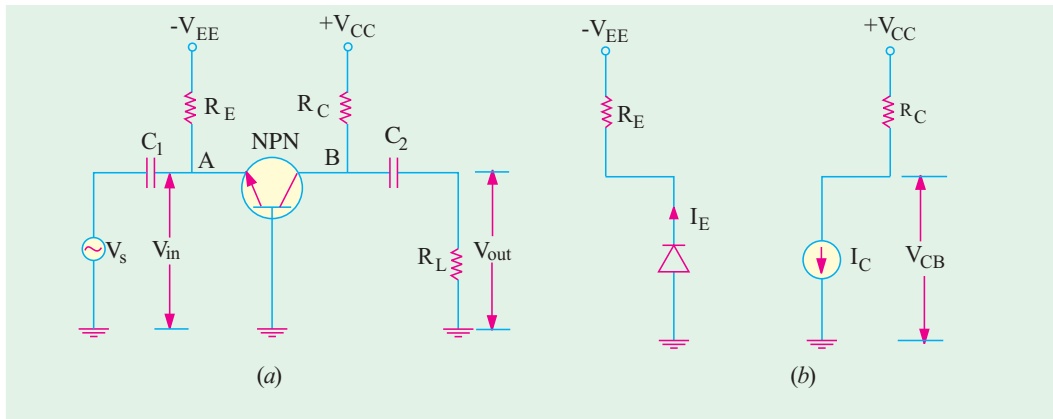


Fig. 59.7

emitter. It produces small fluctuations in transistor voltage and currents in the output circuit. It will be seen that ac output voltage is amplified because it is more than  $V_s$ .

#### (a) DC Equivalent Circuit

For drawing dc equivalent circuit, following procedure should be adopted :

- (i) short all ac sources *i.e.* reduce them to zero,
- (ii) open all capacitors because they block dc.

If we do this, then as seen from Fig. 59.7 (a), neither emitter current can pass through  $C_1$  nor collector current can pass through  $C_2$ . These currents are confined to their respective resistances  $R_E$  and  $R_C$  (earlier we had been designating it as resistance  $R_L$ ).

Here,  $I_C \cong I_E$  and  $V_{CB} = V_{CC} - I_C R_L$

Hence, the dc equivalent circuit becomes as shown in Fig. 59.7 (b).

#### (b) AC Equivalent Circuit

For drawing ac equivalent circuit, following procedure is adopted :

- (i) all dc sources are shorted *i.e.* they are treated as ac ground,
- (ii) all coupling capacitors like  $C_1$  and  $C_2$  in Fig. 59.7 (a) are shorted and
- (iii) emitter diode is replaced by its a.c resistance  $r_{ac} = r_e'$

$$r_e' = \frac{25\text{mV}}{I_E}$$

where  $I_E$  is dc emitter current in mA.

As seen by the input a.c signal, it has to feed  $R_E$  and  $r_e'$  in parallel [Fig. 59.8 (a)]. As looked



from point A in Fig. 59.7 (a),  $R_E$  is grounded through  $V_{EE}$  which has been shorted and  $r'_e$  is grounded via the base.

Similarly, collector has to feed  $R_C$  and  $R_L$  which are connected in parallel across it  $i.e$  at point B in Fig. 59.7 (a). The ac signal in collector sees an output load resistance of  $r_L = R_C \parallel R_L$ .

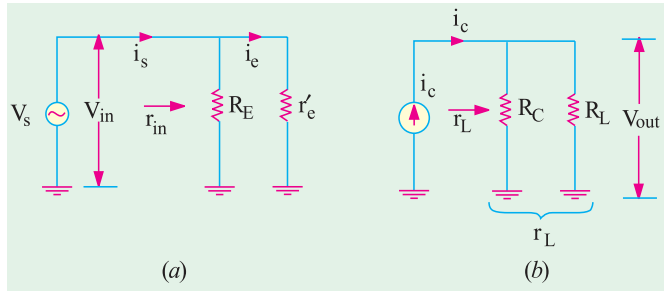


Fig. 59.8

Hence, ac equivalent circuit is as shown in Fig. 59.8 (b). Here, collector diode itself has been shown as a current source.

Following two points are worth noting :

- (i) changes in collector signal current are very nearly equal to changes in emitter signal current. Hence,  $\Delta i_c \cong \Delta i_e$ ,
- (ii) directions of ac currents shown in the circuit diagram are those which correspond to positive half-cycle of the a.c input voltage. That is why  $i_c$  is shown flowing upwards in Fig. 59.8 (b).

(c) Principal Operating Characteristics

1. Input Resistance

As seen from Fig. 59.8 (a), the input resistance of the circuit (or stage) is given by

$$r_{in} = R_E \parallel r'_e = \frac{r'_e R_E}{R_E + r'_e}$$

In practice,  $R_E$  is always much greater than  $r'_e$  so that parallel combination  $R_E \parallel r'_e \cong r'_e$

$\therefore r_{in} \cong r'_e$  input resistance of the emitter diode

2. AC Load Resistance

The collector load as seen by output ac signal consists of a parallel combination of  $R_L$  and  $R_C$ . This has already been designated as  $r_L$ .

$$\therefore r_L = R_C \parallel R_L$$

It should be carefully noted that it is the output resistance **as seen by collector and not the ac output resistance when looking into the collector.**

**Note.** In case,  $R_L$  has not been connected, then  $r_L = R_C$  (Ex. 59.1)

3. Current Gain

It is given by the ratio  $A_i = \frac{i_c}{i_e}$

4. Voltage Gain

It is given by the ratio

$$A_v = \frac{V_{out}}{V_{in}}$$

$$\text{Now, } V_{in}^* = i_e r_{in} = (1 + \beta) i_b \cdot r_{in}$$

$$V_{out} = i_c r_L = \beta i_b r_L$$

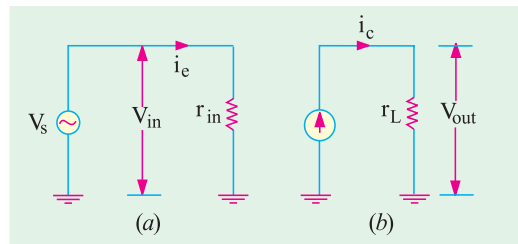


Fig. 59.9

\* Here,  $V_{in}$  equals  $v_s$  because there is no internal resistance of the source. In case it is there, the two will not be equal (Art 59.5).



$$\therefore A_v = \frac{i_b r_L}{(1 + \beta) i_b r_{in}}$$

Taking the ratio  $\beta / (1 + \beta)$  as unity,  $A_v = \frac{r_L}{r_{in}} = \frac{r_L}{r'_e}$

**5. Power Gain**

$$A_p = A_v \cdot A_i$$

When expressed in decibels, it is written as  $G_p = 10 \log_{10} A_p \text{ dB}$ .

When expressed in terms of  $r_{in}$  and  $r_L$ , the a.c equivalent circuit becomes as shown in Fig. 59.9.

**Example 59.1.** For the single-stage CB amplifier shown in Fig. 59.10 (a), find  $r_{in}$ ,  $r_L$ ,  $A_i$ ,  $A_v$  and  $A_p$ . What would be the rms value of the signal voltage across the load if  $v_s$  has an rms value of 1.5 mV? Assume silicon material and transistor  $\alpha = 0.98$ . **(Electronics-II, Madras Univ.)**

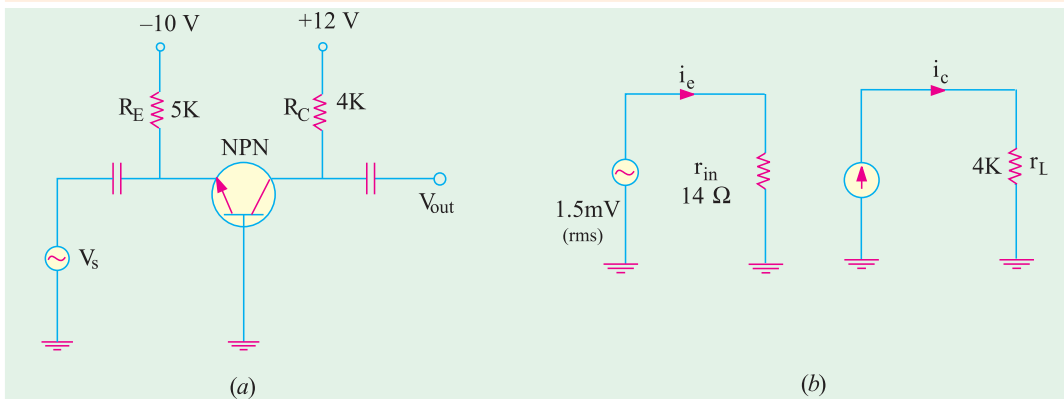


Fig. 59.10

**Solution.** For finding  $I_E$ , let us first find  $I_E$ .

$$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{10 - 0.7}{5K} = 1.86 \text{ mA}$$

$$r'_e = \frac{25 \text{ mV}}{I_E \text{ mA}} = \frac{25}{1.86} = 14$$

(i)  $r_{in} = r'_e \parallel R_E = 14 \Omega \parallel 5K \cong 14 \Omega$

(ii)  $r_L = R_C = 4K$

The ac equivalent circuit becomes as shown in Fig. 59.10 (b).

(iii)  $A_i = \alpha = 0.98$

(iv)  $A_v = \frac{r_L}{r_{in}} = \frac{r_L}{r'_e} = \frac{4K}{14} = 286$

(v)  $A_p = A_i A_v = 0.98 \times 286 = 280$

$G_p = 10 \log_{10} 280 = 24.5 \text{ dB}$

Output ac voltage =  $A_v \times$  input voltage

$\therefore V_{out} = A_v \times V_{in} = 286 \times 1.5 = 429 \text{ mV} = 0.429 \text{ V}$

**Example 59.2.** For the CB amplifier circuit shown in Fig. 59.11 (a), find

- (i) stage  $r_{in}$  (ii)  $r_L$  (iii) a.c output voltage  $V_{out}$  (iv) voltage gain  $A_v$ .  
Take  $V_{BE} = 0.7 \text{ V}$ .

**Solution.**  $I_E = (20 - 0.7)/30K = 0.64 \text{ mA}$ ;  $r'_e = 25/0.64 = 39 \Omega$

(i) stage  $r_{in} = r'_e \parallel R_E = 39 \Omega \parallel 30K \cong 39 \Omega$

(ii)  $r_L = R_L \parallel R_C = 30K \parallel 15K = 10K$



The amplifier circuit alongwith its ac equivalent circuit is shown in Fig. 59.11.

(iii) We will first find the value of  $i_{out}$  as a drop across  $r_L = R_C \parallel R_L$ . For that purpose, we will employ rms values.

$$\text{Now, } i_e = \frac{V_{in}}{r_{in}} = \frac{V_s}{r_{in}} = \frac{1.6}{3.9} = 0.04 \text{ mA}^*$$

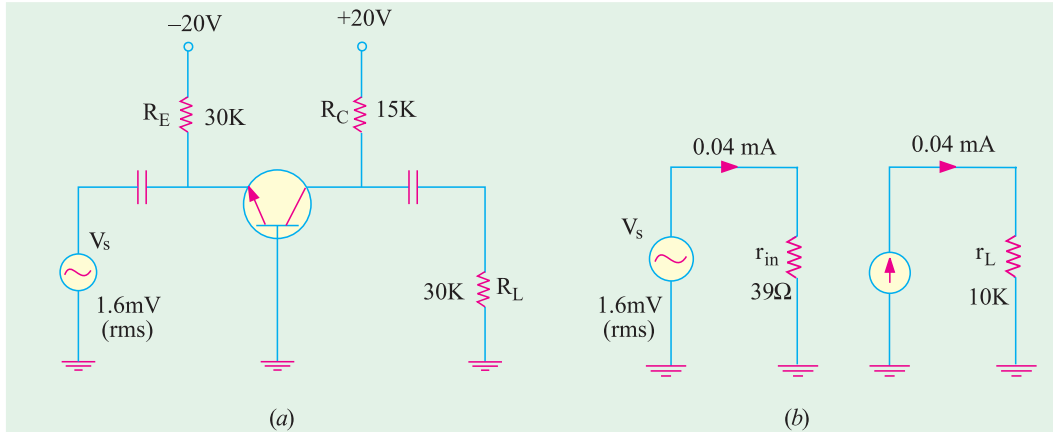


Fig. 59.11

$$i_c = i_e = 0.04 \text{ mA} ; V_{out} = i_c r_L = 0.04 \times 10 \text{ K} = 0.4 \text{ V}$$

$$(iv) A_v = \frac{r_L}{r_{in}} = \frac{r_L}{r_e} = \frac{r_L}{r_e} = \frac{10 \text{ K}}{39} = 250$$

The rms value of ac output voltage may be found with the help of  $A_v$ .

$$\text{Now, } A_v = \frac{V_{out}}{V_{in}} \quad V_{out} = A_v \cdot V_{in} = 250 \cdot 1.6 = 400 \text{ mV} = 0.4 \text{ V}$$

### 59.5. Effect of Source Resistance $R_s$ on Voltage Gain

The voltage gain for the *CB* amplifier circuit has so far been calculated on the assumption that the resistance  $R_s$  of the ac signal source is negligible. The voltage gain will decrease as  $R_s$  increases because more and more of  $V_s$  will drop on  $R_s$  rather than on  $r_{in}$  and so will not appear in the output.

The basic circuit is shown in Fig. 59.12 where  $R_s$  is the internal resistance of the ac signal source.

The ac equivalent circuit is shown in Fig. 59.13 (a). Here,  $R_s$  is in series with  $(r_e' \parallel R_E)$ . The input ac signal voltage  $v_s$  drops on these two series resistors. Obviously,  $V_{in}$  is the drop across  $r_e' \parallel R_E$  and is less than  $V_s$ . Using Proportional Voltage Formula,

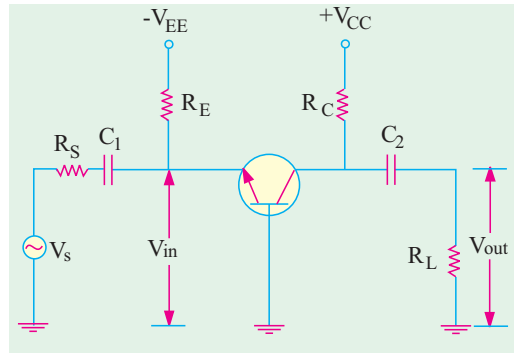


Fig. 59.12

\* Strictly speaking, it is not  $i_e$  but  $i_s$ . a small part of the ac source current goes through  $R_E = 30 \text{ K}$  and the balance (major part) goes through parallel resistance  $r_e'$ . However, if we neglect current through 30 K, then  $i_e = i_s$



$$V_{in} = V_s \frac{r_e' \parallel R_E}{R_s + r_e' \parallel R_E}$$

In practice,  $R_E \gg r_e'$ , so that  $r_e' \parallel R_E \cong r_e'$

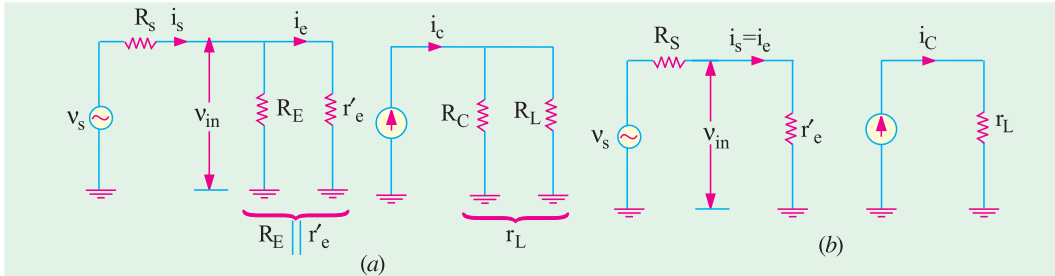


Fig. 59.13

$$\therefore V_{in} = V_s \frac{r_e'}{R_s + r_e'} = \frac{V_s}{1 + \frac{R_s}{r_e'}}$$

As  $R_s$  increases in comparison to  $r_e'$ , the ratio  $R_s/r_e'$  increases thereby making  $v_{in}$  increasingly less than  $V_s$ .

Moreover, as seen from the source, input resistance is

$$R_s + r_e' \parallel R_E \cong R_s + r_e' \quad \therefore \frac{V_{out}}{V_s} = \frac{r_L}{(R_s + r_e')} \frac{r_L}{R_s} \quad \text{--- if } R_s \gg r_e'$$

Making  $R_s$  much larger than  $r_e'$  is called swamping out the emitter diode. This makes voltage gain independent of  $r_e'$  and hence the particular transistor is used. The above gain is different from the voltage gain considered from emitter to collector which is

$$\frac{V_{out}}{V_{in}} = \frac{r_L}{r_{in}} = \frac{r_L}{r_e'}$$

**Example. 59.3.** In the CB amplifier circuit of Fig. 59.14, find  
 (a) voltage gain from source to output      (b) voltage gain from emitter to output  
 (c) approximate value of  $V_{in}$       **(Electronics-I, Patna Univ.)**

**Solution.** (a) The voltage gain from source to output is given by

$$\begin{aligned} \frac{V_{out}}{V_s} &= \frac{r_L}{(R_s + r_e')} \\ r_L &= R_C \parallel R_L = 10 \text{ K} \parallel 2\text{M} \\ &\cong 10 \text{ K}; I_E = 25/25 \text{ K} = 1 \text{ mA} \\ r_e' &= 25/1 = 25 \Omega \\ \frac{V_{out}}{V_s} &= \frac{10 \text{ K}}{(1\text{K} + 25)} = 10 \quad \dots(i) \end{aligned}$$

(b) The voltage gain from emitter to output is

$$\frac{V_{out}}{V_s} = \frac{r_L}{r_{in}}$$

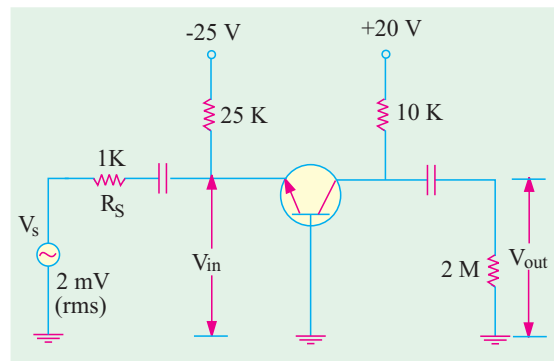


Fig. 59.14





Now,  $r_{in} = R_E // r_e' = 25 \text{ K} // 25 \Omega \cong 25 \Omega$   $\therefore \frac{V_{out}}{V_s} = \frac{10 \text{ K}}{25} = 400 \dots(ii)$

(e) The value of  $V_{in}$  may be found by the following two ways :

(i) As seen from Eq. (ii) above,  $V_{in} = \frac{V_{out}}{400}$  , Now, from Eq. (i) above, we have

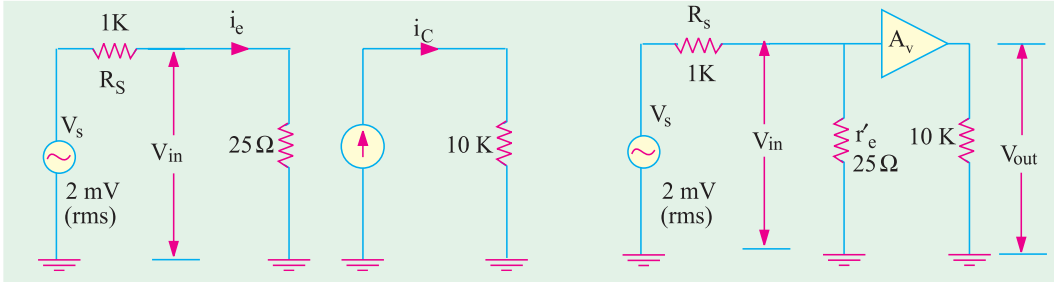


Fig. 59.15

$V_{out} = 10 V_s = 10 \times 2 = 20 \text{ mV}$   $\therefore V_{in} = \frac{20}{400} = 50 \mu\text{V}$

(ii) As seen from the ac equivalent circuit of the amplifier (Fig. 59.15),  $V_s$  is dropped proportionally on the two series resistances  $R_s$  and  $r_e'$  (strictly speaking  $r_e' // R_E$ ). The drop across  $r_{in}$  is called  $V_{in}$ .

$\therefore V_{in} = V_s \frac{r_{in}}{R_s + r_{in}} = 2 \text{ mV} \frac{25}{1000 + 25} = 50 \mu\text{V}$

### 59.6. Equivalent Circuit of a CE Amplifier

Consider the simple CE amplifier circuit of Fig. 59.16 (a) in which base bias has been employed.

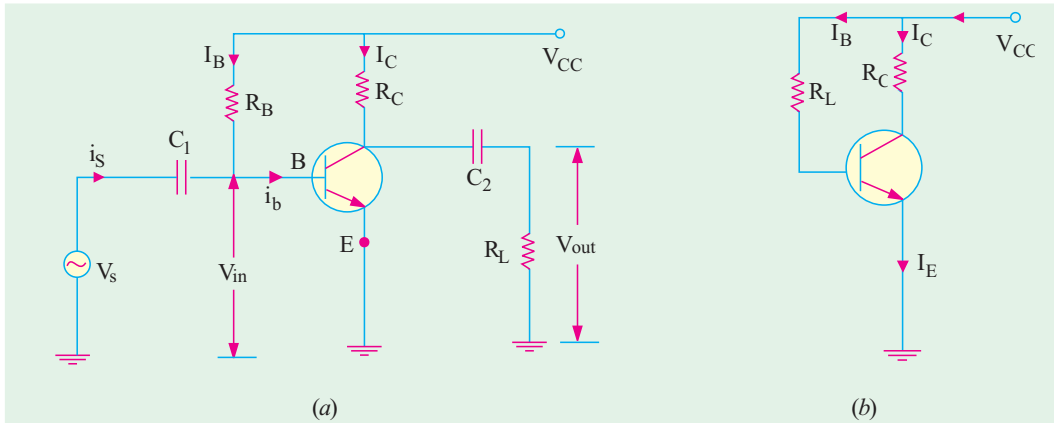


Fig. 59.16

#### (a) DC Equivalent Circuit

For drawing the dc equivalent circuit, same procedure is adopted as given in Art. 59.2. It is shown in Fig. 59.16 (b).

As seen,  $I_B = \frac{V_{CC} - V_{BE}}{R_B}$  ;  $I_C = \beta \cdot I_B$  ;  $I_E \cong I_C \cong \beta I_B$



**(b) AC Equivalent Circuit**

Let us now analyse the ac equivalent circuit given in Fig. 59.17.

As proved in Art. 59.3, the ac resistance as seen by the input signal when looking into the base is  $\beta r_e'$ . It may be called  $r_{in(base)}$  to distinguish it from  $r_{in}$  which is resistance of the CE stage as shown in Fig. 59.17 (a).

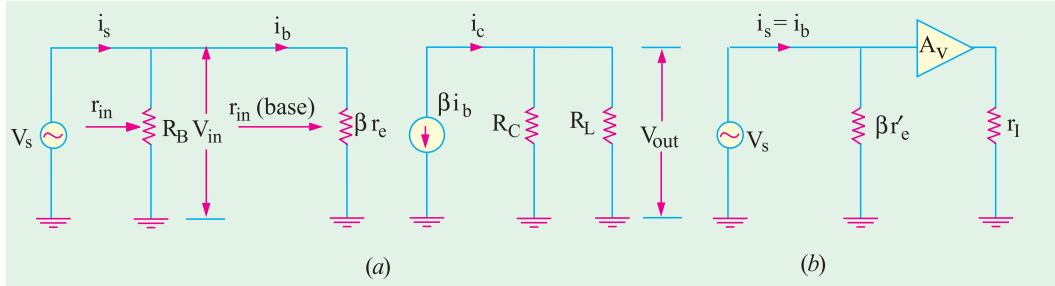


Fig. 59.17

It may be noted that in the absence of ac voltage source resistance  $R_s$ , whole of  $V_s$  acts across  $R_B$  as well as  $\beta r_e'$  because the two are connected in parallel across it.

Another point worth noting is that major part of source current is passes through  $\beta r_e'$  and an extremely small part ( $= v_s/R_B$ ) passes through  $R_s$ . Since  $R_B$  is usually very large, current passing through it can be easily neglected. Hence, as shown in Fig. 59.17 (b),  $i_s = i_b$ .

**(c) Principal Operating Characteristics**

**1. Input Resistance**

As seen from Fig. 59.17, input resistance of the stage is

$$r_{in} = R_B // \beta r_e' \cong \beta r_e' \quad R_B \gg \beta r_e'$$

= input resistance of the base

$$r_{in(stage)} = r_{in(base)}$$

**2. AC Load Resistance.**  $r_L = R_C // R_L$

**3. Current Gain.**  $A_i = \frac{i_c}{i_b}$

**4. Voltage gain.** The voltage gain of the stage or circuit is  $A_v = \frac{v_{out}}{v_{in}}$

Now,  $v_{in} = i_b r_e'$  and  $v_{out} = i_c \cdot r_L = \beta i_b r_L$

$$\therefore A_v = \frac{\beta i_b r_L}{i_b r_e'} = \frac{\beta r_L}{r_e'} \quad \text{--- if } R_B \gg \beta r_e'$$

**5. Power Gain.**  $A_p = A_i \cdot A_v$  and  $G_p = 10 \log_{10} A_v \text{ dB}$

**Example. 59.4.** If in the CE circuit of Fig. 59.16 (a),  $V_{CC} = 20 \text{ V}$ ,  $R_C = 10 \text{ K}$ ,  $R_B = 1 \text{ M}$ ,  $R_L = 1 \text{ M}$ ,  $v_s = 2 \text{ mV}$  and  $\beta = 50$ , find (i)  $i_b$  and  $i_c$  (ii)  $r_{in}$  (iii)  $r_L$  (iv)  $A_n$  (v)  $A_p$  and  $G_p$ .

(Basic Electronics, Bombay Univ. 1991)

**Solution.**  $I_B = 20/1 \text{ M} = 20 \mu\text{A}$ ;  $I_C = \beta I_B = 50 \times 20 \text{ mA} = 1 \text{ mA}$   
 $r_e' = 25 \text{ mV}/1 \text{ mA} = 25 \Omega$ ;  $r_{in(base)} = \beta r_e' = 50 \times 20 = 1250 \Omega$

(i) As seen from Fig. 59.17 (a), ac base current is given by

$$i_b = \frac{v_s}{r_{in(base)}} = \frac{2 \text{ mV}}{1250} = 1.6 \mu\text{A}; i_c = \beta i_b = 50 \times 1.6 = 80 \mu\text{A}$$

(ii) stage  $r_{in} = R_B // \beta r_e' = 1 \text{ M} // 1250 \Omega \cong 1250 \Omega$

(iii)  $r_L = R_L // R_C = 1 \text{ M} // 10 \text{ K} \cong 10 \text{ K}$



(iv)  $A_v = \frac{r_v}{r_e'} \cdot \frac{r_L}{r_e'} = \frac{10\text{ K}}{25} = 400$

(v)  $A_p = r_{in} \cdot A_v = 50 \times 400 = 20,000$ ;  $G_p = 10 \log_{10} 20,000 = 43\text{ dB}$

**Example 59.5.** In the CE amplifier circuit of Fig. 59.18 employing emitter feedback, find :  
 (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  (iv)  $A_p$  and (v)  $G_p$   
 Take transistor  $\beta = 100$ . How will these values change if emitter bypass capacitor is removed ?  
 (Electronics—II, Madras Univ.)

**Solution.** It should be carefully noted that emitter bypass capacitor  $C$  provides **ac ground** to the signal i.e. it shorts out  $R_E$  to ground so far as **ac signal is concerned**. However, it plays its normal role so far as dc quantities are concerned.

Hence, the ac equivalent circuit of the amplifier becomes as shown in Fig. 59.19.

Now,  $I_B = \frac{V_{CC}}{R_B + R_E} = \frac{30}{2\text{ M} + 100 \cdot 10\text{ K}}$   
 $= 10\ \mu\text{A}$ ;  $I_C = \beta I_B$   
 $= 100 \times 10\ \mu\text{A} = 1\ \text{mA}$   
 $I_E \cong I_C = 1\ \text{mA}$ ,  $r_e' = 25/1 = 25\ \Omega$ ;  
 $\beta r_e' = 100 \times 25 = 2500\ \Omega$

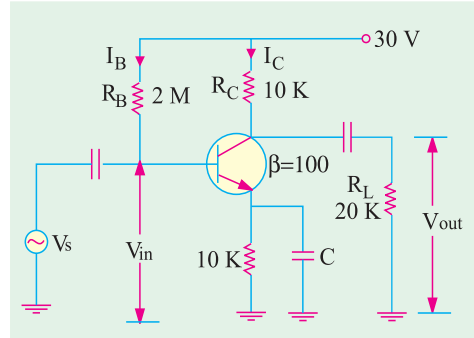


Fig. 59.18

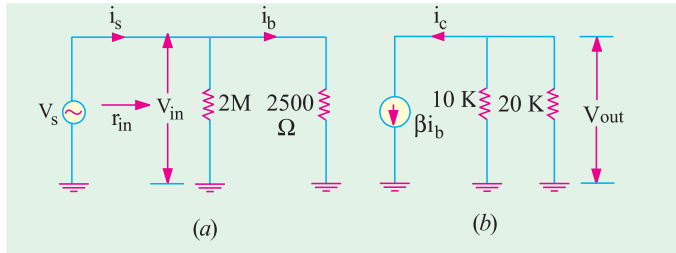


Fig. 59.19

- (i) As seen from Fig. 59.19  
 $r_{in} = R_B \parallel \beta r_e' = 2\text{ M} \parallel 2500\ \Omega \cong 2500\ \Omega$
- (ii)  $r_L = R_C \parallel R_L = 10\ \text{K} \parallel 20\ \text{K} = 6.67\ \text{K}$

(iii)  $A_v = \frac{V_{out}}{V_{in}} = \frac{r_L}{r_e'} = \frac{6.67\ \text{K}}{25} = 267$

(iv)  $A_p = A_i \cdot A_v = 100 \times 267 = 26,700$ ;  $G_p = 10 \log_{10} 26,700 = 44.3\text{ dB}$

**When Emitter Bypass Capacitor is Removed**

When bypass capacitor is removed, ac ground is removed. Now, the ac signal will have to pass through  $R_E$  also. According to  $\beta$ -rule of Art. 57.24, the total emitter resistance referred to base will become  $(1 + \beta)(r_e' + R_E) \cong \beta(r_e' + R_E)$  because  $r_e'$  is in series with  $R_E$  as shown in Fig. 59.20 (a).

Now, the ac equivalent circuit becomes as shown in Fig. 59.20 (b).

Stage  $r_{in} = R_B \parallel \beta(r_e' + R_E) \cong R_B \parallel \beta R_E$   
 It is so because  $R_E$  is much greater than  $r_e'$ .

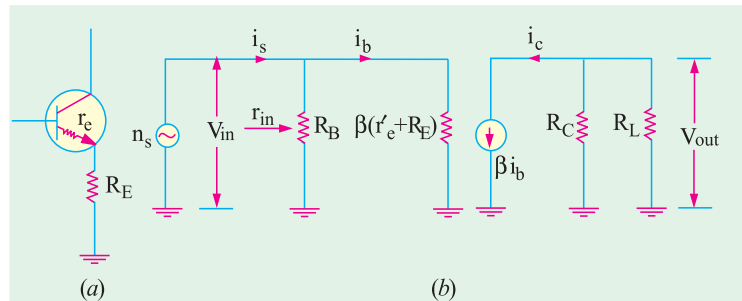


Fig. 59.20



- (i)  $r_e' = 25 \Omega$  — as before
- $\therefore \beta(r_e' + R_E) \cong \beta R_E = 100 \times 10 \text{ K} = 1 \text{ M}$
- $\therefore r_{in} = R_B \parallel \beta R_E = 2 \text{ M} \parallel 1 \text{ M}$
- $= \frac{2}{3} \text{ M} = 666.7 \text{ K}$  — much greater than before
- (ii)  $r_L = 6.67 \text{ K}$  — it remains unchanged
- (iii)  $V_{in} = i_b \beta (r_e' + R_E);$   $V_{out} = i_c r_L = \beta i_b r_L$
- $\therefore A_v = \frac{V_{out}}{V_{in}} = \frac{i_b r_L \beta}{i_b \cdot (r_e' + R_E)} = \frac{r_L}{(r_e' + R_E)} = \frac{r_L}{R_E}$
- $\frac{6.67 \text{ K}}{10 \text{ K}} = 0.667$  — reduced drastically
- (iv)  $A_p = A_i A_v = 100 \times 0.667 = 66.7$  — reduced considerably

It is seen that by removing bypass capacitor, **excessive degeneration has occurred in the amplifier circuit.**

**Example 59.6.** For the circuit shown in Fig. 59.21, compute (i)  $r_{in(base)}$  (ii)  $V_{out}$  (iii)  $A_v$  (iv)  $r_{in}$ . Neglect  $V_{BE}$  and take  $\beta = 200$ .

**Solution.** It may be noted that voltage divider bias has been used in the circuit.

$$V_2 = \frac{15}{15 + 45} \times 30 = 7.5 \text{ V}$$

$$V_E = V_2 - V_{BE} \cong V_2 = 7.5 \text{ V}$$

$$I_E = 7.5 / 7.5 \text{ K} = 1 \text{ mA}$$

$$r_e' = 25 \text{ mV} / 1 \text{ mA} = 25 \Omega$$

The ac equivalent circuit is shown in Fig. 59.22. Since dc source is shorted, 45 K, resistor is ac grounded. On the input side, three resistance become paralleled across  $v_s$  i.e. (i) 15 K (ii) 45 K and (iii)  $\beta r_e'$  or  $r_{in(base)}$ .

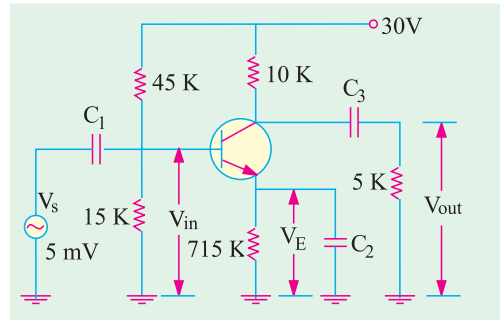


Fig. 59.21

Capacitor  $C_2$  ac grounds the emitter resistance  $R_E$ , so does  $C_2$  to 5 K and  $V_{CC}$  to 10 K.

(i)  $r_{in(base)} = \beta r_e' = 200 \times 25 = 5 \text{ K}$       (ii)  $i_b = 5 \text{ mV} / 5 \text{ K} = 1 \mu\text{A}$

(Obviously,  $i_b$  is not the current which leaves the source but that part of the source current is which enters the base). Now, collector load resistance is  $i_c = \beta i_b = 200 \times 1 = 200 \mu\text{A} = 0.2 \text{ A}$

$r_L = 10 \text{ K} \parallel 5 \text{ K} = 10/3 \text{ K} = 3.33 \text{ K}, V_{out} = i_c r_L = 0.2 \times 3.33 = 0.667 \text{ V}$

(iii)  $A_v = \frac{V_{out}}{V_{in}} = \frac{0.667 \text{ V}}{5 \text{ mV}} = 133$

or  $A_v = \frac{r_L}{r_e'} = \frac{3.33 \text{ K}}{25} = 133$

(iv)  $r_{in}$  means the input ac resistance as seen from the source i.e. from point A in Fig. 59.22. It is different from  $r_{in(base)}$ . Obviously,  $r_{in}$  is equal to the equivalent resistance of three resistances connected in parallel.  $r_{in} = 15 \text{ K} \parallel 45 \text{ K} \parallel 5 \text{ K} = 3.54 \text{ K}$ .

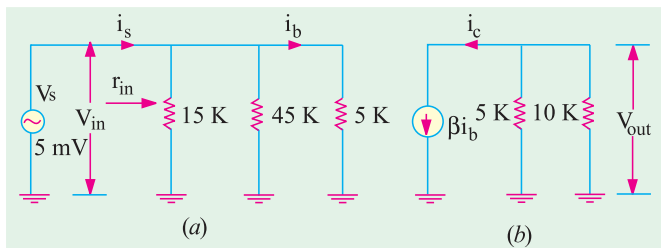


Fig. 59.22



**Example 59.7.** For the CE amplifier circuit of Fig. 59.23, find out (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  (iv)  $A_p$  and (v)  $V_{out}$ . Take transistor  $\beta = 50$  and  $R_S = 0$ . (Applied Electronics-II, Punjab Univ. 1992)

**Solution.**

$$I_E = \frac{V_{EE}}{R_E} = \frac{20}{40} = 0.5 \text{ mA}$$

$$r_e' = 25/0.5 = 50 \Omega$$

$$\beta r_e' = 50 \times 50 = 2500 \Omega$$

(i)  $r_{in} = 10 \text{ K} \parallel 2500 \Omega = 2000 \Omega$

(ii)  $r_L = 20 \text{ K} \parallel 20 \text{ K} = 10 \text{ K}$

(iii)  $A_v = \frac{r_L}{r_e'} = \frac{10 \text{ K}}{50} = 200$

(iv)  $A_p = A_v \cdot A_i = 200 \times 50 = 10,000$

$$G_p = 10 \log_{10} 10000 = 40 \text{ dB}$$

(v)  $V_{out} = A_v \times V_{in} = 200 \times 2 = 40 \text{ mV (r.m.s.)}$

Since  $R_S$  is zero, whole of  $v_s$  appears across the diode base.

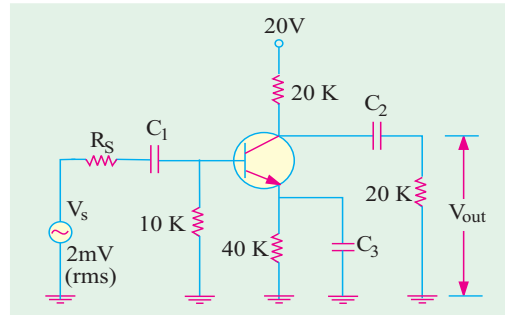


Fig. 59.23

**Example 59.8.** For the single-stage CE amplifier circuit of Fig. 59.24, find approximate value of (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  (iv)  $A_p$  and  $G_p$ . Take transistor  $\beta = 100$ . Use  $r_e' = 50 \text{ mV}/I_E$ . (Electronics-II, Gujarat Univ. 1991)

**Solution.**  $V_2 = 20 \times \frac{5}{5+45} = 2 \text{ V};$

$$I_E = \frac{V_2}{R_E} = \frac{2}{1} = 2 \text{ mA}$$

$$r_e' = 50/2 = 25 \Omega;$$

$$\beta r_e' = 25 \times 100 = 2.5 \text{ K}$$

(i)  $r_{in} = R_1 \parallel R_2 \parallel \beta r_e' = 45 \text{ K} \parallel 5 \text{ K} \parallel 2.5 \text{ K} = 1.6 \text{ K}$

(ii)  $r_L = 5 \text{ K} \parallel 5 \text{ K} = 2.5 \text{ K}$

(iii)  $A_v = \frac{r_L}{r_e'} = \frac{2.5 \text{ K}}{25} = 100$

(iv)  $A_p = A_v \cdot A_i = 100 \times 100 = 10,000$

(v)  $G_p = 10 \log_{10} 10,000 = 40 \text{ dB}$

**Note.**  $R_E$  did not come into picture because it was ac grounded by the bypass capacitor  $C_3$ .

**Example 59.9.** Find the approximate values of the quantities of Ex. 59.8 in case bypass capacitor  $C_3$  in Fig. 59.24 is removed.

**Solution.** (i) In this case,

$$r_{in} = R_1 \parallel R_2 \parallel \beta (r_e' + R_E) = 45 \text{ K} \parallel 5 \text{ K} \parallel 100 (25 + 1 \text{ K}) \\ \cong 45 \text{ K} \parallel 5 \text{ K} \parallel 100 \text{ K} = 5 \text{ K}$$

(ii)  $r_L = 2.5 \text{ K}$  — as before

(iii)  $A_v = \frac{r_L}{(r_e' + R_E)} = \frac{2.5 \text{ K}}{1 \text{ K}} = 2.5$

(Please note the reduction)

(iv)  $A_p = 100 \times 2.5 = 250$

(v)  $G_p = 10 \log_{10} 250 = 24 \text{ dB}$



### 59.7. Effect of Source Resistance $R_S$

Greater the internal resistance of the ac signal source, greater the internal voltage drop and hence lesser the value of  $V_{in}$  because

$$V_{in} = V_s - \text{drop across } R_S.$$

Consider the CE circuit shown in Fig. 59.25 whose ac equivalent circuit is shown in Fig. 59.26.

As seen from Fig. 59.26, on the input side,  $R_S$  is in series with  $R_B \parallel \beta r_e'$ . Hence,  $V_s$  is divided between them in the direct ratio of their resistances. If  $R_S$  is much less than  $R_B \parallel \beta r_e'$ , then

$$V_{in} \cong V_s$$

The voltage gain from base to output is still given by  $\frac{V_{out}}{V_{in}} = \frac{r_L}{r_e'}$

If, in any question,  $V_s$  is given, we will first find out how much of it appears across the base as  $V_{in}$ . Then, for determining  $V_{out}$ , we will simply multiply this  $V_{in}$  by the voltage gain.

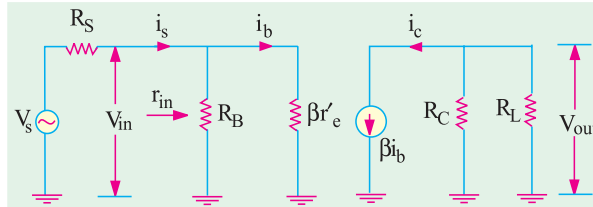


Fig. 59.26

**Solution.** This circuit is similar to that shown in Fig. 59.23 except for the addition of  $R_S$ . As found in Ex. 9.8,  $r_e' = 50 \Omega$ ,

$$\beta r_e' = 2500 \Omega.$$

$$\therefore R_B \parallel \beta r_e' = 10 \text{ K} \parallel 22500 \Omega = 2000 \Omega$$

**(a) When  $R_S = 100 \Omega$**

In this case, 2 mV are dropped across a series combination of  $100 \Omega$  and  $2000 \Omega$ . Drop over  $100 \Omega$  is negligible as compared to that on  $2000 \Omega$ . Hence, it can be presumed that  $V_s = V_{in} = 2 \text{ mV}$ . We have already found that  $A_v = 200$ .

$$\therefore V_{out} = A_v \times V_{in} = 200 \times 2 = \mathbf{400 \text{ mV (rms)}}$$

**(b) When  $R_S = 3 \text{ K}$**

$$\text{In this case, drop across } R_B \parallel \beta r_e' = 2 \text{ K is } V_{in} = V_s \frac{R_B \parallel r_e'}{R_S + (R_B \parallel r_e')} = 2 \frac{2}{3 + 2} = 0.8 \text{ mV}$$

$$\therefore V_{out} = 200 \times 0.8 = \mathbf{160 \text{ mV}}$$

Obviously, as  $R_S$  is increased,  $V_{out}$  is decreased.

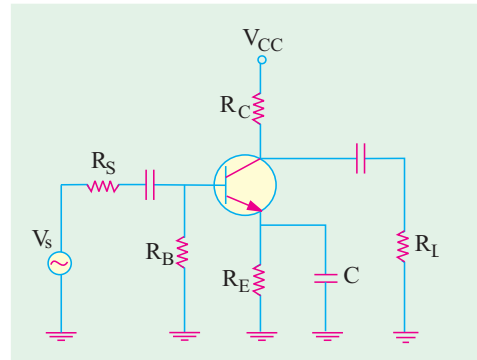


Fig. 59.25

**Example 59.10.** In the CE amplifier circuit of Fig. 59.27, find the rms signal output voltage when  $R_S$  is (a)  $100 \Omega$  and (b)  $3 \text{ K}$ . Take  $b = 50$ .

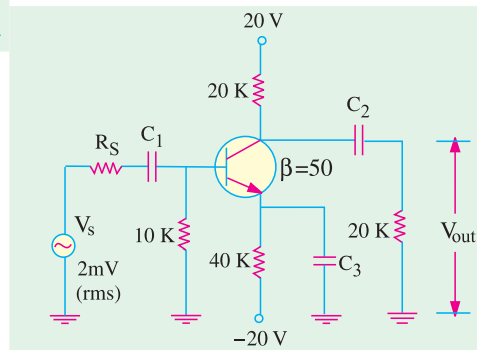


Fig. 59.27

### 59.8. Equivalent Circuit of a CC Amplifier

We will consider the two-supply emitter-bias circuit shown in Fig. 59.28 (a) in which the collector is placed at ac (not dc) ground. The ac input signal is coupled into the base and output signal is taken out of the emitter. This circuit is also called **emitter follower circuit** because the



emitter signal follows the signal at the base both in magnitude and phase.

(a) DC Equivalent Circuit

It is drawn in the usual way by opening all the capacitors and shorting all ac sources. It is shown in Fig. 59.28 (b).

$$I_E = \frac{V_{EE}}{R_E} \frac{V_{BE}}{R_B} / \frac{V_{BE}}{R_E}$$

(b) AC Equivalent Circuit

It is obtained by shorting all ac sources and capacitors and is shown in Fig. 59.29.

(c) Principal Operating Characteristics

1. Input Resistance

The input resistance of the CC stage is given by the parallel combination of  $R_S$  and  $r_{in(base)}$ . Now,  $r_{in(base)}$  is the input resistance looking into the base. It is found to be equal to  $(1 + \beta)(r_e' + r_L) \cong \beta r_L$  i.e.  $\beta$  times the ac load seen by the emitter.

$$\begin{aligned} \therefore r_{in} \text{ or } r_{in(stage)} &= R_B \parallel r_{in(base)} \\ &= R_B \parallel \beta (r_e' + r_L) \cong \beta (r_e' + r_L) \\ &\quad \text{--- when } R_B \text{ is very large} \\ &\cong \beta r_L \quad \text{--- when } r_e' \ll r_L \\ &= \beta R_E \quad \text{--- if } r_L = 0 \end{aligned}$$

2. AC Load Resistance:  $r_L = R_E \parallel R_L$

It is the ac output resistance as seen by the emitter (and not the one when looking into the emitter).

3. Current Gain.  $A_i = \frac{i_e}{i_b} (1 + \beta)$

4. Voltage Gain.  $V_{in} = i_b \cdot r_{in} = i_b \cdot \beta (r_e' + r_L)$   
 $V_{out} = i_e \cdot r_L = \beta i_b \cdot r_L$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{i_b \cdot r_L}{i_b \cdot (r_e' + r_L)} = \frac{r_L}{r_e' + r_L} \cong 1 \quad \text{--- if } r_e' \ll r_L$$

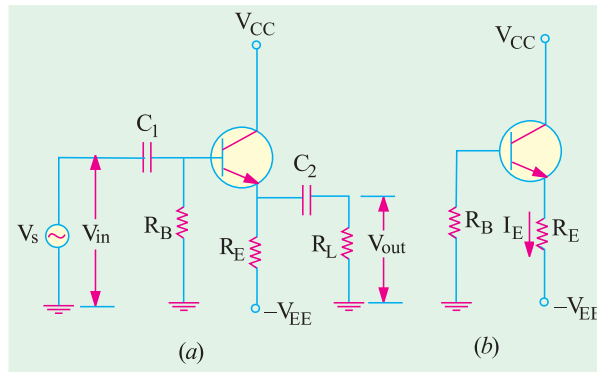


Fig. 59.28

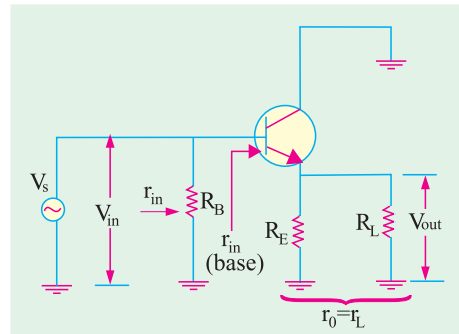


Fig. 59.29

It means that output signal from emitter has the same magnitude as the input signal at the base.

5. Power Gain  $A_p = A_v \cdot A_i = 1 \times (1 + \beta) = (1 + \beta) \cong \beta$   
 $G_p = 10 \log_{10} A_p$  dB

It would be noted from above that main usefulness of the emitter follower is to step up the impedance level i.e. it transforms load impedance to a much higher value. It does not increase the signal voltage. Hence, primary application of emitter follower or CC stage is as an impedance matching device. It offers a higher impedance at the input terminals i.e.  $r_{in}$  and a low output impedance ( $r_L$ ) – something opposite of typical basic transistor amplifier.

Another point worth keeping in mind is that the above formulas are not exact because we have used ideal transistor approximations. However, these formulas do help in rapidly grasping the essential features of an emitter follower. Moreover, they are adequate for preliminary analysis and design.



**Example 59.11.** For the emitter follower shown in Fig. 59.30, find

- (i)  $r_{in}$  or  $r_{in(stage)}$       (ii)  $r_L$       (iii)  $A_v$       (iv)  $A_p$ . Take transistor  $\beta = 50$ .  
 (Electronics, Delhi Univ.)

**Solution.**  $I_E = 20/20 = 1 \text{ mA}$

$$r_e' = 25/1 = 25 \Omega$$

$$r_L = R_E \parallel R_L = 20 \parallel 5 = 4 \text{ K}$$

$$r_{in(base)} = \beta(r_e' + r_L) \cong \beta r_L = 50 \times 4 = 200 \text{ K}$$

$\therefore$  ( $r_e'$  of  $25 \Omega$  has been neglected as compared to  $r_L$  of  $4 \text{ K}$ )

(i)  $r_{in(stage)}$  or  $r_{in} = R_B \parallel r_{in(base)}$   
 $= 400 \text{ K} \parallel 200 \text{ K}$   
 $= 133.3 \text{ K}$

(ii)  $r_L = 20 \text{ K} \parallel 5 \text{ K} = 4 \text{ K}$

(iii)  $A_v \cong 1$  since  $r_e'$  is negligible as compared to  $r_L$ . If it is not, then  $A_v$  could be even less than 1.

(iv)  $A_p = A_i \cdot A_v \cdot \beta \times 1 = \beta = 50$ ;  $G_p = 10 \log_{10} 50 = 17 \text{ dB}$

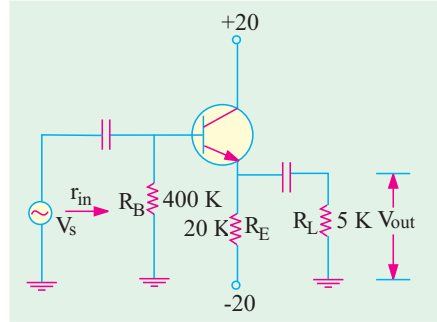


Fig. 59.30

**Example 59.12.** For the beta-stabilized emitter follower circuit of Fig. 59.31, find

- (i)  $r_{in(base)}$       (ii)  $r_{in}$  or  $r_{in(stage)}$       (iii)  $r_L$  and      (iv)  $A_v$ . Take transistor  $\beta = 100$ .  
 (Electronics Technology, Mysore Univ.)

**Solution.**  $V_2 = V_{CC} \frac{R_2}{R_1 + R_2} = 20 \frac{10}{30 + 10} = 5 \text{ V}$

$$I_E = \frac{V_2}{R_E} = \frac{5}{5} = 1 \text{ mA}$$

$$r_e' = 25/1 = 25 \Omega$$

$$r_L = 5 \parallel 5 = 2.5 \text{ K}$$

(i)  $r_{in(base)} = \beta(r_e' + r_L)$   
 $= 100(25 + 2500) = 250 \text{ K}$

(ii)  $r_{in(base)} = R_1 \parallel R_2 \parallel r_{in(base)}$   
 $r_{in(base)} = 30 \text{ K} \parallel 10 \text{ K} \parallel 250 \text{ K} \cong 8 \text{ K}$

(iii)  $r_L = 5 \text{ K} \parallel 5 \text{ K} = 2.5 \text{ K}$

(iv)  $A_p = A_i \cdot A_v \cong 1 \times \beta = 100$ ;  $G_p = 100 \log_{10} 100 = 20 \text{ dB}$

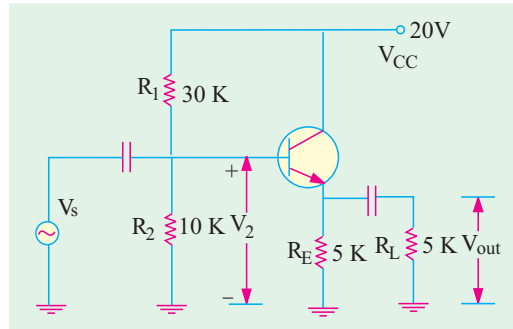


Fig. 59.31

**Example 59.13.** For the CE circuit of Fig. 59.32, find

- (i)  $r_{in}$  or  $r_{in(stage)}$       (ii)  $A_v$       (iii)  $A_p$ . Take transistor  $\beta = 200$

**Solution.**  $I_E = 20/20 = 1 \text{ mA}$        $r_e' = 25/1 = 25 \Omega$

$$r_L = 20 \text{ K} \parallel 50 \Omega = 50 \Omega$$

$$r_{in(base)} = \beta(r_e' + r_L) = 200(25 + 50) = 15 \text{ K};$$

(i)  $r_{in}$  or  $r_{in(stage)} = R_B \parallel r_{in(base)}$   
 $= 100 \text{ K} \parallel 15 \text{ K} = 13 \text{ K}$

(ii) Since  $r_e'$  is not negligible as compared to  $r_L$ , we will use the expression

$$A_v = \frac{r_L}{r_e' + r_L} = \frac{50}{25 + 50} = 0.667$$

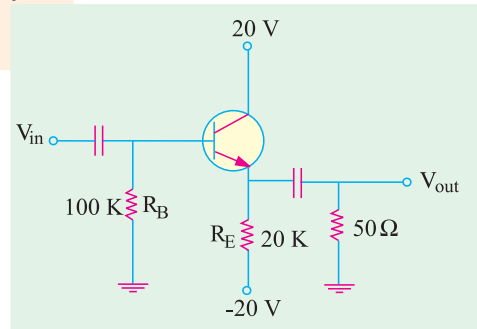


Fig. 59.32





(iii)  $A_p = A_v \cdot A_i = 0.667 \times 200 = 133.3$

**59.9. Small-signal Low-frequency Model or Representation**

Although many transistor representations or models have been suggested and widely used, the equivalent *T*-model is the easiest to understand because, in this representation, component parts retain their identity in all configurations leading to rapid appreciation of a given network.

**59.10. T-Model**

Such models are not in common use today because they do not take into account any gain between input and output.

**(a) CB Circuit**

In Fig. 59.33 is shown the low-frequency *T*-equivalent circuit of a transistor connected in *CB* configuration. It utilizes *T*- or *r*-parameters. All these parameters are ac parameters and are measured under open-circuit conditions. Here,  $r_e$  represents the ac resistance of the forward-biased emitter-base junction. Its value is

$$r_e = \begin{cases} \frac{25 \text{ mV}}{I_E} & \text{— for Ge} \\ \frac{50 \text{ mV}}{I_E} & \text{— for Si} \end{cases}$$

This resistance is fairly small and depends on  $I_E$ . Also,  $r_e$  represents the ac resistance of the reverse-biased *C/B* junction. It is of the order of a few  $\text{M}\Omega$ . Finally,  $r_b$  represents the resistance of the base region which is common to both junctions. Its value depends on the degree of doping. Usually,  $r_b$  is larger than  $r_e$  but much smaller than  $r_c$ .

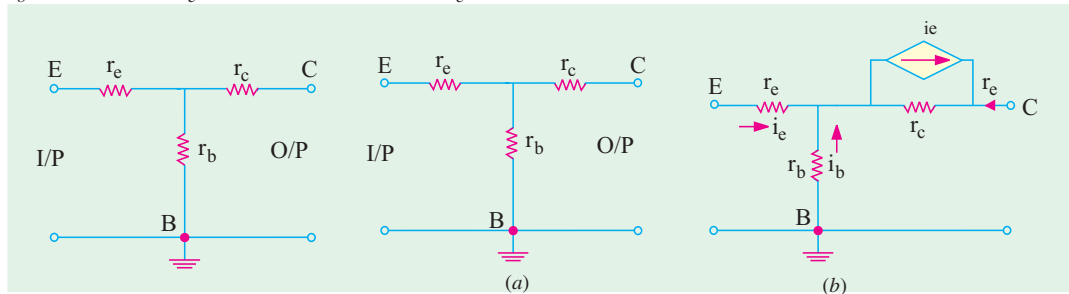


Fig. 59.33

Fig. 59.34

However, circuit shown in Fig. 59.34 (a) is not complete because it does not illustrate the forward current transfer ratio. Since current in the output of a transistor depends on the current at the input, a dependent current-generator in parallel with  $r_c$  must be included as shown in Fig. 59.34 (b). As it is the usual practice, all currents are shown flowing inwards even though some of them may actually be flowing in the opposite direction.

The current generator may be replaced by a voltage generator with the help of Thevenin's theorem as shown in Fig. 59.35 (a). In that case, the *T*-equivalent circuit becomes as shown in Fig. 59.35 (b). The generator has a voltage of  $\alpha i_e r_c = r_{in} i_e$  where  $r_{in} = \alpha r_c$ .

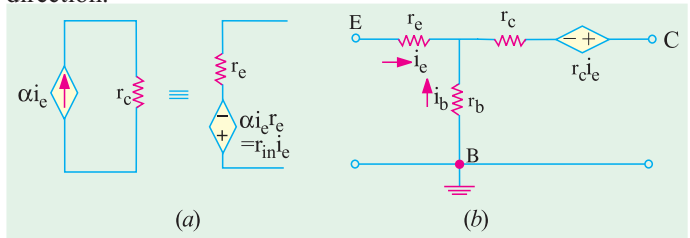


Fig. 59.35

Typical values of different parameters are :

$r_e = 25 \text{ to } 50 \ \Omega$ ;  $r_b = 100 \text{ to } 1000 \ \Omega$ ;  $r_c = 1 \ \text{M}\Omega$



(b) CE Circuit

The  $T$ -equivalent circuit for such a configuration is shown in Fig. 59.36. Whereas circuit shown in Fig. 59.36 contains a parallel current-generator that shown in Fig. 59.37 contains a series-voltage generator.

(c) CC Circuit

The  $T$ -equivalent circuit for such a configuration is shown in Fig. 59.38.

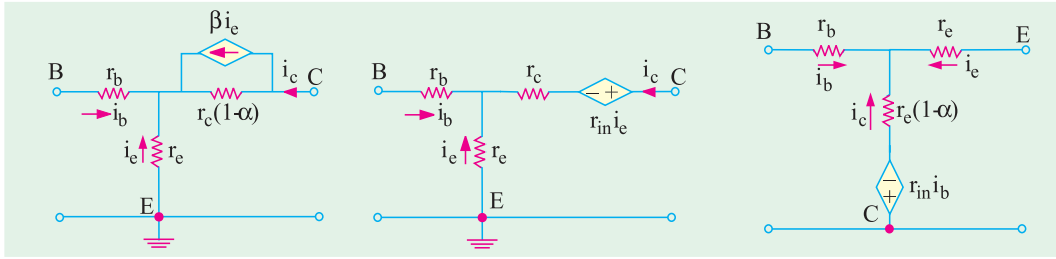


Fig. 59.36

Fig. 59.37

Fig. 59.38

59.11. Formulas for T-Equivalent of a CB Circuit

In Fig. 59.39 is shown a small-signal low-frequency  $T$ -equivalent circuit for  $CB$  configuration. The ac input signal source has a resistance of  $R_s$  and voltage of  $V_s$ .

Of course, dc biasing circuit has been omitted and only ac equivalent shown. The approximate expressions for input and output resistance and voltage and current gains as derived by applying  $KVL$  to the input and output loops are given below without derivation :

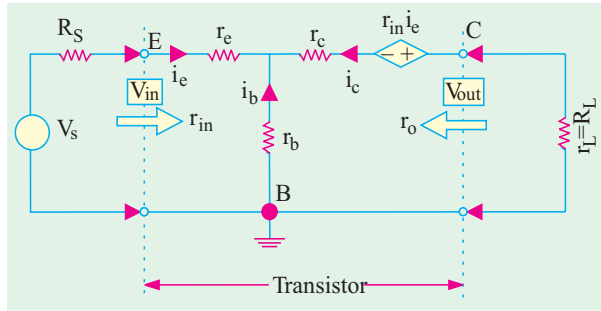


Fig. 59.39

$$1. r_{in} = r_e + r_b(1 - \alpha) + r_e \frac{r_b}{(1 - \alpha)} + r_b \frac{r_c(1 - \alpha)}{r_c + R_L}$$

$$2. r_o = r_c + \frac{r_b r_c}{r_e + r_b + R_s}$$

$$3. A_i = \alpha$$

$$4. A_v = \frac{v_{out}}{v_{in}} = \frac{R_L}{r_e + r_b(1 - \alpha)}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{R_L}{(r_e + R_s) + r_b(1 - \alpha)} \cdot \frac{R_L}{(r_e + R_s)}$$

$$5. A_p = A_i \cdot A_v$$

$$\frac{^2 R_L}{(r_e + r_b(1 - \alpha))}$$

— no source resistance

$$\frac{^2 R_L}{(r_e + R_s) + r_b(1 - \alpha)}$$

— with source resistance

$$G_p = 10 \log_{10} A_p$$



59.12. Formulas for T-Equivalent of a CE Circuit

The low-frequency small-signal T-equivalent circuit for such a configuration is shown in Fig. 59.40.

The approximate expressions for various resistances and gains as found by applying KVL to the input and output loops are given below :

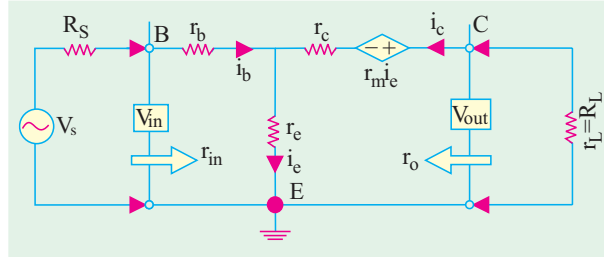


Fig. 59.40

1.  $r_{in} = r_b + \frac{r_e}{(1 - \alpha)}$   
 $= r_b + (1 + \beta)r_e$
2.  $r_o = r_c(1 - \alpha) = \frac{r_e r_c}{r_b + R_S}$
3.  $A_i = \beta \frac{R_L}{(r_b + R_S)(1 - \alpha)}$
4.  $A_v = \frac{V_{out}}{V_{in}} = \frac{r_c R_L}{r_e (r_b + R_S)(1 - \alpha)}$
5.  $A_p = A_v \cdot A_i = \frac{r_c R_L}{(1 - \alpha)[r_e (r_b + R_S)(1 - \alpha)]}$

59.13. Formulas for T-Equivalent of a CC Circuit

The low-frequency T-equivalent circuit for such a configuration is shown in Fig. 59.41. The approximate expressions for various resistances and gains are given below :

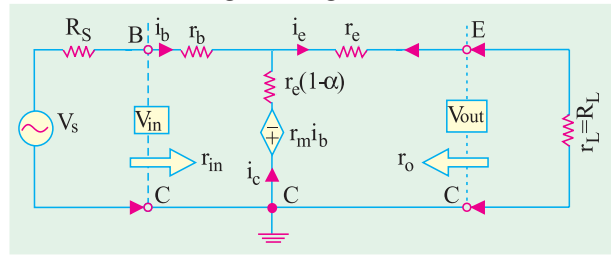


Fig. 59.41

1.  $r_{in} = \frac{R_L}{1 - \alpha} + (1 - \alpha)r_b$
2.  $r_o = r_e + (1 - \alpha)(r_b + R_S)$   
 $= \frac{r_e (r_b + R_S)}{(1 - \alpha)}$
3.  $A_i = \frac{1}{(1 - \alpha)} (1 - \alpha)$
4.  $A_n = 1$
5.  $A_p = \frac{1}{(1 - \alpha)} (1 - \alpha) \quad G_p = 10A_p \text{ dB}$

**Example 59.14.** A junction transistor has  $r_e = 50 \Omega$ ,  $r_b = 1 \text{ K}$ ,  $r_c = 1 \text{ M}$  and  $\alpha = 0.98$ . It is used in common-base circuit with a load resistance of  $10 \text{ K}$ . Calculate the current, voltage and power gains and the input resistance. (Applied Electronics, Punjab Univ. 1993)

**Solution.** (i)  $A_i = \alpha = 0.98$

(ii)  $A_v = \frac{R_L}{r_e + r_b(1 - \alpha)} = \frac{0.98 \cdot 10,000}{50 + 1000(1 - 0.98)} = 140$       (iii)  $A_p = A_v \cdot A_i = 140 \times 0.98 = 137$

(iv)  $r_{in} = r_e + r_b(1 - \alpha) = 50 + 1000(1 - 0.98) = 70 \Omega$

**Example 59.15.** A P-N-P junction transistor is used as a voltage amplifier in the grounded-base circuit with a load resistance being  $300 \text{ K}$  and the internal resistance of the original source being  $200 \Omega$ .

Derive an expression for the voltage gain of the amplifier and calculate its magnitude if the transistor T-network parameters are :  $r_e = 18 \Omega$ ,  $r_b = 700 \Omega$ ,  $r_c = 1 \text{ M}$  and  $r_m = 976 \text{ K}$ .

(Applied Electronics and Circuits, Grad. I.E.T.E.)



**Solution.** Here,  $r_m = \alpha (r_c + r_b) \cong \alpha r_c \quad \therefore \alpha = r_m / r_c = 976 \text{ K} / 1000 \text{ K} = 0.976$

$$A_v = \frac{V_2}{V_1} = \frac{R_L}{r_c} \frac{0.976}{r(1 - )} = \frac{0.976}{18} \frac{300 \times 10^3}{700(1-0.976)} = \mathbf{8,410}$$

$$A_{vg} = \frac{V_2}{V_g} = \frac{R_L}{R_G} \frac{0.976}{r_c} \frac{300 \times 10^3}{r_b(1 - )} = \frac{0.976}{200} \frac{300 \times 10^3}{18 \times 700(1-0.976)} = \mathbf{1247}$$

**Example 59.16.** Calculate the input and output resistance, overall current, voltage and power gains for a CE connected transistor having following r-parameters :

$r_b = 30 \Omega, r_e = 400 \Omega, r_c = 0.75 \text{ M}, \alpha = 0.95, R_L = 10 \text{ K}$  and  $R_S = 400 \Omega$

Also, calculate the power gain in decibels.

(Electronics Technology, Bangalore Univ. 1999)

**Solution. (i)**  $r_{in} = r_b + \frac{r_e}{(1 - \alpha)} = 30 + \frac{400}{(1-0.95)} = \mathbf{8030 \Omega}$

**(ii)**  $r_o = r_e (1 - \alpha) + \frac{r_c r_e}{r_b + r_c + R_S}$   
 $= 750,000(1-0.95) + \frac{0.95 \times 750,000 \times 400}{(30 + 400 + 400)} = 380,870 \mathbf{0.38 \text{ M}}$

**(iii)**  $A_i = \frac{0.95}{1 - 0.95} = 19$

**(iv)**  $A = \frac{R_L}{r_e (r_b + R_S)(1 - \alpha)} = \frac{0.95 \times 10 \times 10^3}{400 + (30 + 400) \times 0.05} = \mathbf{22.5}$

**(v)**  $A_p = A_v \cdot A_i = 22.5 \times 19 = \mathbf{427.5}$

Power gain in decibels,  $G_p = 10 \log_{10} 427.5 = \mathbf{26.3 \text{ dB}}$

### 59.14. What are h-parameters ?

These are **four** constants which describe the behaviour of a two-port linear network. A linear network is one in which resistance, inductances and capacitances remain fixed when voltage across them is changed.

Consider an unknown linear network contained in a black box as shown in Fig. 59.42. As a matter of convention, currents flowing into the box are taken positive whereas those flowing out of it are considered negative.

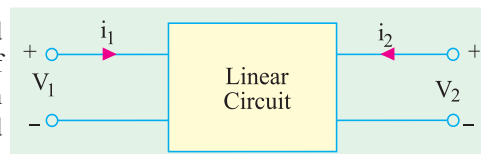


Fig. 59.42

Similarly, voltages are positive from the upper to the lower terminals and negative the other way around.

The electrical behaviour of such a circuit can be described with the help of four hybrid parameters or constants designated as  $h_{11}, h_{12}, h_{21}, h_{22}$ . In this type of double-number subscripts, it is implied that **the first variable is always divided by the other**. The subscript 1 refers to quantities on the input side and 2 to the quantities on the **output side**. The letter 'h' has come from the word **hybrid** which means **mixture** of distinctly different items. These constants are hybrid because they have different units.

Out of the four h-parameters, two are found by short-circuiting the output terminals 2-2 and the other two by open-circuiting the input terminals 1-1 of the circuit.

**(a) Finding  $h_{11}$  and  $h_{21}$  from Short-Circuit Test**

As shown in Fig. 59.43, the output terminals have been shorted so that  $v_2 = 0$ , because no voltage can exist on a short. The linear circuit within the box is driven by an input voltage  $v_1$ . It



produces an input current  $i_1$  whose magnitude depends on the type of circuit within the box.

$$h_{11} = \frac{V_1}{i_1} \quad \text{--- output shorted}$$

$$h_{21} = \frac{i_2}{i_1} \quad \text{--- output shorted}$$

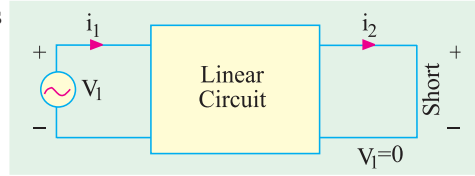


Fig. 59.43

These two constants are known as **forward** parameters.

The constant  $h_{11}$  represents input impedance with output shorted and has the unit of ohm. The constant  $h_{21}$  represents current gain of the circuit with output shorted and has no unit since it is the ratio of two similar quantities.

The voltages and currents of such a two-port network are related by the following sets of equations or  $V/I$  relations.

$$V_1 = h_{11} i_1 + h_{12} V_2 \quad \dots (i)$$

$$i_2 = h_{21} i_1 + h_{22} V_2 \quad \dots (ii)$$

Here, the  $h_s$  are constants for a given circuit but change if the circuit is changed. Knowledge of parameters enables us to find the voltages and currents with the help of the above two equations.

**(b) Finding  $h_{12}$  and  $h_{22}$  from Open-circuit Test**

As shown in Fig. 59.44, the input terminals are open so that  $i_1 = 0$  but there does appear a voltage  $v_1$  across them. The output terminals are driven by an ac voltage  $v_2$  which sets up current  $i_2$ .

$$h_{12} = \frac{V_1}{V_2} \quad \text{--- input open}$$

$$h_{22} = \frac{i_2}{V_2} \quad \text{--- input open}$$

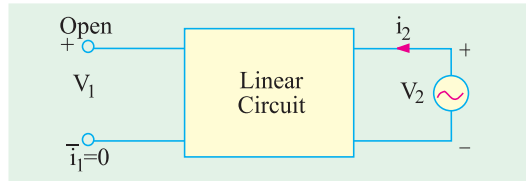


Fig. 59.44

As seen,  $h_{12}$  represents voltage gain (not forward gain which is  $v_2 / v_1$ ). Hence, it has no units. The constant  $h_{22}$  represents admittance (which is reverse of resistance) and has the unit of mho or Siemens,  $S$ . It is actually the admittance looking into the output terminals with input terminals open. Generally, these two constants are also referred to as **reverse parameters**.

Summary of  $h$ -parameters

$h_{11} =$	input impedance	}	with output shorted
$h_{21} =$	forward current gain		
$h_{12} =$	reverse voltage gain	}	with input open
$h_{22} =$	output admittance		

**59.15. Input Impedance of a Two Port Network**

Consider the two-port linear network shown in Fig. 59.45 which has a load resistance  $r_L$  across its output terminals. The voltage source  $V_1$  on the input side drives the circuit and sets up current  $i_1$ . As seen,  $Z_{in} = V_1 / i_1$ . Substituting the value of  $V_1$  from Eq. (i) of Art. 59.14, we get

$$Z_{in} = \frac{V_1}{i_1} = \frac{h_{11} i_1 + h_{12} v_2}{i_1}$$

$$h_{11} = \frac{h_{12} V_2}{i_1} \quad \dots (i)$$

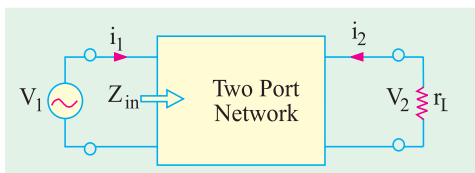


Fig. 59.45



As seen from Fig. 59.45 above  $i_2 = -v_2 / R_L$ \*  
 Substituting this value of  $i_2$  in Eq. (ii) of Art 59.14, we have

$$\frac{-v_2}{R_L} = h_{22} i_1 + h_{22} v_2 \quad \text{or} \quad \frac{v_2}{i_1} = \frac{h_{21}}{h_{22} + 1/R_L}$$

Substituting this value in Eq. (i) above, we have

$$Z_{in} = h_{11} + \frac{h_{12} \cdot h_{21}}{h_{22} + 1/R_L}$$

### 59.16. Voltage Gain of a Two Port Network

The voltage gain of such a circuit (Fig. 59.45) is  $A_v = v_2 / v_1$ . Now  $v_1 = i_1 \cdot Z_{in}$ . Hence,  
 $A_v = v_2 / i_1 \cdot Z_{in}$ .

Substituting the value of  $v_2 / i_1$  as found earlier in Art. 59.15, we get

$$A_v = \frac{h_{21}}{Z_{in}(h_{22} + 1/R_L)}$$

**Example. 59.17.** Find the  $h$ -parameters of the circuit shown in Fig. 59.46 (a).

**Solution.** First of all, let us find the forward parameters  $h_u$  and  $h_{21}$ . For that purpose, a short is put across the output terminals as shown in Fig. 59.46 (b).

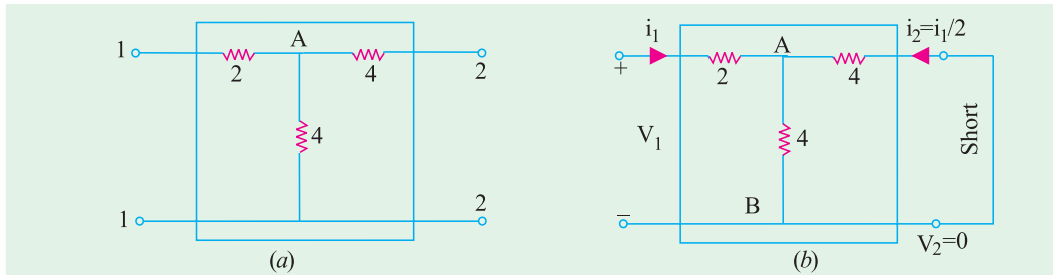


Fig. 59.46

(i) The input impedance of the network as viewed from input terminals is

$$h_{11} = 2 + 4 \parallel 4 = 4 \Omega$$

(ii) As seen from Fig. 9.46 (b), input current  $i_1$  divides into two equal parts at point A. The output current  $i_2 = -i_1/2$  (negative sign has been taken because actually it is flowing out of the box).

$$\therefore h_{21} = \frac{i_2}{i_1} = \frac{-i_1/2}{i_1} = \frac{1}{2} = -0.5$$

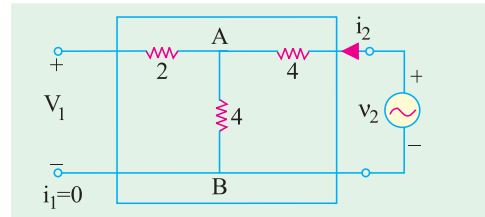


Fig. 59.47

(iii) Now, for finding reverse parameters, we will keep input terminals open and apply  $v_2$  across output terminals as shown in Fig. 59.47. It will produce a current  $i_2$  which will produce equal drops across the two  $4 \Omega$  resistors. The voltage which appears across input terminals as  $v_1$  is the drop across the vertical  $4 \Omega$  resistor connected at point A. Hence,  $v_1 = v_2 / 2$ .

$$\therefore h_{12} = \frac{v_1}{v_2} = \frac{v_2/2}{v_2} = 0.5$$

\* The negative sign is used because actual load current is opposite to that shown in figure.



The input impedance of the network when viewed from output terminals with input terminals open is  $4 + 4 = 8 \Omega$ .

$$\therefore h_{22} = 1/8 = \mathbf{0.125 \text{ Siemens (i.e. mho)}}$$

Hence, for the network shown in Fig. 59.46 (a), the  $h$ -parameters are as under :

$$h_{11} = 4 \Omega \quad h_{21} = -0.5 \quad h_{12} = 0.5 \quad \text{and} \quad h_{22} = 0.125 \text{ S}$$

### 59.17. The $h$ -parameter Notation for Transistors

While using  $h$ -parameters for transistor circuits, their numerical subscripts are replaced by the first letters for defining them.

$$\left. \begin{aligned} h_{11} = h_i &= \text{input impedance} \\ h_{21} = h_f &= \text{forward current gain} \end{aligned} \right\} \text{output shorted}$$

$$\left. \begin{aligned} h_{12} = h_r &= \text{reverse voltage gain} \\ h_{22} = h_o &= \text{output admittance} \end{aligned} \right\} \text{input open}$$

A second subscript is added to the above parameters to indicate the particular configuration.

For example, for  $CE$  connection, the four parameters are written as :

$$h_{ie}, h_{fe}, h_{re} \quad \text{and} \quad h_{oe}$$

Similarly, for  $CB$  connection, these are written as  $h_{ib}, h_{fb}, h_{rb}, h_{op}$  and for  $CC$  connection as  $h_{ic}, h_{fc}, h_{rc}$  and  $h_{oc}$ .

### 59.18. The $h$ -parameters of an Ideal Transistor

As stated earlier, every linear circuit has a set of parameters associated with it, which fully describe its behaviour. When small ac signals are involved, a transistor behaves like a linear device because its output ac signal **varies directly as the input signal**. Hence, for small ac signals, each transistor has its own characteristic set of  $h$ -parameters or constants.

The  $h$ -parameters depend on a number of factors such as

1. transistor type
2. configuration
3. operating point
4. temperature
5. frequency

These  $h$ -parameters can be found experimentally or graphically. The parameters  $h_i$  and  $h_r$  are determined from input characteristics of the  $CE$  transistor whereas  $h_f$  and  $h_o$  are found from output characteristics.

### 59.19. The $h$ -parameters of an Ideal CB Transistor

In Fig. 59.48 (a), a  $CB$ -connected transistor has been shown connected in a black box. Fig., 59.48 (b) gives its equivalent circuit. It should be noted that no external biasing resistors or any signal source has been shown connected to the transistor.

#### (i) Forward Parameters

The two forward  $h$ -parameters can be found from the circuit of Fig. 59.49 (a) where a short has been put across the output. The input impedance is simply  $r_e$ .

$$\therefore h_{ib} = r_e$$

The output current equals the input current  $i_e$ . Since it flows out of the box, it is taken as negative. The forward current gain is

$$h_{fb} = \frac{-i_e}{i_e} = -1$$

(It also called the ac  $\alpha$  of the  $CB$  circuit.)

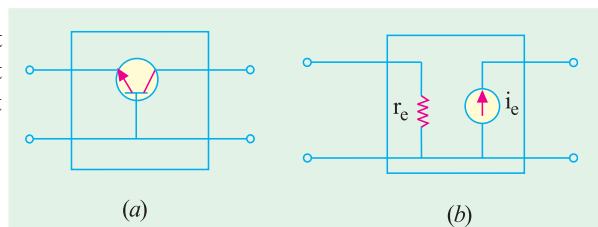


Fig. 59.48

#### (ii) Reverse Parameters

The two reverse parameters can be found from the circuit diagram of Fig. 59.49 (b). When input terminals are open, there can be no ac emitter current. It means that ac current source (inside the box)



has a value of zero and so appears as an 'open'. Because of this open, no voltage can appear across input terminals, however, large  $v_2$  may be. Hence,  $v_1 = 0$ .

$$\therefore h_{rb} = \frac{v_1}{v_2} = \frac{0}{v_2} = 0$$

Similarly, the impedance, looking into the output terminals is infinite. Consequently, its admittance ( $= 1/\infty$ ) is zero.

$$\therefore h_{ob} = 0$$

**Summary**

The four  $h$ -parameters of an ideal transistor connected in  $CB$  configuration are

$$h_{ib} = r_e ; \quad h_{fb} = -1, \quad h_{rb} = 0 ; \quad h_{ob} = 0$$

The equivalent hybrid circuit is shown in Fig. 59.50.

**Note.** In an actual transistor,  $h_{rb}$  and  $h_{ob}$  are not zero but have some finite value though extremely small (ranging from  $10^{-7}$  to  $10^{-6}$ ).

In reality, output impedance is not infinity but very high so that  $h_{ob}$  is extremely small. Similarly, there is some amount of feedback between the output and the input circuits (even when open) though it is very small. Hence,  $h_{rb}$  is very small.

**59.20. The  $h$ -parameters of an Ideal CE Transistor**

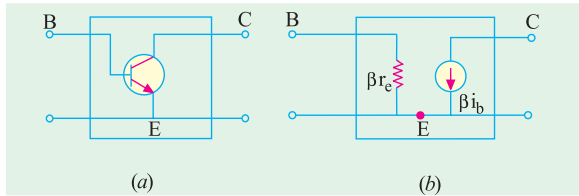


Fig. 59.51

Fig. 59.51 (a) shows a  $CE$ -connected ideal transistor contained in a black box whereas 59.51 (b) shows its ac equivalent circuit in terms of its  $\beta$  and resistance values.

**(a) Forward Parameters**

The two forward  $h$ -parameters can be found from the circuit of Fig. 59.52 (a)

where output has been shorted. Obviously, the input impedance is simply  $\beta r_e$ .

$$\therefore h_{ie} = \beta r_e$$

The forward current gain is given by

$$h_{fe} = \frac{i_2}{i_1} = \frac{i_c}{i_b}$$

(It is also called the ac beta of the  $CE$  circuit)

**(b) Reverse Parameters**

These can be found by reference to the circuit of Fig. 59.52 (b) where input terminals are open but output terminals are driven by an ac voltage source  $v_2$ . With input terminals open, there can be no base current so that  $i_b = 0$ . If  $i_b = 0$ , then collector current source has zero value and looks like an open. Hence, there can be no  $v_1$  due to this open.

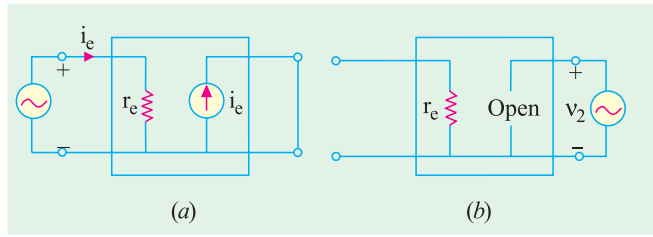


Fig. 59.49

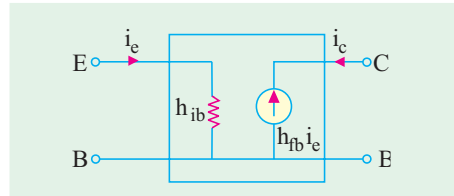


Fig. 59.50

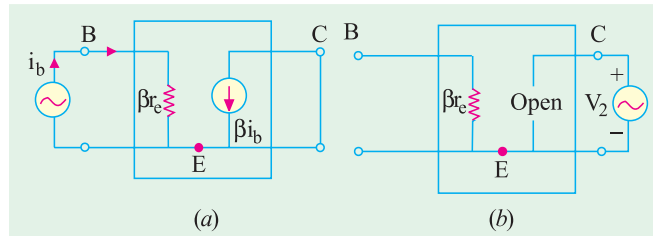


Fig. 59.52





$$\therefore h_{re} = \frac{v_1}{v_2} = \frac{0}{v_2} = 0$$

Again, the impedance looking into the output terminals is infinite so that conductance is zero.

$$\therefore h_{oe} = 0$$

Hence, the four  $h$ -parameters of an ideal transistor connected in  $CE$  configuration are :

$$h_{ie} = \beta r_e; \quad h_{fe} = \beta, \quad h_{re} = 0; \quad h_{oe} = 0.$$

The hybrid equivalent circuit of such a transistor is shown in Fig. 59.53.

**Note.** In practice,  $h_{re}$  and  $h_{oe}$  are not exactly zero but quite small for the same reasons as given in the Note to Art 59.19.

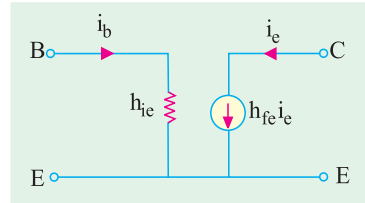


Fig. 59.53

### 59.21. Approximate Hybrid Equivalent Circuits

So far, we did not take into account the following two factors which exist in an actual transistor (as opposed to an ideal one).

- (i) because of the transistor's non-unilateral behaviour, there is a 'feedback' of the output voltage into the input voltage. This feedback is represented by a voltage-controlled generator  $h_{rb}v_2$  as shown in Fig. 59.54 and 59.55.

By definition, an ideal amplifier is one which responds only to signals applied to its input terminals. It should not do the reverse *i.e.* reproduce at the input any portion of the ac signal applied at the output. Such an ideal one-way device is called a unilateral device. A real transistor cannot be unilateral because of unavoidable interaction between its input and output circuits (after all, it consists of a single piece of a crystal). Therefore, not only its output responds to its input but, to a lesser degree, its input also responds to its output.

- (ii) even when input circuits is open, there is some effective value of conductance when looking into the transistor from its output terminals. It is represented by  $h_0$ .

We will now draw **low-frequency small-signal** hybrid equivalent circuits after taking into account the 'feedback' voltage generator and output admittance.

#### (a) Hybrid CB Circuit

In Fig. 59.54 (a) is shown an  $NPN$  transistor connected in  $CB$  configuration. Its ac equivalent circuit employing  $h$ -parameters is shown in Fig. 59.54 (b). The  $V/I$  relationships are given by the following two equations.

$$\begin{aligned} v_{eb} &= h_{ib} i_e + h_{rb} v_{cb} \\ i_e &= h_{fb} i_c + h_{ob} v_{cb} \end{aligned}$$

These equations are self-evident because applied voltage across input terminals must equal the drop over  $h_{ib}$  and the generator voltage. Similarly, current  $i_c$  in the output terminals must equal the sum of two branch currents.

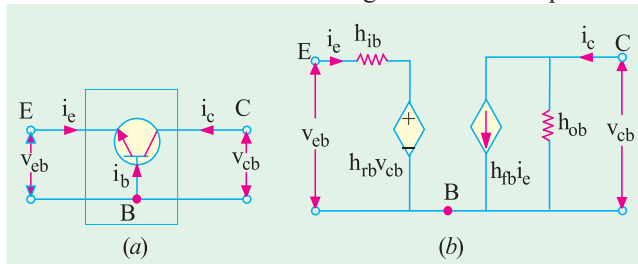


Fig. 59.54

As per current convention stated earlier (Art. 59.14), collector  $i_c$  is shown flowing **inwards** though actually this current flows **outwards** as shown by the arrow inside the ac current source. Similarly, ac voltage polarities have been taken by considering upper terminal positive and lower one as negative (please remember that the dc biasing rule of Art. 59.12 does not apply here).

It may be noted that no external dc biasing resistor or ac voltage sources have been connected to the equivalent circuit as yet.

Incidentally, it may be noted that the ac equivalent circuit contains a Thevenin's circuit in the input and a Norton's circuit in the output. It is all the more a reason to call it a hybrid equivalent circuit.



**(b) Hybrid CE Circuit**

The hybrid equivalent of the transistor alone when connected in CE configuration is shown in Fig. 59.55 (b). Its  $V/I$  characteristics are described by the following two equations.

$$\begin{aligned} v_{be} &= h_{ie} i_b + h_{re} v_{ce} \\ i_e &= h_{fb} i_b + h_{oe} v_{ce} \end{aligned}$$

We may connect signal input source across its input terminals and load resistance across output terminals (Art. 59.22).

**(c) Hybrid CC Circuit**

The hybrid equivalent of a transistor alone when connected in CC configuration is shown in Fig. 59.56 (b). Its  $V/I$  characteristics are defined by the following two equations :

$$\begin{aligned} v_{bc} &= h_{ie} i_b + h_{re} v_{ce} \\ i_e &= h_{fc} i_b + h_{oc} v_{ce} \end{aligned}$$

We may connect signal input source across ioutput terminals  $BC$  and load resistance across output terminals  $EC$  to get a CC amplifier.

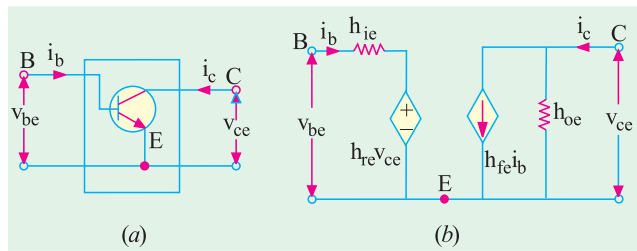


Fig. 59.55

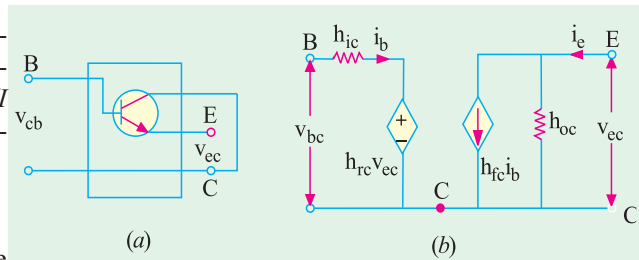


Fig. 59.56

**59.22. Transistor Amplifier Formulae Using  $h$ -parameters**

As shown in Fig. 59.57, if we add a signal source across input terminals 1-1 of a transistor and a load resistor across its output terminals 2-2, we get a small-signal, low-frequency hybrid model of a transistor amplifier. It is valid for all the three configurations and holds good for all types of load whether a resistance or an impedance.

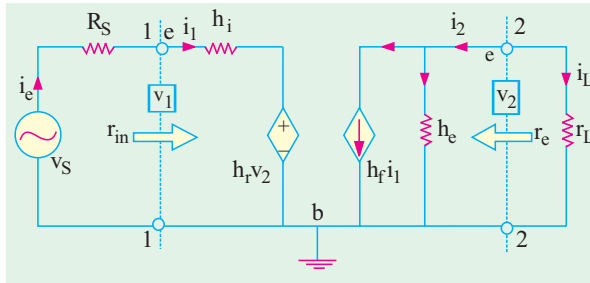


Fig. 59.57

We will now find expressions for its gains and impedances.

Before undertaking the above derivations, let us consider different components

in the hybrid model of Fig. 59.57. The input resistance looks like a resistance ( $h_i$ ) in series with a voltage generator ( $h_r v_2$ ). This generator represents the voltage feed-back from the output to the input circuit. It is known as voltage-controlled generator because its value is determined by  $v_2$  (as  $h_r$  is a dimensionless constant). The output circuit also has two components (*i*)  $h_o$  component which represents the conductance as seen from output terminals and (*ii*) the current-controlled generator ( $h_f i_1$ ) which simulates the transistor's ability to amplify. The parameter  $h_f$  is a dimensionless constant.

The above model can be described mathematically by using the following two equations :

**Input Circuit**

$$\begin{aligned} v_1 &= \text{sum of voltage drops from } a \text{ to } b \\ &= h_i i_1 + h_r v_2 \end{aligned} \quad \dots(i)$$

**Output Circuit**

$$\begin{aligned} i_2 &= \text{sum of currents leaving junction } c \\ &= h_f i_1 + h_o v_2 \end{aligned} \quad \dots(ii)$$



Now,  $v_2 = -i_2 r_L$ . Substituting this value in Eq. (ii) above, we have

$$i_2 = h_f i_1 - h_0 i_2 r_L \quad \dots \text{(iii)}$$

Eq. (i) and (iii) can now be used to find various gains of a transistor.

**(i) Current Gain**

It is given by  $A_i = i_2/i_1$

Dividing both sides of Eq. (iii) by  $i_1$ , we get

$$\frac{i_2}{i_1} = h_f - h_0 \frac{i_2}{i_1} r_L \quad \text{or} \quad A_i = h_f - h_0 A_i r_L$$

$$\therefore A_i = \frac{h_f}{1 - h_0 r_L}$$

If  $r_L = 0$  or  $h_0 r_L \ll 1$ , then  $A_i = h_f$

**Current Gain Taking  $R_S$  into Account**

The source current is not the transistor input current because  $i_1$  partly flows along  $R_S$  and partly along  $r_{in}$ .

To illustrate this point, consider the Norton's equivalent of the source (Fig. 59.58). The overall current gain  $A_{is}$  is given by

$$A_{is} = \frac{i_2}{i_s} = \frac{i_2}{i_1} \frac{i_1}{i_s} = A_i \frac{i_1}{i_s}$$

As seen from Fig. 9.58

$$\frac{i_1}{i_s} = \frac{R_s}{r_{in} + R_s} \quad \text{or} \quad \frac{i_1}{i_s} = \frac{R_s}{r_{in} + R_s}$$

$$\therefore A_{is} = A_i \cdot R_s / (r_{in} + R_s)$$

**(ii) Input Impedance**

It is defined as the resistance when looking into the amplifier from its input terminals. Hence,  $r_{in} = v_1/i_1$ .

From Eq. (i) above, we have

$$r_{in} = \frac{v_1}{i_1} = \frac{h_i i_1 + h_r v_2}{i_1} = h_i + h_r \cdot \frac{v_2}{i_1}$$

Substituting the value of  $v_2 = -i_2 r_L = -A_i i_1 r_L$ , we get

$$r_{in} = h_i - h_i A_i r_L = h_i \left[ 1 - \frac{h_f h_r r_L}{h_0 r_L} \right] = h_i \left[ \frac{h_0 - h_f h_r}{h_0} \right]$$

$$\frac{h_i}{1 - h_0 r_L} \quad \text{where} \quad \Delta h = h_i h_0 - h_f h_r$$

- if  $h_r$  or  $r_L$  is very small.

It is seen that  $r_{in}$  depends on  $r_L$  i.e. ac resistance of the load across output terminals of the transistor.

**(iii) Voltage Gain**

$A_v = v_2/v_1$ . It is also known as the internal voltage gain of the transistor. It is different from  $A_{vs} = v_2/v_s$  which is the gain from the source to the output terminals and is known as stage gain or overall gain.

As seen from above,  $v_2 = -A_i i_2 r_L$  and  $v_1 = i_1 r_{in}$

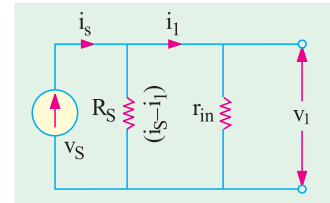


Fig. 59.58



$$\begin{aligned} \therefore A_v &= \frac{v_2}{v_1} = \frac{i_2 A_i r_L}{i_L r_{in}} = A_i \frac{r_L}{r_{in}} \\ &= \frac{h_f}{1 - h_o r_L} \cdot \frac{r_L (1 - h_o r_L)}{h_i h_r r_L} = \frac{h_f \cdot r_L}{h_i h_r r_L} \\ &= \frac{h_f r_L}{h_i (1 - h_o r_L)} = h_f \cdot \frac{r_L}{h_i} \end{aligned}$$

Overall voltage gain is

$$A_{vs} = \frac{v_2}{v_s} = \frac{v_2}{v_1} \cdot \frac{v_1}{v_s} = A_v \frac{v_1}{v_s}$$

Now,  $v_s$  drops over series combination of  $R_S$  and  $r_{in}$ .

Drop across  $r_{in}$  constitutes  $v_1$ . Hence,  $v_1 = v_s \times r_{in} / (R_S + r_{in})$

$$\therefore \frac{v_1}{v_s} = \frac{r_{in}}{(R_S + r_{in})} \quad \text{or} \quad A_{vs} = A_v \frac{r_{in}}{(R_S + r_{in})}$$

As seen, if  $R_S = 0$ ,  $A_{vs} = A_v$

Value of  $A_{vs}$  may also be obtained by adding  $R_S$  to  $h_i$  in the expression for  $A_v$ .

**(iv) Output Impedance**

It is defined as  $r_o = \frac{v_2}{i_2} |_{v_s = 0}$  or  $g_o = \frac{i_2}{v_2}$

Dividing both sides of Eq. (ii) by  $v_2$ , we get

$$g_o = h_f \cdot \frac{i_2}{v_2} = h_o \quad \dots (iv)$$

Taking  $v_s = 0$  and then applying KVL to the input circuit in Fig. 59.57, we get

$$-i_1(h_i + R_S) - h_r v_2 = 0 \quad \text{or} \quad i_1/v_2 = -h_r / (h_i + R_S)$$

Substituting this value in Eq. (iv) above, we have

$$g_o = h_o \cdot \frac{h_f h_r}{(h_i + R_S)} = \frac{h_o R_S}{(h_i + R_S)} \quad \therefore r_o = \frac{1}{g_o} = \frac{h_i + R_S}{h_o R_S}$$

It is seen that  $r_{in}$  depends on  $r_L$  whereas  $r_o$  depends on  $R_S$ .

If  $R_S$  is very large (i.e. circuit is driven by a current source) or  $h_r$  is negligible, then  $r_o \cong 1/h_o$ .

**(v) Power Gain**

$$A_p = \frac{P_2}{P_1} = \frac{v_2 i_2}{v_1 i_1} = A_v A_i = A_i^2 \frac{r_L}{r_{in}}$$

The above formulae are summarized below :

(i)  $A_i = \frac{h_f}{1 - h_o r_L} = h_f$

$A_i = \frac{h_f R_S}{(1 - h_o r_L)(r_{in} + R_S)} = \frac{h_o R_S}{(r_{in} + R_S)}$

(ii)  $r_{in} = h_i \frac{h_f h_r r_L}{1 - h_o r_L} = h_i$

(iii)  $A_v = \frac{h_f h_r}{h_i} \frac{r_L}{h_r r_L} = \frac{h_f r_L}{h_i}$ ;  $A_{vs} = \frac{h_f r_L}{(h_i + R_S)}$



(iv)  $r_o = \frac{h_i R_S}{h_o R_S h} = \frac{1}{h_o} \frac{h_i / R_S}{h / R_S} = \frac{1}{h_o}$

**59.23. Typical Values of Transistor *h*-parameters**

In the table below are given typical values for each parameter for the broad range of transistors available today in each of the three configurations.

Parameter	CB	CE	CC
$h_i$	25 Ω	1 K	1 K
$h_r$	$3 \times 10^{-4}$	$2.5 \times 10^{-4}$	$\cong 1$
$h_f$	-0.98	50	-50
$h_o$	$0.5 \times 10^{-6}$ S	$25 \times 10^{-6}$ S	$25 \times 10^{-6}$ S

**59.24. Approximate Hybrid Formulas**

The approximate hybrid formulas for the three connections are listed below. These are applicable when  $h_o$  and  $h_r$  are very small and  $R_S$  is very large. The given values refer to transistor terminals. The values of  $r_{in(stage)}$  or  $r_{in}'$  and  $r_{o(stage)}$  will depend on biasing resistors and load resistance respectively.

Item	CE	CB	CC
$r_{in}$	$h_{ie}$	$h_{ib}$	$h_{ic} + h_{fe}R_L$
$r_o$	$\frac{1}{h_{oc}}$	$\frac{1}{h_{oB}}$	$\frac{h_{ie}}{h_{fc}}$
$A_i$	$h_{fe} = \beta$	$-h_{fb} \cong 1$	$-h_{fe} \cong \beta$
$A_v$	$\frac{h_{ie}R_C}{h_{is}}$	$\frac{f_{fb}R_C}{h_{ib}}$	1

**59.25. Common Emitter *h*-parameter Analysis**

The *h*-parameter equivalent of the CE circuit of Fig. 59.59 (a) is shown in Fig. 59.59 (b). In Fig. 59.59 (a), no emitter resistor has been connected.

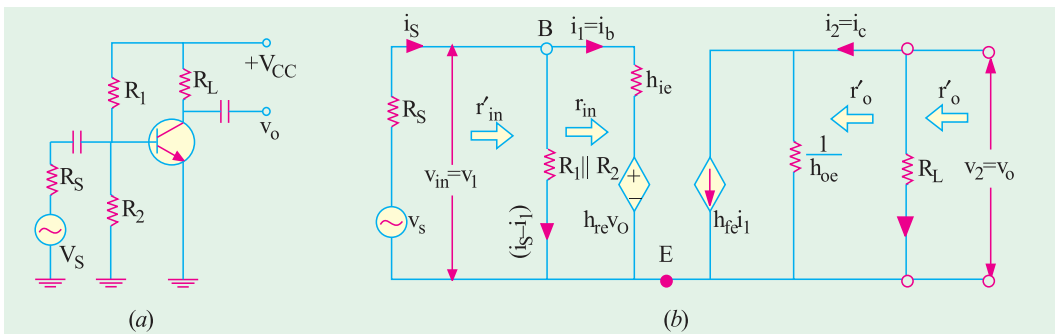


Fig. 59.59

However, Fig. 59.60 shows the CE circuit with an emitter resistor  $R_E$ .



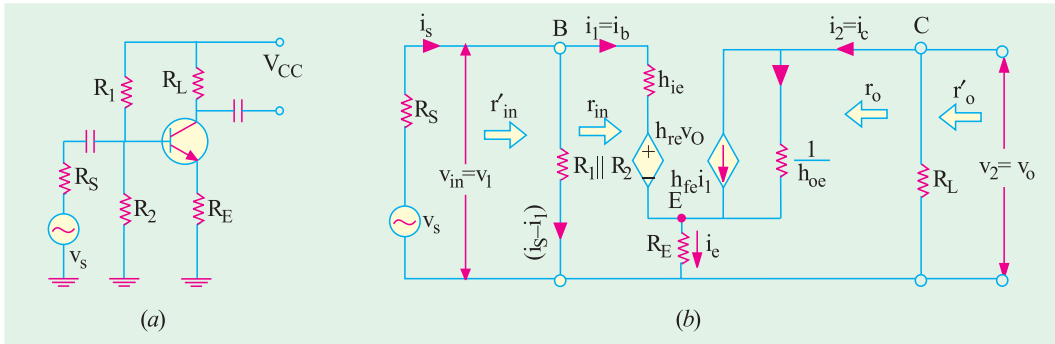


Fig. 59.60

We will now derive expressions for voltage and current gains for both these circuits.

**1. Input Impedance**

When looking into the base-emitter terminals of the transistor,  $h_{ie}$  is in series with  $h_{re}$  no. For a CE circuit,  $h_{re}$  is very small so that  $h_{re} v_o$  is negligible as compared to the drop over  $h_{ie}$ . Hence,  $r_{in} \cong h_{ie}$ .

Now, consider the circuit of Fig. 59.60. Again ignoring  $h_{re} v_o$ , we have

$$\begin{aligned} v_1 &= h_{ie} i_b + i_e R_E = h_{ie} i_b + (i_b + i_c) R_E \\ &= h_{ie} i_b + i_b R_E + h_{fe} i_b R_E \quad ((i_c = h_{fe} i_b)) \\ &= i_b [h_{ie} + R_E (1 + h_{fe})] \end{aligned}$$

$$\therefore r_{in} = r_{in(base)} = \frac{v_1}{i_1} = \frac{v_1}{i_b} [h_{ie} + (1 + h_{fe}) R_E] *$$

$$r_{in} \text{ or } r_{in(base)} = R_1 \parallel R_2 \parallel r_{in(base)}$$

**2. Output Impedance**

Looking back into the collector and emitter terminals of the transistor in Fig. 59.59 (b),  $r_o \cong 1/h_{oe}$ .

$$\text{As seen, } r_o' \text{ or } r_{o(stage)} = r_o \parallel R_L = (1/h_{oe}) \parallel R_L \quad (r_o \parallel R_L)$$

Since  $1/h_{oe}$  is typically 1 M or so and  $R_L$  is usually much smaller,  $r_o' \cong R_L = r_L$

**3. Voltage Gain**

$$A_v = \frac{v_2}{v_1} = \frac{v_o}{v_{in}} \quad \text{— Fig. 9.59 (b)}$$

Now,  $v_o = -i_c R_L$  and  $v_{in} \cong i_b h_{ie}$

$$\therefore A_v = \frac{i_c R_L}{i_b h_{ie}} = \frac{i_c}{i_b} \cdot \frac{R_L}{h_{ie}} = \frac{h_{fe} R_L}{h_{ie}}$$

Now, consider Fig. 59.60 (b). Ignoring  $h_{re} v_o$ , we have from the input loop of the circuit  $v_{in} = i_b [h_{ie} + R_E (1 + h_{fe})]$  —proved above

$$\therefore A_v = \frac{v_o}{v_{in}} = \frac{i_c R_L}{i_b [h_{ie} + R_E (1 + h_{fe})]} = \frac{h_{fe} R_L}{h_{ie} (1 + h_{fe}) R_E}$$

$$\frac{R_L}{R_E} \quad \text{— if } (1 + h_{fe}) R_E \gg h_{ie}$$

\* The above result could also be obtained by applying  $\beta$ -rule (Art. 9..24)



In general,

**4. Current Gain**

$$A_i = \frac{i_2}{i_1} = \frac{h_{fe}}{1 + h_{oe}r_L} h_{fe} \quad \text{--- if } h_{oe}r_L \ll 1$$

$$A_{is} = \frac{h_{fe} \cdot R_1 \parallel R_2}{r_{in} \parallel R_1 \parallel R_2}$$

**5. Power Gain**

$$A_p = A_v \times A_i$$

**59.26. Common Collector *h*-parameter Analysis**

The CC transistor circuit and its *h*-parameter equivalent are shown in Fig. 59.61.

One can make quick approximations of CC gains and impedance if one remembers that  $h_{re} = 1$  i.e. all of  $v_o$  is fed back to the input (Art. 59.23).

**1. Input Impedance**

$$\begin{aligned} v_{in} &= i_b h_{ic} + h_{re} v_o = i_b h_{ic} + v_o = i_b h_{ic} + i_e R_L \\ &= i_b h_{ic} + h_{fe} i_b R_L = i_b (h_{ic} + h_{fe} R_L) \end{aligned}$$

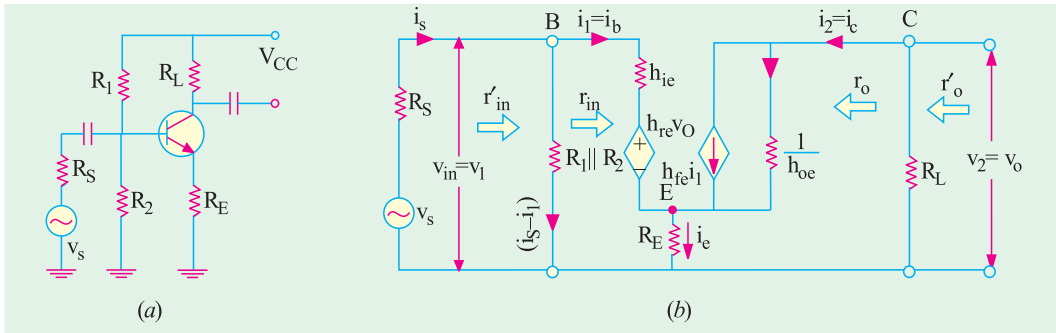


Fig. 59.61

$$\therefore r_{in} = \frac{v_{in}}{i_b} = h_{ic} + h_{fe} R_L$$

As seen,  $r_{in(stage)} = r_{in(base)} \parallel R_1 \parallel R_2 = r_{in(base)} \parallel R_B$  where  $R_B = R_1 \parallel R_2$

**2. Output Impedance**

$$r_o = \left. \frac{v_2}{i_2} \right|_{v_s=0} = \left. \frac{v_o}{i_c} \right|_{v_s=0}$$

Now,  $i_e \cong i_c = h_{fe} i_b = h_{fc} i_b$

Since  $v_s = 0$ ,  $i_b$  is produced by  $h_{re} v_o = v_o$

Hence, considering the input circuit loop, we get

$$i_b = \frac{v_o}{h_{ic} (R_S \parallel R_1 \parallel R_2)} = \frac{v_o}{h_{ic} R_S \parallel R_B}$$

$$i_c = h_{fc} i_b = \frac{h_{fc} v_o}{h_{ic} (R_S \parallel R_B)}$$

where  $R_B = R_1 \parallel R_2$



$$\therefore r_o = \frac{V_o}{i_e} = \frac{h_{ic} (R_S \parallel R_1 \parallel R_2)}{h_{fe}}$$

Also,  $r_o'$  or  $r_{o(stage)} = r_o \parallel R_L$

### 3. Voltage Gain

$$A_v = \frac{v_2}{v_1} = \frac{v_o}{v_{in}}$$

Now,  $v_o = i_e R_L = h_{fe} i_b R_L$  and  $i_b = (v_{in} - v_o) / h_{ic}$

$$v_o = \frac{h_{fe} R_L}{h_{ic}} (v_{in} - v_o) \quad \text{or} \quad v_o = 1 \frac{h_{fe} R_L}{h_{ic}} \frac{h_{fe} R_L v_{in}}{h_{ic}}$$

$$\therefore A_v = \frac{v_o}{v_{in}} = \frac{h_{fe} R_L / h_{ic}}{1 + h_{fe} R_L / h_{ic}} \approx 1$$

### 4. Current Gain

$$A_i = \frac{i_2}{i_1} = \frac{i_e}{i_b} = h_{fe}; \quad A_{is} = \frac{h_{fe} R_B}{r_{in} \parallel R_B}$$

where  $R_B = R_1 \parallel R_2$

## 59.27. Conversion of $h$ -parameters

Transistor data sheets generally specify the transistor in terms of its  $h$ -parameters for  $CB$  connection i.e.  $h_{ib}$ ,  $h_{fb}$ ,  $h_{rb}$  and  $h_{ob}$ . If we want to use the transistor in  $CE$  or  $CC$  configuration we will have to convert the given set of parameters into a set of  $CE$  or  $CC$  parameters. Approximate conversion formulae are tabulated over leaf :

Table No. 59.2		
From $CB$ to $CE$	From $CE$ to $CB$	From $CE$ to $CC$
$h_{ie} = \frac{h_{ib}}{1 + h_{fb}}$	$h_{ie} = \frac{h_{ie}}{1 + h_{fe}}$	$h_{ic} = h_{ic}$
$h_{oe} = \frac{h_{ob}}{1 + h_{fb}}$	$h_{ob} = \frac{h_{oe}}{1 + h_{fe}}$	$h_{oc} = h_{oe}$
$h_{fe} = \frac{h_{fb}}{1 + h_{fb}}$	$h_{fb} = \frac{h_{fe}}{1 + h_{fe}}$	$h_{fe} = -(1 + h_{fe})$
$h_{re} = \frac{h_{ib} h_{ob}}{1 + h_{fb}} \approx h_{rb}$	$h_{rb} = \frac{h_{ie} h_{oe}}{1 + h_{fe}} \approx h_{re}$	$h_{re} = 1 - h_{re} \approx 1$

**Example 59.18.** A transistor used in  $CB$  circuit has the following set of parameters.  
 $h_{ib} = 36 \Omega$ ,  $h_{fb} = 0.98$ ,  $h_{rb} = 5 \times 10^{-4}$ ,  $h_{ob} = 10^{-6}$  Siemens  
 With  $R_S = 2 K$  and  $R_C = 10 K$ , calculate (i)  $r_{in(base)}$  (ii)  $r_{out}$  (iii)  $A_i$  and (iv)  $A_v$ .  
 (Applied Electronics-I; Punjab Univ. 1991)

### Solution. Approximate Values

- (i)  $r_{in} = h_{ib} = 36 \Omega$
- (ii)  $r_o = \frac{1}{h_{ob}} = \frac{1}{10^{-6}} = 1M$
- (iii)  $A_i = h_{fb} = -0.98$
- (iv)  $A_v = \frac{h_{fb}}{h_{ib}} R_C = \frac{0.98}{36} \times 10K = 272$





More Accurate Values

$$\begin{aligned}
 (i) \quad r_{in(base)} &= h_{ib} \frac{h_{rb}h_{fb}}{h_{ob} + 1/r_L} && \text{---Art 9.22} \\
 &= 36 \frac{0.98 \cdot 5 \cdot 10^4}{10^6 + 1/10 \cdot 10^3} && (\because r_L = R_C \text{ since there is no } R_L) \\
 &= 36 + 4.9 = \mathbf{40.9 \Omega}
 \end{aligned}$$

It is the input resistance at transistor terminals.

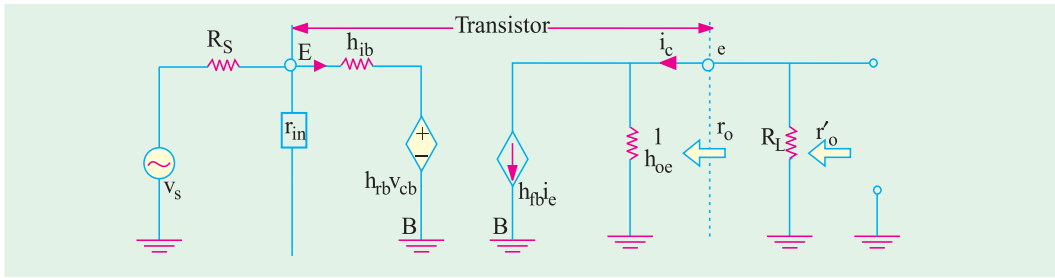


Fig. 59.62

$$(ii) \quad r_o = \frac{h_{ib} R_S}{h_o(h_{ib} + R_S)} \frac{36 \cdot 2000}{10^6(36 + 2000)} \frac{2000}{(0.98) \cdot 5 \cdot 10^4} = \mathbf{0.8 \text{ M}}$$

It is the output resistance at transistor terminals.

$$(iii) \quad A_i = \frac{h_{fb}}{1 + h_{ob}r_L} = \frac{0.98}{1 + 10^6 \cdot 10^4} = \mathbf{-0.97}$$

$$(iv) \quad A_v = \frac{h_{fb}r_L}{h_{ib}(1 + h_{ob}r_L)} = \frac{h_{fb}r_L}{h_{ib} + h_{ob}r_L} \quad (\because r_L = R_C)$$

Here, ac load  $r_L = R_C = 10 \text{ K} = 10^4 \Omega$

$$\therefore A_v = \frac{(0.98) \cdot 10^4}{36(1 + 10^6 \cdot 10^4)} = \frac{9800}{36(10^6 + 10^4)} = \mathbf{249}$$

**Example 59.19.** A transistor used in CE connection (Fig. 59.63) has the following set of  $h$ -parameters :  $h_{ir} = 1 \text{ K}$ ,  $h_{fe} = 100$ ,  $h_{re} = 5 \times 10^{-4}$  and  $h_{oc} = 2 \times 10^{-5} \text{ S}$ . With  $R_S = 2 \text{ K}$  and  $R_C = 5 \text{ K}$ , determine

- (i)  $r_{in}$  (ii)  $r_o$  (iii)  $A_i$  and (iv)  $A_v$

**Solution.**

$$\begin{aligned}
 (i) \quad r_{in} &= h_{ie} \frac{h_{fb}r_L}{h_o + 1/r_L} \\
 &= 1000 \frac{5 \cdot 10^4 \cdot 100}{2 \cdot 10^5 + 1/5 \cdot 10^3} \\
 &= \mathbf{723 \Omega}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad r_o &= \frac{h_{ie} R_S}{(h_{ie} + R_S)h_{oe} + h_{fe} \cdot h_{re}} = \frac{1000 \cdot 2000}{(1000 + 2000) \cdot 2 \cdot 10^{-5} + 100 \cdot 5 \cdot 10^{-4}} \\
 &= 30,000 \Omega = \mathbf{0.03 \text{ M}}
 \end{aligned}$$

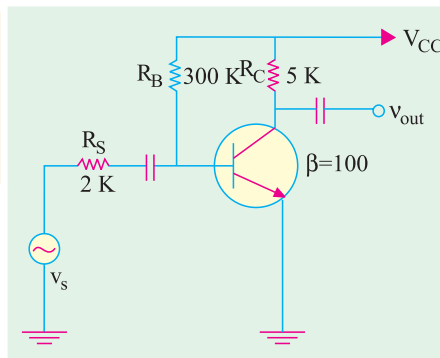


Fig. 59.63



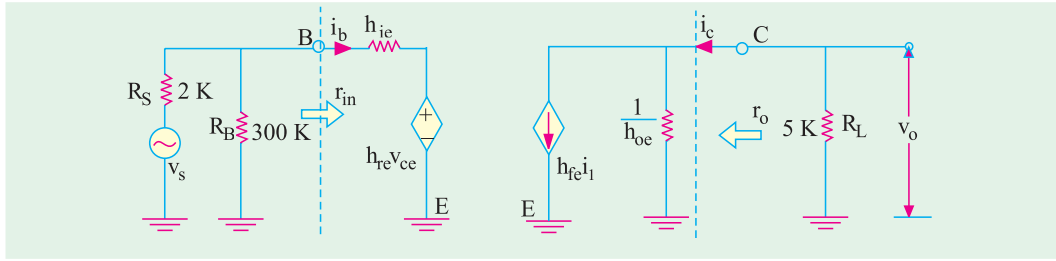


Fig. 59.64

(iii)  $A_i = \frac{h_{fe}}{1 + \frac{R_S}{h_{ie}}} \cdot \frac{100}{1 + \frac{2}{10^5} + \frac{5}{10^3}}$  **91**

(iv)  $A_i = \frac{h_{fe} r_L}{(h_{ie} + R_S) (1 + \frac{1}{h_{oe} \cdot r_L}) + h_{fe} h_{re} r_L}$   

$$\frac{100 \cdot 5 \cdot 10^3}{(100 + 2000)(1 + \frac{1}{2 \cdot 10^5} + \frac{5}{10^3}) + 100 \cdot 5 \cdot 10^4 \cdot 5 \cdot 10^3} = -164$$

The negative sign indicates that there is 180° phase shift between the input and output ac signals. Obviously, it is the overall (or circuit) voltage gain and not the voltage gain of the transistor alone.

**Example 59.20.** In the CE circuit shown in Fig. 59.65, the transistor parameters are :  
 $h_{ie} = 2 \text{ K}$ ,  $h_{fe} = 100$ ,  $h_{re} = 5 \times 10^{-4}$ ,  $h_{oe} = 2 \times 10^{-5} \text{ S}$   
 Calculate (i)  $r_{in(base)}$ , (ii)  $r_{in(stage)}$ , (iii)  $r_o$ , (iv)  $r_{o(stage)}$ , (v)  $A_i$  and (vi)  $A_v$ .  
**(Electronics-1, Karnataka Univ.)**

**Solution.** The hybrid equivalent circuit is shown in Fig. 59.66. We will use the approximate formulas given in Art. 59.24.

- (i)  $r_{in(base)} = h_{ie} = 2 \text{ K}$
- (ii)  $r_{in(stage)} = 2 \text{ K} \parallel 250 \text{ K} = 1.98 \text{ K}$
- (iii)  $r_o = 1 / h_{oe} = 1/2 \times 10^{-5} = 50 \text{ K}$

It is the output impedance of the transistor only.

(iv)  $r_{o(stage)} = r_o' = 50 \text{ K} \parallel 5 \text{ K} = 4.54 \text{ K}$

The impedance takes into account the collector load.

(v)  $A_i \cong h_{fe} = 100$

(vi)  $A_v = \frac{h_{fe} r_L}{h_{ie}} = \frac{100 \cdot 5}{2} = -250$

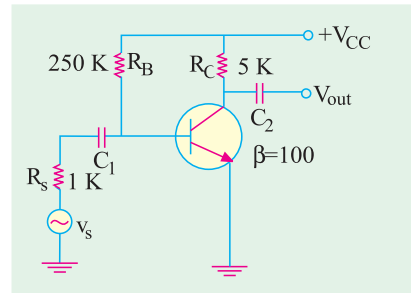


Fig. 59.65

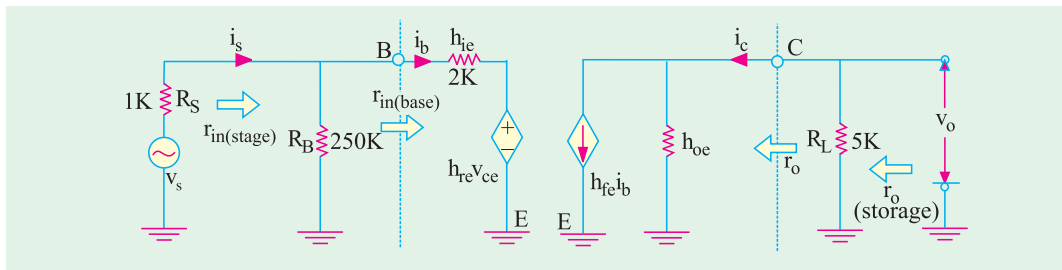


Fig. 59.66



**Example 59.21.** Determine the various gains of the circuit of Fig. 59.67 if an emitter resistance of 0.5 K is included in the circuit. **(Applied Electronics, Punjab Univ. 1991)**

**Solution.** The CE circuit with  $R_E$  included is shown in Fig. 59.47. Different performance characteristics of the circuit are as under :

- (i)  $r_{in(base)} = h_{ie} (1 + h_{fe}) R_E$   
 $= 2 + 101 \times 0.5 = 52.5 \text{ K}$
- (ii)  $r_{in(stage)} = R_B \parallel r_{in} = 250 \parallel 52.5$   
 $= 43.3 \text{ K}$
- (iii)  $r_o = 1/h_{oe} = 50 \text{ K}$
- (iv)  $r_{o(stage)} = r_o' = 50 \text{ K} \parallel 5 \text{ K}$   
 $= 4.54 \text{ K}$
- (v)  $A_v = h_{fe} = 100$
- (vi)  $A_v = \frac{h_{fe} r_L}{h_{ie} (1 + h_{fe}) R_E} = \frac{100 \times 5}{2 \times 50.5} = -9.5$

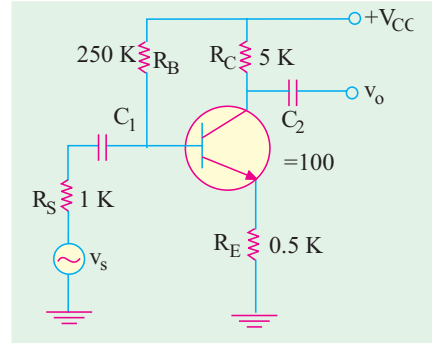


Fig. 59.67

The value is reduced from 250 to 9.5.

**Example 59.22.** The transistor of Fig. 59.68 has the following set of h-parameters :

$$h_{ie} = 2\text{K}, h_{fe} = 50, h_{rs} = 4 \times 10^{-4}, h_{oe} = 25 \times 10^{-6} \text{ Siemens}$$

Determine (i)  $r_{in(base)}$  (ii)  $r_{in(stage)}$  (iii)  $r_o$  (iv)  $r_{o(stage)}$  and (v)  $A_v$ .

**(Electronics-I, Patna Univ. 1991)**

**Solution.** we will use the formula derived in Art. 59.25.

- (i)  $r_{in(base)} = h_{ie} (1 + h_{fe}) R_E = 2 (1 + 50) \times 5$   
 $= 257 \text{ K}$
- (ii)  $r_{in(stage)} = r_{in(base)} \parallel R_1 \parallel R_2 = 257 \parallel 80 \parallel 40$   
 $= 24 \text{ K}$
- (iii)  $r_o = 1/h_{oe} = 1/25 \times 10^{-6} = 40 \text{ K}$
- (iv)  $r_{o(stage)} = r_o \parallel r_L$  where  $r_L = 15 \text{ K} \parallel 30 \text{ K}$   
 $= 10 \text{ K} = 40 \text{ K} \parallel 10 = 8 \text{ K}$
- (v)  $A_v = \frac{h_{fe} R_L}{h_{ie} (1 + h_{fe}) R_E} = \frac{50 \times 10}{257} = -2.9$

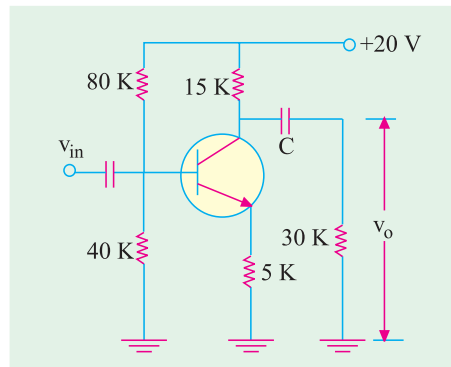


Fig. 59.68

**Example 59.23.** Draw a hybrid small-signal model for a transistor in CE configuration.

A single source with an open circuit voltage of 1 mV and internal impedance of 600 W is connected to the input of transistor AC 125 in CE configuration. The small-signal parameters measured at  $V_{CB} = -5 \text{ V}$  and  $I_E = 2 \text{ mA}$ , are as follows :

$$h_{ie} = 1.7 \text{ K}, h_{re} = 6.5 \times 10^{-4}, h_{fe} = 125, h_{oe} = 80 \mu\text{S}.$$

Calculate the input and output impedances and signal amplification for a load of 5.6 K. Also, find the value of the signal voltage at the output. **(Electronics-I, Gwalior Univ.)**

**Solution.**  $\Delta h = 1.7 \times 10^3 \times 80 \times 10^{-6} - 125 \times 6.5 \times 10^{-4} = 0.055$

$$(i) \quad r_{in} = \frac{h_{ie}}{1 - \Delta h} \parallel \frac{h_{fe} R_L}{h_{oe} R_L} = \frac{1700}{1 - 0.055} \parallel \frac{5.6 \times 10^3}{10^{-6} \times 5.6 \times 10^3} = 55 \times 10^3 \parallel 10^3 = 950 \text{ W}$$

-Art. 59.22



(ii)  $r_o = \frac{h_{ie} R_S}{h_{oe} R_S + h} = \frac{1700 \cdot 600}{80 \cdot 10^6 + 600 \cdot 0.055} = \frac{2300}{0.103} = 22.3 \text{ K}$

(iii)  $A_v = \frac{h_{fe} R_L}{h_{ie} R_L + h} = \frac{125 \cdot 5600}{1700 + 308} = -348.6$

The negative sign merely indicates that there is phase reversal of 180° between the output and input voltages.

Now, 1 mV signal voltage is divided between  $r_{in}$  and  $R_S$ . The input voltage  $v_{in}$  is that which drops over  $r_{in} = 1 \times r_{in} / (r_{in} + R_S) = 1 \times 1387 / (1387 + 600) = 0.698 \text{ mV}$

∴ output voltage =  $-348.6 \times 0.698 = -243.3 \text{ mV}$ .

**Example 59.24.** The transistor of Fig. 59.69 has the following set of h-parameters :  
 $h_{ie} = 2 \text{ K}$ ,  $h_{fe} = 100$ ,  $h_{re} = 5 \times 10^{-4}$ ,  $h_{oe} = 2.5 \times 10^{-5} \text{ S}$   
 Find the voltage gain and the ac impedance of the stage.  
 (Electronics-II, Bombay Univ. 1992)

**Solution.** Using somewhat exact formulas given in Art. 59.22, we have

$$r_{in(base)} = h_{ie} \frac{h_{fe} h_{re}}{h_{oe} + 1/r_L}$$

Now, collector load

$$r_L = 10 \text{ K} \parallel 30 \text{ K} = 7.5 \text{ K}$$

$$\therefore r_{in(base)} = 200 \frac{100 \cdot 5 \cdot 10^{-4}}{2.5 \cdot 10^{-5} + 1/7.5 \cdot 10^3} = 2000 - 316 = 1684 \Omega$$

The ac input impedance of the stage i.e. impedance when looking into point B is

$$r_{in(base)} = r_{in(base)} \parallel R_1 \parallel R_2 = 1.684 \parallel 50 \parallel 25 = 1.53 \text{ K}$$

$$A_v = \frac{h_{fe}}{r_{in(base)} (h_{oe} + 1/r_L)} \quad \text{Now, } r_L = 10 \text{ K} \parallel 30 \text{ K} = 7.5 \text{ K}$$

$$\therefore A_v = \frac{100}{0.184 (2.5 \cdot 10^{-5} + 1/7500)} = -375$$

Obviously,  $R_E$  does not come into the ac picture because it is ac grounded by the bypass capacitor.

**Example 59.25.** In the CC circuit of Fig. 59.70, the transistor parameter are  $h_{ic} = 2 \text{ K}$  and  $h_{fc} = 100$ . Calculate the circuit input and output impedance and voltage, current and power gains.  
 (Electronic Technology, Bangalore Univ.)

**Solution.**  $r_{in} \cong h_{ic} + h_{fc} R_L = 2 + 100 \times 5 = 502 \text{ K}$

$$r_{in(stage)} = R_1 \parallel R_2 \parallel r_{in} = 10 \parallel 10 \parallel 502 = 4.95 \text{ K}$$

$$r_o = \frac{h_{ie} (R_S \parallel R_1 \parallel R_2)}{h_{fe}} = \frac{2 (1 \parallel 10 \parallel 100)}{100} = 28.3 \Omega$$

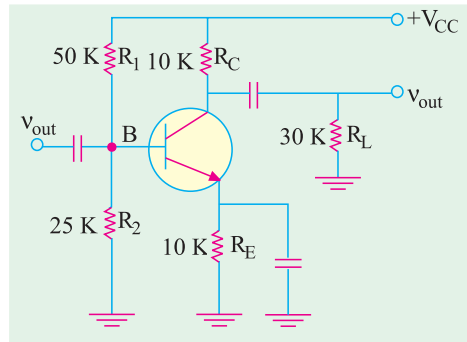


Fig. 59.69

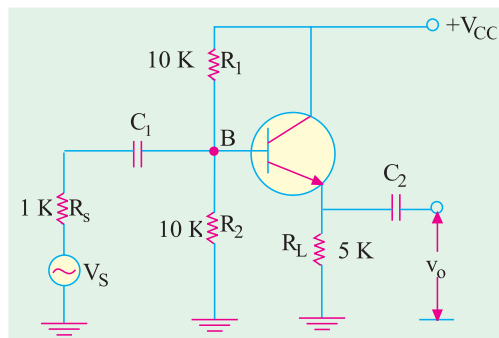


Fig. 59.70



$$r_{o(stage)} = r_o \parallel R_L = 28.3 \Omega \parallel 5K = 28.1 \Omega$$

$$A_v \cong 1 \text{ and } A_i = h_{fe} = 100$$

**Example 59.26.** A transistor with  $h_{ie} = 1.5 K$  and  $h_{fe} = 75$  is used in an emitter follower circuit where resistances  $R_1$  and  $R_2$  are used for normal biasing. Calculate (i)  $A_i$  (ii)  $r_{in}$  (iii)  $r_o$  and (iv)  $A_v$  if  $R_E = 860 \Omega$ ,  $R_1 \parallel R_2 = 20K$  and  $R_S = 1K$ . **(Electronic Engg.; Indore Univ. 1992)**

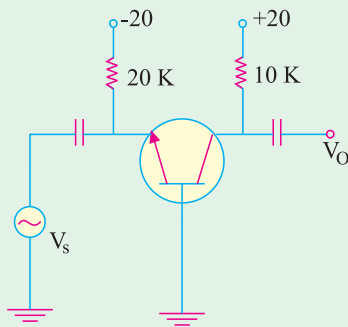
**Solution.** Let us first convert the given CE values of  $h$ -parameters into their equivalent CC values with the help of Table No. 59.2. It is seen that  $h_{ic} = h_{ie} = 1.5 K$  and  $h_{fc} = (1 + h_{fe}) = 76$ .

(i)  $A_i = h_{fe} = 76$                       (ii)  $r_{in} = h_{ic} + h_{fc} R_L = 1.5 + 76 \times 0.860 = 66.9 K$

(iii)  $r_o = \frac{h_{ic} (R_S \parallel R_1 \parallel R_2)}{h_{fe}} = \frac{1.5 \cdot 1 \parallel 20}{76} = 32.3 \Omega$       (iv)  $A_v \cong 1$

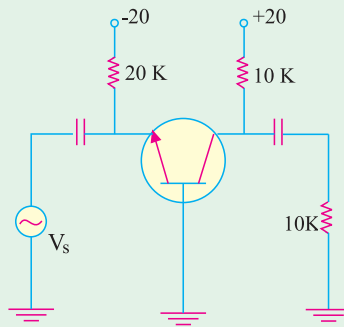
**Tutorial Problems No. 59.1**

- Using ideal transistor approximations for the single-stage CB amplifier of Fig. 59.71, find (i) stage  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  and (iv)  $A_p$ . Take transistor  $\alpha = -0.99$ . **[(i) 25  $\Omega$  (ii) 10 K (iii) 400 (iv) 396]**
- For the single-stage CB amplifier circuit shown in Fig. 59.72, find (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  and (iv)  $A_p$



**Fig. 59.71**

Transistor  $\alpha = -0.9$ . Take  $G_e$  transistor material. **[(i) 25  $\Omega$  (ii) 5 K (iii) 200 (iv) 196]**

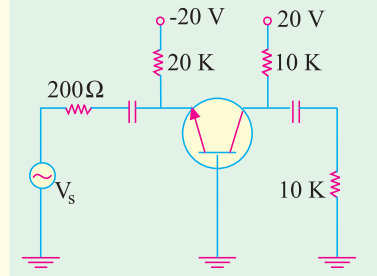


**Fig. 59.72**

- For the CB amplifier circuit of Fig. 59.73, find approximate values of
  - stage  $r_{in}$
  - $r_L$
  - $A_v = v_{out} / v_{in}$
  - $A_{v\sigma} = v_{out} / v_s$
  - $A_p$
  - $G_p$

Take transistor  $\alpha = -0.98$

**[(i) 25  $\Omega$  (ii) 5K (iii) 200 (iv) 22.2 (v) 196 (vi) 22.9 dB]**



**Fig. 59.73**

- Find  $r_{in}$ ,  $r_L$ ,  $A_p$ ,  $A_v$  and  $G_p$  for the CE amplifier shown in Fig. 9.74.

**[(i) 1250  $\Omega$  (ii) 5K (iii) 50 (iv) 200 (v) 30 dB]**



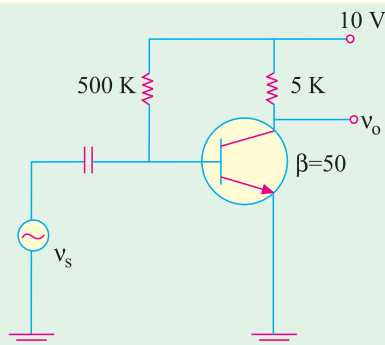


Fig. 59.74

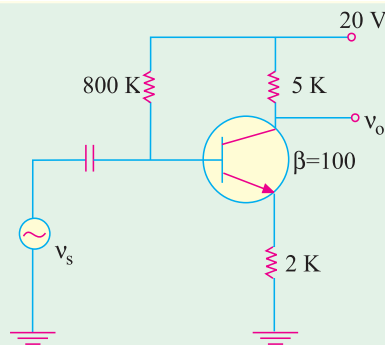


Fig. 59.75

5. For the single-stage CE amplifier circuit of Fig. 59.75, find  
 (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  (iv)  $A_p$  and (v)  $G_p$   
 Take  $\beta = 100$  and use  $r_e = 50 \text{ mV}/I_E$ . **[(i) 160 K (ii) 5K (iii) 2.5 (iv) 250 (v) 24 dB]**
6. For the  $C_E$  amplifier of Fig. 59.76, find approximate values of  
 (i)  $r_{in}$  (ii)  $r_L$  (iii)  $A_v$  (iv)  $A_p$  and  $G_p$ .  
 Take transistor  $\beta = 50$  and use  $r_e = 50 \text{ mV}/I_E$ . **[(i) 2K (ii) 10K (iii) 200 (iv) 10,000 (v) 40 dB]**
7. For the emitter follower circuit shown in Fig. 59.77, find  
 (i)  $r_{in(base)}$  (ii)  $r_{in(stage)}$   
 (iii) stage  $A_v$  (iv)  $A_p$  in decibels  
 Take  $\beta = 100$  and use  $r_e = 25 \text{ mV}/I_E$ . **[(i) 10K (ii) 9.52K (iii) 0.75 (iv) 18.75 dB]**

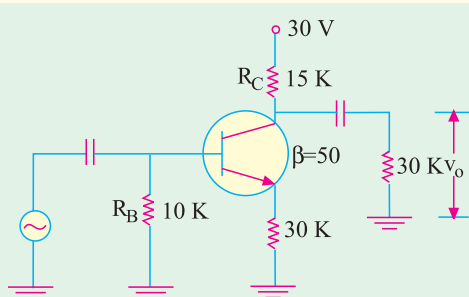


Fig. 59.76

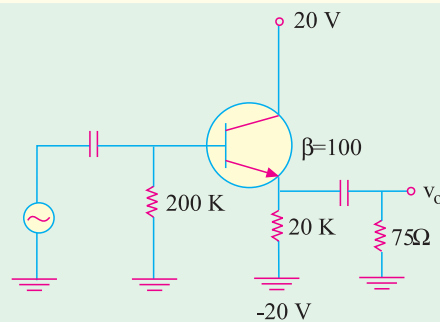


Fig. 59.77

8. An NPN silicon transistor is connected as a  $C_E$  amplifier with load  $R_L$  and a source resistance  $R_S$ . The  $h$ -parameters are :  
 $h_{ie} = 1.1 \text{ K}$ ,  $h_{re} = 2.5 \times 10^{-4}$ ,  $h_{fe} = 50$  and  $h_{oe} = 25 \mu\text{S}$   
 If  $R_L = 10\text{K}$  and  $R_S = 1\text{K}$ , find the various gains and input and output impedances.  
 Derive the relations used.  
**[(i)  $A_i = 40$ ,  $A_{is} = 20$ ,  $r_{in} = 1\text{K}$ ,  $A_v = -400$ ,  $A_{vs} = -200$ ,  $r_o = 52.5 \text{ K}$ ,  $A_p = 16,000$ ,  $G_p = 42 \text{ dB}$ ]**
9. Calculate the gain of a common-emitter transistor amplifier whose hybrid parameters are:  
 $h_{ie} = 1100 \text{ ohms}$ ,  $h_{re} = 2.5 \times 10^{-4}$ ;  $h_{fe} = 50$ ,  $h_{oe} = 25 \mu\text{S}$ ,  $R_L = 5\text{K}$ .  
 Derive any formula used. **(-213) (Electronic Engg., Indore Univ.)**
10. A CE amplifier has  $h_{ie} = 1.1 \text{ K}$ ,  $h_{fe} = 50$ ,  $h_{re} = 2.5 \times 10^{-4}$ ;  $h_{oe} = 25 \mu\text{S}$ ,  $R_S = R_L = 500 \text{ ohm}$ . Calculate the output impedance and voltage gain.  
**(Applied Electronics-I, Punjab Univ. Dec.)**
11. A junction transistor has the following  $h$ -parameters  $h_{ie} = 2000 \text{ ohm}$ ,  $h_{re} = 15 \times 10^{-4}$ ,  $h_{fe} = 49$ ,  $h_{oe} = 50 \mu\text{S}$ . Determine the current gain, voltage gain, input resistance and output resistance of the CE amplifier if the



load resistance is 10K and source resistance is 600 ohm. Derive the expressions used.

(Applied Electronics-I, Punjab May)

12. A CE amplifier has  $h_{ie} = 1.1 \text{ K}$ ,  $h_{fe} = 50$ ,  $h_{re} = 2.5 \times 10^{-4}$ ,  $h_{oe} = 25 \text{ } \mu\text{S}$ ,  $R_S = R_L = 1\text{K}$ . Calculate the output impedance and voltage gain.

(Applied Electronics-I, Punjab Univ. Dec.)

13. A junction transistor has the following  $h$ -parameters  $h_{ie} = 1\text{k}\Omega$ ,  $h_{re} = 1$ ,  $h_{fe} = -50$ ,  $h_{oe} = 25\text{mA/V}$ . This transistor is connected to a source of internal resistance 600 ohm and a load of 40 k $\Omega$ . Calculate the current gain, voltage gain, input resistance and output resistance of the amplifier. Derive the expressions used.

(Applied Electronics-I, Punjab Univ. June)

14. A transistor connected in CE configuration has following  $h$ -parameters.

$$\begin{aligned} h_{ie} &= 1.1 \text{ k}\Omega & h_{re} &= 2.5 \times 10^{-4} \\ h_{fe} &= 50 & h_{oe} &= 25 \text{ } \mu \text{ Siemens} \\ \text{and} & & r_s = r_L &= 1 \text{ k}\Omega \end{aligned}$$

Calculated current gain, input impedance and voltage gain.

(Electronics Engg. Bangalore Univ. 2001)

OBJECTIVE TESTS – 59

1. In an ac amplifier, larger the internal resistance of the ac signal source
  - (a) greater the overall voltage gain
  - (b) greater the input impedance
  - (c) smaller the current gain
  - (d) smaller the circuit voltage gain.
2. The main use of an emitter follower is as
  - (a) power amplifier
  - (b) impedance matching device
  - (c) low-input impedance circuit
  - (d) follower of base signal.
3. An ideal amplifier is one which
  - (a) has infinite voltage gain
  - (b) responds only to signals at its input terminals
  - (c) has positive feedback
  - (d) gives uniform frequency response.
4. The smallest of the four  $h$ -parameters of a transistor is
  - (a)  $h_i$
  - (b)  $h_r$
  - (c)  $h_o$
  - (d)  $h_f$
5. The voltage gain of a single-stage CE amplifier is increased when
  - (a) its ac load is decreased
  - (b) resistance of signal source is increased
  - (c) emitter resistance  $R_E$  is increased.
  - (d) ac load resistance is increased.
6. When emitter bypass capacitor in a CE amplifier is removed, its ..... is considerably reduced.
  - (a) input resistance
  - (b) output load resistance
  - (c) emitter current
  - (d) voltage gain
7. Unique features of a CC amplifier circuit is that it
  - (a) steps up the impedance level
  - (b) does not increase signal voltage
  - (c) acts as an impedance matching device
  - (d) all of the above.
8. The input impedance  $h_{11}$  of a network with output shorted is given by the ratio
  - (a)  $v_1/i_1$
  - (b)  $v_1/v_2$
  - (c)  $i_2/i_1$
  - (d)  $i_2/v_2$
9. The  $h$ -parameters of a transistor depend on its
  - (a) configuration
  - (b) operating point
  - (c) temperature
  - (d) all of the above
10. The output admittance  $h_o$  of an ideal transistor connected in CB configuration is ..... siemens.
  - (a) 0
  - (b)  $1/r$
  - (c)  $1/\beta r_e$
  - (d)  $-1$ .
11. A transistor has  $h_{fe} = 100$ ,  $h_{ie} = 5.2 \text{ K } \Omega$ , and  $r_{bb} = 0$ . At room temperature,  $V_T = 26 \text{ mV}$ , the collector current,  $|I_C|$  will be.
  - (a) 10 mA
  - (b) 5 mA
  - (c) 1 mA
  - (d) 0.5 mA

ANSWERS

1. (d) 2. (b) 3. (b) 4. (c) 5. (d) 6. (d) 7. (d) 8. (a) 9. (d) 10. (a) 11. (a)



## ROUGH WORK

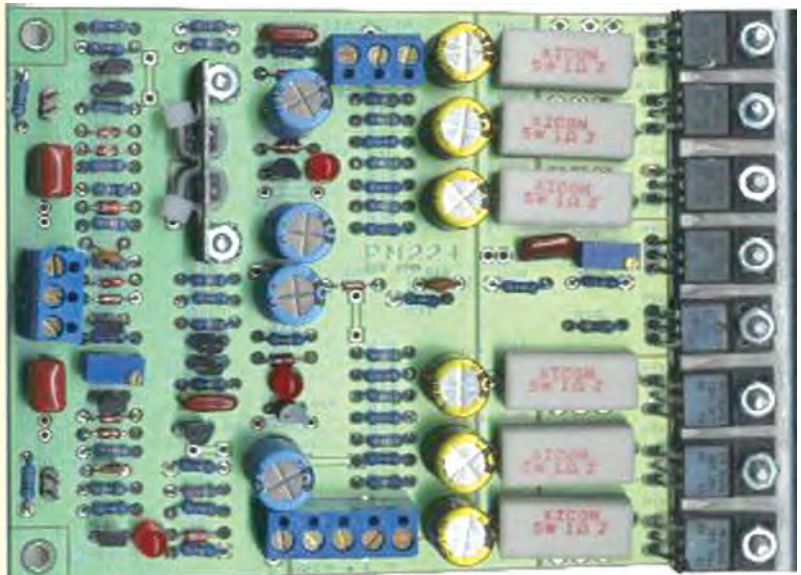


# CHAPTER 60

## Learning Objectives

- Classification of Amplifiers
- Common Base (CB) Amplifier
- Common Emitter (CE) Amplifier
- Common Collector (CC) Amplifier
- Comparison of Amplifier Configurations
- Class-A Amplifier
- Power Rectangle
- Power Efficiency
- Class-B Amplifier
- Maximum Values
- Class-B Push-pull Amplifier
- Crossover Distortion
- Class-B Amplifier
- Class-C Amplifier
- Tuned Amplifier
- Distortion in Amplifiers
- Noise
- The Decibel System
- Value of 1 dB
- Cause of Amplifier Gain Variations
- Miller Effect
- Cut-off Frequencies of Cascaded Amplifiers
- The  $f_T$  of a Transistor
- Relation between  $f_{\alpha}$ ,  $f_{\beta}$  and  $f_T$
- Gain-bandwidth Product

## SINGLE-STAGE TRANSISTOR AMPLIFIERS



Single stage amplifier analysis is of great value in understanding the practical amplifier circuits

### 60.1. Classification of Amplifiers

Linear amplifiers are classified according to their mode of operation *i.e.* the way they operate according to a predetermined set of values. Various amplifier descriptions are based on the following factors :

1. **As based on its input**
  - (a) small-signal amplifier
  - (b) large-signal amplifier
2. **As based on its output**
  - (a) voltage amplifier
  - (b) power amplifier
3. **As based on its frequency response**
  - (a) audio-frequency (AF) amplifier
  - (b) intermediate-frequency (IF) amplifier
  - (c) radio-frequency (RF) amplifier
4. **As based on its biasing conditions**
  - (a) class-A
  - (b) class-AB
  - (c) class-B
  - (d) class-C
5. **As based on transistor configuration**
  - (a) common-base (CB) amplifier
  - (b) common-emitter (CE) amplifier
  - (c) common-collector (CC) amplifier

The description *small-signal, class-A, CE, voltage amplifier* means that input signal is small, biasing condition is class-A, transistor configuration is common-emitter and its output concerns voltage amplification.

We will first take up the basic working of a single-stage amplifier *i.e.* an amplifier having one amplifying element connected in *CB, CE* and *CC* configuration.

### 60.2. Common Base (CB) Amplifier

Both Fig. 60.1 and 60.2 show the circuit of a single-stage *CB* amplifier using *NPN* transistor. As seen, input ac signal is injected into the emitter-base circuit and output is taken from the collector-base circuit. The *E/B* junction is forward-biased by  $V_{EE}$  whereas *C/B* junction is reverse-biased by  $V_{CC}$ . The *Q*-point or dc working conditions are determined by dc batteries along with resistors  $R_E$  and  $R_C$ . In other words, values of  $I_E$ ,  $I_B$  and  $V_{CB}$  are decided by  $V_{CC}$ ,  $V_{EE}$ ,  $R_E$  and  $R_C$ . The voltage  $V_{CB}$  is given by the equation  $V_{CB} = V_{CC} - I_C R_C$ .

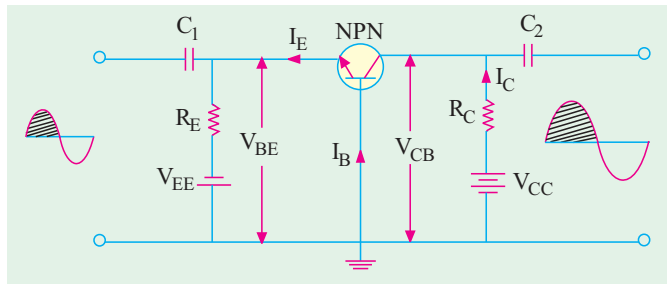


Fig. 60.1

When no signal is applied to the input circuit, the output just *sits at the Q-point so that there is no output signal*.

Let us now see what happens when we apply an ac signal to the *E/B* junction via a coupling capacitor  $C_1$  (which is assumed to offer no reactance to the signal).

#### Circuit Operation

When positive half-cycle of the signal is applied, then

1. forward bias is **decreased** because  $V_{BE}$  is already negative with respect to the ground as per biasing rule of Art. 60.3.
2. consequently,  $I_B$  is **decreased**.
3.  $I_E$  and hence  $I_C$  are **decreased** (because they are both nearly  $\beta$  times the base current).
4. the drop  $I_C R_C$  is **decreased**.
5. hence,  $V_{CB}$  is **increased** as seen by the equation given above.



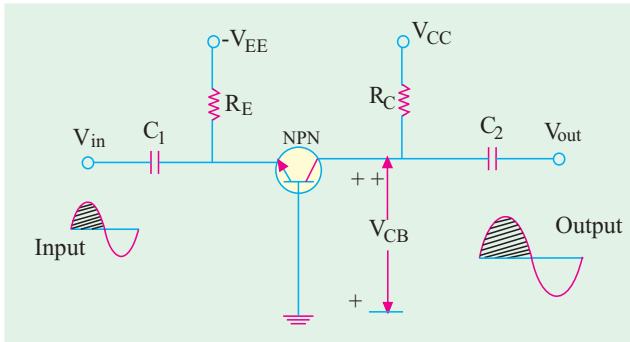


Fig. 60.2

It means that a positive output half-cycle is produced.

Since a **positive-going** input signal produces a **positive-going** output signal, there is no phase reversal between the two.

Voltage amplification in this circuit is possible by reason of relative input and output circuitry rather than current gain ( $\alpha$ ) which is always less than unity. The input circuit has low resistance whereas output circuit has very large resistance. Although

changes in input and output currents are the same, the ac drop across  $R_L$  is very large. Hence, changes in  $V_{CB}$  (which is the output voltage) are much larger than changes in input ac signal. Hence, the voltage amplification.

### 60.3. Various Gains of a CB Amplifier

#### 1. Input Resistance

The ac input resistance of the transistor is given by the emitter junction resistance

$$r_e = \frac{25\text{mV}}{I_E} \quad \text{or} \quad \frac{50\text{mV}}{I_E}$$

As seen from the ac equivalent circuit (Fig. 10.3)

$$r_{in} = r_e \parallel R_E$$

#### 2. Output Resistance

$$r_o = R_C$$

— Fig. 60.2

If a load resistance  $R_L$  is connected across output terminals, then

$$r_o = R_C \parallel R_L$$

— Fig. 60.3

It is called the output resistance of the stage and is written as  $r_{o(stage)}$  or  $r_o'$

#### 3. Current Gain $A_i = \alpha$

#### 4. Voltage Gain $A_v = \frac{r_o}{r_{in}} = \frac{r_o}{r_e}$

#### 5. Power Gain $A_p = A_v \cdot A_i$

The decibel gain is given by  $G_p = 10 \log_{10} A_p \text{ dB}$

### 60.4. Characteristics of a CB Amplifier

Common-base amplifier has

1. very low input resistance (30 – 150  $\Omega$ ),
2. very high output resistance (upto 500 K),
3. a current gain  $\alpha < 1$ ,
4. large voltage gain of about 1500,
5. power gain of upto 30 dB,
6. no phase reversal between input and output voltages.

### Uses

One of the important uses of a *CB* amplifier is in matching a low-impedance circuit to a high-impedance circuit.

It also has high stability of collector current with temperature changes.



**Example 60.1.** For the single-stage CB amplifier shown in Fig. 60.3 (a), find  
 (a) stage input resistance, (b) stage output resistance,  
 (c) current gain, (d) voltage gain of the stage,  
 (e) stage power gain in dB,  
 Assume  $\alpha = 1$ . Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$

(Basic Electronics, Bombay Univ. 1991)

**Solution.** The ac equivalent circuit is shown in Fig. 60.3 (b).

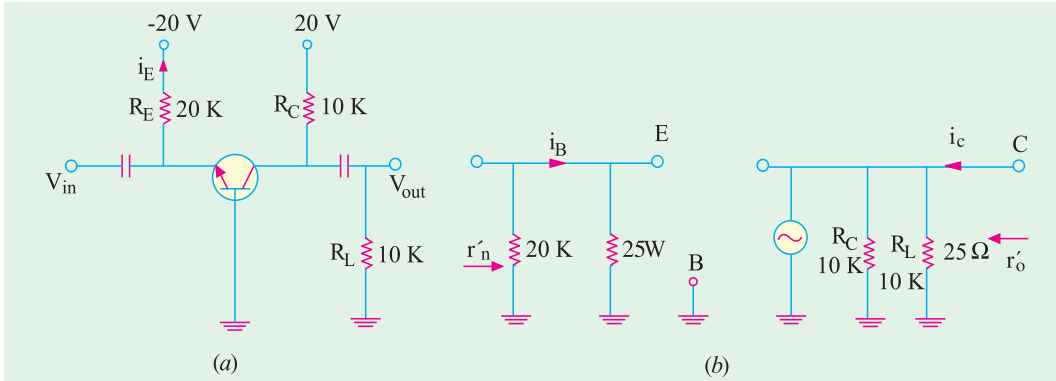


Fig. 60.3

$$I_E = 20/20 = 1\text{mA} \quad r_e = 25/I_E = 25 \Omega$$

(a) stage  $r_{in} = r_e \parallel R_E = 25 \Omega \parallel 20\text{K} \cong 25 \Omega$   
 (b) stage  $r_o = R_C \parallel R_L \quad \therefore r_o' = 10\text{K} \parallel 10\text{K} = 5\text{K}$   
 (c)  $A_i = \alpha = 1$  (d)  $A_v = \frac{r_o'}{r_e} = \frac{5000}{25} = 200$   
 (e)  $A_p = A_v \cdot A_i = 200 \times 1 = 200$   
 $G_p = 10 \log_{10} A_p \text{ dB} = 10 \log_{10} 200 = 23 \text{ dB}$

### 60.5. Common Emitter (CE) Amplifier

Fig. 60.4 and 60.5 show the circuit of a single-stage CE amplifier using an NPN transistor. Here, base is the driven element. The input signal is injected into the base-emitter circuit whereas output signal is taken out from the collector-emitter circuit. The E/B junction is forward-biased by  $V_{BB}$  and C/B junction is reversed-biased by  $V_{CC}$  (in fact, same battery  $V_{CC}$  can provide

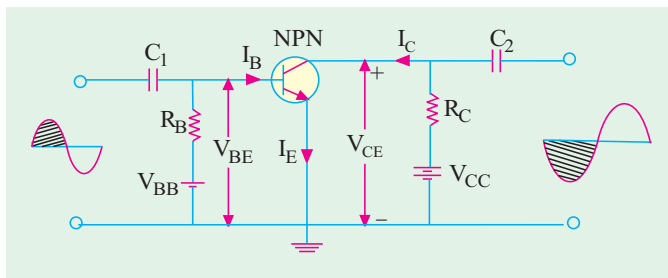


Fig. 60.4

dc power for both base and collector as in Fig. 60.5). The Q-point or working condition is determined by  $V_{CC}$  together with  $R_B$  and  $R_C$ . The dc equation is (Fig. 60.5).

$$I_B \cong V_{BB}/R_B \quad \text{and} \quad V_{CE} = V_{CC} - I_C R_C \quad \text{---neglecting } V_{BE}$$

$$I_C = \beta I_B$$

Now, let us see what happens when an ac signal is applied at the input terminals of the circuit.



**Circuit Operations**

When positive half-cycle of the signal is applied (Fig. 60.4)

1.  $V_{BE}$  is **increased** because it is already positive w.r.t. the ground as per biasing rule of Art 6.3.
2. it leads to increase in forward bias of base-emitter junction
3.  $I_B$  is **increased** somewhat
4.  $I_C$  is increased by  $\alpha$  times the **increased** in  $I_B$ .
5. drop  $I_C R_C$  is **increased** considerably and consequently.
6.  $V_{CE}$  is decreased as seen from the equation given above.

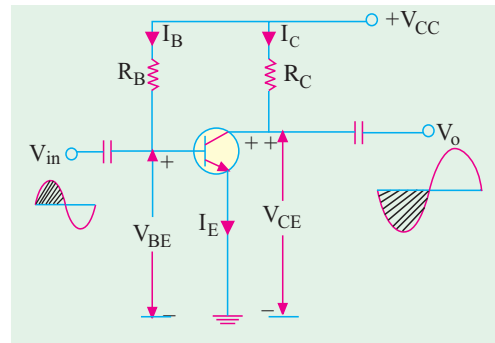


Fig. 60.5

Hence, negative half-cycle of the output is obtained. It means that a positive-going input signal becomes a negative going output signal as shown in Fig. 60.4 and 60.5.

**60.6. Various Gains of a CE Amplifier**

The ac equivalent of the given circuit (Fig. 60.5) is similar to the one shown in Fig. 60.6 (b).

**1. Input Resistance**

When viewed from base, ac resistance of the emitter junction is  $\beta r_e$ . As seen, from Fig. 60.6 (b), circuit input resistance is

$$r_{in}' = R_B \parallel \beta r_e \quad \text{---remember } \beta\text{-rule}$$

$$\cong \beta r_e \quad \text{---when } R_B \gg \beta r_e$$

It is called input resistance of the stage i.e.  $r_{in(stage)}$ .

**2. Output Resistance** :  $r_o = R_C$  ---Fig. 60.5

However, if a load resistor  $R_L$  is connected across the output terminals (Fig. 60.6), then

$$r_o = R_C \parallel R_L = r_L \quad \text{--- Fig. 60.6 (b)}$$

It is called output resistance of the stage and is written as  $r_{o(stage)}$  or  $r_o'$ .

**3. Current Gain** :  $A_i = \beta$

**4. Voltage Gain** :  $A_v = \beta \cdot \frac{r_o'}{r_{in}'} = \beta \cdot \frac{r_o}{\beta r_e} \cong \frac{r_o'}{r_e}$  ---if  $R_B \gg \beta r_e$

It is the stage voltage gain,

**5. Power Gain** :  $A_p = A_v \cdot A_i = \beta \cdot \frac{r_o'}{r_e}$  ;  $G_p = 10 \log_{10} A_p \text{ dB}$

**60.7. Characteristics of a CE Amplifier**

A CE transistor amplifier has the following characteristics :

1. it has moderately low input resistance (1 K to 2 K),
2. its output resistance is moderately large (50 K or so),
3. its current gain ( $\beta$ ) is high (50–300),
4. it has very high voltage gain of the order of 1500 or so,
5. it produces very high power gain of the order of 10,000 times or 40 dB,
6. it produces **phase reversal** of input signal i.e. input and output signals are  $180^\circ$  out of phase with each other.

**Uses**

Most of the transistor amplifiers are of CE type because of large gains in voltage, current and



power. Moreover, their input and output impedance characteristics are suitable for many applications.

**Example. 60.2.** For the single-stage CE amplifier circuit shown in Fig. 60.6 (a), calculate (a)  $r_{in}$  (b)  $r_o$  (c)  $A_i$  (d)  $A_v$  and (e)  $G_p$ . Take transistor  $\beta = 50$ . Neglect  $V_{BE}$  and take  $r_e = 25 \text{ mV}/I_E$ .

**Solution.**  $I_B = \frac{20}{1\text{M}} = 20 \mu\text{A}; \quad I_C = \beta I_B = 50 \times 20 = 1 \text{ mA}$

$r_e = 25/1 = 25 \Omega, \quad \beta r_e = 50 \times 25 = 1250 \Omega$

The ac equivalent circuit is shown in Fig. 60.6 (b).

(a)  $r_{in}' = 1\text{M} \parallel 1250 \Omega \cong 1250 \Omega$

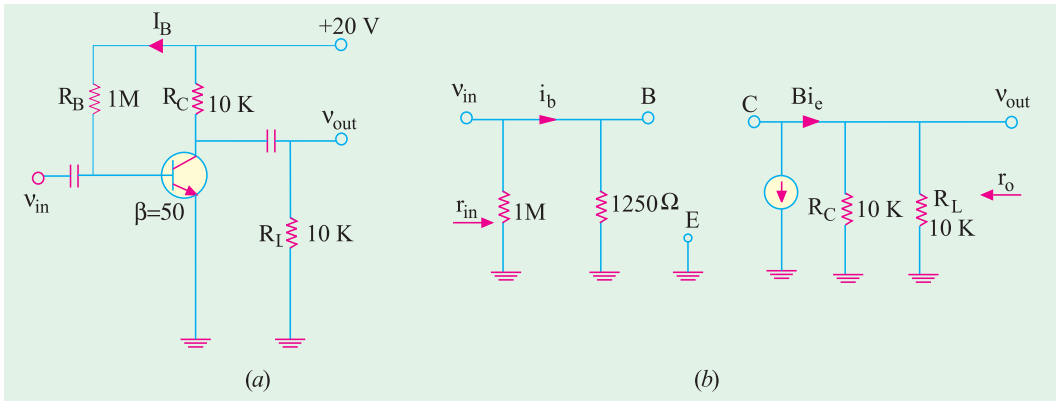


Fig. 60.6

Obviously, it is the input resistance of the stage and not that of the transistor alone.

(b)  $r_o' = R_C \parallel R_L = 10\text{K} \parallel 10\text{K} = 5\text{K}$

(c)  $A_i = 50$

(d)  $A_v = \frac{5\text{K}}{25\Omega} = 200$

(e)  $G_p = 10 \log_{10} 200 = 23 \text{ dB}$

### 60.8. Common Collector (CC) Amplifier

Fig. 60.7 and 60.8 show the circuit of a single-stage CC amplifier using an NPN transistor. The input signal is injected into the base-collector circuit and output signal is taken out from the emitter-collector circuit. The E/B junction is forward-biased by  $V_{EE}$  and C/B junction is reverse-biased by  $V_{CC}$ . The quiescent values of  $I_B$  and  $I_E$  are set by  $V_{CC}$  and  $V_{EE}$  together with  $R_B$  and  $R_E$ . As seen from Fig. 60.8.

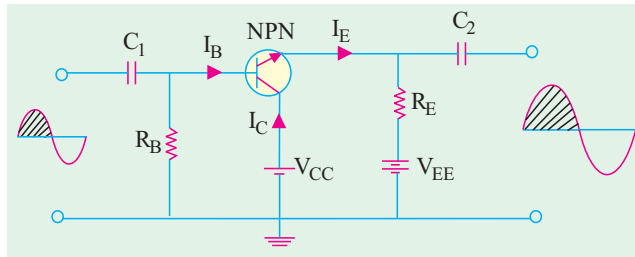


Fig. 60.7

$$I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B / \beta} \cong \frac{V_{EE}}{R_E + R_B / \beta}$$

Let us now see what happens when an ac signal is applied across the input circuit.

#### Circuit Operation

When positive half-cycle of the signal is applied, then



1. forward bias is **increased** since  $V_{BE}$  is positive w.r.t. collector i.e. ground,
2. base current is **increased**,
3. emitter current is **increased**,
4. drop across  $R_E$  is **increased**,
5. hence, output voltage (*i.e.* drop across  $R_E$ ) is **increased**.

Consequently, we get positive half-cycle of the output.

It means that a **positive-going** input signal results in a **positive going** output signal and, consequently, the input and output signals are in phase with each other as shown in Fig. 60.8.

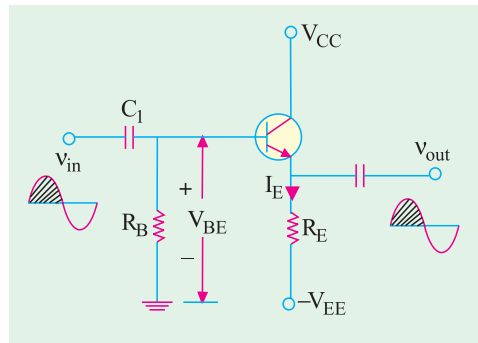


Fig. 60.8

### 60.9. Various Gains of a CC Amplifier

The ac equivalent circuit of the CC amplifier (Fig. 60.8) is given in Fig. 60.9.

1.  $r'_{in} = R_B \parallel \beta (r_e + r_o)$
2.  $r'_o = R_E \parallel R_L$
3.  $A_i = \beta$
4.  $A_v = \frac{r'_o}{r'_o + r_e}$

Since usually  $r'_o \gg r_e$ ,  $A_v \cong \frac{r'_o}{r'_o} = 1$

$$A_p = A_v A_i,$$

$$G_p = 10 \log_{10} A_p \text{ dB}$$

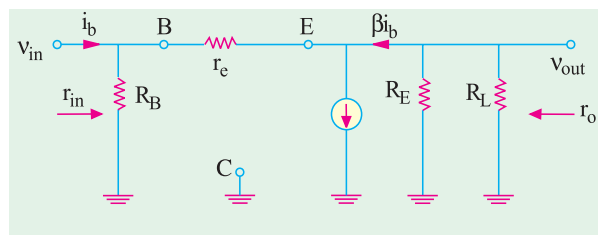


Fig. 60.9

### 60.10. Characteristics of a CC Amplifier

A CC amplifier has the following characteristics :

1. high input impedance (20-500 K),
2. low output impedance (50-1000  $\Omega$ ),
3. high current gain of  $(1 + \beta)$  *i.e.* 50 – 500,
4. voltage gain of less than 1,
5. power gain of 10 to 20 dB,
6. no phase reversal of the input signal.

### 60.11. Uses

The CC amplifiers are used for the following purposes :

1. for impedance matching *i.e.* for connecting a circuit having high output impedance to one having low input impedance;
2. for circuit isolation;
3. as a two-way amplifier since it can pass a signal in either direction;
4. for switching circuits.

**Example 60.3.** For the CC amplifier circuit of Fig. 60.10 (a), compute

- (i)  $r'_o$  (ii)  $r'_{in}$  (iii)  $A_v$  and (iv)  $A_p$

Take transistor  $\beta = 100$ . Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$

(Electronics-I, M.S. Univ. 1991)

**Solution.**  $I_E = \frac{20}{20} = 1 \text{ mA}; \quad r_e = \frac{25}{1} = 25 \Omega$

The ac equivalent circuit is shown in Fig. 60.10 (b).



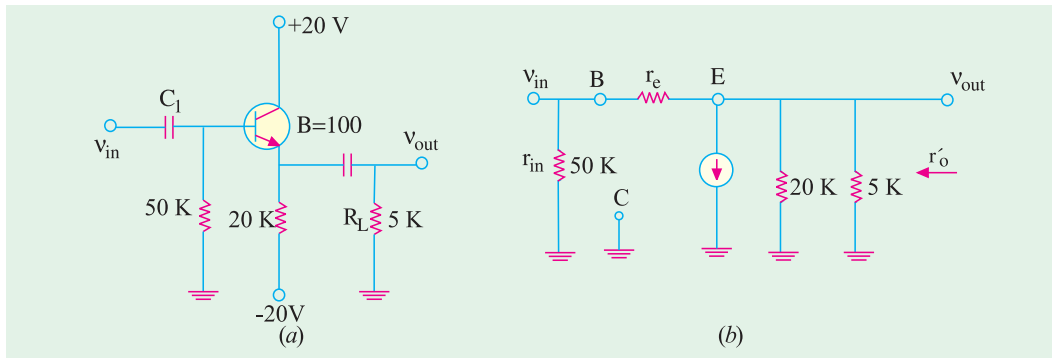


Fig. 60.10

- (i)  $r_o' = R_E \parallel R_L = 20\text{K} \parallel 5\text{K} = 4\text{K}$
- (ii)  $r_{in}' = R_B \parallel \beta (r_e + r_o) \cong R_B \parallel \beta r_o' = \frac{50 \times 400}{450} = 44.4\text{K}$
- (iii)  $A_v = \frac{4k}{4k + 25\Omega}$
- (iv)  $A_p = A_v \cdot A_i = 1 \times 100 = 100$

### 60.12. Comparison of Amplifier Configurations

The basic building blocks of transistor amplifiers are single-stage common-base, common-emitter and common-collector circuits. The choice of configuration and type of transistor for a given application will depend largely upon the desired input and output impedances, voltage, current and power gains and frequency response. For convenience, these values are tabulated in Table No. 10.1. Here the input and output parameters are resistances and apply only to *low-frequency conditions*.

Table No. 60.1			
Type of Circuit			
Characteristic	Common base	Common emitter	Common collector
Current gain	nearly unity ( $\alpha$ )	high ( $\beta$ )	highest ( $1 + \beta$ )
Voltage gain	high	very high	nearly unity
Power gain	moderate	highest	lowest
Input impedance	lowest	moderate	highest
Output impedance	highest	moderate	lowest
Phase reversal	No	Yes	No

In each case, the amplifying action depends on low-power input circuits controlling the high-power output circuits.

In the common-base circuit, although the input and output currents are nearly equal, the low-impedance emitter circuit absorbs for less power as compared to that which is available at the high-impedance collector.

The low base current flowing into the common-emitter circuit (where the impedance is a few kilohms) gives rise to much higher collector current flowing out of the high-impedance output circuit.

The common-collector circuit with approximately equal input and output resistances requires a low input current to control much larger output current.





### 60.13. Amplifier Classification Based on Biasing Conditions

This classification is based on the amount of transistor bias and amplitude of the input signal. It takes into account the portion of the cycle for which the transistor conducts. The three main classifications are :

#### (a) Class-A Amplifier

In this case, the transistor is so biased that output current flows for the full-cycle of the input signal ( $360^\circ$ ) as shown in Fig. 60.11 (a). In other words, the transistor remains *FR*-biased throughout the input cycle. Hence, its conduction angle is  $360^\circ$ .

#### (b) Class-B Amplifier

In this case, the transistor bias and the amplitude of input signal are such that output current flows for only half-cycle ( $180^\circ$ ) of the input signal. It means that transistor stays *FR*-biased for half the input cycle. The transistor conduction angle equals  $180^\circ$ .

#### (c) Class-C Amplifier

In this case, transistor bias and signal amplitude are such that output current flows for appreciably less than half-cycle of the input signal *i.e.* upto  $120^\circ$  or  $150^\circ$  angle of conduction as shown in 60.11 (c). In other words, transistor remains *FR*-biased for less than half the cycle.

#### (d) Class-AB Amplifier

The characteristics of such an amplifier lie in-between those of class-A and class-B. Here, biasing conditions are such that output current flows for appreciably more than half but less than the entire cycle *i.e.* current flows for more than  $180^\circ$  but less than  $360^\circ$ .

### 60.14. Graphic Representation

Since common-emitter is the most versatile and widely-used configuration, we will use CE output characteristic curves to differentiate between main classes of amplifiers.

Fig. 60.12 shows the biasing condition for class-A operation. It is seen that *Q*-point is located at the centre of the load line so that output (collector) current flows for the complete cycle of the input signal (conduction angle of  $360^\circ$ ). Because of centred *Q*-point, the positive and negative swings of the input signal are confined to linear portions of the load line. In this linear region, equal changes in input base current produce equal changes in output (collector) current and voltage. Hence, output is an exact replica of the input. If the amplitude of the input signal is so large as to drive the *Q*-point closer to either cut-off or saturation region (where base current lines are not equally spaced), the output

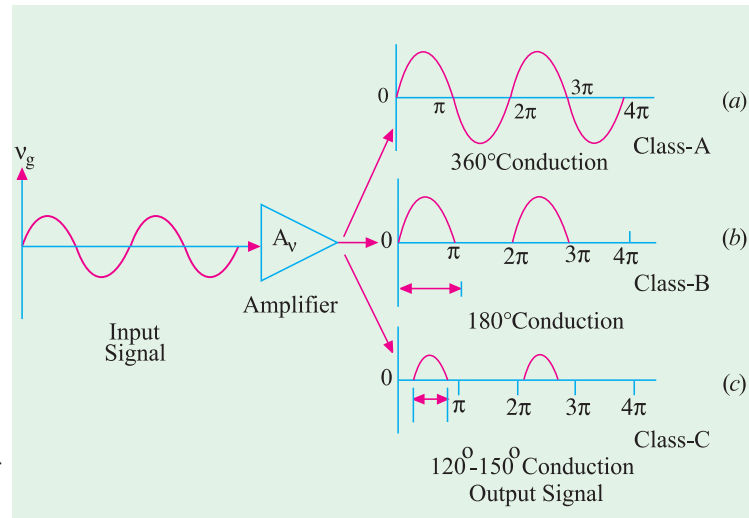


Fig. 60.11

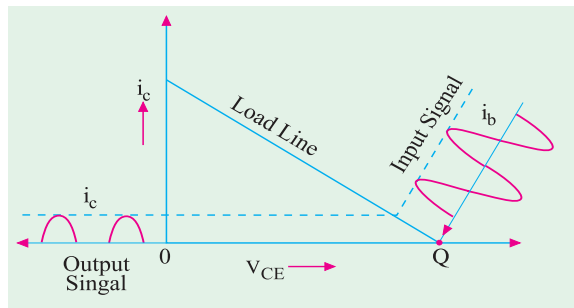


Fig. 60.12

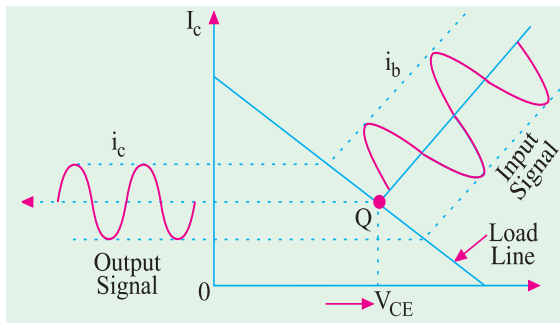


Fig. 60.13

will not be an exact replica of the input *i.e.* output signal will be distorted (Art. 60.30).

Fig. 60.13 shows biasing conditions for class-B operation. In this case, *Q*-point is located at cut-off point or very close to it. It is seen that the output current only flows for positive half-cycles of the input signal. For negative half-cycles, there is no output (collector) current. Obviously, output current flows only for 180° during the positive half-cycles of the input signal.

Fig. 60.14 shows biasing conditions for class-C operation. Here, the transistor is biased well into cut-off. In this case, (collector) current flows only for a part (upto 150°) of the positive half-cycle of the input signal when transistor comes out of the cut-off. During negative half-cycles of the input, the transistor remains deep in cut-off.

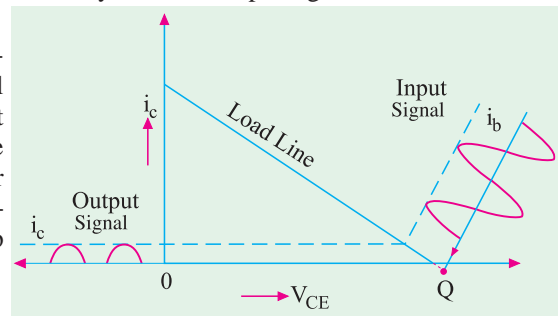


Fig. 60.14

### 60.15. Class-A Amplifier

It is one which has centred *Q*-point so that the transistor operates only over the linear region of its load line.

Another way of defining is that it is an amplifier in which the output current flows during the entire cycle of the input signal *i.e.* it has a conduction angle of 360°. In other words, the transistor remains FR-biased throughout the input cycle.

#### Characteristics

1. Since the transistor operates over the linear portion of the load line, the output waveform is exactly similar to the input waveform. Hence, class-A amplifiers are characterised by a high fidelity of the output. They are used where linearity or freedom from distortion is the prime requisite.
2. Since its operation is restricted only over a small central region of the load line, this amplifier is meant only for amplifying input signals of small amplitude. Large signals, will shift the *Q*-point into non-linear regions near saturation or cut-off and produce distortion (Art. 60.30).
3. Due to the limitation of the input signal amplitude, ac power output per active device (*i.e.* transistor) is small.
4. The overall efficiency of the amplifier circuit is

$$= \frac{\text{ac power delivered the load}}{\text{total power delivered by dc supply}} = \frac{\text{average ac power output}}{\text{average dc power input}}$$

5. The collector efficiency of a transistor is defined as

$$= \frac{\text{average ac power output}}{\text{average dc power input to transistor}}$$

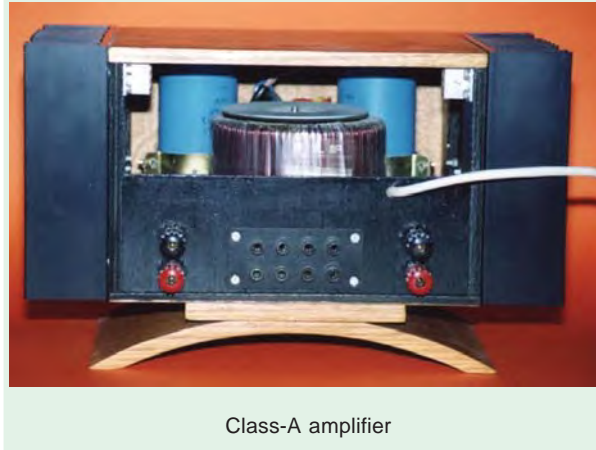
The maximum possible collector efficiency of a class – A amplifier with resistive load is 50%



- 6. In case an output transformer is used, the maximum possible overall efficiency and maximum possible collector efficiency for a class-A amplifier are both 50%.

**60.16. Power Distribution in a Class-A Amplifier**

Fig. 60.15 (a) shows a CE connected transistor which forms the active element of a single-stage class-A amplifier. Fig. 60.15 (b) shows its output characteristic with a centred Q-point. When ac input signal is applied, Q-point shifts up and down from its central position. The output current will also increase or decrease from its quiescent (or no-signal) value  $I_{CQ}$ . Similarly, collector-emitter voltage  $V_{CE}$  will increase or decrease from its quiescent value  $V_{CEQ}$ . So long as signal variations are confined to linear region of the load line, average value of collector current is  $I_{CQ}$  because positive and negative input signal swings will produce equal changes in  $I_{CQ}$ .



Class-A amplifier

Hence, total average dc power drawn by the circuit from collector battery  $V_{CC}$  is

$$P_{in(dc)} = V_{CC} \cdot I_{CQ}$$

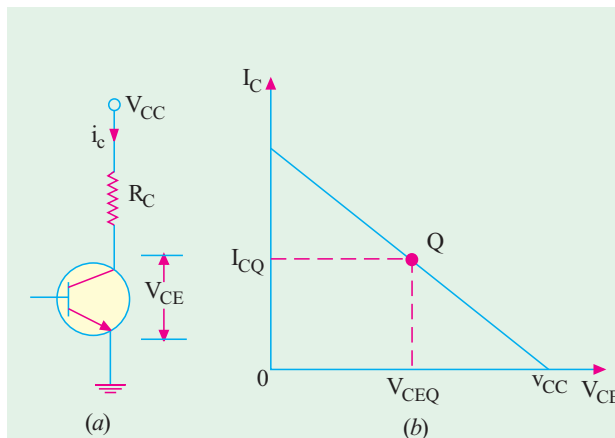


Fig. 60.15

Now, this power goes to supply the following :

- (i) heat dissipated by the load resistor  $R_C$  connected to the collector
- (ii) the balance  $P_{tr(dc)}$  is given to the transistor.

It is further subdivided into :

- (a) ac power developed across the load resistor which constitutes the ac power output

$$P_{o(ac)} = I^2 R_C = \frac{V^2}{R_C} = \frac{V_m^2}{2R_C}$$

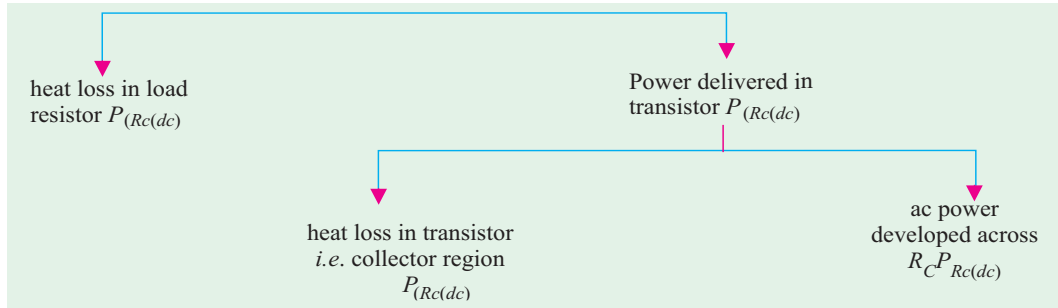
where  $I$  is the rms value of the ac output current through the load,  $V$  is rms value of ac voltage and  $V_m$  is its maximum value.

- (b) power dissipated (in the form of heat) by the transistor itself *i.e.* its collector region. It may be called  $P_{c(dc)}$ .

Since, under zero-signal condition, there is no ac output power, all the power given to the transistor is wasted as heat. Hence, **a transistor dissipates maximum power under zero-signal condition.**

The power flow diagram of the transistor is as follows :





**60.17. Power Rectangle**

The output or collector characteristic of a CE-connected transistor is shown in Fig. 60.16 with centred  $Q$ -point for class-A operation. When input signal is applied, the  $Q$ -point shifts to positions  $Q_1$  and  $Q_2$  alternately. The output current varies around its quiescent value from maximum value of  $I_{C(max)}$  to minimum value  $I_{C(min)}$ . Similarly, collector-emitter voltage varies from maximum value of  $V_{CE(max)}$  to  $V_{CE(min)}$  around its quiescent value of  $V_{CEQ}$ .

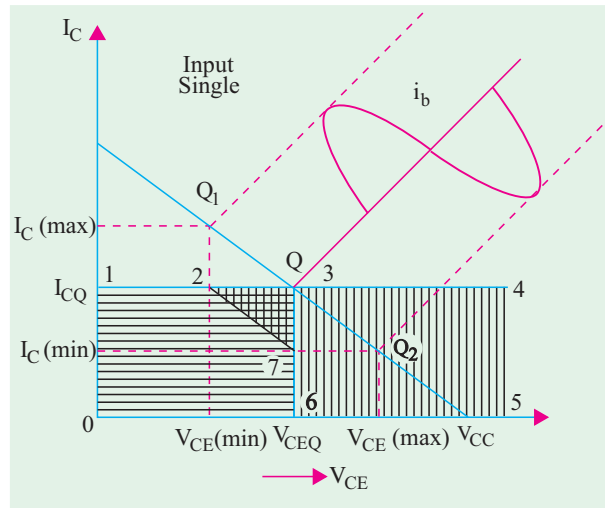


Fig. 60.16

Let us construct the various rectangles as shown in Fig. 60.16.

1. rectangle 0-1-4-5  
 $= V_{CC} \times I_{CQ}$   
 $=$  total average power supplied to the circuit by  $V_{CC}$  battery  
 $= P_{in(dc)}$

Now,  $V_{CC} - V_{CEQ}$   
 $=$  voltage drop across load resistor  $R_C$

2. rectangle 3-4-5-6  $= (V_{CC} - V_{CEQ}) I_{CQ}$   
 $=$  power lost as heat in load resistor  $R_C = I_{CQ}^2 R_C$

Now,  $V_{CEQ}$  is the voltage drop across the transistor itself *i.e.* potential difference between its collector and emitter as shown in Fig. 60.15 (b).

3. rectangle 0-1-3-6  $= V_{CEQ} \cdot I_{CQ} =$  power delivered to transistor  $= P_{tr(dc)}$
4. triangle 2-3-7  $=$  ac power across  $R_C = P_{o(ac)} = I^2 R_C$

It can also be proved that

$$P_{o(ac)} = \frac{V_{CE(max)} - V_{CE(min)}}{2\sqrt{2}} \times \frac{I_{C(max)} - I_{C(min)}}{2\sqrt{2}}$$

$$= \frac{[V_{CE(max)} - V_{CE(min)}] \times [I_{C(max)} - I_{C(max)}]}{8}$$

5. area 0-1-2-7-6  $=$  power dissipated by the collector region of the transistor  
 $= P_{c(dc)}$

Obviously, when there is no input signal, there is no output signal. Hence,  $P_{o(ac)} = 0$ , which means that area of triangle 2-3-7 is zero. In that case, the transistor collector will have to dissipate



maximum power equal to the area of the rectangle 0-1-3-6. It is called **worst-case** condition of the transistor and equals quiescent power.

### 60.18. Power Efficiency

Fig. 60.17. shows the power rectangles and triangles of a class-A amplifier in a very simple way.

$$P_{in(dc)} = \text{areas } A + B + C$$

$$P_{o(ac)} = \text{area } B$$

$$P_{c(dc)} = \text{area } C$$

$$P_{Rc(dc)} = \text{area } A$$

Amplifier efficiency or circuit efficiency or overall efficiency.

$$\eta_{overall} = \frac{B}{A+B+C} = \frac{P_{o(ac)}}{P_{in(dc)}}$$

The collector efficiency is given by

$$\eta_{overall} = \frac{B}{B+C} = \frac{P_{o(ac)}}{P_{tr(dc)}}$$

Incidentally, it can be proved that  $\eta_{overall} = 25 \left[ \frac{V_{CE(max)} - V_{CE(min)}}{V_{CC}} \right]$

Now, maximum value of  $V_{CE} = V_{CC}$  and minimum value is = 0

$$\therefore \text{maximum } \eta_{overall} = 25 \left[ \frac{V_{CC} - 0}{V_{CC}} \right] \% = 25\%$$

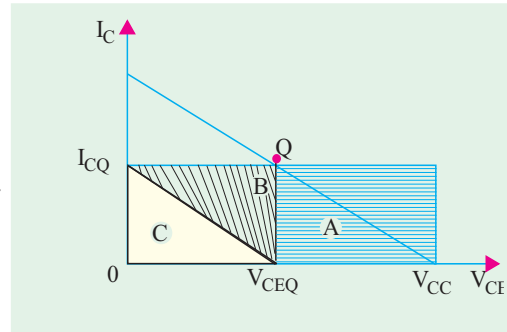


Fig. 60.17

No Input Signal		Max. Input Signal	
DC Power dissipated in Load		DC Power dissipated in Load	
DC Power dissipated in Transistor		Average AC Power	Average Power
		Power	Dissipated in Transistor

Fig. 60.18

### Summary

In a direct-coupled class-A amplifier

1. maximum ac power equals half the power transistor can dissipate (or one-fourth of the total dc input power);
2. maximum ac power the transistor can dissipate is half the dc input power supplied by the battery;
3. the dc input power equals 4 times the ac output in load or twice the maximum power dissipated by the transistor.

The above facts have been illustrated with the help of power square shown in Fig. 60.18.



60.19. Maximum AC Power in Load

Fig. 60.19 (a) shows part of a base-driven CE amplifier with a collector load of  $R_C$ . Fig. 60.19 (b) shows its ac load line with centered Q-point. For the time being, we will neglect limitations of cut-off and saturation and assume (at least theoretically) that voltage and current swings down to zero are possible. Obviously, under such conditions, we will get the maximum possible ac output power.

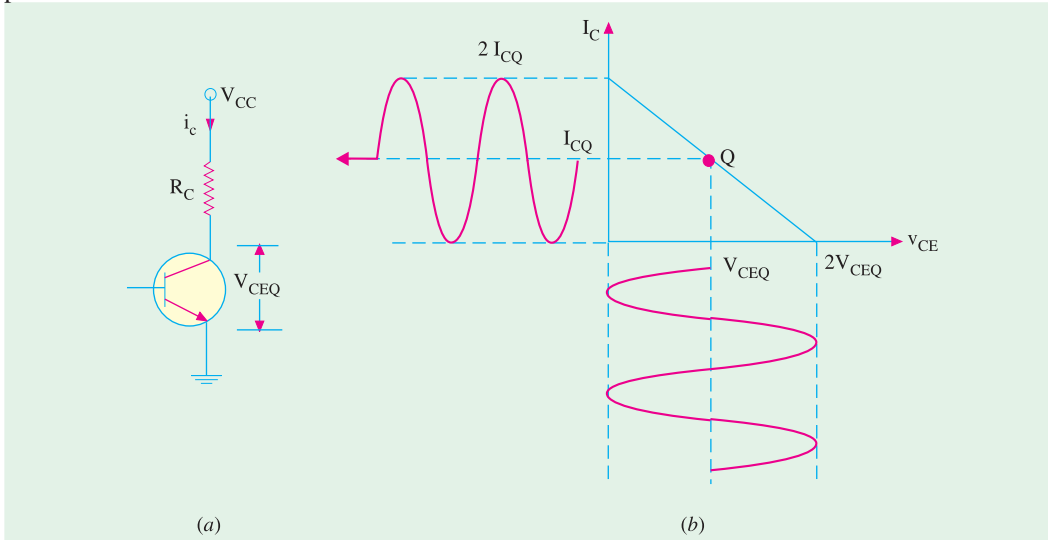


Fig. 60.19

As seen, collector current is a sine wave having peak-to-peak value of  $2 I_{CQ}$  or a maximum value of  $I_{CQ}$  or an rms value of  $I_{CQ} / \sqrt{2}$ . Since it passes through  $R_C$ , it delivers an ac power of

$$P_{o(ac)} = (I_{CQ} / \sqrt{2})^2 R_C = 0.5 I_{CQ}^2 R_C$$

Similarly, ac collector-emitter voltage is a sine wave of peak-to-peak value  $2 V_{CEQ}$  or peak value  $V_{CEQ}$  or rms value  $V_{CEQ} / \sqrt{2}$ . Since this sine wave is applied across  $R_C$ , the ac power can also be expressed as

$$P_{o(ac)} = \left( \frac{V_{CEQ}}{\sqrt{2}} \right)^2 / R_C = \frac{0.5 V_{CEQ}^2}{R_C}$$

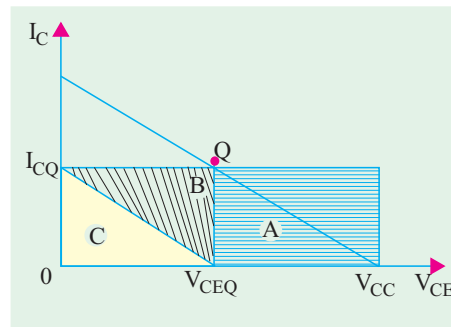


Fig. 60.20

**Note.** If there had been an unbypassed resistor  $R_E$  in the emitter, then ac load  $R_{ac}$  or  $r_L = R_C + R_E$ . Then, we would have used this value rather than  $R_C$  alone.

As seen from power rectangle diagram of Fig. 60.20, under maximum ac power condition

$$\text{area } B = \text{area } C$$

$$\text{Also, area } A = \text{area } B + \text{area } C = 2B$$

$$\therefore \text{maximum } \eta_{\text{overall}} = \frac{B}{A + B + C}$$

$$= \frac{B}{2B + 2B} = \frac{B}{4B} = 0.25 \text{ or } 25\%$$

**Example 60.4.** The 2N 1491 power transistor is used in a CE amplifier meant for class-A operation. If its zero-signal power dissipation is 10 W and ac output power is 3.5 W, find (a) collector efficiency and (b) power rating of the transistor



**Solution. (i)**  $\eta_{coll} = \frac{P_{o(ac)}}{P_{tr(dc)}} = \frac{3.5}{10} = 0.35 = 35\%$

**(ii)** Since zero-signal condition represents worst case condition for the transistor, it means that all the 10 W power is dissipated by it. Hence, transistor power rating is **10 W**.

**Example 60.5.** For the class-A, CE amplifier circuit of Fig. 60.21,  $V_{CEQ} = 10\text{ V}$  and  $I_{CQ} = 500\text{ mA}$ . If collector i.e. output current varies by  $\pm 250\text{ mA}$  when an input signal is applied at the base, compute

- (i)** total dc power taken by the circuit, **(ii)** dc power dissipated by the collector load,  
**(iii)** ac power developed across the load, **(iv)** power delivered to the transistor;  
**(v)** dc power wasted in transistor collector, **(vi)** overall efficiency.  
**(vii)** collector efficiency.

**Solution. (i)**  $P_{in(dc)} = V_{CC} \cdot I_{CQ} = 20 \times 500$   
 $= 10^4\text{ mW} = 10\text{ W}$

**(ii)**  $P_{Re(dc)} = I_{CQ}^2 R_C = 0.5^2 \times 20 = 5\text{ W}$

**(iii)**  $P_{o(ac)} = P_{Re(dc)}$

Now, maximum value of output ac current is  $250\text{ mA} = 0.25\text{ A}$ .

Hence, rms value  $I = 0.25/\sqrt{2}\text{ A}$ .

$\therefore P_{o(ac)} = (0.25/\sqrt{2})^2 \times 20 = 0.625\text{ W}$

**(iv)**  $P_{tr(dc)} = 10 - 5 = 5\text{ W}$

**(v)**  $P_{c(dc)} = 5 - 0.625 = 4.375\text{ W}$

**(vi)**  $\eta_{overall} = \frac{0.625}{5} \times 100 = 6.25\%$

**(vii)**  $\eta_{coll} = \frac{0.625}{5} \times 100 = 12.5\%$

It is seen that due to resistive load in the collector, efficiency of the circuit is very low.

**Example 60.6.** For a class-A amplifier,  $V_{CE(max)} = 25\text{ V}$ ,  $V_{CE(min)} = 5\text{ V}$ ,  $V_{CC} = 30\text{ V}$ . Find its overall efficiency.

**Solution.** As seen from Art. 10.18

$$\% \eta_{overall} = 25 \left[ \frac{V_{CE(max)} - V_{CE(min)}}{V_{CC}} \right] = 25 \left( \frac{25 - 5}{30} \right) = 16.7$$

## 60.20. Transformer-Coupled Class-A Amplifier

The main reason for the poor efficiency of a **direct-coupled** class-A amplifier is the large amount of dc power that the resistive load in collector must dissipate. This problem can be solved by using a suitable transformer for coupling the load (say, a speaker) to the amplifier stage as shown in Fig. 60.22. Since the load is not **directly** connected to the collector terminal, the dc collector current does not pass through it. In an ideal transformer, primary winding resistance is zero. Hence, dc power loss in the load is zero. In practice, however, there is a small dc resistance of the primary winding which does absorb some power though much less than a direct-coupled load.

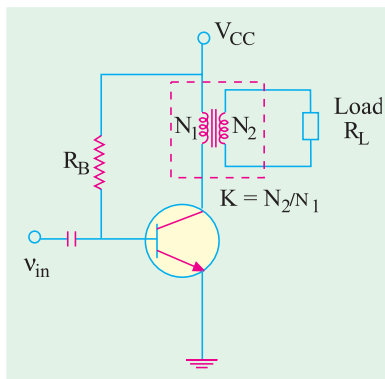


Fig. 60.22

In short, what the transformer does is to substitute **ac load** in place of **ohmic** or dc load.

The secondary load  $R_L$  when referred to primary becomes



$$R_L' = R_L / K^2 = a^2 R_L$$

where  $K = \text{voltage transformation ratio} = N_2/N_1 = V_2/V_1$   
 $a = \text{turns ratio } N_1/N_2 = 1/K$

Since  $a$  is usually made much more than unity or  $K$  is much less than unity,  $R_L'$  can be made to look much bigger than what it actually is

In an ideal transformer, there is no primary drop, hence  $V_{CC} = V_{CEQ}$ . Now, all the power supplied by  $V_{CC}$  is delivered to the transistor. Hence, the overall and collector efficiencies become equal.

$$\eta_{\text{overall}} = \frac{P_{o(ac)}}{V_{CC} I_{CQ}} = \frac{P_{o(ac)}}{V_{CEQ} \cdot I_{CQ}}$$

**Power Diagram**

The power distribution for a transformer-coupled class-A amplifier is shown in Fig. 60.23. Since load loss is zero, the triangle A of Fig. 60.17 is non-existent. Moreover, the load line of Fig. 60.17 becomes a.c. load line in Fig. 60.23. Consequently,  $V_{CEQ} = V_{CC}$ . Area of triangle B equals that of triangle C. Also, sum of A and B represents power delivered to the transistor.

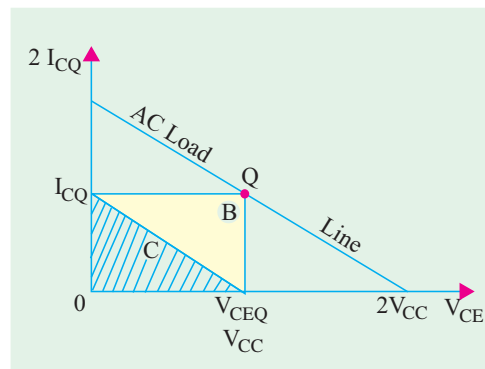


Fig. 60.23

$$\therefore \eta_{\text{overall}} = \frac{B}{B+C} = \frac{B}{2B} = 0.5 \text{ or } 50\%$$

It can be proved that maximum possible overall efficiency and the maximum possible collector efficiency of a class-A amplifier using an output transformer are both 50%.

In general, overall efficiency of a transformer-coupled class-A amplifier is given by

$$\% \eta = 50 \left[ \frac{V_{CE(max)} - V_{CE(min)}}{2V_{CC}} \right]^2 = 50 \left[ \frac{V_{CE(max)} - V_{CE(min)}}{V_{CE(max)} + V_{CE(min)}} \right]^2$$

Obviously, larger the value of  $V_{CE(max)}$  and smaller that of  $V_{CE(min)}$  closer the efficiency to the theoretical limit of 50%.

**Proof**

The conditions for the development of maximum ac power are shown in Fig. 60.24. It must be kept in mind that here

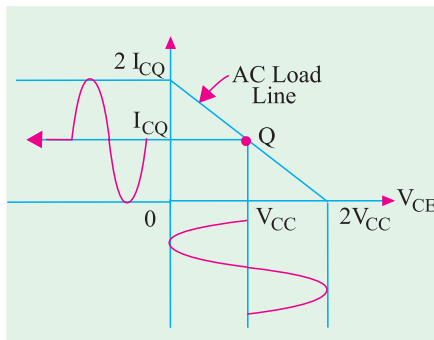


Fig. 60.24

Now, average power delivered by the battery

$$P_{in(dc)} = V_{CC} \cdot I_{CQ}$$

Whole of the power is given to the transistor.

rms output voltage =  $\frac{V_{CC}}{\sqrt{2}}$

rms output current =  $\frac{I_{CQ}}{\sqrt{2}}$

$$\therefore \text{maximum } P_{o(ac)} = \frac{V_{CC}}{\sqrt{2}} \times \frac{I_{CQ}}{\sqrt{2}} = \frac{1}{2} V_{CC} \cdot I_{CQ}$$

= half the input ac power





Hence, maximum value of overall efficiency

$$\eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} = \frac{V_{CC} \cdot I_{CC} / 2}{V_{CC} \cdot I_{CQ}} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

$$\eta_{coll} = \frac{P_{o(ac)}}{P_{tr(dc)}}$$

**Example 60.7.** The optimum load resistance for a certain transistor is 200  $\Omega$ . What is the turns ratio ( $N_1/N_2$ ) of a transformer required to couple an 8- $\Omega$  loud-speaker to the transistor ?

**Solution.**  $R_L' = a^2 R_L \quad \therefore \quad 200 = a^2 \times 8 \quad \text{or} \quad a = 5$

Hence, it should be a 5 : 1 step-down transformer.

**Example 60.8.** In a transformer-coupled class-A amplifier  $V_{CE(max)} = 27 \text{ V}$  and  $V_{CE(min)} = 3 \text{ V}$ . Compute its overall efficiency.

**Solution.**  $\eta_{overall} = 50 \left( \frac{27 - 3}{27 + 3} \right)^2 = 32\%$

**Example 60.9.** For the transformer-coupled optimally-biased class-A amplifier shown in Fig. 10.25, find

(a) transformer turns ratio      (b) collector current      (c) transistor power rating

**Solution.** As shown in Fig. 60.25, if  $V_2$  is the rms secondary voltage, then

$$P_{load} \text{ or } P_{o(ac)} = \frac{V_2^2}{R_L} = \frac{(V_{2m} / \sqrt{2})^2}{R_L} = \frac{V_{2m}^2}{R_L}$$

where  $V_{2m}$  is the maximum value of secondary voltage.

$$\therefore V_{2m} = \sqrt{2 R P_{load}} = \sqrt{2 \times 4 \times 4.5} = 6 \text{ V}$$

Now, peak, value of primary voltage is

$$V_{1m} = a \cdot V_{2m} = 6a$$

This also represents the peak value of collector voltage.

For optimally-biased circuit, peak value of collector-emitter voltage equals  $V_{CC} = 30 \text{ V}$ .

$$\therefore 6a = 30 \text{ or } a = 5 \quad \therefore \quad N_1/N_2 = 5$$

(a) Hence, it is a step-down transformer with a turns ratio of **5 : 1**

(b) The overall efficiency is 50%. Hence, for a load power of 4.5 W, total dc input power supply must be 4.5/0.5 = 9 W

$$\therefore V_{CC} I_C = P_{in(dc)} = 9 \text{ W} \quad \text{or} \quad 30 I_C = 9000 \text{ mW} \quad \therefore \quad I_C = 300 \text{ mA}$$

(c) Worst case condition for the transistor is when there is no input signal. In that case, transistor has to dissipate the entire input. Therefore, transistor power rating = **9W**.

**Example 60.10.** For the optimally-biased transformer-coupled class-A amplifier, maximum collector current change is 100 mA. Find the power transferred to a 4-W speaker load if it is (a) directly-coupled (b) transformer-coupled to the transistor. Find also the turn ratio of the transformer required. (Applied Electronics, Kerala Univ.)

**Solution.** (a) Direct-coupled Load [Fig. 60.26 (a)]

It should be remembered that for an optimally-biased class-A amplifier, peak change in collector-emitter voltage equals  $V_{CC} = 10 \text{ V}$ .

$$\text{RMS value of ac output current } I = (100 / \sqrt{2}) \text{ mA}$$

Average power delivered to the speaker

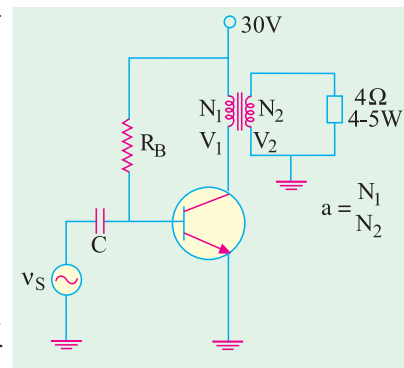


Fig. 60.25



$$= I^2 R_L = \left( \frac{100}{\sqrt{2}} \times 10^{-3} \right)^2 \times 4 = 0.02 \text{ W} = \mathbf{20 \text{ mW}}$$

**(b) Transformer-coupled Load**

Now,  $\Delta V_{CE} = V_{CC} = 10$

V :  $\Delta I_{CQ} = 100 \text{ mA}$

If  $R_L'$  is the secondary load reflected into the primary, then

$$R_L' = \frac{\Delta V_{CE}}{\Delta I_{CQ}} = \frac{10 \text{ V}}{100 \text{ mA}} = 100 \Omega$$

Now,  $a^2 R_L = R_L'$   
 $\therefore a^2 \times 4 = 100$ ,  
 $a = 5$

Hence, it is a step-down transformer with a turns ratio of 5 : 1

Peak value of secondary voltage =  $10 \times 1/5 = 2 \text{ V}$

Peak value of secondary current =  $2 / 4 = 0.5 \text{ A}$

RMS value =  $0.5 / \sqrt{2} \text{ A}$

$$\text{Average power transferred to the speaker} = \left( \frac{0.5}{\sqrt{2}} \right)^2 \times 4 = 0.5 \text{ W} = \mathbf{500 \text{ mW}}$$

Obviously, power transferred has increased  $500/20 = 25$  times i.e.  $a^2 = (N_1/N_2)^2$  times where  $a$  is transformer primary-to-secondary turn ratio i.e.  $a = N_1/N_2$ .

**Example 60.11.** A class-A amplifier shown in Fig. 60.27 operates from  $V_{CC} = 20 \text{ V}$ , draws a no-signal current of  $5 \text{ A}$  and feeds a load of  $40 \Omega$  through a step-up transformer of  $N_2/N_1 = 3.16$ . Find

- (a) whether the amplifier is properly matched for maximum power transfer,
- (b) maximum ac signal power output, (c) maximum dc power input,
- (d) conversion efficiency at maximum signal input.

(Electronic Circuits, Gauhati Univ. 1990)

**Solution.** (a) Here,  $a = N_1/N_2 = 1/3.16$   
 $R_L' = a^2 R_L = (1/3.16)^2 \times 40 = \mathbf{4 \Omega}$

Also,  $R_L' = \frac{V_{CC}}{I_{CQ}} = \frac{20}{5} = \mathbf{4 \Omega}$

Since, the two resistance values are the same, the amplifier is properly-matched.

(b) As seen from Art. 60.20, Maximum value of

$$P_{o(ac)} = \frac{1}{2} V_{CC} I_{CQ}$$

$$= \frac{1}{2} \times 20 \times 5 = \mathbf{50 \text{ W}}$$

(c) The dc power input  
 $= V_{CC} I_{CQ} = 20 \times 5 = \mathbf{100 \text{ W}}$

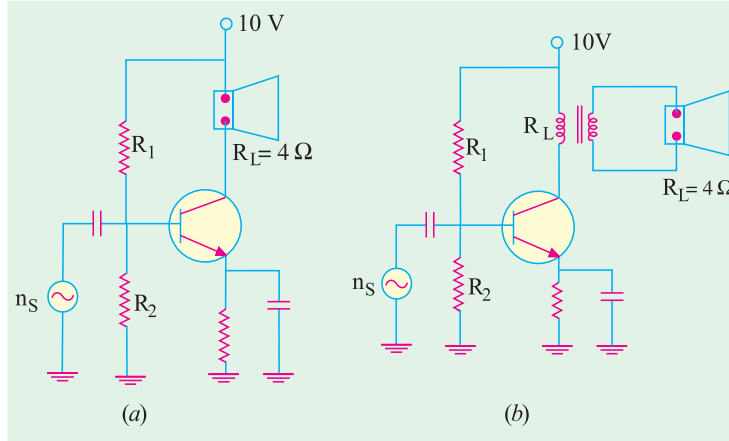


Fig. 60.26

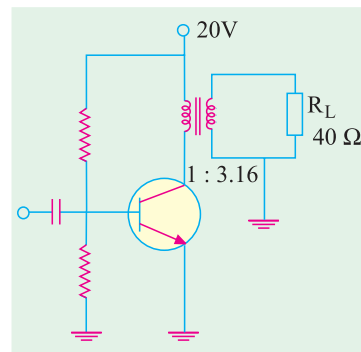


Fig. 60.27



$$(d) \quad \eta = \frac{50}{100} \times 100 = 50\%$$

### 60.21. Class-B Amplifier

Biasing condition for class-B operation has been shown both on current transfer characteristic and output characteristic for CE configuration in Fig. 60.28.

As seen, the transistor has been biased to cut-off. It remains FR-biased for only half-cycle of the input signal. Hence, its conduction angle is only  $180^\circ$ . It is obvious that with zero signal, its collector current is zero.

#### Characteristics

1. Since negative half-cycles are totally absent from the output, the signal distortion is high as compared to class-A amplifiers.
2. Since input voltage is large (upto  $V_{CC}$ ), voltage amplification is reduced.
3. Zero-signal input represents worst condition for class-A amplifiers but best condition for class-B amplifiers.
4. In class-B amplifiers, transistor dissipates more power with increase in signal strength but opposite is the case in class-A amplifiers.
5. Average current in class-B operation is less than in class-A, hence power dissipated is less. Consequently, maximum circuit (or overall) efficiency of a class-B amplifier is 78.5% when peak signal makes  $V_{CE(min)} = 0$ . In general

$$\eta_{overall} = 78.5 \left[ 1 - \frac{V_{CE(min)}}{V_{CC}} \right] \%$$

6. The half-sinewave type of collector current contains very pronounced *even harmonics* (particularly the second one). These impulses can be rendered useful either by employing push-pull circuit (Art. 60.24) or by using tuned amplifiers (Art. 60.29).

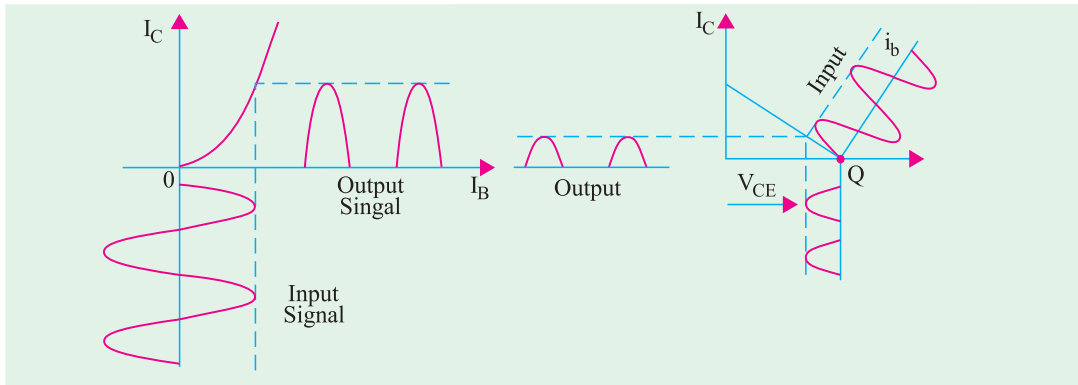


Fig. 60.28

### 60.22. Power Relations for Class-B Operation

(i) **Input DC Power** :  $P_{in(dc)} = V_{CC} I_{dc}$

where  $I_{dc}$  is the average or dc current drawn from the supply.

If  $I_{C(max)}$  is the maximum or peak value of collector or output current, then

$$I_{dc} = \frac{I_{C(max)}}{\pi} \quad \therefore \quad P_{in(dc)} = V_{CC} \frac{I_{C(max)}}{\pi}$$

(ii) **DC Power Loss in Load** :  $P_{Rc(dc)} = I_{dc}^2 R_C$  where  $I_{dc} = I_{C(max)}/\pi$



(iii) **AC Power Output in Load** :  $P_{o(ac)} = I^2 R_C = \frac{V^2}{R_C} = \frac{V_m^2}{2R_C}$

where  $I$  = rms value of output ac current  
 $V$  = rms value of output ac voltage  
 $V_m$  = maximum value of output ac voltage

(iv) **DC Power Loss in Collector Region or Transistor**

$$P_{c(d)} = P_{in(dc)} - P_{Rc(dc)} - P_{o(ac)}; \quad \eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}}$$

**60.23. Maximum Values**

The operation for maximum signal input is shown in Fig. 60.29.

Here,

$$I_{dc} = \frac{I_{C(max)}}{\pi} \quad \therefore P_{in(dc)} = V_{CC} \cdot I_{dc} = \frac{V_{CC} \cdot I_{C(max)}}{\pi}$$

RMS value of output or collector current

$$= \frac{I_{C(max)}}{\sqrt{2}}$$

RMS value of output voltage =  $\frac{V_{CC}}{\sqrt{2}}$

Hence, ac output power during half-cycle

$$= \frac{1}{2} \times \frac{V_{CC}}{\sqrt{2}} \frac{I_{C(max)}}{\sqrt{2}}$$

The factor  $\frac{1}{2}$  comes in because power is produced during one half-cycle only.

$$P_{o(ac)} = \frac{1}{4} \cdot V_{CC} \cdot I_{C(max)}$$

$$\therefore \eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} = \frac{\pi}{4} = 0.785 \text{ or } 78.5\%$$

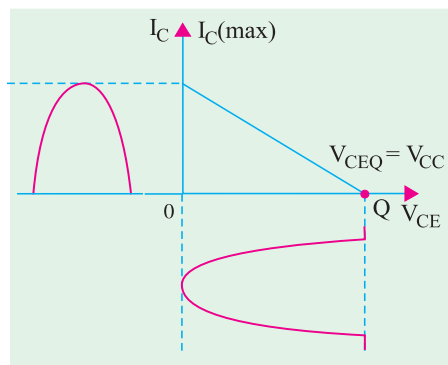


Fig. 60.29

**Example 60.12.** In a class-B amplifier,  $V_{CE(min)} = 2V$  and  $V_{CC} = 15V$ . Find its overall efficiency

**Solution.** As seen from Art 60.21,  $\% \eta = 78.5 \left(1 - \frac{2}{15}\right) = 68\%$

**60.24. Class-B Push-Pull Amplifier**

It employs two identical transistors operating as a *single-stage of amplification*. As shown in Fig. 60.30, the base of the two CE-connected transistors have been connected to the opposite ends of the secondary of the input transformer  $T_1$  and collectors to the opposite ends of the primary of the output transformer  $T_2$ . For getting a balanced circuit, the two emitters have been returned to the centre tap of  $T_1$  secondary and  $V_{CC}$  connected to the centre tap on the primary of  $T_2$ . Since zero bias is required for cut-off, the two bases have been earthed. A push-pull amplifier is sometimes referred to as *balanced amplifier*.



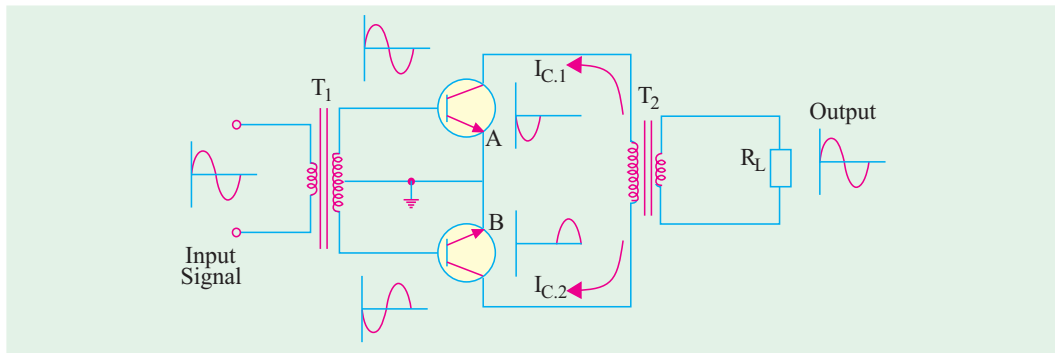


Fig. 60.30

### Principle of Operation

It is seen from Fig. 60.30 that transistor  $A$  and  $B$  are driven by two input signals which are  $180^\circ$  out of phase with each other. These two signals are produced by  $T_1$ .

Transistor  $A$  takes positive half-cycles of the signal whereas  $B$  handles negative half-cycles. When the two outputs are combined, an almost undistorted output waveform is produced as seen from Fig. 60.31.

In Fig. 60.31, the transfer characteristic of transistor  $B$  has been plotted upside down with respect to that of  $A$  in order to get the combined output.

Detailed operation is as under :

During the positive half-cycle of the signal,  $A$  is turned ON because its base is driven positive. It draws collector current  $I_{C-1}$  in the upward direction from  $V_{CC}$ . Meanwhile, transistor  $B$  remains OFF because its base has negative voltage. Hence,  $I_{C-2} = 0$ . Obviously, one positive half-cycle of the output signal appears across secondary load  $R_L$  of  $T_2$ .

During negative half-cycle of the input signal,  $B$  conducts whereas  $A$  remains OFF. Hence,  $I_{C-2}$  is taken by  $B$  but  $I_{C-1} = 0$ . Now, negative output half-cycle is produced across  $R_L$  because  $I_{C-2}$  is pulled down through secondary of  $T_2$ . It is obvious that in the absence of input signal, neither  $A$  nor  $B$  draws any collector current. Hence, there is no drain on  $V_{CC}$  battery.

### Advantages

1. It has high efficiency, theoretical limit being 78.5%. It is primarily due to the fact that there is **no power drawn by the circuit under zero-signal condition**.
2. Since in push-pull arrangement,  $180^\circ$  phase difference exists between even-order harmonics produced by each transistor, they cancel out thereby giving an almost distortion-free output. This automatic cancellation of all even-order harmonics from the output current makes class-B push-pull amplifiers highly desirable for **communication sound equipment**.
3. This double-ended class-B amplifier provides practically four times the power supplied by a single-ended amplifier provided signal load resistance remains the same.

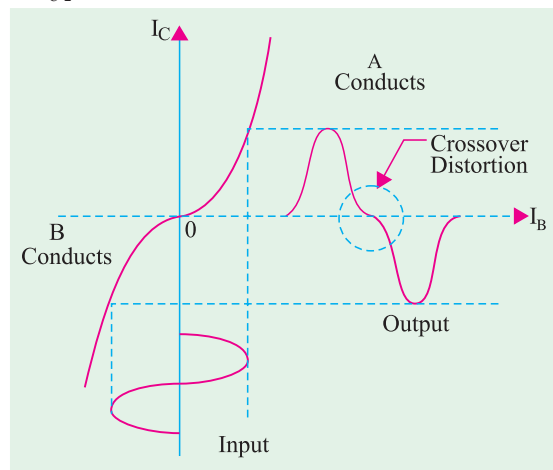


Fig. 60.31



4. The dc components of the two collector currents through the two halves of the primary of  $T_2$  flow in **opposite directions** so that net dc magnetisation of the transformer core is almost nil. Hence, distortions due to the effect of dc magnetic saturation of the transformer core are eliminated. Moreover, smaller size core can be used due to very small net core flux.

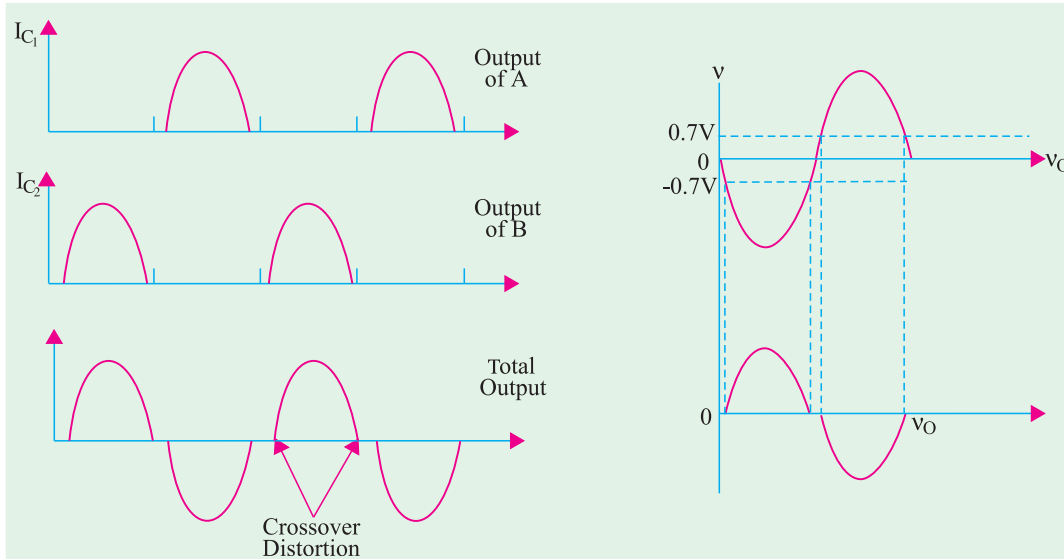


Fig. 60.32

Fig. 60.33

### Uses

Class-B push-pull amplifiers are extensively used for audio work in portable record players, as stereo amplifiers and in high-fidelity radio receivers.

### 60.25. Crossover Distortion

In class-B push-pull operation, there is severe distortion at very low signal level because

- (a) bases of the transistors do not turn ON at 0 V but at 0.3 V for Ge and 0.7 V for Si
- (b) there is non-linearity in the low-signal area.

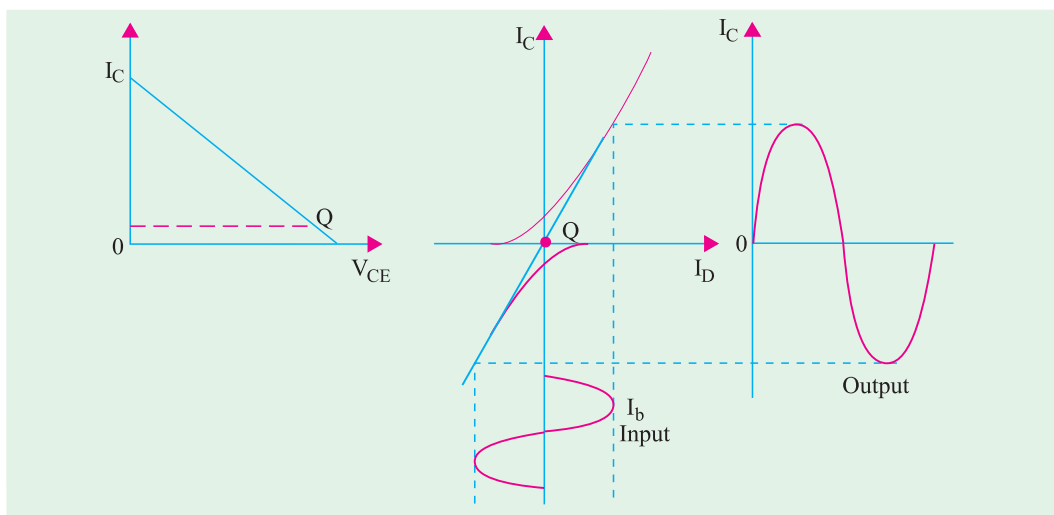


Fig. 60.34



In simple words, crossover distortion occurs as a result of one transistor cutting off before the other begins conducting. The effect is illustrated in Fig. 60.32. For silicon transistors, there is  $2 \times 0.7 = 1.4$  V *dead zone* on the input signal within which neither transistor is turned ON and output is zero (Fig. 60.33).

The distortion so introduced is called *crossover distortion* because it occurs during the time operation *crosses over from one transistor to the other in the push-pull amplifier*. The same was shown earlier in Fig. 60.31 by using transfer characteristics of the two transistors.

Crossover distortion can be eliminated by applying slight forward bias to each emitter diode. It, in effect, means locating the *Q*-point of each transistor slightly above cut-off as shown in Fig. 60.34 so that each one operates for more than one half-cycle. Strictly speaking, it results in class-AB operation because each transistor may operate for about  $200^\circ$  (instead of  $180^\circ$ ). However, for all practical purposes, it is still regarded as class-B push-pull operation.

### 60.26. Power Efficiency of Push-Pull Amplifiers

Consider the circuit shown in Fig. 60.35.

Since the two transistors are identical,  $I_{C1} = I_{C2} = I_C$   
 $\therefore I_C = I_m / \pi$

Total dc supply current  $I_{dc}$  for the two transistors is

$$I_{dc} = 2I_C = 2 \frac{I_m}{\pi}$$

$$\therefore P_{in(dc)} = V_{CC} \times I_{dc}$$

$$= \frac{2I_m}{\pi} V_{CC}$$

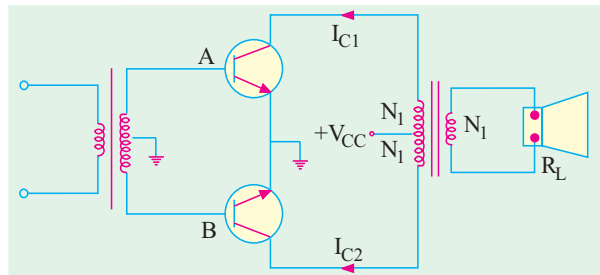


Fig. 60.35

For the sake of simplicity, we will assume transformer turns ratio as  $(N_1 + N_1) : N_1$  so that for each transistor, turns ratio is  $N_1 : N_1$  i.e. 1 because each uses half the secondary of the output transformer.

If  $V_m$  is the peak voltage at either collector, then peak load voltage is also  $V_m$ . The peak current in the load is  $I_m$ . Then, ac power delivered to the load is

$$P_{o(ac)} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = \frac{V_m I_m}{2}$$

The total collector dissipation for the two transistors is

$$2.P_{c(dc)} = P_{in(dc)} - P_{o(ac)} = \frac{2I_m V_{CC}}{\pi} - \frac{V_m I_m}{2} = 2I_m \left( \frac{V_{CC}}{\pi} - \frac{V_m}{4} \right)$$

$$\eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} \times 100 = \frac{V_m I_m / 2}{2I_m / \pi} \times 100 = \frac{\pi}{4} \cdot \frac{V_m}{V_{CC}} \times 100$$

Under ideal condition of maximum power in the load,  $V_m = V_{CC}$

$$\therefore \text{maximum } \eta_{overall} = \frac{\pi}{4} \times 100 = 78.5\%$$

Same is the value of collector efficiency.

### 60.27. Complementary Symmetry Push-Pull Class-B Amplifier

The push-pull amplifier discussed in Art. 60.24 suffers from two disadvantages :

- (i) it requires a bulky and expensive output transformer



- (ii) it requires two out-of-phase input signals which necessitates an input centre-tapped transformer or phase inverter. It makes the driver circuitry quite complicated.

The complementary symmetry amplifier eliminates these two disadvantages while retaining the advantages of push-pull configuration.

As we know, a standard class-B push-pull amplifier requires two power transistors of the *same type* with closely-matched parameters. But the chief requirement of a complementary amplifier is a pair of closely-matched but *oppositely-doped* power transistors. The term 'complementary' arises from the fact that one transistor is *PNP* type and the other is of *NPN* type. They have *symmetry i.e.* both are made with the same material and technology and have the same maximum rating.

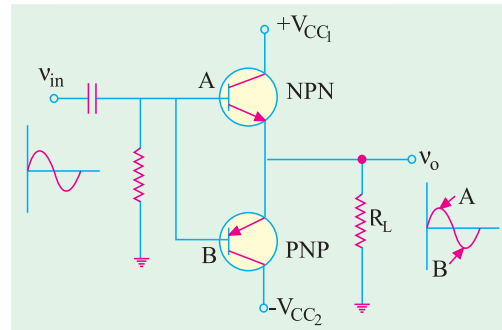


Fig. 60.36

An elementary complementary symmetry class-B push-pull amplifier is diagrammed in Fig. 60.36. The two transistors are complementary to each other and operate as emitter-follower amplifiers. The input is capacitively-coupled whereas output is direct-coupled.

With no input signal, neither transistor conducts and, therefore, current through  $R_L$  is zero.

When input signal is positive-going, transistor A is biased into conduction whereas B is driven into cut-off. When the signal is negative-going, A is turned OFF while B conducts. Obviously, this circuit is a push-pull amplifier because *turning one transistor ON turns the other OFF*.

The circuit possesses the essential characteristics of an emitter follower *i.e.* unity voltage gain, no phase inversion and input impedance much higher than output impedance.

The circuit shown in Fig. 60.36 requires two dc supply batteries. Only one dc battery would be enough if '*totem-pole*' circuit configuration is used.

Due to the elimination of transformer both the high and low-frequency responses of the circuit are extended apart from reduced cost and weight.

### 60.28. Class-C Amplifier

In such amplifiers, the active device *i.e.* the transistor is biased much beyond cut-off. Hence,

1. output current flows only during a part of the possible half-cycle of the input signal,
2. there is no output current flow during any part of the negative half-cycle of the input signal,
3. output signal has hardly any resemblance with the input signal. It consists of short pulses only,
4. class-C amplifiers have high circuit efficiency of about 85 to 90%.

Because of his distortion, class-C amplifiers are not used for audio-frequency work. They are used for high-power output at radio frequencies (*i.e.* RF amplifiers) where harmonic distortion can be removed by simple circuits. In reality, they are used as high-frequency power switchers in radio transmitters rather than as amplifiers.

### 60.29. Tuned Amplifier

The gain of a transistor amplifier depends directly on the value of its load impedance. Such a high impedance can be obtained by using a high- $Q$  tuned or resonant  $LC$  circuit as load (Fig. 60.37). The frequency response curve of the amplifier assumes the same shape as the resonance curve of the tuned circuit. Obviously, only a narrow band of frequencies around the resonant frequency  $f_o$  would be amplified well whereas other frequencies would be discriminated against.

Non-linear distortion is eliminated because of high selectivity of the load impedance. Hence, output is nearly sinusoidal. With the removal of distortion, high amplifier efficiency can be achieved by operating the transistor in its nonlinear region.





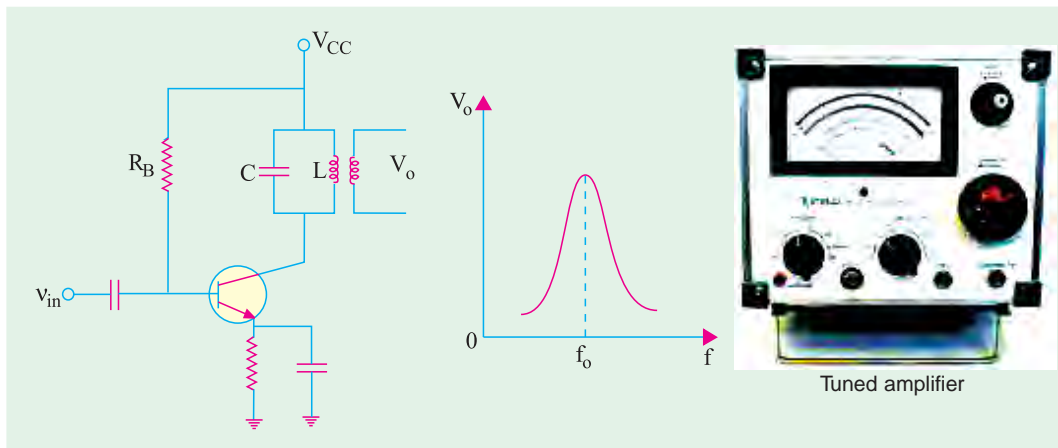


Fig. 60.37

### 60.30. Distortion in Amplifiers

Amplifiers are supposed to produce an output which does not differ from the input in any respect except amplitude *i.e.* the output is expected to be larger than the input. In actual practice, it is impossible to construct such an ideal amplifier whose output is an exact duplication or replica of the input. The output is always found to differ from the input either in its waveform or frequency content. This difference between the output and input of an amplifier is called distortion.

The amplifier distortions may be divided into two broad categories depending on the region of the characteristic used by the transistor and the associated circuit and device reactances.

#### (a) non-linear distortion

This occurs when transistor operates in the non-linear region of its characteristic.

- (i) when we visualize the signal in time domain, it is called amplitude distortion or waveform distortion;
- (ii) when we think of the signal in the frequency domain, it is called harmonic distortion.; This distortion occurs when input signal is of one frequency, say, a pure sine wave.
- (iii) Intermodulation (IM) distortion—when input signal has more than one frequency (like speech).

Non-linear distortion occurs in the case of large-signal inputs when the active device is driven into the non-linear regions of its characteristic.

#### (b) linear distortion

It occurs even when the active device is *working on linear part of its characteristic with small-signal inputs*. It is primarily due to frequency-dependent reactances associated with the circuit or active device itself and occurs when input signal is composite *i.e.* has signals of different frequencies (say, fundamental and its harmonics). However, output contains no frequencies other than those at the input. It may be further subdivided into :

- (i) *frequency distortion*—due to unequal amplification of different frequencies present in the input signal;
- (ii) *phase or delay distortion*—due to unequal phase shift of various signal components.

### 60.31. Non-Linear Distortion

Fig. 60.38 gives time-domain view of this distortion when it is called *amplitude distortion*. As seen, the positive half-cycle of the input signal has been amplified more than its negative half-cycle. Consequently, waveshape of the output signal differs from that of the input signal. As shown below, it is due to the appearance of new frequencies (called harmonics) at the output which are not present in the input.



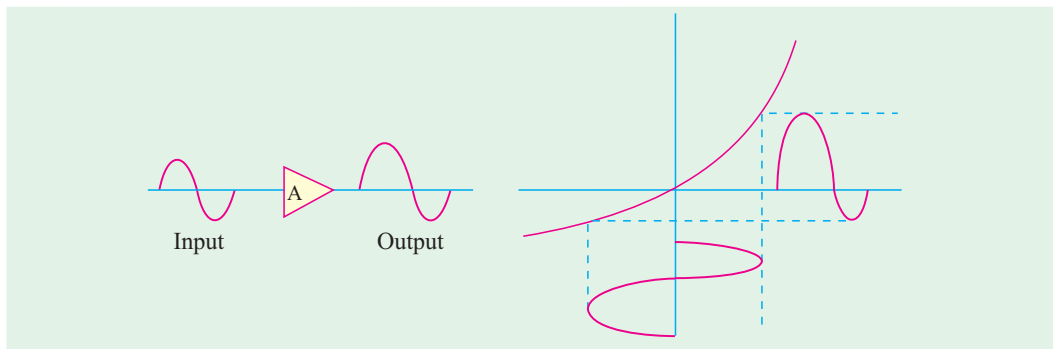


Fig. 60.38

The frequency-domain view of non-linear distortion is shown in Fig. 60.39. The input is a single-frequency ( $f_1$ ) signal but output signal contains dc component and different harmonics *i.e.* frequencies which are harmonically related to each other. These harmonics are an integral multiple of the input signal frequency. Consequently, the output is distorted—the magnitude of distortion depending on the strength and number of these harmonics.

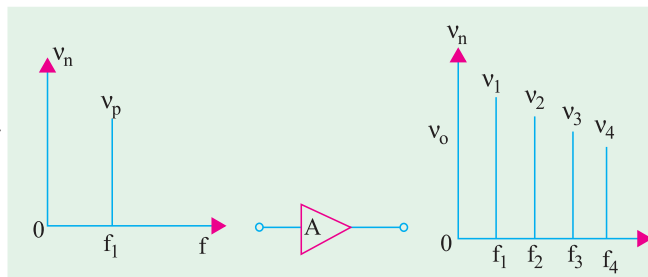


Fig. 60.39

In audio amplifiers used for amplification of speech or music, lesser the harmonic distortion, the better. For speech, harmonic distortion should not exceed 10% otherwise intelligibility will suffer. High-fidelity amplifiers have harmonic and *IM* distortion of less than 1 per cent.

### 60.32. Intermodulation Distortion

It occurs when input signal (like speech) consists of *more than one frequency*. It is also a type of non-linear distortion which generates frequency components not harmonically related to the signal frequencies. Suppose, an input signal contains two frequencies  $f_1$  and  $f_2$ . The output signal will contain their harmonics *i.e.*  $f_1, 2f_1, 3f_1$  etc. and  $f_2, 2f_2, 3f_2$  etc. In addition, there would be components  $(f_1 + f_2)$  and  $(f_1 - f_2)$  and also the sum and difference of the harmonics. These sum and difference frequencies are called intermodulation (*IM*) frequencies and are quite undesirable in amplifiers because they subtract from original intelligence. Since *IM* frequencies are not harmonically related to the signal, they are easily detected by human ear as noise. Hence, great care is taken to minimize them particularly in hi-fi audio amplifiers.

### 60.33. Frequency Distortion

It occurs even when the device is working with small-signal inputs over linear region of its characteristic. It is basically due to change in the amplifier gain with frequency *i.e.* the different input signal frequencies are amplified by different amounts.

Suppose the input signal has two frequencies of 50 Hz and 100 Hz. If they are equally amplified, their resultant output wave is as shown in Fig. 60.40.

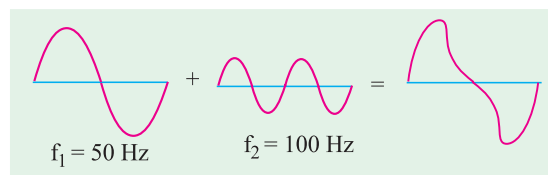


Fig. 60.40



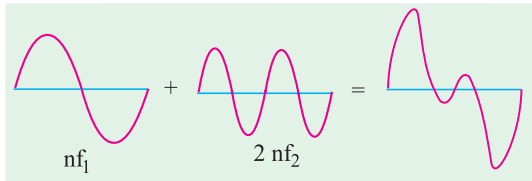


Fig. 60.41

Suppose that the amplifier amplifies the first frequency  $n$  times and the second one  $2n$  times. Then their frequency-distorted output would be as shown in Fig. 60.41.

This distortion is due to the various frequency-dependent reactances (both capacitive and inductive) associated either with the circuit or the active device itself. In the case of audio signals

the frequency distortion leads to a change in the quality of sound. Hence, in the design of untuned or wide-band amplifiers, special steps are taken to reduce variation of gain with signal frequency.

### 60.34. Phase or Delay Distortion

Phase distortion is said to take place when phase angles between the component waves of the output are not the same as the corresponding angles of the input. These changes in phase angles are also due to frequency-dependent capacitive and inductive reactances associated with the circuit and the active device of the amplifier.

As seen from Fig. 60.42, there has been a phase shift in the third harmonic. Hence, the resultant wave so obtained is entirely different from the input wave.

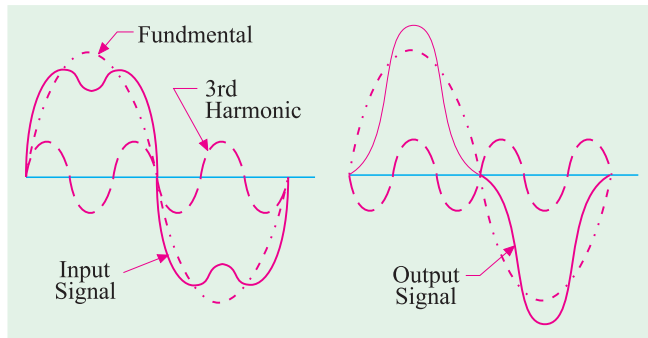


Fig. 60.42

This type of distortion which is due to the non-uniform phase shift of different frequency components, is difficult to eliminate. It should be noted that if all the frequency components in a signal are shifted in phase by an integral multiple of  $180^\circ$ , the resultant output waveform is not changed, although the polarity of the wave may be altered but changes in polarity do not constitute distortion.

Fortunately, the human ear is unable to distinguish phase difference (though eye can) and is thus not sensitive to this distortion. Consequently, phase distortion is of no practical significance in audio amplifiers. But in amplifiers used in television sets (video amplifiers) and other systems where ear is not the final receiver, elimination of phase distortion is important.

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### 60.35. Noise

In general, it may be defined as any kind of unwanted signal not derived from or related to the input signal. Just as distortion is the limiting factor in the amplification of large signals, noise is the limiting factor in the case of small signals.

The signal-to-noise (S/N) ratio should be high for good signal intelligibility. Since all amplifiers contribute some noise of their own to the signal being processed, their output S/N is bound to be less than their input S/N. High quality amplifiers are designed to have an output S/N as close to input S/N as possible. The amplifier performance is measured in terms of noise factor given by

$$\text{Noise factor, } F = \frac{\text{input } S/N}{\text{output } S/N} = \frac{S_i/N_i}{S_o/N_o}; \text{ obviously, } F \text{ is greater than unity.}$$

When expressed in decibels, the noise factor is called noise figure (NF).

$$\therefore NF = 10 \log_{10} F \text{ dB}$$

If an amplifier could be built which generated no noise of its own, then

- (i)  $S_i/N_i = S_o/N_o$
- (ii)  $F = 1$
- (iii)  $NF = 0 \text{ dB}$

If  $G$  is the power gain of the amplifier, then output and input signal powers are related by



$$G = \frac{S_o}{S_i} \quad \text{i.e.} \quad S_o = G.S_i$$

The output noise  $N_o = GN_i + N_A$

where  $N_i$  = input noise power

$N_A$  = noise power generated by amplifier itself

**Example 60.13.** The signal input to a small-signal amplifier consists of 50  $\mu\text{W}$  of signal power and 0.5  $\mu\text{W}$  of noise power. The amplifier generates an internal noise power of 50  $\mu\text{W}$  and has a gain of 20 dB. For this amplifier, compute

(a) input S/N, (b) output S/N, (c) noise factor, (d) noise figure.

(Electronics & Commun. Engg. Pune Univ. 1991)

**Solution.** (a)  $\frac{S_i}{N_i} = \frac{50}{0.5} = 100$

(b) Now,  $10 \log_{10} G = 20 \text{ dB}$  or  $10 \log_{10} G = 2$  or  $G = 10^2 = 100$   
 $S_o = GS_i = 100 \times 50 = 5000 \mu\text{W}$

$$N_o = GN_i + N_A = 100 \times 0.5 + 50 = 100 \mu\text{W} \quad \therefore \frac{S_o}{N_o} = \frac{5000}{100} = 50$$

(c)  $F = \frac{S_i / N_i}{S_o / N_o} = \frac{100}{50} = 2$

(d)  $N_f = 10 \log_{10} 2 \text{ dB} = 10 \times 0.3 = 3 \text{ dB}$

### 60.36. The Decibel System

The decibel system of measurement is widely used in audio, radio, TV and instrument industries for comparing two voltage or power levels. It is based on the established fact that an individual's response to *seeing or hearing is nonlinear*. It has been found that the changes in power and audio levels are related logarithmically. For example, when power is increased from 4 W to 16 W, the audio level does not increase  $16/4 = 4$  times but by a factor of 2 ( $\because 4^2 = 16$ ). Similarly, for a power change from 4 W to 64 W, the audio level changes by a factor of 3 and not by  $64/4 = 16$ . In logarithmic form, the relationship can be written as

$$\text{increase in audio level} = \log_{10} 64 = 3.$$

While comparing two powers, it is common practice to choose the log base of 10. Suppose, we want to compare any two powers  $P_1$  and  $P_2$ . The simple method is to take their ratio  $P_2/P_1$  and to state how many times  $P_2$  is bigger or smaller than  $P_1$  which is used as reference power. In the decibel system, we take the log of this ratio.

$$\therefore \text{power level} = 10 \log_{10} R_2/P_1 \text{ bel}$$

Since bel is too large, we use decibel (dB) instead. Remembering that 1 bel = 10 decibel, the power level becomes

$$= 10 \log_{10} P_2/P_1$$

Similarly, if  $P_i$  is the input power of an amplifier and  $P_o$  its power output, then power gain of the amplifier in decibels is

$$= 10 \log_{10} P_o/P_i \text{ dB}$$

Also, if the output power of an amplifier changes from  $P_1$  to  $P_2$ , the power level change is

$$= 10 \log_{10} P_2/P_1$$

It is obvious that dB is the unit of power change (*i.e.* increase or decrease) and not of power itself.

Another point worth remembering is that 20 dB is not twice as much power as 10 dB.



**Example 60.14.** An amplifier has an input signal of 16 V peak-to-peak and an input impedance of 320 K. It gives an output voltage of 8 V peak-to-peak across a load resistor of 4 W. Calculate the dB power gain of the amplifier. (Electronics-I, Gujarat Univ.)

**Solution.** 
$$P_i = \frac{V_i^2}{R_i} = \frac{(16/2\sqrt{2})^2}{320 \text{ K}} = 100 \mu\text{W}$$

where  $V_i = \text{rms value of input voltage} = \frac{V_{i \times p-p}}{2\sqrt{2}}$

If  $V_o$  is the rms value of the output voltage, then

$$P_o = \frac{V_o^2}{R_o} = \frac{(8/2\sqrt{2})^2}{4} = 2 \text{ W}$$

$\therefore$  power amplification,  $\frac{P_o}{P_i} = \frac{2 \text{ W}}{100 \mu\text{ W}} = 20,000$

decibel power gain =  $10 \log_{10} 20,000 = 10 \times 4.4 = 43 \text{ dB}$

### 60.37. Other Expressions for Power Gain

Suppose  $P_o$  and  $P_i$  are the respective output and input powers of an amplifier,  $R_o$  and  $R_i$  are its output and input resistances and  $V_o$  and  $V_i$  the rms values of output and input voltages, then

$$P_o = \frac{V_o^2}{R_o} \quad \text{and} \quad P_i = \frac{V_i^2}{R_i}$$

Power amplification,  $A_p = P_o/P_i$ . But power gain is given by

$$\begin{aligned} G_p &= 10 \log_{10} P_o / P_i = 10 \log_{10} \frac{V_o^2 / R_o}{V_i^2 / R_i} = 10 \log_{10} (V_o / V_i)^2 \cdot R_i / R_o \\ &= \left[ \log_{10} (V_o / V_i)^2 + \log_{10} R_i / R_o \right] = 10 \log_{10} (V_o / V_i)^2 + 10 \log_{10} R_o / R_i \\ &= 20 \log_{10} V_o / V_i + 10 \log_{10} R_i / R_o \end{aligned}$$

Also  $P_o = I_o^2 R_o$  and  $P_i = I_i^2 R_i$

$\therefore$  power gain,  $G_p = 10 \log_{10} P_o / P_i = 10 \log_{10} I_o^2 R_o / I_i^2 R_i = 10 \left[ \log_{10} (I_o / I_i)^2 R_o / R_i \right]$

$$= 10 \left[ 2 \log_{10} I_o / I_i + \log_{10} R_o / R_i \right] = 20 \log_{10} I_o / I_i + 10 \log_{10} R_o / R_i \text{ dB}$$

Of course, if  $R_o = R_i$ , then  $\log_{10} R_o / R_i = \log_{10} 1 = 0$

Hence, in that case, power gain,  $G_p = 20 \log_{10} I_o / I_i \text{ dB}$

### 60.38. Voltage and Current Levels

Though decibel was initially defined as the unit of power level, it can also be used with voltage and current levels.

Suppose, an amplifier has an input voltage of  $V_i$  and gives an output voltage of  $V_o$ . Then, its voltage amplification is  $A_v = V_o / V_i$  but its decibel voltage gain is

$$G_v = 20 \log_{10} A_v \text{ dB} = 20 \log_{10} V_o / V_i$$

It should be carefully noted that multiplying factor of 20 has been used and not of 10 as is done for finding power level. In fact, it implies that  $R_o$  has been taken equal to  $R_i$  in the power gain equation derived in Art 60.37.

Similarly, current amplification is  $A_i = I_o / I_i$ . However, current gain of the amplifier is

$$G_i = 20 \log_{10} I_o / I_i \text{ dB}$$



**Example 60.15.** The input and output voltages of a network are 16 V and 8V respectively. If input impedances are equal, find the voltage gain.

**Solution.**  $G_v = 20 = \log_{10} V_o/V_i = 20 \log_{10} \frac{8}{16}$   
 $= 20 [\log_{10} 8 - \log_{10} 16] = 20 (0.90 - 1.2) = 20 \times (-0.3) = -6 \text{ dB.}$

### Alternative Method

When the voltage ratio is less than 1, its log is negative which is often difficult to handle. In such cases, it is best to invert the fraction and then make the result negative. The above problem could be solved thus :

$$G_v = -20 \log_{10} \frac{1}{16/8} = -20 \times 0.3 = -6 \text{ dB}$$

**Example 60.16.** A microphone delivers 30 mV to the 300  $\Omega$  input of an amplifier. The ac power delivered to an 8- $\Omega$  speaker is 18 W. What is the power gain of the amplifier ?

**Solution.** We may use any one of the equations given above.

$$G_p = 20 \log_{10} V_o/V_i + 10 \log_{10} R_i/R_o$$

Now,  $P_o = \frac{V_o^2}{R_o}$  or  $18 = \frac{V_o^2}{8} \therefore V_o = \sqrt{8 \times 18} = 12 \text{ V}$

$$\therefore G_p = 20 \log_{10} 12\text{V}/30\text{mV} + 10 \log_{10} 300/8 = 20 \log_{10} 400 + 10 \log_{10} 37.5$$

$$= 20 \times 2.6 + 10 \times 1.57 = 67.7 \text{ dB.}$$

**Example 60.17.** The output power of an amplifier is 100 mW when the signal frequency is 5 kHz. When the frequency is increased to 25 kHz, the output power falls to 50 mW. Calculate the dB change in power. (Electronics, Gujarat Univ. 1991)

**Solution.** The decibel change in power level

$$= 10 \log_{10} 50/100 = 10 \log_{10} \frac{1}{2} = -10 \log_{10} 2 = -10 \times 0.3 = -3 \text{ dB.}$$

**Example 60.18.** The output voltage of an amplifier is 10 V at 5 kHz and 7.07 V at 25 kHz. What is the decibel change in output power level ?

**Solution.** Since changes in voltage are across the same output resistance, the decibel change in power is

$$= 20 \log_{10} V_2/V_1 = 20 \log_{10} 7.07/10 \text{ dB} = -20 \log_{10} 10/7.07 = -20 \log_{10} 1.414 \text{ dB}$$

$$= -20 \times 0.15 = -3 \text{ dB}$$

It is seen from Ex. 60.17 and Ex. 60.18 that output decibel power falls by a 3 dB when

(i) absolute value of power falls to half its original value

or

(ii) output voltage falls to 0.707 or  $1/\sqrt{2}$  of its original value.

**Example 60.19.** A certain radio receiver delivers an output power of 3.6 W.

(i) what would be the decibel gain if power output is increased to 7.2 W ?

(ii) what power output would be required to produce a power gain of 10 dB ?

**Solution.** (i)  $G_p = 10 \log_{10} P_2/P_1 = 10 \log_{10} 7.2/3.6 = 3 \text{ dB}$

(ii) Here,  $G_p = 10$ ,  $P_1 = 3.6 \text{ W}$ ,  $P_2 = ?$

$$\therefore 10 = 10 \log_{10} P_2/3.6 \therefore \log_{10} P_2/3.6 = 1 \text{ or } \frac{P_2}{3.6} = 10^1 = 10$$

$$\therefore P_2 = 3.6 \times 10 = 36 \text{ W}$$

### 60.39. Characteristics of the Decibel System

1. a decibel is a measure of **ratio** and not of **an amount**. It tells us how many times one quantity is greater or lesser with respect to another or the reference quantity. It does not measure the actual (or absolute) voltage or power but only their **changes**;



2. decibel is non-linear *i.e.* 20 dB is not twice as much power or voltage as 10 dB;
3. this log-based system allows a tremendous range of power ratios to be encompassed by using only two-digit numbers. For example  
1 dB = 1.26 : 1 power ratio and 50 dB = 100,000 : 1 power ratio;
4. total dB of a cascaded amplifier can be found by simply adding the stage dBs.

#### 60.40. Value of 1 dB

It can be proved that 1 dB represents the log of two powers which *have a ratio of 1.26*.

$$1 \text{ dB} = 10 \log_{10} P_2/P_1$$

$$\therefore \log_{10} P_2/P_1 = \frac{1}{10} = 0.1 \quad \text{or} \quad \frac{P_2}{P_1} = 10^{0.1} = 1.26$$

Hence, +1 dB represents an increase in power of 26%.

#### 60.41. Zero Decibel Reference Level

By now, it should be clear that decibel does not measure any physical quantity like voltage or power etc., but merely ratio of two physical quantities. For determining the power levels of various powers like  $P_1, P_2, P_3$  etc. it is essential to fix some reference power with which their ratios can be taken. If we fix  $P_0$  as the reference power, then different power levels would be

$$10 \log_{10} P_1/P_0 \quad \text{and} \quad 10 \log_{10} P_2/P_0 \text{ etc.}$$

Obviously,  $P_0$  *cannot be 0 watt i.e.* 0 watt cannot be taken as 0 dB because, in that case, power level of  $P_1$  would be

$$= 10 \log_{10} P_1/0 = \infty$$

In fact, any power compared with zero power is infinity. Hence, decibels would not be defined if zero watt is taken as zero reference level.

Following three zero reference levels are in common use :

1. Zero dB refers to 6 mW dissipated in a 500  $\Omega$  resistive load.

The reference voltage value corresponding to 0 dB is

$$P = V^2/R \quad \text{or} \quad 6 \times 10^{-3} = V^2/500 \quad \text{or} \quad V = 1.73 \text{ V}$$

2. Zero dB refers to 1 mW dissipated in 600  $\Omega$ .

Here,  $1 \times 10^{-3} = V^2/600$ ;  $V = 0.774 \text{ V}$

3. Zero dB refers to a 1-mW dissipation.

It is written as dBm indicating that it uses 1 mW as a reference.

This reference does not depend on any particular load impedance value.

Calculations are performed by using the relation.

$$G_p = 10 \log_{10} P_2/0.001 \text{ dB}_{\text{in}}$$

#### 60.42. Variations in Amplifier Gain with Frequency

If the input voltage of an amplifier is kept constant but its frequency is varied, it is found that the amplifier gain

- (i) remains practically constant over a sizable range of mid-frequencies,
- (ii) decreases at low as well as high frequencies.

A typical frequency- versus-gain curve is shown in Fig. 60.43. While analyzing this curve, three value of frequency are important

- (i) mid-frequency range,
- (ii) lower cut-off frequency,  $f_1$ ,
- (iii) upper cut-off frequency,  $f_2$ .

The lower and upper cut-off frequencies are defined as those frequencies.



(a) where voltage gain of the amplifier decreases to 0.707 times the mid-frequency gain *i.e.* to

$$\frac{1}{\sqrt{2}} A_{v(mid)} = 0.707 A_{v(mid)}$$

or

(b) in terms of power, where power amplification of the amplifier decreases to half its value at mid-frequencies *i.e.*

$$A_{p,1}$$

or  $A_{p,2} = \frac{1}{2} A_{p(mid)}$

(c) in terms of decibels, where power gain **falls by 3 dB**.

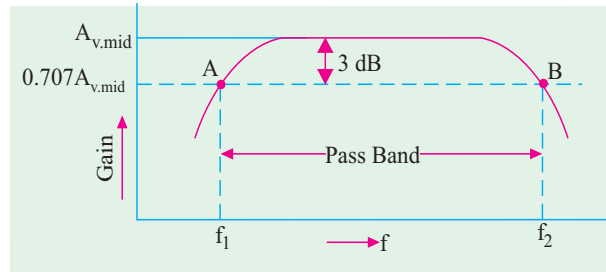


Fig. 60.43

That is why the two cut-off frequencies are referred to as

- (i) -3 dB frequencies or
- (ii) down 3 dB frequencies or
- (iii) 3 dB loss frequencies.

The points A and B in Fig. 60.43 are called 3-dB points or (sometime) as **minus 3 dB points**. The cut-off frequencies are also called roll-off frequencies because at these frequencies the amplifier gain starts rolling down from its midband value or maximum value.

The frequency span between these two cut-off frequencies is called the passband or bandwidth (BW) of the amplifier.

$$\therefore \Delta f = f_2 - f_1 = \text{band width (BW)} = \text{passband}$$

All frequencies lying between  $f_1$  and  $f_2$  are amplified almost equally.

For maximum bandwidth, the stray capacitances in the amplifier must be kept to the minimum.

### 60.43. Causes of Amplifier Gain Variations

The primary cause of gain variation in amplifiers is the presence of capacitances, some of which are connected in series along the signal path and some in parallel.

The different types of external capacitances present in an amplifier circuit are :

1. coupling and bypass capacitors—usually of large capacitance value,
2. internal or inter-element capacitances of the transistor [Fig. 60.44 (a)] and stray wiring capacitance [Fig. 60.44 (b)].

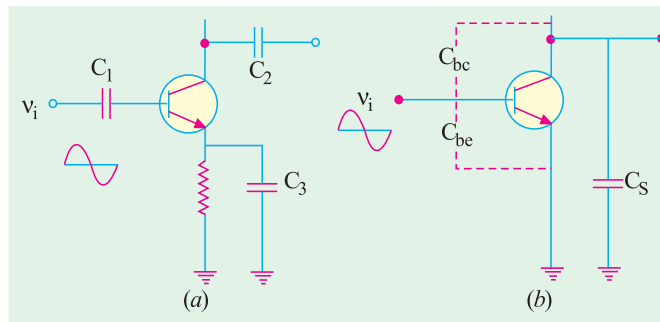


Fig. 60.44

The coupling and bypass capacitors [ $C_1$ ,  $C_2$  and  $C_3$  in Fig. 60.44 (a)] are series-connected whereas interelement capacitances and stray capacitances are parallel-connected

[Fig. 60.44 (b)] to the signal path. At mid-frequencies,  $C_1$ ,  $C_2$  and  $C_3$  act almost as ‘shorts’ but  $C_{bc}$  and  $C_{be}$  act as ‘open’. Hence, their effect on mid-frequencies is negligible.

However, at low frequencies, series-connected capacitors *i.e.* coupling and by-pass capacitors offer relatively large reactance thereby dropping off a large part of the input signal. Hence, amplifier gain starts decreasing as frequency is lowered.





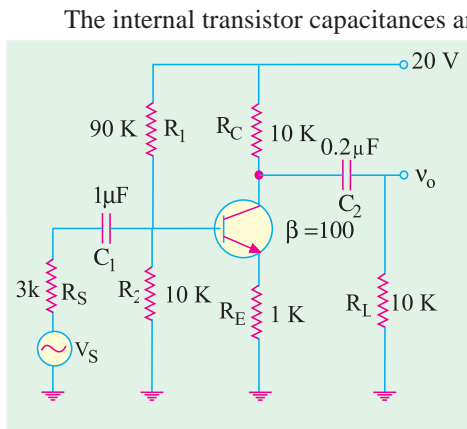


Fig. 60.45

The internal transistor capacitances and stray capacitances are (i) small and (ii) parallel-connected to the signal path. At low frequencies, they offer very high reactance and so act as effective ‘open’. But with increase in frequency, their reactance keeps decreasing till at very high frequencies, they almost short or shunt the ac signal to ground both at the input and output ends. This explains why amplifier gain starts decreasing at high frequencies.

In summary

- (i) series-connected coupling and bypass capacitors cause decrease in amplifier gain at low frequencies,
- (ii) parallel-connected internal transistor capacitances and stray wiring capacitances cause decrease in amplifier gain at high frequencies.

**Example 60.20.** For the RC-coupled circuit of Fig. 60.45, calculate the lower cut-off frequency (i) at  $C_1$  (ii) at  $C_2$  and (iii) for the amplifier.

**Solution.** The low-frequency cut-off for each coupling capacitor is given by

$$f_1 = \frac{1}{2\pi C R_{eq}}$$

where  $R_{eq}$  is the resistance ‘seen’ by the capacitor on its right and left.

(i)  $f_1$  at  $C_1$

For finding the resistance ‘seen’ by  $C_1$ , the dc and ac sources are shorted out because they have negligible resistance. The circuit becomes as shown in Fig. 60.46.

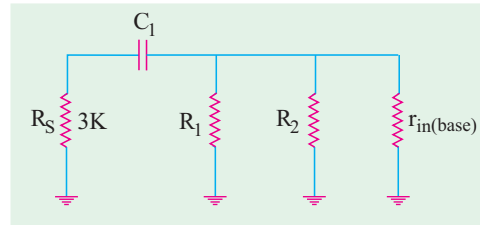


Fig. 60.46

$$R_{eq} = R_S + R_1 \parallel R_2 \parallel r_{in(base)}$$

Now,  $r_{in(base)} = \beta(r_e + R_E) \cong \beta R_E = 100 \text{ K}$

$$\therefore R_{eq} = 3\text{K} + 90\text{K} \parallel 10\text{K} \parallel 100\text{K}$$

$$= 3 + 8.26 = 11.26 \text{ K}$$

$$\therefore f_1 = \frac{1}{2\pi \times 11.26 \times 10^3 \times 1 \times 10^{-6}} = 40 \text{ Hz}$$

(ii)  $f_1$  at  $C_2$

Let us first find the resistance ‘seen’ by  $C_2$ . On one side, it sees  $R_L = 10 \text{ K}$  load resistor to the ground and on the other side, it sees  $R_C$  to ground in parallel with the resistance seen looking into transistor collector.

Since C/B junction is reverse-biased, its resistance is very high. If we consider it as ‘open’, the equivalent circuit becomes as shown in Fig. 60.47.

$$\therefore R_{eq} = R_L + R_C = 10 + 10 = 20 \text{ K}$$

$$\therefore f_1 = \frac{1}{2\pi \times 20 \times 10^3 \times 0.2 \times 10^{-6}} = 40 \text{ Hz}$$

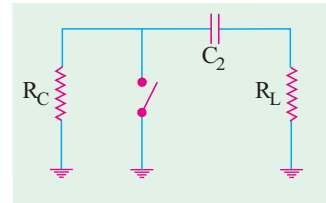


Fig. 60.47

(iii) Since cut-off frequency for  $C_2$  occurs at 40 Hz while cut-off for  $C_1$  occurs way down at 14 Hz,  $C_2$  determines the lower cut-off frequency for the amplifier i.e. **14 Hz**.



**60.44. Miller Effect**

According to this effect, when viewed from input base terminal of the CE-connected transistor, the capacitance  $C_{bc}$  appears as  $(1 + A_v) C_{bc}$  i.e. it is amplified by a factor of  $(1 + A_v)$ . In fact, Miller effect takes into account the feedback from the collector to base and vice-versa due to  $C_{bc}$ .

**Proof**

When  $V_i$  is applied to the transistor's base in Fig. 60.44 (b), the change in collector voltage is

$$\Delta V_C = -A_v \cdot V_i$$

The negative sign is due to phase shift inherent in a CE amplifier. As seen,  $V_C$  is reduced by  $(A_v \cdot V_i)$  when base voltage is increased by  $V_i$ . Hence, total reduction in collector base voltage is

$$\Delta V_{CB} = V_i + A_v \cdot V_i = V_i(1 + A_v)$$

This also represents the change of voltage across  $C_{bc}$  because it is connected across collector and base. Using  $Q = CV$ , the charge supplied to the input of the circuit is

$$Q = C_{bc} \times \Delta V_{CB} = C_{bc} \times (1 + A_v) V_i = (1 + A_v) C_{bc} \times V_i$$

Hence,  $C_{bc}$  appears as  $(1 + A_v) C_{bc}$  when looked from the input side of the circuit.

Incidentally, the total input capacitance to the transistor is

$$C_{in} = C_{be} \parallel (1 + A_v) C_{bc} = C_{be} + (1 + A_v) C_{bc}$$

At high frequencies,  $C_{in}$  reduces the input impedance of the circuit and affects the frequency response (Ex. 60.22).

**Example 60.21.** A CE-connected amplifier has  $C_{bc} = 4 \text{ pF}$ ,  $C_{be} = 10 \text{ pF}$  and  $r_e = 50 \Omega$ . If circuit load resistor is  $10 \text{ K}$ , calculate the value of  $C_{in}$ .

**Solution.**  $A_v \cong \frac{R_E}{r_e} = \frac{100 \text{ K}}{50 \Omega} = 200 \quad \therefore C_{in} = 10 + (1 + 200)4 = 814 \text{ pF}$

**Example 60.22.** Calculate the upper cut-off frequency of the CE amplifier shown in Fig. 60.48. Given the input wiring capacitance  $C_{wi} = 40 \text{ pF}$ ,  $C_{bc} = 8 \text{ pF}$ ,  $C_b = 10 \text{ pF}$  and  $\beta = 100$ .

(Electronic Engg-I, Osmania Univ.)

**Solution.**  $f_2 = \frac{1}{2\pi R_{eq} C_{in}}$

The voltage amplification of the amplifier is

$$A_v \cong \frac{R_C \parallel R_L}{R_L} = \frac{20 \text{ K} \parallel 20 \text{ K}}{400 \Omega} = 25$$

The total capacitance from base to ground is

$$C_{in} = C_{wi} + C_{be} + (1 + A_v) C_{bc} = 40 + 10 + (1 + 25) \times 8 = 258 \text{ pF}$$

Now, let us determine the resistance 'seen' by  $C_{in}$ . If we look to the right, a resistance of  $\beta(r_e + R_E) \cong \beta R_E$  is seen while to the left  $R_1 \parallel R_2 \parallel R_S$  is seen—all in parallel.

$$\therefore R_{eq} = R_1 \parallel R_2 \parallel R_S \parallel \beta R_E = 45 \text{ K} \parallel 5 \text{ K} \parallel 10 \text{ K} \parallel 40 \text{ K} = 2.88 \text{ K}$$

$$\therefore f_2 = \frac{1}{2\pi \times 2.88 \times 10^3 \times 258 \times 10^{-12}} = 214 \text{ kHz}$$

**60.45. Cut-Off Frequencies of Cascaded Amplifiers**

Cascading of stages gives higher amplification but narrow bandwidth because the product of the two remains almost constant. The approximate values of composite lower and upper cut-off frequencies of the cascaded amplifier having  $n$  identical stages are

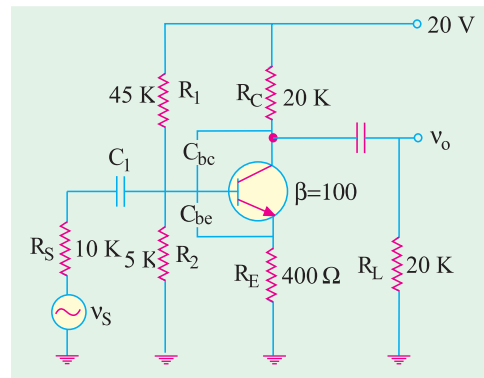


Fig. 60.48



$$f_{1,n} = 1.1\sqrt{n} \times f_1 \quad \text{per stage} = \frac{f_1 \text{ per stage}}{\sqrt{(2^{1/n} - 1)}}$$

$$f_{2,n} = \frac{f_2 \text{ per stage}}{1.1\sqrt{n}} = \sqrt{(2^{1/n} - 1)} \times f_2 \text{ per stage}$$

The reduced bandwidth of the cascaded amplifier is  $= f_{2n} - f_{1,n}$

### 60.46. Transistor Cut-off Frequencies

Even if no external stray capacitances were present in an amplifier, there would still be an upper limit on its frequency response due to

- (i) internal or interelement capacitances of the transistor and
- (ii) transit time of charge carriers across the transistor junctions and through the semiconductor material.

This limitation is expressed in terms of

- (i) alpha cut-off frequency ( $f_\alpha$ ) and (ii) beta cut-off frequency ( $f_\beta$ )

The two are defined below.

### 60.47. Alpha Cut-off Frequency

It is found that at high frequencies, the value of transistor  $\alpha$  begins to fall. This decrease in  $\alpha$  is related to the transit time effect of the charge carriers as they move from the emitter to collector.

The alpha cut-off frequency  $f_\alpha$  is that high frequency at which the  $\alpha$  of a *CB*-connected transistor becomes 0.707 of its low-frequency value (usually 1 kHz).

For example, if value of  $\alpha$  at 1 kHz is 0.98, then its value at  $f_\alpha$  would be  
 $= 0.707 \times 0.98 = 0.693$

It means that at  $f_\alpha$ , the collector current  $I_c$  would be only 0.693 of the emitter current rather than 0.98  $I_E$ .

It is found that  $f_\alpha$  is

- (a) *inversely proportional to the square of the base width,*
- (b) *directly proportional to the minority carrier mobility.*

In this regard, NPN transistors are superior to PNP type because electrons have greater mobility than holes. For decreasing the base transit time, base should be as thin as possible.

It may be noted that alpha cut-off frequency of any given transistor is always greater than its beta cut-off frequency  $f_\beta$ . In fact,  $f_\alpha \cong \beta f_\beta$ .

### 60.48. Beta Cut-off Frequency

It is that high frequency at which the  $\beta$  of a *CE*-connected transistor drops to 0.707 of its low-frequency (1 kHz) value.

### 60.49. The $f_T$ of a Transistor

It is another high-frequency characteristic of a transistor.

It is that high frequency of a *CE*-connected transistor where its  $\beta$  drops to unity *i.e.*  $\beta = 1$ . This frequency is much larger than  $f_\beta$  but less than  $f_\alpha$ . For example, for a typical transistor, their values may be  $f_\beta = 6$  MHz,  $f_T = 300$  MHz and  $f_\alpha = 345$  MHz.

### 60.50. Relation Between $f_\alpha$ , $f_\beta$ and $f_T$

(i) For simple junction transistors  $f_\alpha = 1.2 f_T$  and (ii)  $f_\beta = \frac{f_T}{\beta}$ , where  $\beta$  refers to its low-frequency value.

### 60.51. Gain-Bandwidth Product

At its name indicates, it is the product of the gain and bandwidth of an amplifier. It is very useful in comparing the performance capability of various circuits.

For any amplifier, gain-bandwidth product (GBP) is constant and is equal to  $f_T$ .



If, for example,  $\beta = 1$ , at a frequency of 6 MHz, then  $f_T = 6 \text{ MHz}$ .

$\therefore$  gain-bandwidth product = **6 MHz**.

Hence, for a given  $f_T$  in an amplifier, increased gain may be obtained only at the expense of its bandwidth.

**Example 60.23.** A transistor has  $f_\alpha = 8 \text{ MHz}$  and  $\beta = 80$ . When connected as an amplifier, it has stray capacitance of  $100 \text{ pF}$  at the output terminal. Calculate its upper 3 dB frequency when  $R_L$  is (a)  $10 \text{ K}$  and (b)  $100 \text{ K}$ .

**Solution.** It should be remembered that stray capacitance would reduce the amplifier gain by 3-dB when

capacitive reactance = output resistance

$$\text{i.e.} \quad \frac{1}{2\pi f_s C_s} = R_L \quad \text{or} \quad f_s = \frac{1}{2\pi C_s R_L}$$

First, let us find the value of  $f_\beta$  for comparison with  $f_s$ .

$$f_\beta = \frac{f_1}{80} = \frac{8 \text{ MHz}}{80} = 100 \text{ kHz}$$

(a)  $R_L = 10 \text{ K}$

$$f_2 = \frac{1}{2\pi C_s R_L} = \frac{1}{2\pi \times 100 \times 10^{-12} \times 10 \times 10^3} = 159 \text{ kHz}$$

Obviously, before this frequency is reached, cut-off would have been achieved at  $f_\beta = 100 \text{ kHz}$ .

Hence,  $f_2 = f_\beta = 100 \text{ kHz}$ .

(b)  $R_L = 100 \text{ K}$

$$\text{In this case, } f_s = \frac{10}{100} \times 159 = 15.9 \text{ kHz}$$

In this case, cut-off would be achieved much earlier at 15.9 kHz before  $f_\beta$  is reached

$$\therefore f_2 = 15.9 \text{ kHz.}$$

### Tutorial Problems No. 60.1

- An amplifier raises the power level of its  $5\text{-}\mu\text{W}$  input signal by 30 dB. What is the output power? **[5 mW]**
- An attenuation network provides an output of  $5 \mu\text{W}$  with an input of 5 mW. Calculate the decibel loss of the network. **[-30 dB]**
- What is the decibel difference between a 100 kW and 500 kW radio transmitters? **[6.98 dB]**
- The noise level of a certain tape recording is 30 dB below the signal level. If the signal power is 5 mW, calculate the noise power. **[5 mW]**
- An amplifier rated at 72-W output is connected to an  $8 \Omega$  speaker
  - what input power is required for full power output if power gain is 30 dB,
  - what is the input voltage for rated output if amplifier voltage gain is 40 dB.**[ (a) 72  $\mu\text{W}$  (b) 240 mV ]**
- The characteristics of a certain audio amplifier are such that it gives a voltage amplification of 10 at 100 Hz, 30 at 3 kHz and 60 at 10 kHz. Taking the amplification at 3 kHz as the reference level, calculate the loss or gain in decibels at the other two frequencies. **[-9.54 dB, 6.02 dB]**
- An amplifier with full power rating of 100 W drives a speaker load of  $16 \Omega$ . The hum-level rating of the amplifier is 80 dB below its full-power rating. Calculate



- (i) hum-level in the load,  
 (ii) voltage produced by the hum across the load.

[(i) 1  $\mu$ W (ii) 4 mW]

8. Derive the voltage, current and power gain relationships and then find the voltage gain of the single-stage amplifier shown in Fig. 60.49. An NPN transistor that has  $h_{ie} = 500 \Omega$ ,  $h_{re}$  negligible,  $h_{fe} = 150$ , and  $h_{oe} = 50 \mu S$  is used. Neglect all coupling and stray capacitances.

Comment on the various methods available for biasing the base and discuss their merits.

[- 545]

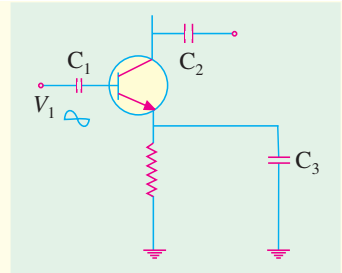


Fig. 60.49

### OBJECTIVE TESTS – 60

- A CB amplifier has very low input resistance because
  - low emitter ac resistance  $r_e'$  shunts all other resistances
  - it handles small input signals
  - emitter bulk resistance is small
  - its base is at ac ground.
- CE amplifier is characterised by
  - low voltage gain
  - moderate power gain
  - signal phase reversal
  - very high output impedance.
- A CC amplifier has the highest
  - voltage gain
  - current gain
  - power gain
  - output impedance.
- In a CC amplifier, voltage gain
  - cannot exceed unity
  - depends on output impedance
  - is dependent on input signal
  - is always constant.
- In a class-A amplifier, conduction extends over  $360^\circ$  because Q-point is
  - located on load line
  - located near saturation point
  - centred on load line
  - located at or near cut-off point.
- The circuit efficiency of a class-A amplifier can be increased by using
  - low dc power input
  - direct-coupled load
  - low-rating transistor
  - transformer-coupled load.
- In a class-A amplifier, worst-case condition occurs with
  - zero signal input
  - maximum signal input
  - high load resistance
  - transformer coupling.
- The output of a class-B amplifier
  - is distortion-free
  - consists of positive half-cycle only
  - is like the output of a full-wave rectifier
  - comprises short-duration current pulses.
- The maximum overall efficiency of a transformer-coupled class-A amplifier is – per cent.
  - 78.5
  - 25
  - 50
  - 85
- A transistor audio amplifier is found to have an overall efficiency of 70 per cent. Most probably, it is a ..... amplifier.
  - class-B push-pull
  - single-stage class-C
  - transformer-coupled class-A
  - direct-coupled class-A
- The main purpose of using transformer coupling in a class-A amplifier is to make it more
  - distortion-free
  - bulky
  - costly
  - efficient.
- A class-B push-pull amplifier has the main advantage of being free from
  - any circuit imbalances
  - unwanted noise
  - even-order harmonic distortion
  - dc magnetic saturation effects.
- Crossover distortion occurs in ..... amplifiers.
  - push-pull
  - class-A
  - class-B
  - class AB
- The maximum overall efficiency of a class-B push-pull amplifier cannot exceed – per cent.
  - 100
  - 78.5
  - 50
  - 85
- The circuit of a class B push-pull amplifier is shown in Fig. 60.50. If the peak output voltage,  $V_o$  is 16 V, the power drawn from the dc source would be
  - 10 W
  - 16 W
  - 20 W
  - 32 W
- The dissipation at the collector is zero in the quiescent state and increases with excitation in the case of a
  - class A series-fed amplifier
  - class A transistor coupled amplifier
  - class AB amplifier
  - class B amplifier
- Class AB operation is often used in power (large signal) amplifiers in order to,



- (a) get maximum efficiency  
(b) remove even harmonics  
(c) overcome cross-over distortion  
(d) reduce collector dissipation
18. The main use of a class-C amplifier is  
(a) as an RF amplifier  
(b) as stereo amplifier  
(c) in communication sound equipment  
(d) as distortion generator.
19. If a class C power amplifier has an input signal with frequency of 200 kHz and the width of collector current pulses of  $0.1\mu\text{s}$ , then the duty cycle of the amplifier will be  
(a) 1% (b) 2% (c) 10% (d) 20%
20. The primary cause of linear distortion in amplifiers is  
(a) change of gain with frequency  
(b) unequal phase shift in component frequencies  
(c) reactances associated with the circuit and active amplifying element  
(d) inherent limitations of the active device.
21. An amplifier is said to suffer from distortion when its output is  
(a) low  
(b) different from its input  
(c) noisy  
(d) larger than its input.
22. While discussing amplifier performance, noise is defined as any kind of unwanted signal in the output which is  
(a) unrelated to the input signal  
(b) derived from the input signal  
(c) not generated by the amplifier  
(d) due to associated circuitry.
23. An ideal amplifier has  
(a) noise figure of less than 1 dB  
(b) noise factor of unity  
(c) output S/N more than input S/N  
(d) noise figure of more than 0 dB.
24. The decibel is a measure of  
(a) power (b) voltage  
(c) current (d) power level.
25. When power output of an amplifier doubles, the increase in its power level is ..... decibels.  
(a) 2 (b) 20 (c) 3 (d) 10
26. When output power level of a radio receiver increases by 3 dB, its absolute power changes by a factor of  
(a) 2 (b) 10 (c) 1/2 (d) 3.
27. Zero watt cannot be chosen as zero decibel level because  
(a) it is impossible to measure zero watt  
(b) it is too small  
(c) every power compared with it would be zero  
(d) it would be impossible to define a decibel.
28. A minus 3 dB point on the gain versus frequency curve of an amplifier is that point where  
(a) signal frequency drops to half the mid-band frequency  
(b) voltage amplification becomes half of its maximum value  
(c) power falls to half its maximum value  
(d) upper cut-off frequency becomes twice the lower cut-off frequency
29. The bandwidth of an amplifier may be increased by  
(a) decreasing the capacitance of its by pass capacitors  
(b) minimizing its stray capacitances  
(c) increasing input signal frequency  
(d) cascading it.
30. Lower cut-off frequency of an amplifier is primarily determined by the  
(a) interval capacitances of the active device used  
(b) stray capacitance between its wiring and ground  
(c) ac  $\beta$  value of its active devices  
(d) capacitances of coupling and bypass capacitors.
31. The main reason for the variation of amplifier gain with frequency is  
(a) the presence of capacitances, both external and internal  
(b) due to interstage transformers  
(c) the logarithmic increase in its output power  
(d) the Miller effect.
32. The gain-bandwidth product of an amplifier is given by  
(a)  $f_2 - f_1$  (b)  $f_\alpha - f_\beta$   
(c)  $f_T$  (d)  $\beta f_\alpha$
33. A Circuit which resonates at 1MHz has a of 100. Bandwidth between half-power points is  
(a) 10 kHz (b) 100 kHz  
(c) 10 Hz (d) 100 Hz
- (UPSC Engg. Services 2002)*  
*(Hint :  $BW = F_o/a$ )*

ANSWERS

1. (a) 2. (c) 3. (b) 4. (a) 5. (c) 6. (d) 7. (a) 8. (b) 9. (c) 10. (a) 11. (d)  
12. (c) 13. (a) 14. (b) 15. (b) 16. (d) 17. (a) 18. (a) 19. (d) 20. (c) 21. (b) 22. (a)  
23. (b) 24. (d) 25. (c) 26. (a) 27. (d) 28. (c) 29. (b) 30. (d) 31. (a) 32. (c) 33. (a)

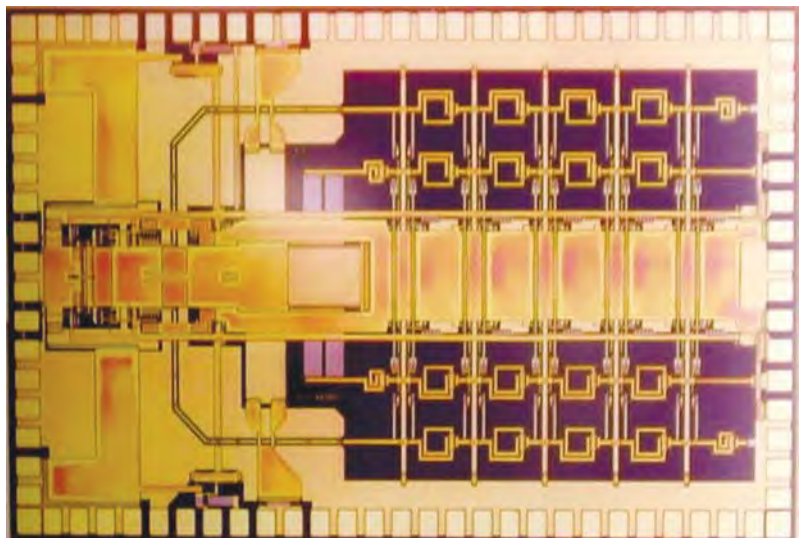


# CHAPTER 61

## Learning Objectives

- General
- Amplifier Coupling
- RC-coupled Two-stage Amplifier
- Impedance-coupled Two-stage Amplifier
- Advantages of Impedance Coupling
- Transformer-coupled Two-stage Amplifier
- Advantages of Transformer Coupling
- Frequency Response
- Direct-coupled Two-stage Amplifier Using Similar Transistors
- Direct-coupled Amplifier Using Complementary Symmetry of Two Transistors
- Darlington Pair
- Advantages of Darlington Pair
- Comparison Between Darlington Pair and Emitter Follower
- Special Features of a Differential Amplifier
- Common Mode Input
- Differential Amplifier

## MULTISTAGE AND FEEDBACK AMPLIFIERS



In a multistage amplifier, a number of single amplifiers are connected in a cascade arrangement *i.e.* the output of first stage is connected to the input of second stage.

61.1. General

Often, the voltage amplification or power gain or frequency response obtained with a single stage of amplification is insufficient to meet the requirements of either a composite electronic circuit or a load device. Hence, two or more single stages of amplification are frequently used to achieve greater voltage or current amplification or both. In such cases, the output of one stage serves as input of the next stage as shown in Fig. 61.1. Such amplifiers may be divided into following two categories :

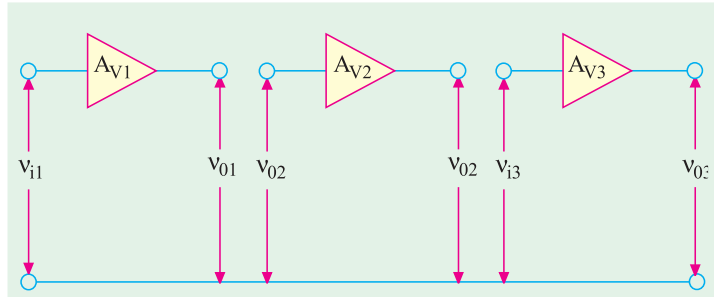


Fig. 61.1

Such amplifiers may be divided into following two categories :

(a) Cascaded Amplifiers

In these amplifiers, each stage as well as the type of interstage coupling used are identical.

(b) Compound Amplifiers

In these amplifiers, each stage may be different from the other (one may be CE and the other may be CC stage) and also different types of interstage couplings may be employed.

As stated above, in cascaded amplifiers, the output ac voltage of the first stage becomes the input voltage of the second stage and the ac output of the second stage becomes the input of the third stage and so on. The overall voltage gain of the cascaded amplifier is equal to the **product** (not the sum) of the gain of the individual stages.

$$\therefore A_v = A_{v1} \times A_{v2} \times A_{v3} \times \dots$$

However, when the voltage gain is expressed in decibels (dB), then the overall decibel gain of the multistage amplifier is equal to the **sum** of the dB gains of the individual stage *i.e.*

$$G = G_1 + G_2 + G_3 + \dots$$

Similarly, the overall current amplification is given by

$$A_i = A_{i1} \times A_{i2} \times A_{i3} \times \dots$$

The overall power gain is given by

$$A_p = A_v \cdot A_i \quad \text{and} \quad G_p = 10 \log_{10} A_p \text{ dB}$$

Suppose in a two-stage cascaded amplifier, first stage has a voltage amplification of 2000 (dB gain of the  $20 \log_{10} 2000 = 66 \text{ dB}$ ) and second stage has corresponding values of 1000 (60 dB). If the ac output of first stage is fed into the second stage, the overall amplification would theoretically become  $= 1000 \times 2000 = 2 \times 10^6$  which corresponds to a dB gain of  $(60 + 66) = 126 \text{ i.e. } 20 \log_{10} 2 \times 10^6 = 20 \times 6.3 = 126$ . The above result would be true only when we **neglect the loading effect of first stage by the second stage**. It would be approximately true so long as the impedance looking into the input of second stage is much greater than the output impedance of the first stage. Otherwise, the overall gain would be much less.

61.2. Amplifier Coupling

All amplifiers need some **coupling network**. Even a single-stage amplifier has to be coupled to the input and output devices. In the case of multistage systems, there is **interstage** coupling. The type of coupling used determines the characteristics of the cascaded amplifier. In fact, amplifiers are classified according to the coupling network used.





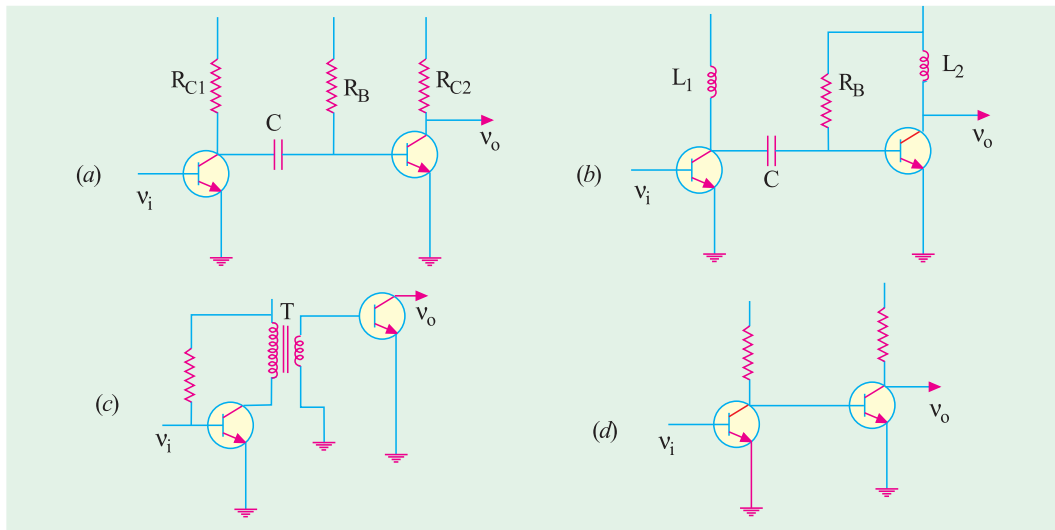


Fig. 61.2

The four basic methods of coupling are :



Modern coupling transformer

### 1. Resistance-Capacitance (RC) Coupling

It is also known as *capacitive* coupling and is shown in Fig. 61.2 (a). Amplifiers using this coupling are known as *RC-coupled* amplifiers. Here, *RC* coupling network consists of two resistors  $R_{C1}$  and  $R_{C2}$  and one capacitor  $C$ . The connecting link between the two stages is  $C$ . The function of the *RC*-coupling network is two-fold :

- (a) to pass ac signal from one stage to the next,
- (b) to block the passage of dc voltages from one stage to the next.

### 2. Impedance Coupling or Inductive Coupling

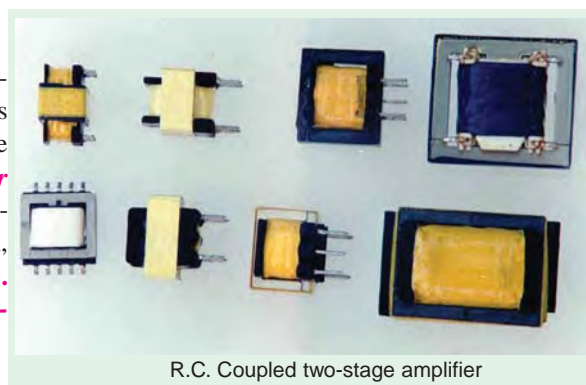
It is also known as choke-capacitance coupling and is shown in Fig. 61.2 (b). Amplifiers using this coupling are known as *impedance-coupled* amplifiers. Here, the coupling network consists of  $L_1$ ,  $C$  and  $R_B$ . The impedance of the coupling coil depends on (i) its inductance and (ii) signal frequency.

### 3. Transformer Coupling

It is shown in Fig. 61.2 (c). Since secondary of the coupling transformer conveys the ac component of the signal directly to the base of the second stage, there is *no need for a coupling capacitor*. Moreover, the secondary winding also provides a base return path, hence there is *no need for a base resistance*. Amplifiers using this coupling are called *transformer-coupled amplifiers*.

### 4. Direct Coupling

It is shown in Fig. 61.2 (d). This coupling is used where it is desirable to connect the load directly in series with the output terminal of the active circuit element. The examples of such load



R.C. Coupled two-stage amplifier

devices are (i) headphones (ii) loud-speakers (iii) dc meters (iv) relays and (v) input circuit of a transistor etc. Of course, direct coupling is permissible only when

- (i) dc component of the output does not disturb the normal operation of the load device,
- (ii) device resistance is so low that it does not appreciably reduce the voltage at the electrodes.

### 61.3. RC-coupled Two-stage Amplifier

Fig. 61.3 shows a two-stage RC-coupled amplifier which consists of two single-stage transistor amplifiers using the CE configuration. The resistors  $R_2$  and  $R_3$  and capacitor  $C_2$  form the coupling network.  $R_2$  is collector load of  $Q_1$  and  $R_4$  is that of  $Q_2$ . Capacitor  $C_1$  couples the input signal whereas  $C_3$  couples out the output signal.  $R_1$  and  $R_3$  provide dc base bias.

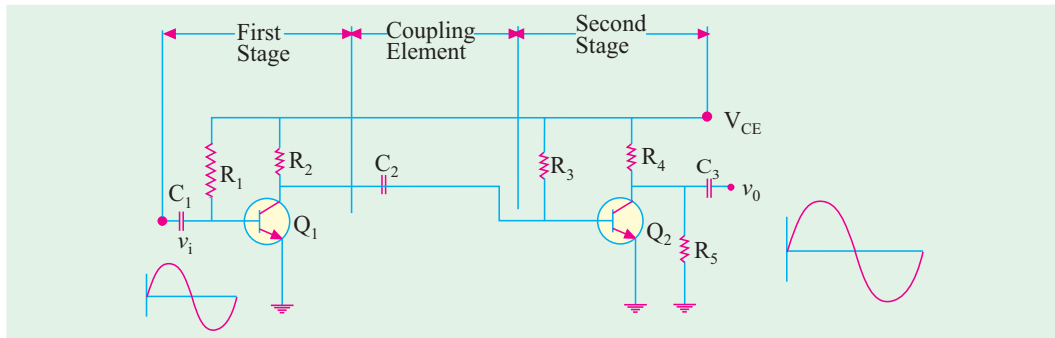


Fig. 61.3

#### (i) Circuit Operation

The brief circuit operation is as under :

1. the input signal  $v_i$  is amplified by  $Q_1$ . It is phase **reversed** (usual with  $C_E$  connection);
2. the amplified output of  $Q_1$  appears across  $R_2$ ;
3. the output of the first stage across  $R_2$  is coupled to the input at  $R_3$  by coupling capacitor  $C_2$ . This capacitor is also sometimes referred to as **blocking capacitor** because it blocks the passage of dc voltages and currents;

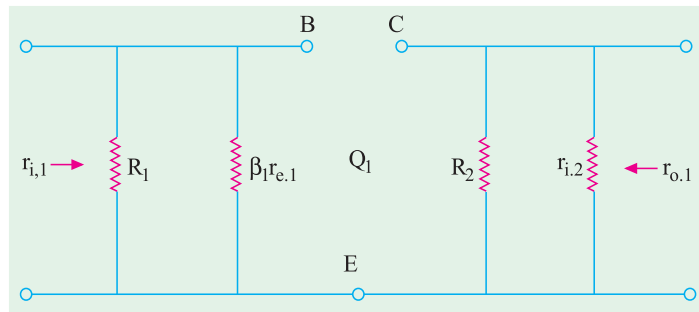


Fig. 61.4

4. the signal at the base of  $Q_2$  is further amplified and **its phase is again reversed**;
5. the ac output of  $Q_2$  appears across  $R_4$ ;
6. the output across  $R_4$  is coupled by  $C_3$  to load resistor  $R_5$ ;
7. the output signal  $v_o$  is the **twice-amplified replica of the input signal  $v_i$** . It is in phase with  $v_i$  because it has been reversed twice.

#### (ii) AC Equivalent Circuit

The ac equivalent circuits for the two stages have been shown separately in Fig. 61.4 and Fig. 61.5 respectively.

If Fig. 61.4,  $r_{i,1} = R_1 \parallel \beta_1 \cdot r_{e,1}$

It is the input impedance of the first stage and not  $r_{in(base)}$ .

The output impedance of the first stage is

$$r_{0,1} = R_2 \parallel r_{i,2}$$

It is so because the input of the second stage forms a part of the output of the first stage.

As seen from Fig. 61.5.

$r_{i,2} = R_3 \parallel \beta_2 \cdot r_{e,2} \cong \beta_2 r_{e,2}$ , where  $r_{e,1}$  and  $r_{e,2}$  are ac junction resistances of the two transistors and are given by

$$r_{e,1} = \frac{25 \text{ mV}}{I_{E,1}} \text{ or } \frac{50 \text{ mV}}{I_{E,1}} \text{ and } r_{e,2} = \frac{25 \text{ mV}}{I_{E,2}} \text{ or } \frac{50 \text{ mV}}{I_{E,2}}$$

The output impedance of  $Q_2$  is  $r_{0,2} = R_4 \parallel R_5$

**(iii) Voltage Gain**

$$r_{i,2} = \beta_2 r_{e,2} \quad \text{Now} \quad A_{v,2} = \beta_2 \frac{r_{0,2}}{\beta_2 \cdot r_{i,2}} = \frac{r_{0,2}}{r_{e,2}}$$

$$\text{Also, } r_{i,2} = \beta_2 r_{e,2} \quad \therefore \quad A_{v,2} = \beta_2 \frac{r_{0,2}}{\beta_2 \cdot r_{e,2}} = \frac{r_{0,2}}{r_{e,2}}$$

The voltage gain of the first stage is also given by a similar equation

$$A_{v,1} = \frac{r_{0,1}}{r_{e,1}}$$

**Example 61.1.** For the two-stage RC-coupled low-level audio amplifier shown in Fig. 61.6, compute the following :

- (i)  $r_i$  (ii)  $A_{v,1}$  (iii)  $A_{v,2}$  and (iv)  $A_v$  in dB. Neglect  $V_{BE}$  and take  $r_e = 25 \text{ mV}/I_E$ .

(Electronic Circuits, Mysore Univ.)

**Solution.** The input impedance of the cascaded amplifier is

$$(i) \quad r_i = R_1 \parallel \beta_1 \cdot r_{e,1}$$

For finding  $r_{e,1}$ , we need  $I_{E,1}$

which approximately equals  $I_{C,1}$

Now,  $I_{C,1} = \beta \cdot I_{B,1}$

Also,  $I_{B,1} = 12/R_1 = 12/0.6 \text{ M} = 20 \mu\text{A}$

$$\therefore I_{C,1} = 100 \times 20 = 2000 \mu\text{A} = 2 \text{ mA}$$

$$\therefore I_{E,1} = 2 \text{ mA}$$

$$\therefore r_{e,1} = 25/2 = 12.5 \Omega ;$$

$$\beta_1 r_{e,1} = 100 \times 12.5 = 1250$$

$$\therefore r_i = R_1 \parallel \beta_1 \cdot r_{e,1} = 0.6 \text{ M} \parallel 1250 \Omega \cong 1250 \Omega$$

$$(ii) \quad A_{v,1} = \frac{r_{0,1}}{r_{e,1}} \quad \text{Now, } r_{0,1} = R_2 \parallel r_{i,2}$$

$$r_{i,2} = R_3 \parallel \beta_2 r_{e,2} = 0.6 \text{ M} \parallel 1250 \Omega \cong 1250 \Omega$$

$$r_{0,1} = 5 \text{ K} \parallel 1250 \Omega = 1000 \Omega ; r_{e,1} = 12.5 \Omega \quad \therefore A_{v,1} = 1000/12.5 = 80$$

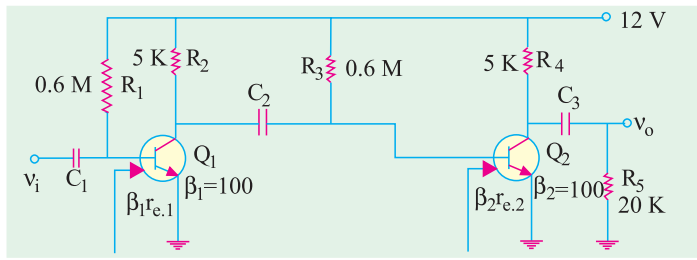


Fig. 61.5

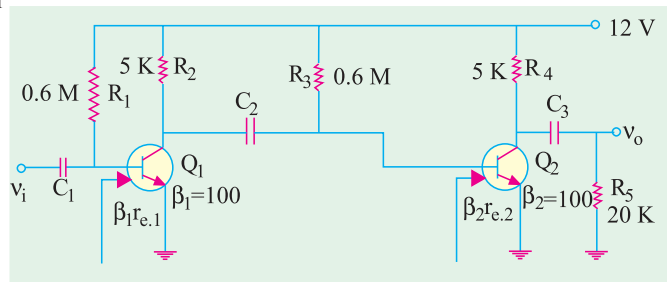


Fig. 61.6

(iii)  $A_{v2} = \frac{r_{0.2}}{r_{e.2}}$

Now,  $r_{0.2} = R_4 \parallel R_5 = 5 \text{ K} \parallel 20 \text{ K} = 4 \text{ K}$ ,  $I_{E.2} = 2 \text{ mA}$  — same as  $I_{E.1}$

$r_{e.2} = 25/2 = 12.5 \text{ } \Omega$   $\therefore A_{v2} = 4000/12.5 = 320$

(iv)  $A_v = A_{v.1} \times A_{v.2} = 80 \times 320 = 25,600$

(v)  $G_v = 20 \log_{10} A_v \text{ dB} = 20 \log_{10} 25,600 = 88 \text{ dB}$ .

**Example 61.2.** For the two-stage RC-coupled amplifier shown in Fig. 61.7 compute the following :

(i)  $r_i$ , (ii)  $A_{v.1}$ , (iii)  $A_{v.2}$ , (iv)  $A_v$  in decibels.

Take  $\beta_1 = \beta_2 = 100$ . Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$

(Applied Electronics-I, Punjab Univ. 1991)

**Solution.** (i) The input impedance of the stage is  $r_i = R_1 \parallel \beta_1 r_{e.1}$

It should be noted that  $R_6$  does not come into the picture because it has been ac grounded by  $C_4$ . However, it would affect the dc emitter current.

$$I_{E.1} = \frac{V_{CC}}{R_6 + R_1 / \beta_1} = \frac{25}{10,000 + 1.5 \times 10^6 / 100} = 1 \text{ mA}$$

$r_{e.1} = 25/1 = 25 \text{ } \Omega$ ;  $\beta_1 r_{e.1} = 100 \times 25 = 2500 \text{ } \Omega$

$r_i = 1.5 \text{ M} \parallel 2500 \text{ } \Omega \cong 2500 \text{ } \Omega$

(ii)  $A_{v.1} = \frac{r_{0.1}}{r_{e.1}}$

Now,  $r_{0.1} = R_2 \parallel r_{i.2}$  and  $r_{i.2} = R_3 \parallel \beta_2 r_{e.2}$

Now,  $I_{E.2} = \frac{25}{10,000 + 1.5 \times 10^6 / 100} = 1 \text{ mA}$

$\therefore r_{e.2} = 25/1 = 25 \text{ } \Omega$ ;  $\beta_2 r_{e.2} = 2500 \text{ } \Omega$

$r_{i.2} = 1.5 \text{ M} \parallel 2500 \text{ } \Omega$   
 $\cong 2500 \text{ } \Omega$

$r_{0.1} = 5 \text{ K} \parallel 2.5 \text{ K}$   
 $= 1,667 \text{ } \Omega$

$\therefore A_{v.1} = 1667/25 = 66.7$

(iii)  $A_{v.2} = \frac{r_{0.2}}{r_{e.2}}$

Now,  $r_{0.2} = R_4 \parallel R_5$   
 $= 5 \text{ K} \parallel 20 \text{ K} = 4 \text{ K}$

$\therefore A_{v.2} = 4000/25 = 160$

(iv)  $A_v = A_{v.1} \times A_{v.2}$   
 $= 66.7 \times 160 = 10,672$ ;

$G_v = 20 \log_{10} 10,672 = 80.3 \text{ dB}$

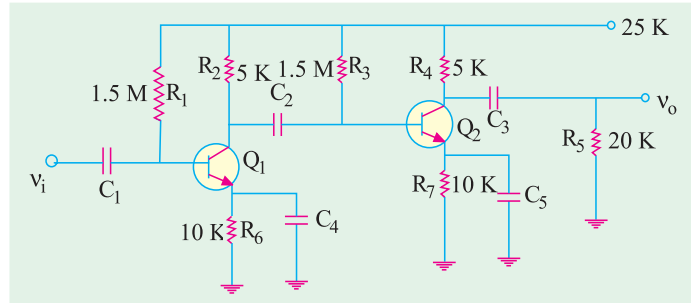


Fig. 61.7

**Example 61.3.** Compute the overall voltage amplification for the two-stage RC-coupled amplifier shown in Fig. 61.8. Express the answer in decibels. Neglect  $V_{BE}$  and use  $r_e = 50 \text{ mV}/I_E$ . Take  $\beta_1 = \beta_2 = 100$ .

(Electronics-I, Allahabad Univ. 1990)

**Solution.** For finding the overall gain, we will have to find each stage gain.

(i)  $A_{v1} = \frac{r_{0.1}}{r_{e.1}}$

Now,  $r_{0.1} = R_3 \parallel r_{i.2}$  and  $r_{i.2} = R_5 \parallel R_6 \parallel \beta_2 r_{e.2}$

Now,  $r_{e.2} = 50/I_{E.2}$

Drop across  $R_6$  =  $20 \times \frac{5}{5+45} = 2V$   
 It also represents the approximate drop across  $R_8$ .  
 $\therefore I_{E2} = 2/1\text{ K} = 2\text{ mA}$   
 $\therefore r_{e,2} = 50/2 = 25\ \Omega$ .  
 $\beta_2 r_{e,2} = 100 \times 25 = 2.5\text{ K}$   
 $r_{i,2} = 45\text{ K} \parallel 5\text{ K}$   
 $\parallel 2.5\text{ K} = 1.6\text{ K}; \therefore r_{o,1} = 5\text{ K} \parallel 1.6\text{ K} = 1.2\text{ K}$   
 $\therefore r_{e,1} = 50/I_{E,1} = 50/2 = 25\ \Omega$   
 $\therefore A_{v,1} = 1200/25 = 48$

$A_{v,2} = \frac{r_{o,2}}{r_{c,2}}; r_{o,2} = R_7 \parallel R_9 = 5\text{ K} \parallel 20\text{ K} = 4\text{ K}$   
 $\therefore A_{v,2} = 4000/25 = 160 \therefore A_v = 48 \times 160 = 7,680; G_v = 20 \log_{10} 7,680 = 77.6\text{ dB}$

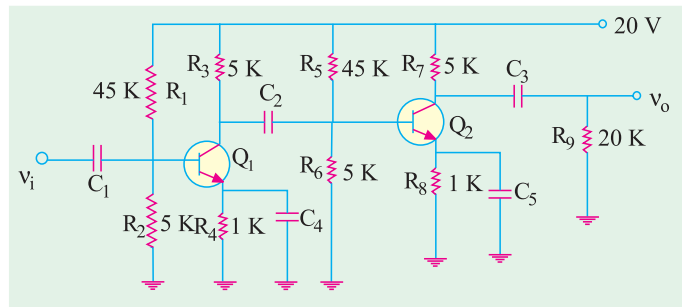


Fig. 61.8

### 61.4. Advantages of RC Coupling

1. It requires no expensive or bulky components and no adjustments. Hence, it is small, light and inexpensive.
2. Its overall amplification is higher than that of the other couplings.
3. It has minimum possible nonlinear distortion because it does not use any coils or transformers which might pick up undesirable signals. Hence, there are no magnetic fields to interfere with the signal.
4. As shown in Fig. 61.9, it has a very flat frequency *versus* gain curve *i.e.* it gives uniform voltage amplification over a wide range from a few hertz to a few megahertz because resistor values are independent of frequency changes.

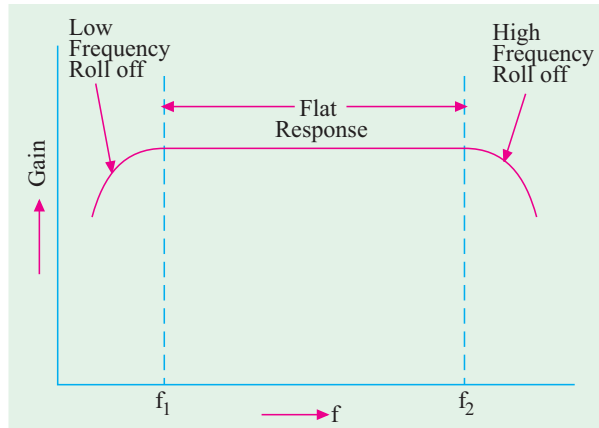


Fig. 61.9

As seen from Fig. 61.9, amplifier gain falls off at very low as well as very high frequencies. At low frequencies, the fall in gain (called *roll-off*) is due to capacitive reactance of the coupling capacitor between the two stages. The high-frequency roll-off is due to output capacitance of the first stage, input capacitance of the second stage and the stray capacitance.

The only drawback of this coupling is that due to large drop across collector load resistors, the collectors work at relatively small voltages unless higher supply voltage is used to overcome this large drop.

### 61.5. Impedance-coupled Two Stage Amplifier

The circuit is shown in Fig. 61.10. The coupling network consists of  $L$ ,  $C_2$  and  $R_3$ . The only basic difference between this circuit and the one shown in Fig. 61.3 is that inductor  $L$  has replaced the resistor  $R_2$ .

**(i) AC Equivalent Circuit**

The ac equivalent circuit (at mid-frequency) of the cascaded amplifier has been shown in Fig. 61.11. Because of mid-frequency range, effects of all capacitances have been ignored.

**(ii) Circuit Operation**

The operation of this circuit is the same as that of the RC-coupled circuit described earlier.

**(iii) Voltage Gain**

It is given by the product of two stage gains  $A_v = A_{v1} \times A_{v2}$

Now,  $A_{v1} = \frac{Z_{0,1}}{r_{e1}}$  where  $Z_{0,1} = X_L \parallel r_{i,2}$  and  $r_{i,2} = R_3 \parallel \beta_2 r_{e,2}$

In case,  $X_L \gg r_{i,2}$ , then,  $Z_{0,1} \cong r_{i,2}$

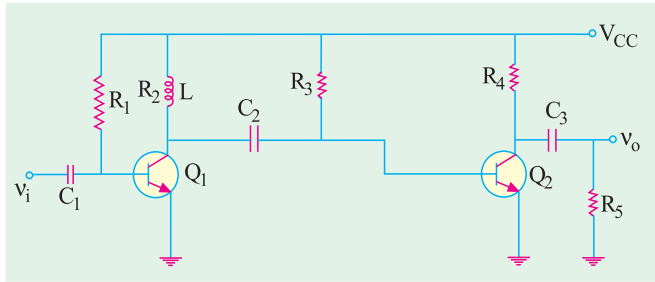


Fig. 61.10

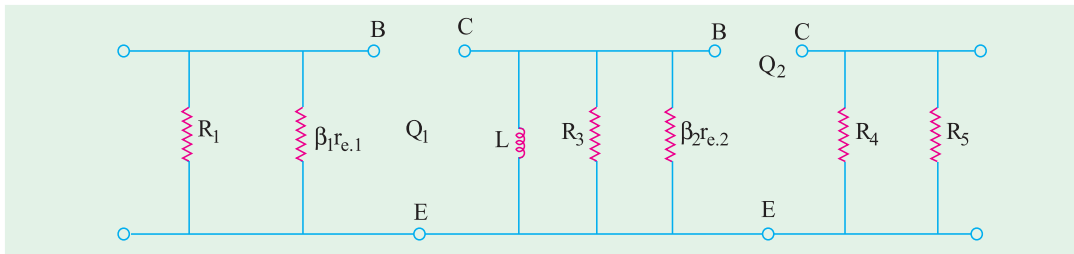


Fig. 61.11

$\therefore A_{v1} = \frac{r_{i,2}}{r_{e1}}$  and  $A_{v2} = \frac{r_{o,2}}{r_{e2}}$

$\therefore A_v = A_{v1} \times A_{v2}$

**Example 61.4.** For the impedance-coupled two-stage amplifier shown in Fig. 61.12, compute the values of

- (i)  $A_{v,2}$ ,
  - (ii)  $A_{v,1}$  at 4 kHz and
  - (iii)  $A_v$  in dB.
- Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$ . Take  $\beta_1 = \beta_2 = 100$ .

(Industrial Electronics, Calcutta Univ. 1991)

**Solution.** (i)  $A_{v,2} = \frac{r_{o,2}}{r_{e,2}}$

Now,  $r_{o,2} = R_4 \parallel R_5 = 8 \text{ K} \parallel 24 \text{ K} = 6 \text{ K}$

$I_{B2} = 12/1.2\text{M} = 10 \mu\text{A}$ ,  $I_{C,2} = \beta_2 \cdot I_{B,2} = 100 \times 10 = 1000 \mu\text{A} = 1 \text{ mA}$

$\therefore I_{E2} \cong 1 \text{ mA}$

$\therefore r_{e,2} = 25/1 = 25 \Omega$

$A_{v,2} = \frac{6,000}{25} = 240$

(ii)  $A_{v,1} = \frac{Z_{0,1}}{r_{e,1}} \cong \frac{r_{i,2}}{r_{e,1}}$

Now,  $r_{e,1} = 25 \Omega$  – equal to  $r_{e,2}$

$X_L = 2\pi fL$   
 $= 2\pi \times 4 \times 10^3 \times 1$   
 $= 25,130 \Omega$   $r_{i,2} = R_3$

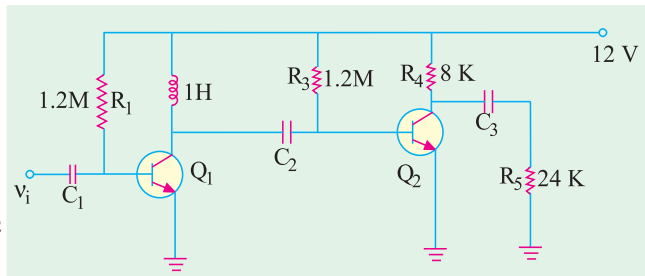


Fig. 61.12

$$\parallel \beta_2 \cdot r_{e,2} = 1.2\text{M} \parallel 2500 \Omega \cong 2500 \Omega$$

Obviously,  $X_L \gg r_{i,2}$  thus justifying the above approximation.

$$\therefore A_{v,1} = 2500/25 = \mathbf{100}$$

$$\text{(iii)} \quad A_v = 100 \times 240 = 24,000, G_v = 20 \log_{10} 24,000 = \mathbf{87.6 \text{ dB}}$$

### 61.6. Advantages of Impedance Coupling

The biggest advantage of this coupling is that there is hardly any dc drop across L so that low collector supply voltages can be used.

However, it has many disadvantages :

1. It is larger, heavier and costlier than RC coupling.
2. In order to prevent the magnetic field of the coupling inductor from affecting the signal, the inductor turns are wound on a closed core and are also shielded.
3. Since inductor impedance depends on frequency, the frequency characteristics of this coupling are not as good as those of BC coupling. The flat part of the frequency *versus* gain curve is small (Fig. 61.13).

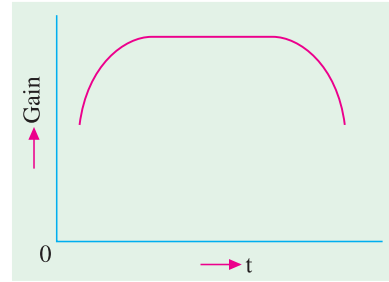


Fig. 61.13

At low frequencies, the gain is low due to large capacitance offered by the coupling capacitor just as in  $R_C$  coupled amplifiers. The gain *increases with frequency* till it levels off at the middle frequencies of the audio range.

At relatively high frequencies, gain drops of again because of the increased reactance.

Hence, impedance coupling is rarely used beyond audio range.

### 61.7. Transformer-coupled Two Stage Amplifier

The circuit for such a cascaded amplifier is shown in Fig. 61.14.  $T_1$  is the coupling transformer whereas  $T_2$  is the output transformer.  $C_1$  is the input coupling capacitor whereas  $C_2, C_3$  and  $C_4$  are the bypass capacitors. Resistors  $R_1$  and  $R_2$  as well as  $R_4$  and  $R_5$  form voltage divider circuits whereas  $R_3$  and  $R_6$  are the emitter-stabilizing resistors.

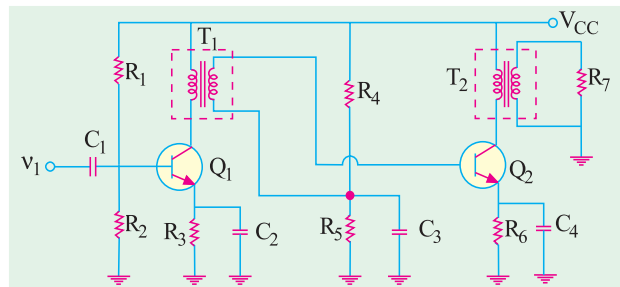


Fig. 61.14

When input signal is coupled through  $C_1$  to the base of  $Q_1$ , it appears in an amplified form in the primary of  $T_1$ . From there, it is passed on to the secondary by magnetic induction. Moreover,  $T_1$  provides dc isolation between the input and output circuits. The secondary of  $T_1$  applies the signal to the base of  $Q_2$  which appears in an amplified form in the primary of  $T_2$ .

From there, it is passed on to the secondary by magnetic induction and finally appears across the matched load  $R_7$ .

#### (ii) Voltage Gain

$$A_{v,1} = \frac{r_{o,1}}{r_{e,1}} \quad \text{Now } r_{o,1} = a^2 r_{i,2} \quad \text{— where } a = N_1 / N_2 \text{ for } T_1$$

$$r_{i,2} = R_4 \parallel R_5 \parallel \beta_2 \cdot r_{e,2}$$

Similarly,  $A_{e,2} = \frac{r_{0,2}}{r_{e,2}}$   
 where  $r_{e,2} = a^2 R_7$

**Example 61.5.** For the transformer-coupled two-stage amplifier shown in Fig. 61.15, calculate (i)  $A_{v,1}$  (ii)  $A_{v,2}$  and (iii)  $A_v$  in dB.

Neglect  $V_{BE}$  and use  $r_e = 50 \text{ mV}/I_E$ . Take  $\beta_1 = \beta_2 = 50$  and treat the transformers as ideal ones.

(Electronics, Indore Univ.)

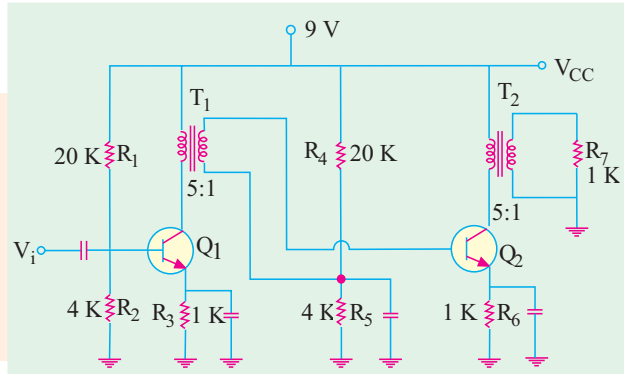


Fig. 61.15

**Solution.** Here, for each transformer  $a = 5$

Drop across  $R_2 = 9 \times 4/24 = 1.5 \text{ V}$

Drop across  $R_3 \cong 1.5 \text{ V}$  and

$r_{e,1} = 50/1.5 = 33.3 \Omega$

and  $r_{e,2} = 33.3 \Omega$

$\beta_2 \cdot r_{e,2} = 50 \times 33.3 = 1665 \Omega$

$r_{i,2} = R_4 \parallel R_5 \parallel \beta_2 \cdot r_{e,2}$   
 $= 20 \text{ K} \parallel 4 \text{ K} \parallel 1665 \Omega = 1110 \Omega$

$r_{0,1} = a^2 \cdot r_{i,2} = 5^2 \times 1110 = 27,750 \Omega$

(i)  $A_{v,1} = \frac{r_{0,1}}{r_{e,1}} = \frac{27,750}{33.3} = 830$

(ii)  $A_{v,2} = \frac{r_{0,2}}{r_{e,2}} = \frac{25 \times 100}{33.3} = 750$

(iii)  $A_v = 830 \times 750 = 622,500$  ;  $G_v = 20 \log_{10} 622,500 = 116 \text{ dB}$

$I_{E,1} = 1.5/1 = 1.5 \text{ mA}$

— same as  $r_{e,1}$

### 61.8. Advantages of Transformer Coupling

1. The operation of a transformer-coupled system is basically *more efficient* because of low dc resistance of the primary connected in the collector circuit,
2. It provides a *higher* voltage gain,
3. It provides *impedance matching* between stages which is desirable for maximum power transfer. Typically, the input impedance of a transistor stage is less than its output impedance. Hence, secondary impedance of the interstage (or coupling) transformer is typically lower than the primary impedance.

This coupling is effective when the final amplifier output is fed to a low-impedance load. For example, the impedance of a typical loud-speaker varies from  $4 \Omega$  to  $16 \Omega$  whereas output impedance of a transistor stage is several hundred ohms. Use of an output audio transformer can avoid the bad effects of such a mismatch.

### Disadvantages

1. The coupling transformer is costly and bulky particularly when operated at audio frequencies because of its heavy iron core,
2. At radio frequencies, the inductance and winding capacitance present lot of problems,
3. It has poor *frequency response* because the transformer is frequency sensitive. Hence, the frequency range of the transformer-coupled amplifiers is limited.
4. It tends to introduce '*hum*' in the output,



### 61.9. Frequency Response

The characteristics of a coupling transformer are that (i) it introduces inductances in both the input and output circuits (ii) leakage inductance exists between the primary and secondary windings and (iii) both windings introduce shunting (distributed) capacitance especially at high frequencies.

A typical gain *versus* frequency curve for a transformer-coupled amplifier is shown in Fig. 61.16.

It is seen that

1. There is decrease in gain at low frequencies and
2. Also there is decrease in gain at high frequencies except for the *resonant rise* in gain at resonant frequency of the tuned circuit formed by inductance and winding capacitance in the circuit.

The output voltage is equal to ac collector current multiplied by the primary reactance of the coupling transformer. Since at low frequencies, primary reactance is small, the gain is less.

At high frequencies, the distributed capacitance existing between different turns of the winding acts as a *bypass capacitor* and so reduces the output voltage and hence the gain. The peak or exaggerated gain occurs due to resonance or tuning effect of inductance and distributed capacitance which *form a tuned circuit*.

Moreover, there is frequency distortion *i.e.* all frequencies are not amplified equally. In fact, the flat response part of the curve is small as compared to *RC* coupling. However, transformer-coupled amplifiers can be designed to have a flat frequency response curve and excellent fidelity over the entire audio frequency range.

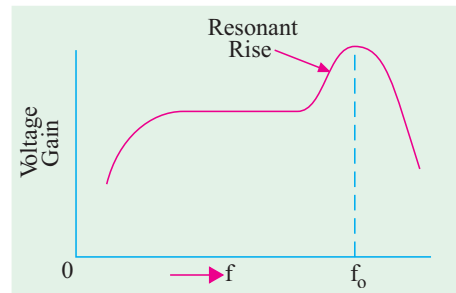


Fig. 61.16

### 61.10. Applications

Transformer coupling is often employed in the last stage of a multistage amplifier where concerted effort is made to maximise power transfer by perfect impedance matching.

**Example 61.6.** In a multistage transformer-coupled amplifier, the output impedance of the first stage is 5 K and the input impedance of the second stage is 1 K. Determine the primary and secondary inductances of the transformer for perfect impedance matching at  $f = 2000$  Hz. If one turn gives an inductance of  $10 \mu\text{H}$ , find the number of primary and secondary turns.

(Electronic Engg.-I, Osmania Univ. 1991)

**Solution.** It should be clearly understood that primary has to match with the output impedance of the first stage and secondary with the input of the second stage.

$$\therefore X_{L_p} = \text{output of 1st stage or } 2\pi f L_p = 5000$$

$$\therefore L_p = 5000/2\pi \times 2000 = \mathbf{0.4 \text{ H}}$$

$$\text{Also, } X_{L_s} = \text{input of 2nd stage}$$

$$\therefore 2\pi f L_s = 1000 ; L_s = 1000/2\pi \times 2000 = \mathbf{0.08 \text{ H}}$$

Now, inductance of a coil varies as the square of its turns.

$$\therefore L \propto N^2 = kN^2$$

$$\text{When, } N = 1, L = 10 \mu\text{H} \quad \therefore 10 \times 10^{-6} = k \times 1^2 \quad \text{or } k = 10^{-5}$$

$$\text{For primary winding } 0.4 = 10^{-5} N_p^2 \times 5 \quad \text{or } N_p = \mathbf{632}$$

$$\text{For secondary winding, } L_s = k N_s^2 \quad \text{or } 0.08 = 10^{-5} N_s^2 \quad \text{or } N_s = \mathbf{89}$$

As seen, it is a nearly 7 : 1 step-down transformer.

61.11. Direct-coupled Two-stage Amplifier Using Similar Transistors

These amplifiers operate without the use of frequency-sensitive components like capacitors, inductors and transformers etc. They are especially suited for amplifying

- (a) ac signals with frequencies as low as a fraction of a hertz,
- (b) change in dc voltages.

Fig. 61.17 shows the circuit of such an amplifier which uses two similar transistors each connected in the CE mode. Both stages employ direct coupling (i) collector of  $Q_1$  is connected directly to the base of  $Q_2$  and (ii) load resistor  $R_2$  is connected to the collector of  $Q_2$ . The resistor  $R_1$  establishes the forward bias of  $Q_1$  and also indirectly that of  $Q_2$ .

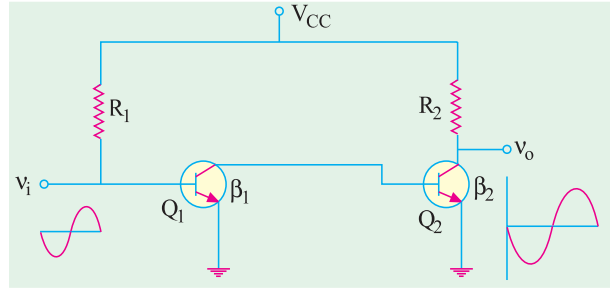


Fig. 61.17

Any signal current at the base of  $Q_1$  is amplified  $\beta_1$  times and appears at the collector of  $Q_1$  and becomes base signal for  $Q_2$ . Hence, it is further amplified  $\beta_2$  times. Obviously, signal current gain of the amplifier is

$$A_i = \beta_1 \times \beta_2 = \beta^2$$

— if transistors are identical

(i) AC Equivalent Circuit

The ac equivalent circuit is shown in Fig. 61.18

(ii) Voltage Gain

The first stage i.e.  $Q_1$  does not produce any voltage gain i.e.  $A_{v1} = 1$  as proved below :

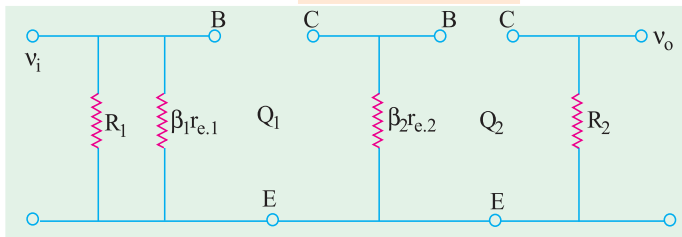


Fig. 61.18

$$A_{v1} = \frac{r_{o1}}{r_{e1}} = \frac{\beta_2 \cdot r_{e2}}{r_{e1}} \quad \dots (i)$$

As seen from Fig. 61.17,  $I_{E2} \cong \beta_2 \cdot I_{B2}$

$$\text{Also } I_{B2} = I_{C1} = I_{E1} \quad \therefore I_{E2} = \beta_2 \cdot I_{E1} \text{ or } \beta_2 = \frac{I_{E2}}{I_{E1}}$$

$$\text{Also } r_{e1} = \frac{25}{I_{E1}} \quad \text{and} \quad r_{e2} = \frac{25}{I_{E2}}$$

Substituting all these values in Eq. (i) above, we get

$$A_{v1} = \frac{I_{E2}}{I_{E1}} \times \frac{25}{I_{E2}} \times \frac{I_{E1}}{25} = 1$$

It proves the statement made above.

$$\text{Now, } A_{v2} = \frac{r_{o2}}{r_{e2}} \quad \text{and} \quad A_v = A_{v1} \times A_{e2} = A_{v2}$$

(iii) Advantages

1. The circuit arrangement is very simple since it uses **minimum number of components**.
2. It is quite **inexpensive**.
3. It has the outstanding ability to **amplify direct current** (i.e. as dc amplifier) and **low-frequency signals**.

4. It has **no coupling or by-pass capacitors** to cause a drop in gain at low frequencies. As seen from Fig. 61.19, the frequency-response curve is flat upto upper cut-off frequency determined by stray wiring capacitance and internal transistor capacitances.

**(iv) Disadvantages**

1. It cannot amplify high-frequency signals.
2. It has poor temperature stability.

It is due to the fact that any variation in base current (due to temperature changes) in one stage is amplified in the following stage (or stages) thereby shifting the Q-point. However, stability can be improved by using emitter-stability resistor (Fig. 61.21).

**(v) Applications**

Some of the applications of direct-coupled amplifiers are in

1. regulator circuits of electronic power supplies,
2. pulse amplifiers
3. differential amplifiers,
4. computer circuitry,
5. electronic instruments.

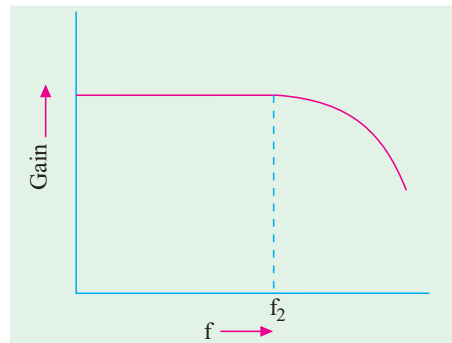


Fig. 61.19

**Example 61.7.** For the direct-coupled amplifier of Fig. 61.20, calculate

- (a) current gain ,
- (b) voltage gain of first stage,
- (c) voltage gain of second stage,
- (d) overall voltage gain in dB,
- (e) overall power gain in dB,
- (f) input resistance.

Neglect  $V_{BE}$  and use  $r_e = 50 \text{ mV}/I_E$ .

**Solution.** (a)  $A_i = \beta_1 \beta_2 = 100 \times 50 = 5000$  (b)  $A_{v,1} = 1$

— Art 61.11

(c)  $A_{v,2} = \frac{r_{o,2}}{r_{e,2}}$

Now,  $r_{e,2} = \frac{50}{I_{E,2}}$

Let us find the value of  $I_{E,2}$  starting from the value of  $I_{B,1}$ . As seen from Fig. 61.20,

$I_{B,1} = 12/1.2\text{M} = 10 \mu\text{A}$   
 $I_{C,1} = \beta_1 \cdot I_{B,1} = 100 \times 10$   
 $= 1000 \mu\text{A}$

$I_{E,1} = I_{C,1} = 1000 \mu\text{A}$   
 $= 1 \text{ mA}$

$I_{B,2} = I_{C,1} = 1 \text{ mA}$   
 $I_{C,2} = \beta_2 \cdot I_{B,2} = 50 \times 1$   
 $= 50 \text{ mA}$

$\therefore I_{E,2} = 50 \text{ mA}$   
 and  $r_{e,2} = 50/50 = 1 \Omega$   
 Also,  $r_{e,2} = R_2 = 200 \Omega$

$\therefore A_{v,2} = \frac{200}{1} = 200$

(d)  $A_n = 1 \times 200 = 200$  ;  $G_v = 20 \log_{10} 200 = 46 \text{ dB}$

(e)  $A_p = A_v \cdot A_i = 200 \times 5,000 = 10^6$  ;  $G_p = 20 \log_{10} 10^6 = 60 \text{ dB}$

(f)  $r_i = R_1 \parallel \beta_1 \cdot r_{e,1}$

$r_{e,1} = \frac{50}{I_{E,1}} = \frac{50}{1} = 50 \Omega$   $\therefore r_i = 1.2 \text{ M} \parallel (50 \times 100) \cong 5 \text{ K}$

— Fig. 61.18

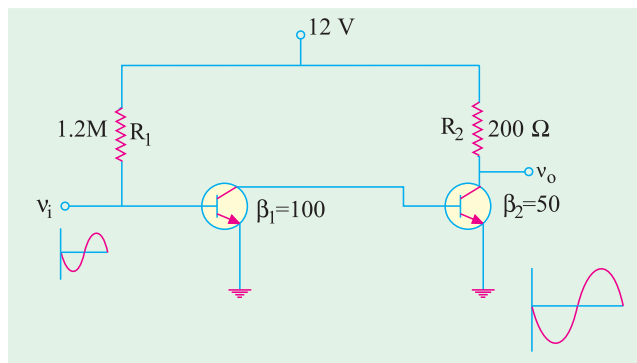


Fig. 61.20

**Example 61.8.** For the emitter-stabilized direct-coupled amplifier of Fig. 61.21, find  
 (i)  $A_{v,1}$ , (ii)  $A_{v,2}$ , (iii)  $A_v$  and (iv)  $r_i$ . Neglect  $V_{BE}$  and use  $r_e = 50 \text{ mV}/I_E$

(Electronics-I, Mysore Univ. 1992)

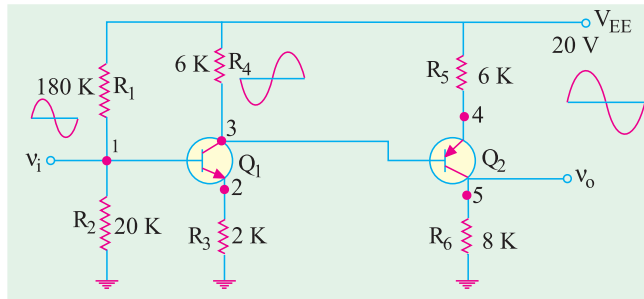


Fig. 61.21

**Solution.** As seen, emitter resistors  $R_3$  and  $R_5$  have been used to improve temperature stability. The voltage divider  $R_1 - R_2$  together with  $R_3$  determines the emitter current of  $Q_1$ . The resistor  $R_4$  has dual function (i) it acts as load resistor for  $Q_1$  and (ii) it establishes base bias of  $Q_2$ .

(i) Since unbypassed resistor  $R_3$  is present

$$A_{v,1} = \frac{r_{0,1}}{r_{e,1} + R_3}$$

As seen from the ac equivalent diagram of Fig. 61.22

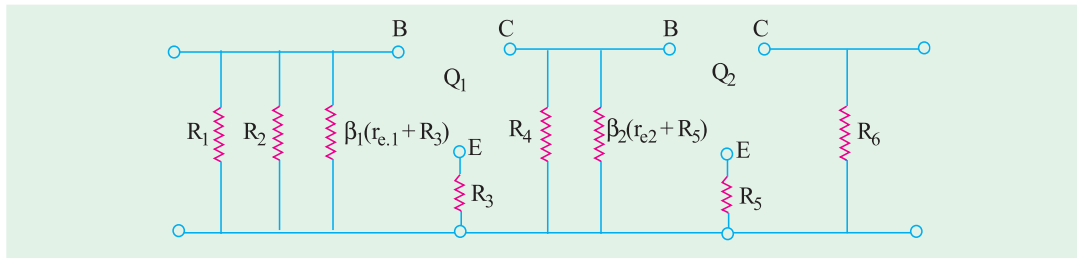


Fig. 61.22

$$r_{0,1} = R_4 \parallel \beta_2 \cdot (r_{e,2} + R_5)$$

Now 
$$r_{e,2} = \frac{50}{I_{E,2}}$$

For finding  $I_{E,2}$ , let us first find  $I_{E,1}$

Drop across 
$$R_2 = 20 \times \frac{20}{180 + 20} = 2 \text{ V}$$

Same is the drop across  $R_3$  since  $V_{BE}$  has been neglected.

$\therefore I_{E,1} = 2 \text{ V} / 1 \text{ K} = 2 \text{ mA}$

Now, 
$$I_{E,2} = \frac{V_{CC} - I_{E,1} R_4}{R_5} = \frac{20 - 2 \times 8}{2} = 2 \text{ mA}$$

$\therefore r_{e,2} = 50 / 2 = 25 \text{ } \Omega$  and  $\beta_2 (r_{e,2} + R_5) \cong 200 \text{ K}$

$r_{0,1} = 8 \text{ K} \parallel 200 \text{ K} = 7.7 \text{ K}$ ;  $r_{e,1} = \frac{50}{2} = 25 \text{ } \Omega$

$\therefore A_{v,1} = \frac{7,700}{(25 + 1000)} = 7.5$

(ii) 
$$A_{v,2} = \frac{r_{0,2}}{(r_{e,2} + R_5)}; r_{0,2} = R_6 = 6000 \text{ } \Omega \therefore A_{v,2} = \frac{6,000}{(25 + 2000)} \cong 3$$

(iii) 
$$A_v = 7.5 \times 3 = 22.5$$

(iv) As seen from Fig. 61.22

$$r_i = R_1 \parallel R_2 \parallel \beta_1 (r_{e,1} + R_3) = 180 \text{ K} \parallel 20 \text{ K} \parallel 100 \text{ K} \text{ ---neglecting } r_{e,1}$$
  

$$= 15.25 \text{ K}$$

**Example. 61.9.** In the circuit of Fig. 61.21, find dc voltage at points marked 1, 2, 3 and 4. Neglect  $V_{BE}$

If a 1-V dc signal at input 1 changes by 0.01 V, what would be the voltage variation at the output?

**Solution.**  $V_1 =$  drop across  $R_2 = 2\text{ V}$   
 $V_2 \cong V_1 = 2\text{ V}; \quad V_3 = V_{CC} - I_{C,1} R_3 = 20 - 2 \times 8 = 4\text{ V}$   
 $V_4 \cong V_3 = 4\text{ V}; \quad V_5 = V_{CC} - I_{C,2} R_6 = 20 - 2 \times 6 = 8\text{ V}$   
 $\Delta v_o = A_v \cdot \Delta v_i = 22.5 \times 0.01 = 0.225\text{ V} = 225\text{ mV}.$

### 61.12. Direct-coupled Amplifier Using Complementary Symmetry of Two Transistors

In this case, an NPN transistor is directly-coupled to its complementary i.e. a PNP transistor. Fig. 61.23 shows a two-stage cascaded amplifier using two complementary transistors connected in CE configuration.

The circuit differs from that shown in Fig. 61.17 in the following three ways :

1. It uses **complementary** transistors rather than **similar** ones,
2. Instead of  $V_{CC}$   $V_{EE}$  power battery has been used,
3. Output is taken directly from terminal of load resistor  $R_2$ .

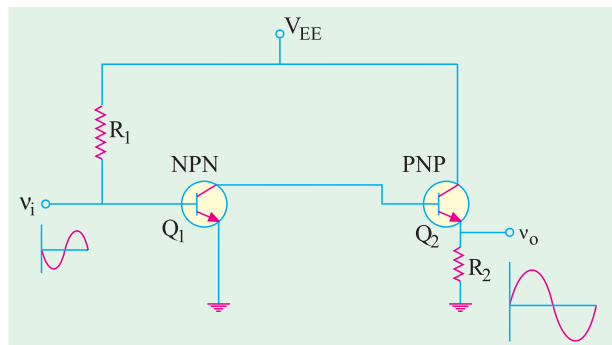


Fig. 61.23

#### More Practical Circuit

A more practical circuit of the above type is shown in Fig. 61.24. Here, bias current of  $Q_1$  is determined by voltage divider  $R_1 - R_2$  and  $R_3$ . As before,  $R_4$  performs two functions :

1. It acts as load for  $Q_1$  and
2. Establishes bias voltage for  $Q_2$

The emitter resistors  $R_3$  and  $R_6$ , as usual, meant to improve amplifier stability.

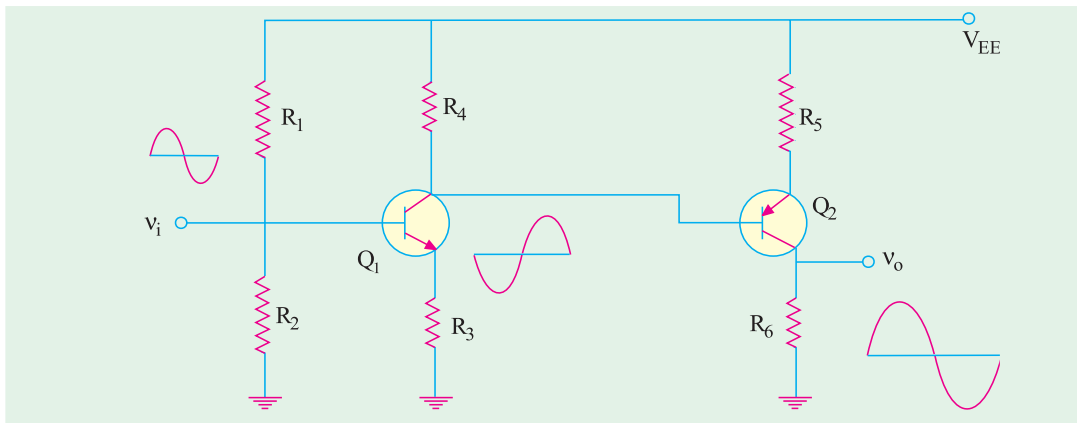


Fig. 61.24

#### (i) Circuit Operation

When a positive-going signal is applied to the base of  $Q_1$ , then

1. its base current **increases**,

2. hence, its collector current increases ( $I_C = \beta I_B$ ), voltage drop across  $R_4$  increases,
4. consequently, voltage at the collector of  $Q_1$  and the base of  $Q_2$  becomes *less positive* or in other words, *more negative*,
5. hence, a negative-going signal is applied to the base of  $Q_2$ .  
The negative-going signal applied to the base of  $Q_2$  causes,
  1. an *increase* in its forward bias (remember, it is a *PNP* transistor),
  2. an *increase* in collector current      3. an *increase* in the voltage developed across  $R_6$ ,
  4. an amplified *positive going* output signal at  $R_6$ .

Hence, it is seen that a signal applied to the input of a two-stage complementary amplifier appears at the output in an amplified form and *of the same polarity*.

### Voltage Gain

It is the same as for the circuit of Fig. 61.17.

**Example 61.10.** For the complementary symmetry circuit of Fig. 61.25, find (a)  $A_v$  (b)  $V_1, V_2, V_3, V_4$  and  $V_5$ . Neglect  $V_{BE}$  and assume  $R_3, R_5 \gg r_e$ .

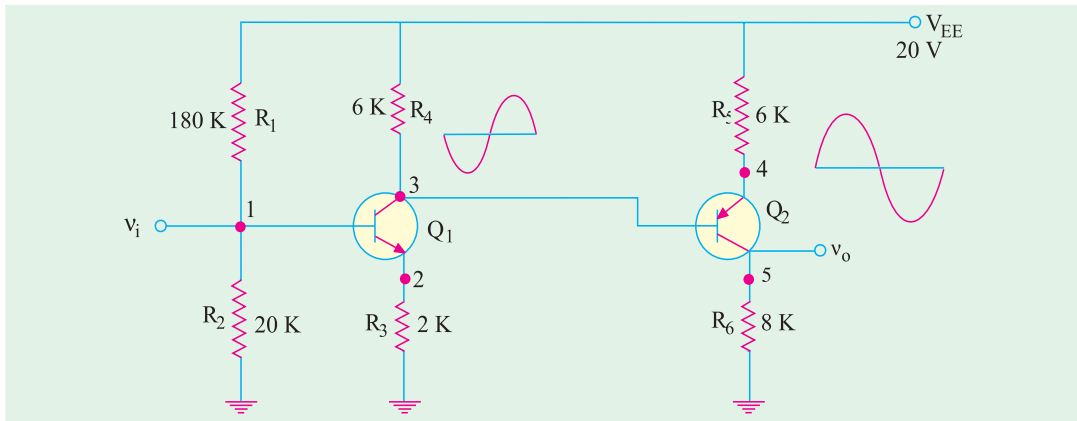


Fig. 61.25

**Solution.** (a) The overall voltage gain is

$$A_v = A_{v,1} \times A_{v,2} \cong \frac{R_4}{R_3} \times \frac{R_6}{R_5} = \frac{6}{2} \times \frac{6}{8}$$

(b) 
$$V_1 = V_{EE} \frac{R_2}{R_1 + R_2} = 2V \qquad \therefore V_2 \cong V_1 = 2V$$

$$\therefore I_{E1} = \frac{2V}{R_3} = \frac{2V}{2K} = 1mA, \qquad I_{C1} \cong I_{E1} = 1mA$$

$$V_3 = V_{EE} - I_{C1} R_4 = 20 - 1 \times 6 = 14V, \qquad V_4 \cong V_3 = 14V;$$

$$I_{C,2} = \frac{20 - 14}{6K} = 1mA$$

$$= \frac{20 - 14}{6K} = 1mA$$

$$\therefore I_{E2} \cong I_{C2} = 1mA$$

$$\begin{aligned}\therefore V_5 &= I_{E,2} \times R_6 \\ &= 1 \times 8 = 8 \text{ V}\end{aligned}$$

### 61.13. Darlington Pair

It is the name given to a *pair of similar* transistors so connected that emitter of one is directly joined to the base of the other as shown in Fig. 61.26 (a). Obviously, the emitter current of  $Q_1$  becomes the base current of  $Q_2$ .

Darlington pairs are commercially mounted in a single package that has only three leads :

base, collector and emitter as shown in Fig. 61.26 (b). It often forms a *double CC stage* in multistage amplifiers. It is so because a Darlington connection can be considered equivalent to two *cascaded emitter followers*.

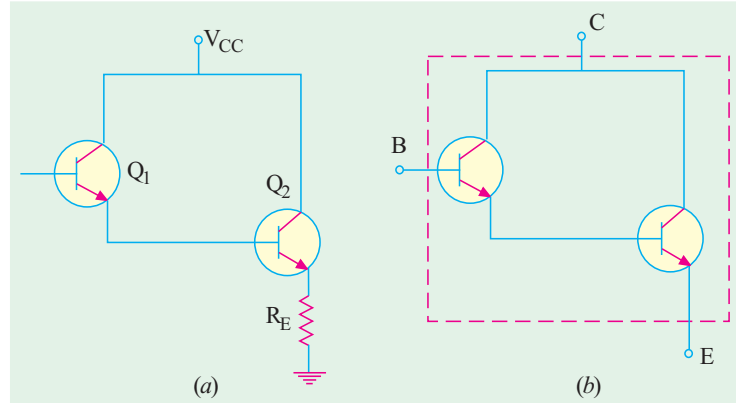


Fig. 61.26

### Main Characteristics

#### (i) Current Gain

It can be proved that current gain of a Darlington pair is  $(1 + \beta_1)(1 + \beta_2) = (1 + \beta)^2 \cong \beta^2$  if the transistors are *identical* (i.e.  $\beta_1 = \beta_2$ ).

#### Proof

$$I_{B,2} = I_{E,1} = (1 + \beta_1) I_{B,1} \cong \beta_1 I_{B,1}$$

$$I_{E,2} \cong \beta_2 \cdot I_{B,2} = \beta_1 \beta_2 I_{B,1} \quad \therefore \quad A_i = \frac{I_{E,2}}{I_{B,1}} = \beta_1 \beta_2 = \beta^2$$

It means that a Darlington pair behaves like a *single transistor having a beta* of  $\beta^2$ .

#### (ii) Input Impedance

In Fig. 61.26 (a), the input impedance seen from the base of  $Q_2$  is

$$r_{i,2} = \beta_2 (r_{e,2} + R_E) \cong \beta_2 R_E$$

Input impedance as seen from the base of  $Q_1$  is

$$r_{i,1} = \beta_1 (r_{e,1} + r_{i,2}) = \beta_1 r_{e,1} + \beta_1 r_{i,2} = \beta_1 r_{e,1} + \beta_1 \beta_2 R_E \cong \beta_1 \beta_2 R_E$$

or  $r_{in(base)} \text{ of } Q_1 = \beta_2 R_E$

**Note.** If there is a load resistance  $R_L$  coupled to the emitter of  $Q_2$ , then

$$r_{i,1} = \beta^2 (R_E \parallel R_L) = \beta^2 r_E$$

As seen, load impedance  $R_E$  has been transformed into  $\beta^2 R_E$ . Obviously, a Darlington pair is capable of high input impedance. In fact, whenever a load causes a severe loss in voltage gain (loading effect), it is usual to step up load impedance *via* a *FET* stage, a single *CC* stage or Darlington pair when much greater impedance transformation is required.

#### (iii) Voltage Gain

Assuming  $r_{e,1} = r_{e,2} = r_e$  we have

$$A_v \cong \frac{R_E}{r_e + R_E} = \frac{1}{1 + \frac{r_e}{R_E}} \cong 1$$

— as in an emitter follower

### 61.14. Advantages of Darlington Pair

1. It can be readily formed from two adjacent transistors in an IC.
2. It has enormous **impedance transformation capability i.e.** it can transform a low-impedance load into a high impedance load. Hence, it is used in a high-gain operational amplifier which depends on very high input impedance for its operation as an integrator or summing amplifier in analogue applications.
3. It uses very few components.
4. It provides very high  $\beta$ -value.

### 61.15. Comparison Between Darlington Pair and Emitter Follower

We will refer to Fig. 9.28 and Fig. 61.26.

1. Input impedance of Darlington pair is  $\beta^2 R_E$  whereas that of emitter follower is  $\beta R_E$  (Art. 9.8).
2. Current gain of Darlington pair is  $\beta^2$  whereas that of emitter follower is  $\beta$ .
3. However, voltage gains of the two are identical.

**Example. 61.11.** For the Darlington pair shown in Fig. 61.27, calculate the value of 1.  $\beta$ , 2. input impedance, 3. voltage amplification.

Assume  $\beta_1 = \beta_2 = 100$  and  $R_L \gg (r_{e1} + r_{e2})$ .

**Solution.** The approximate values are

as under :

1.  $\beta$  of Darlington pair  
 $= \beta_1 \times \beta_2$   
 $= 100 \times 100 = \mathbf{10,000}$
2.  $r_E = R_E \parallel R_L$   
 $= 10 \text{ K} \parallel 500 \Omega = 475 \Omega$   
 $r_{in(base)} \text{ of } Q_1 \cong \beta^2 r_E$   
 $= 10,000 \times 475 = 4.75 \text{ M}$   
 $r_i = \text{input impedance of the pair}$   
 $= R_B \parallel r_{in(base)} \text{ of } Q_1$   
 $= 1 \text{ M} \parallel 4.75 \text{ M} = 0.826 \text{ M} = \mathbf{826 \text{ K}}$
3.  $A_v \cong 1$

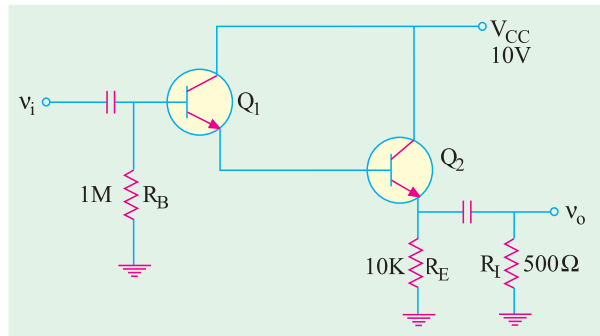


Fig. 61.27

**Example 61.12.** A CE amplifier stage shown in Fig.61.28 is to drive a  $100 \Omega$  load to  $10 \text{ V}_{p-p}$  level. An input signal of  $1 \text{ V}_{p-p}$  is available. Find out if the stage is overloaded or not. Also, find out if this overloading has been avoided by using a Darlington pair as a buffer between  $Q_1$  and the load as shown in Fig. 61.29. Take  $\beta_1 = \beta_2 = 50$ .

**Solution.** The approximate voltage gain of  $Q_1$  is given by

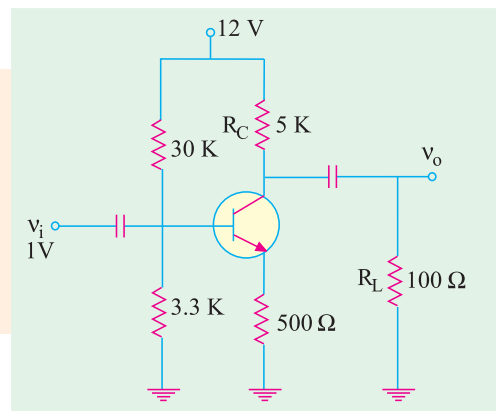


Fig. 61.28



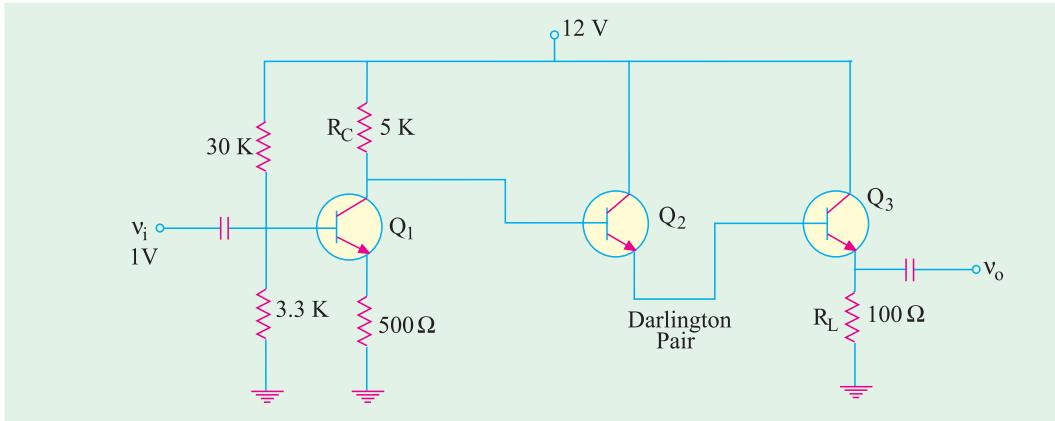


Fig. 61.29

$$A_v = \frac{r_L}{r_E} = \frac{R_C \parallel R_L}{R_E} = \frac{5\text{K} \parallel 100\Omega}{500\Omega} \cong \frac{1}{5}$$

$$\therefore v_o = 1 \times \frac{1}{5} = 0.2 \text{ mV}$$

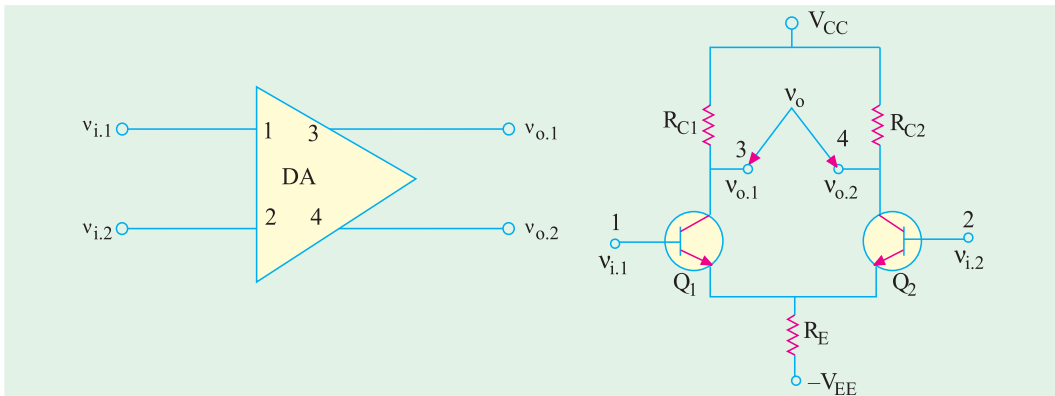


Fig. 61.30

Obviously, *due to overloading of the stage*, severe attenuation has occurred. Hence, the amplifier stage cannot, *by itself*, drive the heavy load.

By using a Darlington pair, a 100 Ω load has been transformed to

$$\beta^2 R_L = 2500 \times 100 = 250 \text{ K}$$

$$\text{Now, } r_L = R_C \parallel \beta^2 R_L = 5 \text{ K} \parallel 250 \text{ K}$$

$$\cong 5 \text{ K} \quad \therefore A_v = \frac{5 \text{ K}}{500 \Omega} = 10$$

This also represents total voltage gain because voltage gain of Darlington pair is one.

$$\therefore v_o = A_v \times v_i = 10 \times 1 = 10 \text{ Vp-p}$$

As seen, now the stage would be able to drive the load since the design goal of 10 V<sub>p-p</sub> has been met.

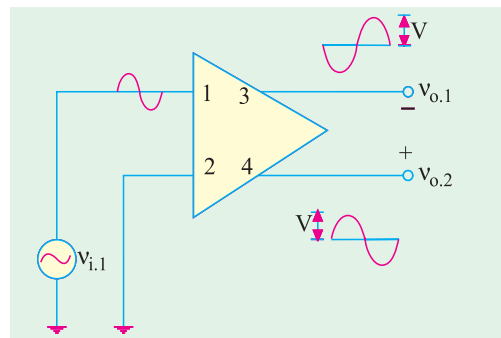


Fig. 61.31

### 61.16. Special Features of a Differential Amplifier

It consists of two basic *CE* amplifiers having their *emitters directly-coupled to each other*. Fig. 61.30 shows both the block diagram and the circuit diagram of such an amplifier.

As seen, it has two separate input terminals 1 and 2 and two separate output terminals 3 and 4. Voltages may be applied to either or both input terminals and output may be taken from either or both output terminals.

There are certain specific phase relationships between both input and both output terminals as discussed below.

#### (a) Single-ended Operation

In Fig. 61.31, input signal  $v_{i,1}$  is applied to terminal 1 with terminal 2 grounded.

It is seen that an *amplified* and *inverted* output signal is obtained at terminal 3 (phase inversion of a *CE* amplifier) but an *equally-amplified* and *in-phase* signal appears across output terminal 4. The differential output voltage has the polarity shown in the figure.

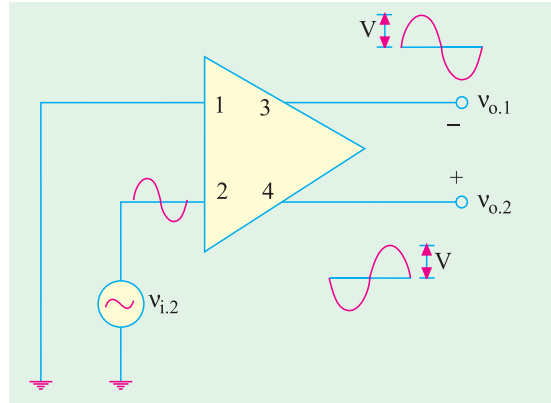


Fig. 61.32



Differential amplifier

As shown in Fig. 61.32, when input signal  $v_{i,2}$  is applied to input terminal 2, an amplified and inverted signal appears at output terminal 4 whereas *equally-amplified* but *in-phase* signal appears at terminal 3.

In summary, we can say that input at any of the two terminals causes outputs at both terminals 3 and 4. The two output are opposite in phase but of equal amplitude.

#### (b) Double-ended Operation

Fig 61.33 illustrates the double ended mode of operation when two input signals of opposite phase are applied to the two input terminals.

Input signal at each input terminal causes signals to appear at both output terminals. The resultant output signals have a peak value of  $2V$  – twice the value for single-ended operation.

However, if two in-phase and equal signals were applied at the two input terminals, the resultant output signal at each output terminal would be zero as shown in Fig. 61.34. It means that output *between* the collectors would be zero.

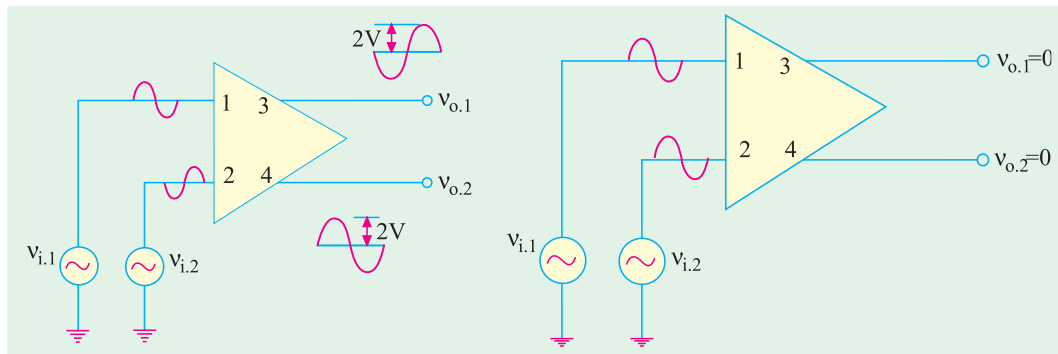


Fig. 61.33

Fig. 61.34

If  $v_{i,1}$  and  $v_{i,2}$  change by *exactly* the same amount, even then output voltage between terminals 4 and 3 remains zero because of symmetry. Only when  $v_{i,1}$  and  $v_{i,2}$  differ from each other, we get an output voltage. When  $v_{i,1}$  is more positive than  $v_{i,2}$ , the output terminal 4 is more positive than terminal 3.

**(c) Inverting and Non-inverting Inputs**

When positive  $v_{i,1}$  acts alone, it produces a differential output voltage with terminal 4 positive with respect to terminal 3 as shown in Fig. 61.35. That is why the input terminal 1 is called non-inverting input terminal. However, when positive  $v_{i,2}$  acts alone, the output voltage is inverted i.e. terminal 3 becomes positive with respect to terminal 4. That is why input terminal 2 is called *inverting terminal*.

**61.17. Common Mode Input**

Fig. 61.36 illustrates the common-mode input of a differential amplifier i.e. when similar or same input signal is applied to both inputs. If the two halves of the diff-amp are identical, the ac output voltage will be zero. The diff-amp is then said to be perfectly balanced.

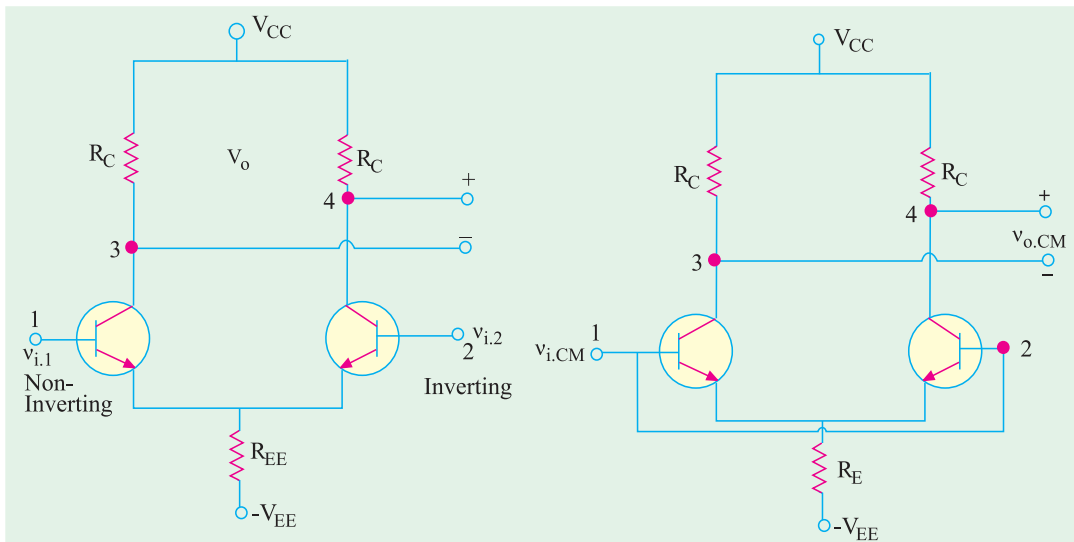


Fig. 61.35

Fig. 61.36

The common-mode rejection ratio (CMMR) is defined as

$$CMMR = \frac{A \times v_{i,cm}}{v_{o,cm}}$$

Suppose,  $v_{i,cm} = 1$  V. In a perfectly-balanced and symmetrical diff-amp, the output should be zero. But, in practice, there is always a small output signal because of *non-symmetry*. Suppose,  $A = 100$  and  $v_{no,em} = 0.01$  V. Then

$$CMMR = \frac{100 \times 1 \text{ V}}{0.01 \text{ V}} = 10,000 = 20 \log_{10} 10,000 = \mathbf{80 \text{ dB}}$$

Larger the value of CMMR (i.e. smaller the value of  $v_{o,cm}$ ), better the diff-amp. Ideally, a common-mode input should produce zero output voltage. Hence, an ideal diff-amp has a CMMR of *infinity*.

**Example 61.13.** Calculate the approximate output voltage for the diff-amp of Fig. 61.37 which uses only a single-ended non-inverting input of  $n-1 = 1$  mV. Take  $r_e = 25$  mV/ $I_E$  and neglect  $V_{BE}$

**Solution.**  $I_E \cong V_{EE}/R_E = 12/6 = 2$  mA

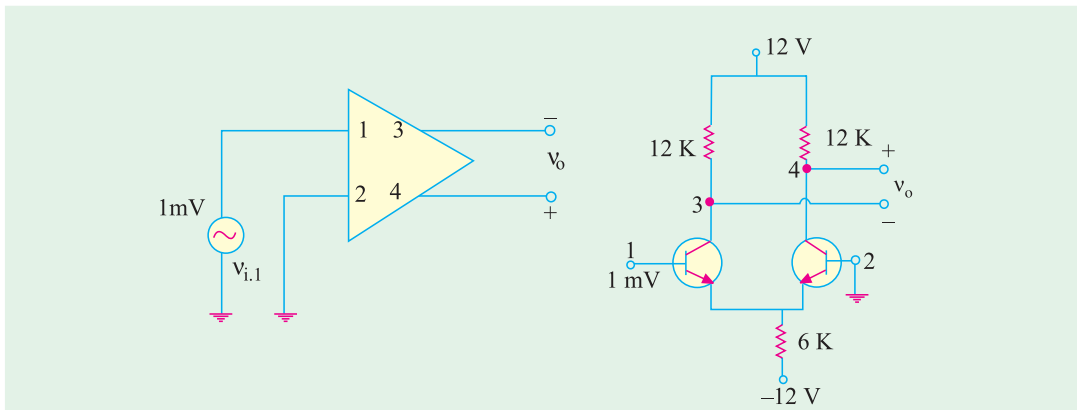


Fig. 61.37

The current is divided equally between the two transistors so that  $I_{E,1} = I_{E,2} = 1 \text{ mA}$ .  
Voltage amplification of each half is approximately given by  $A = R_C / r_e$

Now,  $r_e = 25/1 = 25 \Omega$

$$\therefore A = \frac{12,000}{25} = 480$$

$$\therefore v_o = A \times v_{i,1} = 480 \times 1$$

$$= \mathbf{480 \text{ mV}}$$

The ac output signal is in phase with the input.

**Example 61.14.** If a differential input signal of 1 mV is applied to the diff-amp shown in Fig. 61.38, calculate the output voltage. Neglect  $V_{BE}$  and take  $r_e = 25 \text{ mV}/I_E$

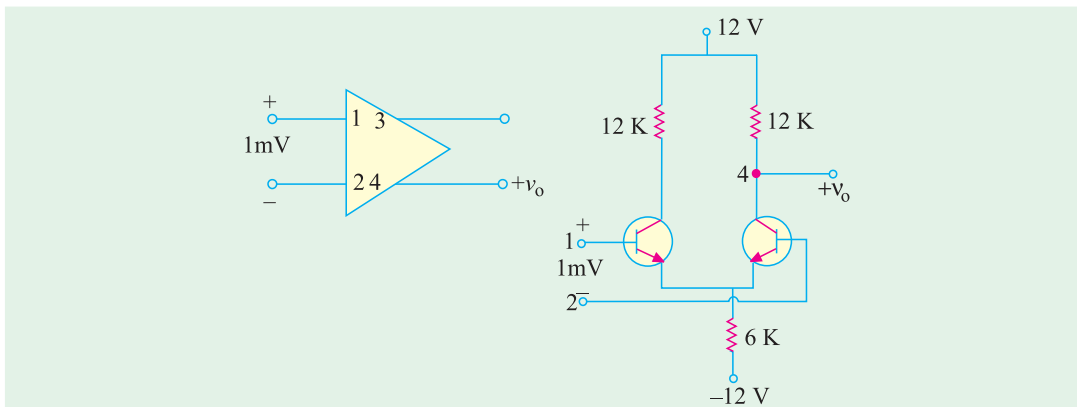


Fig. 61.38

**Solution.**

Here,  $v_{i,1} - v_{i,2} = 1 \text{ mV}$

As seen from Ex. 61.13,

$$A = \mathbf{480}$$

$$\therefore v_o = A (v_{i,1} - v_{i,2}) = 480 \times 1 = \mathbf{480 \text{ mV}}$$

**Example 61.15.** A differential input signal of 1 mV is applied to the diff-amp of Fig. 61.39 when used in single-ended output mode. Calculate the approximate value of output voltage. Neglect  $V_{BE}$  and take  $r_e = 25 \text{ mV}/I_E$ .

**Solution.** Here, again 1 mV is the differential input signal i.e.  $v_{i,1} - v_{i,2} = 1 \text{ mV}$

Since single-ended mode is being used,  
 $A = 480/2 = 240$

$$\therefore v_o = A(v_{i,1} - v_{i,2}) = 240 \times 1 = 240$$

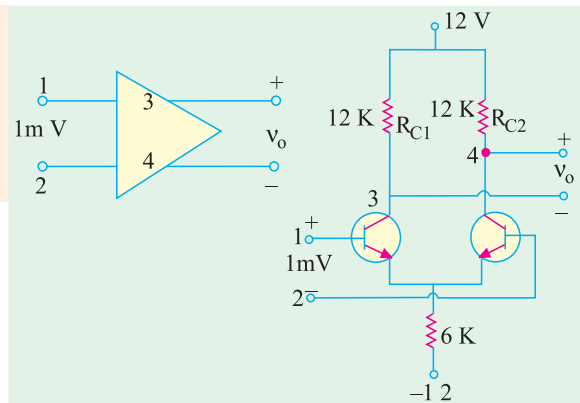


Fig. 61.39

### 61.18. Differential Amplifier

Fig. 61.40 shows the circuit of a differential or difference amplifier. As seen

1. it contains two CE amplifiers,
2. it uses only resistors and transistors
3. it is directly-coupled (emitter-to-emitter) amplifier,
4. it can accept two inputs by means of  $T_1$  and ground and also  $T_2$  and ground,
5. it can provide two separate outputs by means of  $T_3$  and ground and  $T_4$  and ground,
6. it can provide a single output between  $T_3$  and  $T_4$  i.e. differential output.

#### Circuit Operation

We will consider a balanced differential amplifier in which  $Q_1$  and  $Q_2$  are identical and their associated components are matched. In that case, each amplifier stage produces same voltage gain.  $A = r_o/r_e$

In Fig. 61.40.  $r_o = R_2$  or  $R_3$

The output voltage between terminals  $T_3$  and  $T_4$  is

$$v_o(T_3 - T_4) = A(v_{i,1} - v_{i,2})$$

where,  $A =$  voltage gain of each stage

#### Advantages

1. It uses no frequency-dependent coupling or bypassing capacitors. All that it requires is resistors and transistors both of which can be easily integrated on a chip. Hence, it is extensively used in linear IC<sub>5</sub>.
2. It can compare any two signals and **detect any difference**. Thus, if two signals are fed into its inputs, identical in every respect except that one signal has been slightly distorted, then only the difference between the two signals i.e. distortion will be amplified.
3. It gives higher gain than two cascaded stages of ordinary direct coupling.
4. It provides very **uniform amplification** of signal from dc upto very high frequencies.

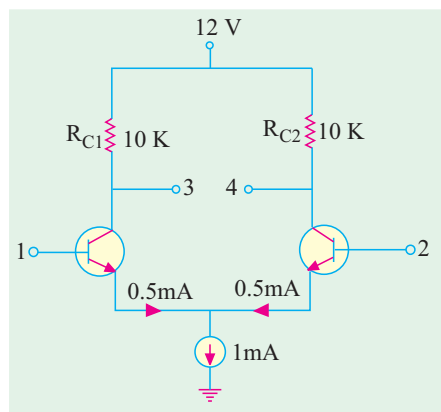


Fig. 61.40

5. It provides isolation between input and output circuits.
6. It is almost a universal choice for amplifying dc.
7. It finds a wide variety of applications such as amplification, mixing, signal generation, amplitude modulation, frequency multiplication and temperature compensation etc.

**Example 61.16.** Calculate the single-ended and differential gain of the diff-amp shown in Fig. 61.41. Use  $r_e = 25 \text{ mV}/I_E$ .

**Solution.** The current of 1 mA from a constant-current source divides into two equal parts.

$$I_{E,1} = I_{E,2} = 1/2 = 0.5 \text{ mA}$$

$$r_{e,1} = r_{e,2} = \frac{25}{0.5} = 50 \Omega$$

Hence, single-ended voltage gain is

$$A = \frac{r_0}{r_e} = \frac{R_C}{r_e} = \frac{10 \text{ K}}{50 \Omega} = 200$$

Hence, each stage has a voltage gain of 200.

If we consider differential (double-ended) gain, its value is twice i.e.  $2 \times 200 = 400$ .

**Example 61.17.** For the diff-amp shown in Fig. 61.41, voltage gain of each stage is 200,  $v_{i,1} = 30 \text{ mV}$  and  $v_{i,2} = 20 \text{ mV}$ . Find the voltages between

(i)  $T_3$  and ground, (ii)  $T_4$  and ground, (iii)  $T_3$  and  $T_4$ .

What are the polarities of output terminals  $T_3$  and  $T_4$ ?

(Basic Electronics, Bombay Univ.)

**Solution.** As stated earlier in Art. 61.18.

$$(i) \quad v_0(T_3) = A_1 \times v_{i,1} = 200 \times 30 \text{ mV} = 6 \text{ V}$$

$$(ii) \quad v_0(T_4) = A_2 \times v_{i,2} = 200 \times 20 \text{ mV} = 4 \text{ V}$$

$$(iii) \quad v_0(T_3 - T_4) = A(v_{i,1} - v_{i,2}) = 200(30 - 20) \text{ mV} = 2 \text{ V}$$

Since  $v_{i,1} > v_{i,2}$ ;  $i_{c,1} R_2 > i_{c,2} R_3$ , hence  $T_4$  will be positive with respect to  $T_3$ .

**Example 61.18.** Calculate the overall voltage gain of the two-stage RC coupled amplifier shown in Fig. 61.42. Neglect  $V_{BE}$  and take  $\beta_1 = \beta_2 = 100$ .

**Solution.** We will first find gain of  $Q_2$  and then multiply it with that of  $Q_1$  to find the overall gain.

DC voltage from base to ground for  $Q_2 =$  drop across 40 K

$$= 30 \times 40 / (40 + 80) = 10 \text{ V}$$

Hence,  $I_{E,2} \cong 10 \text{ V} / 10 \text{ K} = 1 \text{ mA}$

Assuming silicon transistor

$$r_{e,2} = 50 / 1 \text{ mA} = 50 \Omega$$

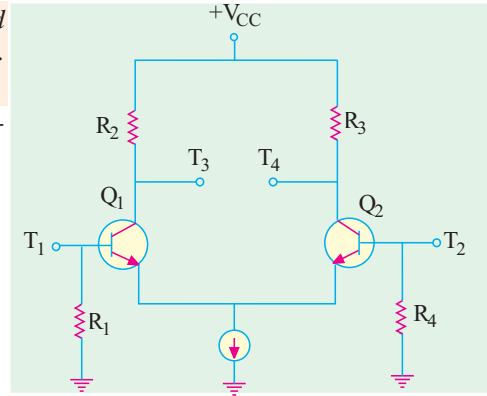


Fig. 61.41

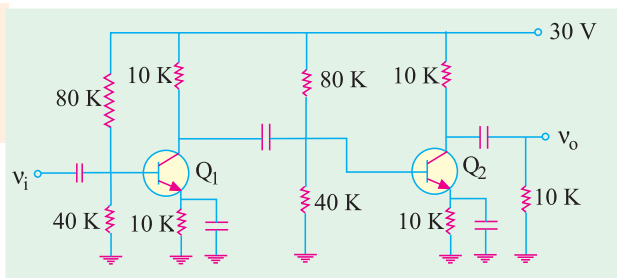


Fig. 61.42

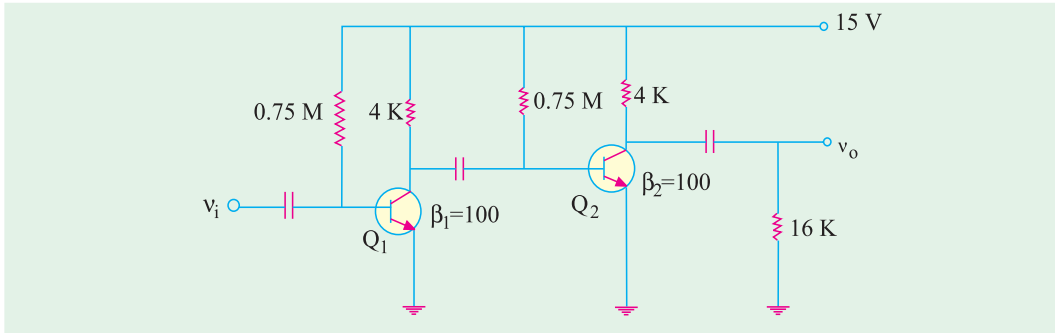


Fig. 61.43

$$\therefore A_{v2} = \frac{r_{L2}}{r_{e2}} = \frac{10\text{K} \parallel 10\text{K}}{50} = 100$$

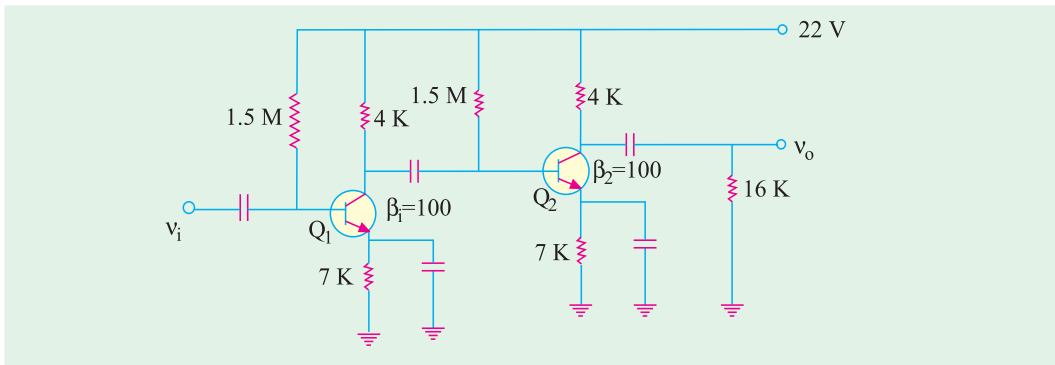


Fig. 61.44

For finding  $A_{v1}$ , we must first calculate the ac load resistance  $r_L$  as seen by  $Q_1$ . It equals the parallel combination of 10 K, 80 K, 40 K and  $\beta r_e = 100 \times 50 = 5\text{ K}$  (because it forms part of the load on  $Q_1$ )\*.

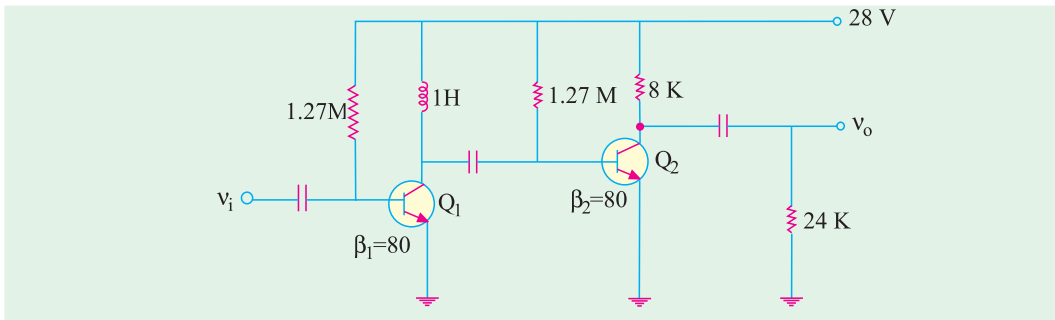


Fig. 61.45

$$\therefore r_{L1} = 10\text{ K} \parallel 80\text{ K} \parallel 40\text{ K} \parallel 5\text{ K} \cong 3\text{ K}$$

$$\therefore A_{v1} = r_{L1}/R_{e1} = 3\text{ K}/50\ \Omega = 60$$

$$\therefore A = A_{v1} \times A_{v2} = 100 \times 60 = \mathbf{6000}$$

\* (Emitter resistance does not come into the picture because it has been ac grounded by the capacitor.)

## Tutorial Problems No. 61.1

1. For the two-stage  $R_C$ -coupled amplifier shown in Fig. 61.43 calculate the approximate values of
- (a) voltage gain for the first stage, (b) voltage gain for the second stage,  
 (c) voltage gain for the amplifier, (d) ac input resistance of the amplifier.  
 Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$  [(a) 76.2 (b) 256 (c) 19,500 or 85.8 dB (d) 1250  $\Omega$ ]

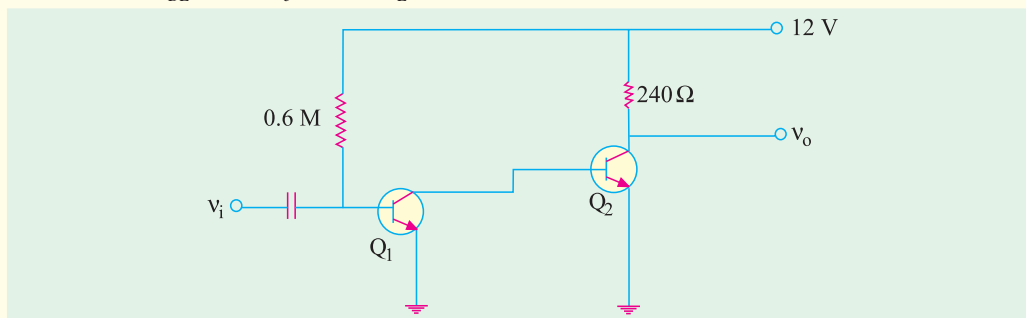


Fig. 61.46

2. For the two-stage  $RC$ -coupled amplifier using base and emitter bias and shown in Fig. 61.44, calculate approximate values of
- (i)  $A_{v,1}$  (ii)  $A_{v,2}$  (iii)  $A_v$  (iv)  $r_i$   
 Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$  [(i) 61.6 (ii) 128 (iii) 7,900 or 78 dB (iv) 2.6 K]
3. Fig. 61.45 shows the circuit of a two-stage impedance-coupled amplifier. Compute the approximate values of
- (i) voltage gain of 1st stage,  
 (ii) voltage gain of the 2nd stage ; both at  $f = 2 \text{ kHz}$ ,  
 (iii) voltage gain of the amplifier.  
 Neglect  $V_{BE}$  and take  $r_e = 25 \text{ mV}/I_E$ . [(i) 80 (ii) 420 (iii) 33,600 or 90.5 dB]
4. For the direct-coupled amplifier shown in Fig. 61.46, calculate the approximate value of
- (i)  $A_i$ , (ii)  $A_{v,1}$ , (iii)  $A_{v,2}$ , (iv)  $A_v$ , (v)  $A_p$  and (vi)  $r_d$   
 Neglect  $V_{BE}$  and assume  $r_e = 25 \text{ mV}/I_E$ . Also,  $\beta_1 = 50$  and  $\beta_2 = 25$ .  
 [(i) 1250 (ii) 1 (iii) 240 (iv) 240 (v) 300,000 or 55 dB (vi) 1250  $\Omega$ ]

## OBJECTIVE TESTS – 61

- The decibel gain of a cascaded amplifier equals the
  - product of individual gains
  - sum of individual gains
  - ratio of stage gains
  - product of voltage and current gains.
- If two stages of a cascaded amplifier have decibel gains of 60 and 30, then overall gain is ..... dB.
  - 90
  - 1800
  - 2
  - 0.5
- Cascading two amplifiers will result in
  - reduction in overall gain and increase in overall bandwidth.
  - reduction in overall gain and reduction in overall bandwidth
  - increase in overall gain increase in overall bandwidth
  - increase in overall gain and reduction in overall bandwidth
- The overall bandwidth of two identical voltage amplifiers connected in cascade will
  - remain the same as that of a single stage
  - be worse than that of a single stage
  - be better than that of a single stage
  - be better if stage gain is low and worse if stage gain is high.
- $RC$  coupling is popular in low-level audio amplifiers because it
  - has better low frequency response
  - is inexpensive and needs no adjustments
  - provides an output signal in phase with the input signal
  - needs low voltage battery for collector supply.



6. Frequency response characteristic at a single-stage RC coupled amplifier is shown in Fig. 61.47. The fall in gain at both ends of the characteristic is due to
- transistor shunt capacitances
  - bypass and coupling capacitances of the circuit
  - transistor shunt capacitances of the lower end and bypass and coupling capacitances at the higher end

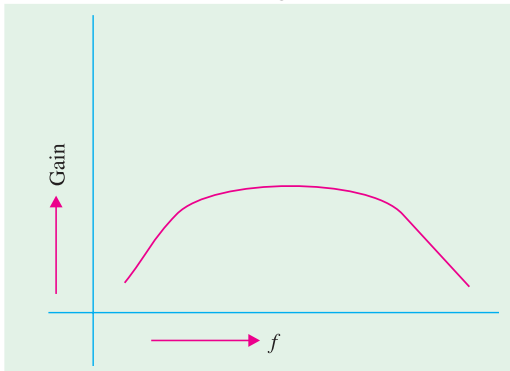


Fig. 61.47

- transistor shunt capacitances at the higher end and bypass and coupling capacitances at the lower end.
7. The most desirable feature of transformer coupling is its
- higher voltage gain
  - wide frequency range
  - ability to provide impedance matching between stages
  - ability to eliminate hum from the output.
8. A transformer coupled amplifier would give
- maximum voltage gain
  - impedance matching
  - maximum current gain
  - larger bandwidth
9. In multistage amplifiers, direct coupling is especially suited for amplifying
- high frequency ac signals
  - changes in dc voltages
  - high-level voltages
  - sinusoidal signals.
10. The outstanding characteristic of a direct-coupled amplifier is its
- utmost economy
  - temperature stability
  - avoidance of frequency-sensitive components
  - ability to amplify direct current and low-frequency signals.
11. A signal may have frequency components which lie in the range of 0.001 Hz to 10 Hz. Which one of the following types of couplings should be chosen in a multistage amplifier designed to amplify this signal ?
- RC coupling
  - transformer coupling
  - direct coupling
  - double-tuned transformer
12. Darlington pairs are frequently used in linear ICs because they
- do not require any capacitors or inductors
  - have enormous impedance transformation capability
  - can be readily formed from two adjacent transistors
  - resemble emitter followers.
13. When same input signal is applied to both the inputs of an ideal diff-amp, the output
- is zero
  - depends on its *CMMR*
  - depends on its voltage gain
  - is determined by its symmetry.
14. The common-mode rejection ratio of an ideal diff-amp is
- zero
  - infinity
  - less than unity
  - greater than unity.
15. One of the advantages of a Darlington pair is that it has enormous ..... transformation capacity.
- voltage
  - current
  - impedance
  - power
16. A Darlington pair and an emitter follower have the same
- input impedance
  - current gain
  - voltage gain
  - power gain.
17. Which of the following mode of operation is possible with a differential amplifier ?

- (a) single-ended input
  - (b) differential input
  - (c) common-mode input
  - (d) all of the above.
- 18.** The gain of a Darlington amplifier pair is determined by ..... the beta values.
- (a) subtracting
  - (b) adding
  - (c) dividing
  - (d) multiplying
- 19.** The amplifier in which the emitter and collector leads of one transistor are connected to the base and collector leads of a second transistor is called an ..... amplifier.
- (a) push-pull
  - (b) Darlington
  - (c) differential
  - (d) complementary

**ANSWERS**

- 1. (b) 2. (a) 3. (c) 4. (c) 5. (b) 6. (d) 7. (c) 8. (b) 9. (b) 10. (d) 11. (c)**  
**12. (c) 13. (a) 14. (b) 15. (c) 16. (c) 17. (d) 18. (d) 19. (b)**

# CHAPTER 62

## Learning Objectives

- Feedback Amplifiers
- Principle of Feedback Amplifiers
- Advantages of Negative Feedback
- Gain Stability
- Decreased Distortion
- Feedback Over Several Stages
- Increased Bandwidth
- Forms of Negative Feedback
- Shunt-derived Series-fed Voltage Feedback
- Current-series Feedback Amplifier
- Voltage-shunt Negative Feedback Amplifier
- Current-shunt Negative Feedback Amplifier
- Noninverting Op-amp with Negative Feedback
- Effect of Negative Feedback on  $R_{in}$  and  $R_{out}$
- $R_{in}$  and  $R_{out}$  of Inverting Op-amp with Negative Feedback

## FEEDBACK AMPLIFIER



A feedback amplifier is one in which a fraction of the amplifier output is fed back to the input circuit

### 62.1. Feedback Amplifiers

A feedback amplifier is one in which a fraction of the amplifier output is fed back to the input circuit. This partial dependence of amplifier output on its input helps to control the output. A feedback amplifier consists of two parts : an amplifier and a feedback circuit.

**(i) Positive feedback**

If the feedback voltage (or current) is so applied as to increase the input voltage (*i.e.* it is in phase with it), then it is called positive feedback. Other names for it are : **regenerative or direct** feedback.

Since positive feedback produces excessive distortion, it is seldom used in amplifiers. However, because it increases the power of the original signal, it is used in oscillator circuits.

**(ii) Negative feedback**

If the feedback voltage (or current) is so applied as to reduce the amplifier input (*i.e.* it is 180° out of phase with it), then it is called negative feedback. Other names for it are : **degenerative or inverse** feedback.

Negative feedback is frequently used in amplifier circuits.

### 62.2. Principle of Feedback Amplifiers

For an ordinary amplifier *i.e.* one without feedback, the voltage gain is given by the ratio of the output voltage  $V_o$  and input voltage  $V_i$ . As shown in the block diagram of Fig. 62.1, the input voltage  $V_i$  is amplified by a factor of  $A$  to the value  $V_o$  of the output voltage.

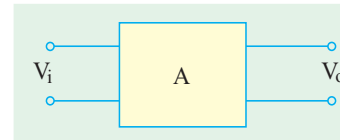


Fig. 62.1

$$\therefore A = V_o / V_i$$

This gain  $A$  is often called **open-loop** gain.

Suppose a feedback loop is added to the amplifier (Fig. 62.2). If  $V_o'$  is the output voltage with feedback, then a fraction  $\beta^*$  of this voltage is applied to the input voltage which, therefore, becomes  $(V_i \pm \beta V_o')$  depending on whether the feedback voltage is in phase or antiphase with it. Assuming positive feedback, the input voltage will become  $(V_i + \beta V_o')$ . When amplified  $A$  times, it becomes  $A(V_i + \beta V_o')$ .

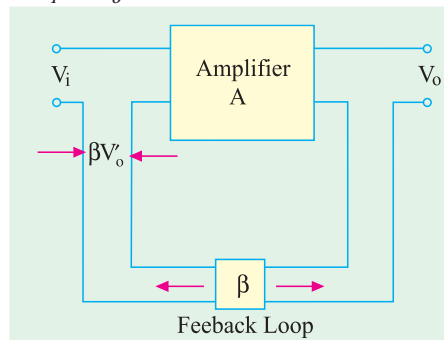


Fig. 62.2

$$\begin{aligned} \therefore A(V_i + \beta V_o') &= V_o' \\ \text{or } V_o'(1 - \beta A) &= AV_i \end{aligned}$$

The amplifier gain  $A'$  with feedback is given by

$$A' = \frac{V_o'}{V_i} = \frac{A}{1 - \beta A}$$

$$\therefore A' = \frac{A}{1 - \beta A} \quad \text{— positive feedback}$$

$$= \frac{A}{1 - (-\beta A)} = \frac{A}{1 + \beta A}$$

— negative feedback

The term ' $\beta A$ ' is called **feedback factor** whereas  $\beta$  is known as **feedback ratio**. The expression  $(1 \pm \beta A)$  is called **loop gain**. The amplifier gain  $A'$  with feedback is also referred to as **closed-loop gain** because it is the gain obtained after the feedback loop is closed. The sacrifice factor is defined as  $S = A/A'$ .

---

\* It may please be noted that it is not the same as the  $\beta$  of a transistor (Art.57.9)

**(a) Negative Feedback**

The amplifier gain with negative feedback is given by  $A' = \frac{A}{(1+\beta A)}$

Obviously,  $A' < A$  because  $|1 + \beta A| > 1$ .

Suppose,  $A = 90$  and  $\beta = 1/10 = 0.1$

Then, gain without feedback is 90 and with negative feedback is

$$A' = \frac{A}{1+\beta A} = \frac{90}{1+0.1 \times 90} = 9$$

As seen, negative feedback reduces the amplifier gain. That is why it is called **degenerative** feedback. A lot of voltage gain is sacrificed due to negative feedback. When  $|\beta A| \gg 1$ , then

$$A' \cong \frac{A}{\beta A} \cong \frac{1}{\beta}$$

It means that  $A'$  depends only on  $\beta$ . But it is very stable because it is not affected by changes in temperature, device parameters, supply voltage and from the aging of circuit components etc. Since resistors can be selected very precisely with almost zero temperature-coefficient of resistance, it is possible to achieve highly precise and stable gain with negative feedback.

**(b) Positive Feedback**

The amplifier gain with positive feedback is given by

$$A' = \frac{A}{1-\beta A} \quad \text{Since } |1 - \beta A| < 1, A' > A$$

Suppose gain without feedback is 90 and  $\beta = 1/100 = 0.01$ , then gain with positive feedback is

$$A' = \frac{90}{1 - (0.01 \times 90)} = 900$$

Since positive feedback increases the amplifier gain. It is called **regenerative** feedback. If  $\beta A = 1$ , then mathematically, the gain becomes infinite which simply means that there is an output without any input! However, electrically speaking, this cannot happen. What actually happens is that the amplifier becomes an oscillator which supplies its own input. In fact, two important and necessary conditions for circuit oscillation are

1. the feedback must be positive,
2. feedback factor must be unity *i.e.*  $\beta A = +1$ .

**62.3. Advantages of Negative Feedback**

The numerous advantages of negative feedback outweigh its only disadvantage of reduced gain.

Among the advantages are :

1. higher fidelity *i.e.* more linear operation,
2. highly stabilized gain,
3. increased bandwidth *i.e.* improved frequency response,
4. less amplitude distortion,
5. less harmonic distortion,
6. less frequency distortion,
7. less phase distortion,
8. reduced noise,
9. input and output impedances can be modified as desired.

**Example 62.1.** In the series-parallel (SP) feedback amplifier of Fig. 62.3, calculate

- (a) open-loop gain of the amplifier,
- (b) gain of the feedback network,
- (c) closed-loop gain of the amplifier,
- (d) sacrifice factor,  $S$ .

(Applied Electronics-I, Punjab Univ. 1991)

**Solution.** (a) Since 1 mV goes into the amplifier and 10 V comes out

$$\therefore A = \frac{10\text{V}}{1\text{mV}} = 10,000$$

- (b) The feedback network is being driven by the output voltage of 10 V.  
 $\therefore$  Gain of the feedback network

$$= \frac{\text{output}}{\text{input}} = \frac{250\text{mV}}{10\text{V}} = 0.025$$

- (c) So far as the feedback amplifier is concerned, input is  $(250 + 1) = 251\text{ mV}$  and final output is 10 V. Hence, gain with feedback is

$$A' = 10\text{ V}/251\text{ mA} = 40$$

- (d) The sacrifice factor is given by

$$S = \frac{A}{A'} = \frac{10,000}{40} = 250$$

By sacrificing so much voltage gain, we have improved many other amplifier quantities. (Art. 62.3)

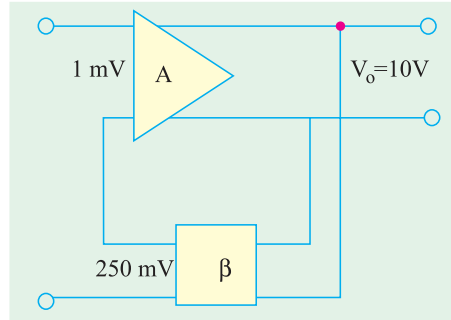


Fig. 62.3

**Example 62.2.** Calculate the gain of a negative feedback amplifier whose gain without feedback is 1000 and  $\beta = 1/10$ . To what value should the input voltage be increased in order that the output voltage with feedback equals the output voltage without feedback ?

**Solution.** Since  $|\beta A| \gg 1$ , the closed-loop gain is  $A' \cong \frac{1}{\beta} \cong \frac{1}{1/10} = 10$

The new increased input voltage is given by

$$V_i' = V_i(1 + \beta A) = 50(1 + 0.04 \times 100) = 250\text{ mV}$$

**Example 62.3.** In a negative-feedback amplifier,  $A = 100$ ,  $\beta = 0.04$  and  $V_i = 50\text{ mV}$ . Find

- (a) gain with feedback, (b) output voltage,  
 (c) feedback factor, (d) feedback voltage. (Applied Electronics, AMIEE, London)

**Solution.** (a)  $A' = \frac{A}{1 + \beta A} = \frac{100}{1 + 0.04 \times 100} = 20$

(b)  $V_o' = A' V_i = 20 \times 50\text{ mV} = 1\text{ V}$  (c) feedback factor =  $\beta A = 0.04 \times 100 = 4$

(d) Feedback voltage =  $\beta V_o' = 0.04 \times 1 = 0.04\text{ V}$

**Example 62.4.** An amplifier having a gain of 500 without feedback has an overall negative feedback applied which reduces the gain to 100. Calculate the fraction of output voltage feedback. If due to ageing of components, the gain without feedback falls by 20%, calculate the percentage fall in gain without feedback. (Applied Electronics-II, Punjab Univ. 1993)

**Solution.**  $A' = \frac{A}{1 + \beta A} \quad \therefore \quad 1 + \beta A = \frac{A}{A'}$

$$\therefore \beta = \frac{1}{A'} - \frac{1}{A} = \frac{1}{100} - \frac{1}{500} = 0.008$$

Now, gain without feedback = 80% of 500 = 400

$$\therefore \text{New } A' = \frac{400}{1 + 0.008 \times 400} = 95.3$$

Hence, change in the gain with feedback in the two cases =  $100 - 95.3 = 4.7$

$$\therefore \text{Percentage fall in gain with feedback is } = \frac{4.7}{100} \times 100 = 4.7\%$$

**Example 62.5.** An amplifier with negative feedback has a voltage gain of 100. It is found that without feedback an input signal of 50 mV is required to produce a given output whereas with feedback, the input signal must be 0.6 V for the same output. Calculate the value of voltage gain without feedback and feedback ratio. (Bangalore University 2001)

**Solution.**  $V_o' = AV_i = 100 \times 0.6 = 60 \text{ V}$  and  $V_o = AV_i$

Since the output voltage with and without feedback are required to be the same,

$$\therefore 60 = A \times 50 \text{ mV}, \quad \therefore A = \frac{60}{50 \text{ mV}} = 1200$$

The amplifier gain with feedback,

$$A' = \frac{A}{1 + \beta A} \quad \text{or} \quad \beta = \frac{A - A'}{AA'} = \frac{1200 - 100}{1200 \times 100} = 0.009$$

#### 62.4. Gain Stability

The gain of an amplifier with negative feedback is given by  $A' = \frac{A}{1 + \beta A}$

Taking logs of both sides, we have  $\log_e A' = \log_e A - \log_e(1 + \beta A)$

Differentiating both sides, we get

$$\frac{dA'}{A'} = \frac{dA}{A} - \frac{\beta \cdot dA}{1 + \beta A} = dA \left( \frac{1}{A} - \frac{\beta}{1 + \beta A} \right) = \frac{1}{1 + \beta A} \frac{dA}{A} = \frac{(dA/A)}{1 + \beta A}$$

If  $\beta A \gg 1$ , then the above expression becomes

$$\frac{dA'}{A'} = \frac{1}{\beta A} \cdot \frac{dA}{A}$$

**Example 62.6.** An amplifier has an open-loop gain of 400 and a feedback of 0.1. If open-loop gain changes by 20% due to temperature, find the percentage change in closed-loop gain. (Electronics-III, Bombay 1991)

**Solution.** Here,  $A = 400$ ,  $\beta = 0.1$ ,  $dA/A = 20\% = 0.2$

$$\text{Now, } \frac{dA'}{A'} = \frac{1}{\beta A} \cdot \frac{dA}{A} = \frac{1}{0.1 \times 400} \times 20\% = 0.5\%$$

It is seen that while the amplifier gain changes by 20%, the feedback gain changes by only 0.5% i.e. an improvement of  $20/0.5 = 40$  times

#### 62.5. Decreased Distortion

Let the harmonic distortion voltage generated within the amplifier change from  $D$  to  $D'$  when negative feedback is applied to the amplifier.

$$\text{Suppose } D' = x D \quad \dots (i)$$

The fraction of the output distortion voltage which is feedback to the input is

$$\beta D' = \beta x D$$

After amplification, it become  $\beta x D_A$  and is antiphase with original distortion voltage  $D$ .

Hence, the new distortion voltage  $D'$  which appears in the output is

$$D' = D - \beta x D A \quad \dots (ii)$$

From (i) and (ii), we get

$$xD = D - \beta x D A \quad \text{or} \quad x = \frac{1}{1 + \beta A}$$

Substituting this value of  $x$  in Eq. (i) above, we have  $D' = \frac{D}{1 + \beta A}$

It is obvious from the above equation that  $D' < D$ . In fact, negative feedback reduces the amplifier distortion by the amount of loop gain *i.e.* by a factor of  $(1 + \beta A)$ .

However, it should be noted that improvement in distortion is possible only when the distortion is produced by the **amplifier itself**, not when it is already present in the input signal.

### 62.6. Feedback Over Several Stages

Multistage amplifiers are used to achieve greater voltage or current amplification or both. In such a case, we have a choice of applying negative feedback to improve amplifier performance. Either we apply some feedback across each stage or we can put it in one loop across the whole amplifier.

A multistage amplifier is shown in Fig. 62.4. In Fig. 62.4 (a) each stage of the  $n$ -stage amplifier has a feedback applied to it. Let  $A$  and  $\beta_1$  be the open-loop gain and feedback ratio respectively of each stage and  $A_1$  the overall gain of the amplifier. Fig. 62.4 (b) shows the arrangement where  $n$  amplifiers have been cascaded in order to get a total gain of  $A^n$ . Let the overall feedback factor be  $\beta_2$  and the overall gain  $A_2$ . The values of the two gains are given as

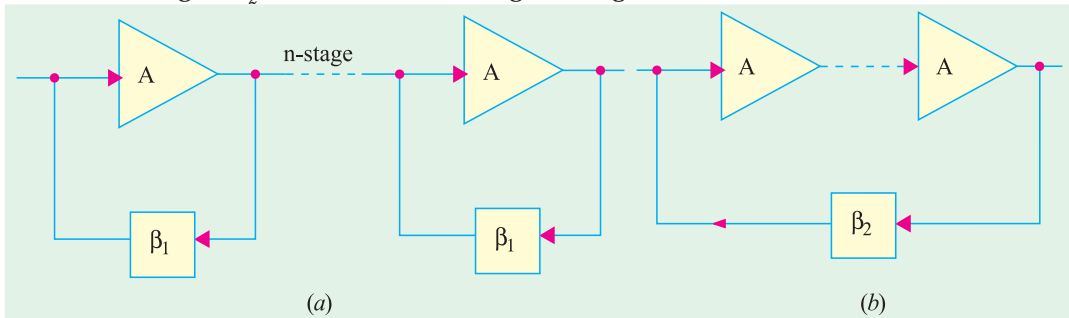


Fig. 62.4

$$A_1 = \left( \frac{A}{1 + A\beta_1} \right)^n \quad \text{and} \quad A_2 = \frac{A^n}{1 + A^n\beta_2} \quad \dots (i)$$

Differentiating the above two expressions, we get

$$\frac{dA_1}{A_1} = \frac{n}{1 + A\beta_1} \cdot \frac{dA}{A} \quad \text{and} \quad \frac{dA_2}{A_2} = \frac{n}{1 + A^n\beta_2} \cdot \frac{dA}{A}$$

For the two circuits to have the same overall gain,  $A_1 = A_2$ . Hence, from Eqn. (i) above, we get  $(1 - 1\beta)^n = 1 + A^n\beta_2$

$$\therefore \frac{dA_2 / A_2}{dA_1 / A_1} = \frac{1}{(1 + A\beta)^{n-1}}$$

If  $n = 1$ , then the denominator in the above equation becomes unity so that fractional gain variations are the same as expected. However, for  $n > 1$  and with  $(1 + A\beta_1)$  being a normally large quantity, the expression  $dA_2/A_2$  will be less than  $dA_1/A_1$ . It means that the overall feedback would appear to be beneficial as far as stabilizing of the gain is concerned.

**Example 62.7.** An amplifier with 10% negative feedback has an open-loop gain of 50. If open-loop gain increases by 10%, what is the percentage change in the closed-loop gain?

(Applied Electronics-I, Punjab Univ. 1991)

**Solution.** Let  $A_1'$  and  $A_2'$  be the closed-loop gains in the two cases and  $A_1$  and  $A_2$  the open-loop gains respectively.

$$(i) \quad A_1' = \frac{A_1}{1 + \beta A_1} = \frac{50}{1 + 0.1 \times 50} = 8.33$$

$$(ii) \quad \text{When open-loop gain changes by 10\%, then } A_2 = 50 + 0.1 \times 50 = 55$$



$$\therefore A_2' = \frac{A_2}{1 + \beta A_2} = \frac{55}{1 + 0.1 \times 55} = 8.46$$

$\therefore$  Percentage change in closed-loop gain is

$$= \frac{A_2' - A_1}{A_1} \times 100 = \frac{8.46 - 8.33}{8.33} \times 100 = \mathbf{1.56\%}$$

**Example 62.8.** Write down formulae for (i) gain (ii) harmonic distortion of a negative feedback amplifier in terms of gain and distortion without feedback and feedback factor. If gain without feedback is 36 dB and harmonic distortion at the normal output level is 10%, what is (a) gain and (b) distortion when negative feedback is applied, the feedback factor being 16 dB.

(Electronic Engg. II, Warangal 1991)

**Solution.** For first part, please refer to Art. 62.6. Distortion ratio is defined as the ratio of the amplitude of the largest harmonic to the amplitude of the fundamental.

$$A_f = A' = \frac{A}{1 + \beta A}$$

$$\text{Now, dB gain} = 20 \log_{10} A \quad \therefore \quad 36 = 20 \log_{10} A, A = 63$$

$$\text{dB feedback factor} = 20 \log_{10} \beta A \quad 16 = 20 \log_{10} \beta A \quad \text{or} \quad \beta A = 6.3$$

$$\text{(a) } A_f = A/(1 + \beta A) = 63/(1 + 6.3) = \mathbf{6.63 \text{ or } 18.72 \text{ dB}}$$

$$\text{(b) } D' = 10 \text{ per cent}/(1 + 6.3) = \mathbf{1.4 \text{ per cent}}$$

**Example 62.9.** The overall gain of a two-stage amplifier is 150. The second stage has 10% of the output voltage as negative feedback and has -150 as forward gain. Calculate (a) gain of the first stage (b) the second harmonic distortion, if the second stage introduces 5% second harmonic without feedback. Assume that the first stage does not introduce distortion.

(Electronics-II, Madras Univ. 1992)

$$\text{Solution. (a) For second stage } D_2 = \frac{D_2}{1 + \beta A_2} = \frac{0.05}{1 + 150 \times 0.1} = \mathbf{0.31\%}$$

(b) For the second stage, gain with feedback is

$$A_2' = \frac{A_2}{1 + \beta A_2} = \frac{150}{1 + 150 \times 0.1} = 9.38$$

$$\text{Now, } A_1 \times A_2' = 150; \quad A_1 = 150/9.38 = \mathbf{16}$$

**Example 62.10.** Determine the effective gain of a feedback amplifier having an amplification without feedback of  $(-200 - j300)$  if the feedback circuit adds to the input signal, a p.d. which is 0.5 percent of the output p.d. and lags a quarter of a cycle behind it in phase. Explain whether the feedback in this case is positive or negative. (Applied Electronics-II, Punjab Univ. 1992)

$$\text{Solution. } A = -200 - j300 = 360 \angle -123.7^\circ$$

The feedback voltage  $V_\beta$  is 0.5 percent of the output voltage and lags  $90^\circ$  behind it.

$$\therefore V_\beta = \left( \frac{0.5}{100} \angle -90^\circ \right) V_0$$

$$\therefore \beta = \frac{V_\beta}{V_0} = \frac{0.5}{100} \angle -90^\circ = -j0.005$$

$$\therefore \beta A = (-200 - j300)(-j0.005) = -1.5 + j1.0$$

In general, the stage gain with feedback is given by

$$A' = \frac{A}{1 - \beta A} = \frac{360 \angle -123.7^\circ}{1 - (-1.5 + j1.0)} = \frac{360 \angle -123.7^\circ}{2.69 \angle -21.8^\circ} = 134 \angle -102^\circ$$

Since both the magnitude and the phase shift of the amplifier are reduced by feedback, the feedback must be negative.

**Example 62.11.** An amplifier has a gain of 100 and 5 per cent distortion with an input signal of 1 V. When an input signal of 1 V is applied to the amplifier, calculate

- (i) output signal voltage, (ii) distortion voltage, (iii) output voltage

**Solution.** (i) Signal output voltage  $V_{os} = AV_i = 100 \times 1 = 100 \text{ V}$

(ii) Distortion voltage =  $DV_o = 0.05 \times 100 = 5 \text{ V}$

(iii) Amplifier output voltage  $V_o = V_{os} + D = 100 + 5 = 105 \text{ V}$

### 62.7. Increased Bandwidth

The bandwidth of an amplifier without feedback is equal to the separation between the 3 dB frequencies  $f_1$  and  $f_2$ .

$$\therefore BW = f_2 - f_1$$

where  $f_1$  = lower 3 dB frequency, and  $f_2$  = upper 3 dB frequency.

If  $A$  is its gain, the gain-bandwidth product is  $A \times BW$ .

Now, when negative feedback is applied, the amplifier gain is reduced. Since the gain-bandwidth product has to remain the same in both cases, it is obvious that the bandwidth must increase to compensate for the decrease in gain. It can be proved that with negative feedback, the lower and upper 3 dB frequencies of an amplifier become.

$$(f')_1 = \frac{f_1}{(1 + \beta A)} \text{ and } (f')_2 = f_2(1 + \beta A)$$

As seen from Fig. 62.5,  $f'_1$  has decreased whereas  $f'_2$  has increased thereby giving a wider separation or bandwidth. Since gain-bandwidth product is the same in both cases.

$$\therefore A \times BW = A' \times BW' \text{ or } A(f_2 - f_1) = A'(f'_2 - f'_1)$$

**Example 62.12.** An RC-coupled amplifier has a mid-frequency gain of 200 and a frequency response from 100 Hz to 20 kHz. A negative feedback network with  $\beta = 0.02$  is incorporated into the amplifier circuit. Determine the new system performance.

(Electronic Circuits, Mysore Univ. 1990)

**Solution.** 
$$A' = \frac{A}{1 + \beta A} = \frac{200}{1 + 0.02 \times 200} = 40 \text{ Hz}$$

$$f'_1 = \frac{f_1}{1 + \beta A} = \frac{100}{1 + 0.02 \times 200} = 20 \text{ Hz}$$

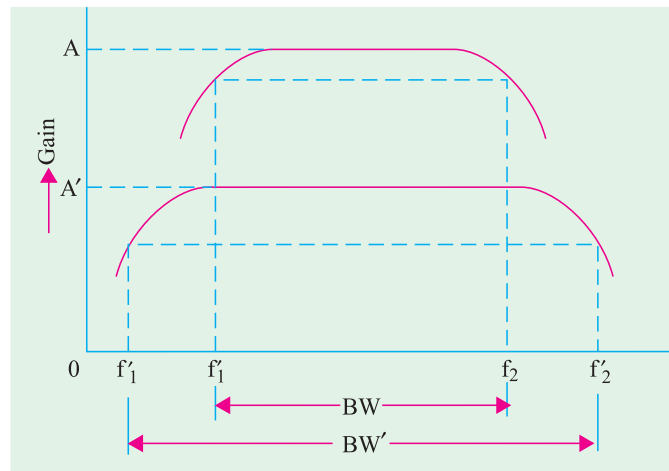


Fig. 62.5

$$f_2' = f_0 (1 + \beta A) = 20(1 + 0.02 \times 200) = \mathbf{100 \text{ Hz}}$$

$$dW = f_2' - f_1' \cong \mathbf{100 \text{ kHz}}$$

Incidentally, it may be proved that gain-bandwidth product remains constant in both cases.

$$dW = f_2 - f_1 \cong 20 \text{ kHz}$$

$$A \times dW = 200 \times 20 = 4000 \text{ kHz ;}$$

$$A' \times dW' = 40 \times 100 = 4000 \text{ kHz}$$

As expected, the two are equal.

### 62.8. Forms of Negative Feedback

The four basic arrangements for using negative feedback are shown in the block diagram of Fig. 62.6. As seen, both voltage and current can be feedback to the input either in series or in parallel. The output voltage provides input in Fig. 62.6 (a) and (b). However, the input to the feedback network is derived from the output current in Fig. 62.6 (c) and (d).

#### (a) Voltage-series Feedback

It is shown in Fig. 62.6 (a). It is also called **shunt-derived series-fed feedback**. The amplifier and feedback circuit are connected series-parallel. Here, a fraction of the output voltage is applied in series with the input voltage via the feedback. As seen, the input to the feedback network is in parallel with the output of the amplifier. Therefore, so far as  $V_o$  is concerned, output resistance of the amplifier is reduced by the shunting effect of the input to the feedback network. It can be proved that

$$R_o' = \frac{R_o}{(1 + \beta A)}$$

Similarly,  $V_i$  sees two circuit elements in series :

- (i) the input resistance of the amplifier and
- (ii) output resistance of the feedback network.

Hence, input resistance of the amplifier as a whole is increased due to feedback. It can be proved that

$$R_i' = R_i (1 + \beta A)$$

In fact, **series feedback always increases the input impedance by a factor of  $(1 + \beta A)$ .**

#### (b) Voltage-shunt Feedback

It is shown in Fig. 62.6 (b). It is also known as **shunt-derived shunt-fed feedback** i.e. it is parallel-parallel (PP) prototype. Here, a small portion of the output voltage is coupled back to the input voltage parallel (shunt).

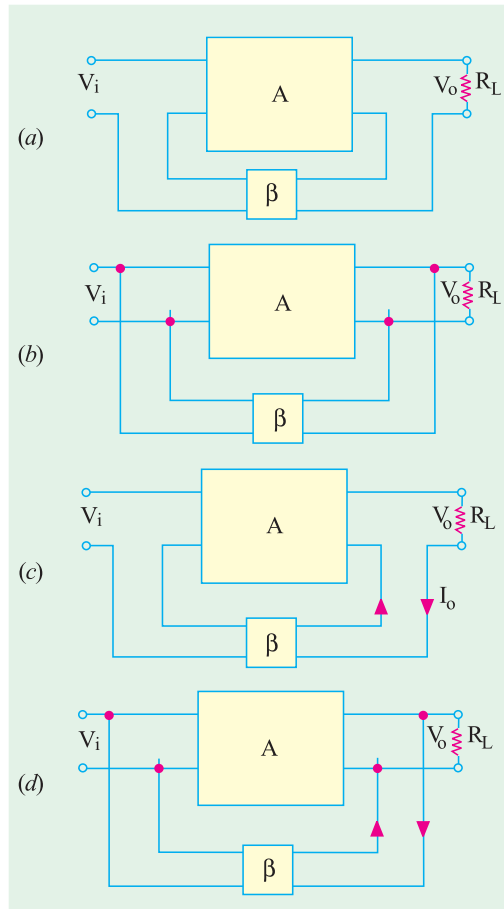


Fig. 62.6



Shunt Voltage

Since the feedback network shunts both the output and input of the amplifier, it decreases both its output and input impedances by a factor of  $1/(1 + \beta A)$

A shunt feedback *always decreases input impedance.*

**(c) Current-series Feedback**

It is shown in Fig. 62.6 (c). It is also known as *series-derived series-fed feedback*. As seen, it is a series-series (SS) circuit. Here, a part of the output current is made to feedback a proportional voltage in series with the input. Since it is a series pick-up and a series feedback, both the input and output impedances of the amplifier are increased due to feedback.

**(d) Current-shunt Feedback**

It is shown in Fig. 62.6 (d). It is also referred to as *series-derived shunt-fed feedback*. It is a parallel-series (PS) prototype. Here, the feedback network picks up a part of the output current and develops a feedback voltage in parallel (shunt) with the input voltage. As seen, feedback network shunts the input but is in series with the output. Hence, output resistance of the amplifier is increased whereas its input resistance is decreased by a factor of loop gain.

The effects of negative feedback on amplifier characteristics are summarized below :

Characteristics	Type of Feedback			
	Voltage series	Voltage shunt	Current series	Current shunt
Voltage gain	decreases	decreases	decreases	decreases
Bandwidth	increases	increases	increases	increases
Harmonic Distortion	decreases	decreases	decreases	decreases
Noise	decreases	decreases	decreases	decreases
Input Resistance	increases	decreases	increases	decreases
Output Resistance	decreases	decreases	increases	increases

**62.9. Shunt-derived Series-fed Voltage Feedback**

The basic principle of such a voltage-controlled feedback is illustrated by the block diagram of Fig. 62.7. Here, the feedback voltage is derived from the voltage divider circuit formed of  $R_1$  and  $R_2$ .

As seen, the voltage drop across  $R_1$  forms the feedback voltage  $V_f$

$$\therefore V_f = V_o \frac{R_1}{R_1 + R_2} = \beta V_o$$

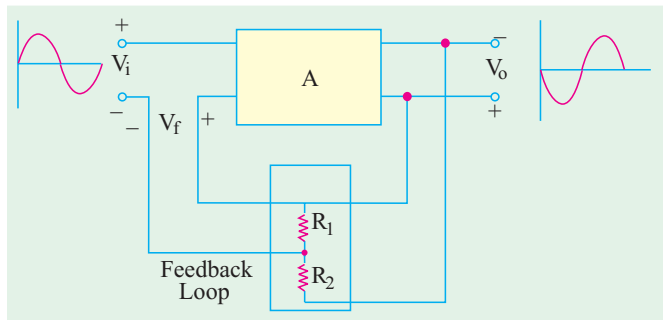


Fig. 62.7

**Example 62.13.** In the voltage-controlled negative feedback amplifier of Fig. 62.8, calculate (a) voltage gain without feedback (b) feedback factor (c) voltage gain with feedback. Neglect  $V_{BE}$  and use  $r_e = 25 \text{ mV}/I_E$ .

**Solution. (a)**  $A = \frac{r_L}{r_e} = \frac{R_3}{r_e}$

Now,  $I_B = \frac{15\text{V}}{1.5\text{M}} = 10\mu\text{A}$   
 $I_E = \beta I_B = 100 \times 10 = 1\text{mA}$   
 $r_e = 25/1 = 25\Omega$  ;  
 $A = \frac{10\text{K}}{25\Omega} = 400$

(b)  $\beta = \frac{R_1}{R_1 + R_2} = \frac{1.5 \times 10^6}{(1.5 + 10) \times 10^6} = 0.13$   
 $\therefore \beta A = 0.13 \times 400 = 52$

(c)  $A' = \frac{A}{1 + \beta A} = \frac{400}{1 + 52} = 7.55$

**62.10. Current-series Feedback Amplifier**

Fig. 62.9 shows a series-derived series-fed feedback amplifier circuit. Since the emitter resistor is unbypassed, it effectively provides current-series feedback. When  $I_E$  passes through  $R_E$ , the feedback voltage drop  $V_f = I_E R_E$  is developed which is applied in phase opposition to the input voltage  $V_i$ . This negative feedback reduces the output voltage  $V_o$ . This feedback can, however, be eliminated by either removing or bypassing the emitter resistor.

It can be proved that

$$\beta = \frac{R_E}{R_C} ; \quad A = \frac{R_C}{r_e + R_E} ; \quad A = \frac{R_C}{r_e}$$

**Example 62.14.** For the current-series feedback amplifier of Fig. 62.10, calculate (i) voltage gain without feedback, (ii) feedback factor, (iii) voltage gain with feedback. Neglect  $V_{BE}$  and use  $r_e = 25\text{ mV}/I_E$ . (Electronics-I, Madras Univ. 1990)

**Solution. (i)**  $A = \frac{R_C}{r_e}$   
 Now,  $I_E = \frac{V_{CC}}{R_E + R_B/\beta}$   
 $= \frac{10}{1 + 900/100} = 1\text{mA}$   
 $\therefore r_e = 25/I_E = 25\Omega$   
 $\therefore A = \frac{10\text{K}}{25\Omega} = 400$

(ii)  $\beta = \frac{R_E}{R_C} = \frac{1}{10} = 0.1$   
 $\beta A = 0.1 \times 400 = 40$

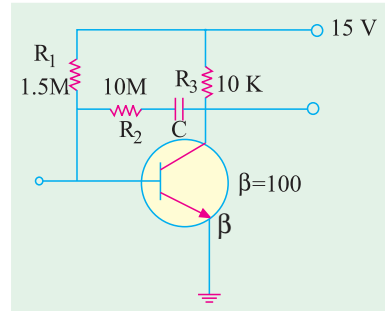


Fig. 62.8

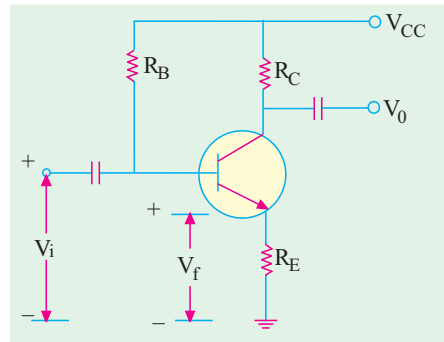


Fig. 62.9

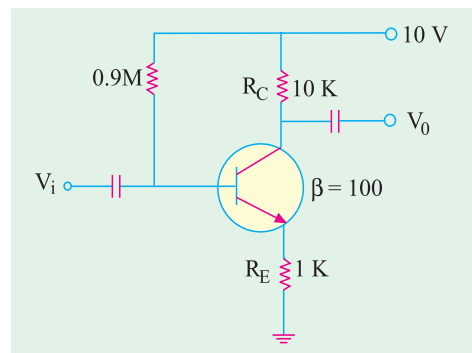


Fig. 62.10

(iii) 
$$A = \frac{R_C}{r_e + R_E} = \frac{10,000}{20 + 1000} = 9.756$$

or 
$$A = \frac{A}{1 + \beta A} = \frac{400}{1 + 400} = 9.756$$

**62.11. Voltage-shunt Negative Feedback Amplifier**

The circuit of such an amplifier is shown in Fig. 62.11. As seen, a portion of the output voltage is coupled through  $R_E$  in parallel with the input signal at the base. This feedback stabilizes the overall gain while decreasing both the input and output resistances. It can be proved that  $\beta = R_C/R_F$ .

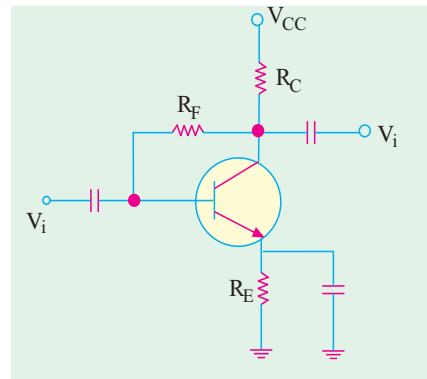


Fig. 62.11

**62.12. Current-shunt Negative Feedback Amplifier**

The two-stage amplifier employing such a feedback is shown in Fig. 62.12. The feedback circuit (consisting of  $C_F$  and  $R_F$ ) samples the output current and develops a feedback voltage in parallel with the input voltage. The unbypassed emitter resistor of  $Q_2$  provides current sensing.

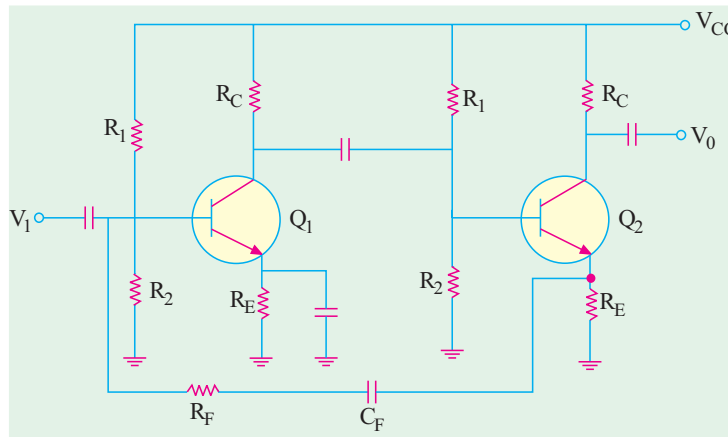


Fig. 62.12

The polarity of the feedback voltage is such that it provides the negative feedback.

**Example 62.15.** Calculate  $A$ ,  $r_{in(stage)}$  and  $I_{o(stage)}$  of the cascaded amplifier shown in Fig. 62.13 with and without voltage series feedback. The transistor parameters are :  $h_{fe} = 100$ ,  $h_{ie} = 2 K$  and  $h_{oe} = 0$ . **(Applied Electronics-I, Punjab Univ. 1992)**

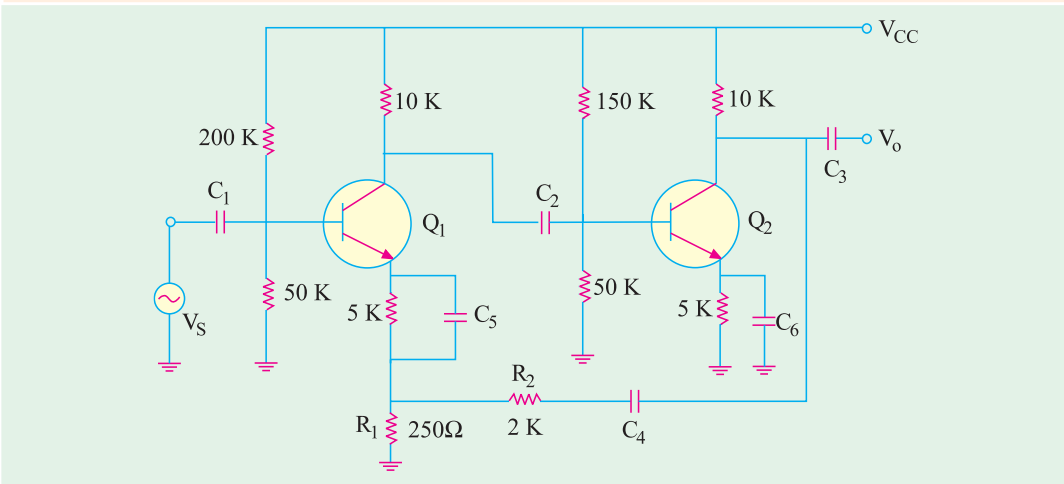


Fig. 62.13

**Solution. (i) Without Feedback.** The  $r_{in(base)}$  for  $Q_1$  is  $= h_{ie} = 2\text{K}$ . Same is the value for  $Q_2$ .

Also,  $r_{in(stage)}$  or  $r_{i-1}$  for  $Q_1 = 200\text{ K} \parallel 50\text{ K} \parallel 2\text{ K} = 1.9\text{ K}$

$r_{o,2}$  or  $r_{L,2}$  for  $Q_2 = 10\text{ K} \parallel (2.0 + 0.25)\text{ K} = 1.83\text{ K}$

$r_{o,1}$  or  $r_{L,1}$  for  $Q_1 = 10\text{ K} \parallel$

$150\text{ K} \parallel 50 \parallel 2\text{ K} = 1.6\text{ K}$

$$\therefore A_{v1} = \frac{h_{fe,1} r_{o,1}}{h_{ie}} = \frac{h_{fe,1} r_{L,1}}{h_{ie}}$$

$$= \frac{100 \times 1.6}{2} = 80$$

$$A_{v2} = \frac{h_{fe,2} r_{o,2}}{h_{ie}} = \frac{h_{fe,2} r_{L,2}}{h_{ie}} = \frac{100 \times 1.83}{2} = 92$$

Overall gain,  $A_v = A_{v1} \cdot A_{v2} = 80 \times 92 = \mathbf{7360}$

**(ii) With Feedback**

The feedback factor,  $\beta = \frac{R_1}{R_1 + R_2} = \frac{0.25}{0.25 + 2.0} = \frac{1}{9}$

$$r_{o2f} = \frac{r_{o2}}{1 + \beta A} = \frac{1.83}{1 + (1/9) \times 7360} = \mathbf{2.2\ \Omega}$$

$$r_{i,1f} = r_{i-1} (1 + \beta A) = 1.9 \times 819 = \mathbf{1556\text{ K}}$$

$$A_f = \frac{A}{(1 + \beta A)} = \frac{7360}{819} = \mathbf{8.9}$$

**Example 62.16.** In the two-stage  $R_C$  coupled amplifier (Fig. 62.14) using emitter feedback, find the overall gain. Neglect  $V_{BE}$  and take  $\beta_1 = \beta_2 = 100$ .

**Solution.** In this amplifier circuit, voltage gain has been stabilized to some extent with the help of  $500\ \Omega$  unbypassed emitter resistance. This  $500\ \Omega$  resistance swamps out  $r_e$ .

$$\therefore A_{v,2} = \frac{r_{L,2}}{r_e + r_E} \cong \frac{r_{L,2}}{r_E} = \frac{10\text{K} \parallel 10\text{K}}{500\ \Omega} = 10$$

Now,  $\beta r_E = 100 \times 500 = 50\text{ K}$

$$r_{i-2} = 80\text{ K} \parallel 40\text{ K} \parallel 50\text{ K}$$

$$r_{L,1} = R_{C,1} \parallel r_{i,2}$$

$$= 10\text{ K} \parallel 80\text{ K} \parallel 40\text{ K} \parallel 50\text{ K} = 6.3\text{ K}$$

$$\therefore A_{v,1} = \frac{r_{L,1}}{r_E} = \frac{6.3 \times 10^3}{500} = 12.6$$

$$\therefore A = 10 \times 12.6 = \mathbf{126}$$

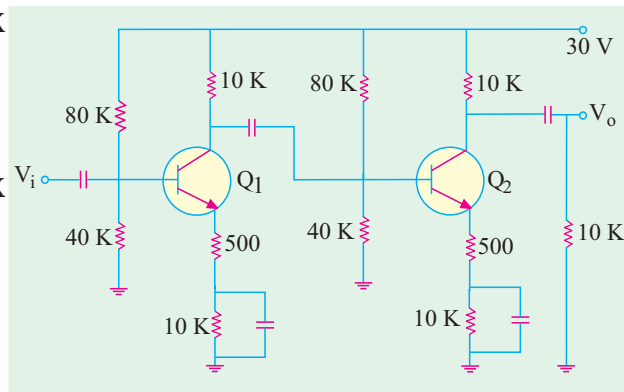


Fig. 62.14

62.13. Noninverting Op-amp With Negative Feedback

The closed-loop noninverting op-amp circuit using negative feedback is shown in Fig. 62.15. The input signal is applied to the noninverting input terminal. The output is applied back to the input terminal through the feedback network formed by  $R_i$  and  $R_f$ .

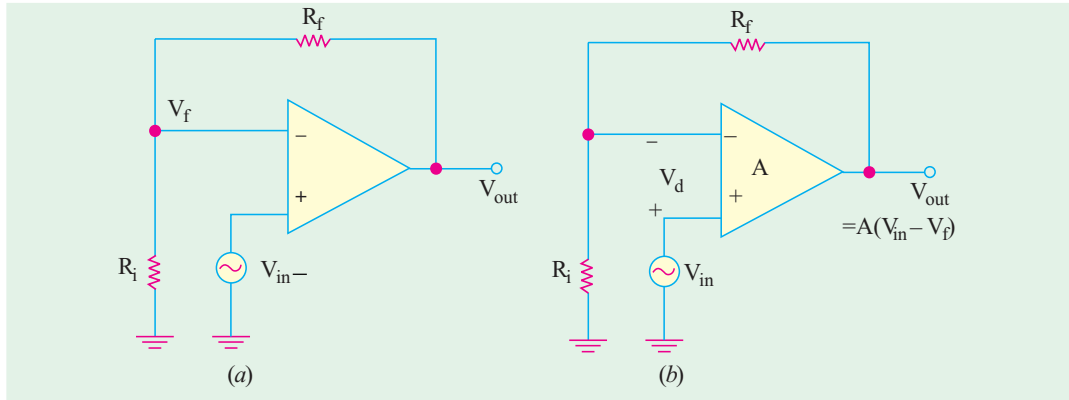


Fig. 62.15

The op-amp acts as both the difference circuit and the open-loop forward gain. The differential input to the op-amp is  $(V_{in} - V_f)$ . This differential voltage is amplified  $A$  times and an output voltage is produced which is given by

$$V_{out} = A v_d = A(V_{in} - V_f) ; \quad \text{where } A \text{ is the open-loop gain of the op-amp}$$

Since,  $(R_i + R_f)$  acts as voltage divider across  $V_{out}$ ,

$$\therefore V_f = V_{out} \frac{R_i}{R_i + R_f}$$

Now,  $\beta = R_i / (R_i + R_f)$ , hence  $V_f = \beta V_{out}$

Substituting this value in the above equation, we get

$$V_{out} = A(V_{in} - \beta V_{out}) \text{ or } V_{out}(1 + \beta A) = A V_{in}$$

Hence, voltage gain  $A'$  with negative feedback is

$$A' = \frac{V_{out}}{V_{in}} = \frac{A}{1 + \beta A} = \frac{A}{1 + A R_i / (R_i + R_f)}$$

If  $A$  is so large that 1 can be neglected as compared to  $\beta A$ , the above equation becomes

$$A' = \frac{A}{\beta A} = \frac{1}{\beta} = \frac{R_i + R_f}{R_i}$$

It is seen that closed-loop gain of a noninverting op-amp is essentially independent of the open-loop gain.

**Example 62.17.** A certain noninverting op-amp has  $R_i = 1K$ ,  $R_f = 99 K$  and open-loop gain  $A = 500,000$ . Determine (i)  $\beta$ , (ii) loop gain, (iii) exact closed-loop gain and (iv) approximate closed-loop gain if it is assumed that open-loop gain  $A = \infty$ .

(Power Electronics, AMIE 1991)

**Solution.** (i)  $\beta = \frac{R_i}{R_i + R_f} = \frac{1}{1 + 99} = 0.01$ , (ii) loop gain =  $\beta A = 500,000 \times 0.01 = 5000$

(iii)  $A' = \frac{A}{1 + \beta A} = \frac{500,000}{1 + 5000} = 9998$



(iv) approx.  $A' = \frac{1}{\beta} = \frac{1}{0.01} = 100$

It is seen that the gain changes by about 0.02%.

**62.14. Effect of Negative Feedback on  $R_{in}$  and  $R_{out}$**

In the previous calculations, the input impedance of an op-amp was considered to be infinite and its output resistance as zero. We will now consider the effect of a finite input resistance and a non-zero output resistance. Since the two effects are different and their values differ by several order of magnitude, we will focus on each effect individually.

**(a)  $R_{in}$  of Noninverting Op-amp**

For this analysis, it would be assumed that a small differential voltage  $V_d$  exists between the two inputs of the op-amp as shown in Fig. 62.16. It, in effect, means that neither the input resistance of the op-amp is assumed to be infinite nor its input current zero.

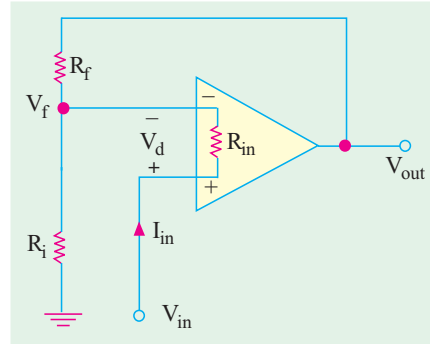


Fig. 62.16

Now,  $V_d = V_{in} - V_f$  or  $V_{in} = V_d + V_f = V_d + \beta V_{out}$   
 Also,  $V_{out} = A \cdot V_d$  where  $A$  is the open-loop gain of the op-amp.  
 $\therefore V_{in} = V_d + A\beta V_{out} = (1 + \beta A) V_d = (1 + \beta A) I_{in} R_{in}$  - ( $\because V_d = I_{in} R_{in}$ )  
 where  $R_{in}$  is the open-loop impedance of the op-amp (i.e. without feedback)

$\therefore R_{in} = \frac{V_{in}}{I_{in}} = (1 + \beta A) R_{in}$

where  $R_{in}$  is the closed-loop input resistance of the non-inverting op-amp.

It will be seen that the closed-loop input resistance of the non-inverting op-amp is much greater than the input resistance without feedback.

**(b)  $R'_{out}$  of Noninverting Op-amp**

An expression for  $R'_{out}$  would be developed with the help Fig. 62.17. Using *KVL*, we get

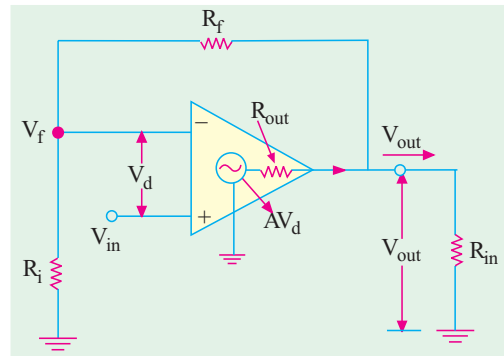


Fig. 62.17

$V_{out} = AV_d - I_{out} R_{out}$   
 Now,  $V_d = (V_{in} - V_f)$  and neglecting  $I_{out} R_{out}$  as compared to  $AV_d$  we have  
 $V_{out} = A(V_{in} - V_f) = A(V_{in} - \beta V_{out})$   
 or  $AV_{in} = (1 + \beta A) V_{out}$

If, with negative feedback, output resistance of the noninverting op-amp is  $R'_{out}$  then  $V_{out} = I_{out} R'_{out}$

Substituting this value in the above equation, we get

$AV_{in} = (1 + \beta A) I_{out} R'_{out}$  or  $\frac{AV_{in}}{I_{out}} = (1 + \beta A) R'_{out}$

The term on the left is the internal output resistance  $R_{out}$  of the op-amp because without feedback,  $AV_{in} = V_{out}$

or  $R_{out} = (1 + \beta A) R'_{out}$  or  $R'_{out} = \frac{R_{out}}{(1 + \beta A)}$

Obviously, output resistance  $R'_{out}$  with negative feedback is much less than without feedback (i.e.  $R_{out}$ ).

**Example 62.18. (a)** Calculate the input and output resistance of the op-amp shown in Fig. 62.18. The data sheet gives :  $R_{in} = 2M$ ,  $R_{out} = 75 \Omega$ . and  $A = 250,000$   
**(b)** Also, calculate the closed-loop voltage gain with negative feedback.

**(Industrial Electronics, Mysore, Univ. 1992)**

**Solution. (a)** The feedback ratio  $\beta$  is given by

$$\beta = \frac{R_f}{R_i + R_f} = \frac{10}{10 + 200} = \frac{10}{210} = 0.048$$

$$R'_{in} = (1 + \beta A) R_{in} = (1 + 250,000 \times 0.048) \times 2 = 24,002 M$$

$$R'_{out} = \frac{R_{out}}{1 + \beta A} = \frac{75}{1 + 12000} = 0.006 \Omega$$

**(b)**  $A' = 1/\beta = 1/0.048 = 20.8$

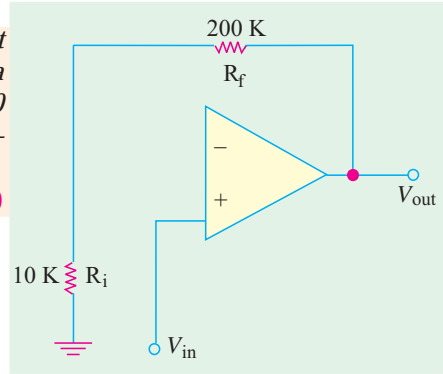


Fig. 62.18

**62.15.  $R_{in}$  and  $R_{out}$  of Inverting Op-amp with Negative Feedback**

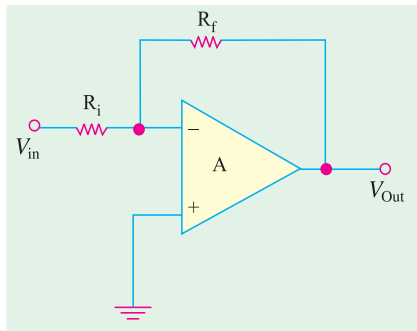


Fig. 62.19

The input resistance  $R_{in}$  of the inverting op-amp with negative feedback will be found by using Fig. 62.19. Since both the input signal and the negative feedback are applied to the inverting terminal. Miller's theorem will be applied to this configuration. According to this theorem, the effective input resistance of an amplifier with a feedback resistor from output to input is given by

$$R_{in(Miller)} = \frac{R_f}{1 + A} \quad \text{and} \quad R_{out(Miller)} = R_f \left( \frac{A}{1 + A} \right)$$

The Miller equivalent of the inverting op-amp is shown in Fig. 62.20 (a)

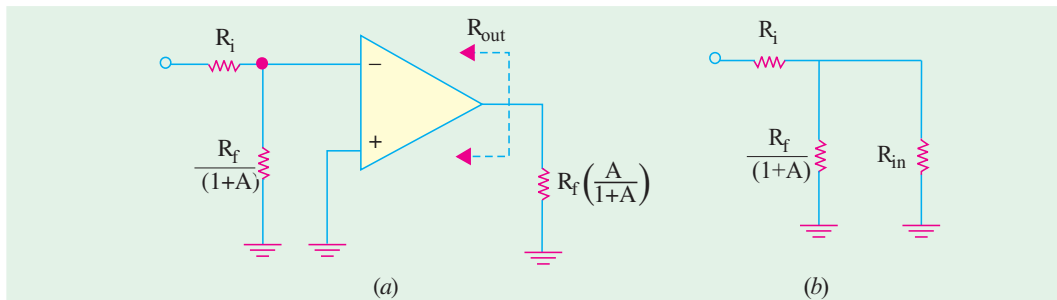


Fig. 62.20

As shown in Fig. 62.20 (b), the Miller input resistance appears in parallel with the internal resistance of the op-amp (without feedback) and  $R_i$  appears in series with this

$$\therefore R_{in} = R_i + \frac{R_f}{(1 + A)} \parallel R_{in}$$

Typically, the term  $R_f(1 + A)$  is much less than  $R_{in}$  of an open-loop op-amp. Hence,

$$\frac{R_f}{(1 + A)} \parallel R_{in} \cong \frac{R_f}{1 + A}$$

Moreover,  $A \gg 1$ , hence,  $\therefore R'_{in} \cong R_i + \frac{R_f}{A}$

Now,  $R_i$  appears in series with  $(R_f/A)$  and if  $R_i \gg R_f/A$ , we have,  $R'_{in} \cong R_i$   
As seen from Fig. 62.20 (b), Miller output resistance appears in parallel with  $R_{out}$  of the op-amp.

$$\therefore R'_{out} = R_f \left( \frac{A}{1 + A} \right) \parallel R_{out}$$

Normally,  $A \gg 1$  and  $R_f \gg R_{out}$  so that  $R'_{out}$  simplifies to  $R'_{out} = R_{out}$

**Example 62.19.** For the inverting op-amp circuit of Fig. 62.21, find (a) input and output resistances and (b) the closed-loop gain. The op-amp has the following parameters :

$$A = 100,000, \quad R_{in} = 5 \text{ M}\Omega, \text{ and } R_{out} = 50 \Omega$$

**Solution.** (a)  $R'_{in} \cong R_i \cong 2 \text{ k}\Omega$   
 $R'_{out} \cong R_{out} = 50 \Omega$

(b)  $A' = \frac{R_f}{R_i} = -\frac{100}{2} = 50$

The negative sign indicates the inherent sign inversion in the process.

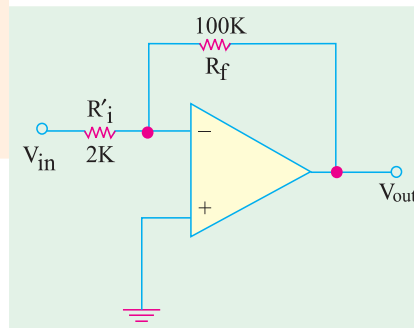


Fig. 62.21

### Tutorial Problems No. 62.1

- For the series-parallel feedback amplifier shown in Fig. 62.22. Calculate
  - open-loop gain,
  - gain of feedback loop,
  - closed-loop gain,
  - sacrifice factor.

**[(i)  $10^6$  (ii) 0.025 (iii) 40 (iv) 25.000]**

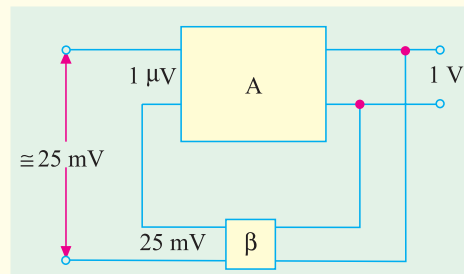


Fig. 62.22

- A negative-feedback amplifier has the following parameters :  
 $A = 200, \beta = 0.02$  and  $V_i = 5 \text{ mV}$   
Compute the following :
  - gain with feedback,
  - output voltage,
  - feedback factor,
  - feedback voltage.

**[(i) 40 (ii) 200 mV (iii) 4 (iv) 8 mV]**

- An amplifier has an open-loop gain of 500 and a feedback of 0.1. If open-loop gain changes by 25% due to temperature etc., find the percentage change in closed-loop gain. **[0.5%]**
- An RC-coupled amplifier has a mid-frequency gain of 400 and lower and upper 3-dB frequencies of 100 Hz and 10 kHz. A negative feedback network with  $\beta = 0.01$  is incorporated into the amplifier circuit. Calculate

- (i) gain with feedback,  
(ii) new bandwidth. [(i) 80 (ii) 75 kHz]
5. In an amplifier with constant signal input of 1 volt, the output falls from 50 to 25 V when feedback is applied. Calculate the fraction of the output which is fed back. If, due to ageing, the amplifier gain fell to 40, find the percentage reduction in stage gain  
(i) without feedback (ii) with the feedback connection. [0.02% (i) 20% (ii) 11.12%]
6. An amplifier has a gain of 1000 without feedback. Calculate the gain when 0.9 per cent of negative feedback is applied. If, due to ageing, gain without feedback falls to 800, calculate the percentage reduction in gain (a) without feedback and (b) with feedback. Comment on the significance of the results of (a) and (b) and state two other advantages of negative feedback. [100 (a) 20% (b) 2.44%](City & Guilds, London)
7. The open-loop gain of an amplifier is  $1000 \angle 70^\circ$  and the feedback factor is  $-0.02 \angle 20^\circ$ . Calculate the amplifier gain with negative feedback. What is the limiting value of  $\beta$  to make the amplifier unstable? [49.9  $\angle -17.1^\circ$ ; 0.001  $\angle -70^\circ$ ] (I.E.E. London)
8. When voltage feedback is applied to an amplifier of gain 100, the overall stage gain falls to 50. Calculate the fraction of the output voltage fed back. If this fraction is maintained, calculate the value of the amplifier gain required if the overall stage gain is to be 75. [0.01; 300] (City & Guilds, London)
9. An amplifier having a gain of 100 has 9 per cent voltage negative feedback applied in series with the input signal. Calculate the overall stage with feedback.  
If a supply voltage variation causes the gain with feedback to fall by 10 percent, determine the percentage change in gain with feedback. [10; 52.6%] (City & Guilds, London)
10. If the gain of an amplifier without feedback is  $(800 - j100)$  and the feedback network of  $\beta = -1/(40 - j20)$  modifies the output voltage to  $V_{fb}$  which is combined in series with the signal voltage, determine the gain of the amplifier with feedback. [38.3 - j18.3] (I.E.R.E., London)
11. Give three reasons for using negative feedback.  
In Fig. 62.23, the box represents an amplifier of gain  $-1000$ , input impedance  $500 \text{ k}\Omega$  and negligible output impedance.  
Calculate the voltage gain and input impedance of the amplifier with feedback. [- 9.9, 50.5 M $\Omega$ ]
12. An amplifier with negative feedback has a voltage gain of 100. It is found that without feedback an input signal of 50 mV is required to produce a given output; whereas with feedback, the input signal must be 0.6V for the same output. Calculate the value of voltage gain without feedback and feedback ratio.

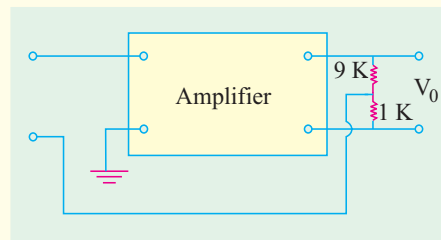


Fig. 62.23

(Electronics Engg., Bangalore Univ. 2001)

## OBJECTIVE TESTS – 62

1. The advantage of using negative feedback in an amplifier is that its gain can be made practically independent of
- temperature changes
  - age of components
  - frequency
  - all of the above.

2. Feedback in an amplifier always helps to
- control its output
  - increase its gain
  - decrease its input impedance
  - stabilize its gain.
3. The only drawback of using negative feedback in amplifiers is that it involves
- gain sacrifice
  - gain stability
  - temperature sensitivity
  - frequency dependence.
4. Closed-loop gain of a feedback amplifier is the gain obtained when
- its output terminals are closed
  - negative feedback is applied
  - feedback loop is closed
  - feedback factor exceeds unity.
5. A large sacrifice factor in a negative feedback amplifiers leads to
- inferior performance
  - increased output impedance
  - characteristics impossible to achieve without feedback
  - precise control over output.
6. Negative feedback in an amplifier
- lowers its lower 3 dB frequency
  - raises its upper 3 dB frequency
  - increases its bandwidth
  - all of the above.
7. Regarding negative feedback in amplifiers which statement is WRONG ?
- it widens the separation between 3 dB frequencies
  - it increases the gain-bandwidth product
  - it improves gain stability
  - it reduces distortion.
8. Negative feedback reduces distortion in an amplifier only when it
- comes as part of input signal
  - is part of its output
  - is generated within the amplifier
  - exceeds a certain safe level.
9. An amplifier with no feedback has a gain-bandwidth product of 4 MHz. Its closed-loop gain is
- 100 kHz
  - 160 MHz
  - 10 MHz
  - 20 kHz.
10. The shunt-derived series-fed feedback in an amplifier
- increases its output impedance
  - decreases its output impedance
  - increases its input impedance
  - both (b) and (c).
11. A feedback amplifier has a closed gain of  $-200$ . It should not vary more than 50% despite 25% variation in amplifier gain  $A$  without feedback. The value of  $A$  is
- 800
  - $-800$
  - 1000
  - $-1000$
12. The gain of a negative feedback amplifier is 40 dB. If the attenuation of the feedback path is 50 dB, then the gain of the amplifier without feedback is
- 78.92
  - 146.32
  - 215.51
  - 317.23
13. In a common emitter amplifier, the unbypassed emitter resistor provides
- voltage-shunt feedback
  - current-series feedback
  - negative-voltage feedback
  - positive-current feedback
40. The new band-width is
- 100 kHz
  - 160 MHz
  - 10 MHz
  - 20 kHz.

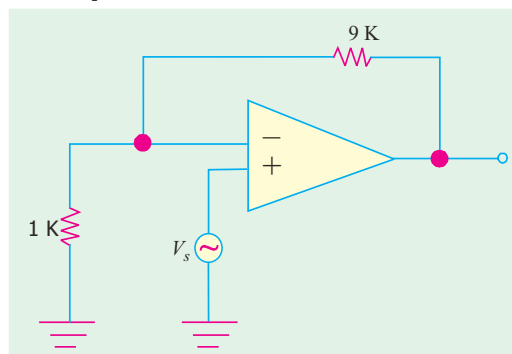


Fig. 62.24

14. The OP-AMP circuit shown in Fig. 62.24 has an input impedance of  $M\Omega$  and an open-loop gain of  $10^5$ . The output impedance seen by the source  $V_s$  is

- (a)  $10^{11}\Omega$
- (b)  $10^{10}\Omega$
- (c)  $10\text{ k}\Omega$
- (d)  $1\text{ k}\Omega$

15. An OP-AMP with an open-loop gain of 10,000,  $R_{in} = 2\text{ K}\Omega$  and  $R_o = 500\Omega$  is used in the non-inverting configuration shown in Fig. 62.25. The output resistance  $R_o'$  is

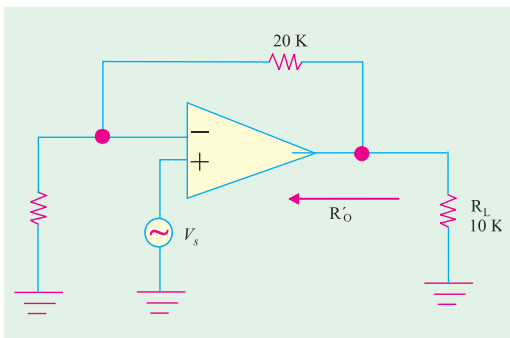


Fig. 62.25

- (a)  $250.5\Omega$
  - (b)  $21\Omega$
  - (c)  $2\Omega$
  - (d)  $0.998\Omega$
16. The feedback used in the circuit shown in Fig. 62.25 can be classified as
- (a) shunt-series feedback
  - (b) shunt-shunt feedback
  - (c) series-shunt feedback
  - (d) series-series feedback

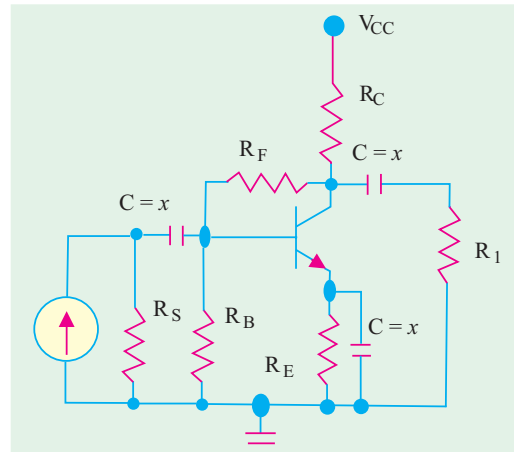


Fig. 62.25

### ANSWERS

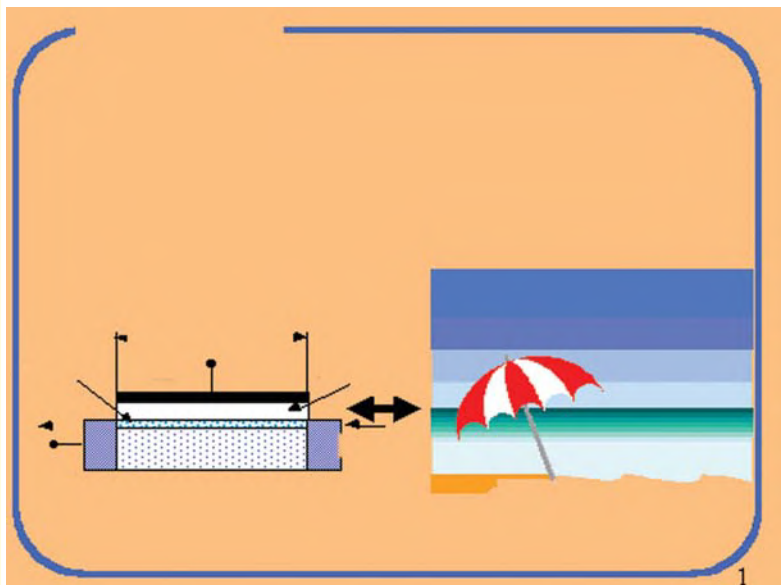
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|--------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a)  | 3. (a)  | 4. (c)  | 5. (c)  | 6. (d)  | 7. (b)  | 8. (c)  |
| 9. (a) | 10. (d) | 11. (d) | 12. (b) | 13. (b) | 14. (b) | 15. (d) | 16. (b) |

# CHAPTER 63

## Learning Objectives

- What is a FET
- Junction FET (JFET)
- Static Characteristics of a JFET
- JFET Drain Characteristic with  $V_{GS} = 0$
- Characteristics with External Bias
- Transfer Characteristic
- Small Signal JFET Parameters
- D.C. Biasing of a JFET
- DC Load Line
- Common Source JFET Amplifier
- JFET Amplifier Gains
- Advantages of FETs
- MOSFET or IGFET—DE MOSFET
- Schematic Symbols for a DEMOSFET
- Static Characteristics of a DEMOSFET
- Enhancement-only N-Channel MOSFET
- Biasing E-only MOSFET—FET Amplifiers
- FET Applications
- MOSFET Handling

## FIELD EFFECT TRANSISTORS



Field Effect Transistor as an Electronic Flute



### 63.1. What is a FET ?

The acronym 'FET' stands for **field effect transistor**. It is a three-terminal unipolar solid-state device in which current is controlled *by an electric field* as is done in vacuum tubes. Broadly speaking, there are two types of FETs :

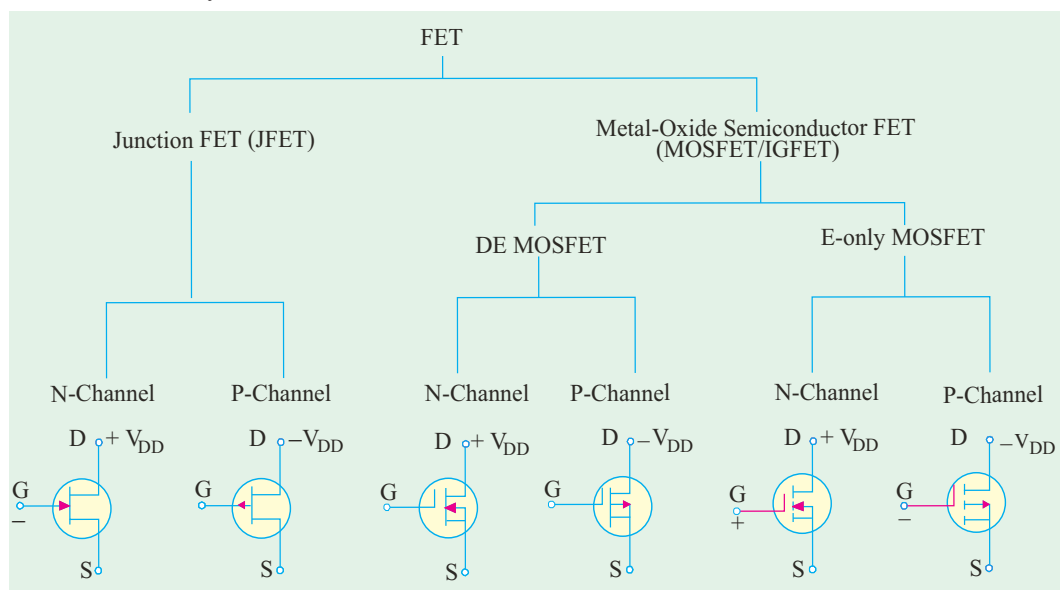
- (a) junction field effect transistor (JFET)
- (b) metal-oxide semiconductor FET (MOSFET)

It is also called insulated-gate FET (IGFET). It may be further subdivided into :

- (i) depletion-enhancement MOSFET *i.e.* DEMOSFET
- (ii) enhancement-only MOSFET *i.e.* E-only MOSFET

Both of these can be either *P*-channel or *N*-channel devices.

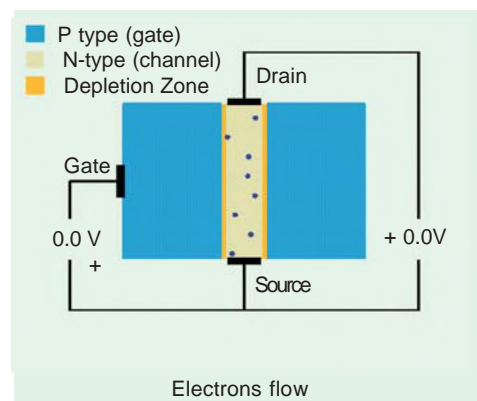
The FET family tree is shown below :



### 63.2. Junction FET (JFET)

#### (a) Basic Construction

As shown in Fig. 63.1, it can be fabricated with either an *N*-channel or *P*-channel though *N*-channel is generally preferred. For fabricating an *N*-channel JFET, first a narrow bar of *N*-type semiconductor material is taken and then two *P*-type junctions are diffused on opposite sides of its middle part [Fig. 63.1 (a)]. These junctions form two *P-N* diodes or **gates** and the area between these gates is called **channel**. The two *P*-regions are internally connected and a single lead is brought out which is called **gate terminal**. Ohmic contacts (direct electrical connections) are made at the two ends of the bar—one lead is called **source terminal** *S* and the other **drain terminal** *D*. When potential difference is established between drain and source, current flows along the length of the 'bar' through the channel located between the two *P*-regions. The current consists of **only majority carriers** which, in the present case, are electrons. *P*-channel JFET is



+



similar in construction except that it uses *P*-type bar and two *N*-type junctions. The majority carriers are holes which flow through the channel located between the two *N*-regions or gates.

Following FET notation is worth remembering:

1. **Source.** It is the terminal through which majority carriers enter the bar. Since carriers come from it, it is called the source.
2. **Drain.** It is the terminal through which majority carriers *leave* the bar *i.e.* they are drained out from this terminal. The drain-to-source voltage  $V_{DS}$  drives the drain current  $I_D$ .
3. **Gate.** These are two internally-connected heavily-doped impurity regions which form two *P-N* junctions. The gate-source voltage  $V_{GS}$  reverse-biases the gates.
4. **Channel.** It is the space between two gates through which majority carriers pass from source-to-drain when  $V_{DS}$  is applied.

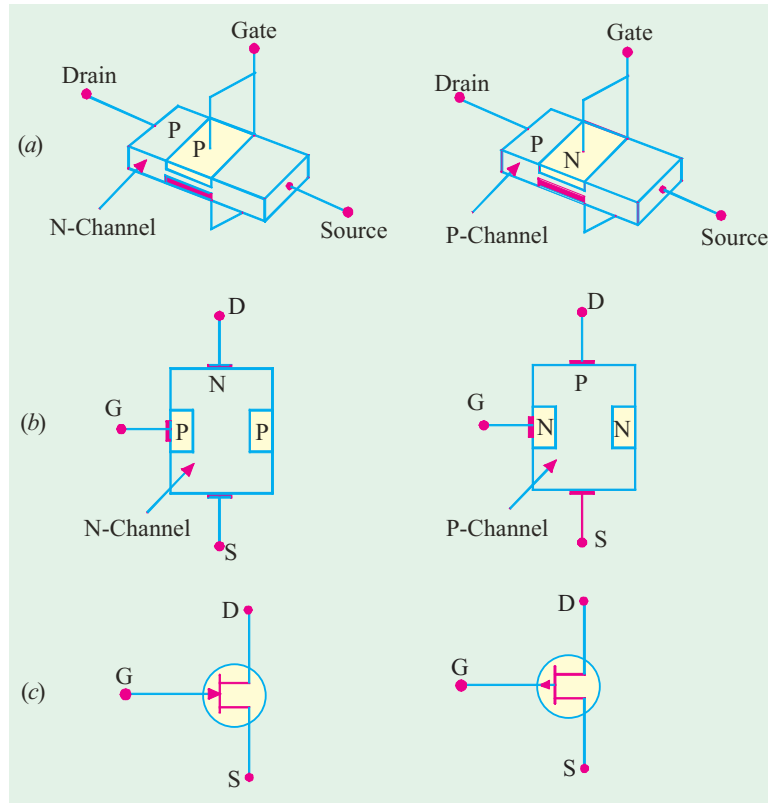


Fig. 63.1

Schematic symbols for *N*-channel and *P*-channel JFET are shown in Fig. 63.1 (c). It must be kept in mind that **gate arrow always points to N-type material.**

**(b) Theory of Operation**

While discussing the theory of operation of a JFET, it should be kept in mind that

1. Gates are always reversed-biased. Hence, gate current  $I_G$  is practically zero.
2. The source terminal is always connected to that end of the drain supply which provides the necessary charge carriers. In an *N*-channel JFET, source terminal *S* is connected to the negative end of the drain voltage supply (for obtaining electrons). In a *P*-channel JFET, *S* is connected to the positive end of the drain voltage supply for getting holes which flow through the channel.

Let us now consider an *N-channel* JFET and discuss its working when either  $V_{GS}$  or  $V_{DS}$  or both are changed.

**(i) When  $V_{GS} = 0$  and  $V_{DS} = 0$**

In this case, drain current  $I_D = 0$ , because  $V_{DS} = 0$ . The depletion regions around the *P-N* junctions are of equal thickness and symmetrical as shown in Fig. 63.2 (a)

**(ii) When  $V_{GS} = 0$  and  $V_{DS}$  is increased from zero**

For this purpose, the JFET is connected to the  $V_{DD}$  supply as shown in Fig. 63.2 (b). The electrons (which are the majority carriers) flow from *S* to *D* whereas conventional drain current  $I_D$  flows through the channel from *D* to *S*. Now, the gate-to-channel bias at any point along the channel is  $= |V_{DS}| + |V_{GS}|$  *i.e.* the numerical sum of the two voltages. In the present case, external bias  $V_{GS} = 0$ .



Hence gate-channel reverse bias is provided by  $V_{DS}$  alone. Since the value of  $V_{DS}$  keeps decreasing (due to progressive drop along the channel) as we go from  $D$  to  $S$ , the gate-channel bias also decreases accordingly. It has maximum value in the drain-gate region and minimum in the source-gate region. Hence, depletion regions penetrate more deeply into the channel in the drain-gate region than in the source-gate region. This explains why the depletion regions become wedge shaped when  $V_{DS}$  is applied [Fig. 63.2 (b)]

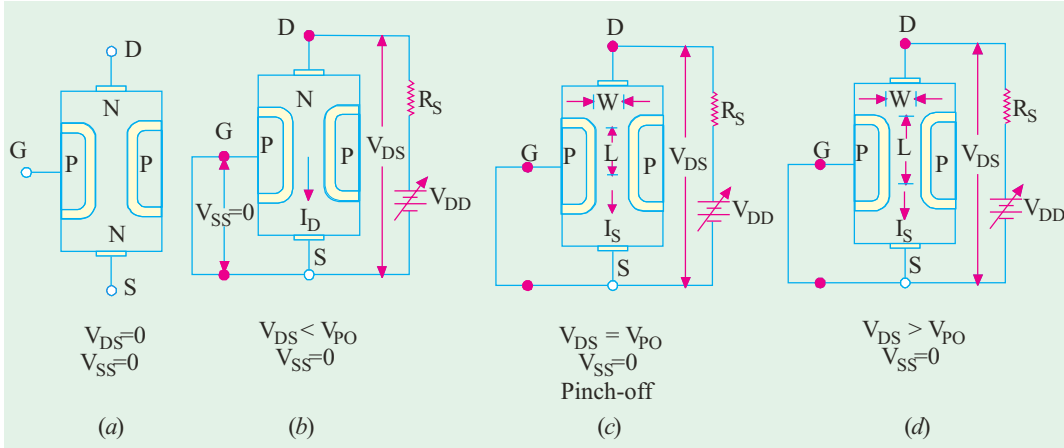


Fig. 63.2

As  $V_{DS}$  is gradually increased from zero,  $I_D$  increases proportionally as per Ohm's law. It is found that for small initial values of  $V_{DS}$ , the  $N$ -type channel material acts like a resistor of constant value. It is so because  $V_{DS}$  being small, the depletion regions are not large enough to have any significant effect on channel cross-section and, hence, its resistance. Consequently,  $I_D$  increases linearly as  $V_{DS}$  is increased from zero onwards (Fig. 63.5).

The ohmic relationship between  $V_{DS}$  and  $I_D$  continues till  $V_{DS}$  reaches a certain critical value called **pinch-off voltage**  $V_{PO}$  when drain current becomes constant at its maximum value called  $I_{DSS}$ . The  $SS$  in  $I_{DSS}$  indicates that the gate is shorted to source to make sure that  $V_{GS} = 0$ . This current is also known as **zero-gate-voltage drain current**. It is seen from Fig. 63.2(c) that under pinch-off conditions, separation between the depletion regions near the drain end reaches a minimum value  $W$ . It should, however, be carefully noted that **pinch-off does not mean 'current-off'**. In fact,  $I_D$  is maximum at pinch-off.

When  $V_{DS}$  is increased beyond  $V_{PO}$ ,  $I_D$  remains constant at its maximum value  $I_{DSS}$  upto a certain point. It is due to the fact that further increase in  $V_{DS}$  (beyond  $V_{PO}$ ) causes more of the channel on the source end to reach the minimum width as shown in Fig. 63.2 (d). It means that the channel width does not increase, instead its length  $L$  increases. As more of the channel reaches the minimum width, the resistance of the channel increases at the same rate at which  $V_{DS}$  increases. In other words, increase in  $V_{DS}$  is neutralized by increases in  $R_{DS}$ . Consequently,  $I_D = (V_{DS} / R_{DS})$  remains unchanged even though  $V_{DS}$  is increased. Ultimately, a certain value of  $V_{DS}$  (called  $V_{DSO}$ ) is reached when JFET breaks down and  $I_D$  increases to an excessive value as seen from drain characteristic of Fig. 63.5.

**(iii) When  $V_{DS} = 0$  and  $V_{GS}$  is decreased from zero**

In this case, as  $V_{GS}$  is made more and more negative, the gate reverse bias increases which increases the thickness of the depletion regions. As negative value of  $V_{GS}$  is increased, a stage comes when the two depletion regions touch each other as shown in Fig. 63.3. In this condition, the channel is said to be cut-off. This value of  $V_{GS}$  which cuts off the channel and

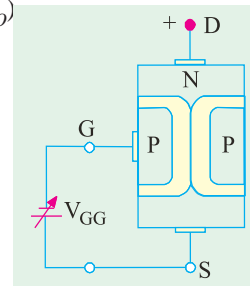


Fig. 63.3



hence the drain current is called  $V_{GS(off)}$ .\*

It may be noted that  $V_{GS(off)} = -V_{PO}$  or  $|V_{PO}| = |V_{GS(off)}|$ . As seen from Fig. 63.6 because  $V_{PO} = 4$  V,  $V_{GS(off)} = -4$  V. Obviously, their absolute values are equal.

**(iv) When  $V_{GS}$  is negative and  $V_{DS}$  is increased**

As seen from Fig. 63.6, as  $V_{GS}$  is made more and more negative, values of  $V_p$  as well as breakdown voltage are decreased.

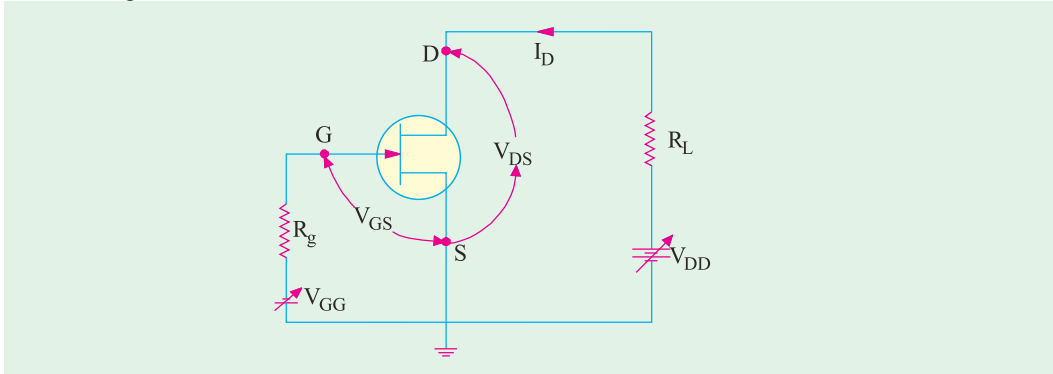


Fig. 63.4

**Summary.** Summarizing the above, we have that

(i) keeping  $V_{GS}$  at a fixed value (either zero or negative), as  $V_{DS}$  is increased,  $I_D$  initially increases till channel pinch-off when it becomes almost constant and finally increases excessively when JFET breaks down under high value of  $V_{DS}$ . As  $V_{GS}$  is kept fixed at progressively higher negative values, the values of  $V_p$  as well as breakdown voltage decrease.

(ii) keeping  $V_{DS}$  at a fixed value, as  $V_{GS}$  is made more and more negative,  $I_D$  decreases till it is reduced to zero for a certain value of  $V_{GS}$  called  $V_{GS(off)}$ .

Since gate voltage controls the drain current, JFET is called a **voltage-controlled** device. A P-channel JFET operates exactly in the same manner as an N-channel JFET except that current carriers are holes and polarities of both  $V_{DD}$  and  $V_{GS}$  are reversed.

Since only one type of majority carrier (either electrons or holes) is used in JFETs, they are called **unipolar devices** unlike bipolar junction transistors (BJTs) which use both electrons and holes as carriers.

### 63.3. Static Characteristics of a JFET

We will consider the following two characteristics:

**(i) drain characteristic**

It gives relation between  $I_D$  and  $V_{DS}$  for different values of  $V_{GS}$  (which is called running variable).

**(ii) transfer characteristic**

It gives relation between  $I_D$  and  $V_{GS}$  for different values of  $V_{DS}$ .

We will analyse these characteristics for an N-channel JFET connected in the common-source mode as shown in Fig. 63.4. We will first consider the drain characteristic when  $V_{GS} = 0$  and then when  $V_{GS}$  has any negative value upto  $V_{GS(off)}$ .

### 63.4. JFET Drain Characteristic With $V_{GS} = 0$

Such a characteristic is shown in Fig. 63.5 and has been already discussed briefly in Art. 63.2. It can be subdivided into following four regions :

#### 1. Ohmic Region OA

This part of the characteristic is linear indicating that for low values of  $V_{DS}$ , current varies directly with voltage following Ohm's Law. It means that JFET behaves **like an ordinary resistor** till

\* It has negative value for an N-channel JFET but a positive value for a P-channel JFET.



point A (called knee) is reached.

**2. Curve AB**

In this region,  $I_D$  increases at reverse square-law rate upto point B which is called **pinch-off point**. This progressive decrease in the rate of increase of  $I_D$  is caused by the square law increase in the depletion region at each gate upto point B where the two regions are closest without touching each other. The drain-to-source voltage  $V_{DS}$  corresponding to point B is called **pinch-off voltage  $V_p^*$** . But it is essential to remember that “**pinch-off**” does not mean “**current-off**”.

**3. Pinch-off Region BC**

It is also known as **saturation region** or ‘**amplified**’ region. Here, JFET operates as a constant-current device because  $I_D$  is relatively independent of  $V_{DS}$ . It is due to the fact that as  $V_{DS}$  increases, channel resistance also increases proportionally thereby keeping  $I_D$  practically constant at  $I_{DSS}$ . It should also be noted that the reverse bias required by the gate-channel junction is supplied entirely by the voltage drop across the channel resistance due to flow of  $I_{DSS}$  and none by external bias because  $V_{GS} = 0$ .

Drain current in this region is given by Shockley's equation

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 = I_{DSS} \left( 1 - \frac{V_{GS}}{V_{GS(off)}} \right)^2$$

It is the normal operating region of the JFET when used as an amplifier.

**4. Breakdown Region**

If  $V_{DS}$  is increased beyond its value corresponding to point C (called avalanche breakdown voltage), JFET enters the breakdown region where  $I_D$  increases to an excessive value. This happens because the reverse-biased gate-channel P-N junction undergoes avalanche breakdown when small changes in  $V_{DS}$  produce very large changes in  $I_D$ .

It is interesting to note that increasing values of  $V_{DS}$  make a JFET behave first as a resistor (ohmic region), then as a constant-current source (pinch-off region) and finally, as a constant-voltage source (breakdown region).

**63.5. JFET Characteristics With External Bias**

Fig. 63.6 shows a family of  $I_D$  versus  $V_{DS}$  curves for different values of  $V_{GS}$ . It is seen that as the negative gate bias voltage is increased

(i) pinch off voltage is reached at a lower value of  $I_D$  than when  $V_{GS} = 0$ .

(ii) value of  $V_{DS}$  for breakdown is decreased.

When an external bias of, say, -1V is applied between the gate and source, the P-N junctions become reverse-biased even when  $I_D = 0$ . Hence,

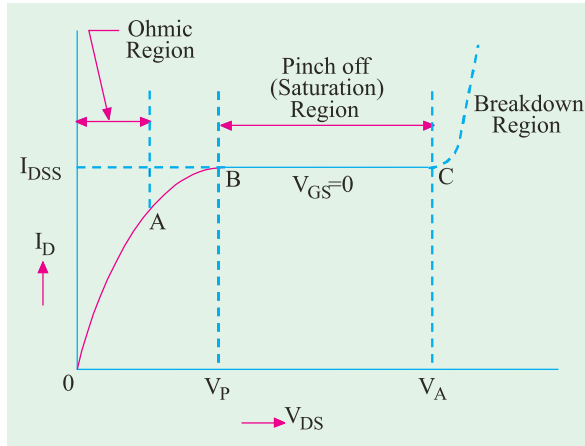


Fig. 63.5

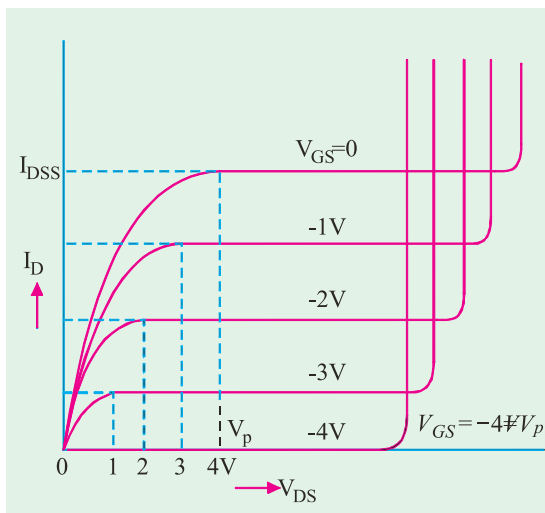


Fig. 63.6

\* It is numerically equal to  $V_{GS(off)}$  i.e.  $V_p = |V_{GSC(off)}|$



the depletion regions are already formed which penetrate the channel to a certain extent.

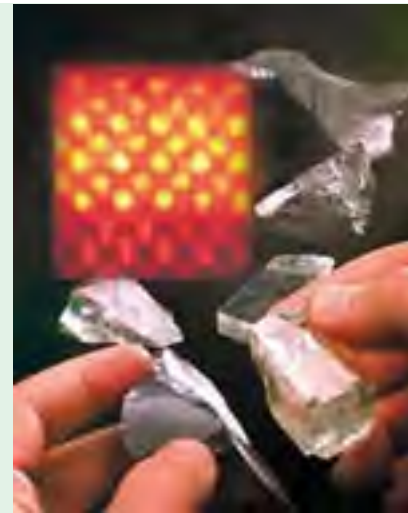
The amount of reverse bias required to be produced by  $I_D$  would, obviously, be decreased by 1V. In other words, a smaller voltage drop along the channel (*i.e.* smaller than when  $V_{GS} = 0$ ) will increase the depletion regions to the point where they will pinch off the current. Consequently,  $V_p$  is reached at a lower  $I_D$  value than when  $V_{GS} = 0$ .

Now, let us see why value of  $V_{DS}$  for breakdown is decreased as the negative gate bias voltage is increased. It is simply due to the fact that  $V_{GS}$  keeps adding to the reverse bias at the junction produced by current flow.

It is seen that with  $V_{GS} = 0$ ,  $I_D$  saturates at  $I_{DSS}$  and the characteristic shows  $V_p = 4V$ . When an external bias of  $-1 V$  is applied, gate-channel junctions still require  $-4 V$  to achieve pinch-off (remember,  $V_{GS} = -V_p$ ). It means that a 3V drop is now required along the channel instead of the previous 4V. Obviously, this 3V drop can be achieved with a lower value of  $I_D$ . Similarly, when  $V_{GS}$  is  $-2V$  and  $-3V$ , pinch-off is achieved with 2 V and 1 V respectively along the channel.

These drops of 2 V and 1V are obtained with further-reduced values of  $I_D$ . As seen, when  $V_{GS} = -4 V$  (*i.e.* numerically equal to  $V_p$ ), no channel drop is required. Hence,  $I_D$  is zero.

In general,  $V_p = V_{DS(P)} - V_{GS}$  where  $V_{DS(P)}$  is the pinch-off value of  $V_{DS}$  for a given value of  $V_{GS}$ .



This micrograph shows a mock field effect transistor with a layer of crystalline strontium titanate instead of silicon dioxide as the gate electrode.

### 63.6. Transfer Characteristic

It is a plot of  $I_D$  versus  $V_{GS}$  for a constant value of  $V_{DS}$  and is shown in Fig. 63.7. It is similar to the transconductance characteristics of a vacuum tube or a transistor. It is seen that when  $V_{GS} = 0$ ,  $I_D = I_{DSS}$  and when  $I_D = 0$ ,  $V_{GS} = V_p$ . The transfer characteristic approximately follows the equation.

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_p} \right)^2 = I_{DSS} \left( 1 - \frac{V_{GS}}{V_{GS(off)}} \right)^2$$

The above equation can be written as

$$V_{GS} = V_{GS(off)} \left( 1 - \sqrt{\frac{I_D}{I_{DSS}}} \right)$$

This characteristic can be obtained from the drain characteristics by reading off  $V_{GS}$  and  $I_{DSS}$  values for different values of  $V_{DS}$ .

### 63.7. Small Signal JFET Parameters

The various parameters of a JFET can be obtained from its two characteristics. The main parameters of a JFET when connected in common-source mode are as under :

**(j) AC Drain Resistance,  $r_d$**

It is the ac resistance between drain and source terminals when JFET is operating **in the pinch-off region**. It is given by

$$r_d = \frac{\text{change in } V_{DS} - V_{GS}}{\text{change in } I_D} \text{ constant or } r_d = \frac{\Delta V_{DS}}{\Delta I_D} \Big|_{V_{GS}}$$

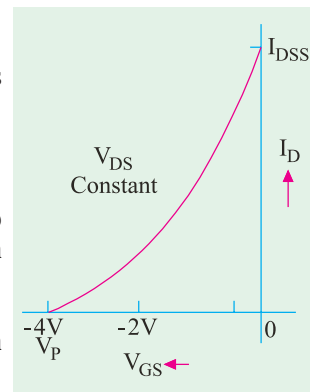


Fig. 63.7



An alternative name is **dynamic drain resistance**. It is given by the slope of the drain characteristic in the pinch-off region. It is sometimes written as  $r_{ds}$  emphasizing the fact that it is the resistance from drain to source. Since  $r_d$  is usually the output resistance of a JFET, it may also be expressed as an output admittance  $y_{os}$ . Obviously,  $y_{os} = 1/r_d$ . It has a very high value.

**(ii) Transconductance,  $g_m$**

It is simply the slope of transfer characteristic.

$$g_m = \frac{\text{change in } I_D}{\text{change in } V_{GS}} \Big|_{V_{DS} \text{ constant}} \text{ or } g_m = \frac{\Delta I_D}{\Delta V_{GS}} \Big|_{V_{DS}}$$

Its unit is Siemens (S) earlier called **mho**. It is also called **forward transconductance ( $g_{fs}$ )** or **forward transadmittance  $y_{fs}$**

The transconductance measured at  $I_{DSS}$  is written as  $g_{mo}$ .

**Mathematical Expression for  $g_m$**

The Shockley equation\* is  $I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$

Differentiating both sides, we have

$$\frac{dI_D}{dI_{DSS}} = 2I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right) \left(-\frac{1}{V_P}\right) \text{ or } g_m = -\frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P}\right)$$

$$\text{When } V_{GS} = 0, \quad g_m = g_{mo} \quad \therefore \quad g_{mo} = -\frac{2I_{DSS}}{V_P}$$

$$\text{From the above two equations, we have } g_m = g_{mo} \left(1 - \frac{V_{DSS}}{V_P}\right) = g_{mo} \sqrt{\frac{I_D}{I_{DSS}}}$$

**(iii) Amplification Factor,  $\mu$**

It is given by  $\mu = \frac{\text{change in } V_{DS}}{\text{change in } V_{GS}} \Big|_{I_D \text{ constant}}$  or  $\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}} \Big|_{I_D}$

It can be proved from above that

$$\mu = g_m \times r_d = g_{fs} \times r_d$$

**(iv) DC Drain Resistance,  $R_{DS}$**

It is also called the static or ohmic resistance of the channel. It is given by

$$R_{DS} = \frac{V_{DS}}{I_D}$$

**Example 63.1.** For an N-channel JFET,  $I_{DSS} = 8.7 \text{ mA}$ ,  $V_P = -3 \text{ V}$ ,  $V_{GS} = -1 \text{ V}$ . Find the values of **(i)  $I_D$  (ii)  $g_{mo}$  (iii)  $g_m$**  **(Basic Electronics, Bombay Univ., 1985)**

$$\text{Solution. (i) } I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 8.7 \left(1 - \frac{-1 \text{ V}}{-3 \text{ V}}\right)^2 = \mathbf{3.87 \text{ A}}$$

$$\text{(ii) } g_{mo} = \frac{-2I_{DSS}}{V_P} = \frac{-2 \times 8.7}{-3} = \mathbf{5.8 \text{ mS}}$$

$$\text{(iii) } g_m = g_{mo} \left(1 - \frac{V_{GS}}{V_P}\right) = 5.8 \left(1 - \frac{-1}{-3}\right) = \mathbf{3.87 \text{ mS}}$$

\* Because of the squared term in the equation, JFET and MOSFET are referred to as square-law devices.



### 63.8. DC Biasing of a JFET

A JFET may be biased by using either

1. a separate power source  $V_{GG}$  as shown in Fig. 63.8 (a),
2. some form of self-bias as shown in Fig. 63.8 (b),
3. source bias as in Fig. 63.8 (c),
4. voltage divider bias as in Fig. 63.8 (d).

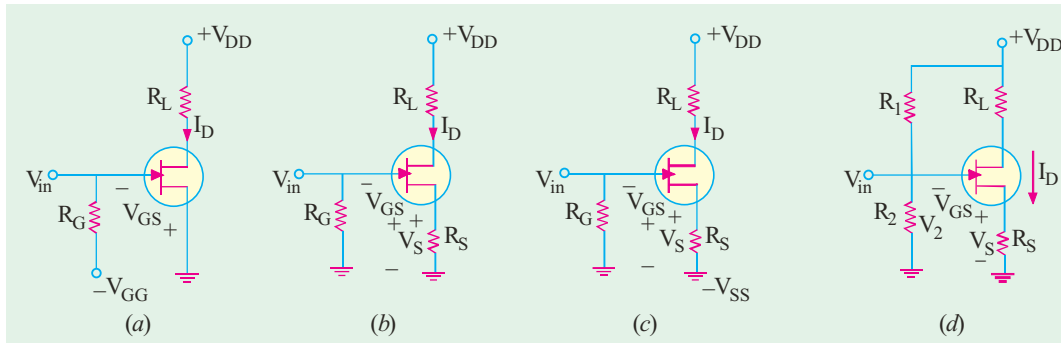


Fig. 63.8

The circuit of Fig. 63.8 (b) is called self-bias circuit because the  $V_{GS}$  bias is obtained from the flow of JFET's own drawn current  $I_D$  through  $R_S$ .

$$\therefore V_S = I_D R_S \text{ and } V_{GS} = -I_D R_S$$

The gate is kept at this much negative potential with respect to the ground.

The addition of  $R_G$  in Fig. 63.8 (b), does not upset this dc bias for the simple reason that no gate current flows through it (the gate leakage current is almost zero). Hence, gate is essentially at **dc ground**. Without  $R_G$ , gate would be kept 'floating' which could collect charge and ultimately cut-off the JFET.

The resistance  $R_G$  additionally serves the purpose of avoiding short-circuiting of the ac input voltage,  $v_{in}$ . Moreover, in case leakage current is not totally negligible,  $R_G$  would provide it an escape route. Otherwise, the leakage current would build up static charge (voltage) at the gate which could change the bias or even destroy the JFET.

Fig. 63.8 (c) shows the *source bias* circuit which employs a self-bias resistor  $R_S$  to obtain  $V_{GS}$ . Here,  $V_{SS} = I_D R_S + V_{GS}$  or  $V_{GS} = V_{SS} - I_D R_S$ .

Fig. 63.8 (d) shows the familiar voltage divider bias. In this case,  $V_2 = V_{GS} + I_D R_S$  or  $V_{GS} = V_2 - I_D R_S$

$$\text{Since, } V_2 = V_{DD} \frac{R_2}{R_1 + R_2} \quad \therefore V_{GS} = V_{DD} \frac{R_2}{R_1 + R_2} - I_D R_S$$

**Example 63.2.** Find the values of  $V_{DS}$  and  $V_{GS}$  in Fig. 63.9 for  $I_D = 4 \text{ mA}$ .

(Applied Electronics-I, Punjab Univ. 1992)

**Solution.**

$$V_S = I_D R_S = 4 \times 10^{-3} \times 500 = 2.0 \text{ V}$$

$$V_D = V_{DD} - I_D R_L = 12 - 4 \times 1.5 = 6 \text{ V}$$

$$\therefore V_{DS} = V_D - V_S = 6 - 2 = 4 \text{ V}$$

Since  $V_G = 0$ ,  $V_{GS} = V_G - V_S = 0 - 2.0 = -2.0 \text{ V}$

### 63.9. DC Load Line

The dc load line for a JFET can be easily drawn by remembering the following two points :

(i) At  $I_D = 0$ ,  $V_{DS} = V_{DD}$

(ii) At  $V_{DS} = 0$ ,  $I_D = \frac{V_{DD}}{R_L}$



The  $Q$ -point is generally situated at the middle point of the load line (for class-A operation) so that

$$V_{DSQ} = \frac{1}{2}V_{DD}$$

$$\text{Also, } I_{DSQ} = \frac{\frac{1}{2}V_{DD}}{R_s + R_L}$$

**Example 63.3.** For the circuit of Fig. 63.11, find the values of  $V_{DSQ}$  and  $V_{GS}$  assuming centrally-located  $Q$ -point and zero gate current.

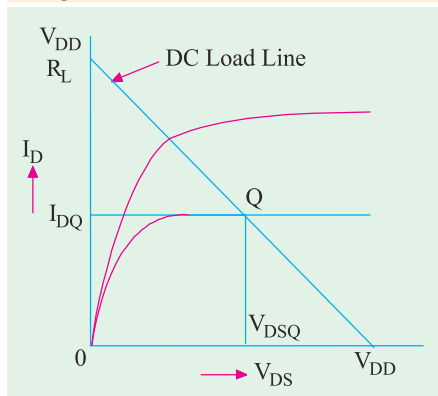


Fig. 63.10

**Solution.** Since  $I_G = 0$ , dc circuit is not disturbed.

$$V_{DS} = \frac{1}{2}V_{DD} = \frac{12}{2} = 6V$$

The balance of 6 V drops across series combination of  $R_L$  and  $R_S$ .

$$\therefore I_D = \frac{6}{150 + 450} = 10 \text{ mA}$$

$$\therefore V_{GS} = -I_D R_S = -10 \times 150 = -1.5 \text{ V}$$

It is obvious that gate is 1.5 V negative with respect to the source which is the common point. Incidentally, in common source connection of a JFET, **gate is the most negative point in the entire circuit.**

**Example 63.4.** What values of  $R_S$  and  $R_L$  are required for the circuit of Fig. 63.12 for setting up an approximate mid-point bias? The JFET parameters are :  $I_{DSS} = 16 \text{ mA}$ ,  $V_{GS(off)} = -8 \text{ V}$  and  $V_D = \frac{1}{2}V_{DD}$ .

**Solution.** It should be noted that as found from Shockley's equation, for mid-point bias,  $I_D \cong I_{DSS}/2$  and  $V_{GS} = V_{GS(off)}/4$ . Hence, for mid-point bias

$$I_D \cong 16/2 = 8 \text{ mA}$$

$$V_{GS} = \frac{1}{4}V_{GS(off)} = -\frac{8}{4} = -2 \text{ V}$$

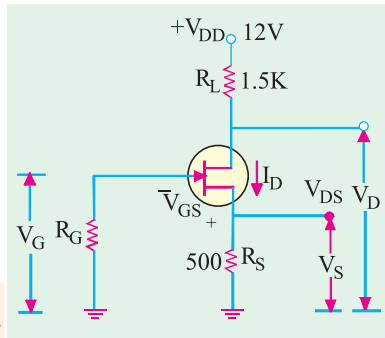


Fig. 63.9

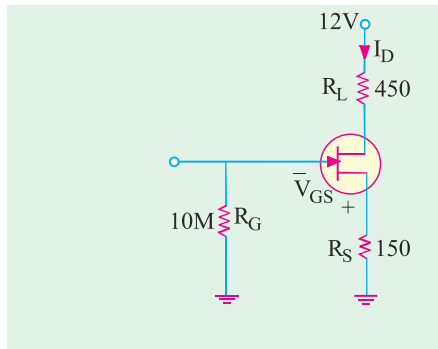


Fig. 63.11

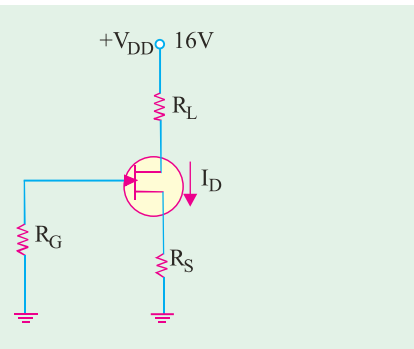


Fig. 63.12





$$R_S = \frac{V_{GS}}{I_D} = \frac{2 \text{ V}}{8 \text{ mA}} = 250 \Omega$$

Now,  $V_D = V_{DD} - I_D R_L$ ,  $R_L = \frac{V_{DD} - V_D}{I_D} = \frac{16 - 8}{8 \text{ mA}} = 1000 \Omega$

**Example 63.5.** Determine the quiescent value of  $V_{GS}$ ,  $I_D$  and  $V_{DS}$  for the JFET circuit of Fig. 63.13 given that  $I_{DSS} = 10 \text{ mA}$ ,  $R_S = 5 \text{ K}$  and  $V_p = -5 \text{ V}$ .

(Electronic Devices & Circuits, Pune Univ. 1991)

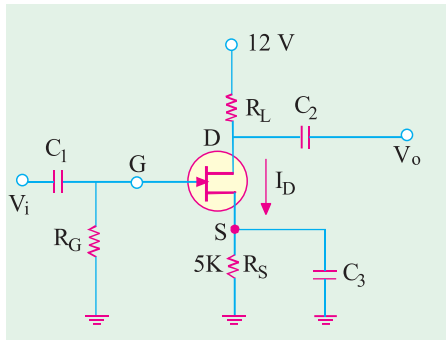


Fig. 63.13

**Solution.** Since  $I_S \cong I_D$ ,  $V_{GS} = -I_D R_S = -5000 I_D$

$$\begin{aligned} \text{Now, } I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2 = 10 \times 10^{-3} \left(1 - \frac{V_{GS}}{-5}\right)^2 \\ &= 10 \times 10^{-3} (1 + 0.2 V_{GS})^2 \end{aligned}$$

Substituting this value in the above equation, we get

$$V_{GS} = -5000(10 \times 10^{-3}) (1 + 0.2 V_{GS})$$

Expanding and rearranging the above, we have

$$2V_{GS}^2 + 21V_{GS} + 50 = 0$$

$$\therefore V_{GS} = -3.65 \text{ V} \quad \text{or} \quad -6.85 \text{ V}$$

Rejecting the higher value because it is more than  $V_p$ , we have  $V_{GS} = -3.65 \text{ V}$

$$\therefore -3.65 = -5000 I_D \quad \therefore I_D = 0.73 \text{ mA}$$

$$V_D = V_{DD} - I_D R_L = 12 - 0.73 \times 2 = 1.54 \text{ V}$$

$$V_S = I_D R_S = 0.73 \times 5 = 3.65 \text{ V} \quad \therefore V_{DS} = V_D - V_S = 10.54 - 3.65 = 6.89 \text{ V}$$

### 63.10. Common Source JFET Amplifier

A simple circuit for such an amplifier is shown in Fig. 63.14. Here,  $R_G$  serves the purpose of providing leakage path to the gate current,  $R_S$  develops gate bias,  $C_3$  provides ac ground to the input signal and  $R_L$  acts as drain load.

#### Working

When **negative-going** signal is applied to the input

1. gate **bias is increased**,
2. depletion regions are **widened**,
3. channel resistance is **increased**,
4.  $I_D$  is **decreased**,
5. drop across  $R_L$  is **decreased**,
6. Consequently, a **positive-going** signal becomes available at the output through  $C_2$  in Fig. 63.14.

When **positive-going signal** is applied at the input, then in a similar way, a **negative-going** signal becomes available at the output.

It is seen that there is a **phase inversion** between the input signal at the gate and output signal at the drain.

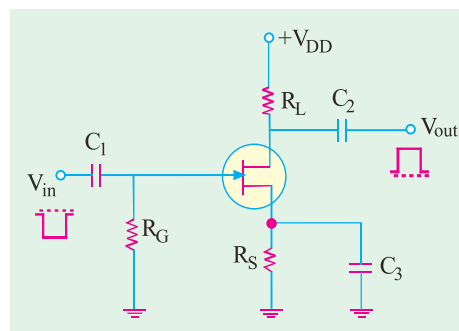


Fig. 63.14



63.11. JFET Amplifier Gains

We will now find the expressions for voltage gain, input resistance, output resistance and input capacitance of a JFET amplifier when connected in different modes. The different capacitance due to P-N junction and channel are shown in Fig. 63.15

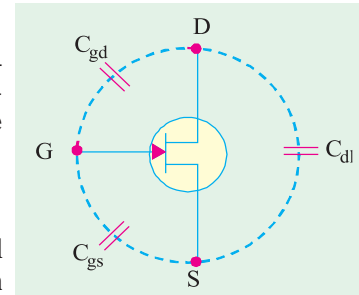


Fig. 63.15

(a) Common Source JFET Amplifier (Fig. 63.16)

(i) Input Resistance  $r'_i = R_G \parallel R_{GS}$

In an ideal JFET,  $R_{GS}$  is infinite because  $I_G = 0$ . In an actual device, however,  $R_{GS}$  is not actually infinite but extremely high (100 M or so) as compared to  $R_G$ . Hence,  $r'_i \cong R_G$ .

(ii) Output Resistance  $r'_o = r_d \parallel R_L \cong R_L$  — if  $r_d \gg R_L$

(iii) Voltage Gain

$$V_o = i_d \times r_d \parallel R_L \quad \text{Now, } i_d = -g_m \times v_i$$

$$\therefore V_o = -g_m V_i \times (r_d \parallel R_L)$$

$$\therefore A_v = \frac{V_o}{V_i} = \frac{-g_m V_i \times (r_d \parallel R_L)}{V_i} = \frac{-g_m r_d R_L}{r_d + R_L} = \frac{-\mu R_L}{r_d + R_L}$$

Also,  $A_v = -g_{fs} \times (r_d \parallel R_L)$

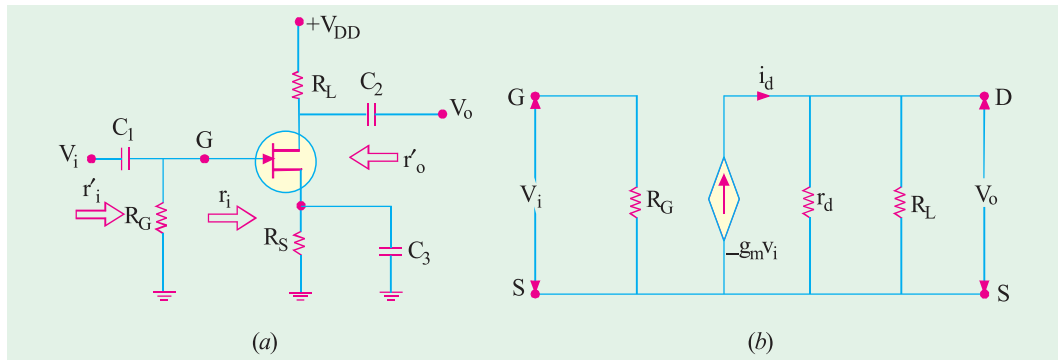


Fig. 63.16

(iv) Input Capacitance

The input capacitance is  $C_{gs}$  which is increased because of Miller effect.

$$\therefore C_i = C_{gs} + (1 - A_v) C_{gd}$$

It is the large value of  $C_{gd}$  which is harmful in high-frequency work.

**Example 63.6.** The common-source amplifier of Fig. 63.16 (a) has  $r_d = 100 \text{ K}$ ,  $R_L = 10 \text{ K}$ ,  $g_m = 3000 \mu\text{S}$ ,  $C_{gs} = 3 \text{ pF}$  and  $C_{gd} = 1.5 \text{ pF}$ . Compute its (i)  $A_v$  and (ii)  $C_i$ .

**Solution. (i)** 
$$A_v = \frac{-g_m r_d R_L}{r_d + R_L}$$

$$= \frac{-3000 \times 10^{-6} \times 100 \times 10^3 \times 10 \times 10^3}{(100 \times 10^3) + (10 \times 10^3)} = -27.3$$

**(ii)** 
$$C_i = C_{gs} + (1 - A_v) C_{gd}$$

$$= 3 + (1 + 27.3) \times 1.5 = 45.5 \text{ pF.}$$

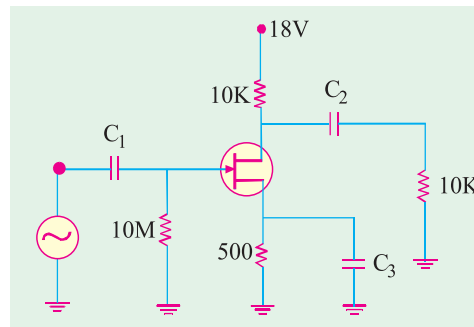


Fig. 63.17



**Example 63.7.** The JFET shown in Fig. 63.17 has  $g_m = 3000 \mu S$  and  $r_{ds} = 100 K$ . Calculate the voltage gain of the CS amplifier circuit. **(Basic Electronics, Bombay Univ. 1992)**

**Solution.** As seen from Art. 63.11 (a)

$$A_v = -g_m \times (r_{ds} \parallel r_L)$$

Now,  $r_L = 10 K \parallel 10 K = 5 K$

$\therefore r_{ds} \parallel r_L = 100 K \parallel 5 K = 4.76 K$

$\therefore A_v = -3000 \times 10^{-6} \times 4.76 = -14.3$

**(b) Common Drain JFET Amplifier**

In the common-drain circuit (also called source follower), the load resistance is in series with the source terminal. There is no drain resistor as shown in Fig. 63.18 (a). The input signal is applied to the gate through the capacitor  $C_1$  and the output is taken out from the source via  $C_2$ . The common-drain equivalent circuit is shown in Fig. 63.18 (b). The current generator is  $g_m V_{gs}$ , where  $v_{gs} = (V_i - V_o)$ . Moreover,  $R_G = R_1 \parallel R_2$ .

**(i) Voltage Gain**

$$V_o = i_d \times (r_d \parallel R_L)$$

Since,  $i_d = g_m V_{gs} = g_m (V_i - V_o)$ ,

$$\therefore V_o = g_m (V_i - V_o) \times (r_d \parallel R_L) = g_m (V_i - V_o) \frac{r_d R_L}{r_d + R_L}$$

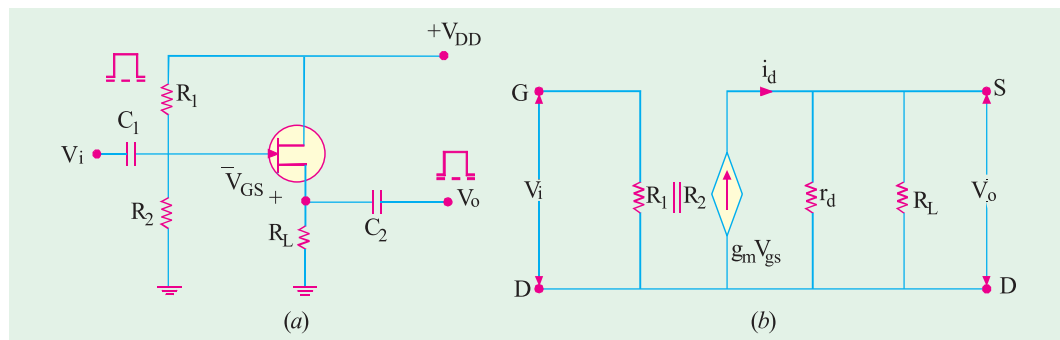


Fig. 63.18

Solving for  $V_o$ , we get

$$V_o = g_m V_i \frac{r_d R_L}{r_d + R_L + g_m r_d R_L} \quad \dots(i)$$

$$\therefore A_v = \frac{V_o}{V_i} = \frac{r_d R_L}{r_d + R_L + g_m r_d R_L} \cong 1 \quad \text{— if } g_m r_d R_L \gg (r_d + R_L)$$

**(ii) Input Resistance**

$$r'_{in} = R_1 \parallel R_2 \quad \text{— for circuit of Fig. 63.18 (a) only.}$$

**(iii) Output Resistance**

In Eq. (i) above,  $g_m V_i$  is a current which is directly proportional to  $V_i$  and  $(r_d R_L) / (r_d + R_L + g_m r_d R_L) = a$  resistance,  $r'_o$ .

$$\therefore r'_o = \frac{r_d R_L}{r_d + R_L (1 + g_m r_d R_L)} = \frac{[r_d / (1 + g_m r_d)] \times R_L}{[r_d / (1 + g_m r_d)] + R_L}$$

$$= \frac{r_d}{1 + g_m r_d} \parallel R_L \cong \frac{1}{g_m} \parallel R_L$$

The above result helps us to draw the modified equivalent circuit of Fig. 63.19 for the common-drain amplifier.

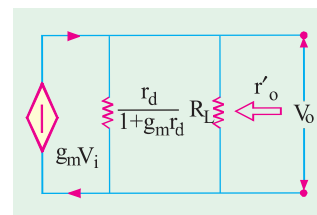


Fig. 63.19



**Example 63.8.** The common-drain circuit of Fig. 63.18 (a) uses a JFET having the following parameters :

$$r_d = 100 \text{ K}, \quad g_m = 3000 \text{ } \mu\text{S} \text{ and} \quad R_L = 10 \text{ K}$$

Calculate (i)  $A_v$  and (ii)  $r'_o$ .

**Solution.** (i)  $A_v = g_m \frac{r_d R_L}{r_d + R_L + g_m r_d R_L}$

$$= 3000 \times 10^{-6} \frac{100 \times 10^3 \times 10 \times 10^3}{(100 \times 10^3) + (10 \times 10^3) + (3000 \times 10^{-6} \times 100 \times 10^3 \times 10 \times 10^3)} = 0.965$$

(ii)  $r'_o = R_L \parallel \frac{r_d}{1 + g_m r_d}$

Now,  $\frac{r_d}{1 + g_m r_d} = \frac{100 \times 10^3}{1 + 3000 \times 10^{-6} \times 100 \times 10^3} = 330 \text{ } \Omega$

$$r'_o = 10 \text{ K} \parallel 330 \text{ } \Omega \cong 320 \text{ } \Omega$$

**(c) Common Gate JFET Amplifier**

In this amplifier configuration, input signal is applied to the source terminal and output is taken from the drain as shown in Fig. 63.20 (a). The gate is grounded,  $R_L$  is in series with drain and a source resistance  $R_S$  is included in the circuit across which  $V_i$  is dropped.

The ac equivalent circuit is shown in Fig. 63.20 (b) where current source is connected between the

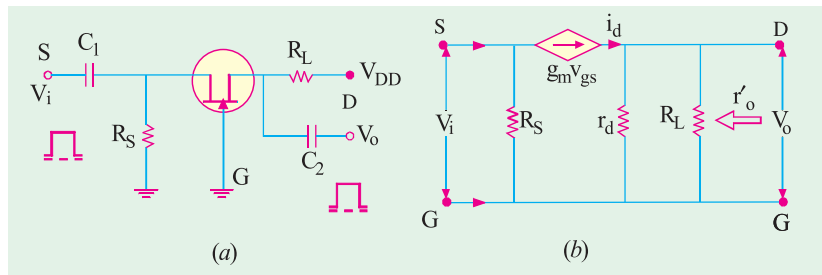


Fig. 63.20

**drain and the source**

**terminals as always.** However, since source and drain are the input and output terminals respectively,  $(g_m V_{gs})$  appears between the input and the output.

**(i) Voltage Gain**

$$V_o = i_d \times (r_d \parallel R_L) = i_d \cdot \frac{r_d R_L}{r_d + R_L}$$

Now,  $i_d = g_m V_{gs} = g_m V_i \quad \therefore V_o = g_m V_i \cdot \frac{r_d R_L}{r_d + R_L} \quad \therefore A_v = \frac{V_o}{V_i} = \frac{g_m r_d R_L}{r_d + R_L}$

It is the same as the gain for a common source amplifier except that it is a positive quantity. It means that  $V_o$  and  $V_i$  are in phase as shown in Fig. 63.20 (a).

**(ii) Input Resistance**

If we ignore current through  $R_S$ , then as seen from the ac equivalent circuit of Fig. 63.20 (b), input current is  $i_d$ . Now,  $i_d = g_m V_{gs} = g_m V_i$ .

The input resistance of the device i.e. JFET is  $r_i = \frac{V_i}{i_d} = \frac{V_i}{g_m V_i} = \frac{1}{g_m}$



The input resistance of the circuit is  $r_i' = r_i \parallel R_S = \frac{1}{g_m} \parallel R_S$

Actually speaking,  $r_d$  and  $R_L$  are also involved in  $r_i'$  but their effect is negligible.

**(iii) Output Resistance**

As seen from the equivalent circuit of Fig. 63.20 (b),  $r_o' = r_d \parallel R_L$ .

Here, only input capacitance of importance is  $C_{gs}$ . Hence, this circuit has low input capacitance as compared to common-source circuit where input capacitance is increased due to Miller effect.\*

**63.12. Advantages of FETs**

FETs combine the many advantages of both BJTs and vacuum tubes. Some of their main advantages are :

1. high input impedance,
2. small size,
3. ruggedness,
4. long life,
5. high frequency response,
6. low noise,
7. negative temperature coefficient, hence better thermal stability,
8. high power gain,
9. a high immunity to radiations,
10. no offset voltage when used as a switch (or chopper),
11. square law characteristics.

The only disadvantages are :

1. small gain-bandwidth product,
2. greater susceptibility to damage in handling them.

**Tutorial Problems No. 63.1**

1. For a particular N-channel JFET,  $V_{GS(off)} = -4$  V. What would be the value of  $I_D$  when  $V_{GS} = -6$  V? **[zero]**
2. For the N-channel JFET shown in Fig. 63.21,  $V_p = 8$  V and  $I_{DSS} = 12$  mA. What would be the value of (i)  $V_{DS}$  at which pinch-off begins and (ii) value of  $I_D$  when  $V_{DS}$  is above pinch-off but below the breakdown voltage? **[3V ; 12 mA]**
3. The data sheet of a JFET indicates that it has  $I_{DSS} = 15$  mA and  $V_{GS(off)} = -5$  V. Calculate the value of  $I_D$  when  $V_{GS}$  is (i) 0, (ii) -1 V and (iii) -4 V. **[(i) 15 mA (ii) 9.6 mA (iii) 0.6 mA]**

4. The data sheet of a JEET gives the following information.  
 $I_{DSS} = 20$  mA,  $V_{GS(off)} = -8$  V.  
 and  $g_{mo} = 4000$   $\mu$ S.  
 Calculate the value of  $I_D$  and  $g_m$  for  $V_{GS} = -4$  V. **[5 mA ; 2000  $\mu$ S]**

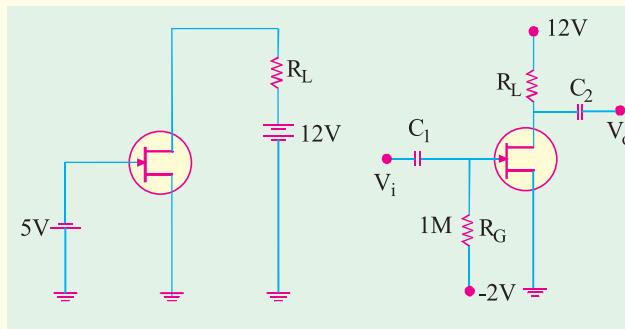


Fig. 63.21

Fig. 63.22

5. For a JEET,  $I_{DSS} = 5$  mA and  $g_{mo} = 4000$   $\mu$ S. Calculate (i)  $V_{GS(off)}$  and (ii)  $g_m$  at mid-point bias. **[(i) -5 V (ii) 3000  $\mu$ S]**
6. At a certain point on the transfer characteristics of an N-channel JFET, following values are read :  $I_{DSS} = 8.4$  mA,  $V_{GS} = -0.5$  V and  $V_p = -3.0$  V. Calculate (i)  $g_{mo}$  and (ii)  $g_m$  at the point. **[(i) 5600  $\mu$ S (ii) 4670  $\mu$ S]**
7. For the JFET circuit of Fig. 63.22,  $I_{DSS} = 9$  mA and  $V_p = -3$  V. Find the value of  $R_L$  for setting the value of  $V_{DS}$  at 7 V. **[5 K]**  
**(Hint :  $V_{GS} = -2$  V as  $I_G = 0$ )**

\* Miller effect occurs only where output is antiphase with input as CS amplifier circuit.

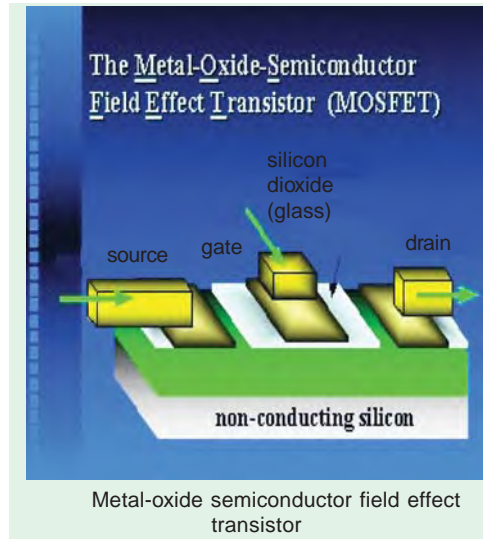


63.13. MOSFET or IGFET

It could be further subdivided as follows :

(i) Depletion-enhancement MOSFET or DE MOSFET

This MOSFET is so called because it can be operated in both *depletion* mode and *enhancement* mode by changing the polarity of  $V_{GS}$ . When negative gate-to-source voltage is applied, the *N*-channel DE MOSFET operates in the depletion mode. However, with positive gate voltage, it operates in the enhancement mode. Since a channel exists between drain and source,  $I_D$  flows even when  $V_{GS} = 0$ . That is why DE MOSFET is known as **normally-ON MOSFET**.



(ii) Enhancement-only MOSFET

As its name indicates, this MOSFET operates only in the enhancement mode and has no depletion mode. It works with *large positive* gate voltages only. It differs in construction from the DE MOSFET in that structurally *there exists no channel between the drain and source*. Hence, it does not conduct when  $V_{GS} = 0$ . That is why it is called **normally-OFF MOSFET**.

In a DE MOSFET,  $I_D$  flows even when  $V_{GS} = 0$ . It operates in depletion mode with negative values of  $V_{GS}$ . As  $V_{GS}$  is made more negative,  $I_D$  decreases till it ceases when  $V_{GS} = V_{GS(off)}$ . It works in enhancement mode when  $V_{GS}$  is positive as shown in Fig. 63.24 (b).

In the case of E-only MOSFET,  $I_D$  flows only when  $V_{GS}$  exceeds  $V_{GS(th)}$  as shown in Fig. 63.30 (c).

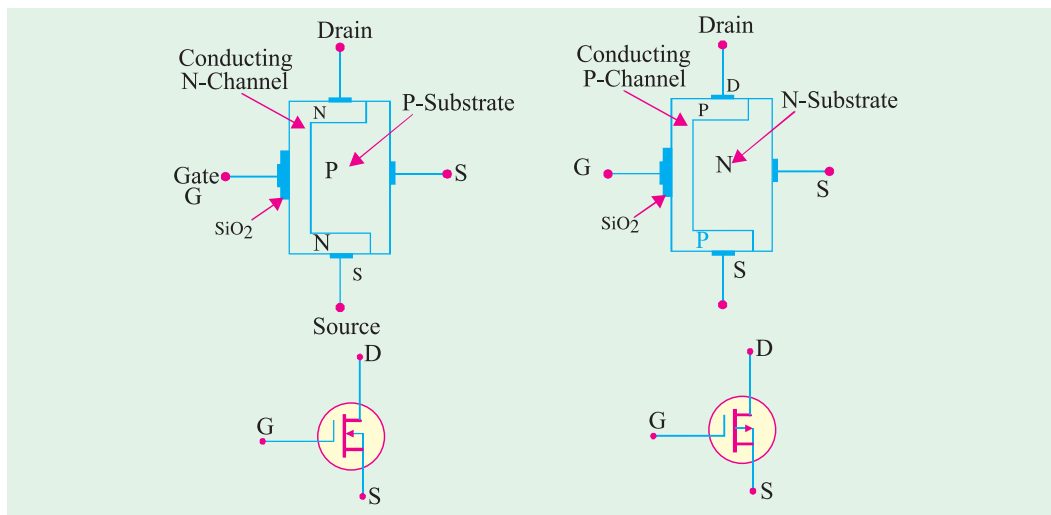


Fig. 63.23

63.14. DE MOSFET

(a) Construction

Like JFET, it has source, gate and drain. However, as shown in Fig. 63.23, its gate is insulated from its conducting channel by an ultra-thin metal-oxide insulating film (usually of silicon dioxide  $SiO_2$ ). Because of this insulating property, MOSFET is alternatively known as insulated-gate field-



effect transistor (IGFET or IGT). Here also, gate voltage controls drain current but main difference between JFET and MOSFET is that, in the latter case, we can apply **both positive and negative voltages to the gate because it is insulated from the channel**. Moreover, the gate,  $\text{SiO}_2$  insulator and channel form a parallel-plate capacitor. Unlike JFET, a DE MOSFET has only one P-region or N-region called substrate. Normally, it is shorted the **source internally**. Fig. 63.23 shows both P-channel and N-channel DE MOSFETs along with their symbols.

**(b) Working**

**(i) Depletion Mode of N-channel DE MOSFET**

When  $V_{GS} = 0$ , electrons can flow freely from source to drain through the conducting channel which exists between them. When gate is given negative voltage, it **depletes** the N-channel of its

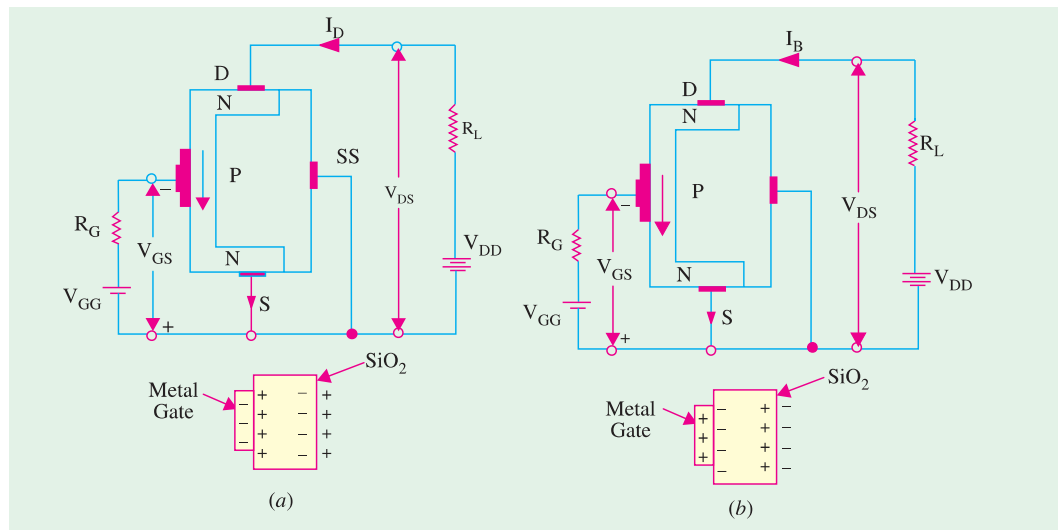


Fig. 63.24

electrons by including positive charge in it as shown in Fig. 63.24 (a). Greater the negative voltage on the gate, greater is the reduction in the number of electrons in the channel and, consequently, lesser its conductivity. In fact, too much negative gate voltage called  $V_{GS(off)}$  can cut-off the channel. Hence, with negative gate voltage, a DE MOSFET behaves like a JFET.

For obvious reasons, negative-gate operation of a DE MOSFET is called its depletion mode operation.

**(ii) Enhancement Mode of N-channel DE MOSFET**

The circuit connections are shown in Fig. 63.24 (b). Again, drain current flows from source to drain even with zero gate bias. When positive voltage is applied to the gate, the input gate capacitor is able to create free electrons in the channel which increases  $I_D$ . As seen from the enlarged view of the gate capacitor in Fig. 63.24 (b), free electrons are induced in the channel **by capacitor action**. These electrons are added to those already existing there. This increased number of electrons **increases** or **enhances** the conductivity of the channel. As positive gate voltage is increased, the number of induced electrons is increased, so conductivity of the source-to-drain channel is increased and, consequently, increasing amount of current flows between the terminals. That is why, positive gate operation of a DE MOSFET is known as its **enhancement mode** operation.

Since gate current in both modes is negligibly small, input resistance of a MOSFET is incredibly high varying from  $10^{10} \Omega$  to  $10^{14} \Omega$ . In fact, MOSFET input current is the leakage current of the capacitor unlike the input current for JFET which is the leakage current of a reverse-biased P-N junction.



**63.15. Schematic Symbols for a DE MOSFET**

Schematic symbol of an *N*-channel normally-ON or DE MOSFET is shown in Fig. 63.25. The gate looks like a metal plate. The arrow is on the substrate and **towards the N-channel**. When *SS* is connected to an external load, we have a 4-terminal device as shown in Fig. 63.25 (a) but when it is internally shorted to *S*, we get a 3-terminal device as shown in Fig. 63.25 (b). Fig. 63.25 (c) shows the symbol for a *P*-channel DE MOSFET.

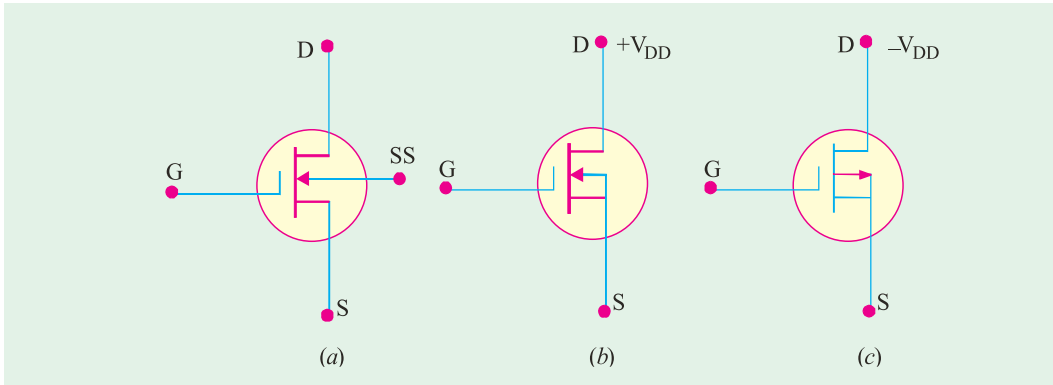


Fig. 63.25

**63.16. Static Characteristics of a DE MOSFET**

In Fig. 63.26 are shown the drain current and transfer characteristics of a common-source *N*-channel DE MOSFET for  $V_{GS}$  varying from +2 V to  $V_{GS(off)}$ .

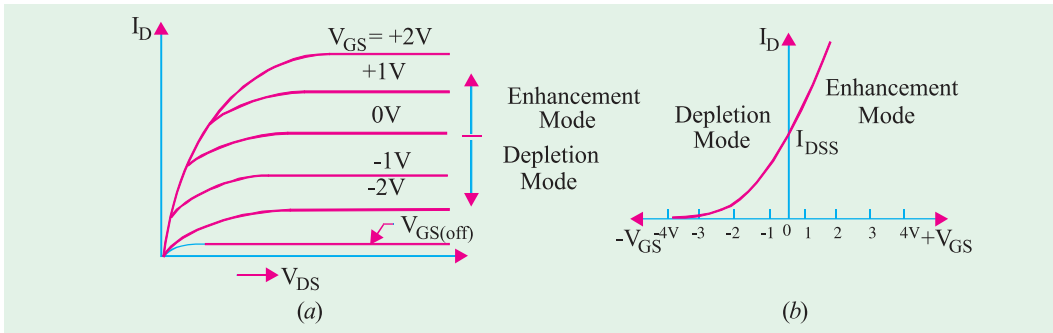


Fig. 63.26

It acts in the enhancement mode when gate is **positive** with respect to source and in the depletion mode when gate is **negative**. As usual,  $V_{GS(off)}$  represents the gate-source voltage which cuts off the source-to-drain current. The transfer characteristic is shown in Fig. 63.26 (b). For a given  $V_{DS}$ ,  $I_D$  flows even when  $V_{GS} = 0$ . However, keeping  $V_{DS}$  constant, as  $V_{GS}$  is made more negative,  $I_D$  keeps decreasing till it becomes zero at  $V_{GS} = V_{GS(off)}$ . When used in the enhancement mode,  $I_D$  increases as  $V_{DS}$  is increased positively.

**Example 63.9.** For the *N*-channel zero-biased DE MOSFET circuit of Fig. 63.27, calculate  $V_{DS}$  if  $I_{DSS} = 10 \text{ mA}$  and  $V_{GS(off)} = -6 \text{ V}$ .

(Electronics-I, Bangalore Univ. 1992)

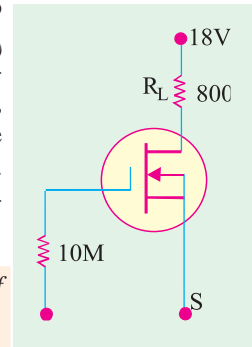


Fig. 63.27





**Solution.** Since  $I_D = I_{DSS} = 10 \text{ mA}$   
 $\therefore V_{DS} = V_{DD} - I_{DSS} R_L$   
 $= 18 - 10 \times 10^{-3} \times 800 = 10 \text{ V}$

**63.17. Enhancement-only N-Channel MOSFET**

This *N*-channel MOSFET (also called NMOS) finds wide application in digital circuitry. As shown in Fig. 63.28 in the NMOS, the *P*-type substrate extends all the way to the metal-oxide layer. Structurally, *there exists no channel between the source and drain*. Hence, an NMOS can never operate with a negative gate voltage because it will induce *positive* charge in the space between the drain and source which will not allow the passage of electrons between the two. Hence, it operates with *positive gate voltage only*.

The normal biasing polarities of this *E*-only MOSFET (both *N*-channel and *P*-channel) are shown in Fig. 63.29. With  $V_{GS} = 0$ ,  $I_D$  is non-existent even when some positive  $V_{DD}$  is applied. It is found that for getting significant amount of drain current, we have to apply sufficiently high positive gate voltage. This voltage is found to produce a thin layer of free electrons very close to the metal-oxide film which stretches all the way from source to drain. The thin layer of *P*-substrate touching the metal-oxide film which provides channel for electrons (and hence acts like *N*-type material) is called *N-type inversion layer or virtual N-channel* [Fig. 63.29 (a)].

The minimum gate-source voltage which produces this *N*-type inversion layer and hence drain current is called threshold voltage  $V_{GS(th)}$  as shown in Fig. 63.30 (c). When  $V_{GS} < V_{GS(th)}$ ,  $I_D = 0$ . Drain current starts only when  $V_{GS} > V_{GS(th)}$ . For a given  $V_{DS}$ , as  $V_{GS}$  is increased, virtual channel deepens and  $I_D$  increases. The value of  $I_D$  is given by  $I_D = K(V_{GS} - V_{GS(th)})^2$  where  $K$  is a constant which depends on the particular MOSFET. Its value can be determined from the data sheet by taking the specified value of  $I_D$  called  $I_{D(ON)}$  at the given value of  $V_{GS}$  and then substituting the values in the above equation. Fig. 63.30 (a)

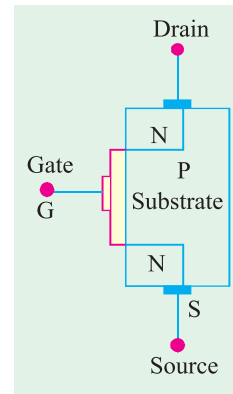


Fig. 63.28

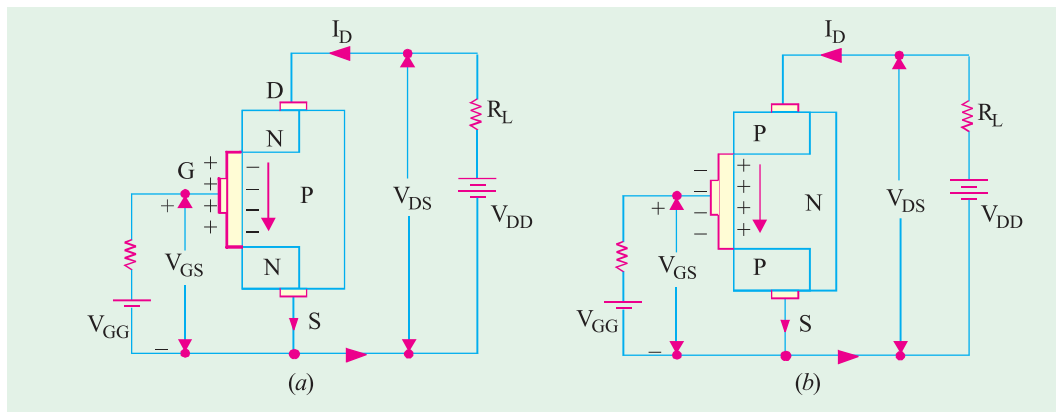


Fig. 63.29

shows the schematic symbol for an *E*-only *N*-channel MOSFET whereas Fig. 63.30 (b) shows its typical drain characteristics. As usual, arrow on the substrate points to the *N*-type material and the vertical line (representing-channel) is broken as a reminder of the normally-OFF condition.



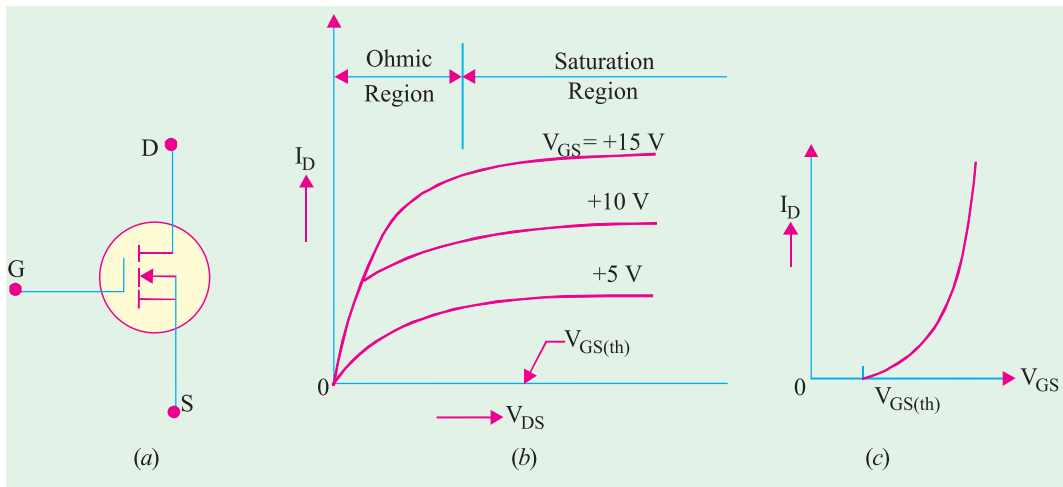


Fig. 63.30

A P-channel E-only MOSFET (PMOS) is constructed like NEMS except that all the P- and N- regions are interchanged. It operates with **negative** gate voltage only.

Differentiating the above-given drain current equation with respect to  $V_{GS}$ , we get

$$\frac{dI_D}{dV_{GS}} = g_m = 2K (V_{GS} - V_{GS(th)})$$

**Transfer Characteristics**

It is shown in Fig. 63.30 (c).  $I_D$  flows only when  $V_{GS}$  exceeds threshold voltage  $V_{GS(th)}$ . This MOSFET does not have an  $I_{DSS}$  parameter as do the JFET and DE MOSFET.

**63.18. Biasing E-only MOSFET**

As stated earlier, enhancement-only MOSFET must have a  $V_{GS}$  greater than  $V_{GS(th)}$ . Fig. 63.31 shows two methods of biasing an N-channel E-MOSFET. In either case, the purpose is to make the gate voltage more positive than the source by an amount exceeding  $V_{GS(th)}$ . Fig. 63.31 (a) shows drain-feedback bias whereas Fig. 63.31 (b) shows voltage-divider bias.

Considering the circuit of Fig. 63.31 (b), we have

$$V_{GS} = V_2 = V_{DD} \frac{R_2}{R_1 + R_2}$$

and  $V_{DS} = V_{DD} - I_D R_L$

where  $I_D = K(V_{GS} - V_{GS(th)})^2$ .

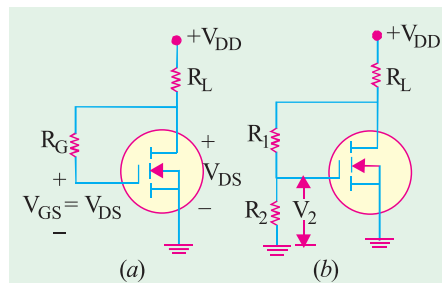


Fig. 63.31

**Example 63.10.** The data sheet of the E-MOSFET shown in Fig. 63.32 gives  $I_{D(ON)} = 4 \text{ mA}$  at  $V_{GS} = 10 \text{ V}$  and  $V_{GS(th)} = 5 \text{ V}$ . Calculate  $V_{GS}$  and  $V_{DS}$  for the circuit. (Applied Electronics, Punjab Univ. 1991)

**Solution.**  $V_{GS} = V_2 = 25 \times (9/15) = 15 \text{ V}$   
Let us now find the value of K.

$$K = \frac{I_{D(ON)}}{(V_{GS} - V_{GS(th)})^2} = \frac{4}{(10 - 5)^2} = 0.16 \text{ mA/V}^2$$

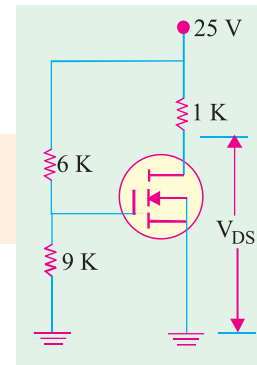


Fig. 63.32



Value of  $I_D$  for  $V_{GS}=15\text{ V}$  is given by

$$I_D = K (V_{GS} - V_{GS(th)})^2 = 0.16 (15 - 5)^2 = 16\text{ mA}$$

Now,  $V_{DS} = V_{DD} - I_D R_L = 25 - 16 \times 1 = 9\text{ V}$

**Example 63.11.** An N-channel E-MOSFET has the following parameters:

$I_{D(ON)} = 4\text{ mA}$  at  $V_{GS} = 10\text{ V}$  and  $V_{GS(th)} = 5\text{ V}$

Calculate its drain current for  $V_{GS} = 8\text{ V}$ .

**Solution.**  $K = \frac{I_{D(ON)}}{(V_{GS} - V_{GS(th)})^2} = \frac{4\text{mA}}{(10\text{V} - 5\text{V})^2} = 0.16\text{mA/V}^2$

Now, using this value of  $K$ ,  $I_D$  can be found thus :

$$I_D = K (V_{GS} - V_{GS(th)})^2 = 0.16 (8 - 5)^2 = 1.44\text{ mA}$$

### 63.19. FET Amplifiers

We will consider the DE MOSFET and E-MOSFET separately.

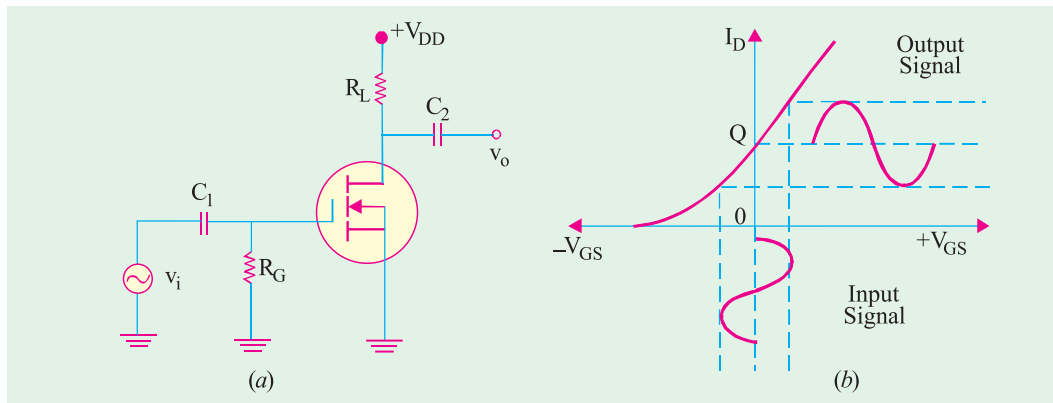


Fig. 63.33

#### (i) DE MOSFET Amplifier

Fig. 63.33 (a) shows a zero-biased N-channel DE MOSFET with an ac source capacitively-coupled to its gate. Since gate is at 0 volt dc and source is at ground,  $V_{GS} = 0$ . Fig. 63.33 (b) shows the transfer characteristic.

The input ac signal voltage  $V_i$  causes  $V_{GS}$  to swing above and below its zero value thus producing a swing in  $I_D$  as shown in Fig. 63.33 (b). The negative swing in  $V_{GS}$  produces depletion mode and  $I_D$

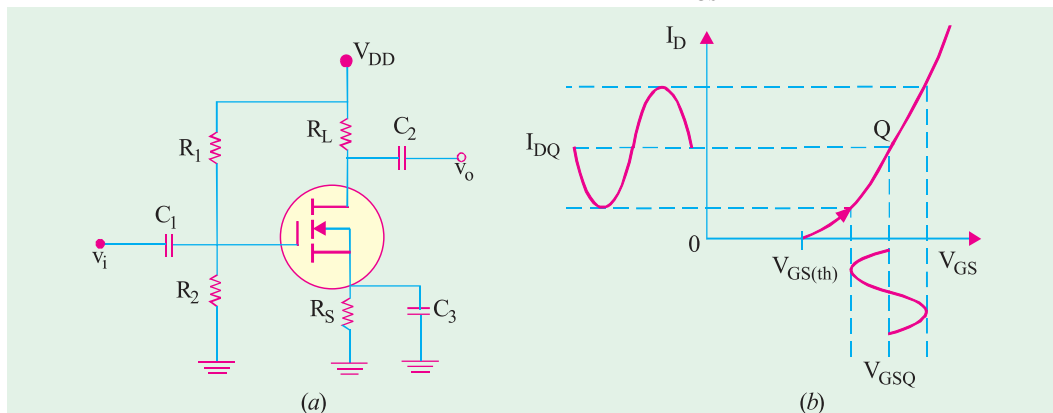


Fig. 63.34



is decreased. A positive swing in  $V_{GS}$  produces enhancement mode so that  $I_D$  is increased. This leads to a large swing in drop across  $R_L$  and can be taken out via  $C_2$  as  $V_o$ .

**(ii) E-MOSFET Amplifier**

In Fig. 63.34 (a) is shown a voltage-divider biased  $N$ -channel E-MOSFET having an ac signal source coupled to its gate. The gate is biased with a positive voltage such that  $V_{GS}$  exceeds  $V_{GS(th)}$ .

As shown in Fig. 63.34 (b), the signal voltage produces a swing in  $V_{GS}$  below and above its  $Q$ -point value. This, in turn, causes a swing in  $I_D$  and hence in  $I_D R_L$  which gives rise to no.

**Example 63.12.** The  $N$ -channel E-MOSFET used in the common-source amplifier of Fig. 63.35 has the following parameters :

$I_{D(ON)} = 4 \text{ mA}$  at  $V_{DS} = 10 \text{ V}$ ,  $V_{GS(th)} = 4 \text{ V}$  and  $g_m = 5000 \mu\text{S}$ .  
Calculate  $V_{GS}$ ,  $I_D$ ,  $V_{DS}$  and  $v_o$ . (Elect. and Electronic Engg., Annamalai Univ. 1992)

**Solution.**

$$V_{GS} = V_{DD} \frac{R_2}{R_1 + R_2} = 16 \times \frac{30}{80} = 6 \text{ V}$$

Now,  $K = \frac{I_{D(NO)}}{(V_{GS} - V_{GS(th)})^2} = \frac{4}{(10 - 4)^2} = 0.16 \text{ mA/V}^2$

$$\therefore I_D = K (V_{GS} - V_{GS(th)})^2 = 0.16(6 - 4)^2 = 0.64 \text{ mA}$$

$$\therefore V_{DS} = V_{DD} - I_D R_L = 16 - 0.64 \times 5 = 12.8 \text{ V}$$

Now,  $A_v = g_m (r_d \parallel R_L) \cong g_m R_L$

$$= 5000 \times 10^{-6} \times 5 \times 10^3 = 25$$

$$\therefore V_o = A_v v_i = 25 \times 100 = 2500 \text{ mV} = 2.5 \text{ V}$$

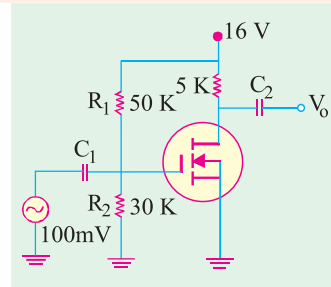


Fig. 63.35

**Example 63.13.** The parameters of the enhancement-only NMOS shown in Fig. 63.36 are  $V_{GS(th)} = 2 \text{ V}$  and  $K = 2 \times 10^{-4} \text{ A/V}^2$ . Calculate the values of  $I_D$  and  $V_{DS}$ .

**Solution.** Since drain is directly returned to gate,  $V_{GS} = V_{DS}$ . As seen from the figure

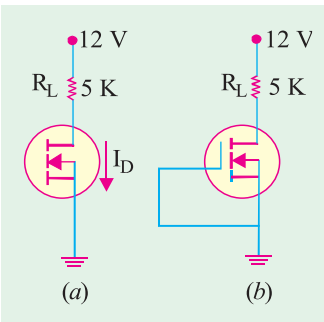


Fig. 63.36

$$V_{DS} = V_{DD} - I_D R_L = 12 - 5 \times 10^3 I_D$$

$$\therefore I_D = (12 - V_{DS}) / 5 \times 10^3 \text{ A}$$

Now,  $I_D = K (V_{GS} - V_{GS(th)})^2 = K (V_{DS} - 2)^2$

or  $\frac{12 - V_{DS}}{5 \times 10^3} = 2 \times 10^{-4} (V_{DS} - 2)^2$  or  $V_{DS}^2 - 3V_{DS} - 8 = 0$

$$\therefore V_{DS} = \frac{3 \pm \sqrt{9 + 32}}{2} = \frac{3 \pm \sqrt{41}}{2} = 4.7 \text{ V}$$

$$\therefore I_D = (12 - 4.7) / 5 \times 10^3 = 1.5 \text{ mA}$$

In Fig. 63.36 (b), since gate has been directly returned to ground,  $V_{GS} = 0$ . Hence,  $I_D = 0$ , because  $V_{GS} \ll V_{GS(th)}$ . With no current,  $V_{DS} = V_{DD} = 12 \text{ V}$ .

**Example 63.14.** A MOSFET has a drain resistance,  $R_L$  of  $44 \text{ k}\Omega$  and operates at  $20 \text{ kHz}$ . Calculate the voltage gain of this device as a single stage amplifier. The MOSFET parameters are :  $g_m = 1.6 \text{ mA/V}$ ,  $r_d = 100 \text{ k}\Omega$ ,  $C_{gs} = 3.0 \text{ pf}$ ,  $C_{ds} = 1.0 \text{ pF}$  and  $C_{gd} = 2.8 \text{ pF}$ .

(U.P.S.C. Engg. Services, 1996)

**Solution.**  $A_v = g_m (r_d \parallel R_L)$

$$= 1.6 \times 10^{-3} (100 \times 10^3 \parallel 44 \times 10^3)$$

$$= 1.6 \times 10^{-3} \times 30.56 \times 10^3$$

$$= 48.9$$



**63.20. FET Applications**

FETs can be used in almost every application in which bipolar transistors can be used. However, they have certain applications which are exclusive to them :

1. As input amplifiers in oscilloscopes, electronic voltmeters and other measuring and testing equipment because their high  $r_{in}$  *reduces loading effect to the minimum.*
2. In logic circuits where it is kept OFF when there is zero input while it is turned ON with very little power input.
3. For mixer operation of *FM* and *TV* receivers.
4. As voltage-variable resistor (*VVR*) in operational amplifiers and tone controls etc.
5. Large-scale integration (*LSI*) and computer memories because of very small size.

**63.21. MOSFET Handling**

MOSFETs require very careful handling particularly when *out of circuit*. In circuit, a MOSFET is as rugged as any other solid-state device of similar construction and size.

It is essential not to permit any stray or static voltage on the gate otherwise the ultra-thin  $\text{SiO}_2$  layer between the channel and the gate will get ruptured. Since gate-channel junction looks like a capacitor with extremely high resistance, it requires only a few electrons to produce a high voltage across it. Even *picking up a MOSFET by its leads can destroy it*. Generally, grounding rings are used to short all leads of a MOSFET for avoiding any voltage build up between them. These grounding or shorting rings are removed only after MOSFET is securely wired into the circuit. Sometimes, conducting foam is applied between the leads instead of using shorting rings. As shown in Fig. 13.37, some MOSFETs have back-to-back Zener diodes internally formed to protect them against stray voltages.

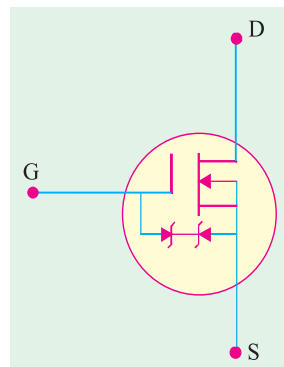


Fig. 63.37

**Tutorial Problems No. 63.2**

1. For a certain DE MOSFET,  $I_{DSS} = 10 \text{ mA}$  and  $V_{GS(off)} = -8 \text{ V}$ . Calculate  $I_D$  when  $V_{GS}$  is (i)  $-4 \text{ V}$  and (ii)  $+4 \text{ V}$ . [ (i) 2.5 mA (ii) 22.5 mA ]
2. The data sheet of a certain zero-biased DE MOSFET gives  $I_{DSS} = 15 \text{ mA}$  and  $V_{GS(off)} = 5 \text{ V}$ . What is the value of drain current ? [15 mA]
3. A certain *E*-only *N*-channel MOSFET has the following parameters :  
 $I_{D(on)} = 4 \text{ mA}$  at  $V_{GS} = 8 \text{ V}$  and  $V_{GS(th)} = 2 \text{ V}$   
 Calculate  $I_D$  for  $V_{GS} = 6 \text{ V}$  [1.78 mA]

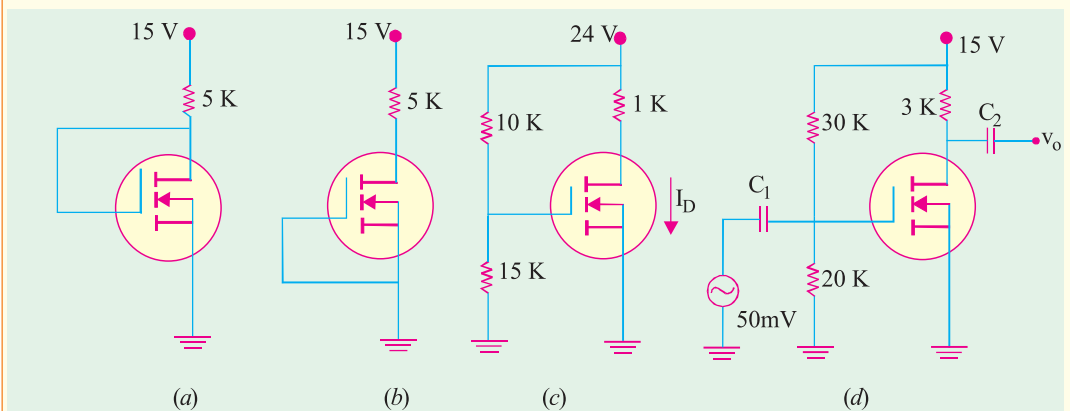


Fig. 63.38



4. The parameters of the  $E$ -only,  $N$ -channel MOSFET shown in Fig. 63.38 (a) are :  $V_{GS(th)} = 2 \text{ V}$  and  $K = 0.3 \text{ mA/V}^2$ . Determine the values of  $I_D$  and  $V_{DS}$ . If the gate is connected as shown in Fig. 63.38 (b), what will be the new values of  $I_D$  and  $V_{DS}$ ? **[(i) 2.07 mA (ii) 4.63 V ; 0 A, 15 V]**
5. The data sheet for the  $E$ -only  $N$ -channel MOSFET of Fig. 63.38 (c) gives  $I_{D(on)} = 3 \text{ mA}$  at  $V_{GS} = 10 \text{ V}$  and  $V_{GS(th)} = 5 \text{ V}$ . Calculate the values of  $V_{GS}$  and  $V_{DS}$ . **[14.4 V ; 13.4 V]**
6. The amplifier circuit of Fig. 63.38 (d) used an  $E$ -only  $N$ -channel MOSFET having the following parameters :  $I_{D(on)} = 5 \text{ mA}$  at  $V_{GS} = 10 \text{ V}$ ,  $V_{GS(th)} = 4 \text{ V}$ ,  $g_m = 5500 \mu\text{S}$ . Calculate  $V_{GS}$ ,  $I_D$ ,  $V_{DS}$  and  $v_o$ . **[6 V ; 0.556 mA ; 12.2 V ; 0.825 V]**
7. A field-effect transistor has a small-signal equivalent circuit with input resistance =  $1000 \text{ M}\Omega$ , forward transfer conductance =  $4 \text{ mS}$  and output conductance =  $100 \mu\text{S}$  when at the operating point,  $V_{DS} = +4 \text{ V}$ ,  $I_D = 2 \text{ mA}$ ,  $V_{GS} = -2 \text{ V}$ .  
Draw the circuit that you would use for a single-stage voltage amplifier. Describe the use and specify the value of as many components as possible if a  $30 \text{ V}$  supply was available.  
What voltage gain would you expect when the output was unloaded? Give reasons which might account for not getting this gain exactly.  
**[Drain load = 12 k $\Omega$ , Source Bias Resistor = 1 k $\Omega$ , - 21.8]**
8. A field-effect transistor is used as a voltage amplifier and with a load resistor of  $40 \text{ k}\Omega$ , a gain of 40 is obtained. If the load resistance is halved, the voltage gain drops to 30. Calculate the output resistance and the mutual conductance of the transistor.  
Briefly compare the advantages and limitations of the field-effect transistor with the bipolar transistor. **[20 k $\Omega$ , 3 mS]**

### OBJECTIVE TESTS – 63

- A FET consists of a
  - source
  - drain
  - gate
  - all of the above.
- FETs have similar properties to
  - PNP transistors
  - NPN transistors
  - thermonic valves
  - unijunction transistors.
- For small values of drain-to-source voltage, JFET behaves like a
  - resistor
  - constant-current source
  - constant-voltage source
  - negative resistance
- In a JFET, the *primary* control on drain current is exerted by
  - channel resistance
  - size of depletion regions
  - voltage drop across channel
  - gate reverse bias.
- After  $V_{DS}$  reaches pinch-off value  $V_p$  in a JFET, drain current  $I_D$  becomes
  - zero
  - low
  - saturated
  - reversed.
- In a JFET, as external bias applied to the gate is increased
  - channel resistance is decreased
  - drain current is increased
  - pinch-off voltage is reached at lower values of  $I_D$ .
  - size of depletion regions is reduced.
- In a JFET, drain current is maximum when  $V_{GS}$  is
  - zero
  - negative
  - positive
  - equal to  $V_p$
- The voltage gain of a given common- source JFET amplifier depends on its
  - input impedance
  - amplification factor
  - dynamic drain resistance
  - drain load resistance.
- A JFET has the disadvantage of
  - being noisy
  - having small gain-bandwidth product
  - possessing positive temperature coefficient
  - having low input impedance.
- A JFET can be cut-off with the help of
  - $V_{GS}$
  - $V_{DS}$
  - $V_{DG}$
  - $V_{DD}$ .



11. The drain source voltage at which drain current becomes nearly constant is called  
 (a) barrier voltage  
 (b) breakdown voltage  
 (c) pick-off voltage  
 (d) pinch-off voltage
12. The transconductance ' $g_m$ ' of a JFET is equal to  
 (a)  $-\frac{2I_{DSC}}{V_p}$   
 (b)  $\frac{2}{|V_p|} \sqrt{I_{DSS} I_D}$   
 (c)  $-\frac{2I_{DSS}}{V_p} \left(1 - \frac{V_{GS}}{V_p}\right)$   
 (d)  $\frac{I_{DSS}}{V_p} \left(1 - \frac{V_{GS}}{V_p}\right)$
13. An FET source follower circuit has of 2 millimho and of 50 k  $\Omega$ . If the source resistance  $R_s$  is 1k  $\Omega$ , the output resistance of the amplifier will be  
 (a) 330  $\Omega$  (b) 450  $\Omega$   
 (c) 500  $\Omega$  (d) 1 k  $\Omega$
14. A DE MOSFET differs from a JFET in the sense that it has no  
 (a) channel (b) gate  
 (c) P-N junctions (d) substrate.
15. For the operation of enhancement only N-channel MOSFET, value of gate voltage has to be  
 (a) high positive (b) high negative  
 (c) low positive (d) zero.
16. The extremely high input impedance of a MOSFET is primarily due to the  
 (a) absence of its channel  
 (b) negative gate-source voltage  
 (c) depletion of current carriers  
 (d) extremely small leakage current of its gate capacitor
17. The main factor which makes a MOSFET likely to break down during normal hand- ling is its  
 (a) very low gate capacitance  
 (b) high leakage current  
 (c) high input resistance  
 (d) both (a) and (c).
18. The main factor which differentiates a DE MOSFET from an E-only MOSFET is the absence of  
 (a) insulated gate (b) electrons  
 (c) channel (d) P-N junctions.
19. The polarity of  $V_{GS}$  for E-only MOSFET is  
 (a) positive  
 (b) negative  
 (c) zero  
 (d) depends on P-or N-channel.
20. A transconductance amplifier has  
 (a) high input impedance and low output impedance  
 (b) low input impedance and high output impedance  
 (c) high input and output impedances  
 (d) low input and output impedances
21. The threshold voltage of an n-channel enhancement mode MOSFET is 0.5 V, when the device is biased at a gate voltage of 3V, pinch-off would occur at a drain voltage of  
 (a) 1.5 V (b) 2.5 V  
 (c) 3.5 V (d) 4.5 V
22. The zero gate bias channel resistance of a junction field-effect transistor is 750 and the pinch-off voltage is 3V. For a gate bias of 1.5 V and very low drain voltage, the device would behave as a resistance of  
 (a) 320  $\Omega$  (b) 816  $\Omega$   
 (c) 1000  $\Omega$  (d) 1270  $\Omega$
23. A MOSFET rated for 15 A, carries a periodic current as shown in Fig. 63.39. the ON state resistance of the MOSFET is 0.15  $\Omega$ . The average ON state loss in the MOSFET is

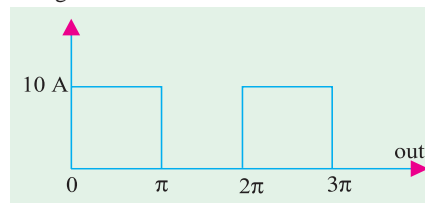


Fig. 63.39

- (a) 33.8 W (b) 15.0 W  
 (c) 7.5 W (d) 3.8 W

ANSWERS

1. (d) 2. (b) 3. (a) 4. (d) 5. (c) 6. (c) 7. (a) 8. (d) 9. (b) 10. (a) 11. (d)  
 12. (c) 13. c 14. (c) 15. (a) 16. (d). 17. (d) 18. (c) 19. (d) 20. (a) 21. (c) 22. (b)  
 23. (c)



## ROUGH WORK

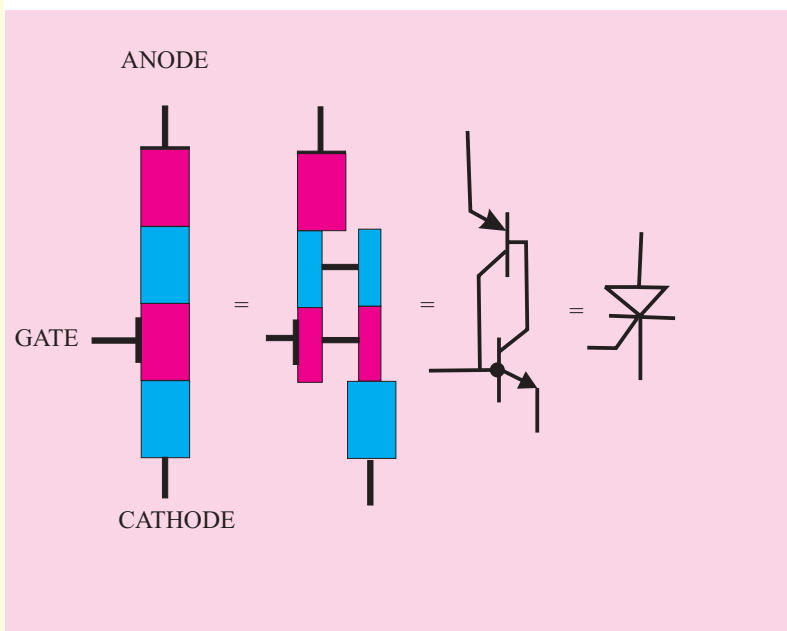


# CHAPTER 64

## Learning Objectives

- What are Breakdown Devices ?
- Unijunction Transistor
- UJT Relaxation Oscillator
- Programmable UJT(PUT)
- Silicon Controlled Rectifier
- Comparison between Transistors and Thyristors
- Transient Effects in an SCR
- Phase Control
- Theft Alarm
- Emergency Lighting System
- Light Activated SCR (LASCR)
- The Shockley Diode
- Triac
- Diac
- Silicon Controlled Switch (SCS)

## BREAKDOWN DEVICES



↑ The Silicon Controlled Rectifier, usually referred to as an SCR, is one of the family of semiconductors that includes transistors and diodes

### 64.1. What are Breakdown Devices ?

These are solid-state devices whose working depends on the *phenomenon of avalanche breakdown*. They are sometimes referred to by the generic name of *thyristor* which is a semiconductor switch whose bistable action depends on *P-N-P-N* regenerative feedback. We will discuss the following devices :

1. Unijunction Transistor (UJT).
2. Silicon Controlled Rectifier (SCR).
3. Light Activated SCR (LASCR).
4. Triac (short for 'triode ac').
5. Diac (short for 'diode ac').
6. Silicon Controlled Switch (SCS).



Cat-Transistor

These devices have two or more junctions and can be switched ON or OFF at *an extremely fast rate*. They are also referred to as *latching* devices. A latch is a kind of switch which initially once closed, remains closed until someone opens it.

### 64.2. Unijunction Transistor

Basically, it is a three-terminal silicon diode. As its name indicates, it has only one *P-N* junction. It differs from an ordinary diode in that it has *three* leads and it differs from a FET in that it has *no ability to amplify*. However, it has the ability *to control a large ac power with a small signal*. It also exhibits a negative resistance characteristic which makes it useful as an oscillator.

#### (a) Construction

It consists of a *lightly-doped silicon* bar with a heavily-doped *P*-type material alloyed to its one side (closer to  $B_2$ ) for producing single *P-N* junction. As shown in Fig. 64.1 (a), there are three terminals : one emitter,  $E$  and two bases  $B_2$  and  $B_1$  at the top and bottom of the silicon bar. The emitter leg is drawn at an angle to the vertical and arrow points in the direction of *conventional* current when UJT is in the conducting state.

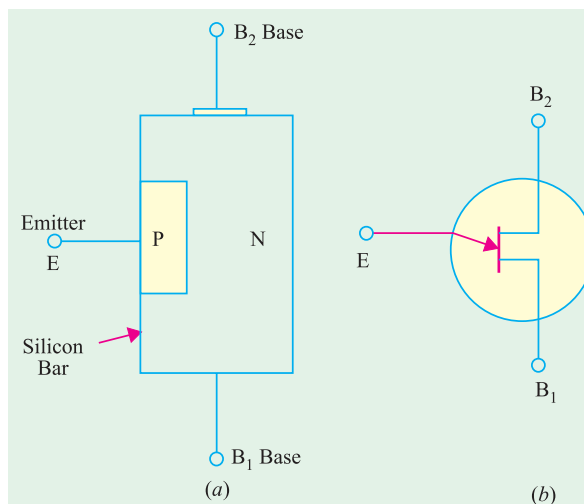


Fig. 64.1

#### (b) Interbase Resistance ( $R_{BB}$ )

It is the resistance between  $B_2$  and  $B_1$  *i.e.* it is the total resistance of the silicon bar from one end to the other with emitter terminal open [Fig. 64.2 (a)].

From the equivalent circuit of Fig. 64.2 (b), it is seen that  $R_{BB} = R_{B2} + R_{B1}$

It should also be noted that point  $A$  is such that  $R_{B1} > R_{B2}$ . Usually,  $R_{B1} = 60\%$  of  $R_{B1}$ . The resistance  $R_{B1}$  has been shown as a variable resistor because its value varies inversely as  $I_E$ .

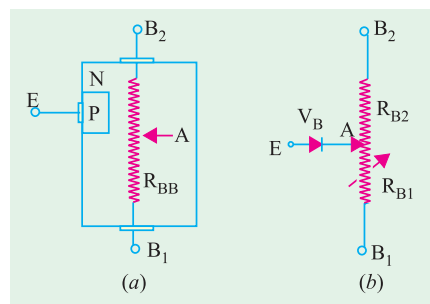


Fig. 64.2

**(c) Intrinsic Stand-off Ratio**

As seen from Fig. 64.3 (a), when a battery of 30 V is applied across  $B_2 B_1$ , there is a progressive fall of voltage over  $R_{BB}$  provided  $E$  is open. It is obvious from Fig. 64.3 (b) that emitter acts as a voltage-divider tap on fixed resistance  $R_{BB}$ .

With emitter open,  $I_1 = I_2$ , the interbase current is given by Ohm's Law.

$$I_1 = I_2 = V_{BB} / R_{BB}$$

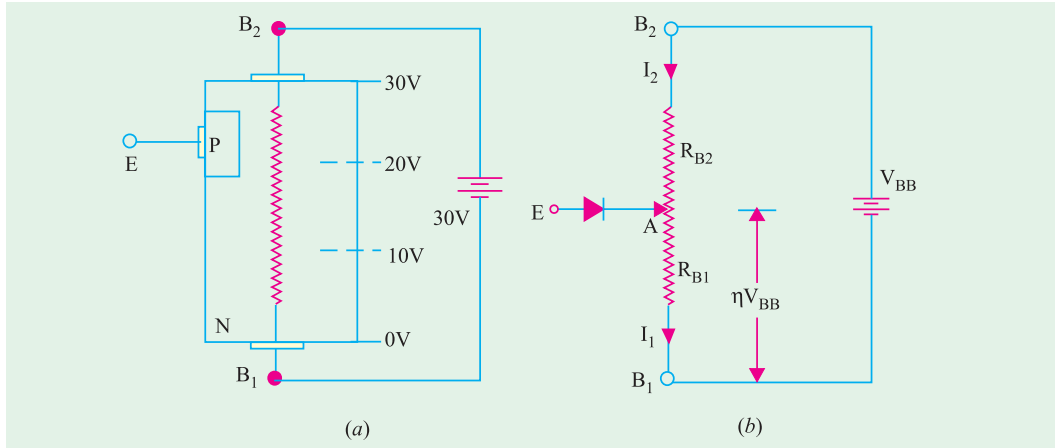


Fig. 64.3

For example, if  $V_{BB} = 30 \text{ V}$  and  $R_{BB} = 15 \text{ K}$ ,  $I_1 = I_2 = 2 \text{ mA}$ .

It may be noted that part of  $V_{BB}$  is dropped over  $R_{B2}$  and part on  $R_{B1}$ . Let us call the voltage drop across  $R_{B1}$  as  $V_A$ . Using simple voltage divider relationship,

$$V_A = V_{BB} \frac{R_{B1}}{R_{B1} + R_{B2}}$$

The voltage division factor is given a special symbol ( $\eta$ ) and the name of '**intrinsic stand-off ratio**'.

$$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}} \quad \therefore V_A = \eta V_{BB}$$

The intrinsic stand-off ratio is the property of the UJT and is always less than unity (0.5 to 0.85). If  $V_{BB} = 30 \text{ V}$  and  $\eta = 0.6$ , then potential of point A with respect to point  $B_1 = 0.6 \times 30 = 18 \text{ V}$ . The remaining 12 V drop across  $R_{B2}$ .

**(d) Operation**

When  $V_{BB}$  is switched on,  $V_A$  is developed and **reverse-biases the junction**. If  $V_B$  is the barrier voltage of the  $P-N$  junction, then total reverse bias voltage is

$$= V_A + V_B = \eta V_{BB} + V_B$$

Value of  $V_B$  for Si is 0.7 V.

It is obvious that emitter junction will not become forward-biased unless its applied voltage  $V_E$  exceeds  $(\eta V_{BB} + V_B)$ . This value of  $V_E$  is called **peak-point voltage**  $V_P$  (Fig. 64.4). When  $V_E = V_P$ , emitter (peak current),  $I_P$  starts to flow through  $R_{B1}$  to ground (i.e.  $B_1$ ). The UJT is then said to have been **-fired** or turned ON. Due to the flow of  $I_E (= I_P)$  through  $R_{B1}$ , number of charge carriers in  $R_{B1}$  is increased which **reduces its resistance**. As  $\eta$  depends on  $R_{B1}$ , **its value is also decreased**.

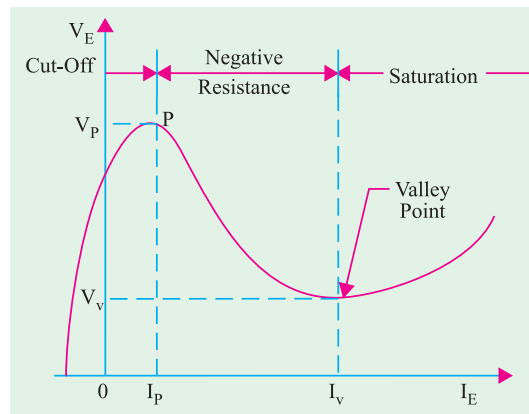


Fig. 64.4

Hence, we find that as  $V_E$  and hence  $I_E$  increases (beyond  $I_p$ ),  $R_{B1}$  decreases,  $\eta$  decreases and  $V_A$  decreases. This decrease in  $V_A$  causes more emitter current to flow which causes a further reduction in  $R_{B1}$ ,  $\eta$  and  $V_A$ . Obviously, the process is regenerative.  $V_A$  as well as  $V_E$  quickly drop as  $I_E$  increases. Since,  $V_E$  decreases when  $I_E$  increases, the UJT **possesses negative resistance**. Beyond the valley point, UJT is in saturation and  $V_E$  increases very little with an increasing  $I_E$ .

It is seen that only terminals  $E$  and  $B_1$  are the active terminals whereas  $B_2$  is the bias terminal *i.e.* it is meant only for applying external voltage across the UJT.

Generally, UJT is triggered into conduction by applying a suitable positive pulse at its emitter. It can be brought back to OFF state by applying a negative trigger pulse.

**(e) Applications**

One unique property of UJT is that it can be triggered by (or an output can be taken from) **any one of its three terminals**. Once triggered, the emitter current  $I_E$  of the UJT increases regeneratively till it reaches a limiting value determined by the external power supply. Because of this particular behaviour, UJT is used in a variety of circuit applications. Some of which are :

1. phase control
2. switching
3. pulse generation,
4. sine wave generator
5. sawtooth generator
6. timing and trigger circuits,
7. voltage or current regulated supplies.

**Example 64.1.** A given silicon UJT has an interbase resistance of 10 K. It has  $R_{B1} = 6 K$  with  $I_E = 0$ . Find

- (a) UJT current if  $V_{BB} = 20 V$  and  $V_E$  is less than  $V_p$ ,
- (b)  $\eta$  and  $V_A$ ,
- (c) peak point voltage,  $V_p$ .

(Applied Electronics-I, Punjab Univ. 1990)

**Solution.** (a) Since  $V_E < V_p$ ,  $I_E = 0$ , because  $P-N$  junction is reverse-biased.

$$\therefore I_1 = I_2 = \frac{V_{BB}}{R_{BB}} = \frac{20}{10K} = 2mA$$

$$(b) \eta = \frac{R_{B1}}{R_{BB}} = \frac{6}{10} = 0.6; V_A = \eta V_{BB} = 0.6 \times 20 = 12V$$

$$(c) V_p = \eta V_{BB} + V_B = 12 + 0.7 = 12.7 V$$

**64.3. UJT Relaxation Oscillator**

The relaxation oscillator shown in Fig. 64.5 consists of a UJT and a capacitor  $C$  which is charged through  $R$  as  $V_{BB}$  is switched on.

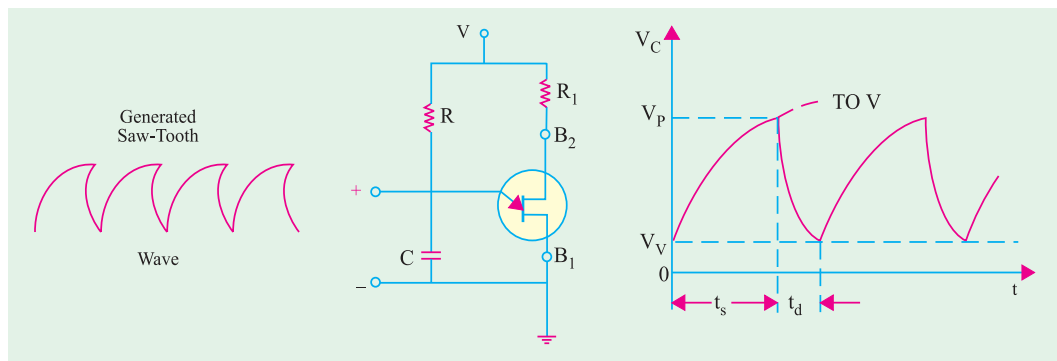


Fig. 64.5

When the capacitor voltage  $V_C$  reaches in time  $t_s$  the value of  $V_p$ , the UJT fires and rapidly discharges  $C$  via  $B_1$  till the voltage falls below the minimum value  $V_v$ . The device then cuts off and

C starts to charge again. This cycle is repeated continuously thus generating a sawtooth waveform across C.

The inclusion of external resistances  $R_2$  and  $R_1$  in series with  $B_2$  and  $B_1$  (Fig. 64.6) provides **spike waveforms**. When the UJT fires, the sudden surge of current through  $B_1$  causes a drop across  $R_1$  which produces positive going spikes. Also, at the time of firing, fall of  $V_{BB1}$  causes  $I_2$  to increase rapidly which generates negative going spikes across  $R_2$  as shown in Fig. 64.6.

By switching over to different capacitors, frequency of the output waveform can be changed as desired.

### Condition for Turn-ON and Turn-OFF

For satisfactory working of the above oscillator, following two conditions for the turn-on and turn-off of the UJT must be met. To ensure turn-on,  $R$  must not limit  $I_E$  at peak point to a value less than  $I_p$ . It means that

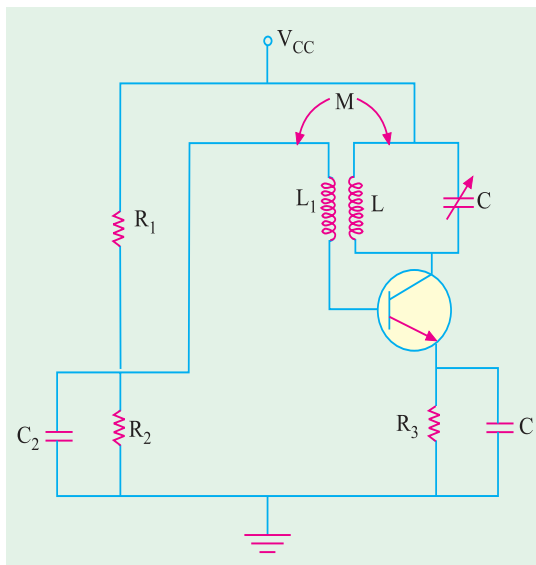


Fig. 64.6

$$V_{BB} - V_p > I_p R \text{ or } R < \frac{V_{BB} - V_p}{I_p}$$

To ensure turn-off of the UJT at valley point,  $R$  must be large enough to permit  $I_E$  (at valley point) to decrease below the specified value of  $I_v$ . In other words, drop across  $R$  at valley point must be less than  $I_v R$ . Hence, condition for turn-off is

$$V_{BB} - V_v < I_v R \text{ or } R > \frac{V_{BB} - V_v}{I_v}$$

Hence, for reliable turn-on and turn-off of the UJT,  $R$  must be in the range

$$\frac{V_{BB} - V_p}{I_p} > R > \frac{V_{BB} - V_v}{I_v}$$

It should be noted that charging time constant of the capacitor for voltage  $V$  is  $T = CR$  whereas discharging time constant is  $T_d = CR_{B1}$ .

The time required to charge upto  $V_p$  (called ramp rise time) is  $t_s = T \log_e (V - V_v) / (V - V_p)$ .

Similarly, time required by the capacitor to discharge from  $V_p$  to  $V_v$  is  $t_d \cong T_d \log_e V_p / V_v$ . The frequency of oscillation is given by  $f = 1 / (t_s + t_d)$ .

**Example 64.2.** The windshield wiper motor of an automobile is controlled by a UJT with  $\eta = 0.6$ . The capacitor has a value of  $50 \mu\text{F}$  and the charging resistor is a series combination of  $50 \text{ K}$  resistor and a  $500 \text{ K}$  potentiometer. Determine the minimum and maximum number of blade strokes per minute possible with this arrangement.

**Solution.** The least value of time constant is  $= 50,000 \times 50 \times 10^{-6} = 2.5$  second.

Maximum value of time constant when whole of potentiometer resistance is used is  $= (50 + 500) \times 10^3 \times 50 \times 10^{-6} = 27.5$  second

Maximum blade strokes per minute  $= 60 / 2.5 = 24$ .

Minimum blade strokes per minute  $= 60 / 27.5 = 2.2$ .

**Example 64.3.** The oscillator circuit shown in Fig. 64.5 uses a UJT with  $R_{BB} = 10 \text{ K}$ ,  $\eta = 0.6$ ,  $V_B = 0.7 \text{ V}$ ,  $V = 50 \text{ V}$ ,  $R_1 = 90 \text{ K}$ ,  $R = 100 \text{ K}$  and  $C = 0.05 \mu\text{F}$ . When UJT is in conduction,  $R_{B1} = 10 \Omega$  and  $V_v = V_B$ . Find (i) ramp rise time,  $t_s$  (ii) approximate discharge time,  $t_d$  and (iii) frequency of oscillation.

**Solution.**  $V_{BB} = V \cdot R_{BB} / (R_{BB} + R_1) = 50 \times 10 / 100 = 5 \text{ V}$   
 $V_P = \eta V_{BB} + V_B = 0.6 \times 5 + 0.7 = 3.7 \text{ V}$   
 $T = CR = 0.05 \times 10^{-6} \times 100 \times 10^3 = 5 \text{ ms}$

(i) The capacitor charges from  $V_C = V_B = 0.7 \text{ V}$  to  $V_P$  towards  $V$  in the time  $t_s$  given by  $t_s = T \log_e (V - V_V) / (V - V_P) = 5 \log_e (50 - 0.7) / (50 - 3.7) = \mathbf{0.315 \text{ ms}}$ .

(ii) The discharge time,  $t_d = CR_{B1} = 0.05 \times 10 = 0.5 \mu\text{s}$ . The time taken by  $C$  to discharge from  $V_P$  to  $V_V (= 0.7 \text{ V})$  towards 0 volt is

$$T_d \cong t_d \log_e V_P / V_V = 0.5 \log_e 3.7 / 0.7 = \mathbf{1.66 \mu\text{s}}$$

(iii)  $f = 1 / (t_s + t_d) = 1 / (0.315 + 0.0016) = \mathbf{3.158 \text{ kHz}}$ .

### 64.4. Programmable UJT (PUT)

Like a SCR, it is also a four-layer or PNPN device with a gate  $G$  as shown in Fig. 64.7. However, its gate is connected to the  $N$ -region adjacent to the anode  $A$ . This  $P$ - $N$  junction controls the ON and OFF states of the  $PUT$ . The gate  $G$  is always biased positive with respect to cathode  $K$ . When anode voltage exceeds gate voltage by about  $0.7 \text{ V}$ , the  $P$ - $N$  junction  $J_1$  becomes forward-biased and the  $PUT$  turns ON. When the anode voltage falls below this level, the  $PUT$  is turned OFF.

As shown in Fig. 64.8 (a), gate bias can be adjusted to any bias level with the help of an external voltage divider circuit  $R_2 - R_3$ . Whenever anode voltage exceeds this **programmable level**, the  $PUT$  turns ON.

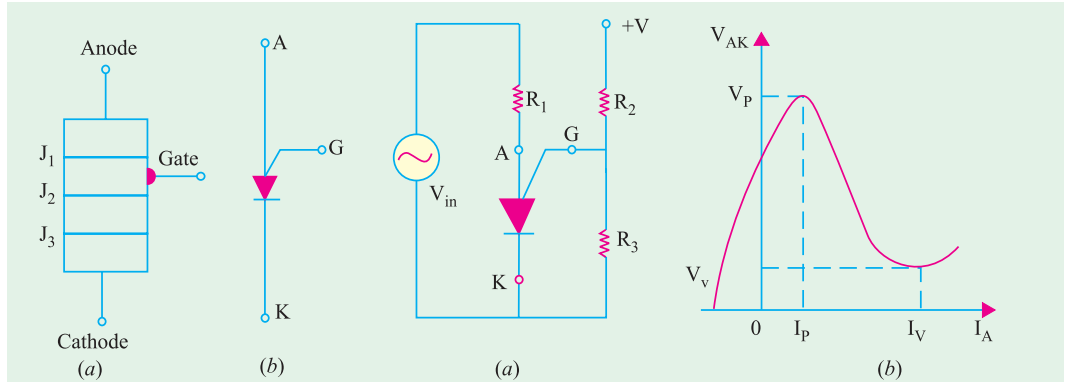


Fig. 64.7

Fig. 64.8

Fig. 64.8 (b) shows the plot of anode-to-cathode voltage  $V_{AK}$  versus anode current  $I_A$ . It is similar to the  $V/I$  characteristic of a  $UJT$ . Hence,  $PUT$  replaces  $UJT$  in many applications, one such application is as relaxation oscillator shown in Fig. 64.9 (a).

Since,  $R_2 = R_3$ ,  $V_G = 12 / 2 = 6 \text{ V}$ . When dc voltage is applied, the  $PUT$  is off but  $C$  starts charging towards  $+12 \text{ V}$  through  $R_1$  [Fig. 64.9 (b)]. When  $V_C$  exceeds  $(V_G + 0.7 \text{ V})$ , the  $PUT$  turns ON and, at the same time,  $C$  starts discharging rapidly through the low ON-resistance of the  $PUT$  and  $R_4$ . Consequently, a voltage spike is developed across  $R_4$  during the discharge. As soon as  $C$  discharges, the  $PUT$  turns OFF and the charging cycle starts all over again as described above.



Silicon controlled rectifier and power diode

### 64.5. Silicon Controlled Rectifier

It is one of the prominent members of the thyristor family. It is a four-layer or PNPN device. Basically, *it is a rectifier with a control element*. In fact, it consists of *three diodes* connected back-to-back with a gate connection. It is widely used as a switching device in power control applications. It can control loads by switching current OFF and ON up to many thousand times a second. It can switch ON for variable lengths of time, thereby delivering selected amount of power to the load. Hence, it possesses the advantages of a rheostat and a switch with none of their disadvantages.

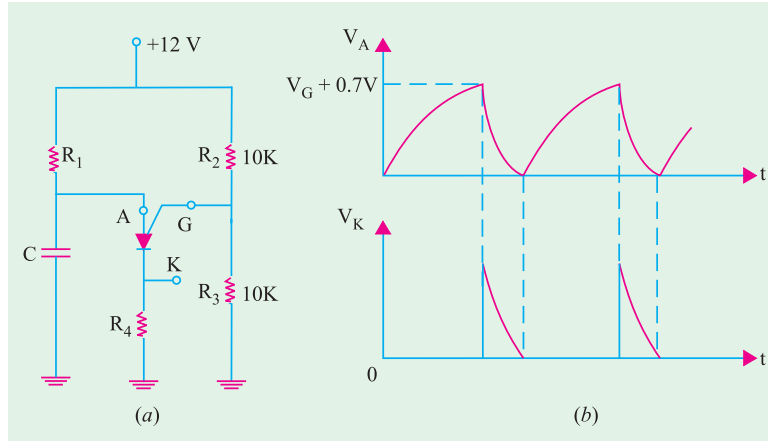


Fig. 64.9

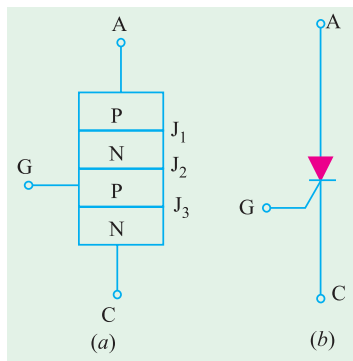


Fig. 64.10

#### (a) Construction

As shown in Fig. 64.10 (a), it is a three terminal four-layer transistor, the layers being alternately of P-type and N-type silicon. The three junctions are marked  $J_1$ ,  $J_2$  and  $J_3$  whereas the three terminals are : anode (A), cathode (C) and gate (G) which is connected to the *inner P-type* layer. The function of the gate is to control the firing of SCR. The schematic symbol is shown in Fig. 64.10 (b).

Since, they conduct large currents, junction areas of SCRs are very large. Commonly used stud-mounted units have their anode connected directly to the stud for good heat dissipation whereas larger units are of 'pillow' type in which many units are stacked in series and held in a pressurized clamp.

#### (b) Biasing

With the polarity of  $V$  as shown in Fig. 64.11 (a), the junctions  $J_1$  and  $J_3$  become forward-biased whereas  $J_2$  is reverse-biased. Hence, no current (except leakage current) can flow through the SCR.

In Fig. 64.11 (b), polarity of  $V$  has been reversed. It is seen that, now, junctions  $J_1$  and  $J_3$  become reverse-biased and only  $J_2$  is forward-biased. Again, there is no flow of current through the SCR.

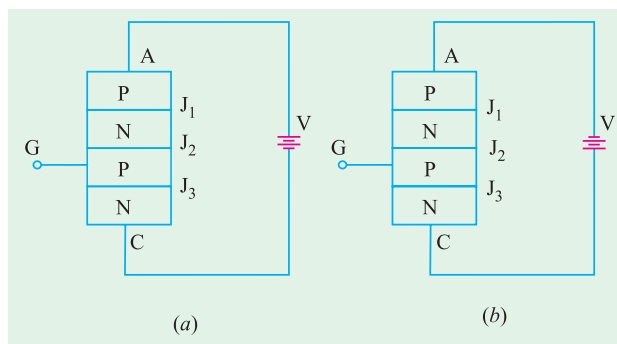


Fig. 64.11

#### (c) Operation

In Fig. 64.11 (a), current flow is blocked due to reverse-biased junction  $J_2$ . However, when anode voltage is increased, a certain critical value called forward breakover voltage  $V_{BO}$  is reached when  $J_2$  breaks down and SCR switches suddenly to a highly conducting state. Under this condition,

SCR offers very little forward resistance ( $0.01 \Omega - 1.0 \Omega$ ) so that voltage across it drops to a low value (about 1 V) as shown in Fig. 64.12 and current is limited only by the power supply and the load resistance. Current keeps flowing indefinitely until the circuit is opened briefly.

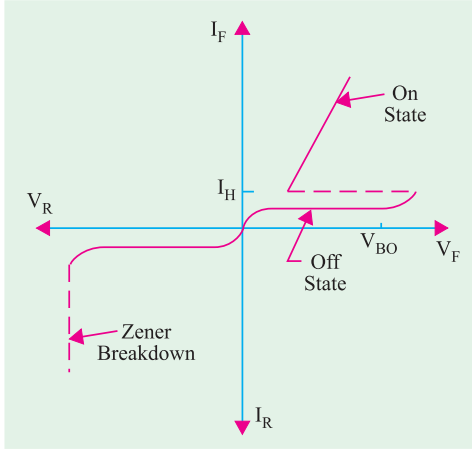


Fig. 64.12

With supply connection as in Fig. 64.11 (b), the current through the SCR is blocked by the two reverse-biased junctions  $J_1$  and  $J_3$ . When  $V$  is increased, a stage comes when Zener breakdown occurs which may destroy the SCR (Fig. 64.12). Hence, it is seen that SCR is a unidirectional device unlike triac *which is bi-directional*.

**(d) Two Transistor Analogy**

The basic operation of a SCR can be described by using two transistor analogy. For this purpose, SCR is split into two 3-layer transistor structures as shown in Fig. 64.13 (a). As seen, transistor  $Q_1$  is a PNP transistor whereas  $Q_2$  is an NPN device interconnected together. It will also be noted from Fig. 64.13 (b) that

- (i) collector current of  $Q_1$  is also the base current of  $Q_2$  and
- (ii) base current of  $Q_1$  is also the collector current of  $Q_2$ .

Suppose that the supply voltage across terminals A and C is such that reverse-biased junction  $J_2$  starts breaking down. Then, current through the device begins to rise. It means that  $I_{E1}$  begins to increase.

Then,

1.  $I_{C1}$  increases (remember  $I_C = \alpha I_E$ );
2. since  $I_{C1} = I_{B2}$ ,  $I_{B2}$  also increases;
3. hence,  $I_{C2}$  increases (remember  $I_C = \beta I_B$ );
4. now,  $I_{C2} = I_{B1}$ , hence  $I_{B1}$  increases;
5. consequently, both  $I_{C1}$  and  $I_{E1}$  increase.

As seen, a regenerative action takes place whereby an initial increase in current produces further increase in the same current. Soon, maximum current is reached limited by external resistances. The two transistors are fully turned ON and voltage across the two transistors falls to a very low value. Typical turn-ON times for an SCR are 0.1 to 1.0  $\mu s$ .

It can be proved that if  $I_G$  is the gate current of the SCR and  $\alpha_1$  and  $\alpha_2$ , the current gains of the PNP and NPN transistors respectively, then anode current is given by

$$I_A = \frac{\alpha_2 I_G}{1 - (\alpha_1 + \alpha_2)}$$

**(e) Firing and Triggering**

Usually, SCR is operated with an anode voltage *slightly less* than the forward breakover voltage  $V_{BO}$  and is triggered into conduction by a low-power gate pulse. Once switched ON, gate has no further control on the device current. Gate signals can be (a) dc firing signals [Fig. 64.14 (a)] or (b) pulse signals [Fig. 64.14 (b)].

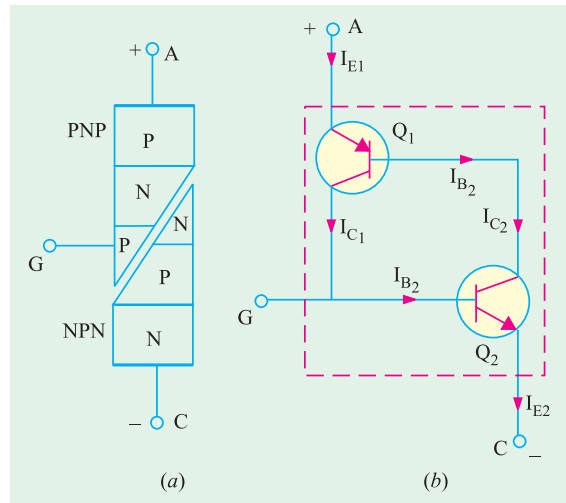


Fig. 64.13



In Fig. 64.14 (a) with  $S$  open, SCR does not conduct and the lamp is out. When  $S$  is closed momentarily, a positive voltage is applied to the gate which forward-biases the centre  $P$ - $N$  junction.

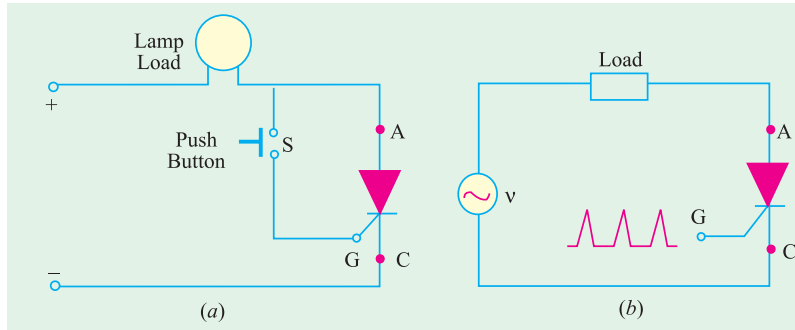


Fig. 64.14

As a result, SCR is pulsed into conduction and the lamp lights up. SCR will remain in the conducting state until the supply voltage is removed or reversed. Fig. 64.14 (b) shows triggering by timed pulses obtained from a pulse source.

We have discussed above the most common method of SCR triggering *i.e.* gate triggering. However, other available triggering methods are as under :

### 1. Thermal Triggering

In this case, the temperature of the forward-biased junction is increased till the reverse-biased junction breaks down.

### 2. Radiation Triggering

Here, triggering is achieved with the help of charge carriers which are produced by the bombardment of the SCR with external high-energy particles like neutrons or protons.

### 3. Voltage Triggering

In this case, the voltage applied across the anode and cathode of the SCR is increased which decreases the width of the depletion layer at the reverse-biased junction leading to its collapse.

### 4. $dv/dt$ Triggering

In this case,  $dv/dt$  is made more than the value of the critical rate of rise of the voltage.

#### (f) Turning OFF

As stated earlier, once 'fired', SCR remains ON *even when triggering pulse is removed*. This ability of the SCR to remain ON even when gate current is removed is referred to as **latching**. In fact, SCR belongs to a class of devices known as **latching devices**.

By now, it is clear that an SCR cannot be turned OFF by simply removing the gate pulse. Number of techniques are employed to turn an SCR off. These are :

1. anode current interruption.
2. reversing polarity of anode-cathode voltage as is done each half-cycle by  $v$  in Fig. 64.14 (b);
3. reducing current through SCR below the holding current  $I_H$  (Fig. 64.12). It is also called **low-current dropout**.

#### (g) Applications

Main application of an SCR is as a **power control device**. It has been shown above that when SCR is OFF, its current is negligible and when it is ON, its voltage is negligible. Consequently, it never dissipates any appreciable amount of power even when controlling substantial amounts of load power. For example, one SCR requires only 150 mA to control a load current of 2500 A. Other common areas of its application include.

- |                     |                              |                      |
|---------------------|------------------------------|----------------------|
| 1. relay controls,  | 2. regulated power supplies, | 3. static switches,  |
| 4. motor controls,  | 5. inverters,                | 6. battery chargers, |
| 7. heater controls, | 8. phase control.            |                      |

SCRs have been designed to control powers upto 10 MW with individual ratings as high as 2000 A at 1.8 kV. Its frequency range of application has been extended to about 50 kHz.

**Example 64.4.** The two-transistor analogy of an SCR has the following data :  
 gain of PNP transistor = 0.4 ; gain of NPN transistor = 0.5 ;  
 gate current = 50 mA.

Calculate the anode current of the device.

**Solution.** Here,  $\alpha_1 = 0.4$  ;  $\alpha_2 = 0.5$  and  $I_G = 50 \text{ mA} = 0.05 \text{ A}$

$$\text{anode current, } I_A = \frac{\alpha_2 I_G}{1 - (\alpha_1 + \alpha_2)} = \frac{0.5 \times 0.05}{1 - (0.4 + 0.5)} = 0.25 \text{ A} = \mathbf{250 \text{ mA}}$$

### 64.6. Comparison Between Transistors and Thyristors

Table No. 64.1 gives the comparison between transistors and thyristors.

Table No. 64.1		
Sl.No.	Transistors	Thyristors
1.	3-layers, 2-junction devices	4-layer, 2- or more junction devices
2.	fast response	very fast response
3.	high efficiency	very high efficiency
4.	highly reliable	very highly reliable
5.	small voltage drop	very small voltage drop
6.	long life	very long life
7.	small to medium power ratings	very small to very large power ratings
8.	require a continuous flow of current to remain in conducting state	require only a small pulse for triggering and thereafter remaining in conducting state.
9.	low power consumption	very low power consumption
10.	low control capability	high control capability
11.	small turn-ON and turn-OFF time	very small turn-ON and turn-OFF time

**Example 64.5.** A 250  $\Omega$  resistor is connected in series with the gate of an SCR as shown in Fig. 64.15. The gate current required for firing the SCR is 8 mA. Calculate the value of the input voltage  $V_{in}$  required for causing the SCR to break down.

(Basic Electronics, Osmania Univ. 1993)

**Solution.** The value of  $V_{in}$  should be such as to (i) overcome the barrier voltage of 0.7 V and (ii) cause 8 mA current to flow through 250  $\Omega$  resistor.

$$V_{in} = V_{GC} + I_G R = 0.7 + 8 \times 10^{-3} \times 250 = \mathbf{2.7 \text{ V}}$$

### 64.7. Transient Effects in an SCR

We will consider the following two effects :

**(i) di/dt Effect**

This effect is produced due to a high initial rate-of-rise of the anode current when an SCR is just switched ON and results in the formation of a **local hot spot near the gate connection** as explained below :

When a triggering pulse is applied to the gate of an SCR, the holes are injected into the P-region where they crowd together and form an initial conduction zone over a small part of the junction  $J_2$  before spreading the conduct throughout the whole area of junction. If the anode current is allowed to rise very rapidly (as would be the case for resistive or capacitive loads), this high current will be forced to flow through this small conduction zone until the conduction has spread through the entire junction. This may result in local hot-spots in the junction which are likely to damage the SCR permanently.

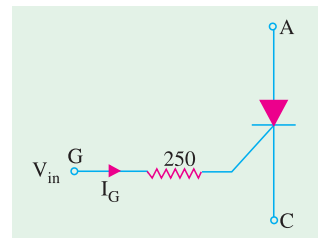


Fig. 64.15

The maximum allowable anode current  $di/dt$  can be increased (and, hence, turn-on time of an SCR decreased) by using specially-designed gate-connection geometries which result in a more rapid distribution of charge throughout the gate region.

(ii)  $dv/dt$  Effect

It is found that sometimes an SCR unwantedly turns ON by itself during sudden changes of the applied anode potential at a time when there is no gate current applied and the SCR is supposed to be blocking. This false triggering is due to the capacitance possessed by the large-area junction  $J_2$  (Fig. 64.10). When rate-of-rise of the applied anode voltage  $dv/dt$  is very high, the capacitive charging current may become high enough to initiate switch-on even in the absence of external gate current. False triggerings due to the  $dv/dt$  are prevented by using a ‘snubber circuit’.

64.8. Phase Control

In the phase control circuit of Fig. 64.16, gate triggering current is derived from the supply itself. The variable resistance  $R$  limits the gate current during positive half-cycles of the supply. If  $R$  is adjusted to a low value, SCR will trigger almost immediately at the commencement of the positive half-cycle of the input.

If, on the other hand,  $R$  is set to a high resistance, SCR may not switch ON until the peak of the positive half-cycle. By adjusting  $R$  between these two extremes, SCR can be

made to switch ON somewhere between the commencement and peak of the positive half-cycle *i.e.* between  $0^\circ$  and  $90^\circ$ .

It is obvious that if  $I_G$  is not enough to trigger the SCR at  $90^\circ$ , then the device will not trigger at all because  $I_G$  has maximum value then. This operation is sometimes referred to as *half-wave variable-resistance phase control*. It is an effective method of controlling the load power.

The purpose of diode  $D$  is to protect the gate from negative voltage which would otherwise be applied to it during the negative half-cycle of the input.

It is seen from Fig. 64.16 that at the instant of SCR switch-ON, gate current flows through  $R_L$ ,  $R$  and  $D$ . Hence, at that *instant*

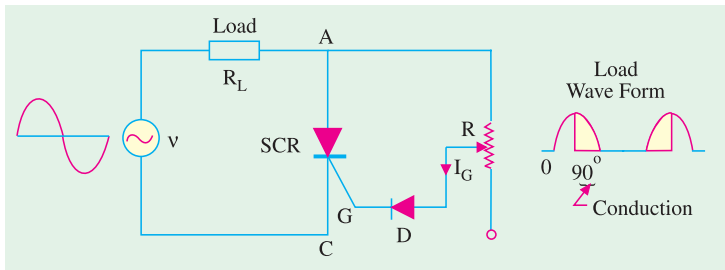


Fig. 64.16

$$v = I_G R_L + I_G R + V_D + V_G \quad \therefore R = \frac{v - V_D - V_G - I_G R_L}{I_G}$$

**Example 64.6.** The circuit of Fig. 64.16 is connected to an ac supply  $v = 50 \sin \theta$  and  $R_L = 50 \Omega$ . Gate current is  $100 \mu A$  and  $V_G = 0.5 V$ . Determine the range of adjustment of  $R$  for the SCR to be triggered between  $30^\circ$  and  $90^\circ$ . Take  $V_D = 0.7 V$ .

**Solution.** (i)

$\theta = 30^\circ$

Now,  $v = 50 \sin \theta = 50 \sin 30^\circ = 25 V$

$\therefore R = \frac{25 - 0.7 - 0.5 - (100 \times 10^{-6} \times 50)}{100 \times 10^{-6}} = 238K$

(ii)

$\theta = 90^\circ$

$v = 50 \sin 90^\circ = 50 V$

$R = \frac{50 - 0.7 - 0.5 - 0.005}{100 \times 10^{-6}} = 488K$

### 64.9. Theft Alarm

The circuit shown in Fig. 64.17 can be used to protect a car tape deck or a radio receiver from theft. The switch  $S$  is located at some concealed point in the car and is kept *closed*. Since gate  $G$  is grounded through the tape deck, the SCR is OFF and the horn is silent. If the tape deck is removed,  $G$  is no longer grounded. Instead, it gets connected to the car battery through  $R$ . Consequently, gate current is set up which fires the SCR. As a result, the horn starts blowing and continues to do so until  $S$  is opened.

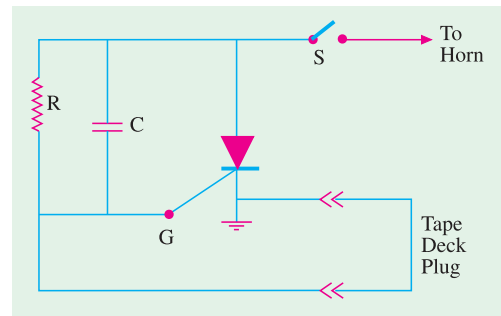


Fig. 64.17

### 64.10. Emergency Lighting System

SCRs find application in circuits that maintain lighting by using a backup battery in case of ac power failure. Fig. 64.18 shows a centre-tapped full-wave rectifier used for providing power to a low-voltage lamp. So long as ac power is available, the battery is charged *via* diode  $D_3$  and resistor  $R_1$  [Fig. 64.18 (a)].

With ac power ON, the capacitor  $C$  charges to the peak value of the full-wave rectified ac voltage *i.e.* to  $12.4 \times 1.414 = 17.5$  V. Same is the voltage of the SCR cathode  $K$ .

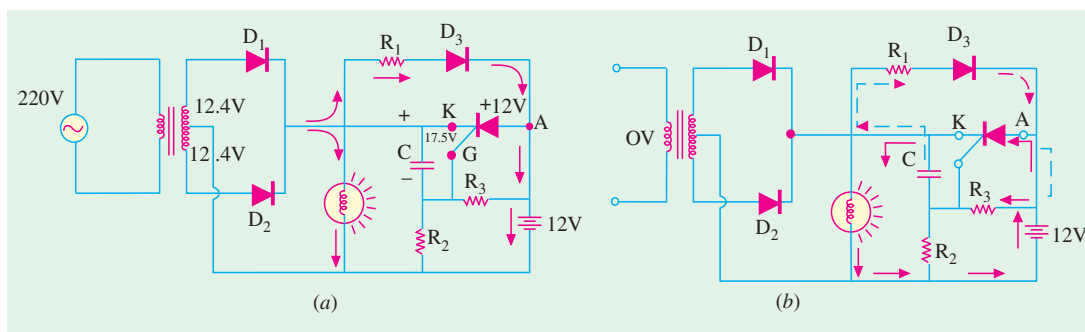


Fig. 64.18

Since voltage of SCR anode  $A$  is less than that of  $K$ , the SCR does not conduct. The SCR gate  $G$  is at a voltage determined by voltage divider  $R_3$ – $R_2$ . Under these conditions, the lamp is run by the ac supply and SCR is OFF.

When ac power is interrupted :

- (i) the capacitor  $C$  discharges through the closed path  $R_1$ ,  $D_3$  and  $R_3$  shown by dotted arrows;
- (ii) the cathode voltage decreases thereby making it less positive than anode;
- (iii) this triggers SCR into conduction which allows the battery current to pass through the lamp thus maintaining illumination.

When ac supply is restored,  $C$  recharges and the SCR turns OFF. The battery starts recharging again.

### 64.11. Light Activated SCR (LASCR)

It is just an ordinary SCR except that *it can also be light-triggered*. Most LASCRs also have a gate terminal for being triggered by an electrical pulse just as a conventional SCR. Fig. 64.19 shows the two LASCR symbols used commonly.

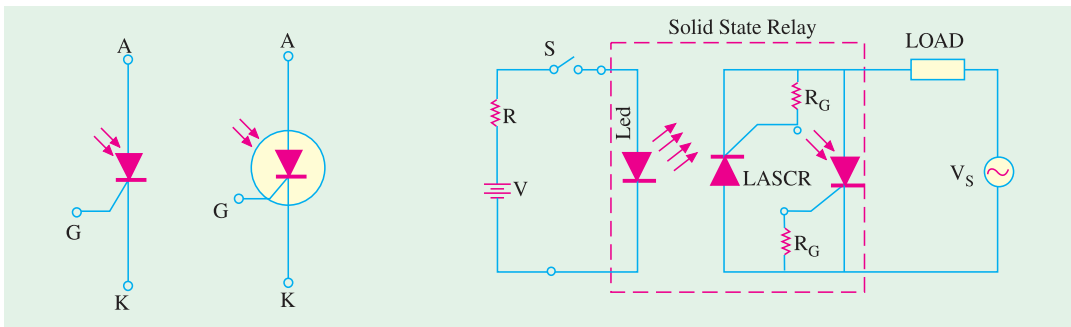


Fig. 64.19

Fig. 64.20

LASCrs are manufactured mostly in relatively low-current ranges and are used for triggering larger SCRs and triacs. They are used in optical light controls, relays, motor control and a variety of computer applications. Some LASCrs have clear windows in their cases so that light sources from other devices can be coupled to them. Many have the light source device encapsulated in the same package so that a relay is formed. Since the relay action does not require direct electrical connection, such relays are often used to couple signals into very high voltage equipment and other dangerous locations. Fig. 64.20 shows the connection of such a solid-state relay. Two LASCrs are connected in reverse parallel in order to obtain conduction in both half-cycles of the applied ac voltage  $V_s$ . A single LED is used to trigger both LASCrs. Bias resistors are used to reduce the light sensitivity of the gates and prevent sporadic triggering during off-periods. Usually, all the three active devices and the two bias resistors  $R_G$  are encapsulated in the same package.

### 64.12. The Shockley Diode\*

It is a *two-terminal four-layer* or *PNPN* device as shown in Fig. 64.21 along with its schematic symbol. It is essentially a low-current *SCR without a gate*. For switching the diode ON, its anode-to-cathode voltage ( $V_{AK}$ ) must be increased to forward switching voltage ( $V_s$ ) which is the equivalent of SCR forward breakover voltage. Like an SCR, it also has a holding current. The *PNPN* structure can be represented by an equivalent circuit consisting of a *PNP* transistor and an *NPN* transistor. One application of the diode is as a relaxation oscillator.

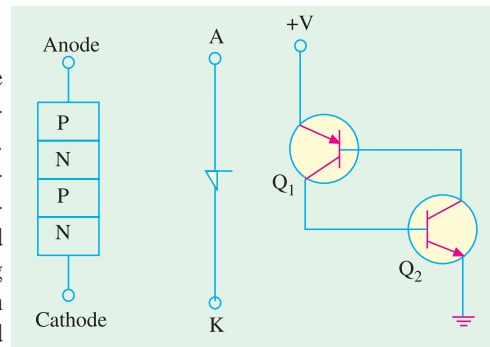


Fig. 64.21

### 64.13. Triac

It is a *5-layer bi-directional device* which can be triggered into conduction by both *positive* and *negative* voltages at its anodes and with both *positive and negative* triggering pulses at its gate. It behaves like two SCRs *connected in parallel, upside down with respect to each other*. That is, the anode of one is tied to the cathode of the other and their gates are directly tied together. Hence, anode and gate voltages applied in either direction will fire a triac because they would fire at least one of the two SCRs which are in opposite directions.

Since a triac responds to both positive and negative voltages at the anode, the concept of cathode used for an SCR is dropped. Instead, the two electrodes are called anodes  $A_1$  and  $A_2$ .

\* After the name of its inventor William Shockley.

**1. Construction**

As shown in Fig. 64.22 (a), a triac has three terminals  $A_1$ ,  $A_2$  and  $G$ . As seen, gate  $G$  is closer to anode  $A_1$ . It is clear from Fig. 64.22 (b), that a triac is nothing but **two inverse parallel-connected**

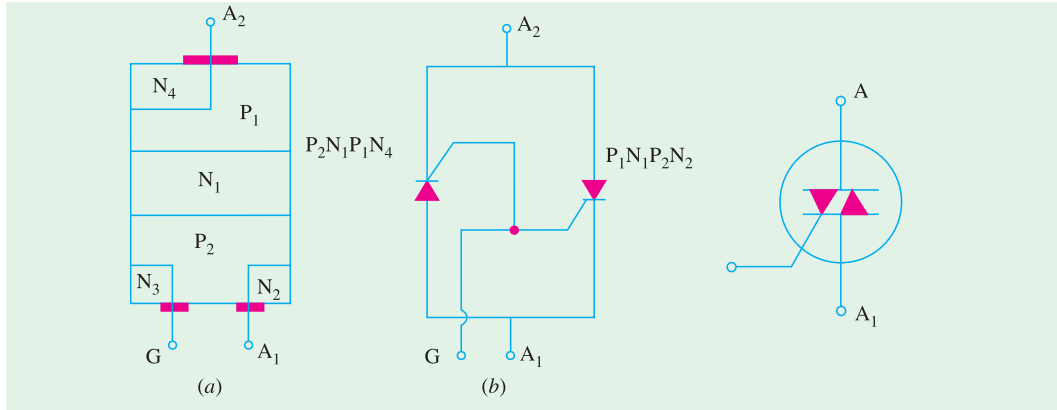


Fig. 64.22

Fig. 64.23

SCRs with a **common gate terminal**. As seen, it has six doped regions. Fig. 64.23 shows the schematic symbol which consists of two inverse-connected SCR symbols.

**2. Operation**

**(a) When  $A_2$  is Positive**

When positive voltage is applied to  $A_2$ , path of current flow is  $P_1-N_1-P_2-N_2$ . The two junctions  $P_1-N_1$  and  $P_2-N_2$  are forward-biased whereas  $N_1-P_2$  junction is blocked. The gate can be given either positive or negative voltage to turn ON the triac as explained below.

**(i) positive gate**

A positive gate (with respect to  $A_1$ ) forward-biases the  $P_2-N_2$  junction and the breakdown occurs as in a normal SCR.

**(ii) negative gate**

A negative gate forward-biases the  $P_2-N_3$  junction and current carriers injected into  $P_2$  turn on the triac.

**(b) When  $A_1$  is Positive**

When positive voltage is applied to anode  $A_1$ , path of current flow is  $P_2-N_1-P_1-N_4$ . The two junctions  $P_2-N_1$  and  $P_1-N_4$  are forward-biased whereas junction  $N_1-P_1$  is blocked. Conduction can be achieved by giving either positive or negative voltage to  $G$  as explained below.

**(i) positive gate**

A positive gate (with respect to  $A_1$ ) injects current carriers by forward-biasing  $P_2-N_2$  junction and thus initiates conduction.

**(ii) negative gate**

A negative gate injects current carriers by forward-biasing  $P_2-N_3$  junction thereby triggering conduction.

It is seen that there are four triac-triggering modes, two each for the two anodes.

Low-current dropout is the only way to open a triac.

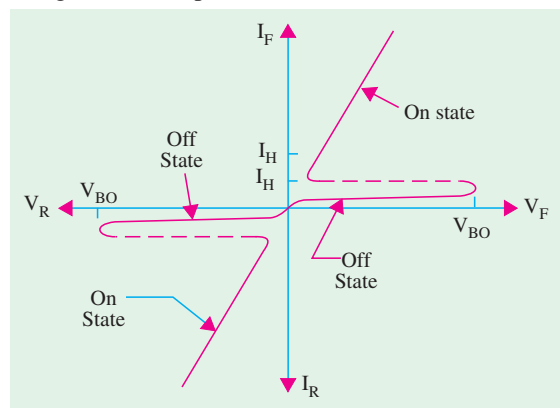


Fig. 64.24

**3. V/I Characteristics**

Typical characteristics of a triac are shown in Fig. 64.24.

As seen, triac exhibits same forward blocking and forward conducting characteristics as an SCR but for *either polarity of voltage applied to the main terminal*. Obviously, a triac has latch current in either direction.

**4. Applications**

One fundamental application of triac is shown in Fig. 64.25. Here, it is used to control ac power to a load by switching ON and OFF during positive and negative half-cycles of the input ac power.

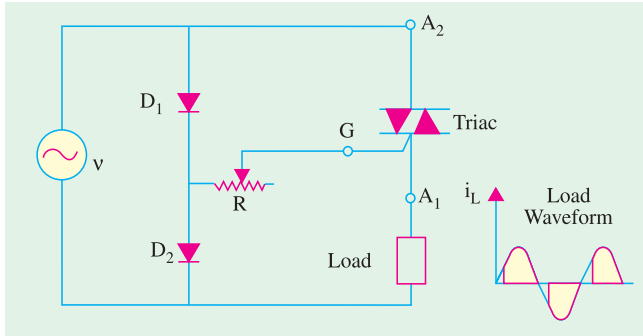


Fig. 64.25

During positive half-cycle of the input, diode  $D_1$  is forward biased,  $D_2$  is reverse-biased and gate is positive with respect to  $A_1$ . By adjusting  $R$ , the point at which conduction commences can be varied.

Diac-triac combination for ac load power control is shown in Fig. 64.26. Firing control of diac is achieved by adjusting  $R$ .

Other applications of a triac include.

1. as static switch to turn ac power OFF and ON;
2. for minimizing radio interference;
3. for light control;
4. for motor speed control etc.

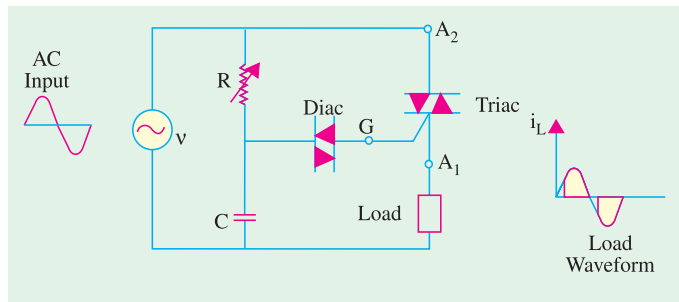


Fig. 64.26

The only disadvantage of triac is that it takes comparatively longer time to recover to OFF state. Hence, its use is limited to ac supply frequencies of upto 400 Hz.

**64.14. Diac**

To put it simply, a diac is nothing else but a *triac without its gate terminal* as shown in Fig.

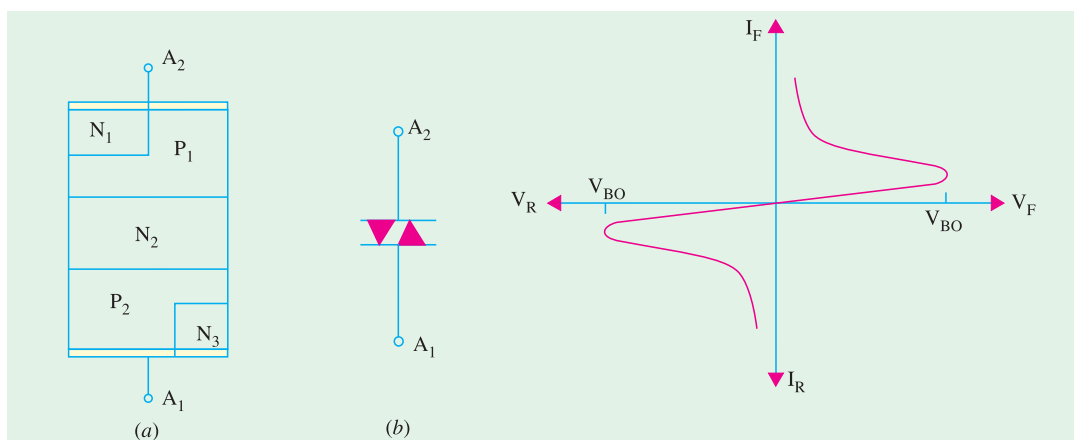


Fig. 64.27

Fig. 64.28

64.27 (a). Its equivalent circuit is a pair of inverted four layer diodes. Its schematic symbol is shown in Fig. 64.27 (b). As seen, **it can break down in either direction.**

When anode  $A_1$  is positive, the current path is  $P_2-N_2-P_1-N_1$ . Similarly, when  $A_2$  is positive, the current flow path is  $P_1-N_2-P_2-N_3$ . Diac is designed to trigger triacs or provide protection against over-voltages.

The operation of a diac can best be explained by imaging it as **two-diodes connected in series**. Voltage applied across it in either direction turns ON one diode, reverse-biasing the other. Hence, it can be switched from OFF to ON state for either polarity of the applied voltage.

The characteristic curve of a typical diac is shown in Fig. 64.28. It resembles the letter Z since diac breaks down in either direction.

As stated above, diac has symmetrical bi-directional switching characteristics. Because of this feature, diacs are frequently used as triggering devices in triac phase control circuits used for light dimming, universal motor speed control and heat control etc.

### 64.15. Silicon Controlled Switch (SCS)

It is a four-layer, four-terminal *PNPN* device having anode  $A$ , cathode  $C$ , anode gate  $G_1$  and cathode gate  $G_2$  as shown in Fig. 64.29. In fact, **it is a low-current SCR with two gate terminals**. The two transistor equivalent circuit is shown in Fig. 64.30.

#### Switching ON and OFF

The device may be switched ON or OFF by a suitable pulse is applied at either gate. As seen from Fig. 64.30, a negative pulse is required at anode gate  $G_1$  to turn the device ON whereas positive pulse is needed to turn it OFF as explained below.

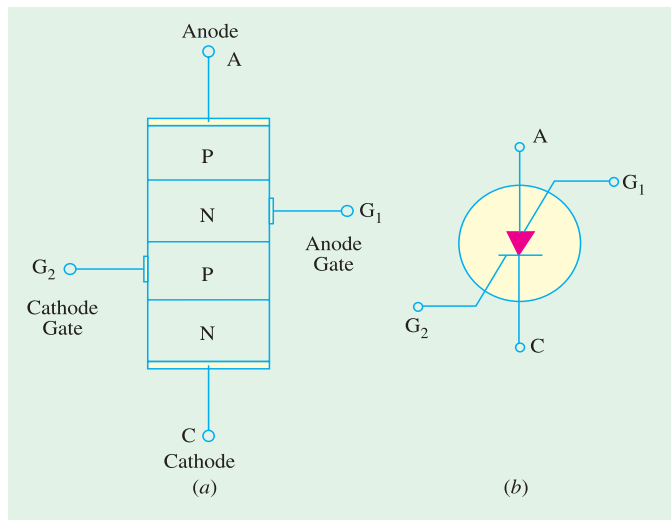


Fig. 64.29

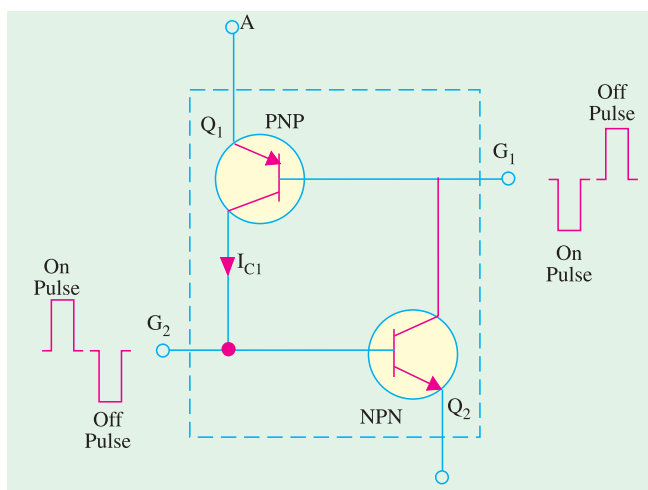


Fig. 64.30

Similarly, at cathode gate  $G_2$ , a negative pulse is required to switch the device OFF and a positive pulse to turn it ON.

As seen from Fig. 64.30, when a negative pulse is applied to  $G_1$ , it forward-biases  $Q_1$  (being *PNP*) which is turned ON. The resulting heavy collector current  $I_C$ , being the base current of  $Q_2$ , turns it ON. Hence, SCS is switched ON. A positive pulse at  $G_1$  will reverse bias E/B junction of  $Q_1$  thereby switching the SCS OFF.

#### V/I Characteristics

The V/I characteristics of an SCS are essentially the same **as those for the SCR** (Fig. 64.13).



As compared to an SCR, an SCS has *much reduced turn-OFF time*. Moreover, it has higher control and triggering sensitivity and a more predictable firing situation.

## Applications

The more common areas of SCS applications are as under :

1. in counters, registers and timing circuits of computers,
2. pulse generators,
3. voltage sensors,
4. oscillators etc.

## OBJECTIVE TESTS – 64

1. A unijunction transistor has
  - (a) anode, cathode and a gate
  - (b) two bases and one emitter
  - (c) two anodes and one gate
  - (d) anode, cathode and two gates.
2. Which semiconductor device acts like a diode and two resistors ?
  - (a) SCR
  - (b) triac
  - (c) diac
  - (d) UJT.
3. A UJT has  $R_{BB} = 10\text{ K}$  and  $R_{B2} = 4\text{ K}$ . Its intrinsic stand-off ratio is
  - (a) 0.6
  - (b) 0.4
  - (c) 2.5
  - (d) 5/3.
4. An SCR conducts appreciable current when its ..... with respect to cathode.
  - (a) anode and gate are both negative
  - (b) anode and gate are both positive
  - (c) anode is negative and gate is positive
  - (d) gate is negative and anode is positive.
5. After firing an SCR, the gating pulse is removed. The current in the SCR will
  - (a) remains the same
  - (b) immediately fall to zero
  - (c) rise up
  - (d) rise a little and then fall to zero.
6. An SCR may be turned OFF by
  - (a) interrupting its anode current
  - (b) reversing polarity of its anode- cathode voltage
  - (c) low-current dropout
  - (d) all of the above.
7. A triac behaves like two
  - (a) inverse parallel-connected SCRs with common gate
  - (b) diodes in series
  - (c) four-layer diodes in parallel
  - (d) resistors and one diode.
8. A triac can be triggered into conduction by
  - (a) only positive voltage at either anode
  - (b) positive or negative voltage at either anode
  - (c) positive or negative voltage at gate
  - (d) both (b) and (c).
9. A diac is equivalent to a
  - (a) pair of SCRs
  - (b) pair of four-layer SCRs
  - (c) diode and two resistors
  - (d) triac with two gates.
10. An SCS has
  - (a) four layers and three terminals
  - (b) three layers and four terminals
  - (c) two anodes and two gates
  - (d) one anode, one cathode and two gates.
11. An SCS may be switched ON by a
  - (a) positive pulse at its anode
  - (b) negative pulse at its cathode
  - (c) positive pulse at its cathode gate  $G_2$
  - (d) positive pulse at its anode gate  $G_1$ .
12. The  $dv/dt$  effect in an SCR can result in
  - (a) high rate-of-rise of anode voltage
  - (b) increased junction capacitance
  - (c) false triggering
  - (d) low capacitive charging current.
13. The  $di/dt$  effect in an SCR leads to the formation of
  - (a) local hot spots
  - (b) conduction zone
  - (c) charge spreading zone
  - (d) none of the above.
14. SCR turns OFF from conducting state to blocking state on
  - (a) reducing gate current
  - (b) reversing gate voltage
  - (c) reducing anode current below holding current value
  - (d) applying ac to the gate
15. When a thyristor is negatively biased,
  - (a) all the three junctions are negatively biased
  - (b) outer junctions are positively biased and the inner junction is negatively biased.

- (c) outer junctions are negatively biased and the inner junction is positively biased
  - (d) the junction near the anode is negatively biased and the one near the cathode is positively biased.
- 16.** A LASCR is just like a conventional SCR except that it
- (a) cannot carry large current
  - (b) can also be light-triggered
  - (c) has no gate terminal
  - (d) cannot be pulse-triggered.
- 17.** The minimum value of current required to maintain conduction in an SCR is called its ..... current.
- (a) commutation
  - (b) holding
  - (c) gate trigger
  - (d) breakover
- 18.** Diacs are primarily used as
- (a) pulse generators
  - (b) triggering devices
  - (c) surge protection devices
  - (d) power thyristors.

**ANSWERS**

- 1.** (b)    **2.** (d)    **3.** (a)    **4.** (b)    **5.** (a)    **6.** (d)    **7.** (a)    **8.** (d)    **9.** (b)    **10.** (d)    **11.** (c)  
**12.** (c)    **13.** (a)    **14.** (c)    **15.** (c)    **16.** (b)    **17.** (b)    **18.** (b)

# CHAPTER 65

## Learning Objectives

- What is an Oscillator?
- Classification of Oscillators
- Damped and Undamped Oscillations
- Oscillatory Circuit
- Essentials of a Feedback LC Oscillator
- Tuned Base Oscillator
- Tuned Collector Oscillator
- Hartley Oscillator
- FET Hartley Oscillator
- Colpitts Oscillator
- Clapp Oscillator
- FET Colpitts Oscillator
- Crystal Controlled Oscillator
- Transistor Pierce Crystal Oscillator
- FET Pierce Oscillator
- Phase Shift Principle
- RC Phase Shift Oscillator
- Wien Bridge Oscillator
- Pulse Definitions
- Basic Requirements of a Sawtooth Generator
- UJT Sawtooth Generator
- Multivibrators (MV)
- Astable Multivibrator
- Bistable Multivibrator (BMV)
- Schmitt Trigger
- Transistor Blocking Oscillator

## SINUSOIDAL AND NON-SINUSOIDAL OSCILLATORS



An oscillator is an electronic device used for the purpose of generating a signal. Oscillators are found in computers, wireless receivers and transmitters, and audiofrequency equipment particularly music synthesizers

### 65.1. What is an Oscillator ?

An electronic oscillator may be defined in any one of the following four ways :

1. It is a circuit which converts dc energy into ac energy at a very high frequency;
2. It is an electronic source of alternating current or voltage having sine, square or sawtooth or pulse shapes;
3. It is a circuit which generates an ac output signal without requiring any externally applied input signal;
4. It is an unstable amplifier.

These definitions exclude electromechanical alternators producing 50 Hz ac power or other devices which convert mechanical or heat energy into electric energy.



Oscillator

### 65.2. Comparison Between an Amplifier and an Oscillator

As discussed in Chapter-10, an amplifier produces an output signal whose waveform is similar to the input signal but whose power level is generally high. This additional power is supplied by

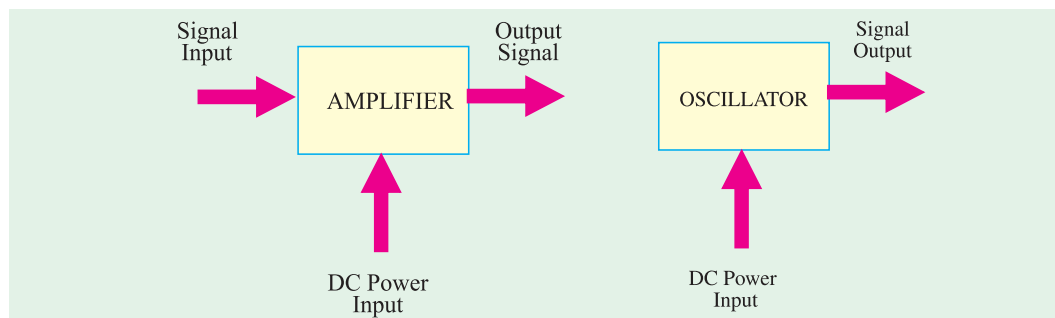


Fig. 65.1

the external dc source. Hence, an amplifier is essentially an energy converter *i.e.* it takes energy from the dc power source and converts it into ac energy at signal frequency. The process of energy conversion is controlled by the input signal. If there is no input signal, there is no energy conversion and hence there is no output signal.

An oscillator differs from an amplifier in one basic aspect : the oscillator **does not require an external signal** either to start or maintain energy conversion process (Fig. 65.1). It keeps producing an output signal so long as the dc power source is connected.

Moreover, the frequency of the output signal is determined by the passive components used in the oscillator and can be varied at will.

### 65.3. Classification of Oscillators

Electronic oscillators may be broadly divided into following two groups :

- (i) Sinusoidal (or harmonic) oscillators—which produce an output having sine waveform;
- (ii) Non-sinusoidal (or relaxation) oscillators—they produce an output which has square, rectangular or sawtooth waveform or is of pulse shape.

Sinusoidal oscillators may be further subdivided into :

- (a) Tuned-circuits or LC feedback oscillators such as Hartley, Colpitts and Clapp etc.;
- (b) RC phase-shift oscillators such as Wien-bridge oscillator;
- (c) Negative-resistance oscillators such as tunnel diode oscillator;
- (d) Crystal oscillators such as Pierce oscillator;

(e) Heterodyne or beat-frequency oscillator (BFO).

The active devices (bipolars, FETs or unijunction transistors) in the above mentioned circuits may be biased class-A, B or C. Class-A operation is used in high-quality audio frequency oscillators. However, radio frequency oscillators are usually operated as class-C.

#### 65.4. Damped and Undamped Oscillations

Sinusoidal oscillations produced by oscillators may be (i) damped or (ii) undamped.

##### (i) Damped Oscillations

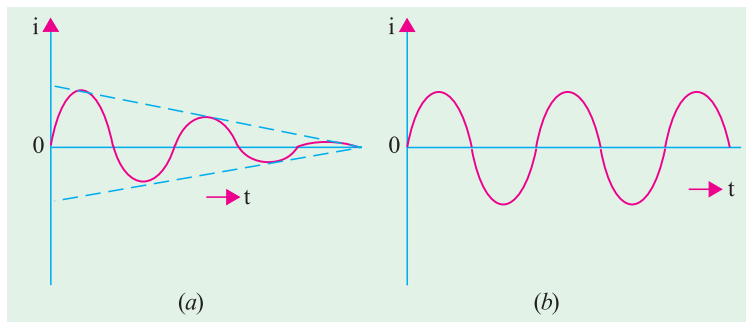


Fig. 65.2

Oscillations whose amplitude keeps decreasing (or decaying) with time are called **damped** or **decaying** oscillations. The waveform of such oscillations is shown in Fig. 65.2 (a). These are produced by those oscillator circuits in which  $I^2 R$  losses take place continuously during each oscillation without any arrangement for compensating the same. Ulti-

mately, the amplitude of the oscillations decays to zero when there is not enough energy to supply circuit losses. However, the frequency or time-period remains constant because **it is determined by the circuit parameters.**

Sinusoidal oscillators serve a variety of functions in telecommunications and in electronics. The most important application in telecommunication is the use of sine waves as carrier signal in both radio and cable transmissions.

Sine wave signals are also used in frequency response testing of various types of systems and equipment including analogue communication channels, amplifiers and filters and closed-loop control systems.

##### (ii) Undamped Oscillations

Oscillations whose amplitude remains constant *i.e.* does not change with time are called undamped oscillations. These are produced by those oscillator circuits which have no losses or if they have, there is provision for compensating them. The constant-amplitude and constant-frequency sinusoidal waves shown in Fig. 65.2 (b) are called **carrier waves** and are used in communication transmitters for transmitting low-frequency audio information to far off places.

#### 65.5. The Oscillatory Circuit

It is also called *LC* circuit or tank circuit. The oscillatory circuit (Fig. 65.3) consists of two reactive elements *i.e.* an inductor and a capacitor. Both are capable of storing energy. The capacitor stores energy in its electric field whenever there is potential difference across its plates. Similarly, a coil or an inductor stores energy in its magnetic field whenever current flows through it. Both *L* and *C* are supposed to be loss-free (*i.e.* their *Q*-factors are infinite).

As shown in Fig. 65.3 (a), suppose the capacitor has been fully-charged from a dc source. Since *S* is open, it cannot discharge through *L*. Now, let us see what happens when *S* is closed.

1. When *S* is closed [Fig. 65.3 (b)] **electrons** move from plate *A* to plate *B* through coil *L* as shown by the arrow (or conventional current flows from *B* to *A*). This electron flow reduces the strength of the electric field and hence the amount of energy stored in it.
2. As electronic current starts flowing, the self-induced emf in the coil opposes the current flow. Hence, rate of discharge of electrons is somewhat slowed down.
3. Due to the flow of current, magnetic field is set up which stores the energy given out by

the electric field [Fig. 65.3 (b)].

4. As plate *A* loses its electrons by discharge, the electron current has a tendency to die down and will actually reduce to zero when all excess electrons on *A* are driven over to plate *B* so that both plates are reduced to the same potential. At that time, there is no electric field but the magnetic field has maximum value.
5. However, due to self-induction (or electrical inertia) of the coil, more electrons are transferred to plate *B* than are necessary **to make up the electron deficiency there**. It means that now plate *B* has more electrons than *A*. Hence, capacitor becomes charged again though in opposite direction as shown in Fig. 65.3 (c).
6. The magnetic field *L* collapses and the energy given out by it is stored in the electric field of the capacitor.
7. After this, the capacitor starts discharging in the opposite direction so that, now, the electrons move from plate *B* to plate *A* [Fig. 65.3 (d)]. The electric field starts collapsing whereas magnetic field starts building up again though in the opposite direction. Fig. 65.3 (d) shows the condition when the capacitor becomes fully discharged once again.
8. However, these discharging electrons overshoot and again an excess amount of electrons flow to plate *A*, thereby charging the capacitor once more.
9. This sequence of charging and discharging continues. The to and fro motion of electrons between the two plates of the capacitor constitutes an oscillatory current.

It may be also noted that during this process, the electric energy of the capacitor is converted into magnetic energy of the coil and **vice versa**.

These oscillations of the capacitor discharge are damped because energy is dissipated away gradually so that their amplitude becomes zero after sometime. There are two reasons for the loss of the energy :

(a) Some energy is lost in the form of heat produced in the resistance of the coil and connecting wires ;

(b) and some energy is lost **in the form of electromagnetic (EM) waves** that are radiated out from the circuit through which an oscillatory current is passing.

Both these losses subtract energy from the circuit with the result that circuit current decreases gradually till it becomes zero. The waveform of the oscillatory discharge is similar to that shown in Fig. 65.2 (a).

### 65.6. Frequency of Oscillatory Current

The frequency of time-period of the oscillatory current depends on two factors :

(a) **Capacitance of the Capacitor**

Larger the capacitor, greater the time required for the reversal of the discharge current *i.e.* lower its frequency.

(b) **Self-inductance of the Coil**

Larger the self-inductance, greater the internal effect and hence longer the time required by the current to stop flowing during discharge of the capacitor.

The frequency of this oscillatory discharge current is given by

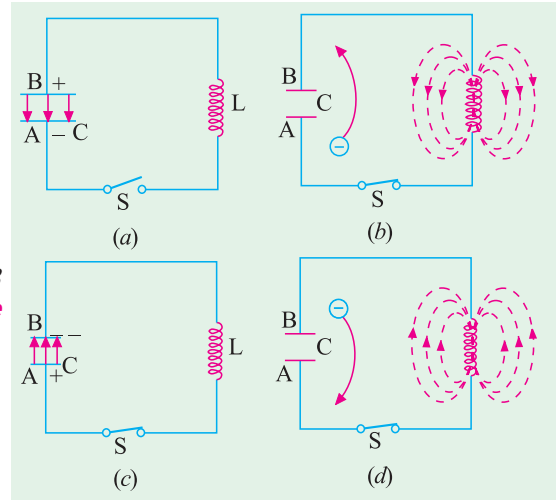


Fig. 65.3

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{159}{\sqrt{LC}} \text{ kHz}$$

where  $L$  = self-inductance in  $\mu H$  and  $C$  = capacitance in  $\mu F$

It may, however, be pointed out here that damped oscillations so produced are not good for radio transmission purpose because of their **limited range and excessive distortion**. For good radio transmission, we need undamped oscillations which can be produced if some additional energy is supplied in correct phase and correct direction to the  $LC$  circuit for making up the  $I^2R$  losses continually occurring in the circuit.

### 65.7. Frequency Stability of an Oscillator

The ability of an oscillator to maintain a constant frequency of oscillation is called its frequency stability. Following factors affect the frequency stability :

#### 1. Operating Point of the Active Device

The  $Q$ -point of the active device (*i.e.* transistor) is so chosen as to confine the circuit operation on the linear portion of its characteristic. Operation on non-linear portion varies the parameters of the transistor which, in turn, affects the frequency stability of the oscillator.

#### 2. Inter-element Capacitances

Any changes in the inter-element capacitances of a transistor particularly the collector- to-emitter capacitance cause changes in the oscillator output frequency, thus affecting its frequency stability. The effect of changes in inter-element capacitances, can be neutralized by adding a swamping capacitor across the offending elements—the added capacitance being made part of the tank circuit.

#### 3. Power Supply

Changes in the dc operating voltages applied to the active device shift the oscillator frequency. This problem can be avoided by using regulated power supply.

#### 4. Temperature Variations

Variations in temperature cause changes in transistor parameters and also change the values of resistors, capacitors and inductors used in the circuit. Since such changes take place slowly, they cause a slow change (called drift) in the oscillator output frequency.

#### 5. Output Load

A change in the output load may cause a change in the  $Q$ -factor of the  $LC$  tuned circuit thereby affecting the oscillator output frequency.

#### 6. Mechanical Vibrations

Since such vibrations change the values of circuit elements, they result in changes of oscillator frequency. This instability factor can be eliminated by isolating the oscillator from the source of mechanical vibrations.

### 65.8. Essentials of a Feedback LC Oscillator

The essential components of a feedback  $LC$  oscillator shown in Fig. 65.4 are :

1. A resonator which consists of an  $LC$  circuit. It is also known as frequency-determining network ( $FDN$ ) or tank circuit.
2. An amplifier whose function is to amplify the oscillations produced by the resonator.

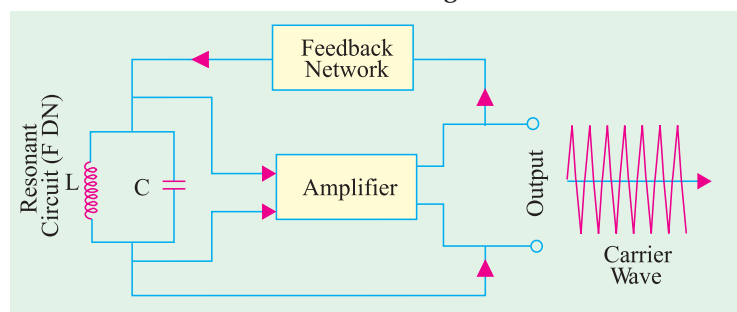


Fig. 65.4

3. A positive feedback network (*PFN*) whose function is to transfer part of the output energy to the resonant *LC* circuit in **proper phase**. The amount of energy fed back is sufficient to meet  $I^2R$  losses in the *LC* circuit.

The essential condition for maintaining oscillations and for finding the value of frequency is

$$\beta A = 1 + j0 \quad \text{or} \quad \beta A \angle \phi = 1 \angle 0$$

It means that

- (i) The feedback factor or loop gain  $|\beta A| = 1$ ,  
 (ii) The net phase shift around the loop is  $0^\circ$  (or an integral multiple of  $360^\circ$ ). In other words, **feedback should be positive.**

The above conditions form Barkhausen criterion for maintaining a steady level of oscillation at a specific frequency.

Majority of the oscillators used in radio receivers and transmitters use tuned circuits with positive feedback. Variations in oscillator circuits are due to the different way by which the feedback is applied. Some of the basic circuits are :

1. **Armstrong or Tickler or Tuned-base Oscillator** — it employs inductive feedback from collector to the tuned *LC* circuit in the base of a transistor.
2. **Tuned Collector Oscillator**—it also employs inductive coupling but the *LC* tuned circuit is in the collector circuit.
3. **Hartley Oscillator**—Here feedback is supplied inductively.
4. **Colpitts Oscillator**—Here feedback is supplied capacitively.
5. **Clapp Oscillator**—It is a slight modification of the Colpitts oscillator.

### 65.9. Tuned Base Oscillator

Such an oscillator using a transistor in *CE* configuration is shown in Fig. 65.5. Resistors  $R_1$ ,  $R_2$  and  $R_3$  determine the dc bias of the circuit. The parallel  $R_3$ — $C_2$  network in the emitter circuit is a stabilizing circuit to prevent signal degeneration. As usual,  $C_1$  is the dc blocking capacitor. The mutually-coupled coils  $L_1$  and  $L$  forming primary and secondary coils of an *RF* transformer provide the required feedback between the collector and base circuits. The amount of feedback depends on the coefficient of coupling between the two coils. The *CE* connected transistor itself provides a phase shift of  $180^\circ$  between its input and output circuits. The transformer provides another  $180^\circ$  phase shift and thus producing a **total phase shift of  $360^\circ$  which is an essential condition for producing oscillations.**

The parallel-tuned *LC* circuit connected between base and emitter is the frequency determining network (*FDM*) i.e. it generates the oscillations at its resonant frequency.

#### Circuit Action

The moment switch *S* is closed, collector current is set up which tends to rise to its quiescent value. This increase in  $I_C$  is accompanied by :

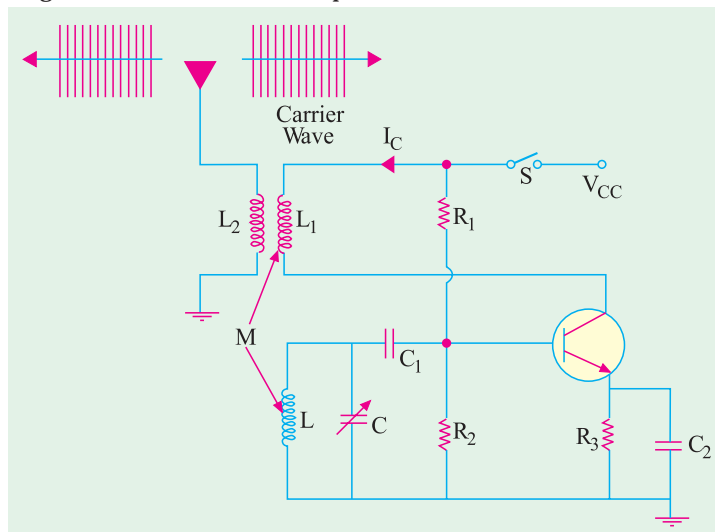


Fig. 65.5



1. An expanding magnetic field through  $L_1$  which links with  $L$  and
2. An induced *e.m.f.* called feedback voltage in  $L$ .

Two immediate reactions of this feedback voltage are:

- (i) Increase in emitter-base voltage (and base current) and
- (ii) A further increase in collector current  $I_C$ .

It is followed by a succession of cycles of

1. An increase in feedback voltage,
2. An increase in emitter-base voltage and
3. An increase in  $I_C$  until saturation is reached.

Meanwhile,  $C$  gets charged. As soon as  $I_C$  ceases to increase, magnetic field of  $L_1$  ceases to expand and thus no longer induces feedback voltage in  $L$ . Having been charged to maximum value,  $C$  starts to discharge through  $L$ . However, decrease in voltage across  $C$  causes the following sequence of reactions :

1. A decrease in emitter-base bias and hence in  $I_B$ ,
2. A decrease in  $I_C$ ;
3. A collapsing magnetic field in  $L_1$ ;
4. An induced feedback voltage in  $L$  though, this time, in **opposite** direction;
5. Further decrease in emitter-base bias and so on till  $I_C$  reaches its cut-off value.

During this time, the capacitor having lost its original charge, again becomes fully charged though with **opposite** polarity. Transistor being in cut-off, the capacitor will again begin to discharge through  $L$ . Since polarity of capacitor charge is opposite to that when transistor was in saturation, the sequence of reactions now will be

1. An increase in emitter-base bias,
2. An increase in  $I_C$ ,
3. An expanding magnetic field in  $L_1$ ,
4. An induced feedback voltage in  $L$ ,
5. A further increase in emitter-base bias and
6. So on till  $I_C$  increases to its saturation value.

This cycle of operation keeps repeating so long as enough energy is supplied to meet losses in the  $LC$  circuit.

The output can be taken out by means of a third winding  $L_2$  magnetically coupled to  $L_1$ . It has approximately the same waveform as collector current.

The frequency of oscillation is equal to the resonant frequency of the  $LC$  circuit.

### 65.10. Tuned Collector Oscillator

Such an oscillator using a transistor in  $CE$  configuration is shown in Fig. 65.6.

#### (i) Frequency Determining Network (FDN)

It is made up of a variable capacitor  $C$  and a coil  $L$  which forms primary winding of a step-down transformer. The combination of  $L$  and  $C$  forms an oscillatory tank circuit to set the frequency of oscillation.

Resistors  $R_1$ ,  $R_2$  and  $R_3$  are used to dc bias the transistor. Capacitors  $C_1$  and  $C_2$  act to bypass  $R_3$  and  $R_2$  respectively so that they have no effect on the ac operation of the circuit. Moreover,  $C_2$  provides ac ground for transformer secondary  $L_1$ .

#### (ii) Positive Feedback

Feedback between the collector-emitter circuit and base-emitter circuit is provided by the transformer secondary winding  $L_1$  which is mutually-coupled to  $L$ . As far as ac signals are concerned,  $L_1$  is connected to emitter *via* low-reactance capacitors  $C_2$  and  $C_1$ .

Since transistor is connected in  $CE$  configuration, it provides a phase shift of  $180^\circ$  between its

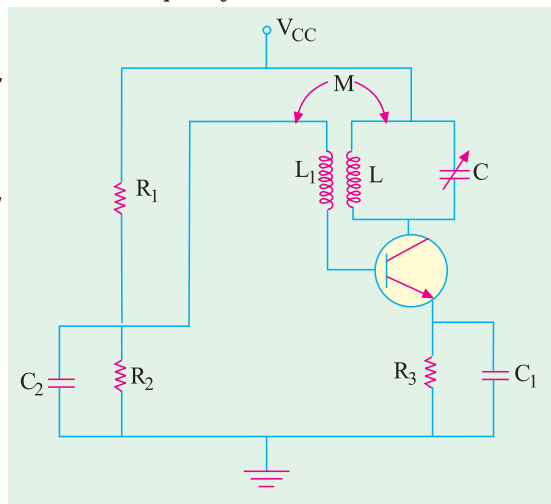


Fig. 65.6

input and output circuits. Another phase shift of 180° is provided by the transformer thus producing a total phase shift of 360° between the output and input voltages resulting in **positive feedback** between the two.

**(iii) Amplifying Action**

The transistor amplifier provides sufficient gain for oscillator action to take place.

**(iv) Working**

When the supply is first switched on, a transient current is developed in the tuned LC circuit as the collector current rises to its quiescent value. This transient current initiates natural oscillations in the tank circuit. These natural oscillations induce a small emf into  $L_1$  by mutual induction which causes corresponding variations in base current. These variations in  $I_B$  are amplified  $\beta$  times and appear in the collector circuit. Part of this amplified energy is used to meet losses taking place in the oscillatory circuit and **the balance is radiated out in the form of electromagnetic waves.**

The frequency of oscillatory current is almost equal to the resonant frequency of the tuned circuit.

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$$

**65.11. Tuned Drain Oscillator (FET)**

The basic circuit is illustrated in Fig. 65.7. It is similar to the tuned collector oscillator of Fig. 65.6. Because of its high input impedance and high voltage amplification, a FET can be used to construct very simple and efficient oscillator circuit. This frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \sqrt{\left(1 + \frac{R}{r_d}\right)}$$

where  $r_d$  = ac drain resistance

The value of mutual inductance required for maintaining oscillations is  $M = \frac{r_d RC + L}{\mu}$ .

**Example 65.1.** A tuned-collector oscillator has a fixed inductance of 100  $\mu$ H and has to be tunable over the frequency band of 500 kHz to 1500 kHz. Find the range of variable capacitor to be used. **(Principles of Telecom. Engg. Pune Univ.)**

**Solution.** Resonant frequency is given by

$$f_0 = 1/2\pi\sqrt{LC} \quad \text{or} \quad C = 1/4\pi^2 f_0^2 L$$

where  $L$  and  $C$  refer to the tank circuit.

**When fo = 500 kHz**

$$C = 1/4\pi^2 \times (500 \times 10^3)^2 \times 100 \times 10^{-6} = 1015 \text{ pF}$$

**When fo = 1500 kHz**

$$C = 1015/(1500/500)^2 = 113 \text{ pF}$$

Hence, capacitor range required is **113 – 1015 pF**

**Example 65.2.** The resonant circuit of a tuned-collector transistor oscillator has a resonant frequency of 5 MHz. If value of capacitance is increased by 50%, calculate the new resonant frequency.

**Solution.** Using the equation for resonant frequency, we have

$$5 \times 10^6 = 1/2\pi\sqrt{LC} \quad \text{--- 1st case}$$

$$f_0 = 1/2\pi\sqrt{L \times 1.5C} \quad \text{--- 2nd case}$$

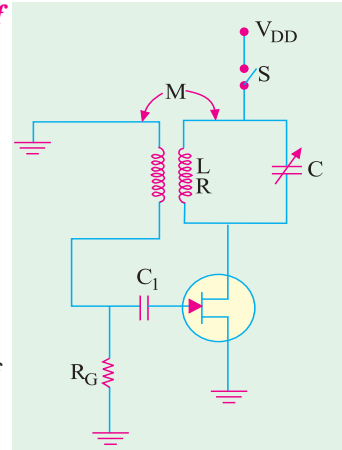


Fig. 65.7

$$\therefore \frac{f_o}{5 \times 10^6} = \frac{1}{\sqrt{1.5}} \quad \text{or} \quad f_o = 4.08 \text{ MHz}$$

### 65.12. Hartley Oscillator

In Fig. 65.8 (a) is shown a transistor Hartley oscillator using  $CE$  configuration. Its general principle of operation is similar to the tuned-collector oscillator discussed in Art. 65.10.

It uses a single tapped-coil having two parts marked  $L_1$  and  $L_2$  instead of two separate coils. So far as ac signals are concerned, one side of  $L_2$  is connected to base *via*  $C_1$  and the other to emitter *via* ground and  $C_3$ . Similarly, one end of  $L_1$  is connected to collector *via*  $C_2$  and the other to common emitter terminal *via*  $C_3$ . In other words,  $L_1$  is in the output circuit *i.e.* collector-emitter circuit whereas  $L_2$  is in the base-emitter circuit *i.e.* input circuit. These two parts are inductively-coupled and form an auto-transformer or a split-tank inductor. Feedback between the output and input circuits is accomplished through autotransformer action which also introduces a phase reversal of  $180^\circ$ . This phase reversal

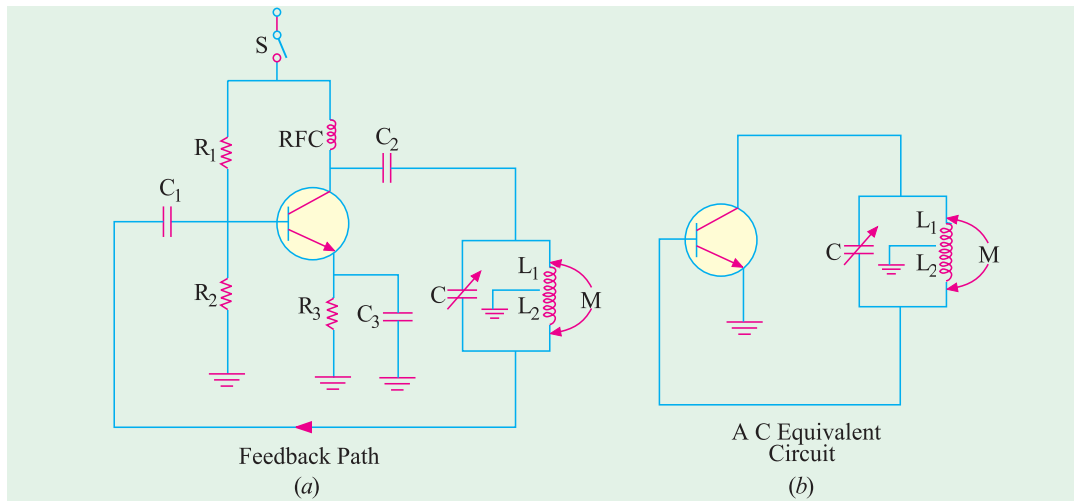


Fig. 65.8



Hartley oscillator

between two voltages occurs because they are taken from **opposite ends** of an inductor ( $L_1 - L_2$  combination) with respect to the tap which is tied to common transistor terminal *i.e.* emitter which is ac grounded *via*  $C_3$ . Since transistor itself introduces a phase shift of  $180^\circ$ , the total phase shift becomes  $360^\circ$  thereby making the **feedback positive or regenerative** which is essential for oscillations (Art 65.8). As seen, positive feedback is obtained from the tank circuit and is coupled to the base *via*  $C_1$ . The feedback factor is given by the ratio of turns in  $L_2$  and  $L_1$  *i.e.* by  $N_2/N_1$  and its value ranges from 0.1 to 0.5. Fig. 65.8 (b) shows the equivalent circuit of Hartley oscillator.

Resistors  $R_1$  and  $R_2$  form a voltage divider for providing the base bias and  $R_3$  is an emitter swamping resistor to add stability to the circuit. Capacitor  $C_3$  provides ac ground thereby preventing any signal degeneration while still providing temperature stabilisation. Radio-frequency choke (RFC) provides dc load for the collector and also keeps ac currents out of the dc supply  $V_{CC}$

When  $V_{CC}$  is first switched on through  $S$ , an initial bias is established by  $R_1 - R_2$  and oscillations are produced because of positive feedback from the  $LC$  tank circuit ( $L_1$  and  $L_2$  constitute  $L$ ). The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where } L = L_1 + L_2 + 2M$$

The output from the tank may be taken out by means of another coil coupled either to  $L_1$  or  $L_2$ .

**Example 65.3.** Calculate the oscillation frequency for the transistor Hartley oscillator circuit (refer to Fig. 65.8). Given the circuit values:  $L_{RFC} = 0.5 \text{ mH}$ ,  $L_1 = 750 \text{ } \mu\text{H}$ ,  $L_2 = 750 \text{ } \mu\text{H}$ ,  $M = 150 \text{ } \mu\text{H}$  and  $C = 150 \text{ pF}$ .

**Solution.**  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  where  $L = L_1 + L_2 + 2M$   
 $\therefore L = 750 \text{ } \mu\text{H} + 750 \text{ } \mu\text{H} + 2 \times 150 \text{ } \mu\text{H} = 1800 \text{ } \mu\text{H}$   
 and  $f_0 = \frac{1}{2\pi\sqrt{1800\text{ } \mu\text{H} \times 150\text{ pF}}} = \mathbf{320\text{ kHz}}$

**Example 65.4.** In an Hartley oscillator if  $L_1 = 0.1 \text{ mH}$  and mutual inductance between the coils equal to  $20 \text{ } \mu\text{H}$ . Calculate the value of capacitor  $C$  of the oscillating circuit to obtain frequency of  $4110 \text{ kHz}$ . (Bangalore University 2001)

**Solution.**  $L = L_1 + L_2 + 2M = 0.1 \text{ mH} + 10 \text{ } \mu\text{H} + 20 \text{ } \mu\text{H} = 130 \text{ } \mu\text{H}$   
 Now the resonant frequency is given by  
 $f_0 = \frac{1}{2\pi\sqrt{LC}}$  or  $C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times 4110^2 \times 130\text{ } \mu\text{H}}$   
 $C = \mathbf{11.5 \text{ pF}}$

**65.13. FET Hartley Oscillator**

The basic circuit is shown in Fig. 65.9.  $R_G$  is the gate biasing resistor. There is mutual induction between the two parts  $L_1$  and  $L_2$  of the coil.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where  $L = L_1 + L_2 + 2M$

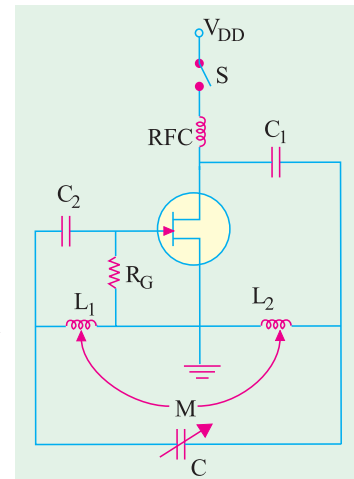


Fig. 65.9

**Example 65.5.** Calculate the oscillator frequency for a FET Hartley oscillator (refer to Fig. 65.9), for the following circuit values:  $C = 250 \text{ pF}$ ,  $L_1 = 1.5 \text{ mH}$ ,  $L_2 = 1.5 \text{ mH}$ , and  $M = 0.5 \text{ mH}$ .

**Solution.**  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  where  $L = L_1 + L_2 + 2M$   
 $\therefore L = 1.5 \text{ mH} + 1.5 \text{ mH} + 2 \times 0.5 \text{ mH} = 4 \text{ mH}$   
 and  $f_0 = \frac{1}{2\pi\sqrt{4 \text{ mH} \times 250 \text{ pF}}} = \mathbf{159.1 \text{ kHz}}$

**65.14. Colpitts Oscillator**

This oscillator is essentially the same as Hartley oscillator except for one difference. Colpitts oscillator uses **tapped capacitance** whereas Hartley oscillator uses **tapped inductance**\*. Fig. 65.10 (a)

\* As an aid to memory, remember that Hartley begins with letter H for Henry i.e. coil and Colpitts begins with C for Capacitor.

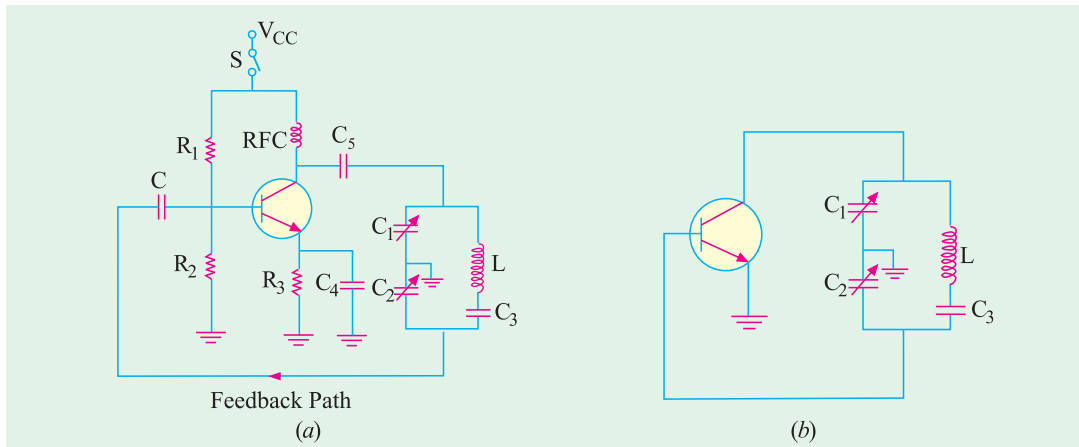


Fig. 65.10

shows the complete circuit with its power source and dc biasing circuit whereas Fig. 65.10 (b) shows its ac equivalent circuit. The two series capacitors  $C_1$  and  $C_2$  form the voltage divider used for providing the feedback voltage (the voltage drop across  $C_2$  constitutes the feedback voltage). The feedback factor is  $C_1/C_2$ . The minimum value of amplifier gain for maintaining oscillations is

$$A_{v(\min)} = \frac{1}{C_1/C_2} = \frac{C_2}{C_1}$$

The tank circuit consists of two ganged capacitors  $C_1$  and  $C_2$  and a single fixed coil. The frequency of oscillation (which does not depend on mutual inductance) is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

Transistor itself produces a phase shift of  $180^\circ$ . Another phase shift of  $180^\circ$  is provided by the capacitive feedback thus giving a total phase shift of  $360^\circ$  between the emitter-base and collector-base circuits.

Resistors  $R_1$  and  $R_2$  form a voltage divider across  $V_{CC}$  for providing base bias,  $R_3$  is for emitter stabilisation and  $R_{FC}$  provides the necessary dc load resistance  $R_C$  for amplifier action. It also prevents ac signal from entering supply dc  $V_{CC}$ . Capacitor  $C_5$  is a bypass capacitor whereas  $C_4$  conveys feedback from the collector-to-base circuit.

When  $S$  is closed, a sudden surge of collector current **shock-excites** the tank circuit into oscillations which are sustained by the feedback and the amplifying action of the transistor.

Colpitts oscillator is widely used in commercial signal generators upto 1 MHz. Frequency of oscillation is varied by gang-tuning the two capacitors  $C_1$  and  $C_2$ .

**Example 65.6.** Determine the circuit oscillation frequency for a transistor Colpitts oscillator shown in Fig. 65.10(a). Given,

$$L = 100 \mu\text{H}, L_{RFC} = 0.5 \text{ mH}, C_1 = 0.005 \mu\text{F}, C_2 = 0.01 \mu\text{F}, C_6 = 10 \mu\text{F}$$

**Solution.** For a transistor Colpitts oscillator, the oscillation frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore C = \frac{0.005 \mu\text{F} \times 0.01 \mu\text{F}}{0.005 \mu\text{F} + 0.01 \mu\text{F}} = 3.3 \text{ nF}$$

$$\text{and } f_0 = \frac{1}{2\pi \times \sqrt{100 \mu\text{H} \times 3.3 \text{ nF}}} = 277 \text{ kHz}$$

**Example 65.7.** For the Colpitts oscillator circuit shown in Fig. 65.11, find the values of

- (a) feedback fraction,
  - (b) minimum gain to sustain oscillations,
  - (c) emitter resistor  $R_E$ .
- (Electronics-I, Bangalore Univ.)

**Solution.** (a) The voltage drop across  $C_2$  is feedback to the input circuit. feedback fraction

$$= \frac{\text{drop across } C_2}{\text{drop across } C_1} = \frac{IX_2}{IX_1} = \frac{C_1}{C_2} = \frac{0.018}{0.16} = \mathbf{0.11}$$

(b)  $A_{v(min)} = \frac{1}{\text{feedback fraction}} = \frac{1}{0.11} = \mathbf{9}$

(c) Now,  $A_v \cong \frac{R_C}{R_E} \quad \therefore \quad R_E = \frac{R_C}{A_v} = \frac{1500}{9} = \mathbf{167 \Omega}$

It should be noted that the above calculations do not take into account the losses in the coil and the loading effect of the amplifier input impedance.

### 65.15. Clapp Oscillator

It is a variation of Colpitts oscillator and is shown in Fig. 65.12 (a). It differs from Colpitts oscillator in respect of capacitor  $C_3$  only which is joined in series with the tank inductor. Fig. 65.12 (b) shows the ac equivalent circuit.

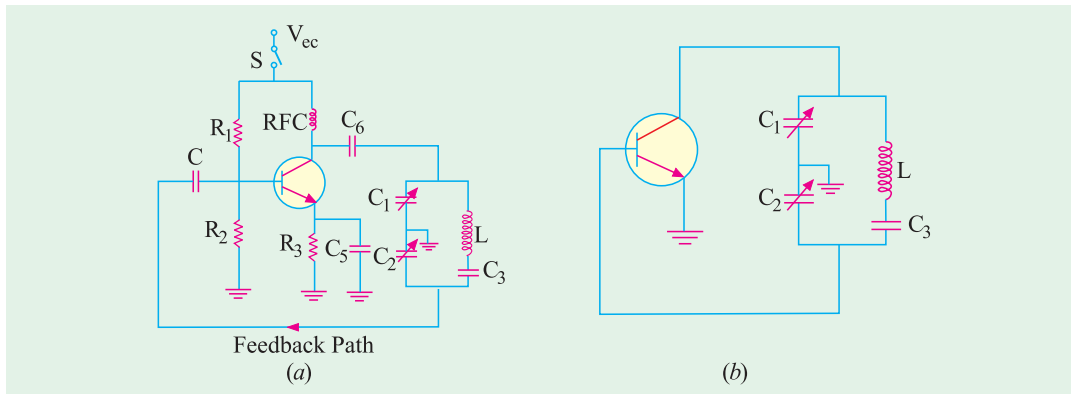


Fig. 65.12

Addition of  $C_3$  (i) improves frequency stability and (ii) eliminates the effect of transistor's parameters on the operation of the circuit.

The operation of this circuit is the same as that of the Colpitts oscillator.

The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

### 65.16. FET Colpitts Oscillator

The circuit is shown in Fig. 65.13. It is similar to the transistor circuit of Fig. 65.10. Here,  $R_G$  is the gate biasing resistor. The radio-frequency coil choke ( $R_{FC}$ ) performs two functions :

- (i) It keeps ac current out of the dc drain supply and
- (ii) It provides drain load.

As seen,  $C_1$  is in the input circuit whereas  $C_2$  is in the output circuit.

The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad ; \quad \text{where, } C = \frac{C_1 C_2}{C_1 + C_2}$$

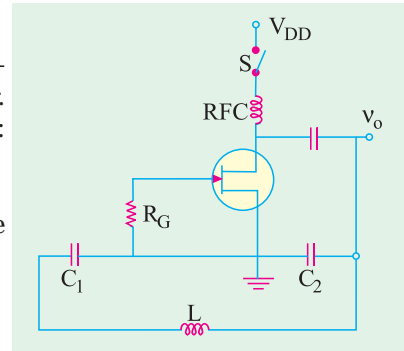


Fig. 65.13

**Example 65.8.** Determine the circuit oscillation frequency for the FET Colpitts oscillator (refer to Fig. 65.13), given  $C_1 = 750 \text{ pF}$ ,  $C_2 = 2500 \text{ pF}$  and  $L = 40 \text{ }\mu\text{H}$ .

**Solution.** For a Colpitts oscillator, the oscillation frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where } C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore C = \frac{750 \text{ pF} \times 2500 \text{ pF}}{750 \text{ pF} + 2500 \text{ pF}} = 576.9 \text{ pF}$$

$$\text{and } f_0 = \frac{1}{2\pi \times \sqrt{40 \mu\text{H} \times 576.9 \text{ pF}}} = 1.048 \text{ MHz}$$

### 65.17. Crystals

For an exceptionally high degree of frequency stability, use of crystal oscillators is essential. The crystal generally used is a finely-ground wafer of translucent quartz (or tourmaline) stone held between two metal plates and housed in a package about the size of a postal stamp. The crystal wafers are cut from the crude quartz in two different ways. The method of 'cutting' determines the crystal's natural resonant frequency and its temperature coefficient. When the wafer is cut so that its flat surface are perpendicular to its **electrical axis**, it is called an **X-cut** crystal (Fig. 65.14). But if the wafer is so cut that its flat surfaces are perpendicular to its **mechanical axis**, it is called **Y-cut** crystal.

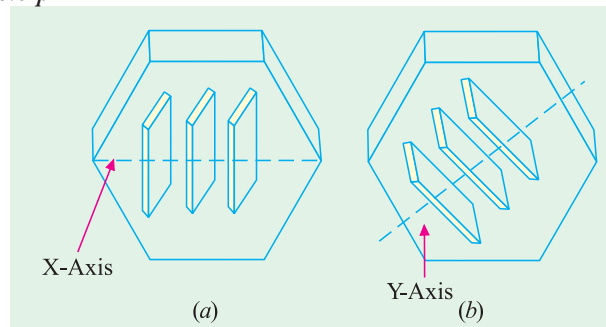


Fig. 65.14

(a) **Piezoelectric Effect**

The quartz crystal described above has peculiar properties. When mechanical stress is applied across its two opposite faces, a **potential difference is developed across them**. It is called **piezoelectric effect**. Conversely, when a potential difference is applied across its two opposite faces, it causes the crystal to **either expand or contract**. If an alternating voltage is applied, the crystal wafer is set into vibrations. The frequency of vibration is equal to the resonant frequency of the crystal as determined by its structural characteristics. Where the frequency of the applied ac voltage equals the natural resonant frequency of the crystal, the amplitude of vibration will be maximum. As a general rule, thinner the crystal, **higher its frequency of vibration**.

#### (b) Equivalent Electrical Circuit

The electrical equivalent circuit of the crystal is shown in Fig. 65.15 (b). It consists of a series  $RLC_1$  circuit in parallel with a capacitor  $C_2$ .

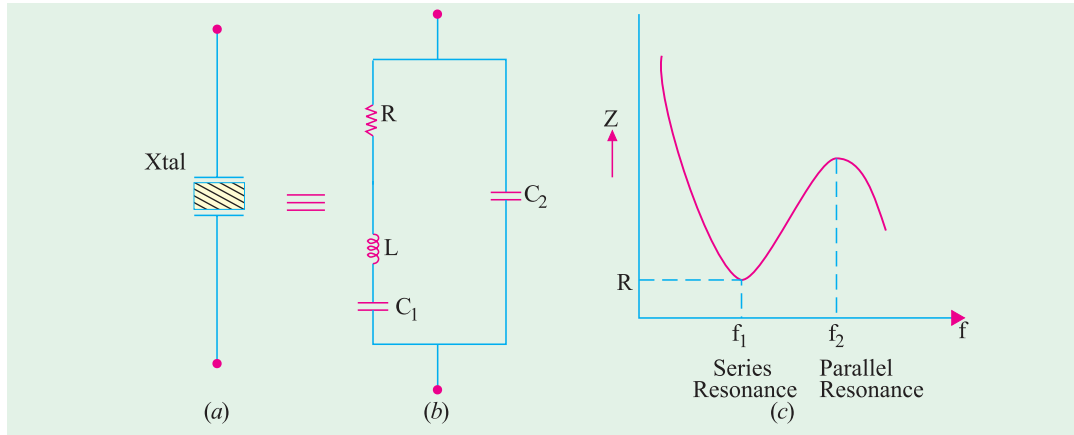


Fig. 65.15

The circuit has two resonant frequencies :

(i) one is the lower series resonance frequency  $f_1$  which occurs when  $X_L = X_{C1}$ . In that case,  $Z = R$  as shown in Fig. 65.15 (c).

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

(ii) the other is the parallel resonance frequency  $f_2$  which occurs when reactance of the series leg equals the reactances of  $C_2$ . At this frequency, the crystal offers very high impedance to the external circuit.

$$f_2 = \frac{1}{2\pi\sqrt{LC}}; \quad \text{where } C = \frac{C_1 C_2}{C_1 + C_2}$$

The impedance versus frequency graph of the crystal is shown in Fig. 65.15 (c). Crystals are available at frequencies of 15 kHz and above. However, at frequencies above 100 MHz, they become so small that handling them becomes a problem.

**(c) Q-factor**

The equivalent inductance of a crystal is very high as compared to either its equivalent capacitance or equivalent resistance. Because of high  $L/R$  ratio, the  $Q$ -factor of a crystal circuit is 20,000 as compared to a maximum of about 1000 for high-quality  $LC$  circuits. Consequently, greater frequency stability and frequency discrimination are obtained because of extremely high  $Q$  (upto  $10^6$ ) and high  $L/R$  ratio of the series  $RLC_1$  circuit.

**(d) Temperature Coefficient**

Temperature variations affect the resonant frequency of a crystal. The number of cycles change per million cycles for a  $1^\circ\text{C}$  change in temperature is called the **temperature coefficient** (TC) of the crystal. It is usually expressed in parts per million (ppm) per  $^\circ\text{C}$ . For example, a TC of 10 ppm per  $^\circ\text{C}$  means that frequency variation is 0.001 per cent\* per  $^\circ\text{C}$  change in temperature. It can also be expressed as 10 Hz/MHz/ $^\circ\text{C}$ . When kept in temperature-controlled ovens, crystal oscillators have frequency stability of about  $\pm 1$  ppm.

Usually, X-cut crystals have negative TC whereas Y-cut crystals have positive TC.

**Example 65.9.** A 600 kHz X-cut crystal when calibrated at  $50^\circ\text{C}$  has a TC of 20 ppm/ $^\circ\text{C}$ . What will be its resonant frequency when its temperature is raised to  $60^\circ\text{C}$ ?

**Solution.** A TC of 20 ppm/ $^\circ\text{C}$  is the same thing as 20 Hz/MHz/ $^\circ\text{C}$ . Since temperature rise is  $10^\circ\text{C}$  and original frequency is 600 kHz = 0.6 MHz, the total change as calculated by ratio propor-

\* Since 10 ppm means a change of 10 in  $10^6$ , hence percentage change is

$$= \frac{10}{10^6} \times 100 = 10^{-3} = 0.001$$



tion is =  $20 \times 10 \times 0.6 = 120$  Hz.

Since TC is negative, the new resonance frequency of the crystal is  
 =  $600,000 - 120 = 599,880 = \mathbf{599.88 \text{ kHz}}$ .

**Example 65.10.** A certain X-cut quartz crystal resonates at 450 kHz. It has an equivalent inductance of 4.2 H and an equivalent capacitance of 0.0297 pF. If its equivalent resistance is 60 Ω, calculate its Q-factor.

**Solution.** 
$$Q = \frac{\omega L}{R} = \frac{2\pi fL}{R}$$

$$= \frac{2\pi \times 450 \times 10^3 \times 4.2}{600} = \mathbf{19,790}$$

**Example 65.11.** The parameters of a crystal oscillator equivalent circuit are :  $L_1 = 0.8\text{H}$ ;  $C_1 = 0.08 \text{ pF}$ ,  $R = 5 \text{ k}\Omega$  and  $C_2 = 1.0 \text{ pF}$ . Determine the resonant frequencies  $f_1$  and  $f_2$

(UPSC Engg. Services 1999)

**Solution.** 
$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{0.8 \times 0.08 \times 10^{-12}}} = \frac{1}{2\pi \times 0.253 \times 10^{-6}}$$

$$= \frac{1}{1.5895 \times 10^{-6}} = 0.629 \times 10^6 \text{ Hz} = \mathbf{629 \text{ kHz}}$$

$$f_2 = \frac{1}{2\pi\sqrt{LC}} \text{ where } C = \frac{C_1 C_2}{C_1 + C_2} = \frac{0.08 \times 1}{0.08 + 1} = 0.074 \text{ pF}$$

$$\therefore f_2 = \frac{1}{2\pi\sqrt{0.8 \times 0.074 \times 10^{-12}}} = \frac{1}{2\pi \times 0.0243 \times 10^6} \text{ (Hz)}$$

$$f_2 = \frac{1}{2\pi\sqrt{0.8 \times 0.074 \times 10^{-12}}} = \frac{1}{2\pi \times 0.0243 \times 10^6} \text{ (Hz)}$$

$$= 0.654 \times 10^6 \text{ Hz} = \mathbf{654 \text{ kHz}}$$

**65.18. Crystal Controlled Oscillator**

Fig. 65.16, shows the use of a crystal to stabilise the frequency of a tuned-collector oscillator which has a crystal (usually quartz) in the feedback circuit.

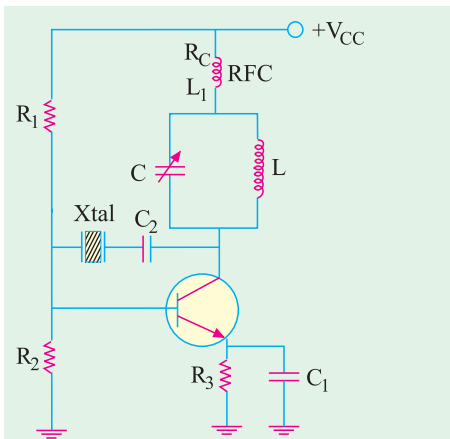


Fig. 65.16



Crystal Controlled oscillator

The LC tank circuit has a frequency of oscillation

$$f_0 = 1/2\pi\sqrt{LC}$$

The circuit is adjusted to have a frequency nearabout the desired operating frequency but the exact frequency is set by the crystal and stabilized by the crystal. For example, if natural frequency of vibration of the crystal is 27 MHz, the LC circuit is made to resonate at this frequency.

As usual, resistors  $R_1$ ,  $R_2$  and  $R_3$  provide a volt-

age-divider stabilised dc bias circuit. Capacitor  $C_1$  by-passes  $R_3$  in order to maintain large gain.  $RFC$  coil  $L_1$  prevents ac signals from entering dc line whereas  $R_C$  is the required dc load of the collector. The coupling capacitor  $C_2$  has negligible impedance at the operating frequency but prevents any dc link between collector and base. Due to extreme stability of crystal oscillations, such oscillators are widely used in communication transmitters and receivers where frequency stability is of prime importance.

**65.19. Transistor Pierce Crystal Oscillator**

A typical circuit originally suggested by Pierce is shown in Fig. 65.17. Here, the crystal is excited in the series-resonance mode because it is connected as a series element in the feedback path from collector to the base. Since, in series resonance, crystal impedance is the smallest, the amount of positive feedback is the largest. The crystal not only provides the feedback but also the necessary phase shift.

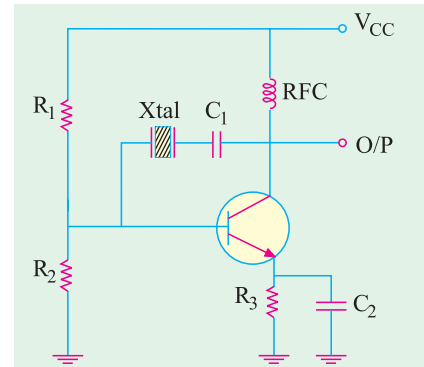


Fig. 65.17

As usual,  $R_1$ ,  $R_2$  and  $R_3$  provide a voltage-divider stabilized dc bias circuit.  $C_2$  bypasses  $R_3$  to avoid degeneration. The  $RFC$  coil provides dc collector load and also prevents any ac signal from entering the dc supply. The coupling capacitor  $C_1$  has negligible reactance at circuit operating frequency but **blocks any dc flow between collector and base**. The oscillation frequency equals the series-resonance frequency of the crystal and is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

— Fig 65.15 (b)

**Advantages**

1. It is a very simple circuit because no tuned circuit other than the crystal itself is required.
2. Different oscillation frequencies can be obtained by simply replacing one crystal with another. It makes it easy for a radio transmitter to work at different frequencies.
3. Since frequency of oscillation is set by the crystal, it remains unaffected by changes in supply voltage and transistor parameters etc.

**65.20. FET Pierce Oscillator**

It is shown in Fig. 65.18. Use of FET is desirable because its high input impedance results in light loading of the crystal which

1. results in good stability and
2. does not lower the  $Q$ -value.

The circuit shown in Fig. 65.18 is essentially a Colpitts oscillator in which

- (i) crystal has replaced the inductor and
- (ii) inherent FET junction capacitance provide the split capacitance as shown in the figure.

The circuit can be made to operate at different frequencies simply by plugging in different crystals between points A and B of the circuit.

The circuit oscillation frequency is given by the series-resonance frequency of the crystal (Art. 65.17)

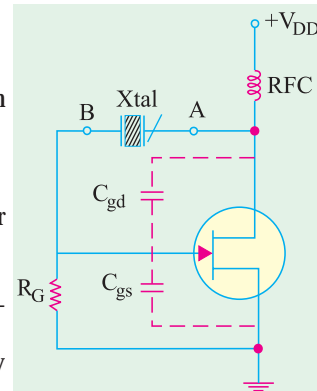


Fig. 65.18

**65.21. Phase Shift Principle**

Tuned circuits are **not an essential requirement for oscillation**. What is essential is that there should be a  $180^\circ$  phase shift around the feedback network\* and loop gain should be greater than unity. The  $180^\circ$  phase shift in the feedback signal can be achieved by using a suitable  $R-C$  network

\* Total phase shift required is  $360^\circ$ . However, the balance of  $180^\circ$  is provided by the active device of the amplifier itself.

consisting of three or four  $R$ - $C$  sections. The sine wave oscillators which use  $R$ - $C$  feedback network are called **phase-shift oscillators**.

### 65.22. RC Phase Shift Oscillator

Fig. 65.19, shows a transistor phase-shift oscillator which uses a three-section  $R$ - $C$  feedback network for producing a total phase shift of  $180^\circ$  (i.e.  $60^\circ$  per section) in the signal fed back to the base. Since  $CE$  amplifier produces a phase reversal of the input signal, total phase shift becomes  $360^\circ$  or  $0^\circ$  which is essential for **regeneration and hence for sustained oscillations**.

Values of  $R$  and  $C$  are so selected that each  $RC$  section produces a phase advance of  $60^\circ$ . Addition of a fourth section improves oscillator stability. It is found that phase shift of  $180^\circ$  occurs **only at one frequency** which becomes the oscillator frequency.

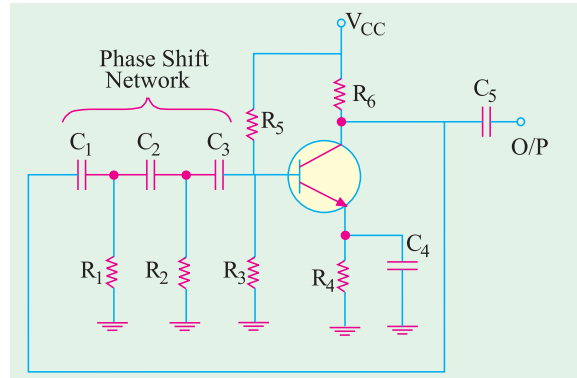


Fig. 65.19

#### (a) Circuit Action

The circuit is set into oscillations by any random or chance variation caused in the base current by

- (i) noise inherent in a transistor or
- (ii) minor variation in the voltage of the dc source.

This variation in the base current

1. is amplified in the collector circuit,
2. is then fed back to the  $RC$  network  $R_1C_1$ ,  $R_2C_2$  and  $R_3C_3$ ,
3. is reversed in phase by the  $RC$  network,
4. is next applied to the base in phase with initial change in base current,
5. and hence is used to sustain cycles of variations in collector current between saturation and cut-off values.

Obviously, the circuit will stop oscillating the moment phase shift differs from  $180^\circ$ .

As is the case with such transistor circuits (i) voltage divider  $R_5$  -  $R_6$  provides dc emitter-base bias, (ii)  $R_6$  controls collector voltage and (iii)  $R_4$ ,  $C_4$  provide temperature stability and prevent ac signal degeneration. The oscillator output voltage is capacitively coupled to the load by  $C_5$ .

#### (b) Frequency of Oscillation

The frequency of oscillation for the three-section  $RC$  oscillator when the three  $R$  and  $C$  components are equal is roughly given by

$$f_0 = \frac{1}{2\pi\sqrt{6} \cdot RC} \text{ Hz} = \frac{0.065}{RC} \text{ Hz}$$

Moreover, it is found that value of  $\beta$  is  $1/29$ . It means that amplifier gain must be more than 29 for oscillator operation.

#### (c) Advantages and Disadvantages

1. Since they do not require any bulky and expensive high-value inductors, such oscillators are well-suited for frequencies below 10 kHz.
2. Since only one frequency can fulfil Barkhausen phase-shift requirement, positive feedback occurs only for one frequency. Hence, pure sine wave output is possible.

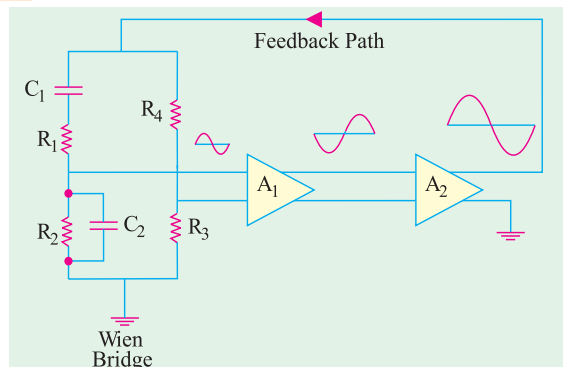


Fig. 65.20

3. It is not suited to variable frequency usage because a large number of capacitors will have to be varied. Moreover, gain adjustment would be necessary every time frequency change is made.
4. It produces a distortion level of nearly 5% in the output signal.
5. It necessitates the use of a high  $\beta$  transistor to overcome losses in the network.

**Example 65.12.** It is desired to design a phase-shift oscillator using a BJT and  $R = 10\text{ k}\Omega$ . Select the value of  $C$  for oscillator operation at 1 kHz.

**Solution.**  $f_0 = \frac{0.065}{RC}$  or  $C = \frac{0.065}{Rf_0} = \frac{0.065}{10\text{ K} \times 1\text{ KHz}}$   
 $= 0.0065 \times 10^{-6}\text{ F} = 6.5\text{ nF}$

### 65.23. Wien Bridge Oscillator

It is a low-frequency (5 Hz — 500 kHz), low-distortion, tunable, high-purity sine wave generator, often used in laboratory work. As shown in the block diagrams of Fig. 65.20 and Fig. 65.21, this oscillator uses two CE-connected RC-coupled transistor amplifiers and one RC-bridge (called Wien bridge) network to provide feedback. Here,  $Q_1$  serves as amplifier-oscillator and  $Q_2$  provides phase reversal and additional amplification. The bridge circuit is used to control the phase of the feedback signal at  $Q_1$ .

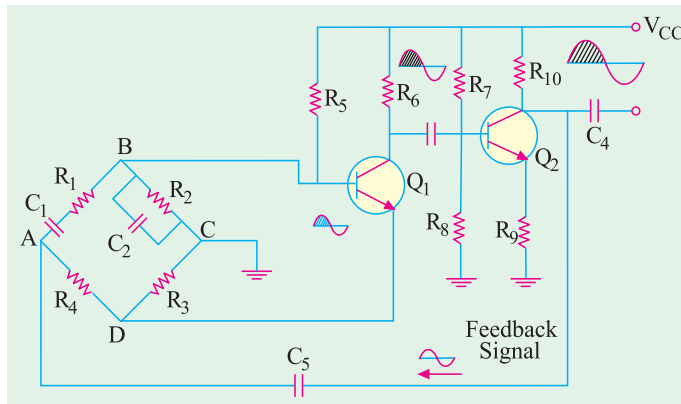


Fig. 65.21

#### (a) Phase Shift Principle

Any input signal at the base of  $Q_1$  appears in the amplified but phase-reversed form across collector resistor  $R_6$  (Fig. 65.21). It is further inverted by  $Q_2$  in order to provide a total phase reversal of  $360^\circ$  for positive feedback. Obviously, the signal at  $R_{10}$  is an amplified replica of the input signal at  $Q_1$  and is of the same phase since it has been inverted twice. We could feed this signal back to the base of  $Q_1$  directly to provide regeneration needed for oscillator operation. But because  $Q_1$  will amplify signals over a wide range of frequencies, direct coupling would result in poor frequency stability. By adding the Wien bridge, oscillator becomes sensitive to a signal of **only one particular frequency**. Hence, we get an oscillator of good frequency stability.

#### (b) Bridge Circuit Principle

It is found that the Wien bridge would become balanced at the signal frequency for which phase shift is exactly  $0^\circ$  (or  $360^\circ$ ),

The balance conditions are

$$\frac{R_4}{R_3} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}} \quad \text{or} \quad f_0 = \frac{1}{2\pi \sqrt{R_1 C_1 R_2 C_2}}$$

If  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$ , then  $f_0 = \frac{1}{2\pi RC}$  and  $\frac{R_4}{R_3} = 2$

#### (c) Circuit Action

Any random change in base current of  $Q_1$  can start oscillations. Suppose, the base current of  $Q_1$  is increased due to some reason. It is equivalent to applying a positive going signal to  $Q_1$ . Following sequence of events will take place :

1. An amplified but phase-reversed signal will appear at the collector of  $Q_1$ ;
2. A still further amplified and twice phase-reversed signal will appear at the collector of  $Q_2$ . Having been inverted twice, this output signal will be in phase with the input signal at  $Q_1$ ;
3. A part of the output signal at  $Q_2$  is fed back to the input points of the bridge circuit (point A-C). A part of this feedback signal is applied to emitter resistor  $R_3$  where it produces degenerative effect. Similarly, a part of the feedback signal is applied across base-bias resistor  $R_2$  where it produces regenerative effect.

At the rated frequency  $f_0$ , effect of **regeneration is made slightly more than that of degeneration** in order to maintain continuous oscillations.

By replacing  $R_3$  with a thermistor, amplitude stability of the oscillator output voltage can be increased.

#### (d) Advantages

Such a circuit has

1. highly stabilized amplitude and voltage amplification,
2. exceedingly good sine wave output,
3. good frequency stability.

**Example 65.13.** Calculate the resonant frequency of a Wien Bridge oscillator (shown in Fig. 65.21) when  $R = 10 \text{ k}\Omega$  and  $C = 2400 \text{ pF}$ .

**Solution.**  $f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10 \text{ K} \times 2400 \text{ pF}} = 6.63 \text{ kHz}$

**Example 65.14.** Design RC elements of a Wien Bridge oscillator (shown in Fig. 65.21), for operation at 2.5 kHz.

**Solution.** Using equal values of  $R_1$  and  $R_2$ , let us select  $R = 100 \text{ k}\Omega$  (you can choose any value in kilo ohms). Then from the relation,

$$f_0 = \frac{1}{2\pi RC} \quad \text{we get} \quad C = \frac{1}{2\pi f_0 R} = \frac{1}{2\pi \times 2.5 \text{ kHz} \times 100 \text{ K}}$$

or  $C = 636 \text{ pF}$

### 65.24. Non-sinusoidal Waveforms

Any waveform whose shape is different from that of a standard sine wave is called **non-sinusoidal waveform**. Examples are: square, rectangular, sawtooth, triangular waveforms and pulses as shown in Fig. 65.22.

#### (a) Pulses

Fig. 65.22 (a) shows a pulse train *i.e.* a stream of pulses at regular intervals. A pulse may, in general, be defined as a voltage or current that changes rapidly from one level of amplitude to another *i.e.* **it is an abrupt discontinuity in voltage or current**. These pulses are extensively used in digital electronics.

#### 1. Mark-to-Space Ratio (MSR)

$$MSR = \frac{\text{pulse width}}{\text{time between pulses}} = \frac{1 \mu\text{s}}{4 \mu\text{s}} = 0.25$$

— Fig. 65.22 (a)

Hence, mark-to-space ratio of the pulse shown in Fig. 65.22 (a) is 1 : 4.

This name has come from early morse-code transmission systems where a pulse was used to cause a pen to mark the paper.

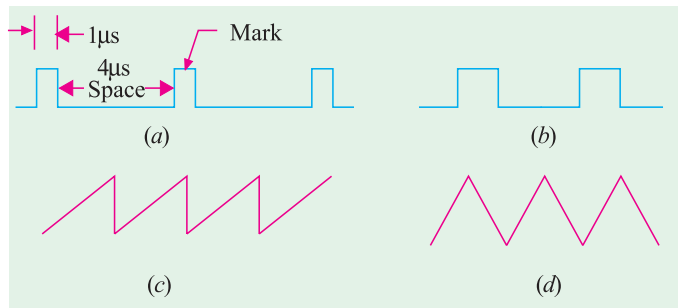


Fig. 65.22

### 2. Pulse Repetition Time (PRT)

It may be defined as *the time between the beginning of one pulse and that of the other*.

As seen from Fig. 65.22 (a),  $PRT = 5 \mu s$

### 3. Pulse Repetition Frequency (PRF)

It is given by *the number of pulses per second*.

$$PRF = \frac{1}{PRT} = \frac{1}{5 \mu s} = \frac{10^6}{5} = 200,000 \text{ Hz} = 200 \text{ kHz}$$

Pulse circuits find applications in almost all electronic-based industries. Various types of pulse code modulations are employed in communication systems whereas radars utilize pulses to track targets. Digital computers require circuits that can be switched very rapidly between two states by using appropriate pulses.

#### (b) Square Wave

It is shown in Fig. 65.22 (b) and is, in fact, a pulse waveform with a mark-to-space ratio of 1:1. Such square waves or pulses are used

1. for audio frequency note generation,
2. for digital electronic switching as in computers, 3, in radars,
4. as synchronizing pulses in TV,
5. for switching of high-power electronic circuits such as thyristor circuits.

#### (c) Sawtooth Wave

It is shown in Fig. 65.22 (c). Such waves are used

1. in the scanning circuits of cathode-ray tubes (CRT),
2. in timing circuits where the time for the wave to proceed from one level to another is measured, such as that produced in an integrating circuit.

#### (d) Triangular Wave

It is shown in Fig. 65.22 (d). Such waves are often used

1. in scanning circuits where a uniform left-to-right scan is required as in computer displays,
2. for audio frequency note generation,
3. in timing circuits for electronic applications.

## 65.25. Classification of Non-sinusoidal Oscillators

Those oscillators which generate waveforms other than sine waveform are called **non-sinusoidal oscillators** or **relaxation oscillators**. Non-sinusoidal waveforms include: square, rectangular, sawtooth and pulse-shaped waves as shown in Fig. 65.22.

A relaxation oscillator may be defined *as a circuit in which voltage or current changes abruptly from one value to another* and which continues to oscillate between these two values as long as dc power is supplied to it.

We will consider the following three types of such oscillators :

1. Sawtooth generators
2. Blocking oscillators
3. Multivibrators (MV)

## 65.26. Pulse Definitions

Due to capacitive effects in a transistor (or to circuit elements external to it), its **output does not directly follow its input**. For example, if we apply a square input pulse to its *E/B* junction, some amount of time lapses be-

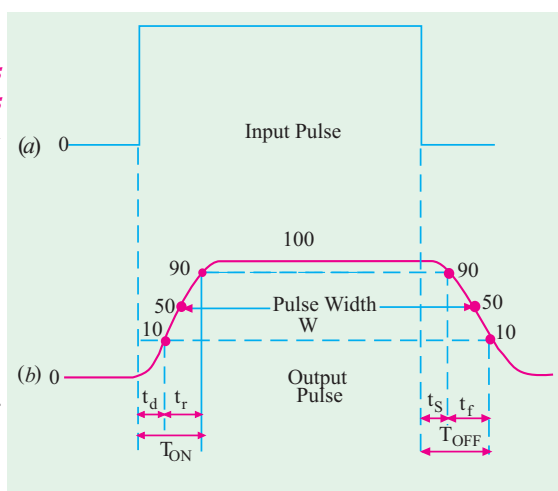


Fig. 65.23

fore  $I_C$  starts to rise. Similarly, when the input becomes zero, there is some time lapse before  $I_C$  starts to decrease. There is always some time delay between the application of input and change in the output. In order to measure how quickly the output changes *i.e.* in order to define the switching (*i.e.* OFF/ON) characteristics of a transistor, we will define the following few terms. It will be assumed that perfect square wave, as shown in Fig. 65.23 (a), has been applied at the input. The output wave and the various time delays are shown in Fig. 65.23 (b).

**1. Time delay,  $t_d$**

It is the time interval between the beginning of the input pulse and the time the output voltage (or current) reaches 10 per cent of its maximum value.

It depends on (i) depletion region capacitances, (ii) turn-on base current and (iii) value of transistor  $\beta$ .

**2. Rise time,  $t_r$**

It is the time taken by the output voltage (or current) to rise from 10% to 90% of its maximum value.

It primarily depends on diffusion capacitance  $C_D$  of the transistor (Art. 1.4).

**3. Turn-on time  $T_{ON}$**

It is equal to the sum of the delay time and rise time *i.e.*  $T_{ON} = t_d + t_r$

**4. Storage time,  $T_s$**

It is the time interval between the end of the input pulse (trailing edge) and the time when output voltage (or current) falls to 90% of its initial maximum value.

It depends on the degree of saturation. Deeper the transistor is driven into saturation, more the stored charge that has to be removed and hence longer the storage time. That is why non-saturated switching is often preferred.

**5. Fall time,  $t_f$**

It is the time interval during which the output voltage (or current) falls from 90% of its maximum value to 10%.

In simple words, it is the time interval between 90% and 10% levels of the output pulse.

**6. Turn-off time,  $T_{OFF}$**

It is equal to the sum of storage time and fall time *i.e.*  $T_{OFF} = t_s + t_f$

For a fast switching transistor,  $T_{ON}$  and  $T_{OFF}$  must be of the order of nanoseconds.

**7. Pulse Width,  $W$**

It is the time duration of the output pulse measured between two 50% levels of the rising and falling waveform.

### 65.27. Basic Requirements of a Sawtooth Generator

The essential requirements of a sawtooth generator are :

1. a dc power source,
2. a switching device (neon tube, thyratron, thyristor, UJT etc.),
3. a capacitor,
4. a resistor.

#### Circuit Action

The  $V/I$  characteristics of the  $RC$  circuit play an important role in the operation of such a generator. By restricting the time interval equal to the time constant  $\lambda = CR$ , only the rising portion of the characteristic ( $OA$  in Fig. 65.24) which is almost a straight line, is utilized. For periods of time greater than  $\lambda$ , the rising portion of the characteristic is no longer a straight line and hence cannot be utilized.

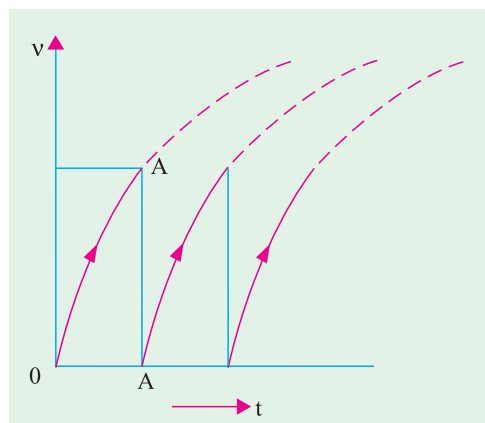


Fig. 65.24

The frequency of the wave is given by the reciprocal of time which elapses between the two waves. In Fig. 65.24,  $f = 1/\lambda$ .

**65.28. UJT Sawtooth Generator**

The circuit is shown in Fig. 65.25. It consists of a power source, a unijunction transistor and an R-C network.

**Circuit Action**

When S is initially closed, following chain of events takes place :

1. a small current is set up through  $R_2$  and  $R_1$  via  $B_2$  and  $B_1$  and an initial reverse bias is established across the  $E/B_1$  junction;
2. at the same time, C begins to get charged through  $R_E$  and voltage across it increases exponentially with time towards the target voltage V;
3. when capacitor voltage equals the emitter firing (or peak point) voltage  $V_p$ ,  $E/B_1$  junction becomes forward-biased and the emitter goes into the negative region of its characteristic;
4. being forward-biased,  $E/B_1$  junction offers very low resistance. Hence, C starts discharging through  $B_1$  and  $R_1$  at a rate determined mainly by  $E/B_1$  junction resistance and  $R_1$ ;
5. as capacitor voltage approaches zero, the  $E/B_1$  junction again becomes reverse-biased and so stops conducting;
6. we revert to the initial state where C begins to charge and the whole cycle of circuit actions is repeated.

The emitter voltage waveform is shown in Fig. 65.25 (b). As seen,  $V_E$  rises exponentially towards the target voltage V but drops to a very low value after it reaches the value  $V_p$  due to sudden conduction through  $E/B_1$  junction. Since  $R_E$  is large (10 K or so) charging rate is comparatively slow but discharge is much quicker since  $R_1$  is very small (50  $\Omega$  or less). **This slow charge and fast discharge produces a sawtooth wave.**

The time required for  $v_E$  to rise to  $V_p$  is given by

$$T = kRC \quad \text{where } k = \log_e^{1/(1-\eta)} \text{ and}$$

$$\eta = \text{intrinsic stand-off ratio} = \frac{R_{B1}}{R_{B1} + R_{B2}} \quad \dots \text{Art 64.2}$$

Here,  $R_{B1}$  and  $R_{B2}$  are the internal inter-base resistances.

The frequency of oscillation of the UJT or of the output sawtooth wave is

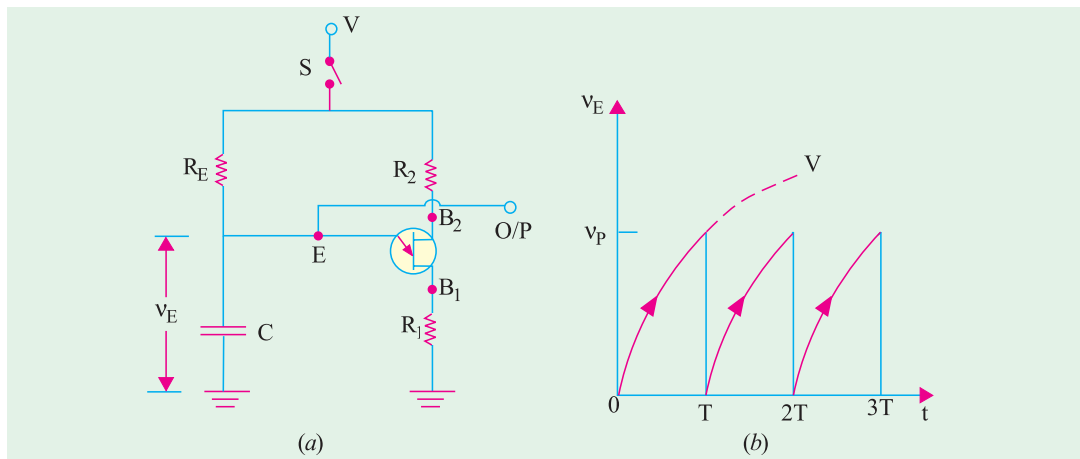


Fig. 65.25

$$f_0 = \frac{1}{T} = \frac{1}{kRC}$$



Its amplitude is determined primarily by applied voltage  $V$  and  $V_P$ .

### Applications

Sawtooth voltage waves are commonly used as

1. sweep voltages at the picture tubes of TV receivers,
2. as sweep voltages of the viewing screens of oscilloscopes and radar equipment.

**Example 65.15.** For the UJT oscillator circuit shown in Fig. 65.25,  $R_E = 10\text{ K}$ ,  $\eta = 0.75$ . If the required oscillator frequency is 1 kHz, find the value of  $C$ .

(Components and Devices, Pune Univ.)

**Solution.**  $k = \log_e 1/(1 - 0.75) = \log_e 4 = 2.3 \log_{10} 4 \cong 1.4$

$$\text{Now, } f_0 = \frac{1}{kRC} \quad \therefore \quad 1 \times 10^3 = \frac{1}{1.4 \times 10 \times 10^3 \times C} \quad \text{or} \quad C = 0.07 \mu\text{F}$$

### 65.29. Multivibrators (MV)

These devices are very useful as pulse generating, storing and counting circuits. They are basically **two-stage amplifiers with positive feedback from the output of one amplifier to the input of the other**. This feedback (Fig. 65.26) is supplied in such a manner that one transistor is driven to saturation and the other to cut-off. It is followed by new set of conditions in which the saturated transistor is driven to cut-off and the cut-off transistor is driven to saturation.

There are three basic types of MVs distinguished by the type of coupling network employed.

1. astable multivibrator (AMV),
2. monostable multivibrator (MMV),
3. bistable multivibrator (BMV).

The first one is the *non-driven type* whereas the other two are the *driven type* (also called triggered oscillators).

#### 1. Astable Multivibrator (AMV)

It is also called **free-running relaxation oscillator**. It has no stable state but only two quasi-stable (half-stable) states between which it keeps oscillating continuously of its own accord without any external excitation.

In this circuit, neither of the two transistors reaches a stable state. When one is ON, the other is OFF and they continuously switch back and forth at a rate depending on the  $RC$  time constant in the circuit. Hence, it oscillates and produces pulses of certain mark-to-space ratio. Moreover, two outputs ( $180^\circ$  out of phase with each other) are available.

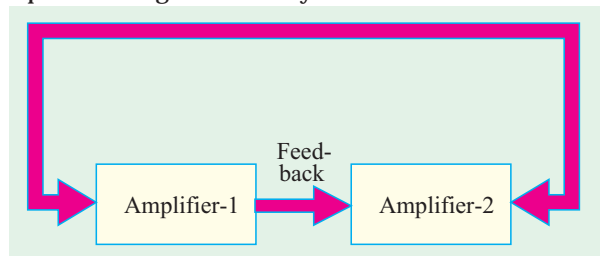


Fig. 65.26

It has two energy-storing elements *i.e. two capacitors*.

#### 2. Monostable Multivibrator (MMV)

It is also called a **single-shot** or **single swing** or a **one-shot** multivibrator. Other names are: **delay multivibrator and univibrator**. It has

(i) one absolutely stable (stand-by) state and (ii) one quasi-stable state.

It can be switched to the quasi-stable state by an external trigger pulse but it returns to the stable condition after a time delay determined by the value of circuit components. It supplies a single output pulse of a desired duration for every input trigger pulse.



Multivibrator

It has one energy-storing element *i.e.* **one-capacitor**.

### 3. Bistable Multivibrator (BMV)

It is also called Eccles-Jordan or flip-flop multivibrator. It has **two absolutely stable states**. It can remain in either of these two states unless an external trigger pulse switches it from one state to the other. Obviously, **it does not oscillate**. It has **no energy storage element**.

Detailed discrete circuits for the above MVs are discussed below after listing their uses.

### 65.30. Uses of Multivibrators

Some of their uses are :

1. as frequency dividers,
2. as sawtooth generators,
3. as square wave and pulse generators,
4. as a standard frequency source when synchronized by an external crystal oscillator,
5. for many specialised uses in radar and TV circuits,
6. as memory elements in computers.

### 65.31. Astable Multivibrator

Fig. 65.27 shows the circuit of a symmetrical collector-coupled AMV using two similar transistors. It, in fact, consists of two *CE* amplifier stages, each providing a feedback to the other. The feedback ratio is unity and positive because of  $180^\circ$  phase shift in each stage. Hence, the circuit oscillates. Because of the very strong feedback signal, the transistors are driven either to saturation or to cut-off (they do not work on the linear region of their characteristics).

The transistor  $Q_1$  is forward-biased by  $V_{CC}$  and  $R_1$  whereas  $Q_2$  is forward-biased by  $V_{CC}$  and  $R_2$ . The collector-emitter voltages of  $Q_1$  and  $Q_2$  are determined respectively by  $R_{L1}$  and  $R_{L2}$  together with  $V_{CC}$ . The output of  $Q_1$  is coupled to the input of  $Q_2$  by  $C_2$  whereas output of  $Q_2$  is coupled to  $Q_1$  by  $C_1$ .

Note that it is not essential to draw the coupling leads at  $45^\circ$  to the vertical as shown but it is usually done because it helps to identify the circuit immediately as MV.

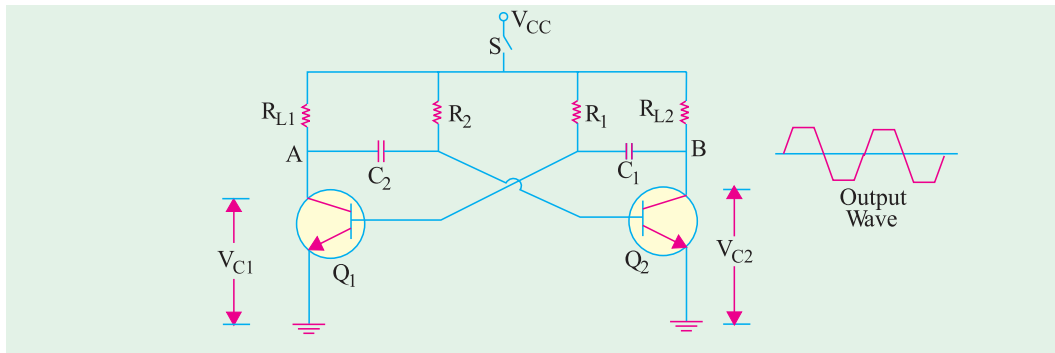


Fig. 65.27

The output can be taken either from point *A* or *B* though these would be phase-reversed with respect to each other as shown in Fig. 65.27.

### Circuit Operation

The circuit operation would be easy to understand if it is remembered that due to feedback (i) when  $Q_1$  is ON,  $Q_2$  is OFF and (ii) when  $Q_2$  is ON,  $Q_1$  is OFF.

When the power is switched on by closing *S*, one of the transistors will start conducting before the other does (or slightly faster than the other). It is so because characteristics of no two seemingly similar transistors can be exactly alike. Suppose that  $Q_1$  starts conducting before  $Q_2$  does. The feedback system is such that  $Q_1$  will be very rapidly driven to saturation and  $Q_2$  to cut-off.

The following sequence of events will occur :

1. Since  $Q_1$  is in saturation, whole of  $V_{CC}$  drops across  $R_{L1}$ . Hence,  $V_{C1} = 0$  and point  $A$  is at zero or ground potential.
2. Since  $Q_2$  is in cut-off *i.e.* it conducts no current, there is no drop across  $R_{L2}$ . Hence, point  $B$  is at  $V_{CC}$ .
3. Since  $A$  is at 0 V,  $C_2$  starts to charge through  $R_2$  towards  $V_{CC}$ .
4. When voltage across  $C_2$  rises sufficiently (*i.e.* more than 0.7 V), it biases  $Q_2$  in the forward direction so that it starts conducting and is soon driven to saturation.
5.  $V_{C2}$  decreases and becomes almost zero when  $Q_2$  gets saturated. The potential of point  $B$  decreases from  $V_{CC}$  to almost 0 V. This potential decrease (negative swing) is applied to the base of  $Q_1$  through  $C_1$ . Consequently,  $Q_1$  is pulled out of saturation and is soon driven to cut-off.
6. Since, now, point  $B$  is at 0 V,  $C_1$  starts charging through  $R_1$  towards the target voltage  $V_{CC}$ .
7. When voltage of  $C_1$  increases sufficiently,  $Q_1$  becomes forward-biased and starts conducting. In this way, the whole cycle is repeated.

It is seen that the circuit alternates between a state in which  $Q_1$  is ON and  $Q_2$  is OFF and a state in which  $Q_1$  is OFF and  $Q_2$  is ON. The time in each state depends on  $RC$  values. Since each transistor is driven alternately into saturation and cut-off the voltage waveform at either collector (points  $A$  and  $B$  in Fig. 65.27) is essentially a square waveform with a peak amplitude equal to  $V_{CC}$  (Fig. 65.28).

### Switching Times

It can be proved that off-time for  $Q_1$  is  $T_1 = 0.69 R_1 C_1$  and that for  $Q_2$  is  $T_2 = 0.69 R_2 C_2$ .

Hence, total time-period of the wave is

$$T = T_1 + T_2 = 0.69 (R_1 C_1 + R_2 C_2)$$

If  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$  *i.e.* the two stages are symmetrical, then  $T = 1.38 RC$

### Frequency of Oscillation

It is given by the reciprocal of time period,

$$\therefore f = \frac{1}{T} = \frac{1}{1.38 RC} = \frac{0.7}{RC}$$

### Minimum Values of $\beta$

To ensure oscillations, the transistors must saturate for which minimum values of  $\beta$  are as under :

$$\beta_1 = \frac{R_1}{R_{L1}} \quad \text{and} \quad \beta_2 = \frac{R_2}{R_{L2}}$$

$$\text{If } R_1 = R_2 = R \text{ and } R_{L1} = R_{L2} = R_L \text{ then } \beta_{\min} = \frac{R}{R_L}$$

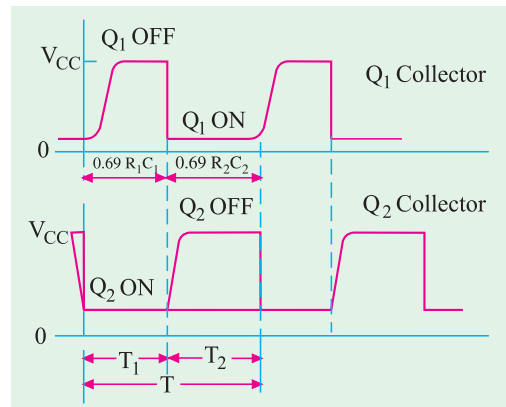


Fig. 65.28

**Example 65.16.** Determine the period and frequency of oscillation for an astable multivibrator with component values:  $R_1 = 2 \text{ K}$ ,  $R_2 = 20 \text{ K}$ ,  $C_1 = 0.01 \mu\text{F}$  and  $C_2 = 0.05 \mu\text{F}$ .

$$\begin{aligned} \text{Solution. } T_1 &= 0.69 \times 2 \text{ K} \times 0.01 \mu\text{F} = 13.8 \mu\text{s} \\ \text{and } T_2 &= 0.69 \times 20 \text{ k} \times 0.05 \mu\text{F} = 690 \mu\text{s} \\ \therefore T &= T_1 + T_2 = 13.8 \mu\text{s} + 690 \mu\text{s} = 703.8 \mu\text{s} \end{aligned}$$

$$\therefore f_0 = \frac{1}{703.8 \mu\text{s}} = 1.42 \text{ kHz}$$

**Example 65.17.** In the AMV circuit of Fig. 65.27,  $R_1 = R_2 = 10\text{ K}$ ,  $C_1 = C_2 = 0.01\ \mu\text{F}$  and  $R_{L1} = R_{L2} = 1\text{ K}$ . Find

- (a) frequency of circuit oscillation, (b) minimum value of transistor  $\beta$ .

(Digital Electronics, Bombay Univ.)

**Solution.** (a)  $T_1 = T_2 = 0.69 \times 10 \times 10^3 \times 0.01 \times 10^{-6} = 69\ \mu\text{s}$

$$\therefore T = T_1 + T_2 = 2 \times 69 = 138\ \mu\text{s} \quad \therefore$$

$$f_0 = \frac{1}{138 \times 10^{-6}} = 7.25\ \text{kHz}$$

(b)  $\beta_{\min} = \frac{R}{R_L} = \frac{10 \times 10^3}{1 \times 10^3} = 10$

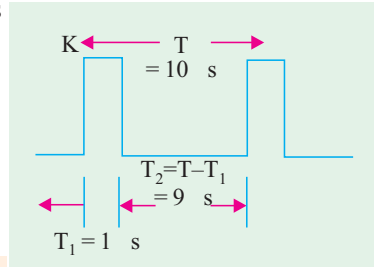


Fig. 65.29

**Example 65.18.** Determine the value of capacitors to be used in an astable multivibrator to provide a train of pulses  $1\ \mu\text{s}$  wide at a repetition rate of  $100\ \text{kHz}$ . Given  $R_1 = R_2 = 10\ \text{K}$ .

**Solution.** Fig. 65.29 shows the waveform to be generated by the astable multivibrator. Note that the desired pulse width is actually the time interval  $T_1 = 1\ \mu\text{s}$  and the time period  $T$  which equals  $10\ \mu\text{s}$  (i.e.  $1/100\ \text{kHz}$ ) is the desired repetition time.

Now  $T_1 = 0.69 R_1 C_1, \quad \therefore C_1 = \frac{1\ \mu\text{s}}{0.69 \times 10\text{K}} = 145\text{pF}$

and  $T_2 = 0.69 R_2 C_2, \quad \therefore C_2 = \frac{9\ \mu\text{s}}{0.69 \times 10\text{K}} = 1304\ \text{pF}$

### 65.32. Monostable Multivibrator (MMV)

A typical MMV circuit is shown in Fig. 65.30. Here,  $Q_1$  is coupled to  $Q_2$  base as in an AMV but the other coupling is different. In this multivibrator, a single narrow input trigger pulse produces a single rectangular pulse whose amplitude, pulse width and wave shape depend upon the values of circuit components rather than upon the trigger pulse.

#### Initial Condition

In the absence of a triggering pulse at  $C_2$  and with  $S$  closed,

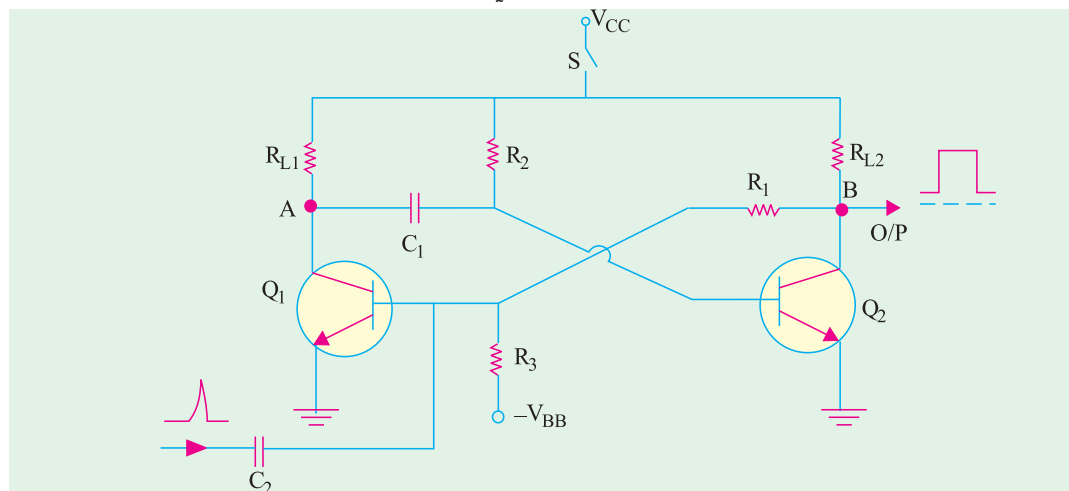


Fig. 65.30

1.  $V_{CC}$  provides reverse bias for  $C/B$  junctions of  $Q_1$  and  $Q_2$  but forward-bias for  $E/B$  junction of  $Q_2$  only. Hence,  $Q_2$  conducts at saturation.
2.  $V_{BB}$  and  $R_3$  reverse bias  $Q_1$  and keep it cut off.
3.  $C_1$  charges to nearly  $V_{CC}$  through  $R_{L1}$  to ground by the low-resistance path provided by saturated  $Q_2$ .

As seen, the initial stable state is represented by

- (i)  $Q_2$  conducting at saturation and (ii)  $Q_1$  cut-off.

### When Trigger Pulse is Applied

When a trigger pulse is applied to  $Q_1$  through  $C_2$ ,  $MMV$  will switch to its opposite unstable state where  $Q_2$  is cut-off and  $Q_1$  conducts at saturation. The chain of circuit actions is as under :

1. If positive trigger pulse is of sufficient amplitude, it will override the reverse bias of the  $E/B$  junction of  $Q_1$  and give it a forward bias. Hence,  $Q_1$  will start conducting.
2. As  $Q_1$  conducts, its collector voltage falls due to voltage drop across  $R_{L1}$ . It means that potential of point  $A$  falls (negative-going signal). This negative-going voltage is fed to  $Q_2$  via  $C_1$  where it decreases its forward bias.
3. As collector current of  $Q_1$  starts decreasing, potential of point  $B$  increases (positive-going signal) due to lesser drop over  $R_{L2}$ . Soon,  $Q_2$  comes out of conduction.
4. The positive-going signal at  $B$  is fed via  $R_1$  to the base of  $Q_1$  where it increases its forward bias further. As  $Q_1$  conducts more, potential of point  $A$  approaches 0 V.
5. This action is cumulative and ends with  $Q_1$  conducting at saturation and  $Q_2$  cut-off.

### Return to Initial Stable State

1. As point  $A$  is at almost 0 V,  $C_1$  starts to discharge through saturated  $Q_1$  to ground.
2. As  $C_1$  discharges, the negative potential at the base of  $Q_2$  is decreased. As  $C_1$  discharges further,  $Q_2$  is pulled out of cut-off.
3. As  $Q_2$  conducts further, a negative-going signal from point  $B$  via  $R_1$  drives  $Q_1$  into cut-off.

Hence, the circuit reverts to its original state with  $Q_2$  conducting at saturation and  $Q_1$  cut-off. It remains in this state till another trigger pulse comes along when the entire cycle repeats itself.

As shown in Fig. 65.30, the output is taken from the collector of  $Q_2$  though it can also be taken from point  $A$  of  $Q_1$ . The width of this pulse is determined by the time constant of  $C_1 R_2$ . Since this MV produces one output pulse for every input trigger pulse it receives, it is called **mono or one-shot multivibrator**:

The width or duration of the pulse is given by  $T = 0.69 C_1 R_2$

It is also known as the one-shot period.

### Uses

1. The falling part of the output pulse from  $MMV$  is often used to trigger another pulse generator circuit thus producing a pulse delayed by a time  $T$  with respect to the input pulse.
2.  $MMV$  is used for regenerating or rejuvenating old and worn out pulses. Various pulses used in computers and telecommunication systems become somewhat distorted during use. An  $MMV$  can be used to generate new, clean and sharp pulses from these distorted and used ones.

**Example 65.19.** A 20 kHz, 75% duty cycle square ( $t_p$ ) wave is used to trigger continuously, a monostable multivibrator with a triggered pulse duration of  $5\mu s$ . What will be the duty cycle of the waveform at output ( $B$ ) of the monostable multivibrator (refer to Fig. 65.30).

**Solution.** Time period of the square wave

$$T = \frac{1}{f} = \frac{1}{20 \text{ kHz}} = 50 \mu\text{s}$$

Since the duty cycle of the square wave is 75%, therefore the time interval during which the input waveform is at a higher voltage level is,  $0.75 \times 50 \mu\text{s} = 37.5 \mu\text{s}$ . Fig. 65.31 (a) shows a sketch of the input waveform which is used to trigger the monostable multivibrator.

Now the monostable multivibrator is triggered once each time a new pulse arrives. The monostable multivibrator remains triggered only for a duration,  $t_p = 5 \mu\text{s}$ . A sketch of the waveform at the output (B) of the monostable multivibrator is as shown in Fig. 65.31(b).

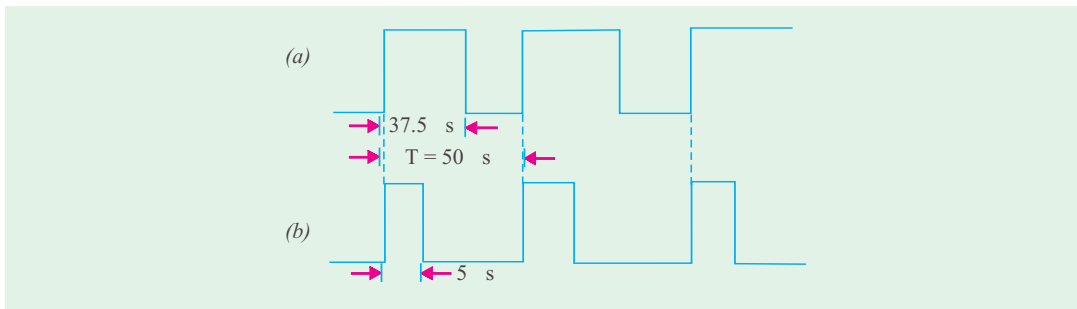


Fig. 65.31

**Example 65.20.** A monostable multivibrator is required to convert a 100 kHz, 30% duty cycle square wave to a 100 kHz, 50% duty cycle square wave. Find the values of  $R_2$  and  $C_2$ .

**Solution.** Fig. 65.32 (a) shows a sketch of the input waveform. In order to convert it to a square waveform with 50% duty cycle, we want that the monostable multivibrator must remain triggered for 50% of the time period.

Now time period of 100 kHz square wave,

$$T = \frac{1}{f} = \frac{1}{100 \text{ kHz}} = 10 \mu\text{s}$$

$\therefore$  The duration for which the monostable multivibrator must remain triggered =  $50\% \times 10 \mu\text{s} = 5 \mu\text{s}$ .

Fig. 65.32 (b) shows a sketch of the required waveform at the output of the monostable multivibrator.

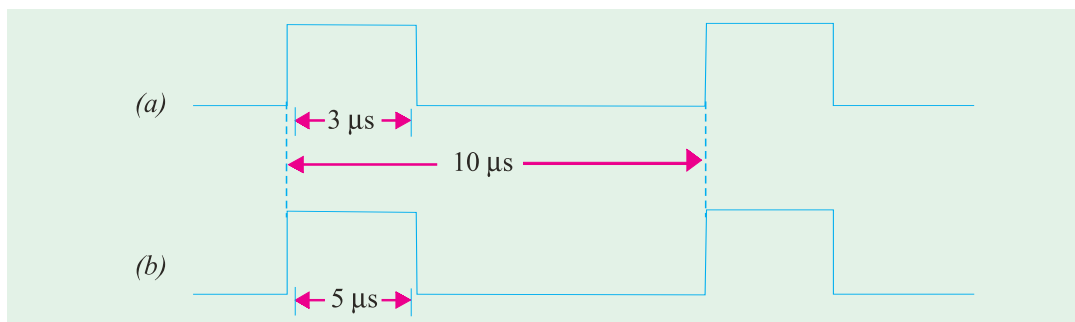


Fig. 65.32

Now width or duration of the pulse,

$$T = 0.69 C_1 R_2$$

Let us select  $C_1 = 0.001 \mu\text{F}$ , then from the above equation.

\* You can choose any value of  $C$  which could result in the value of  $R_2$  in kilohms. The reason for this is that we want the current in the circuit to be limited to few milliamperes.

$$R_2 = \frac{T}{0.69 C_1} = \frac{5 \mu\text{s}}{0.69 \times 0.001 \mu\text{F}} = 7.2 \text{ K}$$

### 65.33. Bistable Multivibrator (BMV)

The basic circuit is shown in Fig. 65.33. As stated earlier, it has **two absolutely stable** states. It can stay **in one of its two states indefinitely** (as long as power is supplied) changing to the other state only when it receives a trigger pulse from outside. When it receives another triggering pulse, only then it goes back to its original state. Since one trigger pulse causes the *MV* to 'flip' from one state to another and the next pulse causes it to '**flop**' back to its original state, the *BMV* is also popularly known as '**flip-flop**' circuit.

The *BMV* circuit shown in Fig. 65.33 differs from the *AMV* circuit of Fig. 65.27 in the following respects :

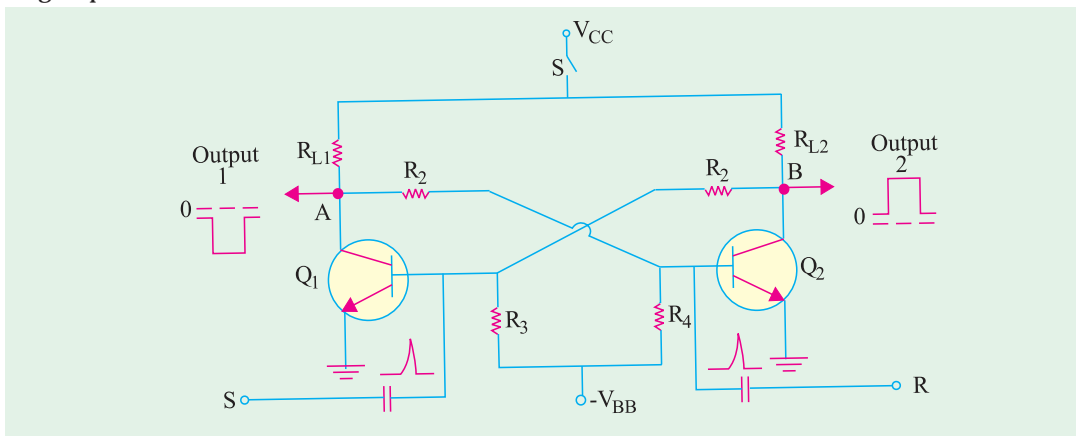


Fig. 65.33

1. the base resistors are not joined to  $V_{CC}$  but to a common source  $-V_{BB}$ ,
2. the feedback is coupled through two resistors (not capacitors).

#### Circuit Action

If  $Q_1$  is conducting, then the fact that point *A* is at nearly 0 V makes the base of  $Q_2$  negative (by the potential divider  $R_2 - R_4$ ) and holds  $Q_2$  off.

Similarly, with  $Q_2$  OFF, the potential divider from  $V_{CC}$  to  $-V_{BB}$  ( $R_{L2}, R_1, R_3$ ) is designed to keep base of  $Q_1$  at about 0.7 V ensuring that  $Q_1$  conducts. It is seen that  $Q_1$  holds  $Q_2$  OFF and  $Q_3$  holds  $Q_1$  ON.

Suppose, now, a positive pulse is applied momentarily to *R*, it will cause  $Q_2$  to conduct. As collector of  $Q_2$  falls to zero, it cuts  $Q_1$  OFF and, consequently, the *BMV* switches over to its other state.

Similarly, a positive trigger pulse applied to *S* will switch the *BMV* back to its original state.

#### Uses

1. in timing circuits as a frequency divider,
2. in counting circuits,
3. in computer memory circuits.

### 65.34. Schmitt Trigger

The Schmitt trigger (after the name of its inverter) is a **binary circuit** and **closely resembles an MV**. It has two bistable states and the magnitude of the input voltage determines which of the two is possible. It is also called emitter-coupled binary oscillator because positive feedback occurs by coupling through emitter resistor  $R_E$ .

### The Quiescent Condition

As shown in Fig. 65.34, it consists of two similar transistors  $Q_1$  and  $Q_2$  coupled through  $R_E$ . Resistors  $R_1$ ,  $R_3$  and  $R_4$  form a voltage divider across  $V_{CC}$  and  $-V_{BB}$  which places a small positive voltage (forward bias) on the base of  $Q_2$ . Hence, when power is first switched ON,  $Q_2$  starts conducting. The flow of its current through  $R_E$  places a small reverse bias on the base of  $Q_1$ , thereby cutting it OFF. Consequently, collector of  $Q_1$  rises to  $V_{CC}$ . This positive voltage, coupled to the base of  $Q_2$  through  $R_3$ , drives  $Q_2$  into saturation and holds it there.

Hence, in the initial static or quiescent condition of the Schmitt trigger,

1.  $Q_2$  is in saturation,
2.  $Q_1$  is cut-off,
3. collector of  $Q_2$  is at 0 V,
4. collector of  $Q_1$  is at  $V_{CC}$ .

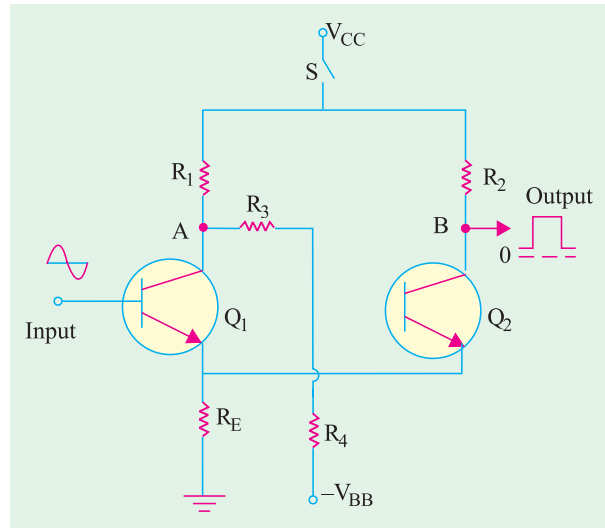


Fig. 65.34

### Circuit Action

Suppose, positive half-cycle of the input ac voltage is applied to the trigger input first. Let us further suppose that this positive voltage is sufficient to overcome the reverse bias on the base of  $Q_1$  placed there by the voltage drop across  $R_E$ . Then, the chain of events that follows is as under :

1.  $Q_1$  comes out of cut-off and starts to conduct;
2. as it does so, its collector voltage drops (swings negative);
3. this negative-swinging voltage coupled to the base of  $Q_2$  via  $R_3$  reduces its forward bias and hence its emitter current;
4. with reduced emitter current, voltage drop across  $R_E$  is reduced;
5. consequently, reverse bias of  $Q_1$  is further lowered and it conducts more heavily;
6. as a result, collector voltage of  $Q_1$  falls further, thereby driving  $Q_2$  still closer to cut-off.

This process is cumulative and ends up with

- (a)  $Q_1$  conducting at saturation with its collector voltage almost zero;
- (b)  $Q_2$  becoming cut-off with its collector voltage nearly  $V_{CC}$ .

### Negative Half-cycle of the Input Voltage

Now, when the negative half-cycle of the input voltage is applied

1.  $Q_1$  becomes reverse-biased. Consequently, its collector current falls and collector voltage rises (i.e. potential of point A increases towards  $V_{CC}$ );
2. this positive-swinging voltage is coupled to the base of  $Q_2$  through  $R_3$  and, as a result,  $Q_2$  is driven to saturation;
3. this re-establishes the original conditions of
  - (a)  $Q_1$  cut off with collector voltage at  $V_{CC}$  and
  - (b)  $Q_2$  at saturation with collector voltage at 0 V.



It completes one cycle. This cycle is repeated as the input voltage rises and falls again. Hence, each cycle of the Schmitt trigger produces a positive-going pulse at its output which is taken out from the collector of  $Q_2$  i.e. from point  $B$  in Fig. 65.34.

### Output Pulse Width

It depends on the time during which  $Q_2$  is conducting. It, in turn, depends on the input voltage, within the limits imposed by emitter resistor  $R_E$ .

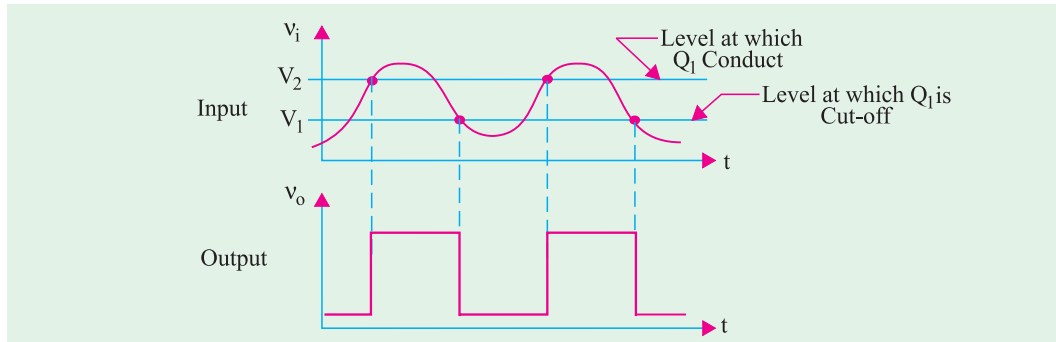


Fig. 65.35

### Uses

1. It is frequently used for wave-shaping purposes. As shown in Fig. 65.35, it can convert inputs with any waveshape into output pulses having rectangular or square waveshapes. That is why Schmitt trigger is often called a 'squaring' circuit or a 'squarer' circuit.
2. It can reshape worn-out pulses by giving them sharp leading and trailing edges.
3. Since a change of state occurs whenever the input crosses a trigger point, the Schmitt trigger is often used as a **level detector** i.e. as a pulse height discriminator.

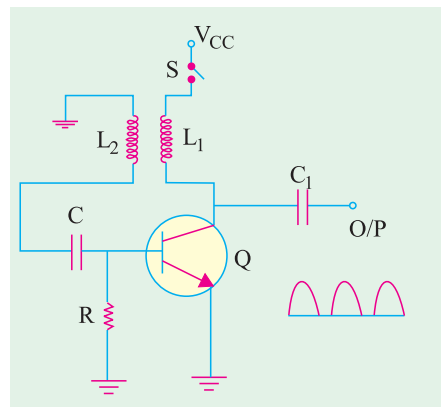


Fig. 65.36

### 65.35. Transistor Blocking Oscillator

The basic circuit is shown in Fig. 65.36. When  $S$  is closed, base current rises rapidly due to forward-bias placed by  $V_{CC}$  on the  $E/B$  junction of  $Q$ . It causes a corresponding increase in its collector current. This rising flow of  $I_C$  through  $L_1$  produces an induced *e.m.f.* in  $L_2$ . Coupling between  $L_1$  and  $L_2$  is such that lower end of  $L_2$  becomes positive and the grounded end negative.

The positive voltage from  $L_2$  is applied to the base of  $Q$  through  $C$ . It further increases the forward-bias of the  $E/B$  junction which leads to further increase in  $I_C$ . Since this process is cumulative,  $Q$  is quickly driven to saturation. At that point, there is no further increase in  $I_C$  and hence, no induced *e.m.f.* in  $L_2$  to be applied to  $Q$ .

Now,  $C$  which had been charged earlier, places a negative charge on the base of  $Q$  which reverse-biases its  $E/B$  junction and ultimately drives it to cut-off.

The transistor remains at cut-off as  $C$  now starts to discharge through  $R$ . When sufficient amount of charge leaks off  $C$  so that reverse bias of  $E/B$  junction is removed and forward bias is re-established,  $Q$  comes out of cut-off and its collector current starts rising once again. Then, the entire cycle of operation is repeated. As shown, the output consists of sharp and narrow pulses.

Tutorial Problems No. 65.1

1. A tuned collector oscillator circuit is tuned to operate at 22 kHz by a variable capacitor set to 2 nF. Find the value of tuned circuit inductance. **(0.026 H)**
2. A tuned collector oscillator operates at 2.2 MHz frequency. At what frequency will it work if its tuned circuit capacitance is reduced by 50% ? **(3.11 MHz)**
3. In a transistorized Hartley oscillator, the tank circuit has the capacitance of 100 pF. The value of inductance between the collector and tapping point is 30 μH and the value of inductance between the tapping point and the transistor base is 100 μH. Determine the frequency of oscillators. Neglect the mutual inductance. **(2.9 MHz)**
4. For the transistor Hartley oscillator circuit shown in Fig. 65.37, find the frequency of operation. Neglect the mutual inductance between the coils. **(73.1 kHz)**

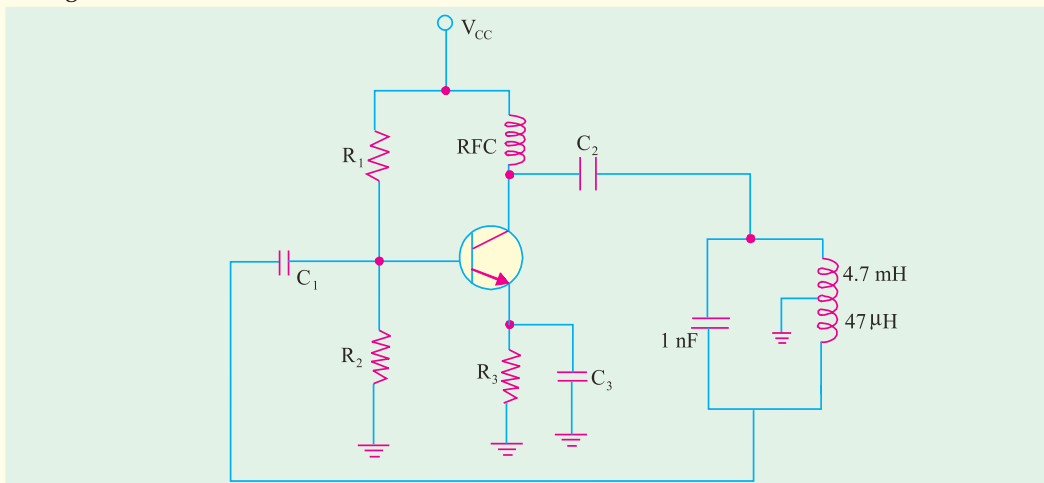


Fig. 65.37

5. A transistor Hartley oscillator is designed with  $L_1 = 2$  mH,  $L_2 = 20$  μH and a variable capacitance. Determine the range of capacitance values if the frequency of operation is varied from 950 kHz to 2050 kHz. **(2.98 pF to 13.9 pF)**

6. In a transistor Colpitts oscillator,  $C_1 = 0.001$  μF,  $C_2 = 0.01$  μF and  $L = 5$  μH. Find the required gain for oscillation and the frequency of oscillations. **(0.91 nF and 2.37 MHz)**

7. Determine the frequency of oscillations for the transistor Colpitts oscillator circuit shown in Fig. 65.38. **(24.4 kHz)**

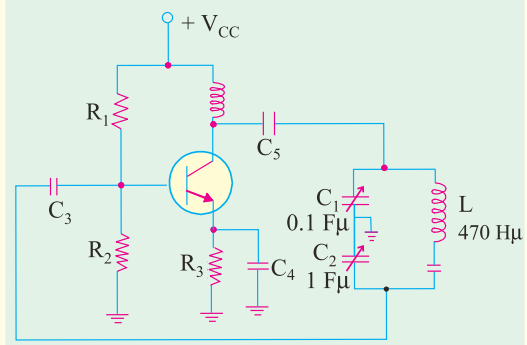


Fig. 65.38

8. A Colpitts oscillator is designed with  $C_1 = 100$  pF,  $C_2 = 7500$  pF. The inductance is variable. Determine the range of inductance values if the frequency of oscillation is to vary between 950 kHz and 2050 kHz. **(61 μH to 284 μH)**

9. The frequency of oscillation of a Colpitts oscillator is given by,

$$f_0 = \frac{1}{2\pi \sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}}$$

where  $L$ ,  $C_1$  and  $C_2$  are the frequency-determining components. Such a circuit operates at 450 kHz with  $C_1 = C_2$ . What will be the oscillation frequency if the value of  $C_2$  is doubled.

**(389.7 kHz)**

10. Calculate the frequency of oscillations for the Clapp oscillator shown in Fig. 65.39. (734.5 kHz)
11. A crystal has the following parameters :  $L = 0.33$  H,  $C_1 = 0.065$  pF,  $C_2 = 1$  pF and  $R = 5.5$  K $\Omega$ . Find the series resonant frequency and  $Q$ -factor of the crystal. (1.09 MHz and 411)
12. A Wien Bridge oscillator is used for operation at  $f_0 = 10$  kHz. The value of  $R$  is 100 K, find the value of capacitor,  $C$ . Assume  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$ . (159 pF)
13. A  $RC$  phase shift oscillator has  $R = 1$  k $\Omega$  and  $C = 0.01$   $\mu$ F. Calculate the frequency of oscillations

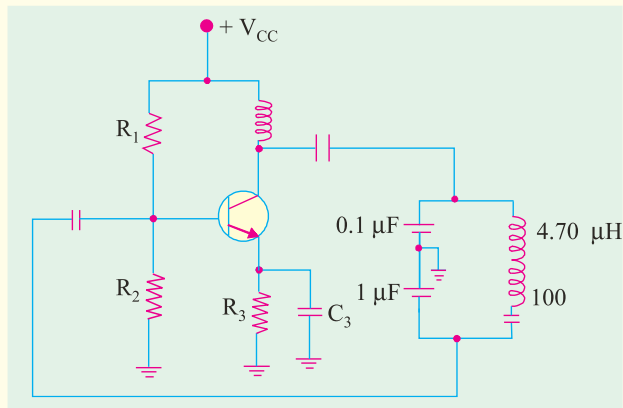


Fig. 65.39

- (Electronics Engg, Annamalai Univ. 2002)
14. Calculate the frequency and the duty cycle of the output of stable multivibrator and draw the waveform obtained across the capacitor  $C_T$  as shown in Fig. 65.40. (Electronics Engg, Bangalore Univ. 2001)
15. In an Hartley oscillator if  $L_1 = 0.1$  mH,  $L_2 = 10$   $\mu$ H and mutual inductance between the coils equal to 20  $\mu$ H. Calculate the value of capacitor  $C$  of the oscillatory circuit to obtain frequency of 4110 kHz and also find the condition for sustained oscillations. (Electronics Engg: Bangalore Univ. 2001)
16. Calculate the frequency of oscillation of a colpitt oscillator with  $C_1 = C_2 = 400$  pF and  $L = 2$  mH. (Electronics Engg., Bangalore Univ. 2002)
17. In an R-C phase shift oscillator  $R = 5000$   $\Omega$  and  $C = 0.1$  MF. Calculate the frequency of oscillations. (Electronics Engg., Bangalore Univ. 2002)

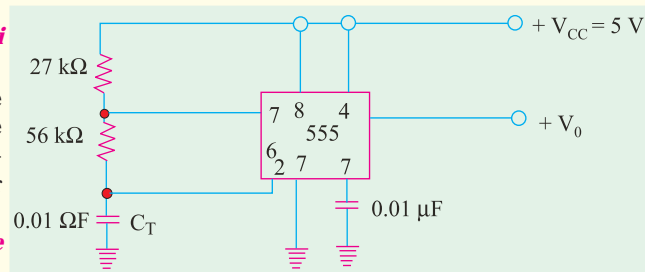


Fig. 65.40

## OBJECTIVE TESTS – 65

- An electronic oscillator is
  - just like an alternator
  - nothing but an amplifier
  - an amplifier with feedback
  - a converter of ac to dc energy.
- The frequency of oscillation of an elementary  $LC$  oscillatory circuit depends on
  - coil resistance
  - coil inductance
  - capacitance
  - both (b) and (c).
- For sustaining oscillations in an oscillator
  - feedback factor should be unity
  - phase shift should be  $0^\circ$
  - feedback should be negative
  - both (a) and (b).
- If Barkhausen criterion is not fulfilled by an oscillator circuit, it will
  - stop oscillating
  - produce damped waves continuously
  - become an amplifier
  - produce high-frequency whistles.
- In a transistor Hartley oscillator
  - inductive feedback is used
  - untapped coil is used
  - entire coil is in the output circuit
  - no capacitor is used
- A Hartley oscillator is used for generating
  - very low frequency oscillation
  - radio-frequency oscillation
  - microwave oscillation
  - audio-frequency oscillation

7. A Colpitts oscillator uses  
 (a) tapped coil  
 (b) inductive feedback  
 (c) tapped capacitance  
 (d) no tuned LC circuit
8. In RC phase-shift oscillator circuits.  
 (a) there is no need for feedback  
 (b) feedback factor is less than unity  
 (c) pure sine wave output is possible  
 (d) transistor parameters determine oscillation frequency.
9. Wien bridge oscillator is most often used whenever  
 (a) wide range of high purity sine waves is to be generated  
 (b) high feedback ratio is needed  
 (c) square output waves are required  
 (d) extremely high resonant frequencies are required.

10. The RC network shown in Fig 65.40 can provide a maximum theoretical phase shift of  
 (a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$

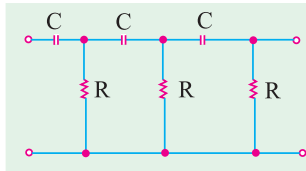


Fig. 65.41

- (UPSC Engineering Services 2002)
11. A CE amplifier can be converted into oscillator by  
 (a) providing adequate positive feedback  
 (b) phase shifting the output by  $180^\circ$  and feeding this phase-shifted output to the input.  
 (c) using only a series tuned circuit as a load on the amplifier  
 (d) using a negative resistance device as a load on the amplifier  
 (e) all of the above

- (UPSC Engineering Service, 2002)
12. The primary advantage of a crystal oscillator is that  
 (a) it can oscillate at any frequency  
 (b) it gives a high output voltage  
 (c) its frequency of oscillation remains almost constant  
 (d) it operates on a very low dc supply voltage.

13. Non-sinusoidal waveforms  
 (a) are departures from sine waveform  
 (b) have low mark-to-space ratio  
 (c) are much easier to generate  
 (d) are unfit for digital operation.

14. A relaxation oscillator is one which  
 (a) has two stable states  
 (b) relaxes indefinitely  
 (c) produces non-sinusoidal output  
 (d) oscillates continuously.
15. A square pulse has a mark-to-space ratio of  
 (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4.
16. Apart from a dc power source, the essential requirements of a sawtooth generator are  
 (a) a resistor (b) a capacitor  
 (c) a switching device (d) all of the above.
17. Which of the following statement is WRONG ?  
 In a multivibrator  
 (a) output is available continuously  
 (b) feedback between two stages is 100%  
 (c) positive feedback is employed  
 (d) when one transistor is ON, the other is OFF.
18. An MMV circuit  
 (a) has no stable state  
 (b) gives two output pulses for one input trigger pulse  
 (c) returns to its stand-by states automatically  
 (d) has no energy-storage element.
19. A BMV circuit  
 (a) has two unstable states  
 (b) has one energy-storage element  
 (c) switches between its two states automatically  
 (d) is not an oscillator.
20. The digital circuit using two inverters shown in Fig. 65.42 will act as  
 (a) a bistable multi-vibrator  
 (b) an astable multi-vibrator  
 (c) a monostable multi-vibrator  
 (d) an oscillator

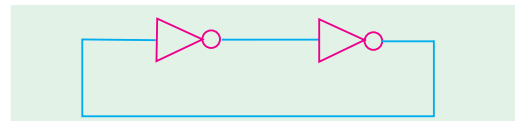


Fig. 65.42

21. In the Schmitt trigger circuit shown in Fig. 65.43 if  $V_{CE(sat)} = 0.1 V$ , the output logic low level ( $V_{OL}$ ) is  
 (a) 1.25 V  
 (b) 1.35 V  
 (c) 2.50 V  
 (d) 5.00 V

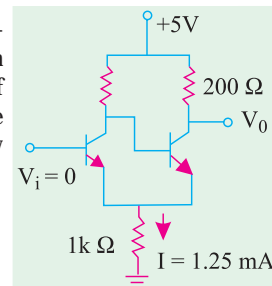


Fig. 65.43

ANSWERS

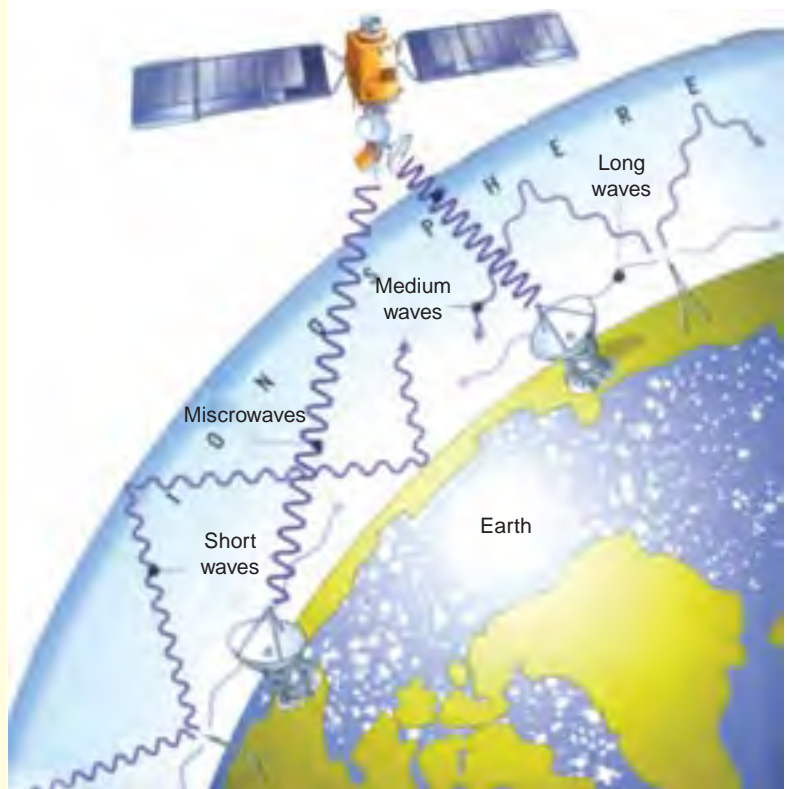
1. (c) 2. (d) 3. (d) 4. (a) 5. (a) 6. (b) 7. (c) 8. (c) 9. (a) 10. (b) 11. (b)  
 12. (c) 13. (a) 14. (c) 15. (a) 16. (d) 17. (a) 18. (c) 19. (a) 20. (a) 21. (b)

# CHAPTER 66

## Learning Objectives

- What is a Carrier Wave?
- Radio Frequency Spectrum
- Sound
- Need for Modulation
- Radio Broadcasting
- Modulation
- Methods of Modulation
- Amplitude Modulation
- Percent Modulation
- Upper and Lower Sidebands
- Mathematical Analysis of a Modulated Carrier Wave
- Power Relation in an AM Wave
- Forms of Amplitude Modulation
- Generation of SSB
- Methods of Amplitude Modulation
- Modulating Amplifier Circuit
- Frequency Modulation
- Modulation Index
- Deviation Ratio
- Percent Modulation
- FM Sidebands
- Modulation Index and Number of Sidebands
- Demodulation or Detection
- Essentials of AM Detection
- Transistor Detectors for AM Signals
- Quadrature Detector
- Frequency Conversion
- Standard Superhet AM Receiver
- FM Receiver
- Comparison between AM and FM
- The Four Fields of FM

## MODULATION AND DEMODULATION



The radio waves travel through the air, bounce off a layer of the atmosphere called the ionosphere, or are relayed by satellite

### 66.1. Introduction

For successful transmission and reception of intelligence (code, voice, music etc.) by the use of radio waves, two processes are essential :

- (i) modulation and (ii) demodulation.

Speech and music etc. are sent thousands of kilometres away by a radio transmitter. The scene in front of a television camera is also sent many kilometres away to viewers. Similarly, a Moon probe or Venus probe checking its environments, sends the information it gathers millions of kilometres through space to receivers on earth. In all these cases, the carrier is the high-frequency radio wave. The intelligence i.e. sight, sound or other data collected by the probe is impressed on the radio wave and is carried along with it to the destination.

Modulation is the process of **combining the low-frequency signal with a very high-frequency radio wave called carrier wave (CW)**. The resultant wave is called **modulated carrier wave**. This job is done at the transmitting station.

Demodulation is the process of separating or **recovering the signal from the modulated carrier wave**. It is just the opposite of modulation and is performed at the receiving end.

### 66.2. What is a Carrier Wave?

It is a high-frequency undamped radio wave produced by radio-frequency oscillators (Chapter 65). As seen from Fig. 66.1, the output of these oscillators is first amplified and then passed on to an antenna. This antenna radiates out these high-frequency (electromagnetic) waves into space. These waves have constant amplitude and travel with the velocity of light. They are inaudible *i.e.* by themselves they cannot produce any sound in the loudspeaker of a receiver. As their name shows, their job is to **carry the signal** (audio or video) from transmitting station to the receiving station. The resultant wave is called **modulated carrier wave**.

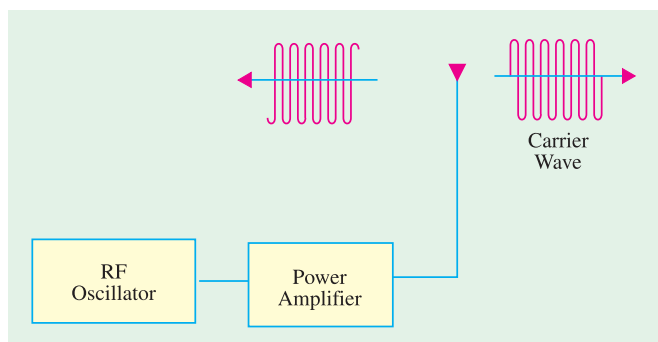


Fig. 66.1

### 66.3. Radio Frequency Spectrum

Radio frequencies used by different communication systems extend from very low frequencies to extra high frequencies as tabulated below along with their acronym abbreviations.

Table No. 66.1			
Frequency	Designation	Abbreviation	Uses
3-30 kHz	very low frequency	VLF	long distance telegraphy broadcasting
30-300 kHz	low frequency	LF	long distance point-to-point service, navigational aids, sound broadcasting and line carrier systems.
300 kHz-3MHz	medium frequency	MF	sound broadcasting, ship-shore services and line carrier systems
3-30 MHz	high frequency	HF	medium and long-distance point-to-point services, sound broadcasting, linear carrier systems.



30-300 MHz	very high frequency	VHF	short-distance communication, TV and sound broadcasting, radar
300 MHz-3GHz	ultra high frequency	UHF	outer-space radio communication,
3-30 GHz	super high frequency	SHF	point-to-point microwave communication systems and radar.
30-300 GHz	extra high frequency	EHF	

Generally, none of the frequencies above 300 GHz is classified as radio waves.

### 66.4. Sound

It is a sort of disturbance which requires some physical medium for its propagation. Human voice consists of a series of compressions and rarefactions which travel through air with a velocity of about 345 m/s. The frequency range of human voice is from 20-4000 Hz which lies within the audible range of 20 to 20,000 Hz. Variations in human voice can be converted into corresponding variations in electric current with the help of a microphone as shown in Fig.66.2.

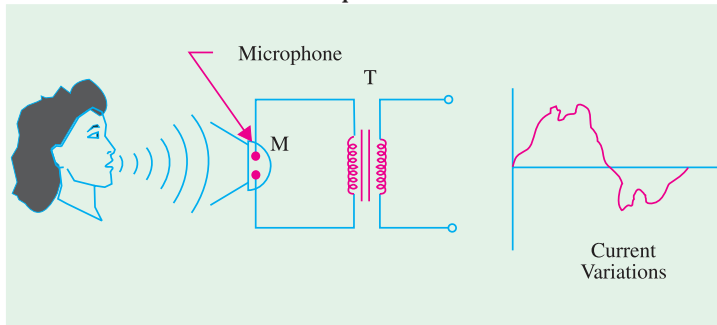


Fig. 66.2

When a sound wave strikes the microphone, it produces AF sound current. The positive half-cycles of sound current are produced by the compressions and negative half cycles by rarefactions.

As seen, human voice does not produce pure sinusoidal current because it does not consist of one frequency alone. It is quite complex and can be analysed to consist of a fundamental (or lowest frequency) and its integral multiple frequencies called **overtones** or **harmonics**.

We are interested in two main characteristics of sound :

- (i) **Intensity**—It is the energy content of the wave. It depends on its amplitude. In fact, intensity of a wave is directly proportional to the square of its amplitude *i.e.*  $I \propto a^2$ . Sensation of loudness felt by a listener depends directly on the intensity of the wave falling on his ears.
- (ii) **Frequency**—It produces the sensation called pitch. Audible sounds have a frequency range from 20 Hz to 20,000 Hz.

Though every sound has complex frequency structure, we will consider only single-frequency sound whose current wave is a pure sine wave as shown in Fig. 66.3. It will be used as the modulating signal when discussing the process of modulation.

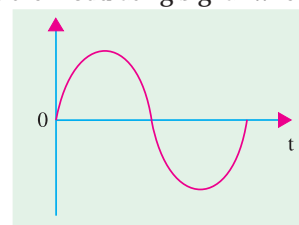


Fig. 66.3

### 66.5. Need for Modulation

Sometimes, beginners question the necessity of modulation *i.e.* using a carrier wave to carry the low-frequency signal from one place to another. Why not transmit the signals directly and save lot of both-eration? Unfortunately, there are three main hurdles in the process of such direct transmission of audio-frequency signals :

1. They have relatively short range,
2. If everybody started transmitting these low-frequency signals directly, mutual interference will render all of them ineffective
3. Size of antennas required for their efficient radiation would be large i.e. about 75 km as explained below.

For efficient radiation of a signal, the minimum length of an antenna is one quarter wavelength ( $\lambda/4$ ). The antenna length  $L$  is connected with the frequency of the signal wave by the relation  $L = 75 \times 10^6/f$  metres. For transmitting an audio signal of  $f = 1000$  Hz,  $L = 75 \times 10^6/10^3 = 75,000$  m = 75 km ! In view of this immense size of antenna length, it is impractical to radiate audio-frequency signals directly into space.

Hence, the solution lies in modulation which enables a low-frequency signal to travel very large distances through space with the help of a high-frequency carrier wave. These carrier waves need reasonably-sized antennas and produce no interference with other transmitters operating in the same area.

### 66.6. Radio Broadcasting

Let us see how radio broadcasting stations broadcast speech or music etc. from their broadcasting studios.

First, the speech or music which consists of a series of compressions and rarefactions is translated into a tiny varying electric current with the help of a crystal microphone. The frequency of variations of this current lies in the audio-range, hence it is known as audio frequency signal.

The audio-frequency signal cannot be radiated out from the antenna directly because transmission at audio-frequencies is not practical. For this purpose, oscillations of very high frequency or radio-frequency are produced with the help of any one of the oscillators discussed in Chapter 15. The electromagnetic waves so produced are of constant amplitude but of extremely high frequency. These waves, which are neither seen nor heard, travel through space with the velocity of light i.e.  $3 \times 10^8$  m/s (approx). The audio-frequency signal which is to be broadcast, is then superimposed on the RF waves, which are known as carrier waves (because they carry A.F. signal through space to distant places). In a way, the carrier waves can be likened to a horse and the audio-frequency signal to a rider. The process by which AF signal or information is impressed on the carrier wave is known as modulation. The horse and rider travel through space. At the receiving end, they strike the receiving aerial and enter the receiver which separates the horse from the rider. The horse i.e. carrier wave is returned and the rider i.e. audio-frequency signal is converted back into sound. This process by which the R.F. waves and A.F. waves are separated is known as **detection** or **demodulation** (because it is the reverse of modulation).



### 66.7. Modulation

It is the process of combining **an audio-frequency (AF) signal with a radio frequency (RF) carrier wave**. The AF signal is also called a **modulating wave** and the resultant wave produced is called **modulated wave**.

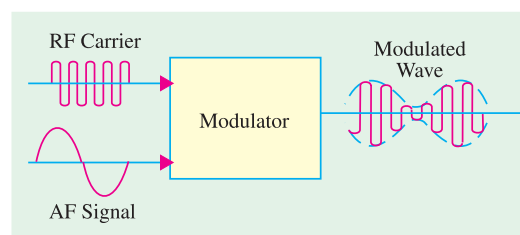


Fig. 66.4



During modulation, some characteristic of the carrier wave is varied in time with the modulating signal and is accomplished by combining the two.

### 66.8. Methods of Modulation

The mathematical expression for a sinusoidal carrier wave is

$$e = E_C \sin (\omega_c t + \phi) = E_C \sin (2 \pi f_c t + \phi)$$

Obviously, the waveform can be varied by any of its following three factors or parameters :

1.  $E_C$ — the amplitude,
2.  $f_c$ — the frequency,
3.  $\phi$  — the phase.

Accordingly, there are three types of sine-wave modulations known as :

#### 1. Amplitude Modulation (AM)

Here, the information or *AF* signal changes the amplitude of the carrier wave without changing its frequency or phase.

#### 2. Frequency Modulation (FM)

In this case, the information signal changes the frequency of the carrier wave without changing its amplitude or phase.

#### 3. Phase Modulation (PM)

Here, the information signal changes the phase of the carrier wave without changing its other two parameters.

### 66.9. Amplitude Modulation

In this case, the **amplitude** of the carrier wave is varied in proportion to the **instantaneous amplitude** of the information signal or *AF* signal. Obviously, the amplitude (and hence the intensity) of the carrier wave is changed **but not its frequency**. Greater the amplitude of the *AF* signal, greater the fluctuations in the amplitude of the carrier wave.

The process of amplitude modulation is shown graphically in Fig. 66.5. For the sake of simplicity, the *AF* signal has been assumed sinusoidal [Fig. 66.5 (a)]. The carrier wave by which it is desired to transmit the *AF* signal is shown in Fig. 66.5 (b). The resultant wave called modulated wave is shown in Fig. 66.5 (c).

The function of the modulator is to mix these two waves in such a way that (a) is transmitted along with (b). All stations broadcasting on the standard broadcast band (550-1550 kHz) use *AM* modulation.

If you observe the envelope of the modulated carrier wave, you will realize that it is an exact replica of the *AF* signal wave.

In summary

(i) **fluctuations** in the amplitude of the carrier wave depend on the **signal amplitude**,

(ii) **rate** at which these fluctuations take place depends on the **frequency** of the audio signal.

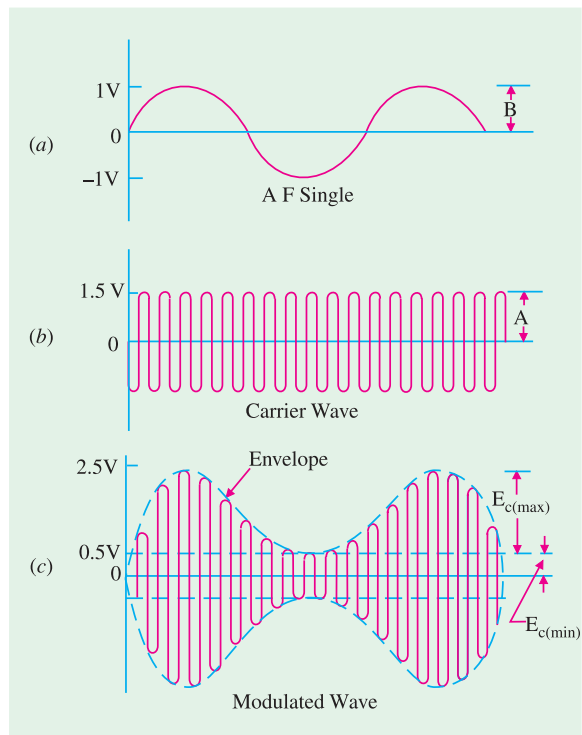


Fig. 66.5

### 66.10. Percent Modulation

It indicates the degree to which the AF signal modulates the carrier wave

$$m = \frac{\text{maximum value of signal wave}}{\text{maximum value of carrier wave}} \times 100 = \frac{\text{signal amplitude}}{\text{carrier amplitude}} \times 100$$

$$= \frac{B}{A} \times 100 \quad \text{—Fig. 66.5}$$

The ratio B/A expressed as a fraction is called **modulation index (MI)**

$$m = M.I. \times 100$$

From Fig. 66.5, it is seen that  $B = 1$  V and  $A = 1.5$  V

$$\therefore m = \frac{1}{1.5} \times 100 = 66.7\%$$

Modulation may also be defined in terms of the values referred to the modulated carrier wave.

$$m = \frac{E_{c(max)} - E_{c(min)}}{E_{c(max)} + E_{c(min)}} \times 100$$

where  $E_{c(max)}$  and  $E_{c(min)}$  are the maximum and minimum values of the amplitude of the **modulated carrier wave**.

Again, from Fig. 66.5 we see that

$$m = \frac{2.5 - 0.5}{2.5 + 0.5} \times 100$$

$$= \frac{2}{3} \times 100 = 66.7\%$$

Fig. 66.6 shows a modulated wave with different degrees of modulation. As before, both the signal and carrier waves are assumed to be sine waves. Smallest value of  $m = 0$  i.e. when amplitude of the modulating signal is zero. It means that  $m = 0$  for an unmodulated carrier wave. Maximum value of  $m = 1$  when  $B = A$ . Value of  $m$  can vary anywhere from 0 to 100% without introducing distortion. Maximum undistorted power of a radio transmitter is obtained when  $m = 100\%$ . If  $m$  is less than 100 per cent, power output is reduced though the power content of the carrier is not. Modulation in excess of 100 per cent produces severe distortion and interference (called splatter) in the transmitter output.

**Example 66.1** A modulated carrier wave has maximum and minimum amplitudes of 750 mV and 250 mV. Calculate the value of percentage modulation.

**Solution.** The modulated wave is shown in Fig. 66.7.

Here,  $E_{c(max)} = 750$  mV and  
 $E_{c(min)} = 250$  mV

$$\therefore m = \frac{E_{c(max)} - E_{c(min)}}{E_{c(max)} + E_{c(min)}} \times 100$$

$$= \frac{750 - 250}{750 + 250} \times 100 = 50\%$$

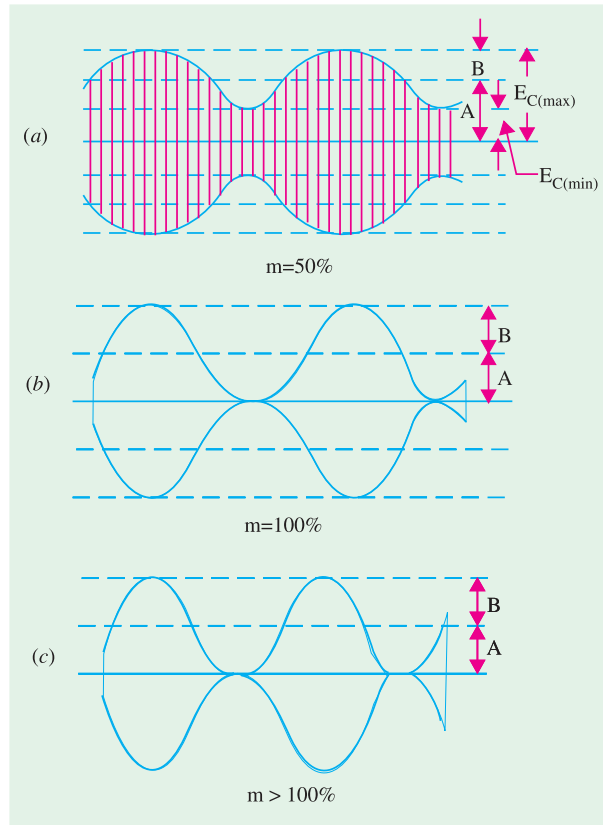


Fig. 66.6

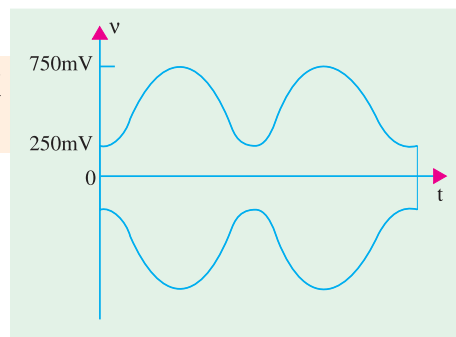


Fig. 66.7

### 66.11. Upper and Lower Side Frequencies

An unmodulated carrier wave consists of only one single-frequency component of frequency  $f_c$ . When it is combined with a modulating signal of frequency  $f_m$ , heterodyning action takes place.

As a result, two additional frequencies called **side frequencies** are produced. The AM wave is found to consist of three frequency components :

1. The original carrier frequency component,  $f_c$ .
2. A higher frequency component ( $f_c + f_m$ ). It is called the sum component.
3. A lower frequency component ( $f_c - f_m$ ). It is called the difference component.

The two new frequencies are called the **upper-side frequency (USF)** and **lower-side frequency (LSF)** respectively and are symmetrically located around the carrier frequency. The modulating frequency remains unchanged but does not appear in the amplifier output because the amplifier's load presents practically zero impedance to this low frequency.

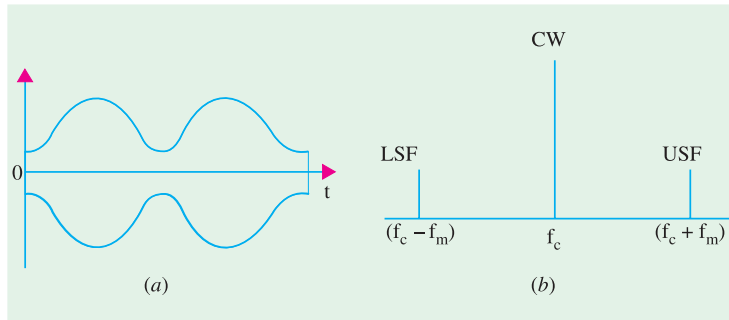


Fig. 66.8

These are shown in time domain in Fig. 66.8 (a) and in frequency domain in Fig. 66.8 (b). The amplitude of the side frequencies depends on the value of  $m$ . The amplitude of each side frequency =  $mA/2$  where  $A$  is the amplitude of unmodulated carrier wave.

**Example 66.2.** A 10-MHz sinusoidal carrier wave of amplitude 10 mV is modulated by a 5 kHz sinusoidal audio signal wave of amplitude 6 mV. Find the frequency components of the resultant modulated wave and their amplitudes.

(Electronics & Comm. Engg.; Madras Univ. 1991)

**Solution.** Here,  $f_c = 10$  MHz and  $f_m = 5$  kHz = 0.005 MHz. The modulated carrier contains the following frequencies :

1. original carrier wave of frequency  
 $f_c = 10$  MHz
2. USF of frequency  
 $= 10 + 0.005 = 10.005$  MHz
3. LSF of frequency  
 $= 10 - 0.005 = 9.995$  MHz

The frequency spectrum is shown in Fig. 66.9

$$\text{Here, } m = \frac{B}{A} = \frac{6}{10} = 0.6 = 0.6$$

Amplitude of  $LSF = USF = mA/2 = 0.6 \times 10/2 = 3$  mV as shown.

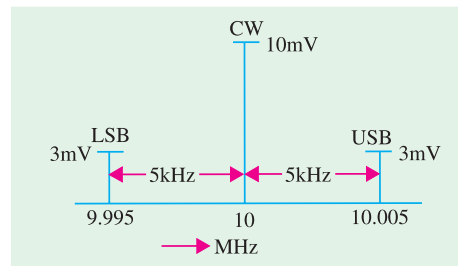


Fig. 66.9

### 66.12. Upper and Lower Sidebands

In Art 66.11, it was assumed that the modulating signal was composed of one frequency component only. However, in a broadcasting station, the modulating signal is the human voice (or music) which contains waves with a frequency range of 20-4000 Hz. Each of these waves has its own LSF and USF. When combined together, they give rise to an upper-side **band (USB)** and a lower-side **band (LSB)** as shown in Fig. 66.10. The USB, in fact, contains all sum components of the signal and carrier frequency whereas LSB contains their difference components.

The channel width (or bandwidth) is given by the difference between extreme frequencies i.e. between maximum frequency of *USB* and minimum frequency of *LSB*. As seen,

Channel width =  $2 \times$  maximum frequency of modulating signal =  $2 \times f_{m(max)}$

**Example 66.3.** An audio signal given by  $15 \sin 2\pi (2000 t)$  amplitude-modulates a sinusoidal carrier wave  $60 \sin 2\pi (100,000) t$ .

Determine :

- (a) modulation index, (b) percent modulation, (c) frequencies of signal and carrier,
- (d) frequency spectrum of the modulated wave.

(Electronics & Telecom Engg., Jadavpur Univ. 1991)

**Solution.** Here,  $B = 15$  and  $A = 60$

(a)  $M.I. = \frac{B}{A} = \frac{15}{60} = 0.25$

(b)  $m = M.I. \times 100 = 0.25 \times 100 = 25\%$

(c)  $f_m = 2000 \text{ Hz}$  — by inspection of the given equation

$f_c = 100,000 \text{ Hz}$  — by inspection of the given equation

(d) The three frequencies present in the modulated *CW* are

(i)  $100,000 \text{ Hz} = 100 \text{ kHz}$

(ii)  $100,000 + 2000 = 102,000 \text{ Hz} = 102 \text{ kHz}$

(iii)  $100,000 - 2000 = 98,000 \text{ Hz} = 98 \text{ kHz}$

**Example 66.4.** A bandwidth of 15 MHz is available for AM transmission. If the maximum audio signal frequency used for modulating the carrier is not to exceed 15 kHz, how many stations can broadcast within this band simultaneously without interfering with each other?

(Electronics & Telecom Engg.; Pune Univ. 1991)

**Solution.** *BW* required by each station

$$= 2 f_{m(max)} = 2 \times 15 = 30 \text{ kHz}$$

Hence, the number of station which can broadcast within this frequency band without interfering with one another is

$$= \frac{15 \text{ MHz}}{30 \text{ kHz}} = 500$$

**Example 66.5.** In a broadcasting studio, a 1000 kHz carrier is modulated by an audio signal of frequency range, 100-5000 Hz. Find (i) width or frequency range of sidebands (ii) maximum and minimum frequencies of *USB* (iii) maximum and minimum frequencies of *LSB* and (iv) width of the channel.

(Electronics & Comm. Engg. IERE, London)

**Solution.** (i) Width of sideband =  $5000 - 100 = 4900 \text{ Hz}$

(ii) Max. frequency of *USB*  
 $= 1000 + 5 = 1005 \text{ kHz}$

Min. frequency of *USB*  
 $= 1000 + 0.1 = 1000.1 \text{ kHz}$

(iii) Max. frequency of *LSB*  
 $= 1000 - 0.1 = 999.9 \text{ kHz}$

Min. frequency of *LSB*  
 $= 1000 - 5 = 995 \text{ kHz}$

(iv) Width of channel =  $1005 - 995 = 10 \text{ kHz}$

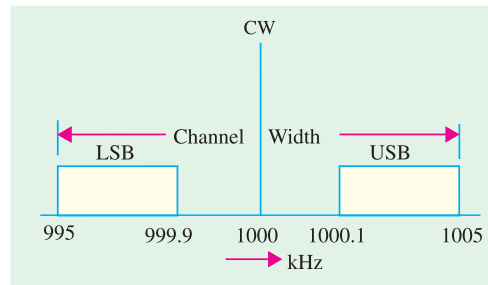


Fig. 66.10

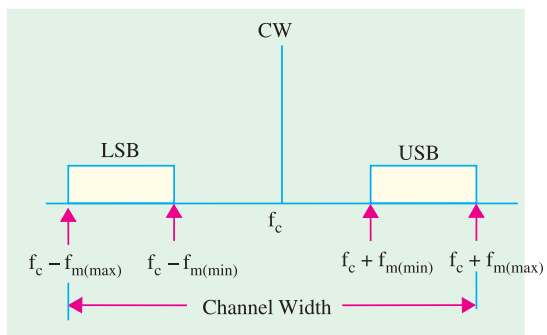


Fig. 66.11

66.13. Mathematical Analysis of a Modulated Carrier Wave

The equation of an unmodulated carrier wave is

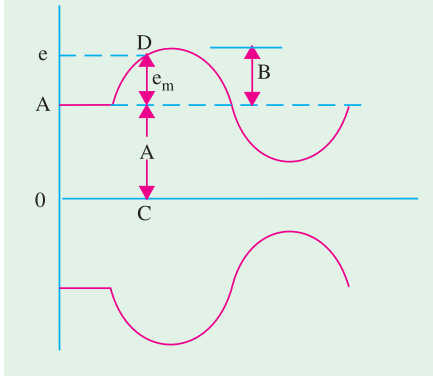


Fig. 66.12

$e_c = E_c \sin 2\pi f_c t = A \sin 2\pi f_c t = A \sin \omega t$   
 where  $A$  is the constant amplitude of the carrier wave and  $\omega = 2\pi f_c$

Let the equation of the single-frequency sinusoidal modulating signal be

$$e_m = E_m \sin 2\pi f_m t = B \sin 2\pi f_m t = B \sin \pi t \quad \text{— where } p = 2\pi f_m$$

As seen from Fig. 66.12, the amplitude of the modulated carrier wave at any instant is

$$= A + e_m \quad (\because A \text{ is constant}) \\ = (A + B \sin \pi t)$$

Hence, its **instantaneous** value is given by

$$e = (A + B \sin pt) \sin \omega t = A \sin \omega t + B \sin \omega t \cdot \sin pt$$

$$= A \sin \omega t + \frac{B}{2} \cdot 2 \sin \omega t \cdot \sin pt = A \sin \omega t + \frac{B}{2} [\cos(\omega - p)t - \cos(\omega + p)t] *$$

$$= A \sin \omega t + \frac{B}{2} \cos(\omega - p)t - \cos(\omega + p)t$$

$$= A \sin 2\pi f_c t + \cos 2\pi (f_c - f_m)t - \cos 2\pi (f_c + f_m)t$$

As seen from Art. 66.10,  $m = B/A$  or  $B = mA$

$$\therefore e = A \sin 2\pi f_c t + \cos 2\pi (f_c - f_m)t - \cos 2\pi (f_c + f_m)t$$

It is seen that the modulated wave contains three components :

- (i)  $A \sin 2\pi f_c t$  — the original carrier wave
- (ii)  $\frac{mA}{2} \cos 2\pi (f_c + f_m)t$  — upper side frequency
- (iii)  $\frac{mA}{2} \cos 2\pi (f_c - f_m)t$  — lower side frequency

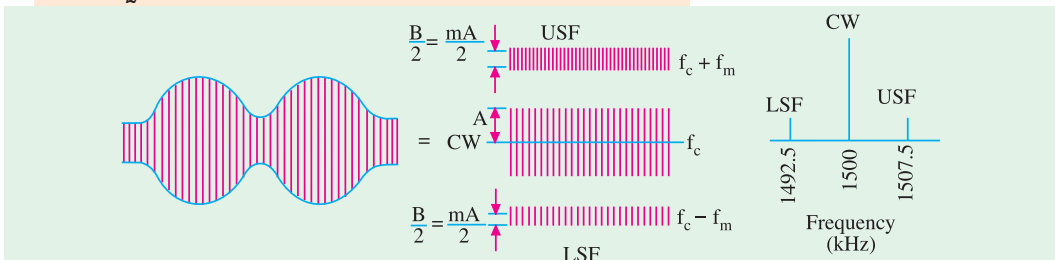


Fig. 66.13

These three frequencies are not a mathematical fiction but they actually exist. In fact, with the help of a narrow band filter, we can separate side frequencies from the carrier wave.

**Example 66.6.** The tuned circuit of the oscillator in an AM transmitter uses a 40 mH coil and a 1 nF capacitor. If the carrier wave produced by the oscillator is modulated by audio frequencies upto 10 kHz, calculate the frequency band occupied by the side bands and channel width.

(Electronics & Telecom. Engg., Calcutta Univ. 1990)

**Solution.** The resonant frequency is given by

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40 \times 10^{-6} \times 1 \times 10^{-9}}} = 796 \text{ kHz}$$

\*  $2 \sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$

The frequency range occupied by the sidebands is from **786 kHz to 806 kHz**. The channel width

$$= 2 \times 10 = \mathbf{20 \text{ kHz}}$$

### 66.14. Power Relations in an AM Wave

As discussed in Art. 66.13, a modulated carrier wave consists of the following three components :

1. original carrier wave of amplitude  $A$
2. *USF* wave of amplitude  $(mA/2)$
3. *LSF* wave of amplitude  $(mA/2)$

Now, power radiated out by a wave through an antenna is proportional to (amplitude)<sup>2</sup>.

$$\text{Carrier power, } P_C \propto A^2 = KA^2$$

$$\text{USB power, } P_{USB} \propto \left(\frac{B}{2}\right)^2 = \frac{KB^2}{4}; \text{ LSB power, } P_{LSB} \propto \left(\frac{B}{2}\right)^2 = \frac{KB^2}{4}$$

$$\text{Total sideband power } P_{SB} = 2 \times \frac{KB^2}{4} = \frac{KB^2}{2}$$

$$\text{Total power radiated out from the antenna is } P_T = P_C + P_{SB} = KA^2 + \frac{KB^2}{2}$$

$$\text{Substituting } B = mA, \text{ we get } P_T = KA^2 + \frac{KB^2}{2} (mA)^2 = KA^2 \left(1 + \frac{m^2}{2}\right)$$

$$\text{Now } P_C = KA^2$$

$$(i) \quad \therefore P_T = P_C \left(1 + \frac{m^2}{2}\right) = P_C \left(\frac{2 + m^2}{2}\right) \quad (ii) \quad P_C = P_T = P_T \left(\frac{2}{2 + m^2}\right)$$

$$(iii) \quad P_{SB} = P_T - P_C = P_C \left(1 + \frac{m^2}{2}\right) - P_C = \frac{m^2}{2} P_C = \left(\frac{m^2}{2 + m^2}\right) P_T$$

$$(iv) \quad P_{USB} = P_{LSB} = \frac{1}{2} P_{SB} = \frac{m^2}{4} P_C = \frac{1}{2} \left(\frac{m^2}{2 + m^2}\right) P_T$$

Let us consider the case when  $m = 1$  or 100%

$$(i) \quad P_T = 1.5 P_C = 1.5 \times \text{carrier power} \quad (ii) \quad P_C = \frac{2}{3} P_T = \frac{2}{3} \times \text{total power radiated}$$

$$(iii) \quad P_{USB} = P_{LSB} = \frac{1}{4} P_C = 25\% \text{ of carrier power}$$

$$(iv) \quad P_{SB} = \frac{1}{2} P_C = 50\% \text{ of carrier power} \quad (v) \quad \frac{P_{USB}}{P_T} = \frac{0.25 P_C}{1.5 P_C} = \frac{1}{6}$$

It means that **single side-band contains 1/6th of the total power radiated out by the transmitter**. That is why single-side band (*SSB*) transmission is more power efficient.

**Example 66.7.** The total power content of an AM wave is 1500 W. For a 100 percent modulation, determine

- (i) power transmitted by carrier, (ii) power transmitted by each side band.

(Electronics & Comm. Engg., Kerala Univ. 1991)

**Solution.**  $P_T = 1500 \text{ W}, P_C = ? \quad P_{USB} = ? \quad P_{LSB} = ?$

$$(i) \quad P_C = P_T \left(\frac{2}{2 + m^2}\right) = P_T \left(\frac{2}{2 + 1^2}\right) = \frac{2}{3} P_T = \frac{2}{3} \times 1500 = \mathbf{1000 \text{ W}}$$

$$(ii) P_{USB} = P_{LSB} = \frac{m^2}{4} P_C = \frac{1^2}{4} \times 1000 = 250 \text{ W}$$

$$\text{As seen, } P_{USB} = \frac{1}{6} P_T$$

**Example 66.8.** The total power content of an AM wave is 2.64 kW at a modulation factor of 80%. Determine the power content of

(i) carrier; (ii) each sideband. (Electronics & Comm., Roorkee Univ. 1991)

$$\text{Solution. (i) } P_C = P_T \left( \frac{2}{2+m^2} \right) = 2.64 \times 10^3 \left( \frac{2}{2+0.8^2} \right) = 2000 \text{ W}$$

$$(ii) P_{USB} = P_{LSB} = \frac{m^2}{4} = \frac{0.8^2}{4} \times 2000 = 320 \text{ W}$$

**Example 66.9.** A transmitter used for radio telephone has an unmodulated carrier power of 10 kW and can be modulated to a maximum of 80 percent by a single-frequency signal before overloading. Find the value to which carrier power can be increased if a 50 percent modulation limit is imposed. (Electronics & Comm. Engg. Andhra Univ.)

$$\text{Solution. } P_T = P_C \left( \frac{2+m^2}{2} \right) = 10 \left( \frac{2+0.8^2}{2} \right) = 13.2 \text{ kW}$$

Now, when  $m = 0.5$ ,  $P_T$  is still 13.2 kW. Hence, new value of carrier power is given by

$$13.2 = P_C \left( \frac{2+0.5^2}{2} \right); \quad P_C = 11.73 \text{ kW}$$

It is seen that  $P_C$  can be increased from 10 kW to 11.73 kW with a total power limit of 13.2 kW and  $m = 0.5$

**Example 66.10.** A certain transmitter radiates 10 kW of power with the carrier unmodulated and 11.8 kW with the carrier sinusoidally modulated.

(a) Find the modulation factor;

(b) If another wave modulated to 40% is also transmitted, calculate the radiated power.

$$\text{Solution. (a) } 11.8 = 10 (1 + m^2/2); \quad m = 0.6 \text{ or } 60\%$$

$$(b) P_T = 10 \left( 1 + \frac{0.6^2}{2} + \frac{0.4^2}{2} \right) = 12.6 \text{ kW}$$

### 66.15. Forms of Amplitude Modulation

As shown in Art. 66.11, **one carrier** and **two sidebands** are produced in AM generation. It is found that it is not necessary to transmit all these signals to enable the receiver to reconstruct the original signal. Accordingly, we may attenuate or altogether remove the carrier or any one of the sidebands without affecting the communication process. The advantages would be

1. less transmitted power and
2. less bandwidth required

The different suppressed component systems are :

(a) **DSB-SC**

It stands for **double-sideband suppressed carrier** system [Fig. 66.14 (a)]. Here, carrier component is suppressed thereby saving enormous amount of power. As seen from Art. 66.14, carrier signal contains 66.7 per cent of the total transmitted power for  $m = 1$ , Hence, power saving amounts to 66.7% at 100% modulation.

(b) **SSB-TC**

As shown in Fig. 66.14 (b), in this case, one sideband is suppressed but the other sideband and carrier are transmitted. It is called **single sideband transmitted carrier** system. For  $m = 1$ , power saved is 1/6 of the total transmitted power.

(c) SSB-SC

This is the **most dramatic suppression of all** because it suppresses one sideband and the carrier and transmits only the remaining sideband as shown in Fig. 66.14 (c). In the standard or double-sideband full-carrier (DSB-FC) AM, carrier conveys **no information but contains maximum power**. Since the two sidebands are exact images of each other, they carry the same audio information. Hence, **all information is available in one sideband only**. Obviously carrier is **superfluous** and one side band is **redundant**. Hence, one sideband and the carrier can be discarded with no loss of information. The result is SSB signal. The advantage of SSB-SC system are as follows :

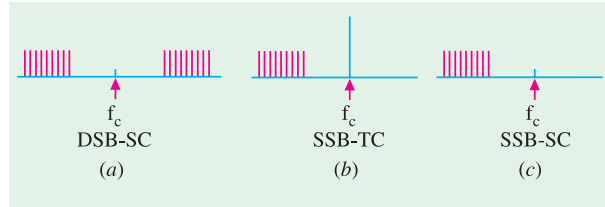


Fig. 66.14

1. Total saving of 83.3% in transmitted power (66.7% due to suppression of carrier wave and 16.6% due to suppression of one sideband). Hence, power is conserved in an SSB transmitter.
2. Bandwidth required is reduced by half i.e. 50%. Hence, twice as many channels can be multiplexed in a given frequency range.
3. The size of power supply required is very small. This fact assumes vital importance particularly in a spacecraft.
4. Since the SSB signal has narrower bandwidth, a narrower passband is permissible within the receiver, thereby limiting the noise pick up.

However, the main reason for wide spread use of DSB-FC (rather than SSB-SC) transmission in broadcasting is the relative simplicity of its modulating equipment.

66.16. Generation of SSB

Three main systems are employed for the generation of SSB.

1. Filter method, 2. Phase cancellation method, 3. The 'third' method.
- All these methods use some form of balanced modulator to suppress the carrier.

**Example 66.11.** In an AM wave, calculate the power saving when the carrier and one sideband are suppressed corresponding to

- (i)  $m = 1$                       (ii)  $m = 0.5$                       (Electronics and Comm. Engg., Osmania Univ.)

**Solution.** (i) When  $m = 1$

$$P_T = P_C \left( 1 + \frac{m^2}{2} \right) = 1.5 P_C ; \quad P_{LSB} = P_{USB} = \frac{m^2}{4} P_C = 0.25 P_C$$

$$\therefore \text{saving} = P_T - P_{USB} = 1.5 P_C - 0.25 P_C = 1.25 P_C$$

$$\text{saving} = \frac{1.25}{1.5 P_C} \times 100 = 83.3\%$$

(ii) When  $m = 0.5$

$$P_T = P_C \left( 1 + \frac{m^2}{2} \right) = P_C \left( \frac{1 + 0.5^2}{2} \right) = 1.125 P_C$$

$$P_{USB} = P_{LSB} = \frac{m^2}{4} P_C = \frac{0.5^2}{4} P_C = 0.0625 P_C$$

$$\therefore \text{saving} = \frac{1.125 P_C - 0.0625 P_C}{1.125 P_C} \times 100 = 94.4\%$$



**66.17. Methods of Amplitude Modulation**

There are two methods of achieving amplitude modulation :

- (i) Amplifier modulation,
- (ii) Oscillator modulation.

Block diagram of Fig. 66.15 illustrates the basic idea of amplifier modulation.

Here, carrier and *AF* signal are fed to an amplifier and the result is an *AM* output . The modulation process takes place in the active device used in the amplifier.

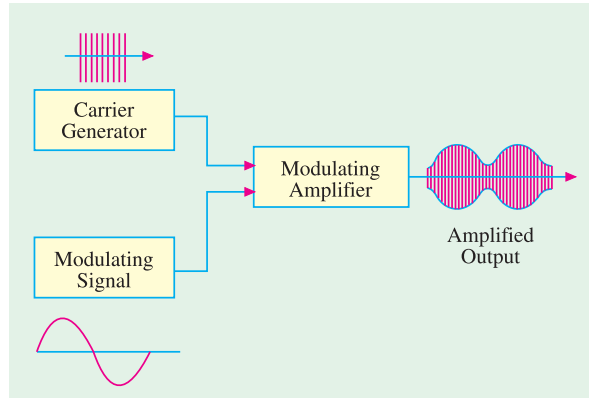


Fig. 66.15

**66.18. Block Diagram of an AM Transmitter**

Fig. 66.16 shows the block diagram of a typical transmitter. The carrier wave is supplied by a crystal-controlled oscillator at the carrier frequency. It is followed by a tuned buffer amplifier and an *RF* output amplifier.

The source of *AF* signal is a microphone. The audio signal is amplified by a low level audio amplifier and, finally, by a power amplifier. It is then combined with the carrier to produce a modulated carrier wave which is ultimately radiated out in the free space by the transmitter antenna as shown.

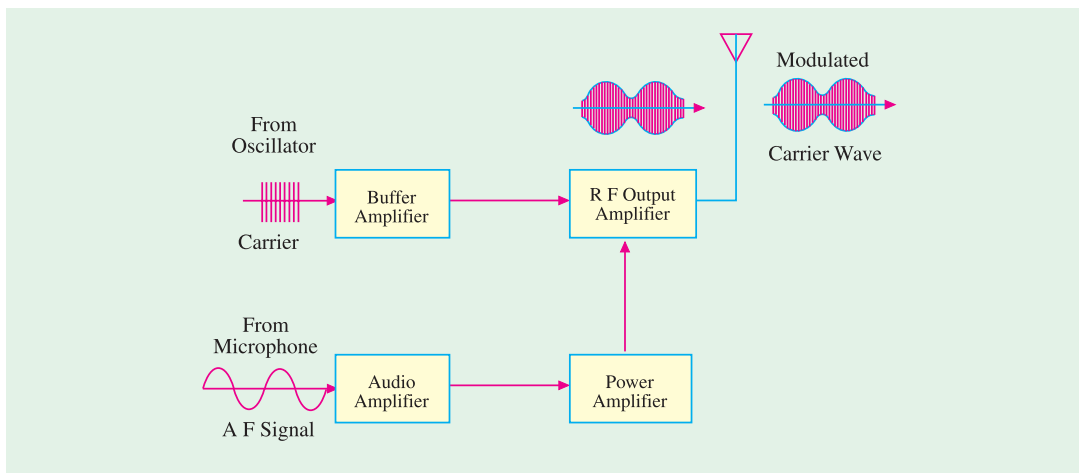


Fig. 66.16

**66.19. Modulating Amplifier Circuit**

It is shown in Fig. 66.17. The carrier signal from a crystal oscillator is coupled to the base of *Q* through transformer *T<sub>1</sub>*. The low-frequency modulating signal is also coupled to the base of *Q* by capacitor *C<sub>1</sub>*. The voltage divider *R<sub>1</sub> – R<sub>2</sub>*, as usual, provides proper forward bias for the transistor whereas *C<sub>2</sub>* and *C<sub>3</sub>* are bypass capacitors.

The modulating signal applied across *R<sub>2</sub>* causes variations in the base bias in step with its own variations. The amplitude of the transistor *RF* output changes according to the changes in *AF* modulating signal. The *L<sub>1</sub> C<sub>4</sub>* circuit is kept tuned to the carrier frequency. The amplitude-modulated

carrier or *RF* current in  $L_1$  induces similar current in  $L_2$  which is connected to an antenna. Finally, the *AM* carrier wave is radiated out in space by the transmitting antenna.

### 66.20. Frequency Modulation OR CAD Simulation Diagram.

As the name shows, in this modulation, it is only the **frequency of the carrier which is changed and not its amplitude.**

The **amount of change** in frequency is determined by the **amplitude** of the modulating signal whereas **rate of change** is determined by the **frequency** of the modulating signal. As shown in Fig. 16.18, in an *FM* carrier, information (or intelligence) is carried as variations in its frequency. As seen, frequency of the modulated carrier increases

as the signal amplitude increases but decreases as the signal amplitude decreases. It is at its highest frequency (point *H*) when the signal amplitude is at its maximum positive value and is at its lowest frequency (point *L*) when signal amplitude has maximum negative value. When signal amplitude is zero, the carrier frequency is at its normal frequency  $f_0$  (also called **resting or centre** frequency.)

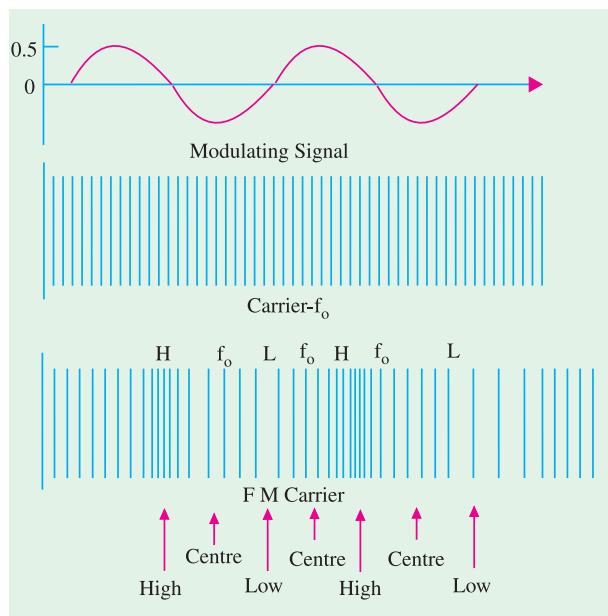


Fig. 66.18

swing between its maximum frequency and lowest frequency 1000 times per second. If  $f_m = 2$  kHz, the rate of frequency swing would be twice as fast :

- In short, we have established two important points about the nature of frequency modulation:
- (i) The **amount** of frequency deviation (or shift or variation) depends on the **amplitude** (loudness) of the audio signal. **Louder the sound, greater the frequency deviation and vice-versa.**

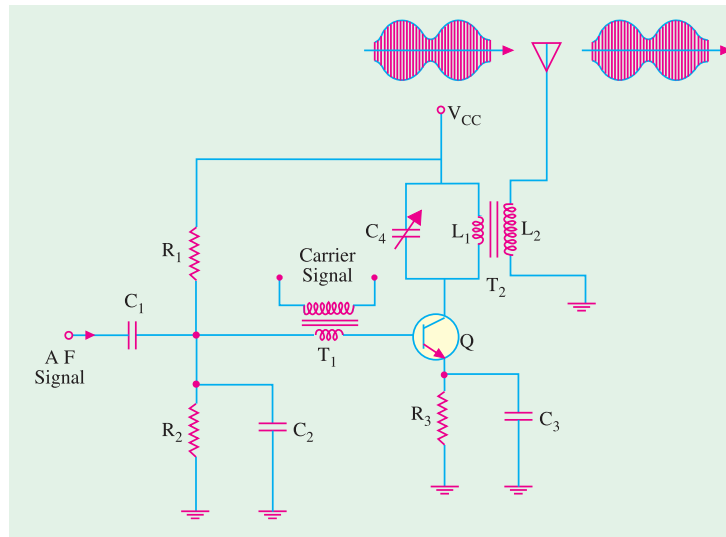


Fig. 66.17

As the name shows, in this modulation, it is only the **frequency of the carrier which is changed and not its amplitude.**

The fact that amount of change in frequency depends on signal amplitude is illustrated in Fig. 66.19 where *R* stands for resting frequency. Here, signal amplitude is almost double of that in Fig. 66.18 **though its frequency is the same.** This **louder** signal causes **greater** frequency change in modulated carrier as indicated by **increased bunching and spreading** of the waves as compared with relatively weaker signal of Fig. 66.18.

The **rate** at which frequency shift takes place depends on the signal frequency as shown in Fig. 66.20. For example, if the modulating signal is 1 kHz, then the modulated carrier will

However, for the purposes of *FM* broadcasts, it has been internationally agreed to restrict maximum deviation to **75 kHz** on each side of the centre frequency for sounds of maximum loudness. Sounds of lesser loudness are permitted **proportionately less frequency deviation**.

- (ii) The **rate** of frequency deviation depends on the **signal frequency**.

### 66.21. Frequency Deviation and Carrier Swing

The frequency of an *FM* transmitter without signal input is called the **resting frequency** or **centre frequency** ( $f_0$ ) and is the allotted frequency of the transmitter. In simple words, it is the carrier frequency on which a station is allowed to broadcast. When the signal is applied, the carrier frequency deviates up and down from its resting value  $f_0$ .

This change or shift either above or below the resting frequency is called frequency deviation ( $\Delta f$ )

The total variation in frequency from the lowest to the highest is called carrier swing (CS). Obviously,

$$\text{carrier swing} = 2 \times \text{frequency deviation of } CS = 2 \times \Delta f$$

A maximum frequency deviation of 75 kHz is allowed for commercial *FM* broadcast stations in the 88 to 168 MHz *VHF* band. Hence, *FM* channel width is  $275 = 150$  kHz. Allowing a 25 kHz guard band on either side, the channel width becomes  $= 2(75 + 25) = 200$  kHz (Fig. 66.21). This guard band is meant to prevent interference between adjacent channels. However, a maximum frequency deviation of 25 kHz is allowed in the sound portion of the TV broadcast.

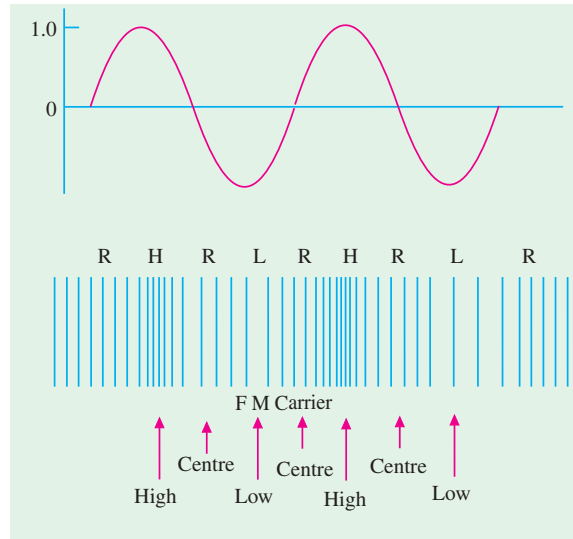


Fig. 66.19

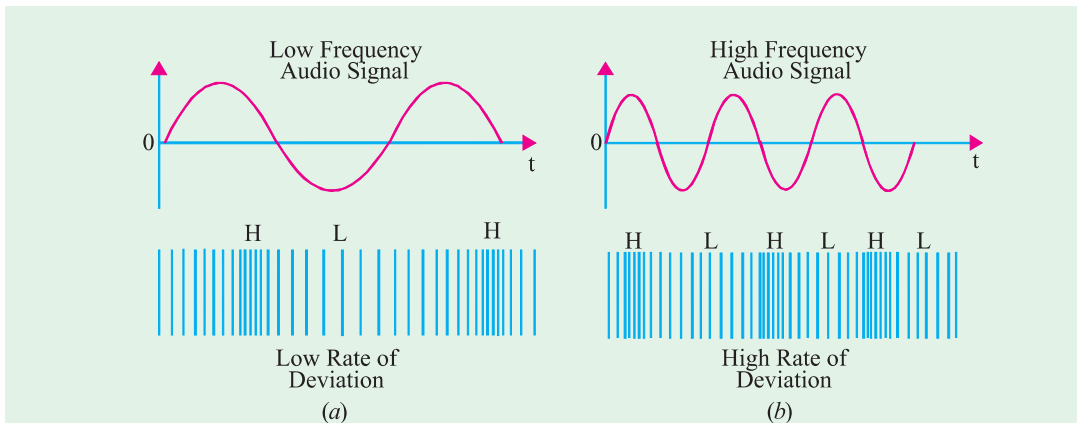


Fig. 66.20

In *FM*, the highest audio frequency transmitted is 15 kHz.

Consider an *FM* carrier of resting frequency 100 MHz. Since  $(\Delta f)_{max} = 75$  kHz, the carrier frequency can swing from the lowest value of 99.925 MHz to the highest value of 100.075 MHz. Of course, deviations lesser than 75 kHz corresponding to relatively softer sounds are always permissible.

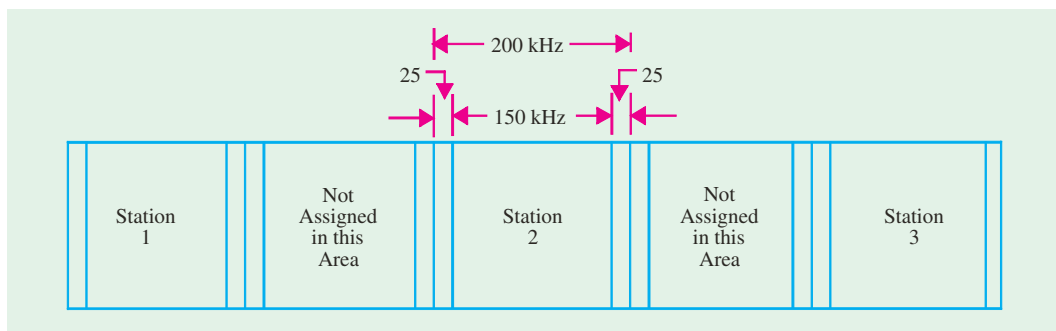


Fig. 66.21

### 66.22. Modulation Index

It is given by the ratio  $m_f = \frac{\text{frequency deviation}}{\text{modulation frequency}} = \frac{\Delta f}{f_m}$

Unlike amplitude modulation, this **modulation index can be greater than unity**. By knowing the value of  $m_f$  we can calculate the number of significant sidebands and the bandwidth of the *FM* signal.

### 66.23. Deviation Ratio

It is the worst-case modulation index in which maximum permitted frequency deviation and maximum permitted audio frequency are used.

$$\therefore \text{deviation ratio} = \frac{(\Delta f)}{f_{m(\max)}}$$

Now, for *FM* broadcast stations,  $(\Delta f)_{\max} = 75 \text{ kHz}$  and maximum permitted frequency of modulating audio signal is 15 kHz.

$$\therefore \text{deviation ratio} = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

$$\text{For sound portion of commercial TV deviation ratio} = \frac{25 \text{ kHz}}{15 \text{ kHz}} = 1.67$$

### 66.24. Percent Modulation

When applied to *FM*, this term has slightly different meaning than when applied to *AM*. In *FM*, it is given by the ratio of actual frequency deviation to the maximum allowed frequency deviation.

$$m = \frac{(\Delta f)_{\text{actual}}}{(\Delta f)_{\max}}$$

Obviously, 100% modulation corresponds to the case when actual deviation equals the maximum allowable frequency deviation. If, in some case, actual deviation is 50 kHz, then

$$m = \frac{50}{75} = \frac{2}{3} = 0.667 = \mathbf{66.7\%}$$

Value of  $m = 0$  corresponds to zero deviation *i.e.* unmodulated carrier wave. It is seen from the above equation that  $m \propto (\Delta f)_{\text{actual}}$ . It means that when **frequency deviation (*i.e.* signal loudness) is doubled, modulation is doubled**.

**Example 66.12.** What is the modulation index of an *FM* carrier having a carrier swing of 100 kHz and a modulating signal of 5 kHz?

**Solution.**  $CS = 2 \times \Delta f$   
 $\therefore \Delta f = \frac{CS}{2} = \frac{100}{2} = 50 \text{ kHz} \therefore m_f = \frac{\Delta f}{f_m} = \frac{50}{5} = 10$

**Example. 66.13.** An FM transmission has a frequency deviation of 18.75 kHz. Calculate percent modulation if it is broadcast  
 (i) in the 88-108 MHz band (ii) as a portion of a TV broadcast.  
 (Elect. and Comm. Engg., Madurai Kamaraj Univ. 1990)

**Solution.** (i) For this transmission band,

$(\Delta f)_{max} = 75 \text{ kHz} \therefore m = \frac{18.75}{75} \times 100 = 25\%$

(ii) In this case,  $(\Delta f)_{max} = 25 \text{ kHz} \therefore m = \frac{18.75}{25} \times 100 = 75\%$

**Example 66.14.** An FM signal has a resting frequency of 105 MHz and highest frequency of 105.03 MHz when modulated by a signal of frequency 5 kHz. Determine

- (i) frequency deviation, (ii) carrier swing, (iii) modulation index,  
 (iv) percent modulation, (v) lowest frequency reached by the FM wave.  
 (Electronics and Comm. Engg., Osmania Univ. 1992)

**Solution.** (i)  $\Delta f = 105.03 - 105 = 0.03 \text{ MHz} = 30 \text{ kHz}$

(ii)  $CS = 2 \times \Delta f = 2 \times 30 = 60 \text{ kHz}$  (iii)  $m_f = \frac{30}{5} = 6$

(iv)  $m = \frac{30}{5} \times 100 = 40\%$  (v) **lowest frequency** =  $105 - 0.03 = 104.97 \text{ kHz}$

### 66.25. FM Sidebands

In FM, when a carrier is modulated, a number of sidebands are formed.\* Though theoretically their number is infinite, their strength becomes negligible after a few sidebands. They lie on both sides of the centre frequency spaced  $f_m$  apart as shown in Fig.66.22. Sidebands at equal distances from  $f_0$  have equal amplitudes. If  $f_0$  is the centre frequency and  $f_m$  the frequency of the modulating signal, then FM carrier contains the following frequencies :

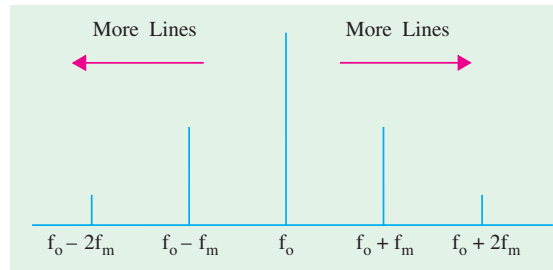


Fig. 66.22

- (i)  $f_0$  (ii)  $f_0 \pm f_m$  (iii)  $f_0 \pm 2 f_m$  (iv)  $f_0 \pm 3 f_m$  and so on

The bandwidth occupied by the spectrum is  $BW = 2nf_m$  where  $n$  is the highest order of the significant sideband.

Another approximate expression for spectrum bandwidth is  $BW = 2(1 + m)f_m$

Now,  $m_f = \frac{\Delta f}{f_m}$ , hence  $BW = 2(\Delta f + f_m)$

This expression is based on the assumption that sidebands having amplitudes less than 5% of the unmodulated carrier wave are negligible or when  $m_f$  is at least 6.

### 66.26. Modulation Index and Number of Sidebands

It is found that the number of sidebands

1. depends **directly** on the amplitude of the modulating signal,
2. depends **inversely** on the frequency of the modulating signal.

\* An AM signal has only two side frequencies for each modulating frequency (Art. 16.11)

Since frequency deviation is directly related to the amplitude of the modulating signal, the above two factors can be combined in one factor called **modulation index**.

Hence, number of sidebands depends on  $m_f = \Delta f / f_m$

Obviously, the number of pairs of sidebands

(i) **increases** as frequency deviation (or amplitude of modulating signal) **increases**.

(ii) **increases** as the modulating signal frequency **decreases**.

Table No. 66.2

$m_f$	0.5	1	2	3	4	5	6	7
$n$	2	3	4	6	7	8	9	11
$BW = 2nf_m$	$4f_m$	$6f_m$	$8f_m$	$12f_m$	$14f_m$	$16f_m$	$18f_m$	$22f_m$
$m_f$	8	9	10	11	12	13	14	15
$n$	12	13	14	15	16	17	18	19
$BW = 2nf_m$	$24f_m$	$26f_m$	$28f_m$	$30f_m$	$32f_m$	$34f_m$	$36f_m$	$38f_m$

Table 66.2 summarizes the relation between the number (n) of sidebands on either side of the carrier and the corresponding bandwidth of the spectrum.

**Example 66.15.** A 5 kHz audio signal is used to frequency-modulate a 100 MHz carrier causing a frequency deviation of 20 kHz. Determine

- (i) modulation index                      (ii) bandwidth of the FM signal.

**Solution.** (i)  $m_f = \frac{\Delta f}{f_m} = \frac{20}{5} = 4$

As seen from Table 66.2  $BW = 14f_m = 14 \times 5 = 70 \text{ kHz}$

**Note.** We cannot use the alternate expression for  $BW$  given in Art. 66.25 above because  $m_f < 6$

**Example 66.16.** In an FM circuit, the modulation index is 10 and the highest modulation frequency is 20 kHz. What is the approximate bandwidth of the resultant FM signal ?

(Applied Electronics, Bombay Univ. 1990)

**Solution.** Since the value of  $m_f$  is more than 6, we will use the expression

$$BW = 2(\Delta_f + f_m)$$

Now,  $m_f = \frac{\Delta f}{f_m}$  or  $10 = \frac{\Delta f}{20} \quad \therefore \quad \Delta_f = 200 \text{ kHz}$

$\therefore BW = 2(200 + 20) = 440 \text{ kHz}$

### 66.27. Mathematical Expression for FM Wave

The unmodulated carrier is given by  $e_c = A \sin 2\pi f_0 t$

The modulating signal frequency is given by  $e_m = B \sin 2\pi f_m t$

The modulated carrier frequency  $f$  swings around the resting frequency  $f_0$  thus

$$f = f_0 + \Delta_f \cdot \sin 2\pi f_m t$$

Hence, equation for the frequency-modulated wave becomes

$$\begin{aligned} e &= A \sin 2\pi f t = A \sin [2\pi(f_0 + \Delta_f \cdot \sin 2\pi f_m t) t] \\ &= A \sin (2\pi f_0 t + \frac{\Delta f}{f_m} + \cos 2\pi f_m t) = A \sin (2\pi f_0 t + m_f \cos 2\pi f_m t) \end{aligned}$$

### 66.28. Demodulation or Detection

When the  $RF$  modulated waves, radiated out from the transmitter antenna, after travelling through space, strike the receiving aerials, they induce very weak  $RF$  currents and voltages in them. If these high-frequency currents are passed through headphones or loudspeakers, they produce no effect on them because all such sound-producing devices are unable to respond to such high frequencies due to large inertia of their vibrating discs etc. Neither will such  $RF$  currents produce any effect on human ear because their frequencies are much beyond the audible frequencies (20 to 20,000 Hz approximately). Hence, it is necessary to demodulate them first in order that the sound-producing devices may be actuated by audio-frequency current similar to that used for modulating the carrier wave at the broadcasting station.

This process of **recovering  $AF$  signal from the modulated carrier wave is known as demodulation or detection.**

The demodulation of an  $AM$  wave involves two operations :

- (i) rectification of the modulated wave and
- (ii) elimination of the  $RF$  component of the modulated wave.

However, demodulation of an  $FM$  wave involves three operations (i) conversion of frequency changes produced by modulating signal into corresponding amplitude changes, (ii) rectification of the modulating signal and (iii) elimination of  $RF$  component of the modulated wave.

### 66.29. Essentials of AM Detection

For recovering the  $AF$  waveform from modulated wave (a mixture of  $AF$  wave and  $RF$  carrier), it is essential to find some way of reducing (or better, eliminating) one half of the modulated wave. The result of this elimination (or rectification) would be that the average value of the wave would not be zero because, now, the impulse would be all in one direction as shown in Fig. 66.23. If this new wave is now passed through a headphone shunted by a suitable capacitor, then  $AF$  wave will pass through the headphone whereas the  $RF$  wave will be by-passed by the capacitor (because the high inductance of magnet coils of the headphones will offer tremendous impedance to  $RF$  currents). Hence, two will become separated.

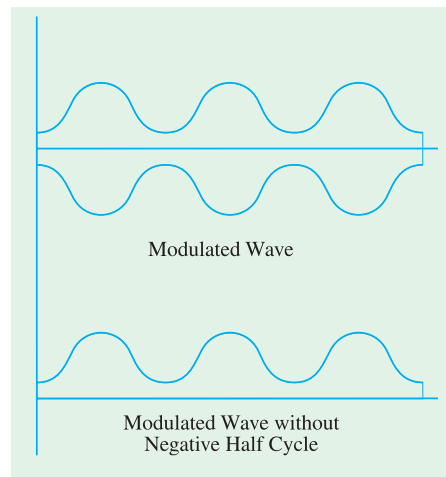


Fig. 66.23

### 66.30. Diode Detector for AM Signals

Diode detection is also known as **envelope-detection** or **linear detection**. In appearance, it looks like an ordinary half-wave rectifier circuit with capacitor input as shown in Fig. 66.24. It is called **envelope detection** because it recovers the  $AF$  signal envelope from the composite signal. Similarly, diode detector is called **linear detector** because its output **is proportional to the voltage of the input signal\***.

#### Circuit Action

Of the various  $RF$  voltages induced in the receiver aerial, only those having the **same** frequency as the resonant frequency of  $LC$  circuit are tuned in due to electromagnetic induction between coils  $L_1$  and  $L$ . By varying  $C$ , the resonant frequency of the  $LC$  circuit can be varied and hence  $RF$  signal of any desired frequency can be tuned in. This input signal is rectified by the diode and passed on to the low-pass filter  $RC_f$ .

\* A detector circuit in which the rectified output is proportional to the square of the input signal voltage is called square-law or non-linear detector.

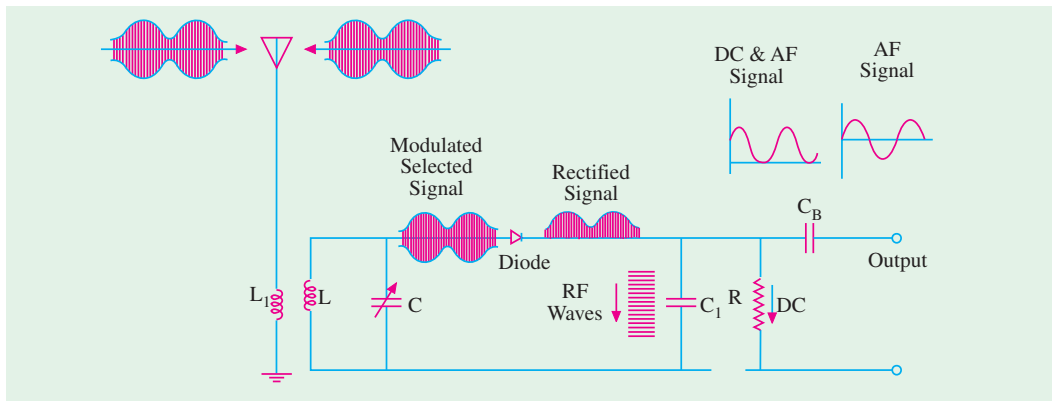


Fig. 66.24

The *RF* carrier wave is filtered out by low-reactance capacitor  $C_1$  which is called *RF* filter capacitor or *RF* by-pass capacitor. The dc component of the remaining signal is shunted out through  $R$  because it cannot pass through blocking capacitor  $C_B$ . But the low, frequency *AF* signal can easily get through  $C_B$  and becomes available across the output. When passed through a suitable device, say, a headphone, the original sound can be heard.

**Advantages**

Diode detectors are extensively used in AM broadcast receivers because they have the following advantages :

1. They can handle comparatively large input signals;
2. They can be operated as linear or power detectors;
3. They rectify with negligible distortion and, hence, have good linearity;
4. They are well-adapted for use in simple automatic-gain control circuits.

**Disadvantages**

However, the disadvantages are that

1. they do not have the ability to amplify the rectified signal by themselves as is done by a transistor detector (Art. 66.31). However, it is not a very serious drawback since signal amplification can be affected both before and after rectification;
2. while conducting, the diode consumes some power which reduces the  $Q$  of its tuned circuit as well as its gain and selectivity.

**66.31. Transistor Detectors for AM Signals**

Transistors can be used as detector amplifiers *i.e.* both for rectification and amplification. As shown in Fig. 66.25, the *RF* signal is applied at the base-emitter junction where rectification takes place. The amplification of the recovered signal takes place in the emitter-collector circuit.

It should be noted that when used as a detector, the transistor has to be biased at cut-off or operated as class-B. As is usual,  $C_2$  ac grounds  $R_3$  whereas voltage divider  $R_1 - R_2$  establishes proper bias which holds the transistor just at cut-off.

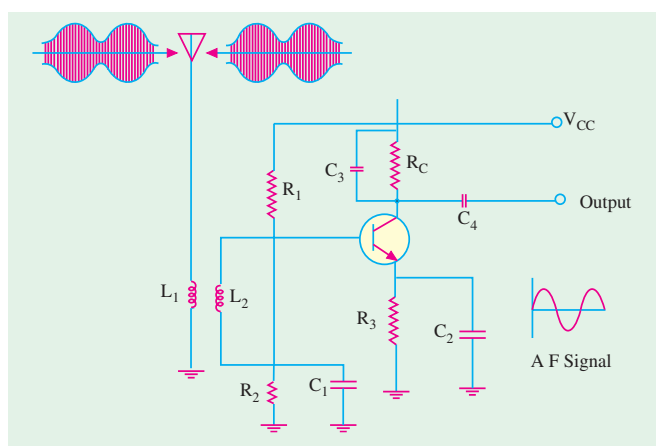


Fig. 66.25



Obviously, only positive half-cycles of the modulated *RF* input signal will drive the transistor into conduction, whereas negative half-cycles would be cut-off. Thus, rectification of the input signal takes place in the base-emitter circuit. The small variations in base current produced by the positive half-cycle of the input signal produce large ( $\beta$  time) variations in the collector current. Hence, an amplified version of the rectified signal appears across  $R_C - C_3$  combination.  $C_3$  eliminates *RF* component so that only *AF* signal voltage drops across  $RC$ . The capacitor  $C_4$  permits *AF* signal to pass through but blocks its dc component.

**66.32. FM Detection**

As discussed earlier, an *FM* carrier signal contains information (or intelligence we wish to convey) in the form of frequency variations above and below the centre frequency of the carrier. For recovering the information, we must first convert the *FM* signal in such a way that it appears as a modulated *RF* voltage across the diode. A simple method of converting frequency variations into voltage variations is to make use of the principle that reactance (of coil or capacitor) varies with frequency. When an *FM* signal is applied to an inductor, the current flowing through it varies in amplitude according to the changes in frequency of the applied signal. Now, changes in frequency of the *FM* signal depend on the amplitude of the modulating *AF* signal. Hence, the current in the inductor varies as per the amplitude of the original modulating signal. In this way, frequency changes in *FM* signal are converted into amplitude changes in current. These changes in current when passed through a resistor produce corresponding changes in voltage.

Hence, we find that, ultimately, frequency variations in *FM* signal are converted into voltage changes. Also, there exists a linear relation between the two – something essential for distortion-less demodulation.

*FM* demodulation may be carried out with the help of (i) ratio detector and (ii) quadrature detector.

**66.33. Quadrature Detector**

This detector depends on the frequency/phase relationship of a tuned circuit. It uses only one tuned circuit and is becoming increasingly popular in the integrated *FM* strips.

**Theory**

Let us first consider the general principle. A sinusoidal current is given by the equation

$$i = I_m \sin \theta = I_m \sin \omega t$$

Suppose, it flows through a circuit shown in Fig. 66.27 (a). The voltage  $V_L$  across the inductor (assumed pure) leads the current  $I$  by  $90^\circ$

$$\therefore V_L = V_L \cos \omega t$$

The voltage  $V_z$  across the parallel tuned circuit will be in phase with  $I$  at resonance. However, at frequencies slightly different ( $\pm 1\%$ ) from the resonant frequency, the phase angle  $f$  will be given by

$$\tan \phi = \frac{yQR}{R} = yQ \quad \text{assuming } f_0/f \approx 1. \quad \text{— Fig.66.26}$$

Here 
$$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \frac{f}{f_0} - \frac{f_0}{f} \approx \frac{2\Delta f}{f_0}$$

where  $\Delta f = f - f_0$

$f_0$  = resonant frequency

$f$  = slightly off-resonance frequency

Hence, equation for  $V_z$  is given by  $V_z = V_z \sin(\omega t - f)$

When the two voltages  $V_L$  and  $V_z$  are applied as inputs to a multiplier, the output voltage  $V_0$  is found to be proportional to their product as shown in Fig. 66.27(b).

$$\begin{aligned} V_0 &\propto V_L \cdot V_z \propto \cos \omega t \cdot \sin(\omega t - f) \\ &\propto \sin(2\omega t + f) + \sin \phi \end{aligned}$$

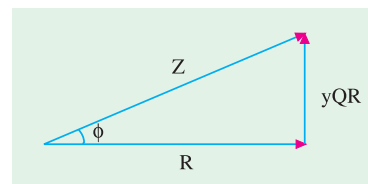


Fig. 66.26

A low-pass filter is used to reject the double frequency component  $\sin(2\omega t + \phi)$  and select only the low-frequency  $\sin \phi$  component.

$$\begin{aligned} \therefore V_0 &\propto \sin \phi \propto \tan \phi && \text{— since } \phi \text{ is very small} \\ &\propto y\phi \propto y \propto 2 \cdot \frac{\Delta f}{f_0} && \text{— where } V_m \text{ is the modulating AF voltage.} \end{aligned}$$

Now,  $\Delta f = f - f_0 \propto k V_m$   
 $\therefore V_0 \propto V_m$

It shows that output voltage  $v_0$  is proportional to the original modulating signal voltage.

### 66.34. Frequency Conversion

Let us first consider its necessity for radio broadcasting purposes and the basic principle of heterodyning action.

#### 1. Need

It is very difficult to design amplifiers which give uniformly high gain over a wide range of radio frequencies used in commercial broadcast stations. However, it is possible to design high-gain uniform amplification over a narrow band of comparatively lower frequencies called intermediate frequencies (*IF*). Hence, it is necessary to convert the modulated *RF* carrier into modulated *IF* carrier by using a frequency converter.

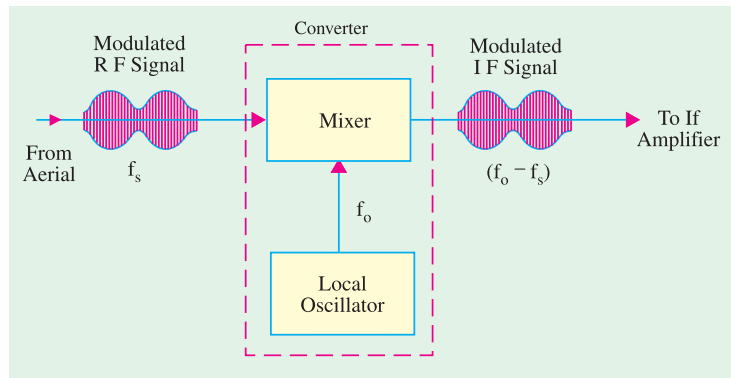


Fig. 66.28

This *IF* signal is then amplified by narrow-band *IF* amplifiers and passed on to the *AM* detectors.

#### 2. Basic Principle

The frequency conversion can be achieved by utilizing the heterodyne principle. For this purpose, the modulated *RF* signal is mixed (in a mixer) with an unmodulated *RF* signal produced by local oscillator as shown in Fig. 66.28.

The oscillator and the mixer may be either two separate devices or may be combined into one device called converter. The process of combining two ac signals of different frequencies in order to obtain a signal of new frequency is called heterodyning action.

#### 3. Heterodyning Action

Suppose the carrier signal of frequency  $f_s$  is heterodyned with another signal of frequency  $f_0$ , then two additional signals are produced whose frequencies are :

- (i)  $f_0 + f_s$  — the sum component
- (ii)  $f_0 - f_s$  — the difference component

Usually, the sum frequency is removed by bandpass filtering. The difference frequency (also called **beat frequency**) is retained and forms the *IF* frequency in *AM* receivers.

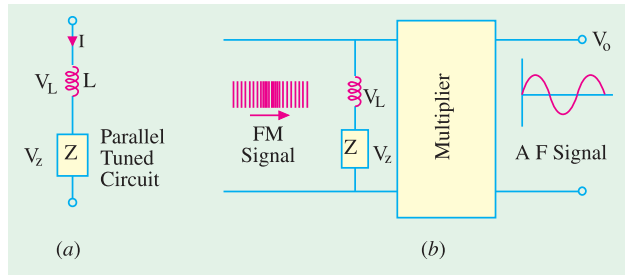


Fig. 66.27

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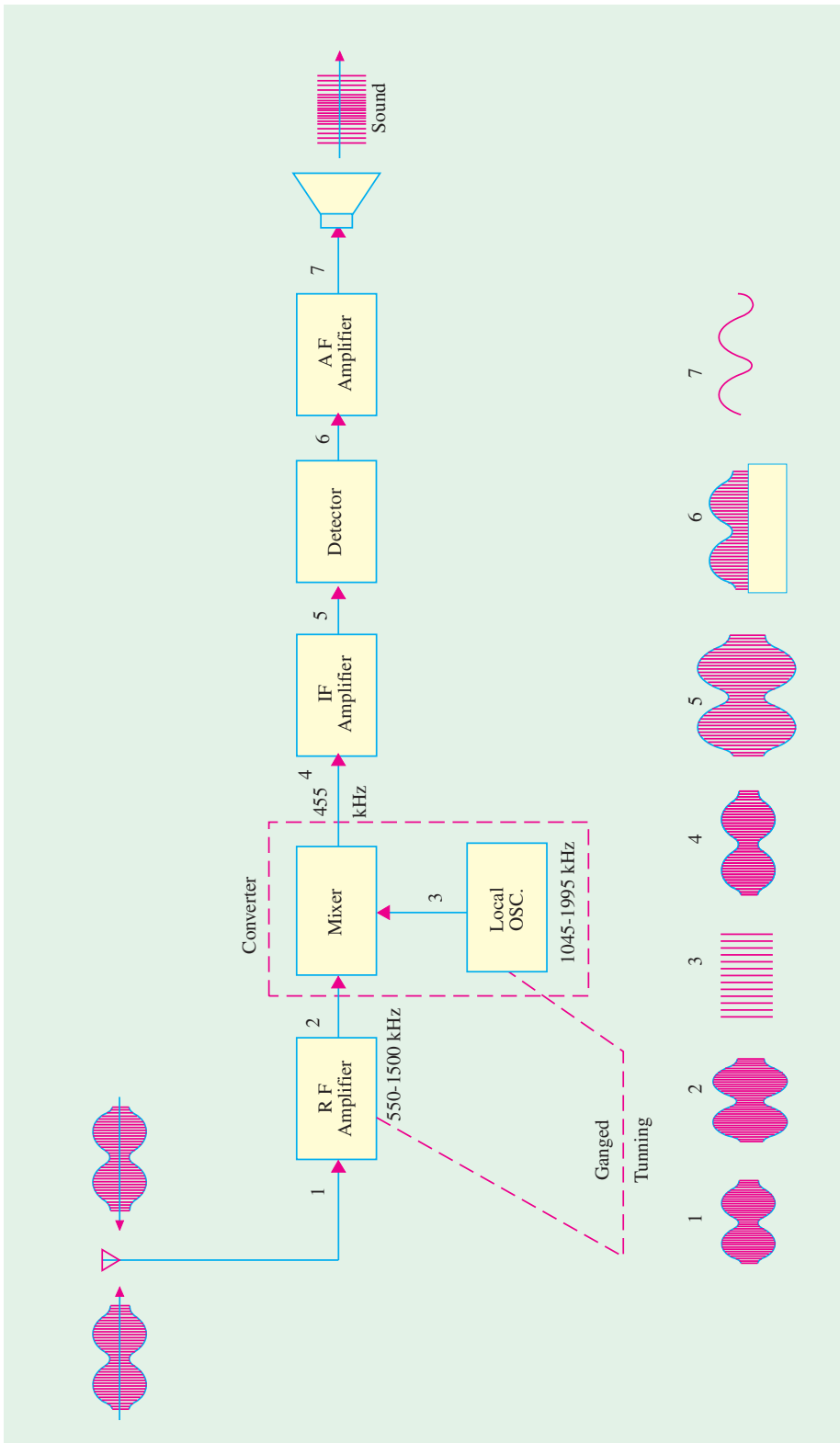


Fig. 66.29

Suppose, an AM carrier signal of frequency 1500 kHz is mixed with an unmodulated carrier (produced by local oscillator) of frequency 1955 kHz. Then, following two frequencies are produced

- (i) 3455 kHz and (ii) 455 kHz

The **higher frequency** is generally filtered out leaving behind the difference frequency of 455 kHz which forms the *IF* frequency.

### 66.35. Superhetrodyne AM Receiver

It is also referred to as superhet and is extensively used in modern AM receivers. Its block diagram with signals is shown in Fig. 66.29. The operation of this receiver is as under :

- (i) Let us assume that the incoming signal frequency is 1500 kHz. It is first amplified by the *R.F.* amplifier.
- (ii) Next, it enters a mixer circuit which is so designed that it can conveniently combine two radio frequencies—one fed into it by the *R.F.* amplifier and the other by a local oscillator.
- (iii) The local oscillator is an *RF* oscillator whose frequency of oscillation can be controlled by varying the capacitance of its capacitor. In fact, the tuning capacitor of the oscillator is ganged with the capacitor of the input circuit so that the difference in the frequency of the selected signal and oscillator frequency is always constant. Usually, the difference is maintained at 455 kHz. If signal frequency is 1,500 kHz, then oscillator frequency can be either 1,955 or 1,045 kHz. Let us suppose that it is 1,955 kHz. In fact, local oscillator frequency is always higher than the frequency of the incoming signal.
- (iv) When two alternating currents of these two different frequencies are combined in the mixer transistors, then phenomenon of beats is produced. In the present case, the beat frequency is  $1955 - 1500 = 455$  kHz. Since this frequency is lower than the signal frequency but still above the range of audio frequencies, it is called **intermediate frequency (IF)**.
- (v) The 455 kHz output of the mixer is then passed on to the *IF* amplifier which is **fixed-tuned** to 455 kHz frequency. In practice, one or more stages of *IF* amplification may be used.
- (vi) The output of *IF* amplifier is demodulated by a detector which provides the audio signal.
- (vii) This audio signal is amplified by the audio-frequency (*AF*) amplifier whose output is fed to a loud-speaker which reproduces the original sound.

### 66.36. Standard Superhet AM Receiver

Fig. 66.30 shows the block diagram of a standard superhet AM receiver alongwith its seven-transistor circuit diagram. Usually, transistor radios are all-wave radios having two or more bands. With the help of a band switch, the coils and trimmers of any one band can be connected to the circuit at will. For the sake of simplicity, the coils and trimmers of one band only have been shown in Fig. 66.30. The operation of various stages and the functions of various radio components used in the radio circuit of Fig. 66.30 are described below.

#### 1. Frequency Changer or Converter

It is the first stage\* in a transistor radio and has two sections—oscillator and mixer. A single silicon transistor  $Q_1$  performs both the above functions.  $L_4$  is the oscillator coil whereas  $L_2$  is the signal input coil. Resistors  $R_1$  and  $R_2$  form a voltage divider for giving base bias to  $Q_1$

The signals intercepted by the aerial are picked up by coil  $L_1$  which is tuned to the transmission frequency of the desired station with the help of ganged capacitor  $C_1$  and its fine-tuning trimmer  $C_2$ . This signal is then induced into the secondary winding  $L_2$  which passes it on to the base of  $Q_1$  via  $C_3$ . The local oscillations produced by  $L_4$  and ganged capacitor  $C_7$  are given to the emitter of  $Q_1$  via  $C_4$

As seen,  $Q_1$  has been used in *CE* configuration for producing oscillations. The positive feedback required for producing oscillations is given from the collector of  $Q_1$  to its emitter through  $L_3$ ,  $L_4$  and  $C_4$ .

\* Sometimes, there is an RF stage before it (Fig. 66.29)

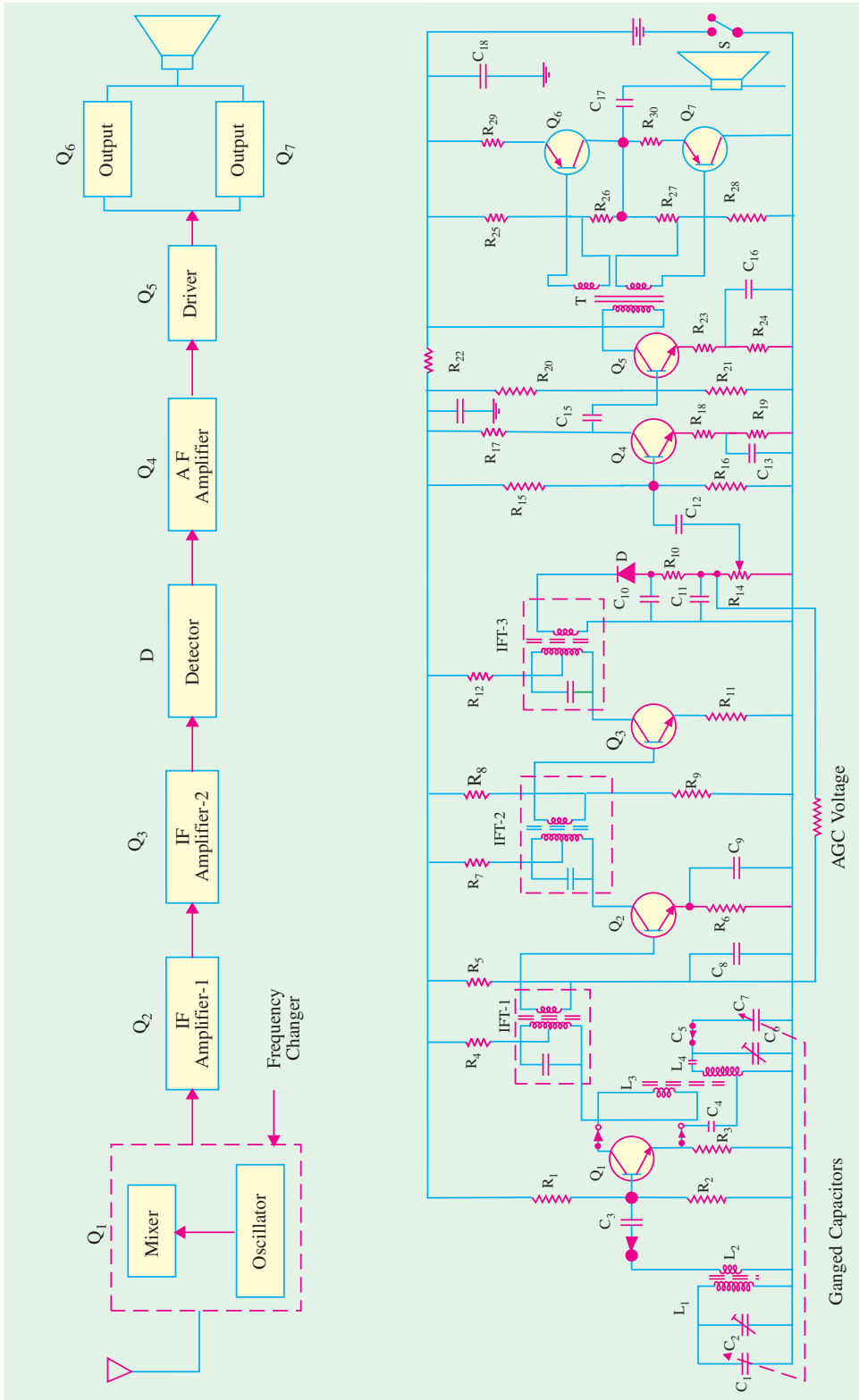


Fig. 66.30

The transistor  $Q_1$  mixes the incoming signal frequency  $f_s$  and oscillation frequency  $f_0$  thereby producing the sum and difference frequencies of  $(f_0 + f_s)$  and  $(f_0 - f_s)$  respectively. The difference frequency called *IF* frequency is then passed on to the *IF* transformer IFT-1 connected in the collector of  $Q_1$ .

## 2. IF Amplifier

As the name indicates, its function is to amplify the *IF* signal obtained from the frequency changer. Usually, two stages are used to obtain sufficient gain. The gain of the first stage is controlled by the automatic gain control circuit (*AGC*) with the help of dc voltage drop across  $R_{10}$ . This dc voltage is obtained from the detector diode. Three fixed-tuned *IF* transformers are used—one at the collector of the frequency changer and the other two for the two *IF* amplifier stages. These transformers are also used as collector load as well as for inter-stage-coupling.

Resistor  $R_5$  gives base bias to  $Q_2$  whereas capacitor  $C_8$  decouples the *AGC* voltage. As usual,  $R_8$  and  $R_9$  form the voltage divider circuit for giving base bias to  $Q_3$ .

The amplified *IF* signal obtained from IFT-2 is further amplified by IFT-3 of the second *IF* amplifier stage before it is passed on to the detector diode.

## 3. Detector Circuit

It consists of a diode\* which rectifies the *IF* signal. Resistor  $R_{13}$ , capacitors  $C_{10}$  and  $C_{11}$  together filter out *RF*. The audio signal is given to the following audio section through potentiometer  $R_{14}$  which is also used as a volume control. Capacitor  $C_{12}$  blocks the dc component in the rectified signal. This dc voltage is used for *AGC* via  $R_{10}$ . The main purpose of using *AGC* is to minimize the variations in sound due to changes in signal strength (because of fading) and to prevent overloading of second *IF* stage because of a very strong signal from a close-by transmitter.

## 4. Audio Frequency (AF) Amplifier

The audio section consists of three stages *i.e.* audio amplifier, driver and output which feeds a loud speaker. The audio-amplifiers amplify the audio signal given to  $Q_4$  through volume control and  $C_{12}$ .  $R_{15}$  and  $R_{16}$  are biasing resistors whereas  $R_{19}$  is the emitter resistor with a by-pass capacitor  $C_{13}$ . The amplified signal is developed over collector load resistor  $R_{17}$ . This amplified signal is capacitively-coupled to the driver stage through  $C_{14}$ .

## 5. Driver

The driver stage further amplifies the audio signal obtained from the previous *AF* amplifier till it is large enough to drive the output stage. Transformer  $T$  (having two secondary windings) is used for coupling to the output stage. The emitter resistor of  $Q_5$  is split into two parts ( $R_{23}$  and  $R_{24}$ ) with only  $R_{24}$  being by-passed by  $C_{16}$ . This provides some negative feedback and reduces distortion.

## 6. Output Stage

The function of this stage is to amplify the audio signal received from the driver stage and hence provide sufficient power to drive the loudspeaker. Class-B push-pull amplifier employing transistors  $Q_6$  and  $Q_7$  is used for economizing battery consumption. Resistors  $R_{25}$  and  $R_{26}$  provide base bias to  $Q_6$  and  $R_{27}$  and  $R_{28}$  to  $Q_7$  whereas  $R_{29}$  and  $R_{30}$  are the emitter resistors of the two respective transistors. The audio signal is given via  $C_{17}$  to the loudspeaker which converts it into sound similar to the one transmitted by the broadcasting transmitter.

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\* Sometimes, transistors are used as detectors.

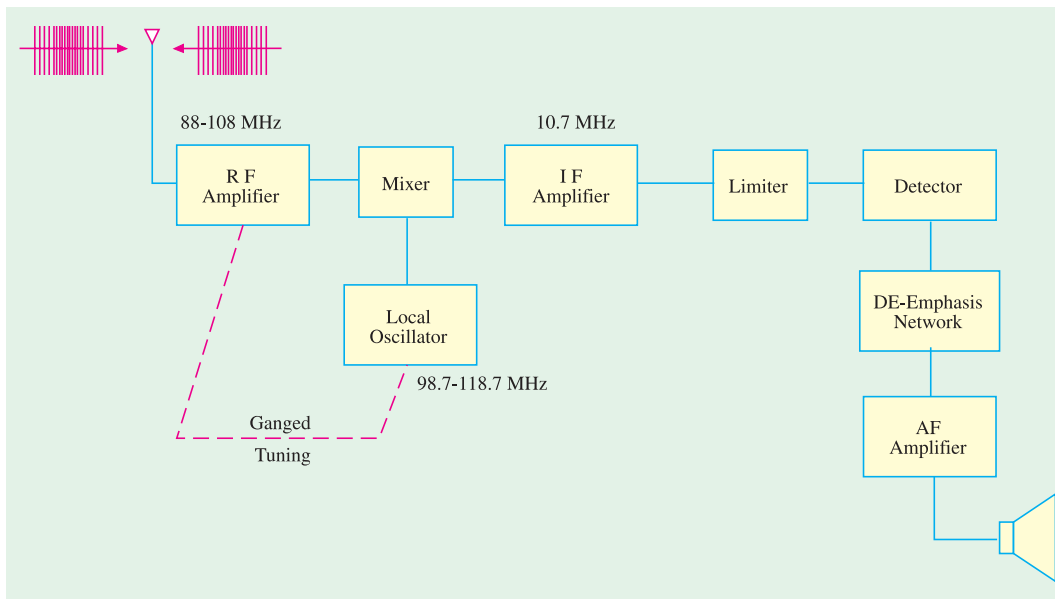


Fig. 66.31

### 66.37. FM Receiver

The block diagram for such a receiver is shown in Fig. 66.31. These receivers operate in the *VHF* band of 88-108 MHz and have an *IF* of 10.7 MHz with a bandpass of  $2 \times 75 \text{ kHz} = 150 \text{ kHz}$ . Like *AM* receivers, such receivers also have a frequency converter and *IF* amplifiers.

The function of the **limiter** is to remove all **amplitude variations** (caused by noise) from *IF* signal which might have crept into the *FM* signal. This removal of amplitude variations is necessary for distortionless demodulation. **Limiter is a sort of clipping circuit.**

The de-emphasis network **reduces the amplitude of high frequencies in the audio signal** which was earlier increased by the pre-emphasis network at the transmitting station. It serves to re-establish the **tonal balance** of the speech or music etc. lost in the pre-emphasis. Without it, the sound signal would have a heavy treble effect. It is, in fact, a low-pass filter having a time constant of  $75 \mu\text{s}$ .

### 66.38. Comparison Between AM and FM

Frequency modulation (*FM*) has the following advantages as compared to amplitude modulation (*AM*) :

1. All transmitted power in *FM* is useful whereas in *AM* most of it is in carrier which **erves no useful purpose.**
2. It has high signal-to-noise (S/N) ratio. It is due to two reasons : firstly, there happens to be less noise at *VHF* band and secondly, *FM* receivers are fitted with amplitude limiters which remove amplitude variations caused by noise.
3. Due to 'guard-band' there is hardly any adjacent-channel interference.
4. Since only transmitter **frequency** is modulated in *FM*, only fraction of a watt of audio power is required to produce 100% modulation as compared to high power required in *AM*.

However, *FM* has the following disadvantages :

1. It requires **much wider channel** — almost 7 to 15 times as large as needed by *AM*.
2. It requires complex and expensive transmitting and receiving equipment.
3. Since *FM* reception is limited to only line of sight, area of reception for *FM* is much smaller than for *AM*.

## 66.39. The Four Fields of FM

There are four major areas of application for FM transmission :

1. First use is in FM broadcast band 88-108 MHz with 200 kHz channels in which commercial FM stations broadcast programmes to their listeners.
2. Second use is in TV. Though video signal is amplitude-modulated, sound is transmitted by a separate transmitter which is frequency-modulated.
3. Third use is in the mobile or emergency services which transmit voice frequencies (20-4000 Hz) only.
4. Fourth use is in the amateur bands where again only voice frequencies are transmitted.

## Tutorial Problems No. 66.1

1. A carrier wave has an amplitude of 500 mV. A modulating signal causes its amplitude to vary from 200 mV to 800 mV. What is the percentage modulation? [60%]
2. A 100 MHz carrier having an amplitude of 50 V is amplitude-modulated by a 5 kHz audio signal having an amplitude of 20 V. Find
  - (i) modulation index,
  - (ii) percent modulation,
  - (iii) components of modulated wave,
  - (iv) the amplitudes of sidebands.
 [(i) 0.4 (ii) 40 (iii) 100 MHz, 100.005 MHz, 99.995 MHz (iv) 10 V]
3. An audio signal given by  $30 \sin(2\pi \times 2500t)$  is used for modulating a carrier wave given by the equation  $60 \sin(2\pi \times 200,000t)$ . Find
  - (i) percent modulation,
  - (ii) frequencies of the signal and the carrier,
  - (iii) frequency spectrum of the modulated wave.
 [(i) 50 (ii) 2500 Hz, 200,000 Hz (iii) 200; 202.5; 197.5 kHz]
4. How many AM broadcast stations can be accommodated in a 5 MHz bandwidth if each station transmits a signal modulated by an audio signal having a maximum frequency of 5 kHz. [500]
5. A 100 kHz bandwidth is to accommodate 5 AM broadcasts simultaneously. What is the maximum modulating frequency permissible for each station? [5 kHz]
6. Total power content of an AM signal is 3000 W. For 100 percent modulation, calculate
  - (i) carrier power
  - (ii) power in each sideband
 [(i) 2 kW (ii) 0.5 kW]
7. The total power content of an AM wave is 1320 W. What is the percent modulation if each sideband contains 160 W? [0.8]
8. An SSB-SC signal contains 2 kW. Find power contained in
  - (i) carrier
  - (ii) sideband.
 [(i) 0 (ii) 2 kW]
9. As compared to DSB-FC, Calculate the power saving in SSB-SC when percent modulation is (i) 1 (ii) 0.8 (iii) 0.5
 [(i) 83.3% (ii) 87.87% (iii) 94.4%]
10. What is the modulation index of an FM carrier having a carrier swing of 120 kHz and a modulating signal of 10 kHz. [6]
11. An FM signal has a resting frequency of 100 MHz and the highest frequency of 100.05 MHz when modulated by an audio signal of 5 kHz. Determine
  - (i) frequency deviation,
  - (ii) carrier swing,
  - (iii) modulation index,
  - (iv) percent modulation.
 [(i) 50 kHz (ii) 100 kHz (iii) 10 (iv) 66.7%]
12. A 10 kHz audio signal is used to frequency-modulate a 100 MHz carrier causing a frequency deviation of 50 kHz. Determine
  - (i) modulation index,
  - (ii) bandwidth of FM signal.
 [(i) 5 (ii) 160 kHz]



13. A carrier of  $1\text{MHz}$  with  $400\text{ Watt}$  of its power is amplitude modulated with a sinusoidal signal of  $2500\text{ Hz}$ . The depth of modulation is  $75\%$ . Calculate the side band frequencies, the band width, the power in the side bands and the total power in the modulated wave.  
[(*Electronics Engg. Bangalore Univ. 2004*)]
14. The total power content of an AM wave is  $2.64\text{ kW}$  at a modulation factor of  $80\%$ . Determine the power content of (i) carrier (ii) each side band.  
[(*Electronics Engg. Bangalore Univ. 2002*)]
15. A  $100\text{MHz}$  carrier wave is frequency modulated by a  $10\text{ kHz}$  sinusoidal modulating signal. If the maximum frequency deviation is  $50\text{ kHz}$ . Find the modulation index.  
[(*Electronics Engg. Bangalore Univ., 2002*)]
16. A  $500\text{ W}$ ,  $1\text{MHz}$  carrier is amplitude modulated with a sinusoidal signal of  $1\text{ kHz}$ . The depth of modulation is  $60\%$ . Calculate the band width, power in the side bands and the total power transmitted.  
[(*Electronics Engg., Bangalore Univ., 2003*)]
17. A carrier signal has a peak amplitude of  $100\text{V}$ . Modulation index is  $40\%$ . Power is developed across a load of  $100\ \Omega$ . Determine the value of transmitted power.  
(*Electronics Engg. Bangalore Univ., 2003*)

### OBJECTIVE TESTS – 66

- The main purpose of modulation is to
  - combine two waves of different frequencies
  - achieve wave-shaping of the carrier wave
  - transmit low-frequency information over long distances efficiently
  - produce sidebands.
- Demodulation
  - is performed at the transmitting station
  - removes side-bands
  - rectifies modulated signal
  - is opposite of modulation.
- In amplitude modulation
  - carrier frequency is changed
  - carrier amplitude is changed
  - three sidebands are produced
  - fidelity is improved.
- $100\%$  modulation is produced in AM when carrier
  - frequency equals signal frequency
  - frequency exceeds signal frequency
  - amplitude equals signal amplitude
  - amplitude exceeds signal amplitude.
- For a given carrier wave, maximum undistorted power is transmitted when value of modulation is
  - 1
  - 0.8
  - 0.5
  - 0.
- In an AM wave with  $100\%$  percent modulation, each sideband carries — of the total transmitted power.
  - one-half
  - one-sixth
  - one-third
  - two-third
- Given a carrier frequency of  $100\text{ kHz}$  and a modulating frequency of  $5\text{ kHz}$ , the bandwidth of AM transmission is \_\_\_ kHz.
  - 5
  - 200
  - 10
  - 20
- When modulation of an AM wave is decreased
  - percentage carrier power is decreased
  - percentage carrier power is increased
  - total transmitted power is increased
  - percentage sideband power is unaffected.
- In AM transmission, power content of the carrier is maximum when  $m$  equals
  - 0
  - 1
  - 0.8
  - 0.5.
- In AM transmission with  $m = 1$ , suppression of carrier cuts power dissipation by a factor of
  - 6
  - 2
  - 3
  - 4
- As compared to DSB-FC  $100\%$  modulated transmission, power saving in SSB-SC system is \_\_\_\_\_ per cent.
  - 94.4
  - 50
  - 100
  - 83.3
- In an AM transmission with  $100\%$  modulation,  $66.7\%$  of power is saved when \_\_\_\_\_ is/are suppressed.
  - carrier
  - carrier and USB
  - carrier and LSB
  - USB and LSB
- In an AM system, full information can be conveyed by transmitting only
  - the carrier
  - the upper sideband
  - the lower sideband
  - any one sideband.

14. A 10 MHz carrier of peak value 10 V is amplitude modulated by a 10 kHz signal of amplitude 6 V. The amplitude of each side band frequency is  
 (a) 3 V (b) 4 V  
 (c) 5 V (d) 6 V
15. The block diagram of an AM receiver is shown in Fig. 66.32. The missing block and its location are respectively

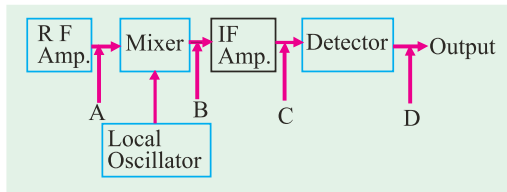


Fig. 66.32

- (a) Band - pass filter at A  
 (b) High-pass filter at B  
 (c) Band - pass filter at C  
 (d) Low-pass filter (audio amplifier) at D.
16. In FM transmission, amplitude of the modulating signal determines  
 (a) rate of frequency variations  
 (b) amount of frequency shift  
 (c) tonal balance of transmission  
 (d) distance of broadcast.
17. In FM, when frequency deviation is doubled  
 (a) modulation is doubled  
 (b) modulation is halved  
 (c) carrier swing is halved  
 (d) modulation index is decreased.
18. In an FM broadcast in VHF band, channel width is \_\_\_\_\_  
 (a) 75 (b) 25  
 (c) 88 (d) 200
19. Modern FM receivers use de-emphasis circuit for  
 (a) reducing high frequency noise  
 (b) reducing the amplitude of high frequencies in the audio signal  
 (c) increasing the amplitude of higher modulating frequencies  
 (d) making demodulation easy
20. One of the serious disadvantages of FM transmission is its  
 (a) high static noise  
 (b) limited line-of-sight range  
 (c) expensive equipment  
 (d) adjacent channel interference.
21. In India, the maximum frequency deviation is about, commercial FM broadcasting signals is about,  
 (a) 5 kHz (b) 15 kHz  
 (c) 7.5 kHz (d) 200 kHz
22. The correct sequence of subsystems in an FM receiver is  
 (a) mixer, RF amplifier, limiter, IF amplifier, discriminator, audio amplifier  
 (b) RF amplifier, mixer, IF amplifier, limiter, discriminator, audio amplifier  
 (c) RF amplifier, mixer, limiter, discriminator, IF amplifier, audio amplifier  
 (d) mixer, IF amplifier, limiter, audio amplifier, discriminator.

### ANSWERS

1. (c) 2. (d) 3. (b) 4. (c) 5. (a) 6. (b) 7. (c) 8. (b) 9. (a) 10. (c) 11. (d)  
 12. (a) 13. (d) 14. (a) 15. (d) 16. (b) 17. (a) 18. (d) 19. (b) 20. (b) 21. (d) 22. (b)

# CHAPTER 67

## Learning Objectives

- What is an Integrated Circuit ?
- Advantages of ICs
- Drawbacks of ICs
- Scale of Integration
- Classification of ICs by Structure
- Comparison between Different ICs
- Classification of ICs by Function
- Linear Integrated Circuits (LICs)
- Manufacturer's Designation of LICs
- Digital Integrated Circuits
- IC Terminology
- Semiconductors Used in Fabrication of ICs and Devices
- How ICs are Made?
- Material Preparation
- Crystal Growing and Wafer Preparation
- Wafer Fabrication
- Oxidation
- Etching
- Diffusion
- Ion Implantation
- Photomask Generation
- Photolithography
- Epitaxy
- Metallization and Interconnections
- Testing, Bonding and Packaging
- Semiconductor Devices and Integrated Circuit Formation
- Popular Applications of ICs

## INTEGRATED CIRCUITS



↑ Jack Kilby would justly be considered one of the greatest electrical engineers of all time for one invention; the monolithic integrated circuit, or microchip. He went on to develop the first industrial, commercial and military applications for this integrated circuits- including the first pocket calculator (pocketronic) and computer that used them

### 67.1. Introduction

Electronic circuitry has undergone tremendous changes since the invention of a triode by Lee De Forest in 1907. In those days, the active components (like triode) and passive components (like resistors, inductors and capacitors etc.) of the circuits were **separate and distinct units** connected by soldered leads.

With the invention of the transistor in 1948 by W.H. Brattain and I. Bardeen, the electronic circuits became considerably reduced in size. It was due to the fact that a transistor was not only cheaper, more reliable and less power consuming but was also much smaller in size than an electron tube. To take advantage of small transistor size, the passive components too were greatly reduced in size thereby making **the entire circuit very small**. Development of printed circuit boards (*PCBs*) further reduced the size of electronic equipment by eliminating bulky wiring and tie points.

In the early 1960s, a new field of **microelectronics** was born primarily to meet the requirements of the Military which wanted to reduce the size of its electronic equipment to approximately one-tenth of its then existing volume. This drive for extreme reduction in the size of electronic circuits has led to the development of microelectronic circuits called **integrated circuits (ICs)** which are so small that their actual construction is done by technicians using high powered microscopes.

### 67.2. What is an Integrated Circuit ?

To put it very briefly, an integrated circuit (*IC*) is **just a packaged electronic circuit**.

A more detailed definition is as under :

An *IC* is a complete electronic circuit in which both the active and passive components are fabricated on a tiny single chip of silicon.

**Active components** are those which have the ability to produce gain. Examples are : transistors and *FETs*.

**Passive components** or devices are those which do not have this ability. Examples are : resistors, capacitors and inductors.

*ICs* are produced by the same processes as are used for manufacturing individual transistors and diodes etc. In such circuits, different components are isolated from each other by isolation diffusion within the crystal chip and are interconnected by an aluminium layer that serves as wires.

A discrete circuit, on the other hand, is one **that is built by connecting separate components**. In this case, each component is produced separately and then all are assembled together to make the electronic circuit.

J.S. Kilby of Texas Instruments was the first person to develop (in 1959) an integrated circuit — a single monolithic silicon chip in which active and passive components were fabricated by successive deposition, etching and diffusions. He was soon followed by Robert Noyce of Fairchild who successfully fabricated a complete *IC* including the interconnections on a single silicon chip. Since then the evolution of this technology is fast-paced.

### 67.3. Advantages of ICs

As compared to **standard printed circuits** which use discrete components, *ICs* have the following advantages :

#### 1. Extremely small physical size

Often the size is thousands of times smaller than a discrete circuit. The various components and their interconnections are distinguishable only under a powerful microscope.

### 2. Very small weight

Since many circuit functions can be packed into a small space, complex electronic equipment can be employed in many applications where weight and space are critical, such as in aircraft or space-vehicles.

### 3. Reduced cost

It is a major advantage of *ICs*. The reduction in cost per unit is due to the fact that many identical circuits can be built simultaneously on a single wafer—this process is called **batch fabrication**. Although the processing steps for the wafer are complex and expensive, the large number of resulting integrated circuits make the ultimate cost of each *IC* fairly low.

### 4. Extremely high reliability

It is perhaps the **most important** advantage of an *IC* and is due to many factors. Most significant factor is the absence of soldered connections. Another is the need for fewer interconnections—the major cause of circuit failures. Small temperature rise due to low power consumptions of *ICs* also improves their reliability. In fact, an *IC* logic gate has been found to be 100,000 times more reliable than a vacuum tube logic gate and 100 times more reliable than a transistor logic gate.

Obviously, higher reliability means that *ICs* will work for longer periods without giving any trouble—something most desirable from both military and consumer application point of view.

### 5. Increased response time and speed

Since various components of an *IC* are located close to each other **in** or **on** a silicon wafer, the time delay of signals is reduced. Moreover, because of the short distances, the chance of stray electrical pickup (called parasitic capacitance) is practically nil. Hence it makes them very suitable for small signal operation and high frequency operation. As a result, the response time or the operating speed of the system is improved.

### 6. Low power consumption

Because of their small size, *ICs* are more suitable for low power operation than bulky discrete circuits.

### 7. Easy replacement

*ICs* are hardly ever repaired because in case of failure, it is more economical to replace them than to repair them.

### 8. Higher yield

The **yield** is the percentage of usable devices. Because of the batch fabrication, the yield is very high. Faulty devices usually occur because of some defect in the silicon wafer or in the fabrication steps. Defects in silicon wafer can occur because of lattice imperfection and strains introduced in crystal growth, cutting and handling of the wafers. Usually such defects are extremely small, but their presence can ruin devices built on or around. Reducing the size of each device greatly increases the chance for a given device to be free of such defects. The same is true for fabrication defects such as the presence of a dust particle on the photolithographic mask.

## 67.4. Drawbacks of ICs

The integrated circuits suffer from the following drawbacks :

1. coils or inductors cannot be fabricated,
2. *ICs* function at fairly low voltages,
3. they handle only limited amount of power,
4. they are quite delicate and cannot withstand rough handling or excessive heat.

However, the advantages of *ICs* far outweigh their disadvantages or drawbacks.

### 67.5. Scale of Integration

Level of integration in *ICs* has been increasing ever since they were developed some three and a half decades back. The number of electronic circuits or components that can be fitted into a standard size *IC* has been dramatically increasing with each passing year. In fact, whole **electronic systems** rather than just **a circuit** are incorporated in one package.

An approximate method of classifying the amount of circuit or component density is as follows:

#### 1. SSI—*small scale integration*

In this case, the number of **circuits** contained in one *IC* package is less than 12 (or number of **components** is less than 50).

#### 2. MSI—*medium scale integration*

Here, number of circuits per package is between 13 and 99 (or number of components is between 50 and 5000).

#### 3. LSI—*large scale integration*

In this case, circuit density is between 100 and 9,999 (or component density is between 5000 and 100,000).

#### 4. VLSI—*very large scale integration*

Here the number of circuits per package is between 10,000 to 99,999 (or number of components is between 100,000 – 1,000,000).

#### 5. ULSI—*ultra large scale integration*

In this case, the circuit density is between 100,000 to 999,999 (or component density is between 1,000,000 – 10,000,000).

#### 6. GSI—*Giga scale integration*

Here the number of circuits per package is 1,000,000 or more (or number of components are over 100,000,000).

In summary,

SSI	<	12	] circuits per chip
MSI		12 – 99	
LSI		100 – 9,999	
VLSI		10,000 – 99,999	
ULSI		100,000 – 999,999	
GSI	>	1,000,000	

### 67.6. Classification of ICs by Structure

Structurally speaking, *ICs* can be classified into the following three types :

#### 1. Monolithic Integrated Circuits

The word 'monolithic' means 'single stone' or more appropriately 'a single-solid structure'. In this *IC*, all circuit components (both active and passive) are fabricated inseparably within a single continuous piece of silicon crystalline material called **wafer (or substrate)**. All components are **atomically** part of the same chip. Transistors, diodes and other passive components are fabricated at appropriate spots in the substrate using epitaxial diffusion technique.

Component interconnections are provided on the surface of the structure and external connecting wires are taken out to



Monolithic IC instrument

the terminals. It is a complete circuit requiring no 'add ons'.

Despite some of its distinct disadvantages, monolithic ICs are in wide use because for mass production, monolithic process has been found to be the most economical.

## 2. Thick and Thin-Film ICs

The essential difference between thick-film and thin-film ICs *is not their relative thickness but the method* of depositing the film. Both have similar appearance, properties and general characteristics though they both differ in many respects from monolithic ICs. These ICs are not formed **within** a silicon wafer but **on** the surface of an insulating substrate such as glass or a ceramic material. Moreover, **only passive components** (resistors, capacitors) are formed through thick or thin-film techniques on the insulating surface. The active elements (transistors, diodes) are added externally as **discrete elements** to complete a functional circuit. These discrete active components are frequently produced by using the monolithic process.

As stated above, the primary difference between the thick and thin film techniques is the process used for forming passive components and the metallic conduction pattern.

### (a) Thin-film ICs

Such circuits are constructed by depositing films (typically 0.1 to 0.5  $\mu\text{m}$ ) of conducting material through a mask on the surface of a substrate made of glass or ceramic. Resistors and conductors are formed by varying the width and thickness of the film and by using materials of different resistivity. Capacitors are produced by sandwiching an insulating oxide film between two conducting films. Small inductors can be made by depositing a spiral formation of film. The active components like transistors and diodes etc. are externally added and inter-connected by wire bonds.

Following two methods are used to produce thin films :

#### (i) vacuum evaporation

In this method, the vaporised material is deposited through a set of masks on the glass or ceramic substrate contained in vacuum.

#### (ii) cathode sputtering

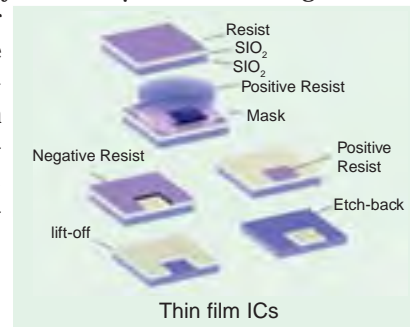
In this method, atoms from a cathode made of the desired film material are deposited on the substrate which is located between the cathode and the anode.

### (b) Thick-film ICs

Such type of integrated circuits are sometimes referred to as **printed** thin-film circuits. They are so called because silk-screen printing techniques are employed to create the desired circuit pattern on the surface of the substrate. The screens are made of fine stainless steel wire mesh and the 'inks' are pastes (of pulverised glass and aluminium) which have conductive, resistive or dielectric properties. After printing, the circuits are high-temperature fired in a furnace to fuse the films to the insulating substrate. As with thin-film ICs, active elements are added externally as discrete components.

## 3. Hybrid or Multichip ICs

As the name implies, such circuits are formed either by inter-connecting a number of individual chips or by a combination of film and monolithic IC techniques. In such ICs, active components are first formed within a silicon wafer (using monolithic technique) which is subsequently covered with an insulating layer such as  $\text{SiO}_2$ . Film techniques are then employed to form passive components on the  $\text{SiO}_2$  surface. Connections are made from the film to the monolithic structure through 'windows' cut in the  $\text{SiO}_2$  layer.



### 67.7. Comparison Between Different ICs

Each type of IC has its own advantages and disadvantages.

Monolithic circuits have **the advantage of lowest cost and highest reliability**. However they have the following disadvantages :

1. isolation between components is poorer,
2. range of values of passive components used in the circuits is comparatively small,
3. inductors cannot be fabricated,
4. they afford no flexibility in circuit design because for making any changes in the circuit, **a new set of masks is required**.

The film circuits have the advantage of forming passive components with broader range of values and reduced tolerances as compared to monolithic circuits. Isolation between their components is also better since they are deposited on a substrate that is an insulator. Use of external discrete active components allows greater flexibility in circuit design. These circuits also give better high frequency performance than monolithic circuits.

However, they suffer from the disadvantages of

1. not being able to fabricate active components,
2. comparatively higher cost and
3. larger physical size.

The chief advantage of multichip ICs is their greater flexibility but they are **too expensive** for mass production and **have least reliability**. Hence, such circuits are generally used as prototypes for monolithic ICs.

The various integrated circuits are arranged in the form of an 'IC' tree of Fig. 67.1.

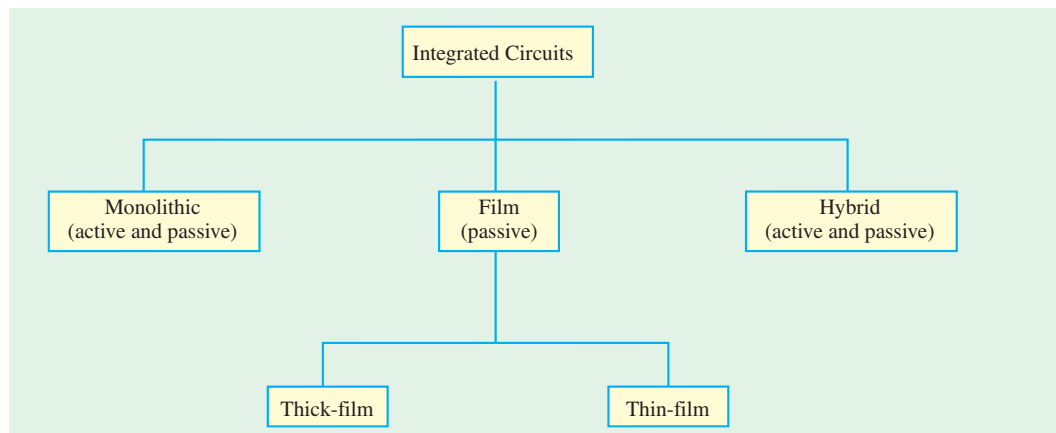


Fig. 67.1

### 67.8. Classification of ICs By Function

The earlier classification of ICs was based on **their method of construction**. However, the integrated circuits can also be classified according to their general function. The two most important categories are :

1. linear and
2. digital

The same fact has been shown in Fig. 67.2.

Examples of linear ICs are :

1. *BEL CA-3020*—used as multipurpose wide-band power amplifier.

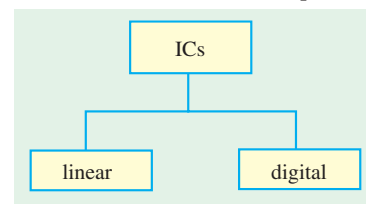


Fig. 67.2



- 2.** *BEL CA-3065*—it is a monolithic *IC* which combines a multistage *IF* amplifier, limiter, an *FM* detector, an electronic attenuator, a Zener diode regulated power supply and an audio amplifier. In fact, this *IC* provides a high performance multistage sub-system of a TV receiver. It is available in 14-pin dual-in-line package.

Both are manufactured by Bharat Electronics Ltd., Bangalore.

- 3.** *SSD 710*—is a linear *IC* used as Differential Comparator

It is manufactured by Solid State Devices, Syed Abdullah Road, Bombay.

An example of a digital *IC* is

- 1.** *BEL 7400*—is a *TTLIC* gate which provides designer with one of the gating logic necessary to design medium-speed digital control and data processing systems. It is available in 14-pin dual-in-line plastic package.

### 67.9. Linear Integrated Circuits (LICs)

*LICs* are also referred to as analog *ICs* because their inputs and outputs can take on a continuous range of values and the outputs are generally proportional to the inputs. As compared to digital *ICs*, *LICs* are used much less. But *LICs* are quickly displacing their discrete circuit counterparts in many applications as their cost becomes competitive. They also possess much higher reliability because so many external connections (major source of circuit failure) are eliminated. *LICs* find wide use in military and industrial applications as well as in consumer products. They are frequently used in



Linear ICs

- |                                   |   |
|-----------------------------------|---|
| <b>1.</b> operational amplifiers, | <b>2.</b> small-signal amplifiers,            |
| <b>3.</b> power amplifiers,       | <b>4.</b> <i>RF</i> and <i>IF</i> amplifiers, |
| <b>5.</b> microwave amplifiers,   | <b>6.</b> multipliers,                        |
| <b>7.</b> voltage comparators,    | <b>8.</b> voltage regulators etc.             |

Operational amplifier is by far the most versatile form for an *LIC* and is discussed separately.

### 67.10. Manufacturer's Designation of LICs

Each manufacturer assigns a specific code and type number to the *LICs* produced by him. For example, an internally-compensated op-amp 741 produced by Fairchild is designated as  $\mu A$  741. Here,  $\mu A$  is the identifying code used by Fairchild. Many other manufacturers produce *LICs* similar to 741 but use their own code while retaining the same type number. For example, Fairchild's original  $\mu A$  741 is manufactured by other manufacturers with the following codes :

- |                                  |    |                |
|----------------------------------|----|----------------|
| <b>1.</b> National Semiconductor | —— | <i>LM</i> 741  |
| <b>2.</b> Motorola               | —— | <i>MC</i> 1741 |
| <b>3.</b> RCA                    | —— | <i>CA</i> 3741 |
| <b>4.</b> Texas Instruments      | —— | <i>SN</i> 5274 |

It is seen that the last three digits in each manufacturer's designation are the same *i.e.* 741. All these op-amps have the same specifications. Hence, in practice, the manufacturer's code number is often ignored and all such *LICs* are referred to as 741.

Many *LICs* are available in different classes such as A, B, C, E, S and SC. For example, main classes of 741 are as under :

741	——	Military grade op-amp
741 C	——	Commercial grade op-amp
741 A	——	Improved version of 741
741 E	——	Improved version of 741 C
741 S	——	Military grade op-amp with higher slew rate
741 SE	——	Commercial grade op-amp with higher slew rate

### 67.11. Digital Integrated Circuits

About 80 per cent of the *IC* market has been captured by digital *ICs* which are mostly utilized by the computer industry. Digital *ICs* lend themselves easily to monolithic integration because a computer **uses a large number of identical circuits**. Moreover, such circuits employ relatively few capacitors and values of resistances, voltages and currents are low.

Digital *ICs* contain circuits whose input and output voltages are limited to **two** possible levels—low or high. It is so because **digital signals are usually binary**. Sometimes, digital circuits are referred to as switching circuits. Digital *ICs* include circuits such as

1. logic gates
2. flip-flops
3. counters
4. clock-chips
5. calculator chips
6. memory chips
7. microprocessors ( $\mu P$ ) etc.

### 67.12. IC Terminology

Some of the common terms used in fabricating integrated circuits are defined below :

1. **Bonding** – attaching the die on the ceramic substrate and then connecting the leads to the package.
2. **Chip** – an extremely small part of a silicon wafer on which *IC* is fabricated. A photograph of the wafer containing hundreds of chips (or dice) and a drawing of a chip are shown in Fig. 67.3. The identical chips, each of which may vary in area from 10 to over 100 mm<sup>2</sup>, may contain up to several million devices.

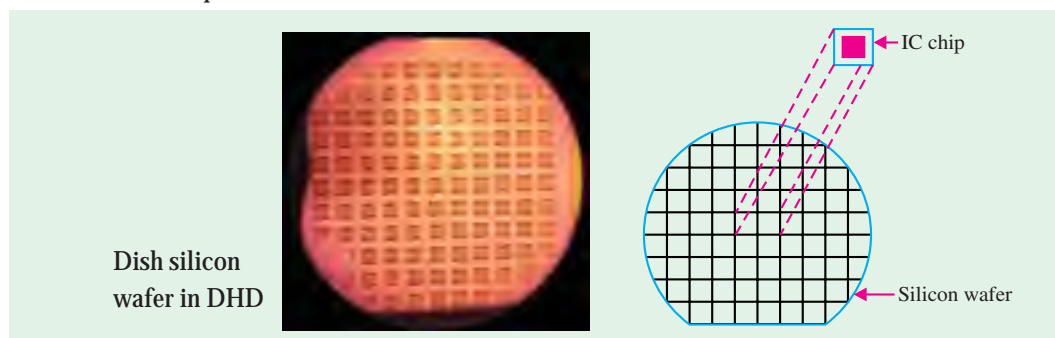


Fig. 67.3

3. **Circuit probing** – testing the electrical performance of each *IC* chip with the help of a microscope and multi-point probe.
4. **Die** – same as a chip.
5. **Diffusion** – a process that consists of the introduction of impurities into selected regions of a wafer to form junctions.
6. **Encapsulation** – putting a cap over the *IC* and sealing it in an inert atmosphere.

7. **Epitaxy** – a process of the controlled growth of a crystalline doped layer of silicon on a single crystal substrate.
8. **Etching** – a process of selective removal of regions of a semiconductor, metal or silicon dioxide.
9. **Mask** – a glass plate with desired pattern for diffusion or metallization. Usually a single mask is not sufficient to fabricate an *IC*.
10. **Metallization** – a process for providing ohmic contacts and interconnections by evaporating aluminium over the chip.
11. **Photolithography** – a process to transfer geometrical pattern from the mask to the surface of the wafer.
12. **Photoresist** – a light-sensitive material that hardens when exposed to ultraviolet light.
13. **Wafer** – a thin disk of semiconductor in which number of *ICs* are fabricated simultaneously.

### 67.13. Semiconductors Used in Fabrication of ICs and Devices

The fabrication of *ICs* has been based on the use of silicon (Si) as the premier semiconductor. Two other semiconductors used for *IC* fabrication are germanium and gallium arsenide (GaAs). But these semiconductors present special problems for device fabrication as discussed below.

Gallium arsenide has very attractive electrical properties but its crystals have a high density of defects which limit the performance of devices made from it. Moreover gallium arsenide is more difficult to grow in single crystal form. Both silicon and germanium do not suffer from these problems. On the plus side, gallium arsenide has an electron velocity that is larger than silicon. Because of this gallium arsenide devices are faster than silicon devices. Also, gallium arsenide has a lower saturation electric field than silicon. Because of this, the gallium arsenide devices have lower power-delay product. Devices made from substrates of gallium arsenide have lower parasitic capacitances. This property contributes to their speed advantage over the silicon devices. Another advantage of gallium arsenide results from direct band gap which makes it possible to provide certain functions not possible in silicon such as **coherent and incoherent light emission**.

A major advantage of silicon, in addition to its abundant availability in the form of sand, is that it is possible to form a superior stable oxide ( $\text{SiO}_2$ ). This oxide has superb insulating properties and provides an essential and excellent ingredient in the fabrication and protection of devices or *ICs*. On the other hand, germanium oxide is unsuited for device applications. The intrinsic resistivity of germanium is  $47 \Omega\text{-cm}$ , while that of silicon is  $230,000 \Omega\text{-cm}$ . The low resistivity of germanium would have precluded the fabrication of rectifying devices with high breakdown voltages. Thus high-voltage rectifying devices and certain infrared sensing devices are practical with silicon. Finally, there is an economic consideration and that is electronic grade germanium is now more costly than silicon. Thus, at present time, silicon remains the major semiconductor in the fabrication of *ICs*.

Some other semiconductors used in semiconductor industry are gallium phosphide (GaP), gallium nitride (GaN), zinc sulphide (ZnS), indium antimonide (InSb), compound of cadmium and selenium (CdSe). GaAs, GaP and GaN are used in high-speed devices and devices requiring emission and absorption of light such as lasers and light emitting diodes (LEDs). ZnS is used as fluorescent material such as those in television screens. In Sb and CdSe are used as light detectors.

### 67.14. How ICs are Made ?

The *ICs* are manufactured in four distinct stages (refer to Fig. 67.4). These are (1) material preparation, (2) crystal growing and wafer preparation, (3) wafer fabrication and (4) testing, bonding and packaging. All these stages are discussed one by one in the following pages.

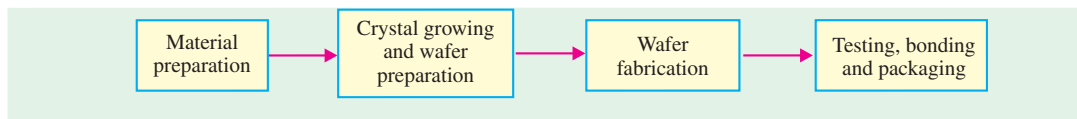


Fig. 67.4

### 67.15. Material Preparation

Silicon, as an element is not found in nature. However, it is found abundantly in nature in the form of silicon dioxide, which constitutes about 20% of earth's crust. Silicon is commonly found as quartz or sand. A number of processes are required to convert sand into pure silicon with a polycrystalline structure. Fig. 67.5 shows the different processes involved in the preparation of polycrystalline silicon from sand. As seen from this figure, the sand is allowed to react with a gas produced from the burning of carbon (coal, coke and wood chips). This produces silicon with 98% purity. Next silicon is further purified in a reactor to produce electronic-grade polycrystalline silicon.

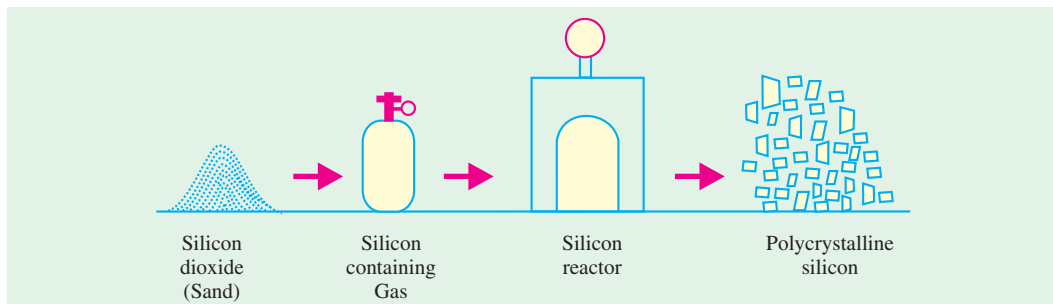


Fig. 67.5

### 67.16. Crystal Growing and Wafer Preparation

The polycrystalline silicon is composed of many small crystals having random orientation and containing many defects. For silicon to be used in the fabrication of ICs, it must be nearly perfect and crystalline in nature. We, therefore, now need to produce single crystals of silicon. This is done by a process called crystal growth. There are two methods to carry out the crystal growth : (i) the **Czochralski** and (ii) the **flat zone** process. The Czochralski process prepares virtually all the silicon used for IC fabrication. The flat zone process is used to prepare crystals for fabricating high-power, high voltage semiconductor devices.

**The Czochralski process.** The equipment used for single crystal growth (called puller) is as shown in Fig. 67.6. The puller has three main components : (i) a furnace which includes quartz crucible, a rotation mechanism (clockwise as shown), and a radio frequency (RF) heating element, (ii) a crystal pulling mechanism which includes a seed holder and a rotation mechanism (counter-clockwise), and (iii) an ambient control which includes an argon gas source, a flow control and an exhaust system. In addition the puller has a computer system to control process parameters such as temperature, crystal diameter, pull rate and rotation speed.

To grow crystals, the polycrystalline silicon is placed in the crucible. The furnace is heated to a temperature of 1690 K which is slightly greater than the melting point (1685 K) of silicon. A precisely controlled amount of dopant (boron or phosphorus) is added to the melt to make the silicon as P-type or N-type. A suitable oriented seed crystal (*i.e.*, a small highly perfect crystal) is suspended over the crucible in a seed holder. The seed is inserted into the melt and a small portion of it is

allowed to melt. The seed is rotated and pulled up very slowly, while at the same time, the crucible is rotated in the opposite direction. The molten silicon attaches itself to the seed and it becomes identical to the seed in structure and orientation. As the seed is pulled up, the material that is attached to the seed solidifies (*i.e.*, freezes). Its crystal structure becomes the same as that of the seed and a larger crystal is formed. Thus using this method, cylindrical single crystal bars (called ingots) of silicon are produced.

The desired diameter of the silicon ingot is obtained by controlling both the temperature and the pulling speed. In the final step, when the bulk of the melt has been grown, the crystal diameter is decreased until there is a point contact with the melt. The resulting ingot is cooled and is removed to be made into thin discs called **wafers**. The ingots have diameters as large as 200 mm with the latest ones approaching 300 mm. The ingot length is of the order of 1000 mm.

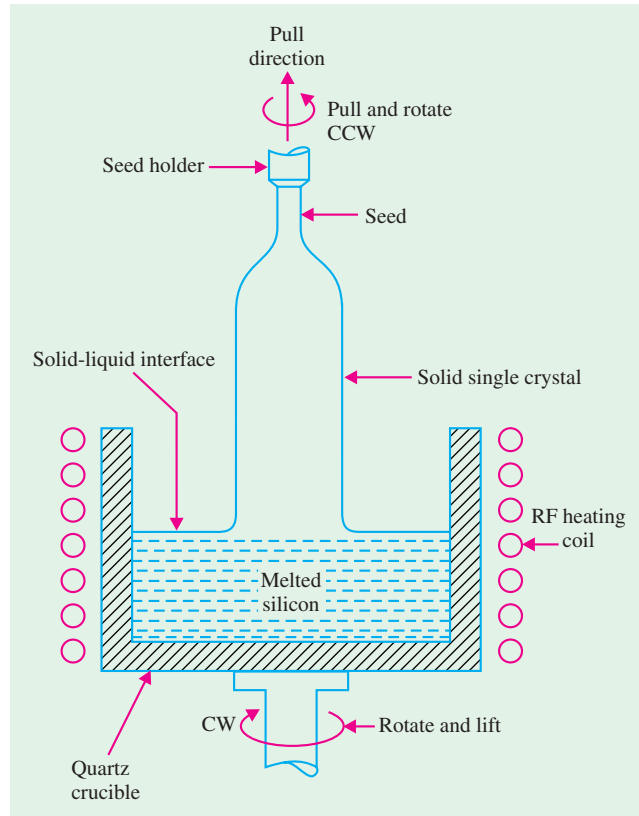


Fig. 67.6

**Wafer preparation:** In this stage the ingot surface is ground throughout to an exact diameter and the top and bottom portions are cut off. After that one or more flat regions are ground along the length of the ingot. These flat regions mark the specific crystal orientation of the ingot and conductivity type (*i.e.*, *P*-type or *N*-type) of silicon material. Refer to Fig. 67.7.

Notice that the ingot is marked with two flats regions : (i) the larger flat (called primary flat) and (ii) the smaller flat or secondary flat regions. The primary flat allows a mechanical locator in automatic processing equipment to position the wafer and to orient the devices relative to the crystal in a specific manner. The secondary flat regions are used to identify the orientation and conductivity type of the crystal. The conductivity of the wafer could be either *P*-type or *N*-type and the crystal orientation,  $\{100\}$  or  $\{111\}$  (refer to Art 50.16 to know more about crystal orientation). If there is no secondary flat as shown in Fig. 67.7 (a), or in words the primary and the secondary flat are superimposed, the wafer is identified as *P*-type with  $\{111\}$ -crystal orientation. If the secondary flat is  $45^\circ$  with respect to the primary flat as shown in Fig. 67.7 (b), it is  $\{111\}$  *N*-type wafer. However, if the secondary flat is  $90^\circ$  with respect to the primary flat as shown in Fig. 67.7 (c), it is  $\{100\}$  *P*-type wafer and if the secondary flat is at  $180^\circ$  with respect to the primary flat as shown in Fig. 67.7 (d), it is  $\{100\}$  *N*-type wafer.

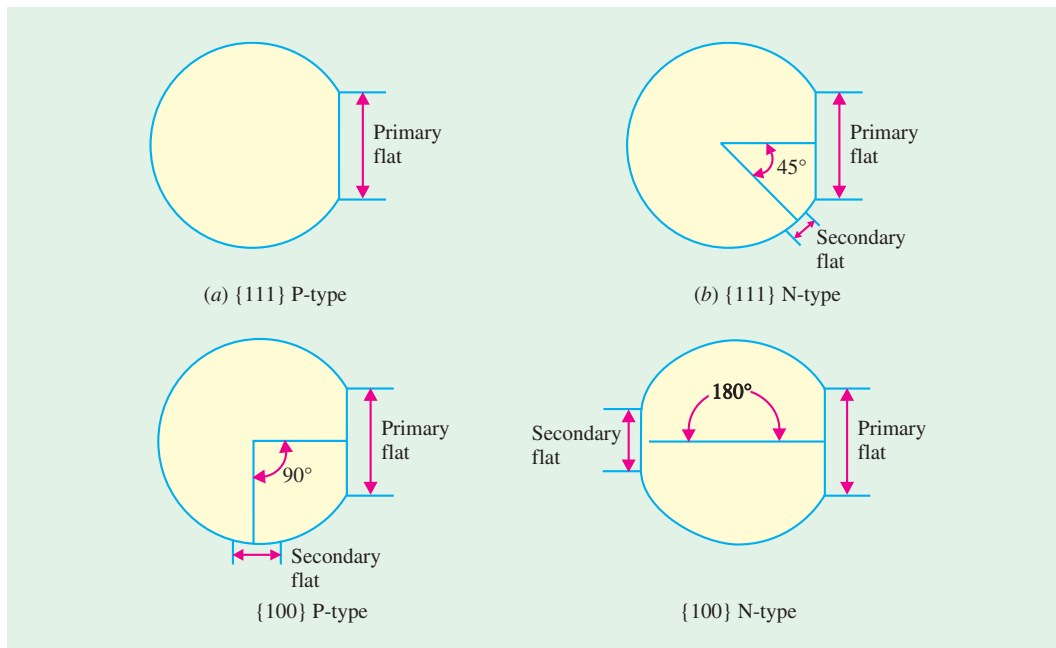


Fig. 67.7

The semiconductor industry uses {111} wafers for fabricating *ICs* with bipolar transistor technology and {100} wafers for metal-oxide semiconductor (MOS) circuits. The choice of wafer conductivity (*i.e.*, *P*-type or *N*-type) depends upon the actual process used for fabricating the *ICs*. Once the orientations are done, the ingot is sliced into wafers by a high-speed **diamond saw**. The wafer thickness varies from 0.4 to 1.0 mm.

After slicing, both sides of the wafer are lapped to produce typical flatness uniformity within  $2\ \mu\text{m}$ . The lapped orientation usually leaves the surface and edges of the wafer, damaged and contaminated. These can be removed by a process called chemical etching. The next step is to polish the wafer surface to a mirror-like finish. Finally the wafers are cleaned, rinsed and dried for use in fabrication of *ICs*. It is interesting to note that the final wafer thickness is about one-third less than after the slicing.

### 67.17. Wafer Fabrication

Following is the category of the processes that are used in the fabrication of *ICs* :

- |   |                      |                 |
|---|----------------------|-----------------|
| (i) oxidation                             | (ii) etching         | (iii) diffusion |
| (iv) ion implantation                     | (v) photolithography | (vi) epitaxy    |
| (vii) metallization and interconnections. |                      |                 |

We will study each process separately and apply some of these later to the formation of a diode bipolar transistor and metal oxide semiconductor field effect transistor. The basic fabrication process is called **planar process**, *i.e.*, a process in which the introduction of impurities and metallic interconnections is carried out from the top of the wafer. A major advantage of the planar process is that each fabrication step is applied to all identical circuits and each of the many wafers at the same time.

It is important to initially emphasize that the fabrication requires an extremely clean environment in addition to the precise control of temperature and humidity.

### 67.18. Oxidation

The process of oxidation consists of growing a thin film of silicon dioxide ( $\text{SiO}_2$ ) on the surface of a silicon wafer. Silicon dioxide has several uses :

1. to serve as a mask against implant or diffusion of dopant into silicon,
2. to provide surface passivation,
3. to isolate one device from another,
4. to act as a component in MOS structures.

Fig. 67.8 shows oxide layer grown on the surface of silicon substrate. The commonly used silicon dopants, such as boron, phosphorus, arsenic, and antimony, have very low diffusion coefficients (*i.e.*, they diffuse with great difficulty) in silicon dioxide. Because of this reason, silicon dioxide is used as a shield against infiltration of these dopants. On the other hand, these dopants diffuse easily if the surface is silicon.

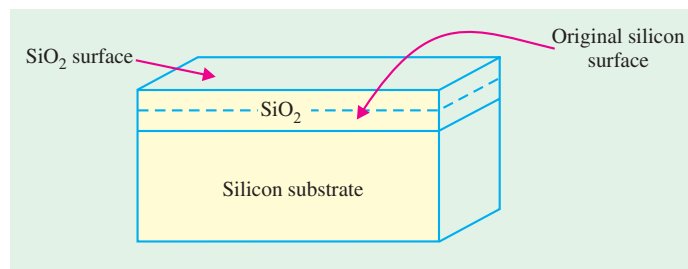


Fig. 67.8

Several techniques have been developed for forming oxide layers. Some of these are thermal oxidation, vapour phase technique [chemical vapour deposition (CVD)], and plasma oxidation. However, thermal oxidation is the more commonly used technique in *IC* processing.

Fig. 67.9 shows a thermal oxidation system. As seen, the oxidation is accomplished by placing the silicon wafers vertically into a quartz boat in a quartz tube. The quartz boat is slowly passed through a resistance heated furnace, in the presence of oxygen, operating at a temperature of about  $1000^\circ\text{C}$ . The oxidizing agent may be *dry* by using dry oxygen or be using a mixture of water vapour and oxygen. A computer controls the whole operation in the thermal oxidation system. The operation include regulating the gas flow sequence, automatic insertion and removal of wafers and the furnace temperature.

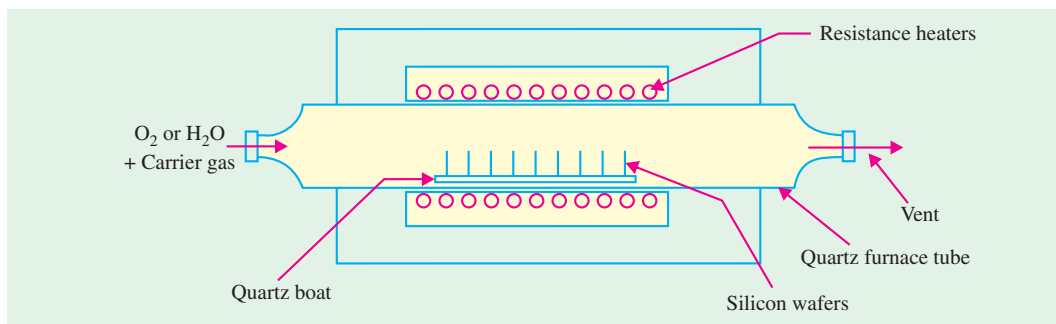


Fig. 67.9

It will be interesting to note that the oxide growth in the dry process is much *slower* but it produces an oxidized layer that has *excellent* electrical properties. For relatively thin oxides such as gate oxide in a MOSFET (typically 10 nm), dry oxidation is used. However, for thicker oxides ( $\geq 500$  nm) such as field oxides in MOS integrated circuits and for bipolar devices, wet oxidation is used to provide both isolation and passivation.

Once the silicon dioxide layer has been formed on the surface of the wafer, it is selectively removed (etched) from those surfaces where impurities are to be introduced and kept as a shield, for the underlying silicon surface, where no dopants are to be allowed.

Oxide layers are relatively free from defects and provide stable and reliable electrical properties.

### 67.19. Etching

Etching is the process of selective removal of regions of a semiconductor, metal or silicon dioxide. There are two types of etching : *wet* and *dry*. In wet etching, the wafers are immersed in a chemical solution at a predetermined temperature. In this process, the material to be etched is removed equally in all directions (*i.e.*, etching is isotropic). Because of this, some material is etched from the regions where it is to be left. This becomes a serious problem when dealing with small dimensions.

In dry (or plasma) etching, the wafers are immersed in gaseous plasma created by a radio-frequency electric field applied to a gas such as argon. The gas breaks down and becomes ionized. Electrons are initially released by field emission. These electrons gain kinetic energy from the field, collide with, and transfer energy to gas molecules, which result in generating ions and electrons. The newly generated electrons collide with other gas molecules and the avalanche process continues throughout the gas, forming plasma. The wafer to be etched is placed on an electrode and is subjected to the bombardment of its surface by gas ions. As a result, the transfer of momentum from the ions to the atoms removes atoms at or near the surface to be etched.

Dry etching also called reactive ion etching (RIE) is directional (or anisotropic). In other words, the material is removed only from those regions where it is required. Most modern processes use only dry etching to produce fine line patterns needed for VLSI integrated circuits.

### 67.20. Diffusion

This process consists of the introduction of impurities into selected regions of a wafer to form junctions. Diffusion occurs in two steps : the *pre-deposition* and the *drive-in* diffusion. In the pre-deposition step, a high concentration of dopant atoms is introduced at the silicon surface by a vapour that contains the dopant at a temperature of 1000°C. More recently, a more accurate method of pre-deposition known as *ion implantation* is used.

Pre-deposition tends to produce, a shallow but heavily doped layer, near the silicon surface. Drive-in is used to drive the impurity atoms deeper into the surface, without adding any more impurities. Fig. 67.10 shows the graph of doping profiles, (*i.e.*, impurity concentration versus depth into the substrate) during the pre-deposition and the drive-in steps of diffusion. The doping profiles indicate that the impurity concentration decreases monotonically from the surface of the substrate. The profiles of the dopant distribution is determined mainly by the temperature and diffusion time. Note that the impurity concentration drops from *OA* (during pre-deposition step) to *OB* at the surface of the substrate after the drive-in step. However the depth into the substrate of impurity has increased from *OC* to *OD* as required.

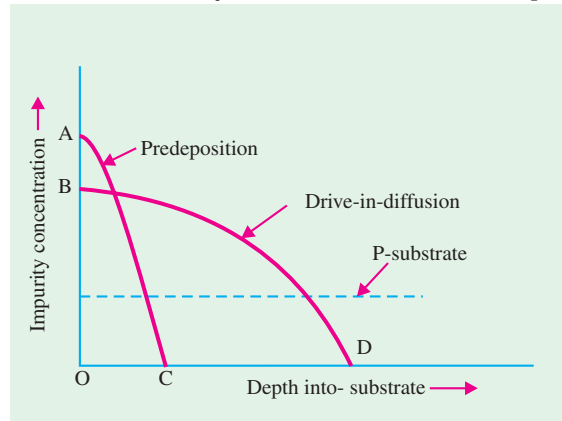


Fig. 67.10

Common dopants are boron for *P*-type layers and phosphorus, antimony and arsenic for *N*-type layers. However, diffusion is rarely performed using pure elements themselves. Rather, compounds of elements are used and impurities may be introduced from either solid, liquid, or gaseous substances.



Fig. 67.11 shows the equipment used in diffusion. The wafers are placed in a quartz boat within a quartz furnace tube. The furnace is heated by resistance heaters surrounding it. To introduce an impurity, phosphorus for example, phosphorus oxychloride ( $\text{POCl}_3$ ) is placed in a container either inside the quartz tube in a region of relatively low temperature or in a container outside the furnace at a temperature that maintains its liquid form. For a *P*-type dopant, boron is used. The proper vapour pressure is maintained by a control of the temperature. Nitrogen and oxygen gas are made to pass over the container. These gases react with the silicon, forming a layer on the surface of the wafer that contains silicon, oxygen and phosphorus. At the high temperature of the furnace, phosphorus easily diffuses into the silicon.

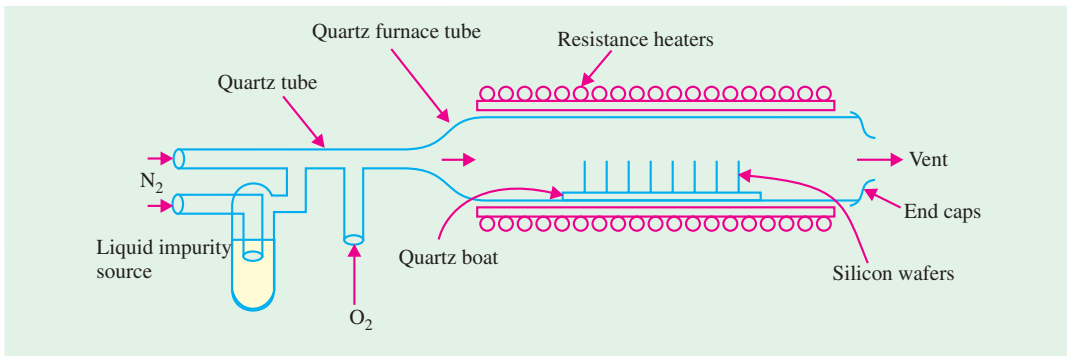


Fig. 67.11

In order that the dopant may be diffused deeper into silicon, the drive-in step follows. This is done at a higher temperature of about  $1100^\circ\text{C}$  inside a furnace similar to that used for pre-deposition, except that no dopant is introduced into the furnace. The higher temperature, causes the dopant atoms to move into silicon more quickly. Diffusion depth is controlled by the time and temperature of the drive-in process. By precise control of the time and temperature, accurate junction depths of fraction of a micron can be obtained. Fig. 67.12 shows the diffusion of dopant into silicon.

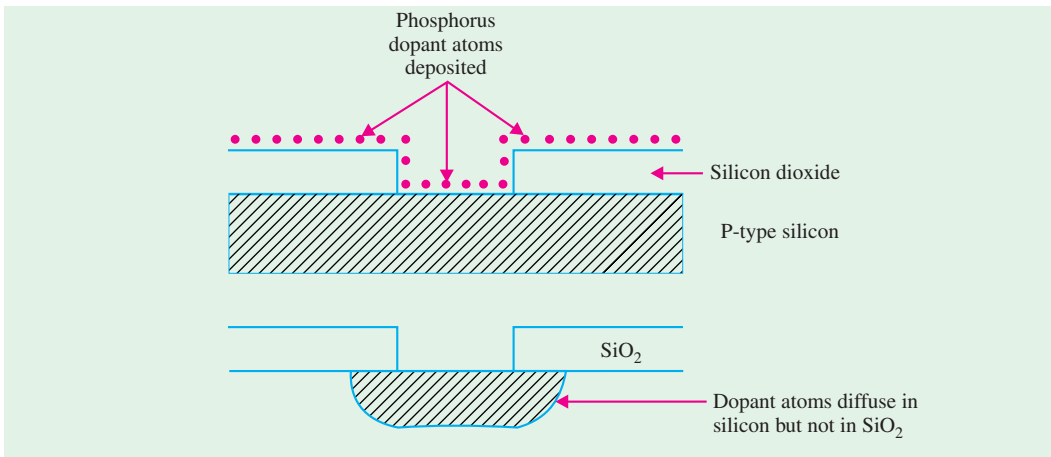


Fig. 67.12

### 67.21. Ion Implantation

This is a process of introducing dopants into selected areas of the surface of the wafer by bombarding the surface with high-energy ions of the particular dopant.

Fig. 67.13 shows a typical ion implantation equipment. To generate ions, such as those of phosphorus, an arc discharge is made to occur in a gas, such as phosphine ( $\text{PH}_3$ ), that contains the dopant.

The ions are then accelerated in an electric field so that they acquire energy of about 20 keV and are passed through a strong magnetic field. The ions are further accelerated so that their energy reaches several hundred keV or MeV, whereupon they are focused on and strike the surface of the silicon wafer.

As is the case with diffusion, the ion beam is made to penetrate only into selected regions of the wafer by a process of masking (discussed later). On entering the wafer, the ions collide with silicon atoms and lose their energy. The depth of penetration of ions in ion implantation is about 0.1 to 1  $\mu\text{m}$ . The higher the energy of ions and the smaller their mass, the greater is the depth of penetration.

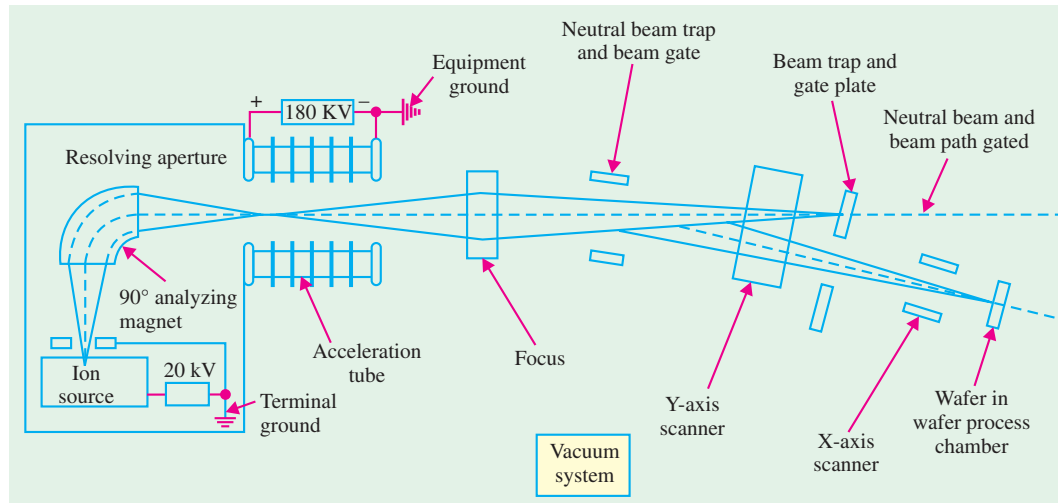


Fig. 67.13

#### **Advantages of ion implantation over diffusion :**

1. Doping levels can be precisely controlled since the incident ion beam can be accurately measured as an electric current.
2. The depth of the dopant can be easily regulated by control of the incident ion velocity. It is capable of very shallow penetrations.
3. Extreme purity of the dopant is guaranteed.
4. The doping uniformity across the surface can be accurately controlled.
5. Because the ions enter the solid as a directed beam, there is very little spread of the beam, thus the doping area can be clearly defined.
6. Since ion implantation is carried out at room temperature, wafers do not face temperature stress. In addition photo-resist can be used as mask for masking impurities, thus there is no need to grow thick masking oxides.

#### **Disadvantages of ion implantation over diffusion :**

1. The ion implantation may create considerable damage to the crystal structure because of the collisions of the high-energy ions with the silicon. Such damage results in inferior performance of ICs made by this process. If the damage is not extensive, the process of **annealing** restores the structure.
2. High initial investment and operational cost of the equipment (> US \$ 1 million).
3. Uses very toxic gases for some of the dopants such as phosphorus and arsenic.

### 67.22. Photomask Generation

The whole process of IC fabrication consists of identifying selected regions of each circuit of the wafer surface into which identical dopant or metallic interconnections are made, while protecting other regions of the wafer surface. To carry out one of the many fabrication processes, a separate

mask is required for each operation whose function is to expose the selected regions and protect the others. There may be hundreds of identical dies (or ICs) on a wafer with each circuit containing hundreds of thousands or millions of devices. Identical steps are carried out simultaneously for each process. For each process, separate mask is needed.

The mask production starts with a drawing using a computer-assisted graphics system with all the information about the drawing stored in digital form. Commands from the computer are prepared that drive a pattern generator, which uses an electron beam to write the particular pattern, for one or several dies, on a glass plate covered with thin chromium film. When the glass plate is prepared, it is called **reticle**. A mask usually refers to a glass plate that contains a pattern for the whole wafer. The reticle pattern is projected onto the wafer and a **wafer stepper** is used that reduces the reticle circuit onto the photo-resist covered wafer that steps across over the surface until the entire array of circuits is built up.

It may be noted that the use of a single mask for all the circuits on a wafer is not feasible for printing very small ( $< 1 \mu\text{m}$ ) features because of alignment problems. However, the use of single mask is still in use for fabricating simple digital and analog circuits such as light emitting diodes (LEDs).

### 67.23. Photolithography

It is a process in which the geometrical pattern on the glass plate (called reticle) is transferred to the surface of the wafer. This is done to open identical windows so that the diffusion (or the ion implantation) process may take place in all identical regions of the same IC and for all ICs on the wafer. As an illustration we assume that the first reticle is used over an oxidized surface as shown in Fig. 67.14 (a).

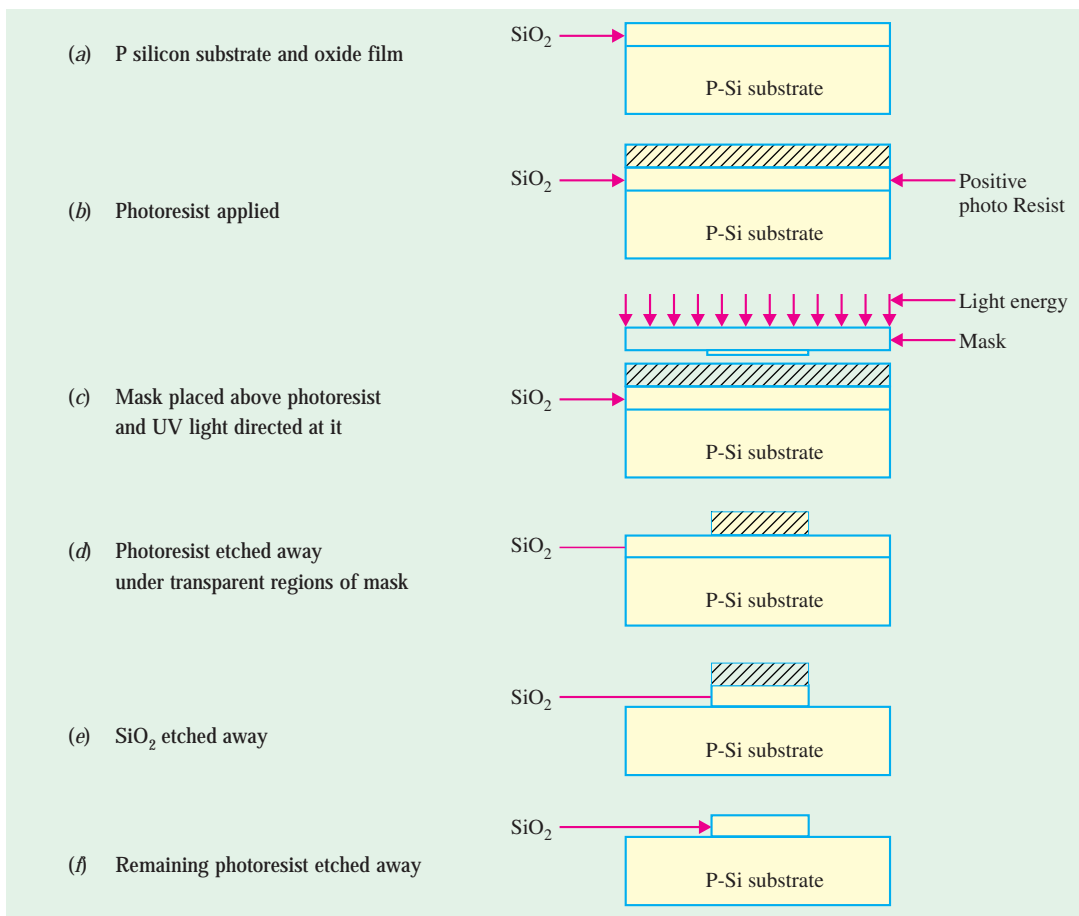
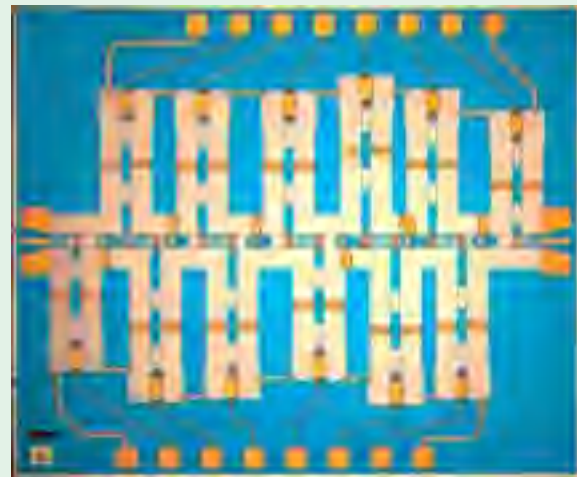


Fig. 67.14

To transfer the patterns, the wafer is coated with light-sensitive material called **photoresist**. By applying small amount (about 1 ml) of the photoresist to the wafer surface and spinning the wafer very rapidly, a uniform film of about 1  $\mu\text{m}$  thick is formed over the oxidized surface of the wafer [refer to Fig. 67.14 (b)]. After this the following steps are taken to open a window on the surface of the wafer :

1. The wafer is baked at 100°C to solidify the photoresist on the wafer.
2. The glass plate is placed on the wafer and aligned by the computer control.
3. The glass plate is exposed to ultraviolet (UV) light with the transparent parts of the glass plate passing the light on to the wafer. The photoresist under the opaque regions of the glass plate is unaffected [refer to Fig. 67.14 (c)].
4. The exposed photoresist is chemically removed by dissolving it in an organic solvent and exposing the silicon dioxide underneath [refer to Fig. 67.14 (d)].
5. The exposed silicon dioxide is then etched away using hydrofluoric acid. The hydrofluoric acid dissolves silicon dioxide and not silicon. The regions under the opaque part of the glass plate are still covered by the silicon dioxide and the photoresist [refer to Fig. 67.14 (e)].
6. The photoresist under the opaque regions of the glass plate is removed using a proper solvent and the silicon dioxide is exposed [refer to Fig. 67.14 (f)].

All surfaces in Fig. 67.14 (f) are protected except those covered by silicon only in which diffusion or ion implantation is to take place. It may be noted that the surfaces covered by silicon dioxide do not permit any entry of dopants.



The photolithograph is used to add circuit elements to the wafer. The wafer is coated with layer of a chemical called photoresist.

The photoresist used in the explanation above is called a **positive photoresist**. Thus a positive photoresist is that which allows the windows to be opened wherever the UV light passes through the **transparent** parts of the mask. On the other hand a photoresist which remains on the surface, when exposed to UV light, and windows are opened under the **opaque** parts of the mask, is called a **negative photoresist**. The negative photoresist is limited to a resolution of 2 to 3  $\mu\text{m}$ . On the other hand, the positive photoresist sacrifice adhesion and simplicity of development to achieve higher resolution.

The method of using UV light has a practical limit for transferring patterns on the silicon. It can be used for the patterns where the linewidths are above 1 to 2  $\mu\text{m}$ . In order to transfer small patterns (*i.e.*, those with below 1  $\mu\text{m}$  linewidth), very short wavelength radiation such as electron-beam or X-rays are used.

#### 67.24. Epitaxy

Epitaxy or **Epitaxial growth** is the process of the **controlled growth** of a crystalline doped layer of silicon on a single crystal substrate. The process of diffusion and ion implantation, which

were earlier described, produce a layer at the surface that is of higher doping density than that which existed before the dopant was added. It is not possible by these methods to produce, at the surface a layer of lower concentration than exists there. This can however, be accomplished by the method of epitaxy. In the processes of diffusion and ion implantation, a dopant is driven into a substrate of doped silicon. In epitaxy, a layer of doped silicon is deposited on top of the surface of the substrate. Normally this single crystal layer has different type of doping from that of the substrate.

Epitaxy is used to deposit  $N$  or  $N^+$  (*i.e.*, heavily doped  $N$ -type) silicon, which is impossible to accomplish by diffusion. It is also used in isolation between bipolar transistors where  $N^-$  (*i.e.*, lightly doped  $N$ -type) is deposited on  $P$ -type layer. It may also be used to improve the surface quality of an  $N$ -type substrate by depositing  $N$ -type material over it. A variety of methods are used to grow the epitaxial layer. These methods include vapour-phase epitaxy (VPE), liquid-phase epitaxy (LPE), and molecular beam epitaxy (MBE). The system for growing an epitaxial layer using vapour-phase epitaxy (VPE) is shown in Fig. 67.15.

In this system, silicon wafers are placed in a long boat-shaped crucible made of graphite. The boat is placed in a long cylindrical quartz tube, which has inlets and outlets for the gases. The tube is heated by induction using the heating coils wound around the tube.

All the chemicals that are introduced and that take part in the reactions are in the form of gases, hence the process is known as Chemical Vapour Deposition (CVD). The epitaxial layer is grown from the vapour phase onto the silicon, which is in the solid state. The thickness of the layer varies from 3 to 30  $\mu\text{m}$  and the thickness of the layer and its doping content are controlled to an accuracy of less than 2 percent. The reactions are carried out at a temperature of approximately 1200°C. The high temperature is necessary so that the dopant atoms can acquire a sufficient amount of energy to allow them to move into the crystal to form covalent bonds and become extension of the single crystal. Because the layer is grown on the substrate, epitaxy is a growth technique where the crystal is formed without reaching the melting point of silicon.

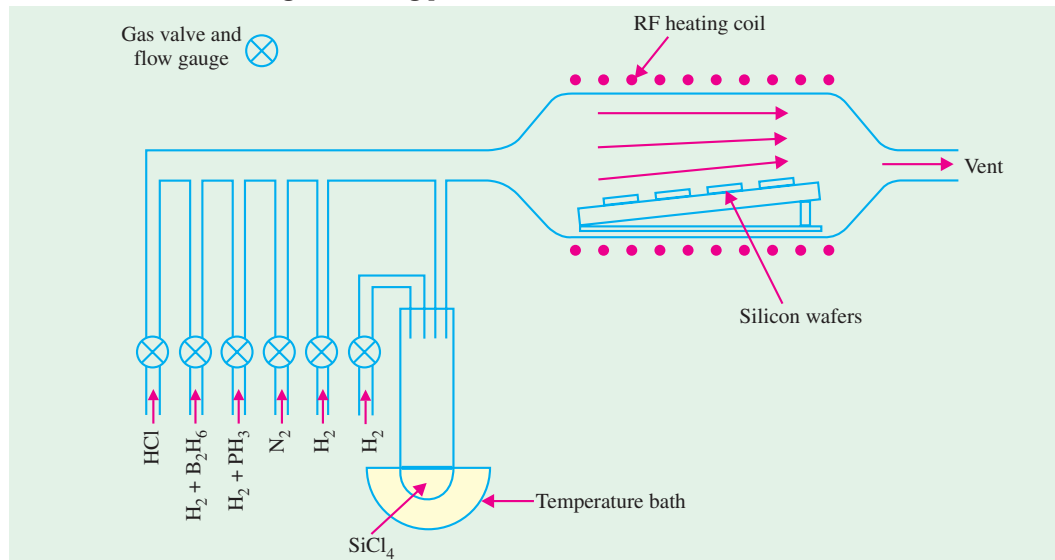


Fig. 67.15

### 67.25. Metallization and Interconnections

After all the fabrication steps of an  $IC$  are completed, it becomes necessary to provide metallic interconnections for the  $IC$  and for external connections to the  $IC$ . The requirement that must be met by the interconnections is that they have low resistance to minimize both the voltage drops on the lines as well as the capacitances between the lines so as to reduce delay times. The connections must

also make **ohmic contacts** to semiconductors in the devices such as *P* and *N* regions of a *PN* junction diode. An ohmic contact is one that exhibits a very low resistance, allowing currents to pass easily in both directions through the contact.

The high conductivity of aluminium makes it the metal of obvious choice, particularly in silicon-based devices. It also has the following advantages :

1. easy to evaporate
2. can be easily etched
3. not expensive,
4. adheres well to silicon dioxide.

There are variety of processes for depositing aluminium on silicon substrates but the following three are important from the subject point of view : (1) resistance heating, (2) electron beam heating and (3) sputtering.

**Resistance heating:** In this process, the source of the heated element and the silicon substrate are located in an evacuated chamber. The source is a small piece of aluminium attached to a coil of tungsten, which serves as a heater [refer to Fig. 67.16 (a)]. The heated element with a high melting point remains solid while the aluminium is vaporized. The aluminium atoms travel to the substrate where they condense, depositing an aluminium layer on the surface of the silicon. A photolithographic masking and etching method is used to remove the metal from the regions where it is not wanted. A typical interconnection between two diffused layers is as shown in Fig. 67.16 (b).



Several cycles of photolithography etching and doping are performed producing multiple layers of circuit elements on the wafer.

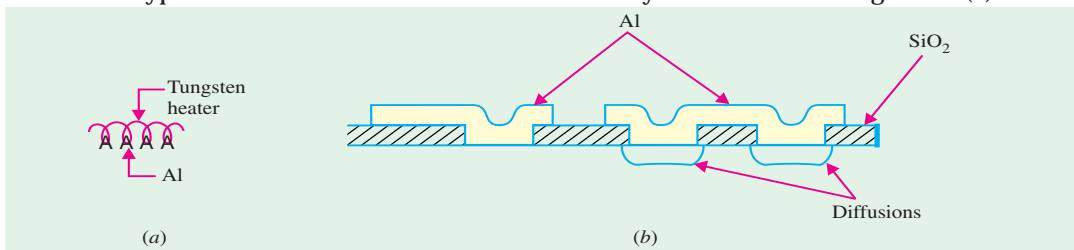


Fig. 67.16

In another method, the evaporation source kept in a boron nitride crucible is heated by radio frequency induction as shown in Fig. 67.17 (a). High deposition rates are possible through this process. However, the crucible may contaminate the metal.

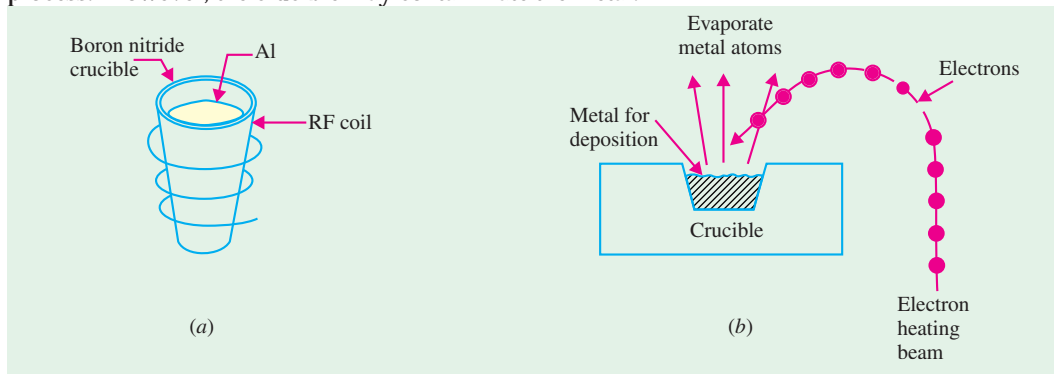


Fig. 67.17

**Electron-beam heating:** In this process, aluminium in a crucible is placed into a vacuum chamber together with the substrate. The aluminium is subjected to a high intensity electron beam formed by an electron gun, which vaporizes the aluminium as shown in Fig. 67.17 (b). This causes the aluminium to travel to the wafer. By the use of mask and photolithography, the aluminium is deposited on the identified regions on the wafer surface.

**Sputtering:** In this process, the material to be deposited is placed in a container maintained at low pressure in the vicinity of the substrate. The material to be deposited is labelled the cathode or the target, while the anode is the substrate. A DC or a radio-frequency high voltage is applied between anode and cathode. This high voltage ionizes the inert gas in the chamber. The ions are accelerated to the cathode (in this operation the anode is usually grounded) where, by impact with the aluminium target, atoms of aluminium are vaporized. A gas of aluminium atoms is generated and deposited on the surface of the wafer.

Following the deposition of aluminium, the silicon wafers are placed in a furnace to solidify the connections so that low resistance metallic contacts are made.

The interconnections between the elements of an *IC* are made by aluminium lines having a thickness of about 0.5  $\mu\text{m}$ . These are laid on top of the silicon dioxide layer, which covers the surface of the wafer. By using photolithography, openings are made in silicon dioxide so that the aluminium layer is connected to the silicon or to the ohmic contact on the silicon. In very complicated integrated circuits, it is necessary to have two or three vertically stacked layers of interconnections separated by silicon dioxide layers. The interconnecting lines terminate at aluminium pads (called bonding pads) from which connections to the outside are made.

### 67.26. Testing, Bonding and Packaging

The individual *IC* chip must be connected properly to outside leads and packaged in a way that is convenient for use in a larger circuit or a system. Since the devices are handled individually once they are separated from the wafer, bonding and packaging are expensive processes.

**Testing** – After the wafer of monolithic circuits has been processed and the final metallization pattern defined, it is placed in a holder under a microscope and is aligned for testing by a machine called **multiple-point probe**. The probe contacts the various pads on an individual circuit, and a series of tests are made to verify the electrical properties of the device in a very short time. After all the circuits are tested, the wafer is removed from the testing machine, sawed between the circuits, and broken apart. Then each die that passed the test is picked up and placed in the package.

**Bonding** – It consists of two steps. In the first step the back of the die is mechanically attached to an appropriate mount medium such as ceramic substrate, multi-layer-ceramic package or metal lead frame. The two common die bonding methods are **hard solders** and **polymers**. In the second step, the bond pads on the circuit side of the die are connected by wires to the package as shown in Fig. 67.18. The three common schemes of interconnection to the chip bond pads are (i) **wire bonding**, (ii) **tape-automated bonding (TAB)**, and (iii) **flip-chip solder bonding**. Wire bonding is further split into two different processes: **thermosonic** and **thermocompression**. The further detail of these processes is beyond the scope of this book.

**Packaging** – The final step in *IC* fabrication is packaging the device in a suitable medium that can protect it from environment of its intended application. In most cases this means the surface of the device must be isolated from the moisture and contaminants and the bonds and other elements must be protected from corrosion and mechanical shocks. The modern *ICs* are mounted in pack-

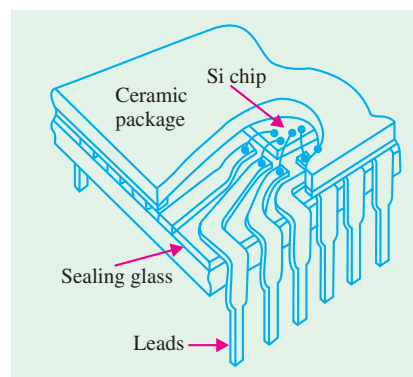


Fig. 67.18

ages with many output leads. In one version, chip is mounted on a stamped metal lead frame and wire bonding is done between the chip and the leads. The package is formed by applying a ceramic or plastic case and trimming away the unwanted parts of the lead frame.

Fig. 67.19 shows various types of the packaging of ICs used these days. As seen from this figure, the packages can be broadly classified into the following two categories :

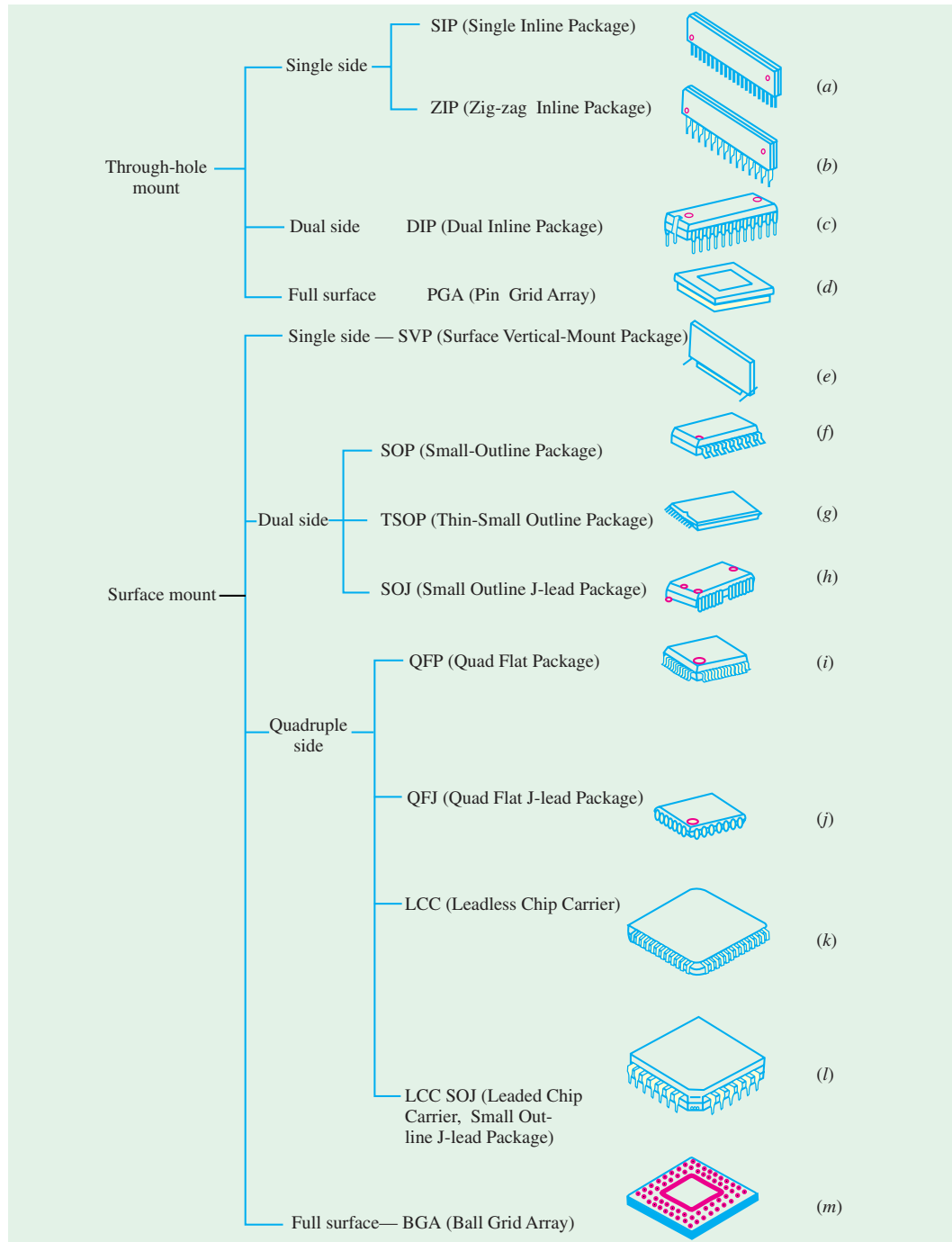


Fig. 67.19



1. **through-hole mount** that involve inserting the package pins through holes on the printed circuit board (PCB) before soldering [Fig. 67.19 (a) through (d)].
2. **surface mount type** where the leads do not pass through holes in the PCB. Instead, surface-mounted package leads are aligned to electrical contacts on the PCB, and are connected simultaneously by solder reflow [Fig. 67.19 (e) through (m)]. As refer to the picture shown in Fig 67.20.

Most packages can be made using ceramic or plastic. The ICs are hermetically sealed for protection from the environment. The pins can be on one side (single in-line or zigzag pattern of leads), two sides (dual in-line package or DIP), or four sides of the package (quad package). Most advanced packages have leads distributed over a large portion of the surface of the package as in through-hole pin grid arrays (PGAs) or surface-mounted ball grid arrays (BGAs).

### 67.27. Semiconductor Devices and Integrated Circuit Formation

There are literally thousands of different semiconductor device structures. They have been developed to achieve specific performances, either as discrete components or in ICs. Accordingly there are many different structures. However, there are basic structures required for each of the major device and circuit types.

1. **Resistors**: Most of the resistors in ICs are formed by the same processes that are used to form devices (i.e., a sequence of oxidation, masking and doping operation). Fig. 67.21 (a) shows a resistor made of a P-type region diffused into an N-type epitaxial layer. Notice the metallic contacts made at the two ends of the epitaxial layer. The section of the resistor, as dictated by the diffusion, is very nearly rectangular in shape, as shown in Fig. 67.21 (b). The value of a resistor (in ohms) is given by :

$$R = \rho \cdot l/a \text{ or } \rho \cdot l/w \cdot d$$

where  $\rho$  is the resistivity of the layer in ohm-cm,  $l$  is the length of the resistive region, and  $a$  is the cross-sectional area of the resistive region. The value of  $a$  is equals to  $w \cdot d$  where  $w$  is the width and  $d$  is the depth of the resistive region.



Fig. 67.20

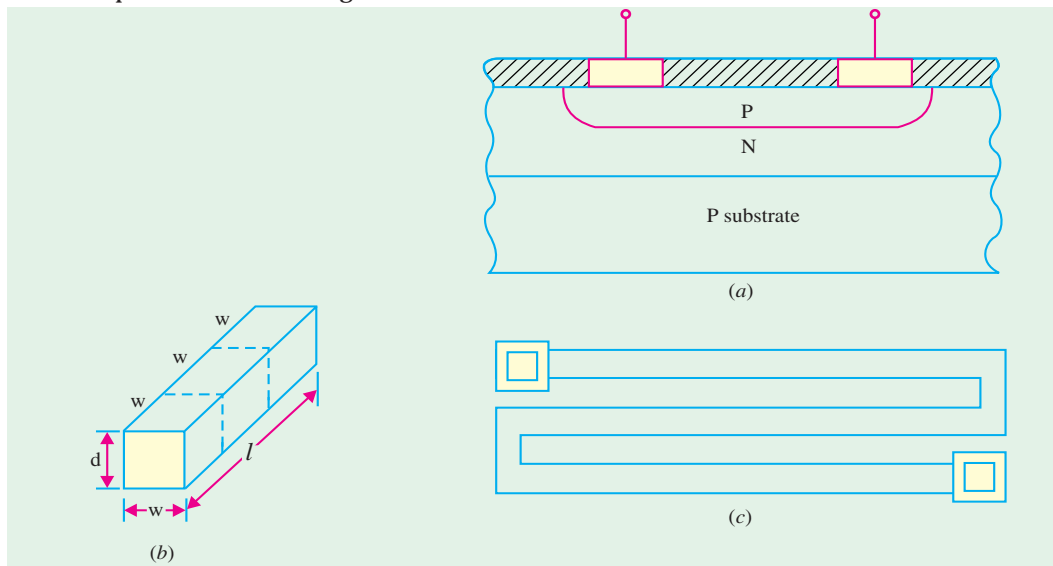


Fig. 67.21

Resistance in monolithic circuits are defined by the term called **sheet resistance**. The sheet resistance is the resistance of a square region having  $w = l$  and has units of ohms per square. Assuming a sheet resistance of 100 to 200 ohms per square, practical resistors may have values ranging from 100 to several kilohms. Higher resistance values are obtained by using a meander pattern as shown in Fig. 67.21 (c). The major problem with the resistors of high values is that they tend to occupy a large area on the chip. For example, a resistor of 50 k $\Omega$  uses up an area of the wafer that may be occupied by hundreds of the transistors. Ion-implantation instead of diffusion can be used to make resistors with precise values of resistance.

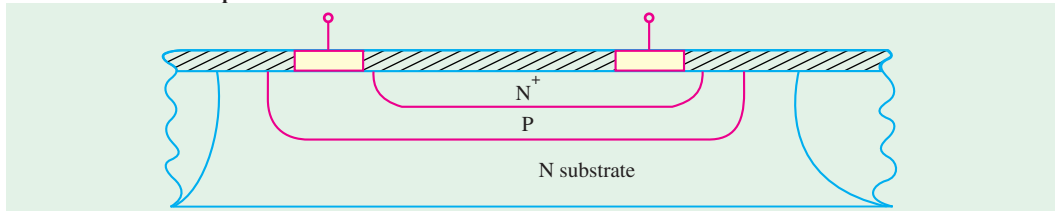


Fig. 67.22

**2. Capacitors:** One type of capacitor in monolithic circuits is made by using the capacitance formed between the  $P$  and  $N$  regions of a reverse-biased diode. Such a capacitor is shown in Fig. 67.22. These capacitors are formed by using the same diffusion processes that are used to form devices. Bipolar transistors are made of three regions in which either of the two  $PN$  junctions may be used as capacitors whereby the breakdown voltage of the capacitor may vary considerably from one to the other. The disadvantage of the junction capacitors is the dependence of the capacitance on the voltage applied to the junction. Capacitors that are voltage-independent can be formed from metal insulator  $N^+$  (*i.e.*, heavily-doped) semiconductor layers as used in MOS structures.

In dense circuits, an oxide/nitride/oxide dielectric sandwich is used. The combination film has a lower dielectric constant, allowing a capacitor area smaller than a conventional silicon dioxide capacitor.

Capacitors can also be fabricated by creating a trench etched vertically into the wafer surface (refer to Fig. 67.23). The trenches are etched either isotropically with wet etching or anisotropically with dry etching techniques. The trench side walls are oxidised and the centre of the trench is filled with deposited polysilicon. The final structure is “wired” from the surface, with the silicon and polysilicon serving as two electrodes and silicon dioxide as a dielectric material.

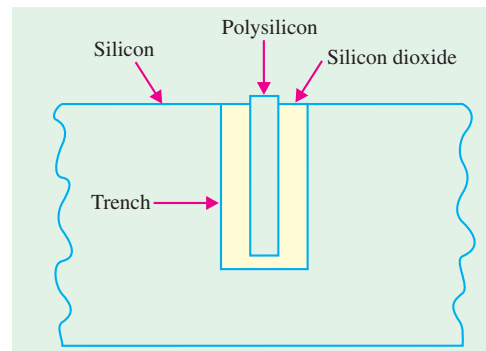


Fig. 67.23

The trenched capacitors are useful when preservation of wafer surface is main criteria. Another alternative to conserve wafer surface area is to build stacked capacitors on the wafer surface. This effort has been driven by the need for small high dielectric capacitors for dynamic random access memory (DRAM) circuits.

**3. Diodes:** In integrated circuits, where all the interconnections and device terminals are made at the surface, a diode is formed in the following two ways :

- (a) In bipolar circuits from a bipolar transistor by placing a **short-circuit** between two of the three terminals of the transistor (*i.e.*, collector to base or emitter to base). Thus there are emitter-base diodes and base-collector diodes.
- (b) In MOS circuits, most of the diodes are formed with the source-drain doping step.

However, in order to make a discrete  $PN$  junction diode, we will discuss the various steps involved in the fabrication.

The starting material is a heavily doped  $N$ -type ( $N^+$ ) substrate about  $150\ \mu\text{m}$  thick. A layer of  $N$ -type silicon ( $1$  to  $5\ \mu\text{m}$ ) is grown on the substrate by epitaxy as shown in Fig. 67.24 (a). Then a layer of silicon dioxide ( $\text{SiO}_2$ ) is deposited by oxidation as shown in Fig. 67.24 (b). Next, the surface is coated with positive photoresist as shown in Fig. 67.24 (c). Then a mask is placed on the surface of silicon, aligned and exposed to ultraviolet light (UV) as shown in Fig. 67.24 (d). Next, mask is removed, resist is removed and silicon dioxide layer under the exposed resist is etched as shown in Fig. 67.24 (e). Then boron is diffused to form  $P$ -type region as shown in Fig. 67.24 (f). Note that boron diffuses easily in silicon but not in silicon dioxide. Next, a thin aluminium film is deposited over the surface as shown in Fig. 67.24 (g). Then the metallized area is covered with resist another mask is used to identify areas where metal is to be preserved. Wafer surface is etched to remove unwanted metal. Resist is then dissolved and we get a structure as shown in Fig. 67.24 (h). Finally, contact metal is deposited on the back surface and ohmic contacts are made by heat treatment.

**4. Bipolar Transistor:** Let us consider the fabrication of an NPN bipolar junction transistor (BJT) on a silicon wafer. While our discussion will be focussed on the BJT, it is understood that the surface of the whole wafer is being processed.

The starting material is a lightly doped  $P$ -type wafer. We will consider a small area on this wafer where we will form a BJT. The base on which the transistor is made is known as the substrate. The function of the substrate is to act as a mechanical support for the device. The reason for the use of a  $P$ -type substrate for the NPN transistor will be clarified when the term isolation is discussed. The substrate has a resistivity of  $3\text{-}10\ \Omega\ \text{cm}$  with a thickness between  $500$  and  $700\ \mu\text{m}$  for wafers having diameters over  $100\ \text{mm}$ .

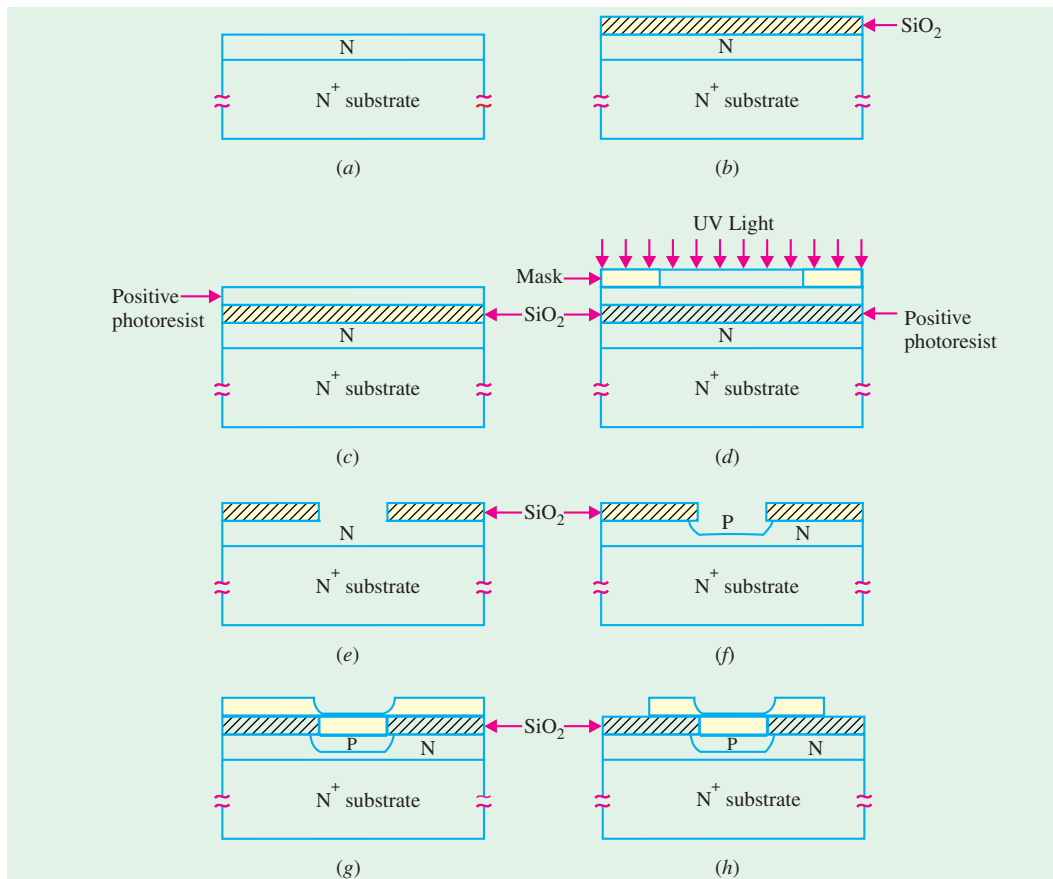


Fig. 67.24

Fig. 67.25 shows the fabrication steps up to, and including the metal contacts to the three regions (i.e., emitter, base and collector).

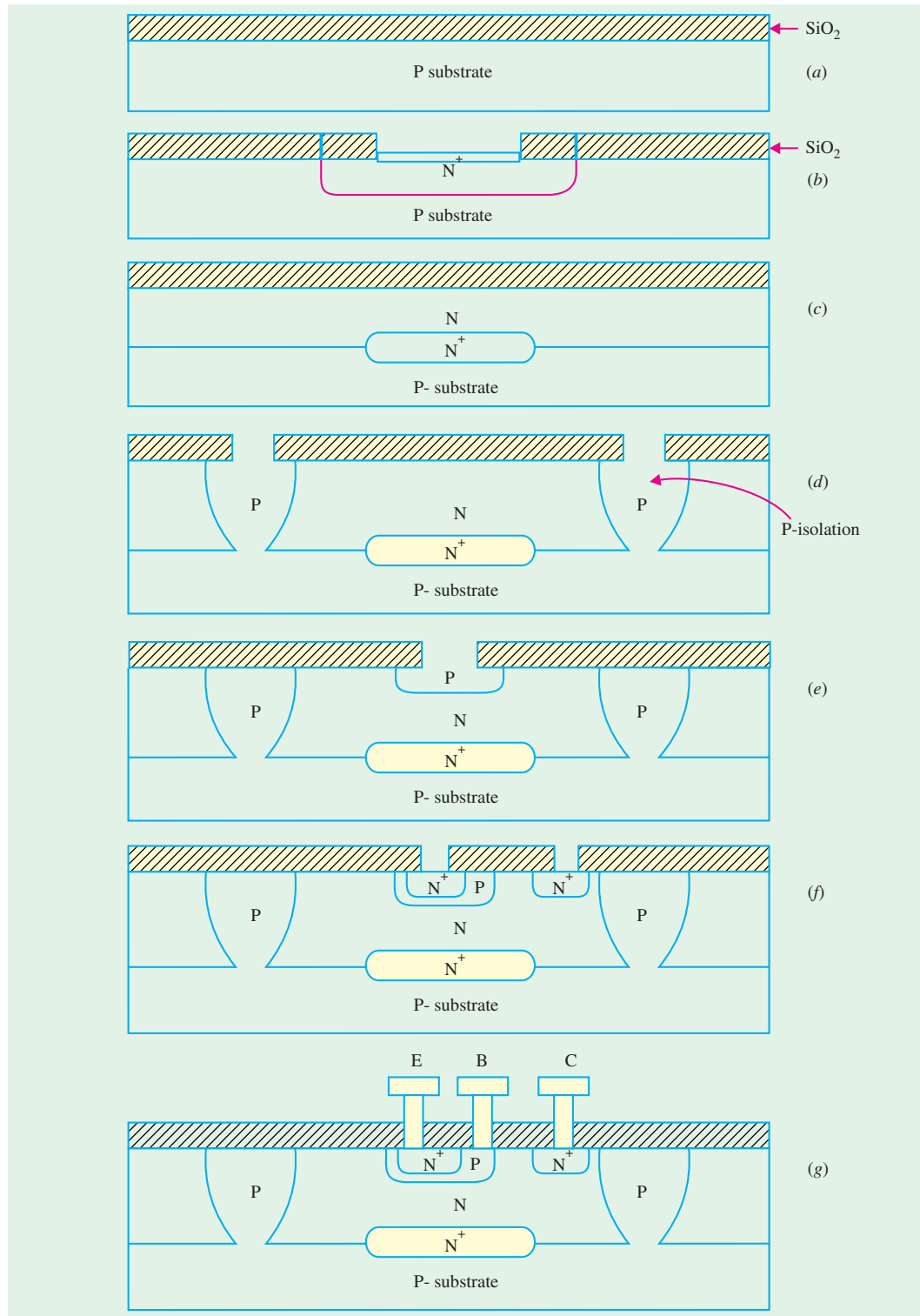


Fig. 67.25

First a layer of silicon dioxide ( $\text{SiO}_2$ ), about  $0.5\ \mu\text{m}$  thick, is deposited on the surface of the substrate by thermal oxidation as shown in Fig. 67.25 (a). Using the **first mask** and the photolithographic process, windows are opened in the oxide for the **buried layer**. The  $\text{N}^+$  (*i.e.*, heavily doped  $N$ -type) buried layer is diffused to a depth of about  $3\ \mu\text{m}$  as shown in Fig. 67.25 (b). This layer serves to collect the carriers that have crossed the base on their way to the collector terminal. It serves as a sub-collector and is used to reduce the collector ohmic-resistance. After the buried layer is diffused, the wafer is stripped of all oxide to permit the next deposition. It is to be noted that during the subsequent high-temperature processes, the buried layer tends to diffuse out.

The next operation is the deposition of a phosphorus-doped  $N$ -type **epitaxial layer** on the whole wafer. This layer has a resistivity of  $0.1$  to  $1\ \Omega\ \text{cm}$ . The thickness of this layer is  $0.5$  to  $5\ \mu\text{m}$  for high-speed digital circuit applications and  $10$ - $20\ \mu\text{m}$  for linear analog circuits. A layer of silicon dioxide ( $\text{SiO}_2$ ) about  $0.5$  to  $1\ \mu\text{m}$  thick is grown thermally on the surface of the epitaxial layer as shown in Fig. 67.25 (c).

Since the collector of NPN transistor is  $N$ -type, and so are the collectors of the adjacent transistors, there is an obvious need to isolate the collectors from each other. The **second mask** is used to etch windows for this isolation regions, which are formed by the subsequent diffusion of boron extending from the surface down to the substrate as shown Fig. 67.25 (d). An  $N$ -type epitaxial layer separates the isolation regions, thus serving as the tub in which each transistor is formed. The diffusion of the isolation region is followed by oxidation of the wafer surface.

The **third mask** is used to open a window for the  $P$ -type base of the transistor,  $P$ -type diffusion or ion implantation is driven to form the base to a depth of about  $2$ - $3\ \mu\text{m}$  as shown in Fig. 67.25 (e). This is followed by the deposition of an oxide layer. The **fourth mask** is used to open windows in the oxide for the  $\text{N}^+$  emitter and the collector contacts. The phosphorus or arsenic diffusion is driven to a depth of about  $2\ \mu\text{m}$  as shown in Fig. 67.25 (f). The need for  $\text{N}^+$  collector is to form a good ohmic contact. The ohmic contact permits easy current flow in both directions. To form a good ohmic contact to an  $N$  material, an  $\text{N}^+$  region is needed between the metal on the top and the  $N$  region. Following the  $\text{N}^+$  diffusion, an oxidation layer is formed over the entire wafer surface.

The **fifth mask** is used to open windows for the formation of metallic contacts to the bipolar transistor terminals. Then an aluminium film  $0.5$  and  $1\ \mu\text{m}$  thick is deposited, by evaporation or sputtering, on the top surface of the *IC* wafer. Assuming that the complete circuits are to be formed on the surface of the wafer, the **sixth mask** is used to define the interconnection pattern in the circuits. These interconnections are etched into the metal that has been deposited on the surface, as shown in Fig. 67.25 (g).

In order to protect the surface of the wafer from moisture and chemical contamination, a silicon nitride ( $\text{Si}_3\text{N}_4$ ), called a **passivation layer**, is deposited on the surface. Contacts to the integrated circuits are made on pads that are located on the periphery of the *IC* chip. Since the *IC* chip will be bonded to an *IC* package, connections are to be made from the package leads to the bonding pads on the *IC* chip.

The **seventh mask** is used to define the bonding holes over the aluminium pads for external connections. Following the seven masks, the circuits are tested by a computer-controlled system and all faulty chips are identified and marked. The wafer is then sawed into chips, which are bonded on *IC* packages. Gold wires about  $25\ \mu\text{m}$  in diameter are used to connect the package leads to the bonding pads on the chip.

Fig. 67.26 shows an additional detail on the buried layer. It indicates the path taken by the carriers on their way from the emitter to the base and to the collector. This path is considerably longer than the path in the discrete BJT shown in Fig. 67.26 (a). Because of this the collector series resistance, labelled the **parasitic resistance**, is quite large and is of the order of hundreds of ohms. To reduce this resistance, we have placed the low resistivity buried layer in Fig. 67.26 (b) into the path of

the carriers which acts as a subcollector. The use of the buried layer reduces the collector resistance by as much as a factor of 20. The result of this is to improve gain-bandwidth product of the transistor by the same factor.

**5. MOSFET:** Let us consider the fabrication of Enhancement mode  $N$ -channel metal oxide semiconductor field-effect transistor. The starting material is a lightly doped  $P$ -type silicon substrate having a resistivity of about  $5 \Omega \text{ cm}$ . The doping density is determined by the drain-substrate breakdown voltage of 20-30 V. A thin (about 20 nm) layer of silicon dioxide is formed over the substrate to provide stress relief to the wafer. Next a thin (about 20 nm) layer of silicon nitride ( $\text{Si}_3\text{N}_4$ ) is deposited by the chemical vapour deposition (CVD) process on top of the silicon dioxide. The silicon dioxide permits selective oxidation so that a thick oxide (about 500 nm) can be formed in the field region. These operations result in Fig. 67.27 (a).

The **first mask** defines the field-effect transistor areas, so that the silicon dioxide and the silicon nitride are chemically etched out except where the transistor is formed.

The next step is to diffuse (or ion-implant) boron in the field-regions to form  $P^+$  (*i.e.* heavily doped  $P$ -type) islands in the substrate. The function of  $P^+$  islands is to help increase the threshold voltage,  $V_T$  and prevent the formation of "**parasitic**" transistors (or electrical cross talk) between adjacent devices on the wafer. This is followed by the formation of a field oxide layer (about 500 nm thick) over the  $P^+$  implanted region as shown in Fig. 67.27 (b). The field oxide layer also helps to increase  $V_T$ .

The remaining silicon nitride and silicon dioxide are etched away and an ultra thin (about 5 to 10 nm) layer of  $\text{SiO}_2$  is grown over the transistor area (not above the field oxide). This forms the gate-oxide of the transistor. Refer to Fig. 67.27 (c). Next, a layer of heavily doped (typically  $N^+$ ) polysilicon is deposited over the entire wafer surface. This is shown in Fig. 67.27 (d).

The **second mask** defines the gate region, whereupon the polysilicon is etched away except over the gate as shown in Fig. 67.27 (e). The heavily doped polysilicon region above the gate behaves electrically like a metal electrode.

By ion-implantation and using the polysilicon gate and the field oxide, as the mask, the source and drain  $N^+$  regions are formed as shown in Fig. 67.27 (e). The  $N^+$  layers literally diffuse a sufficient distance to ensure proper alignment so that the channel length is well defined. The dopants do not penetrate the field oxide. A thin layer of silicon dioxide is then grown, by the CVD process over the wafer.

The **third mask** is used to open windows for the metal contacts to the transistor regions as shown in Fig. 67.27 (f). The thin layer of silicon dioxide is etched away and aluminium is deposited over the surface of the wafer by evaporation or sputtering. The **fourth mask** defines the interconnection pattern that is etched in the aluminium, as shown in Fig. 67.27 (g). This is followed by the deposition of a protective passivation layer of phosphosilicate glass (called  $P$ -glass) over the entire surface of the wafer.

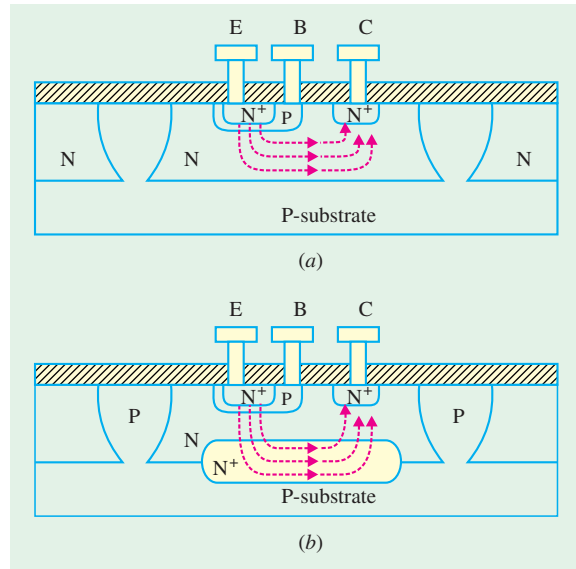


Fig. 67.26

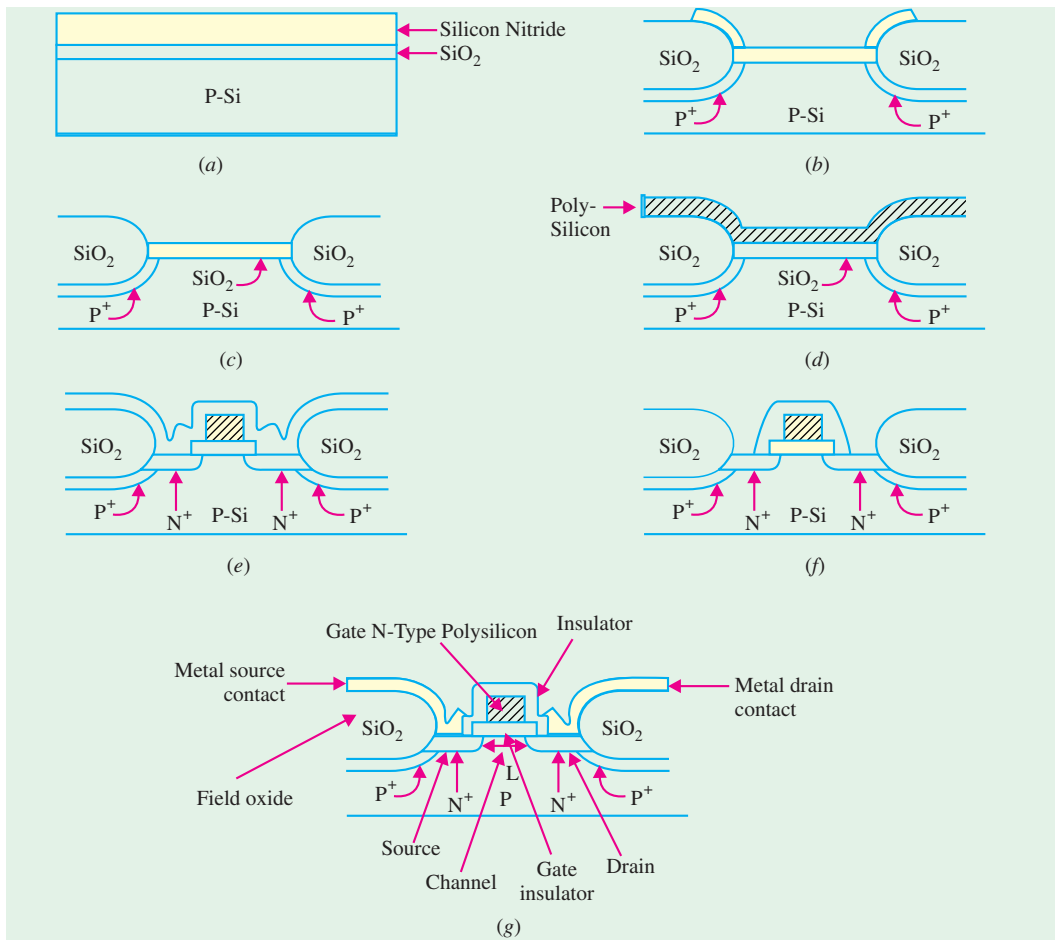


Fig. 67.27

Just as with the BJT, the **fifth mask** is used to open windows, so that bonding wires can be connected to the pads on the IC chip.

### 67.28. Comparison of ICs based on MOS and Bipolar Transistor Technology

1. The ICs based on MOS are less costly to fabricate as compared to those using bipolar transistor. The reason for this is that MOS devices are self-isolating. Bipolar transistors require tubs to isolate devices from each other on one integrated circuit. Isolation in MOS devices is provided by heavy doping and a thick oxide in the regions between adjacent devices.

2. MOS circuits consume less DC power as compared to bipolar transistor circuits.

3. The MOS transistor has lower value of transconductance ( $g_m$ ) as compared to the bipolar transistor. This feature makes the bipolar transistor ICs superior to MOS circuits for analog circuit applications.

4. For the same channel length and the base width, the limiting cut-off frequency of the MOS transistor is better than a bipolar transistor. Therefore the MOS transistor has a higher bandwidth than the bipolar transistor.

5. The packing density of MOS ICs is at least 10 times more than that for bipolar ICs. Also, a MOS resistor occupies less than 1% of the area of a conventional diffused resistor. This high packing density makes MOS ICs especially suited for LSI, VLSI and ULSI circuits.

The main disadvantage of MOS ICs is their slower speed as compared to bipolar ICs. Hence they do not compete with bipolar ICs in ultrahigh-speed applications.

However, due to their (i) low cost, (ii) low power consumption and (iii) high packing density, MOS ICs are widely produced by semiconductor manufacturing industry. The MOS ICs are available as calculator chips, memory chips, microprocessors ( $\mu$ P), single-chip computers etc.

### 67.29. Popular Applications of ICs

There are plenty of electronic products in the market, which used integrated circuits extensively. One of the widely accepted applications is the digital watch, which can display hours, minutes, seconds, day and month. Another popular application is electronic calculator which can perform various functions like addition, subtraction, multiplication and division. There are also advanced scientific calculators that are programmable and can display the graphs.

Modern electronic products such as pocket PC, personal digital assistant (PDA), MP3 players, digital cameras, digital camcorders, mobile phones, digital dictionaries and digital translators, CD (compact disk) players, DVD (digital versatile disk) players etc. also make use of ICs extensively. With a Pocket PC, you can read and send e-mail, edit Word, Excel and Outlook files, listen to digital music, read electronic books (eBooks), track your finances, and browse the web. The digital camera shown in Fig. 67.28 can take pictures and store on the floppy diskette or memory stick. A personal digital assistant (PDA) as shown in Fig. 67.29 is a handheld computer which allows the user to organise calendar, contacts or tasks. These little electronic wonders might put office secretary out of a job.

Electronic games have evolved from microelectronics and computer knowhow. Video games that have to be hooked up to home TV have become very popular. Hand held PCs and mobile phones also come with games that require no TV hook-up.



Fig. 67.28

(courtesy Sony Corporation, Japan)

Fig. 67.29

(courtesy : 3Com, USA)

### OBJECTIVE TESTS – 67

1. First integrated circuit chip was developed by
  - (a) C.V. Raman
  - (b) W.H. Brattain
  - (c) J.S. Kilby
  - (d) Robert Noyce.
2. An integrated electronic circuit is
  - (a) a complicated circuit
  - (b) an integrating device
  - (c) much costlier than a single transistor
  - (d) fabricated on a tiny silicon chip.



3. Most important advantage of an IC is its  
(a) easy replacement in case of circuit failure  
(b) extremely high reliability  
(c) reduced cost  
(d) low power consumption.
4. In monolithic ICs, all components are fabricated by ..... process.  
(a) evaporation  
(b) sputtering  
(c) diffusion  
(d) oxidation.
5. Monolithic ICs are fabricated within a  
(a) soft stone  
(b) single stone  
(c) silicon layer  
(d) ceramic base.
6. As compared to monolithic ICs, film ICs have the advantage of  
(a) better high-frequency response  
(b) much reduced cost  
(c) smaller size  
(d) less flexibility in circuit design.
7. Monolithic technique is ideally suited for fabricating ..... ICs.  
(a) thin-film  
(b) digital  
(c) linear  
(d) thick-film
8. In the context of IC fabrication, metallisation means  
(a) connecting metallic wires  
(b) forming interconnecting conduction pattern and bonding pads  
(c) depositing  $\text{SiO}_2$  layer  
(d) covering with a metallic cap.
9. Monolithic transistors are formed in the epitaxial  $N$ -layer  
(a) in one operation  
(b) by evaporation process  
(c) by successive impurity diffusions  
(d) by oxidation.
10. The major component of a MOS IC is a/an  
(a) FET (b) MOSFET  
(c) BJT (d) SCR.
11. Processing of MOS ICs is less expensive than bipolar ICs primarily because they  
(a) use cheaper components  
(b) need no component isolation  
(c) require much less diffusion steps  
(d) have very high packing density.
12. In an integrated circuit, the  $\text{SiO}_2$  layer provides  
(a) electrical connection to the external circuit  
(b) physical strength  
(c) isolation  
(d) conducting path.
13. The foundation on which an IC is built is called an  
(a) insulator (b) base  
(c) wafer (d) plate
14. Monolithic IC construction uses  
(a) discrete components  
(b) high-value resistors  
(c) connecting wires  
(d) extensive number of components

## ANSWERS

1. (c) 2. (d) 3. (b) 4. (c) 5. (c) 6. (a) 7. (b) 8. (b) 9. (c) 10. (b) 11. (c) 12. (c)  
13. (c) 14. (d)

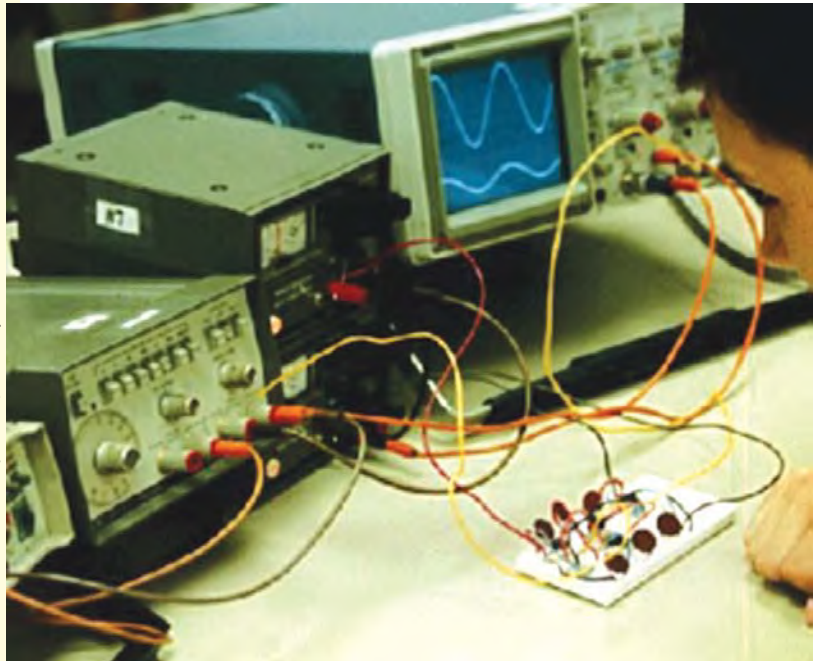
## ROUGH WORK

# CHAPTER 68

## Learning Objectives

- What is an OP-AMP?
- OP-AMP Symbol
- Polarity Conventions
- Ideal Operational Amplifier
- Virtual Ground and Summing Point
- Why  $V_i$  is Reduced to Almost Zero?
- OP-AMP Applications
- Linear Amplifier
- Unity Follower
- Adder or Summer
- Subtractor
- Integrator
- Differentiator
- Comparator
- Audio Amplifier
- OP-Amp Based Oscillator Circuits
- OP-Amp Based Wien Bridge Oscillator
- OP-Amp Based Crystal Oscillator
- A Triangular-wave Oscillator
- A Voltage- controlled Sawtooth Oscillator (VCO)
- A Square-wave Relaxation Oscillator
- High-impedance Voltmeter
- Active Filters
- Low-pass Filter
- High-pass Filter
- Band-pass Filter
- Notch Filter

## OP-AMP AND ITS APPLICATIONS



The operational amplifier was designed to perform mathematical operations. Although now superseded by the digital computer, op-amps are a common feature of modern analog electronics

### 68.1 What is an OP-AMP ?

It is a very high-gain, high- $r_{in}$  directly-coupled negative-feedback amplifier which can amplify signals having frequency ranging from **0 Hz to a little beyond 1 MHz**. They are made with different internal configurations in linear ICs. An *OP-AMP* is so named because it was originally designed to perform mathematical operations like summation, subtraction, multiplication, differentiation and integration etc. in analog computers. Present day usage is much wider in scope but the popular name *OP-AMP* continues.

Typical uses of *OP-AMP* are : scale changing, analog computer operations, in instrumentation and control systems and a great variety of phase-shift and oscillator circuits. The *OP-AMP* is available in three different packages (i) standard dual-in-line package (*DIL*) (ii) *TO-5* case and (iii) the flat-pack.

Although an *OP-AMP* is a complete amplifier, it is so designed that external components (resistors, capacitors etc.) can be connected to its terminals to change its external characteristics. Hence, it is relatively easy to tailor this amplifier to fit a particular application and it is, in fact, due to this versatility that *OP-AMPs* have become so popular in industry.

An *OP-AMP IC* may contain two dozen transistors, a dozen resistors and one or two capacitors.

#### Example of OP-AMPs

1.  $\mu\text{A 709}$ —is a high-gain operational amplifier constructed on a single silicon chip using planar epitaxial process.  
It is intended for use in dc servo systems, high-impedance analog computers and in low-level instrumentation applications.  
It is manufactured by Semiconductors Limited, Pune.
2. [*LM 108 – LM 208*]— Manufactured by Semiconductors Ltd. Bombay,
3. *CA 741 CT* and *CA 741 T*—these are high-gain operational amplifiers which are intended for use as (i) comparator, (ii) integrator, (iii) differentiator, (iv) summer, (v) dc amplifier, (vi) multivibrator and (vii) bandpass filter.  
Manufactured by Bharat Electronics Ltd (*BEL*), Bangalore.

### 68.2. OP-AMP Symbol

Standard triangular symbol for an *OP-AMP* is shown in Fig. 68.1 (a) though the one shown in Fig. 68.1 (b) is also used often. In Fig. 68.1 (b), **the common ground line has been omitted**. It also does not show other necessary connections such as for dc power and feedback etc.

The *OP-AMP's* input can be single-ended or double-ended (or differential input) depending on whether input voltage is applied to one input terminal only or to both. Similarly, amplifier's output can also be either single-ended or double-ended. The most common configuration is **two input terminals and a single output**.

All *OP-AMPs* have a minimum of five terminals :

- |                                   |                                   |
|-----------------------------------|-----------------------------------|
| 1. inverting input terminal,      | 2. non-inverting input terminal,  |
| 3. output terminal,               | 4. positive bias supply terminal, |
| 5. negative bias supply terminal. |                                   |

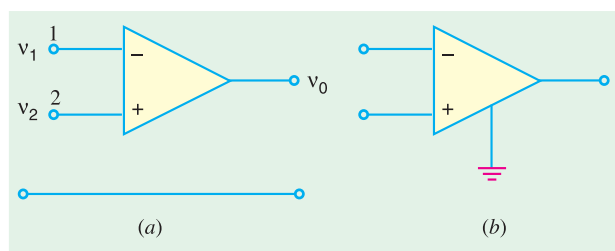


Fig. 68.1

### 68.3. Polarity Conventions

In Fig. 68.1 (b), the input terminals have been marked with minus (–) and plus (+) signs. These are meant to **indicate the inverting and non-inverting terminals only** [Fig. 68.2]. It simply means that a signal applied at negative input terminal will appear amplified but **phase-inverted at the output terminal** as shown in Fig. 68.2 (b). Similarly, signal applied at the positive input terminal will appear amplified and **inphase** at the output. Obviously, these plus and minus polarities **indicate phase reversal only**. It does not mean that voltage  $v_1$  and  $v_2$  in Fig. 68.2 (a) are negative and positive respectively. Additionally, it also does not imply that a **positive input voltage has to be connected to the plus-marked non-inverting terminal 2 and negative input voltage to the negative-marked inverting terminal 1**.

In fact, the amplifier can be used ‘either way up’ so to speak. It may also be noted that all input and output voltages are referred to a common reference usually the ground shown in Fig. 68.1 (a).

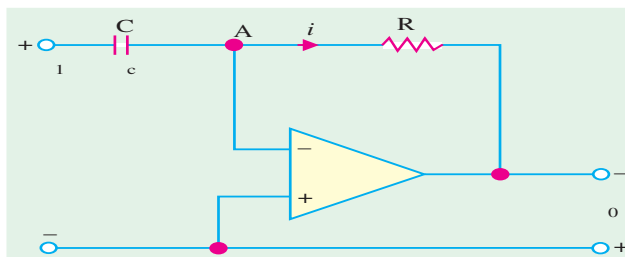


Fig. 68.2

### 68.4. Ideal Operational Amplifier

When an *OP-AMP* is operated without connecting any resistor or capacitor from its output to any one of its inputs (*i.e.*, without feedback), it is said to be in the **open-loop condition**. The word ‘open loop’ means that **feedback path or loop is open**. The specifications of *OP-AMP* under such condition are called open-loop specifications.

An ideal *OP-AMP* (Fig. 68.3) has the following characteristics :

1. its open-loop gain  $A_v$  is **infinite** *i.e.*,  $A_v = -\infty$
2. its input resistance  $R_i$  (measured between inverting and non-inverting terminals) is **infinite** *i.e.*,  $R_i = \infty$  ohm
3. its output resistance  $R_o$  (seen looking back into output terminals) is **zero** *i.e.*,  $R_o = 0 \Omega$
4. it has **infinite bandwidth** *i.e.*, it has flat frequency response from dc to infinity.

Though these characteristics cannot be achieved in practice, yet an ideal *OP-AMP* serves as a convenient reference against which real *OP-AMPs* may be evaluated.

Following additional points are worth noting :

1. infinite input resistance means that input current  $i = 0$  as indicated in Fig. 68.3. It means that an ideal *OP-AMP* is a voltage-controlled device.
2.  $R_o = 0$  means that  $v_o$  is not dependent on the load resistance connected across the output.
3. though for an ideal *OP-AMP*  $A_v = \infty$ , for an actual one, it is extremely high *i.e.*, about  $10^6$ . However, it does not mean that 1 V signal will be amplified to  $10^6$  V at the output. Actually, the maximum value of  $v_o$  is limited by the basis supply voltage, typically  $\pm 15$  V. With  $A_v = 10^6$  and  $v_o = 15$  V the maximum value of input voltage is limited to  $15/10^6 = 15 \mu\text{V}$ . Though 1  $\mu\text{V}$  in the *OP-AMP*, can certainly become 1 V.

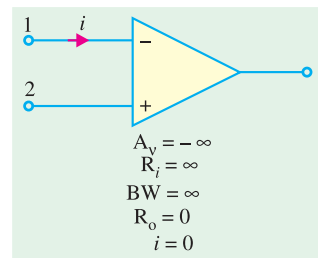


Fig. 68.3

### 68.5. Virtual Ground and Summing Point

In Fig. 68.4 is shown an *OP-AMP* which employs **negative feedback** with the help of resistor  $R_f$  which feeds a portion of the output to the input.

Since input and feedback currents are algebraically added at point A, it is called the **summing point**.

The concept of **virtual ground** arises from the fact that input voltage  $v_i$  at the inverting terminal of the *OP-AMP* is forced to such a small value that, for all practical purposes, it may be assumed to be zero. Hence, point A is essentially at ground voltage and is referred to as **virtual ground**. Obviously, **it is not the actual ground**, which, as seen from Fig. 68.4, is situated below.

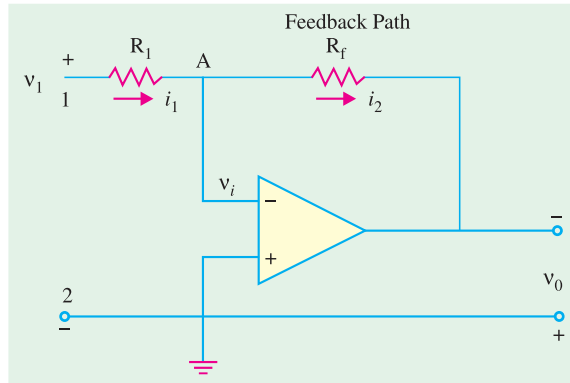


Fig. 68.4

### 68.6. Why $V_i$ is Reduced to Almost Zero ?

When  $v_1$  is applied, point A attains some positive potential and at the same time  $v_0$  is brought into existence. Due to negative feedback, some fraction of the output voltage is fed back to point A antiphase with the voltage already existing there (due to  $v_1$ ).

The algebraic sum of the two voltages is almost zero so that  $v_i \cong 0$ . Obviously,  $v_i$  will become exactly zero when **negative feedback voltage at A is exactly equal to the positive voltage produced by  $v_1$  at A**.

Another point worth considering is that there exists a virtual short between the two terminals of the *OP-AMP* because  $v_i = 0$ . It is virtual because no current flows (remember  $i = 0$ ) despite the existence of this short.

### 68.7. OP-AMP Applications

We will consider the following applications :

1. as scalar or linear (*i.e.*, small-signal) constant-gain amplifier both inverting and non-inverting,
2. as unity follower,
3. Adder or Summer
4. Subtractor,
5. Integrator
6. Differentiator
7. Comparator.

Now, we will discuss the above circuits one by one assuming an ideal *OP-AMP*.

### 68.8. Linear Amplifier

We will consider the functioning of an *OP-AMP* as constant-gain amplifier both in the inverting and non-inverting configurations.

#### (a) Inverting Amplifier or Negative Scale.

As shown in Fig. 68.5, noninverting

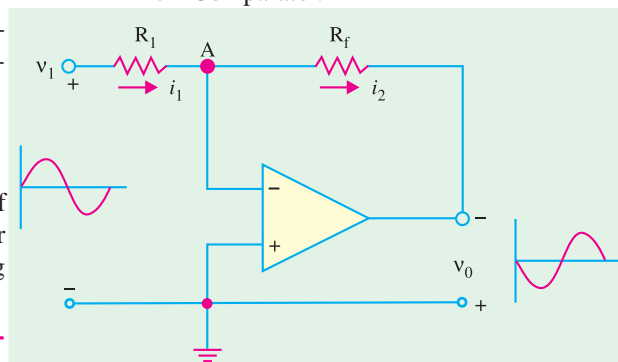


Fig. 68.5

terminal has been grounded, whereas  $R_1$  connects the input signal  $v_1$  to the inverting input. A feedback resistor  $R_f$  has been connected from the output to the inverting input.

**Gain**

Since point A is at ground potential\*,  $i_1 = \frac{v_{in}}{R_1} = \frac{v_1}{R_1}$

$$i_2 = \frac{-v_0}{R_f} \text{ Please note -ve sign}$$

Using *KCL* (Art. 2.2) for point A,

$$i_1 + (-i_2) = 0 \text{ or } \frac{v_1}{R_1} + \frac{v_0}{R_f} = 0 \text{ or } \frac{v_0}{R_f} = -\frac{v_1}{R_1} \text{ or } \frac{v_0}{v_1} = -\frac{R_f}{R_1}$$

$$\therefore A_v = -\frac{R_f}{R_1} \text{ or } A_v = -K \text{ Also, } v_0 = -Kv_{in}$$

It is seen from above, that closed-loop gain of the inverting amplifier depends on the ratio of the two external resistors  $R_1$  and  $R_f$  and is independent of the amplifier parameters.

It is also seen that the *OP-AMP* works as a negative scaler. It scales the input *i.e.*, it multiplies the input by a minus constant factor  $K$ .

**(b) Non-inverting Amplifier or Positive Scaler**

This circuit is used when there is need for an output which is equal to the input multiplied by a positive constant. Such a positive scaler circuit which uses negative feedback but provides an output that equals the input multiplied by a positive constant is shown in Fig. 68.6.

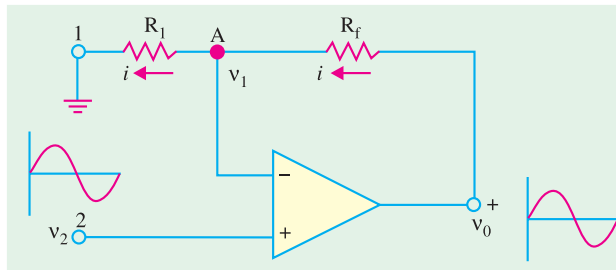


Fig. 68.6

Since input voltage  $v_2$  is applied to the non-inverting terminal, the circuit is also called **non-inverting amplifier**.

Here, polarity of  $v_0$  is the same as that  $v_2$  *i.e.*, both are positive.

**Gain**

Because of virtual short between the two *OP-AMP* terminals, voltage across  $R_1$  is the input voltage  $v_2$ . Also,  $v_0$  is applied across the series combination of  $R_1$  and  $R_f$ .

$$\therefore v_{in} = v_2 = iR_1, v_0 = i(R_1 + R_f)$$

$$\therefore A_v = \frac{v_0}{v_{in}} = \frac{i(R_1 + R_f)}{iR_1} \text{ or } A_v = \frac{R_1 + R_f}{R_1} = \left(1 + \frac{R_f}{R_1}\right)$$

**Alternative Derivation**

As shown in Fig. 68.7, let the currents through the two resistors be  $i_1$  and  $i_2$ .

The voltage across  $R_1$  is  $v_2$  and that across  $R_f$  is  $(v_0 - v_2)$ .

$$\therefore i_1 = \frac{v_2}{R_1} \text{ and } i_2 = \frac{v_0 - v_2}{R_f}$$

Applying *KCL* to junction A, we have

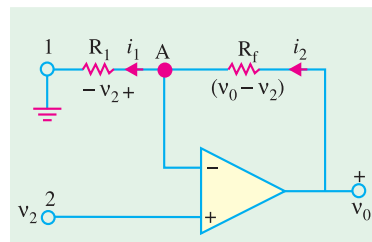


Fig. 68.7

\* If not, then  $i_1 = \frac{v_1 - v_i}{R_1}$  and  $i_2 = \frac{v_0 - v_i}{R_f}$

$$(-i_1) + i_2 = 0 \quad \text{or} \quad \frac{v_2}{R_1} + \frac{(v_0 - v_2)}{R_f} = 0$$

$$\therefore \frac{v_0}{R_f} = v_2 \left( \frac{1}{R_1} + \frac{1}{R_f} \right) = v_2 \frac{R_1 + R_f}{R_1 R_f}$$

$$\therefore \frac{v_0}{v_2} = \frac{R_1 + R_f}{R_1} \quad \text{or} \quad A_v = 1 + \frac{R_f}{R_1} \quad \text{---as before}$$

**Example 68.1.** For the inverting amplifier of Fig. 68.5,  $R_1 = 1 \text{ K}$  and  $R_f = 1 \text{ M}$ . Assuming an ideal OP-AMP amplifier, determine the following circuit values :

- (a) voltage gain, (b) input resistance, (c) output resistance

**Solution.** It should be noted that we will be calculating values of the circuit and not for the OP-AMP proper.

(a) 
$$A_v = -\frac{R_f}{R_1} = -\frac{1000 \text{ K}}{1 \text{ K}} = -1000$$

(b) Because of virtual ground at A,  $R_{in} = R_1 = 1 \text{ K}$

(c) Output resistance of the circuit equal the output resistance of the OP-AMP i.e., **zero ohm**.

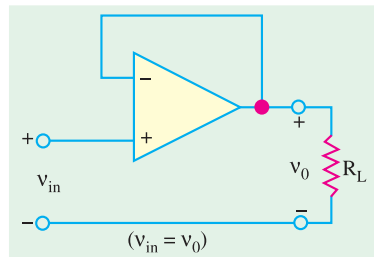


Fig. 68.8

### 68.9. Unity Follower

It provides a gain of unity without any phase reversal. It is very much similar to the emitter follower (Art 68.8) except that its gain is very much closer to being exactly unity.

This circuit (Fig. 68.8) is useful as a buffer or isolation amplifier because it allows, input voltage  $v_{in}$  to be transferred as output voltage  $v_0$  while at the same time preventing load resistance  $R_L$  from loading down the input source. It is due to the fact that its  $R_i = \infty$  and  $R_o = 0$ .

In fact, circuit of Fig. 68.8 can be obtained from that of Fig. 68.6 by putting

$$R_1 = R_f = 0$$

### 68.10. Adder to Summer

The adder circuit provides an output voltage proportional to or equal to the algebraic sum of two or more input voltages each multiplied by a constant gain factor. It is basically similar to a scaler (Fig. 68.5) except that it has more than one input. Fig. 68.9 shows a three-input inverting adder circuit. As seen, **the output voltage is phase-inverted**.

#### Calculations

As before, we will treat point A as virtual ground

$$i_1 = \frac{v_1}{R_1} \quad \text{and} \quad i_2 = \frac{v_2}{R_2}$$

$$i_3 = \frac{v_3}{R_3} \quad \text{and} \quad i = -\frac{v_0}{R_f}$$

Applying KCI to point A, we have

$$i_1 + i_2 + i_3 + (-i) = 0$$

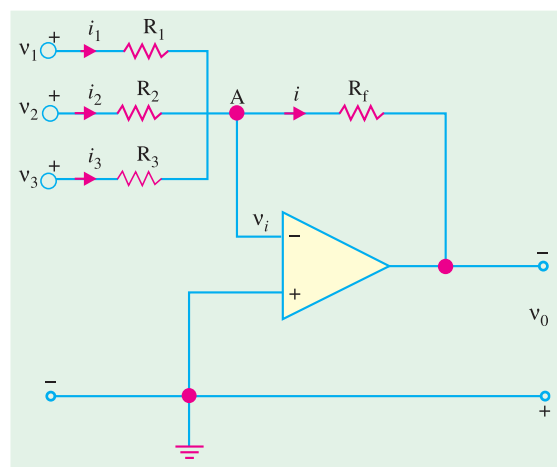


Fig. 68.9



or 
$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} - \left(\frac{-v_0}{R_f}\right) = 0$$

∴ 
$$v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$

or 
$$v_0 = -(K_1 v_1 + K_2 v_2 + K_3 v_3)$$

The overall negative sign is unavoidable because we are using the inverting input terminal.

If  $R_1 = R_2 = R_3 = R$ , then

$$v_0 = -\frac{R_f}{R} (v_1 + v_2 + v_3) = -K (v_1 + v_2 + v_3)$$

Hence, output voltage is proportional to (*not equal to*) the **algebraic sum** of the three input voltages.

If  $R_f = R$ , then output exactly equals the sum of inputs. However, if  $R_f = R/3$

then 
$$v_0 = -\frac{R/3}{R} (v_1 + v_2 + v_3) = -\frac{1}{3} (v_1 + v_2 + v_3)$$

Obviously, the output is equal **to the average of the three inputs**.

### 68.11. Subtractor

The function of a subtractor is to provide an output proportional to or equal to the difference of two input signals. As shown in Fig. 68.10 we have to apply the inputs at the inverting as well as non-inverting terminals.

#### Calculations

According to Superposition Theorem (Art. 2.17)  $v_0 = v_0' + v_0''$  where  $v_0'$  is the output produced by  $v_1$  and  $v_0''$  is that produced by  $v_2$ .

Now, 
$$v_0' = -\frac{R_f}{R_1} \cdot v_1 \quad \dots \text{Art 67.37 (a)}$$

$$v_0'' = \left(1 + \frac{R_f}{R_1}\right) v_2 \quad \dots \text{Art 67.37 (b)}$$

∴ 
$$v_0 = \left(1 + \frac{R_f}{R_1}\right) v_2 - \frac{R_f}{R_1} \cdot v_1$$

Since  $R_f \gg R_1$  and  $R_f/R_1 \gg 1$ , hence

$$v_0 \cong \frac{R_f}{R_1} (v_2 - v_1) = K (v_2 - v_1)$$

Further, If  $R_f = R_1$ , then

$$v_0 = (v_2 - v_1) = \text{difference of the two input voltages}$$

Obviously, if  $R_f \neq R_1$ , then a scale factor is introduced.

**Example 68.2.** Find the output voltages of an OP-AMP inverting adder for the following sets of input voltages and resistors. In all cases,  $R_f = 1\text{ M}$ .

$$v_1 = -3\text{ V}, v_2 = +3\text{ V}, v_3 = +2\text{ V}; R_1 = 250\text{ K}, R_2 = 500\text{ K}, R_3 = 1\text{ M}$$

[Electronic Engg. Nagpur Univ. 1991]

**Solution.**

$$v_0 = -(K_1 v_1 + K_2 v_2 + K_3 v_3)$$

$$K_1 = \frac{R_f}{R_1} = \frac{1000\text{ K}}{250\text{ K}} = 4, K_2 = \frac{1000}{500} = 2, K_3 = \frac{1\text{ M}}{1\text{ M}} = 1$$

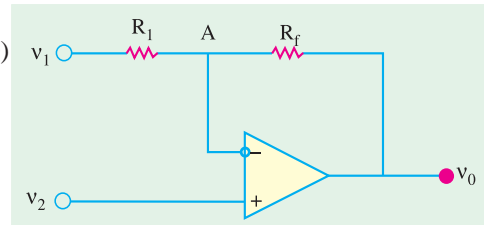


Fig. 68.10

$$\therefore v_0 = -[(4 \times -3) + (2 \times 3) + (1 \times 2)] = +4\text{V}$$

**Example 68.3.** In the subtractor circuit of Fig. 68.10,  $R_1 = 5\text{ K}$ ,  $R_f = 10\text{ K}$ ,  $v_1 = 4\text{ V}$  and  $v_2 = 5\text{ V}$ . Find the value of output voltage.

**Solution.** 
$$v_0 = \left(1 + \frac{R_f}{R_1}\right)v_1 - \frac{R_f}{R_1}v_2 = \left(1 + \frac{10}{5}\right)4 - \frac{10}{5} \times 5 = +2\text{V}$$

**Example 68.4.** Design an OP-AMP circuit that will produce an output equal to  $-(4v_1 + v_2 + 0.1v_3)$ . Write an expression for the output and sketch its output waveform when  $v_1 = 2 \sin \omega t$ ,  $v_2 = +5\text{ V dc}$  and  $v_3 = -100\text{ V dc}$ . [Banglore University 2001]

**Solution.** 
$$v_0 = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right] \quad \dots(1)$$

and also 
$$v_0 = -(4v_1 + v_2 + 0.1v_3) \quad \dots(2)$$

Comparing equations (1) and (2), we find,

$$\frac{R_f}{R_1} = 4, \quad \frac{R_f}{R_2} = 1, \quad \frac{R_f}{R_3} = 0.1$$

Therefore if we assume  $R_f = 100\text{ K}$ , then  $R_1 = 25\text{ K}$ ,  $R_2 = 100\text{ K}$  and  $R_3 = 10\text{ K}$ . With these values of  $R_1$ ,  $R_2$  and  $R_3$ , the OP-AMP circuit is as shown in Fig. 68.11(a)

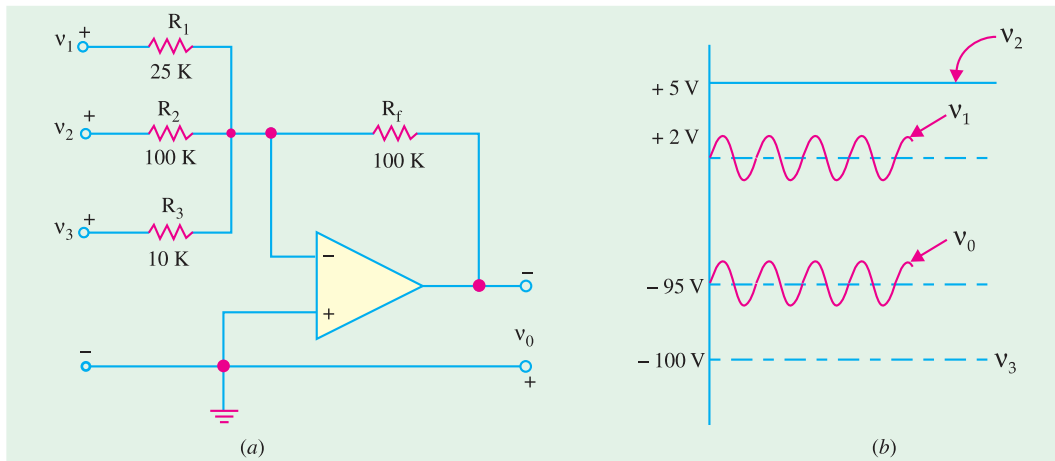


Fig. 68.11

With the given values of  $v_1 = 2 \sin \omega t$ ,  $v_2 = +5\text{ V}$ ,  $v_3 = -100\text{ V dc}$ , the output voltage,  $v_0 = 2 \sin \omega t + 5 - 100 = 2 \sin \omega t - 95\text{ V}$ . The waveform of the output voltage is sketched as shown in Fig. 68.11 (b).

### 68.12. Integrator

The function of an integrator is to provide an output voltage which is proportional to the integral of the input voltage.

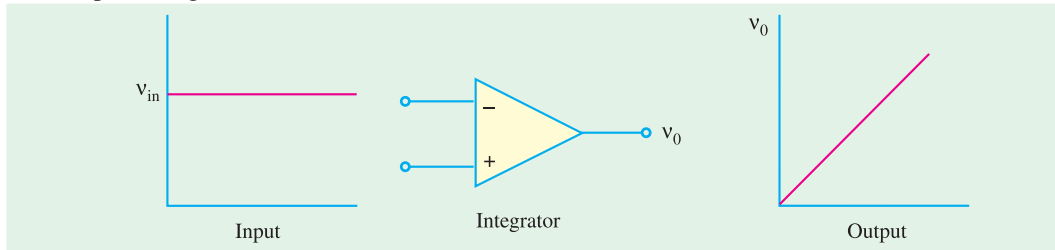


Fig. 68.12

A simple example of integration is shown in Fig. 68.12 where input is dc level and its integral is **a linearly-increasing ramp output**. The actual integration circuit is shown in Fig. 68.13. This circuit is similar to the scaler circuit of Fig. 68.5 except that **the feedback component is a capacitor C instead of a resistor R<sub>f</sub>**.

**Calculations**

As before, point A will be treated as virtual ground.

$$i_1 = \frac{v_1}{R}; i_2 = -\frac{v_0}{X_C} = -\frac{v_0}{1/j\omega C} = -\frac{v_0}{1/sC} = -s C v_0$$

where  $s = j \omega$  in the Laplace notation.

Now  $i_1 = i_2$  ...Art. 68.26 (a)

$$\therefore \frac{v_1}{R} = -s C v_0$$

$$\therefore \frac{v_0}{v_{in}} = \frac{v_0}{v_1} = -\frac{1}{s CR} \quad \dots(i)$$

$$\therefore A_v = -\frac{1}{s CR}$$

Now, the expression of Eq. (i) can be written in time domain as

$$v_0(t) = -\frac{1}{40 \pi} (\cos 2000 \pi t - 1)$$

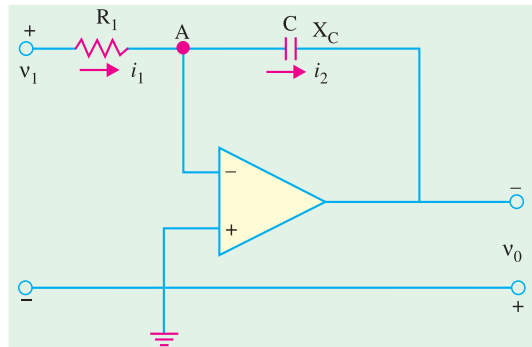


Fig. 68.13

It is seen from above that output (right-hand side expression) is an integral of the input, with an inversion and a scale factor of 1/CR.

This ability to integrate a given signal enables an analog computer solve differential equations and to set up a wide variety of electrical circuit analogs of physical system operation. For example, let

$R = 1 \text{ M}$  and  $C = 1 \mu\text{F}$ . Then

$$\text{scale factor} = -\frac{1}{CR} = -\frac{1}{10^6 \times 10^{-6}} = -1$$

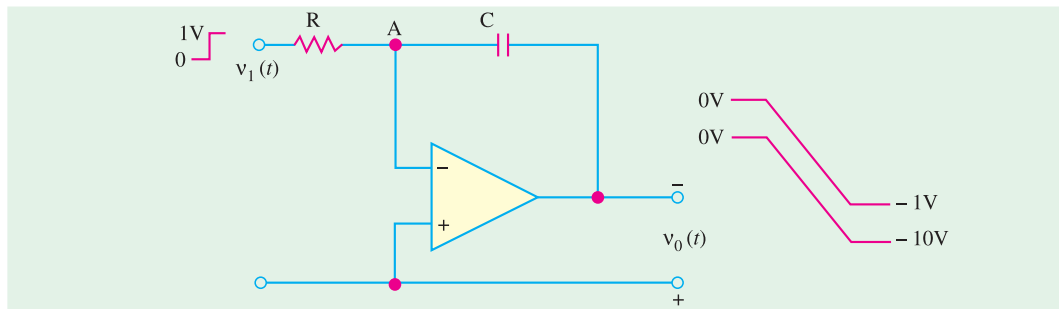


Fig. 68.14

As shown in Fig. 68.14 the input is a step voltage, whereas output is a ramp (or linearly-changing voltages) with a scale multiplier of -1. However, when  $R = 100 \text{ K}$ , then

$$\text{scale factor} = -\frac{1}{10^5 \times 10^{-6}} = -10$$

$$\therefore v_0(t) = -10 \int v_1(t) . dt$$

It is also shown in Fig. 68.14. Of course, we can integrate more than one input as shown below in Fig. 68.15. With multiple inputs, the output is given by

$$v_0(t) = - \left[ K_1 \int v_1(t) dt + K_2 \int v_2(t) dt + K_3 \int v_3(t) dt \right]$$

where 
$$K_1 = \frac{1}{CR_1}, K_2 = \frac{1}{CR_2} \text{ and } K_3 = \frac{1}{CR_3}$$

Fig. 68.15 (a) shows a summing integrator as used in an analog computer. It shows all the three resistors and the capacitor. The analog computer representation of Fig. 68.15 (b) indicates only the scale factor for each input.

**Example 68.5.** A 5-mV, 1-kHz sinusoidal signal is applied to the input of an OP-AMP integrator of Fig. 64.37 for which  $R = 100 \text{ K}$  and  $C = 1 \text{ } \mu\text{F}$ . Find the output voltage.

[Electronic & Comm. Engg. Kurukshetra Univ. 1990]

**Solution.** Scale factor = 
$$-\frac{1}{CR} = \frac{1}{10^5 \times 10^{-6}} = -10$$

The equation for the sinusoidal voltage is

$$v_1 = 5 \sin 2 \pi ft = 5 \sin 2000 \pi t$$

Obviously, it has been assumed that at  $t = 0, v_1 = 0$

$$\begin{aligned} \therefore v_0(t) &= -10 \int_0^t 5 \sin 2000 \pi t = -50 \left| \frac{-\cos 2000 \pi t}{2000} \right|_0^t \\ &= -\frac{1}{40 \pi} (\cos 2000 \pi t - 1) \end{aligned}$$

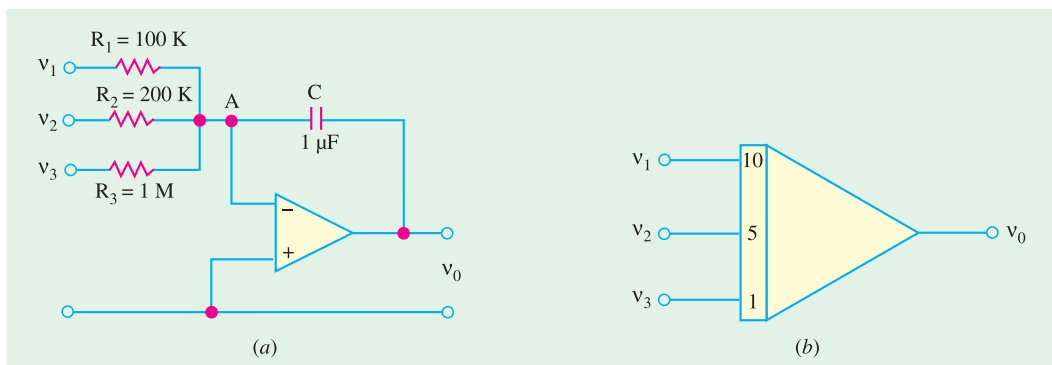


Fig. 68.15

### 68.13. Differentiator

Its function is to provide an output voltage which is *proportional to the rate of the change of the input voltage*. It is an inverse mathematical operation to that of an integrator. As shown in Fig. 68.16, when we feed a differentiator with linearly-increasing ramp input, we get a constant dc output.

#### Circuit

Differentiator circuit can be obtained by interchanging the resistor and capacitor of the integrator circuit of Fig. 68.13.

Let  $i$  = rate of change of charge

$$= \frac{dq}{dt}$$

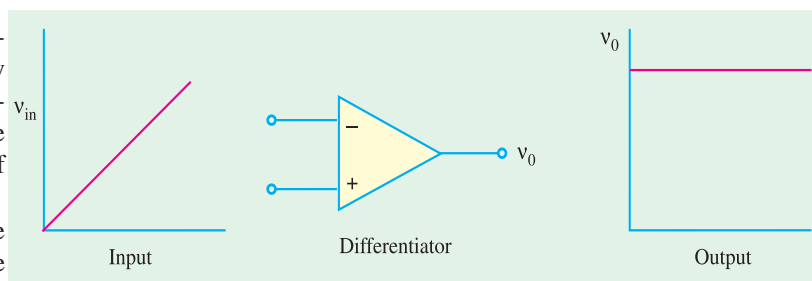


Fig. 68.16

Now,  $q = Cv_c$

$$\therefore i = \frac{d}{dt}(Cv_c) = C \frac{dv_c}{dt}$$

Taking point A as virtual ground

$$v_0 = -iR = -\left(C \cdot \frac{dv_c}{dt}\right)R = -CR \cdot \frac{dv_c}{dt}$$

As seen, output voltage is proportional to the derivate of the input voltage, the constant of proportionality (i.e., scale factor) being  $(-RC)$ .

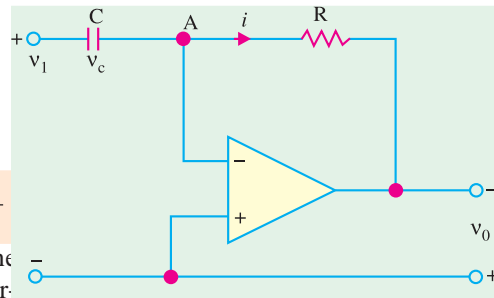


Fig. 68.17

**Example 68.6.** The input to the differentiator circuit of Fig. 68.17 is a sinusoidal voltage of peak value of 5 mV and frequency 1 kHz. Find out the output if  $R = 1000\text{ K}$  and  $C = 1\text{ }\mu\text{F}$ .

**Solution.** The equation of the input voltage is

$$v_1 = 5 \sin 2\pi \times 1000 t = 5 \sin 2000 \pi t \text{ mV}$$

$$\text{scale factor} = CR = 10^{-6} \times 10^5 = 0.1$$

$$v_0 = 0.1 \frac{d}{dt} (5 \sin 2000 \pi t) = (0.5 \times 2000 \pi) \cos$$

$$2000 \pi t = 1000 \pi \cos 2000 \pi t \text{ mV}$$

As seen, output is a cosinusoidal voltage of frequency 1 kHz and peak value 1000  $\pi$  mV.

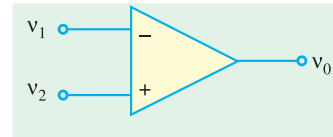


Fig. 68.18

### 68.14. Comparator

It is a circuit which compares two signals or voltage levels. The circuit is shown in Fig. 68.18 and (like that of the unity follower) is the simplest because it needs no additional external components.

If  $v_1$  and  $v_2$  are equal, then  $v_0$  should ideally be zero. Even if  $v_1$  differs from  $v_2$  by a very small amount,  $v_0$  is large because of amplifier's high gain. Hence, circuit of Fig. 68.18 can detect very small changes which is another way of saying that it compares two signals,

### 68.15. Audio Amplifier

As a matter of fact, in most communication receivers, the final output stage is the audio amplifier. The ideal audio amplifier will have the following characteristics :

1. High gain
2. Minimum distortion in the audio frequency range (i.e., 20 Hz to 20 kHz range).
3. High input resistance (or impedance).
4. Low output resistance (or impedance) to provide optimum coupling to the speaker.

The use of OP-AMP in an audio amplifier will fulfill the requirements listed above very nicely. An OP-AMP audio amplifier is shown in Fig. 68.19.

Note that the OP-AMP is supplied only for +V volt power supply, the -V terminal is grounded. Because of this, the output will be between the limits of (+V - 1) volts and +1 volt approximately. Also notice the use of a coupling capacitor  $C_2$  between the OP-AMP and speaker. This capacitor is necessary to reference the speaker signal around ground. The capacitor  $C_s$ , is included in the  $V_{CC}$  line to prevent any transient current caused by the operation of OP-AMP from being coupled back to  $Q_1$  through the power supply. The high gain requirements is accomplished by the combination of two amplifier stages. The high  $R_m$ /Low  $R_{out}$  of the audio amplifier is accomplished by the OP-AMP itself, as the low distortion characteristic.

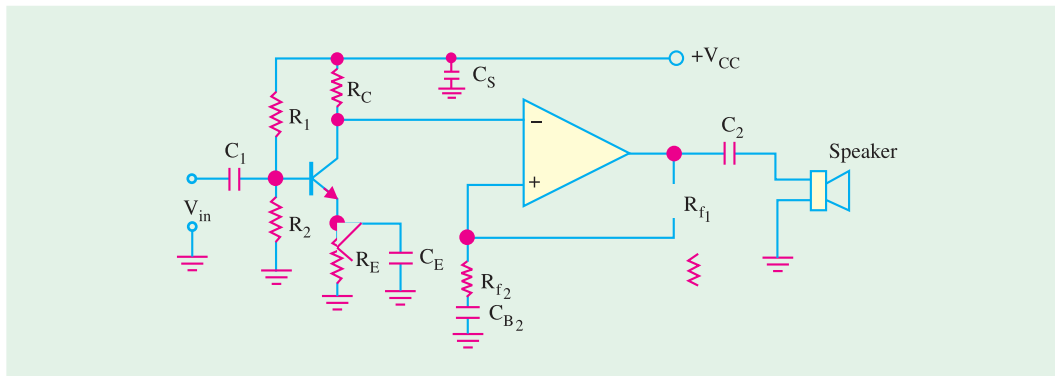


Fig. 68.19

### 68.16. OP-AMP Based Oscillator Circuits

We have already discussed about sinusoidal oscillators in Chapter 15. There we defined the oscillator as a circuit that produces an output waveform without any external signal source. The only input to an oscillator is the dc power supply. As such, the oscillator can be viewed as being a signal generator. We have also discussed in the same chapter, about the different types of oscillator circuits (like Wien Bridge oscillator, Colpitts Oscillator and Crystal Oscillator) using bipolar junction and field-effect transistor. Now we still study these oscillator circuits using *OP-AMP*.

### 68.17. OP-AMP Based Wien Bridge Oscillator

Fig. 68.20 shows the basic version of a Wien Bridge oscillator. The circuit uses an *OP-AMP* and *RC* bridge circuit. Note the basic bridge connection carefully. Resistors  $R_1$ ,  $R_2$  and capacitors  $C_1$  and  $C_2$  form the frequency adjustment elements while resistors  $R_3$  and  $R_4$  form part of the feedback path. The *OP-AMP* output is connected as the bridge input as points 'A' and 'C'. The bridge circuit output at points 'B' and 'D' is the input to the *OP-AMP*. In a typical Wien Bridge oscillator,  $R_1 = R_2 = R$ , and  $C_1 = C_2 = C$ . This means that the two *RC* circuits will have the same cut-off frequency. The cut-off frequency is given by  $(1/2 \pi RC)$ .

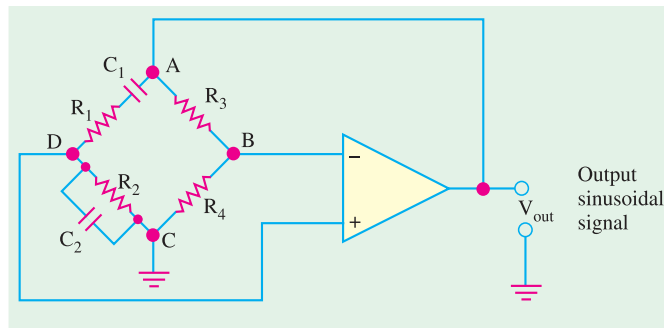


Fig. 68.20

**Example 68.7.** Fig. 68.21 shows the circuit of a Wien-Bridge Oscillator using *OP-AMP* as an amplifier. Notice the components  $R_p$ ,  $C_p$ ,  $R_2$ ,  $C_2$ ,  $R_3$  and  $R_4$  are connected in the bridge configuration in the same way as shown in Fig. 68.22.

Calculate the frequency of the Wien Bridge oscillator.

**Solution.** The frequency of oscillations,

$$f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 51 \text{ K} \times 0.001 \mu\text{F}} = 3.12 \text{ kHz}$$

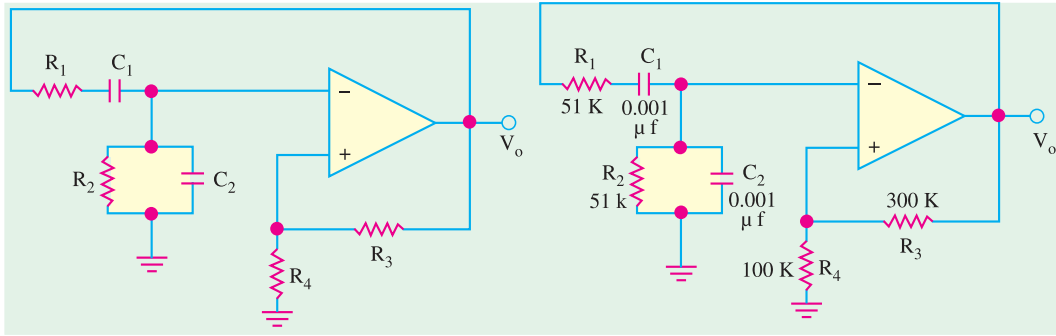


Fig. 68.21

Fig. 68.22

**68.18. OP-AMP Based Colpitts Oscillator**

An *OP-AMP* based Colpitts oscillator is as shown in Fig. 68.23. Here the *OP-AMP* provides the basic amplification needed while a *LC* feedback network of Colpitts configuration sets the oscillator frequency.

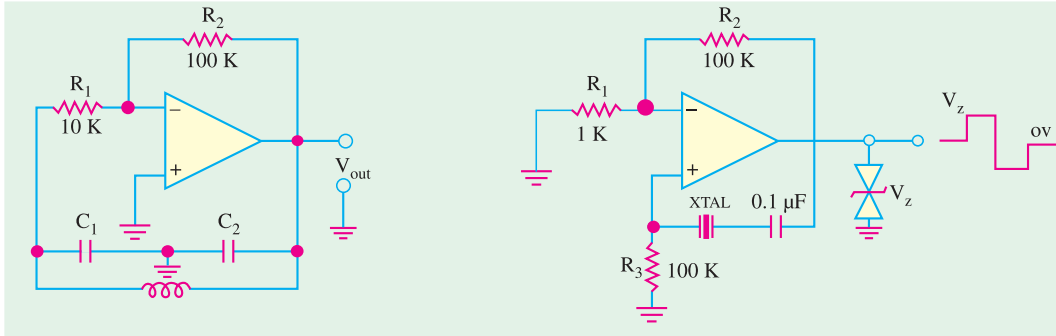


Fig. 68.23

Fig. 68.24

**68.19. OP-AMP Based Crystal Oscillator**

An *OP-AMP* can be used in a crystal oscillator as shown in Fig. 68.24. The crystal is connected in the series resonant path and operates at the crystal series resonant frequency. The present circuit has high gain so that an output square-wave results are shown in the figure. A pair of Zener diodes is shown at the output to provide output amplitude at exactly the Zener voltage ( $V_Z$ ).

**68.20. A Triangular-Wave Oscillator**

Fig. 68.25 shows an *OP-AMP* circuit to generate a triangular-wave from a square-wave. The circuit makes use of two *OP-AMP* : one of them is used as a comparator and the other as an integrator. The operation of the circuit is as given below :

To begin with, let us assume that the output voltage of the comparator is at its maximum negative level. This output is connected to the inverting input

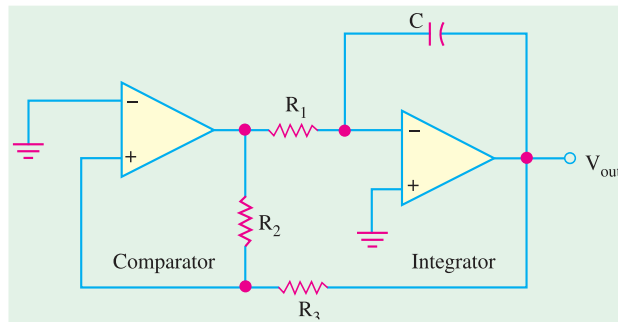


Fig. 68.25

of the integrator through resistor,  $R_1$ . This produces a positive-going ramp on the output of the integrator. When the ramp voltage reaches the upper trigger point (UTP), the comparator switches to its maximum positive level. This positive level causes the integrator ramp to change to a negative going direction. The ramp continues in this direction until the lower trigger point (LTP) of the comparator is reached. At this point, the comparator output switches back to its maximum negative level and the cycle repeats. This action is shown in Fig. 68.26.

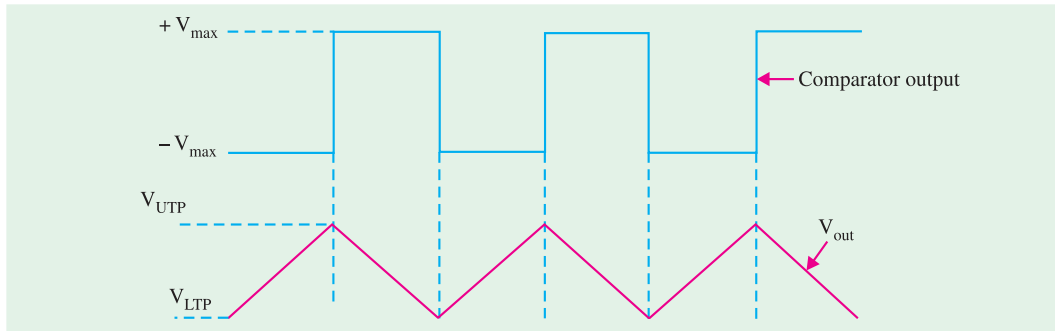


Fig. 68.26

Since the comparator produces square-wave output, therefore, circuit shown in Fig. 68.25 can be used as both a triangular-wave oscillator and a square-wave oscillator. Devices of this type are commonly known as function generator because they produce more than one output function. The output amplitude of the square-wave is set by the output swing of the comparator. While the output amplitude of the triangular-wave is set by the resistors  $R_2$  and  $R_3$  by establishing the UTP and LTP voltages according to the following formulas :

$$V_{UTP} = +V_{max} \left( \frac{R_3}{R_2} \right)$$

$$V_{LTP} = -V_{max} \left( \frac{R_3}{R_2} \right)$$

It may be noted that the comparator output levels,  $+V_{max}$  are equal. The frequency of both waveforms depend on the  $R_1 C$  time constant as well as the amplitude-setting resistors  $R_2$  and  $R_3$ . By varying  $R_1$ , the frequency of oscillation can be adjusted without changing the output amplitude.

$$f = \frac{1}{4 R_1 C} \left( \frac{R_2}{R_3} \right)$$

### 68.21. A Voltage-Controlled Sawtooth Oscillator (VCO)

The voltage-controlled oscillator (VCO) is an oscillator whose frequency can be changed by a variable dc control voltage. The VCOs can be either sinusoidal or nonsinusoidal. One way to build a voltage-controlled sawtooth oscillator is shown in Fig. 68.27 (a). This circuit makes use of an *OP-AMP* integrator that uses switching device called programmable unijunction transistor (abbreviated as PUT) in parallel with the feedback capacitor to terminate each ramp at a prescribed level and effectively “reset” the circuit.

The programmable unijunction transistor (PUT) is a three terminal device *i.e.*, it has an anode, a cathode and a gate terminal. The gate is always biased positively with respect to the cathode. When the anode voltage exceeds the gate voltage by approximately 0.7 V, the PUT turns on and acts as a forward biased diode. When the anode voltage falls below this level, the PUT turns off. Also, the value of current must be above the holding value to maintain conduction.



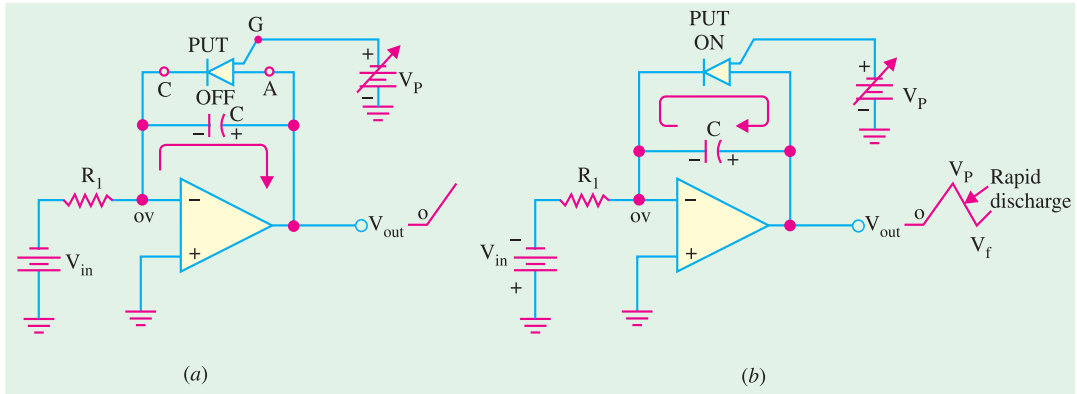


Fig. 68.27

The operation of the circuit may be explained as below. The negative dc input voltage,  $-V_{IN}$ , produces a positive-going ramp on the output. During the time that ramp is increasing the circuit acts as a regular integrator. When the ramp voltage (*i.e.*, voltage at PUT anode) exceeds the gate voltage by 0.7 V, the PUT turns on. This forces the capacitor to discharge rapidly as shown in Fig. 68.27 (b). However, the capacitor does not discharge completely to zero because of the PUT's forward voltage,  $V_F$ . Discharging of the capacitor continues until the current through PUT drops below the holding value. At this point, the PUT turn off and capacitor begins to charge again, thus generating a new output ramp. The cycle repeats and the resulting output is a repetitive sawtooth waveform as shown in the figure. It may be noted that the sawtooth amplitude and period can be adjusted by varying the PUT gate voltage.

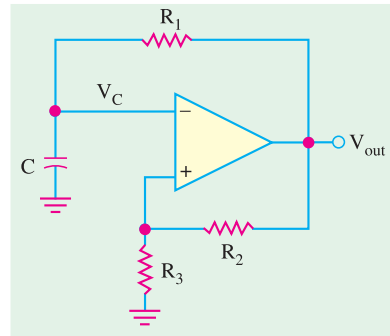


Fig. 68.28

The frequency of smooth curve is given by the relation :

$$f = \frac{|V_{IN}|}{R_1 C} \left( \frac{1}{V_P - V_F} \right)$$

It is evident from the above equation that the frequency depends upon the time constant " $R_1 C$ " of the integrator and the peak voltage set by the PUT. The time period of the sawtooth wave is the reciprocal of the frequency, *i.e.*,

$$T = \frac{V_P - V_F}{|V_{IN}|/R_1 C}$$

### 68.22. A Square-wave Relaxation Oscillator

Fig. 1.28 shows the circuit of a basic relaxation oscillator. Its operation depends upon charging and discharging of a capacitor. Notice that the *OP-AMP*'s inverting input is the capacitor voltage and the noninverting input is a portion of the output fed-back through resistors  $R_2$  and  $R_3$ .

When the circuit is first turned on, the capacitor is uncharged. Because of this, the inverting input is at 0 V. This makes the output a positive maximum, and capacitor begins to charge towards  $V_{out}$  through  $R_1$ . When the capacitor voltage reaches a value equal to the feedback voltage on the noninverting input, the switches to the maximum negative stage. At this point, the capacitor begins to discharge from  $+V_f$  toward  $-V_f$ . When the capacitor voltage reaches  $-V_f$  the *OP-AMP* switches back to the maximum positive state. This action continues to repeat and a square wave input is obtained as

shown in Fig. 68.29.

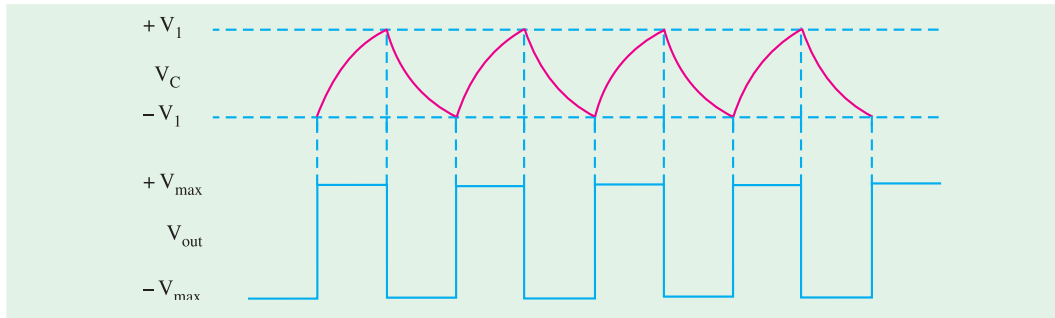


Fig. 68.29

### 68.23. High-impedance Voltmeter

Fig. 68.30 shows the circuit of a high impedance voltmeter. In such a circuit, the closed loop-gain depends on the internal resistance of the meter  $R_M$ . The input voltage will be amplified and the output voltage will cause a proportional current to flow through the meter. By adding a small series potentiometer in the feedback loop, the meter can be calibrated to provide a more accurate reading.

The high input impedance of the *OP-AMP* reduces the circuit loading that is caused by the use of the meter. Although this type of circuit would cause some circuit loading, it would be much more accurate than a VOM (Volt-Ohm-Meter) with an input impedance of 20 K/V.

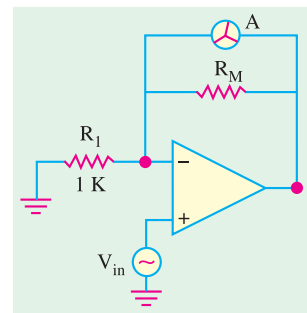


Fig. 68.30

### 68.24. Active Filters

We have already discussed in Chapter 10, about tuned amplifiers. Such amplifiers are designed to amplify only those frequencies that are within certain range. As long as the input signal is within the specified range, it will be amplified. If it goes outside of this frequency range, amplification will be drastically reduced. The tuned amplifier circuits using *OP-AMP* are generally referred to as active filters. Such circuits do not require the use of inductors. The frequency response of the circuit is determined by resistor and capacitor values.

A filter circuit can be constructed using passive components like resistors and capacitors. But an active filter, in addition to the passive components makes use of an *OP-AMP* as an amplifier. The amplifier in the active filter circuit may provide voltage amplification and signal isolation or buffering.

There are four major types of filters namely, low-pass, filter-high-pass filter, and band-pass filter and band-stop or notch filter. All these four types of filters are discussed one by one in the following pages.

### 68.25. Low-Pass Filter

A filter that provides a constant output from dc up to a cut-off frequency ( $f_{OH}$ ) and then passes no signal above that frequency is called an ideal low-pass filter. The ideal response of a low-pass filter is as shown in Fig. 68.31 (a). Notice that the response shows that the filter has a constant output (indicated by a horizontal line AB) from dc or zero frequency up to a cut-off frequency ( $f_{OH}$ ). And beyond  $f_{OH}$ , the output is zero as indicated by the vertical line 'BC' in the figure.

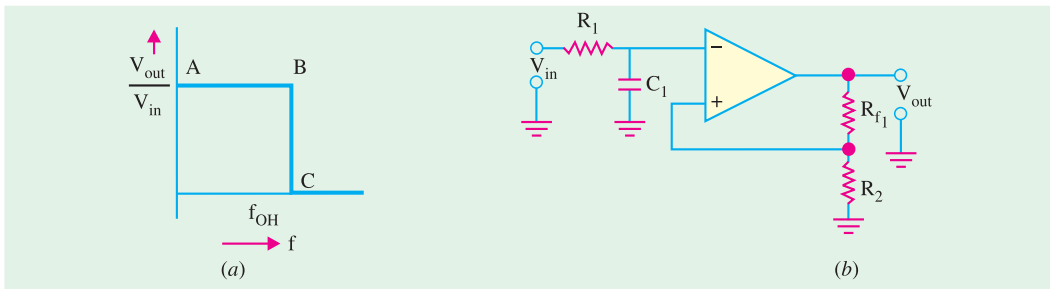


Fig. 68.31

Fig. 68.31 (b) shows the circuit of a low-pass active filter using a single resistor and capacitor. Such a circuit is also referred to as first-order (or single-pole) low-pass filter. It is called first-order because it makes use of a single resistor and a capacitor. The response of such a first-order low pass filter is as shown in Fig. 68.32. Notice that the response below the cut-off frequency ( $f_{OH}$ ) shows a constant gain (indicated by a horizontal line ‘AB’)

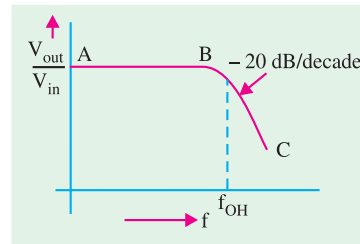


Fig. 68.32

However, beyond the cut-off frequency, the gain does not reduce immediately to zero as expected in Fig. 68.31 (a) but reduces with a slope of 20 dB/decade (means that the output voltage reduces by a factor of 100 when the frequency increases by a factor of 10). The voltage gain for a low-pass filter below the cut-off frequency ( $f_{OH}$ ) is given by the relation.

$$A_v = 1 + \frac{R_3}{R_1}$$

And the cut-off frequency is determined by the relation

$$f_{OH} = \frac{1}{2\pi R_1 C_1}$$

It is possible to connect two sections of the filter together as shown in Fig. 68.33 (a). Such a circuit is called second-order (or two-pole) low pass filter. Fig. 68.33 configuration of the second-order low-pass filter.

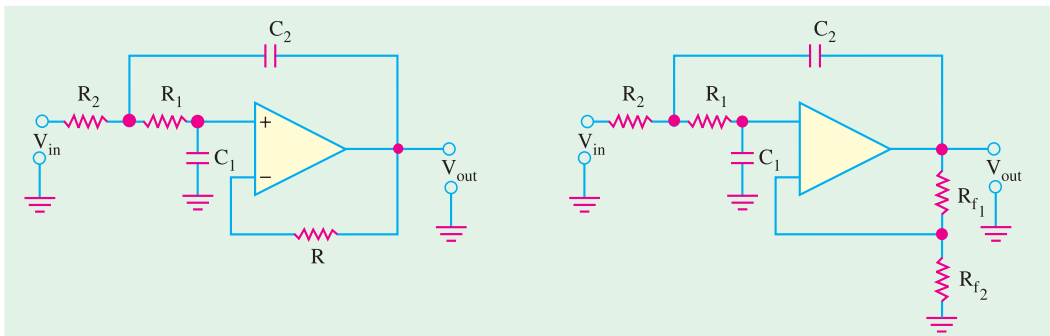


Fig. 68.33

Each circuit shown in Fig. 68.33 has two RC circuits,  $R_1 - C_1$  and  $R_2 - C_2$ . As the operating frequency increases beyond  $f_2$ , each circuit will be dropping the closed-loop gain by 20 dB, giving a total roll-off rate of 40 dB/decade when operated above  $f_2$ . The cut-off frequency for each of the circuit is given by,

$$f_2 = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$

### 68.26. High-pass Filter

As a matter of a fact, there is very little difference between the high-pass filter and the low-pass filter. Fig. 68.34 (a) shows the circuit of a first order (or single-pole) high-pass filter and Fig. 68.34 (b), the circuit of a second-order (or two-pole) high-pass filter. Notice that the only thing that has changed is the position of the capacitors and resistors. The value of cut-off frequencies  $f_1$  and  $f_2$  is obtained by using the same equations we used for low-pass filter.

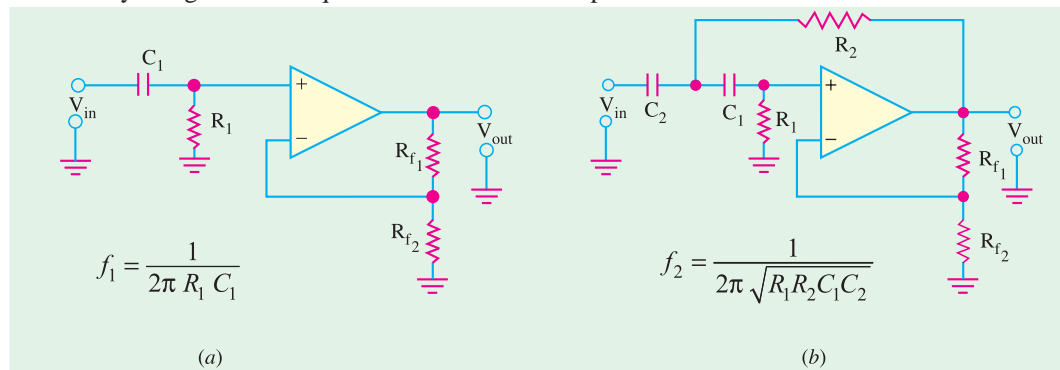


Fig. 68.34

Fig. 68.35 shows the gain versus frequency response of a high-pass filter. Notice that the solid line indicates the ideal response while the dashed line, corresponds to the actual response of the filter circuit. The ideal curve indicates that the filter has a zero output for the frequencies below  $f_{OL}$  (indicated by the line 'AB'). And beyond  $f_{OL}$ , it has a constant output. The actual response curve may correspond to the roll-off gain by 20 dB/decade for first order to 40 dB/decade for second-order low-pass filters.

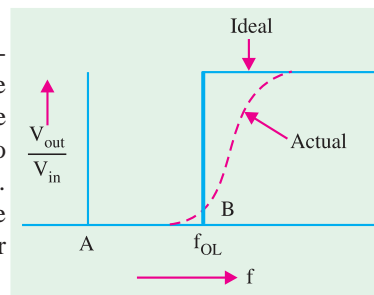


Fig. 68.35

### 68.27. Band-pass Filters

A band-pass filter is the one that is designed to pass all frequencies within its bandwidth. A simple way to construct a band-pass filter is to cascade a low-pass filters and a high-pass filter as shown in Fig. 68.36. The first stage of the band-pass filter will pass all frequencies that are below its cut-off value,  $f_2$ . All the frequencies passed by the first stage will head into the second stage. This stage will pass all frequencies above its value of  $f_1$ . The result of this circuit action is as shown in Fig. 68.37. Note that the only frequencies that all will pass through the amplifier are those that fall within the pass band of both amplifiers. The values of  $f_1$  and  $f_2$  can be obtained by using the relations,  $1/2\pi R_1 C_1$  and,  $1/2\pi R_2 C_2$ . Then bandwidth,

$$BW = f_2 - f_1$$

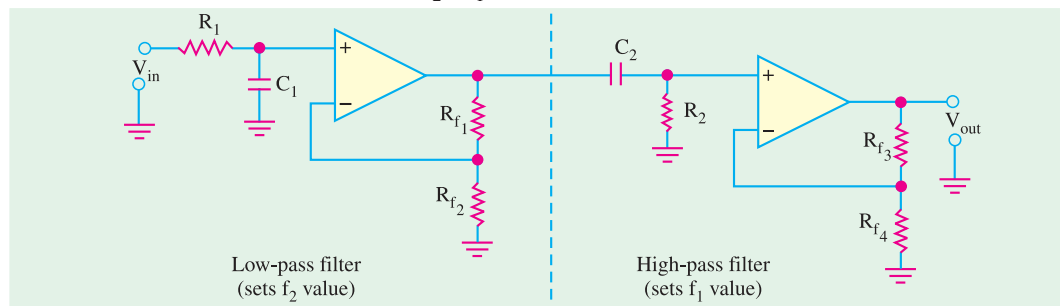


Fig. 68.36

And the centre frequency,

$$f_0 = f_1 \cdot f_2$$

The Quality-factor (or  $Q$ -factor) of the band-pass filter circuit.

$$Q = \frac{f_0}{BW}$$

**68.28. Notch Filter**

The notch filter is designed to block all frequencies that fall within its bandwidth Fig. 68.38 (a) shows a block diagram and 68.38 (b), the gain versus frequency response curve of a multistage notch filter.

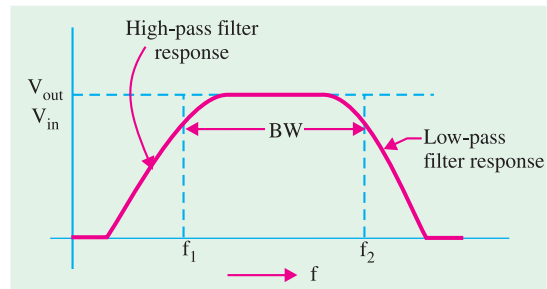


Fig. 68.37

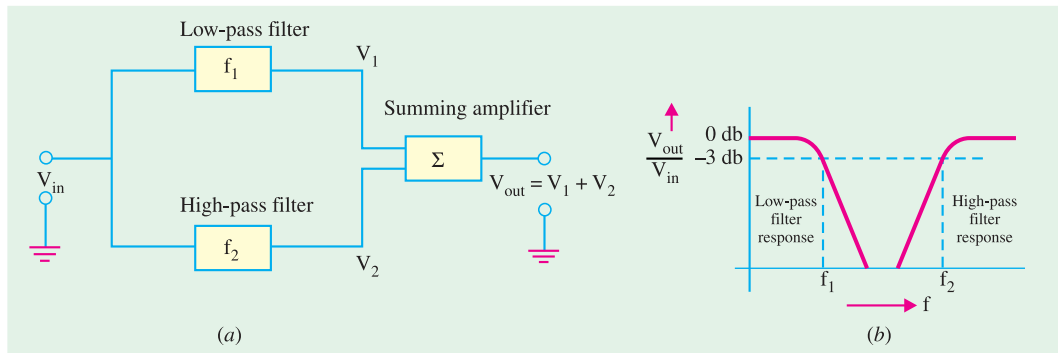


Fig. 68.38

The block diagram shows that the circuit is made up of a high-pass filter, a low-pass filter and a summing amplifier. The summing amplifier produces an output that is equal to a sum of the filter output voltages. The circuit is designed in such a way so that the cut-off frequency,  $f_1$  (which is set by a low-pass filter) is lower in value than the cut-off frequency,  $f_2$  (which is set by high-pass filter). The gap between the values of  $f_1$  and  $f_2$  is the bandwidth of the filter.

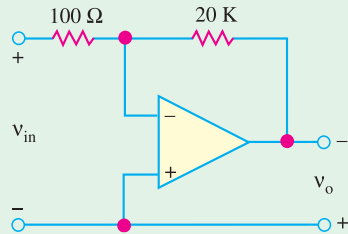
When the circuit input frequency is lower than  $f_1$ , the input signal will pass through low-pass filter to the summing amplifier. Since the input frequency is below the cut-off frequency of the high-pass filter,  $v_2$  will be zero. Thus the output from the summing amplifier will equal the output from the low-pass filter. When the circuit input frequency is higher than  $f_2$ , the input signal will pass through the high-pass filter to the summing amplifier. Since the input frequency is above the cut-off frequency of the low-pass filter,  $v_1$  will zero. Now the summing amplifier output will equal the output from the high-pass filter.

It is evident from the above discussion that frequencies below  $f_1$  and those above  $f_2$ , have been passed by the notch filter. But when the circuit frequency between  $f_1$  and  $f_2$ , neither of the filters will produce an output. Thus  $v_1$  and  $v_2$  will be both zero and the output from the summing amplifier will also be zero.

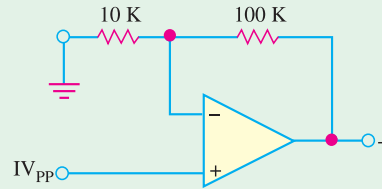
The frequency analysis of the notch filter is identical to the band-pass filter. First, determine the cut-off frequencies of the low-pass and the high-pass filters. Then using these calculated values, determine the bandwidth, center frequency and  $Q$  values of the circuit.

**Tutorial Problems No. 68**

1. Determine the maximum allowable value of  $v_{in}$  for the circuit shown in Fig. 68.39. Assume that the gain of the amplifier is 200. (40 mV)

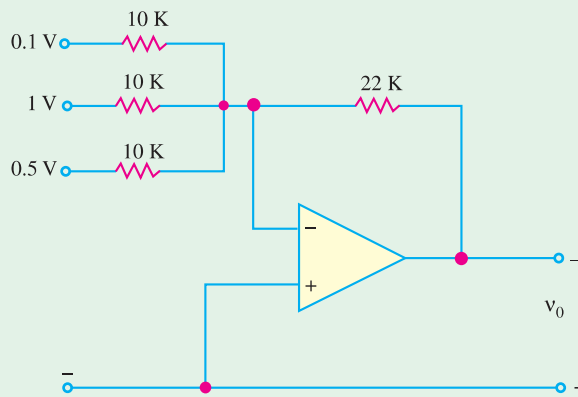


**Fig. 68.39**



**Fig. 68.40**

2. For the non-inverting amplifier shown in Fig. 68.40, find the voltage gain of the circuit. (11)  
 3. Determine the value of output voltage for the summing circuit shown in Fig. 68.41. (3.52 V)



**Fig. 68.41**

**OBJECTIVE TESTS – 68**

1. An OP-AMP can be classified as ..... amplifier.
  - (a) linear
  - (b) low- $r_{in}$
  - (c) positive feedback
  - (d) RC-coupled.
2. An ideal OP-AMP has
  - (a) infinite  $A_v$
  - (b) infinite  $R_i$
  - (c) zero  $R_o$
  - (d) all the above.
3. OP-AMP have become very popular in industry mainly because
  - (a) they are dirt cheap
  - (b) their external characteristics can be changed to suit any application
  - (c) of their extremely small size
  - (d) they are available in different packages.
4. Since input resistance of an ideal OP-AMP is infinite
  - (a) its output resistance is zero
  - (b) its output voltage becomes independent of load resistance
  - (c) its input current is zero
  - (d) it becomes a current-controlled device.
5. The gain of an actual OP-AMP is around
  - (a) 1,000,000
  - (b) 1000
  - (c) 100
  - (d) 10,000
6. When an input voltage of 1 V is applied to an OP-AMP having  $A_v = 10^6$  and bias supply of +15 V, the output voltage available is
  - (a)  $15 \times 10^6$  V
  - (b)  $10^6$  V
  - (c) 15  $\mu$ V
  - (d) 15 V.

7. An inverting amplifier has  $R_f = 2\text{ M}$  and  $R_1 = 2\text{ K}$ . Its scale factor is  
 (a) 1000 (b)  $-1000$   
 (c)  $10^{-3}$  (d)  $-10^{-3}$
8. In an inverting amplifier, the two input terminals of an ideal OP-AMP are at the same potential because  
 (a) the two input terminals are directly shorted internally  
 (b) the input impedance of the OP-AMP is infinity  
 (c) common-mode rejection ratio is infinity  
 (d) the open-loop gain of the OP-AMP is infinity.
9. The open-loop gain of an operational amplifier is  $10^7$ . An input signal of  $1\text{ mV}$  is applied to the inverting input with the non-inverting input connected to the ground. The supply voltages are  $\pm 10\text{ V}$ . The output of the amplifier will be  
 (a)  $+100\text{ V}$   
 (b)  $-100\text{ V}$   
 (c)  $+10\text{ V}$  (approximately)  
 (d)  $-10\text{ V}$  (approximately)
10. The output voltage of the circuit shown in Fig. 68.42 is

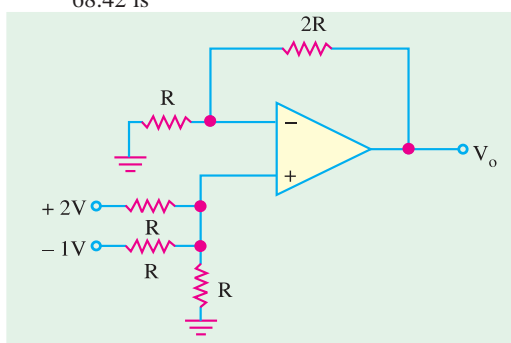


Fig. 68.42

- (a)  $1.0\text{ V}$  (b)  $1.5\text{ V}$   
 (c)  $2.0\text{ V}$  (d)  $2.5\text{ V}$
11. In the circuit shown in Fig. 68.43, the value of output,  $v_o$  is  
 (a)  $+3\text{ V}$  (b)  $-3\text{ V}$   
 (c)  $-7\text{ V}$  (d)  $+7\text{ V}$

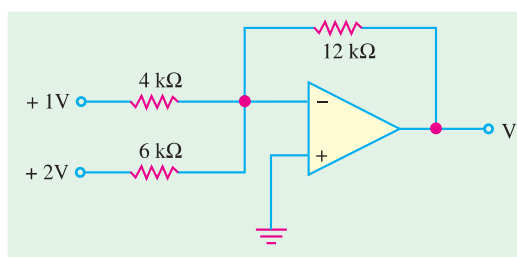


Fig. 68.43

12. When in a negative scaler, both  $R_1$  and  $R_f$  are reduced to zero, the circuit functions as  
 (a) integrator (b) subtractor  
 (c) comparator (d) unity follower.
13. The two input terminals of an OP-AMP are known as  
 (a) positive and negative  
 (b) differential and non-differential  
 (c) inverting and non-inverting  
 (d) high and low.
14. The purpose of comparator is to  
 (a) amplify an input voltage  
 (b) detect the occurrence of a changing input voltage  
 (c) maintain a constant output when the dc input voltage changes.  
 (d) produce a change in input voltage when an input voltage equals the reference voltage.
15. The OP-AMP comparator circuit uses  
 (a) positive feedback  
 (b) negative feedback  
 (c) regenerative feedback  
 (d) no feedback
16. The feedback path in an OP-AMP integrator consists of  
 (a) a resistor  
 (b) a capacitor  
 (c) a resistor and a capacitor in series  
 (d) a resistor and a capacitor in parallel.
17. The feedback path in an OP-AMP differentiator consists of  
 (a) a resistor  
 (b) a capacitor  
 (c) a resistor and a capacitor in series  
 (d) a resistor and a capacitor in parallel.

ANSWERS

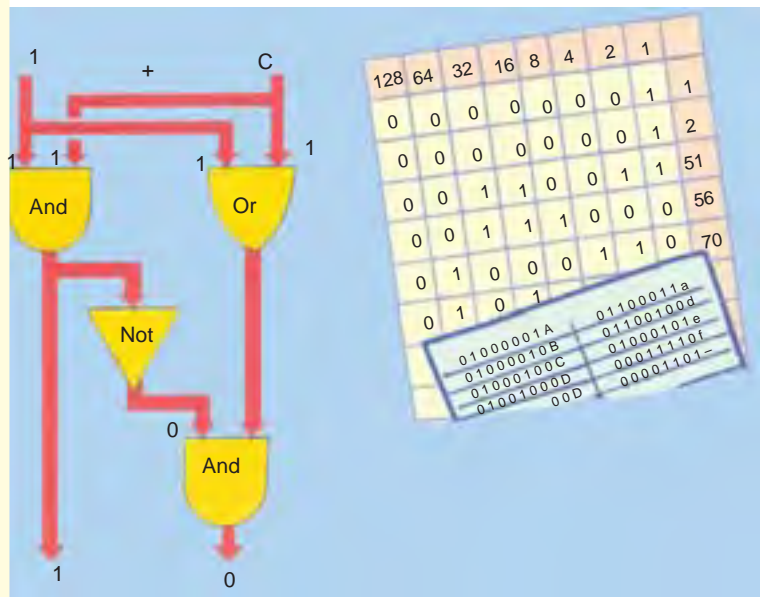
1. (a) 2. (d) 3. (b) 4. (c) 5. (a) 6. (d) 7. (b) 8. (b) 9. (b) 10. (c) 11. (d) 12. (d)  
 13. (c) 14. (d) 15. (d) 16. (b) 17. (a)

# CHAPTER 69

## Learning Objectives

- Number Systems
- The Decimal Number System
- Binary Number System
- Binary to Decimal Conversion
- Binary Fractions
- Double-Dadd Method
- Decimal to Binary Conversion
- Shifting the Place Point
- Binary Operations
- Binary Addition
- Binary Subtraction
- Complement of a Number
- 1's Complemental Subtraction
- 2's Complemental Subtraction
- Binary Multiplication
- Binary Division
- Shifting a Number to Left or Right
- Octal Number System
- Octal to Decimal Conversion
- Decimal to Octal Conversion
- Binary to Octal Conversion
- Octal to Binary Conversion
- Binary to Hexadecimal Conversion
- Decimal to Hexadecimal Conversion
- Hexadecimal to Decimal Conversion
- Digital Coding
- Binary Coded Decimal (BCD) Code
- Octal Coding
- Hexadecimal Coding
- Excess-3 Code
- Gray Code
- Excess-3 Gray Code
- ASCII Code

## NUMBER SYSTEMS AND CODES



Logic gates use switches that control the flow of an electrical current. '1' is 'true' and '0' is 'false'. The columns in the binary system have the values 1, 2, 4, 8, 16, 32 and so on.



### 69.1. Number Systems

The number systems are used quite frequently in the field of digital electronics and computers. However the type of number system used in computers could be different at different stages of the usage. For example, when a user key-in some data into the computer, s(he), will do it using decimal number system *i.e.* the system we all have used for several years for doing arithmetic problems. But when the information goes inside the computer, it needs to be converted to a form suitable for processing data by the digital circuitry. Similarly when the data has to be displayed on the monitor for the user, it has to be again in the decimal number system. Hence the conversion from one number system to another one is an important topic to be understood.

There are four systems of arithmetic which are often used in digital circuits. These systems are:

1. **Decimal**—it has a base (or radix) of 10 *i.e.* it uses 10 different symbols to represent numbers.
2. **Binary**—it has a base of 2 *i.e.* it uses only two different symbols.
3. **Octal**—it has a base of 8 *i.e.* it uses eight different symbols.
4. **Hexadecimal**—it has a base of 16 *i.e.* it uses sixteen different symbols.

All these systems use the same type of **positional notation** except that

- decimal system uses powers of 10
- binary system uses power of 2
- octal system uses powers of 8
- hexadecimal system uses powers of 16.

Decimal numbers are used to represent quantities which are outside the digital system. Binary system is extensively used by digital systems like digital computers which operate on binary information. Octal system has certain advantages in digital work because it requires less circuitry to get information into and out of a digital system. Moreover, it is easier to read, record and print out octal numbers than binary numbers. Hexadecimal number system is particularly suited for microcomputers.

### 69.2. The Decimal Number System

We will briefly recount some important characteristics of this more-familiar system before taking up other systems. This system has a base of 10 and is a **position-value system** (meaning that value of a digit depends on its **position**). It has following characteristics :

#### (i) Base or Radix

It is defined as *the number of different digits which can occur in each position in the number system.*

The decimal number system has a base of 10 meaning that it contains ten unique symbols (or digits). These are : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Any one of these may be used in each position of the number.

Incidentally, it may be noted that we call it a decimal (10's) system although *it does not have a distinct symbol of 10*. As is well-known, it expresses 10 and any number above 10 as a combination of its ten unique symbols.

#### (ii) Position Value

The absolute value of each digit is fixed but its **position value** (or place value or weight) is determined by its **position** in the overall number. For example, position value of 3 in 3000 is not the same as in 300. Also, position value of each 4 in the number 4444 is different as shown in Fig. 69.1.

Similarly, the number 2573 can be broken down as follows :

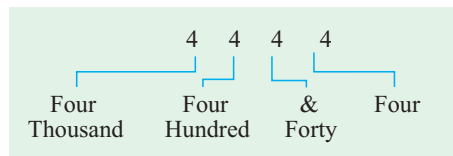


Fig. 69.1

$$2573 = 2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0$$

It will be noted that in this number, 3 is the **least significant digit (LSD)** whereas 2 is the **most significant digit (MSD)**.

Again, the number 2573.469 can be written as

$$2573.469 = 2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2} + 9 \times 10^{-3}$$

It is seen that position values are found by raising the base of the number system (*i.e.* 10 in this case) to the power of the position. Also, powers **are numbered to the left of the decimal point starting with 0 and to the right of the decimal point starting with -1.**

### 69.3. Binary Number System

Like decimal number (or denary) system, it has a radix and it also uses the same type of position value system.

#### (i) Radix

Its base or radix is **two** because it uses only two digits 0 and 1 (the word 'binary digit' is contracted to **bit**). All binary numbers consist of a string of 0s and 1s. Examples are 10, 101 and 1011 which are read as one-zero, one-zero-one and one-zero-one-one to avoid confusion with decimal numbers. Another way to avoid confusion is to add a subscript of 10 for decimal numbers and of 2 for binary numbers as illustrated below.

$10_{10}$ ,  $101_{10}$ ,  $5742_{10}$  — decimal number and  $10_2$ ,  $101_2$ ,  $110001_2$  — binary numbers.

It is seen that the subscript itself is in decimal. It may be noted that binary numbers need **more places for counting because their base is small**



Binary numbers represent all values within computers

#### (ii) Position Value

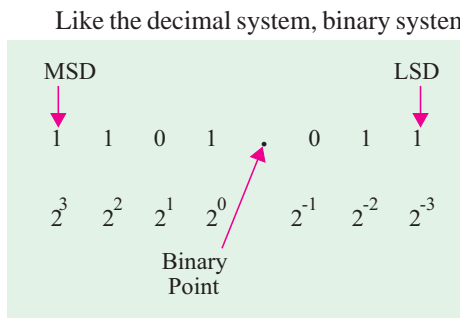


Fig. 69.2

Like the decimal system, binary system is also positionally-weighted. However, in this case, the position value of each bit corresponds to some power of 2. In each binary number, the value increases in powers of 2 starting with 0 to the left of the binary point and decreases to the right of the binary point starting with power of -1. The position value (or weight) of each bit along with a 7-bit binary number 1101.011 is shown in Fig. 69.2.

As seen, the fourth bit to the left of binary point **carries the maximum weight** (*i.e.* it has the highest value) and is called most significant digit (*MSD*). Similarly, the third bit to the right of the binary point is called

least significant digit (*LSD*). The decimal equivalent of the binary number may be found as under

$$\begin{aligned} 1101.011_2 &= (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3}) \\ &= 8 + 4 + 0 + 1 + 0 + \frac{1}{4} + \frac{1}{8} = 13.375_{10} \end{aligned}$$

As stated earlier, position values of different bits are given by **ascending powers of 2** to the **left** of binary point and by **descending power** of 2 to the **right** of binary point. The different digit positions of a given binary number have the following decimal weight (Fig. 69.3)

Binary numbers are used extensively by all digital systems primarily due to the nature of electronics itself. The bit 1 may be represented by

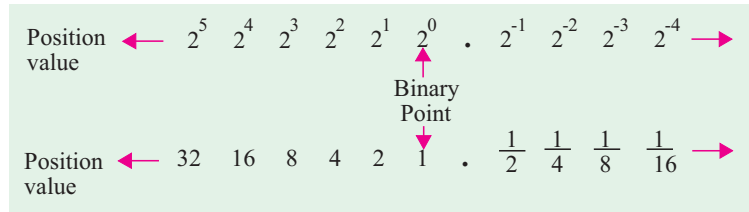


Fig. 69.3

a saturated (fully-conducting) transistor, a light turned ON, a relay energised or a magnet magnetised in a particular direction. The bit 0, on the other hand, can be represented as a cut-off transistor, a light turned OFF, a relay de-energised or a magnet magnetised in the opposite direction. In such cases, there are only two values which a device can assume.

### 69.4. Binary to Decimal Conversion

Following procedure should be adopted for converting a given binary integer (whole number) into its equivalent decimal number :

- Step 1.** Write the binary number *i.e.* all its bits in a row.
- Step 2.** Directly under the bits, write 1, 2, 4, 8, 16, .....starting from *right to left*.
- Step 3.** Cross out the decimal weights which lie under 0 bits.
- Step 4.** Add the remaining weights to get the decimal equivalent.

**Example 69.1.** Convert  $11001_2$  to its equivalent decimal number.

**Solution.** The four steps involved in the conversion are as under

<b>Step 1.</b>	1	1	0	0	1
<b>Step 2.</b>	16	8	4	2	1
<b>Step 3.</b>	16	8	<del>4</del>	<del>2</del>	1
<b>Step 4.</b>	16 + 8 + 1 = 25		∴		$11001_2 = 25_{10}$

It is seen that the number contains 1 sixteen, one eight, 0 four's, 0 two's and 1 one. Certain decimal and binary equivalent numbers are tabulated below in Table No. 69.1

Table No. 69.1

Decimal	Binary	Decimal	Binary	Decimal	Binary
1	1	11	1011	21	10101
2	10	12	1100	22	10110
3	11	13	1101	23	10111
4	100	14	1110	24	11000
5	101	15	1111	25	11001
6	110	16	10000	26	11010
7	111	17	10001	27	11011
8	1000	18	10010	28	11100
9	1001	19	10011	29	11101
10	1010	20	10100	30	11110

### 69.5. Binary Fractions

Here, procedure is the same as for binary integers except that the following weights are used for different bit positions .

●	$2^{-1}$	$2^{-2}$	$2^{-2}$	$2^{-4}$	→
↑	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	→
Binary Point					

**Example 69.2.** Convert the binary fraction 0.101 into its decimal equivalent.

**Solution.** The following four steps will be used for this purpose.

Step 1.	0	●	1	0	1
Step 2.			$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Step 3.			$\frac{1}{2}$	<del><math>\frac{1}{4}</math></del>	$\frac{1}{8}$
Step 4.			$\frac{1}{2} + \frac{1}{8} = 0.625$		
∴			$0.101_2 = 0.625_{10}$		

**Example 69.3.** Find the decimal equivalent of the 6-bit binary number 101.101<sub>2</sub>.

<b>Solution.</b>	1	0	1	1	●	0	1	
	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$		
	4	<del>2</del>	1	$\frac{1}{2}$	<del><math>\frac{1}{4}</math></del>	$\frac{1}{8}$	$= 5 + \frac{1}{2} + \frac{1}{8} = 5.625$	
∴	$101.101_2 = 5.625_{10}$							

### 69.6. Double-Dadd Method

This method of converting binary integers into decimal equivalents is much simpler and quicker than the method given in Art. 69.4 especially in the case of large numbers. Following three steps are involved :

1. Double the first bit to the extreme left and add this doubled value to the next bit on the right.
2. Double the sum obtained and add the doubled value to the next bit.
3. Continue step 2 until the last bit has been added to the previously-doubled sum.

The conversion of 11001<sub>2</sub> is shown in Fig. 69.4. It is seen that 11001<sub>2</sub> = 25<sub>10</sub>

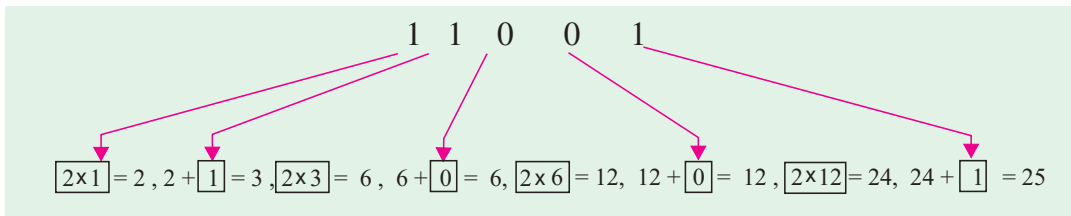


Fig. 69.4

Using double-dadd method, let us convert 111010<sub>2</sub> into its binary equivalent.

1.  $2 \times 1 = 2,$       add next bit 1 so that  $2 + 1 = 3$
2.  $2 \times 3 = 6,$       add next bit 1 so that  $6 + 1 = 7$
3.  $2 \times 7 = 14,$      add next bit 0 so that  $14 + 0 = 14$
4.  $2 \times 14 = 28,$     add next bit 1 so that  $28 + 1 = 29$

5.  $2 \times 29 = 58$ , add next bit 0 so that  $58+0 = 58$   
 $\therefore 111010_2 = 58_{10}$

### 69.7. Decimal to Binary Conversion

#### (a) Integers

Such conversion can be achieved by using the so-called **double-dabble method**. It is also known as **divide-by-two** method. In this method, we progressively divide the given decimal number by 2 and write down the remainders after each division. These remainders taken in the *reverse order* (i.e. from bottom-to-top) form the required binary number. As an example, let us convert  $25_{10}$  into its binary equivalent.

$25 \div 2 = 12 + \text{remainder of } 1$		TOP
$12 \div 2 = 6 + \text{remainder of } 0$		↑
$6 \div 2 = 3 + \text{remainder of } 0$		↑
$3 \div 2 = 1 + \text{remainder of } 1$		↑
$1 \div 2 = 0 + \text{remainder of } 1$		↑
$\therefore 25_{10} = 11001_2$		BOTTOM

The above process may be simplified as under :

Successive Divisions	Remainders	
$2 \overline{) 25}$		
$2 \overline{) 12}$	1	↑
$2 \overline{) 6}$	0	
$2 \overline{) 3}$	0	
$2 \overline{) 1}$	1	
$2 \overline{) 0}$	1	

Reading the remainders from bottom to top, we get  $25_{10} = 11001_2$

It may also be put in the following form :

$25 \div 2 = 12 + 1$		
$12 \div 2 = 6 + 0$		↓
$6 \div 2 = 3 + 0$		↓
$3 \div 2 = 1 + 1$		↓
$1 \div 2 = 0 + 1$		↓
$\therefore \text{ decimal } 25 =$	1 1 0 0 1	binary

#### (b) Fractions

In this case, **Multiply-by-two** rule is used i.e. we multiply each bit by 2 and record the carry in the integer position. These carries taken in the **forward** (top-to-bottom) direction gives the required binary fraction.

Let us convert  $0.8125_{10}$  into its binary equivalent.

$0.8125 \times 2 = 1.625$		$= 0.625$ with a carry of 1
$0.625 \times 2 = 1.25$		↓
$0.25 \times 2 = 0.5$		↓
$0.5 \times 2 = 1.0$		↓
$\therefore 0.8125_{10} = 0.1101_2$		↓

\* It is so because there is 0 at the interger position i.e. to the left of the decimal point.

Please note that *we have to add the binary point from our side*. Let us now convert  $0.77_{10}$  into its binary equivalent.

$0.77 \times 2 = 1.54$	$= 0.54$	with a carry of 1	↓
$0.54 \times 2 = 1.08$	$= 0.08$	with a carry of 1	
$0.08 \times 2 = 0.16$	$= 0.16$	with a carry of 0	
$0.16 \times 2 = 0.32$	$= 0.32$	with a carry of 0	
$0.32 \times 2 = 0.64$	$= 0.64$	with a carry of 0	
$0.64 \times 2 = 1.28$	$= 0.28$	with a carry of 1	

We may stop here but the answer would be approximate.  $\therefore 0.77_{10} \cong .110001_2$

**Example 69.4.** Convert  $25.625_{10}$  into its binary equivalent.

**Solution.** We will do the conversion in two steps (i) first for the integer and (ii) then for the fraction.

<p><b>(a) Integer</b></p> $25 \div 2 = 12 + 1$ $12 \div 2 = 6 + 0$ $6 \div 2 = 3 + 0$ $3 \div 2 = 1 + 1$ $1 \div 2 = 0 + 1$ $\therefore 25_{10} = 11001_2$	↑	<p><b>(b) fraction</b></p> $0.625 \times 2 = 1.25 = 0.25 + 1$ $0.25 \times 2 = 0.5 = 0.5 + 0$ $0.5 \times 2 = 1.0 = 0.0 + 1$ $\therefore 0.625_{10} = 0.101_2$	↓
--	---	--	---

Considering the complete number, we have  $25.625_{10} = 11001.101_2$

Obviously, binary system needs more bits to express the same number than decimal system.

### 69.8. Shifting the Place Point

In a decimal number if the decimal point is moved one place to the right, **the number is multiplied by 10**. For example, when decimal point in 7.86 is shifted one place to the right, it becomes 78.6 *i.e.* it increases the value of the number 10 times. Moving the decimal point one place to the left reduces its value to one-tenth.

In binary numbers, shifting the binary point by one place multiplies or divides the number by 2. For example,  $111.0_2$  is equal to  $7_{10}$  but  $1110.0_2$  is  $14_{10}$ . As seen, 7 is doubled to 14 by moving the binary point one place to the right.

Similarly,  $11.1_2$  is  $(2 + 1 + \frac{1}{2}) = 3.5_{10}$ . Hence,  $111.0_2$  is halved to  $3.5_{10}$  by moving its binary point one place to the left.

### 69.9. Binary Operations

We will now consider the following four binary operations :

1. addition
2. subtraction
3. multiplication
4. division

Addition is the most important of these four operations. In fact, by using **'complements'**, subtraction can be reduced to addition. Most digital computers subtract by complements. It leads to reduction in hardware because only adding type of circuits are required. Similarly, multiplication is nothing but repeated addition and, finally, division is nothing but repeated subtraction.

### 69.10. Binary Addition

Addition is simply the manipulation of numbers for combining physical quantities. For example, in the decimal number system,  $2 + 3 = 5$  means the combination of  $\bullet \bullet$  with  $\bullet \bullet \bullet$  to give a total of  $\bullet \bullet \bullet$ . Addition of binary numbers is similar to the decimal addition.

Following points will help in understanding the rules of binary addition.

1. When 'nothing' is combined with 'nothing', we get nothing.

Binary representation of the above statement is :  $0 + 0 = 0$

2. When nothing is combined with ●, we get ●.

In binary language  $0 + 1 = 1$

3. Combining ● with nothing, gives ●.

The binary equivalent is  $1 + 0 = 1$

4. When we combine ● with ●, we get ●●.

The binary representation of the above is  $1 + 1 = 10$

It should be noted that the above sum is not 'ten' but 'one-zero' i.e. it represents ●● and not ●●●●●●●●. In other words, it is  $10_2$  which represents decimal 2. *It is not decimal ten.*

The last rule is often written as  $1 + 1 = 0$  with a carry of 1

The above rules for binary addition can be summarized as under :

$$\begin{array}{ll} 0 + 0 = 0 & 0 + 1 = 1 \\ 1 + 0 = 1 & 1 + 1 = 0 \quad \text{with a carry of 1} \quad \text{or } = 10_2 \end{array}$$

It is worth noting that 'carry-overs' are performed in the same manner as in decimal arithmetic. The rules of binary addition could also be expressed in the form of a table as shown below

$$\begin{array}{ccc} \underline{0 \ 1} & & \underline{0 \ 1} \\ 0 \ 0 \ 1 & \text{or} & 0 \ 0 \ 1 \\ 1 \ 1 \ 10_2 & & 1 \ 1 \ 0 \quad \text{with a carry 1} \end{array}$$

As an illustration, let us add 101 and 110.

$$\begin{array}{r} \begin{array}{l} 1 \ 0 \ 1 \\ + \ 1 \ 1 \ 0 \\ \hline 1 \ 0 \ 1 \ 1 \end{array} \quad \begin{array}{l} \text{--- first column} \\ \text{--- second column} \\ \text{--- third column} \end{array} \quad \begin{array}{l} 1 + 0 = 1 \\ 0 + 1 = 1 \\ 1 + 1 = 10 \end{array} \end{array}$$

(i.e. 0 with carry 1)

Similarly,

$$\begin{array}{r} \begin{array}{l} \text{1 carry} \\ 1 \ 1 \ 1 \\ + \ 1 \ 1 \ 0 \\ \hline 1 \ 1 \ 0 \ 1 \end{array} \quad \begin{array}{l} \text{--- 1st column} \\ \text{--- 2nd column} \\ \text{--- 3rd column} \end{array} \quad \begin{array}{l} 1 + 0 = 1 \\ 1 + 1 = 0 \quad \text{with carry 1} \\ 1 + 1 + \text{carry of 1} \\ = 10 + 1 = 11_2 \end{array} \end{array}$$

Let us consider one more example :

$$\begin{array}{cccc} & \leftarrow & \leftarrow & \leftarrow \\ \begin{array}{r} \text{1 carry} \\ 1 \ 0 \ 1 \ 1 \\ + \ 1 \ 0 \ 0 \ 1 \\ \hline 0 \end{array} & \begin{array}{r} \text{1 1 carry} \\ 1 \ 0 \ 1 \ 1 \\ + \ 1 \ 0 \ 0 \ 1 \\ \hline 0 \ 0 \end{array} & \begin{array}{r} \text{1 1 carry} \\ 1 \ 0 \ 1 \ 1 \\ + \ 1 \ 0 \ 0 \ 1 \\ \hline 1 \ 0 \ 0 \end{array} & \begin{array}{r} \text{1 1 carry} \\ 1 \ 0 \ 1 \ 1 \\ + \ 1 \ 0 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 0 \ 0 \end{array} \end{array}$$

Hence, we find from the above examples that the only two possible combinations with a carry are :

- (a)  $1 + 1 =$  sum of 0 with a carry of 1.  
It is binary 10 i.e.  $10_2$  which equals decimal 2.
- (b)  $1 + 1 +$  carry of 1 = a sum of 1 with a carry of 1.  
It equals binary 11 i.e.  $11_2$  or decimal 3.

**Example 69.5.** Add  $110011_2$  to  $101101_2$  (Digital Electronics, Bombay Univ. April)

**Solution.**

$$\begin{array}{r} 110011 \\ 101101 \\ \hline 1100000 \end{array}$$

1. **first column :**  $1 + 1 = 0$  with a carry of 1. Hence, we put down zero there and carry 1 to the second column.
2. **second column :**  $1 + 0 = 1$ . But combined with carry 1 from first column, it gives  $1 + 1 = 0$  and a carry of 1. Hence, we put down 0 there and carry 1 further to the third column.
3. **third column :**  $0 + 1 = 1$ . Again, when it is combined with carry of 1 from the second column, we get  $1 + 1 = 0$  with a carry of 1. Hence, again, we put down 0 and carry 1 to the fourth column.
4. **fourth column :** Here,  $0 + 1 = 1$ . When combined with carry 1 from third column, we get  $1 + 1 = 0$  with a carry of 1. Hence, we put down 0 there and carry 1 to the fifth column.
5. **fifth column :** Here,  $1 + 0 = 1$ . When combined with the carry 1 from fourth column, we get  $1 + 1 = 0$  with a carry of 1. Hence, we put down 0 there and carry 1 to the sixth column.
6. **sixth column :** Here, it is a case of  $1 + 1 + \text{carry of } 1 = 11_2$  as stated earlier in (b) above.

### 69.11. Binary Subtraction

It is also performed in a manner similar to that used in decimal subtraction. Because binary system has only two digits, binary subtraction requires more borrowing operations than decimal subtraction. The four rules for binary subtraction are as under:

1.  $0 - 0 = 0,$                       2.  $1 - 0 = 1,$                       3.  $1 - 1 = 0,$
4.  $0 - 1 = 1$  with a borrow of 1 from the next column of the minuend

or  $10 - 1 = 1$

The last result represents  $\bullet - \bullet - \bullet - \bullet$  which makes sense.

While using Rule 4, it should be borne in mind that borrow reduces the remaining minuend by 1. It means that a borrow will cause a 1 in the next column to the left in the minuend to become 0. If the next column also happens to contain 0, it is changed to a 1 and the succeeding 0s in the minuend are changed to 1s until a 1 is found which is then changed to a 0.

**Example 1**

Let us subtract  $0101_2$  from  $1110_2$ . The various steps are explained below :

$\emptyset$ 1 borrow	$\emptyset$ 1	$\emptyset$ 1	$\emptyset$ 1
$\begin{array}{r} 1110 \\ -0101 \\ \hline 1 \end{array}$	$\begin{array}{r} 1110 \\ -0101 \\ \hline 01 \end{array}$	$\begin{array}{r} 1110 \\ -0101 \\ \hline 001 \end{array}$	$\begin{array}{r} 1110 \\ -0101 \\ \hline 1001 \end{array}$

**Explanation**

1. In the first column, since we cannot subtract 1 from 0, we borrow 1 from the next column to the left. Hence, we put down 1 in the answer and change the 1 of the next left column to a 0.
2. We apply Rule 1 to next column *i.e.*  $0 - 0 = 0$
3. We apply Rule 3 to the 3rd column *i.e.*  $1 - 1 = 0$
4. Finally, we apply Rule 2 to the last *i.e.* fourth column *it.*  $1 - 0 = 1$

As a check, it may be noted that talking in terms of decimal numbers, we have subtracted 5 from 14. Obviously, the answer has to be 9 ( $1001_2$ ).



**Example 2.**

Let us now try subtracting  $0001_2$  from  $1000_2$ .

<i>Step 1</i>	<i>Step 2</i>	<i>Step 3</i>	<i>Step 4</i>
1	1 1	01 1	01 1
1 0 0 0	1 0 0 1	1 0 0 1	1 0 0 1
$\underline{-0001}$	$\underline{-0001}$	$\underline{-0001}$	$\underline{-0001}$
1	1	1	0 1 1 1

Since there happened to be a 0 in the second column, it was changed to 1. Again, there was a 0 in the third column, so it was also changed to a 1. Finally, we met a 1 in the fourth column which was changed to a 0 and the final answer was written down as shown above.

**Example 69.6.** Subtract  $0111_2$  from  $1001_2$  (Digital Computations, Punjab Univ. 1991)

<b>Solution.</b>	1 0 0 1	1st column	:	$1 - 1 = 0$
	$\underline{-0111}$	2nd column	:	$0 - 1 = 1$ with a borrow of 1
	0 0 1 0	3rd column	:	1 (after borrow) - 1 = 0
		4th column	:	0 (after borrow) - 0 = 0

**Example 69.7.** Subtract  $01011_2$  from  $10110_2$  (Computer Technology, Pune Univ.)

**Solution.** Step-wise the solution is as under :

<i>Step 1</i>	<i>Step 2</i>	<i>Step 3</i>	<i>Step 4</i>	<i>Step 5</i>
0	0 0	0 0	0 0 0	0 0 0
1 0 1 1 0	1 0 1 1 0	1 0 1 1 0	1 0 1 1 0	1 0 1 1 0
$\underline{-01011}$	$\underline{-01011}$	$\underline{-01011}$	$\underline{-01011}$	$\underline{-01011}$
1	1 1	0 1 1	1 0 1 1	0 1 0 1 1

**69.12. Complement of a Number**

In digital work, two types of complements of a binary number are used for **complemental subtraction** :

**(a) 1's complement**

The 1's complement of a binary number is obtained by changing its each 0 into a 1 and each 1 into a 0. It is also called **radix-minus-one complement**. For example, 1's complement of  $100_2$  is  $011_2$  and of  $1110_2$  is  $0001_2$ .

**(b) 2's complement**

The 2's complement of a binary number is obtained by adding 1 to its 1's complement.

$$2\text{'s complement} = 1\text{'s complement} + 1$$

It is also known as **true complement**. Suppose we are asked to find 2's complement of  $1011_2$ . Its 1's complement is  $0100_2$ . Next, add 1 to get  $0101_2$ . Hence, 2's complement of  $1011_2$  is  $0101_2$ .

The complement method of subtraction reduces **subtraction to an addition process**.

This method is popular in digital computers because

1. only adder circuits are needed thus simplifying the circuitry,
2. it is easy with digital circuits to get the complements.

**69.13. 1's Complemental Subtraction**

In this method, instead of subtracting a number, we add its 1's complement to the minuend. The last carry (whether 0 or 1) is then added to get the final answer. The rules for subtraction by 1's complement are as under :

1. compute the 1's complement of the subtrahend by changing all its 1s to 0s and all its 0s to 1s.
2. add this complement to the minuend
3. perform the end-around carry of the last 1 or 0
4. if there is no end-around carry (*i.e.* 0 carry), then the answer must be recomplemented and a negative sign attached to it.
5. if the end-around carry is 1, no recomplementing is necessary.

Suppose we want to subtract  $101_2$  from  $111_2$ . The procedure is as under :

$$\begin{array}{r}
 111 \\
 + 010 \quad \leftarrow \text{1's complement of subtrahend } 101 \\
 \hline
 1001 \\
 \quad \underline{1} \quad \leftarrow \text{end-round carry} \\
 010
 \end{array}$$

As seen, we have removed from the addition sum the 1 carry in the last position and added it onto the remainder. It is called *end-around carry*.

Let us now subtract  $1101_2$  from  $1010_2$ .

$$\begin{array}{r}
 1010 \\
 + 0010 \quad \leftarrow \text{1's complement of } 1101 \\
 \hline
 1100 \\
 \text{NO CARRY}
 \end{array}$$

As seen, there is no end-around carry in this case. Hence, as per Rule 4 given above, answer must be recomplemented to get 0011 and a negative sign attached to it. Therefore, the final answer becomes—0011.

Finally, consider the complemental subtraction of  $1110_2$  from  $0110_2$ .

$$\begin{array}{r}
 0110 \\
 + 0001 \quad \leftarrow \text{1's complement of } 1110_2 \\
 \hline
 0111 \\
 \text{NO CARRY}
 \end{array}$$

As seen, there is no carry. However, we may add an extra 0 from our side to make it a 0 carry as shown below.

$$\begin{array}{r}
 0110 \\
 + 0001 \quad \leftarrow \text{1's complement of subtrahend} \\
 \hline
 00111 \\
 \quad \underline{0} \quad \leftarrow \text{end-around carry} \\
 00111
 \end{array}$$

After recomplementing, it becomes 1000. When negative sign is attached, the final answer becomes  $-1000_2$ .

**Example 69.8.** Using 1's complemental method, subtract  $01101_2$  from  $11011_2$ .

**Solution.**

$$\begin{array}{r}
 11011 \\
 + 10010 \quad \leftarrow \text{1's complement of subtrahend} \\
 \hline
 101101 \\
 \quad \underline{1} \quad \leftarrow \text{end-around carry} \\
 01110
 \end{array}$$

Since end-around carry is 1, we take the final answer as it is (Rule 5).

**Example 69.9.** Use 1's complement to subtract  $11011_2$  from  $01101_2$ .

(Computer Science, Allahabad Univ.)

**Solution.**

$$\begin{array}{r} 01101 \\ + 00100 \quad \leftarrow \text{1's complement of } 11011_2 \\ \hline 10001 \quad \rightarrow -01110 \\ \text{NO CARRY} \end{array}$$

Since there is no final carry, we recompute the answer and attach a minus sign to get the final answer  $-01110_2$ .

### 69.14. 2's Complement Subtraction

In this case, the procedure is as under :

1. find the 2's complement of the subtrahend,
2. add this complement to the minuend,
3. drop the final carry,
4. if the carry is 1, the answer is positive and needs no recomplementing,
5. if there is no carry, recompute the answer and attach minus sign.

**Example 69.10.** Using 2's complement, subtract  $1010_2$  from  $1101_2$ .

**Solution.** The 1's complement of 1010 is 0101. The 2's complement is  $0101 + 1 = 0110$ . We will add it to 1101.

$$\begin{array}{r} 1101 \\ + 0110 \quad \leftarrow \text{2's complement of } 1010_2 \\ \hline 10011 \\ \text{DROP} \end{array}$$

The final answer is  $0011_2$ .

**Example 69.11.** Use 2's complement to subtract  $1101_2$  from  $1010_2$ .

(Digital Computations, Punjab Univ. 1992)

**Solution.** The 1's complement of 1101 is 0010. The 2's complement is 0011.

$$\begin{array}{r} 1010 \\ + 0011 \quad \leftarrow \text{2's complement of } 1101_2 \\ \hline 1101 \\ \text{NO CARRY} \end{array}$$

In this case, there is no carry. Hence, we have to recompute the answer. For this purpose, we first subtract 1 from it to get 1100. Next, we recompute it to get 0011. After attaching the minus sign, the final answer becomes  $-0011_2$ .

Talking in terms of decimal numbers, we have subtracted 13 from 10. Obviously the answer is  $-3$ .

### 69.15. Binary Multiplication

The procedure for this multiplication is the same as for decimal multiplication though it is comparatively much easier. The four simple rules are as under:

1.  $0 \times 0 = 0$ ,
2.  $0 \times 1 = 0$ ,
3.  $1 \times 0 = 0$ ,
4.  $1 \times 1 = 1$ .

The rules of binary multiplication could be summarized in the form of a table as shown.

$$\begin{array}{r} 01 \\ 00 \\ \hline 101 \end{array}$$

As in the decimal system, the procedure is

1. copy the multiplicand when multiplier digit is 1 but not when it is 0
2. shift as in decimal multiplication
3. add the resulting binary numbers according to the rules of binary addition.

**Example 69.12.** Multiply  $111_2$  by  $101_2$  using binary multiplication method. (Electronics-1, Indore Univ. 1991)

**Solution.**

$$\begin{array}{r}
 111 \\
 \times 101 \\
 \hline
 111 \\
 000 \quad \text{--- shift left, no add} \\
 111 \quad \text{--- shift left and add} \\
 \hline
 100111
 \end{array}$$

**Example 69.13.** Multiply  $1101_2$  by  $1100_2$ . (Digital Electronics, Bombay Univ. 1990)

**Solution.**

$$\begin{array}{r}
 1101 \\
 \times 1100 \\
 \hline
 0000 \\
 0000 \\
 1101 \\
 1101 \\
 \hline
 10011100
 \end{array}$$

**Example 69.14.** Multiply  $1111$  by  $0111_2$ .

**Solution.** This example has been included for the specific purpose of explaining how to handle the addition if multiplication results in *columns with more than two 1s*.

1. Result of the first column is 1.
2. In the second column, addition of  $1 + 1 = 10_2$ . Hence, we put down 0 there and carry 1 to the third column.
3. In the third column,  $1 + 1 + 1 + 1 = 100_2$  (decimal 4). We keep one 0 there, put the second 0 in fourth column and pass on 1 to the fifth column.
4. In the fourth column,  $1 + 1 + 1 + 0 = 11$ , (decimal 3). Hence, one 1 is kept there and the other 1 is passed on to the fifth column.
5. In the fifth column,  $1 + 1 + 1 + 1 = 100_2$  (decimal 4). Again, one 0 is retained there, second 0 is passed on to the sixth column and 1 to the seventh column.
6. In the sixth column,  $1 + 0 = 1$ .
7. The seventh column already has 1 given by the addition of the fifth column.

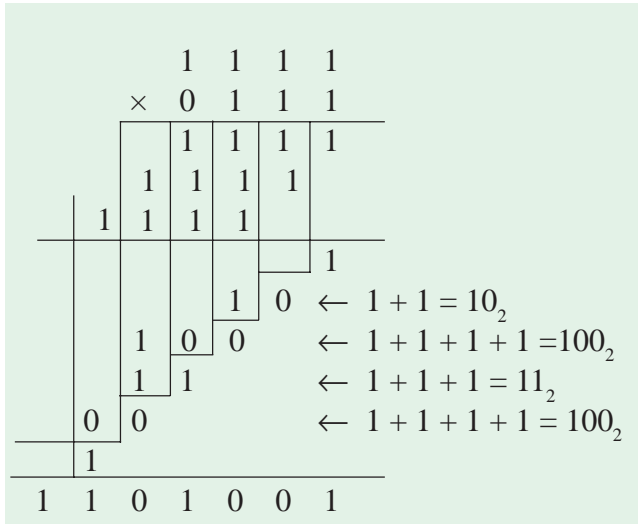


Fig. 69.5

### 69.16. Binary Division

It is similar to the division in the decimal system. As in that system, here also division by 0 is meaningless. Rules are :

1.  $0 \div 1 = 0$  or  $\frac{0}{1} = 0$ ,
2.  $1 \div 1 = 1$  or  $\frac{1}{1} = 1$ .

**Example 69.15.** Carry out the binary division  $11001 \div 101$

**Solution.**

$$\begin{array}{r} 101 \\ 101 \overline{) 11001} \\ \underline{101} \phantom{00} \\ 10 \phantom{00} \\ \underline{101} \phantom{00} \\ 000 \end{array}$$

After we bring down the next 0 bit, the number 10 so formed is not divisible by 101. Hence, we put a 0 in the quotient. Therefore, the answer is  $101_2$  (decimal 5).

Incidentally, it may be noted that the dividend  $25_{10}$  and divisor is  $5_{10}$  so that the result of division, as expected, is  $5_{10}$ .

**Example 69.16.** Divide  $11011_2$  by  $100_2$  (Digital Computations, Punjab Univ. 1990)

**Solution.**

$$\begin{array}{r} 110.11 \\ 100 \overline{) 11011} \\ \underline{100} \phantom{00} \\ 101 \phantom{00} \\ \underline{100} \phantom{00} \\ 11 \phantom{00} \\ \underline{110} \phantom{00} \\ 100 \phantom{00} \\ \underline{100} \phantom{00} \\ 000 \end{array}$$

$$\begin{array}{r} 6.75 \\ 4 \overline{) 27} \\ \underline{24} \phantom{00} \\ 30 \phantom{00} \\ \underline{28} \phantom{00} \\ 20 \phantom{00} \\ \underline{20} \phantom{00} \\ 00 \end{array}$$

### 69.17. Shifting a Number to Left or Right

Shifting binary numbers one step to the left or right corresponds respectively to multiplication or division by decimal 2.

When binary number  $101100_2$  ( $44_{10}$ ) is shifted one step **to the left**, it becomes  $1011000_2$  which is  $88_{10}$  *i.e.* it is doubled. If the given number is shifted one step **to the right**, it becomes  $10110_2$  which is  $22_{10}$ . Obviously, the number is halved.

### 69.18. Representation of Binary Numbers as Electrical Signals

As seen from above, any binary number can be represented as a string of 0s and 1s. However, it is fine for paper and pencil calculations only. Practical problem is how to apply the desired binary information to logic circuits in digital computers. For that purpose, two types of electrical signals are selected to represent 1 and 0. Since speed and accuracy are of primary importance in digital circuits, the two electrical signals chosen to represent 1 and 0 must meet very rigid requirements.

1. they must be suitable for use in high-speed circuitry,
2. the signals should be very easy to tell apart,
3. they must be hard to confuse with each other.

The second and third statements may look alike but they, in fact, are not so. It is found that all transistor circuits distort, to some extent, the electrical signals that pass through them. Sometime, these distorted signals can look confusingly alike. Hence, this effect of distortion or degradation has to be kept in mind while selecting the two signals.

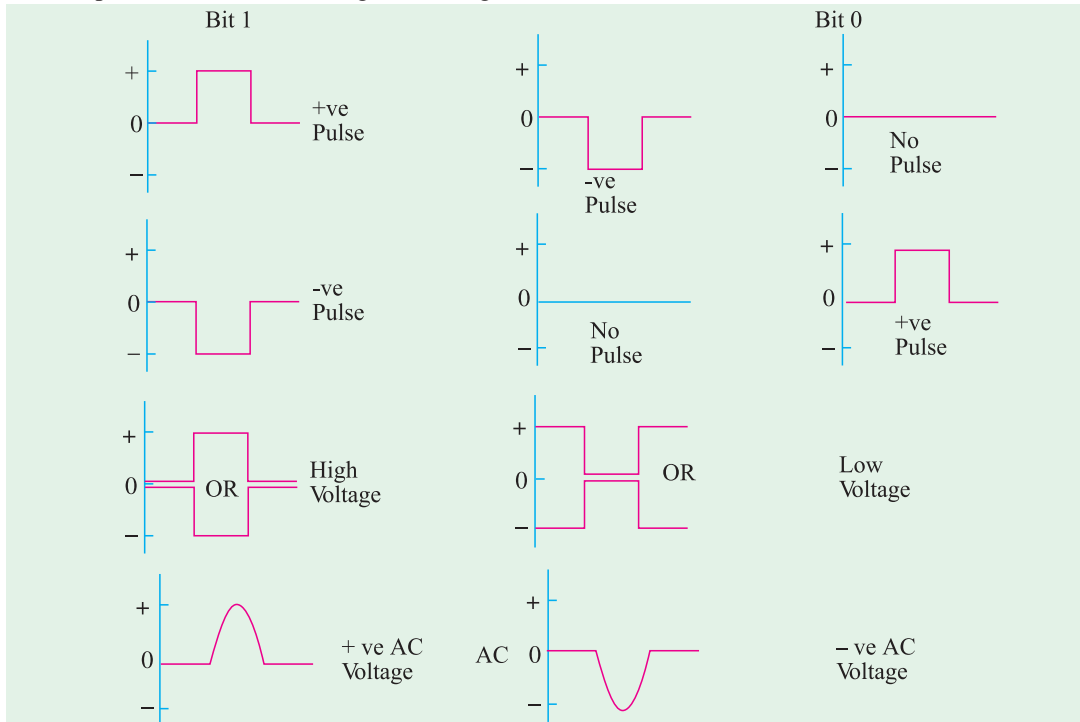


Fig. 69.6

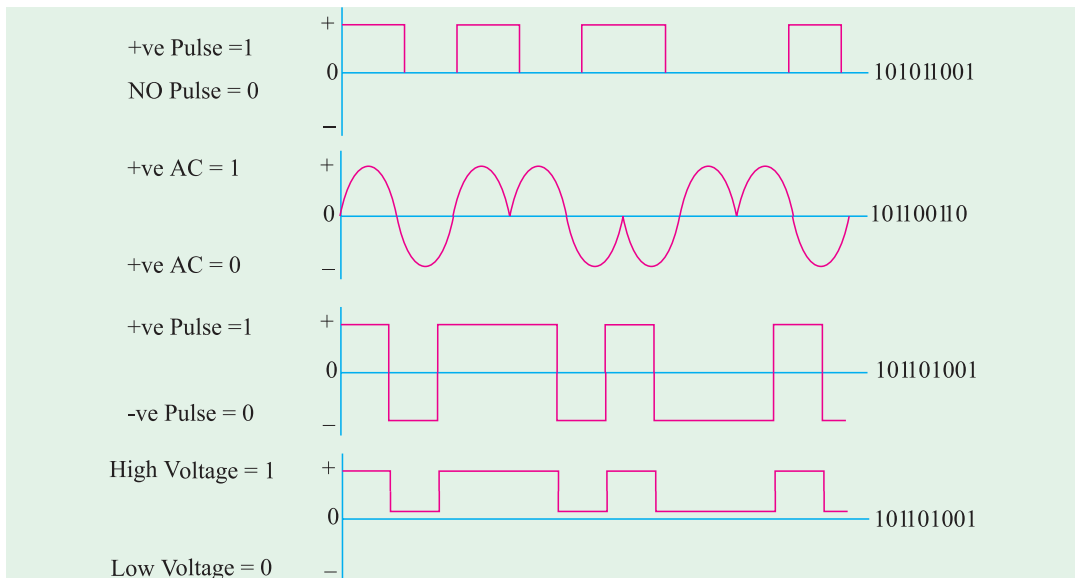


Fig. 69.7

In Fig. 69.6 are shown several signal pairs that meet the above requirements. It will be noted that it is impossible to distort a positive pulse (representing 1) to look like the no pulse or negative pulse (representing 0).

Fig. 69.7 shows how signal pairs can be used to represent different binary numbers.

### 69.19. Octal Number System

#### (i) Radix or Base

It has a base of 8 which means that it has eight distinct counting digits :

0, 1, 2, 3, 4, 5, 6, and 7

These digits 0 through 7, have exactly the same physical meaning as in decimal system.

For counting beyond 7, **2-digit combinations are formed taking the second digit followed by the first, then the second followed by the second and so on.** Hence, after 7, the next octal number is 10 (second digit followed by first), then 11 (second digit followed by second) and so on. Hence, different octal numbers are :

0,	1,	2,	3,	4,	5,	6,	7,
10,	11,	12,	13,	14,	15,	16,	17,
20,	21,	22,	23,	24,	25,	26,	27,
30,	31,	32,	...	...	...	...	...

#### (ii) Position Value

The position value (or weight) for each digit is given by different powers of 8 as shown below:

$$\leftarrow 8^3 \quad 8^2 \quad 8^1 \quad 8^0 \quad \cdot \quad 8^{-1} \quad 8^{-2} \quad 8^{-3} \rightarrow$$

↑  
octal point

For example, decimal equivalent of octal 352 is

$$\begin{array}{ccccccc} 3 & 5 & 2 & \cdot & & & 0 \\ 8^2 & 8^1 & 8^0 & & & & \end{array}$$

$$64 \quad 8 \quad 1 = 3 \times 64 + 5 \times 8 + 2 \times 1 = 234_{10}$$

or  $352_8 = 3 \times 8^2 + 5 \times 8^1 + 2 \times 8^0 = 192 + 40 + 2 = 234_{10}$

Similarly, decimal equivalent of octal 127.24 is

$$\begin{aligned} 127.24_8 &= 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 2 \times 8^{-1} + 4 \times 8^{-2} \\ &= 64 + 16 + 7 + \frac{2}{8} + \frac{4}{64} = 87.3125_{10} \end{aligned}$$

### 69.20. Octal to Decimal Conversion

Procedure is exactly the same as given in Art. 68.4 except that we will use digit of 8 rather than 2.

Suppose, we want to convert octal 206.104<sub>8</sub> into its decimal equivalent number. The procedure is as under :

$$\begin{array}{ccccccc} 2 & 0 & 6 & \cdot & 1 & 0 & 4 \\ 8^2 & 8^1 & 8^0 & & 8^{-1} & 8^{-2} & 8^{-3} \end{array}$$

$\therefore 206.104_8 = 2 \times 8^2 + 6 \times 8^0 + \frac{1}{8} + \frac{4}{8^3} = 128 + 6 + \frac{1}{8} + \frac{1}{128} = \left(134 \frac{17}{128}\right)_{10}$

### 69.21. Decimal to Octal Conversion

The **double-dabble** method (Art 69.7) is used with 8 acting as the **multiplying** factor for **integers** and the **dividing** factor for **fractions**.

Let us see how we can convert  $175_{10}$  into its octal equivalent.

$175 \div 8 = 21$	with 7 remainder	↑
$21 \div 8 = 2$	with 5 remainder	
$2 \div 8 = 0$	with 2 remainder	

Taking the remainders in the **reverse order**, we get  $257_8$ .  $\therefore 175_{10} = 257_8$

Let us now take decimal fraction 0.15. Its octal equivalent can be found as under:

$0.15 \times 8 = 1.20 = 0.20$	with a carry of 1	↓
$0.20 \times 8 = 1.60 = 0.60$	with a carry of 1	
$0.60 \times 8 = 4.80 = 0.80$	with a carry of 4	
$\therefore 0.15_{10} \cong 114_8$		

As seen, here carries have been taken in the **forward direction** *i.e.* from top to bottom.

Using positional notation, the first few octal numbers and their decimal equivalents are shown in Table 69.2.

Table No. 69.2					
Octal	Decimal	Octal	Decimal	Octal	Decimal
0	0	12	10	24	20
1	1	13	11	25	21
2	2	14	12	26	22
3	3	15	13	27	23
4	4	16	14	30	24
5	5	17	15	31	25
6	6	20	16	32	26
7	7	21	17	33	27
10	8	22	18	34	28
11	9	23	19	35	29

### 69.22. Binary to Octal Conversion

The simplest procedure is to use **binary-triplet** method. In this method, the given binary number is arranged into groups of 3 bits starting from the octal point and then each group is converted to its equivalent octal number. Of course, where necessary, extra 0s can be added **in front** (*i.e.* left end) of the binary number to complete groups of three.

Suppose, we want to convert  $101011_2$  into its octal equivalent. Converting the bits into groups of three, we have  $\therefore$

$101$	$011$	
Now, $101_2$ is 5 octal and $011$ is 3 octal.		
$\therefore$	$101$	$011$
	↓	↓
	5	3
		$\therefore 101\ 011_2 = 53_8$

Now, take  $111110111_2$ . We will first split it into groups of **three bits** (space is left between the groups for easy reading). Then, each group is given its octal number as shown below.

$111$	$110$	$111$	
↓	↓	↓	
7	6	7	$\therefore 111\ 110\ 111_2 = 767_8$



Finally, take the example of a mixed binary number  $10101.11_2$ . Here, we will have to add one 0 in front of the integral part as well as to the fractional part

$$\begin{array}{ccc} 010 & 101 & 110 \\ \downarrow & \downarrow & \downarrow \\ 2 & 5 & 6 \end{array} \quad \therefore \quad 10101.11_2 = 25.6_8$$

The equivalence between binary triplets and octal numbers is given in Table 69.3.

Binary	Octal	Binary	Octal
000	0	1010	12
001	1	1011	13
010	2	1100	14
011	3	1101	15
100	4	1110	16
101	5	1111	17
110	6	10000	20
111	7	10001	21
1000	10	10010	22
1001	11	10011	23

### 69.23. Octal to Binary Conversion

The procedure for this conversion is just the opposite of that given in Art 69.22. Here, each digit of the given octal number is converted into its equivalent binary triplet. For example, to change  $75_8$  into its binary equivalent, proceed as under:

$$\begin{array}{cc} 7 & 5 \\ \downarrow & \downarrow \\ 111 & 101 \end{array} \quad \therefore \quad 75_8 = 111\ 101_2$$

Similarly,  $74.562_8$  can be converted into binary equivalent as under:

$$\begin{array}{ccccc} 7 & 4 & \cdot & 5 & 6 & 2 \\ \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow \\ 111 & 100 & & 101 & 110 & 010 \end{array} \quad \therefore \quad 74.562_8 = 111\ 100.101\ 110\ 010_2$$

Incidentally, it may be noted that number of digits in octal numbers is **one-third of that in equivalent binary numbers**. In the present case, it is five versus fifteen.

### 69.24. Usefulness of Octal Number System

We have already discussed the octal number system and conversion from the binary and decimal numbers to octal and vice versa. The ease with which conversions can be made between octal and binary makes the octal system attractive as a “shorthand” means of expressing large binary numbers. In computer work, binary number with up to 64 bits are not uncommon. These binary numbers, as we shall see, do not always represent a numerical quantity but are often some type of code that conveys non numerical information. In computers, binary numbers might represent :

1. actual numerical data
2. numbers corresponding to a location called (address) in memory,
3. an instruction code

- 4. a code representing alphabetic and other non numerical characters,
- 5. group of bits representing the status of devices internal or external to the computer

When dealing with a large quantity of binary numbers of many bits, it is convenient and more efficient for us to write the numbers in octal rather than binary. However keep in mind that the digital circuits and systems work strictly in binary. We use octal numbers only as a convenience for the operators of the system.

### 69.25. Hexadecimal Number System

The characteristics of this system are as under :

- 1. it has a base of 16. Hence, it uses sixteen distinct counting digits 0 through 9 and A through F as detailed below :  
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- 2. place value (or weight) for each digit is in **ascending powers** of 16 for integers and **de-ascending powers** of 16 for fractions.

The chief use of this system is in connection with **byte-organised machines**. It is used for specifying addresses of different binary numbers stored in computer memory.

### 69.26. How to Count Beyond F in Hex Number System ?

As usual, we resort to **2-digit combinations**. After reaching **F**, we take the second digit followed by the first digit, then second followed by second, then second followed by third and so on. The first few 'hex' numbers and their decimal equivalents are given in Table 69.4.

Hexadecimal	Decimal	Hexadecimal	Decimal	Hexadecimal	Decimal
0	0	B	11	16	22
1	1	C	12	17	23
2	2	D	13	18	24
3	3	E	14	19	25
4	4	F	15	1A	26
5	5	10	16	1B	27
6	6	11	17	1C	28
7	7	12	18	1D	29
8	8	13	19	1E	30
9	9	14	20	1F	31
A	10	15	21	20	32
				21	33

### 69.27. Binary to Hexadecimal Conversion

The simple method is to split the given binary number into **4-bit groups** (supplying 0s from our own side if necessary) and then give each group its 'hex' value as found from Table 69.5.

Binary	Hex.	Binary	Hex.	Binary	Hex.
0000	0	0110	6	1100	C
0001	1	0111	7	1101	D
0010	2	1000	8	1110	E
0011	3	1001	9	1111	F
0100	4	1010	A	10000	10
0101	5	1011	B	10001	11

Let us see how we would convert  $10001\ 100_2$  into its hexadecimal equivalent number. We will first split the given binary number into 4-bit groups and then give each group its proper value from Table 69.5.

$$\begin{array}{cc} 1000 & 1100 \\ \downarrow & \downarrow \\ 8 & C \end{array} \quad \therefore \quad 1000\ 1100_2 = 8C_{16}$$

Let us now consider  $1011010111_2$ . Following the above procedure, we have

$$\begin{array}{ccc} 0010 & 1101 & 0111 \\ \downarrow & \downarrow & \downarrow \\ 2 & D & 7 \end{array} \quad \therefore \quad 1011010111_2 = 2D7_{16}$$

It is seen that two 0s have been added to complete the 4-bit groups.

### 69.28. Hexadecimal to Binary Conversion

Here, the procedure is just the reverse of that given in Art. 69.27. Each hexadecimal digit is converted into its equivalent 4-bit binary.

Suppose, we want to convert  $23A_{16}$  into its binary equivalent. It can be done as given below:

$$\begin{array}{ccc} 2 & 3 & A \\ \downarrow & \downarrow & \downarrow \\ 0010 & 0011 & 1010 \end{array} \quad \therefore \quad 23A_{16} = 0010\ 0011\ 1010_2$$

Incidentally, hex numbers contain **one-fourth** the number of bits contained in the equivalent binary number. Optionally, we could drop off the two 0s in front of the binary equivalent.

### 69.29. Decimal to Hexadecimal Conversion

Two methods are available for such a conversion. One is to go from decimal to binary and then to hexadecimal. The other method called **hex dabble** method is similar to the double-dabble (or divide-by-two) method of Art 69.7 except that we use 16 (instead of 2) for successive divisions. As an example let us convert decimal 1983 into hexadecimal by consulting Table No. 69.4 for remainders.

Hence, $1983_{10} = 7BF_{16}$	$1983 \div 16 = 123 + 15 \rightarrow F$	↑
	$123 \div 16 = 7 + 11 \rightarrow B$	
	$7 \div 16 = 0 + 7 \rightarrow 7$	

### 69.30. Hexadecimal to Decimal Conversion

Two methods are available for such a conversion. One is to convert from hexadecimal to binary and then to decimal. The other direct method is as follows :

Instead of using powers of 2, use power of 16 for the weights. Then, sum up the products of hexadecimal digits and their weights to get the decimal equivalent. As an example, let us convert  $F6D9$  to decimal.

$$\begin{aligned} F6D9 &= F(16^3) + 6(16^2) + D(16^1) + 9(16^0) = 15 \times 16^3 + 6 \times 16^2 + 13 \times 16^1 + 9 \times 16^0 \\ &= 61,440 + 1536 + 208 + 9 = 63,193_{10} \end{aligned}$$

**Example 69.17.** Find the binary, octal and hexadecimal equivalents of the following decimal numbers (i) 32 (ii) 256 (iii) 51. (Digital Computations, Punjab Univ. May 1990)

**Solution. (i) Decimal number 32**

As seen from Art : 10–7	32	16	8	4	2	1	
	1	0	0	0	0	0	$\therefore 32_{10} = 100000_2$
For octal conversion :	100	000					$\therefore 32_{10} = 40_8$
	↓	↓					
	4	0					

For hexadecimal conversion  $0010 \quad 0000$   
 $\downarrow \quad \downarrow$   
 $2 \quad 0 \quad \therefore 32_{10} = 20_{16}$

As will be seen, the binary number has been divided into two 4-bit groups for which purpose two 0s have been added to the left.

(ii) In the same way, it can be found that  $256_{10} = 100000000_8 = 100_{16}$

(iii) Also  $51_{10} = 110011_2 = 63_8 = 33_{16}$

**Example 69.18.** Convert the following numbers to decimal

(i)  $(11010)_2$

(ii)  $(AB60)_{16}$

(iii)  $(777)_8$

(Digital Computations, Punjab Univ. 1992)

**Solution.** (i) For binary to decimal conversion, we will follow the procedure given in Art. 69.4.

$\therefore 11010_2 = 16 + 8 + 2 = 26_{10}$

1	1	0	1	0
16	8	4	2	1
16	8	4	2	1

(ii) Following the procedure given in Art. 69.30, we have,

$$AB60_{16} = A(16^3) + B(16^2) + 6(16^1) + 0(16^0) = 10 \times 16^3 + 11 \times 16^2 + 6 \times 16^1 + 0 \times 16^0 = 43,872_{10}$$

(iii) As per the procedure given in Art. 69.20,  $777_8 = 7(8^2) + 7(8^1) + 7(8^0) = 511_{10}$ .

**Example 69.19.** A computer is transmitting the following groups of bytes (each consisting of 8-bits) to some output device. Give the equivalent octal and hexadecimal listings.

1000	1100	0011	1010
0010	1110	1001	0101
0101	1111	1011	0110
0111	1011	0101	1011

**Solution.**

Binary		Octal			Hexadecimal	
1000	1100	10	001	100	1000	1100
		2	1	4	8	C
0010	1110	00	101	110	0010	1110
		0	5	6	2	E
0101	1111	01	011	111	0101	1111
		1	3	7	5	F
0111	1011	01	111	011	0111	1011
		1	7	3	7	B
0011	1010	00	111	010	0011	1010
		0	7	2	3	A
1001	0101	10	010	101	1001	0101
		2	2	5	9	5
1011	0110	10	110	110	1011	0110
		2	6	6	B	6
0101	1011	01	011	011	0101	1011
		1	3	3	5	B

Hence, the groups of given memory bytes when expressed in different number systems become as under :

<i>Binary</i>	<i>Octal</i>	<i>Hexadecimal</i>
10001100	214	8C
00101110	056	2E
0101111	137	5F
01111011	173	7B
00111010	072	3A
10010101	225	95
10110110	266	B6
01011011	133	5B

It is clear from above that so far as the computer operator (or programmer) is concerned, it is much easier to handle this data when expressed in octal or hexadecimal system than in the binary system. For example, it is much easier and less error-prone to write the hexadecimal 8C than the binary 10001100 or 6AF than 011010101111. Of course, when the need arises, the operator can easily convert from octal or hexadecimal to binary.

### 69.31. Digital Coding

In digital logic circuits, each number or piece of information is defined by an equivalent combination of binary digits. A complete group of these combinations which represents numbers, letters or symbols is called a *digital code*.

Codes have been used for security reasons so that others may not be able to read the message even if it is intercepted. In modern digital equipment, codes are used to represent and process numerical information. The choice of a code depends on the function or purpose it has to serve. Some codes are suitable where arithmetic operations are performed whereas others have high efficiency *i.e.* they give more information using fewer bits.

In certain applications, use of one code or the other simplifies and reduces the circuitry required to process the information. By limiting the switching circuitry, reliability of the digital system is increased. Of continuing importance are other codes which allow for error detection or correction. These codes enable the computers to determine whether the information that was coded and transmitted is received correctly and, if there is an error, to correct it. Since coding itself is a detailed subject, only few of the more familiar codes will be discussed.

### 69.32. Binary Coded Decimal (BCD) Code

It is a binary code in which *each* decimal digit is represented by a group of four bits. Since the right-to-left weighting of the 4-bit positions is 8-4-2-1, it is also called an 8421 code. It is a weighted numerical code. As said above, here each decimal digit from 0 through 9 requires a 4-bit binary-coded number. For example, the decimal number 35 in *BCD* code is 0101. The coding of ten decimal digits is given in Table No. 69.6. Lest you think that *BCD* code is the same thing as binary numbers, consider the following.

In the binary system, ten is represented by 1010 but in *BCD* code, it is 0001 0000. Seventeen in binary is 10001 but in *BCD* code, it is 0001 0111. See the difference ! Actually, the confu-

Table No. 69.6

<i>Decimal</i>	<i>BCD</i>
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

sion is due to the fact that the first nine numbers in *BCD* and binary are exactly similar (Table No. 69.6). After that, they become quite different (Table No. 69.7).

Decimal	BCD			
26	0010	0110		
59	0101	1001		
673	0110	0111	0011	
2498	0010	0100	1001	1000

It should be realized that with four-bits, sixteen numbers ( $2^4$ ) can be represented although in the *BCD* code only ten of these are used. The following six combinations are *invalid* in the *BCD* code : 1010, 1011, 1100, 1101, 1110 and 1111.

The main advantage of *BCD* code is that it can be read and recognised easily although special adders are needed for arithmetic operations.

Any decimal number can be expressed in *BCD* code by replacing each decimal digit by the appropriate 4-bit combination. Conversely, a *BCD* number can be easily converted into a decimal number by dividing the coded number into groups of four bits (starting with *LSB*) and then writing down the decimal digit represented by each four-bit group.

**Example 69.20.** Write the decimal number 369 in *BCD* code.

**Solution.** For writing this 3-digit number in *BCD*, the value of each digit must be replaced by its 4-bit equivalent from the *BCD* code. From Table No. 69.6, we get

$$3 = 0011, \quad 6 = 0110, \quad 9 = 1001$$

$$\therefore 369_{10} = 001101101001_{BCD}$$

**Example 69.21.** Typically digital thermometers use *BCD* to drive their digital displays. How many *BCD* bits are required to drive a 3-digit thermometer display? What 12 bits are sent to display for a temperature of 157 degrees.

**Solution.** There are 12 *BCD* bits required to drive a 3-digit thermometer display because each *BCD* digit is represented by a group of four bits.

In order to display a temperature of 157 degrees, we know that we have to send 12-bits. These bits can be determined by replacing each decimal digit by its equivalent four bit binary. Thus,

$$\begin{array}{ccc} 1 & 5 & 7 \\ \downarrow & \downarrow & \downarrow \\ 0001 & 0101 & 0111 \end{array}$$

$$\therefore 157_{10} = 000101010111_{BCD}$$

**Example 69.22.** Find the equivalent decimal value for the *BCD* code number 0001010001110101. (Applied Electronics, A.M.I.E.E., London)

**Solution.** Starting from the *LSB*, the given number can be divided into groups of four bits as 0001 0100 0111 0101. As seen from Table No. 69.6,

$$\begin{array}{ll} 0001 = 1, & 0100 = 4, \\ 0111 = 7 & \text{and } 0101 = 5 \end{array}$$

Hence,  $0001010001110101_{BCD} = 1475_{10}$

**Example 69.23. (a)** Convert the hexadecimal number F8E6 to the corresponding decimal number.

- (b) Convert the decimal number 2479 to the corresponding hexadecimal number.
- (c) Encode the following decimal numbers into 8421 *BCD* numbers (i) 59 (ii) 39 and (iii) 584.
- (d) Decode the following 8421 *BCD* numbers (i) 0101 (ii) 0111.

(Digital Computations, Punjab Univ. 1990)

**Solution. (a)**  $F8E6 = F(16^3) + 8(16^2) + E(16^1) + 6(16^0)$   
 $= 15 \times 16^3 + 8 \times 16^2 + 14 \times 16^1 + 6 \times 16^0 = 63,71810$

(b) We would use the hex-dabble method explained in Art 69.29

$$\begin{array}{r} 2479 \div 16 = 154 + 15 \rightarrow F \\ 154 \div 16 = 9 + 10 \rightarrow A \\ \therefore 2479_{10} = 9AF_{16} \quad 9 \div 16 = 0 + 9 \rightarrow 9 \end{array}$$

(c) As seen from Table No. 69.6

5	9	3	9	5	8	4
↓	↓	↓	↓	↓	↓	↓
0101	1001	0011	1001	0101	1000	0100

(d) Again, consulting Table No. 69.6, we have

0101	0111
↓	↓
5	7

### 69.33. Octal Coding

It involves grouping the bits in *three's*. For example,  $(1756)_8 = (001\ 111\ 101\ 110)_2 = (001111101110)_2$ . Similarly, the 24-bit number stored in the computer memory such as 101 010 011 100 010 111 000 110 can be read in the octal as

101	010	011	100	010	111	000	110
↓	↓	↓	↓	↓	↓	↓	↓
5	2	3	4	2	7	0	6

Apart from ease of recognition and conversion to binary, one important feature of the octal code is that its numbers are straight binary numbers which can be manipulated mathematically. For example, octal 25 expressed in octal code is 010 101 which can be read as binary 010101

$$\begin{aligned} (25)_8 &= 2 \times 8^1 + 5 \times 8^0 = (21)_{10} \\ (010101)_2 &= 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (21)_{10} \end{aligned}$$

You might recall that in *BCD* code, the resulting number is always a 4-bit group and a special adder is needed to convert it into decimal. In octal coding, 3-bit grouping is used but the resulting binary number can be considered a single number in natural binary form.

### 69.34. Hexadecimal Coding

The advantage of this coding is that four bits are expressed by a single character. However, the disadvantage is that new symbols have to be used to represent the values from 1010 to 1111 binary. As seen from Table No. 69.5, the binary number 1010 0101 is hex number A5. Similarly, hexadecimal number C7 is  $(11000111)_2$ . To prove that the resulting binary number is the same as the hexadecimal value, consider the following example:

$$\begin{aligned} (3D)_{16} &= 3 \times 16^1 + D \times 16^0 = 3 \times 16 + 13 \times 1 = (61)_{10} \\ (3D)_{16} &= (00111101)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (61)_{10} \end{aligned}$$

### 69.35. Excess-3 Code

It is an unweighted code and is a modified form of *BCD*. It is widely used to represent numerical data in digital equipment. It is abbreviated as XS-3. As its name implies, each coded number in XS-3 is *three larger* than in *BCD* code. For example, six is written as 1001. As compared to *BCD*, the XS-3 has poorer recognition but it is more desirable for arithmetic operations. A few numbers using Excess-3 code are given in Table 69.8.

Table No. 69.8			
Decimal	XS-3		
3			0110
26		0101	1001
629	1001	0101	1100
3274	0110	0101	1010 0111

**69.36. Gray Code**

It is an unweighted code for numbers 0 through 9 and is largely used in mechanical switching systems. As seen from Table No. 69.9, only a single bit changes between each successive word. Because of this, the amount of switching is minimized and the reliability of the switching system is improved.

Decimal	Gray	XS-3 Gray
0	0000	0010
1	0001	0110
2	0011	0111
3	0010	0100
4	0110	0100
5	0111	1100
6	0101	1101
7	0100	1111
8	1100	1110
9	1101	1010

**69.37. Excess-3 Gray Code**

It is shown in Table No. 69.9 and is the original gray code shifted by three binary combinations. It exhibits the same properties as the Gray Code.

**Example 69.24.** Express the number  $43_{10}$  in XS-3 code.

**Solution.** Let us first represent each decimal digit by its 4-bit XS-3 code.

$$4 = 0111, \quad 3 = 0110 \quad \therefore 43_{10} = 01110110_{XS-3}$$

**Example 69.25.** The number 0110 1001 is expressed in XS-3 code. What is its decimal value ?

**Solution.** Starting from least significant bit (LSB), the given number is first separated into groups of four and then each group is replaced by its equivalent value i.e. actual value decreased by 3.

$$0110 = 6 - 3 = 3; \quad 1001 = 9 - 3 = 6$$

$$\therefore 01101001_{XS-3} = (36)_{10}$$

**69.38. Other Codes**

Some of the other codes which are presently popular are given below:

**(a) 4-bit codes**

The different 4-bit weighted BCD codes for decimal numbers 0 through 9 in use are : 5421,  $2^*421$ , 7421, 7421, etc. and are tabulated below:

Decimal	5421	$2^*421$	7421	$74\bar{2}1$
0	0000	0000	0000	0000
1	0001	0001	0001	0111
2	0010	0010	0010	0110
3	0011	0011	0011	0101
4	0100	0100	0100	0100
5	1000	1011	0101	1010
6	1001	1100	0110	1001
7	1010	1101	1000	1000
8	1011	1110	1001	1111
9	1100	1111	1010	1110

**(b) 5-bit Codes**

(i) **2-out-of-5 codes** is an unweighted BCD code and allows easy error detection. It has been used in communications and telephone operation.

(ii) **51111 Code** is a weighted BCD code and is much easier to operate with electronic circuitry.

(iii) **Shift-counter (Johnson) Code** is an unweighted BCD code and because of its pattern is easily operated on with electronic circuitry.



- (c) **7-bit Biquinary Code**— it uses a group of seven bits to represent decimal numbers and has code features which provide easy error detection and ease of operation.
- (d) **Ring-counter Code**— it is also called 10-bit code because it uses a group of 10 bits to represent a decimal number. Though it requires as many as 10 positions, the ease of error detection with the code and of operating electronic circuits to implement the code make it quite attractive.
- (e) **Alphanumeric Code**— In addition to numerical data, a computer must be able to handle non-numerical information used in input/output (I/O) processing. In other words, a computer should recognize codes that represent letters of the alphabet, punctuation marks, and other special characters as well as numbers. These codes are called alphanumeric codes. A complete alphanumeric code would include the 26 lowercase letters, 26 uppercase letters, 10 numeric digits, 7 punctuation marks, and anywhere from 20 to 40 other characters, such as +, -, /, #, \$, “, and so on. We can say that an alphanumeric code represents all of the various characters and functions that are found on a computer keyboard. The most widely used alphanumeric code is the American standard code for Information Interchange (ASCII). Another similarly I/O-oriented code is EBCDIC (Extended Binary Coded Decimal Interchange Code).

### 69.39. ASCII Code

The ASCII code (Pronounced “askee) is a seven-bit code, and so it has  $2^7$  (=128) possible code groups. This is more than enough to represent all of the standard keyboard characters as well as control functions such as the (RETURN) and LINEFEED) functions. Table No. 69.11 shows a partial listing of the ASCII code. In addition to the binary code group for each character, the table gives the octal and hexadecimal equivalents. The complete list of the ASCII code is given in the Appendix.

Table No. 69.11

Character	7-Bit ASCII	Octal	Hex	Character	7-Bit ASCII	Octal	Hex
A	100 0001	101	41	Y	101 1001	131	59
B	100 0010	102	42	Z	101 1010	132	5A
C	100 0011	103	43	0	011 0000	060	30
D	100 0100	104	44	1	011 0001	061	31
E	100 0101	105	45	2	011 0010	062	32
F	100 0110	106	46	3	011 0011	063	33
G	100 0111	107	47	4	011 0100	064	34
H	100 1000	110	48	5	011 0101	065	35
I	100 1001	111	49	6	011 0110	066	36
J	100 1010	112	4A	7	011 0111	067	37
K	100 1011	113	4B	8	011 1000	070	38
L	100 1100	114	4C	9	011 1001	071	39
M	100 1101	115	4D	blank	010 0000	040	20
N	100 1110	116	4E	.	010 1110	056	2E
O	100 1111	117	4F	(	010 1000	050	28
P	101 0000	120	50	+	010 1011	053	2B
Q	101 0001	121	51	\$	010 0100	044	24
R	101 0010	122	52	*	010 1010	052	2A

S	101 0011	123	53	)	010 1001	051	29
T	101 0100	124	54	—	010 1101	055	2D
U	101 0101	125	55	/	010 1111	057	2F
V	101 0110	126	56	,	010 1100	054	2C
W	101 0111	127	57	=	011 1101	075	3D
X	101 1000	130	58	<RETURN	000 1101	015	0D
				<LINEFEED>	000 1010	012	0A

**Example. 69.26.** The following is a message encoded in ASCII code. What is the message ?  
 1001000 1000101 1001100 1001100 1001111

**Solution.** Convert each seven-bit code to its equivalent hexadecimal number. The resulting values are:

48 45 4C 4C 4F

Now locate these hexadecimal values in Table No. 19.11 and determine the character represented by each. The results are:

“H E L L O”.

### Tutorial Problems No. 69.1

- Find the decimal equivalents of the following binary numbers:  
 (a) 101 (b) 1001 (c) 10.011 [(a) 510 (b) 910 (c) 3.037510]
- What are the decimal equivalents of the following binary numbers ?  
 (a) 1111 (b) 10100 (c) 11011001 (d) 10011001  
 [(a) 1510 (b) 2010 (c) 10910 (d) 15310]
- Express the following binary numbers into their equivalent decimal numbers :  
 (a) 11.01 (b) 101.11 (c) 110.01 [(a) 3.2510 (b) 5.7510 (c) 6.2510]
- Convert the following decimal numbers into their binary equivalents:  
 (a) 25 (b) 125 (c) 0.85  
 [(a) 110012 (b) 11111012 (c) 0.1101102]
- What are the binary equivalents of the following decimal numbers ?  
 (a) 27 (b) 92 (c) 64  
 [(a) 110112 (b) 10111002 (c) 10000002]
- Perform the following binary additions:  
 (a) 1011 + 1001 (b) 1011 + 110 (c) 101101 + 1101101 (d) 1011.01 + 1001.11  
 (e) 0.0011 + 0.1110 (f) 1111 + 111 + 1111  
 [(a) 101002 (b) 100012 (c) 11000112 (d) 101012 (e) 1.00012 (f) 11010012]
- Add the following binary numbers 1011 and 1001  
 [10100](Digital Computations, Punjab Univ. Dec.)
- Perform the following binary subtractions :  
 (a) 1101 – 1011 (b) 111 – 101 (c) 1000 – 11 (d) 101011 – 10010  
 [(a) 00102 (b) 0102 (c) 010012 (d) 0110012]
- Carry out the following subtractions using binary number system:  
 (a)  $64_{10} - 32_{10}$  (b)  $128_{10} - 64_{10}$  (c)  $93.5_{10} - 42.75_{10}$  (d)  $7_{10} - 11_{10}$   
 [(a) 1000002 (b) 10000002 (c) 110010.112 (d)  $\bar{A}1002$ ]
- Subtract the following binary numbers: 100011 – 111010  
 [–10111](Digital Computations, Punjab Univ., Dec. 1984)
- Find the 1’s complements of the following binary numbers:  
 (a) 01101 (b) 1101 (c) 1001 (d) 1010  
 [(a) 100102 (b) 00102 (c) 01102 (d) 01012]
- What are 2’s complements of the following binary numbers ?  
 (a) 1011 (b) 11011 (c) 11011.01 (d) 10011.11  
 [(a) 01012 (b) 001012 (c) 00100.112 (d) 01100.012]

13. Use the 1's and 2's complements of the following binary subtractions :  
 (i)  $1111 - 1011$  (ii)  $110011 - 100101$  (iii)  $100011 - 111010$   
 [(i) 0100 (ii) 1110 (iii) -10111] (*Digital Computations, Punjab Univ. Dec.*)
14. Multiply the following binary numbers:  
 (a)  $1100 \times 101$  (b)  $10101 \times 101$  (c)  $10111 \times 101$  (d)  $1110 \times 111$   
 [(a) 1111002 (b) 11010012 (c) 11100112 (d) 11000102]
15. Perform the following binary divisions :  
 (a)  $11011 \div 100$  (b)  $1110011 \div 101$  (c)  $1100010 \div 111$   
 [(a) 110.112 (b) 101112 (c) 11102]
16. Convert the following binary numbers into their octal equivalents :  
 (a) 1001 (b) 11011 (c) 10 101 111 (d) 1101.0110111  
 (e) 11 111 011 110 101  
 [(a) 118 (b) 338 (c) 258 (d) 15.3348 (e) 373658]
17. Convert the undergiven octal numbers into their binary equivalents:  
 (a) 13 (b) 11 (c) 713 (d) 3674  
 [(a) 0112 (b) 10012 (c) 111 001 0112 (d) 11 110 111 1002]
18. Convert the following numbers :  
 (a)  $357_8$  to decimal (b)  $6421_8$  to decimal (c)  $1359_{10}$  to octal (d)  $7777_{10}$  to octal  
 [(a) 23910 (b) 334510 (c) 25178 (d) 171418]
19. Convert the following real numbers to the binary numbers:  
 (i) 12.0 (ii) 25.0 (iii) 0.125  
 [(i) 1100 (ii) 11001 (iii) 0.001] (*Digital Computations, Punjab Univ.*)
20. Convert the following binary numbers into their equivalent hexadecimal numbers:  
 (a) 1101 0111 (b) 1010 0110 (c) 1001 1110 (d) 1100 1111  
 [(a) D716 (b) A616 (c) 9E16 (d) CF16]
21. Convert the following decimal numbers to binary numbers by converting them to octal, then to binary.  
 (i) 850 (ii) 7563  
 [(a) 1101010010 (ii) 11101100010111] (*Digital Computations, Punjab Univ. Dec.*)
22. Convert the following numbers to decimal  
 (i)  $(11010)_2$  (ii)  $(777)_8$   
 [(i) 26 (ii) 511] (*Digital Computations, Punjab Univ. June*)
23. What are the binary equivalents of the following hexadecimal numbers ?  
 (a) 9F (b) A2 (c) ED (d) 27  
 [(a) 1001 11112 (b) 1010 00102 (c) 1110 11012 (d) 0011 01112]
24. Convert the decimal number 4397 and the octal number 2735 to hexadecimal numbers.  
 [(i) 5DD (ii) 12D] (*Power & Digital Electronics, Punjab Univ. Dec. 1984*)
25. Convert the following decimal numbers into BCD code  
 (a) 18 (ii) 92 (iii) 321 and (iv) 4721  
 [(i) 0001 1000 (ii) 1001 0010 (iii) 0011 0010 0001 (iv) 0100011100100001]
26. Find the decimal values for the BCD-coded numbers  
 (i) 0110 1000 (ii) 0111 0100 1001 and (iii) 1000 0100 0111 0110  
 [(i) 68 (ii) 749 (iii) 8476]
27. Convert the following hexadecimal numbers into binary numbers:  
 (i) E 5 (ii) B4D (iii) 7AF4  
 (*Digital Computations, Punjab Univ. May*)  
 [(i) 11100101 (ii) 101101001101 (iii) 0111 1010 1111 0100]
28. Encode the following decimal numbers into 8421 numbers.  
 (i) 45 (ii) 732 (iii) 94,685  
 (*Digital Computations, Punjab Univ. May*)  
 [(i) 0100 0101 (ii) 111 0011 0010 (iii) 001 0100 0110 1000 0101]
29. Decode the following 8421 BCD numbers :  
 (i) 0011 1000 0111 (ii) 1001 0110 0111 1000 0111 0011  
 [(i) 387 (ii) 967,873] (*Digital Computations, Punjab Univ. Dec.*)



- 21.** Octal coding involves grouping the bits in  
(a) 5's                    (b) 7's  
(c) 4's                    (d) 3's.
- 22.** In Excess-3 code each coded number is .....than in BCD code.  
(a) four larger    (b) three smaller  
(c) three larger    (d) much larger.
- 23.** Which numbering system uses numbers and letters as symbols ?  
(a) decimal            (b) binary  
(c) octal                (d) hexadecimal
- 24.** To convert a whole decimal number into a hexadecimal equivalent, one should divide the decimal value by.....  
(a) 2                      (b) 8  
(c) 10                    (d) 16.

**ANSWERS**

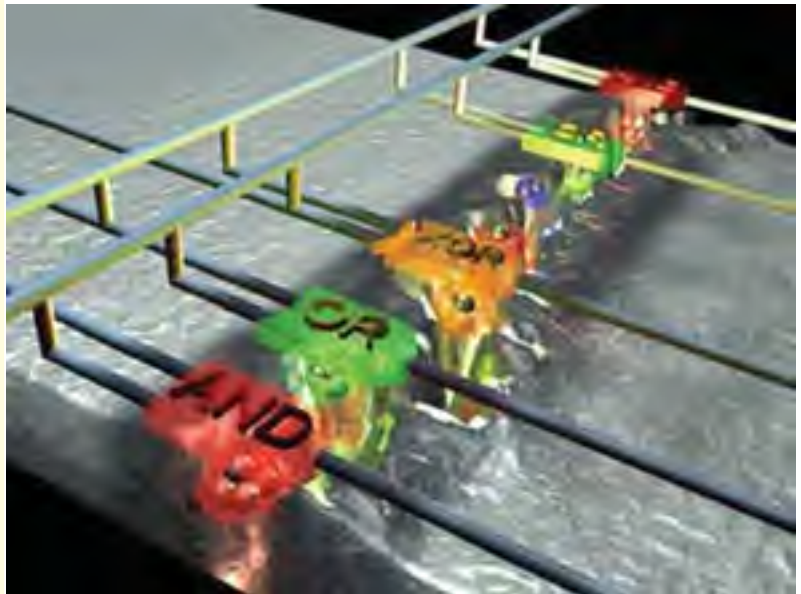
- 1. (a)    2. (a)    3. (b)    4. (b)    5. (a)    6. (d)    7. (b)    8. (d)    9. (c)    10. (a)**  
**11. (d)    12. (a)    13. (c)    14. (b)    15. (d)    16. (a)    17. (a)    18. (a)    19. (c)    20. (a)**  
**21. (d)    22. (c)    23. (d)    24. (a)**

# CHAPTER 70

## Learning Objectives

- Definition of a Logic Gate
- Positive and Negative Logic
- The OR Gate
- Equivalent Relay Circuit of an OR Gate
- Diode OR Gate
- Transistor OR Gate
- OR Gate Symbolizes Logic Addition
- Three Input OR Gate
- Exclusive OR Gate
- The AND Gate
- Equivalent Relay Circuit of an AND Gate
- Diode AND Gate
- Transistor AND Circuit
- AND Gate Symbolizes Logic Multiplication
- The NOT Gate
- The NOT Operation
- Bubbled Gates
- The NOR Gate
- The NAND Gate
- The XNOR Gate
- Logic Gate at Glance
- Digital Signals Applied to Logic Gates
- Sequential Logic Circuits
- Adders and Subtractors
- Half Adder
- Full Adder
- Parallel Binary Adder
- Half Subtractor
- Full Subtractor

## LOGIC GATES



↑ A logic gate is a circuit that has one or more input signals but only one output signal. All logic gates can be analysed by constructing a truth table

+

### 70.1. Definition of a Logic Gate

A logic gate is an electronic circuit **which makes logic decisions**. It has one output and one or more inputs. The output signal appears only for certain combinations of input signals. Logic gates are the basic building blocks from which most of the digital systems are built up. They implement the hardware logic function based on the logical algebra developed by George Boole which is called Boolean algebra in his honour. A unique characteristic of the Boolean algebra is that variables used in it **can assume only one of the two values** *i.e.* either 0 or 1. Hence, every variable is either a 0 or a 1.

These gates are available today in the form of various IC families. The most popular families are: transistor-transistor logic (*TTL*), emitter-coupled logic (*ECL*), metal-oxide-semiconductor (*MOS*) and complementary metal-oxide-semiconductor (*CMOS*).

In this chapter, we will consider the *OR*, *AND*, *NOT*, *NOR*, *NAND*, exclusive *OR* (*XOR*) and exclusive *NOR* (*XNOR*) gates along with their truth tables.

### 70.2. Positive and Negative Logic

In computing systems, the number symbols 0 and 1 represent two possible states of a circuit or device. It makes no difference if these two states are referred to as *ON* and *OFF*, *CLOSED* and *OPEN*, *HIGH* and *LOW PLUS* and *MINUS* or *TRUE* and *FALSE* depending on the circumstances. Main point is that **they must be symbolized by two opposite conditions**.

In positive logic, a 1 represents

1. an *ON* circuit
2. a *CLOSED* switch
3. a *HIGH* voltage
4. a *PLUS* sign
5. a *TRUE* statement

Consequently, a 0 represents

1. an *OFF* circuit
2. an *OPEN* switch
3. a *LOW* voltage
4. a *MINUS* sign
5. a *FALSE* statement.

In negative logic, just opposite conditions prevail.

Suppose, a digital system has two voltage levels of 0V and 5V. If we say that symbol 1 stands for 5V and symbol 0 for 0V, then we have positive logic system. If, on other hand, we decide that a 1 should represent 0V and 0 should represent 5V, then we will get negative logic system.

Main point is that in **positive logic, the more positive** of the two voltage levels represents the 1 while in negative logic, **the more negative** voltage represents the 1. Moreover, it is not essential that a 0 has to be represented by 0V although in some cases the two may coincide. Suppose, in a circuit, the two voltage levels are 2V and 10V. Then for positive logic, the 1 represents 10V and the 0 represents 2V (*i.e.* lesser of the two voltages). If the voltage levels are –2V and –8V, then, in positive logic, the 1 represents –2V and the 0 represents –8V (*i.e.* lesser of the two voltages).

Unless stated otherwise, we will be using only **positive logic** in this chapter.

### 70.3. The OR Gate

The electronic symbol for a two-input *OR* gate is shown in Fig. 70.1 (a) and its equivalent switching circuit in Fig. 70.1 (b). The two inputs have been marked as *A* and *B* and the output as *X*. It is worth reminding the reader that as per Boolean algebra, the three variables *A*, *B* and *X* can have only one of the two values *i.e.* either 0 or 1.

#### Logic Operation

The *OR* gate has an **output of 1 when either A or B or both are 1**.

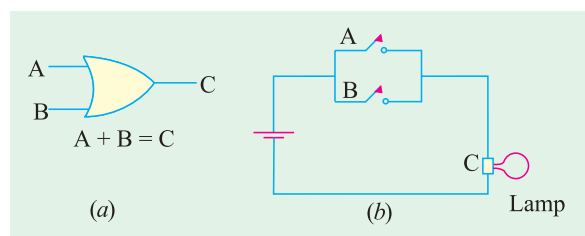


Fig. 70.1

In other words, it is an **any-or-all** gate because an output occurs when any or all the inputs are present.

As seen from Fig. 70.1 (b), the lamp will light up (logic 1) when either switch A or B or both are closed.

Obviously, the output would be 0 **if and only if both its inputs are 0**. In terms of the switching conditions, it means that lamp would be OFF (logic 0) only when both switches A and B are OFF.

The OR gate represents the Boolean equation  $A + B = X$

The meaning of this equation is that X is true when either A is true or B is true or both are true. Alternatively, it means that output X is 1 when either A or B or both are 1.

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Fig. 70.2

The above logic operation of the OR gate can be summarised with the help of the truth table given in Fig. 70.2. A truth table may be defined as a table which **gives the output state for all possible input combinations**. The OR Table 70.1 gives outputs for all possible AB inputs of 00, 01, 10 and 11.

We may interpret the truth table as follows:

When both inputs are 0 (switches are OPEN), output X is 0 (lamp is OFF). When A is in logic state 0 (switch A is OPEN) but B is in logic state 1 (switch B is CLOSED), the output X is logic state 1 (lamp is ON). Lamp would be also ON when A is CLOSED and B is OPEN. Of course, lamp would be ON when both switched are CLOSED. It is so because an OR gate is equivalent to a **parallel circuit in its logic function**.

Another point worth remembering is that the above OR gate is called **inclusive OR** gate because **it includes the case when both inputs are true**.

### 70.4. Equivalent Relay Circuit of an OR Gate

In Fig. 70.3, the relay contacts have been wired **in parallel**. When +5V is applied to A, relay  $K_1$  is energised and pulls M down thereby closing the contact. Hence, supply voltage of +5V appears at the output X.

Similarly, when +5V are applied to input B,  $K_2$  is energised and pulls N down thereby bringing X in contact with S. Of course, when both A and B are at +5V, X is at +5V. Incidentally, when inputs at A and B are 0, X is also 0.

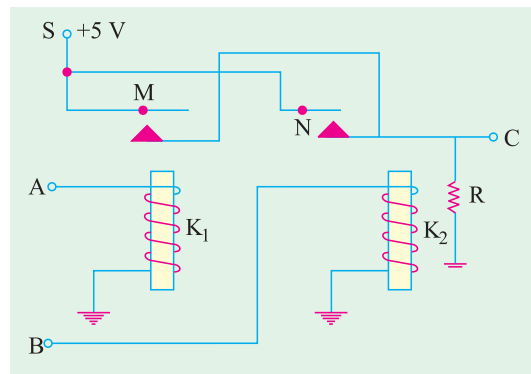


Fig. 70.3

### 70.5. Diode OR Gate

Fig. 70.4. shows the diode OR gate consisting of two ideal diodes  $D_1$  and  $D_2$  connected in parallel across the output X.

1. When A is at +5V,  $D_1$  is forward-biased and hence conducts. The circuit current flows via R dropping 5V across it. In this way, point X achieves potential of +5V.
2. When +5V is applied to B,  $D_2$  conducts causing point X to go to +5V.
3. When both A and B are +5V, the drop across R is 5V because voltages of A and B **are in parallel**.

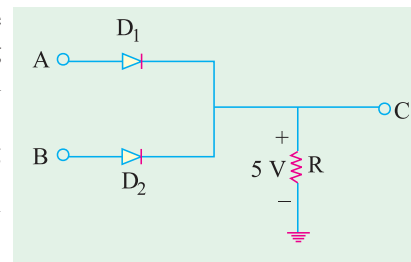


Fig. 70.4



Again, point X is driven to +5 V.

- Obviously, when there is no voltage either at A or B, output X remains 0.

### 70.6. Transistor OR Gate

Fig. 70.5 illustrates a possible transistor OR gate consisting of three interconnected transistors  $Q_1$ ,  $Q_2$ , and  $Q_3$  supplied from a common supply  $V_{cc} = +5$  V.

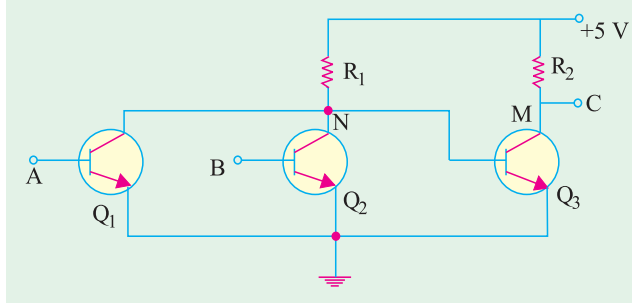


Fig. 70.5

- When +5 V is applied to A,  $Q_1$  is forward-biased and so it conducts. Assuming that  $Q_1$  is saturated, entire  $V_{cc} = 5$  V drops across  $R_1$  thus causing N to go to ground. This, in turn, cuts off  $Q_3$  thereby causing X to go to  $V_{cc}$  i.e. +5V.

- When +5 V is applied to B,  $Q_2$  conducts thereby driving N to ground i.e. 0V. With no forward bias on its base,  $Q_3$  is cut-off thus driving X again to  $V_{cc}$  i.e. +5 V.

- If both inputs A and B are grounded,  $Q_1$  and  $Q_2$  are cut-off driving N to +5 V. As a result,  $Q_3$  becomes forward-biased and conducts fully. In that case, entire  $V_{cc}$  drops across  $R_2$  driving M and hence X to ground.

### 70.7. OR Gate Symbolizes Logic Addition

According to Boolean algebra, OR gate performs **logical addition**. Its truth table can be written as given below:

It must be clearly understood that '+' sign in Boolean algebra **does not stand for the addition** as understood in the ordinary or numerical algebra. In symbolic logic, the '+' sign indicates **OR** operation whose rules are given above. In logic algebra,  $A + B = X$  means that if A is true **OR** B is true, then X will be true. **It does not mean here that sum of A and B equals X.** The other symbols used for '+' sign are U and V. Hence, the above equation could also be written as  $AUB = X$  or  $AVB = X$ .

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

The meaning of the last three logic additions is that output is 1 when either input A or B or both are 1. The first addition implies that output is 0 **only when both inputs are 0.**

The meaning of the '+' sign often becomes clear from the context as shown below:

- $1 + 1 = 2$  — decimal addition
- $1 + 1 = 10$  — binary addition
- $1 + 1 = 1$  — OR addition

We can put the above OR laws in more general terms

- $A + 1 = 1$
- $A + 0 = A$
- $A + A = A$  — not 2A

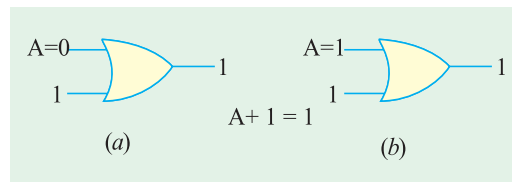


Fig. 70.6

**(i)  $A + 1 = 1$**

As we know, A can have two values: 0 or 1. When A is 0, then we have  $0 + 1 = 1$  as shown in Fig. 70.6 (a).

When A = 1, then the above expression becomes :  $1 + 1 = 1$  as shown in Fig. 70.6 (b), Hence, we find that **irrespective of the value of A.**

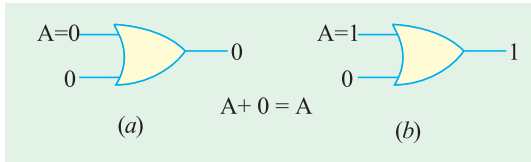


Fig. 70.7

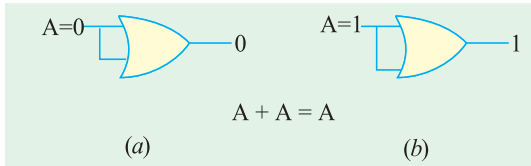


Fig. 70.8

$A + 1 = 1$

(ii)  $A + 0 = A$

If  $A = 0$ , then  $0 + 0 = 0$  i.e. output is 0 which is correct and is shown in Fig. 70.7 (a). The output is what the value of  $A$  is.

As shown in Fig. 70.7 (b), when  $A = 1$ , output is 1 because  $1 + 0 = 1$ . Again, output is what the value of  $A$  is.

(iii)  $A + A = A$

With  $A$  set to 0, the output is 0 because  $0 + 0 = 0$  as shown in Fig. 70.8 (a).

With  $A$  set to 1, the output is 1 because  $1 + 1 = 1$  as shown in Fig. 70.8 (b). Obviously, the output in both cases is  $A$ .

### 70.8. Three Input OR Gate

The electronic symbol for a 3-input (fan-in of 3) **inclusive OR** gate is shown in Fig. 70.9. As is usual in logic algebra, the inputs  $A, B, C$  as well as the output  $X$  can have only one of the two values i.e. 0 or 1.

**Truth Table**

It is shown in Table 70.2. Following points are worth noting:

1. The number of rows in the table is  $2^3 = 8$  i.e. there are eight ways of combining the three inputs. In general, the number of horizontal rows is  $2^n$  where  $n$  is the number of inputs.
2. In first column  $A$ , logic values alternate between 0 and 1 every four rows twice.
3. The second input column  $B$  alternates between 0 and 1 every two rows four times.
4. The third input column  $C$  alternates between 0 and 1 every other row eight times.

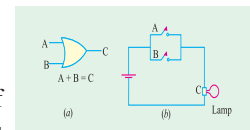


Fig. 70.9

The truth table symbolizes the Boolean equation  $A + B + C = X$  which means that output  $X$  is 1 when **either  $A$  or  $B$  or  $C$  is 1 or all are 1**. Alternatively,  $X$  is true when either  $A$  or  $B$  or  $C$  is true or all are true.

**vvhh**

Its electronic symbol is shown in Fig. 70.10 (a) and its equivalent switching circuit in Fig. 70.10 (b).

In this gate, output is 1 if its either input **but not both**, is 1. In other words, it has an output 1 **when its inputs are different**.

The output is 0 only when inputs **are the same**.

To put it a bit differently, this logic gate has output 0 **when inputs are either all 0 or all 1**.

Table No. 70.2

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

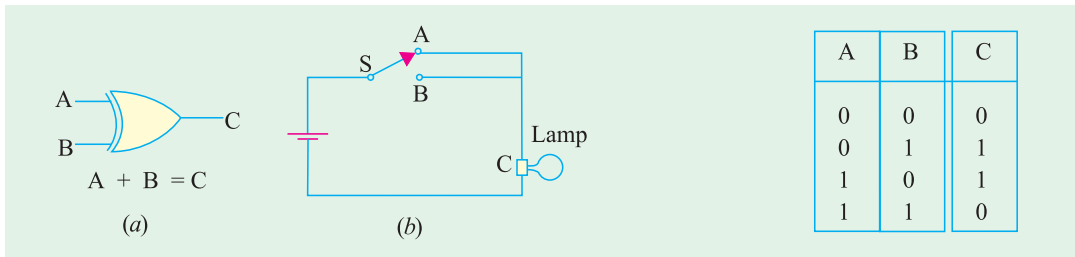


Fig. 70.10

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

Fig. 70.11

This gate works on the Boolean equation  $A \oplus B = X$

The circle around plus (+) sign is worth noting.

The circuit is also called an *inequality comparator* or detector because it produces an output only **when the two inputs are different**.

**Explanation**

The *inclusive OR* gate exemplifies the everyday usage of the word *OR* which stands for one or the other or both. Take the following statement:

*To qualify for a competition, you might have to subscribe to a magazine OR belong to a club.* Obviously, there is no bar on your doing both. But now take *exclusive* statement:

*You can be rich OR you can be poor.*

Obviously, you cannot be both at the same time.

The change-over switching *circuit* of Fig. 70.10 (b) simulates the *exclusive OR (XOR)* gate. Switch positions A and B will individually light up the lamp but a combination of A and B is not possible.

The truth table for a 2-input *XOR* gate is given in Table No.70.3. It is instructive to compare it with that for an *inclusive OR* gate (Table 70.1).

**70.10. The AND Gate**

The electronic (or logic) symbol for a 2-input *AND* gate is shown in Fig. 70.12 (a) and its equivalent switching circuit in Fig. 70.12 (b). It is worth reminding the readers once again that the three variables A, B, C can have a **value of either 0 or 1**.

**Logic Operation**

1. The *AND* gate gives an output only **when all its inputs are present**.
2. The *AND* gate has a 1 output when both A **and** B are 1. Hence, this gate is an **all-or-nothing** gate whose output occurs only when all its inputs are present.
3. In True/False terminology, the output of an *AND* gate will be **true** only if **all its inputs are true**. Its output would be false if **any of its inputs is false**.

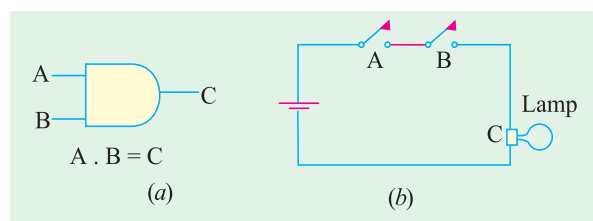


Fig. 70.12

The *AND* gate works on the Boolean algebra

$$A \times B = X \text{ or } A . B = X \text{ or } AB = X$$

It is a **logical** multiplication and is different from the **arithmetic** multiplication. Often the sign ‘×’ is replaced by a dot which itself is generally omitted as shown above. The logical meaning of the above equation is that

1. output  $X$  is 1 only when both  $A$  and  $B$  are 1.
2. output  $X$  is true only when both  $A$  and  $B$  are true.

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Fig. 70.13

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

ABC = X  
Fig. 70.14

As seen from Fig. 70.12 (b), the lamp would be ON when both switches  $A$  and  $B$  are closed. Even when one switch is open, the lamp would be OFF. Obviously, an AND gate is equivalent to a series switching circuit.

**Truth Table** Fig. 70.13 shows truth table for a 2-input AND gate and Fig. 70.14 gives the same for a 3-input AND gate.

As seen,  $X$  is at logic 1 only when all inputs are at logic 1, not otherwise. The procedure for writing down the first three columns is the same as explained in Art. 70.8 earlier.

### 70.11. Equivalent Relay Circuit of an AND Gate

The AND gate can be physically realized with the help of relay circuit shown in Fig. 70.15. Here, the two relay contacts have been wired in series.

When +5 V is applied to both input circuits, relays  $K_1$  and  $K_2$  are energized thereby pulling  $M$  and  $N$  downwards which brings  $C$  in contact with the supply point  $S$ . Hence,  $X$  goes to +5 V.

It is obvious that energizing only one relay will not make  $X$  go to +5 V.

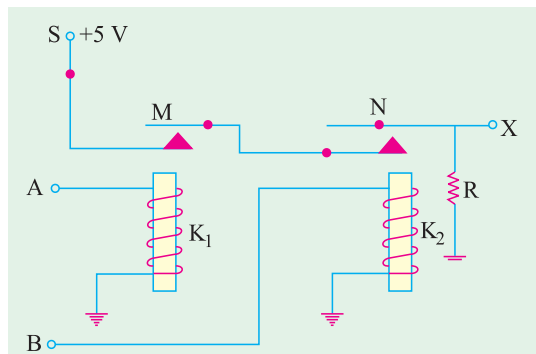


Fig. 70.15

### 70.12. Diode AND Gate

It is shown in Fig. 70.16. Its logical operation is as under :

1. When  $A$  is at 0 V, diode  $D_1$  conducts and the supply voltage of +5 V drops across  $R$ . Consequently, point  $N$  and hence point  $X$  are driven to 0 V. Therefore, the output  $C$  is 0.
2. Similarly, when  $B$  is at 0 V,  $D_2$  conducts thereby driving  $N$  and hence  $X$  to ground.
3. Obviously, when both  $A$  and  $B$  are at 0 V, both diodes conduct and, again, the output  $X$  is 0.
4. There is no supply current and hence no drop across  $R$  only when both  $A$  and  $B$  are at +5 V. Only in that case, the output  $X$  goes to supply voltage of +5 V.

### 70.13. Transistor AND Circuit

It is shown in Fig. 70.17. When both  $A$  and  $B$  are at +5 V, the two transistors  $Q_1$  and  $Q_2$  conduct. The current so produced drops the supply voltage of +5 V across  $R_1$  thereby driving point  $N$  and hence base of  $Q_3$  to ground or 0V. This cuts off  $Q_3$  so that  $X$  goes to supply voltage of +5 V.

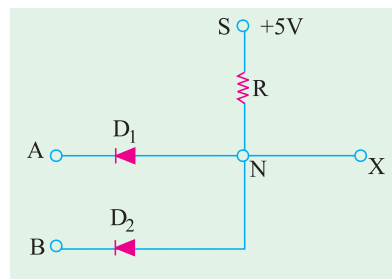


Fig. 70.16

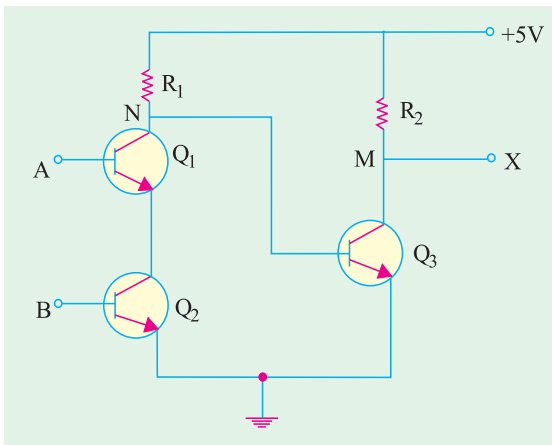


Fig. 70.17

Obviously, there is an output at  $X$  only when there is an input at  $A$  and  $B$ .

If either  $A$  or  $B$  is at  $0\text{ V}$ , then  $Q_1$  or  $Q_2$  will be cut off and no drop will take place across  $R_1$ . Hence, point  $N$  will go to supply voltage of  $+5\text{ V}$ . Consequently,  $Q_3$  will conduct and whole of supply voltage will be dropped across  $R_2$ . As a result, point  $M$  and hence output  $X$  will go to  $0\text{ V}$ .

### 70.14. AND Gate Symbolizes Logic Multiplication

According to Boolean algebra, the *AND* gate performs logical multiplication on its inputs as given below:

- $0.0 = 0$
- $0.1 = 0$
- $1.0 = 0$
- $1.1 = 1$

In general, we can put the laws of Boolean multiplication in the following form:

$$A.1 = A \qquad A.0 = 0$$

$$A.A = A \qquad \text{--- not } A^2$$

The above identities can be verified by giving values of  $0$  and  $1$  to  $A$ .

1.  **$A.1 = A$**       When  $A = 0$   
 then  $0.1 = 0$       —Fig. 70.18 (a)  
 When  $A = 1$   
 then  $1.1 = 1$       —Fig. 70.18 (b)

It is seen that in each case, output has the same value as that of  $A$ .

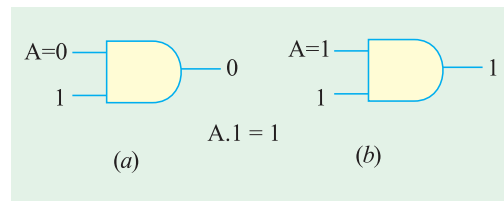


Fig. 70.18

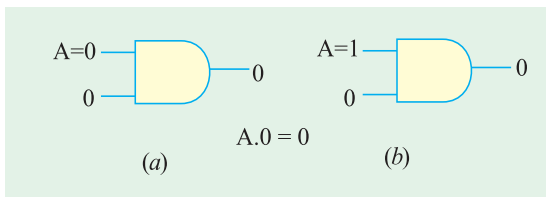


Fig. 70.19

2.  **$A.0 = 0$**   
 When  $A = 0$   
 then  $0.0 = 0$       —Fig. 70.19 (a)  
 When  $A = 1$   
 then  $1.0 = 0$       —Fig. 70.19 (b)

It is seen that output is always  $0$  *whatever the value of  $A$ .*

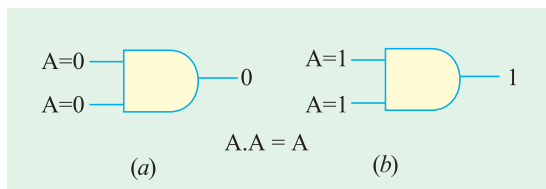


Fig. 70.20

3.  **$A.A = A$**   
 When  $A = 0$ , then  $0.0 = 0$  — Fig 70.20 (a)  
 When  $A = 1$ , then  $1.1 = 1$  — Fig. 70.20 (b)

It is seen that output always *takes on the value of  $A$ .*

### 70.15. The NOT Gate

It is so called because *its output is NOT the same as its input*. It is also called an *inverter* because it inverts the input signal. It has **one** input and **one** output as shown in Fig. 70.21 (a). All it does is to invert (or complement) the input as seen from its truth table of Fig. 70.21 (b).

The schematic symbol for inversion is a small circle as shown in Fig. 70.21 (a). The logical symbol for inversion or negation or complementation is a bar over the function to indicate the opposite state.

Sometimes, a prime is also used as  $A'$ . For ex-

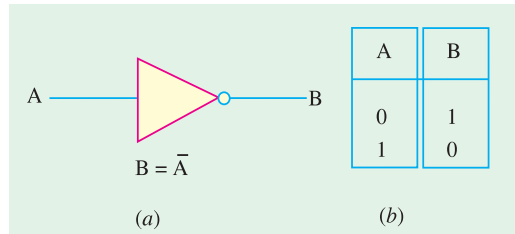


Fig. 70.21

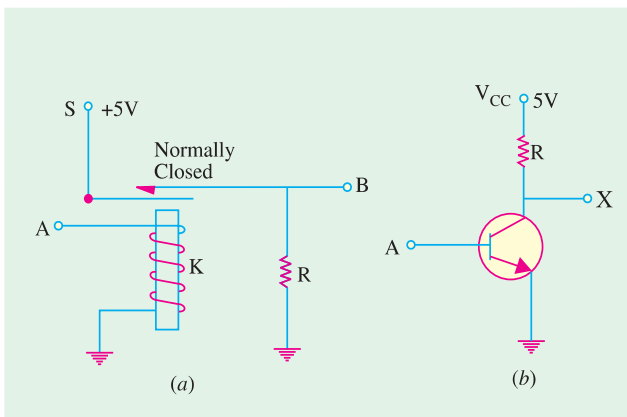


Fig. 70.22

ample,  $\bar{A}$  means not-A. Similarly,  $\overline{(A + B)}$  means the complement of (A + B).

### 70.16. Equivalent Circuits for a NOT Gate

The relay circuit of Fig. 70.22 (a) is a physical realization of the complementation operation of the Boolean algebra. When +5 V is applied to input A, K is energised and opens the *normally-closed* contact thereby driving output X to 0 V. Of course, when A is at 0V, X

has the supply voltage of +5 V applied to it because the relay contact is normally closed.

In the equivalent transistor circuit of Fig. 70.22 (b) when +5V is applied to A, the transistor will be fully turned ON, drawing maximum collector current. Hence, whole of  $V_{CC} = 5$  V will drop across R thereby sending X to 0 V. With 0 V applied at A, the transistor will be cut OFF and the output X, therefore, will go to  $V_{CC}$  i.e. +5 V. Obviously, in each case, **output is the opposite of input**.

### 70.17. The NOT Operation

It is a *complementation* operation and its symbol is an **overbar**. It can be defined as under:

As stated earlier,  $\bar{0}$  means taking the negation or complement of 0 which is 1.

$$\begin{aligned} \bar{0} &= 1 \\ \bar{1} &= 0 \end{aligned}$$

It should also be noted that complement of a value can be taken repeatedly. For example,

$$\bar{\bar{1}} = \bar{0} = 1 \quad \text{or} \quad \bar{\bar{0}} = \bar{1} = 0$$

As seen double complementation gives the original value as shown in Fig. 70.23.

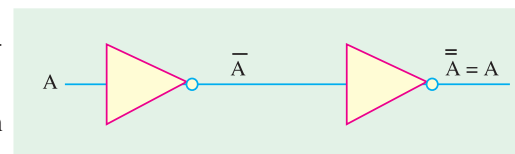


Fig. 70.23

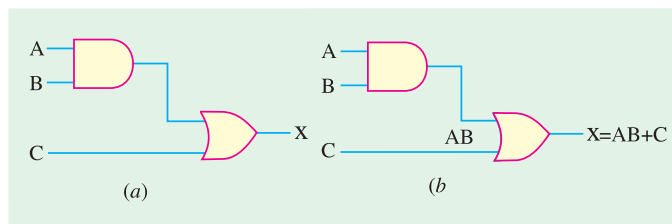


Fig. 70.24

**Example 70.1.** Find the Boolean equation for the output  $X$  of Fig. 70.24 (a). Evaluate  $X$  when (i)  $A = 0, B = 1, C = 1$  (ii)  $A = 1, B = 1, C = 1$ . [Computer Engg., Pune Univ. 1991]

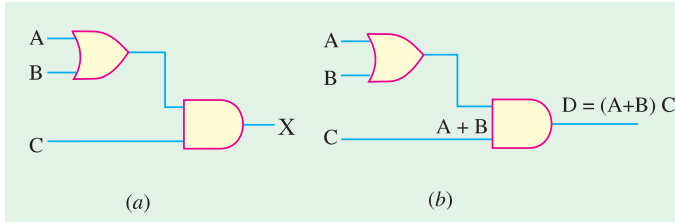


Fig. 70.25

**Solution.** The output of the AND gate is  $AB$ . It then becomes one of the inputs for the 2 input OR gate. When  $AB$  is ORed with  $C$ , we get  $(AB+C)$ .

$$\therefore X = AB + C$$

—Fig. 70.24 (b)

(i)  $X = 0.1 + 1 = 0 + 1 = 1$

—Art 70.7

(ii)  $X = 1.1 + 1 = 1 + 1 = 1$

**Example 70.2.** Find the Boolean expression for the output of Fig. 70.25 (a) and evaluate it when (i)  $A = 0, B = 1, C = 1$ , (ii)  $A = 1, B = 1, C = 0$ .

**Solution.** The output of the OR gate is  $(A + B)$ . Afterwards, it becomes the input of the AND gate. When ANDed with  $C$ , it becomes  $(A + B).C$ .

$$\therefore X = (A + B).C \quad \text{—Fig. 70.25 (b)}$$

(i)  $X = (0 + 1).1 = 1.1 = 1$

(ii)  $X = (1 + 1).0 = 1.0 = 0$

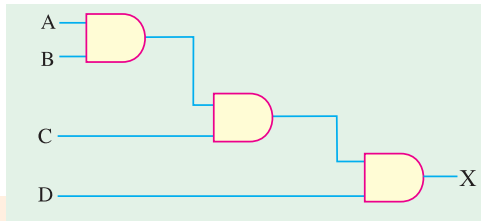


Fig. 70.26

**Example 70.3.** Find the Boolean expression for the output of Fig. 70.26 and compute its value when  $A = B = C = 1$  and  $X = 0$ .

(Digital Computations, Punjab Univ. 1990)

**Solution.** The circuit is made up of three AND gates. Obviously, it is equivalent to a single 4-input AND gate i.e. an AND gate with a fan-in of four.

Output of the first gate is  $AB$ , that of the second is  $ABC$  and that of the third is  $ABCD$ . Hence, final output is  $X = ABCD$ .

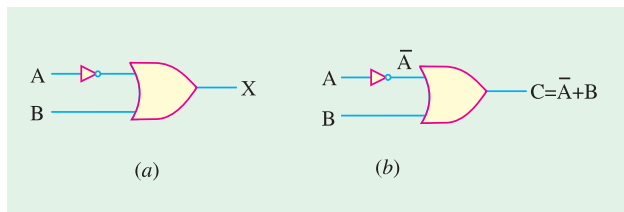


Fig. 70.27

Substituting the given values, we get

$$X = 1.1.1.0 = 1.1.0 = 1.0 = 0$$

**Example 70.4.** Find the Boolean expression for the output  $X$  of Fig. 70.27 (a) and compute its value when

(i)  $A = 0, B = 1$

(ii)  $A = 1, B = 0$

**Solution.** As seen, one of the inputs to the OR gate is inverted i.e.  $A$  becomes  $\bar{A}$  as shown in Fig. 70.27 (b). Hence, output is given by  $X = \bar{A} + B$

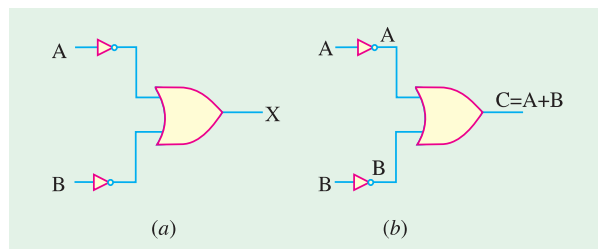


Fig. 70.28

(i)  $X = \bar{0} + 1 = 1 + 1 = 1$  (ii)  $X = \bar{1} + 0 = 0 + 0 = 0$

**Example 70.5.** What is the Boolean expression for the output X of Fig. 70.28 (a) ? Compute the value of X when

(i)  $A = 0, B = 0$  (ii)  $A = 1, B = 1$

**Solution.** As seen, in this case, both inputs to the OR gate have been inverted. Hence as shown in Fig. 70.28 (b), the inputs become  $\bar{A}$  and  $\bar{B}$ . Therefore, Boolean expression for the output becomes  $X = \bar{A} + \bar{B}$ .

(i)  $X = \bar{0} + \bar{0} = 1 + 1 = 1$

(ii)  $X = \bar{1} + \bar{1} = 0 + 0 = 0$

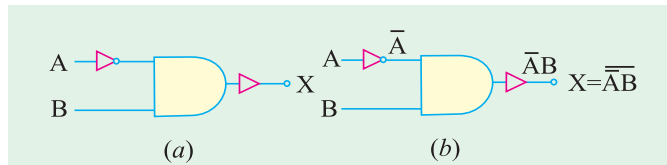


Fig. 70.29

**Example 70.6.** Write down the Boolean equation for the output X of Fig. 70.29 (a). Compute its value when

(i)  $A = 0, B = 0$

(ii)  $A = 0, B = 1$

(iii)  $A = 1, B = 0$

(iv)  $A = 1, B = 1$

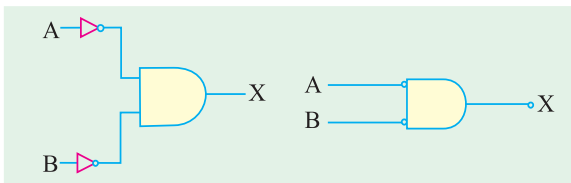


Fig. 70.30

**Solution.** As seen, inputs to the AND gate are A and B [Fig. 70.29 (a)]. The output of the AND gate is  $A.B$ . However, this output is inverted by the second inverter connected in the output. Hence, final output equation is

$X = \overline{A.B}$

(i)  $X = \overline{0.0} = \overline{1.0} = \bar{0} = 1$

(ii)  $X = \overline{0.1} = \overline{1.1} = \bar{1} = 0$

(iii)  $X = \overline{1.0} = \overline{0.0} = \bar{0} = 1$

(iv)  $X = \overline{1.1} = \overline{0.1} = \bar{0} = 1$

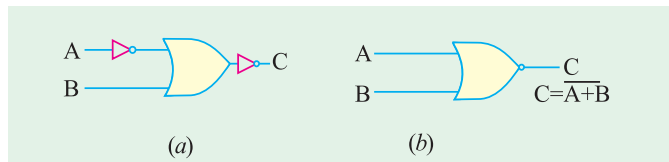


Fig. 70.31

While evaluating expressions of the above type, you must remember the following two points :

1. take the NOT i.e. inversion of the individual term first.
2. When a NOT or inversion is applied to more than one term (like 1.0), you should work out the OR (or AND) operation first and then take the NOT of the result so obtained.

### 70.18. Bubbled Gates

A bubbled gate is one whose inputs are NOTed or inverted i.e. it is a negated gate. Fig. 70.30 (a) shows an AND gate whose both input are inverted.

In practice, instead of this logic symbol, the one shown in Fig. 70.30 (b) is widely used. As seen, the inverter triangles have been eliminated and the two small circles or bubbles have been moved to the inputs of the gate. Such a gate is called a **bubbled AND** gate, the bubbles acting as a reminder of the inversion or complementation that takes place before ANDing the inputs.

It would be shown later that a bubbled AND gate is equivalent to a NOR gate.

Table 70.6

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Fig. 70.32



Similarly, a bubbled OR gate is equivalent to a AND gate.

### 70.19. The NOR Gate

In fact, it is a NOT-OR gate. It can be made out of an OR gate by connecting an inverter in its output as shown in Fig. 70.31 (a).

The output equation is given by

$$X = \overline{A + B}$$

A NOR function is just the reverse of the OR function.

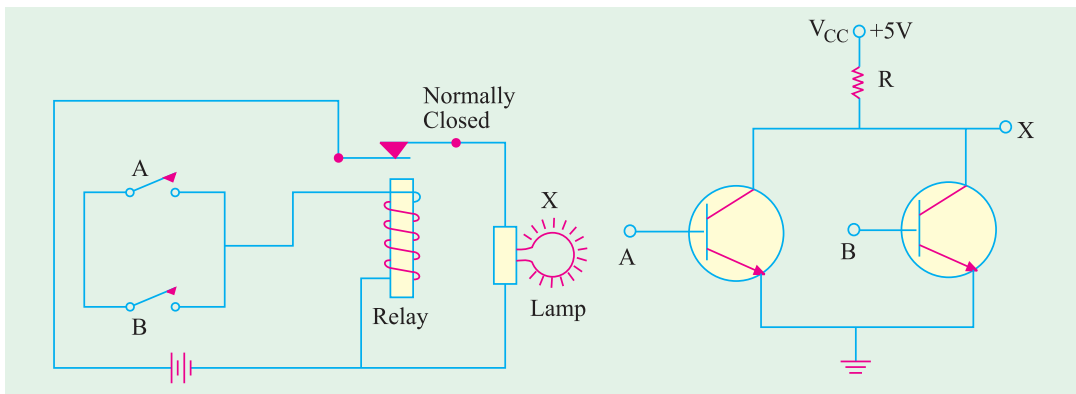


Fig. 70.33

Fig. 70.34

#### Logic Operation

A NOR gate will have an output of 1 **only when all its inputs are 0**. Obviously, if any input is 1, the output will be 0. Alternatively, in a NOR gate, output is **true only when all inputs are false**.

The truth table for a 2-input NOR gate is shown in Fig. 70.32. It will be observed that the output X is just the reverse of that shown in Fig. 70.2.

The equivalent relay circuit for a NOR gate is shown in Fig. 70.33.

It is seen that the lamp glows under 00 input condition only but not under 01, 10, 11 input conditions.

The transistor equivalent of the NOR gate is shown in Fig. 70.34. As seen, output X is 1 only when both transistors are cut-off *i.e.* when  $A = 0$  and  $B = 0$ . For any other input condition like 01, 10 and 11, one or both transistors saturate forcing point X to go to ground.

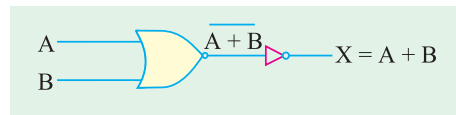


Fig. 70.35

### 70.20. NOR Gate is a Universal Gate

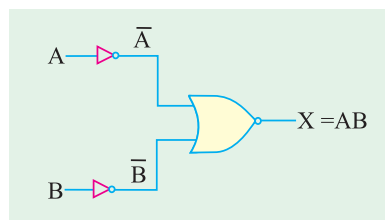


Fig. 70.36

It is interesting to note that a NOR gate can be used to realize the basic logic functions : OR, AND and NOT. That is why it is often referred to as a **universal** gate. Examples are given below:

#### 1. As OR Gate

As shown in Fig. 70.35, the output from NOR gate is  $A + B$ . By using another inverter in the output, the final output is inverted and is given by  $X = \overline{A + B}$  which is the logic function of a normal OR gate.

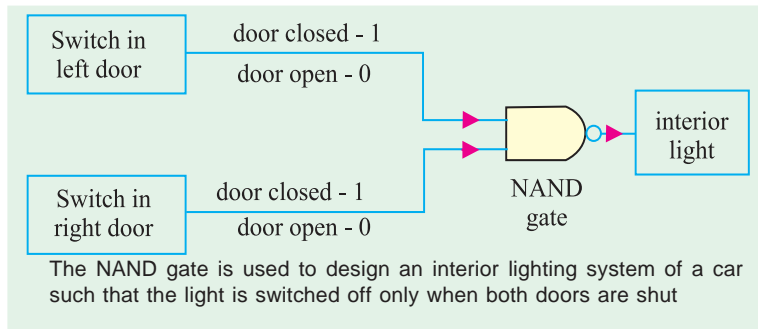
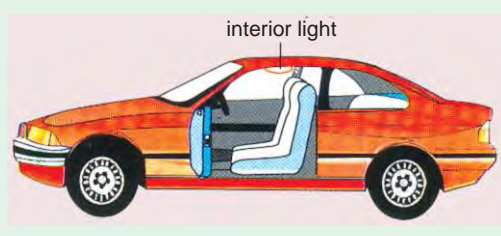
**2. As AND Gate**

Here, two inverters have been used, one for each input (Fig. 70.36). The inputs have, thus, been **inverted before they are applied** to the NOR gate.

The output is  $\bar{A} + \bar{B}$  which can be proved (with the help of De Morgan's theorem) to be equal to  $AB$ .

Incidentally, we could have used a bubbled NOR gate for the above purpose.

**3. As NOT Gate**



The two inputs have been tied together as shown in Fig. 70.37 (a). The output is  $\bar{A} + \bar{A}$  which can be proved to be equal to  $A$  with the help of De Morgan's theorem. Instead of the first symbol, the second symbol shown in Fig.

70.37 (b) is widely used where only single input has been used.

Here is a different way of making OR and AND gates. Fig. 70.38 (a) shows how we can use NOR gates to produce an OR gate. Similarly, Fig. 70.38 (b) shows the formation of an AND gate from three NOR gates. Knowledge of De Morgan's theorem is needed to understand their logic operation.

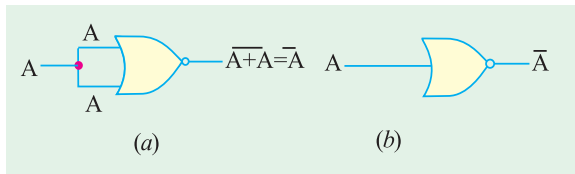


Fig. 70.37

**70.21. The NAND Gate**

It is, in fact, a NOT-AND gate. It can be obtained by connecting a NOT gate in the output of an AND gate as shown in Fig. 70.39. Its output is given by the Boolean equation.

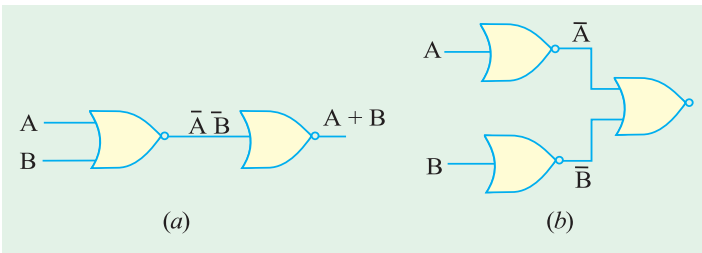


Fig. 70.38

This gate gives an output of 1 if its **both inputs are not 1**. In other words, it gives an output 1 if **either A or B or both are 0**.

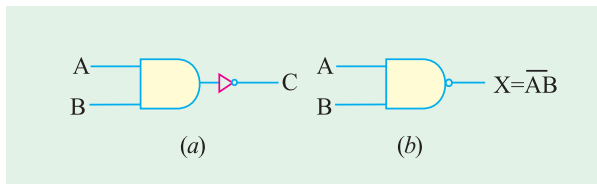


Fig. 70.39

The truth table for a 2-input NAND gate is given in Fig. 70.40. It is just the opposite of the truth for AND gate. It is so because NAND gate performs reverse function of an AND gate.

The equivalent relay circuit of a NAND gate is shown in Fig. 70.41.

It is seen that lamp glows under input conditions of 00, 01, 10 but not under 11 input condition

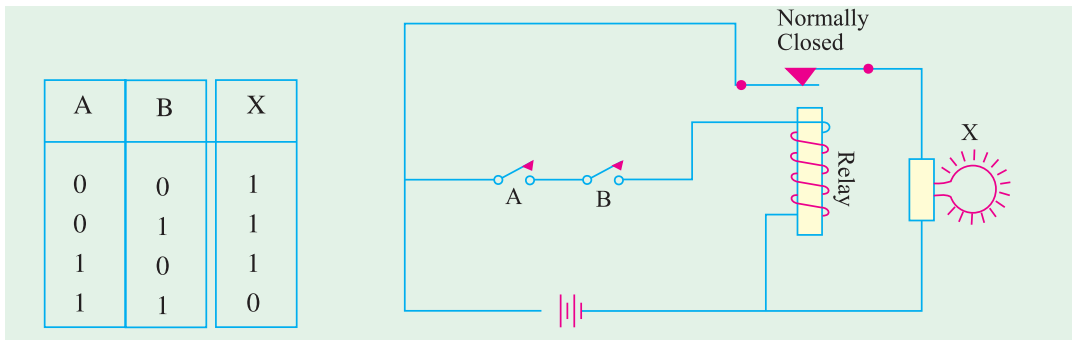


Fig. 70.40

Fig. 70.41

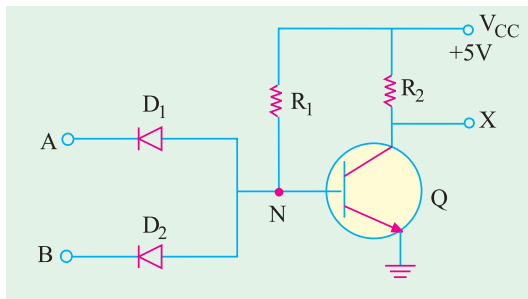


Fig. 70.42

when both switches *A* and *B* are *ON*. The diode-transistor equivalent of a *NAND* gate is shown in Fig. 70.42.

It is seen that point *N* would be driven to ground when either *D*<sub>1</sub> or *D*<sub>2</sub> or both *D*<sub>1</sub> and *D*<sub>2</sub> conduct. It represents input conditions of 10, 01 and 11\*. Under such conditions, *Q* is cut off and hence *X* goes to *V*<sub>CC</sub> meaning logic 1 state. Only time *X* is 0 is when *A* = 1 and *B* = 1 (i.e. input voltages at *A* and *B* are at +5V) so that *N* is +5 V and *Q* is saturated.

**70.22. NAND Gate is a Universal Gate**

*NAND* gate is also called universal gate because it can perform all the three logic functions of an *OR* gate, *AND* gate and inverter as shown below.

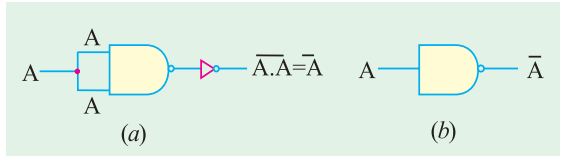


Fig. 70.43

As shown in Fig. 70.43 (a), a *NOT* gate can be made out of a *NAND* gate by connecting its two inputs together. When a *NAND* gate is used as a *NOT* gate, the logic symbol of Fig. 70.43 (b) is employed instead.

The use of two *NAND* gates to produce an *AND* gate is shown in Fig. 70.44 (a).

Similarly, Fig. 70.44 (b) shows how *OR* gate can be made out of three *NAND* gates. The *OR* function may not be clear from the figure because we need De Morgan's theorem to prove that  $\overline{\overline{A} \overline{B}} = A + B$ .

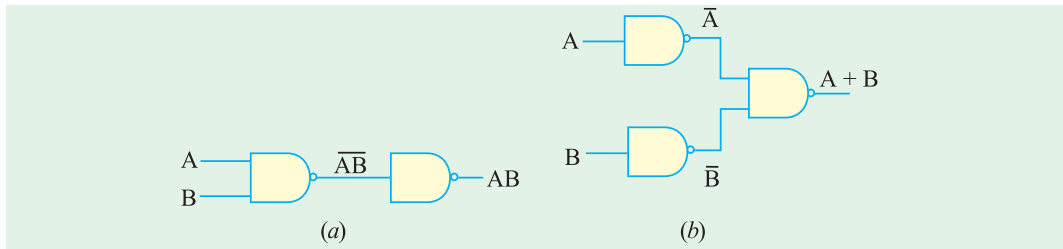


Fig. 70.44

\* In this case,  $V_A = V_B = 0 V$

### 70.23. The XNOR Gate

It is known as a not-*XOR* gate i.e.  $\overline{XOR}$  gate. Its logic symbol and truth table are shown in Fig. 70.45.

Its logic function and truth table are *just the reverse of those for XOR gate* (Art 70.9).

This gate has an output 1 if **its both inputs are either 0 or 1**. In other words, for getting an output, its both inputs should be **at the same logic level** of either 0 or 1. Obviously, it produces **no** output if its two inputs are at the **opposite** logic level.

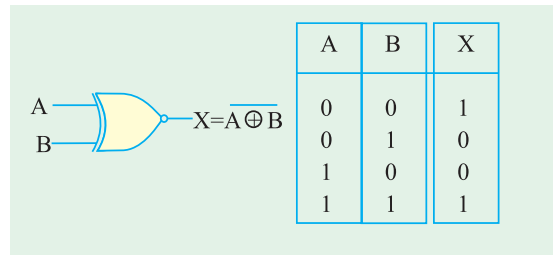


Fig. 70.45

### 70.24. Logic Gates at a Glance

In Fig. 70.46 is shown the summary of all the 2-output logic gates considered so far along with their truth tables.

Following points should prove helpful when writing these truth tables:

1. In first column *A*, logic values alternate between 0 and 1 every two rows
2. In second column *B*, logic values alternate every other row
3. Column *X* is filled up as per the logic function it performs

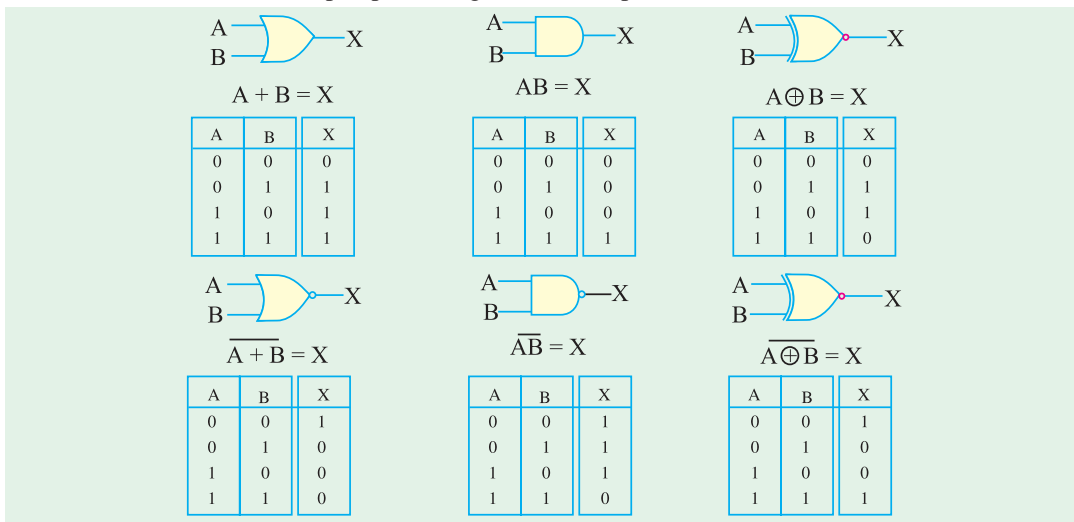


Fig. 70.46

4. Truth tables for *NOR*, *NAND* and *XNOR* (or  $\overline{XOR}$ ) gates are *just the opposite* of those for *OR*, *AND* and *XOR* gates.

**Example 70.7.** An electrical signal is expressed as 101011. Explain its meaning. If this signal is applied to a NOT gate, what would be the output signal ?

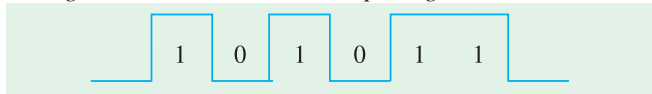


Fig. 70.47

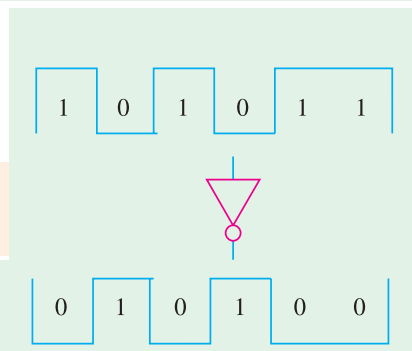


Fig. 70.48

**Solution.** The signal represents binary number  $101011_2$ . It is electrically represented as a train of pulses. Taking positive logic, 1 will represent high voltage and 0 will represent low (or zero) voltage as shown in Fig. 70.47.

When such a signal is applied to a *NOT* gate, it would be inverted or complemented as shown in Fig. 70.48.

The *NOT* output will represent the binary number  $010100_2$ .

**Example 70.8.** Two electrical signals represented by  $A = 101101$  and  $B = 110101$  are applied to a 2-input *AND* gate. Sketch the output signal and the binary number it represents.

**Solution.** The pulse trains corresponding to  $A$  and  $B$  are shown in Fig. 70.49.

Remember that in an *AND* gate,  $C$  is 1 only when both  $A$  and  $B$  are 1. It is an *all-or-nothing gate*. The output can be found in different time intervals as under :

- |    |              |   |             |
|----|--------------|---|-------------|
| 1. | 1st interval | : | $1 + 1 = 1$ |
| 2. | 2nd interval | : | $0 + 1 = 0$ |
| 3. | 3rd interval | : | $1 + 0 = 0$ |
| 4. | 4th interval | : | $1 + 1 = 1$ |
| 5. | 5th interval | : | $0 + 0 = 0$ |
| 6. | 6th interval | : | $1 + 1 = 1$ |

Hence, output of the *AND* gate is  $100101_2$ . It is sketched in Fig. 70.49.

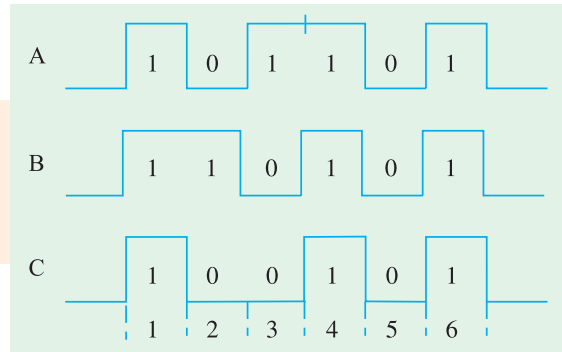


Fig. 70.49

**Example 70.9.** Convert the Boolean expression  $(AB + C)$  into a logic circuit using different logic gates. (Computer Engg. Pune Univ. 1992)

**Solution.** In such cases, it is best to start with the *output and work towards the input*. As seen,  $C$  has been *OR*ed with  $AB$ . Hence, the output gate must be a 2-input *OR* gate as shown in Fig. 70.50 (a).

Now, term  $AB$  is an *AND* function. Hence, we need an *AND* gate with inputs  $A$  and  $B$ . The complete logic circuit is shown in Fig. 70.50 (b).

**Example 70.10.** Design logic hardware based on the Boolean expression  $(A + \bar{B}C)$ .

**Solution.** We will work from *output to input*. It is seen that the last gate is a 2-input *OR* gate with inputs of  $A$  and  $\bar{B}C$ . It is shown in Fig. 70.51 (a).

Since  $\bar{B}$  has been *AND*ed with  $C$ , it requires an *AND* gate as shown in Fig. 70.51 (b). For inversion of  $B$ , a *NOT* gate has been used as shown in Fig. 70.51 (c).

**Example 70.11.** Design a logic circuit whose output is given by the Boolean expression  $(A + B). \bar{AB}$ . (Computer Science, Allahabad Univ. 1992)

**Solution.** Working from output to input, we find that the output gate has to be a 2-input *AND* gate with inputs of  $(A + B)$  and  $\bar{AB}$ . The first step of the circuit design is shown in Fig. 70.52 (a). It is also seen that the input to the entire circuit consists of  $A$  and  $B$  only.

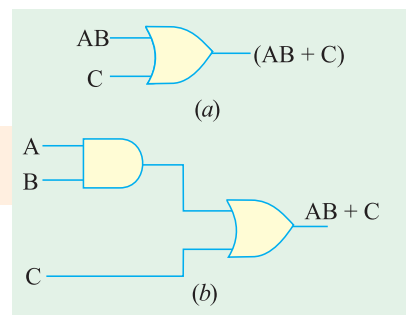


Fig. 70.50

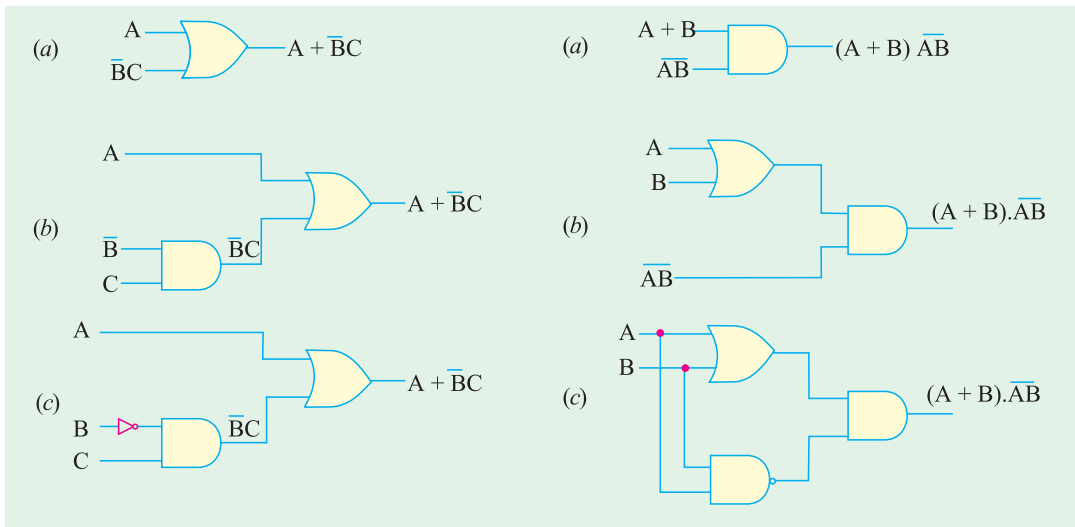


Fig. 70.51

Fig. 70.52

The input of  $(A + B)$  has been obtained with the help of an *OR* gate as shown in Fig. 70.52 (b).

Finally, a *NAND* gate is connected **in parallel** with the *OR* gate for getting its inputs of  $A$  and  $B$  and thereafter for supplying an output of  $\overline{AB}$ . The complete circuit is shown in Fig. 70.52 (c).

### 70.25. Digital Signals Applied to Logic Gates

A binary digital signal applied to a logic gate is nothing else but the application of a time sequence of 1's and 0's. The response of *AND* and *OR* gates to various periodic digital signals is shown in Fig. 70.53.

Fig. 70.54 shows how two waveforms can be *OR*ed by using two input gates.

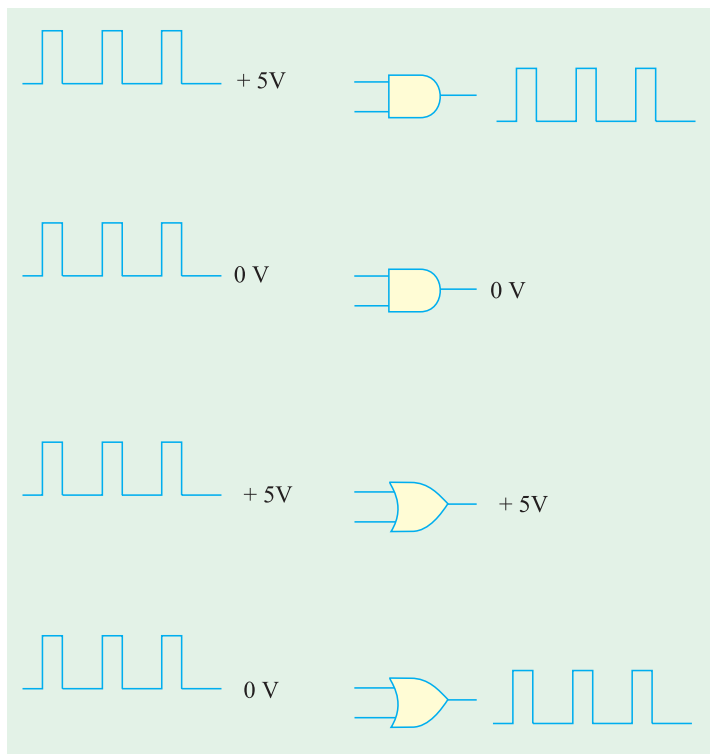


Fig. 70.53

### 70.26. Applications of Logic Gates

Range of application of the logic gates is very wide but the main headings would include.

1. to build more complex devices like binary counters etc.,
2. for decision making in automatic control of machines and various industrial processes,
3. in calculators and computers,
4. in digital measuring techniques,

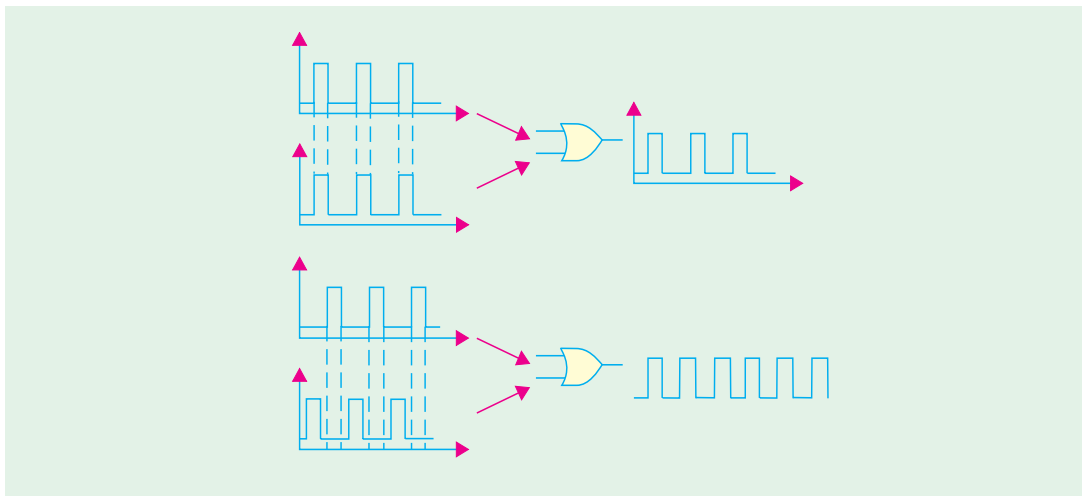


Fig. 70.54

- 5. in digital processing of communications,
- 6. in musical instruments, games and domestic appliances etc.

Sometimes it is more convenient to show the digital signals or pulse waveforms without the  $x$ - and  $y$ -axes. The examples below will follow this practice.

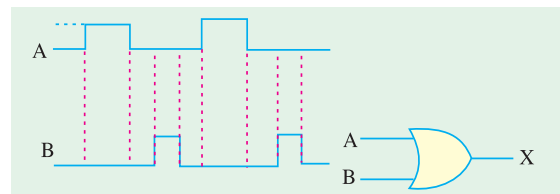


Fig. 70.55

**Example 70.12.** Fig. 70.55 shows an OR gate with two input waveforms. What is the resulting output waveform ?

**Solution.** Remember that the output of an OR gate is 1 when either or both inputs are 1. Therefore, we can sketch the output waveform,  $C$  as shown in Fig. 70.56.

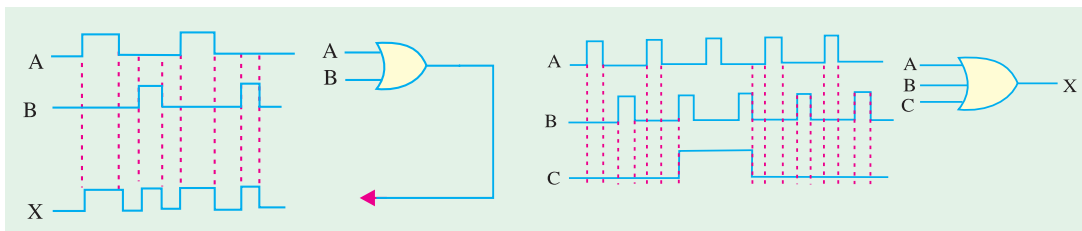


Fig. 70.56

Fig. 70.57

**Example 70.13.** For a three-input OR gate shown in Fig. 70.57 determine the output waveform.

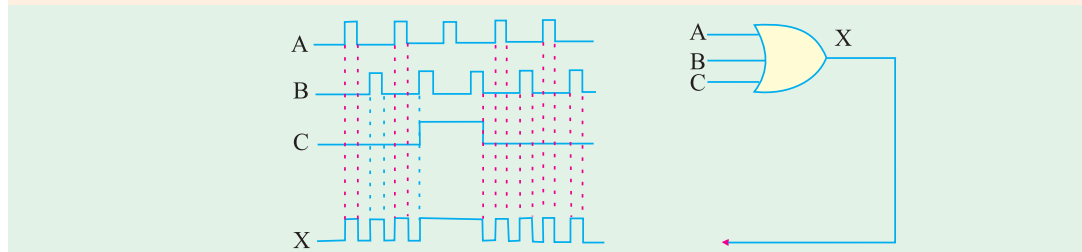


Fig. 70.58

**Solution.** Remember the output of a three-input OR gate is 1 when one or more of the inputs are

1. Therefore, we can sketch the output waveform,  $D$  as shown in Fig. 70.58.

**Example 70.14.** Fig. 70.59 shows a 2-input AND gate with waveforms  $A$  and  $B$ . Sketch the resulting output waveform.

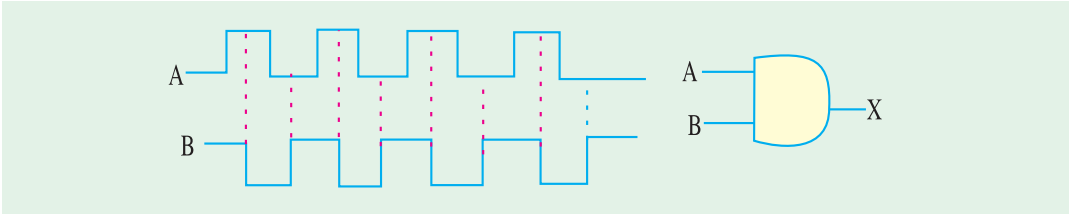


Fig. 70.59

**Solution.** Remember the AND gate produces an output 1 only when all its inputs are present. Thus the output of AND gate is 1 when both  $A$  and  $B$  are 1. Its output is 0 when any of its inputs is 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.60.

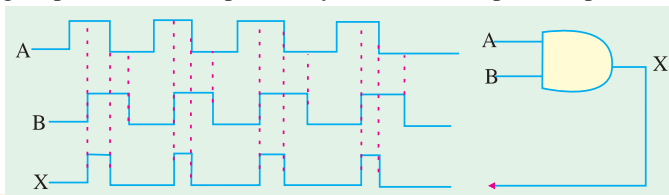


Fig. 70.60

**Example 70.15.** For a 3-input AND gate with waveforms  $A$ ,  $B$  and  $C$  at its inputs as shown in Fig. 70.61, determine the resulting output waveform.

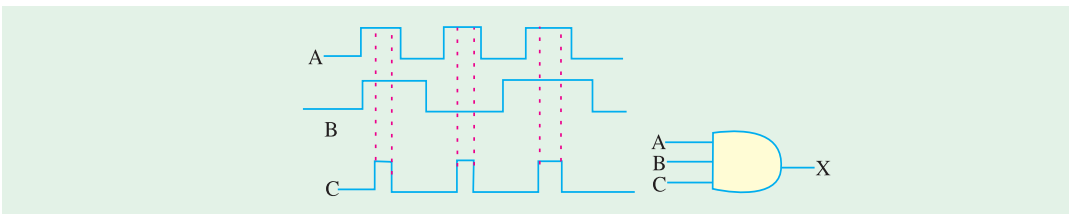


Fig. 70.61

**Solution.** A 3-input AND gate produces an output 1 only when all the three inputs  $A$ ,  $B$  and  $C$  are 1. Its output is 0 when any one of three inputs is 0. Using this concept, the resulting output waveform is as shown in Fig. 70.62.

**Example 70.16.** Fig. 70.63 shows the two waveforms applied to the NOR gate inputs. Sketch the resulting output waveform.

**Solution.** Remember, a NOR gate will have an output of 1 only when all its inputs are 0. Obviously, if any input is 1, the output will be 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.64.

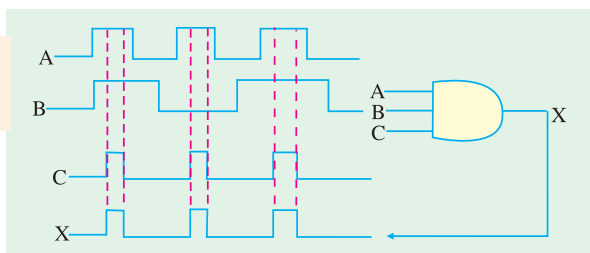


Fig. 70.62

**Example 70.17.** Sketch the output waveform for a 3-input NOR gate shown in Fig. 70.65. Showing the proper time relationship to the inputs.

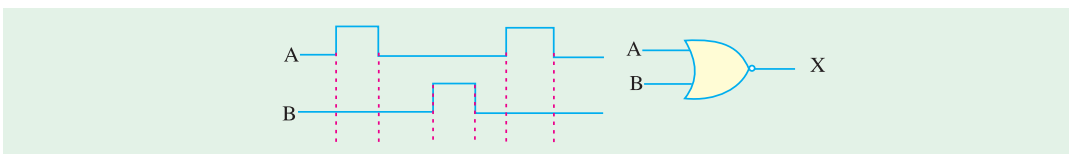


Fig. 70.63



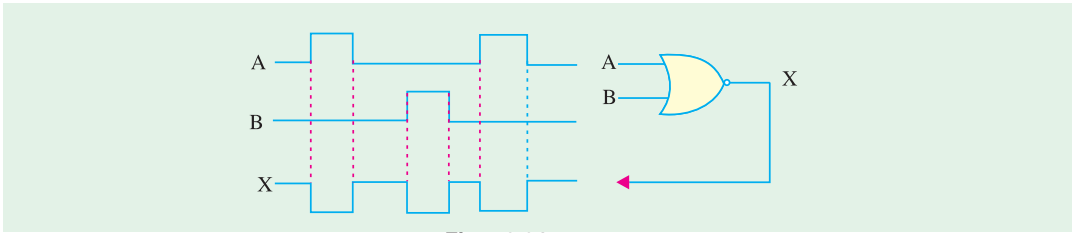


Fig. 70.64

**Solution.** Remember, a 3-input *NOR* gate will have an output of 1 only when all the three inputs *A*, *B* and *C* are 0. Obviously if any one of the three inputs *A*, *B* and *C* or all are 1, the output will be 0.

Using this concept, we can sketch the output waveform as shown in Fig. 70.66.

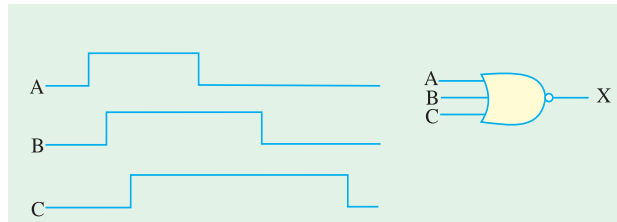


Fig. 70.65

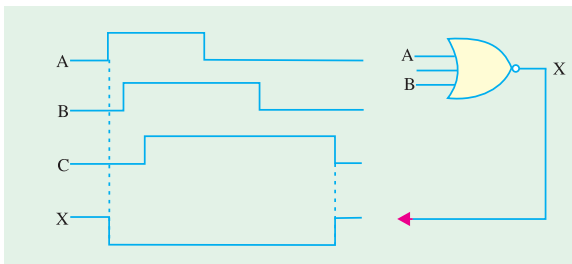


Fig. 70.66

**Example 70.18.** Fig. 70.67 shows the two waveforms *A* and *B* applied to the *NAND* gate inputs. Determine the resulting output waveform.

**Solution.** Remember, the output of *NAND* gate is 1 if either *A* or *B* or both are 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.68.

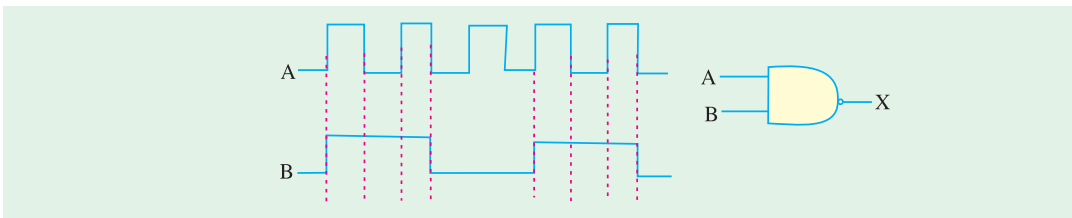


Fig. 70.67

**Example 70.19.** Sketch the output waveform for a 3-input *NAND* gate shown in Fig. 70.69

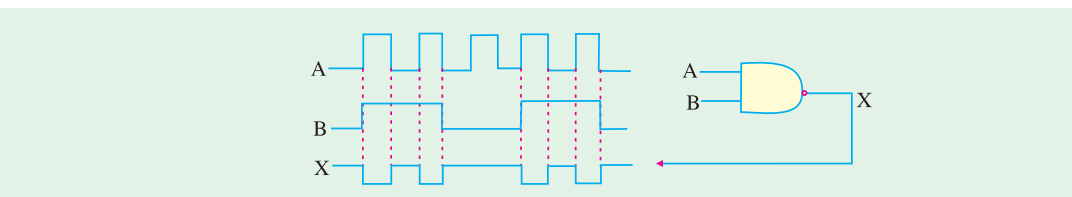


Fig. 70.68

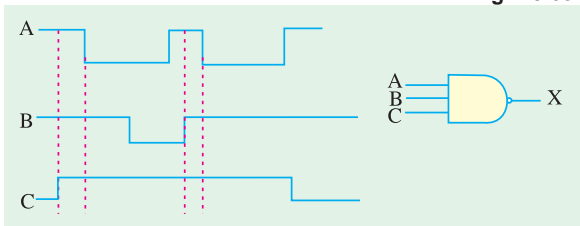


Fig. 70.69

**Solution.** Remember, the output of a 3-input *NAND* gate is 1 only if either any one of the inputs *A*, *B* and *C* or all are 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.70.

**Example 70.20.** The waveforms A and B are applied as an input to the XOR and XNOR gate as shown in Fig. 70.71. Determine the output waveforms of these logic gates.

**Solution.** In exclusive-OR (XOR) gate, output is 1 if its either input but not both, is 1. In other words, it has an output 1 when its inputs are different. The

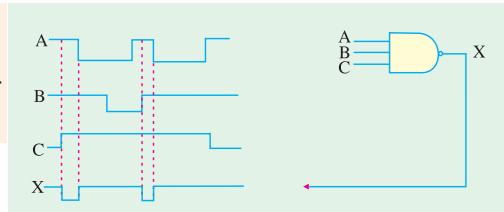


Fig. 70.70

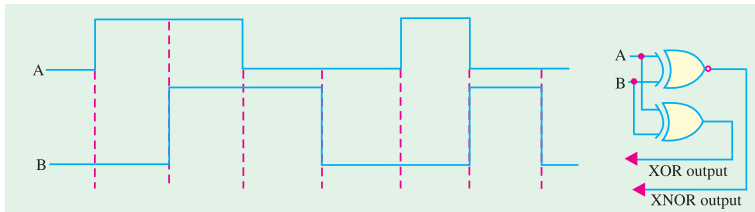


Fig. 70.71

output is 0 only when inputs are the same. Using this concept, we can sketch the XOR output waveform as shown in Fig. 70.72.

In XNOR gate, output is 1 if its both inputs are either 0 or 1. It produces 0 output if its two inputs are at the opposite logic level. Using this concept, we can sketch the XNOR output waveform as shown in Fig. 70.72

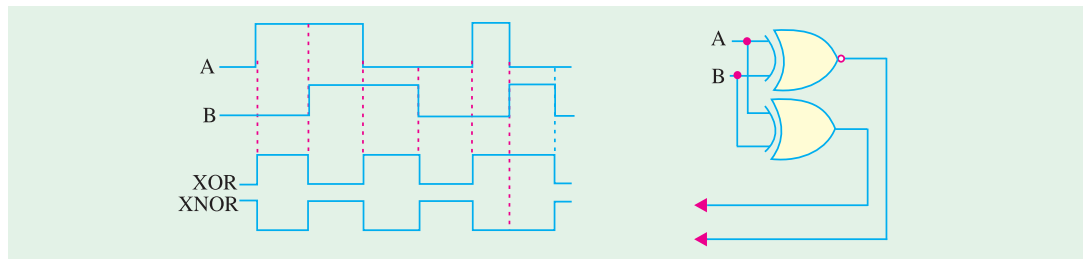


Fig. 70.72

### 70.27. Combinational Logic Circuit

It is a circuit built from various logic gate combinations. The circuit possesses a set of inputs, a *memoryless* logic network to operate on the inputs and a set of outputs as shown in Fig. 70.73 (a). The output from a combinational logic circuit depends solely on the *present* input values and not on the previous ones. Moreover, output combinational networks are used to make logical decisions and control the operation of different circuits in digital electronic systems. For a given set of input conditions, the output of such a circuit is the same. Consequently, a truth table can fully describe the

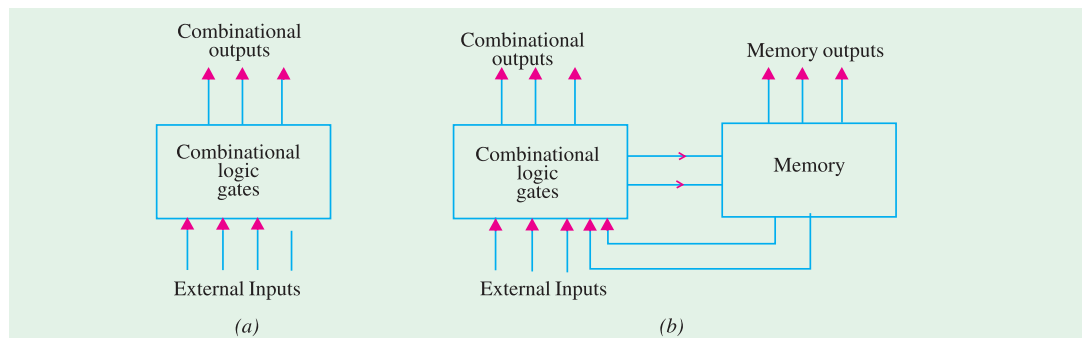


Fig. 70.73

operation of such a circuit. Examples of such a circuit are : decoders, adders, multiplexer and demultiplexers etc.

### 70.28. Sequential Logic Circuits

Such circuits have inputs, logic network, outputs and a memory as shown in Fig. 70.73 (b). Their present output depends not only on their present inputs but also on the previous logic states of the outputs.

Examples of such circuits are a variety of latches and flip-flops. Sequential logic circuits may be either synchronous or asynchronous. The synchronous sequential circuits are built to operate at a clocked rate whereas asynchronous ones are without clocking.

### 70.29. Adders and Subtractors

The logical gates discussed so far can be used for performing arithmetical functions like addition, subtraction, multiplication and division in electronic calculators and digital instruments. In the central processing unit (CPU) of a computer, these arithmetic functions are carried out by the arithmetic and logic unit (ALU). The logic functions used generally are XOR, OR and AND. We will consider the following:

1. Half Adder—it is 1-bit adder and carries out binary addition with the help of XOR and AND gates. It has two inputs and two outputs.
2. Full Adder—It has three inputs and can add three bits at a time. It is made up of two half adders and one OR gate. These adders can also perform subtraction by the method of 1's and 2's complements.
3. Half Subtractor—it uses one XOR and one AND gate.
4. Full Subtractor—it employs two half subtractors and one OR gate.

### 70.30. Half Adder

It can add 2 binary digits at a time and produce a 2-bit data i.e. sum and carry according to the binary addition rules (Art. 70.10).

#### Block Diagram

It is shown in Fig. 70.74. As can be seen, it has two inputs for applying the two binary digits to be added. As is well known, binary

addition of two bits always produces 2-bit output data i.e. one SUM and one CARRY. For example, (1 + 1) gives a sum of 0 and a carry of 1. Also, (0 + 0) gives the sum 0 and carry 0. That is why the adder has two outputs : one for SUM and the other for CARRY.

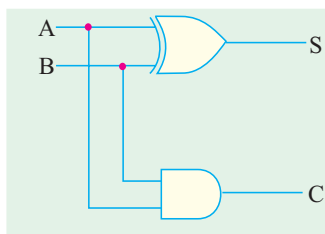


Fig. 70.75

Table 70.7

Input		Output	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Fig. 70.74

Truth Table 70.7 lists the two columns of input, one of SUM and one of CARRY. The SUM output has the same logic pattern as when A is XORed with B. In fact, to add it to XOR. Also, the CARRY output has the same logic pattern as when A is ANDed with B. That is why a half-adder can be formed from a combination of one XOR gate and one AND gate as shown in Fig. 70.75.

The circuit is called half-adder because it cannot accept a carry-in from previous additions. For that purpose, we need a 3-input adder called full-adder.

Incidentally, the logical equations for the *SUM* and *CARRY* are  $S = A \oplus B$  and  $C = A.B$

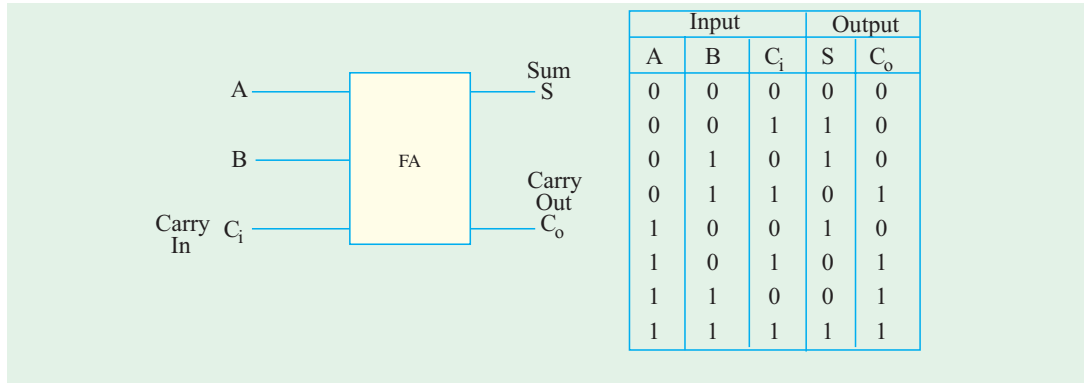


Fig. 70.76

### 70.31. Full Adder

As shown in the block diagram of Fig. 70.76, it has *three inputs and two outputs*. It can add 3 digits (or bits) **at a time**. The bits *A* and *B* which are to be added come from the two registers and the third input comes from the carry generated by the previous addition. It produces two outputs; *SUM* and *CARRY-OUT*.

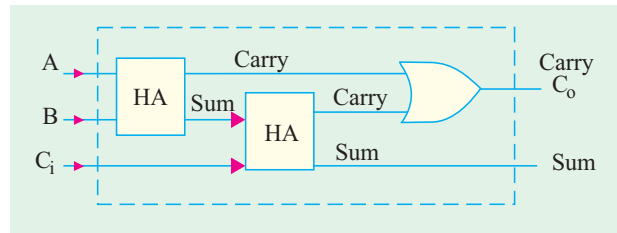


Fig. 70.77

The truth table 70.9 gives all possible input/output relationships for the full adder. *A* and *B* are the inputs from the respective digits of the registers to be added and *C<sub>i</sub>* is the input for any carry generated by the previous stage. The *SUM* output gives binary addition of *A*, *B* and *C<sub>i</sub>*. The other output generates the carry *C<sub>o</sub>* to be added to the next stage.

The full-adder can be constructed from two half-adders and one *OR* gate (Fig. 70.77).

**Working.** Let us illustrate, with the help of two examples, how this full adder adds three bits.

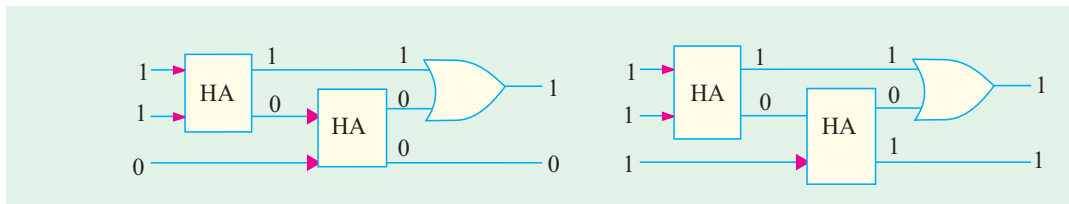


Fig. 70.78

Fig. 70.79

(i) **A = 1, B = 1, Ci = 0**

The full adder with these three inputs is shown in Fig. 70.78. First half adder gives a sum of 0 and a carry of 1. The second *HA* gives a sum of 0 with a carry of 0. The final output is : *SUM* 0, *CARRY* 1. As we know from the rules of binary addition,  $1 + 1 + 0 = 10_2$  (*i.e* decimal 2).

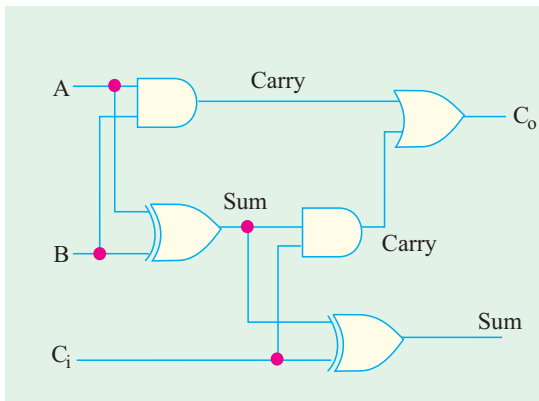


Fig. 70.80

(ii)  $A = 1, B = 1, C = 1$

As detailed in Fig. 70.79, we get a final *SUM* 1 with a *CARRY* 1. The result conforms to the binary addition :  $1 + 1 + 1 = 11_2$  (i.e. decimal 3)

**Detailed Circuit**

Fig. 70.80 shows more circuit details of a full adder. The two half adders have been replaced by their *XOR* and *AND* gates. The final carry is given by the *OR* gate and final sum by the *XOR* gate of the second adder.

**70.32. Parallel Binary Adder**

For adding two 4-bit numbers, we need 4 full adders connected in parallel as shown in Fig. 70.81. The two numbers being added are  $A_3 A_2 A_1 A_0$  and  $B_3 B_2 B_1 B_0$  and their sum is  $S_4 S_3 S_2 S_1 S_0$ .

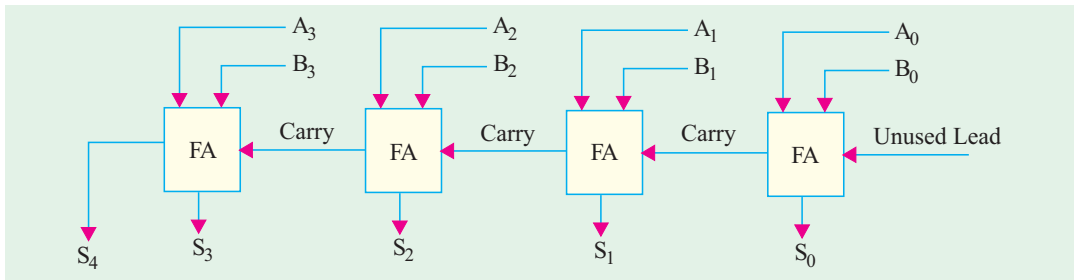


Fig. 70.81

The first adder could be a half adder though we may use a full adder but leave its *CARRY-IN* lead unconnected. As seen, different bits are fed to the four adders from two parallel registers which hold these bits. The final *SUM* appears as a 5-digit display.

**Operation**

The actual operation may be better understood with the help of the diagram of Fig. 70.82. Suppose, we want to add the following two 4-bit numbers.

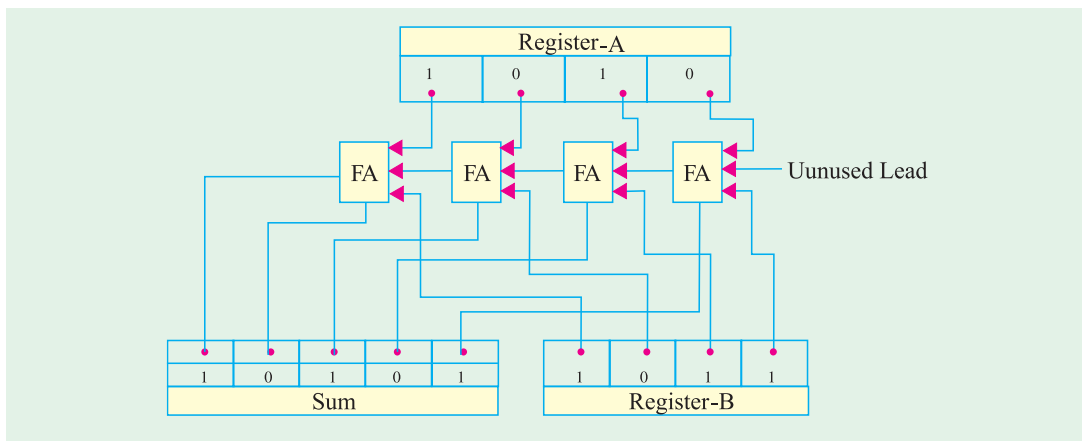


Fig. 70.82

$$\begin{array}{r} 1010 \\ +1011 \\ \hline 10101 \end{array} \qquad \begin{array}{r} 10 \\ +11 \\ \hline 21 \end{array}$$

The first adder performs 0 + 1 binary addition, giving a sum of 1 and a carry of 0. The two bits 0 and 1 are supplied *simultaneously* from the two registers *A* and *B*. The sum 1 appears on the display panel and carry 0 is passed on to the next full adder.

The next adder adds 1 + 1 + 0 carry = sum 1 with a carry 1. The third adder performs 0 + 0 + 1 carry = 1 with carry 0. The fourth adder adds 1 + 1 + 0 carry = sum 0 with carry 1 both of which appear on the display unit. Hence, the final addition of the two numbers appears as 10101.

It may be noted that the largest binary numbers that can be added by this parallel adder are 1111 and 1111 which give a sum of 11110<sub>2</sub> (decimal 30). To increase its capacity, more full adders may be connected at the left end of Fig. 70.82. For example, for adding 6-bit numbers, we will have to add two more adders thus making a total of *six*.

Table 70.10

Input		Output	
A	B	W	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

Fig. 70.83

### 70.33. Half Subtractor

It can subtract only two binary digits *at a time* and produce an output of a difference and a borrow. As shown in the block diagram of Fig. 70.83, it has two inputs and two outputs.

The operation of a half subtractor is based on the rules of binary subtraction illustrated in the truth Table 70.10 for all possible input/output combinations. The difference output in the fourth column has the same logic pattern as when *A* is *XOR*ed with *B* (same was the case for *SUM* in Art. 70.30). Hence, we can use an *XOR* gate to get the difference of two bits. The borrow output in the third column can be obtained by *AND*ing *A* with *B*.

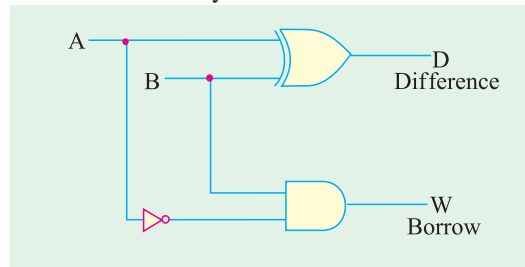


Fig. 70.84

The circuit for a half subtractor is shown in Fig. 70.84.

As mentioned earlier, the logical equations for the difference and borrow are given by

$$D = A \oplus B \text{ and } W = \bar{A} B$$

### 70.34. Full Subtractor

As shown in the block diagram of Fig. 70.85, it has three inputs and two outputs. As explained above, half subtractor can handle only 2 bits at a time and can be used for the least significant column of a subtraction problem. A full subtractor can, however, take care of higher-order columns.

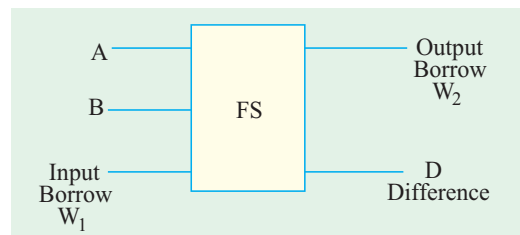


Fig. 70.85

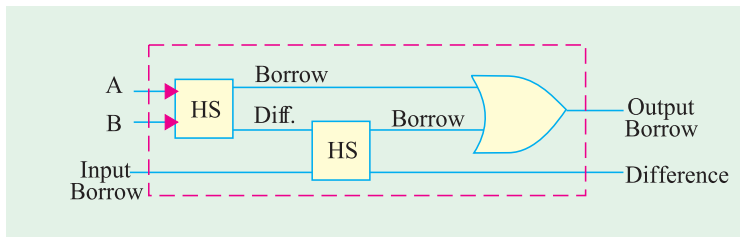


Fig. 70.86

As shown in Fig. 70.86, a full subtractor consists of two half subtractors and one OR gate.

It may be remarked here that by cascading 4 full subtractors, we can directly subtract 4-bit numbers *i.e.* we can subtract  $B_3 B_2 B_1 B_0$  from  $A_3 A_2 A_1 A_0$ .

**Tutorial Problems No. 70.1**

- Find the Boolean equation for the output of the logic circuit shown in Fig. 70.87. What would be the output if  $A = 1, B = 0, C = 1, D = 1$   $[X = AB + CD; 1]$

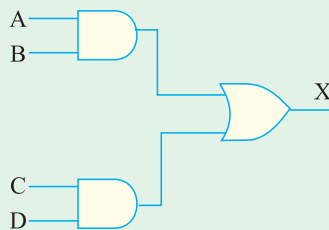


Fig. 70.87

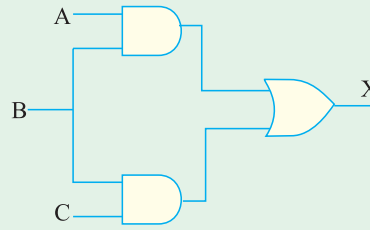


Fig. 70.88

- What is the output equation of the logic circuit shown in Fig. 70.88? Evaluate the output if  $A = 1, B = 1, C = 0$ .  $[X = AB + (B + C); 1]$
- After finding the Boolean equation for the circuit shown in Fig. 70.89, compute the output if  $A = 1, B = 0, C = 1, D = 0$ .  $[X = (A + B)(C + D); 1]$

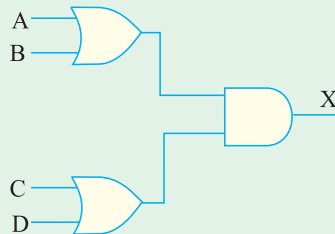


Fig. 70.89

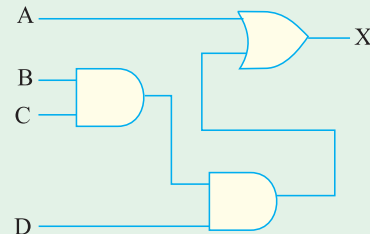


Fig. 70.90

- Translate the hardware shown in Fig. 70.90 into a Boolean expression. Compute the value of the output if  $A = 0, B = 1, C = 0, D = 1$ .  $[X = A + BCD; 0]$
- What is the Boolean expression for the logic diagram shown in Fig. 70.91? Evaluate its output if  $A = 1, B = 1$ , and  $C = 1$ .  $[X = AB + C; 1]$

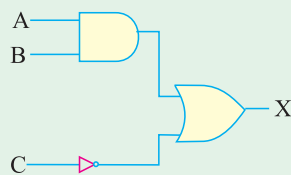


Fig. 70.91

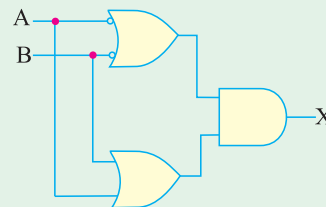


Fig. 70.92

6. Find Boolean expression for the logic circuit of Fig. 70.92. What is the output if  $A = 1, B = 1$  ?

$$[X = (\bar{A} + \bar{B}) \cdot (A + B) ; 0]$$

7. Give the logic functions performed by the circuits shown in Fig. 70.93 (a), (b) and (c).

$$[(a) X = \bar{A}B \cdot (\bar{C} + D) \quad (b) X = \bar{A}B \cdot C \quad (c) (A + B)CD]$$

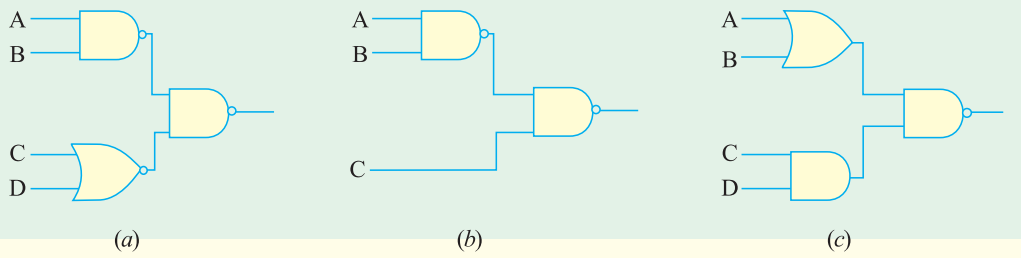


Fig. 70.93

8. State the logic functions of the circuits shown in Fig. 70.94 (a), (b) and (c).

$$[(a) (A + B) \cdot (CD) \quad (b) AB + CD \quad (c) (A + B) \cdot C \cdot D \cdot E]$$

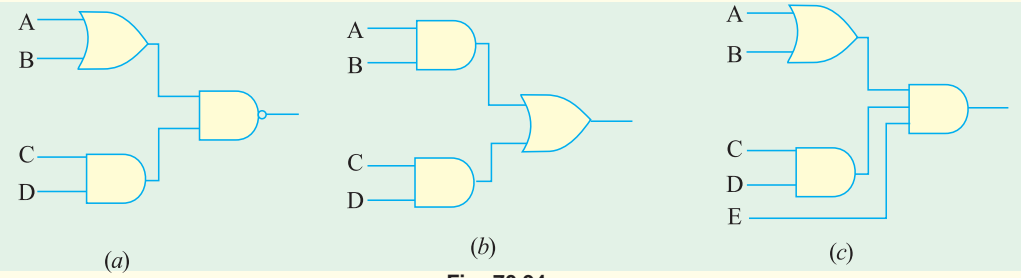


Fig. 70.94

9. State the logic functions performed by the circuits of Fig. 70.95 (a), (b) and (c).

$$[(a) \bar{A}B \cdot (\bar{C} + D) \quad (b) \bar{A}B + C \quad (c) A + B]$$

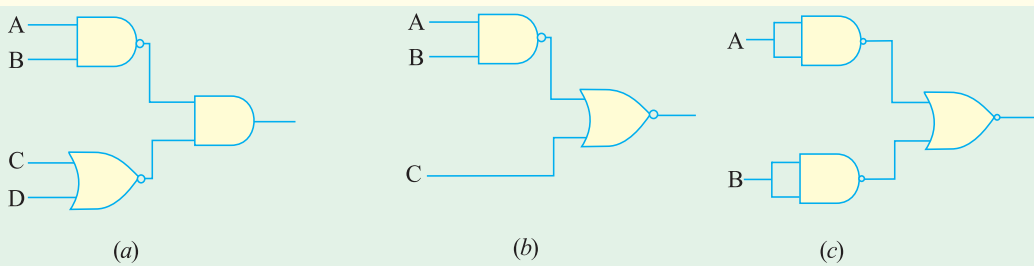


Fig. 70.95

10. Write the logic functions for the circuits shown in Fig. 70.96 (a), (b) and (c).

$$[(a) A(B + C) + CD \quad (b) \bar{A} + B \cdot A \cdot C \quad (c) A \bar{B} + \bar{C}D]$$

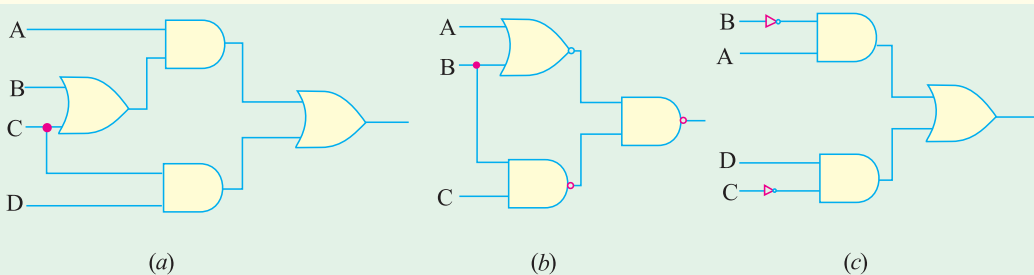


Fig. 70.96



11. What would be the output signal if two signals  $A = 101011_2$  and  $B = 110101_2$  are applied to the inputs of an *AND* gate? [100 001]2
12. What would be the output signal if two input binary signals given by  $A = 100101$  and  $B = 110110$  are applied to (a) *OR* gate (b) *NAND* gate and (c) *XNOR* gate? [(a) 1101112 (b) 0110112 (c) 1011002]
13. Sketch the output waveform at *C*-for a 2-input *OR* gate shown in Fig. 70.97 and with the given *A* and *B* input waveforms.

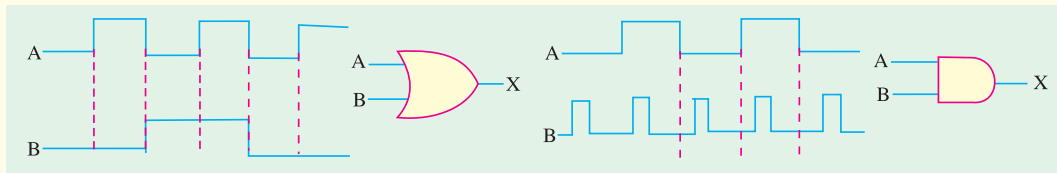


Fig. 70.97

Fig. 70.98

14. Sketch the output waveform of a 2-input *AND* gate shown in Fig. 70.98 with the input waveforms *A* and *B*.
15. Sketch the output waveform at *D* for a 3-input *AND* gate shown in Fig. 70.99, with the given *A*, *B* and *C* input waveforms.

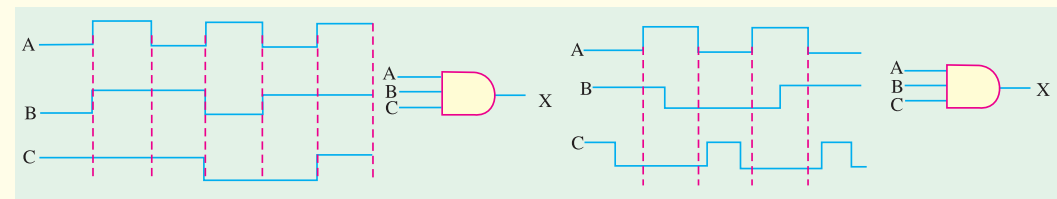


Fig. 70.99

Fig. 70.100

16. Sketch the output waveform at *D* for a 3-input *AND* gate shown in Fig. 70.100, with the given *A*, *B* and *C* input waveforms.

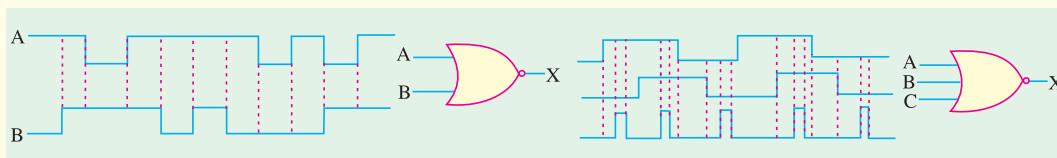


Fig. 70.101

Fig. 70.102

17. Sketch the output waveform at *C* for a 2-input *NOR* gate shown in Fig. 70.101, with the given *A* and *B* input waveforms.
18. Sketch the output waveform at *D* for a 3-input *NOR* gate shown in Fig. 70.102, with the given *A*, *B* and *C* input waveforms.

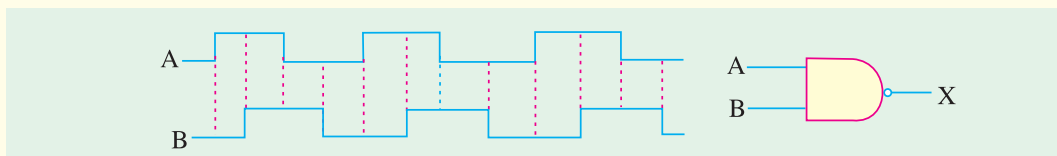


Fig. 70.103

19. Sketch the output waveform at *C* for a 2-input *NAND* gate shown in Fig. 70.103, with the given *A* and *B* input waveforms.

20. Sketch the output waveform at  $D$  for a 3-input *NAND* gate shown in Fig. 70.104, with the given  $A$ ,  $B$  and  $C$  input waveforms.

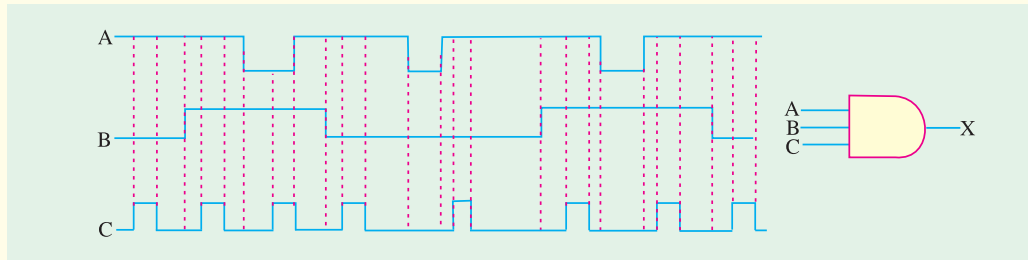


Fig. 70.104

## OBJECTIVE TESTS – 70

- A logic gate is an electronic circuit which
  - makes logic decision
  - allows electron flow only in one direction
  - works on binary algebra
  - alternates between 0 and 1 values.
- In positive logic, logic state 1 corresponds to
  - positive voltage
  - higher voltage level
  - zero voltage level
  - lower voltage level.
- In negative logic, the logic state 1 corresponds to
  - negative voltage
  - zero voltage
  - more negative voltage
  - lower voltage level.
- The voltage levels of a negative logic system
  - must necessarily be negative
  - may be negative or positive
  - must necessarily be positive
  - must necessarily be 0V and -5V
- The output of a 2-input *OR* gate is zero only when its
  - both inputs are 0
  - either input is 1
  - both inputs are 1
  - either input is 0.
- An *XOR* gate produces an output only when its two inputs are
  - high
  - low
  - different
  - same.
- An *AND* gate
  - implements logic addition
  - is equivalent to a series switching circuit
  - is an any-or-all gate
  - is equivalent to a parallel switching circuit.
- When an input electrical signal  $A = 10100$  is applied to a *NOT* gate, its output signal is
  - 01011
  - 10101
  - 10100
  - 00101.
- The only function of a *NOT* gate is to
  - stop a signal
  - recomplement a signal
  - invert an input signal
  - act as a universal gate.
- A *NOR* gate is ON only when all its inputs are
  - ON
  - positive
  - high
  - OFF.
- For getting an output from an *XNOR* gate, its both inputs must be
  - high
  - low
  - at the same logic level
  - at the opposite logic levels.
- In a certain 2-input logic gate, when  $A = 0$ ,  $B = 0$ , then  $C = 1$  and when  $A = 0$ ,  $B = 1$ , then again  $C = 1$ . It must be ..... gate.
  - XOR*
  - AND*
  - NAND*
  - NOR*
- The logic symbol shown in Fig. 70.105 represents
  - single-output *AND* gate
  - NAND* gate
  - NAND* gate used as *NOT* gate
  - NOR* gate.

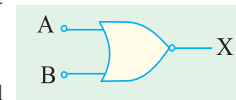


Fig. 70.105

14. The output from the logic gate shown in Fig. 70.106 will be available when inputs ..... are present.

- (a) A and C
- (b) B and C
- (c) A, B and C
- (d) A and B

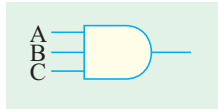


Fig. 70.106

15. To get an output 1 from circuit of Fig. 70.107, the input must be A B C

- (a) 0 1 0
- (b) 1 0 0
- (c) 1 0 1
- (d) 1 1 0

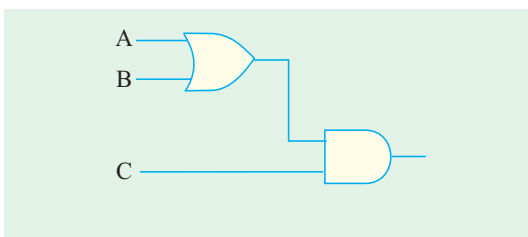


Fig. 70.107

16. Which of the following logic gates in Fig. 70.108 will have an output of 1 ?

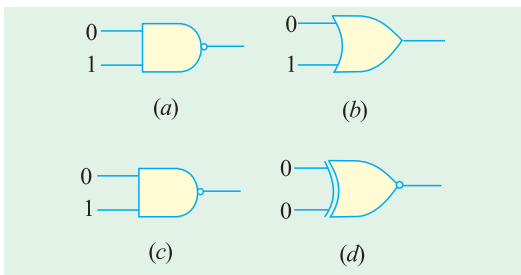


Fig. 70.108

17. A half-adder can be constructed from

- (a) two XNOR gates only
- (b) one XOR and one OR gate with their outputs connected in parallel
- (c) one XOR and one OR gate with their inputs connected in parallel
- (d) one XOR gate and one AND gate

18. The digital equivalent of an electric series circuit is the ..... gate.

- (a) NOR
- (b) NAND
- (c) OR
- (d) AND

19. Which of the following represents analog data ?

- (a) ON and OFF states
- (b) 0 and 1
- (c) 0V and 5V
- (d) 1.5, 3.2, 4 and 5V

20. The logic gate which produces a 0 or low-level output when one or both of the inputs are 1 is called ..... gate.

- (a) AND
- (b) OR
- (c) NOR
- (d) NAND

21. The output X of the gated network shown in Fig. 70.109 is

- (a)  $\overline{AB} \cdot \overline{CD} \cdot \overline{EF}$
- (b)  $\overline{AB} + \overline{CD} + \overline{EF}$
- (c)  $AB + CD + EF$
- (d)  $(A + B)(C + D)(E + F)$

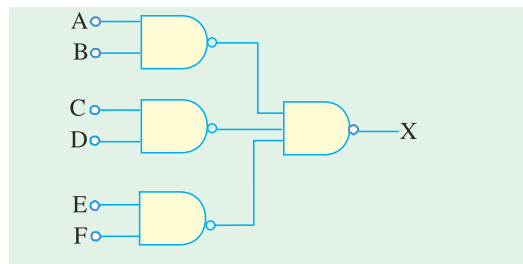


Fig. 70.109

22. The digital circuit shown in Fig 70.110 generates a modified clock pulse at the output. Choose the correct output waveform from the options given below. (GATE; 2004)

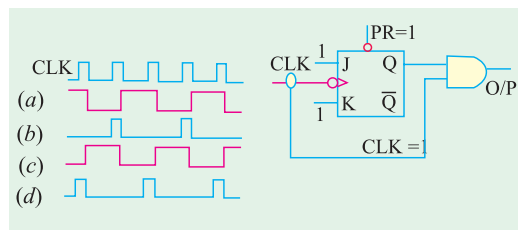


Fig. 70.110

ANSWERS

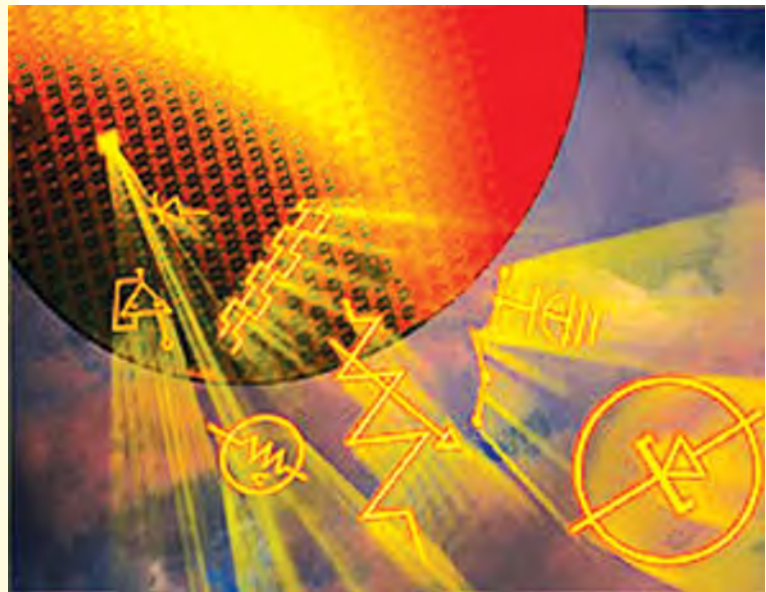
- 1. (a)    2. (b)    3. (d)    4. (b)    5. (a)    6. (c)    7. (b)    8. (a)    9. (c)    10. (d)
- 11. (c)    12. (c)    13. (d)    14. (c)    15. (c)    16. (d)    17. (c)    18. (d)    19. (d)    20. (c)
- 21. (c)    22. (d)

# CHAPTER 71

## Learning Objectives

- Unique Feature of Boolean Algebra
- Laws of Boolean Algebra
- Equivalent Switching Circuits
- DeMorgan's Theorem's
- The Sum-of-Products (SOP) Form
- The Standard SOP Form
- The Standard POS Form
- The Karnaugh Map
- The Four-variable Karnaugh Map
- Square Adjacency in Karnaugh Map
- Mapping Directly on Karnaugh Map from a Truth Table
- "Don't Care" Conditions
- Main Logic Families
- Saturated and Non-saturated Logic Circuits
- DC supply voltage
- Noise Immunity
- Noise Margin-Power Dissipation
- Power Dissipation versus Frequency
- Propagation Delay
- Speed-Power Product
- RTL Circuit
- DTL Circuit
- TTL Circuit
- TTL Sub-families
- ECL Circuit
- MOS family

## BOOLEAN ALGEBRA AND LOGIC FAMILIES



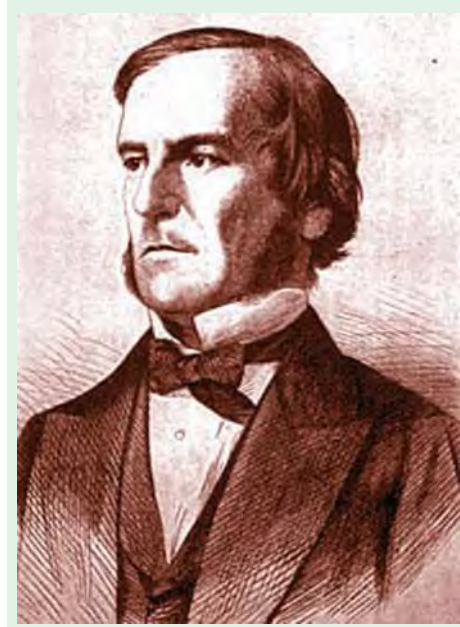
DNA Logic gates have been developed and are the first step towards creating a computer that has a structure of that of electronic PC

### 71.1. Introduction

Boolean algebra, named after its pioneer George Boole (1815-1864) is the algebra of logic presently applied to the operation of computer devices. The rules of this algebra are based on human reasoning. It originated from the study of how we reason, what lines of reasoning are valid and what constitutes proof etc.

Starting with his investigation of the laws of thought, Boole developed in 1854 a mathematical system of logic in which he expressed truth functions as *symbols* and then manipulated these symbols to arrive at a conclusion. His new system was not the **ordinary numerical algebra** we know from our high school days but a totally new system called **logic algebra**. For example, in Boolean algebra  $A + A = A$  and not  $2A$  as is the case in ordinary algebra.

Boolean algebra remained in the realm of philosophy till 1938 when Claude E. Shannon used it to solve relay logic problems. As we know, all thinking and logic is concerned with finding answers to binary or two-valued questions like : is it good or bad, right or wrong, true or false etc. This binary nature of logic is exactly like the binary working of relay and switching circuits where relay is either energised or not, light is ON or OFF or pulse is present or not. Because of its very logical nature, Boolean algebra is ideal for the design and analysis of logic circuits used in computers. Moreover, it provides an economical and straight forward way of describing computer circuitry and complicated switching circuits. As compared to other mathematical tools of analysis and design, Boolean algebra has the advantages of simplicity, speed and accuracy.



George Boole (1815–1864)

### 71.2. Unique Feature of Boolean Algebra

As we know, the different variables used in ordinary algebra can have *any value* including plus and minus values. There is no restriction on the value they can assume. For example, in the equation  $2x + 3y = z$ , the variables  $x$ ,  $y$  and  $z$  can take on any value available in the entire field of real numbers.

However, the variables used in Boolean algebra have a unique property *i.e.* they can assume **only one of the two possible values of 0 and 1**. Each of the variable used in a logical or Boolean equation can assume only the value 0 or 1. For example, in the logical equation  $A + B = C$ , **each** of the three variables  $A$ ,  $B$  and  $C$  can have only the values of either 0 or 1. **This point must be clearly taken note of by the reader for easy understanding of the laws of Boolean algebra.**

### 71.3. Laws of Boolean Algebra

As started earlier, Boolean algebra is a system of Mathematics based on *logic*. It has its own set of fundamental laws which are necessary for manipulating different Boolean expressions.

#### 1. OR Laws

These four laws have already been discussed in the previous chapter. These are

Law 1.  $A + 0 = A$

Law 3.  $A + A = A$

Law 2.  $A + 1 = 1$

Law 4.  $A + \bar{A} = 1$

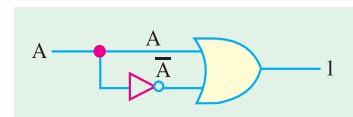


Fig. 71.1

The expression given in Law 4 can be understood with the help of Fig. 71.1. Consider the following two possibilities :

- (i) When  $A = 0, \bar{A} = 1 \quad \therefore A + \bar{A} = 0 + 1 = 1$
- (ii) When  $A = 1, \bar{A} = 0 \quad \therefore A + \bar{A} = 1 + 0 = 1$

**2. AND Laws**

- Law 5.  $A \cdot 0 = 0$       Law 7.  $A \cdot A = A$
- Law 6.  $A \cdot 1 = A$       Law 8.  $A \cdot \bar{A} = 0$

The expression for Law 8 can be easily understood with the help of the logic circuit of Fig. 71.2. Consider the following two possibilities :

- (i) When  $A = 0, \bar{A} = 1 \quad \therefore A \cdot \bar{A} = 0 \cdot 1 = 0$
- (ii) When  $A = 1, \bar{A} = 0 \quad \therefore A \cdot \bar{A} = 1 \cdot 0 = 0$

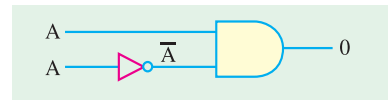


Fig. 71.2

**3. Laws of Complementation**

- Law 9.  $\bar{0} = 1$       Law 10.  $\bar{1} = 0$
- Law 11. if  $A = 0$ , then  $\bar{A} = 1$       Law 12. if  $A = 1$ , then  $\bar{A} = 0$
- Law 13.  $A = \bar{\bar{A}}$

**4. Commutative Laws**

These laws allow *change in the position* of variables in *OR* and *AND* expressions.

- Law 14.  $A + B = B + A$       Law 15.  $A \cdot B = B \cdot A$

These two laws express the fact that the order in which a combination of terms is performed does not affect the final result of the combination.

**5. Associative Laws**

These laws allow removal of brackets from logical expression and regrouping of variables.

- Law 16.  $A + (B + C) = (A + B) + C$
- Law 17.  $(A + B) + (C + D) = A + B + C + D$       Law 18.  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

**6. Distributive Laws**

These laws permit factoring or multiplying out of an expression.

- Law 19.  $A(B + C) = AB + AC$       Law 20.  $A + BC = (A + B)(A + C)$
- Law 21.  $A + \bar{A} \cdot B = A + B$

**7. Absorptive Laws**

These enable us to reduce a complicated logic expression to a simpler form by absorbing some of the terms into existing terms.

- Law 22.  $A + AB = A$       Law 23.  $A \cdot (A + B) = A$
- Law 24.  $A \cdot (\bar{A} + B) = AB$

The above laws can be used to prove any given Boolean identity and also for simplifying complicated expressions.

**71.4. Equivalent Switching Circuits**

The equivalent circuits to illustrate some of the *OR* and *AND* laws given above are shown in Fig. 71.3.

(i) Fig. 71.3 (a) illustrates  $A + 1 = 1$ . Here, lower switch is permanently closed representing 1. Hence, value of the *OR* function is 1 (*i.e.* it is *ON*) whatever the value of *A*.

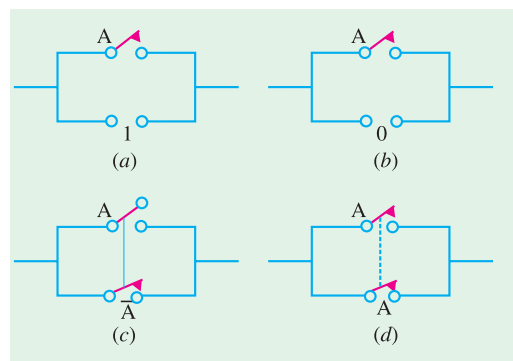


Fig. 71.3

- (ii) Fig. 71.3 (b) represents  $A + 0 = A$ . Here, function value is determined by  $A$  alone.
- (iii) In Fig. 71.3 (c) when  $A$  opens,  $A$  closes and *vice versa*. Obviously, whatever the position of  $A$ , the circuit would always be *ON* proving that  $A + \bar{A} = 1$ .
- (iv) Fig. 71.3 (d) proves  $A + A = A$ . It shows that final result depends on the value of  $A$  alone. If  $A = 0$ , the two switches are open, hence circuit is *OFF*. If  $A = 1$ , both switches are closed. Hence, circuit is *ON*.

(v) In Fig. 71.4 (a), the circuit is permanently *OFF* irrespective of the value of  $A$ . It is due to **permanent** open (0) in the circuit. Hence, it proves  $A \cdot 0 = 0$ .

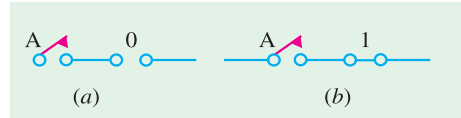


Fig. 71.4

(vi) Fig. 71.4 (b) shows that circuit conditions will depend solely on the position of the switch. If  $A = 1$ , circuit is *ON* (1) and when  $A = 0$ , circuit is *OFF* (0). It is all due to the presence of a **permanent** short (1) in the series circuit. Hence, everything depends on  $A$ .

**Example 71.1.** Prove the following Boolean identity :  $AC + ABC = AC$

**Solution.** Taking the left hand side expression as  $y$ , we get

$$y = AC + ABC = AC(1 + B)$$

Now  $1 + B = 1$  —Law 2

$\therefore y = AC \cdot 1 = AC$  —Law 6

$\therefore AC + ABC = AC$

**Example 71.2.** Determine the logic expression for the output  $Y$ , from the truth table shown in Fig. 71.5. Simplify and sketch the logic circuit for the simplified expression.

**Solution.** There are two 1s in the output column of the given truth table. The corresponding binary values are 001 and 101. These values are converted into product terms as follows :

$$001 \rightarrow \bar{A} \bar{B} C \quad \text{and} \quad 101 \rightarrow A \bar{B} C.$$

Inputs			Output
A	B	C	Y
0	0	0	0
0	0	1	1 $\rightarrow \bar{A} \bar{B} C$
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1 $\rightarrow A \bar{B} C$
1	1	0	0
1	1	1	0

Fig. 71.5

The resulting expression for the output,

$$Y = \bar{A} \bar{B} C + A \bar{B} C$$

$$= (\bar{A} + A) \bar{B} C \quad \dots(\text{Law 19})$$

$$= 1 \cdot \bar{B} C \quad \dots(\text{Law 4})$$

$$= \bar{B} C \quad \dots(\text{Law 6})$$

Fig. 71.6 shows the logic circuit to implement the simplified logic expression for the output. As seen it is formed by ANDING the variables  $\bar{B}$  and  $C$ . The  $\bar{B}$  can be obtained by inverting  $B$ .

**Example 71.3.** Prove the following Boolean identity :  $(A + B)(A + C) = A + BC$

**Solution.** Putting the left hand side expression equal to  $y$ , we get

$$\begin{aligned}
 Y &= (A + B)(A + C) \\
 &= AA + AC + AB + BC && \text{---Law 19} \\
 &= A + AC + AB + BC && \text{---Law 7} \\
 &= A + AB + AC + BC \\
 &= A(1 + B) + AC + BC && \text{---Law 19} \\
 &= A + AC + BC && \text{---Law 2} \\
 &= A(1 + C) + BC && \text{---Law 19} \\
 &= A + BC && \text{---Law 2}
 \end{aligned}$$

$$\therefore (A + B)(A + C) = A + BC$$

**Example 71.4.** Prove the following identity :  $A + \bar{A}B = A + B$

**Solution.** Let the left-hand side expression be put equal to  $Y$ .

$$\begin{aligned}
 Y &= A + \bar{A}B = A \cdot 1 + \bar{A}B && \text{---Law 6} \\
 &= A(1 + B) + \bar{A}B && \text{---Law 12} \\
 &= A \cdot 1 + AB + \bar{A}B && \text{---Law 19} \\
 &= A + BA + B\bar{A} && \text{---Law 6 and 15} \\
 &= A + B(A + \bar{A}) && \text{---Law 19} \\
 &= A + B \cdot 1 && \text{---Law 4} \\
 &= A + B && \text{---Law 6}
 \end{aligned}$$

$$\therefore A + \bar{A}B = A + B$$

**Example 71.5.** Prove the following Boolean identity :  $(A + B)(A + \bar{B})(\bar{A} + C) = AC$

(Digital Computations, Punjab Univ. 1990)

**Solution.** Let the left-hand side expression be represented by  $Y$ .

$$\begin{aligned}
 \therefore Y &= (A + B)(A + \bar{B})(\bar{A} + C) = (AA + A\bar{B} + BA + B\bar{B})(\bar{A} + C) \\
 &= (A + AB + A\bar{B})(\bar{A} + C) = [A(1 + B) + A\bar{B}](\bar{A} + C) && \because B\bar{B} = 0 \\
 &= (A + A\bar{B})(\bar{A} + C) = A(1 + \bar{B})(\bar{A} + C) \\
 &= (A \cdot 1)(\bar{A} + C) = A(\bar{A} + C) = A\bar{A} + AC = AC && \because A\bar{A} = 0
 \end{aligned}$$

**Example 71.6.** Prove the following Boolean identity :

$$ABC + A\bar{B}C + AB\bar{C} = A + (B + C)$$

(Digital Electronic Systems-I, Kurukshetra Univ. 1991)

**Solution.** Equating the left-hand side expression to  $Y$ , we have

$$\begin{aligned}
 Y &= ABC + A\bar{B}C + AB\bar{C} = AC(B + \bar{B}) + AB\bar{C} \\
 &= AC + AB\bar{C} && \text{---Law 4} \\
 &= A(C + B\bar{C}) \\
 &= A(C + B) && \text{---Law 21} \\
 &= A(B + C) && \text{---Law 14}
 \end{aligned}$$

Hence, it proves the given identity.

**Example 71.7.** Simplify the following Boolean Expression :

$$A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC + ABC + A\bar{B}C$$

(Digital Computations, Punjab Univ. 1992)

**Solution.** Bringing together those terms which have two common letters, we get

$$Y = A\bar{B}\bar{C} + ABC + A\bar{B}C + A\bar{B}C + \bar{A}BC$$



$$\begin{aligned}
 &= AB(\bar{C} + C) + A\bar{B}(\bar{C} + C) + \bar{A}BC \\
 &= AB + A\bar{B} + \bar{A}BC \quad \text{---Law 4} \\
 &= A(B + \bar{B}) + \bar{A}BC \quad \text{---Law 4} \\
 &= A + \bar{A}BC = A + BC \quad \text{---Law 21}
 \end{aligned}$$

**Example 71.8.** Simplify the following expression and show the minimum gate implementation.

$$Y = A \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + B \cdot \bar{C} \cdot D$$

**Solution.** As seen from OR and AND laws of Art 67.3,  $A + \bar{A} = 1$  and  $A \cdot 1 = A$

$$\begin{aligned}
 \therefore Y &= B\bar{C}\bar{D}(A + \bar{A}) + B\bar{C}D = B\bar{C}\bar{D} \cdot 1 + B\bar{C}D \\
 &= B \cdot \bar{C} \cdot \bar{D} + B \cdot \bar{C} \cdot D = B \cdot \bar{C} \cdot (\bar{D} + D) = B \cdot \bar{C} \cdot 1 = B \cdot \bar{C}
 \end{aligned}$$

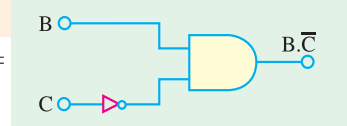


Fig. 71.7

Minimum gate implementation is shown by the circuit of Fig. 71.7

**Example 71.9.** Simplify the following Boolean expression and draw the logic circuits for the simplified expressions.

(a)  $Y = \bar{A}BC + A\bar{B}C + ABC + B\bar{C}$       (b)  $Y = \bar{B}(A + C) + C(\bar{A} + B) + AC$

**Solution.** (a)  $Y = \bar{A}BC + A\bar{B}C + ABC + B\bar{C} = BC(A + \bar{A}) + AC(\bar{B} + B) + B\bar{C}$   
 $= BC + AC + B\bar{C} = B(C + \bar{C}) + AC = AC + B$

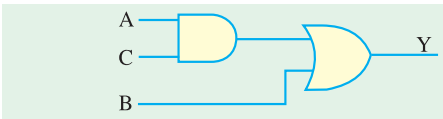


Fig. 71.8

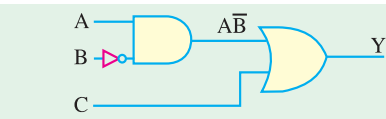


Fig. 71.9

(b)  $Y = \bar{B}(A + C) + C(\bar{A} + B) + AC = A\bar{B} + \bar{B}C + \bar{A}C + BC + AC$   
 $= A\bar{B} + C(\bar{B} + \bar{A} + B + A) = A\bar{B} + C \cdot 1 = A\bar{B} + C$

**Example 71.10.** Using truth table, prove that  $A + \bar{A}B = A + B$  and illustrate the equivalence with the help of a switching circuit.

**Solution.** Since there are only two variables  $A$  and  $B$ , their number of possible combination is  $2^2 = 4$  in terms of 0 and 1.

Table No. 71.1					
A	B	$\bar{A}$	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

As seen,  $\bar{A}$  is the negative of  $A$ . In the fourth column of Table 71.1,  $\bar{A}$  has been ANDed with  $B$ . In the fifth column,  $A$  has been ORed with  $\bar{A}B$ . The values in last column have been obtained by ORing  $A$  with  $B$ . By comparing results of column 5 to 6, the equivalence between the two statements can be proved.

Switching circuit of Fig. 71.10 (a) represents  $(A + \bar{A}B)$ . In this circuit, when  $A$  is open,  $\bar{A}$  is closed and vice versa. It can be shown that this circuit becomes closed with either  $A$  or  $B$  is closed.

(i) when  $A$  is closed, then  $\bar{A}$  opens. Circuit is completed via the upper branch.

(ii) keeping  $A$  open, when we close  $B$ , the circuit again becomes closed via lower branch because  $\bar{A}$  is already closed (due to  $A$  being open).

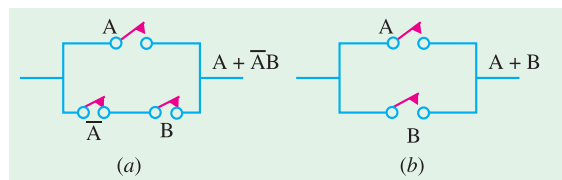


Fig. 71.10

Hence, all that we have to do for closing the circuit of Fig. 71.10 (a) is to close either switch  $A$  or  $B$ . It is exactly what circuit of Fig. 71.10 (b) does.

**Example 71.11.** Simplify the expression :  $(AB + C)(AB + D)$

**Solution.** Let  $Y = (AB + C)(AB + D)$

$$\begin{aligned}
 &= ABAB + ABD + ABC + CD && \text{---Law 19} \\
 &= AAB + ABD + ABC + CD \\
 &= AB + ABD + ABC + CD && \text{---Law 7} \\
 &= AB(1 + D) + ABC + CD = AB + ABC + CD && \text{---Law 2} \\
 &= AB(1 + C) + CD = AB + CD
 \end{aligned}$$

$\therefore (AB + C)(AB + D) = AB + CD$

### 71.5. DE Morgan’s Theorem

These two theorems (or rules) are a great help in *simplifying complicated logical expressions*. The theorems can be stated as under :

Law 25.  $\overline{A + B} = \bar{A} \cdot \bar{B}$     Law 26.  $\overline{A \cdot B} = \bar{A} + \bar{B}$

The first statement says that *the complement of a sum equals the product of complements*. The second statement says that *the complement of a product equals the sum of the complements*. In fact, it allows transformation from a sum-of-products from to a product-of-sum from.

As seen from the above two laws, the procedure required for taking out an expression from under a NOT sign is as follows :

1. complement the given expression i.e., remove the overall NOT sign
2. change all and ANDs to ORs and all the ORs to ANDs.
3. complement or negate all individual variables.

As an illustration, take the following example

$$\begin{aligned}
 \overline{A + BC} &= A + BC && \text{---step 1} \\
 &= A(B + C) && \text{---step 2} \\
 &= \bar{A}(\bar{B} + \bar{C}) && \text{---step 3}
 \end{aligned}$$

Next, consider this example,

$$\begin{aligned}
 \overline{(\bar{A} + B + \bar{C})(\bar{A} + B + C)} &= (\bar{A} + B + \bar{C})(\bar{A} + B + C) && \text{---step 1} \\
 &= \bar{A}B\bar{C} + \bar{A}BC && \text{---step 2} \\
 &= \overline{\overline{\bar{A}B\bar{C}}} + \overline{\overline{\bar{A}BC}} && \text{---step 3} \\
 &= A\bar{B}C + A\bar{B}\bar{C}
 \end{aligned}$$

This process is called **demorganization**. It should, however, be noted that opposite procedure would be followed in bringing an expression under the NOT sign. Let us bring the expression  $\bar{A} + \bar{B} + \bar{C}$  under the NOT sign.

$$\begin{aligned}
 \overline{\bar{A} + \bar{B} + \bar{C}} &= \overline{\overline{\overline{\bar{A} + \bar{B} + \bar{C}}}} && \text{---step 3} \\
 &= \overline{\overline{A + B + C}} && \text{---step 2} \\
 &= \overline{ABC} && \text{---step 1} \\
 &= \overline{ABC}
 \end{aligned}$$

Fig. 71.11 shows the circuits to illustrate De Morgan’s theorems.

As seen, basic logic function can be either an OR gate or an AND gate.

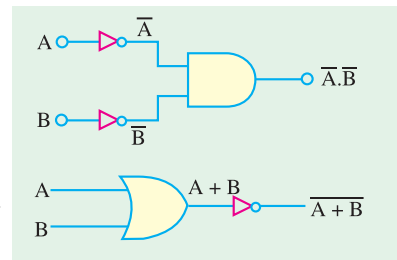


Fig. 71.11

**Example 71.12.** Demorganize the expression :  
 $(A + B)(C + D)$

**Solution.** The procedure is as explained above.

$$\begin{aligned} \overline{(A + B)(C + D)} &= \overline{(A + B)(C + D)} && \text{---step 1} \\ &= \overline{AB} + \overline{CD} && \text{---step 2} \\ &= \overline{AB} + \overline{CD} && \text{---step 3} \end{aligned}$$

**Example 71.13.** Simplify each of the following expressions using De Morgan's theorems :

- (a)  $\overline{A(B + C)D}$   
 (b)  $\overline{(M + N)(\overline{M} + N)}$       (c)  $\overline{\overline{AB} \overline{C} D}$

**Solution.** Please note that there is more than one way of simplifying the expressions given in part (a), (b) and (c).

(a) 
$$\begin{aligned} \overline{A(B + C)D} &= \overline{A(B + C)D} && \dots \text{step 1} \\ &= \overline{A + (B + C) + D} && \dots \text{step 2} \\ &= \overline{A + B + C + D} && \dots \text{step 3} \\ &= \overline{A + B + C + D} && \dots(\because \overline{B + C} = \overline{B} \overline{C}) \end{aligned}$$

(b) 
$$\begin{aligned} \overline{(M + N)(\overline{M} + N)} &= \overline{(M + N)(\overline{M} + N)} && \dots \text{step 1} \\ &= \overline{M\overline{N}} + \overline{\overline{M}N} && \dots \text{step 2} \\ &= \overline{M\overline{N}} + \overline{\overline{M}N} && \dots \text{step 3} \\ &= \overline{M\overline{N}} + \overline{\overline{M}N} && \dots \text{step 4} \end{aligned}$$

(c) 
$$\begin{aligned} \overline{\overline{AB} \overline{C} D} &= \overline{\overline{AB} \overline{C} D} && \dots \text{step 1} \\ &= \overline{\overline{AB} \overline{C}} + \overline{D} && \dots \text{step 2} \\ &= \overline{\overline{AB} \overline{C}} + \overline{D} && \dots \text{step 3} \\ &= \overline{\overline{AB} \overline{C}} + \overline{D} && \dots(\because \overline{\overline{AB} \overline{C}} = \overline{\overline{AB}} \overline{\overline{C}}) \end{aligned}$$

The term  $\overline{\overline{AB} \overline{C}}$  can be simplified further by De Morganising the term  $\overline{\overline{AB}}$  as  $\overline{\overline{A} + \overline{\overline{B}}}$ . Thus

$$\overline{\overline{\overline{AB} \overline{C}}} = \overline{\overline{AB} \overline{C}} + \overline{D} = (\overline{A} + \overline{B}) \overline{C} + \overline{D} = \overline{A} \overline{C} + \overline{B} \overline{C} + \overline{D}$$

**Example 71.14.** Find switching circuits for the following logic expressions :

- (i)  $A \cdot (B + C)$                       (ii)  $A \overline{B} + CD$                       (iii)  $(\overline{A} B + AC) \overline{C}$

(Industrial Electronics, City & Guilds, London)

**Solution.** (i)  $A \cdot (B + C)$

Here, switches  $B$  and  $C$  have been *O*Red i.e. connected in parallel. This parallel circuit is connected in series with switch  $A$  because  $(B + C)$  has been *A*ND ed with  $A$ . hence, the circuit becomes as shown in Fig. 71.12 (a). As seen, it is a series-parallel circuit.



De Morgan (1806–1871)

(ii)  $\overline{AB} + CD$

Here,  $\overline{AB}$  has been ORed with  $CD$ . We can easily make out that a 2-branched circuit is needed for this logic expression. One branch contains switch  $A$  in series with  $\overline{B}$  (with one contact point shows raised to indicate negation) and the other branch contains two series-connected switches  $C$  and  $D$  as show in Fig. 71.12 (b).

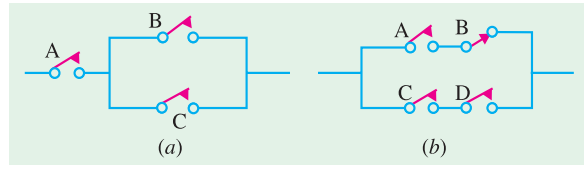


Fig. 71.12

(iii)  $(\overline{AB} + AC)\overline{C}$

From the look of it, we can make out that it consists of a series-parallel circuit as shown in Fig. 71.13.  $\overline{A}B$  has been ORed with  $AC$  i.e.  $\overline{A}B$  and  $AC$  are in parallel. Of course,  $\overline{A}$  and  $B$  are in series in one branch where as  $A$  and  $C$  are in series in the other branch. Both these parallel branches are in series with  $\overline{C}$ .

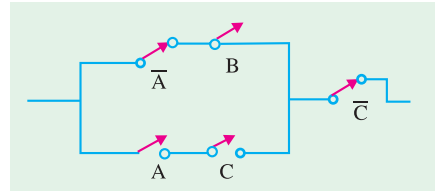


Fig. 71.13

**Example 71.15.** Prove that 3-input NAND gate of Fig. 71.14 (a) is equivalent to the bubbled AND gate of Fig. 71.14 (b).

**Solution.** The output of NAND gate is  $\overline{ABC}$  and that of bubbled OR gate is  $\overline{A} + \overline{B} + \overline{C}$ . We have to show that the above two expressions are equivalent.

De Morgan's theorem can be used to prove the above equivalence,

$$\begin{aligned} \overline{ABC} &= \overline{ABC} && \text{---step 1} \\ &= \overline{A + B + C} && \text{---step 2} \\ &= \overline{A} + \overline{B} + \overline{C} && \text{---step 3} \end{aligned}$$

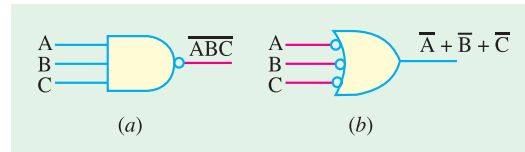


Fig. 71.14

71.6. Duals

Basic duality underlies all Boolean algebra. *Each expression has its dual which is as true as the original expression.* For getting the dual of a given Boolean expression, the procedure is to convert

1. all 1s to 0s and all 0s to 1s.
2. all ANDs to ORs and all ORs to ANDs.

The dual so obtained *is also found to be true.*

Some of the Boolean relations and their duals are given in Table 71.2.

**Example 71.16.** Design a logic circuit whose output is HIGH only when a majority of the inputs  $A, B$  and  $C$  are HIGH.

**Solution.** Since there are three inputs,  $A, B$  and  $C$ , therefore whenever two or more than two (i.e. a majority) inputs are HIGH, the output is HIGH. This situation can be represented in the form of a truth table as shown in Fig. 71.15.

Table No. 71.2			
Relation		Dual Relation	
$A \cdot 0$	$= 0$	$A + 1$	$= 1$
$A \cdot A$	$= A$	$A + A$	$= A$
$A \cdot \overline{A}$	$= 0$	$A + \overline{A}$	$= 1$
$A \cdot 1$	$= A$	$A + 0$	$= A$
$A \cdot (A + B)$	$= A$	$A + AB$	$= 1$
$A \cdot (\overline{A} + B)$	$= AB$	$A + \overline{A}B$	$= A + B$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$\bar{A}BC$

$A\bar{B}C$

$AB\bar{C}$

$ABC$

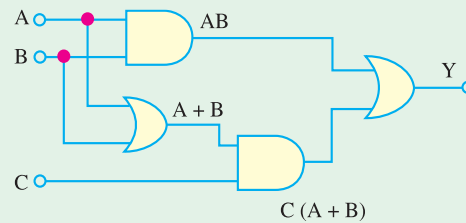


Fig. 71.15

Fig. 71.16

There are four 1s in the output column of the truth table. The corresponding binary values are 011, 101, 110 and 111 respectively. Converting these values into product terms and summing up all the terms, we get

$$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$$

Adding the term  $ABC$  two times from our side in the Boolean expression for the output,

$$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC \quad (\because ABC + ABC + ABC = ABC)$$

Bringing together those terms which have two common letters, we get,

$$\begin{aligned} Y &= AB\bar{C} + ABC + A\bar{B}C + ABC + \bar{A}BC + ABC \\ &= AB(\bar{C} + C) + AC(\bar{B} + B) + BC(\bar{A} + A) \\ &= AB + AC + BC \quad (\because \bar{C} + C = \bar{B} + B = \bar{A} + A = 1) \\ &= AB + C(A + B) \end{aligned}$$

The logic circuit that produces the output  $Y = AB + C(A + B)$  is as shown in Fig. 71.16.

Alternatively :

You can also arrange the equation,

$$Y = AB + AC + BC$$

as  $Y = A(B + C) + BC \quad \dots (i)$

or  $Y = (A + C)B + AC \quad \dots (ii)$

If you implement equation (i) or (ii) using AND and OR logic gates, the number of logic gates is used will still be the same (4).

**Example 71.17.** Determine the Boolean expression for the logic circuit shown in Fig. 71.17. Simplify the Boolean expression using Boolean Laws and De Morgan's theorem. Redraw the logic circuit using the simplified Boolean expression.

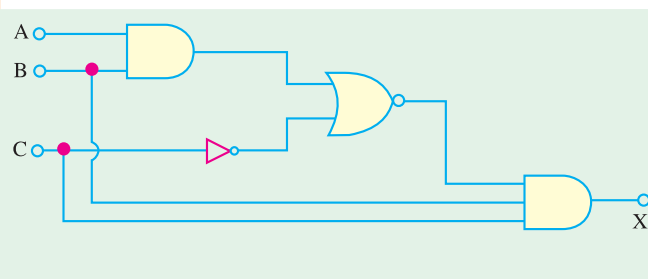


Fig. 71.17

**Solution.** The output of the given circuit can be obtained by determining the output of each logic gate while working from left to right.

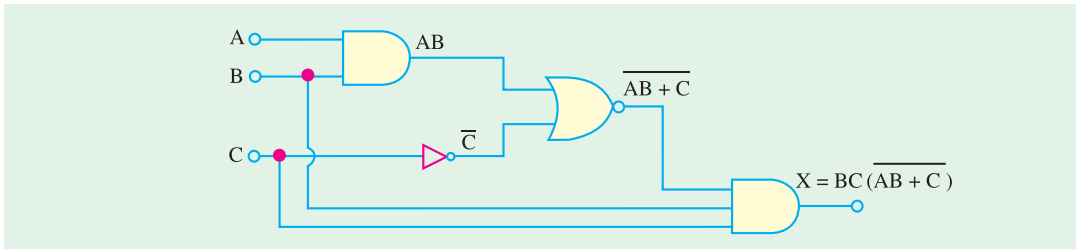


Fig. 71.18

As seen from the logic circuit shown in Fig. 71.18. The output of the circuit is,

$$X = BC \overline{(AB + C)}$$

The output, X can be simplified by De Morganizing the term  $\overline{(AB + C)}$  as shown below.

$$\begin{aligned} BC \overline{(AB + C)} &= BC(AB + \overline{C}) \quad \dots \text{step 1} \\ &= BC(A + B) \cdot \overline{C} \quad \dots \text{step 2} \\ &= BC(\overline{A} + \overline{B}) \cdot \overline{C} \quad \dots \text{step 3} \\ &= BC(\overline{A} + \overline{B})C \quad \dots \text{Law 13} \\ &= BC(\overline{A} + \overline{B}) \quad \dots \text{Law 7} \\ &= \overline{A}BC + BC\overline{B} \\ &= \overline{A}BC + 0 = \quad \dots \text{Law 8} \\ &= \overline{A}BC \quad \dots \text{Law 1} \end{aligned}$$

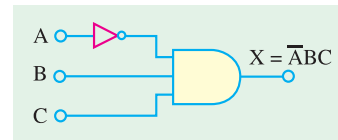


Fig. 71.19

The logic circuit with a simplified Boolean expression  $X = \overline{A}BC$  is as shown in Fig. 71.19.

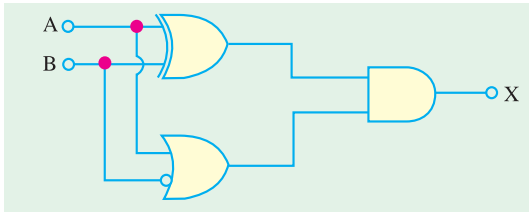


Fig. 71.20

**Example 71.18.** Determine the output X of a logic circuit shown in Fig. 71.20. Simplify the output expression using Boolean Laws and theorems. Redraw the logic circuit with the simplified expression.

**Solution.** The output of the given logic circuit can be obtained by determining the output of each logic gate while working from left to right.

As seen from Fig. 71.21, the output,

$$\begin{aligned} X &= (\overline{A}B + A\overline{B})(A + \overline{B}) \\ &= \overline{A}BA + A\overline{B}A + \overline{A}B\overline{B} + A\overline{B}\overline{B} \\ &= 0 + A\overline{B}A + 0 + A\overline{B}\overline{B} \quad \dots \text{Law 8} \\ &= A\overline{B} + A\overline{B} \quad \dots \text{Law and Law 1} \\ &= A\overline{B} \quad \dots \text{Law 3} \end{aligned}$$

Using the simplified Boolean expression, the logic circuit is as shown in Fig. 71.22.

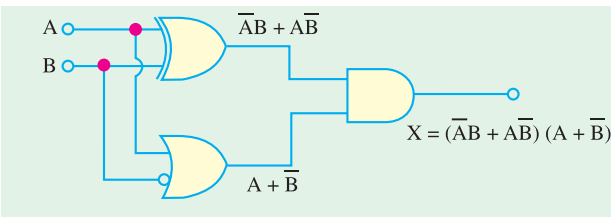


Fig. 71.21

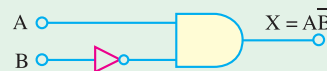


Fig. 71.22

**Example 71.19.** Consider the logic circuit shown in Fig. 71.23. Determine the Boolean expression at the circuit output, simplify it. From the simplified Boolean expression, find which logic gate is redundant in the given logic circuit.

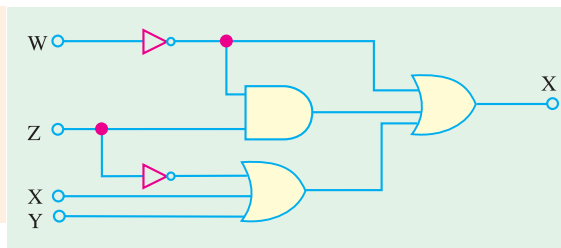


Fig. 71.23

**Solution.** As seen from Fig. 71.24, the logic circuit output,

$$X = \bar{W} + \bar{W}Z + XY\bar{Z}$$

The expression can be simplified as follows :

$$\begin{aligned} &= \bar{W} + \bar{W}Z + XY\bar{Z} && \text{---Law 19} \\ &= \bar{W}(1 + Z) + XY\bar{Z} \\ &= \bar{W} + XY\bar{Z} && \text{---Law 2} \end{aligned}$$

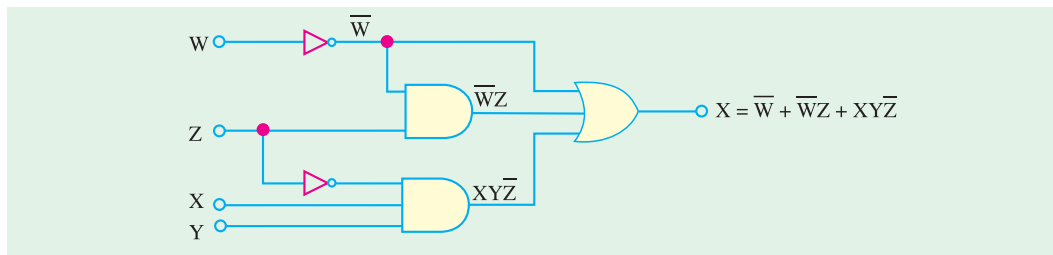


Fig. 71.24

From the simplified Boolean expression of the output  $X = \bar{W} + XY\bar{Z}$ , and the actual output  $\bar{W} + \bar{W}Z + XY\bar{Z}$ , we find that the two-input AND gate (producing the term  $\bar{W}Z$ ) is redundant.

**Example 71.20.** Determine the output of the logic circuit shown in Fig. 71.25. Simplify the output Boolean expression and sketch the logic circuit.

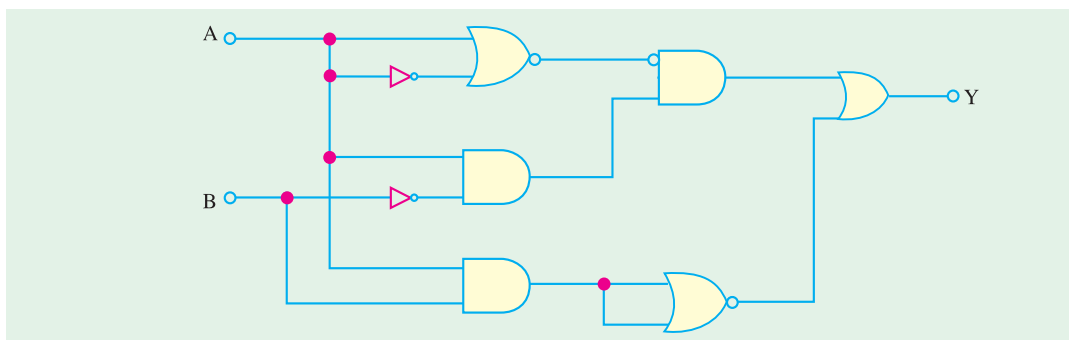


Fig. 71.25

**Solution.** The output of the circuit can be obtained by determining the output of each logic gate while working from left to right.

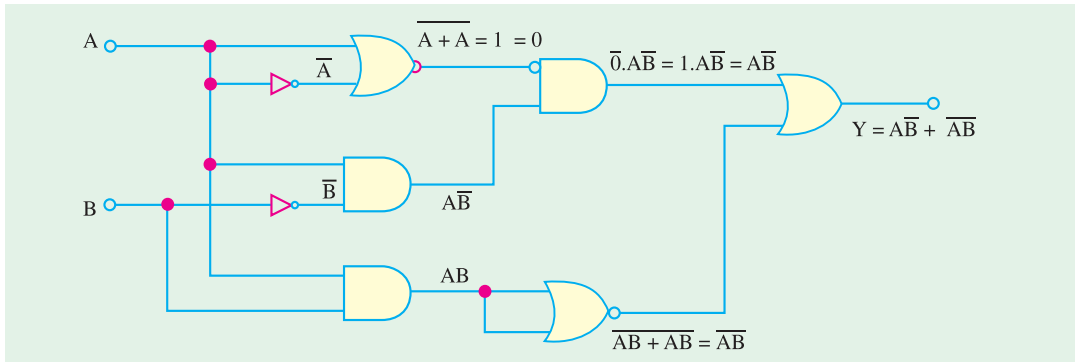


Fig. 71.26

As seen from the circuit shown in Fig. 71.26, we find that the output,

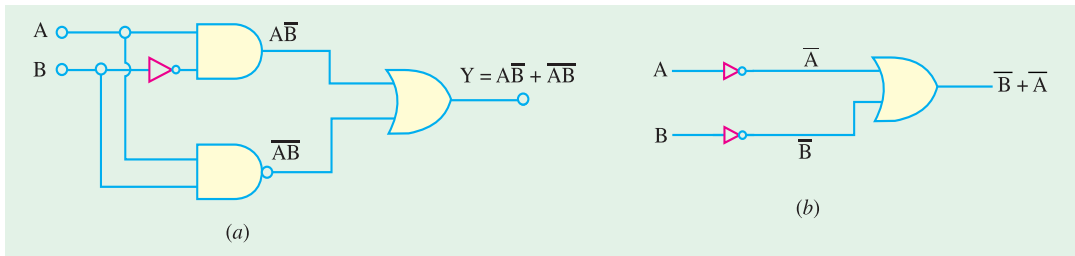


Fig. 71.27

$$y = A\bar{B} + \bar{A}B$$

The sketch of a logic circuit for the simplified Boolean expression is as shown in Fig. 71.27.

**Alternatively :**

$$\begin{aligned} y &= A\bar{B} + \bar{A}B \\ &= A\bar{B} + (\bar{A} + \bar{B}) \\ &= (A\bar{B} + \bar{B}) + \bar{A} \\ &= (A+1)\bar{B} + \bar{A} \\ &= \bar{B} + \bar{A} \end{aligned}$$

The logic circuit to implement this logic equation is as shown in Fig. 71.27(b). Notice the difference in terms of the number and type of logic gates used in the circuits shown in Fig. 71.27. So when you are simplifying and designing logic circuits, it is always possible to have more than one solution.

The circuit shown in Fig. 71.27 shows that it makes use of one inverter (or NOT gate), one AND gate, one NAND gate and one OR gate. In other words, there are four different types of logic gates. However the logic circuit of Fig. 71.27 (b) makes use of only three logic gates (two inverters and one OR gate).

### 71.7. Standard Forms of Boolean Expressions

All Boolean expressions, regardless of their form, can be converted into either of the two following standard forms :



1. Sum-of-products (*SOP*) form and
2. Product-of-sums (*POS*) form.

The standardization of Boolean expressions makes their evaluation, simplification and implementation much more systematic and easier. Now we shall discuss these two standard forms in more detail.

### 71.8. The Sum-of-Products (SOP) Form

A product term is a term consisting of the product (or Boolean multiplication) of literals (*i.e.* variables or their complements). When we add two or more product terms, the resulting expression is called, sum-of-products (*SOP*) expression. Some examples of sum-of-products expressions are  $\overline{A}B + \overline{A}BC$ ,  $\overline{A}BC + \overline{A}CD + \overline{A}BCD$ ,  $\overline{A}B + AC + \overline{A}B C$

Sometimes, it is convenient to define the set of variables contained in the expression (in either complemented or uncomplemented form) as a *domain*. For example, domain of the expression  $\overline{A}B + AB$  is the set of variables  $A$  and  $B$ . Similarly the domain of the expression  $\overline{A}BC + ABC + A\overline{B}CD$  is the set of variables  $A, B, C$  and  $D$ .

Any logic expression can be changed into *SOP* form by applying Boolean algebra laws and theorems. For example, the expression  $A(BC + D)$  can be converted to *SOP* form by applying the distributive law :

$$A(BC + D) = ABC + AD$$

### 71.9. The Standard SOP Form

So far, we have seen *SOP* (sum-of-products) expressions in which some of the product terms do not contain all the variables in the domain of the expression. For example, the expression,  $\overline{A}BC + A\overline{C}D + A\overline{B}CD$  has a domain made up of the variables  $A, B, C$  and  $D$ . But notice that the first two terms contains only three variables, *i.e.*  $D$  or  $\overline{D}$  is missing from the first term and  $B$  or  $\overline{B}$  is missing from the second term.

A standard *SOP* expression is defined as an expression in which all the variables in the domain appear in each product term. For example  $\overline{A}BCD + A\overline{B}\overline{C}D + A\overline{B}CD$  is a standard *SOP* expression. Standard *SOP* expressions are important in constructing truth tables and in karnaugh map simplification method.

It is very straightforward to convert non-standard product term to a standard *SOP* using Boolean algebra. Each product term in the *SOP* expression that does not contain all the variables in the domain has to be expanded to standard form to include all the variables in the domain and their complements. As stated below, a nonstandard *SOP* expression is converted into standard form using Boolean algebra law 4 :  $A + \overline{A} = 1$  *i.e.* a variable added to its complement equals 1.

**Step 1.** Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms. It is possible because we know that we can multiply anything by 1 without changing its value.

**Step 2.** Repeat step 1 until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form. Note that in converting a product term to a standard form, the number of product terms is doubled for each missing variable.

For example, suppose we want to convert the Boolean expression  $\overline{A}BC + A\overline{C}D + A\overline{B}CD$  to a standard *SOP* form. Then following the above procedure we proceed as below :

$$\begin{aligned} & \overline{A}BC(D + \overline{D}) + A(B + \overline{B})\overline{C}D + A\overline{B}CD \\ & = \overline{A}BCD + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D + A\overline{B}C\overline{D} + A\overline{B}CD \end{aligned}$$

The expression given above is a standard *SOP* expression.

### 71.10. The Product-of-sums (POS) Form

The sum term is a term consisting of the sum (or Boolean addition) of literals (*i.e.* variables or their complements). When we multiply two or more sum terms, the resulting expression is called product-of-sums (*POS*). Some examples of *POS* form are  $(A + \bar{B})(\bar{A} + B + C)$ ,  $(\bar{A} + B + C)(A + \bar{C} + D)(A + \bar{B} + C + D)$  and  $(\bar{A} + \bar{B})(A + C)(\bar{A} + \bar{B} + C)$ .

It may be carefully noted that a *POS* expression, can contain a single-variable term as in  $\bar{A}(A + \bar{B} + C)(B + \bar{C} + \bar{D})$ . In *POS* expression, a single overbar cannot extend over more than one variable, although more than one variable in a term can have an overbar.

### 71.11. The Standard POS Form

So far, we have seen *POS* (product-of-sums) expressions in which some of the sum terms do not contain all the variables in the domain of the expression. For example, the expression,  $(\bar{A} + B + C)(A + \bar{C} + D)(A + \bar{B} + C + D)$  has a domain made up of variables, *A*, *B*, *C* and *D*. Notice that the complete set of variables in the domain is not represented in the first two terms of the expression, *i.e.* *D* or  $\bar{D}$  is missing in the first term and *B* or  $\bar{B}$  is missing in the second term.

A standard *POS* expression is defined as an expression in which all the variables in the domain appear in each sum term. For example  $(\bar{A} + B + C + D)(A + B + \bar{C} + D)(A + \bar{B} + C + D)$  is a standard *POS* form. Any nonstandard *POS* expression can be converted to a standard form using Boolean algebra.

Each sum term in an *POS* expression that does not contain all the variables in the domain can be expanded to standard form to include all variables in the domain and their complements. As stated below : a nonstandard *POS* expression is converted into a standard form using Boolean algebra Law 8 :  $A \cdot \bar{A} = 0$ , *i.e.* a variable multiplied by its complemented equals 0.

**Step 1.** Add to each nonstandard product term a term made up of the product of a missing variable and its complement. This results in two sum terms. This is possible because we know that we can add 0 to anything without changing its value.

**Step 2.** Apply law 20, *i.e.*  $A + BC = (A + B)(A + C)$

**Step 3.** Repeat Step 1 until all resulting sum terms contain all variables in the domain in either complemented or uncomplemented form.

For example we want to convert the Boolean expression,

$$(\bar{A} + B + C)(A + \bar{C} + D)(A + \bar{B} + C + D)$$

into a standard *POS* form. Then following the above procedure, we proceed as below :

$$\begin{aligned} & (\bar{A} + B + C + D\bar{D})(A + B\bar{B} + \bar{C} + D)(A + \bar{B} + C + D) \\ & = (\bar{A} + B + C + D)(\bar{A} + B + C + \bar{D})(A + B + \bar{C} + D)(A + \bar{B} + \bar{C} + D)(A + \bar{B} + C + D) \end{aligned}$$

The expression given above is a standard *POS* expression

### 71.12. The Karnaugh Map

The Karnaugh map (or simply a *K*-map) is similar to a truth table because it presents all the possible values of input variables and the resulting output for each value. However, instead of being organised into columns and rows like a truth table, the Karnaugh map is an array of squares (or cells) in which each square represents a binary value of the input variables. The squares are arranged in a way so that simplification of given expression is simply a matter of grouping the squares. Karnaugh maps can be used for expression with two-three, four, and five variable Karnaugh maps to illustrate the principles. Karnaugh map with five-variables is beyond the scope of this book. For higher number of

variables, a Quine-McClusky method can be used. This method is also beyond the scope of this book.

The number of squares in a Karnaugh map is equal to the total number of possible input variable combinations (as is the number of rows in a truth table). For two variables, the number of square is  $2^2 = 4$ , for three variables, the number of squares is  $2^3 = 8$  and for four variables, the number of squares is  $2^4 = 16$ .

### 71.13. The Two-variable Karnaugh Map

Fig. 71.28 (a) shows a two-variable Karnaugh map. As seen, it is an array of four squares. In this case,  $A$  and  $B$  are used for two variables although any other two letters could be used. The binary values of  $A$  (*i.e.* 0 and 1) are indicated along the left side as  $\bar{A}$  and  $A$  (notice the sequence) and the binary values of  $B$  are indicated across the top as  $\bar{B}$  and  $B$ . The value of a given square is the value of  $A$  at the left in the same row combined with the value of  $B$  at the top in the same column. For example, a square in the upper left corner has a value of  $\bar{A}\bar{B}$  and a square in the lower right corner has a value of  $AB$ . Fig. 71.28 (b) shows the standard product terms represented by each square in the Karnaugh map.

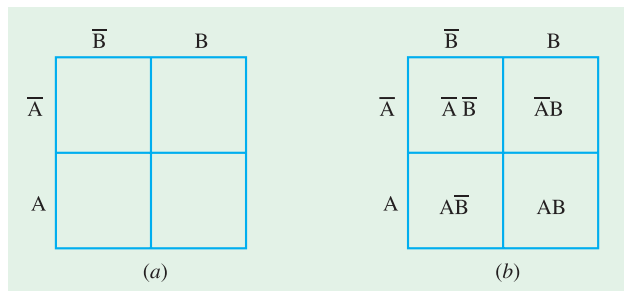


Fig. 71.28

### 71.14. The Three-variable Karnaugh Map

Fig. 71.29 (a) shows a three-variable Karnaugh map. As seen it is an array of eight squares. In this case,  $A$ ,  $B$  and  $C$  are used for the variables although any other three letters could be used. The value of  $A$  and  $B$  are along the left side (notice the sequence carefully) and the values of  $C$  are across the top.

The value of a given square is the values of  $A$  and  $B$  at the left in the same row combined with the value of  $C$  at the top in the same column. For example, a square in the upper left corner has a value of  $\bar{A}\bar{B}\bar{C}$  and a square in the bottom right corner has a value of  $A\bar{B}C$ . Fig. 71.29 (b) shows the product terms that are represented by each square in the Karnaugh map.

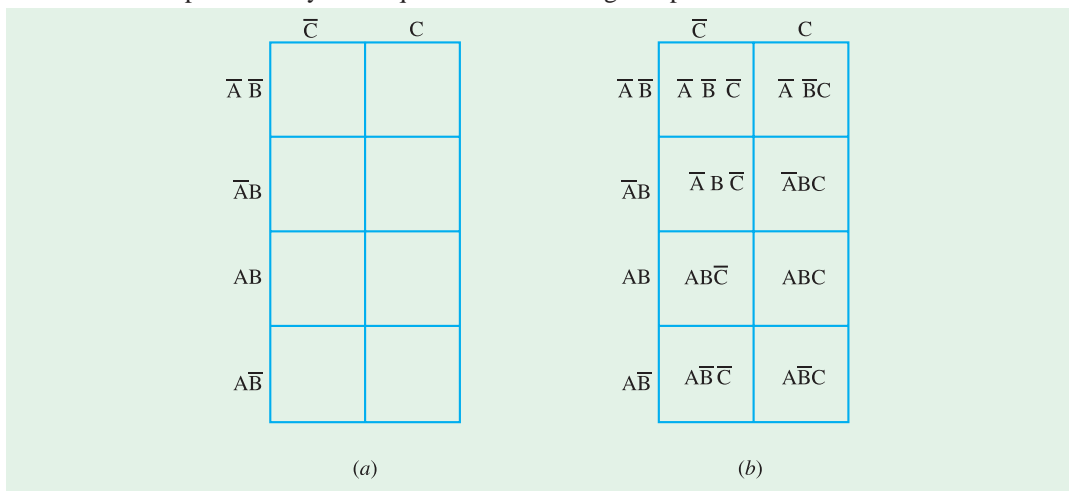


Fig. 71.29

### 71.15. The Four-variable Karnaugh Map

Fig. 71.30 (a) shows a four-variable Karnaugh map. As seen, it is an array of sixteen squares. In this case  $A, B, C$  and  $D$  are used for the variables. The values of  $A$  and  $B$  are along the left side, and the values of  $C$  and  $D$  are across the top. The sequence of the variable values may be noted carefully.

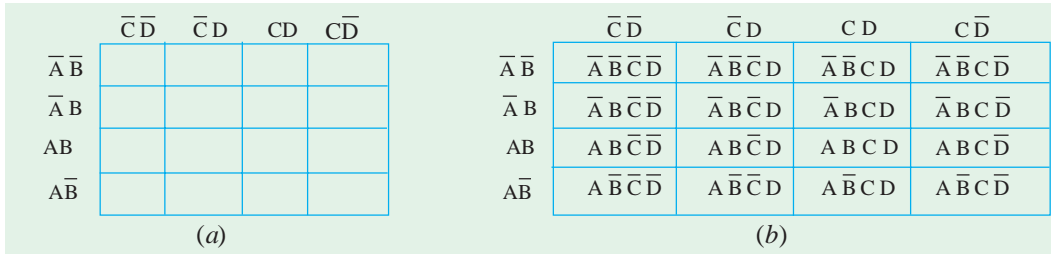


Fig. 71.30

The value of a given square is the values of  $A$  and  $B$  at the left in the same row combined with the values of  $C$  and  $D$  at the top in the same column. For example, a square in the upper right corner has a value  $\bar{A}\bar{B}C\bar{D}$  and a square in the lower left corner has a value  $ABC\bar{D}$ . Fig. 71.30 (b) shows the standard product terms that are represented by each square in the four-variable Karnaugh map.

### 71.16. Square Adjacency in Karnaugh Map

We have already discussed a two-variable Karnaugh map, a three-variable Karnaugh map and a four-variable Karnaugh map. Now we shall discuss the concept of square adjacency in a Karnaugh map.

It will be interesting to know that the squares in a Karnaugh map are arranged in such a way that there is only a single-variable change between adjacent squares. Adjacency is defined as a single-variable change. It means the squares that differ by only one variable are adjacent. For example, in a three-variable Karnaugh map shown in Fig. 71.29 (b), the  $\bar{A}B\bar{C}$  square is adjacent to  $\bar{A}\bar{B}\bar{C}$  square, the  $\bar{A}BC$  square and the  $ABC\bar{C}$  square. It may be carefully noted that square with values that differ by more than one variable are not adjacent. For example, the  $\bar{A}B\bar{C}$  square is not adjacent to the  $\bar{A}\bar{B}C$  square, the  $ABC$  square, the  $A\bar{B}\bar{C}$  square or the  $A\bar{B}C$  square. In other words, each square is adjacent to the squares that are immediately next to it on any of its four sides. However, a square is not adjacent to the squares that diagonally touch any of its corners.

It may also be noted that squares in the top row are adjacent to the corresponding squares in the bottom row and squares in the outerleft column are adjacent to the corresponding squares in the outer right column. This is called “wraparound” adjacency because we can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder. Fig. 71.31 (a) and (b) shows the square adjacencies with a three-variable and a four-variable Karnaugh maps respectively.

Notice the square adjacencies in a four variable Karnaugh map :

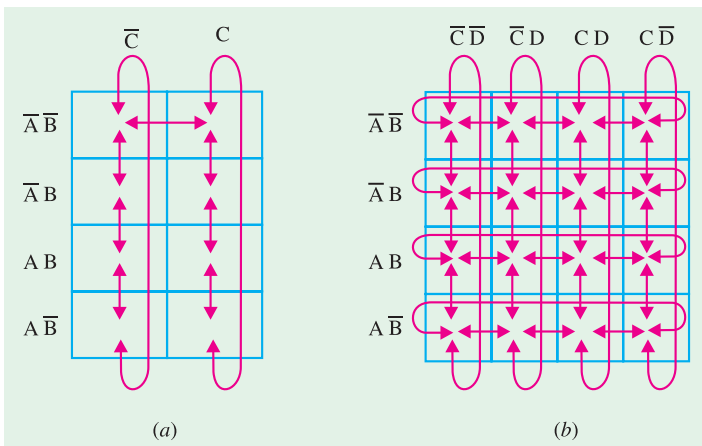


Fig. 71.31

Here for example, the square  $\bar{A}\bar{B}\bar{C}\bar{D}$  is adjacent to  $\bar{A}\bar{B}\bar{C}D$  square,  $\bar{A}\bar{B}C\bar{D}$  square,  $A\bar{B}\bar{C}\bar{D}$  square and  $A\bar{B}C\bar{D}$  square. Similarly  $\bar{A}B\bar{C}\bar{D}$  square is adjacent to  $\bar{A}\bar{B}\bar{C}\bar{D}$  square,  $\bar{A}B\bar{C}\bar{D}$  square,  $\bar{A}BC\bar{D}$  square and  $ABC\bar{D}$  square.

### 71.17. Mapping a Standard SOP Expression on the Karnaugh Map

Consider a SOP (sum-of-products) expression.  $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$ . In order to map this expression on the Karnaugh map, we need a three variable Karnaugh map because the given expression has three variables  $A, B,$  and  $C$ . Then select the first product term  $\bar{A}\bar{B}\bar{C}$  and enter 1 in the corresponding square (i.e. the first row and the first column) as shown in Fig. 71.32.

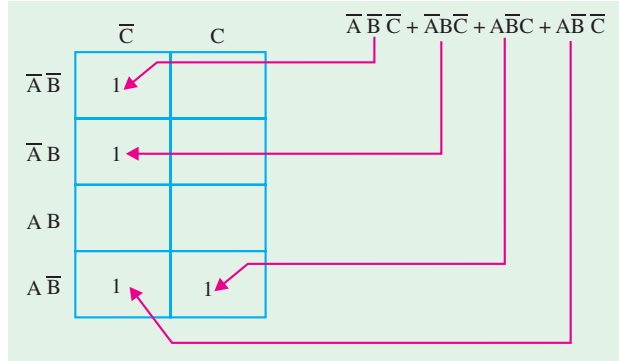


Fig. 71.32

Similarly, for the second product term,  $\bar{A}B\bar{C}$  place a 1 in the second row and first column. Repeat this process for the other two product terms, i.e.  $\bar{A}\bar{B}C$  and

$A\bar{B}\bar{C}$ . The squares that do not have 1 are the squares for which the expression is 0. Usually when working with sum-of-products expressions, the 0s are left off the map.

**Example 71.21.** Map the following SOP expression on the Karnaugh map:  $\bar{A}\bar{B}C + \bar{A}BC + ABC + ABC$ .

**Solution.** Sketch a three variable Karnaugh map as shown in Fig. 71.33. Select the first product term  $\bar{A}\bar{B}C$  and enter 1 in the cor-

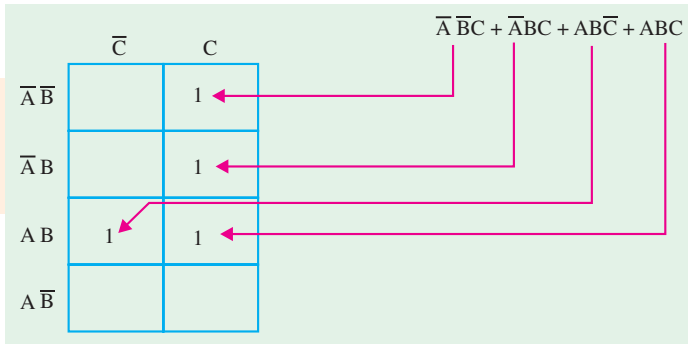


Fig. 71.33

responding square. Similarly enter 1 for the other product terms in the given SOP expression. Check that number of 1s in the Karnaugh map is equal to the number of product terms in the given SOP expression.

**Example 71.22.** Map the following standard sum-of-products (SOP) expression on a Karnaugh map:

$$\bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + ABCD$$

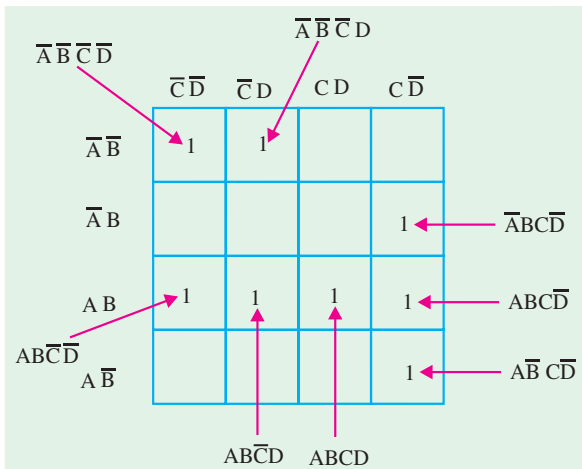


Fig. 71.34

**Solution.** Sketch a four variable Karnaugh map. Select the first product term  $\bar{A} B C \bar{D}$  from the given *SOP* expression and enter 1 in the corresponding square as shown in Fig. 71.34. Similarly enter 1 for the other product terms in the given *SOP* expression. Check that number of 1s in the Karnaugh Map is equal to the number of product terms in the given *SOP* expression.

### 71.18. Mapping a Nonstandard SOP Expression on the Karnaugh Map

A nonstandard sum-of-products (*SOP*) expression is a one that has product terms with one or more missing variables. In such a case, Boolean expression must first be converted to a standard form by a procedure explained in Art. 71.9.

Let us consider an example to illustrate the procedure for mapping a nonstandard *SOP* expression on the Karnaugh map. Suppose we have the *SOP* expression :

$$\bar{A} + \bar{A}\bar{B} + ABC\bar{C}$$

As seen, this expression is obviously not in standard form because each product term does not have three variables. The first term is missing two variables, the second term is missing one variable and the third term is standard. In order to convert the given nonstandard *SOP* expression to a standard form, we multiply the first product term by  $B + \bar{B}$  and  $C + \bar{C}$ , and the second term by  $C + \bar{C}$ . Expanding the resulting expression, we get,

$$\begin{aligned} & \bar{A}(B + \bar{B})(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + ABC\bar{C} \\ &= \bar{A}BC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC\bar{C} \end{aligned}$$

Rearranging the expression for our convenience, we get

$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC\bar{C} + A\bar{B}\bar{C} + ABC\bar{C}$$

This expression can be mapped on a three-variable Karnaugh map as shown in Fig. 71.35.

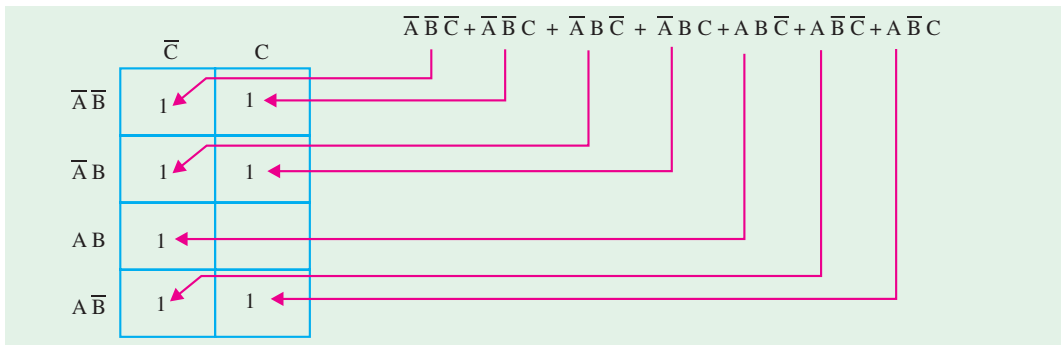


Fig. 71.35

### 71.19. Simplification of Boolean Expression Using Karnaugh Map

The process that results in an expression containing the minimum number of possible terms with the minimum number of variables is called **simplification** or **minimization**. After the *SOP* (or the Boolean) expression has been mapped on the Karnaugh map, there are three steps in the process of obtaining a minimum *SOP* expression. The three steps are : (a) grouping the 1s, (b) determining the product term for each group and (c) summing the resulting product terms.

(a) **Grouping the 1s** : We can group the 1s on the Karnaugh map according to the following rules by enclosing those adjacent squares containing 1s. The objective is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8 or 16 squares. In the case of two-variable Karnaugh map, 4 squares is the maximum group, for three-variable map, 8 squares are the maximum group and so on.
2. Each square in the group must be adjacent to one or more squares in that same group but all squares in the same group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the Karnaugh map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include non-common 1s.

(b) **Determining the Product Term for each group** : Following are the rules that are applied to find the minimum product terms and the minimum sum-of-products expression :

1. **Group the squares that have 1s** : Each group of squares containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are known as contradictory variables.
2. In order to determine the minimum product term for each group, we need to look at the standard methodology for three-variable and four-variable Karnaugh map respectively.
  - (i) **For a three-variable K-map** : (1) for 1-square, group we get a three-variable product term, (2) for a 2-square group, we get a two-variable product term, (3) for a 4-square product term, we get a one-variable product term.
  - (ii) **For a four-variable K-map** : (1) For a 1-square group, we get a four-variable product term, (2) for a 2-square group, we get a three-variable product term, (3) for a 4-square group, we get a two-variable product term and (4) for a 8-square group, we get a one-variable product term.

(c) **Summing the resulting product terms** : When all the minimum product terms are derived from the Karnaugh map, these are summed to form the minimum sum-of-products expression.

**Note** : In some cases, there may be more than one way to group the 1s to form the product terms. Whatever be the way, the minimal expression must have the same number of product terms and each product term, the same number of Boolean variables.

The examples given below will help you to understand and apply the simplification of the SOP expression using Karnaugh map.

**Example 71.23.** Simplify the following Boolean expression using the Karnaugh mapping technique :

$$X = \bar{A}B + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$$

**Solution.** The first step is to map the given Boolean expression on the Karnaugh map. Notice that there are three variables  $A$ ,  $B$  and  $C$  in the Boolean expression, therefore we need a three-variable Karnaugh map.

The Boolean expression to be mapped is,

$$X = \bar{A}B + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$$

Note that the given Boolean expression is a nonstandard SOP expression because the first product term  $\bar{A}B$  has the variable  $C$  missing in it. This can be converted into a standard SOP form by modifying the expression as below.

$$\begin{aligned} X &= \bar{A}B \cdot 1 + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} \\ &= \bar{A}B(C + \bar{C}) + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} && \dots(C + \bar{C} = 1) \\ &= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} && \dots(1) \end{aligned}$$

Equation (1) can be mapped on the Karnaugh map as shown in Fig. 71.36. In order to simplify the given expression, the 1s can be grouped together as shown by the loop around the 1s. The four 1s in the first column are grouped together and the term we get is  $\bar{C}$ . This is because of the fact that the squares within this group contain both  $A$  and  $\bar{A}$  and  $B$  and  $\bar{B}$ , so these variables are eliminated. Similarly, the two 1s in the second row are grouped together and the term we get is  $\bar{A}B$ . This is because of the fact that squares in this group contain both  $C$  and  $\bar{C}$  which is eliminated. Summing up the two-product terms, the simplified expression is,

$$X = \bar{A}B + \bar{C}.$$

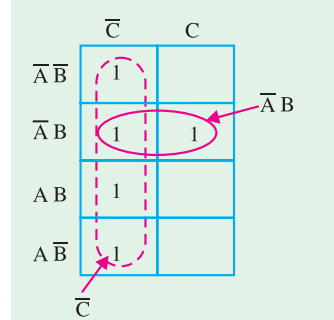


Fig. 71.36

**Example 71.24.** Simplify the following SOP expression using the Karnaugh mapping procedure :

$$X = \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + ABCD$$

**Solution.** First of all, notice that the given SOP expression is already in the standard form i.e. all the product terms in the given expression have all the four variables  $A, B, C$  and  $D$ .

Next sketch a four-variable Karnaugh map. Select the first product term ( $\bar{A}B\bar{C}D$ ) from the given expression and enter 1 in the corresponding square as shown in Fig. 71.37. Similarly enter 1 for the other product terms in the given SOP expression to complete the mapping. In order to simplify the given SOP expression, the 1s can be grouped together as shown by the loop around the 1s. The four 1s in the second column are grouped together and the product term we get is  $\bar{C}D$ . This is because of the fact that squares within this group contain both  $A$  and  $\bar{A}$  and  $B$  and  $\bar{B}$ , so these variables are eliminated.

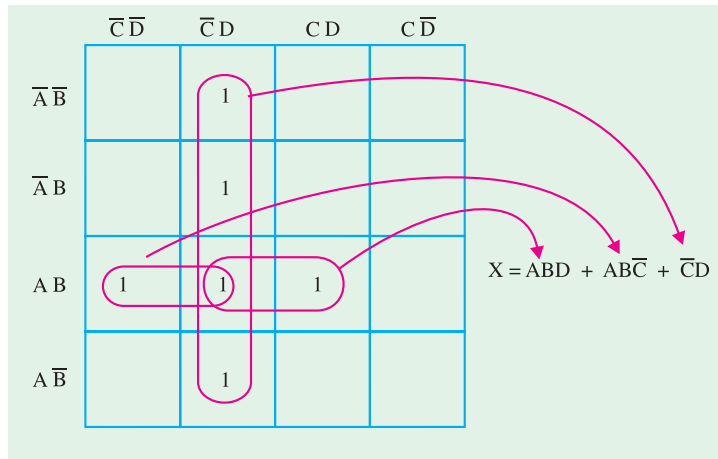


Fig. 71.37

The two 1s in the first and second columns can be grouped together. This group contains both  $D$  and  $\bar{D}$ , so this variable is eliminated and the resulting product term is  $A\bar{B}C$ . Similarly the two 1s in the second and third columns can be grouped together. This group contains both  $C$  and  $\bar{C}$ , so this variable is eliminated and the resulting product term is  $ABD$ .

The resulting minimal or simplified SOP expression is obtained by summing up the three product terms  $\bar{C}D, A\bar{B}C$  and  $ABD$  as shown below :

$$X = ABD + A\bar{B}C + \bar{C}D.$$

**Example 71.25.** Simplify the following SOP expression using the Karnaugh mapping technique.

$$X = B\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + \bar{A}BCD + ABCD$$

**Solution.** First of all, notice that the given SOP expression is in the nonstandard SOP form because the first product term ( $B\bar{C}\bar{D}$ ) has a variable  $A$  or  $\bar{A}$  missing in it. Let us convert the given SOP expression into a standard SOP form as shown below :



$$\begin{aligned}
 X &= 1 \cdot B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD \\
 &= (\bar{A} + A)B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD \quad \dots(\because \bar{A} + A = 1) \\
 &= \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD
 \end{aligned}$$

This expression can be mapped on to the four-variable Karnaugh by entering 1 for each product term in the corresponding square as shown in Fig. 71.38.

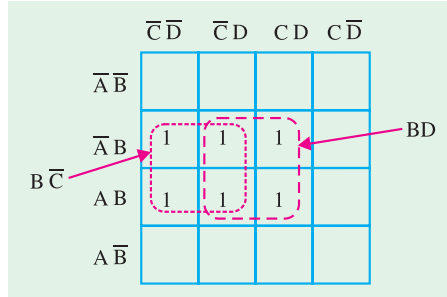


Fig. 71.38

In order to simplify the SOP expression, the 1s can be grouped together as shown by the loop around the 1s. The four 1s looped together form the first and second columns, contain both A and  $\bar{A}$  and D and  $\bar{D}$ , so these variables are eliminated and the resulting product term is  $B\bar{C}$ . Similarly, the four 1s looped together form the second and the third column contain both A and  $\bar{A}$  and C and  $\bar{C}$ , so these variables are eliminated and the resulting product term is

$BD$ . The resulting simplified SOP expression is the sum of the product terms  $B\bar{C}$  and  $BD$ , i.e.,

$$X = B\bar{C} + BD$$

**Example 71.26.** Fig. 71.39 shows a Karnaugh map of a sum-of-products (SOP) function. Determine the simplified SOP function. (UPSC Civil Services 2000)

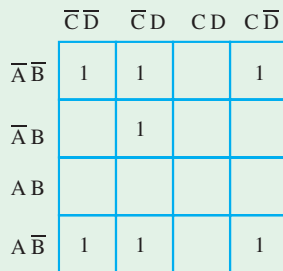


Fig. 71.39

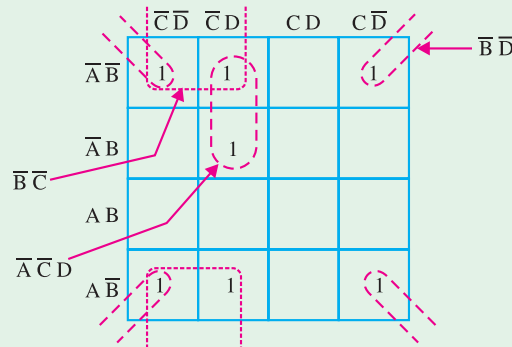


Fig. 71.40

**Solution.** The grouping of 1s is as shown in the Fig. 71.40. Notice the “wrap around” four-square group that includes the 1s on four corners of the Karnaugh map. This group produces a product term  $\bar{B}\bar{D}$ . This is determined by observing that the group contains both A and  $\bar{A}$  and C and  $\bar{C}$ , so these variables are eliminated.

Another group of four with “wrap-around” adjacency is formed from the top and the bottom rows of the Karnaugh map. This group overlaps with the previous group and produces a product term  $\bar{B}\bar{C}$ . This is determined by observing that this group contains both A and  $\bar{A}$  and D and  $\bar{D}$ , so these variables are eliminated.

The remaining 1 is absorbed in a overlapping group of two squares. This group produces a three-variable term  $\bar{A}\bar{C}D$ . This is determined by observing that this group contains both B and  $\bar{B}$ , so this variable is eliminated. This resulting simplified SOP function is the sum of the product terms  $\bar{B}\bar{D} + \bar{B}\bar{C}$  and  $\bar{A}\bar{C}D$ , i.e.

$$X = \bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}\bar{C}D$$

**71.20. Mapping Directly on Karnaugh Map from a Truth Table**

It is possible to map directly on Karnaugh map from a truth table. Recall that a truth table gives the output of a Boolean expression for all possible input variable combinations. Let us illustrate direct mapping through an example of a Boolean expression and its truth table representation.

Let  $X = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$ . Then its truth table can be indicated as shown in Fig. 71.41 (a) and its Karnaugh mapping is shown in Fig. 71.41 (b). Notice in the truth table that the output X is 1 for four different input variable combinations.

It is evident from the Fig. 71.41 (a) and (b) that truth table and Karnaugh map are simply different ways to represent a logic function.

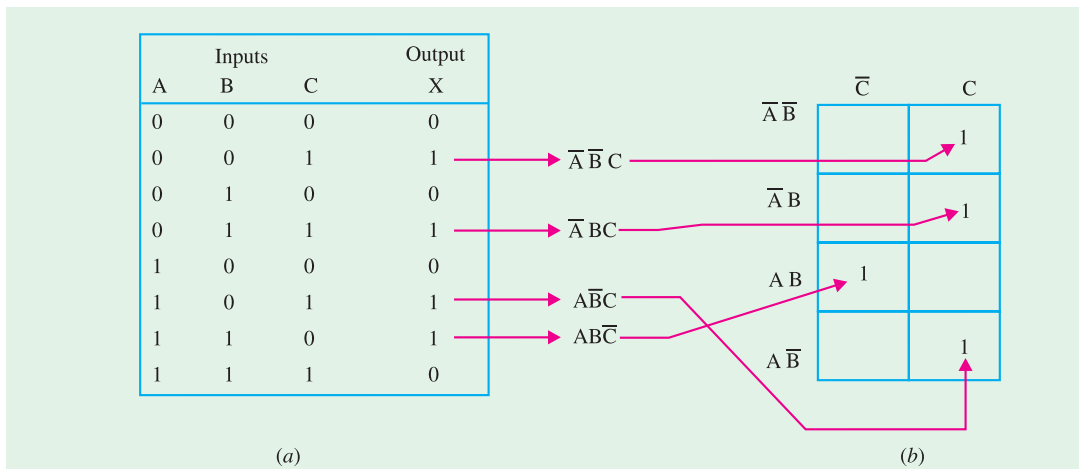


Fig. 71.41

**Example 71.27.** Implement the following Boolean expression using minimum number of 3-input NAND gates.

$$f(A, B, C, D) = \Sigma(1, 2, 3, 4, 7, 9, 10, 12)$$

(UPSC Engg. Services 1991)

**Solution.** The given Boolean function indicates that its output is 1 corresponding to the terms indicated within the expression i.e., 1, 2, 3, 4, 7, 9, 10 and 12. This is shown in Fig. 71.42 (a). We can map these values directly on to the four-variable Karnaugh map as shown in Fig. 71.42 (b). In order to simplify the Boolean expression represented on the Karnaugh map, group the 1s as shown in the Fig. 71.42 (b). The group of two squares in the first column produces a product term  $B\bar{C}\bar{D}$ . This is determined by observing that the group contains both A and  $\bar{A}$ , so this variable is eliminated. Another group of two squares in the second column produces term  $\bar{B}CD$ . The variable A is eliminated because the group contains both A and  $\bar{A}$ .

Another group of two squares in the third column produces the term  $\bar{A}CD$ . The variable B is eliminated because the group contains both B and  $\bar{B}$ . Still another group of two squares in the fourth column produces the term  $\bar{B}C\bar{D}$ . The variable A is eliminated because the group contains both A and  $\bar{A}$ . Thus the resulting simplified expression,

$$f(A, B, C, D) = B\bar{C}\bar{D} + \bar{B}CD + \bar{A}CD + \bar{B}C\bar{D}$$

This can be implemented using 3-input NAND gate as shown in Fig. 71.43.

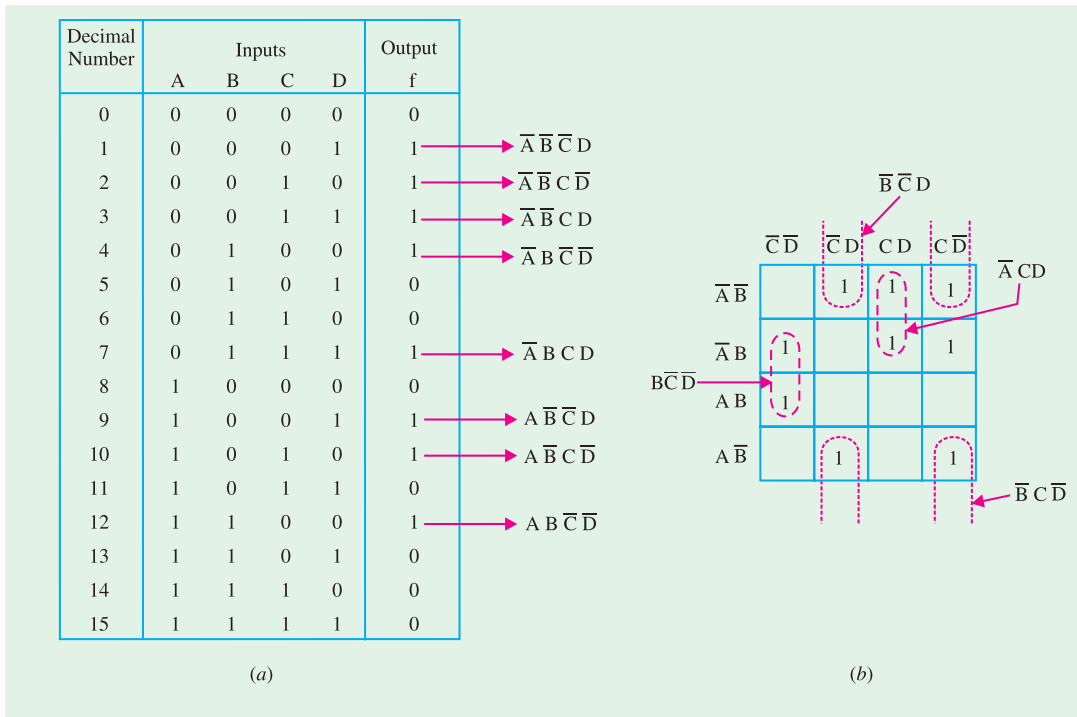


Fig. 71.42

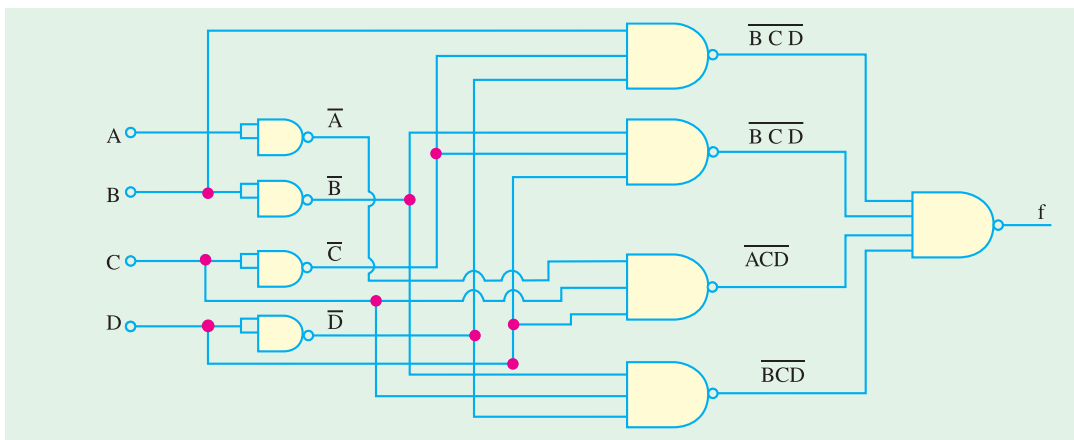


Fig. 71.43

### 71.21. “Don’t Care” conditions

In digital systems design sometimes a situation arises in which some input variable conditions are not allowed. For example in a *BCD* (binary coded decimal) code, there are six invalid combinations : 1010, 1011, 1100, 1101, 1110 and 1111. Since these unallowed states will never occur in an application involving the *BCD* code, they can be treated as “don’t care” terms with respect to their effect on the output. That is, for these “don’t care” terms either 1 or 0 may be assigned to the output.

Now, we shall discuss as how the “don’t care” terms can be used to advantage on the Karnaugh map for simplifying the Logic equations.

Consider for example, a combinational circuit which products a ‘1’ output corresponding to a

BCD input equal and greater than 6. The output is 0 corresponding to a BCD input less than 6. The truth table for this situation is as shown in Fig. 71.44 (a).

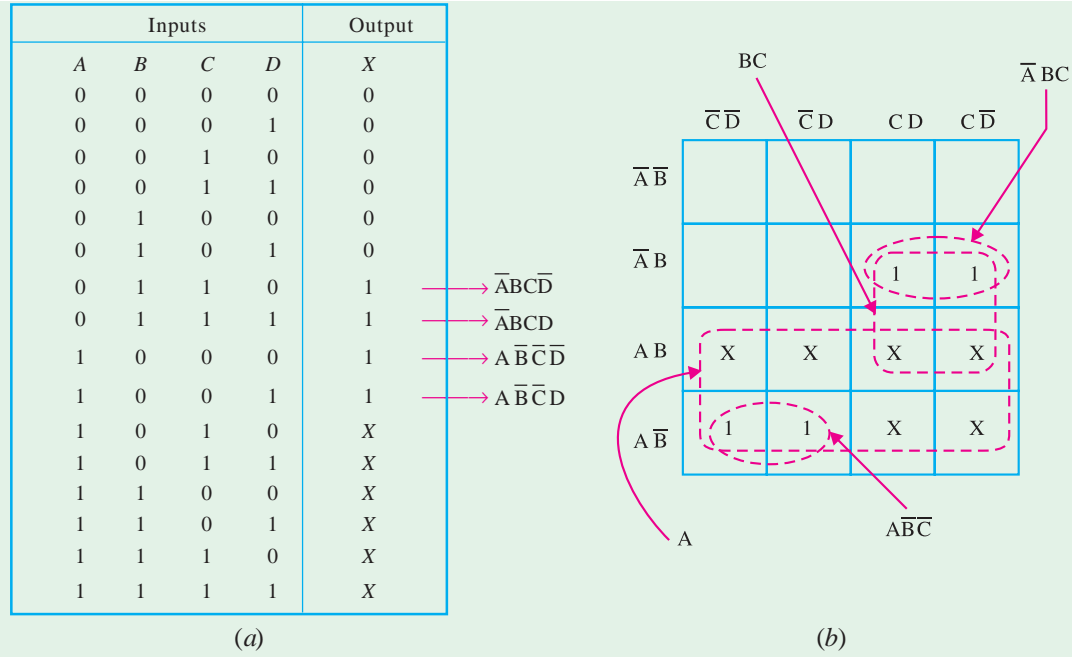


Fig. 71.44

We know that we can place 1s directly from the truth table on the Karnaugh map. Similarly we can place X for the “don’t care” entries directly on the Karnaugh map as shown in Fig. 71.44 (b). When grouping the 1s, Xs can be treated as 1s make a larger grouping or as 0s, if they cannot be used to advantage. Recall the larger the group of 1s the simpler the resulting term will be. Taking advantage of the “don’t care” and using them as 1s, the resulting expression for the output is  $A + BC$ . However, if the “don’t cares” are not used as 1s, the resulting expression is  $AB\bar{C} + \bar{A}BC$ . Thus we can see the advantage of using “dont care” terms to get the simplest logic expression.

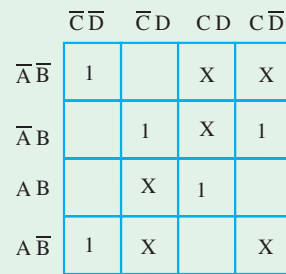


Fig. 71.45

**Example 71.28.** Consider the Karnaugh map shown in Fig. 71.45. Determine the logic function represented by the map and simplify it in the minimal form. (UPSC Engg. Services 1997)

**Solution.** We know that when grouping the 1s, Xs can be treated as 1s to make a larger grouping or as 0s if they cannot be used to advantage.

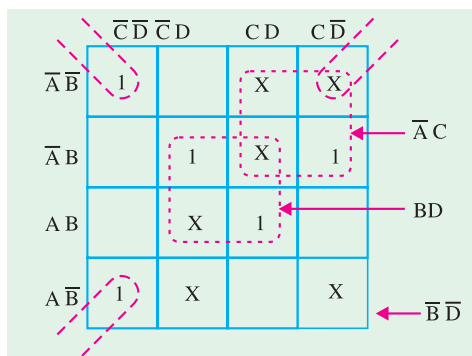


Fig. 71.46

Recall, the larger the group of 1s, the simpler the resulting term will be. Taking advantage of the Xs and using them as 1s, the grouping of 1s and Xs is as shown in Fig. 71.46.

Notice the “wrap-around” four-square group that includes the 1s and Xs on four corners of the Karnaugh map. This group produces a product term  $\bar{B}\bar{D}$ . This is determined by observing that the group contains both A and  $\bar{A}$  and C and  $\bar{C}$ , so these variables are eliminated. Another group of four squares containing 1s and Xs around the centre of Karnaugh map is formed.

This group produces a product term  $BD$ . This is determined by observing that the group contains both  $A$  and  $\bar{A}$  and  $C$  and  $\bar{C}$ , so these variables are eliminated.

Another group of four containing 1s and Xs is formed near the top right corner of Karnaugh map. This group overlaps with the previous group and produces a product term  $\bar{A}C$ . This is determined by observing that the group contains both  $B$  and  $\bar{B}$  and  $D$  and  $\bar{D}$ , so these variables are eliminated.

The resulting simplified logic function is the sum of the three product terms:  $\bar{B}\bar{D}$ ,  $BD$  and  $\bar{A}C$ , i.e.,

$$X = \bar{B}\bar{D} + BD + \bar{A}C$$

## 71.22. Main Logic Families

Most digital systems are designed by combining various logic functions discussed in Chapter 19. All these logic circuits are available in IC modules and are divided into many 'families'. Each family is classified by abbreviations which indicate the type of logic circuit used. For example, *RTL* means resistor-transistor logic. We will discuss the following seven transistor logic families although the first two are, at present, of historic interest only.

1. **Resistance-transistor logic (RTL)** : it was the first family group of logic circuits to be developed and packaged in IC form in early 1960s;
2. **Diode-transistor logic (DLT)** : It followed RTL in late 1960s;
3. **Transistor-transistor logic (TTL) OR ( $T^2L$ )** : was introduced in the early 1970 s;
4. **Schottky TTL** : was introduced to improve the speed of TTL;
5. **Emitter-coupled logic (ECL)** : It is fastest logic line currently available;
6. **Integrated-injection logic ( $I^2L$ )** : It is one of the latest of the bipolar types of logic;
7. **Complementary metal-oxide semiconductor (CMOS)** : It has the lowest power dissipation of the currently-available logic circuits.

The various logic families discussed above possess different characteristics as detailed below.

## 71.23. Saturated and Non-saturated Logic Circuits

Those logic circuits in which transistors are driven into saturation are called **saturated** logic circuits or simply **saturated logic**. Those circuits which avoid saturation of their transistors are designed **non-saturated logic**.

The disadvantage of saturated logic is the delay that occurs when the transistors is brought out of saturation. When a transistor is saturated, its base is flooded with carriers. Even when base voltage is switched off, the base remains flooded for some time till all carriers leave it. The time required by the carriers to leave the base is called **saturation delay time ( $t_s$ )**. Obviously, saturated logic circuits have low switching speeds whereas non-saturated type are much faster. *TTL* is the example of a saturated logic whereas *ECL* represents a non-saturated logic.

## 71.24. Basic Operating Characteristics and Parameters of Logic Families

When we work with digital ICs from different logic families, we should be familiar with, not only their logical operation but also with the basic operational properties. Following are the important basic operational properties important from the subject point of view.

- |                         |                              |                       |
|-------------------------|------------------------------|-----------------------|
| 1. DC supply voltage.   | 2. TTL and CMOS logic levels | 3. Noise immunity     |
| 4. Noise margin.        | 5. Power dissipation.        | 6. Propagation delay. |
| 7. Speed-power product. | 8. Loading and fan-out.      |                       |

Now we will describe all the above operational characteristics one by one in the following pages.

## 71.25. DC Supply Voltage

The standard value of the dc supply voltage for *TTL* (i.e., transistor-transistor logic) and CMOS (i.e., complementary metal-oxide semiconductor) device is +5V. For simplicity, the dc supply voltage is usually omitted from the logic circuits. But in practice, it is connected to the  $V_{CC}$  or  $V_{DD}$  pin of an IC

package and the ground is connected to the *GND* pin of an *IC* package. Both the voltage and ground are distributed internally to all the logic gates with the package as shown in Fig. 71.47 (a). The connections for the single logic gate are as shown in Fig. 71.47 (b).

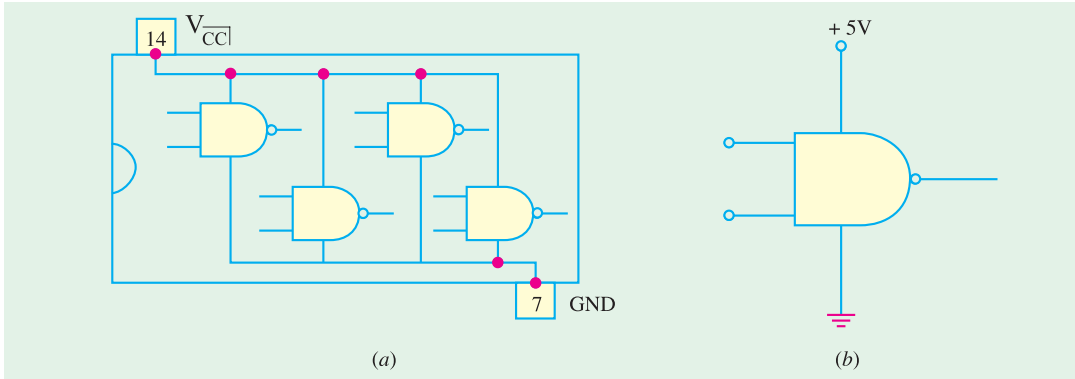


Fig. 71.47

**71.26. TTL and CMOS Logic Levels**

For TTL circuits, the range of input voltages, that can represent a valid *LOW* (logic 0) and a valid *HIGH* (logic 1) is as shown in Fig. 71.48 (a). As seen from this diagram, the range of input voltages that can represent a valid *LOW* (logic 0) is from 0 to 0.8 V. The *LOW* input voltage is indicated by the symbol  $V_{IL}$ . The lower limit for  $V_{IL}$  is represented by  $V_{IL(min)}$  and the higher limit for  $V_{IL}$ , by  $V_{IL(max)}$ . The range of input voltages that can represent a valid *HIGH* (logic 1) is from 2 V to  $V_{CC}$  (usually 5 V). The *HIGH* input voltage is indicated by symbol  $V_{IH}$ . The lower limit for  $V_{IH}$  is represented by  $V_{IH(min)}$  and the higher limit for  $V_{IH}$  by  $V_{IH(max)}$ . Note that the range of values between 0.8 V and 2 V is called *indeterminate*. This means that when an input voltage is in this range, it can be interpreted as a *HIGH* or *LOW* by the logic circuit. Therefore TTL logic gates cannot be operated reliably when input voltages are in this range.

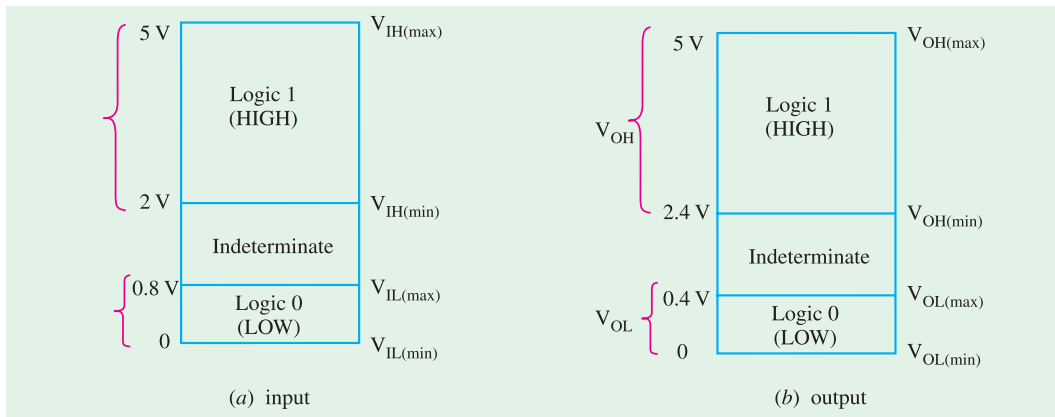


Fig. 71.48

Fig. 71.48 (b) shows the range of *TTL* output voltages that can represent valid *HIGH* (logic 1) and valid *LOW* (logic 0). As seen the range for logic 1 output is from 2.4 V to 5 V and logic 0 output is from 0 to 0.4 V. Note again that the range of values between 0.4 V and 2.4 V is indeterminate. Also note that the output voltage for logic 1 is indicated by the symbol  $V_{OH}$ . The lower limit for  $V_{OH}$  is represented by  $V_{OH(min)}$  and the higher limit for  $V_{OH}$  by  $V_{OH(max)}$ . Similarly the output voltage for logic 0 is indicated by the symbol  $V_{OL}$ . The lower limit for  $V_{OL}$  is represented by  $V_{OL(min)}$  and the higher limit for  $V_{OL}$  by  $V_{OL(max)}$ .

It may be noted from Fig. 71.48 (a) and (b) that the minimum HIGH output voltage,  $V_{OH(\min)}$  is greater than the minimum HIGH input voltage  $V_{IH(\min)}$ . On the other hand, the maximum LOW output voltage,  $V_{OL(\max)}$  is less than the maximum LOW input voltage,  $V_{IL(\max)}$ .

The input and output voltages for a device from HCMOS (*i.e.* High-speed CMOS) logic family for  $V_{DD} = 5\text{ V}$  as shown in Fig. 71.49 (a) and (b) respectively.

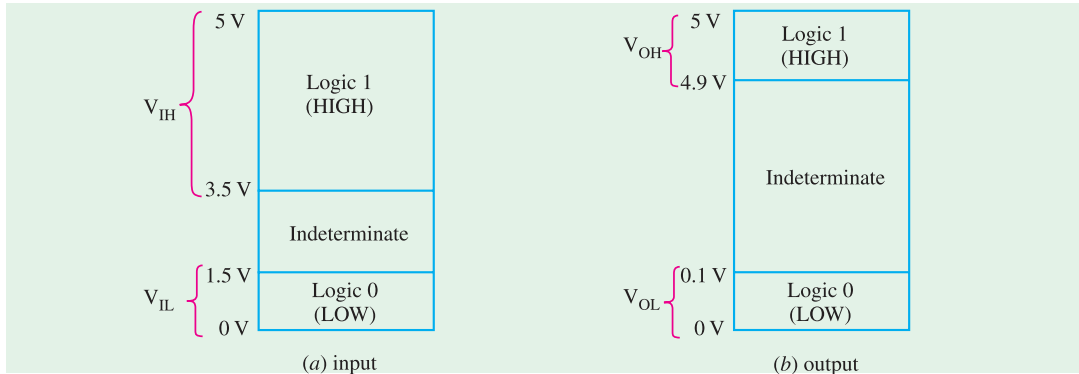


Fig. 71.49

Notice that the input and output voltage values in the HCMOS device are different from that of a TTL device. In a HCMOS device,  $V_{IL(\max)}$  is 1.5 V (its value is 0.8 V in TTL),  $V_{IH(\min)}$  is 3.5 V (its value is 2 V in TTL),  $V_{OL(\max)}$  is 0.1 V (its value is 0.4 V in TTL),  $V_{OH(\min)}$  is 4.9 V (its value is 2.4 V in TTL).

### 71.27. Noise Immunity

The noise immunity of a logic circuit refers to the circuit's ability to tolerate noise without causing a false change in its output voltage. The noise voltage is produced by stray electric and magnetic fields on the connecting wires between logic circuits. Sometimes, too much noise voltage cause the voltage at the input of the logic circuit to drop below  $V_{IH(\min)}$  or rise above  $V_{IL(\max)}$ . This could produce unpredictable operation in a logic circuit.

### 71.28. Noise Margin

A quantitative measure of a circuit's noise immunity is called noise margin. It is expressed in volts. There are two values of noise margin specified for a given logic circuit as described below.

1. The high level noise margin ( $V_{NH}$ )
2. The low level noise margin ( $V_{NL}$ )

These parameters are shown in Fig. 71.50 and are given by the equations,

$$V_{NH} = V_{OH(\min)} - V_{IH(\min)}$$

$$V_{NL} = V_{IL(\max)} - V_{OL(\max)}$$

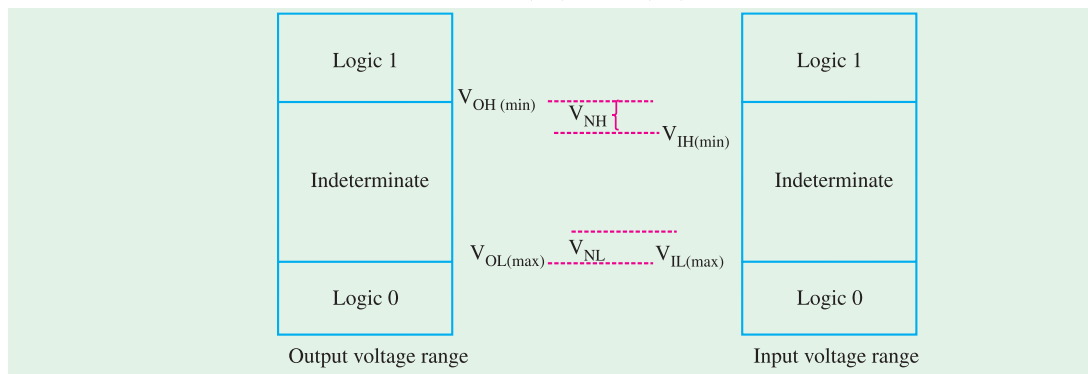


Fig. 71.50

**Example 71.29.** Table shows the input/output voltage specifications for the standard TT family.

Parameter	Min (V)	Typical (V)	Max (V)
$V_{OH}$	2.4	3.4	—
$V_{OL}$	—	0.2	0.4
$V_{IH}$	2.0	—	—
$V_{IL}$	—	—	0.8

Using these values, find (a) the maximum value of noise spike that can be tolerated when a HIGH output is driving an input, (b) the maximum value of noise spike when a LOW output is driving an input.

**Solution.** The maximum value of the noise spike, when driven by a HIGH output,

$$V_{NH} = V_{OH(\min)} - V_{IH(\min)} = 2.4 - 2.0 = 0.4 \text{ V}$$

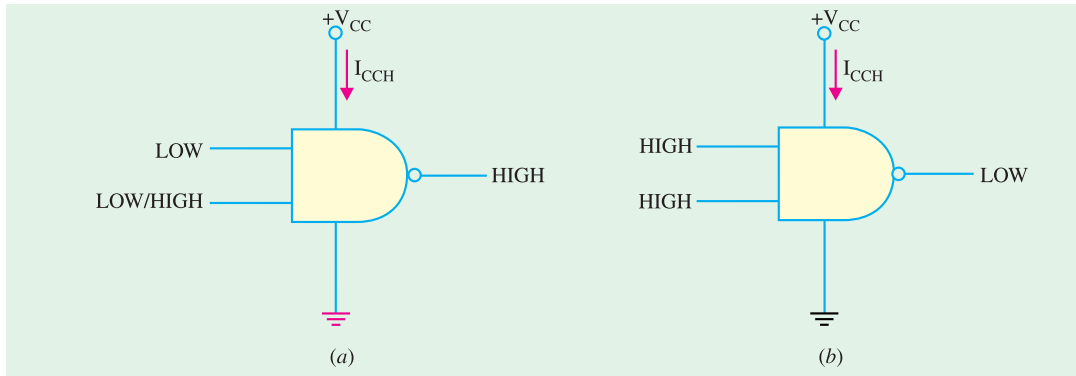
and the maximum value of the noise spike, when driven by a LOW output,

$$V_{NL} = V_{IL(\max)} - V_{OL(\max)} = 0.8 - 0.4 = 0.4 \text{ V}$$

It is observed that a TTL gate is immune to 0.4 V of poise for both the HIGH and LOW input states.

### 71.29. Power Dissipation

As a matter of fact, all logic gates draw current from the dc supply voltage for its normal operation. When the logic gate is in the HIGH output state, it draws an amount of current,  $I_{CCH}$ , as shown in Fig. 71.51 (a) and when in LOW output state, it draws an amount of current  $I_{CCL}$  as shown in Fig. 71.51 (b).



**Fig. 71.51**

The power dissipation of a logic gate is given by the product of the dc supply voltage ( $V_{CC}$ ) and the amount of current drawn from the supply (i.e.,  $I_{CCH}$  or  $I_{CCL}$ ). Thus power dissipation is given by :

$$P_D = V_{CC} \cdot I_{CCH} \text{ or } V_{CC} \cdot I_{CCL}$$

For example, if  $I_{CCH}$  is 2.5 mA when  $V_{CC}$  is 5 V, the power dissipation,

$$P_D = V_{CC} \cdot I_{CCH} = 5 \text{ V} \times 2.5 \text{ mA} = 12.5 \text{ mW.}$$

Usually, the logic gate operates with the inputs, which keep on changing with time (i.e., the input is pulsed). Accordingly the output of a logic gate also switches back and forth between HIGH and LOW. Because of this, the amount of current drawn from the dc supply also varies between  $I_{CCH}$  and  $I_{CCL}$ . In such a situation, we calculate the average power dissipation. The average power dissipation depends upon the duty cycle and is usually specified for a duty cycle of 50%, the output is HIGH half the time and LOW the other half. Therefore average supply current is :



$$I_{CC} = \frac{I_{CCH} + I_{CCL}}{2}$$

and the average power dissipation is,

$$P_D = V_{CC} \cdot I_{CC}$$

**Example 71.30.** A TTL logic gate draws 2 mA when its output is HIGH and 3.5 mA when its output is LOW. Calculate the average power dissipation if the supply voltage is 5 V and the logic gate is operated on 50% duty cycle.

**Solution.** The average supply current,

$$I_{CC} = \frac{I_{CCH} + I_{CCL}}{2} = \frac{2\text{mA} + 3.5\text{mA}}{2} = 2.75\text{ mA}$$

∴ The average power dissipation,

$$P_D = V_{CC} \cdot I_{CC} = 5 \times 2.75\text{ mA} = 13.7\text{ mW}$$

### 71.30. Power Dissipation versus Frequency

Fig. 71.52 shows a graph of power dissipation versus frequency for a TTL and a CMOS logic gate. As seen from this graph, the power dissipation in a TTL circuit is essentially constant over its range of operating frequencies. However, the power dissipation in a CMOS circuit is frequency dependent. That is, it is extremely low under zero frequency (or dc) conditions. But the power dissipation increases as the frequency increases. For example, the power dissipation of a typical TTL logic gate is a constant 2 mW. On the other hand, power dissipation of typical CMOS logic gate is 0.0025 mW under static (or dc) conditions and 0.17 mW at 100 kHz.

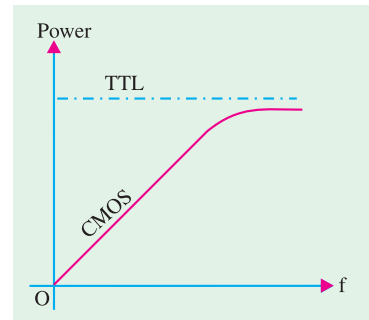


Fig. 71.52

### 71.31. Propagation Delay

When a signal passes (*i.e.*, propagates) through a logic circuit, it always experiences a finite time delay as shown in Fig. 71.53. It shows that the change in output level occurs after a short time, (called the propagation delay time), later than the change in input level that caused it. There are two propagation delay times specified for logic gates.

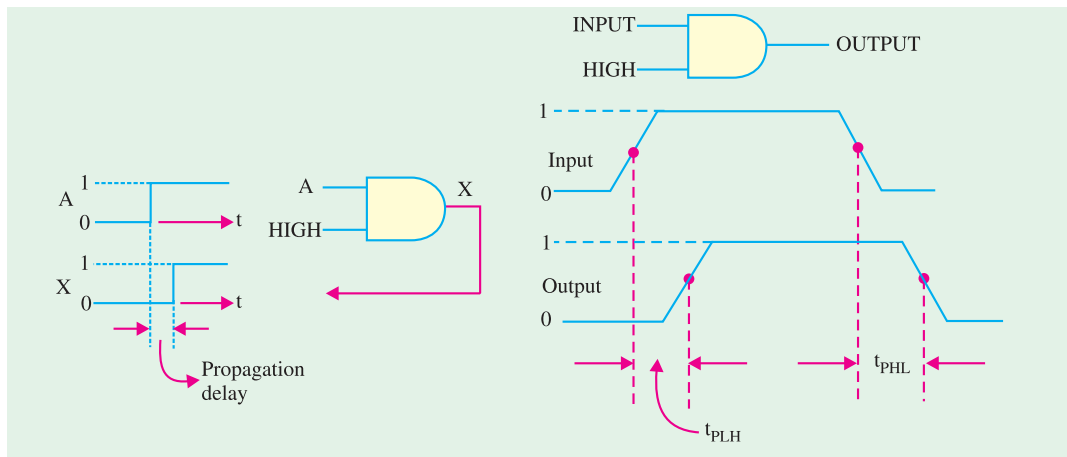


Fig. 71.53

Fig. 71.54

1.  $t_{PLH}$  – It is the time interval between a designated point on the input pulse and the corresponding point on the output pulse when the output is changing from LOW to HIGH (or 0 to 1) as shown in Fig. 71.54.
2.  $t_{PHL}$  – It is the time interval between a designated point on the input pulse and the corresponding point on the output pulse when the output is changing from HIGH to LOW (or 1 to 0) as shown in Fig. 71.54.

It may be noted that the propagation delay times are indicated in Fig. 71.54 with 50% points on the pulse edges used as references.

The propagation delay time of a logic gate limits its maximum operating frequency. The greater the propagation delay time of a logic gate, the lower is its maximum operating frequency. This means a high speed logic gate is a one that has a small propagation delay time. For example, a logic gate with a delay time of 2 ns is faster than a logic gate that has a delay time of 10 ns.

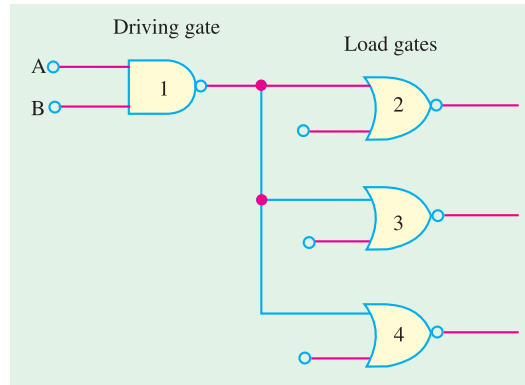


Fig. 71.55

### 71.32. Speed-Power Product

It provides a basis for comparison of logic circuits when the propagation delay and power dissipation are important considerations in the selection of the type of logic family to be used in certain application. The speed power product is expressed in picojoule (pJ). It may be noted carefully that the lower the speed-product, the better is the value. Typically the CMOS family has much lower value of speed-power product as compared to the TTL logic family. For example, a typical CMOS logic family has a speed-power product of 1.5 pJ at 100 Hz while a typical TTL has speed-power product of 20 pJ.

### 71.33. Loading and Fan-out

When the output of any logic gate is connected to one or more inputs of other logic gates, a load on the driving gate is created. This is shown in Fig. 71.55. Here the output of a logic gate (labeled as 1) is connected to the inputs of 3 other logic gates (labeled as 2, 3, and 4). Note that the logic gate labeled 1 is called a **driving gate** while the logic gates labeled as 2, 3 and 4 are called **load gates**.

In any logic family, there is a limit to the number of load gate inputs that a given logic gate can drive. This limit is called the **fan-out** of the logic gate. Now we will study the loading and fan-out in TTL and CMOS logic families in more detail.

**Loading and fan-out in TTL family :** A TTL gate that acts as a driving gate **sources** (i.e., supplies) current to a load gate input in the logic HIGH state and **sinks** (i.e., receives) current from the load gate in the logic LOW state. Fig. 71.56 (a) illustrates the current sourcing and (b) shows current sinking in the logic gates.

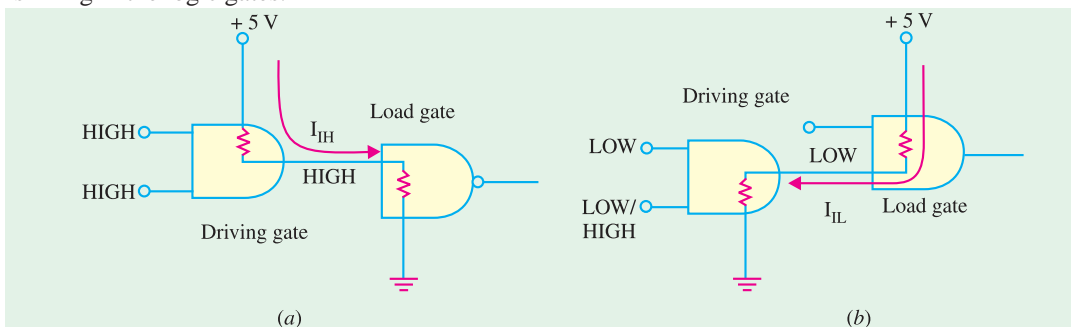


Fig. 71.56

Note that  $I_{IH}$  is the current supplied by the driving gate to the load gate when the input of the load gate is HIGH similarly.  $I_{IL}$  is the current received by the driving gate from the load gate when the input of the load gate is LOW.

As more and more number of load gates are connected to the driving gate, the loading on the driving gate increases. The total source current increases with each load gate input that is added as illustrated in Fig. 71.57. As the source current increases, the internal voltage drop of the driving gate increases  $V_{OH}$ . This causes the voltage drop  $V_{OH}$  to decrease.

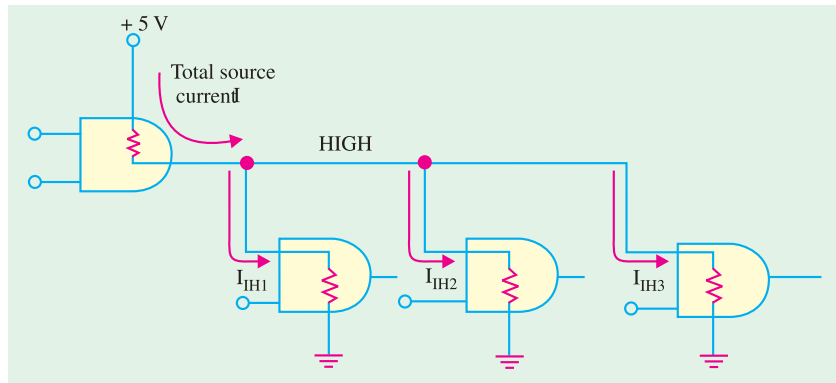


Fig. 71.57

If a large number of load gate inputs are connected, drops below  $V_{OH(\min)}$ . As a result of this, HIGH level noise margin is reduced, thus affecting the specified operating characteristics of the gate. Moreover, as the total source current increases, the power dissipation of the driving gate increases.

The maximum number of load gate inputs that can be connected without affecting the specified operational characteristics of the driving gate is called **fan-out**. Its value is important for designing logic circuits. For example, the standard *TTL* has a fan-out of 10. One input of the same logic family as the driving gate is referred to as a **unit load**. This we can also say that a standard *TTL* has a fan-out of 10 unit loads.

The total sink current also increases with each load gate input that is added as shown in Fig. 71.58. As this current increases, the internal voltage drop of the driving gate increases, causing  $V_{OL}$  to increase. If a large number of loads are added,  $V_{OL}$  exceeds  $V_{OL(\max)}$  and the LOW-level margin is reduced.

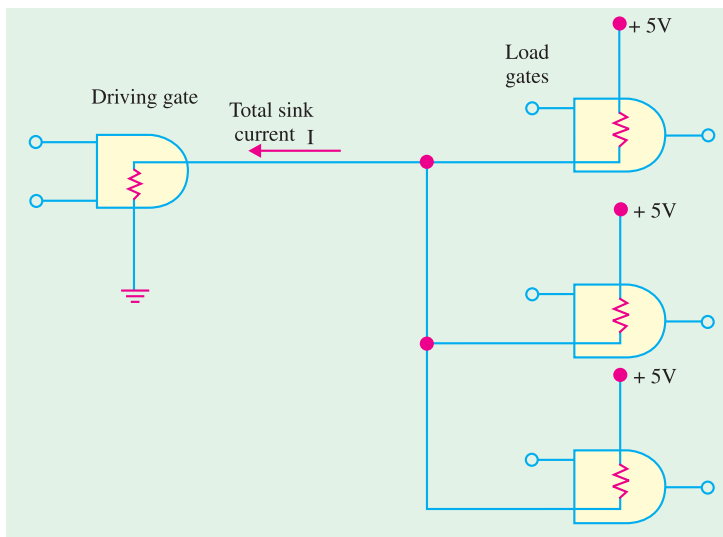


Fig. 71.58

As a matter of fact, in *TTL*, the current sinking capability (in the LOW state) is the limiting factor in determining the fan-out.

**Loading and fan-out in CMOS:** Loading in *CMOS* logic family differs from that in *TTL*. It is because of the fact that the field-effect transistors used in *CMOS* logic family present a predominantly capacitive load to the driving gate as shown in Fig. 71.59. In this case, the limitations are the charging and discharging times associated with the output resistance of the driving gate and the input capacitance of the load gates.

When the output of the driving gate is HIGH, the input capacitance of the load gate is charging through the output resistance of the driving gate. When the output of the driving gate is LOW, the

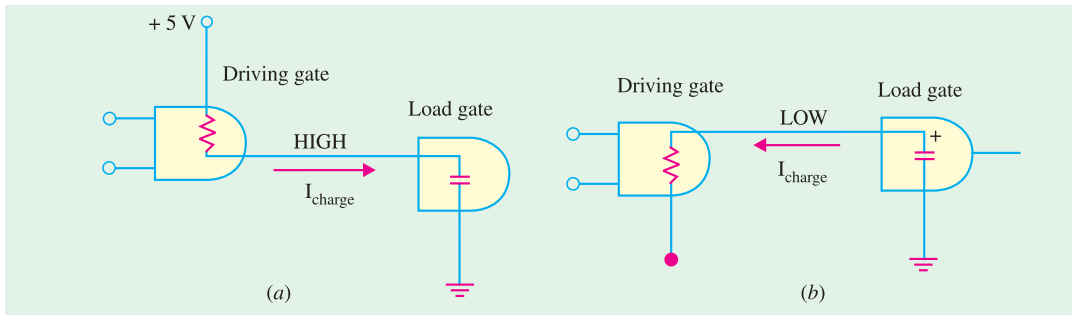


Fig. 71.59

capacitance is discharging. When more and more number of load gate inputs are added to the driving gate output, the total capacitance increases because the input capacitances effectively appear in parallel. This increase in capacitance increases the charging and discharging times. As a result, the maximum operating frequency of the logic gate is reduced. Thus fan-out in a CMOS logic family depends upon the operating frequency. The smaller the number of load gate inputs, the greater the maximum operating frequency.

**71.34. RTL Circuit**

It is a **saturated logic**. It uses only transistors and resistors as circuit elements and also resistances in the input to each base. This family is based on the NOR circuit shown in Fig. 71.60. All other members of the family are made up of NOR cells or variations on them.

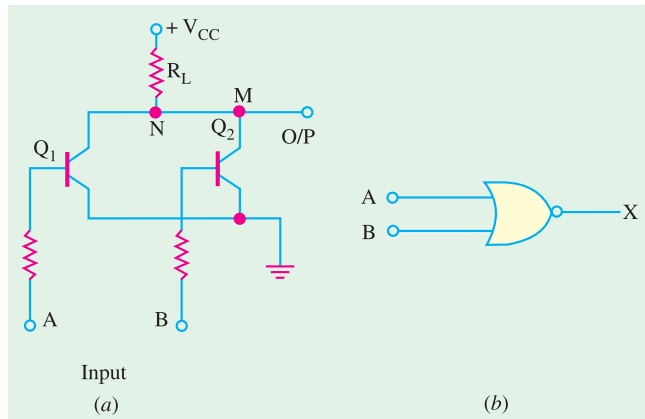


Fig. 71.60

**Circuit Operation**

We will assume ideal transistors. When both inputs A and B are 0 V (or logic 0) both transistors are turned OFF, hence point M goes to +V<sub>CC</sub> so that output is logic 1.

If either or both input terminals are at +V<sub>CC</sub> i.e. are high (or logic 1), one or both transistors would be fully turned ON (i.e. saturate) thereby reducing the voltage of point N to almost 0 V. Hence, output would be at logic 0.

It is seen that the output is at logic 1 only when **both inputs are at logic 0** — the NOR logic functions as shown in Fig. 71.60 (b).

The RTL family has the following characteristics :

1. relatively slow speed,
2. low fan-out of 6 and a fan-in of 4,
3. poor noise immunity,
4. expensive since resistors are required to be fabricated,
5. cannot operate at speed above 4 MHz.

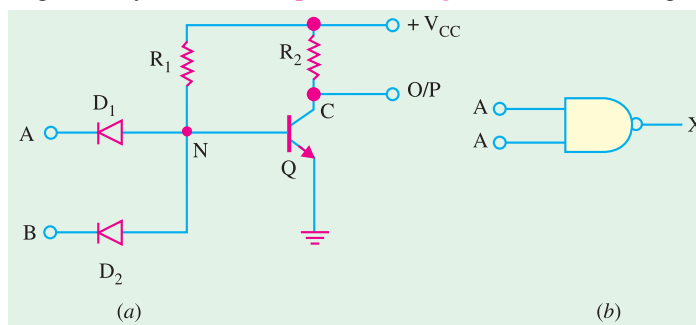


Fig. 71.61

### 71.35. DTL Circuit

It is a *saturated* logic because transistors between cut-off and saturation. It was the next family to be introduced after *RTL*. It consists of diodes, resistors and transistors. The basic gate of this family performs *NAND* function. As shown in Fig. 71.61 (a) the circuit basically consists of a diode *AND* gate followed by a transistor inverter which leads to a *NAND* gate.

#### Circuit Operation

1. When both  $D_1$  and  $D_2$  have positive voltage applied to them (logic 1), neither conducts and  $Q$  is turned *ON* by the current provided by  $V_{CC}$  through  $R_1$ . Since  $Q$  becomes saturated, point  $C$  is brought to 0V (logic 0). Hence output goes logic 0.
2. If either or both inputs are at 0V (logic 0), the associated diode will conduct driving point  $N$  to ground *i.e.* 0V. Since there is no base voltage for  $Q$ , it will be cut *OFF* thereby driving point  $C$  and hence output to  $V_{CC}$  *i.e.* logic 1.

It is seen that output is low (a logic 0) only when all inputs are high—the condition for a *NAND* gate.

The *DTL* family is characterised by

1. relatively lower speed,
2. Comparatively better noise immunity,
3. propagation delay of 30 ns,
4. a fan-out of 5.

### 71.36. TTL Circuit

It is a *saturated logic*. It is the most widely used circuit line since early 1970s because of its speed, good fan-out and easy interface with other digital circuitry. The unique feature of this circuit is that it used *multiple-emitter transistor* at input which replaces the input diodes of the *DTL*. The number of emitters is equal to the number of inputs of the logic circuit (limited to 8). Since a multi-emitter transistor is small in area than the diodes it replaces, the yield from a wafer is increased. Moreover, smaller area results in *lower capacitance* to the substrate, thereby wide selection of circuit modules ranging from simple gates and flip-flops in *SSI* circuit series through various registers in computers in *MSI* circuit series to micro-processor bit-slice chips in the *LSI* series.

#### Basic Circuit

The basic circuit of the *TTL* family is the *NAND* gate cell shown in Fig. 71.62. However, at present *NOR*, *OR* and *AND* gate configurations have also been added to the series.

#### Circuit Operation

1. If both inputs  $A$  and  $B$  are high (logic 1),  $E/B$  junction of  $Q_1$  is *reverse-biased* so that it has no emitter current. Hence  $Q_1$  is *OFF*. However, its  $C/B$  junction is *forward-biased* supplying base current to  $Q_2$  from  $V_{CC}$  via  $R_1$ . As a result transistor  $Q_2$  is turned fully *ON* (*i.e.* it becomes saturated) driving point  $N$  to 0V. Hence, output is a logic 0.
2. When either or both inputs are at 0V (logic 0), the associated  $E/B$  junction becomes forward-biased. The value of  $R_1$  is so selected as to ensure that  $Q$  is turned fully *ON*. The voltage at point  $M$  falls to 0V with the result the base current for  $Q_2$  is reduced to zero. Hence,  $Q_2$  is cut *OFF* driving point  $N$  and the output to logic 1.

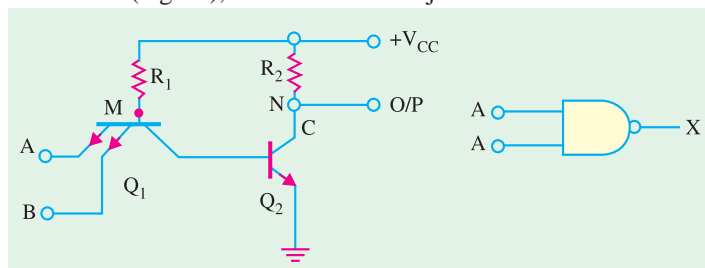


Fig. 71.62

### Totem Pole Output

The basic circuit of Fig. 71.62 is never used in practice. Its modified version with an added output stage is in common use (Fig. 71.63). This extra output stage is often known as *totem-pole* stage because the three components  $Q_3$ ,  $Q_4$  and  $D$  are stacked one on top of the other in the manner of a totem-pole. The circuit action is as follows :

#### 1. When Input is High

In this case, the two input terminals have *positive* voltage (logic 1). The  $E/B$  junction is reverse-biased because of which there is no emitter current. Hence,  $Q_1$  is *OFF*. Since  $C/B$  junction of  $Q_1$  is forward-biased, base current of  $Q_2$  flows from  $V_{CC}$  through  $R_1$ . Hence,  $Q_2$  is turned *ON*. As a result, potential of point  $N$  falls so much that  $Q_3$  is turned *OFF*. At the same time  $Q_4$  is turned *ON* by the voltage drop across  $R_3$ . Now, when  $Q_4$  is *ON*, its collector potential (*i.e.* potential of point  $C$ ) is nearly that of its emitter. Hence, output is low *i.e.* at logic 0.

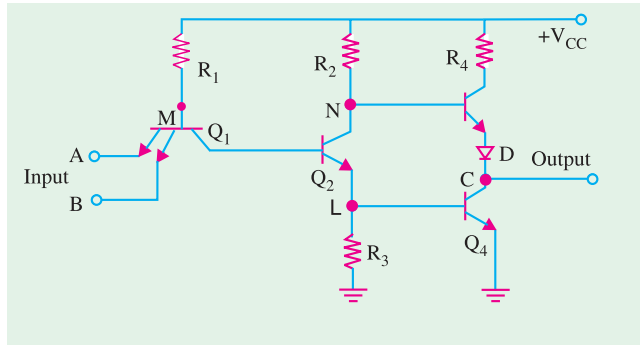


Fig. 71.63

In short, when inputs are at logic 1,  $Q_1$  is *OFF*,  $Q_2$  is *ON*,  $Q_3$  is *OFF* and  $Q_4$  is *ON* because of which output is logic 0.

#### 2. When Input is Low

If any of the two inputs or both are low (logic 0),  $Q_1$  turns *ON* and potential of its collector (point  $M$ ) falls. Hence,  $Q_2$  is turned *OFF*, grounding its emitter and the base of  $Q_4$  so that  $Q_4$  is also turned *OFF*.

Since  $N$  is at  $V_{CC}$ , it turns  $Q_3$  *ON*. The potential of point  $C$  is  $V_{CC}$  minus drop in  $R_4$ ,  $Q_3$  and  $D$ . Since these drops do not amount to much, output is at logic 1.

It may be noted that when *even-numbered transistors are ON, the odd-numbered ones are OFF and vice-versa*.

The function of diode  $D$  in Fig. 71.63 is to prevent both  $Q_3$  and  $Q_4$  from being turned *ON* simultaneously. If both were to be *ON* at the same time, they would offer low impedance to the supply which will draw excessive current and produce large noise ‘spikes’ in the output. It may also be noted that the addition of a pair of totem pole transistor increases the operating speed and output current capability of this circuit. The standard *TTL*-family has

1. greater speed than *DTL*,
2. less noise immunity (0.4 V),
3. average propagation delay per gate of 9 ns,
4. average power dissipation of 10mW,
5. a fan-out of 10 meaning one output can drive 10 other *TTL* inputs,

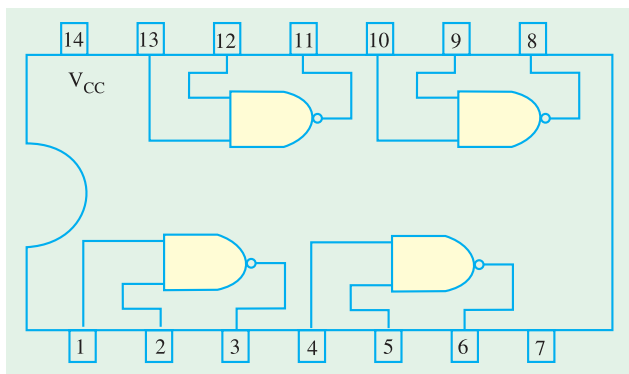


Fig. 71.64

Fig. 71.64 shows a pin-out of a *BEL* 7400 IC referred to as ‘quad (quadrupole). 2-input *NAND* gate chip’. As seen, there are four separate

2-input *NAND* gates of which any or all may be used at any point in time. It is manufactured by Bharat Electronics LTD. (BEL) Bangalore, India.

### 71.37. TTL Sub-families

*TTL* has several *sub-families* having different speed and lower dissipation characteristics as detailed below :

1. **74L00** series—the letter *L* standing for low power consumption. It has an average power dissipation of 1 mW per gate but an average propagation delay of 33 ns.
2. **74H00** series—the letter *H* standing for higher speed. It has a propagation delay of 6 ns but average power dissipation of 23 mW/gate.
3. **74S00**—the letter *S* representing Schottky. It has the highest speed because its average propagation delay is just 3 ns per gate. However, its average power dissipation is 23 mW/gate.
4. **74LS00**—It is called low-power Schottky *TTL*. It has an average propagation delay of 9.5 ns and an average power dissipation of 2 mW.
5. **74AS00** series—The letter *A* representing Advanced and *S* standing for schottky. It is called advanced schottky *TTL* series. It is the fastest *TTL* series.
6. **74ALS00** series—It is called Advanced Low-power Schottky *TTL* series. The *74ALS* series has the lower speed-power product and the lowest gate power dissipation of all the *TTL* series.
7. **74F00** series —The letter *F* standing for fast. This logic family uses a new *IC* fabrication technique to reduce interdevice capacitances to achieve reduced propagation delays. It has a propagation delay of 3 ns and a power consumption of 6 mW.

Table 71.3 summarizes the important characteristics of the each of the *TTL* sub-families.

Characteristic	<i>TTL</i> sub-family					
	74	74S	74LS	74AS	74ALS	74F
Propagation delay (ns)	9	3	9.5	1.7	4	3
Power Dissipation (mW)	10	20	2	8	1.2	6
Speed-power product (pJ)	90	60	19	13.6	4.8	18
Fan-out	10	20	20	40	20	33

### 71.38. ECL Circuit

The *ECL* also called current-mode logic (*CML*), has the **highest speed** of any of the currently-available logic circuits. It is primarily due to the fact that transistors never operate fully saturated or cut-off. That is why *ECL* is known as **non-saturated logic**. The latest *ECL* series has propagation delay time varying from 0.1ns to 0.8 ns. However, power dissipation is increased since one transistor is always in the active region.

Another feature of *ECL* is that it provides two outputs which are always complement of each other (Fig. 71.65). It is so because the circuit operation is based on a differential amplifier.

This family is particularly suited to monolithic fabrication techniques because logic levels are function of resistor ratios.

#### Circuit Operation

The basic circuit shown in Fig. 71.65 is combined *OR/NOR* circuit and is operated from a  $V_{EE} = -5.2$  V supply. A built-in constant-current source provides current to the emitters. Strictly speaking,

logic 1 is represented by  $-0.9\text{ V}$  (less negative) and logic 0 by  $-1.75\text{ V}$  (more negative). Please note that it is a *positive* logic. In negative logic, the functions would be *AND/NAND*. A reference voltage of  $-1.29\text{ V}$  is applied to the base of  $Q_3$  from a built-in temperature-compensated reference voltage source (*RVS*).

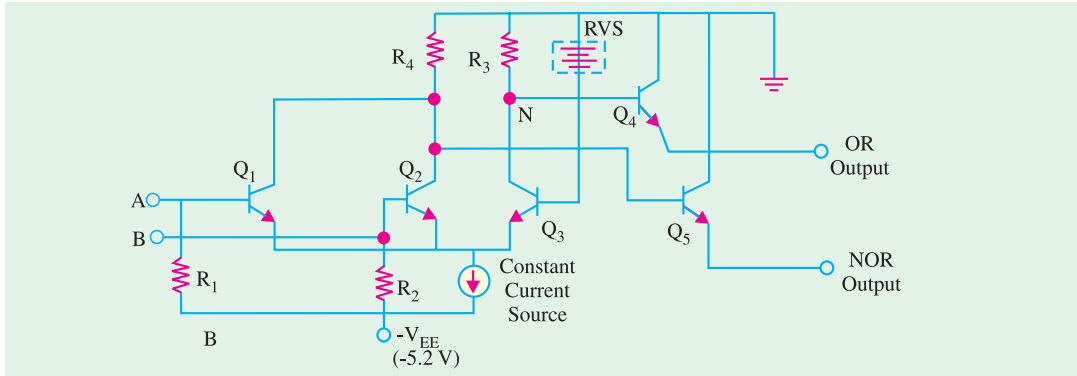


Fig. 71.65

**1. When both inputs are logical 0 i.e.  $-1.75\text{ V}$**

In this case, base potential of  $Q_3$  is less negative (or more positive) than the base potential of either  $Q_1$  or  $Q_2$ . Hence,  $Q_3$  conducts whilst  $Q_1$  and  $Q_2$  do not. Only enough base current is drawn by  $Q_3$  from *RVS* so as to remain out of saturation. The collector current of  $Q_3$  develops a voltage of  $-1.0\text{ V}$  across  $R_3$  which makes  $Q_4$  to conduct. Transistor  $Q_4$  gives an output voltage at the emitter of about  $-1.0 - 0.7 = -1.7\text{ V}$  which represents logic 0. Since collector potentials of  $Q_1$  and  $Q_2$  are nearly zero (because they are cut off), the output voltage at the emitter of  $Q_5$  is  $0 - 0.7 = -0.7\text{ V}$  which is a logic 1. Obviously, the two outputs are complements of each other.

**2. When either input A or B is at logical 1 i.e.  $-0.9\text{ V}$**

In that case, the associated transistor (either  $Q_1$  or  $Q_2$ ) is turned *ON* while  $Q_3$  is turned *OFF*. The collector potentials of  $Q_1/Q_2$  and are opposite of the previous case. Hence, now  $Q_5$  output is at logical 1 and  $Q_4$  output is at logical 0.

Typical characteristics of an *ECL* family are :

1. propagation delay time per gate of  $0.3\text{ ns}$  (meaning extremely fast speed),
2. power dissipation of  $25\text{ mW}$ ,
3. fan-out of 25 to 50,
4. noise margin from about  $0.2$  to  $0.25\text{ V}$ .

It will be interesting to know that *ECL* family of *ICs* does not include a wide range of general purpose logic devices as do the *TTL* and *CMOS* logic families. *ECL* does include complex, special purpose *ICs* used in applications such as high-speed data transmission, high-speed memories and high-speed arithmetic units. The relatively low noise margins and high power drain of *ECL* are disadvantages compared with *TTL* and *CMOS*. Another drawback is its negative power supply voltage and logic levels which are not compatible with those of other logic families. This makes it difficult to use *ECL* devices in conjunction with *TTL* and/or *CMOS ICs*. Special level-shifting (also called interface) circuits must be connected between *ECL* devices and the *TTL* (or *CMOS*) devices on both input and output.

**71.39. I<sup>2</sup>L Circuit**

It is the latest entry into the bipolar *saturated* logic field. It uses no biasing and loading resistors at all! Resistors require lot of power and space on an *IC* chip. Hence, their elimination results in *higher density circuits operating at much reduce power*. Because of its *high speed* and *less power dissipa-*



tion, it is used in large computers. Such circuits are also used where high packing density is of prime consideration as in digital wrist watches.  $I^2L$  chips are capable of microwatt power dissipation yet can provide high currents when necessary to drive LED displays. Another feature of  $I^2L$  (integrated injection logic) is that it is easy to fabricate.

### Circuit Operation

$I^2L$  NOR logic circuit is shown in Fig. 71.66. Here, transistors  $Q_1$  and  $Q_2$  act as current sources to the bases of  $Q_3$  and  $Q_4$  respectively.

If  $A$  input goes low, the current to the base of  $Q_2$  will be shorted to ground which will result in  $Q_2$  being turned OFF. The input  $B$  controls  $Q_4$  in a similar way.

If  $A$  is high, base current flowing to  $Q_2$  will turn in ON, making output  $C$  low. Same would be the case when  $B$  is high.

It is obvious that output would be high only when **both inputs**  $A$  and  $B$  are low.

The output would be low when either  $A$  or  $B$  or both are high *i.e.* NOR logic function.

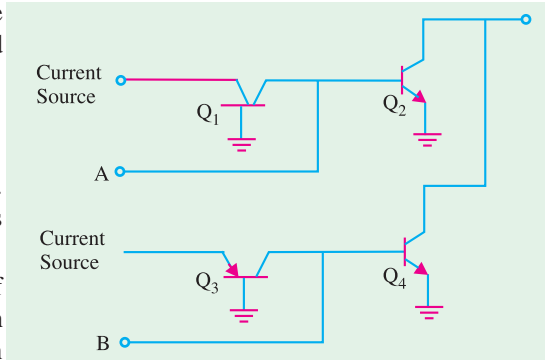


Fig. 71.66

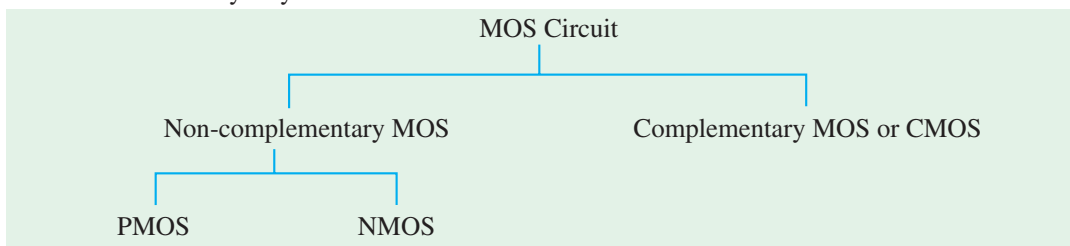
### 71.40. MOS Family

It does not use bipolar transistors but just Enhancement-only MOSFETs (Art. 71.1). There are two kinds of digital MOS circuits.

(a) one which uses MOSFETs of one polarity either all of  $N$ -type called NMOS or all of  $P$ -type called PMOS but not both on the same chip,

(b) the other which employs both  $N$ -type and  $P$ -type MOSFETs on the same chip. It is called complementary MOS (CMOS).

The MOS family may be subdivided as under :



Since a FET requires small area, it is possible to fabricate a large number of MOS circuits on a single small chip. Gating arrays with thousands of gates and flip-flops are manufactured in standard containers and are often used in IC memories and microprocessors.

Followings are some of the advantages of MOS ICs over the bipolar ICs (i.e. TTL, ECL etc.)

1. The MOS IC is relatively simple and inexpensive to fabricate.
2. The MOS device size is small and it consumes less power. Because of the small size, the MOS ICs can accommodate a much larger number of circuit elements on a single chip than bipolar IC in the area of large scale integration. This makes them especially well-suited for complex ICs such as microprocessor, memory chips etc.
3. MOS digital ICs normally do not use the IC resistor elements that take up so much of the chip area or bipolar ICs.

The continuous improvement in *MOSIC* technology has led to the device that are faster than *TTL* devices. Consequently *MOS* devices (especially *CMOS*) have become dominant in the *SSI* (small-scale integration) and *MSI* (medium-scale integration) market.

The major disadvantage of *MOS* devices is that they are susceptible to static-electricity damage. Although we can minimize it by adopting proper handling procedures, yet *TTL* is still more durable for laboratory experimentation. As a result, we are likely to see *TTL* devices used in education as long as they are available.

**71.41. PMOS Circuit**

A *PMOS NAND* gate is shown in Fig. 71.67 (a). As seen, there are no resistors in the circuit. The gate consists of two *E* only *MOSEFETs*  $Q_1$  and  $Q_2$  as logic elements and the third one  $Q_3$  as load resistors. When  $-V_{DD}$  (say  $-12\text{ V}$ ) is applied, *MOSFETs* will be turned *ON* and when  $0\text{ V}$  is applied they would be turned *OFF*. Hence, with positive logic,  $0\text{ V}$  would be 1 and  $-12\text{ V}$  would be 0 since 1 is assigned to the most positive voltage.

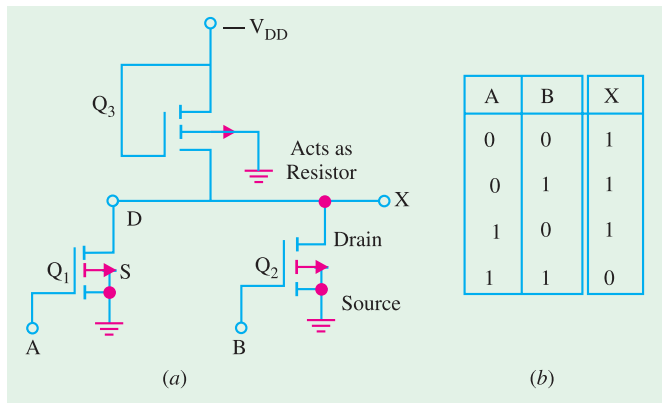


Fig. 71.67

**Circuit Operation**

1. If any of the two inputs *A* or *B* is at logic 0 (*i.e.*  $-12\text{ V}$ ), the concerned *MOSFET* would turn *ON* thereby offering a low resistance from drain to source thus causing the output to be nearly  $0\text{ V}$  *i.e.* a logic 1.
2. The output can be at  $-12\text{ V}$  *i.e.* logic 0 only when both inputs *A* and *B* are at  $0\text{ V}$  *i.e.* logic 1. This is the *NAND* function as shown by the truth table of Fig. 71.67 (b).

Incidentally, the positive logic *NAND* gate of Fig. 71.67 (a) would be the *negative logic NOR* gate since the two are identical.

**71.42. NMOS Circuit**

In Fig. 71.68 (a) is shown a two-input *NOR* gate circuit consisting of two *MOSFETs*  $Q_1$  and  $Q_2$  acting as logic elements and  $Q_3$  as a load resistor. For positive logic,  $0\text{ V}$  will be logic 0 and positive voltage ( $+V_{DD}$ ) will be logic 1.

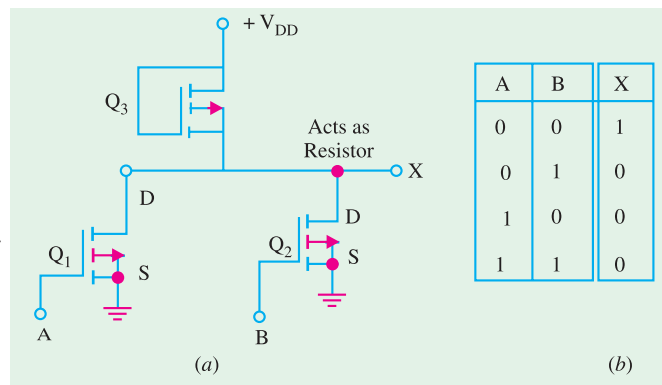


Fig. 71.68

**Circuit Operation**

1. If any of the inputs *A* or *B* is at logic 1, the corresponding *MOSFET* will conduct causing the output to go low *i.e.* logic 0.
2. If both inputs *A* and *B* are at logic 0, then both *MOSFETs* will be *OFF* driving the output to logic 1.

This is *NOR* function as shown by the truth table of Fig. 71.68 (b).

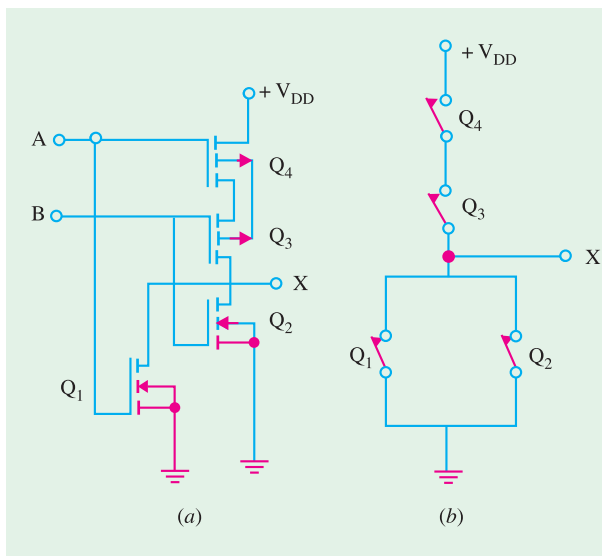


Fig. 71.69

Fig. 71.69 (a) shows a CMOS NOR circuit which has two *N*-channel MOSFETs  $Q_1$  and  $Q_2$  and two *P*-channel MOSFETs  $Q_3$  and  $Q_4$ . The two inputs  $A$  and  $B$  switch between  $+V_{DD}$  (logic 1) and ground (logic 0).

### Circuit Operation

1. Let  $A = B = \text{logic } 1$  i.e. have positive voltage. In that case,  $Q_1$  and  $Q_2$  are *ON* (closed switches) but  $Q_3$  and  $Q_4$  are *OFF* and act as open switches as shown in Fig. 71.69 (b). Hence, output  $C$  is logic 0.
2. If either  $A$  or  $B$  is at logic 1, then the associated *N*-channel MOSFET ( $Q_1$  or  $Q_2$ ) is turned *ON* but the associated *P*-channel MOSFET ( $Q_3$  or  $Q_4$ ) is turned *OFF*. Since either  $Q_3$  or  $Q_4$  would be *OFF* [Fig. 71.69 (b)], output  $C$  would be at logic 0.
3. When both  $A$  and  $B$  are at logic 0,  $Q_3$  and  $Q_4$  would be *ON* but  $Q_1$  and  $Q_2$  would be *OFF*—just the opposite of that in Fig. 71.69 (b). Hence output  $C$  would be at logic 1 (remember, voltage across an open equals the supply voltage—Art. 1.21).

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Fig. 71.70

The above logic represents a *NOR* function as shown in the truth table of Fig. 71.70. It would be observed from Fig. 71.69 that in each combination of  $A$  and  $B$ , there is at least **one open switch** between  $+V_{DD}$  and ground. Hence, the gate draws **only leakage current from supply for any static state**. However, when the gate switches from one level to another, some power is consumed because two MOSFETs are **partly ON** at the same time. Because of this reason, power dissipated by CMOS circuit is a **function of input signal frequency**. Higher the frequency, greater the power dissipation.

### 71.44. CMOS Sub-families and their characteristics

The CMOS family of ICs competes directly with TTL in the small and medium-scale integration. As CMOS technology has produced better and better performance characteristics, it has gradually taken over the field that has been dominated by TTL for so long. As a matter of fact, TTL devices will be around for a long time, but more and more new equipment is using CMOS logic circuits. These days, the CMOS ICs provide all the logic-functions that are available in TTL. Some special-purpose functions provided in CMOS ICs are not provided even by TTL. Before we look at the various CMOS

### 71.43. CMOS Circuit

CMOS logic circuits use both PMOS and NMOS devices in the same circuit. It gives the advantage of **drastic decrease in power dissipation** (12 nW per gate) and **increase in speed of operation**. In fact, it has the **lowest power dissipation** amongst different logic families. It has very high packing density i.e. larger number of circuits can be placed on a single chip. As a result, it is extensively used in VLSI circuits such as on-chip computers and memory systems. The recent silicon-on-sapphire MOS (SOSMOS) is 2 to 4 times faster than the standard CMOS. Hence, they are being widely used for everything from electronic watches and calculators to micro processors.

subfamilies, let us define few terms that are used when *ICs* from different families or series are to be used together or as replacements for one another.

**(a) Pin-to-pin Compatible :** Two *ICs* are said to be pin-to-pin compatible when three pin configurations are the same. For example, pin 14 on both *ICs* is  $V_{CC}$  supply. Pin 7 on both is ground etc.

**(b) Functionally Equivalent :** Two *ICs* are said to be functionally equivalent when the logic function they perform are exactly the same. For example, both contain four two-input *AND* gates, or two-input *NAND* gates etc.

**(c) Electrically compatible :** Two *ICs* are said to be electrically compatible when they can be connected directly to each other without taking any special measures to ensure proper operation.

Let us now study the different *CMOS* subfamilies.

- 5000/14000 series :** The *CMOS* 4000 series or the improved 4000 *B* is the oldest series and was first introduced by *RCA*. This series is functionally equivalent to 14000 series from Motorola. The *CMOS* devices in 4000/14000 series have a very low power dissipation and can operate over a wide range of supply voltage (*i.e.* from 3 to 15 V). These devices are very slow as compared to *TTL* and other *CMOS* subfamilies. The 40H00 series was designed to be faster than 4000 series. It did overcome some of the speed limitations, but it is still much slower than *LSTTL* series. The 4000 series have very low output current capability. The devices in 4000 series are not pin-compatible or electrically compatible with any *TTL* series.
- 74C00 series :** This is pin-compatible as well as functionally equivalent to *TTL* devices having the same device number. For example, 74C04 is a HEX inverter that has the same pin configuration as the *TTL* 7404 HEX inverter *IC*. The performance characteristics of 74C00 series are the same as those of 4000 series.
- 74HC/HCT00 series :** The letter *H/HC* is standing for high-speed *CMOS* series. This series has a speed which is 10 times faster than of 74LS00 series devices. It has also a higher current capability than that of 74C00 series. The 74HC/HCT00 series *ICs* are pin-compatible with and functionally equivalent to *TTL ICs* with the same device number. This series has become the most widely used series.
- 74AC/ACT 00 series :** This series is referred as advanced *CMOS* logic. It is functionally equivalent to the *TTL* series but not pin-compatible.
- 74AHC00 series :** This series is referred to as advanced high-speed *CMOS* logic. It is faster, and has lower power dissipation. The devices in this series are 3 times faster and can be used as direct replacements for *HC* series devices.
- 74-BiCMOS series :** These days, the *IC* manufacturers have developed a new logic series called *BiCMOS* logic. This series combines the best features of bipolar and *CMOS* logic *i.e.* low power characteristics of *CMOS* and high speed characteristics of bipolar circuits. *BiCMOS ICs* are available only in those functions that are used in microprocessor interfacing and memory applications such as latches, buffers, drivers and transceivers.
- 74-Low Voltage series :** A new series of logic using a nominal supply voltage of 3.3 V has been developed to meet the extremely low power design requirements of battery powered and hand held devices. These *ICs* are being designed into the circuits of notebook computers, mobile radios, hand-held video games, telecommunication equipment, personal digital assistant (*PDA*) and high performance work station computers.

Table 71.4 shows some of the common Low-voltage families identified by the suffixes as indicated :

Table 71.4

Suffixes	Low-voltage series
<i>LV</i>	Low-voltage <i>HCMOS</i>
<i>LVC</i>	Low-voltage <i>CMOS</i>
<i>LVT</i>	Low-voltage technology
<i>ALVC</i>	Advanced Low-voltage <i>CMOS</i>
<i>HLL</i>	High speed Low-power Low-voltage

The power consumption of *CMOS* logic ICs decreases approximately with the square of the supply voltage. On the other hand, the propagation delay increases slightly at this reduced voltage. However the speed is restored and even increased by using finer geometry and sub-micron (<1 $\mu$ m) *CMOS* technology that is tailored for low-power and low-voltage applications.

8. **74 AHC/AHCT series :** This is an enhanced version of 74 *HC/HCT*00 series. It provides superior speed and low power consumption. The 74AHC series devices have half the static power consumption, one-third the propagation delay, high output drive current, and can operate at  $V_{CC}$  of 3.3 to 5 V.

Table 71.5 shows a comparison of the important characteristics of the *CMOS* logic families. It is evident from this table that the devices from 74AHC/AHCT series has the lowest propagation delay and the smallest value of power dissipation.

Table 71.5

S. No.	Characteristic	CMOS logic family			
		4000 B	74 HC/HCT	74 BiCMOS	74 AHC/AHCT
1.	Propagation delay (ns)	50	8	2.9	3.7
2.	Power dissipation static (mW)	0.001	0.0025	0.0003–7.5	0.000009
3.	Speed-power product (pw-s)	105	15	0.00087 to 22	—

### Tutorial Problems No. 71.1

1. Prove the following identities :

$$(i) AB + A\bar{B} = A \quad (ii) A + A\bar{B} = A + B \quad (iii) (A + B)(A + \bar{B}) = A$$

2. Simplify the following Boolean expressions :

$$(i) A\bar{A}C \quad (ii) ABCD + ABD \quad (iii) ABCD + A\bar{B}CD$$

$$[(i) 0, (ii) ABD, (iii) ACD]$$

3. Simplify the following Boolean functions :

$$(i) A + \bar{A}B + AB \quad (ii) \bar{A}B + \bar{A}B + AB\bar{A}B \quad (iii) (AB + C)(AB + D)$$

$$(iv) A + \bar{B}C(A + B\bar{C}) \quad (v) A[B + C(\overline{AB + AC})] \quad (vi) (A + B) \cdot (A + C)$$

$$[(i) A + B \quad (ii) 1 \quad (iii) AB + CD \quad (iv) A \quad (v) AB \quad (vi) A + B.C \quad (vii) \bar{B}]$$

4. Simplify the following Boolean expressions :

(i)  $A\bar{B}C + AB\bar{C} + ABC$  (ii)  $\bar{A}\bar{C} + B\bar{C} + \bar{A}BC + ABC$

(iii)  $AD + A\bar{B}\bar{C} + \bar{B}C\bar{D} + \bar{A}CD + \bar{A}BC\bar{D}$

(iv)  $A\bar{B}\bar{D} + AB\bar{C}\bar{D} + ABC\bar{D} + \bar{A}BC\bar{D}$

**[(i)  $A(B+C)$  (ii)  $B + \bar{A}\bar{C}$  (iii)  $AD + \bar{A}\bar{B} + \bar{A}\bar{C}$  (iv)  $A\bar{B}\bar{D} + A\bar{D} + C\bar{D}$ ]**

5. Use truth tables to verify the following identities :

(i)  $\bar{A} + B = \bar{A} \cdot B$  (ii)  $A + BC = (A + B)(A + C)$

(iii)  $A(\bar{A} + \bar{B}) = AB$

6. Write the logic equation for the switching circuit of Fig. 71.71.

**$[A(B + C + D + E)]$**

7. Write down the logical expression which describes the working of the circuit of Fig. 71.72.

**$[AB(CD)]$**

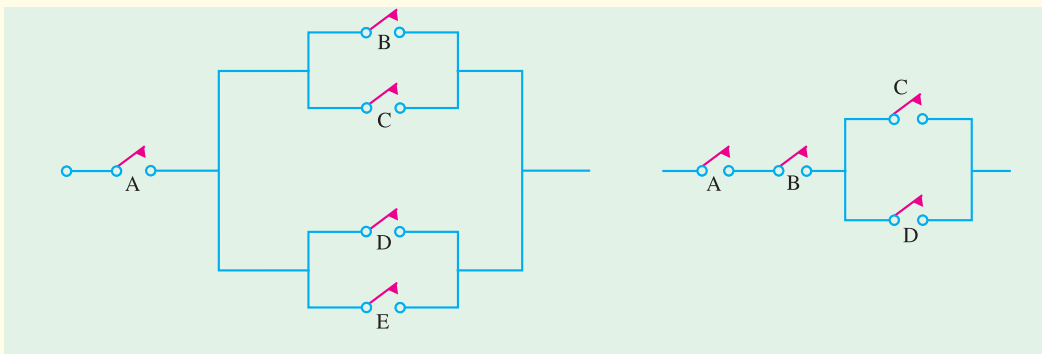


Fig. 71.71

Fig. 71.72

8. Simplify the following Boolean expressions :

(i)  $y = ABC\bar{C} + AB\bar{C} + AB\bar{C} + \bar{A}\bar{C}\bar{B} + \bar{A}\bar{B}$

(ii)  $y = ABC\bar{D} + ABC\bar{D} + ABC + ABD + CD$

**[(i)  $\bar{A}\bar{C} + \bar{B}$  (ii)  $AB + CD$ ]**

9. Design a logic circuit whose output is HIGH only when majority of inputs A, B and C are LOW.

10. Determine the Boolean expression for the logic circuit shown in Fig. 71.73. Simplify the Boolean expression using Boolean Laws and De Morgan's theorem. Redraw the logic circuit using the simplified Boolean expression.

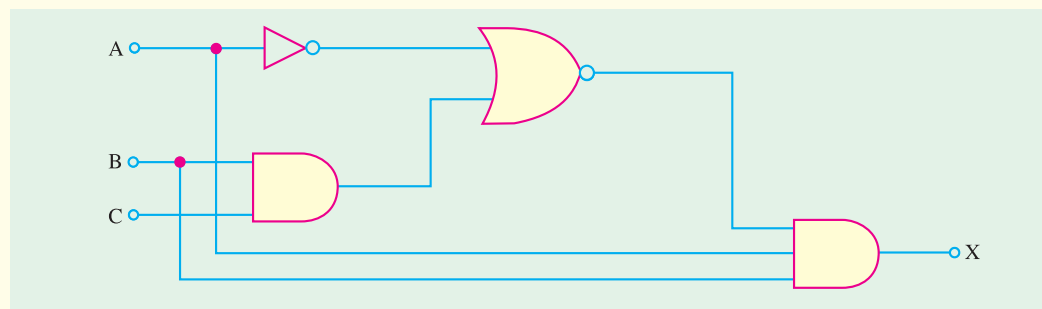


Fig. 71.73

11. Determine the output, X of a logic circuit shown in Fig. 71.74. Simplify the output expression using Boolean Laws and theorems. Redraw the logic circuit with the simplified expression.

**$(\bar{B}\bar{C}D + AC)$**

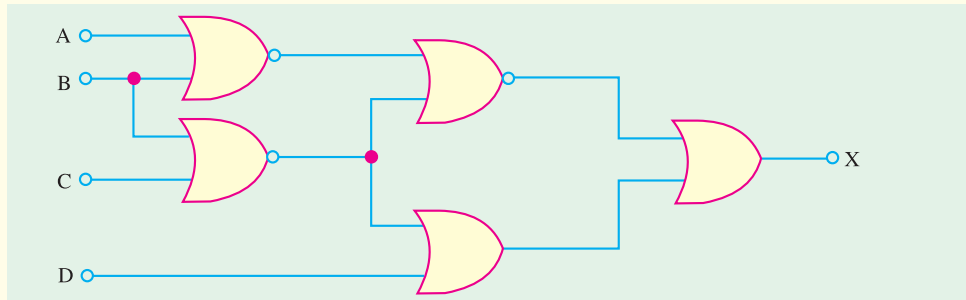


Fig. 71.74

12. Consider the logic circuit shown in Fig. 71.75. Determine the Boolean expression at the circuit output X, simplify it. Using the simplified expression, redraw the logic circuit.

$$A(\overline{B} + C)$$

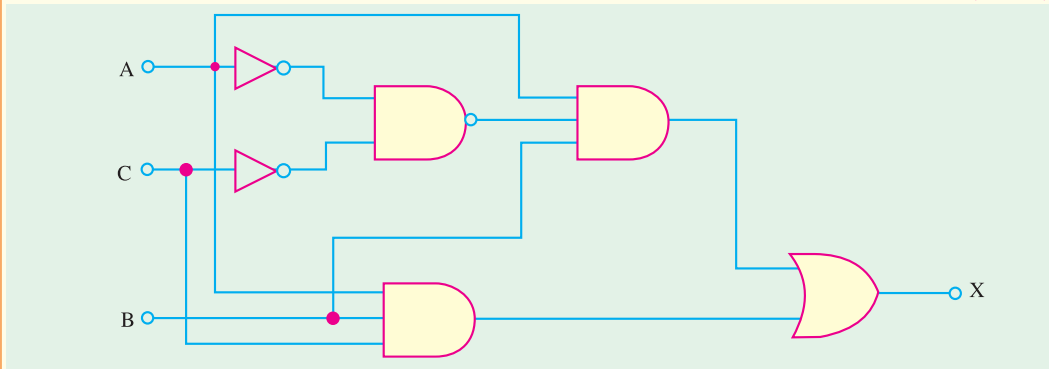


Fig. 71.75

13. Fig. 71.76 (a) shows a three-variable Karnaugh map. Group the 1s and hence obtain the minimized Boolean expression.

$$(AB + BC + \overline{A} \overline{B} C)$$

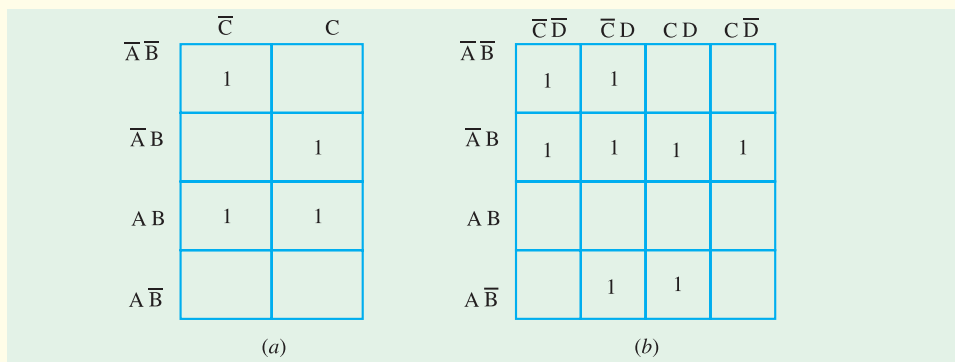


Fig. 71.76

14. Fig. 71.76 (b) shows a four-variable Karnaugh-map. Group the 1s and hence obtain the minimized Boolean expression.
15. A truth table has output 1s for these inputs :  $ABCD = 0011, ABCD = 0110, ABCD = 1000$  and  $ABCD = 1100$ , and 0s for the other inputs. Draw the Karnaugh map and find the simplified Boolean equation for the truth table.
16. Determine the minimized expression for the Karnaugh map shown in Fig. 71.77 (a).

$$(\overline{A} B + \overline{A} \overline{B} C + \overline{A} \overline{B} D)$$

$$(\overline{A} C D + \overline{A} B C D + \overline{A} B C \overline{D})$$

$$(B \overline{C} + BD + \overline{B} C)$$

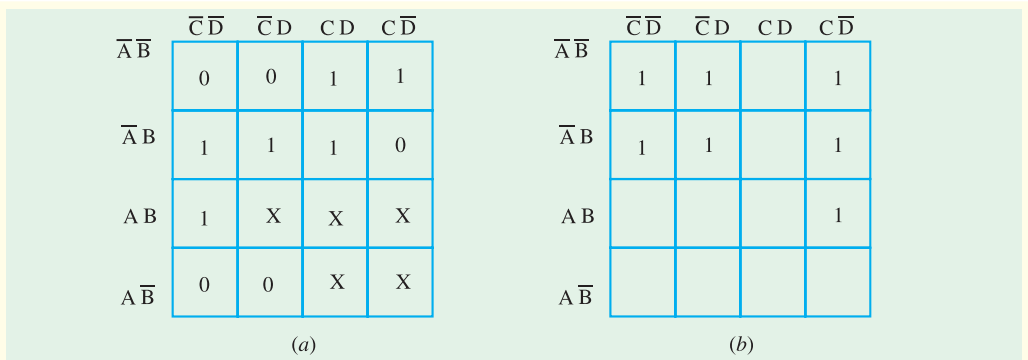


Fig. 71.77

17. Determine the simplified Boolean expression for the Karnaugh map shown in Fig. 71.77 (b).

(UPSC Engg. Services 1995)  $(\bar{A}\bar{C} + \bar{A}\bar{D} + ABC)$

OBJECTIVE TESTS – 71

- Boolean algebra is essential based on
  - symbols
  - logic
  - truth
  - numbers
- The first person who used Boolean algebra for the design of relay switching circuits was
  - Aristotle
  - Boole
  - Shannon
  - Ramanujam
- Different variables used in Boolean algebra can have values of
  - 0 or 1
  - low or high
  - true or false
  - On or OFF.
- According to the algebra of logic,  $(A + \bar{A})$  equals
  - A
  - 1
  - 0
  - $\bar{A}\bar{A}$ .
- According to the absorptive Laws of Boolean algebra, expression  $(A + A\bar{B})$  equals
  - A
  - B
  - AB
  - A
- When we demorganize  $\overline{A\bar{B}}$ , we get
  - $\overline{AB}$
  - $\bar{A} + \bar{B}$
  - $A + B$
  - $\overline{AB}$ .
- The dual of the statement  $(A + 1) = 1$  is
  - $A \cdot 1 = A$
  - $A \cdot 0 = 0$
  - $A + A = A$
  - $A \cdot A = 1$ .
- The expression  $\overline{ABC}$  can be simplified to
  - $\bar{A} \cdot \bar{B} \cdot \bar{C}$
  - $AB + BC + CA$
  - $AB + \bar{C}$
  - $\bar{A} + \bar{B} + \bar{C}$ .
- While obtaining minimal sum-of-products expression,
  - all don't cares are ignored
  - all don't cares are treated as logic 1s
  - all don't cares are treated as logic 0s
  - only such don't cares that aid minimization are treated as logic 1s.
- In a saturated bipolar logic circuit, transistor operate
  - in deep cut-off
  - over active region
  - in saturation
  - just short of saturation
- Saturated logic circuits have inherently
  - short saturation delay time
  - low switching speed
  - higher power dissipation
  - lower noise immunity.
- Noise margin is expressed in
  - decibel
  - watt
  - volt
  - phon
- DTL family employs
  - resistors and transistors
  - diode and resistor
  - diode and transistors
  - diodes, resistors and transistors.
- The chief advantage of Schottky TTL logic family is its least
  - power dissipation
  - propagation delay
  - fan-in
  - noise immunity.
- The main advantage claimed for ECL family of logic gates is its
  - very large fan-in
  - use of negative power supply voltage
  - extremely low propagation times
  - least power dissipation.
- Special feature of an  $I^2L$  logic circuit is that it
  - uses only high-value resistors



- (b) dissipates negligible power  
 (c) is a bipolar saturated logic  
 (d) is easy to fabricate  
 (e) uses no biasing and loading resistors.
17. A unique advantages feature of *CMOS* logic family is its  
 (a) use of *NMOS* circuits  
 (b) power dissipation in nanowatt range  
 (c) speed  
 (d) dependence on frequency for power dissipation.
18. *CMOS* circuits are extensively used for on-chip computers mainly because of their extremely  
 (a) low power dissipation  
 (b) large packing density  
 (c) high noise immunity  
 (d) low cost.
19. The main advantage of a *CMOS* logic family over the *TTL* family is its  
 (a) much reduced power  
 (b) increased speed of operation  
 (c) extremely low cost  
 (d) series base resistor
20. *CMOS* logic family uses only  
 (a) *MOSFETs* and resistors  
 (b) *NMOS* circuits  
 (c) *MOSFETs* (d) bipolar transistors.
21. Power is drawn by a *CMOS* circuit only when  
 (a) its output is high (b) its output is low  
 (c) it switches logic levels  
 (d) in static state.
22. The most obvious identifying feature of a *TTL* gate is its  
 (a) large fan-out (b) high power dissipation  
 (c) interconnected transistors  
 (d) multiemitter input transistor.
23. In digital circuits shottky transistors are preferred over normal transistor because of their  
 (a) lower propagation delay  
 (b) higher propagation delay  
 (c) lower power dissipation  
 (d) higher power dissipation
24. A unique operting feature of *ECL* circuit is its  
 (a) very high speed  
 (b) high power dissipation  
 (c) series base resistor  
 (d) compatibility with other logic series.
25. The fan-in a logic gate refers to the number of  
 (a) input devices that can be connected  
 (b) input terminals (c) output terminals  
 (d) circuits output can drive
26. Which of the following statements regarding *ICs* is not correct,  
 (a) *ECL* has the least propagation delay  
 (b) *TTL* has the largest fan out  
 (c) *CMOS* has the biggest noise margin  
 (d) *TTL* has the lowest power consumption.
27. The Boolean equation for *NOR* gate is  
 (a)  $\overline{AB} = C$  (b)  $\overline{A + B} = C$   
 (c)  $A + B = C$  (d)  $\overline{A + B} = C$ .
28. The Boolean equation for a *NAND* gate is  
 (a)  $\overline{A + B} = C$  (b)  $\overline{AB} = C$   
 (c)  $A + B = C$  (d)  $\overline{A + B} = C$
29. The logic function of an inverter is  
 (a)  $B = A$  (b)  $B = \overline{A}$   
 (c)  $B = \overline{\overline{A}}$  (d) None of these
30. The simplified form of the Boolean expression  $Y = (\overline{A}.BC + D) (\overline{A}.D + \overline{B}.C)$  can be written as  
 (a)  $\overline{A}.D + \overline{B}.C.D$  (b)  $AD + B.C.D$   
 (c)  $(\overline{A} + D)(\overline{B}.C + \overline{D})$  (d)  $A.\overline{D} + BC.\overline{D}$

(GATE; 2004)

### ANSWERS

1. (b) 2. (c) 3. (a) 4. (b) 5. (a) 6. (c) 7. (b) 8. (d) 9. (d) 10. (c) 11. (b) 12. (c)  
 13. (d) 14. (b) 15. (c) 16. (e) 17. (b) 18. (b) 19. (a) 20. (c) 21. (c) 22. (d) 23. (a) 24. (a)  
 25. (b) 26. (d) 27. (d) 28. (d) 29. (b) 30. (a)

### Learning Objectives

- Flip-Flop (FF)
- Latch
- NAND Gate Latch
- NOR Gate Latch
- Clocked Signals
- Some Main Ideas Common to Clocked Flip-Flops
- Clocked S-C Flip-Flop
- Clocked J-K Flip-Flop
- Clocked D Flip-Flop
- Parallel Transfer of Data Using D-Flip-Flops
- D Latch (Transparent Latch)
- Clocked J-K Flip-Flop with Asynchronous Inputs
- Flip-Flop Timing Parameters
- IC Flip-Flop Timing Values
- Applications of Flip-Flop
- Flip-Flop Synchronization
- Parallel Data Transfer
- Frequency Division
- Counting
- Schmitt Trigger Devices
- Types of one-shots
- Retriggerable One-Shot
- Actual One-Shot Devices
- Clock Generator Circuits
- Schmitt Trigger Oscillator
- 555 Timer Used as an Astable Multivibrator

# FLIP-FLOPS AND RELATED DEVICES



A flip-flop is made up of logic gates. The flip-flops are extensively used as a memory cell in static random access memory of a computer

### 72.1. Introduction

The output logic levels of combinational logic circuits at any instant of time are dependent on the logic levels present at the inputs at that time. Any prior input-logic level conditions have no effect on the present outputs. This is due to the fact that combinational logic circuits have no *memory*. However, it will be interesting to know that most digital systems are made up of both combinational logic circuits and memory elements.

The most important memory element is the flip-flop (abbreviated as FF). The FF is made up of an assembly of logic gates. It may be noted that even though a logic gate, by itself, has no storage capability, but several such logic gates can be connected together in ways that permit information to be stored. Several different gate arrangements are used to produce these flip-flops.

The flip-flops are used extensively as a memory cell in static random access memory (SRAM) of a computer.

### 72.2. Flip-Flop (FF)

We have already discussed in the last article that flip-flop (abbreviated as FF) is made up of logic gates and it permits information to be stored in it. Fig. 72.1 (a) shows a general type of symbol used for a flip-flop.

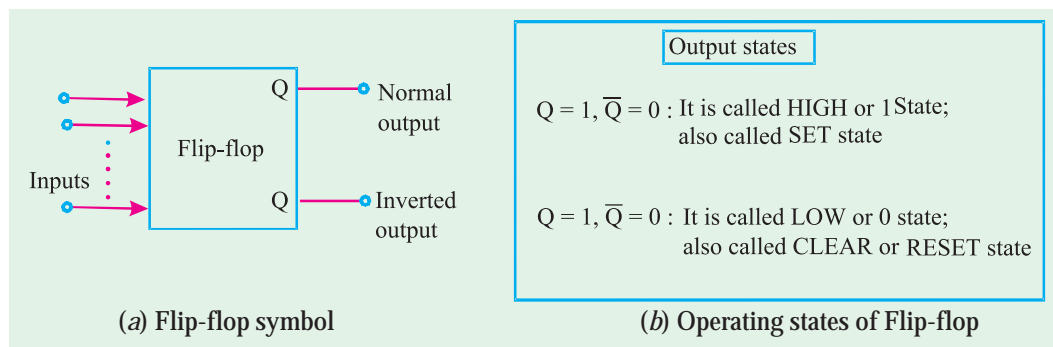


Fig. 72.1

As seen from this diagram, the flip-flop has two outputs labeled as  $Q$  and  $\bar{Q}$  that are inverse (or complement) of each other. The  $Q$  output is called the *normal* flip-flop output and  $\bar{Q}$  is the *inverted* flip-flop output. It may be carefully noted that whenever we refer to the state of a flip-flop, we are referring to the state of its *normal* ( $Q$ ) output. For instance if we say flip-flop is in the HIGH state, we mean that  $Q = 1$ . On the other hand if we say flip-flop is in LOW state, we mean that  $Q = 0$ . It is automatically understood that the  $\bar{Q}$  state will always be *inverse* of  $Q$ , i. e., if  $Q = 1$ ,  $\bar{Q} = 0$ , and if  $Q = 0$ ,  $\bar{Q} = 1$ .

Fig. 72.1 (b) shows the two possible operating states of a flip-flop. One possible state is  $Q = 1$ ,  $\bar{Q} = 0$  and the other  $Q = 0$ ,  $\bar{Q} = 1$ . The state  $Q = 1$ ,  $\bar{Q} = 0$  is called HIGH state or 1 state. It is also called *SET* state. Whenever, the inputs to a flip-flop cause it to go to the  $Q = 1$  state, we call this *setting* the flip-flop or the flip-flop is set.

In a similar way, the state  $Q = 0$ ,  $\bar{Q} = 1$  is called LOW state or 0 state. It is also called *RESET* or *CLEAR* state. Whenever, the inputs to a flip-flop cause it to go to the  $Q = 0$  state, we call this *resetting or clearing* the flip-flop. In the later part of the chapter we will see that many flip-flops will have a *SET* input and/or *RESET* (*CLEAR*) input that is used to drive the flip-flop into a specific output state.

It may be noted from the flip-flop symbol (Fig. 72.1 (a)) that a flip-flop can have one or more inputs. These inputs can be used to switch the flip-flop back and forth between its two possible output states.

It will be shown later that most flip-flop inputs need only to be momentarily activated (or pulsed) in order to cause a change in the output state. The flip-flop output will remain in the new state even after the input pulse is over. This is flip-flop's memory characteristic.

It will be interesting to know that a flip-flop is also known as a latch and a bistable multivibrator. The term **“latch”** is used for certain types of flip-flops that will be described later. The term bistable multivibrator is the more technical name but flip-flop is the one used more frequently among the engineers and technologists.

### 72.3. Latch

A latch is the most basic type of flip-flop circuit. It can be constructed using NAND or NOR gates. Accordingly the latches are of two types :

1. NAND gate latch and
2. NOR gate latch

Both these types of latch are discussed one by one in the following pages.

### 72.4. NAND Gate Latch

**Construction.** Fig. 72.2 shows a latch constructed from NAND gates. It is called NAND gate latch or simply NAND latch. As seen from this diagram, the two NAND gates are cross-coupled. This means output of NAND-1 is connected to one of the inputs of NAND-2. Similarly, the output of NAND-2 is connected to one of the inputs of NAND-1. The NAND gate latch outputs are labeled as  $Q$  and  $\bar{Q}$  respectively. Under normal conditions, these outputs will always be the inverse (or complement) of each other. The latch has two inputs namely SET and CLEAR. The SET input sets  $Q$  output of the latch to 1 state while the CLEAR input sets the  $Q$  output to the 0 state.

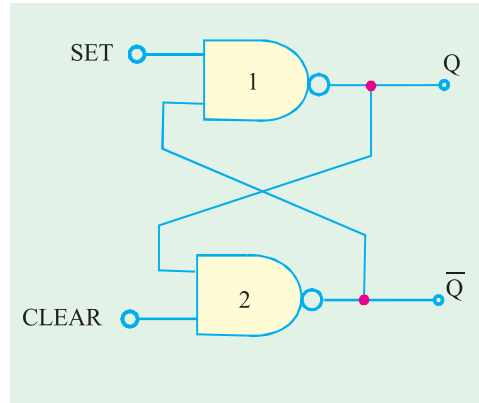


Fig. 72.2. NAND gate latch.

**Operation.** Let us now understand the operation of NAND gate latch. Normally, the SET and CLEAR inputs of the latch are resting in the HIGH state. Whenever we want to change the latch outputs, one of the two inputs will be pulsed LOW. Let us begin our study by showing that there are two equally likely output states when SET = CLEAR = 1. One possibility is shown in Fig. 72.3 (a) where we have  $Q = 0$  and  $\bar{Q} = 1$ . With  $Q = 0$ , the inputs to the NAND-2 are 0 and 1, which produces  $\bar{Q} = 1$ . The 1 from  $\bar{Q}$  causes NAND-1 to have 1 at both inputs to produce a 0 output at  $Q$ . Thus we find that a LOW at the NAND-1 output produces a HIGH at NAND-2 output which in turn keeps the NAND-1 output LOW.

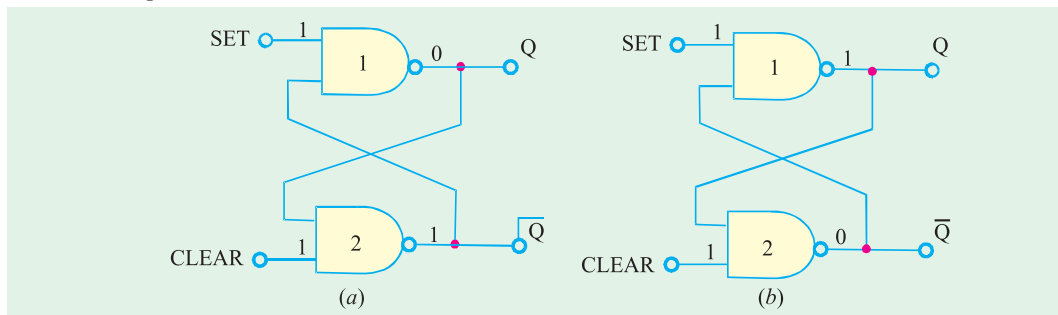


Fig. 72.3. Illustrating the two possible output states of NAND gate latch.

Fig. 72.3 (b) shows the second possibility. Here  $Q = 1$  and  $\bar{Q} = 0$ . As seen from this figure, the HIGH from NAND-1 produces a LOW at the NAND-2 output, which in turn keeps the NAND-1 output HIGH. Thus there are two possible output states when SET = CLEAR = 1. However which one of these two states exist will depend on what has occurred previously at the inputs. This is discussed below in more detail where we will take up three situations : (1) setting the latch, (2) clearing the latch and simultaneous setting and clearing the latch.

**1. Setting the Latch.** We have already mentioned above that if SET = CLEAR = 1, the output  $Q$  can be in two possible states i.e.  $Q = 0$  or  $Q = 1$ . Now we will study as what happens when the SET input is momentarily pulsed LOW while CLEAR is kept HIGH.

Fig. 72.4 (a) shows what happens when  $Q = 0$  prior to the occurrence of the pulse. As SET input is pulsed LOW at time ' $t_0$ ',  $Q$  will go HIGH. This will force  $\bar{Q}$  to go LOW so that NAND-1 has now two LOW-inputs. Thus when SET returns to 1 state at  $t_1$ , the NAND-1 output remains HIGH which in turn keeps the NAND-2 output LOW.

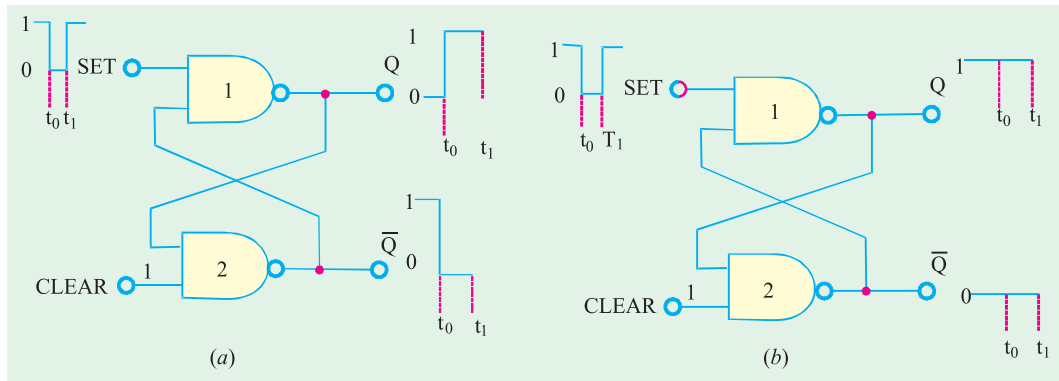


Fig. 72.4. Setting the NAND gate latch

Fig. 72.4 (b) shows what happens when  $Q = 1$  and  $\bar{Q} = 0$ . Prior to the application of the SET pulse. Since  $\bar{Q} = 0$  is already keeping the NAND-1 output HIGH, the LOW pulse at SET will not change anything. Thus when SET returns HIGH, the latch outputs are still in the  $Q = 1$ ,  $\bar{Q} = 0$  state (i.e no change in states).

**2. Clearing the latch.** Now we will study what happens when CLEAR input is pulsed LOW while SET is kept HIGH. Fig. 72.5 (a) shows the situation when  $Q = 0$  and  $\bar{Q} = 1$  prior to the application of the pulse. Since  $Q = 0$  is already keeping the NAND-2 output HIGH, the LOW pulse at CLEAR will not have any effect. When CLEAR returns HIGH, the latch outputs are still  $Q = 0$  and  $\bar{Q} = 1$ .

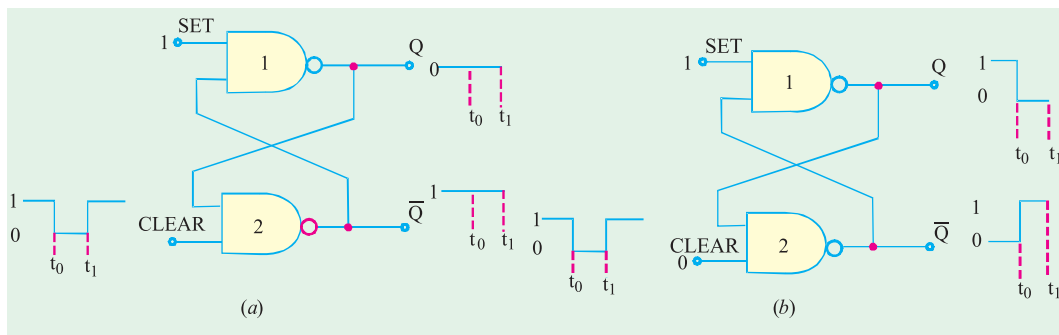


Fig 72.5. Clearing the NAND gate latch.

Fig 72.5 (b) shows the situation where  $Q = 1$  prior to the occurrence of the CLEAR pulse. As CLEAR is pulsed LOW AT  $t_0$ ,  $\bar{Q}$  will go HIGH and this HIGH forces  $Q$  to go LOW because NAND-2 now has two LOW inputs. Thus when CLEAR returns HIGH at  $t_1$ , the NAND-2 output remains HIGH. This in turn keeps the NAND-1 output LOW.

**3. Simultaneous setting and clearing.** If the SET and CLEAR inputs are simultaneously pulsed LOW, this will produce HIGH levels at both NAND outputs so that  $Q = \bar{Q} = 1$ . This is an undesirable condition as the two outputs are supposed to be inverse of each other. Moreover, when the SET and CLEAR inputs return HIGH, the resulting output state will depend on which input returns HIGH first. Simultaneous transitions back to 1 state will produce unpredictable results. Because of these reasons the SET = CLEAR = 0 condition is normally not used for the NAND latch and is considered as invalid condition.

Inputs		Output
SET	CLEAR	
1	1	No change
0	1	$Q = 1$
1	0	$Q = 0$
0	0	$Q = \bar{Q} = 1$ ( Invalid)

Fig. 72.6. Truth table for NAND gate latch.

The operation of NAND gate latch may be summarized in the form of a truth table as shown in Fig 72.6. Each line of the truth table is described as below :

- 1. SET = CLEAR = 1.** This condition is the normal resting state of the latch. The  $Q$  and  $\bar{Q}$  outputs will remain in the same state in which they were prior to this input condition.
- 2. SET = 0, CLEAR = 1.** This condition will always cause the output to go to  $Q = 1$  state where it will remain even after SET returns HIGH. This is called setting the latch.
- 3. SET = 1, CLEAR = 0.** This condition will always produce  $Q = 0$ . The output will remain in this state even after CLEAR returns HIGH. This is called clearing or resetting the latch.
- 4. SET = CLEAR = 0.** This condition tries to set and clear the latch simultaneously. It produces invalid results and should not be used.

**72.5. Alternative Representations of NAND Gate Latch**

We have already discussed in the last article about the NAND gate latch operation. It was mentioned that both SET and CLEAR inputs are active-LOW. Further the SET input will set  $Q = 1$  when SET goes LOW. On the other hand, the CLEAR input will clear  $Q = 0$  when CLEAR goes LOW. Because of this reason NAND gate latch is often drawn using the alternative representation for each NAND gate as shown in Fig 72.7 (a). (Recall from Art 3-41 that a NAND gate is equivalent to a bubbled OR gate). The bubbles on the inputs of OR gate as well as labelling of the signals as  $\overline{\text{SET}}$  and  $\overline{\text{CLEAR}}$  indicate the active-LOW status of these inputs.

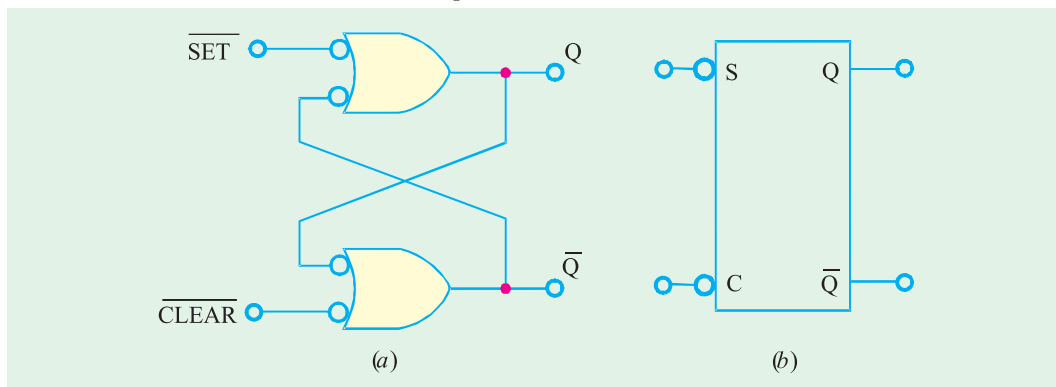


Fig. 72.7. Alternative representations of NAND gate latch.

Fig. 72.7 (b) shows a block representation. The  $S$  and  $C$  labels represent the SET and CLEAR inputs. While the bubbles at  $S$  and  $C$  inputs indicate the active LOW nature of these inputs. So remember whenever we use this block symbol, it represents a NAND gate latch.

**Note.** The action of **Clearing** a flip-flop or a latch is also called **resetting**. Both these terms (*i.e.* clearing or resetting) is used interchangeably in the field of digital electronics. Thus a SET-CLEAR latch can also be referred to as SET-RESET latch ( or simply  $S - R$  latch).

**Example 72.1.** The waveforms shown in Fig 72.8 (a) are applied to the inputs of the NAND latch shown in Fig. 72.8 (b). Assume that initially  $Q = 0$  and determine the  $Q -$  waveform.

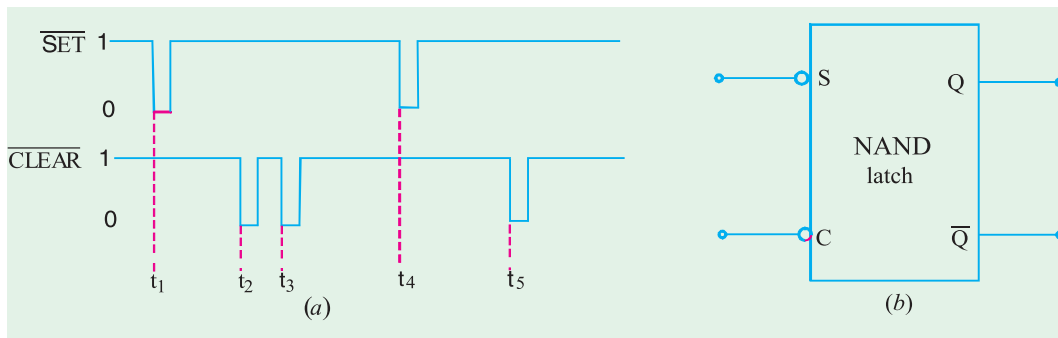


Fig. 72.8

**Solution** Given :

In order to determine the waveform at  $Q$ -output, refer to the truth table shown in Fig. 72.6 and the input waveforms. Notice that prior to  $t = t_1$ ,  $\overline{SET} = \overline{CLEAR} = 1$  and  $Q = 0$ . At  $t = t_1$ ,  $\overline{SET}$  goes 0 and  $\overline{CLEAR}$  stays at 1. Referring to the truth table in Fig. 72-6, we find that if  $\overline{SET} = 0$ ,  $\overline{CLEAR} = 1$ ,  $Q = 1$ . Therefore the  $Q$ - output is indicated as 1 at  $t = t_1$  in Fig. 72.9.

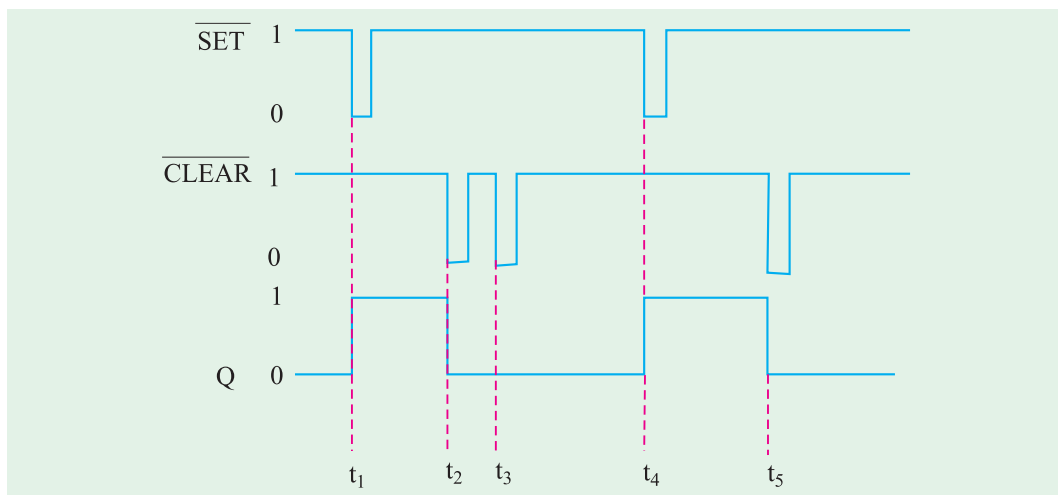


Fig. 72.9

At  $t = t_2$ ,  $\overline{SET} = 1$ , and  $\overline{CLEAR}$  goes 0. Again referring to the truth table shown in Fig. 72.6, we find that  $Q = 0$ , therefore we sketch the  $Q$  output as 0 in Fig. 72.9. Similarly, at  $t = t_3$ ,  $\overline{SET} = 1$ , and  $\overline{CLEAR} = 0$ . Since there is no change in inputs, the  $Q$ -output also remains at 0 as indicated in Fig. 72.9. At  $t = t_4$ ,  $\overline{SET}$  goes 0, and  $\overline{CLEAR}$  is 1, therefore the output goes 1. At  $t = t_5$ ,  $\overline{SET} = 1$  and  $\overline{CLEAR}$  goes 0, therefore output goes 0. The complete  $Q$ -output waveform is shown in Fig. 72.9.

**Example 72.2.** If the  $\overline{SET}$  and  $\overline{CLEAR}$  waveforms shown in Fig 72.10 (a) are applied to the inputs of NAND latch shown in Fig. 72.10 (b) determine the waveform that will be observed on the  $Q$  output. Assume that initially  $Q = 0$ .

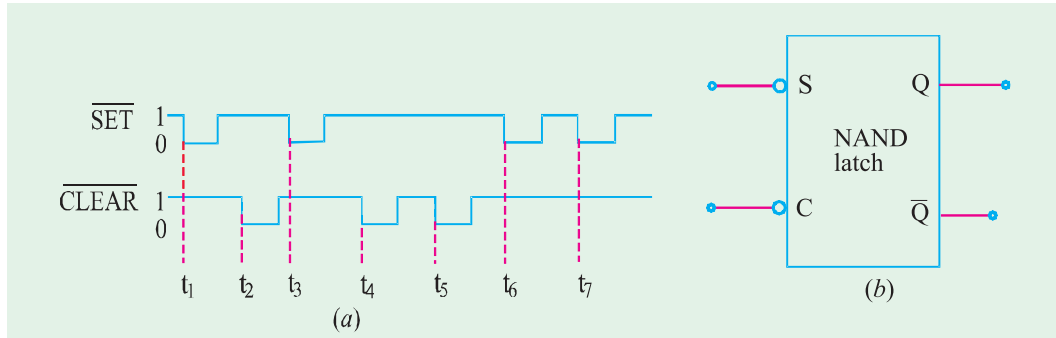


Fig. 72.10

**Solution :**

In order to determine, the  $Q$ -output, let us refer to the truth table shown in Fig. 72.6 and the input waveforms. At  $t = t_1$ ,  $\overline{SET}$  goes from 1 to 0,  $\overline{CLEAR} = 1$ , so the output  $Q$  goes from 0 to 1 as shown in Fig 72.11. At  $t = t_2$ ,  $\overline{SET} = 1$ ,  $\overline{CLEAR}$  goes from 1 to 0, so does the  $Q$ -output (i.e. it also goes from 1 to 0). At  $t = t_3$ ,  $\overline{SET}$  goes from 1 to 0,  $\overline{CLEAR} = 1$ , therefore  $Q$ -output goes 1, At  $t = t_4$ ,  $\overline{SET} = 1$ ,  $\overline{CLEAR}$  goes to 0, therefore  $Q$ -output goes 0 and so on. The complete waveform at  $Q$ -output is shown in Fig. 72.11.

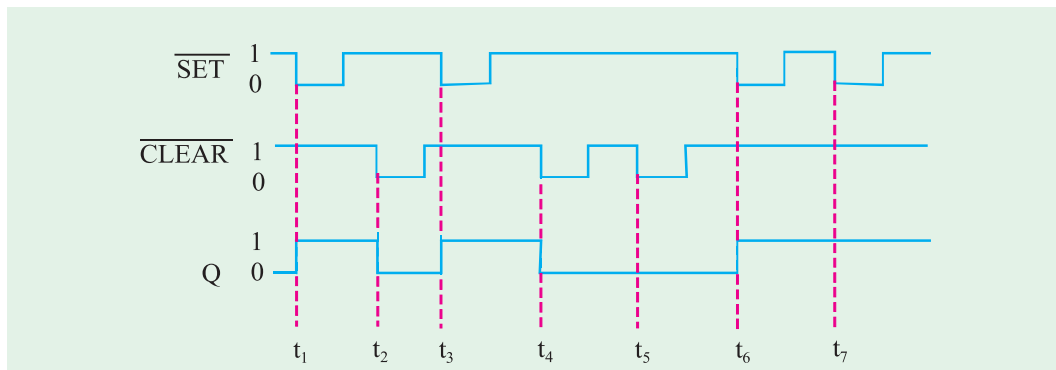


Fig. 72.11

**72.6. Application of NAND Latch to Debounce a Mechanical Switch**

Consider a mechanical switch with contact points  $A$  and  $B$  connected to  $V_{cc}$  (+ 5V) supply and ground respectively as shown in the Fig. 72.12 (a). It has been found experimentally that when the switch moves from contact position  $A$  to  $B$ , it produces several output voltage transitions as shown in Fig. 72.12 (b). It is due to a phenomenon called **switch bounce** i.e. the switch makes and breaks contact with contact  $B$  several times before coming to rest on contact  $B$ . In a similar manner, when the switch moves from contact position  $B$  to  $A$ , it again produces several output voltage transitions before coming to rest on contact  $A$ .



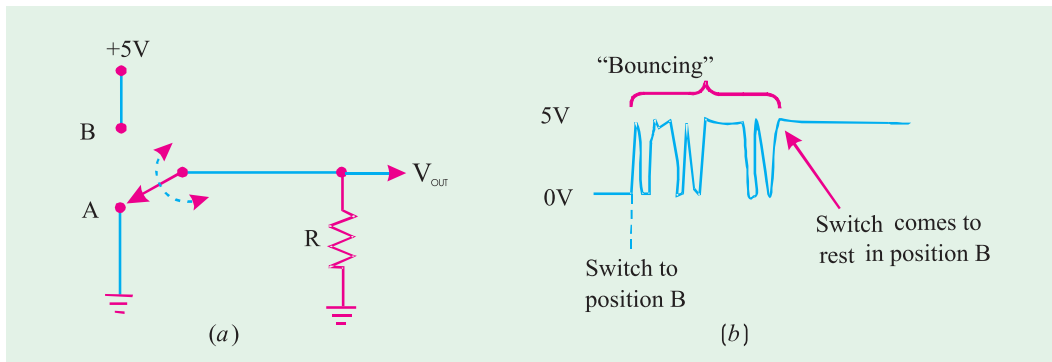


Fig. 72.12. Illustrating Switch Contact bounce

As a matter of fact, the multiple transitions on the output signal generally last only for few milliseconds. But such transitions are unacceptable in many applications. A NAND latch can be used as shown in Fig. 72.13 (a) to eliminate the switch bounce.

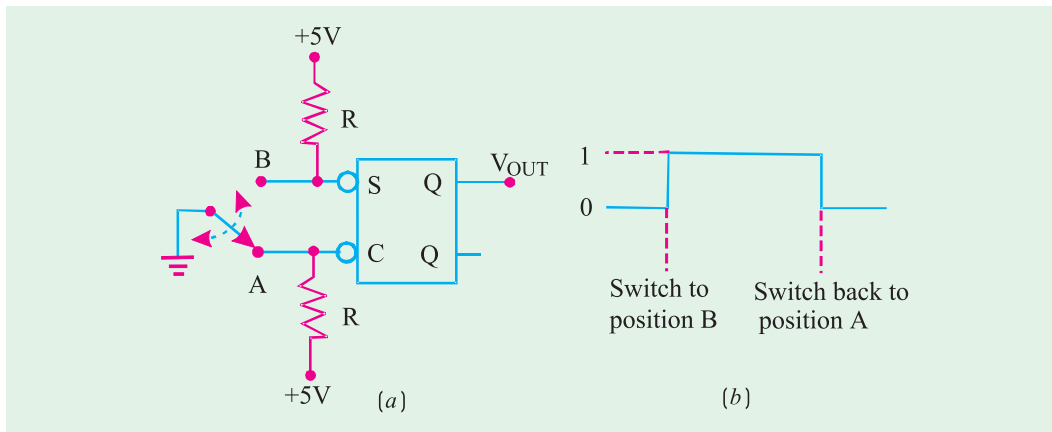


Fig. 72.13. Application of NAND latch to eliminate switch contact bounce.

The operation of the switch debouncing circuit may be understood as follows : Let us assume that initially the switch is resting in position *A* so that  $\overline{\text{CLEAR}}$  input is LOW and  $Q = 0$ . When the switch is moved to position *B*,  $\overline{\text{CLEAR}}$  will go HIGH and a LOW will appear on the  $\overline{\text{SET}}$  input as the switch first makes contact. This will set  $Q = 1$  within few nanoseconds. Now if the switch bounces off contact *B*,  $\overline{\text{SET}}$  and  $\overline{\text{CLEAR}}$  will both be HIGH and  $Q$  will not be affected, *i.e.* it will stay HIGH. Thus nothing will happen at  $Q$  as the switch bounces on and off contact *B* before finally coming to rest in position *B* as shown in Fig. 72.13 (b).

In a similar manner, when the switch is moved from position *B* back to position *A*, it will place a LOW on the  $\overline{\text{CLEAR}}$  input as it first makes contact. This clears  $Q$  to the LOW state. It will remain there ever if the switch bounces on and off contact *A* several times before coming to rest.

It is evident from the above discussion that the output at  $Q$  will consists of single transition each time the switch is moved from one position to the other.

### 72.7. NOR Gate Latch

Fig. 72.14 (a) shows a latch constructed from two cross-coupled NOR gates. It is called NOR gate latch or simply the NOR latch. As seen in the figure, the arrangement is similar to the

NAND gate latch (shown in Fig 72.2 on page 2637) except that the  $Q$  and  $\bar{Q}$  outputs have reversed positions.

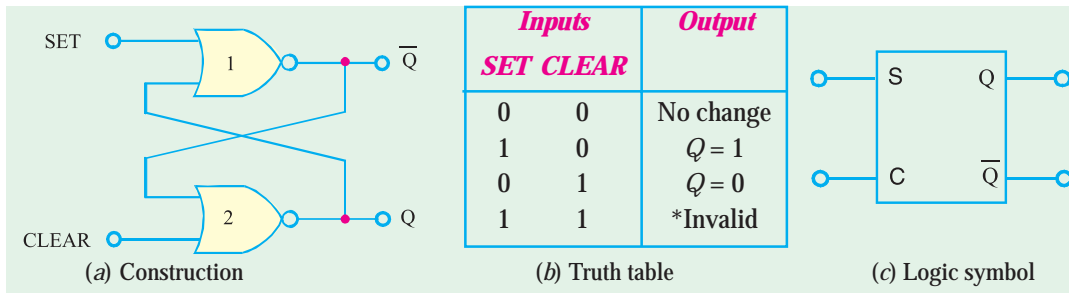


Fig. 72.14. NOR gate latch.

The detailed operation of the NOR latch can be understood exactly in the same manner as for the NAND latch. The results are given in the truth table as shown in Fig 72.14 (b) and are summarized as follows :

1. **SET = CLEAR = 0.** This condition is the normal resting state for the NOR latch. It has no effect on the output state. In other words,  $Q$  and  $\bar{Q}$  will remain in the same state in which they were prior to this input condition.
2. **SET = 1, CLEAR = 0.** This condition will always cause the output to go to  $Q = 1$  state where it will remain even after SET returns to 0.
3. **SET = 0, CLEAR = 1.** This condition will always cause the output to go to  $Q = 0$  state where it will remain even after CLEAR returns to 0.
4. **SET = 1, CLEAR = 1.** This condition tries to set and clear the latch simultaneously. It produces invalid results and should not be used.

It may be carefully noted that the NOR latch operates exactly in the same manner as the NAND latch. However, the SET and CLEAR inputs are active-HIGH rather than active-LOW. Moreover the normal resting state is SET = CLEAR = 0. Further the output  $Q$  will be set HIGH by a HIGH pulse on the SET input and it will be cleared LOW by a HIGH pulse on the CLEAR input. Fig 72.14 (c) shows the logic symbol for the NOR latch. Notice that there are no bubbles on the S and C inputs (unlike NAND latch) which indicates that these inputs are active-HIGH.

**Example 72.3.** The waveforms shown in Fig. 72.15 (a) are applied to the inputs of the NOR latch shown in Fig. 72.15 (b). Assume that initially,  $Q = 0$  and determine the  $Q$ -waveform.

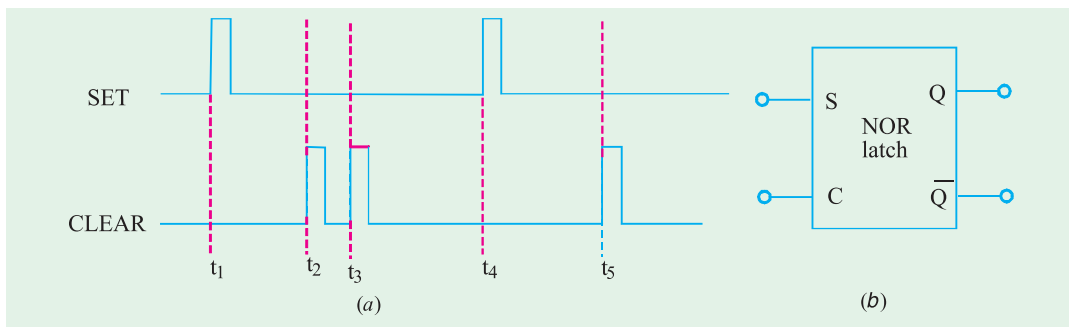


Fig. 72.15

**Solution :**

In order to determine the  $Q$ -waveform, we will refer to the truth table of the NOR latch (shown in

Fig. 72.14 (b) page 2643) and the SET and CLEAR waveforms of Fig. 72.15 (a). At  $t = t_1$ , SET goes 1 and CLEAR is 0, therefore  $Q$ -output also goes 1. At  $t = t_2$ , SET = 0 and CLEAR goes 1, therefore the  $Q$ -output goes 0. At  $t = t_3$ , SET = 0 and CLEAR goes 1 again, therefore the  $Q$ -output remains 0. At  $t = t_4$ , SET goes 1 and CLEAR is 0, therefore  $Q$ -output goes 1. At  $t = t_5$ , SET = 1 and CLEAR goes 1, therefore  $Q$ -output goes 0. The complete sketch of the  $Q$ -waveform along with SET and CLEAR waveforms is as shown in Fig. 72.16.

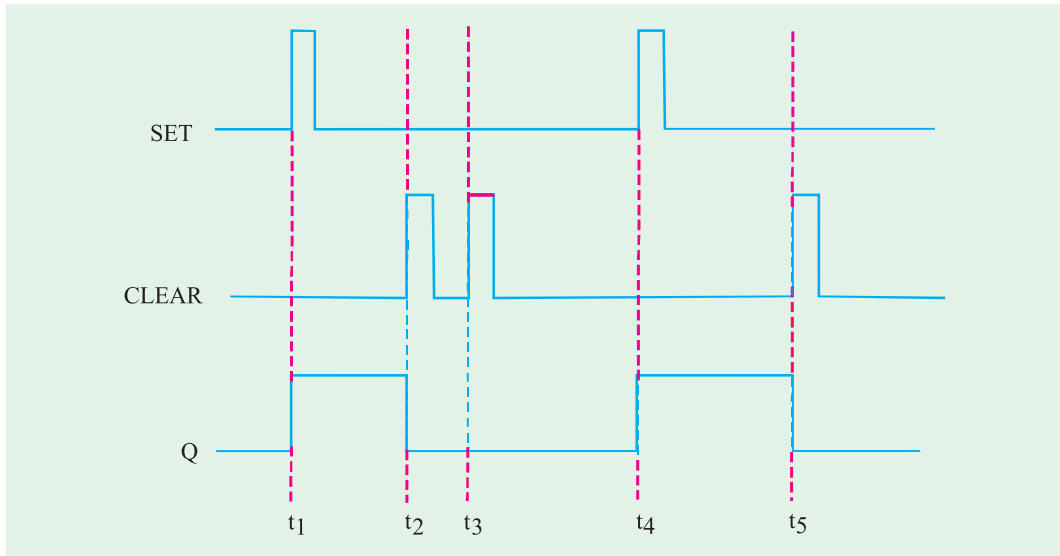


Fig. 72.16

**Example 72.4.** If the SET and CLEAR waveforms shown in Fig. 72.16. (a) are applied to the inputs of NOR latch shown in Fig. 72.17. (b), determine the waveform that will be observed on the  $Q$ -output. Assume that initially  $Q = 0$ .

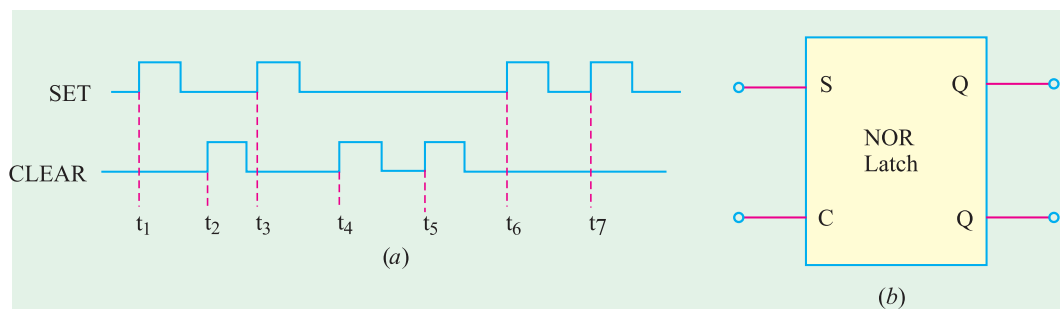


Fig. 72.17

**Solution :**

In order to determine the  $Q$ -waveform, we will refer to the truth table of the NOR latch (shown in Fig. 72.14 (b) page 2643) and the SET and CLEAR waveforms of Fig. 72.17. At  $t = t_1$ , SET goes 1 and CLEAR = 0, therefore  $Q$  goes 1. At  $t = t_2$ , SET = 0 and CLEAR goes 1, therefore  $Q$  goes 0. At  $t = t_3$ , SET = 1 again, and CLEAR = 0, therefore  $Q$  goes 1 again. At  $t = t_4$ , SET = 0. and CLEAR = 1, therefore  $Q$  goes 0 and so on. A complete sketch of  $Q$  waveform along with input waveforms is shown in Fig. 72.18.

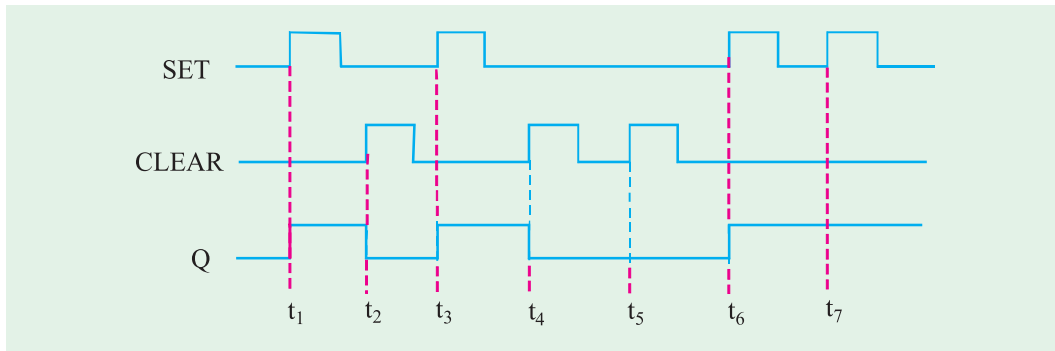


Fig. 72.18

## 72.8. Clocked Signals

Strictly speaking, the digital systems are of the following two types :

**1. Asynchronous systems.** In these systems, the outputs of logic circuits can change state any time one or more of the inputs change. Generally, an asynchronous system is more difficult to design and troubleshoot than a synchronous system.

**2. Synchronous systems.** In these systems, the exact time at which the output can change states are determined by a signal commonly called a **clock**.

The clock signal is generally a rectangular pulse train as shown in Fig. 72.19 (a) or a square wave as shown in Fig. 72.19 (b). The clock signal is distributed to all parts of the digital system and most of the system outputs can change state only when the clock makes a transition. The transitions are more commonly referred to as **edges** and are pointed out in Fig. 72.19.

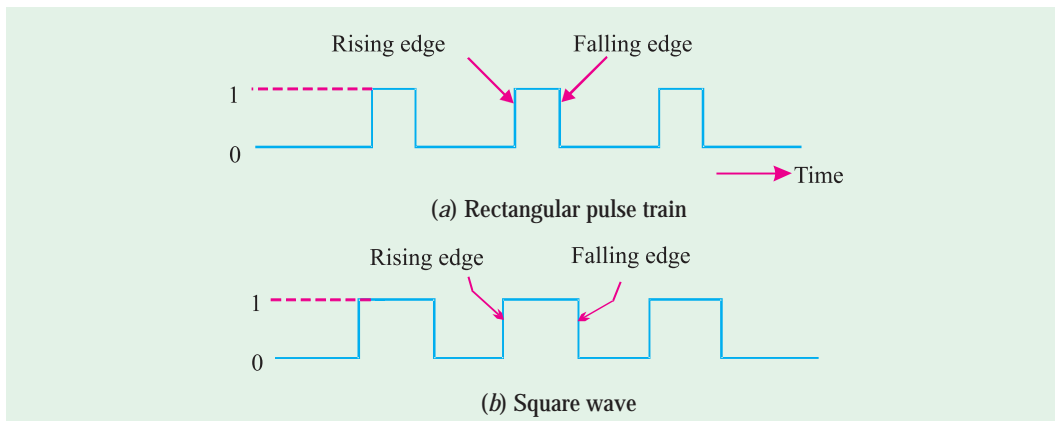


Fig. 72.19

It may be noted that when the clock changes from 0 to 1, this is called rising edge or **positive-going transition** (PGT). On the other hand, when the clock changes from 1 to 0, this is called falling edge or **negative going transition** (NGT).

As a matter of fact, most digital systems are principally synchronous (although there are always some asynchronous parts). It is due to the fact that synchronous circuits are easier to design and troubleshoot. The synchronization is accomplished through the use of **clocked flip-flops** that are designed to change states on one or the other of the clock's transitions.

### 72.9. Some Main Ideas Common to Clocked Flip-Flops

There are several types of flip-flops that are used in a wide range of applications in the field of digital electronics. Before we begin our study of the different clocked flip-flops, let us describe the main ideas that are common to all of them.

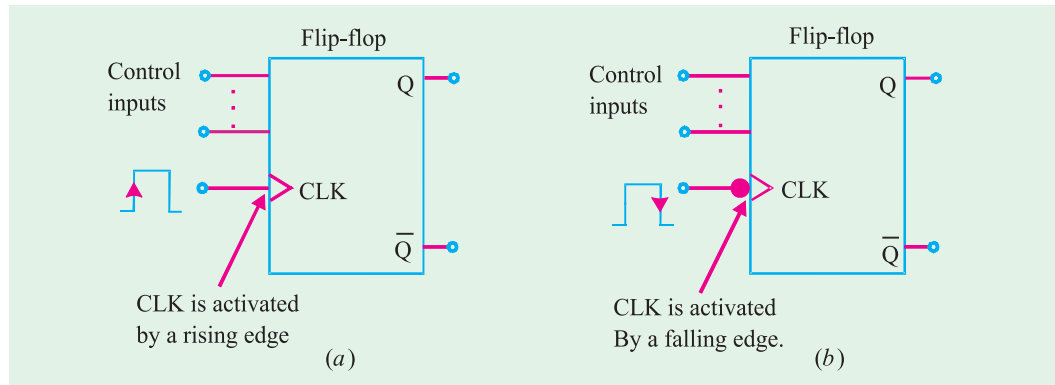


Fig 72.20. Clocked Flip-Flops.

Fig. 72.20. (a) and (b) shows the logic symbols for a typical clocked flip-flop. As seen, a clocked flip-flop has a clock input and some control inputs. There are described below in more detail.

**CLK Input.** Clocked flip-flops have a **clock** input that is labeled as CLK, CK or CP. However, we will normally use CLK as shown in Fig. 72.20 (a) and (b). In most clocked flip-flops, the CLK input is **edge-triggered**. This means the CLK input is activated by a signal transition. The edge-triggered activation is indicated by the presence of a small triangle on the CLK input. This contrasts with the latches, which are level-triggered.

Fig. 72.20 (a) shows a flip-flop with a small triangle on its CLK input to indicate that the input is activated only when a rising edge occurs. It may be noted that no other part of the input pulse will have an effect on the CLK input.

Fig 72.20 (b) shows a flip-flop symbol which has a bubble (a small circle) as well as a triangle on its CLK input. This signifies that CLK input is activated **only** when a falling edge occurs. Again note that no other part of the input pulse will have an effect on the CLK input.

**Control Inputs.** Clocked flip-flops also have one or more **control inputs** that can have various names depending on their operation. The control inputs will have no effect on  $Q$  until the active clock transition occurs. In other words, their effect is synchronized with the signal applied to CLK. Because of this reason, the control inputs are referred to as **synchronized control inputs**.

### 72.10. Setup and Hold Times in Clocked Flip-Flops

Strictly speaking, there are two timing requirements that must be met if a clocked flip-flop is to respond reliably to its control inputs when the active CLK edge occurs. These two requirements are : (1) set up time,  $t_s$  and (2) hold time,  $t_H$ . Both these requirements are discussed below.

**1. Set up time,  $t_s$ .** It is the time interval immediately preceding the active edge of the CLK signal during which the control input must be maintained at the proper level. The situation is illustrated in Fig 72.21 (a) for a flip-flop that triggers on the rising edge. Usually, the IC manufacturers specify the minimum allowable set up time,  $t_s$  (min). If this time requirement is not met, the flip-flop may not respond reliably when the clock edge occurs.

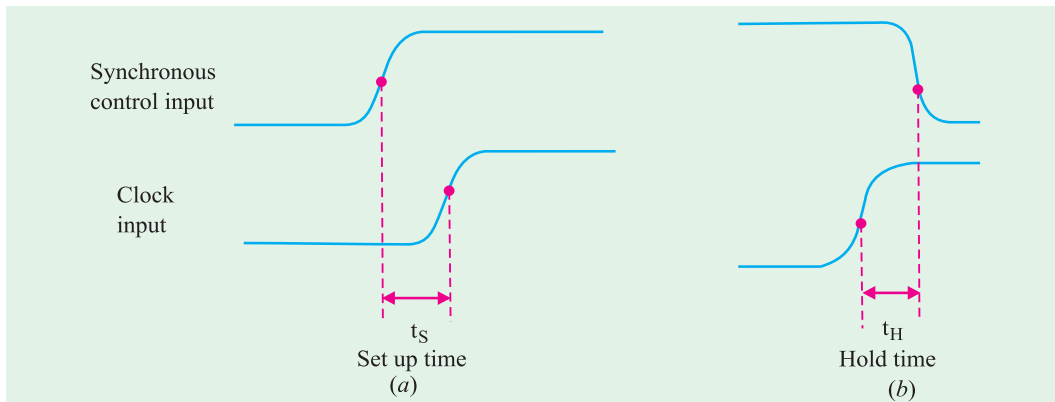


Fig. 72.21. Illustrating the set-up and hold time requirements, for a flip-flop that triggers on the rising edge.

**2. Hold time,  $t_H$ .** It is the time interval immediately following the active edge of the CLK signal during which the synchronous control input must be maintained at the proper level. Refer to Fig. 72.21 (b) usually the IC manufacturers specify the minimum acceptable value of hold time,  $t_H$  (min). If this requirement is not met, the flip-flop will not trigger reliably.

It is evident from the above discussion that in order to ensure that a clocked flip-flop will respond properly when the active clock edge occurs, the control inputs must be stable (*i.e.* unchanging) for at least a time interval.

1.  $t_s$  (min) prior to the active clock edge.
2.  $t_H$  (min) after the active clock edge.

A typical value of  $t_s$  (min) is in the range of 5 to 50 ns whereas hold times are generally from 0 to 10 ns. It may be carefully noted that the set up and hold times are measured between the 50 per cent points on the edges.

The set up and hold time requirements are extremely important in synchronous systems. This is due to the reason that there will be many situations where the synchronous control inputs to a flip-flop are changing at approximately the same time as the CLK input.

### 72.11. Clocked S - C Flip-Flop

Fig. 72.22 (a) shows the logic symbol for a clocked S - C flip-flop that is triggered by the rising edge of the clock signal. In other words this flip-flop can change output states only when a signal applied to its clock input makes a transition from 0 to 1. The S and C inputs control the state of the flip-flop in the same manner as discussed earlier for the NOR latch. But it may be noted that the flip-flop does not respond to these inputs until the occurrence of the rising edge of the clock signal.

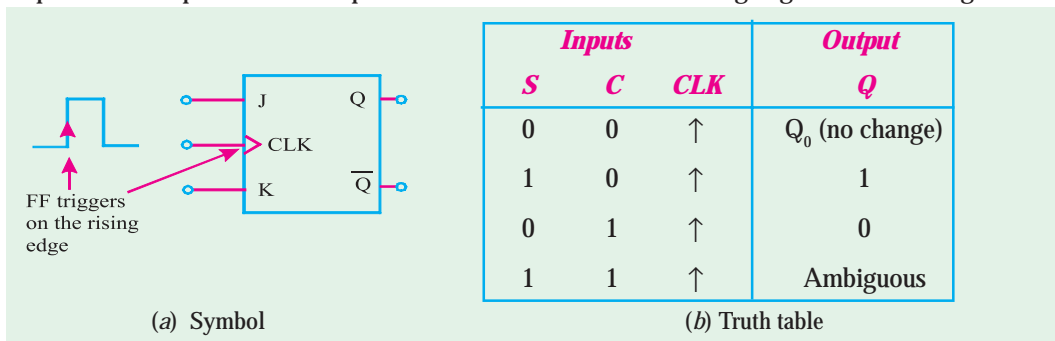


Fig. 72.22. Clocked S-C Flip-Flop.

Fig. 72.22 (b) shows the truth table for a clocked  $S$ - $C$  flip-flop that is triggered by the rising edge of the clock signal. The truth table indicates how the flip-flop output will respond to the rising edge at the CLK input for the various combinations of  $S$  and  $C$  inputs. The up arrow ( $\uparrow$ ) in the truth table indicates that the rising edge is required at CLK input. The label  $Q_0$  indicates the level at  $Q$  prior to the rising edge. This nomenclature is used quite often by IC manufacturers in their IC data sheets/manuals.

The operation of  $S$ - $C$  flip-flop may be understood with the help of input and output waveforms shown in Fig 72.23. Assuming that the setup and hold time requirements are being met in all cases, the waveforms can be analysed as follows :

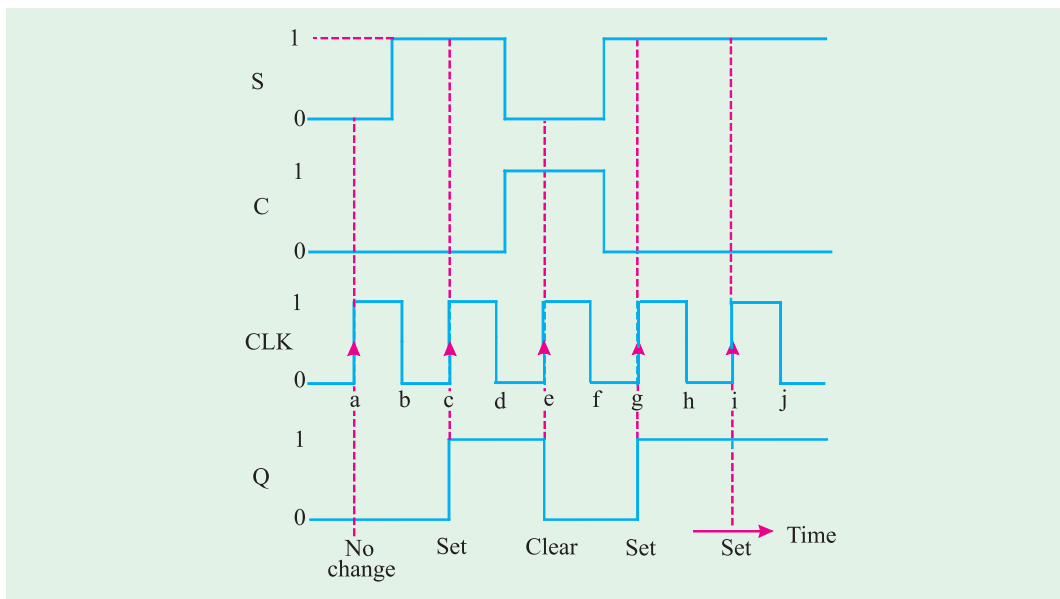


Fig. 72.23. Typical input and output waveforms of a  $S$ - $C$  flip-flop.

1. Notice that initially all the inputs ( $S$ ,  $C$  and  $CLK$ ) are 0 and the  $Q$  output is assumed to be zero, *i.e.*  $Q_0 = 0$ .
2. When the rising edge of the first clock pulse occurs (refer to point 'a'), both the  $S$  and  $C$  inputs are 0, so the flip-flop output is not affected and remains in the  $Q = 0$  state (*i.e.*  $Q = Q_0 = 0$ ).
3. At the occurrence of the rising edge of the second clock pulse (refer to point 'c'), the  $S$  input is now HIGH, with  $C$  input still LOW. This causes the flip-flop output to set to 1 state.
4. At the occurrence of the rising edge of the third clock pulse (refer to point 'e'), the  $S$  input is LOW with  $C$  input HIGH. This causes the flip-flop output to clear to 0 state.
5. The rising edge of the fourth clock pulse sets the flip-flop output to the  $Q = 1$  state (refer to point 'g') because  $S = 1$  and  $C = 0$ .
6. The fifth clock pulse also finds that  $S = 1$  and  $C = 0$  at the rising edge (refer to point 'i'). This situation will produce HIGH output. But since  $Q$  is already HIGH, so it remains in that state.

It may be carefully noted from the waveforms shown in Fig. 72.23, that the flip-flop is not affected by the falling edge of the clock pulses. It may also be noted that  $S$  and  $C$  input levels have no effect on the flip-flop except upon the occurrence of a rising edge of the clock signal.

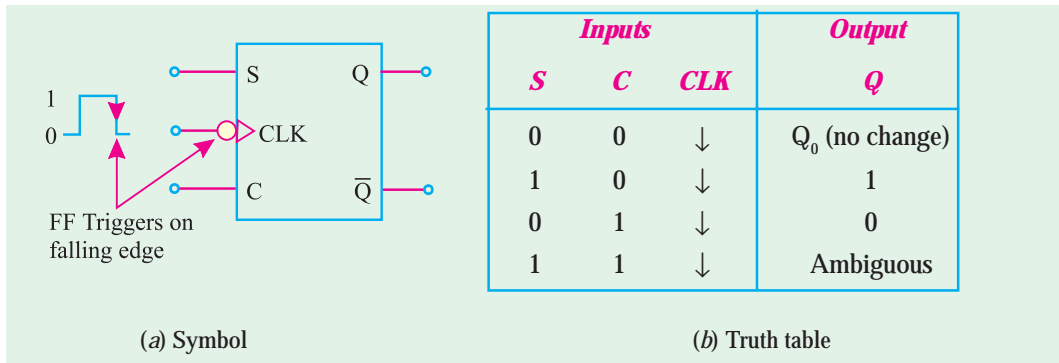


Fig. 72.24. Falling-edge triggered S-C flip-flop.

Fig. 72.24 (a) shows the symbol and 72.24 (b) the truth table for a clocked S - C flip-flop that triggers on the falling edge of the CLK input. Notice the presence of a small circle and a triangle on the CLK input of the flip-flop. These indicate that this flip-flop will trigger only when CLK input goes from 1 to 0. This flip-flop operates in the same way as the rising-edge flip-flop except that the output can change states only on the falling edge of the clock pulses (refer to points b, d, f, h and j in Fig. 72.23). In actual practice both the rising-edge and falling-edge triggered flip-flops are used in digital systems.

**Example 72.5.** Fig. 72.25 (a) shows the SET and CLEAR waveforms applied at the inputs of the clocked S-C flip-flop shown in Fig. 72.25 (b)

Sketch the waveforms at Q and  $\bar{Q}$  outputs of the flip-flop. Assume that initially  $Q = 0$  and  $\bar{Q} = 1$ .

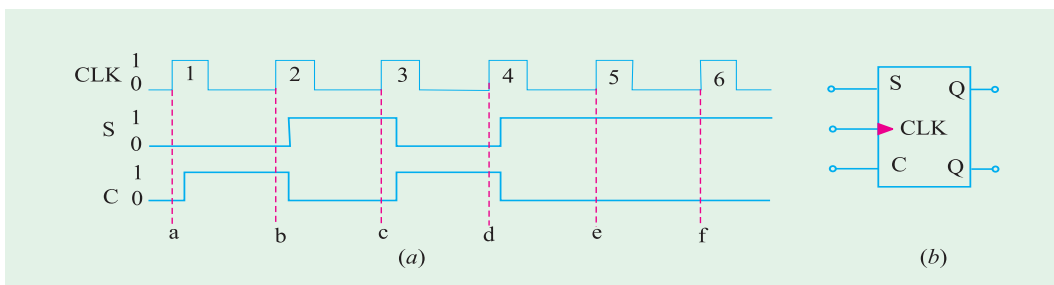


Fig. 72.25

**Solution.** Given :

In order to determine the Q-output, we will refer to the waveforms applied at the flip-flop inputs shown in Fig. 72.25 (a) and its truth table shown in Fig. 72.24 (b). At point 'a',  $S = C = 0$ , therefore  $Q_0 = 0$ , At point 'b',  $S = 0$  and  $C = 1$ , therefore Q remains 0. At point 'c',  $S = 1$  and  $C = 0$ , therefore  $Q = 1$ . At point 'd',  $S = 0$  and  $C = 1$ , therefore  $Q = 0$ . At point 'e',  $S = 1$ ,  $C = 0$ , therefore  $Q = 1$ . At point 'f', S and C inputs remain the same, therefore Q also remains 1. The sketch of Q-output along with the CLK, S and C waveforms is as shown in Fig. 72.26. The  $\bar{Q}$ -output waveform is determined by inverting the Q-waveform.



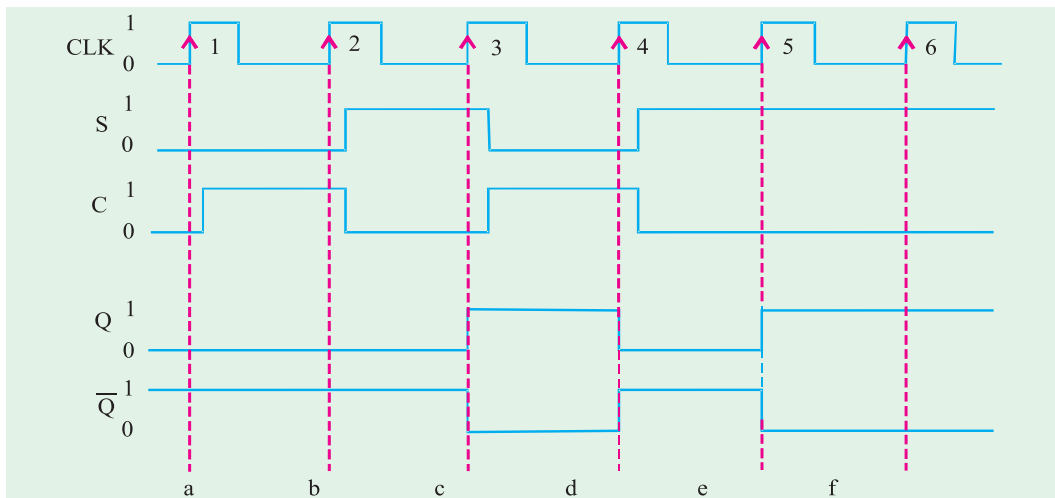


Fig. 72.26

### 72.12. Internal Circuitry of an Edge-triggered S-C Flip-Flop

Fig. 72.26 shows a simplified version of an internal circuitry of an edge triggered *S-C* flip-flop. Notice that the circuit contains the following three sections :

1. A basic NAND latch
2. A pulse steering circuit and
3. An edge-detector circuit

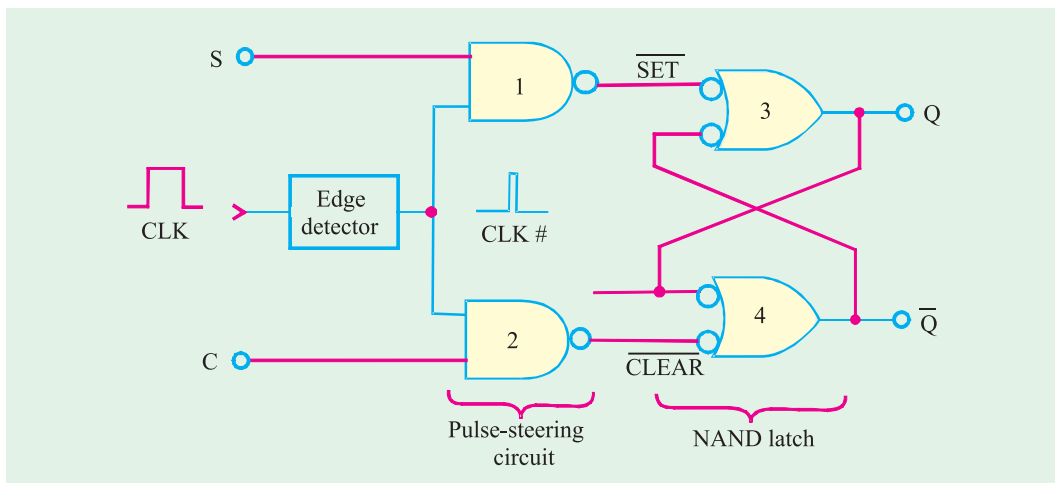


Fig. 72.27. Internal circuitry of an edge-triggered S-C flip-flop.

As seen from the Fig. 72.27, the edge detector produces a narrow positive-going spike (CLK #). The pulse-steering circuit “steers” (or moves) the spike through to the SET and CLEAR input of the latch in accordance with the levels present at *S* and *C* inputs. For example with *S* = 1 and *C* = 0, the spike is inverted and passed through NAND gate-1 to produce a LOW pulse at the SET input of the latch that sets *Q* = 1. With *S* = 0 and *C* = 1, the spike is inverted and passed through NAND gate-2 to produce a LOW pulse at the CLEAR input of the latch that resets *Q* = 0.

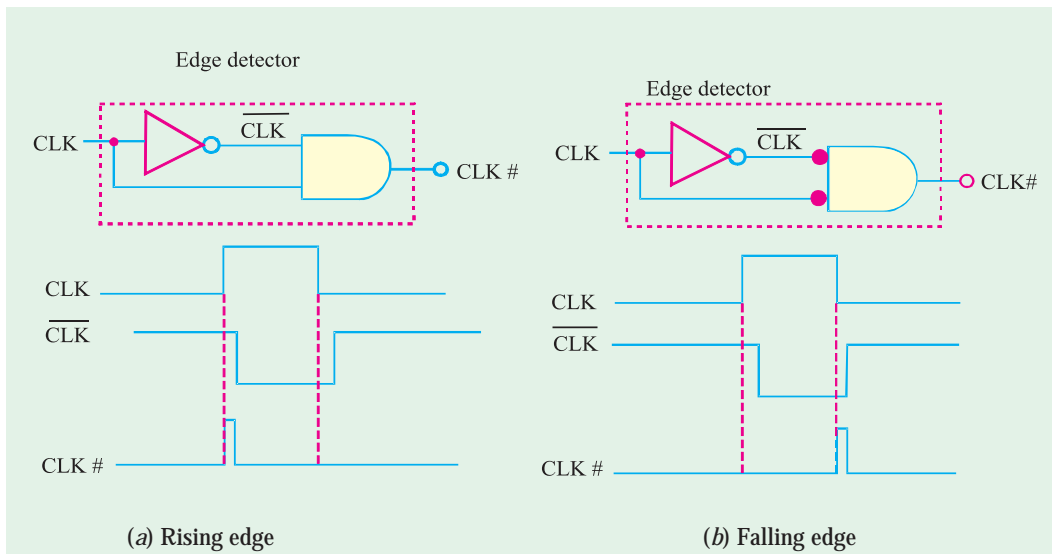


Fig. 72.28. Implementation of edge-detector circuits used in edge triggered flip-flops.

Fig. 72.28 shows the implementation of edge-detector circuits used in edge-triggered flip-flops. Thus Fig. 72.28 (a) shows how the spike is generated for edge-triggered flip-flops that trigger on the rising edge of the CLK pulse. As seen from this figure, the INVERTER produces an output with a delay of a few nanoseconds. Because of this the transitions of  $\overline{CLK}$  occur a little bit after that of CLK. Next the AND gate produces an output spike that is HIGH only for the few nanoseconds when both CLK and  $\overline{CLK}$  are HIGH. As a result of this, we get a narrow pulse at edge detector output (CLK #) which occurs on the rising edge of the CLK.

Similarly Fig 72.28(b) produces CLK # on the falling edge of CLK for flip-flops that are to trigger on the falling edge.

It may be noted that since the CLK # signal is HIGH for only a few nanoseconds, the Q output is affected by the levels at S and C only for a short time during and after the occurrence of the active edge of CLK. This gives the flip-flop its edge-triggered property.

### 72.13. Clocked J-K Flip-Flop

Fig. 72.28 (a) shows the symbol and (b) the truth table for a clocked J-K flip-flop that is triggered by the rising edge of the clock signal. The J and K inputs control the state of the flip-flop in the same manner as the S and C inputs do for the S-C flip-flop. However there is one major difference—the J = K = 1 condition in J-K flip-flop does not result in an ambiguous output unlike S = C = 1 in S-C flip-flop for, J = K = 1 condition, the J-K flip-flop will always go to its opposite state upon the rising edge of the clock signal. This is called **trigger mode**. In this mode, if both J and K are left HIGH, the flip-flop will change states (i.e. toggle) for each rising edge of the clock.

The operation of J-K flip-flop for each combination of J and K is summarized in Fig. 72.29 (b) Notice that the truth table is the same except for J = K = 1 condition. This condition results in  $Q = \overline{Q}_0$  which means that the new value of Q will be the inverse of the value it had prior to the rising edge of the clock pulse. It is called toggle operation.

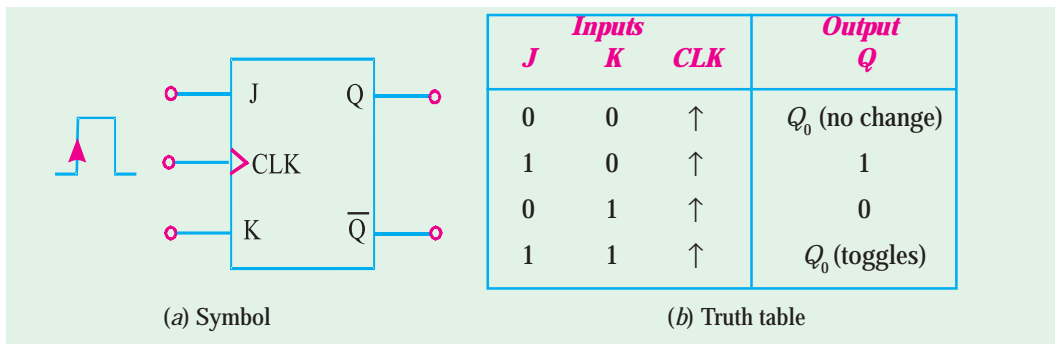


Fig. 72.29. Rising edge triggered J-K flip-flop.

In order to understand the operation of J-K flip-flop, let us consider the  $J$ ,  $K$  and CLK waveforms as shown in Fig. 72.30. Assume that set up and hold time requirements are met. The operation may be explained as below.

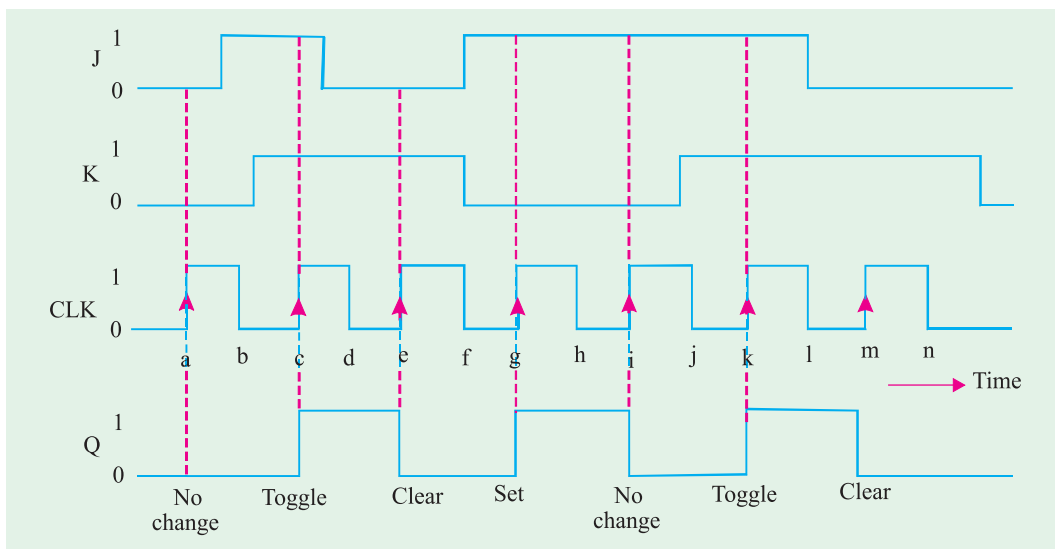


Fig. 72.30

- Initially all the inputs are 0, and the  $Q$  output is also assumed to be 0, i.e.  $Q_0 = 0$ .
- When the rising edge of the first clock pulse occurs (refer to point  $a$ ), the  $J = K = 0$  condition exists. Thus the flip-flop does not change its output state, i.e.  $Q = Q_0 = 0$ .
- When the rising edge of the second clock pulse occurs (refer to point  $c$ ), the  $J = K = 1$  condition exists. Thus the flip-flop toggles to its opposite state i.e.  $Q = \overline{Q_0} = \overline{0} = 1$ .
- When the rising edge of the third clock pulse occurs (refer to point  $e$ ),  $J = 0$  and  $K = 1$  condition exists. Thus the flip-flop is cleared to the  $Q = 0$  state.
- When the rising edge of the fourth clock pulse occurs (refer to point  $g$ ),  $J = 1$  and  $K = 0$  condition exists. This condition sets the output  $Q$  to 1 state.
- When the rising edge of fifth clock pulse occurs (refer to point  $i$ ),  $J = 1$  and  $K = 0$  condition exists. This is the condition that sets the output  $Q$  to 1 state. However since  $Q$  is already 1, so it will remain there. Hence no change in the output state.
- When the rising edge of sixth clock pulse occurs (refer to point  $k$ ),  $J = K = 1$  condition exists. This condition causes the flip-flop to toggle to its opposite state.

- 8. When the rising edge of seventh clock pulse occurs (refer to point m),  $J = 0$  and  $K = 1$  condition exists. This condition causes the flop-flop to clear to  $Q = 0$  state.

It may be noted from the waveforms that the flip-flop is not affected by the falling edge of the clock pulses. Also the  $J$  and  $K$  input levels have no effect except the occurrence of the rising edge of the clock signal. The  $J$  and  $K$  inputs by themselves cannot cause the flip-flop to change states.

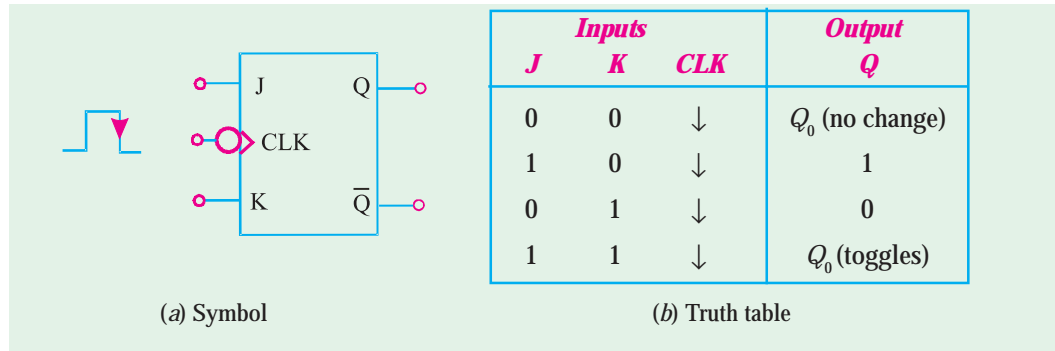


Fig. 72.31. Falling edge triggered J-K flip-flop.

Fig. 72.31 shows the symbol and 72.31 (b) the truth table for a clocked  $J$ - $K$  flip-flop that triggers on the falling edge of the clock pulse. The small circle on the CLK input indicates that the flip-flop will trigger when the CLK input goes from 1 to 0. This flip-flop operates in the same way as the rising edge of the flip-flop of Fig. 72.27 except that the output can change states only on the falling edge of CLK signal (i.e. points b, d, f, h, j, l and n). As a matter of fact, both polarities of edge-triggered  $J$ - $K$  flip-flops are in common usage in the field of digital electronics.

Strictly speaking, the  $J$ - $K$  flip-flop is much more versatile than the  $S$ - $C$  flip-flop because it has no ambiguous states. The  $J = K = 1$  condition, which produces the toggling operation, finds extensive use in all types of binary counters.

**Example 72.6.** Fig 72.32 (a) shows the waveforms applied at  $J$ ,  $K$  and CLK inputs of the clocked  $J$ - $K$  flip-flop shown in Fig. 72.32 (b). Sketch the  $Q$  output waveform Assume  $Q = 0$  initially.

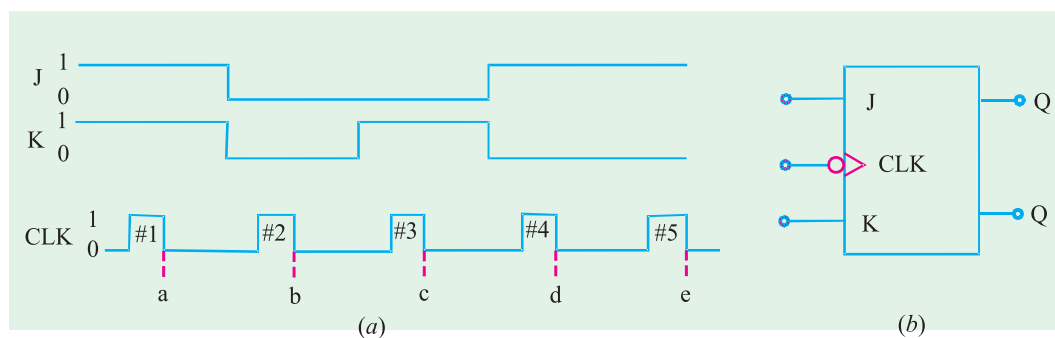


Fig. 72.32

**Solution :**

Notice that flip-flop shown in Fig. 72.32 (b) is a clocked  $J$ - $K$  flip-flop. Also notice the presence of a small circle at the CLK input of the flip-flop. This indicates that the flip-flop will trigger corresponding to the falling edge of the clock pulse. So you need to identify the states of  $J$  and  $K$  inputs

corresponding to the falling edge points of the CLK pulse waveform *i.e.* points *a, b, c, d* and *e* shown in Fig. 72.32.

Using the truth table of clocked *J-K* flip-flop shown in Fig. 72.31, and the given waveforms the *Q*-waveform sketch is shown in Fig. 72.33. On the arrival of first CLK pulse at point '*a*'  $J = K = 1$ , the *a* output does not change, *i.e.* it stays at  $Q = 1$  level. On the arrival of third CLK pulse *J-K* flip-flop will toggle, *i.e.*, its *Q*-output changes from 0 to 1. On the arrival of second CLK pulse, at point '*b*',  $J = K = 0$  the *Q*-at point '*c*'  $J = 0, K = 1$ , the *Q* output goes 0. On the arrival of fourth CLK pulse, at point '*d*'  $J = 1, K = 0$ , the *Q*-output goes 1. On the arrival of fifth CLK pulse at point '*e*'  $J = 1, K = 0$ , the *Q*-output remains at 1 level.

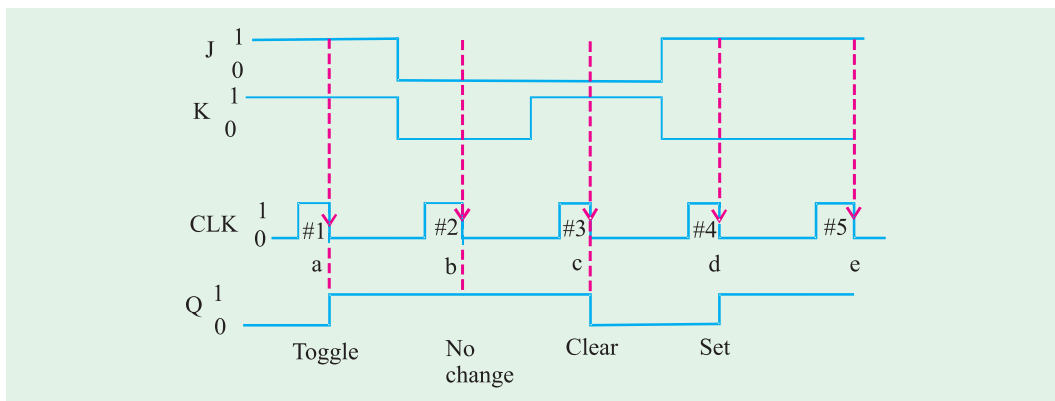


Fig. 72.33

**Example 72.7.** What will be the output waveform *Q* of a *J-K* flip-flop if the following waveforms are applied at the input? Assume the flip-flop triggers at the falling edge of clock pulse.

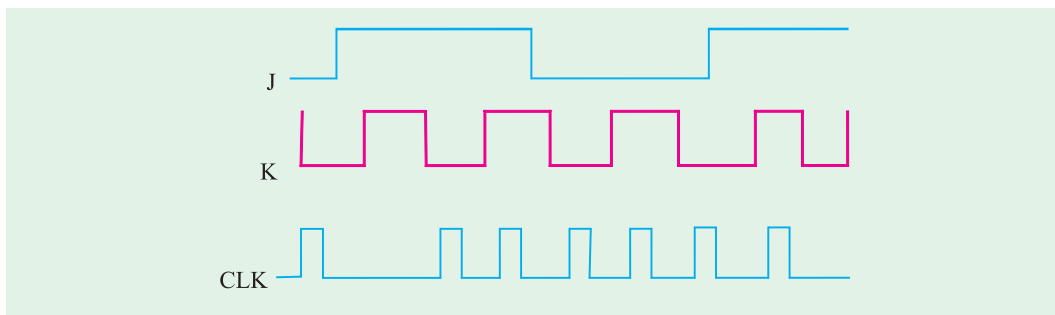


Fig. 72.34

( Grad. IETE Dec. 1996)

**Solution :**

Recall the truth table of a clocked *J-K* flip-flop, the flip-flop triggers corresponding to the logic levels at the *J* and *K* inputs. If  $J = K = 1$ , the flip-flop output remains in its previous state. If  $J = 1, K = 0$  the flip-flop output goes 1, if  $J = 0, K = 1$ , the flip-flop goes LOW. However, if  $J = K = 1$ , the flip-flop output toggles. Keeping it in mind, we can sketch the *Q*-output waveform as shown in Fig. 72.35.

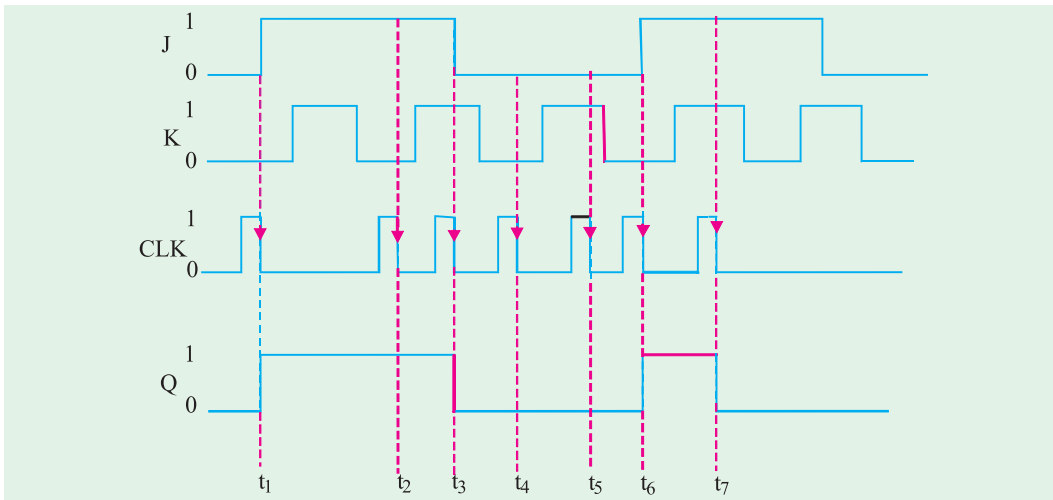


Fig. 72.35

Notice that at  $t_1$ ,  $J = 1, K = 0$  therefore the  $Q$ -output goes 1. At  $t_2$ ,  $J = 1, K = 0$  so the  $Q$ -output remains 1. At  $t_3$ ,  $J$  goes 0,  $K = 1$ , therefore the  $Q$ -output goes 0. At  $t_4$ ,  $J$  is still 0,  $K$  is still 1, therefore the  $Q$ -output stays 0. At  $t_5$ ,  $J$  is still 0,  $K$  is still 1, therefore the  $Q$ -output stays 0. At  $t_6$ ,  $J$  goes 1,  $K$  is 0, so the  $Q$ -output goes 1. At  $t_7$  both  $J$  and  $K$  are 1, therefore the  $Q$ -output toggles and goes 0, as shown in the Fig. 72.35.

### 72.14. Internal Circuitry of an Edge-triggered $J$ - $K$ Flip-Flop

Fig. 72.36 shows a simplified version of the internal circuitry of an edge-triggered  $J$ - $K$  flip-flop. As seen, the flip-flop contains the same three sections as the  $S$ - $C$  triggered flip-flop (refer to Fig 72.27 page 2646), i.e.,

1. A basic NAND gate latch
2. A pulse steering circuit and
3. An edge detector circuit

The only difference between the internal circuitry of edge-triggered  $J$ - $K$  flip-flop and that of  $S$ - $C$  flip flop is that in  $J$ - $K$  flip-flop circuit,  $Q$  and  $\bar{Q}$  outputs are fed back to the pulse-steering NAND gates.

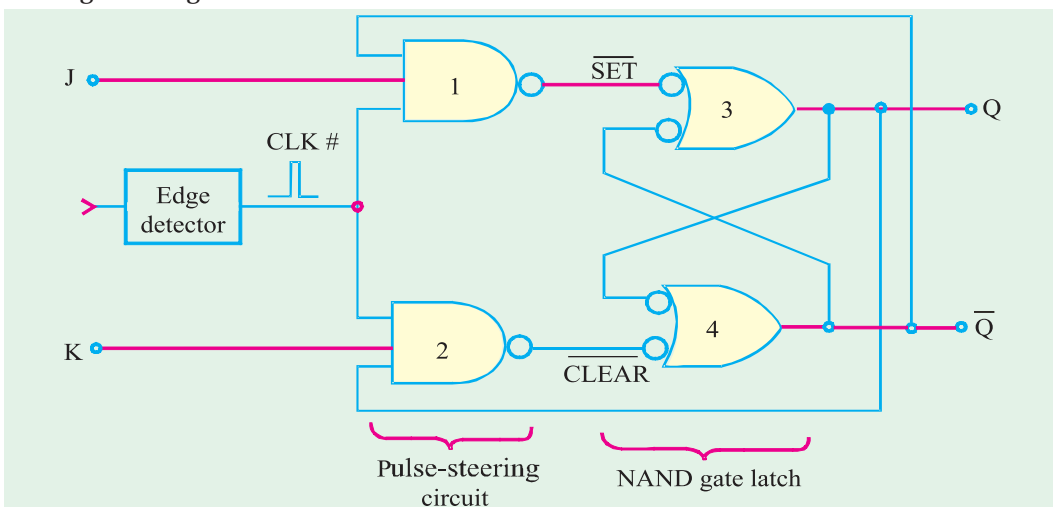


Fig 72.36. A simplified version of an internal circuitry of an edge-triggered  $J$ - $K$  flip-flop.

It is because of this feedback connection that J-K flip-flop gives the toggle operation for  $J = K = 1$  condition.

Let us examine the toggle condition in more detail. Assume that  $J = K = 1$  and that  $Q = 0$  when a CLK pulse occurs. With  $Q = 0$  and  $\bar{Q} = 1$ , NAND gate 1 will steer CLK # (inverted) to  $\overline{SET}$  input of the NAND gate latch to produce  $Q = 1$ .

If we assume  $Q = 1$ . When a CLK pulse occurs, NAND gate 2 will steer CLK # (inverted) to the  $\overline{CLEAR}$  input of the latch to produce  $Q = 0$ . It is evident from the above discussion that  $Q$  always ends up in the opposite state.

**Note.** It may be carefully noted that in order for toggle operation to work, the CLK # pulse must be very narrow. It must return to 0 before the  $Q$  and  $\bar{Q}$  outputs toggle to their new states. Otherwise the new states of  $Q$  and  $\bar{Q}$  will cause the CLK # pulse to toggle the latch outputs again.

### 72.15. Clocked D Flip-Flop

Fig. 72.37 (a) shows the symbol and 72.37 (b) the truth table for a clocked D flip-flop that triggers on the rising edge of the clock pulse. Notice that this flip-flop has only one synchronous control input,  $D$  which stands for data.

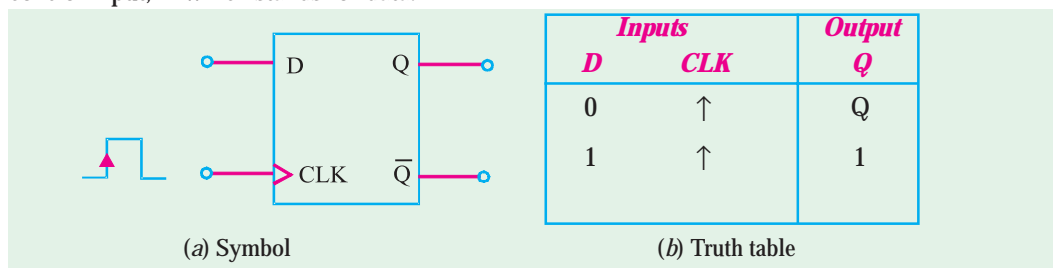


Fig. 72.37. Clocked D flip-flop.

The operation of the clocked D flip-flop is very simple. The output  $Q$  will go to the same that is present on the D input when the rising edge occurs at the CLK. In other words, the level present at D will be stored in the flip-flop at the instant the rising edge occurs.

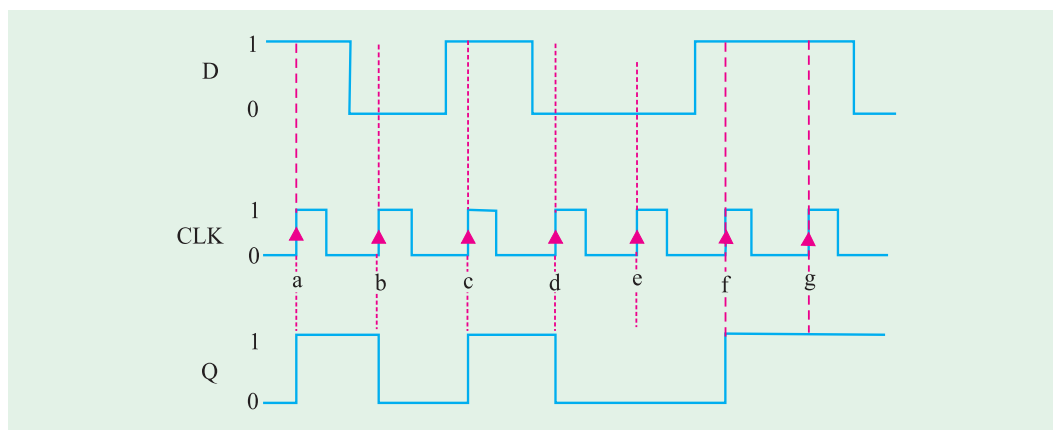


Fig. 72.38. Input and output waveforms to illustrate the operation of clocked D flip-flop.

In order to understand the operation of flip-flop in more detail, consider the waveforms at  $D$  and  $CLK$  input as shown in Fig. 72.38. Assume that  $Q$  is initially 0.

1. When the first rising edge of the CLK pulse occurs (refer to point 'a'), the *D* input is 1, therefore *Q* will go to 1 state. Even though the *D* input level changes between the points 'a' and 'b', it has no effect on *Q*. The output *Q* is storing the 1 that was on *D* at point 'a'.
2. When the second rising edge of the CLK pulse occurs (refer to point 'b'), *Q* goes to 0 state since *D* is 0 at that time. The output *Q* stores this 0 value until the rising edge of the third CLK pulse (at point 'c') causes *Q* to go to 1 since *D* is 1 at that time.
3. In a similar manner, the *Q* output takes on the levels present at *D* when the rising edges occur at points 'd', 'e', 'f' and 'g'. Notice that *Q* stays 0 at point 'e' because *D* is still 0.

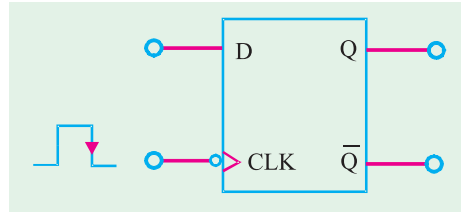


Fig. 72.39. Symbol for a falling-edge triggered *D* flip-flop.

A falling-edge triggered *D* flip-flop operates in the same way as the rising edge triggered *D* flip-flop. However, the difference is that *Q* will take on the value of *D* when a falling edge occurs at the CLK.

A falling-edge triggered *D* flip-flop operates in the same manner as the *D* flip-flop discussed above except that *Q* will take on the value of *D* when a falling edge occurs at the CLK input. The symbol for *D* flip-flop that triggers on the falling edge has a bubble on the CLK input as shown in Fig. 72.39.

IC 7474 is an example of clocked *D* flip-flop. It contains two rising edge triggered *D* flip-flops. The pin configuration and some other specification can be found in the data sheet of the device.

### 72.16. Implementation of *D* Flip-Flop from a *J-K* Flip-Flop

An edge-triggered *D* flip-flop can be obtained easily by adding a single INVERTER to an edge-triggered *J-K* flip-flop as shown in Fig. 72.40. A similar approach can be used to convert a *S-C* flip-flop to a *D* flip-flop.

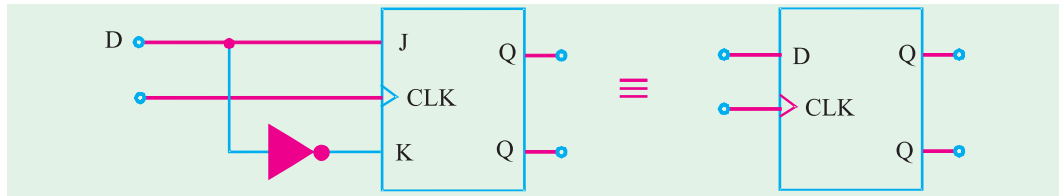


Fig. 72.40. Implementation of *D* flip-flop from a *J-K* flip-flop.

**Example 72.8.** Fig 72.41 shows a *D* flip-flop with the input and clock waveforms applied at their respective inputs. Determine the *Q* (or output) waveform.

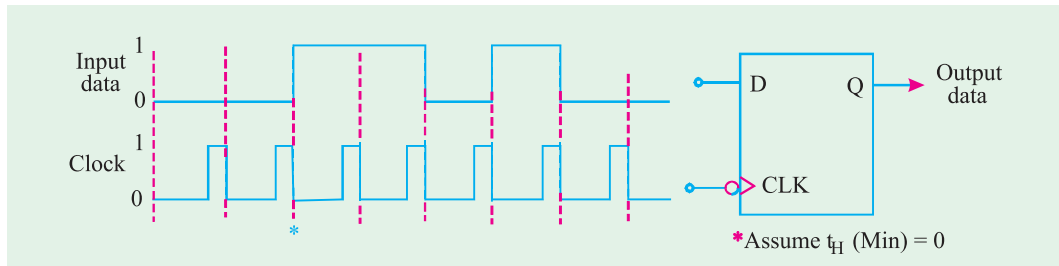


Fig. 72.41



**Solution :**

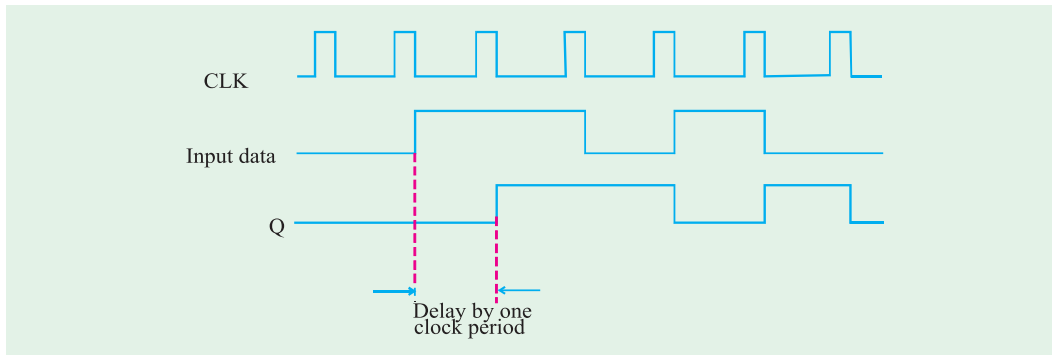


Fig. 72.42

**Note.** The output ( $Q$ ) is delayed from the input by one clock period. This is an advantage as a  $D$ -Flip-flop is used sometimes to delay a binary waveform so that the binary information appears at the output a certain amount of time after it appears at the  $D$  input.

It is also possible to delay the input by two clock periods. This can be achieved by connecting  $Q$  to the  $D$  input of a second flip-flop and connect the clock signal to the second flip-flop. The output of the second flip-flop will be delayed by 2 clock periods from the input data.

**Example 72.9.** An edge-triggered  $D$  Flip-flop can be made to operate in the toggle mode by connecting it as shown in Fig. 72.43. Assume  $Q = 0$  initially and determine the  $Q$  (output) waveform.

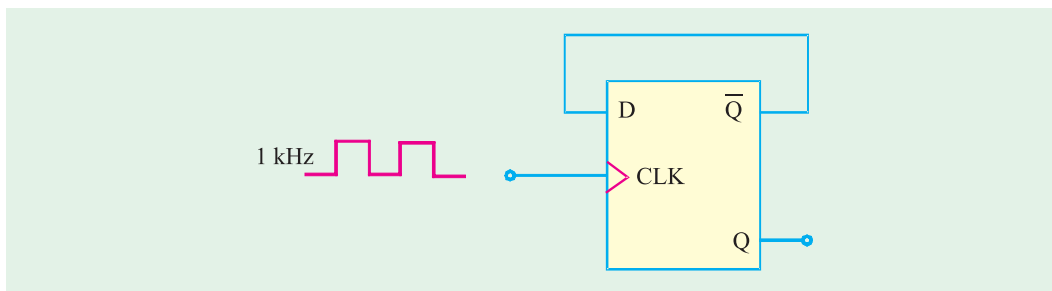


Fig. 72.43

**Solution :** The clock is 1 kHz waveform. The output waveform can be obtained from the input as shown in Fig. 72.44.

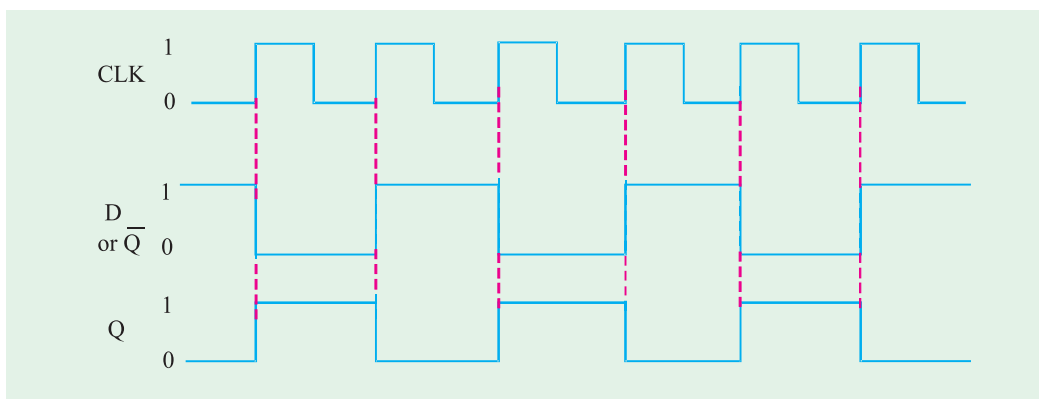


Fig. 72.44

Notice that the output is a square wave of half-the frequency of input *i.e.* 7200 kHz.

**Note.** A *D*- flip-flop with  $\bar{Q}$  tied to its *D*-input as shown in Fig 72.43 is also known as *T* flip-flop where *T* stands for Toggle (or trigger).

### 72.17. Parallel Transfer of Data Using *D*-Flip-Flops

We have already discussed in Art 72.15 that the *Q*-output of a *D* flip-flop is the same as the *D* input. Let us now study the usefulness of this flip-flop.

Fig. 72.45 shows an application of *D* flip-flop used for parallel transfer of binary data from *X*, *Y*, *Z* – the three outputs of a combinational logic circuit to the outputs  $Q_1$ ,  $Q_2$ , and  $Q_3$  of the *D* flip-flops for storage. The transfer occurs upon application of TRANSFER pulse to the common CLK inputs. The flip-flops can store these values for subsequent processing.

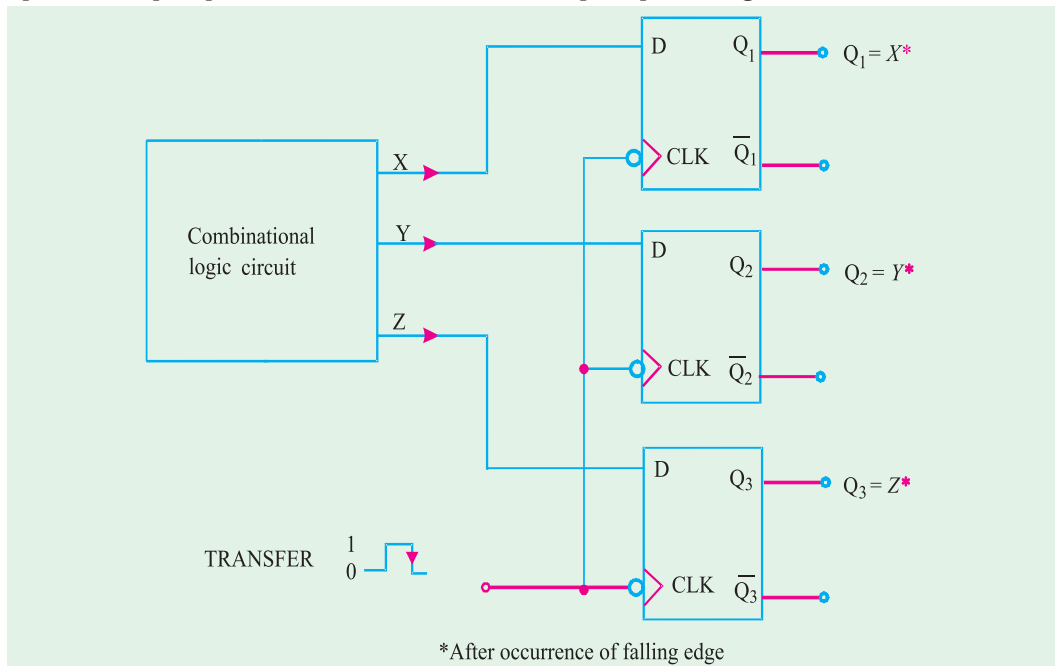


Fig. 72.45. Illustrating parallel transfer of data using *D* flip-flops.

### 72.18. *D* Latch (Transparent Latch)

We have already discussed in Art. 72.15 about an edge – triggered *D* flip-flop. Such a flip-flop uses an edge-detector circuit to ensure that the output will respond to the *D* input only when the active transition of the clock occurs. If this edge detector circuit is not used, the resultant circuit operates somewhat differently. It is called a *D* latch and has the arrangement shown in Fig 72.46 (a).

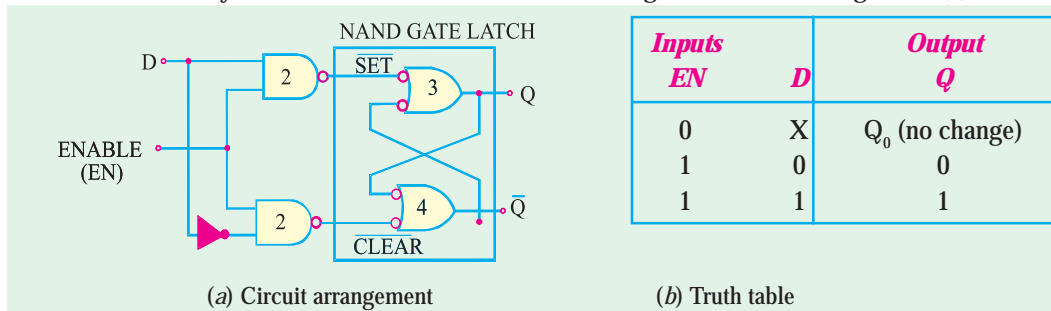


Fig. 72.46. *D* latch.

As seen from Fig. 72.46. (a), the circuit contains a NAND gate latch, and the steering NAND gates without the edge-detector circuit. The common input to the steering gates is called an enable input (abbreviated as EN) rather than a clock input because its effect on  $Q$  and  $\bar{Q}$  outputs is not restricted to occurring only on its transitions. The operation of the  $D$  latch may be explained as follows:

1. When EN is LOW, whatever be the value of  $D$  input, it is inhibited from affecting the NAND gate latch. It is due to the reason that the outputs of both the steering gates will be held HIGH. Thus the  $Q$  and  $\bar{Q}$  outputs will stay at whatever level they had just before EN went LOW. In other words, the outputs are “latched” to their current level and cannot change while EN is LOW even if  $D$  changes. This is represented as  $Q_0$  in the truth table. Refer to Fig. 72.46 (b).
2. When EN is HIGH, the  $D$  input will produce a LOW at either  $\overline{SET}$  or the  $\overline{CLEAR}$  inputs of the NAND gate latch to cause  $Q$  to become the same level as  $D$ . If  $D$  changes while EN is HIGH,  $Q$  will follow the change exactly. In other words, while  $EN = 1$ , the  $Q$  output will look exactly like  $D$ . Because of this reason, the  $D$  latch in this mode is called “transparent”.

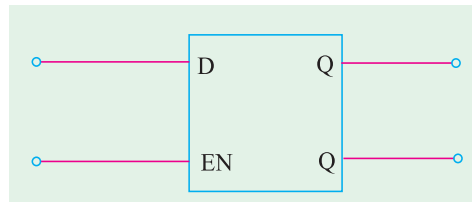


Fig. 72.47. Logic symbol for a D-latches.

The operation is summarized in the truth table shown in Fig. 72.46 (b). The logic symbol for the  $D$ -latch is shown in Fig. 72.47. It may be noted that even though the EN input operates in the same way as CLK input of an edge-triggered flip-flop, there is no small triangle on the EN input. This is because the small triangle symbol is used strictly for inputs that can cause an output change only when an edge occurs. So remember the  $D$  latch is not an edge-triggered device.

IC 7475 is an example of  $D$ -latch. It contains four transparent  $D$ -latches.

**Example 72.10.** Fig. 72.48 shows the waveforms applied at the  $D$  and EN inputs of a  $D$  latch. Sketch the output waveform at  $Q$ -output Assume initially  $Q = 0$

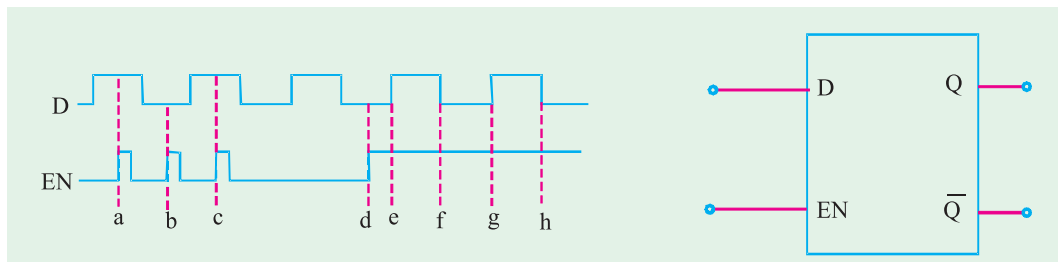


Fig. 72.48

**Solution.**

Using the truth table of  $D$ -latch shown in Fig. 72.46, and the input waveforms, the output waveform at  $Q$  can be obtained and is sketched as shown in Fig. 72.49. Notice that at point ‘a’, EN goes 1, and  $D$  is 1, therefore a output goes 1. At point ‘b’ EN is 1, but  $D = 0$ , therefore  $Q$  goes 0, At point ‘c’, EN is 1, therefore  $Q$  goes 1 and so on.

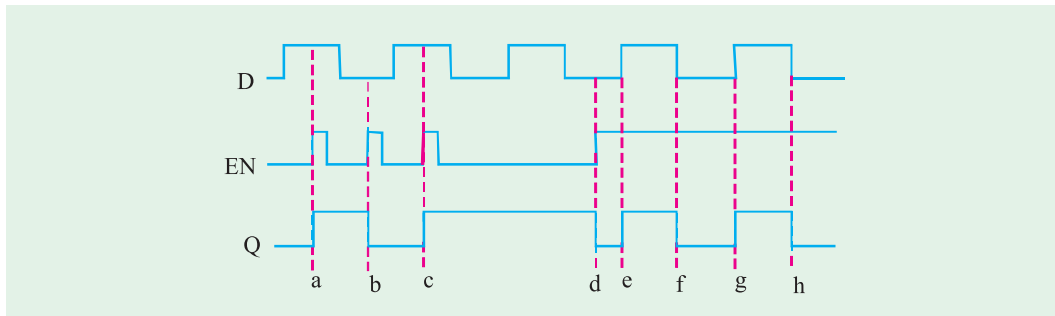


Fig. 72.49

### 72.19. Clocked $J$ - $K$ Flip-Flop with Asynchronous Inputs

Strictly speaking for the clocked flip-flops, the  $S$ ,  $C$ ,  $J$ ,  $K$  and  $D$  inputs discussed in the previous articles are referred to as **control inputs**. These inputs are also known as **synchronous inputs**. These are called synchronous because their effect on the flip-flop output is synchronized with the CLK (*i.e.* clock) input. As discussed earlier, the synchronous control inputs must be used in conjunction with a signal to trigger the flip-flop.

As a matter of fact, most clocked flip-flops also have one or more **asynchronous** inputs. These inputs operate independently of the synchronous inputs and clock input. The asynchronous inputs of a flip-flop can be used to set its output to 1 state or clear to the 0 state at any time regardless of the conditions at the other inputs. In other words, the asynchronous inputs can be used to **override** all the other inputs in order to place the flip-flop output in one state or the other. Because of this reason, the asynchronous inputs are also called **override inputs**.

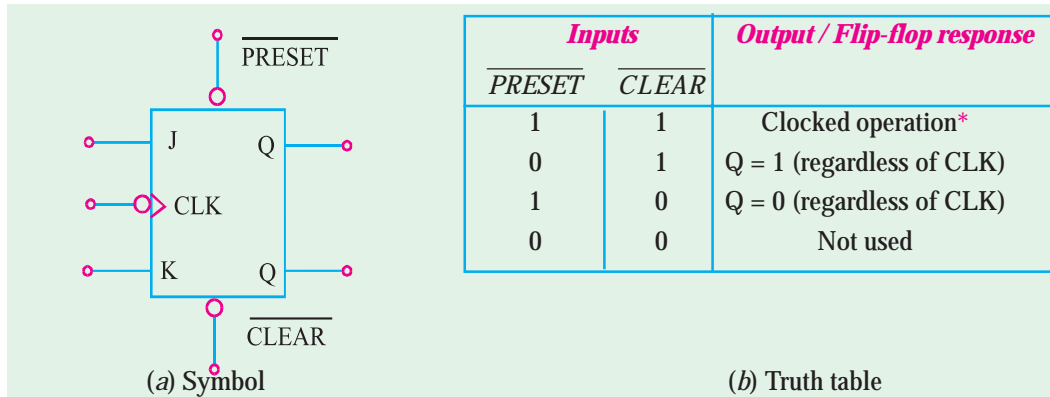


Fig. 72.50

Fig. 72.50 shows a clocked  $J$ - $K$  flip-flop with two asynchronous inputs namely (1)  $\overline{PRESET}$  and (2)  $\overline{CLEAR}$ . Both these inputs are active-LOW. This is indicated by the bubbles on the flip-flop symbol. Fig 72.50 (b) shows the truth table for the clocked  $J$ - $K$  flip-flop with asynchronous inputs. Let us study the various cases given in the truth table :

1.  $\overline{PRESET} = \overline{CLEAR} = 1$ . This condition indicates that both the asynchronous inputs are inactive and the flip-flop is free to respond to the  $J$ ,  $K$  and CLK inputs. In other words, the flip-flop operates as a normal clocked flip-flop.
2.  $\overline{PRESET} = 0$ ,  $\overline{CLEAR} = 1$ . This condition indicates that the asynchronous input,  $\overline{PRESET}$  is activated. Because of this, the output,  $Q$  is immediately set to 1, no matter what conditions are present at the  $J$ ,  $K$  and CLK inputs. It may be carefully noted that the CLK input cannot affect the flip-flop while  $\overline{PRESET} = 0$ .

3.  $\overline{\text{PRESET}} = 1, \overline{\text{CLEAR}} = 0$ . This condition indicates that the asynchronous input,  $\overline{\text{CLEAR}}$  is activated. Because of this, the output,  $Q$  is immediately set to 0, no matter what conditions are present at the J, K and CLK inputs. It may be carefully noted that the CLK input cannot affect the flip-flop while  $\overline{\text{CLEAR}} = 0$ .
4.  $\overline{\text{PRESET}} = \overline{\text{CLEAR}} = 0$ . This condition should not be used because it can produce an ambiguous response.

It will be interesting to know that the  $\overline{\text{PRESET}}$  and  $\overline{\text{CLEAR}}$  inputs respond to dc levels. This means that if a constant 0 is held on the  $\overline{\text{PRESET}}$  input, the flip-flop will remain in the  $Q = 1$  state regardless of what is occurring at the other inputs. Similarly, a constant 0 at the  $\overline{\text{CLEAR}}$  input holds the flip-flop in  $Q = 0$  state. Thus the asynchronous inputs can be used to hold the flip-flop in a particular state for any desired interval of time. However, in actual practice, the asynchronous inputs are used to set or clear the flip-flop to the desired state by application of a momentary pulse.

Many clocked flip-flops that are commercially available as ICs, will have both  $\overline{\text{PRESET}}$  and  $\overline{\text{CLEAR}}$ . However some ICs will have only  $\overline{\text{CLEAR}}$  input. Some other ICs will have asynchronous inputs that are active-HIGH. For these ICs, the flip-flop symbol would not have a bubble on the asynchronous inputs.

ICs 7476 and 74LS76 are popular  $J$ - $K$  flip-flops with asynchronous inputs preset and clear. The preset is designated by  $\overline{S}_D$  and clear by  $\overline{R}_D$ . The IC 7476 is a rising edge triggered while 74LS76 is a falling edge triggered device. Each package contains two  $J$ - $K$  flip-flops.

**Example 72.11.** Fig. 72.51 (a) shows the logic symbol for a  $J$ - $K$  flip-flop that responds to the falling edge on its clock pulse and has active-LOW asynchronous inputs. The  $J$  and  $K$  inputs are tied HIGH.

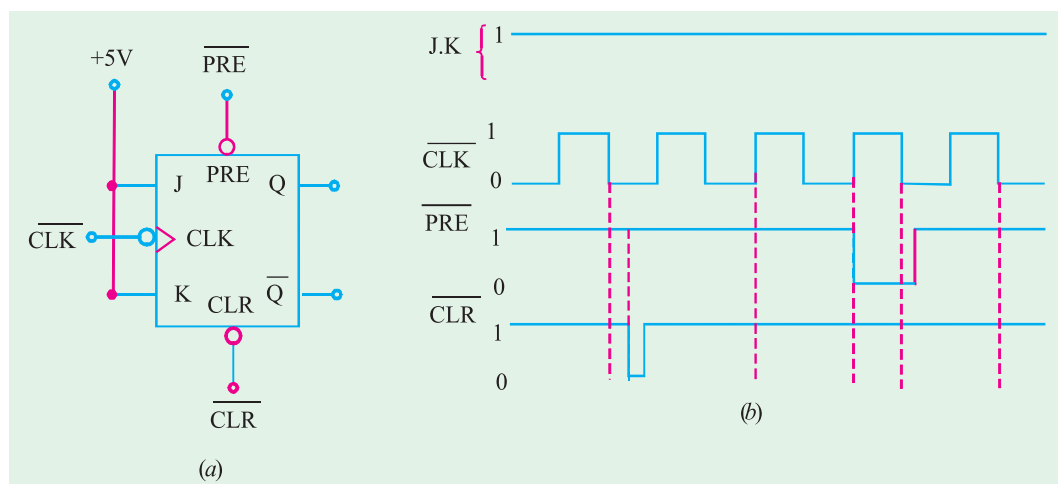


Fig. 72.51

Determine the  $Q$ -output in response to the waveforms shown in Fig. 72.51 (b). Assume that initially  $Q = 0$ .

**Solution.** Notice that the  $J$  and  $K$  are tied to +5V. The waveforms shown in Fig. 72.51 (b) are applied at the inputs of the  $J$ - $K$  flip flop.

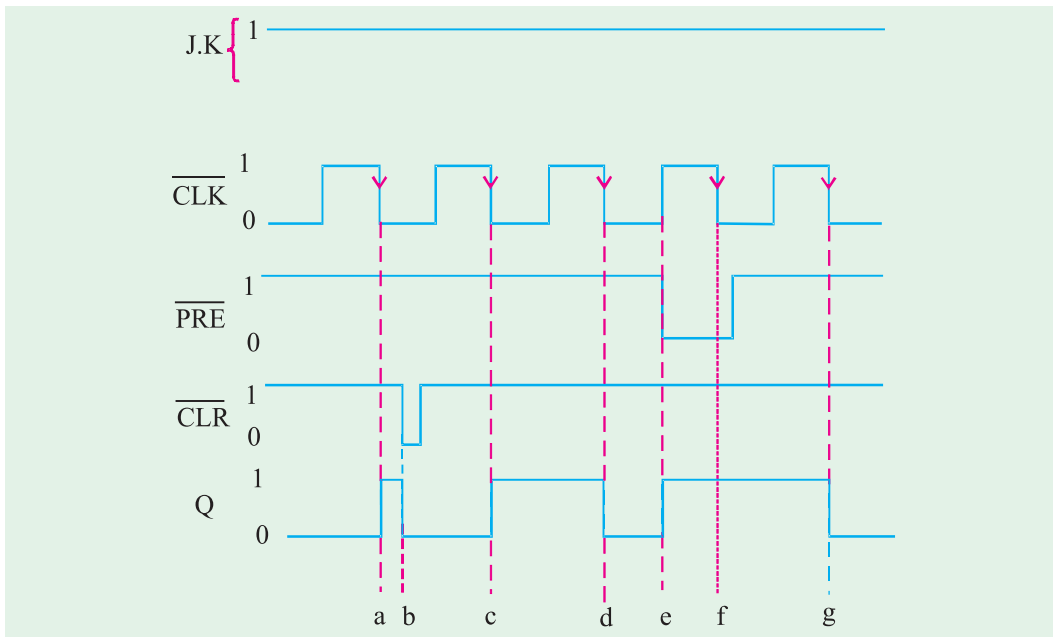


Fig. 72.52

In order to sketch the  $Q$ -output waveform, recall the truth table of  $J$ - $K$  flip-flop indicated in Fig. 72.31 (b) and Fig. 72.50 (b). Notice that when  $\overline{\text{PRE}}=1$  and  $\overline{\text{CLR}}=1$ , the  $J$ - $K$  flip-flop will trigger at the falling edge of the clock pulse. Thus at point 'a',  $\overline{\text{PRE}}=\overline{\text{CLR}}=1$ ,  $J=K=1$ , therefore the  $Q$ -output toggles *i.e.* it goes from 0 to 1. At point 'b'  $\overline{\text{CLR}}$  goes 0, therefore  $Q$  goes 0. At point 'c',  $\overline{\text{PRE}}=\overline{\text{CLR}}=1$ , therefore under normal operation  $J$ - $K$  flip-flop toggles *i.e.*  $Q$  goes from 0 to 1 again. At point 'd' there is no change in  $\overline{\text{PRE}}$  or  $\overline{\text{CLR}}$  inputs. So the  $Q$ -output toggles again, *i.e.*  $Q$  goes 0. At point 'e'  $\overline{\text{PRE}}$  goes 0, therefore  $Q$ -output goes 1. At point 'g',  $\overline{\text{PRE}}=\overline{\text{CLR}}=1$ , and  $Q$  toggles to 0. The complete  $Q$ -output waveform is shown in Fig. 72.52 along with the input waveforms.

**Example 72.12.** Determine the  $Q$ -output for the JK flip-flop shown in Fig. 72.53, Assume that  $Q = 0$  initially and remember that the asynchronous inputs override all other inputs.

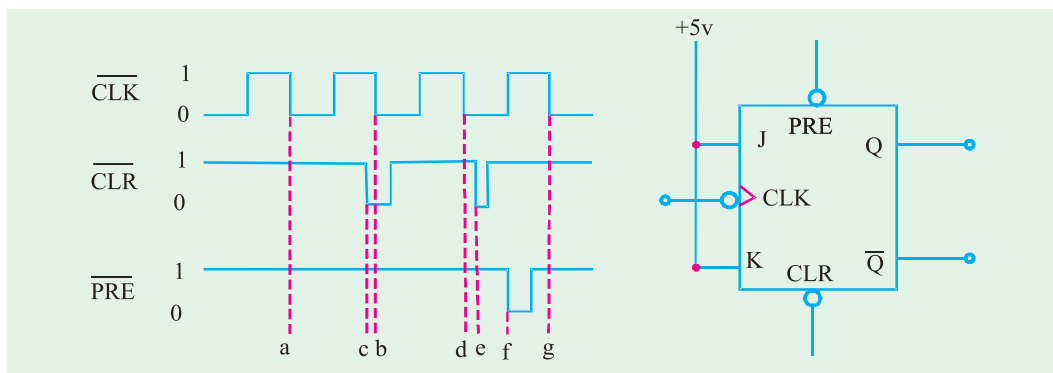


Fig. 72.53

**Solution :**

The waveform at the  $Q$ -output can be obtained by recalling the truth-table of a  $JK$  flip-flop. Shown in Fig. 72.31 (b) and 72.50 (b).

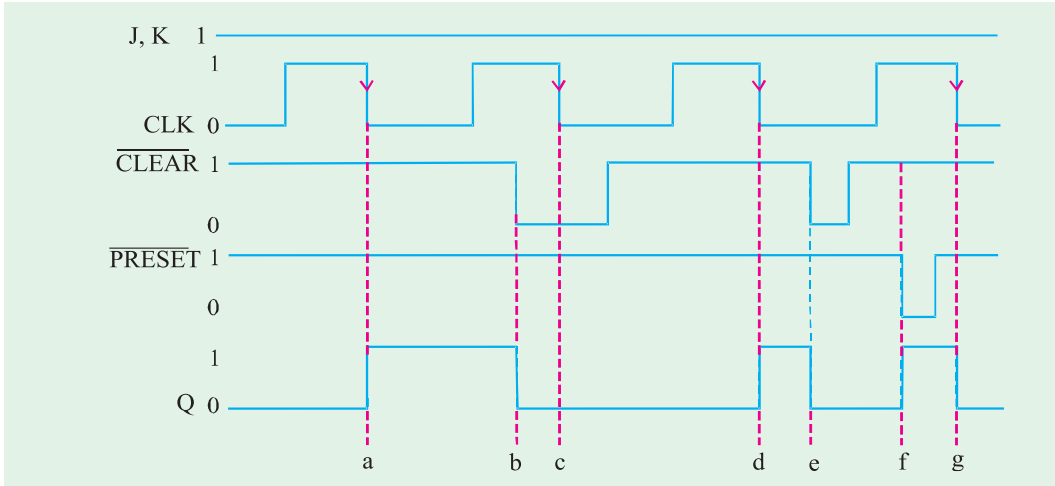


Fig. 72.54

Notice that at point 'a',  $J = K = 1$ ,  $\overline{\text{PRESET}} = \overline{\text{CLEAR}} = 1$ , the falling edge of CLK pulse toggles the  $J$ - $K$  flip-flop  $Q$  output to 1. At point 'b'  $\overline{\text{CLEAR}}$  goes 0, therefore irrespective of the other inputs,  $Q$ -output goes 0. At point 'c' the output remains 0 because  $\overline{\text{CLEAR}}$  is still 0. At point 'd',  $\overline{\text{PRESET}} = \overline{\text{CLEAR}} = 1$ ,  $J = K = 1$ , the  $J$ - $K$  flip-flop output toggles on the arrival of falling edge of CLK, i.e.  $Q$  goes 1. At point 'e',  $\overline{\text{CLEAR}}$  goes 0 again, therefore  $Q$ -output goes 0. At point 'f',  $\overline{\text{PRESET}}$  goes 0, therefore  $Q$ -output goes 1. Finally at point 'g',  $\overline{\text{CLEAR}} = \overline{\text{PRESET}} = 1$ ,  $J = K = 1$ , the output toggles on the arrival of falling edge of CLK pulse, i.e.  $Q$  goes 0.

The complete sketch of  $Q$ -output waveform along with other input waveforms is shown in Fig. 72.54.

**Example 72.13.** In the circuit of Fig. 72.55, inputs  $A$ ,  $B$  and  $C$  are all initially LOW. Output  $Y$  is supposed to go HIGH only when  $A$ ,  $B$  and  $C$  goes HIGH in a certain sequence.

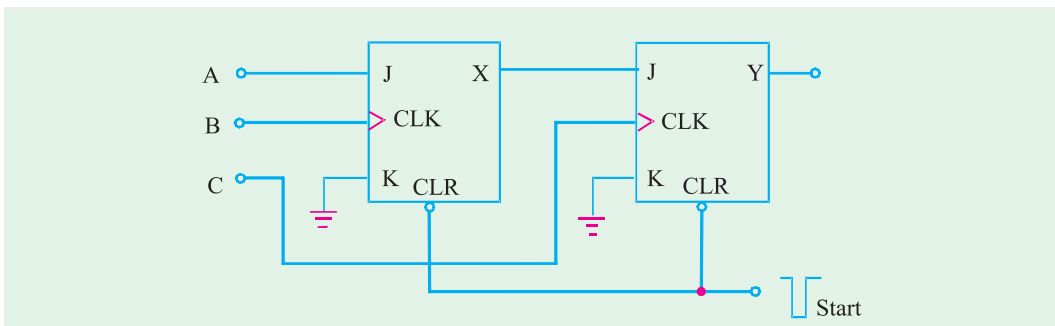


Fig. 72.55

- (a) *Determine the sequence that will make Y go HIGH.*  
 (b) *Explain why the START pulse is needed.*

**Solution.**

**(a) Sequence at the inputs (A, B and C)**

We know that Y can go HIGH only when C goes HIGH while X is already HIGH.  
 X can go HIGH only if B goes HIGH while A is HIGH.

*Thus the correct sequence that makes Y go HIGH is A, B, C.*

**(b) Need for START pulse**

We know that the outputs X and Y need to be cleared to 0 before applying the A, B, C signals. To clear the outputs, we need a negative going START pulse at CLR input. (notice that CLR input of JK flip-flop is active LOW).

## 72.20. Alternative Designations for Asynchronous Inputs

We have already discussed in the last article about the asynchronous clocked *J-K* flip-flop. These flip-flops are available as ICs and have two asynchronous inputs namely  $\overline{\text{PRESET}}$  and  $\overline{\text{CLEAR}}$  (or PRESET and CLEAR). IC manufacturers have not agreed on what nomenclature is used for these asynchronous inputs. The most common designations are  $\overline{\text{PRE}}$  (short for  $\overline{\text{PRESET}}$ ) and  $\overline{\text{CLR}}$  (short for  $\overline{\text{CLEAR}}$ ). The designations  $\overline{S}_D$  (direct SET) and  $\overline{R}_D$  (direct RESET) are also used. However we will use the labels  $\overline{\text{PRE}}$  and  $\overline{\text{CLR}}$  to represent asynchronous inputs. Whenever these asynchronous inputs are active-HIGH, we will not use the overbar to indicate their active-HIGH status *i.e.* PRE and CLR.

## 72.21. Flip-Flop Timing Parameters

The performance, operating requirements and limitations of flip-flops are specified by several timing parameters found on the data sheet for the device refer to Appendix A. Generally the timing parameters are applicable to all CMOS and TTL flip-flops.

**1. Setup and Hold Times.** These have already been discussed in Art 72.10 (page 2637). They represent requirements that must be met for reliable flip-flop triggering. The manufacturer's IC data sheet will always specify the minimum values of  $t_S$  and  $t_H$ . For example, IC 7474 has set up time of 20 ns and hold time of 5 ns. On the other hand 74HC112 has set up time of 25ns and zero hold time.

**2. Propagation delay.** It is the interval of time required after an input signal has been applied for the resulting output change to occur. Fig. 72.56 (a) illustrates the propagation delays that occur in response to a rising edge of the CLK input when *Q*-output changes from 0 to 1. Similarly Fig. 72.56 (b) illustrates the propagation delay that occur in response to the rising edge of the CLK input when *Q*-output changes from 1 to 0. It may be noted that these delays are measured between the 50% points (between logic 0 and logic 1 voltage levels) on the input and output waveforms. The same type of delays occur in response to the signals on a flip-flops asynchronous inputs (*i.e.* PRESET and CLEAR). The manufacturer's data sheets usually specify propagation delays in response to all inputs and they usually specify the maximum values for  $t_{PLH}$  and  $t_{PHL}$ . Notice that  $t_{PLH}$  is the delay going from logic LOW to HIGH level whereas  $t_{PHL}$  is the delay going from logic HIGH to LOW level.



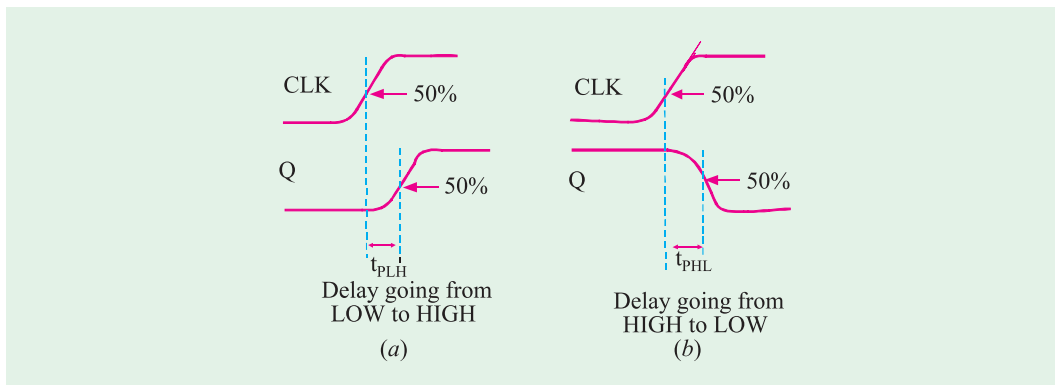


Fig. 72.56

**3. Maximum Clock Frequency ( $f_{max}$ ).** This is the highest frequency that may be applied to the CLK input of a flip-flop and still have it triggered reliably. The value of  $f_{max}$  limit will vary from flip-flop to flip-flop even with flip-flops having the same device number. For example,  $f_{max}$  for IC7474 is 15 MHz, for 74LS112, it is 30 MHz, for 74C74, it is 5 MHz and for 74HC112, it is 20 MHz.

**4. Clock Pulse HIGH and LOW Times.** The manufacturer will also specify the minimum time duration that the CLK signal must remain LOW before it goes HIGH {represented as  $t_w(L)$ } and the minimum time that CLK must be kept HIGH before it returns LOW (represented as  $t_w(H)$ ). These times are illustrated in Fig. 72.57 (a) Failure to meet these minimum time requirements can result in unreliable triggering. It may be noted that these time values are measured between the halfway (50%) points on the signal transitions.

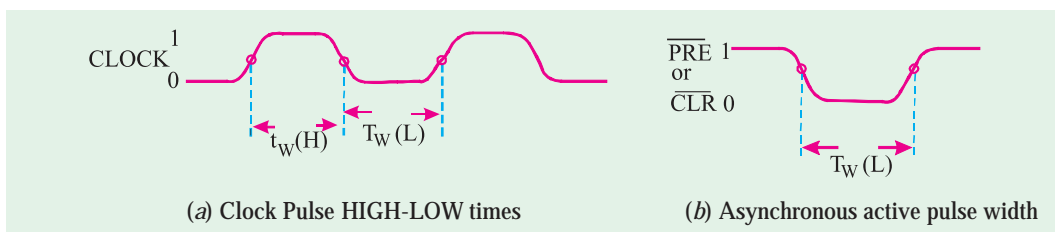


Fig. 72.57

**5. Asynchronous Active Pulse Width.** It is the minimum time duration that a PRESET or CLEAR input must be kept in its active state in order to set or clear the flip-flop reliably. Fig. 72.57 (b) shows  $t_w(L)$  for active-LOW asynchronous inputs.

**6. Clock Transition Times.** Strictly speaking, the rise and fall times of a clock waveform should be kept very short for reliable triggering. If the clock signal takes too long to change from one logic level to the other, the flip-flop may trigger erratically or not at all. IC manufacturers usually do not list a maximum transition time requirement for each flip-flop integrated circuit. Instead, it is usually given as a general requirement for all ICs within a given logic family. For example, the transition times should generally be  $\leq 50$  ns for TTL devices and  $\leq 200$  ns for CMOS. These requirements will vary among the different manufacturers and among the various subfamilies within the broad TTL and CMOS logic-families.

## 72.22. IC Flip-Flop Timing Values

We have already discussed the various flip-flop timing parameters in the last article. Table 72.1 lists all these timing values for the various flip-flops such as 7474, 74LS112, 74C74 and 74HC112. Notice that 7474 is a dual edge triggered *D* flip-flop (Standard TTL) device whereas 74LS74 is a dual edge-triggered *J-K* flip-flop (low-power schottky TTL) device. Similarly 74C74 is a dual edge-triggered *D* flip-flop (metal-gate CMOS) device whereas 74HC112 is a dual edge-triggered *J-K* flip-flop (high-speed CMOS) device.

Parameter (Times in ns)	TTL		CMOS	
	7474	74LS112	74C74	74HC112
$t_S$	20	20	60	25
$t_H$	5	0	0	0
$t_{PHL}$ from $CLK$ to $Q$	40	24	200	31
$t_{PLH}$ from $CLK$ to $Q$	25	16	200	31
$t_{PHL}$ from $\overline{CLR}$ to $Q$	40	24	225	41
$t_{PLH}$ from $\overline{PRE}$ to $Q$	25	16	225	41
$t_w(L)$ $CLK$ LOW time	37	15	100	25
$t_w(H)$ $CLK$ HIGH time	30	20	100	25
$t_w(L)$ at $\overline{PRE}$ or $\overline{CLR}$	30	15	60	25
$f_{Max}$ in MHz	15	30	5	20

Following are some of the important points which may be noted carefully from Table 72.1.

1. The 74HC series of CMOS devices has timing values that are comparable to those of TTL devices. This can be observed by reading the values below the 7474 column and the values below 74HC112 column.
2. The 74C series devices are much slower than the 74HC series. This can be observed by comparing all the minimum timing requirements below the 74C74 and 74HC112 column.
3. All the flip-flops have nonzero set up time requirement.
4. All of the flip-flops have very low hold-time ( $t_H$ ) requirements *e.g.* The IC7474 has  $t_H = 5$  ns whereas IC74LS112, 74C74 and 74HC112 has  $t_H = 0$ .

**Example 72.14.** The data sheet of a certain flip-flop specifies that the minimum HIGH time  $t_w(H)$  for the clock pulse is 30 ns and the minimum LOW time  $t_w(L)$  is 37 ns. What is the maximum operating frequency?

**Solution.** Given :  $t_w(H) = 30$  ns =  $30 \times 10^{-6}$  s and  $t_w(L) = 37$  ns =  $37 \times 10^{-6}$  s.

We know that the minimum time period,

$$\begin{aligned} t_{min} &= t_w(H) + t_w(L) \\ &= (30 \times 10^{-6} + 37 \times 10^{-6}) = 47 \times 10^{-6} \text{ s} \end{aligned}$$

and the maximum operating frequency,

$$\begin{aligned} f_{max} &= \frac{1}{t_{min}} = \frac{1}{47 \times 10^{-6} \text{ ns}} = 21.276 \times 10^6 \text{ Hz} \\ &= \mathbf{21.276 \text{ MHz.}} \end{aligned}$$

**Example 72.15.** Using the flip-flop timing values shown in Table 72.2, determine the followings :

Parameter (time in ns)	TTL		CMOS	
	7474	74LS112	74C74	74HC112
$t_{PHL}$ from CLK to Q	40	24	200	31
$t_{PLH}$ from CLK to Q	25	16	200	31
$t_w(L)$ at $\overline{PRE}$ or $\overline{CLR}$	30	15	60	25

- (a) Assume that initially  $Q = 0$ . How long it can take for  $Q$  to go HIGH when a rising edge appears at the CLK input of IC7474?
- (b) Assume that initially  $Q = 1$ . How long it can take for  $Q$  to go LOW in response to the  $\overline{CLR}$  input of an IC 74HC112?
- (c) What is the narrowest pulse that should be applied to the  $\overline{CLR}$  input of the IC 74LS112 flip-flop to clear  $Q$  reliably?

**Solution.**

- (a) The rising edge will cause the  $Q$  output to go from LOW to HIGH. The delay from CLK to  $Q$  is listed as  $t_{PLH} = 25$  ns for IC7474.
- (b) For the IC 74HC112, the time required for  $Q$  to go from HIGH to LOW in response to  $\overline{CLR}$  input is listed as  $t_{PHL} = 31$  ns.
- (c) For the IC 74HC112, the narrowest pulse at the  $\overline{CLR}$  input is listed as  $t_w(L) = 15$  ns.

### 72.23. Potential Timing Problems in Flip-Flop Circuits

As a matter of fact, in many digital circuits, the output of one flip-flop is connected either directly or through logic gates to the input of another flip-flop and both flip-flops are triggered by the same clock signal. This situation leads to a potential timing problem. In order to understand this problem, consider a situation shown in Fig. 72.58. Here the output of flip-flop (1)  $Q_1$  is connected to the  $J$  input of flip-flop (2). Notice that both the flip-flops are clocked by the same signal at their CLK inputs.

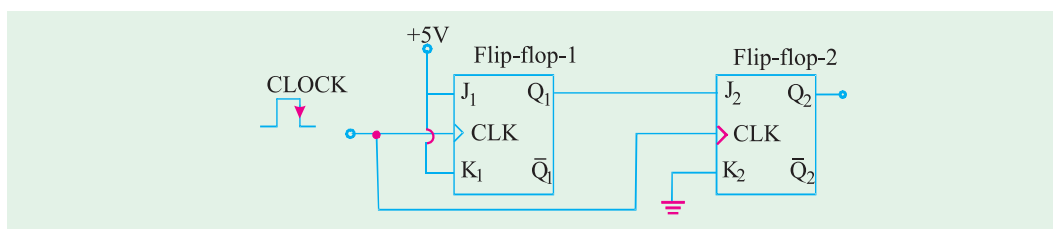


Fig. 72.58

Let us understand the potential timing problem now. Since  $Q_1$  will change on the rising edge of the clock pulse, the  $J_2$  input will be changing as it receives the same rising edge. This could lead to an unpredictable response at  $Q_2$ .

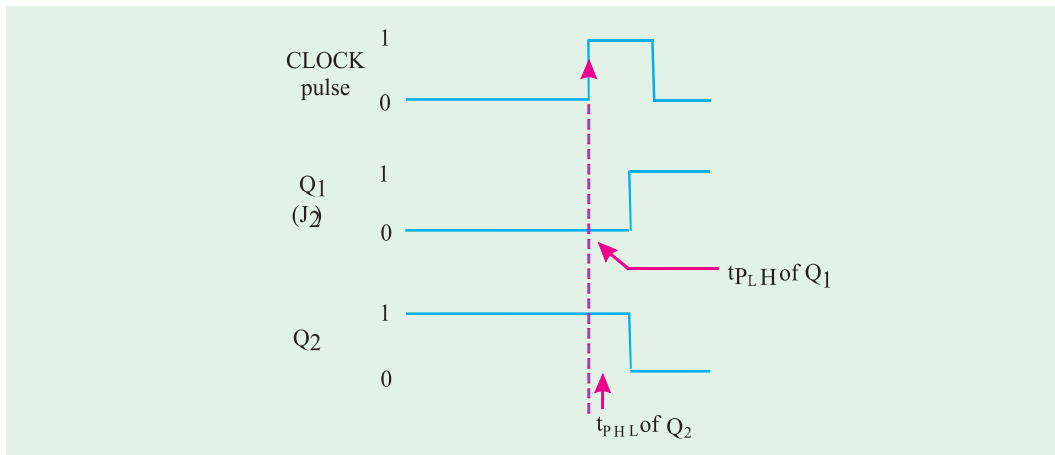


Fig. 72.59

Assume that initially  $Q_1 = 0$  and  $Q_2 = 1$ . Further the flip-flop (1) has  $J_1 = K_1 = +5V$  (i.e. HIGH) and flip-flop (2) has  $J_2 = Q_1$  and  $K_2 = 0$  prior to the occurrence of rising edge of the clock pulse. When the rising edge occurs,  $Q_1$  will toggle to the HIGH state but it will not actually go HIGH until after its propagation delay,  $t_{PLH}$  as shown in Fig. 72.59. The same rising edge will reliably clock  $Q_2$  to the LOW state provided that  $t_{PLH}$  is greater than  $Q_2$ 's hold time requirement,  $t_H$  as shown in the figure. If this condition is not met, the response of  $Q_2$  will be unpredictable.

Since all edge-triggered flip-flops have hold time ( $t_H$ ) requirement that is 5 ns or less. Therefore for those flip-flops, situations like that shown in Fig. 72.59 will not be a problem.

Thus in all of the flip-flop circuits that you will study in this book, we will assume that flip-flop's hold time requirement is short enough to respond reliably according to the following rule:

**The flip-flop output will go to a state determined by the logic levels present at its synchronous control inputs just prior to the active clock edge.**

Applying the above stated rule to the circuit shown in Fig. 72.59, it says that the output of flip-flop (2),  $Q_2$  will go to a state determined by  $J_2 = 1$  and  $K_2 = 0$  condition that is present just prior to the occurrence of the rising edge of the clock pulse. The fact that  $J_2$  is changing in response to the same rising edge has no effect.

## 72.24. Applications of Flip-Flop

Although there is a wide variety of applications of edge-triggered (clocked) flip-flops, yet the following are important from the subject point of view :

1. Flip-flop synchronization
2. Data storage and transfer
3. Serial data transfer-shift registers
4. Frequency division
5. Counting

All these applications are discussed one by one in the following pages.

## 72.25. Flip-Flop Synchronization

Strictly speaking, most digital systems are principally synchronous in their operation. It means, most of the signals will change states in synchronism with the clock transitions. However, in many cases, there will be an external signal that is not synchronised to the clock. In other words such an external signal is an asynchronous signal. Such signals often occur as a result of a human operator actuating an input switch at some random time relative to the clock signal. This randomness can

produce unpredictable and undesirable results. Now we will study how a flip-flop can be used to synchronize the effect of an asynchronous input.

Fig. 72.60 shows a situation where a signal  $X$  is generated from a debounced switch that is actuated by an operator. The switch output  $A$  goes high when the operator actuates the switch and goes LOW when the operator releases the switch. The switch output  $X$  is used as an input to control the passage of the control signal through the AND gate so that clock pulses appear at output  $Y$  only as long as  $X$  is HIGH.

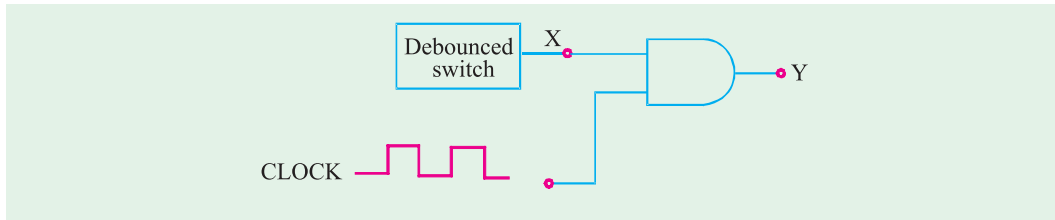


Fig. 72.60

The problem with the circuit shown in Fig. 72.60 is that signal  $X$  is asynchronous. That is it can change states at any time relative to the clock signal because the exact times when the operator actuates or releases the switch are essentially *random*. This can produce partial clock pulses at output  $Y$  if either transition of  $X$  occurs while the clock signal is HIGH. It is shown in Fig. 72.61. This type of output is often not acceptable. So a method for preventing the appearance of partial pulses at  $Y$  must be developed. One possible solution is shown in Fig. 72.61.

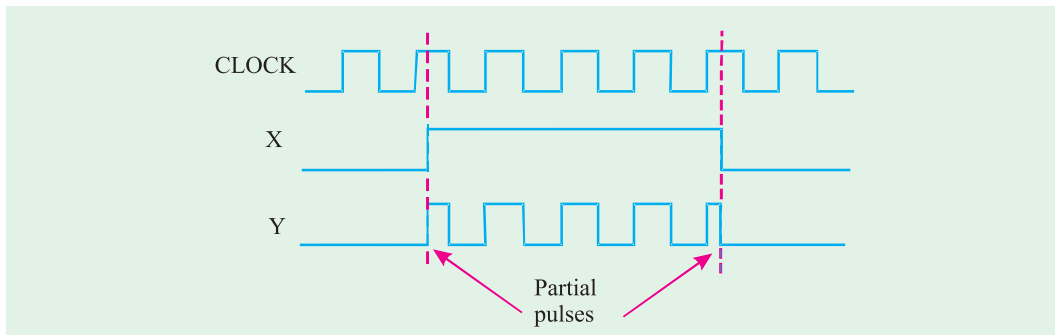


Fig. 72.61

As seen in Fig. 72.62, the  $X$  signal is connected to the  $D$  input of a flip-flop. The flip-flop is clocked by the falling edge of the clock signal. Thus when  $X$  goes HIGH,  $Q$  will not go HIGH until the next falling edge of the clock at time  $t_1$ . This HIGH at  $Q$  will enable the AND gate to pass subsequent complete clock pulses to  $Y$  as shown in Fig. 72.61.

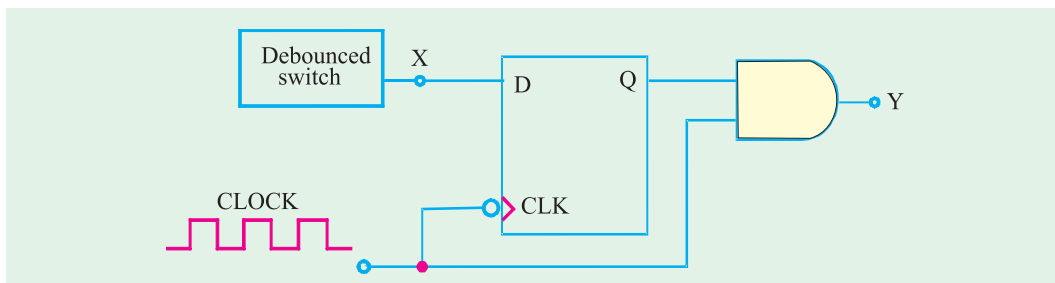


Fig. 72.62

Similarly, when  $X$  goes LOW,  $Q$  will not go LOW until the next falling edge of the clock at  $t_2$ . Thus the AND gate will not inhibit clock pulses until clock pulse that ends at  $t_2$  has passed through to  $Y$ . Therefore, output  $Y$  contains only complete pulses (see Fig. 72.63).

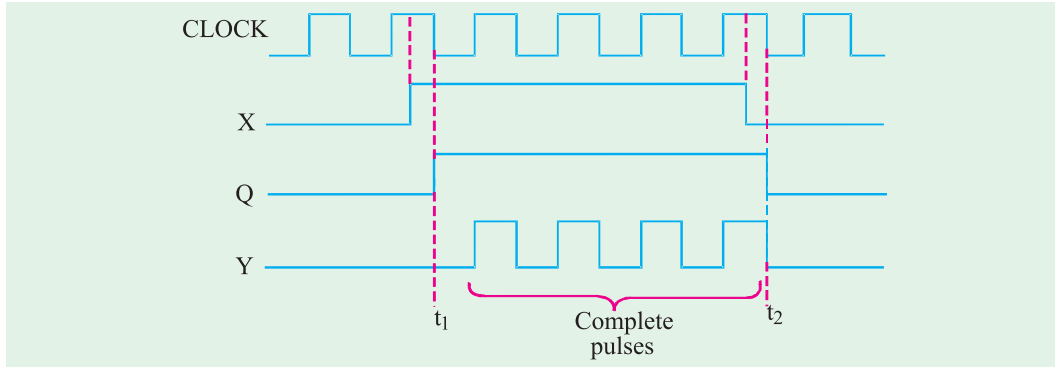


Fig. 72.63

### 72.26. Data Storage and Transfer

The most common use of flip-flops in the field of digital electronics, is for the storage of data or information. The data may represent numerical values (e.g. binary numbers, BCD -binary coded decimal numbers or any data that have been encoded in binary). These data are generally stored in groups of flip-flops called **registers**. The operation most often performed on data that are stored in a flip-flop or a register is the **data transfer** operation. This operation involves the transfer of data from one flip-flop (or register) to another. The data transfer is of two types: (a) synchronous and (b) asynchronous.

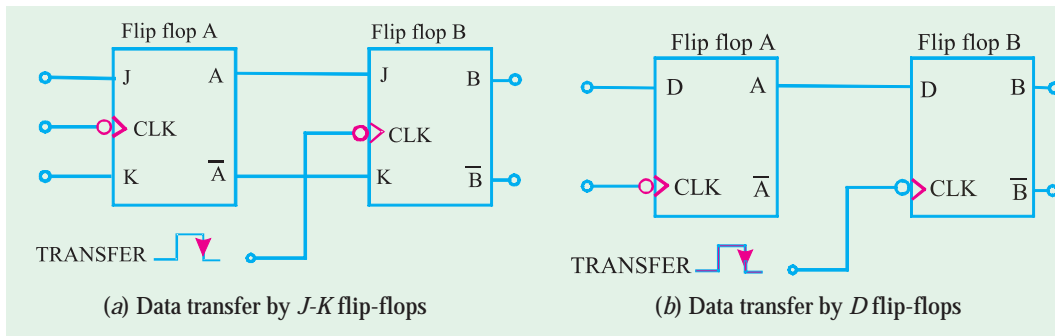


Fig. 72.64

Fig. 72.64 (a) and (b) shows how synchronous data transfer operation can be accomplished between two flip-flops using J-K and D flip-flops respectively. In each case, the logic value that is currently stored in flip-flop A is transferred to flip-flop B upon the falling edge of the TRANSFER pulse. Thus after this falling edge, the B output will be the same as the A output.

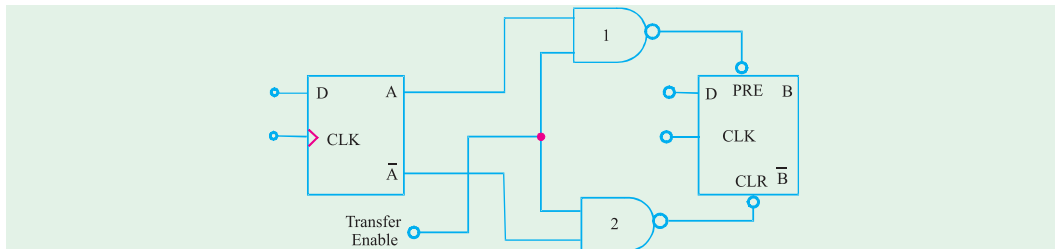


Fig. 72.65. Illustrating asynchronous data transfer operation.

The data transfer illustrated in Fig. 72.64 is called synchronous transfer because the synchronous control and CLK inputs of a flip-flop are used to perform the transfer operation. Fig. 72.64 shows the data transfer that can be obtained using the asynchronous inputs of a flip-flop. This method called asynchronous transfer can be accomplished using the PRESET and CLEAR inputs of any flip-flop. Notice that flip-flop outputs  $A$  and  $\bar{A}$  are connected to PRESET and CLEAR through two-input NAND gates 1 and 2 respectively.

In this method, the asynchronous inputs (*i.e.* PRESET and CLEAR) respond to LOW levels. The operation of the circuit may be explained as follows :

When the **Transfer Enable** line is held LOW, the two NAND outputs are kept HIGH, with no effect on the flip-flop outputs. But when the **Transfer Enable** line is made HIGH, one of the NAND outputs will go LOW depending on the state of  $A$  and  $\bar{A}$  outputs. This LOW will either set or clear the flip-flop  $B$ .

### 72.27. Parallel Data Transfer

We have already discussed in the last article that flip-flops can be used for data storage as well as for transfer of data from one flip-flop to another. This idea can be further extended for transfer of data from one group of flip-flops (called register  $A$ ) to another group of flip-flops (called register  $B$ ). Register  $A$  consists of four flip-flops  $A_1, A_2, A_3$  and  $A_4$ . Similarly register  $B$  also consists of four flip-flops labelled as  $B_1, B_2, B_3$  and  $B_4$  upon application of falling edge of a TRANSFER pulse, the logic level stored in  $A_1$  is transferred to  $B_1, A_2$  is transferred to  $B_2, A_3$  is transferred to  $B_3$  and  $A_4$  is transferred to  $B_4$ . It may be noted that the transfer of contents from register  $A$  to  $B$  occurs at the same time. Because of this reason it is called synchronous transfer or parallel data transfer.

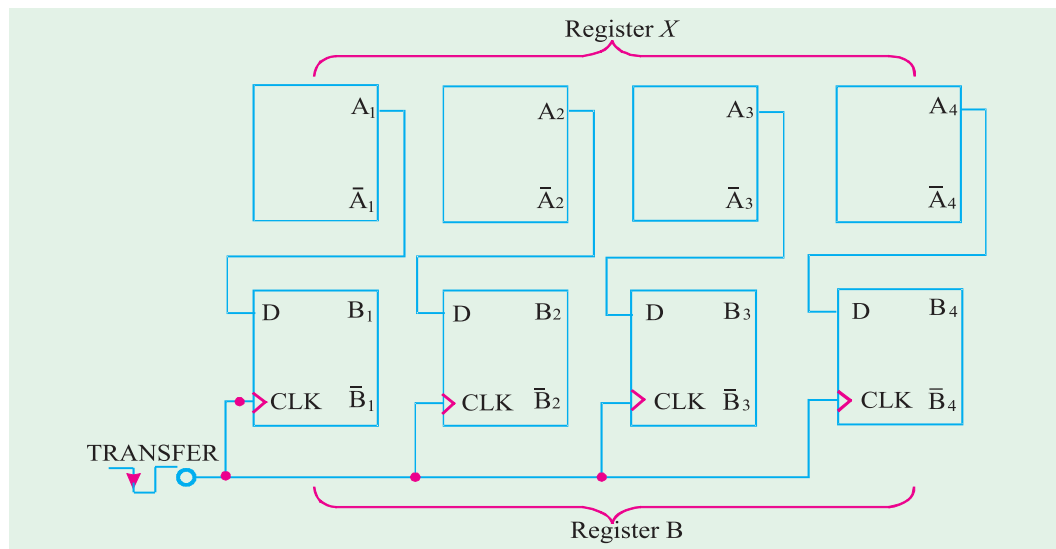


Fig. 72.66. Illustrating parallel data transfer from register  $X$  into register  $Y$

It may be carefully noted that parallel data transfer does not change the contents of the register that is the source of data. For example in Fig. 72-66 if  $A_1 A_2 A_3 A_4 = 1011$  and  $B_1 B_2 B_3 B_4 = 0100$  prior to the occurrence of the TRANSFER pulse, then both registers will be holding 1011 after the TRANSFER pulse. Another point is that the group of four flip-flops indicated above is an example of a basic register used for data storage. In digital systems, data are normally stored in groups of bits (usually 8, 16, 32, 64 ...) that represent numbers, codes or other information.

**Example 72.16. (a)** Draw a circuit diagram for the synchronous parallel transfer of data from one three-bit register to another using J-K Flip-flops. **(b)** Repeat for asynchronous parallel transfer.

**Solution. (a)** Fig. 72.67 shows the circuit diagram for the synchronous parallel transfer of data from one three-bit register X to another register Y using *J-K* flip-flops. Notice that the contents of  $X_1$ ,  $X_2$ , and  $X_3$  are transferred simultaneously into  $Y_1$ ,  $Y_2$ , and  $Y_3$ . It may be carefully noted that the circuit allows the parallel transfer of data from the normal outputs as well as the complemented outputs of the flip-flops constituting register X.

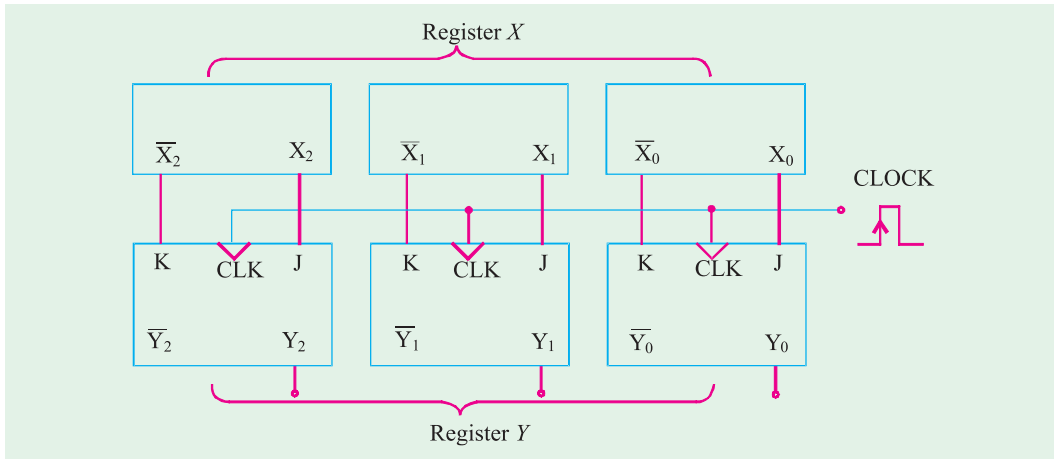


Fig. 72.67. Synchronous parallel transfer of data from one three-bit register to another

**(b)** Fig. 72-68 shows the circuit diagram for the asynchronous parallel data transfer from one three-bit register to another using *J-K* flip-flops. The circuit allows data transfer from either normal outputs  $X_0, X_1$ , and  $X_2$  or the complemented outputs  $\bar{X}_1, \bar{X}_2$  and  $\bar{X}_3$ .

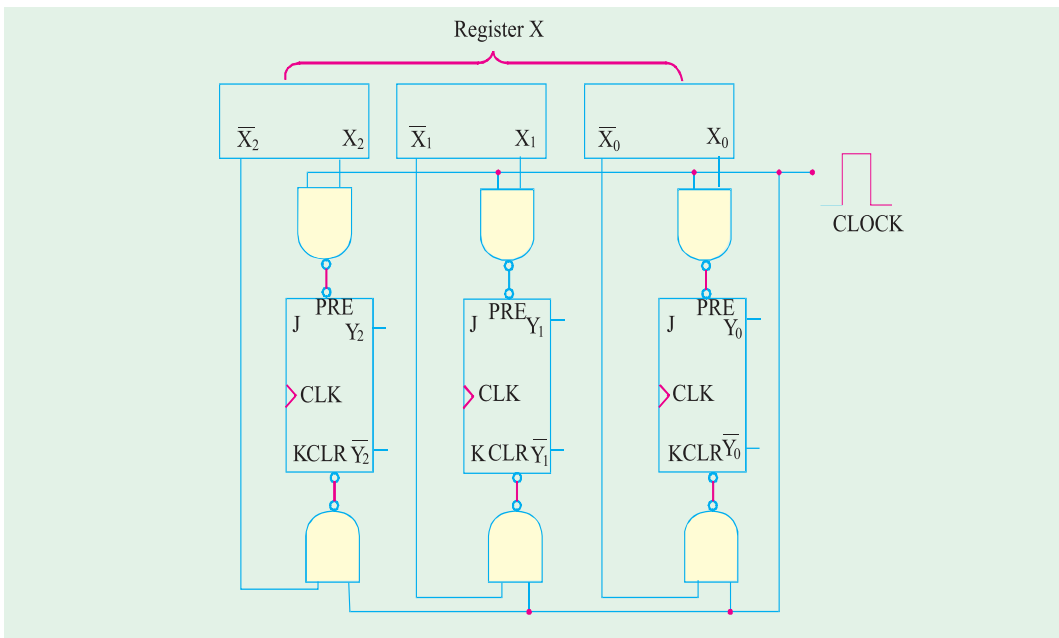


Fig. 72.68. Asynchronous parallel transfer of data from one three-bit register to another.



## 72.28. Serial Data Transfer : Shift Registers

We have already discussed in the last article that a **register** is basically a group of flip-flops to store data. But there is no interconnection between the flip-flops. A **shift register** is a group of flip-flops arranged in such a way that the binary numbers stored in the flip-flops are shifted from one flip-flop to the next for every clock pulse. Fig. 72.69 shows a four-bit shift register using *J-K* flip-flops. Notice that the *J* and *K* inputs of flip-flops '3' are fed by DATA IN waveform. The *J* and *K* inputs of flip-flop '2' are fed by the  $X_3$  and  $\bar{X}_3$ . Similarly *J* and *K* inputs of flip-flop '1' are fed by the  $X_2$  and  $\bar{X}_2$  and *J* and *K* inputs of flip-flop '0' are fed by the  $X_1$  and  $\bar{X}_1$ . The CLK input of all the four flip-flops are connected together to a common input which receives shift pulses. Incidentally notice that all the flip-flops trigger on the falling edge of the CLK pulse.

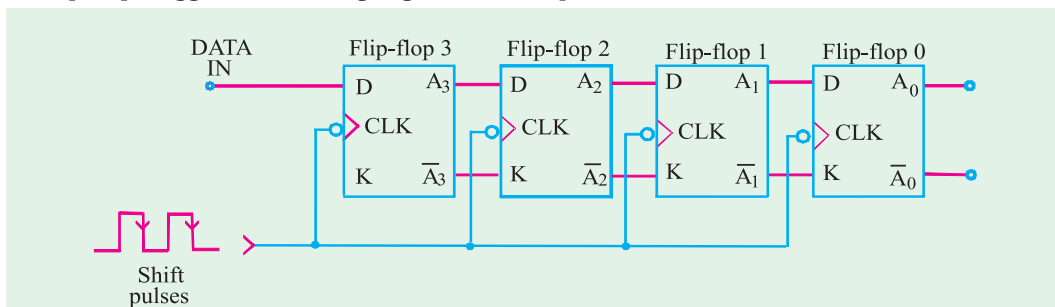


Fig. 72.69. Four-bit shift register.

The operation of the shift register may be understood from the explanation given below. Let us assume that initially all the flip-flops are in the 0 state and input of flip-flop '3' is fed by the DATA IN waveform as shown in Fig. 72.70. The other waveforms shown in Fig. 72.70 indicate how the input data are shifted from left to right from flip-flop to flip-flop as shift pulses are applied. When the first falling edge occurs at  $t_1$ , each of the flip-flop outputs  $A_2$ ,  $A_1$  and  $A_0$  will have  $D = 0$  condition present at its inputs because of the state of the flip-flop on its left. The flip-flop '3' will have  $D = 1$ , because of DATA-IN. Thus at  $t_1$  only  $A_3$  will go HIGH while all the other flip-flop outputs remain LOW. When the second falling edge occurs at  $t_2$ , flip-flop '3' will have  $D = 0$  because of DATA-IN. The Flip-flop '2' will have  $D = 1$  because of current HIGH at  $A_3$ . Flip-flops '1' and '0' will still have  $D = 0$ . Thus at  $t_2$ , only flip-flop '2' output  $A_2$  will go HIGH, flip-flop output  $A_3$  will go LOW, and  $A_1$  and  $A_0$  will remain LOW.

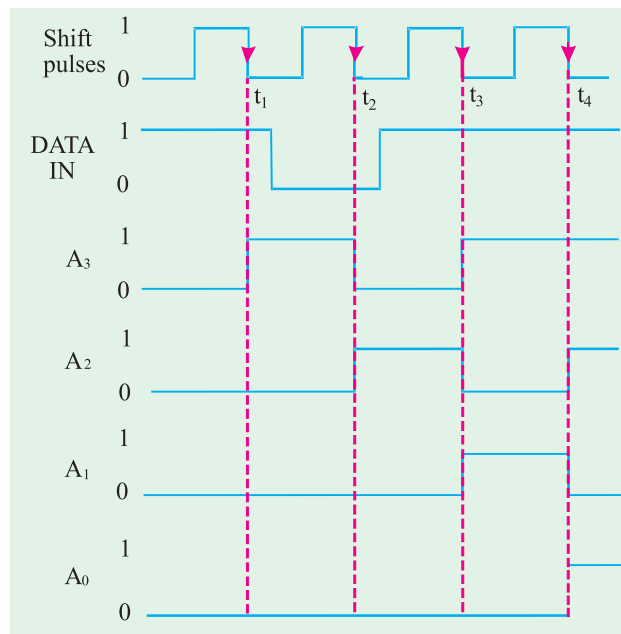


Fig. 72.70. Illustrating serial data transfer.

A similar reasoning can be used to determine how the waveforms change at  $t_3$  and  $t_4$ . It may be carefully noted that on each falling edge of the shift pulses, each flip-flop output takes on the level

that was present at the output of the flip-flop on its left just *prior* to the falling edge. Of course, flip-flop '3' output  $X_3$  takes on the level that was present at DATA IN just prior to the falling edge.

**Note.** In the shift register arrangement discussed above, it is necessary that the flip-flops have a very small hold time requirement. This is because of the fact that there are times when the  $J$  and  $K$  inputs are changing at about the same time as the CLK edge. For example, the flip-flop '3' output,  $X_3$  changes from 1 to 0 in response to the falling edge at  $t_2$ , causing the  $J$  and  $K$  inputs of flip-flop '2' output  $X_2$  to change while its CLK input is changing. Actually because of the propagation delay of flip-flop '3', the  $J$  and  $K$  inputs of flip-flop '2' won't change for a short time after the falling edge. Because of this reason, a shift register should be implemented using edge-triggered flip-flops that have a  $t_H$  value less than one CLK-to-output propagation delay. This later requirement is easily satisfied by most modern edge triggered flip-flops.

### 72.29. Serial Data Transfer Between Registers

We have already discussed in the last article that a shift register is a group of flip-flops arranged in such a manner that the binary numbers stored in the flip-flops are shifted from one flip-flop to the next after every clock pulse. Now we will study as how data can be serially transferred (or shifted) from one register to the other.

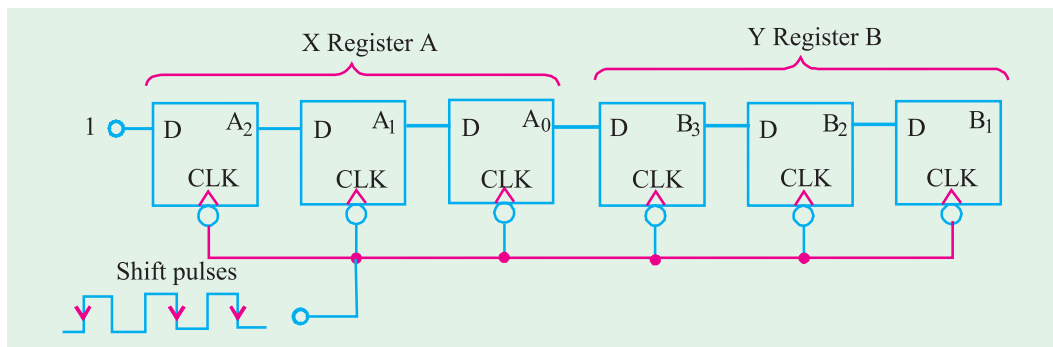


Fig. 72.71

Fig. 72.71 shows three-bit shift registers connected in such a manner that the contents of the A-register will be serially transferred (shifted) into register B. Notice how  $A_0$ , output of last flip-flop of register A is connected to the  $D$ -input of the first flip-flop of register B. The CLK input of all the flip-flops is connected together at one point where the shift pulses are applied.

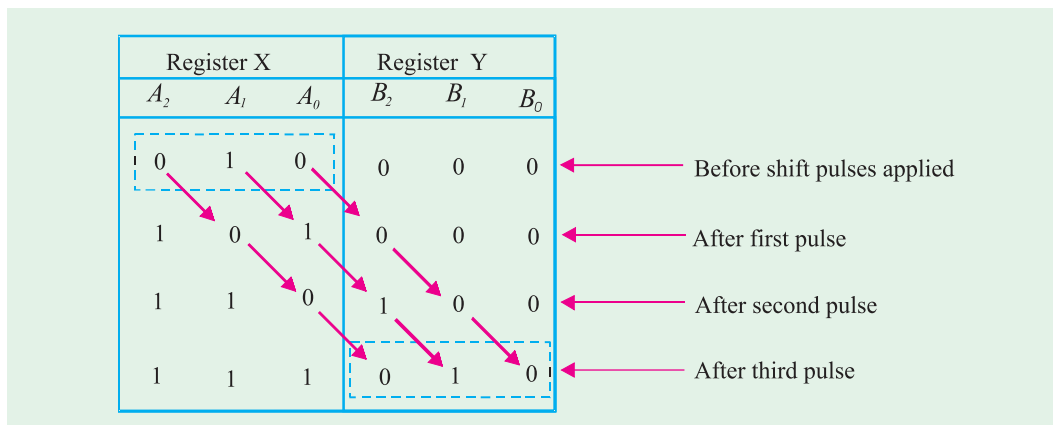


Fig. 72.72

As the shift pulses are applied, the data transfer takes place as follows:

$$A_2 \rightarrow A_1 \rightarrow A_0 \rightarrow B_2 \rightarrow B_1 \rightarrow B_0$$

The flip-flop output  $A_2$  will go to a state determined by its D input. For discussion purpose let us suppose that  $D$  is held HIGH, so that  $A_2$  will go HIGH on the first pulse and will remain there. Further let us assume that before any shift pulses are applied, the contents of A register are 010. (*i.e.*  $A_2 = 0$ ,  $A_1 = 1$ ,  $A_0 = 0$ ) and the B register is at 000. As the shift pulse are applied one after the another, the states of each flip-flop output change are indicated in table as shown in Fig. 72.72. Following are some of the points which may be carefully noted from this table.

1. On the falling edge of each clock pulse, each flip-flop takes on the value that was stored in the flip-flop on its left prior to the occurrence of the pulse.
2. After first pulse,  $A_2$  is set to 1, the 0 that was initially in  $A_2$  is in  $A_1$ , the 1 that was initially in  $A_1$  is in  $A_0$ , the 0 that was initially in  $A_0$  is in  $B_2$ , the 0 that was initially in  $B_2$  is in  $B_1$ , the 0 that was initially in  $B_1$  is in  $B_0$ . The 0 that was initially in  $Y_0$  is lost.
3. After second pulse,  $A_2$  is still at 1 (because  $D$  is HIGH), the 1 that was previously in  $A_2$  is in  $A_1$ , the 0 that was previously in  $A_1$  is in  $A_0$ , the 1 that was previously in  $A_0$  is in  $B_2$ , the 0 that was previously in  $B_2$  is in  $B_1$  and 0 that was previously in  $B_1$  is in  $B_0$ .
4. After third pulse,  $A_2$  is still at 1 (because  $D$  is held HIGH), the 1 that was previously in  $A_2$  is in  $A_1$ , the 1 that was previous in  $A_1$  is in  $A_0$ , the 0 that was previously in  $A_0$  is in  $B_2$ , the 1 that was previously in  $B_2$  is in  $B_1$ , the 0 that was previously in  $B_1$  is in  $B_0$ .

It is evident from the above discussion that after these shift pulses, the 010 stored in the A register has now been shifted to B register. The register A is now at 111. Notice that it has lost its original data.

#### Notes.

1. The flip-flops shown in Fig. 72.71 can also be connected easily so that data shifts from right to left. As a matter of fact, there is no general advantage of shifting in one direction over the other. The direction chosen by the digital system designer depends upon the nature of application.
2. In serial data transfer (refer to Fig. 72.71) of  $N$  bits of data requires  $N$  clock pulses. This means three bits of data required three pulses, four bits requires four pulses and so on. However in parallel transfer, all the data is transferred simultaneously upon the occurrence of a single transfer pulse (refer to Fig. 72.66). In other words it does not matter how many bits are being transferred. It is obvious that parallel transfer is much faster than serial transfer using shift registers. However, parallel transfer requires more interconnections between the sending register (A) and receiving register (B) than does the serial transfer. This is an important consideration when the sending and receiving registers are at a distance from each other. The choice of either parallel or serial transmissions depends on the particular system application and specifications. In actual practice, a combination of two types is used to take advantage of the speed of parallel transfer and economy and simplicity of serial transfer.

### 72.30. Frequency Division

Many applications require frequency division. For example a quartz (or digital) watch makes use of a quartz crystal to generate a very stable oscillator frequency. This frequency is usually 1MHz or more. In order to advance the "seconds" display once every second, the oscillator frequency is divided by a value that will produce a very stable and accurate 1Hz output frequency. The frequency division can be achieved by using flip-flops.

Fig. 72.73 (a) shows a single  $J$ - $K$  flip-flop connected to toggle (*i.e.*  $J = K = 1$ ). When the clock pulses are applied at its CLK input, the  $Q$  output is a square wave with one-half the frequency of the clock input as shown in Fig. 72.73 (b). Thus a single flip-flop can be used as a divide-by-2 device. As

seen from the figure, the flip-flop changes state on each falling edge of the clock pulse. This results in an output that changes at half the frequency of the clock waveform.

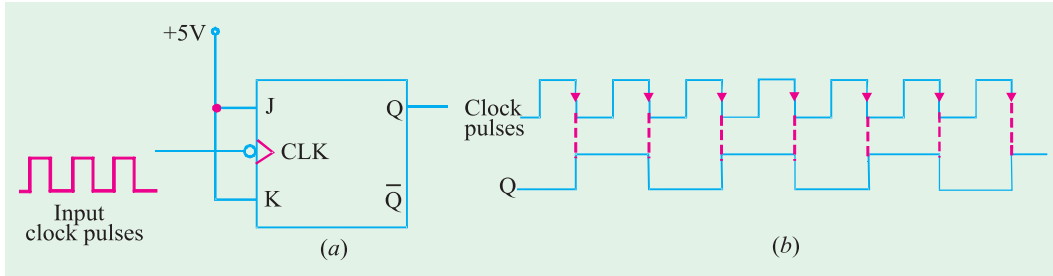


Fig. 72.73

Further division of the clock frequency can be achieved by using the output of one flip-flop as the clock input to the second flip-flop as shown in Fig. 72.74. Notice that the frequency of  $Q_0$  output is divided by 2 by flip-flop 1. Therefore the  $Q_1$  output is one-fourth the frequency of the original clock input. It may be carefully noted that propagation delays are not shown on the timing diagrams.

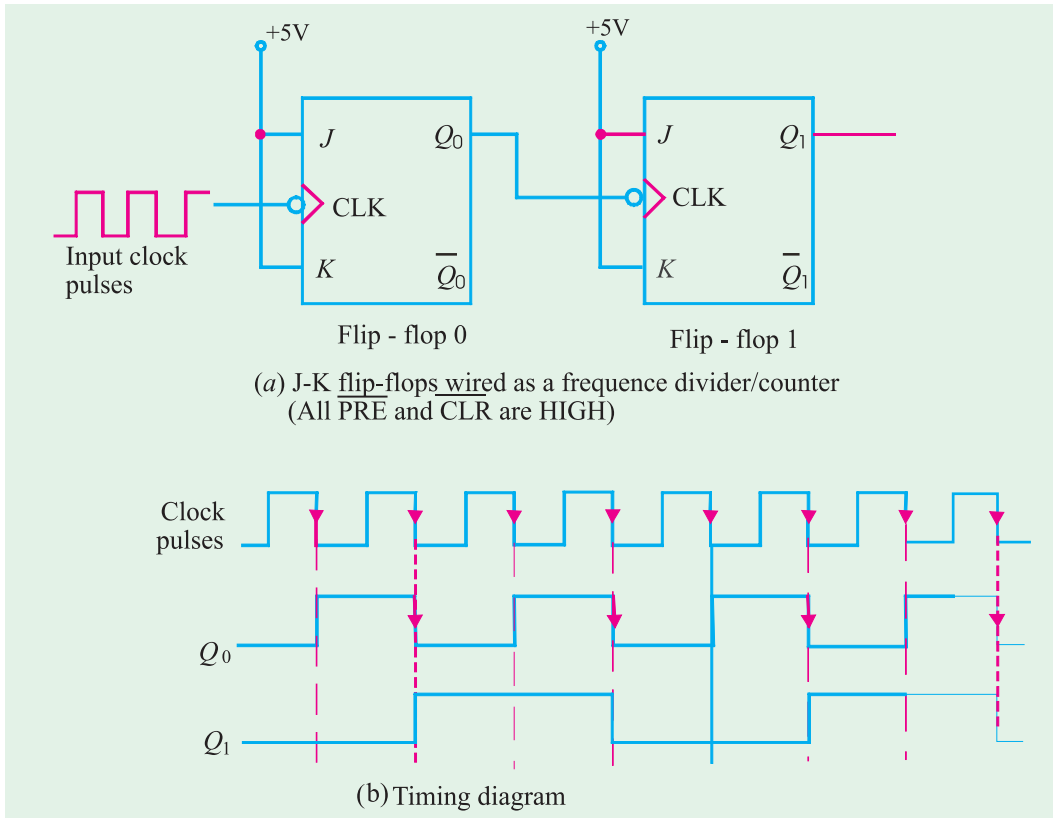


Fig. 72.74

By connecting the flip-flops in the way described above, a frequency division of  $2^N$  is achieved, where  $N$  is the number of flip-flops. For example, three flip-flops divide the clock frequency by  $2^3=8$ , four flip-flops divide the clock frequency by  $2^4=16$  and so on.

**Example 72.17.** How many flip-flops are required to divide a frequency by thirty-two.

**Solution.** We know that a single flip-flop can divide the input frequency by 2. Two flip-flops

connected together can divide the frequency by 4. In general a frequency division of  $2^N$  can be achieved by using  $N$  number of flip-flops. Thus,

$$2^N = 32 \Rightarrow N = 5 \quad \text{Ans.}$$

**Example 72.18.** Fig. 72.74 shows three J-K flip-flops wired as frequency divider.

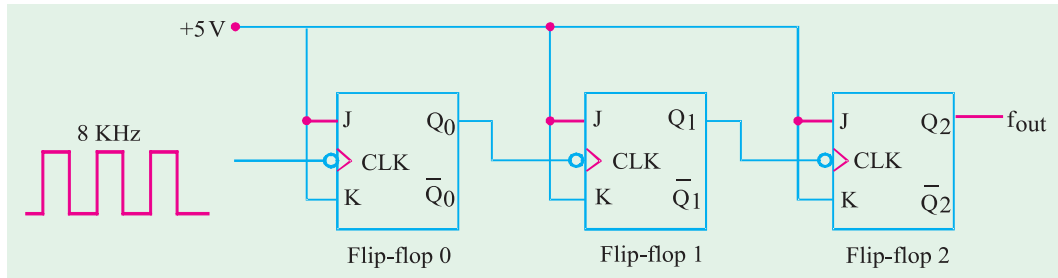


Fig. 72.75

Sketch the frequency waveform at  $Q_2$  output when an 8 kHz square wave input is applied at CLK input of the flip-flop 'D'.

**Solution.** Note that each flip-flop is connected to toggle (i.e.  $J = K = 1$ ). Since these are falling edge triggered flip-flops, the outputs change on the falling edge of clock pulse as shown in Fig. 72.76. There is one output pulse at  $Q_2$  for every eight input pulses, so the output frequency is  $f_{out} = 8 \text{ kHz} / 8 = 1 \text{ kHz}$ . Fig 72.76 shows the waveforms at  $Q_0$  and  $Q_1$  outputs also.

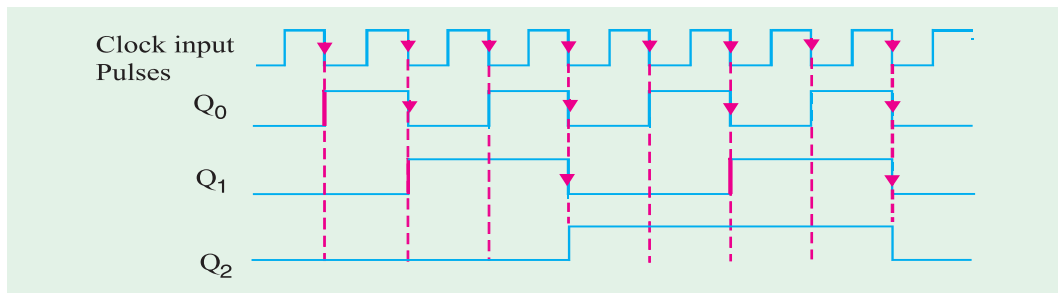


Fig. 72.76

### 72.31. Counting

Another important application of flip-flops is in digital counters. These are covered in more detail in chapter 7 on counters and Registers Fig 72.77 illustrates the concept of a 2-bit counter. Here two J-K flip-flops are wired to toggle (i.e.  $J = K = 1$ ). Both the flip-flops are initially RESET. Flip-flop 0 toggles on the falling edge of each clock pulse. The  $Q$ -output of flip-flop 0 clocks flip-flop 1, so that each time  $Q_0$  makes a HIGH-to-LOW transition, flip-flop 1 toggles. The resulting  $Q_0$  and  $Q_1$  waveforms are shown in the figure.

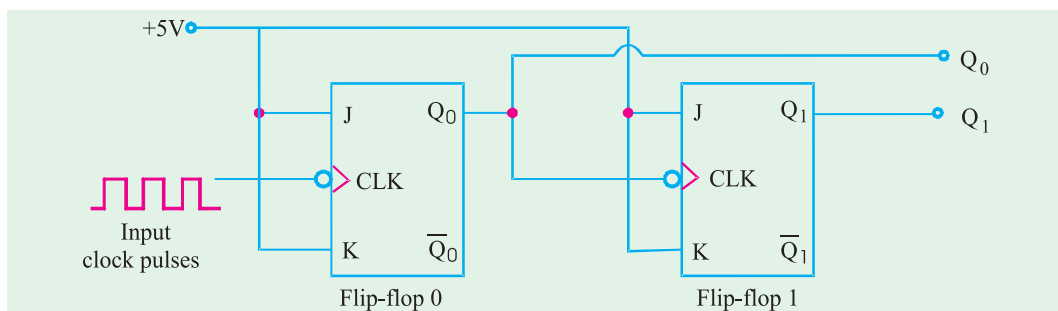


Fig. 72.77. J-K flip-flops wired as a 2-bit counter

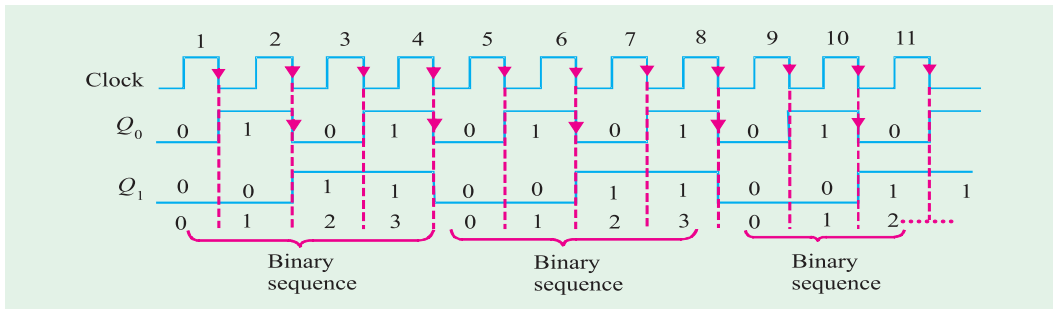


Fig. 72.78

Examine the sequence of  $Q_0$  and  $Q_1$  in Fig. 72.77. Prior to clock pulse 1,  $Q_0 = 0$  and  $Q_1 = 0$ , after clock pulse 1,  $Q_0 = 1$  and  $Q_1 = 0$ , after clock pulse 2,  $Q_0 = 0$  and  $Q_1 = 0$ , and after clock pulse 3,  $Q_0 = 1$  and  $Q_1 = 1$ . If we take  $Q_0$  as the least significant bit, a two bit sequence is produced as the flip-flops are clocked. This binary sequence repeats every four clock pulses as shown in the timing diagram of Fig. 72.78. Thus flip-flops are counting in sequence from 0 to 3 (i.e. 00, 01, 10 and 11) and then recycling back to 0 to begin the counting sequence again.

**Example 72.19.** Fig. 72.80 shows a J-K flip-flops wired as a 3-bit counter.

Sketch the output waveforms at  $Q_0$ ,  $Q_1$  and  $Q_2$ . Also show the binary sequence represented by these waveforms.

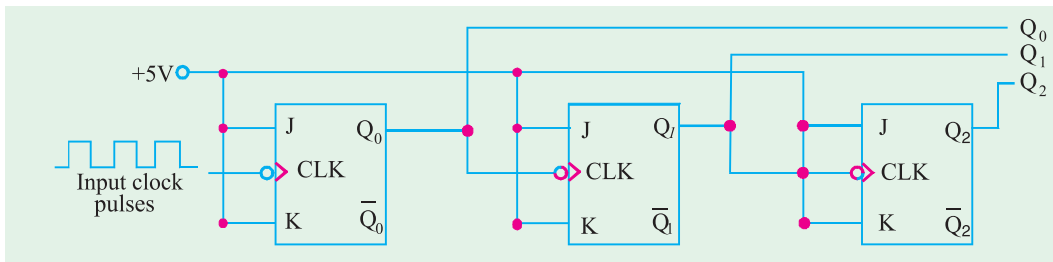


Fig. 72.79

**Solution.** Fig. 72.80 shows the output waveforms at  $Q_0$ ,  $Q_1$  and  $Q_2$ . Notice that the outputs change on the falling edge of the clock pulses. The outputs go through the binary sequence 000, 001, 010, 011, 100, 101, 110, 111, 000, 001, 010, 100 as indicated.

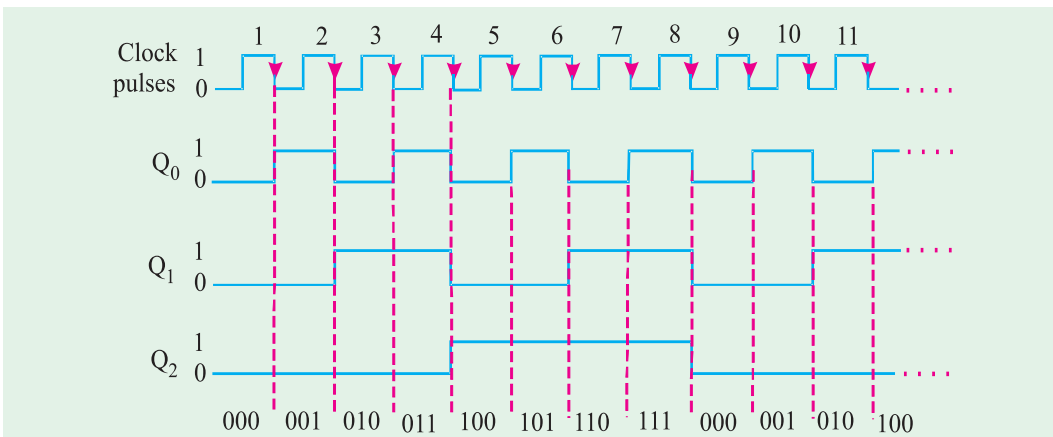


Fig. 72.80

### 72.32. Schmitt-Trigger Devices

Strictly speaking, a Schmitt-trigger circuit is not classified as a flip-flop but it does exhibit a type of memory characteristic that makes it useful in certain special situations. One of those situations is shown in Fig. 72.81 (a). Here a standard INVERTER is being driven by a logic input that has relatively slow transition times. When these transition times exceed the maximum allowed values, the outputs of logic gates and INVERTERS may produce oscillations as the input signal passes through the indeterminate range (refer to Fig. 72.81 (b)). The same input conditions can also produce erratic triggering of flip-flops.

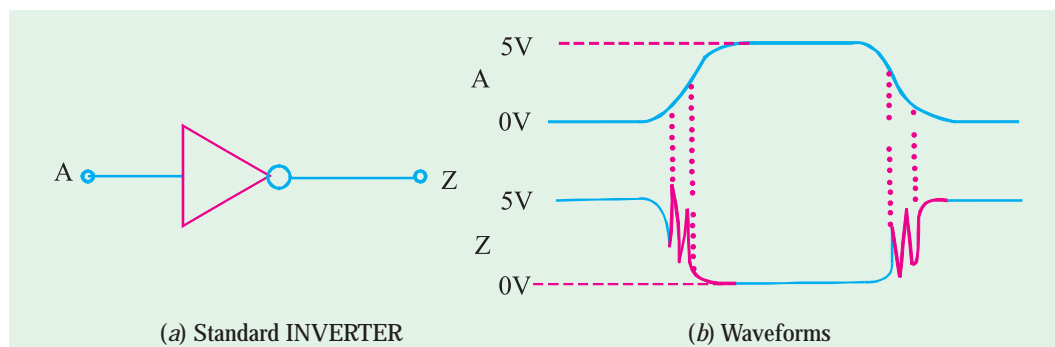


Fig. 72.81

A device that has a Schmitt-trigger type of input is designed to accept slow changing signals and produce an output that has oscillation free transitions. The output will generally have a very rapid transition times (typically 10ns) that are independent of the input signal characteristics. Fig. 72.82 (a) shows the symbol for a Schmitt-trigger INVERTER and (b) shows its response to a slow-changing input.

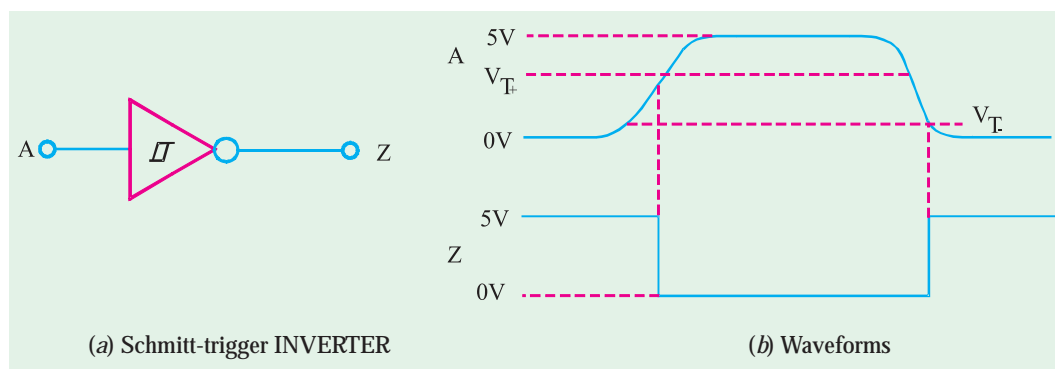


Fig. 72.82

As seen from Fig. 72.82 (b), we find that the output does not change from HIGH to LOW until the input exceeds the rising-edge threshold voltage,  $V_{T+}$ . Once the output goes LOW, it will remain there even when the input drops back below  $V_{T+}$  (this is its memory characteristic) until it drops all the way down below the falling edge threshold voltage,  $V_{T-}$ . The values of the two threshold voltages will vary from one logic family to another. But  $V_{T-}$  will always be less than  $V_{T+}$ .

It is evident from the above discussion that schmitt trigger is a special device that is used to transform slowly changing waveforms into sharply defined, jitter-free output signals. They are useful for changing clock edges that may have slow rise and fall times into straight vertical edges.

There are several ICs with Schmitt-trigger inputs. The 7414, 74LS14 and 74HC14 are hex INVERTER ICs with Schmitt-trigger inputs. On the other hand 7413, 74LS13 and 74HC13 are dual four-input NANDs with Schmitt-trigger inputs.

### 72.33. One-Shot (Monostable Multivibrator)

A one-shot (abbreviated as *OS*) is also referred to as monostable multivibrator. It is a digital circuit that is somewhat related to a flip flop. For instance, like a flip-flop, the one-shot has two inputs,  $Q$  and  $\bar{Q}$  which are inverse (or complement) of each other. However, unlike the flip-flop, the one-shot has only one stable output state (normally,  $Q = 0$  and  $\bar{Q} = 1$ ). The one-shot remains in this stable state until it is triggered by an input signal. Once triggered, the one-shot outputs switch to the opposite state ( $Q = 1, \bar{Q} = 0$ ). This state is called quasi-stable (i.e. unstable) state. The one-shot remains in this quasi-stable state for a fixed period of time,  $t_p$  which is determined by the RC time constant. The RC time constant in turn is determined by the values of external components connected to one-shot. After a time,  $t_p$  the one-shot outputs return to their resting state until triggered again.

Fig. 72.83 (a) shows the logic symbol for one-shot. As seen from this figure, it has one input labeled as trigger input ( $T$ ), a normal output ( $Q$ ), and an inverted output ( $\bar{Q}$ ). It needs two external components: resistor ( $R_T$ ) and a capacitor ( $C_T$ ) for its operation. The values of  $R_T$  and  $C_T$  determines the exact value of time period ( $t_p$ ) for which the one-shot switches to its quasi stable state. The value of  $t_p$  can vary from several nanoseconds to several tens of seconds. Fig. 72.83 (b) shows the stable state (where  $Q = 0, \bar{Q} = 1$ ) and the quasi-stable state (where  $Q = 1$  and  $\bar{Q} = 0$ ).

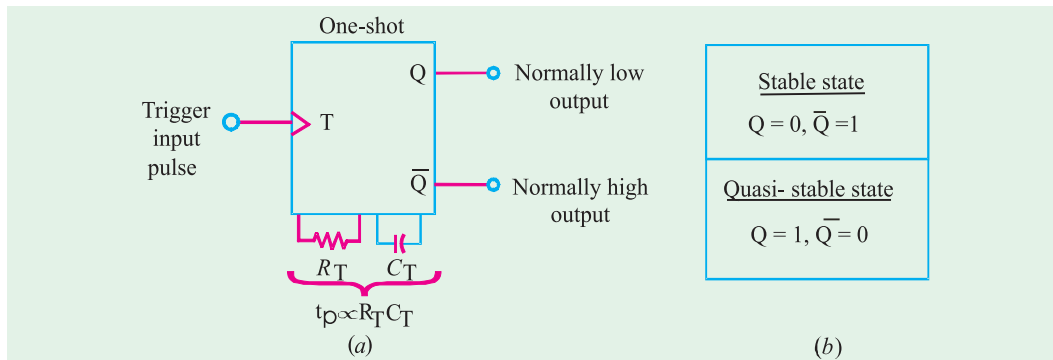


Fig. 72.83. One-shot symbol and Output states.

### 72.34. Types of One-Shots

Depending upon the operation, the one-shot is of the following two types :

1. Non-retriggerable one-shot and
2. Retriggerable one-shot

Both the type of one-shots are available in the IC form. Now we shall study both these one-shots one by one in the following pages.

### 72.35. Non-Retriggerable One-Shot

We have already discussed in the last article that one-shot (or a monostable multivibrator) is a device with only one stable state. It is normally in its stable state and will change to its quasi-stable state only when triggered. Once it is triggered, the one-shot remains in its quasi-stable state for a fixed



period of time and then automatically returns to its stable state. The time that the device remains in the quasi-stable state determines the pulse width ( $t_p$ ) of the output.

Fig. 72.84 (a) shows the waveforms of a trigger input signal  $T$  and output of one-shot. Notice that the rising edge of the trigger signal will trigger the one-shot to its quasi-stable state. The one-shot will remain in the quasi-stable state for a time,  $t_p$  after which it automatically returns to its stable state.

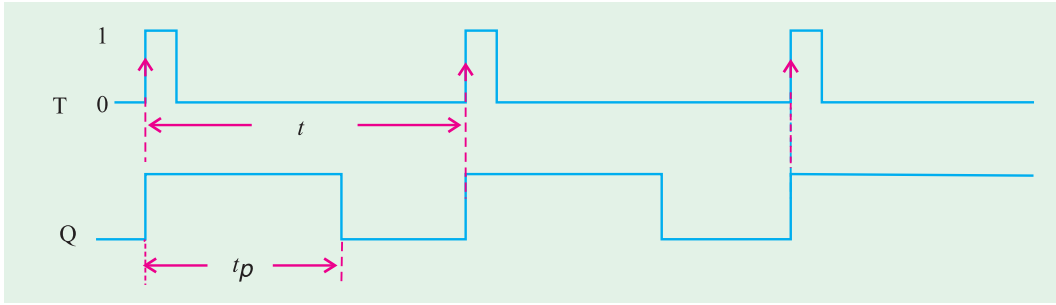


Fig. 72.84

It may be noted from the diagram shown above that one-shot is being triggered at intervals greater than its pulse width (*i.e.*  $t > t_p$ ).

Now we will consider a situation where the one-shot is being triggered at intervals less than its pulse width (*i.e.*  $t < t_p$ ). Refer to Fig. 72.85. Notice that the rising edges of the  $T$  signal at points 'b' has no effect on the one-shot because it has already been triggered to the quasi-stable state by the rising edge at point 'a'. The one-shot must return to the stable state before it can be triggered. The same argument is valid for the rising edge at point 'd'. It has no effect on the one-shot because it has already been triggered by the rising edge at point 'c'.

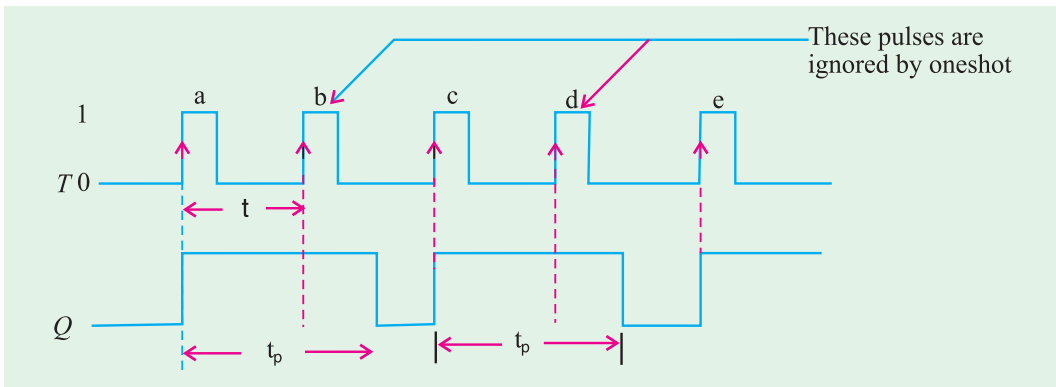


Fig. 72.85

It may be carefully noted that one-shot output pulse duration ( $t_p$ ) is always the same regardless of the input pulses. The pulse duration  $t_p$  depends only on the values of external components  $R_T$  and  $C_T$  and the internal one-shot circuitry. In a typical monoshot, the value of  $t_p$  is given by the relation.

$$t_p = 0.7 R_T C_T$$

**Example 72.20.** Fig. 72.84 shows three non-retriggerable one-shots connected in a timing chain that produces three sequential output pulses. Note the "1" in front of the pulse on each one-shot symbol to indicate non-retriggerable operation.

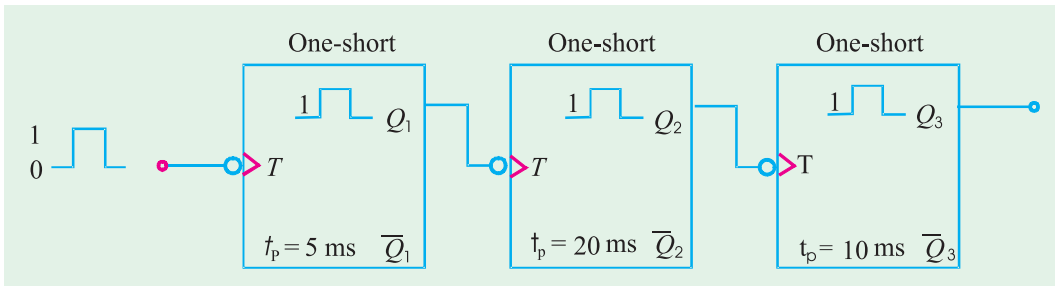


Fig. 72.86

Draw a timing diagram showing the relationship between the input pulse and the three one-shot outputs. Assume an input pulse duration of 10 ms.

**Solution.** Notice that all the one-shots trigger at the falling edge of the input signal. To begin with, the first one-shot triggers at the falling edge of the input signal and the second one-shot triggers at the falling edge of  $Q_1$ . The third one-shot triggers at the falling edge of  $Q_2$ .

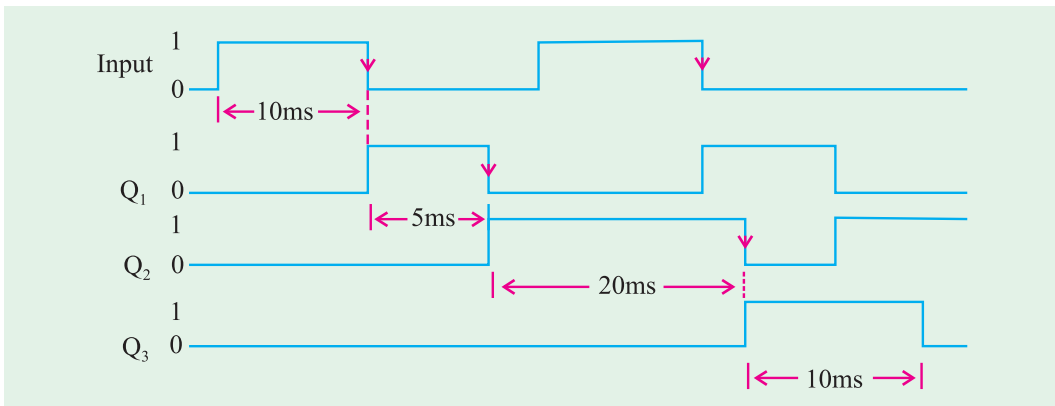


Fig. 72.87

Fig. 72.87 shows a sketch of the timing waveforms at  $Q_1$ ,  $Q_2$  and  $Q_3$  along with the input trigger waveform.

**Example 72.21.** Sketch the waveforms at  $Q$  and  $\bar{Q}$  outputs of a one-shot which is being triggered by a signal shown in Fig. 72.88. Assume that one-shot triggers at the rising edge of the trigger signal and has a  $t_p = 1.5$  ms.

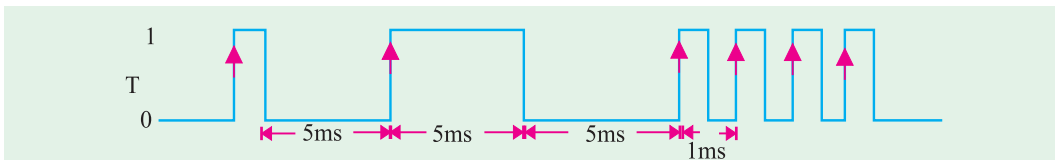


Fig. 72.88

**Solution.** Fig. 72.89 shows the waveforms at the one-shot outputs  $Q$  and  $\bar{Q}$  in response to the trigger signal ( $T$ ). Notice that the rising edges at points  $a$ ,  $b$ ,  $c$  and  $e$  of the  $T$  signal, will trigger one-shot to its quasi-stable state. The one-shot will remain in the quasi-stable state for a time,  $t_p$  after which it automatically returns to its stable state.

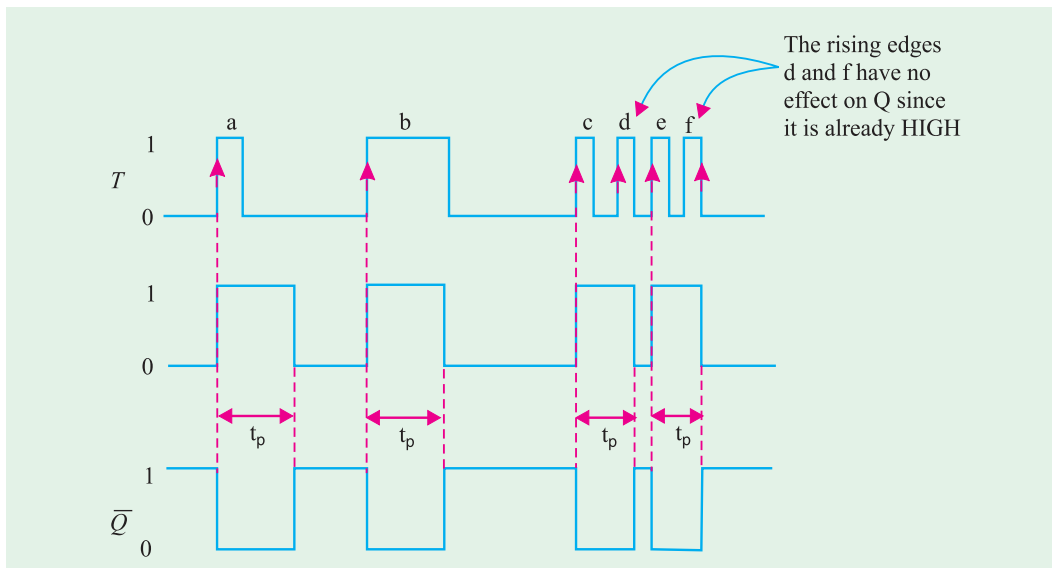


Fig. 72.89

The rising edges of the  $T$  signal at points 'd' and 'f' have no effect on the one-shot because it has already been triggered to the quasi-stable state. The one-shot must return to the stable state before it can be triggered.

### 72.36. Retriggerable One-Shot

This one-shot usually operates in the same manner as the non-retriggerable one-shot except for one major difference. The major difference is that retriggerable one-shot can be retriggered while it is in the quasi-stable state, and it will begin a new  $t_p$  interval. Fig. 72.90 compares the response of both types of one-shot using a  $t_p$  of 2 ms interval.

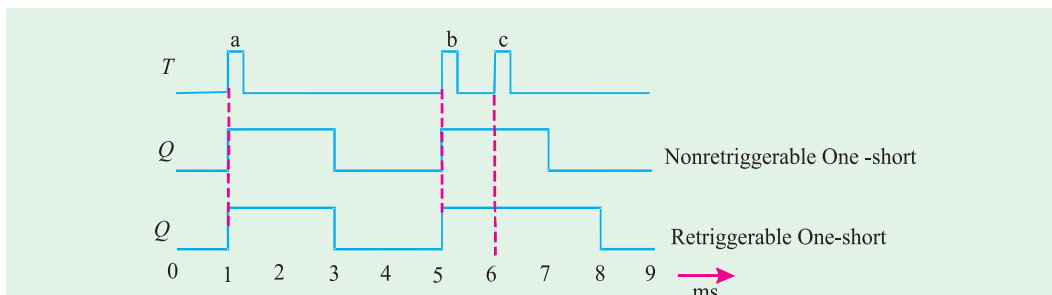


Fig. 72.90

As seen from the diagram, both types of one-shot respond to the first trigger pulse indicated as 'a' at  $t = 1$  ms by going HIGH for 2 ms and then returning LOW. The second trigger pulse indicated as 'b' at  $t = 5$  ms triggers both one-shots to the HIGH state. The third trigger pulse indicated as 'c', at  $t = 6$  ms has no effect on the non-retriggerable one-shot because it is already in the quasi-stable state. However, this trigger pulse will retrigger the retriggerable one-shot to begin a new  $t_p = 2$  ms interval. Thus it will stay HIGH for 2 ms after this third trigger pulse.

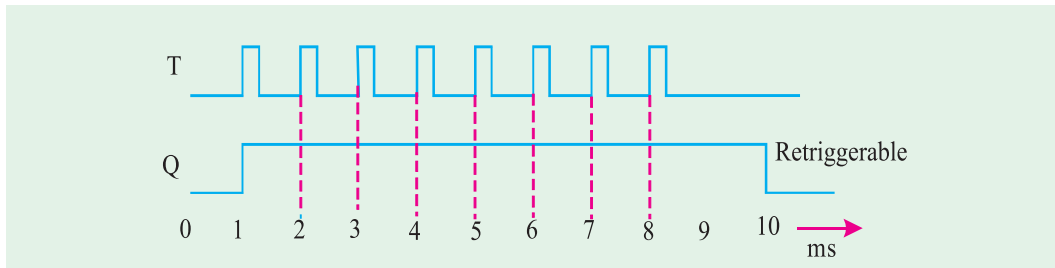


Fig. 72.91

It is evident from the discussion above that a retriggerable one-shot begins a new  $t_p$  interval each time a trigger pulse is applied, regardless of the current state of its  $Q$ -output. As a matter of fact, trigger pulses can be applied at a rate fast enough that one-shot will always be retriggered before the end of the  $t_p$  interval and  $Q$ -output will remain HIGH. Such a situation is shown in Fig. 72.91. As seen from this diagram, eight pulses are applied every 1 ms. The  $Q$ -output does not return LOW until 2 ms after the last trigger pulse.

**Example 72.22.** A retriggerable one-shot can be used as a pulse-frequency detector that detects when the frequency of a pulse input is below a predetermined value. A simple example of this application is shown in Fig. 72.92. The operation begins by momentarily closing switch SW1.

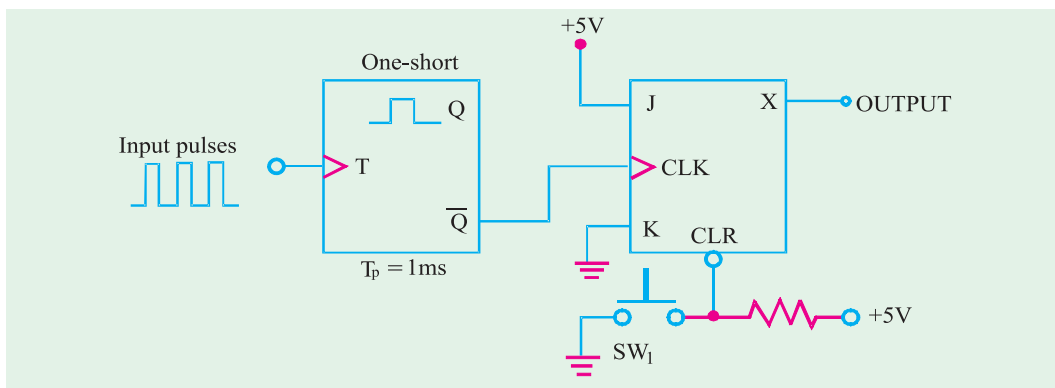


Fig. 72.92

(a) Describe how the circuit responds to input frequencies above 1 kHz. (b) Describe how the circuit responds to input frequencies below 1 kHz. (c) How would you modify the circuit to detect when the input frequency drops below 50 kHz?

**Solution.**

- (a) **Circuit response to input frequencies above 1 kHz.** Closing the switch SW1, clears  $X$  to Zero. Since the one-shot has  $t_p = 1$  ms, the one-shot will be retriggered before the end of the  $t_p$  interval for frequencies greater than 1 kHz. Thus  $\bar{Q}$  will stay LOW. As a result of this, there is no clock input to  $JK$  flip-flop and hence  $X$  will remain LOW.
- (b) **Circuit response to input frequencies below 1 kHz.** If the input frequency falls below 1 kHz, the  $\bar{Q}$  will return HIGH before the one-shot is triggered again. This positive-going-trigger will clock  $JK$  flip-flop and  $X$  will change to HIGH.
- (c) **Modification of the circuit to detect frequencies below 50 kHz.** In order to detect frequencies below 50 kHz, we will have to change  $t_p$  of the one-shot to  $1/50$  kHz = 20 ms.

## 72.37. Actual One-Shot Devices

There are several one-shot ICs that are available commercially in both the non-retriggerable and retriggerable versions. Table 72.3 shows some of the most popular one-shot ICs available commercially. As seen from the table 72.3 we find that 74121 and 74221 ICs are non-retriggerable one-shots. Whereas 74122 and 74123 ICs retriggerable one-shots.

Table 72.3. Actual one-shot devices

Device	Description
74121	A single non-retriggerable one-shot IC in standard TTL series.
74221	A dual non-retriggerable one-shot IC in standard TTL series.
74LS221	A dual non-retriggerable one-shot IC in Low-power Schottky TTL series
74HC221	A dual non-retriggerable one-shot IC in high-speed CMOS series
74122	A single retriggerable one-shot IC in standard TTL series
74LS122	A single retriggerable one-shot in IC low-power Schottky TTL series.
74123	A dual retriggerable one-shot IC in standard TTL series
74LS123	A dual retriggerable one-shot IC in lower-power Schottky TTL series
74HC123	A dual retriggerable one-shot IC in high-speed CMOS series.

Fig. 72.93 shows the logic symbol for the 74121 non-retriggerable one-shot IC. As seen from the diagram, the 74121 IC contains internal logic gates to allow inputs  $\bar{A}_1$ ,  $\bar{A}_2$ , and B to trigger the device in a variety of ways. The B input is a Schmitt-Trigger type of input that is allowed to have slow transition times and still reliably trigger the one-shot. The pins labelled  $R_{EXT}$ ,  $R_{EXT}/C_{EXT}$  are used to connect an external resistor and capacitor to achieve the desired output pulse duration.

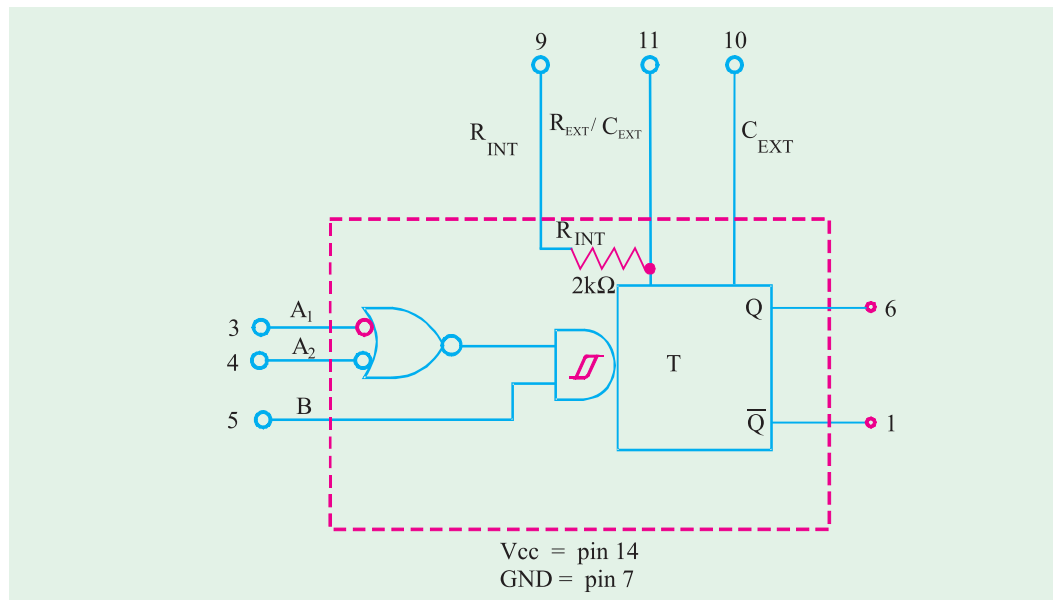


Fig. 72.93. Logic symbol of 74121.

A typical pulse width of 30 ns is produced when no external components are used and the internal timing resistor ( $2\text{ k}\Omega$ ) is connected to  $V_{CC}$ . The pulse width can be set anywhere between 30 ns and 28 ns by the use of external components, (i.e.  $R_{EXT}$  and  $C_{EXT}$ ). Mathematically the pulse width.

$$t_p = 0.7 R C_{EXT} \dots(i)$$

where  $R$  is either  $R_{INT}$  or  $R_{EXT}$ . Notice that if  $R$  is in kilohms ( $k\Omega$ ) and  $C_{EXT}$  is in picofarads (PF), the output pulse width ( $t_p$ ) is in nanoseconds. Further if  $R$  is in kilohms ( $k\Omega$ ) and  $C_{EXT}$  is in microfarads ( $\mu F$ ), the output pulse width ( $t_p$ ) is in microseconds ( $\mu S$ ).

Table 72.4 shows the truth table of 74121. Notice that for the first four rows of the table, the Q-output is LOW.

Table 72.4

Inputs			Output	
$A_1$	$\bar{A}_2$	B	Q	$\bar{Q}$
L	X	H	L	L
X	L	H	L	L
X	X	L	L	L
H	H	X	L	L
H	↓	H	⌋	⌋
↓	H	H	⌋	⌋
↓	↓	H	⌋	⌋
L	X	↑	⌋	⌋
X	L	↑	⌋	⌋

H = High voltage level  
 L = Low voltage level  
 X = Don't case  
 ↑ = Rising edge  
 ↓ = Falling edge

It is because of the fact that no trigger pulse is applied at any one of the three inputs. In the next five lines of the truth table, notice that we have a trigger pulse at one of the inputs ( $A_1$ ,  $A_2$  or B) and the other two inputs are connected LOW or HIGH.

Fig. 72.94 (a) shows the logic symbol and 72.94 (b) the truth table for 74123 a dual retriggerable one-shot. As seen from the diagram, the 74123 IC contains two retriggerable one-shots. Each retriggerable one-shot contains  $\bar{A}_1$ ,  $B_1$  and  $R_D$  to trigger the device in a variety of ways.

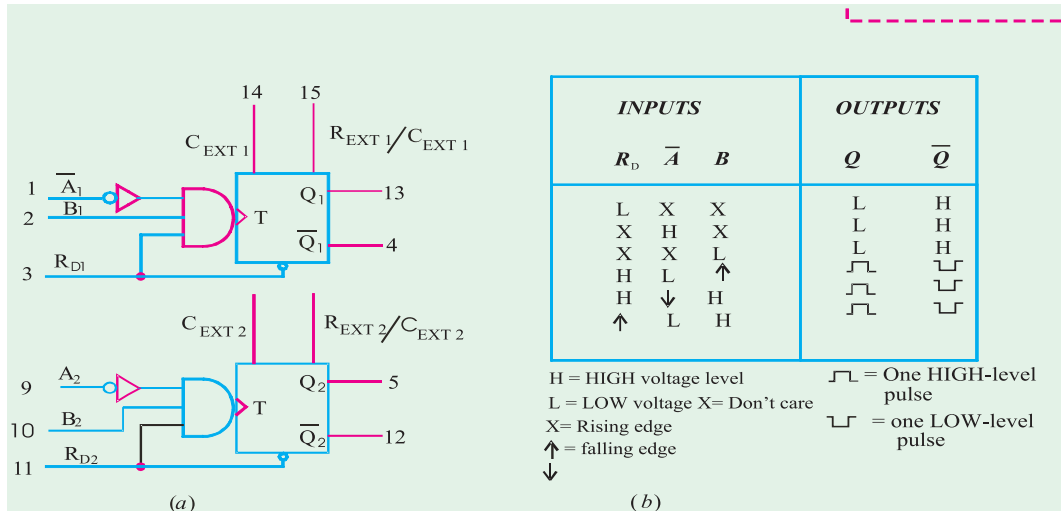


Fig. 72.94

A minimum pulse width of approximately 45 ns is obtained with no external components. Wider pulse widths are achieved by using external components. A general formula for calculating the values of these components for a specified pulse width ( $t_p$ ), provided  $C_{EXT} > 1000$  pF,

$$t_p = 0.28 R C_{EXT} \left( 1 + \frac{0.7}{R} \right) \dots (ii)$$

where 0.28 is a constant determined by a particular type of one-shot,  $R$  is either the internal or external resistor,  $C_{EXT}$  is in pF and  $t_p$  is in ns.

**Notes.**

1. While selecting the value of  $R_{EXT}$ , care should be taken that its value must be in kilohms ( $k\Omega$ ) so that the circuit current is in milli amperes (mA). If the resistor value is chosen in ohms ( $\Omega$ ), the circuit current will be in amperes which the IC will not be able to handle it. On the other hand, if the value of  $R_{EXT}$  is in megohms ( $M\Omega$ ), the current will be in micro amperes ( $\mu A$ ) which may be too small for internal operation of 74121. Moreover, resistances with values in the range of megohms are susceptible to electrostatic noise.
2. For a desired pulse width  $t_w$ , we can determine the values of external components in two ways.
  - (i) Arbitrarily select the value of capacitor,  $C_{EXT}$  and then using equation  $t_w = 0.7 R_{EXT} C_{EXT}$  determine the value of  $R_{EXT}$  be in microfarads ( $\mu F$ ).
  - (ii) Arbitrarily select the value of  $R_{EXT}$  and then using the equation,  $t_w = 0.7 R_{EXT} C_{EXT}$ , determine the value of  $C_{EXT}$ .
  - (iii) If  $t_w$  is required to be in nanoseconds, choose  $R_{EXT}$  in kilohms ( $k\Omega$ ) so that  $C_{EXT}$  is in picofarads. However if the desired  $t_w$  is required to be in milliseconds, still select  $R_{EXT}$  is in kilohms ( $k\Omega$ ), but  $C_{EXT}$  will now be in microfarads ( $\mu F$ ). Care should be taken while selecting the component values in the sense that the components are available only in certain standard values.
3. While selecting the value of  $C_{EXT}$  care should be taken that it is much larger than any stray capacitance that might be encountered in a typical electronic circuit. Values of capacitance less than 100 pF (0.0001  $\mu F$ ) may be unsuitable because it is not uncommon for there to be 50 pF of stray capacitance between traces in a printed-circuit board.

**Example 72.23.** A certain digital circuit requires a one-shot with a pulse width of 10ms. Determine the values of external components to be connected with an IC 74121 to produce the desired pulse width. Also show the connections of the external components to 74121.

**Solution.**

Let  $R_{EXT}$  = The value of external resistor and

$C_{EXT}$  = The value of external capacitor.

Then we know that the pulse width for a 74121 IC ( $t_p$ ),

$$10 \times 10^{-3} = 0.7 R_{EXT} C_{EXT} \quad \dots (i)$$

Since the desired pulse width is in milli seconds, let us arbitrarily select  $C_{EXT} = 1 \mu F$ . Then from equations (i).

$$10 \times 10^{-3} = 0.7 \times R_{EXT} \times (1 \times 10^{-6})$$

$$\therefore R_{EXT} = \frac{10 \times 10^{-6}}{0.7 (1 \times 10^{-6})} = 14.28 \times 10^{-3} \Omega = 14.28 k\Omega$$

In order to select a resistor value of 14.28  $k\Omega$ , we can use a 10  $k\Omega$  fixed resistor with a 5  $k\Omega$  potentiometer.

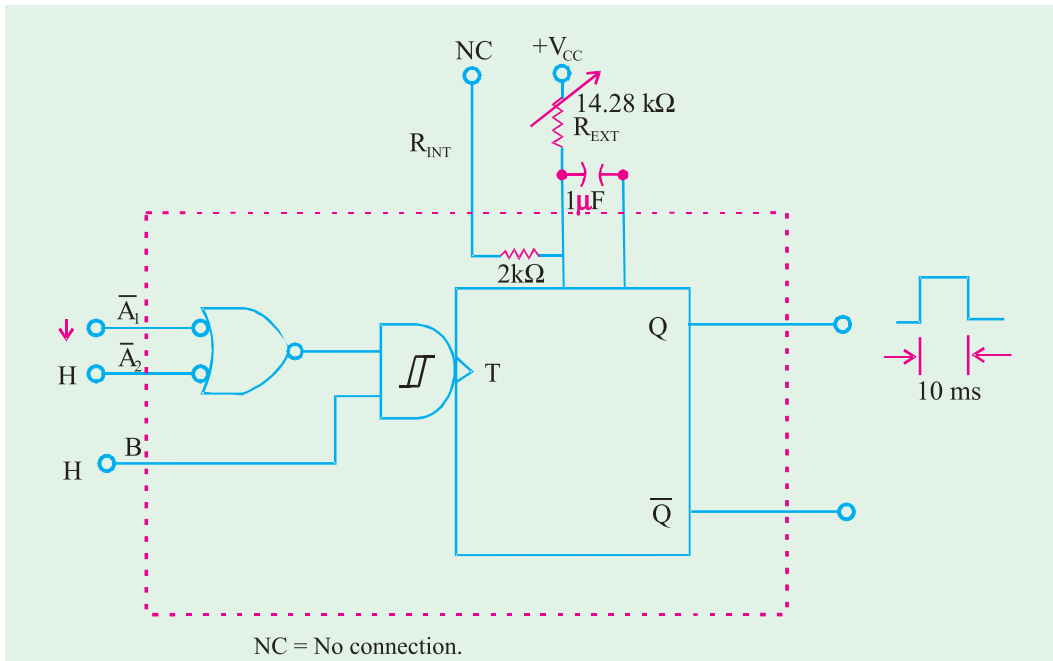


Fig. 72.95

Fig. 72.95 shows a 14.28 kΩ resistor with 1 μF capacitor connected to the 74121 IC for producing a desired pulse width of 10 ms. Notice the settings on  $\bar{A}_1$ ,  $\bar{A}_2$  and  $B$  inputs of 74121 as well.

**Example 72.24.** Design a circuit using a 74121 to convert a 50 kHz, 80% duty cycle square wave to a 50 kHz, 50% duty cycle square wave.

**Solution.**

Fig. 72.96 shows the given square wave. Notice that it has a time period of 1/50 kHz (= 20 μs) and has a pulse width of 80% of 20 μs (= 16 μs).

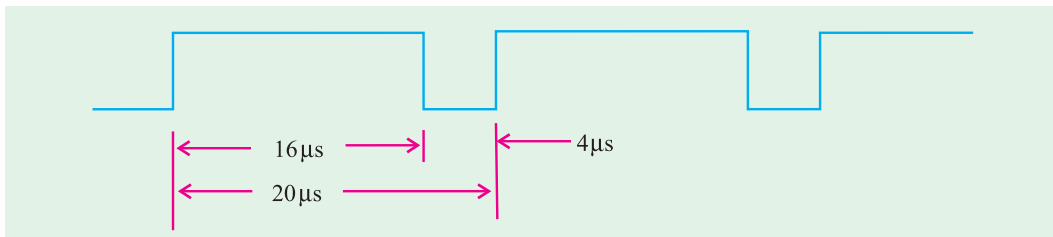


Fig. 72.96

Now we have to stretch the 4 μs negative pulse to 10 μs to make the duty cycle 50%. If we use the falling edge on the negative pulse to trigger  $\bar{A}_1$  input to a 74121 and set the output pulse width ( $t_p$ ) to 10 μs, we should have the solution as shown in 72.97 (b). The output will be taken from  $\bar{Q}$  because it provides a negative pulse when triggered.

Let  $R_{EXT}$  = The value of external resistor and  
 $C_{EXT}$  = The value of external capacitor,

Then the pulse width ( $t_p$ )

$$10 \times 10^{-6} = 0.7 R_{EXT} C_{EXT}$$

Let us arbitrarily select  $C_{EXT} = 0.001 \mu F$  (equal to 1000pf); then

$$10 \mu s = 0.7 R_{EXT}$$



or

$$R_{EXT} = \frac{10 \times 10^{-6}}{0.7 \times 0.001 \times 10^{-6}}$$

$$= 14.4 \text{ k}\Omega$$

In order to select a resistor of 14.4 kΩ , we can choose a fixed resistor of 10 k Ω with a 5 k Ω potentiometer.

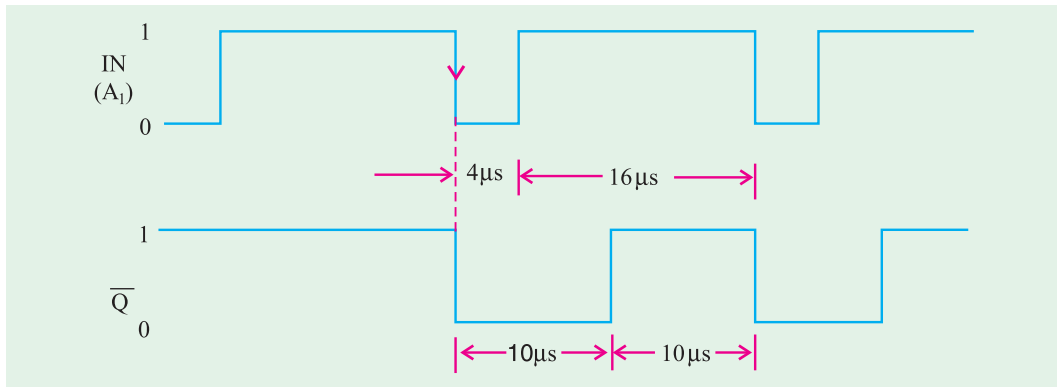


Fig. 72.97

Fig. 72.98 shows the circuit connections for producing the desired waveform. Notice that the input waveform is applied at  $\bar{A}_1$  input while  $\bar{A}_2$  and B are tied HIGH. The output is obtained from  $\bar{Q}$ .

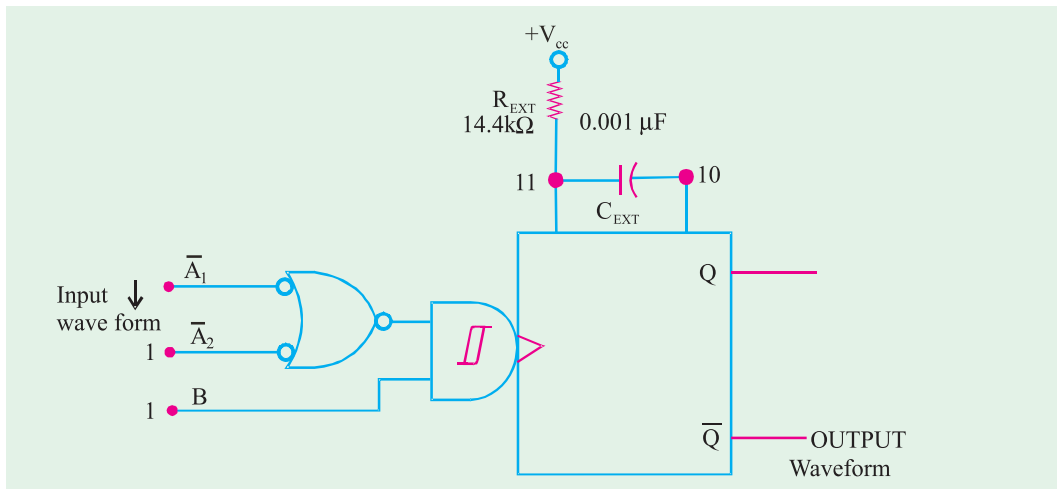


Fig. 72.98

**Example 72.25.** Determine the value of  $R_{EXT}$  and  $C_{EXT}$  that will produce a pulse width of 50 μS when connected to a 74123.

**Solution.**

Let  $R_{EXT}$  = The value of external resistor and

$C_{EXT}$  = The value of external capacitor

Then we know that the pulse width for a 74123 IC ( $t_p$ ) ,

$$50 \times 10^{-6} = 0.28 R_{EXT} C_{EXT} \left( 1 + \frac{0.7}{R_{EXT}} \right) \quad \dots (i)$$

Let us arbitrarily select  $C_{EXT} = 0.01 \mu\text{F}$  (recall that  $C_{EXT}$  has to be greater than 1000 pF), then, from equation (i),

$$50 \times 10^{-6} = 0.28 R_{EXT} (0.01 \times 10^{-6}) \left( 1 + \frac{0.7}{R_{EXT}} \right)$$

$$= 0.28 (R_{EXT} + 0.7) (0.01 \times 10^{-6})$$

$$\therefore R_{EXT} + 0.7 = \frac{50 \times 10^{-6}}{0.28 \times (0.01 \times 10^{-6})} = 17857 \Omega$$

or  $R_{EXT} = 17857 - 0.7 = 17856.3 \Omega = 17.9 \text{ k} \Omega$

Thus we can use a fixed resistor of 15 k and 5 k potentiometer to select  $R_{EXT} = 17.9 \text{ k} \Omega$ .

### 72.38. Clock Generator Circuits

We have already discussed in Art. 72.2 that flip-flop has two stable states due to which they are known as bistable multivibrators. Further in Art. 72.33 we have discussed that one-shot has one stable state due to which they are referred to as monostable multivibrators. Now we shall discuss a third type of multivibrator which has no stable states. Such a multivibrator is called an astable or free-running multivibrator. An astable multivibrator switches back and forth (*i.e.* oscillates) between two unstable states without any external triggering.

There are several types of astable multivibrators that are in common use but the following three are important from the subject point of view;

1. Schmitt-Trigger oscillator
2. 555 Timer used as astable oscillator.
3. Crystal-controlled clock generator.

Now we shall discuss all the three oscillators one by one in the following pages.

### 72.39. Schmitt-Trigger Oscillator

Fig. 72.99 shows a Schmitt-trigger INVERTER connected as an oscillator. The signal at  $V_{OUT}$  is an approximate square wave with a frequency that depends upon the values of  $R$  and  $C$  values. The relationship between the frequency and  $RC$  values is shown in Fig. 72.100 for three different Schmitt-trigger INVERTERS.

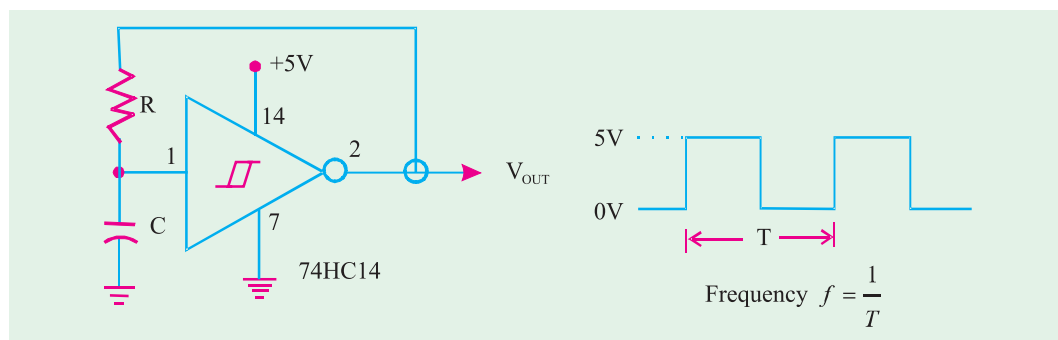


Fig. 72.99

As seen from this diagram, for 7414 Schmitt-trigger INVERTER, the frequency is given by,  $f = 0.8/RC$ . Notice that for this IC, the value of  $R \leq 500 \Omega$ . For 74LS14 IC, the frequency of the output square wave is again  $0.8/RC$  but the value of  $R \leq 2 \text{ k}\Omega$ . On the other hand for 74HC14, the frequency of the output square wave is  $1.2/RC$ , where the value of  $R$  must be  $10 \text{ M}\Omega$ .

IC	Frequency	Remarks
7414	$\approx 0.8/RC$	( $R \leq 500 \Omega$ )
74LS14	$\approx 0.8/RC$	( $R \leq 2 \text{ k}\Omega$ )
74HC14	$\approx 1.2/RC$	( $R \leq 10 \text{ M}\Omega$ )

Table 72.5

### 72.40. 555 Timer Used as an Astable Multivibrator

The 555 IC is a very popular, general purpose timer IC. It can be connected as a one-shot or as an astable multivibrator or as a free running oscillator. Fig. 72.100 shows how external components can be connected to a 555 so that it operates as a free-running oscillator. The output of the oscillator is a repetitive rectangular waveform as shown in Fig. 72.100 (b).

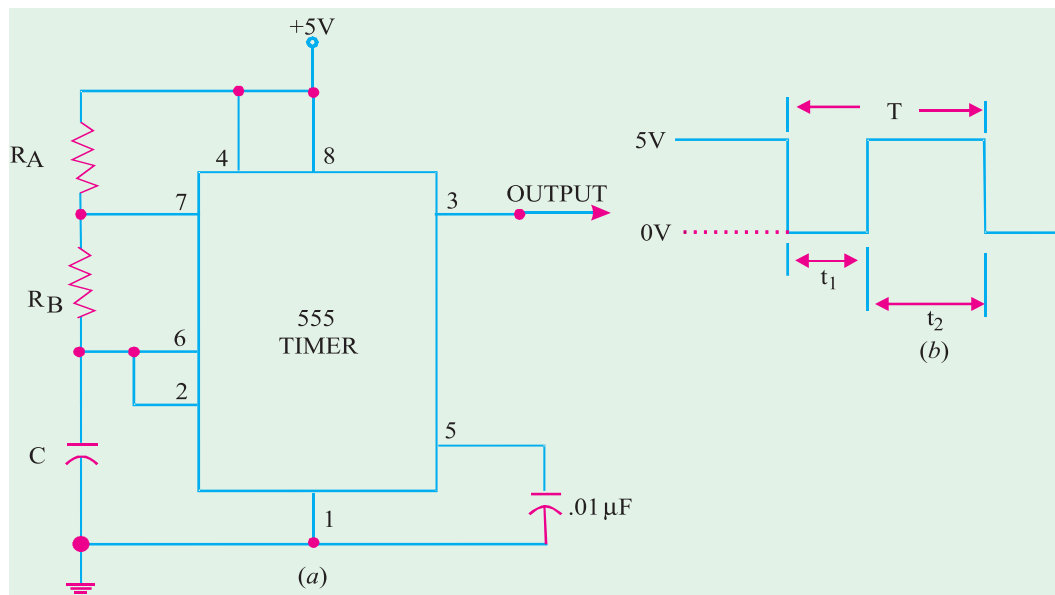


Fig. 72.100. 555 timer used as an astable multivibrator.

As seen from the diagram, the output switches between two logic levels with the time intervals ( $t_1$  and  $t_2$ ) at each level. These two time intervals are determined by the  $R$  and  $C$  values.

The time interval  $t_1$  is given by the equation,

$$t_1 = 0.7 R_B C \quad \dots (i)$$

and the time interval,

$$t_2 = 0.7 (R_A + R_B) C \quad \dots (ii)$$

$\therefore$  The time period, of the output waveform

$$T = t_1 + t_2$$

Substituting the values of  $t_1$  and  $t_2$  from equations (i) and (ii), the time period,

$$\text{or } T = 0.7 R_B C + 0.7 (R_A + R_B) C \\ = 0.7 (R_A + 2 R_B) C$$

The frequency of the output waveform,

$$f = \frac{1}{T} = \frac{1}{0.7 (R_A + 2 R_B) C}$$

$$= \frac{1.44}{(R_A + 2 R_B) C}$$

Further, the duty cycle of the output waveform,

$$\text{duty cycle} = \frac{t_2}{T} \times 100 \%$$

It may be noted from equations (i) & (ii) that  $t_1$  and  $t_2$  cannot be equal unless  $R_A$  is made zero. This cannot be done without producing excess current through the device. This means, it is impossible to produce an output waveform with a perfect 50% duty cycle. However, it is possible to get very close to 50% duty cycle (i.e.  $t_1 \approx t_2$ ) by selecting,

$$R_B \gg R_A$$

It may be carefully noted that the value of  $R_A$  must be greater than  $1\text{ k}\Omega$  and the value of  $C$  must be greater than  $500\text{ pF}$ .

**Example 72.26.** Fig. 72.101 shows the circuit of a 555 timer used as an astable multivibrator.

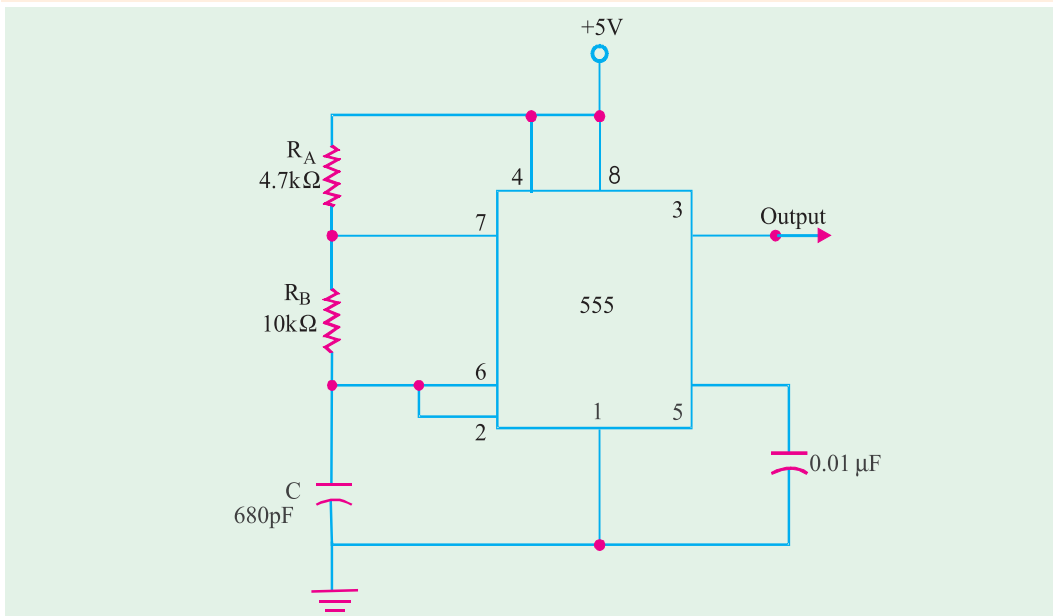


Fig. 72.101

Determine the values  $t_1$ ,  $t_2$ , frequency and duty cycle of the output waveform.

**Solution.**

$$t_1 = 0.7 R_B C = 0.7 \times (10 \times 10^3) \times (680 \times 10^{-12})$$

$$= 4.76 \times 10^{-6} \text{ s} = 4.76 \mu\text{s}$$

$$t_2 = 0.7 (R_A + R_B) C$$

$$= 0.7 \times ((4.7 \times 10^3) + (10 \times 10^3)) \times (680 \times 10^{-12})$$

$$= 6.997 \times 10^{-6} \text{ s} \approx 6.997 \mu\text{s}$$

$$T = t_1 + t_2 = 4.76 + 6.997 = 11.757 \mu\text{s}$$

$$f = \frac{1}{T} = \frac{1}{11.757 \times 10^{-6}} = 8.5 \times 10^4 = 85 \text{ kHz}$$

$$\text{Duty cycle} = \frac{t_2}{T} \times 100\% = \frac{6.997}{11.757} \times 100\% = 59.5 \%$$

Tutorial Problems No. 72.1

- The waveforms of Fig 72.100 are applied to the inputs of a NAND gate latch shown in Fig.72.102. Assume that initially  $Q = 0$ , and determine the waveform at  $Q$ -output.

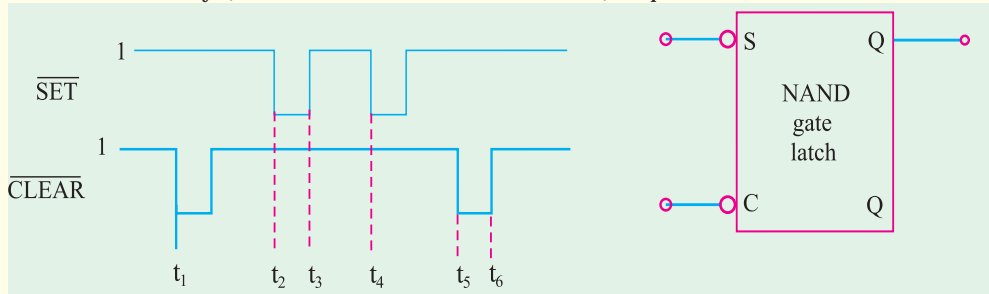


Fig. 72.102

- The waveforms of Fig. 72.103 (a) are applied to the inputs of a NOR gate latch shown in Fig. 72.103 (b). Assume that initially  $Q = 0$  and determine the waveform at  $Q$  output.

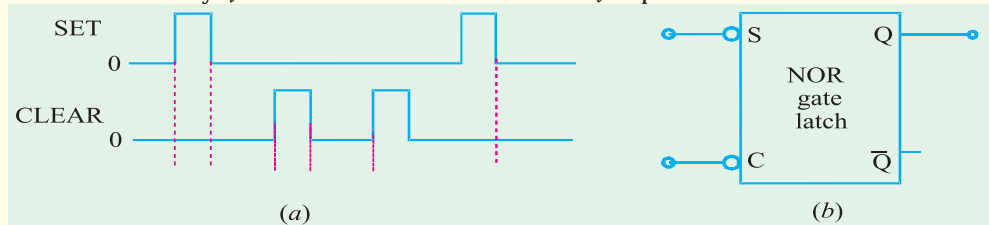


Fig. 72.103

- Apply the  $J$ - $K$  and CLK waveforms to a flip-flop shown in Fig. 72.104. Assume that  $Q = 0$  initially. Sketch the output waveform.

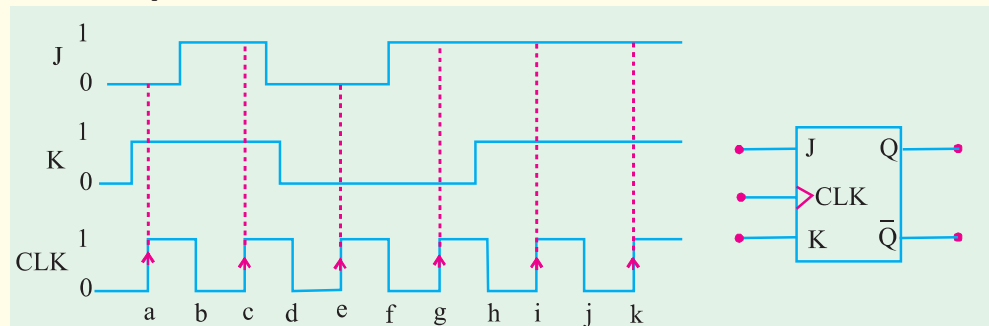


Fig. 72.104

- Determine the output waveform for a positive-edge triggered  $D$  flip-flop for the  $J$ ,  $K$  and CLK waveforms as shown in Fig. 72.105.

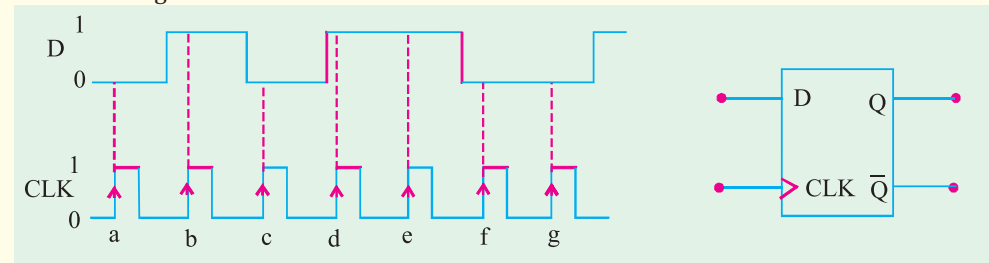


Fig. 72.105

5. Determine the  $Q$  waveform for a  $D$  latch with ENABLE (EN) and DATA ( $D$ ) input waveforms shown in Fig. 72.106. Assume  $Q = 0$  initially.

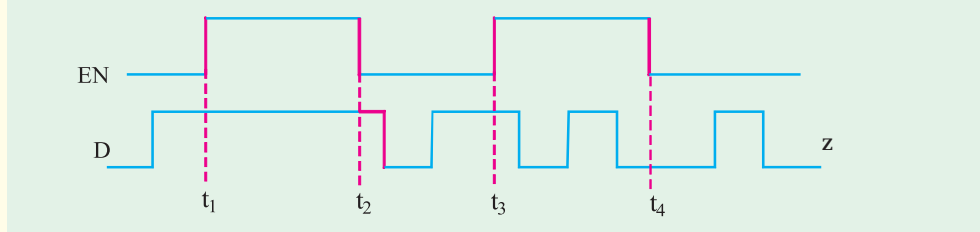


Fig. 72.106

6. Fig. 72.107 (a) shows the symbol for a  $J-K$  flip-flop that responds to a NGT (*i.e.* negative going trigger) on its clock input and has asynchronous inputs. Sketch the output waveform in response to the  $\overline{\text{CLK}}$ ,  $\overline{\text{PRE}}$  and  $\overline{\text{CLR}}$  waveforms as shown in Fig. 72.107 (b).

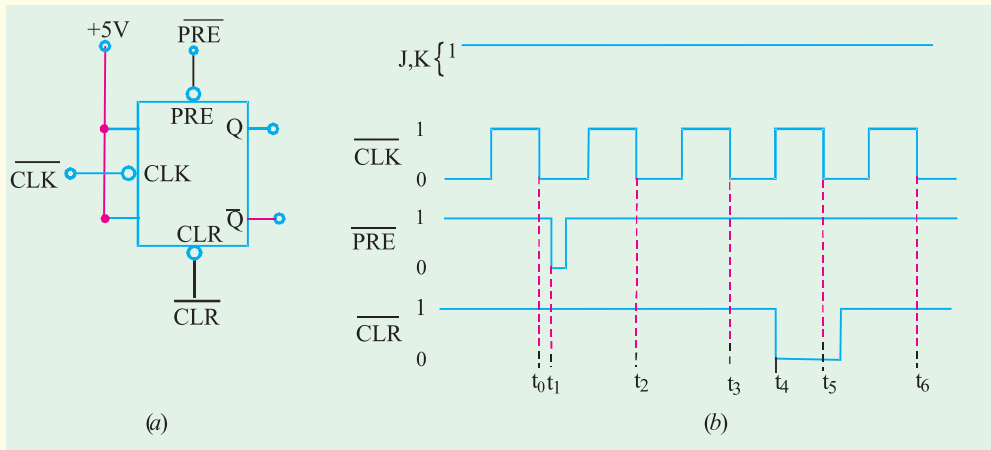


Fig. 72.107

7. Determine the  $Q$  output for a negative edge triggered  $J-K$  flip-flop for the input waveforms shown in Fig. 72.108. Assume  $t_H = 0$  and that  $Q = 0$  initially.

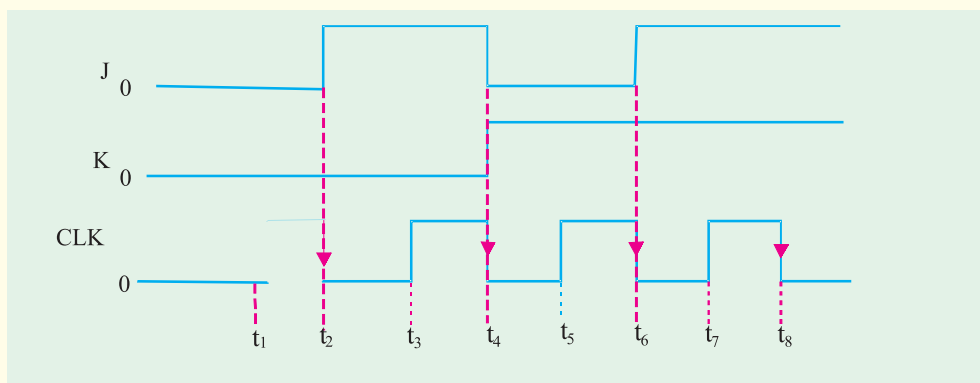


Fig. 72.108

8. A 20-kHz clock signal is applied to a  $J-K$  flip-flop with  $J = K = 1$ . Sketch the output waveform and determine its frequency. (Ans 10 kHz)

9. In a J-K flip-flop,  $J = K = 1$ . A 1 MHz square wave is applied at its clock input. It has a propagation delay of 50 ns. Draw the input square wave and the output waveform expected at  $Q$ . Show the propagation delay time.
10. A D flip-flop has following datasheet information :  
Set up-time = 5 ns; Hold time = 10 ns; Propagation time = 15 ns  
How far ahead of the triggering clock edge must the data be applied.
11. Draw the block diagram of a 4-bit shift register using D flip-flops. If initially all the flip-flop outputs are in zero state, prepare the truth table when the input sequence is 1, 1, 0, 1, 0. Draw the above shift register using J-K flip-flops only.
12. Consider a 4-bit shift register using J-K flip-flops. Assume that initially,  $Q_0 = 1$ ,  $Q_1 = 1$ ,  $Q_2 = 1$  and  $Q_3 = 1$ . Sketch the output of each flip-flop if an input sequence 101101 is applied to  $D_0$  synchronously with the clock ( $Q_0$  is LSB).
13. Calculate the frequency and the duty cycle of the 555 astable multivibrator output for  $C = 0.001 \mu\text{F}$ ,  $R_A = 2.2 \text{ k}\Omega$  and  $R_B = 100 \text{ k}\Omega$ .  
(Ans 50.5%)

### OBJECTIVE TESTS – 72

1. If a NAND latch has a 1 on the SET input and a 0 on the CLEAR input, then the SET input goes to 0, the latch will be:  
(a) HIGH (b) LOW  
(c) Invalid (d) None of these
2. The invalid state of a NAND latch occurs when  
(a)  $S = 1$ ,  $C = 0$   
(b)  $S = 0$ ,  $C = 1$   
(c)  $S = 1$ ,  $C = 1$   
(d)  $S = 0$ ,  $C = 0$
3. The invalid state of a NOR latch occurs when  
(a)  $S = 1$ ,  $C = 0$   
(b)  $S = 0$ ,  $C = 1$   
(c)  $S = 1$ ,  $C = 1$   
(d)  $S = 0$ ,  $C = 0$
4. Like a latch, the flip-flop belongs to a category of logic circuits known as  
(a) monostable multivibrators  
(b) astable multivibrators  
(c) bistable multivibrators  
(d) none of these
5. Which of the following flip-flops is used as a latch?  
(a) J-K flip-flop  
(b) S-C flip-flop  
(c) D flip-flop  
(d) T flip-flop
6. A flip-flop which can have an uncertain output state is :  
(a) J-K flip-flop  
(b) S-C flip-flop  
(c) D flip-flop  
(d) T flip-flop
7. The purpose of a clock input to a flip-flop is to  
(a) clear the device  
(b) set the device  
(c) always cause the output to change the states  
(d) cause the output to assume a state dependent on the controlling (S - C, J-K or D) inputs.
8. A feature that distinguishes the J-K flip-flop from an S-C flip-flop is the  
(a) toggle condition  
(b) preset input  
(c) type of clock  
(d) clear input.
9. A J-K flip-flop is in the toggle condition when  
(a)  $J = 1$ ,  $K = 0$   
(b)  $J = 1$ ,  $K = 1$   
(c)  $J = 0$ ,  $K = 0$   
(d)  $J = 0$ ,  $K = 1$
10. A J-K flip-flop with  $J = 1$  and  $K = 1$  has a 10 kHz clock input. The  $Q$  = output is,  
(a) constantly LOW

- (b) constantly HIGH
  - (c) a 5 kHz square wave
  - (d) a 10 kHz square wave
11. For an edge-triggered *D* flip-flop,
- (a) a change in the state of the flip-flop can occur only at a clock pulse edge
  - (b) the state that flip-flop goes to depends on the *D* input
  - (c) the output follows the input at each clock pulse
  - (d) all of these answers
12. The flip-flop shown in Fig. 72.109 logically behaves as
- (a) a *D* flip-flop
  - (b) a *J-K* flip-flop
  - (c) a *T* flip-flop
  - (d) an *R-S* flip-flop

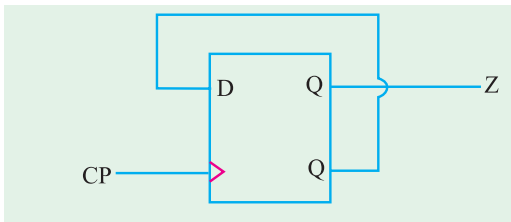


Fig. 72.109

(Grad. I.E.T.E. June, 1997)

13. A *J-K* flip-flop is a device to
- (a) divide the frequency by 2
  - (b) divide the frequency by 4
  - (c) generate waveform of same frequency as that of the input
  - (d) cannot be used for frequency division.
14. A 1  $\mu$ s pulse can be converted into a 1 ms pulse by using
- (a) a monostable multivibrator
  - (b) an astable multivibrator
  - (c) a bistable multivibrator
  - (d) a *J-K* flip-flop
- (U.P.S.C. Engg. Services, 1999)
15. The following is not a sequential circuit
- (a) *J-K* flip-flop
  - (b) counter
  - (c) full-adder
  - (d) shift register
- (Dip. I.E.T.E., Dec. 1996)
16. A one = shot is a type of
- (a) monostable multivibrator

- (b) astable multivibrator
  - (c) bistable multivibrator
  - (d) timer
17. An astable multivibrator
- (a) requires a periodic trigger input
  - (b) has one stable states
  - (c) is an oscillator
  - (d) produces a non-periodic pulse output.
18. A 1 msec pulse can be converted to a 10 msec pulse by using:
- (a) an astable multivibrator
  - (b) a monostable multivibrator
  - (c) a bistable multivibrator
  - (d) a *J-K* flip-flop
- (U.P.S.C. Engg. Services 1990)
19. A retriggerable one shot is one which
- (a) can be triggered only once
  - (b) has two quassi-stable states
  - (c) cannot be triggered until full pulse has been outputted.
  - (d) is capable of being triggered while the output is being generated.
20. The output of the circuit shown in Fig. 72.110 will be
- (a) delayed pulses
  - (b) squarewaves
  - (c) triangular waves
  - (d) trapezoidal waves.

(U.P.S.C Engg. Services 1997)

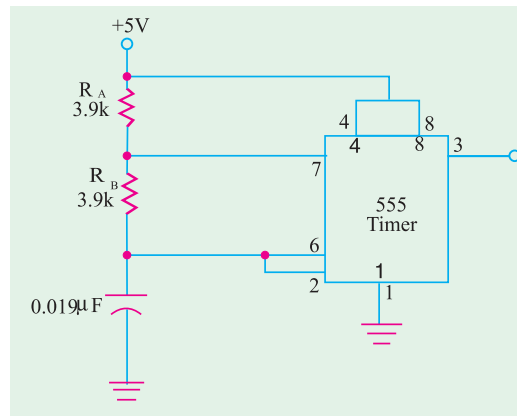


Fig. 72.110



21. The input waveform  $V_i$  and the output waveform  $V_o$  of a Schmitt NAND are shown in Fig 72.111. The duty cycle of the output waveform is

- (a) 100 %
- (b) 85.5 %
- (c) 72.2 %
- (d) 25 %

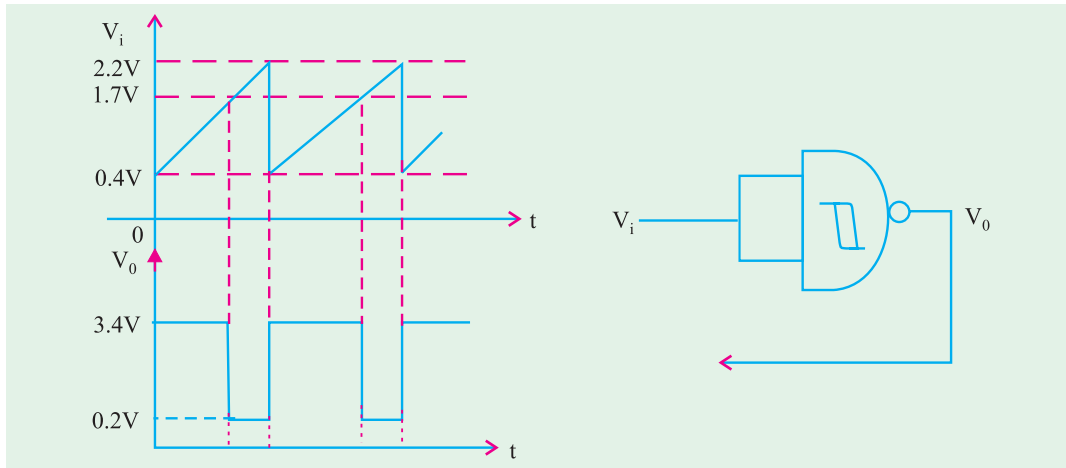


Fig. 72.111

(U.P.S.C Engg. Services. 1999)

22. Which of the following circuit is used for production of delays?

- (a) astable multivibrator
- (b) bistable multivibrator
- (c) monostable multivibrator
- (d) schmitt-trigger

23. A Schmitt-trigger can be used as a

- (a) comparator only
- (b) square-wave generator only

(c) flip-flop only

(d) comparator, square wave generator or flip-flop.

24. A 555 IC timer can be used to operate as

- (a) as monostable multivibrator
- (b) an astable multivibrator
- (c) a voltage controlled oscillator
- (d) all of the above

### ANSWERS

- |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (d)  | 3. (c)  | 4. (c)  | 5. (b)  | 6. (b)  |
| 7. (d)  | 8. (a)  | 9. (b)  | 10. (c) | 11. (d) | 12. (c) |
| 13. (a) | 14. (a) | 15. (c) | 16. (a) | 17. (c) | 18. (b) |
| 19. (d) | 20. (b) | 21. (c) | 22. (c) | 23. (a) | 24. (d) |

# CHAPTER 73

## Learning Objectives

- Introduction
- Analog and Digital Instruments
- Functions of Instruments
- Electronic versus Electrical Instruments
- Essentials of an Electronic Instrument
- The Basic Meter Movement
- Characteristics of Moving Coil Meter Movement
- Variation of Basic Meter Movement
- Converting Basic Meter to DC Ammeter
- Multirange Meter
- Measurement of Current
- Loading Effect of a Voltmeter
- Ohmmeter
- The Multimeter
- Rectifier Type AC Meter
- Electronic Voltmeters
- The Digital Voltmeter (DVM)
- Cathode Ray Tube (CRT)
- Normal Operation of a CRO
- Dual Trace CRO—Dual Beam CRO
- Lissajous Figures
- Applications of a CRO
- The Q Meter
- Logic Analysers
- Signal Generators
- Audio Generators
- Pulse Generators
- RF Generators
- Frequency Synthesizer
- IEEE-488 General Purpose Interface Bus (GPIB) Instruments

## ELECTRONIC INSTRUMENTS



The Digital Voltmeter

### 73.1. Introduction

Electronic instrumentation is such an interesting field that it combines elements of technologies ranging from the nineteenth to the twenty first centuries. Modern computer-based instrumentation is now evident in every reasonably equipped laboratory and workshop and in catalogs and advertisements of all of the manufacturers. Yet at the root of many space-age instruments is circuitry, such as the wheatstone bridge that is found in nineteenth-century textbooks. Although newer techniques are still in widespread use in new as well as old instruments. In this chapter on electronic instruments you will find both types discussed.

The scientific and technological progress of any nation depends on its ability to measure, calculate and finally, estimate the unknown. Also, the success of an engineer or technician is judged by his ability to measure precisely and to correctly interpret the circuit performance. There are three ways of making such measurements :

- (a) **by mechanical means**—like measuring gas pressure by Bourdon pressure gauge.
- (b) **by electrical means**—like measuring potential difference with an electrical voltmeter.
- (c) **by electronic means**—which is a very sensitive way of detecting the measured quantity because of amplification provided by the active electron device.

The electronic instruments generally have higher sensitivity, faster response and greater flexibility than mechanical or electrical instruments in indicating, recording and, where required, in controlling the measured quantity.

### 73.2. Analog and Digital Instruments

The deflection type instruments with a scale and movable pointer are called *analog* instruments. The deflection of the pointer is a function of (and, hence, analogous to) the value of the electrical quantity being measured.

Digital instruments are those which use logic circuits and techniques to obtain a measurement and then display it in numerical-reading (digital) form. The digital readouts employ either *LED* displays or liquid crystal displays (*LCD*).

Some of the advantages of digital instruments over analog instruments are as under :

1. easy readability
2. greater accuracy
3. better resolution
4. automatic polarity and zeroing

### 73.3. Functions of Instruments

Functionally, different instruments may be divided into the following three categories :

#### 1. Indicating instruments

These are the instruments which indicate the instantaneous value of quantity being measured, at the time it is being measured. The indication is in the form of pointer deflection (analog instruments) or digital readout (digital instruments). Ammeters and voltmeters are examples of such instruments.

#### 2. Recording instruments

Such instruments provide a graphic record of the variations in the quantity being measured over a selected period of time. Many of these instruments are electromechanical devices which use paper charts and mechanical writing instruments such as an inked pen or stylus.

Electronic recording instruments are of two types :

- (a) **null type**—which operate on a comparison basis.
- (b) **galvanometer type**—which operate on deflection type.

### 3. Controlling instruments

These are widely used in industrial processes. Their function is to control the quantity being measured with the help of information feed back to them by monitoring devices. This class forms the basis of automatic control systems (automation) which are extensively employed in science and industry.

#### 73.4. Electronic Versus Electrical Instruments

Both electrical and electronic instruments measure electrical quantities like voltage and current etc. Purely electrical instruments do not have any built-in amplifying device to increase the amplitude of the quantity being measured. The common dc voltmeter based on moving-coil meter movement is clearly an electrical instrument.

The electronic instruments always include in their make-up some active electron device such as vacuum tube, semiconductor diode or an integrated circuit etc.

As seen, the main distinguishing factor between the two types of instruments is the presence of an electron device in the electronic instruments. Of course, movement of electrons is common to both types, their main difference being that control of electron movement is more effective in electronic instruments than in electrical instruments.

Although electronic instruments are usually more expensive than their electrical counterparts, they offer following advantages for measurements purposes :

1. since electronic instruments can amplify the input signal, they possess very high sensitivity *i.e.* they are capable of measuring extremely small (low-amplitude) signals,
2. because of high sensitivity, their input impedance is increased which means less loading effect when making measurements,
3. they have greater speed *i.e.* faster response and flexibility,
4. they can monitor remote signals.

#### 73.5. Essentials of an Electronic Instrument

As shown Fig. 73.1, an electronic instrument is made up of the following three elements :

##### 1. Transducer

It is the first sensing element and is required only when measuring a non-electrical quantity, say, temperature or pressure. Its function is to convert the non-electrical physical quantity into an electrical signal.

Of course, a transducer is not required if the quantity being measured is already in the electrical form.

##### 2. Signal Modifier

It is the second element and its function is to make the incoming signal suitable for application to the indicating device.

For example, the signal may need amplification before it can be properly displayed. Other types of signal modifiers are : voltage dividers for reducing the amount of signal applied to the indicating device or wave shaping circuits such as filters, rectifiers or chopper etc.

##### 3. Indicating Device

For general purpose instruments like voltmeters, ammeters or ohm meters, the indicating device is usually a deflection type meter as shown in Fig. 73.1. In digital readout instruments, the indicating device is of digital design.

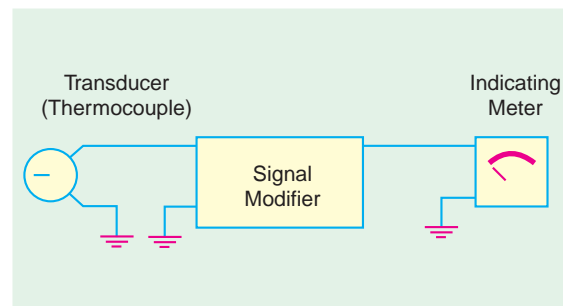


Fig. 73.1

### 73.6. Measurement Standards

All instruments, whether electrical or electronic, are calibrated at the time of manufacture against a measurement standard.

#### 1. International Standards

These are defined by international agreement and are maintained at the international Bureau of Weights and Measurements in Paris.

#### 2. Primary Standards

These are maintained at national standards laboratories in each country. They are not available for use outside these laboratories. Their principal function is to calibrate and verify the secondary standards used in industry.

#### 3. Secondary Standards

These are the basic reference standards used by industrial laboratories and are maintained by the particular industry to which they belong. They are periodically sent to national laboratory for calibration and verification against primary standards.

#### 4. Working Standards

These are the main tools of a measurement laboratory and are used to check and calibrate the instrument used in the laboratory.

### 73.7. The Basic Meter Movement

It is also called D' Arsonval meter movement or a permanent-magnet moving-coil (PMMC) meter movement. Since it is widely used in electronic instruments, it is worthwhile to discuss its construction and principle of operation.

#### 1. Construction

As shown in Fig. 73.2, it consists of a permanent horse-shoe magnet with soft iron pole pieces attached to it. Between the two pole-pieces is situated a cylinder-shaped soft iron core around which moves a coil of fine wire wound on a light metal frame. The metal frame is mounted in jewel bearings

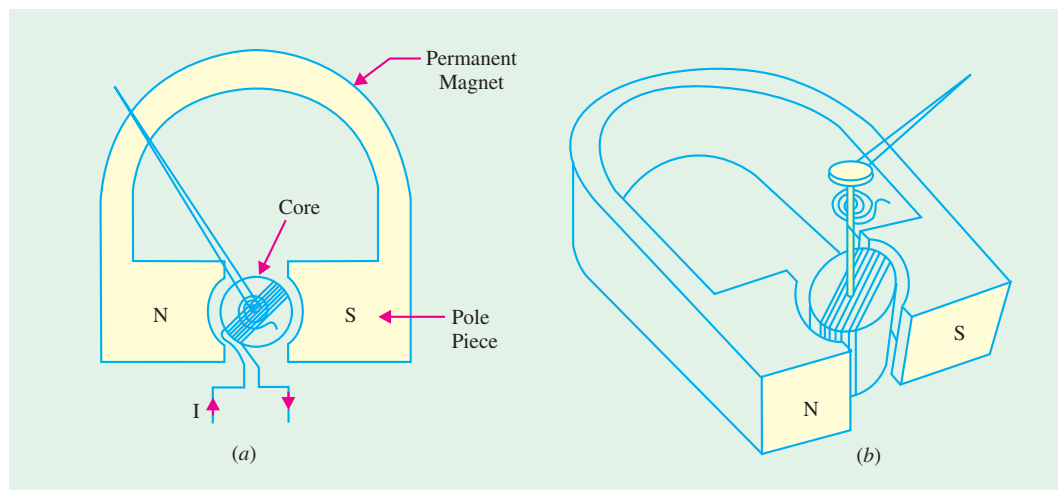


Fig. 73.2

so that it can rotate freely. A light pointer attached to the moving coil moves up-scale as the coil rotates when current is passed through it. The rotating coil is prevented from continuous rotation by a spring which provides restoring torque.

The moving coil movement described above is being increasingly replaced by tautband move-

ment in which the moving coil and the pointer are suspended between bands of spring metal so that the restoring force is torsional. The bands perform two functions (i) they support the coil and (ii) they provide restoring torque thereby eliminating the pivots and jewels used with coil spring movement.

As compared to pivoted movement, the taut-band has the advantages of

1. greater sensitivity *i.e.* small full-scale deflection current
2. ruggedness,
3. minimal friction,
4. easy to manufacture.

## 2. Principle of Operation

This meter movement works on the *motor* principle and is a current-responding device. The deflection of the pointer is directly proportional to the amount of current passing through the coil.

When direct current flows through the coil, the magnetic field so produced reacts with the field of the permanent magnet. The resultant force turns the coil alongwith its pointer. The amount of deflection is directly proportional to the amount of current in the coil. Hence, their scale is linear. With correct polarity, the pointer reads up-scale to the right whereas incorrect polarity forces the pointer off-scale to the left.

## 73.8. Characteristics of Moving Coil Meter Movement

We will discuss the following three characteristics :

- (i) full-scale deflection current ( $I_m$ ),
- (ii) internal resistance of the coil ( $R_m$ ),
- (iii) sensitivity ( $S$ ).

### 1. Full-scale Deflection Current ( $I_m$ )

It is the current needed to deflect the pointer all the way to the right to the last mark on the calibrated scale. Typical values of  $I_m$  for D' Arsonval movement vary from 2  $\mu\text{A}$  to 30 mA.

It should be noted that for smaller currents, the number of turns in the moving coil has to be *more* so that the magnetic field produced by the coil is strong enough to react with the field of the permanent magnet for producing reasonable deflection of the pointer. Fine wire has to be used for reducing the weight of the moving coil but it increases its resistance. Heavy currents need thick wire but lesser number of turns so that resistance of the moving coil is comparatively less. The schematic symbol is shown in Fig. 73.3.

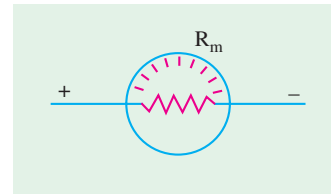


Fig. 73.3

### 2. Internal Resistance ( $R_m$ )

It is the dc ohmic resistance of the wire of the moving coil. A movement with smaller  $I_m$  has higher  $R_m$  and *vice versa*. Typical values of  $R_m$  range from 1.2  $\Omega$  for a 30 mA movement to 2 k $\Omega$  for a 50  $\mu\text{A}$  movement.

### 3. Sensitivity ( $S$ )

It is also known as *current sensitivity or sensitivity factor*. It is given by the reciprocal of full-scale deflection current  $I_m$ .

$$\therefore S = \frac{1}{I_m} \text{ ohm/volt.}$$

For example, the sensitivity of a 50- $\mu\text{A}$  meter movement is

$$S = \frac{1}{50 \mu\text{A}} = \frac{1}{50 \times 10^{-6}} \Omega/\text{V} = 20,000 \Omega/\text{V} = 20 \text{ k}\Omega/\text{V}$$

The above figure shows that a full-scale deflection of  $50\ \mu\text{A}$  is produced whenever  $20,000\ \Omega$  of resistance is present in the meter circuit *for each volt of applied voltage*. It also represents the ohms-per-volt rating of the meter. The sensitivity of a meter movement depends on the strength of the permanent magnet and number of turns in the coil. Larger the number of turns, smaller the amount of current required to produce full-scale deflection and, hence, higher the sensitivity. A high current sensitivity means a high quality meter movement. It also determines the lowest range that can be covered when the meter movement is modified as an ammeter (Art 73.10) or voltmeter (Art 73.12)

### 73.9. Variations of Basic Meter Movement

The basic moving-coil system discussed in Art 73.7 can be converted into an instrument to measure dc as well as ac quantities like current, voltage and resistance etc. Without any modification, it can carry a maximum current of  $I_m$  can withstand a maximum dc voltage  $v = I_m R_m$ .

#### 1. DC instruments

- (a) it can be made into a dc ammeter, milliammeter or micrommeter by adding a suitable shunt resistor  $R_{sh}$  in parallel with it as shown in Fig. 73.4 (a),
- (b) it can be changed into a dc voltmeter by connecting a multiplier resistor  $R_{mult}$  in series with it as shown in Fig. 73.4 (b),

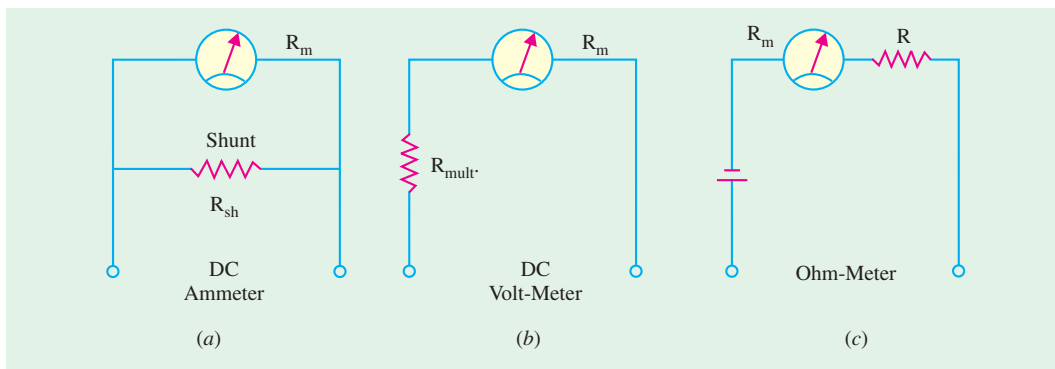


Fig. 73.4

- (c) it can be converted into an ohmmeter with the help of a battery and series resistor  $R$  as shown in Fig. 73.4 (c).

#### 2. AC Instruments

- (a) it can be changed into an ac audio-frequency ammeter or voltmeter by simply adding an extra rectifier as shown in Fig. 73.5 (a).

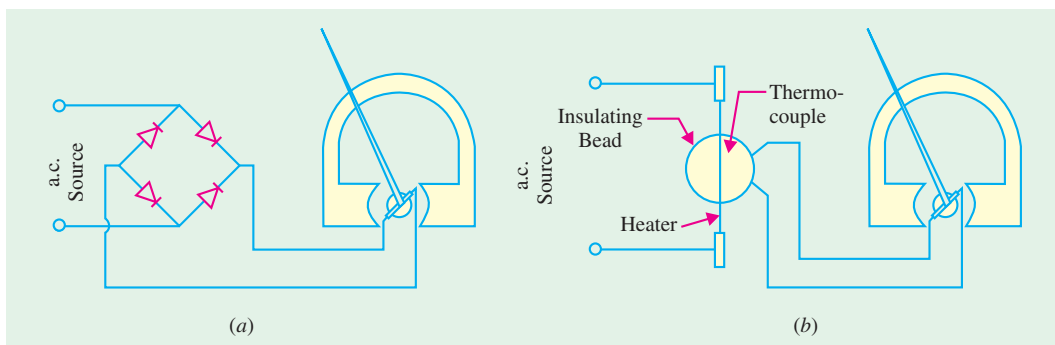
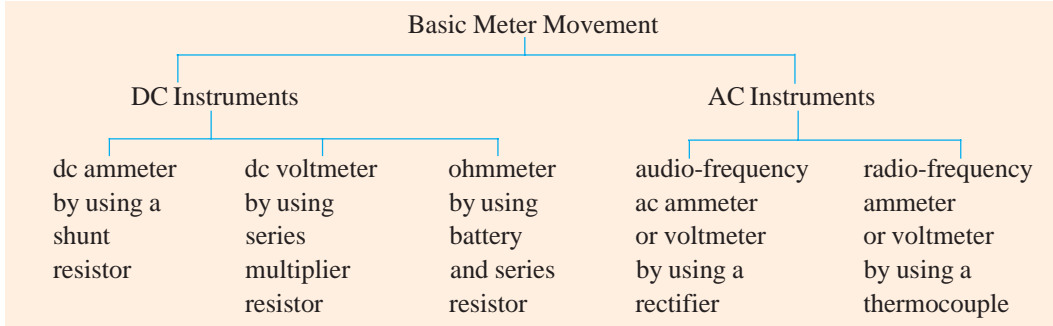


Fig. 73.5

- (b) it can be converted into a radio frequency ammeter or voltmeter by adding a thermocouple as shown in Fig. 73.5 (b).

The above modifications of the basic meter movement have been tabulated below :



### 73.10. Converting Basic Meter to DC Ammeter

As stated earlier and again shown in Fig. 73.6 (a), the basic meter movement can carry a maximum current of  $I_m$  i.e. its full-scale deflection current. However, its current range can be increased (i.e. multiplied) to any value by connecting a low resistance (called shunt resistance  $R_{sh}$ ) in parallel with it as shown in Fig. 73.6 (b). The shunted meter works as an ammeter with an extended range.

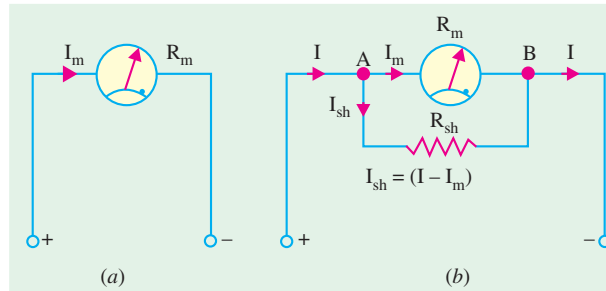


Fig. 73.6

Suppose, we want to measure a line current of  $I$  with the help of this meter movement. Obviously, the value of  $R_{sh}$  should be such as to shunt or bypass a current of  $(I - I_m)$ . As seen, range extension is from  $I_m$  to  $I$ . The ratio  $I/I_m = n$  is known as the **multiplying power or multiplying factor** of the shunt. It means that a shunt allows the meter to measure current  $I$  which is  $n$  times larger than  $I_m$ .

#### Value of $R_{sh}$

In Fig. 73.6 (b), voltage across the meter and the shunt is the same because they are joined in parallel.

$$\begin{aligned} \therefore I_m R_m &= I_{sh} \cdot R_{sh} = (I - I_m) R_{sh} \\ \therefore R_{sh} &= \frac{I_m}{(I - I_m)} \cdot R_m = \frac{1}{(I/I_m - 1)} \cdot R_m \quad \therefore R_{sh} = \frac{R_m}{(n - 1)} \end{aligned}$$

Hence,  $n$  is the multiplying factor of the shunt. It is seen that larger the value of  $n$  i.e. greater the range extension required, smaller the shunt resistance needed. Incidentally, it may be noted that the resistance of the **shunted** meter is

$$= R_m \parallel R_{sh} = \frac{R_m R_{sh}}{R_m + R_{sh}}$$

It is much less than either  $R_m$  or  $R_{sh}$

**Example 73.1.** It is required to convert a 5-mA meter with 20  $\Omega$  internal resistance into a 5-A ammeter. Calculate

- (a) the value of shunt resistance required
- (b) multiplying factor of the shunt.



**Solution.** Here,  $I = 5\text{ A}$ ,  $I_{sh} = 5\text{ mA} = 0.005\text{ A}$ ,  $R_m = 20\ \Omega$

$$(a) \quad R_{sh} = \frac{I_m}{(I - I_{sh})} \cdot R_m = \frac{0.005}{(5 - 0.005)} \times 20 = \mathbf{0.02\ \Omega \text{ (approx)}}$$

$$(b) \quad n = \frac{I}{I_{sh}} = \frac{5}{0.005} = \mathbf{1000}$$

**Note.**  $R_{sh} = \frac{R_m}{(n - 1)} = \frac{20}{(1000 - 1)} = \frac{20}{999} = 0.02\ \Omega$  —as found above

Fig. 73.7 shows such an ammeter connected in a load circuit.

### 73.11. Multirange Meter

The shunt resistance discussed above gives only a single range ammeter. By using universal shunt (also called Ayrton shunt), we can obtain a multirange ammeter as shown in Fig. 73.8.

It is seen that by changing the switch position from *A* to *B* to *C* and finally to *D*, the current range can be extended as desired.

#### 1. Switch at A

Here, the meter is unshunted and so can read up to its full-scale deflection current of 1 mA only.

#### 2. Switch at B

In this case,  $R_1$  shunts the meter and extends its range to 10 mA *i.e.* increases it ten times.

Since  $n = 10 \quad \therefore R_1 = \frac{R_m}{(n - 1)} \cong \frac{100\ \Omega}{(10 - 1)} = 11.11\ \Omega$

#### 3. Switch at C

Here,  $R_2$  shunts  $R_m$  and extends meter range from 1 mA to 0.1 A *i.e.* to 100 mA. Obviously,

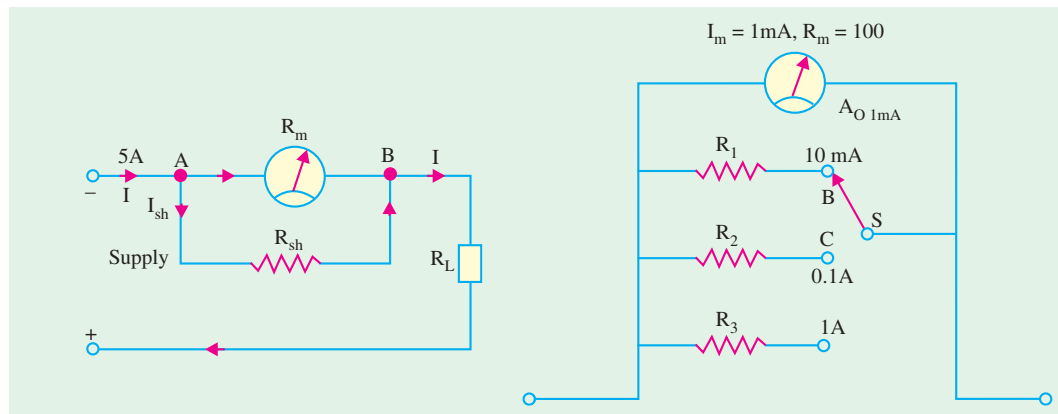


Fig. 73.7

Fig. 73.8

$$n = \frac{100}{1} = 100 \quad \therefore R_2 = \frac{100\ \Omega}{(100 - 1)} = 1.01\ \Omega$$

#### 4. Switch at D

In this case,  $R_3$  shunts  $R_m$  and extends the current range of the meter from 1 mA to 1.0 A *i.e.* 1.0 A to 1000 mA. Hence  $n = 1000/1 = 1000$ .

$$\therefore R_3 = \frac{100\ \Omega}{(1000 - 1)} = 0.1001\ \Omega$$

Incidentally, it may be noted that greater the range extension, smaller the shunt resistance.

**Alternative Method**

An alternative circuit for range extension is shown in Fig. 73.9. It is called ‘add on’ method of shunting the meter because resistances can be added one after another for changing the range. Unlike in Fig. 73.8, there is no possibility of the meter being in the circuit without any shunt.

As seen, the universal shunt consists of three resistances  $R_1$ ,  $R_2$  and  $R_3$ . How they are connected as a shunt is determined by the switch position. When  $S$  is at position  $A$ , the combination  $(R_1 + R_2 + R_3)$  becomes connected across  $R_m$ . When  $S$  is at position  $B$ ,  $(R_2 + R_3)$  become connected in parallel across  $(R_1 + R_m)$  and so on.

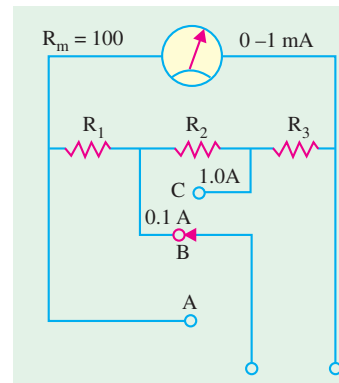


Fig. 73.9

**1. Switch at A**

In this case, multiplying factor  $n = 10 \text{ mA}/1 \text{ mA} = 10$

$$\therefore (R_1 + R_2 + R_3) = \frac{R_m}{(n-1)} = \frac{100}{9} \Omega \quad \dots(i)$$

**2. Switch at B**

Here,  $(R_2 + R_3)$  become in parallel with  $(R_1 + R_m)$  or  $(R_1 + 100)$ . Also,  $n = 100/1 = 100$

$$\therefore R_2 + R_3 = \frac{R_1 + 100}{99} \quad \dots(ii)$$

**3. Switch at C**

In this position,  $R_3$  is in parallel with  $(R_1 + R_2 + 100)$  and  $n = 1000/1 = 1000$

$$\therefore R_3 = \frac{R_1 + R_2 + 100}{999} \quad \dots(iii)$$

Solving for  $R_1$ ,  $R_2$  and  $R_3$  from Eq. (i), (ii) and (iii) we have

$$R_1 = 10 \Omega, R_2 = 1 \Omega \text{ and } R_3 = 1/9 \Omega$$

**73.12. Measurement of Current**

While measuring current flowing in a circuit, following two points must be kept in mind :

1. The current meter must be connected in series with the circuit where current is to be measured (Fig. 73.7). The full circuit current cannot flow through the meter unless it is made a series component.
2. The dc meter must be connected with the correct polarity for the pointer to read up-scale to the right. Reversed polarity deflects the pointer down-scale to the left forcing it against the stop which can sometime bend the pointer.

**73.13. Converting Basic Meter to DC Voltmeter**

The basic meter movement can measure a maximum voltage of  $I_m R_m$  which is very small [Fig. 73.10 (a)]. However, its voltage range can be extended to any value by connecting a large resistance in series with it as shown in Fig. 73.10 (b). The series resistance is also called **multiplier resistance** because it multiplies the voltage reading capability of the meter many times. It is usually connected inside the voltmeter case.

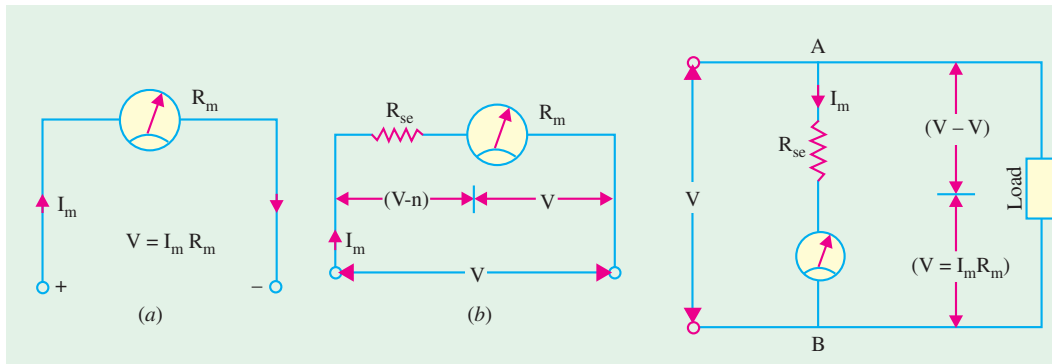


Fig. 73.10

Fig. 73.11

But it should be noted that the voltmeter is connected in parallel with the load across which the voltage is to be measured (Fig. 73.11).

**Value of  $R_{se}$**

Suppose, it is desired to extend the voltage range of the meter from  $v$  to  $V$ . The ratio  $V/v$  is known as the **voltage multiplication**. As seen from Fig. 73.11, drop across  $R_{se}$  is  $(V - v)$  and current through it is the same as meter current *i.e.*  $I_m$

$$\therefore I_m R_{se} = (V - v) \quad \dots(i)$$

$$\therefore R_{se} = \frac{V - v}{I_m} = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

The voltage multiplication ( $m$ ) can be found from Eq. (i) above,

Dividing both sides by  $v$ , we get

$$\frac{I_m R_{se}}{v} = V_v - 1$$

$$\therefore \frac{V}{v} = 1 + \frac{I_m R_{se}}{v} = 1 + \frac{I_m R_{se}}{I_m R_m}$$

$$\therefore V_v = \left(1 + \frac{R_{se}}{R_m}\right) \quad \therefore m = \left(1 + \frac{R_{se}}{R_m}\right)$$

It is seen that for a given meter, higher the series resistance, greater the voltage range extension.

**Example 73.2.** A  $50\text{-}\mu\text{A}$  meter movement with an internal resistance of  $k\Omega$  is to be used as dc voltmeter of range  $50\text{ V}$ . Calculate the

- (a) multiplier resistance required and
- (b) voltage multiplication.

**Solution. (a)**

$$R_{se} = \frac{V}{I_m} - R_m$$

$$= \frac{50}{50 \times 10^{-6}} - 1000$$

$$= 10^6 - 1000 = 999,000 \Omega = \mathbf{999\text{ k}\Omega}$$

**73.14. Multirange DC Voltmeter**

A multirange voltmeter with ranges of  $0\text{--}5\text{ V}$ ,  $0\text{--}25\text{ V}$  and  $0\text{--}50\text{ V}$  is shown in Fig. 73.12. Different values of resistors  $R_1$ ,  $R_2$  and  $R_3$  can be found in the same way as in Art. 73.11. It would be found that for the meter movement shown in figure

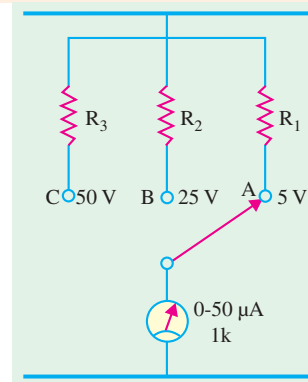


Fig. 73.12

$$R_1 = 99 \text{ K}, R_2 = 499 \text{ K}, R_3 = 999 \text{ K}$$

It is seen that higher the voltage range greater the multiplier resistance required (in almost the same proportion as the ranges).

### 73.15. Loading Effect of a Voltmeter

When the voltmeter resistance is not high as compared to the resistance of the circuit across which it is connected, the measured voltage becomes less. The decrease in voltage may be negligible or it may be appreciable depending on the sensitivity (ohms-per-volt rating) and input resistance of the voltmeter. It is called voltmeter *loading effect* because the voltmeter loads down the circuit across which it is connected. Since input resistance of electronic voltmeter is very high (10 MΩ or more), loading is not a problem in their case.

Consider the circuit shown in Fig. 73.13 in which two 15-K resistors are connected in series across a 100-V dc source. The drop across each is 50V. Now, suppose, that a 30-K voltmeter is connected across  $R_2$  to measure voltage drop across it. Due to loading effect of the voltmeter, the reading is reduced from 50V to 40V as explained below. As seen from Fig. 73.13 (b), combined resistance of  $R_2$  and voltmeter is  $15 \text{ K} \parallel 30 \text{ K} = 10 \text{ K}$ .

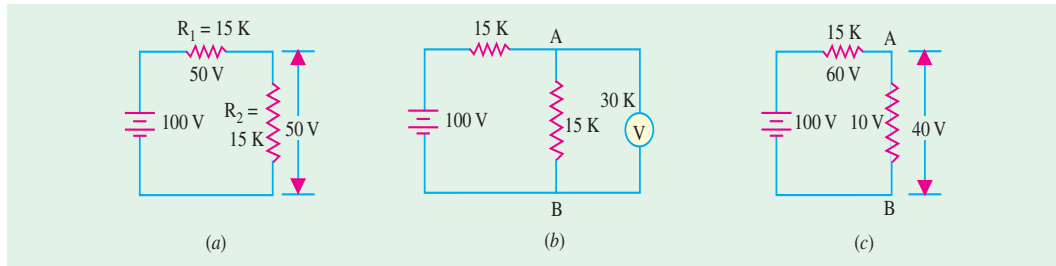


Fig. 73.13

$$\text{drop across } 10 \text{ K} = \frac{10}{10 + 15} \times 100 = 40\text{V}$$

Loading effect can be minimized by using a voltmeter whose resistance is as high as possible as compared to that of the circuit across which it is connected.

#### Correction Formula

The loading effect can be neutralized by using the following formula :

$$V_{corr} = V_{means} + \frac{R_1 R_2}{R_v (R_1 + R_2)} \cdot V_{means}$$

where

$V_{corr}$  = corrected voltage reading

$V_{meas}$  = measured voltage reading

$R_v$  = voltmeter resistance

$R_1, R_2$  = voltage dividing resistances in the circuit

In the above case,

$$V_{corr} = 40 + \frac{15 \times 15}{30 (15 + 15)} \times 40 = 40 + 10 = 50 \text{ V}$$

### 73.16. Ohmmeter

The basic meter movement can be used to measure resistance it is combined with a battery and a current-limiting resistance as shown in Fig. 73.14 (a). In that case, it is known as an ohmmeter.

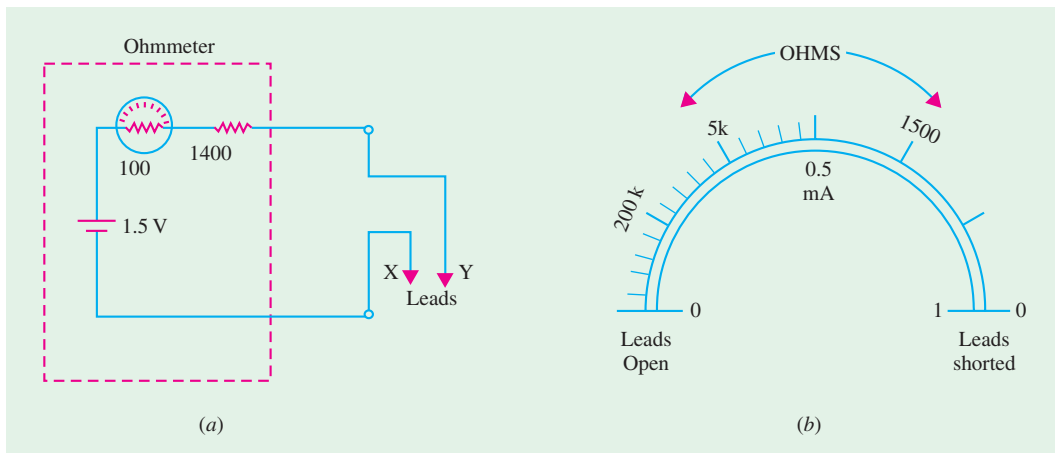


Fig. 73.14

For measuring resistance, the ohm-meter leads  $X$ - $Y$  are connected across the unknown resistance after switching off the power in the circuit under test. Only in that case, the ohmmeter battery can provide current for the meter movement. Since the amount of current depends on the amount of external resistance, the meter scale can be calibrated in ohms (instead of mA).

When the leads  $X$ - $Y$  are shorted, meter current is  $1.5\text{ V}/(100 + 1400)\ \Omega = 1\text{ mA}$ . The meter shows full-scale deflection to the right. The ohmmeter reading corresponds to  $0\ \Omega$  because external resistance is zero. When leads  $X$ - $Y$  are open *i.e.* do not touch each other, meter current is zero. Hence, it corresponds to infinite resistance on the ohmmeter scale.

Following points about the ohmmeter are worth noting :

1. the resistance scale is non-linear *i.e.* it is expanded at the right near zero ohm and crowded at the left near infinite ohm. This nonlinearity is due to the reciprocal function  $I = V/R$ ;
2. the ohmmeter reads up-scale regardless of the polarity of the leads because direction of current is determined by the internal battery;
3. at half-scale deflection, external resistance equals the internal resistance of the ohmmeter.
4. the test leads should be shorted and 'ZERO OHMS' control adjusted to bring the pointer to zero on each range.



Fig. 73.15. Digital micro-ohmmeter



Fig. 73.16. Digital milli-ohmmeter

Fig. 73.15 shows a digital micro-ohmmeter having a range of  $1\mu\Omega - 2\text{ k}\Omega$  with  $3\frac{1}{2}$  digit, 7-segment LED display. It has a basic accuracy of  $\pm 0.2\% \pm 1$  digit and is based on a design using MOS LSI ICs and glass epoxy PCB.

Fig. 73.16 shows a battery-operated portable digital milli-ohmmeter having a measurement range of  $200\text{ m}\Omega - 2\text{ k}\Omega$  with an accuracy of  $\pm 0.5\% \pm 1$  digit. It has a  $3\frac{1}{2}$  digit 7-segment LED display.

### 73.17. The Multimeter

It is extensively used in cable industry, motor industry, transformer and switchgear industry. It is also called volt-ohm-milliammeter (VOM). It is a general purpose instrument having the necessary circuitry and switching arrangement for measuring ac/dc voltage or ac/dc current or resistance. It is simple, compact and portable because the only power it uses is the battery for the ohm-meter.

Multimeters may be of analog type (Fig. 73.17) or digital type (Fig. 73.18). The analog type is of the pointer and scale type *i.e.* it uses the basic D' Arsonval meter movement. However digital multimeters (DMMs) are becoming increasingly popular because of their easy readability, numerical display and improved accuracy.



Fig. 73.18

Courtesy : Fluke Corporation

It has been designed for speed, accuracy and reliability.

### 73.18. Rectifier Type AC Meter

The D' Arsonval meter movement can be used for measuring alternating quantities provided a rectifier is added to the measuring circuit. A similar rectifier arrangement is found as part of AC VOLTS function in multimeters (Art. 73.17). Such as meters are more widely used than either (costly but more accurate) dynamometer type or more delicate thermal and hotwire type.



Fig. 73.17. A digital multimeter

Fig. 73.17 shows the photograph of an analog multimeter designed primarily for electrical, electronic, radio and TV engineers and technicians. It sells under the brand name of Motwane Multimeter 8 X Mark-III.

It is a 5-function, 30-range meter which measures high ac/dc voltages from 0 to 2.5 kV and ac/dc currents from 0 to 10 A. Its three resistance ranges cover from 0 to 20 M $\Omega$ . It is reputed for its excellent reliability, operational simplicity and easy portability.

Fig. 73.18 depicts a digital multimeter which can measure dc voltage upto 1000 V, ac voltages upto 750 V, ac/dc currents from 15  $\mu$ A to 10 A and resistances from 0  $\Omega$  to 100 M $\Omega$ . It has a 5 digit multifunction vacuum fluorescent display allowing the user to measure two different parameters of the same signal from one test connection. The user can also view both measurements at the same time.

In Fig. 73.19 is shown a hand-held autoranging, digital multimeter (DMM) having high contrast, 4 digit LCD read-out. It has been designed for speed, accuracy and reliability.



Fig. 73.19. Courtesy : Fluke Corporation

**(a) With Half-Wave Rectifier**

The circuit of an ac Voltmeter using half-wave diode recifier is shown in Fig. 73.20. Here, a half-wave rectifier has been combined in series with a dc meter movement.

When used as a dc voltmeter (*i.e.* without rectifier) it would have (say, for example) a range of 10V. However, if an ac voltages of rms value 10V is applied across input terminals *AB*, it would read 4.5 V.

It is so because an *ac* voltage of rms value 10 V has a peak value of  $10 \times \sqrt{2} = 14 \text{ V}$  and an average value of  $0.636 \times 14 = 9 \text{ V}$ . Since in the half-wave rectified output, one half-cycle is absent, the average for the full cycle is  $9/2 = 4.5 \text{ V}$ . The meter movement will, therefore read 4.5 V *i.e.* 45% of the dc value. It may also be noted that ac sensitivity of a half-wave ac meter is only 45 per cent of the dc sensitivity.

**(b) With Full-Wave Rectifier**

The circuit is shown in Fig. 73.21. In this case, the meter reading would be 90% of rms input voltage *i.e.* 90% of the dc value.

**73.19. Electronic Voltmeters**

A *VOM* can be used to measure voltages but it lacks both sensitivity and high input resistances. Moreover, its input resistance is different for each range. The electronic voltmeter (*EVM*), on the

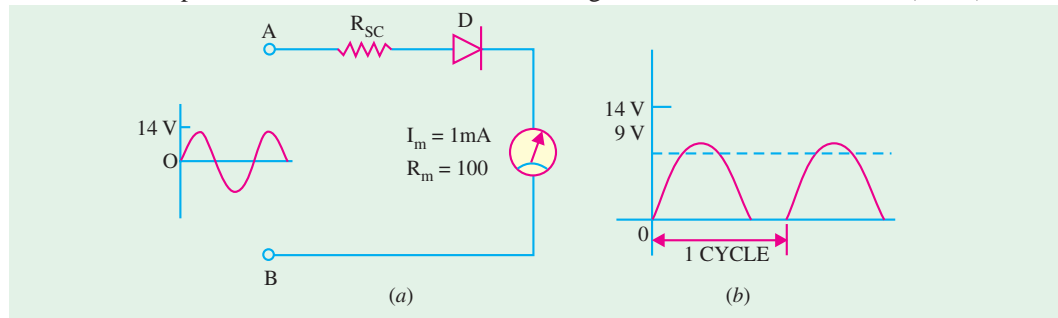


Fig. 73.20

other hand, has input resistance ranging from 10 MΩ to 100 MΩ, thus producing less loading of the circuit under test than the *VOM*. Another advantage of *EVM* is that its input resistance remains constant over all ranges.

Two types of voltmeters are in use today (i) analog and (ii) digital. However, a distinction must be made between a digital instrument and an instrument with digital readout. A digital instrument is one which uses internal circuitry of digital design. A digital readout instrument is one whose measuring circuitry is of analog design but the indicating device is of digital design.

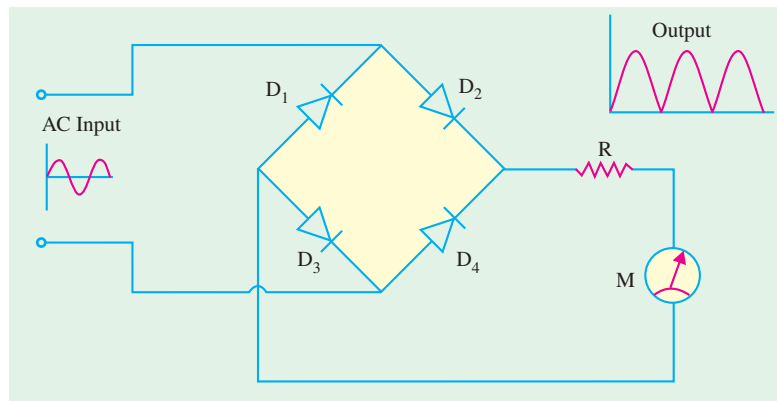


Fig. 73.21

The electronic voltmeters go by a variety of names reflecting the technology used.

- (i) vacuum-tube voltmeter (*VTVM*)—it uses vacuum tubes with deflection meter movement,
- (ii) digital voltmeters like transistor voltmeter (*TVM*) and *FET* voltmeter (*FETVM*).

### 73.20. Direct Current FET VM

The schematic diagram of a *FET VM* using difference amplifier is shown in Fig. 73.22. The two *FETs* are identical so that increase in the current of one *FET* is offset by corresponding decrease in the source current of the other. The two *FETs* form the lower arms of the balanced bridge circuit whereas the two drain resistors  $R_D$  form the upper arms. The meter movement is connected across the drain terminals of the *FETs*.

The circuit is balanced under zero-input-voltage condition provided the two *FETs* are identical. In that case, there would be no current through  $M$ . Zero-Adjust potentiometer is used to get null deflection in case there is a small current through  $M$  under zero-signal condition.

Full-scale calibration is adjusted with the help of variable resistor  $R$ .

When positive voltage is applied to the gate of  $F_1$ , some current flows through  $M$ . The magnitude of this current is found to be proportional to the voltage being measured. Hence, meter is calibrated in volts to indicate input voltage.

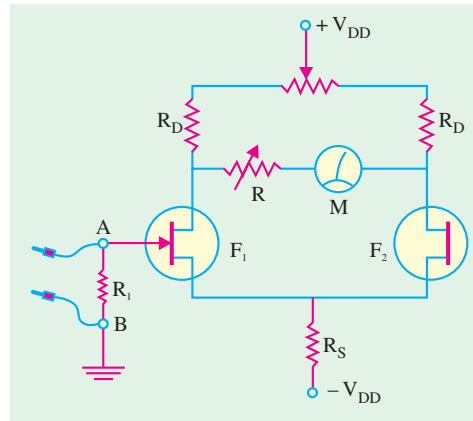


Fig. 73.22

### 73.21. Electronic Voltmeter for Alternating Currents

The block diagram of such an *EVM* for ac measurements is shown in Fig. 73.23 where voltage divider allows range selection. The amplifier provides the necessary gain to establish voltmeter sensitivity as well as high input impedance. The negative feed-back ensures stability and accurate overall gain.

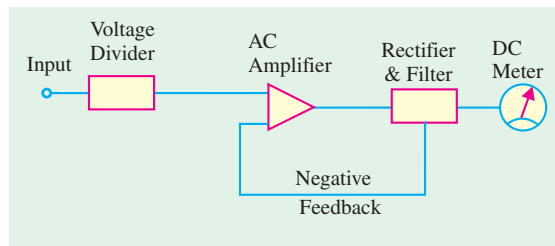


Fig. 73.23

### 73.22. The Digital Voltmeter (DVM)

Such a voltmeter displays measurements of dc or ac voltages as discrete numerals instead of pointer deflections on a continuous scale as in analog instruments. As compared to other voltmeters, a *DVM* offers the advantages of :

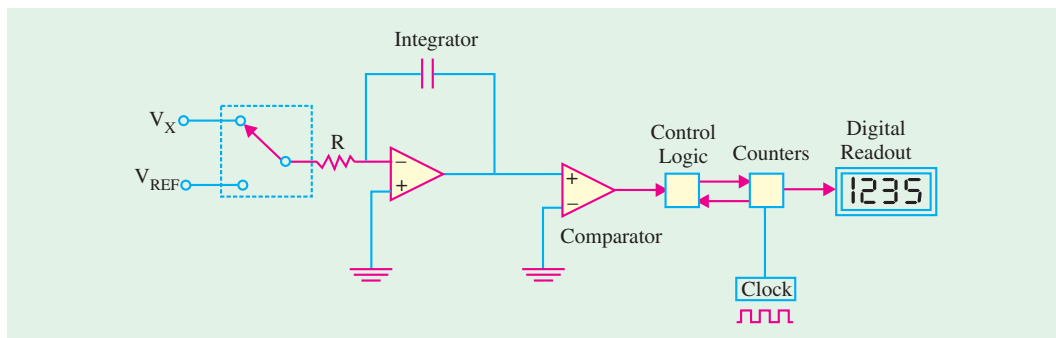


Fig. 73.24

1. greater, speed,
2. higher accuracy and resolution,
3. no parallax,
4. reduced human error,
5. compatibility with other digital equipment for further processing and recording.



With the development and perfection of IC modules, the size and power requirement of *DVMs* have reduced to a level where they can compete with conventional analog instrument both in price and portability.

The block diagram of a *DVM* based on dual-slope technique is shown in Fig. 73.24. The dual-slope analog-digital (*A - D*) converter consists of five basic blocks : an Op-Amp used as an integrator, a level comparator, a basic clock (for generating timing pulses), a set of decimal counters and a block of logic circuitry.

The unknown voltage  $V_x$  is applied through switch *S* to the integrator for a known period of time *T* as shown in Fig. 73.25. This period is determined by counting the clock frequency in decimal counters. During time period *T*, *C* is charged at a rate proportional to  $V_x$ .

At the end of time interval *T*, *S* is shifted to the reference voltage  $V_{ref}$  of *opposite* polarity. The capacitor charge begins to decrease with time and results in a down-ward linear ramp voltage. During the second period a known voltage (*i.e.*  $V_{ref}$  is observed for an unknown time (*t*). This unknown time *t* is determined by counting timing pulses from the clock until the voltage across the capacitor reaches its basic reference value (reference may be ground or any other basic reference level). From similar triangles of Fig. 73.25.

$$\frac{V_x}{T} = \frac{V_{ref}}{t} \quad \therefore V_x = \frac{V_{ref}}{t} \times V_{ref}$$

The count after *t* which is proportional to the input voltage  $V_x$  is displayed as the measured voltage.

By using appropriate signal conditioners, currents, resistances and ac voltages can be measured by the same instrument.

*DVMs* are often used in data processing systems or data logging systems. In such systems, a number of analog input signals are scanned sequentially by an electronic system and then each signal is converted into an equivalent digital value by the *A/D* converter in the *DVM*. The digital value is then transmitted to a printer alongwith the information about the input line from which the signal has been derived. The whole data is then printed out. In this way, a large number of input signals can be automatically scanned or processed and their values either printed or logged.

Fig. 73.26 shows a portable digital dc micro-voltmeter (Agronic-112). It has a measurement range of 1  $\mu$ V - 1000 V with an accuracy of  $\pm 0.2\% \pm 1$  digit. It uses latest *MOS LSI ICs* and glass epoxy *PCB*. It has 3½ digit, 7-segment *LED* display and is widely-used by the testing and servicing departments of industries, research laboratories, educational institutions and service centres.

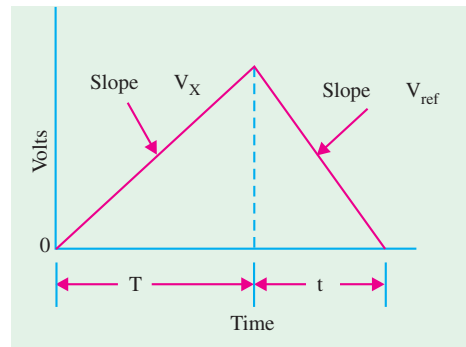


Fig. 73.25



Fig. 73.26. Digital Voltmeter

### 73.23. Cathode-Ray Oscilloscope (CRO)

It is generally referred to as oscilloscope or scope and is the basic tool of an electronic engineer and technician as voltmeter, ammeter and wattmeter are those of an electrical engineer or electrician.

The *CRO* provides a two-dimensional visual display of the signal waveshape on a screen thereby allowing an electronic engineer to 'see' the signal in various parts of the circuit. It, in effect, gives the electronic engineer an eye to 'see' what is happening inside the circuit itself. It is only by 'seeing' the signal waveforms that he/she

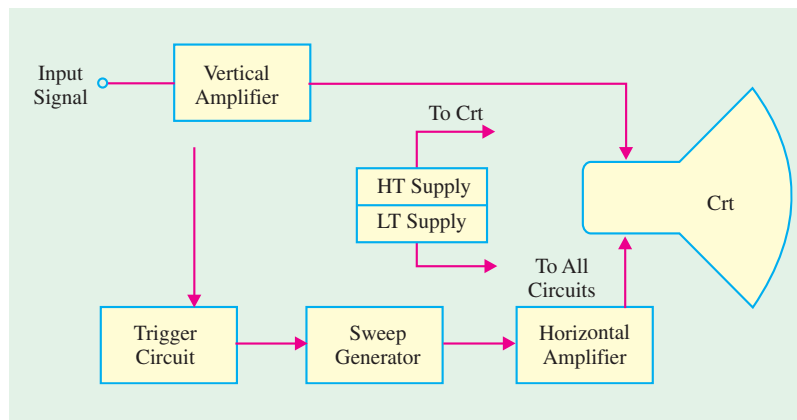


Fig. 73.27

can correct errors, understand mistakes in the circuit design and thus make suitable adjustments.

An oscilloscope can display and also measure many electrical quantities like ac/dc voltage, time, phase relationships, frequency and a wide range of waveform characteristics like rise-time, fall-time and overshoot etc. Non-electrical quantities like pressure, strain, temperature and acceleration etc. can also be measured by using different transducers to first convert them into an equivalent voltage.

As seen from the block diagram of an oscilloscope (Fig. 73.27), it consists of the following major sub-systems :

1. **Cathode Ray Tube (CRT)**—it displays the quantity being measured.
2. **Vertical amplifier**—it amplifies the signal waveform to be viewed.
3. **Horizontal amplifier**—it is fed with a sawtooth voltage which is then applied to the X-plates.
4. **Sweep generator**—produces sawtooth voltage waveform used for horizontal deflection of the electron beam.
5. **Trigger circuit**—produces trigger pulses to start horizontal sweep.
6. **High and low-voltage power supply.**

The operating controls of a basic oscilloscope are shown in Fig. 73.28.

The different **terminals** provide.

1. horizontal amplifier input,
2. vertical amplifier input,
3. sync. input,
4. Z-axis input,
5. external sweep input.

As seen, different **controls** permit adjustment of

1. **Intensity**—for correct brightness of the trace on the screen,
2. **Focus**—for sharp focus of the trace.
3. **Horizontal centering**—for moving the pattern right and left on the screen.
4. **Vertical centering**—for moving the pattern up and down on the screen.
5. **Horizontal gain (also Time/div or Time/cm)**—for adjusting pattern width.
6. **Vertical gain (also volt/div or volt/cm)**—for adjusting pattern height.
7. **Sweep frequency**—for selecting number of cycles in the pattern.
8. **Sync. voltage amplitude**—for locking the pattern.

The different **switches** permit selection of :

1. sweep type,
2. sweep range,
3. sync. type

A CRO can operate upto 500 MHz, can allow viewing of signals within a time span of a few nanoseconds and can provide a number of waveform displays simultaneously on the screen. It also has the ability to hold the displays for a short or long time (of many hours) so that the original signal may be compared with one coming on later.

### 73.24. Cathode Ray Tube (CRT)

It is the ‘heart’ of an oscilloscope and is very similar to the picture tube in a television set.

#### Construction

The cross-sectional view of a general-purpose electrostatic deflection CRT is shown in Fig. 73.29. Its four major components are :

1. an electron gun for producing a stream of electrons,
2. focussing and accelerating anodes-for producing a narrow and sharply-focussed beam of electrons,
3. horizontal and vertical deflecting plates-for controlling the path of the beam,
4. an evacuated glass envelope with a phosphorescent screen which produces bright spot when struck by a high-velocity electron beam.



A photograph of Cathode ray tube

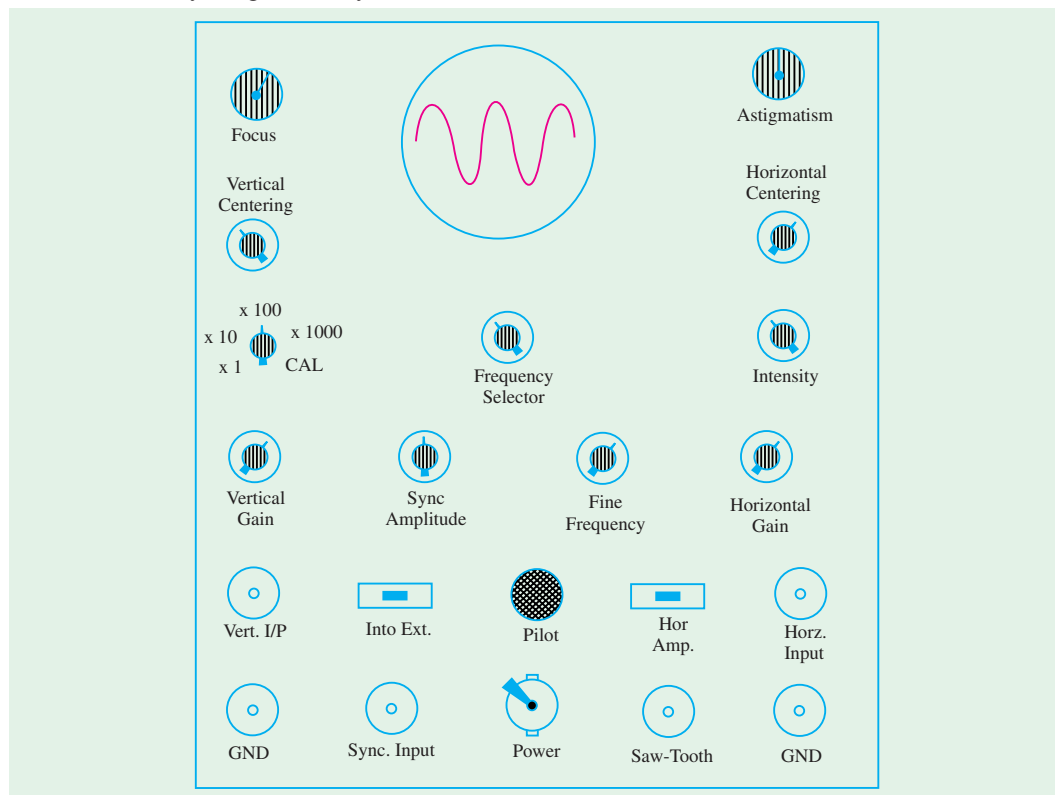


Fig. 73.28

As shown, a CRT is a self-contained unit like any electron tube with a base through which leads are brought out for different pins.

**1. Electron Gun Assembly**

The electron gun assembly consists of an indirectly-heated cathode  $K$ , a control grid  $G$ , a pre-accelerator anode  $A_1$ , focussing anode  $A_2$  and an accelerating anode  $A_3$ . The sole function of the

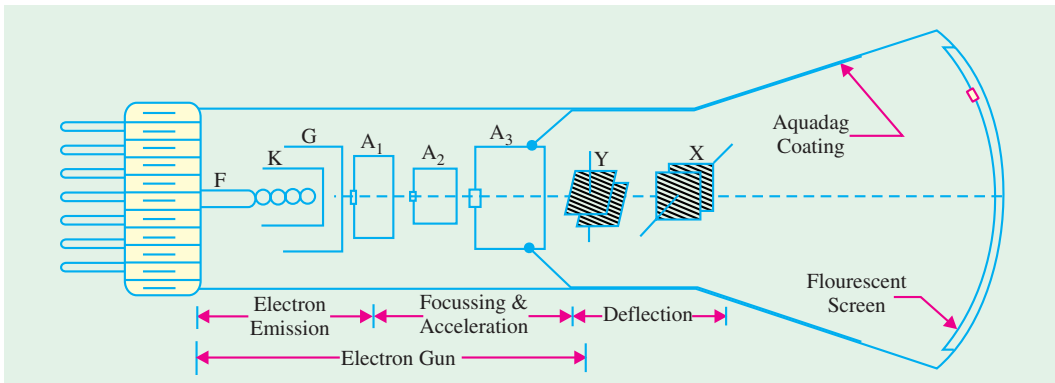


Fig. 73.29

electrons gun assembly is to provide a focussed beam of electrons which is accelerated towards the flourescent screen. The electrons are given off by thermionic emission from the cathode. The control grid is a metallic cylinder with a small aperture in line with the cathode and kept at a negative potential with respect to  $K$ . The number of electrons allowed to pass through the grid aperture (and, hence, the beam current) depends on the amount of the control grid bias. Since the intensity (or brightness) of the spot  $S$  on the screen depends on the strength of beam current, the knob controlling the grid bias is called the *intensity control*.

The anodes  $A_1$  and  $A_3$ , which are both at positive potential with respect to  $K$ , operate to accelerate the electron beam (Fig. 73.30). The cylindrical focussing anode  $A_2$ , being at negative potential, repels electrons from all sides and compresses them into a fine beam. The knob controlling the potential of  $A_2$  provides the *focus control*.

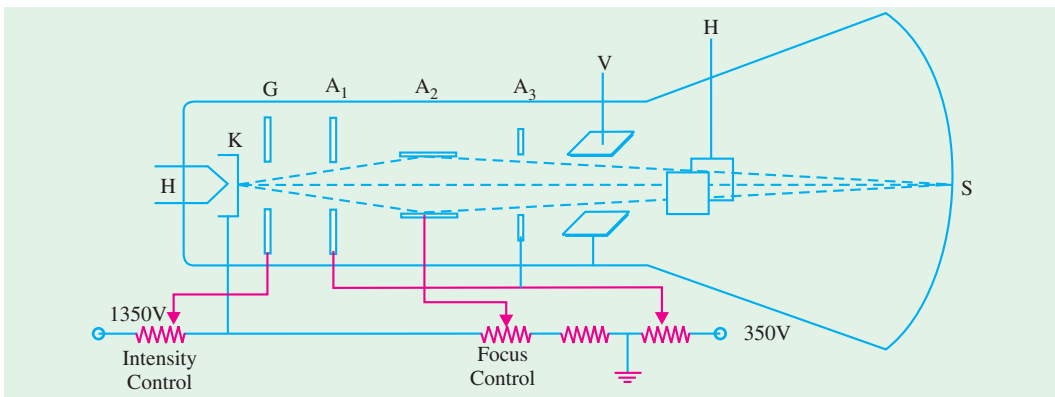


Fig. 73.30

**2. Deflecting Plates**

Two sets of deflecting plates are used for deflecting the thin pencil-like electronic beam both in the vertical and horizontal directions. The first set marked  $Y$  (nearer to the gun) is for vertical deflection and  $X$ -set is for horizontal deflection. When no potential is applied across the plates, beam passes between both sets of plates undeflected and produces a bright spot at the centre of the screen.

If upper  $Y$ -plate is given a positive potential, the beam is deflected upwards depending on the value of the applied potential. Similarly, the beam (and hence the spot) deflects downwards when lower  $Y$ -plate is made positive. However, if an **alternating** voltage is applied across the  $Y$ -plates, the spot keeps moving up and down thereby producing a vertical luminous trace on the screen due to persistence of vision. The maximum displacement of the spot from its central position is equal to the amplitude of the applied voltage.

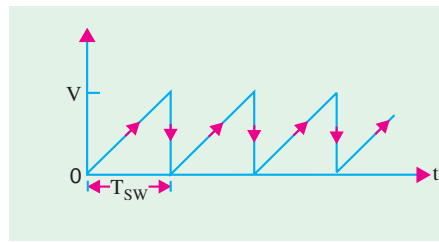


Fig. 73.31

The screen spot is deflected horizontally if similar voltages are applied to the  $X$ -plates. The dc potentials on the  $Y$ - and  $X$ -plates are adjustable by means of **centring controls**.

It must be remembered that the signal to be displayed on the screen is always applied across the  $Y$ -plates. The voltage applied across  $X$ -plates is a ramp voltage *i.e.* a voltage which increases linearly with time. It has a sawtooth wave-form as shown in Fig. 73.31. It is also called horizontal time-base or sweep voltage. It has a sweep time of  $T_{sw}$ .

### 3. Glass Envelope

It is funnel-shaped having a phosphor-coated screen at its flared end. It is highly-evacuated in order to permit the electron beam to traverse the tube easily. The inside of the flared part of the tube is coated with a conducting graphite layer called Aquadag which is maintained at the same potential as  $A_3$ . This layer performs two functions (i) it accelerates the electron beam after it passes between the deflecting plates and (ii) collects the electrons produced by secondary emission when electron beam strikes the screen. Hence, it prevents the formation of negative charge on the screen.

The screen itself is coated with a thin layer of a fluorescent material called phosphor. When struck by high-energy electrons, it glows. In other words, it absorbs the kinetic energy of the electrons and converts it into light—the process being known as **fluorescence**. That is why the screen is called **fluorescent screen**. The colour of the emitted light depends on the type of phosphor used.

### 73.25. Deflection Sensitivity of a CRT

Fig. 73.32 shows the upward deflection of an electron beam when it passes between the vertical or  $Y$ -plates of a CRT. The beam deflects upwards because the upper  $Y$ -plate has been made positive with respect to the lower plate. Reversing the polarity of the **applied** voltage would, obviously, cause the beam to deflect downwards.

The vertical deflection of the beam is

$$y = \frac{1}{2} \cdot D \cdot \frac{l}{D} \cdot \frac{V_d}{V_A}$$

where  $V_A$  is the accelerating voltage applied to the electrons which make up the electron beam.

The deflection sensitivity of a CRT is defined as the vertical deflection of the beam on the screen per unit deflecting voltage.

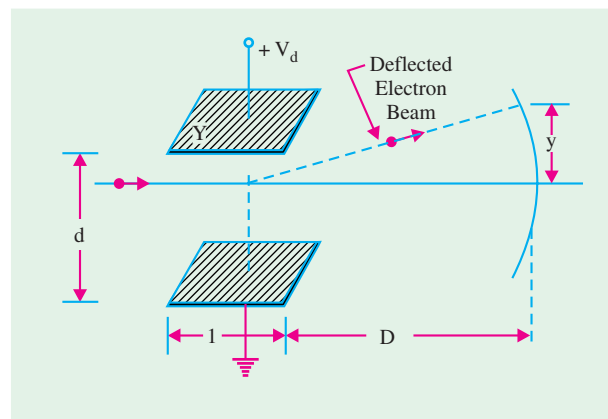


Fig. 73.32

$$\therefore S = \frac{y}{V_d}$$

Using the above equation, we get  $S = \frac{lD}{2dV_A}$

The deflection factor which is defined as the reciprocal of deflection sensitivity is given by  $G = 1/S$ .

Substituting the value of  $S$  from above

$$G = 2 \cdot \frac{d}{l} \cdot \frac{V_A}{D} \text{ volt/metre}$$

### 73.26. Normal Operation of a CRO

The signal to be viewed or displayed on the screen is applied across the  $Y$ -plates of a  $CRT$ . But to see its waveform or pattern, it is essential to spread it out horizontally from left to right. It is achieved by applying a sawtooth voltage wave (produced by a time base generator) to  $X$ -plates. Under these conditions, the electron beam would move uniformly from left to right thereby graphic vertical variations of the input signal versus time. Due to repetitive tracing of the viewed waveform, we get a continuous display because of persistence of vision. However, for getting a stable stationary display on the screen, it is essential to synchronize the horizontal sweeping of the beam (sync) with the input signal across  $Y$ -plates. The signal will be properly synced only when its frequency equals the sweep-generator frequency.

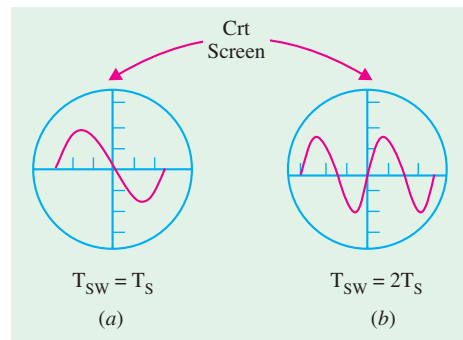


Fig. 73.33

In general, for proper synchronization of time-base with the signal, the condition is

$$T_{sw} = n T_s$$

where  $T_s$  the time-period of the signal and  $n$  is an integer.

If  $n = 1$ , then  $T_{sw} = T_s$  i.e. time-periods of the sweep voltage and input signal voltage are equal, then one cycle of the signal would be displayed as shown in Fig. 73.33 (a).

On the other hand, if  $T_{sw}$  is twice  $T_s$ , then two cycles of the signal voltage would be displayed as shown in Fig. 73.33 (b). Obviously, three full cycles of the input voltage would be spread out on the screen when  $T_{sw} = 3 T_s$ .

#### Internal Synchronization

The periodic sawtooth voltage which is applied to  $X$ -plates for horizontal sweep (or scan) of the beam across the screen is usually provided by the unijunction relaxation oscillator. When the sawtooth voltage falls abruptly to zero, the beam experience no horizontal deflection and hence flies back almost instantly to the original (central) position.

The usual method of synchronizing the input signal is to use a portion of the input signal to trigger the sweep generator so that the frequency of the sweep signal is locked or synchronized to the input signal. It is called internal sync. because the synchronization is obtained by internal wiring connection as shown in the block the diagram of Fig. 73.34.

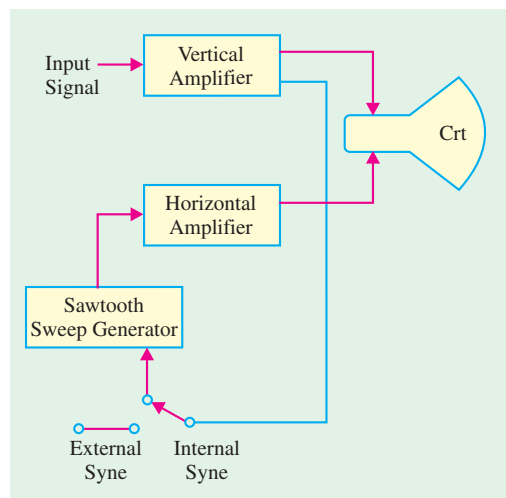


Fig. 73.34

### 73.27. Triggered and Non-Triggered Scopes

Oscilloscopes may be classified into two basic types :

1. triggered sweep type.
2. recurrent sweep (free-running) type.

Triggered oscilloscopes, being more sophisticated, are generally used in industrial laboratories and plants, in engineering and technical school laboratories and in all those applications which require study of low- and high-frequency waveforms, for accurate measurement of time and timing relationships etc.

A non-triggered oscilloscope is generally used in servicing work where a certain amount of waveform error can be tolerated and bandwidth requirements are limited to a few MHz.

The sweep (or ramp) generator which produces sawtooth voltage for  $X$ -deflection plates is present in both types of scopes. In non-triggered oscilloscopes, this generator runs continuously (recurrent sweep) and the control and calibration of the sweep is based on the repetition frequency of the sweep. For producing a stable stationary display, the sweep frequency has to be forced into synchronization with the input signal on the  $Y$ -plates. This is done by manually adjusting the free-running sweep frequency to a value very close to signal frequency (or some submultiple of it) and then depending on the internal sync signal (derived from the input) for locking the sweep generator into exact step. Unfortunately, this method is limited to the display of signals which have constant frequency and amplitude. Hence voice or music signals from a microphone cannot be displayed on this scope because it has to be readjusted for each for new change in frequency. Moreover, a free-running or recurrent time base cannot display less than one complete cycle of the input signal on the scope screen. On the other hand, triggered time base can be adjusted to pick out a small part of a waveform which can then be expanded horizontally for evaluation of waveform details.

The triggered oscilloscope is provided with a triggered (or driven) sweep. Here the input signal is caused to generate pulses that trigger the sweep thereby ensuring that the sweep is necessarily in step with the trigger that drives it. Hence, screen display remains stable in spite of variations in the frequency or amplitude of the input signal. It means that there is automatic mode of triggering in such scopes. Consequently, input signals of very short duration can be displayed for the simple reason that sweep is initiated by a trigger pulse derived from the waveform under observation.

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Fig. 73.35 illustrates a triggered oscilloscope having a bandwidth 0-6 MHz, vertical sensitivity of 10 mV/div and horizontal sweep rate varying from 0.2  $\mu$ s/div to 0.1 s/div.



Fig. 73.35. A non-triggered oscilloscope

### 73.28. Dual Trace CRO

Such oscilloscopes are used extensively by industrial firms and research laboratories. They produce a dual-trace display by means of electronic switching of two separate input signals. As shown in the block diagram of Fig. 73.36, there are two vertical input circuits marked channel A and B with identical pre-amplifiers. The outputs of these preamplifiers are fed to an electronic switch which alternately connects them to the main vertical amplifier of the oscilloscope. In this type of scope, there is only one electron beam. The electronic switch is also capable of selecting a variety of display modes.

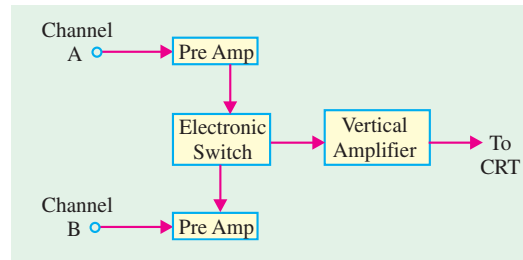


Fig. 73.36

In Fig. 73.37 is shown a dual trace oscilloscope (type VOS-26) which has a band-width of 5-15 MHz, vertical sensitivity of 10 mV/cm to 30 V/cm and calibrated sweep speed from 0.3  $\mu$ s/cm to 10 ms/cm. It is a very sensitive yet simple oscilloscope which assures long life and easy maintenance.

### 73.29. Dual Beam CRO

Such a CRO has two sets of vertical deflection plates and has two electronic beams which produce two separate traces on the scope screen by using the same set of horizontal deflection plates. This scope makes it possible to observe two time-related wave forms at different points *i.e.* the electronic circuit.

Such a scope does not have the same number of display modes as the dual-trace scope yet it is ideally suited for different input signals.



Fig. 73.37. Dual Beam CRO

### 73.30. Storage Oscilloscope

It can retain a CRT display for 10 to 150 hours after the pattern is first produced on the screen. It uses the phenomenon of secondary electron emission to build up and store electrostatic charges on the surface of an insulated target. Such oscilloscopes are especially useful.

1. For real-time observation of events that occur only once.
2. For displaying the waveform of a very low-frequency signal.

Fig. 73.38 shows a 4 channel storage oscilloscope with 400 MHz bandwidth. It has a standard floppy disk drive which makes the saving of screen images or data to a disk, simple. The disk can then be inserted into your personal computer (PC) for importing to desk top publishing or spreadsheet programs. The storage oscilloscopes find their application in biophysics/biomedical research, audio system measurement and analysis, power supply and power-related design, electrophysical and electromechanical system design etc.



Fig. 73.38. 4 channel storage oscilloscope



### 73.31. Sampling CRO

It is specifically meant to observe very high frequency repetitive electric signals by using the sampling technique. Such high-frequency signals cannot be viewed by conventional oscilloscopes because its frequency range is limited by the gain-bandwidth product of its vertical amplifier. The sampling technique 'slows down' the signal frequency many thousands of times thereby making it easier to view it on the screen.

The oscilloscope shown in Fig. 73.38 is a sampling cathode ray oscilloscope. Its sample rate is 100 M samples per second.

### 73.32. Digital Readout CRO

It provides digital readout of the signal information such as voltage or time etc. in addition to the conventional CRT display. It consists of a high-speed laboratory CRO and an electronic counter, both contained in one cabinet.

### 73.33. Handheld Battery Operated Oscilloscope

Fig. 73.39 shows a handheld battery operated oscilloscope Model THS 720 P manufactured by Tektronix Corporation. It has built in 3-3/4 digit digital multimeter (DMM) with \*data logger and power analyser. It has a bandwidth of over 100 MHz and the sampling rate is as high as 500 M samples per second. The oscilloscope and power analyser can operate simultaneously and independently on the same or separate signals. This type of an oscilloscope is extremely useful for electric/power electronics measurements. Examples of such measurements are : (1) testing and verifying correct operation of motors (2) checking transformer efficiency, (3) verifying power supply performance, (4) measuring the effect of neutral current etc.



Fig. 73.39. Courtesy : Tektronix Corporation

### 73.34. Lissajous Figures

Lissajous figures (or patterns) are named in honour of the French scientist who first obtained them geometrically and optically. They illustrate one of the earliest uses to which the CRO was put.

Lissajous patterns are formed when two sine waves are applied simultaneously to the vertical and horizontal deflecting plates of a CRO. The two sine waves may be obtained from two audio oscillators as shown in Fig. 73.40. Obviously, in this case, *a sine wave sweeps a sine-wave input signal*. The shape of the Lissajous pattern depends on

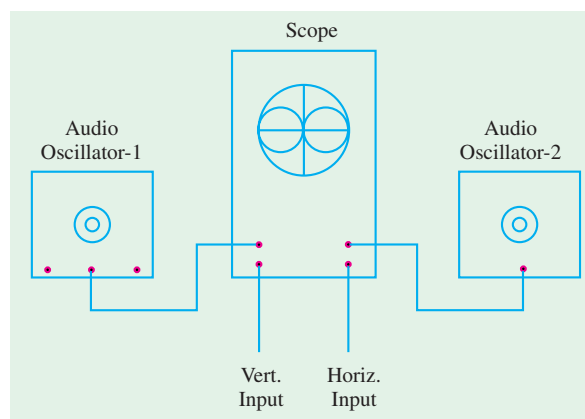


Fig. 73.40

\* Data logger is a system which can acquire data and store it in a memory.

the frequency and phase relationship of the two sine waves.

Two sine waves of the *same* frequency and amplitude may produce a straight line, an ellipse or a circle depending on their phase difference (73.41).

In general, the shape of Lissajous figures depends on (i) amplitude, (ii) phase difference and (iii) ratio of frequency of the two waves.

Lissajous figures are used for (i) determining an unknown frequency by comparing it with a known frequency (ii) checking audio oscillator with a known-frequency signal and (iii) checking audio amplifiers and feedback networks for phase shift.

### 73.35. Frequency Determination with Lissajous Figures

The unknown signal is applied across one set of deflecting plates and a known signal across the other. By studying the resultant Lissajous pattern, unknown frequency can be found.

Depend on the frequency ratio, the various patterns obtained are shown in Fig. 73.42. The ratio of the two frequencies is given by

$$\frac{f_H}{f_V} = \frac{\text{No. of points of horizontal tangency}}{\text{No. of points of vertical tangency}}$$

In Fig. 73.41 (a), there is one point of tangency along the horizontal as well as vertical axis. Hence,  $f_H = f_V$  i.e. the signals have the same frequency. In Fig. 73.42 (e)  $f_H/f_V = 3/2$ . In other words  $f_H = 1.5 f_V$ .

It should be noted in passing that this method of frequency determination has limitations and is being discarded gradually because low-cost digital frequency counters are becoming increasingly available in the market Fig. 73.43. The two main limitations of this method are as under :

- (i) the numerator and denominator of the frequency ratio must be whole numbers,
- (ii) the maximum ratio of frequencies that can be used is 10 : 1. Beyond that, the Lissajous patterns become too complex to analyse.

Fig. 73.43 shows a 10-digit digital frequency counter Model No. PM 6685 manufactured by Fluke Corporation. This can measure frequencies from 10 Hz to 300 MHz. This is an ideal instrument for R and D laboratories, testing, servicing and even outside the lab environment such as in base station transmitters of large telecommunication networks like GSM.

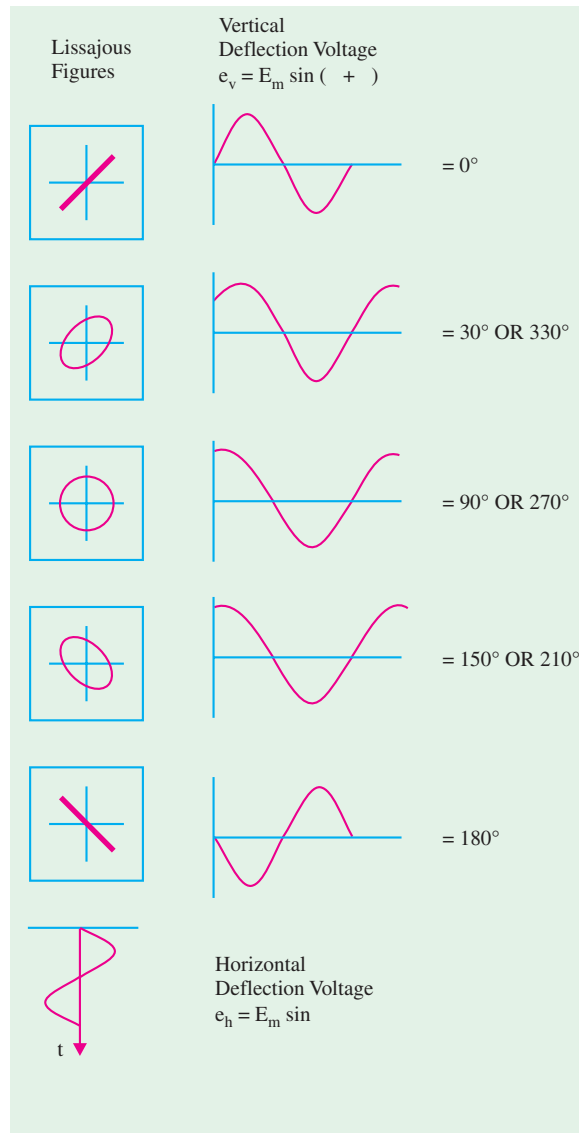


Fig. 73.41

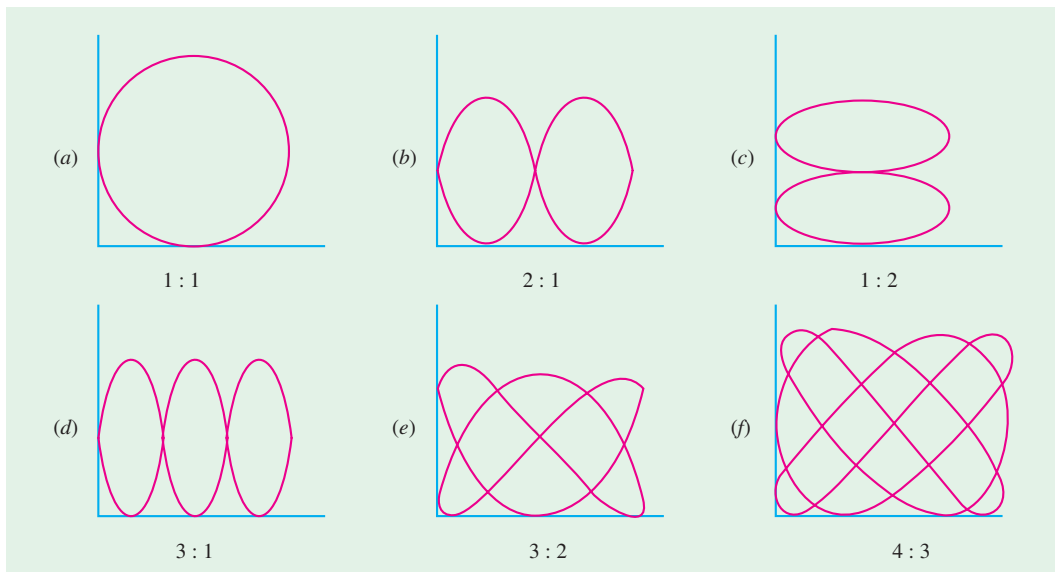


Fig. 73.42

### 73.36. Applications of a CRO

As stated earlier, no other instrument in electronic industry is as versatile as a *CRO*. In fact, a modern oscilloscope is the most useful single piece of electronic equipment that not only removes guess work from technical troubleshooting but makes it possible to determine the trouble quickly. Some of its uses are as under :

#### (a) In Radio Work

1. to trace and measure a signal throughout the *RF*, *IF* and *AF* channels of radio and television receivers.
2. it provides the only effective way of adjusting *FM* receivers, broadband high-frequency *RF* amplifiers and automatic frequency control circuits;
3. to test *AF* circuits for different types of distortions and other spurious oscillations;
4. to give visual display of waveshapes such as sine waves, square waves and their many different combinations;
5. to trace transistor curves
6. to visually show the composite synchronized TV signal
7. to display the response of tuned circuits etc.

#### (b) Scientific and Engineering Applications

1. measurement of ac/dc voltages,
2. finding B/H curves for hysteresis loop,
3. for engine pressure analysis,
4. for study of stress, strain, torque, acceleration etc.,
5. frequency and phase determination by using Lissajous figures,
6. radiation patterns of antenna,
7. amplifier gain,



Fig. 73.43

(Courtesy : Fluke Corporation)

- 8. modulation percentage,
- 9. complex waveform as a short-cut for Fourier analysis,
- 10. standing waves in transmission lines etc.

### 73.37. The Q Meter

This instrument is designed to measure some of the electrical properties of coils and capacitors by measuring the  $Q$ -value of an  $R-L-C$  circuit.

#### 1. Construction

As shown in Fig. 73.44, it essentially consists of

- (i) a frequency-calibrated continuously-variable  $RF$  oscillator,
- (ii) a calibrated variable capacitor  $C$ ,
- (iii)  $VTVM$  which is calibrated to read  $Q$  directly.

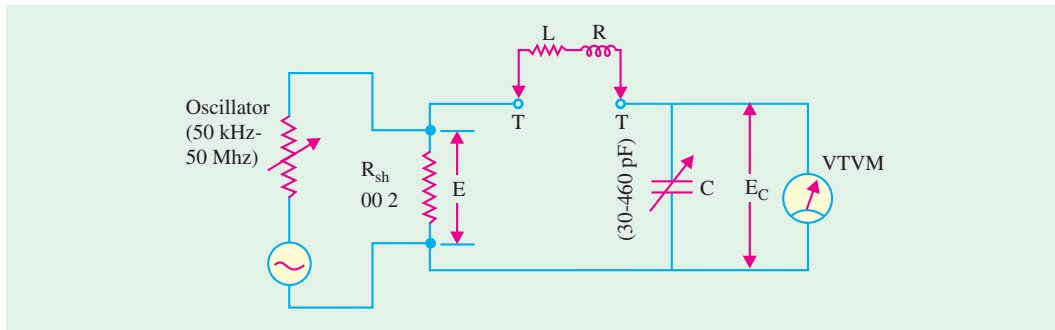


Fig. 73.44

#### 2. Principle of Operation

The basic principle used in  $Q$ -meter is the resonant rise of the voltage across the capacitor in an  $R.L.C$  circuit. The condition for series resonance (Art. 14.10) is

$$X_L = X_C \text{ and } E = IR$$

The value of circuit  $Q$  is

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$

If the applied voltage  $E$  is constant, then  $Q \propto E_C$ . Hence, by measuring voltage drop across  $C$  under resonant conditions,  $Q$  can be found. Alternatively,  $VTVM$  can be calibrated directly in terms of  $Q$  (rather than voltage).

As seen from Fig. 73.44, the oscillator delivers current to a very small ( $0.02 \Omega$ ) shunt resistance  $R_{sh}$  thereby developing a voltage  $E$  across it. It becomes the applied voltage for the  $RLC$  circuit and corresponds to the source voltage  $E$  of Fig. 73.45.

It is measured by a thermocouple meter marked 'Multiply  $Q$  by'. The voltage  $E_C$  across variable  $C$  is measured by the  $VTVM$ . The value of  $Q$ -factor is given by  $Q = E_C/E$ .

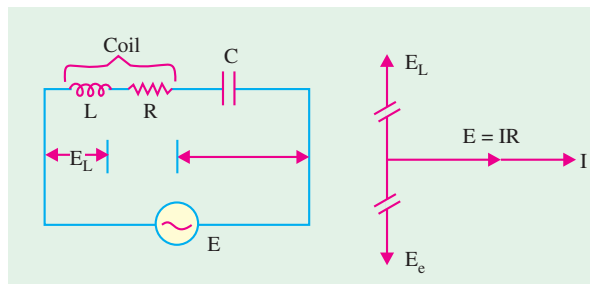


Fig. 73.45

For making measurement, the unknown coil is connected to the test terminals  $TT$  of the instrument and the circuit is tuned to resonance (Fig. 73.44)

- (i) either by setting the oscillator to a given frequency and varying  $C$ ,

(ii) or by keeping value of  $C$  constant and adjusting the oscillator frequency.

The reading on the  $VTVM$  must be multiplied by the index setting of the 'Multiply  $Q$  by' meter in order to obtain the actual value of  $Q$ .

### 3. Applications

Some of the specialized uses of this instrument are to measure

1.  $Q$  of a coil,
2. inductance and capacitance,
3. distributed capacitance of a coil,
4.  $Q$  and  $p.f.$  of a dielectric material,
5. mutual inductance of coupled circuits,
6. coefficient of coupling,
7. critical coupling,
8. reactance and effective resistance of an inductor at operating frequency,
9. bandwidth of a tuned circuit etc.

The above list does not exhaust the number of its possible applications. It has been very truly said that if ever an  $RF$  problem exists, a  $Q$  meter can always provide the answer.

### 73.38. Logic Analysers

The oscilloscope is probably the best tool for the development of analog or digital system design. It can be used to examine the waveforms and determine the voltage and rise time of the analog or digital signals. But the oscilloscope has two limitations especially when used in digital system design : (1) high speed random pulses can not be observed easily (2) oscilloscope cannot monitor a few signal lines simultaneously. For example, in a commonly available oscilloscope, the maximum number of inputs are four.

For these reasons, the logic analysers has been developed. It operates on a slightly different principle than that of an oscilloscope. Because there are many signal lines in a digital system (such as a microprocessor based system), the data is changing rapidly on each line, a logic analyser must take a *snap shot* of the activities on the lines and store the logic state of each signal in memory for each cycle of the system clock. The conditions under which the snapshot is taken are determined by triggering circuits, which can respond to various combinations of events.

The logic analyser enables the activity of many digital signal points to be recorded simultaneously and then examined in detail. The information is recorded with respect to a clock signal to determine whether they are HIGH or LOW with respect to a defined threshold voltage. This information is stored in memory and is then available for detailed analysis via the logic analyser's display. The clock signal can be internally or externally generated.

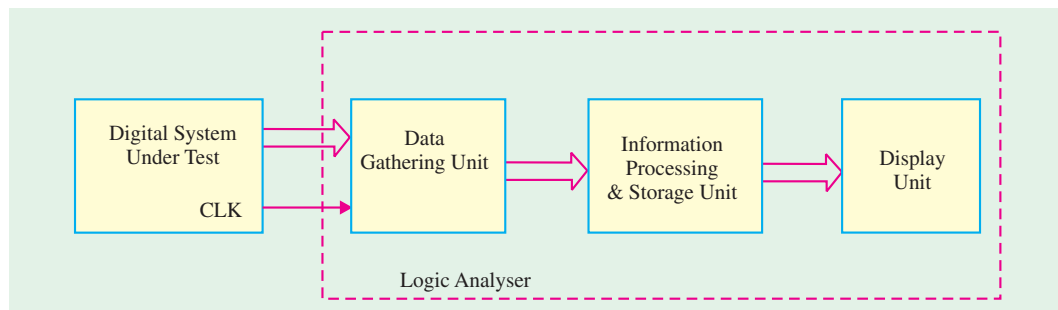


Fig. 73.46

Fig. 73.46 shows a block diagram of a typical logic analyser. It has a data gathering unit, information processing and storage unit and a display unit. The data gathering unit has (1) a pod slots for carrying data from the digital system under test to the logic analyser and (2) a key pad. The key pad is used to enter commands and set up the parameters that the logic analyser will use. The display unit is a cathode ray tube (CRT) that displays the command menu for the operator and also displays the output data.

#### Applications

- ◆ hardware/software debugging
- ◆ parametric/mixed signal testing
- ◆ hardware simulation and stimulus-response testing
- ◆ complex debugging with deep memory.

Fig. 73.47 shows a family of logic analysers, available from Tektronix corporation. Each logic analyser has at least 34 channels, 4-channel digitizing oscilloscope, off-line analysis capability for viewing data and creating setups on a separate PC.



Fig. 73.47. Logic analysers

#### Applications.

Logic analyser is a very powerful tool in the field of microprocessor based system development. Some of its major applications in this area are :

1. Hardware debug and verification.
2. Processor/bus debug and verification.
3. Embedded software integration, debug and verification.

### 73.39. Spectrum Analysers

The spectrum analyser is an instrument that brings together a superhetrodyne radio receiver with a swept frequency local oscillator and an oscilloscope to present a display of amplitude versus frequency.

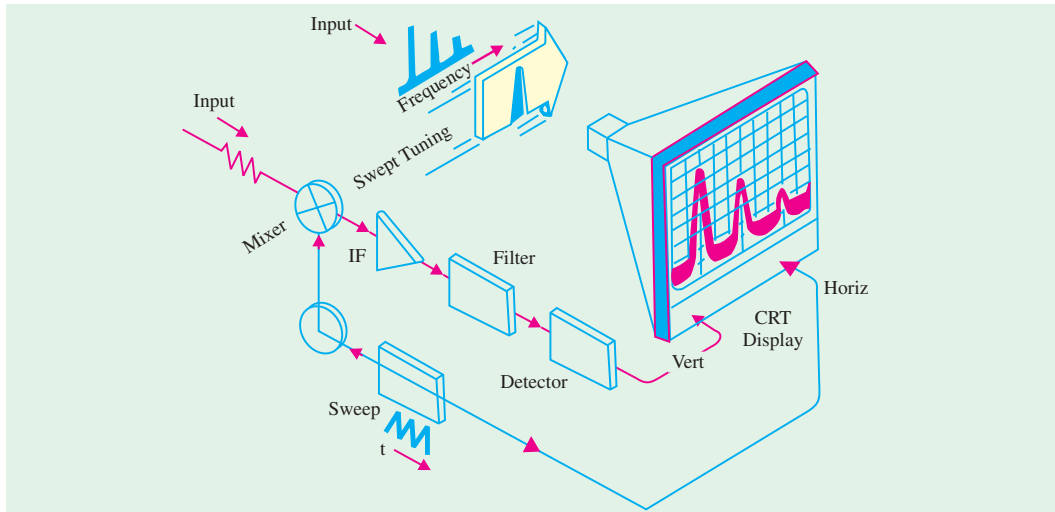


Fig. 73.48

(Courtesy Hewlett Packard)

Fig. 73.48 shows a simple block diagram of a spectrum analyser. As seen, the spectrum analyser is actually a superhetrodyne receiver in which local oscillator is a sweep generator. A low frequency saw-tooth wave is applied to both the sweep oscillator and the horizontal deflection plates of the CRT, producing a horizontal deflection that is a function of frequency. The lowest frequency is represented by left side of the trace while the highest frequency is represented by the right side of the trace. The sweep is from left to right.

The input signal is mixed with local oscillator to produce the *IF* (*i.e.* intermediate-frequency or difference) signal. The bandwidth of the *IF* amplifier is relatively narrow, so the output signal at the detector will have a strength that is proportional to the frequency that the local oscillator is converting to the *IF* at that instant. The display will then contain “poles” that represent the amplitudes of the various input frequency components.

There are several spectrum analysers available in the market manufactured by the companies like Rhode & Schwarz (Tektronix), Hewlett Packard (now called Agilent Technologies), and so on. Fig. 73.49 shows two commercially available spectrum analyses.

The spectrum analyser shown in Fig. 73.49 (a) is Model *FSE 30* manufactured by Rhode & Schwarz but marked by Tektronix Corporation.

It has a frequency range from 20 Hz to 76.5 GHz, a bandwidth from 1 Hz to 10 MHz. Another spectrum analyser Model No. 3066 shown in Fig. 73.49 (b) is a real-time instrument. It has a frequency range from *DC* to 5 MHz, a bandwidth from *DC* to 3 GHz. The real-time spectrum analyser take a very different approach compared to traditional sweeping spectrum analysers. Rather than acquiring one frequency step at a time, the real time spectrum analyser captures a block of frequencies all at once.

It is possible to use computers to do spectrum analysis of the signals. There is a variety of software available over the internet from several companies. Some softwares can be downloaded free of cost from the companies web-sites.

### Applications

The spectrum analyser is used to :

1. check the spectral purity of signal sources.
2. evaluate local electromagnetic interference (*EMI*) problems.
3. do site surveys prior to installing radio receiving or transmitting equipment.
4. test transmitters.
5. analyse signatures.

## 73.40. Signal Generators

A signal generator is an instrument that provides a controlled output waveform or signal for use in **testing, aligning** or in **measurements** on other circuits or equipment. The signal generators can be classified into the following categories :

- |                           |                             |
|---------------------------|-----------------------------|
| 1. Audio generators       | 2. Function generators      |
| 3. Pulse generators       | 4. <i>RF</i> generators     |
| 5. Frequency synthesizers | 6. Other signal generators. |

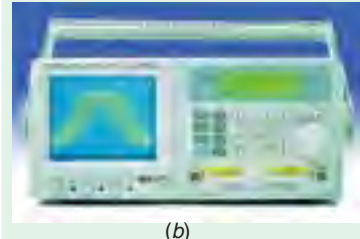
## 73.41. Audio Generators

The audio generators covers the frequency range 20 Hz to 20 kHz, although few models produce signals up to 100 kHz. Audio generators always produce **pure sine** waves and most also produce square waves. They uses a 600  $\Omega$  output impedance and produce output levels from  $-40$  dB mW to  $+4$  dB mW.

Two methods of frequency selection are typically used in audio signal generators. **continuous** and **step**. On the continuous type of a dial, we turn a knob to the desired frequency. Many such audio



(a)



(b)

Fig. 73.49

(Courtesy : Rhode & Schwarz and Tektronics Corporation)

generators have a scale that reads 20 to 200 (or alternatively 2 to 20) and a *range* selector switch determines whether the output frequencies will be 20 to 200 Hz, 200 to 2000 Hz or 2000 to 20,000 Hz. In a step-tuned generator, these controls are replaced by a *rotary* or *pushbutton* switch bank. As many as four decode switches might be used, although three is a more common number. These will be marked 0 through 100, 0 through 10 and 0.1 through 1.0 in decade steps. A multiplier switch determines whether the actual frequency will be X1, X10, X100 or X1000 the frequency indicated on the selector switches.

Fig. 73.50 shows a block diagram of an audio signal generator. The audio oscillator section is usually an RC phase-shift oscillator (or a Wien Bridge oscillator) circuit. A power amplifier stage provides buffering between the load and the oscillator and it develops the output signal amplitude. The ac voltmeter at the output is strictly optional, but in some models it is used with a *level control* to set precisely the input signal to the attenuator. Not all quality audio signal generators use this feature. So the lack of an ac output meter is not, in itself, indication of quality. In some models, an audio digital frequency counter is used ahead of the attenuator to provide digital display of the output frequency.



Audio signal generator

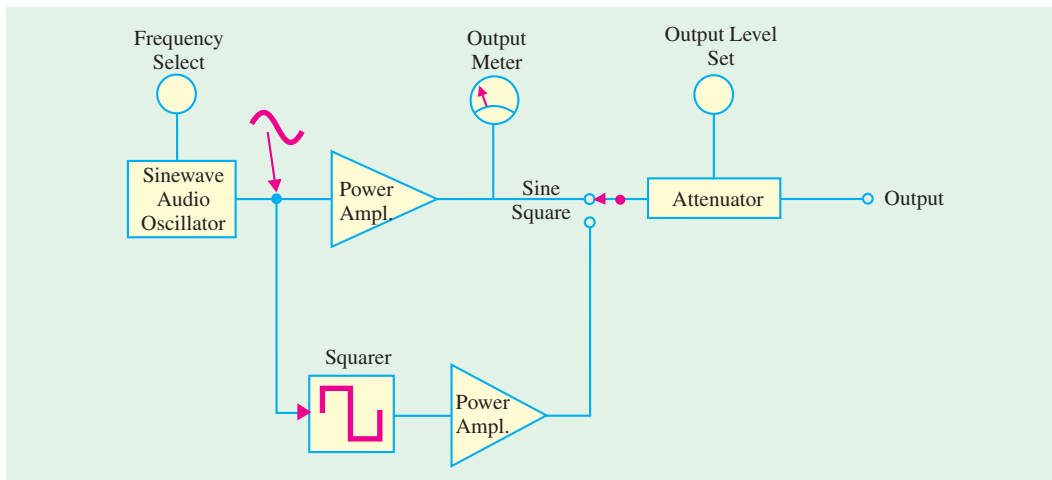


Fig. 73.50

### Applications

The audio generators are basically used to test the amplitude and frequency response of audio amplifiers.

### 73.42. Function Generators

These generators typically, cover at least the same frequency range as audio signal generators (*i.e.* 20 Hz to 20 kHz) but most modern designs have extended frequency ranges. A very common frequency range for function generators is 0.01 Hz to 3 MHz.



The major difference between a function generator and an audio generator is in the number of output waveforms. The audio signal generator produces only sine waves and square waves. While almost all function generators produce these basic waveforms plus triangular waves. Besides this, some function generators also produce sawtooth, pulse and non-symmetrical square waves. Fig. 73.51 shows the controls of a typical function generator.



Fig. 73.51

Fig. 73.52 shows a simple block diagram of a function generator. The major parts of a function generator are schmitt trigger, integrator, sine-wave converter and an attenuator. The schmitt trigger converts a slowly varying input signal to a square wave signal. This square wave signal is available at the output as well as it is also connected to the integrator as an input through a potentiometer ( $R$ ). The potentiometer is used to adjust the frequency of the output signal. The frequency range is adjusted by selecting the appropriate capacitor connected in the integrator circuit.

The sine-wave converter is a six-level (or more) diode-resistor loading circuit.

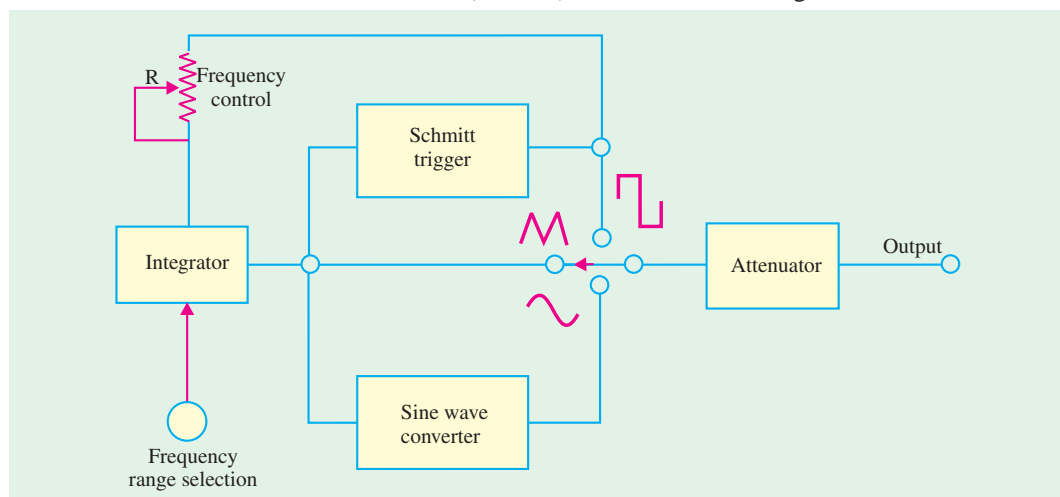


Fig. 73.52

Let us see how a simple diode-resistor circuit shown in Fig. 73.53 (a) is used to convert a triangular wave into a square wave.

Note that if diodes  $D_1$  and  $D_2$  and resistors  $R_3$  and  $R_4$  were not present in the circuit, of Fig. 73.53 (a),  $R_1$  and  $R_2$  would simply behave as a voltage divider. In this case the output from the circuit would be an attenuated version of the triangular wave :

$$V_0 = V_i \frac{R_2}{R_1 + R_2}$$

With diodes  $D_1$  and  $D_3$  in the circuit,  $R_1$  and  $R_2$  still behave as a simple voltage divider until the voltage drop across  $R_2$ ,  $V_{R_2}$  exceeds  $+V_1$ . At this point  $D_1$  becomes forward biased, and  $R_3$  is effectively in parallel with  $R_2$ . Now,

$$V_0 = V_i \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}$$

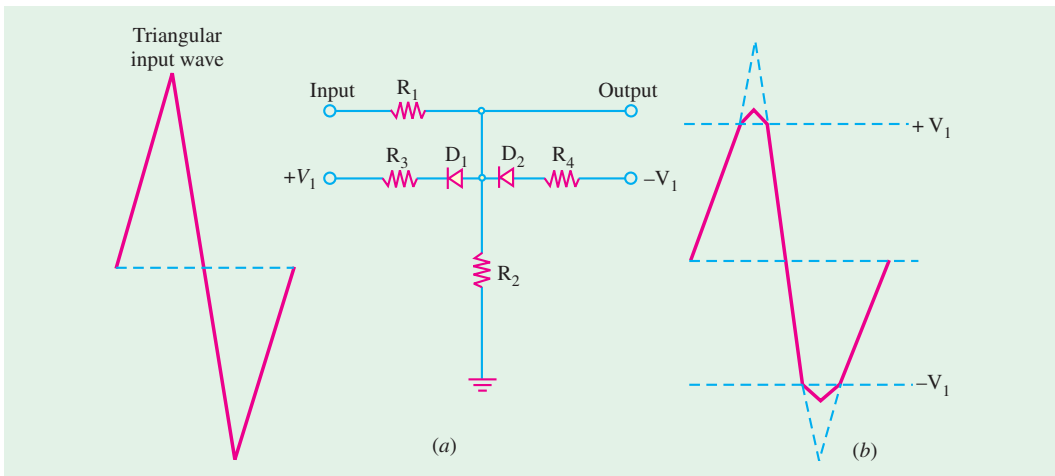


Fig. 73.53

Output voltage levels above  $+V_1$  are attenuated to a greater extent than levels below  $+V_1$ . Consequently, the output voltage rises less steeply than without  $D_1$  and  $R_3$  in the circuit (refer to Fig. 73.53 (b)). When the output falls below  $+V_1$ , the diode  $D_1$  is reverse biased. As a result of this,  $R_3$  is no longer in parallel with  $R_2$ , and the attenuation is once again  $R_2/(R_1 + R_2)$ . Similarly during the negative half cycle of the input, the output voltage,

$$V_0 = V_i \frac{R_2}{R_1 + R_2}$$

until  $V_0$  goes below  $-V_1$ . Then  $D_2$  becomes forward biased, putting  $R_4$  in parallel with  $R_2$  and making,

$$V_0 = V_i \frac{R_2 \parallel R_4}{R_1 + R_2 \parallel R_4}$$

With  $R_3 = R_4$ , the negative half-cycle of the output is similar in shape to the positive half-cycle. If we employ six or more diodes, all connected via resistors to different bias voltage levels (refer to Fig. 73.54 (a)), a good sine-wave approximation can be achieved. With six diodes, the three positive bias voltage levels and three negative bias voltage levels, the slope of the output wave changes three times during each quarter cycle. Assuming correctly selected bias voltages and resistor values, the output wave shape is as shown in Fig. 73.54 (b).

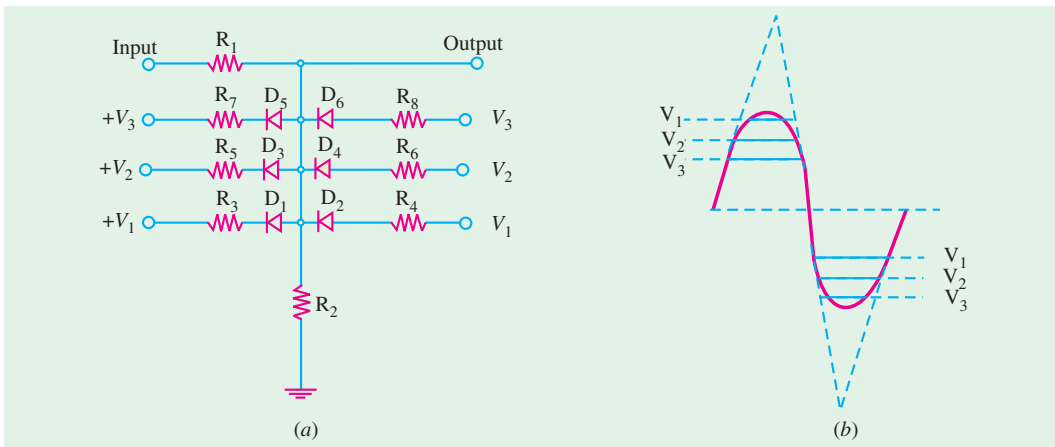


Fig. 73.54

### Applications

The function generator is an essential equipment for an electronic laboratory to generate signals to test a variety of analog and digital system during the design phase as well as to trouble shoot such systems.

Fig. 73.55 shows the picture of a Tektronix function generators Model No. CFG 253. This model has a frequency range form 0.03 Hz to 3 MHz. In addition to sine, square and triangular waves it can also produce *TTL* signals. Another function generator from Tektronix (Model No. CFG280 has a wide frequency range form 0.01 Hz to 11 MHz. It has a built in frequency counter with a range from 1 Hz to 100 MHz.



**Fig. 73.55**

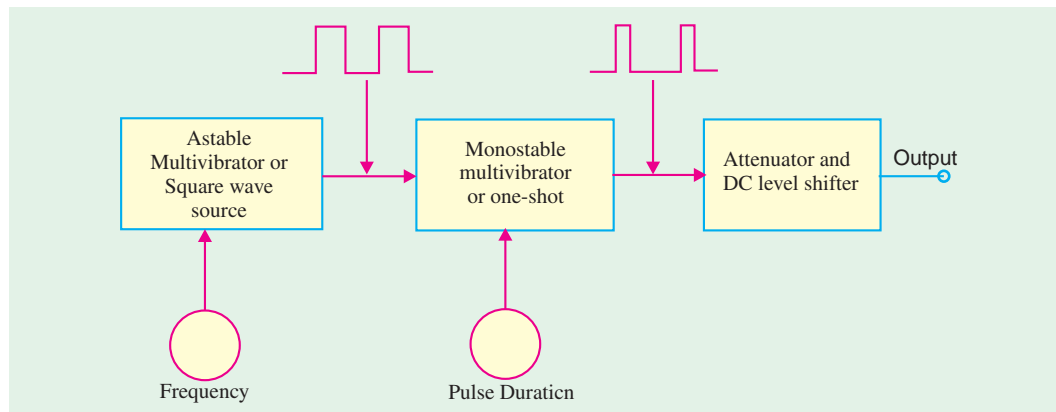
(Courtesy : Tektronix Corporation)

### Applications

The function generators can be employed in a variety of applications in the area of product design, training, manufacturing production test, field repair, bench calibration and repair, laboratory and research, and education. Mainly they are used for testing amplifiers, filter and digital circuits.

### 73.43. Pulse Generators

Fig. 73.56 shows the block diagram of a pulse generator. As seen, an astable multivibrator generates square waves. This is used to trigger monostable multivibrator (*i.e.* one-shot). The pulse repetition rate is set by the square-wave frequency. The one-shot triggers on the leading edge of the square-wave and produces one output pulse for each input cycle. The duration of each output pulse is set by the on-time of the one-shot. It may be very short or may approach the period of the square wave. The attenuator facilities output amplitude control and dc level shifting.



**Fig. 73.56**

A typical pulse generator will allow the user to select the repetition rate, duration, amplitude and number of output pulses to be output in a given burst. The most common frequency range is from 1 Hz to 50 MHz. The pulse width is adjustable from 10 ns to over 10 ms and the output is variable from 3 mV to 30 V.

Fig. 73.57 shows a pulse generator from Fluke Corporation Model No. PM5786. This instrument has a frequency range from 1 Hz to 125 MHz. The output pulse width can



**Fig. 73.57.** (Courtesy Fluke Corporation)

be varied from 8 ns to 100 ms. The instrument can also be used to generate pulse bursts. The output voltage level can be adjusted up to 10 V.

#### Applications

The pulse generators are used extensively to test :

1. Memory circuits
2. Shift registers
3. Counters
4. Other digital components, subsystems and systems.

### 73.44. RF Generators

A radio frequency (*RF*) signal generator has a sinusoidal output with a frequency somewhere in the range of 100 kHz to 40 GHz region. Fig. 73.58 shows the block diagram of an *RF* generator. As soon, the instrument consists of an *RF* oscillator, an amplifier, a calibrated attenuator and an output level meter.

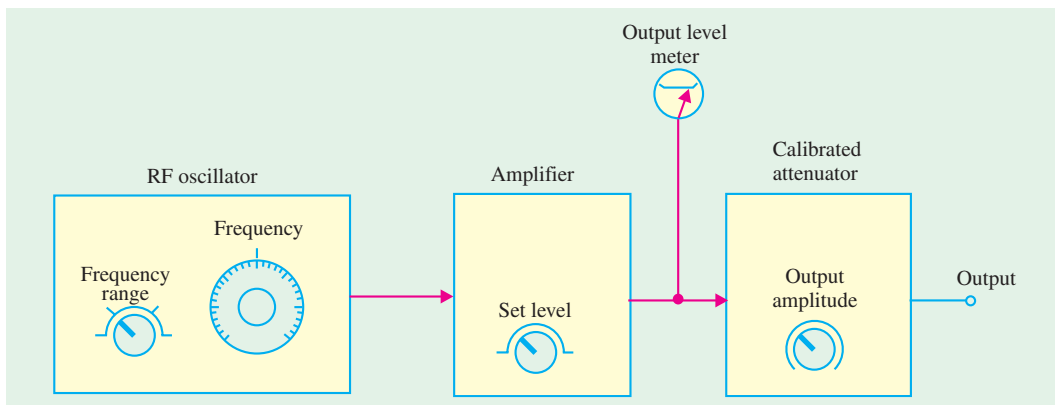


Fig. 73.58

The *RF* oscillator has a continuous frequency control and a frequency range switch, to set the output to any desired frequency. The amplifier includes an output amplitude control. This allows the voltage applied to the attenuator to be set to a calibration point on the output level meter. The output level must always be **reset** to this calibration point everytime the frequency is changed. This is necessary to ensure that the output voltage levels are correct, as indicated on the calibrated attenuator.

The oscillator circuit used in an *RF* generator is usually either a Hartley Oscillator or Colpitts oscillator. Most *RF* signal generators include facilities for amplitude modulation and frequency modulation of the output. Switches are provided on the front panel to allow the user to select no modulation as well as internal or external *AM* or *FM* modulation. It may be noted that each section of the *RF* generator is **shielded** by enclosing it in a metal box. The whole system is then completely shielded. The purpose of shielding is (1) to prevent *RF* interference between the components and (2) to prevent the emission of *RF* energy from any point except the output terminals. As a matter of fact, even the power line is decoupled by means of *RF* chokes and capacitors to prevent *RF* emission from it.

Fig. 73.59 (a) shows an analog *RF* generator Model No. SML01 manufactured by Tektronics Corporation. This



Fig. 73.59

(Courtesy Fluke Corporation)

instrument is a general purpose signal generator and is available at low cost. It has a wide frequency range from 9 kHz to 3.3 GHz. Another RF generator Model No. SMP04 shown in Fig. 73.59 (b) from Tektronix Corporation is a high precision signal source. It has a wide frequency range from 0.01 GHz to 40 GHz. This instrument can produce, AM-, FM-, phase- and pulse modulated signals as well.

### Applications

The RF signal generators are widely used in the area of radar and communication, research and development laboratories, education and training, electromagnetic interference (EMI) testing and material testing. Their main applications are :

1. To perform variety of tests on radio transmitters and receivers.
2. To test the amplitude and frequency response of RF amplifiers during the design phase.

### 73.45. Frequency Synthesizer

It is another type of RF generator that uses *phase-locked loop (PLL)* to generate output frequencies over a wide range. The most common range is from 1 MHz to 160 MHz. Fig. 73.60 shows a simple block diagram of a frequency synthesizer. As seen, the major components of the frequency synthesizer are : voltage controlled oscillator (VCO), divide-by-N counter, phase detector, crystal oscillator, low-pass filter and a square-wave circuit.

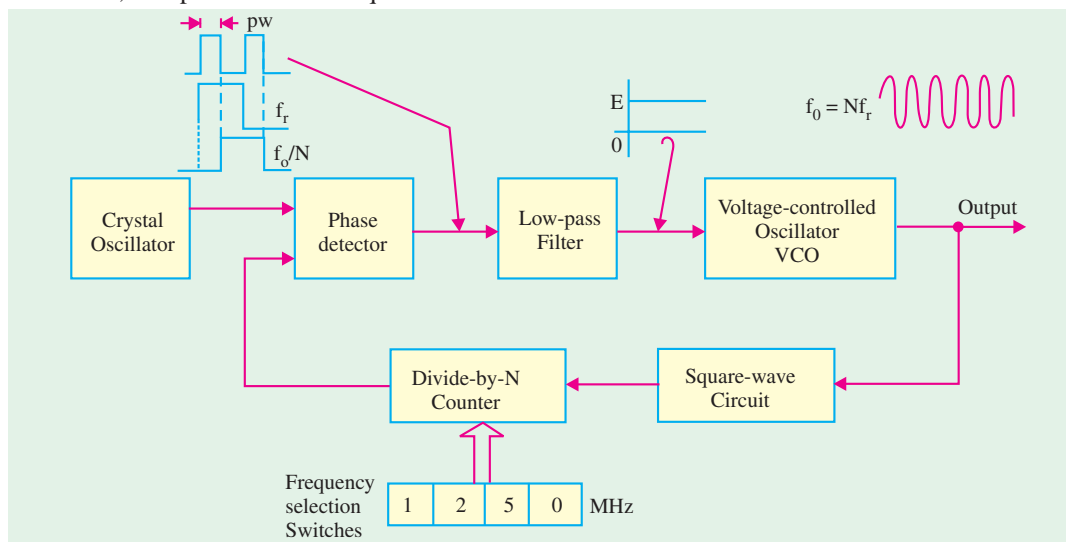


Fig. 73.60

The output of the crystal oscillator (a reference frequency,  $f_r$ ), is fed into one input of a phase detector. The other input of a phase detector has another square-wave applied to it as shown in the figure. The frequencies of these two square waves is identical but there is a phase difference ( $\phi$ ) between them. The output of the phase detector is a pulse waveform with pulse width controlled by the phase difference. The output of the phase detector is applied to the low-pass filter which converts it into a dc voltage,  $E$ . The dc voltage,  $E$  is used as the control voltage for the VCO and it determines the output frequency of VCO. The output of VCO is fed to a circuit that converts it into a square wave for triggering a digital divide-by-N counter. The divide-by-N counter divides the VCO frequency by a number set by a bank of switches. These switches may be push buttons with digital readouts or they may be thumb-wheel type which indicate their position numerically. The switches are connected in such a way that the displayed number is the factor  $N$  by which the output frequency is divided before being applied to the phase detector. The switches allow the user to obtain frequency which is any integer multiple of the crystal oscillator frequency.

### Applications

The frequency synthesizer is used in almost same areas as the RF signal generators.

### 73.46. Other Signal Generators

There are some signal generators that do not fit well in various pres-established categories. Some of these signal generators are as discussed below :

**1. RF Markers :** These devices are usually crystal controlled and have a fixed output frequency for use as a reference. These are used to calibrate TV signals.

**2. Digitally Programmable Test Oscillators :** These instruments can have extremely wide frequency range although some versions have much narrow range also. The set frequency can be programmed through the front panel keypad or via a computer interface input such as IEEE-488 general purpose interface bus (commonly known as GPIB).

**3. Arbitrary Waveform Generators :** These instruments allow the user to design and generate virtually any desired waveform. The arbitrary waveform generator is quite useful to perform a variety of tests on communication equipment. For example, a modulated signal that varies over the entire bandwidth and amplitude range of the equipment shown in Fig. 73.61 could be created for testing purpose. Noise could also be superimposed upon the signal and gaps might be introduced between waveform bursts, to investigate the response of the system. Once such a waveform has been designed, it could be stored in the instrument memory and can be recalled repeatedly for production testing.

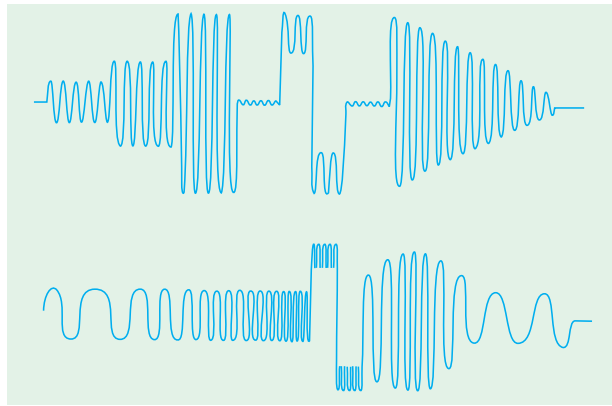


Fig. 73.61

Fig. 73.62 shows an arbitrary waveform generator from Agilent Technologies Model No. 33250A. This instrument can generate real world signals up to 80 MHz. It has a capability to display the waveforms in colour. It can be used as a function generator/pulse generator as well. The instrument has a GPIB/LAN interfaces.

### Applications

The arbitrary waveform generators are used extensively in the following areas :

1. Communications design and test for producing (a) arbitrary IF based signals and (b) standard waveforms for communication.
2. Mixed signal design and test.
3. Disk drive read/write design and test.
4. Real word simulations.
5. High-speed low filter data and clock pulse generation.



Fig. 73.62 (Courtesy : Agilent Technologies)

### 73.47. IEEE-488 General Purpose Interface Bus (GPIB) Instruments

These days automatic test equipment (ATE) is one of the leading methods for testing electronic equipment in factory production and troubleshooting situations. The basic method is to use a programmable digital computer to control a bank of test instruments. The bank of instruments can be configured for a special purpose or for general use. For example, we could select a particular line-up of equipment needed to test a broadcast audio console and provide a computer program to do a

variety of measurements such as gain, frequency response, total harmonic distortion etc. The other possibility could be to do a generalized test set. This method is adopted by number of industries who have many electronic devices or systems to test. There is a main bank of electronic test equipment adapters to make the devices (or systems) under test interconnect with the system and a computer program to do a variety of measurements such as gain, frequency response, total harmonic distortion etc. The other possibility could be to do a generalized test set. This method is adopted by number of industries who have many electronic devices or systems to test. There is a main bank of electronic test equipment adapters to make the devices (or systems) under test interconnect with the system and a computer program for each type of equipment. Such an approach reduces the test equipment cost drastically.

The Institution of Electrical and Electronic Engineers (*IEEE*) has laid out a specification titled *IEEE* standard Digital Interface for programmable instrumentation or *IEEE-488*. This specification provides details for a standard interface between a computer and instruments. The *IEEE-488* bus or General purpose interface bus ( *GPIB*) is a tool that is based on the *IEEE* specifications. The Hewlett-Packard interface bus (*HPiB*) is a proprietary version of the *IEEE-488* bus.

The digital signals on the *IEEE-488* bus are generally similar to *TTL* (transistor-transistor logic), *i.e.* a logic *LOW* is less than 0.8 V and a logic *HIGH* is greater than 2.0 V. The digital signals can be connected to the instruments through a multiconductor cable up to 20 metres in length provided that an instrument load is placed every 2 metres. Most *IEEE-488/GPIB* systems operate unrestricted to 250 kilobytes per second or faster with some restrictions.

There are two basic configurations for the *IEEE-488/GPIB* system : (1) linear and (2) star. In the linear configuration shown in Fig. 73.63 (a), a tap-off to the next instrument is taken from the previous one in series. On the other hand, in star configuration shown in Fig. 73.63 (b), the instruments are connected from a central point.

Fig. 73.64 shows the basic structure of *IEEE-488/GPIB* system. The figure indicates four different devices (*i.e.* computer, frequency counter, signal generator and digital multimeter) connected to the bus. The *IEEE-488/GPIB* system itself consists of three major buses : (1) general interface management (*GIM*) bus. (2) data I/O (*DIO*) bus and (3) data byte transfer (*DBT*) bus.

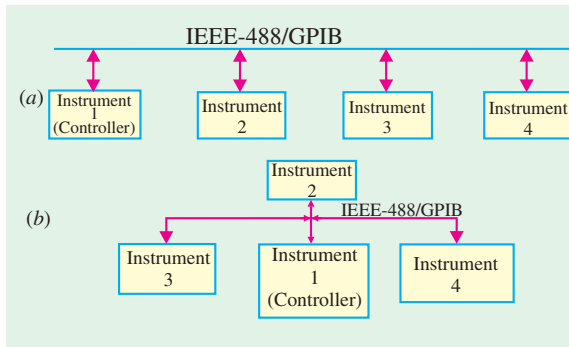


Fig. 73.63

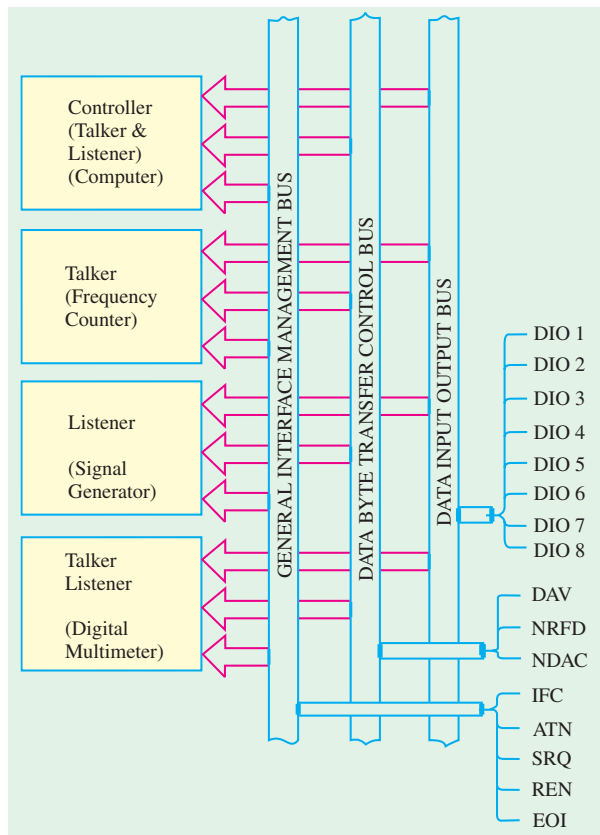


Fig. 73.64

**1. General Interface Management (GIM) bus :** This bus coordinates the whole system and ensures an orderly flow of data over the data input/output (DIO) bus. This bus has a number of signals explained below.

**IFC (Interface Clear Signal) :** This signal is used by the controller to place all devices in a predefined quiescent or standby condition.

**ATN (Attention Signal) :** This signal is used by the computer/controller to let the system know how data on the DIO bus lines are to be interpreted and which device is to respond to the data.

**SRQ (Service Request Signal) :** This signal is used by a device on the system to ask the controller for attention. It is basically an interrupt request signal.

**REN (Remote Enable Signal) :** This signal is used by the controller to select between two alternate sources of device programming data.

**EOI (End or Identify Signal) :** This signal is used by talkers for the following two purposes : (1) it will follow the end of a multiple byte sequence of data in order to indicate that the data are now finished. (2) It is also used in conjunction with the ATN signal for polling the system.

**2. Data I/O (DIO) bus :** This bus is a bidirectional 8-bit data bus that carries data, interface messages and device-dependent messages between the controller, talkers and listeners. This bus sends asynchronously in byte-serial format.

**3. Data Byte Transfer (DBT) bus:** This bus controls the sending of data along the DIO bus. There are three signals on this bus as explained below :

**DAV (Data Valid Signal) :** This signal indicates the availability and validity of data on the line. If the measurement is not finished, for example, the DAV signal will be false.

**NRFD (Not Ready For Data Signal) :** This signal lets the controller know whether or not the specific device addressed is in a condition to receive data.

**NDAC (Not Data Accepted Signal) :** This signal is used to indicate to the controller whether or not the device accepted the data sent to it over the DIO bus.

Each signal line in the bus has a circuit similar to the one shown in Fig. 73.65. As seen from this circuit, each signal line has a pull-up and pull-down resistors, receiver, and driver-circuits. Besides this each signal line has also a shunt protection diode and a stray capacitance.

Fig. 73.66 shows the 7-bit binary signals used in the IEEE-488/GPIB system for ASCII and

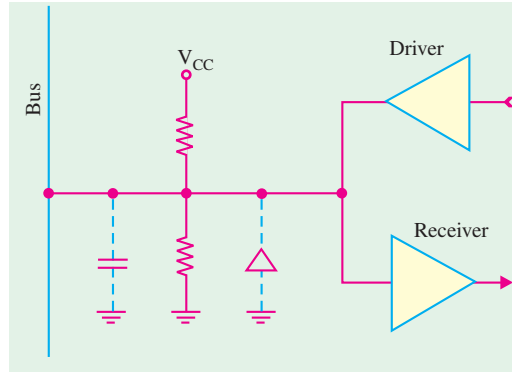


Fig. 73.65

BITS		ASCII & GPIB CODE CHART																															
B7	B6	CONTROL				NUMBERS SYMBOLS				UPPER CASE				LOWER CASE																			
B4	B3	B2	B1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25				
0	0	0	0	NUL	0	20	DLE	16	SP	0	60	16	100	0	120	16	140	0	160	16	P	112											
0	1	0	0	GTL	21	16	IL0	41	1	61	17	101	1	121	17	141	1	161	17														
0	1	1	0	SOH	11	17	DC1	21	33	31	49	41	65	51	81	61	97	71	q	113													
0	1	1	1	STX	22	18	DC2	22	42	2	62	18	102	2	122	18	142	2	162	18													
1	0	0	0	ETX	3	13	DC3	43	3	63	19	103	3	123	19	143	3	163	19														
1	0	0	1	SDC	24	DC4	44	4	64	20	104	4	124	20	144	4	164	20															
1	0	1	0	EOT	4	14	DC4	44	4	64	20	104	4	124	20	144	4	164	20														
1	0	1	1	PPC	25	PPU	45	5	65	21	105	5	125	21	145	5	165	21															
1	1	0	0	ENQ	5	25	NAK	25	37	35	53	45	69	55	85	65	101	75	u	117													
1	1	0	1	ACK	6	26	SYN	26	46	6	66	22	106	6	126	22	146	6	166	22													
1	1	1	0	BEL	7	27	ETB	27	47	7	67	23	107	7	127	23	147	7	167	23													
1	1	1	1	GET	7	17	SPE	23	37	39	57	47	71	57	87	67	103	77	w	119													
1	1	0	0	BS	8	18	CAN	24	28	40	38	56	48	72	58	88	68	104	78	x	120												
1	1	0	1	TCT	9	19	EM	25	29	41	39	57	49	73	53	89	69	105	79	y	121												
1	1	1	0	LF	10	32	SUB	32	52	10	72	26	112	10	132	26	152	10	172	26													
1	1	1	1	VT	11	33	ESC	26	46	11	73	27	113	11	133	27	153	11	173	27													
1	1	0	0	FF	11	33	FS	27	47	11	73	27	113	11	133	27	153	11	173	27													
1	1	1	0	CR	13	35	GS	29	49	13	75	29	115	13	135	29	155	13	175	29													
1	1	1	0	SO	15	36	RS	30	50	15	76	30	116	15	136	30	156	15	176	30													
1	1	1	1	E	15	36	RS	30	50	15	76	30	116	15	136	30	156	15	176	30													
1	1	1	1	SI	15	37	US	31	51	15	77	31	117	15	137	31	157	15	177	31													
					15	1F	31	2F	47	3F	63	4F	79	5F	95	6F	117	7F	127														
					ADDRESS	UNIVERSAL	COMMANDS	COMMANDS	LISTEN	ADDRESSES	TALK	ADDRESSES	SECONDARY	ADDRESSES	OR	COMMANDS																	

Fig. 73.66

**KEY**

25	PPU	GPIB Code	Tektronix® Ref : ANSI STD X3. 4-1977 IEEE STD 488-1978, ISO STD 646-1973
NAK	1	ASCII character decimal	



*GPIB* message codes.

The signals defined for the three buses in the *IEEE-488/GPIB* systems are implemented as conductors in a system interface cable. Each *IEEE-488/GPIB* compatible instrument will have a female 36-pin Amphenol-style connector on the rear panel. The pin-out definitions are given in Table 73.1.

The devices connected to *IEEE-488/GPIB* system (*i.e.* computer, frequency counter, signal generator and digital multimeter) are categorised as controller, listener and/or talker.

**Table 73.1**

<i>Pin No</i>	<i>Signal Lin</i>	<i>Pin No</i>	<i>Signal Line</i>
1	<i>DIO 1</i>	13	<i>DIO 5</i>
2	<i>DIO 2</i>	14	<i>DIO 6</i>
3	<i>DIO 3</i>	15	<i>DIO 7</i>
4	<i>DIO 4</i>	16	<i>DIO 8</i>
5	<i>EOI</i>	17	<i>REN</i>
6	<i>DAV</i>	18	Ground (6)
7	<i>NRFD</i>	19	Ground (7)
8	<i>NDAC</i>	20	Ground (8)
9	<i>IFC</i>	21	(Ground 9)
10	<i>SRQ</i>	22	(Ground 10)
11	<i>ATN</i>	23	(Ground 11)
12	Shield	24	Digital Ground

**1. Controller :** Its function is to communicate device addresses and other inface buses to instruments in the system.

**2. Listener :** Its function is to receive commands from other instruments (usually the controller) when the correct address is placed on the bus. The listener acts on the message received but does not send back any data to the controller. The signal generator shown in Fig. 73.22 is an example of a listener.

**3. Talker :** Its function is to respond to the message sent to it by the controller. The frequency counter shown in Fig. 73.62 is an example of a talker.

There is also a combination device that accepts commands from the controller to set up ranges, tasks etc. and then returns data back over the *DIO* bus to the controller. The digital multimeter shown in Fig. 73.62 is an example of this category.

The *IEEE-488* was introduced to the electronic industry in 1977. Since then it has evolved to *IEEE-488.1* in 1987 and further to *IEEE-488.2* in 1990. At present the system allows the control upto 14 instruments and it has data transfer rate greater than 1 M bytes/s.

### 73.48. VXI bus

The *VXI* bus is another fast growing platform for instrumentation systems. It was introduced in 1987 and since then it has experienced tremendous growth and acceptance around the world. *VXI* uses a mainframe chassis with a maximum of 13 slots to hold **modular instruments on plug-in boards**. Fig. 73.67 shows an example of system using *VXI* instruments. The *VXI* backplane includes



**Fig. 73.67**

the 32-bit VME data bus, high performance instrumentation buses for precision timing and synchronization between instrument components, standardized initialization and resource management to ease configuration.

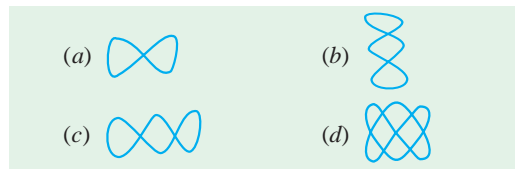
### OBJECTIVE TESTS – 73

- Digital instruments are those which
  - have numerical readout
  - use LED or LCD displays
  - have a circuitry of digital design
  - use deflection type meter movement.
- The main difference between the electronic and electrical instruments is that an electronic instrument contains
  - an electronic device
  - a transducer
  - a digital readout
  - electrons.
- The essential elements of an electronic instrument are
  - transducer
  - signal conditioner
  - indicating device
  - all of the above.
- The current sensitivity of a meter is expressed in
  - ampere
  - ohm/ampere
  - ohm/volt
  - ampere/division.
- The basic meter movement can be converted into an ohmmeter by connecting a .....with it.
  - high resistance in series
  - low resistance in parallel
  - battery in series
  - battery and a variable resistance in series
- The D' Arsonval meter movement can be converted into an audio-frequency ac ammeter by adding a ..... to it.
  - thermocouple
  - rectifier
  - chopper
  - transducer.
- In a linear meter, half-scale deflection occurs when there is ... per cent of the rated current through its coil
  - 100
  - 25
  - 50
  - 75
- A 0-1 mA meter has a sensitivity of
  - 1 k  $\Omega$ /V
  - 1 mA
  - 1 k  $\Omega$
  - 1000 A.
- A moving coil instrument has a resistance of 10  $\Omega$  and takes 40 mA to produce full-scale deflection. The shunt resistance required to convert this instrument for use as an ammeter of range 0 to 2 A is
  - 0.1021  $\Omega$
  - 0.2041  $\Omega$
  - 0.2561  $\Omega$
  - 0.4210  $\Omega$
- A moving coil ammeter has a fixed shunt of 0.02 ohm resistance. If the coil resistance of the meter is 1000  $\Omega$ , a potential difference of 500 mV is required across it for full-scale deflection. Under this condition, the current in the shunt would be
  - 2.5 A
  - 25 A
  - 0.25 A
  - 0.025 A
- It is desired to convert a 0-1000  $\mu$ A meter movement, with an internal resistance of 100  $\Omega$  into a 0-100 mA meter. The required value of shunt resistance is about
  - 1  $\Omega$
  - 10  $\Omega$
  - 99  $\Omega$
  - 100  $\Omega$
- Loading effect is principally caused by... instruments
  - high resistance
  - low-sensitivity
  - high-sensitivity
  - high-range
- A multimeter is used to measure
  - resistance
  - current
  - voltage
  - all of the above
- A sinusoidal voltage of rms value 10 V is applied to a D' Arsonval movement connected in series with a half-wave rectifier. It will show a reading of... volt
  - 9
  - 4.5
  - 10
  - 7.7
- A VTVM produces negligible loading effect on a circuit under test primarily because
  - it virtually drawn no current from the circuit
  - of its very high internal resistance
  - it uses high vacuum tubes
  - it is a null deflection instrument.
- In a 3½ digit voltmeter, the largest number that can be read is
  - 0999
  - 1999
  - 4999
  - 9999
- A 3½ digit voltmeter having a resolution of 100 mV can be used to measure maximum voltage of
  - 100 V
  - 200 V
  - 1000 V
  - 5000 V
- The signal to be observed on the screen of an oscilloscope is applied
  - across its X-plates
  - across its Y-plates
  - to the horizontal amplifier
  - to the trigger circuit.
- When a 30 V dc is applied to the vertical deflection plates of a cathode ray tube, the bright

spot moves 1 cm away from the centre. If 30 V (rms) ac is applied, then the movement of the spot will be nearly

- (a) 1 cm (b) 1.5 cm  
(c) 2 cm (d) 3 cm

20. Production of a steady stationary display of a signal waveform on the scope screen is due to  
(a) persistence of vision  
(b) fluorescent material of the screen  
(c) proper sync. between the signal and the sweep generator  
(d) electrostatic focussing of the electron beam.
21. Two sinusoidal signals of frequency  $f$  and  $3f$  are applied at  $x$  and  $y$  inputs respectively to an oscilloscope. Which one of the following patterns can be observed on the screen ?



22. The  $X$ - and  $Y$ -inputs of a  $CRO$  are respectively  $V \sin \omega t$  and  $-V \sin \omega t$ . The resulting Lissajous pattern will be  
(a) a straight line (b) a circle  
(c) an ellipse (d) a figure of eight
23. The deflection sensitivity of a  $CRT$  depends inversely on the  
(a) length of the vertical deflecting plates  
(b) distance between screen and deflecting plates  
(c) deflecting voltage  
(d) separation between  $Y$ -plates.

24. Two complete signal cycles would be displayed on the screen of a scope when time-period of the sweep generator is ..... the signal time period.

- (a) half (b) twice  
(c) equal (d) thrice

25. A non-triggered oscilloscope is one which  
(a) has no sweep generator  
(b) cannot produce a stable stationary screen display  
(c) has a continuously running time-base generator  
(d) can display a portion of the input signal wave form.

26. A dual-trace  $CRO$  has  
(a) one electron gun (b) two electron guns  
(c) one electron gun and one two-pole switch  
(d) two electron guns and one two-pole switch

27. The operation a  $Q$ -meter is based on  
(a) self-induction (b) series resonance  
(c) mutual induction (d) eddy currents.

28. The resolution of a logic analyser is  
(a) maximum number of input channels  
(b) the minimum duration of the glitch it can capture  
(c) its internal clock period  
(d) the minimum amplitude of the input signal it can display

29. A spectrum analyser can be described as  
(a) voltage selective frequency meter  
(b) current selective frequency meter  
(c) frequency selective voltmeter  
(d) None of these

ANSWERS

1. (c) 2. (a) 3. (d) 4. (c) 5. (d) 6. (b) 7. (c) 8. (a) 9. (b) 10. (b)  
11. (a) 12. (b) 13. (d) 14. (b) 15. (b) 16. (a) 17. (c) 18. (a) 19. (a) 20. (c)  
21. (b) 22. (a) 23. (d) 24. (b) 25. (c) 26. (c) 27. (b) 28. (a) 29. (d)