Electrical Power Systems

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New Academic Science

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To

My Parents

Wife

and

Children

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Preface

"Electrical Power System" has been written primarily for the undergraduate students in Electrical Engineering. The book covers conventional topics like the basics of power systems, line constant calculations, performance of lines, corona, mechanical design of overhead lines etc., and the more advanced topics like load flows studies, economic load dispatch, optimal power flows, state estimation in power systems etc. The book covers a very wide spectrum of electrical power system studies which is normally not available in one single book. The book is so comprehensibly written that at least five to six courses on power systems can be designed.

It has been the constant endeavour of the author to understand the difficulties of his students in the classroom and accordingly prepare the lecture notes after consulting various journals and books on electrical power systems. The present book is an outcome of these notes and some research work the author carried out.

Chapters 1 to 9 deal into the electrical and mechanical design of overhead and underground transmission networks. The analysis and performance of the system in terms of line constant calculations, efficiency and regulations, corona loss and interference of power lines with communication networks have been studied. A chapter on mechanical design of lines gives in a nutshell all the important aspects of erection of overhead lines. Chapter 5 on HVDC transmission discusses combined characteristics of rectifiers and inverters. Various controls like constant ignition angle, constant extinction angle, and constant currents have been discussed. Advantages and disadvantages of HVDC *vs* HVAC have been explained. Also, the role of HVDC link in improving system stability has been discussed.

Chapter 10 is devoted to the study of voltage-reactive power problems on transmission lines.

Chapter 11 defines an effectively grounded system, discusses and compares various systems of neutral grounding.

Chapter 12 describes transients in power systems. Travelling waves on transmission lines, capacitance switching and lightning phenomenon have been discussed.

Chapter 13 discusses calculation of symmetrical and asymmetrical fault conditions on the system, concept of infinite bus and short circuit capacity of a bus.

Electric Power System is the most capital intensive and the most complex system ever developed by man. Not only that the system should be operated most effectively and efficiently, any abnormality in the operation of the system must be detected fast and reliable operation of the protective system must be ascertained. Protective relays is the subject of Chapter 14 of the book wherein various types of relays from conventional electromechanical relays to digital protective relays have been discussed.

Chapter 15 presents material on the conventional circuit breakers like air break C.B., oil C.B., airblast C.B. etc., and the more advanced ones like the vacuum C.B. and SF_6 circuit breakers.

With the higher and higher operating voltages the impulse insulation levels of the system are increasing. Chapter 16 is devoted to the insulation problems of the system and the solutions in terms of coordinating the insulation levels economically of various equipments on the system have been discussed.

Chapter 17 deals into power system synchronous stability for a single machine connected to an infinite bus and multi-machine systems. Various techniques have been explained using algorithms and flow charts.

With the advent of digital computers and modern methods of network solution, it has now been possible to analyse the present day large interconnected systems with greater accuracy and short computational effort. Various techniques of load flow solutions of large networks have been discussed and explained using flow charts in Chapter 18. Various techniques have been compared in terms of their complexities and computational efforts.

With the advancement in technology of generation and load dispatching it has been possible to maintain the cost of electrical energy almost same even though the cost of fuel and other components have multiplied over the years. Chapter 19 on economic load dispatching discusses some of the classical techniques which even today are being used by the electric utilities. The techniques have been explained with the help of flow charts, algorithms and suitable examples.

Chapter 20 deals into the load frequency control or automatic generation control problems of the system.

The economics of a.c. power transmission has always forced the planning engineers to transmit as much power as possible through existing transmission lines. The need for higher index of reliability, the availability of hydro-power over long distances from the load centres, the difficulty of acquiring right-of-way for new transmission lines (the so-called corridor crisis) and the increased pressure to maximise the utilisation of both new and existing lines has helped to motivate the development and application of compensation system. Chapter 21 on compensation in power system discusses elaborately both the series and shunt compensation of overhead lines. The concept of FACTS (Flexible A.C. Transmission Systems) has also been introduced.

The voltage stability also known as load stability is now a major concern in planning and operation of electric power system. Chapter 22 on power system voltage stability discusses various factors which lead to this problem and methods to improve the voltage stability of the system.

State estimation is the process of determining a set of values by making use of the measurements made from the system and since the measurements may not be precise due to inherent errors associated with measuring devices, statistical methods have been discussed in Chapter 23, using the line power flows and maximum likelihood criterion have been discussed in detail with a number of solved problems. Techniques to detect and identify bad data during measurements have also been discussed.

Unit commitment is a way out to suggest just sufficient number of generating units with sufficient generating capacity to meet a given load demand economically with sufficient reserve capacity to meet any abnormal operating conditions. This aspect has been nicely dealt with suitable examples in Chapter 24.

Chapter 25 deals into economic scheduling of hydro-thermal plants and optimal power flows including the multi-objective optimal power flows.

Appendix A on formulation of bus impedance matrix is given which is very useful for the analysis of the system, especially for short circuit studies. Power transmission and synchronous machines as power systems elements have been discussed in Appendices B and C respectively.

A suitable number of problems have been solved to help understand the relevant theory. At the end of each chapter unsolved problems with their answers have been suggested for further practice. At the end, a large number of multiple choice questions have been added to help the reader to test himself. An extensive bibliography will help the reader to locate detailed information on various topics of his interest.

Any constructive suggestions for the improvement of the book will be gratefully acknowledged.

Last but not the least, I wish to express my gratitude to my wife Usha, daughter Meenu and son Sandeep for their patience and encouragement during the preparation of the book.

C.L. WADHWA

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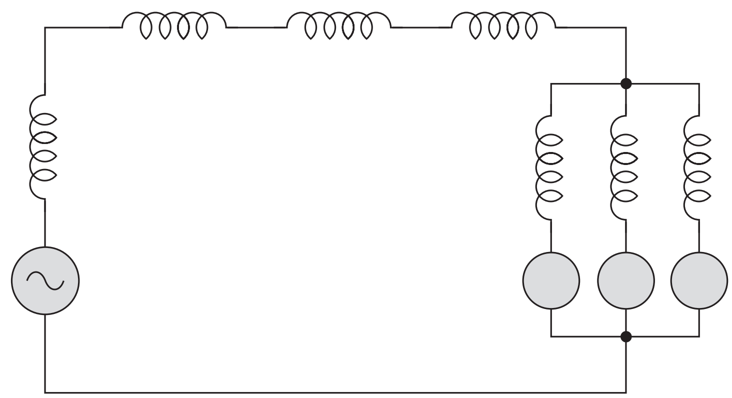
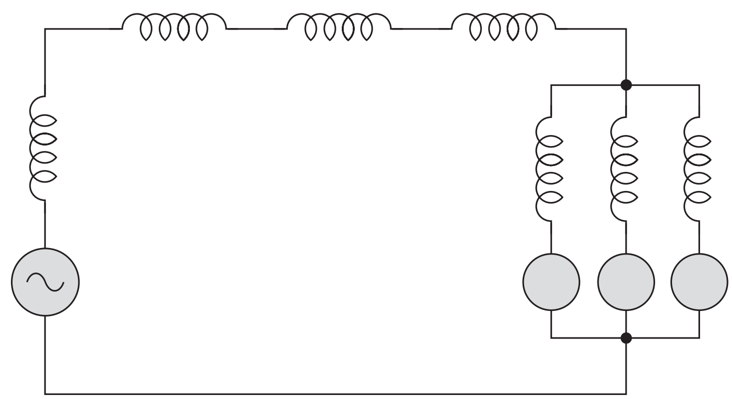


1

FUNDAMENTALS OF POWER SYSTEMS



1

Fundamentals of Power Systems

INTRODUCTION

The three basic elements of electrical engineering are resistor, inductor and capacitor. The resistor consumes ohmic or dissipative energy whereas the inductor and capacitor store in the positive half cycle and give away in the negative half cycle of supply the magnetic field and electric field energies respectively. The ohmic form of energy is dissipated into heat whenever a current flows in a resistive medium. If I is the current flowing for a period of t seconds through a resistance of R ohms, the heat dissipated will be I^2Rt watt sec. In case of an inductor the energy is stored in the form of magnetic field. For a coil of L henries and a current of I amperes flowing, the energy stored is given by $\frac{1}{2}LI^2$. The energy is stored between the metallic plates of the capacitor in the form of electric field and is given by $\frac{1}{2}CV^2$, where C is the capacitance and V is the voltage across the plates.

We shall start with power transmission using 1-ϕ circuits and assume in all our analysis that the source is a perfect sinusoidal with fundamental frequency component only.

1.1 SINGLE-PHASE TRANSMISSION

Let us consider an inductive circuit and let the instantaneous voltage be

$$v = V_m \sin \omega t \tag{1.1}$$

Then the current will be $i = I_m \sin (\omega t - \phi)$, where ϕ is the angle by which the current lags the voltage (Fig. 1.1).

The instantaneous power is given by

$$\begin{aligned} p = vi &= V_m \sin \omega t \,.\, I_m \sin (\omega t - \phi) \\ &= V_m I_m \sin \omega t \sin (\omega t - \phi) \qquad (1.2) \\ &= \frac{V_m I_m}{2} [\cos \phi - \cos (2\omega t - \phi)] \end{aligned}$$

The value of p is positive when both v and i are either positive or negative and represents the rate at which the energy is being consumed by the load. In this case the current flows in the direction of voltage drop. On the other hand power is negative when the current flows in the direction of voltage rise which means that the energy is being transferred from the load into the network to which it is connected. If the circuit is purely reactive the voltage and current will be 90° out of phase and hence the power will have equal positive and negative half cycles and the average value will be zero. From equation (1.2) the power pulsates around the average power at double the supply frequency.

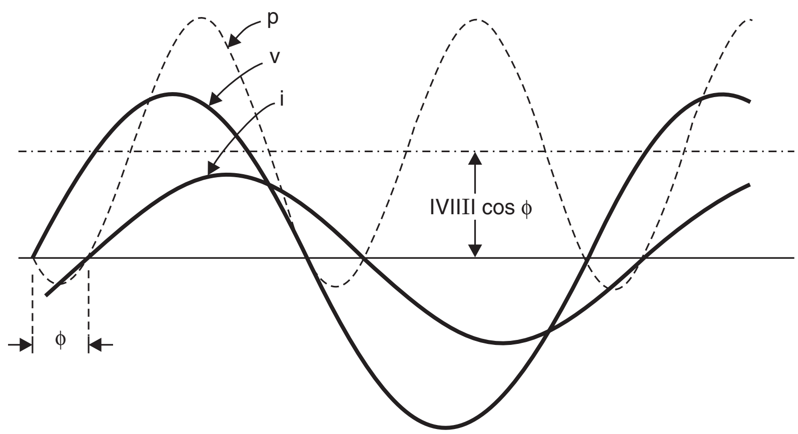


Fig. 1.1 Voltage, current and power in single phase circuit.

Equation (1.2) can be rewritten as

$$p = \underbrace{VI \cos \phi\,(1 - \cos 2\omega t)}_{\text{I}} - \underbrace{VI \sin \phi \sin 2\omega t}_{\text{II}} \tag{1.3}$$

We have decomposed the instantaneous power into two components (Fig. 1.2).

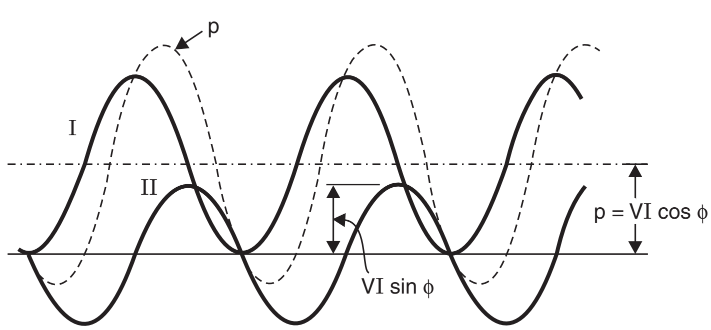


Fig. 1.2 Active, reactive and total power in a single phase circuit.

(*i*) The component P marked I pulsates around the same average power $VI \cos \phi$ but never goes negative as the factor $(1 - \cos 2\omega t)$ can at the most become zero but it will never go negative. We define this average power as the real power P which physically means the useful power being transmitted.

(*ii*) The component marked II contains the term sin ϕ which is negative for capacitive circuit and is positive for inductive circuit. This component pulsates and has zero as its average value. This component is known as reactive power as it travels back and forth on the line without doing any useful work.

Equation (1.3) is rewritten as

$$p = P(1 - \cos 2\omega t) - Q \sin 2\omega t \tag{1.4}$$

Both P and Q have the same dimensions of watts but to emphasise the fact that Q represents a nonactive power, it is measured in terms of voltamperes reactive *i.e.*, $V\,Ar$.

The term Q requires more attention because of the interesting property of sin ϕ which is – ve for capacitive circuits and is +ve for inductive circuits. This means a capacitor is a generator of positive reactive $V\,Ar$, a concept which is usually adopted by power system engineers. So it is better to consider a capacitor supplying a lagging current rather than taking a leading current (Fig. 1.3).

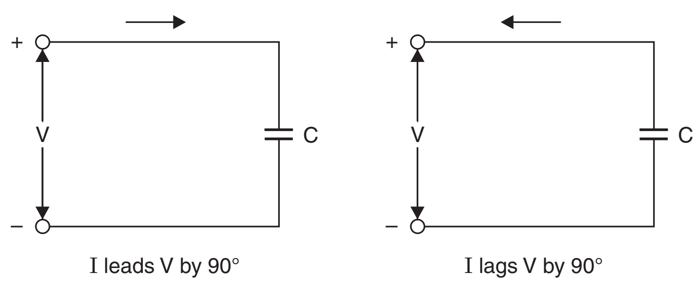


Fig. 1.3 V-I relations in a capacitor.

Consider a circuit in which an inductive load is shunted by a capacitor. If Q is the total reactive power requirement of the load and Q' is the reactive power that the capacitor can generate, the net reactive power to be transmitted over the line will be $(Q - Q')$. This is the basic concept of synchronous phase modifiers for controlling the voltage of the system. The phase modifier controls the flow of reactive power by suitable excitation and hence the voltage is controlled. The phase modifier is basically a synchronous machine working as a capacitor when overexcited and as an inductor when underexcited.

It is interesting to consider the case when a capacitor and an inductor of the same reactive power requirement are connected in parallel (Fig. 1.4).

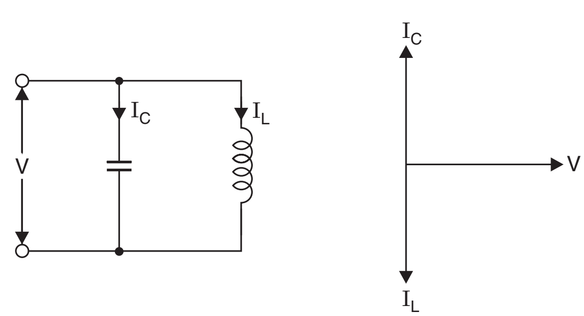


Fig. 1.4 Power flow in L-C circuit.

The currents I_L and I_C are equal in magnitude and, therefore, the power requirement is same. The line power will, therefore, be zero. Physically this means that the energy travels back and forth between the capacitor and the inductor. In one half cycle at a particular moment the capacitor is fully charged and the coil has no energy stored. Half a voltage cycle later the coil stores maximum energy and the capacitor is fully discharged.

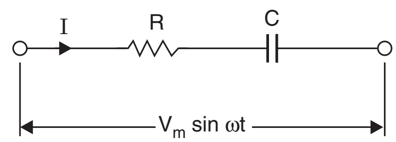


Fig. 1.5 Relationship between electric field energy and reactive power.

The following example illustrates the relationship between the reactive power and the electric field energy stored by the capacitor. Consider an *RC* circuit (Fig. 1.5).

From Fig. 1.5

$$I = \frac{V}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{V\omega C}{\sqrt{R^2\omega^2C^2 + 1}} \tag{1.5}$$

and if voltage is taken as reference *i.e.*, $v = V_m \sin \omega t$, the current

$$i = I_m \sin(\omega t + \phi)$$

$$\therefore \quad i = \frac{V_m\omega C}{\sqrt{R^2\omega^2C^2 + 1}} \,.\, \sin(\omega t + \phi) \tag{1.6}$$

where $$\sin\phi = \frac{I/\omega C}{\sqrt{I^2R^2 + (I/\omega C)^2}} = \frac{1}{\sqrt{R^2\omega^2C^2 + 1}} \tag{1.7}$$

Now $$\text{reactive power } Q = VI \sin\phi \tag{1.8}$$

Substituting for I and $\sin\phi$, we have

$$Q = V \,.\, \frac{V\omega C}{\sqrt{R^2\omega^2C^2 + 1}} \,.\, \frac{1}{\sqrt{R^2\omega^2C^2 + 1}} = \frac{V^2\omega C}{R^2\omega^2C^2 + 1} \tag{1.9}$$

$$\therefore \quad \text{Reactive power} = \frac{V^2\omega C}{R^2\omega^2C^2 + 1}$$

Now this can be related with the electric energy stored by the capacitor. The energy stored by the capacitor

$$W = \tfrac{1}{2}Cv^2 \tag{1.10}$$

Now $$v = \frac{1}{C}\int i\, dt = \frac{1}{C}\frac{V_m\omega C}{\sqrt{R^2\omega^2C^2 + 1}} \,.\, \frac{\cos(\omega t + \phi)}{\omega} = \frac{V_m \cos(\omega t + \phi)}{\sqrt{R^2\omega^2C^2 + 1}} \tag{1.11}$$

$$\therefore \quad W = \tfrac{1}{2}C \,.\, \frac{V_m^2 \cos^2(\omega t + \phi)}{R^2\omega^2C^2 + 1} = \frac{V^2\cos^2(\omega t + \phi)}{R^2\omega^2C^2 + 1} \tag{1.12}$$

$$\frac{dW}{dt} = \frac{V^2}{R^2\omega^2C^2 + 1} \,.\, 2\cos(\omega t + \phi) \,.\, \sin(\omega t + \phi) \,.\, \omega C$$

$$= \frac{V^2\omega C}{R^2\omega^2C^2 + 1} \,.\, \sin 2(\omega t + \phi)$$

$$= Q \sin 2(\omega t + \phi) \tag{1.13}$$

From this it is clear that the rate of change of electric field energy is a harmonically varying quantity with a frequency double the supply frequency and has a peak value equal to Q.

In an R-L circuit the magnetic field energy and reactive power in a coil are similarly related.

1.2 THE 3-PHASE TRANSMISSION

Assuming that the system is balanced which means that the 3-phase voltages and currents are balanced. These quantities can be expressed mathematically as follows:

$$\begin{aligned}
V_a &= V_m \sin \omega t \\
V_b &= V_m \sin (\omega t - 120°) \\
V_c &= V_m \sin (\omega t + 120°) \\
i_a &= I_m \sin (\omega t - \phi) \\
i_b &= I_m \sin (\omega t - \phi - 120°) \\
i_c &= I_m \sin (\omega t - \phi + 120°)
\end{aligned} \tag{1.14}$$

The total power transmitted equals the sum of the individual powers in each phase.

$$\begin{aligned}
p &= V_a i_a + V_b i_b + V_c i_c \\
&= V_m \sin \omega t I_m \sin (\omega t - \phi) + V_m \sin (\omega t - 120°)\, I_m \sin (\omega t - 120° - \phi) \\
&\qquad + V_m \sin (\omega t + 120°) I_m \sin (\omega t + 120° - \phi) \\
&= VI[2 \sin \omega t \sin (\omega t - \phi) + 2 \sin (\omega t - 120°) \sin (\omega t - 120° - \phi) \\
&\qquad + 2 \sin (\omega t + 120°) \sin (\omega t + 120° - \phi)] \\
&= VI[\cos \phi - \cos (2\omega t - \phi) + \cos \phi - \cos (2\omega t - 240° - \phi) \\
&\qquad + \cos \phi - \cos (2\omega t + 240° - \phi)] \\
&= 3VI \cos \phi
\end{aligned} \tag{1.15}$$

This shows that the total instantaneous 3-phase power is constant and is equal to three times the real power per phase *i.e.*, $p = 3P$, where P is the power per phase.

In case of single phase transmission we noted that the instantaneous power expression contained both the real and reactive power expression but here in case of 3-phase we find that the instantaneous power is constant. This does not mean that the reactive power is of no importance in a 3-phase system.

For a 3-phase system the sum of three currents at any instant is zero, this does not mean that the current in each phase is zero. Similarly, even though the sum of reactive power instantaneously in 3-phase system is zero but in each phase it does exist and is equal to $VI \sin \phi$ and, therefore, for 3-ϕ the reactive power is equal to $Q_{3\phi} = 3VI \sin \phi = 3Q$, where Q is the reactive power in each phase. It is to be noted here that the term $Q_{3\phi}$ makes as little physical sense as would the concept of 3-phase currents $I_{3\phi} = 3I$. Nevertheless the reactive power in a 3-phase system is expressed as $Q_{3\phi}$. This is done to maintain symmetry between the active and reactive powers.

1.3 COMPLEX POWER

Consider a single phase network and let

$$V = |V|e^{j\alpha} \text{ and } I = |I|e^{j\beta} \tag{1.16}$$

where α and β are the angles that V and I subtend with respect to some reference axis. We calculate the real and reactive power by finding the product of V with the conjugate of I *i.e.*,

$$\begin{aligned} S = VI^* &= |V|e^{j\alpha}\ |I|e^{-j\beta} = |V|\ |I|e^{j(\alpha-\beta)} \\ &= |V|\ |I| \cos(\alpha - \beta) + j|V|\ |I| \sin(\alpha - \beta) \end{aligned} \tag{1.17}$$

Here the angle $(\alpha - \beta)$ is the phase difference between the phasor V and I and is normally denoted by ϕ.

$$\begin{aligned} \therefore \qquad S &= |V|\ |I| \cos\phi + j|V|\ |I| \sin\phi \\ &= P + jQ \end{aligned} \tag{1.18}$$

The quantity S is called the complex power. The magnitude of $S = \sqrt{P^2 + Q^2}$ is termed as the apparent power and its units are volt-amperes and the larger units are kVA or MVA. The practical significance of apparent power is as a rating unit of generators and transformers, as the apparent power rating is a direct indication of heating of machine which determines the rating of the machines. It is to be noted that Q is positive when $(\alpha - \beta)$ is positive *i.e.*, when V leads I *i.e.*, the load is inductive and Q is –ve when V lags I *i.e.*, the load is capacitive. This agrees with the normal convention adopted in power system *i.e.*, taking Q due to an inductive load as +ve and Q due to a capacitive load as negative. Therefore, to obtain proper sign for reactive power it is necessary to find out VI^* rather than V^*I which would reverse the sign for Q as

$$\begin{aligned} V^*I &= |V|e^{-j\alpha}\ |I|e^{j\beta} = |V|\ |I|e^{-j(\alpha-\beta)} \\ &= |V|\ |I| \cos(\alpha - \beta) - j|V|\ |I| \sin(\alpha - \beta) \\ &= |V|\ |I| \cos\phi - j|V|\ |I| \sin\phi \\ &= P - jQ \end{aligned} \tag{1.19}$$

1.4 LOAD CHARACTERISTICS

In an electric power system it is difficult to predict the load variation accurately. The load devices may vary from a few watt night lamps to multi-megawatt induction motors. The following category of loads are present in a system:

(*i*)	Motor devices	70%
(*ii*)	Heating and lighting equipment	25%
(*iii*)	Electronic devices	5%

The heating load maintains constant resistance with voltage change and hence the power varies with (voltage)^2 whereas lighting load is independent of frequency and power consumed varies as $V^{1.6}$ rather than V^2.

For an impedance load *i.e.,* lumped load

$$P = \frac{V^2}{R^2 + (2\pi fL)^2} \,.\, R$$

and

$$Q = \frac{V^2}{R^2 + (2\pi fL)^2} \,.\, (2\pi fL) \tag{1.20}$$

From this it is clear that both P and Q increase as the square of voltage magnitude. Also with increasing frequency the active power P decreases whereas Q increases.

The above equations are of the form

$$\begin{aligned} P &= P\,[f,\ |V|] \\ Q &= Q\,[f,\ |V|] \end{aligned} \tag{1.21}$$

Composite loads which form a major part of the system load are also function of voltage and frequency and can, in general, be written as in equation (1.21). For this type of load, however, no direct relationship is available as for impedance loads. For a particular composite load an empirical relation between the load, and voltage and frequency can be obtained. Normally we are concerned with incremental changes in P and Q as a function of incremental changes in $|\ V\ |$ and f. From equation (1.21).

$$\Delta P \simeq \frac{\partial P}{\partial |V|} \,.\, |\Delta V| + \frac{\partial P}{\partial f} \,.\, \Delta f$$

and

$$\Delta Q \simeq \frac{\partial Q}{\partial |V|} \,.\, |\Delta V| + \frac{\partial Q}{\partial f} \,.\, \Delta f \tag{1.22}$$

The four partial derivatives can be obtained empirically. However, it is to be remembered that whereas an impedance load P decreases with increasing frequency, a composite load will increase. This is because a composite load mostly consists of induction motors which always will experience increased load, as frequency or speed increases.

The need for ensuring a high degree of service reliability in the operation of modern electric systems can hardly be over-emphasized. The supply should not only be reliable but should be of good quality *i.e.*, the voltage and frequency should vary within certain limits, otherwise operation of the system at subnormal frequency and lower voltage will result in serious problems, especially in case of fractional horse-power motors. In case of refrigerators reduced frequency results into reduced efficiency and high consumption as the motor draws larger current at reduced power factor. The system operation at subnormal frequency and voltage leads to the loss of revenue to the suppliers due to accompanying reduction in load demand. The most serious effect of subnormal frequency and voltage is on the operation of the thermal power station auxiliaries. The output of the auxiliaries goes down as a result of which the generation is also decreased. This may result in complete shut-down of the plant if corrective measures like load shedding is not resorted to. Load shedding is done with the help of under-frequency relays which automatically disconnect blocks of loads or sectionalise the transmission system depending upon the system requirements.

1.5 THE PER UNIT SYSTEM

In a large interconnected power system with various voltage levels and various capacity equipments it has been found quite convenient to work with per unit (p.u.) system of quantities for analysis purposes rather than in absolute values of quantities. Sometimes per cent values are used instead of p.u. but it is always convenient to use p.u. values. The p.u. value of any quantity is defined as

$$\frac{\text{The actual value of the quantity (in any unit)}}{\text{The base or reference value in the same unit}}$$

In electrical engineering the three basic quantities are voltage, current and impedance. If we choose any two of them as the base or reference quantity, the third one automatically will have a base or reference value depending upon the other two *e.g.*, if V and I are the base voltage and current in a system, the base impedance of the system is fixed and is given by

$$Z = \frac{V}{I}$$

The ratings of the equipments in a power system are given in terms of operating voltage and the capacity in kVA. Therefore, it is found convenient and useful to select voltage and kVA as the base quantities. Let V_b be the base voltage and kVA_b be the base kilovoltamperes, then

$$V_{\text{p.u.}} = \frac{V_{\text{actual}}}{V_b}$$

The base current $= \dfrac{kVA_b \times 1000}{V_b}$

$\therefore$ p.u. current $= \dfrac{\text{Actual current}}{\text{Base current}} = \dfrac{\text{Actual current}}{kVA_b \times 1000} \times V_b$

Base impedance $= \dfrac{\text{Base voltage}}{\text{Base current}}$

$$= \frac{{V_b}^2}{kVA_b \times 1000}$$

$\therefore$ p.u. impedance $= \dfrac{\text{Actual impedance}}{\text{Base impedance}}$

$$= \frac{Z \,.\, kVA_b \times 1000}{{V_b}^2} = \frac{Z \,.\, MVA_b}{(kV_b)^2}$$

This means that the p.u. impedance is directly proportional to the base kVA and inversely proportional to square of base voltage. Normally the p.u. impedance of various equipments corresponding to its own rating voltage and kVA are given and since we choose one common base kVA and voltage for the whole system, therefore, it is desired to find out the p.u. impedance of the various equipments corresponding to the common base voltage and kVA. If the individual quantities are $Z_{\text{p.u. old}}$, kVA_{old} and V_{old} and the common base quantities are $Z_{\text{p.u. new}}$, kVA_{new} and V_{new}, then making use of the relation above,

$$Z_{\text{p.u. new}} = Z_{\text{p.u. old}} \cdot \frac{kVA_{\text{new}}}{kVA_{\text{old}}} \cdot \left(\frac{V_{\text{old}}}{V_{\text{new}}}\right)^2 \qquad (1.23)$$

This is a very important relation used in power system analysis.

The p.u. impedance of an equipment corresponding to its own rating is given by

$$Z_{\text{p.u.}} = \frac{IZ}{V}$$

where Z is the absolute value of the impedance of the equipment. It is seen that the p.u. representation of the impedance of an equipment is more meaningful than its absolute value *e.g.*, saying that the impedance of a machine is 10 ohms does not give any idea regarding the size of the machine. For a large size machine 10 ohms appears to be quite large, whereas for small machines 10 ohms is very small. Whereas for equipments of the same general type the p.u. volt drops and losses are in the same order regardless of size.

With p.u. system there is less chance of making mistake in phase and line voltages, single phase or three phase quantities. Also the p.u. impedance of the transformer is same whether referred on to primary or secondary side of the transformer which is not the case when considering absolute value of these impedances. This is illustrated below:

Let the impedance of the transformer referred to primary side be Z_p and that on the secondary side be Z_s, then

$$Z_p = Z_s \left(\frac{V_p}{V_s}\right)^2$$

where V_p and V_s are the primary and secondary voltages of the transformer.

Now

$$\begin{aligned} Z_{p\ \text{p.u.}} &= \frac{Z_p I_p}{V_p} = Z_s \left(\frac{V_p}{V_s}\right)^2 \cdot \frac{I_p}{V_p} \\ &= Z_s \cdot \frac{V_p I_p}{V_s^2} = Z_s \cdot \frac{V_s I_s}{V_s^2} = \frac{Z_s I_s}{V_s} \\ &= Z_{s\ \text{p.u.}} \end{aligned}$$

From this it is clear that the p.u. impedance of the transformer referred to primary side $Z_{p\ \text{p.u.}}$ is equal to the p.u. impedance of the transformer referred to the secondary side $Z_{s\ \text{p.u.}}$. This is a great advantage of p.u. system of calculation.

The base values in a system are selected in such a way that the p.u. voltages and currents in system are approximately unity. Sometimes the base kVA is chosen equal to the sum of the ratings of the various equipments on the system or equal to the capacity of the largest unit.

The different voltage levels in a power system are due to the presence of transformers. Therefore, the procedure for selecting base voltage is as follows: A voltage corresponding to any part of the system could be taken as a base and the base voltages in other parts of the circuit, separated from the original part by transformers is related through the turns ratio of the transformers. This is very important. Say, if the base voltage on primary side is V_{pb} then on the secondary side of the transformer the base voltage will be $V_{sb} = V_{pb}(N_s/N_p)$, where N_s and N_p are the turns of the transformer on secondary and primary side respectively.

The following example illustrates the procedure for selecting the base quantities in various parts of the system and their effect on the p.u. values of the impedances of the various equipments.

Example 1.1: A 100 MVA, 33 kV 3-phase generator has a subtransient reactance of 15%. The generator is connected to the motors through a transmission line and transformers as shown in Fig. E1.1*a*. The motors have rated inputs of 30 MVA, 20 MVA and 50 MVA at 30 kV with 20% subtransient reactance. The 3-phase transformers are rated at 110 MVA, 32 kV, Δ/110 kV Y with leakage reactance 8%. The line has a reactance of 50 ohms. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding p.u. values.

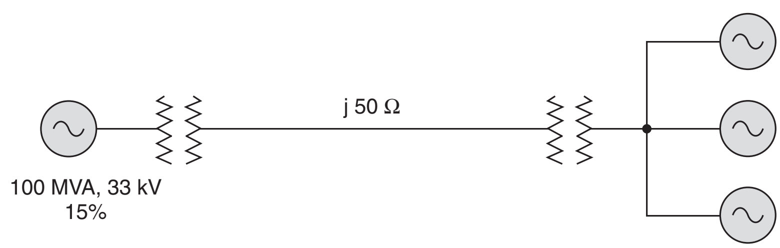


Fig. E1.1*a*

Solution: Assuming base values as 100 MVA and 33 kV in the generator circuit, the p.u. reactance of generator will be 15%. The base value of voltage in the line will be

$$33 \times \frac{110}{32} = 113.43 \text{ kV}$$

In the motor circuit,

$$113.43 \times \frac{32}{110} = 33 \text{ kV}$$

The reactance of the transformer given is 8% corresponding to 110 MVA, 32 kV. Therefore, corresponding to 100 MVA and 33 kV the p.u. reactance will be (using Eq. 1.23).

$$0.08 \times \frac{100}{110} \times \left(\frac{32}{33}\right)^2 = 0.06838 \text{ p.u.}$$

$$\text{The p.u. impedance of line} = \frac{50 \times 100}{(113.43)^2} = 0.3886 \text{ p.u.}$$

$$\text{The p.u. reactance of motor 1} = 0.2 \times \frac{100}{30} \times \left(\frac{30}{33}\right)^2 = 0.5509 \text{ p.u.}$$

$$\text{motor 2} = 0.2 \times \frac{100}{20} \times \left(\frac{30}{33}\right)^2 = 0.826 \text{ p.u.}$$

$$\text{motor 3} = 0.2 \times \frac{100}{50} \times \left(\frac{30}{33}\right)^2 = 0.3305 \text{ p.u.}$$

The reactance diagram for the system is shown in Fig. E1.1*b*.

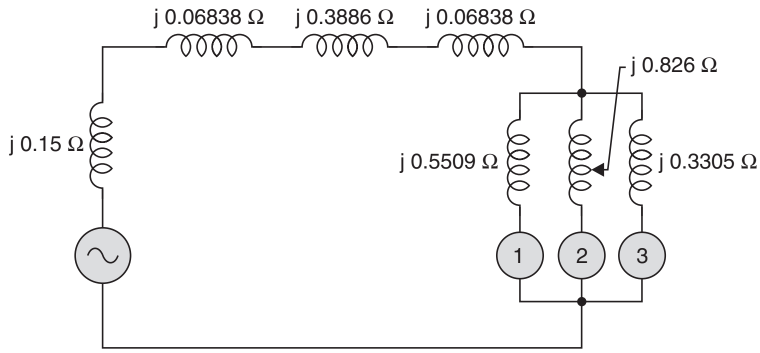


Fig. E1.1*b* Reactance diagram for Example 1.1.

PROBLEMS

1.1. Two generators rated at 10 MVA, 13.2 kV and 15 MVA, 13.2 kV are connected in parallel to a busbar. They feed supply to two motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 kV. Assuming base quantities as 50 MVA and 13.8 kV draw the reactance diagram. The per cent reactance for generators is 15% and that for motors is 20%.

1.2. Three generators are rated as follows: Generator 1–100 MVA, 33 kV, reactance 10%; Generator 2–150 MVA, 32 kV, reactance 8%; Generator 3–110 MVA, 30 kV, reactance 12%. Determine the reactance of the generator corresponding to base values of 200 MVA, 35 kV.

1.3. A 3-bus system is given in Fig. P1.3. The ratings of the various components are listed below:

Generator 1 = 50 MVA, 13.8 kV, X″ = 0.15 p.u.

Generator 2 = 40 MVA, 13.2 kV, X″ = 0.20

Generator 3 = 30 MVA, 11 kV, X″ = 0.25

Transformer 1 = 45 MVA, 11 kV, Δ/110 kV Y, X = 0.1 p.u.

Transformer 2 = 25 MVA, 12.5 kV, Δ/115 kV Y, X = 0.15 p.u.

Transformer 3 = 40 MVA, 12.5 kV, Δ/115 kV Y, X = 0.1 p.u.

The line impedances are shown in Fig. P1.3. Determine the reactance diagram based on 50 MVA and 13.8 kV as base quantities in Generator 1.

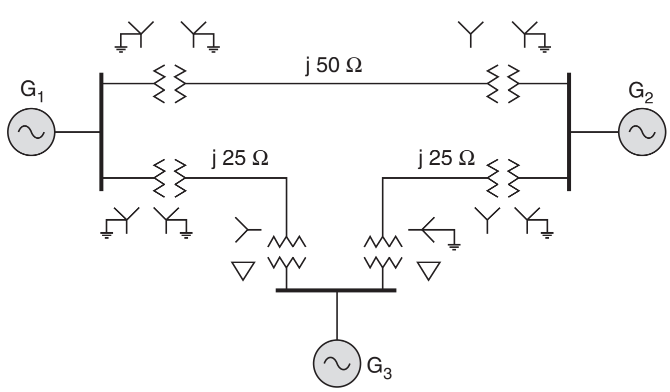


Fig. P1.3

1.4. Explain clearly the concept of reactive power in single phase and three phase circuits.

1.5. Explain clearly how the magnetic field energy and the reactive power in an inductive circuit are related.

1.6. Explain clearly what you mean by good quality supply and discuss the effect of bad supply on the performance of the system.

1.7. Explain the p.u. system of analysing power system problems. Discuss the advantages of this method over the absolute method of analysis.

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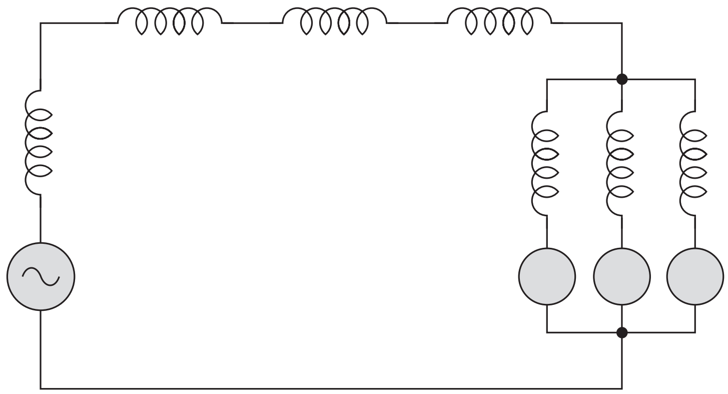
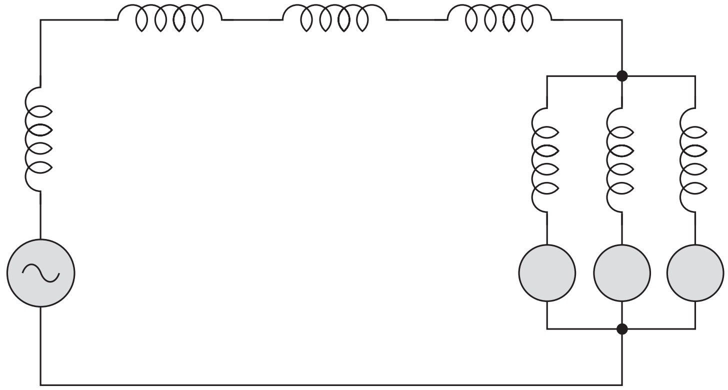


2

LINE CONSTANT CALCULATIONS



2

Line Constant Calculations

INTRODUCTION

An electric transmission line can be represented by a series combination of resistance, inductance and shunt combination of conductance and capacitance. These parameters are symbolized as *R, L, G* and *C* respectively. Of these *R* and *G* are least important in the sense that they do not affect much the total equivalent impedance of the line and hence the transmission capacity. They are of course very much important when transmission efficiency and economy are to be evaluated as they completely determine the real transmission line losses.

The resistance of a conductor is given by

$$R = \frac{\text{Power loss in conductor}}{I^2} \text{ ohms} \tag{2.1}$$

where R is the effective resistance of the conductor and I the current flowing through the conductor. The effective resistance is equal to the d.c. resistance of the conductor only if the current is uniformly distributed throughout the section of the conductor. The difference in the d.c. resistance and effective resistance to frequencies less than 50 Hz is less than 1% for copper conductors of section less than 350,000 circular mils. The loss on the overhead lines is due to (*i*) ohmic loss in the power conductors, (*ii*) corona loss and (*iii*) leakage at the insulators which support the lines at the towers. This leakage loss is different from the leakage in cables as in cables the leakage is uniformly distributed along its length, whereas the leakage on overhead lines is limited only to the insulators. This could be represented as conductance uniformly distributed along the line. Since the corona loss and the leakage over the insulators is negligibly small under normal operating conditions, the conductance between the conductors of an overhead line is assumed to be zero.

2.1 MAGNETIC FLUX DENSITY

A current carrying conductor produces a magnetic field which is in the form of closed circular loops around the conductor. The relation of the magnetic field direction to the current direction can be easily remembered by means of the right hand rule. With the thumb pointing in the direction of the current, the fingers of the right hand encircling the wire point in the direction of the magnetic field.

According to Biot-Savart's law, the magnetic flux density at any point P as produced by a current carrying element shown in Fig. 2.1 is given by

$$dB = \frac{\mu}{4\pi} \frac{Idl \sin \theta}{r^2} \tag{2.2}$$

where dB = infinitesimal flux density at point P,

I = current in the element,

dl = length of element,

θ = angle between current direction and radius vector to P, and

r = radius vector.

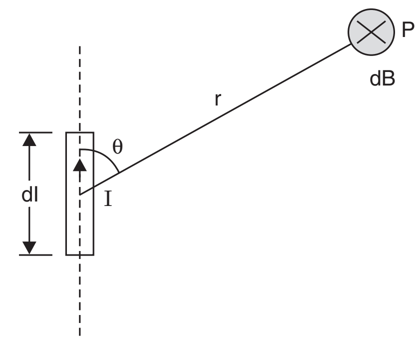


Fig. 2.1 Flux density to a current carrying element.

In order to determine the magnetic flux density B due to a long, straight or curved conductor, we assume that the conductor is made up of infinitesimal lengths dl and B is given by

$$B = \frac{\mu I}{4\pi} \int \frac{\sin \theta}{r^2}\, dl \tag{2.3}$$

The integration is carried out over the length of the conductor.

If relation (2.3) is made use of in evaluating the magnetic flux density B at any point due to an infinite conductor, it is given by

$$B = \frac{\mu I}{2\pi R} \tag{2.4}$$

where R = radial distance of the point from the conductor. The direction of the flux density is normal to the plane containing the conductor and the radius vector R.

If B is now integrated around a path of radius R enclosing the wire once (Fig. 2.2), we have

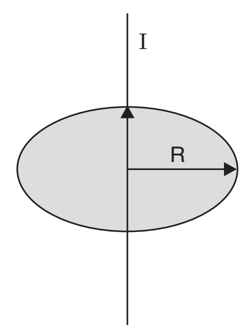


Fig. 2.2 Ampere's law: Line integral of H over a closed path.

$$\oint Bdl = \frac{\mu I}{2\pi R} \oint dl$$

$$= \frac{\mu I}{2\pi R} . 2\pi R = \mu I$$

or

$$\oint Hdl = I \text{ as } H = \frac{B}{\mu} \tag{2.5}$$

In words it states that the line integral of H around a single closed path is equal to the current enclosed. This is known as Ampere's law. If the path of integration encloses N number of turns of wire, each with a current I in the same direction, then

$$\int Hdl = NI \tag{2.6}$$

These relations are very much useful in evaluating the flux linkages and hence the inductance of a given system of conductors.

Variation of the current in the conductors causes a change in the number of flux linkages. According to Faraday's laws of electromagnetic induction, this change in flux linkages induces a voltage in the conductors which is proportional to the rate of change of flux linkages.

2.2 INDUCTORS AND INDUCTANCE

An inductor is a device which stores energy in a magnetic field. By definition, the inductance L of an inductor is the ratio of its total magnetic flux linkages to the current I through the inductor or

$$L = \frac{N\psi_m}{I} = \frac{\lambda}{I} \tag{2.7}$$

This definition is satisfactory for a medium for which the permeability is constant. However, the permeability of ferrous media is not constant and for such cases the inductance is defined as the ratio of the infinitesimal change in flux linkage to the infinitesimal change in current producing it, *i.e.,*

$$L = \frac{d\lambda}{dI} \tag{2.8}$$

The unit of inductance is the henry.

Mutual inductance between two circuits is defined as the flux linkages of one circuit due to the current in the second circuit per ampere of current in the second circuit. If the current I_2 produces λ_{12} flux linkages with circuit 1, the mutual inductance is

$$M_{12} = \frac{\lambda_{12}}{I_2} \text{ henries} \tag{2.9}$$

The phasor voltage drop in circuit 1 caused by the flux linkages of circuit 2 is

$$V_1 = j\omega M_{12} I_2 = j\omega\lambda_{12} \text{ volts.} \tag{2.10}$$

2.3 MAGNETIC FIELD INTENSITY DUE TO A LONG CURRENT CARRYING CONDUCTOR

Let us consider a long current carrying conductor with radius R as shown in Fig. 2.3. We will consider here that the current is uniformly distributed across the section of the conductor. The flux linkages here will be both due to internal flux and external flux. The magnetic field intensity due to the current distribution inside the conductor is calculated as follows:

Consider a cylinder with radius $r < R$. The current enclosed by the cylinder will be

$$I' = I\left(\frac{r}{R}\right)^2. \tag{2.11}$$

where I is the current through the conductor.

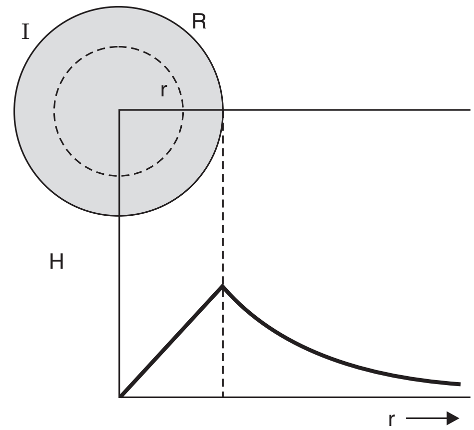


Fig. 2.3 Variation of H due to current in the conductor for r ≤ R and r > R.

Therefore, the magnetic field intensity at a distance r due to this current, using Ampere's Law,

$$H_r = \frac{I'}{2\pi r} = I\left(\frac{r}{R}\right)^2 \frac{1}{2\pi r} = \frac{Ir}{2\pi R^2} \tag{2.12}$$

which means that the magnetic field intensity inside the conductor is directly proportional to the distance from the centre of the conductor.

Now consider a cylinder with radius $r > R$. Applying Ampere's Law,

$$H = \frac{I}{2\pi r}$$

which means H is inversely proportional to r outside the conductor. The variation of H as a function of r is shown in Fig. 2.3. It can be shown that the magnetic field density (energy volume density)

$$W_e = \frac{1}{2}\mu H^2$$

From this and the distribution of magnetic field intensity as shown in Fig. 2.3, the following observations are made:

(*i*) Although the volume of the conductor is comparatively small, the field densities are of high magnitude, and the magnetic field energy stored in the conductor is not small.

(*ii*) The presence of the earth will affect the magnetic field geometry insignificantly.

2.4 INDUCTANCE OF TWO-WIRE (1-ϕ) TRANSMISSION LINE

By definition inductance is the flux linkages per ampere (Fig. 2.4). So the objective is to find out the flux linkages to this system of conductors. Now there are two flux linkages: (*i*) due to internal flux, and (*ii*) due to external flux.

Internal flux linkages: In order to determine the internal flux linkages, we start with the magnetic field intensity H at any distance $r < R$.

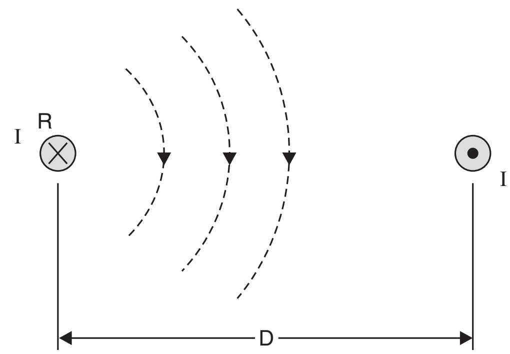


Fig. 2.4 Magnetic field due to one conductor of a 1-ϕ transmission line.

$$H = \frac{Ir}{2\pi R^2} \tag{2.13}$$

$$\therefore \quad B = \mu H = \mu_0 H = \frac{\mu_0 I}{2\pi R^2} \,.\, r \quad (\text{as } \mu_r = 1) \text{ for conductors.}$$

This flux density as we see is varying with r. We can assume this to be constant over an infinitesimal distance dr. The flux lines are in the form of circles concentric to the conductor. Therefore, the flux lines passing through the concentric cylindrical shells of radii r and $r + dr$,

$$d\phi = B \,.\, \text{Area normal to flux density } B$$
$$= Bdrl$$

where l is the length of wire.

In case the inductance per unit is desired, $l = 1$ metre.

$$\therefore \quad d\phi = Bdr$$
$$= \frac{\mu_0 I}{2\pi R^2}\, rdr$$

Now flux linkages = Flux × No. of turns.

Here since only a part of the conductor ($r < R$) is being enclosed by the flux lines $d\phi$,

$$\therefore \quad d\lambda = d\phi \left(\frac{r^2}{R^2}\right)$$
$$= \frac{\mu_0 I}{2\pi R^2}\, rdr \,\frac{r^2}{R^2}$$

$$\therefore \quad \text{Total internal flux linkages } \lambda = \int_0^R d\lambda$$
$$= \frac{\mu_0 I}{2\pi R^4} \int_0^R r^3 dr$$
$$= \frac{\mu_0 I}{8\pi} \tag{2.14}$$

From this it is clear that the flux linkage due to internal flux is independent of the size of the conductor.

External flux linkages: These flux linkages are due to the flux lines outside the conductor. There will be no flux line that encloses both the conductors. This is because for any distance $r > D$ the total current enclosed is zero (single phase line *i.e.*, one conductor is a 'go' conductor and the other 'return'). The magnetic field intensity H due to one conductor at any distance $R \le r < D$,

$$H = \frac{I}{2\pi r}$$

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r} \quad (\mu_r = 1 \text{ as the medium is air})$$

The flux density B can be considered uniform over a distance dr. Therefore, as in case of internal flux, the flux lines passing through the concentric cylindrical shells with radii r and $(r + dr)$ will be (per unit length)

$$d\phi = B.dr.1$$

Since this flux encloses only one conductor, therefore, the number of turns enclosed by this flux is one.

$$\therefore \quad d\lambda = d\phi.1$$

$$= B.dr.1.1$$

$$= \frac{\mu_0 I}{2\pi r} dr$$

Therefore, the total external flux linkages due to current flow in one conductor,

$$\lambda = \int_R^{D-R} d\lambda$$

The lower limit is because we measure the distances from the centre of the conductor and external flux begins from the surface of the conductor and this extends up to the surface of the other conductor and, therefore, the upper limit $(D - R)$

$$\lambda = \frac{\mu_0 I}{2\pi} \int_R^{D-R} \frac{dr}{r}$$

$$= \frac{\mu_0 I}{2\pi} \ln \frac{D-R}{R}$$

Since R is small as compared to D *i.e.* $R << D$,

$$D - R \simeq D$$

$$\therefore \quad \lambda = \frac{\mu_0 I}{2\pi} \ln \frac{D}{R}$$

$\therefore$ Total flux linkages due to one conductor

= Total internal flux linkages + Total external flux linkages

$$= \frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln \frac{D}{R} \qquad ...(2.15)$$

$\therefore$ Total flux linkage due to both the conductors $= 2\left[\frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln \frac{D}{R}\right]$

$\therefore$ Inductance L per unit length = $\left[\frac{\mu_0}{4\pi} + \frac{\mu_0}{\pi}\ln\frac{D}{R}\right]$ Henry/metre (2.16)

Since $\mu_0 = 4\pi \times 10^{-7}$,

$$L = \left[1 + 4\ln\frac{D}{R}\right] \times 10^{-7} \text{ Henry/metre}$$

$$= 4 \times 10^{-7} \times \left[\frac{1}{4} + \ln\frac{D}{R}\right] \text{ Henry/metre} \quad (2.17)$$

since $\ln e^{1/4} = \frac{1}{4}$

$$\therefore \quad L = 4 \times 10^{-7} \times \left(\ln e^{1/4} + \ln\frac{D}{R}\right)$$

$$= 4 \times 10^{-7} \ln\frac{D}{Re^{-1/4}}$$

$$= 4 \times 10^{-7} \ln\frac{D}{R'} \text{ Henry/metre} \quad (2.18)$$

The radius R' is that of a fictitious conductor assumed to have no internal flux linkages but with the same inductance as the actual conductor with radius R. The quantity $e^{-1/4} = 0.7788$. The multiplying factor of 0.7788 to adjust the radius in order to account for internal flux linkages applies only to solid round conductors.

2.5 FLUX LINKAGES OF ONE CONDUCTOR IN A GROUP OF CONDUCTORS

Let us now find out the flux linkages of one conductor due to current flowing in the conductor itself and the current flowing in the other conductors. It is assumed here that the sum of the currents in the various conductors is zero. The system of conductors is shown in Fig. 2.5.

Theoretically, the flux due to a conductor is extending from the centre of the conductor right up to infinity. We will assume here that P is a point very far from the group of the conductors, the flux linkages will extend up to this point and the distances are as shown in Fig. 2.5. The objective here is to calculate the flux linkages of say, conductor 1 due to the current I_1, carried by the conductor itself and flux linkage to conductor 1 due to the current carried by conductors 2, 3, ..., n.

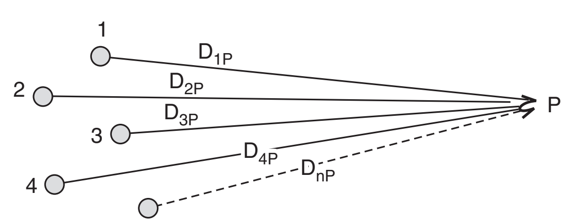


Fig. 2.5 Cross-sectional view of a group of n conductors. Point P is remote from the group of conductors.

The flux linkage of conductor 1 due to the current I_1 including the internal flux linkages

$$\lambda_{1p_1} = \frac{\mu_0 I_1}{8\pi} + \frac{\mu_0 I_1}{2\pi} \ln \frac{D_{1p}}{R_1}$$

$$= 2 \times 10^{-7} I_1 \ln \frac{D_{1p}}{R_1'} \tag{2.19}$$

The flux linkages λ_{1p_2} to conductor 1 due to current in conductor 2 are

$$\lambda_{1p_2} = 2 \times 10^{-7} I_2 \ln \frac{D_{2p}}{D_{12}} \tag{2.20}$$

It is to be seen that flux due to conductor 2 that lies between conductor 2 and 1 will not link conductor 1 and therefore the distances involved are D_{2p} and D_{12}.

The flux linkages of conductor 1 due to all the conductors

$$\lambda_{1p} = 2 \times 10^{-7} \left[I_1 \ln \frac{D_{1p}}{R_1'} + I_2 \ln \frac{D_{2p}}{D_{12}} + \ldots\ldots + I_n \ln \frac{D_{np}}{D_{1n}} \right]$$

$$= 2 \times 10^{-7} \left[I_1 \ln \frac{1}{R_1'} + I_2 \ln \frac{1}{D_{12}} + \ldots\ldots + I_n \ln \frac{1}{D_{1n}} \right]$$

$$+ 2 \times 10^{-7} [I_1 \ln D_{1p} + I_2 \ln D_{2p} + \ldots\ldots + I_n \ln D_{np}] \tag{2.21}$$

Since $I_1 + I_2 + \ldots\ldots + I_n = 0$,

$\therefore$ $$I_n = -(I_1 + I_2 + \ldots\ldots + I_{n-1})$$

Substituting this in the second term of equation (2.21),

$$\lambda_{1p} = 2 \times 10^{-7} \left[I_1 \ln \frac{1}{R_1'} + I_2 \ln \frac{1}{D_{12}} + \ldots\ldots + I_n \ln \frac{1}{D_{1n}} \right]$$

$$+ 2 \times 10^{-7} [I_1 \ln D_{1p} + I_2 \ln D_{2p} + \ldots\ldots + I_{n-1} \ln D_{(n-1)p}$$
$$- I_1 \ln D_{np} - I_2 \ln D_{np} \ldots\ldots - I_{n-1} \ln D_{np}]$$

$$= 2 \times 10^{-7} \left[I_1 \ln \frac{1}{R_1'} + I_2 \ln \frac{1}{D_{12}} + \ldots\ldots + I_n \ln \frac{1}{D_{1n}} \right]$$

$$+ 2 \times 10^{-7} \left[I_1 \ln \frac{D_{1p}}{D_{np}} + I_2 \ln \frac{D_{2p}}{D_{np}} + \ldots\ldots + I_{n-1} \ln \frac{D_{(n-1)p}}{D_{np}} \right] \tag{2.22}$$

Now P is a point very far from the group of the conductors, the ratios

$$\frac{D_{1p}}{D_{np}} \simeq \frac{D_{2p}}{D_{np}} \simeq \ldots\ldots \simeq \frac{D_{(n-1)p}}{D_{np}} = 1$$

$\therefore$ $$\ln \frac{D_{1p}}{D_{np}} \simeq \ln \frac{D_{2p}}{D_{np}} \simeq \ldots\ldots \simeq \ln \frac{D_{(n-1)p}}{D_{np}} = 0$$

The net flux linkages λ_{1p}, therefore, are

$$\lambda_{1p} = 2 \times 10^{-7} \left[I_1 \ln \frac{1}{R_1{}'} + I_2 \ln \frac{1}{D_{12}} + ... + I_n \ln \frac{1}{D_{1n}} \right] \text{wb-turns/metre} \tag{2.23}$$

2.6 INDUCTANCE OF 3-φ UNSYMMETRICALLY SPACED TRANSMISSION LINE

Consider a single circuit 3-φ system (Fig. 2.6) having three conductors a, b and c carrying currents I_a, I_b and I_c respectively. The three conductors are unsymmetrically placed *i.e.*, $a \neq b \neq c$ and each has a radius of R metres.

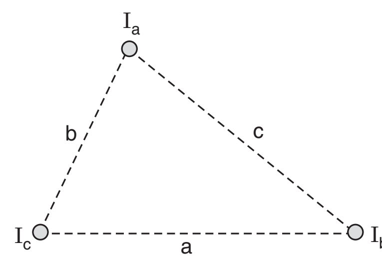


Fig. 2.6 3-φ transmission line with unsymmetrical spacing.

The flux linkage of conductor a due to I_a, I_b and I_c from equation (2.23),

$$\lambda_a = 2 \times 10^{-7} \left[I_a \ln \frac{1}{R'} + I_b \ln \frac{1}{c} + I_c \ln \frac{1}{b} \right]$$

Similarly,

$$\lambda_b = 2 \times 10^{-7} \left[I_a \ln \frac{1}{c} + I_b \ln \frac{1}{R'} + I_c \ln \frac{1}{a} \right]$$

$$\lambda_c = 2 \times 10^{-7} \left[I_a \ln \frac{1}{b} + I_b \ln \frac{1}{a} + I_c \ln \frac{1}{R'} \right]$$

Now taking I_a as reference

$$I_b = k^2 I_a \text{ and } I_c = k I_a$$

where $k = (-0.5 + j0.866)$

Substituting these values of I_b and I_c in the expression for λ_a,

$$\lambda_a = 2 \times 10^{-7} \left[I_a \ln \frac{1}{R'} + I_a (-0.5 - j0.866) \ln \frac{1}{c} + I_a (-0.5 + j0.866) \ln \frac{1}{b} \right]$$

$$\therefore \quad L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \left[\ln \frac{1}{R'} - \ln \frac{1}{\sqrt{bc}} - j \frac{\sqrt{3}}{2} \ln \frac{b}{c} \right] \tag{2.24}$$

Similarly, $$L_b = 2 \times 10^{-7} \left[\ln \frac{1}{R'} - \ln \frac{1}{\sqrt{ac}} - j \frac{\sqrt{3}}{2} \ln \frac{c}{a} \right] \tag{2.25}$$

and $$L_c = 2 \times 10^{-7} \left[\ln \frac{1}{R'} - \ln \frac{1}{\sqrt{ab}} - j \frac{\sqrt{3}}{2} \ln \frac{a}{b} \right] \tag{2.26}$$

It is clear from the expressions for inductances of conductors a, b and c that the three inductances are unequal and they contain imaginary term which is due to the mutual inductance.

In case the transmission line is transposed *i.e.*, each conductor takes all the three positions of the conductors, each position for one third length of the line as shown in Fig. 2.7. The average value of the inductance

$$L = \frac{L_a + L_b + L_c}{3}$$

$$= \frac{1}{3}\left[2 \times 10^{-7}\left(3 \ln \frac{1}{R'} - \ln \frac{1}{abc} - j\frac{\sqrt{3}}{2} \ln 1\right)\right]$$

$$= 2 \times 10^{-7} \ln \frac{\sqrt[3]{abc}}{R'} \text{ Henry/metre} \tag{2.27}$$

For symmetrical spacing $a = b = c = d$,

$$L = 2 \times 10^{-7} \ln \frac{d}{R'} \text{ Henry/metre.} \tag{2.28}$$

2.7 TRANSPOSITION OF POWER LINES

By transposition of conductors is meant the exchanging of position of the power conductors at regular intervals along the line, so that each conductor occupies the original position of every other conductor over an equal distance.

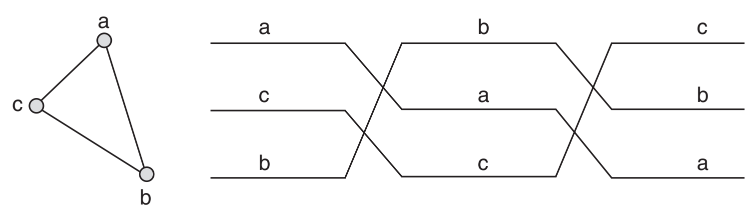


Fig. 2.7 Transposition of conductors.

A complete transposition cycle is shown in Fig. 2.7. If the spacing is unsymmetrical, even though the system operates under balanced condition, voltage drops of different magnitude will be there in the three conductors due to unequal inductance of the three phases. Also, due to unsymmetrical spacing, the magnetic field external to the conductors is not zero, thereby causing induced voltages in adjacent electrical circuits, particularly telephone lines, that may result in telephone interference. To reduce this effect to a minimum the conductors are transposed as shown in Fig. 2.7. It is enough to transpose either power line or the communication lines. Under balanced operating condition, the magnetic field linking an adjacent telephone line is shifted 120° in time phase with each rotation of the conductor positions in the transposition cycle. Over the length of the one complete transposition cycle of power line, the net voltage induced in the telephone line is zero as it is the sum of three induced voltages which are displaced by 120° in time phase. Under unbalanced conditions, of course, where power currents flow in the earth or in overhead ground wires (zero sequence currents), voltages will be induced in communication lines and interference will take place.

Modern power lines are normally not transposed. The transposition, however, may be affected at the intermediate switching station. It is to be noted that the difference in the inductances of the three phases is negligibly small due to asymmetrical spacing and the inductance of the untransposed line is taken equal to the average value of the inductance of one phase of the same line correctly transposed.

2.8 COMPOSITE CONDUCTORS

For transmission lines operating at high voltages normally stranded conductors are used. These conductors are known as composite conductors as they compose of two or more elements or strands electrically in parallel. The conductors used for transmission lines are stranded copper conductors, hollow copper conductors, ACSR conductors, copper weld and copper weld-copper conductors. By using different proportion of steel and aluminium strands different tensile and current carrying capacity conductors can be obtained. By the use of a filler such as a paper, between the outer aluminium strands and the inner steel strands, a conductor of large diameter can be obtained for use in high voltages. This type of conductor is known as expanded ACSR. Sometimes hollow conductors are used to increase the effective diameter of the conductor so as to reduce corona loss and hence radio interference level. A typical hollow copper conductor (Anaconda) consists of a twisted copper '*I*' beam as a core about which strands of copper wire are wound. The '*I*' beam is twisted in a direction opposite to that of the inner layer of strands.

Aluminium conductor steel reinforced (ACSR) which combine the lightness, electrical conductivity and rustlessness of aluminium with the high tensile strength of steel are now employed as overhead conductors on every kind of system, low voltage distribution to the most important long distance transmission lines of the world. The reasons for this can be summarised as follows:

1. Aluminium conductor steel reinforced (ACSR) are normally cheaper than copper conductors of equal resistance and this economy is obtained without sacrifice of efficiency, of reliability or of length of useful life.

2. The superior mechanical strength of ACSR can be utilized by using spans of larger lengths which results in smaller number of supports for a particular length of transmission.

3. A reduction in the number of supports involves a corresponding reduction in the total cost of insulators, foundations' erection and incidentally the costs of maintenance, replacements and stores are similarly reduced.

4. The increase in span length is beneficial in another way. It is well known that the vast majority of shut downs in the operation of an overhead line arise at points of supports, due to faulty insulators, flash-overs by birds and so on. Hence a reduction in the number of points of supports will correspondingly reduce the risk of outages.

5. Corona losses are reduced because of the larger diameter of the conductor.

6. These conductors are corrosion resistant and are useful under unfavourable conditions of industrial atmosphere and severe condition of exposure such as may occur on the sea coast.

The conductivity of an aluminium conductor steel reinforced is taken as that of the aluminium portion alone and though the steel core may add slightly to the current carrying capacity, this is usually neglected. The specific resistance of hard drawn aluminium is approximately 1.6 times that of normal hard drawn copper and the sectional area of an aluminium conductor must, therefore, be 1.6 times that of the equivalent copper. In order to obtain the overall diameter of a stranded conductor, multiply the wire diameter (diameter of one strand) D by the appropriate constant in the table below.

No. of wires of equal diameter	3	4	7	12	19	37	61	91
Dia-constant	2.155	2.41	3	4.155	5	7	9	11

2.9 INDUCTANCE OF COMPOSITE CONDUCTORS

An expression for the inductance of composite conductors will be derived. The composite conductors consist of two groups of conductors each having m and n number of strands respectively as shown in Fig. 2.8. The current is assumed to be equally divided amongst the strands. One group of conductors act as a 'go' conductor for the single-phase line and the other as the 'return'. The current per strand is I/m ampere in one group and $-I/n$ ampere in the other.

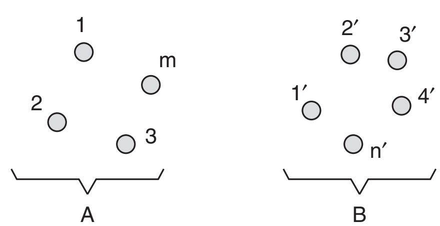


Fig. 2.8 Inductance of composite conductors—1-φ transmission line.

Using equation (2.23), the flux linkage of strand 1 in conductor A is given by

$$\lambda_1 = 2 \times 10^{-7}\frac{I}{m}\left[\ln\frac{1}{R'} + \ln\frac{1}{D_{12}} + \ldots\ldots + \ln\frac{1}{D_{1m}}\right]$$

$$- 2 \times 10^{-7}\frac{I}{n}\left[\ln\frac{1}{D_{11}'} + \ln\frac{1}{D_{12}'} + \ldots\ldots + \ln\frac{1}{D_{1n}'}\right]$$

$$= 2 \times 10^{-7} I \ln\frac{\sqrt[n]{D_{11}'D_{12}' \ldots\ldots D_{1n}'}}{\sqrt[m]{R' D_{12}D_{13} \ldots\ldots D_{1m}}} \text{ wb-turns/metre}$$

$$L_1 = \frac{\lambda_1}{I/m} = 2m \times 10^{-7}\ln\frac{\sqrt[n]{D_{11}'D_{12}' \ldots\ldots D'_{1n}}}{\sqrt[m]{R' D_{12}D_{13} \ldots\ldots D_{1m}}} \tag{2.29}$$

Similarly the inductance of filament 2 in conductor A

$$L_2 = \frac{\lambda_2}{I/m} = 2m \times 10^{-7}\ln\frac{\sqrt[n]{D_{21}'\,D_{22}'\,D_{23}' \ldots\ldots D_{2n}'}}{\sqrt[m]{R' D_{21}D_{23} \ldots\ldots D_{2m}}} \tag{2.30}$$

The average inductance of m strands in conductor A

$$L_{av} = \frac{L_1 + L_2 + \ldots\ldots + L_m}{m}$$

Since all the strands of conductor A are electrically parallel, the inductance of conductor A will be

$$L_A = \frac{L_{av}}{m} = \frac{L_1 + L_2 + \ldots\ldots + L_m}{m^2} \quad (2.31a)$$

Substituting the values of L_1, L_2, ..., L_m in equation (2.31a)

$$L_A = 2 \times 10^{-7} \ln \frac{\sqrt[mn]{(D'_{11}D'_{12} \ldots D_{1n})(D'_{21}D'_{22} \ldots D'_{2n}) \ldots (D'_{m1}D'_{m2} \ldots D'_{mn})}}{\sqrt[m^2]{(R' D_{12}D_{13} \ldots D_{1m})(R' D_{21}D_{23} \ldots D_{2m}) \ldots (R' D_{m1}D_{m2} \ldots D_{mm})}} \quad (2.31b)$$

The mnth root of the product of the mn distances between m strands of conductor A and n strands of conductor B is called geometric mean distance (GMD) and is denoted as D_m and the m^2th root of m^2 distances *i.e.*, the distances of the various strands from one of the strands and the radius of the same strand, the distances of such m groupings constitute m^2 terms in the denominator, is called the geometric mean radius (GMR) or self GMD and is denoted as D_s. The expression for inductance of conductor A consisting of m strands from equation (2.31b) becomes

$$L_A = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \text{ Henry/metre}$$

The inductance of conductor B can also be similarly obtained and the total inductance of the composite conductors is $L = L_A + L_B$.

Example 2.1: What will be the equivalent radius of a bundle conductor having its part conductors of radius 'r' on the periphery of a circle of dia 'd' if the number of conductors is 2, 3, 4, 6 ?

Solution: Let the equivalent radius or geometric mean radius be ρ_0, then for two conductors

$$\rho_0 = (rd)^{1/2} = r^{1/2} d^{1/2}$$

When there are three conductors

$$\rho_0 = (rd'd')^{1/3} = r^{1/3} d^{2/3} \left(\frac{3}{4}\right)^{1/3}$$

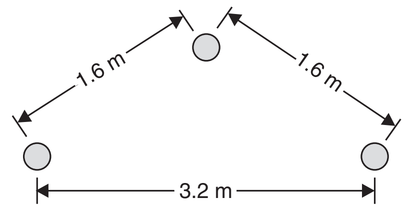
For four conductors

$$\rho_0 = \left(r \frac{d}{\sqrt{2}} \cdot \frac{d}{\sqrt{2}} d\right)^{1/4} = r^{1/4} d^{3/4} \left(\frac{1}{2}\right)^{1/4}$$

For six conductors

$$\rho_0 = r^{1/6} \left(\frac{d}{2}\right)^{5/6} 6^{1/6} = \left\{6r\left(\frac{d}{2}\right)^5\right\}^{1/6} \qquad \textbf{Ans.}$$

Example 2.2: Determine the inductance of a 3-phase line operating at 50 Hz and conductors arranged as follows. The conductor diameter is 0.8 cm.



Solution: The self GMD of the conductor

$$= \frac{0.7788 \times 0.8}{2 \times 100} = 0.003115 \text{ metres.}$$

The mutual GMD of the conductor

$$= \sqrt[3]{1.6 \times 3.2 \times 1.6} = 2.015 \text{ metres.}$$

$$\therefore \quad \text{Inductance per km} = 2 \times 10^{-4} \ln \frac{2.015}{0.003115}$$

$$= 2 \times 6.472 \times 10^{-4} \text{ Henry/km}$$

$$= 1.294 \text{ mH/km.} \quad \textbf{Ans.}$$

Example 2.3: A conductor consists of seven identical strands each having a radius of r. Determine the factor by which r should be multiplied to find the self GMD of the conductor.

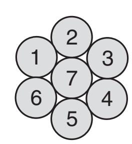
Solution: From the figure shown here

$$D_{11} = r$$

$$D_{12} = D_{16} = 2r = D_{17}$$

$$D_{14} = 4r$$

$$D_{13} = D_{15} = \sqrt{D_{14}^{\,2} - D_{45}^{\,2}}$$



as the conductors through 6 lie at the circumference of a circle.

$$\therefore \quad D_{13} = D_{15} = \sqrt{16r^2 - 4r^2} = \sqrt{12r^2} = 2\sqrt{3r}$$

$$\therefore \quad D_{s_1} = \sqrt[7]{r.2r.2\sqrt{3r}.4r.2\sqrt{3r}.2r.2r}$$

$$= \sqrt[7]{r^7.128 \times 3} = \sqrt[7]{384}.r$$

$$D_{s_7} = \sqrt[7]{2r.r.2r.2r.2r.2r.2r}$$

$$= \sqrt[7]{64r}$$

$$\therefore \quad D_s = \sqrt[7]{D_{s_1}\ D_{s_2}\ D_{s_3} \ldots D_{s_7}} = r\,.\sqrt[49]{384^6 \times 64}$$

$$= \sqrt[49]{2.04 \times 10^{17}}\ .\ r$$

$$D_{s_{eq}} = \sqrt[7]{0.7788}\ D_s = \sqrt[7]{0.7788}\ .\ \sqrt[49]{2.05 \times 10^{17}}\ .\ r$$

$$= 2.176r \quad \textbf{Ans.}$$

Example 2.4: Determine the inductance of a 1-ϕ transmission line consisting of three conductors of 2.5 mm radii in the 'go' conductor and two conductors of 5 mm radii in the, return, conductor. The configuration of the line is as shown in Fig. E.2.4.

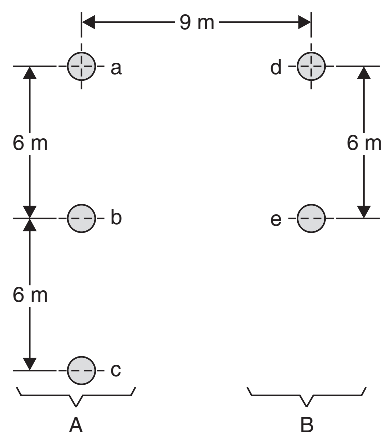


Fig. E.2.4

Solution: The self GMD of conductor in group A = 0.001947 m. The self GMD of conductor in group A,

$$D_{SA} = \sqrt[9]{0.001947 \times 6 \times 12 \times 0.001947 \times 6 \times 6 \times 0.001947 \times 6 \times 12}$$

$$= 0.4809 \text{ metres}$$

$$D_{SB} = \sqrt{5 \times 10^{-3} \times 0.7788 \times 6} = 0.1528 \text{ m}$$

$$D_{ae} = \sqrt{9^2 + 6^2} = 10.81 \text{ m}$$

$$D_{cd} = \sqrt{12^2 + 9^2} = 15 \text{ m}$$

$$D_{MA} = D_{MB} = \sqrt[6]{9 \times 10.81 \times 10.81 \times 9 \times 15 \times 10.81}$$

$$= 10.74 \text{ metres}$$

$$\therefore \quad \text{Inductance } L_A = 2 \times 10^{-7} \ln \frac{10.74}{0.4809}$$

$$= 0.62 \text{ mH/km}$$

$$L_B = 2 \times 10^{-7} \ln \frac{10.74}{0.1528} = 0.8 \text{ mH/km}$$

$\therefore$ Total inductance per km = 1.42 mH/km. **Ans.**

2.10 INDUCTANCE OF DOUBLE CIRCUIT 3-ϕ LINE

The double circuit line consists of three conductors in each circuit (Fig. 2.9). The three conductors correspond to three phases, a, b, c and a', b', and c'. Conductors a and a' are electrically parallel and constitute one phase. Similarly conductors b, b' and c, c' form other phases.

This means there are two conductors (strands) per phase.

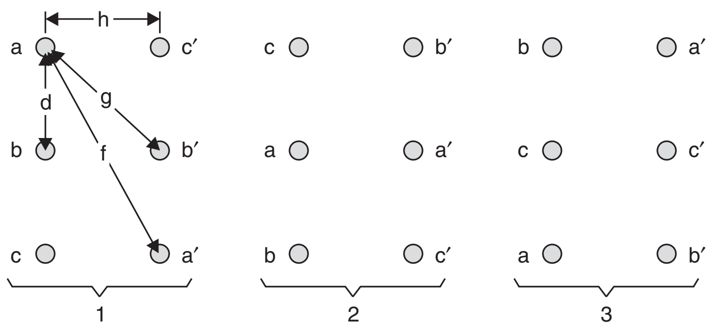


Fig. 2.9 Transposed double circuit line.

Since the conductors are not symmetrically placed, to calculate the inductance of the line, the conductors should be transposed. The three positions have been indicated in Fig. 2.9.

The GMD of the conductors in phase 'a' with the conductors in other two phases in position 1,

$$\text{GMD}_1 = (d \cdot 2d \cdot h \cdot g)^{1/4} = 2^{1/4}\, d^{1/2}\, g^{1/4}\, h^{1/4}$$

The GMD in the second position

$$\text{GMD}_2 = (d \cdot d \cdot g \cdot g)^{1/4} = d^{1/2}\, g^{1/2}$$

Similarly $$\text{GMD}_3 = (d \cdot 2d \cdot h \cdot g)^{1/4} = 2^{1/4}\, d^{1/2}\, g^{1/4}\, h^{1/4}$$

The equivalent GMD of the system is given by

$$\begin{aligned}\text{GMD} &= \{2^{1/4}\, d^{1/2}\, g^{1/4}\, h^{1/4}\, d^{1/2}\, g^{1/2}\, 2^{1/4}\, d^{1/2}\, g^{1/4}\, h^{1/4}\}^{1/3}\\ &= 2^{1/6}\, d^{1/2}\, g^{1/3}\, h^{1/6}\end{aligned}$$

Self GMD of phase aa' conductors in position 1,

$$\text{GMR}_1 = \text{Self GMD}_1 = \sqrt{r' f}$$

Self GMD in position 2.

$$\text{GMR}_2 = \sqrt{r' h}$$

and $$\text{GMR}_3 = \sqrt{r' f}$$

The equivalent $$\begin{aligned}\text{GMR} &= (r'^3 f^2 h)^{1/6}\\ &= r'^{1/2} f^{1/3}\, h^{1/6}\end{aligned}$$

$$\text{Inductance per phase} = 2 \times 10^{-7} \ln \frac{\text{GMD}}{\text{GMR}} \text{ Henry/metre/phase}$$

$$= 2 \times 10^{-7} \ln \frac{2^{1/6} d^{1/2} g^{1/3} h^{1/6}}{r'^{1/2} f^{1/3} h^{1/6}}$$

$$= 2 \times 10^{-7} \ln 2^{1/6} \left(\frac{d}{r'}\right)^{1/2} \left(\frac{g}{f}\right)^{1/3} \text{ H/metre/phase} \tag{2.32}$$

Here the conductors of two phases are placed diagonally opposite rather than in the same horizontal plane, in all the three positions. By doing this the self GMD of the conductors is increased whereas the GMD reduced, thereby the inductance per phase in lowered.

Example 2.5: Determine the inductance per km of a transposed double circuit 3-ϕ line shown in Fig. E.2.5. Each circuit of the line remains on its own side. The dia of the conductor is 2.532 cm.

Solution: Refer to Fig. E.2.5.

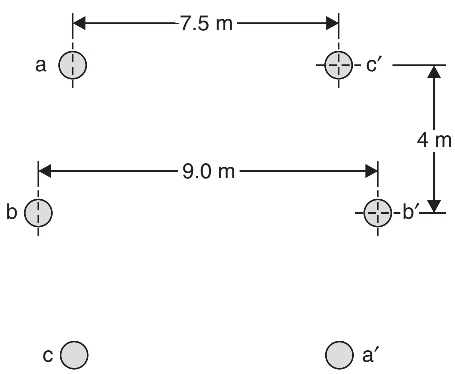


Fig. E.2.5

Self GMD of each conductor $= 1.266 \times 0.7788$ cm

$= 0.00986$ metre

$$D_{bc} = D_{ab} = \sqrt{4^2 + 0.75^2} = 4.0697 \text{ m}$$

$$D_{ab'} = \sqrt{4^2 + 8.25^2} = 9.1685 \text{ m}$$

$$D_{aa'} = \sqrt{8^2 + 7.5^2} = 10.965 \text{ m}$$

$$D_{m_1} = \sqrt[4]{4.0697 \times 8 \times 7.5 \times 9.168} = 6.878$$

$$D_{m_2} = \sqrt[4]{4.0697 \times 4.0697 \times 9.1685 \times 9.1685} = 6.1084$$

$$D_{m_3} = D_{m_1} = 6.878$$

$$\therefore \quad D_m = \sqrt[3]{D_{m_1} D_{m_2} D_{m_3}} = 6.61 \text{ m}$$

Self GMD of each phase $D_{s_1} = \sqrt{0.00986 \times 10.965}$

$= 0.3288 = D_{s_3}$

$$D_{s_2} = \sqrt{0.00986 \times 9} = 0.29789$$

$\therefore$ $D_s = \sqrt[3]{D_{s_1} D_{s_2} D_{s_3}} = 0.318 \text{ m}$

$\therefore$ $\text{Inductance} = 2 \times 10^{-4} \ln \dfrac{6.61}{0.318}$ H/km/phase

$= 0.606$ mH/km/phase. **Ans.**

Example 2.6: Determine the inductance of the double circuit line shown in Fig. E.2.6. The self GMD of the conductor is 0.0069 metre.

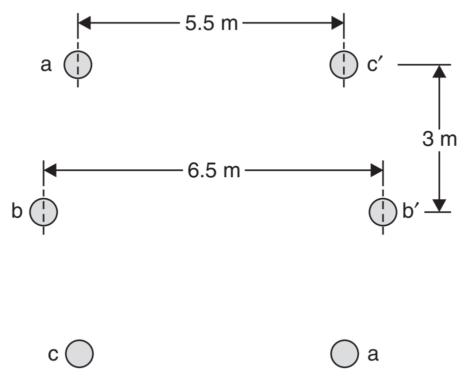


Fig. E.2.6

Solution: $D_{ab} = D_{bc} = \sqrt{3^2 + 0.5^2} = 3.04 \text{ m}$

$$D_{ac} = 6 \text{ m}$$

$$D_{ab'} = \sqrt{3^2 + 6^2} = 6.708 \text{ m}$$

$$D_{aa'} = \sqrt{6^2 + 5.5^2} = 8.14 \text{ m}$$

$$D_{m_1} = \sqrt[4]{3.04 \times 6 \times 5.5 \times 6.708} = 5.09 \text{ m} = D_{m_3}$$

$$D_{m_2} = \sqrt[4]{3.04 \times 3.04 \times 6.708 \times 6.708} = 4.515 \text{ m}$$

$\therefore$ $D_m = 4.89 \text{ m}$

Also $D_{s_1} = \sqrt{0.0069 \times 8.14} = 0.2370 = D_{s_3}$

$$D_{s_2} = 0.2117, D_s = 0.228 \text{ m}$$

$\text{Inductance} = 2 \times 10^{-7} \ln \dfrac{4.89}{0.228} = 0.613$ mH/km. **Ans.**

Example 2.7: Determine the inductance per km per phase of a single circuit 460 kV line using two bundle conductors per phase as shown in Fig. E.2.7. The dia of each conductor is 5.0 cm.

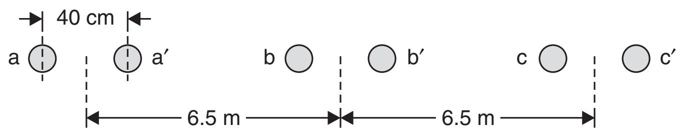


Fig. E.2.7

Solution: Assuming the effect of transposition to be negligibly small,

$$D_s = \sqrt{0.025 \times 0.4 \times 0.7788} = 0.08825$$

$$D_m = \sqrt[3]{6.5 \times 13.0 \times 6.5} = 8.19 \text{ m}$$

$$\therefore \quad \text{Inductance per km/phase} = 2 \times 10^{-4} \ln \frac{8.19}{0.08825}$$

$$= 0.906 \text{ mH/km/phase.} \quad \textbf{Ans.}$$

2.11 CONCEPT OF GEOMETRIC MEAN DISTANCE

Geometric mean distance is a mathematical concept used for the calculation of inductance. By definition the geometric mean distance of a point with respect to a number of points is the geometric mean of the distances between that point and each of the other points (Fig. 2.10).

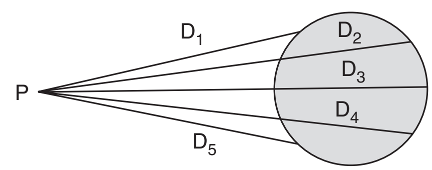


Fig. 2.10 Distances from an external point to five points on a circle.

The geometric mean distance of point P with respect to five points on the circle is

$$\text{GMD}_p = \sqrt[5]{D_1 D_2 D_3 D_4 D_5}$$

In case the number of points on the circle are increased to infinity, it can be seen intuitively that the geometric mean distance between the point P and the infinite points on the circle will be the geometric mean of all the distances and will correspond to the distance between the point P and centre of the circle.

The concept of GMD is applicable to areas also. The GMD between two circular areas will be the distance between the centres of the two areas and so on.

The GMD method does not apply strictly to non-homogeneous conductors such as ACSR or when the current is not uniformly distributed over the section of the conductor. An approximate value of inductance for ACSR conductors can be calculated by assuming negligible current in the steel strands.

2.12 BUNDLED CONDUCTORS

For voltages in excess of 230 kV, it is in fact not possible to use a round single conductor. Instead of going in for a hollow conductor it is preferable to use more than one conductor per phase which is known as bundling of conductors. A bundle conductor is a conductor made up of two or more sub-conductors and is used as one phase conductor. It is found that the increase in transmission capacity justifies economically the use of two conductor bundles on 220 kV lines. The following are the advantages in using bundle conductors:

1. Reduced reactance.
2. Reduced voltage gradient.
3. Reduced corona loss.
4. Reduced radio interference.
5. Reduced surge impedance.

The reactance of the bundle conductors is reduced because the self GMD of the conductors is increased and as we know reactance = $K \ln \dfrac{\text{GMD}}{\text{GMR}}$ and as GMR is increased the reactance is reduced.

Theoretically, there is an optimum sub-conductor spacing for bundle conductors that will give minimum gradient on the surface of a sub-conductor and hence highest disruptive voltage. For a two conductor bundle, the equation for maximum gradient at the surface of a sub-conductor is

$$g = \frac{V\left(1+\dfrac{2r}{s}\right)}{2r \ln \dfrac{d}{\sqrt{rs}}} \tag{2.33}$$

where s is the separation between the sub-conductors. Because of the effect of the sub-conductors on each other, the gradient at the surface of a sub-conductor is not uniform. (It varies cosinusoidal manner from a maximum at a point on the outside surface on the line of centres, to a minimum at the corresponding point on the inside surface.) The optimum spacing between sub-conductors for reducing voltage gradient is eight to ten times the diameter of the conductor regardless of the number of sub-conductors per phase.

Since the voltage gradient is reduced by using bundled conductors the radio interference is also reduced.

Finally we know that surge impedance of a line is given by $\sqrt{L/C}$, where L is the inductance and C is the capacitance per unit length of the line. Since by bundling, the self GMD is increased, the inductance is reduced and capacitance increased, as a result the surge impedance is reduced. This in turn means that the maximum power that can be transmitted is increased. Therefore, for large power transmission at higher voltages bundled conductors should be used.

The procedure for calculating the reactance of the bundled conductor is same as for composite conductors. The basic difference between a composite conductor and bundled conductor is that the sub-conductors of a bundled conductor are separated from each other by a distance of almost 30 cms or more and the wires of a composite conductor touch each other.

2.13 SKIN AND PROXIMITY EFFECT

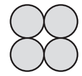
When direct current flows in the conductor, the current is uniformly distributed across the section of the conductor whereas flow of alternating current is non-uniform, with the outer filaments of the conductor carrying more current than the filaments closer to the centre. This results in a higher resistance to alternating current than to direct current and is commonly known as skin effect. This effect is more, the more is the frequency of supply and the size of the conductor. A conductor could be considered as composed of very thin filaments. The inner filaments carrying currents give rise to flux which links the inner filaments only where as the flux due to current carrying outer filaments enclose both the inner as well as the outer filaments (Art. 2.4). The flux linkages per ampere to inner strands is more as compared to outer strands. Hence the inductance/impedance of the inner strands is greater than those of outer strands which results in more current in the outer strands as compared to the inner strands. This non-uniformity of flux linkage is the main cause of skin effect.

The alternating magnetic flux in a conductor caused by the current flowing in a neighbouring conductor gives rise to circulating currents which cause an apparent increase in the resistance of a conductor. This phenomenon is called proximity effect. In a two-wire system more lines of flux link elements farther apart than the elements nearest each other. Therefore, the inductance of the elements farther apart is more as compared to the elements near each other and the current density is less in the elements farther apart than the current density in the elements near each other. The effective resistance is, therefore, increased due to non-uniform distribution of current. The proximity effect is pronounced in case of cables where the distance between the conductors is small whereas for overhead lines with usual spacing the proximity effect is negligibly small.

PROBLEMS

2.1. Show that the inductance per unit length of an overhead line due to internal flux linkages is constant and is independent of size of conductor.

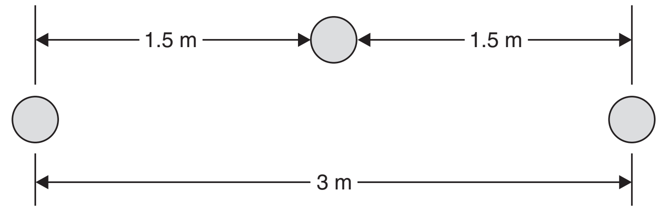
2.2. Determine the self GMD of the following types of conductors in terms of the radius r of an individual strand.



(i)

(ii)

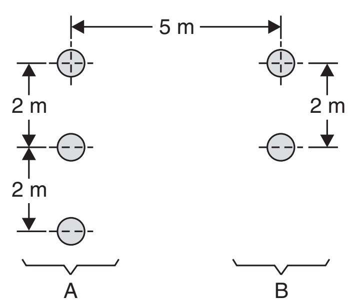
2.3. A single circuit 3-phase line operated at 50 Hz is arranged as follows. The conductor diameter is 0.6 cm. Determine the inductance and inductive reactance per km. Prove the formula used.



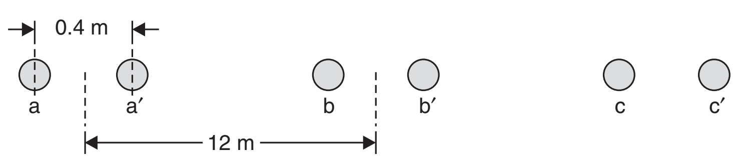
2.4. Derive expressions for the inductance of a 3-phase line with conductors untransposed. What is the significance of imaginary term in the expression for inductance ? Hence derive the expression for inductance for a completely transposed line.

2.5. Derive an expression for the flux linkages of one conductor in a group of n conductors carrying currents whose sum is zero. Hence derive an expression for inductance of composite conductors of a 1-phase line consisting of m strands in one conductor and n strands in the other conductor.

2.6. Determine the inductance of a 1-phase transmission line having the following arrangement of conductors. One circuit consists of three wires of 2 mm dia each and the other circuit two wires of 4 mm dia each.

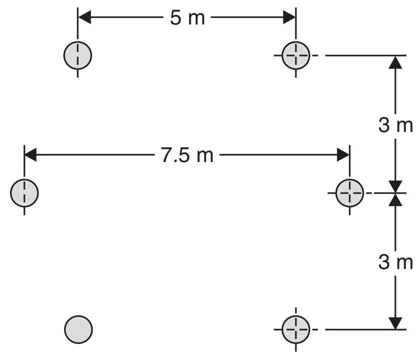


2.7. Determine the inductance per km of a 3-phase transmission line having conductors per phase and arranged as shown in figure.

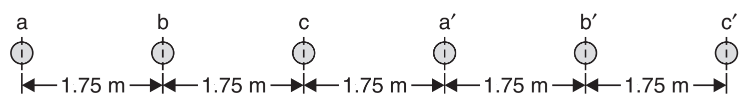


The dia of each conductor is 25 mm and carries 50% of the phase current.

2.8. Determine the inductance per km of a double circuit 3-phase line as shown in figure below. The transmission line is transposed within each circuit and each circuit remains on its own side. The dia of each conductor is 15 mm. Explain why the given arrangement is better as compared to when conductors of the same phase are placed in the same horizontal plane.



2.9. Determine the inductance per km/phase of a double circuit 3-ϕ line. The radius of each conductor is 15 mm.



2.10. Explain the concept of self GMD and mutual GMD for evaluating inductance of transmission lines.

2.11. What are ACSR conductors ? Explain the advantages of ACSR conductors when used for overhead lines.

2.12. What are bundled conductors ? Discuss the advantages of bundled conductors when used for overhead lines.

2.13. Explain clearly the 'skin effect' and 'proximity effect' when referred to overhead lines.

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1. Electromagnetics, Kraus.
2. Elements of Power System Analysis, W.D. Stevenson Jr., McGraw-Hill, 1962.
3. The Transmission and Distribution of Electrical Energy, H. Cotton, The English University Press Ltd., 1958.
4. Principles of Electric Power Transmission, H. Waddicor, Chapman and Hill, London, 5th ed., 1964.
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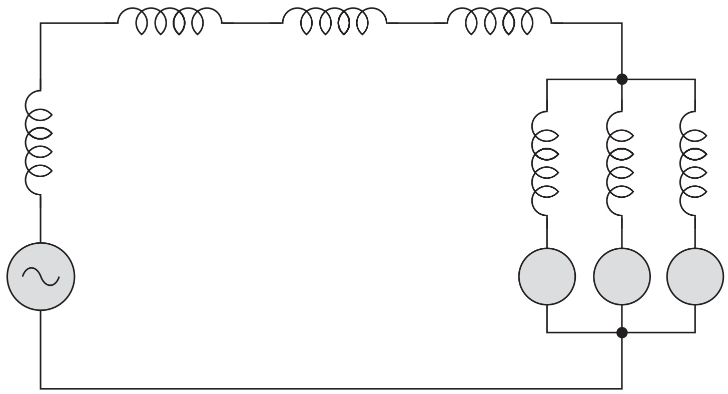
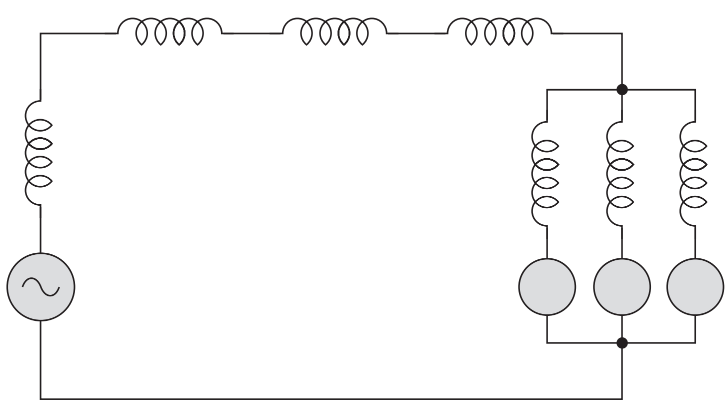


3

CAPACITANCE OF TRANSMISSION LINES



3

Capacitance of Transmission Lines

INTRODUCTION

The flow of current through a conductor gives rise to a magnetic field and charging of conductor results in an electric field. A charge if brought in the vicinity of this electric field experiences a force. The intensity of this field at any point is defined as the force per unit charge and is termed as electric field intensity designated as E. The units of this field are newton per coulomb or volts per metre. The direction of electric field intensity is the same as the direction of the force experienced by the unit charge. Since we are here concerned with the transmission line conductors it is better to know this electric field due to infinite line of charge.

3.1 ELECTRIC FIELD OF AN INFINITE LINE OF CHARGE

Consider the field produced by a thin line of charge as shown in Fig. 3.1. Let a positive charge ρ_L coulomb per metre be uniformly distributed along the infinitesimally thin line of infinite length.

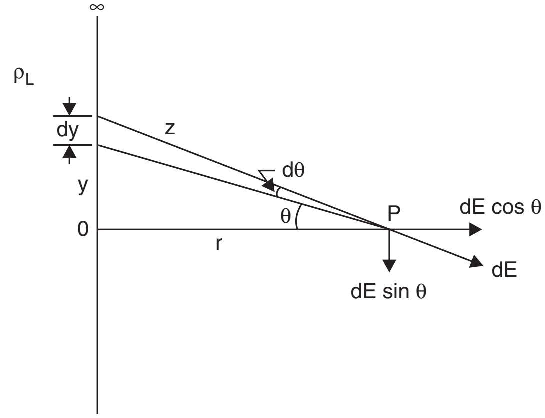


Fig. 3.1 Infinite line charge: Electric field intensity at P.

It is required to find out electric field intensity E at P due to infinite line charge. Take an infinitesimal charge $\rho_L \,.\, dy$ which could be considered a point charge. The electric field intensity dE at P due to this charge is given by

$$dE = \frac{\rho_L\, dy}{4\pi\varepsilon_0 z^2}\,\bar{a}_z \tag{3.1}$$

where $\bar{a}_z$ is the unit vector along z.

Now this infinitesimal intensity dE has two components, one $dE \cos\theta$ along r-axis and another $dE \sin\theta$ normal to r-axis. If we take another element $\rho_L\, dy$ symmetrically below r-axis it can be seen that due to this charge the electric field intensity at P will be added along r-axis whereas that normal to r will be subtracted *i.e.*, the electric field intensity due to both the elements at P will be $2\, dE \cos\theta$.

Therefore, total intensity at P due to infinite line of charge

$$E_r = \frac{2\rho_L}{4\pi\varepsilon_0}\int \frac{dy}{z^2}\cdot\frac{r}{z}$$

Since $\quad z^2 = r^2 + y^2$

$$E_r = \frac{\rho_L}{2\pi\varepsilon_0}\int_0^\infty \frac{r\,dy}{(r^2+y^2)^{3/2}}$$

$$= \frac{\rho_L r}{2\pi\varepsilon_0}\int_0^\infty \frac{dy}{(r^2+y^2)^{3/2}}$$

r is taken outside the sign of integration since it is taken as a constant distance from the line charge.

Substituting $\quad y = r\tan\theta$

$$dy = r\sec^2\theta\, d\theta$$

and the limits will be, for $y = 0$, $\quad \theta = 0$

and $\quad$ for $y = \infty = r\tan\theta$, $\theta = \pi/2$

$$E_r = \frac{\rho_L r}{2\pi\varepsilon_0}\int_0^{\pi/2}\frac{r\sec^2\theta\, d\theta}{r^3\sec^3\theta}$$

$$= \frac{\rho_L}{2\pi\varepsilon_0 r}\int_0^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{\rho_L}{2\pi\varepsilon_0 r}\left[\sin\theta\right]_0^{\pi/2}$$

$$= \frac{\rho_L}{2\pi\varepsilon_0 r} \tag{3.2}$$

From this it is clear that the field intensity due to a line charge at a point P is proportional to the linear charge density and is inversely proportional to the distance of the point P from the line charge, and the direction is along r-axis. This relation also holds good when the length of the charge is large as compared with the distance r from the charge. In case of a transmission line the distance of separation of the wires is very small is compared with the length of the line and, therefore, for this

$$E = \frac{\rho_L}{2\pi\varepsilon_0 r}.$$

3.2 POTENTIAL DIFFERENCE BETWEEN TWO POINTS DUE TO A LINE CHARGE

The potential at any distance r from the charge is the work done in moving a unit positive charge from infinity to that point and the potential difference between two points at distances r_1 and r_2 is the work done in moving a unit positive charge from r_2 to r_1 as shown in Fig. 3.2 or it is the line integral of the electric field intensity between points r_2 and r_1.

$$V = -\int_{r_2}^{r_1} E_r \, dr \tag{3.3}$$

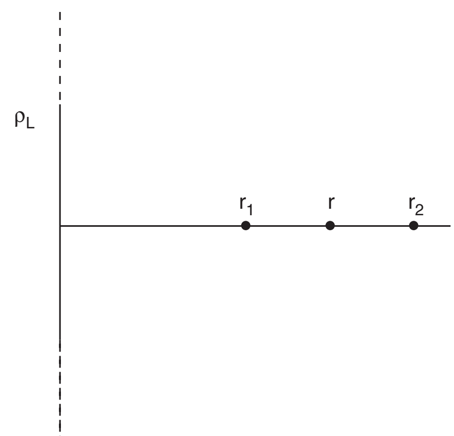


Fig. 3.2 Potential due to infinite line of charge.

Here E_r is taken as negative because the unit charge is to be moved against the direction of the electric field intensity E_r. Now substituting for E_r from equation (3.2).

$$V = -\frac{\rho_L}{2\pi\varepsilon_0}\int_{r_2}^{r_1} \frac{dr}{r}$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0} \ln \frac{r_2}{r_1} \tag{3.4}$$

3.3 TWO INFINITE LINES OF CHARGE

Consider the two infinite line charges as shown in Fig. 3.3. The line charge density of one is ρ_L coulomb/metre and that of the other is $-\rho_L$ C/metre and say they are separated by a distance $2s$. Let 'O' be the origin and the centre point between the charges. Since the charges are of opposite polarity a plane passing through '*O*' will be the neutral plane and, therefore, '*O*' is taken as the origin for voltage calculation. It is required to find out the potential of point $P(x, y)$ with respect to '*O*' (the neutral point or zero potential point) due to the two infinite line charges. The potential at P due to positive linear charge using equation (3.4).

$$V_+ = \frac{\rho_L}{2\pi\varepsilon_0} \ln \frac{s}{r_2} \tag{3.5}$$

and the potential due to negative charge

$$V_- = -\frac{\rho_L}{2\pi\varepsilon_0} \ln \frac{s}{r_1} \tag{3.6}$$

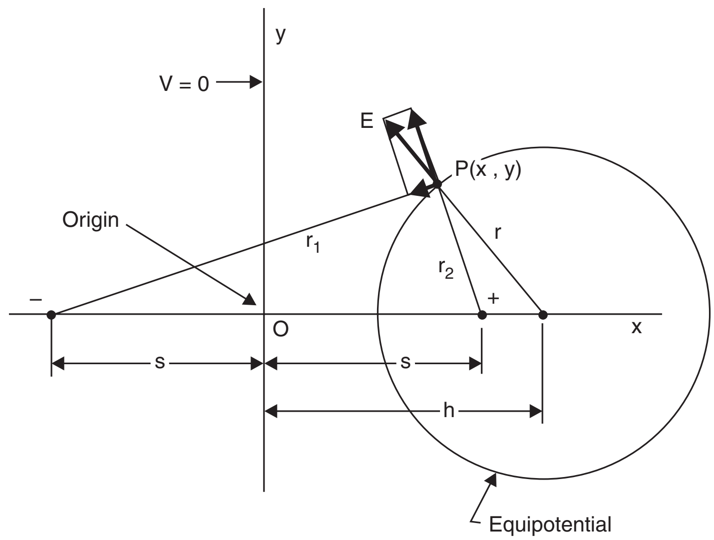


Fig. 3.3 Two infinite lines of charges.

Total potential at P is $\quad V = V_+ + V_-$

$$V = \frac{\rho_L}{2\pi\varepsilon_0} \ln \frac{r_1}{r_2} \tag{3.7}$$

Now it is required to find out the locus of the equipotential lines.

To find out that, equation (3.7) is written in the form

$$\ln \frac{r_1}{r_2} = \frac{2\pi\varepsilon_0 V}{\rho_L}$$

or

$$\frac{r_1}{r_2} = \exp\,(2\pi\varepsilon_0 V/\rho_L) \tag{3.8}$$

Now for a particular voltage V at P, the term on the right side of equation (3.8) is constant.

$$\therefore \qquad \frac{r_1}{r_2} = K \tag{3.9}$$

From Fig. 3.3

$$r_1 = \sqrt{(s+x)^2 + y^2}$$

and $$r_2 = \sqrt{(s-x)^2 + y^2}$$

Substituting these relations in equation (3.9)

$$\frac{\sqrt{(s+x)^2 + y^2}}{\sqrt{(s-x)^2 + y^2}} = K$$

Squaring both the sides and rearranging we get

$$x^2 - 2xs\,\frac{K^2+1}{K^2-1} + s^2 + y^2 = 0 \tag{3.10}$$

For completing the square add on both the sides of equation (3.10), the term $s^2\,\frac{(K^2+1)^2}{(K^2-1)^2}$; we get

$$\left(x - \frac{K^2+1}{K^2-1}\,s\right)^2 + y^2 = \left(\frac{2Ks}{K^2-1}\right)^2 \tag{3.11}$$

which represents an equation to a circle

$$(x-h)^2 + (y-g)^2 = r^2 \tag{3.12}$$

where $$r = \frac{2Ks}{K^2-1} \tag{3.13}$$

and centre at

$$x = h = \frac{K^2+1}{K^2-1}\,s,\; y = g = 0 \tag{3.14}$$

An equipotential line corresponding to voltage V at P is drawn in Fig. 3.3. For higher potentials, K increases and it can be seen from the equation of the equipotential lines, r decreases *i.e.*, r approaches zero and h approaches 's' so that the equipotentials are smaller circles with their centres more nearly at the line of charge.

3.4 CAPACITANCE OF A 1-φ TRANSMISSION LINE

Before an expression for the capacitance is derived we define the following terms:

Capacitor: It is an electrical device which consists of two conductors separated by a dielectric medium and is used for storing electrostatic energy.

Capacitance: The capacitance of a capacitor is the ratio of the charge on one of its conductors to the potential difference between the conductors.

We make use of this definition of capacitance and other results derived previously in this Chapter for finding out the capacitance of the transmission lines.

Consider a 1-φ transmission line as shown in Fig. 3.4. Let a fixed potential V be applied between the conductors so that the charge per unit length of each conductor is ρ_L coulomb per metre. The length of the line is very large as compared with the distance of separation h of the conductors, and radius r of each conductor is very small as compared to the distance of separation. It is to be noted that the charge ρ_L coulomb/metre is distributed on the surface of the conductor which is non-uniformly distributed over the surface such that it has higher density on the adjacent sides of the conductors. This charge distribution can be considered as a line charge as in the previous section. The surface of the conductor represents an equipotential surface with circular cross-section and radius equal to r. So the objective will be to find out the equivalent line charge distribution for a system of two conductors with operating voltage V, distance of separation h and radius of the equipotential surface r. This equivalent charge distribution, as can be seen from Fig. 3.4 and the results of the previous section, will be a line charge ρ_L coulomb/metre separated by a distance s, where s can be obtained from any of the equations (3.13) or (3.14).

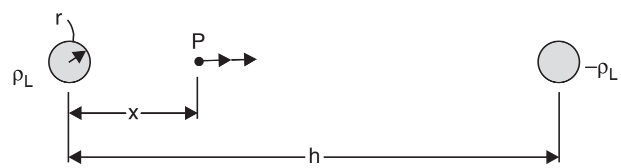


Fig. 3.4 1-ϕ transmission line.

Refer to section 3.3. V is a function of K and is given by

$$V = \frac{\rho_L}{\pi\varepsilon_0} \ln K$$

From this $$C = \frac{\rho_L}{V} = \frac{\pi\varepsilon_0}{\ln K} \text{ F/metre}$$

The value of K can be obtained from equations (3.13) and (3.14) by eliminating s from these equations.

$$r = \frac{2Ks}{K^2 - 1}$$

$$h = \frac{K^2 + 1}{K^2 - 1} s$$

or $$\frac{h}{r} = \frac{K^2 + 1}{2K}$$

or $$K^2 - 2K \frac{h}{r} + 1 = 0$$

or $$K = \frac{2\frac{h}{r} \pm \sqrt{\frac{4h^2}{r^2} - 4}}{2} = \frac{h}{r} \pm \sqrt{h^2/r^2 - 1} \tag{3.15}$$

Since the capacitance of a given system is constant only one of the two values of K is to be used. Since $\frac{h}{r} >> 1$,

$$K \simeq \frac{h}{r} + \frac{h}{r} \tag{3.16}$$

Since $K \neq 0$ only positive sign is taken into account. Therefore

$$C = \frac{\pi\varepsilon_0}{\dfrac{h}{r} + \sqrt{\dfrac{h^2}{r^2} - 1}} \text{ F/metre} \tag{3.17}$$

The expression for capacitance obtained above is very accurate. However, it could be assumed that the charge is uniformaly distributed which is not very far from the actual condition for power system problems where $\dfrac{h}{r} >> 1$. The derivation is much more simplified and is as follows (refer to Fig. 3.4):

Since the charge is assumed to be uniformly distributed over the surface of the conductor, this could be considered as concentrated along the axis on conductor. The electric field intensity at point P due to ρ_L is

$$E_+ = \frac{\rho_L}{2\pi\varepsilon_0 x} \tag{3.18}$$

and is directed along $\bar{a}_x$. Similarly electric field intensity at P due to $-\rho_L$

$$E_- = \frac{\rho_L}{2\pi\varepsilon_0 (h-x)} \tag{3.19}$$

along $\bar{a}_x$ again, as this time the force experienced by a unit positive charge at P will be towards the negative charge (force of attraction).

Total electric field intensity at P

$$E = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{h-x}\right] \tag{3.20}$$

The potential difference between the conductors

$$\begin{aligned}
V &= -\int_{h-r}^{r} E\, dx \\
&= \frac{\rho_L}{2\pi\varepsilon_0}\int_r^{h-r}\left(\frac{1}{x} + \frac{1}{h-x}\right) dx \\
&= \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln x - \ln (h-x)\right]_r^{h-r} \\
&= \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln (h-r) - \ln r - \ln \{h-(h-r)\} + \ln (h-r)\right] \\
&= \frac{\rho_L}{2\pi\varepsilon_0} 2\ln\frac{h-r}{r} = \frac{\rho_L}{\pi\varepsilon_0}\ln\frac{h-r}{r}
\end{aligned} \tag{3.21}$$

Since $h >> r$, $h - r \simeq h$.

$$\therefore \qquad V = \frac{\rho_L}{\pi\varepsilon_0}\ln\frac{h}{r}$$

or
$$C = \frac{\rho_L}{V} = \frac{\pi\varepsilon_0}{\ln h / r} \text{ F/metre} \tag{3.22}$$

Equation (3.22) corresponds to the expression for capacitance of a single phase transmission line. Compare this expression with the expression for inductance equation (2.17) of a single phase transmission line. Equation for inductance contains a constant term corresponding to the internal flux linkages whereas since charges reside on the surface of the conductor, similar term is absent in the capacitance expression. As a result of this, the radius in the expression for capacitance is the actual outside radius of the conductor whereas for inductance equation (2.18) the radius is the self GMD of the conductor. *The concept of self GMD is applicable for inductance calculation and not for the capacitance.*

Sometimes it is required to know the capacitance between one conductor and a neutral point between them which will be defined as the charge on one of the conductors per unit of voltage difference between the neutral and the conductor. This means the capacitance of one conductor with respect to the neutral plane is two times the capacitance of the single-phase line (Fig. 3.5).

$$C_{an} = 2C_{ab} = \frac{2\pi\varepsilon_0}{\ln \frac{h}{r}}$$

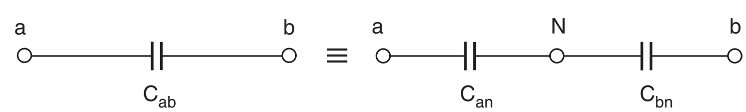


Fig. 3.5

3.5 CAPACITANCE OF A 3-PHASE UNSYMMETRICALLY SPACED TRANSMISSION LINE

For an untransposed line the capacitances between conductor to neutral of the three conductors are unequal. In transposed lines the average capacitance of each conductor to neutral is the same as the capacitance to neutral of any other phase. The dissymmetry of the untransposed line is slight for the usual transmission lines and, therefore, the calculations for capacitance are carried out as though the lines were completely transposed. The three positions of the conductors are shown in Fig. 3.6.

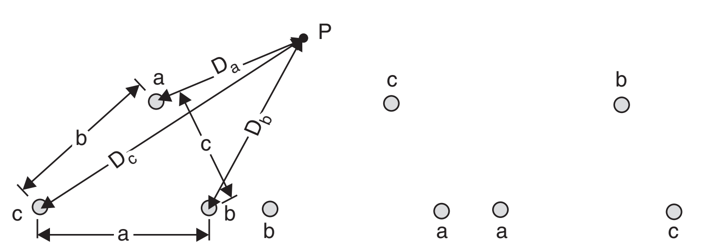


Fig. 3.6 Unsymmetrically spaced transposed 3-phase transmission line.

Since the potential due to a linear charge is a linear function of the charge it follows that the potentials of more than one charges are linearly superposable.

Considering Fig. 3.6, let a point P be at a large distance D from the system such that D_a, D_b and D_c are approximately same. It is required to find out the potential of conductor a due to charges ρ_a, ρ_b and ρ_c per unit length of the conductors. Since it is a 3-phase balanced system, taking ρ_a as the reference charge,

$$\rho_b = \rho_a \angle -120 \text{ and } \rho_c = \rho_a \angle 120$$

The potential of conductor 'a' with respect to point P due to the charge on the conductor itself,

$$V_{aa} = \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{D_a}{r} \tag{3.23}$$

Similarly, the potential of conductor 'a' due to the charges ρ_b and ρ_c respectively are

$$\frac{\rho_b}{2\pi\varepsilon_0} \ln \frac{D_b}{c} \text{ and } \frac{\rho_c}{2\pi\varepsilon_0} \ln \frac{D_c}{b}$$

$$\therefore \qquad V_a' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a}{r} + \rho_b \ln \frac{D_b}{c} + \rho_c \ln \frac{D_c}{b}\right] \tag{3.24}$$

Similarly, the potential of conductor a in the other two positions is given by

$$V_a'' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_b}{r} + \rho_b \ln \frac{D_c}{a} + \rho_c \ln \frac{D_a}{c}\right] \tag{3.25}$$

and

$$V_a''' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_c}{r} + \rho_b \ln \frac{D_a}{b} + \rho_c \ln \frac{D_b}{a}\right] \tag{3.26}$$

The average voltage of phase a with respect to point P

$$V_a = \frac{V_a' + V_a'' + V_a'''}{3}$$

$$= \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a D_b D_c}{r^3} + \rho_b \ln \frac{D_a D_b D_c}{abc} + \rho_c \ln \frac{D_a D_b D_c}{abc}\right] \tag{3.27}$$

Now $\qquad \rho_a + \rho_b + \rho_c = 0$

$\therefore \qquad \rho_b + \rho_c = -\rho_a$

Substituting this in the expression (3.27),

$$V_a = \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a D_b D_c}{r^3} + (\rho_b + \rho_c) \ln \frac{D_a D_b D_c}{abc}\right]$$

$$= \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a D_b D_c}{r^3} - \rho_a \ln \frac{D_a D_b D_c}{abc}\right]$$

$$= \frac{\rho_a}{6\pi\varepsilon_0} \ln \frac{abc}{r^3}$$

$$= \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{\sqrt[3]{abc}}{r}$$

$$= \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{\text{GMD}}{r}$$

or $$C = \frac{\rho_a}{V_a} = \frac{2\pi\varepsilon_0}{\ln \dfrac{\text{GMD}}{r}} \text{ F/metre} \quad (3.28)$$

Since the conductors b and c also occupy the same three positions as occupied by conductor a, the average voltage of the conductors is same and, therefore, the capacitance is also the same.

For a symmetrical spacing of the conductors,

$$a = b = c = h$$

$$\therefore \quad C = \frac{2\pi\varepsilon_0}{\ln \dfrac{h}{r}} \quad (3.29)$$

Example 3.1: Determine the capacitance and the charging current per km when the transmission line of example 2.2 is operating at 132 kV.

Solution: The radius of conductor = 0.4 cm.

The mutual GMD of conductors, D_m = 2.015 metres.

$$\therefore \quad \text{Capacitance per phase per metre} = \frac{2\pi\varepsilon_0}{\ln \dfrac{2.015}{0.4} \times 10^2} \text{ F/metre}$$

$$= \frac{10^{-9}}{18 \times \ln \dfrac{201.5}{0.4}} = 8.928 \text{ pF/metre}$$

$$= 8.928 \times 10^{-12} \times 10^3 \text{ F/km}$$

$$= 8.928 \times 10^{-9} \text{ F/km}$$

The charging current $$= \frac{132 \times 1000}{\sqrt{3}} \times 8.928 \times 10^{-9} \times 314$$

$$= 0.2136 \text{ amp/km.}$$ **Ans.**

3.6 CAPACITANCE OF A DOUBLE CIRCUIT LINE

Normally two configurations of conductors are used: (*i*) hexagonal spacing, and (*ii*) flat vertical spacing. First of all an expression of capacitance for hexagonal spacing is derived.

Hexagonal Spacing

Since the conductors of the same phase are connected in parallel the charge per unit length is the same (Fig. 3.7). Also, because of the symmetrical arrangement the phases are balanced and the conductors of each phase are also balanced if the effect of ground is neglected. Therefore, the transposition of conductors is not required.

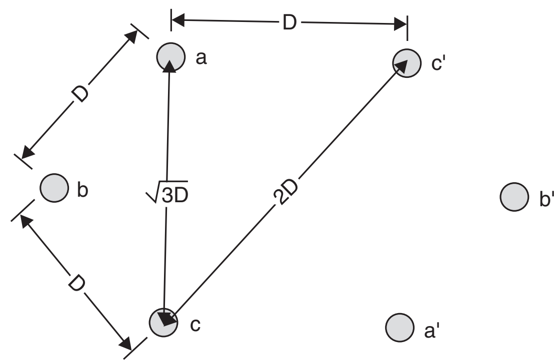


Fig. 3.7 Double circuit line—Hexagonal spacing.

Assume a point P very far from the system of conductors such that the distances of the conductors from P are almost same. It is to be noted here that point P corresponds to almost zero potential. The potential of conductor a with respect to point P due to the charge on the conductor itself and the charges on conductors b, c, a', b' and c' is given by

$$V_a = \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{D_a}{r} + \frac{\rho_b}{2\pi\varepsilon_0} \ln \frac{D_b}{D} + \frac{\rho_c}{2\pi\varepsilon_0} \ln \frac{D_c}{\sqrt{3}D} + \frac{\rho_{a'}}{2\pi\varepsilon_0} \ln \frac{D_{a'}}{2D} + \frac{\rho_{b'}}{2\pi\varepsilon_0} \ln \frac{D_{b'}}{\sqrt{3}D} + \frac{\rho_{c'}}{2\pi\varepsilon_0} \ln \frac{D_{c'}}{D} \tag{3.30}$$

Since $\rho_a = \rho_{a'}$, $\rho_b = \rho_{b'}$ and $\rho_c = \rho_{c'}$

$$V_a = \frac{1}{2\pi\varepsilon_0}\left[\rho_a\left(\ln\frac{D_a}{r} + \ln\frac{D_{a'}}{2D}\right) + \rho_b\left(\ln\frac{D_b}{D} + \ln\frac{D_{b'}}{\sqrt{3}D}\right) + \rho_c\left(\ln\frac{D_c}{\sqrt{3}D} + \ln\frac{D_{c'}}{D}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln (D_a D_{a'}) + \rho_b \ln (D_b D_{b'}) + \rho_c \ln (D_c D_{c'}) + \rho_a \ln \frac{1}{2Dr} + \rho_b \ln \frac{1}{\sqrt{3D^2}} + \rho_c \ln \frac{1}{\sqrt{3D^2}}\right]$$

$$= \frac{1}{2\pi\varepsilon_0} (\rho_a + \rho_b + \rho_c) \ln (D_a D_{a'}) + \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{2Dr} + (\rho_b + \rho_c) \ln \frac{1}{\sqrt{3D^2}}\right] \tag{3.31}$$

Since $D_a D_{a'} = D_b D_{b'} = D_c D_{c'}$.

Also since $\rho_a + \rho_b + \rho_c = 0$,

$$V_a = \frac{1}{2\pi\varepsilon_0} \rho_a \left(\ln \frac{1}{2Dr} - \ln \frac{1}{\sqrt{3D^2}}\right)$$

$$V_a = \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{\sqrt{3}D}{2r}$$

or
$$C = \frac{\rho_a}{V_a} = \frac{2\pi\varepsilon_0}{\ln \frac{\sqrt{3}D}{2r}} \text{ F/metre/conductor.} \tag{3.32}$$

Equation (3.32) represents an expression for the capacitance of conductor a alone, whereas there are two conductors per phase a and a'. Therefore, the capacitance of the system per phase will be twice the capacitance of one conductor to neutral, *i.e.*,

$$C = \frac{4\pi\varepsilon_0}{\ln \frac{\sqrt{3}D}{2r}} \text{ F/metre/phase} \tag{3.33}$$

Here expression for capacitance for phase a has been derived. Since the conductors of different phases are symmetrically placed, the expression for capacitance for other phases will also be the same.

Flat Vertical Spacing

Refer to the system of conductors in Fig. 3.8. The conductors of different phases are not symmetrically placed; therefore, the derivation of capacitance expression will require the transposition of conductors as shown in Fig. 3.8.

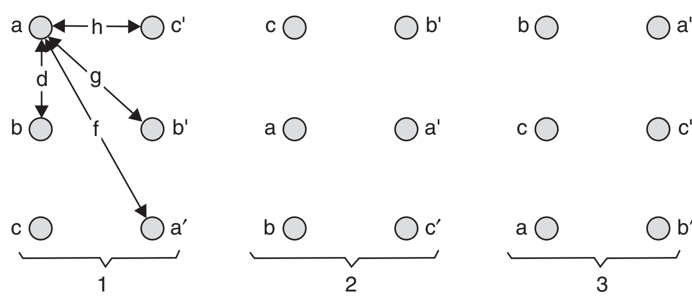


Fig. 3.8 Double circuit flat vertical spacing, transposed line.

It is required to find out average voltage of conductor a in the three different positions due to the charge on conductor a and the conductors b, c, a', b' and c'. For this we again assume a point very far from the system of conductors such that $D_a \simeq D_b \simeq D_c \simeq D_{a'} \simeq D_{b'} \simeq D_{c'}$.

Since point P is at a very large distance from the system of conductors, the potential of point P is approximately zero. The potential of conductor a in position 1.

$$V_a' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_a}{r} + \rho_b \ln \frac{D_b}{d} + \rho_c \ln \frac{D_c}{2d} + \rho_{a'} \ln \frac{D_{a'}}{f} + \rho_{b'} \ln \frac{D_{b'}}{g} + \rho_{c'} \ln \frac{D_{c'}}{h}\right] \tag{3.34}$$

Using the relations $D_a \simeq D_b \simeq D_c \simeq D_{a'} \simeq D_{b'} \simeq D_{c'}$, $\rho_a + \rho_b + \rho_c = 0$, and $\rho_a = \rho_{a'}$, $\rho_b = \rho_{b'}$, $\rho_c = \rho_{c'}$.

$$V_a' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{rf} + \rho_b \ln \frac{1}{dg} + \rho_c \ln \frac{1}{2dh}\right] \tag{3.35}$$

The potential of conductor a in position 2.

$$V_a'' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{rh} + \rho_b \ln \frac{1}{dg} + \rho_c \ln \frac{1}{dg}\right] \tag{3.36}$$

The potential of conductor a in position 3.

$$V_a''' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{rf} + \rho_b \ln \frac{1}{2dh} + \rho_c \ln \frac{1}{dg}\right] \tag{3.37}$$

The average potential of conductor a in three positions

$$V_a = \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{r^3 f^2 h} + \rho_b \ln \frac{1}{2d^3 g^2 h} + \rho_c \ln \frac{1}{2d^3 g^2 h}\right]$$

$$= \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{1}{r^3 f^2 h} - \rho_a \ln \frac{1}{2d^3 g^2 h}\right]$$

Since $\rho_a + \rho_b + \rho_c = 0$,

$$\therefore \quad V_a = \frac{\rho_a}{6\pi\varepsilon_0} \ln \frac{2d^3 g^2 h}{r^3 f^2 h}$$

$$= \frac{\rho_a}{2\pi\varepsilon_0} \ln \frac{3\sqrt{2}\, d}{r}\left(\frac{g}{f}\right)^{2/3}$$

$$\therefore \quad C = \frac{\rho_a}{V_a} = \frac{2\pi\varepsilon_0}{\ln \sqrt[3]{2}\, \frac{d}{r}\left(\frac{g}{f}\right)^{2/3}} \text{ F/metre/conductor} \tag{3.38}$$

The capacitance C per phase of the system

$$C = \frac{4\pi\varepsilon_0}{\ln \sqrt[3]{2}\, \frac{d}{r} \cdot \left(\frac{g}{f}\right)^{2/3}} \text{ F/metre/phase.} \tag{3.39}$$

Example 3.2: Determine the capacitance and the charging current per km when the transmission line of example 2.5 operates at 220 kV, dia of conductor = 2.5 cm.

Solution: The mutual GMD of the circuit will be same as calculated in example 2.5. The procedure for evaluating self GMD is same as in case of problem 2.5 except that for r', r is used, as the electric charge resides on the surface of the conductor unlike the magnetic flux which is present inside the conductor.

Mutual GMD = 6.61 metres

$$D_{s_1} = \sqrt{1.25 \times 10^{-2} \times 10.965} = 0.3702 \text{ metre} = D_{s_3}$$

$$D_{s_2} = \sqrt{1.25 \times 10^{-2} \times 9} = 0.3354 \text{ metre}$$

$$\therefore \qquad D_s = \sqrt[3]{D_{s_1} D_{s_2} D_{s_3}} = \sqrt[3]{0.045965899} = 0.3582 \text{ metre}$$

$$\therefore \quad \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \dfrac{6.61}{0.3582}} = 0.019056\ \mu\text{F/km}$$

$$\therefore \quad \text{Charging current per km} = \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01905 \times 10^{-6}$$

$$= 0.76 \text{ amp/km.} \quad \textbf{Ans.}$$

Example 3.3: Determine the capacitance and charging current per km of the line of example 2.7 if the line operates at 220 kV, dia = 4.5 cms.

Solution: The mutual GMD of the system is same as in example 2.7 *i.e.*, GMD = 8.19 metres.

$$D_s = \sqrt{2.25 \times 10^{-2} \times 0.4} = 0.094868 \text{ metre}$$

$$\therefore \quad \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \dfrac{8.19}{0.094868}} = 0.01246\ \mu\text{F}$$

$$\text{The charging current per km} = \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01246 \times 10^{-6}$$

$$= 0.497 \text{ amp.} \quad \textbf{Ans.}$$

3.7 EFFECT OF EARTH ON THE CAPACITANCE OF CONDUCTORS

The electric flux lines due to an isolated (effect of earth neglected) positively charged conductor emanate from the conductor and terminate on to an imaginary conductor placed at infinity. The electric flux lines and the equipotential lines are orthogonal to each other. In case the effect of earth is taken into account the distribution of flux lines will change remarkably. The earth is considered to be conducting and an equipotential plane of infinite extent. Therefore, these flux lines are forced to cut the surface of the earth orthogonally. The positive charge on the conductor induces negative charge on the earth surface. This distribution of charge on the surface of the earth should be replaced by an equivalent charge for the calculation of electric field potential and other related quantities due to this isolated charged conductor. The method of images due to Kelvin refers to the replacement of a surface distribution of charge on a conducting surface by suitable charges. Since earth is an equipotential plane which is possible only if we assume the presence of an imaginary conductor below the surface of the earth at a depth equal to the height of the actual conductor above the surface of the earth. Also the charge on the conductor should be opposite to the charge on the actual conductor. Thus the earth can be replaced for the calculation of capacitance by a fictitious charged conductor with charge equal and opposite to the charge on the actual conductor and at a depth below the surface of the earth as the height of the actual conductor above earth. This imaginary conductor is called the image of the actual conductor.

Capacitance of single conductor: It is required to calculate the capacitance of this conductor to ground. As discussed above, the earth can be replaced by a fictitious conductor as shown in Fig. 3.9. This means the single conductor with the earth is equivalent to a single-phase transmission line. The capacitance for a single-phase transmission line from equation (3.22) is given as

$$C = \frac{\pi\varepsilon_0}{\ln (2h / r)} \tag{3.40}$$

$\therefore$ The capacitance C of the conductor with reference to ground

$$C = \frac{2\pi\varepsilon_0}{\ln (2h / r)} \text{ F/metre} \tag{3.41}$$

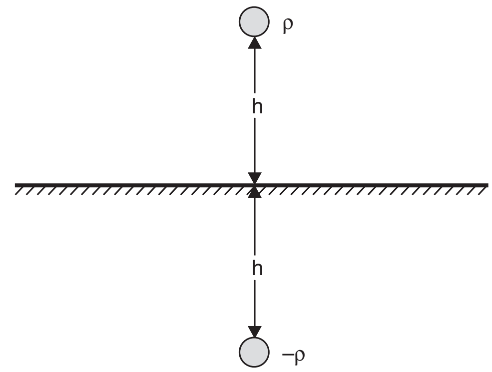


Fig. 3.9 One conductor line and its image.

3.7.1 Effect of Earth on the Capacitance of Single-phase Transmission Line

Conductors a and b constitute the transmission line and the conductors a' and b' their corresponding images (Fig. 3.10).

Assuming that the conductors a and b are running physically parallel to earth and are at the same heights above the ground, then $D_{aa'} = D_{bb'}$ and $D_{ab'} = D_{ba'}$.

We calculate the potential of conductor a with respect to point P which is very far away from the system, due to the charges, viz., charge on the conductor a and its image and conductor b and its image.

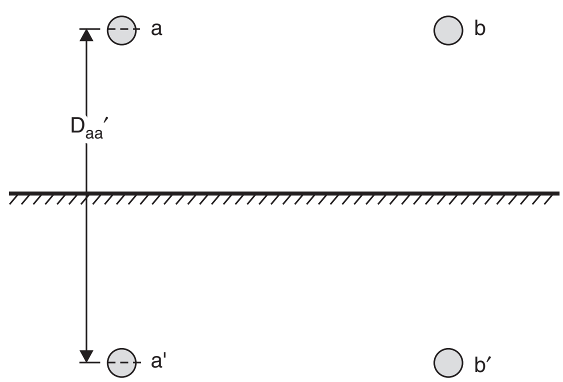


Fig. 3.10 Single-phase line and its image.

$$V_a = \frac{\rho}{2\pi\varepsilon_0} \ln \frac{D_a}{r} - \frac{\rho}{2\pi\varepsilon_0} \ln \frac{D_{a'}}{D_{aa'}} - \frac{\rho}{2\pi\varepsilon_0} \ln \frac{D_b}{D_{ab}} + \frac{\rho}{2\pi\varepsilon_0} \ln \frac{D_{b'}}{D_{ab'}}$$

$$= \frac{\rho}{2\pi\varepsilon_0} \ln \frac{D_{ab}}{r} \frac{D_{aa'}}{D_{ab'}} \tag{3.42}$$

$$\therefore \qquad \frac{\rho}{V_a} = C = \frac{2\pi\varepsilon_0}{\ln \dfrac{D_{ab}}{r} \dfrac{D_{aa'}}{D_{ab'}}} \tag{3.43}$$

Therefore, the capacitance of the single phase transmission line will be

$$C = \frac{\pi\varepsilon_0}{\ln \dfrac{D_{ab}}{r} \dfrac{D_{aa'}}{D_{ab'}}}$$

From the expression for capacitance it is clear that since the ratio $\dfrac{D_{aa'}}{D_{ab'}} < 1$, the effect of earth on the capacitance of the system is to increase it. However, normally the distance of separation between the conductors is much smaller than the height of the conductor above the ground; therefore, the ratio $\dfrac{D_{aa'}}{D_{ab'}} \simeq 1$ and for all practical purposes the effect of earth can be neglected.

3.7.2 Effect of Earth on the Capacitance of a 3-phase Line

Since the conductors along with their images are unsymmetrically spaced the capacitance calculation will be made by transposing the lines (Fig. 3.11).

Again assuming a point P very far from the system the potential of conductor a in position 1.

$$V_a' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a\left(\ln\frac{D_a}{r} - \ln\frac{D_{a'}}{D_{aa'}}\right) + \rho_b\left(\ln\frac{D_b}{D_{ab}} - \ln\frac{D_{b'}}{D_{ab'}}\right) + \rho_c\left(\ln\frac{D_c}{D_{ac}} - \ln\frac{D_{c'}}{D_{ac'}}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln\frac{D_{aa'}}{r} + \rho_b \ln\frac{D_{ab'}}{D_{ab}} + \rho_c \ln\frac{D_{ac'}}{D_{ac}}\right] \tag{3.44}$$

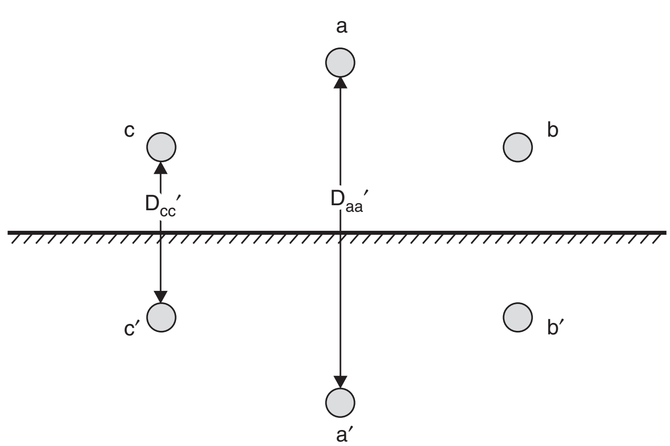


Fig. 3.11 3-phase transmission line and its image.

Potential of a in position 2.

$$V_a'' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_{bb'}}{r} + \rho_b \ln \frac{D_{bc'}}{D_{bc}} + \rho_c \ln \frac{D_{ba'}}{D_{ba}}\right] \tag{3.45}$$

Similarly potential of a in position 3.

$$V_a''' = \frac{1}{2\pi\varepsilon_0}\left[\rho_a \ln \frac{D_{cc'}}{r} + \rho_b \ln \frac{D_{ca'}}{D_{ca}} + \rho_c \ln \frac{D_{cb'}}{D_{cb}}\right] \tag{3.46}$$

The average voltage of conductor a,

$$V_a = \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{D_{aa'}D_{bb'}D_{cc'}}{r^3} + \rho_b \ln \frac{D_{ab'}D_{bc'}D_{ca'}}{D_{ab}D_{bc}D_{ca}} + \rho_c \ln \frac{D_{ac'}D_{ba'}D_{cb'}}{D_{ac}D_{ba}D_{cb}}\right] \tag{3.47}$$

Since $D_{ab'} = D_{ba'}$, $D_{bc'} = D_{cb'}$ and $D_{ca'} = D_{ac'}$.

$$V_a = \frac{1}{6\pi\varepsilon_0}\left[\rho_a \ln \frac{D_{aa'}D_{bb'}D_{cc'}}{r^3} - \rho_a \ln \frac{D_{ab'}D_{bc'}D_{ca'}}{D_{ab}D_{bc}D_{ca}}\right]$$

$$= \frac{\rho_a}{6\pi\varepsilon_0} . \ln \frac{D_{ab}D_{bc}D_{ca}}{r^3} \cdot \frac{D_{aa'}D_{bb'}D_{cc'}}{D_{ab'}D_{bc'}D_{ca'}}$$

$$\therefore \qquad C = \frac{\rho_a}{V_a} = \frac{2\pi\varepsilon_0}{\ln \dfrac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{r} \sqrt[3]{\dfrac{D_{aa'}D_{bb'}D_{cc'}}{D_{ab'}D_{bc'}D_{ca'}}}} \text{ F/metre} \tag{3.48}$$

It can be seen from the expression that the effect of earth is to increase the capacitance. But normally the height of the conductors is large as compared to the distance of separation between the conductors and, therefore, for all practical purposes the effect of earth on the capacitances can be neglected.

PROBLEMS

3.1. Do you get a constant term in the expression for capacitance as in case of inductance ? Give reasons.

3.2. Derive an expression for the capacitance per km of a single phase line taking into account the effect of ground.

3.3. What is method of images ? Derive an expression for the capacitance per unit length of a 3-phase line completely transposed. What is the effect of earth on the capacitance of the line ?

3.4. Determine the capacitance and charging current per unit length of the line when the arrangement of the conductors is as shown in Fig. P.3.4.

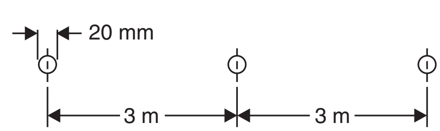


Fig. P.3.4

The operating voltage is 132 kV.

3.5. Determine the capacitance and charging current per unit length of the line when the arrangement of the conductor is shown in Fig. P.3.5. The line is completely transposed.

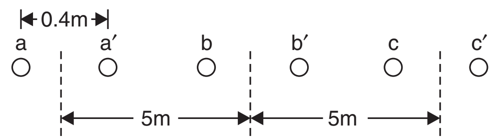


Fig. P.3.5

The dia of conductor is 15 mm, and operating voltage is 220 kV.

3.6. A 3-phase double circuit line is shown in Fig. P.3.6. The diameter of each conductor is 2 cm. Determine the capacitance and charging current per km length of the line, assume that the line is transposed and the operating voltage 220 kV.

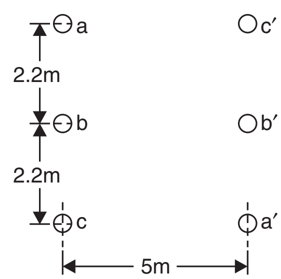


Fig. P.3.6

3.7. Determine the capacitance and charging current per km length of the double circuit 3-phase line as shown in Fig. P.2.8. The transmission line is transposed within each circuit and each circuit remains on its own side. The dia of each conductor is 15 mm and operating voltage 220 kV.

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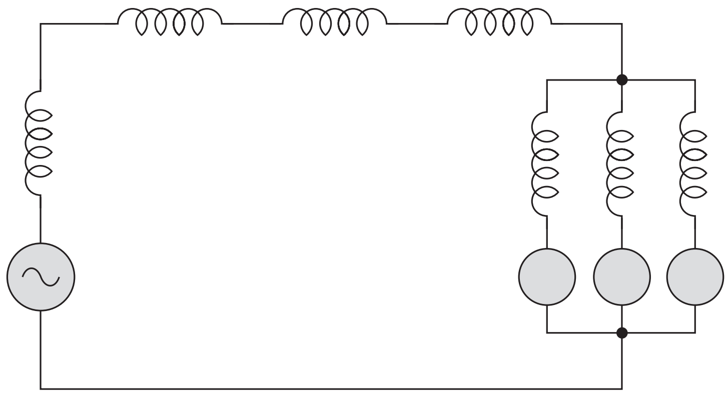
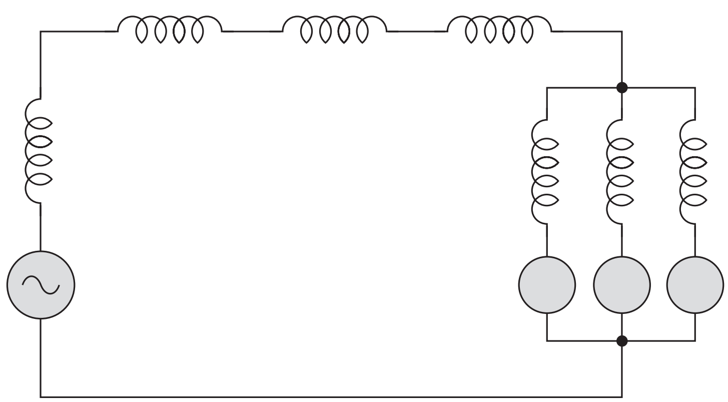


4

PERFORMANCE OF LINES



4

Performance of Lines

INTRODUCTION

Definition: By performance of lines is meant the determination of efficiency and regulation of lines.

The efficiency of lines is defined as

$$\% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power sent from the sending end}} \times 100$$

$$\% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end + losses}} \times 100$$

The end of the line where load is connected is called the receiving end and where source of supply is connected is called the sending end.

The regulation of a line is defined as the change in the receiving end voltage, expressed in per cent of full load voltage, from no load to full load, keeping the sending end voltage and frequency constant. Expressed mathematically,

$$\% \text{ regulation} = \frac{V_r' - V_r}{V_r} \times 100 \tag{4.1}$$

where V_r' is the receiving end voltage under no load condition and V_r the receiving end voltage under full load condition. It is to be noted here that V_r' and V_r are the magnitudes of voltages.

4.1 REPRESENTATION OF LINES

A transmission line is a set of conductors being run from one place to another supported on transmission towers. Such lines, therefore, have four distributed parameters, series resistance and inductance, and shunt capacitance and conductance. It will be shown later on in this chapter that the voltages and currents vary harmonically along the line with respect to the distance of the point under consideration. This observation is very important in representing the lines of different lengths. It is to be noted that the electrical power is being transmitted over the overhead lines at approximately the speed of light. In order to get one full wave variation of voltage or current on the line the length of the line for 50 Hz supply will be given by

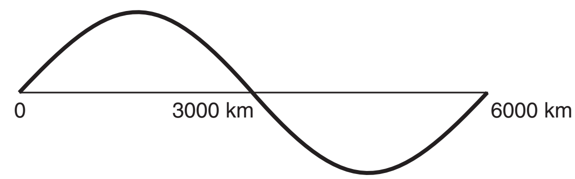


Fig. 4.1 Voltage distribution of 50 Hz supply.

$$f \cdot \lambda = v$$

where f is frequency of supply, λ is the wavelength *i.e.*, the length of the line in this case and v the velocity of the wave *i.e.*, the velocity of light.

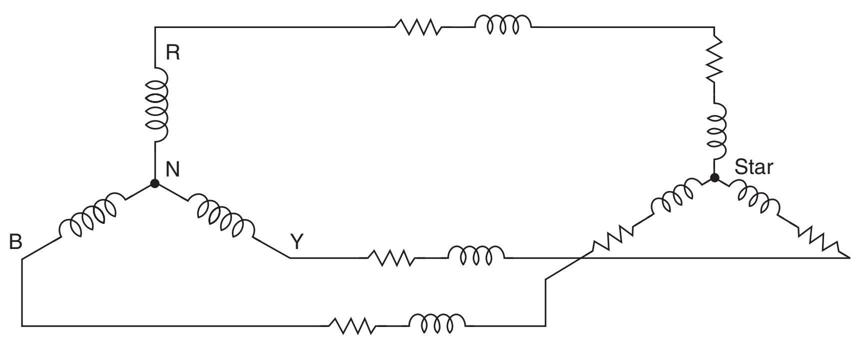
Substituting for $f = 50$ and $v = 3 \times 10^8$ m/sec.,

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{50} = 6 \times 10^6 \text{ metres}$$

$$= 6000 \text{ km}.$$

This means that if the length of the line is 6000 km the voltage or current wave at the two ends of the line will be as shown in Fig. 4.1.

(a) Single-line diagram of a 3-phase system



(b) 3-phase diagram of (a)

Fig. 4.2

For line lengths less than about 160 km, the voltage or current variation on the line is not much and it can be said that for line length of about 160 km the parameters could be assumed to be lumped and not distributed. Such lines are known as electrically short transmission lines. In power systems these electrically short transmission lines are again categorised as short transmission lines and medium transmission lines. The lines up to about 80 km are termed as short transmission lines where the effect of shunt capacitance is neglected and the lines above 80 km and below 160 km length are termed as medium length lines. For medium length lines the shunt capacitance can be assumed to be lumped at the middle of the line or half of the shunt capacitance may be considered to be lumped at each end of the line. The two representations of medium length lines are termed as nominal-T and nominal-π respectively. For line lengths more than 160 km the parameters are distributed and rigorous calculations are required to be made except in certain cases where lines up to 250 km can be analysed using nominal-π representation.

A typical 3-phase system is shown in Fig. 4.2. A 3-phase star load is connected to the generator through a 3-phase transmission system. The 3-phase system is normally balanced system irrespective of the fact that the conductors are not transposed, as the untransposed conductors introduce slight dissymmetry which can be ignored for all practical purposes.

It is known that the sum of all the currents in a balanced polyphase network is zero and, therefore, the current through the wire connected between the star point of the load and neutral of the system is zero. This means that the star point of the load and neutral of the system are at the same potential.

A 3-phase balanced system can, therefore, be analysed on single-phase basis in which the neutral wire is of zero impedance. The equivalent single-phase representation of Fig. 4.2. is shown in Fig. 4.3.

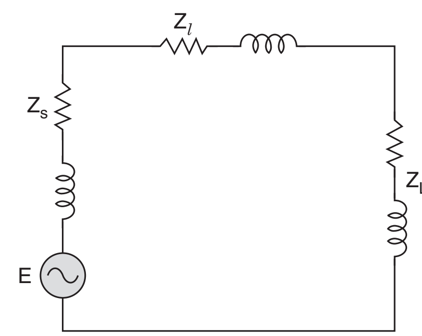


Fig. 4.3 Single phase representation of 3-phase balanced system.

4.2 SHORT TRANSMISSION LINE

The equivalent circuit and vector diagram for a short transmission line are shown in Fig. 4.4(*a*) and (*b*) respectively.

The vector diagram is drawn taking I_r, the receiving end current, as the reference.

From the vector diagram,

$$V_s \cos \phi_s = V_r \cos \phi_r + I_r R \tag{4.2a}$$

$$V_s \sin \phi_s = V_r \sin \phi_r + I_r X \tag{4.2b}$$

Squaring and adding equations (4.2*a*) and (4.2*b*),

$$V_s^2 = V_r^2 + 2I_r R V_r \cos \phi_r + 2I_r X V_r \sin \phi_r + I_r^2 (R^2 + X^2) \tag{4.3}$$

$$V_s = V_r \sqrt{1 + \frac{2I_r R \cos \phi_r}{V_r} + \frac{2I_r X \sin \phi_r}{V_r} + \frac{{I_r}^2}{{V_r}^2}(R^2 + X^2)}$$

In practice the last term under the square root sign is generally negligible; therefore,

$$V_s = V_r \left\{1 + \left(\frac{2I_r R}{V_r} \cos \phi_r + \frac{2I_r X}{V_r} \sin \phi_r\right)\right\}^{1/2} \tag{4.4}$$

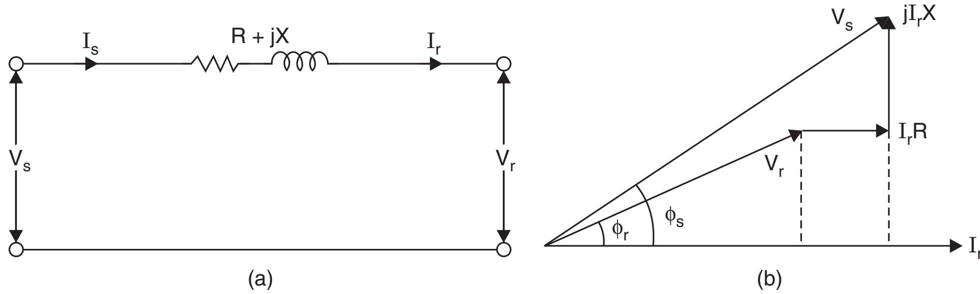


Fig. 4.4 Short-transmission line: (*a*) equivalent circuit, (*b*) phasor diagram.

The terms within the simple brackets is small as compared to unity. Using binomial expansion and limiting only to second term,

$$V_s \simeq V_r + I_rR \cos \phi_r + I_rX \sin \phi_r \tag{4.5}$$

Here V_s is the sending end voltage corresponding to a particular load current and power factor condition. It can be seen from the equivalent circuit of short line that the receiving end voltage under no load V_r' is the same as the sending end voltage under full load condition, *i.e.*,

$$V_r' = V_s$$

$$\therefore \qquad \% \text{ regulation} = \frac{V_s - V_r}{V_r} \times 100$$

$$= \left(\frac{I_rR}{V_r} \cos \phi_r + \frac{I_rX}{V_r} \sin \phi_r \right) \times 100 \tag{4.6}$$

or

$$\text{regulation per unit} = \frac{I_rR}{V_r} \cos \phi_r + \frac{I_rX}{V_r} \sin \phi_r$$

$$= v_r \cos \phi_r + v_x \sin \phi_r \tag{4.7}$$

where v_r and v_x are the per unit values of resistance and reactance of the line. It will be shown later on in this chapter that in a four terminal passive network the voltage and current on the receiving end and sending end are related by the following pair of equations:

$$V_s = AV_r + BI_r \tag{4.8}$$

$$I_s = CV_r + DI_r \tag{4.9}$$

where *A, B, C, D* are called the constants of the network. The transmission line is also a four-terminal network and it is now desired to find these constants for short transmission line.

Before these constants are determined it is desirable to understand what these constants are.

From equation (4.8),

$$A = \left. \frac{V_s}{V_r} \right|_{I_r = 0}$$

This means A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimensionless.

$$B = \left.\frac{V_s}{I_r}\right|_{V_r = 0}$$

B is the voltage impressed at the sending end to have one ampere at the short circuited receiving end. This is known as transfer impedance in network theory.

From equation (4.9),

$$C = \left.\frac{I_s}{V_r}\right|_{I_r = 0}$$

C is the current in amperes into the sending end per volt on the open-circuited receiving end. It has the dimension of admittance.

$$D = \left.\frac{I_s}{I_r}\right|_{V_r = 0}$$

D is the current at the sending end for one ampere of current at the short circuited receiving end. The constants *A, B, C* and *D* are related for a passive network as follows:

$$AD - BC = 1$$

This relation provides a good check on the values of these constants.

The sending end voltage and current can be written from the equivalent network as

$$V_s = V_r + I_r Z \tag{4.10}$$

$$I_s = I_r \tag{4.11}$$

Comparing the coefficients of equations (4.10) and (4.11) with equations (4.8) and (4.9) respectively, the constants for short transmission line are

$$A = 1$$

$$B = Z$$

$$C = 0$$

$$D = 1$$

Checking the values of *A, B, C* and *D* with the relation

$$AD - BC = 1$$

$$1 \cdot 1 - Z \cdot 0 = 1$$

So, the values calculated are correct for a short transmission line.

The *ABCD* constants can be used for calculation of regulation of the line as follows:

Normally the quantities P, I_r and $\cos \phi_r$ at the receiving end are given and of course the *ABCD* constant. Then determine sending end voltage using relation

$$V_s = AV_r + BI_r$$

To determine V_r' the no load voltage at the receiving end, equation (4.8) is made use of

$$V_r' = \frac{V_s}{A}, \text{ when } I_r = 0$$

$$\% \text{ regulation} = \frac{V_s / A - V_r}{V_r} \times 100$$

is thus evaluated.

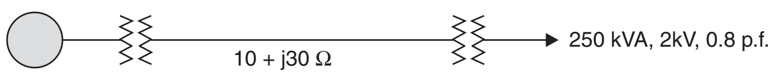
To determine % η of transmission, the following relation is made use of:

$$\% \eta = \frac{\text{Power received at the receiving end}}{\text{Power received at the receiving end + losses}} \times 100$$

$$= \frac{P}{P + 3I_r^2 R} \times 100$$

where R is the resistance per phase of the line.

Example 4.1: Determine the voltage at the generating station and the efficiency of transmission for the following 1-phase system:



Transformer ratio 2 kV/11 kV. The resistance on l.v. side = 0.04 ohm and h.v. side 1.3 ohm. Reactance on l.v. and h.v. side is 0.125 ohm and 4.5 ohm.

Solution: The transmission line equivalent impedance when referred to l.v. side will be

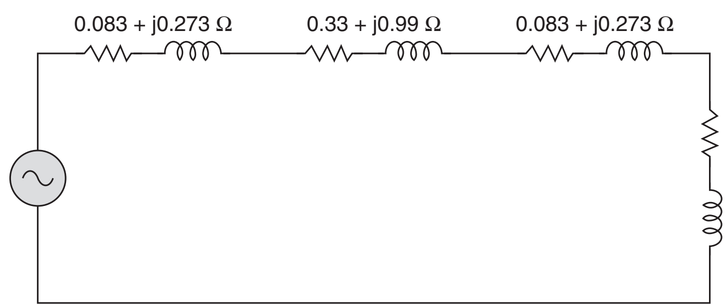
$$Z = 10 \times \left(\frac{2}{11}\right)^2 + j30 \times \left(\frac{2}{11}\right)^2$$

$$= (0.33 + j0.99)$$

$$\text{Transformer impedance} = 0.04 + 1.3 \times \left(\frac{2}{11}\right)^2 + j0.125 + j4.5 \times \left(\frac{2}{11}\right)^2$$

$$= (0.083 + j0.273)$$

The equivalent circuit for the total system



Load 250 kVA, 2 kV, 0.8 p.f.

The line current $= \frac{250 \times 1000}{2000} = 125$ amps.

The line loss $= I^2R = 125^2 \times 0.496 = 7.7$ kW

The output $= 250 \times 0.8 = 200$ kW

$$\therefore \quad \% \eta = \frac{200}{200 + 7.7} \times 100 = 96.3\%. \quad \textbf{Ans.}$$

Taking I_r as the reference, the sending end voltage

$$\begin{aligned} V_s &= (V_r \cos \phi_r + IR) + j(V_r \sin \phi_r + IX) \\ &= (2000 \times 0.8 + 125 \times 0.496) + j(2000 \times 0.6 + 125 \times 1.536) \\ &= 2168 \text{ volts.} \quad \textbf{Ans.} \end{aligned}$$

Example 4.2: A load of three impedances each $(6 + j9)$ is supplied through a line having an impedance of $(1 + j2)$ ohm. The supply voltage is 400 volts 50 Hz. Determine the power input and output when the load is (*i*) star connected and, (*ii*) delta connected.

Solution: When load is star connected:

The line to neutral voltage $= \dfrac{400}{\sqrt{3}} = 231$ volts

The impedance per phase $= (6 + j9) + (1 + j2)$

$= (7 + j11)$ ohm.

$\therefore$ Line current $= \dfrac{231}{7 + j11} = 17.7$ amp

Power input $= 3 \times 17.7^2 \times 7 = 6591$ watts

Power output $= 3 \times 17.7^2 \times 6 = 5649$ watts

When load is mesh connected: For the same impedance $(6 + j9)$, the equivalent star impedance will be

$$\frac{1}{3}(6 + j9) = (2 + j3) \text{ ohm.}$$

The impedance per phase $= (2 + j3) + (1 + j2) = (3 + j5)$

$\therefore$ Line current $= \dfrac{231}{3 + j5} = 39.6$ amps.

Power input $= 3 \times 39.6^2 \times 3 = 14124.9$ watts

Power output $= 3 \times 39.6^2 \times 2 = 9416$ watts. **Ans.**

From the above problem it is clear that for a particular supply voltage and particular load impedance the power consumed is more when the load is delta connected than when it is star connected.

Example 4.3: A 3-phase 50 Hz transmission line has conductors of section 90 mm^2 and effective diameter of 1 cm and are placed at the vertices of an equilateral triangle of side 1 metre. The line is 20 km long and delivers a load of 10 MW at 33 KV and p.f. 0.8. Neglect capacitance and assume temperature of 20°C. Determine the efficiency and regulation of the line.

Solution: The inductance of the line

$$\begin{aligned} &= 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre} \\ &= 2 \times 10^{-7} \ln \frac{100}{0.5} = 10.59 \times 10^{-7} \text{ H/metre} \end{aligned}$$

$\therefore$ Inductance of 20 km length of line $= 2.119 \times 10^{-2}$ H

Inductive reactance = 6.65 ohm

Now resistance of copper conductor at 20°C is $\frac{1}{58}$ per metre length when the section is 1 mm^2.

$\therefore$ Resistance = $\frac{1}{58} \times \frac{20 \times 1000}{90}$ = 3.83 ohm

The current = $\frac{10 \times 1000}{\sqrt{3} \times 33 \times 0.8}$ = 218.68 amps

Since the capacitance is to be neglected, the receiving end current is same as the sending end and, therefore,

the loss on the line = 3 × 218.68^2 × 3.83 = 0.549 MW

$$\therefore \qquad \eta = \frac{\text{output}}{\text{output + loss}} = \frac{10}{10 + 0.549} = 0.9479$$

To determine the voltage regulation we determine the sending end voltage

$$\begin{aligned} V_s &= (V_r \cos \phi_r + I_r R) + j(V_r \sin \phi_r + I_r X) \\ &= (19052 \times 0.8 + 218.68 \times 3.83) + j(19052 \times 0.6 + 218.68 \times 6.65) \\ &= 16079 + j12885 \\ V_s &= 20605 \text{ volts} \end{aligned}$$

Since it is a short line, the voltage regulation will be

$$\% \text{ regulation} = \frac{20605 - 19052}{19052} \times 100 = 8.15\%. \quad \textbf{Ans.}$$

Example 4.4: A 400 V, 3-phase 4-wire service mains supplies a star connected load. The resistance of each line is 0.1 ohm and that of neutral 0.2 ohm. The load impedances are Z_R = (6 + j9), Z_Y = 8 ohms and Z_B = (6 – j8). Calculate the voltage across each load impedance and current in the neutral. Phase sequence RYB.

Solution: Since it is a 3-phase, 4-wire system, the current in each line can be found out considering each phase independent of each other and then we add all the three currents to find out the current in the neutral.

Taking phase R as reference,

$$\begin{aligned} I_R &= \frac{400 + j0.0}{\sqrt{3}(6.3 + j9.0)} \\ &= \frac{231(6.3 - j9.0)}{120.69} = 12.06 - j17.22 \\ I_Y &= \frac{231 \angle -120}{8.3} = 27.83(-0.5 - j0.866) \\ &= -13.92 - j24.1 \\ I_B &= \frac{231 \angle 120}{(6.3 - j8)} = \frac{231(6.3 + j8) \angle 120}{103.69} = 2.23(6.3 + j8) \angle 120 \\ &= (14.05 + j17.84)(-0.5 + j0.866) = -22.47 + j3.24 \end{aligned}$$

The neutral current $= I_R + I_Y + I_B = (-24.33 - j38.08)$ amps

$= 45.18$ amps.

Voltage across phase R impedance $= (12.06 - j17.22)(6 + j9)$

$= 72.36 + 154.98 + j108.54 - j103.32$

$= 227.4$ volts. **Ans.**

Voltage across phase Y impedance $= (-13.92 - j24.1)(8)$

$= 226.65$ volts. **Ans.**

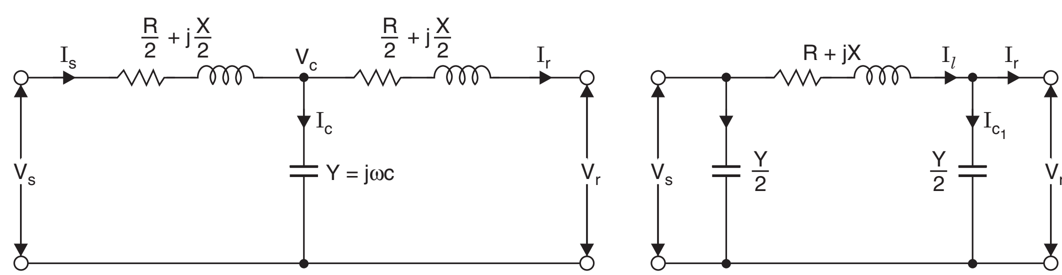
Voltage across phase B impedance $= (-22.47 + j3.24)(6 - j8)$

$= -134.82 + 25.92 + j19.44 + j179.76$

$= 227.02$ volts. **Ans.**

4.3 MEDIUM LENGTH LINES

It has been mentioned previously that transmission lines with lengths between 80 km and 160 km are categorised as medium length lines where the parameters are assumed to be lumped. The shunt capacitance is either assumed to be concentrated at the middle of the line or half of the total capacitance is concentrated at each end of the line. The two configurations are known as nominal-*T* and nominal-π respectively. The nominal circuits are shown in Figs. 4.5 (*a*) and (*b*).



(a) Nominal-T (b) Nominal-π

Fig. 4.5

It is to be noted that the two representations are approximate to the exact representation of the actual line. Also the two representations are not equivalent as can be seen by using the star-delta transformations. However, they are good enough for practical purposes and do not involve much error.

4.3.1 Nominal-*T*

The vector diagram for lagging power factor load is shown in Fig. 4.6. While analysing the medium length lines using nominal-*T*, it is preferable to take receiving end current as the reference vector as the calculations become relatively easier as compared to taking V_r as the reference.

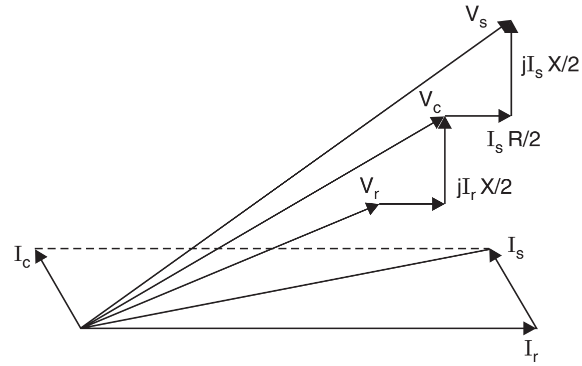


Fig. 4.6 Phasor diagram for nominal-*T*.

For calculating regulation of the line refer to Fig. 4.5(*a*). The objective first is to calculate V_s which is done as follows:

$$V_c = (|V_r| \cos \phi_r + j.|V_r| \sin \phi_r) + I_r \left(\frac{R}{2} + j\,\frac{X}{2}\right) \tag{4.12}$$

$$I_c = j\omega C V_c \tag{4.13}$$

$$I_s = I_c + I_r = I_r + j\omega C V_c \tag{4.14}$$

$$V_s = V_c + I_s \left(\frac{R}{2} + j\,\frac{X}{2}\right)$$

$$= (|V_r| \cos \phi_r + j|V_r| \sin \phi_r) + I_r \left(\frac{R}{2} + j\,\frac{X}{2}\right) + I_s \left(\frac{R}{2} + j\,\frac{X}{2}\right) \tag{4.15}$$

To calculate regulation it is required to calculate V_r' the receiving end no load voltage keeping V_s as calculated above fixed in magnitude. The nominal-*T* circuit for this condition reduces to the following:

From Fig. 4.7,

$$V_r' = \frac{|V_s|\left(-\dfrac{j}{\omega C}\right)}{\dfrac{R}{2} + j\,\dfrac{X}{2} - \dfrac{j}{\omega C}} \tag{4.16}$$

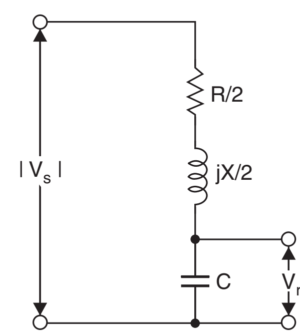


Fig. 4.7 Equivalent circuit under no load.

Now the regulation for nominal-*T* can be obtained as %

$$\text{regulation} = \frac{V_r' - V_r}{V_r} \times 100$$

To determine efficiency of the line it is suggested to make use of the following formula:

$$\%\,\eta = \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end + loss}} \times 100$$

The other formula is

$$\%\,\eta = \frac{\text{Power delivered at the receiving end}}{\text{Power sent at the sending end}}$$

A small error in evaluating phase angle between sending end voltage and current will lead to inaccurate calculation of efficiency. Therefore, it is suggested to make use of the first formula.

$$\% \, \eta = \frac{P}{P + 3\dfrac{R}{2}(I_r^{\,2} + I_s^{\,2})} \times 100$$

where P is the 3-phase power delivered at the receiving end, R is the resistance per phase.

In order to determine A, B, C, D constants for nominal-T (Fig. 4.5).

$$V_c = V_r + I_r \frac{Z}{2}$$

$$I_c = V_c Y$$

$$I_s = I_r + I_c = I_r + V_c Y = I_r + \left(V_r + I_r \frac{Z}{2}\right) Y$$

$$V_s = V_c + I_s \frac{Z}{2} = V_r + I_r \frac{Z}{2} + \left\{I_r + \left(V_r + I_r \frac{Z}{2}\right) Y\right\} . \frac{Z}{2}$$

$$= V_r \left(1 + \frac{YZ}{2}\right) + I_r \left(\frac{Z}{2} + \frac{Z}{2} + \frac{YZ^2}{4}\right)$$

$$= V_r \left(1 + \frac{YZ}{2}\right) + I_r \left(Z + \frac{YZ^2}{4}\right)$$

$$V_s = V_r \left(1 + \frac{YZ}{2}\right) + I_r Z \left(1 + \frac{YZ}{4}\right) \tag{4.17}$$

$$I_s = I_r \left(1 + \frac{YZ}{2}\right) + V_r Y$$

$$= YV_r + \left(1 + \frac{YZ}{2}\right) I_r \tag{4.18}$$

Writing down the voltage and current equation,

$$V_s = AV_r + BI_r \tag{4.19}$$

$$I_s = CV_r + DI_r \tag{4.20}$$

Comparing the coefficients of equations (4.17) to (4.20)

$$A = 1 + \frac{YZ}{2}$$

$$B = Z \left(1 + \frac{YZ}{4}\right)$$

$$C = Y$$

$$D = \left(1 + \frac{YZ}{2}\right)$$

From above it is clear that $A = D$ and

$$AD - BC = \left(1 + \frac{YZ}{2}\right)^2 - YZ\left(1 + \frac{YZ}{4}\right)$$

$$= 1 + \frac{Y^2Z^2}{4} - YZ - YZ - \frac{Y^2Z^2}{4}$$

$$= 1 \qquad (4.21)$$

Therefore, the constants as obtained above are correct.

4.3.2 Nominal-π

The circuit and its vector diagrams are shown in Figs. 4.8 (*a*) and (*b*).

For nominal-π it is desirable to take receiving end voltage as the reference vector. Refer to Fig. 4.8 (*b*) for calculating V_s.

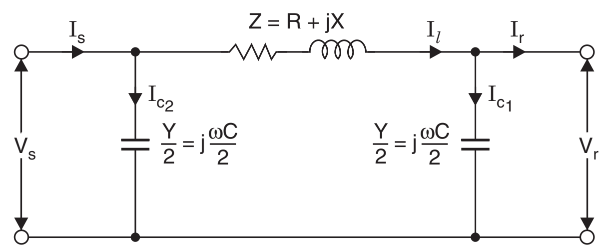


Fig. 4.8 (*a*) Nominal-π.

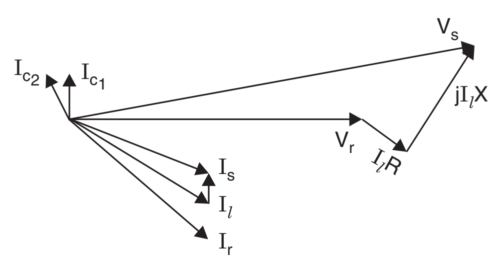


Fig. 4.8 (*b*) Phasor diagram for nominal-π.

$$I_{c_1} = jV_r \frac{\omega C}{2}$$

$$I_l = |I_r| (\cos\phi_r - j\sin\phi_r) + jV_r \frac{\omega C}{2}$$

$$V_s = V_r + I_l Z$$

$$= V_r + \left\{|I_r|(\cos\phi_r - j\sin\phi_r) + jV_r \frac{\omega C}{2}\right\}(R + jX) \qquad (4.22)$$

and
$$I_s = I_l + I_{c_2} = I_l + jV_s \frac{\omega C}{2}$$

$$I_s = |I_r| (\cos \phi_r - j \sin \phi_r) + jV_r \frac{\omega C}{2}$$

$$+ j \frac{\omega C}{2} \left[V_r + \left\{ |I_r| (\cos \phi_r - j \sin \phi_r) + jV_r \frac{\omega C}{2} \right\} (R + jX) \right] \quad (4.23)$$

Having calculated the sending end voltage, it is required to find out no load receiving end voltage for regulation keeping sending end voltage constant in magnitude. The nominal-π circuit for this reduces to Fig. 4.9.

$$V_r' = \frac{|V_s| \left(-\frac{2j}{\omega C} \right)}{R + jX - \frac{j}{\omega C / 2}}$$

Therefore % regulation $= \dfrac{V_r' - V_r}{V_r} \times 100$

and $$\% \eta = \frac{P}{P + 3I_l^2 R} \times 100$$

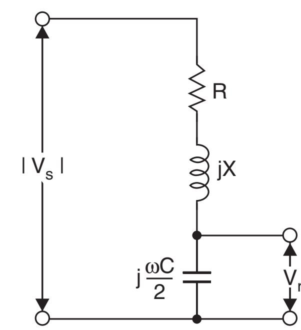


Fig. 4.9 Equivalent circuit under no load.

To determine *A, B, C, D* constants for nominal-π refer to Fig. 4.8(*a*).

$$I_{c_1} = V_r \frac{Y}{2}$$

$$I_l = I_r + I_{c_1} = I_r + V_r \frac{Y}{2}$$

$$V_s = V_r + I_l Z = V_r + \left(I_r + V_r \frac{Y}{2} \right) Z$$

$$= \left(1 + \frac{YZ}{2} \right) V_r + ZI_r \quad (4.24)$$

$$I_s = I_l + I_{c_2} = I_l + V_s \frac{Y}{2} = I_r + V_r \frac{Y}{2} + \left\{ V_r \left(1 + \frac{YZ}{2} \right) + ZI_r \right\} \frac{Y}{2}$$

$$= V_r \left(Y + \frac{Y^2 Z}{4} \right) + \left(1 + \frac{YZ}{2} \right) I_r \quad (4.25)$$

Comparing the coefficients of equations (4.24) and (4.25) with equations (4.19) and (4.20),

$$A = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y \left(1 + \frac{YZ}{4} \right)$$

$$D = \left(1 + \frac{YZ}{2} \right)$$

From above it is clear that

$$A = D$$

and $$AD - BC = 1 + \frac{Y^2Z^2}{4} + YZ - YZ - \frac{Y^2Z^2}{4} = 1$$

which means that the values of A, B, C and D are correct.

Example 4.5: Determine the efficiency and regulation of a 3-phase, 100 km, 50 Hz transmission line delivering 20 MW at a p.f. of 0.8 lagging and 66 kV to a balanced load. The conductors are of copper, each having resistance 0.1 ohm per km, 1.5 cm outside dia, spaced equilaterally 2 metres between centres. Neglect leakance and use (*i*) nominal-*T*, and (*ii*) nominal-π method.

Solution: Total resistance of line 100 × 0.1 = 10 ohms.

The inductance of the line $= 2 \times 10^{-7} \times 100 \times 1000 \ln \frac{200}{0.75}$ H

$= 11.17 \times 10^{-2}$ H

∴ Inductive reactance $= 314 \times 11.17 \times 10^{-2} = 35.1$ ohm

The capacitance/phase $$= \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \frac{200}{0.75}} \times 100 \times 1000$$

$= 9.954 \times 10^{-7} = 0.9954$ μF.

Nominal-T method: The nominal-*T* circuit for the problem is given below:

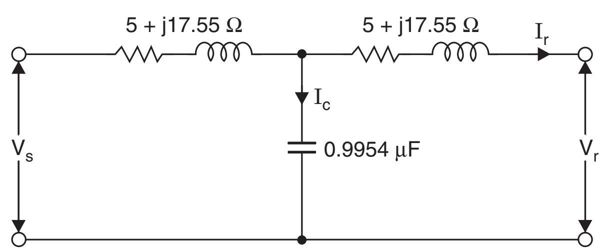


Fig. E.4.5(*a*)

$$I_r = \frac{20 \times 1000}{\sqrt{3} \times 66 \times 0.8} = 218.68 \text{ amps}$$

$$V_r = \frac{66 \times 1000}{\sqrt{3}} = 38104 \text{ volts}$$

Taking I_r as the reference, the voltage across the condenser will be

$$V_c = (38104 \times 0.8 + 218.68 \times 5) + j(38104 \times 0.6 + 218.68 \times 17.55)$$
$$= 31576 + j26700$$

The current $I_c = j\omega CV_c = j314(31576 + j26700) \times 0.9954 \times 10^{-6}$

$= j9.87 - 8.34$

∴ $I_s = 218.68 + j9.87 - 8.34 = 210.34 + j9.87$

$= 210.57$ amps

$\therefore \qquad V_s = V_c + I_s \dfrac{Z}{2}$

$= 31576 + j26700 + (210.34 + j9.87)(5 + j17.53)$

$= 31576 + 1051 - 173 + j26700 + j3691 + j49.35$

$= 32454 + j30440$

$\therefore \qquad |V_s| = 44495$ volts

The no load receiving end voltage will be

$$\frac{|V_s|(-j3199)}{5 + j17.55 - j3199} = \frac{44495(-j3199)}{5 - j3181} = 44746 \text{ volts}$$

$$\therefore \qquad \% \text{ regulation} = \frac{44746 - 38104}{38104} \times 100 = 17.4\%. \quad \textbf{Ans.}$$

To determine η we evaluate transmission line losses as follows:

$$3[218.68^2 \times 5 + 210.57^2 \times 5] = 1382409 \text{ watts} = 1.3824 \text{ MW}$$

$$\therefore \qquad \% \, \eta = \frac{20}{20 + 1.3824} \times 100 = 93.5\%. \quad \textbf{Ans.}$$

Nominal-π method: The nominal-π circuit for the problem is as follows:

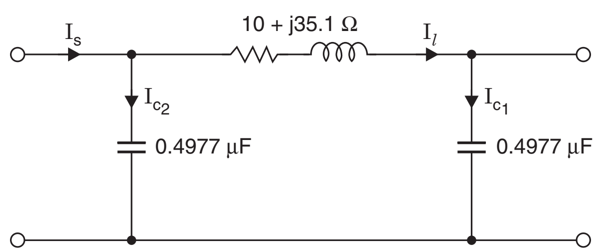


Fig. E.4.5(*b*)

For nominal-π it is preferable to take receiving end voltage as the reference phasor. The current $I_r = 218.68\,(0.8 - j0.6)$.

Current $\qquad I_{c_1} = j\omega C V_r = j314 \times 0.4977 \times 10^{-6} \times 38104 = j5.95$ amp

$\therefore \qquad I_l = I_r + I_{c_1} = 174.94 - j131.20 + j5.95 = 174.94 - j125.25$

$\therefore \qquad V_s = \mathrm{V}_r + I_l Z = 38104 + (174.94 - j125.25)(10 + j35.1)$

$= 38104 + 1749.4 - j1252.5 + j6140 + 4396$

$= 44249 + j4886$ volts

$\therefore \qquad |V_s| = 44518$ volts

The no load receiving end voltage will be

$$\frac{44518(-j6398)}{10 + j35.1 - j6398} = \frac{44518(-j6398)}{10 - j6363} = 44762 \text{ volts}$$

$$\therefore \qquad \% \text{ regulation} = \frac{44762 - 38104}{38104} \times 100 = 17.47\%$$

The line current $I_l = 215.15$

$\therefore$ $$\text{Loss} = 3 \times 215.15^2 \times 10 = 1.388 \text{ MW}$$

$\therefore$ $$\% \, \eta = \frac{20 \times 100}{21.388} = 93.5\%. \quad \textbf{Ans.}$$

4.4 LONG TRANSMISSION LINES

So far electrically short transmission lines less than 160 km in length have been considered wherein the parameters are assumed to be lumped. In case the lines are more than 160 km long, for accurate solutions the parameters must be taken as distributed uniformly along the length as a result of which the voltages and currents will vary from point to point on the line. Consider Fig. 4.10 for analysis.

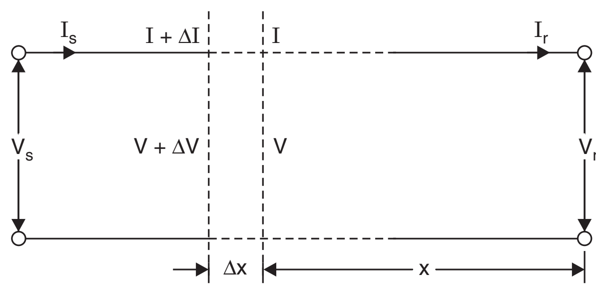


Fig. 4.10 Long transmission line.

Let z = series impedance per unit length

y = shunt admittance per unit length

l = length of the line

$Z = zl$ = total series impedance

$Y = yl$ = total shunt admittance

For clearity the elemental length dx is redrawn (Fig. 4.11).

For analysis we shall take the receiving end as the reference for measuring distances. Take an elemental length dx of the line at a distance of x from the receiving end. Say the voltage and current at a distance x are V and I and at a distance $x + dx$, $V + \Delta V$ and $I + \Delta I$ respectively.

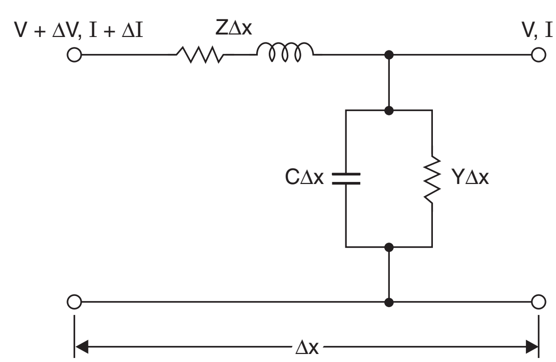


Fig. 4.11 Equivalent of a differential length of a line.

$$\Delta V = Iz \, \Delta x$$

$$\Delta I = Vy \, \Delta x \qquad (4.26)$$

From equation (4.26)

$$\frac{\Delta V}{\Delta x} = Iz$$

$$\frac{\Delta I}{\Delta x} = Vy \qquad (4.27)$$

which in the limit when $\Delta x \to 0$ reduce to

$$\frac{dV}{dx} = Iz \tag{4.28}$$

$$\frac{dI}{dx} = Vy \tag{4.29}$$

Differentiating equation (4.28), we get

$$\frac{d^2V}{dx^2} = z\,\frac{dI}{dx} = z\,.\,y\,.\,V$$

or

$$\frac{d^2V}{dx^2} - zyV = 0 \tag{4.30}$$

The solution of this equation is

$$V = A \exp(\sqrt{yz}.x) + B \exp(-\sqrt{yz}.x) \tag{4.31}$$

Now, from equations (4.28) and (4.31) let

$$Z_c = \sqrt{\frac{z}{y}} \text{ and } \gamma = \sqrt{yz} = \alpha + j\beta \tag{4.32}$$

where Z_c is known as characteristic impedance and γ the propagation constant.

The equations (4.31) and (4.32) are rewritten as

$$V = Ae^{\gamma x} + Be^{-\gamma x} \tag{4.33}$$

$$I = \frac{I}{Z_c}(Ae^{\gamma x} - Be^{-\gamma x}) \tag{4.34}$$

Two constants are to be determined, hence two boundary conditions should be known. As mentioned previously the receiving end voltage and current are known.

$\therefore$ At $x = 0$,

$$V = V_r \quad \text{and} \quad I = I_r$$

Substituting these values in equations (4.33) and (4.34),

$$V_r = A + B$$

$$I_r = \frac{1}{Z_c}(A - B)$$

$$A = \frac{V_r + I_r Z_c}{2} \text{ and } B = \frac{V_r - I_r Z_c}{2}$$

Substituting the values of A and B in equations (4.33) and (4.34), we obtain

$$V = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r - I_r Z_c}{2} e^{-\gamma x} \tag{4.35}$$

and

$$I = \frac{1}{Z_c}\left[\frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x}\right] \tag{4.36}$$

As mentioned previously V and I are the voltage and current at any point distant x from the receiving end. It can be seen very easily from the above expression that V and I (magnitude and phase) are functions of the distance x, receiving end voltage V_r and current I_r and the parameters of the line, which means they vary as we move from receiving end towards the sending end.

Before we proceed further to determine the equivalent circuit for a long transmission line it looks imperative to understand the physical significance of the voltage and current equations (4.35) and (4.36). The quantities Z_c and γ are complex.

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L}{g + j\omega C}}$$

For a lossless line $r = 0$, $g = 0$,

$$Z_c = \sqrt{\frac{L}{C}}$$

a pure resistance, and this is known as surge impedance of the line. When dealing with high frequencies or surges normally the losses are neglected and, therefore, the characteristic impedance becomes the surge impedance. Surge impedance loading of a line is the power transmitted when the line is terminated through a resistance equal to surge impedance. The approximate value of surge impedance for overhead lines is 400 ohms and that for cables is about 40 ohms. The phase angle of Z_c for transmission lines is usually between 0° and –15°. A line terminated in its characteristic impedance is called a flat line or an infinite line. The latter term arises from the fact that a line of infinite length cannot have a reflected wave.

The lower value of surge impedance in case of cables is due to the relatively large capacitance and low inductance of the cables.

The propagation constant $\gamma = \alpha + j\beta$; the real part is known as attenuation constant and the quadrature component β the phase constant and is measured in radians per unit length.

The equation (4.35) becomes

$$V = \frac{V_r + I_r Z_c}{2} e^{\alpha x} . e^{j\beta x} + \frac{V_r - I_r Z_c}{2} e^{-\alpha x} . e^{-j\beta x} \tag{4.37}$$

The first term in the above expression is called the incident voltage wave as its value increases as x is increased. Since we are taking receiving end as the reference and as x increases the value of voltage increases that means a voltage wave decreases in magnitude as it travels from the sending end towards the receiving end, that is why this part of the voltage in the above expression is called incident voltage. For similar reason the second part is called the reflected voltage. At any point along the line, voltage is the sum of these two components *i.e.*, sums of incident and reflected voltages.

As the current expression is similar to the voltage, the current can also be considered as sum of incident and reflected current waves.

The equations for voltage and currents can be rearranged as follows:

$$V = V_r . \frac{e^{\gamma x} + e^{-\gamma x}}{2} + I_r Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$= V_r \cosh \gamma x + I_r Z_c \sinh \gamma x \tag{4.38}$$

and

$$I = \frac{1}{Z_c}\left[V_r \frac{e^{\gamma x} - e^{-\gamma x}}{2} + I_r Z_c \frac{e^{\gamma x} + e^{-\gamma x}}{2}\right]$$

$$= \frac{1}{Z_c} [V_r \sinh \gamma x + I_r Z_c \cosh \gamma x]$$

$$= \frac{V_r}{Z_c} \sinh \gamma x + I_r \cosh \gamma x \tag{4.39}$$

Rewriting these equations for $x = l$, where $V = V_s$ and $I = I_s$

$$V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l \tag{4.40}$$

$$I_s = V_r \frac{\sinh \gamma l}{Z_c} + I_r \cosh \gamma l \tag{4.41}$$

These two equations relate the sending end voltage and current with the receiving end quantities. We have said previously that these quantities are related by the general equations.

$$V_s = AV_r + BI_r \tag{4.19}$$

$$I_s = CV_r + DI_r \tag{4.20}$$

where A, B, C and D are such that

$$A = D$$

and $$AD - BC = 1$$

Comparing the coefficients of the equations (4.40) and (4.41) with equations (4.19) and (4.20) respectively,

$$A = \cosh \gamma l$$

$$B = Z_c \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_c}$$

and $$D = \cosh \gamma l$$

From this it is clear that

$$A = D = \cosh \gamma l$$

and $$AD - BC = \cosh^2 \gamma l - Z_c \sinh \gamma l \cdot \frac{\sinh \gamma l}{Z_c} = 1.$$

Example 4.6: A single circuit 50 Hz, 3-phase transmission line has the following parameters per km:

$$R = 0.2 \text{ ohm}, L = 1.3 \text{ mH and } C = 0.01 \text{ μF}$$

The voltage at the receiving end is 132 kV. If the line is open at the receiving end, find the rms value and phase angle of the following:

(*i*) The incident voltage to neutral at the receiving end (reference).

(*ii*) The reflected voltage to neutral at the receiving end.

(*iii*) The incident and reflected voltages to neutral at 120 km from the receiving end.

Solution: The series impedance per unit length of the line

$$z = r + jx = (0.2 + j1.3 \times 314 \times 10^{-3}) = (0.2 + j0.408)$$

$$= 0.454 \angle 63.88°$$

The shunt admittance $= j\omega C = j314 \times 0.01 \times 10^{-6}$

$= 3.14 \times 10^{-6} \angle +90°$

The characteristic impedance $Z_c = \sqrt{\dfrac{z}{y}} = \sqrt{\dfrac{0.454}{0.314} \times 10^5 \angle 63.88 - 90°}$

$= 380 \angle -13.06°$

$$\gamma = \sqrt{yz} = \sqrt{0.314 \times 0.454 \times 10^{-6}} \angle (90 + 63.88)°/2$$

$= (0.2714 + j1.169) \times 10^{-3}$

$= (\alpha + j\beta)$

The receiving end line to neutral voltage V_r

$$= \frac{132 \times 1000}{\sqrt{3}} = 76200 \text{ volts}$$

and receiving end current under open circuited condition $I_r = 0$

(*i*) The incident voltage to neutral at the receiving end ($x = 0$)

$$= \frac{V_r + I_r Z_c}{2}$$

Since it is no load condition, $I_r = 0$.

$\therefore$ Incident voltage $= \dfrac{V_r}{2} = \dfrac{76200}{2} = 38100$ volts

(*ii*) Similarly, the reflected voltage to neutral at the receiving end

$$\frac{V_r - I_r Z_c}{2} = \frac{V_r}{2} = 38100 \text{ volts}$$

(*iii*) The incident voltage at a distance of 120 km from the receiving end

$$V_r^+ = V_r \exp(\alpha x) \exp(j\beta x)$$

$$= 76.2 \exp(0.2714 \times 120 \times 10^{-3}) \exp(j1.169 \times 120 \times 10^{-3})$$

$$= 78.7 \angle 8.02°$$

$$V_r^- = 76.2 \exp(-\alpha x) \exp(-j\beta x) = 76.2 \exp(-0.0325) \exp(-j0.140)$$

$$= 73.76 \angle -8.02°$$

The reflected voltage at a distance of 120 km from the receiving end

$$= \frac{73.76}{2} \angle -8.02° = 36.88 \angle -8.02° \text{ kV.}$$ **Ans.**

The incident voltage at a distance of 120 km from the receiving end

$$= \frac{78.7}{2} \angle 8.02° = 39.35 \angle 8.02°$$

Example 4.7: Determine the efficiency of the line in the Example 4.6 if the line is 120 km long and delivers 40 MW at 132 kV and 0.8 p.f. lagging.

Solution: Receiving end current $I_r = \dfrac{40 \times 1000}{\sqrt{3} \times 132 \times 0.8}$

$= 218.7$ amps.

From the previous example

$$Z_c = 380 \angle -13.06°$$

For 120 km length of line,

$$e^{\alpha x} e^{j\beta x} = 1.033 \angle 8.02°$$

and

$$e^{-\alpha x} e^{-j\beta x} = 0.968 \angle -8.02°$$

Taking V_r as the reference,

$$I_r = 218.7 \angle -36.8°$$

$$V_s^+ = \frac{V_r + I_r Z_c}{2} e^{\alpha x} e^{j\beta x}$$

$$= \frac{76200 + 380 \times 218.7 \angle -13.06 \angle -36.8}{2} \times 1.033 \angle 8.02°$$

$$= 74.63 \angle -18°$$

$$V_s^- = \frac{V_r - I_r Z_c}{2} e^{-\alpha x} e^{-j\beta x}$$

$$= \frac{76200 - 380 \times 218.7 \angle -49.86}{2} \times 0.968 \angle -8.02°$$

$$= 32.619 \angle 62.37 \text{ kV}$$

$$V_s = V_s^+ + V_s^- = 74.63 \angle -18° + 32.619 \angle 62.37°$$

$$= 86077 + j5751 = 86.26 \angle 3.82°$$

Now

$$I_s = \frac{V_r / Z_c + I_r}{2} e^{\alpha x} e^{j\beta x} - \frac{V_r / Z_c - I_r}{2} e^{-\alpha x} e^{-j\beta x}$$

$$= \frac{V_s^+}{Z_c} - \frac{V_s^-}{Z_c} = \left(\frac{74.63 \angle -18°}{380 \angle -13.06°} - \frac{32.619 \angle 62.37°}{380 \angle -13.06°} \right) \text{kA}$$

$$= 200.39 \angle -29.9°$$

Power at the sending end $= 3 \times |V_s| \, |I_s| \cos \phi_s$

$$= 3 \times 86.26 \times 200.39 \cos 33.72$$

$$= 43.132 \text{ MW}$$

$$\therefore \qquad \% \, \eta = \frac{40}{43.132} \times 100 = 92.7\%$$

Example 4.8: Determine the *ABCD* parameters of the line of example 4.6 and verify the sending end quantities found in Example 4.7.

Solution: $\gamma l = (0.2714 + j1.169)120 \times 10^{-3} = 0.03254 + j0.1402$

$$A = \cosh \gamma l = \cosh (0.03254 + j0.1402)$$

$$= \cosh 0.03254 \cos 0.1402 + j \sinh 0.03254 \sin 0.1402$$

$$= 0.99 + j0.004435 = 0.99 \angle 0.26°$$

$$B = Z_c \sinh \gamma l$$

$$\sinh \gamma l = \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l$$

$$= \sinh 0.03254 \cos 0.1402 + j \cosh 0.03254 \sin 0.1402$$

$$= 0.031958 + j0.1386$$
$$= 0.1422 \angle 77°$$

$\therefore$ $$B = Z_c \sinh \gamma l = 380 \angle -13.06° \times 0.1422 \angle 77°$$
$$= 54.03 \angle 64°$$

$\therefore$ $$V_s = AV_r + BI_r$$
$$= 0.99 \angle 0.26 \times 76200 + 54.03 \angle 64° \times 218.7 \angle -36.8°$$
$$= 75438 + 11772 \angle 27.2°$$
$$= 85908 + j5380$$
$$= 86.07 \angle 3.588°.$$

Example 4.9: Determine the sending end voltage and efficiency using nominal-π and nominal-T methods for the problem 4.6.

Solution: *Nominal-π method:*

The resistance of the line $= 0.2 \times 120 = 24$ ohms

The inductive reactance $= 1.3 \times 10^{-3} \times 120 \times 314 = 48.98\ \Omega$

The capacitance $= 0.01 \times 10^{-6} \times 120 = 1.2\ \mu F$

The nominal-π circuit will be

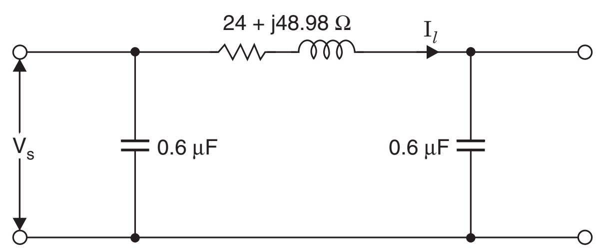


Fig. E.4.9(*a*)

Taking receiving end voltage as reference,

$$I_r = 218.7(0.8 - j0.6) = 174.96 - j131.22$$

$$I_{c_1} = j314 \times 0.6 \times 10^{-6} \times 76200 = j14.356 \text{ amp}$$

$\therefore$ $$I_l = I_{c_1} + I_r = 174.96 - j116.86 = 210.39 \angle 33.73°$$

$\therefore$ $$V_s = 76200 + (174.96 - j116.86)(24 + j48.38)$$
$$= 76200 + 4199 + j8596 - j2804 + 5723$$
$$= 86122 + j5765$$
$$= 86314 \angle 3.82° \text{ volts}$$

The loss $= 3 \times 210.39^2 \times 24 = 3.187$ MW

$\therefore$ $$\% \eta = \frac{40 \times 100}{43.187} = 92.69\%$$

Nominal-*T* method: The nominal-*T* circuit will be

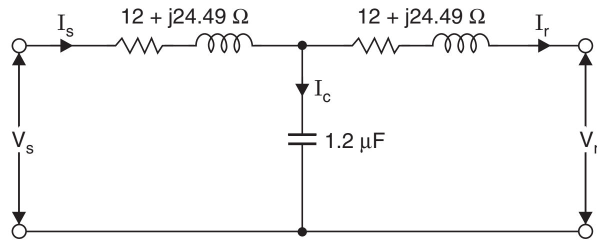


Fig. E.4.9(*b*)

Taking receiving end current as reference,

$$V_c = 76200(0.8 + j0.6) + 218.7(12 + j24.49)$$
$$= 60960 + j45720 + 2624 + j5356$$
$$= 63584 + j51076$$
$$I_c = j314 \times 1.2 \times 10^{-6}(63584 + j51076)$$
$$= j23.95 - 19.24$$
$$\therefore \quad I_s = 218.7 + j23.95 - 19.24 = 199.46 + j23.95$$
$$= 200.89 \angle 6.8°$$
$$V_s = 63584 + j51076 + (199.46 + j23.95)(12 + j24.49)$$
$$= 63584 + j51076 + 2393 + j4884.7 + j287.4 - 586.5$$
$$= 65390 + j56248$$
$$= 86.25 \angle 40.70°. \quad \textbf{Ans.}$$
$$\text{The loss} = 3 \times 12(200.89^2 + 218.7^2) = 3.174 \text{ MW}$$
$$\therefore \quad \% \ \eta = \frac{40}{43.174} \times 100 = 92.64\%. \quad \textbf{Ans.}$$

The Equivalent Circuit Representation of a Long Line

It has been mentioned previously that for lengths more than 160 km the parameters are assumed to be distributed and for which we have got the voltage and currents in the previous section. We will now derive equivalent-π and equivalent-*T* circuits for such long lines. The nominal-π and nominal-*T* circuits do not represent the lines exactly because they do not account for the parameters of the lines being uniformly distributed.

Equivalent-π Representation

Let us assume that the two terminal conditions *i.e.*, the sending end and receiving end voltage and currents can be related with the following equivalent network given in Fig. 4.12.

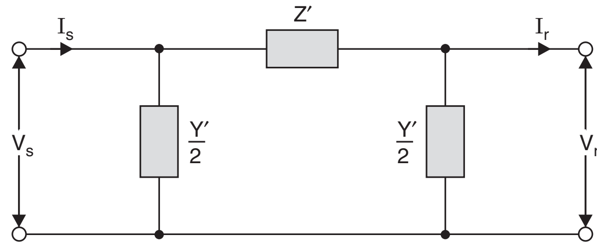


Fig. 4.12 Equivalent circuit of a long line.

From the network the following relations can be derived:

$$V_s = \left(1 + \frac{Y'Z'}{2}\right) V_r + Z'I_r \tag{4.42}$$

$$I_s = Y'\left(1 + \frac{Y'Z'}{4}\right) V_r + \left(1 + \frac{Y'Z'}{2}\right) I_r \tag{4.43}$$

Comparing equations (4.42) and (4.43) with equations (4.40) and (4.41) respectively,

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \tag{4.44}$$

$$Z' = Z_c \sinh \gamma l \tag{4.45}$$

$$Y'\left(1 + \frac{Y'Z'}{4}\right) = \frac{\sinh \gamma l}{Z_c} \tag{4.46}$$

and

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \tag{4.47}$$

Considering equation (4.45),

$$\begin{aligned} Z' &= Z_c \sinh \gamma l \\ &= \sqrt{\frac{z}{y}} \,.\, l \, \frac{\sinh \gamma l}{\sqrt{yz}.l} \,.\, \sqrt{yz} \\ &= zl \, \frac{\sinh \gamma l}{\gamma l} \\ &= Z \, \frac{\sinh \gamma l}{\gamma l} \end{aligned}$$

This means to get the equivalent series impedance the lumped impedance Z should be multiplied by $(\sinh \gamma l)/\gamma l$. Now to get the shunt arm of the equivalent-π circuit, we substitute Z' in equation (4.44).

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l$$

$$1 + \frac{Y'}{2} Z_c \sinh \gamma l = \cosh \gamma l$$

$$\frac{Y'}{2} Z_c \sinh \gamma l = \cosh \gamma l - 1 = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} - \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$$

or
$$Y'Z_c \sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2} = 2 \sinh^2 \frac{\gamma l}{2}$$

or
$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh \frac{\gamma l}{2} = \sqrt{\frac{y}{z}} \cdot \frac{\sqrt{yz}.l}{2} \cdot \frac{\tanh \gamma l / 2}{\sqrt{yz} l / 2}$$

$$= \frac{Y}{2} \frac{\tanh \gamma l / 2}{\gamma l / 2}$$

where Y is the total shunt admittance. This means to get the shunt arm of the equivalent-π, the shunt arm of the nominal-π should be multiplied by $\dfrac{\tanh \gamma l}{\gamma l}$.

The equivalent-π circuit can be represented as shown in Fig. 4.13.

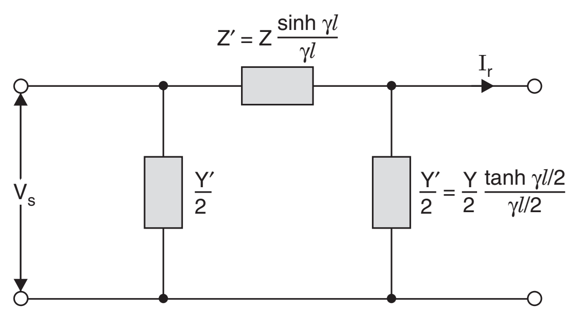


Fig. 4.13 Equivalent-π representation.

Equivalent-T Representation of Long Line

To determine the equivalent-T circuit consider the following network in Fig. 4.14.

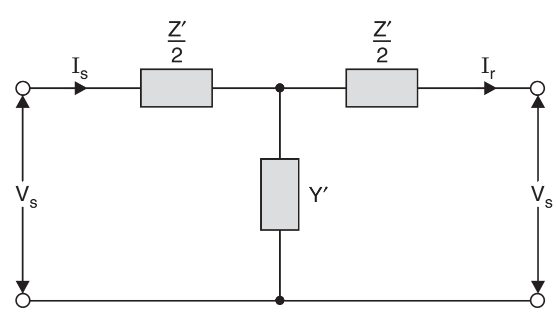


Fig. 4.14 Equivalent-T representation.

From the network the following relations can be derived (Refer to equations (4.17) and (4.18))

$$V_s = \left(1 + \frac{Y'Z'}{2}\right) V_r + Z' \left(1 + \frac{Y'Z'}{4}\right) I_r \tag{4.48}$$

$$I_s = Y'V_r + \left(1 + \frac{Y'Z'}{2}\right) I_r \tag{4.49}$$

Comparing equations (4.48) and (4.49) with equations (4.40) and (4.41),

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \tag{4.50}$$

$$Z'\left(1 + \frac{Y'Z'}{4}\right) = Z_c \sinh \gamma l \tag{4.51}$$

$$Y' = \frac{\sinh \gamma l}{Z_c} \tag{4.52}$$

and

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \tag{4.53}$$

To determine the shunt branch of the equivalent-T circuit consider equation (4.52).

$$Y' = \frac{1}{Z_c} \sinh \gamma l = \sqrt{\frac{y}{z}} \cdot \sqrt{yz} \cdot l \frac{\sinh \gamma l}{\sqrt{yz}\, l}$$

$$= yl \frac{\sinh \gamma l}{\gamma l}$$

$$= Y \frac{\sinh \gamma l}{\gamma l} \tag{4.54}$$

This means to get the shunt branch of the equivalent-T, the shunt branch of the nominal-T (lumped shunt admittance) should be multiplied by the factor $\dfrac{\sinh \gamma l}{\gamma l}$.

To get the series impedance of the equivalent-T, equation (4.52) is substituted in equation (4.50).

$$1 + \frac{Z'}{2} \frac{\sinh \gamma l}{Z_c} = \cosh \gamma l$$

$$\frac{Z'}{2} \cdot 2 \cdot \frac{\sinh \gamma l/2}{Z_c} \cdot \cosh \frac{\gamma l}{2} = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} - \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$$

$$\frac{Z'}{2} \cdot 2 \cdot \frac{\sinh \gamma l/2 \cdot \cosh \gamma l/2}{Z_c} = 2 \sinh^2 \frac{\gamma l}{2}$$

$$\frac{Z'}{2} = Z_c \tanh \frac{\gamma l}{2}$$

$$= \sqrt{\frac{z}{y}} \cdot \frac{\sqrt{yz}.l}{2} \cdot \frac{\tanh \gamma l/2}{\sqrt{yz}\,.\, l/2}$$

$$= \frac{Z}{2} \frac{\tanh \gamma l/2}{\gamma l/2} \tag{4.55}$$

This means to get the series branch of the equivalent-T circuit, the series branch of the nominal-T (lumped series impedance) should be multiplied by the factor $\dfrac{\tanh \gamma l/2}{\gamma l/2}$.

The equivalent-T representation is given in Fig. 4.15.

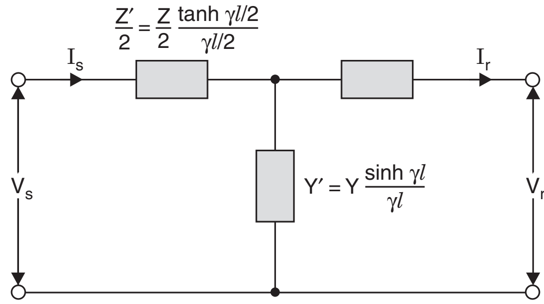


Fig. 4.15 Equivalent-*T* representation.

It is to be noted that since the ratio of tanh $\gamma l/2$ to $\gamma l/2$ and sinh γl to γl is almost equal to unity for small values of γl, the nominal circuits represent the medium length lines quite accurately. This fact can be proved by expanding the constants of the equivalent circuits.

$$D = A = \cosh \gamma l = 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \ldots\ldots$$

$$B = Z_c \sinh \gamma l = \sqrt{\frac{z}{y}}\left[\sqrt{yz}\, l + \frac{(yz)^{3/2}}{6} l^3 + \frac{(yz)^{5/2}}{120} l^5 + \ldots\ldots\right]$$

$$= Z\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \ldots\ldots\right]$$

Similarly

$$C = \frac{\sinh \gamma l}{Z_c} = Y\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \ldots\ldots\right)$$

$$= Y + \frac{Y^2Z}{6} + \frac{Y^3Z^2}{120} + \ldots\ldots \tag{4.56}$$

Since Y^2 is very small, $\frac{Y^2Z}{6} \simeq \frac{Y^2Z}{4}$.

Therefore, taking into account only the first order term we have

$$A = 1 + \frac{YZ}{2} = D$$

$$B = Z$$

$$C = Y + \frac{Y^2Z}{4} = Y\left(1 + \frac{YZ}{4}\right)$$

which are the *A, B, C* and *D* parameters of nominal-π representation.

4.5 *ABCD* CONSTANTS

We know that, the sending end quantities *i.e.*, V_s and I_s are given by

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

Similar relations for V_r and I_r can be found from these equations as follows:

Multiply equation (4.19) by C and (4.20) by A.

$$CV_s = CAV_r + BCI_r \tag{4.57}$$

$$AI_s = ACV_r + ADI_r \tag{4.58}$$

Subtracting equation (4.57) from (4.58),

$$AI_s - CV_s = (AD - BC)\, I_r$$

Since $AD - BC = 1$ and $A = D$,

$$I_r = -\,CV_s + DI_s \tag{4.59}$$

Next to eliminate I_r from equations (4.19) and (4.20), multiply equation (4.19) by D and (4.20) by B.

$$DV_s = ADV_r + BDI_r \tag{4.60}$$

$$BI_s = BCV_r + BDI_r \tag{4.61}$$

Subtracting equation (4.61) from (4.60),

$$DV_s - BI_s = (AD - BC)\, V_r$$

Again $AD - BC = 1$ and $V_r = DV_s - BI_s$ (4.62)

Proof for the relation AD – BC = 1: Consider Fig. 4.16(a) where a two-terminal pair network with parameters A, B, C and D is connected to an ideal voltage source with zero internal impedance at one end and the other end is short circuited.

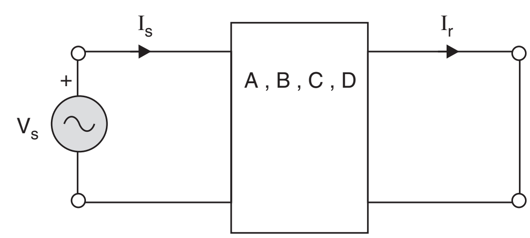


Fig. 4.16(*a*) Two-terminal pair network.

The equation representing this condition is

$$V_s = E = 0 + BI_2$$

or

$$I_2 = \frac{E}{B} \tag{4.63}$$

Now we short circuit the sending end and connect the generator at the receiving end as shown in Fig. 4.16(b). The positive directions of flow of current are shown in the figures.

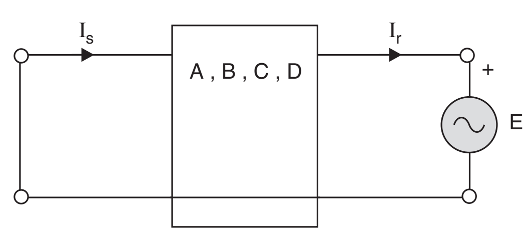


Fig. 4.16(*b*) Source at the receiving end.

$$\therefore \quad 0 = AE + BI_r \tag{4.64}$$

Since transmission line is a linear passive bilateral network, therefore

$$I_s = -\,I_2 = CE + DI_r \tag{4.65}$$

Eliminating I_r from equations (4.64) and (4.65) we obtain

$$-\,I_2 = CE + D\,\frac{-\,AE}{B} \tag{4.66}$$

Since from equation (4.63) $I_2 = \dfrac{E}{B}$, equation (4.66) becomes

$$\frac{-E}{B} = CE + D\frac{-AE}{B}$$

or
$$-\frac{1}{B} = C - D\frac{A}{B}$$

or
$$-1 = BC - AD$$

or
$$AD - BC = 1 \tag{4.67}$$

As is said earlier that if *A, B, C, D* parameters are calculated independently, equation (4.67) gives a check on the accuracy of the values calculated.

Constants for Two Networks in Tandem

Two networks are said to be connected in tandem when the ouput of one network is connected to the input of the other network. Let the constants of these networks be A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 which are connected in tandem as shown in Fig. 4.17. These two networks could be two transmission lines or a transformer connected to a transmission line etc.

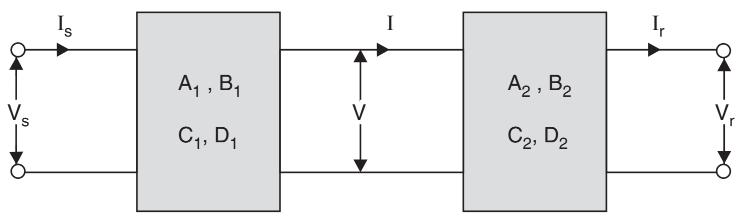


Fig. 4.17 Two networks in tandem.

The net constants of the system relating the terminal conditions can be found as follows:

$$V = D_1V_s - B_1I_s \tag{4.68}$$

$$I = -C_1V_s + A_1I_s \tag{4.69}$$

and
$$V = A_2V_r + B_2I_r \tag{4.70}$$

$$I = C_2V_r + D_2I_r \tag{4.71}$$

From equations (4.68) and (4.70) and equations (4.69) and (4.71) respectively,

$$D_1V_s - B_1I_s = A_2V_r + B_2I_r \tag{4.72}$$

$$-C_1V_s + A_1I_s = C_2V_r + D_2I_r \tag{4.73}$$

Multiplying equation (4.72) by A_1 and (4.73) by B_1 and adding the resulting equations:

$$(A_1D_1 - B_1C_1)V_s = (A_1A_2 + B_1C_2)V_r + (A_1B_2 + B_1D_2)I_r \tag{4.74}$$

Multiplying equation (4.72) by C_1 and (4.73) by D_1 and adding the resulting equations,

$$(A_1D_1 - B_1C_1)I_s = (A_2C_1 + C_2D_1)V_r + (B_2C_1 + D_1D_2)I_r \tag{4.75}$$

Since $A_1D_1 - B_1C_1 = 1$, the constants for the two networks in tandem are

$$\left.\begin{aligned} A &= A_1A_2 + B_1C_2 \\ B &= A_1B_2 + B_1D_2 \\ C &= A_2C_1 + C_2D_1 \\ D &= B_2C_1 + D_1D_2 \end{aligned}\right\} \tag{4.76}$$

The relation is given in matrix form as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

If network 2 is at the sending end and 1 is at the receiving end the overall constants for the two networks in tandem can be obtained by interchanging the subscripts in equations.

Constants for Two Networks in Parallel

In case two networks are connected in parallel as shown in Fig. 4.18, the constants for the overall networks can be obtained as follows:

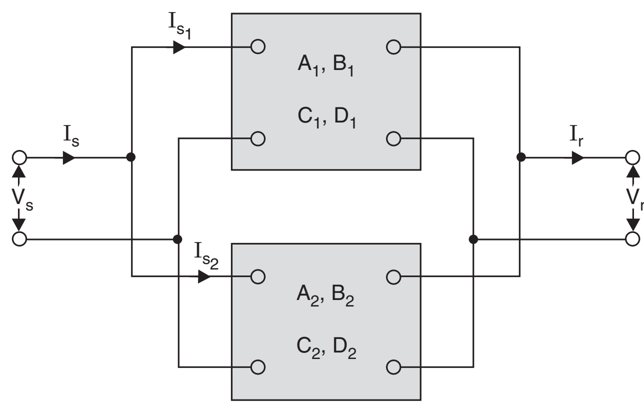


Fig. 4.18 Two networks in parallel.

The derivation is based on the fact that transmission line is a reciprocal network (symmetrical network) and when two reciprocal networks are connected in parallel, the resulting network is also reciprocal (The resulting network is not reciprocal in case the two networks are connected in tandem).

Writing the equations for the terminal conditions,

$$V_s = A_1 V_r + B_1 I_{r_1} \tag{4.77}$$

$$V_s = A_2 V_r + B_2 I_{r_2} \tag{4.78}$$

Since the overall expression required is

$$V_s = AV_r + BI_r \tag{4.79}$$

where $I_r = I_{r_1} + I_{r_2}$.

Therefore, multiplying equations (4.77) and (4.78) by B_2 and B_1 respectively and adding, we get

$$(B_1 + B_2)V_s = (A_1B_2 + A_2B_1)V_r + B_1B_2(I_{r_1} + I_{r_2})$$

or

$$V_s = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} V_r + \frac{B_1B_2}{B_1 + B_2} I_r \tag{4.80}$$

Comparing the coefficients of equations (4.79) and (4.80), we get

$$A = \frac{A_1B_2 + A_2B_1}{B_1 + B_2}$$

and
$$B = \frac{B_1B_2}{B_1 + B_2} \tag{4.81}$$

Since transmission line is a symmetrical network,

$$A = D = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} = \frac{D_1B_2 + D_2B_1}{B_1 + B_2} \tag{4.82}$$

Also since transmission line is a two terminal pair network,

$$AD - BC = 1 \tag{4.83}$$

Using relations (4.81), (4.82) and (4.83) we obtain

$$C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \tag{4.84}$$

Measurement of A, B, C, D Constants

If a transmission line is already erected, the constants can be measured by conducting the open circuit and short circuit tests at the two ends of the line as follows:

In Fig. 4.19, the connection diagrams for conducting O.C. and S.C. test on the sending end are shown. Similar connections are made for performing these tests on the receiving end side. Before proceeding further in the determination of the constants the following impedances are defined:

Z_{so} = Sending end impedance with receiving end open-circuited,

Z_{ss} = Sending end impedance with receiving end short-circuited,

Z_{ro} = Receiving end impedance with sending end open-circuited,

Z_{rs} = Receiving end impedance with sending end short-circuited.

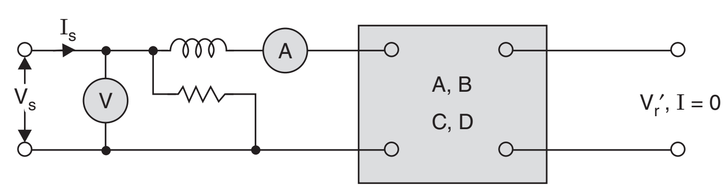


Fig. 4.19(*a*) Open-circuit tests.

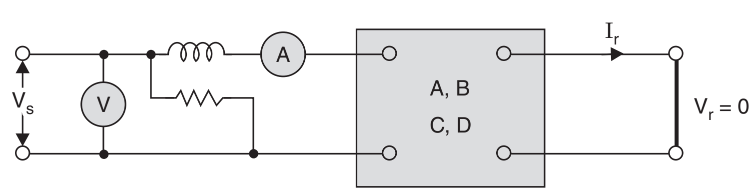


Fig. 4.19(*b*) Short-circuit tests.

Using equations

$$V_s = AV_r + BI_r \tag{4.19}$$

$$I_s = CV_r + DI_r \tag{4.20}$$

for making impedance measurements on the sending end side, we get

$$Z_{so} = \frac{V_s}{I_s} = \frac{A}{C} \text{ for } I_r = 0 \text{ (Open Circuit test)} \tag{4.85}$$

and

$$Z_{ss} = \frac{V_s}{I_s} = \frac{B}{D} \text{ for } V_r = 0 \text{ (Short Circuit test)} \tag{4.86}$$

It is to be noted here that the impedances are complex quantities, the magnitudes are obtained by the ratio of the voltages and currents and the angle is obtained with the help of Wattmeter reading.

To determine the impedances on the receiving end the following equations are made use of:

$$V_r = DV_s - BI_s \tag{4.62}$$

$$I_r = -CV_s + AI_s \tag{4.59}$$

Since the direction of sending end current according to the above equation enters the network whereas while performing the tests on receiving end side, the direction of the current will be leaving the network, therefore, these equations become

$$V_r = DV_s + BI_s \tag{4.87}$$

$$-I_r = -CV_s - AI_s \quad \text{or} \quad I_r = CV_s + AI_s \tag{4.88}$$

Therefore, for sending end open-circuits $I_s = 0$

$$Z_{ro} = \frac{V_r}{I_r} = \frac{D}{C} \tag{4.89}$$

and for short-circuit $V_s = 0$

$$Z_{rs} = \frac{V_r}{I_r} = \frac{B}{A} \tag{4.90}$$

From equations (4.89) and (4.90), we obtain

$$Z_{ro} - Z_{rs} = \frac{D}{C} - \frac{B}{A} = \frac{AD - BC}{AC} = \frac{1}{AC}$$

$$\frac{Z_{ro} - Z_{rs}}{Z_{so}} = \frac{1}{AC}\,\frac{C}{A} = \frac{1}{A^2}$$

or

$$A = \sqrt{\frac{Z_{so}}{Z_{ro} - Z_{rs}}} \tag{4.91}$$

$$Z_{rs} = \frac{B}{A}$$

or

$$B = AZ_{rs} = Z_{rs}\sqrt{\frac{Z_{so}}{Z_{ro} - Z_{rs}}} \tag{4.92}$$

$$Z_{so} = \frac{A}{C}$$

or $$C = \frac{A}{Z_{so}} = \frac{1}{Z_{so}} \sqrt{\frac{Z_{so}}{Z_{ro} - Z_{rs}}} \tag{4.93}$$

$$Z_{ro} = \frac{D}{C}$$

or $$D = CZ_{ro} = \frac{Z_{ro}}{Z_{so}} \sqrt{\frac{Z_{ro}}{Z_{ro} - Z_{rs}}}$$

$$= Z_{ro} \sqrt{\frac{1}{Z_{so}(Z_{ro} - Z_{rs})}} \tag{4.94}$$

Since for a symmetric network $Z_{ro} = Z_{so}$

$$D = A = \sqrt{\frac{Z_{so}}{Z_{ro} - Z_{rs}}} \tag{4.95}$$

Example 4.10: Determine the sending end voltage current, power and power factor for a 160 km section of 3-phase line delivering 50 MVA at 132 kV and p.f. 0.8 lagging. Also find the efficiency and regulation of the line. Resistance per line 0.1557 ohm per km, spacing 3.7 m, 6.475 m, 7.4 m transposed. Evaluate the *A, B, C, D* parameters also. Diameter 1.956 cm.

Solution: $R = 0.1557 \times 160 = 24.9\ \Omega$

$$\text{GMD} = \sqrt[3]{3.7 \times 6.475 \times 7.4}$$

$$= 5.6 \text{ metre.}$$

$$\text{Inductance} = 2 \times 10^{-7} \left(\ln \frac{560}{0.978} \right) \times 160 \times 1000$$

$$= 0.2032 \text{ H}$$

or $$X_L = 63.8$$

$$C = \frac{2\pi \in_0}{\ln \dfrac{560}{0.978}} \times 160 \times 1000$$

$$= \frac{1}{36\pi} \times 10^{-9} \left(\frac{2\pi}{\ln \dfrac{560}{0.978}} \right) \times 160 \times 1000$$

$$= \frac{10^{-9}}{18} \times \frac{160000}{\ln \left(\dfrac{560}{0.978} \right)}$$

$$= 1399 \times 10^{-9} \text{ F}$$

$$= 1.399\ \mu\text{F}$$

$$Z = \sqrt{0.1557^2 + 0.39875^2} = 0.428\underline{/68.67^\circ}$$

$$j\omega C = j \times 314 \times \frac{1.399}{160} \times 10^{-6}$$

$$= j\,2.745 \times 10^{-6}$$

$$Z_c = \sqrt{\frac{0.428}{2.745} \times 10^{+6}}\,/\underline{(68.67 - 90)}$$

$$= 394.8/\underline{-10.66°}$$

$$\gamma = \sqrt{0.428 \times 2.745 \times 10^{-6}}\,/\underline{(90 + 68.67)}$$

$$= 1.084 \times 10^{-3}/\underline{79.335°}$$

$$= (0.2 + j1.0653) \times 10^{-3}$$

Now
$$\gamma l = (0.2 + j1.0653) \times 10^{-3} \times 160$$

$$= 0.0320 + j0.17$$

$$A = \cosh \gamma l = \cosh (0.032 + j0.17)$$

$$= \cosh 0.032 \cos 0.17 + \sinh 0.032 \sin 0.17$$

$$= 0.9855 + j0.032 \times 0.16926$$

$$= 0.9855 + j0.005417$$

$$= 0.9855/\underline{0.32°}$$

$$B = Z_c \sinh \gamma l$$

$$\sinh \gamma l = \sinh 0.032 \cos 0.17 + j \cosh 0.032 \sin 0.17$$

$$= 0.032 \times 0.9835 + j0.9855 \times 0.17$$

$$= 0.0315 + j0.167535$$

$$= 0.17047/\underline{79.35°}$$

$\therefore$
$$B = 394.8 \times 0.17047/\underline{79.35 - 10.66}$$

$$= 67.3/\underline{68.69}$$

$$C = \frac{\sinh \gamma l}{Z_c} = \frac{0.17047}{394.8}$$

$$= 4.3 \times 10^{-4}/\underline{90°}$$

Taking V_r as reference, we have

$$I_r = \frac{50{,}000}{\sqrt{3} \times 132} = 218.68 \text{ A}$$

$$V_s = 0.9855/\underline{0.32°} \times 76.208 + 67.3/\underline{68.69°} \times 218.68/\underline{-36.87°} \times 10^{-3}$$

$$= 75.103/\underline{0.32°} + 14.717/\underline{31.82°}$$

$$= 75.103 + 12.505 + j7.76$$

$$= 87.608 + j7.76$$

$= 87.95\angle 5.06°$

or V_s line to line $= 152.34$ kV. **Ans.**

$$I_s = 4.3 \times 10^{-4}\angle 90° \times 76.208 \times 10^3 + 0.9855\angle 0.32° \times 218.68\angle -36.87°$$

$$= 32.76\angle 90° + 215.51\angle -36.55°$$

$$= j\,32.76 + 173.12 - j\,128.34$$

$$= 173.12 - j\,95.58$$

$$= 197.75\angle -28.9°. \quad \textbf{Ans.}$$

$\therefore$ Sending end power $= 3 \times 87.95 \times 197.75\angle 33.96°$

$= 43278$ kW

and sending end p.f. = 0.829. **Ans.**

No load voltage at the receiving end

$$= \frac{V_s}{A} = \frac{87.95}{0.9855} = 89.24$$

$$\therefore \quad \% \text{ regulation} = \frac{89.24 - 76.208}{76.208} \times 100$$

$= 17.1\%$. **Ans.**

$$\% \text{ efficiency} = \frac{50{,}000 \times 0.8}{43278} \times 100$$

$= 92.4\ \%$. **Ans.**

4.6 FERRANTI-EFFECT

When a long line is operating under no load or light load condition, the receiving end voltage is greater than the sending end voltage. This is known as Ferranti-effect. This phenomenon can be explained with the following reasonings:

(*i*) Assume no load condition. The equation (4.37)

$$V = \frac{V_r + I_r Z_c}{2} e^{\alpha x} e^{j\beta x} + \frac{V_r + I_r Z_c}{2} e^{-\alpha x} e^{-j\beta x}$$

reduces to:

$$V_s = \frac{V_r}{2} e^{\alpha l} e^{j\beta l} + \frac{V_r}{2} e^{-\alpha l} e^{-j\beta l}$$

when $x = l$ and $I_r = 0$.

At $l = 0$

$$V_r = \frac{V_r}{2} + \frac{V_r}{2}$$

As l increases, the incident component of sending end voltage increases exponentially and turns the vector anti-clockwise through an angle βl, whereas the reflected part of sending end voltage decreases by the same amount and is rotated clockwise through the same angle βl.

The sum of these two components of sending end voltage gives a voltage which is smaller than V_r.

(*ii*) A simple explanation of Ferranti-effect can be given by approximating the distributed parameters of the line by lumped impedance as shown in Fig. 4.20 (*a*).

Since usually the capacitive reactance of the line is quite large as compared to the inductive reactance, under no load or lightly loaded condition the line current is of leading p.f. The phasor diagram is given below for this operating condition.

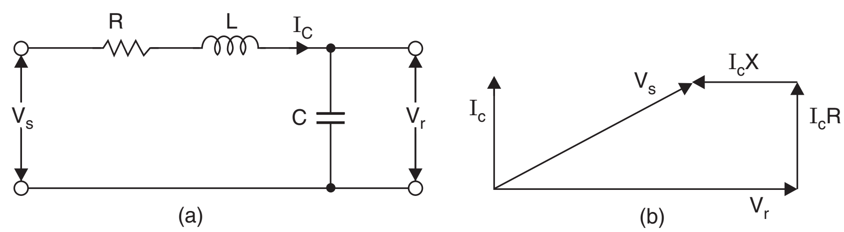


Fig. 4.20 (*a*) Line representation (Lumped) under no load condition (*b*) Its phasor diagram.

The charging current produces drop in the reactance of the line which is in phase opposition to the receiving end voltage and hence the sending end voltage becomes smaller than the receiving end voltage.

Yet another way of explaining the Ferranti-effect is based on the net reactive power flow on the line. It is known that if the reactive power generated at a point is more than the reactive power absorbed, the voltage at that point becomes higher than the normal value and vice versa. The inductive reactance of the line is a sink for the reactive power whereas the shunt capacitances generate reactive power. In fact, if the line loading corresponds to the surge impedance loading, the voltage is same everywhere as the reactive power absorbed then equals the reactive power generated by the line. The SIL, therefore, gives definite meaning to the terms lightly loaded or fully loaded lines. If the loading is less than SIL, the reactive power generated is more than absorbed, therefore, the receiving end voltage is greater than the sending end voltage. This explains, therefore, the phenomenon due to Ferranti-effect.

PROBLEMS

4.1. Determine the sending end voltage, current, power factor of a 1-phase 50 Hz, 76.2 kV transmission delivering a load of 12 MW at 0.8 p.f. The line constants are R = 25 ohm, inductance 200 mH and capacitance between lines 2.5 μF. Also determine the regulation and η of transmission. Use nominal-π method. Draw phasor diagram.

4.2. A 3-phase 4-wire 400/231 volt, 50 Hz system has a balanced 3-phase motor load of 20 kW at a power factor 0.8 lagging and 1-phase loads 25 A at unity p.f., 45 A at p.f. 0.9 leading and 25 A at p.f. 0.8 lagging, the phase sequence being in the order given. Determine the current in each line and neutral and the capacitance required across each phase to obtain unity p.f. in each line.

4.3. A 400 V 3-phase 4-wire system supplies the following loads: Phase R-40 A at p.f. 0.8 lagging, phase Y-30 A at unity p.f. and phase B-20 A at 0.8 leading. The resistance of each conductor is 0.2 ohm and of the neutral 0.4 ohm. Determine the load voltages.

4.4. A 400 V, 3-phase, 4-wire system supplies the following loads: Phase *R*-20 kVA at p.f. 0.8 lag, phase *Y*-20 kVA at 0.8 lead p.f. and phase *B*-20 kVA at unity p.f. The resistance of each line is 0.2 ohm and of the neutral 0.4 ohm. Calculate the current in the neutral wire and the load voltages.

4.5. The phase turns ratios of transformers *A* and *B* as shown in the diagram are 3 : 1 amd 2 : 1. Determine the no load line voltage on each side of *A* and *B*; also the line current when the load current is 1210 amps. Neglect line drop and magnetising current.

Fig. P.4.5

4.6. Determine the efficiency and regulation of a 3-phase, 50 Hz, 150 kms long transmission line having three conductors spaced 3.5 metres delta formation when the receiving end delivers 25 MVA at 120 kV and p.f. 0.9 lagging. The resistance of the conductor is 0.25 ohm per km and the effective dia is 0.75 cm. Neglect leakance and use (*i*) nominal-*T*, (*ii*) nominal-π, and (*iii*) exact solution methods.

4.7. A 3-phase 50 Hz transmission line has resistance, inductance and capacitance per phase of 10 ohm, 0.1 H and 0.9 μF respectively and delivers a load of 35 MW at 132 kV and 0.8 p.f. lag. Determine the efficiency and regulation of the line using (*i*) nominal-*T*, (*ii*) nominal-π and (*iii*) exact solution methods.

4.8. A short 3-phase transmission line has a series line impedance per phase of (20 + *j*50) ohm. The line delivers a load of 50 MW at 0.7 p.f. lag. Determine the regulation of the line and the *A, B, C, D* parameters of the line. If the same load is delivered at 0.7 p.f. lead, determine the regulation of the line. System voltage 220 kV.

4.9. Find the *A, B, C, D* parameters of a 3-phase, 80 km, 50 Hz transmission line with series impedance of (0.15 + *j*0.78) ohm per km and a shunt admittance of $j5.0 \times 10^{-6}$ ohm per km.

4.10. Determine both *T* and π equivalents for the line of problem 4.9. Also determine the propagation constant and the surge impedance of the line.

4.11. Determine *A, B, C, D* parameters of the line 400 km long having per unit impedance and admittance as in problem 4.9 assuming (*i*) the line could be represented by nominal-*T* or nominal-π, and (*ii*) the exact representation.

4.12. Determine the series and shunt parameters for the (*i*) equivalent-π, (*ii*) equivalent-*T* circuit for the 400 km long line of problem 4.11.

4.13. Determine the efficiency and regulation of the line of problem 4.11 when in delivers a load of 125 MW at 0.8 p.f. lag and 400 kV.

4.14. Differentiate between a nominal-*T* and equivalent-*T* representation of a transmission line.

4.15. Explain clearly the 'Ferranti effect' with a phasor diagram.

4.16. Explain the classification of lines based on their length of transmission.

4.17. Explain how you obtain, *A, B, C, D* parameters of a model of a long transmission line in the laboratory.

4.18. What is meant by 'Natural loading' of lines ? Explain with reasons whether the economic loading for (*i*) overhead, and (*ii*) underground lines are more/less than their natural loadings.

4.19. Derive for a long line the sending end voltage and current relations in terms of receiving end voltage and current and the parameters of the line.

4.20. Derive equivalent parameters of two transmission lines when they are connected in (*a*) tandem and (*b*) parallel.

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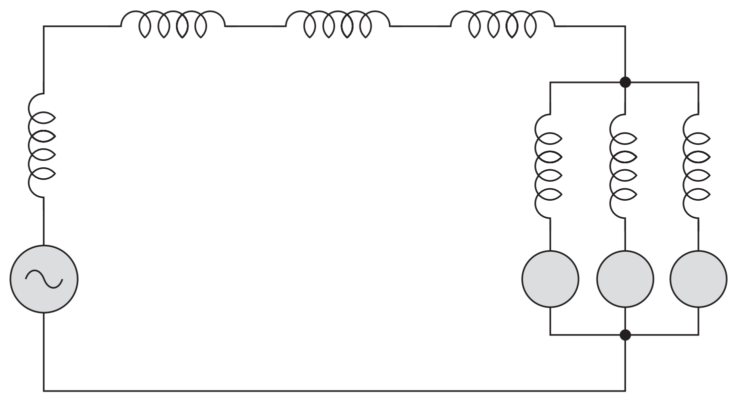

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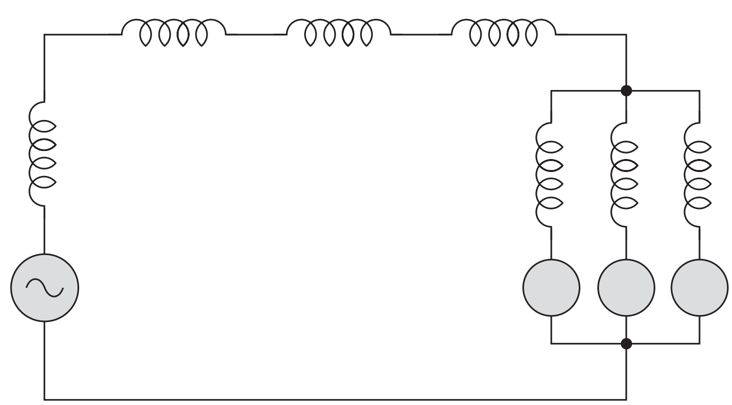
HIGH VOLTAGE d.c. TRANSMISSION



5

High Voltage d.c. Transmission

INTRODUCTION

The use of d.c. for day to day application is much older than that of a.c. The first Central Electric Station was installed by Edison in New York in 1882 which operated at 110 V d.c. It is of interest to know as to why then a.c. almost replaced all d.c. lines and why direct current again is being used for some high voltage transmission lines.

The use of transformer for transmitting power over longer distances and at higher voltages justified the use of a.c. especially where the electric energy was to be harnessed from water power which usually is available far from the load centres. The polyphase induction motors which serve the majority of industrial and residential purposes are simpler and rugged in construction and cheaper as compared to d.c. motors of the same ratings. The commutators of d.c. machines impose limitation on voltage, speed and size due to the commutation problem (sparking). For operating a machine at high voltage, a relatively large diameter commutator is required which restricts the speed of the machine due to the centrifugal force and a low speed machine is heavier and costlier than a high speed machine of equal rating. The use of steam turbines which have a higher efficiency at high speed made the use of a.c. generator superior as compared to d.c. generators. For all such reasons power was generated, transmitted, distributed and consumed as alternating current. If, however, some applications needed the use of d.c., alternating current was converted to direct current locally by motor-generator sets, rotary convertors or by mercury arc rectifiers.

The supporters of d.c., however, did not forget the advantages of d.c. transmission. They suggested that there are strong technical reasons at least for two cases where the use of direct current transmission be resorted to. However, generation use and even most transmission and distribution may be done by a.c.

(*i*) Because of large charging currents, the use of high voltage a.c. for underground transmission over longer distances is prohibited. The transmission of power using d.c. has no such limitation.

(*ii*) Parallel operation of a.c. with d.c. which increases the stability limit of the system or interconnection of two large a.c. systems by a d.c. transmission tie line. Here the d.c. line is an asynchronous link between two rigid (frequency constant) systems where otherwise slight difference in frequency of the two large systems would produce serious problems of power transfer control in the small capacity link.

The Historic Thury System named after a French engineer René Thury who designed the system requires for d.c. transmission a large number of series wound generators driven by prime movers, to be connected in series for high voltage at the sending end of the line, and at the receiving end a comparable number of series wound d.c. motors can be again connected in series to drive low voltage d.c. or a.c. generators. The system operated at constant current. Switching and instrumentation was simple. An ammeter and a voltmeter were the only instruments required. The Thury System worked well for transfer of small powers. For large power, of course, the limitation of d.c. machines came in the way and therefore better convertors than motor generator sets were required.

Extensive research has been carried out especially in Sweden for the development of high voltage convertors. Thyristors of ratings 50 kV and 100 amperes have been developed and now there are many countries in the world where the transmission of power over longer distances and high voltages is being done by d.c.

A d.c. transmission line requires convertor at each end. At the sending end a.c. is converted into d.c. and at the receiving end it is converted back to a.c. for use.

5.1 RECTIFICATION

A valve normally conducts in one direction only from anode to cathode and while it is conducting there is a small drop of volts across it. While analysing the rectifier circuits, the valves, the transformers are assumed to be ideal *i.e.*, without any voltage drop and the d.c. load is assumed to have infinite inductance from which it follows that the direct current is constant *i.e.*, free from ripples.

Transformer secondary can be connected to give 3-phase, 6-phase and 12-phase supply to the rectifier valves. The larger the number of phases, lower is the ripple content in the d.c. output. But 6-phase connection is found to be sufficiently good from all practical viewpoints.

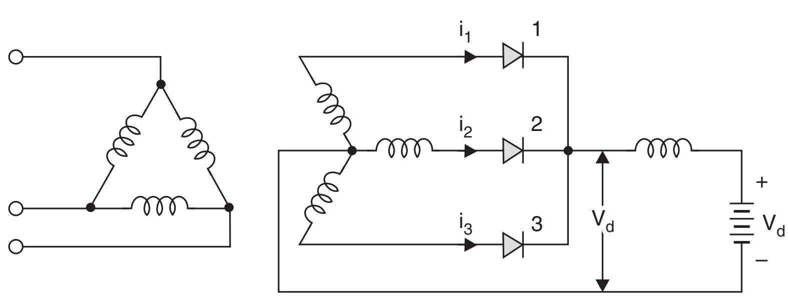


Fig. 5.1 3-phase rectifier.

To begin with, a 3-phase arrangement will be described but analysis will be done for a general n-phase system. The 3-phase system is the simplest convertor circuit but is not practical because the direct current in the secondary windings saturates the transformer core. This could be avoided by using zig-zag connections. The 3-phase system as shown in Fig. 5.1 is, however, useful in explaining other connections.

Figure 5.2 shows the current and voltage wave-form in the three phases of the supply transformer. When grid control is not used, conduction will take place between the cathode and the anode of highest potential and, therefore, the output voltage is indicated by the thick line and the current output will be continuous. From the voltage wave-form it is clear that the change-over from one anode to the other takes place at an electrical angle calculated as follows:

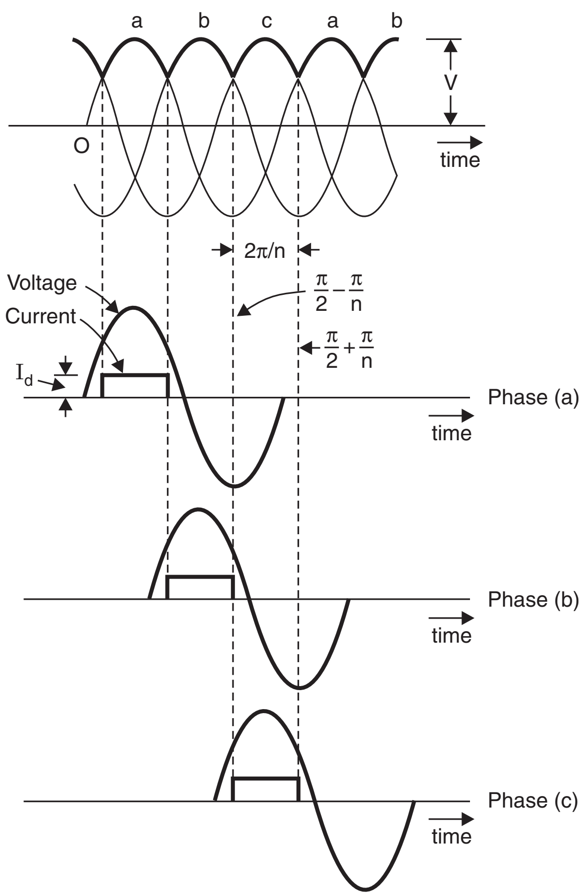


Fig. 5.2 Wave-forms of anode voltage and rectified current in each phase.

Taking point '*O*' as the reference, the conduction starts from 30° and continues up to 150° *i.e.,* ($\pi/2 - \pi/3$) to ($\pi/2 + \pi/3$) *i.e.*, in general for an n-phase or n-anode system the change-over takes place at ($\pi/2 - \pi/n$) and conduction continues up to ($\pi/2 + \pi/n$). Now since conduction takes place only during the positive half cycle, the average value of the d.c. voltage will be

$$V_0 = \frac{1}{2\pi/n} \int_{\pi/2 - \pi/n}^{\pi/2 + \pi/n} V_m \sin\theta \, d\theta = -\frac{nV_m}{2\pi} \Big[\cos\theta \Big]_{\pi/2 - \pi/n}^{\pi/2 + \pi/n}$$

$$= \frac{V_m \sin \pi/n}{\pi/n} \tag{5.1}$$

For 3ϕ, $n = 3$, and

$$V_0 = \frac{V_m \sin \pi/3}{\pi/3} = \frac{3V_m}{\pi} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi} V_m = 0.83V_m \tag{5.2}$$

For 6ϕ, $n = 6$, and

$$V_0 = \frac{V_m \sin \pi/6}{\pi/6} = \frac{3V_m}{\pi} \tag{5.3}$$

The wave of anode current is a rectangular pulse of height I_d and length 120°. Its average value is $I_d/3$ and the r.m.s. value $I_d/\sqrt{3} = 0.577I_d$.

The transformer secondary current is the same as the anode current. The current in actual practice can't reduce to zero instantly nor it can rise to a finite value instantly because of the finite inductance of the system. Hence two anodes conduct simultaneously over a period known as the commutation period or overlap period (overlap angle γ). Say initially anode a is conducting. When anode b commences to conduct, it short circuits the a and b phases which results in zero current in a and I_d in b finally. This is shown in Fig. 5.3.

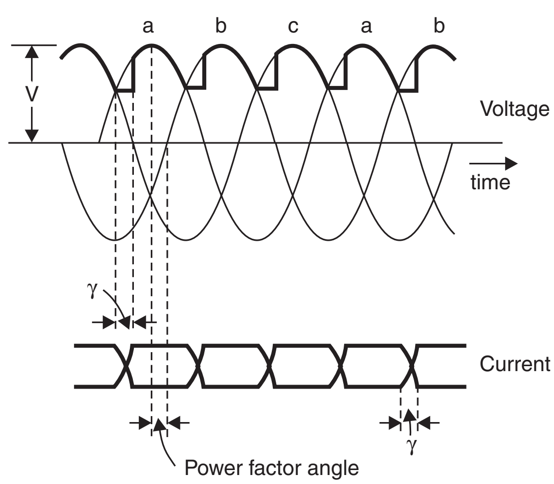


Fig. 5.3 Voltage and current waveforms with commutation angle γ.

The instant of conduction of an anode can be controlled by applying a suitable pulse at a suitable instant to a third electrode known as grid which is placed in between the cathode and anode. Once the conduction starts, the grid of course loses control over the conduction process. Fig. 5.4 shows the use of grid control for the firing of the anodes. Say a positive pulse is applied to the grid such that the conduction is delayed by an angle α.

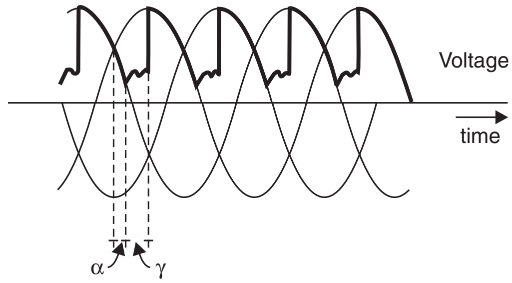


Fig. 5.4 Voltage waveform with grid control angle α.

When the delay is α, considering n-phase system the average output voltage will be

$$V_0' = \frac{1}{2\pi/n}\int_{(\pi/2)-(\pi/n)+\alpha}^{(\pi/2)+(\pi/n)+\alpha} V_m \sin\theta\, d\theta = \frac{nV_m}{2\pi}\Big[-\cos\theta\Big]_{(\pi/2)-(\pi/n)+\alpha}^{(\pi/2)+(\pi/n)+\alpha}$$

$$= \frac{nV_m}{2\pi}\left[\sin\left(\frac{\pi}{n}+\alpha\right)+\sin\left(\frac{\pi}{n}-\alpha\right)\right]$$

$$= \frac{nV_m}{\pi}\sin\frac{\pi}{n}\,.\cos\alpha$$

$$= V_0\cos\alpha \qquad (5.4)$$

This means the d.c. output voltage with grid control is obtained by multiplying the d.c. output voltage without control with cosine of the angle by which the firing is delayed.

For calculating the d.c. output voltage when overlap is to be considered, refer to Fig. 5.5.

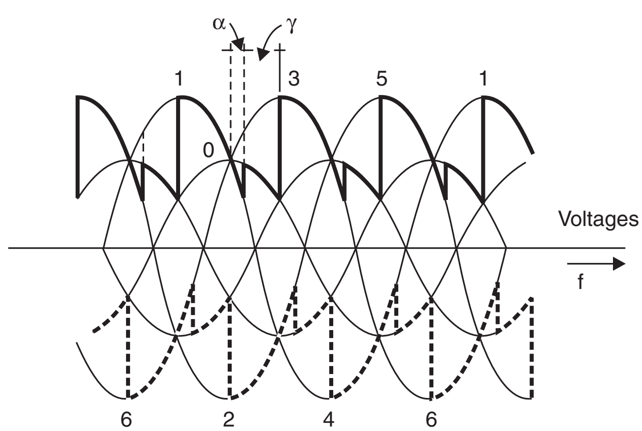


Fig. 5.5 Voltage waveform with grid control α and overlap γ in case of bridge connection.

The d.c. voltage is given by the d.c. voltage without overlap (with grid control) minus the average value of the hatched area during the overlap period. We find out the hatched area which is equal to any ordinate y of the hatched area integrated over the period α to α + γ, *i.e.,*

$$\Delta V_d = \int_{\alpha}^{\alpha+\gamma} y\, d\theta = \int_{\alpha}^{\alpha+\gamma}\left(e_b - \frac{e_a+e_b}{2}\right) d\theta$$

$$= \int_{\alpha}^{\alpha+\gamma} \frac{e_b - e_a}{2}\, d\theta$$

The limits α and α + γ are made clear from the vector diagram (Fig. 5.6).

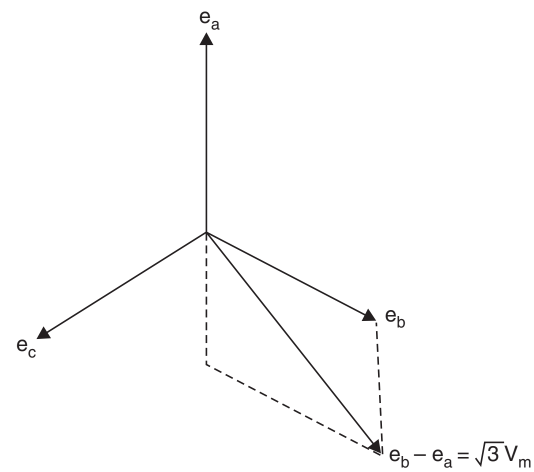


Fig. 5.6 Phasor diagram for evaluating limits of hatched area.

Here $(e_b - e_a)$ leads e_c by 90° *i.e.*, the value of $e_b - e_a$ is zero at point '*O*' in Fig. 5.5 and with respect to point '*O*' the angles to calculate the hatched area are α and α + γ. Therefore,

$$\Delta V_d = \int_{\alpha}^{\alpha+\gamma} \frac{\sqrt{3}}{2} V_m \sin\theta\, d\theta$$

$$= -\frac{\sqrt{3}}{2} V_m \Big[\cos\theta\Big]_{\alpha}^{\alpha+\gamma} = \frac{\sqrt{3}}{2} V_m [\cos\alpha - \cos(\alpha+\gamma)] \tag{5.5}$$

The average value of this drop = $\dfrac{1}{2\pi/n}$ [Area].

For 3-phase, $n = 3$ and the average value of the area will be

$$\frac{3}{2\pi} \cdot \frac{\sqrt{3}}{2} V_m [\cos\alpha - \cos(\alpha+\gamma)] \tag{5.6}$$

∴ The d.c. output with overlap for $n = 3$ will be

$$V_0 \cos\alpha - \frac{V_0}{2}[\cos\alpha - \cos(\alpha+\gamma)]$$

$$V_d = \frac{V_0}{2}[\cos\alpha + \cos(\alpha+\gamma)] \tag{5.7}$$

5.2 THE 3-PHASE BRIDGE RECTIFIER OR GRAETZ CIRCUIT

The bridge rectifier is the most practical circuit used for converting a.c. into d.c. for HVDC transmission. For a given alternating voltage the output direct voltage is doubled as the two anodes conduct simultaneously and hence the power is doubled. There is no current in the windings of the transformer bank and the r.m.s. current is less than twice that of the 3-phase circuit; thereby the winding is used efficiently. For waveform and the bridge circuit refer to Fig. 5.7.

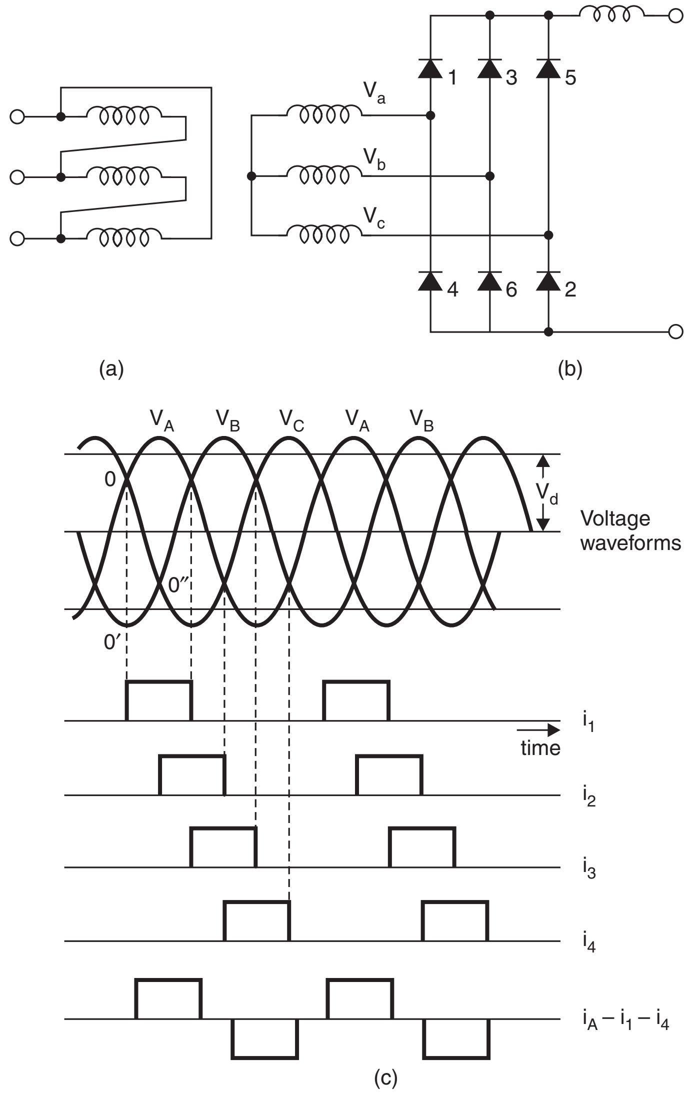


Fig. 5.7 (*a*) Bridge rectifier circuit; (*b*) Voltage waveform; (*c*) Current waveform.

The sequence of operation of the bridge circuit can be explained as follows: Let V_a be the most positive at the beginning of the sequence say point *O* in Fig. 5.7 (*b*). Corresponding to this point V_b is most negative; therefore, the conduction will take place between phase *a* and *b* from *a* to *b*. The rectifiers will be 1 and 6 (Fig. 5.7 (*a*)). V_b continues to be most negative from *O′* to *O″* and after *O″*, V_c becomes most negative and then conduction takes place between phases *a* and *c* from *a* to *c* through the rectifiers 1 and 2. Next diode 3 takes over from 1 and current returns through 2. The complete sequence of the diodes conducting is, therefore, 1 and 6, 1 and 2, 3 and

2, 3 and 4, 5 and 4, 5 and 6 and 1 and 6 again. The grid control and overlapping will modify the magnitude of voltage and can be taken into account as in the case of simple 3-phase circuit.

The output voltage for a bridge circuit can be obtained by either doubling the voltage of the simple 3-phase circuit or by using the line voltage in the formula for six diodes, 6-phase rectification.

We know that the output voltage of a 3-phase circuit is $\frac{3\sqrt{3}}{2\pi} V_m$. Therefore, for a bridge circuit it will be

$$\frac{3\sqrt{3}}{\pi} V_m$$

The output voltage for an n-phase circuit is

$$\frac{V_m \sin \pi/n}{\pi/n}$$

$\therefore$ For 6-phase circuit $n = 6$ and maximum value of voltage is $\sqrt{3}V_m$.

Substituting these values,

$$V_0 = \frac{\sqrt{3}V_m \sin \pi/6}{\pi/6} = \frac{\sqrt{3}V_m}{\pi} \,.\, 6 \,.\, \tfrac{1}{2} = 3\sqrt{3}\,\frac{V_m}{\pi}. \tag{5.8}$$

Example 5.1: A bridge connected rectifier is fed from 220 kV/110 kV transformer with primary connected to 220 kV. Determine the d.c. output voltage when the commutation angle is 15° and the delay angle (*a*) 0°, (*b*) 30° and (*c*) 45°.

Solution: The d.c. output voltage is given by

$$V_d = \frac{V_0}{2} [\cos \alpha + \cos (\alpha + \gamma)]$$

where $$V_0 = \frac{3\sqrt{2}}{\pi} V_L = \frac{3\sqrt{2} \times 110}{\pi} = 148.60 \text{ kV}$$

(*a*) For $\alpha = 0°$

$$V_d = \frac{148.60}{2} [\cos 0° + \cos 15°]$$
$$= 146.06 \text{ kV.} \quad \textbf{Ans.}$$

(*b*) For $\alpha = 30°$

$$V_d = \frac{148.60}{2} [\cos 30° + \cos 45°]$$
$$= 116.87 \text{ kV.} \quad \textbf{Ans.}$$

(*c*) For $\alpha = 45°$

$$V_d = 74.30[\cos 45° + \cos 60°]$$
$$= 89.68 \text{ kV.} \quad \textbf{Ans.}$$

Example 5.2: A bridge connected rectifier operates with $\alpha = 30°$ and $\gamma = 15°$. Determine the necessary line secondary voltage of the rectifier transformer which is normally rated at 220/110 kV, if it is required to obtain a d.c. output voltage of 100 kV. Also determine the tap ratio required.

Solution:

$$V_d = \frac{3\sqrt{2}}{2\pi} V_L[\cos\alpha + \cos(\alpha+\gamma)]$$

$$100 = \frac{3\sqrt{2}}{2\pi} V_L[\cos 30° + \cos 45°]$$

or $\quad V_L = 94.115$ kV **Ans.**

$\therefore$ The tap ratio $= \dfrac{94.115}{110} = 0.85.$ **Ans.**

Current Relationship in a Bridge Circuit

In case of a bridge circuit, two valves conduct simultaneously. These two valves correspond to two different phases *i.e.*, two phases are short circuited. Let L be the inductance in henries for each phase and i_s be the current at any instant; then the equation describing the circuit will be

$$2L\frac{di_s}{dt} = \sqrt{3}V_m \sin\omega t$$

or
$$\frac{di_s}{dt} = \sqrt{3}\frac{V_m}{2L}\sin\omega t\, dt \quad \text{or} \quad i_s = -\sqrt{3}\frac{V_m}{2L}\cdot\frac{1}{\omega}\cos\omega t + A$$

At the beginning when $\omega t = \alpha$, $i_s = 0$ and at the end when $\omega t = \alpha + \gamma$, $i_s = I_d$.

$$\therefore \quad A = \frac{\sqrt{3}V_m}{2\omega L}\cos\alpha$$

and
$$I_d = \frac{\sqrt{3}V_m}{2\omega L}[\cos\alpha - \cos(\alpha+\gamma)]$$

$$= \frac{V_L}{\sqrt{2}X}[\cos\alpha - \cos(\alpha+\gamma)] \quad (5.9)$$

where V_L is the r.m.s. line to line voltage.

Now for the bridge circuit

$$V_0 = \frac{3\sqrt{3}V_m}{\pi}$$

$$\therefore \quad \sqrt{3}V_m = \frac{\pi V_0}{3}$$

$$\therefore \quad I_d = \frac{\pi V_0}{6X}[\cos\alpha - \cos(\alpha+\gamma)] \quad (5.10)$$

We know that, the bridge output voltage after taking into account grid control and overlap γ is

$$V_d = \frac{V_0}{2}[\cos\alpha + \cos(\alpha+\gamma)]$$

Here V_0 is the bridge rectifier voltage without grid control and overlap.

Now adding the two equations (5.7) and (5.10),

$$\frac{2V_d}{V_0} + \frac{6XI_d}{\pi V_0} = 2\cos\alpha$$

or
$$\frac{V_d}{V_0} + \frac{3XI_d}{\pi V_0} = \cos\alpha$$

or
$$V_d = V_0 \cos\alpha - \frac{3XI_d}{\pi} \tag{5.11}$$

Figure 5.8 shows the equivalent circuit represented by equation (5.11). It is to be noted that the drop $3XI_d/\pi$ represents to voltage drop due to commutation and not a physical resistance drop. V_d can be varied by varying the V_0 which in turn can be varied by changing the tap change of the transformer and by changing α.

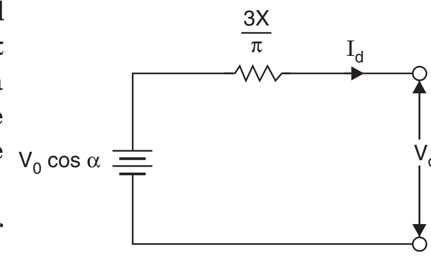


Fig. 5.8 Equivalent circuit represent ing operation of a bridge rectifier.

In fact there are various circuits used for rectification, of which, the best converter circuit for high voltage d.c. transmission is the 3-phase bridge circuit. This has the following advantages:

(*i*) The transformer connections are very simple. It does not require any trapping. The secondary connection may be connected in *Y* or in delta.

(*ii*) For a given power output, the rating of the transformer secondary is less than any other circuits. Therefore, the rating of the primary of the transformer is less than any other circuit.

(*iii*) For a given output voltage, the PIV of the valves is only half that of any of the other circuits and therefore for a given PIV the output voltage is twice that of some other circuits.

(*iv*) Arc backs can be suppressed by grid control and a bypass valve.

Example 5.3: If the rectifier in Example 5.2 delivers 800 A, determine the effective reactance per phase.

Solution: $V_d = V_0 \cos\alpha - \dfrac{3I_dX}{\pi}$

$$100{,}000 = \frac{3\sqrt{2} \times 94.115}{\pi} \times 0.866 \times 1000 - \frac{3 \times 800X}{\pi}$$

or $X = 13.22\ \Omega$. **Ans.**

5.3 INVERSION

In case of valves the conduction takes place in only one direction and, therefore, the current in a converter cannot be reversed. With rectifier operation the output current I_d and output voltage V_d are such that power is absorbed by a load. For inverter operation it is required to transfer power from the direct current to the alternating current system which can only be obtained by the reversal of the average direct voltage. The voltage then opposes the current as in a d.c.

motor and is called a countervoltage. Therefore, for inversion, an alternating voltage system must exist on the primary side of the transformer and grid control of the converter is essential.

When grid control is used we know that output voltage is $V_0 \cos \alpha$ and becomes zero when $\alpha = 90°$ and reverses when $90° < \alpha < 180°$. This means the voltage becomes negative when grid control angle α lies between 90° and 180° and the applied direct voltage from the rectifier forces current through the valves against this negative voltage or back voltage. Triggering beyond 180° results in the a.c. systems being connected to the d.c. source in such polarity that the flow of SCR current will be aided rather than opposed, thus allowing a short circuit current of damaging proportions to build up in the d.c. system. The converter at the receiving end (inverter) thus receives power and inverts. The inverter 3-phase bridge circuit along with its voltage and current waveforms is shown in Fig. 5.9.

Commutation from valve 3 to valve 1 is possible only when phase c is positive with respect to a and the current changeover must be complete before N by a time δ_0 equal to the deionized time of the valves. It can be seen from the current waveforms that the current supplied by the inverter to the a.c. system leads the voltage and hence the inverter may be considered as a generator of leading vars or an absorber of lagging vars.

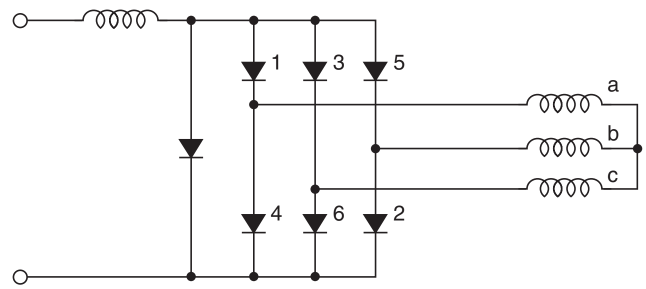


Fig. 5.9 (*a*) Inverter bridge connection.

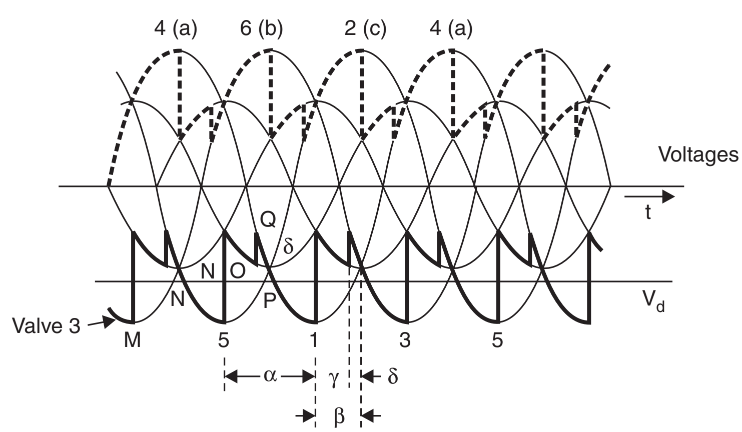


Fig. 5.9 (*b*) Voltage waveforms.

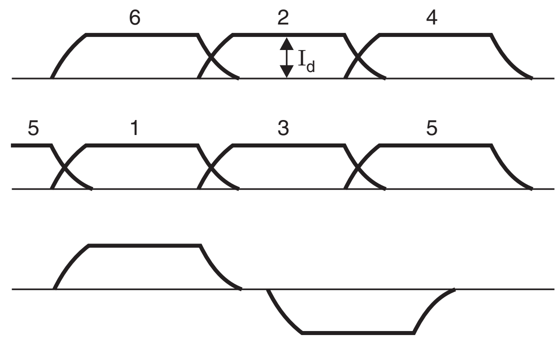


Fig. 5.9 (*c*) Current waveforms.

Refer to Fig. 5.9 (*b*) for the operation of the bridge inverter. Valve 3 is triggered at *M* and as the cathode is held negative to the anode by the applied direct voltage V_d, current flows which is limited only by the circuit impedance. The cathode and anode of valve 3 are at the same potential if the arc drop is neglected. When time *N* is reached, the anode to cathode open circuit voltage is zero and the valve tries to stop conduction. Because of the large inductance of the transformer, conduction in valve 3 continues until time 0 when valve 5 is triggered. Since the anode to cathode voltage of 5 is greater than that of 3, valve 5 will conduct but for a time 5 and 3 will conduct simultaneously and the current is gradually transferred from 3 to 5 until at *Q* valve 3 stops conducting. It is very essential to trigger valve 5 before the time *P*, otherwise valve 3 will continue to conduct as it is being subjected to a positively rising voltage and with that the inversion process will break down.

In rectifier theory the ignition angle is denoted as α and is the angle by which ignition is delayed from the instant at which the commutating voltage is zero and increasing. In case of inverter the ignition angle β is defined as $(\pi - \alpha)$ and β is equal also to $(\gamma + \delta)$ where δ and γ are shown in Fig. 5.9 (*b*).

Rewriting the converter equations (5.10) and (5.7),

$$I_d = \frac{\pi V_0}{6X} [\cos \alpha - \cos (\alpha + \gamma)]$$

and
$$V_d = \frac{V_0}{2} [\cos \alpha + \cos (\alpha + \gamma)]$$

Substituting $\alpha = 180 - \beta$ and $\gamma = \beta - \delta$, we obtain

$$I_d = \frac{\pi V_0}{6X} [-\cos \beta - \cos (\pi - \beta + \beta - \delta)]$$

$$= \frac{\pi V_0}{6X} [-\cos \beta + \cos \delta]$$

$$= -\frac{\pi V_0}{6X} [\cos \beta - \cos \delta]$$

and
$$V_d = \frac{V_0}{2} [-\cos \beta - \cos \delta] = -\frac{V_0}{2} [\cos \beta + \cos \delta]$$

$\therefore$ We obtain

$$V_d = -[V_0 \cos \delta - I_d R_c] \tag{5.12}$$

and $$V_d = -[V_0 \cos\beta + I_d R_c] \tag{5.13}$$

where $R_c = \dfrac{3X}{\pi}$.

Therefore two equivalent circuits are obtained for the bridge circuit as shown in Fig. 5.10.

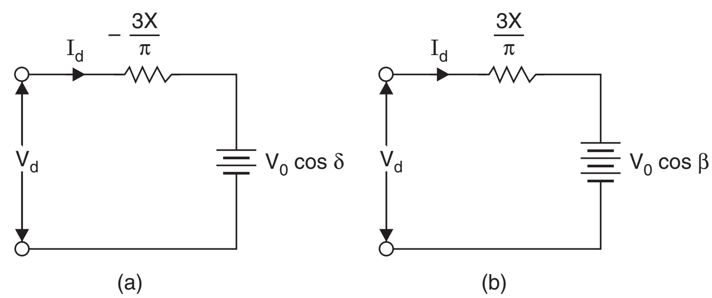


Fig. 5.10 (*a*) Equivalent circuit of inverter for constant δ in terms of angle δ.
(*b*) Equivalent circuit of inverter for constant β in terms of angle β.

Example 5.4: A d.c. link has a loop resistance of 10 Ω and is connected to transformers giving secondary voltage of 120 kV at each end. The bridge connected converters operate as follows:

Rectifier: $\alpha = 15°$, $X = 15\ \Omega$

Inverter: $\delta_0 = 10°$, $\gamma = 15°$, $X = 15\ \Omega$

Allow 5° margin on δ_0 for δ.

Calculate the direct current delivered if the inverter operates on constant β control.

Solution:

$$I_d = \frac{V_{or} - V_{oi}}{R}$$

where V_{or} and V_{oi} are the rectifier and inverter d.c. output voltages and R the loop resistance.

$$V_{or} = V_0 \cos\alpha - \frac{3I_d X}{\pi}$$

$$= \frac{3\sqrt{2} \times 120}{\pi} \cos 15° - \frac{45 I_d}{\pi}$$

$$V_{oi} = V_0 \cos\beta + \frac{3I_d X}{\pi} \quad [\text{here } \beta = (\delta + \gamma)]$$

$$= \frac{3\sqrt{2} \times 120}{\pi} \cos 25° + \frac{45 I_d}{\pi}$$

$\therefore$ $$I_d R = 10 I_d = V_{or} - V_{oi}$$

$$= \frac{3\sqrt{2} \times 120}{\pi} [\cos 15° - \cos 25°] \times 1000 - \frac{90 I_d}{\pi}$$

or $$I_d \left(10 + \frac{90}{\pi}\right) = 9664$$

or $$I_d = 249.96 \text{ amp.} \quad \textbf{Ans.}$$

5.4 KINDS OF d.c. LINKS

D.C. lines are classified as follows: (1) Monopolar lines, (2) Bipolar lines and (3) Homopolar lines. As the name suggests monopolar lines are those in which the line has one conductor only and the earth is used as the return conductor (refer Fig. 5.11).

The line is normally operating with negative polarity as the corona loss and the radio interference are reduced. The bipolar lines have two conductors—one operating with +ve polarity and the other negative polarity. There are two converters of equal voltage rating and connected in series at each end of the d.c. line. Refer to Fig. 5.11 (*b*). The rating of the bipolar line is expressed as ± 650 kV for example and is pronounced as plus and minus 650 kV. The junction of the converters may be grounded at one end or at both the ends. If it is grounded at both the ends each line can be operated independently.

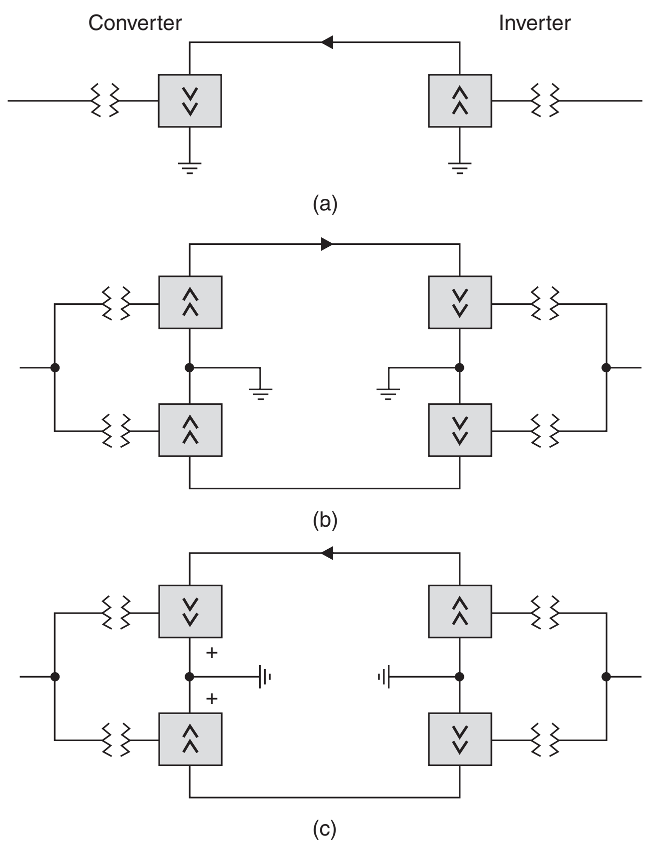


Fig. 5.11 Kinds of d.c. links: (*a*) Monopolar, (*b*) Bipolar, and (*c*) Homopolar lines.

The homopolar lines have two or more conductors having the same polarity usually negative for the reason of corona and radio interference and always operate with ground as the return.

5.5 PARALLEL AND SERIES CONNECTION OF THYRISTORS

In case of HVDC transmission, the voltage and current levels are so high that a single thyristor cannot meet these requirements. Under such circumstances, it is essential to use more than one thyristors in parallel to obtain increased current requirements and, in series, to achieve higher voltage.

Parallel Connections

When thyristors are connected in parallel, the current sharing between them may not be equal. The thyristor with lower dynamic resistance will take more current resulting in further reduction in resistance and further increasing the flow of current through it. The process is cumulative till the thyristor gives way.

For parallel operation of thyristors, it is desirable that the finger voltage of various devices should be same, the latching current level of all the devices is such that when gate pulse is applied, all of them will turn on and remain on when the gate pulse is removed. Also, the holding current of various devices should not differ much.

In order to nullify the difference in characteristics of the thyristors due to slightly different turn-on time, finger voltage, holding current, latching current etc. which results in unequal current sharing, it is desirable to insert suitable value of inductance in each thyristor circuit.

Series Connection of Thyristors

In order to obtain higher d.c. voltage for HVDC transmission, two or more than two thyristors are to be connected in series. In case, the thyristors have different leakage resistances, the thyristor with higher leakage resistance will have larger voltage drop across it. High resistances of equal values are connected across the thyristors.

5.6 POWER FLOW IN *HVDC* TRANSMISSION SYSTEM

The equivalent circuit of a d.c. transmission system under steady state operating condition is shown in Fig. 5.12.

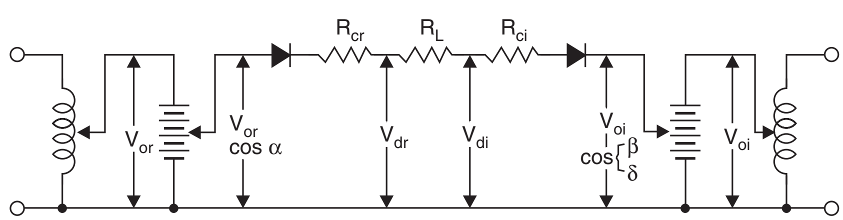


Fig. 5.12 Equivalent circuit of a d.c. transmission link.

The current I_d in the line is given by

$$I_d = \frac{V_{or} \cos \alpha - V_{oi} \cos (\beta \text{ or } \delta)}{R_{cr} + R_L \pm R_{ci}} \tag{5.14}$$

where R_L is the line resistance, R_{cr} and R_{ci} are the fictitious rectifier and inverter resistances. If the inverter operates with constant ignition angle in the expression for I_d, cos β and + R_{ci} are used otherwise for constant extinction angle δ, cos δ and – R_{ci} are used. Here in our study we consider constant ignition angle β operation of inverter as ignition angle β can be controlled directly whereas δ is controlled indirectly through controlling β to values computed from the direct current I_d, the commutating voltage and the desired extinction angle. From the equation (5.14), it is clear that the current I_d is proportional to the difference of the two internal voltages (rectifier and inverter voltages) and is controlled by regulating these voltages as the resistances in the denominator of the expression for I_d are practically fixed for a given system.

Internal voltages can be controlled by any one or both of the following methods:

(*i*) Grid Control. (*ii*) Tap Change Control.

Small changes in voltages are adjusted using grid control as it is quite fast (about 5 ms) and large changes are brought about by tap changes which are inherently slow (about 5 sec. per step). Both these methods are used cooperatively at each terminal for voltage control and hence control of I_d and power flow.

From equation (5.14) it is clear that I_d and hence the difference of internal voltages are always positive as the thyristors can conduct only in one direction. Therefore, if it is desired to reverse the direction of power transmission, the polarity of the direct voltages at both ends of the line must be reversed while maintaining the sign of their algebraic difference. Inverter then acts as a rectifier and the rectifier as an inverter. It is to be noted that the terminal voltage of the rectifier is always greater in absolute value than that of the inverter, although it is lesser algebraically in the event of negative voltage.

5.6.1 Comparison between Constant Current/Constant Voltage System

Power flow in an HVDC system can be regulated by the following methods:

(*i*) Constant Current, variable voltage.

(*ii*) Constant Voltage, variable current systems.

In case of constant current system, all the loads and power sources are connected in series. A load is taken out of the circuit by short circuiting it by a switch and a source is taken out of the circuit by first reducing its e.m.f. to zero and then short circuiting it. In constant voltage system, various loads and sources are connected in parallel. A load or a source is taken out of the circuit by opening the switch in the corresponding branch. Whereas the constant current system was used in the past for street lighting and on some of the earlier d.c. transmission projects, the constant voltage system is almost universally used these days in a.c. transmission and distribution networks.

Most of the HVDC projects to date are two terminal networks, therefore, the distinction between series and parallel connection of the converter and inverter disappears. The comparison between the constant current and constant voltage system is, therefore, made on the following grounds:

(*i*) The limitation of variation of current due to faults on the d.c. line or converter or due to variation in a.c. voltages.

(*ii*) The energy losses and efficiency. On a constant current system, the short circuit currents are limited to theoretically full load current but practically at the most two times the full load currents. However, in case of constant voltage a.c. systems the fault currents are as high as 20 times the full load current as the current is limited by the effective impedance of the system. On constant voltage d.c. system fault currents would be much greater, as these are limited only by circuit resistance.

As regards losses I^2R losses are relatively larger in a constant current system (always full load losses) as compared to the constant voltage system where the losses are proportional to square of the power transmitted. As the system operates for a short time at its rated power, the daily or annual energy loss is much less in constant voltage system. The opposite is true of those losses which are a function of operating voltage such as corona and dielectric losses. These are more for a constant voltage system as compared to constant current system. In practice, however, the voltage dependent losses are always much less than the current dependent losses.

Thus, consideration of fault levels favour the constant current system whereas the energy loss favours the constant voltage systems. In the past it was possible to operate the system either as constant current or constant voltage system. However, with advancement in technology it is now possible with the help of automatic controls to operate the system combining the best features of the two systems.

In case of HVDC transmission it is desirable to have a high power factor of the system for the following reasons:

(*i*) For a given current and voltage of the thyristor and transformers, the power rating of the converters is high.

(*ii*) The stresses on the thyristors and damping circuits are reduced.

(*iii*) For the same power to be transmitted the current rating of the system is reduced and also the copper losses in the a.c. lines are reduced.

(*iv*) In a.c. lines the voltage drop is reduced.

The p.f. on the a.c. side can be improved by using shunt capacitors. However, this involves cost both for the capacitors and the switching devices.

On the d.c. side, the p.f. of the converter is given as

$$\cos \phi = \frac{V_d}{V_{do}} = \frac{1}{2} [\cos \alpha + \cos (\alpha + \gamma)]$$

for a rectifier and for an inverter it is given as

$$\cos \phi = \frac{1}{2} [\cos \delta + \cos (\delta + \gamma)]$$

Fig. 5.13 shows the phase voltages and one phase current of a rectifier at two values of control angle $\alpha = 0$ and $\alpha = 30°$, assuming negligible commutating reactance. It is clear that with $\alpha = 0°$ the power factor angle ϕ representing the phase shift between the fundamental of the current waveform and the corresponding phase voltage is zero. The power factor angle increases as α increases. From the expression and the Fig. 5.13, it is clear that the power factor angle increases with increase in firing angle α. In practice and under normal condition α is kept near 15° for the following reasons:

(*i*) To ensure that all the thyristors of a bridge will be fired at the same instant in time.

(*ii*) To allow a small margin for an immediate small power increase, if it is dictated by the rectifier grid control regulator.

It is concluded that smaller the firing angle, the smaller will be the *V Ar* requirements of the rectifier as then sin ϕ is smaller.

Similar to rectifier operation, the p.f. angle ϕ increases with increase in angle δ in case of inverter. Therefore, for $\delta = 0$, the *V Ar* demand of the inverter will be minimum and for $\beta > 0$ the current leads the voltage and the inverter consumes lagging *Vars*.

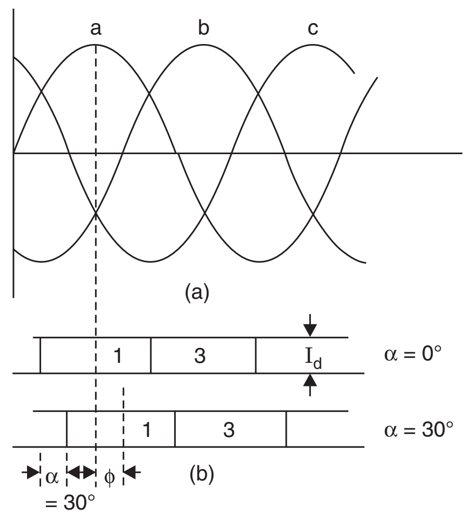


Fig. 5.13 Rectifier operation (*a*) Voltage waveform (*b*) Current waveform.

5.7 CONSTANT IGNITION ANGLE β CONTROL

Refer to Fig. 5.12. The voltage Vd_i from the rectifier circuit is given as

$$V_{di} = V_{or} \cos \alpha - I_d(R_{cr} + R_L) \tag{5.15}$$

and from the inverter circuit, it is given as

$$V_{di} = V_{oi} \cos \beta + I_d R_{ci} \tag{5.16}$$

These equations have been plotted on the operation diagram of Fig. 5.14. In order that current I_d flows, the open circuit voltage of the rectifier must be higher than the open circuit back voltage of the inverter. The point of intersection (I_d, V_{di}) of these characteristics gives the operating point. The inverter operation under such condition (constant β) is not satisfactory for the following reasons:

(*a*) The extinction angle may be larger than required, involving unnecessary *V Ar* demand.

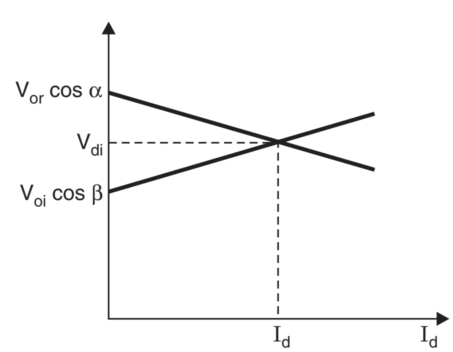


Fig. 5.14 Operation diagram.

(*b*) In case there is voltage dip on the a.c. side, the life of thyristors is endangered due to commutation failure.

This is shown in Fig. 5.15. Suppose that normal current I_d is flowing and that β is so adjusted that $\delta > \delta_0$ as shown in Fig. 5.15(*a*). If, now, the voltage on a.c. side experiences a symmetrical dip and assuming that the current does not change substantially, the same current has to be commutated at reduced voltage. This requires that the shaded areas of the two figures should be equal. As a result δ of Fig. 5.15 (*b*) is reduced below δ_0 with subsequent commutation failure. However, if β was kept large in anticipation of voltage dip, the inverter *VAr* requirements would be excessive.

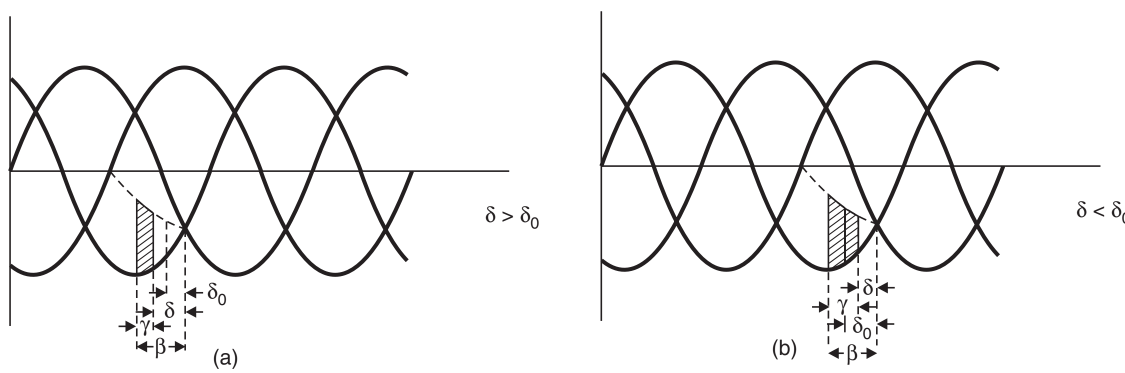


Fig. 5.15 Symmetrical reduction of a.c. system voltage.
(*a*) $\delta > \delta_0$; (*b*) $\delta < \delta_0$

5.8 CONSTANT EXTINCTION ANGLE δ CONTROL

This control requires a fast compounding device. The current I_d expression, for an inverter, describes the nature of the control problem.

$$I_d = \frac{\pi V_o}{6x} (\cos \delta - \cos \beta)$$

In order to have certain value of I_d, for a particular value of V_o and δ, value of β can be obtained from the expression above. If β is continuously computed and the thyristors are fired accordingly, there is a substantial risk of commutation failure for the following reasons:

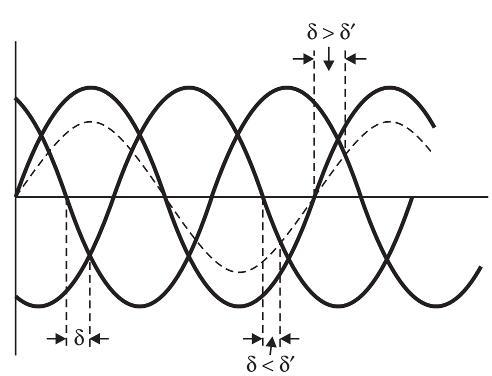


Fig. 5.16 Asymmetrical reduction of a.c. system voltage.

(*i*) The above expression of I_d holds good only for 3-phase symmetrical voltages. A shunt fault say a line to line fault will not only reduce the commutating voltage but also increase or decrease the available commutation angle as shown in Fig. 5.16.

(*ii*) In case of a dip in voltage on the a.c. side, the rate of change of current I_d may be very large despite the fact that large smoothing reactors have been incorporated in the system.

The constant extinction angle controller consists of a separate computer for each group of thyristors connected to the same phase. This is desired as the voltage of different phases become unbalanced during an asymmetrical fault.

Each computer continuously computes and provides an output signal when the correct instant for firing a thyristor has arrived for safe commutation. The computer has to monitor the following quantities:

(*i*) The amplitude of the commutating voltage which is the voltage between two phases, the phase on which the conducting thyristor is connected and the phase on which the next thyristor to fire is connected. This voltage is responsible for circulating commutating current when the two thyristors are conducting simultaneously.

(*ii*) The phase of the commutating voltage.

(*iii*) The magnitude of the direct current.

(*iv*) The rate of change of the direct current.

5.9 CONSTANT CURRENT CONTROL

Constant Current Controller performs the following operations:

(*i*) Measures the system current I_d.

(*ii*) Compares it with a reference current I_{ds}.

(*iii*) Computes $(I_{ds} - I_d)$ and amplifies the error signal $(I_{ds} - I_d)$.

(*iv*) The output error signal controls α in case of rectifier and δ in case of inverter in proper direction to reduce the error.

If the measured current is more than the reference, α must be increased in case of rectifier to decrease the open circuit voltage of the rectifier. The difference between the open circuit voltage of the rectifier and the inverter is thereby decreased and the current I_d is decreased proportionally.

However, in case of an inverter, if the measured current is more than the reference current, the open circuit voltage of the inverter must be increased instead of being decreased as in a rectifier in order to decrease the difference of the open circuit voltages. This refers, however, to the absolute value of the inverter voltage. If we consider the inverter voltage to be negative, which is usual if the same converter sometimes rectifies and at other times inverts, the algebraic value of inverter voltage must be decreased as in a rectifier and to accomplish this, α must be increased, as in a rectifier. The graph $\cos \alpha$ *vs* α in the range $0 \le \alpha \ge \pi$ is monotonic where the algebraic value of $\cos \alpha$ increases with decrease in α *i.e.,* the algebraic value of open circuit voltage $V_o \cos \alpha$ increases with decrease in α. This means that the same constant current controller can be used for a given converter without change of connections during both rectification and inversion.

5.10 ACTUAL CONTROL CHARACTERISTICS

Consider the system of Fig. 5.12. Let the rectifier be equipped with a constant current regulator and the inverter with a constant extinction angle (CEA) regulator. The constant current regulator characteristic is

I_d = Constant, a vertical line and the constant extinction angle characteristic

$$V_{dr} = V_{oi} \cos \beta + (R_L - R_{ci})I_d \tag{5.17}$$

where V_{dr} and I_d are the sending end voltage and current respectively. These characteristics have been drawn in Fig. 5.17. The constant current characteristic should ideally be a vertical line but in practice it has a high negative slope. Assuming that the commutation resistance R_{ci} is somewhat greater than the line resistance R_L, the constant extinction angle characteristic due to the inverter is a straight line with a small negative slope and, intercept as $V_{oi} \cos \beta$ as shown in Fig. 5.17 by the line *CD*. The operating point of the rectifier inverter system is the point of intersection (*G*) of the operating characteristics of the rectifier and the inverter. The constant current characteristic can be shifted horizontally by adjusting the current setting of the current regulator. If the current measured is less than the current setting, the regulator advances the firing angle of the rectifier thyristor (α is decreased), thereby the internal voltage of the rectifier is increased and the current I_d is increased. On the other hand, if the current measured is more than the current setting, the firing angle is further delayed (α is increased) thereby the internal voltage of the rectifier is decreased and the current I_d is decreased. Similarly, the inverter characteristic can be raised or lowered by means of the tap-changer on the transformer at the inverter station which varies the alternating voltage on the thyristor side. As the tap is changed, the constant extinction angle control restores the desired value of β. The internal direct voltage at the inverter is changed in proportion to the alternating voltage since $\cos \beta$ is constant and this changes the direct current I_d which, however, is quickly restored to the set value by the current regulator at the rectifier station by changing the internal voltage at the station. The d.c. reactor on the line tend to prevent rapid changes in current, thus easing the duty of the current regulators. From Fig. 5.17, it is clear that if the rectifier characteristics were perfectly vertical and the inverter characteristic perfectly horizontal, it could be said that the rectifier controls the direct voltage of the line. However, in practice, each control affects both current and voltage although it affects one of them more and the other less.

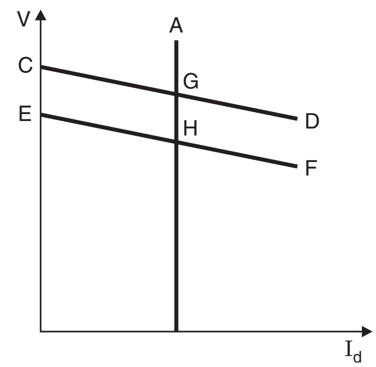


Fig. 5.17 Control characteristics.

Suppose the inverter voltage is increased, in order to keep the line current constant, the rectifier voltage must be increased by the same amount. If the increase in inverter voltage is small, the rectifier voltage can be increased by advancing the firing *i.e.,* decreasing α (electronic control) and thus the increase is brought about very quickly. However, if the inverter voltage is increased by the tap changer, the rectifier voltage should also be changed by its own tap changer. It has already been reported that the ignition angle α usually lies between 10°–20°.

With this the p.f. of the converter is high and if small change in voltage is required, it is obtained very fast through firing angle control over this small interval.

So far we have considered the behaviour of the controls for slow changes of voltage. We now consider the rapid reduction in voltage due to shunt faults on a.c. system. Consider Fig. 5.17. Let the inverter voltage reduce. As a result the inverter characteristic is shifted downward from *CD* to *EF* and the new operating point is *H*. The system, thus operates at the same current but at reduced voltage. As a result the power transmitted is reduced in proportion to voltage. If the dip in voltage is momentary due to a transient shunt fault, the initial conditions would be restored soon, otherwise, the inverter tap changer is operated to increase the voltage to normal value.

In order to ensure that the d.c. line current does not go below a certain predetermined value, it is generally desirable to provide constant current control for the inverter as well. For a system in which power is reversible, converters have to be provided with both C.E.A. and constant current controls. The combination of C.E.A. and Constant Current Controls can be represented by the block diagram in Fig. 5.18 (*a*) and the operating characteristic of the converter provided with such a control is shown in Fig. 5.18 (*b*). Let the current setting of this Constant Current Control Output be I_{ds}. If the line current measured is more than I_{ds}, the Constant Current Control output will be zero and the inverter advance angle β will correspond to the output from the C.E.A. Control only *i.e.,* the inverter will operate at the minimum required angle of advance β.

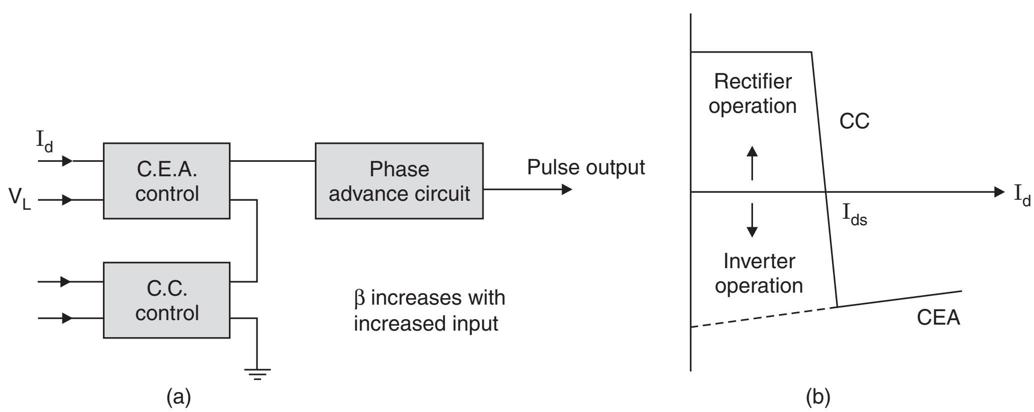


Fig. 5.18 (*a*) Block diagram of CC and CEA Controls.
(*b*) Characteristics of system as in (*a*).

The output voltage is then given by

$$V_d = \frac{3\sqrt{2}}{\pi} V_L \cos \delta - \frac{3X}{\pi} I_d \tag{5.18}$$

If the measured current is less than the setting of the converter, the constant current control will provide some output in order to increase the angle β more than the required minimum which increases the voltage V_d and brings the current back to its set value I_{ds}.

Consider Fig. 5.19. Suppose the rectifier and the inverter are associated with both the Constant Current Control and Constant ignition/extinction angle controls. Characteristics *ABC* and $A'B'C$ represent the normal and reduced voltage operation of the rectifier respectively. DEF is the characteristic of the inverter.

Now consider a dip in a.c. voltage at the rectifier station. As a result the d.c. voltage suffers a proportionate dip and the rectifier characteristic shifts from ABC to $A'B'C$ (Fig. 5.19). If the inverter were not associated with the Constant Current Control, it can be seen from Fig. 5.19 that the new rectifier characteristic ($A'B'C$) would not intersect the constant voltage characteristic (GD) of the inverter. Consequently, the current and power drop to zero after a short delay due to d.c. reactors. Because of the Constant Current regulator associated with inverter, the new rectifier characteristic intersects the inverter characteristic at I. It may now be said that the rectifier controls the direct voltage whereas the inverter the direct current—a situation contrary to normal operating condition.

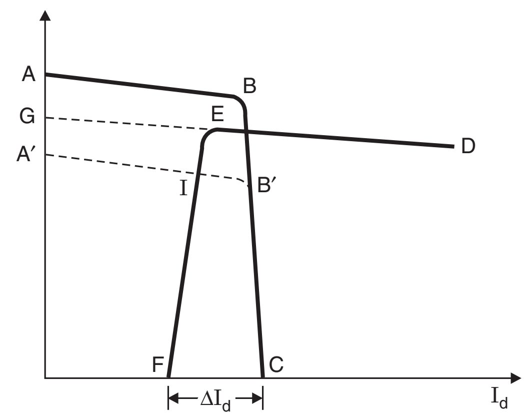


Fig. 5.19 Constant current and CIA/CEA characteristics.

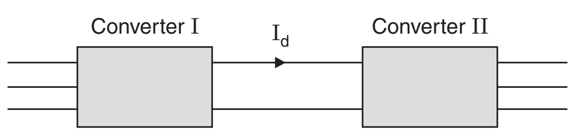
The constant current characteristic of the inverter is set at a lower current than that of the rectifier. The difference between the current setting of the rectifier and that of the inverter is called the current margin and is denoted by ΔI_d. It is generally 15% of the rated current. In fact, the current margin should be sufficient to avoid the intersection of the two constant current characteristics, otherwise this will lead to erratic operation of the two regulators.

When there is dip in rectifier voltage, the power transmitted is reduced not only in proportion to the dip in voltage but also because of reduction in current by an amount ΔI_d. Thus a voltage dip at the rectifier end reduces the power more than an equal dip at the inverter end. This, however, is much better than having power suddenly reduced to zero. If the dip in power is not desired, the current settings of the current regulators are increased. In order to avoid erratic operation of the regulators, the increase in current setting is first effected at the rectifier and then at the inverter station. The whole process, however, requires only a few tenths of a second. Later if the measured current exceeds the new current setting by a similar amount the setting is reduced by that amount first at the inverter station and a little later at the rectifier, again to avoid intersection of the constant current characteristics. In either case, the current margin is first increased and later decreased to its normal value. In this way, there is no danger of accidentally changing the sign of the current margin and thus suddenly reversing the power.

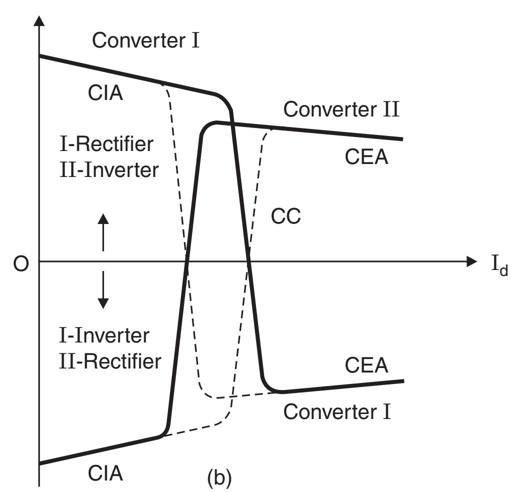
Under abnormal condition *i.e.*, low rectifier voltage condition, the rectifier current control finds the current to be less than its setting and hence tries to bring it to its setting by increasing the rectifier voltage, by decreasing ignition angle delay. It is unable to do so, however, either because the delay is already zero or because the minimum control overrides the current control. The other possibility to increase the current is to reduce inverter voltage. Therefore, the inverter operates at a higher extinction angle than the minimum specified value. This happens because the current regulator fires the inverter thyristor before the C.E.A. Control has a chance to do so.

Under normal operating condition or low inverter voltage, the inverter current regulator sees that the measured current is more than its own setting, it tries to lower the current by raising the inverter voltage. For this, it must decrease the extinction angle δ by decreasing the ignition advance angle β. It cannot do because the C.E.A. regulator is already igniting the thyristors before the current regulator would do so.

In many d.c. transmission lines each converter sometimes works as a rectifier and at other times as an inverter. At times both the converters may be expected to operate as inverters *e.g.*, de-energization of a d.c. line. Therefore, each converter is given a combined characteristic as shown in Fig. 5.20 consisting of three linear portions C.I.A, C.C. and C.E.A.



(a)



(b)

Fig. 5.20 Reversal of power (*a*) Block diagram (*b*) Control characteristics.

The power is transmitted from Converter I to Converter II as per the thick line characteristics of Fig. 5.20. *i.e.*, Converter I acts as a rectifier whereas Converter II as the inverter. If the direction of power is to be reversed *i.e.*, Converter II to work as rectifier and Converter I to work as an inverter, Characteristics are changed to those shown by dotted line.

Here the polarity of direct voltage is changed keeping the direction of current unchanged. Both the converters are given the same current setting but at the converter designated as inverter, a signal equal to current margin is subtracted from that current setting, making it a smaller current setting. When it is desired to reverse the direction of power, the margin signal should be transferred to the converter that becomes the inverter.

In order to reverse the voltage polarity and hence the flow of power, the shunt capacitance of the line must be first discharged and then recharged with the opposite polarity. This process implies a greater current at the end of the line, initially the inverter than at the end, initially the rectifier. The difference of terminal currents, however, cannot exceed the current margin. Hence the shortest time of voltage reversal is

$$T = C \frac{\Delta V_d}{\Delta I_d} \tag{5.19}$$

where C is the line capacitance, ΔV_d is the algebraic change of direct voltage and ΔI_d is the current margin.

5.11 FREQUENCY CONTROL

In case of a.c. systems frequency can be controlled by adjusting steam input to the prime movers. If the frequency is high, the steam input to the prime mover is decreased temporarily and the K.E. is drawn on to supply the required electrical output. On the other hand if the frequency is low, steam input to the prime movers in increased and the excess of mechanical power input over electrical power output goes into increase of kinetic energy which results into increased speed and frequency.

The frequency of the a.c. system can also be controlled if it is connected to a d.c. system which has its power rating comparable or slightly greater than that of a.c. system. The frequency can be controlled both at the receiving end of the system and/or the sending end of the system. A frequency discriminator circuit is used at the terminal where the frequency is to be controlled. Its output signal is proportional to the frequency deviation from its normal value and is used as an error signal to advance or retard the firing of converters. If the frequency is low and if it is an inverter circuit, the ignition of the thyristors should be advanced so that the received power is increased. On the other hand, if it is a rectifier circuit, the ignition should be delayed so that the power at the sending end is decreased. Hence a reversing switch is required on the output of the discriminator. The same error signal in transmitted from the discriminator to the distant terminal. There is no change in the error signal at the rectifying converter but at the inverter a marginal signal is subtracted from it, so that even though the frequency is correct, it appears low regardless of whether the frequency controlled network is importing or exporting. In either case, the inverter advances its ignition angle in a vain bid to increase the frequency and as a result reaches the minimum allowable extinction angle and thus determines the direct voltage. In case of low voltage at the rectifier, limiting the line voltage the inverter does take over the frequency control, even though there is then a small frequency error due to the marginal signal. The frequency control is analogous to the current control already discussed. The converter with lower voltage, controls the direct voltage of the line and the one with higher voltage controls the frequency.

5.12 REACTIVE *VAr* REQUIREMENTS OF *HVDC* CONVERTERS

A d.c. line itself does not require reactive power and voltage drop on the line is only the IR drop where I is the d.c. current. The converters at both ends of the line, however, draw reactive power from the a.c. system. It varies with the transmitted power and is approximately half of the power at each end. It is independent of the length of the line. Filters are provided on the a.c. side of the converters for HVDC transmission to reduce harmonic currents and voltages on the a.c. side which would otherwise produce interference with other equipment or with communication lines. These filters are of large size and involve considerable cost. But fortunately these can also provide a major part of the leading reactive power required by the converter. The magnitude of reactive power supplied by the filter is greatly influenced by the position of the a.c. filters. Filters on d.c. side are used to smoothen the d.c. output and thus these are effective in reducing interference with communication lines. The cost of these filters is much less compared to those of a.c. filters. Filters usually have one or more L.C. turned circuits with a fairly high Q say 25 or more and these are designed from the considerations of steady sine waves obtained by Fourier analysis.

Relations between a.c. and d.c. Quantities

These relations hold good between the a.c. quantities at the point where the voltage waves are sinusoidal and the d.c. quantities.

Assuming the losses in the Converter Circuit to be negligible, the a.c. power must equal the d.c. power *i.e.,*

$$3VI_L \cos \phi = V_d I_d = I_d V_o \cos \alpha \tag{5.20}$$

Where V is the r.m.s. line to neutral a.c. voltage and I_L the r.m.s. value of the fundamental frequency component of alternating line current. The line current has the wave shape as shown in Fig. 5.21.

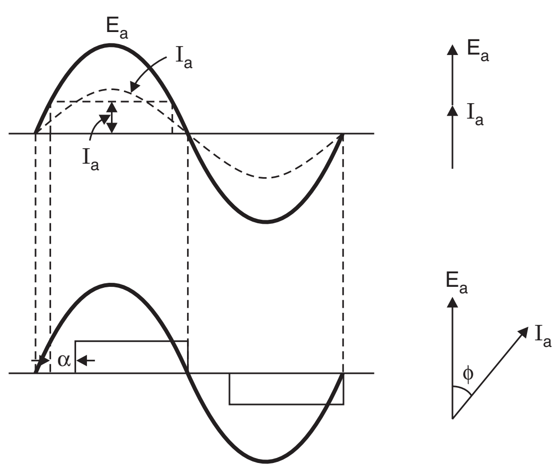


Fig. 5.21 Relation between ignition delay and phase displacement.

The current has positive and negative rectangular pulses of height I_d and width $\frac{2\pi}{3}$ radians. This shape is independent of α as long as there is no over lap. By Fourier series analysis the crest value of the fundamental component of this wave is

$$\sqrt{2} I_L = \frac{2}{\pi} \int_{-\pi/3}^{\pi/3} I_d \cos\theta \, d\theta = \frac{2}{\pi} I_d \Big[\sin\theta \Big]_{-\pi/3}^{\pi/3}$$

$$= \frac{2\sqrt{3}}{\pi} I_d$$

or
$$I_L = \frac{\sqrt{6}}{\pi} I_d \qquad (5.21)$$

Substituting the values of I_L and V_o in equation (5.20), we have

$$\frac{3V_L}{\sqrt{3}} \frac{\sqrt{6}}{\pi} I_d \cos\phi = \frac{3\sqrt{2}}{\pi} V_L I_d \cos\alpha$$

or
$$\frac{3\sqrt{2}}{\pi} V_L I_d \cos\phi = \frac{3\sqrt{2}}{\pi} V_L I_d \cos\alpha$$

or
$$\cos\phi = \cos\alpha \qquad (5.22)$$

where $\cos\phi$ is the displacement factor or vector power factor and ϕ is the angle by which the fundamental line current lags the line-to-neutral source voltage.

From equations (5.20) and (5.21), it is clear that the converter operates as a transformer which has a variable voltage ratio depending upon the ignition delay angle and has a fixed current ratio.

Figure 5.21 shows that when $\alpha = 0$, the fundamental component of sine wave current is in phase with the line to neutral source voltage. As ignition angle α is increased the displacement angle between the fundamental component of current and the line to neutral voltage increases and the current lags behind the voltage. Thus, the converter which may be a rectifier or an inverter draws reactive power from the a.c. system. The rectifier takes lagging Vars from the a.c. system and inverter also takes lagging Vars from the system but it is sometimes said to deliver leading Vars to the a.c. system. In this regard a rectifier can be considered as an induction motor and an inverter an induction generator. Both the induction generator and inverter work best when connected in parallel with synchronous machines and shunt capacitors. Normally a converter will absorb lagging Vars from the a.c. system but imagine a converter in which the thyristors have been replaced by synchronously controlled switches which will close on negative instead of positive α, the converter could be made to deliver lagging Vars instead of consuming reactive power.

When overlap is considered ($\gamma < 60°$), it is found from equation (5.7) that

$$V_d = V_o \left[\frac{\cos\alpha + \cos(\alpha + \gamma)}{2} \right]$$

or
$$V_d = \frac{3\sqrt{6}}{\pi} \frac{(\cos\alpha + \cos(\alpha + \gamma))}{2} V$$

and substituting this in equation (5.20), we have

$$\frac{3V_L}{\sqrt{3}} \frac{\sqrt{6}}{\pi} I_d \cos \phi = \frac{3\sqrt{6}}{\pi} \frac{V_L}{\sqrt{3}} \frac{[\cos \alpha + \cos (\alpha + \gamma)]}{2} I_d$$

or
$$\cos \phi = \left[\frac{\cos \alpha + \cos (\alpha + \gamma)}{2}\right] \tag{5.23}$$

and
$$I_L \cos \phi = \frac{\sqrt{6}}{\pi} I_d \left[\frac{\cos \alpha + \cos (\alpha + \gamma)}{2}\right] \tag{5.24}$$

A harmonic analysis of the current wave shows that

$$I_L \sin \phi = \frac{\sqrt{6}}{\pi} I_d \frac{2\gamma + \sin 2\alpha - \sin 2(\alpha + \gamma)}{4[\cos \alpha - \cos (\alpha + \gamma)]} \tag{5.25}$$

Here γ is in radians. The phasor fundamental current is given as

$$I_L = I_L \cos \phi - jI_L \sin \phi \tag{5.26}$$

The reactive power on the a.c. side is given as

$$Q = 3VI_L \sin \phi = P \tan \phi$$

where
$$\tan \phi = \frac{2\gamma + \sin 2\alpha - \sin 2(\alpha + \gamma)}{[\cos 2\alpha - \cos 2(\alpha + \gamma)]} \tag{5.27}$$

However, it is to be noted that there is no reactive power on the d.c. side of the converter. Fig. 5.22 shows a typical arrangement including a shunt harmonic filter and reactive power compensator on the transformer primary.

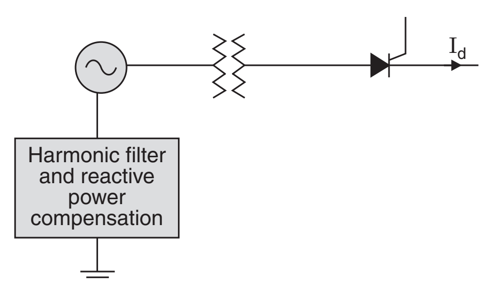


Fig. 5.22 Typical a.c./d.c. system.

A shunt filter is designed such that it presents a low impedance to all the a.c. harmonic currents and capacitive impedance to fundamental frequency. As a result, the voltage across the filter is constrained to be substantially a fundamental frequency sine wave. Consequently, for analytical purposes, the theory developed above for calculation of reactive power can be used. The voltage V across the filter is to be taken as the commutation e.m.f. and transformer leakage inductance alone as commutation inductance provided e.m.f. V is taken as reference phasor.

If more than one similar bridge connected thyristor groups per converter is used with separate group transformers having parallel connected primaries, then active and reactive powers simply add when referred to the common primary bus-bar. This holds good independent of the phase shift produced by the group transformer connections provided firing angles and d.c. currents are similar.

Reactive power can be supplied either using a synchronous or a static capacitor or a combination of the two. Filtering is always required. A larger filter than minimum is less costly than a minimum filter plus shunt capacitors and gives better filtering. In the event of a temporary fault in the a.c. or d.c. sides, a local synchronous capacitor tends to maintain a.c. voltage due to its inertia so that an inverter can continue to contribute power to the a.c. system in order to preserve stability. A static capacitor, however, does not have this effect. The impedance of the synchronous capacitor increases with frequency and is, therefore, useless for harmonic filtering. Also, synchronous capacitor, besides being noisy, it is costlier and has greater losses than a static capacitor. The reactive power of a synchronous capacitor can be controlled whereas a static capacitor can supply only a fixed amount of Vars. However, whereas a static capacitor can be made for any voltage a synchronous capacitor is usually of low voltage say 33 kV which, therefore, requires special provision usually in the form of tertiary winding on the converter transformer. Therefore, in regard to choice of type of reactive *VAr* compensator, following recommendations are made:

1. A static capacitor is always required for filtering and for this purpose its rating is about 20 to 30% of the power rating of the converter. Switching by sections is to be avoided.
2. A synchronous capacitor is, for stability reasons, a must in a system where the a.c. terminal voltage reduces below the minimum by 30% for over 0.5 sec. due to the sudden blocking of the station.
3. In case the installed generating capacity in the nearby plant is not sufficient synchronous capacitors are connected to low voltage tertiary windings of the converter transformer.

5.13 PARALLEL OPERATION OF d.c. LINK WITH AN a.c. NETWORK

In case of a d.c. link the power to be transmitted depends upon the four control parameters V_r, V_i, α and β, all of which can be controlled more or less independently over a desired range. Thus when a d.c. system is operated in parallel with an a.c. system (Fig. 5.23) following objectives can be achieved:

(*a*) Constant current flow.

(*b*) Constant power flow.

(*c*) Constant angle between the a.c. bus bar voltages.

(*b*) Constant voltage at either end.

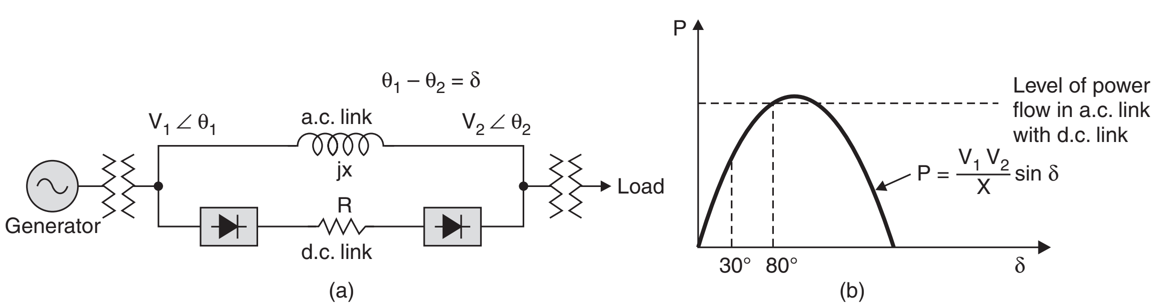


Fig. 5.23 (*a*) a.c. and d.c. connected in parallel (*b*) Power angle diagram for a.c. link.

Controls as suggested at (*a*) and (*b*) have already been discussed. Control at (*c*) helps in improving transient stability of the system. Better utilisation of the a.c. transmission lines can result. It is known that in case of a.c. lines power transmitted is given by the expression

$$P = \frac{V_1 V_2}{X} \sin \delta \tag{5.28}$$

where V_1, V_2 are the voltages at the two ends of the line, X the inductive reactance and δ is the phase angle between V_1 and V_2. Usually a.c. lines are operated at an angle δ of about 30° in order to allow a margin for additional power flow which is sufficient to meet transient fluctuation in the load or to meet sudden changes in system conditions such as shunt faults. However, if the a.c. system is connected to a d.c. link, the a.c. line can be operated at a much greater angle say 78–80° [Fig. 5.23 (*b*)] which represents an increase of 95% in the transmission capacity of the line. In order to achieve this increase in power transmission it would be necessary for d.c. link to be controlled either by a signal proportional to δ or by measurement of the a.c. power flow. In both cases a signal proportional to rate of change of the controlling parameter will be required to achieve good stabilised flow on the d.c. link. Under normal operating condition, the power flow through d.c. link would be small and hence the grid angle α, of the rectifier large so that when required during abnormal condition such as a shunt fault or a sudden increase in load, when power transmission through a.c. line decreases, the power flow can be opened up very quickly by decreasing the grid control angle to a suitable value. Facilities would also be necessary for reversal of power flow in the link due to sudden drop in the sending end voltage, particularly if these are due to a.c. system faults.

In order to meet objective (*d*) listed above it is desirable to use synchronous capacitors or/and Static Capacitor rather than use a d.c. link for the purpose. Voltage control may, however, be a fringe benefit from a d.c. link installed for other purposes, especially if the compensating equipment installed with the link can be controlled to maintain system voltage.

5.14 GROUND RETURN

HVDC transmission lines use ground or sea water as the return conductor either continuously (monopolar) or for short times of emergency (bipolar). These return paths are called ground return even if sea water is used as a return path. For the same length of transmission the resistance offered by the ground in case of d.c. is much less as compared to a.c. transmission because the d.c. spreads over a very large cross sectional area in both depth and width as compared to a.c. or transient currents. In fact the earth resistance in case of d.c. is independent of the length (for long lines) and equals the sum of the electrode resistances. Since the resistance in case of d.c. is low as compared to a.c. there is low power loss in comparison with a metallic line conductor of economical size and equal length if the ground electrodes are properly designed.

A line with ground return (monopolar) is more economical than a bipolar line because the ground return saves most of the cost of one metallic conductor and the losses in it. Besides this, there are two more definite advantages of using ground as the return.

The first advantage is that a d.c. line can be built in two stages if the initial load requirement demands. Initially it will operate as a monopolar line with ground as return and later on in the second stage it can be built as a bipolar line. Thus a considerable part of the total investment can be deferred until the second stage.

The second advantage is the reliability of the system *i.e.*, in the event of an outage of one conductor of the bipolar line, it can be operated temporarily at almost half of its rated power by the use of the healthy line and the ground. For this reason the reliability of a bipolar line is equal to that of a double circuit 3-phase line although it has only two conductors instead of six for 3-phase line.

The ground return lines have the following disadvantages:

(*i*) The ground currents cause electrolytic corrosion of buried or immersed metallic structures.

(*ii*) It is difficult to design ground electrodes for low resistance and low cost of installation and maintenance.

(*iii*) Ground currents cause dangerous step and touch voltages.

(*iv*) The ground currents interfere with the operation of other services such as a.c. power transmission, ships' compasses and railway signals.

5.15 CIRCUIT BREAKING

It is easy to interrupt a.c. currents because of their natural zeros. Since d.c. is a steady unidirectional current it does not have a natural zero and therefore it is difficult to interrupt large d.c. currents at high voltages.

The d.c. transmission projects till this date are two terminal projects and it is not difficult to interrupt the fault currents. The faults on the d.c. line or in the converters are cleared by using the control grids of the converter valves to stop the direct current temporarily.

The a.c. transmission lines also were radial initially. But later on with the increase in demand the requirement of low cost energy and of higher reliability, these transmission lines turned into complex networks. The lack of d.c. breakers has inhibited the networking of d.c. lines. The transient faults can be cleared using grid control, but permanent faults can be cleared using a combination of grid control, fault locators and isolating switches. Reasonable proposals have been made for clearing faults on such lines by running the whole system to zero using grid control, opening switches to isolate the faulty section and then raising the voltage back to normal. The time taken for this sequence of operation is approximately equal to the rapid reclosure of a.c. circuit breakers.

The requirement for d.c. circuit breaking is not to break the actual short circuit currents but to interrupt load currents in circuits at high potential with respect to ground because the short circuit currents can be limited to normal load currents using the grid control. If such switches could be developed, lines could be switched into or out of an unfaulted network without running the voltage down. Some such switches have been suggested wherein an artificial zero of current is created through the contacts of the switch by the oscillatory discharge of a capacitor. The crest value of the oscillatory currents should be greater than the direct current to be interrupted.

A schematic diagram of such a switch is shown in Fig. 5.24.

A is a normally open contact whereas M and B are normally closed contacts. As a result the capacitor C is charged to line voltage through the high resistance R. When it is desired to interrupt the current I_d, the operating mechanism opens contact B and closes A, thus initiating the oscillations in the circuit consisting of M, A, C and L and immediately afterwards the contact M opens which interrupt the current at a current zero such as P as shown in Fig. 5.24 (*b*). After this, contact A is opened and B closed.

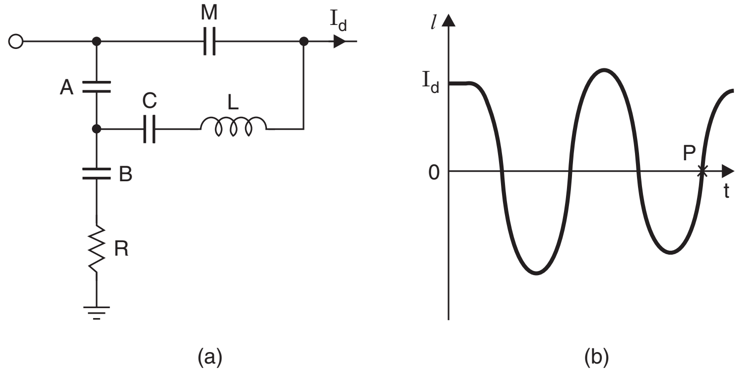


Fig. 5.24 (*a*) Schematic diagram of a d.c. switch (oscillatory discharge).
(*b*) Current waveform through *M*.

Another switch proposed is as shown in Fig. 5.25.

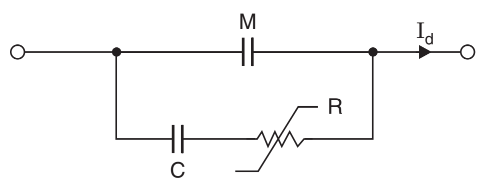


Fig. 5.25 Schematic diagram of a d.c. switch (Nonlinear resistor).

Here M is the main contact which is normally closed and C is a capacitor which is normally uncharged. When it is desired to interrupt current I_d, the contacts M open, thereby a part of the current is diverted to the capacitor and as a result the current to be interrupted by the contact M is small. The rate of rise of recovery voltage across M is $\frac{dV_c}{dt} = \frac{I_d}{C}$. The non-linear resistor R absorbs energy without greatly adding to the voltage across M.

5.16 ADVANTAGES OF d.c. TRANSMISSION

There is a fundamental difference between the transmission of power in a d.c. and in an a.c. system. In an a.c. system power is given by $P = \frac{E_1 E_2}{X} \sin \delta$, where E_1 and E_2 are line voltages at the two ends, δ the electrical angle between E_1 and E_2 and X is the line reactance whereas in d.c. the power is given by

$$P = \frac{Ed_1 - Ed_2}{R} Ed_2$$

where Ed_1 and Ed_2 are the d.c. voltages at the two ends and R is the line resistance.

From this it is clear that the d.c. power is proportional to the difference of the line voltages and thus will vary much more with the voltages than in the case of the a.c. transmission, where the power is proportional to the product of the line end voltages.

Line Circuit: The line construction is simpler as compared to a.c. transmission. A single conductor line with ground as return can be compared with a 3-phase single circuit line. Hence the line is relatively cheaper and has the same reliability as that of a 3-phase single circuit line because 3-phase lines cannot operate, except for a short time when there is a single line to ground fault or a *L-L* fault as this creates unbalancing in the voltages and hence interfere with the communication lines and other sensitive apparatus on the system. It is claimed that a bipolar d.c. line has the same reliability index as a two-circuit 3-phase line having six line conductors.

Power per Conductor: For transmitting power both on a.c. and d.c. circuits let us assume that the two lines have the same number of conductors and insulators. Assuming that the current is limited by temperature rise, the direct current equals the r.m.s. alternating current. Since the crest voltage in both cases is same for the insulators the direct voltage is $\sqrt{2}$ times the r.m.s. alternating voltage.

The power per conductor in case of d.c. is

$$P_d = V_d I_d$$

and the power per conductor in a.c. is

$$P_a = V_a I_a \cos \phi$$

where I_a and I_d are the currents per conductor and V_a and V_d the line to ground voltages and $\cos \phi$ the power factor.

Now since $V_d = \sqrt{2} V_a$ and $I_a = I_d$

$$\frac{P_d}{P_a} = \frac{V_d I_d}{\frac{V_d}{\sqrt{2}} \cdot I_d \cos \phi} = \frac{\sqrt{2}}{\cos \phi}$$

since $\cos \phi \le 1.0$, the power per conductor in case of d.c. is more as compared to a.c.

Power per Circuit: Let us compare the power transmission capabilities of a 3-phase single circuit line and a bipolar line. The power capabilities of the respective circuits are

$$P_d = 2p_d \text{ and } P_a = 3p_a$$

where p_d and p_a are the power transmitted per conductor of d.c. and a.c. lines. The ratio

$$\frac{P_d}{P_a} = \frac{2p_d}{3p_a} = \frac{2V_d I_d}{3V_a I_a \cos \phi} = \frac{2V_d I_d}{\frac{3}{\sqrt{2}} V_d I_d \cos \phi}$$

$$= \frac{2\sqrt{2}}{3 \cos \phi} = \frac{2.828}{3 \cos \phi}$$

Normally $\cos \phi < 1$ and is of the order of 0.9. Therefore, the power transmission capability of the bipolar line is same as that of the 3-phase single circuit line. The d.c. line is cheaper and simpler as it requires two conductors instead of three and hence 2/3 as many insulators, and the towers are cheaper and narrower and hence a narrow right of way could be used.

No Charging Current: In case of a.c. the charging current flows in the cable conductor, a severe decrease in the value of load current transmittable occurs if thermal rating is not to be exceeded; in the higher voltage range lengths of the order of 32 km create a need for drastic derating. A further current loading reduction is caused by the appreciable magnitude of dielectric losses at high voltages. Since in case of d.c. the charging current is totally absent the length of transmission is not limited and the cable need not be derated.

No Skin Effect: The a.c. resistance of a conductor is somewhat higher than its d.c. resistance because in case of a.c. the current is not uniformly distributed over the section of the conductor. The current density is higher on the outer section of the conductor as compared to the inner section. This is known as skin effect. As a result of this the conductor section is not utilized fully. This effect is absent in case of d.c.

No Compensation Required: Long distance a.c. power transmission is feasible only with the use of series and shunt compensation, applied at intervals along the line. For such lines shunt compensation (shunt reactors) is required to absorb the line charging kVAs during light load conditions and series compensation (use of series capacitors) for stability reasons. Since d.c. line operate at unity power factor and charging currents are absent no compensation is required.

Less Corona Loss and Radio Interference: The corona loss is directly proportional to ($f + 25$), where f is the frequency of supply. f being zero in case of d.c., the corona losses are less as compared to a.c. Corona loss and radio interference are directly related and hence radio interference in case of d.c. is less as compared to a.c. Also corona and radio interference slightly decrease by foul weather conditions (snow, rain or fog) in case of d.c. whereas they increase appreciably in case of a.c. supply.

Higher Operating Voltages Possible: The modern high voltage transmission lines are designed based on the expected switching surges rather than the lightning voltages because the former are more severe as compared to the latter. The level of switching surges due to d.c. is lower as compared to a.c. and hence, the same size of conductors and string insulators can be used for higher voltages in case of d.c. as compared to a.c. In cables, where the limiting factor is usually the normal working voltage the insulation will withstand a direct voltage higher than that of alternating voltage, which is already 1.4 times the r.m.s. value of the alternating voltage.

No Stability Problem: For a two machine system the power transmitted from one machine to another through a lossless system is given by

$$P = \frac{E_1 E_2}{X} \sin \delta$$

where X is the inductive reactance between the machines. The longer the length of the line, the higher is the value of X and hence lower will be the capability of the system to transmit power from one end to the other. With this the steady state stability limit of the system is reduced. The transient state stability limit is normally lower than the steady state; therefore with longer lines used for transmission, the transient stability also becomes very low. A d.c. transmission line does not have any stability problem in itself because d.c. operation is an asynchronous operation of the machines. In fact two separate a.c. systems interconnected only by a d.c. link do not operate in synchronism even if their nominal frequencies are equal and they can operate at different nominal frequencies *e.g.*, one operating at 60 Hz and the other at 50 Hz.

Low Short Circuit Currents: The interconnection of a.c. system through an a.c. system increases the fault level to the extent that sometimes the existing switchgear has to be replaced. However, the interconnection of a.c. system with d.c. links does not increase the level so much and is limited automatically by the grid control to twice its rated current. As a result of this fault d.c. links do not draw large currents from the a.c. system.

5.17 DISADVANTAGES

However, the d.c. transmission has certain disadvantages as well which are listed below:

Expensive Converters: The converters required at both ends of the line have proved to be reliable but they are much more expensive than the conventional a.c. equipments. The converters have very little overload capacity and they absorb reactive power which must be supplied locally. The converters produce lot of harmonics both on d.c and a.c. sides which may cause interference with the audio-frequency communication lines. Filters are required on the a.c. side of each converter for diminishing the magnitude of harmonics in the a.c. networks. These also increase the cost of the converters.

Voltage Transformation: The power transmitted can be used at lower voltage only. Voltage transformation is not easier in case of d.c. and hence it has to be done on the a.c. side of the system.

Circuit breaking for multi-terminal lines is difficult.

5.18 CABLES

It is well known that, the a.c. transmission through cables is limited in distance due to the charging current. The charging kVA of 3-phase single circuit cables per km are

1250 kVA at 132 kV

3125 kVA at 220 kV

9375 kVA at 400 kV

It is clear that enormous amount of charging kVA are required; therefore, if a.c. transmission by cables is required, the charging current has to be absorbed at intermediate stations if distances exceed the following:

64 km at 132 kV

40 km at 220 kV

24 km at 400 kV

Since in case of d.c. the charging current is absent, there is no distance limitation on transmission by underground or undersea cables.

5.19 ECONOMIC DISTANCES FOR d.c. TRANSMISSION

The cost per unit length of a d.c. line is much less as compared to a.c. line for the same power capacity and comparable reliability whereas the cost of terminal equipment is much more in case of d.c. (converters and inverters) than in case of a.c. (Transformers). If we plot the variation of cost of power as a function of distance of transmission, the variation is given by the curves in Fig. 5.26. The vertical intercept of each curve is the cost of the terminal equipment and the slope is the cost per unit length of the line and that of other accessories whose cost varies with distance. The curve for a.c. transmission intersects that of d.c. at an abscissa called a breakeven point which means if the distance of transmission is more than the breakeven point distance, it is preferable to use d.c., otherwise a.c. should be used.

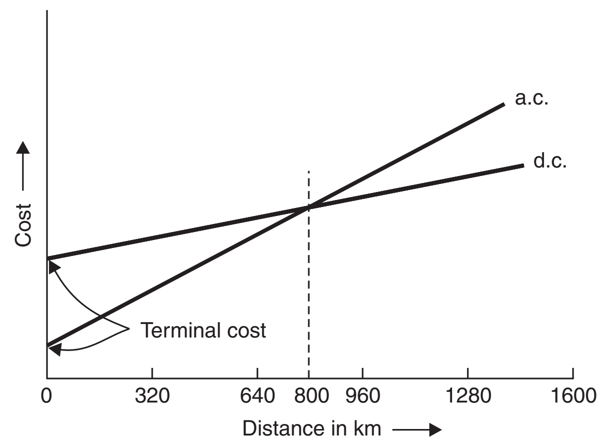


Fig. 5.26 Comparative costs of a.c. and d.c. overhead lines vs distance.

There is hardly any scope for reducing the cost of transformers used for a.c. transmission whereas lot of progress has been made in the development of converting devices and the breakeven distances are reducing with further development of these devices.

The d.c. transmission links so far used mostly are based on the factors like long river crossings, frequency conversions and asynchronous ties between large a.c. systems. To give an approximate idea of economic distance of transmission using d.c. is that for 400 km of distance the power to be transmitted should be at least 100 MW.

PROBLEMS

5.1. A bridge connected rectifier is fed from 238 kV/110 kV transformer from 238 kV supply. Calculate the direct voltage output when the commutation angle is 20° and delay angle

(*a*) 0° (*b*) 30° (*c*) 60°.

5.2. It is required to obtain a direct voltage of 100 kV from a bridge-connected rectifier operating with $\alpha = 30°$ and $\gamma = 15°$. Calculate the necessary line secondary voltage of the rectifier transformer which is normally rated at 345 kV/150 kV. Calculate the tap ratio required.

5.3. Show that in a 3-phase bridge rectifier operating with no delay and with 60° overlap, the direct current is half of the crest value of the line short circuit current on the valve side of the transformer bank.

5.4. If the rectifier in Problem 5.2 delivers a current of 500 amps, determine the effective reactance per phase.

5.5. A d.c. link has a loop resistance of 5 ohm and is connected to transformers giving secondary voltage of 110 kV at each end. The bridge connected converters operate as follows:

Rectifier: $\alpha = 15°$ Inverter: $\delta_0 = 10°$

$X = 10\ \Omega$ $\gamma = 15°$

Allow 5° margin on δ_0 for δ

$X = 10\ \Omega$

Determine the direct current delivered if the inverter operates on constant β.

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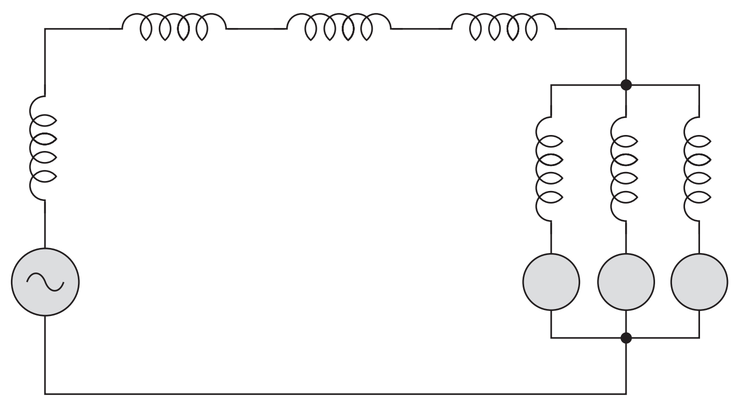
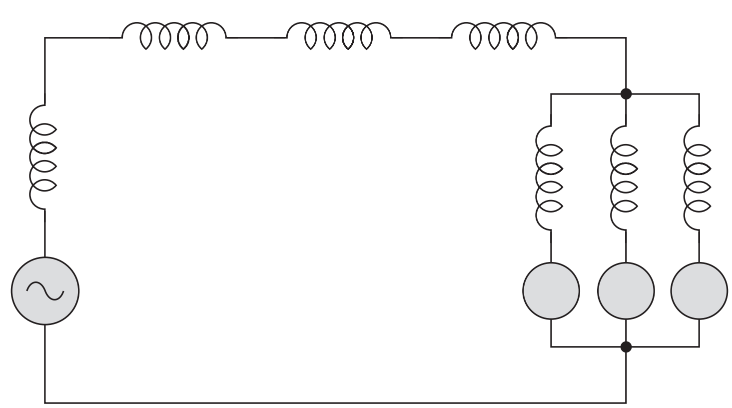


6

CORONA



6

Corona

INTRODUCTION

Corona phenomenon is the ionization of air surrounding the power conductor. Free electrons are normally present in free space because of radioactivity and cosmic rays. As the potential between the conductors is increased, the gradient around the surface of the conductor increases. Assume that the spacing between the conductors is large as compared with the diameter of the conductors. The free electrons will move with certain velocity depending upon the field strength. These electrons will collide with the molecules of air and in case the speed is large, they will dislodge electrons from these molecules, thereby the number of electrons will increase. The process of ionization is thus cumulative and ultimately forms an electron avalanche. This results in ionization of the air surrounding the conductor. In case the ratio of spacing between conductors to the radius of the conductor is less than 15, flash over will occur between the conductors before corona phenomenon occurs. Usually for overhead lines this ratio is far more than this number and hence flash-over can be regarded as impossible.

Corona phenomenon is, therefore, defined as a self-sustained electric discharge in which the field intensified ionization is localized only over a portion of the distance between the electrodes.

When a voltage higher than the critical voltage is applied between two parallel polished wires, the glow is quite even. After operation for a short time, reddish beads or tufts form along the wire, while around the surface of the wire there is a bluish white glow. If the conductors are examined through a stroboscope, so that one wire is always seen when at a given half of the wave, it is noticed that the reddish tufts or beads are formed when the conductor is negative and a smoother bluish white glow when the conductor is positive. The a.c. corona, viewed through a stroboscope, has the same appearance as direct current corona. As corona phenomenon is initiated, a hissing noise is heard and ozone gas is formed which can be detected by its characteristic odour.

6.1 CRITICAL DISRUPTIVE VOLTAGE

Consider a single-phase transmission line (Fig. 6.1). Let r be the radius of each conductor and d the distance of separation such that $d >> r$. Since it is a single-phase transmission line, let q be the charge per unit length on one of the conductors and hence $-q$ on the other. If the operating voltage is V, the potential of conductor A with respect to neutral plane N will be $V/2$ and that of B will be $-V/2$. Consider a point P at a distance x where we want to find the electric field intensity. Bring a unit positive charge at P.

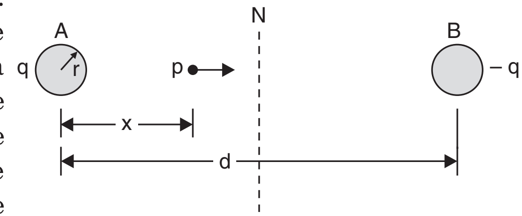


Fig. 6.1 1-ϕ transmission line.

The field due to A will be repulsive and that due to B will be attractive; thereby the electric field intensity at P due to both the line charges will be additive and it will be

$$E_x = \frac{q}{2\pi\varepsilon_0 x} + \frac{q}{2\pi\varepsilon_0 (d-x)} = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right]$$

The potential difference between the conductors

$$V = -\int_{d-r}^{r} E_x dx = \int_{r}^{d-r} \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right] dx$$

$$= \frac{q}{2\pi\varepsilon_0}\Big[\ln x - \ln (d-x)\Big]_r^{d-r}$$

$$= \frac{q}{2\pi\varepsilon_0} . 2 \ln \frac{d-r}{r} = \frac{q}{\pi\varepsilon_0} \ln \frac{d-r}{r} \tag{6.1}$$

Since r is very small as compared to d, $d - r \simeq d$.

$$\therefore \quad V = \frac{q}{\pi\varepsilon_0} \ln \frac{d}{r} \tag{6.2}$$

Now gradient at any point x from the centre of the conductor A is given by

$$E_x = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right]$$

$$= \frac{q}{2\pi\varepsilon_0} \cdot \frac{d}{x(d-x)}$$

Substituting for q from the above equation,

$$q = \frac{\pi\varepsilon_0 V}{\ln \dfrac{d}{r}}$$

$$E_x = \frac{\pi\varepsilon_0 V}{\ln \dfrac{d}{r}} \cdot \frac{1}{2\pi\varepsilon_0} \cdot \frac{d}{x(d-x)}$$

$$= \frac{V}{2 \ln \frac{d}{r}} \cdot \frac{d}{x(d-x)}$$

$$= \frac{V' d}{x(d-x) \ln \frac{d}{r}} \tag{6.3}$$

Here V' is the line to neutral voltage of the system. In case of 3-phase system

$$V' = \frac{V_L}{\sqrt{3}}$$

where V_L is the line to line voltage.

From the expression for the gradient it is clear that for a given transmission system the gradient increases as x decreases *i.e.*, the gradient is maximum when $x = r$, the surface of the conductor, and this value is given by

$$g_{\max} = E_r = E_{\max} = \frac{V' d}{r(d-r) \ln \frac{d}{r}}$$

$$\simeq \frac{V'}{r \ln \frac{d}{r}}$$

or

$$V' = r g_{\max} \ln \frac{d}{r} \tag{6.4}$$

Critical disruptive voltage is defined as the voltage at which complete disruption of dielectric occurs. This voltage corresponds to the gradient at the surface equal to the breakdown strength of air. This dielectric strength is normally denoted by g_0 and is equal to 30 kV/cm peak at NTP *i.e.*, 25°C and 76 cm of Hg.

At any other temperature and pressure

$$g_0' = g_0 \cdot \delta \tag{6.5}$$

where δ is the air density correction factor and is given by

$$\delta = \frac{3.92b}{273 + t} \tag{6.6}$$

where b is the barometric pressure in cm of Hg and t the temperature in °C.

Therefore, the critical disruptive voltage is given by

$$V' = r g_0 \, \delta \ln \frac{d}{r} \text{ kV} \tag{6.7}$$

In deriving the above expression, an assumption is made that the conductor is solid and the surface is smooth. For higher voltages ACSR conductors are used. The cross-section of such a conductor is a series of arcs of circles each of much smaller diameter than the conductor as a whole. The potential gradient for such a conductor will, in consequence, be greater than for the equivalent smooth conductor, so that the breakdown voltage for a stranded conductor will be somewhat less than for a smooth conductor. The irregularities on the surface of such a conductor are increased further because of the deposition of dust and dirt on its surface and the breakdown voltage is further reduced. An average value for the ratio of breakdown voltage for such a conductor and a smooth conductor lies between 0.85 to unity and is denoted by m_0. Suitable values of m_0 are given below:

Polished wires	1.0
Roughened or weathered wires	0.98 to 0.93
Seven strand cable	0.87 to 0.83
Large cables with more than seven strands	0.90 approx.

The final expression for the critical disruptive voltage after taking into account the atmospheric conditions and the surface of the conductor is given by

$$V' = rg_0\delta m_0 \ln \frac{d}{r} \text{ kV} \tag{6.8}$$

When the voltage applied corresponds to the critical disruptive voltage, corona phenomenon starts but it is not visible because the charged ions in the air must receive some finite energy to cause further ionization by collisions. For a radial field, it must reach a gradient g_v at the surface of the conductor to cause a gradient g_0, a finite distance away from the surface of the conductor. The distance between g_v and g_0 is called the energy distance. According to Peek this distance is equal to $(r + 0.301\sqrt{r}\,)$ for two parallel conductors and $(r + 0.308\sqrt{r}\,)$ for co-axial conductors. From this it is clear that g_v is not constant as g_0 is, and is a function of the size of the conductor.

$$g_v = g_0\delta\left(1+\frac{0.3}{\sqrt{r\delta}}\right) \text{ kV/cm for two wires in parallel.} \tag{6.9}$$

Also if V_v is the critical visual disruptive voltage, then

$$V_v = g_v r \ln \frac{d}{r}$$

or
$$g_v = \frac{V_v}{r \ln \frac{d}{r}} = g_0\delta\left(1+\frac{0.3}{\sqrt{r\delta}}\right)$$

or
$$V_v = rg_0\delta\left[1+\frac{0.3}{\sqrt{r\delta}}\right]\ln\frac{d}{r} \text{ kV} \tag{6.10}$$

In case the irregularity factor is taken into account,

$$V_v = g_0 m_v\, \delta r\left[1+\frac{0.3}{\sqrt{r\delta}}\right]\ln\frac{d}{r}$$

$$= 21.1 m_v\, \delta r\left[1+\frac{0.3}{\sqrt{r\delta}}\right]\ln\frac{d}{r} \text{ kV r.m.s.} \tag{6.11}$$

where r is the radius in cms. The irregularity factor m_v has the following values:

m_v = 1.0 for polished wires

= 0.98 to 0.93 for rough conductor exposed to atmospheric severities

= 0.72 for local corona on stranded conductors.

Since the surface of the conductor is irregular, the corona does not start simultaneously on the whole surface but it takes place at different points of the conductor which are pointed and this is known as local corona. For this $m_v = 0.72$ and for decided corona or general corona $m_v = 0.82$.

Example 6.1: Find the critical disruptive voltage and the critical voltages for local and general corona on a 3-phase overhead transmission line, consisting of three stranded copper conductors spaced 2.5 m apart at the corners of an equilateral triangle. Air temperature and pressure are 21°C and 73.6 cm Hg respectively. The conductor dia, irregularity factor and surface factors are 10.4 mm, 0.85, 0.7 and 0.8 respectively.

Solution: The critical disruptive voltage is given by

$$V_d = 21.1\, m\, \delta r \ln \frac{d}{r}$$

where $\delta = \dfrac{3.92b}{273+t} = \dfrac{3.92 \times 73.6}{273+21} = \dfrac{3.92 \times 73.6}{294} = 0.9813$

$$V_d = 21.1 \times 0.85 \times 0.9813 \times 0.52 \ln \frac{250}{0.52} = 56.5 \text{ kV}$$

or the critical disruptive line to line voltage = $56.5 \times \sqrt{3} = 97.89$ kV **Ans.**

The visual critical voltage is given by

$$V_v = 21.1\, m\, \delta r \left(1 + \frac{0.3}{\sqrt{r\delta}}\right) \ln \frac{d}{r}$$

Here $m = 0.7$ for local corona

$= 0.8$ for decided corona or general corona

Now $\sqrt{r\delta} = \sqrt{0.52 \times 0.9813} = 0.71433$

$\therefore$ V_v for local corona $= 21.1 \times 0.7 \times 0.9813 \times 0.52(1 + 0.42) \ln \dfrac{d}{r}$

$= 10.7 \times 6.175$

$= 66.07$ kV

The line to line voltage will be $66.0725.\sqrt{3} = 114.44$ kV.

The visual critical voltage for general corona will be

$$114.44 \times \frac{0.8}{0.7} = 130.78 \text{ kV} \quad \textbf{Ans.}$$

Example 6.2: A conductor with 2.5 cm dia is passed centrally through a porcelain bushing $\varepsilon_r = 4$ having internal and external diameters of 3 cm and 9 cm respectively. The voltage between the conductor and an earthed clamp surrounding the porcelain is 20 kV r.m.s. Determine whether corona will be present in the air space round the conductor.

Solution: Let $g_{1\,max}$ be the maximum gradient on the surface of the conductor and $g_{2\,max}$ the maximum gradient on the inner side of the porcelain

$$g_{1\,max} = \frac{q}{2\pi\varepsilon_0 r}$$

$$g_{2\,\max} = \frac{q}{2\pi\varepsilon_0\varepsilon_r r_1}$$

$$\therefore \qquad g_{1\,\max}\, r = g_{2\,\max}\, \varepsilon_r r_1$$

$$g_{1\,\max} \times 1.25 = g_{2\,\max} \times 4 \times 1.5$$

$$\therefore \qquad g_{1\,\max} = 4.8 g_{2\,\max}$$

or
$$g_{2\,\max} = \frac{g_{1\max}}{4.8} = 0.208 g_{1\,\max}$$

Now
$$20 = g_{1\,\max}\, r \ln \frac{1.5}{1.25} + g_{2\,\max} \times 1.5 \ln \frac{4.5}{1.5}$$

$$= 1.25 g_{1\,\max} \ln \frac{1.5}{1.25} + 0.208 g_{1\,\max} \times 1.5 \ln \frac{4.5}{1.5}$$

$$= 0.228 g_{1\,\max} + 0.3427 g_{1\,\max}$$

$$= 0.570 g_{1\,\max}$$

$$\therefore \qquad g_{1\,\max} = \frac{20}{0.570} = 35 \text{ kV/cm.}$$

Since the gradient exceeds 21.1 kV/cm, corona will be present.

Example 6.3: Determine the critical disruptive voltage and corona loss for a 3-phase line operating at 110 kV which has conductor of 1.25 cm dia arranged in a 3.05 metre delta. Assume air density factor of 1.07 and the dielectric strength of air to be 21 kV/cm.

Solution: The disruptive critical voltage

$$V = 21\, m\, \delta r \ln \frac{d}{r}$$

$$= 21 \times 1.07 \times 0.625 \ln \frac{305}{0.625}$$

$$= 21 \times 1.07 \times 0.625 \times 6.19 = 87 \text{ kV} \quad \textbf{Ans.}$$

The line to line voltage is $87\sqrt{3} = 150.6$ kV.

Since the operating voltage is 110 kV, the corona loss will be absent.

Corona loss zero. **Ans.**

Example 6.4: A single phase overhead line has two conductors of dia 1 cm with a spacing of 1 metre between centres. If the dielectric strength of air is 21 kV/cm, determine the line voltage for which corona will commence on the line.

Solution: The disruptive critical voltage (phase)

$$V_d = 21\delta r \ln \frac{d}{r} = 21.1 \times 0.5 \ln \frac{100}{0.5}$$

$$= 21 \times 0.5 \times 5.2983 = 55.6 \text{ kV.} \quad \textbf{Ans.}$$

6.2 CORONA LOSS

The ions produced by the electric field result in space charges which move round the conductor. The energy required for the charges to remain in motion is derived from the supply system.

The space surrounding the conductor is lossy. In order to maintain the flow of energy over the conductor in the field wherein this additional energy would have been otherwise absent, it is necessary to supply this additional loss from the supply system. This additional power is referred to as corona loss.

An experimental set up (Fig. 6.2) can be arranged to measure corona loss in case of d.c. in a concentric cylinder case.

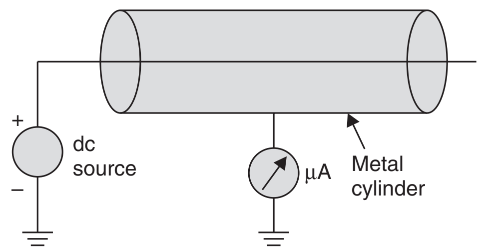


Fig. 6.2 Corona loss measurement with d.c. source.

Since the phenomenon is resistive, the loss will be *VI* watt. Peek made a number of experiments to study the effect of various parameters on the corona loss and he deduced an empirical relation.

$$P = 241 \times 10^{-5} \frac{(f + 25)}{\delta} \sqrt{\frac{r}{d}} \, (V_p - V_0)^2 \text{ kW/km/phase} \tag{6.12}$$

where f is the frequency of supply, δ the air density correction factor, V_p the operating voltage in kV and V_0 the critical disruptive voltage. The equation derived is for a fair weather condition. The approximate loss under foul weather condition is obtained by taking V_0 as 0.8 times the fair weather value. As a matter of fact, with perfectly smooth and cylindrical conductors no corona loss occurs until visual critical voltage is reached when the loss suddenly takes a definite value as calculated by the above formula. It then follows the quadratic law for higher voltages. The empirical relation as derived by Peek has certain limitations and gives correct results only if the supply frequency lies between 25 to 120 Hz, the conductor radius is greater than 0.25 cm and the ratio $\frac{V_p}{V_0} > 1.8$. Also a small error in m_0, the irregularity factor, will lead to wrong results when using this formula.

Factors Affecting Corona Loss

The following are the factors that affect corona loss on overhead transmission lines:

(*i*) Electrical factors,

(*ii*) Atmospheric factors, and

(*iii*) Factors connected with the conductors.

The factors are discussed one by one in the sequence.

Electrical Factors: Frequency and waveform of supply: Referring to the expression (6.12) for corona loss it is seen that corona loss is a function of frequency. Thus higher the frequency of supply the higher are corona losses. This shows that d.c. corona loss is less as compared with a.c. corona. Actually because of corona phenomenon in case of a.c. third harmonics are always present and hence the frequency is not only 50 Hz but it contains 3rd harmonic component also. Hence the corona loss is still large as compared with 50 Hz alone.

Field Around the Conductor: The field around the conductor in addition to being a function of the voltage, depends upon the configuration of the conductors, *i.e.,* whether they are placed in vertical configuration, delta formation etc. Say if the formation is horizontal the field near the middle conductor is large as compared to the outer conductors *i.e.,* the critical disruptive voltage is lower for the middle conductors and hence the corona loss on the middle conductor is more as compared with the two outer conductors. The height of the conductors from the ground has its effect on corona loss. The smaller the height, the greater the corona loss.

When lines are irregularly spaced, the surface gradients of the conductors and hence the corona losses if any are unequal.

Atmospheric Factors: Pressure and temperature effect: From the expression for loss (6.12) it is clear that it is a function of air density correction factor δ which appears directly in the denominator of the expression and indirectly in the value of critical disruptive voltage.

$$V_0 = 21.1 m_0 \, \delta r \ln \frac{d}{r} \text{ kV}$$

The lower the value of δ the higher the loss; because loss is $\alpha(V - V_0)^2$, the lower the value of δ, the lower the value of V_0 and hence higher the value of $(V - V_0)^2$, where V is the operating voltage in kV. This shows that the effect of δ on corona loss is very serious. For lower values the pressure should be low and temperature higher. It is for this reason that the corona loss is more on hilly areas than on plain areas.

Dust, Rain, Snow and Hail Effect: The particles of dust clog to the conductor; thereby the critical voltage for local corona reduces which increases corona loss. Similarly, the bad atmospheric conditions such as rains, snow and hailstorm reduce the critical disruptive voltage and hence increase the corona loss.

Factors Connected with the Conductor: Diameter of the Conductor: From the expression (6.12) for corona loss it can be seen that the conductor size appears at two places and if other things are assumed constant,

$$\text{loss} \propto \sqrt{\frac{r}{d}}$$

and

$$\text{loss} \propto (V - V_0)^2$$

It appears from the first relation that loss is proportional to the square root of the size of the conductor, *i.e.,* larger the dia of the conductor larger will be the loss. But from the second expression as V_0 is approximately directly proportional to the size of the conductor, hence larger the size of the conductor larger will be the critical disruptive voltage and hence smaller will be the factor $(V - V_0)^2$. It is found in practice that the effect of the second proportionality is much more than the first on the corona losses, and hence larger the size of the conductor lower is the corona loss.

Number of Conductors/Phases: For operating voltage 380 kV and above it is found that one conductor per phase gives large corona loss and hence large radio interference (*RI*) level which interferes with the communication lines which normally run parallel to the power lines. This problem of large corona loss is solved by using two or more than two conductors per phase which is known as bundling of conductors. By bundling the conductors the self GMD of the conductors is increased thereby; the critical disruptive voltage is increased and hence corona loss is reduced.

Profile of the Conductor: By this is meant the shape of the conductor whether cylindrical, flat, oval etc. Because of field uniformity in case of cylindrical conductor the corona loss is less in this as compared to any other shape.

Surface Conditions of the Conductors: The conductors are exposed to atmospheric conditions. The surface would have dirt etc. deposited on it which will lower the disruptive voltage and increase corona loss.

Heating of the Conductor by Load Current: The heating of the conductor by the load current has an indirect reducing effect on the corona loss. Without such heating the conductor would tend to have a slightly lower temperature than the surrounding air. In the absence of heating, dew in the form of tiny water drops would form on the conductor in foggy weather or at times of high humidity, which induces additional corona. The heating effect of the load current is, however, large enough to prevent such condensation.

During rains, the heating of the conductor has no influence on the corona loss but, after the rain it accelerates the drying of the conductor surface. The time during which the water drops remain on the surface is reduced and the loss is also reduced.

For long transmission lines which pass through routes of varying altitudes, the average value of corona loss is obtained by finding out the corona loss per km at a number of points and then an average is taken out.

Methods of Reducing Corona Loss

These losses can be reduced by using

(*i*) large dia conductors,

(*ii*) hollow conductors, and

(*iii*) bundled conductors.

It has already been discussed how large dia and bundled conductors reduce the corona losses. The idea of using the hollow conductors is again the same *i.e.*, to have a large diameter without materially adding to its weight. In one of the designs one or more layers of copper wires are stranded over a twisted *I*-beam core. Another design consists of tongued and grooved copper segments spiralled together to form a self-supporting hollow tube. This conductor has a smooth surface. Expanded steel cored aluminium conductors which incorporate plastic or fibrous spacing material have also been proposed. Lines using the above types of conductors are more expensive than those using the conventional type and the economic limit to the conductor diameter appears to be somewhat between 3.75 and 5 cms. These special conductors are more effective in reducing corona. Losses during fair weather conditions and there may not be the same degree of improvement during bad weather conditions.

Example 6.5: Determine the corona characteristics of a 3-phase line 160 km long, conductor diameter 1.036 cm, 2.44 m delta spacing, air temperature 26.67°, altitude 2440 m, corresponding to an approximate barometric pressure of 73.15 cm, operating voltage 110 kV at 50 Hz.

Solution: Radius of conductor = $\frac{1.036}{2}$ = 0.518 cm

The ratio $\frac{d}{r} = \frac{2.44}{0.518} \times 100 = 471$

and $\sqrt{\frac{r}{d}} = \sqrt{\frac{1}{471}} = 0.046075$

$$\delta = \frac{3.92b}{273+t} = \frac{3.92 \times 73.15}{273+26.67} = 0.957$$

Assuming a surface irregularity factor 0.85, the critical disruptive voltage

$$V_d = 21.1 \times 0.85\delta r \ln \frac{d}{r}$$
$$= 21.1 \times 0.85 \times 0.957 \times 0.518 \ln 471$$
$$= 54.72 \text{ kV line to neutral}$$

The visual critical voltage $V_v = 21.1 m_v \delta r \left(1+\frac{0.3}{\sqrt{r\delta}}\right) \ln \frac{d}{r}$

Assuming a value of $m_v = 0.72$,

$$V_v = 21.1 \times 0.72 \times 0.957 \times 0.518 \left(1+\frac{0.3}{\sqrt{0.518 \times 0.957}}\right) \ln 471 = 66 \text{ kV}$$

The power loss = $241 \times 10^{-5} \frac{f+25}{\delta} \sqrt{\frac{r}{d}} (V - V_d)^2$ kW/phase/km

$$= 241 \times 10^{-5} \times \frac{75}{0.957} \times 0.046075(63.5 - 54.72)^2$$
$$= 0.671 \text{ kW/phase/km}$$

or $= 107.36$ kW/phase

or $= 322$ kW for three phases.

The corona loss under foul weather condition will be when the disruptive voltage is taken as 0.8 × fair weather value, *i.e.,*

$$V_d = 0.8 \times 54.72 = 43.77 \text{ kV}$$

∴ Loss per phase/km will be

$$241 \times 10^{-5} \frac{75}{0.957} 0.046075(63.5 - 43.77)^2 = 3.3875 \text{ kW/km/phase}$$

or 542 kW/phase

or Total loss = 1626 kW for all the three phases. **Ans.**

6.3 LINE DESIGN BASED ON CORONA

It is desirable to avoid corona loss on power lines under fair weather conditions. Bad weather conditions such as rain sleet greatly increase the corona loss and also lower the critical voltage of the line. On account of the latter effect, it is not practical to design high voltage lines which will be corona-free at all times. If the lines are designed without corona even during bad weather conditions, the size of the towers and the conductors will be uneconomical. Since the bad weather conditions in a particular region prevail only for a very short duration of the year, the average corona loss throughout the year will be very small. A typical transmission line may have a fair weather loss of 1 kW per 3-phase mile and foul weather loss of 20 kW per three phase mile.

6.3.1 Disadvantages of Corona

(*i*) There is a definite loss of power even though it is not much during fair weather condition.
(*ii*) When corona is present the effective capacitance of the conductors is increased because the effective dia of the conductor is increased. This effect increases the flow of charging current. Because of corona triple frequency currents flow through the ground in case of a grounded system and they give rise to a voltage of triple frequency in an ungrounded system. These triple frequency currents and voltages interfere with the communication circuits due to electromagnetic and electrostatic induction effects.

6.3.2 Advantages of Corona

It reduces the magnitude of high voltage steep fronted waves due to lighting or switching by partially dissipating as a corona loss. In this way it acts as a safety valve to some extent.

6.4 RADIO INTERFERENCE

Radio interference is the adverse effect introduced by corona on wireless broadcasting. The corona discharges emit radiation which may introduce noise signals in the communication lines, radio and television receivers. It is mainly due to the brush discharges on the surface irregularities of the conductor during positive half cycles. This leads to corona loss at voltages lower than the critical voltages. The negative discharges are less troublesome for radio reception. Radio interference is considered as a field measured in microvolts per metre at any distance from the transmission line and is significant only at voltages greater than 200 kV. There is gradual increase in *RI* level till the voltage is such that measurable corona loss takes place. Above this voltage there is rapid increase in *RI* level. The rate of increase is more for smooth and large diameter conductors. The amplitude of *RI* level varies inversely as the frequency at which the interference is measured. Thus the services in the higher frequency band *e.g.*, television, frequency modulated broadcasting, microwave relay, radar etc. are less affected. Radio interference is one of the very important factors while designing a transmission line.

6.5 INDUCTIVE INTERFERENCE BETWEEN POWER AND COMMUNICATION LINES

It is a common practice to run communication lines along the same route as the power lines since the user of electrical energy is also the user of electrical communication system. The transmission lines transmit bulk power at relatively higher voltages. These lines give rise to electromagnetic and electrostatic fields of sufficient magnitude which induce currents and voltages respectively in the neighbouring communication lines. The effects of extraneous currents and voltages on communication systems include interference with communication service *e.g.*, superposition of extraneous currents on the true speech currents in the communication wires, hazard to person and damage to apparatus due to extraneous voltages. In extreme cases the effect of these fields may make it impossible to transmit any message faithfully and may raise the potential of the apparatus above the ground to such an extent as to render the handling of the telephone receiver extremely dangerous.

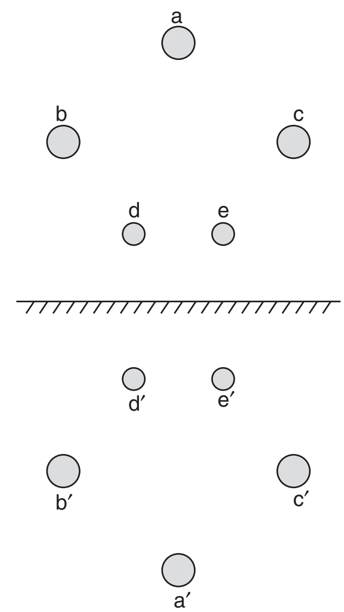


Fig. 6.3 3-phase single circuit power line, communication line and their images.

Electromagnetic Effects: Consider Fig. 6.3. a, b and c are the power conductors of a 3-phase single circuit line on a transmission tower and d and e are the conductors of a neighbouring communication line running on the same transmission towers as the power conductors or a neighbouring separate line. Let the distances between power conductors and communication conductors be D_{ad}, D_{ae}, D_{bd}, D_{be}, D_{cd} and D_{ce} respectively and the currents through power conductors be I_a, I_b and I_c respectively such that $I_a + I_b + I_c = 0$. The flux linkage to conductor d due to current I_a in conductor a will be $\psi_{ad} = 2 \times 10^{-7} I_a \ln \dfrac{\infty}{D_{ad}}$. Similarly, the flux linkage to conductor e due to current I_a in conductor a

$$\psi_{ae} = 2 \times 10^{-7} I_a \ln \frac{\infty}{D_{ae}}$$

$\therefore$ Mutual flux linkage between conductor d and e due to current I_a will be

$$\psi_{ad} - \psi_{ae} = 2 \times 10^{-7} I_a \ln \frac{D_{ae}}{D_{ad}}$$

or mutual inductance $M_a = \dfrac{\psi_{ad} - \psi_{ae}}{I_a} = 2 \times 10^{-7} \ln \dfrac{D_{ae}}{D_{ad}}$ H/metre

Similarly M_b and M_c the mutual inductances between conductor b and the loop de and between conductor c and the loop de respectively are given as

$$M_b = 2 \times 10^{-7} \ln \frac{D_{be}}{D_{bd}} \text{ H/metre}$$

$$M_c = 2 \times 10^{-7} \ln \frac{D_{ce}}{D_{cd}} \text{ H/metre}$$

These mutual inductances are due to fluxes which have a phase displacement of 120°; therefore, the net effect of the magnetic field will be

$$M = M_a + M_b + M_c$$

where **M** is the net mutual inductance which is the phasor sum of the three inductances.

If I is the current in the power conductors and f is the supply frequency, the voltage induced in the communication conductors d and e will be $V = 2\pi f\, MI$ volts per m.

It is to be noted that larger the distance between the power conductors and the communication conductors, smaller is the value of mutual inductance and since the current through the power conductors is displaced by 120°, there is appreciable amount of cancellation of the power frequency voltages. But the presence of harmonics and multiples of third harmonics will not cancel as they are in phase in all the power conductors and, therefore, are dangerous for the communication circuits. Also, since these harmonics come within audio frequency range, they are dangerous for the communication circuits.

Electrostatic Effects: Consider again Fig. 6.3. Let q be the charge per unit length of the power line. The voltage of conductor d due to charge on conductor can be obtained by considering the charge on conductor a and its image on the ground. Let conductor a be at a height h_a from the ground. Therefore, the voltage of conductor d will approximately be

$$V_{ad} = \frac{q}{2\pi\varepsilon_0} \int_{h_a}^{D_{ad}} \left[\frac{1}{x} + \frac{1}{(2h_a - x)} \right] dx$$

$$= \frac{q}{2\pi\varepsilon_0} \left[\ln \frac{2h_a - x}{x} \right]_{D_{ad}}^{h_a} = \frac{q}{2\pi\varepsilon_0} \left[\ln \frac{2h_a - D_{ad}}{D_{ad}} \right]$$

Now from the geometry the voltage of conductor a is $V_a = \dfrac{q}{2\pi\varepsilon_0} \ln \dfrac{2h_a}{r}$, where r is the radius of conductor a.

∴ Substituting for q in the expression for V_{ad} above, we get

$$V_{ad} = \frac{2\pi\varepsilon_0 V_a}{\ln \dfrac{2h_a}{r}} \cdot \frac{1}{2\pi\varepsilon_0} \ln \frac{2h_a - D_{ad}}{D_{ad}}$$

$$= V_a \cdot \frac{\ln \dfrac{2h_a - D_{ad}}{D_{ad}}}{\ln \dfrac{2h_a}{r}}$$

Similarly, we can obtain the potential of conductor d due to conductors b and c and hence the potential of conductor d due to conductors a, b and c will be

$$V_d = V_{ad} + V_{bd} + V_{cd}$$

Similarly, the potential of conductor e due to conductors a, b and c can be obtained.

PROBLEMS

6.1. Determine the corona characteristics of a 3-phase, 50 Hz, 132 kV transmission line 100 km long running through terrain at an altitude of 600 metres, temp. of 30°C and barometric pressure 74 cm. The conductors are 1.5 cm diameter and spaced with equilateral spacing of 2.75 metres. Assume surface irregularity factor of 0.9 and $m_v = 0.75$.

6.2. A 3-phase, 50 Hz, 132 kV transmission line consists of conductors of 1.17 cm dia and spaced equilaterally at a distance of 3 metres. The line conductors have smooth surface with value for $m = 0.96$. The barometric pressure is 72 cm of Hg and temperature of 20°C. Determine the fair and foul weather corona loss per km per phase.

6.3. A 3-phase, 50 Hz, 138 kV transmission line has conductors in equilateral formation spaced 2.5 metres apart. The conductor diameter is 1.04 cm and the surface factor is 0.85. The air pressure and temperature are 74 cm of Hg and 21°C respectively. Determine the critical visual voltage for corona and the corona loss per km per phase of the line, $m_v = 0.72$.

6.4. A single phase transmission line has conductors of diameter 1.25 cm and spaced 2.5 metres apart. Derive an expression for the potential gradient at any point on a line joining the centres of the conductors if the operating voltage of line is 60 kV. Calculate the voltage at which corona will start.

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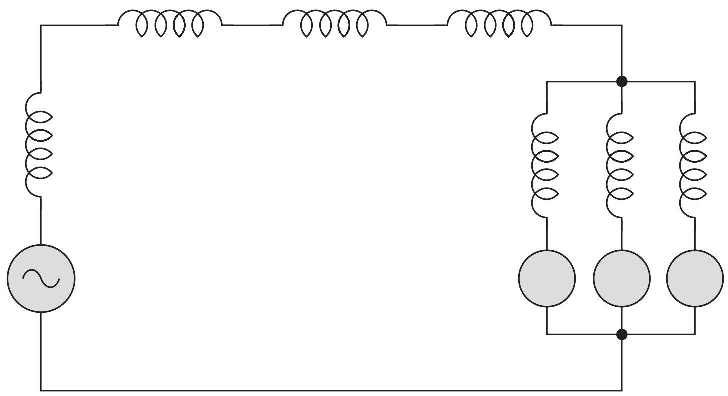
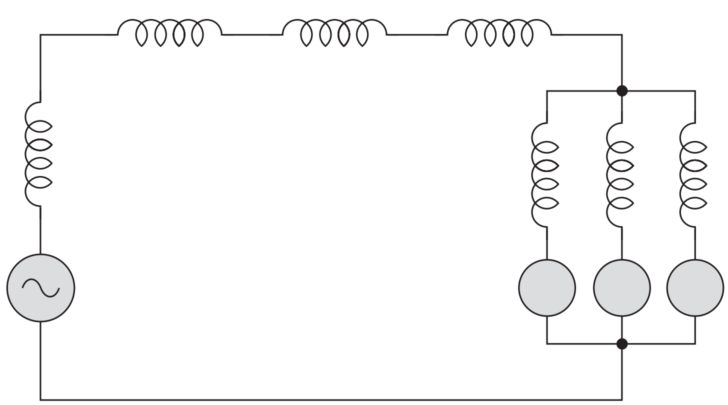


7

MECHANICAL DESIGN OF TRANSMISSION LINES



7

Mechanical Design of Transmission Lines

INTRODUCTION

The transmission line conductors are supported on transmission line towers. The supports are of the following types:

1. Steel poles.
2. Reinforced concrete poles.
3. Broad-base steel lattice structure towers.

Normally for short spans and voltages up to 33 kV, the first two types of supports are used whereas for long spans and higher voltages the broad-base steel lattice structures are used.

When a perfectly flexible wire of uniform weight is hung between the two horizontal supports, it will form a catenary.

7.1 THE CATENARY CURVE

Let the conductor be strung between the supports A and B (Fig. 7.1) and l is the distance between the support, w the weight per unit length of the wire, T_0 the tension in the wire at the lowest point H of the wire in kg and OX and OY are the axes drawn from the origin O. The location of O is c units below the lowest point H such that $T_0 = wc$ or $c = T_0/w$. ψ is the angle subtended by T with the horizontal axis. Consider the equilibrium of the small length s of the wire up to point $P(x, y)$. Three forces are acting on this length of the wire:

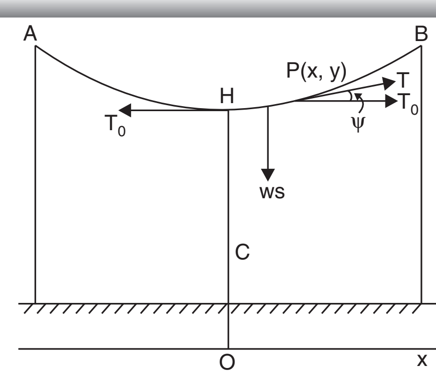


Fig. 7.1 The catenary curve.

(*i*) The horizontal tension $T_0 = wc$, (*ii*) The vertical weight ws,

(*iii*) The tension T.

From Fig. 7.1 it is clear that

$$T \cos \psi = T_0 = wc \tag{7.1}$$

and

$$T \sin \psi = ws \tag{7.2}$$

From equations (7.1) and (7.2)

$$\tan \psi = \frac{dy}{dx} = \frac{ws}{wc} = \frac{s}{c} \tag{7.3}$$

Now for a differential length

or

$$ds = \sqrt{dx^2 + dy^2} \tag{7.4}$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{7.5}$$

Substituting for $\frac{dy}{dx}$ from (7.3) into (7.5),

$$\frac{ds}{dx} = \sqrt{1 + \frac{s^2}{c^2}} = \frac{\sqrt{c^2 + s^2}}{c}$$

or

$$\frac{cds}{\sqrt{c^2 + s^2}} = dx$$

Let

$$s = c \sinh \theta, \; ds = c \cosh \theta \; d\theta \tag{7.6}$$

$$\frac{c \,.\, c \cosh \theta \,.\, d\theta}{c \cosh \theta} = dx$$

or

$$cd\theta = dx$$

or

$$c\theta = x + A$$

Now for $x = 0$, $s = 0$, $\therefore \quad \theta = 0$.

Substituting for x and θ in the equation above,

$$0 = A$$

$$c\theta = x$$

or

$$\theta = \frac{x}{c}$$

From equation (7.6), we obtain

$$\theta = \sinh^{-1} \frac{s}{c}$$

$$\therefore \qquad \sinh^{-1} \frac{s}{c} = \frac{x}{c}$$

or

$$\frac{s}{c} = \sinh \frac{x}{c}$$

or $$s = c \sinh \frac{x}{c} \tag{7.7}$$

Now from equations (7.3) and (7.7), we have

$$\frac{dy}{dx} = \sinh \frac{x}{c}$$

$$dy = \sinh \frac{x}{c}\, dx$$

$$y = c \cosh \frac{x}{c} + B \tag{7.8}$$

From Fig. 7.1, $x = 0, y = c$

Substituting this condition in equation (7.8) for evaluating B,

$$c = c + B$$

$\therefore$ $$B = 0$$

$$y = c \cosh \frac{x}{c} \tag{7.9}$$

Equations (7.7) and (7.9) represent a catenary and they give the relationships between the length of the wire measured from the lowest point H and the vertical height of any point $P(x, y)$ as measured above the origin O respectively in terms of the distance x as measured from O along OX .

Expanding the terms sinh x/c and cosh x/c, the equations (7.7) and (7.9) become

$$s = c \,.\, \frac{x}{c} + \frac{x^3}{3!\,c^3} + \frac{x^5}{5!\,c^5} + \ldots \tag{7.10}$$

$$v = c \,.\, 1 + \frac{x^2}{2!\,c^2} + \frac{x^4}{4!\,c^4} + \ldots \tag{7.11}$$

These expressions can be approximated by taking the first two terms in case the span length is not large as compared with c.

After approximations the equations (7.10) and (7.11) can be rewritten as

$$s = x + \frac{x^3}{6c^2} \tag{7.12}$$

$$y = c + \frac{x^2}{2c}. \tag{7.13}$$

7.2 SAG TENSION CALCULATIONS

Tension T: To calculate tension T at any point $P(x, y)$ on the wire, use is made of equations (7.1) and (7.2). Squaring and adding equations (7.1) and (7.2), we get

$$T^2 \cos^2 \psi + T^2 \sinh^2 \psi = w^2c^2 + w^2s^2$$

$$T^2 = w^2(s^2 + c^2)$$

$$= w^2 \left(c^2 \sinh^2 \frac{x}{c} + c^2 \right)$$

$$= w^2c^2(1 + \sinh^2 x/c)$$

$$= w^2c^2 \cosh^2 x/c$$

$$T = wc \cosh x/c$$

$$T = wy \tag{7.14}$$

From equation (7.14) it is clear that the tension in the wire at any point $P(x, y)$ in the wire is the product of the y-coordinate of the point and the weight per unit length of the wire.

Sag d: The sag d at point $P(x, y)$ is the vertical distance between the point P and the lowest point H. To calculate the sag, equation (7.13) is used

$$y = c + \frac{x^2}{2c}$$

$$y - c = \frac{x^2}{2c} = \text{sag } d$$

Now this sag is maximum when $x = l/2$.

$$d = \frac{l^2}{8c}$$

$$d = \frac{l^2}{8c} = \frac{wl^2}{8T} = \frac{wl^2}{8fA} = \frac{l^2\delta}{8f} \tag{7.15}$$

where f = stress corresponding to tension T,

A = area of cross section of the conductor, and

$\delta = \dfrac{w}{A}$ constant. δ is the density of the conductor material and is, therefore, constant for a particular material.

Length L of the Conductor: Using equation (7.12),

$$s = x + \frac{x^3}{6c^2}$$

Substituting $x = (l/2)$ to get the length of the conductor between the point H and the support end A or B,

$$\frac{L}{2} = \frac{l}{2} + \frac{l^3}{48c^2}$$

or

$$L = l + \frac{l^3}{24c^2}$$

Substituting for $c = f_i\,\delta$,

$$L = l + \frac{l^3\delta^2}{24f^2} \tag{7.16}$$

We have used the relation $T = wc$ instead of $T_0 = wc$. This approximation is true for small values of sag and it can be assumed that when d is small, the tension T is approximately uniform throughout the wire.

Having derived the basic equations for a wire strung between two supports we are now ready to design the transmission lines. An overhead line must be designed from the view point of worst probable loads rather than the worst possible loads because the cost of the overhead line will become very large if designed on the basis of worst possible conditions.

The sag to be allowed in a conductor at the time of erection *i.e.*, still air and relatively higher temperature must be such that in bad weather conditions which are a combination of wind and lower temperatures (snow or ice coating), a specified maximum tension for the conductor is not exceeded. The problem can be restated as follows:

Given the maximum tension which must not be exceeded under specified severe conditions of wind, ice or other loading at a specified temperature, to determine the sag and tension at some other conditions of loading and temperature, including the still air and higher temeratures.

Effect of Wind and Ice Loading: As discussed earlier the severe conditions are the wind and ice loadings. Under this condition the per unit length of the wire experiences the following loading: (*i*) the weight of the conductor w acting vertically downwards, (*ii*) the ice loading w_i acting vertically downwards, and (*iii*) the wind loading w_a acting horizontally.

Ice Loading: Let r be the radius of the conductor and t the thickness of ice (Fig. 7.2). The volume of ice per unit length

$$= \pi\{(r + t)^2 - r^2\} \cdot 1$$
$$= \pi(2rt + t^2) \cdot 1$$

If ρ is the density of ice (910 kg/m³), the weight of ice per unit length of conductor

$$w_i = \pi\rho(2rt + t^2) \text{ kg/metre}$$

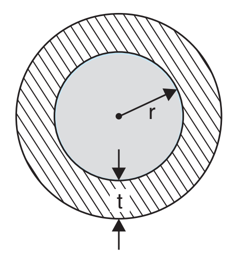


Fig. 7.2 Ice-coated conductor.

Wind Loading: Let p be the wind pressure in kg/m²; assuming the ice coating of thickness t, the projected area per unit length on which the wind is acting is

$$(r + t) \cdot 1 \text{ sq. metres}$$

∴ Wind loading w_a per unit length will be

$$w_a = 2(r + t) \cdot p \text{ kg/metre}$$

Total vertical loading $= w + w_i$

Total loading $\quad W = \sqrt{(w + w_i)^2 + w_a^2}$

The loading factor $\quad q = W/w$

Let the two conditions *i.e.*, the severe conditions of wind and ice loading and the still air be characterised by the following:

Wind and Ice Loading:

L_1 = length of the conductor between the supports

$W/w = q_1$ = loading factor

f_1 = maximum working stress in the conductor in kg/cm²

t_1 = ambient temperature.

Still Air and Higher Temperature:

L_2 = length of the conductor between the supports

q_2 = loading factor which is unity under these conditions

f_2 = maximum working stress in the conductor in kg/cm^2 corresponding to f_1 under wind and ice loading condition

t_2 = ambient temperature

E = modulus of elasticity in kg/cm^2

α = coefficient of linear expansion.

The problem now is, given f_1 calculate f_2 which is required for stringing the conductor during fair weather (still air) conditions.

Length L_2 from equation (7.12),

$$s = x + \frac{x^3}{6c^2}$$

for $x = l/2$

$$\frac{L_2}{2} = \frac{l}{2} + \frac{l^3}{48c^2} \quad \text{or} \quad L_2 = l + \frac{l^3}{24c^2}$$

Now with loading different from w the value of $c = T/W$, where W is the total loading in kg/metre

$$c = \frac{fA}{W} = \frac{f\,.\,Aw}{Ww} = \frac{f}{\delta q}$$

Substituting this value of c in the expression for L_2,

$$L_2 = l + \frac{l^3\delta_2^2 q_2^2}{24f_2^2}$$

As said earlier under this condition of standstill air and higher temperature, $q_2 = 1$.

Similarly,
$$L_1 = l + \frac{l^3\delta_1^2 q_1^2}{24f_1^2}$$

In order to relate f_2 with f_1, one possibility is to find out some relation between the two lengths L_1 and L_2. Now due to higher temperature the length under standstill condition is $l \times \alpha(t_2 - t_1)$ metres more than under ice loading conditions but due to increased sag at higher temperatures the stress in the material is reduced from f_1 to f_2 and hence there is contraction of length at higher temperature than lower temperature.

Therefore,
$$L_2 = L_1 + l\alpha(t_2 - t_1) - \frac{f_1 - f_2}{E}\,l$$

It is to be noted here that little error is introduced if l is taken instead of L for the last term on the right hand side of the above equation.

Now substituting for L_2 and L_1,

$$l + \frac{l^3\delta_2^2 q_2^2}{24f_2^2} = l + \frac{l^3\delta_1^2 q_1^2}{24f_1^2} + l\alpha(t_2 - t_1) - \frac{f_1 - f_2}{E}\,l$$

or $$f_1 - \frac{l^2\delta^2 q_1^2}{24 f_1^2} E = f_2 - \frac{l^2\delta^2 q_2^2}{24 f_2^2} E + \alpha(t_2 - t_1)E$$

The quantities on the left hand of the above expression are known so that putting this equal to K we have

$$K = f_2 - \frac{l^2\delta^2 q_2^2}{24 f_2^2} E + \alpha(t_2 - t_1)E$$

or $$K - \alpha(t_2 - t_1)E = f_2 - \frac{l^2\delta^2 q_2^2}{24 f_2^2} . E$$

Again the quantity on the left hand side is known and let this be equal to N; we have

$$N = f_2 - \frac{l^2\delta^2 q_2^2}{24 f_2^2} E$$

or $$f_2^{\,2}(f_2 - N) = \frac{l^2\delta^2 q_2^2}{24} E$$

The quantity on the right hand side of the above expression is known and let this be equal to M; we then have

$$f_2^2(f_2 - N) = M \tag{7.17}$$

This is a cubic equation in f_2. This equation can be solved on a slide rule as follows: Set the cursor on scale A corresponding to the figure M. Make a suitable guess of the solution and set the end of the slide at this guessed value on scale D. If the figure on scale B under the cursor is equal to $(f_2 - N)$, where f_2 is the guessed value, the guess is correct, otherwise have a fresh guess and proceed until the requirement is met.

The procedure for evaluating f_2 is summarized as follows:

1. Evaluate the loading factors q_1 and q_2 for the two conditions of load from

$$q = \frac{\sqrt{(w + w_i)^2 + w_a^2}}{w}$$

2. Calculate K from the expression

$$K = f_1 - \frac{l^2\delta^2 q_1^2 E}{24 f_1^2}$$

3. Evaluate N and M from the expressions

$$N = K - \alpha t E \text{ and } M = \frac{l^2\delta^2 q_2^2 E}{24}$$

4. Evaluate f_2 from the cubic equation

$$f_2^2(f_2 - N) = M$$

5. The sag is then evaluated from the expression

$$d = \frac{l^2\delta^2 q_2^2}{8 f_2}$$

Example 7.1: An overhead line has the following data:

Span length 160 metres, conductor dia 0.95 cm, weight per unit length of the conductor 0.65 kg/metre. Ultimate stress 4250 kg/cm^2, wind pressure 40 kg/m^2 of projected area. Factor of safety 5.

Calculate the sag.

Solution: Factor of safety = 5

$$\text{The working stress} = \frac{4250}{5} = 850 \text{ kg/cm}^2$$

$$\text{The area of section of the conductor} = \frac{\pi}{4} . d^2 = \frac{\pi}{4} \times 0.95^2 = 0.7084 \text{ sq. cm}$$

The wind pressure per unit length of the conductor = $40 \times 0.95 \times 10^{-2} = 0.38$ kg/cm

The weight of conductor per unit length = 0.65 kg/metre

$$\begin{aligned}
\text{The total effective weight} &= \sqrt{0.65^2 + 0.38^2} \\
&= \sqrt{0.4225 + 0.1444} \\
&= \sqrt{0.5669} \\
&= 0.7529 \text{ kg/metre} \\
\text{Working tension} &= 850 \times 0.7084 = 602.5 \text{ kg} \\
c &= \frac{T}{W} = \frac{602.5}{0.7529} = 800 \text{ metres} \\
d &= \frac{l^2}{8c} = \frac{160 \times 160}{8 \times 800} = 4 \text{ metres.} \quad \textbf{Ans.}
\end{aligned}$$

Example 7.2: A transmission line conductor having a dia of 19.5 mm weights 0.85 kg/m. The span is 275 metres. The wind pressure is 39 kg/m^2 of projected area with ice coating of 13 mm. The ultimate strength of the conductor is 8000 kg. Calculate the maximum sag if the factor of safety is 2 and ice weighs 910 kg/m^3.

Solution: The overall dia of the conductor with ice coating = 1.95 + 2.6 = 4.55 cm

The projected area per metre length of conductor = $4.55 \times 1 \times 10^{-2}$ sq. metre

The wind load per metre length = $4.55 \times 10^{-2} \times 39 = 1.77$ kg/metre

$$\begin{aligned}
\text{The area of section of ice} &= \pi[(r + t)^2 - r^2] \\
&= \pi[22.75^2 - 9.75^2] \\
&= \pi(517.56 - 95.06) \\
&= 1327.32 \text{ sq. mm} \\
&= 1.327 \times 10^{-3} \text{ sq. metre}
\end{aligned}$$

$\therefore$ Ice weight per metre length = $1.327 \times 10^{-3} \times 910 = 1.207$ kg/metre

$$\begin{aligned}
\text{The total weight } W &= \sqrt{(0.85 + 1.207)^2 + 1.77^2} \\
&= \sqrt{4.23125 + 3.1329} \\
&= 2.71369 \text{ kg/metre}
\end{aligned}$$

$$\text{The working tension of the conductor} = \frac{8000}{2} = 4000 \text{ kg}$$

$$\therefore \qquad c = \frac{T}{W} = \frac{4000}{2.71369} = 1474 \text{ metres}$$

$\therefore$ The maximum sag $= \dfrac{l^2}{8c} = \dfrac{275^2}{8 \times 1474} = 6.4$ metres. **Ans.**

Example 7.3: Determine the sag at 32.2°C and 65.5°C in an 8 SWG copper conductor erected on a 45.7 metre span length. The wind pressure is 48.82 kg/sq. metre of projected area at a temperature of 4.5°C, weight of wire is 0.1156 kg/metre. The working stress shall not exceed $\frac{1}{2}$ the ultimate tensile strength.

Modulus of elasticity	$= 1.26 \times 10^4$ kg/mm^2
Coefficient of linear expansion	$= 16.6 \times 10^{-6}$ per °C
Ultimate stress of the conductor	$= 42$ kg/mm^2
Dia of conductor	$= 4.1$ mm

Solution:

Cross-section of conductor	$= 13.2$ sq. mm
Projected area of conductor per unit length	$= 4.1 \times 10^{-3} \times 1$
Wind loading per metre length	$= 4.1 \times 10^{-3} \times 48.82 = 0.2$ kg/m
$\therefore$ Effective load per metre length	$= \sqrt{0.1157^2 + 0.2^2} = 0.23$ kg

$\therefore$ Loading factor $q_1 = \dfrac{0.23}{0.115} = 2.0$

Now $\dfrac{w}{A} = \dfrac{0.1157}{13.2} = 8.765 \times 10^{-3}$ kg/metre/sq. mm

Working stress $f_1 = 21$ kg/mm^2

$$T_1 = f_1 A = 21 \times 13.2 = 277 \text{ kg}$$

$$c = \frac{T_1}{W} = \frac{277}{0.23} = 1205 \text{ metres}$$

Sag at this temperature (4.5°C) $= \dfrac{l^2}{8c} = \dfrac{45.7 \times 45.7}{8 \times 1205} = 0.2166$ metres

Sag at 32.2°C:

The difference in temperature = 32.2°C – 4.5°C = 27.7°C

$$K = f_1 - \frac{l^2 \delta^2 q_1^2 E}{24 f_1^{\,2}} = 21 - \frac{45.7^2 \times 8.765^2 \times 10^{-6} \times 2^2 \times 1.26 \times 10^4}{24 \times 21 \times 21}$$

$$= 21 - 0.764 = 20.236$$

$$\alpha t E = 16.6 \times 10^{-6} \times 27.7 \times 1.26 \times 10^4 = 579 \times 10^{-2} = 5.790$$

Now $$\frac{l^2 \delta^2 q_2^2 E}{24} = \frac{45.7^2 \times 8.765^2 \times 10^{-6} \times 1.26 \times 10^4}{24}$$

$$= 84.23$$

Writing the equation

$$f_1 - \frac{l^2\delta^2 q_1^2 E}{24 f_1^{\,2}} = f_2 - \frac{l^2\delta^2 q_2^2 E}{24 f_2^{\,2}} + \alpha t E$$

$$20.236 = f_2 - \frac{84.23}{f_2^{\,2}} + 5.79$$

$$f_2^{\,3} - 84.23 - 14.44 f_2^{\,2} = 0$$

$$f_2^{\,2}(f_2 - 14.44) = 84.23$$

$$f_2 = 14.83 \text{ kg/mm}^2$$

$$\therefore \quad T = 14.83 \times 13.2 = 195 \text{ kg}$$

$$\therefore \quad c = \frac{T}{w} = \frac{195}{0.1157} = 1691 \text{ metres}$$

$$\therefore \quad d = \frac{l^2}{8c} = \frac{45.7 \times 45.7}{8 \times 1691} = 0.1543 \text{ metres} \quad \textbf{Ans.}$$

Similarly sag corresponding to 65.5°C can be obtained. The reader will find this to be equal to 0.264 metre.

7.3 SUPPORTS AT DIFFERENT LEVELS

When transmission lines are run on steep inclines, the two ends *A* and *B* of the towers will not be at the same height. The shape of the wire strung between the support will form a part of the catenary and, therefore, the lowest point of the catenary will not lie in the middle of the span. Referring to Fig. 7.3, *A* and *B* are the support ends. To have complete catenary, extend the curve *BHA* to *A′* such that *A′* is at the same level as *B*. *H* is the mid-point of *A′B* and hence is the lowest point. Let *H* be at a horizontal distance of *a* units from *A* and *b* units from *B*. The difference in levels of supports *A* and *B* is *h* units. The sag *d* is as shown.

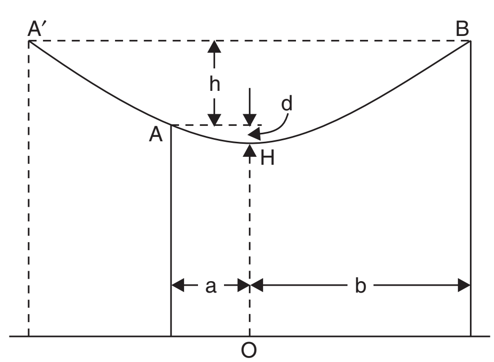


Fig. 7.3 Supports at unequal level.

The objective to begin with is to find out *a* and *b*. This is done as follows:

From equation (7.13)

$$y = c + \frac{x^2}{2c}$$

Taking O as the origin,

At A, $x = a$, $y = c + d$

At B, $x = b$, $y = c + d + h$

Substituting these in equation (7.13),

$$c + d = \frac{a^2}{2c} \tag{7.18}$$

and
$$c + d + h = \frac{b^2}{2c} \tag{7.19}$$

Using these equations,

$$h = \frac{b^2 - a^2}{2c} = \frac{(b+a)(b-a)}{2c} = \frac{l(b-a)}{2c}$$

Now
$$b - a = (a + b - 2a) = (l - 2a)$$

∴
$$h = \frac{l(l-2a)}{2c}$$

or
$$\frac{2ch}{l} = l - 2a$$

or
$$2a = l - \frac{2ch}{l}$$

or
$$a = \frac{l}{2} - \frac{ch}{l} \tag{7.20}$$

and
$$b = \frac{l}{2} + \frac{ch}{l} \tag{7.21}$$

From equation (7.20) it is clear that in case $\frac{ch}{l} > \frac{l}{2}$, a becomes negative which can be illustrated by Fig. 7.4.

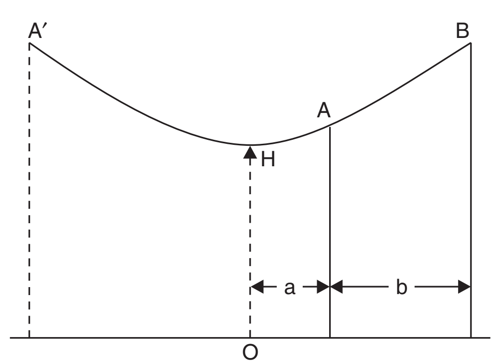


Fig. 7.4 Supports at unequal level $\frac{ch}{l} > \frac{l}{2}$.

Having calculated a and b, the sag and the length of the conductor can be calculated using equations (7.13) and (7.12) respectively.

$$\text{sag } d = \frac{a^2}{2c} = \left(\frac{l}{2} - \frac{ch}{l}\right)^2 \cdot \frac{l}{2c} = \left(\frac{l}{2} - \frac{fh}{l\delta q}\right)^2 \cdot \frac{\delta q}{2f}$$

$$= \frac{q\delta}{2f}\left[\frac{l^2}{4} + \frac{f^2h^2}{l^2q^2\delta^2} - \frac{lfh}{lq\delta}\right]$$

$$= \frac{q\delta l^2}{8f} + \frac{fh^2}{2q\delta l^2} - \frac{h}{2} \qquad (7.22)$$

The length $L_A = a + \dfrac{a^3}{6c^2}$ and $L_B = b + \dfrac{b^3}{6c^2}$

$\therefore$ Total length $L = L_A + L_B = (a + b) + \dfrac{a^3 + b^3}{6c^2}$

$$= (a + b) + \frac{(a+b)(a^2 - ab + b^2)}{6c^2}$$

$$= l\left[1 + \frac{a^2 - ab + b^2}{6c^2}\right]$$

$$= l + \frac{q^2\delta^2 l^3}{24f^2} + \frac{h^2}{2l}$$

Similarly other calculations can be made as for the case when the supports are at the same height.

Example 7.4: An overhead line at a river crossing is supported from two towers of heights 30 metres and 90 metres above water level with a span of 300 metres. The weight of the conductor is 1 kg/metre and the working tension is 2000 kg. Determine the clearance between the conductor and the water level mid-way between the towers.

Solution: The working tension is 2000 kg and w = 1 kg/metre.

$\therefore$ $c = \dfrac{T}{w} = \dfrac{2000}{1}$ = 2000 metres

Now h = 90 – 30 = 60 metres

$$a = \frac{l}{2} - \frac{ch}{l} = 150 - \frac{2000 \times 60}{300} = 150 - 400 = -250 \text{ m}$$

$\therefore$ b = 550 metres

$$d_1 = \frac{a^2}{2c} = \frac{250 \times 250}{2 \times 2000} = 15.625 \text{ metres.}$$

The sag at 400 metres

$$d_2 = \frac{400^2}{2c} = \frac{400 \times 400}{2 \times 2000} = 40 \text{ metres}$$

Therefore, height of the mid-point with respect to A (fig. 7.4)

= 40 – 15.625 = 24.375 metres

Therefore, the clearance between the conductor and the water level mid-way between the towers will be 30 + 24.375 = 54.375 metres. **Ans.**

7.4 STRINGING CHART

After calculating f_2 from equation (7.17), the value of d_2, the sag, can be calculated using equation (7.15). Various values of f_2 and d_2 are calculated using equations (7.17) and (7.15) repeatedly for different temperatures. The curves of tension and sag versus temperature are called the stringing charts and are useful while erecting the transmission line conductors for adjusting the sag and tension properly.

7.5 SAG TEMPLATE

Normally there are two types of supports being used.

(*i*) The standard or straight run or intermediate tower.

(*ii*) The angle or anchor or tension tower.

While the straight run towers are used for straight runs and normal conditions, the angle towers are used at angles, terminals and other points where a considerable amount of unbalanced pull may be thrown on the supports. The angle towers are, therefore, designed to withstand heavy loadings as compared to standard towers.

In order to locate the position of the towers, the first step is to know a suitable value for the support height and if there are no special guiding factors which dictate this choice, several alternatives may be tried.

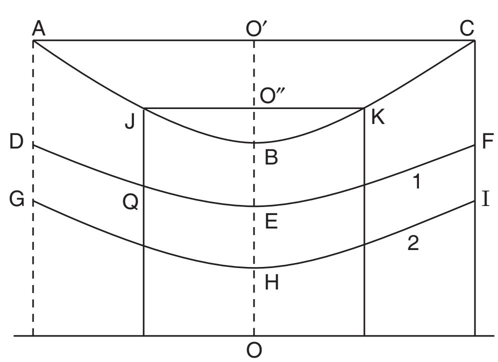


Fig. 7.5 Sag template: (1) ground clearance line, and (2) tower footing line.

The next step is to plot a sag template on a piece of transparent paper which consists of a set of curves as shown in Fig. 7.5. The horizontal and vertical distances represent the span lengths and sags respectively. GHI is the tower footing line (2), *i.e.*, this line gives the location of the footing of the tower. DEF is the ground clearance line (1) *i.e.*, the minimum clearance of the power conductor from the ground. This clearance to ground will depend upon the operating voltage and is given, according to Indian Electricity Rules, in the following table:

Minimum clearance to ground for overhead lines

Voltage between lines	*Minimum height (metres)*	*Remarks*
Less than 650 d.c.	5.8	Across public roads
or	5.2	Other positions
325 V a.c.	4.6	Inaccessible areas to vehicles
Less than 66 kV	6.0	
Between 66 kV and 110 kV	6.3	
Between 110 kV and 165 kV	6.6	
Exceeding 165 kV	6.9	

Curve *ABC* is such that with a span length of *AC,* the maximum sag of the conductor would be *O′B* and with span *JK*, the maximum sag is *O″B*. The curve *DEF* is obtained from *ABC* by subtracting ordinates from *ABC* equal to the minimum ground clearance required and curve *GHI* is obtained from *ABC* by setting off from *ABC* a distance representing the height of the standard tower from the point of attachment of the lowest conductor to the ground level.

It is then clear that if such a transparent template is placed upon a profile map of the route, as indicated such that the ground clearance line just touches the profile as at *Q* then points *G* and *I*, where the 'support foot' line cuts the profile, will indicate the position of the towers represented by *GA* and *IC*. The curve *ABC* will represent the actual shape and position of the lowest conductor, and since *JQ* represents the correct ground clearance, the conductor nowhere approaches the ground by more than the safe amount.

In the particular case shown the points of conductor support are upon the same horizontal level, but the same process applies when the route is a steeply sloping one, and the shape and position of the conductor will always be represented by the curve *ABC* as shown in Fig. 7.5.

7.6 EQUIVALENT SPAN

It is clear from above that the location of the towers depends upon the profile of the land along which transmission line is to be run, which means the span lengths between structures may not be equal. When successive spans are unequal, changes in load or temperature will bring unequal changes in tension in the different spans.

It is very tedious to make calculation of sag and tension for each and every span and then to make adjustment while erecting the transmission line, and, in any case the difference in tensions in the various spans will be automatically equalized by the deflection from the vertical of the insulator strings in case of suspension type of insulators.

It is, therefore, necessary to have calculations assuming uniform tension in the conductor between two tension towers. It can be shown mathematically that the variations in the tension with variation in load or temperature are within reasonable limits, the same as those which would occur in the same circumstances in a hypothetical span of one particular length, termed the equivalent span. This span length is calculated from the formula

$$\text{Equivalent span length } L = \sqrt{\frac{l_1^3 + l_2^3 + l_3^3 + \ldots}{l_1 + l_2 + l_3 + \ldots}}$$

where l_1, l_2, l_3, ... are the lengths of the individual spans between the two tension towers. For preparing a sag template, this value of the span is made use of.

It is to be noted that the method of sag template for locating towers should not be used for long spans as well as where the slope of the profile is very steep. In such cases it is desirable to make actual calculations for sag and tension.

7.7 STRINGING OF CONDUCTORS

After the transmission line towers of suitable heights are fixed to the ground and the insulator strings are attached to the cross-arms, the next step is the stringing of the conductors. The stringing of conductors is divided into three parts:

1. Paying out the conductors from the drums and hauling them over snatch-blocks hung from the cross-arm on a level with the suspension clamps at the ends of the insulator strings.

2. Pulling up the conductors, still hanging in the snatch-blocks to the correct tension and adjusting the sag.

3. Transferring the conductors from the snatch blocks to the insulator clamps.

While stringing conductors the tension in the conductor could be recorded with a dynamometer but it is always desirable to make final adjustments by sag measurements as the tension recorded by the meter may be substantially higher than the tension in the conductor at the far end of the section due to friction in the pulley-blocks employed on the cross-arms.

A 1″ × 2″ batten, painted white, is fixed horizontally to each of two adjacent towers at the desired level of the lowest point of one conductor. A climber on one tower keeps his line of sight between the battens, and the conductor is slowly let down until he signals that the sag is correct. Only the lowest conductor need be sagged this way, the others can be adjusted parallel.

7.8 VIBRATION AND VIBRATION DAMPERS

The overhead transmission lines experience vibrations in the vertical plane and are of two types. However, these are not to be confused with the much slower swing of the conductors in the horizontal plane due to simple wind deflection:

1. Aeoline vibrations,

2. Galloping or dancing of conductors (vibration).

The first one is of high frequency and low amplitude. The frequency is of the order of 5 to 40 c/s and the amplitude of the order of 2 cms to 5 cms with a loop length of 1 metre to 10 metres. It appears to be a condition of their formation that the wind velocity shall be quite low, 4 to 15 km/hr, that it shall be steady and free from gusts, uniform over a wide area and free from eddies.

The galloping or dancing of conductors are the low frequency high amplitude vibrations of the conductors. The frequency is of the order of $\frac{1}{4}$ to $1\frac{1}{2}$ cycle/sec and the amplitude is about 6 metres. Whereas the former are forced vibrations, the latter are of the self-excited type. These vibrations are caused due to the wind when it blows past a non-circular conductor. Once this starts it builds within itself and the vibrations may become very large and may result in flexure fatigue in the conductors or there may be flashover between conductors of different phases. This may lead to outages in the systems.

Aeoline Vibrations: While these vibrations are not of themselves harmful, however, they may give rise to troubles at points where free vibration is restricted. At a heavy anchor clamp, *e.g.*, a travelling wave will be reflected back with a momentary bending of the cable about the mouth of the clamp, and even the provision of a large radius at the mouth is not a complete safeguard, since the wave will result in impact between the conductor and radius which, continuously repeated, may damage the wires. A light anchor clamp very freely supported will reduce this hazard considerably and, in cases where vibration is not severe, will in probability provide adequate protection at dead-end points. If the clamp has very low inertia so that it can vibrate with the conductor without substantial time lag, the bending of the conductor as well as the impact between the conductor and clamp will be greatly diminished, thus resulting in less danger of conductor failure.

Besides the design of the clamp and accessories, the vibration problem has been solved by the use of special devices which fall into two groups (*a*) reinforcement or armour rods, and (*b*) dampers.

Within the first group will fall the Varney system of reinforcement, in which the conductor, where it lies in the suspension clamp, is surrounded by a spiral layer of small round rods, preferably tapered at each end, and appreciably larger in diameter than the individual wires of the conductor. These rods, in effect, give at the point of suspension a stranded cable of much larger diameter than the actual conductor, so that they provide a resistance to bending equal to that of a much larger conductor; whereas the energy which must be absorbed by this bending is only that produced by the wind acting on the small conductor itself. Armour rods also provide excellent protection against flashover and have been used for this purpose in various cases where no vibration was experienced or anticipated.

A true damper, however, is a device specially designed to absorb the vibrational energy, and thereby prevent the occurrence of any vibration at all. Many designs have been put forward, among which is the stock-bridge damper, an extremely simple, nevertheless effective device for suppressing high frequency vibrations. It consists of two hollow weights of special shape fixed at either end of a length of flexible steel cable which is itself fastened to the conductor at its midpoint by means of an aluminium clamp (Fig. 7.6).

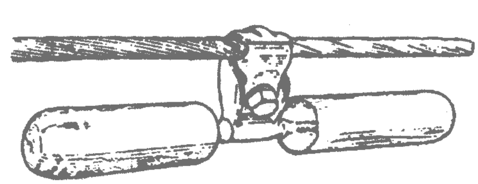


Fig. 7.6 A Stock bridge damper.

The weights are of galvanized iron or in the smaller sizes of zinc and the flexible cable is protected by enclosing it in a water-tight flexible aluminium tube. Among the special features of the design is the method by which the clamp is attached to the conductor. An efficient grip is obtained by the use of only one bolt, and the ease of application is such that these dampers may, if necessary, be attached to a line while it is alive, making use of special long handled tools.

Two dampers are required for each point of suspension of the conductor, one on either side, clamped to the conductor at appropriate distances, depending on the conductor size. Each span of conductor will thus contain two dampers and this is ample for normal conditions. For very long spans, however, it may be desirable to install additional dampers.

The damping action of the stock-bridge device is due to the dissipation of the vibrational energy of the conductor by hysteresis and inter-strand friction in the flexible damper cable. Vibration in the main cable causes relative motion between the central clamp and the weights of the damper, resulting in the bending of the flexible cable through a magnified arc and a consequent absorption of energy. The damping effect is automatic, the first tremor of vibration in the conductor being damped out before the amplitude is able to build up to a measurable magnitude. Some degree of vibration in the conductor must occur before the damper is brought into operation, but the amount is extremely small, and a conductor fitted with a damper remains quiescent except for a barely perceptive quiver.

Dancing of Conductors: To damp these oscillations it is required to make the conductor circular. For stranded conductors PVC tape is wrapped to make the conductor circular. This method is useful only when sleet formation is not there on the conductor. For this situation, the sleet or ice coating could be reduced by increasing the I^2R loss on the conductor which of course is not a practical solution to the problem.

PROBLEMS

7.1. Derive expressions for sag and tension in a power conductor strung between two supports at equal heights taking into account the wind and ice loadings also.

7.2. Derive the expressions for sag and tension when the supports are at unequal heights.

7.3. The transmission line is designed based on worst probable conditions and not worst possible conditions. Why ?

7.4. Derive an expression for the stress in the conductor during fair weather condition in terms of worst probable conditions using the usual notation for the various quantities.

7.5. What is a stringing chart ? Explain clearly the procedure adopted for stringing the power conductors on the supports.

7.6. What is a sag-template ? Explain how this is useful for location of towers and stringing of power conductors.

7.7. Describe the vibration of power conductors and explain the methods used to damp out these vibrations.

7.8. An overhead line has a conductor of cross-section 2.5 cm^2 hard drawn copper and a span length of 150 metres. Determine the sag which must be allowed if the tension is not to exceed one-fifth of the ultimate strength of 4175 kg/cm^2 (*a*) in still air, and (*b*) with a wind pressure of 1.3 kg/metre and an ice coating of 1.25 cms. Determine also the vertical sag in the latter case.

7.9. An overhead conductor consists of 7 strands of silicon-bronze having an ultimate strength of 10,000 kg/cm^2 and an area of 2.5 cm^2 when erected between supports 650 metres apart and having a 20 metre difference in level, determine the vertical sag which must be allowed so that the factor of safety shall be 5. Assume the wire weighs 2 kg/metre, ice loading 1 kg/metre and wind loading is 1.75 kg/metre.

7.10. An overhead line has the following data: Span length 185 metres. Difference in levels of supports 6.5 metres, conductor dia 1.82 cm, weight per unit length of conductor 2.5 kg/metre, wind pressure 49 kg/m^2 of projected area. Maximum tensile stress of the conductor 4250 kg/cm^2. Factor of safety 5. Calculate the allowable sag in metres at the lower support.

7.11. A transmission line conductor at a river crossing is supported from two towers at heights of 45 metres and 75 metres above water level. The span length is 300 metres. Weight of the conductor 0.85 kg/metre. Determine the clearance between the conductor and water at a point midway between towers if the tension in the conductor is 2050 kg.

7.12. An overhead line having a conductor of dia 10 mm and a span length of 150 metres has a sag of 3.5 metres at – 5°C with 10 mm thick ice coating and a wind pressure of 40 kg/m^2 of projected area. $E = 127 \times 10^4$ kg/cm^2, $\alpha = 16.6 \times 10^{-6}$/C, ice density 910 kg/m^3, copper density 8850 kg/m^3. Determine the temperature at which the sag will remain the same under fair weather conditions.

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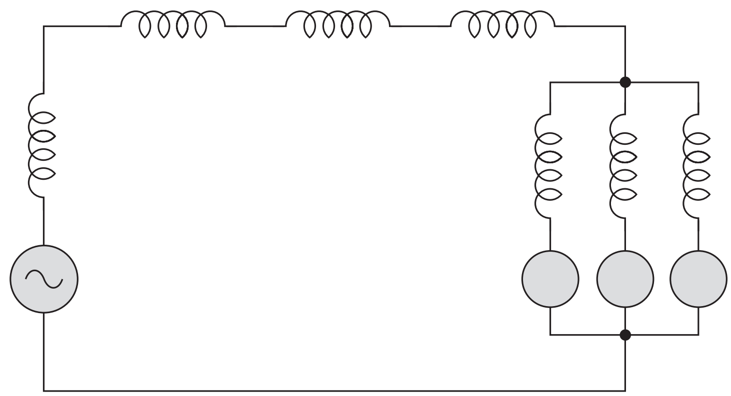
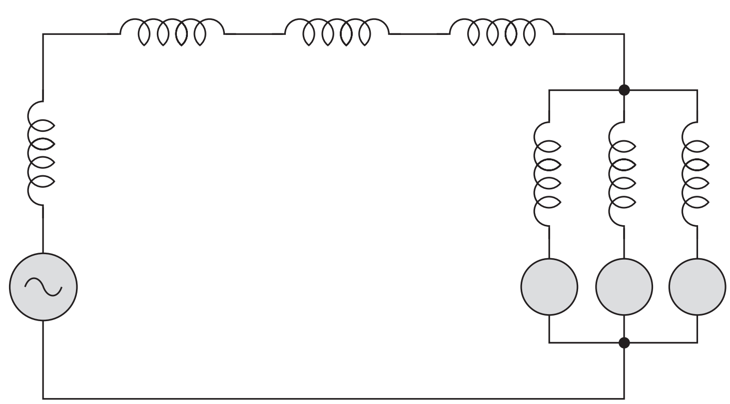


8

OVERHEAD LINE INSULATORS



8

Overhead Line Insulators

INTRODUCTION

The insulators for overhead lines provide insulation to the power conductor from the ground. The insulators are connected to the cross arm of the supporting structure and the power conductor passes through the clamp of the insulator. These insulators are mainly made of either glazed porcelain or toughened glass. The materials used for porcelain are silica 20% feldspar 30% and clay 50%. The porcelain should be ivory white, sound and free from defects. It should be vitrified because the presence of pores or air in the porcelain will lower down its dielectric strength. Any sealed air impurity will also lower the dielectric strength of porcelain. It is, therefore, desirable that porcelain to be used for insulators should be air-free and impervious to the entrance of liquids and gases. The dielectric strength of porcelain should be 15 kV to 17 kV for every one-tenth inch thickness. Normally it is difficult to manufacture homogeneous porcelain and, therefore, for a particular operating voltage two, three or more pieces construction is adopted in which each piece is glazed separately and then they are cemented together. Porcelain is mechanically strong, less affected by temperature and has minimum leakage problem.

Toughened glass is also sometimes used for insulators because it has higher dielectric strength (35 kV for one-tenth inch thickness) which makes it possible to make use of single piece construction, whatever be the operating voltage. Glass being transparent, it is very easy to detect any flaw like trapping of air etc. It has lower coefficient of thermal expansion and; as a result, the strains due to temperature changes are minimized. The major drawback of glass is that moisture condenses very easily on its surface and hence its use is limited to about 33 kV.

The design of the insulators is such that the stress due to contraction and expansion in any part of the insulator does not lead to any defect. It is desirable not to allow porcelain to come in direct contact with a hard metal screw thread. Normally cement is used between metal and the porcelain. It is seen that cement so used does not cause fracture by expansion or contraction.

8.1 TYPES OF INSULATORS

There are three types of insulators used for overhead lines:

(*i*) Pin type,

(*ii*) Suspension type, and

(*iii*) Strain type.

Pin type insulator (Fig. 8.1) consists of a single or multiple shells (petticoats or rain sheds) adapted to be mounted on a spindle to be fixed to the cross arm of the supporting structure. Multiple shells are provided in order to obtain sufficient length of leakage path so that the flash over voltage between the power conductor and the pin of the insulator is increased. The design of the shells is such that when the uppermost shell is wet due to rain the lower shells are dry and provide sufficient leakage resistance. It is desirable that the horizontal distance between the tip of the lowermost shell should be less as compared with the vertical distance between the same tip and the cross-arm, otherwise in case of an arc-over, the discharge will take place between the power conductor and cross-arm rather than power conductor and the pin of the insulator; thereby, the cross-arm will have to be replaced rather than the insulator. It is to be noted that the power conductor passes through the groove at the top of the insulator and is tied to the insulator by the annealed wire of the same material as the conductor. The pin type insulators are normally used upto 33 kV. In any case it is not desirable to use them beyond 50 kV as the cost of such insulators then increases much faster than the voltage. The cost beyond 50 kV is given by

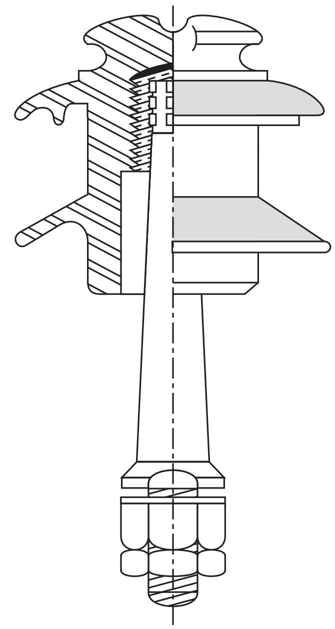


Fig. 8.1 Pin type insulator.

$$\text{Cost} \propto V^x$$

where $x > 2$.

The insulators and its pin should be sufficiently mechanically strong to withstand the resultant force due to combined effect of the weight of the conductor, wind pressure and ice loading if any per span length.

The pin type of insulators are uneconomical beyond 33 kV operating voltage. Also the replacement of these insulators is expensive. For these reasons for insulating overhead lines against higher voltages, suspension insulators (Fig. 8.2) are used. These insulators consist of one or more insulator units flexibly connected together and adapted to be hung for the cross-arm of the supporting structure and to carry a power conductor at its lowest extremity. Such composite units are known as string insulators. Each insulator is a single disc-shaped piece of porcelain grooved on the undersurface to increase the surface leakage path between the metal cap at the top and the metal pin at the bottom of the insulator. The cap at the top is recessed so that it can take the pin of another unit and in this way a string of any required number of insulators can be built. The cap and the pin are secured to the insulator by means of cement. The standard unit is 10″ × $5\frac{3}{4}$″ in size. The diameter is taken as 10″ as it gives optimum spark over to puncture voltage ratio. Increasing the diameter further increases the flash over or spark over voltage but it lowers the above ratio which is undesirable.

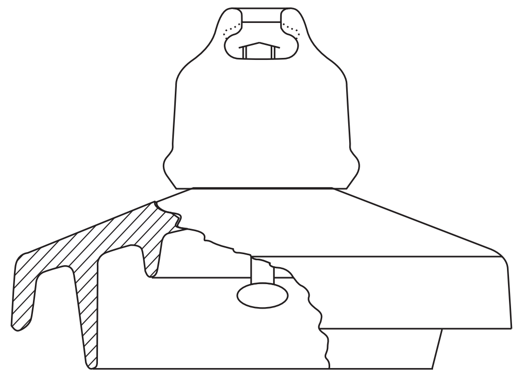


Fig. 8.2 Suspension type insulators.

Suspension insulators being free to swing, the clearances required between the power conductor and the supporting structure are more as compared to pin type insulators. This means the length of the cross arm for suspension insulators is more as compared with the pin type.

The suspension insulators, in addition to being economical as compared to pin type for voltages more than 33 kV, have the following further advantages:

1. Each insulator is designed for 11 kV and hence for any operating voltage a string of insulators can be used. For example, for 132 kV transmission, the number of insulators required is 12 (maximum).

2. In case of failure of one of the units in the string, only that particular unit needs replacement rather than the whole string.

3. Since the power conductor and string swing together in case of wind pressure, the mechanical stresses at the point of attachment are reduced as compared with the pin type of insulator where because of the rigid nature of the attachment fatigue and ultimate brittleness of the wire result.

4. The operating voltage of the existing transmission can be increased by adding suitable number of discs in the string instead of replacing all the insulators as is necessary in case of pin type insulators.

The strain insulators are exactly identical in shape with the suspension insulators. These strings are placed in the horizontal plane rather than the vertical plane as is done in case of suspension insulators (discs are in vertical plane in case of string insulators). These are used to take the tension of the conductors at line terminals, at angle towers, at road crossings and at junction of overhead lines with cables. These insulators are, therefore, known as tension or strain insulators. For low voltages of the order of 11 kV, shackle insulators are used. But for higher voltages a string of insulators is used. Whenever the tension in the conductor is very high as at long river crossings etc., sometimes two, even three, strings of insulators in parallel have been used.

8.2 POTENTIAL DISTRIBUTION OVER A STRING OF SUSPENSION INSULATORS

The thumb rule for finding the number of insulator discs for a particular operating voltage is to have one disc for every 11 kV. This does not mean that the voltage across the discs of the string is uniformly distributed. This is because of the capacitances formed between the metal parts of the insulators and the tower structure. These capacitances could be made negligibly small by increasing the distance between the insulators and the tower structure which requires larger lengths of cross arms. This will result into bigger size of the towers and hence it becomes uneconomical. Therefore, in practice the insulators are not very far from the tower structure and hence these capacitances affect the voltage distribution across the string. The capacitance of each unit is known as mutual capacitance. Fig. 8.3 represents an equivalent circuit for a string of 4 insulator discs.

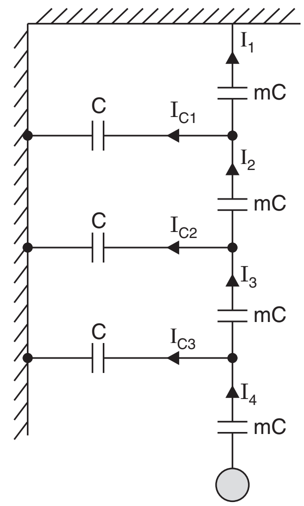


Fig. 8.3 Potential distribution over a string of four insulators.

Let $$m = \frac{\text{Mutual capacity}}{\text{Capacitance to ground}} = \frac{mC}{C}$$

Here capacity to ground is the capacitance of metal part of the insulator disc to the tower structure.

Since the insulator discs are identical, each disc is represented by its mutual capacity mC. Let V be the operating voltage and V_1, V_2, V_3 and V_4 the voltage drops across the units starting from the cross arm towards the power conductor.

$$V = V_1 + V_2 + V_3 + V_4 \tag{8.1}$$

The objective is to find out the voltage across each disc as a multiple of the operating voltage and to compare the voltage across each unit. From the diagram it is clear that

$$I_2 = I_1 + I_{c_1}$$

$$= V_1 m\omega C + V_1 \omega C \tag{8.2}$$

where ω is the supply angular frequency

or $$I_2 = V_1 C\omega(m + 1)$$

$$V_2 m\omega C = V_1 C\omega(m + 1)$$

or $$V_2 = \frac{V_1}{m}(m+1) = \frac{m+1}{m} V_1 \tag{8.3}$$

Also $$I_3 = I_2 + I_{c_2}$$

$$= V_2 m\omega C + (V_1 + V_2)\omega C$$

$$V_3 m\omega C = V_2 \omega C(m + 1) + V_1 \omega C$$

Substituting for V_2 in terms of V_1 gives

$$V_3 m\omega C = \frac{m+1}{m} V_1 \omega C(m + 1) + V_1 \omega C$$

$$= V_1 \omega C \left[\frac{(m+1)^2}{m} + 1 \right]$$

$$= V_1 \omega C \frac{(m^2 + 3m + 1)}{m}$$

or
$$V_3 = V_1 \left[\frac{m^2 + 3m + 1}{m^2} \right] \quad (8.4)$$

Similarly V_4 can also be expressed in terms of V_1 as follows:

$$I_4 = I_3 + I_{c_3}$$

$$V_4 m \omega C = V_1 \omega C \left[\frac{m^2 + 3m + 1}{m} \right] + \omega C \, [V_1 + V_2 + V_3]$$

$$= V_1 \omega C \left[\frac{m^2 + 3m + 1}{m} \right] + \omega C \left[V_1 + \frac{m+1}{m} V_1 + \frac{m^2 + 3m + 1}{m^2} V_1 \right]$$

$$= V_1 \omega v \left[\frac{m^2 + 3m + 1}{m} + \frac{3m^2 + 4m + 1}{m^2} \right]$$

or
$$V_4 = V_1 \left[\frac{m^2 + 3m + 1}{m^2} + \frac{3m^2 + 4m + 1}{m^3} \right] \quad (8.5)$$

So we have expressed V_2, V_3 and V_4 in terms of V_1 and the ratio of the capacities *i.e.*, m.

Now
$$V = V_1 + V_2 + V_3 + V_4$$

Therefore, since m is known, V_1 can be expressed in terms of V and from this V_2, V_3 and V_4 can be obtained.

Normally the value of $m > 1$. Let $m = 5$. With this if there is a string of four insulators as shown in Fig. 8.3,

$$V_2 = \frac{m+1}{m} V_1 - \frac{6}{5} V_1$$

$$V_3 = \frac{m^2 + 3m + 1}{m^2} V_1$$

$$= \frac{41}{25} V_1$$

Similarly
$$V_4 = \left[\frac{41}{25} + \frac{75 + 20 + 1}{125} \right] V_1$$

This shows that $V_1 < V_2 < V_3 < V_4$

This means the voltage drop across the unit nearest the cross arm is minimum and it goes on increasing as we go towards the power conductor. The voltage drop across the unit nearest the power conductor is maximum.

This conclusion can be drawn looking at the diagram without going into mathematics. Since the mutual capacity of each disc is same and the current through the topmost unit is minimum, the voltage drop across that unit will be minimum. As we go down the unit towards the power conductor the current goes on increasing being the maximum in the lowest unit; the voltage drop is maximum there.

It is clear that the lowermost unit in a string of insulators is fully stressed or utilized. As we go towards the cross arm the units are less stressed as compared to their capacity and hence they are not utilized fully. String efficiency is a measure of the utilization of material in the string and is defined as

$$\eta = \frac{\text{Voltage across the string}}{n \times \text{Voltage across the unit near the power conductor}}$$

or

$$\eta = \frac{\text{S.O.V. for the string}}{n \times \text{S.O.V. of one disc}}$$

where n is the number of insulators in the string and S.O.V. represents the spark over voltage.

The potential distribution across the insulator string can be obtained in an easier way as follows. Figure 8.4 is modified slightly so as to keep symmetry in writing mathematical expression.

At junction n,

$$I_{n+1} = I_n + I_{C_n}$$

$$v_{n+1}\omega mC = v_n \omega mC + V_n \omega C$$

$$v_{n+1} = \frac{V_n}{m} + v_n \qquad (8.6)$$

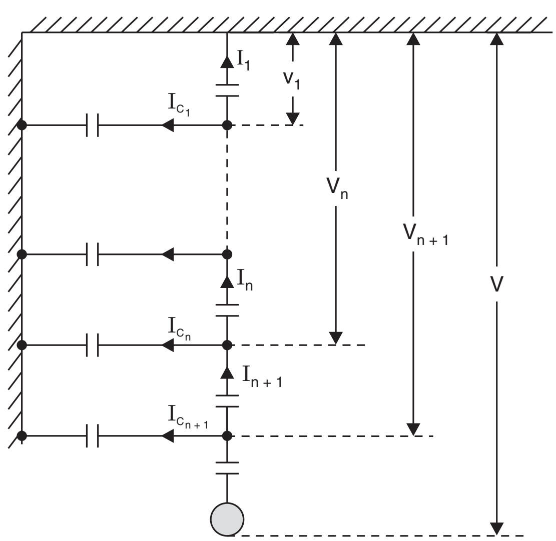


Fig. 8.4 Potential distribution over a string of (n + 1) units equal capacities.

Here V_n is the voltage across n units from the top and v_n is the drop across nth unit. With this formula voltage drop across any unit can be obtained in terms of v_n. Since it is known that the total voltage across the string is the sum of voltages across all the units (which have been obtained in terms of v_n), the value of v_n can be obtained and hence by back substitution the value of voltage across every unit can be calculated.

The procedure can be explained by the following example:

Let $m = 5$, no. of units = 5 and total operating voltage is 66 kV line to ground. Using the relation

$$v_{n+1} = \frac{V_n}{m} + v_n$$

$$v_2 = \frac{V_1}{m} + v_1$$

Since $V_1 = v_1$,

$$v_2 = v_1 \left(1 + \frac{1}{m}\right) = \left(1 + \frac{1}{5}\right) v_1 = 1.2v_1$$

$$= \frac{V_2}{m} + v_2 = \frac{v_1 + v_2}{5} + 1.2v_1$$

$$= \frac{v_1 + 1.2v_1}{5} + 1.2v_1$$

$$= 0.44v_1 + 1.2v_1$$

$$= \frac{8.2}{5} v_1 = 1.64v_1$$

$$v_4 = \frac{V_3}{m} + v_3 = \frac{V_2 + v_3}{m} + v_3$$

$$= \frac{v_1 + v_2}{m} + \frac{v_3}{m} + v_3$$

$$= \frac{2.2v_1 + 1.64v_1}{5} + 1.64v_1$$

$$= \frac{3.84}{5} v_1 + 1.64v_1$$

$$= (0.768 + 1.64)\, v_1$$

$$= 2.408v_1$$

$$v_5 = \frac{V_4}{m} + v_4$$

$$= \frac{V_3 + v_4}{m} + 2.408v_1$$

$$= \frac{3.84v_1 + 2.408v_1}{5} + 2.408v_1$$

$$= \frac{6.248}{5} v_1 + 2.408v_1$$

$$= (1.2496 + 2.408)v_1$$

$$= 3.6576v_1$$

$$\therefore \quad V = v_1 + v_2 + v_3 + v_4 + v_5$$
$$= v_1(1 + 1.2 + 1.64 + 2.408 + 3.6576)$$
$$= 9.9056 v_1$$
$$v_1 = \frac{66}{9.9056} = 6.67 \text{ kV}$$
$$v_2 = 1.2 \times 6.67 = 8 \text{ kV}$$
$$v_3 = 1.64 \times 6.67 = 10.92 \text{ kV}$$
$$v_4 = 2.408 \times 6.67 = 16.03 \text{ kV}$$
$$v_5 = 3.657 \times 6.67 = 24.3 \text{ kV}$$
$$V = v_1 + v_2 + v_3 + v_4 + v_5 = 66 \text{ kV}$$
$$\therefore \quad \% \text{ string efficiency} = \frac{66 \times 100}{5 \times 24.3} = 54.4\%$$

It can be seen that the voltage drops across various units will tend to be equal in case the value of m is large. In case of high voltage lines since the clearance between the conductor and the tower structure should be more to avoid flash over under normal operating condition, the value of m will go on increasing with operating voltage. This is because mutual capacity being fixed the ground capacitance goes on decreasing with larger clearances and hence the ratio of the two capacitances goes on increasing.

8.3 METHODS OF EQUALISING THE POTENTIAL

1. *Selection of m*: One of the methods for equalising the potential drop across the various units of the string is to have a larger value of m which as is said earlier needs longer cross arms and hence taller supporting structures and hence it is uneconomical to go beyond certain value of the length of cross arm. It has been found that the value of $m = 10$ is about the maximum which may be obtained.

2. *Grading of Units*: It can be seen from the Fig. 8.4 that unequal distribution of voltage is due to the leakage current from the insulator pin to the tower structure. This current can't be eliminated. The other possibility is that disc of different capacities could be used such that the product of their capacitive reactance and the current flowing through the respective unit is same. This requires that the unit nearest the cross arm should have the minimum capacitance (maximum capacitive reactance) and as we go towards the power conductor the capacitance should increase. By this grading it can be shown that complete equality of voltage can be obtained. Refer to Fig. 8.5. Here again the capacitance between the metal work and the power conductor is neglected.

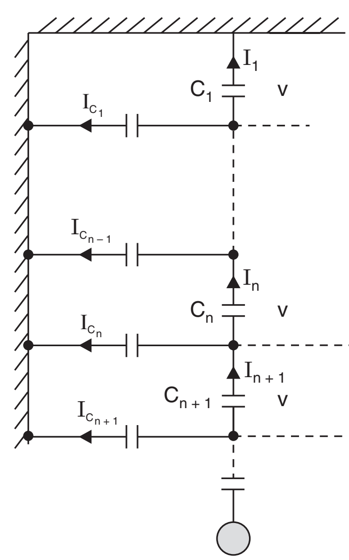


Fig. 8.5 Potential distribution over a string of (n+1) insulators—unequal capacities.

Here the ground capacitances C are of equal value whereas the mutual capacities are different. Since we assume the equal voltage drop across the various units,

$$I_{c_1} = \frac{I_{c_n}}{n}$$

$$nv\omega C = I_{c_n}$$

At junction n

$$I_{n+1} = I_n + I_{c_n}$$

$$v\omega C_{n+1} = v\omega C_n + nv\omega C$$

$$v\omega C_{n+1} = v\omega (C_n + nC)$$

or

$$C_{n+1} = C_n + nC \tag{8.7}$$

From this it is clear that if the capacitance of one unit is fixed the capacitance of other units can be found for equal distribution of voltage across the units of the string.

This means that in order to carry out unit grading, units of different capacities are required. This requires large stocks of different sized units, which is uneconomical and impractical.

Therefore, this method is normally not used except for very high voltage lines.

3. *Static Shielding*: In case of unit grading we used units of different capacities, so that the flow of different currents through the respective units produces equal voltage drop. In static shielding the idea is to cancel exactly the pin to tower charging currents so that the same current flows through the units of identical capacities to produce equal voltage drops across each unit. The arrangement is shown in Fig. 8.6. In this method a guard ring or grading ring is connected round to the power conductor such that this surrounds the bottom unit.

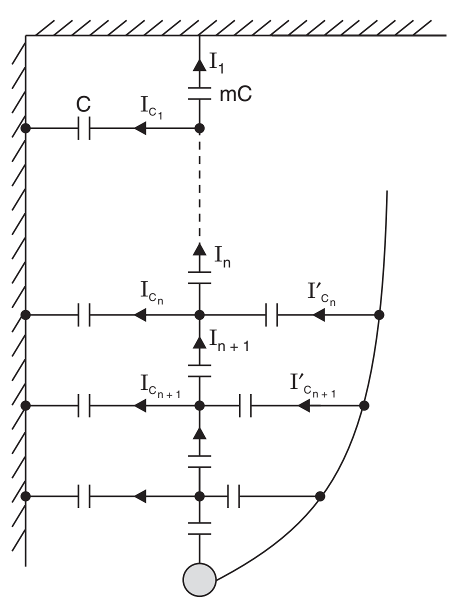


Fig. 8.6 Static shielding.

Since identical units are being used their mutual capacities are equal. Similarly the ground capacitances are equal. The design of the ring should be such that this gives rise to the capacitances which will cancel exactly the charging current in that particular section, such that

$$I_{n+1} = I_n$$

and
$$I_{c_n} = I'_{c_n}$$

$$nv\omega C = (V - nv)\omega C_n$$

where V is the operating voltage and C_n is the capacitance between the guard ring and the pin of the nth unit.

Let $V = kv$, where k is the number of units used. Then

$$\omega nvC = (k - n)v\omega C_n$$

or
$$nC = (k - n)C_n$$

or
$$C_n = \frac{n}{k - n} C \tag{8.8}$$

In order to obtain perfect equal distribution of voltage the capacitance of the guard ring with respect to the pins of the insulators can be given by the expression above. In practice it is very difficult to achieve this condition. Nevertheless this method is normally used and advantage is gained partially. It has been found that for a 14 unit string the voltage across the bottom unit without guard ring was 18.3% of the operating voltage and with guard ring it was 11.8% which is a great improvement.

Grading ring serves two purposes: (*i*) equalisation of voltage drop across the units, and (*ii*) when used with arcing horn (which is fixed at the top end of the string) it protects the insulator string from flashover whenever an over voltage (under normal or abnormal condition) appears between the tower structure and the power conductor. The combination of the two provides path through the air medium to discharge the energy contained in the abnormal voltage and thus the insulator string is saved.

Whenever a transmission line is seen through the areas where there is lot of smoke or a chemical industry of where the frequency of lightning strokes is large, special design insulators are normally used.

Example 8.1: Determine the maximum voltage that the string of the suspension insulators in Fig. E.8.1 can withstand if the maximum voltage per unit is 17.5 kV.

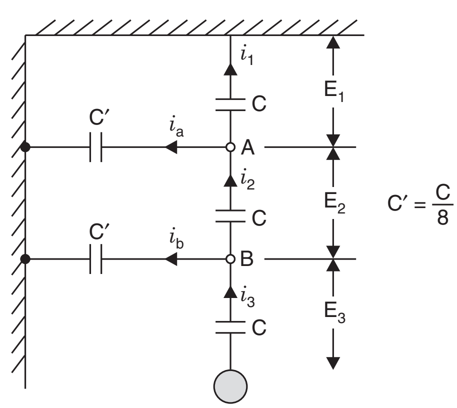


Fig. E.8.1

Solution: Let the voltages across the various units be E_1, E_2 and E_3 as shown such that $E = E_1 + E_2 + E_3$, where E is the desired withstand voltage of the string. Applying Kirchhoff's current law at A,

$$i_2 = i_1 + i_a = E_1\omega C + E_1\omega \frac{C}{8} = E_1\omega C\left(1+\frac{1}{8}\right) = \frac{9}{8} E_1\omega C$$

$$= E_2\omega C$$

or
$$E_2 = \frac{9}{8} E_1$$

Similarly at B,

$$i_3 = i_2 + i_b$$

$$E_3\omega C = E_2\omega C = \frac{E_1\omega C}{8} + \frac{E_2\omega C}{8}$$

$$= E_2\omega C\left[1+\frac{1}{8}\right] + \frac{E_1\omega C}{8}$$

$$= \frac{9}{8}\times\frac{9}{8}\,\omega C E_1 + \frac{E_1\omega C}{8}$$

$$= \left(\frac{81}{64}+\frac{1}{8}\right)\omega C E_1 = \frac{89}{64}\,\omega C E_1$$

$$E_3 = \frac{89}{64} E_1$$

It can be seen that the voltage across the line unit *i.e.*, unit near the power conductor is maximum.

$\therefore$
$$E_3 = \frac{89}{64} E_1 = 17.5 \text{ kV}$$

or
$$E_1 = 17.5 \times \frac{64}{89} = 12.58 \text{ kV}$$

$$E_2 = \frac{9}{8} E_1 = \frac{9}{8}\times 12.58 = 14.15 \text{ kV}$$

$\therefore$
$$E = E_1 + E_2 + E_3 = 12.58 + 14.15 + 17.5 = 44.23 \text{ kV}. \quad \textbf{Ans.}$$

Example 8.2: Determine the voltage across each disc of suspension insulators as a percentage of the line voltage to earth. The self and capacitance to ground of each disc is C and $0.2C$ respectively. The capacitance between the link pin and the guard ring is $0.1C$. (b) If the capacitance to the line of the lower link pin were increased to $0.3C$ by means of a guard ring, determine the redistribution of voltage. Also determine the string efficiency in each case.

Solution: (a) Let E_1, E_2 and E_3 be the voltage drops across the discs as shown in Fig. E. 8.2. Applying Kirchhoff's current law at node A, we have

$$I_2 + I_x = I_1 + I_a$$

$$E_2\omega C + (E_2 + E_3)\omega 0.1C - E_1\omega C - 0.2E_1\omega C = 0$$

$$E_2 + (E_2 + E_3)\, 0.1 - E_1 - 0.2E_1 = 0$$

$$1.1E_2 + 0.1E_3 - 1.2E_2 = 0$$

or $$1.2E_1 - 1.1E_2 - 0.1E_3 = 0 \quad (1)$$

Again writing equation at node B

$$I_3 + I_y - I_2 - I_b = 0$$

$$E_3C\omega + E_3 0.1\omega C - E_2\omega C - (E_1 + E_2)\ 0.2C = 0$$

$$E_3 + 0.1E_3 - 1.2E_2 - 0.2E_1 = 0 \quad (2)$$

$$0.2E_1 + 1.2E_2 - 1.1E_3 = 0$$

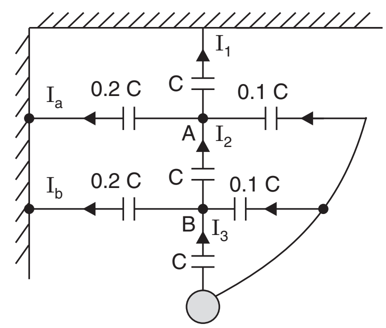


Fig. E.8.2

There are three unknowns with two equations. We divide both of them by E_3 and rewrite them as

$$12x - 11y = 1 \quad (1a)$$

$$2x + 12y = 11 \quad (2a)$$

where $x = \dfrac{E_1}{E_3}$ and $y = \dfrac{E_2}{E_3}$.

$$12x - 11y = 1$$

$$12x + 72y = 66$$

$$83y = 65$$

$$y = \frac{65}{83} = 0.783 = \frac{E_2}{E_3}$$

$\therefore$ $$2x = 11 - 12 \times \frac{65}{83} = 11 - 9.39 \text{ or } x = 0.801 = \frac{E_1}{E_3}$$

$\therefore$ $$E_2 = 0.783E_3 \text{ and } E_1 = 0.801E_3$$

Now total voltage is

$$E = E_1 + E_2 + E_3 = 0.801E_3 + 0.783E_3 + E_3 = 2.584E_3$$

$\therefore$ $$E_3 = \frac{E}{2.584} \times 100 = 38.70\%$$

$$E_2 = 0.783 \times 38.70 = 30.3\%$$

$$E_1 = 0.801 \times 38.7 = 31\%$$

The % string efficiency $= \dfrac{1}{3 \times 0.387} \times 100 = 86.1\%$

(b) If the capacitance to the line of the lower pin is increased to $0.3C$ the equation at node A is unchanged whereas that at B now becomes

$$0.2E_1 + 1.2E_2 - 1.3E_3 = 0 \quad (3)$$

or $$2x + 12y = 13 \quad (3a)$$

or $$12x - 11y = 1 \quad (1a)$$

$$12x + 72y = 78$$

$$12x - 11y = 1$$

$$83y = 77$$

$$y = 0.9277$$

$$2x = 13 - 12 \times 0.9277$$

or $$x = 0.9337$$

$\therefore$ $$\frac{E_1}{E_3} = 0.9337 \quad \text{and} \quad \frac{E_2}{E_3} = 0.9277$$

$$E_1 = 0.9337E_3 \text{ and } E_2 = 0.9277E_3$$

$$E = E_1 + E_2 + E_3 = 0.9337E_3 + 0.9277E_3 + E_3$$

$$E_3 = 0.3494E \text{ or } 34.94\%$$

$$E_2 = 0.9277 \times 34.94\% = 32.42\%$$

$$E_1 = 0.9337 \times 34.94 = 32.62\%$$

The % string efficiency $= \dfrac{1}{3 \times 0.3494} \times 100 = 95.4\%.$

Example 8.3: A string of eight suspension insulators is to be fitted with a grading ring. If the pin to earth capacitances are all equal to C, find the values of line to pin capacitances that would give a uniform voltage distribution over the string.

Solution: For voltage distribution to be uniform (see Fig. E.8.3)

$$I_a = I_A$$

$$I_b = I_B \text{ and so on.}$$

Also $$E_1 = E_2 = E_3 = \ldots = E_8$$

$$I_a = \omega C \frac{E}{8} \text{ and } I_A = C_1\omega \frac{7E}{8}$$

and $$\omega C \frac{E}{8} = \omega C_1 \times \frac{7E}{8}$$

or $$C_1 = \frac{C}{7}$$

Similarly $$I_b = I_B$$

$$I_b = \frac{2E}{8} \omega C$$

and $$I_B = \frac{6E}{8} \omega C_2$$

$$\omega C = 3C_2\omega \quad \text{or} \quad C_2 = \frac{C}{3}$$

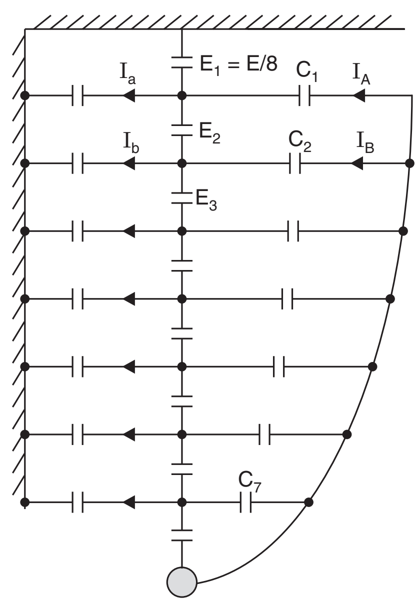


Fig. E.8.3

Similarly other results can be obtained. It will be seen that the other values are $\dfrac{3C}{5}, C, \dfrac{5C}{3}$, $3C$ and $7C$ respectively.

PROBLEMS

8.1. Each conductor of a 33 kV, 3-phase system is suspended by a string of three similar insulators, the capacitance of each disc is nine times the capacitance to ground. Calculate the voltage across each insulator. Determine the string efficiency also.

8.2. A string of eight suspension insulators is to be graded to obtain uniform distribution of voltage across the string. If the capacitance of the top unit is 10 times the capacitance to ground of each unit, determine the capacitance of the remaining seven units.

8.3. A string of six insulator units has mutual capacitance 10 times the capacitance to ground. Determine the voltage across each unit as a fraction of the operating voltage. Also determine the string efficiency.

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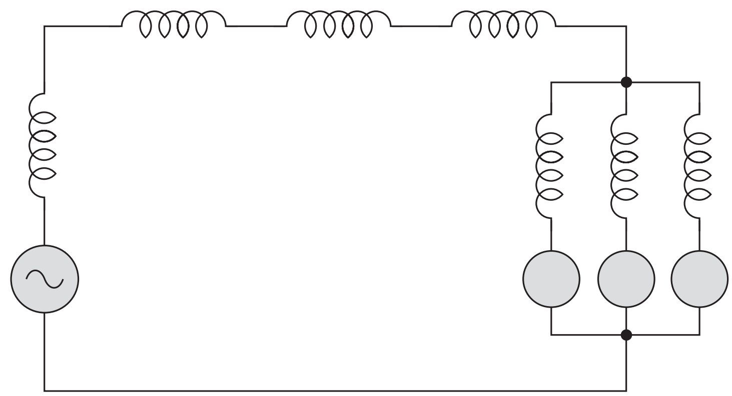
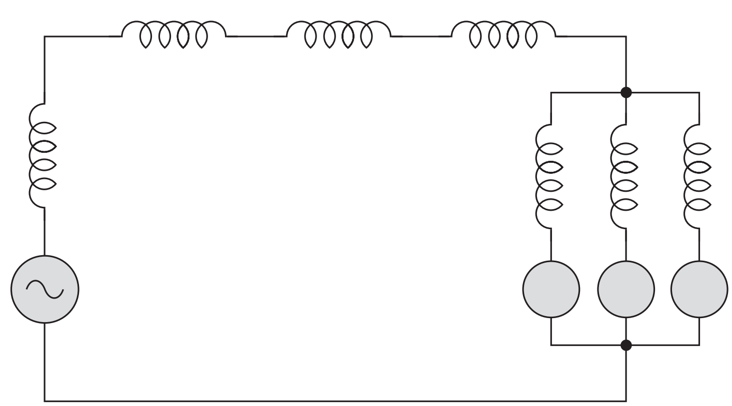


9

INSULATED CABLES



9

Insulated Cables

INTRODUCTION

All electric cables consist of three essential points.

(*a*) The conductor for transmitting electrical power.

(*b*) The insulation, an electrical insulating medium, needed to insulate the conductor from direct contact with earth or other objects, and

(*c*) External protection against mechanical damage, chemical or electro-chemical attack, fire or any other dangerous effects external to the cable.

Copper conductor has extensively been used for cables but of late aluminium is being used to a considerable extent. To obtain flexibility a number of wires are made up into a strand which makes it easier to handle, less liable to kink and break and to a large extent eliminates risk of the conductor breaking through the dielectric. The wires in a stranded conductor are twisted together to form lays. The successive layers usually are stranded in opposite direction *i.e.,* if the wires of one layer have a right-handed lay, the next layer has a left-handed lay. Standard stranding consists of 6 wires around 1, then 12 wires around 6, followed by 18, 24 and so on. A stranded conductor is expressed as 19/0.1 where the first number strands for the number of strands used and the second number corresponds to the diameter of each strand in mm. Sometimes the second number given corresponds to the gauge of the strand used *e.g.* 3/20 which means a cable with three strands each of 20 SWG.

9.1 THE INSULATION

The main requirements of the insulating materials used for cable are:

1. High insulation resistance.
2. High dielectric strength.
3. Good mechanical properties *i.e.,* tenacity and elasticity.

4. It should not be affected by chemicals around it.

5. It should be non-hygroscopic because the dielectric strength of any material goes very much down with moisture content.

Vulcanized Rubber

Rubber in its natural form is highly insulating but it absorbs moisture readily and gets oxidized into a resinous material; thereby it loses insulating properties. When it is mixed with sulphur alongwith other carefully chosen ingredients and is subjected to a particular temperature it changes into vulcanized rubber which does not absorb moisture and has better insulating properties than even the pure rubber. It is elastic and resilient.

The electrical properties expected of rubber insulation are high break-down strength and high insulation resistance. In fact the insulation strength of the vulcanized rubber is so good that for lower voltages the radial thickness is limited due to mechanical consideration.

The physical properties expected of rubber insulation are that the cable should withstand normal hazards of installation and it should give trouble-free service.

Vulcanized rubber insulated cables are used for wiring of houses, buildings and factories for low power work.

There are two main groups of synthetic rubber material : (*i*) general purpose synthetics which have rubber-like properties and (*ii*) special purpose synthetics which have better properties than the rubber *e.g.* fire resisting and oil resisting properties. The four main types are: (*i*) butyl rubber, (*ii*) silicon rubber, (*iii*) neoprene, and (*iv*) styrene rubber.

Butyl Rubber: The processing of butyl rubber is similar to that of natural rubber but it is more difficult and its properties are comparable to those of natural rubber. The continuous temperature to which butyl rubber can be subjected is 85°C whereas for natural rubber it is 60°C. The current rating of butyl insulated cables is approximately same as those of paper or PVC insulated cables. Butyl rubber compound can be so manufactured that it has low water absorption and offers interesting possibilities for a non-metallic sheathed cable suitable for direct burial in the ground.

Silicon Rubber: It is a mechanically weak material and needs external protection but it has high heat resistant properties. It can be operated at temperatures of the order of 150°C. The raw materials used for the silicon rubber are sand, marsh gas, salt, coke and magnesium.

Neoprene: Neoprene is a polymerized chlorobutadiene. Chlorobutadiene is a colourless liquid which is polymerized into a solid varying from a pale yellow to a darkish brown colour. Neoprene does not have good insulating properties and is used up to 660 V a.c. but it has very good fire resisting properties and therefore it is more useful as a sheathing material.

Styrene Rubber: Styrene is used both for insulating and sheathing of cables. It has properties almost equal to the natural rubber.

Polyvinyl Chloride (PVC)

It is a polymer derived generally from acetylene and it can be produced in different grades depending upon the polymerization process. For use in cable industry the polymer must be compounded with a plasticizer which makes it plastic over a wide range of temperature. The grade of PVC depends upon the plasticizer. PVC is inferior to vulcanized in respect of elasticity and insulation resistance. PVC material has many grades.

General Purpose Type: It is used both for sheathing and as an insulating material. In this compound monomeric plasticizers are used. It is to be noted that a V.R. insulated PVC sheathed cable is not good for use.

Hard Grade PVC: These are manufactured with less amount of plasticizer as compared with general purpose type. Hard grade PVC are used for higher temperatures for short duration of time like in soldering and are better than the general purpose type. Hard grade cannot be used for low continuous temperatures.

Heat Resisting PVC: Because of the use of monomeric plasticizer which volatilizes at temperature 80°C–100°C, general purpose type compounds become stiff. By using polymeric plasticizers it is possible to operate the cables continuously around 100°C.

PVC compounds are normally costlier than the rubber compounds and the polymeric plasticized compounds are more expensive than the monomeric plasticized ones. PVC is inert to oxygen, oils, alkalis and acids and, therefore, if the environmental conditions are such that these things are present in the atmosphere, PVC is more useful than rubber.

Polythene

This material can be used for high frequency cables. This has been used to a limited extent for power cables also. The thermal dissipation properties are better than those of impregnated paper and the impulse strength compares favourably with an impregnated paper-insulated cable. The maximum operating temperature of this cable under short circuits is 100°C.

Cross-linked Polythene: The use of polythene for cables has been limited by its low melting point. By cross-linking the molecules, in roughly the same way as vulcanising rubber, a new material is produced which does not melt but carbonizes at 250° to 300°C. By using chemical process it has been made technically possible to cross-link polythene in conventional equipment for the manufacture of rubber. This is why the product is said to be "vulcanised" or "cross-linked" polythene.

The polythene is inert to chemical reactions as it does not have double bonds and polar groups. Therefore, it was thought that polythene could be cross-linked only through special condition, *e.g.*, by irradiating polythene with electrons, thereby it could be given properties of cross-linking such as change of tensile strength and better temperature stability. Many irradiation processes have been developed in the cable making industry even though large amounts of high energy radiations are required and the procedure is expensive:

Polythene can also be irradiated with ultraviolet light, after adding to it a smal quantity of ultraviolet sensitive material such as benzophenone. Under the influence of ultraviolet light on benzophenone a radical is formed of the same type as in the decomposition of peroxide by the radical mechanism. Organic peroxides have also been used successfully to crosslink the polythene.

Impregnated Paper

A suitable layer of the paper is lapped on the conductor depending upon the operating voltage. It is then dried by the combined application of heat and vacuum. This is carried out in a hermetically sealed steam heated chamber. The temperature is 120°–130°C before vacuum is created. After the cable is dried an insulating compound having the same temperature as that of the chamber is forced into the chamber. All the pores of the paper cable are completely filled with this compound. After impregnation the cable is allowed to cool under the compound so that the void formation due to compound shrinkage is minimized. After this metal sheath is applied.

In case of pre-impregnated type the papers are dried and impregnated before they are applied on the conductor.

The compound used in case of impregnated paper is a semifluid and when the cables are laid on gradients the fluid tends to move from higher to lower gradient. This reduces the compound content at higher gradients and may result in void formation at higher gradients. This is very serious for cables operating at voltages higher than 3.3 kV. In many cases the failures of the cables have been due to the void formation at the higher levels or due to the bursting of the sheath at the lower levels because of the excessive internal pressure of the head of compound.

Protective Coverings

A cotton braid is applied over the insulated conductor and is then impregnated with a compound, which is water and weather proof.

The rubber insulated cables are covered with a lead alloy sheath and is used for fixed installation inside or outside buildings in place of braided and compound finished cable in conduit.

Cables are protected against mechanical damage by armouring the cables with steel tapes or galvanized steel wires. A bedding of compounded fibrous material under the armour is used to provide a cushion between the sheath and the tapes or wires. Since aluminium is much stronger than lead and can withstand the rigors normally associated with cable installation, cables with aluminium sheaths are not armoured. Another exceptions for armouring are the single core cables for a.c. voltages because of the power loss in the armouring. If at all armouring is necessary, non-magnetic materials should be used. This reduces the losses but they still remain quite large. Steel tape is the cheapest material for armouring a cable and is useful for damage against direct blows or abrasion. This is used normally for cables with conductor diameter more than half an inch. Galvanized steel wires are used for longer length of the cables and is therefore more suitable for installation where longitudinal stresses are involved. Lead sheaths are used where cables are subjected to vibrations.

Both lead and aluminium sheaths are prone to corrosive attack which may be caused by chemical, bacteriological and/or electrolytic action. In case of slight corrosion hazards serving consisting of one PVC tape, one self-vulcanized rubber tape, one PVC tape and one bituminized hessian tape may be used. For severe corrosive conditions the serving used is the same as used for less corrosive actions except that two self-vulcanizing rubber tapes and two bituminized hessian tapes are required.

9.2 EXTRA HIGH VOLTAGE CABLES

The dielectric material surrounds the conductor and we know that every dielectric material has certain dielectric strength which, if exceeded, will result in rupture of the dielectric. In general the disruptive failure can be prevented by designing the cable such that the maximum electric stress (which occurs at the surface of the conductor) is below that required for short time puncture of the dielectric. In case the potential gradient is taken a low value, the overall size of the cable above 11 kV becomes relatively large. Also, if the gradient is taken large to reduce the overall size of the cable the dielectric losses increase very much which may result in thermal breakdown of the cable. So a compromise between the two has to be made and normally the value of working stress is taken about one-fifth of the breakdown value for design purposes.

Electrostatic Stresses in Single Core Cable

Let r be the radius of the conductor, R the inner radius of the sheath, ε the permittivity of the dielectric, λ the charge per unit length, V the potential of the conductor with respect to the sheath and g the gradient at a distance x from the centre of the conductor within the dielectric material. $g = \dfrac{\lambda}{2\pi\varepsilon x} = E$, where E is the electric field intensity.

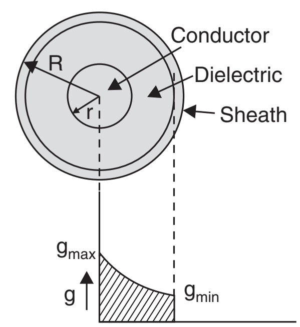


Fig. 9.1 Electric stress in a single core cable.

Now
$$V = -\int_R^r E\,dx = \int_r^R \frac{\lambda}{2\pi\varepsilon x}\,dx$$

$$= \frac{\lambda}{2\pi\varepsilon} \ln \frac{R}{r} \qquad (9.1)$$

Since
$$g = \frac{\lambda}{2\pi\varepsilon x},$$

∴
$$g = \frac{V}{x \ln \dfrac{R}{r}} \qquad (9.2)$$

From the above equation (9.2) for gradient it is clear that the gradient is maximum when $x = r$ that is it is maximum at the surface of the conductor and its value is given by

$$g_{\max} = \frac{V}{r \ln \dfrac{R}{r}} \qquad (9.3)$$

and the gradient is minimum at the inner radius of the sheath where it is given by

$$g_{\min} = \frac{V}{R \ln \dfrac{R}{r}} \qquad (9.4)$$

In order to keep a fixed overall size of the cable (R) for a particular operating voltage V, there is a particular value of the radius of the conductor which gives minimum gradient at the surface of the conductor. The objective here is to find the minimum value of $g_{\max}$ *i.e.,* to maximise.

$f(r) = r \ln \dfrac{R}{r}$ since V is fixed.

$$\frac{df(r)}{dr} = -r \cdot \frac{r}{R} \cdot \frac{R}{r^2} + \ln \frac{R}{r} = 0$$

or
$$\ln \frac{R}{r} = 1$$

or $$\frac{R}{r} = e \tag{9.5}$$

From this it is clear that to have minimum value of gradient at the surface of the conductor the inner radius of the sheath and the conductor radius are related by the equation (9.5). A plot of the gradient at the surface of the conductor and the ratio r/R is given in Fig. 9.2.

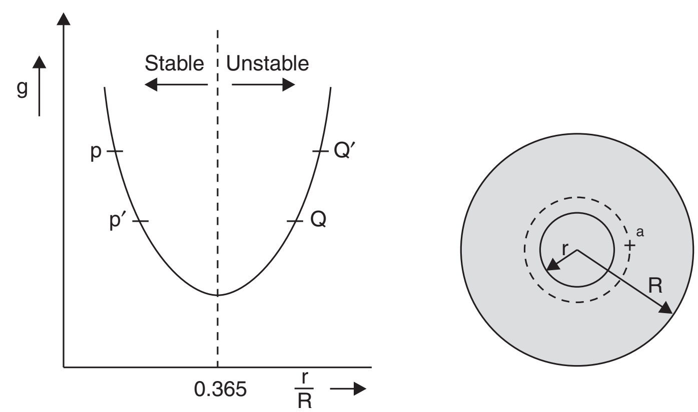


Fig. 9.2 Variation of g as a function of r/R.

Here study is made of the stable operation of the cable for particular ratios r/R *i.e.,* what ratio of r/R leads to stable operation of the cable and what ratios will lead to unstable operation. Say the ratio r/R corresponds to the point Q on the curve in Fig. 9.2. Now due to some manufacturing defects say a thin film of air surrounding the conductor is trapped. Let the thickness of this film be a units. Since the working dielectric strength of the insulating material is taken about 40-50 kV/cm to which now air surrounding the conductor is stressed, which will get ionized, therefore, the effective radius of the conductor will now be $(r + a)$ units and the ratio will be $(r + a)/R$. Corresponding to this ratio the operating point now shifts to Q' *i.e.,* the stress to which the dielectric material is subjected is increased and this may finally lead to rupture of the material. This situation will arise for all operating points to the right of the minimum point on the curve in Fig. 9.2.

Let us now take a cable with ratio r/R such that it corresponds to point P on the curve *i.e.,* left to the minimum point. Say, again due to similar reasons if the radius becomes $(r + a)$ and the ratio $(r + a)/R$ the operating point shifts to the point P' where the dielectric material is subjected to a relatively smaller electric stress than at point P. Therefore it can be seen that for all ratios r/R less than the minimum $1/e$ the cable operates satisfactorily. This means for satisfactory operation of the cable

$$\frac{r}{R} < \frac{1}{e}$$

or $$\frac{R}{r} > e$$

Now if this principle is used for the design of cables then we see that there will be large difference between the stress at the surface of the conductor and the stress at the inner radius of the sheath, which means the dielectric material will not be fully utilised.

Example 9.1: Determine the economic overall diameter of a 1-core cable metal sheathed for a working voltage of 85 kV if the dielectric strength of the insulating material is 65 kV/cm.

Solution: For economic size the ratio of the outer dia to the conductor dia should be *e*.

$$\therefore \qquad V = r\, g_{\max} \ln \frac{r_1}{r} = r\, g_{\max} \ln e = r\, g_{\max}$$

where r is the radius of the conductor in cm.

$$\therefore \qquad 85 = 65r$$

or

$$r = \frac{85}{65} = 1.3 \text{ cm}$$

$\therefore$ dia of the conductor = 2 × 1.3 = 2.6 cm

and dia of the sheath = 2.6e = 7.07 cm. **Ans.**

9.3 GRADING OF CABLES

By grading of a cable is meant the distribution of dielectric material such that the difference between the maximum gradient and the minimum is reduced, thereby a cable of the same size could be operated at higher voltages or for the same operating voltage a cable of relatively smaller size could be used.

There are two methods of grading:

1. Capacitance grading where more than one dielectric material is used.

2. Intersheath grading where the same dielectric material is used but potentials at certain radii are held to certain values by interposing thin metal sheaths.

Capacitance Grading

Let λ be the charge per unit length. If we have one single dielectric material the gradient at any radius x will be

$$g = \frac{\lambda}{2\pi\varepsilon x}$$

where ε is the permittivity of the material. If we could use an infinite number of materials with varying permittivities given by

$$\varepsilon = \frac{k}{x}$$

the gradient at any radius x now becomes (Fig. 9.4)

$$g = \frac{\lambda}{2\pi \,.\, \frac{k}{x} \,.\, x} = \frac{\lambda}{2\pi k} = \text{constant}$$

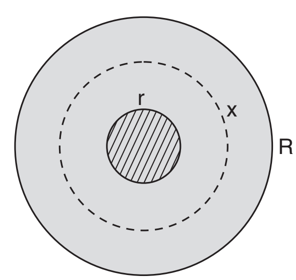


Fig. 9.3 Capacitance grading.

i.e., for a particular operating voltage the overall size of the cable is minimum. This looks quite all right but practically it is impossible to have infinite number of dielectric materials with varying permittivities as given above. Normally two or three materials are used. Let there be three materials with permittivities ε_1, ε_2 and ε_3 placed at radii r, r_1 and r_2 respectively (Fig. 9.5).

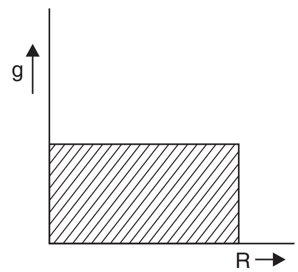


Fig. 9.4 Gradient with infinite number of materials.

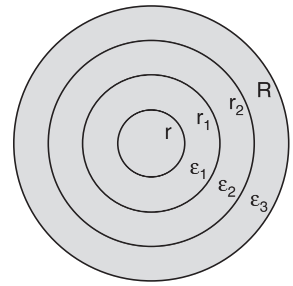


Fig. 9.5 Single core cable with three materials.

Let the dielectric strength and working stresses of this material be G_1, G_2, G_3 and g_1, g_2 and g_3 respectively. The objective now is to find out the locations of these materials with respect to the conductor of the cable. We can't keep any material anywhere we like. There must be some criterion, otherwise the results of grading may be offset. There are two possibilities:

(*i*) The factor of safety for all the materials be same, thereby the working stress of the various materials different.

(*ii*) The same working stress for different materials.

(*i*) The gradient at the surface of the conductor will be

$$\frac{\lambda}{2\pi\varepsilon_1 r} = \frac{G_1}{f}$$

where f is the factor of safety.

The gradient at radius $r_1 = \dfrac{\lambda}{2\pi\varepsilon_2 r_1} = \dfrac{G_2}{f}$

The gradient at radius $r_2 = \dfrac{\lambda}{2\pi\varepsilon_3 r_2} = \dfrac{G_3}{f}$

From these three relations,

$$\lambda = 2\pi\varepsilon_1 r\,\frac{G_1}{f} = 2\pi\varepsilon_2 r_1\,\frac{G_2}{f} = 2\pi\varepsilon_3 r_2\,\frac{G_3}{f}$$

or $$\varepsilon_1 r G_1 = \varepsilon_2 r_1 G_2 = \varepsilon_3 r_2 G_3$$

Since $r < r_1 < r_2$, $$\varepsilon_1 G_1 > \varepsilon_2 G_2 > \varepsilon_3 G_3 \tag{9.6}$$

This means the material with highest product of dielectric strength and permittivity should be placed nearest to the conductor and the other layers should be in the descending order of the product of dielectric strength and permittivity. So this is one arrangement of the dielectric materials.

(*ii*) The second alternative as is said earlier is when all the materials are subjected to the same maximum stress.

With this arrangement,

$$g_{\max} = \frac{\lambda}{2\pi\varepsilon_1 r} = \frac{\lambda}{2\pi\varepsilon_2 r_1} = \frac{\lambda}{2\pi\varepsilon_3 r_2}$$

or $$\varepsilon_1 r = \varepsilon_2 r_1 = \varepsilon_3 r_2$$

Again since $r < r_1 < r_2$,

$$\varepsilon_1 > \varepsilon_2 > \varepsilon_3 \qquad (9.7)$$

The dielectric material with highest permittivity should be placed nearest the conductor and other layers will be in the descending order of their permittivities.

The distribution of voltage using capacitance grading (same stress) is shown in Fig. 9.6.

Total operating voltage (hatched area) of the cable if g_{max} is the working stress,

$$V = g_{max}\, r \ln \frac{r_1}{r} + g_{max}\, r_1 \ln \frac{r_2}{r_1} + g_{max}\, r_2 \ln \frac{R}{r_2}$$

$$= g_{max}\left[r \ln \frac{r_1}{r} + r_1 \ln \frac{r_2}{r_1} + r_2 \ln \frac{R}{r_2}\right] \text{volts} \qquad (9.8)$$

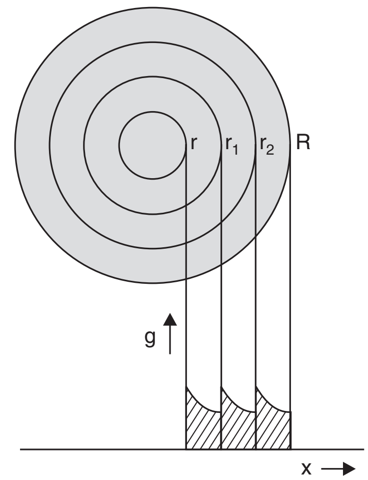


Fig. 9.6 Capacitance grading-voltage distribution.

Intersheath Grading

An auxiliary transformer is used to maintain the metal sheath and the power conductor at certain potentials; thereby the stress distribution is forced to be different from the one which it would be without the intersheaths. The objective now here is to show that the gradient with intersheath will be smaller than the gradient without intersheath for the same overall radius and the operating voltage. Since a homogeneous material is being used the maximum value of the stress at various intersheaths is same.

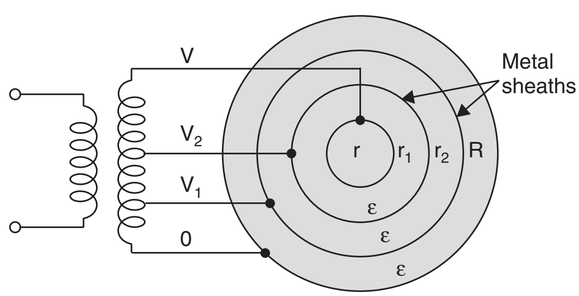


Fig. 9.7 Intersheath grading.

Let the thickness of the materials be such that

$$\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha$$

With this arrangement, the gradient at the surface of the conductor

$$g_{max} = \frac{V - V_2}{r \ln \dfrac{r_1}{r}} \qquad (9.9)$$

Similarly gradients at radii r_1 and r_2 respectively are

$$\frac{V_2 - V_1}{r_1 \ln \frac{r_2}{r_1}} \text{ and } \frac{V_1}{r_2 \ln \frac{R}{r_2}}$$

Since g_{max} are same at the various radii,

$$\frac{V - V_2}{r \ln \frac{r_1}{r}} = \frac{V_2 - V_1}{r_1 \ln \frac{r_2}{r_1}} = -\frac{V_1}{r_2 \ln \frac{R}{r_2}}$$

or
$$\frac{V - V_2}{r \ln \alpha} = \frac{V_2 - V_1}{r_1 \ln \alpha} = \frac{V_1}{r_2 \ln \alpha}$$

or
$$\frac{V - V_2}{r} = \frac{V_2 - V_1}{r_1} = \frac{V_1}{r_2} \tag{9.10}$$

We want to compare the gradients under the two conditions; therefore, we must express them in terms of V, r and α. To find the gradient with intersheath we express V_1 in terms of V_2 and then V_2 in terms of V.

To determine V_1 in terms of V_2, from equation (9.10) we have

$$\frac{V_2 - V_1}{r_1} = \frac{V_1}{r_2}$$

$$\frac{V_2}{r_1} = V_1 \left[\frac{1}{r_1} + \frac{1}{r_2}\right] = V_1 \left[\frac{r_1 + r_2}{r_1 r_2}\right]$$

or
$$V_2 = V_1 \left[1 + \frac{r_1}{r_2}\right] = V_1 \left[1 + \frac{1}{\alpha}\right]$$

∴
$$V_1 = V_2 \left[\frac{\alpha}{1 + \alpha}\right] \tag{9.11}$$

To express V_2 in terms of V from equation (9.10) we have

$$\frac{V - V_2}{r} = \frac{V_2 - V_1}{r_1}$$

or
$$V - V_2 = \frac{V_2 - V_1}{\alpha}$$

or
$$V - V_2 = \frac{V_2}{\alpha} - \frac{1}{\alpha}\left[V_2 \cdot \frac{\alpha}{1 + \alpha}\right] = \frac{V_2}{\alpha} - \frac{V_2}{1 + \alpha} = \frac{V_2}{\alpha + \alpha^2}$$

or
$$V = V_2 + \frac{V_2}{\alpha + \alpha^2} = V_2 \left[\frac{1 + \alpha + \alpha^2}{\alpha(1 + \alpha)}\right]$$

or
$$V_2 = V \cdot \frac{\alpha(1 + \alpha)}{1 + \alpha + \alpha^2} \tag{9.12}$$

Now substituting for V_2 in equation (9.9) for gradient, we have

$$g_{\max} = \frac{V - V_2}{r \ln \alpha} = \frac{V - V\dfrac{(\alpha + \alpha^2)}{(1 + \alpha + \alpha^2)}}{r \ln \alpha}$$

$$= \frac{V}{r \ln \alpha} \cdot \frac{1}{1 + \alpha + \alpha^2} \tag{9.13}$$

Now the gradient at the surface of the conductor without intersheath

$$g = \frac{V}{r \ln R/r} = \frac{V}{3r \ln \alpha} \tag{9.14}$$

Therefore,
$$\frac{g_{\max}}{g} = \frac{3}{1 + \alpha + \alpha^2} \tag{9.15}$$

From the geometry of the cable $\alpha > 1$, therefore, the gradient with intersheath is lower than without intersheath for the same overall size and operating voltage of the cable. This is what we intended to prove. This means that a cable of a particular size can be operated for higher voltages or for a particular voltage the size of the cable can be reduced. The voltage of the cable with this intersheath arrangement is given by

$$V = g_{\max} \left[r \ln \frac{r_1}{r} + r_1 \ln \frac{r_2}{r_1} + r_2 \ln \frac{R}{r_2} \right]$$

$$= g_{\max} \ln \alpha \, [r + r_1 + r_2] \tag{9.16}$$

There can be other arrangements of intersheaths as well *e.g.,* the insulating material thickness between successive intersheaths is constant, *i.e.,*

$$r_1 = r + d,\ r_2 = r + 2d \text{ and } R = r + 3d$$

The grading theory is more of theoretical interest than practical for the following reasons. Capacitance grading is difficult of non-availability of materials with widely varying permittivities and secondly with time the permittivities of the materials may change as a result this may completely change the potential gradient distribution and may even lead to complete rupture of the cable dielectric material at normal working voltage.

In case of intersheath, there is possibility of damage of intersheath during laying operation and secondly since charging current flows through the intersheath which in case of a long cable may result in overheating.

For these reasons the modern practice is to avoid grading in favour of oil and gas filled cables.

Example 9.2: A single core lead covered cable is to be designed for 66 kV to earth. Its conductor radius is 0.5 cm and its three insulating materials *A, B* and *C* have relative permittivities of 4, 2.5 and 4.0 with maximum permissible stresses of 50, 30 and 40 kV/cm respectively. Determine the minimum internal diameter of the lead sheath. Discuss the arrangement of the insulating materials.

Solution: In order to have minimum internal diameter of the lead sheath and consistency in the electric stresses, it is desired that the material to be placed near the surface of the conductor should be one which has maximum of the product of electric stress and the permittivity *i.e.*, in this case the material A has the product $4 \times 50 = 200$ as the maximum and will be placed near the surface of the conductor and material C what has the product $4 \times 40 = 160$ will be placed next and then materials B.

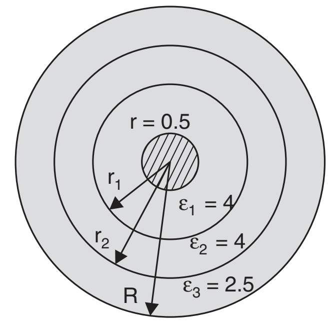


Fig. E.9.2

Here $r = 0.5$ cm

Let q be the charge per unit length of the cable. Then

$$g_{1\,\max} = \frac{q}{2\pi\varepsilon_0\varepsilon_1 r}$$

$$g_{2\,\max} = \frac{q}{2\pi\varepsilon_0\varepsilon_2 r_1}$$

$$g_{3\,\max} = \frac{q}{2\pi\varepsilon_0\varepsilon_3 r_2}$$

From these equations

$$q = 2\pi\varepsilon_0\varepsilon_1 r g_{1\,\max} = 2\pi\varepsilon_0\varepsilon_2 r_1 g_{2\,\max} = 2\pi\varepsilon_0\varepsilon_3 r_2 g_{3\,\max}$$

or $$\varepsilon_1 r g_{1\,\max} = \varepsilon_2 r_1 g_{2\,\max} = \varepsilon_3 r_2 g_{3\,\max}$$

or $$\frac{r_1}{r} = \frac{\varepsilon_1 g_{1\max}}{\varepsilon_2 g_{2\max}}$$

or $$r_1 = \frac{200}{160} \times 0.5 = \frac{2.5}{4} = 0.625 \text{ cms}$$

Similarly, $$\frac{r_2}{r_1} = \frac{\varepsilon_2 g_{2\max}}{\varepsilon_3 g_{3\max}} = \frac{4 \times 40}{2.5 \times 30}$$

or $$r_2 = 0.625 \times \frac{160}{75} = 1.33 \text{ cms}$$

Now $$V = 66 = r g_{1\,\max} \ln \frac{r_1}{r} + r_1 g_{2\,\max} \ln \frac{r_2}{r_1} + r_2 g_{3\,\max} \ln \frac{R}{r_2}$$

$$= 0.5 \times 50 \ln \frac{0.625}{0.5} + 0.625 \times 40 \ln \frac{1.33}{0.625} + 1.33 \times 30 \ln \frac{R}{1.33}$$

or $$66 = 25 \ln \frac{0.625}{0.5} + 25 \ln \frac{1.33}{0.625} + 39.9 \ln \frac{R}{1.33}$$

$$= 5.578 + 18.88 + 39.9 \ln \frac{R}{1.33}$$

$$39.9 \ln \frac{R}{1.33} = 41.54$$

$$\ln \frac{R}{1.33} = 1.0411635$$

or $$\frac{R}{1.33} = 2.83251$$

or $D = 7.53$ cms. **Ans.**

Example 9.3: A conductor of 1 cm dia passes centrally through a porcelain cylinder of internal dia 2 cms and external dia 7 cms. The cylinder is surrounded by a tightly fitting metal sheath. The permittivity of porcelain is 5 and the peak voltage gradient in air must not exceed 34 kV/cm. Determine the maximum safe working voltage.

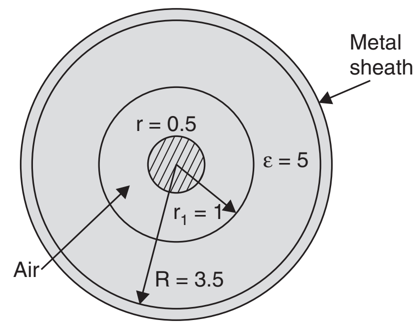


Fig. E.9.3

Solution: The configuration is given below:

Let q be the charge per unit length of the conductor.

$$g_{1\,\max} = \frac{q}{2\pi\varepsilon_0 r}$$

Also $$g_{2\,\max} = \frac{q}{2\pi\varepsilon_0\varepsilon_r r_1}$$

$\therefore$ $$rg_{1\,\max} = g_{2\max}\,\varepsilon_r r_1$$

or $$g_{2\,\max} = \frac{rg_{1\max}}{\varepsilon_r r_1} = \frac{0.5\times 34}{5\times 1} = 3.4 \text{ kV/cm}$$

$$V = rg_{1\max}\ln\frac{r_1}{r} + r_1 g_{2\,\max}\ln\frac{R}{r_1}$$

$$= 0.5\times 34\ln\frac{1}{0.5} + 1\times 3.4\ln\frac{3.5}{1}$$

$$= 11.7835 + 4.2594$$

$$= 16.04 \text{ kV peak}$$

$\therefore$ $V = 11.34$ kV r.m.s. **Ans.**

Example 9.4: A 66 kV concentric cable with two inter-sheaths has a core diameter 1.8 cm. Dielectric material 3.5 mm thick constitutes the three zones of insulation. Determine the maximum stress in each of the three layers if 20 kV is maintained across each of the inner two.

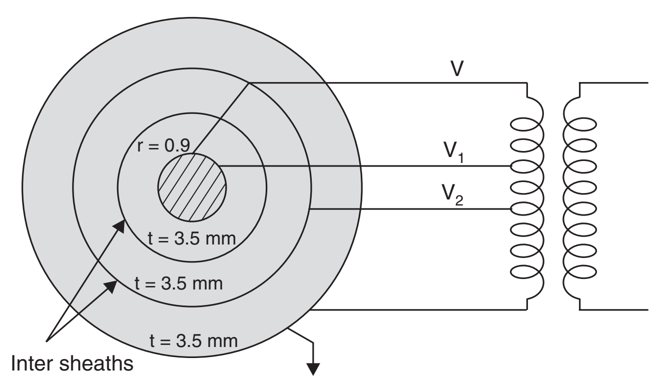


Fig. E.9.4

Solution: Refer to Fig. E.9.4.

The overall radius of the cable

$$0.9 + 3 \times 0.35 = 0.9 + 1.05 = 1.95 \text{ cms.}$$

Let $g_{1\,max}$ be the maximum stress on the surface of the conductor,

$g_{2\,max}$ the maximum stress on the first intersheath,

$g_{3\,max}$ the maximum stress on the second intersheath.

$$V_1 - V_2 = g_{2\,max}\, r_1 \ln \frac{r_2}{r_1} = 0.3085\, g_{2\,max} = 20$$

$$V - V_1 = g_{1\,max}\, r \ln \frac{r_1}{r} = g_{1\,max} \times 0.9 \ln \frac{1.25}{0.9} = 0.2956\, g_{1\,max}$$

$$V_2 = g_{3\,max}\, r_2 \ln \frac{r_3}{r_2} = 0.3165\, g_{3\,max}$$

Now $\quad V - V_1 = 20 \text{ kV} = 0.2956\, g_{1\,max}$

$\therefore \quad g_{1\,max} = \dfrac{20}{0.2956} = 67.6 \text{ kV/cm}$

Also, $\quad V_1 - V_2 = 20 \text{ kV} = 0.3085\, g_{2\,max}$

$\therefore \quad g_{2\,max} = \dfrac{20}{0.3085} = 64.83 \text{ kV/cm}$

and $\quad V_2 = 66 - 40 = 26 \text{ kV} = 0.3165\, g_{3\,max}$

$\therefore \quad g_{3\,max} = 82 \text{ kV/cm.}$ **Ans.**

9.4 INSULATION RESISTANCE OF A CABLE

The usual load current flows through the core of the cable whereas leakage current *i.e.*, the current which is not useful flows radially *i.e.*, from the conductor to the sheath through the dielectric material (Fig. 9.8). The flow of leakage current is shown by dotted lines. The resistance of any material is given by

$$R = \rho \frac{l}{A}$$

where ρ is the specific resistance of the material, l the length of the current path and A is the cross section normal to the flow of current. In case of a cable since the area of section increases as we go from the core to the sheath we first write an expression for the insulation resistance of an annular cylinder with radii x and $(x + dx)$ units as measured from the centre of the core.

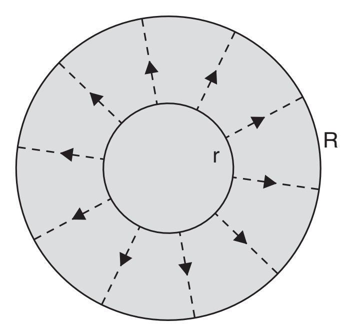


Fig. 9.8 Insulation resistance of a cable.

$$dR = \rho \frac{ax}{2\pi x\,.\,1}$$

Here unit in the denominator represents the unit length of the cable *i.e.*, dR represents the differential leakage resistance for unit length of the cable.

$$R = \frac{\rho}{2\pi} \int_r^R \frac{dx}{x}$$

$$= \frac{\rho}{2\pi} \ln \frac{R}{r} \text{ ohms/metre length} \tag{9.17}$$

In case the length of the cable is l unit the leakage resistance

$$R = \frac{\rho}{2\pi l} \ln \frac{R}{r} \text{ ohms}$$

It is to be noted that whereas the resistance of the core of the cable is directly proportional to the length of the cable, the leakage resistance is inversely proportional to the length of the cable.

9.5 CAPACITANCE OF A SINGLE CABLE

A single core cable is in effect an electrostatic capacitor because it has two electrodes, the core of the cable and the sheath separated by a dielectric material (Fig. 9.9). Let λ be the charge per unit length. By definition capacitance is the ratio of the charge on one of the electrodes to the potential difference between the electrodes.

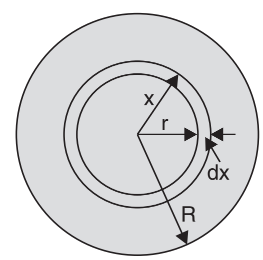


Fig. 9.9 Capacitance of a 1-core cable.

From equation (9.1),

$$\text{Voltage } V = \frac{\lambda}{2\pi\varepsilon} \ln \frac{R}{r}$$

$$\therefore \quad \frac{\lambda}{V} = C = \frac{2\pi\varepsilon}{\ln R/r} \text{ F/metre} \tag{9.18}$$

It is to be noted here that the capacitance of a cable is much more important than that of an overhead line because of the nearness of the conductors to one another and to the earthed sheath. Also the permittivity of the dielectric material is higher than that of air.

Capacitance of a 3-Core Cable

If we could assume that the dielectric is uniform between the core and the sheath, it is possible to calculate the capacitance of a 3-core cable. But normally it is not so and, therefore, it is desirable to find the capacitance by measurements. In a 3-core cable, sheath is at earth potential and the three conductors at supply potentials. There are six capacitances formed between these systems. Three capacitances are between the sheath and the conductors and the other three capacitances between the conductors (Fig. 9.10).

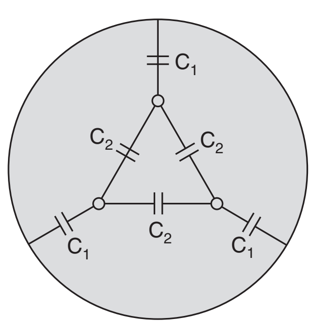


Fig. 9.10 Capacitances of a 3-core cable.

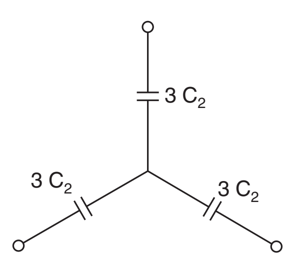


Fig. 9.11 Star equivalent of delta.

Let C_1 be the capacitance between sheath and the conductor and C_2 the capacitance between each conductor. It is desirable to connect this system of capacitors into an equivalent star connection. The equivalent star of a delta connection (capacitance between conductors) will be as shown in Fig. 9.11.

Since the star point is at sheath potential and the other terminals correspond to the conductors of the cable, the whole system of capacitors can be reduced to the following star system of capacitors (Fig. 9.12 (*a*) and (*b*)).

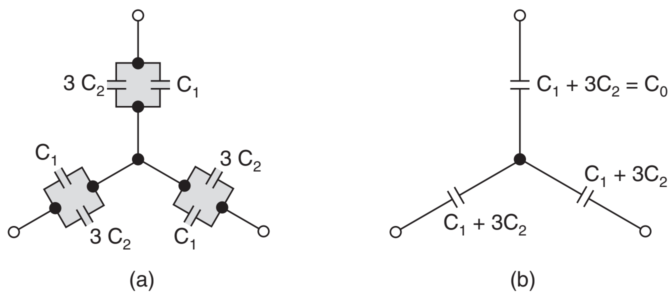


Fig. 9.12 (*a*) and (*b*) Equivalent capacitance of a 3-core cable.

There are two unknowns C_1 and C_2 to find out the capacitance per phase of the cable. We need to make two measurements: (*i*) bunch the three cores and measure the capacitance between the bunched conductors and the sheath. Let this be C_x given by $C_x = 3C_1$ as shown in Fig. 9.13 (*a*); and (*ii*) connect any two cores to the sheath and measure the capacitance between the remaining conductor and the sheath. Let this be C_y given by Fig. 9.13(*b*). The equivalent circuit will be $C_y = C_1 + 2C_2$. From these two measurements,

$$C_1 = \frac{C_x}{3}$$

and

$$C_2 = \frac{1}{2}\,(C_y - C_1)$$

$$= \frac{1}{2}\left[C_y - \frac{C_x}{3}\right]$$

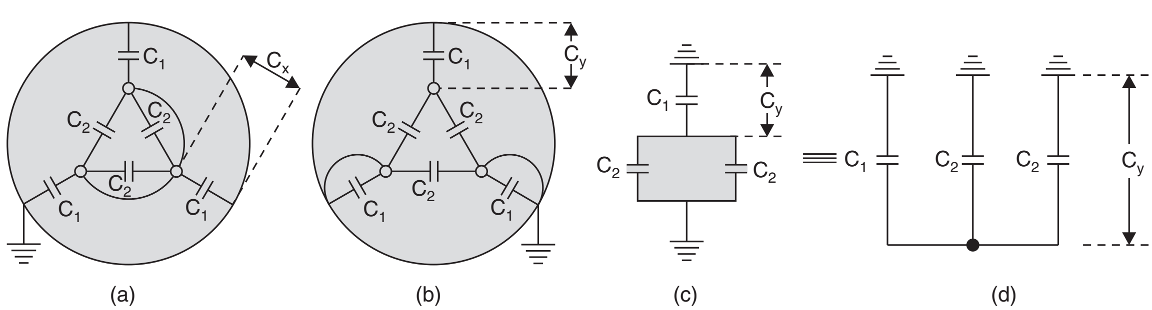


Fig. 9.13 (*a*) and (*b*) Capacitance calculations by measurement ;
(*c*) and (*d*) Equivalent of 9.13 (*b*).

Since the capacitance per phase as from Fig. 9.12 (*b*) is given by

$$\begin{aligned} C_0 &= C_1 + 3C_2 \\ &= \frac{C_x}{3} + \frac{3}{2}\left(C_y - \frac{C_x}{3}\right) \\ &= \frac{C_x}{3} + \frac{3}{2}C_y - \frac{C_x}{2} \\ &= \frac{3}{2}C_y - \frac{C_x}{6} \end{aligned} \tag{9.19}$$

In case the test figures are not available, the following empirical formula due to Simon gives an approximate value of capacitance for circular conductors

$$C_0 = \frac{0.0299\varepsilon_r}{\ln\left[1 + \frac{T+t}{d}\left\{3.84 - 1.70\frac{t}{T} + 0.52\frac{t^2}{T^2}\right\}\right]} \ \mu\text{F/km} \tag{9.20}$$

where ε_r = relative permittivity of the dielectric,

d = conductor diameter,

t = belt insulation thickness, and

T = conductor insulation thickness.

all in the same units. The main uncertainty in this formula is that of the value of ε_r. An average value of 3.5 may be taken for calculation.

Example 9.5: The capacitance of a 3-core lead sheathed cable measured between any two of the conductors with sheath earthed is 0.19 μF per km. Determine the equivalent star connected capacity and the kVA required to keep 16 kms of the cable charged when connected to 20 kV, 50 Hz supply.

Solution: The equivalent circuit is shown in Fig. E. 9.5.

Since the capacitance measured is 3.04 μF between the conductors, the capacitance per phase will be

$$2 \times 3.04 = 6.08\ \mu\text{F}$$

3-phase MVA required

$$= V^2\omega C = 20^2 \times 314 \times 6.08 \times 10^{-6}$$

$$= 0.763 \text{ MVA or } 763 \text{ kVA.} \quad \textbf{Ans.}$$

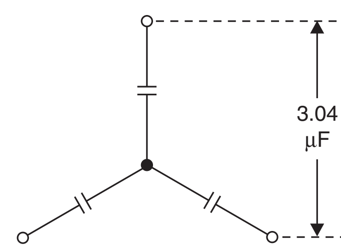


Fig. E.9.5

Example 9.6: A 3-phase, 3-core, metal sheathed cable gave the following results on test for capacitance:

(*i*) Capacitance between two conductors bunched with the sheath and the third conductor 0.4 μF per km.

(*ii*) Capacitance between bunched conductors and sheath 0.625 μF/km.

Determine the capacitance (*a*) between any two conductors, and (*b*) between any two bunched conductors and the third conductor if the sheath is insulated. (*c*) Also calculate the charging current per phase per km. when it is connected to 10 kV, 50 Hz supply.

Solution: From Figs. 9.13 (*a*) and 9.13 (*b*),

$$C_x = 3C_1 = 0.625$$

and $$C_y = C_1 + 2C_2 = 0.4$$

and from equation (9.19),

$$C_0 = \frac{3}{2} C_y - \frac{C_x}{6} = \frac{3}{2} 0.4 - \frac{0.625}{6} = 0.496 \text{ μF/km}$$

∴ capacitance between any two conductors = 0.248 μF/km. **Ans.**

(*ii*) From the measurement

$$C_1 = 0.208 \text{ μF/km}$$

and $$C_2 = 0.096 \text{ μF/km}$$

The equivalent circuit for measuring capacitance between two bunched conductors and the third conductor will be as in Fig. E.9.6.

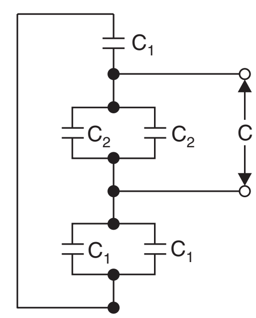


Fig. E.9.6

The equivalent capacitance C will be

$$C = 2C_2 + \frac{2}{3} C_1$$

Substituting the values for C_1 and C_2, the capacitor

$$C = 0.33 \text{ μF/km.} \quad \textbf{Ans.}$$

(*iii*) The charging current per phase per km will be

$$\frac{V}{\sqrt{3}} \omega C_0 \times 10^3 \text{ amps} = \frac{10}{\sqrt{3}} \times 314 \times 0.496 \times 10^{-6} \times 10^3$$

$$= 0.899 \text{ A.} \quad \textbf{Ans.}$$

9.6 HEATING OF CABLES

The temperature rise of a body depends upon the rate of generation and dissipation of heat by the body. If the rate of generation is greater than the rate of dissipation, the temperature goes on rising and vice versa.

In case of an underground cable the sources of heat generation are

(*i*) core loss, *i.e.*, copper loss in the core of the cable,

(*ii*) the dielectric loss, and

(*iii*) sheath losses,

and the heat is dissipated through the dielectric to the ground and finally to the atmosphere.

Core Loss: In order to find out the core loss the value of the resistance of the cable is calculated as follows:

(*i*) Knowing the resistance of the conductor at ambient temperature, 20°C, the resistance is calculated assuming an operating temperature of 65°C.

$$R_{65} = R_{20} (1 + \alpha t)$$

where α is the temperature coefficient of the conductor material and t is the difference in temperature which, in this case, is

$$t = 65°C - 20°C = 45°C$$

(*ii*) Since the effective area of section of the cable is smaller than the actual physical section, the effective resistance of the cable is larger. A factor of 1.02 is multiplied to get the resitance.

(*iii*) The length of the outermost strand is greater than the central strand. The effect of stranding on the resistance is obtained by multiplying the resistance as calculated according to the length of the central strand by a factor of 1.02.

Having calculated thus the resistance of the cable the core loss is calculated as I^2R where I is the current carried by the cable.

Dielectric Loss. The cable is a sort of capacitor with the core and the sheath forming the two plates of the condenser separated by dielectric material. The equivalent circuit for this system is represented by a parallel combination of leakage resistance R and a capacitance C. The equivalent circuit with its phasor diagram is given in Fig. 9.14. The loss in the dielectric is due to the loss in the equivalent leakage resistance.

$$P = \frac{V^2}{R}$$

From phasor diagram, $$\frac{V/R}{V\omega C} = \tan\delta$$

or $$\frac{V}{R} = V\omega C \tan\delta$$

$\therefore$ $$P = V^2\omega C \tan\delta$$

where δ is the dielectric loss angle and ω is the power supply frequency. Since δ is normally very small,

$$\tan\delta = \delta$$

$\therefore$ $$P = V^2\omega C\delta \text{ watts, where } \delta \text{ is in radians} \tag{9.21}$$

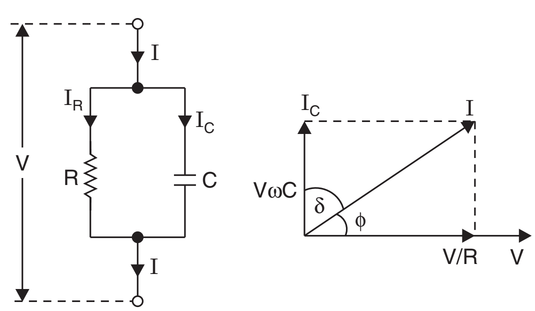


Fig. 9.14 (*a*) Equivalent of a cable, (*b*) Phasor diagram of (*a*).

From the phasor diagram, the power factor angle of the dielectric is given by

$$\phi = 90 - \delta$$

$$\therefore \qquad \cos\phi = \cos(90 - \delta) = \sin\delta$$

The power factor of a dielectric is a function of the temperature of the dielectric and also depends upon the voltage stress to which the dielectric is stressed.

Variation of Dielectric Power Factor with Temperature

The variation of dielectric power factor with temperature of a cable operating at normal voltage is given in Fig. 9.15. The variation roughly follows a *V* shape, it decreases with increase in temperature to a minimum value and rises again with increase of temperature. The minimum point lies somewhere between 30°C and 60°C depending upon the type of impregnating compound.

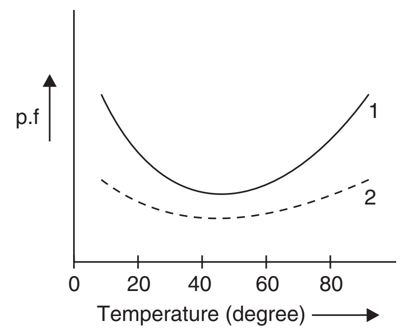


Fig. 9.15 Variation of power factor with temperature:
(1) water-washed paper and
(2) deionized water-washed paper.

It is said earlier that the operating temperature of a paper insulated cable is about 65°C which is to the right of the minimum point on the *V* curve. Around the operating temperature, if the temperature is further increased due to overloads or other reasons, this will increase the dielectric losses further giving a larger heat generation. The rise in temperature will also increase the temperature gradient between the cable and the atmosphere which will result in greater heat dissipation. If the rate of heat dissipation is less as compared with the heat generation, the temperature will continue to increase until the dielectric overheats and fails electrically. This is known as thermal instability.

Fortunately, action taken to reduce loss angle generally flattens the loss angle/temperature curve as in curve 2 of Fig. 9.15 and reduces the tendency towards thermal instability.

Variation of Dielectric Power Factor with Voltage

In case of solid type of cables when the stresses are high, dielectric loss does not vary directly as square of the voltage; rather the losses are more due to the ionization (corona loss) at weak points in the insulation. As a result there is increase of dielectric power factor. The weak points may be in the form of moisture in the insulation or more generally the presence of void formation. A void is a space which may be between the core papers and the conductor or sheath, or may lie as, more or less, flat films between one layer of paper and another. This space instead of being filled with compound contains air or some other gases at low pressures. Since the dielectric strength of air is smaller than the normal working stress of the dielectric, such a space is liable to ionization.

Figure 9.16 shows the variation of dielectric power factor as a function of electric stress. Since the electric stress near the surface of the conductor is maximum, the voids near the surface are the first to break down and the ionization then spreads progressively through the whole insulation; the voids near the sheath are the last to breakdown.

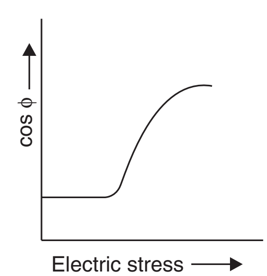


Fig. 9.16 Variation of p.f. with electric stress.

Sheath Losses. When single core cables are used for a.c. transmission, the current flowing through the core of the cable gives rise to a pulsating magnetic field which when links with the sheath, induces voltage in it. This induced voltage sets up currents under certain conditions in the sheaths and this results in sheath losses. Since the sheath currents are proportional to the cable core currents, the sheath losses are also proportional to the conductor losses. If λ is the ratio of sheath loss to the conductor loss, the equivalent a.c. resistance of the cable will be $R_{eq} = R(1 + \lambda)$ where R is the resistance of the core of the cable.

According to Cramp and Calder Wood the sheath currents can be divided into two kinds:

1. Sheath eddy currents; these are the currents which flow entirely in the sheath of the same cable.

2. Sheath circuit currents which flow from the sheath of one cable to the sheath of another cable.

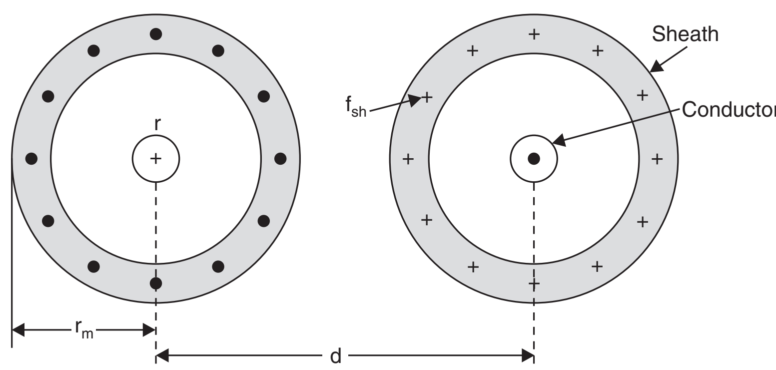


Fig. 9.17 Single phase underground line—Sheath losses.

The first type of currents will flow through the sheath when the sheaths of the two cables are not connected at both ends or when they are connected only at one end because the currents do not find a closed path through the sheaths of the two cables, whereas the second type of currents flow when the sheaths of both the cables are electrically connected at both the ends. This is known as bonding of the cables.

We first of all derive an expression for the voltage induced in the sheath of individual cable in a two cable system. Now the voltage induced in the sheath due to a current I in the core of the cable will be

$$V = \omega MI$$

where M is the mutual inductance between the core of the cable and the sheath. Mutual inductance M between the sheath and the core by definition is the flux linkages that link both the core and the sheath due to the current I in the core of the cable, per ampere of the current carried by the core. The flux due to I in one conductor extends upto the centre of the other conductor and the flux lines that enclose both the core and sheath of the same cable extend from the centre of the other cable to the mean radius of the sheath of the cable (Fig. 9.17). Any flux line between r and r_m encloses only the conductor and not the sheath.

$$M = 2 \times 10^{-7} \ln \frac{d}{r_m} \text{ H/metre} \tag{9.22}$$

$$\therefore \qquad V_s = \omega MI = \left(2\omega I \ln \frac{d}{r_m}\right) \times 10^{-7} \text{ volts/metre} \tag{9.23}$$

If the sheaths are bonded at one end, the voltage between the two sheaths at the far end will be

$$2V_s = 2\omega MI = 4 \times 10^{-7} \text{ w}I \ln \frac{d}{r_m} \text{ volts/metre} \tag{9.24}$$

In case of a short circuit, the currents I are of large magnitude which may result in high voltages between the sheaths and in case the sheaths are not bonded, these high voltages may result in sparking between the sheaths and then pitting the surface of the sheaths of the cables.

Arnold has suggested an approximate formula that gives the sheath losses due to sheath eddy currents.

$$\text{Sheath eddy current loss} = I^2 \left\{\frac{3\omega^2}{R_s}\left(\frac{r_m}{d}\right)^2 \times 10^{-18}\right\} \text{ watts/cm/phase} \tag{9.25}$$

where I = current per conductor in amps,

r_m = mean radius of sheath,

d = spacing between conductors, and

R_s = sheath resistance.

These losses are usually negligible as they form only about 2% of the core losses.

Sheath Circuit Currents

Because of the high voltages induced between the sheaths when they are unbonded during short circuit conditions, it is usually considered good practice to bond the sheaths at both the ends.

Each cable of a single phase transmission line can be considered as an air core transformer with loose magnetic coupling and ratio as 1/1. The core of the cable acts as the primary and the sheath as the secondary of the transformer. The equivalent circuit is given in Fig. 9.18.

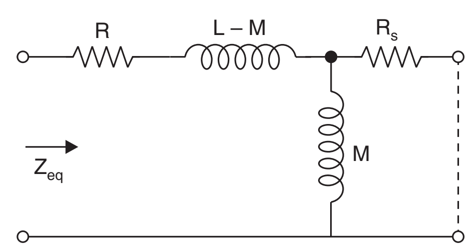


Fig. 9.18 Equivalent of a cable—Air core transformer.

Here R is the resistance of the core of the cable, $(L - M)$ the leakage inductance of the core, M the mutual inductance between the core and the sheath and R_s is the sheath resistance. The expression for self and mutual inductance in terms of the geometry of the cable is given by (Fig. 9.17)

$$L = 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre}$$

$$M = 2 \times 10^{-7} \ln \frac{d}{r_m} \text{ H/metre}$$

$$\therefore \qquad L - M = 2 \times 10^{-7} \ln \frac{r_m}{r} \text{ H/metre} \tag{9.26}$$

Since the leakage flux (the flux that does not link the core) due to the current in the sheath is zero, therefore, the leakage inductance on the secondary side is not shown. The secondary side is shorted through the dotted line to represent bonding of the cable sheaths (secondary of the equivalent air core transformer).

The equivalent impedance Z_{eq} as seen through the primary,

$$Z_{eq} = R + j\omega(L - M) + \frac{R_s j\omega M}{R_s + j\omega M}$$

$$Z_{eq} = R + j\omega(L - M) + \frac{jR_s\omega M(R_s - j\omega M)}{R_s^2 + \omega^2 M^2}$$

$$= R + \frac{\omega^2 M^2 R_s}{R_s^2 + \omega^2 M^2} + j\omega L + j\omega M \left\{ \frac{R_s^2}{R_s^2 + \omega^2 M^2} - 1 \right\}$$

$$= R + \frac{\omega^2 M^2 R_s}{R_s^2 + \omega^2 M^2} + j\omega \left\{ L - M \frac{\omega^2 M^2}{R_s^2 + \omega^2 M^2} \right\} \tag{9.27}$$

$$\therefore \qquad R_{eq} = R + \frac{\omega^2 M^2 R_s}{R_s^2 + \omega^2 M^2} \tag{9.28}$$

and

$$L_{eq} = L - M \frac{\omega^2 M^2}{R_s^2 + \omega^2 M^2} \tag{9.29}$$

It is seen that by bonding the cable at both the ends the equivalent resistance of the system is increased whereas the inductance is reduced.

The sheath losses are given by $I^2 \dfrac{\omega^2 M^2 R_s}{R_s^2 + \omega^2 M^2}$.

In order to reduce the sheath losses and thereby if it is required to increase the current carrying capacity of a cable, sheaths of the three single core cables in a 3-phase system are cross bonded as shown in Fig. 9.19.

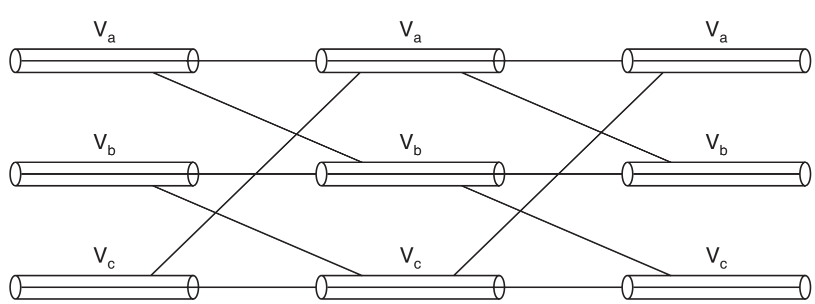


Fig. 9.19 Basic cross bonded system.

Since the three voltages are 120° apart and the cable sheaths are cross bonded as shown above the net voltage will be zero in case the cables are placed at the vertices of an equilateral triangle. Due to the asymmetry of the cable arrangement, the three voltages in series will not be quite balanced and some voltage will appear which, no doubt relatively small in magnitude, drives appreciable current and hence results in sheath losses.

In order to completely eliminate the sheath losses, a successful method is to transpose the cables as in case of overhead lines along with cross bonding (Fig. 9.20).

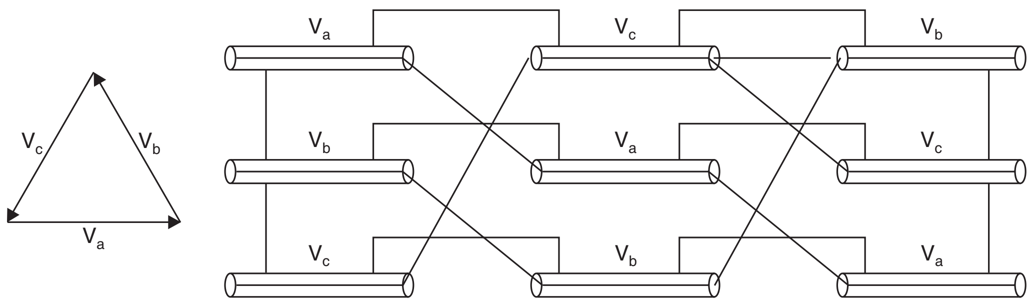


Fig. 9.20 Practical cross bonded system.

The voltage in the sheaths are now balanced and thereby there is no residual voltage which could circulate sheath currents and therefore they are absent.

Example 9.7: Three single core lead sheathed cables carry three-phase current of 400 A. The nominal conductor area of the cores is 1.25 sq. cm, the sheath thickness is 0.152 cm and the diameter over the sheath is 2.28 cms. They are supported in equilateral formation with a distance between cable centres of 5.08 cms. Calculate the induced e.m.f. in each sheath when the cable is 2 km long and supply is 50 Hz.

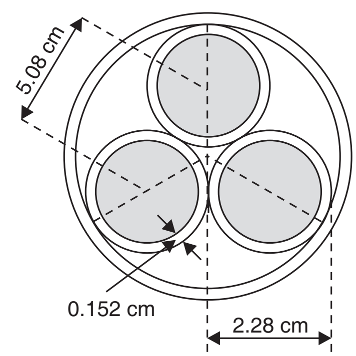


Fig. E.9.7

Solution: From the figure, the mean radius of the sheath

$$= \frac{2.28}{2} - \frac{0.152}{2} = \frac{2.28 - 0.152}{2}$$
$$= 1.06 \text{ cm.}$$

$\therefore$ The mutual inductance $= 2 \times 10^{-7} \ln \dfrac{d}{r_m}$ H/metre

$= 2 \times 10^{-7} \ln \dfrac{5.08}{1.06}$

$= 3.134 \times 10^{-7}$ H/metre

$\therefore$ For 2 km length it will be $= 2 \times 10^3 \times 3.134 \times 10^{-7}$ H

$= 6.268 \times 10^{-4}$ H

$\therefore$ Voltage induced ωIM $= 314 \times 6.268 \times 10^{-4} \times 400$ volts

$= 78.72$ volts. **Ans.**

Example 9.8: If the lead sheath resistance in the previous problem is 2.14 ohm per km and that of the conductor is 0.1625 ohm/km, determine the ratio of sheath loss to core loss of the cable.

Solution: The ratio of

$$\frac{\text{Sheath loss}}{\text{Core loss}} = \frac{R_s M^2 \omega^2}{R(R_s^{\,2} + M^2 \omega^2)}$$

$$= \frac{2 \times 2.14 \times 314^2 \times 6.268^2 \times 10^{-8}}{2 \times 0.1625\,((2 \times 2.14)^2 + 314^2 \times 6.268^2 \times 10^{-8})}$$

$= 0.0277.$ **Ans.**

9.7 CURRENT RATING OF A CABLE

The capital investment on underground transmission for some countries is in terms of hundreds of millions of rupees. It is, therefore, of great importance to determine the optimum current carrying capacity of the cables. The following factors decide the safe continuous current in a cable:

1. The maximum permissible temperature at which the insulation surrounding the conductor can be operated.

2. The method of heat dissipation through the cable.

3. The installation conditions and the ambient conditions.

As is said earlier that the temperature of a cable rises when the heat generated is greater than the heat dissipated. The allowable temperature values are:

VIR and PVC insulated cables	60°C
Impregnated paper insulated cables:	
1. Oil filled and gas-pressure cables	85°C
2. 33 kV solid type cables, armoured	65°C
3. 22 kV screened cables, armoured	65°C
4. 11 kV belted cables, armoured	65°C

Heat dissipates through the insulation, metal sheath cable bedding and servings and finally into the surrounding earth or air. The heat flow due to a three-single-core cable laid direct into the ground is shown in Fig. 9.21.

Similarly, the mechanism of heat-flow in a 3-phase belted type cable is shown in Fig. 9.22.

It is seen that the heat flow in a cable is similar to the leakage current flow *i.e.* both of them flow radially out from the core of the cable through the dielectric material, the sheath, bedding and serving and finally to the ground.

In an electric circuit, current is given by Ohm's law

$$I = \frac{V}{R} = \frac{\text{Potential difference in volts}}{\text{Resistance in ohms}}$$

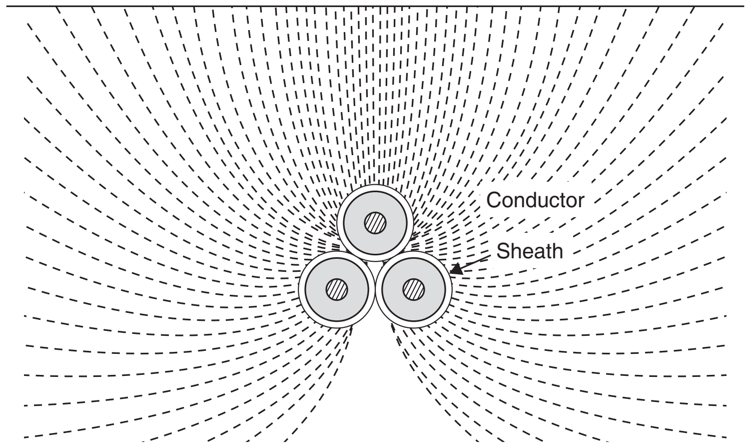


Fig. 9.21 Heat flow due to a three-single-core cable laid into the ground.

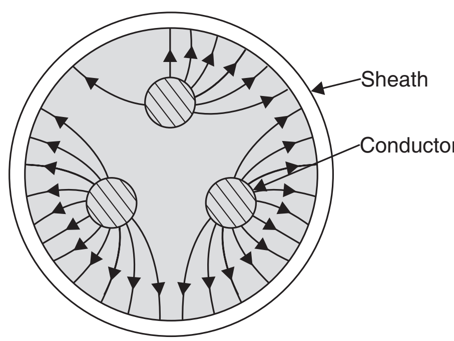


Fig. 9.22 Mechanism of heat flow in a 3-phase cable.

Similarly heat flow H is given by

$$H = \frac{\text{Temperature difference in } ^\circ\text{C}}{\text{Thermal resistance in thermal ohms}} \tag{9.30}$$

One thermal ohm is defined as the difference in degrees °C between opposite faces of a 1 cm cube produced by the flow of 1 watt of heat and is, therefore, expressed in °C/watt/cm

$$\text{Thermal resistance} = g\,\frac{l}{A}$$

where g = thermal resistivity of the material,

l = length of the heat flow path, and

A = section through which heat flows.

The equivalent circuit for heat flow of a 3-phase cable is shown in Fig. 9.23. The three cores form one pole as they have the same temperature and the earth's surface, which is again an isothermal surface, forms another pole so that the total temperature difference is the difference between the conductor and the ambient temperatures. From the conductors the heat passes through the individual dielectric materials which will form three parallel paths S_1, S_2, S_3, where S_1, S_2, S_3 represent the thermal resistances of the dielectric materials. From this heat flows through the thermal resistance S_4 of bedding, S_5 of servings and finally G of the earth to the ambient temperature. Here the metal parts like the metal sheath and armouring have negligible thermal resistance and they are taken as sources of heat.

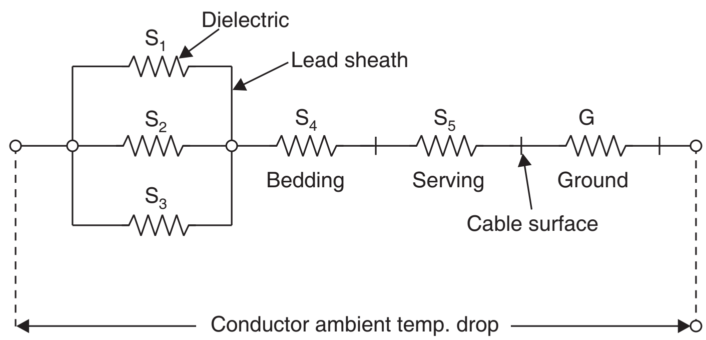


Fig. 9.23 Equivalent circuit for heat flow of a 3-phase cable.

From this it is clear that for determination of current carrying capacity of a cable, we must know the thermal resistances of the various components.

Thermal Resistance of a Single Core Cable

Here we derive an expression for the thermal resistance of the dielectric material of a single core cable (Fig. 9.24). Let r be the radius of the conductor and R the inner radius of the sheath. Since the heat flow is radial, the thermal resistance of an annulus of thickness dx and length l metre at a distance x from the centre of the conductor

$$dS = g \cdot \frac{dx}{2\pi x \cdot l}$$

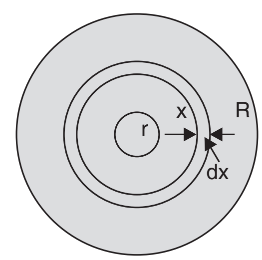


Fig. 9.24 A single core cable.

where g is the thermal resistivity of the dielectric,

$$S = \frac{g}{2\pi}\int_r^R \frac{dx}{x} = \frac{g}{2\pi} \ln \frac{R}{r} \text{ thermal ohms/m} \qquad (9.31)$$

The usual values for g are

Oil filled cable: 4.5 thermal ohm/m

Impregnated pressure cable: 5.5 thermal ohm/m.

Thermal Resistance of a 3-Core Cable

Because of the non-uniformity of the dielectric material it is difficult to derive an expression for the thermal resistance of a 3-core cable. Simon has suggested an empirical relation based on the experimental work which gives fairly accurate results.

$$S = \frac{g}{6\pi}\left(0.85 + \frac{0.2t}{T}\right) \ln\left[\left(4.15 - \frac{1.1t}{T}\right)\left(\frac{T+t}{r}\right) + 1\right] \qquad (9.32)$$

where T = thickness of conductor insulation,

t = thickness of belt insulation, and

r = conductor radius.

One more empirical relation is also available which gives the results accurate within 10 per cent.

$$S = \frac{g}{6\pi} \ln \frac{R^6 - a^6}{3R^3 a^2 r} \text{ thermal ohms/m} \tag{9.33}$$

where a is the radius of the circle at which the centres of the conductor cross sections 'lie' and 'R' is the outer dielectric radius.

Thermal Resistance of the Ground

The thermal resistivity of the soil is highly dependent on moisture content. The presence of trees overhanging the cable route will decrease the normal moisture content for the particular type of soil. Assuming the ground to be an isothermal plane and that the ground is homogeneous the thermal resistance of the ground is given by

$$G = \frac{g}{2\pi} \ln \frac{2h}{R} \text{ thermal ohms/m} \tag{9.34}$$

where g is the thermal resistivity of the ground, h the depth of cable axis below ground, R radius over the lead sheath. In practice it is found that the value of g determined in the laboratory must be multiplied by a correction factor of 2/3 in order to obtain the actual value. Therefore, the amended formula is

$$G = \frac{g}{3} \ln \frac{2h}{R} \text{ thermal ohms/m} \tag{9.35}$$

Calculation of Current Rating

Having known the thermal resistance of the various components it is now possible to calculate the current rating of the cable under steady state conditions, *i.e.*, when the current load is continuous or loading which keeps steady state thermal conditions. Neglecting the dielectric losses, let θ_m be the maximum permissible temperature of the core of the cable, θ_s the sheath temperature and θ_a the ambient temperature. The heat generated in the core of the cable will pass through the dielectric medium whereas through the bedding, serving and the ground the heat flow is sum of the heat generated in the core and the sheath. With these observations, the following relations hold good

$$nI^2R = \frac{\theta_m - \theta_s}{S_1} \tag{9.36}$$

where θ_s is sheath temperature, n the number of cores, R the resistance of each core and I is the current in each core, an expression for which is required here. S_1 is the thermal resistance of the dielectric. In case the ratio of sheath loss to core is λ, the heat flowing through bedding, serving and the ground will be $(1 + \lambda)\, nI^2R$ and the following relation will hold good;

$$(1 + \lambda)\, nI^2R = \frac{\theta_s - \theta_a}{S_4 + S_5 + G} \tag{9.37}$$

Since normally θ_s is not known, eliminating θ_s from the two equations (9.36) and (9.37),

$$\theta_m - \theta_s = nI^2RS_1$$

$$\theta_s - \theta_a = (1 + \lambda)\, nI^2R\, \{S_4 + S_5 + G\}$$

Adding the two equations (9.36) and (9.37),

$$\theta_m - \theta_a = nI^2R\,\{S_1 + (1 + \lambda)(S_4 + S_5 + G)\}$$

$$\therefore \qquad I = \sqrt{\frac{\theta_m - \theta_a}{nR\,\{S_1 + (1+\lambda)\,(S_4 + S_5 + G)\}}}\,.$$

9.8 OVERHEAD LINES VERSUS UNDERGROUND CABLES

Electric energy can be transmitted from one place to another through either the overhead lines or the underground cables. The inductance is more predominant in case of overhead lines whereas capacitance is in case of underground cables.

1. The large charging current on very high voltage cables limits the use of cable for long length transmission. Where a long distance transmission is required, overhead transmission lines are used.

2. The conductor in the overhead line is less expensive than the underground cable. The size of the conductor for the same power transmission is smaller in case of overhead lines than the cables because of the better heat dissipation in overhead lines.

3. The insulation cost is more in case of cables than the overhead lines. Overhead lines use bare conductors supported on steel towers insulated from the towers through the porcelain insulators. There is sufficient spacing between the conductor depending upon the operating voltage; the air between the conductors provides insulation. The insulation in underground cables is provided by various wrappings of high grade paper tapes. A metal sheath is applied over the insulation to prevent moisture from entering the insulation. Oil or inert gas is introduced to fill the voids. Storage vessels containing a reservoir of the oil or gas are installed at intervals along the route of the cable to take up the expansion and contraction of the oil or gas in the cable. Thus for high voltage underground transmission, the insulation problem is quite complicated and expensive.

4. The erection cost of an overhead line is much less than the underground cable.

There are certain situations where underground cables are used notwithstanding the cost. They are:

(*i*) Underground cables give greater safety to the public, less interference with amenities and better outlook to the city.

(*ii*) For power station and substation, connections or a link in overhead lines.

(*iii*) For submarine crossings.

9.9 TYPES OF CABLES

Cables are classified depending upon the material used for insulation such as paper, rubber or asbestos. Paper tapes of about 10 cms to 15 cms thickness can be wound on to a conductor in successive layers to achieve a required operating voltage and is used for voltages of 10 kV and above. In the mass-impregnation construction the paper is lapped on in its natural state and is then thoroughly dried by the combined application of heat and vacuum. It is then impregnated with insulating compound. The cable is heated in a hermetically sealed steam-heated vessel to a temperature of 120°–130°C before vacuum is applied. The compound to be used for impregnation is heated to almost 120°C in a separate vessel and is then admitted in the cable vessel. The compound fills all the pores in the paper and all the spaces in the cable assembly. After impregnation the cable is allowed to cool down in the compound in order to minimize void formation due to shrinkage. The metal sheath is then applied.

In case of pre-impregnated construction the papers are dried and impregnated before application to the conductor and after that there is no drying or impregnation process. The cables are further subdivided into solid, oil-filled or gas-filled types depending upon how the paper insulation is impregnated.

For mass impregnated cables when they are laid on a gradient, the compound used for impregnation tends to migrate from the higher to lower level. Thus voids are formed in the cable at the higher level and because of higher pressure of oil in the lower level cable, the compound will try to leak out. For voltages more than 10 kV, it is the void formation which has been responsible for breakdown.

Three-phase solid paper insulation cables are of two types: (*i*) the belted type and, (*ii*) shielded type. The belted type consists of three separately insulated conductors with an overall insulating tape enclosing all the three conductors and finally the metallic sheath is applied. The major disadvantage of belted type construction is that the electric stress is not purely radial. The existence of tangential stresses forces a leakage current (not the charging current) to flow along the layers of paper and the loss of power sets up local heating. It is to be noted that the resistance and dielectric strength of laminated paper is much less along the layers as compared to that across the layers. The local heating of the dielectric may result in breakdown of the material. The breakdown phenomenon due to tangential electric stress is shown in Fig. 9.25.

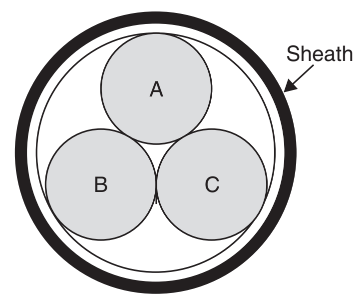


Fig. 9.25 Breakdown of a 3-phase belted cable.

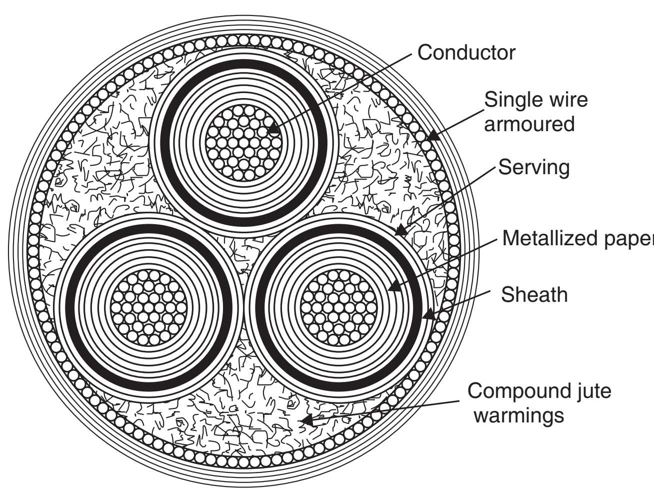


Fig. 9.26 Cross section of shielded cable.

The tangential stresses are eliminated in case of the shielded construction. In this each conductor is individually insulated and covered with a thin metallic non-magnetic shielding tape. The three shields are in contact with each other and the three conductors behave as three single phase conductors. The three conductors are then cabled together with an additional shield wrapped round them. There is no belt insulation provided but it is lead covered and armoured. All the four shields and the lead sheath are at earth potential and, therefore, the electric stresses are radial only; thereby, the tangential stresses are completely eliminated. The 3-phase shielded construction cable is shown in Fig. 9.26.

The following are the methods for elimination of void formation in the cables:

(*i*) The use of low viscosity mineral oil for the impregnation of the dielectric and the inclusion of oil channels so that any tendency of void formation (due to cyclic heating and cooling of impregnant) is eliminated.

(*ii*) The use of inert gas at high pressure within the metal sheath and in direct contact with the dielectric.

The first method is used in oil-filled cables. Oil ducts are provided within the cable itself and they communicate with oil tanks provided at suitable locations along the cable route so as to accommodate any changes in the oil volume during heating and cooling process (Fig. 9.27).

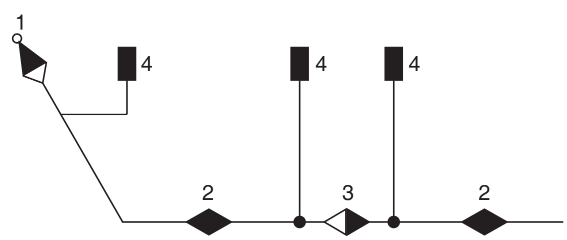


Fig. 9.27 Diagram of route of oil filled cable: 1. End coupling; 2. Connecting coupling; 3. Stopper coupling and 4. Pressure tank.

Single phase oil filled cables consist of a concentric stranded conductor built around an open helical spring core which serves as a channel for the flow of oil. The cable is insulated and sheathed in the same manner as the solid type cables. The 3-phase cables are normally of the shielded design type and consist of three oil channels composed of helical springs that extend through the cable in spaces normally occupied by filler material (Fig. 9.28). Another design of three-core oil filled cable is the flat type as shown in Fig. 9.29. The flat sides are reinforced with metallic tapes and binding wires so that during increase in pressure of oil, due to heating, the flat side is deformed and the section of the cable becomes slightly elliptical. Yet another construction of 3-core oil filled cables uses 3-core paper insulated cable without a lead sheath. The cable is pulled into a steel pipe which then is filled with oil. Pumps are then used to maintain a specified oil pressure and allow it to expand and contract with the loading cycle.

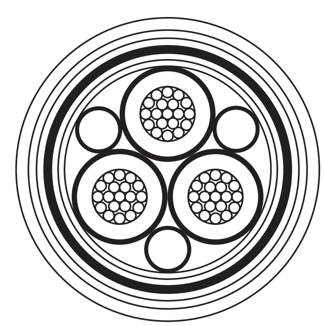


Fig. 9.28 3-core oil filled compact sector conductor.

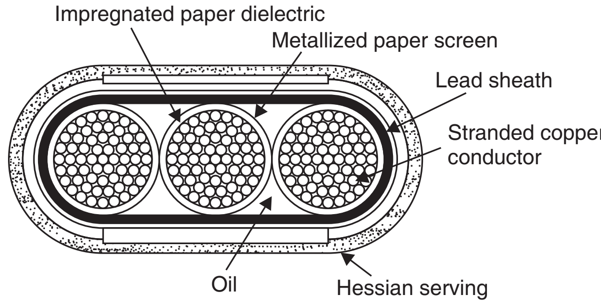


Fig. 9.29 Flat oil filled pressure cable.

Leakage or oil in these cables is a very serious problem. Automatic signalling is, therefore, installed to indicate the fall in oil pressure in any of the phases. Oil filled cables require relatively smaller amount of insulation as compared to solid type for the same operating voltage and are recommended for all voltages ranging between 66 kV and 400 kV.

To obviate the disadvantages of oil filled cables in terms of expansion and contraction of oil during loading cycles, the gas filled cables are used which have a self-contained compensating arrangement within the confines of the lead sheath. The compression cable is fundamentally a solid type construction with two important modifications; (*i*) the cable cross section is non-circular and (*ii*) the sheath thickness is reduced to allow the cable to breathe more easily. The cable is then surrounded with an envelope and the space between the two is filled with an inert gas at a nominal pressure of 14 kg/cm^2 which compresses the cable dielectric via the diaphragm sheath. During heating, the cable compound expands and travels radially through the dielectric and a space is provided by it by movement of the sheath, the non-circular shape becomes circular there. When the cable cools down, the gas pressure acting via the metallic sheath, forces the compound back into the paper insulation.

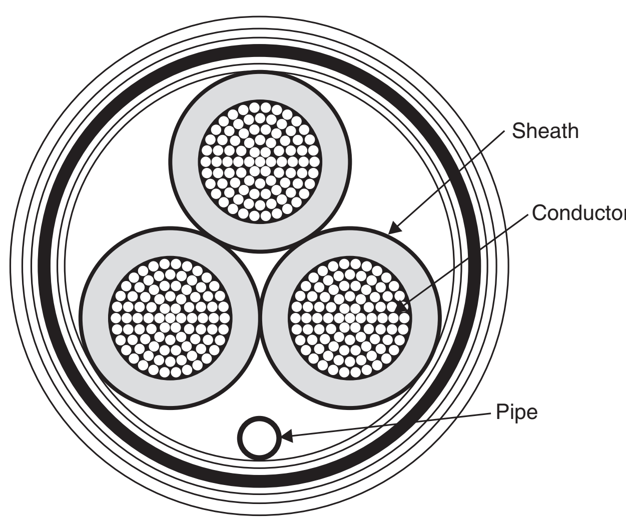


Fig. 9.30 3-phase impregnated pressure cable.

The gas cushion cable consists of stranded conductor, paper insulated, screened, lead sheathed, metallic reinforced and with a rubber-containing water proof covering. A continuous gas space throughout the length of the cable is provided. The inert gas introduced is at high pressure within the lead sheath and in contact with the dielectric in order to suppress gaseous ionization.

The impregnated pressure cable is similar to solid type except that provision is made for longitudinal gas flow. The cable has a mass-impregnated insulation design and is maintained under a gas pressure of 14 kg/cm^2. In single core cables the sheath clearance is about 0.175 cm, and in 3-core cables about 0.075 cm. In case of 3-core cables, a lead gas channel pipe is provided which is located in the space normally occupied by the filler (Fig. 9.30). The object of this pipe is to provide low resistance path between joints.

Because of the good thermal characteristic and high dielectric strength of the gas SF_6, it is used for insulating the cables also. SF_6 gas insulated cables can be matched to overhead lines and can be operated corresponding to their surge impedance loading. These cables can be used for transporting thousands of MVA even at UHV whereas the conventional cables are limited to 1000 MVA and 500 kV.

PROBLEMS

9.1. Determine the operating voltage of a single core cable of dia 2 cm and having three insulating material of permittivities 5, 4, 3. The overall diameter of the cable is 5 cms and the maximum working stress is 40 kV/cm. Compare the operating voltage with the voltage if the cable were not graded and the material with same working stress was used.

9.2. A single core cable has an inner diameter of 5 cms and a core diameter of 1.5 cm. Its paper dielectric has a working maximum dielectric stress of 60 kV/cm. Calculate the maximum permissible line voltage when such cables are used on a 3-phase power system.

9.3. Show that for a concentric cable of given dimensions and given maximum potential gradient in the dielectric, the maximum permissible voltage between the core and the sheath is independent of the permittivity of the insulating material.

9.4. What is 'void formation' in a cable ? How does this affect the performance of a cable ? What steps are taken to prevent the formation of these voids ?

9.5. A 1-phase concentric cable 5 km long has a capacitance of 0.2 μF per km, the relative permittivity of the dielectric being 3.5. The diameter of the inner conductor is 1.5 cm and the supply voltage is 66 kV at 50 Hz. Calculate the inner diameter of the outer conductor, the rms voltage gradient at the surface of the inner conductor and the rms value of the charging current.

9.6. Describe with a neat sketch, the construction of a 3-core belted type cable. Discuss the limitations of such a cable.

9.7. The capacitances of a 3-core cable belted type are measured and found to be as follows:

(*i*) between 3-cores bunched together and the sheath 8 μF.

(*ii*) between conductor and the other two connected together to the sheath 5 μF.

Calculate the capacitance to neutral and the total charging kVA when the cable is connected to a 11 kV 50 Hz 3-phase supply.

9.8. An *H*-type cable, 40 km long has a capacitance per km between any two-conductors of 0.15 μF. The supply voltage is 3-phase 33 kV at 50 Hz. Determine the charging current.

9.9. A 3-phase metal sheathed cable one km long gave the following results on a test for capacitance:

(*i*) Capacitance between two conductors bunched with the sheath and the third conductor 0.5 μF.

(*ii*) Capacitance between bunched conductors and sheath 1 μF. With the sheath insulated, find the capacitance (*a*) between any two cores, (*b*) between any two bunched conductors and the third conductor, and (*c*) calculate the charging current per phase per km when connected to 11 kV, 50 Hz supply.

9.10. A 3-phase underground cable consists of 3 single core cables each of radius 0.75 cm and spaced 5 cm apart in equilateral formation. The diameter of the lead sheath is 2.3 cm and the sheath thickness 0.15 cm. The specific resistance of lead is $22.0 \times 10^{-6}\ \Omega$ cm at the working temperature and the conductor resistance 0.162 ohm per km at 65°C. For a cable length of 1.6 km and a load of 200 amps determine (*i*) the ratio of sheath loss, to core loss and (*ii*) the induced voltage without bonding.

9.11. A single core metal sheathed cable operating at 66 kV is to be graded by means of a metallic intersheath: (*a*) Determine the diameter of the intersheath and the voltage at which it must be maintained in order to obtain the minimum overall cable diameter D. The dielectric strength of the material is 50 kV/cm. (*b*) Compare the conductor and outside diameters with those of an ungraded cable of the same material under the same condition.

9.12. Determine the maximum working voltage of a single core lead sheathed cable having a conductor 1 cm dia and sheath of 5 cm dia inside. Two insulating materials with permittivities and maximum stresses 4, 2.5 and 60 kV/cm and 50 kV/cm respectively are used.

9.13. The inner and outer dia of a cable are 3 cms and 8.5 cms. The cable is insulated with two materials having permittivities of 5 and 3 respectively with corresponding stresses of 38 kV/cm and 28 kV/cm. Calculate the radial thickness of each insulating layer and the safe working voltage of the cable.

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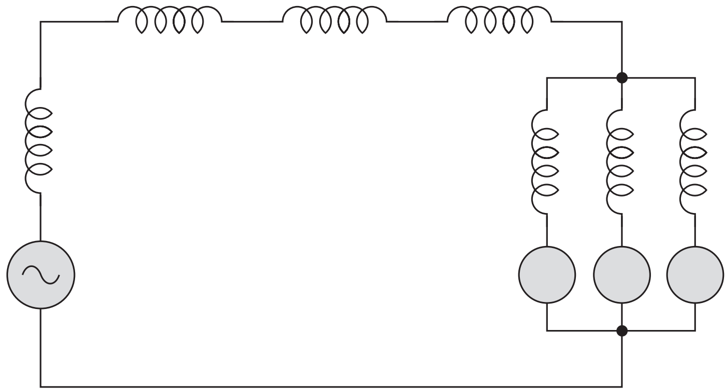
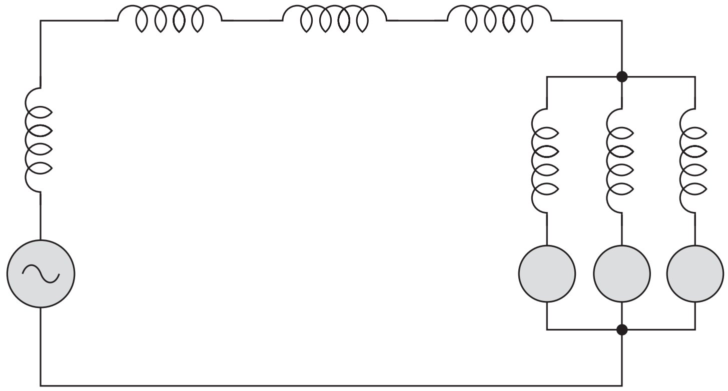


10

VOLTAGE CONTROL



10

Voltage Control

INTRODUCTION

A power system is said to be well designed if it gives a good quality of reliable supply. By good quality is meant the voltage levels within the reasonable limits. Practically all the equipments on the power systems are designed to operate satisfactorily only when the voltage levels on the system correspond to their rated voltages or at the most the variations are within say 5%. If the voltage variation is more than a prespecified value, the performance of the equipments suffers and the life of most of the equipment also is sacrificed. The picture on a television set starts rolling if the voltage is below a certain level, the fluorescent tube refuses to glow if the voltage is below a certain level. The torque of an induction motor (which forms about 70% of the total load on the system) varies as square of the terminal voltage and so on. Thus the necessity of controlling the voltage on the system is very much strong.

When power is supplied to a load through a transmission line keeping the sending end voltage constant, the receiving end or load voltage undergoes variations depending upon the magnitude of the load and the power factor of the load. The higher the load with smaller power factor the greater is the voltage variation. The voltage variation at a node is an indication of the unbalance between the reactive power generated and consumed by that node. If the reactive power generated is greater than consumed, the voltage goes up and vice versa. Whenever the voltage level of a particular bus undergoes variation this is due to the unbalance between the two vars at that bus.

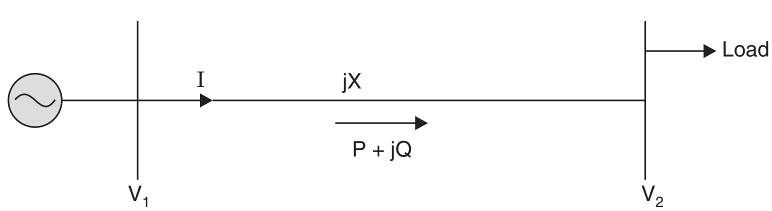


Fig. 10.1 Load connected to the source through a line.

To understand this problem refer to Fig. 10.1 where node one is a generator node with reference voltage V_1 and node two is the load node with voltage V_2. The two bus bars are interconnected through a short line.

Assuming the interconnector to be lossless (R = 0) and the voltage V_1 constant (by adjusting the excitation of the generator), the following relations hold good:

$$V_2 = V_1 - IZ \tag{10.1}$$

$$V_1^* I = P - jQ \text{ (assuming inductive load)} \tag{10.2}$$

From equation (10.2),

$$I = \frac{P - jQ}{V_1^*}$$

$$V_1^* = V_1,\ V_1 \text{ being the reference vector}$$

Substituting for I in equation (10.1),

$$V_2 = V_1 - j\,\frac{P - jQ}{V_1}\,X$$

$$= \left(V_1 - \frac{Q}{V_1}X\right) - j\,\frac{P}{V_1}X \tag{10.3}$$

The vector diagram for this relation is given in Fig. 10.2.

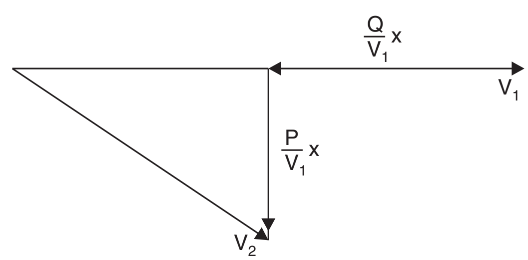


Fig. 10.2 Phasor diagram for system in Fig. 10.1.

From the above it is clear that the load voltage V_2 is not affected much due to the real component of the load P as it is normal to the vector V_1 whereas the drop due to reactive component of load is directly subtracted from the voltage V_1. Assuming the voltage drop due to real power negligible, the voltage drop is directly proportional to the reactive power Q. The relation is given by

$$V_2 = V_1 - \frac{Q}{V_1}X \tag{10.4}$$

In order to keep the receiving end voltage V_2 fixed for a particular sending end voltage V_1, the drop $(Q/V_1)X$ must remain constant. Since, in this the only variable quantity is Q, it is this reactive vars which must be locally adjusted to keep this quantity fixed *i.e.*, let Q be the value of reactive vars which keeps V_2 to a specified value, any deviation in Q at node 2 must be locally adjusted. The local generation can be obtained by connecting shunt capacitors or synchronous capacitors and/or shunt inductors (for light loads or capacitive loads).

Referring again to equation (10.4), in order to keep V_2 constant for fixed V_1, another possibility is that the product QX be kept constant. This is achieved by introducing series capacitors which will reduce the net reactance of the system. Since the voltage variation will be more for larger loads (larger reactive power), the variation could be controlled by switching in suitable series capacitors.

10.1 METHODS OF VOLTAGE CONTROL

The methods for voltage control are the use of (*i*) Shunt capacitors; (*ii*) Series capacitors; (*iii*) Synchronous capacitors; (*iv*) Tap changing transformers; and (*v*) Booster transformers.

The first three methods could also be categorised as reactive var injection methods.

In earlier times the voltage control was done by adjusting the excitation of the generator at the sending end. The larger the reactive power required by the load the more is the excitation to be provided at the sending end. This method worked well in small isolated system where there was no local load at the sending end. Also there are limits for the excitation as well. Excitation below a certain limit may result in unstability (if this machine is connected to a synchronous load) of the system and excitation above certain level will result in overheating of the rotor. Therefore, in any case, the amount of regulation by this method is limited by the permissible voltage rise at the sending end and by the difficulty of designing efficient generating plant when the range of excitation is so wide.

Before we discuss the various methods in detail for voltage control it seems imperative to know the various sources and sinks of reactive power in a power system.

Sources and Sinks (Generation and Absorption) of Reactive Power

Transmission Lines: Let the transmission line be loaded such that the load current is I amperes and load voltage V volts; assuming the transmission line to be lossless, the reactive power absorbed by the transmission line will be

$$I^2\omega L$$

where ω is the supply angular frequency and L the inductance of the line. Due to the shunt capacitance of the line, the reactive vars supplied by the line are

$$V^2\omega C$$

where C is the shunt capacitance of the line. In case the reactive vars supplied by the line are equal to the reactive vars absorbed,

$$I^2\omega L = V^2\omega C$$

or

$$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n \tag{10.5}$$

Dimensionally the ratio V to I is the impedance and, therefore, Z_n is called the natural impedance of the line and the loading condition in which the vars absorbed are equal to the vars generated by the line is called the surge impedance loading (SIL) and it is here where the voltage throughout the length of the line is same *i.e.* if the transmission line is terminated by a load corresponding to its surge impedance the voltage at the load is constant. In case $I^2\omega L > V^2\omega C$ the voltage will sag and if $I^2\omega L < V^2\omega C$ (light load condition) the voltage will rise (see

Fig. 10.3). Normally the loading is greater than the SIL and, therefore, the condition $I^2\omega L > V^2\omega C$ exists and the net effect of the line will be to absorb (sink) the reactive vars. Under light load conditions the effect of shunt capacitors is predominating and the line will work as vars generator (source).

Transformers: The equivalent circuit of a transformer for power frequency is given in Fig. 10.4.

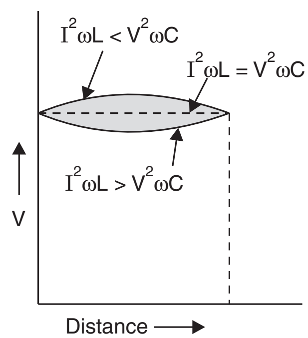


Fig. 10.3 Variation of voltage as a function of distance of line.

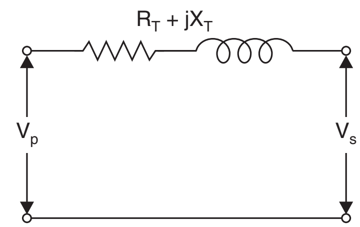


Fig. 10.4 Equivalent circuit of a transformer.

It is clear that the transformers always absorb reactive power. Let X_T be the per unit reactance of a transformer with kVA as volt ampere rating and kV as the voltage rating.

Since by definition

$$\text{Per unit reactance} = \frac{\text{Actual reactance } X \cdot I}{V}$$

Actual reactance $\quad X = X_T \cdot \dfrac{V}{I}$

Now $\quad I = \dfrac{\text{kVA}}{\sqrt{3}\ \text{kV}}$

$\therefore \quad X = \dfrac{\sqrt{3}\ X_T \cdot \text{kV}^2 \cdot 1000}{\text{kVA}}$

The reactive power loss $= 3I^2X$

$$= \frac{3\,\text{kVA}^2}{3\,\text{kV}^2}\,\frac{\sqrt{3}\cdot X_T \cdot \text{kV}^2\ .\ 1000}{\text{kVA}}$$

$$= \sqrt{3}\ \text{kVA} \cdot X_T\ \text{kVArs} \qquad (10.6)$$

The above expression gives the VArs consumed by the transformer when it is loaded to its full capacity.

Cables: Cables have very small inductance and relatively very large capacitance because of the nearness of the conductors, larger size of the conductors and the dielectric material used has a relative permittivity greater than unity. They are, therefore, generators of reactive power.

Synchronous Machines: It is known that the power transmitted from a generator bus to an infinite bus bar is given by

$$P = \frac{|E||V|}{X} \sin \delta \qquad (10.7)$$

where E = generator voltage,

V = infinite bus bar voltage,

X = the reactance of the unit, and

δ = angle between E and V.

Similar relation for the reactive power for a round rotor machine is given by

$$Q = \frac{|V||E|}{X} \cos \delta - \frac{|V|^2}{X} \qquad (10.8)$$

The above formula tells that if

$$E \cos \delta > |V|$$

then $Q > 0$ and the generator produces reactive power *i.e.*, it acts as a capacitor. This inequality is generally satisfied when the generator is over excited. Since cos δ = cos $(-\delta)$ the inequality is true for both as when machine is working as a generator or as a motor. Therefore, it can be said that an over-excited synchronous machine produces reactive power and acts as a shunt capacitor.

Similarly when $|E| \cos \delta < |V|$, $Q < 0$ *i.e.*, negative and the machine consumes reactive power from the system. Consequently an under-excited machine acts as a shunt coil. This characteristic of the machine *i.e.* it draws leading or lagging current depending upon the excitation is nicely shown as a V curve in Fig. 10.5.

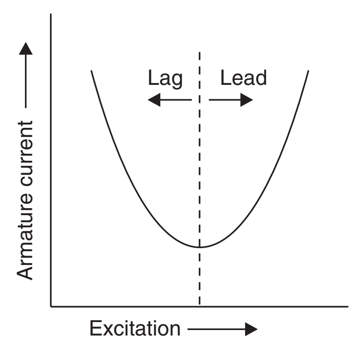


Fig. 10.5 V-curves of synchronous machine.

When the machine is used over-excited it is known as synchronous capacitor and the special feature of the machine is that then it is run under no load condition; thereby $\delta = 0$ in the inequality and $|E| > |V|$ *i.e.*, Q can be continuously and simply controlled by controlling $|E|$ *i.e.*, by varying the d.c. excitation. In case the inequality is satisfied both ways during the operation of the machine, it is then known as synchronous phase modifier.

A synchronous phase modifier has a smaller shaft and bearing and higher speeds as compared to a synchronous motor used for mechanical loads. A synchronous phase modifier has a higher overall efficiency as compared with a synchronous motor. Standard machines are designed for full load output at leading power factor and can carry about 50% of their rated capacity when the p.f. is lagging. A machine designed to operate at full load for lagging p.f. is physically larger, is more expensive and has greater losses.

Shunt Capacitors and Reactors

As is said earlier the shunt capacitors are used across an inductive load so as to supply part of the reactive vars required by the load so that the reactive vars transmitted over the line are reduced, thereby the voltage across the load is maintained within certain desirable limits. Similarly, the shunt reactors are used across capacitive loads or lightly loaded lines to absorb some of the leading vars again to control the voltage across the load to within certain desirable limits. Capacitors are connected either directly to a bus bar or through a tertiary winding of the main transformer and are disposed along the route to minimize the voltage drop and the losses. The disadvantage of the use of shunt capacitor or reactor is that with the fall of voltage at a particular node the correction vars are also reduced *i.e.*, when it is most needed, its effectiveness falls. Similarly, on light loads when the corrective vars required are relatively less, the capacitor output is large.

Series Capacitors

If a static capacitor is connected in series with the line, it reduces the inductive reactance between the load and the supply point and the voltage drop is approximately

$$IR \cos \phi_r + I(X_L - X_c) \sin \phi_r \qquad (10.9)$$

It is clear from the vector diagram (Fig. 10.6) that the voltage drop produced by an inductive load can be reduced particularly when the line has a high X/R ratio. In practice X_c may be so chosen that the factor $(X_L - X_c) \sin \phi_r$ becomes negative and numerically equal to $R \cos \phi_r$ so that the voltage drop becomes zero. The ratio X_c/X_L expressed as a percentage is usually referred to as the percentage compensation.

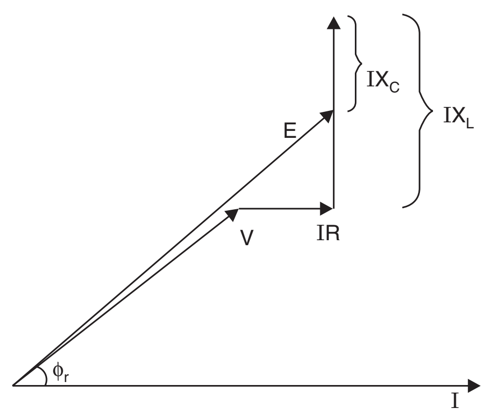


Fig. 10.6 Phasor diagram when series capacitor is connected on a line.

If I is the full load current and X_c is the capacitive reactance of the series capacitor then the drop across the capacitor is IX_c and the VAr rating is I^2X_c. The voltage boost produced by the series capacitor

$$\Delta V = IX_c \sin \phi_r \qquad (10.10)$$

One drawback of series capacitors is the high overvoltage produced across the capacitor terminals under short circuit conditions. The drop across the capacitor is I_fX_c, where I_f is the fault current which is of the order of 20 times the full load current under certain circuit condition. A spark gap with a high speed contactor is used to protect the capacitor under these conditions.

Comparison between Series and Shunt Capacitors

(*i*) The voltage boost due to a shunt capacitor is evenly distributed over the transmission line whereas the change in voltage between the two ends of the series capacitor where it is connected, is sudden. The voltage drop along the line is unaffected.

(*ii*) Let Q_c' be the reactive power of the shunt capacitor, E_r the receiving end voltage and X the reactance of the line; the current through the capacitor will be Q_c'/E_r and the drop due to this current in the line will be $(Q_c'/E_r)X$.

Similarly let Q_c be the rating of the series capacitor I, the line current and $\sin \phi_r$ the sine of the power factor angle of the load. The drop across the series capacitor will be $(Q_c/I) \sin \phi_r$ since the magnitude of the voltage across the capacitor is Q_c/I.

For a typical load with p.f. 0.8 lag, $\sin \phi_r = 0.6$ and assume $IX/E_r = 0.1$.

For equality of voltage boost with the two applications

$$\frac{Q_c' \, X}{E_r} = \frac{Q_c \sin \phi_r}{I} \tag{10.11}$$

or
$$\frac{Q_c'}{Q_c} = \frac{\sin \phi_r}{IX / E_r} = \frac{0.6}{0.1} = 6$$

It is evident that for the same voltage boost the reactive power capacity of a shunt capacitor is greater than that of a series capacitor.

(*iii*) The shunt capacitor improves the p.f. of the load whereas the series capacitor has little effect on the p.f.

(*iv*) For long transmission lines where the total reactance is high, serves capacitors are effective for improvement of system stability.

Synchronous Capacitors

A great advantage of the synchronous capacitor is its flexibility for use for all load conditions because it supplies vars when over-excited, *i.e.* during peak load conditions and it consumes vars when under-excited during light load conditions.

There is smooth variation of reactive vars by synchronous capacitors as compared with step by step variation by the static capacitors.

Synchronous machines can be overloaded for short periods whereas static capacitors cannot. For large outputs the synchronous capacitors are much better than the static capacitors from economic viewpoint because otherwise a combination of shunt capacitors and reactors is required which becomes costlier and also the control is not smooth as is achieved with synchronous capacitors.

The main disadvantage of the synchronous capacitor is the possibility of its falling out of step which will thus produce a large sudden change in voltage. Also these machines add to the short circuit capacity of the system during fault condition.

A transmission line is said to be a constant voltage or a regulated line if its receiving end voltage is controlled by varying the reactive power at the receiving end when the sending end voltage is kept constant. Other systems where the reactive power available at the receiving end corresponds to the reactive power requirements of the load are termed as unregulated systems.

Tap Changing Transformers

The main job of a transformer is to transform electric energy from one voltage level to another. Almost all power transformers on transmission lines are provided with taps for ratio control *i.e.*, control of secondary voltage. There are two types of tap changing transformers:

(*i*) Off-load tap changing transformers.

(*ii*) On-load (under-load) tap changing transformers.

The tap changing transformers do not control the voltage by regulating the flow of reactive vars but by changing the transformation ratio, the voltage in the secondary circuit is varied and voltage control is obtained. This method is the most popular as it can be used for controlling voltages at all levels.

Figure 10.7 refers to the off-load tap changing transformer which requires the disconnection of the transformer when the tap setting is to be changed. The modern practice is to use on-load tap changing transformer which is shown in Fig. 10.8. In the position shown the voltage is a maximum and since the currents divide equally and flow in opposition through the coil between Q_1 and Q_2, the resultant flux is zero and hence minimum impedance. To reduce the voltage, the following operations are required in sequence : (*i*) open Q_1; (*ii*) move selector switch S_1 to the next contact; (*iii*) close Q_1; (*iv*) open Q_2; (*v*) move selector switch S_2 to the next contact; and (*vi*) close Q_2.

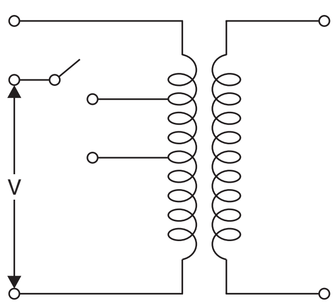


Fig. 10.7 Off-load tap changing transformer.

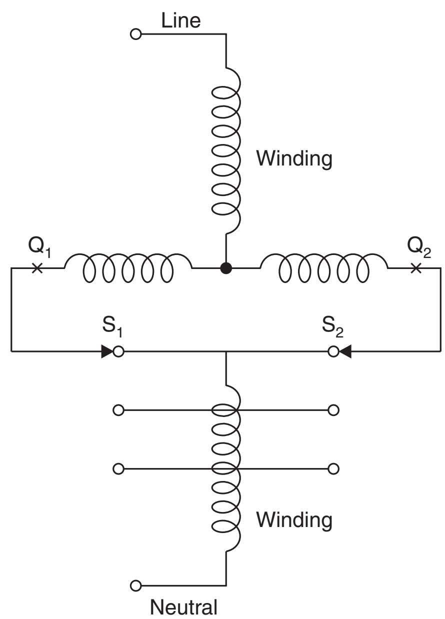


Fig. 10.8 On-load tap changing transformer.

Thus six operations are required for one change in tap position. The voltage change between taps is often 1.25 per cent of the nominal voltage where nominal voltages are the voltages at the ends of the transmission line and the actual voltages are t_sV_1 and t_rV_2 where t_s and t_r are the fractions of the nominal transformation ratios, *i.e.*, the tap ratio/nominal ratio.

Consider the operation of a radial transmission line with tap changing transformers at both the ends as shown in Fig. 10.9. It is desired to find out the tap changing ratios required to completely compensate for the voltage drop in the line. We assume here that the product of t_s and t_r is unity as this ensures that the overall voltage level remains of the same order and that the minimum range of taps on both transformers is used.

From Fig. 10.9, we have

$$t_sV_1 = t_rV_2 + IZ \tag{10.12}$$

We know that the approximate line drop is given as

$$IZ = \Delta V = v_r \cos \phi + v_x \sin \phi \tag{10.13}$$

$$= IR \cos \phi + IX \sin \phi$$

$$= R \cdot I \cos \phi + X \cdot I \sin \phi$$

$$= \frac{R \cdot P}{V_r} + \frac{X \cdot Q}{V_r}$$

$$= \frac{RP + XQ}{t_r V_2} \qquad (10.14)$$

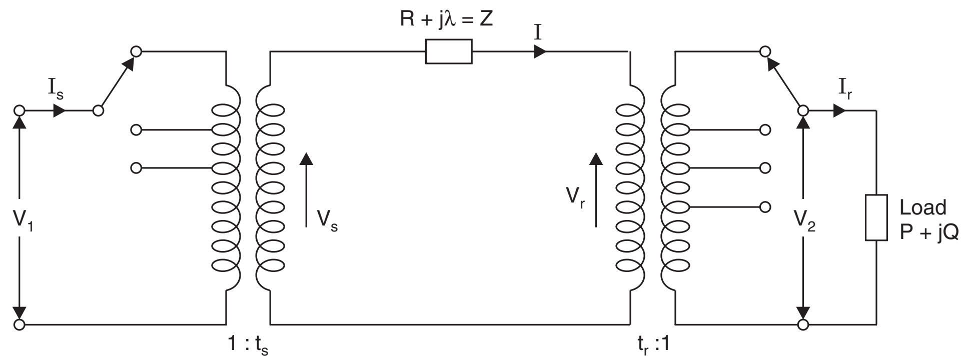


Fig. 10.9 Radial transmission line with on-load tap changing transformer at both the ends.

$$\therefore \qquad t_s V_1 = t_r V_2 + \frac{RP + XQ}{t_r V_2} \qquad (10.15)$$

$$t_s = \frac{1}{V_1}\left[t_r V_2 + \frac{RP + XQ}{t_r V_2}\right] \qquad (10.16)$$

Now as
$$t_s t_r = 1 \qquad (10.17)$$

$$t_s = \frac{1}{V_1}\left[\frac{V_2}{t_s} + \frac{RP + XQ}{V_2 / t_s}\right]$$

or
$$t_s^2 = \frac{V_2}{V_1} + \left(\frac{RP + XQ}{V_2 V_1}\right) t_s^2$$

or
$$t_s^2 \left[1 - \frac{RP + XQ}{V_1 V_2}\right] = \frac{V_2}{V_1} \qquad (10.18)$$

From the equation (10.18), for particular values of V_2 and V_1 and the load requirements P and Q, the value of t_s can be obtained.

The tap changing operation is normally motor operated. A closed loop control of the secondary voltage level is possible.

Booster Transformers: The two-winding load tap changing transformer performs two functions, transforming the voltage and bucking or boosting the voltage whereas the booster transformer performs the latter function only. It can be installed at a sub-station as an additional equipment if voltage regulation is further found to be necessary or it can be installed as a separate piece of equipment at any intermediate point in the line. The latter application may be desirable on economical or technical grounds to increase the voltage at an intermediate point in a line rather than at the ends as with tap changing transformer.

For small outputs and voltages upto 2000 volts, the simplest booster consists of an auto transformer with necessary tappings, whereas for higher voltages and larger sizes it is necessary to utilize on-load tap changing gear and also to perform the switching in an isolated circuit, the voltage of which is only a fraction of the line voltage. One method is to energize the primaries of the boosting transformers by means of a regulating transformer, the secondary of which is provided with tappings along with tap changing gear as shown in Fig. 10.10. The voltage changes are made by means of a motor operated controller and arrangements are made to reverse the connections to the primaries of the regulating transformers so that both buck and boost can be obtained. The sensing device for voltage variation should be sensitive to current rather than voltage as the current varies 100% from no load to full load whereas the voltage varies only by 10% or so.

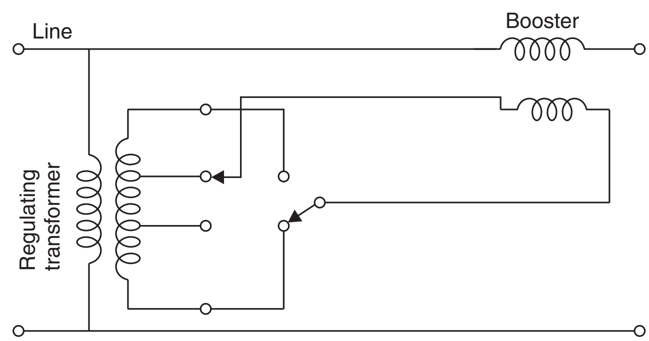


Fig. 10.10 Booster transformer along with regulating transformer.

The following are the advantages of booster transformer:

(*i*) The transformer can be used at any intermediate point in the system.

(*ii*) When it is used along with a fixed ratio transformer it can be taken out for inspection or overhaul without affecting much the system.

(*iii*) The rating of the booster is the product of the current and the injected voltage and is hence only about 10% of that of a main transformer.

The disadvantages of the booster, when it is used in conjunction with the main transformer, are

(*i*) The two are more expensive than a transformer with on-load tap changing gear.

(*ii*) They are less efficient due to the losses in the booster.

(*iii*) They take more floor space.

The booster transformers are normally used in distribution feeders where the cost of tap changing transformer is very high.

Example 10.1: In the radial transmission system shown in Fig. E.10.1 all per unit values are referred to the voltage bases shown and 100 MVA. Determine the total power, active and reactive, supplied by the generator and the p.f. at which the generator must operate.

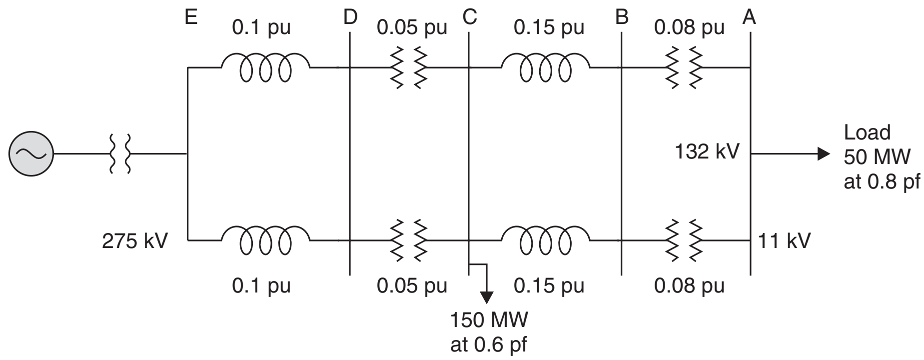


Fig. E.10.1

Solution: Nominal voltages are assumed and voltage drops in the circuits are neglected.

To find out the total power active and reactive we add up all the active powers connected to the system and reactive power connected to the system plus the reactive power losses in the lines and the transformers. Taking voltage as 1.0 p.u. everywhere,

Power at bus A = Active power 0.5 p.u. and reactive power 0.375 p.u.

The total reactance between bus C and A = 0.075 + 0.04 = 0.115 p.u.

$$\therefore \qquad I^2X \text{ loss} = \frac{P^2 + Q^2}{V^2} X = \frac{0.5^2 + 0.375^2}{1^2} \times 0.115 = 0.0449 \text{ p.u.}$$

Active power tapped at bus C = 1.5 p.u. and reactive power 2.0 p.u.

Total active power between E and C = 0.5 + 1.5 = 2.0 p.u.

Reactive power between E and C = 0.375 + 0.0449 + 2.0

= 2.4199 p.u.

Total reactance between E and C = 0.05 + 0.025 = 0.075 p.u.

$$\therefore \qquad I^2X \text{ loss} = \frac{2^2 + 2.4199^2}{1^2} \times 0.075 = 0.73919 \text{ p.u.}$$

$\therefore$ Total active power supplied by the generator = 200 MW

and the reactive power supplied = 315.9 MW

and the p.f. of the generator = 0.5349. **Ans.**

Example 10.2: A 230 kV line is fed through 33/230 kV transformer from a constant 33 kV supply. A single line diagram of the 3-phase system is shown in Fig. E.10.2. The impedance of the line and transformers at 230 kV is (30 + j80) ohms. Both the transformers are equipped with tap changing facilities which are so arranged that the product of the two off nominal settings is unity. If the load on the system is 150 MW at 0.9 p.f., determine the settings of the tap changers required to maintain the voltage of the load bus bar at 33 kV.

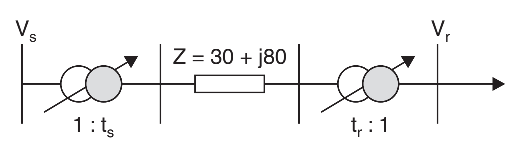


Fig. E.10.2

Solution: It is desired to have $V_s = V_r$ and $t_s t_r = 1.0$.

The load is 150 MW and 72.65 MVAr

From equation (10.18) we have

$$t_s^2\left(1-\frac{RP+XQ}{V_sV_r}\right)=\frac{V_r}{V_s}=1.0$$

Substituting the values we have

$$t_s^2\left(1-\frac{30\times\frac{150}{3}\times10^6+80\times\frac{72.65}{3}\times10^6}{\left(\frac{230}{\sqrt{3}}\right)^2\times10^6}\right)=1.0$$

or $$t_s = 1.11 \text{ p.u.} \quad \textbf{Ans.}$$

10.2 DETERMINATION OF SYNCHRONOUS PHASE MODIFIER CAPACITY

Consider the generalized equation

$$V_s = AV_r + BI_r \tag{10.19}$$

where V_s and V_r are the sending and receiving end voltages, I_r the receiving end current and A, B are the system constants which may include the transformers also. The equation above is represented by the phasor diagram (Fig. 10.11).

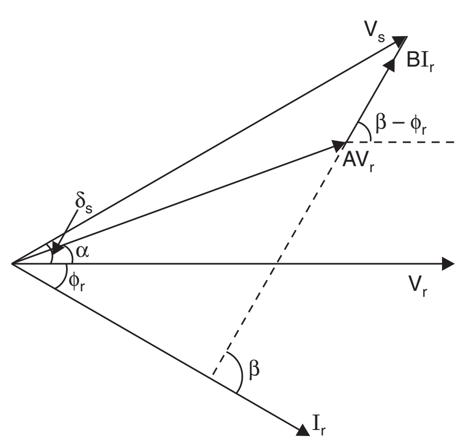


Fig. 10.11 Phasor diagram to represent equation (10.19).

Taking V_r as the reference the above equation is re-written as

$$V_s \angle\delta_s = AV_r \angle\alpha + BI_r \angle(\beta-\phi_r) \tag{10.20}$$

$$= AV_r \cos\alpha + jAV_r \sin\alpha + BI_r \cos(\beta-\phi_r) + jBI_r \sin(\beta-\phi_r) \tag{10.21}$$

$$V_s^2 = A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r \cos\alpha\cos(\beta-\phi_r) + 2ABV_rI_r \sin\alpha\sin(\beta-\phi_r)$$

$$= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r \cos(\alpha-\beta+\phi_r)$$

$$= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r [\cos(\alpha-\beta)\cos\phi_r - \sin(\alpha-\beta)\sin\phi_r] \tag{10.22}$$

Now since $P_r = V_r I_r \cos \phi_r$ and $Q_r = V_r I_r \sin \phi_r$, substituting these in the expression above,

$$V_s^2 = A^2 V_r^2 + B^2 I_r^2 + 2ABP_r \cos (\alpha - \beta) - 2ABQ_r \sin (\alpha - \beta) \tag{10.23}$$

Also since

$$I_r = I_p - jI_q,\ I_r^2 = I_p^2 + I_q^2$$

and

$$I_p = \frac{P_r}{V_r},\ I_q = \frac{Q_r}{V_r}$$

$\therefore$

$$V_s^2 = A^2 V_r^2 + B^2 \left(\frac{P_r^2}{V_r^2} + \frac{Q_r^2}{V_r^2} \right)$$
$$+ 2ABP_r \cos (\alpha - \beta) - 2ABQ_r \sin (\alpha - \beta) \tag{10.24}$$

In a certain system normally A, B, α, β, P_r, Q_r and V_r are known; it is required to find out the sending end voltage. The above expression (10.24) can be made use of for the purpose.

Or sometimes the sending end and receiving end voltages are fixed and A, B, α, β, P_r and Q_r (load) are given; it is required to find out the capacity of the phase modifier. In this case the required quantity is Q_r. It is to be noted here that for this problem Q_r is the net reactive power at the receiving end and not the reactive power for the load as in the first type of problem as stated above. So if the net reactive power required to maintain certain voltages at the two ends is known, the capacity of the phase modifier can be determined.

Graphical Method (Power Circle Diagram)

In the previous section we have studied analytical method of determining the capacity of phase modifiers for certain system conditions. We will here discuss graphical methods which are easier to work with for such problems. First of all we will describe here receiving, end power circle diagram.

Receiving End Power Circle Diagram: Consider again equation in general circuit constants

$$V_s = AV_r + BI_r$$

The phasor diagram for this expression is given in Fig. 10.11.

In this phasor diagram except for I_r all other phasors represent voltages. We are interested in studying the power diagram, that too receiving end power diagram. The voltage phasor diagram must be multiplied by suitable value of current. If we multiply equation (10.19) by V_r/B we get as

$$\frac{V_s V_r}{B} = \frac{AV_r^2}{B} + V_r I_r \tag{10.25}$$

We find that the last term in the expression represents the volt-amperes at the receiving end; this is what is required. Since V_r is taken as the reference, the effect of multiplying the equation (10.19) by V_r/B will be to change the magnitude of all the phasors in Fig. 10.11 by $|V_r|/|B|$ and rotate them clockwise through an angle $\angle(0 - \beta°)$ *i.e.*, $-\beta°$. As a result of this, Fig. 10.11 becomes Fig. 10.12. In Fig. 10.12 the origin is shifted from 0 to n for the reason which is clear from the Fig. 10.12 itself.

Now when origin is shifted to n and phasor BI_r is to be rotated through $-\beta°$, this phasor will subtend an angle $-\phi_r$ with the horizontal axis. V_r^2/B will subtend an angle $-\beta$ with the horizontal axis. Now with respect to V_r^2/B other phasors AV_r^2/B and V_sV_r/B are drawn as shown in Fig. 10.12.

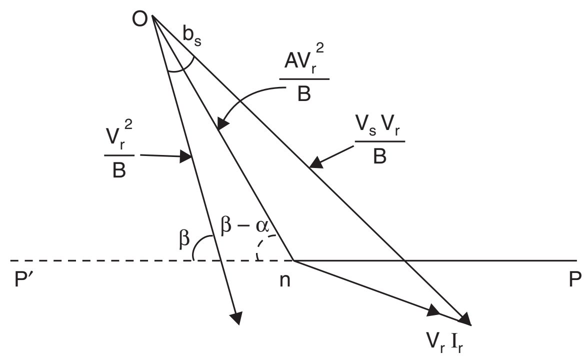


Fig. 10.12 Phasor diagram of Fig. 10.11 multiplied by V_r/B.

The horizontal component $V_rI_r \cos \phi_r$ of V_rI_r along np gives the active component of power and the vertical component $V_rI_r \sin \phi_r$ the reactive component.

The phasor diagram in Fig. 10.12 corresponds to an inductive load. Let V_r subtend an angle θ_1 and I_r an angle θ_2 with respect to some reference axis as shown in Fig. 10.13.

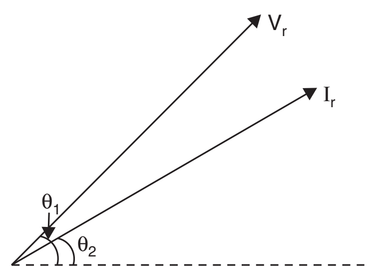


Fig. 10.13 Phasor diagram for an inductive load.

$$V_r = |V_r| \angle\theta_1 \quad \text{and} \quad I_r = |I_r| \angle\theta_2$$

$$\begin{aligned} V_rI_r^* &= |V_r|\ |I_r| \angle(\theta_1 - \theta_2) \\ &= |V_r|\ |I_r|\ [\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2)] \\ &= P + jQ \end{aligned}$$

So we see that when the load is inductive the reactive power is positive. Of course if the product of conjugate of V_r and I_r is taken then it willl be

$$V_r^* I_r = P - jQ$$

But most power system engineers prefer to use positive sign to indicate lagging vars and we will follow this convention here. In order to follow this convention the phasor diagram in Fig. 10.12 will have to be rotated through 180° along pp' and thus will now become as in Fig. 10.14.

However, it is to be noted that the phasor diagram of current and voltage are not affected by the convention used for the sign of reactive power.

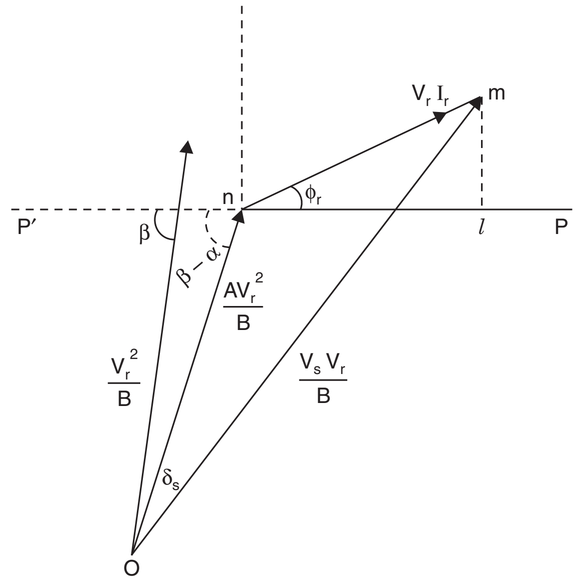


Fig. 10.14 Phasor diagram of Fig. 10.12 rotated through 180° along pp′.

Let us now understand some properties of this power diagram. For a particular receiving end voltage the location of point O is fixed and this forms the centre of the receiving end power circle diagram. The radius of the circle corresponds to Om and has different values for different sending end voltages. Therefore, for a particular receiving end voltage and different sending end voltages we get concentric circles with centre at O. Point m corresponds to the operating point at the receiving end of the transmission line. From the diagram it is seen that angle δ_s is between V_sV_r/B and V_r^2/B *i.e.*, the angle is between V_s and V_r as V_r/B is common in both V_sV_r/B and V_r^2/B. This angle δ_s is known as the load angle or torque angle.

For different receiving end voltage, it can be seen that the diagrams will be eccentric circles.

As is said earlier, the following two types of problems can be solved using receiving end power circle diagram:

1. Given P_r, ϕ_r, V_r and line constants, determine sending end voltage. For this problem proceed as follows:

Normally in a 3-phase system, 3-phase power is specified and L-L voltage is given. The power circle diagram that we have obtained we started with phase quantities. We could make use of 3-phase quantities also and in that case the power will be 3-phase power and voltage line to line. The procedure we are going to describe is say on per phase basis.

(*a*) Let P be the 3-phase power and V_L the line to line voltage at the receiving end, then

$$P_r = \frac{P}{3} \text{ and } V_r = \frac{V_L}{\sqrt{3}}$$

(*b*) Calculate $\dfrac{|A||V_r^2|}{|B|}$.

(*c*) Now looking at the relative values of P_r and $\frac{|A||V_r^2|}{|B|}$ choose a suitable scale.

(*d*) Draw a horizontal line and fix a point n on this line. From this point draw a line subtending an angle ϕ_r as shown in Fig. 10.14. Then after reducing P_r to scale cut the horizontal line at l by an amount equal to P_r. Draw a vertical line such that it cuts the slanted line (at angle ϕ_r) at m. Thus the operating point m is obtained.

(*e*) Now from the point n, draw a line no equal to $\frac{|A||V_r^2|}{|B|}$ (reduced to scale) at angle $(\beta - \alpha)$ in the third quadrant.

(*f*) Measure the length Om. Convert this to MVA or kVA depending upon the scale chosen. Then

$$Om \times \text{scale} = \frac{|V_s||V_r|}{|B|}$$

Here $|V_r|$ and $|B|$ are known, $|V_s|$ can be obtained. This $|V_s|$ is the line to neutral voltage. The sending end line to line voltage will be $\sqrt{3}\, V_s$.

The first problem as we see corresponds to an unregulated system where sending end voltage can take any value depending upon the load condition.

In case we want to fix up both the sending end and receiving end voltages it becomes a constant voltage or a regulated system in which we have to instal some reactive power injecting device at the receiving end. The second problem is then defined as follows:

2. Given V_s, V_r, P_r, ϕ_r and line constants, determine the capacity of the phase modifier (Fig. 10.15).

For this problem repeat the procedure from (*a*) to (*c*) as in the previous problem.

Calculate $\frac{|V_s||V_r|}{|B|}$ and draw to scale an arc of a circle with centre at O. This arc can intersect the vertical load line ml at any one of the three positions m' as shown in Fig. 10.15 *i.e.*, above m or in between m and the horizontal line or below the horizontal line.

If m' lies above m the phase modifier is said to be under-excited and if it lies below m, the phase modifier is said to be over-excited. In all cases m' gives the condition at the receiving end.

The capacity of the phase modifier in all cases will be mm'. The VArs requirements of the load are fixed and are equal to ml. Therefore, the division of VArs in the three situations is as follows:

(*i*) When m' is above m. The capacity of the phase modifier is mm'. The VArs transmitted over the line are $m'l$, *i.e.*, in order to have sending end voltage corresponding to this operating point, transmission line has to transmit not only the VArs required by the load but it has to supply VArs to the synchronous phase modifier equal to mm' *i.e.*, the phase modifier takes the lagging VArs from the system which means it is under-excited.

(*ii*) When m' lies between m and l. In order to meet the VArs requirements of the load mm' is supplied by the phase modifier and $m'l$ have to be transmitted over the line. The phase modifier is over-excited.

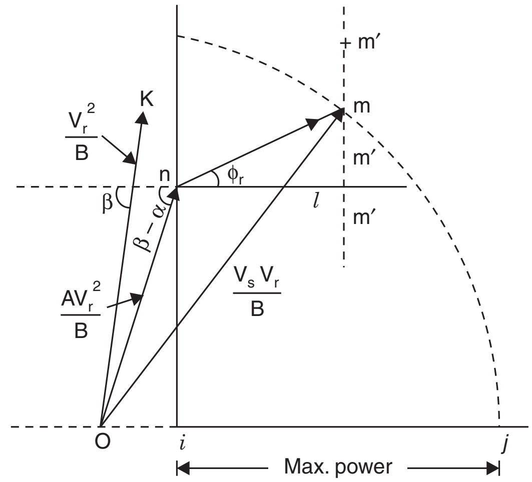


Fig. 10.15 Power circle diagram (receiving end) indicating various operating conditions.

(*iii*) When m' lies below the horizontal axis. The capacity of the phase modifier is mm'. Here the phase modifier not only supplied VArs to the load but it supplies lm' VArs to the transmission line also to get this operating point. The phase modifier is over-excited.

The power factor of the load is fixed and is given by cos ϕ_r. The power factor of the transmission line at the receiving end will depend upon the position of the operating point m' with respect to the horizontal axis. The power factor angle in all cases is the angle between the line nm' and the horizontal axis. If the point m' lies above the horizontal axis the power factor is lagging and if it lies below the horizontal axis it is leading.

To find out the load angle or torque angle δ_s, draw a horizontal line passing through O and then from O draw a line subtending an angle β. This line corresponds to $|V_r^2|/|B|$. Cut this line to scale equal to $|V_r^2|/|B|$. The angle between Ok and Om' gives the torque angle for regulated systems and for unregulated systems the angle between Ok and Om is the torque angle δ_s.

Example 10.3: The generalized circuit constants of a transmission line are as follows:

$$A = D = 0.895 \angle 1.4°, B = 182.5 \angle 78.6° \text{ ohms}$$

(*i*) If the line supplies a load of 50 MW at 0.9 p.f. and 215 kV, find the sending end voltage and hence the regulation of the line.

(*ii*) For a load of 80 MW at 0.9 p.f. lag, 215 kV, determine the reactive power supplied by the line and by the synchronous capacitor if the sending end voltage is 236 kV. Also determine the p.f. of the line at the receiving end.

(*iii*) Determine the maximum power that can be transmitted if the sending and receiving end voltages are as in (*ii*).

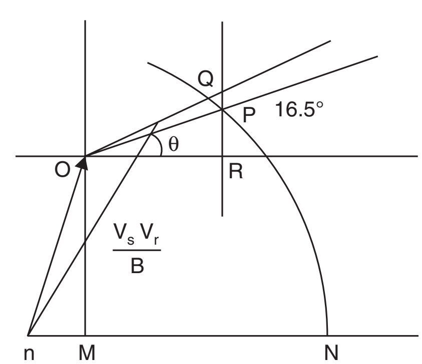


Fig. E.10.3

Solution: (*i*) $\dfrac{AV_r^2}{B} = \dfrac{0.895 \times 215^2}{182.5}$ = 226.7 MVA

Assume a scale of 1 cm = 50 MW.

$$\beta - \alpha = 78.6 - 1.4 = 77.2°$$

$$\cos^{-1} 0.9 = 25.84°$$

To scale, $$\frac{AV_r^2}{B} = \frac{226.7}{50} = 4.5 \text{ cm}$$

$$\frac{V_s V_r}{B} = 5.3 \text{ cm} = 265 \text{ MW (from the diagram)}$$

Since V_r = 215 kV and B = 182.5,

$$\therefore \qquad V_s = \frac{265 \times 182.5}{215} = 224.9 \text{ kV}$$

$$\therefore \qquad V_r' = \frac{V_s}{A} = \frac{224.9}{0.895} = 251 \text{ kV}$$

$$\therefore \qquad \% \text{ regulation} = \frac{251 - 215}{215} \times 100 = 16.74\%.$$ **Ans.**

(*ii*) When load is 80 MW at 0.9 p.f. and sending end voltage is 236 kV

$$80 \text{ MW} = 1.6 \text{ cm}$$

$$\frac{V_s V_r}{B} = \frac{236 \times 215}{182.5} = 278 \text{ MVA} = 5.56 \text{ cms}$$

RQ from Fig. E.10.3 is the total reactive MVArs required by the load. Out of total *RQ*, *PR* is supplied by the line and *QP* is supplied by the phase modifier.

$$QP = 0.25 \text{ cm} = 12.5 \text{ MV Ar}$$

$$PR = 0.50 \text{ cm} = 25 \text{ MV Ar}$$

$$\cos \theta = 0.958.$$ **Ans.**

(*iii*) The maximum power that can be transmitted corresponds to MN which is 4.55 cms and therefore the maximum power that can be transmitted is 227.5 MW.

10.3 SENDING END POWER CIRCLE DIAGRAM

The sending end power circle diagram is developed on the same line as the receiving end. The voltage equation for this is

$$V_r = DV_s - BI_s \tag{10.26}$$

Taking V_s as the reference, the phasor diagram (Fig. 10.16) to express the above equation is as follows:

Multiplying equation (10.26) by $-V_s/B$ the equation becomes

$$\frac{-V_r V_s}{B} = \frac{-DV_s^2}{B} + V_s I_s \tag{10.27}$$

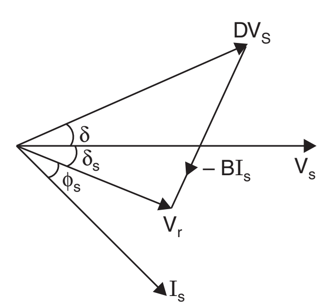


Fig. 10.16 Phasor diagram for equation (10.26)

This operation results in changing the magnitude of all the phasors in Fig. 10.16 by an amount $|V_s|/|B|$ and rotating them through

$$\frac{-V_s}{B} = \frac{|V_s|\angle 180°}{|B|\angle\beta} = \frac{|V_s|}{|B|}\angle(180-\beta)°$$

(180 – β)° clockwise. This results in Fig. 10.17.

For the same reasoning as for the receiving end power circle diagram this diagram in Fig. 10.17 is rotated through 180° along the horizontal axis and results into Fig. 10.18.

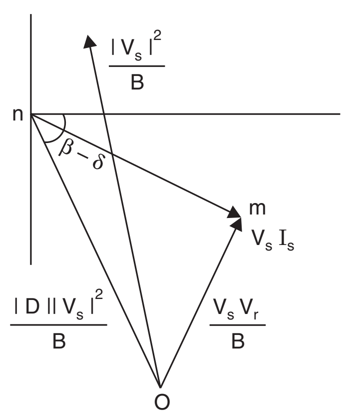


Fig. 10.17 Phasor diagram of Fig. 10.16 multiplied by – V_s/B.

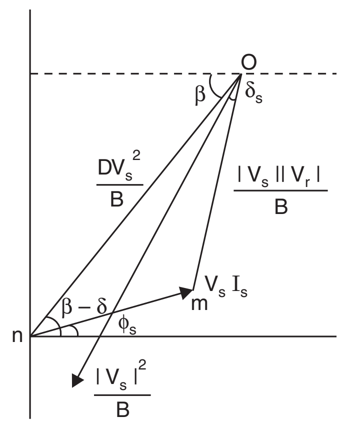


Fig. 10.18 Phasor diagram of Fig. 10.17 rotated through 180° along horizontal axis.

Here *O* is the centre of the sending end power circle and *Om* is the radius of length $(|V_s|\ |V_r|)/|B|$. For a fixed sending end voltage since the location of the centre is fixed, there will be concentric circles for different values of receiving end voltages.

It is possible to find out the maximum power that can be transmitted over the line both at the receiving end and sending end using the receiving end and sending end power circle diagrams respectively. The difference in power at the two ends is known as transmission loss and is due to the losses on the overhead line.

For finding out the maximum power that can be transmitted at the receiving end refer to Fig. 10.15.

Example 10.4: A 3-phase overhead line has per phase resistance and reactance of 6 ohm and 20 ohm respectively. The sending end voltage is 66 kV while the receiving end voltage is maintained at 66 kV by a synchronous phase modifier. Determine the KV Ar of the modifier when the load at the receiving end is 75 MW at p.f. 0.8 lagging; also determine the maximum load that can be transmitted.

Solution: $\alpha = 0°,\ \beta = 73.3°$

$A = 1,\ B = 20.88\ \Omega$

Using equation (10.24) and substituting 3-phase quantities, we have

$$66^2 = 66^2 + 20.88^2\left[\left(\frac{75}{66}\right)^2 + \left(\frac{Q_r}{66}\right)^2\right]$$

$$+\ 2 \times 1 \times 20.88 \times 75 \cos 73.3 + 2 \times 1 \times 20.88 \times Q_r \sin 73.3$$

$$0 = 436\left[1.29 + \left(\frac{Q_r}{66}\right)^2\right] + 3132 \cos 73.3$$

or $$0 = 562.44 + 0.1Q_r^2 + 900 + 40\,Q_r$$

or $$0 = 0.1\,Q_r^2 + 40\,Q_r + 1462.44$$

or $$Q_r^2 + 400\,Q_r + 14624 = 0$$

$$\sqrt{b^2 - 4\,ac} = \sqrt{160000 - 4 \times 14624}$$

$$= 318.6$$

$$Q_r = \frac{-400 + 318.6}{2}$$

$$Q_r = -40.7$$

Since Q_r is negative, the phase modifier supplies 40.7 MV Ar in addition to the MV Ar requirements of the load *i.e.*, the phase modifier capacity is

$$= 40.7 + \frac{75}{0.8} \times 0.6$$

$$= 40.7 + 56.25$$

$$= 96.95 \text{ MV Ar.} \quad \textbf{Ans.}$$

Now maximum power transmitted is given as

$$\frac{V_sV_r}{B} - \frac{AV_r^2}{B} \cos(\beta - \alpha)$$

Here $P_{\max} = \dfrac{V_r^2}{B}(1 - \cos\beta) = 148.67$ MW. **Ans.**

PROBLEMS

10.1. A 3-phase induction motor delivers 500 HP at an efficiency of 90% when the operating p.f. is 0.8 lag. A loaded synchronous motor with a power consumption of 120 kW is connected in parallel with the induction motor. Calculate the necessary kVA and the operating p.f. of the synchronous motor if the overall p.f. is to be unity.

10.2. A 3-phase line having an impedance of $(5 + j20)$ ohm per phase delivers a load of 30 MW at a p.f., of 0.8 lag and voltage 33 kV. Determine the capacity of the phase modifier required to be installed at the receiving end if the voltage at the sending end is to be maintained at 33 kV.

10.3. Determine the tap ratio in problem 2 if the receiving end voltage is to be maintained at 0.92 p.u. of the sending end voltage. The line is fed through a 33/220 kV transformer. Assume $t_st_r = 1$.

10.4. Determine the transformer tap ratios when the receiving end voltage is equal to the sending end voltage, the high voltage line operates at 230 kV and transmits 80 MW at 0.8 p.f. and the impedance of the line is $(40 + j150)$ ohms. Assume $t_st_r = 1$.

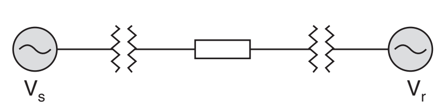


Fig. P.10.4

10.5. A 132 kV line is fed through 33 kV/132 kV transformer from a constant 33 kV supply as shown in Fig. P.10.5. The total impedance of the line and transformers at 132 kV is (25 + *j*60) ohms. Both the transformers are equipped with tap changing facilities which are so arranged that the product of the two off-nominal settings is unity. If the load on the system is 100 MW at 0.6 p.f. lag, calculate the setting of the tap changer required to maintain the voltage of the load bus bars at 33 kV.

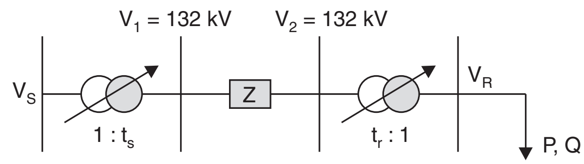


Fig. P.10.5

10.6. A 3-phase line has an impedance of (20 + *j*60) ohm per phase. The sending end voltage is 142 kV while the receiving end voltage is maintained at 132 kV for all loads by an automatic phase modifier. If the kV Ar of the modifier has the same value for zero load as for a load of 50 MW, determine the rating of the modifier and the p.f. of this load.

10.7. A typical line has the following parameters:

$$A = D = 0.96 \angle 1.0°,\ B = 100 \angle 80°.$$

(*i*) If the line supplies a load of 30 MW at 0.8 p.f. lag and 110 kV, find the sending end voltage and hence the regulation of the line.

(*ii*) For a load of 50 MW at 0.8 p.f. lag, 110 kV, find the reactive power supplied by the line and by the synchronous capacitor if the sending end voltage is 120 kV. Also, determine the p.f. of the line at the receiving end.

(*iii*) Find the maximum power that can be transmitted if the sending and receiving end voltages are as in (*ii*).

(*iv*) Find the power and p.f. of the load if the voltages at the two ends are 110 kV with a phase difference of 20°.

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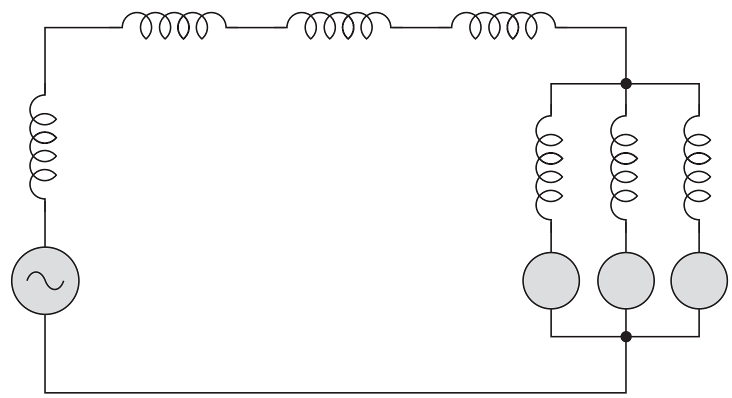
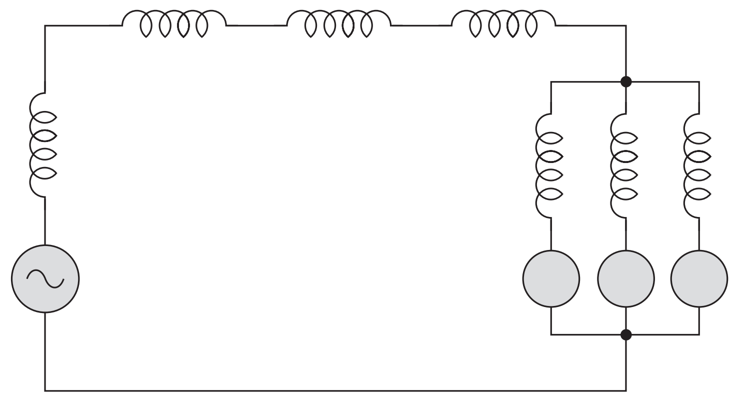


11

NEUTRAL GROUNDING



11

Neutral Grounding

INTRODUCTION

The neutral grounding is an important aspect of power system design because the performance of the system in terms of the short circuits, stability, protection etc. is greatly affected by the state of the neutral. In most of the modern high voltage systems the neutral of the system is solidly grounded *i.e.*, the neutral is connected directly to the ground without any intentional impedance between the neutral and the ground. Generally the neutral of the generator is connected through resistance to limit the stator short circuit current and also for stability reasons. The advantages of neutral grounding are:

(*i*) Voltages of the phases are limited to phase to ground voltages.

(*ii*) The high voltages due to arcing grounds or transient line to ground faults are eliminated.

(*iii*) Sensitive protective relays against line to ground faults can be used.

(*iv*) The over voltages due to lightning are discharged to ground, otherwise there will be positive reflection at the isolated neutral of the system.

The following are the advantages of operating with isolated neutral:

(*i*) It is possible to maintain the supply with a fault on one line.

(*ii*) Interference with communication lines is reduced because of the absence of zero sequence currents.

11.1 EFFECTIVELY GROUNDED SYSTEM

The term effectively grounded is now used instead of the old term solidly grounded system for reason of definition. The AIEE Standard No. 32, May 1947, defines the effective grounding as follows:

A system or a portion of a system can be said to be effectively grounded when for all points on the system or specified portion thereof the ratio of zero sequence reactance to positive sequence reactance (X_0/X_1) is not greater than three and the ratio of zero sequence resistance to positive sequence reactance is not greater than one for any condition of operation and for any amount of generator capacity. The effective grounded systems are less expensive than any other type of grounding for all operating voltages because for such a system the maximum line to ground voltage during a fault does not exceed 80% of the line voltage whereas for all other groundings the voltage of the healthy phases rises to about 100% line-to-line voltage.

11.2 UNGROUNDED SYSTEM

The system has been analysed in Chapter 12. A summary of this is given here.

Under balanced conditions the potential of the neutral is held at ground due to the presence of the shunt capacitance of the system (Fig. 11.1 (*a*)). Under balanced condition the vector diagram is given in Fig. 11.1 (*b*).

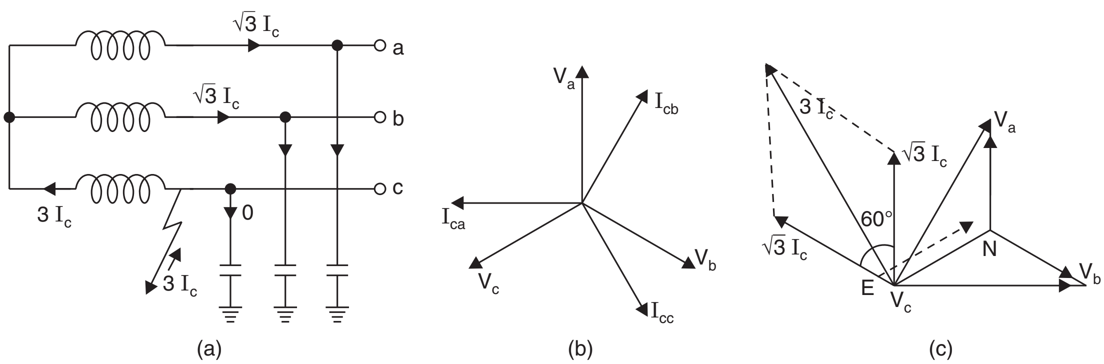


Fig. 11.1 (*a*) Isolated neutral system: fault on phase *c*; (*b*) Phasor diagram for healthy system; and (*c*) Phasor diagram for fault on phase *c*.

In case of a line to ground fault on phase *c* vector diagram becomes as shown in Fig. 11.1 (*c*). A charging current of three times the per phase charging current flows. The voltage of the healthy phases rise to $\sqrt{3}\,V_{ph}$. The presence of inductance and capacitance in the system leads to what is known as Arcing Grounds and the voltage of the system may rise to dangerously high values as explained in Chapter 12. These voltages can be eliminated by connecting an inductance of suitable value between the neutral and the ground. If the value of the inductive reactance is such that the fault current I_L balances exactly the charging current, then the grounding is known as resonant grounding or ground fault neutralizer or Peterson coil.

11.3 RESONANT GROUNDING

It is desired here to calculate the value of inductance such that $3I_C = I_L$ so that theoretically there is no current in the fault or it is so small that the arc will not maintain itself and the fault is extinguished (Fig. 11.2).

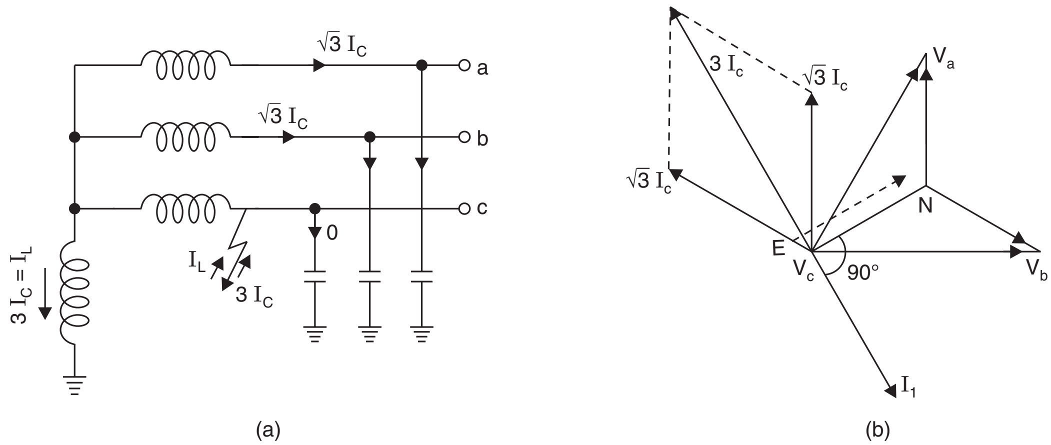


Fig. 11.2 Resonant grounded 3-phase system. Fault on phase *c*, (*b*) Phasor diagram for (*a*).

Let V_{ph} be the line to ground voltage of the system. The voltage of the healthy phases during *L-G* fault on one of the phases will be $\sqrt{3}V_{ph}$. If C is the capacitance to ground of each phase, then the charging current will be $3V_{ph}\omega C$.

If L is the inductance to be connected between the neutral and the ground, then

$$I_L = \frac{V_{ph}}{\omega L}$$

For balance condition

$$I_L = 3V_{ph}\omega C = \frac{V_{ph}}{\omega L}$$

or

$$L = \frac{1}{3\omega^2 C}$$

The use of the resonant grounding will reduce the line interruption due to transient line to ground faults which will not be possible with other forms of grounding. Also the tendency of a single phase to ground fault developing into a two or 3-phase fault will be decreased.

Ground fault neutralizers should not be used where

(*i*) Fully graded insulation transformers are used as the neutrals of such transformers are not sufficiently well insulated.

(*ii*) Auto-transformers having a ratio greater than 0.95 to 1 are used.

The coils of the ground fault neutralizers are ten-minutes time-rated on system where permanent ground faults can be located and removed promptly by ground relays or other suitable means. Otherwise, continuous time-rated neutralizers are used on all other systems. However, if for any reason more current flows through the fault neutralizer a circuit breaker closes after a certain time-lag and the earth-fault current flows through the parallel circuit by-passing the arc suppression coil (Fig. 11.3).

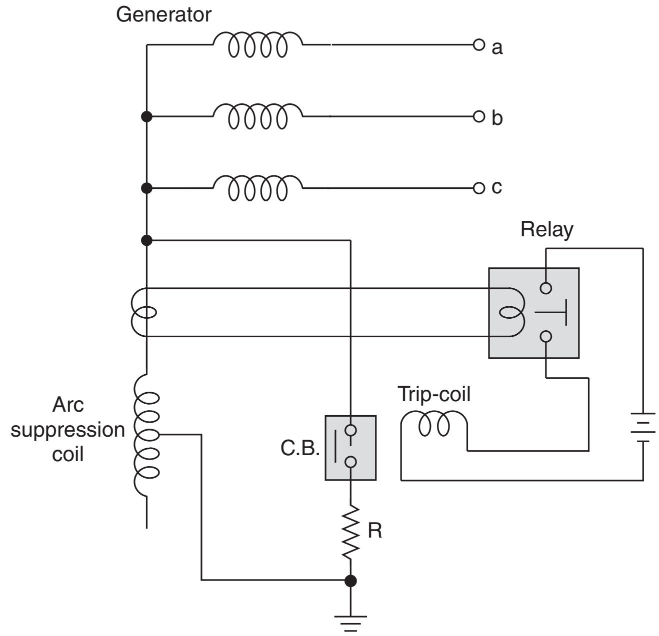


Fig. 11.3 Connection of arc suppression coil.

Circuit breaker (C.B.) is normally open but is closed by the trip coil when the relay operates after a predetermined time. With this the fault current is by-passed through the resistor branch.

Example 11.1: A 132 kV, 3-phase, 50 Hz transmission line 192 km long consists of three conductors of effective diameter 20 mm, arranged in a vertical plane with 4 m spacing and regularly transposed. Find the inductance and kVA rating of the arc suppressor coil in the system.

Solution: The capacitance per phase is given by

$$\frac{2\pi\varepsilon_0}{\ln \frac{d}{r}} \text{ F/metre}$$

$$= \frac{2\pi \times \frac{1}{36\pi} \times 10^{-9}}{\ln \frac{d}{r}} \text{ F/metre}$$

$$= \frac{10^{-9}}{18 \ln \frac{d}{r}} = \frac{10^{-9}}{18 \ln \frac{\sqrt[3]{4 \times 4 \times 8}}{10 \times 10^{-3}}}$$

$$= \frac{10^{-9}}{18 \times 6.2} = \frac{10^{-10}}{1.8 \times 6.2} = 0.896 \times 10^{-11} \text{ F/m}$$

or $\quad 0.896 \times 10^{-11} \times 192 \times 10^3 = 172 \times 10^{-8} \text{ F} = 1.72\ \mu\text{F}$

Now $$\omega L = \frac{1}{3\omega C}$$

or $$L = \frac{1}{3\omega^2 C} = \frac{10^6}{3 \times 314^2 \times 1.72}$$

$$= \frac{10^6 \times 10^{-4}}{5.16 \times 9.86}$$

$$= \frac{100}{5.16 \times 9.86}$$

$$= 1.97 \text{ henry.}$$

$\therefore$ MVA rating of the suppressor coil is

$$\frac{V^2}{3\omega L} = \frac{132 \times 132}{3 \times 314 \times 1.97}$$

$$= 9.389 \text{ MVA per coil.}$$ **Ans.**

Example 11.2: A 50 Hz overhead line has line to earth capacitance of 1 μF. It is decided to use an earth fault neutralizer. Determine the reactance to neutralise the capacitance of (*i*) 100% of the length of the line, (*ii*) 90% length of the line, and (*iii*) 80% of the length of the line.

Solution: (*i*) The inductive reactance of the coil for 100% neutralizer will be

$$\omega L = \frac{1}{3\omega C} = \frac{1}{3 \times 314 \times 1 \times 10^{-6}} = \frac{10^6}{3 \times 314}$$

$$= 1061 \text{ ohms.}$$

(*ii*) The inductive reactance for neutralizing 90% of the capacitance

$$\omega L = \frac{1}{3\omega C} = \frac{10^6}{3 \times 314 \times 1 \times 0.9} = 1179 \text{ ohms}$$

(*iii*) For 80% neutralization the inductive reactance is

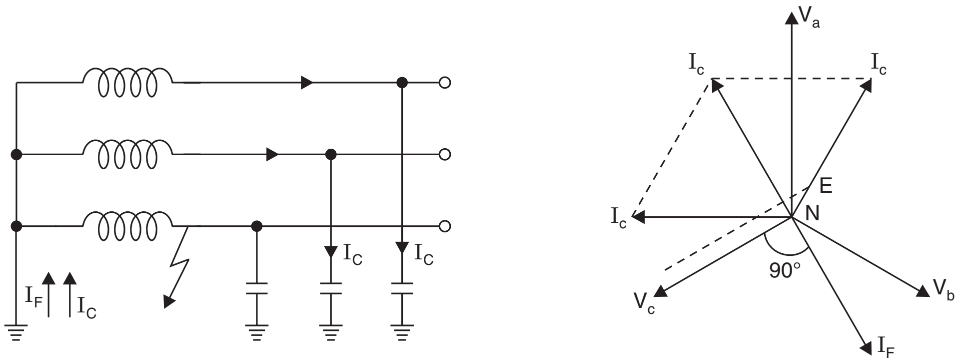
$$\frac{1061}{0.8} = 1326 \text{ ohms.}$$ **Ans.**

11.4 METHODS OF NEUTRAL GROUNDING

There are various methods of grounding the neutral of the system. They are: (*i*) Solid grounding; (*ii*) Resistance grounding; (*iii*) Reactance grounding; (*iv*) Voltage transformer grounding; and (*v*) Zig-zag transformer grounding.

Solid Grounding or Effective Grounding (Fig. 11.4): Consider *L-G* fault on phase *c*. The neutral and terminal *c* are at earth potential. The reversed vector is shown at V_c. The voltage of the healthy phases remains unchanged *i.e.*, phase to ground voltages and the currents are as shown in Fig. 11.4 (*b*). The charging current will be fully eliminated. Since in this system of grounding the voltage of the healthy phases in case of a line to ground fault does not exceed 80% of the *L-L* and is much less as compared to other forms of grounding, the equipments for all voltage classes are less expensive. An 84% lightning arrester instead of 105% can be used.

On system 115 kV and above additional savings are possible because of the transformers with the insulation graded towards the neutral are less costly.

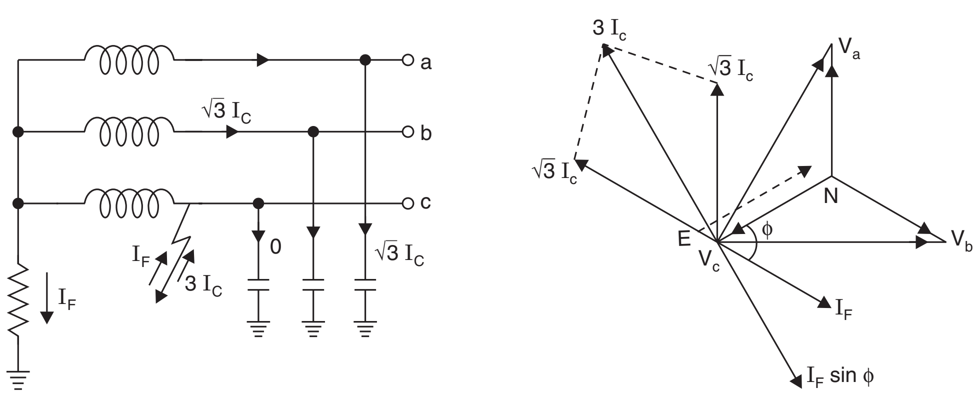


(a) (b)

Fig. 11.4 (*a*) Solidly grounded system (*b*) Phasor diagram.

Resistance Grounding (Fig. 11.5): The value of the resistance commonly used is quite high (in order to limit power loss in resistor during *L-G* fault) as compared with the system reactance. With the increase in operating voltage the value of resistance required for grounding also increases (decreases with capacity) so as to limit (25% of full load current) the short circuit current during line to ground faults. Resistance grounding is normally used where the charging current is small *i.e.*, for low voltage short length overhead lines.

Resistance grounding reduces the arcing ground hazards and it permits ready relaying of ground faults. In certain situations resistance grounding has helped in improving the stability of the system during ground fault by replacing the power dropped, as a result of low voltage, with an approximately equal power loss in the resistor, thus reducing the advance in phase of the generators.



(a) (b)

Fig. 11.5 (*a*) Resistance grounded system, (*b*) Phasor diagram.

Reactance Grounded System: A reactance grounded system is one in which the neutral is grounded through impedance which is highly reactive. In fact whether a system is solidly grounded or reactance grounded depends upon the ratio of X_0/X_1.

For reactance grounded system $\dfrac{X_0}{X_1} > 3.0$.

For solid grounded system $\dfrac{X_0}{X_1} < 3.0$.

When a neutral is solidly grounded, but if $X_0/X_1 > 3$, the system is presumed to be reactance grounded rather than solidly grounded. Reactance grounding lies between effective grounding and resonant grounding. The value of reactance required is to keep currents within safe limits. This method of grounding may be used for grounding the neutral of synchronous motors and capacitors and also for circuits having large charging currents.

Earthing Transformers: If a neutral point is required which otherwise is not available (*e.g.*, delta connection, bus bar points etc.), a zig-zag transformer is used. These transformers do not have secondary winding (Fig. 11.6). Each limb of the transformer has two identical windings wound differentially such that under normal conditions that total flux in each limb is negligibly small and, therefore, the transformer draws very little magnetising current. The grounding transformers are of short time rating usually 10 seconds to 1 minute. Therefore, the sizes of such transformers are small as compard to the power transformers of the same ratings.

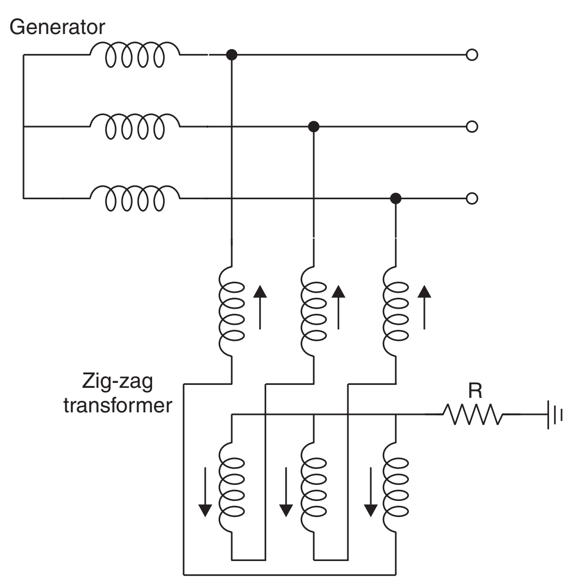


Fig. 11.6 Zig-zag transformer for neutral grounding.

If a zig-zag transformer is not available, a star delta transformer can be used without loading the delta side as shown in Fig. 11.7.

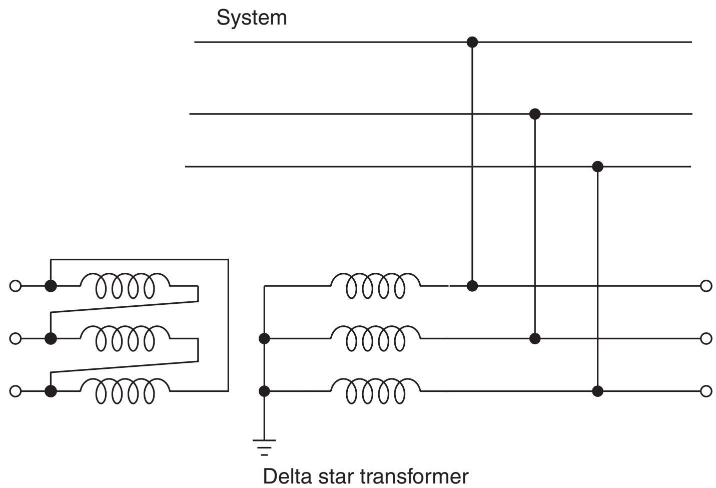


Fig. 11.7 Star delta transformer grounding.

11.5 GENERATOR NEUTRAL BREAKER

When a line to ground fault occurs the generator armature and field circuit breakers are tripped and the input to the prime mover is shut off. With these operations the current through the fault does not necessarily stop immediately because a certain time is required for the generator field flux to decay. The fault current can be reduced to a very low value (as determined by capacitance effects) immediately after the fault, if a generator neutral breaker is employed and it is also tripped simultaneously along with the field and armature breakers (Fig. 11.8). In case the value of the neutral impedance is very high and the fault current is limited, there is no need for a neutral breaker.

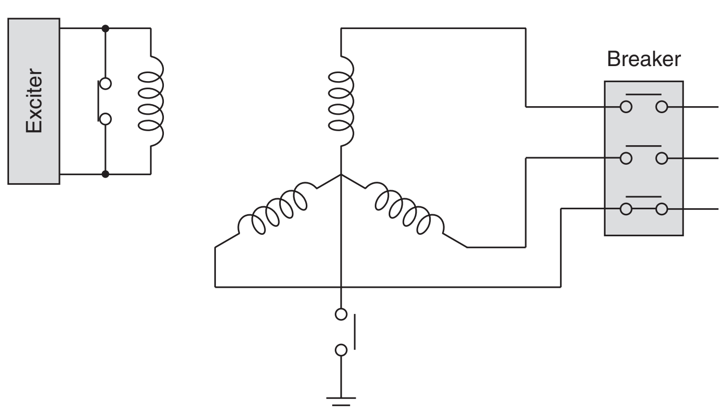


Fig. 11.8 Schematic diagram of generator switching.

11.6 GROUNDING PRACTICE

(*i*) One grounding is normally provided at each voltage level. Between generation and distribution, there are various voltage levels; it is desirable to have ground available at each voltage level.

(*ii*) The generators are normally provided with resistance grounding and synchronous motors or synchronous capacitors are provided with reactance grounding.

(*iii*) Where several generators are connected to a common neutral bus, the bus is connected to ground through a single grounding device. Disconnect switches can be used to ground the desired generators to the neutral bus.

(*iv*) Where several generators are operating in parallel, only one generator neutral is grounded. This is done to avoid the interference of zero sequence currents. Normally two grounds are available in a station but only one is used at a time. The other is used when the first generator is out of service.

(*v*) For low voltages up to 600 volts and for high voltages above 33 kV solid grounding is used whereas for medium voltages between 3.3 kV and 33 kV resistance or reactance grounding is used.

PROBLEMS

11.1. What are the various methods of neutral grounding ? Compare their performance with respect to (*i*) protective relaying, (*ii*) fault levels, (*iii*) stability, (*iv*) voltage levels of power systems.

11.2. Explain the phenomenon of 'Arcing grounds' and suggest the method to minimise the effect of this phenomenon.

11.3. Discuss the advantages of (*i*) grounding the neutral of the system, (*ii*) keeping the neutral isolated.

11.4. A transmission line has a capacitance of 0.1 μF per phase. Determine the inductance of Peterson coil to neutralize the effect of capacitance of (*i*) complete length of line, (*ii*) 97% of the line, (*iii*) 90% length of the line. The supply frequency is 50 Hz.

11.5. A 132 kV, 50 Hz, 3-phase, 100 km long transmission line has a capacitance of 0.012 μF per km per phase. Determine the inductive reactance and kVA rating of the arc suppression coil suitable for the line to eliminate arcing ground phenomenon.

11.6. A 132 kV, 3-phase, 50 Hz overhead line of 100 km length has a capacitance to earth of each line of 0.01 μF per km. Determine inductance and kVA rating of the arc suppression suitable for this line.

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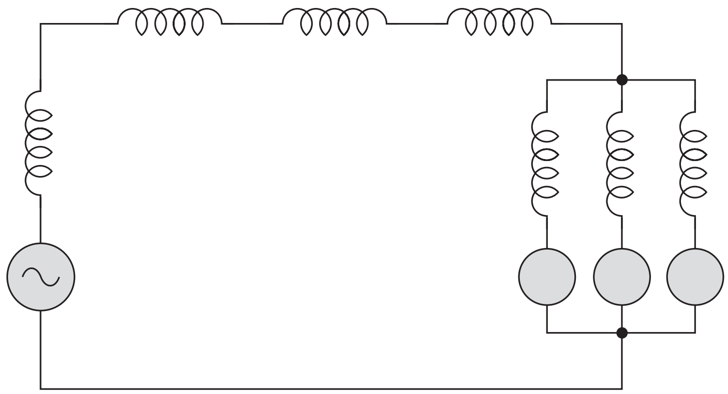
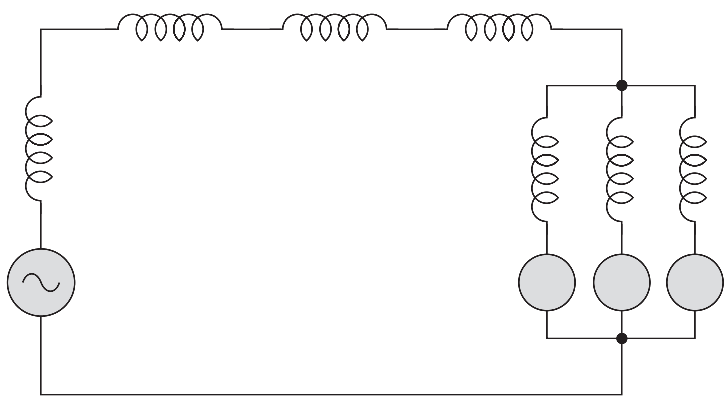


12

TRANSIENTS IN POWER SYSTEMS



12

Transients in Power Systems

INTRODUCTION

Transients phenomenon is an aperiodic function of time and does not last longer. The duration for which they last is very insignificant as compared with the operating time of the system. Yet they are very important because depending upon the severity of these transients, the system may result into black out in a city, shut down of a plant, fires in some buildings, etc.

The power system can be considered as made up of linear impedance elements of resistance, inductance and capacitance. The circuit is normally energized and carries load until a fault suddenly occurs. The fault, then, corresponds to the closing of a switch (or switches, depending upon the type of fault) in the electrical circuit. The closing of this switch changes the circuit so that a new distribution of currents and voltages is brought about. This redistribution is accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high. It is very important to realize that this redistribution of currents and voltages cannot take place instantaneously for the following reasons:

1. The electromagnetic energy stored by an inductance L is $\frac{1}{2}LI^2$, where I is the instantaneous value of current. Assuming inductance to be constant the change in magnetic energy requires change in current which an inductor is opposed by an e.m.f. of magnitude $L \frac{dI}{dt}$. In order to change the current instantaneously $dt = 0$ and therefore $L\frac{dI}{0}$ is infinity, *i.e.,* to bring about instantaneous change in current the e.m.f. in the inductor should become infinity which is practically not possible and, therefore, it can be said that the change of energy in an inductor is gradual.

2. The electrostatic energy stored by a capacitor C is given by $\frac{1}{2}CV^2$, where V is the instantaneous value of voltage. Assuming capacitance to be constant, the change in energy requires change in voltage across the capacitor.

Since, for a capacitor, $\frac{dV}{dt} = \frac{I}{C}$, to bring instantaneous change in voltage, *i.e.,* for $dt = 0$ the change in current required is infinite which again cannot be achieved in practice and, therefore, it can be said that change in energy in a capacitor is also gradual.

There are only two components L and C in an electrical circuit which store energy and we have seen that the change in energy through these components is gradual and, therefore, the redistribution of energy following a circuit change takes a finite time. The third component, the resistance R, consumes energy. At any time, the principle of conservation of energy in an electrical circuit applies, *i.e.*, the rate of generation of energy is equal to the rate of storage of energy plus the rate of energy consumption.

It is clear that the three simple facts, namely,

1. the current cannot change instantaneously through an inductor,
2. the voltage across a capacitor cannot change instantaneously, and
3. the law of conservation of energy must hold good, are fundamental to the phenomenon of transients in electric power systems.

From the above it can be said that in order to have transients in an electrical system the following requirements should be met:

1. Either inductor or capacitor or both should be present.
2. A sudden change in the form of a fault or any switching operation should take place.

There are two components of voltages in a power system during transient period: (*i*) Fundamental frequency voltages, and (*ii*) natural frequency voltages usually of short duration which are superimposed upon the fundamental frequency voltages. There is third component also known as harmonic voltages resulting from unbalanced currents flowing in rotating machines in which the reactances in the direct and quadrature axes are unequal.

Natural frequency voltages appear immediately after the sudden occurrence of a fault. They simply add to the fundamental frequency voltages. Since resultant voltages are of greater importance from a practical viewpoint it will be preferable to speak of the fundamental frequency and natural frequency components simply as a transient voltage. The transient voltages are affected by the number of connections and the arrangements of the circuits.

Transients in which only one form of energy—storage, magnetic or electric is concerned, are called single energy transients, where both magnetic and electric energies are contained in or accepted by the circuit, double energy transients are involved.

12.1 TRANSIENTS IN SIMPLE CIRCUITS

For analysing circuits for transients we will make use of Laplace transform technique which is more powerful and easy to handle the transient problems than the differential equation technique. We will assume here lumped impedances only. The transients will depend upon the driving source also, *i.e.*, whether it is a d.c. source or an a.c. source. We will begin with simple problems and then go to some complicated problems.

1. D.C. Source

(*a*) *Resistance only* (Fig. 12.1 (*a*)): As soon as the switch S is closed, the current in the circuit will be determined according to Ohm's law.

$$I = \frac{V}{R}$$

Now transients will be there in the circuit.

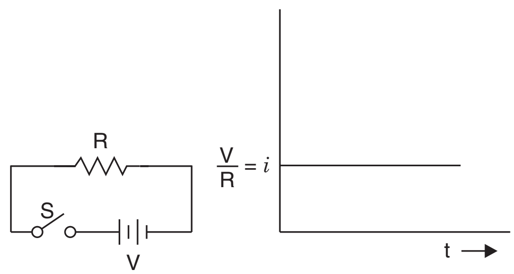


Fig. 12.1(*a*) Resistive circuit.

(*b*) *Inductance only* (Fig. 12.1 (*b*)): When switch S is closed, the current in the circuit will be given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{Ls} = \frac{V}{L} \cdot \frac{1}{s^2}$$

$$i(t) = \frac{V}{L} t$$

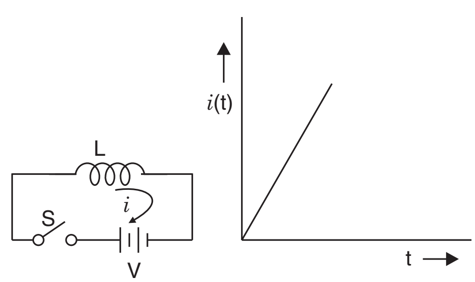


Fig. 12.1 (*b*) Inductive circuit.

This shows that when a pure inductance is switched on to a d.c. source, the current at $t = 0_+$ is zero and this increases linearly with time till for infinite time it becomes infinity. In practice, of course, a choke coil will have some finite resistance, however small; the value of the current will settle down to the value V/R, where R is the resistance of the coil.

(*c*) *Capacitance only* (Fig. 12.1 (*c*)): When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot Cs = VC$$

which is an impulse of strength (magnitude) VC.

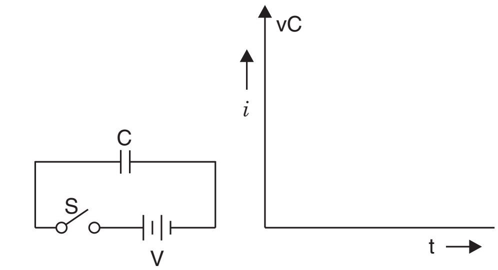


Fig. 12.1 (*c*) Capacitive circuit.

(*d*) *R-L circuit* (Fig. 12.1 (*d*)): When switch *S* is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + Ls} = \frac{V}{s} \cdot \frac{1/L}{s + R/L}$$

$$= \frac{V}{L}\left[\frac{1}{s} - \frac{1}{s + R/L}\right] \cdot \frac{L}{R}$$

$$= \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s + R/L}\right]$$

$$i(t) = \frac{V}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right]$$

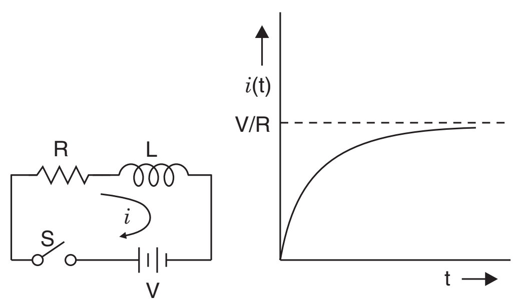


Fig. 12.1(*d*) *R-L* circuit.

The variation of current is shown in Fig. 12.1(*d*). It can be seen from the expression that the current will reach *V/R* value after infinite time. Also it can be seen that the inductor just after closing of the switch behaves as an open circuit and that is why the current at $t = 0_+$ is zero. When $t = L/R$,

$$i(t) = \frac{V}{R}\left(I - \frac{1}{e}\right)$$

$$= I_m \left(I - \frac{1}{e}\right)$$

$$= 0.632\ I_m$$

At time $t = L/R$, the current in the circuit is 63.2% of the maximum value reached in the circuit. This time in seconds is called the time-constant of the circuit. The larger the value of inductance in the circuit as compared with resistance the slower will be the build up of current in the circuit. The energy stored in the inductor under steady state condition will be $\frac{1}{2}LI_m^2$, where $I_m = V/R$.

(*e*) *R-C circuit* (Fig. 12.1(*e*)): After the switch S is closed, current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + 1/Cs}$$

$$= \frac{V}{s} \cdot \frac{(1/RC)Cs}{s + 1/RC} = \frac{V}{R} \cdot \frac{1}{s + 1/RC}$$

$$i(t) = \frac{V}{R} \cdot e^{-t/CR}$$

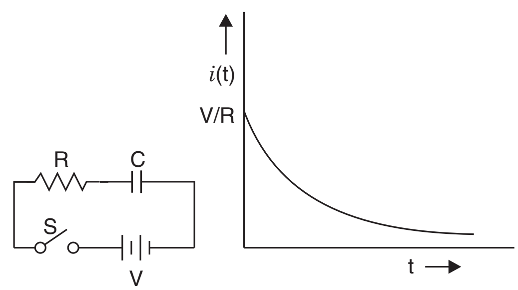


Fig. 12.1(*e*) *R-C* circuit.

It is seen that at $t = 0$, the capacitor acts as a short-circuit to the d.c. source and the current is V/R limited only by the resistance of the circuit. At $t = \infty$ the current in the circuit is zero and the capacitor is charged to a voltage V. The energy stored by the capacitor is $\frac{1}{2}CV^2$.

(*f*) *R-L-C circuit* (Fig. 12.1(*f*)): After the switch S is closed, the current in the circuit is given by

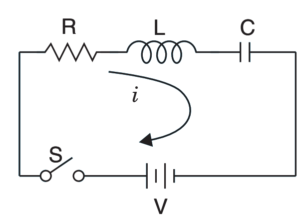


Fig. 12.1(*f*) *R-L-C* circuit.

$$I(s) = \frac{V}{s} \cdot \frac{1}{R + Ls + 1Cs}$$

$$= \frac{V}{s} \cdot \frac{Cs}{RCs + LCs^2 + 1}$$

$$= \frac{V}{s} \cdot \frac{1/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{V}{L} \cdot \frac{1}{\left\{s + \left(\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\}\left\{s + \left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\}}$$

Let $\quad \frac{R}{2L} = a$ and $\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = b$; then

$$I(s) = \frac{V}{L} \cdot \frac{1}{(s + a - b)(s + a + b)}$$

$$= \frac{V}{2bL}\left\{\frac{1}{(s+a-b)} - \frac{1}{(s+a+b)}\right\}$$

$$i(t) = \frac{V}{2bL}\{e^{-(a-b)t} - e^{-(a+b)t}\}$$

There are three conditions based on the value of b:

(*i*) If $\dfrac{R^2}{4L^2} > \dfrac{1}{LC}$, b is real.

(*ii*) If $\dfrac{R^2}{4L^2} = \dfrac{1}{LC}$, b is zero.

(*iii*) If $\dfrac{R^2}{4L^2} < \dfrac{1}{LC}$, b is imaginary.

Case I: When b is real.

The expression for current will be

$$i(t) = \frac{V}{2\sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}}\,.\,L}\left[\exp\left\{-\left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t\right\} - \exp\left\{-\left(\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t\right\}\right]$$

and the variation of current is given in Fig. 12.1(*g*).

Case II: When $b = 0$.

The expression for current becomes

$$i(t) = \frac{V}{2bL}\{e^{-at} - e^{-at}\} \text{ which is indeterminate.}$$

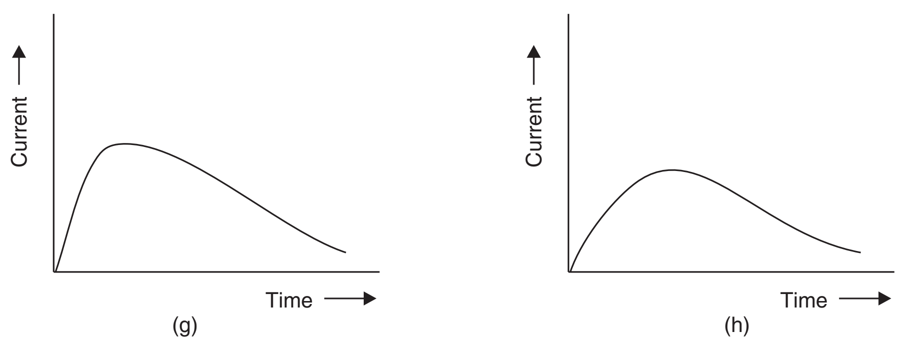


Fig. 12.1(*g*) Waveform when *b* is real (*h*) Waveform when *b* = 0.

Therefore, differentiating $i(t)$ with respect to b gives

$$i(t) = \frac{V}{2L}\,.\,t\{e^{-(a-b)t} + e^{-(a+b)t}\}$$

Now at $b = 0$

$$i(t) = \frac{V}{L}te^{-at} = \frac{V}{L}te^{-(R/2L)t}$$

The variation of current is given in Fig. 12.1(*h*).

Case III: When b is imaginary.

$$i(t) = \frac{V}{2bL}\{e^{-at}\,.\,e^{jkt} - e^{-at}\,.\,e^{-jkt}\} = \frac{V}{2bL}\,e^{-at}\,.\,2\sin kt$$

$$= \frac{V}{2L\sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}}}\,e^{-at}\,.\,2\sin\left(\sqrt{-\frac{R^2}{4L^2} + \frac{1}{LC}}\right)t$$

The wave shape of the current is shown in Fig. 12.1(i).

When b is positive or zero, the variation of current is non-oscillatory whereas it is oscillatory when b is imaginary. Because of the presence of the capacitance, the current in all the three cases dies down to zero value with d.c. source in the circuit.

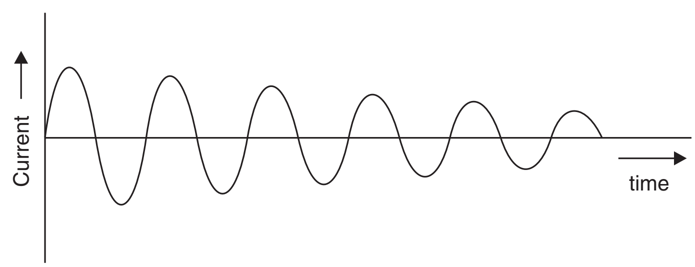


Fig. 12.1(i) Waveform when b is imaginary.

2. A.C. Source

R-L circuit (Fig. 12.2): When switch S is closed, the current in the circuit is given by

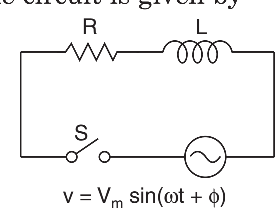


Fig. 12.2 *R-L* circuit connected to an a.c. source.

$$I(s) = \frac{V(s)}{Z(s)} = V_m\left\{\frac{\omega\cos\phi}{s^2+\omega^2} + \frac{s\sin\phi}{s^2+\omega^2}\right\}.\frac{1}{R+Ls}$$

$$= \frac{V_m}{L}\left\{\frac{\omega\cos\phi}{s^2+\omega^2} + \frac{s\sin\phi}{s^2+\omega^2}\right\}.\frac{1}{s+R/L}$$

Let $\dfrac{R}{L} = a$; then

$$I(s) = \frac{V_m}{L}\left\{\frac{\omega\cos\phi}{(s+a)(s^2+\omega^2)} + \frac{s\sin\phi}{(s+a)(s^2+\omega^2)}\right\}$$

Now $$\frac{1}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{1}{s+a} + \frac{a}{s^2+\omega^2} - \frac{s}{s^2+\omega^2}\right\}$$

and $$\frac{s}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{as}{s^2+\omega^2} + \frac{\omega^2}{s^2+\omega^2} - \frac{a}{s+a}\right\}$$

Therefore $$\mathcal{L}^{-1}I(s) = \frac{V_m}{(a^2+\omega^2)L}\left[\omega\cos\phi\left\{e^{-at} + \frac{a}{\omega}\sin\omega t - \cos\omega t\right\}\right.$$

$$\left. + \sin\phi\{a\cos\omega t + \omega\sin\omega t - ae^{-at}\}\right]$$

The equation can be further simplified to

$$i(t) = \frac{V_m}{L\sqrt{a^2 + \omega^2}} \{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)\, e^{-at}\}$$

$$= \frac{V_m}{(R^2 + \omega^2 L^2)^{1/2}} \{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)\, e^{-at}\}$$

where $\theta = \tan^{-1} \dfrac{\omega L}{R}$.

The variation of current is shown in Fig. 12.3.

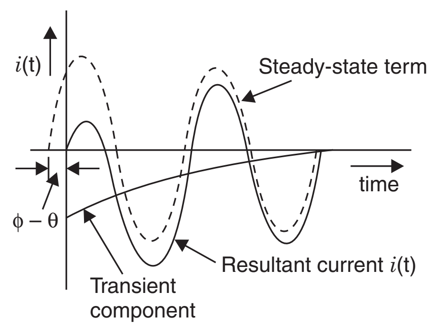


Fig. 12.3 Asymmetrical alternating current.

The first term in the expression above is the steady state sinusoidal variation and the second term is the transient part of it which vanishes theoretically after infinite time. But practically, it vanishes very quickly after two or three cycles. The transient decay as is seen depends upon the time constant $\dfrac{1}{a} = \dfrac{L}{R}$ of the circuit. Also at $t = 0$ it can be seen that the transient component equals the steady state component and since the transient component is negative the net current is zero at $t = 0$. It can be seen that the transient component will be zero in case the switching on of the voltage wave is done when $\theta = \phi$, *i.e.*, when the wave is passing through an angle $\phi = \tan^{-1} \dfrac{\omega L}{R}$. This is the situation when we have no transient even though the circuit contains inductance and there is switching operation also. On the other hand if $\phi - \theta = \pm \pi/2$, the transient term will have its maximum value and the first peak of the resulting current will be twice the peak value of the sinusoidal steady state component.

12.2 3-PHASE SUDDEN SHORT CIRCUIT OF AN ALTERNATOR

The study of 3-phase short circuit of an alternator is almost the same as the previous article except for the fact that in the previous case we assumed the voltage source to be of constant magnitude; here in this case the flux linkages vary and therefore the source is of varying magnitude. This being a 3-phase circuit, the switching angles in the different phases are 120° apart. So there is a good chance that the conditions of $\phi - \theta = \pm \pi/2$ may occur where the d.c.

decaying component may have its maximum value at $t = 0$ and the total current in some phase may be twice the peak value of the steady state current.

Whenever a 3-phase short circuit occurs at the terminals of an alternator, the current in the armature circuit increases suddenly to a large value and since the resistance of the circuit then is small as compared to its reactance, the current is highly lagging and the p.f. is approximately zero. Due to this sudden switching, as analysed in the previous section, there are two components of currents:

(*i*) a.c. component,

(*ii*) d.c. component (decaying).

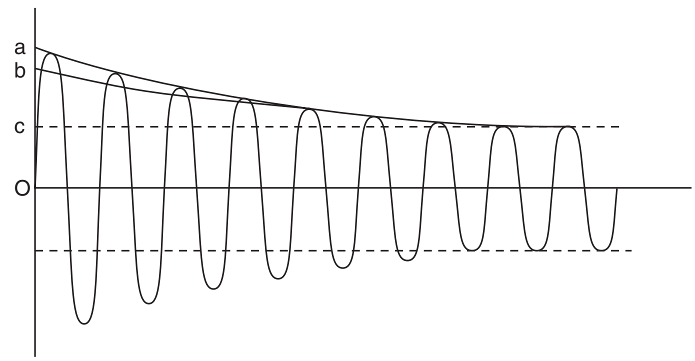


Fig. 12.4 The oscillogram of current variation as a function of time after a 3-phase fault takes place at the terminals of an alternator. The d.c. component is not shown. *Oa*—Subtransient current; *Ob*—Transient current; and *Oc*—Steady state current.

The current oscillogram is shown in Fig. 12.4. The rotor rotates at zero speed with respect to the field due to a.c. component of current in the stator whereas it rotates at synchronous speed with respect to the field due to the d.c. component of current in the stator conductors. The rotor winding acts as the secondary of a transformer for which the primary is the stator winding. Similarly in case the rotor has the damper winding fixed on its poles, the whole system will work as a three winding transformer in which stator is the primary and the rotor field winding and damper windings form the secondaries of the transformer. It is to be noted that the transformer action is there with respect to the d.c. component of current only. The a.c. component of current being highly lagging tries to demagnetise *i.e.*, reduce the flux in air gap. This reduction of flux from the instant of short circuit to the steady state operation cannot take place instantaneously because of the large amount of energy stored by the inductance of the corresponding system. So this change in flux is slow and depends upon the time-constant of the system. In order to balance the suddenly increased demagnetising m.m.f. of the armature current, the exciting current, *i.e.*, the field winding current must increase in the same direction of flow as before the fault. This happens due to the transformer action. At the same time, the current in the damper and the eddy currents in the adjacent metal parts increase in obedience to Lenz's law, thus assisting the rotor field winding to sustain the flux in the air gap.

At the instant of the short-circuit there is mutual coupling between the stator winding, rotor winding and the damper winding and the equivalent circuit is represented in Fig. 12.5(*a*).

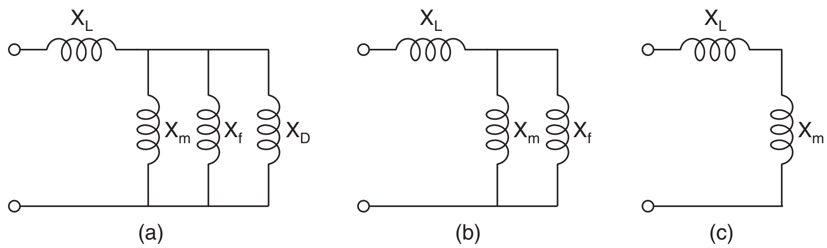


Fig. 12.5 Equivalent circuit of an alternator under (*a*) Subtransient; (*b*) Transient; and (*c*) Steady state conditions.

Since the equivalent resistance of the damper winding when referred to the stator is more as compared to the rotor winding, the time constant of damper winding is smaller than the rotor field winding. Therefore, the effect of damper winding and the eddy current in the pole faces disappears after the first few cycles. Accordingly, the equivalent circuit after first few cycles reduces to the one shown in Fig. 12.5(*b*). After a few more cycles depending upon the time constant of the field winding the effect of the d.c. component will die down and steady state conditions will prevail for which the equivalent circuit is shown in Fig. 12.5(*c*).

The reactance in the initial stage corresponding to Fig. 12.5(*a*) is called the subtransient reactance; corresponding to Fig. 12.5(*b*) it is called as the transient reactance and the steady state reactance is the synchronous reactance (Fig. 12.5(*c*)). It can be seen from the equivalent circuit that the inductance increases as from the initial stage to the final steady state *i.e.*, synchronous reactance > transient reactance > subtransient reactance.

12.3 THE RESTRIKING VOLTAGE AFTER REMOVAL OF SHORT CIRCUIT

The system (Fig. 12.6) consists of an alternator connected to a busbar. The load is removed after a short circuit occurs. It is required to determine the voltage across the circuit breaker during the opening period.

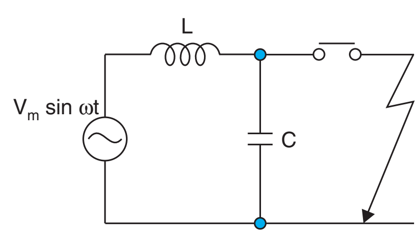


Fig. 12.6 Equivalent circuit to determine the restriking voltage.

The generator is represented by a constant voltage source behind the internal inductance L. The capacitance to ground of the busbars, the bushings etc., is lumped and is represented by C. The following assumptions are made, in addition, for the analysis of the system:

(*i*) The fault is a solid one *i.e.*, there is no arcing.

(*ii*) The magnitude of the positive sequence impedance is assumed to be constant for the period in which the overvoltage is to be determined.

(*iii*) The effects of saturation and corona are neglected which will tend to reduce the overvoltages.

(*iv*) The charging current of the transmission line before the fault, and load currents are neglected.

(*v*) The current interruption takes place at current zero when the voltage passes through maximum value.

(*vi*) The system is assumed to be lossless.

The method used for analysis is known as current cancellation method which means the voltage across the C.B. contact after it opens is the product of the current during the fault and the impedance of the network between the circuit breaker contacts shorting the voltage sources.

$$\text{The fault current} = I(s) = \frac{V(s)}{Z(s)}$$

$$= \frac{V_m}{s} \cdot \frac{1}{Ls}$$

Here we have taken V_m, instead of $V_m \sin \omega t$, because the fault interruption takes place at current zero when the voltage is passing through maximum value V_m.

Now the impedance between the circuit breaker contacts after shortcircuiting the voltage source will be the impedance of the parallel combination of L and C, *i.e.*,

$$Z_0(s) = \frac{Ls \,.\, 1/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1} = \frac{s/C}{s^2 + 1/LC}$$

$$v(s) = I(s)Z_0(s) = \frac{V_m}{s} \cdot \frac{1}{sL} \cdot \frac{s/C}{s^2 + 1/LC}$$

$$= \frac{V_m}{s} \cdot \frac{1}{LC} \cdot \frac{1}{s^2 + 1/LC} = V_m \left[\frac{1}{s} - \frac{s}{s^2 + 1/LC} \right]$$

$$v(t) = V_m[1 - \cos \omega_0 t]$$

where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ or $f_0 = \dfrac{1}{2\pi \sqrt{LC}}$

f_0 is the natural frequency of oscillation.

This variation of voltage across the circuit breaker is shown in Fig. 12.7. The voltage $v(t)$ is called the restriking voltage and it has its first peak value when

$$\omega_0 t = \pi$$

or
$$\frac{1}{\sqrt{LC}} t = \pi$$

or
$$t = \pi \sqrt{LC}$$

and the value is $2V_m$.

At $t = 0$ the value of the voltage is zero.

This type of transient is known as single frequency or energy transient.

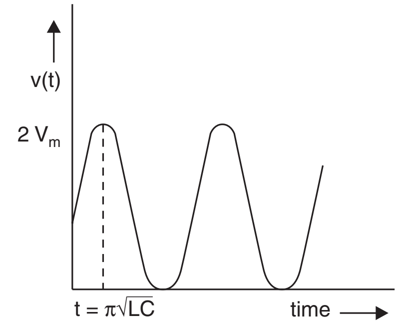


Fig. 12.7 Restriking voltage.

Double Frequency Transient

The simplest circuit to demonstrate the double frequency transients is given in Fig. 12.8. Here L_1 and C_1 are the inductance and stray capacitance on the source side of the breaker and L_2 and C_2 on the load side.

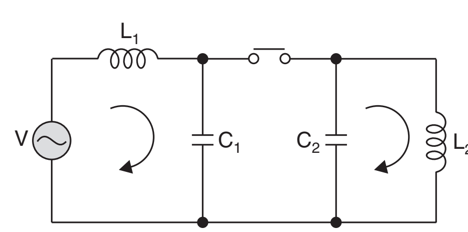


Fig. 12.8 Circuit with double frequency transients.

When the circuit breaker operates, the load is completely isolated from the generator and the two halves of the circuit behave independently. Before the switch operates, the voltage across the capacitors is given by

$$V_c = V \cdot \frac{L_2}{L_1 + L_2}$$

Normally $L_2 > L_1$ and therefore the capacitor voltage is a little less than the source voltage at any time. When the current passes through zero value, the voltage is at its maximum. When the circuit breaker interrupts the current at its zero, the capacitor C_2 will oscillate with L_2 at a natural frequency of

$$f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

and C_1 will oscillate with L_1 at a natural frequency

$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

So opening of the switch will result in double frequency transients in this circuit.

12.4 TRAVELLING WAVES ON TRANSMISSION LINES

So far, we have analysed the transient behaviour of various circuits with lumped parameters. However, there are some parts of a power system where this approach is inadequate. The most obvious example is the transmission line. Here the parameters L, C and R are uniformly distributed over the length of the line. For steady state operation of the line the transmission lines could be represented by lumped parameters but for the transient behaviour of the lines they must be represented by their actual circuits *i.e.*, distributed parameters. We say that for a 50 Hz supply and short transmission line the sending end current equals the receiving end current and the change in voltage from sending end to receiving end is smooth. This is not so when transmission line is subjected to a transient.

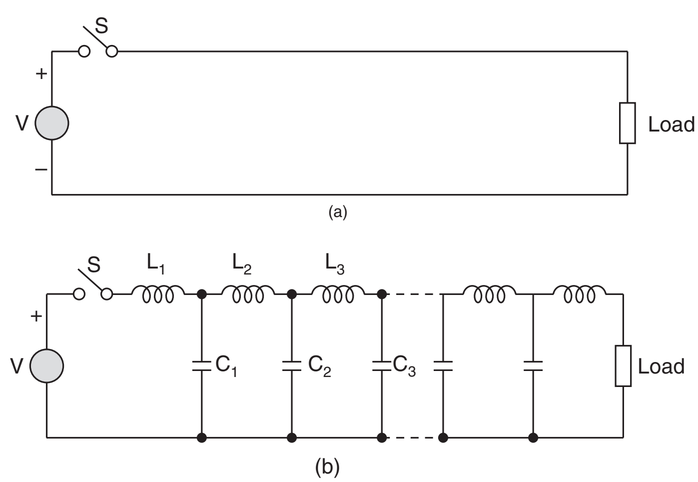


Fig. 12.9 (*a*) Long transmission line, (*b*) Equivalent π-section of a long transmission line.

To understand the travelling wave phenomenon over transmission line consider Fig. 12.9 (*a*). The line is assumed to be lossless. Let L and C be the inductance and capacitance respectively per unit length of the line. The line has been represented in Fig. 12.9 (*b*) by a large number of L and C π-sections. The closing of the switch is similar to opening the valve at the end of a channel, thereby admitting water to the channel from some reservoir behind. When the valve is opened the channel does not get filled up instantaneously. We observe the water advancing down the channel. At any instant the channel ahead of the wave front is dry while that behind is filled with water to the capacity. Similarly, when the switch S is closed the voltage does not appear instantaneously at the other end. When switch S is closed, the inductance L_1 acts as an open circuit and C_1 as short circuit instantaneously. The same instant the next section cannot be charged because the voltage across the capacitor C_1 is zero. So unless the capacitor C_1 is charged to some value whatsoever, charging of the capacitor C_2 through L_2 is not possible which, of course, will take some finite time. The same argument applies to the third section, fourth section and so on. So we see that the voltage at the successive sections builds up gradually. This gradual build up of voltage over the transmission line conductors can be regarded as though a voltage wave is travelling from one end to the other end and the gradual charging of the capacitances is due to the associated current wave.

Now it is desired to find out expressions for the relation between the voltage and current waves travelling over the transmission lines and their velocity of propagation.

Suppose that the wave after time t has travelled through a distance x. Since we have assumed lossless lines whatever is the value of voltage and current waves at the start, they remain same throughout the travel. Consider a distance dx which is travelled by the waves in time dt. The electrostatic flux is associated with the voltage wave and the electromagnetic flux with the current wave. The electrostatic flux which is equal to the charge between the conductors of the line up to a distance x is given by

$$q = VCx \tag{12.1}$$

The current in the conductor is determined by the rate at which the charge flows into and out of the line.

$$I = \frac{dq}{dt} = VC\frac{dx}{dt} \tag{12.2}$$

Here dx/dt is the velocity of the travelling wave over the line conductor and let this be represented by v. Then

$$I = VCv \tag{12.3}$$

Similarly the electromagnetic flux linkages created around the conductors due to the current flowing in them up to a distance of x is given by

$$\psi = ILx \tag{12.4}$$

The voltage is the rate at which the flux linkages link around the conductor

$$V = IL\frac{dx}{dt} = ILv \tag{12.5}$$

Dividing equation (12.5) by (12.3), we get

$$\frac{V}{I} = \frac{ILv}{VCv} = \frac{I}{V} \cdot \frac{L}{C}$$

or $$\frac{V^2}{I^2} = \frac{L}{C}$$

or $$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n \quad (12.6)$$

The expression is a ratio of voltage to current which has the dimensions of impedance and is therefore here designated as surge impedance of the line. It is also known as the natural impedance because this impedance has nothing to do with the load impedance. It is purely a characteristic of the transmission line. The value of this impedance is about 400 ohms for overhead transmission lines and 40 ohms for cables.

Next, multiplying equations (12.3) with (12.5), we get

$$VI = VCv \cdot ILv = VILCv^2$$

or $$v^2 = \frac{1}{LC}$$

or $$v = \frac{1}{\sqrt{LC}} \quad (12.7)$$

Now expressions for L and C for overhead lines are

$$L = 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre}$$

$$C = \frac{2\pi\varepsilon}{\ln \frac{d}{r}} \text{ F/metre}$$

Substituting these values in equation (12.7), the velocity of propagation of the wave

$$v = \frac{1}{\left(2 \times 10^{-7} \ln \frac{d}{r} \cdot \frac{2\pi\varepsilon}{\ln d/r}\right)^{1/2}}$$

$$= \frac{1}{\sqrt{4\pi\varepsilon \,.\, 10^{-7}}} = \frac{1}{\sqrt{4\pi \,.\, \frac{1}{36\pi} \times 10^{-9} \times 10^{-7}}}$$

$$= 3 \times 10^8 \text{ metres/sec.}$$

This is the velocity of light. This means the velocity of propagation of the travelling waves over the overhead transmission lines equals the velocity of light. In actual practice because of the resistance and leakance of the lines the velocity of the travelling wave is slightly less than the velocity of light. Normally a velocity of approximately 250 m/μ sec is assumed. It can be seen from the expression that the velocity of these waves over the cables will be smaller than over the overhead lines because of the permittivity term in the denominator.

Since $\varepsilon = \varepsilon_0\varepsilon_r$ for overhead lines $\varepsilon_r = 1$ whereas for cables where the conductor is surrounded by some dielectric material for which $\varepsilon_r > 1$, the term ε is greater for cables than for overhead lines and therefore the velocity of the waves over the cables is smaller than over the overhead lines.

Let us study the behaviour of these lines to the travelling waves when they reach the other end of the lines or whenever they see a change in the impedance (impedance other than characteristic impedance of the line).

Open-End Line

Consider a line with the receiving end open-circuited as shown in Fig. 12.10.

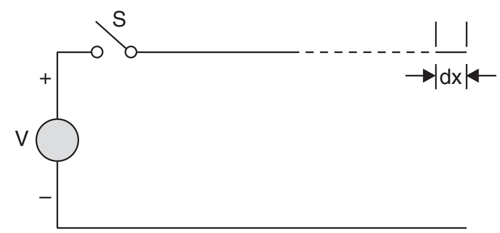


Fig. 12.10 Case of an open-ended line.

When switch S is closed, a voltage and current wave of magnitudes V and I respectively travel towards the open-end. These waves are related by the equation:

$$\frac{V}{I} = Z$$

where Z is the characteristic impedance of the line. Consider the last element dx of the line, because, it is here where the wave is going to see a change in impedance, an impedance different from Z (infinite impedance as the line is open-ended).

The electromagnetic energy stored by the element dx is given by $\frac{1}{2} LdxI^2$ and electrostatic energy in the element dx, $\frac{1}{2} CdxV^2$. Since the current at the open-end is zero, the electromagnetic energy vanishes and is transformed into electrostatic energy. As a result, let the change in voltage be e; then

$$\frac{1}{2} LdxI^2 = \frac{1}{2} Cdxe^2$$

or
$$\left(\frac{e}{I}\right)^2 = \frac{L}{C}$$

or
$$e = IZ = V$$

This means the potential of the open-end is raised by V volts; therefore, the total potential of the open-end when the wave reaches this end is

$$V + V = 2V$$

The wave that starts travelling over the line when the switch S is closed, could be considered as the incident wave and after the wave reaches the open-end, the rise in potential V could be considered due to a wave which is reflected at the open-end and actual voltage at the open-end could be considered as the refracted or transmitted wave and is thus

Refracted wave = Incident wave + Reflected wave

We have seen that for an open-end line a travelling wave is reflected back with positive sign and coefficient of reflection as unity.

Let us see now about the current wave.

As soon as the incident current wave I reaches the open-end, the current at the open end is zero, this could be explained by saying that a current wave of I magnitude travels back over the transmission line. This means for an open-end line, a current wave is reflected with negative sign and coefficient of reflection unity. The variation of current and voltage waves over the line is explained in Fig. 12.11.

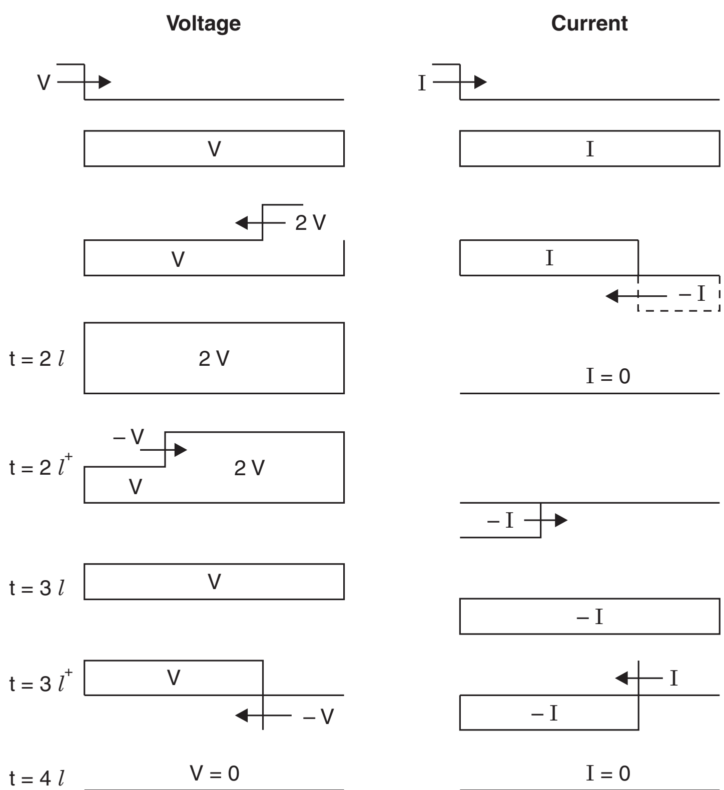


Fig. 12.11 Variation of voltage and current in an open-ended line.

After the voltage and current waves are reflected back from the open-end, they reach the source end, the voltage over the line becomes $2V$ and the current is zero. The voltage at source end cannot be more than the source voltage V therefore a voltage wave of $-V$ and current wave of $-I$ is reflected back into the line (Fig. 12.11). It can be seen that after the waves have travelled through a distance of $4l$, where l is the length of the line, they would have wiped out both the current and voltage waves, leaving the line momentarily in its original state. The above cycle repeats itself.

Short-circuited Line

Consider the line with receiving end short-circuited as shown in Fig. 12.12.

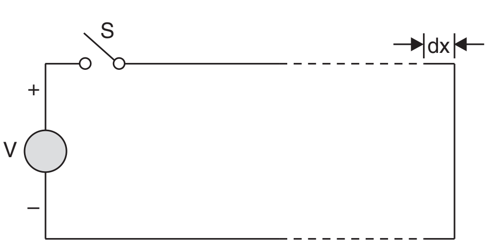


Fig. 12.12 Case of a short-circuited line.

When switch S is closed, a voltage wave of magnitude V and current wave of magnitude I start travelling towards the shorted end. Consider again the last element dx, where the electrostatic energy stored by the element is $\frac{1}{2}CdxV^2$ and electromagnetic energy $\frac{1}{2}LdxI^2$. Since the voltage at the shorted end is zero, the electrostatic energy vanishes and is transformed into electromagnetic energy. As a result, let the change in the current be i; then

$$\frac{1}{2}CdxV^2 = \frac{1}{2}Ldxi^2$$

or
$$V = iZ$$

or
$$i = \frac{V}{Z} = I$$

This means the increase in current is I amperes. As a result the total current at the shorted end, when the current wave reaches the end is $(I + I) = 2I$ amperes. This could be considered due to a reflected current wave of magnitude I amperes. Therefore for a short-circuited end the current wave is reflected back with positive sign and coefficient of reflection as unity. Since the voltage at the shorted end is zero, a voltage wave of $-V$ could be considered to have been reflected back into the line, *i.e.*, the current wave in case of short-circuited end is reflected back with positive sign and with coefficient of reflection as unity, whereas the voltage wave is reflected back with negative sign and unity coefficient of reflection. The variation of voltage and current over the line is explained in Fig. 12.13.

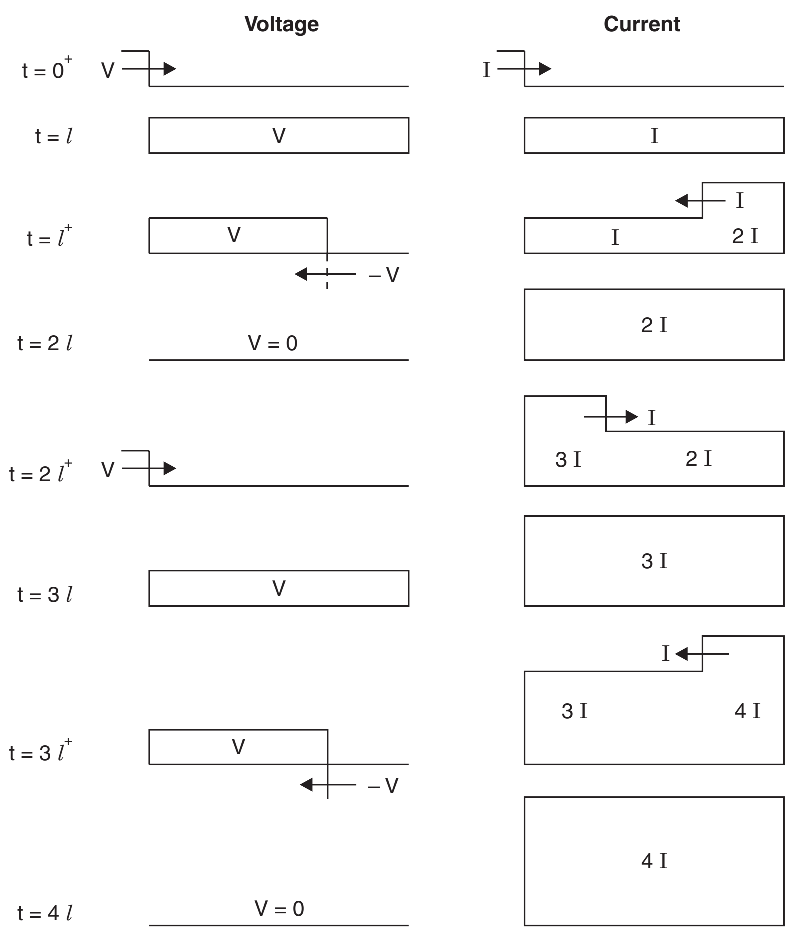


Fig. 12.13 Variation of voltage and current in a short ended line.

It is seen from above that the voltage wave periodically reduces to zero after it has travelled through a distance of twice the length of the line whereas after each reflection at either end the current is built up by an amount $V/Z_n = I$. Theoretically, the reflection will be infinite and therefore the current will reach infinite value. But practically in an actual system the current will be limited by the resistance of the line and the final value of the current will be $I' = V/R$, where R is the resistance of transmission line.

Line Terminated Through a Resistance

Let Z be the surge impedance of the line terminated through a resistance R (Fig. 12.14). It has been seen in the previous sections that whatever be the value of the terminating impedance whether it is open or short circuited, one of the two voltage or current waves is reflected back with negative sign. Also, since the reflected wave travels along the overhead line or over the line along which the incident wave travelled, therefore, the following relation holds good for reflected voltage and current waves.

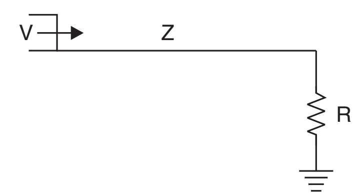


Fig. 12.14 Line terminated through a resistance.

$$I' = -\frac{V'}{Z}$$

where V' and I' are the reflected voltage and current waves. Also,

Refracted or transmitted wave = Incident wave + Reflected wave

Let V'' and I'' be the refracted voltage and current waves into the resistor R, when the incident waves V and I reach the resistance R. The following relations hold good:

$$I = \frac{V}{Z}$$

$$I' = -\frac{V'}{Z}$$

$$I'' = \frac{V''}{R}$$

Since $I'' = I + I'$ and $V'' = V + V'$, using these relations, we have

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} \tag{12.8}$$

$$= \frac{V}{Z} - \frac{V'' - V}{Z} = \frac{2V}{Z} - \frac{V''}{Z}$$

or
$$V'' = \frac{2VR}{Z + R} \tag{12.9}$$

and current
$$I'' = \frac{2V}{R + Z} = \frac{V}{Z} \cdot \frac{2Z}{R + Z} = I \cdot \frac{2Z}{R + Z} \tag{12.10}$$

Similarly substituting for V'' in terms of $(V + V')$, equation (12.8) becomes

$$\frac{V + V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

or $$V' = V \cdot \frac{R - Z}{R + Z} \tag{12.11}$$

and $$I' = -\frac{V'}{Z} = -\frac{V}{Z} \cdot \frac{(R - Z)}{R + Z} \tag{12.12}$$

From the relations above, the coefficient of refraction for current waves

$$= \frac{2Z}{R + Z}$$

and for voltage waves $$= \frac{2R}{R + Z}$$

Similarly, the coefficient of reflection for current waves

$$= -\frac{R - Z}{R + Z}$$

and for voltage waves $$= +\frac{R - Z}{R + Z}$$

Now the two extreme cases can be derived out of this general expression. For open circuit,

$$R \to \infty$$

and coefficient of refraction for current waves

$$\frac{2Z}{\infty + Z} = 0$$

and coefficient of refraction for voltage waves

$$= \frac{2R}{R + Z} = \frac{2}{1 + Z/R} = \frac{2}{1 + Z/\infty} = 2$$

Similarly, coefficient of reflection for current waves

$$= -\frac{R - Z}{R + Z} = -\frac{1 - Z/R}{1 + Z/R} = -1$$

and coefficient of reflection for voltage waves

$$= \frac{R - Z}{R + Z} = 1$$

Similarly, to find out the coefficients of reflection and refraction for current and voltage waves for the short circuit case, the value of $R = 0$ is to be substituted in the corresponding relations as derived in this section.

It is, therefore, seen here that whenever a travelling wave looks into a change in impedance, it suffers reflection and refraction. It is shown below that in case $Z = R$ *i.e.*, the line is terminated through a resistance whose value equals the surge impedance of the line (*i.e.*, no change in the impedance) there will be no reflection and the wave will enter fully into the resistance, *i.e.*, the coefficient of refraction will be unity whereas the coefficient of reflection will be zero.

When $R = Z$, substituting this, the coefficient of reflection for current wave

$$= -\frac{R - Z}{R + Z} = \frac{Z - Z}{Z + Z} = 0$$

and for voltage wave
$$= \frac{R - Z}{R + Z} = 0$$

The coefficient of refraction for current wave

$$= \frac{2Z}{R + Z} = \frac{2Z}{2Z} = 1$$

and for voltage wave
$$= \frac{2R}{R + Z} = 1$$

It is seen that when a transmission line is terminated through a resistance equal to its surge impedance the wave does not suffer reflection and, therefore, such lines could be said to be of infinite length. Such lines are also called as matched lines and the load corresponding to this is known as surge impedance loading or natural impedance loading. Detailed idea about this kind of loading is given in Chapter 4.

Line Connected to a Cable

A wave travels over the line and enters the cable (Fig. 12.15). Since the wave looks into a different impedance, it suffers reflection and refraction at the junction and the refracted voltage wave is given by

$$V'' = V \cdot \frac{2Z_2}{Z_1 + Z_2}$$

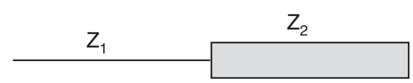


Fig. 12.15 Line connected to a cable.

The other waves can be obtained by using the relations (12.10–12.12). The impedance of the overhead line and cable are approximately 400 ohms and 40 ohms respectively. With these values it can be seen that the voltage entering the cable will be

$$V'' = V \cdot \frac{2 \times 40}{40 + 400} = \frac{2}{11} V$$

i.e., it is about 20% of the incident voltage V. It is for this reason that an overhead line is terminated near a station by connecting the station equipment to the overhead line through a short length of underground cable. Besides the reduction in the magnitude of the voltage wave, the steepness is also reduced because of the capacitance of the cable. This is explained in the next section. The reduction in steepness is very important because this is one of the factors for reducing the voltage distribution along the windings of the equipment. While connecting the overhead line to a station equipment through a cable it is important to note that the length of the cable should not be very short (should not be shorter than the expected length of the wave) otherwise successive reflections at the junction may result in piling up of voltage and the voltage at the junction may reach the incident voltage.

Reflection and Refraction at a T-junction

A voltage wave V is travelling over the line with surge impedance Z_1 as shown in Fig. 12.16. When it reaches the junction, it looks a change in impedance and, therefore, suffers reflection and refraction. Let V_2'', I_2'' and V_3'', I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively. Since Z_2 and Z_3 form a parallel path as far as the surge wave is concerned, $V_2'' = V_3'' = V''$. Therefore, the following relations hold good:

$$V + V' = V''$$

$$I = \frac{V}{Z_1},\ I' = -\frac{V'}{Z_1}$$

$$I_2'' = \frac{V''}{Z_2} \text{ and } I_3'' = \frac{V''}{Z_3}$$

and
$$I + I' = I_2'' + I_3'' \tag{12.13}$$

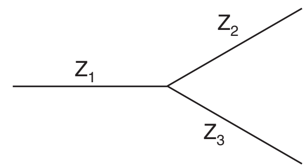


Fig. 12.16 A bifurcated line.

Substituting in equation (12.13) the values of currents

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

Substituting for $V' = V'' - V$,

$$\frac{V}{Z_1} - \frac{V'' - V}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right]$$

or
$$V'' = \frac{2V/Z_1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}} \tag{12.14}$$

Similarly other quantities can be derived.

Example 12.1: A 3-phase transmission line has conductors 1.5 cms in diameter spaced 1 metre apart in equilateral formation. The resistance and leakance are negligible. Calculate (*i*) the natural impedance of the line, (*ii*) the line current if a voltage wave of 11 kV travels along the line, (*iii*) the rate of energy absorption, the rate of reflection and the state and the form of reflection if the line is terminated through a star connected load of 1000 ohm per phase, (*iv*) the value of the terminating resistance for no reflection and (*v*) the amount of reflected and transmitted power if the line is connected to a cable extension with inductance and capacitance per phase per cm of 0.5×10^{-8} H and 1×10^{-6} μF respectively.

Solution: The inductance per unit length

$$= 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre}$$

$$= 2 \times 10^{-7} \ln \frac{100}{0.75}$$

$$= 2 \times 10^{-7} \ln 133.3$$

$$= 2 \times 10^{-7} \times 4.89$$

$$= 9.78 \times 10^{-7} \text{ H/m}$$

The capacitance per phase per unit length

$$= \frac{2\pi\varepsilon}{\ln d/r} \text{ F/metre}$$

$$= \frac{2\pi \times 10^{-9}}{36\pi \ln d/r}$$

$$= \frac{1}{18} \times \frac{10^{-9}}{4.89} = 1.136 \times 10^{-11}$$

$\therefore$ The natural impedance $= \sqrt{\frac{L}{C}}$ ohms

$$= \sqrt{\frac{9.78 \times 10^{-7}}{1.136 \times 10^{-11}}} = 294\ \Omega. \quad \textbf{Ans.}$$

(*ii*) The line current $= \frac{11000}{\sqrt{3} \times 294} = 21.6$ amps. **Ans.**

(*iii*) Since the terminating resistance is of higher value as compared to the value of the surge impedance of the line, the reflection is with a positive sign.

The voltage across the terminating resistance

$$E'' = \frac{2Z_2E}{Z_1 + Z_2}$$

where Z_1 = line surge impedance, Z_2 = terminating impedance, and E = incident voltage.

$$E'' = 2 \times \frac{11000}{\sqrt{3}} \frac{1000}{1294} = 9.8 \text{ kV}$$

$\therefore$ The rate of power consumption $= \frac{3E''^2}{R}$ MW

$$= \frac{3 \times 9.8 \times 9.8}{1000} \times 1000 \text{ kW}$$

$= 288$ kW. **Ans.**

The reflected voltage $E' = \frac{Z_2 - Z_1}{Z_2 + Z_1} E = \frac{1000 - 294}{1294} \times \frac{11}{\sqrt{3}}$ kV

$$= \frac{706}{1294} \times \frac{11}{\sqrt{3}} = 3.465 \text{ kV}$$

$\therefore$ The rate of reflected energy $= \frac{3 \times 3.465^2}{294} \times 1000$ kW

$= 121.8$ kW. **Ans.**

(*iv*) In order that the incident wave when reaches the terminating resistance, does not suffer reflection, the terminating resistance should be equal to the surge impedance of the line, *i.e.*, 294 ohms. **Ans.**

(*v*) The surge impedance of the cable $= \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5 \times 10^{-8}}{10^{-12}}}$

$= 70.7$ ohm

The refracted voltage $= \frac{2 \times 70.7}{294 + 70.7} \times \frac{11}{\sqrt{3}}$

$$= \frac{2 \times 70.7 \times 11}{\sqrt{3} \times 364.7} = 2.46 \text{ kV}$$

$$\text{The reflected voltage } = \frac{70.7 - 294}{364.7} \times \frac{11}{\sqrt{3}}$$

$$= \frac{-223.3 \times 11}{\sqrt{3} \times 364.7} = -3.9 \text{ kV}$$

∴ The refracted and reflected powers are respectively.

$$\frac{3 \times 2.46^2}{70.7} \times 1000 = 256 \text{ kW} \quad \text{and} \quad \frac{3 \times 3.9^2}{294} \times 1000 = 155 \text{ kW}. \quad \textbf{Ans.}$$

Example 12.2: A surge of 15 kV magnitude travels along a cable towards its junction with an overhead line. The inductance and capacitance of the cable and overhead line are respectively 0.3 mH, 0.4 μF and 1.5 mH, 0.012 μF per km. Find the voltage rise at the junction due to the surge.

Solution: In this problem the surge travels from the cable towards the overhead line and hence there will be positive voltage reflection at the junction.

$$\text{The natural impedance of the cable} = \sqrt{\frac{0.3 \times 10^{-3}}{0.4 \times 10^{-6}}}$$

$$= \sqrt{\frac{3 \times 10^{-4}}{0.4 \times 10^{-6}}} = 27.38$$

$$\text{The natural impedance of the line} = \sqrt{\frac{1.5 \times 10^{-3}}{0.012 \times 10^{-6}}}$$

$$= \sqrt{\frac{1.5 \times 10^{-3}}{0.12 \times 10^{-7}}} = 353 \text{ ohms.}$$

The voltage rise at the junction is the voltage transmitted into the overhead line as the voltage is zero before the surge reaches the junction.

$$E'' = \frac{2 \times 353 \times 15}{353 + 27} = \frac{2 \times 353 \times 15}{380} = 27.87 \text{ kV}. \quad \textbf{Ans.}$$

Example 12.3: A surge of 100 kV travelling in a line of natural impedance 600 ohms arrives at a junction with two lines of impedances 800 ohms and 200 ohms respectively. Find the surge voltages and currents transmitted into each branch line.

Solution: The problem deals with a reflection at a T-joint. The various natural impedances are: Z_1 = 600 ohms, Z_2 = 800 ohms, Z_3 = 200 ohms. The surge magnitude is 100 kV.

The surge as it reaches the joint suffers reflection and here the two lines are in parallel; therefore, the transmitted voltage will have the same magnitude and is given by

$$E'' = \frac{2E/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{2 \times 100/600}{\frac{1}{600} + \frac{1}{800} + \frac{1}{200}}$$

$$= \frac{0.333}{(1.67 + 1.25 + 5.0) \times 10^{-3}} = \frac{0.333 \times 10^3}{7.92}$$

$$= \frac{33.3}{7.92} \times 10 = 42.04 \text{ kV.} \quad \textbf{Ans.}$$

The transmitted current in line $Z_2 = \dfrac{42.04 \times 1000}{800}$ amps = 52.55 amps. **Ans.**

The transmitted current in line $Z_3 = \dfrac{42.04 \times 1000}{200}$ amps = 210.2 amps. **Ans.**

Line Terminated Through a Capacitance

We consider here that a d.c. surge of infinite length travels over the line of surge impedance Z and is incident on the capacitor as shown in Fig. 12.17. We are interested in finding out the voltage across the capacitor *i.e.*, the refracted voltage. The refracted voltage, using equation (12.9),

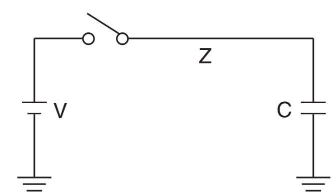


Fig. 12.17 Line terminated through a capacitance.

$$V''(s) = \frac{2.1/Cs}{Z + 1/Cs} \cdot \frac{V}{s} = \frac{2V}{s} \cdot \frac{1}{ZCs + 1}$$

$$= \frac{2V}{s} \cdot \frac{1/ZC}{s + 1/ZC} = 2V\left[\frac{1}{s} - \frac{1}{s + 1/ZC}\right]$$

$$v''(t) = 2V[1 - e^{-t/ZC}] \tag{12.15}$$

The variation of voltage is shown in Fig. 12.18(*b*).

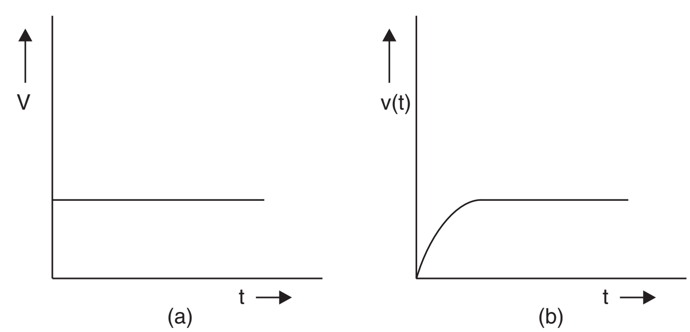


Fig. 12.18 (*a*) Incident voltage and (*b*) Voltage across capacitor.

It is to be noted that since terminating impedance is not a transmission line, therefore, $V''(s)$ is not a travelling wave but it is the voltage across the capacitor C.

Capacitor Connection at a T

The voltage across capacitor is given by the equation

$$V''(s) = \frac{2V/Z_1 s}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + Cs} = \frac{2VZ_2}{s} \cdot \frac{(1/Z_1 Z_2 C)}{\dfrac{(Z_1 + Z_2)}{Z_1 Z_2 C} + s}$$

$$= \frac{2V}{sZ_1C} \cdot \frac{1}{s + \dfrac{Z_1 + Z_2}{Z_1Z_2C}}$$

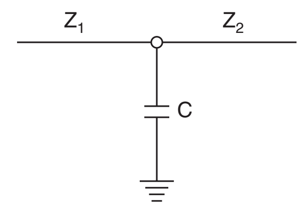


Fig. 12.19 Capacitor connected at *T*.

Let $\dfrac{Z_1 + Z_2}{Z_1Z_2C} = \alpha$; then

$$V''(s) = \frac{2V}{s} \cdot \frac{1/Z_1C}{s + \alpha}$$

or

$$V''(s) = \frac{2V}{s} \cdot \frac{Z_2}{Z_1 + Z_2} \cdot \frac{(Z_1 + Z_2)/Z_1Z_2C}{(s + \alpha)}$$

$$= \frac{2V}{s} \cdot \frac{Z_2}{Z_1 + Z_2} \cdot \frac{\alpha}{s + \alpha} = \frac{2VZ_2}{Z_1 + Z_2}\left[\frac{1}{s} - \frac{1}{s + \alpha}\right]$$

or

$$v''(t) = \frac{2V \, . \, Z_2}{Z_1 + Z_2}\left[1 - \exp\left(-\frac{Z_1 + Z_2}{Z_1Z_2C}\,t\right)\right] \tag{12.16}$$

The variation of the wave is shown in Fig. 12.20.

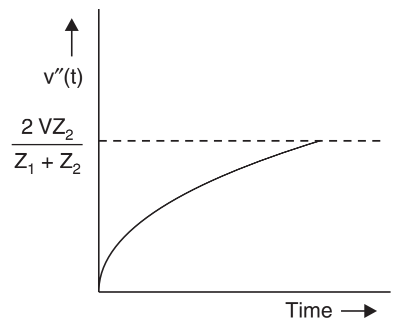


Fig. 12.20 Variation of voltage across the capacitor.

We have assumed in the derivation of the expression for voltage across the capacitor in the previous section that the travelling surge is of infinite length. Let us now derive the expression when the surge is of finite duration say τ (Fig. 12.21). Also, let the magnitude of this wave be V units. The wave could be decomposed into two waves.

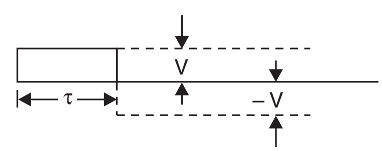


Fig. 12.21 Surge of finite length τ.

Here $\quad f(t) = Vu(t) - Vu(t - \tau)$

$$Vu(t - \tau) = V \text{ for } t \geq \tau$$

$$= 0 \text{ for } t < \tau$$

With this, voltage across the capacitor is given by

$$V''(s) = \mathcal{L}\{f(t)\} \, . \, \frac{2/Z_1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + Cs} = \frac{2V/Z_1s}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + Cs} - \frac{(2V/Z_1s) \, . \, e^{-\tau s}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + Cs}$$

$$v''(t) = 2V \cdot \frac{Z_2}{Z_1 + Z_2}\left[1 - \exp\left(-\frac{Z_1 + Z_2}{Z_1 Z_2 C}\,t\right)\right]$$
$$- \frac{2VZ_2}{Z_1 + Z_2}\left[1 - \exp\left\{-\frac{Z_1 + Z_2}{Z_1 Z_2 C}\,(t - \tau)\right\}\right]$$

The variation of voltage is shown in Fig. 12.22.

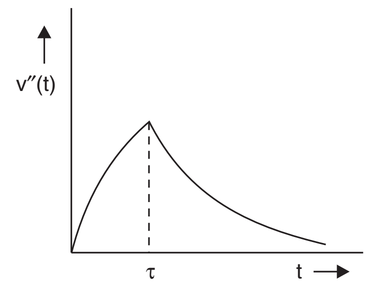


Fig. 12.22 Variation of voltage across the capacitor with finite duration incident surge.

Thus for time $0 < t < \tau$ only the first term in the expression is active and for $t \geq \tau$ both the terms are active. The rise in voltage is maximum at $t = \tau$ when the value will be

$$v''(t) = \frac{2VZ_2}{Z_1 + Z_2}\left[1 - \exp\left(-\frac{Z_1 + Z_2}{Z_1 Z_2 C}\,\tau\right)\right] - \frac{2VZ_2}{Z_1 + Z_2}\,[1 - e^0]$$
$$= \frac{2VZ_2}{Z_1 + Z_2}\left[1 - \exp\left(-\frac{Z_1 + Z_2}{Z_1 Z_2 C}\,\tau\right)\right] \qquad (12.17)$$

It is, therefore, clear that the attenuation in the magnitude of voltage for a short wave is much more rapid than for long wave.

We have seen that the effect of a shunt capacitor is to reduce the steepness and magnitude of the wave reaching an equipment. Since an inductor is dual to a capacitor, an inductor in series of the lines should give the same effect.

Example 12.4: A 500 kV 2 μ sec rectangular surge on a line having a surge impedance of 350 ohms approaches a station at which the concentrated earth capacitance is 3000 pF. Determine the maximum value of the transmitted wave.

Solution: The diagram corresponding to the problem is as follows:

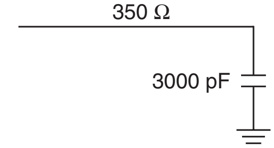


Fig. E.12.4

The maximum value of voltage will be

$$E'' = 2E\left[1 - \exp\left(-\frac{\tau}{ZC}\right)\right]$$
$$= 2 \times 500\left[1 - \exp\left(-\frac{2 \times 10^{-6} \times 10^{12}}{350 \times 3000}\right)\right]$$
$$= 2 \times 500\left[1 - \exp\left(-\frac{2 \times 10^{3}}{350 \times 3}\right)\right]$$
$$= 2 \times 500[1 - e^{-1.9}]$$
$$= 2 \times 500[1 - 0.15]$$

$= 850$ kV. **Ans.**

Example 12.5: An inductance of 800 μH connects two sections of a transmission line each having a surge impedance of 350 ohms. A 500 kV 2 μs rectangular surge travels along the line towards the inductance. Determine the maximum value of the transmitted wave.

Solution: The maximum value of the transmitted surge is given by

$$E'' = E\left[1 - \exp\left(-\frac{2Z}{L}\tau\right)\right]$$

$$= 500\left[1 - \exp\left(-\frac{2 \times 350}{800} \times 2\right)\right]$$

$$= 500[1 - e^{-0.875 \times 2}]$$

$$= 500[1 - e^{-1.750}]$$

$$= 500[1 - 0.173]$$

$$= 413.5 \text{ kV.} \quad \textbf{Ans.}$$

12.5 ATTENUATION OF TRAVELLING WAVES

Let R, L, C and G be the resistance, inductance, capacitance and conductance respectively per unit length of a line (Fig. 12.23). Let the value of voltage and current waves at $x = 0$ be V_0 and I_0. Our objective is to find the values of voltage and current waves when they have travelled through a distance of x units over the overhead line. Let the time taken be t units when voltage and current waves are V and I respectively. To travel a distance of dx, let the time taken be dt. The equivalent circuit for the differential length dx of the line is shown in Fig. 12.24.

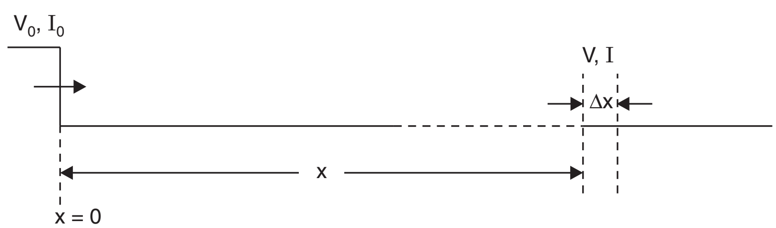


Fig. 12.23 Travelling wave on a lossy line.

The power loss in the differential element is

$$dp = I^2R\,dx + V^2G\,dx \qquad (12.18)$$

Also power at a distance $x \cdot VI = p = I^2Z_n$

Differential power, $\quad dp = -2IZ_n dI \qquad (12.19)$

where Z_n is the natural impedance of the line. Here negative sign has been assigned as there is reduction in power as the wave travels with time.

Equating the equations (12.18) and (12.19),

$$-2IZ_n dI = I^2R\,dx + V^2G\,dx$$

$$= I^2R\,dx + I^2Z_n^2G\,dx$$

or

$$dI = -\frac{I(R + GZ_n^2)}{2Z_n}dx$$

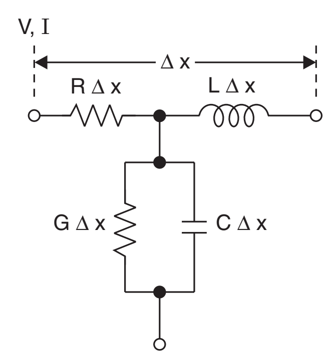


Fig. 12.24 Differential element of transmission line.

or
$$\frac{dI}{I} = -\frac{(R + GZ_n^2)}{2Z_n}\,dx$$

or
$$\ln I = -\left(\frac{R + GZ_n^2}{2Z_n}\right)x + A$$

At $x = 0$, $I = I_0$, $\quad\therefore\quad A = \ln I_0$.

or
$$\ln\frac{I}{I_0} = -\frac{R + GZ_n^2}{2Z_n}\,x = -\,ax$$

where $a = \dfrac{R + GZ_n^2}{2Z_n}$.

$$\therefore \qquad I = I_0 e^{-ax}. \tag{12.20}$$

Similarly it can be proved that $V = V_0 e^{-ax}$. This shows that the current and voltage waves get attenuated exponentially as they travel over the line and the magnitude of attenuation depends upon the parameters of the line. Since the value of resistance depends not only on the size of the conductors but also on the shape and length of the waves. An empirical relation due to Foust and Menger takes into account the shape and length of the wave for calculating the voltage and current at any point on the line after it has travelled through a distance x units and is given as

$$V = \frac{V_0}{1 + KxV_0} \tag{12.21}$$

where x is in kms, V and V_0 are in kV and K is the attenuation constant, of value

K = 0.00037 for chopped waves

= 0.00019 of short-waves

= 0.0001 for long-waves.

Example 12.6: A travelling wave of 50 kV enters an overhead line of surge impedance 400 ohms and conductor resistance 6 ohm per km. Determine (*i*) the value of the voltage wave when it has travelled through a distance of 50 km, and (*ii*) the power loss and the heat loss of the wave during the time required to traverse this distance. Neglect the losses in the insulation and assume a wave velocity of 3×10^5 km per second. Determine the corresponding values for a cable having surge impedance of 40 ohms and relative permittivity 4.

Solution: (*i*) Since the line has some specific resistance, the wave as it travels gets attenuated in magnitude.

The magnitude of the wave is given by

$$e = e_0 \varepsilon^{-1/2}(R/Z + GZ)x$$

where e = the value of voltage when travelled through a distance of x kilometres, R, G the resistance and leakance per kilometre length of the line and Z is the surge impedance, e_0 = initial magnitude of the surge voltage, ε the Naperian base.

Here in this problem e_0 = 50 kV, x = 50 km, R = 6 ohm and Z = 400 ohm and G = 0.0 mhos.

Substituting these values,

$$e = 50\varepsilon^{-1/2}\left(\frac{6}{400}\times 50\right) = 50 \times \varepsilon^{-0.375} = 50 \times 0.69 = 34.5 \text{ kV}$$

(*ii*) The power loss is the instantaneous quantity and is required to be calculated when the wave travels the distance of 50 km where the voltage magnitude is 34.5 kV.

$$\text{The power loss} = \frac{34.5 \times 34.5}{400} \times 1000 \text{ kW} = 2975 \text{ kW}$$

The heat loss is the integrated value of power over the distance (or time) the wave has travelled.

$$\text{Heat loss} = \int_0^t ei\, dt$$

Now $e = e_0\varepsilon^{-1/2}\,\dfrac{Rx}{Z}$ and similarly, $i = i_0\varepsilon^{-1/2}\,\dfrac{R}{Z}x$. Now,

$$x = vt$$

$$\therefore \qquad e = e_0 \,.\, \varepsilon^{-1/2}\,\frac{R}{Z}vt \text{ and } i = i_0\varepsilon^{-1/2}\frac{R}{Z}vt$$

Substituting these values, we get

$$\text{Heat loss} = \int_0^t e_0 i_0\, \varepsilon^{-(R/Z)vt}\, dt$$

where v = the velocity of the wave

$$t = \frac{x}{v} = \frac{50}{3\times 10^5} = 16.67 \times 10^{-5} \text{ sec}$$

and

$$i_0 = \frac{e_0}{Z} = \frac{50\times 1000}{4000} = 125 \text{ amps.}$$

$$\therefore \qquad \text{Heat loss} = -\int_0^{16.67\times 10^{-5}} 50\times 125\varepsilon^{-(R/Z)vt}\, dt$$

$$= -\,50 \times 125 \times \frac{400}{6\times 3\times 10^5}\,[e^{-0.75} - 1]$$

$$= 0.736 \text{ kJ} \quad \text{or} \quad 176 \text{ cal.} \quad \textbf{Ans.}$$

12.6 CAPACITANCE SWITCHING

The switching of a capacitance such as disconnecting a line or a cable or a bank of capacitor poses serious problems in power systems in terms of abnormally high voltages across the circuit breaker contacts. Under this situation the current leads the voltage by about 90°. Assuming that the current interruption takes place when it is passing through zero value the capacitor will be charged to maximum voltage. Since the capacitor is now isolated from the source, it retains its charge as shown in Fig. 12.25 (*c*) and because of trapping of this charge, half a cycle after the current zero the voltage across the circuit breaker contact is $2V$ which may prove to be dangerous and may result in the circuit breaker restrike. This is equivalent to closing the switch suddenly which will result into oscillations in the circuit at the natural frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The circuit condition corresponds to Fig. 12.6. The only difference between the two circuits is that whereas in Fig. 12.25 the capacitor is charged to a voltage V, in Fig. 12.6 it is assumed to be without charge. Therefore, the voltage across the capacitor reaches $3V$. Since the source voltage is V, the voltage across the breaker contacts after another half cycle will be $4V$ which may cause another restrike. This phenomenon may theoretically continue indefinitely, increasing the voltage by successive increments of $2V$. This may result into an external flashover or the failure of the capacitor. This is due to the inability of the circuit breaker to provide sufficient dielectric strength to the contacts to avoid restrikes after they are opened first.

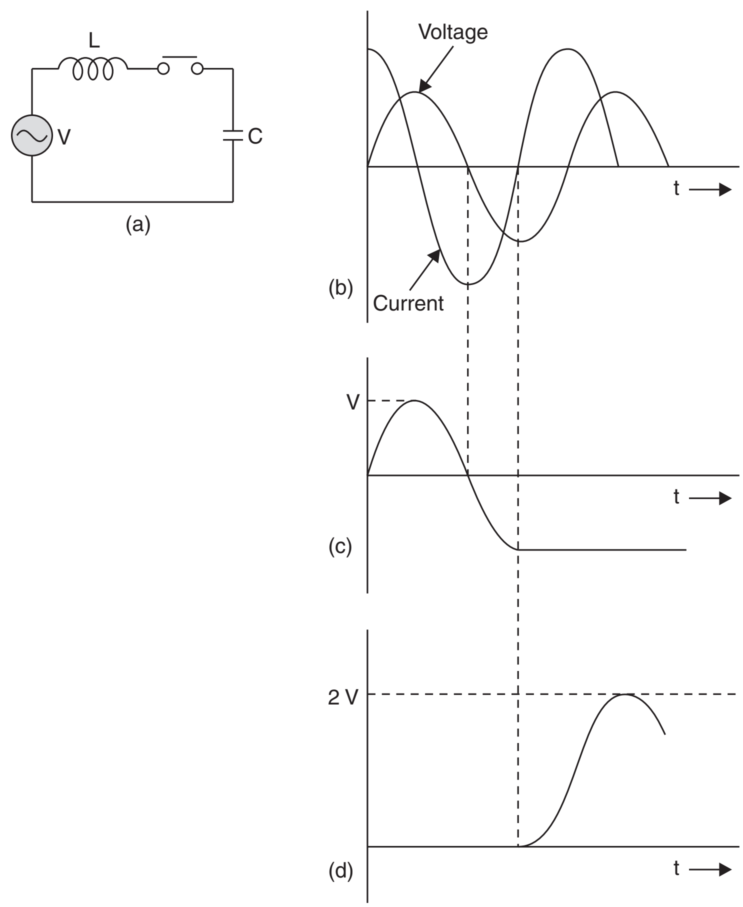


Fig. 12.25 (*a*) Equivalent circuit for capacitor switching; (*b*) System voltage and current; (*c*) Capacitor voltage; (*d*) Voltage across the switch.

This problem is practically solved by using air blast circuit breakers or multibreak breakers.

12.7 OVERVOLTAGE DUE TO ARCING GROUND

Figure 12.26 shows a 3-phase system with isolated neutral. The shunt capacitances are also shown. Under balanced conditions and complete transposed transmission lines, the potential of the neutral is near the ground potential and the currents in various phases through the shunt capacitors are leading their corresponding voltages by 90°. They are displaced from each other by 120° so that the net sum of the three currents is zero (Fig. 12.26(*b*)). Say there is line-to-ground fault on one of the three phases (say phase '*c*'). The voltage across the shunt capacitor of that phase reduces to zero whereas those of the healthy phases become line-to-line voltages and now they are displaced by 60° rather than 120°. The net charging current now is three times the phase current under balanced conditions (Fig. 12.26(*c*)). These currents flow through the fault and the windings of the alternator. The magnitude of this current is often sufficient to sustain an arc and, therefore, we have an arcing ground. This could be due to a flashover of a support insulator. Here this flashover acts as a switch. If the arc extinguishes when the current is passing through zero value, the capacitors in phases *a* and *b* are charged to line voltages. The voltage across the line and the grounded points of the post insulator will be the superposition of the capacitor voltage and the generator voltage and this voltage may be good enough to cause flashover which is equivalent to restrike in a circuit breaker. Because of the presence of the inductance of the generator winding, the capacitances will form an oscillatory circuit and these oscillations may build up to still higher voltages and the arc may reignite causing further transient disturbances which may finally lead to complete rupture of the post insulators.

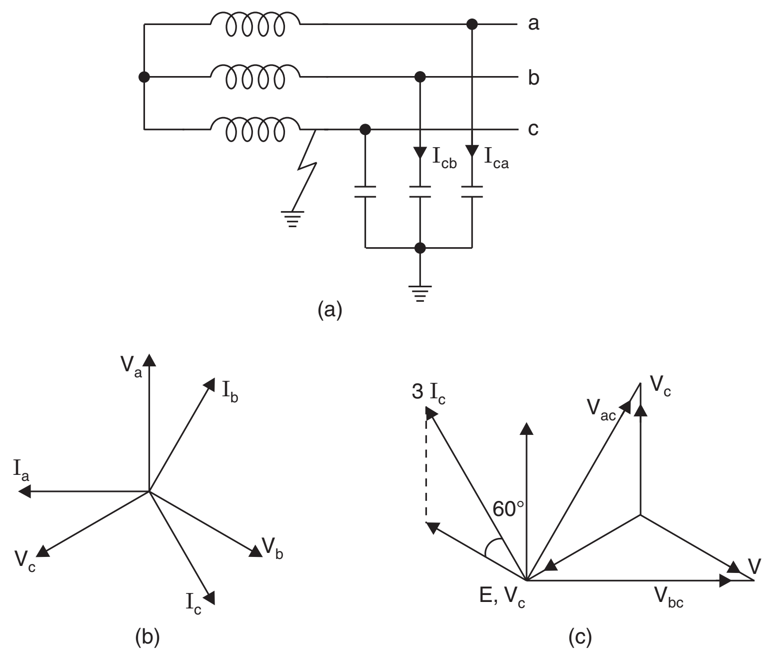


Fig. 12.26 (*a*) 3-phase system with isolated neutral; (*b*) Phasor diagram under healthy condition; (*c*) Phasor diagram under faulted condition.

12.8 LIGHTNING PHENOMENON

Lightning has been a source of wonder to mankind for thousands of years. Schonland points out that any real scientific search for the first time was made into the phenomenon of lightning by Franklin in 18th century.

Before going into the various theories explaining the charge formation in a thunder cloud and the mechanism of lightning, it is desirable to review some of the accepted facts concerning the thunder cloud and the associated phenomenon.

1. The height of the cloud base above the surrounding ground level may vary from 500 to 30,000 ft. The charged centres which are responsible for lightning are in the range of 1000 to 5000 ft.

2. The maximum charge on a cloud is of the order of 10 coulombs which is built up exponentially over a period of perhaps many seconds or even minutes.

3. The maximum potential of a cloud lies approximately within the range of 10 MV to 100 MV.

4. The energy in a lightning stroke may be of the order of 250 kWhr.

5. Raindrops:

(*a*) Raindrops elongate and become unstable under an electric field, the limiting diameter being 0.3 cm in a field of 10 kV/cm.

(*b*) A free falling raindrop attains a constant velocity with respect to the air depending upon its size. This velocity is 800 cm/sec for drops of the size 0.25 cm dia. and is zero for spray. This means that in case the air currents are moving upwards with a velocity greater than 800 cm/sec, no rain drop can fall.

(*c*) Falling raindrops greater than 0.5 cm in dia become unstable and break up into smaller drops.

(*d*) When a drop is broken up by air currents, the water particles become positively charged and the air negatively charged.

(*e*) When an ice crystal strikes with air currents, the ice crystal is negatively charged and the air positively charged.

Wilson's Theory of Charge Separation

Wilson's theory is based on the assumption that a large number of ions are present in the atmosphere. Many of these ions attach themselves to small dust particles and water particles. It also assumes that an electric field exists in the earth's atmosphere during fair weather which is directed downwards towards the earth (Fig. 12.27(*a*)). The intensity of the field is approximately 1 volt/cm at the surface of the earth and decreases gradually with height so that at 30,000 ft it is only about 0.02 V/cm. A relatively large raindrop (0.1 cm radius) falling in this field becomes polarized, the upper side acquires a negative charge and the lower side a positive charge. Subsequently, the lower part of the drop attracts –ve charges from the atmosphere which are available in abundance in the atmosphere leaving a preponderance of positive charges in the air. The upwards motion of air currents tends to carry up the top of the cloud, the +ve air and smaller drops that the wind can blow against gravity. Meanwhile the falling heavier raindrops which are negatively charged settle on the base of the cloud. It is to be noted that the selective action of capturing –ve charges from the atmosphere by the lower surface of the drop is possible. No such selective action occurs at the upper surface. Thus in the original system, both the positive and negative charges which were mixed up, producing essentially a neutral space charge, are now separated. Thus according to Wilson's theory since larger negatively charged drops settle on the base of the cloud and smaller positively charged drops settle on the uper positions of the cloud, the lower base of the cloud is negatively charged and the upper region is positively charged (Fig. 12.27(*b*)).

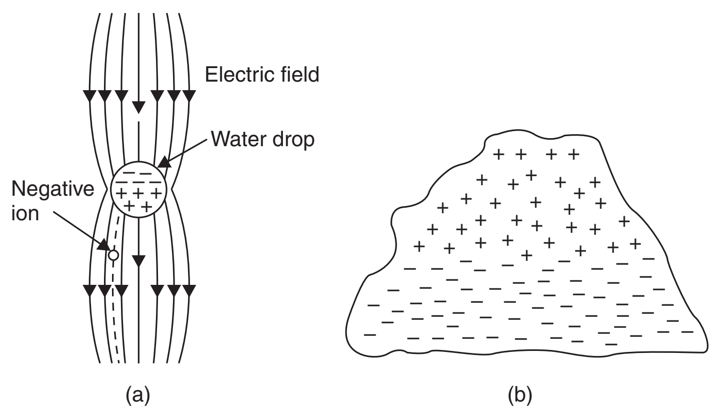


Fig. 12.27 (*a*) Capture of negative ions by large falling drop; (*b*) Charge separation in a thunder cloud according to Wilson's theory.

Simpson's and Scarse Theory

Simpson's theory is based on the temperature variations in the various regions of the cloud. When water droplets are broken due to air currents, water droplets acquire positive charges whereas the air is negatively charged. Also when ice crystals strike with air, the air is positively charged and the crystals are negatively charged. The theory is explained with the help of Fig. 12.28.

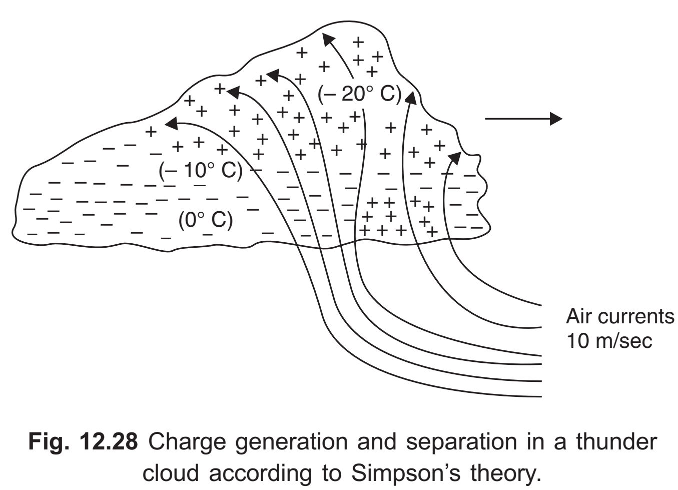


Fig. 12.28 Charge generation and separation in a thunder cloud according to Simpson's theory.

Let the cloud move in the direction from left to right as shown by the arrow. The air currents are also shown in the diagram. If the velocity of the air currents is about 10 m/sec in the base of the cloud, these air currents when collide with the water particles in the base of the cloud, the water drops are broken and carried upwards unless they combine together and fall down in a pocket as shown by a pocket of positive charges (right to bottom region in Fig. 12.28). With the collision of water particles we know the air is negatively charged and the water particles positively charged. These negative charges in the air are immediately absorbed by the cloud particles which are carried away upwards with the air currents. The air currents go still higher in the cloud where the moisture freezes into ice crystals. The air currents when collide with ice crystals the air is positively charged and it goes in the upper region of cloud whereas the negatively charged ice crystals drift gently down in the lower region of the cloud. This is how the charge is separated in a thundercloud. Once the charge separation is complete, the conditions are now set for a lightning stroke.

Mechanism of Lightning Stroke

Lightning phenomenon is the discharge of the cloud to the ground. The cloud and the ground form two plates of a gigantic capacitor and the dielectric medium is air. Since the lower part of the cloud is negatively charged, the earth is positively charged by induction. Lightning discharge will require the puncture of the air between the cloud and the earth. For breakdown of air at STP condition the electric field required is 30 kV/cm peak. But in a cloud where the moisture content in the air is large and also because of the high altitude (lower pressure) it is seen that for breakdown of air the electric field required is only 10 kV/cm. The mechanism of lightning discharge is best explained with the help of Fig. 12.29.

After a gradient of approximately 10 kV/cm is set up in the cloud, the air surrounding gets ionized. At this a streamer (Fig. 12.29(*a*)) starts from the cloud towards the earth which cannot be detected with the naked eye; only a spot travelling is detected. The current in the streamer is of the order of 100 amperes and the speed of the streamer is 0.5 ft/μ sec. This streamer is known as pilot streamer because this leads to the lightning phenomenon. Depending upon the state of ionization of the air surrounding the streamer, it is branched to several paths and this is known as stepped leader (Fig. 12.29(*b*)). The leader steps are of the order of 50 m in length and are accomplished in about a microsecond. The charge is brought from the cloud through the already ionized path to these pauses. The air surrounding these pauses is again ionized and the leader in this way reaches the earth (Fig. 12.29(*c*)).

Once the stepped leader has made contact with the earth it is believed that a power return stroke (Fig. 12.29(*c*)) moves very fast up towards the cloud through the already ionized path by the leader. This streamer is very intense where the current varies between 1000 amps and 200,000 amps and the speed is about 10% that of light. It is here where the –ve charge of the cloud is being neutralized by the positive induced charge on the earth (Fig. 12.29(*d*)). It is this instant which gives rise to lightning flash which we observe with our naked eye. There may be another cell of charges in the cloud near the neutralized charged cell. This charged cell will try to neutralize through this ionised path. This streamer is known as dart leader Fig. 12.29(*e*). The velocity of the dart leader is about 3% of the velocity of light. The effect of the dart leader is much more severe than that of the return stroke.

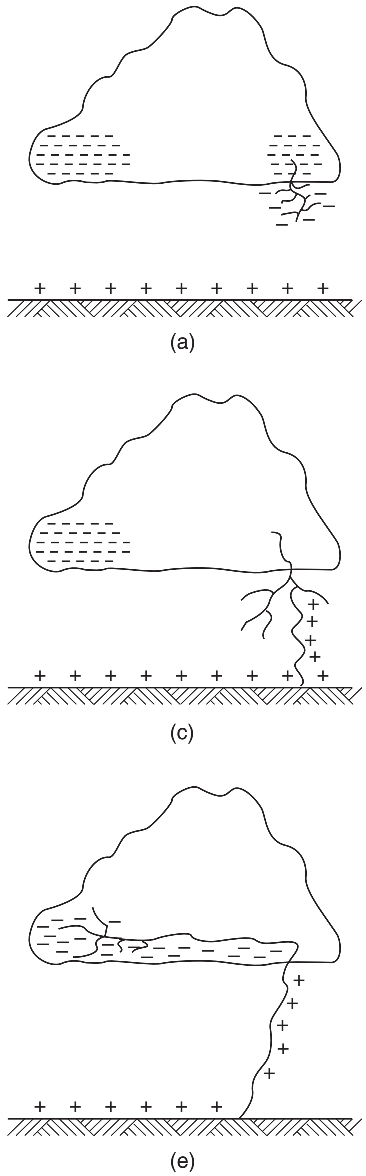


Fig. 12.29 Lightning mechanism

The discharge current in the return streamer is relatively very large but as it lasts only for a few microseconds the energy contained in the streamer is small and hence this streamer is known as cold lightning stroke whereas the dart leader is known as hot lightning stroke because even though the current in this leader is relatively smaller but it lasts for some milliseconds and therefore the energy contained in this leader is relatively larger.

It is found that each thunder cloud may contain as many as 40 charged cells and a heavy lightning stroke may occur. This is known as multiple stroke.

12.9 LINE DESIGN BASED ON LIGHTNING

The severity of switching surges for voltage 400 kV and above is much more than that due to lightning voltages. All the same it is desired to protect the transmission lines against direct lightning strokes. The object of good line design is to reduce the number of outages caused by lightning. To achieve this the following actions are required:

(*i*) The incidence of stroke on to power conductor should be minimised.

(*ii*) The effect of those strokes which are incident on the system should be minimized.

To achieve (*i*) we know that, lightning normally falls on tall objects; thus tall towers are more vulnerable to lightning than the smaller towers. In order to keep smaller tower height for a particular ground clearance, the span lengths will decrease which requires more number of towers and hence the associated accessories like insulators etc. The cost will go up very high. Therefore, a compromise has to be made so that adequate clearance is provided, at the same time keeping longer span and hence lesser number of towers.

With a particular number of towers the chances of incidence of lightning on power conductors can be minimized by placing a ground wire at the top of the tower structure. Refer to article 16.3 for ground wires.

Once the stroke is incident on the ground wire, the lightning current propagates in both the directions along the ground wire. The tower presents a discontinuity to the travelling waves; therefore they suffer reflections and refraction. The system is, then, equivalent to a line bifurcated at the tower point.

We know that, the voltage and current transmitted into the tower will depend upon the surge impedance of the tower and the ground impedance (tower footing resistance) of the tower. If it is low, the wave reflected back up the tower will largely remove the potential existing due to the incident wave. In this way the chance of flash over is eliminated. If, on the other hand, the incident wave encounters a high ground impedance, positive reflection will take place and the potential on the top of the tower structure will be raised rather than lowered. It is, therefore, desired that for good line design high surge impedances in the ground wire circuits, the tower structures and the tower footing should be avoided. Various methods for lowering the tower footing resistances have been discussed in article 16.3.

PROBLEMS

12.1. Given an *RL* circuit with a sudden 50 Hz sinusoidal voltage applied where $R = 20$ ohms, $L = 0.36$ H and voltage $V = 220$ V.

(*a*) The switch is closed at such a time as to permit maximum transient current. What is the instantaneous value of V upon closing the switch ?

(*b*) What is the maximum value of current in part (*a*) ?

(*c*) Let the switch be closed so as to yield minimum transient current. What instantaneous values of V and α correspond to this instant of closing the switch ?

12.2. Determine the relative attenuation occurring in two cycles in the over voltage surge set up on a 132 kV cable fed through an air blast breaker when the breaker opens on a system short circuit.

The breaker uses critical resistance switching. The network parameters are $R = 10$ ohms, $L = 8$ mH and $C = 0.08\ \mu F$.

12.3. Explain with neat diagrams two different theories of charge generation and separation in a thunder cloud.

12.4. Explain with neat sketches the mechanism of lightning discharge.

12.5. Differentiate between a hot lightning stroke and a cold lightning stroke.

12.6. Show that a travelling wave moves with a velocity of light on the overhead line and its speed is proportional to $1/\sqrt{\varepsilon_r}$ on a cable with dielectric material of permittivity ε_r.

12.7. Explain the variation of current and voltage on an overhead line when one end of the line is (*i*) short-circuited, and (*ii*) open-circuited and at the other end a source of constant e.m.f. V is switched in.

12.8. What is a travelling wave ? Explain the development of such a wave on an overhead line.

12.9. An overhead transmission line with surge impedance 400 ohms is 300 km long. One end of this line is short-circuited and at the other end a source of 11 kV is suddenly switched in. Calculate the current at the source end 0.005 sec after the voltage is applied.

12.10. Explain why a short length of cable is connected between the dead end tower and the terminal apparatus in a station. An overhead line with surge impedance 400 ohms is connected to a terminal apparatus through a short length of cable of surge impedance 40 ohms.

A travelling wave of constant magnitude 100 kV and infinite duration originates in the overhead line and travels towards the junction with the cable. Calculate the energy transmitted into the cable during a period of 5 μ sec after the arrival of the wave at the junction.

12.11. An overhead line with inductance and capacitance per km of 1.24 mH and 0.087 μF respectively is connected in series with an underground cable having inductance and capacitance of 0.185 mH/km and 0.285 μF/km. Calculate the values of transmitted and reflected waves of voltage and current at the junction due to a voltage surge of 110 kV travelling to the junction (*i*) along the line towards the cable, and (*ii*) along the cable towards the line.

12.12. An overhead line with surge impedance 400 ohms bifurcates into two lines of surge impedance 400 ohms and 40 ohms respectively. If a surge of 20 kV is incident on the overhead line, determine the magnitudes of voltage and current which enter the bifurcated lines.

12.13. A long overhead line has a surge impedance of 500 ohms and an effective resistance of 6 ohms per km. If a surge of 400 kV enters the line at a certain point, calculate the magnitude of this surge after it has traversed 100 km and calculate the power loss and heat loss of the wave over this distance. Assume velocity of wave as 3×10^8 m/sec.

12.14. A rectangular surge of 2 μ sec duration and magnitude 100 kV travels along a line of surge impedance 500 ohms. The latter is connected to another line of equal impedance through an inductor of 500 μH. Calculate the maximum value of surge transmitted to the second line.

12.15. The effective inductance and capacitance of a faulted system as viewed from the contacts of a breaker are 2.5 mH and 600 pF respectively. Determine the restriking voltage across the breaker contacts when a fault current of 150 amps is chopped.

12.16. What is arcing ground ? Explain its effect on the performance of a power system.

12.17. What is "capacitance switching" ? Explain its effect on the performance of the circuit breaker.

12.18. Derive an expression for the restriking voltage across the circuit breaker contacts. The system consists of an unloaded alternator with neutral solidly grounded.

12.19. Explain clearly the variation of current and impedance of an alternator when a 3-phase sudden short-circuit takes place at its terminals.

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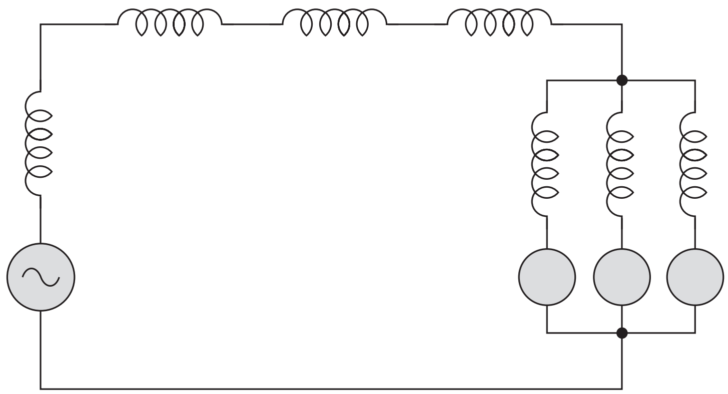
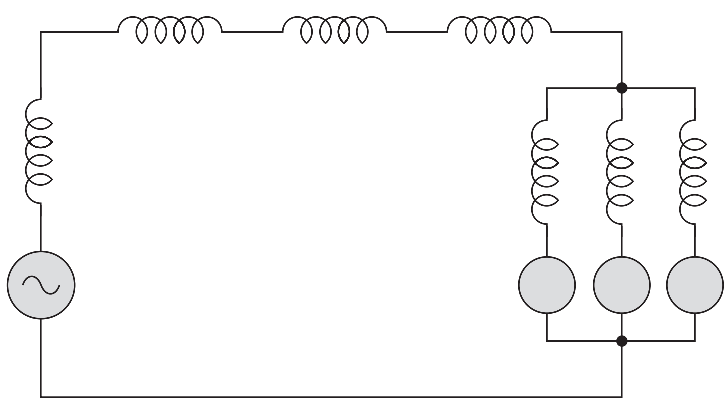


13

SYMMETRICAL COMPONENTS AND FAULT CALCULATIONS



13

Symmetrical Components and Fault Calculations

INTRODUCTION

In 1918, Dr. C.L. Fortescue presented a paper entitled "Method of Symmetrical Coordinates Applied to Solution of Polyphase Networks" at AIEE in which he proved that "a system of n vectors or quantities may be resolved, when n is prime, into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining $(n - 1)$ systems consist of n equi-spaced vectors which with the first mentioned group of equal vectors forms an equal number of symmetrical n-phase systems".

The method of symmetrical components is a general one applicable to any polyphase system.

Because of the widespread use of 3-phase systems and the greater familiarity which electrical engineers have with them, symmetrical component equations will be developed for 3-phase systems.

13.1 3-PHASE SYSTEMS

Any three coplanar vectors V_a, V_b and V_c can be expressed in terms of three new vectors V_1, V_2 and V_3 by three simultaneous linear equations with constant coefficients. Thus

$$V_a = a_{11}V_1 + a_{12}V_2 + a_{13}V_3 \tag{13.1}$$

$$V_b = a_{21}V_1 + a_{22}V_2 + a_{23}V_3 \tag{13.2}$$

$$V_c = a_{31}V_1 + a_{32}V_2 + a_{33} V_3 \tag{13.3}$$

Each of the original vectors has been replaced by a set of three vectors making a total of nine vectors. This has been done to simplify the calculations and to have better understanding of the problem. With this in mind, two conditions should be satisfied in selecting systems of components to replace 3-phase current and voltage vectors:

1. Calculations should be simplified by the use of the chosen systems of components. This is possible only if the impedances (or admittances) associated with the components of current (or voltage) can be obtained readily by calculation or test.

2. The system of components chosen should have physical significance and be an aid in determining power system performance.

According the Fortescue's theorem, the three unbalanced vectors V_a, V_b and V_c can be replaced by a set of three balanced systems of vectors. Therefore, the solution of equations (13.1)–(13.3) is unique. A balanced system of three vectors is one in which the vectors are equal in magnitude and are equi-spaced. The three symmetrical component vectors replacing V_a, V_b and V_c are:

1. Positive sequence component which has three vectors of equal magnitude but displaced in phase from each other by 120° and has the same phase sequence as the original vectors.

2. Negative sequence component which has three vectors of equal magnitude but displaced in phase from each other by 120° and has the phase sequence opposite to the original vectors.

3. Zero sequence component which has three vectors of equal magnitude and also are in phase with each other.

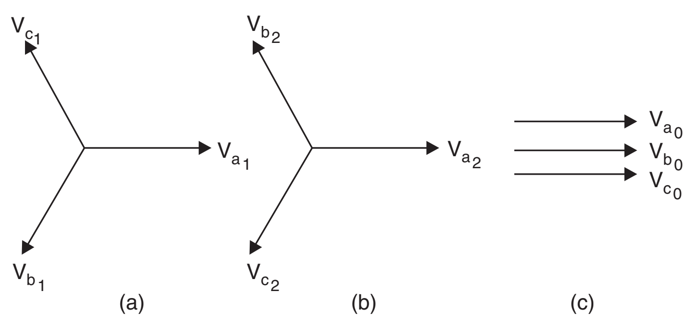


Fig. 13.1 (*a*) Positive sequence component; (*b*) Negative sequence component; (*c*) Zero sequence component.

The components have been shown in Fig. 13.1. The voltage vectors have been designated as V_a, V_b and V_c and the phase sequence is assumed here as a, b, c. The subscripts 1, 2 and 0 are being used to represent positive, negative and zero sequence quantities respectively.

13.2 SIGNIFICANCE OF POSITIVE, NEGATIVE AND ZERO SEQUENCE COMPONENTS

By a positive sequence system of vectors is meant the vectors are equal in magnitude and 120° apart in phase, in which the time order of arrival of the phase vectors at a fixed axis of reference corresponds to the generated voltages. This really means that if a set of positive sequence voltages is applied to the stator winding of the alternator, the direction of rotation of the stator field is the same as the rotor or alternatively if the direction of rotation of the stator field is the same as that of the rotor, the set of voltages are positive sequence voltages. On the contrary if the direction of rotation of the stator field is opposite to that of the rotor, the set of voltages are negative sequence voltages. The zero sequence voltages are single phase voltages and, therefore, they give rise to an alternating field in space. Since the 3-phase windings are 120° apart in space, at any particular instant the three vector fields due to the three phases are 120° apart and, therefore, assuming complete symmetry of the windings, the net flux in the air gap will be zero.

From Fig. 13.1, the following relations between the original unbalanced vectors and their corresponding symmetrical components, can be written:

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} \tag{13.4}$$

$$V_b = V_{b_1} + V_{b_2} + V_{b_0} \tag{13.5}$$

$$V_c = V_{c_1} + V_{c_2} + V_{c_0} \tag{13.6}$$

Assuming phase a as the reference as shown in Fig. 13.1 the following relations between the symmetrical components of phases b and c in terms of phase a can be written. Here use is made of the operator λ which has a magnitude of unity and rotation through 120°, *i.e.*, when any vector is multiplied by λ, the vector magnitude remains same but is rotated anticlockwise through 120°. Thus

$$\lambda = 1\angle 120°$$

In the complex form $\lambda = \cos 120° + j \sin 120°$

$$= -0.5 + j0.866$$

Similarly $\lambda^2 = -0.5 - j0.866$

$$\lambda^3 = 1.0 = 1\angle 360°$$

or $\lambda^3 - 1 = 0$

or $(\lambda - 1)(\lambda^2 + \lambda + 1) = 0$

Since $\lambda \neq 1$ as λ is a complex quantity as defined above,

$\therefore$ $\lambda^2 + \lambda + 1 = 0$

In fact λ is a number which when doubly squared remains λ itself, *i.e.*, $\lambda^4 = \lambda$.

So the important relations that will be frequently required in power system analysis are

$$\lambda = -0.5 + j0.866 = 1.0\angle 120°$$

$$\lambda^2 = -0.5 - j0.866 = 1.0\angle -120°$$

$$\lambda^3 = 1.0\angle 0°$$

$$\lambda^4 = \lambda$$

$$\lambda^2 + \lambda + 1 = 0$$

Now we go back to deriving relations between the symmetrical components of phases b and c in terms of the symmetrical components of phase a.

From Fig. 13.1, $V_{b_1} = \lambda^2 V_{a_1}$

This means in order to express V_{b_1} in terms of V_{a_1}, V_{a_1} should be rotated anti-clockwise through 240°.

Similarly $V_{c_1} = \lambda V_{a_1}$

For negative sequence vectors

$$V_{b_1} = \lambda V_{a_2},\ V_{c_2} = \lambda^2 V_{a_2}$$

For zero sequence vectors

$$V_{b_0} = V_{a_0} = V_{c_0}$$

Substituting these relations in equations (13.4)–(13.6),

$$V_a = V_{a_1} + V_{a_2} + V_{c_0} \qquad (13.7)$$

$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0} \qquad (13.8)$$

$$V_c = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0} \qquad (13.9)$$

Compare equations (13.1)–(13.3) with equations (13.7)–(13.9),

$$a_{11} = a_{12} = a_{13} = 1$$
$$a_{21} = \lambda^2,\ a_{22} = \lambda,\ a_{23} = 1$$
$$a_{31} = \lambda,\ a_{32} = \lambda^2,\ a_{33} = 1$$

The coefficients have been uniquely determined for the 3-phase systems. Equations (13.7)–(13.9) express the phase voltages V_a, V_b and V_c in terms of the symmetrical components of phase a *i.e.*, in case V_{a_1}, V_{a_2} and V_{a_0} are known, the phase voltages V_a, V_b and V_c can be calculated.

Similar relations between the phase currents in terms of the symmetrical components of currents taking phase a as reference hold good and are given below:

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} \qquad (13.7a)$$

$$I_b = \lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0} \qquad (13.8a)$$

$$I_c = \lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0} \qquad (13.9a)$$

Normally the unbalanced phase voltages and currents are known in a system; it is required to find out the symmetrical components. The procedure is as follows:

The problem is: given V_a, V_b, V_c, find out V_{a_1}, V_{a_2} and V_{a_0}. To find out positive sequence component V_{a_1}, multiply equations (13.7), (13.8) and (13.9) by 1, λ and λ^2 respectively and adding them up, it gives

$$\begin{aligned} V_a + \lambda V_b + \lambda^2 V_c &= V_{a_1}(1 + \lambda^3 + \lambda^3) + V_{a_2}(1 + \lambda^2 + \lambda^4) + V_{a_0}(1 + \lambda + \lambda^2) \\ &= 3V_{a_1} + V_{a_2}(1 + \lambda^2 + \lambda) + 0 \\ &= 3V_{a_1} \end{aligned}$$

Since $\quad 1 + \lambda + \lambda^2 = 0$

$$\therefore \quad V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$

For negative sequence component V_{a_2} multiplying equations (13.7), (13.8) and (13.9) by 1, λ^2 and λ respectively and adding,

$$\begin{aligned} V_a + \lambda^2 V_b + \lambda V_c &= V_{a_1}(1 + \lambda^4 + \lambda^2) + V_{a_2}(1 + \lambda^3 + \lambda^3) + V_{a_0}(1 + \lambda^2 + \lambda) \\ &= 3V_{a_2} \end{aligned}$$

$$\therefore \quad V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

For zero sequence component V_{a_0}, add equations (13.7), (13.8) and (13.9)

$$V_a + V_b + V_c = V_{a_1}(1 + \lambda^2 + \lambda) + V_{a_2}(1 + \lambda + \lambda^2) + 3V_{a_0}$$

or

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$

Rewriting these equations,

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) \quad (13.10)$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c) \quad (13.11)$$

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c) \quad (13.12)$$

Similarly these relations for currents are given as

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c) \quad (13.13)$$

In the equations above V_a, V_b and V_c may be the line to ground voltages, line to neutral voltages, line to line voltages at a point in the network or they may be the generated or induced voltages, in fact any set of three voltages revolving at the same rate which may exist in the 3-phase system. Similarly, the three currents could be, phase currents, line currents, the currents flowing into a fault from the line conductors etc.

Example 13.1: The line-to-ground voltages on the high voltage side of a step-up transformer are 100 kV, 33 kV and 38 kV on phases a, b and c respectively. The voltage of phase a leads that of phase b by 100° and lags that of phase c by 176.5°. Determine analytically the symmetrical components of voltage

$$V_a = 100\angle 0°$$

$$V_b = 33\angle -100°$$

$$V_c = 38\angle 176.5°.$$

Solution:

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$

$$= \tfrac{1}{3}[100\angle 0° + 33\angle -100° \,.\, \angle 120° + 38\angle 176.5° \angle -120°]$$

$$= \tfrac{1}{3}[100 + j0.0 + 33\angle 20° + 38\angle 56.5°]$$

$$= \tfrac{1}{3}[151.97 + j42.97] = 50.65 + j14.32. \qquad \textbf{Ans.}$$

$$V_{a_2} = \tfrac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c]$$

$$= \tfrac{1}{3}[100 + j0.0 + 33\angle -220° + 38\angle 296.5°]$$

$$= (30.55 - j4.26). \qquad \textbf{Ans.}$$

Similarly,

$$V_{c_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$

$$= \tfrac{1}{3}[100 + j0.0 + 33\angle -100° + 38\angle 176.5°]$$

$$= \tfrac{1}{3}[56.37 - j30.18]$$

$$= 18.79 - j10.06. \qquad \textbf{Ans.}$$

Example 13.2: The line currents in amperes in phases a, b and c respectively are $500 + j150$, $100 - j600$ and $-300 + j600$ referred to the same reference vector. Find the symmetrical component of currents.

Solution: The line currents are

$$I_a = 500 + j150,\ I_b = 100 - j600 \text{ and } I_c = -300 + j600 \text{ amps}$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

$$= \tfrac{1}{3}[500 + j150 + 100 - j600 - 300 + j600]$$

$$= 100 + j\,50 \text{ amps.} \quad \textbf{Ans.}$$

$$I_{a_1} = \tfrac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$$

$$= \tfrac{1}{3}[500 + j150 + (-0.5 + j0.866)(100 - j600) + (-0.5 - j0.866)(-300 + j600)]$$

$$= \tfrac{1}{3}[1639 + j496.4] = 546.3 + j165.46 \text{ amps.} \quad \textbf{Ans.}$$

$$I_{a_2} = \tfrac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$

$$= \tfrac{1}{3}[500 + j150 + (-0.5 - j0.866)(100 - j600) + (-0.5 + j0.866)(-300 + j600)]$$

$$= \tfrac{1}{3}[146.3 - j65.46]$$

$$= 48.8 - j21.82 \text{ amps.} \quad \textbf{Ans.}$$

13.3 AVERAGE 3-PHASE POWER IN TERMS OF SYMMETRICAL COMPONENTS

The average power

$$P = V_a I_a \cos\phi_a + V_b I_b \cos\phi_b + V_c I_c \cos\phi_c \tag{13.14}$$

$$= V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c$$

$$= (V_{a_1} + V_{a_2} + V_{a_0}) \cdot (I_{a_1} + I_{a_2} + I_{a_0}) + (\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0}) \cdot (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0}) + (\lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}) \cdot (\lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0})$$

Taking first term on the r.h.s.,

$$(V_{a_1} + V_{a_2} + V_{a_0}) \cdot (I_{a_1} + I_{a_2} + I_{a_0})$$

$$= V_{a_1} \cdot I_{a_1} + V_{a_2} \cdot I_{a_2} + V_{a_0} \cdot I_{a_0} + V_{a_1} \cdot I_{a_2} + V_{a_1} \cdot I_{a_0} + V_{a_2} \cdot I_{a_1} + V_{a_2} \cdot I_{a_0} + V_{a_0} \cdot I_{a_1} + V_{a_0} \cdot I_{a_2}$$

Expanding second term on the r.h.s.,

$$(\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0}) \cdot (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0})$$

$$= \lambda^2 V_{a_1} \cdot \lambda^2 I_{a_1} + \lambda^2 V_{a_1} \cdot \lambda I_{a_2} + \lambda^2 V_{a_1} \cdot I_{a_0} + \lambda V_{a_2} \cdot \lambda^2 I_{a_1} + \lambda V_{a_2} \cdot \lambda I_{a_2} + \lambda V_{a_2} \cdot I_{a_0} + V_{a_0} \cdot \lambda^2 I_{a_1} + V_{a_0} \cdot \lambda I_{a_2} + V_{a_0} \cdot I_{a_0}$$

Now the dot product of two vectors does not change when both are rotated through the same angle.

For example,
$$\lambda^2 V_{a_1} \cdot \lambda^2 I_{a_1} = V_{a_1} \cdot I_{a_1}$$
$$\lambda^2 V_{a_1} \cdot \lambda I_{a_2} = \lambda V_{a_1} \cdot I_{a_2}$$

The addition of the terms after expanding and rearranging,

$$\begin{aligned}
P &= 3V_{a_0} \cdot I_{a_0} + 3V_{a_2} \cdot I_{a_2} + 3V_{a_1} \cdot I_{a_1} + V_{a_1} \cdot I_{a_2}(1+\lambda+\lambda^2) \\
&\quad + V_{a_1} \cdot I_{a_0}(1+\lambda+\lambda^2) + V_{a_2} \cdot I_{a_1}(1+\lambda+\lambda^2) + V_{a_2} \cdot I_{a_0}(1+\lambda+\lambda^2) \\
&\quad + V_{a_0} \cdot I_{a_1}(1+\lambda+\lambda^2) + V_{a_0} \cdot I_{a_2}(1+\lambda+\lambda^2) \\
&= 3(V_{a_1} \cdot I_{a_1} + V_{a_2} \cdot I_{a_2} + V_{a_0} \cdot I_{a_0}) \\
&= 3[\,|V_{a_1}|\,|I_{a_1}|\cos\theta_1 + |V_{a_2}|\,|I_{a_2}|\cos\theta_2 \\
&\qquad + |V_{a_0}|\,|I_{a_0}|\cos\theta_0] \qquad (13.15)
\end{aligned}$$

The same power expression can be very easily derived using matrix manipulations.

$$\begin{aligned}
P + jQ &= V_a I_a^* + V_b I_b^* + V_c I_c^* \\
&= [V_a \;\; V_b \;\; V_c]\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*
\end{aligned}$$

Since from equations (13.7), (13.8) and (13.9),

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = AV$$

and
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = (AV)^T = V^T A^T$$

$\therefore$
$$P + jQ = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = [V_{a_0} \;\; V_{a_1} \;\; V_{a_2}]\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Now substituting for the phase currents the corresponding symmetrical components, noting that λ and λ^2 are conjugate,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}^* \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*$$

$\therefore$
$$\begin{aligned}
P + jQ &= [V_{a_0} \;\; V_{a_1} \;\; V_{a_2}]\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^* \\
&= [V_{a_0} \;\; V_{a_1} \;\; V_{a_2}]\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^* \\
&= 3[V_{a_0} \;\; V_{a_1} \;\; V_{a_2}]\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*
\end{aligned}$$

$$= 3[V_{a_0} I_{a_0}^* + V_{a_1} I_{a_1}^* + V_{a_2} I_{a_2}^*]$$

$$\therefore \quad P = 3[\,|V_{a_0}|\,|I_{a_0}|\cos\theta_0 + |V_{a_1}|\,|I_{a_1}|\cos\theta_1 + |V_{a_2}|\,|I_{a_2}|\cos\theta_2].$$

13.4 SEQUENCE IMPEDANCES

So far we have discussed the symmetrical components for the currents, voltages and power. Let us now study something about the sequence impedances of the system. The sequence impedances of an equipment or a component of power system are the positive, negative and zero sequence impedances. They are defined as follows:

The positive sequence impedance of an equipment is the impedance offered by the equipment to the flow of positive sequence currents. Similarly, the negative sequence or zero sequence impedance of the equipment is the impedance offered by the equipment to the flow of corresponding sequence current. The significance of the positive, negative and zero sequence currents has already been discussed in this chapter. For a 3-phase, symmetrical static circuit without internal voltages like transformers and transmission lines, the impedances to the currents of any sequence are the same in the three phases; also the currents of a particular sequence will produce drop of the same sequence or a voltage of a particular sequence will produce current of the same sequence only, which means there is no mutual coupling between the sequence networks. Since for a static device, the sequence has no significance, the positive and negative sequence impedances are equal; the zero sequence impedance which includes the impedance of the return path through the ground, in the general case, is different from the positive and negative sequence impedance. In a symmetrical rotating machine the impedances met by armature currents of a given sequence are equal in the three phases. Since by definition the inductance, which forms a part of impedance, is the flux linkages per ampere, it will depend upon the phase order of the sequence current relative to the direction of rotation of the rotor; positive, negative and zero sequence impedances are unequal in the general case. In fact for a rotating machine, the positive sequence impedance varies, having minimum value immediately following the fault and then increases with time until steady state conditions are reached when the positive sequence impedance corresponds to the synchronous impedance. The variation of the positive sequence impedance for a rotating machine has been discussed in Chapter 12.

Let us represent positive, negative and zero sequence impedances respectively by Z_1, Z_2 and Z_0. We have already mentioned that for the symmetrical systems there is no mutual coupling between the sequence networks. The three-sequence systems can then be considered separately and phase currents and voltages determined by superposing their symmetrical components of current and voltage respectively.

Before we proceed further to use the symmetrical components technique for the analysis of unbalanced conditions in power systems, it is desirable to know the methods for measuring the sequence impedances.

Measurement of Sequence Impedances of Rotating Machines

Measurement of Positive Sequence Impedance: As already mentioned, the positive sequence impedance depends upon the working of the machine, *i.e.*, whether it is working under subtransient, transient or steady state condition. The impedance under steady state condition is known as the synchronous impedance and is measured by the well-known open circuit short circuit test. This impedance is defined as

$$\text{Synchronous impedance in p.u.} = \frac{\begin{array}{c}\text{Field current at rated armature current}\\ \text{on sustained symmetrical short circuit}\end{array}}{\begin{array}{l}\text{Field current at normal open circuit voltage on}\\ \text{the air gap line (}i.e.\text{, the extended straight line}\\ \text{part of the magnetisation curve)}\end{array}}$$

Method of Test for Synchronous Impedance: The machine is run at synchronous speed in proper direction with the help of a prime mover (Fig. 13.2).

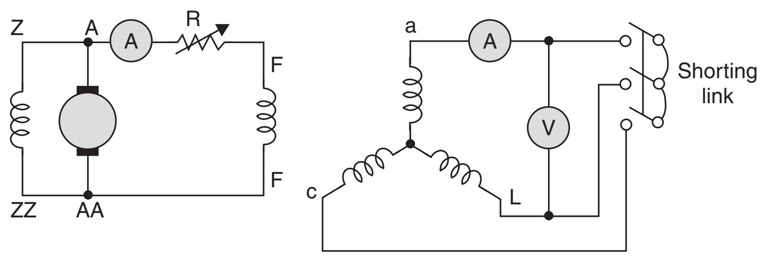


Fig. 13.2 Connection diagram for open circuit and short circuit test on an alternator.

The switch is kept in off position to perform open circuit test. The readings of voltmeter for various field currents are taken. Next the excitation is reduced to minimum by putting the total resistance in the field circuit and the switch is closed to perform short circuit test. Since short circuit test is under unsaturated condition of the machine it will be a linear characteristic passing through the origin and one single reading is enough. The two characteristics are plotted and according to the definition of synchronous impedance the value is calculated from the graph.

Method of Test for Subtransient Reactance: Apply voltage across any two terminals except the neutral with the rotor at rest and short circuited on itself through an ammeter (Fig. 13.3). The rotor is rotated by hand and it will be observed that for a fixed voltage applied, the current in the field varies with the position of the rotor. When the rotor is in the position of maximum induced field current (the direct axis position of rotor), one half the voltage required to circulate rated current is equal to the direct axis subtransient reactance X_d'' in per unit value. If the rotor is in the position of minimum induced field current the quadrature axis subtransient reactance X_q'' is obtained.

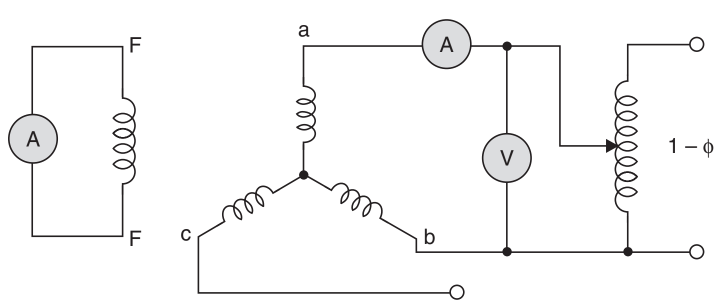


Fig. 13.3 Measurement of subtransient reactance of an alternator.

Measurement of Negative Sequence Reactance: The negative sequence reactance of a machine is the impedance offered to the flow of negative sequence current.

The machine is driven at rated speed and a reduced voltage is applied to circulate approximately the rated current. It is to be noted here that since negative sequence currents flow in this case, there is possibility of hunting which will result in oscillation of the pointer of the ammeter. The mean reading may be taken. The negative sequence impedance is given by

$$Z_2 = \frac{V}{\sqrt{3}I}$$

where V is the voltmeter and I the ammeter reading as shown in the diagram (Fig. 13.4).

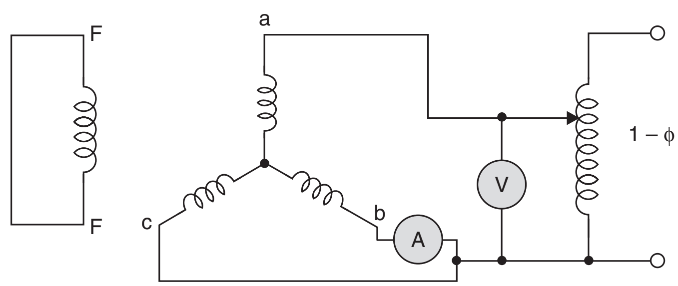


Fig. 13.4 Measurement of negative sequence impedance.

This can be proved mathematically as follows:

From the experiment, since it is similar to a line-to-line fault with alternator unloaded,

$$I_a = 0, I_b = I, I_c = -I$$

$$I_{a_1} = -I_{a_2} \quad \text{and} \quad V_{a_1} = V_{a_2} \quad \text{and} \quad V_{a_0} = 0,\ I_{a_0} = 0 \text{ (see section 13.7.1)}$$

From the measurement, voltage

$$V = V_a - V_b$$

i.e.

$$V = V_{a_1} + V_{a_2} - (\lambda^2 V_{a_1} + \lambda V_{a_2}) = 2V_{a_2} + V_{a_2} = 3V_{a_2}$$

and current in the ammeter

$$I = I_b = \lambda^2 I_{a_1} + \lambda I_{a_2} = (\lambda - \lambda^2) I_{a_2}$$

Now $\quad (\lambda - \lambda^2) = -0.5 + j0.866 + 0.5 + j0.866 = j\sqrt{3} = |\sqrt{3}|\angle 90°$

$\therefore \quad$ Current measured $= I = \sqrt{3}\, I_{a_2}$

Now
$$\frac{V}{\sqrt{3}I} = \frac{V_a - V_b}{\sqrt{3}I_b} = \frac{3V_{a_2}}{\sqrt{3}\cdot\sqrt{3}I_{a_2}} = \frac{V_{a_2}}{I_{a_2}} = Z_2$$

Measurement of Zero Sequence Impedance: Zero sequence impedance is the impedance offered by the machine to the flow of the zero sequence current. This impedance is quite variable and depends upon the distribution, *i.e.*, the pitch and the breadth factors. If the windings were infinitely distributed so that each phase produced a sinusoidal distribution of the m.m.f. then the superposition of the three phases with equal instantaneous currents cancel each other and produce zero field and consequently zero reactance except for slot and end-connection fluxes. The departure from this by introducing chording and breadth factors determines the zero sequence impedance. However, zero sequence impedance is much smaller than positive and negative sequence impedances. The machine must, of course, be star connected for otherwise the term zero sequence impedance has no significance as no zero sequence currents can flow.

The machine (Fig. 13.5) is at standstill and a reduced voltage is applied. The zero sequence impedance $Z_0 = V/3I$.

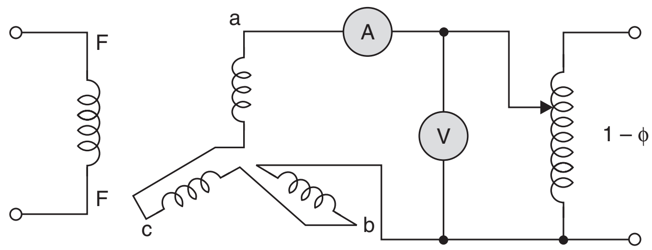


Fig. 13.5 Measurement of zero sequence impedance.

This connection ensures equal distribution of current in the three phases and for this reason is preferable to connecting the three phases in parallel. However, if the six terminals are not available the three phases are connected in parallel and experiment is conducted in the same fashion.

13.5 FAULT CALCULATIONS

Broadly speaking the faults can be classified as:

1. Shunt faults (short circuits).
2. Series faults (open conductor).

Shunt type of faults involve power conductor or conductors-to-ground or short circuit between conductors. When circuits are controlled by fuses or any device which does not open all three phases, one or two phases of the circuit may be opened while the other phases or phase is closed. These are called series type of faults. These faults may also occur with one or two broken conductors. Shunt faults are characterised by increase in current and fall in voltage and frequency whereas series faults are characterised by increase in voltage and frequency and fall in current in the faulted phases.

Shunt type of faults are classified as (*i*) Line-to-ground fault; (*ii*) Line-to-line fault; (*iii*) Double line-to-ground fault; and (*iv*) 3-phase fault. Of these, the first three are the unsymmetrical faults as the symmetry is disturbed in one or two phases. The method of symmetrical components will be utilized to analyse the unbalancing in the system. The 3-phase fault is a balanced fault which could also be analysed using symmetrical components.

The series faults are classified as: (*i*) one open conductor, and (*ii*) two open conductors. These faults also disturb the symmetry in one or two phases and are, therefore, unbalanced faults. The method of symmetrical components can be used for analysing such situations in the system.

Here we will discuss only the shunt type of faults.

Voltage of the Neutral

The potential of the neutral when it is grounded through some impedance or is isolated, will not be at ground potential under unbalanced conditions such as unsymmetrical faults. The potential of the neutral is given as $V_n = -I_n Z_n$, where Z_n is the neutral grounding impedance and I_n the neutral current. Here negative sign is used as the current flows from the ground to the neutral of the system and potential of the neutral is lower than the ground.

For a 3-phase system,

$$
\begin{aligned}
I_n &= I_a + I_b + I_c \\
&= (I_{a_1} + I_{a_2} + I_{a_0}) + (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0}) + (\lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0}) \\
&= I_{a_1}(1 + \lambda + \lambda^2) + I_{a_2}(1 + \lambda + \lambda^2) + 3I_{a_0} \\
&= 3I_{a_0} \qquad (13.16)
\end{aligned}
$$

$$\therefore \quad V_n = -3I_{a_0} Z_n \qquad (13.17)$$

Since the positive sequence and negative sequence components of currents through the neutral are absent, the drops due to these currents are also zero. Also for a balanced set of currents or voltages the neutral is at ground potential; therefore, for positive and negative sequence networks, neutral of the system will be taken as the reference.

Reference of Voltages

The phase voltages at any point in a grounded system and their zero sequence components of voltage will be referred to the ground at that point. The positive and negative sequence components of voltage are referred to neutral. For the positive and negative sequence systems, therefore, the expressions voltage to neutral and voltage to ground may be used interchangeably but for the zero sequence system it is important to distinguish between the two terms.

The analysis here will apply to a symmetrical 3-phase system with dissymmetry only at one point *i.e.*, faults at simultaneously more than one point will not be considered. In a 3-phase system, the unknown quantities are the 3-phase voltage V_a, V_b and V_c and the 3-phase currents I_a, I_b and I_c *i.e.*, there are six unknowns. To determine these quantities, six linearly independent equations are required. In any given problem, certain conditions are required about the unknown quantities and these are the boundary conditions which can be expressed in the form of equation, *e.g.*, if conductor a is faulted to ground at some point, the voltage of this conductor at the faulted point is zero, *i.e.*, $V_a = 0$. It has already been seen that the 3-phase voltages and currents can be expressed in terms of their corresponding three symmetrical components. Therefore, instead of 3-phase voltages and currents being unknown one can say that six symmetrical components V_{a_0}, V_{a_1}, V_{a_2}, I_{a_0}, I_{a_1} and I_{a_2} are unknown. In a 3-phase system, three equations (boundary conditions) can be written in terms of the three unknown phase currents and voltages at the point of dissymmetry. Three more equations are needed for a solution of the six unknowns. The advantage in using the six unknown components instead of the six unknown phase quantities is that the impedances met by the sequence currents can be determined either by calculation or test. This is not usually the case with phase impedances. However, if the phase impedances can also be readily obtained, there may be no advantage in introducing components; in fact, the use of phase quantities may give a simpler solution. The three sequence equations using the sequence generated voltages and the sequence impedances are derived as follows.

13.6 SEQUENCE NETWORK EQUATIONS

These equations will be derived for an unloaded alternator with neutral solidly grounded, assuming that the system is balanced, *i.e.*, the generated voltages are of equal magnitude and displaced by 120°. Consider the diagram (Fig. 13.6).

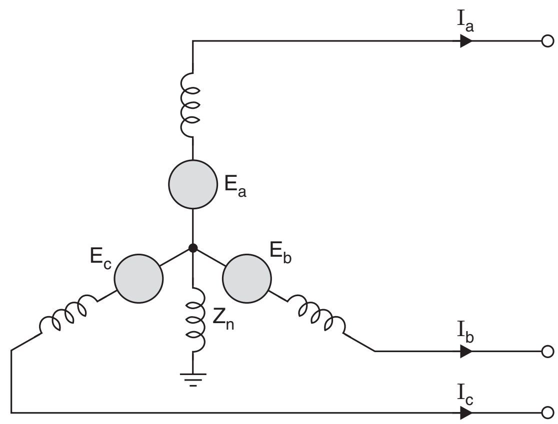


Fig. 13.6 A balanced 3-phase system.

Since the sequence impedances per phase are same for all three phases and we are considering initially a balanced system the analysis will be done on single phase basis. The positive sequence component of voltage at the fault point is the positive sequence generated voltage minus the drop due to positive sequence current in positive sequence impedance (as positive sequence current does not produce drop in negative or zero sequence impedances)

$$V_{a_1} = E_a - I_{a_1} Z_1$$

Similarly, the negative sequence component of voltage at the fault point is the generated negative sequence voltage minus the drop due to negative sequence current in negative sequence impedance (as negative sequence current does not produce drop in positive or zero sequence impedances)

$$V_{a_2} = E_{a_2} - I_{a_2} Z_2$$

Since the negative sequence voltage generated is zero, therefore,

$$E_{a_2} = 0$$

or

$$V_{a_2} = - I_{a_2} Z_2$$

Similarly, for zero sequence voltages

$$E_{a_0} = 0$$

$$V_{a_0} = V_n - I_{a_0} Z_{g_0} = -3 I_{a_0} Z_n - I_{a_0} Z_{g_0} = - I_{a_0} (Z_{g_0} + 3Z_n)$$

where Z_{g_0} is the zero sequence impedance of the generator and Z_n is the neutral impedance.

The three sequence network equations are, therefore,

$$V_{a_1} = E_a - I_{a_1} Z_1 \tag{13.18}$$

$$V_{a_2} = -\,I_{a_1} Z_2 \tag{13.19}$$

$$V_{a_0} = -\,I_{a_0} Z_0 \tag{13.20}$$

where $Z_0 = Z_{g_0} + 3Z_n$ and the corresponding sequence networks for the unloaded alternator are shown in Fig. 13.7.

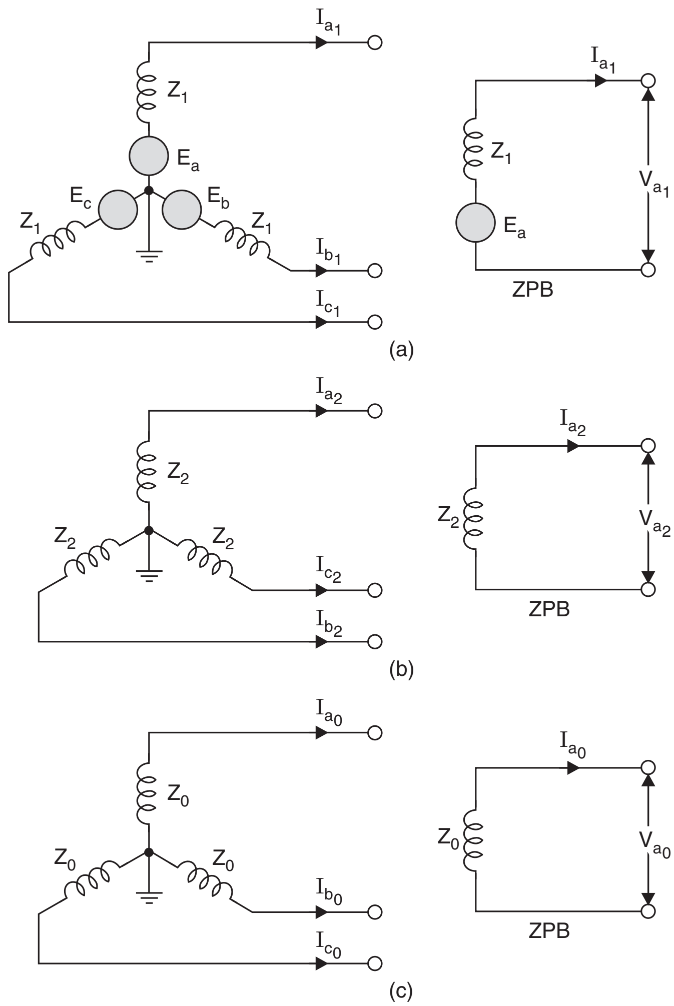


Fig. 13.7 Sequence networks: (*a*) Positive sequence network; (*b*) Negative sequence network; and (*c*) Zero sequence network.

Simultaneous solution of the three sequence equations and the three boundary conditions equations in which the phase quantities have been replaced by their symmetrical components of currents and voltages, will give the six unknown symmetrical components of currents and voltages. Once the symmetrical components of currents and voltages are known the phase currents and voltages can be obtained by using the relation (13.7) through (13.9) respectively. The sequence network equation in matrix notation will be

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} \tag{13.20a}$$

Now we are ready with mathematical tools to analyse various types of shunt faults. For all type of faults the sequence network equations will be as given by equations (13.18)–(13.20) whereas the three equations describing the boundary conditions will be different for different types of faults. The analysis will be done by both the algebraic manipulations and the matrix manipulations for the sake of completeness. We will analyse first of all a system where faults take place on an unloaded alternator with neutral solidly grounded and it is assumed that the faults are also solid so that no impedance is introduced between the fault points. Later on the analysis will be made with (*i*) neutral grounded through some impedance Z_n, and (*ii*) fault having some impedance Z_f.

13.7 SINGLE LINE-TO-GROUND FAULT

The system to be analysed is shown in Fig. 13.8. Let the fault take place on phase *a*. The boundary conditions are

$$V_a = 0 \tag{13.21}$$

$$I_b = 0 \tag{13.22}$$

$$I_c = 0 \tag{13.23}$$

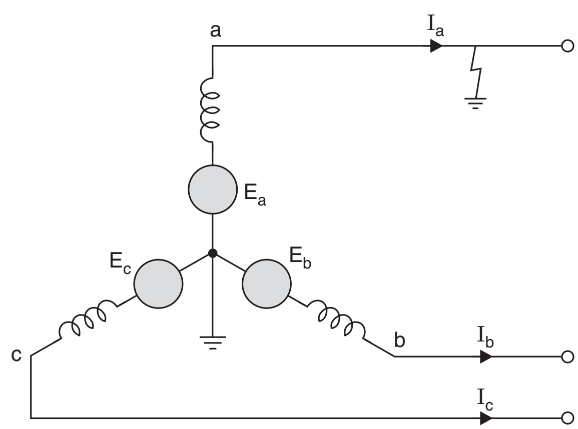


Fig. 13.8 A solidly grounded, unloaded alternator: L-G fault on phase a.

and the sequence network equations are

$$V_{a_0} = -I_{a_0} Z_0 \tag{13.18}$$

$$V_{a_1} = E_a - I_{a_1} Z_1 \tag{13.19}$$

$$V_{a_2} = -I_{a_2} Z_2 \tag{13.20}$$

The solution of these six equations will give six unknowns V_{a_0}, V_{a_1}, V_{a_2} and I_{a_0}, I_{a_1} and I_{a_2}.

From equation (13.13),

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

Substituting the values of I_b and I_c from equations (13.22–13.23),

$$I_{a_1} = I_{a_2} = I_{a_0} = I_a/3 \tag{13.24}$$

Equation (13.21) can be written in terms of symmetrical components

$$V_a = 0 = V_{a_1} + V_{a_2} + V_{a_0} \tag{13.25}$$

Now substituting the values of V_{a_0}, V_{a_1} and V_{a_2} from the sequence network equation,

$$E_a - I_{a_1} Z_1 - I_{a_2} Z_2 - I_{a_0} Z_0 = 0 \tag{13.26}$$

Since $\quad I_{a_1} = I_{a_2} = I_{a_0}$

Equation (13.26) becomes

$$E_a - I_{a_1} Z_1 - I_{a_1} Z_2 - I_{a_1} Z_0 = 0$$

or

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0} \tag{13.27}$$

From equation (13.27) it is clear that to simulate a L-G fault all the three sequence networks are required and since the currents are all equal in magnitude and phase angle, therefore, the three sequence networks must be connected in series. The voltage across each sequence network corresponds to the same sequence component of V_a. The interconnection of the sequence network is shown in Fig. 13.9.

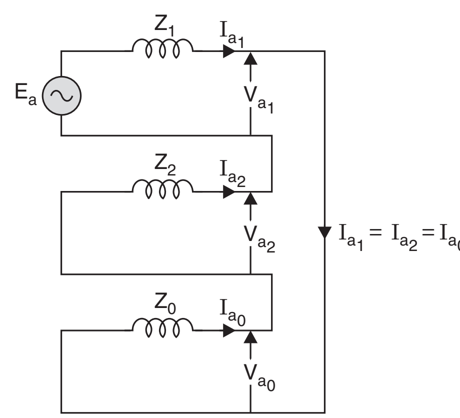


Fig. 13.9 Interconnection of sequence networks for L-G fault.

So far we have calculated $I_{a_1} = I_{a_2} = I_{a_0}$. To calculate the remaining three unknowns V_{a_0}, V_{a_1}, V_{a_2}, use is made of the sequence network equations.

The analysis will now be made using matrix manipulations.

From equation (13.13)

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

Substituting for $I_b = I_c = 0$,

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From this equation $\quad I_{a_0} = I_{a_1} = I_{a_2} = I_a/3$

Substituting equation (13.24) into equation (13.20(a)),

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_1} \\ I_{a_1} \\ I_{a_1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a_1} Z_0 \\ I_{a_1} Z_1 \\ I_{a_1} Z_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} -I_{a_1} Z_0 \\ E_a - I_{a_1} Z_1 \\ -I_{a_1} Z_2 \end{bmatrix}$$

$$\therefore \quad V_{a_0} + V_{a_1} + V_{a_2} = 0 = -I_{a_1} Z_0 + E_a - I_{a_1} Z_1 - I_{a_1} Z_2$$

$$\therefore \quad I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

Now in case of line-to-ground fault the neutral current

$$I_n = I_a = I_{a_1} + I_{a_2} + I_{a_0}$$

and for the same case, $I_{a_1} = I_{a_2} = I_{a_0}$

$$\therefore \quad I_n = 3 I_{a_0}$$

In case the neutral is not grounded the zero sequence impedance Z_0 becomes infinite and, therefore, from equation (13.27),

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + \infty} = 0$$

The same result can be envisaged by looking at the system when the neutral is isolated; there is no return path for the current and, therefore, $I_{a_1} = I_{a_2} = I_{a_0} = 0$. This means that for this system the fault current $I_a = 0$.

Example 13.3: A 25 MVA, 13.2 kV alternator with solidly grounded neutral has a subtransient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u. respectively. A single line to ground fault occurs at the terminals of an unloaded alternator; determine the fault current and the line-to-line voltages. Neglect resistance.

Solution: Normally the positive sequence impedance is greater than the negative sequence but since the given positive sequence impedance corresponds to the subtransient state, it may be less than the negative sequence impedance. The sequence network for a line-to-ground fault current is shown in Fig. E.13.3.

Let the line-to-neutral voltage at the fault point before the fault be 1.0 + j0.0 p.u. For a line-to-ground fault the fault impedance is

$$j0.25 + j0.35 + j0.1 = j0.7$$

$$\therefore \quad I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1 + j0.0}{j0.7} = -j1.428$$

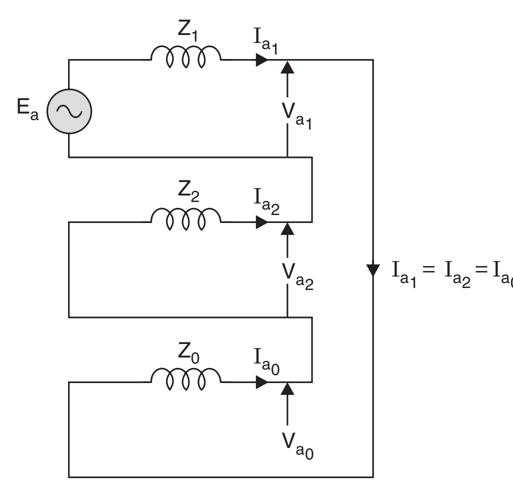


Fig. E.13.3 Interconnection of sequence network.

For a *L-G* fault

$$I_{a_1} = I_{a_2} = I_{a_0} = -j1.428$$

$\therefore$ The p.u. fault current $\quad I_a = I_{a_1} + I_{a_2} + I_{a_0} = 3I_{a_1} = -j4.285$

Let the base quantities be 25 MVA, 13.2 kV, and hence

$$\text{the base current} = \frac{25 \times 1000}{\sqrt{3} \times 13.2} = 1093 \text{ amps}$$

$\therefore$ The fault current in amperes = 1093 × 4.285 = 4685 amps

To find out the voltages, we first find out the sequence components of voltages.

$$V_{a_1} = E_a - I_{a_1} Z_1$$
$$= 1 + j0.0 - (-j1.428)(j0.25)$$
$$= 1 - 0.357 = 0.643$$
$$V_{a_2} = -I_{a_2} Z_2 = -(-j1.428)\,(j0.35)$$
$$= -0.4998$$

Similarly, $\quad V_{a_0} = -I_{a_0} Z_0 = -(-j1.428)\,(j0.1) = 0.1428$

As a numeric check $V_a = 0$. Substituting the values of V_{a_1}, V_{a_2} and V_{a_0},

$$0.643 - 0.4998 - 0.1428 \simeq 0$$
$$V_b = V_{b_1} + V_{b_2} + V_{b_0} \text{ and } V_c = V_{c_1} + V_{c_2} + V_{c_0}$$

Now $\quad V_{b_1} = \lambda^2 V_{a_1} = (-0.5 - j0.866)\,(0.643)$

$$= -0.3215 - j0.5568$$
$$V_{b_2} = \lambda V_{a_2} = (-0.5 + j0.866)(-0.50)$$
$$= (0.25 - j0.433)$$
$$V_{b_0} = V_{a_0} = V_{c_0} = -0.1428$$
$$V_{c_1} = \lambda V_{a_1} = (-0.5 + j0.866)(0.643)$$
$$= -0.3215 + j0.5568$$
$$V_{c_2} = \lambda^2 V_{a_2} = (-0.5 - j0.866)\,(-0.5)$$
$$= 0.25 + j0.433$$

$\therefore \quad V_b = -0.3215 - j0.5568 + 0.25 - j0.433 - 0.1428$

$$= -0.2143 - j0.9898$$

and $\quad V_c = -0.3215 + j0.5568 + 0.25 + j0.433 - 0.1428$

$$= -0.2143 + j0.9898$$

Now the line-to-line voltage

$$V_{ab} = V_a - V_b. \text{ Since } V_a = 0,$$
$$V_{ab} = -V_b = 0.2143 + j0.9898$$
$$V_{ac} = -V_c = 0.2143 - j0.9898$$

and $\quad V_{bc} = V_b - V_c = -j2 \times 0.9898$

$$= -j1.9796$$

Now $\quad V_{ab} = 0.2143 + j0.9898 = \sqrt{(0.4592 + 9.797) \times 10^{-1}}$

$$= \sqrt{10.346 \times 10^{-1}} = \sqrt{1.0346} = 1.0127 \text{ p.u.}$$

The line-to-line voltage will be

$$V_{ab} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.717 \text{ kV}$$

$$V_{ac} = 7.717 \text{ kV}$$

and $$V_{bc} = 1.9796 \times \frac{13.2}{\sqrt{3}} = 15.08 \text{ kV}.$$

Line-to-line Fault

As shown in Fig. 13.10, the line-to-line fault takes place on phases b and c. The boundary conditions are

$$I_a = 0 \tag{13.28}$$

$$I_b + I_c = 0 \tag{13.29}$$

$$V_b = V_c \tag{13.30}$$

and the sequence network equations are given by equations (13.18)–(13.20). The solution of these six equations will give six unknowns.

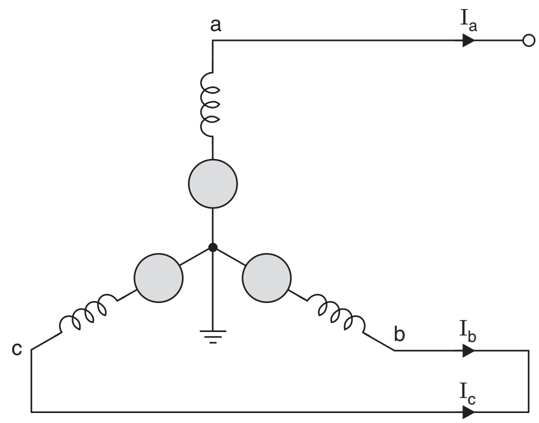


Fig. 13.10 L-L fault on an unloaded and neutral grounded alternator.

Using the relations $\quad I_{a_1} = \frac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$

$$I_{a_2} = \frac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \frac{1}{3}(I_a + I_b + I_c)$$

and substituting for I_a, I_b and I_c

$$I_{a_1} = \frac{1}{3}(0 + \lambda I_b - \lambda^2 I_b)$$

$$= \frac{1}{3}(\lambda - \lambda^2) I_b$$

$$I_{a_2} = \tfrac{1}{3}(0 + \lambda^2 I_b - \lambda I_b)$$

$$= \frac{I_b}{3}(\lambda^2 - \lambda)$$

and $$I_{a_0} = \tfrac{1}{3}(0 + 0) = 0$$

which means for a line-to-line fault the zero-sequence component of current is absent and positive-sequence component of current is equal in magnitude but opposite in phase to negative sequence component of current, *i.e.*

$$I_{a_1} = - I_{a_2} \quad \text{...(13.31)}$$

To simulate *L-L* fault condition zero sequence network is not required and the positive and negative-sequence networks are to be connected in opposition as $I_{a_1} = - I_{a_2}$.

Now from equations (13.8) and (13.9)

$$V_b = V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2}$$

$$V_c = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_1}$$

Substituting these relations in equation (13.30),

$$V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2} = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_2}$$

or $$(\lambda^2 - \lambda) V_{a_1} = (\lambda^2 - \lambda) V_{a_2}$$

$\therefore$ $$V_{a_1} = V_{a_2} \quad \text{...(13.32)}$$

That is, positive-sequence component of voltage equals the negative-sequence component of voltage. This also means that the two sequence networks are connected in opposition. Now making use of the sequence network equation and the equation (13.32),

$$V_{a_1} = V_{a_2}$$

$$E_a - I_{a_1} Z_1 = - I_{a_2} Z_2 = I_{a_1} Z_2$$

or $$I_{a_1} = \frac{E_a}{Z_1 + Z_2}$$

The interconnection of the sequence network for simulation of *L-L* fault is shown in Fig. 13.11.

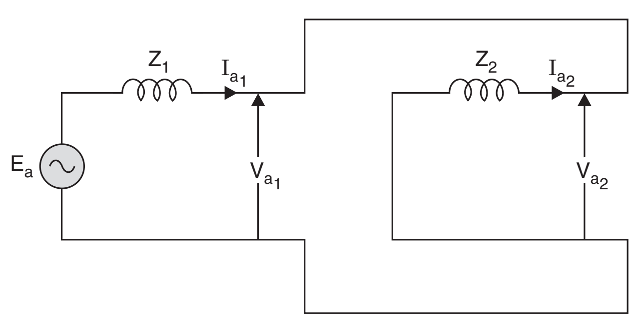


Fig. 13.11 Interconnection of sequence networks for L-L fault.

So far we have calculated I_{a_1}, I_{a_2} and I_{a_0}, we can calculate the three symmetrical components of voltages V_{a_1}, V_{a_2} and V_{a_0} and then using the relations (13.7)–(13.9), the phase currents and voltages can be obtained. It is to be noted here that since $I_{a_0} = 0$, $\therefore$ $V_{a_0} = 0$.

The *L-L* fault can be analysed using matrix manipulation as follows:

Using the relation (14.13) and substituting for I_a, I_b and I_c,

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a_0} = 0,\ I_{a_1} = (\lambda - \lambda^2)I_b \quad \text{and} \quad I_{a_2} = (\lambda^2 - \lambda)I_b$$

$$\therefore \qquad I_{a_1} = -I_{a_2}$$

Again using the relation (13.20*a*) and substituting for V_a, V_b and V_c,

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} V_2 \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c) = 0$$

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_b)$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_b)$$

$$\therefore \qquad V_{a_1} = V_{a_2}$$

The sequence network equations are

$$\begin{bmatrix} 0 \\ V_{a_1} \\ V_{a_1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a_1} \\ -I_{a_1} \end{bmatrix}$$

$$\therefore \qquad V_{a_1} = E_a - I_{a_1} Z_1 = +\, I_{a_1} Z_2$$

$$\therefore \qquad I_{a_1} = \frac{E_a}{Z_1 + Z_2}.$$

The interconnection of the sequence network for simulating *L-L* fault satisfies all the relations derived. We have derived mathematically that zero sequence current will be absent in this case, which can be envisaged physically from the network also. We see that in the system there is only one ground *i.e.*, the grounded neutral of the system and since the fault does not involve ground the zero sequence currents which are single phase currents do not flow *i.e.*, $I_{a_0} = 0$.

Example 13.4: Determine the fault current and the line-to-line voltage at the fault when a line-to-line fault occurs at the terminals of the alternator described in Example 13.3.

Solution: The sequence network for *L-L* fault is shown in Fig. E.13.4. Since the zero sequence network is absent, assuming $(1 + j0.0)$ prefault per unit voltage,

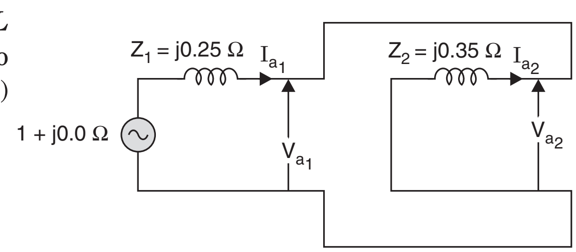


Fig. E.13.4 Sequence network.

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2} = \frac{1 + j0.0}{j0.25 + j0.35}$$

$$= \frac{1 + j0.0}{j0.6} = -j1.667$$

Now for a *L-L* fault

$$I_{a_1} = -I_{a_2} = -j1.667$$

$\therefore$ $$I_{a_2} = j1.667$$

and $$I_{a_0} = 0$$

To find out the fault current, $I_b = -I_c$, we use the following relations:

$$I_b = I_{b_1} + I_{b_2} + I_{b_0} = I_{b_1} + I_{b_2} = \lambda^2 I_{a_1} + \lambda I_{a_2}$$
$$= (-0.5 - j0.866)(-j1.667) + (-0.5 + j0.866)(j1.667)$$
$$= j0.833 - 1.4436 - j0.833 - 1.4436$$
$$= -2.8872 \text{ p.u.}$$

Now base current is 1093 amperes.

$\therefore$ Fault current = 1093 × 2.8872 = 3155.71 amperes

To find out line-to-line voltage we find out the sequence components of voltages

$$V_{a_1} = E_a - I_{a_1} Z_1 = 1 + j0.0 - (-j1.667)(j0.25)$$
$$= 1 - 0.4167 = 0.5833$$

Similarly,

$$V_{a_2} = -I_{a_2} Z_2 = (-j1.667)(j0.35) = 0.5834$$

i.e., $$V_{a_1} = V_{a_2} = 0.5833 \text{ p.u.}$$

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} = V_{a_1} + V_{a_2} = 2 \times 0.5833 = 1.1666 \text{ p.u.}$$
$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2}$$
$$= (-0.5 - j0.866)\,(0.5833) + (-0.5 + j0.866)\,(0.5833)$$
$$= -0.5833$$

and $$V_b = V_c = -0.5833$$

Line voltage

$$V_{ab} = V_a - V_b = 1.1666 - (-0.5833) = 1.7499$$
$$V_{ac} = V_a - V_c = 1.7499$$

and $$V_{bc} = V_b - V_c = 0.0$$

The line-to-line voltage

$$V_{ab} = 1.7499 \times \frac{13.2}{\sqrt{3}} = 13.33 \text{ kV}$$
$$V_{ac} = 13.33 \text{ kV}$$

and $$V_{bc} = 0.0 \text{ kV.} \quad \textbf{Ans.}$$

Double Line to Ground Fault

Double line to ground fault takes place on phases *b* and *c* (Fig. 13.12). The boundary conditions are

$$I_a = 0 \tag{13.33}$$
$$V_b = 0 \tag{13.34}$$
$$V_c = 0 \tag{13.35}$$

and the sequence network equations are given by (13.18)–(13.20).

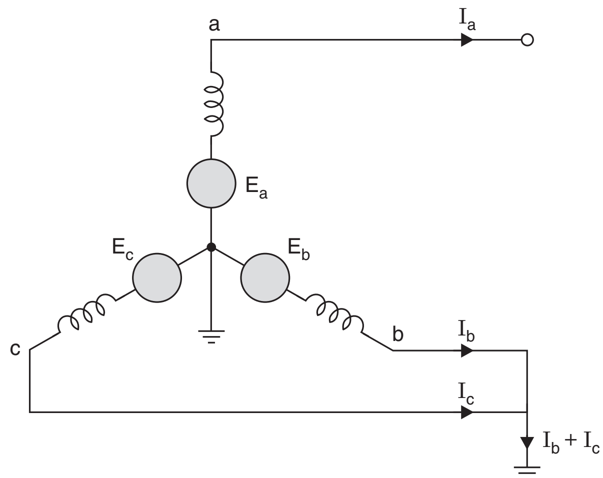


Fig. 13.12 A solidly grounded, unloaded alternator, L-L-G fault.

The solution of these six equations will give the six unknown symmetrical components.

Using the equations (13.10)–(13.12) and substituting for V_a, V_b and V_c from (13.34) and (13.35).

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$
$$= V_a/3$$
$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$
$$= V_a/3$$
$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$
$$= V_a/3$$

i.e.,
$$V_{a_0} = V_{a_1} = V_{a_2} \tag{13.36}$$

Using this relation of voltages and substituting in the sequence network equations

$$V_{a_0} = V_{a_1}$$
$$-I_{a_0} Z_0 = E_a - V_{a_1} Z_1$$

$\therefore$
$$I_{a_0} = -\frac{E_a - I_{a_1} Z_1}{Z_0} \tag{13.37}$$

Similarly
$$V_{a_2} = V_{a_1}$$
$$-I_{a_2} Z_2 = E_a - I_{a_1} Z_1$$

$\therefore$
$$I_{a_2} = -\frac{E_a - I_{a_1} Z_1}{Z_2} \tag{13.38}$$

Now from equation (13.33),

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} = 0$$

Substituting values of I_{a_2} and I_{a_0} from equations (13.38) and (13.37),

$$I_{a_1} - \frac{E_a - I_{a_1}Z_1}{Z_2} - \frac{E_a - I_{a_1}Z_2}{Z_0} = 0$$

Rearranging the terms gives

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0Z_2}{Z_0 + Z_2}} \qquad \ldots(13.39)$$

From equation (13.39) it is clear that all the three sequence networks are required to simulate *L-L-G* fault and also that the negative and zero sequence networks are connected in parallel. The sequence network interconnection is shown in Fig. 13.13.

From equation (13.39) it is clear that the zero and negative sequence networks are first connected in parallel and then in opposition with the positive sequence network. The same has been shown in Fig. 13.13.

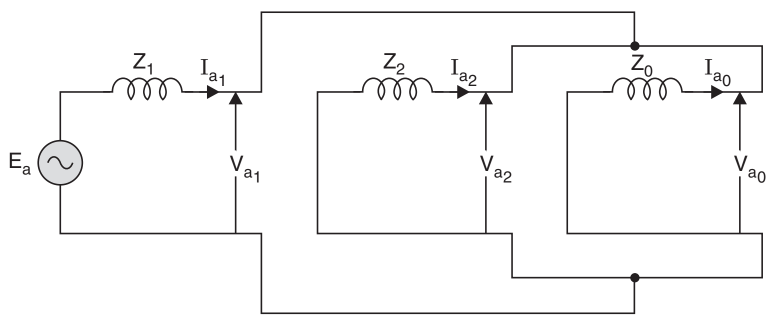


Fig. 13.13 Interconnection of sequence networks for L-L-G fault.

The analysis is made using matrix manipulation.

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$V_{a_0} = V_{a_1} = V_{a_2} = V_a/3$$

Using these relations in the sequence network equations,

$$\begin{bmatrix} V_{a_1} \\ V_{a_1} \\ V_{a_1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}$$

These equations are to be solved for I_{a_0}, I_{a_1} and I_{a_1}.

Rearranging the terms,

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} -V_{a_1} \\ E_1 - V_{a_1} \\ -V_{a_1} \end{bmatrix}$$

or $$AX = B$$

where X is the current vector.

So to find X, pre-multiply this equation by A^{-1}. Therefore,

$$X = A^{-1}B.$$

Now
$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

as it is a diagonal matrix.

Therefore,
$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} -V_{a_1} \\ E_a - V_{a_1} \\ -V_{a_1} \end{bmatrix}$$

or
$$I_{a_0} = -\frac{V_{a_1}}{Z_0} = -\frac{E_a - I_{a_1} Z_1}{Z_0}$$

$$I_{a_2} = -\frac{V_{a_1}}{Z_2} = -\frac{E_a - I_{a_1} Z_1}{Z_2}$$

Use the relation $I_{a_1} + I_{a_2} + I_{a_0} = 0$ and substitute the values of I_{a_0} and I_{a_2} as in equations (13.37) and (13.38) and rearrange the terms. The following is obtained:

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0\, Z_2}{Z_0 + Z_2}}$$

The neutral current

$$\begin{aligned} I_n &= I_b + I_c \\ &= \lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0} + \lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0} \\ &= (\lambda^2 + \lambda) I_{a_1} + (\lambda + \lambda^2) I_{a_2} + 2 I_{a_0} \\ &= -I_{a_1} - I_{a_2} + 2 I_{a_0} \\ &= I_{a_0} + 2 I_{a_0} = 3 I_{a_0} \end{aligned} \tag{13.40}$$

Example 13.5: Determine the fault current and the line-to-line voltages at the fault when a double line-to-ground fault occurs at the terminals of the alternator described in Example 13.4.

Solution: Assuming $(1 + j0.0)$ p.u. as prefault voltage,

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0\, Z_2}{Z_0 + Z_2}} = \frac{1 + j0.0}{j0.25 + \dfrac{j0.1 \times j0.35}{j0.45}} = \frac{1 + j0.0}{j0.25 + j0.0778}$$

$$= \frac{1 + j0.0}{j0.3278} = -j3.0506 \text{ p.u.}$$

Now for L-L-G, $\quad V_{a_1} = V_{a_2} = V_{a_0}$

Also $\quad V_{a_1} = E_a - I_{a_1} Z_1$

To find out I_{a_2} and I_{a_0}, we should first find V_{a_1} and since $V_{a_1} = V_{a_2} = -I_{a_2} Z_2$, I_{a_2} can be obtained.

Similarly, $\quad V_{a_1} = V_{a_0} = -I_{a_0} Z_0$, I_{a_0} can be obtained.

$$V_{a_1} = 1 + j0.0 - (-j3.0506)(j0.25)$$
$$= 1 - 0.7626 = 0.2374$$

$\therefore$ $$V_{a_2} = V_{a_0} = 0.2374$$

and $$I_{a_2} = -\frac{V_{a_2}}{Z_2} = -\frac{0.2374}{j0.35} = j\frac{0.2374}{0.35} = j0.678$$

Similarly, $$I_{a_0} = -\frac{V_{a_0}}{Z_0} = -\frac{0.2374}{j0.1} = j2.374$$

$$I_{a_2} + I_{a_0} = j0.678 + j2.374 = j3.05 = -I_{a_1}$$

Now $$\text{fault current} = I_b + I_c = 3I_{a_0} = 3 \times j2.374 = j7.122 \text{ p.u.}$$

Since base current is 1093 amperes, the fault current will be

$$1093 \times 7.122 = 7784.3 \text{ amperes}$$

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} = 3V_{a_1} = 3 \times 0.2374 = 0.7122$$

and $$V_b = V_c = 0$$

The line-to-line fault voltage,

$$V_{ab} = V_a = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42 \text{ kV}$$

$$V_{ac} = V_a = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42 \text{ kV}$$

$$V_{bc} = 0.0 \text{ kV}$$

3-phase Fault

As shown in Fig. 13.14, the boundary conditions are

$$I_a + I_b + I_c = 0 \tag{13.41}$$

$$V_a = V_b = V_c \tag{13.42}$$

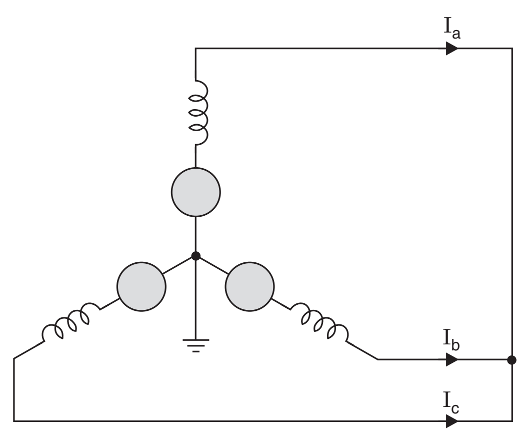


Fig. 13.14 A 3-phase neutral grounded and unloaded alternator 3-phase shorted.

Since $| I_a | = | I_b | = | I_c |$ and if I_a is taken as reference

$$I_b = \lambda^2 I_a \quad \text{and} \quad I_c = \lambda I_a$$

Using the relation

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

and substituting the values of I_b and I_c,

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda^3 I_a + \lambda^3 \mathrm{I}_a)$$

$$= I_a \tag{13.43}$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

Substituting for I_b and I_c in terms of I_a,

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^4 I_a + \lambda^2 I_a)$$

$$= \tfrac{1}{3}(I_a + \lambda I_a + \lambda^2 I_a)$$

$$= \frac{I_a}{3}(1 + \lambda + \lambda^2)$$

$$= 0 \tag{13.44}$$

Similarly,

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

$$= 0 \tag{13.45}$$

which means that for a 3-phase fault zero-as well as negative-sequence components of current are absent and the positive-sequence component of current is equal to the phase current.

Now using the voltage boundary relation,

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) = \tfrac{1}{3}(V_a + \lambda V_a + \lambda^2 V_a)$$

$$= \frac{V_a}{3}(1 + \lambda + \lambda^2) = 0 \tag{13.46}$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

$$= 0 \tag{13.47}$$

$$V_{a_0} = 0 \tag{13.48}$$

Since

$$V_{a_1} = 0 = E_a - I_{a_1} Z_1,$$

$\therefore$

$$I_{a_1} = \frac{E_a}{Z_1} \tag{13.49}$$

The sequence network is shown in Fig. 13.15.

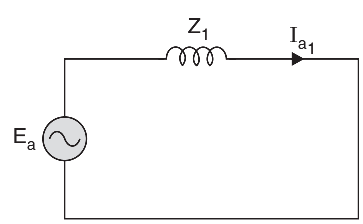


Fig. 13.15 Interconnection of sequence network-3-phase fault.

From the analysis of the various faults, the following observations are made:

1. Positive sequence currents are present in all types of faults.

2. Negative sequence currents are present in all unsymmetrical faults.

3. Zero sequence currents are present when the neutral of the system is grounded and the fault also involves the ground, and magnitude of the neutral current is equal to $3I_{a_0}$.

Since only the positive sequence voltages are generated in the synchronous machine, the question is frequently raised as to the origin of negative and zero sequence voltages that appear throughout the network. It is seen from the analysis that any unbalanced condition gives rise to positive sequence currents and other sequence currents. The negative-and zero-sequence currents produce corresponding drops in their respective networks. These voltages are in general a maximum at the fault point and decrease as the neutral bus is approached.

So far we have studied the various faults on an unloaded alternator with the neutral solidly grounded and the fault is assumed to be solid, *i.e.*, with no fault impedance. Now we will analyse all these faults with neutral impedance Z_n and fault impedance Z_f. Analysis will be made using algebraic manipulations only. Matrix method will not be repeated, the reader can always try the analysis based on the treatment done earlier in this chapter.

13.8 LINE-TO-GROUND FAULT WITH Z_f

The fault impedance is Z_f and the neutral impedance Z_n (Fig. 13.16).

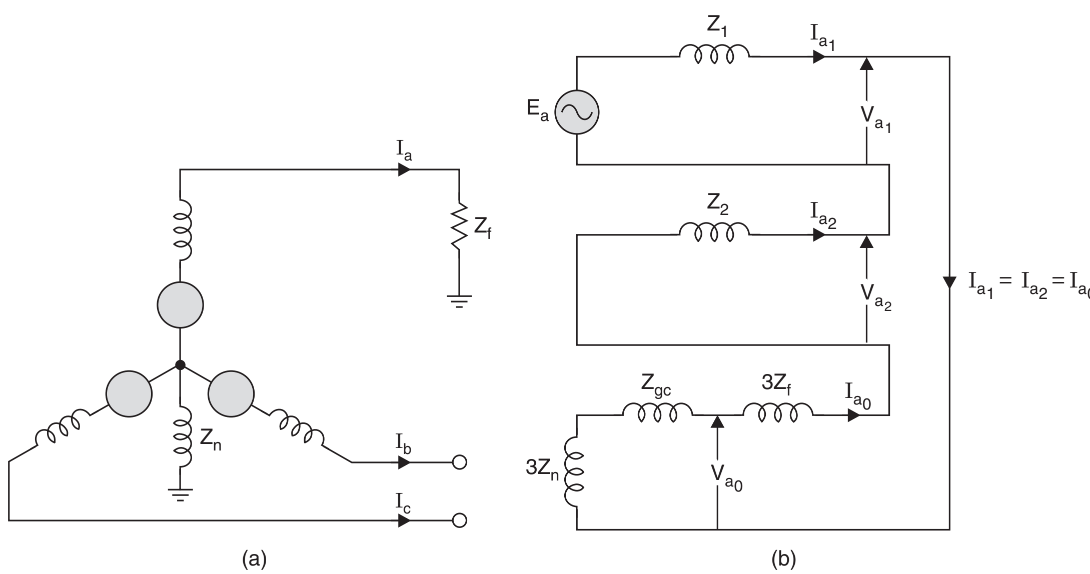


Fig. 13.16 (*a*) A 3-phase unloaded alternator with neutral grounded through impedance Z_n and fault impedance Z_f, L-G fault; (*b*) Interconnection of sequence network for L-G fault.

The boundary conditions are

$$V_a = I_a Z_f$$
$$I_b = 0, I_c = 0$$
$$V_{a_0} = -I_{a_0}(Z_{g_0} + 3Z_n)$$
$$V_{a_1} = E_a - I_{a_1} Z_1,\ V_{a_2} = -I_{a_2} Z_2$$

The solution of these equations gives the unknown quantities.

From equation (13.13) and the boundary condition above,

$$I_{a_1} = I_{a_2} = I_{a_0} = I_a/3$$
$$V_{a_1} + V_{a_2} + V_{a_0} = V_a = 3I_{a_1}(Z_f)$$
$$E_a - I_{a_1} Z_1 - I_{a_1} Z_2 - I_{a_1}(Z_0 + 3Z_n) = 3I_{a_1}(Z_f)$$

$\therefore$
$$E_a = I_{a_1}[Z_1 + Z_2 + \{(Z_0 + 3Z_n) + 3Z_f\}]$$

$\therefore$
$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + (Z_0 + 3Z_n) + 3Z_f} \tag{13.50}$$

Since I_{a_1}, I_{a_2} and I_{a_0} are known, V_{a_1}, V_{a_2} and V_{a_0} can be calculated from the sequence network equations. The sequence network interconnection is shown in Fig. 13.16(*b*).

Line-to-Line Fault with Z_f

The boundary conditions, as shown in Fig. 13.17(*a*), are

$$I_a = 0 \tag{13.28}$$
$$I_b + I_c = 0 \tag{13.29}$$
$$V_b = V_c + I_b Z_f \tag{13.51}$$

and the sequence network equations are

$$V_{a_1} = E_a - I_{a_1} Z_1$$
$$V_{a_2} = -I_{a_2} Z_2$$
$$V_{a_0} = -I_{a_0} Z_0$$

By using equation (13.13), we know that $I_{a_1} = -I_{a_2}$ and $I_{a_0} = 0$.

Using equations (13.8)–(13.9) in equation (13.51),

$$V_b = V_c + I_b Z_f$$
$$V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2} = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_2} + (\lambda^2 I_{a_1} + \lambda I_{a_2}) Z_f$$

or
$$\lambda^2 V_{a_1} - \lambda V_{a_1} = (\lambda^2 - \lambda) V_{a_2} + (\lambda^2 I_{a_1} - \lambda I_{a_1}) Z_f$$

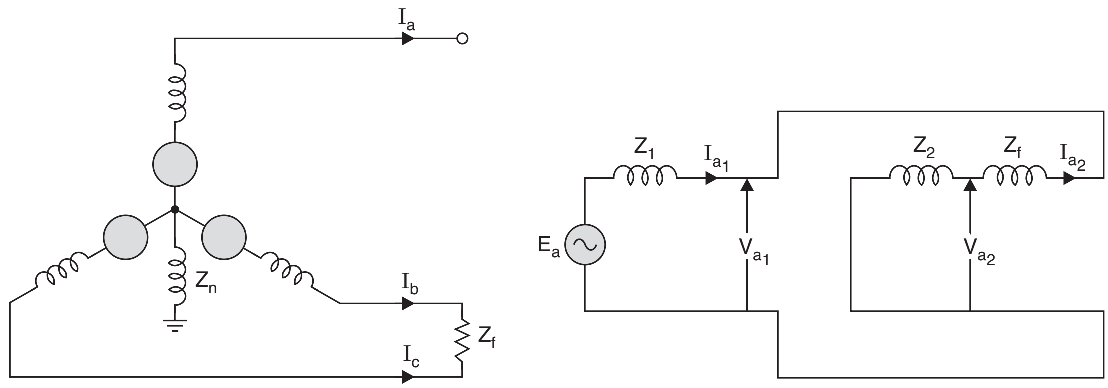
or
$$V_{a_1} = V_{a_2} + I_{a_1} Z_f \tag{13.52}$$

Now substituting for V_{a_1} and V_{a_2} from the sequence network equations,

$$E_a - I_{a_1} Z_1 = -I_{a_2} Z_2 + I_{a_1} Z_f$$
$$E_a - I_{a_1} Z_1 = I_{a_1}(Z_2 + Z_f)$$

or
$$I_{a_1} = \frac{E_a}{Z_1 + (Z_2 + Z_f)} \tag{13.53}$$

The interconnection of the sequence network is shown in Fig. 13.17(*b*).



(a) (b)

Fig. 13.17 (*a*) L-L fault; (*b*) Interconnection of sequence network, fault impedance Z_f, L-L fault.

Double Line-to-Ground Fault

Fault impedance is Z_f and neutral impedance z_n. The boundary conditions, as shown in Fig. 13.18(*a*), are

$$I_b = 0$$

$$V_b = V_c = (I_b + I_c)Z_f \tag{13.54}$$

and the sequence network equations are

$$V_{a_1} = E_a - I_{a_1} Z_1$$

$$V_{a_2} = - I_{a_1} Z_2$$

$$V_{a_0} = - I_{a_0} (Z_0 + 3Z_n)$$

We know that $(I_b + I_c) = 3 I_{a_0}$

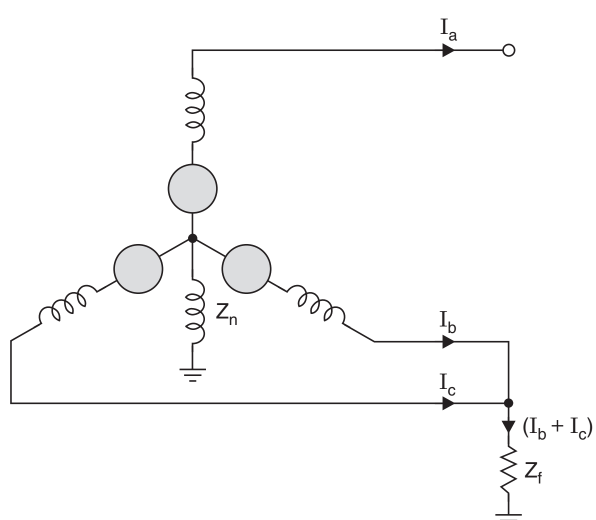


Fig. 13.18 (*a*) L-L-G fault. Fault impedance Z_f and neutral impedance Z_n.

∴ Equation (13.54) becomes

$$V_b = V_c = 3I_{a_0}Z_f$$

∴ $$\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0} = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}$$

or $$V_{a_1} = V_{a_2}$$

Using this relation in equation

$$V_b = 3I_{a_0} Z_f$$

$$\lambda^2 V_{a_1} + \lambda V_{a_1} + V_{a_0} = 3I_{a_0}Z_f$$

or $$-V_{a_1} + V_{a_0} = 3I_{a_0}Z_f$$

or $$V_{a_1} = V_{a_0} - 3I_{a_0}Z_f$$

Substituting for V_{a_1} and V_{a_0} from the sequence equation and expressing I_{a_0} in terms of I_{a_1}, we get

$$E_a - I_{a_1}Z_1 = -I_{a_0}(Z_0 + 3Z_n) - 3I_{a_0}Z_f$$

or $$I_{a_0} = -\frac{E_a - I_{a_1}Z_1}{Z_0 + 3Z_n + 3Z_f}$$

Similarly making use of the relation $V_{a_1} = V_{a_2}$, we express I_{a_2} in terms of I_{a_1}.

$$E_a - I_{a_1}Z_1 = -I_{a_2}Z_2$$

or $$I_{a_2} = -\frac{E_a - I_{a_1}Z_1}{Z_2}$$

Substituting the values of I_{a_2} and I_{a_0} in the equation

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} = 0$$

$$I_{a_1} - \frac{E_a - I_{a_1}Z_1}{Z_2} - \frac{E_a - I_{a_1}Z_1}{Z_0 + 3Z_n + 3Z_f} = 0$$

or $$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_n + 3Z_f)}{Z_2 + Z_0 + 3Z_n + 3Z_f}} \qquad (13.55)$$

The interconnection of the sequence network is shown in Fig. 13.18(*b*).

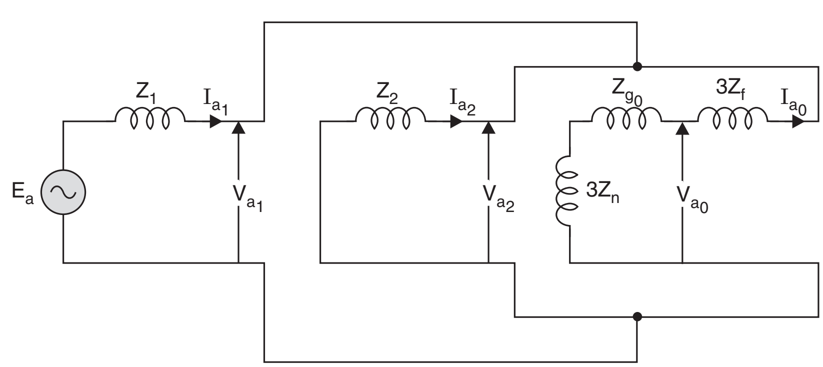


Fig. 13.18 (*b*) Interconnection of sequence networks for Fig. 13.18(*a*).

Before we proceed further to study the faults on an actual system where the alternator may be connected to a transmission line through a transformer or any other interconnected system, we will like to study the sequence network representation of various components like a generator, transformer, a synchronous motor etc.

13.9 SEQUENCE NETWORKS

The positive sequence network is in all respects identical with the usual networks considered. Each synchronous machine must be considered as a source of e.m.f. which may vary in magnitude and phase position depending upon the distribution of power and reactive volt amperes just prior to the occurrence of the fault. The positive sequence voltage at the point of fault will drop, the amount being dependent upon the type of faults; for 3-phase faults it will be zero; for double line-to-ground fault, line-to-line fault and single line-to-ground fault, it will be higher in the order stated.

The negative sequence network is in general quite similar to the positive sequence network except for the fact that since no negative sequence voltages are generated, the source of e.m.f. is absent.

The zero sequence network likewise will be free of internal voltages, the flow of current resulting from the voltage at the point of fault. The impedances to zero sequence current are very frequently different from the positive or negative sequence currents. The transformer and generator impedances will depend upon the type of connections whether star or delta connected; if star, whether grounded or not.

Equivalent circuit for the zero sequence network depends upon the impedances met by the zero sequence currents flowing through the three phases and their sum, $3I_{a_0}$, flowing through the neutral impedance and returning through the ground or a neutral conductor. If there is no complete path for zero sequence currents in a circuit, the zero sequence impedance is infinite. Thus a Y-connected circuit with ungrounded neutral has infinite impedance to zero sequence currents (Fig. 13.19(a)).

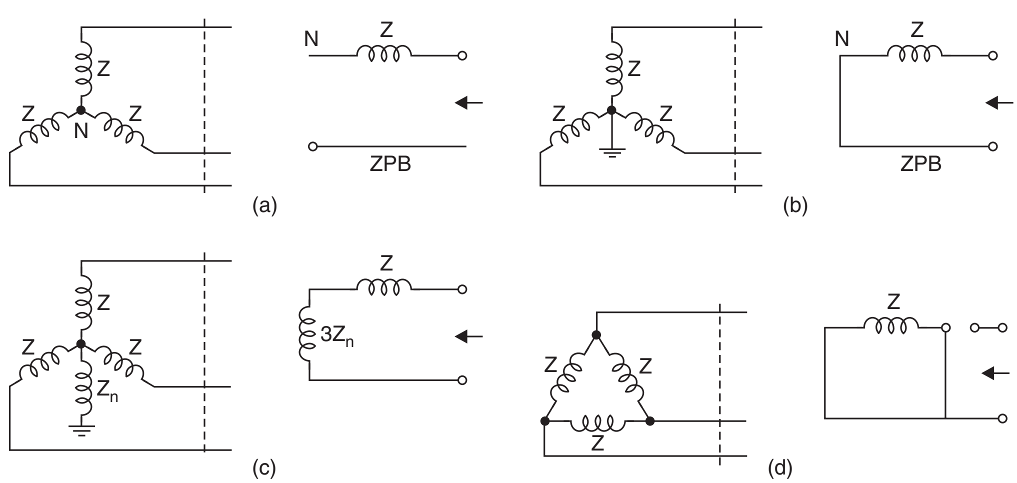


Fig. 13.19 Zero sequence networks for a 3-phase load.

In case the star point is solidly grounded *i.e.*, zero impedance between the neutral and the ground, a zero impedance is connected between the neutral point and the zero potential bus (Fig. 13.19(*b*)).

In case the neutral is grounded through some impedance Z_n, an impedance of $3Z_n$ should be connected between the neutral point and the zero potential bus (Fig. 13.21(*c*)).

A current of $3I_{a_0}$ produces a drop of $3I_{a_0}Z_n$ and to show in the equivalent zero sequence network the same drop where current of I_{a_0} flows, the impedance should be $3Z_n$.

A delta-connected circuit provides no path for zero sequence currents flowing in the line. The zero sequence currents being single phase, circulate within the winding. Hence viewed from its terminals its zero sequence impedance is infinite (Fig. 13.21(*d*)).

The zero sequence equivalent circuits of 3-phase transformers require special attention because of possibility of various combinations. The general circuit for any combination is given in Fig. 13.20 (*a*).

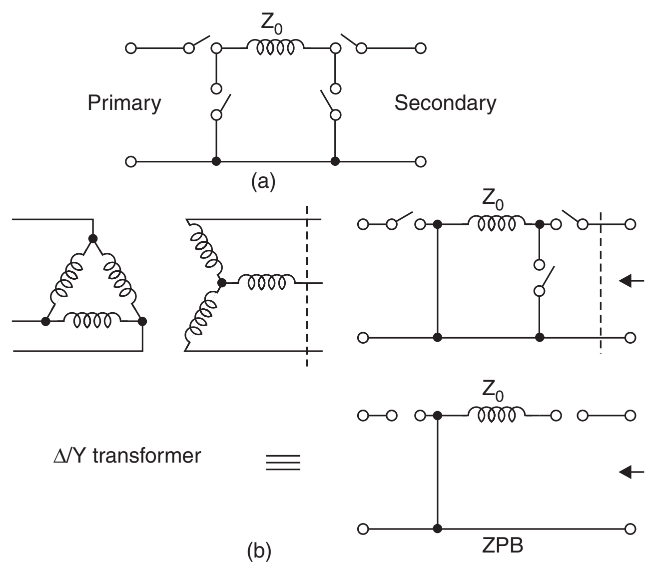


Fig. 13.20 (*a*) Switch arrangements for a transformer
(*b*) Equivalent of Δ/Y.

Z is the zero sequence impedance of the windings of the transformer. These are two series and two shunt switches. See the location of the switches. One series and one shunt switch are for both the sides separately. The series switch of a particular side is closed if it is star grounded and the shunt switch is closed if that side is delta connected, otherwise they are left open.

Say the transformer is Δ/Y connected with star ungrounded (Fig. 13.20(*b*)). Since the primary is delta connected, the shunt switch of primary side is closed and series is left open. The secondary is star ungrounded; therefore, the series switch is left open and shunt switch is also left open.

The zero sequence equivalent circuits for a few more combinations using this rule are drawn in Fig. 13.21.

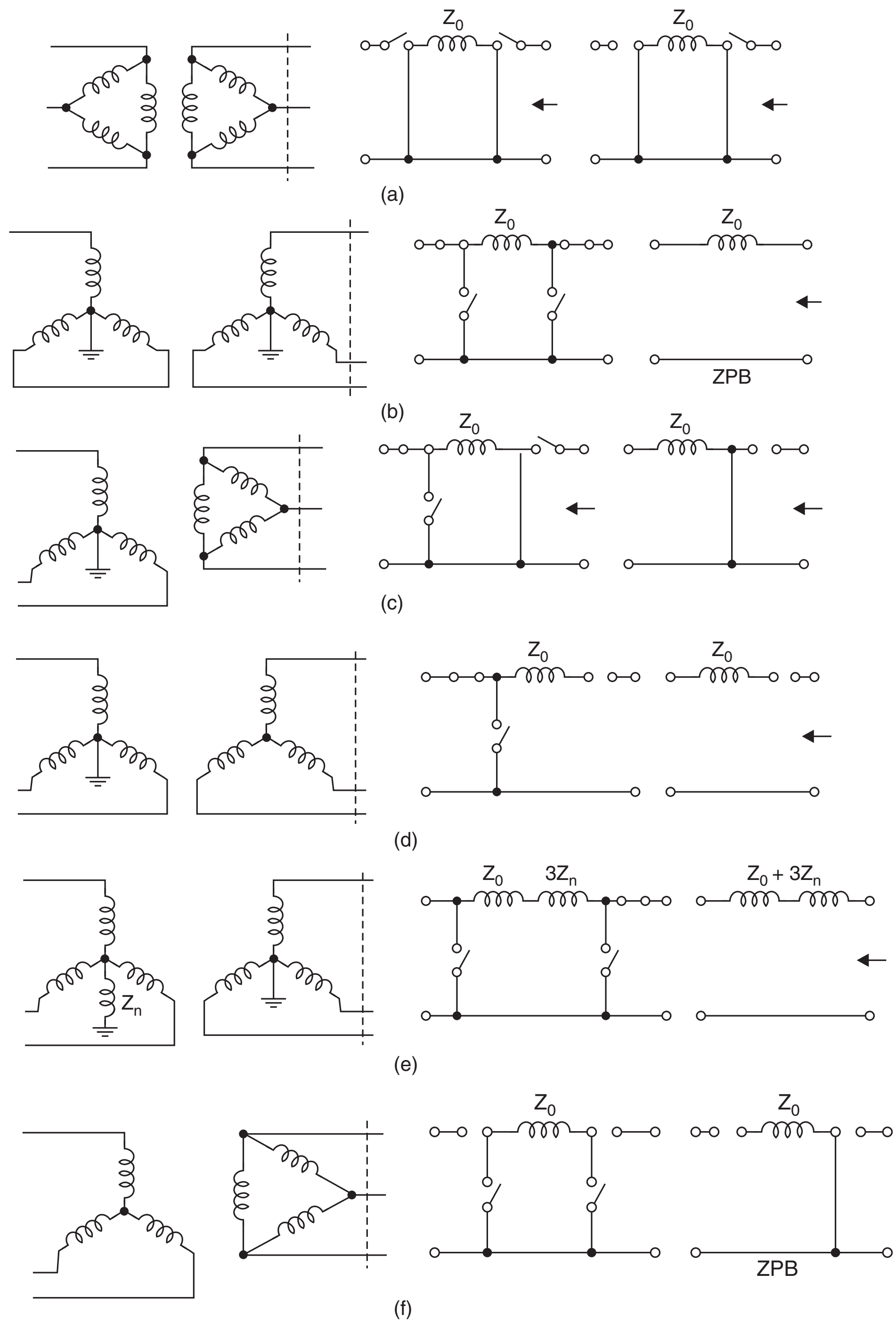


Fig. 13.21 Zero sequence equivalent circuits of transformers.

The reader after having some practice with the switch diagram will be able to draw the equivalent circuit very easily. Now we are ready to analyse the faults on power system.

13.10 FAULTS ON POWER SYSTEMS

The faults are analysed easily by making use of Thevenin's theorem. As the readers know that this theorem can be used for determining the changes that take place in currents and voltages of a linear network when an additional impedance is added between two nodes of the network. The theorem states that:

The changes that take place in the network voltages and currents due to the addition of an impedance (a short circuit) between two network nodes are identical with those voltages and currents that would be caused by an e.m.f. placed in series with the impedance and having a magnitude and polarity equal to the pre-fault voltage that existed between the nodes in question and the impedance as seen between the nodes with all active voltage sources short circuited.

To determine the current and voltage distribution in the system, the distribution in each of the sequence networks must first be determined. The Thevenin's equivalents of positive, negative and zero sequence networks are identical to those of a network of single generator.

Consider the system in Fig. 13.22 for illustration of the application of Thevenin's theorem for determining the equivalent positive, negative and zero sequence networks.

Thevenin's equivalent of positive sequence networks is obtained from the positive sequence network. The Thevenin's equivalent voltage source is the prefault voltage at the fault point and the equivalent impedance $Z_{1\,eq}$ is the impedance as seen between the fault point and the zero potential bus shorting the voltage sources. It is to be noted here that positive sequence impedance of the alternator or the synchronous machine depends upon the state of the machine *i.e.*, whether it is sub-transient, transient or steady state.

Similarly, the Thevenin's equivalent negative and zero sequence networks are obtained from the negative and zero sequence networks respectively. Since the system is balanced, no negative or zero sequence currents are flowing before the fault occurs. The prefault negative and zero sequence voltages at the fault point are zero. Therefore, no e.m.fs. appear in the equivalent circuits. The impedances $Z_{2\,eq}$ and $Z_{0\,eq}$ are measured between the fault point and the reference bus in their respective networks.

In the positive network, the currents throughout the system due to the fault can be added to the load currents before the fault to give the total positive sequence current during the fault. The net fault current is the fault current considering the system under no load condition plus the load current super-imposed over the fault currents.

(a)

Fig. 13.22 (*a*) Single line diagram of a balanced 3-phase system.

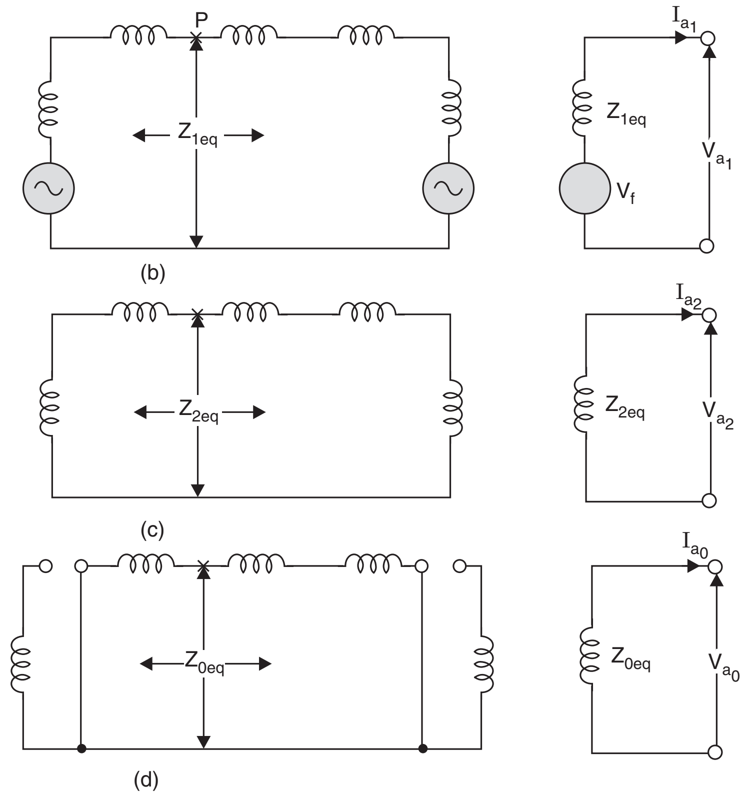


Fig. 13.22 (*b*), (*c*) and (*d*) Thevenin's equivalent of positive, negative and zero sequence networks.

13.11 PHASE SHIFT Δ-Y TRANSFORMERS

The two possible ways of connecting Δ-*Y* transformers are shown in Figs. 13.23 (*a*) and (*b*).

The small letters used refer to the star side and capital letters to the delta side of the transformer. The winding *e′e* on star side corresponds to the *E′E* on the delta side. The primed letters indicate the beginning of the winding and unprimed the finish of the winding. Figs. 13.23 (*c*) and (*d*) give the voltage vector diagram for positive sequence of the connections in (*a*) and (*b*) respectively, neglecting the voltage drop in the transformer. Say vector diagram (*c*) is drawn such that V_{a_1} and V_{CB1} are in phase and the other vectors follow. Similarly, in (*d*), V_{a_1} and V_{BC_1} are in phase. If each voltage is expressed in per unit with its own voltage as the base voltage, V_{BC_1}, V_{a_1} and V_{A_1} in Fig. (*c*) are equal in magnitude, and therefore,

$$V_{A_1} = jV_{BC_1} = jV_{a_1} \tag{13.56}$$

whereas in Fig. (*d*)

$$V_{A_1} = -jV_{BC_1} = -jV_{a_1} \tag{13.57}$$

From the above, it is clear that the line to neutral voltage V_{A_1} on the delta side leads the line to neutral voltage on star side in Fig. (*a*) by 90° whereas in Fig. (*b*) it lags by 90°.

The connection diagram in Figs. (*a*) and (*b*) and their corresponding vector diagrams for positive sequence voltage in Figs. (*c*) and (*d*) relate to the usual transformer connection diagrams and hence if the connection diagram is given, the phase relation between V_{A_1} and V_{a_1} can be determined by inspection. Referring to Figs. (*e*) and (*f*) which are the negative sequence voltage vector diagrams of Figs. (*a*) and (*b*) respectively, we have

For Fig. (*e*),

$$V_{A_2} = -jV_{CB_2} = -jV_{a_2} \tag{13.58}$$

and for Fig. (*f*),

$$V_{A_2} = jV_{BC_2} = jV_{a_2} \tag{13.59}$$

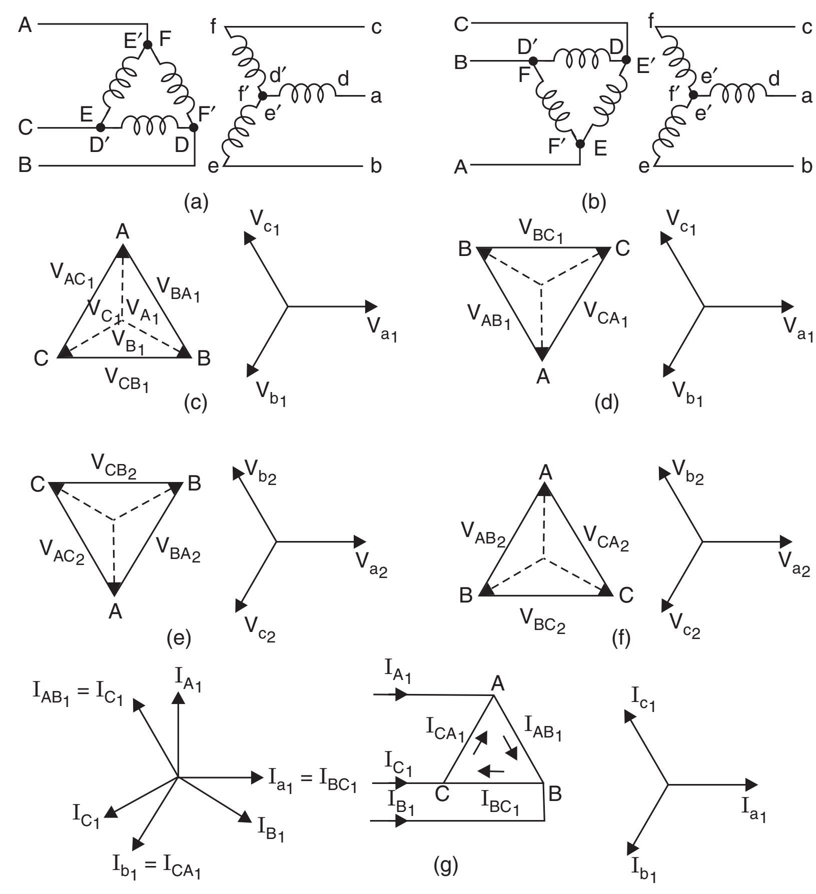


Fig. 13.23 Phase shift in Δ-Y transformer.

It is clear that the phase shift in the negative sequence voltages is in the direction opposite to the shift in phase of the positive sequence voltage for the same connection diagram.

Since the kVA rating of the transformer on the two sides is the same, if we neglect the exciting current, resistance and the voltage drop, it is essential that the shift in phase of positive and negative sequence line currents in passing through a Δ-Y or Y-Δ transformer banks with transformer exciting currents neglected must correspond to the shift in phase of line-to-neutral voltages with the drop neglected.

Referring of Fig. (*g*) which corresponds to the positive sequence current vector diagram of Fig. (*a*), let the currents leave the neutral of the star side and enter the delta side of the transformer. This means in star, the current goes from e' to e whereas in delta it goes from E to E', *i.e.,* from B to C as indicated by the arrow. Arrows on the delta side are used to indicate direction of current flow but do not indicate the direction of phase relation with respect to star currents. Let I_{a_1} be the reference vector and with exciting current neglected I_{BC_1} is in phase with I_{a_1}. Again expressing the line currents in per unit with its own-current as the base current

$$I_{A_1} = -jI_{CB_1} = jI_{BC_1} = jI_{a_1} \tag{13.60}$$

Similarly for negative sequence current,

$$I_{A_2} = -jI_{a_2} \tag{13.61}$$

In fact these current relations can be derived in a different way also. We know that the total input to the transformer as a unit is zero assuming a lossless transformer, *i.e.,* $V_1I_1 + V_2I_2 = 0$. That is

$$V_{A_1} I_{A_1} + V_{a_1} I_{a_1} = 0 \tag{13.62}$$

Now we have from equation (13.56),

$$V_{A_1} = jV_{a_1}$$

Substituting this relation in equation (13.62),

or $$jV_{a_1} I_{A_1} + V_{a_1} I_{a_1} = 0$$

or $$jI_{A_1} = -\, I_{a_1}$$

and $$I_{A_1} = jI_{a_1} \tag{13.60}$$

or $$V_{A_2} I_{A_2} + V_{a_2} I_{a_2} = 0$$

or $$-jV_{a_2} I_{A_2} + V_{a_2} I_{a_2} = 0$$

or $$-jI_{A_2} = -\, I_{a_2}$$

$$I_{A_2} = -jI_{a_2} \tag{13.61}$$

Similarly for the other connections where $V_{A_1} = -\,jV_{a_1}$ and $V_{A_2} = jV_{a_2}$ the current relations can be derived.

It is, therefore, seen that the positive sequence line-to-neutral voltages and line currents are shifted 90° in phase in the same direction in passing through a Y-Δ or Δ-Y transformer whereas the corresponding negative sequence quantities are shifted 90° in the direction opposite to the positive sequence shift.

In case it is desired to know only the magnitude of voltage and currents in a system during faults, we need not consider the phase shift of 90°. If both magnitude and phase relations are required then we must consider the 90° phase shift. To solve the short circuit problems in which the connection of the Δ-Y transformer is not given, any one of the two connections can be assumed. The only difference in the final results will be the sign of the voltages and currents. The sign in one case is plus and in the other it will be minus, the magnitudes will remain same.

Example 13.6: A 30 MVA, 13.8 kV, 3-phase alternator has a subtransient reactance of 15% and negative and zero sequence reactances of 15% and 5% respectively. The alternator supplies two motors over a transmission line having transformers at both ends as shown on the one-line diagram. The motors have rated inputs of 20 MVA and 10 MVA both 12.5 kV with

20% subtransient reactance and negative and zero sequence reactances are 20% and 5% respectively. Current limiting reactors of 2.0 ohms each are in the neutral of the alternator and the larger motor. The 3-phase transformers are both rated 35 MVA, 13.2 Δ-115*Y* kV with leakage reactance of 10%. Series reactance of the line is 80 ohms. The zero sequence reactance of the line is 200 ohms. Determine the fault current when (*i*) *L-G* (*ii*) *L-L*, and (*iii*) *L-L-G* fault takes place at point *P*. Assume V_f = 120 kV.

Solution: The three sequence networks will be as shown in Fig. E.13.6. Assume base of 30 MVA and base voltage of 13.8 kV in generator circuit.

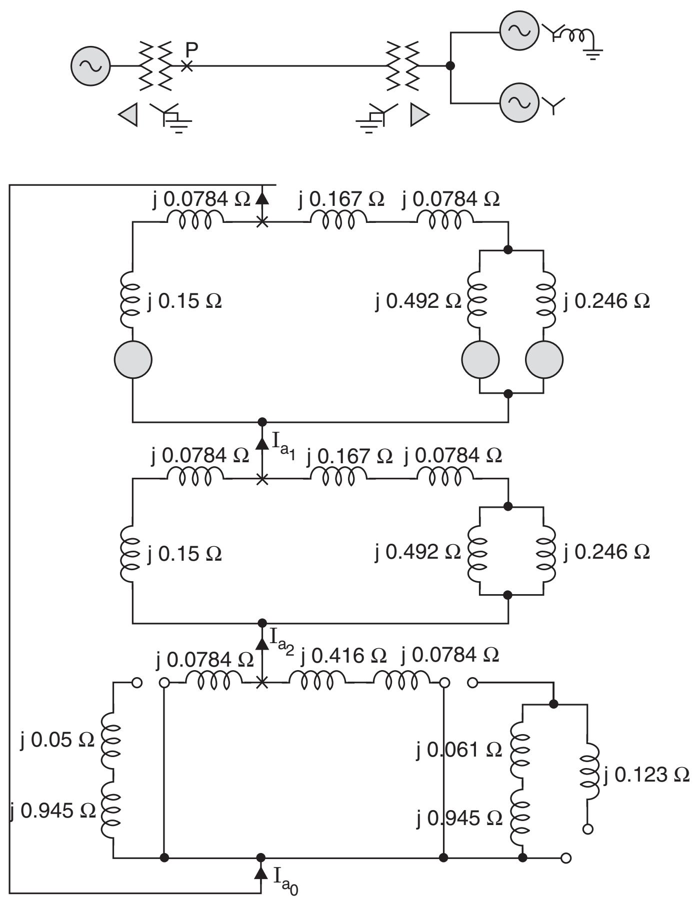


Fig. E.13.6

Positive Sequence Network

The base voltage on the line side of the transformer = $13.8 \times \dfrac{115}{13.2} = 120$ kV

∴ The base voltage on the motor side of the transformer = $120 \times \dfrac{13.2}{115} = 13.8$ kV

The per cent reactance of transformer $= 10 \times \left(\dfrac{13.2}{13.8}\right)^2 \times \dfrac{30}{35} = 7.8423\%$

The per cent reactance of motor $= 20 \times \left(\dfrac{12.5}{13.8}\right)^2 \times \dfrac{30}{20} = 24.6\%$

The per cent reactance of line $= 80 \times \dfrac{30}{120^2} \times 100 = \dfrac{2400}{144} = 16.7\%$

Negative Sequence Network: The network is exactly identical to positive sequence network except for the sources.

Zero Sequence Network

The neutral reactance $= 2 \times 3 \times \dfrac{30}{(13.8)^2} \times 100 = \dfrac{180 \times 100}{(13.8)^2} = 94.5\%$

The zero sequence reactance of line $= 200 \times \dfrac{30}{(120)^2} \times 100 = \dfrac{6000}{144} = 41.6\%$

Once the three sequence networks are ready we analyse different fault conditions as follows:

L-G Fault: The three sequence networks are connected in series, positive sequence impedance between P and ZPB is (when sources are short circuited) $j0.146$. Similarly,

$$\text{Negative sequence impedance} = j0.146$$

$$\text{Zero sequence impedance} = 0.06767$$

$$\text{Total impedance} = j0.3596$$

$$\therefore \qquad I_{a_1} = \frac{1 + j0.0}{j0.35967} = -j2.78 \text{ p.u.} = I_{a_2} = I_{a_0}$$

Fault current $= 3I_{a_1} = -j8.34$

Base current $= \dfrac{30 \times 1000}{\sqrt{3} \times 13.8} = 1255$ amps

or on the line side

Base current $= \dfrac{30 \times 1000}{\sqrt{3} \times 120} = 144.3$ amps

$\therefore$ Fault current $= 144.3 \times 8.34 = 1203$ amps

L-L Fault: Here only positive and negative sequence networks are required.

$$I_{a_1} = \frac{1 + j0.0}{Z_1 + Z_2} = \frac{1 + j0.0}{j0.146 + j0.146} = \frac{1 + j0.0}{j0.292} = -j3.42$$

$$\therefore \qquad I_{a_1} = -I_{a_2} = -j3.424$$

Fault current $\quad I_b = -I_c = \lambda^2 I_{a_1} + \lambda I_{a_2}$ as $I_{a_0} = 0$

$$I_b\,(-0.5 - j0.866)(-j3.424) + (-0.5 + j0.866)(j3.424)$$

$$= j1.712 - 2.965 - j1.712 - 2.965 = 5.9315 \text{ p.u.}$$

$\therefore \qquad$ Fault current $= 5.9315 \times 144.3 = 855.9$ amps

L-L-G Fault: Here

$$I_{a_1} = \frac{1 + j0.0}{j0.146 + \dfrac{j0.146 \times j0.06767}{j0.146 + j0.06767}} = \frac{1 + j0.0}{j0.19224} = -j5.2 \text{ p.u.}$$

$$I_{a_2} = -\frac{I_{a_1} Z_0}{Z_2 + Z_0} = \frac{+j5.2 \times j0.06767}{j0.21367} = j1.647$$

$$I_{a_0} = j3.553$$

The fault current is

$$I_b + I_c = 3I_{a_0} = 3 \times j3.553 \text{ p.u.}$$

∴ The fault current = 3 × 3.553 × 144.3 = 1538 amps.

13.12 REACTORS

Reactor is a coil which has high inductive reactance as compared to its resistance and is used to limit the short circuit current during fault conditions. To perform this function it is essential that magnetic saturation at high current does not reduce the coil reactance. If an iron cored inductor is expected to maintain constant reactance for currents two to three times its normal value it will turn out to be very costly and heavy. Therefore air cored coils having constant inductance are generally used for current limiting reactors.

Air cored reactors are normally of two types: (*i*) oil immersed type, and (*ii*) dry type. Oil immersed reactors can be cooled by any of the means used for cooling the power transformer whereas the dry type are usually cooled by natural ventilation and are sometimes designed with forced-air and heat exchanger auxiliaries. Reactors are usually built as single phase units.

With the increase in interconnection of power system the fault levels are increasing. It is, therefore, necessary to increase the reactance by introducing reactors at strategic points in the system. The following are the various possibilities:

(*i*) *Generator Reactors*: The reactance of modern alternators may be as high as 2.0 p.u. which means even a dead short-circuit at the terminals of the alternator will result in a current less than full load current and, therefore, no external reactor is required for limiting the short circuit current of such a machine. However, if some old machines are being used alongwith the modern alternator, these old machines need the reactors for limiting the short circuit current. The location of reactors is given in Fig. 13.24(*a*).

(*ii*) *Feeder Reactor*: The per unit value of reactance of a feeder based on its ratings may be small but when compared with the rating of the whole system, its value is quite large and hence a small reactor will be effective in limiting the short circuit current should a fault occur close to the generating station. In case this feeder reactor is not there, a fault in such a location would bring the bus bar voltage almost down to zero value and there is a possibility of various generators falling out of step. We know that, to improve the transient stability of a system the critical clearing angle should be as small as possible, *i.e.*, the breakers should be as fast as possible. In order to obtain this situation and at the same time to reduce the current to be interrupted the feeder must be associated with a reactor (Fig. 13.24(*b*)).

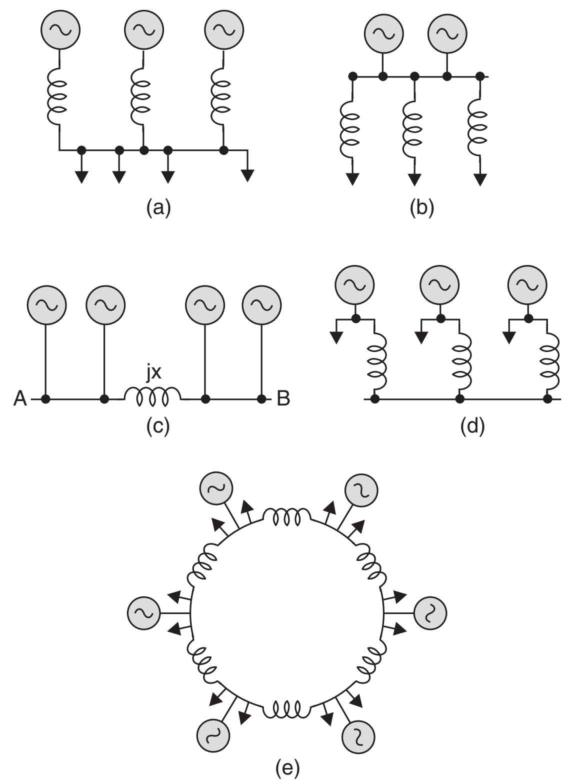


Fig. 13.24 Types of reactors: (*a*) Generator; (*b*) Feeder and (c-e) busbar.

(*iii*) *Busbar reactor*: There are three methods of interconnecting the busbar through the reactors as shown in Fig. 13.24 (*c-e*). The simple method is suitable for plants of moderate output whereas for large-sized plants either the star or ring system of connection is used. It is to be noted that any transfer of power from say section *A* to section *B* of the generators, a difference in potential between the bus section is developed. If the power to be transferred is wattless the difference in voltage between the bus section will be much more as compared to when active power of same magnitude will be transferred. Refer to phasor diagram (Fig. 13.25) for the two conditions when resistance of the system is neglected.

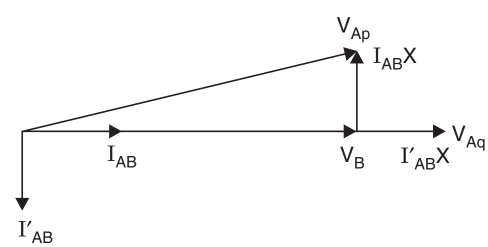


Fig. 13.25 Phasor diagram for Fig. 13.24(*b*).

V_{Ap} is the voltage of bus *A* when active power is transferred and V_{Aq} is the voltage of bus *A* when reactive power of same magnitude is transferred from *A* to *B*. Since the allowable voltage difference between the bus sections is quite limited it is desirable to meet the wattless requirement of load at bus *B* by adjusting excitation of the plant at *B* and the active power requirement can be met by transferring power from *A* to *B*.

In case of the ring arrangement, the current to be transferred between two sections flows through two paths in parallel whereas in tie-bar or star system the current flows through two reactors in series. As a result of this configuration whenever a busbar connection is removed for repairs or maintenance in case of a ring arrangement, the maximum power that can be transferred reduces materially which is not the case in case of tie-bar system. For protection the two arrangements involve almost the same cost, except in the limit, it is advantageous to use the tie-bar system.

Calculation of 3-phase Short-Circuit Currents

The sudden short-circuit of a 3-phase alternator has been discussed in Chapter 12. It is shown there that the impedance of the alternator grows from the instant of short circuit to the steady state condition. Which impedance should be considered for evaluating the short-circuit currents, depends upon whether subtransient, transient or steady state short circuit current is required.

The p.u. impedance of an equipment = $\dfrac{IZ}{V}$

where Z is the impedance of the equipment in ohms and I and V are the rated current and voltage respectively.

Now $$I_{sc} = V/Z$$

$\therefore$ $$Z_{\text{p.u.}} = \frac{IZ}{V} = \frac{I}{I_{sc}} = \frac{IV}{I_{sc}V}$$

If VI is the base or full load volt-amperes and VI_{sc} the short-circuit volt-amperes, then

$$Z_{\text{p.u.}} = \frac{\text{Base or full load volt-amperes}}{\text{Short-circuit volt-amperes}}$$

or $$\text{S.C. MVA} = \frac{\text{Base or full load MVA}}{Z_{\text{p.u.}}}$$

This is the relation that will be used for evaluating the short circuit MVA.

13.13 CONCEPT OF SHORT-CIRCUIT CAPACITY OF A BUS

Consider Fig. 13.26. The diagram shown is a part of a large interconnected system. Assume that a symmetrical short circuit occurs at bus 1.

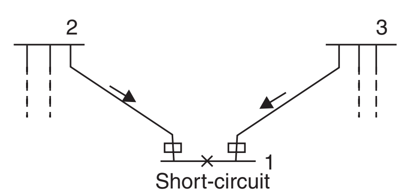


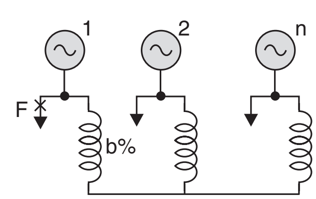
Fig. 13.26 A three-bus system with short-circuit at bus 1.

The prefault voltage of bus is 1 p.u. and as soon as the fault takes place, the voltage of this bus reduces to almost zero. The voltage of the other buses will sag during the short-circuit and the reduction in voltage of various buses is an indication of the "strength" of the network. We normally are interested in knowing this strength and the severity of the short-circuit stresses. Both these objectives are met by a quantity known as short-circuit capacity or fault level of the bus in question. By strength of a bus is meant the ability of the bus to maintain its voltage when a fault takes place at other bus. Of course when a fault takes place at the bus in question, the voltage of this bus will reduce to zero but in case a fault takes place at some other bus then how far the bus in question is able to maintain its voltage is a measure of the strength of the bus. The short-circuit capacity is defined as the product of the magnitude of prefault voltage and post-fault current. Since the strength of a bus is directly related to its short-circuit capacity, the higher the short circuit capacity of the bus the more it is able to maintain its voltage in case of a fault on any other bus. Also it can be seen that higher the short-circuit capacity, lower will be the equivalent impedance as seen between the faulted bus and the zero potential bus of the system. For a bus which is infinitely strong or which has infinite short-circuit capacity will have zero equivalent impedance. In fact such a bus is known as "infinite bus". Such a bus is characterized by a zero equivalent impedance and it is able to maintain constant voltage irrespective of where the short circuit takes place except, of course, for a short circuit on the bus itself, when its voltage will reduce to zero.

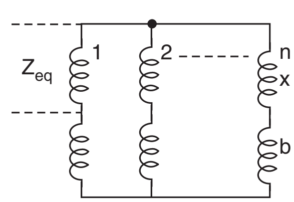
Whenever a short circuit takes place at a bus with higher short-circuit capacity or fault level, high current flows in the bus. This taxes the circuit breaker. The short-circuit stress to which a breaker is subjected is directly related to short-circuit capacity rather than the short-circuit current for two reasons. The first job of the breaker is to extinguish the short-circuit current and once it has extinguished the arc, the breaker contacts must maintain sufficient insulation strength to withstand the voltage (recovery voltage) that appears across them. Since the recovery voltage is 1 p.u. it is logical to rate a breaker for both the post-fault current and prefault voltage, *i.e.*, in terms of short-circuit capacity rather than the short-circuit current.

Example 13.7: A generating station having n section busbars each rated at Q kVA with $x\%$ reactance is connected on the tie-bars system through busbar reactances of $b\%$. Determine the short-circuit kVA if a 3-phase fault takes place on one section. Determine the short-circuit kVA when n is very large.

Solution: The tie-bar system is represented as follows:



Let the fault take place at F. The equivalent circuit will be as follows:



The equivalent impedance Z_{eq} between the zero potential bus and the fault point is

$$\left\{\frac{b+x}{n-1}+b\right\} \parallel x \quad \text{or} \quad \frac{bn+x}{n-1} \parallel x$$

or
$$\frac{1}{Z_{eq}} = \frac{1}{x} + \frac{(n-1)}{(bn+x)}$$

$\therefore$ The short-circuit kVA $= \dfrac{Q}{Z_{eq}} \times 100 = Q\left[\dfrac{1}{x} + \dfrac{(n-1)}{bn+x}\right] \times 100$

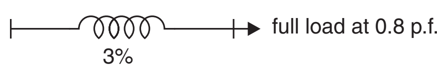
Now, if n is very large,

$$Q\left[\frac{1}{x} + \frac{1-1/n}{b+x/n}\right] = Q\left[\frac{1}{x} + \frac{1}{b}\right]$$

The short-circuit MVA is independent of the number of section. This is the main advantage of tie-bar system. This effectively means that any extension of a large tie-bar interconnected system will not require replacement of the existing switchgear system.

Example 13.8: Determine the percentage increase of busbar voltage required to compensate for the reactance drop when the feeder having a reactance of 3% carries a full load current at a p.f. 0.8 lagging.

Solution: The system is shown below:



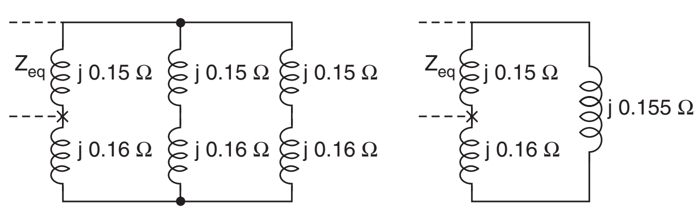
For a series impedance the approximate % drop in volts = $v_r \cos \phi_r + v_x \sin \phi_r$, where v_r and v_x are the per cent resistance and reactance of the series element respectively. Since the feeder has negligible resistance $v_r = 0$.

$\therefore$ Per cent drop of volts = $v_x \sin \phi_r = 3 \times 0.6 = 1.8\%$. **Ans.**

Example 13.9: A small generating station has a busbar divided into three sections. Each section is connected to a tie-bar with reactors each rated at 5 MVA, 0.1 p.u. reactance. A generator of 8 MVA rating and 0.15 p.u. reactance is connected to each section of the busbar. Determine the short-circuit capacity of the breaker if a 3-phase fault takes place on one of the sections of busbar.

Solution: Let the base MVA be 8 MVA, the per unit reactance of the generator be 0.15 p.u. and that of the reactor 0.1 × 8/5 = 0.16 p.u.

The equivalent circuit is as shown below:



The equivalent impedance $Z_{eq} = \dfrac{j0.15 \times j0.315}{j0.465} = j0.1016129$

$\therefore$ Short-circuit capacity $= \dfrac{\text{Base MVA}}{Z_{eq}} = \dfrac{8}{j0.1016129} = 78.73$ MVA.

Example 13.10: Two generating stations having short-circuit capacities of 1200 MVA and 800 MVA respectively and operating at 11 kV are linked by an interconnected cable having a reactance of 0.5 ohm per phase. Determine the short-circuit capacity of each station.

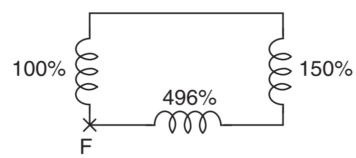
Solution: Assuming base MVA as 1200, the per cent reactance of one generating station is 100% and that of the other is

$$\frac{1200}{800} \times 100 = 150\%$$

The % reactance of the cable is

$$\frac{0.5 \times 1200}{11 \times 11} \times 100 = 496\%$$

When a 3-phase fault takes place at 1200 MVA capacity plant the equivalent circuit will be as follows:

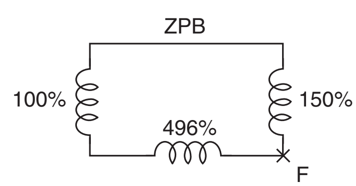


When the fault is at F, fault impedance between F and the neutral bus will be 86.59%.

$\therefore$ The short-circuit MVA of this bus will be as follows:

$$\frac{1200}{86.59} \times 100 = 1386 \text{ MVA.} \quad \textbf{Ans.}$$

For fault at the other station, the equivalent circuit will be as follows:



The equivalent fault impedance between F and neutral bus will be 119.84%.

$\therefore$ The short-circuit MVA will be

$$\frac{1200}{119.84} \times 100 = 1001 \text{ MVA.} \quad \textbf{Ans.}$$

Example 13.11: Determine the fault MVA, if a fault takes place at F in the diagram shown (Fig. E.13.11). The p.u. values of reactance are given with 100 MVA as base. Resistance may be neglected.

In order to draw Fig. E.13.11(c) from (b), we draw two buses neutral N and the fault point bus F and arrange the various elements of (b) between these buses. The other network reductions are quite clear from the figures till we arrive at (g), where the equivalent fault impedance between the neutral bus and the fault point is 0.14 p.u.

$$\therefore \qquad \text{The S.C. MVA} = \frac{100}{0.14} = 714.28 \text{ MVA.} \quad \textbf{Ans.}$$

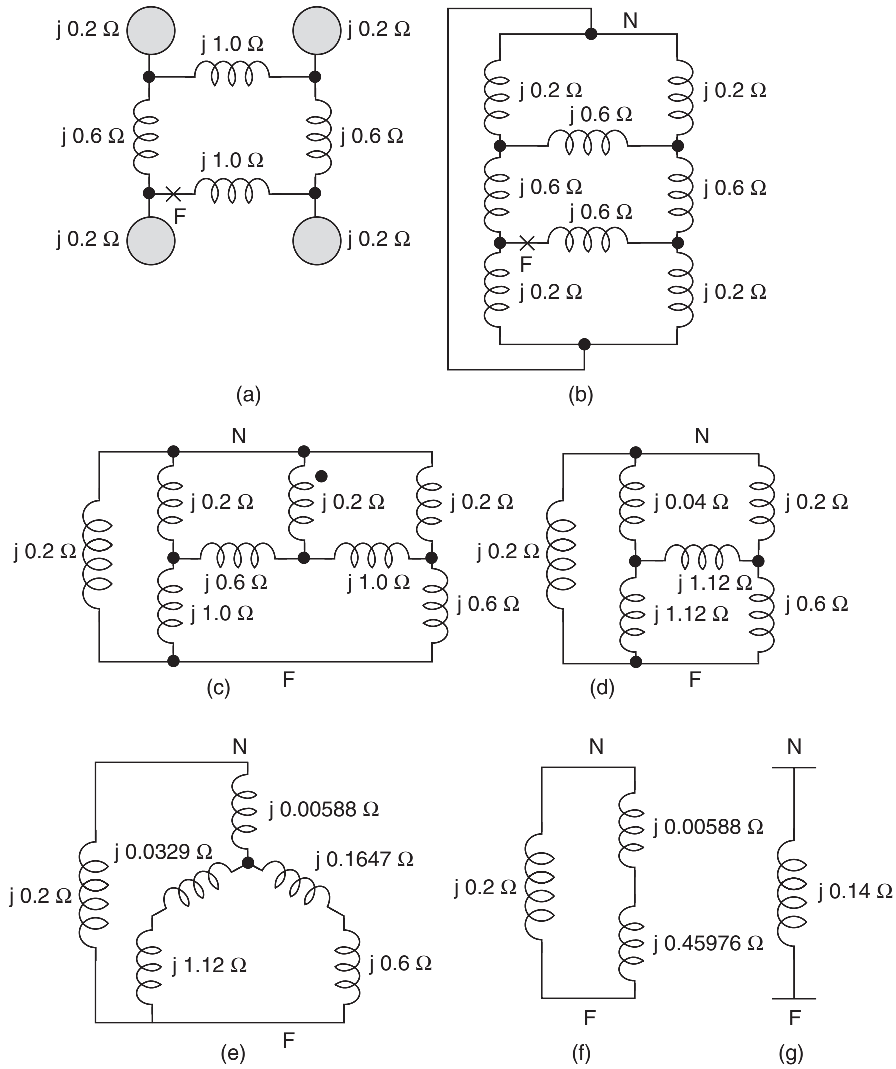


Fig. E.13.11

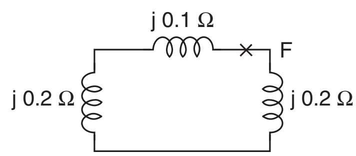
Example 13.12: An alternator and a synchronous motor each rated for 50 MVA, 13.2 kV having subtransient reactance of 20% are connected through a transmission link of reactance 10% on the base of machine ratings. The motor acts as a load of 30 MW at 0.8 p.f. lead and terminal voltage 12.5 kV when a 3-phase fault takes place at the motor terminals. Determine the subtransient current in the alternator, the motor and the fault.

Solution: Taking base quantities as 50 MVA, 13.2 kV,

$$\text{The base current} = \frac{50 \times 1000}{\sqrt{3} \times 13.2} = 2186 \text{ amps}$$

$$\text{The prefault voltage} = \frac{12.5}{13.5} = 0.9469 \text{ p.u.}$$

Take this voltage as the reference.



The fault impedance = $\dfrac{j0.3 \times j0.2}{j0.5}$ = $j0.12$ p.u.

∴ The fault current = $\dfrac{0.9469}{j0.12}$ = – $j7.89$ p.u.

The full load current before the fault takes place = $\dfrac{30 \times 1000}{\sqrt{3} \times 12.5 \times 0.8}$ = 1732 amps

$$\text{p.u. load current} = \frac{1732}{2186} = 0.7923 \angle 36.8°$$
$$= 0.6344 + j0.4746$$

The p.u. fault current supplied by the motor = – $j7.89 \times 3/5$ = – $j4.734$

and that supplied by the generator = – $j7.89 \times 2/5$ = – $j3.156$.

∴ The net current supplied by the generator during fault

$$= -j3.156 + 0.6344 + j0.4746$$
$$= 0.6344 - j2.6814 = 2.755 \text{ p.u.}$$

The net current supplied by the motor – 0.6344 – $j0.4746$ – $j4.734$

$$= (-0.6344 - j5.2086) \text{ p.u.} = 5.247 \text{ p.u.}$$

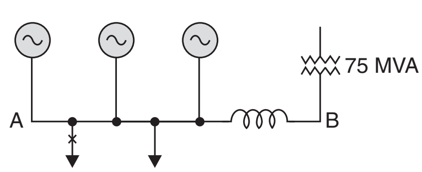
∴ Fault current from the generator = 2.755 × 2186 = 6022 amps.

Fault current from the motor = 5.247 × 2186 = 11470 amps

and fault current = – $j17247$ amps. **Ans.**

Example 13.13: A station operating at 33 kV is divided into sections *A* and *B*. Section *A* consists of three generators 15 MVA each having a reactance of 15% and section *B* is fed from the grid through a 75 MVA transformer of 8% reactance. The circuit breakers have each a rupturing capacity of 750 MVA. Determine the reactance of the reactor to prevent the breakers being overloaded if a symmetrical short circuit occurs on an outgoing feeder connected to *A*.

Solution: The system is given below:

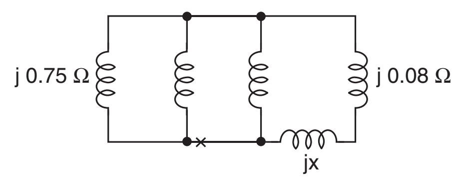


Assume the base MVA as 75.

The p.u. reactance of each generator = 0.15 × $\dfrac{75}{15}$ = $j0.75$

The p.u. reactance of transformer = $j0.08$ p.u.

Let $x\%$ be the reactance of the reactor for base of 75 MVA. The equivalent circuit for a fault of A is as shown in diagram.



The per cent impedance between the fault point and the neutral bus is

$$\frac{0.25(X+0.08)}{0.25+X+0.08} = \frac{0.25X+0.0200}{X+0.33}$$

Now $$\text{S.C. MVA} = \frac{\text{Base MVA}}{\text{p.u. impedance}}$$

or $$750 = \frac{75(X+0.33)}{0.25X+0.02}$$

$$187X + 15.00 = 75X + 24.75$$

$$112X = 9.75$$

$$X = 0.08705 \text{ p.u.}$$

$\therefore$ Actual value of reactance in ohms $= \dfrac{0.08705 \times 33^2}{75}$ = 1.264 ohms. **Ans.**

Example 13.14: A double line to ground fault occurs on phases b and c, at point P in the circuit whose single line diagram is shown in Fig. 13.22(a). Determine the subtransient currents in all phases of machine-1, the fault current and the voltages of machine I and voltages at the fault point. Neglect pre-fault current. Assume that machine-2 is a synchronous motor operating at rated voltage. Both the machines are rated 1.25 MVA, 600 volts with reactances of $X'' = X_2 = 8\%$ and $X_0 = 4\%$. Each 3-phase transformer is rated 1.25 MVA, 600 volts delta/4160 volts star with leakage reactance of 5%. The reactances of transmission line are $X_1 = X_2 = 12\%$ and $X_0 = 40\%$ on a base of 1.25 MVA, 4160 volts.

Solution: Select 600 volts, 1.25 MVA as base quantities in the generator circuit. Since the transformation ratio is 600/4160 volts and the transformer is rated at 1.25 MVA, no change of reactances is required.

From Fig. 13.22, the Thevenin's equivalent impedances are:

$$Z_{1\,eq} = (8 + 5) \parallel (8 + 5 + 12) = 8.55\%$$

$$Z_{2\,eq} = 8.55\%$$

$$Z_{0\,eq} = 5 \parallel 45 = 4.5\%$$

Now $$I_{a_1} = \frac{E_a}{Z_{1\,eq} + \dfrac{Z_{0\,eq} Z_{2\,eq}}{Z_{0\,eq} + Z_{2\,eq}}}$$

$$I_{a_1} = \frac{1.0}{j0.0855 + \dfrac{j0.0855 \times j0.045}{j0.1305}}$$

$$= -j\,8.697 \text{ p.u.}$$

$$I_{a_2} = \frac{I_{a_1} Z_{0\,eq}}{Z_{0\,eq} + Z_{2\,eq}}$$

$$= -\frac{j8.697 \times j0.045}{j0.1305}$$

$$= j\,3.0 \text{ p.u.}$$

$$I_{a_0} = -\frac{j8.697 \times j0.0855}{j0.1305}$$

$$= j\,5.698 \text{ p.u.}$$

$$V_{a_1} = 1.0 - (-j8.697)\,(j0.0855)$$

$$= 1.0 - 0.7436 = 0.2564$$

$$V_{a_2} = -I_{a_2} Z_{2\,eq} = -(j\,3.0)\,(j\,0.0853)$$

$$= 0.2564$$

Similarly $V_{a_0} = 0.2564$

The fault current $I_a = 0$

$$I_b = (-0.5 - j0.866)\,(-j8.697) + (-0.5 + j0.866)\,(j3.0) + j5.698$$

$$= j4.3485 - 7.5316 - j1.5 - 2.598 + j5.698$$

$$= -10.1296 + j\,8.5465$$

$$= 13.25\underline{/139.85}$$

$$I_c = (-0.5 + j\,0.866)\,(-j8.697) + (-0.5 - j0.866)^*\,(j3.0) + j5.698$$

$$= -10.1296 - j\,8.5465$$

$$= 13.25\underline{/220.15}$$

The current supplied by machine 1 are

$$I_{a_1} = -j8.697 \times \frac{25}{38} = -j5.722$$

or $I_{A_1} = jI_{a_1} = 5.722$

$$I_{a_2} = j3 \times \frac{25}{38} = j1.9737$$

or $I_{A_2} = 1.9737$

$\therefore$ $I_A = 5.722 + 1.9737 = 7.6956$

$$I_B = (-0.5 - j0.866)\,(5.722) + (-.05 + j0.866)\,(1.9737)$$

$$= -2.861 - j4.955 - 0.9868 + j1.709$$

$$= -3.8478 - j3.246$$

$$= 5.034\angle 220.15$$

$$I_C = 5.034\angle 139.85$$

Voltages at the fault point

$$V_a = V_{a_1} + V_{a_2} + V_{a_0}$$

$$= 3 \times 0.2564$$

$$= 0.7692 \text{ p.u.}$$

$$V_b = 0 \text{ and } V_c = 0$$

$$V_{ab} = V_a - V_b = 0.2564 - 0 = 0.2564 \text{ p.u.}$$

$$V_{bc} = V_b - V_c = 0.0$$

$$V_{ca} = V_c - V_a = -0.2564 \text{ p.u.}$$

Now the sequence voltages in the generator circuit are

$$V_{A_1} = 1.0 - (5.722)(j0.05)$$

$$= 1 - j0.2861$$

$$V_{A_2} = -1.9737^* (j0.05)$$

$$V_A = 1 - j0.2861 - j0.098685$$

$$= 1 - j0.3848$$

$$= 1.0715\angle 21° \text{ p.u.}$$

$$V_B = (-0.5 - j0.866)(1 - j0.2861) + (-0.5 + j0.866)(-j0.09868)$$

$$= -0.5 + j0.1430 - j0.866 - 0.24776$$

$$= -0.74776 - j0.723$$

$$= 1.04\angle 224$$

$$V_C = (-0.5 + j0.866)(1 - j0.2861) + (-0.5 - j0.866)(-j0.09868)$$

$$= -0.74776 + j0.723$$

$$= 1.04\angle 136°$$

$$V_{AB} = V_A - V_B$$

$$= 1.0 - j0.3848 + 0.74776 + j0.723$$

$$= 1.74776 + j0.3382$$

$$= 1.78\angle 10.95°$$

$$V_{BC} = V_B - V_C$$

$$= -0.74776 - j0.723 + 0.74776 - j0.723$$

$$= -j1.446$$

$$V_{CA} = V_C - V_A$$

$$= -0.74776 - j0.723 - 1 + j0.3848$$

$$= -1.74776 - j0.3382$$

$$= 1.78\angle 190.95 \quad \textbf{Ans.}$$

Example 13.15: A generator supplies a motor through a Y/Δ transformer. The generator is connected to the star side of the transformer. A fault occurs between the motor terminals and the transformer. The symmetrical components of the subtransient current in the motor flowing towards the fault are $I_{a_1} = -0.8 - j2.6$ p.u., $I_{a_2} = -j2.0$ p.u. and $I_{a_0} = -j3.0$ p.u. From the transformer towards the fault $I_{a_1} = 0.8 - j0.4$ p.u., $I_{a_2} = -j1.0$ p.u. and $I_{a_0} = 0$. Assume $X'' = X_2$ for both the motor and the generator. Describe the type of fault. Find (*i*) the pre-fault current if any, in line '*a*' (*ii*) the subtransient fault current in p.u. and (*iii*) the subtransient current in each phase of the generator in p.u.

Solution: The system is shown in Fig. E.13.15.1

Since the currents contain zero sequence components the fault is either *L-G* or *L-L-G*. The total fault current is the sum of fault currents fed from the transformer side and the motor side.

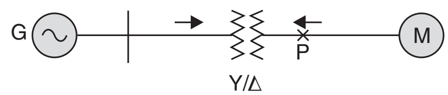


Fig. E.13.15.1

Total positive sequence fault current

$$= -0.8 - j2.6 + 0.8 - j0.4$$

$$= -j3.0$$

Similarly, it is found that total negative sequence and zero sequence fault currents are $I_{a_2} = -j3.0$ and $I_{a_0} = -j3.0$. Since all the three sequence components of current are equal, it is a *L-G* fault.

(*i*) Let the prefault current be $a + jb$ and since for *L-G* fault total I_{a_1} = total I_{a_2} = total $I_{a_0} = -j3.0$ p.u. in the case. The distribution of negative sequence current is $-j2.0$ p.u. from the motor and $-j1.0$ from the generator side *i.e.*, the ratio of the reactance from the two sides is 1 : 2 *i.e.*, it is given as in Fig. E.13.15.2.

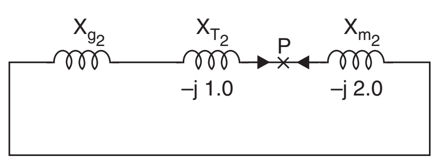


Fig. E.13.15.2

Therefore, positive sequence current supplied by the motor would be $-j2.0$ and that by generator $-j1.0$ if the prefault current is neglected. Now considering the prefault current, we should have positive sequence current supplied by the motor as

$$-j2.0 - (a + jb) = -0.8 - j2.6$$

Separating the real and imaginary quantities, we have $-2 - b = -2.6$

and $$-a = -0.8$$

or $$b = 0.6$$

and $$a = 0.8$$

Therefore, the prefault current is

$$(0.8 + j0.6)$$

(*ii*) The subtransient fault current = $3I_{a_1} = -j9.0$ p.u.

(*iii*) The sequence components from the generator are

$$I_{a_1} = 0.8 - j0.4,\ I_{a_2} = -j1.0,\ I_{a_0} = 0$$

$$I_{A_1} = j(0.8 - j0.4) I_{A_2} = -j(-j1.0)$$

$$I_{A_1} = j0.8 + 0.4 \quad I_{A_2} = -1.0$$

$$I_A = I_{A_1} + I_{A_2} = +j0.8 + 0.4 - 1.0$$

$$= -0.6 + j0.8. \quad \textbf{Ans.}$$

$$I_B = \lambda^2 I_{A_1} + \lambda I_{A_2}$$

$$= (0.5 - j0.866)(j0.8 + 0.4) + (-0.5 + j0.866)(-1.0)$$

$$= -j0.4 - 0.2 + 0.6928 - j0.3464 + 0.5 - j0.866$$

$$= 0.9928 - j1.6124. \quad \textbf{Ans.}$$

$$I_c = \lambda^2 I_{A_1} + \lambda^2 I_{A_2}$$

$$= (-0.5 + j0.866)(0.4 + j0.8) = (-0.5 - j0.866)(-1.0)$$

$$= -0.2 - j0.4 + j0.3464 - 0.6928 + 0.5 + j0.866$$

$$= -0.3928 + j0.8124. \quad \textbf{Ans.}$$

Similarly the currents from the motor side can be computed.

Example 13.16: A transformer is rated at 11 kV/0.4 kV, 500 kVA, 5% reactance. Determine the short circuit MVA of the transformer when connected to an infinite bus.

Solution: Since the transformer is connected to an infinite bus, the p.u. impedance of the circuit will be 0.05 *i.e.*, the p.u. impedance offered by the transformer.

$$\therefore \qquad \text{S.C. MVA} = \frac{0.5}{0.05} = 10 \text{ MVA.} \quad \textbf{Ans.}$$

Example 13.17: Three identical resistors are star connected and rated 2500 volts, 500 kVA as a three phase unit. The resistors are connected to the low-tension side of a Δ/Y transformer. The voltage at the resistor load are

$$|V_{ab}| = 2000 \text{ volts},\ |V_{bc}| = 2800 \text{ volts}$$

and $|V_{ca}| = 2500$ volts. Select base as 2500 volts

500 kVA, find the line voltages and currents in per unit on the delta side of the transformer. Assume that the neutral of the load is not connected to the neutral of the transformer secondary.

Solution: The per unit voltages are

$$|V_{ab}| = \frac{2000}{2500} = 0.8 \text{ p.u.}$$

$$|V_{bc}| = \frac{2800}{2500} = 1.12 \text{ p.u.}$$

$$|V_{ca}| = 1.0 \text{ p.u.}$$

Assuming an angle of 180° of V_{ca} and using the law of cosines.

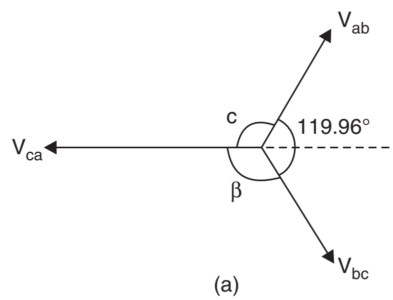


Fig. E.13.17(*a*)

$$1.12^2 = 0.8^2 + 1.0^2 + 2{*}0.8{*}\ 1.0 \cos \alpha$$
$$\alpha = 103.94°$$

Similarly

$$0.8^2 = 1.12^2 + 1.0^2 + 2{*}\ 1{*}\ 1.12 \cos \beta$$
$$\beta = 136.1°$$

The line voltages are

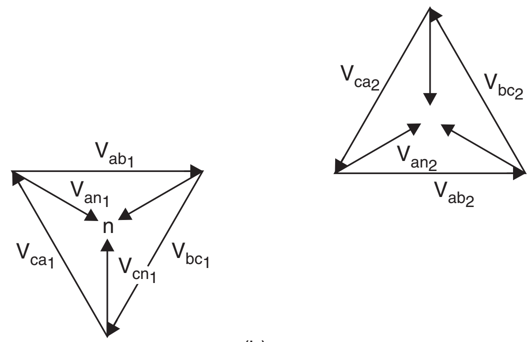
$$\therefore \quad V_{ab} = 0.8\underline{/76.06}$$
$$V_{ca} = 1.0\underline{/180°}$$
$$V_{bc} = 1.12\ \underline{/-43.9°}$$

The symmetrical components of the line voltages are

$$V_{ab_1} = \frac{1}{3}[V_{ab} + \lambda V_{bc} + \lambda^2 V_{ca}]$$
$$= \frac{1}{3}[0.8\underline{/76.06} + 1.12\underline{/-43.9+120} + 1.0\underline{/180+240°}]$$
$$= \frac{1}{3}[0.1927 + j\,0.7764 + 0.2690 + j1.0872 + 0.5 + j\,0.866]$$
$$= 0.3205 + j0.9098$$
$$= 0.9646\underline{/70.59°}$$
$$V_{ab_2} = \frac{1}{3}[0.8\underline{/76.06} + 1.12\underline{/-43.9+240} + 1.0\underline{/180+120}]$$
$$= \frac{1}{3}\ [0.1927 + j0.7764 - 1.0760 - j0.3106 + 0.5 - j0.866]$$
$$= -\,0.1277 - j0.1334$$
$$= 0.1846\underline{/226.25°}$$

As neutral is isolated $V_{ab_0} = 0$.

In order to evaluate the positive and negative sequence components of phase to neutral voltage, we take V_{ab_1} and V_{ab_2} as the reference phases as shown in the following figure.



(b)

Fig. E.13.17(*b*)

From Fig. 13.17(*b*)

$$V_{an_1} = V_{ab_1} \underline{/-30°}$$
$$= 0.9646 \underline{/70.59 - 30}$$
$$= 0.9646 \underline{/40.59} \text{ p.u.}$$

and
$$V_{an_2} = V_{ab_2} \underline{/30°}$$
$$= 0.1846 \underline{/226.25 + 30°}$$
$$= 0.1846 \underline{/256.25}$$

Since each resister has an impedance of $1.0 \underline{/0°}$ p.u.

$$I_{a_1} = \frac{V_{an1}}{1.0\underline{/0°}} = 0.9646 \underline{/40.59}$$

and
$$I_{a_2} = \frac{V_{an_2}}{1.0\underline{/0°}} = 0.1846 \underline{/196.25°}$$

V_{an_1} and V_{an_2} are the voltages on the star connected low voltage side of the transformer. The corresponding voltages on the delta side (high tension) are

$$V_{A_1} = -jV_{an_1} = 0.9646 \underline{/40.59 - 90}$$
$$= 0.9646 \underline{/-49.41}$$
$$= 0.6276 - j0.7325$$
$$V_{A_2} = -jV_{an_2} = 0.1846 \underline{/256.25 + 90}$$
$$= 0.1846 \underline{/-13.75°}$$
$$= 0.1793 - j\,0.04387$$

$\therefore$
$$V_A = V_{A_1} + V_{A_2} = 0.9646 \underline{/-49.41} + 0.1846 \underline{/-13.75°}$$
$$= 0.8069 - j\,0.7763$$
$$= 1.12 \underline{/-43.9}$$
$$V_{B_1} = \lambda^2 V_{A_1} = 0.9646 \underline{/-49.41 + 240}$$
$$= -0.9481 - j\,0.1773$$

$$V_{B_2} = \lambda V_{A_2} = 0.1846\underline{/-13.75 + 120°}$$
$$= -0.0516 + j\,0.1772$$
$$V_B = V_{B_1} + V_{B_2} = 1.0\underline{/180°}$$
$$V_{C_1} = \lambda V_{A_1} = 0.9646\underline{/-49.41 + 120°}$$
$$= 0.3205 + j\,0.9097$$

$\therefore$
$$V_{C_2} = \lambda^2 V_{A2} = 0.1846\underline{/-13.75 + 240°}$$
$$= -0.1276 - j0.1333$$
$$V_C = V_{C_1} + V_{C_2} = 0.1929 + j\,0.7764$$
$$= 0.8\underline{/76.06}$$

Now
$$V_{AB} = V_A - V_B$$
$$= 0.8069 - j0.7763 + 1$$
$$= 1.8069 - j\,0.7763$$
$$= 1.967\underline{/-23.25} \text{ (line to neutral voltage base)}$$
$$= 1.1356 - 23.25 \text{ (line to line voltage base)}$$

Similarly
$$V_{BC} = V_B - V_C$$
$$= -1.0 - 0.1929 - j0.7764$$
$$= -1.1929 - j0.7764$$
$$= 1.423\underline{/213.05°} \text{ p.u. (line to neutral voltage base)}$$
$$= \frac{1.423}{\sqrt{3}}\underline{/213.05}$$
$$= 0.8215\underline{/213.05°} \text{ p.u. (line to line voltage base)}$$
$$V_{CA} = V_C - V_A$$
$$= 0.1929 + j0.7764 - 0.8069 + j0.7763$$
$$= -0.614 + j1.5527$$
$$= 1.6697\underline{/111.57°} \text{ (line to neutral voltage base)}$$
$$= 0.9639\ \underline{/111.57°} \text{ p.u. (line to line voltage base)}$$

As the load impedance in each phase is resistive and one p.u., I_{a_1} and V_{an_1} are found to have identical p.u. values. Similarly I_{a_2} and V_{an_2} are identical in p.u. Therefore, I_A must be identical to V_A expressed in p.u. thus

$$I_A = 1.12\underline{/-43.9}$$
$$I_B = 1.0\ \underline{/180°}$$

and
$$I_C = 0.8\underline{/76.06°}. \quad \textbf{Ans.}$$

PROBLEMS

13.1. The line currents in a 3-phase supply to an unbalanced load are respectively $I_a = 10 + j20$, $I_b = 12 - j10$ and $I_c = -3 - j5$ amperes. The phase sequence is *abc*. Determine the sequence components of currents.

13.2. The voltages across a 3-phase unbalanced load are V_a = 300 V, $V_b = 300 \angle -90°$ V and $V_c = 800 \angle 143.1°$ V respectively. Determine the sequence components of voltages. Phase sequence is *abc*.

13.3. Three 6.6 kV, 12 MVA, 3-phase alternators are connected to a common set of busbars. The positive, negative and zero sequence impedances of each alternator are 15%, 12% and 4.5% respectively. If an earth fault occurs on one busbar, determine the fault current:

(*i*) if all the alternator neutrals are solidly grounded;

(*ii*) if one only of the alternator neutrals is solidly earthed and the others are isolated;

(*iii*) if one of the alternator neutrals is earthed through a reactance of 0.5 ohm and the others are isolated.

13.4. A 3-phase alternator is connected to a star/delta transformer through a transmission line as shown here:

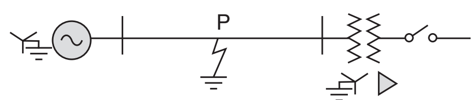


Fig. P.13.4

The positive, negative and zero sequence impedances of the alternator are $j0.1$, $j0.1$ and $j0.05$ p.u. respectively and those of transformer are $j0.05$ p.u. each. A line-to-ground fault occurs at *P* as shown. The respective sequence impedances on the left and right of the fault point are $X'''_L = j0.2$ p.u., $X_{L_2} = j0.2$ p.u. and $X_{L_0} = j0.4$ p.u. and $X_r'' = j0.2$, $X_{r_2} = j0.2$ and $X_{r_0} = j0.4$ p.u. Determine the line current feeding into the fault and voltages at the fault when (*i*) the generator is grounded as shown, (*ii*) the generator neutral is isolated.

13.5. A 50 Hz, 50 MVA, 13.2 kV star grounded alternator is connected to a line through a Δ/*Y* transformer as shown here. The positive, negative and zero sequence impedances of the alternator are $j0.1$, $j0.1$ and $j0.05$ p.u. respectively.

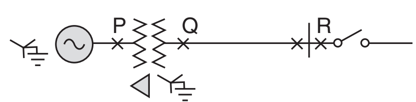


Fig. P.13.5

The transformer rated at 13.2 kV Δ/120 kV *Y*, 50 MVA with star solidly grounded has the sequence impedances of $X'' = X_2 = X_0 = j0.1$ p.u. each. The line impedances between *Q* and *R* are $X'' = j0.03$, $X_2 = j0.03$ and $X_0 = j0.09$ p.u., respectively. Assuming the fault to take place at *P*, determine the subtransient fault current for a (*i*) 3-phase fault, (*ii*) a line-to-ground fault, (*iii*) a line-to-line fault, (*iv*) a double line-to-ground fault. Also express these fault currents as a percentage of 3-phase fault current as calculated in (*i*).

13.6. Solve Problem 13.5 when fault is at point *Q*.

13.7. Solve Problem 13.5 when fault is at point *R*.

13.8. Solve Problem 13.5 when the neutral of the alternator is grounded through an impedance of $j0.2$ ohm.

13.9. A 50 Hz, 13.2 kV, 15 MVA alternator has $X'' = X_2$ = 20% and X_0 = 8% and its neutral is grounded through a reactor of 0.5 ohm. Determine the initial symmetrical r.m.s. current in the ground and in line *c*, when a double line-to-ground fault occurs on phase *b* and *c* and the generator voltage is 12 kV before the fault takes place.

13.10. A 3-phase generator is rated for 60 MVA, 6.9 kV and subtransient reactance $X_d'' = j0.15$ p.u. The generator feeds a motor through a line with impedance of $j0.1$ p.u. on generator rating. The motor is rated at 10 MVA and 6.9 kV with $X_d'' = j0.2$ p.u. on the motor base. The voltage at the terminal of the motor is 1 p.u. and takes a load current of 1.0 p.u. at unity p.f. A symmetrical fault occurs at the motor terminals. Determine the subtransient r.m.s. current at the fault, in the generator and in the motor.

13.11. A 65-MVA star connected 16 kV synchronous generator is connected to a 20 kV/120 kV, 75 MVA Δ/Y transformer. The subtransient reactance X_d'' of the machine is 0.12 p.u. and the reactance of transformer is 0.1 p.u. When the machine is unloaded, a 3-phase fault takes place on the HT side of the transformer. Determine (*i*) the subtransient symmetrical fault current on both sides of the transformer, (*ii*) the maximum value possible of the d.c. current. Assume 1 p.u. generator voltage.

13.12. If in problem 13.11 a 3-phase balanced impedance (on a base of 120 kV and 75 MVA) of $(0.8 + j0.6)$ p.u. ohm is connected across the transformer terminals at 120 kV and a fault takes place beyond the load terminals as shown in Fig. P.13.12, determine the subtransient fault current and the generator current using the Thevenin's theorem. Assume per-fault voltage to be 1.0 p.u.

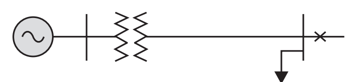


Fig. P.13.12

13.13. Four 50 MVA generators of 15% reactance each are connected via four 35 MVA reactors each of 10% reactance to a common bus bar. The feeders are each connected to the junction of each alternator and its reactor. Determine the rating of each feeder circuit breaker.

13.14. Two 50 MVA, 50 Hz, 11 kV alternators with sub-transient reactance $X'' = j0.1$ p.u. and a transformer of 40 MVA 11 kV/66 kV and reactance of 0.08 p.u. are connected to a bus *A*. Another generator 60 MVA, 11 kV alternator with reactance of 0.12 p.u. is connected to bus *B*. Bus *A* and *B* are interconnected through a reactor of 80 MVA 20 per cent reactance. If a 3-phase fault occurs on the high voltage side of the transformer, calculate the current fed into the fault.

13.15. Two generating stations having short circuit capacities of 1500 MVA and 1000 MVA respectively and operating at 11 kV are linked by an interconnected cable having a reactance of 0.6 ohm per phase, determine the short circuit capacity of each station.

13.16. A 33 kV 3-phase transmission line of resistance 2 ohm and reactance 8 ohm is connected at each end to 2 MVA 33/6.6 kV Δ/Y transformer. The resistance and reactance drops of the transformers are 1% and 3% respectively. Determine the fault current in each section of the system when a 3-phase fault take place on the low voltage side of the step-down transformer. Assume a source with zero impedance.

13.17. Four busbar sections have each a generator of 40 MVA 10% reactance and a busbar reactor of 8% reactance. Determine the maximum MVA fed into a fault on any bus bar section and also the maximum MVA if the number of similar bus bars in sections is very large.

13.18. A 30 MVA, 11 kV generator has subtransient reactance of 10%, supplies power to three identical motors through a transformer as shown in Fig. P.13.18. Each motor is rated for 8 MVA, 6.6 kV with subtransient and transient reactances of $j0.15$ and $j0.25$ p.u. respectively. The transformer is rated for 30 MVA, 11 kV/6.6 kV and leakage reactance 8%. The motor bus bar voltage is 6.6 kV when a 3-phase fault takes place at *F*. Determine (*i*) the subtransient current in the fault, (*ii*) the subtransient current in breaker *B*, (*iii*) the momentary current in breaker *B*, and (*iv*) the current to be interrupted by breaker *B* in 8 cycles.

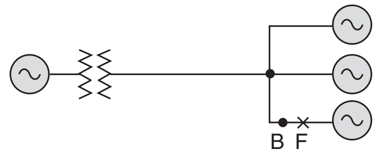


Fig. P.13.18

13.19. A power plant has two generators of 10 MVA, 15% reactance each and two 5 MVA generators of 10% reactance paralleled at a common bus bar from which load is taken through a number of 4 MVA step up transformers each having a reactance of 5%. Determine the short circuit capacity of the breakers on the (*i*) low voltage, and (*ii*) high voltage side of the transformer.

13.20. A 3-phase, 5 MVA, 6.6 kV alternator with a reactance of 8% is connected to a transmission line of series impedance (0.12 + *j*0.48) ohm per km. The transformer is rated at 3 MVA, 6.6 kV/33 kV and reactance 5%. Determine the fault current supplied by the generator operating under no load with voltage 6.9 kV when a 3-phase delta connected fault occurs 15 km along the line with fault impedance between each line being (12 + *j*48) ohms.

13.21. A single line-to-ground fault occurs on phase *a* at point *P* in the circuit whose single line diagram is shown here. Determine the subtransient current in phase *a* of machine 1 and in the fault at *P*. Neglect prefault current. Assume that machine 2 is a synchronous motor operating at rated voltage. Both machines are rated. 1.5 MVA, 600 volts with reactances of $X'' = X_2 = 8\%$ and $X_0 = 4\%$. Each 3-phase transformer is rated 1.25 MVA, 660 volts delta/ 4160 volts star with leakage reactance of 5%. The reactances of transmission line are $X_1 = X_2 = 12\%$ and $X_0 = 40\%$ on a base of 1.25 MVA, 4160 volts.

Fig. P.13.21

13.22. Solve Problem 13.21 when fault is at *R*.

13.23. A 50 Hz, 80 MVA, 11 kV generator has positive, negative and zero sequence impedances of *j*0.4, *j*0.3 and *j*0.1 p.u. respectively. The generator is connected to a busbar *A* through a transformer having $X_1 = X_2 = X_0 = j0.4$ p.u. on 100 MVA base and rated voltage. Determine the ohmic resistance and rating of the earthing resistor such that for a *L-G* fault on busbar *B* the fault current of the generator does not exceed full load current. A reactor of reactance 0.08 p.u. on 100 MVA base is connected between busbars *A* and *B*.

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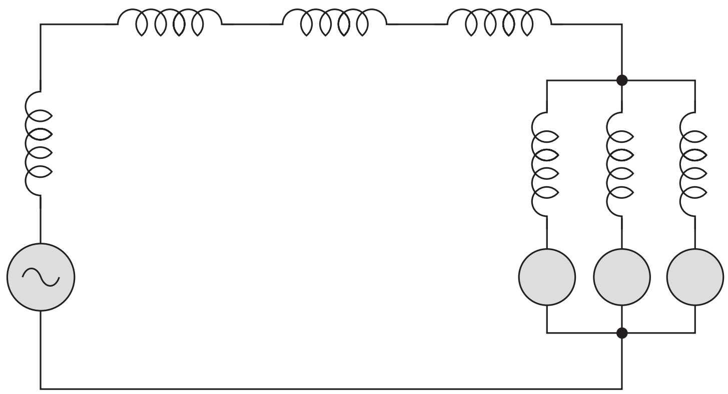
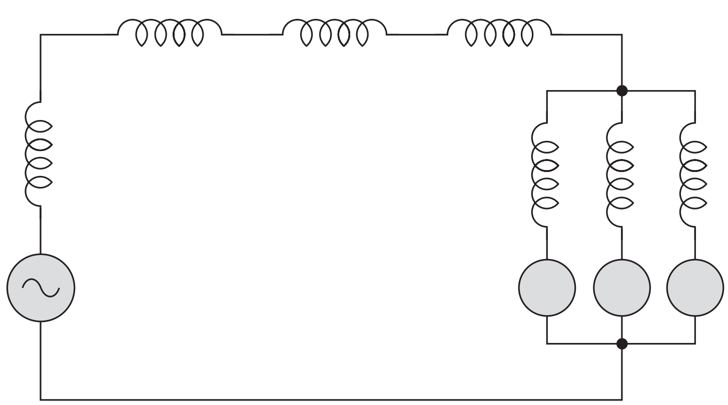


14

PROTECTIVE RELAYS



14

Protective Relays

INTRODUCTION

The capital investment involved in a power system for the generation, transmission and distribution of electrical power is so great that proper precautions must be taken to ensure that the equipment not only operates as nearly as possible to peak efficiencies, but also that it is protected from accidents. The purpose of the protective relays and protective relaying systems is to operate the correct circuit breakers so as to disconnect only the faulty equipment from the system as quickly as possible, thus minimising the trouble and damage caused by faults when they do occur.

The modern power system is very complex and even though protective equipments form 4 to 5% of the total cost involved in the system, they play a very important role in the system design for good quality of reliable supply.

The most severe electrical failures in a power system are shunt faults which are characterized by increase in system current, reduction in voltage, power factor and frequency. The protective relays do not eliminate the possibility of faults on the system, rather their action starts only after the fault has occurred on the system. It would be ideal if protection could anticipate and prevent faults but this is impossible except where the original cause of a fault creates some effects which can operate a protective relay. So far only one type of relay falls within this category, this is the gas detector relay (Buchholz relay) used to protect transformers which operates when the oil level in the conservator pipe of a transformer is lowered by the accumulation of gas caused by a poor connection or by an incipient breakdown of insulation (slowly developing fault).

There are two groups of relaying equipments for protecting any equipment:

1. Primary relaying equipment.
2. Back-up relaying equipment.

Primary relaying is the first line of defence for protecting the equipments whereas the back-up protection relaying works only when the primary relaying equipment fails which means back-up relaying is inherently slow in action. Primary relaying may fail because of failure of any of the following:

(*i*) Protective relays (moving mechanism etc.).

(*ii*) Circuit breaker.

(*iii*) D.C. tripping voltage supply.

(*iv*) Current or voltage supply to the relays.

Since it is required that back-up relays should operate in case primary relays fail, the back-up relays should not have anything common with primary relays. Hitherto, the practice has been to locate the back-up relays at a different station.

A second job of the back-up relays is to act as primary protection in case the primary protection equipment is taken out for repair and maintenance.

14.1 SOME DEFINITIONS

Relay: A relay is an automatic device which senses an abnormal condition in an electric circuit and closes its contacts. These contacts in turn close the circuit breaker trip coil circuit, thereby it opens the circuit breaker and the faulty part of the electric circuit is desconnected from the rest of the healthy circuit.

Pick up Level: The value of the actuating quantity (current or voltage) which is on the threshold (border) above which the relay operates.

Reset Level: The value of current or voltage below which a relay opens its contacts and comes to original position.

Operating Time: The time which elapses between the instant when the actuating quantity exceeds the pick-up value to the instant when the relay contacts close.

Reset Time: The time which elapses between the instant when the actuating quantity becomes less than the reset value to the instant when the relay contact returns to its normal position.

Primary Relays: The relays which are connected directly in the circuit to be protected.

Secondary Relays: The realys which are connected in the circuit to be protected through current and potential transformers.

Auxiliary Relays: Relays which operate in response to the opening or closing of its operating circuit to assist another relay in the performance of its function. This relay may be instantaneous or may have a time delay.

Reach: A distance relay operates whenever the impedance seen by the relay is less than a prespecified value. This impedance or the corresponding distance is known as the reach of the realy.

Underreach: The tendency of the relay to restrain at the set value or the impedance or impedance lower than the set value is known as underreach.

Overreach: The tendency of the relay to operate at impedances larger than its setting is known as overreach.

14.2 FUNCTIONAL CHARACTERISTICS OF A PROTECTIVE RELAY

A protective relay is required to satisfy four basic functional characteristics: (*i*) reliability, (*ii*) selectivity, (*iii*) speed, and (*iv*) sensitivity.

Reliability: The relay should be reliable is a basic requirement. It must operate when it is required. There are various components which go into operation before a relay operates. Therefore, every component and circuit which is involved in the operation of the relay plays an imporant role; for example, lack of suitable current and voltage transformers may result in unreliable operation.

Since the protective relays remain idle most of the time on the power system, proper maintenance will play a vital role in improving the reliable operation of the relay.

Inherent reliability is a matter of design based on long experience. This can be achieved partly by: (*i*) simplicity and robustness in construction, (*ii*) high contact pressure, (*iii*) dust free enclosures, (*iv*) good contact material, (*v*) good workmanship, and (*vi*) careful maintenance.

Selectivity: It is the basic requirement of the relay in which it should be possible to select which part of the system is faulty and which is not and should isolate the faulty part of the system from the healthy one. Selectivity is achieved in two ways: (*i*) unit system of protection, and (*ii*) non-unit system of protection.

Unit system of protection means the one is which the protection responds only to faults within its own zone and does not make note of the conditions elsewhere, *e.g.*, the differential protection of transformers and generators. Here the protection scheme will work only if the fault is in the transformer or the generator respectively.

Non-unit system of protection is one in which the selectivity is obtained by grading the time or current settings of the relays at different locations, all of which may respond to a given fault.

Speed: A protective relay must operate at the required speed. It should neither be too slow which may result in damage to the equipment, nor should it be too fast which may result in undesired operation during transient faults.

The shorter the time for which a fault is allowed to persist on the system, the more load can be transferred between given points on the power system without loss of synchronism. Fig. 14.1 shows the curves which represent the power that can be transmitted as a function of fault clearing time for various types of faults.

It can be seen from the curves that the severest fault is the 3-phase fault and the least severe is the *L-G* fault in terms of transmission of power.

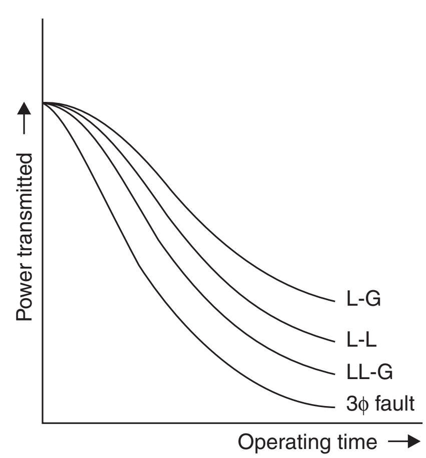


Fig. 14.1 Power transmitted during various faults on a system as a function of relay plus breaker time.

Sensitivity: A relay should be sufficiently sensitive so that it operates reliably when required under the actual conditions in the system which produce the least tendency for operation. It is normally expressed in terms of minimum volt-amperes required for the relay operation.

14.3 OPERATING PRINCIPLES OF RELAYS

Basically there are two different operating principles of relays: (*i*) electromagnetic attraction, and (*ii*) electromagnetic induction.

In the electromagnetic attraction type of relays the operation is obtained by virtue of an armature being attracted to the poles of an electromagnet or a plunger being drawn into a solenoid. These relays can be operated by both d.c. as well as a.c. quantities. With d.c. the torque developed is constant and if this force exceeds a predetermined value the relay operates.

In case of a.c. quantity the force is given by

$$F \propto I^2$$

$$F = K'I^2$$

Let $I = I_m \sin \omega t$; then $$F = K'I_m{}^2 - K' \cos 2\omega t$$

This shows that the force consists of two components, one the constant, independent of time, whereas the other is a function of time and pulsates at double the supply frequency. The total deflecting force, therefore, pulsates at double the frequency. Since the restraining force is constant the net force is a pulsating one which means that the relay armature vibrates at double the power supply frequency. These vibrations will lead to sparking between the contacts and the relay will soon be damaged.

To overcome this difficulty in a.c. electromagnet, the two fluxes producing the force are displaced in time phase so that the resultant deflecting force is always positive and constant. This phase displacement can be achieved either by providing two windings on the electromagnet having a phase shifting network or by putting shading ring on the poles of the magnet as shown in Fig. 14.2. However, the shading ring or coil method is more simple and is widely used.

Induction Relays: The induction relays operate based on the electromagnetic induction principle. Therefore, these relays can be used only on a.c. circuits and not on d.c. circuits. Depending upon the type of rotor being used, these relays are categorised as (*i*) induction disc type, and (*ii*) induction cup type of relays.

In case of induction disc type of relays, disc is the moving element on which the moving contact of relay is fixed whereas in case of induction cup the contact is fixed with the cup. There are two structures available under the induction disc type of relay: (*i*) the shaded pole structure, and (*ii*) the watthour meter structure.

Shaded Pole Structures: As shown in Fig. 14.2, the disc is placed between the shaded and unshaded poles of the relay. The relay consists of an operating coil which is fed by the current proportional to the system current. The air gap flux produced by this flux is split into two out-of-phase components by a shading ring made of copper that encircles part of the pole face of each pole at the air gap. The disc is normally made of aluminium so as to have low inertia and, therefore, requires less deflecting torque for its motion. Sometimes, instead of shading ring, shading coils are used which can be short circuited by the contact of some other relay. Unless the contacts of the other relay are closed, the shading coil remains open and hence no torque can be developed. Such torque control is employed where directional feature is required which will be described later.

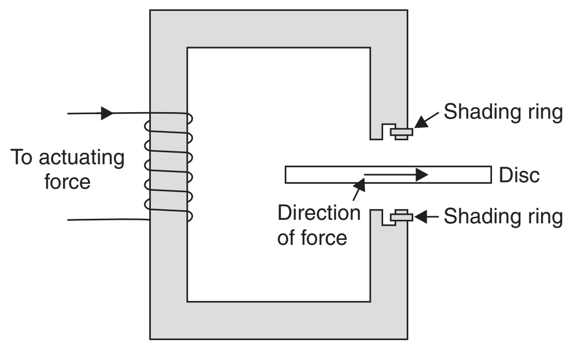


Fig. 14.2 Shaded pole structure.

14.4 TORQUE PRODUCTION IN AN INDUCTION RELAY

It is well known that for producing torque, two fluxes displaced in space and time phase are required. Let these fluxes be

$$\phi_1 = \phi_m \sin \omega t$$
$$\phi_2 = \phi'_m \sin (\omega t + \theta)$$

Flux ϕ_1 is produced by the shaded pole and ϕ_2 by the unshaded. The shaded pole flux lags that by the unshaded pole by angle θ. The two fluxes ϕ_1 and ϕ_2 will induce voltages e_1 and e_2 respectively in the disc due to induction. These voltages will circulate eddy currents in the disc of the relay. Assuming the disc to be non-inductive, these currents will be in phase with their respective voltages. The vector diagram (Fig. 14.3) shows the phase relations between various quantities.

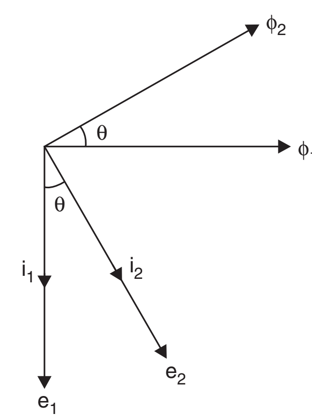


Fig. 14.3 Phasor diagram for an induction relay.

$$e_1 \propto \frac{d\phi_1}{dt}$$
$$\propto \phi_m \omega \cos \omega t$$

and $$e_2 \propto \phi'_m \, \omega \cos (\omega t + \theta)$$

The eddy current $i_1 \propto e_1$.

Assuming same resistance to flow of eddy current,

$$i_2 \propto e_2$$

i.e., $$i_1 \propto \phi_m \omega \cos \omega t$$

and $$i_2 \propto \phi'_m \, \omega \cos (\omega t + \theta)$$

The flux ϕ_1 will interact with eddy current i_2 and ϕ_2 will interact with i_1 and since ϕ_2 is leading ϕ_1 the torque due to ϕ_2 and i_1 will be reckoned as positive whereas that due to ϕ_1 and i_2 as negative. The resultant torque is

$$\begin{aligned} T &\propto \phi_2 i_1 - \phi_1 i_2 \\ &\propto \phi'_m \sin(\omega t + \theta) \,.\, \phi_m \omega \cos \omega t - \phi_m \sin \omega t .\, \phi'_m \,\omega \cos(\omega t + \theta) \\ &\propto \phi_m \phi'_m \sin(\omega t + \theta) \cos \omega t - \phi_m \phi'_m \sin \omega t \cos(\omega t + \theta) \\ &\propto \phi_m \phi'_m \sin \theta \qquad (14.1) \end{aligned}$$

Thus the torque is maximum when the two fluxes are displaced by 90° and since ϕ_2 leads ϕ_1, the rotation of the disc under the poles will be from unshaded pole towards the shaded pole. Also it is seen that the torque is of constant magnitude; therefore, there is no possibility of vibration.

The control torque is provided with the help of a control spring which is attached to the spindle of the disc. As the disc moves towards closing of the contacts, the spring torque increases slightly with the winding of the spring. The relay disc is so shaped that as it turns towards the pick up position (closing of contacts), there is increase in the area of the disc between the poles of the actuating structure which causes increase in eddy currents and, therefore, increase in electrical torque that just balances the increase in the control spring torque. The shape of the disc usually is that of a spiral.

Since the shape of the disc is not perfectly circular, suitable balance weight is provided on that part of the disc which has smaller area.

The damping torque is provided by a permanent magnet of high retentivity steel. The motion of the disc can be controlled by adjusting the position of this magnet.

The minimum torque required for the movement of the disc is fixed for a particular design, *i.e.*, the ampere-turns required are fixed. Therefore, for different pick up current settings, number of turns are changed effectively so as to keep the same ampere-turns. Higher current setting will require smaller number of turns. Selection of the required current setting is by means of a plug setting multiplier which has a single insulated plug. While the plug is withdrawn for adjusting it to a different current setting during on-load condition, maximum current tap is automatically connected, thus avoiding the risk of open circuiting the secondary of the C.T. under load condition.

The operating time of the relay depends upon the distance between the moving contact and the fixed contact of the relay. The distance between the contacts is adjusted by the movement of the disc back stop which is controlled by rotating a knurled moulded wheel at the base of the graduated time multiplier scale. This is known as time multiplier setting. The higher the time multiplier setting the greater is the operating time.

Watt-hour Meter Structure: The construction of this structure is exactly identical to watt-hour meters. The structure (Fig. 14.4) has two separate coils on two different magnetic circuits, each of which produces one of the two necessary fluxes for driving the disc of the relay.

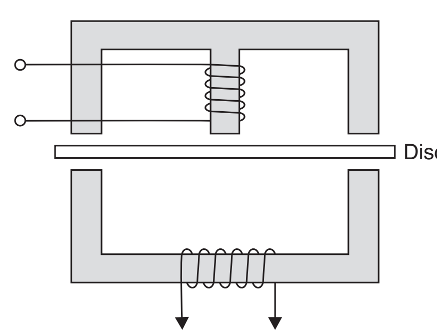


Fig. 14.4 Watt-hour meter structure.

Induction Cup Relays (Fig. 14.5): This relay has four or more electromagnets. A stationary iron core is placed between these electromagnets. The rotor is a hollow cylindrical cup which is free to rotate in the gap between the electromagnets and the stationary iron core. When the electromagnets are energized, they induce voltages in the rotor cup and hence the eddy currents. The eddy currents due to one flux interact with the flux due to the other pole; thereby a torque is produced similar to the induction disc type of relay.

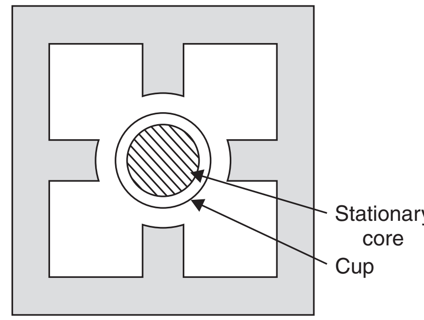


Fig. 14.5 Induction cup structure.

The induction cup type of relays are more sensitive than the induction disc type of relays and are used in high speed relay applications.

The ratio of reset to pick up is inherently high in case of induction relays as compared to attracted armature relays as their operation does not involve any change in the air gap of the magnetic circuit as it is in the case of latter. The ratio lies between 95% and 100%. This is not perfectly 100% because of the friction and imperfect compensation of the control spring torque.

14.5 OVER-CURRENT RELAYS

Depending upon the time of operation the relays are categorized as: (*i*) Instantaneous over-current relay, (*ii*) Inverse time-current relay, (*iii*) Inverse definite minimum time (IDMT) over-current relay, (*iv*) Very inverse relay, and (*v*) Extremely inverse relay.

(*i*) *Instantaneous over-current relay* is one in which no intentional time delay is provided for the operation. The time of operation of such relays is approximately 0.1 sec. This characteristic can be achieved with the help of hinged armature relays. The instantaneous relay is more effective where the impedance Z_s between the source and the relay is small compared with the impedance Z_l of the section to be protected.

(*ii*) *Inverse time-current relay* is one in which the operating time reduces as the actuating quantity increases in magnitude. The more pronounced the effect is the more inverse the characteristic is said to be. In fact, all time current curves are inverse to a greater or lesser degree. They are normally more inverse near the pick up value of the actuating quantity and become less inverse as it is increased. This characteristic can be obtained with induction type of relays by using a suitable core which does not saturate for a large value of fault current. If the saturation occurs at a very early stage, the time of operation remains same over the working range. The characteristic is shown by curve (*a*) in Fig. 14.6 and is known as definite time characteristic.

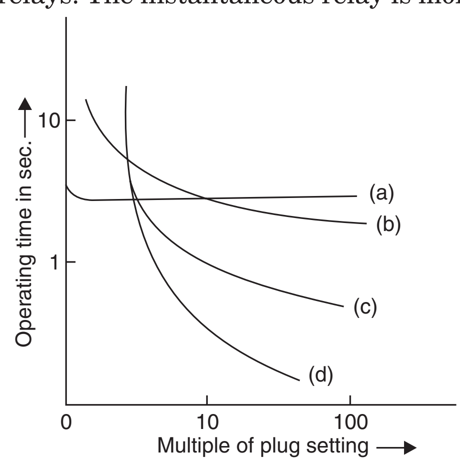


Fig. 14.6 Characteristics of various over-current relays: (*a*) definite time, (*b*) IDMT, (*c*) very inverse, and (*d*) extremely inverse.

(*iii*) *Inverse definite minimum time over-current relay* is one in which the operating time is approximately inversely proportional to the fault current near pick up value and becomes substantially constant slightly above the pick up value of the relay (Fig. 14.6(*b*)). This is achieved by using a core of the electromagnet which gets saturated for currents slightly greater than the pick up current.

(*iv*) *Very inverse relay* is one in which the saturation of the core occurs at a later stage, the characteristic assumes the shape as shown in Fig. 14.6(*c*) and is known as very inverse characteristic. The time-current characteristic is inverse over a greater range and after saturation tends to definite time.

(*v*) *Extremely inverse relay* is one in which the saturation occurs at a still later stage than curve (*c*) in Fig. 14.6. The equation describing the curve (*d*) in the figure is approximately of the form $I^2t = K$, where I is the operating current and t the operating time.

Time Current Characteristics

These curves are normally plotted on log-log graph papers as shown in Fig. 14.7. The ordinate is the operating time and the abscissa the multiple of pick up value of the actuating quantity. The abscissa is taken as multiple of pick up value so that the same curves can be used for any value of pick up, *i.e.*, if the curves are known for pick up value of 2.5 A, then the characteristics remain same for 5 A or 6.25 A or any other pick up value. This is possible with induction type of relays where the pick up adjustment is by coil, because the ampere-turns at pick up are the same for each tap and hence at a given multiple of pick up, the coil ampere-turns and hence the torque are the same regardless of the tap used.

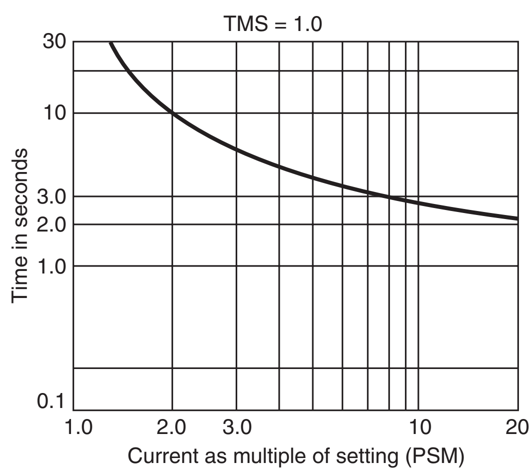


Fig. 14.7 Standard 2.2 sec IDMT curve.

The advantage of plotting the curves on log-log sheets is that if the characteristic for one particular pick up value and one time multiplier setting is known, then the characteristics for any other pick up value and time multiplier settings can be obtained.

The curves are used to estimate not only the operating time of the relay for a given multiple of pick up and time multiplier setting but also it is possible to know how far the relay moving contact would have travelled towards the fixed contacts within any time interval.

This method is also useful in finding out whether the relay will pick up and how long it will take for the operation of the relay when the actuating quantity is changing as for example during the in-rush current period of starting a motor etc.

For most effective use of the characteristics the multiple of pick up should not be less than 1.5 because then the total actuating force is low and any additional friction may not result in operation of the relay or it may take inordinately long time.

The inverse time current relays are non-directional relays and are used for the protection of feeders, transmission lines, transformers, machines and other numerous applications.

Example 14.1: Determine the time of operation of a relay of rating 5 amps, 2.2 sec IDMT and having a relay setting of 125% TMS = 0.6. It is connected to a supply circuit through a C.T. 400/5 ratio. The fault current is 4000 amps.

Solution: The pick up value of the relay is 5 amps but since the relay setting is 125%, therefore, the operating current of the relay is

$$5 \times 1.25 = 6.25 \text{ amps}$$

The plug setting multiplier of the relay,

$$\text{PSM} = \frac{\text{Secondary current}}{\text{Relay current setting}}$$

$$= \frac{\text{Primary current (fault current)}}{\text{Relay current setting} \times \text{CT ratio}}$$

$$= \frac{4000}{6.25 \times 80} = 8$$

From the standard 2.2 sec curve (Fig. 14.7) the operating time for PSM = 8 is 3.2 sec.

Since the TMS is 0.6, the actual operating time of the relay is 1.92 secs. **Ans.**

14.6 DIRECTIONAL OVER-CURRENT RELAYS

The relay consists of two units: (*i*) directional unit; and (*ii*) non-directional or inverse time current unit. The second unit is exactly the same as discussed in the previous section.

The directional unit is a four-pole induction cup unit. Two opposite poles are fed with voltage and the other two poles are fed with current. The voltage is taken as the polarizing quantity. The polarizing quantity is one which produces one of the two fluxes required for production of torque and this quantity is taken as the reference compared with the other quantity which is current here. This means that the phase angle of the polarizing quantity must remain more or less fixed when the other quantity suffers wide changes in phase angle.

In a circuit at a point the current can flow in one direction at a particular instant. Let us say this is the normal direction of flow of current. Under this condition the directional unit will develop negative torque and the relay will be restrained to operate. Now if due to certain changes in the circuit condition, the current flows in opposite direction, the relay will develop positive torque and will operate.

For a directional over-current unit unless the directional unit contacts are closed, the over-current unit is not energized because the operating coil of the over-current unit completes its circuit through the directional unit contacts or if the over-current unit has shading coil on its poles for the production of lagging flux; then the shading coil completes its circuit through the directional unit contacts (Fig. 14.8).

The contacts of the directional unit can be easily removed and if maintenance is required the whole unit can be easily dismantled and re-assembled without altering its characteristics.

The torque developed by a directional unit is given by

$$T = VI \cos(\theta - \tau) - K \tag{14.2}$$

where V = r.m.s. magnitude of the voltage fed to the voltage coil circuit,

I = the r.m.s. magnitude of the current in current coil,

θ = the angle between I and V,

τ = the maximum torque angle (a design quantity), and

K = restraining torque including spring and friction.

Say for a particular installation $(\theta - \tau)$ = constant K_1; then the torque equation becomes

$$T = K_1VI - K$$

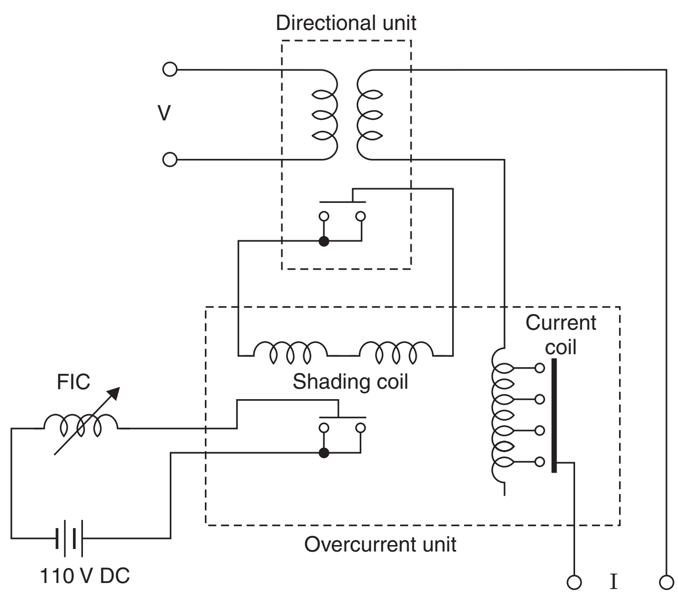


Fig. 14.8 Internal connection diagram of a directional over-current relay.

Under threshold condition when the relay is about to start,

$$T = 0 = K_1VI - K$$

or

$$VI = \frac{K}{K_1} = K' = \text{constant} \tag{14.3}$$

This characteristic is known as a constant product characteristic and is of the form of a rectangular hyperbola as shown in Fig. 14.9.

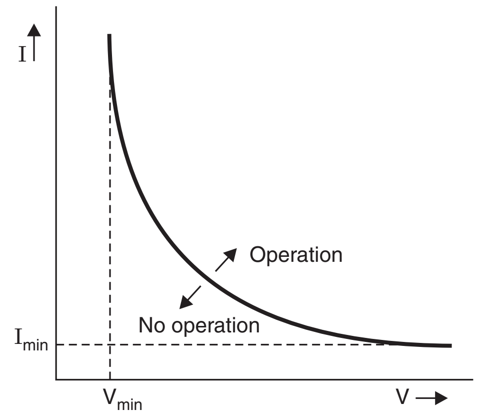


Fig. 14.9 Constant product characteristic of a directional relay.

For the operation of the relay the product of V and I should give a minimum torque which exceeds the friction and spring torque. From the characteristic it is clear that it is not enough to have the product greater than K' but there is a minimum value of voltage and a minimum value of current required for the torque to be developed. The product of any value of voltage and any value of current to exceed K' is not enough. Say A is the location of the directional relay (Fig. 14.10). In case the fault is close to the relay the voltage to be fed to the relay may be less than the minimum voltage required. The maximum distance up to which the voltage is less than the minimum voltage required is known as the dead zone of the directional relay *i.e.*, if the fault takes place within this zone the relay will not operate.

Fig. 14.10 Directional relay used on a line.

Consider the torque equation (14.2) again

$$T = VI \cos(\theta - \tau) - K$$

The phasor diagram is shown in Fig. 14.11. Here ϕ_v is the flux due to the voltage coil and lags behind the voltage by about 60° to 70°. ϕ_1 is the flux due to the current coil. The net torque is produced due to the interaction of ϕ_I and ϕ_v. The torque is maximum when the two fluxes are displaced by 90°. Here dotted line in the phasor diagram represents the desired position of ϕ_I for maximum torque and since V is the reference or polarising quantity and ϕ_v has fixed position with respect to V for a particular design, the angle between the dotted line and the polarising quantity V is known as the maximum torque angle and is normally denoted by τ. This means when the relay current leads the voltage by an angle τ, maximum torque is produced.

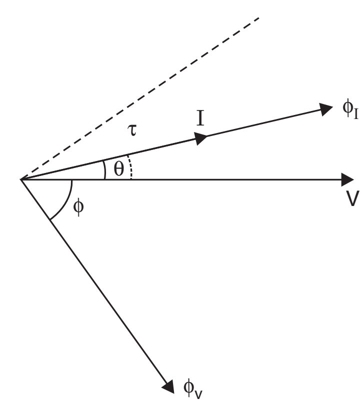


Fig. 14.11 Phasor diagram of a directional relay.

Referring again the torque equation, if V is fixed and under operating condition K is negligible, then

$$I \cos(\theta - \tau) = 0$$

Since I cannot be zero for torque production

$$\cos(\theta - \tau) = 0$$

i.e., $$\theta - \tau = \pm \frac{\pi}{2}$$

$\therefore$ $$\theta = \tau \pm \frac{\pi}{2} \qquad (14.4)$$

This is the equation describing the polar characteristic (Fig. 14.12) of the directional relay.

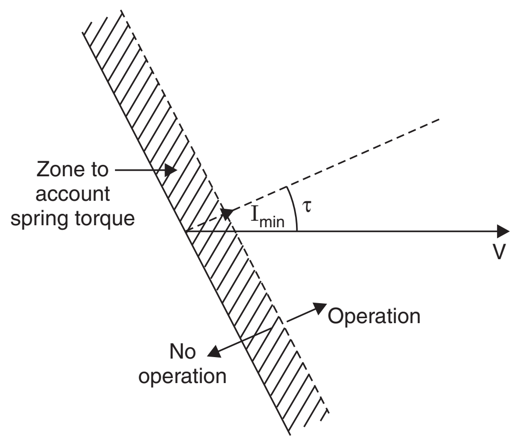


Fig. 14.12 Polar characteristic of directional relay.

The zone between the dotted line and the line parallel to it corresponds to the spring torque. If the current vector lies within these lines the torque developed is less than the spring torque and hence the relay does not operate. If the current crosses the dotted line the spring torque is less than the operating torque and hence the relay operates.

Example 14.2: A cable circuit with an impedance angle of 10° is to be protected by directional over current relays. Specify the connection you use for the directional element and justify by actually working out the connections and the maximum torque angle setting needed for the relay. Specify also the phase shifting network to be used if the relay potential coil has an impedance of 1000 ∠60°. Assume a four-pole cup element for the directional element.

Solution: For the cable the impedance angle under operating condition is 10°. With 30° connection the phase angle between V_a and V_{ac} is 30°. V_{ac} lagging V_a and, therefore, I_a leads V_{ac} by 20°. The relay quantities are current proportional to I_a and voltage proportional to V_{ac} for a fault on phase a. In case of fault on phase a the voltage of this phase up to the relay point becomes quite small and say the phase angle between V_a and V_{ac} becomes 50° instead of 30°; thereby the angle between I_a and V_{ac} becomes 40°. The phasor diagram is shown here (Fig. E.14.2).

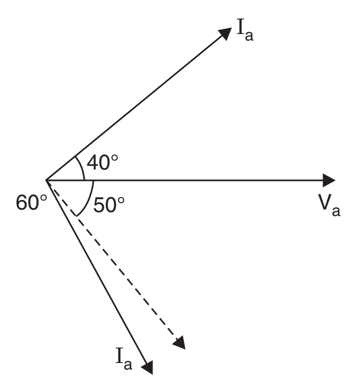


Fig. E.14.2

Voltage V_{ac} is applied to the potential coil which has an impedance angle of 60° and the position of current I_v is shown in the phasor diagram. For torque to be maximum the angle between I_a and I_v should be 90°; therefore, a capacitor of suitable value should be connected such that the impedance angle becomes 50° rather than 60° as shown by a dotted phasor in the phasor diagram. Now

$$1000 \angle 60° = (500 + j860)$$

For angle to be 50°, $\tan^{-1} = \dfrac{X}{R} = 50°$

or $\tan 50° = \dfrac{X}{R}$ or $X = 596$

But we have inductive reactance of 860 Ω.

∴ The capacitive reactance required = 860 – 596 = 264

∴ Value of $C = \dfrac{1}{314 \times 264}$ Farads = 12.1 μF. **Ans.**

14.7 THE UNIVERSAL RELAY TORQUE EQUATION

The universal relay torque equation is given as follows:

$$T = K_1 I^2 + K_2 V^2 + K_3 VI \cos(\theta - \tau) + K \qquad (4.5)$$

By assigning plus or minus signs to some of the terms and letting others be zero and sometimes adding some terms having a combination of voltage and current, the operating characteristics of all types of relays can be obtained. For example, for over-current relay $K_2 = 0$, $K_3 = 0$ and the spring torque will be $-K$. Similarly, for directional relay, $K_1 = 0$, $K_2 = 0$.

Distance Relays

We will study a very interesting and versatile family of relays known as distance relays with the help of universal torque equation. Under this, only a few types of relays will be considered here. They are: (*i*) impedance relays, (*ii*) reactance relays, (*iii*) mho relays.

It is to be noted here that in electrical engineering 'impedance' term can be applied to resistance alone or reactance alone or a combination of the two. In protective relaying, however, these terms have different meanings and hence relays under these names will have different characteristics.

From the universal torque equation putting $K_3 = 0$ and giving negative sign to voltage term, it becomes

$$T = K_1 I^2 - K_2 V^2 \text{ (neglecting spring torque)} \qquad (14.6)$$

This means the operating torque is produced by the current coil and restraining torque by the voltage coil, which means that an impedance relay is a voltage restrained over-current relay.

For the operation of the relay the operating torque should be greater than the restraining torque, *i.e.,*

$$K_1 I^2 > K_2 V^2$$

Here V and I are the voltage and current quantities fed to the relay.

$$\therefore \quad \frac{V^2}{I^2} < \frac{K_1}{K_2}$$

or
$$Z < \sqrt{\frac{K_1}{K_2}}$$

or $\quad Z <$ constant (design impedance)

This means that the impedance relay will operate only if the impedance seen by the relay is less than a prespecified value (design impedance). At threshold condition,

$$Z = \sqrt{\frac{K_1}{K_2}} \tag{14.7}$$

The operating characteristic of an impedance relay on *V-I* diagram is shown in Fig. 14.13.

The initial bend in the characteristic is due to the presence of spring torque.

Normally, the operating characteristics of distance relays are shown on an impedance diagram or *R-X* diagram. This characteristic for an impedance diagram is shown in Fig. 14.14.

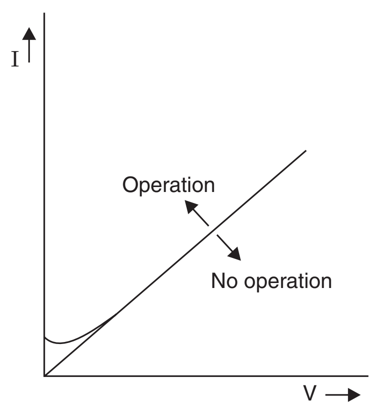


Fig. 14.13 Operating characteristic of an impedance relay on *V-I* diagram.

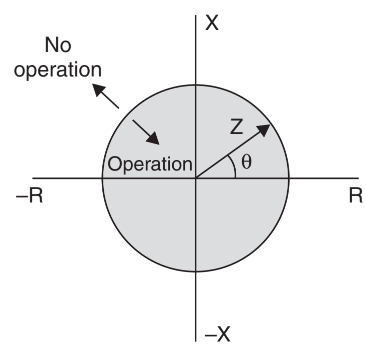


Fig. 14.14 Operating characteristic of an impedance relay on *R-X* diagram.

This is clear from the characteristic that if the impedance as seen by the relay lies within the circle the relay will operate; otherwise, it will not. The position of one value of Z is shown in the figure with angle θ with the +R-axis. This means that the current lags the voltage by angle θ. In case the two were in phase, the Z vector would have coincided with +R-axis. In case the current was lagging the voltage by 180°, the Z vector would coincide with – R-axis. It is to be noted here that – R-axis does not mean here negative resistance axis but the one as explained. When I lags behind V, the Z vector lies in the upper semi-circle and Z lies in the lower when I leads the voltage. Since the operation of the relay is independent of the phase relation between V and I, the operating characteristic is a circle and hence it is a non-directional relay.

The impedance relays normally used are high speed relays. These relays may use a balance beam structure or an induction cup structure.

The directional property to the impedance relay can be given by using the impedance relay along with a directional unit as is done in case of a simple overcurrent relay to work as a directional over current relay. This means the impedance unit will operate only when the directional unit has operated. The characteristic of such a combination will be as shown in Fig. 14.15.

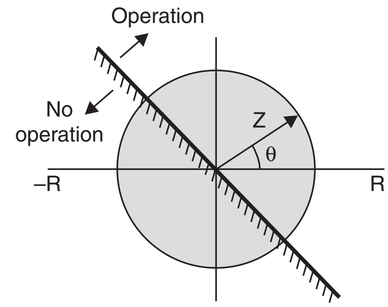


Fig. 14.15 Operating characteristic of an impedance relay with directional unit.

From the characteristic it is clear that if the impedance vector as seen by the relay lies in a zone indicated by the thick line (intersection of straight line and circle) the relay will operate, otherwise, it will not.

Reactance relay: In this relay the operating torque is obtained by current and the restraining torque due to a current-voltage directional element. This means, a reactance relay is an over-current relay with directional restraint. The directional element is so designed that its maximum torque angle is 90°, *i.e.*, $\tau = 90°$ in the universal torque equation.

$$T = K_1 I^2 - K_3 VI \cos(\theta - \tau)$$
$$= K_1 I^2 - K_3 VI \cos(\theta - 90°)$$
$$= K_1 I^2 - K_3 VI \sin\theta \quad (14.8)$$

For the operation of the relay,

$$K_1 I^2 > K_3 VI \sin\theta$$

or
$$\frac{VI}{I^2}\sin\theta < \frac{K_1}{K_3}$$

or
$$Z\sin\theta < \frac{K_1}{K_3}$$

$$X < \frac{K_1}{K_3} \quad (14.9)$$

This means for the operation of the relay the reactance seen by the relay should be smaller than the reactance for which the relay has been designed. The characteristic will be as shown in Fig. 14.16.

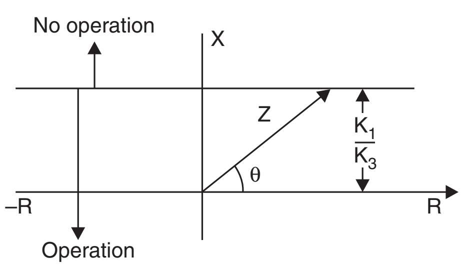


Fig. 14.16 Characteristic of a reactance relay.

This means if the impedance vector head lies on the parallel lines (R-axis and the operating characteristic) this will have a constant X component. The important point about this characteristic is that the resistance component of the impedance has no effect on the operation of the relay. It responds only to the reactance component of the impedance. The relay will operate for all impedances whose heads lie below the operating characteristic whether below or above the R-axis.

This relay as can be seen from the characteristic, is a non-directional relay. This will not be able to discriminate when used on transmission lines, whether the fault has taken place in the section where the relay is located or it has taken place in the adjoining section. It is not possible to use a directional unit of the type used alongwith impedance relay because in that case the relay will operate even under normal load conditions if the system is operating at or near unity power factor condition. Under the condition of high power factor or leading power factor, the impedance seen by the relay is a very low or even negative reactance. The relay that is used to give directional feature to the reactance relay, is known as mho relay or admittance relay which is dealt in the next section.

The mho relay when used alongwith the reactance relay is known as starting relay or starting unit.

The structures used for the reactance relay are

1. Induction cup.
2. Double-induction loop structure.

A typical reactance relay using induction cup structure is shown in Fig. 14.17.

It is a four-pole structure. This has operating, polarising and restraining coils. The operating torque is produced by the interaction of fluxes due to the windings carrying current coils, *i.e.*, interaction of fluxes of poles 1, 2 and 3 and the restraining torque is developed due to the interaction of fluxes due to the poles 1, 3 and 4. The operating torque will be proportional to I^2 and the restraining torque proportional to $VI \cos(\theta - 90°)$. The desired maximum torque angle is obtained with the help of R-C circuits as shown in Fig. 14.17.

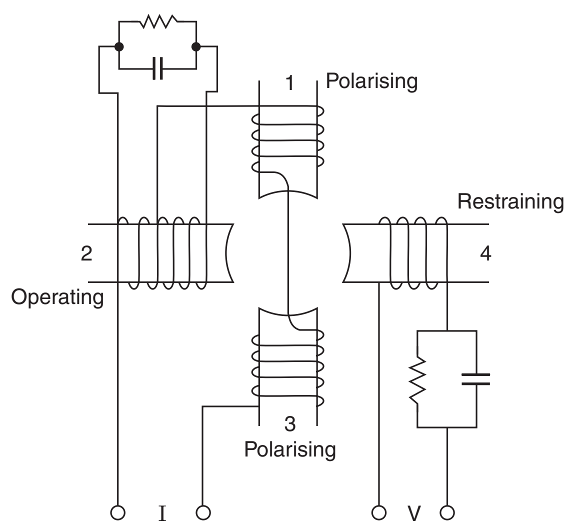


Fig. 14.17 Schematic diagram of a reactance relay.

The mho relay: In this relay the operating torque is obtained by the V-I element and restraining torque due to the voltage element. This means a mho relay is a voltage restrained directional relay. From the universal torque equation

$$T = K_3 VI \cos(\theta - \tau) - K_2 V^2 \tag{14.10}$$

For the relay to operate

$$K_3 VI \cos(\theta - \tau) > K_2 V^2$$

or

$$\frac{V^2}{VI} < \frac{K_3}{K_2} \cos(\theta - \tau)$$

or $$Z < \frac{K_3}{K_2} \cos(\theta - \tau) \tag{14.11}$$

This characteristic, when drawn on an admittance diagram is a straight line passing through the origin and if drawn on an impedance diagram it is a circle passing through the origin as shown in Fig. 14.18.

The relay operates when the impedance seen by the relay falls within this circle. The relay is inherently directional so that it needs only one pair of contacts which makes it fast tripping for fault clearance and reduces the *VA* burden on the current transformers.

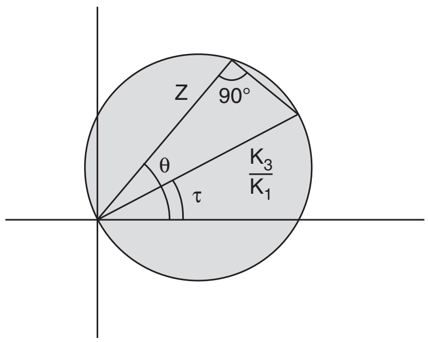


Fig. 14.18 Mho characteristic.

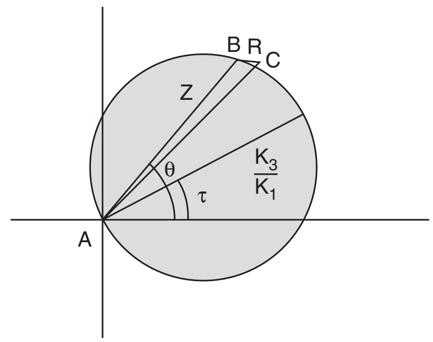


Fig. 14.19 Effect of arc resistance.

The impedance angle of the protected line is normally 60° to 70° which is shown by the line *AB* in Fig. 14.19.

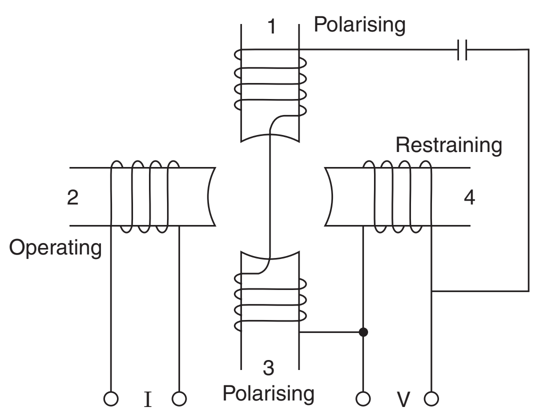


Fig. 14.20 Schematic diagram of a mho relay.

The arc resistance *R* is represented by *BC*. By making τ, the maximum torque angle, equal to or a little less lagging than θ, the circle is made to fit very closely round the fault area so that the relay is an accurate measuring device and does not operate during power swings which may occur on long or heavily loaded lines.

A typical mho relay using induction cup structure is shown in Fig. 14.20.

The operating torque is produced by the inter-action of fluxes due to the poles 1, 2 and 3 and the restraining torque due to the poles 1, 3 and 4.

Effect of Type of Fault

The impedance as seen by the relay will depend upon the type of fault, *e.g.*, if it is a 3-phase fault, the impedance seen by the relay will correspond to the positive sequence impedance of the system and if it is a line-to-ground fault, the impedance seen will correspond to the sum of positive, negative and zero sequence impedances. Thus, actually speaking, a different setting is required for each type of fault. In order that the relay has the same sensitivity for all types of faults it is required that the relay connections should be such that they measure the common impedance in all types of faults, *i.e.*, the positive sequence impedance. This is done by suitable choice of voltage and current coil connections. It is usual to employ three earth-fault measuring, three phase-fault measuring relays—one for each phase and each phase-pair respectively.

Connection for Phase Fault Relays:

These relays will respond to

(*i*) Three phase-fault.

(*ii*) L-L fault.

(*iii*) L-L-G fault.

The relay voltage and current coils are fed as follows:

Relay	*Current* (I_r)	*Voltage* (V_r)
a phase	$I_a - I_b$	$V_a - V_b = V_{ab}$
b phase	$I_b - I_c$	$V_b - V_c = V_{bc}$
c phase	$I_c - I_a$	$V_c - V_a = V_{ca}$

Here suffix r stands for the relay quantities. With these quantities fed to the relay, the relay will measure only the positive sequence impedance for the above mentioned phase faults. Now the impedance measured by the relay is equal to impedance between the fault point and the relay point. For any phase-pair say b and c, we know (Chapter13) that $V_{a_1} = V_{a_2}$ for any type of phase fault. Therefore, the voltage (positive and negative sequence) at the relay location are

$$V_{r_1} = V_{f_1} + I_{a_1} Z_1$$

$$V_{r_2} = V_{f_2} + I_{a_2} Z_2$$

$$V_{r_1} - V_{r_2} = I_{a_1} Z_1 - I_{a_2} Z_2$$

Since for a transmission line $Z_1 = Z_2$,

$$V_{r_1} - V_{r_2} = (I_{a_1} - I_{a_2}) Z_1$$

or
$$Z_1 = \frac{V_{r_1} - V_{r_2}}{I_{a_1} - I_{a_2}} \tag{14.12}$$

Now using symmetrical components

$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0}$$

$$V_c = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}$$

$$\therefore \quad V_b - V_c = (\lambda^2 - \lambda) V_{a_1} + (\lambda - \lambda^2) V_{a_2}$$

Now at the relay location $V_{a_1} = V_{r_1}$ and $V_{a_2} = V_{r_2}$

$\therefore$ $$V_b - V_c = (V_{r_1} = V_{r_2})(\lambda^2 - \lambda)$$

Similarly $$I_b - I_c = (I_{a_1} - I_{a_2})(\lambda^2 - \lambda)$$

From these equations, $$\frac{V_b - V_c}{I_b - I_c} = \frac{V_{r_1} - V_{r_2}}{I_{a_1} - I_{a_2}}$$

and this right hand side equals Z_1 the positive sequence impedance as

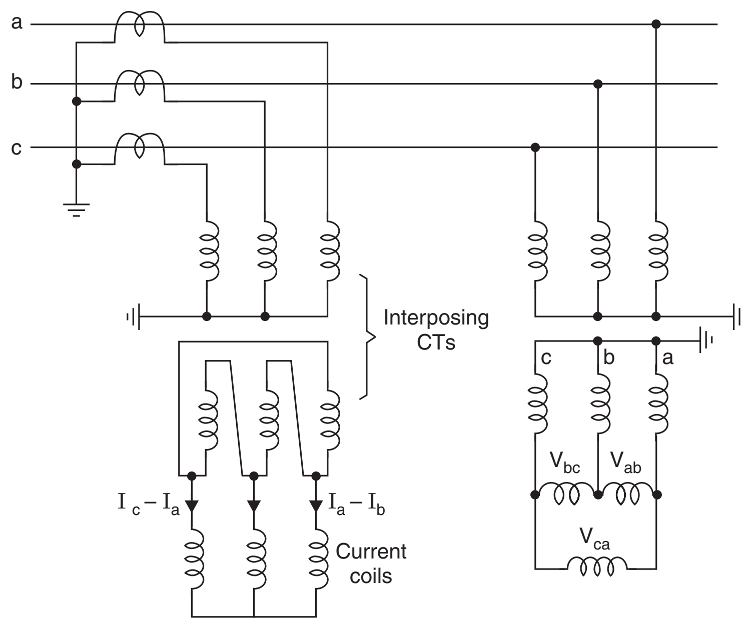


Fig. 14.21 Basic connections of phase fault relays.

derived in equation (14.12), *i.e.,*

$$\frac{V_b - V_c}{I_b - I_c} = \frac{V_{r_1} - V_{r_2}}{I_{a_1} - I_{a_2}} = Z_1 \tag{14.13}$$

This shows that when the relay is fed with the quantities as given in the table (p. 355), the relay looks into only the positive sequence impedance.

The connections for phase faults are shown in Fig. 14.21.

Connections for Earth Fault Relays

Let the fault be on phase a. Since it is a line-to-ground fault, the impedance as seen by the relay will be $(Z_1 + Z_2 + Z_0)$. The voltage upto the relay point will be

$$V_r = I_{a_1} Z_1 + I_{a_2} Z_2 + I_{a_0} Z_0 \tag{14.14}$$

Also $I_a = I_{a_1} + I_{a_2} + I_{a_0}$ and $I_a + I_b + I_c = 3I_{a_0} = I_{\text{res}}$ (say), where I_a, I_b and I_c are the currents during the fault at the relay point and I_{res} is the residual current.

For a transmission line $Z_1 = Z_2$; normally the zero sequence impedance of the line is greater than positive sequence impedance. Let $Z_0 = KZ_1$. Here $K > 1$.

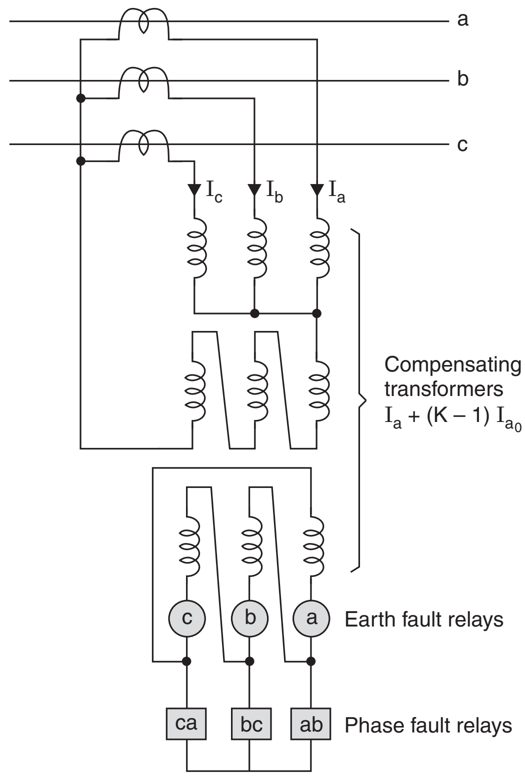


Fig. 14.22 Relay current input arrangement.

After substituting these values in equation (14.14),

$$V_r = I_{a_1} Z_1 + I_{a_2} Z_1 + I_{a_0} KZ_1$$
$$= Z_1\{I_{a_1} + I_{a_2} + I_{a_0} + (K-1) I_{a_0}\}$$
$$= Z_1\{I_a + (K-1) I_{a_0}\}$$
$$= Z_1\{I_a + (K-1) I_{res}/3\}$$

or
$$\frac{V_r}{I_a} = Z_1 + \frac{(K-1)}{3} \frac{I_{res}}{I_a}$$

or
$$\frac{V_r}{I_a + \frac{1}{3}(K-1) I_{res}} = Z_1 \qquad (14.15)$$

From this it is clear that for the relay to respond only to positive sequence impedance the current fed to the relay is $I_a + (K-1)\, \dfrac{I_{res}}{3}$. The arrangement of connections (Fig. 14.22) for current coil is such that the relay has same sensitivity for all types of shunt faults.

14.8 DIFFERENTIAL RELAYS

The differential relay is one that operates when the vector difference of two or more similar electrical quantities exceeds a pre-determined value. This means for a differential relay, it should have: (1) two or more similar electrical quantities, and (2) these quantities should have phase displacement (normally approx. 180°), for the operation of the relay. The name is not due to a particular construction of the relay but is due to the way in which the relay is connected in the circuit.

The most common application of this relay is the current differential type. The simple connection for this type of protection is given in Fig. 14.23.

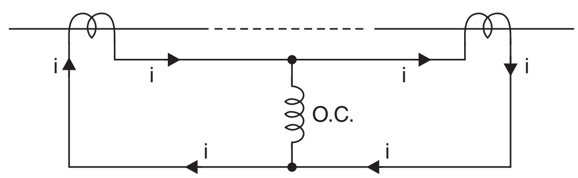


Fig. 14.23 Simple differential protection.

The dotted line represents the equipment to be protected which may be a transformer, an alternator, a bus etc. Two suitable CTs are connected in series as shown with the help of pilot wires. The relay operating coil is connected between the mid-points (equipotential points) of the pilot wire. The voltage induced in the secondary of the CTs will circulate a current through the combined impedance of the pilot wires and the CTs. In case the operating coil is not connected between the equipotential points (which are infinite), there will be difference current (sufficient during through fault condition) through the operating coil of the relay and this may result in maloperation of the relay. When the operating coil of the relay is not connected between the equipotential points, even though the current through each CT is same, the burden on the two CTs is unequal. This causes the heavily loaded CTs to saturate during through fault, thereby causing dissimilarity in the characteristics of the two CTs which results in maloperation of the relay.

Consider Fig. 14.24 for the operating principle of a differential relay. It is expected of the scheme that in case of a fault in the circuit between the two CTs the relay must operate and in case the fault is outside this zone the relay should not operate. Such protection is known as unit protection. When the fault is outside the zone of protection, it is known as external fault or through fault.

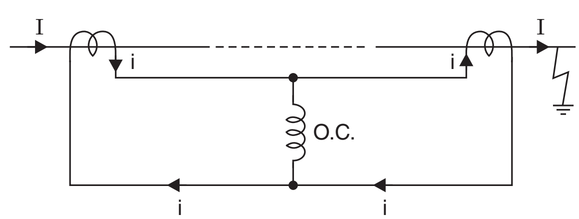


Fig. 14.24 Differential protection during a through fault.

Consider the scheme in Fig. 14.24 for a through fault. The current flowing through the primaries of the two CTs is same (whether the system is fed from one end or both the ends). If the two CTs behave identically for all fault currents, the secondary currents are of the same magnitude and phase. The difference current, therefore, being zero through the operating coil, the relay does not operate. This is a desirable feature.

For an internal fault, consider Fig. 14.25 (*a*) when the circuit is fed from one end and Fig. 14.25 (*b*) when the circuit is fed from both the ends. It can be seen that in both the cases, a current will flow through the operating coil of the relay and it will operate. This form of protection is known as Merz-Price protection.

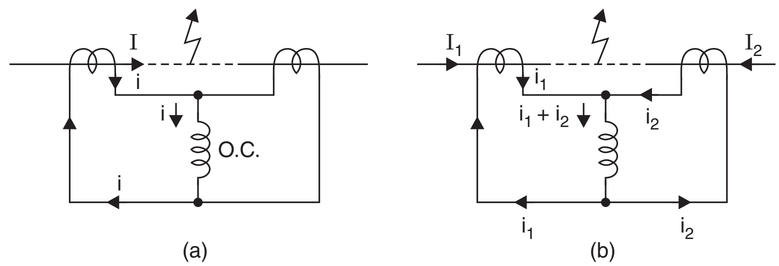


Fig. 14.25 Differential protection for an internal fault: (*a*) fed from one end; and (*b*) fed from both the ends.

The above form of protection was assumed on the fact that the two CTs used were identical. But in practice this is not true. Current transformers of the type normally used do not transform their currents so accurately under transient conditions especially. This is true because the short circuit current is offset, *i.e.*, it contains d.c. components. Suppose the two CTs under normal conditions differ in their magnetic properties slightly in terms of different amounts of residual magnetism or in terms of unequal burden on the two CTs, one of the CTs will saturate earlier during short circuit currents (offset currents) and thus the two CTs will transform their primary current differently even for a through fault condition. This effect is more pronounced especially when the scheme is used for the protection of power transformers.

To accommodate these features, Merz-Price protection is modified by biasing the relay. This is commonly known as biased differential protectior or percentage differential protection and is shown in Fig. 14.26.

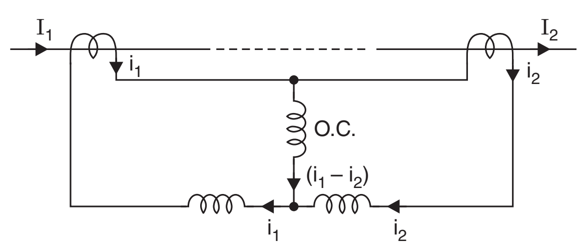


Fig. 14.26 Percentage differential protection.

The relay consists of an operating coil and a restraining coil. The operating coil is connected to the mid-point of the restraining coil. The operating current is a variable quantity because of the restraining coil. Normally, no current flows through the operating coil under through fault condition, but owing to the dissimilarities in CTs, the differential current through the operating coil is $(i_1 - i_2)$ and the equivalent current in the restraining coil is $(i_1 + i_2)/2$.

The torque developed by the operating coil is proportional to the ampere-turns, *i.e.*, $T_0 \propto (i_1 - i_2)n_0$, where n_0 is the number of turns in the operating coil. The torque due to restraining coil $T \propto (i_1 + i_2)\,\frac{n_r}{2}$, where n_r is the number of turns in the restraining coil. At balance

$$(i_1 - i_2)n_0 = (i_1 + i_2)\,\frac{n_r}{2}$$

or
$$\frac{i_1 - i_2}{(i_1 + i_2)/2} = \frac{n_r}{n_0}$$

The operating characteristic is shown in Fig. 14.27.

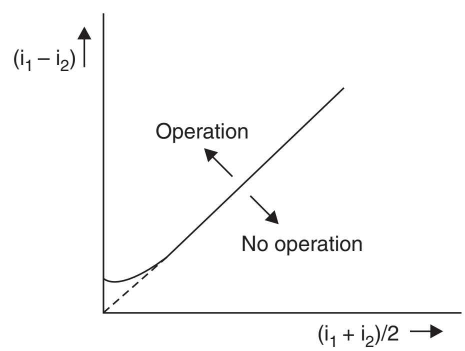


Fig. 14.27 Operating characteristic of a percentage differential relay.

It is clear from the characteristic that except for the effect of the control spring at low currents, the ratio of the differential operating current to the average restraining current is a fixed percentage. This is why it is known as percentage differential relay.

The differential relays described above are known as current balance relays. Another class of relays are the voltage balance relays. Here the CTs at the two ends are connected in opposition as shown in Fig. 14.28.

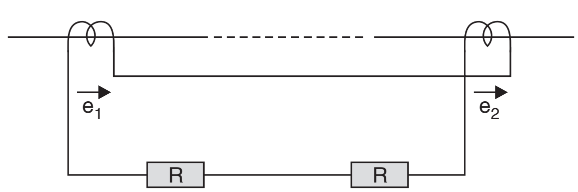


Fig. 14.28 Voltage balance protection.

The relays are connected in series with the pilot wires. The relative polarity of the CTs is such that there is no current through the relays under balanced or through fault conditions. The requirement of CT is that they should induce voltages in the secondary linearly with respect to the current. Since the magnitude of the fault current is very large, in order that the voltage should be a linear function of such large currents the CTs should be air-cored.

The term 'pilot' means the interconnecting channel between the two ends of the equipment or a circuit over which information from one end to the other can be conveyed. Three different types of such channels are in use:

1. Wire pilot.
2. Carrier current pilot.
3. Microwave pilot.

The first one is in the form of a two-wire line, such as a telephone line. The second one for the protective relaying is one in which low voltage high frequency (30 KHz to 200 KHz) currents are transmitted along the conductor of the line (line to be protected) at one end and received at the other end, the earth or ground wire generally acting as the return conductor. A microwave pilot is an ultra high frequency radio system operating above 900 MHz.

A wire pilot is generally economical for distances up to 10 to 15 kms. For a 3-phase transmission line, a summation transformer may be used to combine the three currents and thus use two pilot wires instead of six (two wires per phase). For more than 15 kms, carrier current pilot usually becomes more economical. When for technical reasons (number of channels) carrier channels cannot be used, microwave pilots are used. Pilot wire current differential scheme is normally used for the protection of generators, transformers, buses etc., where the length of the wire required is small. The reasons for not using the current differential relay for transmission line protection are:

(*i*) Cost of pilot wires.

(*ii*) The large voltage drop in the pilot wires requiring better insulation.

(*iii*) The pilot currents and voltages would be excessive for pilot circuits rented from a telephone company.

(*iv*) The likelihood of improper operation owing to C.T. inaccuracies under heavy loading.

14.9 FEEDER PROTECTION

The word feeder here means the connecting link between two circuits. The feeder could be in the form of a transmission line, short, medium or long, or this could be a distribution circuit. The various methods of protecting the feeders are:

1. Overcurrent protection.
2. Distance protection.
3. Pilot relaying protection.

Of these, overcurrent protection is the simplest and cheapest form of protection. It is most difficult to apply and needs readjustment, should a change in the circuit occur. This may even have to be replaced depending upon the circuit conditions.

Overcurrent relaying for distribution circuits besides being simple and cheap provides the following advantages:

(*i*) Very often the relays need not be directional and hence no a.c. voltage source is required.

(*ii*) Two-phase and one earth fault relays are required for the complete protection of three-phase circuits as shown in Fig. 14.29(*a*).

Where distance protection is costly, overcurrent protection is used for phase and ground faults on station service, electric utilities (distribution circuits) and on some subtransmission lines. Overcurrent protection is normally used as back up protection where the primary protection is provided with distance schemes.

The discrimination using overcurrent protection is achieved in the following ways:

(*i*) Time graded system.

(*ii*) Current graded system.

(*iii*) Time-current graded system.

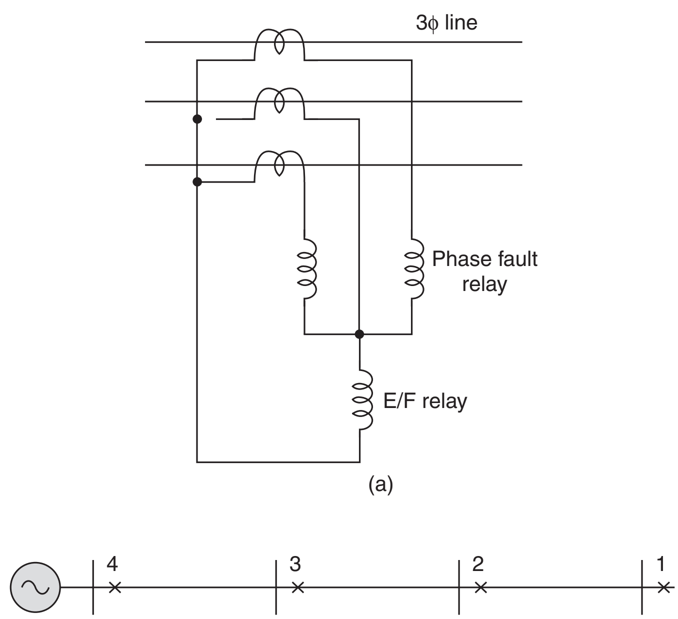


Fig. 14.29 (*a*) Two-phase relays and one earth fault relay.
(*b*) A radial feeder with relays.

Time Graded System

The selectivity is achieved based on the time of operation of the relays. Consider a radial feeder in Fig. 14.29(*b*). The feeder is being fed from one source and has three substations indicated by the vertical lines. The crosses represent the location of the relays. The relays used are simple overcurrent relays. The time of operation of the relays at various locations is so adjusted that the relay farthest from the source will have minimum time of operation and as it is approached towards the source the operating time increases. This is the main drawback of grading the relays in this way because it is required that the more severe a fault is, lesser should be the operating time of the relays whereas in this scheme the operating time increases. The main application of such a grading is done on systems where the fault current does not vary much with the location of the fault and hence the inverse characteristic is not used.

Current Graded System

This type of grading is done on a system where the fault current varies appreciably with the location of the fault. This means as we go towards the source the fault current increases. With this if the relays are set to pick at a progressively higher current towards the source, then the disadvantage of the long time delay that occurs in case of time graded systems can be partially overcome. This is known as current grading.

Since it is difficult to determine the magnitude of the current accurately and also the accuracy of the relays under transient conditions is likely to suffer, current grading alone cannot be used. Usually a combination of the two gradings, *i.e.*, current time grading is used.

Time-Current Grading System

This type of grading is achieved with the help of inverse time overcurrent relays and the most widely used is the IDMT relay. The other inverse characteristics, *e.g.*, very inverse or extremely inverse are also employed depending upon the system requirements. If the IDMT relays are slow at low values of overloads, extremely inverse relays are used and if the fault current reduces substantially as the fault location moves away from the source, very inverse type of relays are used.

Selection of Current Setting

For proper coordination between various relays on a radial feeder, the pick up of a relay should be such that it will operate for all short circuits in its own line and should provide back up protection for short circuits in immediately adjoining line. For back up protection setting, it should be equal to the value of the current when the fault is at the far end of the adjoining section with minimum generation connected to the system. A 3-phase fault under maximum generation gives the maximum fault current and line-to-line fault under minimum generation gives the minimum fault current. The relay must respond between these two extreme limits. On a radial system the current setting of the relay farthest from the source should be minimum and it goes on increasing as we go towards the source. According to Indian Standard specifications the operating value should exceed 1.3 times the setting, *i.e.,*

$$\text{Min short circuit current} \geq 1.3 I_{\text{setting}}$$

Selection of Time Setting

For proper coordination between various relays on a radial feeder the operating time of the relay farthest from the source should be minimum and it should increase as we go towards the source. Referring to Fig. 14.29(*b*), if the time of operation of relay 1 is say T_1, that of the relay 2 say T_2 then $T_2 = T_1 + t$, where t is the time step between successive relays and consists of the time of operation of C.B. at 1, over-travel of relay at 2 and factor of safety time. Here over-travel of relay at 2 means, the travel of the relay at 2 due to inertia of the moving system of the relay even after the fault at location 1 is removed. A suitable value of over-travel is 0.1 sec. Similarly factor of safety time is taken as 0.1 sec. The time grading should be done at the maximum fault currents because at lower values it will automatically have a higher selectivity as the curves are more inverse in that range.

The characteristics of the various IDMT relays used on system in Fig. 14.29(*b*) on a simple graph should look like the ones in thick lines in Fig. 14.30.

If suppose characteristic 2 intersects 1 at a current I, this means that if the current exceeds I amperes relay at 2 will operate faster than 1 which is not desirable if the fault is in the zone of relay 1.

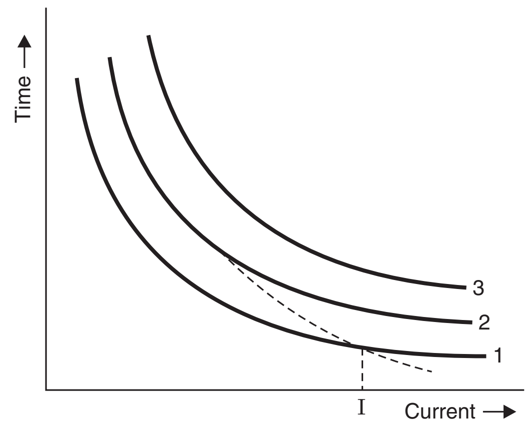
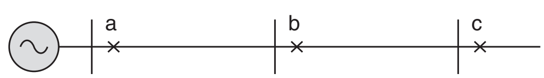


Fig. 14.30 Time-current characteristics for the radial system.

Therefore, for proper coordination the characteristics should not intersect.

Example 14.3: It is required to provide time-current grading for the following system:



Relay point	*CT ratio*	*Fault current*
a	400/5	6000 amps
b	200/5	5000 amps
c	200/5	4000 amps

Use 2.2 sec IDMT characteristic of Example 14.1.

Solution: The secondary current with maximum fault current at c is

$$\frac{4000}{40} = 100 \text{ amps}$$

If 100% setting is used, the PSM $= \frac{100}{5} = 20.$

Corresponding to this the operating time is 2.2 sec. If TMS = 0.1, the operating time of this relay = 2.2 × 0.1 = 0.22. To achieve discrimination between relay at b and at c when the fault takes place just before c or just after c when there is no change in fault current. Let the discriminating time between relays be 0.5 sec which includes the time for operation of relay at c, the operation time of C.B. at c and over-travel of relay b. The operating time of relay at b when fault takes place near c will be 0.22 + 0.5 = 0.72 sec.

The secondary current in the relay at location b when fault takes place near c will be

$$\frac{4000}{40} = 100 \text{ amps}$$

Assuming the current setting of relay 125%, the relay operating current will be 5 × 1.25 = 6.25 amp.

$$\therefore \qquad \text{PSM} = \frac{100}{6.25} = 16$$

The operating time from the curve is 2.5 sec approx. The operating time of relay at b when graded w.r.t. relay at c is 0.72 sec.

$$\therefore \qquad \text{TMS of relay at } b = \frac{0.72}{2.5} = 0.29$$

When fault is near b, the PSM with operating current 6.25 amp is

$$\text{PSM} = \frac{5000}{6.25 \times 40} = 20$$

The operating time corresponding to this PSM = 2.2 sec.

∴ Actual operating time of relay at b = 2.2 × 0.29 = 0.638 sec

Since C.T. ratio at a is 400/5 which is high as compared to relay at b, therefore, the current discrimination is inherent. Let the per cent setting of relay at a be 125%.

∴ the PSM of relay a when fault takes place near b = $\dfrac{5000}{6.25 \times 80} = 10$

The operating time is 3 sec corresponding to PSM = 10 whereas the operating time of relay at a with respect to b will be 0.638 + 0.5 = 1.138 sec.

$$\therefore \qquad \text{TMS} = \frac{1.138}{3} = 0.379$$

When fault is near a, the PSM will be

$$\frac{6000}{6.25 \times 80} = 12$$

The operating time corresponding to PSM = 12 is 2.6 sec.

∴ The actual operating time of relay at a will be

2.6 × 0.379 = 0.985 sec. **Ans.**

Protection of Parallel Feeders

Refer to Fig. 14.31, where two feeders are connected in parallel to increase the transmission capacity. The feeders are again fed from one end.

It is required that in case of a fault on any one of the feeders, that feeder should be isolated from the supply and the load must receive power through the other healthy feeder.

In case of a fault on any one feeder, say 2, the current will be fed to the fault as shown in Fig. 14.31 by the arrows. It is to be seen that as far as the relays near the source are concerned, the direction of current is same as the normal direction of the current, whereas the direction of the current in the relay near the load end of the faulty feeder is reversed. Therefore, for proper coordination the relays near the source-end are non-directional relays whereas relays near the load-end are directional relays. The direction of the current for which the directional relays will operate is indicated by the corresponding arrow heads. In this case as soon as fault takes place in feeder 2, the directional relay in feeder 2 will operate first; thereby the current in feeder 1 corresponds to load current and after some time the non-directional relay in feeder 2 will operate, thereby isolating feeder 2 from the source.

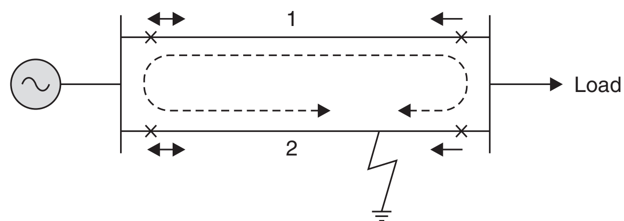


Fig. 14.31 Protection of parallel feeders.

Protection of Ring Mains

As shown in Fig. 14.32, four substations are inter-connected and fed through one source. The relays at A and B are non-directional relays. The coordination can be achieved by opening the ring at A and considering the system as a radial feeder connected to one source (Fig. 14.32(*b*)).

The relays used are directional overcurrent relays with the relay near end A having minimum time of operation. Next open the ring at B as shown in Fig. 14.32(*c*).

The total protection scheme consists of six directional overcurrent relays and two non-directional overcurrent relays. The scheme is shown in Fig. 14.32(*d*).

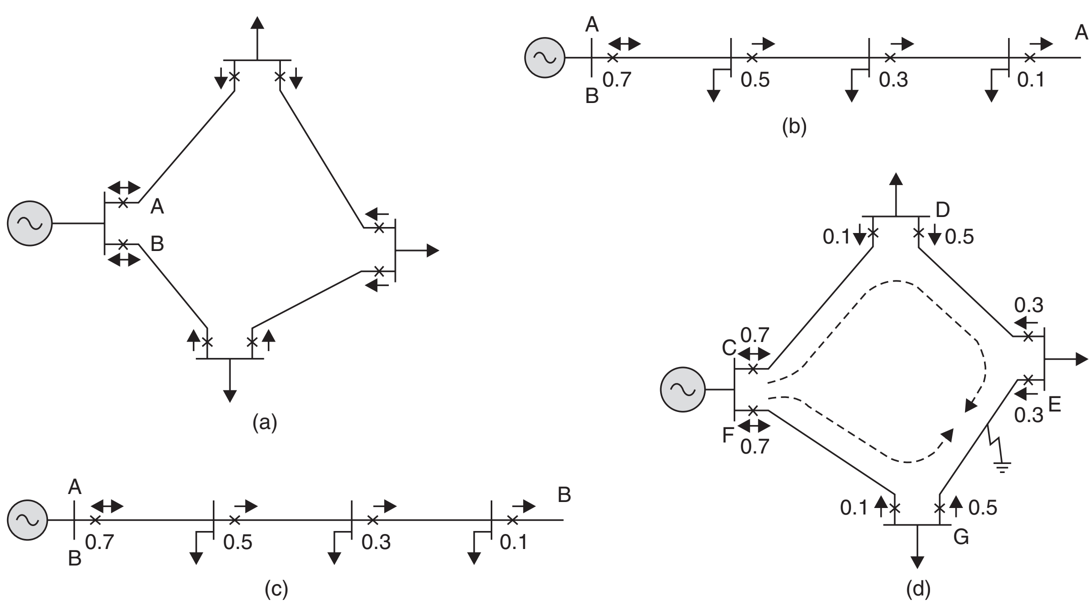


Fig. 14.32 (*a*) Ring mains to be protected; (*b*) Ring opened at *A* and spread; (*c*) Ring opened at *B* and spread; and (*d*) Ring mains with protective scheme.

Consider a fault as shown in Fig. 14.32(*d*). The fault will be fed as shown by long arrows. The relays at locations *CDE* and *FG* will start moving. The relay at *E* will operate first as this has minimum operating time out of these relays; thereby after a time of 0.3 sec. The relays at

C and *D* will reset as the fault current ceases to flow through these relays. Out of relays *F* and *G, G* has smaller operating time and, therefore, relay at *G* will operate first; thereby isolating the feeder *GE* from the source. So far, we have considered the feeders being fed from one end only. In case the flow of power could be reversed at will by connecting more than one sources, it is necessary to consider each feeder separately without any reference to the others and any one form of protection to be described next should be used.

14.10 DISTANCE PROTECTION

Whenever over-current relaying is found slow or is not selective distance protection should be used. Since the fault currents depend upon the generating capacity and system configuration, the distance relays are preferred to the overcurrent relays.

Consider Fig. 14.33 which consists of two line sections *AB* and *CD*; it is desired to provide distance protection scheme.

The protection scheme is divided in three zones. Say for relay at *A*, the three zones are Z_{1a}, Z_{2a} and Z_{3a}. Z_{1a} corresponds to approximately 80% length of the line *AB* and is a high speed zone. No intentional time lag is provided for this zone. The ordinate shown corresponding to Z_{1a} gives the operating time in case the fault takes place in this zone. It is to be noted here that the first zone is extended only up to 80% and not 100% length of the line as the relay impedance measurement will not be very accurate towards the end of the line especially when the current is offset.

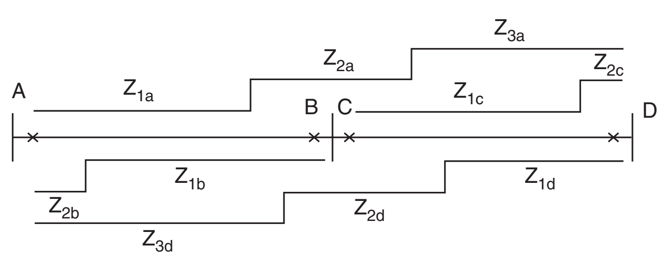


Fig. 14.33 3-zone protection.

Second zone Z_{2a} for relay at *A* covers remaining 20% length of the line *AB* and 20% of the adjoining line. In case of a fault in this section relay at *A* will operate when the time elapsed corresponds to the ordinate Z_{2a}.The main idea of the second zone is to provide protection for the remaining 20% section of the line *AB*. In case of an arcing fault in section *AB* which adds to the impedance of the line as seen by the relay at *A*, the adjustment is such that the relay at *A* will see that impedance in second zone and will operate. This is why the second zone is extended into the adjoining line. The operating time of the second zone is normally about 0.2 to 0.5 second.

The third zone unit at *A* provides back up protection for faults in the line *CD, i.e.*, if there is a fault in the line *CD* and if for some reason the relay at *C* fails to operate then relay at *A* will provide back up protection. The delay time for the third zone is usually 0.4 to 1.00 sec.

In case the feeder is being fed from both the ends and say the fault takes place in the second zone of line AB (20% of the line AB), the relay at B will operate instantaneously (because it lies in the first zone of BA) whereas the fault lies in the second zone of the relay at A. This is undesirable from stability point of view and it is desirable to avoid this delay. This is made possible when the relay at B gives an intertrip signal to the relay at A in order to trip the breaker quickly rather than waiting for zone-2 tripping.

Impedance Relay Protection

It has already been discussed that an impedance relay responds to the impedance seen by the relay. If the impedance seen by the relay is less than its setting the relay operates. The impedance relays are non-directional relays and, therefore, need a directional relay with them. The characteristic of the impedance relays with a directional unit for 3-zone protection is shown in Fig. 14.34(a). While designing the relays; it is usual to make maximum torque angle τ smaller than the impedance angle θ of the line so that the effect of the arc resistance is reduced. The contact circuit for a 3-zone impedance protection is shown in Fig. 14.34(b).

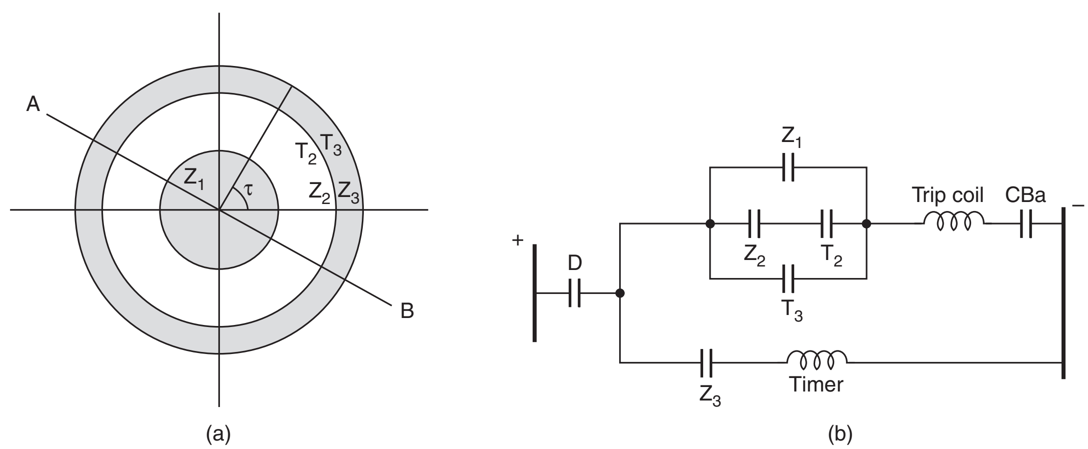


Fig. 14.34(a) Impedance characteristics for 3-zone protection; (b) Contact circuit for 3-zone impedance relay.

The parallel lines in Fig. 14.34(b) represent the contacts of the various units, D-directional unit, Z_1, Z_2, Z_3 the 3-zone units, and T_2, T_3 the timing units. T_2 and T_3 are operating times for zones 2 and 3 respectively.

Since Z_3 unit starts when the fault lies in any of the zones 1, 2 or 3 as the impedance of the fault will be less than Z_3, Z_3 is the starting unit and, therefore, the time unit is placed in series with Z_3 unit.

Now for a fault in zone 1, all the three units will start but since the operating time of unit 1 is smallest, this will operate and the faulty section will be isolated from the source. In case the fault is in second zone, the units Z_2 and Z_3 will start but unit Z_2 will operate in time T_2 and isolate the faulty section from the source.

Reactance Relay Protection

A reactance relay responds only to the reactance component of the impedance. A reactance relay is a non-directional relay and the directional unit of the type used along with the impedance relay cannot be used for the reasons discussed earlier. A mho relay is used as the starting relay along with the reactance relay. Fig. 14.35(a) shows the characteristics of the reactance relays for 3-zone protection.

The mho unit prevents the operation of the reactance units under load conditions. Also it gives protection for the 3rd zone of the scheme.

The contact arrangement for 3-zone protection using reactance relays is given in Fig. 14.35(b).

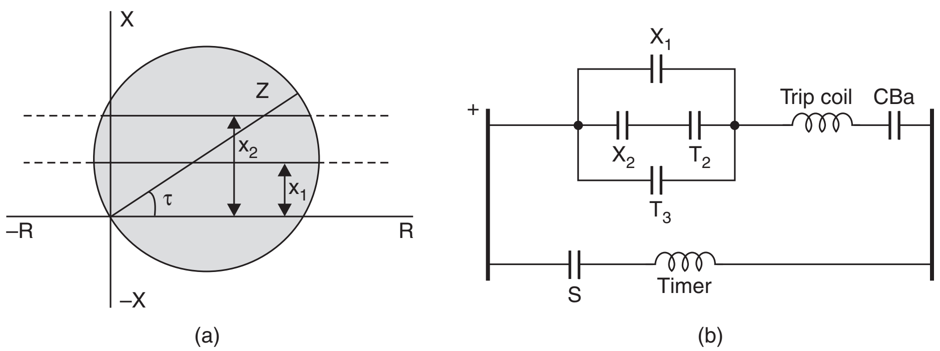


Fig. 14.35 (*a*) Reactance relay characteristic for 3-zone protection; (*b*) Contact circuit for 3-zone reactance relay.

The operation is explained as follows:

The contact circuit is connected between the d.c. supply terminals. In case the fault takes place in the first zone, all the three units X_1, X_2 and S start. Since the operation of X_1 takes the least time, contact X_1 is closed. CBa, the auxiliary contact of the circuit breaker, is a normally closed contact; therefore, trip coil gets energized which in turn operates the circuit breaker, thus isolating the faulty section of the line from the source. Similarly, the operation of the contact circuit can be explained if the fault is in zone 2 or 3.

Mho Relay Protection

Mho relay, inherently being a directional relay, does not need additional unit for the purpose. Fig. 14.36(a) shows the characteristics for 3-zone protection. The contact arrangement is shown in Fig. 14.36(b). The operation of this circuit is similar to the circuits for reactance relays.

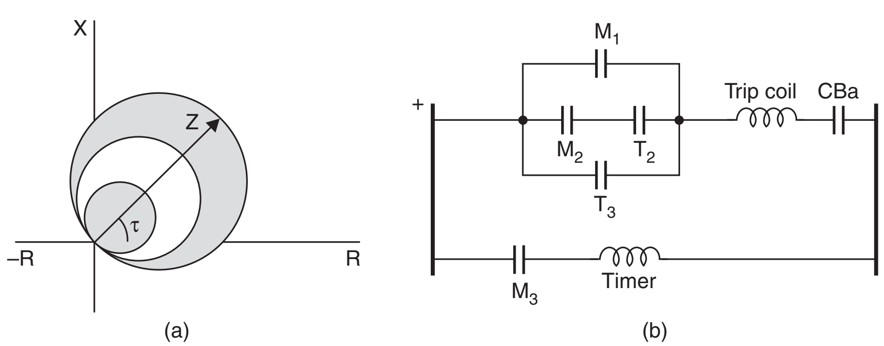


Fig. 14.36 (*a*) Mho relay characteristic for 3-zone protection; (*b*) Contact circuit for 3-zone mho relay.

Power Swings

Under steady state conditions the rotor axis is fixed with respect to the stator reference axis, whereas under disturbed condition which may be due to a fault or a sudden change in load, the rotor swings around the final steady state value, if the fault is not severe and the sudden change in load is not very large as to warrant out of step operation of the synchronous machines. During swinging the rotor angle changes and hence the current changes assuming the voltages to be practically constant. This results in change in impedance. Since the two quantities, voltage and current, are fed to a distance relay, the impedance as seen by the relay keeps on changing To study the variation of impedance with change in rotor angle, we consider the system of Fig. 14.37.

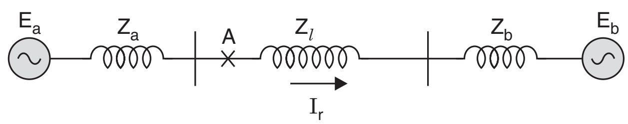


Fig. 14.37 Two machine system connected through a line.

Say, the distance relay is located at A and power flows from E_a to E_b; then, if I_r is the relay current and E_r is the relay voltage,

$$Z_r = E_r/I_r \tag{14.16}$$

where Z_r is the impedance seen by the relay. From Fig. 14.37,

$$I_r = \frac{E_a - E_b}{Z_a + Z_b + Z_l} = \frac{E_a - E_b}{Z_T} \tag{14.17}$$

where $Z_T = Z_a + Z_b + Z_l$ and since $E_r = E_a - I_r Z_a$,

$$\therefore \quad Z_r = \frac{(E_a - I_r Z_a)Z_T}{E_a - E_b} = \frac{E_a Z_T}{E_a - E_b} - \frac{I_r Z_a}{E_a - E_b} Z_T \tag{14.18}$$

Let δ be the angle between E_a and E_b such that E_a leads E_b by an angle δ.

$$Z_r = \frac{1}{1-(E_b/E_a)} \cdot Z_T - Z_a = \frac{Ke^{j\delta}}{Ke^{j\delta} - 1} . Z_T - Z_a \tag{14.19}$$

where $K = \left| \dfrac{E_a}{E_b} \right|$.

Here K is real and can have values equal to, less than or greater than unity.

When $K = 1$, the expression for Z_r becomes

$$Z_r = \frac{e^{j\delta}}{e^{j\delta} - 1} Z_T - Z_a = \left(\frac{Z_T}{2} - Z_a \right) - j\frac{Z_T}{2} \cot \frac{\delta}{2} \tag{14.20}$$

Equation (14.19) represents a family of circles with K as parameter and δ as the variable. The centres of these circles lie on the straight line indicated by Z_T on the R-X diagram. For all values of K greater than unity, the centres of the swing impedance loci circles will be located in the first quadrant and for $K < 1$ the centres will lie in the third quadrant and for $K = 1$, the centres lie on the straight line which also happens to be the perpendicular bisector of the total impedance Z_T.

It can also be shown from equation (14.19) that for a particular value of δ and different values of K, the swing impedance loci are circular arcs for all values of δ except δ = 180° when the locus is a straight line. The two families of swing impedance loci are orthogonal to each other and form the swing impedance chart. Fig. 14.38 shows the swing impedance loci corresponding to K as parameter and δ as variable and we are mostly concerned with this only.

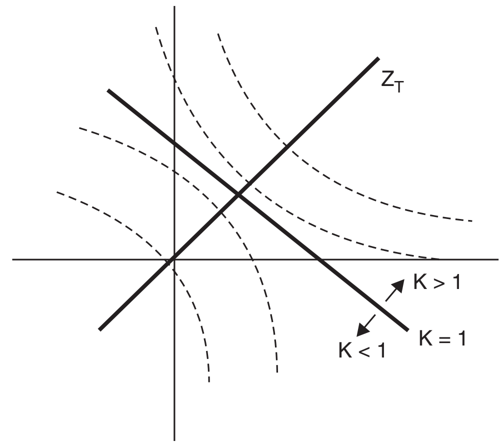


Fig. 14.38 Swing impedance loci.

The phasor drawn from the origin, *i.e.*, the relay location to any point on the impedance chart will represent the impedance seen be the relay. Since the p.u. nodal voltages of the system are practically same, $K = 1$ and hence the effect of power swing is generally seen from the swing impedance locus corresponding to $K = 1$ as illustrated in Fig. 14.39.

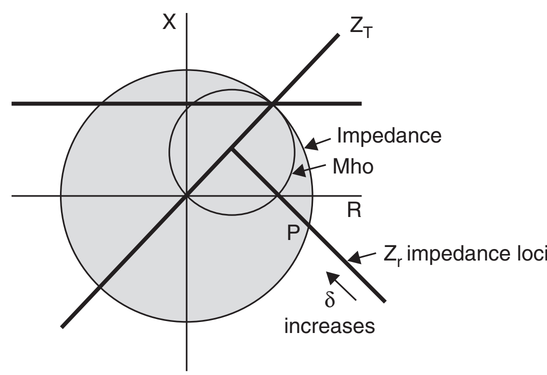


Fig. 14.39 Impedance seen by a distance relay.

As angle δ increases, the impedance as seen by the relay will move along the swing impedance loci during swinging. For certain value of δ, the Z_r line may intersect the impedance characteristic as at P and the relay may trip as the impedance seen by the relay lies within the operating zone even when it is a power swing rather than a fault. It can be seen from Fig. 14.39 that a mho relay has least tendency for operation during swinging as compared to an impedance relay which is highly prone to this operation. It can also be seen from Fig. 14.39 that to avoid maloperation of the distance relays during a swing the angular range of its operation characteristics should be reduced which normally is obtained by using elliptic characteristics (with major axis along Z_T) rather than circular. The elliptic characteristics have been discussed in section 14.16.

Applications of Distance Relays

Since the resistance of the ground is a variable quantity, a ground fault relay should be independent of the resistance. Consequently, reactance relays are normally preferred for ground fault relaying.

For phase fault protection each type has certain advantages and disadvantages. For short transmission lines reactance type relay is used because more of the line can be protected at high speed. This is due to the fact that a reactance relay is practically independent of the arc resistance which may be large compared with the line impedance.

The mho type relay is most suited for long lines where especially there are more chances of severe synchronizing power surges on the system. This does not need any additional equipment to prevent tripping during these surges whereas a reactance relay does need. The mho relay occupies the least space on an *R-X* diagram for a given line section and is, therefore, least affected by abnormal system conditions except the line faults. Since mho relay is most affected by arc resistance, it is used for long lines. This relay is more reliable than the other two because the relay has only one set of contacts.

The impedance relay is less affected from synchronizing power surges as compared to reactance relay and also this relay is less affected from arc resistance as compared with the mho relay. The impedance relay is, therefore, used for protecting medium length transmission lines.

The above are the basic principles for the selection of the distance relays. These need not necessarily be always true. One should use the relays which are best suited for a particular system. This comes only through experience, which, many a time, is a guiding factor for suitable selection of the relay.

14.11 GENERATOR PROTECTION

The following are the various types of faults that can occur on an alternator:

Stator faults: Under this the faults possible are: (*i*) phase-to-phase faults, (*ii*) phase-to ground faults, and (*iii*) inter-turn faults. The danger of these faults is that they may lead to damage the laminations due to heat generated at the point of fault and hence need partial reinsulation and rebuilding of the core which is very costly and time consuming. The phase-to-phase and interturn faults are less common as compared to the phase-to-ground faults.

Rotor faults: There may be ground faults or short between the turns of the field winding, caused by the severe mechanical and thermal stresses acting upon the winding insulation. The field system is not grounded normally and, therefore, a simple line-to-ground fault does not give any fault current. A second fault to earth will short circuit part of the field winding and may thereby produce an unsymmetrical field system which gives rise to unbalanced forces on the rotor and results in excess pressure and bearings and shaft distortion if the fault is not removed quickly. It is, therefore, necessary to know the existence of the first occurrence of earth fault so that corrective measures are taken before the second fault is allowed to occur.

Abnormal running conditions: These conditions involve: (*i*) unbalanced loading, (*ii*) overloading, (*iii*) overspeed, (*iv*) overvoltage, (*v*) failure of prime mover, and (*vi*) loss of excitation.

The unbalanced loading results in circulation of negative sequence currents in the stator winding which gives rise to a rotating magnetic field. This field rotates at double the synchronous speed with respect to the rotor and induces a voltage of double the frequency in the rotor conductor. If the degree of unbalance is large these currents will over-heat the rotor stamping and the field winding.

Overloading of the stator will over-heat the stator winding which may damage the insulation depending upon the degree of overloading. In case of hydraulic generators a sudden loss of load results in overspeeding of the generator because the water flow to the turbine cannot be stopped quickly because of mechanical and hydraulic inertia.

Over-voltages are caused by overspeeding of the generator or due to faulty operation of the voltage regulator.

The failure of prime mover results in motoring of the synchronous generator and thus draws power from the system in case it is not a single generator system. This may lead to a dangerous mechanical condition, if allowed to persist.

The loss of excitation of a generator may result in loss of synchronism and slightly increased generator speed since the power input to machine remains unchanged. The machine, therefore, behaves as an induction generator and draws its exciting current from the system which is equal to its full load rated current. This leads to overheating of the stator winding and rotor body because of currents induced in the rotor body due to slip speed. This situation should not be allowed to continue for long and corrective measures in terms of restoration of excitation or disconnection of alternator, should be taken. The loss of excitation may also lead to pole slipping conditions which result in voltage reduction for outputs above half the rated load.

Rotor Protection

Figure 14.40 shows the modern method of protecting the rotor against earth faults or open circuits. A small power supply is connected to the positive pole of the field circuit. A fault detecting relay and a high resistance to limit the current are connected in series with this circuit. A fault at any point on the field circuit will pass a current of sufficient magnitude through the relay to cause operation. The earth fault relays are instantaneous and are connected to the alarm circuit for indication as a single ground fault does not require immediate attention to the set.

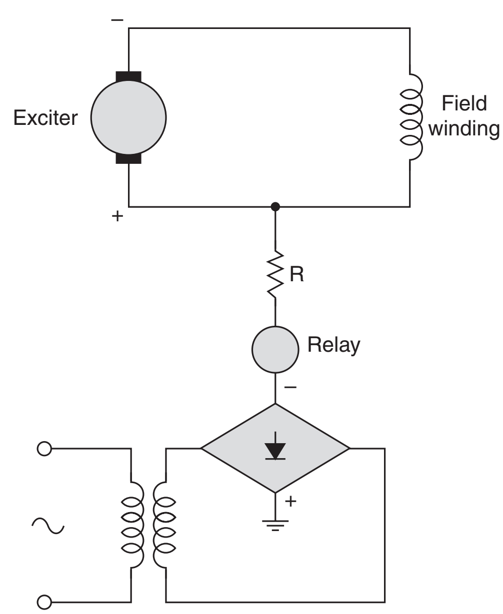


Fig. 14.40 Rotor earth fault protection.

Unbalanced Loading

Figure 14.41 shows the protection of alternators against negative phase sequence currents. The negative sequence current segregating network is used, the output of which is proportional to the generator negative phase sequence current and is fed into a relay with an inverse square law characteristic, *i.e.*, $I^2t = K$ or $t \propto 1/I^2$. The pick up and time delay adjustments are provided such that the relay characteristic can be chosen to match closely any machine characteristic. The relay is connected to trip the generator main breaker. Sometimes an auxiliary alarm relay is provided which gives warning when the maximum continuous permissible negative phase sequence current is exceeded. The relay normally used is an IDMT relay.

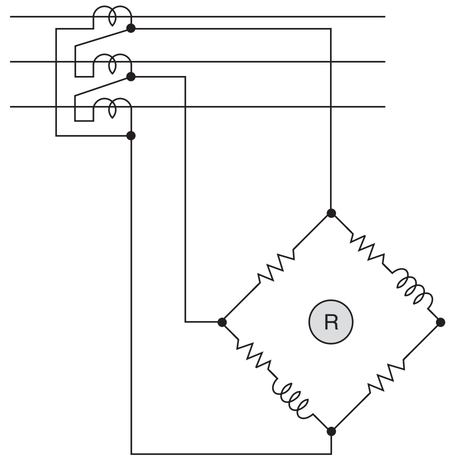


Fig. 14.41 Negative sequence relay.

Overload Protection

The overload operation of the alternator results in overheating of the stator winding. Normally an overcurrent relay with time delay adjustment should serve the purpose. But because the temperature of the winding not only depends upon the overloading but also on the state of the cooling system, if the cooling system fails, the temperature of the stator winding may reach dangerous values even though the alternator is not fully loaded. Also, if an overcurrent relay is used, it has got to be discriminated with respect to other overcurrent relays on the system. Since we are using this relay at the source, it will be the slowest in operation and hence poses a serious problem to the stator windings. The most reliable method will, therefore, be one which senses the temperature of the winding and depending upon the temperature gives an operating signal. The temperature detector coils in the form of thermistors or thermocouples are embedded at various points in the stator winding to give an indication of the temperature condition.

Generators rated below about 50 MW are not provided with temperature detecting devices for overload protection. They are provided with thermal relays which use bimetallic strips as overload sensing device. The heating and cooling characteristics of these strips are matched with the heating and cooling characteristics of the machine to be protected. The thermal relay will also not respond to overheating due to failure of the cooling system.

Overspeed Protection

The speed goes up whenever there is sudden loss of load *i.e.*, there is sudden loss in output of the generator. This reduction in output can be detected using a wattmetric relay at the generator terminals which operates instantaneously to close its contacts. A second relay monitors the steam input to the turbines at a chosen stage and the contacts are held closed when the steam pressure is in the full load region.

Over-voltage Protection

This protection is normally provided for hydroelectric and gas turbine generators and not for steam turbine generators. The protection used is an a.c. over-voltage relay which has a pick-up value of 110% of the normal value and operates instantaneously at about 130% to 150% of the rated voltage. The relay unit should be compensated against the frequency and it should be energized from a potential transformer other than the one used for the automatic voltage regulator. The operation of the relay introduces resistance in the generator or exciter field circuit and if over-voltage still persists, the main generator breaker and the generator or exciter field breaker is tripped.

Failure of Prime Mover

Whenever a prime mover fails, the generator connected to the system starts motoring; thereby it draws electrical power from the system and drives the prime mover. The power taken by the generator under such condition is very low being about 2% for the turbo-alternators and 10% for the engine driven sets. The power factor of the current depends upon the excitation level and hence may be either leading or lagging. The wattmetric relay with directional characteristic is used. The relay must be associated with a time delay relay to prevent tripping due to power swings.

Loss of Excitation

Very large alternators cannot be allowed to run asynchronously for long as the relative motion between the stator field and the rotor induces large currents in the rotor body and, therefore, there is high rate of heating of the rotor surfaces and the loss of excitation scheme is arranged to trip after certain time delay. The protection scheme uses an offset mho relay operated from a.c. current and voltage at the generator terminals. The relay setting is so arranged that the relay operates whenever the excitation goes below a certain value and the machine starts running asynchronously. Fig. 14.42 shows the relay connection and Fig. 14.43 shows the various characteristics on *R-X* diagram.

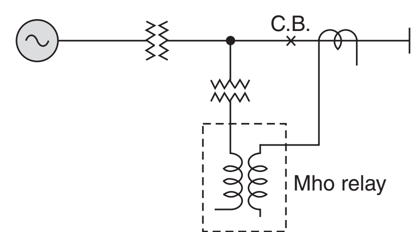


Fig. 14.42 Relay connections for loss of excitation.

It is seen that the impedance as seen by the relay during loss of excitation will swing into the relay characteristic and thus the relay will operate. The loci of impedance for system fault and for power swings is also shown in Fig. 14.43 and it can be seen that for these conditions the relay will not operate.

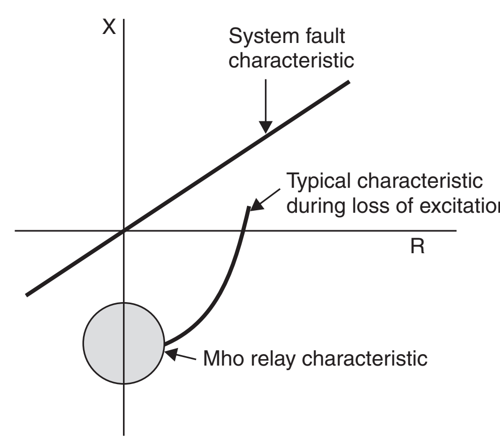


Fig. 14.43 Loss of excitation characteristic.

Under normal operating condition when a synchronous alternator is connected to the grid it supplies lagging reactive power to the system in addition to the active power and the p.f. is lagging and the impedance of the alternator as seen by the relay lies in the first quadrant of the R-X diagram. However, due to failure of excitation, the synchronous alternator now works as an induction generator and it draws lagging reactive power from the grid, of course it supplies active power to the grid and hence it operates at leading p.f. As a result of this, the impedance of the induction generator as seen by the relay shifts into the fourth quadrant of the R-X diagram and this impedance swings into off-set mho relay characteristic as shown in Fig. 14.43 and the relay will operate.

Stator Protection

It is the general practice to provide differential protection for generators above 10 MVA. This form of protection is most suited and should be used if justified economically.

If all the six terminals of a star connected 3ϕ generator are available, the scheme of percentage differential relay shown in Fig. 14.44 (*a*) is provided.

It can be seen that for an external fault the relay does not operate and for an internal fault it does operate.

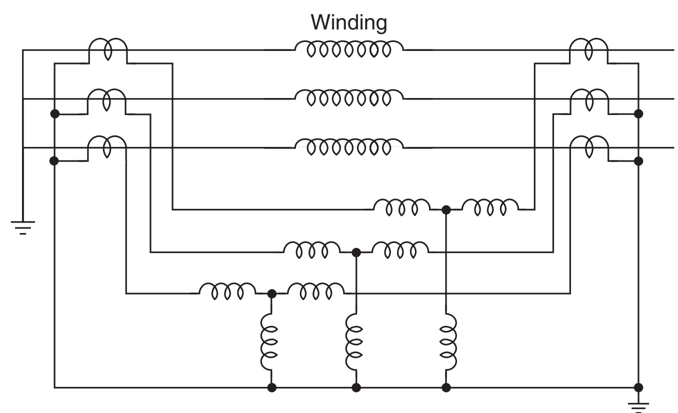


Fig. 14.44 (*a*) Stator protection Y grounded alternator.

In case the generator is delta connected, Fig. 14.44 (*b*) gives the scheme of percentage differential protection.

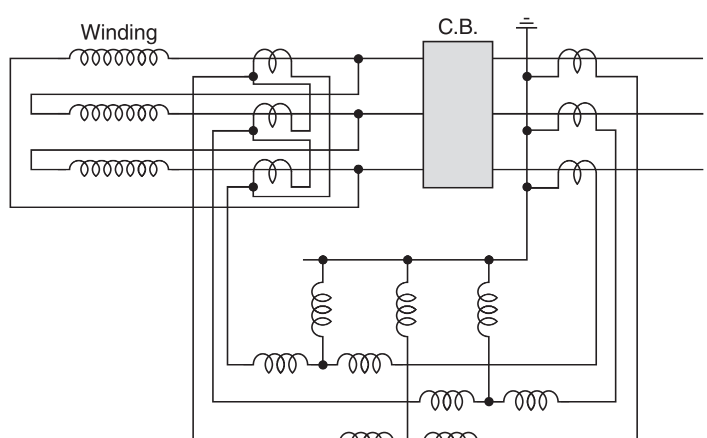


Fig. 14.44 (*b*) Stator protection, Δ connected alternator.

It can be seen again that for an external line-to-line fault, the relays do not operate whereas for an internal fault they will operate.

Restricted Earth Fault Protection

If the star point is not available because it is made inside the generator and if it is grounded through some low impedance, percentage differential relaying for ground faults only can be provided. This protection is known as restricted earth fault protection and is shown below in Fig. 14.44 (*c*).

It can be seen that for an external fault, the current cannot flow through the operating coil and hence the relay does not operate. It is very easy to see from the same scheme that for an internal fault, the current will flow only through the CT in the neutral and not in the winding CTs. Therefore, the current will flow through the operating coil and the relay will operate. The scheme, as is said earlier, can be used only for earth faults and not for phase faults.

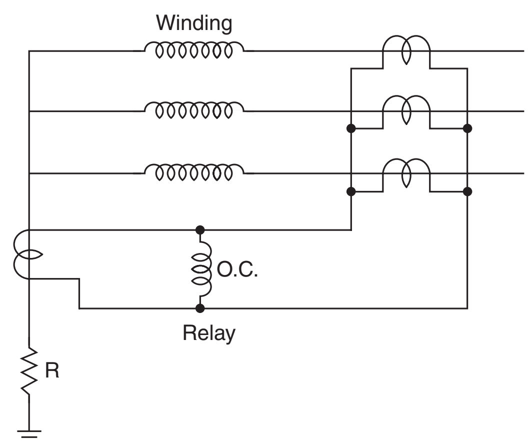


Fig. 14.44 (*c*) Restricted earth fault protection for an alternator.

Protection of Inter-turn Faults

Differential protection cannot be used for inter-turn faults because the currents at the two ends of the winding remain same. Differential protection against inter-turn faults is provided only for machines with multicircuit winding, *i.e.*, having more than one coil per phase.

The method used for protection is known as split-phase relaying. If the number of circuits is even for split-phase protection, they are divided into two equal groups of parallel circuits with a CT for each group. If the number is odd, the number of circuits in each of the two groups will not be equal and the CTs must have different primary current ratings so that under normal conditions their secondary currents will be equal. Split phase relaying will operate for any type of short circuits in the generator winding. The scheme is shown in Fig. 14.45(*a*).

For protection against inter-turn faults of winding having one circuit per phase one of the methods suggested uses a five limb voltage transformer with a tertiary winding. The tertiary winding alongwith the secondary winding are connected to a wattmetric relay. The high voltage winding is connected between the neutral and line terminal of the alternator. The voltage across the tertiary winding is the residual voltage which in normal condition is zero and has some voltage under abnormal condition. This residual voltage circulates currents in the current coil of the wattmetric relay and the secondary of the transformer feeds the potential coil of the relay element. The scheme is shown in Fig. 14.45 (*b*).

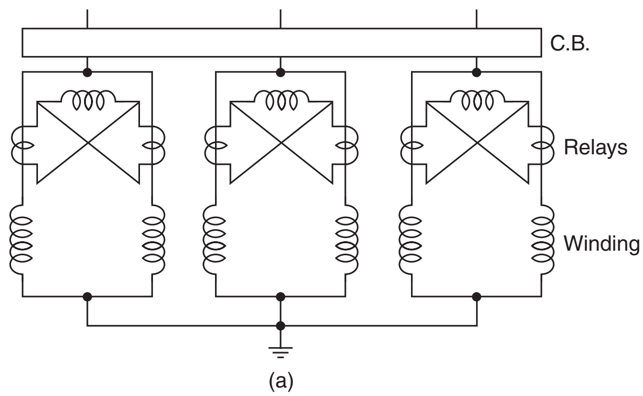


Fig. 14.45 (*a*) Inter-turn fault protection.

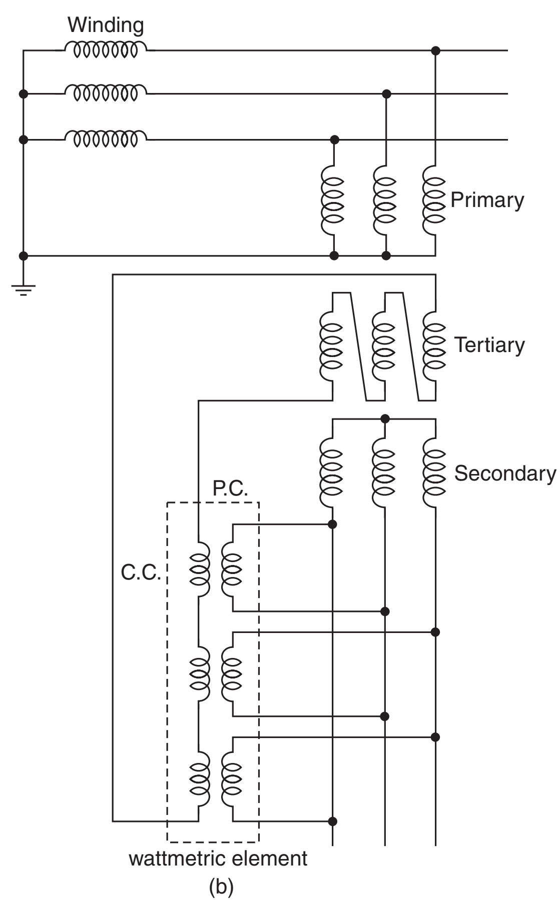
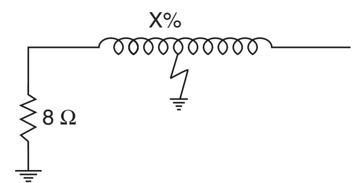


Fig. 14.45 (*b*) Inter-turn fault protection.

Example 14.4: A 6.6 kV, 5 MVA star connected alternator has a reactance of 1.5 ohm per phase and negligible resistance. Merz-Price protection scheme is used which operates when the out of balance current exceeds 25% of the full load current. The neutral of the generator is grounded through a resistance of 8 ohms. Determine the proportion of the winding which remains unprotected against earth fault. Show that the effects of the alternator reactance can be ignored.

Solution: First, we will show that the effect of the alternator reactance can be neglected. Since the reactance of the winding is ∝ square of the number of turns, *i.e.*, $x_G \propto N^2$, let x% be the winding that remains unprotected. The number or turns will be $xN/100$, *i.e.*, the reactance will be proportional to $x^2N^2/100^2$.



∴ If 1.5 ohm is the reactance of the winding, the reactance of the un-protected winding will be $\dfrac{1.5x^2N^2}{100^2}$. Since this reactance is to be added vectorially with resistance of 8 ohms and x being small, the effect of the reactance can be neglected.

The phase voltage will be $\dfrac{6600}{\sqrt{3}}$ = 3810 volts.

The voltage of the unprotected portion $= 3810\dfrac{x}{100}$

The fault current $= 3810\dfrac{x}{100}\dfrac{1}{8}$ amps

The full load current $= \dfrac{5000}{\sqrt{3}\times 6.6} = 437.37$ amps

The out of balance current required for the operation of the relay

$$= 437.37 \times 0.25 = 109.34 \text{ amps.}$$

$$\frac{3810x}{800} = 109.34$$

or $$4.7625x = 109.34$$

or $$x = 22.95\%. \quad \textbf{Ans.}$$

Example 14.5: An alternator rated at 10 kV protected by the balanced circulating current system has its neutral grounded through a resistance of 10 ohms. The protective relay is set to operate when there is an out of balance current of 1.8 amp in the pilot wires, which are connected to the secondary windings of 1000/5 ratio current transformers. Determine (*i*) the per cent winding which remains unprotected, (*ii*) the minimum value of the earthing resistance required to protect 80% of the winding.

Solution: (*i*) The phase voltage of the alternator $= \dfrac{10{,}000}{\sqrt{3}}$ = 5773 volts.

Let $x\%$ be the per cent winding which remains unprotected. The voltage of the unprotected portion of the winding = $5773 \dfrac{x}{100}$. Since the resistance in the neutral is 10 ohms the fault current will be $5773 \dfrac{x}{100} \dfrac{1}{10}$ amp.

The current in the pilot wires will be with a CT of 1000/5 amps ratio

$$= 5773 \, \frac{x}{100} \cdot \frac{1}{10} \cdot \frac{5}{1000} \text{ amps}$$

and this current should be equal to 1.8 amps for the operation of the relay.

$$5773 \, \frac{x}{100} \cdot \frac{1}{10} \cdot \frac{5}{1000} = 1.8$$

or

$$5773x = 3.6 \times 10^5$$

$$x = \frac{36 \times 10^4}{5.773 \times 10^3} = 62.36\%$$

(*ii*) To protect 80% of the winding, the unprotected portion is 20%. The voltage of the unprotected portion

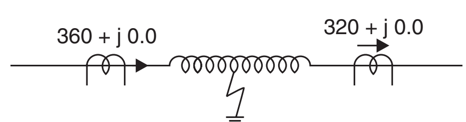
$$5773 \times 0.2 = 1154.6 \text{ volts}$$

Let R be the minimum value of the earthing resistance; the fault current will be $\dfrac{1154.6}{R}$ amp.

The fault current through the pilot wire will be $\dfrac{1154.6}{R} \cdot \dfrac{5}{1000}$ amp and this should equal the operating current of 1.8 amp or $\dfrac{1154.6}{R} \cdot \dfrac{5}{1000} = 1.8$

or

$$R = \frac{1800}{5 \times 1154.6} = 0.3118 \ \Omega. \quad \textbf{Ans.}$$



Example 14.6: The figure above shows the percentage differential relay used for the protection of an alternator winding. The relay has a minimum pick up current of 0.2 ampere and has a percentage slope of 10%. A high resistance ground fault occurs near the grounded neutral end of the generator winding with the current distribution as shown. Assume a CT ratio of 400 : 5; determine whether the relay will operate.

Solution: The difference current = 360 – 320 = 40 amps.

The current in the operating coil = $\dfrac{40 \times 5}{400}$ = 0.5 amp.

The average sum of the two currents = $\dfrac{360 + 320}{2}$ = 340 amps.

The average current through the restraining coil = $\frac{340 \times 5}{400}$ = 4.25 amps.

With 10% slope the operating current will be

$$0.1 \times \text{restraining current} + 0.2 = 0.1 \times 4.25 + 0.2 = 0.625 \text{ amp.}$$

Since the current through the operating coil is 0.5 amp, therefore the relay will not operate.

14.12 PROTECTION OF TRANSFORMERS

Transformers are normally protected against short circuits and over-heating. For short circuits normally percentage differential protection is recommended for transformers rated for more than 1 MVA. For low rating overcurrent relaying is used.

The primary and secondary currents of a transformer are normally different from each other and are related by their turns ratio. These currents are displaced in phase from each other by 30° if the windings are star-delta connected. The differential protection scheme is considered to be suitable if it satisfies the two conditions: (*i*) The relays must not operate under normal load conditions and for through fault (external fault) conditions; and (*ii*) it must operate for severe enough internal fault conditions. In fact, these are the tests that any good protection scheme must satisfy. For differential protection, the vector difference of two currents is fed to the operating coil of the relay. This means for an external fault the line currents of the two CTs should be equal in magnitude and should be in phase opposition so that the difference current is zero.

The CTs on the star side of the power transformer are connected in delta, and on the delta side, they are connected in star as the line currents of star-delta power transformer will be displaced in phase by 30°. It is required that this phase displacement must be nullified by connecting the CTs in that fashion.

Let us take first of all a star-star transformer (Fig. 14.46). When the star point of both the transformers is ungrounded, a line-to-ground fault has no meaning because no fault current

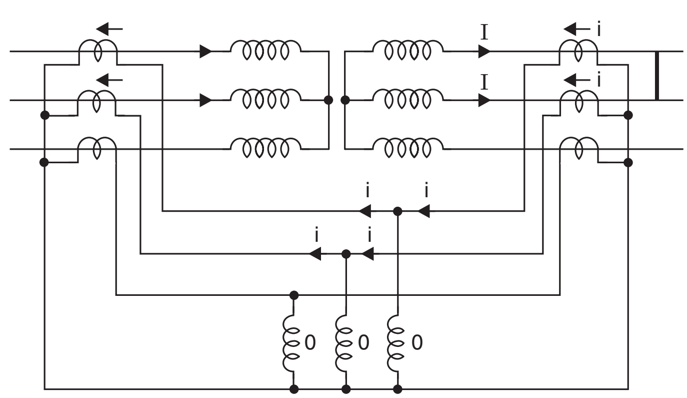


Fig. 14.46 Ungrounded star-star transformer protection, through fault.

can flow. Consider a *L-L* fault as shown (through fault). Let the transformers be of unity ratio. In that case it can be seen that for an external fault there will be no current through the operating coil and, therefore, the relay will not operate which is the desired result. It is seen that when the transformer is star/star the CT can also be connected in star-star in case the transformer star point is not grounded.

Next let us take star/star transformer with one of the star points grounded (Fig. 14.47). Assuming again unity transformation ratio, if the fault current on the secondary side is $3I$, only $2I$ current will be supplied by the primary (isolated star point). This type of connection of secondary is known as zero sequence current generator. From the distribution of the currents as shown in Fig. 14.47, it is clear that even for an external fault the currents circulate through the operating coil of the relay and the relay will operate which is not desirable. This means this scheme of protection is not proper. We, therefore, make use of the thumb rule that the CT be connected in delta if the power transformer is star connected.

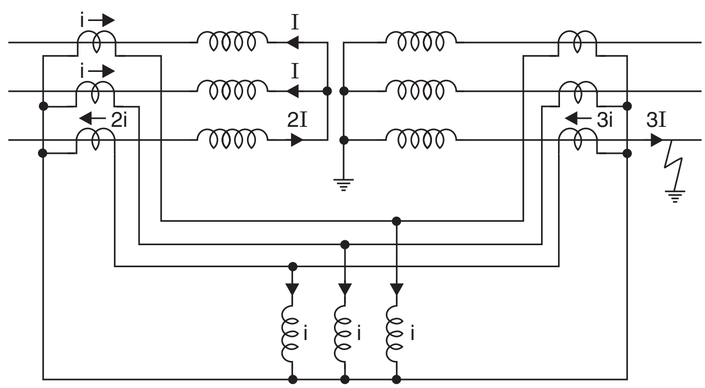


Fig. 4.47 Grounded star/star transformer protection, through fault.

It is seen from Fig. 14.48 that when CTs are delta connected, for an external fault, the relays do not operate. Now for an internal fault the scheme is shown in Fig. 14.49.

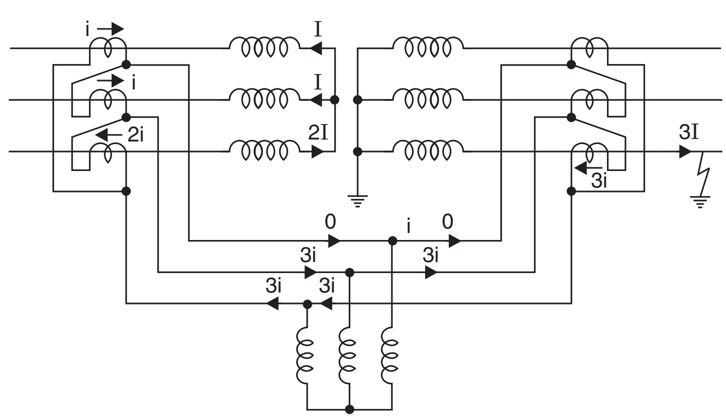


Fig. 14.48 Transformer star/star grounded CT delta connected protection.

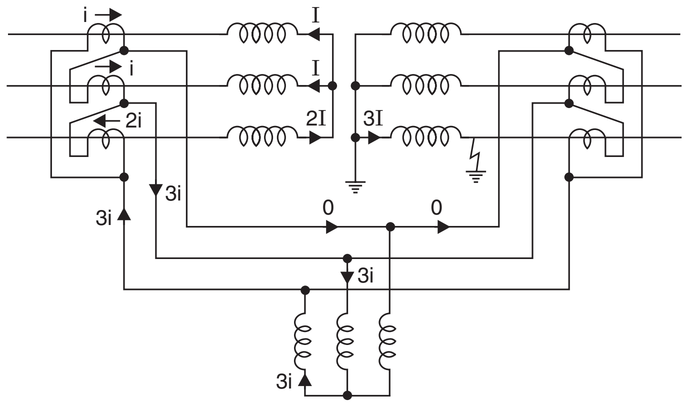


Fig. 14.49 Star/star grounded, internal fault.

It is seen that the relays operate for an internal fault and, therefore, this scheme of protection is satisfactory. Next we take up a delta-star transformer. From the current flow in Fig. 14.50, it is clear that for an external fault the relays do not operate. It can be very easily seen that with the scheme for an internal fault the relays do operate. Therefore, the scheme is satisfactory.

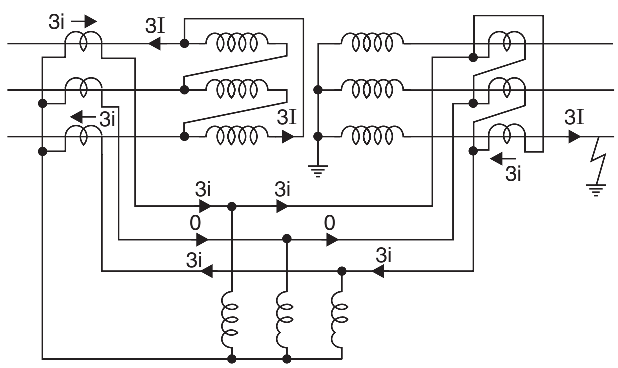


Fig. 14.50 Delta/star grounded transformer protection.

Example 14.7: A 3-phase transformer rated for 33 kV/6.6 kV is connected star/delta and the protecting current transformer on the low voltage side have a ratio of 400/5. Determine the ratio of the current transformer on the HV side.

Solution: Since the LT side is delta connected, the CTs on that side will be star connected. Therefore, if 400 amps is the line current, the CT secondary current is 5 amps. The line current on the star side of the power transformer will be

$$400 \times \frac{6.6}{33} = 80 \text{ amps.}$$

The CTs on the star side are delta connected and the current required on the relay side of the CT is 5 amps. Therefore, the current in the CT secondary (phase current) is $\frac{5}{\sqrt{3}}$.

The CT ratio on the HT side will be $80 : \dfrac{5}{\sqrt{3}}$. **Ans.**

Example 14.8: For a 10 MVA, 132 kV/6.6 kV power transformer with delta-star connections, obtain the number of turns each current transformer should have, for the differential protection scheme to circulate a current of 5 A in the pilot wires.

Solution: The line current on HV side $= \dfrac{10{,}000}{\sqrt{3} \times 132} = 43.73$ amps.

The line current on LV side $= \dfrac{10{,}000}{\sqrt{3} \times 6.6} = 874.75$ amps.

The CT on the delta side (HV side) are star connected.

$\therefore$ The ratio of CT on the LV side is $874.75 : 5/\sqrt{3}$ and the CT ratio on the HV side will be 43.73 : 5. **Ans.**

Example 14.9: A 3-phase 50 HZ, 110 V positive sequence voltage supply is connected to terminals A, B and C of figure shown below. Calculate the values of R_1, R_2 and C such that a current of 1 amp flows through each arm and that there is no potential across the relays. What will be the potential across the relays if the phase sequence of supply is reversed. Assume that the impedance of the relays is very high as compared to the impedance of the components of the network.

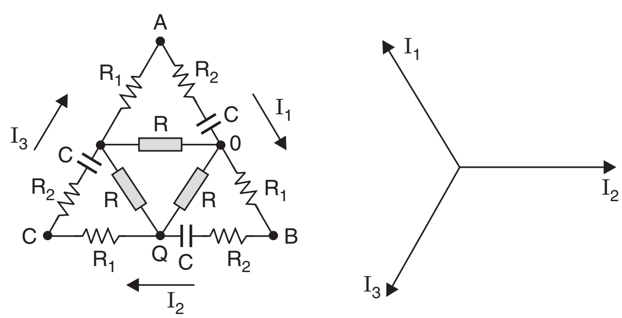


Fig. E.14.9

Solution: Taking current I_2 as reference and applying Kirchhoff's law for voltages for the mesh OBQ, assuming no current flows through OQ.

$$1(-0.5 + j0.866)R_1 + R_2 \,.\, 1 - \frac{j}{\omega C} = 0$$

Since the relay points are at the same potential,

$$-0.5R_1 + j0.866R_1 + R_2 - \frac{j}{\omega C} = 0$$

Separating real and imaginary quantities,

$$0.5R_1 = R_2$$

and
$$0.866R_1 = \frac{1}{\omega C}$$

Also since the current through the arm is 1 amp when a supply of 110 volts is applied,

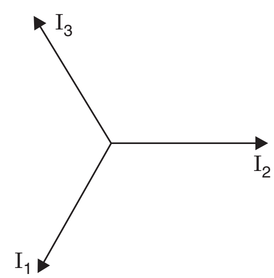
$$\therefore \qquad 1 = \frac{110}{R_1 + R_2 - j/\omega C} = \frac{110}{3R_2 - j\sqrt{3}R_2} = \frac{110}{(3 - j\sqrt{3})R_2}$$

or $$R_2 = \frac{110}{3 - j\sqrt{3}} = 31.75 \text{ ohms.} \quad \textbf{Ans.}$$

$$\therefore \qquad R_1 = 63.50 \text{ ohms.} \quad \textbf{Ans.}$$

and $$C = \frac{1}{0.866 R_1 \omega} = \frac{1}{0.866 \times 63.50 \times 314} = 57.9\ \mu\text{F.} \quad \textbf{Ans.}$$

(*ii*) When the phase sequence is changed, the phasor diagram will be as shown:



If again 1 amp flows through the arm, the voltage across the terminals of the relay will be

$$(-0.5 - j0.866)63.50 + (31.75 - j55) = -31.75 - j55 + 31.75 - j55 = -j110 \text{ volts.} \quad \textbf{Ans.}$$

Therefore, if the sequence is changed the voltage across the terminals of the relay will be as 110 volts.

Biased Differential Protection

Biased differential protection is used in case low fault settings and high operating speeds are to be obtained when the following conditions exist or a power transformer:

(*i*) On-load tap changing.

(*ii*) Magnetising inrush current.

(*iii*) Unmatched CTs.

To take into account the magnetising inrush currents, a high speed biased differential relay incorporating harmonic restraint feature is used. The peak value of the inrush currents may be as high as 6 to 8 times the normal full load currents. Insofar as inrush current affects the operation of transformer differential protection relays, two aspects are of significance: (*i*) The current flows only in one of two windings of the transformer (the primary winding) and, therefore, it is as good as an internal fault as far as protection scheme is concerned. (*ii*) The wave shape of the inrush current differs from the usual fault current in that it contains a high component of second harmonics. It thus follows that a relay designed to detect the second harmonic component in the magnetising inrush current can be made to utilise this as a means of discrimination between inrush condition and the internal fault currents. This is achieved by the use of a second harmonic filter which is arranged to inject an additional bias current in the relay circuit proportional to the second harmonic component. Fig. 14.51 gives the basic circuit of the harmonic restraint relay. Harmonic restraint is obtained from the tuned circuit $X_C X_L$ which permits only currents of fundamental frequency to enter the operating circuit. The restraint coil is energised by a d.c., proportional to bias winding current as well as the d.c. due to harmonics. The d.c. and higher harmonics, mostly second harmonics (in case of inrush currents), are diverted into the rectifier bridge feeding the restraining coil. The relay is adjusted so that it will not operate when the harmonic current exceeds 15% of the fundamental current.

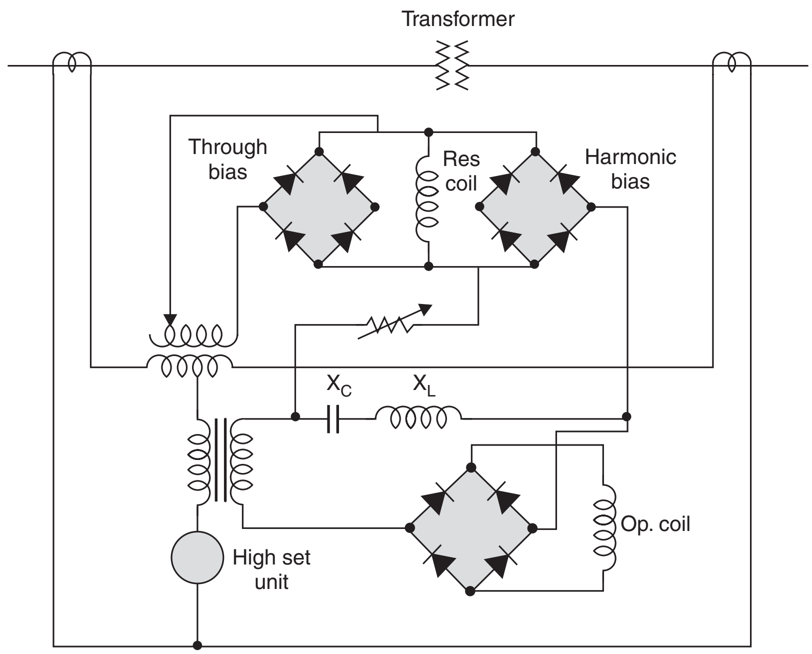


Fig. 14.51 Harmonic restraint relay.

The disadvantage of the harmonic restraint relay is that it will not operate for an internal fault that contains considerable harmonics which may be due to an arc or due to saturation of current transformer. Also, if a fault exists at the time a transformer is energized, harmonics in the magnetising current may prevent the harmonic restraint relay from tripping. For this purpose an instantaneous overcurrent relay in the differential circuit is normally provided which is set above the maximum inrush current and this operates in less than one cycle on internal faults.

The earth fault current in case of a transformer for a given fault position within the winding depends upon the winding connection and the method of neutral grounding.

Consider Fig. 14.52 where the delta-star transformer has 1 : 1 voltage ratio. The line currents in the delta winding will, therefore, be $\sqrt{3}$ times the line currents in the star winding. If the fault is at 100% of the winding from the neutral of the star and I_F is the fault current, then for a fault at x% of the winding, the fault current on the star side is $\frac{x}{100} I_F$, whereas the fault current on the delta side will be $I_F\left(\frac{x}{100}\right)^2 \cdot \frac{1}{\sqrt{3}}$ as the effective turns ratio of primary to secondary now is $\sqrt{3} : \frac{x}{100}$. Thus in this case the earth fault for a given neutral resistance is directly proportional to the percentage of winding (star side) between the neutral and the fault point and on the primary side the fault current is proportional to the square of the per cent winding short circuited.

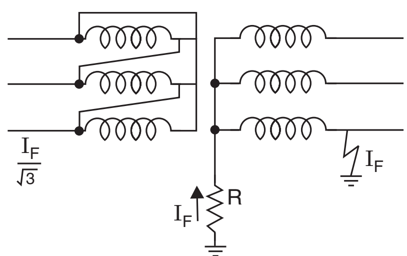


Fig. 14.52 Transformer earth fault for resistance grounded star winding.

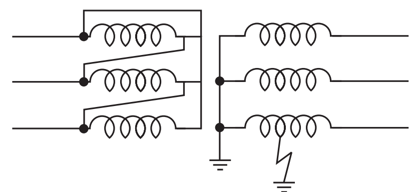


Fig. 14.53 Transformer earth fault for solidly grounded star winding.

Consider Fig. 14.53 where the star winding is solidly grounded. The earth fault current is limited by the impedance of the winding which in turn is proportional to the square of the number of turns of the winding.

The leakage reactance of the faulted winding in terms of reactance per turn increases, the nearer the fault is to the star point, but the reactance of the other winding is effectively reduced owing to the change in transformation ratio so that the fault current becomes minimum at some point near the middle of the winding.

IDMT relays are used to protect the transformer against the external short circuits and the overloads. This protection acts as a back up protection. Extremely inverse characteristics is the ideal as the heating characteristics of the transformer closely resemble the characteristics of these relays. The protection is located on the supply side of the transformer and is arranged to trip both the HV and LV circuit breakers.

Since the overcurrent settings are quite high, there are inherent difficulties in the provision of sensitive earth fault relays. Therefore, separate earth fault protection known as restricted earth fault protection is provided to both windings of the transformer.

Buchholz Relay

Whenever a fault takes place in a transformer the oil of the tank gets overheated and gases are formed. The generation of the gases may be slow or violent depending upon whether the fault is a minor or incipient one or heavy short circuit. The generation of gas is used as a means of fault detection. Buchholz relay is the simplest form of protection which is commonly used for this form of protection in all transformers provided with conservators. It consists of two hinged floats in a metallic chamber, which is connected in the upper side of the pipe run between the oil conservator and the transformer tank. One of the floats is near the top of the chamber and the other opposite the orifice of the pipe to the transformer as shown in Fig. 14.54.

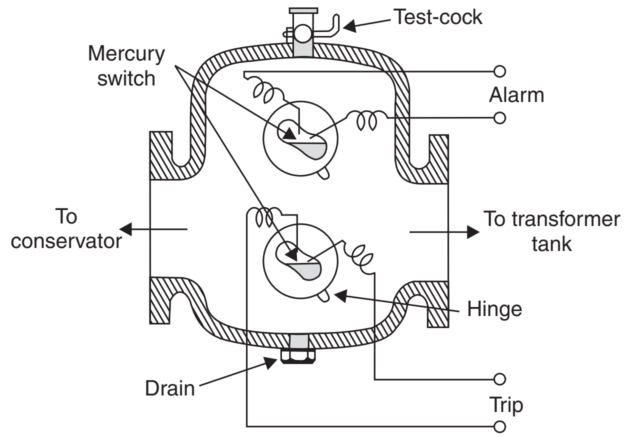


Fig. 14.54 Buchholz relay.

For a minor or incipient fault, the slow generation of gas gives rise to gas bubbles which try to go to the conservator but are trapped in the upper portion of the relay chamber, thereby a fall in oil level takes place. This disturbs the equilibrium of the gas float. The float tilts and the alarm circuit is closed through the mercury switch and the indication is given.

For a heavy fault, large volumes of gases are generated which cause violent displacement of the oil and impinge upon the baffle plates of the lower float and thus the balance of the lower float is disturbed. The lower float is tilted and the contacts are closed which are arranged to trip the transformer.

14.13 TRANSLAY RELAY

It is a voltage balanced system in which the secondary CT voltages (voltages are proportional to the CT secondary current as air-cored CTs are used) at the ends of the feeder are compared. The CTs are connected in opposition (see Fig. 14.55). Associated with the CT at each end is an induction relay. The upper magnet system acts as a quadrature transformer and produces at the pilot terminals a voltage which varies with the primary current. As long as the currents at the two ends are equal, the voltages induced are also equal and hence no current flows in the pilot wires. In case the CTs are of ordinary instrument type where there is possibility of dissymmetry in the characteristics of the CTs at the two ends, compensating devices are provided in the relay to neutralize the effect of unbalancing of the CTs. In case of a through fault or due to asymmetry in CTs under normal conditions the current through the pilot wires is capacitive and, therefore, the flux in the series magnet (due to capacitive current) is in phase with the leakage flux from the upper magnet thereby the net torque on the disc is zero. This is shown in the phasor diagram (Fig. 14.56). Here V is the voltage across the CT secondary and E is the induced voltage across the pilot wires, ϕ_v the flux in the upper magnet, ϕ_c the flux in the lower magnet and I_c the pilot current.

Whenever an internal fault occurs, current flows through the pilot wires because either one of the voltages has reversed in polarity (if the feeder is fed from both the ends) or the voltage at one end has collapsed (if the feeder is fed from one end only). The relay at an end

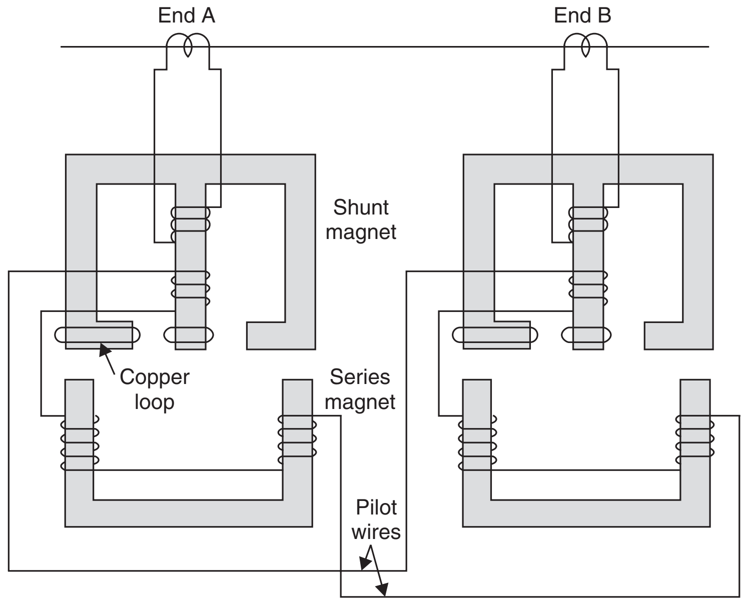


Fig. 14.55 Translay relay applied to 1-phase system.

will operate if there is current in its upper and lower coils and it will not operate at an end with no primary current because there is current only in the lower coil. Under internal fault condition since the pilot wire impedance is mostly resistive, the current through the pilot wire will be in phase with the secondary voltage. The phasor diagram is shown in Fig. 14.57 for this condition. Since the two fluxes are 90° apart approximately, the positive torque is produced and the relay operates.

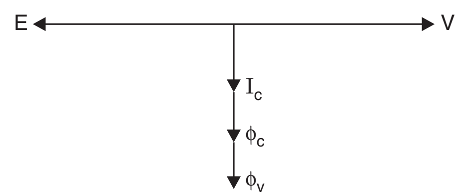


Fig. 14.56 Phasor diagram for through fault.

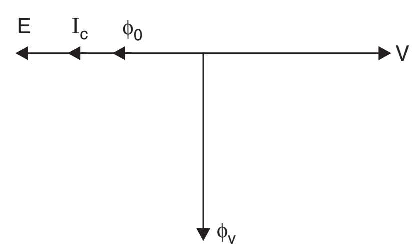


Fig. 14.57 Phasor diagram for an internal fault.

The copper loop fitted to the central limb of the upper electromagnet gives rise to the flux which when interacts with the pilot capacitance current prevents the operation of the relay as indicated by phasor diagram in Fig. 14.56. Bias is obtained by the action of a second copper loop, mounted on an outer limb of the upper magnet. Under normal condition, when current flows in the upper coil only, the relay behaves as a shaded pole type but the torque produced is arranged to act in a reverse *i.e.*, restraining sense. This feature is equivalent to providing restraining coil in a percentage differential relay and prevents the operation of the relay due to mismatching of the CTs and/or any spill current due to through faults.

Translay relay protection when applied to 3-phase system requires a single relay element with a summation transformer as shown in Fig. 14.58.

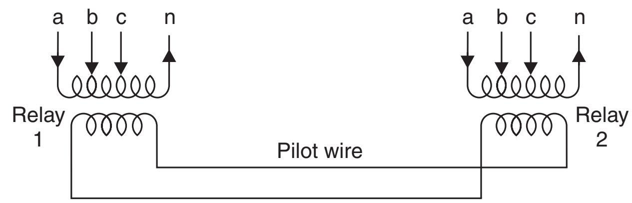


Fig. 14.58 Summation transformer connection for relays.

The system needs only two pilot wires. The operation of the system is as follows:

Under normal condition since the system is balanced, there is no voltage induced in the secondary of upper electromagnet as the sum of three currents at any instant of time is zero. Even if there is any unbalanced loading of the phases, the unbalancing will induce voltage at both ends of the system and since the pilot wire connections are such that these voltages are in opposition and equal in magnitude, normally no current flows through the pilot wires. The operation further is exactly identical to the single phase system.

Summation current transformer is used whenever 3-phase currents at one end of the line are to be compared with currents at the other end of the line. The transformer gives single phase output, the magnitude of which depends upon the nature of fault. The arrangement is shown in Fig. 14.58. For a balanced fault the current through *cn* of the winding is zero. The phase *a* current energizes 1 p.u. turns between *a* and *b* and the phasor sum of I_a and I_b flows in the 1 p.u. turns between *b* and *c*.

14.14 CARRIER CURRENT PROTECTION

Pilot wire protection is usually limited to circuits of length 10 miles or so. For longer overhead lines the power line itself may be used as the channel between terminal equipments. The primary consideration is the coupling of the protective gear to the power line. Coupling between line and earth, even though results in saving in terminal equipment, it introduces additional power losses at the high frequencies normally used and is undesirable if the channel is to be used for purposes other than protection *e.g.*, telegraphy, telephone etc. Experience, therefore, has shown that it is preferable to connect equipments between two phases. Coupling is done through a series LC wide-band filter tuned to the carrier frequency *i.e.*, it allows only carrier frequency to pass through and offers a very high impedance to power frequency. A parallel LC filter provides a drainage path to earth for power frequency currents, thus maintaining the connection point to the h.f. equipment at earth potential. Line traps are provided at the overhead line termination in series with the phases used for signalling. The line traps are tuned to power frequency; whereas they offer high impedance to carrier frequency, thereby they avoid the interference between the carrier signals of the adjacent line sections and provide continuity for the power circuit. These are mounted on the top of the coupling capacitor stacks. The connection from the relay equipment mounted indoors to the outdoor coupling apparatus is via a coaxial cable or a low-loss screened twin cable.

The signal is injected into the power line circuit as shown in Fig. 14.59 through the coupling capacitor. The signal is generated by a transmitter consisting of an electronic oscillator and amplifier with an output usually of about 15 to 20 watts at a frequency between 50 and 500 kHz. Below 50 kHz the size and cost of the coupling components would be too high; above 500 kHz the line losses and hence the signal attenuation would be too great on long lines. 15 watts output has been considered to be sufficient from loss point of view for lines of length 100 miles. Carrier current can be used only on overhead lines because the capacitance of a cable would attenuate the carrier signals to ineffectual values.

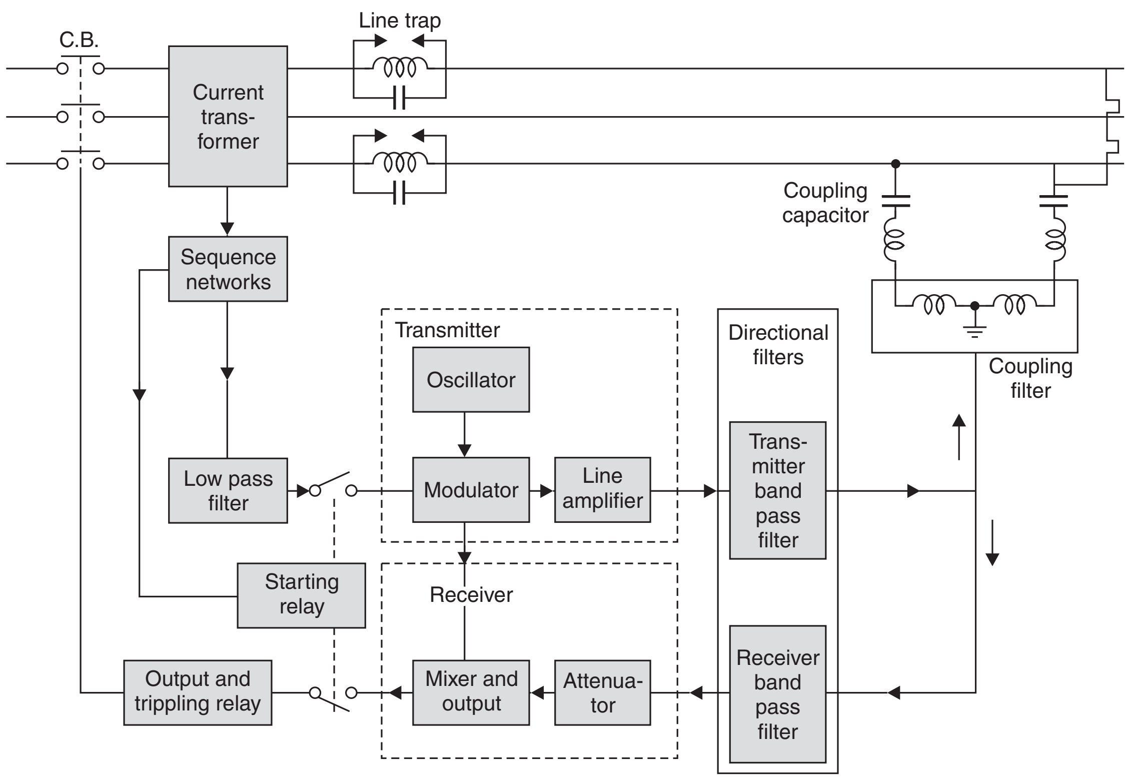


Fig. 14.59 Block diagram of equipment used in carrier phase comparison scheme.

Phase Comparison Scheme

Phase comparison relaying blocks the operation of the relay at both ends of the line whenever the carrier current signals are displaced in time so that there is little or no time interval when a signal is not being transmitted from one end or the other. Tripping of the relays will occur when the signals at the two ends are concurrent and there is time between the consecutive concurrent signals when no signal is being transmitted from both the ends (when feeder is fed from both the ends). To achieve phase comparison on these lines, the line current transformers are so connected that their secondary currents are 180° out of phase when current is flowing in the feeder under both normal and/or external fault condition. When an internal fault takes place, the current at one of the ends reverses and thus the two currents are in phase (when fed from both ends) and, therefore, there is time when no signal is being received and the relay operates. In case the feeder is fed from one end, for an internal fault the current at one of the ends reduces to zero and hence again there is time when no signal is received and the relay will operate. This is illustrated diagrammatically in Fig. 14.60.

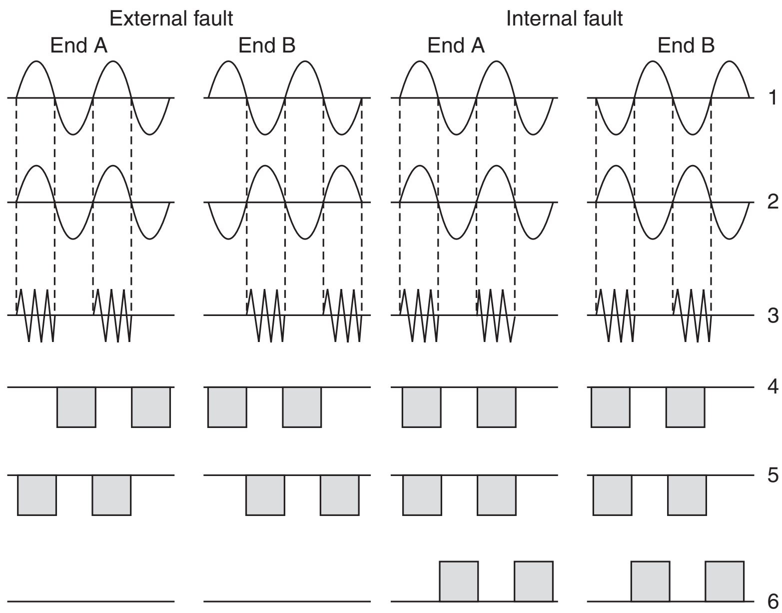


Fig. 14.60 Principle of carrier phase comparison scheme
1. Primary current, 2. Secondary current, 3. Transmitted signal,
4. Received signal, 5. Locally derived signal, and 6. Output.

The operation of this scheme is explained with the help of a block diagram (Fig. 14.59) as follows:

The block diagram shows the equipments required at end *A* of the line. Similar equipment is connected at end *B* of the same line. The line current transformers are connected as summation transformer; thus 3-phase currents are reduced to a single phase quantity and is fed to a sequence network which is sensitive only to negative sequence currents. The output from the sequence network is fed into the starting equipment which operates in two stages known as low set and high set. The differential between the settings of the two relays is such that, on the incidence of a fault, the low set relays at both the terminals operate at a lower current than any of the high set relays. The low set relays start the comparison (phase) process and the high set relays control the tripping circuit.

The contacts of the low set relay allow the 50 Hz output from the sequence network to be fed into the transmitter through a low-pass filter. This 50 Hz input to the transmitter modulates the high frequency input from the oscillator. The output from the modulator is partly fed to the local mixer circuit and partly is amplified through an amplifier and fed to the line through the transmitter band pass filter and the coupling equipment. The transmitted signal enters the receiver circuit through the receiver band pass filter at end *B* after passing through the coupling equipment at that end. It is then attenuated and passed into the mixer circuit.

From end B also a similar signal reaches the end A mixer circuit in exactly the same way. Thus mixer circuits at both the ends receive two signals, one from the local circuit and another from the other end circuit. The phase relation between these signals is so arranged that it is 180° for a normal operation of the system or for an external fault and it is 0° for an internal fault. The receiver output increases as the phase angle between two signals decreases. If the fault current is high enough to operate the high set relay in the starting equipment, the output from the receiver is applied to the output relay, which operates the tripping relays.

In case the feeder is fed from end A only, under internal fault condition, the equipment at A will receive signal from the local circuit only and no signal from end B and since the high set relay at A will receive a high current it will operate and thus the breaker at end A will operate, whereas at end B even though there is output from the mixer circuit but because the high set relay cannot operate, this output from the mixer circuit cannot be fed to the tripping relays and hence the relays at B will not operate.

14.15 COMPARATORS

The job of a relay is to sense any abnormal condition in the system and send a signal to the breaker which in turn disconnects the faulty section of the feeder from the healthy one. The relay does all this by comparing two quantities either in amplitude or in phase. The phase relation and the amplitudes are a function of the system conditions. The device which makes these comparisons is known as a comparator and forms the heart of a relay. The comparator decides the operating characteristics of a relay.

We first of all derive the general threshold equation assuming that there are two inputs S_1 and S_2. These input quantities are derived from the system through the current and voltage transformers and some mixing circuits (see Fig. 14.61).

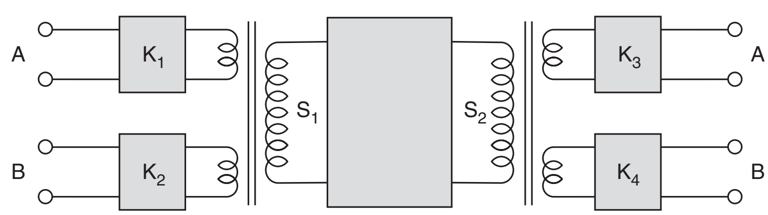


Fig. 14.61 A general comparator.

From Fig. 14.62 let $\quad \bar{S}_1 = K_1\bar{A} + \bar{K}_2\bar{B}$

and $\quad \bar{S}_2 = K_3\bar{A} + \bar{K}_4\bar{B}$

where A and B are the primary system quantities, K_1 and K_3 are the scalar numbers and $\bar{K}_2$ and $\bar{K}_4$ are the complex numbers with θ_2 and θ_4 as the angles. Let the vector A be the reference vector and B lags behind A by angle ϕ. Then equations can be rewritten as

$$S_1 = K_1 | A | + | K_2 | \, | B | \{\cos (\theta_2 - \phi) + j \sin (\theta_2 - \phi)\}$$

and $$S_2 = K_3 | A | + | K_4 | \; | B | \{\cos (\theta_4 - \phi) + j \sin (\theta_4 - \phi)\}$$

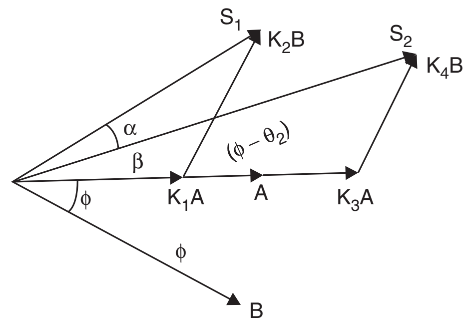


Fig. 14.62 Phasor diagram for a comparator.

If the operating criterion is such that $| S_1 | \geq | S_2 |$ then at the threshold of operation $| S_1 | = | S_2 |$.

$$\{K_1 | A | + | K_2 | \; | B | \cos (\theta_2 - \phi)\}^2 + \{| K_2 | \; | B | \sin (\theta_2 - \phi)\}^2$$
$$= \{ K_3 | A | + | K_4 | \; | B | \cos (\theta_4 - \phi)\}^2 + \{| K_4 | \; | B | \sin (\theta_4 - \phi)\}^2$$

After rearranging the terms, we get

$$(K_1^2 - K_3^2) | A |^2 + 2 | A | \; | B | \{K_1 | K_2 | \cos (\theta_2 - \phi) - K_3 | K_4 | \cos (\theta_4 - \phi)\}$$
$$+ K_2^2 | B |^2 \cos^2 (\theta_2 - \phi) - | K_4 |^2 | B |^2 \cos^2 (\theta_4 - \phi)$$
$$+ | B |^2 \{| K_2 |^2 \sin^2 (\theta_2 - \phi) - | K_4^2 | \sin^2 (\theta_4 - \phi)\} = 0$$

or $$(K_1^2 - K_3^2) | A |^2 + 2 | A | \; | B | \{K_1 | K_2 | \cos (\theta_2 - \phi) - K_3 | K_4 | \cos (\theta_4 - \phi)\}$$
$$+ (| K_2 |^2 - | K_4 |^2) | B |^2 = 0$$

Dividing by $(| K_2 |^2 - | K_4 |^2) | A |^2$ and rearranging the terms,

$$\left| \frac{B}{A} \right|^2 + 2 \left| \frac{B}{A} \right| \frac{(K_1 | K_2 | \cos \theta_2 - K_3 | K_4 | \cos \theta_4) \cos \phi + (K_1 | K_2 | \sin \theta_2 - K_3 | K_4 | \sin \theta_4) \sin \phi\}}{| K_2 |^2 - | K_4 |^2}$$
$$+ \frac{K_1^2 - K_3^2}{| K_2 |^2 - | K_4 |^2} = 0$$

or $$\left| \frac{B}{A} \right|^2 + 2 \left| \frac{B}{A} \right| [A_0 \cos \phi + B_0 \sin \phi] + C_0 = 0$$

where $$A_0 = \frac{K_1 | K_2 | \cos \theta_2 - K_3 | K_4 | \cos \theta_4}{| K_2 |^2 - | K_4 |^2}$$

$$B_0 = \frac{K_1 | K_2 | \sin \theta_2 - K_3 | K_4 | \sin \theta_4}{| K_2 |^2 - | K_4 |^2}$$

and $$C_0 = \frac{K_1^2 - K_3^2}{| K_2 |^2 - | K_4 |^2}$$

The above equation represents an equation to a circle with radius

$$r = \frac{\sqrt{K_1^2 |K_4|^2 + |K_2|^2\ K_3^2 - 2K_1 |K_2|\,K_3\,|K_4| \cos(\theta_2 - \theta_4)}}{|K_2|^2 - |K_4|^2}$$

and coordinates of the centre are (c, ϕ) where

$$c = \frac{\sqrt{K_1^2 |K_2|^2 + K_3^2\,|K_4|^2 - 2K_1 |K_2|\,K_3\,|K_4| \cos(\theta_2 - \theta_4)}}{|K_2|^2 - |K_4|^2}$$

The characteristic is shown in Fig. 14.63.

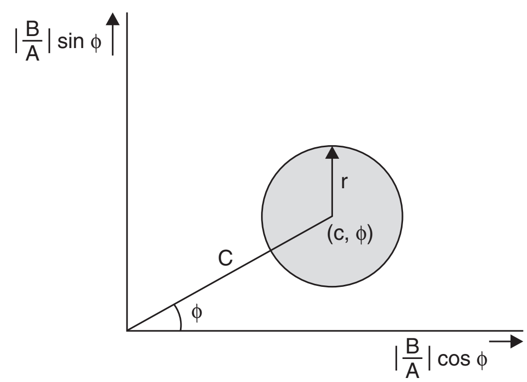


Fig. 14.63 Threshold characteristic of an amplitude comparator.

Analysis for Phase Comparator

Here again we have to compare the two inputs S_1 and S_2, this time their phase relation. Say $S_1 = |S_1| \angle \alpha$ and $S_2 = |S_2| \angle \beta$ with respect to a reference axis. The scalar product of these two vector quantities is maximum when they are in phase and the threshold condition *i.e.*, positive torque will be obtained when $\alpha - \beta = \pm \pi/2$. Under this condition

$$\tan(\alpha - \beta) = \pm \infty$$

or
$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} = \pm \infty$$

or
$$1 + \tan\alpha \tan\beta = 0$$

Substituting for $\tan\alpha$ and $\tan\beta$, the equation becomes

$$1 + \frac{|K_2||B|\sin(\theta_2 - \phi)}{K_1|A| + |K_2||B|\cos(\theta_2 - \phi)} \times \frac{|K_4||B|\sin(\theta_4 - \phi)}{K_3|A| + |K_4||B|\cos(\theta_4 - \phi)} = 0$$

or
$$K_1K_3|A|^2 + K_1|K_4|\,|A|\,|B|\cos(\theta_4 - \phi) + K_3|K_2|\,|A|\,|B|\cos(\theta_2 - \phi)$$
$$+ |B|^2 |K_2|\,|K_4| \cos(\theta_2 - \phi)\cos(\theta_4 - \phi)$$
$$+ |K_2|\,|K_4|\,|B|^2 \sin(\theta_2 - \phi)\sin(\theta_4 - \phi) = 0$$

Dividing the equation by $|K_2||K_4||A|^2 \cos(\theta_2 - \theta_4)$, we get

$$\left|\frac{B}{A}\right|^2 + \frac{\left|\frac{B}{A}\right|\{(K_1|K_4|\cos\theta_4 + |K_2|K_3\cos\theta_2)\cos\phi + (K_1|K_4|\sin\theta_4 + |K_2|K_3\sin\theta_2)\sin\phi\}}{|K_2||K_4|\cos(\theta_2-\theta_4)}$$

$$+ \frac{K_1K_3}{|K_2||K_4|\cos(\theta_2-\theta_4)} = 0$$

or
$$\left|\frac{B}{A}\right|^2 + \left|\frac{B}{A}\right|\{A_0'\cos\phi + B_0'\sin\phi\} + C_0' = 0$$

where
$$A_0' = \frac{K_1|K_4|\cos\theta_4 + |K_2|K_3\cos\theta_2}{|K_2||K_4|\cos(\theta_2-\theta_4)}$$

$$B_0' = \frac{K_1|K_4|\sin\theta_4 + |K_2|K_3\sin\theta_2}{|K_2||K_4|\cos(\theta_2-\theta_4)}$$

and
$$C_0' = \frac{K_1K_3}{|K_2||K_4|\cos(\theta_2-\theta_4)}$$

The above equation represents an equation to a circle with radius

$$r = \frac{\sqrt{K_1^2|K_4|^2 + |K_2|^2 K_3^2 - 2K_1|K_2|K_3|K_4|\cos(\theta_2-\theta_4)}}{2|K_2||K_4|\cos(\theta_2-\theta_4)}$$

and the co-ordinates of the centre are (C, ϕ), where

$$C = \frac{\sqrt{K_1^2|K_4|^2 + |K_2|^2 K_3^2 + 2K_1|K_2|K_3|K_4|\cos(\theta_2-\theta_4)}}{2|K_2||K_4|\cos(\theta_2-\theta_4)}$$

as given in Fig. 14.64.

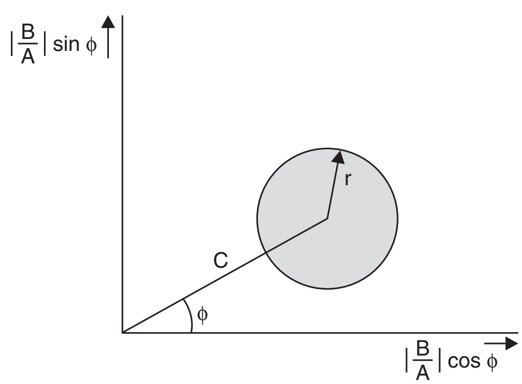


Fig. 14.64 Threshold characteristic of a comparator.

Duality between Amplitude and Phase Comparator

It can be shown with the help of phasor diagrams that if the input quantities to the comparator are changed to the sum and difference of the original two input quantities, an inherent amplitude comparator becomes a phase comparator and vice-versa.

Consider, for example, an amplitude comparator with inputs S_1 and S_2 such that it operates when

$$| S_1 | > | S_2 |$$

Now let the inputs be changed to $| S_1 + S_2 |$ and $| S_1 - S_2 |$ and they are such that $| S_1 + S_2 | > | S_1 - S_2 |$. If these quantities are fed to the amplitude comparator, the comparator essentially compares the phase relation between S_1 and S_2. This is indicated in Fig. 14.65.

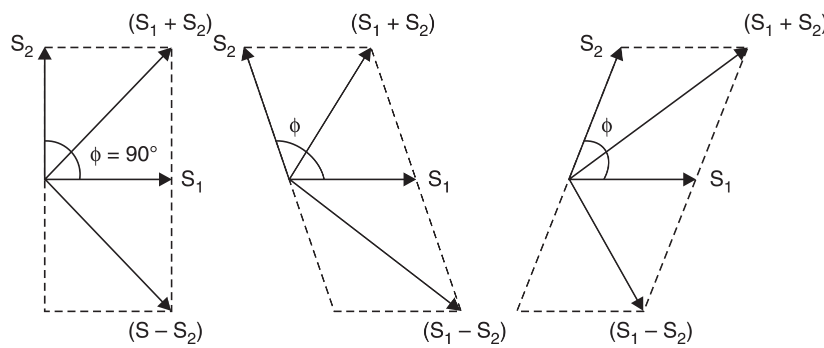


Fig. 14.65 Phase comparison using an amplitude comparator.

We see that the requirement $| S_1 + S_2 | < | S_1 - S_2 |$ puts a condition on the phase relation between S_1 and S_2 *i.e.*, unless phase difference between original phasors S_1 and S_2 exceeds 90° (cosine of angle greater than 90° is negative), $| S_1 + S_2 |$ cannot be less than $| S_1 - S_2 |$. Therefore, the original amplitude comparator with inputs now as $| S_1 + S_2 |$ and $| S_1 - S_2 |$ is a phase comparator, *i.e.*, a converted phase comparator.

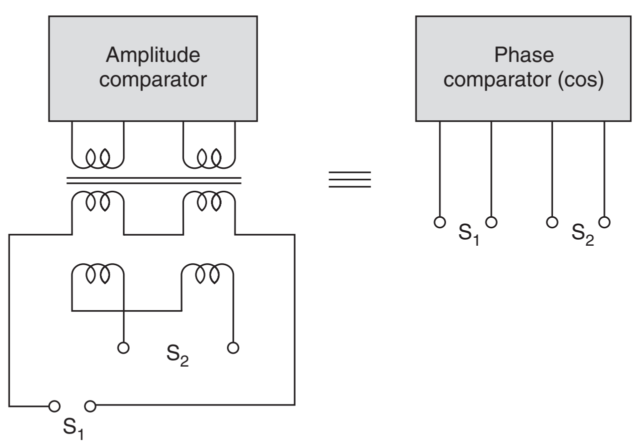


Fig. 14.66 Equivalence of phase comparator.

It is to be noted that the phase comparator in case of static circuits is a cosine comparator as opposed to a sine comparator in case of electromechanical relays.

The amplitude comparison using a phase comparator is explained with the help of phasor diagram (Fig. 14.67) and the schematic diagram of equivalence is shown in Fig. 14.68.

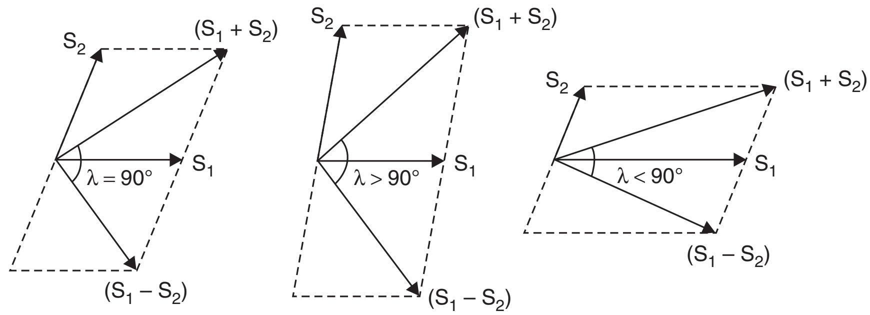


Fig. 14.67 Amplitude comparison using a phase comparator.

From the phasor diagram it is clear that if the original inputs to phase comparator are S_1 and S_2 with such a phase relation that they will operate the relay and if now the inputs are changed to $| S_1 + S_2 |$ and $| S_1 - S_2 |$ and fed to the same phase comparator, the comparator essentially compares the amplitude relation between S_1 and S_2. Unless $| S_1 | > | S_2 |$, the phase relation between $| S_1 + S_2 |$ and $| S_1 - S_2 |$ will not be less than 90° and hence the resultant comparator will be an amplitude comparator, that is it will be a converted amplitude comparator.

Though a given relay characteristic can be obtained using either of the two comparators, consideration of the constants calculated for required characteristics would indicate which type of comparator is preferable. In general an inherent comparator is better than the converted type because if one quantity is very small compared with the other, a small error in the large quantity may cause an incorrect comparison when their sum and difference are supplied as input to the relay.

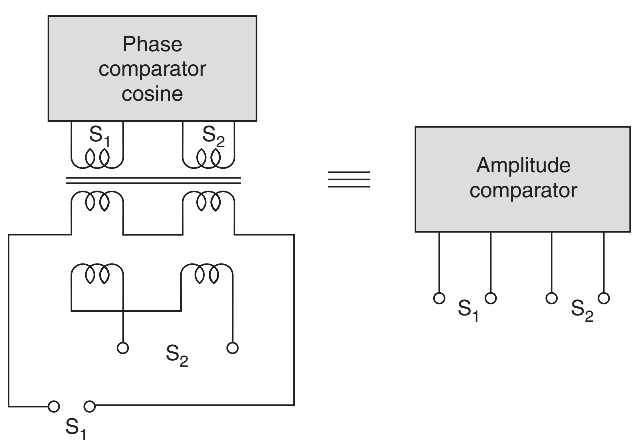


Fig. 14.68 Equivalence of amplitude comparator.

Static Amplitude Comparators

Mainly there are three types of amplitude comparators: (*i*) integrating comparators, (*ii*) instantaneous comparators, and (*iii*) sampling comparators. These are discussed in brief as follows:

Integrating Comparators: These are further classified as (*i*) circulating current type and (*ii*) voltage opposed type.

The basic circuit for the circulating current is shown in Fig. 14.69. The currents are the input signals. The relay will operate whenever $S_1 > S_2$, where $S_1 = Ki_1$ and $S_2 = Ki_2$. The voltage across the relay does not exceed twice the forward voltage drop of one of the rectifiers and this will normally be of the order of 1 volt. The voltage across the relay is given in Fig. 14.70. Ideally the comparator is independent of the phase angle between i_1 and i_2, but in practice the wave shape is dependent on the phase angle. When i_1 and i_2 are out of phase, the difference $(i_1 - i_2)$ has both the positive and negative loops. If $i_1 > i_2$, the positive loop is bigger than the negative and if $i_1 < i_2$, the negative loop is bigger than the positive. When $i_1 = i_2$ the positive and negative loops are equal, each loop occupying $\frac{1}{4}$ of the time duration of one cycle of the input currents. The output wave form is, therefore, a double frequency pulsation.

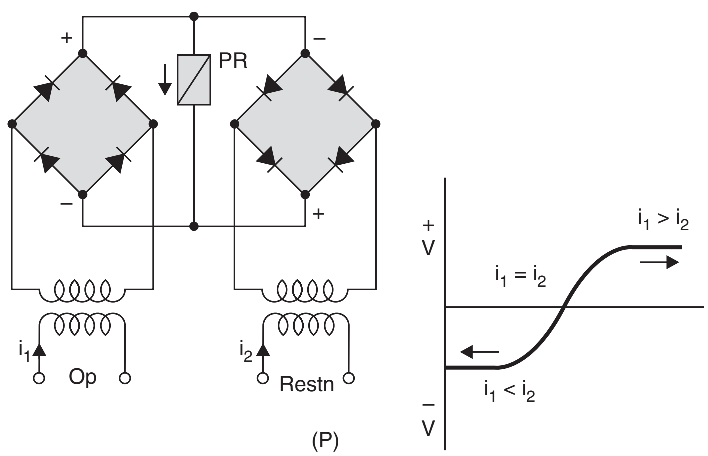


Fig. 14.69 Circulating current comparator.

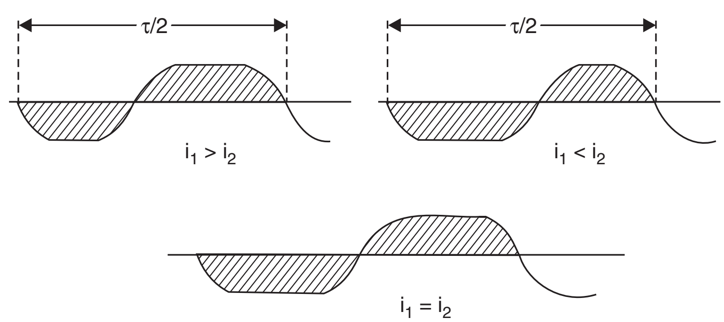


Fig. 14.70 Comparison of outputs.

It is, therefore, desired that the output device should be an integrating device responding to the average area over one cycle of the output wave form.

A static integrating circuit instead of a polarized relay can be used which consists of an averaging, polarity detecting circuit as shown in Fig. 14.71.

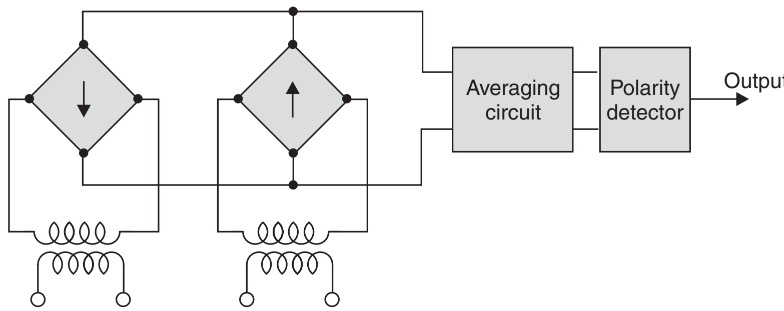


Fig. 14.71 Rectifier bridge comparator with static output device.

The two currents i_1 and i_2 are rectified and their difference $(i_1 - i_2)$ is averaged. If the average value is positive, output is obtained.

The opposed voltage type of comparator works with voltage input signals derived from PTS and is shown in Fig. 14.72. The operation of the relay depends on the average of the difference of the rectified voltages $(V_1 - V_2)$. The bridge is less sensitive at low inputs and the comparator has no limiting action on both voltage and current in the output device.

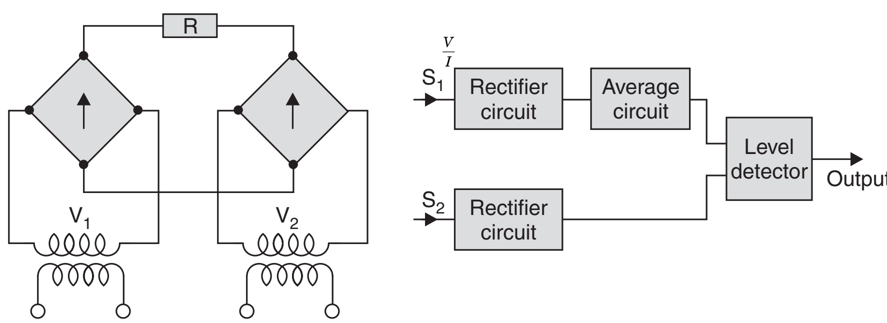


Fig. 14.72 The opposed voltage comparator.

Instantaneous Comparator: These comparators can further be classified as: (*i*) averaging type, and (*ii*) phase splitting type.

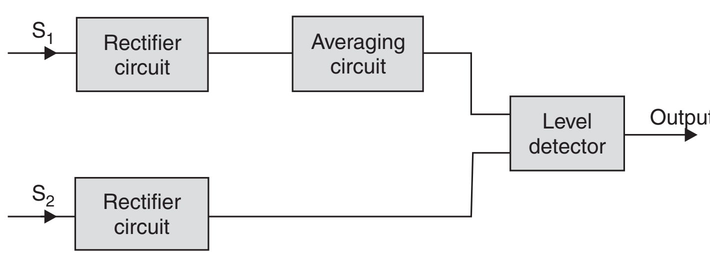


Fig. 14.73 Block diagram of averaging type instantaneous amplitude comparator.

In case of an averaging type the restraining signal is rectified and smoothened completely in order to provide a level of restraint. The operating signal is rectified full wave but is not smoothened. The peak of the operating signal should exceed the restraint level for operation. The block diagram is shown in Fig. 14.73 and the wave shapes are given in Fig. 14.74.

Smoothing is done with the help of a capacitor; as a result there is delay in operation. Better method is phase splitting before rectification *i.e.*, the input is split into six components 60° apart (Fig. 14.75), so that it is smoothed within 5%. In this case both operating and restraining inputs are smoothed out before being compared so that a continuous output signal is obtained. The time of operation is determined by the time constant of the slowest arm of the phase splitting circuit.

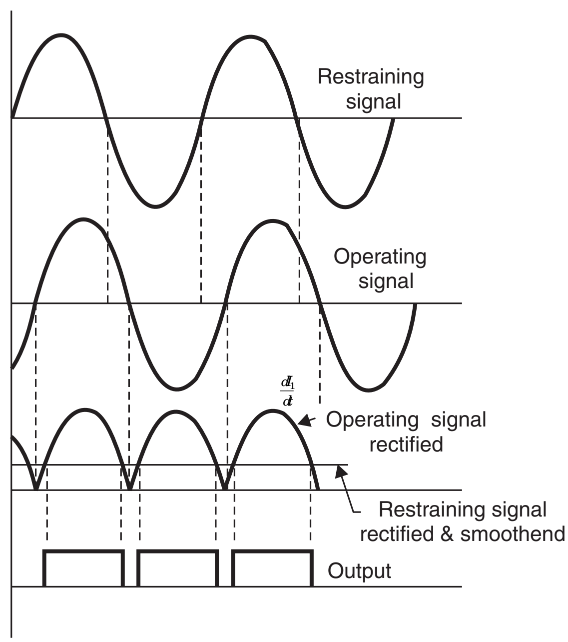


Fig. 14.74 Wave shapes of an instantaneous amplitude comparator.

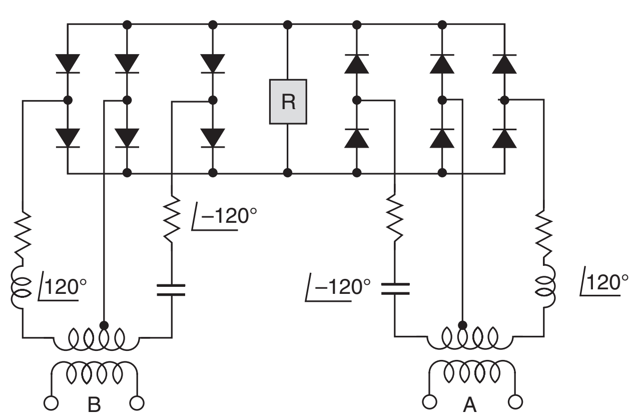


Fig. 14.75 Phase splitting of inputs (six-phase).

Sampling Comparator: In this comparator one or both the signals are sampled at the same instant or at different instants and are compared. When one signal is sampled, it is compared with the signal proportional to its rectified (average) value.

In case of reactance relay the sampled value of voltage is compared with the average value of current when it is passing through zero value. Let ϕ be the p.f. angle of the circuit. When current passes through zero the instantaneous value of voltage will be $V \sin \phi$. The reactance relay operates when $X < K$, where X is the reactance seen by the relay and K is design reactance of the system.

Since $X = Z \sin \phi$,

$$\therefore \quad Z \sin \phi < K$$

or
$$\frac{V}{I} \sin \phi < K$$

$$\frac{V_m}{\sqrt{2}} \sin \phi < KI_{av} \times 1.11$$

or
$$V_m \sin \phi < \sqrt{2}\, KI_{av} \times 1.11 < K'I_{av}$$

The block diagram for the comparator is shown in Fig. 14.76. Reactance relay operation can also be explained when both voltage and current signals are sampled. Voltage is sampled again when the current is passing through zero value and current is sampled after a delay of say α. Again if power factor of the circuit is $\cos \phi$, the voltage signal at the current zero instant will be $V_m \sin \phi_m$ and the current signal after a delay of α will be $I_m \sin \alpha$. For the reactance relay,

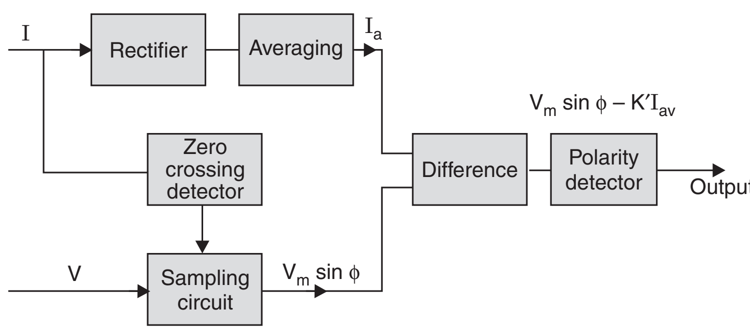


Fig. 14.76 Sampling comparator for reactance relay.

$$\frac{V}{I} \sin \phi < K$$

or
$$V_m \sin \phi < K'I_m \sin \alpha$$

The amplitudes of these two signals are converted into proportional pulse widths and these pulses are compared in an AND gate. In case the two sampled signals are taken at different instants of time, the pulse width representing the one taken first in time sequence is delayed by the time difference between the two sampling instants, before feeding to the AND gate. The scheme is shown in Fig. 14.77.

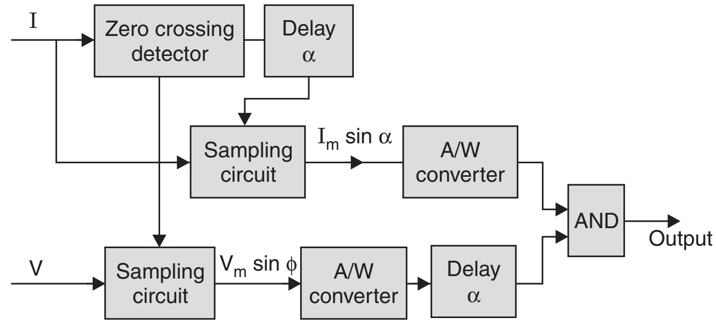


Fig. 14.77 Block diagram when both the signals are sampled.

With the use of sampling techniques, the phase shifting and mixing circuits are eliminated which results in saving in space and cost even though the sampling techniques need a higher degree of sophistication in the relay circuitry.

Phase Comparator

In this type of comparator, the operation of the relay takes place when the phase relation between two inputs S_1 and S_2 varies within certain specified limits. It is the phase relation between the signals that is mainly compared and an output is obtained which operates the tripping relays. Mathematically, the condition of operation is given by

$$-\alpha_1 \leq \theta \leq \alpha_2$$

where θ is the angle by which S_1 lags S_2. If $\alpha_1 = \alpha_2 = 90°$, the comparator is known as cosine comparator and if $\alpha_1 = 0°$ and $\alpha_2 = 180°$ it is known as sine comparator.

There are two types of phase comparators:

(*i*) Coincidence type; and

(*ii*) Vector product type.

Coincidence Type Phase Comparator: Consider two signals S_1 and S_2; their period of coincidence depends upon their phase difference. If the two signals have a phase difference of α, the period of coincidence of such signals is $\psi = (180° - \alpha)$ which means if the operation is desired for a phase angle α less than say + 90°, then coincidence period should be greater than 90°. Thus, the criterion for operation becomes $-90° \leq \alpha \leq 90°$ which is illustrated in Fig. 14.78.

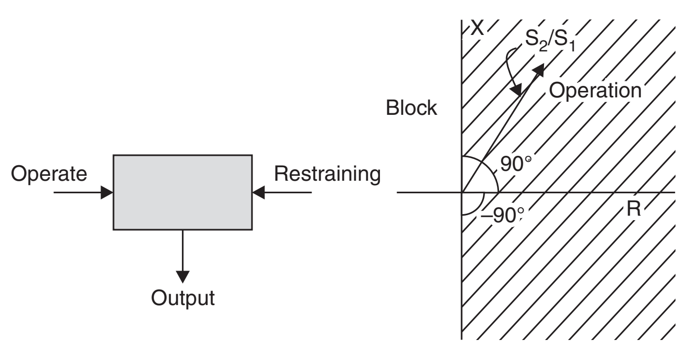


Fig. 14.78 Phase comparator output when angle θ between S_1 and S_2 is within limits – 90° and + 90°.

By measuring the period of coincidence, it is possible to design the circuit to give an output a 'Yes' or a 'No' depending upon the phase relation of the input signal. Some of the techniques employed to measure the period of coincidence are given below:

Block-spike phase comparison: In this method one input is converted into a square wave and the other into a pulse of short duration (known as a spike) at the instant when this input is either passing through zero value or when it is passing through peak value. The squared wave and the spike then are fed into an AND gate and there is an output when the two signals coincide at any time as shown in Fig. 14.79 (*a*). Depending upon the instant of spiking (*i.e.*, whether at zero or peak value) the output is available for different phase differences. With spike derived at peak value, output for $-90° \le \alpha \le 90°$ and with spike derived at zero value output for $0° \le \alpha \le 180°$ is obtained. The main disadvantage of this method is that in case of spurious spike due to any switching or external interference, operation of the relay may take place which is not desirable. Shielding of the circuit against electric and magnetic field is, therefore, essential.

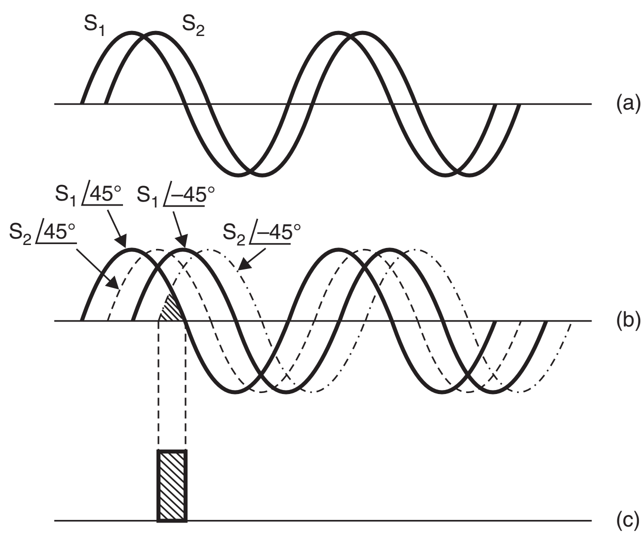


Fig. 14.79 Phase splitting technique.

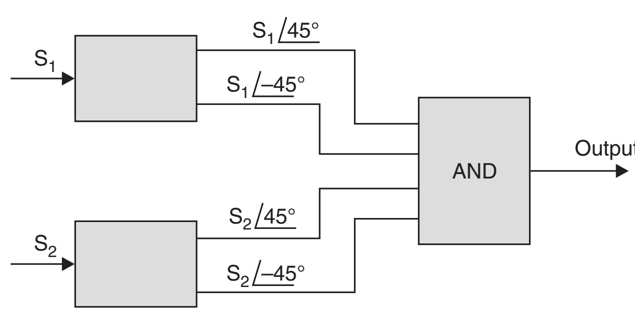


Fig. 14.80 Phase splitting (block diagram).

Phase splitting technique: The method requires splitting of phase of the two input signals, each into two components shifted ± 45° with respect to the original signal. The four components are then fed into an AND gate which gives an output when the four signals are positive simultaneously at any time in the cycle as shown in Fig. 14.79. It can be seen that output will be obtained for $-90° \le \alpha \le 90°$. The block schematic is shown in Fig. 14.80.

Because of the time constants of the phase shift circuit, the method is slower than the block spike method. The time of operation can be reduced to less than half a cycle by using two such comparators for each polarity. This method, however, is not affected by spurious signals.

Integrating phase comparator: The two signals S_1 and S_2 are fed into an AND gate the output of which is integrated to measure the period of coincidence of the two signals. If period of coincidence exceeds 90°, the output is obtained so that the condition is $-90° \le \alpha \le 90°$ for operation. The most common type of AND gate uses diode or transistor coincidence circuit as shown in Fig. 14.81.

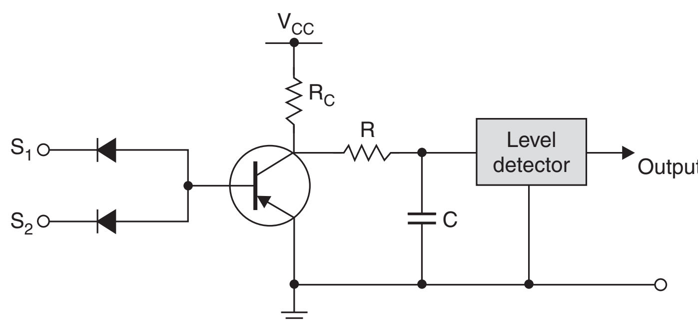


Fig. 14.81 Integrating type phase comparator.

During the positive coincidence period capacitor C charges through R and then discharges suddenly as the coincidence period ends. A level detector compares the voltage built up across the capacitor with a fixed voltage level corresponding to 90° charging period and gives an output if the former exceeds the latter. If we use two comparators for the two polarities (two half cycles), the operating time can be reduced to less than half a cycle.

Integrating type comparator with rectifier type AND gate: The rectifier circuit is shown in Fig. 14.82. The signal i_1 is known as the operating signal and i_2 the gating signal. The device works on the principle that a diode functions as a gate so long as it is kept open by a forward current. Current can flow both in the forward and reverse direction provided the reverse current is less than the forward current.

The gating current is more than two times the operating current. During one half cycle the gating current $i_2/2$ flows through D_1 and D_2; thereby it opens these diodes and allows the current i_1 to flow through D_1 in the forward direction and through D_2 in the reverse direction. The voltage across PQ due to i_1 is say of positive polarity. Since the current $i_2/2$ flows in opposite direction through the resistor, the drop due to gating current is zero. During the next half cycle the gating signal flows through D_3 and D_4 and since i_1 flows in reverse direction, therefore, output of opposite polarity appears across PQ. The output across PQ can be shown to be proportional to cosine of the angle between the two signals. The output is fed to the polarity detection circuit, the *RC* charging circuit and the level detector circuit as in the previous phase comparator. The output is positive during positive coincidence period and negative during anticoincidence period.

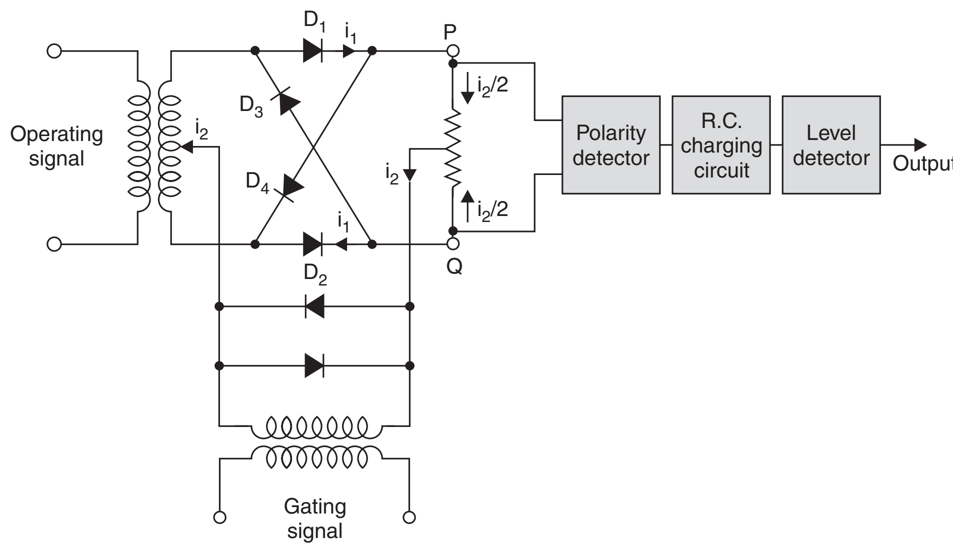


Fig. 14.82 Basic circuit of an integrating phase comparator using rectifier bridge AND gate.

Vector Product Phase Comparator: In these devices an output proportional to the vector product of two input quantities is obtained. These devices operate on the principle of Hall effect and magneto resistivity.

Hall effect comparator: This comparator is based on Hall effect discovered by E.H. Hall. The semiconductors normally used as Hall element are indium antimonide and indium arsenide. Of these the latter is considered as a better Hall element. The basic principle of operation of Hall element is shown in Fig. 14.83. When a current I is passed along X-direction of the Hall element which is placed in a magnetic field in the Y-direction, a voltage known as Hall voltage is induced in the Z-direction across the edges of the element.

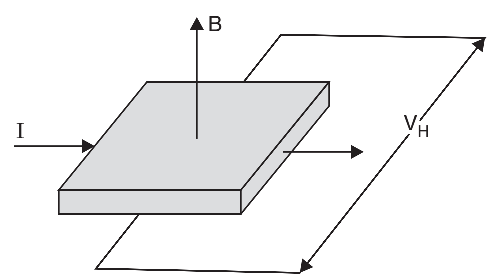


Fig. 14.83 Hall effect phase comparator.

If the two inputs are ϕ and I, and are sinusoidal quantities, given by

$$\phi = \phi_m \sin \omega t$$
$$I = I_m \sin (\omega t - \alpha)$$

the vector product of the two vectors is given by

$$V_H = K\phi I \sin 90° = K\phi_m I_m \sin \omega t \sin (\omega t - \alpha)$$
$$= \frac{K\phi_m I_m}{2} [\cos \alpha - \cos (2\omega t - \alpha)]$$

It is clear from the expression that Hall voltage consists of a d.c. component and a time varying component of double the original frequency. The double frequency component can be eliminated by cross-connecting two Hall elements as shown in Fig. 14.84. The two input signals are the two sinusoidal currents I_1 and I_2.

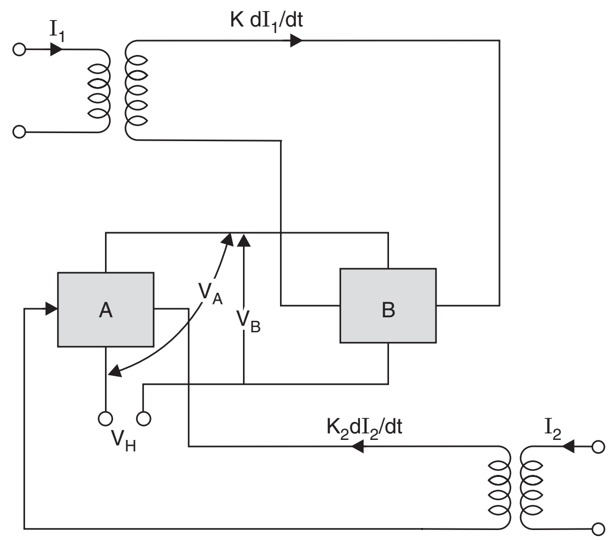


Fig. 14.84 Cross connection of two Hall elements.

Let
$$I_1 = I_{m_1} \sin \omega t$$
$$I_2 = I_{m_2} \sin (\omega t + \alpha)$$

The two fluxes ϕ_A and ϕ_B through the elements A and B are $\phi_A \propto I_1$ and $\phi_B \propto I_2$, and the currents through the elements are $I_A \propto \dfrac{dI_2}{dt}$ and $I_B \propto \dfrac{dI_1}{dt}$.

Since the two elements are so connected that the output voltages oppose each other, therefore, the resultant voltage is given by

$$V_H = V_A - V_B$$
$$\propto I_{m_1} \,.\, \sin \omega t \,.\, I_{m_2} \omega \cos (\omega t + \alpha) - I_{m_2} \sin (\omega t + \alpha) \omega I_{m_1} \cos \omega t$$

or
$$V_H \propto I_{m_1} I_{m_2} \sin \alpha$$

The device thus acts as a sine phase comparator. Because of the high cost of Hall element, large temperature error and low output, this comparator is normally not used.

Magneto-resistivity comparator: When a semiconductor is subjected to a magnetic field its resistivity varies. This effect is known as Gauss effect or magneto resistivity. If a voltage V_1 produces a magnetic field through a semiconductor disc and another voltage V_2 passes current through the disc at right angles to the magnetic field, the current will be proportional to V_1V_2 $\cos \theta$, where θ is the angle between the two voltages *i.e.*, the current is maximum when the two voltages are in phase and zero when they are in quadrature. This type of relay is mostly used in USSR and is considered better than Hall element relay because of simpler construction and circuitry (see Fig. 14.85). Polarizing current is not required and output is relatively higher.

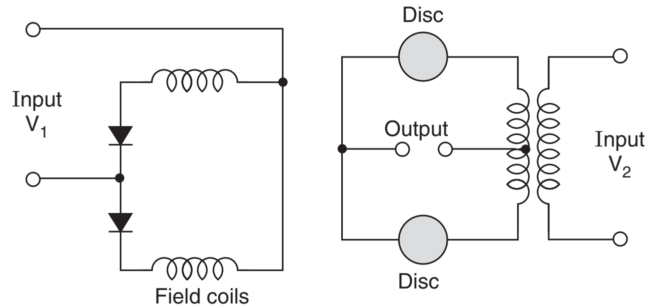


Fig. 14.85 Phase comparator magneto resistivity.

14.16 STATIC RELAYS

The term static relay refers to a relay which incorporates solid state components like transistors, diodes etc., for the measurement or comparison of electrical quantities. The static network is so designed that it gives an output signal in the tripping direction whenever a threshold condition is reached. The output signal in turn operates a tripping device which may be electronic or electromagnetic.

The need for the static relays arose because of the requirement of fast and reliable protective schemes for the modern power systems which is growing both in complexity and fault levels. The scheme should be fast so as to preserve dynamic stability of the system as the character and loading approach design limits. The supply problem associated with the thermionic valves has been solved with the use of semiconductors. The transistors have made it possible to achieve greater sensitivity and at the same time excellent mechanical stability which is not possible with the electromechanical relays. It is to be noted here that it is usually not economical to replace existing electro-mechanical relays with their static counterparts just to reduce maintenance. The protective relays, nowadays, are being fed by iron cored current transformers and hence excessive saturation should be avoided to ensure high speed and discriminative operation. The static relays reduce the burden on the current transformer.

It is interesting to note that the static relays have first been commercially manufactured for the distance and differential protective schemes whereas the much simpler overcurrent relays have not been brought out. The reason behind this is that the distance and differential schemes are more amenable to mathematical analysis whereas the overcurrent characteristics are more of empirical nature. Therefore, a static overcurrent relay cannot compete with the conventional electromechanical relay. With the use of static relays it has been possible to obtain many varied and complex distance protection characteristics which is impossible to obtain with the conventional electro-mechanical relays.

The use of electronic valves for static relays was taken up by Fitzgerald in 1928 who presented a carrier current protection scheme for the transmission line. In spite of the advantages like fast operation, low maintenance, low CT and PT burden offered by the valves, they suffered inherently from the requirements of HT supply, short life, large power consumption, LT supply for the heater elements. These relays could not meet practical requirements and hence never reached the commercial stages.

Transductor relays are magnetic amplifier relays which consist of a control and operating winding. The control winding is energized with d.c. and the operating winding with a.c. The transductor relays are mechanically very simple and are quite reliable. Since the relay rectifies and smoothens a signal, a delay is introduced because of the time constant of the smoothing circuit and the relays are slow and, therefore, are discarded for protection applications.

Rectifier bridge relays, initially used in Germany, revolutionalized the development of static relays. This relay consists of two rectifier bridges and a moving coil or polarized moving iron relay. These will be discussed later on in this chapter.

Transistor relays are the most widely used static relays. In fact when we talk of static relays we generally mean transistor relays. The fact that a transistor can be used both as an amplifying device and as a switching device, makes this component suitable for achieving any functional characteristic. The transistor circuits cannot only perform the essential functions of a relay such as comparison of inputs, summation and integrating them but they also provide necessary flexibility to suit the various relay requirements. The advantages of transistor relays can be summarized as follows:

1. The power consumption is low and hence provides less burden on the CTs and PTs as compared to the conventional electromechanical relays.

2. The relays are fast in operation.

3. No moving parts, hence friction or contact troubles are absent and as a result minimum maintenance is required.

4. The relays have greater sensitivity as amplification of signals can be obtained very easily.

5. The relay has a high reset to pick up ratio and the reset is very quick.

6. The use of printed circuits avoids wiring errors and facilitates rationalization of batch production.

7. It is possible to obtain wide range of characteristics approaching more or less to the ideal requirements.

Transistor relays, however, have the following limitations:

1. The characteristics vary with temperature and ageing.

2. The reliability of the scheme depends upon a large number of small components and their electrical connections.

3. The relays have low short time overload capacity compared with electromechanical relays.

With the advancement of semiconductor manufacturing technology it has been possible to manufacture transistors which are insensitive to temperature variation and ageing and careful design of the static relay circuitry can compensate for the other limitations.

Before we proceed further to study the static relays. A brief introduction of semiconductor devices is given below:

A semiconductor is a material which has its conductivity lying between a good conductor and an insulator. Since these materials are solid and include on moving parts, these are also called solid state devices. Most of the diodes and transistors are made with junctions of large area formed between two different types of silicon. If phosphorous, arsenic or antimony is added as an impurity to the pure silicon metal, an excess of electron is given so that it has *N*-type conductivity wherein the majority carriers are the electrons. If the added impurity is boron, aluminium or indium, holes are created in the pure silicon so that it acquires *P*-type conductivity *i.e.*, the majority carriers are the holes. A rectifier unit can be formed in a single piece of silicon if one end is changed into *P*-type material and the other end is changed into *N*-type material . In this way, a barrier layer or junction appears between the two kinds of material as shown in Fig. 14.86; this is called a junction rectifier or diode and it contains a *PN* junction. A diode is said to be forward biased when *P* terminal is made more positive than the *N*-terminal and current passes easily through the diode if only a small voltage is applied. However, if the applied voltage is reversed so that *P* material is made more negative than *N* material, the diode is said to be reverse biased as shown in Fig. 14.86 (*d*). If the applied voltage is small, the current is also small and is known as leakage current. If the reverse voltage is increased to a large value, known as breakdown voltage, the diode loses its blocking property and a large avalanche current limited only by the external resistance will flow. See Fig. 14.87 for the characteristic of the diode.

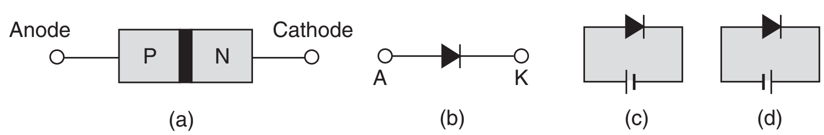


Fig. 14.86 (*a*) Diode; (*b*) Symbol of a diode; (*c*) Diode forward biased; and (*d*) Diode reverse biased.

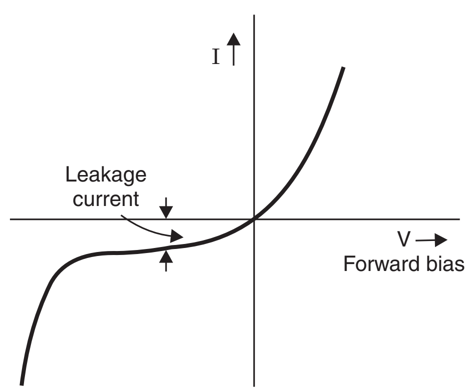


Fig. 14.87 Characteristic of a diode.

Transistor: A single piece of silicon if doped with the same type of impurity at either end and the central section has characteristics different from either end, a transistor is produced. If the end sections are doped with *P*-type material and the central section with *N*-type, a *PNP* transistor is produced. The two ends are emitter and collector and the central section the base. Although both ends may be *P*-type material, the emitter and collector are not interchangeable as the emitter is made with different dimensions and with heavier doping than the collector.

The transistor could be considered as two diodes in series. One diode is the junction between base and emitter which is usually forward biased and the other diode is the junction between base and collector which must be reverse biased (see Fig. 14.88).

This is the kind of bias when the transistor is operating as an amplifier and the mode of operation is known as common emitter as emitter is the common terminal between the input and output terminals. The other two modes of operation are common base and common collector which are shown in Figs. 14.89 and 14.90 respectively.

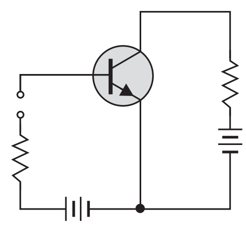


Fig. 14.88 NPN forward biased.

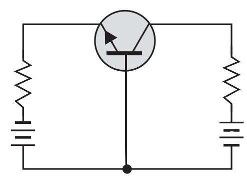


Fig. 14.89 Common base mode.

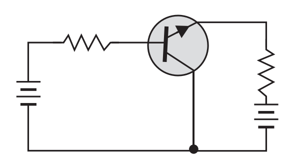


Fig. 14.90 Common collector mode.

Because of the high gain of common emitter circuit, it is most commonly used as an amplifier.

Comparative study of different configurations of transistor amplifiers

	Common emitter	*Common base*	*Common collector*
Voltage gain	High	High	Low
Current gain	High	Low	High
Power gain	High	Medium	Low
Output impedance	Medium	High	Low
Input impedance	Medium	Low	High
Phase shift	180°	0°	0°

Transistor as a switch: Transistor for relaying purposes is more often used as a switch rather than as an amplifier. Here also common emitter connections are most commonly used. For putting the transistor into ON position the emitter-base and the collector-base junctions are forward biased and for OFF both the junctions are reverse biased.

D.C. Amplifiers: When one transistor is not enough to provide required amplification, two or more than two transistors are connected in cascade so that the output of one becomes input to the next and so on till desired amplification is obtained. The cascading may be through some component like the capacitor or the transformer or direct cascading without any component between two stages of amplification. If the input signal is a slowly changing d.c. voltage or low frequency a.c. voltage, a direct connection is made from the collector of T_1 to the base of T_2 as shown in Fig. 14.91. This combination is known as a two-stage direct coupled amplifier.

In Fig. 14.91, it is to be noted that when collector current increases in T_1, the collector current decreases in T_2. This is true when both transistors are NPN or PNP. However, if the

NPN T_2 is replaced by a PNP transistor shown as T_3 in Fig. 14.92, the increase of T_1 collector current still drives point A more negative which increases the flow of electrons through R_3 and from collector to emitter of T_3.

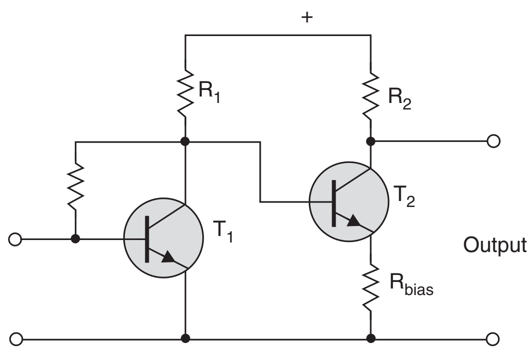


Fig. 14.91 DC amplifier with both NPN transistors.

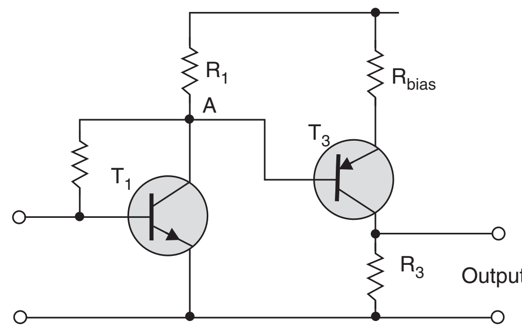


Fig. 14.92 DC amplifier with NPN and PNP transistors.

Darlington Circuit: If two transistors of the same type are directly connected as shown in Fig. 14.93, the emitter of T_1 is connected to base of T_2, the combination has a total of three external connections B, C and E, and acts as a single transistor whose gain equals the T_1 gain × the T_2 gain. In order to handle greater load current T_1 may be connected to two or more than two transistors in parallel as shown in Fig. 14.94. The R_E resistors help to equalize the load current.

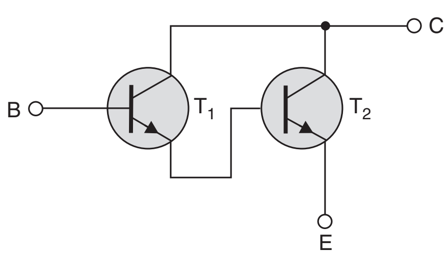


Fig. 14.93 Darlington circuit.

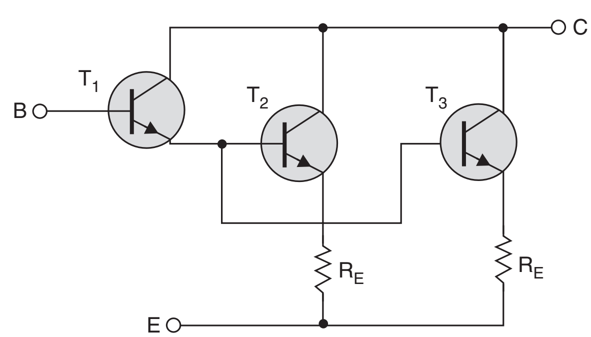


Fig. 14.94 High current Darlington circuit.

Schmitt Trigger Circuit: When a pair of transistors is direct coupled as shown in Fig. 14.95, it provides a sudden turn-on or triggering action by T_2 and occurs at a selected value on a slowly changing signal applied to the base of T_1. So long as the input signal is below the desired trigger point, T_1 has no current flow. The resistances R_2, R_3 and R_7 form a voltage divider across the HT supply so that the T_2 base has a potential of about +7 volts if HT supply is 12 volts and R_2 = 2 kΩ, R_3 = 3 kΩ and R_7 = 7 kΩ; electrons flow through R_6, emitter and collector of T_2 and through R_4. While conducting, the T_2 emitter must have forward bias. The output across R_4 is obtained.

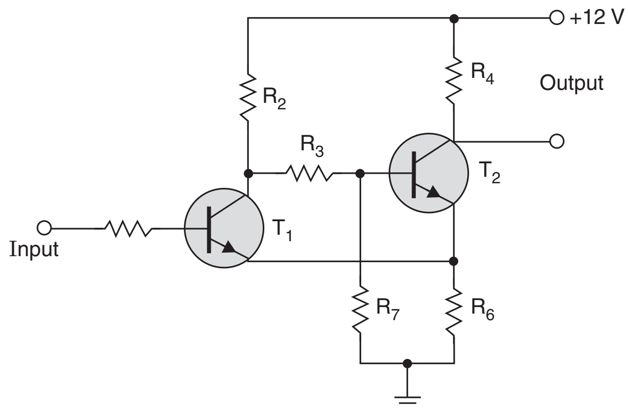


Fig. 14.95 Schmitt trigger circuit.

The Thyristor Family: Thyristor is a bistable semiconductor device, comprising, three or more junctions which can be switched from the "off-state" to the "on-state" or vice-versa. Even though power transistors with high current and voltage ratings are now available, the basic differences in the fabrication and operation of a thyristor and a transistor make it possible for

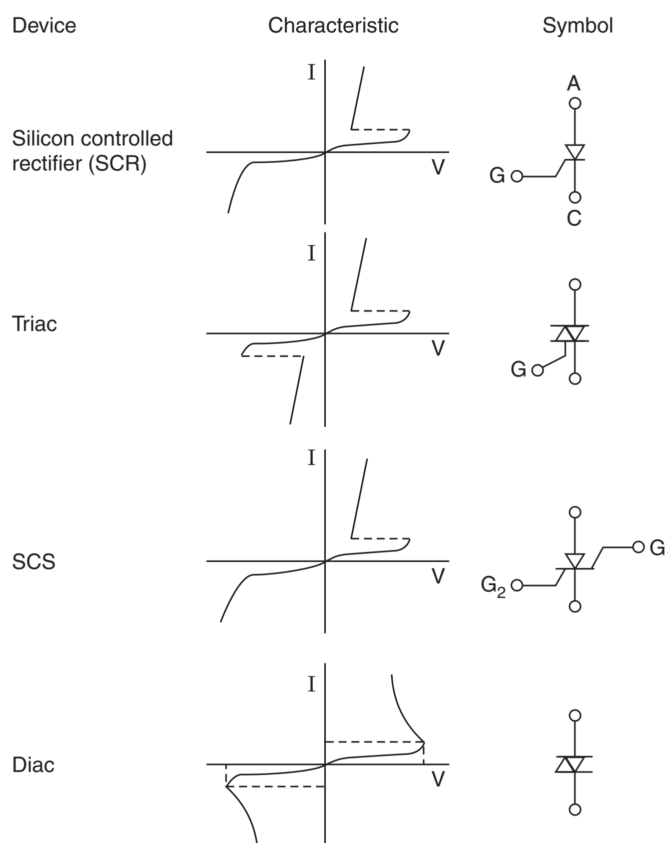


Fig. 14.96 Thyristor family with characteristics and symbols.

the former to have much higher voltage and current ratings for a given size than those of the latter. For conduction, a transistor requires continuous base current whereas for an SCR, a single gate pulse is required for its conduction. Further an SCR is used as a switching device whereas a power transistor is required to operate in the active region in many applications. The other members of the thyristor family are in general low power devices except the triac which is a bilateral device with three terminals and conducts in both the directions. The triac is equivalent to two SCRs connected in anti-parallel. The silicon controlled switch (SCS) is similar to the SCR except for the fact that SCS has two gates and, therefore, can be turned on or off by any of the gates. The diac is a two-terminal, four-layer device which is generally used for triggering triacs. Fig. 14.96 shows a few of these devices and their respective *V-I* characteristics and symbolic representation.

Zener Diode: If the impurities added to PN junction are more than the normal, the breakdown voltage is decreased. Many diodes are made with the purpose of operating often or continuously at a desired value of break-down voltage; such a diode may be used so as to limit or regulate the amount of voltage applied to a load circuit. The characteristic and its application as a voltage regulator are shown in Fig. 14.97.

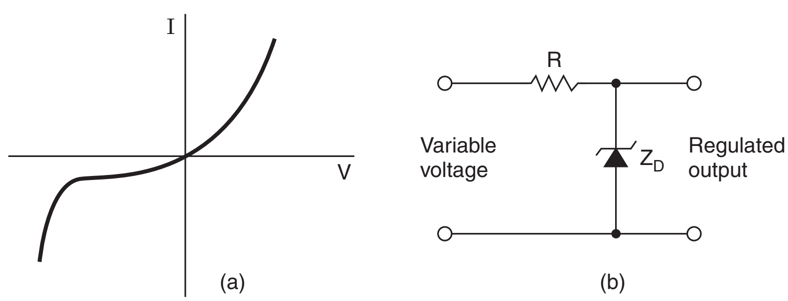


Fig. 14.97 (*a*) Characteristic of Zener diode; and (*b*) Zener diode as voltage regulator.

The Unijunction Transistor (UJT): A unijunction transistor consists of a bar or crystal of *N*-type silicon of high resistance; an ordinary ohmic contact is made at each of the ends which are called base 1 and base 2. The UJT is not like other transistors but is used as a switching device to apply a sudden pulse of power to energize a relay or to fire an SCR. The characteristic of UJT and its application for firing an SCR are shown in Fig. 14.98.

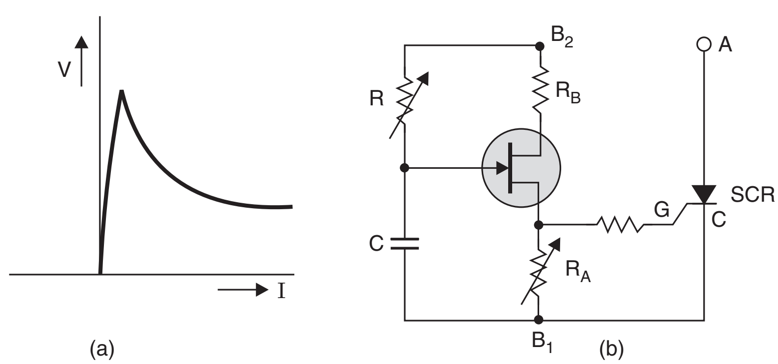


Fig. 14.98 (*a*) Characteristic of UJT; (*b*) SCR firing by UJT.

Time Delay Circuits: Various types of circuits for obtaining time delay depending upon the amount of delay required are given in Fig. 14.99. If the delay required is in terms of few microseconds a delay cable is used; for medium delays of the order of milliseconds a resonant circuit is used whereas for delays of the order of minutes or even hours RC circuits are used.

Level Detectors: The level detector compares an alternating or unsmoothed rectified signal against a d.c. datum. Whenever the peak input exceeds the d.c. datum, an output is there; otherwise it is zero. The simplest form of level detector is shown in Fig. 14.100, where the input voltage must exceed the opposing bias voltage for any output.

Schmitt trigger circuit (Fig. 14.95) is another level detector circuit which is normally used for the purpose.

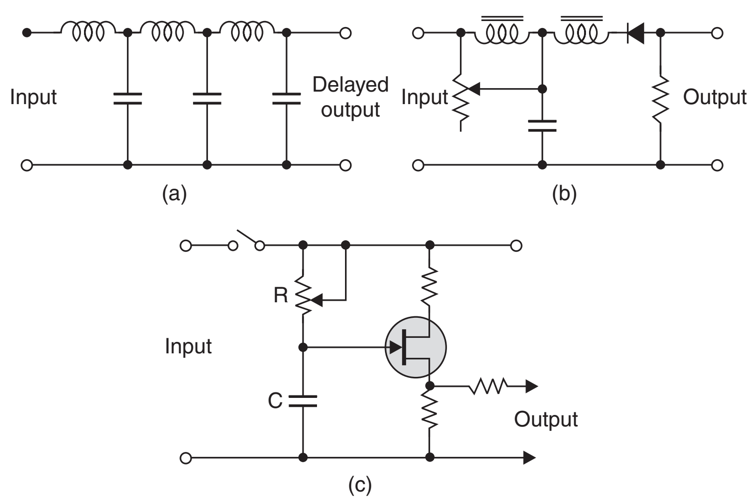


Fig. 14.99 Time delay circuits; (*a*) a delay line; (*b*) a resonant circuit; and (*c*) an R-C circuit.

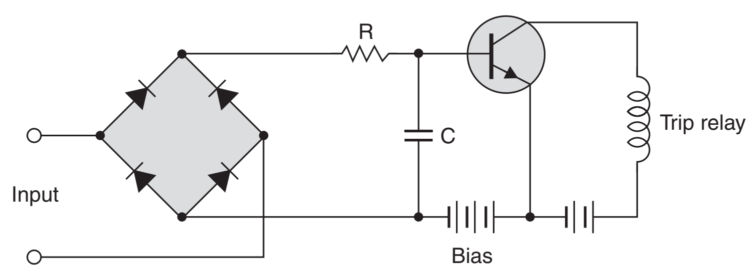


Fig. 14.100 Level detector.

Positive Feedback: When the output of a device is not proportional to the input signal, it is said to be non-linear. This is the result of positive feedback which means output is fed back to the input in phase with the input so that the input is strengthened. This leads to rapid increase of both the input and the output signals which leads to what is known as snap action in electromagnetic relays.

A multivibrator is a circuit constructed by coupling two amplifiers together using strong positive feedback. If in the absence of the triggering pulses, the circuit can remain permanently in only one state, it is monostable; if it can remain permanently in either state, it is bistable; and if the circuit cannot remain permanently in either state, it is astable. The devices need two transistors of equivalent characteristics. Bistable circuits are normally known as flip-flops.

Monostable Multi-vibrator: Refer to Fig. 14.101, where T_2 is coupled to T_1 through the capacitor C. Under equilibrium condition T_2 is ON and V_{BB} causes T_1 to be OFF and the capacitor C_t is charged approximately to V_{CC}. Resistor R_1 is chosen so that T_1 is ON if T_2 happens to be OFF—as a result of a negative going pulse. If this pulse turns T_2 OFF, T_1 turns ON and remains ON so long as T_2 is OFF. When T_1 is ON, the left hand terminal of C_t is effectively grounded and the base of T_2 is driven negative by an amount equal to V_{CC}—T_2 is held OFF by the charge on C_t. This charge decreases as C_t charges towards + V_{CC} through R_t and T_2 turns ON again when the voltage across C_t reaches nearly zero. This turns T_1 OFF and the circuit reverts to its stable state.

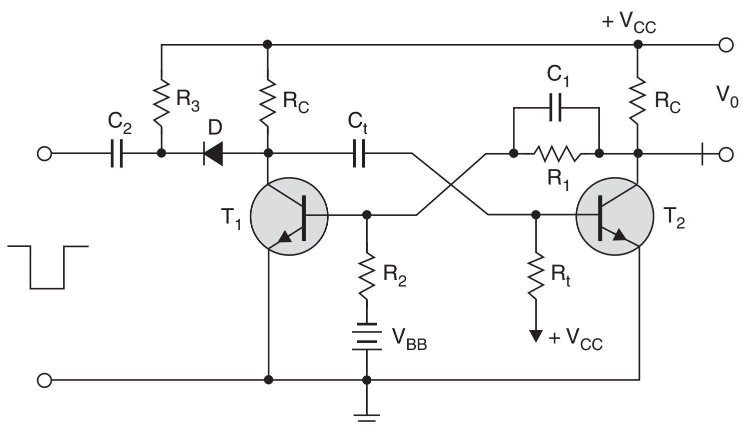


Fig. 14.101 Monostable multi-vibrator.

Bistable Multi-vibrator: Refer to Fig. 14.102 for a bistable multi-vibrator or a flip-flop which consists of two inverters in cascade where the output of the second inverter is fed to the input terminals of the first inverter. Bistable circuit is used as an output element to switch a polarized moving coil relay which in turn switches the auxiliary relays in the trip circuit. The polarized relay is connected in the collector circuit of one of the transistors, say T_1, such that its coil resistance and any additional resistance will equal the value of R_c used in the collector circuit of T_2. A capacitor C is connected across the polarized relay and the additional resistance so that T_2 is ON when switch S is closed. At the time of switching, the base of T_2 is quickly driven to conduction because of the presence of the capacitor C.

If a positive pulse is now applied at the base of T_1, it is driven to conduction and T_2 is cut OFF. As a result of this, the polarized relay is switched ON and it remains in this state till the circuit is reset either by applying a reset positive pulse to the base of T_2 or opening or closing of the switch S.

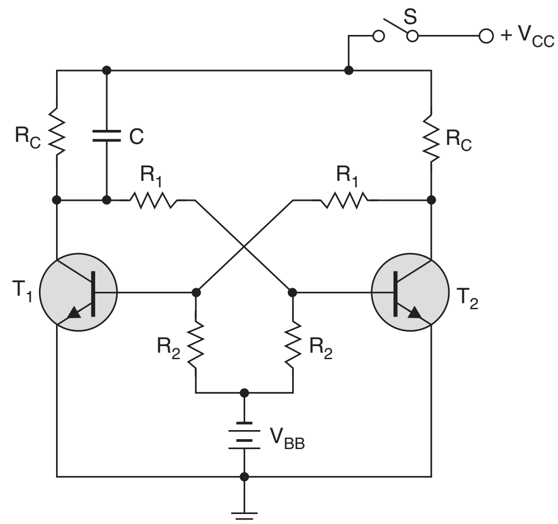


Fig. 14.102 Bistable multi-vibrator.

Astable Multi-vibrator: Refer to Fig. 14.103 wherein the astable multi-vibrator is shown as two transistors identically connected. Due to component tolerances, however, an abrupt application of power causes one transistor to turn on first.

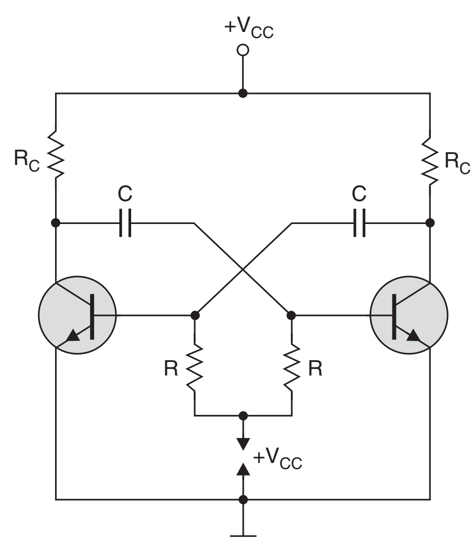


Fig. 14.103 Astable multi-vibrator.

When a transistor turns on, the capacitor connected to its collector drives the base of the other transistor negative turning it off. This is held OFF by the capacitor till its voltage falls to nearly zero. The transistor then turns ON, turning OFF the other one. This process is repeated.

A switching circuit is said to be non-regenerative if an external drive is required to hold the active devices in their desired operating states. The non-regenerative circuit is similar to a simple relay that requires a continuous current to keep its contacts closed. Refer to Fig. 14.104 for a basic non-regenerative switching circuit.

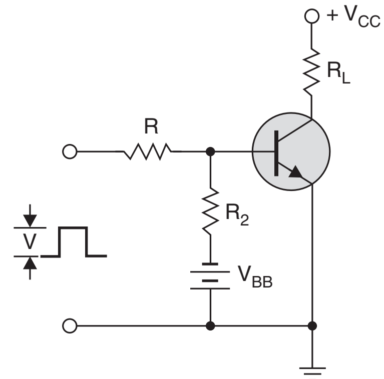
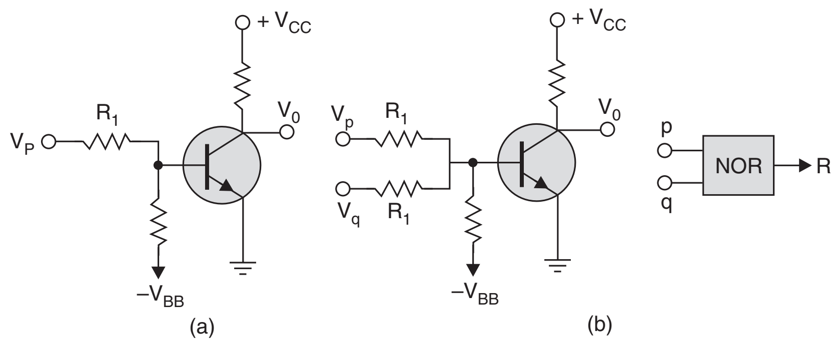


Fig. 14.104 A non-regenerative switch.

Here NPN transistor is turned ON by an input pulse of magnitude V and turned off after the pulse reduces to zero. The base bias V_{BB} aids the transistor to turn off and keeps it in that state till the next pulse appears.

Different logic circuits can be obtained by using the basic non-regenerative circuit.

Figure 14.105 (*a*) is an inverter circuit. There is no output voltage when V_P is present while there is an output when V_p is absent.



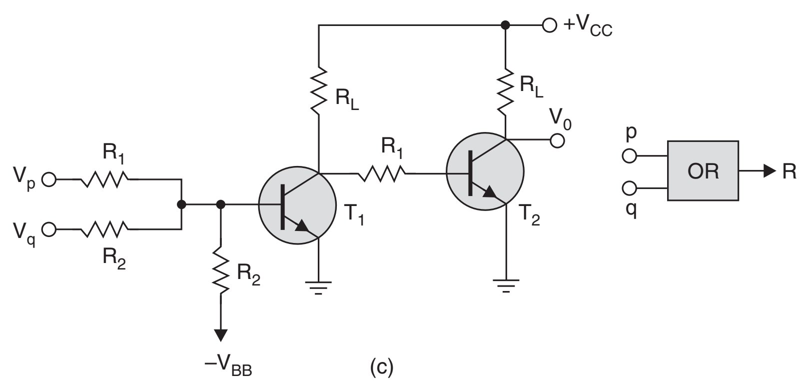


Fig. 14.105 Logic circuits: (*a*) Inverter; (*b*) NOR circuit; (*c*) OR circuit; (*contd.*).

Figure 14.105 (*b*) is a NOR circuit which has no output if either V_p or V_q is present.

Figure 14.105 (*c*) is an OR circuit which is obtained by connecting an inverter circuit at the output of the NOR circuit and thus there is an output if either V_p or V_q is present.

Figure 14.105 (*d*) is a NAND circuit which has no output if V_p and V_q are present.

Figure 14.105 (*e*) is an AND circuit which is obtained by connecting an inverter at the output of the NAND circuit and thus has an output if V_p and V_q are present.

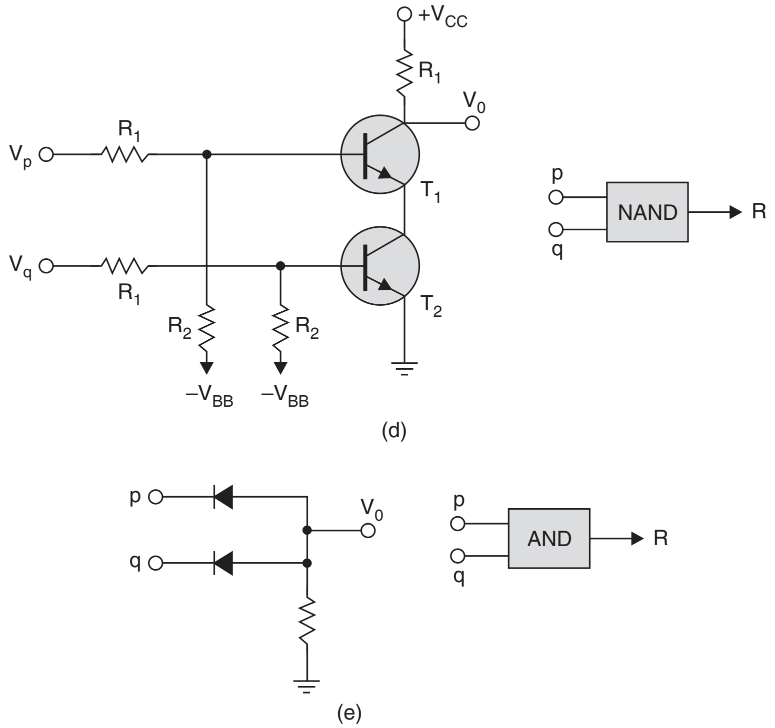


Fig. 14.105 (*d*) NAND circuit; and (*e*) AND circuit.

Overcurrent Relays

The overcurrent relays, even though simplest of all types of electromechanical relays, are the most difficult static relays. This is because the induction disc characteristics of the overcurrent relays (inverse characteristics) are not amenable to simple mathematical analysis. The first static relays developed were the high speed differential relays and the distance relays. Although the static overcurrent relays are complicated in circuit constructions they have the advantages of low CT burden and occupy less space on the relay panel as compared to the electromechanical relays.

Instantaneous Overcurrent Relays: Figs. 14.106 (*a*), (*b*) and (*c*) indicate the basic circuit of the relay, the block diagram of the modified version of the relay and detailed circuit of the relay respectively.

As shown in Fig. 14.106 (*a*), the auxiliary transformer is fed from the main current transformer. The auxiliary transformer or the transactor gives an output voltage proportional to the fault current. The filter circuit R_1C_1 protects the bridge circuit from transient overvoltages. R_2C_2 is a filter circuit to smooth out the output from the bridge circuit. *Th* is a thermistor to give temperature compensation to the transistor T_1, P is the potentiometer for selecting different pick up values. D is the diode to protect the output transistor T_2 from high reverse voltages induced when the inductive output circuit (relay trip coil *TR*) is opened. Initially the transistors T_1 and T_2 are not conducting. Whenever a short circuit takes place, a voltage proportional to short circuit current develops across R_2C_2 and hence between the base and emitter terminals of T_1. Whenever the base voltage of T_1 exceeds the pick up value set by the potentiometer P, the transistor T_1 conducts which in turn conducts T_2 the output transistor. Thus, the trip coil of the breaker is energized. The pick up value of the relay is adjusted both by the tap position of the transactor and the potentiometer P. Refer to Fig. 14.106 (*b*) for the block diagram of the modified version of the instantaneous overcurrent relay. The current proportional to the fault current is fed to the transactor which has an output proportional to this current. This voltage is rectified and partially filtered so as to provide high speed of operation (filtering provides inherent time delay). The partially filtered voltage is limited by a limiter (zener diode) and is then compared against a preset pick up value (again a zener diode) and if it exceeds it, a signal is given to the output transistor through an amplifier. The output transistor conducts and the breaker trip coil is energized.

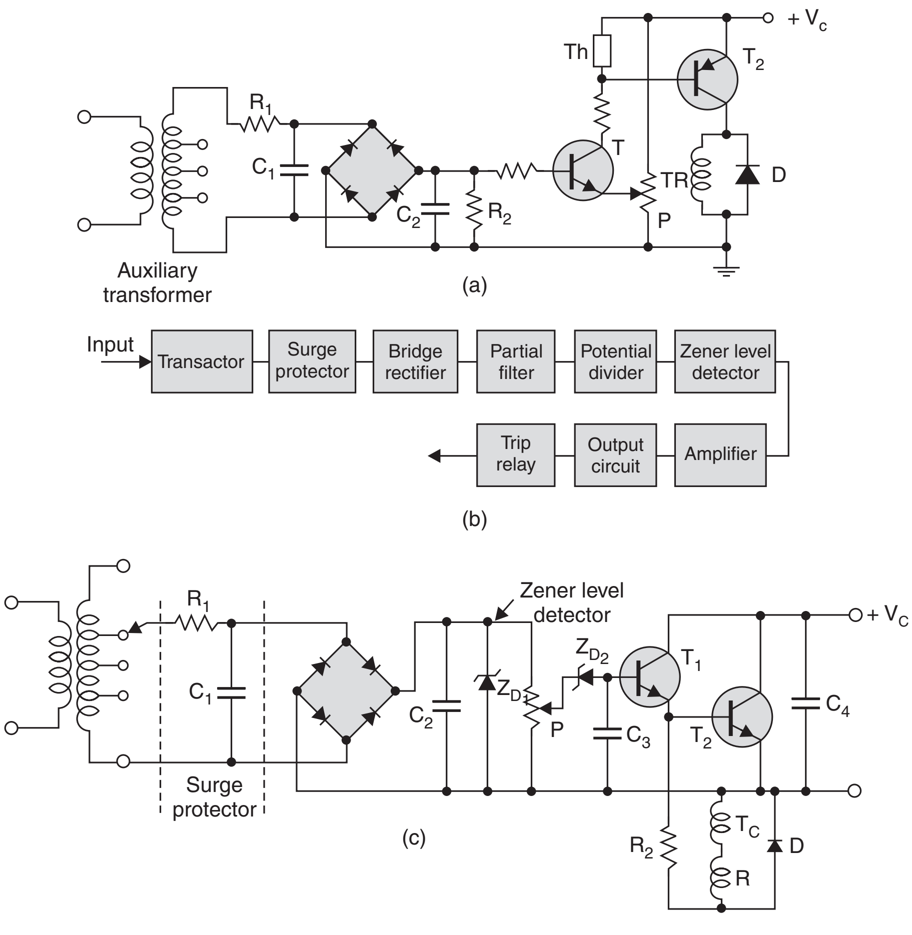


Fig. 14.106 Instantaneous overcurrent relay: (*a*) basic circuit; (*b*) block diagram of improved version; and (*c*) detailed circuit.

The operation of the modified version of the relay is explained as follows. Refer to Fig. 14.106 (*c*). The current proportional to the fault current is fed to the transactor which has an output proportional to this current. R_1C_1 protects the bridge circuit from transient overvoltages. The output voltage from the transactor is rectified and smoothed partially. The zener diode Z_{D_1} acts as a limiter of the rectified voltage to safe value even though the fault current is very high. A fixed portion of the rectified voltage through a potential divider is compared against the breakdown voltage of another zener diode Z_{D_2}. When the rectified voltage is greater than the reference voltage, the transistor T_2 conducts through $T_1 - R_2$ and the trip relay is energized which operates the breaker.

Definite Time Overcurrent Relay: Fig. 14.107 shows the detailed circuit for the definite time overcurrent relay. The time of operation of a definite time overcurrent relay is constant and does not depend upon the severity of fault. The function of the input current is only to initiate the charging of a capacitor and thereafter the circuit acts by itself till the breaker is tripped.

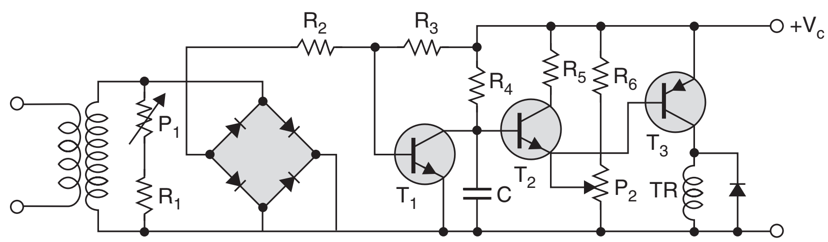


Fig. 14.107 Definite time overcurrent relay.

The operation of the relay with reference to Fig. 14.107 is explained as follows; P_1 is the potentiometer to adjust the pick up value of the relay and P_2 to adjust the operating time of the relay.

Under normal conditions the transistor T_1 is conducting due to the bias applied from the supply voltage V_c through resistor R_3; thereby the capacitor C is short-circuited. Whenever fault current exceeds the pick up value set by the potentiometer P_1, the rectified voltage is applied to the base emitter junction of T_1 through resistance R_2 which thereby is reverse-biased and, therefore, T_1 is switched off. The capacitor C starts charging from the supply voltage V_c through resistance R_4. Since the supply voltage is of constant magnitude (independent of the fault current), the capacitor C is charged through R_4 to a certain voltage in a fixed (definite) time which exceeds the emitter setting of transistor T_2 fixed by the potentiometer P_2. T_2 starts conducting which forces T_3 also to conduct, thereby the trip coil of the breaker is energized and the breaker operates. When healthy conditions are restored the transistor T_1 starts conducting, thus short-circuiting the capacitor C and the relay is reset.

Inverse Time-current Relay: Whereas the input current, in case of definite overcurrent relay controls only the pick up level, in case of inverse time overcurrent relay the input current controls not only the pick up level but also the charging voltage level of the capacitor so that the time of operation depends on the level input current. Fig. 14.108 shows the circuit for an inverse time-overcurrent relay. Under normal conditions the transistor T_1 is conducting due to the bias applied from the supply voltage V_c, through resistor R_4 and P_2; thereby the capacitor C is short-circuited. Whenever fault current exceeds the pick up value set by the potentiometer

P_2 and the transactor tap setting, the transistor T_1 is switched off. The capacitor C starts charging from the input current through the resistor R_3 and potentiometer P_1 by the voltage developed across the resistor R_1. It is to be seen that the basic difference between definite time overcurrent relay and the inverse time-overcurrent relay lies in the source of charging the capacitor. Whereas in case of definite time-overcurrent relay, the charging takes place through a source of constant magnitude and hence the capacitor requires a definite time to charge to a predetermined voltage level, in case of inverse time-overcurrent relay the charging takes place through a source of variable voltage magnitude (depending upon the severity of fault, the more severe a fault is, the more will be the voltage across R_1 and less the time for charging capacitor C to a predetermined voltage level) and hence capacitor gets charged in different times. When the voltage across the capacitor exceeds the value set by the potentiometer P_3, transistor T_2 conducts and thus forces output transistor T_3 to conduct which in turn energizes the trip coil of the breaker and the breaker operates. In this circuit the time multiplier setting is determined by the potentiometers P_1 and P_3 and plug setting multiplier is determined by the tap position of the transactor and the potentiometer P_2.

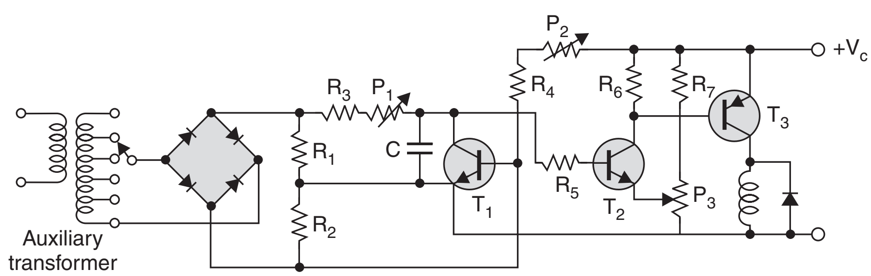


Fig. 14.108 Inverse time-current relay.

Directional Relay

The induction cup type of electromagnetic relays even though are very sensitive have dead zone in their operation. In static directional relays this problem is less serious because static comparators are inherently very sensitive and it is possible to make directional unit reliable down to 1% of system voltage which is well within the minimum fault voltage.

There are two types of comparators used for the purpose. One of these is the Hall effect generator which normally is used by Russian engineers whereas in European countries the rectifier bridge type of comparator is used and it is this comparator which is described below for directional relay.

Rectifier Bridge Phase Comparator: This comparator has already been described in section 14.15 of this chapter. The unit has a maximum output angle near unity p.f. so that for phase faults when quadrature connections (90° connection) are made, the current in the potential circuit must be shifted forward 30° by an RC circuit as shown in Fig. 14.109. For ground fault the current in the potential circuit is made to lag by 45° using an R-L circuit. The maximum output phase angle is adjusted with the help of variable resistor.

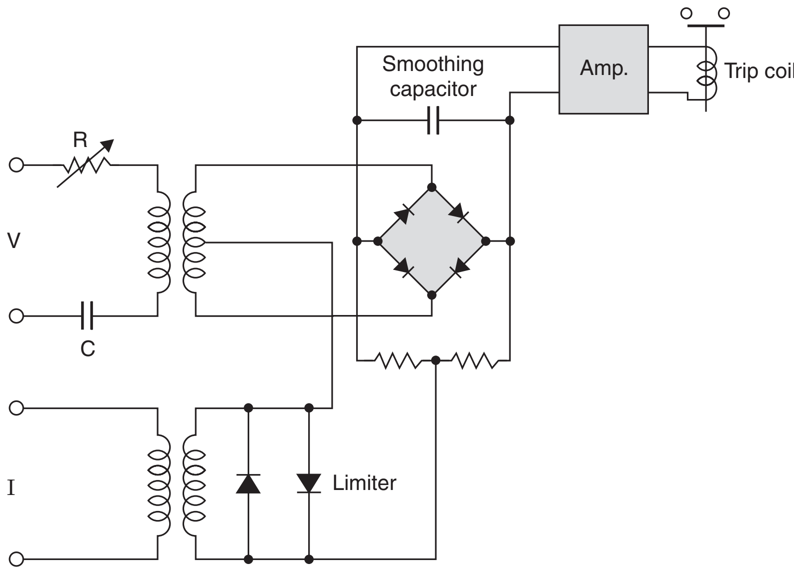


Fig. 14.109 Directional relay using rectifier bridge phase comparator.

Differential Protection

The basic differential scheme is given in Fig. 14.110.

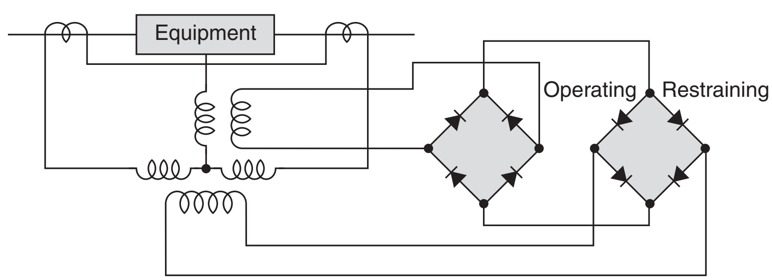


Fig. 14.110 Static differential relay.

The relay operates when

$$K_1 n_0 I_0 > K_2 n_r I_r + K'$$

where n_0 and n_r are the number of turns in the operating and restraining coils respectively and K_1 and K_2 are the design constants and K' the spring control torque.

At the threshold of operation $K' = K_1 n_0 I_{0\ \min}$.

Now equation under threshold condition becomes

$$K_1 n_0 I_0 = K_2 n_r I_r + K_1 n_0 I_{0\ \min}$$

or

$$I_0 = \frac{K_2 n_r}{K_1 n_0} I_r + I_{0\ \min} = K_3 I_r + I_{0\ \min}$$

This is an equation to a straight line of the form $y = mx + c$, the intercept $c = I_{0\ min}$ and the slope $m = K_3$. The characteristic is drawn in Fig. 14.111.

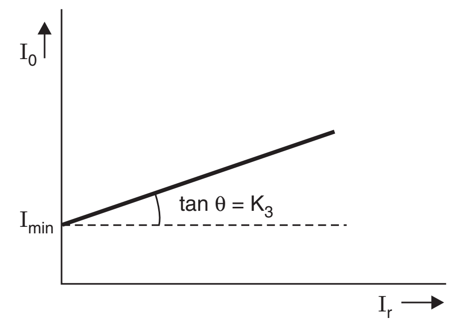


Fig. 14.111 Percentage differential relay characteristic.

Rectifier bridge amplitude comparator is most widely used as a static element for comparing the magnitude of currents or voltages for the differential scheme. Fig. 14.112 (*a*) is a block diagram representation of Fig. 14.112 (*b*), where inputs *P* and *Q* to the comparator are the sum and difference of the currents or voltages to be compared, the output from the comparator is amplified and used to operate the relay. The figure gives a typical circuit for the amplifier used.

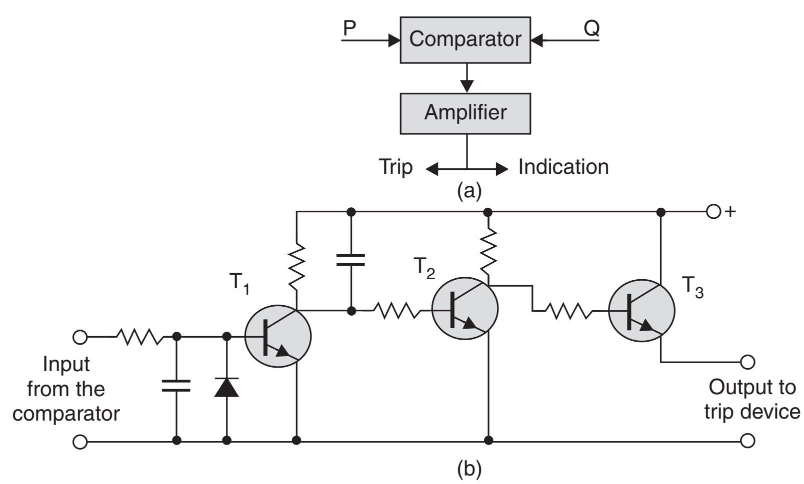


Fig. 14.112 (*a*) Block diagram of differential scheme; (*b*) Typical amplifier circuit (T_2 normally conducting).

The single phase circuit using rectifier bridge corresponding to the block diagram of Fig. 14.112 (*a*) is given in Fig. 14.113.

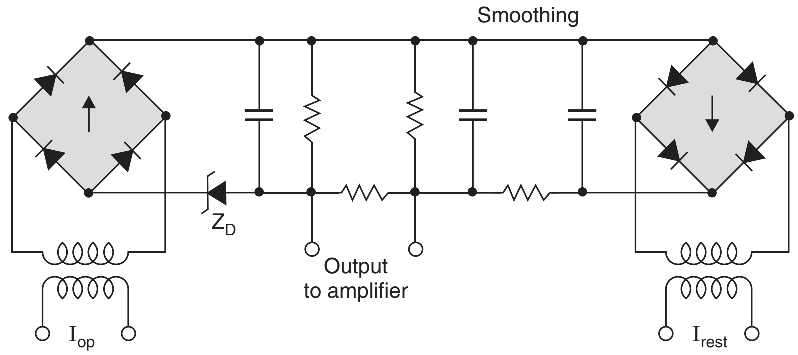


Fig. 14.113 Single phase static comparator scheme for differential protection.

Here Z_D the zener diode is used for limiting the output voltage from the difference CT Figs. 14.114 (*a*) and (*b*) give a scheme for polyphase application. In scheme (*a*) the voltage outputs are utilized from the rectifier bridges whereas in scheme (*b*) current outputs are utilized. Here also in both the cases the zener diode Z_D limits the output from the difference current transformers. The voltage outputs from the restraining (sum of currents) and operating (difference of currents) circuits are the maximum values from the three phases; therefore, the tripping signal is automatically derived from the faulted phase and the restraining signal is based on the through current in the sound phases.

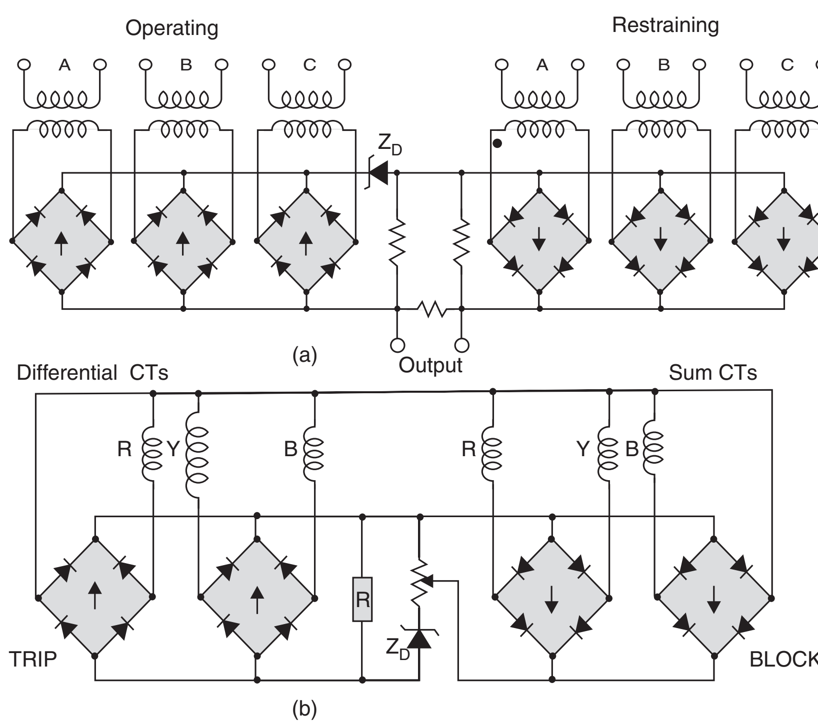


Fig. 14.114 Polyphase differential protection:
(*a*) voltage comparison; and (*b*) current comparison.

Static Distance Relays

Static distance relays since do not have moving parts, they operate much faster and without fear of incorrect tripping as compared to electromagnetic relays. With semiconductor devices it is possible to obtain other distance characteristics than the traditional ones. Static distance relays are accurate over a wider range of fault currents and line lengths and require much lower burden as compared to their counterparts in electromechanical relays.

In a static distance relay the two input quantities must be similar, *e.g.*, two voltages or two currents because they are not electrically separate as they are in case of an electro-mechanical relay, *e.g.*, in an impedance relay magnets are energized by voltage and currents and since the net effect required is a force on a moving mechanism, it can be equally obtained either by a voltage or a current which is not true in case of static devices. Whenever two inputs are compared in a static device circular or straight line distance characteristics are obtained. With more than two inputs more complex characteristics can be obtained. In a current comparator current is obtained from the voltage by connecting an impedance $Z_r \angle \theta$ in series with it. $Z_r \angle \theta$ is the design impedance or a replica of the impedance of the line to be protected when referred to the secondary side of the CT. With this, if the line voltage is V and line current is I, then the current V/Z_r is compared with I. Similarly, in a voltage comparator, the current is turned into a voltage by passing it through the replica impedance Z_r and the drop IZ_r is compared with the line voltage V. Sometimes it may be convenient to compare the two voltages V and IZ_r in a current comparator which is done by connecting resistance in series with each voltage. Fig. 14.115 shows the arrangement of inputs for two-input comparators: (*i*) with current inputs. and (*ii*) with voltage inputs.

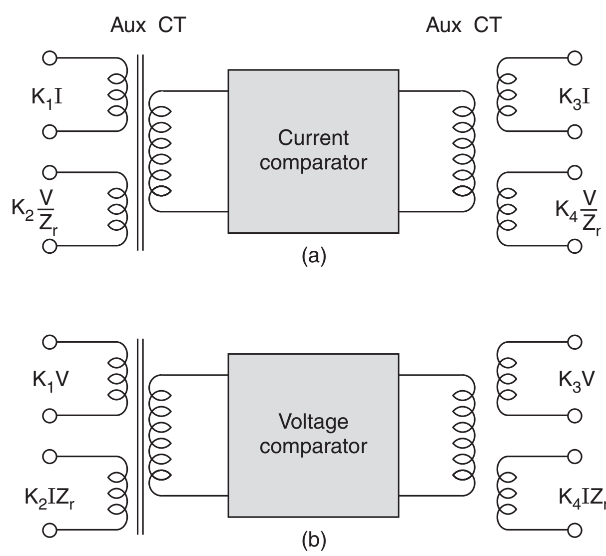


Fig. 14.115 Two-input comparators: 7(*a*) current inputs, (*b*) voltage inputs.

It is to be noted that if there are any transients in the primary current, the same will be reflected in both V and IZ_r and cancel out their (transient) effects on the impedance measurement. Therefore, the concept or use of replica impedance is not only convenient but permits fast tripping also as it eliminates error due to transients in the fault current. A simplified static impedance relay circuit is shown in Fig. 14.116.

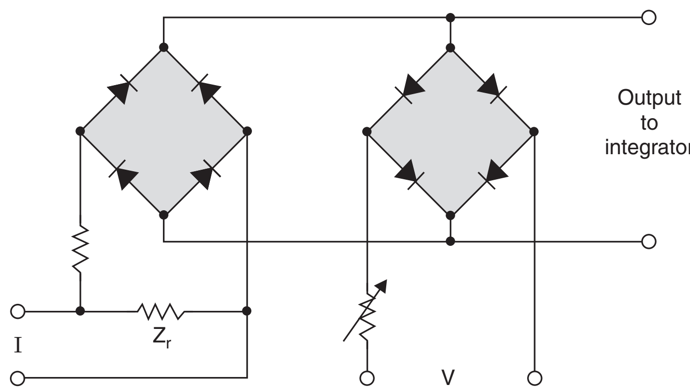


Fig. 14.116 Simplified impedance relay.

The main problem in the impedance relay is to smooth out one of the inputs so that the pick-up does not vary from zero to infinity during the cycle as first the voltage and then the current passes through zero. Normally, the voltage is smoothed as it is easier to do so as compared to the current and this is done by a phase-splitting circuit as described in preceding section (14.15).

Distance Relay Characteristics

These characteristics as we know are normally plotted on an *R-X* diagram but it is sometimes convenient to plot them on *G-B* diagram as well.

We will discuss here again the characteristics in respect of (*i*) directional, (*ii*) impedance (*iii*) angle impedance (*iv*) reactance (*v*) mho relays and the following points should be borne in mind:

1. When only single term quantities are compared corresponding to the current and voltage of the circuit to be protected the characteristic will either be a straight line passing through the origin or a circle with its centre at the origin depending on whether it is a phase or amplitude comparison and whether the characteristic is plotted on impedance or admittance diagram.

2. If one quantity is compared with the sum or the difference of the two quantities, the circle passes through the origin and the straight line is off-set from the origin. The directional relay is a mathematical dual of impedance relay. We now derive the characteristics of various relays including those of directional relay.

(*i*) *Directional Relay:* Phase Comparison: Directional relay is basically a phase comparator which compares the phase relation between V and I and as long as the phase relation $-90° \le \theta \le 90°$ is satisfied the relay operates. The inputs in case of static directional relay are V and IZ_r and the characteristic is

$$Z \,.\, Z_r \cos(\phi - \theta) \ge 0.$$

Since Z and Z_r cannot be zero,

$$\therefore \qquad \cos(\phi - \theta) \ge 0 \quad \text{or} \quad \phi - \theta = \pm \pi/2$$

Amplitude Comparison: The inputs for amplitude comparison will be

$$| V + IZ_r | \text{ and } | V - IZ_r |$$

For operation of the relay

$$| (V + IZ_r) | > | (V - IZ_r) |$$

or $$| Z + Z_r | > | Z - Z_r |$$

and for no operation $$| Z + Z_r | < | Z - Z_r |.$$

This characteristic is similar to the one given in Fig. 14.117 when phase comparison is made.

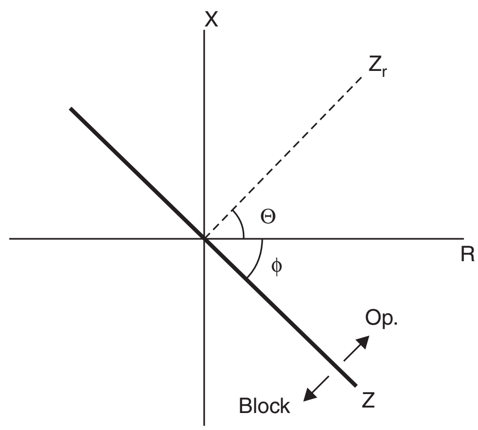


Fig. 14.117 Directional relay characteristic.

(*ii*) *Impedance Relay:* Amplitude comparison: Impedance relay is inherently an amplitude comparator and the inputs for an electromechanical impedance relay are V and I whereas for a static relay these inputs are IZ_r and V.

For relay to operate,

$$| IZ_r | > | V |$$

or $$| Z | < | Z_r | \quad \text{or} \quad R + jX < Z_r$$

For threshold condition $R + jX = Z_r$, which is an equation to a circle on an R-X diagram.

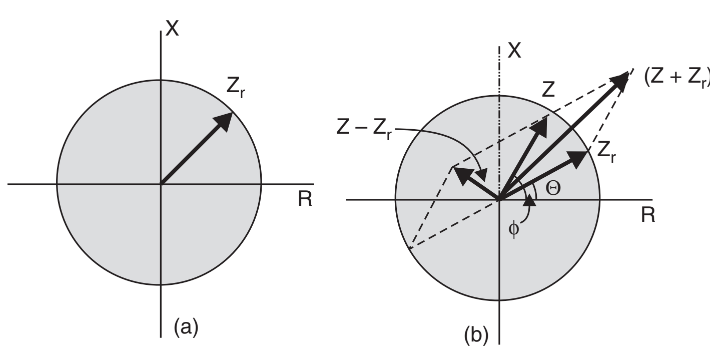


Fig. 14.118 Impedance relay characteristic.

The circle has a centre at the origin and radius is Z_r. The characteristic is shown in Fig. 14.118 (*a*).

Phase comparison: The inputs are $(V + IZ_r)$ and $(V - IZ_r)$. The characteristic is shown in Fig. 14.118 (*b*). It can be seen that as long as Z lies along the circumference of circle with radius Z_r, the two quantities $(Z + Z_r)$ and $(Z - Z_r)$ make angle of ± 90°. This gives the same characteristic as in Fig. 14.118 (*a*).

(*iii*) *Angle Impedance Relay:* Amplitude comparison: The two input quantities are $(2IZ_r - V)$ and V, and for the relay to operate

$$| 2Z_r - Z | > | Z |$$

The characteristic is shown in Fig. 14.119 (*a*).

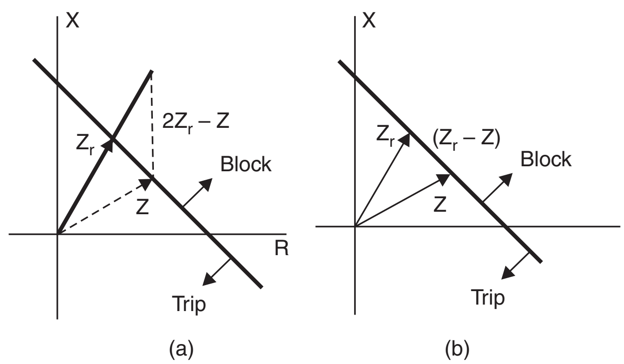


Fig. 14.119 Angle impedance characteristic.

Phase Comparison: For phase comparison the inputs are $(IZ_r - V)$ and IZ_r and for relay to operate the angle between $(Z_r - Z)$ and Z_r should lie within ± 90°. It can be seen that the characteristic is a straight line normal to Z_r. As long as Z lies below the line (Fig. 14.119 (*b*)) the angle between $(Z_r - Z)$ and Z does not go beyond the limits ± 90°.

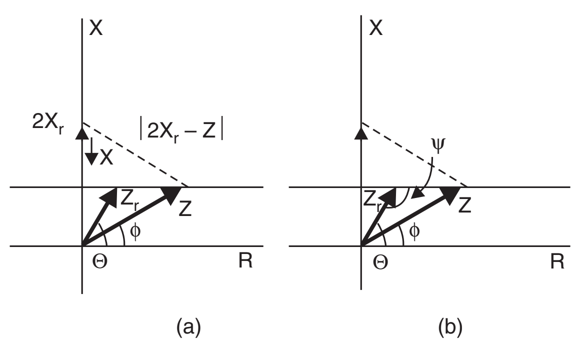


Fig. 14.120 Reactance relay characteristic

(*iv*) *Reactance Relay:* Amplitude comparison: This relay is a particular case of an angle impedance relay in which the reactance component of the impedance is measured and, therefore, the characteristic should satisfy the condition $| 2X_r - Z | > | Z |$.

The two inputs to the comparator are V and $(2IZ_r - 2IR_r - V)$, where R_r is made equal to the resistance of Z_r, thus leaving only its reactive component X_r.

Phase Comparison: The two inputs are IZ_r and $(IZ_r - V)$ as in angle impedance relay. Fig. 14.120 (*b*) shows the relay trips when Z is below the characteristic *i.e.*, when $(\psi + \theta) < 180°$. If Z were purely reactive ψ would be 90° under threshold condition and the relay would trip when $Z \sin \phi < X_r$ on the impedance diagram.

(*v*) *Mho or Angle Admittance Relay:* On an admittance diagram this is a straight line characteristic offset from the origin whereas on an impedance diagram it is a circle passing through the origin. This is the inverse of the angle impedance relay. The two relays are dual of each other. The equation of one type on an impedance diagram corresponds to the equation of other type on admittance diagram and vice-versa.

(*vi*) *Amplitude Comparison:* The two input quantities are $| IZ_r |$ and $| 2V - IZ_r |$. The relay operates when $| 2Z - Z_r | < | Z_r |$. The characteristic is shown in Fig. 14.121. Since Z_r is the diameter of the circle, the relay will operate as long as the fault impedance Z lies within the circle.

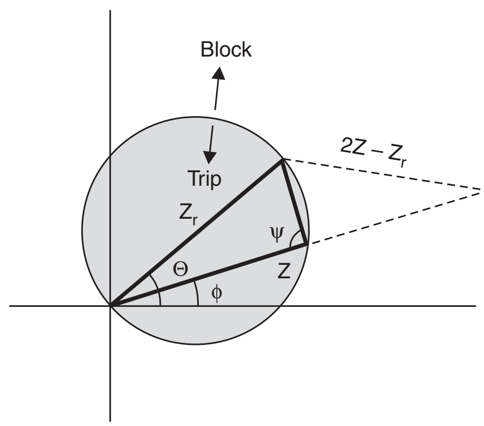


Fig. 14.121 Mho relay characteristic.

Phase Comparison: The two inputs are $| IZ_r - V |$ and V and the relay trips when the phase angle between them is less than 90° *i.e.*, when $90° > \psi > -90°$.

Since mho relay is inherently a directional relay, phase comparator is the more convenient construction.

The following tables summarize the voltage and current inputs to static distance relays.

Characteristic	*Amplitude comparator*		*Phase comparator*	
	Operate	*Restrain*	*Operate*	*Polarize*
Voltage inputs to static distance relay				
Directional	$V + IZ_r$	$V - IZ_r$	IZ_r	V
Impedance	IZ_r	V	$IZ_r - V$	$IZ_r + V$

Angle impedance (ohm)	$2IZ_r - V$	V	$IZ_r - V$	IZ_r
Reactance relay	$2IZ_r - 2IR_r - V$	V	$IZ_r - V$	IZ_r
Angle admittance (mho)	IZ_r	$2V - IZ_r$	$IZ_r - V$	V
Current inputs to static distance relay				
Directional	$I + VY_r$	$VY_r - I$	I	VY_r
Impedance	I	VY_r	$I + VY_r$	$I - VY_r$
Angle impedance	$2I - VY_r$	VY_r	I	$I - VY_r$
Reactance	$2I - 2IR_rY_r - VY_r$	VY_r	I	$I - VY_r$
Angle admittance	I	$2VY_r - I$	$I - VY_r$	VY_r

Conic Characteristics: We have studied in section 14.10, that during power swings there is a possibility of maloperation of the relays having circular or straight line characteristics which could be avoided by using a relay with elliptic characteristics. This characteristic is achieved by a three-input amplitude comparator or by hybrid comparator. The basic circuit for three-input comparator is shown in Fig. 14.122.

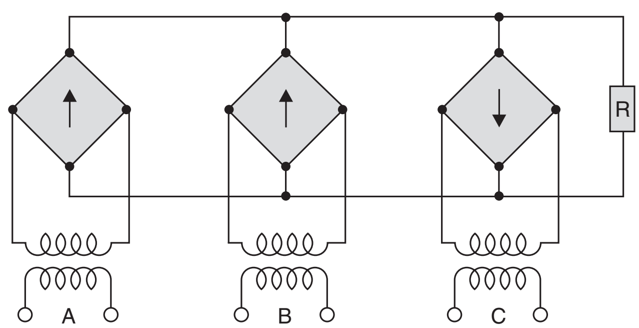


Fig. 14.122 Three-input amplitude comparator.

The three-inputs A, B and C are

$$A = \frac{V_r}{Z_1 + Z_2} - I\,\frac{Z_1}{Z_1 + Z_2}$$

$$B = \frac{V_r}{Z_1 + Z_2} - I\,\frac{Z_2}{Z_1 + Z_2}$$

$$C = I$$

where Z_1 and Z_2 are the design impedances, I the fault current and V_r the voltage at the relay point during fault.

If Z_1 and Z_2 represent the vectors, the tips of which coincide with the foci of the ellipse drawn on complex plane and if the characteristic passes through the origin, then from Fig. 14.123,

$$|\, Z_L - Z_1 \,| + |\, Z_L - Z_2 \,| = |\, Z_1 + Z_2 \,|$$

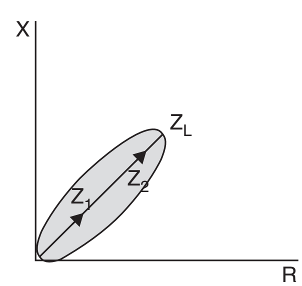


Fig. 14.123 Elliptical characteristic.

Multiplying the above equation by I, we obtain

$$| IZ_L - IZ_1 | + | IZ_L - IZ_2 | = | IZ_1 + IZ_2 |$$

Now since $IZ_L = V_r$, we can write

$$| V_r - IZ_1 | + | V_r - IZ_2 | = | I(Z_1 + Z_2) |$$

Dividing throughout by $| Z_1 + Z_2 |$, we get

$$\left| \frac{V_r}{Z_1 + Z_2} - I \frac{Z_1}{Z_1 + Z_2} \right| + \left| \frac{V_r}{Z_1 + Z_2} - I \frac{Z_2}{Z_1 + Z_2} \right| = | I |$$

This equation represents the operating characteristic of an elliptic relay.

Quadrilateral Characteristic: The characteristic can be obtained by using four relays having straight line characteristics. One of the static arrangements for such a characteristic is by using multi-input comparator.

The four inputs required for a quadrilateral characteristic are

$$S_1 = Z_1 I \angle (\theta_1 - \phi) - K_1 \angle \alpha_1 . V$$

$$S_2 = Z_2 I \angle (\theta_2 - \phi)$$

$$S_3 = Z_3 I \angle (\theta_3 - \phi)$$

and $$S_4 = K_4 \angle \alpha_4 . V$$

where V is the line voltage, I the line current, ϕ the angle between V and I and θ_1, θ_2 and θ_3 are the phase angles of the impedances Z_1, Z_2 and Z_3 respectively which are connected in the current circuit, α_1 and α_4 are the phase shifts of the voltage where required for locating the impedance characteristic.

To enclose the fault area, let

$$Z_2 = X_r, Z_3 = R_r, Z_1 = R_r + jX_r = Z_r$$

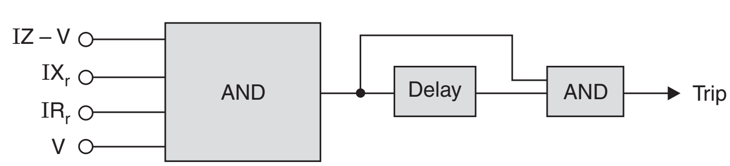
and let $$\alpha_1 = \alpha_4 = 0 \text{ and } K_1 = K_4 = 1$$

the above inputs become

$$S_1 = IZ_r - V, S_2 = IX_r, S_3 = IR_r \text{ and } S_4 = V$$

This gives a composite impedance characteristic as shown in Fig. 14.124 (*g*). The mho circle caused by the intersection of S_1 and S_2 will not interfere with the rectangular tripping area if $Z_r = R_r + jX_r$ because the circle of diameter Z_r goes through the corners of the rectangle bounded by R_r and X_r.

Tripping occurs if all the equations resulting from comparison of all the inputs in pairs are simultaneously satisfied for the length of time set by the delay unit. The rectangle can be rotated through an angle α in the leading direction by phase shifting V or I.



(a)

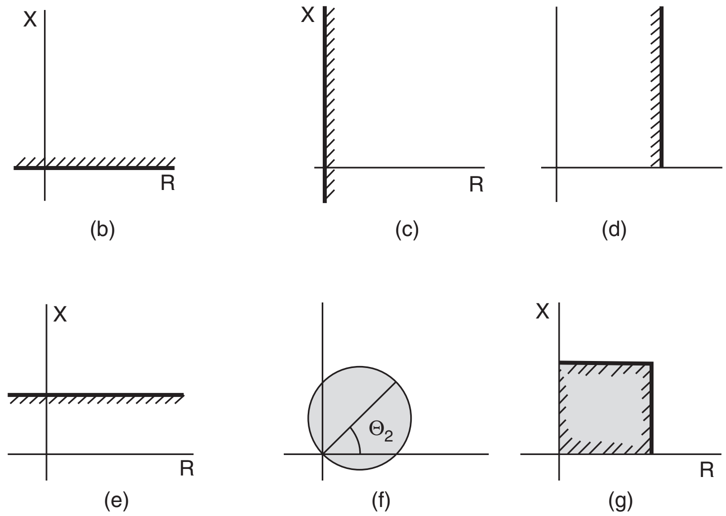


Fig. 14.124 Four input phase comparator: (*a*) Block diagram; (*b*) IX_r and V; (*c*) IR_r and V; and (*d*) IR_r and $(IZ_r - V)$; (*e*) IX_r and $(IZ_r - V)$; (*f*) V and $(IZ_r - V)$; and (*g*) Composite characteristic.

14.17 DIGITAL PROTECTION

With increase in interconnection of power system components, larger sizes of alternators and higher capacity (Higher voltage) transmission lines it has become almost essential to operate the protective relays and CBS as fast as possible to improve the transient stability of power system.

As the transmission operating voltage increases the X/R ratio also increases *e.g.,* for 500 kV line it is 20 and for 765 kV lines it is 27 (Table 21.1). Also for large size alternators the ratio $X/R >> 1$. (We know that larger the X/R ratio of a circuit the longer is the duration of transients in the circuit, whenever a switching operation in the form of a fault or a sudden increase of load takes place.)

The transients consist of a large number of harmonic currents and voltages besides the d.c. component. For protective relaying purposes, since it is the fundamental component of current and voltage that is required, which should be extracted from the transients. Earlier analog filters were used which have an inherent large time delays. Digital filters play an important role in extracting the fundamental components from the transient in about half a cycle. For this reason the digital protection relaying schemes have been developed which are fast in operation and have a higher index of reliability.

With the advent of microprocessors, minicomputers and now *PC*, protective relaying schemes have been developed using on line these devices. The use of these devices has resulted in several advantages such as low burden, faster in operation, low maintenance and not affected by external causes such as vibrations or mechanical shocks. The other advantages are :

1. *Flexibility :* With the same hardware or slight modifications in the hardware, a variety of protection functions *viz*. Various distance relay characteristics (ohm, mho, quadrilateral, parabolic etc.) can be obtained with suitable changes in the software.

2. *Lower Cost:* With advancement in technology and higher level of competition in the manufacture of hardware and software, will bring down the cost of these protective schemes.

3. *Self Checking Capability:* With the proper software control, most of the hardware faults can be diagnosed and properly checked.

4. *Digital Communication:* The microprocessor based relay furnishes easy interface with digital communication equipments.

We will consider here briefly the digital protection of three major and important components of power system, the transmission lines, the generator and the power transformers.

14.17.1. Transmission Lines

In digital distance relaying scheme, the line current and voltage are continuously monitored by *CT* and *PT* respectively. In case of a fault, the digital filter eliminates the higher harmonics and d.c. components and only fundamental component of voltage and current are filtered out which are used to determine the line impedance up to the point of fault.

If R and L are the resistance and inductance of the line from the fault to the relay point and v and i the relay voltage and current respectively then

$$v = Ri + L\frac{di}{dt} \tag{14.21}$$

The impedance of the line can be calculated by

(*i*) Predictive calculation of apparent impedance and

(*ii*) Solution of difference equations methods.

Without going through the mathematical aspect, the results are given as follows :

(*i*) Predictive calculation

$$Z = \left(\frac{V_m{}^2}{I_m{}^2}\right)^{1/2} \text{ and impedance angle}$$

$$\theta = \tan^{-1}\left(\frac{\omega v}{v'}\right) - \tan^{-1}\left(\frac{\omega i}{i'}\right) \tag{14.22}$$

where ω is fundamental angular frequency v and v' are the instantaneous voltage and its derivative respectively.

(*ii*) Solution of difference equations:

$$R = \frac{(V_{k-1} + V_k)(i_{k-1} - i_{k-2}) - (V_{k-1} + V_{k+2})(i_k - i_{k-1})}{(i_{k-1} + i_k)(i_{k-1} - i_{k-2}) - (i_{k-1} + i_{k-2})(i_k - i_{k-1})}$$

and

$$L = \frac{h}{2}\,\frac{(V_{k-1} + V_{k-2})(i_{k-1} + i_k) - (V_{k-1} + V_k)(i_{k-1} + i_{k-2})}{(i_{k-1} + i_k)(i_{k-1} - i_{k-2}) - (i_{k-1} + i_{k+2})(i_k - i_{k-1})} \tag{14.23}$$

where h is the time interval between two successive samples and suffix k indicates the value of V or i during kth interval.

In the software design of the logic the data acquisition system (DAS) simultaneously samples the filtered voltages and currents of the protected line and by the interface system these sampled values are stored in the memory of the work station. After a sample sweep, the voltage and current are compared with its corresponding samples in the previous 50 Hz cycle stored in the microprocessor memory. If any five consecutive samples of current or voltage are found to be more or less than a predetermined set value (*i.e.,* 6.25% = $\frac{1}{16} = \frac{1}{2^4}$ shifting of the bits), a significant disturbance in the system is assumed to have taken place. When the microprocessor detects such a disturbance, it disables the interrupt, determines the peak values of voltages and currents of fundamental frequency and calculates the impedance of the line from the relay point to the fault point both in its magnitude and phase angle using the equation (14.22) or calculates R and L using the equations (14.23). If the fault impedance lies within the requisite $R - X$ characteristic which is stored in the EPROM, the relay operates and finally it sends tripping signal to the circuit breaker.

Hardware Design : Fig. 14.125 shows a simplified diagram of the microprocessor based distance relay. The data acquisition is carried out after removing all the higher harmonics and d.c. transient by Butterworth active band pass (48–52 Hz) filter. The band pass filter output is converted into digital signals by the sample and hold circuit using A/D converter and these signals are fed into the input ports of the microprocessor. The sampling interval of the digital signals is set by a timer which is also controlled by the same microprocessor. The work station which is used for this realisation is an Intel 8086 A based system.

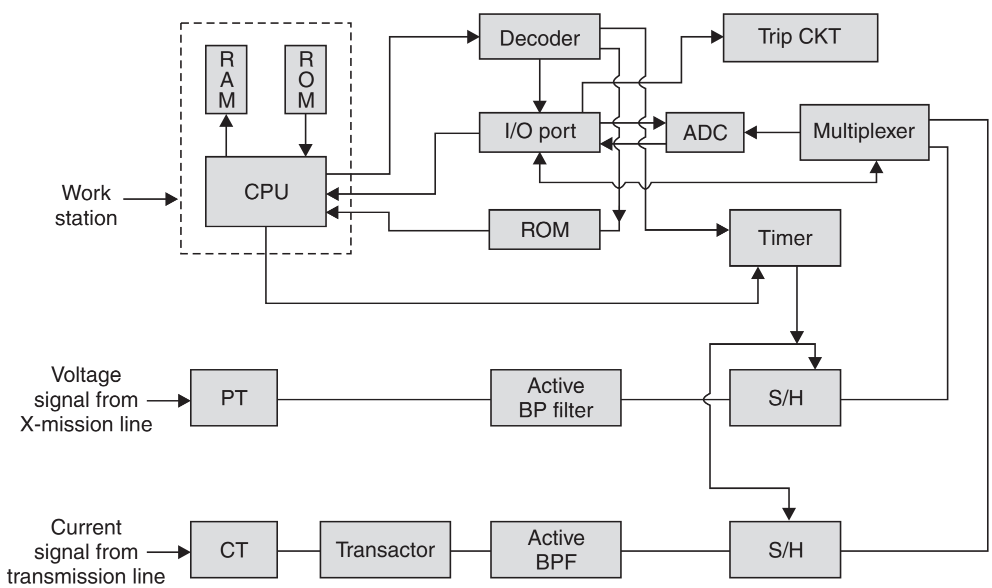


Fig. 14.125 Block diagram of hardware system.

In the sample and hold circuit, the samples are taken at an interval of 0.5 m sec. to obtain 40 samples $\left(\frac{1000}{50 \times 0.5} = 40\right)$ per cycle. For this 8253 programmable timer is used. The sample and hold operation is carried out using NE5537 chip which operates from ±5 V to ±18 V supplies. For A to D conversion CMOS eight channel analog multiplexer IC 4051 is used. The enabling of the multiplexer is done by the INTEL 8255 PPI port as shown in Fig. 14.126.

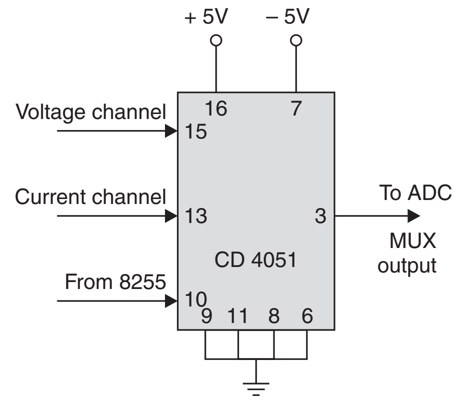


Fig. 14.126 Analog multiplexer circuit.

AD 7574 IC is selected as analog to digital converter which is a CMOS microprocessor compatible 8-bit ADC and uses the successive approximation technique to provide a conversion time of 15 μ sec. AD 7574 is designed to operate as a memory-mapped impact device and can be interfaced like static *RAM*, *ROM* or slow memory. Here it is operated in bipolar operation –10 V to +10 V in slow memory mode.

The tripping circuit consists of LEDs and an open collector NAND gates. These gates are used to indicate the occurrence of faults in different zones. The unused lines of port *A* (8255 *A*) are used to give the trip command to operate a circuit breaker possibly through an SCR or some other triggering device.

14.17.2. Generator Protection

The percentage differential protection using electromechanical relays has already been discussed in article 14.8. For the relay to operate for an internal fault.

$$(i_1 - i_2)n_0 \geq \frac{(i_1 + i_2)n_r}{2}$$

where i_1 is the current entering the stator winding and i_2 the current leaving the stator winding, n_0 is the number of turns of the operating coil and n_r the turns of the restraining coil of the relay. The ratio $\frac{n_r}{n_0} = S$ is known as bias setting and lies within 5 to 10%.

The current i_1 and i_2 are converted into voltages using transactor and these voltages are selected one by one by multiplexer and sampled. *A*/*D* converter digitises these samples and with the help of digital filters, harmonics and d.c. components are filtered out and only r.m.s. values of digital samples $(I_1 - I_2)$ and $(I_1 + I_2)/2$ are stored in the memory of microprocessor. Whenever r.m.s. value of $(I_1 - I_2)$ exceeds S times the r.m.s. value of $(I_1 + I_2)/2$ trip signal is given and the breaker operates.

Another scheme using second harmonic currents induced in the field winding during fault is suggested here. We know that for any unbalanced faults negative sequence currents flow in the stator winding which gives rise to rotating magnetic field with relative velocity $2n_s$ with respect to field winding.

As a result second harmonic currents are induced in the field winding. The presence of second harmonic component in the field circuit more than 0.2 p.u. is an indication of abnormality or fault in the stator circuit or external to the stator circuit. In order to ensure that the fault is internal a reverse power relay is used which shows the direction of current or power flow at the stator terminals. If the second harmonic current in the field winding exceed 0.2 p.u. and if the reverse power relay operates which suggests that there is a severe internal fault in the stator winding for which the relay gives trip signal to the *CB*.

Figure 14.127 shows the scheme for segregating the sequence currents from the phase fault currents. Once the sequence currents are obtain following logic is used to fix up the type of fault:

(*i*) If there is only positive sequence current and negative and zero sequence currents are absent it is a 3-ϕ fault or 3-ϕ to ground fault.

(*ii*) If all the three sequence are present and all are equal it is a *L-G* fault.

(*iii*) If all the three sequence currents are present and if $|I_{a_1}| = |-(I_{a_2} + I_{a_0})|$ it is a *LLG* fault.

(*iv*) If zero sequence is absent and positive and negative sequence currents are equal in magnitude it is a *L-L* fault.

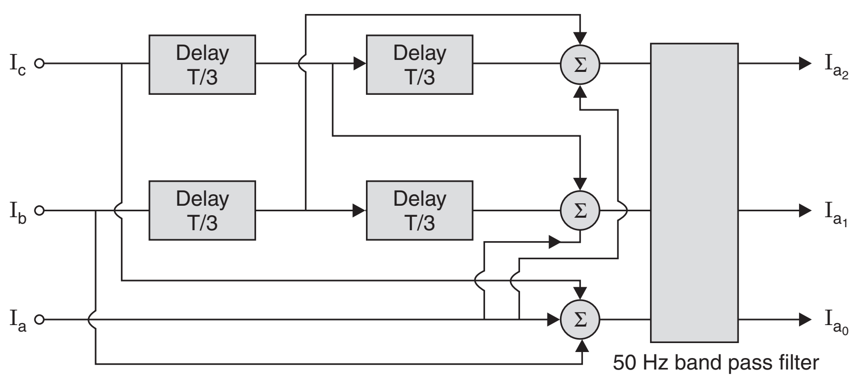


Fig. 14.127 Segregation of sequence currents from phase current.

The relay is realised by using 8085 or 8086 microprocessor. Samples are taken at an interval of 2.5 mS (8 samples per cycle) to make the relay fairly fast. Sampling of positive and negative sequence currents in the armature and field circuit is carried out simultaneously which are then converted into the digital form using *A/D* converter and fed to the input port of the microprocessor. The sampling interval is set by the timer which is also controlled by the microprocessor.

The hardware consists of Data Aquisition System (DAS) and interface with the work station. The DAS consists of an analog multiplexer and *A/D* converter. The sampling is achieved by a 8252 programmable timer, the output of which is fed to the interrupt RST 7.5.

Selection of sequence currents in the armature and field circuit for input to *ADC* is carried out through an analog multiplexer CD 4052 BM. The select input of this multiplexer is driven by two lower bits of upper half of output port *C* of 8255 A device. The start of conversion

(SOC) is generated using the most significant bit of port *C* is 8255 A and is driven to *ADC* 570. The *ADC* converts the currents one by one. The trip signal is the output from port *B* of 8255 A which has been programmed in mode *D* and the type of fault is indicated on the screen.

14.17.3. Transformer Protection

Figure 14.51 shows a harmonic restraint percentage differential relay to avoid the operation of the relay at the time of energization of the transformer when the secondary of the transformer is open. Under this condition, the transformer draws a very large current of the order of 6 to 8 times the normal full load current; of course for a short duration (1 to 2 sec.) which may appear to be an internal short circuit fault: As this in-rush current is rich in second harmonic and this is fed into the restraining coil of the relay and the relay is so adjusted that it will not operate when the second harmonic current exceeds 15% of the fundamental current.

Similarly there could be in rush current in the primary of the transformer whenever there are overvoltages in the system due to either Ferranti effect or load rejection. This in rush current is rich in odd harmonics especially third and fifth harmonics. Since the transformer usually has delta connection, the third harmonic currents circulate within the delta winding and hence are absent in the line of the system and is, therefore, difficult to monitor. Hence, fifth harmonic component of current is monitored to avoid maloperation of the percentage differential relay. It is estimated that whenever fifth harmonic current exceeds 8% of the full load current, the relay should be restrained from operation.

For filtering out the 2nd and 5th harmonic components digital filters are used because of their inherent advantages of high accuracy, high reliability, greater flexibility, performance not attacked by ageing of components etc.

Various digital filters have been designed. Walsh Transform based filter is normally used for protective relaying purpose, because of its faster response as compared to other filters and for protective relay fast and reliable filters are a must. We will not go into the details of various digital filters as it is out of the scope of the book, the readers can refer to some of the references given at the end of the chapter. However, we define here walsh transform as follows:

Any given continuous function periodic with a time period $T = 1$ can be synthesised from a Walsh series, is given by

$$f(t) = \sum_{n=0}^{\infty} W_n W_a l(n, t) \tag{14.24}$$

where
$$W_n = \int_0^1 f(t)\, W_a l(n, t) \tag{14.25}$$

If, however, a function $f(t)$ is specified by N samples $(X_1, X_2, ..., X_N)$ during a time period, we cannot use walsh series for it. But we can define the discrete Walsh transform and its inverse from equations (14.24) and (14.25) as

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} W_k \; W_a l(n, k/N) \tag{14.26}$$

where
$$W_k = \sum_{k=0}^{N-1} X_n \; W_a l(n, k/N) \tag{14.27}$$

for $k, n = 0, 1, 2, ..., N - 1$

The settling time of the filter is less than 20 mS.

Principle of Operation of Relay: The currents I_1 and I_2 from the *CT* secondaries are converted into proportional voltages using air cored transformer *i.e.,* transactors. These are then selected one by one with the help of multiplexer and sampled. According to sampling theorem the sampling frequency must be at least equal to twice the highest frequency that is to be detected by sampling process. In the present scheme 16 samples per cycle is chosen in order that filter harmonic component is also evaluated without causing more attenuation and distortion.

We know that for percentage relay to operate

$$| I_1 - I_2 |_1 \geq S \left| \frac{I_1 + I_2}{2} \right|_1 \tag{14.28}$$

Where I_1 and I_2 are fundamental components of currents. Here subscript 1 to the magnitude of currents represents fundamental component. If we consider the magnetising in rush current due to switching in of an unloaded power transformer (rich in 2nd harmonic) and in rush current due to over voltages rich in 5th harmonic, we have the final tripping criterion as

$$| I_1 - I_2 |_1 \geq S \left| \frac{I_1 + I_2}{2} \right|_1$$

and the final blocking criterion

$$| I_1 - I_2 |_2 \geq 0.15 \; | I_1 - I_2 |_1 \tag{14.29}$$

$$| I_1 - I_2 |_5 \geq 0.08 \; | I_1 - I_2 |_1 \tag{14.30}$$

Here S is a bias factor which varies between 0.1 to 0.4 for power transformer. The above tripping criterion makes the relay in rush current proof. The procedure is summarised as follows:

(*a*) Obtain samples of I_1 and I_2, convert them to digital signals and construct

$$| I_1 - I_2 | \quad \text{and} \quad \left| \frac{I_1 + I_2}{2} \right|$$

(*b*) Extract the r.m.s. values of

$$| I_1 - I_2 |_1, \; | I_1 - I_2 |_2, \; | I_1 - I_2 |_5 \text{ and } \left| \frac{I_1 + I_2}{2} \right|_1$$

(*c*) If the r.m.s. value of 2nd harmonic current is greater than 15% of the fundamental current or if the r.m.s. value of 5th harmonic current is greater than 8% of the r.m.s. value of the fundamental current (Equations 14.29 and 14.30) the relay is restrained to operate. However, if it is a normal internal fault then the r.m.s. value of operating current $| I_1 - I_2 |$ is greater than the r.m.s. value of restraining current multiplied by bias setting *i.e.,*

$$| I_1 - I_2 | > S \left| \frac{I_1 + I_2}{2} \right|$$

the relay operates. The relaying schemes are shown in Fig. 14.128 and Fig. 14.129.

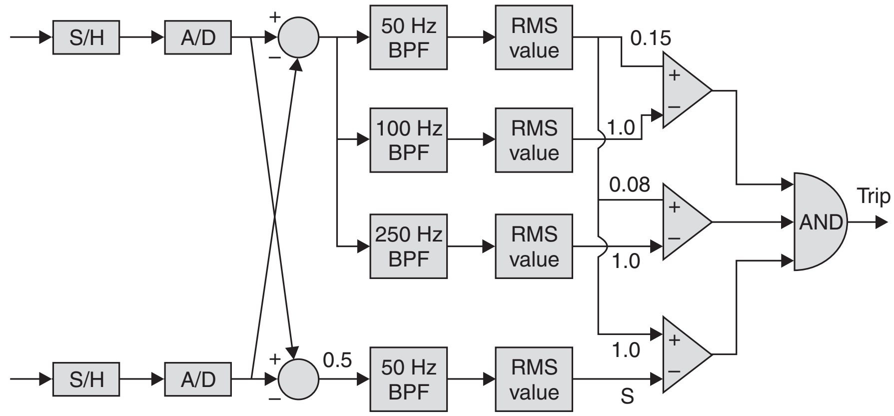


Fig. 14.128 Percentage differential relay.

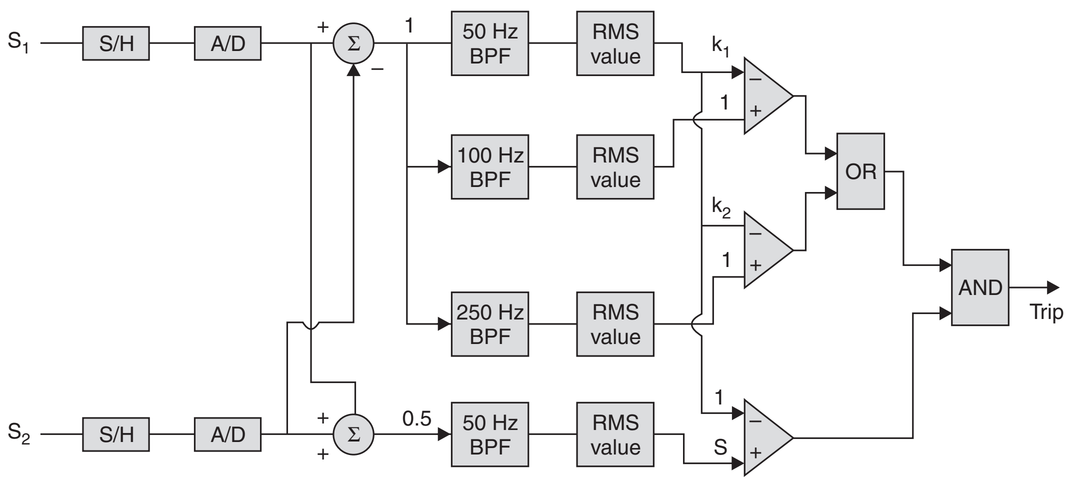


Fig. 14.129 Percentage differential relay.

Hardware and Software Used : The relay is realised using intel 8086 system. Fig. 14.128 and Fig. 14.129 show the simplified diagram of the hardware. Data Aquisition System (DAS) has 4 Channel analog multiplexer (CD 4052 BM) and *A*/*D* converter *AD* 570. Three primary and three secondary currents of the power transformer are obtained through *CT* secondaries and are converted into proportional voltage using transactors. These six voltage signals are fed to the multiplexer. These analog signals from the multiplexer reach the *A*/*D* converter which converts analog signals into an equivalent 12-bit digital signals. The *A*/*D* converter is connected in bipolar mode (± 5 V) to read both the positive and negative values of the analog signal and gives minimum conversion time of about 3 μ sec. A start of conversion (SOC) pulse is generated by timer (8253) and starts the conversion at it's falling edge. The 12-bit output is read by software through port *B* and *C* (lower) of PPI 8255 and is stored in the memory buffer for processing.

The input/output interface, a link between external circuit and the microprocessor consists of programmable interrupt controller (PIC 8259), a programmable peripheral interface (PPI 8255) and a programmable interval timer (PIT 8253).

The microprocessor used is a 16 bit Intel 8086 Vinytics kit which is connected to *PC-XT* through a serial interface using *RS*-232 connector. The kit and *PC* are interfaced by a software package called *PC-KIT*. The software enables easy development of assembly language program on the *PC* using Microsoft Assembler and executing the code after downloading it into the kit *i.e.,* it provides an interactive environment for the kit through *PC-XT*. The relaying scheme is shown in Fig. 14.129.

14.18 FUSES AND HRC FUSES

A fuse is a small piece of wire connected in between two terminals mounted on insulated base and is connected in series with the circuit. The fuse is perhaps the cheapest and simplest form of protection and is used for protecting low voltage equipments against overloads and/or short circuits. The fuse is expected to carry the normal working current safely without overheating and during overloads or short circuits it gets heated up to melting point rapidly. The materials used normally are tin, lead, silver, zinc, aluminium, copper etc. For small values of currents an alloy of lead and tin in the ratio of 37 per cent and 63 per cent respectively is used. For currents more than 15 amperes this alloy is not used as the diameter of the wire will be large and after fusing, the metal released will be excessive. Silver is found to be quite satisfactory as a fuse material because it is not subjected to oxidation and its oxide is unstable. The only drawback is that it is a relatively costlier material. Therefore, for low range current circuits either lead-tin alloy or copper is used.

Definition

Fuse: Fuse is a device used in circuit for protecting electrical equipments against overloads and/or short circuits.

Fuse element or Fuse Wire: It is that part of the fuse device which melts when an excessive current flows in the circuit and thus isolates the faulty device from the supply circuit.

Minimum Fusing Current: Minimum fusing current is a value corresponding to operation in an arbitrary time obtained under prescribed test conditions. Alternatively, it is a value of current corresponding to a chosen value of time indicated on a time/current curve which is itself obtained from prescribed testing condition.

Fuse Rating: It is that value of current which when flows through the element, does not melt it. This value of the fuse is less than the minimum fusing current.

Fusing Factor: This is the ratio of minimum fusing current to the current rating of fusing element, *i.e.,*

$$\text{Fusing factor} = \frac{\text{Minimum fusing current}}{\text{Fuse rating}}$$

The fusing factor is always greater than unity.

Prospective Current: It is defined as the r.m.s. value of current which would flow in a circuit immediately following the fuse when a short circuit occurs assuming that the fuse has been replaced by a link of negligible resistance.

Melting Time or Pre-arcing Time: The time taken from the instant the current that causes a break in the fuse wire starts flowing, to the instant the arc is initiated.

Arcing Time: The time taken from the instant of arc initiation to the instant of arc being extinguished.

Total Operating Time: It is the sum of the pre-arcing and the arcing time. The most commonly used fuse in 'house wiring' and small current circuits is the semi-enclosed or rewirable fuse. Whenever the fuse wire blows off due to overload or short-circuit, the fuse carrier can be pulled out, the new wire can be placed and the supply can be restored. This looks simple and is really very simple only if the wire is replaced by the correct size. For a layman this may prove dangerous if he replaces the fuse wire by some copper wire not to the specification and there is a possibility of burning the equipment. Besides, the fuse wire, since is exposed to atmosphere, it is affected by ambient temperature. The time-current characteristics of such fuse get deteriorated with time and hence are not reliable for discrimination purposes. These fuses are, therefore, mainly used for domestic and lighting loads. For all important and costly equipments operating at low voltages (up to 33 kV) another class of fuse is used which is known as cartridge fuse. These are described below. When the HRC (High Rupturing Capacity) cartridge fuse link was first introduced, it was designed to satisfy two important requirements. The first was to cope up with the increasing rupturing capacity on the supply system and the second was non-deterioration to overcome the serious disadvantages suffered by the types of semi-enclosed fuses.

Construction of HRC Fuse

The HRC fuse consists of a ceramic body usually of steatite, pure silver element, clean silica quartz, asbestos washers, porcelain plugs, brass end-caps and copper tags (see Fig. 14.130). The brass end-caps and copper tags are electro-tinned. The metal end-caps are screwed to the ceramic body by means of special forged screws to withstand the pressure developed under short circuit condition. The contacts are welded to the end-caps. The assembly also includes solder of various types, cement and indicator devices. Deterioration of the fuse must involve a change in one or more of these materials or a change in their structure. Normally the fuse element has two or more sections joined by means of a tin joint. The fuse wire is not in the form of a long cylindrical wire as after it melts, it will form a string of droplets and will result into an arc between the droplets. Afterwards these droplets will also evaporate and a long arc will be struck. The purpose of the tin joint is to limit the temperature of fuse under small overload conditions. The melting point of silver is 960°C while that of pure tin is 230°C. As the circuit is overloaded the melting of tin prevents the silver element from attaining high temperature. The shape of the fuse element depends upon the time-current characteristic required.

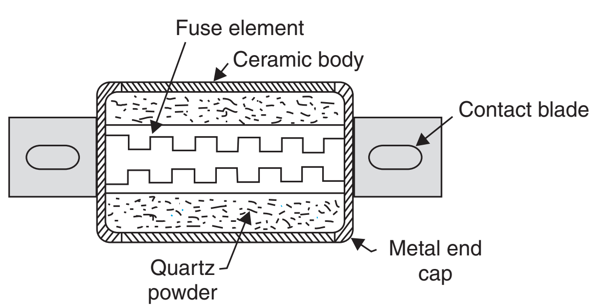


Fig. 14.130 HRC fuse link.

Fuse Operation

When an HRC fuse operates, the element absorbs energy from the circuit and heats until it melts. The heat produced during operation is given by $\int i^2R\,dt$, where R is the instantaneous resistance and i the instantaneous current during the operating time. The fuse element melts before the fault current reaches its peak value. As the element melts, it vaporizes and disperses. This action is then followed by a period of arcing during which chemical reaction between the silver vapour and the quartz powder takes place, which further results into building up a high resistance and reduces the current to zero. Thus the arc is quenched. Generally, the filling powder used is quartz-sand as it can absorb heat at a very high rate and it does not evolve appreciable amount of gas.

Cut-off Current

When an HRC fuse interrupts a heavy fault it exhibits an ability to limit the short circuit current. This ability is referred to as a 'cut-off' as shown in Fig. 14.131 and has the effect of

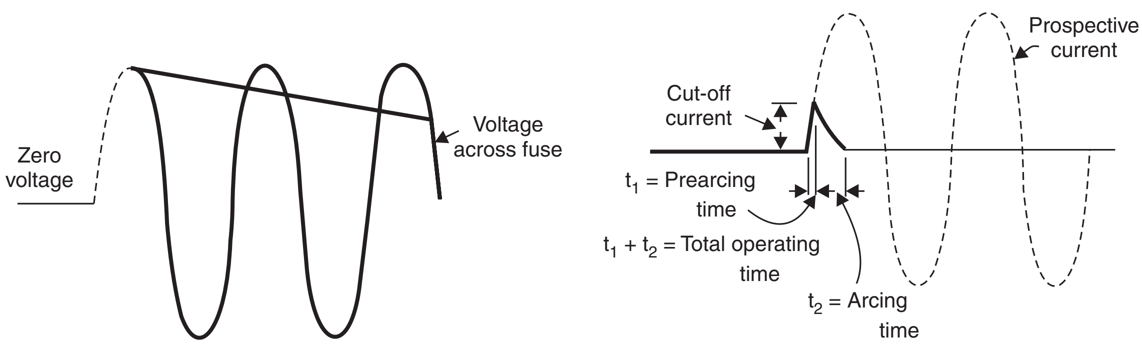


Fig. 14.131 Cut-off characteristic of HRC fuse.

reducing the magnetic and thermal stresses both in the system and within the fuse itself under fault conditions. Cut-off is in fact one of the main reasons why HRC fuse is so successful as a protective device and it is at times preferred over the circuit breaker of low ratings. Due to this property of the HRC fuse the operating time is as low as 1/4th of a cycle. The maximum to which the fault current reaches before the fuse melts is called the cut-off current.

Properties of HRC Fuses

The careful designs of HRC fuses have overcome the disadvantages of the conventional rewirable fuses and their properties are described as follows:

(*i*) *Fast Operation:* The HRC fuse interrrupts the short circuit current long before its maximum value is attained which is not true in case of CBs. This property of the HRC fuse reduces both thermal and magnetic stresses on the equipment to be protected and the short circuit fault is interrupted well within the first quarter of a cycle.

(*ii*) *Rupturing Capacity:* The rupturing capacity of a fuse is expressed in terms of MVA and is equal to the product of service voltage and the r.m.s. value of the prospective current it can handle. Since the fuse melts much before the current reaches prospective value due to its cut-off property, it is clear that a fuse is never called upon to carry a current equal to its rupturing capacity. It is to be noted here that the rupturing capacity of a breaker is different from the rupturing capacity of a fuse.

(*iii*) *Non-deterioration:* This means that all the characteristics of the fuse are maintained throughout its life. As the fuses are called upon to function only once in a while, it is most essential that they should preserve their characteristics throughout their useful life. This also implies that they should not operate inadvertently when carrying normal load currents as so often happens with a rewirable fuse which may fail due to oxidation and reduction of cross-sectional area. This property of HRC fuses is very important and is achieved by the hermetic sealing of the silver element within the fuse body with the help of special cementing and the soldering of the end caps. It has been found that HRC fuses maintain non-deterioration property unimpaired even after approximately 20 years of their manufacture.

(*iv*) *Low-temperature Operation:* This is required to eliminate the deterioration of the fuses and to prevent overheating of associated contacts. This is achieved by employing fabricated elements of pure silver which are specially designed to give a low temperature rise when carrying their full-rated current.

(*v*) *Accurate Discrimination:* By this characteristic is meant that an HRC fuse on a distribution system will isolate the faulty section from the healthy section whenever a fault takes place. In case of an HRC fuse it is found that the time of operation is inversely proportional to the prospective short circuit current over a much wider range of fault condition and, within practical limits, while the values of prospective short circuit current increase, the time of operation will continue to decrease without reaching a definite minimum. This means that a fuse of low current rating will blow before a fuse of a higher rating, no matter how heavy the fault. It is, therefore, desirable while designing the installation from the view point of discrimination to use fuses of the same design and characteristics throughout, which will ensure that time-current characteristics of each succeeding current size will not cross and the characteristics will be parallel to one another up to the maximum values of fault current.

Arc Voltage within Safe Limits: Whenever an inductive circuit is inter-rupted, there is likelihood of large voltages induced. The magnitude of such voltages depend upon the magnitude of the short circuit to be interrupted and the circuit constants. A careful design of the HRC fuse controls these overvoltages and keeps them within safe limits.

Low Cost: It is known that because of the cut-off characteristics of the HRC fuse, for the same rupturing capacity the actual current to be interrupted by an HRC fuse is much less as compared to any other interrupting device and hence it is less expensive as compared to other interrupting devices. It is, therefore, usual to employ a circuit breaker of low rupturing capacity backed up by an HRC fuse where circuit breakers are necessary for other reasons. A combination of these two circuit interrupting devices works as follows. Whenever there is an overload the CB trips whereas for short circuits the HRC fuse operates.

Applications of HRC Fuses

The applications of HRC fuses are enormous but a few very important are: (*i*) protection of cables, (*ii*) protection of bus bars, (*iii*) protection of industrial distribution system, (*iv*) contactor gear for motor control, (*v*) earth faults—both of low and high magnitude, (*vi*) semiconductor rectifiers and (*vii*) aircraft.

It is to be noted that the HRC fuses cover a very wide range of applications. This involves the principles of fuse design in varying degrees. For special application, the parameters of the fuse are defined to close limits. The design of HRC fuses for the same rupturing capacities for protecting an SCR are different from the one for protecting cables.

An HRC fuse rated for 150 amps continuous rating and 200 kA rupturing capacity at 400 V used for protecting a semiconductor device weights about 30 gm whereas an industrial application HRC fuse rated for 100 amps and 250 kA rupturing capacity weights about 200 gms. The HRC fuses have been used for protecting aircraft equipments and offer many advantages not available by alternative means.

Within wide limits HRC fuses are not affected by frequency. For practical purposes, a fuse tested and rated at 50 Hz is satisfactory for 60 Hz duty and vice versa. As frequency tends towards d.c., the interrupting capability of the fuse at the lower and medium overcurrents may be less, because it is in these zones that d.c. duty is the more onerous. A fuse which has been tested and rated at a given frequency will almost invariably safely interrupt short-circuit faults of higher frequency.

14.19 LINEAR COUPLERS

The protective relays are normally not connected directly to the system but these are connected through current and/or potential transformers. With this, the relays have to handle smaller magnitudes of voltages and currents and, therefore, the protective relays become relatively cheaper. During the process of transformation of primary quantities (voltages and currents) to the secondary quantities, certain errors are involved. The primary quantities are not in exact ratio of transformation and also the phase relations are not proper.

14.19.1 Current Transformers

The primary winding of a current transformer is connected in series with the circuit whose current is to be sensed and across the secondary of the current transformer, the operating coil of the relay is connected. The current transformers used in protection are usually primary bar *i.e.*, these have one turn of primary winding *i.e.*, the conductor of the circuit itself forms the primary of the current transformer. Since the primary is connected in series with the power circuit, the voltage drop across its terminals is very small and the primary current is independent of the secondary current contrary to power transformer where the primary current depends upon the secondary current.

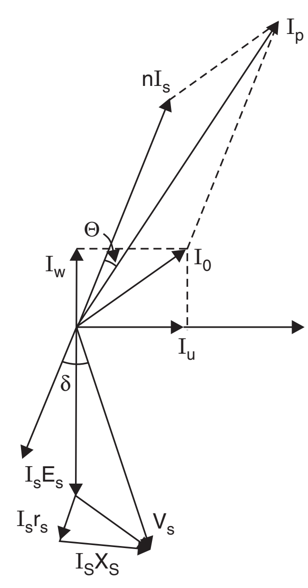


Fig. 14.132 Phasor diagram of a current transformer.

Fig. 14.132 shows phasor diagram for a current transformer.

Here I_μ represent the magnetising current, I_w the working current (iron loss components), I_p the primary, I_s the secondary current and E_s the induced secondary voltage and V_s the secondary terminal voltage.

Let R be actual transformation ratio *i.e.*, $R = \dfrac{I_p}{I_s}$

Using phasor diagram as shown in Fig. 14.132 and after certain algebraic manipulation, it is found that

$$R = n + \frac{I_w \cos \delta + I_\mu \sin \delta}{I_s}$$

where n is the nominal transformation ratio,

i.e., $$n = \frac{\text{Number of secondary turns}}{\text{Number of primary turns}}$$

It can be seen from the above expression that the current ratio R of the transformer differs from the turns ratio n by an amount which depends upon the magnitude of the exciting current of the transformer and upon the current and p.f. of the secondary circuit. The ratio error is defined as

$$\frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}} = \frac{n - R}{R}$$

Similarly phase angle error introduced by the current transformer is defined as the angle by which the secondary current phasor when reversed differs in phase from the primary current. The angle is taken as positive if the reversed secondary current leads the primary current. On very low p.f. the phase angle error may be negative. The phase angle error is approximately given by

$$\theta = \frac{I_\mu \cos \delta - I_\omega \sin \delta}{nI_s}$$

Usually the angle δ is small and hence it can be seen from the expression that the ratio error is largely dependent upon the working component of exciting current whereas the phase angle error is dependent upon the magnetising component of the exciting current.

In order to minimize the exciting ampere turns required to reduce these errors, the core must have small iron loss and a low reluctance path. The flux density should be as low as possible. The length of magnetic path should be as small as is consistent with good mechanical construction and with insulation requirements. This results into low reluctance path. For the same reason, joints in the core should be avoided as far as possible. If these are unavoidable because of a typical construction, these should be made as efficient as possibly by careful assembly. For protective relays where the primary current is very large, the ring form of core is most commonly used. The secondary winding is uniformly wound on the core and the primary is a single bar in the form of the power conductor. Since there is no joint, this construction gives the minimum reluctance path for the magnetic path and very small leakage reactance. To meet these requirements mumetal cores are commonly used because this material has very high permeability, low loss and small retentivity.

It is desirable that the winding should be so designed that they withstand without damage the large electromagnetic forces which are developed when a short circuit takes place on the system. The bar primary ring-core construction is generally recognised as the most satisfactory from this point of view.

The ratio error can be compensated by using one or two turns less than that number which would make turns ratio n equal to the nominal ratio *e.g.*, for 400/5 current transformer of the bar primary type the number of secondary turns would be 79 instead of 80. However, with this the phase angle error is not affected.

From Fig. 14.127 it is seen that as the power factor of the load is reduced, the angle ϕ increases which brings the reversed secondary vector nI_s more and more in phase with I_0. This increases the value of I_p for any given value of I_s and then the transformation ratio $\frac{I_p}{I_s}$ increases. With reduced power factor since reversed nI_s moves more into phase wtih I_0 and hence I_p, the phase angle error is obviously reduced.

For a given value of secondary current if the burden (VA loading) on the CT is increased, the secondary terminal voltage increases which calls for increased secondary induced voltage and hence increased core flux density. The exciting current I_0 is thus increased and this increases the transformation ratio, causing the ratio error to become less positive for any given values of frequency and power factor. Similarly, the phase angle error also increases with increase in secondary burden.

For a given burden and p.f., the secondary induced voltage is constant and is proportional to the product of frequency and flux density. Therefore, an increase in frequency will result in a proportionate decrease in flux density. Thus the effect of increase in frequency on ratio and phase angle error is similar to decrease in burden of the CT.

As mentioned earlier the current in the primary winding of a current transformer is a fixed quantity and is thus not affected by the state of the secondary winding *i.e.*, whether it is shorted or is kept open. Therefore, if a current transformer has its secondary circuit opened when current is flowing in its primary circuit, a very high flux density is produced in the core owing to the absence of opposing ampere turns from the secondary winding. This high flux density results in a very high induced voltage in the secondary winding which may result in damage of insulation and danger to the personnel. Also, if the large magnetising force acting on the core is removed suddenly, the core of the CT may get saturated which will affect the performance of the CT adversely in terms of ratio and phase angle errors and the CT may have to be discarded. For these reasons care should be taken to ensure that whenever primary is connected in a line circuit, the secondary should not be left open.

If due to some reasons the secondary circuit is left open while the primary current is flowing, the transformer should be demagnetised before being used again. Following methods are normally recommended.

A current at least equal to the one which was flowing through the primary when the open circuit took place, is passed. The current is supplied by a motor generator set. The supply to the motor is disconnected but the alternator field is still on. As the alternator slows down, its terminal voltage falls gradually to zero, and the core of the transformer is passed through a large number of cycles of magnetisation of gradually decreasing magnitude and finally reducing to zero.

In yet another method, a very high resistance is connected across the secondary of the current transformer. Rated full load current is then passed through the primary and the secondary resistance is then gradually reduced to zero as uniformy as possible. Thus the magnetisation of the core is reduced to its normal value.

In general the percentage ratio error increases with increase in primary current. The accuracy class of a CT is normally indicated as follows:

15/5 *P,* 10.

Here the first number (15) represents the VA burden of the CT, the second number (5 *P*) the number of times the primary current *i.e.*, 5 *P* means the accuracy is being determined for a current 5 times the normal primary current and the third number (10) represents the percentage composite error. The percentage composite error is defined as

$$\frac{100}{I_p}\sqrt{\frac{1}{T}\int_0^T (ni_s - i_p)^2\, dt}$$

where n = nominal transformation ratio,

i_s = instantaneous secondary current,

i_p = instantaneous primary current,

I_p = r.m.s. value of primary current, and

T = time period of current wave.

The current transformers for protective relaying are mostly of ring construction (bushing type) because it is simple in construction, less expensive and has linear characteristic (input/output). This transformer is built into equipments such as circuit breakers, generators or power transformers, the core being arranged to encircle an insulating bushing through which the power conductor passes (Fig. 14.133). Because of larger cross-section of bushing CT, it is more accurate than other CTs at high multiple of the primary current which exist under fault conditions. This is why this CT has linear characteristic *i.e.,* smaller ratio error for large currents. However, for smaller currents bushing CTs are less accurate because of its larger exciting current.

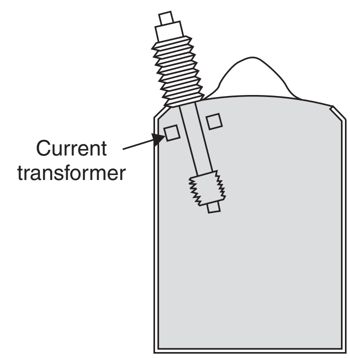


Fig. 14.133 Circuit breaker with current transformer.

In case of protective current transformers we are normally concerned with the ratio error rather than the phase angle error for the following reasons:

(*i*) The power factor of the load (operating coil of a relay) connected to the secondary of a current transformer is generally low and, therefore, the reversed secondary current vector is almost in phase with the exciting current and hence the effect of the exciting current on the phase angle accuracy is negligible.

(*ii*) The phase angle error normally existing in CTS can be tolerated in protective relaying.

The B-H curve of a magnetic material is, in general, given by the curve as shown in Fig. 14.134. Between the points *A* (Ankle point) and *B* (Knee point) the characteristic is linear. If the same stampings are used both for the protective CTS and the measuring CTS, the working range of the protective CT lies between the ankle and the knee point (as a protective CT is expected to transform primary currents linearly on to the secondary side for a relatively large range of currents about 20 times the full load currents) of the characteristic whereas the measuring CT usually operates at a point around the ankle point as the measuring CT is expected to be accurate up to its 120% rated current only. Therefore, if the same material is used for both the CTS *i.e.*, for protective and measuring CTS, it is quite obvious that for the same volt amperes ratings, the protective CT would require larger cross-section and this would be bigger in size.

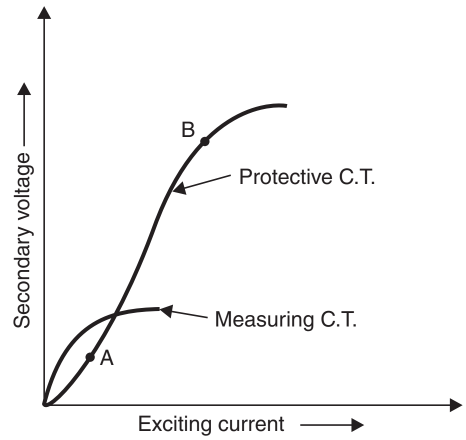


Fig. 14.134 B-H curves of protective and measuring current transformers.

Polarity Marking: In a.c. circuits current is reversing its direction every half a cycle, one might wonder what the significance is of polarity marking. The polarity marking assumes its importance whenever two or more than two CTS are to be interconnected (to get sum or difference of various currents) or a CT and a PT are to be used to produce some desired effect such as torque in a relay (directional relay). If a CT is used in isolation, polarity marking is not necessary.

The relative polarities of CT primary and secondary terminals are identified either by the symbols P_1 and P_2 for the primary and S_1 and S_2 for the secondary terminals or by painted polarity marks. The convention is that when primary current enters the P_1 terminal, secondary current leaves the S_1 terminal as shown by the arrows in Fig. 14.135 or, when current enters the P_2 terminal, it leaves the S_2 terminal. However, when paint is used the terminals corresponding to P_1 and S_1 are identified.

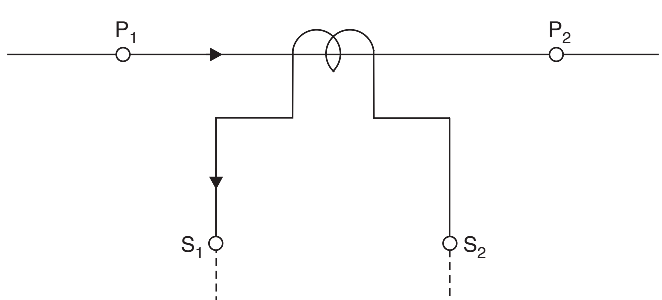


Fig. 14.135 Polarity marking.

14.19.2 Potential Transformers

The potential transformer is similar in construction to the power transformer. The main difference between the two is that whereas the secondary current in the power transformer depends upon the loading conditions, the current in the potential transformer equals its magnetising current. The errors introduced by potential transformers are, in general less serious as compared to the current transformers. Refer to Fig. 14.136 for the phasor diagram of the potential transformer.

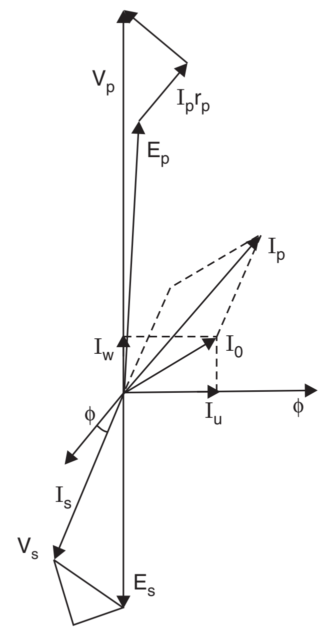


Fig. 14.136 Phasor diagram of a potential transformer.

Here E_p, E_s are primary and secondary induced voltages, V_p, V_s, the corresponding terminal voltages. The load p.f. angle ϕ is usually very small as the load connected across the PT secondary is highly resistive. Here nominal ratio

$$n = \frac{E_s}{E_p}$$

whereas the actual ratio of transformation is $\frac{V_s}{V_p}$. From the phasor diagram it can be proved after certain manipulation that the ratio error.

$$R = \frac{nI_s\,[r_s \cos\phi + X_s \sin\phi] + I_w r_p + I_\mu X_p}{V_s}$$

Similarly, the phase angle error which is defined as the angle between the reversed secondary terminal voltage and the primary voltage, is given by

$$\theta = \frac{I_s}{V_s}\left(X_s \cos\phi - r_s \sin\phi + \frac{I_w\,X_p - I_\mu r_p}{nV_s}\right)$$

where r_p, x_p are the equivalent resistance and reactance referred to the primary side and r_s and x_s correspond to the secondary side.

From the expressions above, it is clear that the ratio and phase angle errors depend upon the resistance and reactance of the transformer windings and also it depends upon the loss component and magnetising components of the exciting current of the transformer.

In order to bring down the errors of the potential transformers

(*i*) the flux density in the core should be as low as possible.

(*ii*) the reluctance of the transformer core should be as low as possible.

(*iii*) the leakage reactance of the two windings should be made small by placing the two windings as close together as is consistent with insulation requirements.

(*iv*) The resistance of the winding should be made small.

(*v*) Turns compensation should be done by adjusting the number of primary and secondary turns so that '*n*' is less than the nominal ratio.

With increase in burden, assuming the secondary voltage to remain constant, the secondary current increases and hence the primary current increases. With this, the primary and secondary voltage drops are increased and, therefore, for a given value of V_P, E_p, E_s and V_s are reduced. The net result is to increase the actual ratio $\frac{V_p}{V_s}$ of the transformer as the burden increases. From the phasor diagram, it is clear that with increase in I_p and I_s the voltage drops in the primary and secondary windings increase and hence the phasor V_s retards whereas V_p advances with respect to the flux phasor ϕ. Hence, the phase angle error also increases with increase in burden.

Similarly it can be seen that the ratio of transformation increases and the phase angle error decreases with reduction in load p.f.

For a given applied voltage, reduction in frequency results in increase of core flux and hence increase in exciting current I_0 which does not influence the transformer ratio seriously. Whereas reduction in frequency results in reduction in phase angle error.

Mainly there are two types of potential transformers:

(*i*) The instrument potential transformers, and

(*ii*) The capacitance potential transformer or potential divider.

The instrument potential transformer is a conventional two winding transformer in which primary is connected to the system directly between phase and ground or between two phases as the case may be and the secondary is connected to the voltage coil of a relay. A capacitance potential transformer is a capacitance potential divider connected between phase and ground of the power circuit.

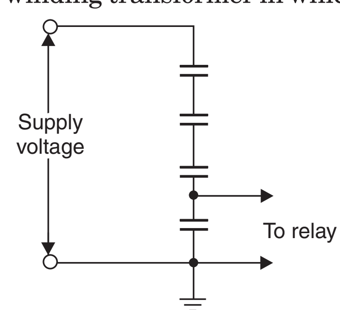


Fig. 14.137 Capacitance potential divider.

The capacitance potential devices used for protective relaying are of two types (*i*) the coupling capacitor potential device and (*ii*) the bushing potential device. The two devices are more or less similar electrically, the main difference being in the formation of the capacitances which in turn affects their rated burden. Whereas the coupling capacitor device consists of a stack of series connected capacitor units and an auxiliary capacitor (Fig. 14.137), the bushing potential device uses the capacitance coupling of a specially constructed bushing of a circuit breaker or power transformer as shown in Fig. 14.138.

Capacitance potential devices are used for potective relaying for operating voltages exceeding 66 kV when these are sufficiently less expensive as compared to the potential transformers. However, the potential devices can prove to be less costly even below 66 kV if a carrier current system is to be used on the power system. because then coupling between the power system (operating at high voltage) and the carrier system (operating at low voltage) can be provided very easily using coupling capacitor potential devices.

Whenever two or more than two parallel lines are running from a common bus, it is desirable to use a single potential transformer connected to the bus rather than a potential device, as a potential transformer will generally have sufficient capacity to supply the protective relaying equipment of all the lines whereas one set of potential devices may not.

Another advantage of potential transformers connected to the bus is when the protective relays make use of 'memory action' for their reliable operation when the fault is close to the circuit breaker (*e.g.*, dead zone in a directional relay). Under these conditions, the relays will get the voltage supply before the line circuit breaker was closed and hence the relays can use the memory action for their operation. Whereas if the voltage source is on the line side of the breaker as is usually the case with the potential devices, the relays will not get any voltage and hence the memory action will not be effective. Therefore, the main relay may not operate and the back up relays at other location will be required to clear the fault affecting the continuity of supply to some other unfaulted section.

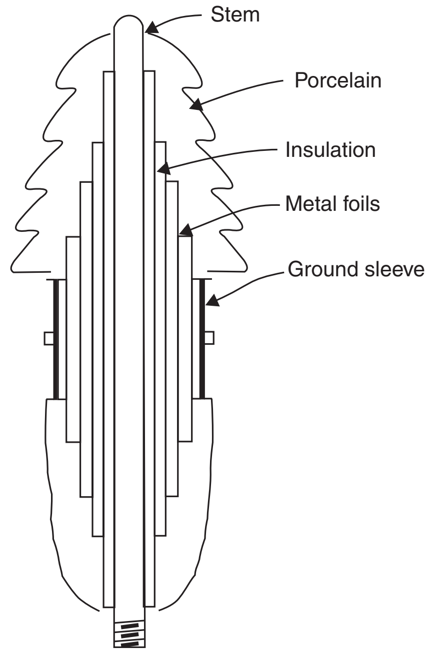


Fig. 14.138 Capacitor type high voltage bushing.

In case of a ringmains system, it is desirable to provide capacitance potential devices on the line side of the breakers of each circuit as there is no satisfactory location for a single set of potential transformer to serve the relays of all circuits.

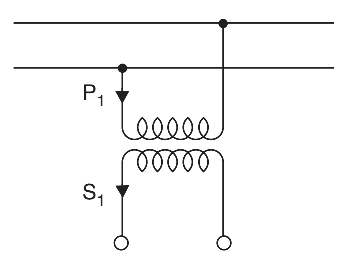


Fig. 14.139 Polarity marking of a P.T.

Polarity Marking: The terminals of the potential transformer are marked to indicate the relative polarities of the primary and secondary windings. The polarity marks have the same significance as for a current transformer, namely, that if the current enters the P_1 terminal, it leaves S_1 terminal (or T_1 terminal of tertiary winding) of the secondary winding as shown in Fig. 14.139. However, in case of capacitance potential devices S_1 and T_1 terminals are marked, the P_1 terminal being obvious from the configuration of the device.

Example 14.10: A 100/5A bar primary current transformer supplies an over current relay set at 25% pick up and it has a burden of 5 VA. Determine the knee point voltage and cross-section of the core if the CT has 50 turns on its secondary and the fault current is 15 times the relay setting. Assume the flux density as 1.4 Wb/m^2.

Solution: The operating current of the relay

$$= 5 \times 0.25 = 1.25 \text{ Amp.}$$

The secondary voltage $= \dfrac{5 \text{ VA}}{1.25} = 4$ volts

The CT secondary voltage when current is 15 times the relay setting = 15 × 4 = 60 volts. The knee voltage must be slightly greater than 60 volts.

Now $E = 4.44\, Bm\, A\, f\, N$

$$\therefore \quad A = \frac{60}{4.44 \times 1.4 \times 50 \times 50}$$

$$= 3.86 \times 10^{-3} \text{ sq. m}$$

$$= 38.6 \text{ sq. cms.}$$

Example 14.11: Determine the VA output of a current transformer having a ratio of 100 : 5 and secondary resistance of 0.1 ohm. The resistance of the connecting lead is 0.1 ohm and the relay burden is 5 VA.

Solution: Here output voltages are 5^2 (0.1 + 0.1) + 5 = 10 VA. **Ans.**

PROBLEMS

14.1. What is a protective relay ? Explain the functional characteristics of a protective relay.

14.2. Explain what is meant by primary protection and back up protection.

14.3. Define the terms: (*i*) Pick up value, (*ii*) Reset value, (*iii*) Operating time, and (*iv*) Reset time.

14.4. What are unit system and non-unit system of protection?

14.5. Show that the torque on the disc of an induction disc relay is maximum when the phase difference between the two fluxes is 90°. Indicate the direction of rotation of the disc with reference to the fluxes under the poles.

14.6. Explain the process of fault clearing with the help of a neat sketch.

14.7. Describe the construction, principle of operation and application of an (*i*) induction disc and (*ii*) induction cup type of relay. What is the ratio of reset to pick up value in case of these relays ?

14.8. Explain why the ratio of reset to pick up should be high.

14.9. Write a short note on the time-current characteristics of an overcurrent relay. Draw these characteristics for the relays used to protect a radial feeder with three substations fed from one end.

14.10. Classify the various types of overcurrent relays and give their applications alongwith approximate characteristics.

14.11. What is an IDMT characteristic ? Explain how this is achieved in practice.

14.12. What is meant by 'directional feature' of a directional overcurrent relay ? Describe the construction, principle of operation and application of a directional overcurrent relay.

14.13. Explain clearly the *V-I* and polar characteristics of a directional relay. Mark clearly the operation and no-operation zones.

14.14. What is meant by 'dead zone' when referred to a directional relay and explain clearly how it is taken care of.

14.15. What is Universal Torque Equation ? Using this equation derive the following characteristics: (*i*) impedance relay; (*ii*) reactance relay; (*iii*) mho relay.

Draw the characteristics and indicate clearly the zones of operation and no-operation.

14.16. Explain how you provide directional feature to (*i*) impedance, and (*ii*) reactance relay. Explain why the directional feature provided for impedance relay cannot be used for a reactance relay.

14.17. Draw schematic diagrams for the (*i*) impedance relay, (*ii*) reactance, relay and (*iii*) mho relay.

14.18. Show mathematically how the distance relays should be connected so that they provide equally sensitive protection against three-phase and phase-to-phase faults. Give the diagram of connections also.

14.19. Explain what is meant by phase fault compensation as applied to distance protection. Why is it necessary and how can it be achieved ? Give the diagram of connections.

14.20. Explain clearly the basic principle of operation of a differential relay. Explain the working of this type of relay for (*i*) an internal fault, and (*ii*) a through fault.

14.21. Compare the merits and demerits of various pilot wire relaying schemes for protecting transmission lines.

14.22. What is meant by per cent bias ? How is this achieved in practice in differential relay ? Under what circumstances is a percentage differential relay preferred over the differential relay ?

14.23. Explain clearly the basic principle of operation of a percentage differential relay for (*i*) internal fault, and (*ii*) through fault.

14.24. Give various schemes of protection for feeders and compare their performance.

14.25. What is meant by (*i*) time-graded, (*ii*) current graded, and (*iii*) time-current graded system ? Explain why time-current graded system is normally preferred over the other systems of protection ?

14.26. Explain clearly how the selection of current and time settings is done in a time-current graded system ?

14.27. Give schemes of protection for a parallel feeder fed from (*i*) one end, and (*ii*) both the ends.

14.28. Give a scheme of protection for a ring main having three substations and fed from one end. Explain whether the same scheme could be used if the ring mains were fed from more than one end.

14.29. What is meant by 3-zone protection ? Give such schemes of protection for (*i*) short length lines, (*ii*) medium length lines, and (*iii*) long lines. Give schematic diagrams of contact circuits and explain their principle of operation for these schemes.

14.30. Explain the carrier system of protection. With a block diagram and neat sketches discuss how the phase comparison scheme can be used for protecting a feeder fed from (*i*) one end, and (*ii*) both the ends. What is the basis for the choice of frequency in power line carrier system ? Explain whether this scheme can be used for the protection of underground cables.

14.31. Explain the principle of Merz-Price system of protection used for power transformers. What are the limitations of this scheme and how are they overcome ?

14.32. Describe, with a neat diagram, a circulating-current protection scheme for a 3-phase, 1 MVA, 11 kV/400 volts delta-star transformer. If the current transformers have a nominal secondary current of 5 amps, calculate their ratios.

14.33. A 3-phase 66/11 kV star-delta connected transformer is protected by Merz-price Protection System. The CTs on the LT side have a ratio of 420/5 amps. Show that the CTs on the HT side will have a ratio of $70 : 5/\sqrt{3}$.

14.34. An IDMT overcurrent relay rated at 5 amp has a current setting of 150% and has a time-multiplier setting of 0.8. The relay is connected in the circuit through a CT having ratio 400/5. Calculate the time of operation of the relay if the circuit carries a fault current of 4800 amps. Assume the relay to have 2.2 sec IDMT characteristic.

14.35. A 13.8 kV, 125 MVA star connected alternator has a synchronous reactance of 1.4 p.u./phase and a negligible resistance. It is protected by a Merz-Price balanced current system which operates when the out of balance current exceeds 10% of the full load current. If the neutral point is earthed through a resistance of 2 ohms, determine what proportion of the winding is protected against earth fault.

14.36. What is restricted earth fault protection for alternators ? Why is this form of protection used for alternators even though it does not provide protection for the complete winding ?

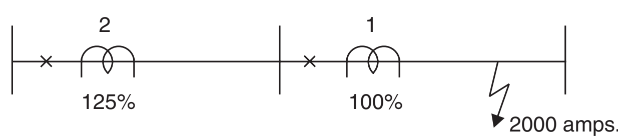
14.37. A 3-phase 33 kV star connected alternator is to be protected using circulating current protection. The pilot wires are connected to the secondary windings of 100/5 ratio current transformer. The protective relay is adjusted to operate with an out-of-balance current of 1 amp in the pilot wires. Determine the (*i*) earthing resistance which will protect 90% of the winding, and (*ii*) the per cent of the winding which would be protected if the earthing resistance is 15 ohm.

14.38. Explain with reasons the connection of CTs for protecting a delta/star transformer. Justify your scheme of protection for (*i*) internal fault, and (*ii*) external fault by showing current distribution in the scheme.

14.39. Describe the construction, principle of operation and applications of 'Buchholz relay. Why is this form of protection an ideal protection scheme ?

14.40. What are the abnormal conditions in a large alternator against which protection is necessary ?

14.41. Determine the time of operation of the relays placed at location No. 1 and 2 assuming that fault current is 2000 amps, CT ratio 200/1, relay 1 set at 100% and 2 at 125% and that the relay No. 1 has a time-multiplier of 0.2. The time grading margin between the relays is 0.5 sec for discrimination. Assume the relay to have 2.2 sec IDMT characteristic.



14.42. Describe schemes of protection for an alternator against inter-turn faults when the number of circuits per phase are (*i*) even, and (*ii*) odd.

14.43. Describe the rotor protection against earth fault.

14.44. What is Translay protection ? Give such a scheme of protection for a three-phase transmission line.

14.45. What is an HRC fuse ? Compare an HRC fuse with a circuit breaker as interrupting device.

14.46. Explain the terms (*i*) Pre-arcing time, (*ii*) Arcing time, (*iii*) Cut-off.

14.47. Describe the construction, principle of operation and application of an HRC fuse.

14.48. Explain the characteristics of an HRC fuse and discuss how they are useful in circuit breaking.

14.49. Explain briefly why digital protection schemes are required for large capacity power system components.

14.50. Discuss with the help of neat diagrams, the hardware and software of the digital protection scheme for transmission lines using distance relays.

14.51. Describe with neat block diagram the constructions and principle of operation of microprocessor based percentage differential relay protection scheme for a large synchronous generator.

14.52. Describe with neat block diagram, the microprocessor based relaying scheme for the protection of synchronous generator by monitoring the field current of the alternator.

14.53. Describe with block diagram the construction and principle of operation of a microprocessor based percentage differential relay scheme for the protection of a power transformer.

14.54. Describe a protection scheme which restrains the operation of the relay during in rush magnetising current and also during in rush magnetising current due to over voltage caused by load rejection. Write tripping and blocking signals for preventing relay operation during in rush current phenomenon for a unit protection scheme for the power transformer.

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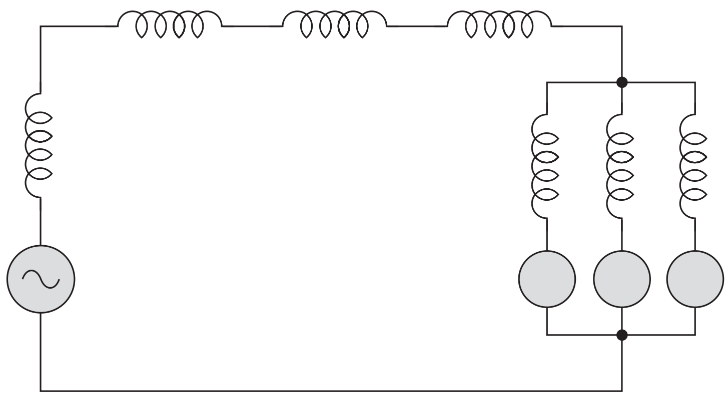

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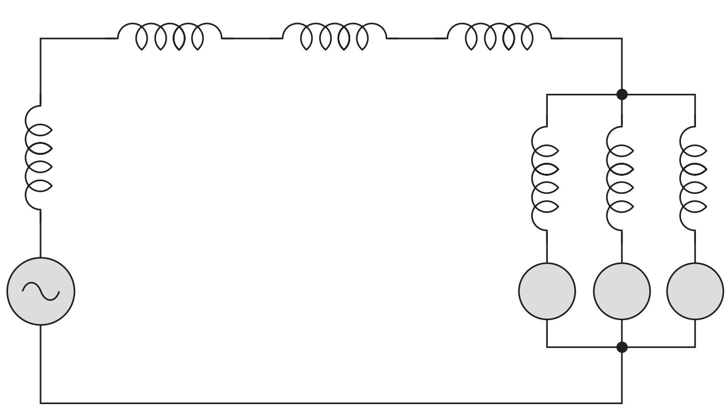
CIRCUIT BREAKERS



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Circuit Breakers

INTRODUCTION

An electrical power system is very complex. It needs some form of switchgear in order that it may be safely and efficiently controlled or regulated under both normal and abnormal operating conditions. A tumbler switch with a fuse serves to control the light and other electrical equipments and is, in a way, a form of switchgear. On the other hand, a C.B. in a station serves exactly the same purpose but it has many added and much more complex features.

The basic construction of any circuit breaker requires the separation of contacts in an insulating fluid which serves two functions here: (1) It extinguishes the arc drawn between the contacts when the C.B. opens. (2) It provides adequate insulation between the contacts and from each contact to earth.

Many insulating fluids are used for arc extinction and the fluid chosen depends upon the rating and type of C.B. The insulating fluids commonly used for C.B. are:

1. Air at atmospheric pressure.
2. Compressed air.
3. Oil which produces hydrogen for arc extinction.
4. Ultra high vacuum.
5. Sulphur hexafluoride (SF_6).

The gases which have been considered for C.B. are: (*i*) simple gases—air, oxygen, hydrogen, nitrogen, carbon dioxide; and (*ii*) electronegative gases—sulphur hexafluoride, arcton.

Properties required of a gas for C.B. use are:

(*i*) High dielectric strength.

(*ii*) Thermal and chemical stability.

(*iii*) Non-inflammability.

(*iv*) High thermal conductivity: This assists cooling of current carrying conductors immersed in the gas and also assists the arc extinction process.

(*v*) Arc extinguishing ability: It should have a low dissociation temperature, a short thermal time constant (ratio of energy contained in an arc column at any instant to the rate of energy dissipation at the same instant) and should not produce conducting products such as carbon during arcing.

(*vi*) Commercial availability at moderate cost: Of the simple gases air is the cheapest and most widely used for circuit breaking. Hydrogen has better arc extinguishing property but it has lower dielectric strength as compared with air. Also if hydrogen is contaminated with air, it forms an explosive mixture. Nitrogen has similar properties as air, CO_2 has almost the same dielectric strength as air but is a better arc extinguishing medium at moderate currents. Oxygen is a good extinguishing medium but is chemically active. SF_6 has outstanding arc quenching properties and good dielectric strength. Of all these gases SF_6 and air are used in commercial gas blast circuit breakers.

Air at atmospheric pressure is 'free' but dry air costs a lot when stored at say 75 atmospheres. The compressed air supply system is a vital part of an air blast C.B. Moisture from the air is removed by refrigeration, by drying agents or by storing at several times the working pressure and then expanding it to the working pressure for use in the C.B. The relative cost of storing the air reduces with increase in pressure. If the air to be used by the breaker is at 35 kg cm^2 it is common to store it at 210 kg/cm^2.

Air has an advantage over the electronegative gases in that air can be compressed to extremely high pressures at room temperature and then its dielectric strength even exceeds that of these gases.

The dielectric strength of SF_6 at normal pressure and temperature is 2 to 3 times that of air and at 2 atm its strength is comparable with the transformer oil. Although SF_6 is a vapour, it can be liquefied at moderate pressure and stored in steel cylinders. Even though SF_6 has better insulating and arc quenching properties than air at an equal pressure, it has the important disadvantage that it cannot be used much above 14 kg/cm^2 unless the gas is heated to avoid liquefaction. Circuit breakers, using SF_6 at 14 kg/cm^2, have heaters installed in the high pressure reservoir. The interrupting capacity of a C.B. is approximately directly proportional to the gas pressure; therefore, it is possible for a compressed-air break to have a higher interrupting capacity than an SF_6 break at the expense of increased gas pressure. The choice between the two gases depends to a large extent on the cost of the complete equipment. During maintenance of an SF_6 filled C.B. the gas is pumped from the breaker into a receiver and stored in liquid form. Metal fluorides which may be formed during the interruption of short circuit currents are removed from the C.B. tank by filtering the gas through activated alumina before the breaker is inspected. When maintenance is complete, the breaker is resealed, dried with nitrogen and evacuated to a few mm Hg pressure to remove air prior to refilling with SF_6. During operation of the C.B. the gaseous decomposition products, which can be toxic in the presence of moisture are removed using activated alumina as an absorber.

15.1 ARC IN OIL

In an oil circuit breaker, the heat of the oil decomposes the oil which boils at 658°K. The gases liberated are approx. (1) Hydrogen 70%; (2) Acetylene 20%; (3) Methane 5%; and (4) Ethylene 5%.

The temperature about the arc is too high for the three last-named gases to exist and the arc itself runs into a mixture of hydrogen, carbon and copper vapour at temperature above 6000°K. The hydrogen being a diatomic gas gets dissociated into the atomic state which changes the characteristics of the arc on account of its associated change in its thermal conductivity. The outcome of this is that the discharge suddenly contracts and acquires an appreciably higher core temperature. In certain cases the thermal ionization may be so great that the discharge runs with a lower voltage which may stop the ionization due to the electric field strength. The transition from the field ionization to thermal ionization is most marked in hydrogen and, therefore, in oil circuit breakers.

Arcs in Air

The arc in an air circuit breaker runs in a mixture of nitrogen, oxygen and copper vapour. When the current is more than 100 amps, these gases get dissociated into atoms and the arc is contracted as explained above. The oxygen gas may remain dissociated even when the current is of the order of 1 ampere.

Initiation of the Arc

The separation of the C.B. contacts which are carrying current gives rise to an arc without changing much the current waveform. Initially when the contacts just begin to separate the magnitude of current is very large but the contact resistance being very small, a small voltage appears across them. But the distance of separation being very very small, a large voltage gradient is set up which is good enough to cause ionization of the particles between the contacts. Also it is known that with the copper contacts which are generally used for the circuit breakers very little thermal ionization can occur at temperature below the melting point. For effective field emission the voltage gradient required is 10^6 V/cm. From this it is clear that the arc is initiated by the field emission rather than the thermal ionization. This high voltage gradient exists only for a fraction of a micro-second. But in this short period a large number of electrons would have been liberated from the cathode and these electrons while reaching anode, on their way would have collided with the atoms and molecules of the gases. Thus each emitted electron tends to create others and these in turn derive energy from the field and multiply. In short, the work done by the initially emitted electrons enables the discharge to be maintained. Finally, if the current is high, the discharge attains the form of an arc having a temperature high enough for thermal ionization, which results in lower voltage gradient. Thus an arc is initiated due to field effect and then maintained due to thermal ionization.

Deionization

As discussed above, the arc consists of ionized particles of gases. This arc can be interrupted if the contact gap could be deionized. This is the basic principle of arc interruption in all circuit breakers. The process of deionization is possible in the following ways: (*i*) high pressure, (*ii*) forced convection and turbulence, and (*iii*) arc splitting.

Forced Convection and Turbulence

When a gas blast is directed along a discharge, efficient cooling is obtained. In case of oil C.B. the hydrogen gas which has better thermal conductivity flows along the discharge. If the gas blast is axially directed, this not only gives cooling action but compels arc to shrink in diameter which in turn raises the temperature of the core of the arc. Because of the gases being generated at high pressure there will be turbulence near the surface of the arc and under certain conditions this effect may be used in the process of deionization especially in the C.B. where gas blast is used for extinction of the arc.

Arc Splitting

There are two methods: (1) The arc is forced into an arrangement of splitters by which the arc is lengthened and the cooling is improved because of contacts with the splitters. (2) The arc is made to split into relatively smaller arcs. The idea here is to ensure that the sum of the cathode-anode voltage drops of short length should be more than the supply voltage; thereby the energy fed to the arc will be reduced.

15.2 ARC INTERRUPTION THEORIES

When a short circuit on a system occurs, the relay gives a signal to the C.B. to trip and isolate the healthy section of the system from the faulted without causing any harm to the system or to itself. There are two methods by which arc interruption is done: (*i*) high resistance method, and (*ii*) low resistance or current zero interruption method.

High resistance method: In this method the arc resistance is increased in time to such a high value that it forces the current to reach zero without possibility of arc being restruck thereafter. The rate at which the resistance is increased or the current is decreased is not abnormal so as to cause harmful induced voltages in the system. The arc resistance may be increased due to any or all of the deionizing methods discussed earlier *i.e.*, cooling, lengthening and splitting of the arc. Because of the resistive nature of the arc discharge, most of the energy in the system will be received by the C.B. Therefore, while designing the C.B., provision of mechanical strength to withstand such sudden release of large quantities of energy must be made. This is the main drawback of this method of arc interruption. This method is, therefore, used for low and medium power a.c. circuit breakers and in d.c. circuit breakers.

Low resistance or current zero interruption: This method is used only in a.c. circuit interruption because there is natural zero of current present in such systems. In case of a 50 Hz supply there are 100 zeros per second. This property of a.c. circuit is exploited for interruption purposes and the current is not allowed to rise again after a zero occurs. Also it is neither necessary nor desirable to cut off the current at any other point on the a.c. wave because this will induce high voltages in the system.

The phenomenon of arc extinction is explained by two theories: (*i*) energy balance theory; and (*ii*) voltage race theory.

Energy balance theory: This theory is based on the fact that if the rate at which the heat generated between the contacts is lower than the rate at which heat between the contacts is dissipated the arc will be extinguished, otherwise it will restrike. The heat generated varies from time to time depending upon the separation of contacts. Initially when the contacts are about to open, the restriking voltage is zero and, therefore, the heat generated is zero. Again when the contacts are fully open, the resistance between the contacts is almost infinite and hence the heat generated is zero. Between these two limits the heat generation reaches a maximum. Now, if the heat so generated could be removed by cooling, lengthening and splitting the arc at a rate faster than the generation the arc is extinguished.

Voltage race theory: The arc, as is said earlier, is due to the ionization of the gap between the contacts. Effectively the resistance in the initial stages is small and as the contacts separate, resistance is increasing. The problem here is to remove the electrons and ions from the contact gap immediately after the current reaches zero. Because it is this stage where the ionization is at minimum and if the ions could be removed either by recombining them into neutral molecules or by sweeping them away by inserting insulation at a rate faster than the rate of ionization, the arc will be interrupted. The recombination can be accelerated by cooling and increasing the pressure in the arc space.

The ionization at current zero depends upon the voltage appearing between the contacts. This voltage is known as restriking voltage which depends upon the power factor and other factors of the circuit like the inductance and capacitance. The expression for voltage is given by (for a lossless system)

$$v = V\left(1 - \cos\frac{t}{\sqrt{LC}}\right)$$

where v = restriking voltage at any instant t, V the value of voltage at the instant of interruption and L and C are the series inductance and shunt capacitance up to the fault point. It can be seen that lower the value of the inductance and capacitance the higher will be the natural frequency of oscillation and more severe will be the effect of restriking voltage. Therefore, a fault near the source is more severe from the view-point of arc interruption as compared to a fault far from the source. It is seen that v is a function of V which in turn depends upon the power factor of the system. In case the system is highly lagging, this voltage will correspond to the peak system voltage. The variation of this voltage is shown in Fig. 15.1(*a*).

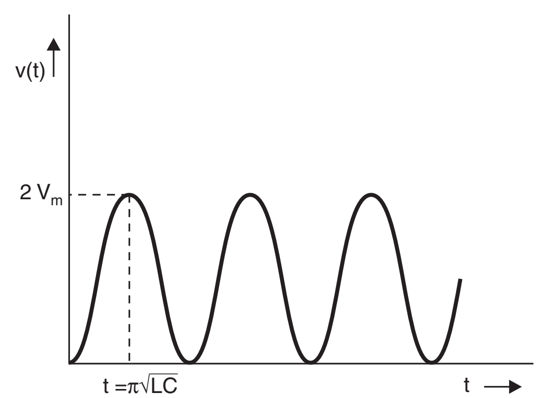


Fig. 15.1 (*a*) Restriking voltage across breaker contacts.

In the analysis in Section 12.3 we assumed that the arc voltage across the breaker contacts is negligible which is true in high voltage circuits where it is usually only a small percentage of the system voltage even though in low voltage circuits it may be much more significant. Fig. 15.1(*a*) shows the restriking voltage across the breaker contacts when arc voltage is neglected and losses are also neglected. If losses are taken into account but arc voltage neglected, the restriking voltage will be damped out depending upon the effective resistance of the system. The restriking voltage across the breaker contacts will then be as shown in Fig. 15.1(*b*).

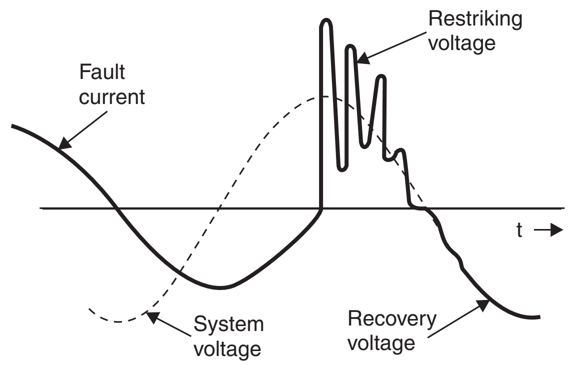


Fig. 15.1(*b*) Restriking voltage when losses are included.

While drawing Fig. 15.1(*b*) we assumed that the current lags behind the voltage during fault by 90°. In actual practice it is not always true. A fault current can have any degree of asymmetry depending upon the time in the cycle at which the fault occurs. The circuit breaker will again interrupt at current zero and the recovery voltage will oscillate about the instantaneous value of the supply voltage which in this case will be $V_m \sin \phi$, where ϕ is the angle by which the current lags the voltage. The restriking voltage, therefore, in this case will be low as compared to when $\phi = 90°$. This is illustrated in Fig. 15.1(*c*).

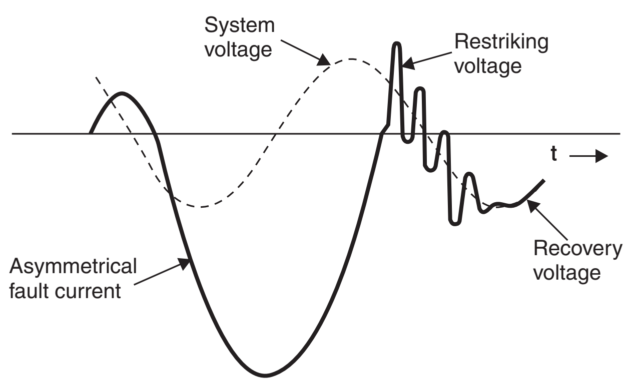


Fig. 15.1(*c*) Restriking voltage when asymmetrical current is interrupted.

If the arc voltage is included into the analysis it will have the effect of increasing the restriking voltage. This effect is offset by a second effect of the arc voltage which is to oppose the current flow and thereby change the phase of the current, bringing it more into phase with the supply voltage; thereby the voltage is not at its peak when the current passes through zero value. This is illustrated in Fig. 15.1(*d*).

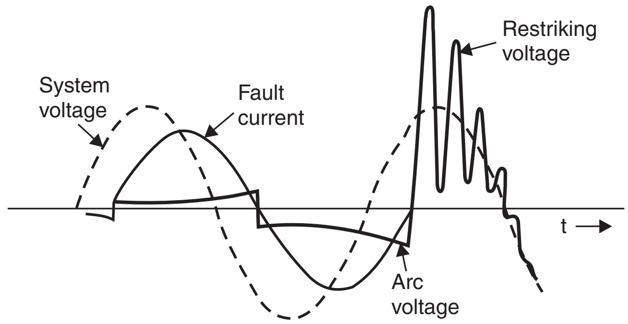


Fig. 15.1(*d*) Restriking voltage including arc voltage.

Restriking voltage: The resultant transient voltage which appears across the breaker contacts at the instant of arc extinction is known as the restriking voltage.

Recovery voltage: The power frequency r.m.s. voltage that appears across the breaker contacts after the transient oscillations die out and final extinction of arc has resulted in all the poles is called the recovery voltage.

Active recovery voltage: It is defined as the instantaneous recovery voltage at the instant of arc extinction.

The instantaneous recovery voltage is given by

$$V_{\text{ar}} = KV_m \sin \phi$$

where $K = 1$ if the three-phase fault is also grounded and $K = 1.5$ if the three-phase fault is isolated.

Rate of Rise of Restriking Voltage (*RRRV*)*:* As shown in Fig 15.1(*a*),

$$\text{The average RRRV} = \frac{\text{Peak value of restriking voltage}}{\text{Time taken to reach to peak value}}$$

$$= \frac{2V_m}{\pi\sqrt{LC}}$$

Rewriting the equation,

$$v = V_m\left(1 - \cos\frac{t}{\sqrt{LC}}\right)$$

The RRRV is given by

$$\frac{dv}{dt} = \frac{V_m}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}}$$

This is maximum when

$$\frac{t}{\sqrt{LC}} = \frac{\pi}{2}$$

or

$$t = \frac{\pi}{2}\sqrt{LC}$$

and the value is $\dfrac{V_m}{\sqrt{LC}}$

The rate at which the restriking voltage rises is, therefore, very important in the arc extinction process because the ionization process will depend upon this rate. Therefore, it is found that if the RRRV is smaller than the rate at which the dielectric between the contacts is developed, the arc will be extinguished; otherwise there will be further restrike. This theory has been advocated by Dr. J. Slepian.

15.3 CURRENT CHOPPING

When a circuit breaker is made to interrupt low inductive currents such as currents due to no load magnetising current of a transformer, it does so even before the current actually passes through zero value especially when the breaker exerts the same deionizing force for all currents within its short circuit capacity. This breaking of current before it passes through the natural zero is termed as current chopping. This current chopping may take place even in breakers which produce varying degree of deionizing force. The effect of a practically instantaneous collapse of the arc current, even of only a few amperes, is potentially very serious from the point of view of over-voltages which may result in the system. Referring to Fig. 15.2, the arc

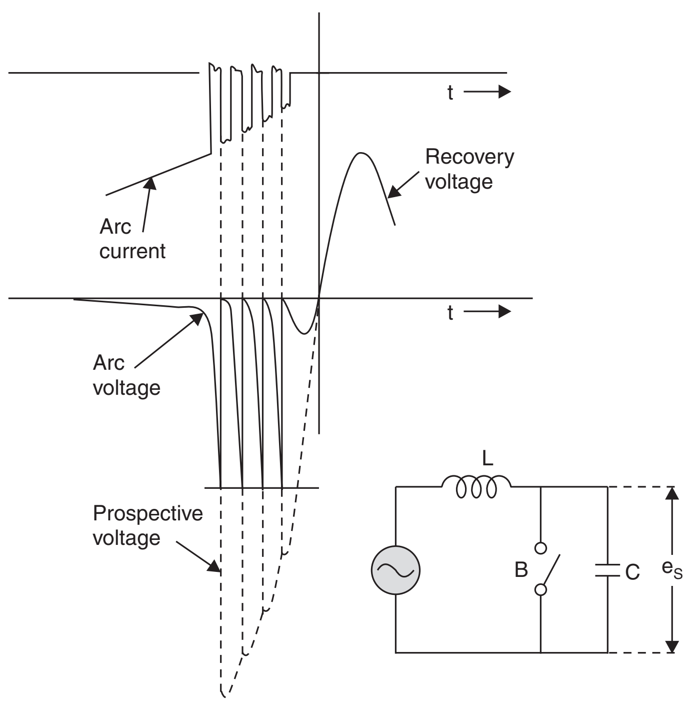


Fig. 15.2 Current chopping.

current is seen to approach zero in normal fashion initially with low arc voltage so that there is virtually no capacitance current. At a certain arc current, because of the large deionizing force, the current suddenly reduces to zero. The current in the arc was flowing from the source through the inductance and the circuit breaker contacts. The energy contained in the electromagnetic field cannot become zero instantaneously. It changes into some other form of energy. The only possibility is the conversion from electromagnetic to electrostatic form of energy *i.e.,* the current is diverted to the capacitor from the arc. If i_a is the instantaneous value of arc current where the chop takes place, the prospective value of voltage to which the capacitor will be charged, will be

$$V = i_a \sqrt{L / C}$$

where L is the series inductance and C the shunt capacitance. This voltage appears across the circuit breaker contacts. Fortunately, the breaker gap restrikes before the voltage is allowed to reach this value (prospective voltage which normally is very high as compared to the system voltage). The deionizing force is still in action and the current will again be chopped. Successive chops may occur as shown in the diagram until a final chop brings the current to a zero prematurely with no further restrike since the gap is now in an advanced stage of deionization.

Resistance Switching

As is seen in the previous section that during current chopping very high voltages may appear across the C.B. contacts and these voltages may endanger the operation of the system. To reduce these voltages, a resistance across the breaker contacts is connected as shown in Fig. 15.3. The shunt resistor performs one or more of the following functions:

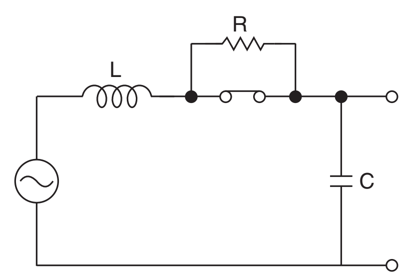


Fig. 15.3 Resistance switching.

(*i*) It reduces the rate of rise of restriking voltage and thus reduces duties of the breaker.

(*ii*) It reduces the transient voltages during switching out inductive or capacitive loads.

(*iii*) In a multi-break C.B. they may be used to help to distribute the transient recovery voltage more uniformly across the several gaps.

To reduce the transient recovery voltage requires a considerably lower value of resistor whereas for voltage equalisation a resistor of relatively high ohmic value will be required. In this case it is required that its resistance be low compared with the reactance of the capacitance, shunting the breaks at the frequency of the recovery transient. It is often necessary to compromise and make one resistor do more than one of these jobs Critical restriking voltage damping is obtained if

$$R = 0.5\sqrt{\frac{L}{C}}$$

Example 15.1: In a system of 132 kV, the line to ground capacitance is 0.01 μF and the inductance is 5 henries. Determine the voltage appearing across the pole of a C.B. if a magnetising current of 5 amps (instantaneous value) is interrupted. Determine also the value of resistance to be used across the contacts to eliminate the restriking voltage.

Solution: This is a case of conversion of electromagnetic energy into electrostatic energy and hence the voltage appearing across breaker contacts is nothing but the voltage across the capacitor which is given by

$$e = i\sqrt{\frac{L}{C}} = 5 \times \sqrt{\frac{5 \times 10^6}{0.01}} = 5 \times 10^4 \sqrt{5}$$

$= 11.18 \times 10^4$ volts or 111.8 kV. **Ans.**

In order to eliminate the transient critically the value of resistance across the breaker contacts required is

$$R = \frac{1}{2}\sqrt{\frac{L}{C}} = 0.5 \times 10^4 \sqrt{5}$$

$= 11180$ ohms

$= 11.18$ kΩ. **Ans.**

Example 15.2: In a short circuit test on a 132 kV 3-phase system, the breaker gave the following results: p.f. of the fault 0.4, recovery voltage 0.95 of full line value; the breaking current is symmetrical and the restriking transient had a natural frequency of 16 kHZ. Determine the rate of rise of restriking voltage. Assume that the fault is grounded.

Solution: The peak value of line to neutral voltage

$$\frac{132}{\sqrt{3}}.\sqrt{2} = 107.75 \text{ kV}$$

Since the recovery voltage is 0.95 times the full line value, the recovery voltage = 107.75 × 0.95 = 102.4 kV. Since the power factor of fault is 0.4, the value of the voltage when the current is zero will be $V_m \sin \theta$, where $\theta = \cos^{-1} 0.4 = 66.42°$ or $\sin \theta = 0.916$.

∴ The active recovery voltage = 102.4 × 0.916

= 93.85 kV

The maximum restriking voltage = 2 × 93.85 = 187.7 kV

$$\therefore \qquad \text{RRRV} = \frac{V}{t}, \text{ where } t = \frac{1}{2f_n} = \frac{10^{-3}}{2 \times 16} \text{ sec}$$

$$\therefore \qquad \text{RRRV} = \frac{187.7 \times 2 \times 16}{10^{-3}} = 32 \times 187.7 \times 10^3 \text{ kV/sec}$$

= 6.0 kV/μsec **Ans.**

Example 15.3. In a short circuit test on a 3-pole, 132 kV C.B. the following observations are made: p.f. of fault 0.4, the recovery voltage 0.90 times full line value, the breaking current symmetrical, the frequency of oscillations of restriking voltage 16 kHz. Assume that the neutral is grounded and the fault does not involve ground, determine the average rate of rise of restriking voltage.

Solution: Peak value of L-G voltage $= \dfrac{132}{\sqrt{3}} \times \sqrt{2} = 107.77$ kV

Instantaneous value of recovery voltage is

$$V_r = KV_m \sin \phi$$

where $K = K_1 K_2$

and K_1 = multiplying factor due to system voltage

K_2 = 1.5 here as fault does not involve ground

$\therefore \qquad V_r = 0.90 \times 1.5 \times 107.77 \times 0.92 = 133.85$ kV

Now $\qquad f_n = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore \qquad \dfrac{1}{\pi\sqrt{LC}} = 2 \times 16 \times 10^3 = 32 \times 10^3$

The average $\qquad \text{RRRV} = \dfrac{2V_r}{\pi\sqrt{LC}} = 2 \times 133.85 \times 32 \times 10^3$ kV/sec

or the average $\qquad \text{RRRV} = 2 \times 133.85 \times 32 \times 10^3 \times 10^{-6}$ kV/μsec

$= 8.566$ kV/μsec. **Ans.**

15.4 OIL CIRCUIT BREAKER

Mineral oil has better insulating properties than air. It is this property of oil which prompted Steinmetz to break current under oil. He immersed an ordinary knife switch in oil and investigated the breaking capacity of the arrangement. Reliability, simplicity of construction and relative cheapness are particular virtues of oil breakers. Oil, however, has the following disadvantages:

1. It is inflammable and may cause fire hazards.
2. There is a possibility of its forming an explosive mixture with air.
3. Because of the production of carbon particles in the oil due to heating, periodical reconditioning or replacement is required.

The following are the categories of oil circuit breakers that have been developed so far:

1. The plain-break oil C.B.
2. The controlled break oil C.B. or bulk oil C.B.
3. Minimum oil C.B.

The first and second categories of breakers are also known as the dead tank construction because the tank is held at earth potential whereas the third category is known as live tank as the tank containing oil is insulated from the ground.

The Plain-break Oil Circuit Breaker

The circuit breaker consists of a metal tank containing oil and encloses two or more contacts (Fig. 15.4). Since large energies are to be dissipated within the tank, a large gaseous pressure is developed. To withstand such a large pressure the tank has to be a strong one. It is usual to make the tank and the top plates either of welded sheet steel or boiler plates. The distance between phases and the clearances between the live metal and the earthed metal are a function of the operating voltage.

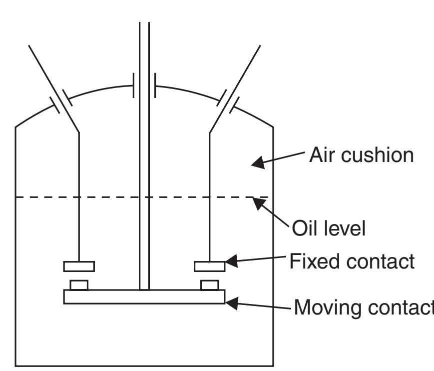


Fig. 15.4 Schematic diagram of a plain-break oil circuit breaker.

An air cushion is necessary between the oil surface and tank cover to accommodate the displaced oil when gas forms around the arc. The air cushion also serves to absorb the mechanical shock of the upward oil movement. The breaker tank should be securely bolted to an adequate foundation, otherwise it may jump out when interrupting very heavy current.

An ample head of oil above the arcing contacts is necessary (*i*) to provide substantial oil pressure at the arc; and (*ii*) to prevent occurrence of the chimney effect. A chimney of gas from the arc to the oil surface is produced which comes in contact with the earthed tank. If this gas is partially ionized and is of low dielectric strength, an arc will strike between the contact and the earthed tank with serious consequences. Therefore, an appreciable amount of oil depending upon the working voltage should always exist between the contact and the tank.

A gas outlet from the tank is essential and some form of vent is fitted in the tank cover. The position of the vent is carefully chosen so that the partially ionized gases which come out of the vent do not harm the personnel and also do not cause flash-over to the neighbouring equipments.

Principle of Operation: The plain-break principle involves the simple process of separating the current carrying contacts under oil with no special control over the resulting arc other than the increase in length caused by the moving contact. The final arc extinction is obtained when a certain critical gap between the contacts is reached, the length of which depends upon the arc current and the recovery voltage.

At the instant of contact separation an arc is established between them. Initially, the separation is very small and a high voltage gradient between the contacts ionizes the oil. The gas obtained from the oil is mainly hydrogen which cannot remain in molecular form and is dissociated in its atomic form releasing lots of heat. With this, the arc core attains a temperature of 5000°K. The mixture of gases occupies a volume about one thousand times that of the oil decomposed. The oil is, therefore, pushed away from the arc and an expanding gas bubble surrounds the arc region Based on energy balance principle, final extinction of arc takes place at a current zero when the power input to the arc is less than that dissipated between the contacts.

From practical point of view the speed of the break should be as high as possible because a certain break distance has to be reached before interruption is likely to occur and the sooner this is achieved the smaller the energy released in the breaker and the less mechanically strong a breaker will have to be designed.

The double break arrangement as shown in Fig. 15.4 is perhaps the most familiar of all oil circuit breakers. The two breaks in series give rapid arc lengthening without the need for a specially fast contact speed, and the total gap distance at the end of stroke can conveniently be made ample. He vertical break principle also permits the use of a cylindrical oil tank requiring relatively low floor area.

The Controlled-break Oil Circuit Breakers

The plain-break circuit breakers are used widely on low voltage d.c. circuits and on low voltage distribution a.c. circuits. For higher voltages they become unduly large in size and require huge amounts of transformer oil. Also it is not suitable for high speed interruption *i.e.*, they cannot be used for auto reclosing.

The primary object of any controlled-break principle is to obtain final extinction consistently, while the contact gap is still short and is approximately equal to the clearance required under oil when in the open position. The arc control pots are shown in Fig. 15.5.

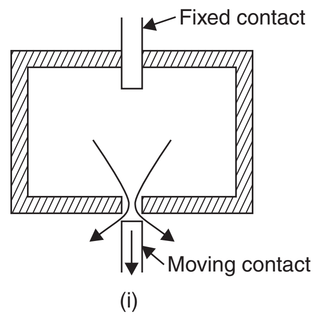


Fig. 15.5 Types of arc control pots: (*i*) axial blast pot, and (*ii*) cross blast pot.

The contacts are enclosed in a chamber made of insulating material and provided with a series of vents on one side of the chamber. Final arc extinction takes place within the chamber which is secured to the fixed contact. The whole assembly is immersed in the oil. There is a small clearance between the throat and the moving contact. Also, in most types, one or more small bleed holes are provided in the upper wall to prevent air being trapped when the breaker is filled with oil.

The arcing conditions in this breaker are different from those of plain-break breakers. The internal space available to the gases which are produced due to the decomposition of oil is little more than that swept out by the moving contact. As is said earlier, the mixture of gases occupies a volume about one thousand times that of oil decomposed, a large pressure is set up between the contacts. As a result the movement of oil is restricted in the chamber and the expansion of internal gas bubble is limited in the pressure chamber. Also the heated gas is forced out of the chamber away from the arc. These two conditions allow much better cooling of the arc which results in higher breaking capacities of these breakers as compared to plain-break breakers. The flow of gases through the vents lengthens the arc and the gases flowing around the arc with high velocity give turbulent condition which increases the energy losses. Also because of the large gas pressures, the mean free path of the electrons and ions is reduced which results in effective deionization. It is seen that the pressures are self-regulated in the sense that higher the breaking currents larger will be the pressures generated and these breakers give their best performance at the highest currents within their ratings.

Various improvements in the design of pressure chambers have been suggested to provide high speed arc interruption especially at currents below the rated maximum. One solution to this problem is to use an intermediate contact between the fixed and moving contacts in the chamber. The important features of the intermediate contact are that its movement is limited and that spring pressure tends to keep it up against the moving contact. Thus when the moving contact starts to withdraw, the intermediate contact follows and a primary arc is drawn between this and the fixed contact (Fig. 15.6). After some time the intermediate contact meets a stop and a second (series) arc is drawn between the intermediate and moving contacts.

The aim here is to extinguish the second arc quickly by using the gas pressure and the oil momentum due to the first arc. This is done by arresting the intermediate contact at a definite short distance and high gas pressure is achieved by providing a small vent. Thus fast and high gas pressures are obtained with safety to the chamber.

A modification of this double break oil C.B. has been developed to give a similar effect. A common cross-bar carries both sets of moving contacts which are so arranged that contact separation at one of the breaks occurs slightly before that in the other break. The arc in the first break creates gas pressure and oil momentum, which are effective at the second break. This second arc, therefore, experiences 'ready-made' deionizing effects as soon as it appears.

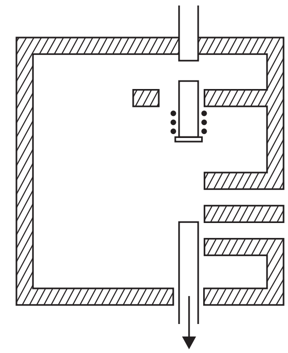


Fig. 15.6 Improved version of control pots.

Minimum Oil Circuit Breakers

One of the important developments in the design of oil C.Bs. has been to reduce the amount of oil needed because the severity of a fire involving an oil switch is to some extent proportional to the volume of oil contained. The other advantages are:

(*i*) Reduction in tank size.

(*ii*) Reduction in total weight.

(*iii*) Reduction in cost.

The use of pressure chamber for arc control in the bulk oil C.B. mentioned in the previous article reduced the volume of oil. But it still requires huge amounts of oil for higher voltages. The minimum oil C.B. uses solid materials for insulating purposes and uses just enough oil for arc quenching. The bulk oil breakers described in the previous section are of the dead tank type because tank is at earth potential whereas the arc interrupting device is enclosed in a tank of insulating material in case of a minimum oil breaker, the whole of which is at line voltage during normal operation and, therefore, these are known as live tank breakers.

The minimum oil C.Bs. can be of self-blast type or external blast type or a combination of the two. In case of self-blast type the gas pressure developed depends upon the current to be interrupted. The higher the current to be interrupted the larger is the gas pressure developed and hence more effective is the breaker for arc quenching. But this puts a limit on the design of the arc chamber for mechanical stresses. With the use of better insulating materials for the arcing chambers such as glass fibre, reinforced synthetic resins etc., the minimum oil C.Bs. are able to meet easily the increased fault levels of the systems. Most of the minimum oil C.Bs. these days are the self-blast type.

There are two different designs of the arcing chambers in terms of the ventings provided: (*i*) axial venting, and (*ii*) radial venting.

In case of axial venting the gases produced sweep the arc in longitudinal direction whereas in case of radial venting they sweep the arc in transverse direction. Since axial venting generates high gas pressures and has high dielectric strength it is used mainly for the interruption of low currents at high voltages. The radial venting is used for interruption of relatively heavy currents at low voltages as the gas pressures developed are low and also the dielectric strength is low. Many a times a combination of both is used so that the arc chamber is equally efficient at low as well as at high currents. Such chambers, however, suffer from the disadvantages of longer arcing periods which can be eliminated by providing oil injection devices in addition to the self-blast. The contacts are usually operated by pull rods or rotating insulators actuated in turn by solenoid or pneumatic mechanisms. This type of C.B. is available up to 8000 MVA at 245 kV with a total break time of 3 to 5 cycles.

15.5 AIR CIRCUIT BREAKERS

The arc interruption in oil is due to the generation of hydrogen gas because of the decomposition of oil. This fact prompted the investigators to study the interruption in air. No doubt, arc interruption properties of hydrogen are much superior to air, but air has several advantages as an arc extinguishing medium as compared to oil. They are:

1. Fire risk and maintenance associated with the use of oil are eliminated.
2. Arcing products in air are generally completely removed whereas oil deteriorates with successive breaking operation. Therefore, the expense of regular oil replacement is avoided.
3. Heavy mechanical stresses set up by gas pressure and oil movement are absent.
4. Relatively inferior arc extinguishing properties of air may be offset by using various principles of arc control and operating air at high pressures.

This is why except for a certain medium range of voltages, air circuit breakers are widely used for the low voltage circuits as well as the highest transmission voltages.

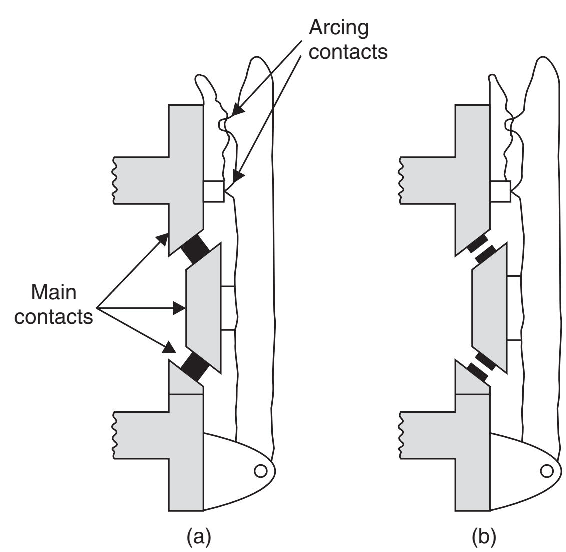


Fig. 15.7 The use of additional contacts for arcing:
(*a*) Fully closed; and (*b*) Main contacts open and arcing contacts closed.

Simple air circuit breakers which do not incorporate any arc-control devices are used for low voltages, below 1 kV. The oil C.Bs. are not used for heavy fault currents on low voltages due to carbonization of oil and unduly rapid current collapse. These breakers usually have two pairs of contacts per phase. The main pair of contacts carries the current under normal operating conditions and is made of copper. The additional pair actually becomes the arcing electrode as the circuit breaker is opened and are made of carbon because the vaporization and distortion of the contacts due to the heat of the arc are confined to these contacts and, therefore, the material used for the contacts should be non-volatile. The main contacts separate while the arcing pair is still in contact and the arc is, therefore, initiated only when the arcing pair separates (Fig. 15.7).

The principle of operation of these breakers is based on the high resistance method discussed earlier.

Arc Chute Air Circuit Breakers: In this case the arc is extinguished by lengthening and increasing the voltage gradient *i.e.*, power loss of the arc. The arc discharge is moved upward by both thermal and electromagnetic effects as shown in Fig. 15.8. This is then driven into a chute consisting of splitters and baffles. The splitters increase the length of the arc even further and the baffles give improved cooling. In this breaker relatively high arc resistance is obtained near current zeros. This effect plays an important role in obtaining high breaking capacity by modifying the circuit p.f. near current zero such that the voltage available to restrike the arc is appreciably less than the peak value. A disadvantage of arc chute principle is the inefficiency at low currents where the electromagnetic fields are weak. The chute itself is not necessarily less efficient in its lengthening and de-ionizing action than at high currents, but the movement of the arc into the chute tends to become slower and high speed interruption is less assured.

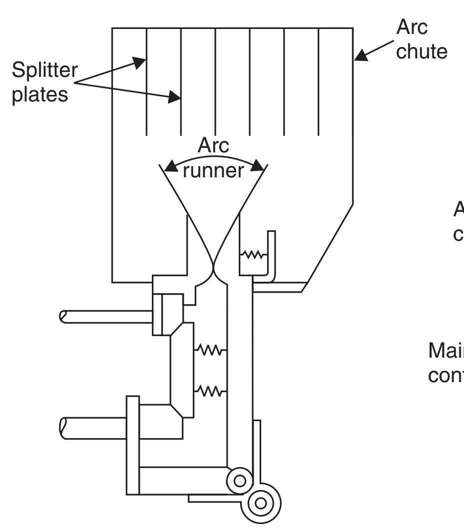


Fig. 15.8 Diagram of an arc chute air circuit breaker.

15.6 AIR BLAST CIRCUIT BREAKERS

The most common method of arc control in air circuit breakers is that of subjecting the arc to high pressure air blast. There are two types of air blast circuit breakers: (*i*) Axial blast types, and (*ii*) Cross blast types.

The designations refer to the direction of the air blast in relation to the arc.

Axial Blast Circuit Breaker

The fixed and moving contacts are held in closed position by spring pressure (Fig. 15.9). The breaker reservoir tank is connected to the arc chamber when a tripping impulse opens the air valve. The air entering the arc chamber exerts pressure on the moving contacts which moves when the air pressure exceeds the spring force. The air moves with sonic velocity near the nozzle and the arc is subjected to high pressure and there is considerable heat loss due to forced convection. With this the diameter of the arc is reduced and the core temperature is very high. The temperature gradients set up within the arc are very steep which results in greater heat losses.

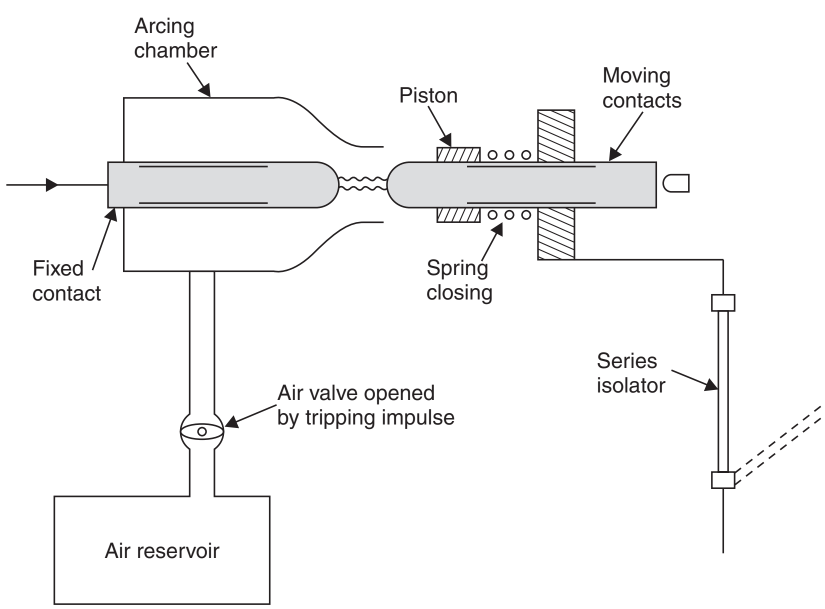


Fig. 15.9 Axial blast air circuit breaker.

When the current passes through zero, the air blast is more effective because the residual column is very narrow and the high rate of heat loss becomes increasingly effective. It is known that with a given arc length and heat loss per unit surface area, the total rate of heat loss is proportional to the arc diameter, whereas the total energy content of the arc is roughly proportional to the square of the diameter. The narrower the residual column, the more effective are the heat losses in reducing the temperature and conductivity. Such conditions may allow the column to recover dielectric strength very rapidly at current zeros.

It is important to note here that the air pressure from the reservoir is maximum initially and falls thereafter. It is known that for a particular reservoir pressure there is a certain optimum contact gap at which the breaking capacity is a maximum. This gap is usually small (in mm) and may reach very quickly if the inertia of the moving parts is kept to a minimum. The shorter the gap, relatively smaller amounts of energy are released in the arcing chamber. The arc is kept in the high velocity blast of air converging into the nozzle throat. The falling reservoir pressure and short optimum gap result in three important features of the axial blast principle.

1. The interruption must take place at the first current zero after the optimum gap has reached otherwise restrikes may take place at subsequent zeros due to falling air pressures. It is to be noted here that the chances of interruption in case of O.C.B. increase if arcing persists beyond the first current zero.

2. The axial blast circuit breaker gives high speed clearance because of the short gap needed for interruption. This is desirable for improving transient stability on high voltage transmission and interconnection networks.

3. The small contact gap after interruption constitutes inadequate clearance for the normal system voltage; therefore, an auxiliary switch known as an isolating switch is incorporated as part of this C.B. and opens immediately after fault interruption to provide the necessary insulation clearance. The moving contact is allowed to return and engage the fixed contact as the air pressure in the chamber falls below the spring pressure. The air pressure on the moving contact must be maintained until the isolator is fully open.

For low voltages the isolating switch is not required and an adequate travel is provided instead for the moving contact.

The arcing time of arc controlled circuit breaker varies considerably depending upon the breaking current. The higher the breaking current (within the rating of the breaker), the smaller the arcing time. The arcing time in case of air blast circuit breaker is independent of the breaking current because of the fixed air pressure and the optimum short gap. The arc duration as a function of breaking current is almost flat as can be seen in Fig. 15.10. The short gap along with an isolating switch gives a total break time of 2 to 5 cycles.

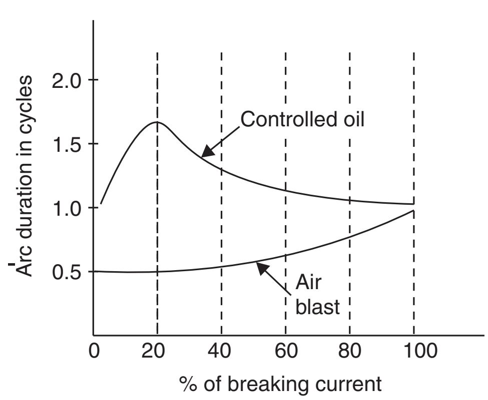


Fig. 15.10 Short circuit performance characteristic.

The operation of the air blast circuit breaker is very much affected by the circuit natural frequency. When the current is passing through zero value the residual column has relatively high resistance which reduces the likelihood of the restriking voltage transient being damped. Now the effect of rate of rise of restriking voltage during this zero current condition is more serious especially where the chance of extinction decreases after the optimum gap has reached. It is to be noted that the chance of extinction in case of oil circuit breaker increases form one current zero to the next. The effect of natural frequency on the performance of the air blast circuit breaker is overcome by shunting the arc with resistors of suitable values.

The Cross Blast Air Circuit Breakers

In this case the blast is directed transversely, across the arc and the physical conditions are different from the axial blast. The cross blast lengthens and forces the arc into a suitable chute and serves rather the same purpose as electromagnetic force in the low voltage air C.B. discussed earlier. The final interruption gap is good enough to provide normal insulation clearance so that a series isolating switch is unnecessary. Consistent high speed operation is not usually obtained to the extent possible with the axial blast air C.Bs.

Air blast C.Bs. can also be of (*i*) live tank type, and (*ii*) dead tank type. Live tank has a metal tank insulated from ground and compressed air is used for insulation between contacts. The tank is supported by a porcelain insulator. In case of dead tank type, the tank is held at ground potential. The breaker contacts are insulated from the tank by compressed air in parallel with solid insulation immersed in the air.

Most of the circuit breakers up to 11 kV are either of the air break type or of the oil break type. Between 11 kV and 66 kV mainly oil C.Bs. are in use while between 132 kV and 275 kV the market is shared by oil (both minimum as well as bulk oil) and gas blast breakers. At the highest system voltages *i.e.*, between 400 kV and above the C.Bs. are of the gas blast type.

15.7 VACUUM CIRCUIT BREAKERS

A vacuum system is one in which the pressure maintained is at a value below the atmospheric pressure and is measured in terms of mm of mercury. One standard atmospheric pressure at 0°C is equal to 760 mm of mercury. One mm of Hg pressure is also known as one torr after the name of Torricelli who was the first to obtain pressures below atmospheric, with the help of mercury barometer. Sometimes 10^{-3} torr is known as one micron. It is now possible to obtain pressures as low as 10^{-8} torr.

In a Townsend type of discharge, in a gas, the mean free path of the particles is small and electrons get multiplied due to various ionization processes and an electron avalanche is formed. In a vacuum of the order of 10^{-5} torr the mean free path is of the order of few metres and thus when the electrodes are separated by a few mm an electron crosses the gap without any collision. Therefore, in a vacuum the current growth prior to breakdown cannot take place due to formation of electron avalanches. However, if it could be possible to liberate gas in the vacuum by some means, the discharge could take place according to Townsend process. Thus, a vacuum arc is different from the general class of low and high pressure arcs. In the vacuum arc the neutral atoms, ions and electrons do not come from the medium in which the arc is drawn but they are obtained from the electrodes themselves by evaporating its surface material. Because of the large mean free path for the electrons, the dielectric strength of the vacuum is a thousand times more than when the gas is used as the interrupting medium. In this range of vacuum the breakdown strength is independent of the gas density and depends only on the gap length and upon the condition of electrode surface. Highly polished and thoroughly degassed electrodes show higher breakdown strength. Contacts get roughened after use and thus the dielectric strength or breakdown strength decreases which can be improved by applying successive high voltage impulses which of course does not change the roughened surface but removes the loosely adhering metal particles from the electrodes which were deposited during arcing. It has been observed that for a vacuum of 10^{-6} torr some of the metals like silver, copper-bismuth etc. attain their maximum breakdown strength when the gap is slightly less than 3 mm. This property of vacuum switches permits the use of short gaps for fast operation.

The Vacuum Arc

The vacuum arc results from the neutral atoms, ions and electrons emitted from the electrodes themselves. As the current carrying contacts are separated, cathode spots are formed depending upon the current to be interrupted. For low currents a highly mobile cathode spot is formed and for large currents a multiple number of cathode spots are formed. These spots constitute the main source of vapour in the arc. The processes involved in drawing the arc will be due to high electric field between the contacts or resistive heating produced at the point of operation or a combination of the two. The cathode surfaces, normally, are not perfectly smooth but have many micro projections. When the contacts are separating, the current flowing in the circuit will be concentrated in these projections as they form the last point of contact. Due to their small area of cross-section, the projections will suffer explosive evaporation by resistive heating and supply sufficient quantity of vapour for the arc formation. Since in case of vacuum breakers the emission occurs only at the cathode spots and not from the entire surface of the cathode, the vacuum arc is also known as cold cathode arc. In cold cathode the emission of electrons could be due to any of the combinations of the following mechanisms: (*i*) Field emission; (*ii*) Thermionic emission; (*iii*) Field and Thermionic emission; (*iv*) Secondary emission by positive ion bombardment; (*v*) Secondary emission by photons; and (*vi*) Pinch effect.

Vacuum Arc Stability

In a.c. circuit the current passes through zero value 100 times in a second. It is desirable to interrupt the current when it is passing through zero value, otherwise over-voltage will be induced due to current chopping. Therefore, it is necessary for successful arc interruption that it be stable for a half cycle duration and particularly it should continue to exist when the current approaches zero. The stability of arc in vacuum depends upon: (*i*) the contact material and its vapour pressure, and (*ii*) circuit parameters such as voltage, current, inductance and capacitance. It has been observed that higher the vapour pressure at low temperature the better is the stability of the arc. There are certain metals like Zn, Bi which show these characteristics and are better electrode materials for vacuum breakers. Besides the vapour pressure, the thermal conductivity of the metal also affects the current chopping level. A good heat conducting metal will cool its surface faster and hence its contact surface temperature will fall which will result into reduction in evaporation rate and arc will be chopped because of insufficient vapour. On the other hand, a bad heat conductor will maintain its temperature and vaporization for a longer time and the arc will be more stable.

Shunt capacitor across the breaker contacts reduces the average life time of the arc. The higher the value of the capacitance, more is the reduction in life time. An inductance, in series, on the other hand increases the life time. Similarly, higher the system operating voltage, the longer is the duration for which the arc exists because more restoring voltage is available to keep the arc burning.

Current Chopping

It is known that current chopping in air and oil C.Bs. occurs due to instability in the arc column whereas in case of vacuum breakers it depends upon the vapour pressure and the electron emission properties of the contact material. It is possible to reduce the current level at which chopping takes place by selecting a contact material which gives out sufficient metal vapour to allow the current to come to a very low value or zero value but it is normally not done as it affects the dielectric strength adversely. Since gas pressure is low in a vacuum switch, the main criterion to limit current chopping is the proper selection of contact material. It has been found that no single metal gives all the desirable properties. A high vapour pressure and low conductivity metal is more desirable to limit the current chopping whereas low vapour pressure metals are more desirable from the arc extinction point of view. Materials having high boiling and melting points have low vapour pressure at high temperatures but are poor conductors whereas metals having low boiling and melting points have high vapour pressure at high temperatures, low electron functions and have good thermal and electrical conductivities. Therefore, to combine these contradictory properties in one single material, composites of two or more metals or a metal and a nonmetal have to be made. Copper-bismuth, silver-bismuth, silver-lead, copper-lead are some of the alloys used as contact materials.

Vacuum Arc-recovery Phenomenon

When the arc interruption is over, the space between the surrounding the electrodes is filled with vapour and plasma. The presence of this residue affects very much the ability of an interrupter to withstand high voltages. The process by which this residue decays and by which the vacuum gap regains its dielectric strength is known as arc recovery phenomenon. At current zero the cathode spot extinguishes within 10^{-8} second and after this the original dielectric strength is established very soon. This quick build-up of dielectric strength is due to the condensing, quick diffusion and of metal vapour to the glass walls in absence of gas molecules. After the arc is interrupted, the recovery strength during the first few micro-seconds is 1 kV/μsec for an arc current of 100 A, as compared with 50 V/μsec in case of air gap.

Construction of Vacuum Breaker

A schematic diagram of the vacuum C.B. is shown in Fig. 15.11. It is a very simple device as compared to an air or an oil C.B. The outer envelope is normally made of glass due to the ease of joining it to the metallic end-caps and also because the glass envelope makes it easy to examine from outside the state of the contacts after the breaker has interrupted the current. This is important since a change from a silvery mirror-like finish to a milky white colour shows that the baffle is losing its vacuum. A sputter shield is provided in between the contacts and the envelope in order to prevent the metal vapour reaching the envelope as it reduces the breakdown strength between the contacts. This is generally made of stainless steel. Inside the sputter shield the breaker has two contacts, one fixed and the other moving contact which moves through a short distance of 5 to 10 mm depending upon the operating voltage. The metallic bellows made of stainless steel is used to move the lower contact. The design of the bellows is very important as the life of the vacuum breaker depends upon the ability of this part to perform repeated operations satisfactorily. The periphery of the end-cap is sealed to the envelope and the fixed contact stem is an integral part of one end-cap. One end of the fixed as well as moving contact is brought out of the chamber for external connections.

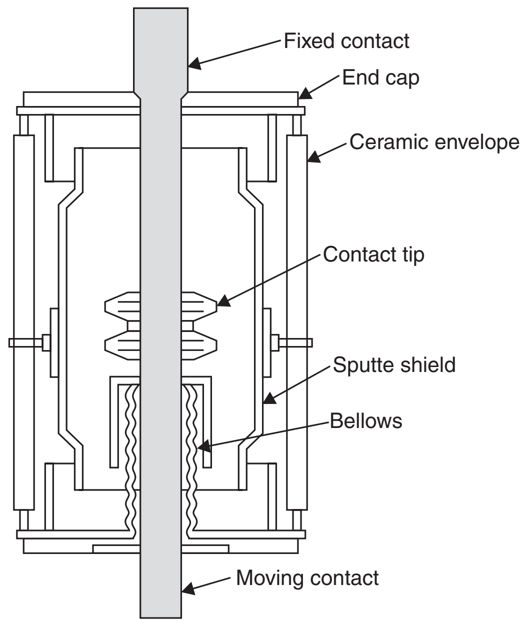


Fig. 15.11 A schematic diagram of a vacuum circuit breaker.

The lower end of the breaker is fixed to a spring-operated or solenoid operated mechanism so that the metallic bellows inside the chamber are moved downward and upwards during opening and closing operation respectively. It is to be noted that the operating mechanism should provide sufficient pressure for a good connection between the contacts and should avoid any bouncing action.

Application of Vacuum Breakers

Because of the short gap and excellent recovery characteristics of vacuum breakers, they can be used where the switching frequency is high and required to be reliable. For low fault interrupting capacities the cost is low as compared to other interrupting devices. The vacuum switches can be used for capacitor switching which is a very difficult task using oil C.Bs. They can be used along with static overcurrent relays and given an overall clearance time of less than 40 m-sec on phase-to-phase faults. There are many applications where a simple load-break switch is not enough and at the same time the devices used should not be costly. They include reactor switching, transformer switching, line dropping, capacitor bank switching. These applications give a fast RRRV and vacuum breakers are the best solutions. Where voltages are high and the current to be interrupted is low, these breakers have definite advantages over the air or oil C.Bs. As the maintenance required is the least, these breakers are most suitable in a country like India where there is a very big complex rural electrification programme. the distribution network is mostly at 11 kV or 33 kV, and therefore, they will be more suitable than any other type of C.Bs.

15.8 SULPHUR HEXAFLUORIDE (SF_6) CIRCUIT BREAKERS

SF_6, as has been discussed earlier, has excellent insulating strength because of its affinity for electrons (electronegativity) *i.e.*, whenever a free electron collides with the neutral gas molecule to form negative ion, the electron is absorbed by the neutral gas molecule. The attachment of the electron with the neutral gas molecule may occur in two ways:

$$SF_6 + e \rightarrow SF_6^-$$

$$SF_6 + e \rightarrow SF_5^- + F$$

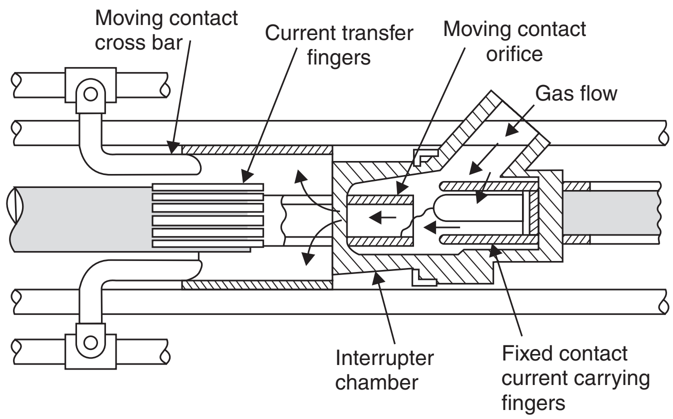
The negative ions formed are relatively heavier as compared to free electrons and, therefore, under a given electric field the ions do not attain sufficient energy to lead cumulative ionization in the gas. Thus, these processes represent an effective way of removing electrons from the space which otherwise would have contributed to form electron avalanche. This property, therefore, gives rise to very high dielectric strength for SF_6. The gas not only possesses a good dielectric strength but it has the unique property of fast recombination after the source energizing the spark is removed. This property of SF_6 makes it very effective in quenching arcs. SF_6 is approximately 100 times as effective as air in quenching arcs. SF_6 has excellent heat transfer properties because its high molecular weight together with its low gaseous viscosity enable it to transfer heat by convection more effectively than the common gases. The arc is thus better interrupted by slowing down the electrons by cooling in case of SF_6 as the arc quenching medium. The thermal time constant of SF_6 is low and as a result the pressures at which it should be stored and used are relatively smaller as compared to air. Also for the same limiting voltage the natural frequency of mains may be greater (almost 100 times) in case of SF_6 as compared to air because of lower time constant of SF_6. This means that SF_6 breakers can withstand severe RRRV and thus are most suitable for short line faults without switching resistors and can interrupt capacitive currents without restriking.

Construction of SF_6 Breaker

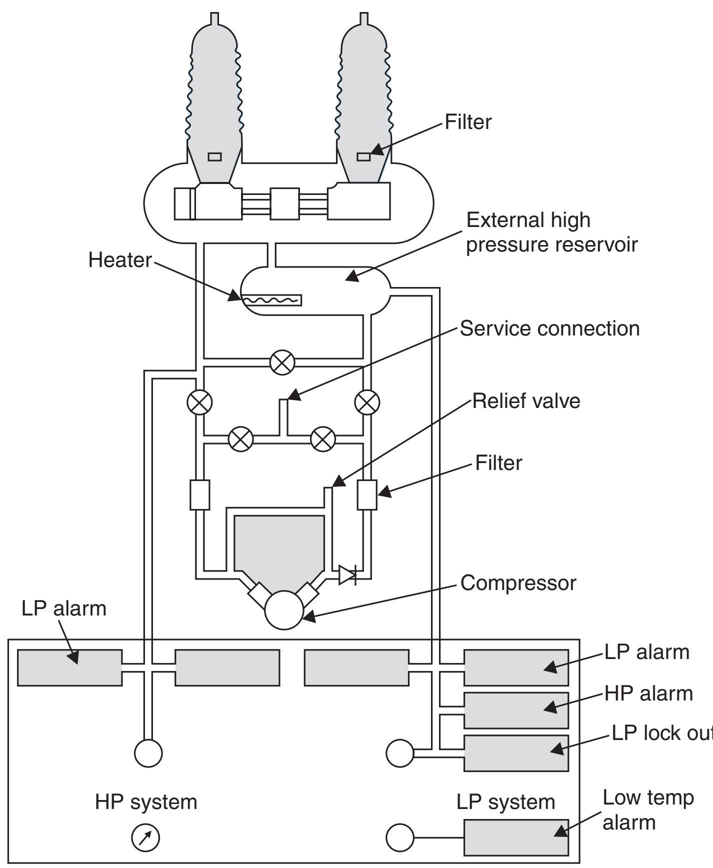
A 132 kV dead tank type SF_6 circuit breaker is shown in Figs. 15.12(*a*) and (*b*). This consist of two parts mainly: (*i*) the interrupter unit, and (*ii*) the gas system.

The Interrupter Unit: This consists of fixed contacts which comprise a set of current carrying fingers and an arcing probe. When the breaker is in the close position, the fingers make contact round the circumference of the moving contact which has the arcing probe enclosed within its hollow end. The contacts are surrounded by interrupting nozzles and a blast shield which controls are displacement and the movement of the hot gas. The moving contact is in the form of hollow nozzle sliding in a second set of spring loaded fingers. Side vents in the moving contact allow the high pressure gas into the main tank. As soon as the moving contact is withdrawn from the fixed finger contacts an arc is drawn between the moving nozzle and the arcing probe. As the contacts move further apart, the arc is extended and attenuated. It is finally extinguished by the gas flow from the high pressure to the low pressure systems.

The Gas System: The closed circuit gas system used in the SF_6 C.Bs. is shown in Fig. 15.12(*b*). Since the gas pressure is very high, lot of care is to be taken to prevent gas



(*a*) Interrupter head



(*b*) The gas system

Fig. 15.12 Dead tank 132 kV SF_6 breaker.

leakages at joints by providing perfect scaling. The low and high pressure system are fitted with low pressure alarms and a set of lock-out switches which give a warning the moment the gas pressure drops below a certain value, because otherwise there will be reduction in the dielectric strength and arc quenching ability of the breaker is endangered. If the danger limit is reached the safety devices immobilise the breaker. The over-riding safety devices see to it that a fault in the control circuit does not permit the compressor to build up excessive pressure in the high pressure reservoir or continue to pump gas into the atmosphere in the event of a major leak. The gas is stored in the high pressure chamber at 16 atmospheres whereas the gas pressure on the low pressure side is 3 atmospheres. The temperature is 20°C. In order to prevent liquefaction of the gas in the high pressure chamber at low temperature, a heater is fitted in the high pressure chamber. A thermostat is set to switch on when the ambient temperature falls below 16°C.

Advantages of SF_6 Breakers

The following are the advantages of SF_6 breakers over the conventional breakers:

1. The current chopping tendency is minimized by using the gas SF_6 at low pressure and low velocity.
2. The closed circuit gas cycle and low velocity operation eliminates the moisture problem and gives noiseless operation of the breaker.
3. Because of the outstanding arc quenching properties of SF_6, the arcing time is small and, therefore, contact erosion is less.
4. No carbon particle is formed during arcing and, therefore, there is no reduction in the dielectric strength of the gas.
5. The circuit breaker performance is not affected due to the atmospheric conditions.
6. Electrical clearances are drastically reduced due to high dielectric strength of SF_6.

15.9 RATING OF CIRCUIT BREAKERS

A circuit breaker has to work under different circumstances. It is rated in terms of (*i*) the number of poles, (*ii*) rated voltage and current, (*iii*) rated frequency, (*iv*) rated making capacity, (*v*) rated symmetrical and asymmetrical breaking capacities, (*vi*) short time rating, and (*vii*) operating duty.

The number of poles per phase of a breaker is a function of the operating voltage.

The voltage levels at various points in a system vary depending upon the system condition and as a result the breaker has to operate under such variable voltage conditions. The breaker is expected to operate at a maximum voltage which normally is higher than the rated nominal voltage.

The rated current of a circuit breaker is the maximum value of current in r.m.s. amperes which it shall carry continuously without exceeding the temperature limits of the various parts of the breaker.

The rated frequency of a breaker is the frequency for which it is designed to operate. Applications at frequencies other than the designed, need special considerations.

The making current is the peak value of the maximum current loop, including d.c. component, in any phase during the first cycle of current when the C.B. is closed (Fig. 15.13). Then making current corresponds to the ordinate I_1. The capacity of a breaker to make currents depends upon its ability to withstand and to close successfully against the effect of electromagnetic forces. The maximum force in any phase is a function of the square of the maximum instantaneous current occurring in that phase on closing. It is, therefore, the practice to specify making current in terms of peak value rather than in terms of r.m.s. value. The making capacity is, therefore, specified by the product of the making current it can make and carry instantaneously at the rated service voltage.

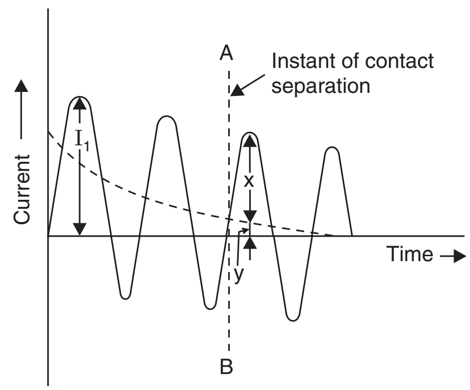


Fig. 15.13 Determination of breaking current at the instant of contact separation.

It is known that in a particular phase the current is maximum right at the instant short circuit takes place, after which the current decreases. The current in the first one or two cycles (depending upon the time constant of the damper winding) is known as subtransient current and in the next 8 to 10 cycles it is known as transient current and finally the steady current where the effect of both damper and field winding dies down. The asymmetry in the current is due to the d.c. component. In case the symmetrical breaking current is known, the making current can be obtained by multiplying this current by $\sqrt{2}$ to get the peak value and again by 1.8 to include the doubling effect (*i.e.,* d.c. component at the first peak is almost equal to the a.c. component).

The breaking current of a breaker depends upon the instant on the current wave when the contacts begin to open. In Fig. 15.13 the contacts start separating at AB. The symmetric breaking current is given by $x/\sqrt{2}$ amp and the asymmetric breaking current is given by

$$\sqrt{\left(\frac{x}{\sqrt{2}}\right)^2 + y^2}$$

The breaking capacity of a breaker is the product of the breaking current and the recovery voltage. The symmetrical breaking capacity is the product of symmetric breaking current and the recovery voltage. Similarly, the asymmetrical breaking capacity is the product of asymmetric breaking current and the recovery voltage.

The short time rated current is the current that can be safely applied, with the C.B. in its normal conditions, for 3 seconds, if the ratio of symmetrical breaking current to normal current is less than 40 or for 1 second otherwise.

These ratings are based on thermal limitations. In case the circuit breaker is not used for auto-reclosing, it must be capable of the following short circuit operating duty:

$$B - 3 - MB - 3 - MB$$

where B is breaking operation, 3 is the duration in minutes, M is the making operation.

Circuit breakers with auto-reclosures should be capable of the following short circuit operating duty:

$$B–D_t–MB$$

Here D_t is the dead time of the breaker in terms of cycles.

Example 15.4: A circuit breaker is rated at 1500 amps, 2000 MVA, 33 kV, 3 sec, 3-phase, oil circuit breaker. Determine the rated normal current, breaking current, making current and short time rating (current).

Solution: The rated normal current is 1500 amps.

Breaking current $= \dfrac{2000}{\sqrt{3 \times 33}} = 34.99$ kV

Making current $= 2.55 \times 34.99 = 89.22$ kA

Short time rating $= 34.99$ kA for 3 sec. **Ans.**

Example 15.5: A generator connected through a 3-cycle C.B. to a transformer is rated 10 MVA, 13.8 kV with reactances of $X_d'' = 10\%$, $X_d' = 15\%$ and $X_d = 100\%$. It is operating at no load and rated voltage when a 3-phase short circuit occurs between the breaker and the transformer. Determine (*i*) the sustained short circuit current in the breaker; (*ii*) the initial symmetrical r.m.s. current in the breaker; (*iii*) the maximum possible d.c. component of the short circuit current in the breaker; (*iv*) the momentary current rating of the breaker; (*v*) the current to be interrupted by the breaker; and (*vi*) the interrupting kVA.

Solution:

(*i*) Since the steady reactance is 100%,

$\therefore$ The steady state short circuit MVA = 10 MVA

$\therefore$ Steady state short circuit current $= \dfrac{10 \times 1000}{\sqrt{3} \times 13.8} = 418$ amps

(*ii*) The initial symmetrical r.m.s. current is the current corresponding to sub-transient state where the % reactance is 10.

$\therefore$ Short circuit MVA $= \dfrac{10}{10} \times 100 = 100$ MVA

$\therefore$ The short circuit current $= \dfrac{100}{\sqrt{3} \times 13.8} = 4180$ amps (r.m.s.)

(*iii*) The maximum possible d.c. component = peak value of the subtransient current $= \sqrt{2} \times 4180 = 5910$ amps.

(*iv*) Momentary current rating = 1.6 × 4180 = 6688 amps

(*v*) Since it is a 3-cycle breaker the current to be interrupted by the breaker = 1.2 × symmetrical breaking current = 1.2 × 4180 = 5019 amps.

(*vi*) The interrupting kVA = $\sqrt{3} \times 13.8 \times 5016$ = 119897 kVA or 119.897 MVA **Ans.**

15.10 TESTING OF CIRCUIT BREAKERS

An equipment when designed to certain specification and is fabricated, needs testing for its performance. The general design is tried and the results of such tests conducted on one selected breaker and are thus applicable to all others of identical construction. These tests are called the type tests. These tests are classified as follows:

1. Short circuit tests:
   (*i*) Making capacity test.
   (*ii*) Breaking capacity test.
   (*iii*) Short time current test.
   (*iv*) Operating duty test.
2. Dielectric tests:
   (*i*) Power frequency test:
      (*a*) One minute dry withstand test.
      (*b*) One minute wet withstand test.
   (*ii*) Impulse voltage dry withstand test.
3. Thermal test.
4. Mechanical test.

Once a particular design is found satisfactory, a large number of similar C.Bs. are manufactured for marketing. Every piece of C.B. is then tested before putting into service. These tests are known as routine tests. With these tests it is possible to find out if incorrect assembly or inferior quality material has been used for a proven design equipment. These tests are classified as: (*i*) operation tests, (*ii*) millivoltdrop tests, (*iii*) power frequency voltage tests at manufacturer's premises, and (*iv*) power frequency voltage tests after erection on site.

We will discuss first the type tests. In that also we will discuss the short circuit tests after the other three tests.

Dielectric Tests

The general dielectric characteristics of any circuit breaker or switchgear unit depend upon the basic design *i.e.,* clearances, bushing materials, etc., upon correctness and accuracy in assembly and upon the quality of materials used. For a C.B. these factors are checked from the viewpoint of their ability to withstand overvoltages at the normal service voltage and abnormal voltages during lightning or other phenomenon.

The test voltage is applied for a period of one minute between (*i*) phases with the breaker closed, (*ii*) phases and earth with C.B. open, and (*iii*) across the terminals with breaker open. With this the breaker must not flashover or puncture. These tests are normally made on indoor switchgear. For such C.Bs. the impulse tests generally are unnecessary because it is not exposed to impulse voltages of a very high order. The high frequency switching surges do occur but the effect of these in cable systems used for indoor switchgear are found to be safely withstood by the switchgear if it has withstood the normal frequency test.

Since the outdoor switchgear is electrically exposed, they will be subjected to overvoltages caused by lightning. The effect of these voltages is much more serious than the power frequency voltages in service. Therefore, this class of switchgear is subjected in addition to power frequency tests, the impulse voltage tests.

The test voltage should be a standard 1/50 μ sec wave, the peak value of which is specified according to the rated voltage of the breaker. A higher impulse voltage is specified for non-effectively grounded system than those for solidly grounded system. The test voltages are applied between (*i*) each pole and earth in turn with the breaker closed and remaining phases earthed, and (*ii*) between all terminals on one side of the breaker and all the other terminals earthed, with the breaker open. The specified voltages are withstand values *i.e.,* the breaker should not flashover for 10 applications of the wave. Normally this test is carried out with waves of both the polarities.

The wet dielectric test is used for outdoor switchgear. In this, the external insulation is sprayed for two minutes while the rated service voltage is applied; the test overvoltage is then maintained for 30 seconds during which no flashover should occur. The effect of rain on external insulation is partly beneficial, insofar as the surface is thereby cleaned, but is also harmful if the rain contains impurities.

Thermal Tests

These tests are made to check the thermal behaviour of the breakers. In this test the rated current through all three phases of the switchgear is passed continuously for a period long enough to achieve steady state conditions. Temperature readings are obtained by means of thermocouples whose hot junctions are placed in appropriate positions. The temperature rise above ambient, of conductors, must normally not exceed 40°C when the rated normal current is less than 800 amps and 50°C if it is 800 amps and above.

An additional requirement in the type test is the measurement of the contact resistances between the isolating contacts and between the moving and fixed contacts. These points are generally the main sources of excessive heat generation. The voltage drop across the breaker pole is measured for different values of d.c. current which is a measure of the resistance of current carrying parts and hence that of contacts.

Mechanical Tests

A C.B. must open and close at the correct speed and perform such operations without mechanical failure. The breaker mechanism is, therefore, subjected to a mechanical endurance type test involving repeated opening and closing of the breaker. B.S. 116 : 1952 requires 500 such operations without failure and with no adjustment of the mechanism. Some manufacturers feel that as many as 20,000 operations may be reached before any useful information regarding the possible causes of failure may be obtained. A resulting change in the material or dimensions of a particular component may considerably improve the life and efficiency of the mechanism.

Short Circuit Tests

These tests are carried out in short circuit testing stations to prove the ratings of the C.Bs. Before discussing the tests it is proper to discuss about the short circuit testing stations.

There are two types of testing stations: (*i*) field type, and (*ii*) laboratory type.

In case of field type stations the power required for testing is directly taken from a large power system. The breaker to be tested is connected to the system. Whereas this method of testing is economical for high voltage C.Bs. it suffers from the following drawbacks:

1. The tests cannot be repeatedly carried out for research and development as it disturbs the whole network.

2. The power available depends upon the location of the testing stations, loading conditions, installed capacity, etc.

3. Test conditions like the desired recovery voltage, the RRRV etc. cannot be achieved conveniently.

In case of laboratory testing the power required for testing is provided by specially designed generators. This method has the following advantages:

1. Test conditions such as current, voltage, power factor, restriking voltages can be controlled accurately.

2. Several indirect testing methods can be used.

3. Tests can be repeated and hence research and development over the design is possible.

The limitations of this method are the cost and the limited power availability for testing the breakers.

Short Circuit Test Plants

The essential components of a typical test plant are represented in Fig. 15.14. The short-circuit power is supplied by specially designed short-circuit generators driven by induction motors. The magnitude of voltage can be varied by adjusting excitation of the generator or the transformer ratio. A plant master-breaker is available to interrupt the test short circuit current if the test breaker should fail. Initiation of the short circuit may be by the master breaker, but is always done by a making switch which is specially designed for closing on very heavy currents but never called upon to break currents. The generator winding may be arranged for either star or delta connection according to the voltage required; by further dividing the winding into two sections which may be connected in series or parallel, a choice of four voltages is available. In addition to this the use of resistors and reactors in series gives a wide range of current and power factors. The generator, transformer and reactors are housed together, usually in the building accommodating the test cells.

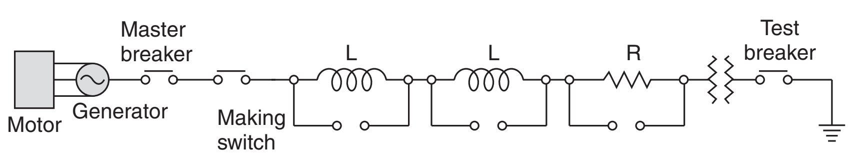


Fig. 15.14 Schematic diagram of a typical test plant.

Generator

The short circuit generator is different in design from the conventional power station. The capacity of these generators may be of the order of 2000 MVA and very rigid bracing of the conductors and coil ends is necessary in view of the high electromagnetic forces possible. The limiting factor for the maximum output current is the electromagnetic force. Since the operation of the generator is intermittent, this need not be very efficient. The reduction of ventilation enables the main flux to be increased and permits the inclusion of extra coil end supports. The machine reactance is reduced to a minimum.

Immediately before the actual closing of the making switch the generator driving motor is switched out and the short circuit energy is taken from the kinetic energy of the generator set. This is done to avoid any disturbance to the system during short circuit. However, in this case it is necessary to compensate for the decrement in generator voltage corresponding to the diminishing generator speed during the test. This is achieved by adjusting the generator field excitation to increase at a suitable rate during the short circuit period.

Resistors and Reactors

The resistors are used to control the p.f. of the current and to control the rate of decay of d.c. component of current. There are a number of coils per phase and by combinations of series and parallel connections, desired value of resistance and/or reactance can be obtained.

Capacitors

These are used for breaking line charging currents and for controlling the rate of re-striking voltage.

Short Circuit Transformer

The leakage reactance of the transformer is low so as to withstand repeated short circuits. Since they are in use intermittently, they do not pose any cooling problem. For voltage higher than the generated voltages, usually banks of single phase transformers are employed. In the short circuit station at Bhopal there are three single phase units each of 11 kV/76 kV. The normal rating is 30 MVA but their short circuit capacity is 475 MVA.

Master C.Bs.

These breakers are provided as back up which will operate, should the breaker under test fail to operate. This breaker is normally air blast type and the capacity is more than the breaker under test. After every test, it isolates the test breaker from the supply and can handle the full short circuit of the test circuit.

Make Switch

The make switch is closed after other switches are closed. The closing of the switch is fast, sure and without chatter. In order to avoid bouncing and hence welding of contacts, a high air pressure is maintained in the chamber. The closing speed is high so that the contacts are fully closed before the short circuit current reaches its peak value.

Test Procedure

Before the test is performed all the components are adjusted to suitable values so as to obtain desired values of voltage, current, rate of rise of restriking voltage, p.f., etc. The measuring circuits are connected and oscillograph loops are calibrated.

During the test several operations are performed in a sequence in a short time of the order of 0.2. sec. This is done with the help of a drum switch with several pairs of contacts which is rotated with a motor. This drum when rotated closes and opens several control circuits according to a certain sequence. In one of the breaking capacity tests the following sequence was observed:

(*i*) After running the motor to a speed the supply is switched off.

(*ii*) Impulse excitation is switched on.

(*iii*) Master C.B. is closed.

(*iv*) Oscillograph is switched on.

(*v*) Make switch is closed.

(*vi*) C.B. under test is opened.

(*vii*) Master C.B. is opened.

(*viii*) Exciter circuit is switched off.

The circuit for direct test is shown in Fig. 15.15.

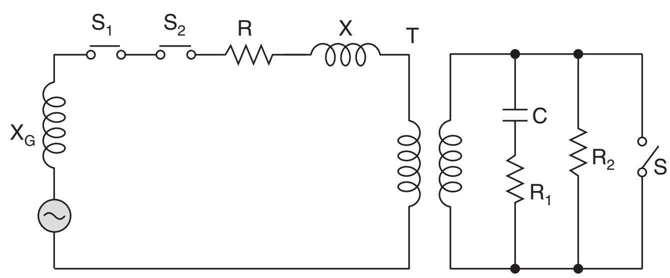


Fig. 15.15 Circuit for direct testing.

Here X_G = generator reactance, S_1 and S_2 are master and make switches respectively. R and X are the resistance and reactance for limiting the current and control of p.f., T is the transformer, C, R_1 and R_2 is the circuit for adjusting the restriking voltage.

For testing, breaking capacity of the breaker under test, master and breaker under test are closed first. Short circuit is applied by closing the making switch. The breaker under test is opened at the desired moment and the breaking current is determined from the oscillograph as explained earlier.

For making capacity test the master and the make switches are closed first and short circuit is applied by closing the breaker under test. The making current is determined from the oscillograph as explained earlier.

For short time current test, the current is passed through the breaker for a short time say 1 second and the oscillogram is taken as shown in Fig. 15.16.

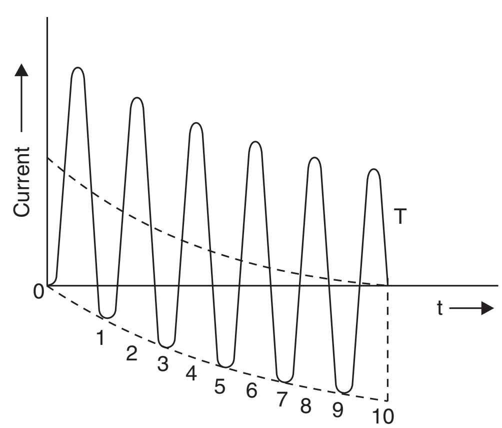


Fig. 15.16 Determination of short time current.

From the oscillogram the equivalent r.m.s. value of short-time current is obtained as follows:

The time interval 0 to T is divided into 10 equal parts marked as 0, 1, 2, ... , 9, 10. Let the r.m.s. value of currents at these instants be $I_0, I_1, I_2, ..., I_9, I_{10}$ (asymmetrical values). From these values, the r.m.s. value of short-time current is calculated using Simpson formula.

$$I = \sqrt{\frac{1}{3}\left[I_0^2 + 4(I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2) + 2(I_2^2 + I_4^2 + I_6^2 + I_8^2 + I_{10}^2)\right]}$$

Operating duty tests are performed according to standard specification unless the duty is marked on the rating plate of the breaker. The tests according to specifications are:

(*i*) *B*–3′–*B*–3′–*B* at 10% of rated symmetrical breaking capacity;

(*ii*) *B*–3′–*B*–3′–*B* at 30% of rated symmetrical breaking capacity;

(*iii*) *B*–3′–*B*–3′–*B* at 60% of rated symmetrical breaking capacity;

(*iv*) *B*–3′–*MB*–3′–*MB* at not less than 100% of rated symmetrical breaking capacity and not less than 100% of rated making capacity. Test duty (*iv*) may be performed as two separate duties as follows:

(*a*) *M*–3′–*M* (Make test);

(*b*) *B*–3′–*B*–3′–*B* (Break test).

(*v*) *B*–3′–*B*–3′–*B* at not less than 100% of rated asymmetrical breaking capacity.

Here *B* and *M* represent breaking and making operations respectively. *MB* denotes the making operation followed by breaking operation without any intentional time lag. 3′ denotes the time in minutes between successive operations of an operating duty.

15.11 AUTORECLOSING

Depending upon the time, for which the faults exist on the system, are classified as follows:

(*i*) Transient fault (*ii*) Semi-permanent fault and (*iii*) Permanent Fault.

The transient fault exists only for a short time and these can be removed faster still if the line is disconnected from the system momentarily so that the arc extinguishes. After the arc is deionised, the line can be reclosed to restore normal service. It is found that about 80% of the faults are transient faults, 12% semi-permanent and 8% are permanent faults. If it is semi-permanent fault, may be due to a twig falling on the power conductor or a bird spanning the power conductors, reclosing could be resorted with some delay so that the cause of the fault could be burnt away during a time delay trip and the line could be reclosed to restore normal service. However, for permanent fault reclosing does not help as it has to be attended and removed, and the line is to be taken out till the fault is cleared. Therefore, if the fault is not cleared after the first reclosure, a double or triple shot reclosing is desired. If the fault still persists, the line is taken out of service.

Autoreclosing could be single phase or three phase. Single phase autoreclosing is resorted when a line to ground fault takes place and reclosed after a predetermined time. For multiphase faults, all three phases are opened and reclosure is not attempted. In single phase autoreclosing the power can still be fed through the healthy phases to the system and the system is less unstable as compared to 3-phase reclosing. In case of three phase autoreclosing, all the three phases are opened independent of the type of fault, be it a single line to ground, or a line to line or 3-phase fault and are reclosed after a pre-determined time. Here during the opening period, no power can be transmitted and hence the system is liable to operate unstably.

PROBLEMS

15.1. Explain how arc is initiated and sustained in a circuit breaker when the circuit breaker contacts separate.

15.2. Discuss the principle of arc interruption in (*i*) an oil C.B.; and (*ii*) air blast circuit breaker.

15.3. Compare the performance and characteristics of (*i*) minimum oil breakers and air blast C.B.; (*ii*) air blast C.B. and bulk oil C.B.

15.4. Explain the terms (*i*) Symmetrical breaking current; (*ii*) Asymmetrical breaking current; and (*iii*) making current. Explain clearly how these currents can be determined from oscillograms taken during short circuit tests on a 3-phase C.B.

15.5. Explain the terms (*i*) restriking voltage; (*ii*) recovery voltage; and (*iii*) RRRV. Derive an expression for the restriking voltage in terms of system voltage, inductance and capacitance, across a C.B. contact when a 3-phase fault takes place. Assume the neutral of the system to be solidly grounded.

15.6. In a short circuit test on a C.B. the following readings were obtained on a single frequency transient:

(*i*) Time to reach the peak restriking voltage 40 μsec;

(*ii*) the peak restriking voltage 100 kV.

Determine the average RRRV and the frequency of oscillation.

15.7. An 11 kV, 50 Hz alternator is connected to a system which has inductance and capacitance per phase of 10 mH and 0.01 μF respectively. Determine (*i*) the maximum voltage across the breaker contacts; (*ii*) Frequency of transient oscillation; (*iii*) the average RRRV; and (*iv*) the maximum RRRV.

15.8. A 66 kV, 50 Hz, 3-phase alternator has an earthed neutral. The inductance and capacitance per phase of the system are 7 mH and 0.01 μF respectively. The short circuit test gave the following results: Power factor of fault 0.25, fault current symmetrical recovery voltage is 90% of full line voltage. Assuming that the fault is isolated from the ground, calculate the RRRV.

15.9. A circuit breaker is rated as 2500 A, 1500 MVA, 33 kV, 3 secs, 3-phase oil C.B. Determine the rated symmetrical breaking current, rated making current, short time rating and rated service voltage.

15.10. Differentiate between type tests and routine tests. What different tests are carried out to prove the ability of a C.B.?

15.11. Describe with the help of a neat diagram the procedure of testing a C.B. in a testing station.

15.12. Describe with the help of neat diagram short circuit testing stations. What are the advantages of laboratory type testing station?

15.13. What are the requirements of the contact material for a vacuum circuit breaker? Why is current chopping not a serious problem with such circuit breakers?

15.14. Describe the construction, principle of operation and application of a vacuum breaker.

15.15. Compare the performance of SF_6 gas with air when used for circuit breaking.

15.16. Explain the process of 'current chopping' in SF_6 breakers.

15.17. Describe the construction, principle of operation and application of SF_6 circuit breaker. How does this breaker essentially differ from an air blast breaker?

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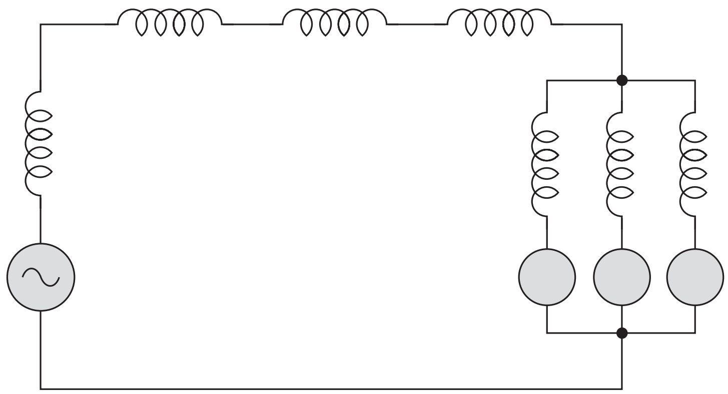
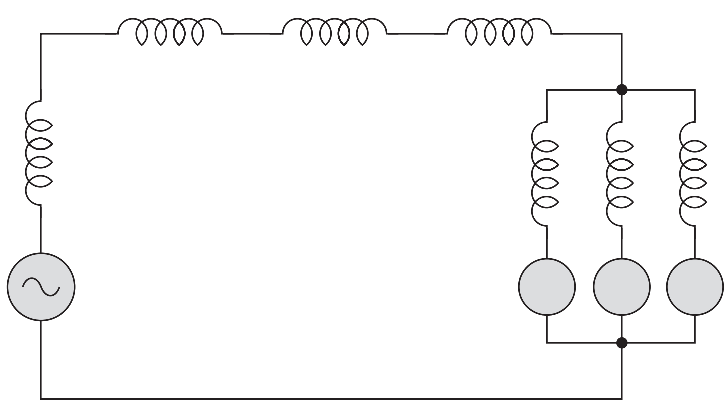


16

INSULATION COORDINATION AND OVERVOLTAGE PROTECTION



16

Insulation Coordination and Overvoltage Protection

INTRODUCTION

Insulation coordination means the correlation of the insulation of the various equipments in a power system to the insulation of the protective devices used for the protection of those equipments against overvoltages. In a power system various equipments like transformers, circuit breakers, bus supports etc. have different breakdown voltages and hence the volt-time characteristics. In order that all the equipments should be properly protected it is desired that the insulation of the various protective devices must be properly coordinated. The basic concept of insulation coordination is illustrated in Fig. 16.1. Curve A is the volt-time Curve of the protective device and B the volt-time curve of the equipment to be protected. Figure 16.1 shows the desired positions of the volt-time curves of the protecting device and the equipment to be protected. Thus, any insulation having a withstand voltage strength in excess of the insulation strength of curve B is protected by the protective device of curve A.

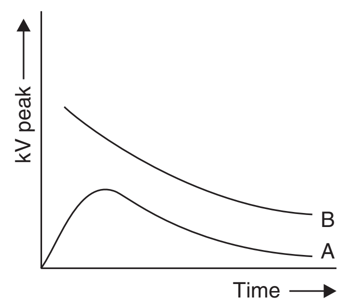


Fig. 16.1 Volt-time curve A (protecting device and) volt-time curve B (device to be protected)

The 'volt-time curve' expression will be used very frequently in this chapter. It is, therefore, necessary to understand the meaning of this expression.

16.1 VOLT-TIME CURVE

The breakdown voltage for a particular insulation or flashover voltage for a gap is a function of both the magnitude of voltage and the time of application of the voltage. The volt-time curve is a graph showing the relation between the crest flashover voltages and the time to flashover for a series of impulse applications of a given wave shape. For the construction of volt-time curve the following procedure is adopted. Waves of the same shape but of different peak values are applied to the insulation whose volt-time curve is required. If flashover occurs on the front of the wave, the flashover point gives one point on the volt-time curve. The other possibility is that the flashover occurs just at the peak value of the wave; this gives another point on the *V-T* curve. The third possibility is that the flashover occurs on the tail side of the wave. In this case to find the point on the *V-T* curve, draw a horizontal line from the peak value of this wave and also draw a vertical line passing through the point where the flashover takes place. The intersection of the horizontal and vertical lines gives the point on the *V-T* curve. This procedure is nicely shown in Fig. 16.2.

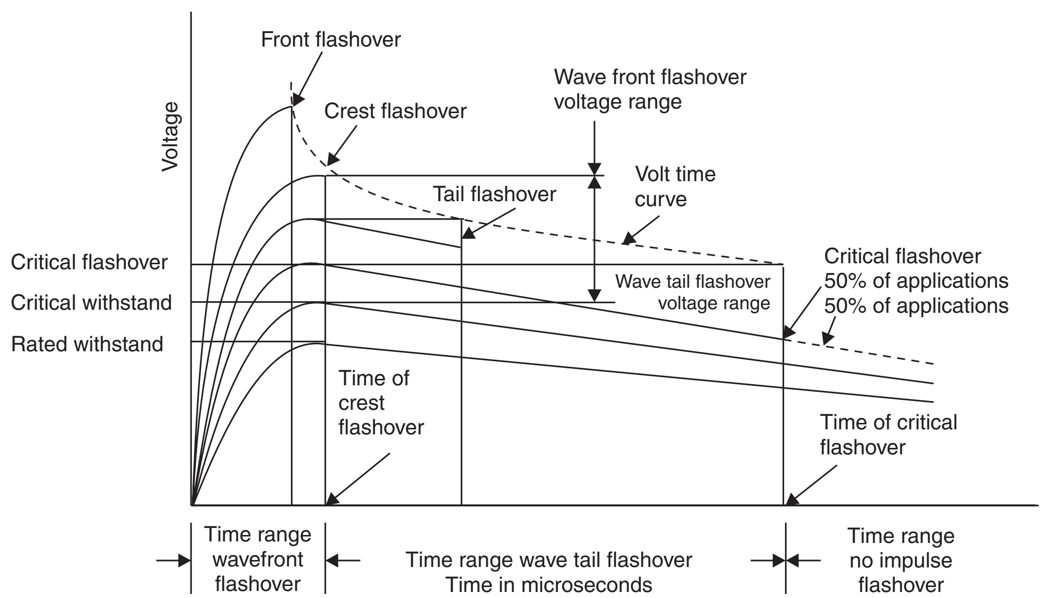


Fig. 16.2 Volt-time curve (construction)

The overvoltages against which coordination is required could be caused on the system due to system faults, switching operation or lightning surges. For lower voltages, normally up to about 345 kV, overvoltages caused by system faults or switching operations do not cause damage to equipment insulation although they may be detrimental to protective devices. Overvoltages caused by lightning are of sufficient magnitude to affect the equipment insulation whereas for voltages above 345 kV it is these switching surges which are more dangerous for the equipments than the lightning surges.

The problem of coordinating the insulation of the protective equipment involves not only guarding the equipment insulation but also it is desired that the protecting equipment should not be damaged.

To assist in the process of insulation coordination, standard insulation levels have been recommended. These insulation levels are defined as follows:

Basic impulse insulation levels (BIL) are reference levels expressed in impulse crest voltage with a standard wave not longer than 1.2/50 μsec wave. Apparatus insulation as demonstrated by suitable tests shall be equal to or greater than the basic insulation level.

The problem of insulation coordination can be studied under three steps:

1. Selection of a suitable insulation which is a function of reference class voltage (*i.e.*, 1.05 × operating voltage of the system). Table 16.1 gives the BIL for various reference class voltages.

Table 16.1 Basic Impulse Insulation Levels

Reference class kV	*Standard basic impulse level kV*	*Reduced insulation levels*
23	150	
34.5	200	
46	250	
69	350	
92	450	
115	550	450
138	650	550
161	750	650
196	900	
230	1050	900
287	1300	1050
345	1550	1300

2. The design of the various equipments such that the breakdown or flashover strength of all insulation in the station equals or exceeds the selected level as in (1).

3. Selection of protective devices that will give the apparatus as good protection as can be justified economically.

The above procedure requires that the apparatus to be protected shall have a withstand test value not less than the kV magnitude given in the second column of Table 16.1, irrespective of the polarity of the wave positive or negative and irrespective of how the system was grounded.

The third column of the table gives the reduced insulation levels which are used for selecting insulation levels of solidly grounded systems and for systems operating above 345 kV where switching surges are of more importance than the lightning surges. At 345 kV, the switching voltage is considered to be 2.7 p.u., *i.e.*, 345 × 2.7 = 931.5 kV which corresponds to the lightning level. At 500 kV, however, 2.7 p.u. will mean 2.7 × 500 = 1350 kV switching voltage which exceeds the lightning voltage level. Therefore, the ratio of switching voltage to operating voltage is reduced by using the switching resistances between the C.B. contacts. For 500 kV, it has been possible to obtain this ratio as 2.0 and for 765 kV it is 1.7. With further increase in operating voltages it is hoped that the ratio could be brought to 1.5. So, for

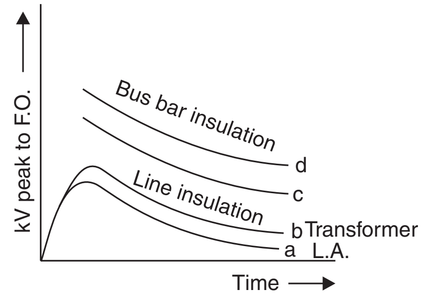


Fig. 16.3 Volt-time curves

switching voltages the reduced levels in third column are used *i.e.*, for 345 kV, the standard BIL is 1550 kV but if the equipment can withstand even 1425 kV or 1300 kV it will serve the purpose. Figure 16.3 gives the relative position of the volt-time curves of the various equipments in a substation for proper coordination. To illustrate the selection of the BIL of a transformer to be operated on a 138 kV system assume that the transformer is of large capacity and its star point is solidly grounded. The grounding is such that the line-to-ground voltage of the healthy phase during a ground fault on one of the phase is say 74% of the normal L-L voltage. Allowing for 5% overvoltage during operating conditions, the arrester rms operating voltage will be $1.05 \times 0.74 \times 138 = 107.2$ kV. The nearest standard rating is 109 kV. The characteristic of such a L.A. is shown in Fig. 16.4. From the figure the breakdown value of the arrester is 400 kV. Assuming a 15% margin plus 35 kV between the insulation levels of L.A. and the transformer, the insulation level of transformer should be at least equal to $400 + 0.15 \times 400 + 35 = 495$ kV. From Fig. 16.4 (or from the table the reduced level of transformer for 138 kV is 550 kV) the insulation level of transformer is 550 kV; therefore a lightning arrester of 109 kV rating can be applied.

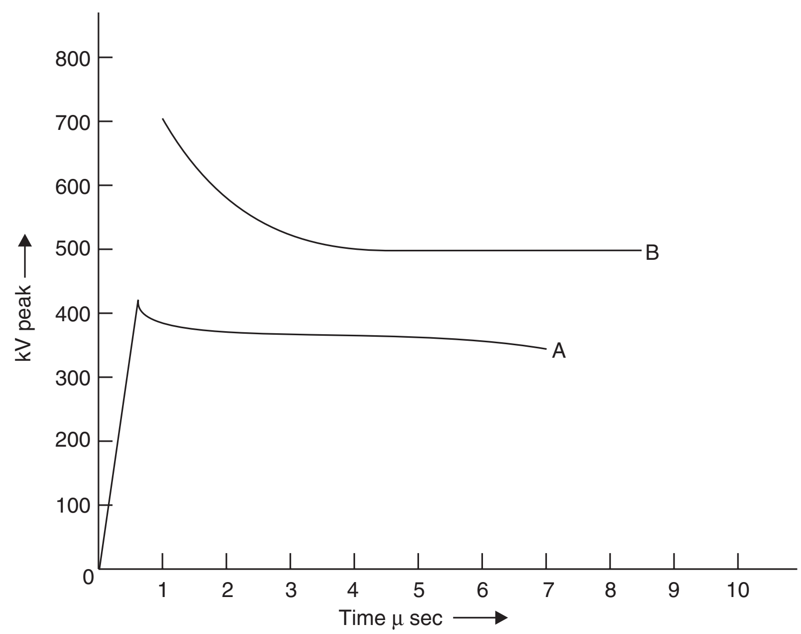


Fig. 16.4 Coordination of transformer insulation with lightning arrester: A—Lightning arrester 109 kV, B—Transformer insulation withstand characteristic.

It is to be noted that low voltage lines are not as highly insulated as higher voltage lines so that lightning surges coming into the station would normally be much less than in a higher voltage station because the high voltage surges will flashover the line insulation of low voltage line and not reach the station.

The traditional approach to insulation coordination requires the evaluation of the highest overvoltages to which an equipment may be subjected during operation and selection of standardized value of withstand impulse voltage with suitable safety margin. However, it is realized that overvoltages are a random phenomenon and it is uneconomical to design plant with such a high degree of safety that they sustain the infrequent ones. It is also known that insulation designed on this basis does not give 100% protection and insulation failure may occur even in well designed plants and, therefore, it is desired to limit the frequency of insulation failures to the most economical value taking into account equipment cost and service continuity. Insulation coordination, therefore, should be based on evaluation and limitation of the risk of failure than on the a *priori* choice of a safety margin.

The modern practice, therefore, is to make use of probabilistic concepts and statistical procedures especially for very high voltage equipments which might later on be extended to all cases where a close adjustment of insulation to system conditions proves economical. The statistical methods even though laborious are quite useful.

16.2 OVERVOLTAGE PROTECTION

The causes of overvoltages in the system have been studied extensively in Chapter 12. Basically, there are two sources: (*i*) external overvoltages due to mainly lightning, and (*ii*) internal overvoltages mainly due to switching operation. The system can be protected against external overvoltages using what are known as shielding methods which do not allow an arc path to form between the line conductor and ground, thereby giving inherent protection in the line design. For protection against internal voltages normally non-shielding methods are used which allow an arc path between the ground structure and the line conductor but means are provided to quench the arc. The use of ground wire is a shielding method whereas the use of spark gaps, and lightning arresters are the non-shielding methods. We will study first the non-shielding methods and then the shielding methods. However, the non-shielding methods can also be used for external overvoltages.

The non-shielding methods are based upon the principle of insulation breakdown as the overvoltage is incident on the protective device; thereby a part of the energy content in the overvoltage is discharged to the ground through the protective device. The insulation breakdown is not only a function of voltage but it depends upon the time for which it is applied and also it depends upon the shape and size of the electrodes used. The steeper the shape of the voltage wave, the larger will be the magnitude of voltage required for breakdown; this is because an expenditure of energy is required for the rupture of any dielectric, whether gaseous, liquid or solid, and energy involves time. The energy criterion for various insulations can be compared in terms of a common term known as Impulse Ratio which is defined as the ratio of breakdown voltage due to an impulse of specified shape to the breakdown voltage at power frequency. The impulse ratio for sphere gap is unity because this gap has a fairly uniform field and the breakdown takes place on the field ionization phenomenon mainly whereas for a needle gap it varies between 1.5 to 2.3 depending upon the frequency and gap length. This ratio is higher than unity because of the nonuniform field between the electrodes. The impulse ratio of a gap of given geometry and dimension is greater with solid than with air dielectric. The insulators should have a high impulse ratio for an economic design whereas the lightning arresters should have a low impulse ratio so that a surge incident on the lightning arrester may be by passed to the ground instead of passing it on to the apparatus.

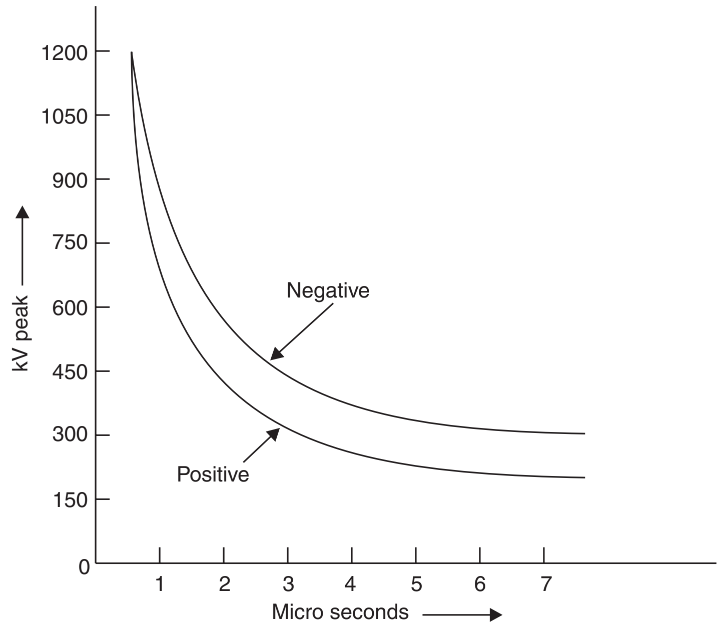


Fig. 16.5 Volt-time curves of gaps for positive and negative polarity.

The volt-time characteristics of gaps having one electrode grounded depend upon the polarity of the voltage wave. From Fig. 16.5 it is seen that the volt-time characteristic for positive polarity is lower than the negative polarity, *i.e.*, the breakdown voltage for a negative impulse is greater than for a positive because of the nearness of earthed metal or of current carrying conductors. For post insulators the negative polarity wave has a high breakdown value whereas for suspension insulators the reverse is true.

Horn Gap

The horn gap consists of two horn shaped rods separated by a small distance. One end of this is connected to the line and the other to the earth as shown in Fig. 16.6, with or without a series resistance. The choke connected between the equipment to be protected and the horn gap serves two purposes: (*i*) The steepness of the wave incident on the equipment to be protected is reduced. (*ii*) It reflects the voltage surge back on to the horn.

Whenever a surge voltage exceeds the breakdown value of the gap a discharge takes place and the energy content in the rest part of the wave is by-passed to the ground. An arc is set up between the gap, which acts like a flexible conductor and rises upwards under the influence of the electromagnetic forces, thus increasing the length of the arc which eventually blows out.

There are two major drawbacks of the horn gap: (*i*) The time of operation of the gap is quite large as compared to the modern protective gear. (*ii*) If used on isolated neutral the horn gap may constitute a vicious kind of arcing ground. For these reasons, the horn gap has almost vanished from important power lines.

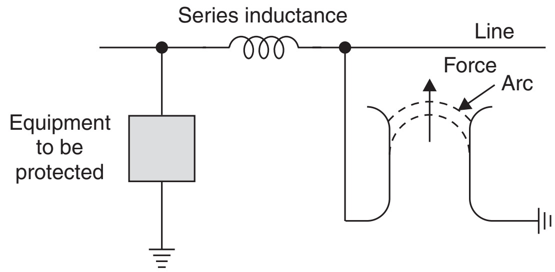


Fig. 16.6 Horn gap connected in the system for protection.

Surge Diverters

The following are the basic requirements of a surge diverter:

(*i*) It should not pass any current at normal or abnormal (normally 5% more than the normal voltage) power frequency voltage.

(*ii*) It should breakdown as quickly as possible after the abnormal high frequency voltage arrives.

(*iii*) It should not only protect the equipment for which it is used but should discharge the surge current without damaging itself.

(*iv*) It should interrupt the power frequency follow current after the surge is discharged to ground.

There are mainly three types of surge diverters: (*i*) Rod gap, (*ii*) Protector tube or expulsion type of lightning arrester, (*iii*) Valve type of lightning arrester.

Rod Gap

This type of surge diverter is perhaps the simplest, cheapest and most rugged one. Fig. 16.7 shows one such gap for a breaker bushing. This may take the form of arcing ring. Fig. 16.8 shows the breakdown characteristics (volt-time) of a rod gap. For a given gap and wave shape of the voltage, the time for breakdown varies approximately inversely with the applied voltage.

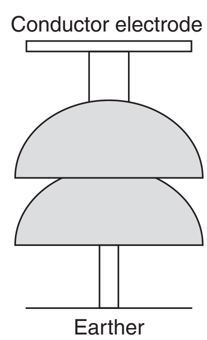


Fig. 16.7 A rod gap.

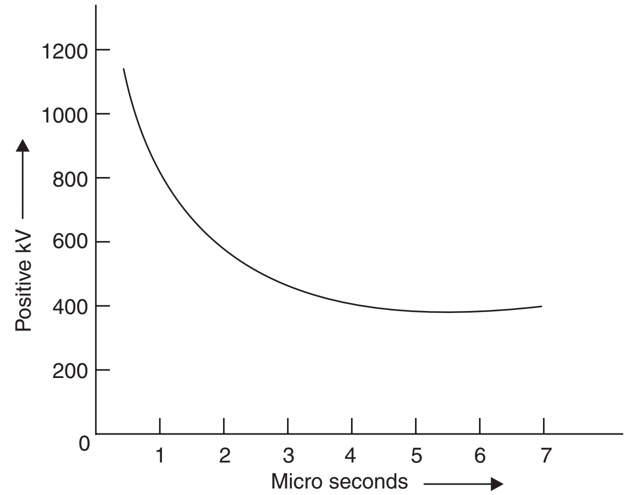


Fig. 16.8 Volt-time characteristic of rod gap.

The times to flashover for positive polarity are lower than for negative polarities. Also it is found that the flashover voltage depends to some extent on the length of the lower (grounded) rod. For low values of this length there is a reasonable difference between positive (lower value) and negative flashover voltages. Usually a length of 1.5 to 2.0 times the gap spacing is good enough to reduce this difference to a reasonable amount. The gap setting normally chosen is such that its breakdown voltage is not less than 30% below the voltage withstand level of the equipment to be protected.

Even though rod gap is the cheapest form of protection, it suffers from the major disadvantage that it does not satisfy one of the basic requirements of a lightning arrester listed at no. (*iv*) *i.e.*, it does not interrupt the power frequency follow current. This means that every operation of the rod gap results in a *L-G* fault and the breakers must operate to de-energize the circuit to clear the flashover. The rod gap, therefore, is generally used as back up protection.

Expulsion Type of Lightning Arrester: An improvement of the rod gap is the expulsion tube which consists of (*i*) a series gap (1) external to the tube which is good enough to withstand normal system voltage, thereby there is no possibility of corona or leakage current across the tube; (*ii*) a tube which has a fibre lining on the inner side which is a highly gas evolving material; (*iii*) a spark gap (2) in the tube; and (*iv*) an open vent at the lower end for the gases to be expelled (Fig. 16.9). It is desired that the breakdown voltage of a tube must be lower than that of the insulation for which it is used. When a surge voltage is incident on the expulsion tube the series gap is spanned and an arc is formed between the electrodes within the tube. The heat of the arc vaporizes some of the organic material of the tube wall causing a high gas pressure to build up in the tube. The resulting neutral gas creates lot of

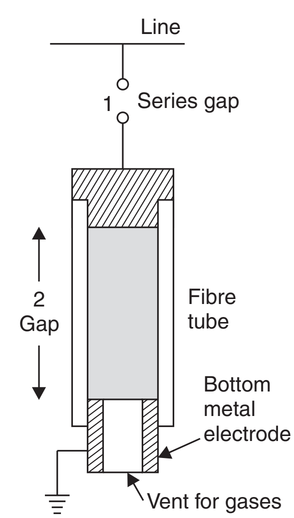


Fig. 16.9 Expulsion type lightning arrester.

turbulence within the tube and is expelled out from the open bottom vent of the tube and it extinguishes the arc at the first current zero. At this instant the rate of build up of insulation strength is greater than the RRRV. Very high currents have been interrupted using these tubes. The breakdown voltage of expulsion tubes is slightly lower than for plain rod gaps for the same spacing. With each operation of the tube the diameter of the tube (fibre lining) increases; thereby the insulation characteristics of the tube will lower down even though not materially. The volt-time characteristics (Fig. 16.10) of the expulsion tube are somewhat better than the rod gap and have the ability to interrupt power voltage after flashover.

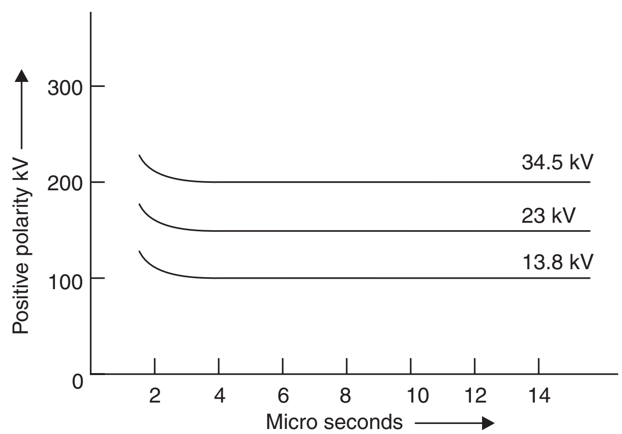


Fig. 16.10 Volt-time characteristic of expulsion gaps.

Valve Type Lightning Arresters: An improved but more expensive surge diverter is the valve type of lightning arrester or a non-linear surge diverter. A porcelain bushing (Fig. 16.11) contains a number of series gaps, coil units and the valve elements of the non-linear resistance material usually made of silicon carbide disc, the latter possessing low resistance to high currents and high resistance to low currents. The characteristic is usually expressed as $I = KV^n$, where n lies between 2 and 6 and K is constant, a function of the geometry and dimension of the resistor. The non-linear characteristic is attributed to the properties of the electrical contacts between the grains of silicon carbide. The discs are 90 mm in dia and 25 mm thick. A grading ring or a high resistance is connected across the disc so that the system voltage is evenly distributed over the discs. The high resistance keeps the inner assembly dry due to some heat generated.

Figure 16.12 shows the volt-ampere characteristics of a non-linear resistance of the required type. The closed curve represents the dynamic characteristic corresponding to the application of a voltage surge whereas the dotted line represents the static characteristic. The voltage corresponding to the horizontal tangent to the dynamic characteristic is known as the residual voltage (IR drop) and is the peak value of the voltage during the discharge of the surge current. This voltage varies from 3 kV to 6 kV depending upon the type of arrester *i.e.,* whether station or line type, the magnitude and wave shape of the discharge current. The spark gaps are so designed that they give an impulse ratio of unity to the surge diverter and as a result they are unable to interrupt high values of current and the follow up currents are limited to 20 to 30 A. The impulse breakdown strength of a diverter is smaller than the residual voltage, and therefore, from the point of view of insulation coordination residual voltage decides the protection level.

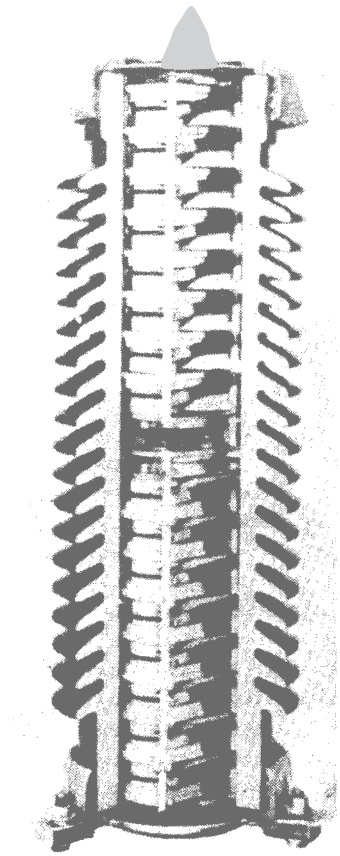


Fig. 16.11 Valve-type lightning arrester.

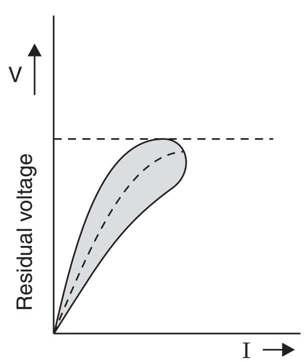


Fig. 16.12 Volt-ampere characteristic of valve-type LA.

The operation of the arrester can be easily understood with the help of Figs. 16.13 (*a*) and (*b*). When a surge voltage is incident at the terminal of the arrester it causes the two gap units to flashover, thereby a path is provided to the surge to the ground through the coil element and the non-linear resistor element. Because of the high frequency of the surge, the coil develops sufficient voltage across its terminals to cause the by-pass gap to flashover. With this the coil is removed from the circuit and the voltage across the LA is the IR drop due to the non-linear element. This condition continues till power frequency currents follow the preionized path. For power frequency the impedance of the coil is very low and, therefore, the arc will become unstable and the current will be transferred to the coil (Fig. 16.13 (*b*)). The magnetic field developed by the follow current in the coil reacts with this current in the arcs of the gap assemblies, causing them to be driven into arc quenching chambers which are an integral part of the gap unit. The arc is extinguished at the first current zero by cooling and lengthening the arc and also because the current and voltage are almost in phase. Thus the diverter comes back to normal state after discharging the surge to the ground successfully.

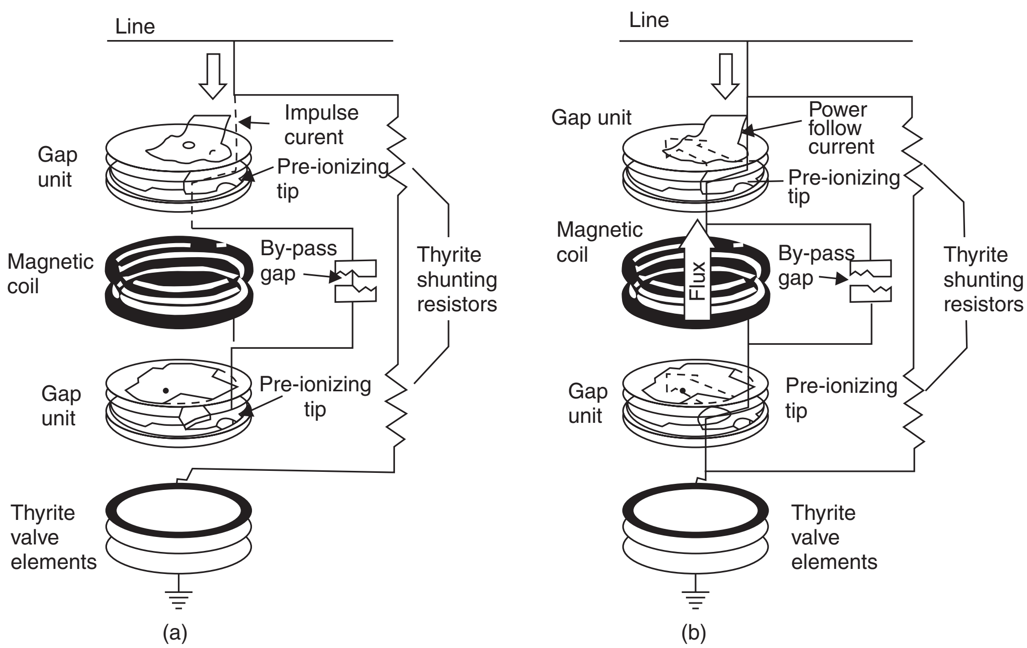


Fig. 16.13 Schematic diagram of valve-type arrester indicating path of (*a*) Surge current, (*b*) Follow current.

Location of Lightning Arresters: The normal practice is to locate the lightning arrester as close as possible to the equipment to be protected. The following are the reasons for the practice: (*i*) This reduces the chances of surges entering the circuit between the protective equipment and the equipment to be protected. (*ii*) If there is a distance between the two, a steep fronted wave after being incident on the lightning arrester, which sparks over corresponding to its spark-over voltage, enters the transformer after travelling over the lead between the two. The wave suffers reflection at the terminal and, therefore, the total voltage at the terminal of the transformer is the sum of reflected and the incident voltage which approaches nearly twice the incident voltage *i.e.,* the transformer may experience a surge twice as high as that of the lightning arrester. If the lightning arrester is right at the terminals this could not occur. (*iii*) If L is the inductance of the lead between the two, and IR the residual voltage of the lightning arrester, the voltage incident at the transformer terminal will be

$$V = IR + L\frac{di}{dt}$$

where di/dt is the rate of change of the surge current.

It is possible to provide some separation between the two, where a capacitor is connected at the terminals of the equipment to be protected. This reduces the steepness of the wave and hence the rate di/dt and this also reduces the stress distribution over the winding of the equipment.

There are three classes of lightning arresters available:

(*i*) **Station type:** The voltage ratings of such arresters vary from 3 kV to 312 kV and are designed to discharge currents not less than 100,000 amps. They are used for the protection of substation and power transformers.

(*ii*) **Line type:** The voltage ratings vary from 20 kV to 73 kV and can discharge currents between 65,000 amps and 100,000 amps. They are used for the protection of distribution transformers, small power transformers and sometimes small substations.

(*iii*) **Distribution type:** The voltage ratings vary from 8 kV to 15 kV and can discharge currents up to 65,000 amperes. They are used mainly for pole mounted substation for the protection of distribution transformers up to and including the 15 kV classification.

Rating of Lightning Arrester: A lightning arrester is expected to discharge surge currents of very large magnitude, thousands of amperes, but since the time is very short in terms of microseconds, the energy that is dissipated through the lightning arrester is small compared with what it would have been if a few amperes of power frequency current had been flown for a few cycles. Therefore, the main considerations in selecting the rating of a lightning arrester is the line-to-ground dynamic voltage to which the arrester may be subjected for any condition of system operation. An allowance of 5% is normally assumed, to take into account the light operating condition under no load at the far end of the line due to Ferranti effect and the sudden loss of load on water wheel generators. This means an arrester of 105% is used on a system where the line to ground voltage may reach line-to-line value during line-to-ground fault condition.

The overvoltages on a system as is discussed earlier depend upon the neutral grounding condition which is determined by the parameters of the system. We recall that a system is said to be solidly grounded only if

$$\frac{R_0}{X_1} \leq 1$$

and

$$\frac{X_0}{X_1} \leq 3$$

and under this condition the line-to-ground voltage during a L-G fault does not exceed 80% of the L-L voltage and, therefore, an arrester of (80% + 0.05 × 80%) = 1.05 × 80% = 84% is required. This is the extreme situation in case of solidly grounded system. In the same system the voltage may be less than 80%; say it may be 75%. In that case the rating of the lightning arrester will be 1.05 × 75% = 78.75%. The overvoltages can actually be obtained with the help of precalculated curves. One set of curves corresponding to a particular system is given in Fig. 16.14.

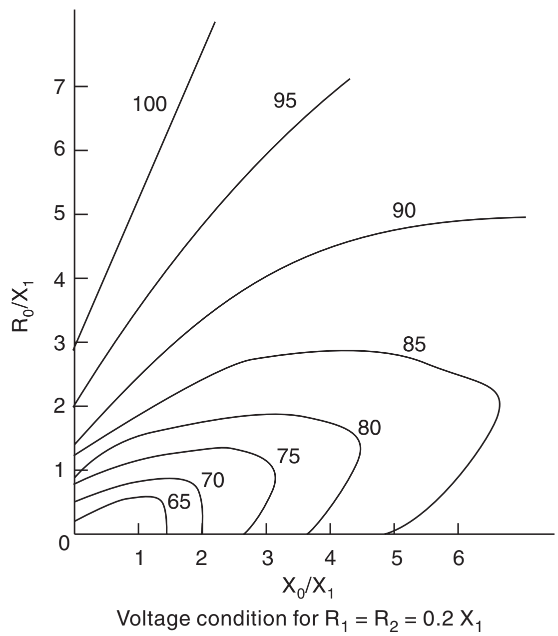


Fig. 16.14 Maximum line-to-ground voltage at fault location for grounded neutral system under any fault condition.

For system grounded through Peterson coil, the overvoltages may be 100% if it is properly tuned and, therefore, it is customary to apply an arrester of 105% for such systems. Even though there is a risk of overvoltages becoming more than 100% if it is not properly tuned, but it is generally not feasible to select arresters of sufficiently high rating to eliminate all risks of arrester damage due to these reasons. The voltage rating of the arrester, therefore, ranges between 75% to 105% depending upon the neutral grounding condition.

So far we have discussed the non-shielding method. We now discuss the shielding method *i.e.*, the use of ground wires for the protection of transmission lines against direct lightning strokes.

16.3 GROUND WIRES

The ground wire is a conductor running parallel to the power conductors of the transmission line and is placed at the top of the tower structure supporting the power conductors (Fig. 16.15 (*a*)). For horizontal configuration of the power line conductors, there are two ground wires to provide effective shielding to power conductors from direct lightning stroke whereas in vertical configuration there is one ground wire. The ground wire is made of galvanized steel wire or in the modern high voltage transmission lines ACSR conductor of the same size as the power conductor is used. The material and size of the conductor are more from mechanical consideration rather than electrical. A reduction in the effective ground resistance can be achieved by other relatively simpler and cheaper means. The ground wire serves the following purposes: (*i*) It shields the power conductors from direct lightning strokes. (*ii*) Whenever a lightning stroke falls on the tower, the ground wires on both sides of the tower provide parallel paths for the stroke, thereby the effective impedance (surge impedance) is reduced and the tower top potential is relatively less. (*iii*) There is electric and magnetic coupling between the ground wire and the power conductors, thereby the changes of insulation failure are reduced.

Protective angle of the ground wire is defined as the angle between the vertical line passing through the ground wire and the line passing through the outermost power conductor (Fig. 16.15 (*a*)) and the protective zone is the zone which is a cone with apex at the location of the ground wire and surface generated by line passing through the outermost conductor. According to Lacey, a ground wire provides adequate shielding to any power conductor that lies below a quarter circle drawn with its centre at the height of ground wire and with its radius equal to the height of the ground wire above the ground. If two or more ground wires are used, the protective zone between the two adjacent wires can be taken as a semicircle having as its diameter a line connecting the two ground wires (Fig. 16.15 (*b*)). The field experience alongwith laboratory investigation has proved that the protective angle should be almost 30° on plain areas whereas the angle decreases on hilly areas by an amount equal to the slope of the hill.

The voltage to which a transmission tower is raised when a lightning strikes the tower, is independent of the operating voltage of the system and hence the design of transmission line against lightning for a desired performance is independent of the operating voltage. The

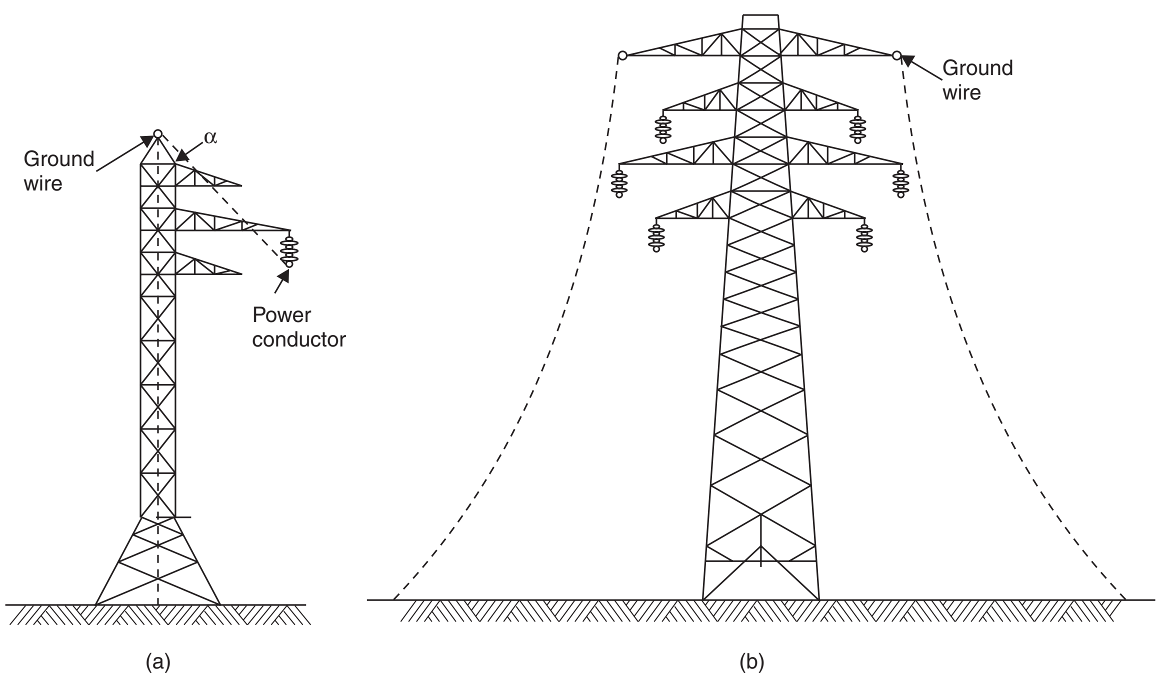


Fig. 16.15 (*a*) Protective angle; (*b*) Protection afforded by two ground wires.

basic requirement for the design of a line based on direct stroke are: (*i*) The ground wires used for shielding the line should be mechanically strong and should be so located that they provide sufficient shield. (*ii*) There should be sufficient clearance between the power conductors themselves and between the power conductors and the ground or the tower structure for a particular operating voltage. (*iii*) The tower footing resistance should be as low as can be justified economically.

To meet the first point the ground wire as is said earlier is made of galvanized steel wire or ACSR wire and the protective angle decides the location of the ground wire for effective shielding. The second factor, *i.e.*, adequate clearance between conductor and tower structure is obtained by designing a suitable length of cross arm such that when a string is given a swing of 30° towards the tower structure the air gap between the power conductor and tower structure should be good enough to withstand the switching voltage expected on the system, normally four times the line-to-ground voltage (Fig. 16.16).

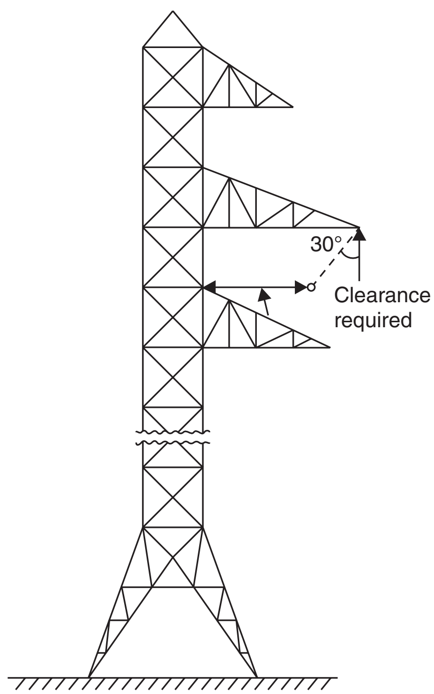


Fig. 16.16 Clearance determination or cross arm length determination.

The clearances between the conductors also should be adjusted by adjusting the sag so that the mid span flashovers are avoided.

The third requirement is to have a low tower footing resistance economically feasible. The standard value of this resistance acceptable is approximately 10 ohms for 66 kV lines and increases with the operating voltage. For 400 kV it is approx. 80 ohms. The tower footing resistance is the value of the footing resistance when measured at 50 Hz. The line performance with regard to lightning depends upon the impulse value of the resistance which is a function of the soil resistivity, critical breakdown gradient of the soil, length and type of driven grounds or counterpoises and the magnitude of the surge current. If the construction of the tower does not give a suitable value of the footing resistance, following methods are adopted.

One possibility could be the chemical treatment of the soil. This method is not practically possible because of the long length of the lines and because this method needs regular check up about the soil conditions. It is not possible to check up the soil conditions at each and every tower of the line which runs in several miles. Therefore, this method is used more for improving the grounds of the substation.

The methods normally used for improving the grounds of transmission towers are the use of (*i*) ground rods, and (*ii*) counterpoises.

Ground Rods

Ground rods are used to reduce the tower footing resistance. These are put into the ground surrounding the tower structure. Fig. 16.17 shows the variation of ground resistance with the length and thickness of the ground rods used. It is seen that the size (thickness) of the rod does not play a major role in reducing the ground resistance as does the length of the rod. Therefore, it is better to use thin but long rods or many small rods.

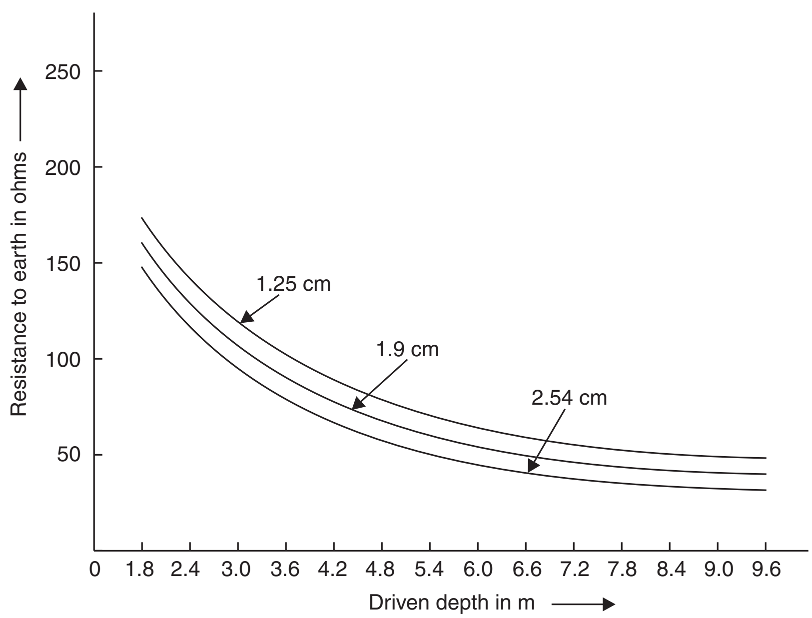


Fig. 16.17 Ground rod resistance as a function of rod length.

Counterpoise

A counterpoise is galvanized steel wire run in parallel or radial or a combination of the two, with respect to the overhead line. The various configurations used are shown in Fig. 16.18.

The corners of the squares indicate the location of the tower legs. The lightning stroke as is incident on the tower, discharges to the ground through the tower and then through the counterpoises. It is the surge impedance of the counterpoises which is important initially and once the surge has travelled over the counterpoise it is the leakage resistance of the counterpoise that is effective. While selecting a suitable counterpoise it is necessary to see that the leakage resistance of the counterpoise should always be smaller than the surge impedance; otherwise, positive reflections of the surge will take place and hence instead of lowering the potential of the tower (by the use of counterpoise) is will be raised.

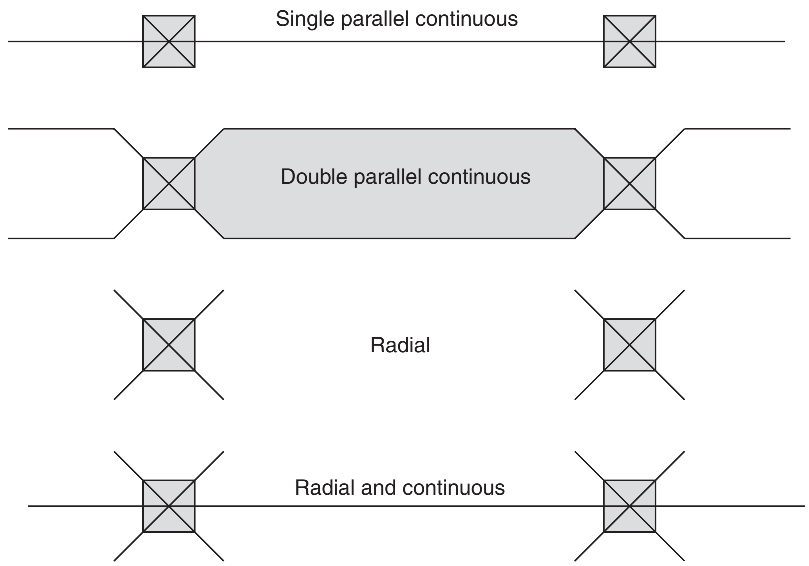


Fig. 16.18 Arrangement of counterpoise.

The leakage resistance of the counterpoise depends upon the surface area, *i.e.*, whether we have one long continuous counterpoise say 1000 m or four smaller counterpoises of 250 m each, as far as the leakage resistance is concerned it is same, whereas the surge impedance of say 1000 m if it is 200 ohms, then it will be 200/4, if there are four counterpoises of 250 m. each, as these four wires will now be connected in parallel. Also if the surge takes say 6 micro-seconds to travel a distance of 1000 m to reduce the surge impedance to leakage impedance, with four of 250 m, it will take 1.5. μ sec, that is, the surge will be discharged to ground faster, the shorter the length of the ground wire. It is, therefore, desirable to have many short counterpoises instead of one long counterpoise. But we should not have too many short counterpoises, otherwise the surge impedance will become smaller than the leakage resistance (which is fixed for a counterpoise) and positive reflections will occur.

The question arises as to why we should have a low value of tower footing resistance. It is clear that, whenever a lightning strikes a power line, a current is injected into the power system. The voltage to which the system will be raised depends upon what impedances the current encounters. Say if the lightning stroke strikes a tower, the potential of the tower will depend upon the impedance of the tower. If it is high, the potential of the tower will also be high which will result in flashover of the insulator discs and result in a line-to-ground fault. The flashover will take place from the tower structure to the power conductor and, therefore, it is known as back flashover,

Surge absorbers: A surge absorber is a device which absorbs energy contained in a travelling wave. Corona is a means of absorbing energy in the form of corona loss. A short length of cable between the equipment and the overhead line absorbs energy in the travelling wave because of its high capacitance and low inductance. Another method of absorbing energy is the use of Ferranti surge absorber which consists of an air core inductor connected in series with the line and surrounded by an earthed metallic sheet called a dissipator. The dissipator is insulated from the inductor by the air as shown in Fig. 16.19.

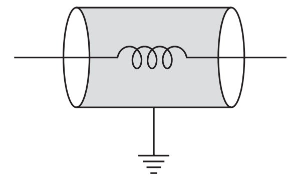


Fig. 16.19 Ferranti surge absorber.

The surge absorber acts like an air cored transformer whose primary is the low inductance inductor and the dissipator acts as the single turn short circuit secondary. Whenever the travelling wave is incident on the surge absorber a part of the energy contained in the wave is dissipated as heat due to transformer action and by eddy currents. Because of the series inductance, the steepness of the wave also is reduced. It is claimed that the stress in the end turns is reduced by 15% with the help of surge absorber.

16.4 SURGE PROTECTION OF ROTATING MACHINE

A rotating machine is less exposed to lightning surge as compared to transformers. Because of the limited space available, the insulation on the windings of rotating machines is kept to a minimum. The main difference between the winding of rotating machine and transformer is that in case of rotating machines the turns are fewer but longer and are deeply buried in the stator slots. Surge impedance of rotating machines in approx. 1000 Ω and since the inductance and capacitance of the windings are large as compared to the overhead lines the velocity of propagation is lower than on the lines. For a typical machine it is 15 to 20 metres/ μ sec. This means that in case of surges with steep fronts, the voltage will be distributed or concentrated at the first few turns. Since the insulation is not immersed in oil, its impulse ratio is approx. unity whereas that of the transformer is more than 2.0.

The rotating machine should be protected against major and minor insulations. By major insulation is meant the insulation between winding and the frame and minor insulation means inter-turn insulation.

The major insulation is normally determined by the expected line-to-ground voltage across the terminal of the machine whereas the minor insulation is determined by the rate of rise of the voltage. Therefore, in order to protect the rotating machine against surges requires limiting the surge voltage magnitude at the machine terminals and sloping the wave front of the incoming surge. To protect the major insulation a special lightning arrester is connected at the terminal of the machine and to protect the minor insulation a condenser of suitable rating is connected at the terminals of the machine as shown in Fig. 16.20.

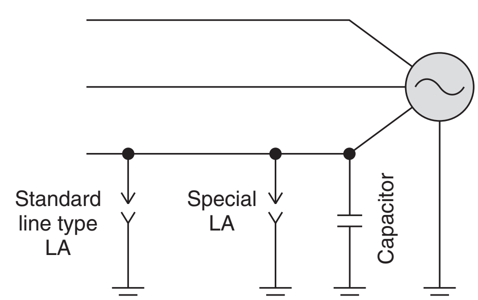


Fig. 16.20 Surge protection of rotating machine.

PROBLEMS

16.1. What are volt-time curves ? What is their significance in power system studies ?

16.2. What are BILS ? Explain their significance in power system studies.

16.3. Describe the construction, principle of operation and applications of (*i*) Rod gaps; (*ii*) Expulsion gap; and (*iii*) Valve type lightning arrester.

16.4. Compare the relative performances of the following: (*i*) Rod gap; (*ii*) Expulsion gap; and (*iii*) Valve type L.A.

16.5. Explain clearly how the rating of a lightning arrester is selected. What is the best location of a lightning arrester and why ?

16.6. What is tower-footing resistance ? What are the methods to reduce this resistance ? Why is it required to have this resistance as low as economically feasible ?

16.7. What are ground rods and counterpoises ? Explain clearly how these can be used to improve the grounding conditions. Give various arrangements of counterpoise.

16.8. "The leakage resistance of a counterpoise should be lower than its surge impedance." Why ?

16.9. What is a ground wire ? Discuss its location with respect to power conductors.

16.10. What are the requirements of a ground wire for protecting power conductors against direct lightning stroke ? Explain how they are achieved in practice.

16.11. Explain the principle of operation of Ferranti surge absorber.

16.12. What are the basic requirements of a lightning arrester ? Differentiate between (*i*) a lightning arrester and a lightning conductor, and (*ii*) a surge diverter and a surge absorber.

16.13. Explain clearly how a lightning arrester is selected for protecting a power transformer.

16.14. Give a scheme of protecting a rotating machine against overvoltages. Explain clearly how the scheme is different from protecting a power transformer.

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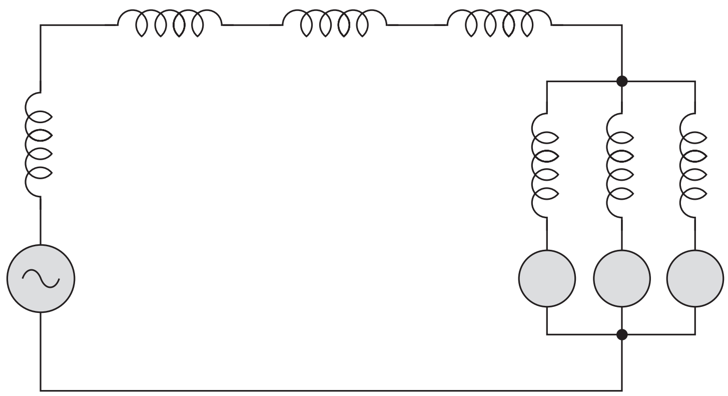
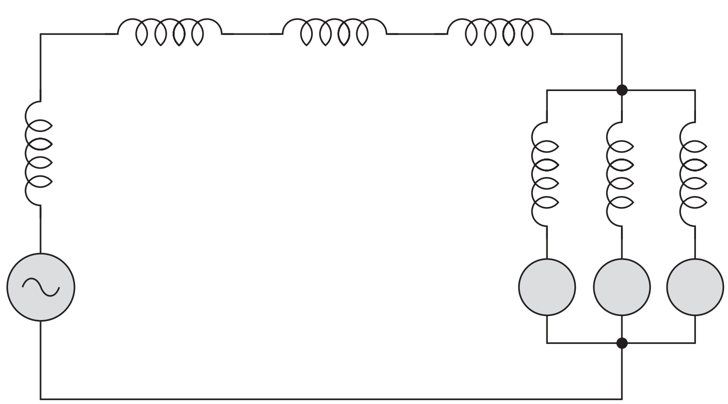


17

POWER SYSTEM SYNCHRONOUS STABILITY



17

Power System Synchronous Stability

INTRODUCTION

A classical problem of maximum power transfer is well known. Consider Fig. 17.1 (*a*). In case the source is a d.c. one, the maximum power is transferred when $Z_I = Z_L$, where Z_I and Z_L are both resistive and in case of an a.c. circuit maximum power is transferred when Z_L = conjugate of $Z_I = Z_I^*$.

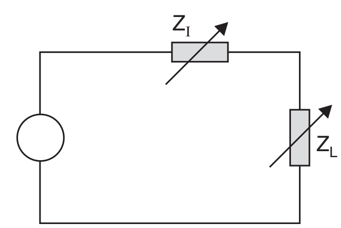


Fig. 17.1 (*a*) Maximum power transfer.

But with this, half of the total power transferred is wasted in the circuit *i.e.*, in R in case of d.c. circuit and in Z in case of a.c. circuit. For a power system engineer this is a highly uneconomical proposition. Normally, a transmission loss of about 15% is permissible.

Further, the power system engineer is faced with a variable load ranging from minimum to maximum at or near a constant value of voltage. Voltage is a very important factor as the light output of a lamp reduces very much when it is operated below a certain rated voltage. The induction motor draws more current for the same torque when operated at lower voltages and under extreme low voltage condition motors may stall under load.

The maximum power transfer problem becomes much more complicated by the presence of synchronous machines in the electric power system. If attempts are made to transfer power more than certain value known as steady state stability limit, the machines may fall out of step and supply to customers may be affected.

The magnitude of power that can be transmitted from a source to asynchronous loads such as heaters, lamps and induction motors depends upon the range of voltage that is available from the source which may be tolerated by the load and the current carrying ability of the various components of the network.

Whereas, when two or more synchronous machines are in operation on the same power system, it is found that a power transfer limit exists even though voltages at both the ends can be held at specified values. If a synchronous motor is connected to a synchronous generator, loss of synchronism results in stalling of synchronous motor and if two generators are connected, loss of synchronism will result in wild fluctuation of current and voltage within the transmission network. The power transfer between the sources is alternatively positive and negative with an average of zero. Under such situation it is imperative to separate the machines by opening the circuit breakers and resynchronizing them.

There are two forms of instability in power systems, the stalling of asynchronous loads (voltage stability or load stability) and the loss of synchronism between synchronous machines. The synchronous stability is again divided into two regimes:

(*i*) Steady state stability.

(*ii*) Transient state stability.

The steady state stability is the stability of the system under conditions of gradual or relatively slow change in load. The load is assumed to be applied at a rate which is slow when compared either with the natural frequency of oscillations of the major parts of the system or with the rate of change of field flux in the rotating machine in response to the change in loading.

The transient state stability refers to the maximum flow of power possible through a point without losing the stability with sudden and large changes in the network conditions such as brought about by faults, by sudden large increment of loads.

Besides the two categories of stabilities as mentioned above, there is a third category of stability known as dynamic stability. When synchronous machines are operated alongwith fast acting voltage regulator, the stability limits of the system are higher than when rather slow acting regulators are used. Dynamic stability also corresponds to slow changes in load as in the case of steady state stability but the main difference between the two is that dynamic stability is made possible by the action of fast acting voltage regulators which are capable of changing the flux at a faster rate than that caused by the system in falling out of step whereas in steady state stability we assume that the regulator acts slowly in order to adjust the terminal voltage to the prescribed value. It is to be noted that during dynamic stability zone the system does not operate on a single power angle curve but the modern fast-acting exciters will change the operating curve during the period under study. A typical curve is shown in Fig. 17.1 (*b*). The power systems are usually not designed to operate in the region of dynamic stability as absolute dependence on voltage regulator performance has not been considered advisable. However, during disturbance and under emergency conditions, the power system can be operated for realizing additional transfer of power by operating it in the dynamic stability zone.

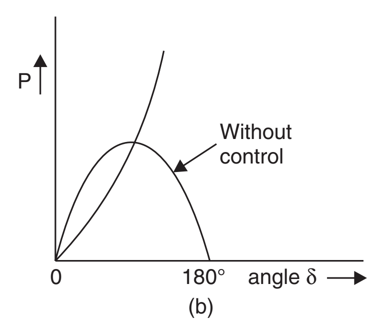


Fig. 17.1 (*b*) Power angle curves with and without excitation control.

17.1 THE POWER FLOW

Consider Fig. 17.2 for the calculation of power flow. All the quantities have been expressed in polar form

$$I = \frac{V_1 \angle \delta - V_2 \angle 0}{Z \angle \theta} = \frac{V_1}{Z} \angle (\delta - \theta) - \frac{V_2}{Z} \angle - \theta$$

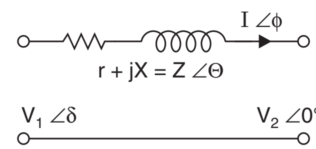


Fig. 17.2 Power flow in a 1-phase line.

Power received is given by

$$P_2 = \text{Re } [V_2 I^*]$$

or

$$P_2 = \text{Re}\left[V_2\left\{\frac{V_1}{Z}\angle(\theta - \delta) - \frac{V_2}{Z}\angle\theta\right\}\right]$$

$$= \frac{V_1 V_2}{Z}\cos(\theta - \delta) - \frac{V_2^2}{Z}\cos\theta \tag{17.1}$$

Let $\theta = 90 - \alpha$

$$\therefore \qquad P_2 = \frac{V_1 V_2}{Z}\cos(90 - \alpha - \delta) - \frac{V_2^2}{Z}\cos(90 - \alpha)$$

$$= \frac{V_1 V_2}{Z}\sin(\alpha + \delta) - \frac{V_2^2}{Z}\sin\alpha \tag{17.2}$$

Now α is a function of the impedance of the line; therefore, the power P_2 received is maximum when $\alpha + \delta = 90°$ or $\delta = (90 - \alpha)$ and the value is given by

$$P_{2\,\max} = \frac{V_1 V_2}{Z} - \frac{V_2^2}{Z}\sin\alpha$$

Also

$$\sin\alpha = \frac{r}{Z}$$

$$\therefore \qquad P_{2\,\max} = \frac{V_1 V_2}{\sqrt{r^2 + x^2}} - \frac{V_2^2}{\sqrt{r^2 + x^2}} \cdot \frac{r}{\sqrt{r^2 + x^2}}$$

and when $V_1 = V_2$,

$$P_{2\,\max} = V_2^2\left[\frac{1}{\sqrt{r^2 + x^2}} - \frac{r}{r^2 + x^2}\right] \tag{17.3}$$

For $P_{2\,\max}$ to be maximum

$$\frac{dP_{2\max}}{dx} = 0 = V_2^2\left[\frac{x}{(r^2 + x^2)^{3/2}} - \frac{2xr}{(r^2 + x^2)^2}\right]$$

or

$$\frac{V_2^2 x}{(r^2 + x^2)^2}\left[\sqrt{r^2 + x^2} - 2r\right] = 0$$

or

$$r^2 + x^2 = 4r^2$$

or

$$x = \sqrt{3}r \tag{17.4}$$

This shows that the maximum power can be transferred from end 1 to end 2 when the reactance of the line is $\sqrt{3}$ times it resistance. Normally, the reactance is quite large as compared to the resistance. The equation shows that it is not necessarily desirable to compensate by series capacitance for all the reactance. Also it is clear that the power can be transferred only if reactance in present. In case reactance is zero power cannot be transmitted.

For a lossless line $r = 0$ and the transmitted power

$$P_2 = \frac{V_1 V_2}{x} \sin \delta \tag{17.5}$$

The equation (17.5) shows that the power transmitted depends upon the system reactance and the angle between the two rotors. The curve drawn between P_2 and δ is known as the power angle curve and is shown in Fig. 17.3.

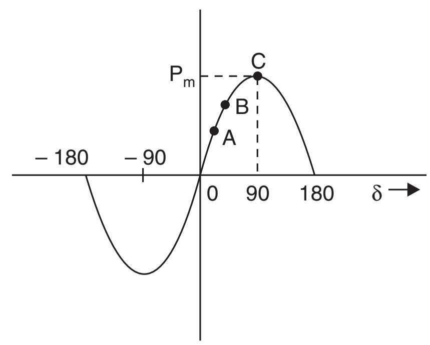


Fig. 17.3 Power angle curve.

The maximum power transmitted is given by

$$P_m = \frac{V_1 V_2}{x}$$

for a given V_1, V_2 and x and occurs at an angle of 90°. The torque angle δ is positive for generator action and negative for motor action. In case of generator action the rotor advances in the direction of rotation whereas for motor action, the rotor retards or falls back opposite to the direction of rotation. The maximum value of power transmitted can be varied by varying V_1, V_2 and x the circuit reactance. The system is stable if and only if for an increase in rotor angle δ the transmitted power also increases, *i.e.*, the $dP/d\delta$ should be positive. It can be seen from Fig. 17.3 that the range where $dP/d\delta$ is positive lies between 90 and –90°. When the tie-line impedance is purely capacitive (negative reactance), the range of angle for delivering power to the system is from 180° to 270° instead of from 0 to 90°. At zero degree with inductive reactance the power transmitted is zero whereas at 180° with capacitive reactance even though the power to be transmitted is zero but a large wattless current will flow which is not desirable and, therefore, normally over compensation of lines (by using series capacitor) is never done.

We study the power angle curve (Fig. 17.3) in detail. Let P be the mechanical input to the generator and the mechanical output from the motor assuming negligible frictional and transmission losses. Say initially this power corresponds to point A on the power angle curve. If a small increment of shaft load is added to the motor, the output power of the motor increases as the speed does not change momentarily whereas the input to the motor remains unchanged. Therefore, there is a net torque on the motor tending to retard it and its speed decreases temporarily. As a result of reduction in motor speed, the rotor angle δ increases and consequently the power input to the motor increases until finally the input and output are again in equilibrium and steady operation takes place at a new point B higher than A on the power angle curve. The gradual addition of load on the motor shaft is possible till the point C is reached on the power angle curve where $P = P_{\max}$ and any further addition of load will result in increase in angle δ but reduction in input power to the motor and, therefore, the motor will decelerate further and it will pull out of step and will probably stall unless it has damper winding which may keep it running as an induction motor. P_m is known as the steady state stability limit of the system which means that it is the maximum power that can be transmitted and synchronism will be lost if an attempt is made to transmit power more than this limit.

The steady state stability limit can be increased by (*i*) increasing the excitation of the motor or generator or both so that the internal e.m.fs. are increased, and (*ii*) reducing the reactance. This is done by either running parallel lines or by using the series capacitors.

17.2 THE SWING EQUATION

Under normal operations, the relative position of the rotor axis and the stator magnetic field axis is fixed. The angle between the two is known as the load angle or torque angle denoted by δ and depends upon the loading of the machine. Larger the loading, larger is the value of the torque angle δ. If some load is added or removed from the shaft of the synchronous machine, the rotor will decelerate or accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins. It is said that the rotor is swinging with respect to the stator field. The equation describing the relative motion of the rotor (load angle δ) with respect to the stator field as a function of time is known as swing equation. If T_s represents the shaft torque and T_e the electromagnetic torque and if these are assumed positive for a generator, the net torque causing acceleration is

$$T_a = T_s - T_e \tag{17.6}$$

and T_a is positive if shaft torque input is greater than the electromagnetic power output. For a motor if T_e the electromagnetic torque input is greater than the shaft torque output the motor rotor will accelerate. A similar relation holds good when expressed in terms of power, *i.e.*,

$$P_a = P_s - P_e$$

where P_a is accelerating power.

Since a synchronous machine is a rotating body, the laws of mechanics apply to this also. We know that power is equal to torque times the angular velocity.

$$P_a = P_a\omega \tag{17.7}$$

Now torque is moment of inertia times the angular acceleration.

$$\therefore \qquad P_a = T_a\omega = I\alpha\omega = M\alpha \tag{17.8}$$

Here ω is the angular velocity in mechanical radians per sec, *i.e.*,

$$\omega = \frac{2\pi n_s}{60}$$

where n_s is the synchronous speed of the machine in r.p.m. and α is the acceleration in mechanical radians/sec^2.

In equations (17.7) and (17.8) ω should be used in mechanical radians/sec only and not electrical radians/sec. I is the moment of inertia in kg-m^2.

From equation (17.8),

$$M = I\omega$$

where M is in joule-sec/mechanical radian.

Since we are interested in studying the rotor motion in terms of electrical degrees or electrical radians we make use of the following relation:

No. of electrical radians or degrees

= No. of mechanical radians or degrees × Number of pairs of poles

If M is to be expressed in joule-sec/electrical radian when ω is in mechanical radians/sec, then

$$M = I\omega/\text{Number of pairs of poles}$$

and if M is to be expressed in joule-sec/electrical degree, then

$$M = I\omega/(\text{Number of pairs of poles} \times 57.32)$$

as 1 radian = 57.32°.

Here M is known as the angular momentum and is expressed in terms of megajoules-seconds per electrical degree if P_a is expressed in megawatts and α is in electrical degrees per second squared. The acceleration α can be expressed in terms of the angular position of the rotor as

$$\alpha = \frac{d^2\theta}{dt^2} \tag{17.9}$$

The angle θ changes continuously with respect to time when a sudden change occurs in the system. The value of θ is given by

$$\theta = \omega_r t + \delta \tag{17.10}$$

where ω_r is the angular velocity of the reference synchronously rotating axis and δ is the angular displacement in electrical degrees from the synchronously rotating reference axis (Fig. 17.4).

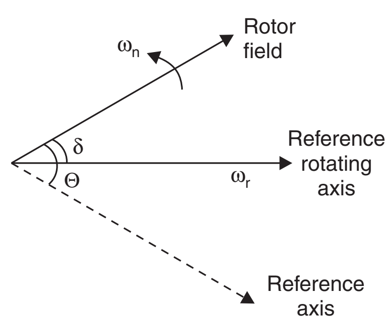


Fig. 17.4 Angular position of rotor with respect to reference axis.

Taking the derivative of equation (17.10),

$$\frac{d\theta}{dt} = \omega_r + \frac{d\delta}{dt} \tag{17.11}$$

and

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \tag{17.12}$$

From equations (17.8), (17.9) and (17.12) we obtain

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_c \tag{17.13}$$

Equation (17.13) is known as the swing equation. The angle δ is the difference between the internal angle of the machine and the angle of the synchronously rotating reference axis which in this case corresponds to the infinite bus. If it is a two machine system two swing equations are required, one for each machine. The torque angle between the two machines depends upon the angles between each machine and the synchronously rotating reference frame.

From equation (17.8) $M = I\omega$ which therefore is not constant but varies somewhat during the swings due to variation in ω. In practice, the change in ω from the normal system angular velocity is not much during swing except of course when the machine falls out of step and, therefore, very little error is involved by the assumption that M is constant and is equal to the value Iw_n, where ω_n is the normal angular velocity of the machine. This value of M is known as the inertia constant of the machine, and is normally used in calculations for stability studies. The inertia constant is truly constant because it is the angular momentum at synchronous speed.

Another important constant which is quite useful in stability studies is denoted by H and is defined as the ratio of the kinetic energy at rated speed to the rated apparent power of the machine, *i.e.*,

$$H = \frac{\text{Stored energy in megajoules}}{\text{Rating in MVA}}$$

H is also sometimes called as inertia constant. A relation between M and H is derived as follows:

Let G be the rating of the machine in MVA and f the frequency of the system. Then by definition

$$G \times H = \text{Stored energy in megajoules}$$

$$= \frac{1}{2} M\omega = \frac{1}{2} M \cdot 2\pi f$$

or

$$M = \frac{GH}{\pi f} \text{ megajoule-second/radian}$$

$$= \frac{GH}{180 f} \text{ megajoule-second/elect. degree}$$

From above it is clear that M depends upon the size of the machine as well as on its type whereas H does not vary widely with size and has a characteristic value or set of values for each class of machines. In this respect H is similar to per cent reactance of machines. Whenever value of H is not known a characteristic value may be used. The value of H is lower in case of water wheel generators as compared to turbo-alternator. Some typical values of H are given in the following table:

Typical values of *H*

Type of machine	*Inertia constant H, MJ/MVA*
Typical values of H	
Water wheel generator	
Slow speed < 200 r.p.m.	2–3
High speed > 200 r.p.m.	2–4

(*Contd.*)...

Synchronous capacitor	
Large	1.25
Small	1.00
Turbine alternator	
Condensing 1800 r.p.m.	9.6
3600 r.p.m.	7.4
Non-condensing 3600 r.p.m.	4.3

The inertia constant H can be expressed in terms of another base MVA in case it is known based on a particular MVA as in case of p.u. reactance. The only difference is that the p.u. reactance corresponding to new base MVA is directly proportional to the MVA whereas the inertia constant is inversely proportional. When several machines at one particular location are to be replaced by one simple equivalent machine, the rating of the equivalent machine is equal to sum of the ratings of several machines and the equivalent M is the sum of the inertia constants M of the individual machines.

Referring back to swing equation,

$$M \frac{d^2\delta}{dt^2} = P_s - P_e$$

P_s is fixed and substituting for P_e for a lossless system the swing equation becomes

$$M \frac{d^2\delta}{dt^2} = P_s - \frac{V_1 V_2}{x} \sin \delta$$

$$= P_s - P_m \sin \delta \tag{17.14}$$

For a multi-machine system solution of several swing equations is required. Normally the point by point method is used for the solution. Even for a single machine system connected to infinite bus with resistance neglected, the formal solution of swing equation is possible only when $P_s = 0$ and by the use of elliptic integrals. The solution of swing equation gives the relation between rotor angle as a function of time and this relation is plotted in terms of curves. If the curves show that the angle between two machine rotors increases without limit, the system is unstable. On the other hand, if the angles initially increase and then start reducing, it is probable though not certain that the system is stable. Many a time, in a multi-machine system one of the machines may stay in step on the first swing and yet go out of step in the subsequent swing. For a two machine system under the assumption of constant input, no damping and constant voltage behind transient reactance, the machines either fall out of step in the first swing or never. Under this condition the two machines are said to be running at standstill with respect to each other. There is a graphical method of determining whether the two machines are running at standstill with respect to each other or not. This method is known as Equal Area Criterion for stability. The use of this method eliminates partially or wholly the calculation of swing curves which thus saves a considerable amount of work. The method is applicable to any two machine systems which satisfies the above assumption. The method is not applicable to a multi-machine system directly. Even if the assumptions made are not strictly true but they do not invalidate the criterion because if these assumptions were not true, they would help the system to restore rather than make it fall out of step *e.g.*, if damping is present, it will damp out the oscillations rather than increasing them.

Example 17.1: A 50 Hz four-pole turbogenerator rated 20 MVA, 13.2 kV has an inertia constant of H = 9.0 kW-sec/kVA. Determine the K.E. stored in the rotor at synchronous speed. Determine the acceleration if the input less the rotational losses is 25000 HP and the electric power developed is 15000 kW. If the acceleration computed for the generator is constant for a period of 15 cycles, determine the change in torque angle in that period and the r.p.m. at the end of 15 cycles. Assume that the generator is synchronized with a large system and has no accelerating torque before the 15 cycle period begins.

Solution: (a) Since

$$H = \frac{\text{Stored energy in megajoules}}{\text{Machine rating in MVA}}$$

$\therefore$ K.E. stored in the rotor in megajoules = H × Machine rating in MVA

$$= 9 \times 20 = 180 \text{ megajoules} \quad \textbf{Ans.}$$

(b) The accelerating power = Mechanical power input – Electrical power output

$$P_a = P_i - P_G$$
$$= 25000 \times 0.735 - 15000$$
$$= 18375 - 15000 = 3375 \text{ kW} = 3.375 \text{ MW}$$

Now the acceleration is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{3.375}{M}$$

$$M = \frac{GH}{\pi f} = \frac{20 \times 9}{\pi \times 50} = 1.146 \text{ megajoules-sec/radian}$$

$$\therefore \quad \frac{d^2\delta}{dt^2} = \frac{3.375}{1.146} = 2.945 \text{ rad/sec}^2 \quad \textbf{Ans.}$$

(c) Again using swing equation,

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \text{constant} = 2.945$$

$$\frac{2d\delta}{dt}\cdot\frac{d^2\delta}{dt^2}\,dt = 2 \times 2.945\,\frac{d\delta}{dt}\,dt$$

$$\left(\frac{d\delta}{dt}\right)^2 = 5.89\delta + A$$

Since at $t = 0$, $\frac{d\delta}{dt} = 0$, $\therefore A = 0$.

$$\therefore \quad \frac{d\delta}{dt} = \sqrt{5.89}.\sqrt{\delta}$$

or

$$\frac{d\delta}{\sqrt{\delta}} = \sqrt{5.89}\,dt$$

$$\delta^{-1/2}d\delta = 2.427\,dt$$

$$2\delta^{1/2} = 2.427t$$

$$\delta^{1/2} = 1.2135t$$

$$\delta = 1.47258 \times t^2$$

Now $$t = \frac{15}{50} = \frac{3}{10} = 0.3 \text{ sec.}$$

$$\delta = 1.47258 \times 0.09 = 0.1325 \text{ radian}$$

$$= 7.5955 \text{ electrical degrees}$$

Now $$\frac{d\delta}{dt} = 2.425\sqrt{\delta} = 2.425\sqrt{0.1325}$$

$$= 0.8827 \text{ rad/sec.}$$

$$= \frac{0.8827}{4\pi} \text{ r.p.s.}$$

or $$\frac{0.8827}{4\pi} \times 60 \text{ r.p.m.} = 4.2 \text{ r.p.m.}$$

∴ Rotor speed at the end of 15 cycles = 1504.2 r.p.m. **Ans.**

17.3 STEADY STATE STABILITY

The study of steady state stability of power system involves the study of dynamics of the system when the rate of application of load is quite slow as compared with the natural frequency of oscillation of the system. The dynamics of the system can be described by swing equation (17.14) which is non linear. In case the changes are small, these equations can be linearized around the initial operating point. Consider a system consisting of a generator connected to an infinite busbar through a lossless network (Fig. 17.5) with initial operating point on the power angle curve as (P_0, δ_0).

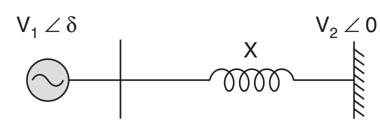


Fig. 17.5 Finite machine connected to an infinite bus.

Say the load is changed (increased) by ΔP, as a result of which the load angle changes by $\Delta\,\delta$ and the swing equation can be linearized around the point (P_0, δ_0) and is given as

$$M\,\frac{d^2\Delta\delta}{dt^2} = P_i - P_e$$

$$= P_i - (P_0 + \Delta P) = -\,\Delta P$$

$$= -\left(\frac{\partial P}{\partial \delta}\right)\bigg|_{\delta_0} .\,\Delta\,\delta \qquad (17.15)$$

Now let $\dfrac{d}{dt} = p$, equation (17.15) reduces to

$$Mp^2\,\Delta\,\delta + \frac{\partial P}{\partial \delta}\bigg|_{\delta_0} .\,\Delta\,\delta = 0$$

or $$\left(Mp^2 + \frac{\partial P}{\partial \delta}\bigg|_{\delta_0}\right)\Delta\,\delta = 0 \qquad (17.16)$$

Equation (17.16) is the characteristic equation with the two roots.

$$p = \pm\left(\frac{-\left.\dfrac{\partial P}{\partial \delta}\right|_{\delta_0}}{M}\right)^{\frac{1}{2}} \tag{17.17}$$

When $\dfrac{\partial P}{\partial \delta}$ is positive $\delta < \dfrac{\pi}{2}$, the two roots are pure imaginary and conjugate, the rotor motion is oscillatory and undamped around δ_0, when $\dfrac{\delta P}{\partial \delta}$ is negative ($\delta > 90°$), both the roots are real, one positive and the other negative respectively and hence the system is unstable. When $\dfrac{\partial P}{\partial \delta} = 0$, $\delta = 90°$ the system is critically stable. The frequency of oscillation of the system is given by the roots of the characteristic equation.

Let us now study the steady state stability of the system when system series resistance and shunt capacitance are included. The power transmitted between bus 1 and 2 is given by (Fig. 10.14).

$$P = \frac{V_1 V_2}{|B|}\cos(\beta - \delta) - \frac{A V_2^2}{|B|}\cos(\beta - \alpha) \tag{17.18}$$

For system to be stable under steady state operation.

$$\frac{\partial P}{\partial \delta} > 0$$

or $$\frac{V_1 V_2}{|B|}\sin(\beta - \delta) > 0$$

or $$\beta > \delta$$

and the system is critically stable $\dfrac{\partial P}{\partial \delta} = 0$

or $$\beta = \delta$$

and the maximum power transfer is given as

$$P_m = \frac{V_1 V_2}{|B|} - \frac{A V_2^2}{|B|}\cos(\beta - \alpha) \tag{17.19}$$

Which is an indication of steady state stability limit. If the resistance is considered but shunt capacitance neglected, we have

$$|B| > X, \beta < 90° \quad \text{and} \quad A = 1.0 \angle 0°$$

i.e., $$\alpha = 0°$$

So that, from equation (17.19)

$$P_m = \frac{V_1 V_2}{|B|} - \frac{A V_2^2}{|B|}\cos\beta \tag{17.20}$$

Comparing equation (17.20) with equation (17.5), since $|B| > X$ and equation (17.20) contains a negative term also, the steady state stability limit is, therefore, lowered and hence equation (17.5) gives optimistic result. It is to be noted, however, that transient stability results are optimistic when resistance is included.

If capacitance is also considered when $|A| < 1$ and $\alpha > 0$ it can be seen that P_m will now have a slightly higher value as compared to when capacitance is neglected but this value is still lower than the one given by equation (17.5).

While deriving equation (17.5) we assumed that the e.m.f. induced behind the reactance of the generator is constant which means the excitation of the generator is kept constant. But in actual practice, it is the terminal voltage which is kept constant and hence with change in load the internal e.m.f. of the generator will change. The steady state stability limit with variable excitation is discussed in article 17.10.

Example 17.2: A 50 Hz synchronous generator is connected to an infinite bus through a line. The p.u. reactances of generator and the line are $j0.3$ p.u. and $j0.2$ p.u. respectively. The generator no load voltage is 1.1 p.u. and that of infinite bus is 1.0 p.u. The inertia constant of the generator is 3 MW-sec/MVA. Determine the frequency of natural oscillations if the generator is loaded to (*i*) 60% and (*ii*) 75% of its maximum power transfer capacity and small perturbation in power is given.

Solution: Since the system is operating initially under steady state condition, a small perturbation in power will make the rotor oscillate. The natural frequency of oscillation is given by

$$f_n = \left\{\left(\frac{\partial P_e}{\partial \delta}\right)\delta_0 / M\right\}^{\frac{1}{2}}$$

(*i*) 60% loading

Now
$$\frac{\partial P_e}{\partial \delta} = \frac{V_1 V_2}{X}\cos\delta$$
$$= \frac{1.1 \times 1}{0.5}\cos\delta_0$$
$$= \frac{1.1 \times 1}{0.5} \times 0.8$$
$$= 1.76$$
$$\left(\text{Since } \sin\delta_0 = \frac{0.6}{1} = 0.6\right)$$
$$M = \frac{GH}{\pi f} = \frac{1 \times 3}{50\pi}$$

$\therefore$
$$f_n = \left(\frac{1.76}{3} \times 50\,\pi\right) = 9.6 \text{ rad/sec.}$$
$$= 1.53 \text{ Hz} \quad \textbf{Ans.}$$

(*ii*) 75% loading

$$\sin\delta_0 = 0.75 \text{ or } \delta_0 = 0.848 \text{ rad}$$
$$\left.\frac{\partial P_e}{\partial \delta}\right|_{\delta_0} = \frac{1.1 \times 1.0}{0.5} \times 0.6614 = 1.455$$

$$\therefore \qquad f_n = \left(\frac{1.455}{3} \times 50\pi\right)^{\frac{1}{2}} = 8{,}726 \text{ rad sec.}$$

$$= 1.39 \text{ Hz} \quad \textbf{Ans.}$$

17.4 EQUAL AREA CRITERION

The equal area criterion is derived using the swing equation for a machine connected to an infinite bus. The swing equation is given as

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e$$

Multiplying both the sides of the equation by $2d\delta/dt$, and integrating with respect to time, we get

$$\int 2M\frac{d^2\delta}{dt^2}.\frac{d\delta}{dt}\,dt = \int 2(P_s - P_e)\frac{d\delta}{dt}\,dt$$

$$M\left(\frac{d\delta}{dt}\right)^2 = 2\int_{\delta_0}^{\delta}(P_s - P_e)\,d\delta$$

or

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta}(P_s - P_e)\,d\delta + c} \qquad (17.21)$$

where δ_0 is the initial torque angle before any disturbance occurs and at this time $d\delta/dt = 0$. The angle δ will stop changing and the machine will again be operating at synchronous speed after a disturbance when $d\delta/dt = 0$ or when

$$\int_{\delta_0}^{\delta}(P_s - P_e)\,d\delta = 0 = \int_{\delta_0}^{\delta} P_a\,d\delta \qquad (17.22)$$

This means that the area under the curve P_a should be zero which is possible only when P_a is both accelerating and decelerating powers, *i.e.*, for a part of the graph $P_s > P_e$ and for the other $P_e > P_s$ (Fig. 17.6). For a generator action $P_s > P_e$ for positive area A_1 and $P_e > P_s$ for negative area A_2 for stable operation. Hence the name equal area criterion. The area A_1 represents the kinetic energy stored by the rotor during acceleration and the area A_2 represents the kinetic energy given up by the rotor to the system and when it is all given up, the machine has returned to its original speed. It is to be noted that the kinetic energy involved in our explanation is fictitious as it is being calculated corresponding to relative speed rather than the actual speed.

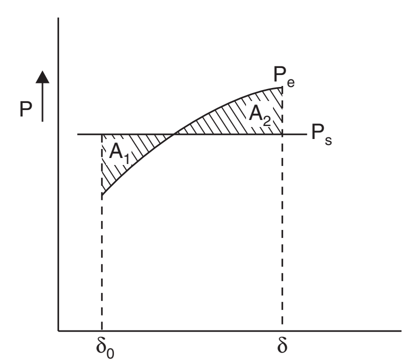


Fig. 17.6 Equal area criterion.

The following passage explains the operation of a synchronous motor using equal area criterion when sudden increase in mechanical load on that motor occurs.

The following points are to be noted with regard to the change in torque-angle whenever a disturbance occurs:

1. There is no change in torque angle when the speed of the rotor is the synchronous speed.

2. The angle increases in case of a motor if $P_s > P_e$ *i.e.*, the mechanical output is more than the electrical input and the speed goes down.

3. The angle decreases if the speed is more than the synchronous speed.

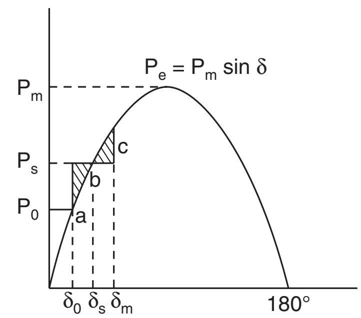


Fig. 17.7 Sudden change of load—equal area criterion.

With these points in view we explain the operations of the motor when there is sudden increase in load (Fig. 17.7). Initially the motor is operating when the torque angle is δ_0 with mechanical output P_0. Let this load be increased to P_s. Momentarily there is no change in angle δ corresponding to electrical input to the motor $P_e < P_s$. Therefore, the motor decelerates (K.E. supplied by the motor) as a result the torque angle increases and P_e starts increasing. At point b, $P_e = P_s$ and, therefore, decelerating force is zero but due to inertia of the rotor, the torque angle goes on increasing. The speed of the rotor beyond b starts increasing. The speed is minimum at b. When the speed equals the synchronous speed beyond point b, say at point c, the rotor angle stops increasing. But between c and b, $P_e > P_s$; therefore, the motor accelerates and the torque angle starts decreasing. The speed goes on increasing till it reaches the point b where this time the speed is maximum and is more than the synchronous speed. Between b and a, $P_s > P_e$; therefore, the rotor starts decelerating but the speed is more than synchronous speed till it reaches the point a where once again the speed equals the synchronous speed and the angle stops decreasing. The speed at a and c is the synchronous speed whereas at b, the speed is below synchronous speed when the rotor oscillates from a to b and is above synchronous speed when it oscillates between c and b. The following table shows the changes in angle, speed, electric power input, mechanical power output and acceleration or deceleration as the machine oscillates between a and c. Finally, of course, the machine will settle at b after the oscillations are damped out.

Position	*Torque angle*	*Motor speed*	*Power*	*Acceleration or deceleration*
At pt. a	$\delta = \delta_o$	$\omega = \omega_n$	$P_e < P_s$	Deceleration
From a to b	Increasing	$\omega < \omega_n$	$P_e < P_s$	Deceleration
At pt. b	$\delta = \delta_s$ (increasing)	$\omega < \omega_n$	$P_e = P_s$	
From b to c	Increasing	$\omega < \omega_n$	$P_e > P_s$	Acceleration
At pt. c	$\delta = \delta_{max}$	$\omega = \omega_n$	$P_e > P_s$	
From c to b	δ decreasing	$\omega > \omega_n$ increasing	$P_e > P_s$	Acceleration
At pt. b	$\delta = \delta_s$	$\omega > \omega_n$ max	$P_e = P_s$	
From pt. b to a	δ decreasing	$\omega > \omega_n$ decreasing	$P_e = P_s$	Deceleration
At pt. a	The cycle repeats itself.			

We have seen that with power angle curve as shown in Fig. 17.7 if the load on the motor shaft is increased suddenly from P_0 to P_s, the system is stable. For this system let us find out the maximum value of the P_s such that the system is critically stable *i.e.*, any attempt to increase P_s beyond this value the system becomes unstable. Referring to Fig. 17.8,

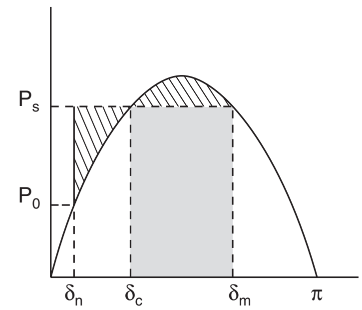


Fig. 17.8 Transient stability limit.

$$P_s = P_m \sin \delta_c = P_m \sin \delta_m$$

$$\therefore \quad \delta_m = (\pi - \delta_c)$$

Here δ_c is known as critical torque angle.

For the two shaded areas to be equal, the following condition should be satisfied:

$$P_s(\delta_m - \delta_0) = \int_{\delta_0}^{\delta_m} P_m \sin \delta \, d\delta \tag{17.23}$$

Also $P_s = P_m \sin \delta_m$.

Substituting this in equation (17.17),

$$P_m \sin \delta_m (\delta_m - \delta_0) = \int_{\delta_0}^{\delta_m} P_m \sin \delta \, d\delta$$

$$P_m \sin \delta_m (\delta_m - \delta_0) = P_m(\cos \delta_0 - \cos \delta_m) \tag{17.24}$$

Here δ_m is the only unknown which can be obtained and hence P_s can be calculated.

Other Applications of Equal Area Criterion

Next we apply the equal area criterion to two different systems of operation: (*i*) sustained line fault, and (*ii*) a line fault cleared after some time by the simultaneous tripping of the breakers at both the ends.

As shown in Fig. 17.9, it is a parallel feeder fed from one end and at the other end is the infinite bus. The power angle curve corresponding to the healthy condition is given by curve A in Fig. 17.10.

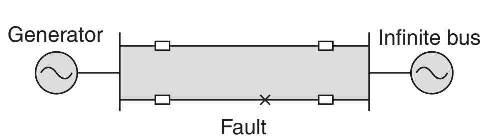


Fig. 17.9 Parallel feeder.

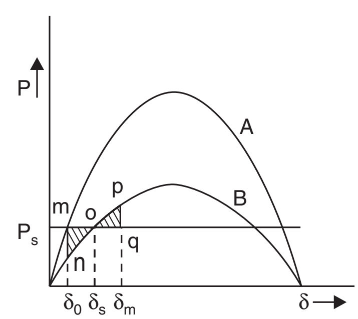


Fig. 17.10 Curve A—power angle curve under healthy condition; Curve B—power angle curve under faulted condition.

P_s is the input to the generator which is assumed constant. Also the voltage behind transient reactance is assumed constant. Curve B represents the power angle curve when a fault occurs at one of the two lines and the breakers operate instantaneously and simultaneously at both the ends on that line. As a result the euqivalent impedance between the bus bars is increased and hence curve B will be lower than curve A. Corresponding to input P_s the torque angle of the generator is δ_0 initially. Now as soon as there is a fault and instantaneously it is cleared, the output of the generator goes down to point n on curve B and since input remains constant which is higher than the output, the rotor accelerates and hence the torque angle increases and the operating point moves along curve B towards o from n. As it reaches o, the accelerating power becomes zero and the speed of the generator is more than the infinite bus and the speed continues to increase. From o to p the rotor experiences deceleration but the speed is more than the speed of the infinite bus till it reaches point p where the relative speed is zero and the torque angle ceases to increase. At point p, the output is more than the input to the generator and hence the rotor decelerates and the speed goes down relative to the infinite bus till it reaches point o where the speed is minimum. This torque angle continues to decrease till it reaches point n where again the speed is equal to the speed of infinite bus. The cycle repeats itself if damping is not present. It is found that in practice because of the damping present the rotor operates at point o on curve B and the torque angle is δ_s. To determine the transient stability limit for this case we should raise the input line P_s such that the area below the line P_s and the curve B and above the line P_s and the curve B at the intersection of P_s and B are equal. This is illustrated in Fig. 17.11. The value of P_s corresponding to this situation is known as the transient stability limit.

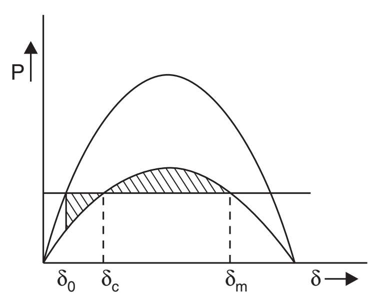


Fig. 17.11 Transient stability limit for system in Fig. 17.9.

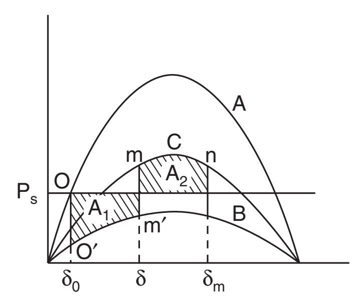


Fig. 17.12 Equal area criterion applied to system of Fig. 17.9 for a fault cleared at an angle $\delta < \delta_0$.

Fault Cleared after Some Time

Curve A in Fig. 17.12 represents the power angle curve corresponding to healthy condition of system in Fig. 17.9. Curve B represents corresponding to fault on one of the two lines and fault allowed to exist for some time. Curve C corresponds to the situation when the faulted line is removed.

Initially for P_s the torque angle is δ_0. At the time of fault, the output of the generator becomes as at O'. Hence, the rotor accelerates and the rotor moves along the curve B up to point m' when the faulted line is removed and the operating point becomes m on curve C where the output is more than the input and the rotor decelerates till the speed becomes equal to the speed of the infinite bus and the torque angle ceases to increase at point n.

From Fig. 17.12 it is clear that the transient stability limit not only depends upon the type of disturbance but it also depends upon the clearing time of the breaker. Faster the breaker operation smaller will be the area A_1 and hence larger will be the transient stability limit.

One more interesting case will be studied here. Consider Fig. 17.13 where generator is connected to an infinite bus through a single feeder.

Say there occurs a three-phase fault on the line temporarily (Fig. 17.14). The power angle curve will correspond to the horizontal axis because power transferred is zero. If the breaker reclose after some time corresponding to clearing angle δ_c when the fault is vanished, the output will be more than the input and hence the rotor decelerates. Finally, if the clearing angle δ_c is such that $A_1 = A_2$, the system becomes stable.

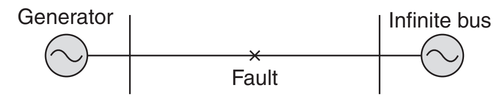


Fig. 17.13 Generator connected to infinite bus through a line.

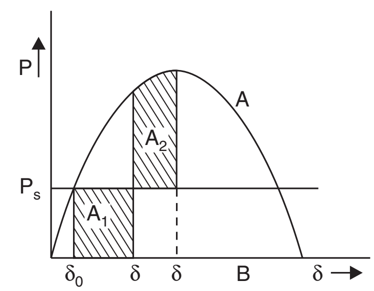


Fig. 17.14 Power angle curves: A—Healthy condition B—3-phase fault.

17.5 CRITICAL CLEARING ANGLE

We see that for any given initial load there is a critical clearing angle. If the actual clearing angle is greater than the critical value the system is unstable, otherwise it is stable. So we now proceed to determine the value of critical clearing angle for a given load.

Let the three power angle curves (Fig. 17.15) be represented as

$A = P_m \sin \delta$ before the fault.

$B = r_1 P_m \sin \delta$ during the fault.

$C = r_2 P_m \sin \delta$ after the fault.

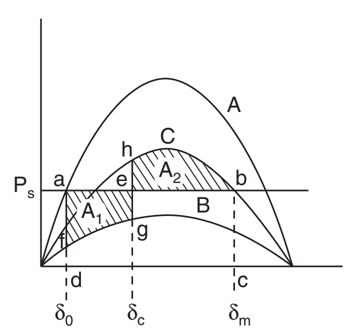


Fig. 17.15 Critical clearing angle.

For transient stability limit, the two areas $A_1 = A_2$ or equivalently the area under the curve *abcd* (a rectangle) should be equal to the area under the curve *dfghbc i.e.,*

$$(\delta_m - \delta_0)P_s = \int_{\delta_0}^{\delta_e} r_1 P_m \sin\delta\, d\delta + \int_{\delta_e}^{\delta_m} r_2 P_m \sin\delta\, d\delta$$

$$= r_1 P_m[\cos\delta_0 - \cos\delta_c] + r_2 P_m[\cos\delta_c - \cos\delta_m]$$

Now substituting $P_s = P_m \sin\delta_0$

$$(\delta_m - \delta_0)P_m \sin\delta_0 = r_1 P_m[\cos\delta_0 - \cos\delta_c] + r_2 P_m[\cos\delta_c - \cos\delta_m]$$

$$(\delta_m - \delta_0)\sin\delta_0 = (r_2 - r_1)\cos\delta_c + r_1\cos\delta_0 - r_2\cos\delta_m$$

$$\therefore \qquad \cos\delta_c = \frac{(\delta_m - \delta_0)\sin\delta_0 - r_1\cos\delta_0 + r_2\cos\delta_m}{r_2 - r_1} \qquad (17.25)$$

Now from the curves,

$$P_s = P_m \sin\delta_0 = r_2 P_m \sin\delta_m = r_2 P_m \sin(\pi - \delta_m)$$

or $$\sin\delta_0 = r_2 \sin(\pi - \delta_m)$$

or $$\delta_m = \pi - \sin^{-1}\left(\frac{\sin\delta_0}{r_2}\right) \qquad (17.26)$$

Thus if r_1, r_2 and δ_0 are known, the critical clearing angle δ_c can be obtained.

Example 17.3: A motor is receiving 25% of the power that it is capable of receiving from an infinite bus. If the load on the motor is doubled, calculate the maximum value of δ during the swinging of the rotor around its new equilibrium position.

Solution:

$\sin\delta_0 = 0.25 \qquad \therefore \quad \delta_0 = 14.48°$

$\sin\delta_c = 0.5 \qquad \delta_c = 30°.$

$\delta_m = ?$

$$(\delta_m - \delta_0)\,0.5 = \int_{\delta_0}^{\delta_m} \sin\delta\, d\delta$$

$$0.5(\delta_m - 0.2527) = (\cos\delta_0 - \cos\delta_m)$$

$$0.5\delta_m = \cos\delta_0 - \cos\delta_m + 0.1263$$

$$0.5\,\delta_m + \cos\delta_m = 0.96823 + 0.1263 = 1.094535$$

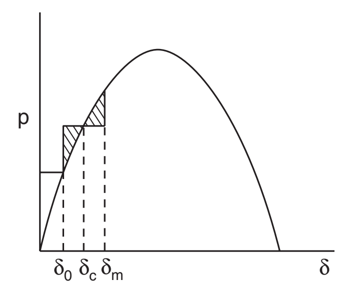


Fig. E.17.3

For solution of this equation we make guess for δ_m such that δ_m should be greater than 30° (as δ_c = 30°). After some trials δ_m is found to be 45°.

Example 17.4: A 50 Hz generator is delivering 50% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and the infinite bus to 500% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 75% of the original maximum value. Determine the critical clearing angle for the condition described.

Solution: Let P_m be the maximum power that can be delivered.

$$P_m \sin\delta_0 = 0.5\,P_m \qquad \text{or} \qquad \delta_0 = 30°$$

During fault the reactance is 500% of the value before the fault

$$\therefore \qquad r_1 = 0.2 \qquad \text{and} \qquad r_2 = 0.75 \text{ (given).}$$

The critical clearing angle δ_c is given by

$$\delta_c = \cos^{-1}\frac{(P_s / P_m)(\delta_m - \delta_0) + r_2\cos\delta_m - r_1\cos\delta_0}{r_2 - r_1}$$

$$0.5P_m = 0.75P_m \sin \delta_m$$

$\therefore$ $$\delta_m = 41.8° \text{ or } \delta_m = 180 - 41.8 = 138.2°$$

or $$\delta_m = 2.412 \text{ radians}$$

Substituting these values in the above expression, we get

$$= \cos^{-1}\frac{0.5(2.412 - 0.5236) - 0.75 \times 0.74547 - 0.2 \times 0.866}{0.55}$$

$$= \cos^{-1} 0.3836$$

$$= 67.44°. \quad \textbf{Ans.}$$

Example 17.5: Fig. E. 17.5 shows a generator connected to a metropolitan system (infinite bus) through high voltage lines. The numbers on the figure indicate the reactances in p.u. Breakers adjacent to a fault on both sides are arranged to clear simultaneously. Determine the critical clearing angle for the generator for a 3-phase fault at the point P when the generator is delivering 1.0 p.u. power. Assume that the voltage behind transient reactance is 1.2 p.u. for the generator and that the voltage at the infinite bus is 1.0 p.u.

Solution: The equivalent circuit during fault is given by Fig. E. 17.5. This is to be reduced further to delta and branch between A' and B' is only required, as branches between $A'P$ and $B'P$ do not absorb any active power.

$$Z_{A'B'} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$

$$= \frac{0.375 \times 0.35 + 0.35 \times 0.0545 + 0.375 \times 0.0545}{0.0545}$$

$$= \frac{0.13125 + 0.019075 + 0.0204375}{0.0545} = j3.1332568 \text{ p.u.}$$

The reactance between generator and the infinite bus during fault is, therefore, $j3.133$ p.u.

The reactance between the generator and infinite bus before the fault

$$j0.3 + \frac{j0.55}{2} + j0.15 = j0.45 + j0.275 = j0.725 \text{ p.u.}$$

The reactance between the generator and infinite bus after the fault is cleared is

$$j0.3 + j0.55 + j0.15 = j1.0$$

The maximum power output is given by $\dfrac{E_1 E_2}{X}$ which is given as

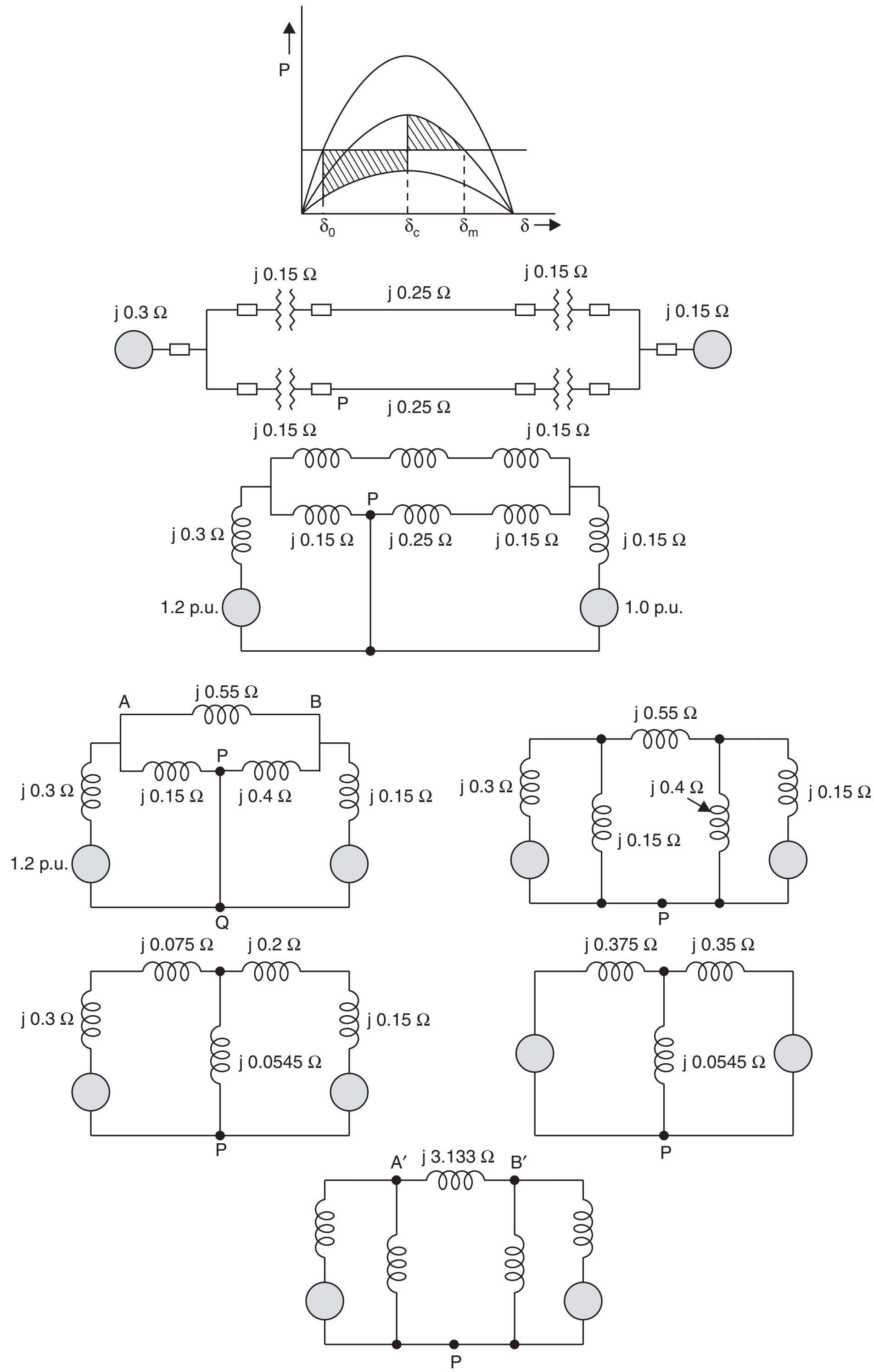
Before the fault $$\frac{1.2 \times 1.0}{0.72} = 1.67 \text{ p.u.}$$

During the fault $$\frac{1.2 \times 1.0}{3.133} = 0.383$$

After the fault $$\frac{1.2 \times 1.0}{1.0} = 1.2 \text{ p.u.}$$

The value of $$r_1 = \frac{0.383}{1.67} = 0.23$$

$$r_2 = \frac{1.2}{1.67} = 0.71856287$$



P
δ0
δc
δm
δ
j 0.15 Ω
j 0.25 Ω
j 0.15 Ω
j 0.3 Ω
j 0.15 Ω
P
j 0.25 Ω
j 0.15 Ω
j 0.15 Ω
j 0.15 Ω
j 0.15 Ω
P
j 0.3 Ω
j 0.15 Ω
j 0.25 Ω
j 0.15 Ω
j 0.15 Ω
1.2 p.u.
1.0 p.u.
A
j 0.55 Ω
B
P
j 0.3 Ω
j 0.15 Ω
j 0.4 Ω
j 0.15 Ω
1.2 p.u.
Q
j 0.55 Ω
j 0.3 Ω
j 0.4 Ω
j 0.15 Ω
j 0.15 Ω
P
j 0.075 Ω
j 0.2 Ω
j 0.3 Ω
j 0.15 Ω
j 0.0545 Ω
P
j 0.375 Ω
j 0.35 Ω
j 0.0545 Ω
P
A′
j 3.133 Ω
B′
P

Fig. E.17.5

Now $P_s = P_m \sin \delta_0$

$$1 = 1.67 \sin \delta_0 \text{ or } \sin \delta_0 = \frac{1}{1.67} = 0.6$$

$$\delta_0 = 36° \text{ or } 0.642 \text{ radian}$$

$$\sin \delta_m = \frac{1.0}{1.2} = 0.833 \text{ or } \delta_m = 56.4° \text{ or } 123.6° = 2.157 \text{ rad.}$$

$$\delta_c = \cos^{-1} \frac{0.6(2.15 - 0.642) - 0.71856 \times 0.5534 - 0.23 \times 0.809}{0.48856}$$

$$= \cos^{-1} \frac{0.9048 - 0.39765 - 0.186}{0.48856} = \cos^{-1} 0.657$$

$$\delta_c = 48.9°.$$

17.6 TWO FINITE MACHINES

A system having two machines can be replaced by an equivalent system having one machine connected to an infinite bus such that the swing equation and swing curves of angular displacement between the two machines are the same for both systems. Let δ_1, M_1, P_{a_1} be the quantities referred to one machine and δ_2, M_2, P_{a_2} for the other machine. The swing equations for the two machines can be written as

$$\frac{d^2\delta_1}{dt^2} = \frac{P_{a_1}}{M_1} = \frac{P_{s_1} - P_{e_1}}{M_1} \tag{17.27}$$

and

$$\frac{d^2\delta_2}{dt^2} = \frac{P_{a_2}}{M_2} = \frac{P_{s_2} - P_{e_2}}{M_2} \tag{17.28}$$

Let $\delta = \delta_1 - \delta_2$ the relative angle between the two rotor axes; then

$$\frac{d^2\delta}{dt^2} = \frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} \tag{17.29}$$

Substituting the values, we have

$$\frac{d^2\delta}{dt^2} = \frac{P_{s_1} - P_{e_1}}{M_1} - \frac{P_{s_2} - P_{e_2}}{M_2} \tag{17.30}$$

Multiplying both the sides by $\dfrac{M_1 M_2}{M_1 + M_2}$ we have

$$\frac{M_1 M_2}{M_1 + M_2} \frac{d^2\delta}{dt^2} = \frac{M_1 M_2}{M_1 + M_2} \left[\frac{P_{s_1} - P_{e_1}}{M_1} - \frac{P_{s_2} - P_{e_2}}{M_2} \right]$$

$$= \frac{1}{M_1 + M_2} [(M_2 P_{s_1} - M_1 P_{s_2}) - (M_2 P_{e_1} - M_1 P_{e_2})]$$

$$= \frac{M_2 P_{s_1} - M_1 P_{s_2}}{M_1 + M_2} - \frac{M_2 P_{e_1} - M_1 P_{e_2}}{M_1 + M_2} \tag{17.31}$$

which can be rewritten as $M \dfrac{d^2\delta}{dt^2} = P_s - P_e$

where $$M = \frac{M_1 M_2}{M_1 + M_2}$$

and $$P_s = \frac{M_2 P_{s_1} - M_1 P_{s_2}}{M_1 + M_2}$$

$$P_e = \frac{M_2 P_{e_1} - M_1 P_{e_2}}{M_1 + M_2}$$

It is clear that the inertia constant of the equivalent machine is a function of the inertia constant of the two actual machines and the law of combination is similar to that of the parallel combination of impedances which means that the inertia constant of the equivalent machine is smaller than the inertia constant of the two individual machines. The equivalent input to the one machine is a function of the inertia constant and individual inputs to the two machine systems. Similarly the equivalent output is a function of the inertia constants and individual outputs of the two machines.

It is to be noted that the law of combination of inertia constants of the equivalent machine corresponding to a group of machines that swing together is different from the one where the machines do not swing together (the above two finite machine case). There the accelerating power of the group is the sum of the accelerating powers of the individual machines and the accelerations of all machines are equal as they swing together. Therefore, the inertia constants combine like impedances in series

$$M_1 \frac{d^2\delta}{dt^2} = P_{a_1}$$

$$M_2 \frac{d^2\delta}{dt^2} = P_{a_2}$$

Since the total accelerating power is $P_{a_1} + P_{a_2}$,

$\therefore$ $$M_1 \frac{d^2\delta}{dt^2} + M_2 \frac{d^2\delta}{dt^2} = P_{a_1} + P_{a_2}$$

or $$(M_1 + M_2)\frac{d^2\delta}{dt^2} = P_{a_1} + P_{a_2} = P_a \tag{17.32}$$

or $$M \frac{d^2\delta}{dt^2} = P_a$$

where $M = (M_1 + M_2)$.

If the network between the two finite machines is reactive the power angle curve is given by

$$P_e = E_1 E_2 Y_{12} \sin \delta \tag{17.33}$$

which is identical to the power angle curve for one machine connected to an infinite bus through a reactance network. Here Y_{12} is the transfer admittance between the two machines.

17.7 POINT-BY-POINT METHOD

The equal area criterion method is useful in determining the critical clearing angle, *i.e.*, the condition when the system will be stable provided the fault is cleared before the rotor angle exceeds the critical clearing angle. The power system engineer is not interested in knowing the clearing angle for stable operation of the system. He wants to know the critical clearing time corresponding to the critical clearing angle (*i.e.*, the time when the rotor would have moved to the critical clearing angle) so that he can design the operating times of the relay and circuit breaker so that the total time taken by them should be less than the critical clearing time for stable operation of the system. the point-by-point method is used for the solution of critical clearing time associated with critical clearing angle. This method can also be used for the solution of multimachine system. The method requires the solution of swing equation by making certain approximations. The equation can be solved on (*i*) digital computer, and (*ii*) analogue computer or by long hand calculation. There are various techniques used for the solution of swing equation on digital computer. The step-by-step or point-by-point method is a conventional, approximate but proven method. This involves the calculations of the rotor angles as time is incremented. The accuracy of the solution depends upon the time increment used in the analysis. As the time interval is decreased the computed swing curve approaches the true curve.

The point-by-point method calculates the change in the angular position of the rotor during a short interval of time. The following assumptions are made during the computational procedure:

(*i*) The accelerating power P_a computed at the beginning of an interval is assumed to be constant from the middle of the preceding interval to the middle of the interval under consideration.

(*ii*) The angular velocity, computed at the middle of an interval, remains constant over the interval. Refer to Fig. 17.16 for better understanding of the assumptions.

The procedure for the first iteration is outlined below:

(*i*) Evaluate the accelerating power P_a,

$$P_{a(0+)} = P_s - P_{e(0+)} \tag{17.34}$$

(*ii*) From the swing equation

$$\frac{d^2\delta}{dt^2} = \alpha_{(0+)} = \frac{P_{a(0+)}}{M}$$

where α is the acceleration. Evaluate α.

(*iii*) The change in angular velocity for the first interval (from Fig. 17.16)

$$\Delta\omega_1 = \alpha_{(0+)}.\ \Delta t \tag{17.35}$$

$$\therefore \qquad \omega_1 = \omega_0 + \Delta\omega_1 = \omega_0 + \alpha_{(0+)}\Delta t \tag{17.36}$$

Here ω_0 is the relative angular velocity and is zero at $t = 0$.

(*iv*) The change in rotor angle for the first interval,

$$\Delta\delta_1 = \Delta\omega_1.\ \Delta t \tag{17.37}$$

$$\therefore \qquad \delta_1 = \delta_0 + \Delta\delta_1 = \delta_0 + \Delta\omega_1.\ \Delta t = \delta_0 + \alpha_{(0+)}(\Delta t)^2 \tag{17.38}$$

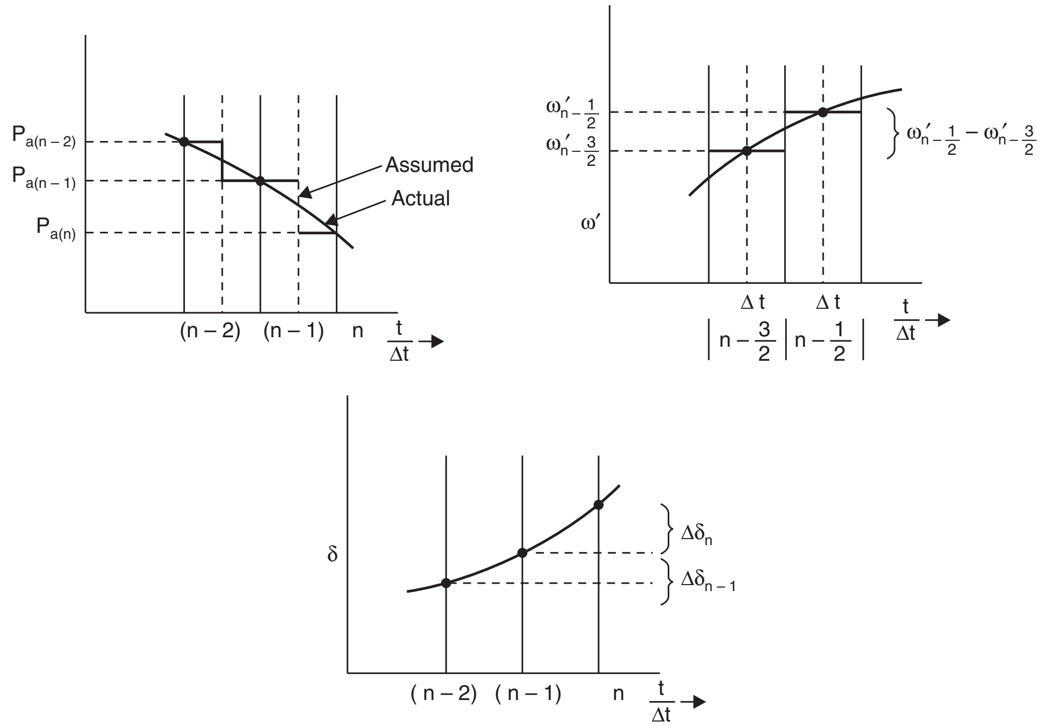


Fig. 17.16 Diagram showing the approximations used in the incremental calculation of P_e, α, ω and δ.

It is to be noted here that for evaluating the acceleration $\alpha_{(0+)}$, the value of $P_{a(0+)}$ should be taken, mean of the values of P_a before the fault and after the fault. Since the accelerating power is zero in the middle of the preceding interval (normal operation), the accelerating power will be half of the accelerating power immediately after the fault.

If the discontinuity occurs due to removal of the fault or due to any switching operation, there are three possibilities:

(*a*) The discontinuity occurs at the beginning of the *i*th interval.

(*b*) The discontinuity occurs at the middle of an interval.

(*c*) The discontinuity occurs at some time other than the beginning or the middle of an interval.

To evaluate P_a under first situation one should use the value corresponding to average values of accelerating powers $P_{a(i-1)-}$ and $P_{a(i-1)+}$ *i.e.,*

$$P_{a(i-1)} = \tfrac{1}{2}(P_{a(i-1)-} + P_{a(i-1)+})$$

where $P_{a(i-1)-}$ is the accelerating power immediately before clearing the fault and $P_{a(1-1)+}$ that immediately after clearing the fault.

To evaluate P_a under second situation no special procedure is required. P_a is taken as the value at the beginning of the interval, *i.e.,*

$$P_a = P_s - \text{Output during the fault}$$

whereas at the beginning of the interval following clearing P_a is given by

$$P_a = P_s - \text{Output after the fault is cleared}$$

by taking value of δ and the angular velocity ω at the beginning of the interval following clearing.

To evaluate P_a under the third situation, a weighted average value of P_a before and after the discontinuity may be used. It is found in practice that such a precise evaluation of P_a is not required as the time interval used in calculation is so short that it is sufficiently accurate to consider the discontinuity to occur either at the beginning or at the middle of an interval and P_a is evaluated as outlined above in the first two possibilities.

The value of δ_1 obtained above, forms one point on the swing curve. For the second interval δ_1 will be used for evaluating the accelerating power P_{a_1} and the procedure will be repeated during the subsequent intervals. The equation for nth interval can be written as

$$P_{a(n-1)} = P_s - P_{e(n-1)} \tag{17.39}$$

$$P_{e(n-1)} = \frac{|E||V|}{X} \sin \delta_{n-1} \tag{17.40}$$

$$\alpha_{n-1} = \frac{P_{a(n-1)}}{M} \tag{17.41}$$

$$\Delta\omega_{n-1/2} = \alpha_{n-1}\,\Delta t \tag{17.42}$$

$$\omega_{n-1/2} = \omega_{n-3/2} + \alpha_{n-1}\,\Delta t \tag{17.43}$$

$$\Delta\delta_n = \omega_{n-1/2}\,\Delta t = (\omega_{n-3/2} + \alpha_{n-1}\,\Delta t)\,\Delta t \tag{17.44}$$

$$= \Delta\delta_{n-1} + \alpha_{n-1}\Delta t^2$$

$$= \Delta\delta_{n-1} + \frac{P_{a(n-1)}}{M}\,\Delta t^2 \tag{17.45}$$

$$\therefore \qquad \delta_n = \delta_{n-1} + \Delta\delta_n \tag{17.46}$$

These equations can be used for plotting the swing curve and hence, the critical clearing time, corresponding to the critical clearing angle, as obtained by using Equal Area Criterion, can be determined and a suitable protecting scheme can be designed to avoid the system from falling out of step.

Example 17.6: Determine the critical fault clearing angle for the network shown when a three-phase fault takes place at B and the breakers at A and B operate simultaneously. The generator is delivering 1.0 p.u. power before the fault takes place. Assuming the inertia constant H = 4.0, (a) determine the critical clearing time, and (b) plot the swing curves both under sustained fault and when the breakers have operated. Use point by point method taking time interval of 0.05 sec.

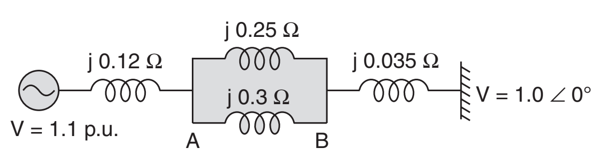


Fig. E.17.6

Solution: (*a*) We calculate the critical clearing angle first by using equation (17.25):

(*i*) When the system is operating normally.

The reactance between the buses

$$= j0.12 + j0.035 + \frac{j0.25 \times j0.3}{j0.55}$$

$$= j0.29136$$

$$P_m = \frac{1.1 \times 1.0}{0.29136} = 3.775 \text{ p.u.}$$

(*ii*) During the fault (3-phase) $P_{m_1} = 0$.

(*iii*) After the breakers operate, the reactance between the machines will be

$$j0.12 + j0.25 + j0.035 = j0.405$$

$$\therefore \quad P_{m2} = \frac{1.1 \times 1.0}{0.405} = 2.716 \text{ p.u.}$$

From these data, $r_1 = 0$

$$r_2 = \frac{2.716}{3.775} = 0.7194$$

Now $P_m \sin \delta_0 = 1.0$

$$3.775 \sin \delta_0 = 1.0$$

or $$\sin \delta_0 = \frac{1}{3.775} \quad \text{or} \quad \delta_0 = 15.36° = 0.2649 \text{ rad.}$$

To obtain δ_m we take the curve after the fault when maximum value of power is 2.716 p.u.

$$\therefore \quad \sin \delta_m = \frac{1}{2.716} = 0.368$$

$$\therefore \quad \delta_m = 180 - 21.6 = 158.4° = 2.763 \text{ radians}$$

Using the equation

$$\cos \delta_c = \frac{(\delta_m - \delta_0) \sin \delta_0 - r_1 \cos \delta_0 + r_2 \cos \delta_m}{r_2 - r_1}$$

and substituting the various values calculated above, we have

$$\cos \delta_c = \frac{(2.763 - 0.2649) \times 0.2649 - 0.7194 \times 0.9297}{0.7194 - 0.0}$$

$$= \frac{0.66095 - 0.6688}{0.7194} = 0.010948$$

$$\delta_c = 90.62$$

(*b*) Swing curve $$M = \frac{GH}{\pi f} = \frac{1.0 \times 4}{180 \times 50} = 0.444 \times 10^{-3} \text{ p.u.}$$

We now calculate the rotor angle for the first interval $\alpha_{(0+)} = \dfrac{P_{a(0+)}}{M}$. Since the power transmitted is zero and the shaft power is unity, the average value of power in the first interval will be $P_{a(0+)} = \dfrac{1.0 - 0.0}{2} = 0.5$.

$\therefore \qquad \alpha_{(0+)} = \dfrac{0.5}{0.444} \times 10^3 = 1125 \text{ deg/sec}^2.$

The change in angular velocity $\Delta\omega_1 = \alpha_{(0+)}\, \Delta t$

$= 1125 \times 0.05 = 56.25$ deg/sec.

Therefore $\qquad \omega_1 = \omega_0 + \Delta\omega_1 = 0.0 + 56.25 = 56.25$ deg/sec.

The change in rotor angle

$$\Delta\delta_1 = \omega_1\, \Delta t = 56.25 \times 0.05 = 2.8125 \text{ degrees}$$

$\therefore$ Rotor angle $\delta_1 = \delta_0 + \Delta\delta_1 = 15.36 + 2.8125 = 18.17°$

It is to be noted here that $\omega_0 = 0$ as this is the angular velocity of the rotor with respect to the reference axis and is zero before the fault takes place, *i.e.*, under normal operation of the machine.

Second interval

$$P_{a_1} = 1 - 0.0 = 1.0$$

$\therefore \qquad \alpha_1 = \dfrac{1.0}{0.444} \times 10^3 = 2250 \text{ deg/sec}^2.$

$$\Delta\omega_2 = \alpha_1\, \Delta t = 2250 \times 0.05 = 112.5 \text{ deg/sec.}$$

$$\omega_2 = \omega_1 + \Delta\omega_2 = 56.25 + 112.5 = 168.75 \text{ deg/sec.}$$

$\therefore \qquad \Delta\delta_2 = \omega_2\, \Delta t = 168.75 \times 0.05 = 8.43°$

$\therefore \qquad \delta_2 = \delta_1 + \Delta\delta_2 = 18.17 + 8.43 = 26.60°$

Since α and $\Delta\omega$ are same during the successive intervals, therefore

$$\omega_3 = \omega_2 + \Delta\omega_3 = 281.25, \quad \delta_3 = 40.66°$$

$$\omega_4 = 393.75 \qquad \delta_4 = 60.34$$

$$\omega_5 = 506.25 \qquad \delta_5 = 85.65$$

$$\omega_6 = 618.75 \qquad \delta_6 = 116.58$$

$$\omega_7 = 731.25 \qquad \delta_7 = 153.14$$

Following table gives the torque angles during various intervals:

Time in secs.	*Angle*
0.0	15.36°
0.05	18.17°
0.10	26.60°
0.15	40.66°
0.20	60.34°
0.25	85.65°
0.30	116.58°
0.35	153.14°

From the above table it is clear that at the end of 0.2 sec, *i.e.*, 10 cycles the rotor angle is 60.34° which is much less than the critical clearing angle and, therefore, if we use a 10 cycle breaker the swing curve after 10 cycles can be calculated as follows:

Since the operation of the circuit breaker will isolate the faulty sections of the system, therefore, the system operates on the power characteristic $P_e = 2.716 \sin \delta$.

To find out accelerating power after the breaker have operated, we take the average of the accelerating powers as there is a discontinuity. The accelerating power before the breakers operate is 1.0 p.u.

The electromagnetic power output from the generator at this instant is $P_e = 2.716 \sin 60.34 = 2.36$ p.u. and since shaft power is 1.0 p.u.

$$\text{Accelerating power} = P_s - P_e = 1.0 - 2.36 = -1.36$$

∴ The average accelerating power will be $\dfrac{1 - 1.36}{2} = -0.18$ p.u. which means during the time 0.2 sec and 0.25 sec, there is reduction in the angular velocity.

The deceleration $= \dfrac{0.18}{0.4444} \times 10^3 = 405$ deg/sec^2.

The change in angular velocity $= 405 \times 0.05 = 20.25$ deg/sec.

∴ The angular velocity $= 393.75 - 20.25 = 373.50$

The change in angle $= 18.67°$

$$\text{Rotor angle } \delta_5 = 60.34 + 18.67 = 79°$$

$$P_e = 2.66 \text{ p.u.}$$

Accelerating power $= P_s - P_e = 1 - 2.66 = -1.66$

Deceleration $= \dfrac{1.66}{0.4444} \times 10^3 = 3735.3$ degree/sec^2.

Change in angular velocity $= 3735.3 \times 0.05 = 186.7$ deg/sec.

Angular velocity $= 373.5 - 186.8 = 186.8$ deg/sec.

Change in angle $= 9.34$

$$\text{Angle } \delta_6 = 79 + 9.34 = 88.34°$$

$$P_{a+} = -1.7148 \text{ p.u.}$$

Deceleration $= 3858$ deg/sec^2.

Change in angular velocity $= 192.9$

Net velocity $= 186.8 - 192.9 = -6.1$ deg/sec.

Change in rotor angle $= -0.3$

$$\delta_7 = 88.34 - 0.3 = 88.04°$$

Deceleration $= \dfrac{-1.71}{0.4444} \times 10^3 = 3857$ deg/sec^2.

∴ Change in angular velocity $= 192.89$

$$\text{Net velocity} = -6.1 - 192.89 = -199 \text{ deg/sec.}$$

Change in rotor angle $= -\,9.95$

Rotor angle $\delta_8 = 88.94 - 9.95 = 78.1°$

$\delta_9 = 58.82°$

It can be easily seen that the system will come back to stable operation.

From the table it can be seen that the critical operating time is

$$0.25 + 0.05\frac{90.62 - 85.65}{116.58 - 85.65} = 0.258 \text{ sec.} \quad \textbf{Ans.}$$

To illustrate the procedure for evaluating swing curve when discontinuity occurs at the middle of an interval, let us assume that the breaker clears the fault at the end of 0.175 sec which occurs at the mid of the interval between 0.15 sec and 0.2 sec.

From the table corresponding to sustained fault, it can be seen that angle at 0.15 sec is 40.66° and the angular velocity $\omega_3 = 281.25$. $P_a = 1.0$ before the fault is cleared.

$$\therefore \quad \frac{P_a}{M} = 2250 \text{ deg/sec}^2,\ \Delta\omega_4 = 112.5,\ \omega_4 = 393.75$$

$$\Delta\delta_4 = 19.68,\ \therefore\ \delta_4 = \delta_3 + 19.68 = 60.34°$$

To evaluate δ_5,

$$P_a = 1 - 2.716 \sin 60.34° = -\,1.360142$$

$$\frac{P_a}{M} = -\,3063.38,\ \Delta\omega_5 = -\,153.169,\ \omega_5 = 240.58$$

$$\Delta\delta_5 = 12.03,\ \delta_5 = 72.37$$

To evaluate δ_6; $\quad P_a = 1 - 2.716 \sin 72.37 = -1.5884$

$$\frac{P_a}{M} = -\,3577.5264,\ \therefore\ \Delta\omega_6 = -178.87,\ \omega_6 = 64.70368$$

$$\Delta\delta_6 = 3.085,\ \delta_6 = 75.45°$$

Similarly other values of δ can be calculated.

17.8 FACTORS AFFECTING TRANSIENT STABILITY

From the swing equation $d^2\delta/dt^2 = P_a/M$ the acceleration of rotor $d^2\delta/dt^2$ is inversely proportional to the moment of inertia of the machine when accelerating power is constant which means higher the moment of inertia the slower will be the change in the rotor angle of the machine and thus allows a longer time for breaker operation to isolate the fault before the machine passes through the critical clearing angle. Higher moment of inertia means the heavier rotor which requires higher SCR. Since the p.u. synchronous impedance is the reciprocal of the SCR ratio, the p.u. impedance becomes smaller with higher values of SCR and, therefore, the short circuit currents in the system increase. Therefore, higher moment of inertia method of improving the transient stability is uneconomical and is normally not used. The methods normally used are

(*i*) Higher system voltage.

(*ii*) Use of parallel lines to reduce the series reactance.

(*iii*) Use of high speed circuit breakers and auto-reclosing breakers.

From equation (17.5) it is clear that an increase in system voltage results in higher value of power P_m that can be transferred between nodes. Since shaft power $P_s = P_m \sin \delta_0$ with higher value of P_m, δ_0 is reduced and, therefore, the difference between the critical clearing angle and the initial angle δ_0 is increased. Therefore, increasing P_m allows the machine to rotate through large angle before it reaches the critical clearing angle which results in greater critical clearing time and the probability of maintaining stability.

Reducing the series reactance by using series capacitor is normally economical for lines of length more than 320 kms. For lines of length less than 320 kms, the objective is achieved by running parallel lines. When parallel lines are used, instead of a single line, some power can be transferred over the healthy line even during a three-phase fault on one of the lines, unless of course when a fault takes place at the paralleling bus when no power can be transferred out the parallelled lines. For other types of faults on one line more power can be transferred during the fault if there are two lines in parallel than can be transferred over a single faulted line. The effect of reducing the series reactance is to increase P_m which, therefore, increases the transient stability limit of a system.

The quicker a breaker operates, the faster the fault is removed from the system and the better is the tendency of the system to restore to normal operating conditions. The use of high speed breakers has materially improved the transient stability of the power systems and does not require any other methods for the purpose.

17.9 THE ROLE OF AUTOMATIC VOLTAGE REGULATOR (AVR) IN IMPROVING STABILITY

A voltage regulator is the heart of the excitation system. The output voltage of the generator changes only when the voltage regulator instructs the excitation system to do so irrespective of the speed of response of the exciter. A regulator senses changes in the output voltage and/or current and causes corrective action to take place. If the regulator is slow *i.e.*, has dead band or backlash or is otherwise insensitive, the system will be a poor one. By dead band of a regulator is meant the time elapsing between the voltage deviation and the return to the prescribed value due to slow action of the regulator. This is shown in Fig. 17.17.

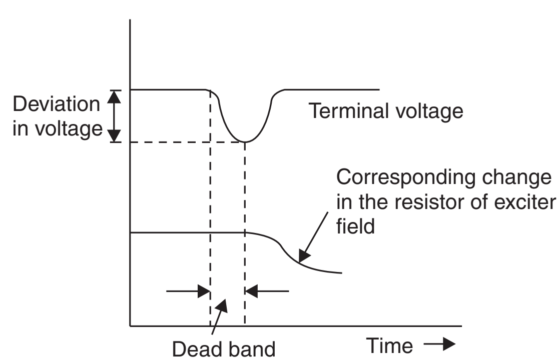


Fig. 17.17 The effect of dead band in a regulator.

Modern trends in the design of power system components have resulted in lower stability margins. The following factors contribute to this trend:

(*i*) Large generator units with lower inertia constant and higher p.u. reactances are being manufactured.

(*ii*) Large interconnected system operating practices with increased dependence on the transmission system to carry greater loading.

These trends have led to the increased dependence on the use of excitation control as a means of improving stability. This has prompted significant technological advances in excitation system.

In earlier systems the voltage regulator was entirely manual. The operator oberved the terminal voltage and adjusted the field rheostat (the voltage regulator) until the desired output conditions were obtained. The speed of this device is of great importance in studying stability. A typical excitation arrangement for the rotor of a synchronous generator is shown in Fig. 17.18.

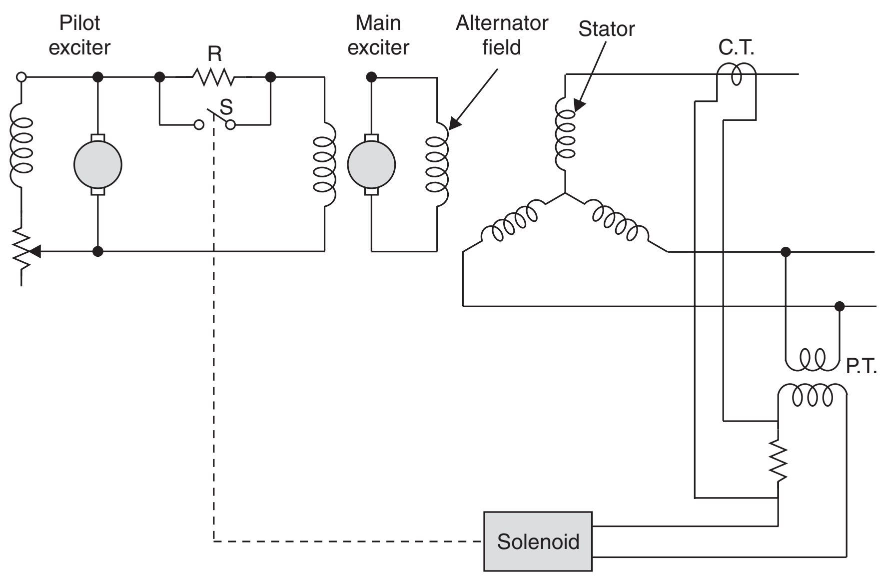


Fig. 17.18 A typical excitation arrangement for an alternator.

The armatures of both the pilot exciter and the main exciter are driven from the main rotor shaft. A fixed resistor R is connected in series with the main exciter field winding which can be short circuited as determined by a voltage sensing mechanism actuated by the terminal voltage and the currents of the alternator. The greater the time for which switch S is closed (R is short circuited) the higher will be the alternator output voltage. Because of the inherent high inductance of the generator field winding, it is difficult to make rapid changes in the field current. This introduces considerable time lag in the control function. One method of obtaining a fast response is to use such a design of the exciter which will give a relatively higher (higher than the one required under steady state condition) maximum voltage when R is short circuited. This voltage is known as the ceiling voltage. This voltage is normally two times the normal steady state voltage of the exciter. With such a design the changes in field current are obtained faster and hence faster control of alternator output voltage is obtained. It is to be noted that the use of pilot exciter along with main exciter has a much faster response than the self-excited main exciter alone since the exciter field control is independent of the exciter output voltage. Because the switch operator is electromechanical, the response may be slow compared to more modern systems.

In most modern systems the voltage regulator is a controller that senses the generator output voltage and/or current and initiates corrective action by changing the exciter control in the desired direction. A typical arrangement for such an excitation system is shown in Fig. 17.19.

The auxiliary control shown in Fig. 17.19 may include a comparator which may be used to set a lower limit on excitation, especially at leading p.f. operation, for prevention of instability due to very weak coupling across the air gap. Other auxiliary controls are sometimes desirable for damping to prevent overshoot, feedback of speed, frequency etc. Some of the amplification and comparison functions in modern regulators consist of solid state active devices. Various configurations have been used but all have generally fast operation with no appreciable time delay compared to other system time constants. The future of solid state voltage regulator is bright because of the inherent reliability, ease of maintenance and low initial cost of these devices. The modern regulators are known as continuously acting because of their faster operation and because they do not have a dead band.

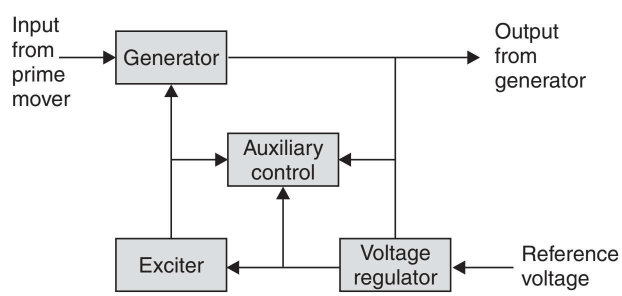


Fig. 17.19 A typical arrangement of excitation components.

The transient stability of a system can be improved if the excitation system has high speed of response and a high ceiling voltage with faster change in excitation and hence boost of internal machine flux; the electrical output of the machine may be increased during the first swing which reduces the accelerating power and results in improved transient performance. The modern excitation system is effective in two ways:

(*i*) It reduces the severity of machine swings when subjected to large impacts by reducing the magnitude of the first swing. (*ii*) It ensures that the subsequent swings are smaller than the first swing.

In the modern large interconnected power system the second effect is very important. There may be certain situations when various modes of oscillations reinforce each other during later swings, which alongwith the inherent weak system damping may cause transient instability after the first swing. With proper compensation a modern excitation system can be very effective in correcting this type of problem.

Generator and Automatic Voltage Regulator Characteristics

The equivalent circuit of a generator will be modified depending upon the action of regulator. Basically, there are three possibilities.

(*i*) Operation with constant excitation and hence constant no load voltage, *i.e.*, no regulator action. This operation of the alternator is represented by an equivalent circuit with synchronous impedance Z_s behind a constant voltage source E.

(*ii*) The other possibility is the use of a regulator which is acting non-continuously, *i.e.*, the terminal voltage varies with load. This is represented by a constant voltage E' behind a transient reactance.

(*iii*) The third possibility is that when the terminal voltage is held constant irrespective of load changes by using continuously acting automatic voltage regulator.

Each of these three possibilities would give different values of maximum power output which depends upon the speed of the automatic voltage regulator. The variation of maximum power is shown in Fig. 17.20 which indicates the possibility of increase in operating range of the synchronous generator. However, it should be noted that due to overheating of the stator winding it may not be possible to utilize the full leading p.f. increase in operating range. It is possible, now, with the use of fast acting automatic voltage regulator to retain in the synchronism of a generator which has passed through its steady state limiting angle of 90°. With the action of the voltage regulator the voltage is increased and hence the system works on a different power angle curve wherein the power instead of falling increases and after $\delta = 90°$ it is maintained and $dP/d\delta$ is still positive.

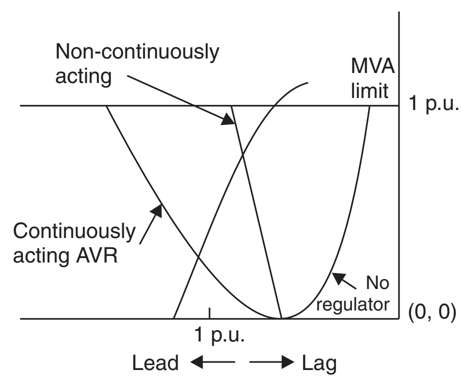


Fig. 17.20 Performance chart of generator with AVR.

17.10 THE EXCITATION SYSTEM

The first step in the sophistication of the primitive excitation system was the introduction of an amplifier in the feed back path which amplified the error signal and made the system fast acting. With the increase in size of the units and growth in the interconnection of the system, the excitation systems have become more and more complex.

With the advent of solid state technology and availability of reliable high current rectifiers a.c. exciters have been developed, the output of which is rectified to provide the d.c. current required by the alternator field through the slip rings. Since the control circuitry is also solid state in these cases, the overall response is quite fast. The excitation is controlled by electronically adjusting the firing angle of the silicon controlled rectifier. The firing angle can be controlled very quickly compared to the other time constants involved.

Another system that has been developed is unique in that it is brushless *i.e.*, it does not require slip rings since the alternator-exciter and diode rectifiers are rotating with the shaft.

With this the problems of cooling and maintenance associated with slip rings, commutators and brushes are thus avoided.

The excitation system consists of pilot exciter, main exciter and the SCRS. The main exciter is an a.c. generator, the armature and field windings of which are located in the stator. The stationary field of the main exciter is fed through a magnetic amplifier which controls and regulates the output voltage of the main alternator. The pilot exciter which is a permanent magnet alternator supplies excitation power to the magnetic amplifiers. The overall response of the system to controlling signals (voltage correction signals) is very fast.

High Frequency Excitation System: The response of the system with alternator rectifier exciters is improved by designing alternators for operation at frequencies higher than that of the main alternator. Recently frequencies of 400 to 500 Hz have been used and excellent response characteristics have been obtained.

The high frequency excitation system is derived from a.c. main exciter (500 Hz) a pilot exciter and a rectifier unit. Both the exciters are mounted on the main shaft of the alternator. The main exciter is an induction type alternator having three phase a.c. winding the field windings on the stator and it has no winding on its rotor. This increases the reliability of the system and reduces the maintenance problem. The pilot exciter is a permanent magnet type which has a rotating field and stationary armature and serves a source of stable supply to power magnetic amplifiers of automatic voltage regulators and manual excitation of main exciter. The rectifier unit is a three phase static convertor which converts the main exciter output and feeds the turbo-alternator field. A schemetic diagram of the system is shown in Fig. 17.21.

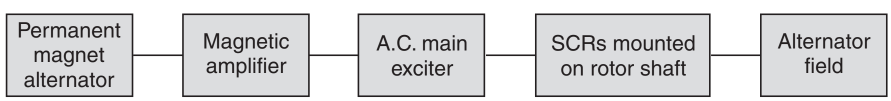


Fig. 17.21 Schematic diagram of high frequency excitation system.

The excitation of the alternator is regulated by varying the main exciter output voltage. The excitation of the main exciter is varied by means of the transistorized automatic voltage regulator having power magnetic amplifiers as its output stage.

Automatic voltage regulator consists of the following components:

(*i*) Power magnetic amplifiers.

(*ii*) Voltage corrector.

(*iii*) Bias circuit.

(*iv*) Feedback circuits.

(*v*) Maximum excitation limiter.

(*vi*) Minimum excitation limiter.

(*vii*) Matching circuit.

Power Magnetic Amplifiers. There are two types of magnetic amplifiers, *viz*., boosting and bucking amplifiers. Both the amplifiers are fed from the pilot exciter. The boosting amplifier provides additional excitation during variation of alternator load and in case of abnormal drop of alternator voltage. The bucking amplifier provides de-excitation when there is sudden drop of load and the generator voltage has a tendency to shoot up.

The amplification factor of the amplifiers is increased by providing internal feed back with the help of SCR connected in series with the a.c. series winding of the amplifier.

Voltage Corrector: It is a solid state device and provides the control current to the magnetic amplifiers. The control current is a function of the alternator terminal voltage and the load current of the alternator. The smaller the terminal voltage and larger the load current the alternator excitation should be increased to bring back the normal terminal voltage. The voltage input to the voltage corrector is the sum of the voltage (*i*) proportional to the terminal voltage and (*ii*) proportional to the load current to the alternator. The former is obtained through potential transformers and the latter across a resistor connected between the terminal of secondary of a current transformer.

Bias Circuit: The bias circuit provides the required shift in the characteristic of amplifiers. The bias circuit is fed from the auxiliary transformer through the SCRS. The current is varied with the help of a rheostat in the circuit.

Feedback Circuit: The feedback circuits (negative feedback) provide automatic regulation during static and dynamic conditions of the alternator.

Maximum Excitation Limiter: This is required to avoid overheating of the rotor of the alternator. This is achieved with the help of a non-linear transformer and a set of magnetic amplifiers. Whenever the output voltage of the main exciter increases, the secondary current of non-linear transformer sharply increases and magnetises a magnetic amplifier which, in turn, demagnetises the boosting magnetic amplifier and magnetises the bucking magnetic amplifier. This results in reduction of excitation current of main exciter, thus limiting the output voltage of the main exciter. Under normal operating conditions the secondary current of the non-linear transformer is negligibly small and can't actuate the magnetic amplifier.

Minimum Excitation Limiter: The excitation of the alternator should not go below a certain minimum value due to stability reasons. The minimum excitation limiter operates the series connected control windings of both the boosting and bucking magnetic amplifiers. The control currents are proportional to main alternator terminal voltage and the alternator load currents. These are fed to the minimum excitation limiter through the potential transformer and the current transformers.

Matching Circuits: The matching circuit consists of two magnetic amplifiers alongwith their bias, feed back and control windings. Whenever there is some problem with the automatic voltage regulator circuit, the smooth change over of the excitation control from AVR to manual is achieved with the help of matching circuit.

17.11 EFFECT OF GROUNDING ON STABILITY

The line-to-ground fault is the most common fault and the magnitude of fault current depends upon the zero, positive and negative sequence impedances. Of these impedances, positive and negative sequence impedances are practically constant. The zero-sequence impedance depends upon the impedance of the grounding resistor or reactor, number of grounding points, the zero-sequence impedance of the lines and of the tower-footings. Normally a *L-G* fault is less severe as compared to other faults due to high value of zero-sequence impedance. The effect of a double line-to-ground fault approaches that of a line-to-line fault and a line-to-ground fault approaches no fault when zero-sequence impedance is appreciably large. Therefore, from stability point of view, the impedance grounding is better than solid grounding especially during ground faults. Either resistors or reactors are used to decrease the severity of ground faults. Reactors are more commonly used as they are cheaper than the equivalent values of resistors.

However, it is to be noted that at some locations resistors are more useful as compared to reactors from stability point of view. Consider the system of Fig. 17.22. In case of a *L-G* fault on the system, the output of the generator is reduced and hence the input to the motor is decreased. For the generator $P_s > P_e$, therefore, the generator accelerates and for the motor the mechanical output is more than the electrical input, hence the motor decelerates. If the fault is closer to the generator, the acceleration of generator is more; similarly if the fault is closer to the motor, the input to the motor is affected more and hence the motor deceleration is more. A resistor in the generator neutral will increase the output of the generator during *L-G* fault and hence the acceleration is reduced. The grounding resistor consumes power during a ground fault and thus exerts braking effect on the synchronous machine which is greater, the closer the fault is to the resistor and the closer the machine is to the fault. A grounding resistor located near a generator is, therefore, beneficial. However, a grounding resistor should neither be used near an actual or equivalent synchronous motor nor it should be used near a synchronous condenser, as such machines already are retarded by faults. In a two-machine system, it is, therefore, advisable to have resistance grounding at the sending end and reactance grounding at the receiving end. it is to be noted that during 3-phase faults neither resistance grounding, nor reactance grounding have any effect on power transfer and hence on the stability of the system.

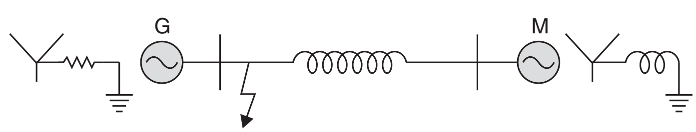


Fig. 17.22 Synchronous generator and a motor connected through a line—*L-G* fault.

17.12 PREVENTION OF STEADY STATE PULL OUT

A synchronous machine when over-excited delivers lagging reactive volt amperes to the system whereas when under-excited it draws lagging reactive volt amperes from the system. The upper excitation limit is governed by the thermal heating of the rotor whereas the lower excitation limit is governed by the stability of the system.

The power transferred between two buses is given by

$$P = \frac{E_1E_2}{X}\sin\delta$$

where E_1 and E_2 are the induced voltages of the two machines and X is the total reactance between the machines. If E_2 is the voltage of the infinite bus which is fixed in magnitude and phase, the lower the excitation

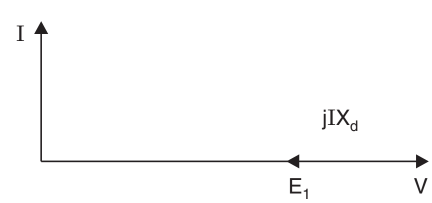


Fig. 17.23 Phasor diagram for under-excited case and P = 0.

of machine 1 the lower will be E_1 and hence the stability limit will be lowered. If the system voltage rises due to large charging current, in order to maintain constant terminal voltage V, machine 1 should be under-excited; thereby the induced voltage E_1 is lowered (Fig. 17.23).

It is to be noted that for a given terminal voltage V and the power P to be transferred, there is some value of Q below which the machine will go into unstable operation. This means that the P and Q delivered from the generator terminals to the system must not go beyond certain predetermined limits. Therefore, there must be some practical method of finding out this lower limit of excitation and giving a signal to the excitation system so that it does not go into unstable region of operation.

In this section an expression for minimum excitation limit (MEL) is derived and some controls have been suggested to hold the excitation system to this limit.

The problem is defined as follows:

Given the terminal voltage V as constant (Fig. 17.24), determine the relationship between P, Q, V and δ allowing E_1 and E_2 to swing. Since we are interested in the pull out condition, δ will be set to 90°.

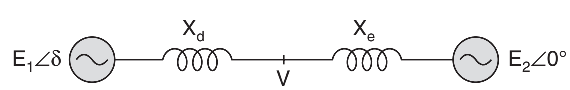


Fig. 17.24 Finite machine connected to an infinite bus through X_e.

From Fig. 17.25,

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan\alpha = \frac{I_p X_d}{V - I_q X_d} \quad \text{and} \quad \tan\beta = \frac{I_p X_e}{V + I_q X_e}$$

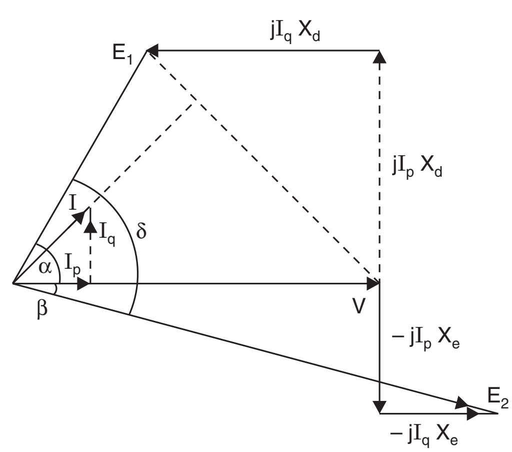


Fig. 17.25 Phasor diagram for system in Fig. 17.24.

Since $(\alpha + \beta) = \delta = 90°$ for extreme pull out condition,

$$\tan(\alpha+\beta) = \tan 90° = \infty$$

or
$$1 - \tan\alpha\tan\beta = 0$$

Substituting values of tan α and tan β, we have

$$1 - \frac{I_p X_d}{V - I_q X_d} \cdot \frac{I_p X_e}{V + I_q X_e} = 0$$

or
$$1 = \frac{I_p^2 X_e X_d}{V^2 + VI_q X_e - VI_q X_d - I_q^2 X_d X_e}$$

Let $P = VI_p$ and $Q = -I_q V$.

Here Q is taken negative as the machine is operating under leading power factor.

Substituting values of P and Q , we have

$$1 = \frac{(P^2 / V^2) X_e X_d}{V^2 + Q(X_d - X_e) - (Q^2 / V^2) X_d X_e}$$

or
$$V^2 + Q(X_d - X_e) - \frac{Q^2}{V^2} X_d X_e - \frac{P^2}{V^2} X_e X_d = 0$$

Multiplying throughout by $V^2/(X_d X_e)$, we get

$$\frac{V^4}{X_d X_e} + V^2 Q \frac{X_d - X_e}{X_e X_d} - Q^2 - P^2 = 0$$

or
$$P^2 + Q^2 - QV^2 \frac{(X_d - X_e)}{X_e X_d} + \frac{V^4}{4}\left(\frac{X_d - X_e}{X_d X_e}\right)^2 = \frac{V^4}{X_d X_e} + \frac{V^4}{4}\left(\frac{X_d - X_e}{X_d X_e}\right)^2$$

or
$$P^2 + \left\{Q - \frac{V^2}{2}\frac{(X_d - X_e)}{X_d X_e}\right\}^2 = \left\{V^2\left(\frac{X_d + X_e}{2X_d X_e}\right)\right\}^2$$

This is an equation to a circle with radius

$$r = V^2 \frac{X_d + X_e}{2X_d X_e} = \frac{V^2}{2}\left(\frac{1}{X_e} + \frac{1}{X_d}\right)$$

and centre at
$$P_c = 0,\ Q_c = \frac{V^2}{2}\frac{X_d - X_e}{X_e X_d} = \frac{V^2}{2}\left[\frac{1}{X_e} - \frac{1}{X_d}\right]$$

For a given power P, the vertical component of E_1 (Fig. 17.25) is constant and equals $I_p X_d$. When $\delta = 90°$, E_1 will be at a minimum and the machine cannot be excited below this level. Therefore, the operation of the machine should be within the pull out circle as shown in Fig. 17.26. The pull out curve gives the relation between power P and the corresponding reactive power (minimum) Q. If the value of Q for a particular P (and hence excitation) is less than as indicated in the curve, the system will be unstable. It is seen that corresponding to $P = 0$ the reactive power (minimum) is V^2/X_d which corresponds to the short circuit reactive volt-amperes of the generator. The dotted curve in Fig. 17.26 which lies above the pull out curve is referred to as the minimum excitation limit curve (MEL). An automatic excitation limit control can be provided which will prevent the machine from going below this dotted curve. With any attempt on the part of the machine to go below MEL curve, the limit control would override the terminal voltage control and allows increased excitation; thereby the machine remains in stable region of operation. Since the MEL curve is above the pull out curve, it provides extra margin for stability. This margin depends upon the requirements of a particular utility but typical values may vary between 10 to 20% of the radius of the circle shown in Fig. 17.26. Another curve known as a loss of field relay curve may be placed in between the pull out curve and the MEL curve. If, for any reason, the excitation level goes below this level, the relay will give signal to either trip a machine or sound an alarm etc.

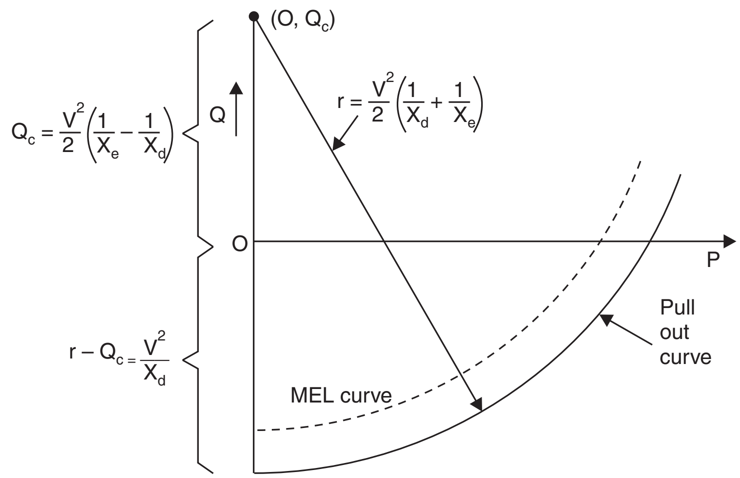


Fig. 17.26 Pullout curve of *Q* vs *P* for a synchronous generator.

Example 17.7: A synchronous generator is connected to an infinite bus through a tie-line. The reactances of generator and the tie line are 1.1 p.u. and 0.4 p.u. respectively both on 100 MVA base. Assume generator terminal voltage of 0.98 p.u. Determine the centre and radius for the pull out curve and also the minimum permissible output vars when the output powers are (*i*) 0.0 (*ii*) 0.25 p.u. and 0.5 p.u.

Solution: The coordinates of the centre are

$$P_c = 0,\ Q_c = \frac{V^2}{2}\left[\frac{1}{0.4} - \frac{1}{1.1}\right] = \frac{0.98^2}{2}\left[\frac{0.7}{0.44}\right]$$

$$= 0.7639 \text{ p.u.}$$

Coordinates of centre (0.0, 0.7639)

$$\text{Radius of circle} = \frac{V^2}{2}\left[\frac{1}{X_d} + \frac{1}{X_e}\right]$$

$$= \frac{0.98^2}{2}\left[\frac{1}{1.1} + \frac{1}{0.4}\right]$$

$$= \frac{0.98^2}{2} \times \frac{1.5}{0.44}$$

$$= 1.637 \text{ p.u.}$$

(*i*) Substituting the values in equation to the circle, we have

$$0.0 + \left\{Q - \frac{0.98^2}{2}\left(\frac{1.1 - 0.4}{0.44}\right)\right\}^2 = \left\{0.98^2\left(\frac{1.5}{2 \times 0.44}\right)\right\}^2$$

or $$Q = 0.7639 = \pm 1.637$$

or $$Q = 2.4, -0.8731 \text{ p.u.}$$

Considering only the –ve sign for minimum excitation, the reactive power is 87.31 MVAr.

(*ii*) $P = 0.25$ p.u. Using same equation

$$0.25^2 + (Q - 0.7639)^2 = (1.637)^2$$

$$(Q - 0.7639)^2 = (1.637)^2 - (0.25)^2 = 1.887 \times 1.387$$

or $$Q - 0.7639 = \pm 1.6178$$

$$Q = -0.8539 \text{ p.u.} \quad \textbf{Ans.}$$

(*iii*) 0.5 p.u.

$$(Q - 0.7639)^2 = (1.637)^2 - (0.5)^2 = 2.137 \times 1.137$$

$$= 2.429769$$

or $$Q - 0.7639 = \pm 1.55877$$

or $$Q = -0.7948 \text{ p.u.} \quad \textbf{Ans.}$$

17.13 MULTI-MACHINE STABILITY—CLASSICAL MODEL

So far we have considered the transient stability study of a finite machine, connected to an infinite bus without a load connected to either a finite machine or an infinite bus. In an actual power system, there will be large number of both load and generator buses. There are two methods of representing load buses—(1) Represent them as they are taken in load flow studies. Here the Y-bus matrix will be exactly identical to the load flow case. (2) The loads are represented by constant admittance. Therefore, the presence of loads is taken care of by the bus admittance matrix and this matrix is different from the one calculated for load flow studies. Since we are interested in the synchronous stability of the system, we need not bother about the load bus current, and our objective is to calculate generator currents only in order to find out the prefault or pretransient induced e.m.f. behind the transient reactance of individual machines. These voltages are required to find out the power transfer between the machines.

The load currents, thus, having been accounted for by the constant admittances, the bus currents consists of the generator currents and the general network equations are

$$\begin{bmatrix} I_G \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{AA} & Y_{AB} \\ Y_{BA} & Y_{BB} \end{bmatrix} \begin{bmatrix} E_G \\ V \end{bmatrix} \quad (17.47)$$

where $$I_G = [I_1, I_2, ..., I_n]^T \quad (17.48)$$

generator current vector ($n \times 1$), 0 is an $m \times 1$ null current vector corresponding to load buses which have been represented by constant admittances and hence the injected current is zero for each load bus.

$$E_G = [E_1, E_2, ..., E_n]^T = \text{Generator internal source voltage column vector } (n \times 1)$$

$$V = [V_{n+1}, V_{n+2}, ..., V_{n+m}]^T = \text{Internal bus voltage column vector } (m \times 1)$$

and Y is an $(n + m) \times (n + m)$ matrix such that Y_{AA} is $n \times n$, Y_{AB} is $n \times m$, Y_{BA} is $m \times n$ and Y_{BB} is $(m \times m)$ matrix.

As is said earlier we are interested in generator currents only, our objective would be to eliminate load nodes. From equation (17.47)

$$[0] = [Y_{BA}]\ [E_G] + [Y_{BB}]\ [V] \tag{17.49}$$

or

$$V = -\ [Y_{BB}]^{-1}\ [Y_{BA}]\ [E_G] \tag{17.50}$$

Also, from equation (17.47)

$$[I_G] = [Y_{AA}]\ [E_G] + [Y_{AB}]\ [V] \tag{17.51}$$

Substituting for $[V]$, from equation (17.50), we have

$$\begin{aligned}[I_G] &= [Y_{AA}][E_G] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}][E_G]\\ &= \{[Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}]\}[E_G]\\ &= [Y][E_G]\\ &= n \times n \text{ reduced admittance matrix.}\end{aligned} \tag{17.52}$$

Example 17.8: A generator is connected to an infinite bus through a double circuit line as shown in Fig. E. 17.8.

Compute the prefault, faulted and post-fault reduced Y matrices. Suppose a three phase fault takes place at the mid-point of one feeder and the breakers operate after certain time.

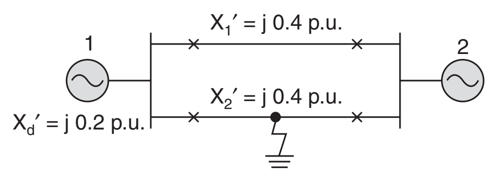
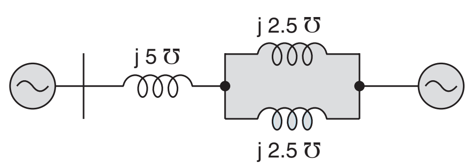


Fig. E.17.8

Solution: The equivalent circuit (pre-fault) is given below:



The full Y-matrix is given as (by inspection)

$$\left[\begin{array}{cc:c} -j5 & 0 & j5.0 \\ 0 & -j5 & j5.0 \\ \hline j5.0 & j5.0 & -j10 \end{array}\right]$$

Here $\quad [Y_{AA}] = \begin{bmatrix} -j5 & 0 \\ 0 & -j5 \end{bmatrix},\ [Y_{AB}] = \begin{bmatrix} j5.0 \\ j5.0 \end{bmatrix}\ [Y_{BA}] = [j5.0, j5.0]$

and $\quad [Y_{BB}] = [j10]$

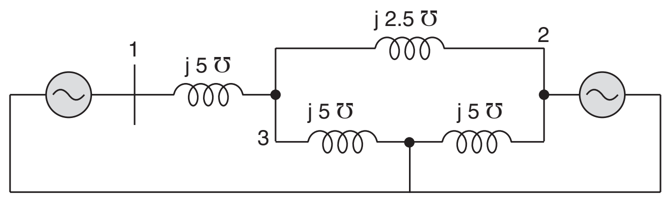
Now $\quad [Y] = [Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}]$

$$= \begin{bmatrix} -j5 & 0 \\ 0 & -j5 \end{bmatrix} - \begin{bmatrix} j5.0 \\ j5.0 \end{bmatrix}\left[\frac{1}{+j10}\right][j5.0 \quad j5.0]$$

$$= \begin{bmatrix} -j5 & 0 \\ 0 & -j5 \end{bmatrix} - \frac{j1}{+10}\begin{bmatrix} -25 & -25 \\ -25 & -25 \end{bmatrix}$$

$$= \begin{bmatrix} -j5 & 0 \\ 0 & -j5 \end{bmatrix} + \frac{1}{10}\begin{bmatrix} j25 & j25 \\ j25 & j25 \end{bmatrix} = \begin{bmatrix} -j2.5 & j2.5 \\ j2.5 & -j2.5 \end{bmatrix}$$

Faulted Case. The equivalent circuit is given as follows:



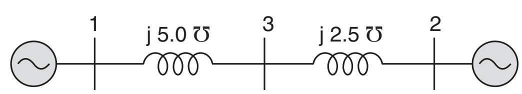
The full matrix (admittance) is given as

$$Y_{\text{full}} = \begin{bmatrix} -j5 & 0 & j5 \\ 0 & -j7.5 & j2.5 \\ j5 & j2.5 & -j12.5 \end{bmatrix}$$

Following the procedure as outlined for prefault, the reduced matrix (admittance) can be shown to be:

$$Y = \begin{bmatrix} -j3 & j1 \\ j1 & -j7 \end{bmatrix}$$

Post-fault: The equivalent circuit under this condition is given as



The full admittance matrix is given as

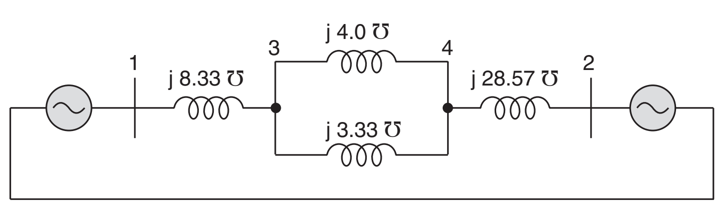
$$\begin{bmatrix} -j5.0 & 0 & j5.0 \\ 0 & -j2.5 & j2.5 \\ j5.0 & j2.5 & -j7.5 \end{bmatrix}$$

and the reduced admittance matrix

$$[Y] = \begin{bmatrix} -j1.667 & j1.667 \\ j1.667 & -j1.667 \end{bmatrix} \quad \textbf{Ans.}$$

Example 17.9: For the system of example 17.9 determine the reduced admittance matrices for prefault, fault and post-fault conditions and hence determine the power-angle characteristics for the three conditions.

Solution: The network under prefault condition is given as follows:



The full admittance matrix

$$\left[\begin{array}{cc:cc} -j8.33 & 0.0 & j8.33 & 0.0 \\ 0.0 & -j28.57 & 0.0 & j28.57 \\ \hline j8.33 & 0.0 & -j15.67 & j7.33 \\ 0.0 & j28.57 & j7.33 & -j35.9 \end{array}\right]$$

Since there are two generators in the system, the reduced matrix will be of order 2 × 2. Therefore, the original matrix has been partitioned as shown above. The reduced admittance matrix

$$[Y] = [Y_{AA}] - [Y_{AB}][Y_{BB}]^{-1}[Y_{BA}]$$

$$[Y_{BB}] = \begin{bmatrix} -j15.67 & j7.33 \\ j7.33 & -j35.9 \end{bmatrix}$$

Its determinant = – 15.67 × 35.9 + 7.33 × 7.33 = – 508.8

$$[Y_{BB}]^{-1} = -\frac{1}{508.8}\begin{bmatrix} -j35.9 & -j7.33 \\ -j7.33 & -j15.67 \end{bmatrix}$$

$$= \begin{bmatrix} j0.07055 & j0.0144 \\ j0.0144 & j0.03079 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j28.57 \end{bmatrix} - \begin{bmatrix} j8.33 & 0.0 \\ 0.0 & j28.57 \end{bmatrix}$$

$$\begin{bmatrix} j0.07055 & j0.0144 \\ j0.0144 & j0.03079 \end{bmatrix}\begin{bmatrix} j8.33 & 0.0 \\ 0.0 & j28.57 \end{bmatrix}$$

$$= \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j28.57 \end{bmatrix} - \begin{bmatrix} j8.33 & 0.0 \\ 0.0 & j28.57 \end{bmatrix}$$

$$\begin{bmatrix} -0.58768 & -0.41408 \\ -0.119952 & -0.87967 \end{bmatrix}$$

$$= \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j20.57 \end{bmatrix} - \begin{bmatrix} -j4.895 & -j3.427 \\ -j3.427 & -j25.13 \end{bmatrix}$$

$$Y = \begin{bmatrix} -j3.435 & j3.427 \\ j3.427 & -j3.44 \end{bmatrix}$$

Now power transmitted between node 1 and 2 from 1 to 2 is given by

$$P_e = E_1 E_2 Y_{12} \sin\delta$$
$$= 1.1 \times 1.0 \times 3.427 \sin\delta$$
$$= 3.7697 \sin\delta$$

During fault condition: Since there is a 3φ fault on bus 3, therefore, voltage of bus 3 reduces to zero and hence elements corresponding to 3rd row and column will be absent. The full matrix during fault will be

$$\left[\begin{array}{cc:c} -j8.33 & 0.0 & 0.0 \\ 0.0 & -j28.57 & j28.57 \\ \hline 0.0 & j28.57 & -j35.9 \end{array}\right]$$

The reduced admittance matrix

$$Y = \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j28.57 \end{bmatrix} - \begin{bmatrix} 0.0 \\ j28.57 \end{bmatrix} \frac{1}{-j35.9} [0.0 \quad j28.57]$$

$$= \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j28.57 \end{bmatrix} - \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & -j22.73 \end{bmatrix}$$

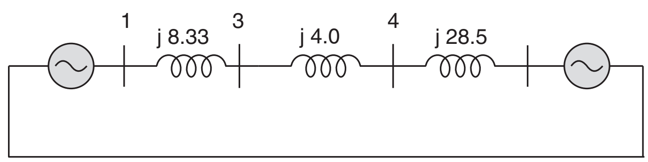
$$= \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j5.83 \end{bmatrix}$$

Hence the power angle characteristic will be

$$P_e = E_1 E_2 Y_{12} \sin \delta$$

$$= 1.1 \times 1.0 \times 0.0 \sin \delta = 0.0$$

Post fault condition: After the fault is cleared, the network reduces to



The full matrix will be

$$\left[\begin{array}{cc:cc} -j8.33 & 0.0 & j8.33 & 0.0 \\ 0.0 & -j28.57 & 0.0 & j28.57 \\ \hline j8.33 & 0.0 & -j12.33 & j4.0 \\ 0.0 & j28.57 & j4.0 & -j32.57 \end{array}\right]$$

Here $$Y_{BB} = \begin{bmatrix} -j12.33 & j4.0 \\ j4.0 & -j32.57 \end{bmatrix}$$

$$Y_{BB}^{-1} = \frac{1}{-385.58} \begin{bmatrix} -j32.57 & -j4.0 \\ -j4.0 & -j12.33 \end{bmatrix}$$

$$= \begin{bmatrix} j0.08447 & j0.010374 \\ j0.010374 & j0.031977 \end{bmatrix}$$

$$Y = \begin{bmatrix} -j8.33 & 0.0 \\ 0.0 & -j28.57 \end{bmatrix} - \begin{bmatrix} j8.33 & 0.0 \\ 0.0 & j28.57 \end{bmatrix}$$

$$\begin{bmatrix} j0.08447 & j0.010374 \\ j0.010374 & j0.031977 \end{bmatrix} \begin{bmatrix} j8.33 & 0.0 \\ 0.0 & j28.57 \end{bmatrix}$$

After simplification it can be seen that the element

$$Y_{12} = j2.4679$$

$\therefore$ The power-angle characteristic for post fault condition

$$P_e = E_1 E_2 Y_{12} \sin \delta$$

$$= 1.1 \times 1.0 \times 2.4679 \sin \delta$$

$$= 2.7147 \sin \delta \quad \textbf{Ans.}$$

In typical transient stability studies, depending upon the state of the system, [Y] will have different entries. The states are, pre-fault, faulted and post-fault. The prefault state involves load flow study and determination of E_G^S alongwith angles δ^s. The faulted state exists at $t = 0$ and persists until the fault is cleared at $t = \tau_c$. For $t = \tau_c$, the post fault [Y] is used. Under the three conditions, since the configuration of the system changes, the entries of [Y] matrix will be different in each case. However, it is to be noted that the Y matrix defined in equation (17.52) includes the generator transient impedance X_d' and the load impedances.

Let $Y = Y_{pq} = G_{pq} - jB_{pq} = Y_{pq}\angle -\theta_{pq}$

and
$$E_p = |E_p| \angle \delta_p$$

The electrical power injected at generator node p

$$P_p = \text{Re}\,[E_p^* I_p]$$
$$= \text{Re}\left\{|E_p| \angle -\delta_p \sum_{q=1}^{n} Y_{pq} |E_q| \angle \delta_q\right\}$$
$$= \sum_{q=1}^{n} |E_p||E_q||Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \tag{17.53}$$
$$p = 1, 2, ..., n$$

From load flow solution, the terminal voltage alongwith its angle, the active and reactive power supplied by the generator are known. For pth node (generator) let these quantities by P_P^0 and Q_P^0. Here superscript 0 represents time $t = 0$ *i.e.*, initial operating condition.

If I_{pp} and I_{pq} are the inphase and quadrature component at bus p, then

$$I_{pp} = \frac{P_p^0}{V_p^0} \quad \text{and} \quad I_{pq} = \frac{Q_p^0}{V_p^0}$$

Let us assume $V_p{}^0$ as the reference, then

$$E_p \angle \delta_p' = \left(V_p^0 + \frac{Q_p^0 X'_{dp}}{V_p^0}\right) + j\,\frac{P_p^0 X_{dp}}{V_p^0} \tag{17.54}$$

∴ The initial generator angle δ_p^0 is obtained by adding the pre-transient voltage angle β_p^0 to δ_p' *i.e.*,

$$\delta_p^0 = \delta_p' + \beta_p \tag{17.55}$$

Here x'_{dp} is the transient reactance of the pth generator.

Similarly if V_i is the load bus voltage of ith node as obtained from load flow solution and if P_i and Q_i are the active and reactive powers of the load, then

$$G_i = \frac{P_i}{V_i^2} \quad \text{and} \quad B_i = \frac{Q_i}{V_i^2}$$

So far we have discussed preliminary preparation one has to make before studying the multi-machine stability of power system.

The assumptions used for two machine system are also assumed valid for a multi-machine system.

1. Mechanical power input to each generator is constant.
2. Saliency of generators is neglected and the transient generator model will be chosen.
3. The e.m.f. behind transient reactance will remain constant throughout the post fault period.
4. All resistances and damping torques are neglected.
5. Loads are represented by constant passive admittances.
6. The variation of frequency is considered to be negligible and hence, the change in load with frequency is neglected.

The swing equation for the pth generator is given by

$$\frac{d^2\delta_p}{dt^2} = (P_s - P_p)/M_p$$

$$= \frac{1}{M_p}\left[P_s - \sum_{q=1}^{n} E_p E_q Y_{pq} \cos\left(\delta_{pq} + \delta_p - \delta_q\right)\right] \quad (17.56)$$

The set of equations (17.56) is a set of n-coupled non-linear second order differential equations. We introduce here state variables to convert each second order swing equation by two coupled first order state differential equation.

Let $x_1 \triangleq = \delta$ (rotor angular position in electrical radian)

$x_2 \triangleq = \dot{\delta} = \frac{d\delta}{dt}$ (rotor angular velocity in electrical radian per sec.)

These state variables form the components of the state vector.

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix}$$

$$\therefore \quad \dot{x}_1 = x_2 = \frac{d\delta}{dt} = \omega - 2\pi f \quad (17.57)$$

$$\dot{x}_2 = \frac{d^2\delta}{dt^2} = \frac{1}{M}\left[P_s - P_p\right] = \omega \quad (17.58)$$

Here state vector $X^T = [\omega_1, \delta_1, \omega_2, \delta_2, ..., \omega_n, \delta_n]$ is a vector of dimension $(2n \times 1)$ where n is the number of generators. The generator power P_p is a function of $\delta(= x_1)$,

$$\therefore \quad \dot{x}_1 = f_1(x_1, x_2)$$

$$\dot{x}_2 = f_2(x_1, x_2)$$

or in vector form $x = f(x)$.

The swing equation expressed in terms of state equations can be solved by (*i*) Modified Euler Method (*ii*) Runge-Kutta method and many other methods. We will study here only the two methods mentioned above. Before we apply these two techniques for the solution of swing equation, we study here their mathematical aspects.

Modified Euler Method: The objective is to solve the equations $x = f(x)$

The independent variable is discretised into time elements t^0, t^1, ..., t^k preferably equidistant.

1. Assume an initial solution X^0 and set the time count $k = 0$.
2. Compute the derivative $\dot{X}^k = f(X^k)$
3. Compute first estimate of the state vector

$$X^{k \pm 1} = X^k + \dot{X}^k \, \Delta t$$

4. Compute second estimate of the state derivative

$$\dot{X}^{k+1} = f\,(X^{k+1})$$

5. Compute the average of the state derivatives

$$\dot{X}^k \, av = \frac{1}{2}\left(\dot{X}^k + \dot{X}^{k+1}\right)$$

6. Evaluate the second and first estimate of the state vector.

$$X^{k+1} = X^k + \dot{X}^k \, av \; \Delta t$$

Print the X^{k+1}.

7. Check if $t > t$ max. If not, increment time $t = t + \Delta t$ *i.e.,* $K = K + 1$ and go to step 2.
8. Terminate the computation.

Fig. 17.27 shows the various steps involved.

Runge-Kutta Method: Consider two first order differential equations in two variables X_1 and X_2 such that

$$\dot{X}_1 = f_1\,(x_1\, x_2)$$
$$\dot{X}_2 = f_2\,(x_1\, x_2)$$

1. Assume a solution x_1^0, x_2^0 and set time count $K = 0$
2. Compute the following eight constants:

$$K_1^k = f_1\,(x_1^k, x_2^k)\,\Delta t$$
$$l_1^k = f_2\,(x_1^k, x_2^k)\,\Delta t$$
$$K_2^k = f_1\left(x_1^k + \frac{1}{2}K_1^k, x_2^k + \frac{1}{2}\,l_1^k\right)\Delta t$$
$$l_2^k = f_2\left(x_1^k + \frac{1}{2}K_1^k, x_2^k + \frac{1}{2}\,l_1^k\right)\Delta t$$
$$K_3^k = f_1\left(x_1^k + \frac{1}{2}K_2^k, x_2^k + \frac{1}{2}\,l_2^k\right)\Delta t$$
$$l_3^k = f_2\left(x_1^k + \frac{1}{2}K_2^k, x_2^k + \frac{1}{2}\,l_2^k\right)\Delta t$$
$$K_4^k = f_1\left(x_1^k + K_3^k\,, x_2^k + \,l_3^k\right)\Delta t$$
$$l_4^k = f_2\left(x_1^k + K_3^k\,, x_2^k + \,l_3^k\right)\Delta t \qquad (17.59)$$

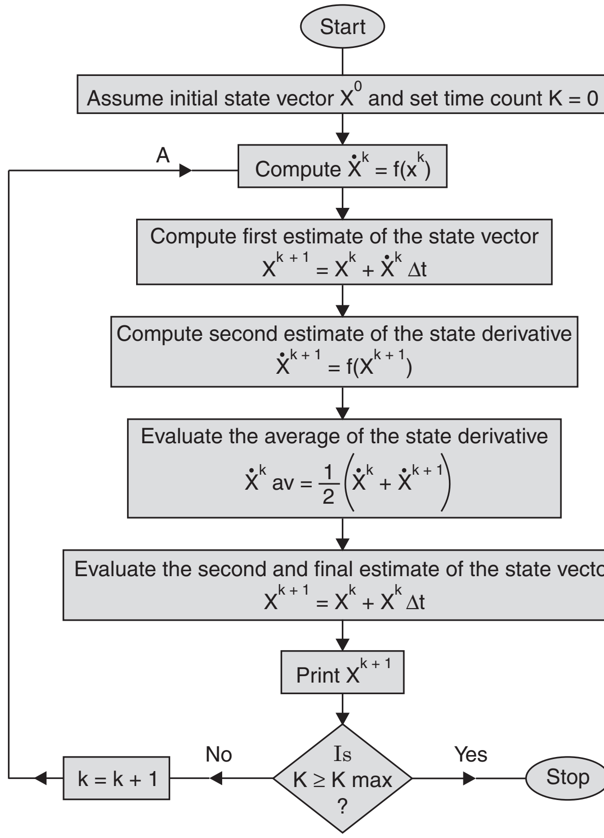


Fig. 17.27 Modified Euler's method.

3. Compute the change in state vector

$$\Delta X_1^k = \frac{1}{6}\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)$$

$$\Delta X_2^k = \frac{1}{6}\left(l_1^k + 2l_2^k + 2l_3^k + l_4^k\right)$$

4. Evaluate the new state vector

$$X_1{}^{k+1} = X_1^k + \Delta K_1^k$$

$$X_2{}^{k+1} = X_2^k + \Delta K_2^k$$

and print the state vector.

5. Advance the time count $K = K + 1$ and check if $K > K$ max. If not go to step 2.
6. Terminate computation.

Runge-Kutta method applied to solution of swing equations for a multi-machine system:

1. Obtain a load flow solution for the pretransient conditions.
2. Calculate the generator internal voltages behind transient reactances using equations (17.54) and (17.55). The state vector δ^s have finite values but $\omega^s = 0$ under pretransient condition.

3. Assume the occurrence of a fault and calculate the reduced admittance matrix for this condition and initialize time count $K = 0$, initialise $j = 0$.
4. Determine the eight constants using equations (17.56) and (17.57) *e.g.*,

$$K_1^k = f_1\,(\delta^k,\ \omega^k)\ \Delta t$$

$$l_1^k = f_2\,(\delta^k,\ \omega^k)\ \Delta t$$

$$K_2^k = f_1\left(\delta^k + \frac{1}{2}K_1^k,\ \omega_1^k + \frac{1}{2}l_1^k\right)\Delta t$$

and so on.

5. Compute the change in state vector.

$$\Delta\delta^k = \frac{1}{6}\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)$$

and

$$\Delta\omega^k = \frac{1}{6}\left(K_1^k + 2l_2^k + 2l_3^k + l_4^k\right)$$

6. Evaluate the new state vector

$$\delta^{k+1} = \delta^k + \Delta\delta^k$$

$\omega^{k+1} = \omega^k + \Delta\omega^k$ and print the state vector.

7. Evaluate the internal voltage behind transient reactance using the relation

$$E_p^{\,k+1} = \left|\,E_p^{\,k}\,\right|\cos\delta_p^{\,k+1} + j\left|\,E_p^{\,k}\,\right|\sin\delta_p^{\,k+1}$$

8. Check if $t < t_c$ yes, $K = K + 1$ and go to step 4.
9. Check if $j = 0$, yes, modify the network data and obtain a new reduced admittance matrix corresponding to post-fault condition. Set $j = j + 1$.
10. Set $K = K + 1$
11. Check if $K < K$ max, yes, go to step 4.
12. Terminate the process.

Computational Algorithm using Modified Euler Method for Power System Problems (Fig. 17.28).

1. Obtain a load flow solution for the pretransient condition.
2. Calculate the generator internal voltages behind transient reactances using equations (17.54) and (17.55). The state vectors have finite values whereas all $\omega^s = 0$ under pre-transient condition.
3. Assume the occurrence of fault and initialize time $K = 0$. Calculate the reduced admittance matrix for this condition. Set Count $J = 0$.
4. Determine the state derivatives using equations (17.56) and (17.57) and calculate the first state estimate.

$$\delta_p^{\,k+1} = \delta_p^{\,k} + \left.\frac{d\delta_p}{dt}\right|_{\dot{k}} \Delta t$$

$$\omega_p^{\,k+1} = \omega_p^{\,k} + \left.\frac{d\omega}{dt}\right|_{\dot{k}} \Delta t$$

5. Second estimate of the variables can be obtained if derivatives at $t = t + \Delta t$ are obtained. For this we must have new values of E^s at time $t = t + \Delta t$, so that the generated powers can be calculated.

$$E_p^{k+1} = \left| E_p \right|^k \cos \delta_p^{k+1} + j \left| E_p \right|^k \sin \delta_p^{k+1}$$

and use these values in equations (17.56) and (17.57) to obtain state derivatives.

6. Determine the average value of the state derivative and obtain the second estimate of state variables and the second estimate of the internal voltage angles and machine angular speeds.

$$\delta_p^{k+1} = \delta_p^k + \left[\frac{\dfrac{d\delta_p^k}{dt} + \dfrac{d\,\delta_p^{k+1}}{dt}}{2} \right] \Delta t$$

$$\omega_p^{k+1} = \omega_p^k + \left(\frac{d\,\omega_p^k/dt + \dfrac{d\,\omega_p^{k+1}}{dt}}{2} \right) \Delta t$$

7. Compute final internal voltages of the generators at the end of $(t + \Delta t)$.

$$E_p^{k+1} = \left| E_p^k \right| \cos \delta_p^{k+1} + j \left| E_p^k \right| \sin \delta_p^{k+1}$$

and print the result.

8. Check if $t < t_c$ (clearing time). If yes, advance time by Δt and go to step 4.
9. Check if $J = 0$, if yes, the nodal admittance matrix is changed corresponding to the post-fault condition and a new reduced admittance matrix is obtained.

   Set $j = j + 1$.
10. Set $K = K + 1$ and $t = t + \Delta t$
11. Check if $t \le t$ max, if yes go to step 4.
12. Terminate the process of computation.

The relation between δ and t for various generators, are thus, obtained and stability or otherwise of the system can be estimated for a particular type of fault and a particular clearing time.

Example 17.10: Consider a system having the following parameters:

$$P_m = 3.0 \text{ p.u.}, \; r_1 P_m = 1.2, \; r_2 P_m = 2.0$$
$$H = 3.0, \; f = 60 \text{ Hz}; \; \Delta t = 0.02 \text{ sec.}$$

$P_e = 1.5$ p.u. Determine the rotor angle and angular frequency at the end of 0.02 sec. using Runge-Kutta and Euler's modified method.

Solution: Runge-Kutta method

Initial angle $\delta_0 = \sin^{-1} \dfrac{1.5}{3.0} = 30° = 0.524$ rad and initial relative angular velocity

$$\omega_0 = \delta = 0.0$$

Now using the equations,

$$\frac{d\delta p}{dt} = \omega_P - 2\pi f = (\omega_P - 376.992)$$

$$\frac{d\omega p}{dt} = \frac{\pi f}{H}(P_s - P_e) = 62.83\ (1.5 - 1.20 \sin \delta)$$

and the relation given by equation (17.59)

$$K_1^0 = \omega_1(0) - 2\pi f = 0$$

$$l_1^0 = 62.83\ (1.5 - 1.2 \sin 0.524 \text{ rad})\ \Delta t = 1.13094$$

$$K_2^0 = (377.5574 - 376.992)\ 0.02 = 0.0113094$$

$$l_2^0 = 62.83\ (1.5 - 1.2 \sin 0.524)\ \Delta t = 1.13094$$

$$K_3^0 = 0.0113094$$

$$l_3^0 = 62.83\ (1.5 - 1.2 \sin 0.5296547) \times 0.02$$

$$= 1.123045$$

$$K_4^0 = 1.123045 * 0.02 = 0.02246$$

$$l_4^0 = 62.83\ (1.5 - 1.2 \sin 0.5353094) \times 0.02$$

$$= 62.83\ (1.5 - 0.6121286) * 0.02$$

$$= 1.115699$$

$$= \frac{1}{6}\ (0.0 + 4 \times 0.0113094 + 0.02246)$$

$$= 0.01128$$

$$\delta_1 = 0.53528$$

$$\Delta\omega = \frac{1}{6}\ (3 \times 1.13094 + 2 \times 1.123045 + 1.115699)$$

$$= \frac{1}{6}\ (3.39282 + 2.246090 + 1.115699)$$

$$= 1.125768$$

$$\therefore \qquad \omega_1 = \omega_0 + \Delta\omega_0 = 0.0 + 1.125768$$

$$= 1.125768$$

and $\delta_1 = 0.53528$ **Ans.**

Now suppose the breakers operate at time 0.15 sec. and typical values of the rotor angle and angular velocity at the end of 0.14 sec., are 1.026 radians and 6.501 rad/sec., determine the two quantities at the end of 1.6 sec., using Runge-Kutta method:

$$\delta^7 = 1.026 \text{ rad } \omega^7 = 6.501 \text{ rad/sec.}$$

The equations to be solved are

$$\dot{\delta}_p = (\dot{\omega}_p(t) - 2\pi f)$$

and

$$\dot{\omega}_p = \frac{\pi f}{H}\ (1.5 - 1.2 \sin \delta)$$

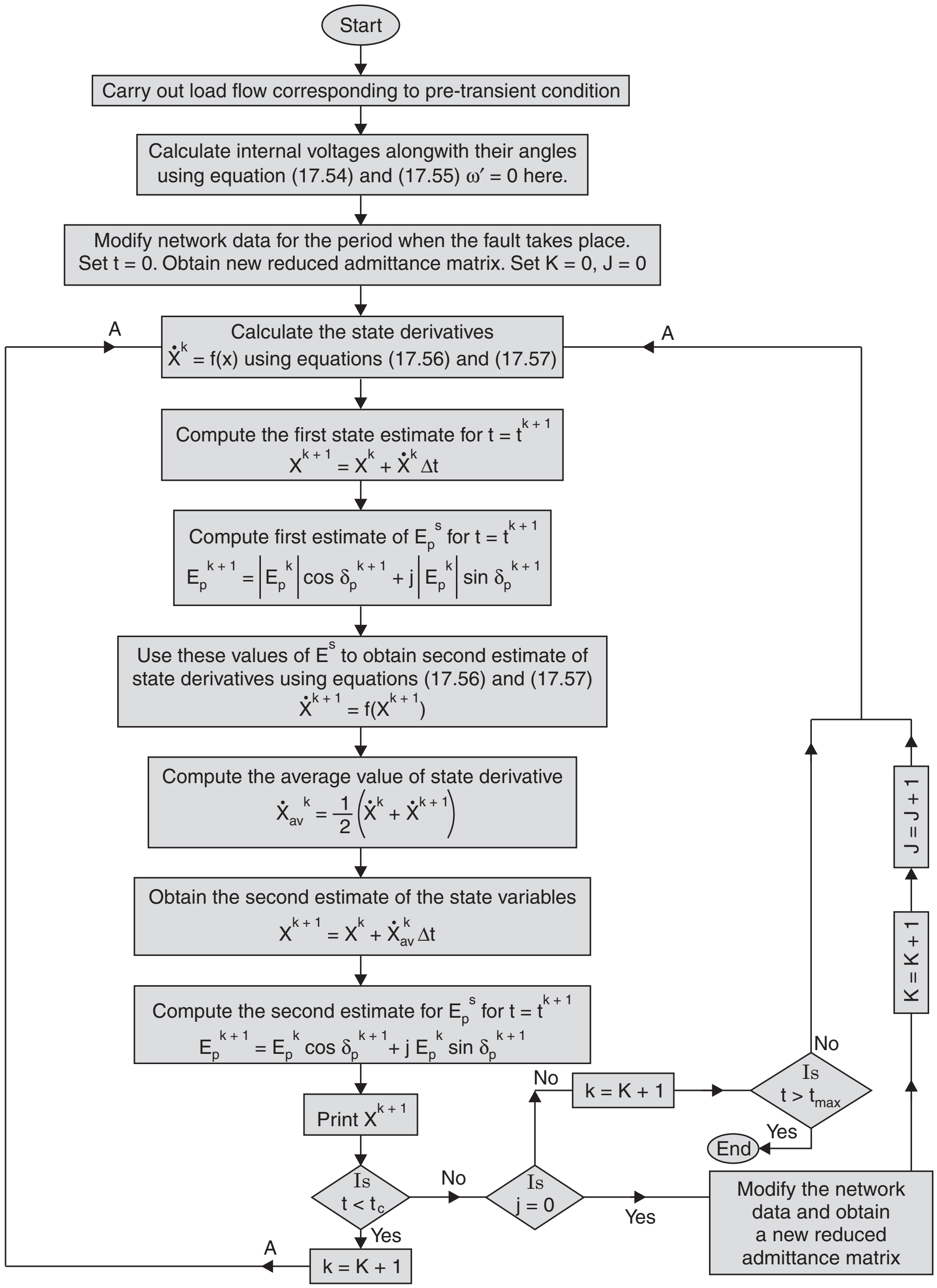


Fig. 17.28 Modified Euler's method for power system problem.

Here ω_p (1.4) = 383.493 = 376.992 + 6.501 and the power angle curve corresponds to fault curve as $t_c > 1.4$ sec.

$$K_1^7 = (383.493 - 376.992) \times 0.02$$
$$= 0.13002$$
$$l_1^7 = 62.83\ (1.5 - 1.2 \sin 1.026) \times 0.02$$
$$= 0.595277$$
$$K_2^7 = (6.501 + 0.297638) \times 0.02$$
$$= 0.1359727$$
$$l_2^7 = 62.83\ (1.5 - 1.2 \sin 1.09101) \times 0.02$$
$$= 0.5472338$$
$$K_3^7 = (6.501 + 0.2736169) \times 0.02$$
$$= 0.1354923$$
$$l_3^7 = 62.83\ (1.5 - 1.2 \sin 1.0939863) \times 0.02$$
$$= 0.545168$$
$$K_4^7 = (6.501 + 0.545168) \times 0.02$$
$$= 0.1409233$$
$$l_4^7 = 62.83\ (1.5 - 1.2 \sin 1.16149) \times 0.02$$
$$= 0.5015371$$
$$\Delta\delta^7 = \frac{1}{6}\ (0.13002 + 2 \times 0.1359727 + 2 \times 0.1346495 + 0.1409233)$$
$$= 0.13536$$

and $$\delta^8 = \delta^7 + \Delta\delta^7 = 1.026 + 0.13536$$
$$= 1.16136 \text{ rad.} \quad \textbf{Ans.}$$
$$\Delta\omega^7 = \frac{1}{6}\ (0.595277 + 2 \times 0.5472338 + 2 \times 0.545168 + 0.5015371)$$
$$= 0.5469362$$

∴ $$\omega^8 = 7.0479 \text{ rad/sec.} \quad \textbf{Ans.}$$

Euler's Modified Method: The equation to be solved are

$$\delta = \omega_p\ (t) - 2\pi f$$
$$\dot{\omega} = 62.83\ (1.5 - 1.2 \sin \delta)$$

The first step is to determine derivatives at $t = 0$

$$\dot{\delta}(0) = 0 \text{ and}$$
$$\dot{\omega} = 62.83\ (1.5 - 1.2 \sin 0.524) = 56.547$$
$$\delta_1^1 = \delta_1^{\ 0} + \frac{d\delta t}{dt}\,\Delta t$$
$$= 0.524 + 0.0 = 0.524$$
$$\omega_1^1 = 56.547 * 0.02 = 1.13094$$

Since there is no change in δ, the value of $E_p^{\ k+1}$ remains same and hence power generated remains same. Again evaluate derivative of δ and angular velocity as evaluated above.

$$\delta = 1.13094$$

$$\dot{\omega} = 56.547$$

Average value of derivatives

$$\delta_{av} = 0.56547 = \frac{0.0 + 1.13094}{2}$$

$$\dot{\omega}_{av} = \frac{56.547 + 56.547}{2} = 56.547$$

∴ New estimate of the variables

$$\delta_0^1 = 0.524 + 0.56547 * 0.02$$

$$= 0.5353 \quad \textbf{Ans.}$$

and $$\omega_1^1 = 0.0 + 56.547 \times 0.02$$

$$= 1.13094 \quad \textbf{Ans.}$$

17.14 LIMITATIONS OF THE CLASSICAL MODEL

The classical model is valid if the study of transient stability is made up to the first swing or for a period of 1 sec. The equipment used for the excitation control is relatively slow and simple. In the past the sizes of power system were such that the period of oscillation during machine dynamic response due to any impact in the system, was small usually less than one second.

Today the system is growing bigger and bigger using lot of interconnections. The system has large inertia but relatively weak ties. This results in longer periods of oscillation during transients. The modern excitation control system is extremely fast. Therefore, the effect of control equipments on the transient stability can't be ignored. There have been instances when the system withstood the first swing after a disturbance took place, but went out of step after several oscillations.

If the transient stability is to be studied for a longer period (longer than 1 sec.) it is unrealistic to assume that the mechanical power will not change. The turbine-governor and the boiler characteristics must be included in the analysis.

It is not correct to represent load by constant admittances. In the constant admittance model, the change in voltage is reflected in the active and reactive powers consumed by the load but the change in frequency (which occurs as a result of sudden short circuit in the system) is not reflected at all in the load power. In an actual power system the reduction in power consumed by the load is not proportional to V^2 but rather less than this as an increase in frequency (generator rotors accelerate during a short circuit) will result in an increase in the load power.

PROBLEMS

17.1. The generalized circuit constants of a nominal circuit representing a 3-phase transmission line are

$$A = D = 0.980 \angle 0.3°$$

$$B = 82.5 \angle 76.0° \ \Omega$$

$$C = 0.0005 \angle 90° \ \mho$$

Determine the steady state stability limit of the line if the sending end and receiving end voltages are held at 220 kV.

17.2. Two power stations A and B are located close together. Station A has four identical generator sets each rated 100 MVA and each having an inertia constant of 9 MJ/MVA whereas station B has three sets each rated 200 MVA, 4MJ/MVA. Calculate the inertia constant of the equivalent machines of both the stations on 150 MVA base.

17.3. A 4-pole, 50 Hz, 22 kV turboalternator has a rating of 100 MVA p.f. 0.8 lag. The moment of inertia of rotor is 8000 kgm^2. Determine M and H.

17.4. A turboalternator with 4-pole, 50 Hz, 80 MW, p.f. 0.8 lag and moment of inertia 40,000 kgm^2 is interconnected via a short transmission line to another alternator with 2-pole, 50 Hz, 100 MW, p.f. 0.8 lag and moment of inertia 10,000 kgm^2. Determine the inertia constant of the single equivalent machine on a base of 100 MVA.

17.5. Station A transmits 50 MW of power to station B through a tie line. The maximum steady state capacity of the line is 100 MW. Determine the allowable sudden load that can be switched on without loss of stability.

17.6. Determine the kinetic energy stored by a 50 MVA, 50 Hz two pole alternator with an inertia constant (H) of 5 kW sec. per KVA. If the machine is running steadily at synchronous speed with a shaft input (minus rotational losses) of 65000 HP when the electrical power developed suddenly changes from its normal value to a value of 40 MW, determine the acceleration or deceleration of the rotor. If the acceleration computed for the generator is constant for a period of 10 cycles, determine the change in torque angle in that period and the r.p.m. at the end of the 10 cycles.

17.7. A generator operating at 50 Hz delivers 1 p.u. power to an infinite bus when a fault occurs which reduces the maximum power transferable to 0.4 p.u. whereas the maximum power transferable before the fault was 1.75 p.u. and is 1.25 p.u. after the fault is cleared. Determine the critical clearing angle. If the inertia constant (H) of the generator is 4 p.u., determine the critical clearing time of the breakers. Plot the swing curve both for sustained fault condition and when the fault is cleared after 5 cycles. Assume a time interval of 0.05 sec.

17.8. Determine the critical clearing angle of the system shown in Fig. E.17.4 when the voltage E = 1.2 p.u. and voltage of infinite bus is 1.0 p.u. Determine the critical clearing time of the breaker if the inertia constant (H) of the system is 3.4 p.u. Plot the swing curves both for the sustained fault condition and when the fault is cleared after 5 cycles. Assume a time interval of (i) 0.05 sec, (ii) 0.025 sec, and (iii) 0.1 sec.

17.9. Differentiate between steady state stability and transient stability of a power system. Discuss the factors that affect (i) steady state stability, and (ii) transient state stability of the system.

17.10. Derive an expression for the maximum power transfer between two nodes. Show that this power is maximum when $X = \sqrt{3R}$, where X is the reactance and R the resistance of the system.

17.11. Derive swing equation and discuss its application in the study of power system stability.

17.12. What is 'equal area criterion' ? Discuss its application and limitation in the study of power system stability.

17.13. Discuss the application of equal area criterion for the system stability study when (i) a sudden increase in load takes place, and (ii) a short circuit on one of the parallel feeders takes place which is cleared after certain time.

17.14. Explain the point by point method of solving the swing equation. Compare this method with the equal area criterion method.

17.15. Discuss the effect of neutral grounding on the stability of power system.

17.16. A synchronous generator is connected to an infinite bus. Derive an expression for the pull out curve under steady state operation. Hence explain what you mean by MEL of the generator. Suggest some suitable controls for the excitation system.

17.17. Given a generator and a system with reactance of X_d = 1.5 p.u. and X_e = 0.2 p.u. both on 100 MVA base. Assume a generator terminal voltage of 0.96 p.u. Determine the centre and radius for the pull out curve and also the minimum permissible output vars when the output power is zero.

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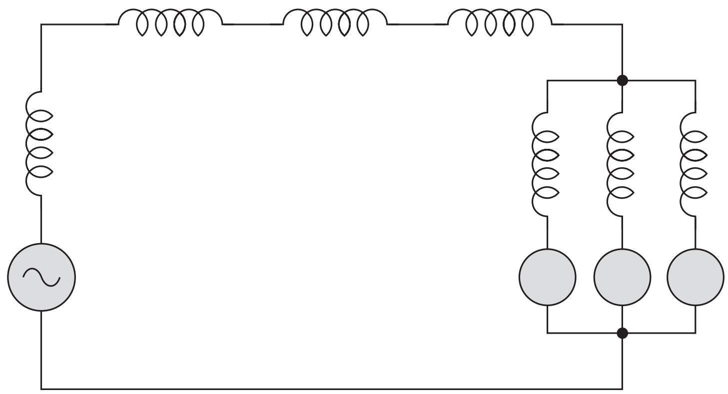

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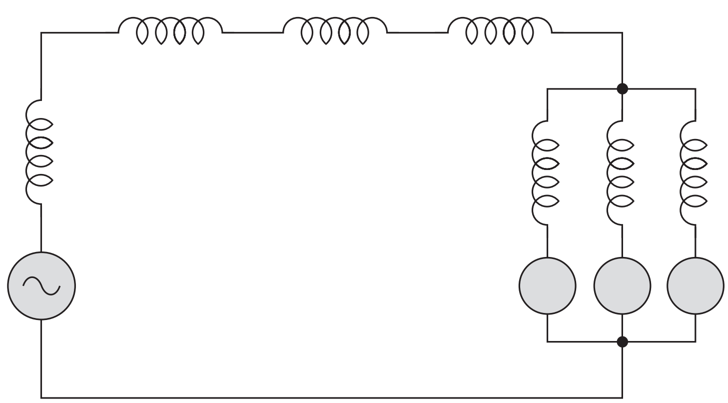
LOAD FLOWS



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Load Flows

INTRODUCTION

Load flow solution is a solution of the network under steady state condition subject to certain inequality constraints under which the system operates. These constraints can be in the form of load nodal voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting power channels (transmission lines). Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand. These analyses require the calculation of numerous load flows under both normal and abnormal (outage of transmission lines, or outage of some generating source) operating conditions. Load flow solution also gives the initial conditions of the system when the transient behaviour of the system is to be studied.

Load flow solution for power network can be worked out both ways according as it is operating under (*i*) balanced, or (*ii*) unbalanced conditions. The following treatment will be for a system operating under balanced conditions only. For such a system a single phase representation is adequate. A load flow solution of the power system requires mainly the following steps:

(*i*) Formulation or the network equations.

(*ii*) Suitable mathematical technique for solution of the equations.

Since we are studying the system under steady state conditions the network equations will be in the form of simple algebraic equations. The load and hence generation are continually changing in a real power system. We will assume here that loads and hence generation are fixed at a particular value over a suitable period of time, *e.g.*, half an hour or so.

18.1 BUS CLASSIFICATION

In a power system each bus or node is associated with four quantities, real and reactive powers, bus voltage magnitude and its phase angle. In a load flow solution two out of the four quantities are specified and the remaining two are required to be obtained through the solution of the equations. Depending upon which quantities have been specified, the buses are classified in the following three categories:

1. *Load Bus:* At this bus the real and reactive components or power are specified. It is desired to find out the voltage magnitude and phase angle through the load flow solution. It is required to specify only P_D and Q_D at such a bus as at a load bus voltage can be allowed to vary within the permissible values *e.g.*, 5%. Also phase angle of the voltage is not very important for the load.

2. *Generator Bus or Voltage Controlled Bus:* Here the voltage magnitude corresponding to the generation voltage and real power P_G corresponding to its ratings are specified. It is required to find out the reactive power generation Q_G and the phase angle of the bus voltage.

3. *Slack, Swing or Reference Bus:* In a power system there are mainly two types of buses: load and generator buses. For these buses we have specified the real power P injections. Now $\sum_{i=1}^{n} P_i$ = real power loss P_L, where P_i is the power injection at the buses, which is taken as positive for generator buses and is negative for load buses. The losses remain unknown until the load flow solution is complete. It is for this reason that generally one of the generator buses is made to take the additional real and reactive power to supply transmission losses. That is why this type of bus is also known as the slack or swing bus. At this bus, the voltage magnitude V and phase angle δ are specified whereas real and reactive powers P_G and Q_G are obtained through the load flow solution. The following table summarises the above discussion:

Bus type	*Quantities specified*	*Quantities to be obtained*
Load bus	P, Q	$\lvert V \rvert, \delta$
Generator bus	$P, \lvert V \rvert$	Q, δ
Slack bus	$\lvert V \rvert, \delta$	P, Q

The phase angle of the voltage at the slack bus is usually taken as the reference. In the following analysis the real and reactive components of voltage at a bus are taken as the independent variables for the load flow equations *i.e.,*

$$V_i \angle \delta_i = e_i + jf_i$$

where e_i and f_i are the real and reactive components of voltage at the ith bus. There are various other formulations wherein either voltage or current or both are taken as the independent variables. The load flow equations can be formulated using either the loop or bus frame of reference. However, from the viewpoint of computer time and memory, the nodal admittance formulation, using the nodal voltages as the independent variables is the most economic.

18.2 NODAL ADMITTANCE MATRIX

The load flow equations, using nodal admittance formulation for a three- bus system (Fig. 18.1), are developed first and then they are generalized for an n-bus system.

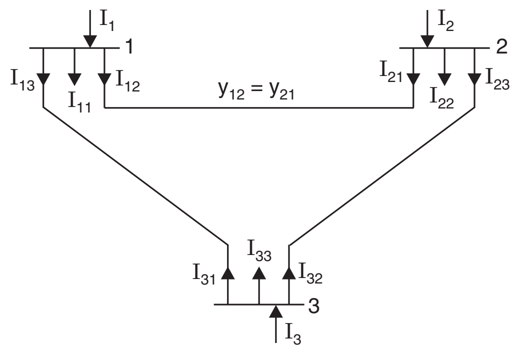

Fig. 18.1 Three-bus system.

At node 1

$$\begin{aligned} I_1 &= I_{11} + I_{12} + I_{13} \\ &= V_1 y_{11} + (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13} \\ &= V_1 y_{11} + V_1 y_{12} - V_2 y_{12} + V_1 y_{13} - V_3 y_{13} \\ &= V_1(y_{11} + y_{12} + y_{13}) - V_2 y_{12} - V_3 y_{13} \\ &= V_1 Y_{11} + V_2 Y_{12} + V_3 Y_{13} \end{aligned} \tag{18.1}$$

Here y_{11} is the shunt charging admittance at bus 1 and ground

$$Y_{11} = y_{11} + y_{12} + y_{13}$$
$$Y_{12} = -y_{12}$$
$$Y_{13} = -y_{13}$$

Similarly nodal current equations for the other nodes can be written as follows:

$$I_2 = V_1 Y_{21} + V_2 Y_{22} + V_3 Y_{23} \tag{18.2}$$
$$I_3 = V_1 Y_{31} + V_2 Y_{32} + V_3 Y_{33} \tag{18.3}$$

These equations can be written in a matrix form as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{18.4}$$

or in compact form these equations can be written as

$$I_p = \sum_{q=1}^{3} Y_{pq} V_q, \; p = 1 \text{ to } 3 \tag{18.5}$$

From this we now write nodal current equation for an n-bus system where each node is connected to all other nodes

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q, \; p = 1, 2, ..., n \tag{18.6}$$

or in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{18.6 a}$$

It can be shown that the nodal admittance matrix is a sparse matrix (a few number of elements are non-zero) for an actual power system. Consider Fig. 18.2.

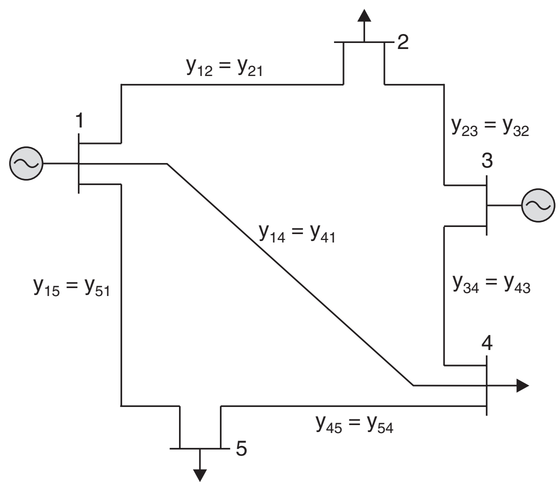

Fig. 18.2 Five-bus system.

The nodal admittance matrix for the above system is as follows:

$$Y_{pq} = \begin{bmatrix} Y_{11} & Y_{12} & 0 & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & 0 & 0 \\ 0 & Y_{32} & Y_{33} & Y_{34} & 0 \\ Y_{41} & 0 & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & 0 & 0 & Y_{54} & Y_{55} \end{bmatrix} \tag{18.7}$$

It can be seen that out of 25 elements, eight elements are zero whereas 17 are non-zero. In a large system of 100 nodes, these non-zero elements may be as small as 2% of the total elements. This is where we see that the computer memory requirements for storing the nodal admittance matrix is very low. It need store only a very few non-zero elements, it need not store the zeros of the matrix. Again the nodal admittance being a symmetric matrix along the leading diagonal, the computer need store the upper triangular nodal admittance matrix only. Thus, the computer memory requirement for storing the nodal admittance is all the more reduced.

If the interconnection between the various nodes for a given system, and the admittance value for each interconnecting circuit are known, the admittance matrix may be assembled as follows:

(*i*) The diagonal element of each node is the sum of the admittances connected to it.

(*ii*) The off-diagonal element is the negated admittance between the nodes.

However, it can be seen that the sum of the elements in each column of the admittance matrix summates to zero which means that nodal admittance matrix is a singular matrix and hence the rows of the matrix are linearly dependent.

We have already justified the necessity of selecting one of the buses as the slack or reference bus based on power balance in the system. Based on admittance matrix approach, it can be said mathematically that one of the buses should be taken as slack, otherwise the nodal matrix is singular and cannot be handled. By taking one of the buses as reference, corresponding row and column are deleted from the nodal admittance matrix and hence the reduced matrix becomes non-singular, which can be handled very easily.

Shunt Branches

Shunt admittances, are added to the diagonal elements corresponding to the nodes at which they are connected. The off-diagonal elements are unaffected. The addition of shunt element in this way may either strengthen or weaken the Y matrix diagonal depending on whether the additional elements add to or subtract from the diagonal terms, representing series admittance summations. Shunt inductances strengthen the diagonal while shunt capacitances weaken it. With this also the singularity of the admittance matrix can be avoided.

Tapped Transformers

The tapped transformers operating at off-nominal tap positions provide means of exchanging reactive power between networks operating at different voltages, and between generators and the network system to which they are connected, it is required to reflect this into power balance equations. Refer to Fig. 18.3. Here t is the per unit off-nominal tap position, either positive or negative, and the sending end of the equivalent circuit corresponds to the node to which the transformer is connected.

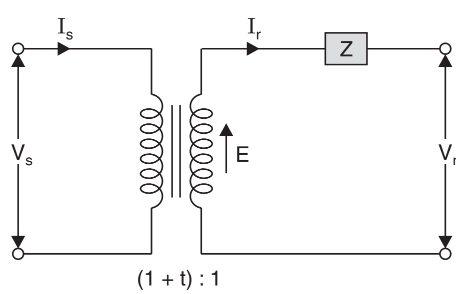

Fig. 18.3 Elementary transformer connection
Z = transformer impedance
t = off-nominal tap position.

From the equivalent circuit,

$$V_s = (1 + t)E \tag{18.8}$$

and

$$V_r = E - I_r Z \tag{18.9}$$

where Z is the per unit impedance of the transformer. From the above two equations eliminating E we have

$$I_r = \frac{1}{Z}\left[\frac{V_s}{1+t} - V_r\right]$$

or
$$I_r = \frac{1}{Z}\left[\frac{V_s - V_r}{1+t} - \frac{tV_r}{1+t}\right] \tag{18.10}$$

Also
$$I_s = I_r/(1 + t)$$

$$= \frac{1}{Z}\left[\frac{V_s}{(1+t)^2} - \frac{V_r}{1+t}\right]$$

or
$$= \frac{1}{Z}\left[\frac{V_s - V_r}{1+t} - \frac{tV_s}{(1+t)^2}\right] \tag{18.11}$$

Equations (18.10) and (18.11) for I_r and I_s have been rearranged so as to suit the equivalent circuit as shown in Fig. 18.4. The equivalent circuit can be used to include the presence of tap

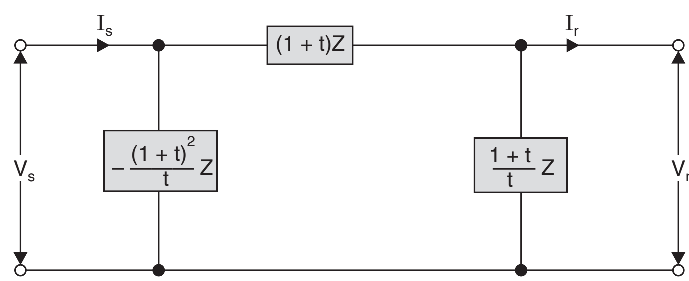

Fig. 18.4 Equivalent circuit of a transformer for nodal admittance matrix.

changing transformer while formulating the nodal admittance matrix. The shunt admittances of the equivalent circuit are added to the Y matrix diagonal elements at the two nodes, and the negated series admittance is the off-diagonal element corresponding to the transformer interconnection. If the transformer connection is included in a transmission circuit as shown in Fig. 18.5, the equivalent circuit representation in the above equation (18.10–18.11) will hold good except that the branch impedance is the sum of the transformer impedance and the circuit impedance. If Z_1 is the line impedance, the expressions for I_r and I_s become

$$I_r = \frac{1}{Z + Z_1}\left[\frac{V_s - V_r}{1+t} - \frac{tV_r}{1+t}\right] \tag{18.12}$$

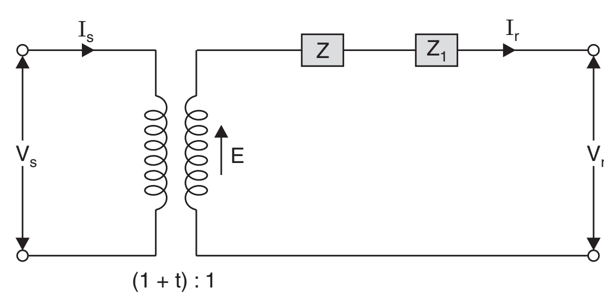

Fig. 18.5 Transformer connected to a line of impedance Z_1.

and $$I_s = \frac{1}{Z + Z_1}\left[\frac{V_s - V_r}{1+t} - \frac{tV_s}{(1+t)^2}\right] \qquad ...(18.13)$$

If the transformer is connected between nodes p and q the equivalent circuit for nodal admittance formulation will be as shown in Fig. 18.6.

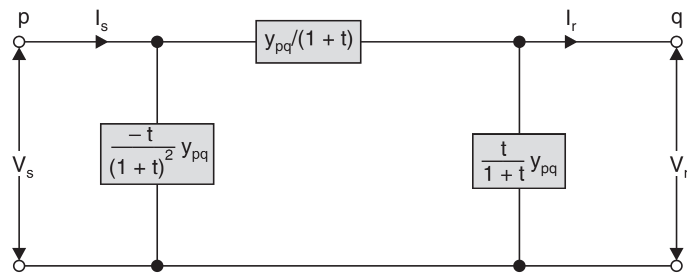

Fig. 18.6 Equivalent of Fig. 18.5 for nodal admittance matrix.

With this we are able to take into account the shunt branches and the effect of tap changing under load transformers on the power flow in the interconnected network. Once these components are included in the nodal admittance matrix we need not bother about them as their presence will be taken care of by the admittance matrix.

18.3 DEVELOPMENT OF LOAD FLOW EQUATIONS

The nodal current equations derived earlier are rewritten below for a n-bus system in equation (18.6).

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q, \; p = 1, 2, ..., n \qquad (18.6)$$

$$I_p = Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \qquad (18.14)$$

or $$V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \qquad (18.15)$$

Now $$V_p^* I_p = P_p - jQ_p \qquad (18.16)$$

or $$I_p = \frac{P_p - jQ_p}{V_p^*} \qquad (18.17)$$

Substituting for I_p in equation (18.15),

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q\right], \; p = 1, 2, ..., n \qquad (18.18)$$

I_p has been substituted by the real and reactive powers because normally in a power system these quantities are specified.

18.4 ITERATIVE METHODS

Equations (18.18) are the load flow equations where bus voltages are the variables. It can be seen that the load flow equations are nonlinear and they can be solved by an iterative method. The iterative methods are

(*i*) Gauss's method,

(*ii*) Gauss-Seidel method.

Before these methods are explained we take a specific example and apply these methods for the solution of the load flow equations. Refer to Fig. 18.2. The five bus system has two generators at buses 1 and 3 and three load buses 2, 4 and 5. The nodal admittance matrix is also given alongwith Fig. 18.2. By using this nodal admittance matrix alongwith the equation (18.18), the load flow equations for the five bus system are written as follows. Assuming bus 1 as the slack bus,

$$V_1 = V_1 \text{ Specified fixed value}$$

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - (Y_{21}V_1 + Y_{23}V_3)\right]$$

$$V_3 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - (Y_{32}V_2 + Y_{34}V_4)\right]$$

$$V_4 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - (Y_{41}V_1 + Y_{43}V_3 + Y_{45}V_5)\right]$$

$$V_5 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^*} - (Y_{51}V_1 + Y_{54}V_4)\right] \tag{18.19}$$

To understand the procedure for solution of these equations we make a simplifying assumption at this stage that all the other buses except bus No. 1 are load buses *i.e.*, buses where P and Q are specified. We will include the presence of PV buses is the next article. The admittances and voltages as used in these equations are complex quantities and the number of nonlinear equations is $(n - 1)$ where n is the total number of buses in the system. The following is the Gauss iterative procedure for solving the equations. The flow chart for the procedure is given in Fig. 18.7.

Gauss Method

1. Assume a flat voltage profile for all nodal voltages except the slack bus 1. Let slack bus voltage be $a + j0.0$. Assume a suitable value of convergence criterion ε, *e.g.*, if the absolute value of the maximum change in voltage between any two consecutive iterations is less than a prespecified tolerance ε, the convergence is achieved and the iterative procedure is terminated.

2. Set iteration count $K = 0$

3. Set bus count $p = 1$

4. Check for the slack bus. If it is not a slack bus go to next step. Since voltage at the slack bus is fixed both in magnitude and phase, it does not vary during iterative procedure and hence go to step 6 if it is a slack bus.

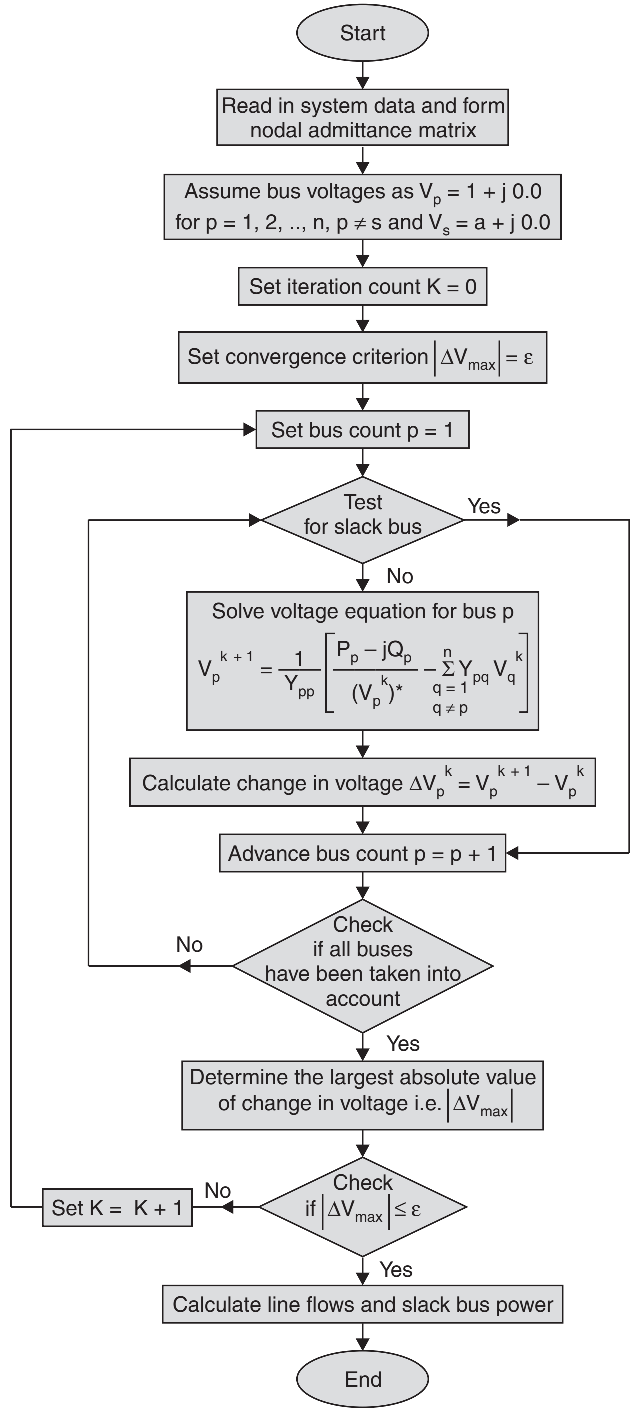

Fig. 18.7 Flow chart for load flow solution using Gauss method (Load bus system only).

5. Calculate the bus voltage V_p^{K+1} using equation (18.18) and the difference in bus voltage $\Delta V_p^K = V_p^{K+1} - V_p^K$.

6. Advance the bus count by 1 to evaluate other values of V_p^{K+1} and ΔV_p^K.

7. Check if all buses have been taken into account. If yes, go to the next step, otherwise go back to step 4.

8. Determine the largest absolute value of change in voltage $|\Delta V|_{max}$.

9. If $|\Delta V|_{max}$ is less than a specified tolerance ε, evaluate line flows and print the voltage and line flows. If not, advance the iteration count $K = K + 1$ and go back to step 3.

Computational feature: It is possible to obtain significant reduction in computer time if all arithmetic operations can be performed in advance when they do not change with the iterations. Since P, Q and Y at a bus do not change with iterations, the term $\dfrac{P_p - jQ_p}{Y_{pp}}$ can be evaluated before hand.

Let
$$A_p = \frac{P_p - jQ_p}{Y_{pp}} \text{ for all } p = 1, ..., n, p \neq s \tag{18.20}$$

Similarly let

$$B_{pq} = \frac{Y_{pq}}{Y_{pp}} \text{ for all } p = 1, 2, ..., n, p \neq s$$

and
$$q = 1, 2, ..., n, q \neq p \tag{18.21}$$

With these simplifications the voltage equation now becomes

$$V_p^{K+1} = \frac{A_p}{(V_p^K)^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} B_{pq} V_p^K,\ p = 1, 2, ... n, p \neq s \tag{18.22}$$

Gauss-Seidel Iterative Method (Including PV Buses)

The bus voltage equations (18.22) can also be solved by the Gauss-Seidel method. In this method the new calculated voltage V_i^{K+1} immediately replaces V_i^K and is used in the solution of the subsequent equations. The equations (18.19) in this case become

$$V_2^{K+1} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^K)^*} - (Y_{21}V_1 + Y_{23}V_3^K)\right]$$

$$V_3^{K+1} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^K)^*} - (Y_{32}V_2^{K+1} + Y_{34}V_4^K)\right]$$

$$V_4^{K+1} = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^K)^*} - (Y_{41}V_1 + Y_{43}V_3^{K+1} + Y_{45}V_5^K)\right]$$

$$V_5^{K+1} = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{(V_5^K)^*} - (Y_{51}V_1 + Y_{54}V_4^{K+1})\right] \tag{18.23}$$

The general load flow equation resultant from Gauss-Seidel method will be as given below:

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^K)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}\, V_q^K\right] \tag{18.24}$$

The second term on the r.h.s. of the above equation is clear because the voltage prior to bus p should correspond to the value as calculated during the current iteration.

The procedure for solution of these equations is outlined below and takes into account the presence of voltage controlled buses in addition to the load buses.

Before we proceed to solve these equations (18.24) the following points must be kept in mind:

1. The voltages and admittances are complex quantities.

2. Buses 2, 4 and 5 are load buses, where P and Q are known quantities whereas bus 3 is a voltage controlled bus where only P is known and Q is not known. Therefore, for solving equation corresponding to a voltage controlled bus Q must first be calculated and is given as

$$Q_p = \sum_{q=1}^{n} \{f_p(e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})\} \tag{18.25}$$

where $\quad Y_{pq} = G_{pq} - jB_{pq}$

and $\quad V_p = e_p + jf_p$

This equation is derived later on in this chapter.

After Q at the voltage controlled bus has been calculated, it should be checked whether this value violates the reactive power generation at the bus *i.e.*, whether it is less than the minimum reactive power generation permissible (because of stability problem) or it is more than the maximum reactive power generation permissible (because of rotor heating). If it violates, the voltage controlled bus is made to act as a load bus for that iteration only and the reactive power to be substituted in the expression for that iteration will correspond to the limit it has violated, *i.e.*, if it is less than Q_{min}, the value to be substituted will be Q_{min}. If the calculated value of Q lies within the limits, this value of Q will be substituted in the expression.

The violation of reactive power limit could be due to the specified voltage either being too low or too high. Since $|V|_{spec}$ can be obtained only by controlling reactive power, therefore, it is possible that we have specified $|V|$ beyond the capability of the reactive power generation of the generator. If the value of Q for a generator bus is calculated after assuming magnitude equal to $|V|_{sp}$ for that bus and phase angle corresponding to the value in that iteration and if Q so obtained violates the limit, the value of voltage should be taken corresponding to the voltage in that iteration and not V_{sp}. Otherwise the voltage should correspond to specified voltage and phase angle as in that iteration.

3. The number of nonlinear equation is $(n - 1)$, where n is the number of buses in the system.

The following is Gauss-Seidel iterative procedure for solving equations:

1. Assume a flat voltage profile $1 + j0.0$ for all nodal voltages except the slack bus 1. Assume a suitable value of ε the convergence criterion *i.e.*, if the absolute value of the maximum

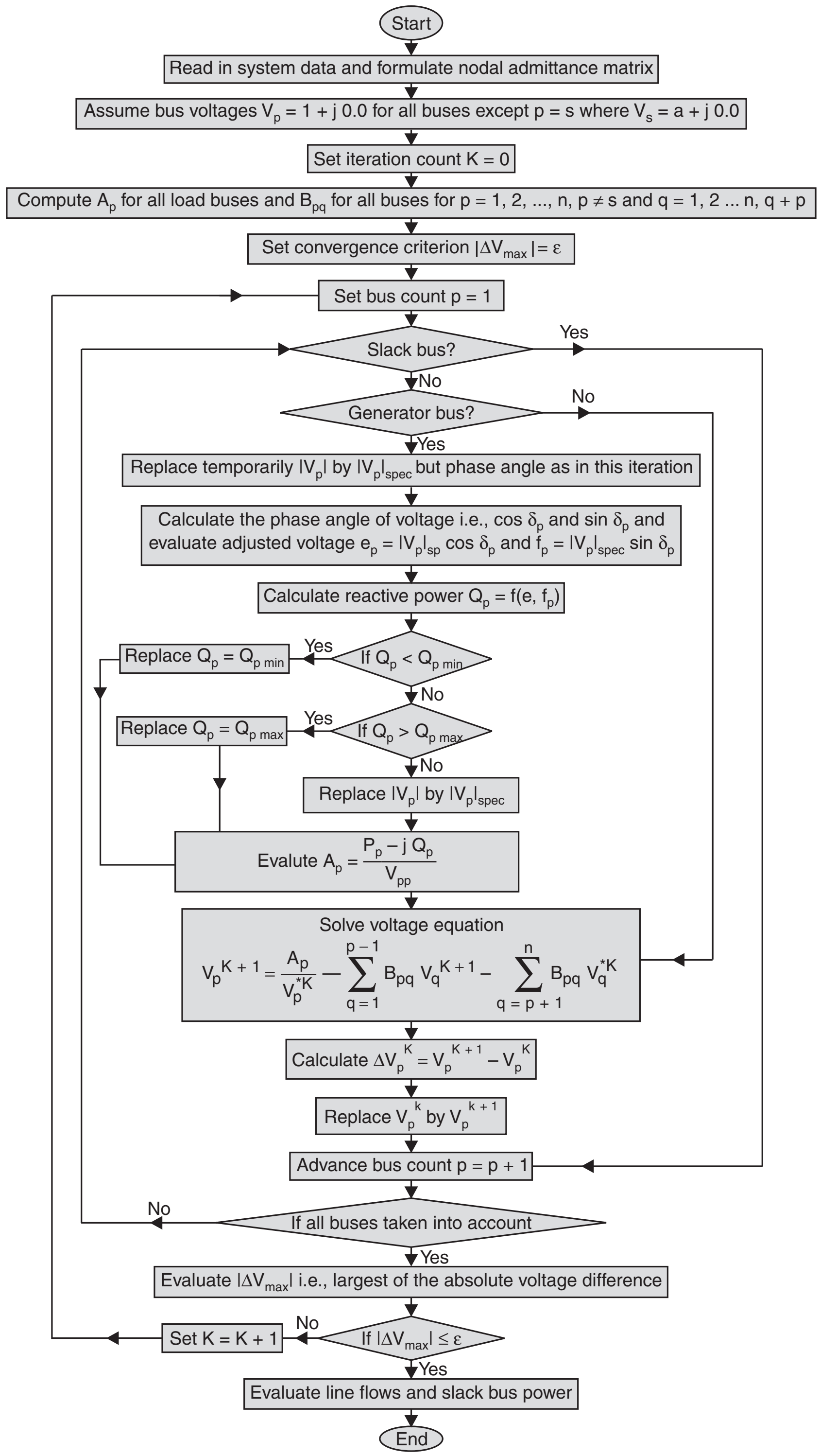

Fig. 18.8 Flow chart for load flow solution using Gauss-Seidel method. Voltage controlled buses included.

change in voltage between any two consecutive iterations is less than a prespecified tolerance ε, the convergence is achieved and the iterative procedure is terminated.

1 (*a*) Set iteration count $k = 0$

1 (*b*) Set bus count $p = 1$

1 (*c*) Check for slack bus. If it is a slack bus, go to step 4 (*a*), otherwise go to next step.

2. Check which of the buses are voltage controlled and which are load buses. For voltage controlled buses go to next statement, otherwise go to step 4.

3. Replace the value of the voltage magnitude of voltage controlled bus in that iteration by the specified value. Keep the phase angle same as in that iteration. Calculate Q for the generator bus. If Q lies within the lower and upper bounds calculate the term $(P - jQ)/V$ for this bus. Repeat this for all voltage controlled buses and calculate this term and substitute this term $(P - jQ)/V$ in the load flow equation corresponding to the voltage controlled bus. Calculate the new value of voltage for the bus. It is to be noted that if there are more than one generator buses, the voltage magnitude of that bus only is replaced by its specified value, while calculating P and Q of a particular bus. The voltage of other generator buses will be corresponding to the value in that iteration.

In case any or all the voltage controlled buses violate the reactive power generation, the bus will be treated as a load bus and the magnitude of the reactive power at this bus will correspond to the limit it has violated, as explained earlier, and the value of the magnitude of voltage will correspond to the value in that iteration (not corresponding to specified voltage), and then go to next step.

4. For bus p evaluate V_p^{K+1} from equation (18.24) and change in voltage

$$\Delta V_p^K = V_p^{K+1} - V_p^K.$$

4 (*a*) Advance the bus count by 1 and check if all the buses have been taken into account. If yes, go to next step, otherwise go to step 1 (*c*).

5. Find out the largest of the absolute value of the change in voltage. If this is less than a prespecified tolerance move on to next step, otherwise go back to step 2.

6. Calculate the injected powers and the line flows using the nodal voltages.

The flow chart for Gauss-Seidel method is given in Fig. 18.8. The flow chart includes the presence of voltage controlled buses and the simplification of the computational procedure.

It is found in practice that the process of convergence due to G-S method is slow *i.e.*, it requires a large number of iterations before a solution is obtained. The process of convergence can be speeded up if the voltage correction during consecutive iteration is modified to

$$V_{p(acc)}^{K+1} = V_p^K + \alpha(V_p^{K+1} - V_p^K)$$

where α is known as acceleration factor and is a real number. A suitable value of α for a particular system can be obtained by running trial load flows. α = 1.6 is a general recommended value for most of the systems. However, it may be noted that a wrong selection of α may result in slower convergence and sometimes even result in divergence from the solution.

Example 18.1: The following is the system data for a load flow solution:

The line admittances:

Bus code	*Admittance*
1–2	2–j8.0
1–3	1–j4.0
2–3	0.666–j 2.664
2–4	1–j4.0
3–4	2–j8.0

The schedule of active and reactive powers:

Bus code	*P*	*Q*	*V*	*Remarks*
1	–	–	1.06	Slack
2	0.5	0.2	1 + j0.0	*PQ*
3	0.4	0.3	1 + j0.0	*PQ*
4	0.3	0.1	1 + j0.0	*PQ*

Determine the voltages at the end of first iteration using Gauss-Seidel method. Take $\alpha = 1.6$.

Solution: The admittance matrix will be as given below:

$$Y_{pq} = \begin{bmatrix} 3 - j12.0 & -2 + j8.0 & -1 + j4.0 & 0.0 \\ -2 + j8.0 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4.0 \\ -1 + j4.0 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8.0 \\ 0.0 & -1 + j4.0 & -2 + j8.0 & 3 - j12.0 \end{bmatrix}$$

The powers for load buses are to be taken as negative and that for generator buses as positive.

For the system given

$$V_2{}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{(3.666 - j14.664)}\left[\frac{-0.5 + j0.2}{1 - j0.0} - 1.06(-2 + j8) - 1.0\right.$$

$$\left.(-0.666 + j2.664) - (-1 + j4.0)1.0\right]$$

$$= (1.01187 - j0.02888)$$

$$V_{2\,\text{acc}}^1 = (1.0 + j0.0) + 1.6\{1.01187 - j0.02888 - 1.0 - j0.0\}$$

$$= 1.01899 - j0.046208 \quad \textbf{Ans.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{(3.666 - j14.664)}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4.0)1.06\right.$$

$$\left. - (-0.666 + j2.664)(1.01899 - j0.046208) - (-2 + j8)(1 + j0.0)\right]$$

$$= 0.994119 - j0.029248$$

$$V_{3\,\text{acc}}^1 = (1 + j0.0) + 1.6[0.994119 - j0.029248 - 1 - j0.0]$$

$$= 0.99059 - j0.0467968 \quad \textbf{Ans.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{(3 - j12)}\left[\frac{-0.3 + j0.1}{1 - j0.0} - (-1 + j4.0)(1.01899 - j0.046208)\right.$$

$$\left. - (-2 + j8)(0.99059 - j0.0467968)\right]$$

$$= 0.9716032 - j0.064684$$

$$V_{4\,\text{acc}}^1 = 1.0 + j0.0 + 1.6[0.9716032 - j0.064684 - 1 - j0.0]$$

$$= 0.954565 - j0.1034944 \quad \textbf{Ans.}$$

Example 18.2: If in Example 18.1, bus 2 is taken as a generator bus with $|V_2| = 1.04$ and reactive power constraint is

$$0.1 \le Q_2 \le 1.0$$

Determine the voltages starting with a flat voltage profile and assuming accelerating factor as 1.0.

Solution: Since bus 2 is taken as a generator bus Q_2 is not specified and $P_2 = 0.5$.

To find V_2^1 we first find Q_2 with $V_2 = 1.04 + j0.0$ as the phase angle of the voltage is 0.0 to begin with.

$$P_2 - jQ_2 = V_2^* \sum_{q=1}^{4} Y_{2q}V_q = V_2^*[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4]$$

$$\therefore \quad Q_2 = -I_m[V_2^{0*}(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4)]$$

$$= -I_m\,[(1.04 - j0.0)(-2 + j8.0)(1.06)$$

$$+ (3.666 - j14.664)(1.04) + (-0.666 + j2.664)(1 + j0.0)$$

$$+ (-1 + j4.0)1.0] = 0.1108$$

Since Q_2 lies within the limits, therefore V_2 will be taken as $|V_2|_{spec}$. and phase angle as in this iteration,

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

Bus 2 being a generator bus, p_2 and Q_2 are to be taken as positive and value of P_2 as the specified and Q_2 as the one calculated above *i.e.,* $Q_2 = 0.1108$.

$$\therefore \qquad V_2 = \frac{1}{(3.666 - j14.664)}\left[\frac{0.5 - j0.1108}{1.04 - j0.0} - (-2 + j8.0)1.06 - (-0.666 + j2.664)1.0 - (-1 + j4.0)1.0\right]$$

$$V_2^1 = 1.0472846 + j0.0291476$$

$$\therefore \qquad \delta = 1.59°$$

and

$$V_2^1 = 1.04 \angle 1.59° = 1.0395985 + j0.02891158$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4)1.06 - (-0.666 + 2.664)(1.0395985 + j0.02891158) - (-2 + j8)(1 + j0.0)\right]$$

$$= 0.9978866 - j0.015607057$$

Similarly V_4^1 can be obtained and it will be found to be

$$V_4^1 = 0.998065 - j0.022336 \quad \textbf{Ans.}$$

Example 18.3: If the reactive power constraint on generator 2 is

$$0.2 \le Q_2 \le 1.0$$

Solve the previous example for voltages at the end of first iteration.

Solution: Since Q_2 calculated corresponding to initial guess $V_2 = 1.04 + j0.0$ is 0.1108 p.u. which is less than the minimum specified, the reactive power generation for bus 2 is fixed at 0.2, the lower limit, and the bus is treated as a load bus for this iteration. Voltage V_2 will be $V_2^0 = 1 + j0.0$ as for all other load buses for this iteration. It is to be noted that the generator bus to be treated as a load bus means that the specified quantities are P and Q and unknown are $|V|$ and δ but P and Q are to be taken as positive as in case of a generator bus in contrast to a load bus where P and Q are taken as negative. Therefore,

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{(P_2 - jQ_2)}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{0.5 - j0.2}{1 - j0.0} - (-2 + j8.0)1.06\right.$$

$$\left. - (-0.666 + j0.2664) - (-1 + j4.0)1.0\right]$$

$$= 1.098221 + j0.030105662$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{\,0}} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4)1.06\right.$$

$$- (-0.666 + j2.664)(1.098221$$

$$\left. + j0.030105662) - (-2 + j8.0)(1 + j0.0)\right].$$

Similarly V_4^1 can be evaluated.

18.5 NEWTON-RAPHSON METHOD

Development of load flow equations: The load flow problem can also be solved by using Newton-Raphson method. The equations for the method are derived as follows:

We know that at any bus p,

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

Let $\qquad V_p = e_p + jf_p \quad \text{and} \quad Y_{pq} + G_{pq} - jB_{pq}$

$$P_p - jQ_p = (e_p + jf_p)^* \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

$$= (e_p - jf_p) \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

Separating the real and imaginary parts we have

$$P_p = \sum_{q=1}^{n} \{e_p(e_q G_{pq} + f_q\ B_{pq}) + f_p\ (f_q G_{pq} - e_q B_{pq})\} \tag{18.26}$$

and

$$Q_p \sum_{q=1}^{n} \{f_p(e_q G_{pq} + f_q\ B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq})\} \tag{18.25}$$

also

$$|\ V_p\ |^2 = e_p^2 + f_p^2 \tag{18.27}$$

These three sets of equations are the load flow equations and it can be seen that they are non-linear equations in terms of the real and imaginary components of nodal voltages. Here the left hand quantities *i.e.*, P_p, Q_p (for a load bus) and P_p and $|V_p|$ for generator bus are specified and e_p and f_p are unknown quantities. For an n-bus system, the number of unknowns are $2(n-1)$ because the voltage at the slack bus is known and is kept fixed both in magnitude and phase. Therefore, if bus 1 is taken as the slack, the unknown variables are $(e_2, e_3, ..., e_{n-1}, e_n, f_2, f_3, ..., f_{n-1}, f_n)$. Thus, to solve the problem for $2(n-1)$ variables we need to solve $2(n-1)$ set of equations.

Newton-Raphson method is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first approximation.

The mathematical background of this method is explained as follows:

Let the unknown variables be $(x_1, x_2, ..., x_n)$ and the specified quantities $y_1, y_2, ..., y_n$. These are related by the set of non-linear equations:

$$\begin{aligned} y_1 &= f_1(x_1, x_2, ..., x_n) \\ y_2 &= f_2(x_1, x_2, ..., x_n) \\ &\vdots \\ y_n &= f_n(x_1, x_2, ..., x_n) \end{aligned} \tag{18.28}$$

To solve these equations we start with an approximate solution $(x_1^0, x_2^0, ..., x_n^0)$. Here superscript zero means the zeroth iteration in the process of solving the above non-linear equations (18.28). It is to be noted that the initial solution for the equations should not be very far from the actual solution. Otherwise, there are chances of the solution diverging rather than converging and it may not be possible to achieve a solution whatever be the computer time utilized. At first glance it may appear to be a great drawback for the Newton-Raphson technique but the problem of initial guess for a power system is not at all difficult. A flat voltage profile *i.e.*, $V_p = 1.0 + j0.0$ for $p = 1, 2, ..., n$ except the slack bus has been found to be satisfactory for almost all practical systems.

The equations are linearized about the initial guess. We will expand first equation $y_1 = f_1$ and the result for the other equations will follow.

Assume $\Delta x_1^0, \Delta x_2^0, ..., \Delta x_n^0$ are the corrections required for $x_1^0, x_2^0, ..., x_n^0$ respectively for the next better solution. The equation $y_1 = f_1$ will be

$$\begin{aligned} y_1 &= f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, ..., x_n^0 + \Delta x_n^0) \\ &= f_1(x_1^0, x_2^0, ..., x_n^0) + \Delta x_1^0 \left.\frac{\partial f_1}{\partial x_1}\right|_{x^0} + \Delta x_2^0 \left.\frac{\partial f_1}{\partial x_2}\right|_{x^0} + ... + \Delta x_n^0 \left.\frac{\partial f_1}{\partial x_n}\right|_{x^0} + \phi_1 \end{aligned}$$

where ϕ_1 is function of higher order of Δx^s and higher derivatives which are neglected according to Netwon-Raphson method. In fact it is this assumption which needs the initial solution to be close to the final solution. If all the equations are linearized and arranged in a matrix form, we have

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0, \ldots, x_n^0) \\ y_2 - f_2(x_1^0, x_2^0, \ldots, x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0, \ldots, x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \tag{18.29}$$

$$B = J \,.\, C$$

Here J is the first derivative matrix known as the Jacobian matrix. The solution of the equations requires calculation of left hand vector B which is the difference of the specified quantities and calculated quantities at $(x_1^0, x_2^0, \ldots, x_n^0)$. Similarly J is calculated at this guess. Solution of the matrix equation gives $(\Delta x_1^0, \Delta x_2^0, \ldots, \Delta x_n^0)$ and the next better solution is obtained as follows:

$$x_1^1 = x_1^0 + \Delta x_1^0$$
$$x_2^1 = x_2^0 + \Delta x_2^0$$
$$x_n^1 = x_n^0 + \Delta x_n^0$$

The better solution is now available and is

$$(x_1^1, x_2^1, x_3^1, \ldots, x_n^1)$$

With these values the process is repeated till (*i*) the largest (in magnitude) element in the left column of the equations is less than a prespecified value or (*ii*) the largest element in the column vector $(\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$ is less than a prespecified value.

When referred to a power system problem (assuming there is only one generator bus which is taken as slack bus and all other buses are load buses), the above set of linearized equations become

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \cdots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \cdots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \cdots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \cdots & \frac{\partial Q_3}{\partial f_n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \cdots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \cdots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}$$

$$2(n-1) \times 1 \qquad\qquad 2(n-1) \times 2(n-1) \qquad\qquad 2(n-1) \times 1 \tag{18.30}$$

In short form it can be written as

$$\begin{bmatrix} \Delta P \\ -- \\ \Delta Q \end{bmatrix} = \left[\begin{array}{c:c} J_1 & J_2 \\ --- & --- \\ J_3 & J_4 \end{array}\right] \begin{bmatrix} \Delta e \\ -- \\ \Delta f \end{bmatrix}$$

In case the system contains all types of buses, the set of equations can be written as

$$\begin{bmatrix} \Delta P \\ -- \\ \Delta Q \\ -- \\ |\Delta V_p|^2 \end{bmatrix} = \left[\begin{array}{c:c} J_1 & J_2 \\ --- & --- \\ J_3 & J_4 \\ --- & --- \\ J_5 & J_6 \end{array}\right] \begin{bmatrix} \Delta e \\ -- \\ \Delta f \end{bmatrix}$$

The elements of the Jacobian matrix can be derived from the three load flow equations (18.25) to (18.27).

The off-diagonal elements of J_1 are

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq},\ q \neq p \tag{18.31}$$

and the diagonal elements of J_1 are

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + f_p B_{pp} - f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_p G_{pq} + f_q B_{pq})$$

$$= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}) \tag{18.32}$$

The off-diagonal elements of J_2 are

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq},\ q \neq p \tag{18.33}$$

and the diagonal elements of J_2 are

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq}) \tag{18.34}$$

The off-diagonal elements of J_3 are

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq},\ q \neq p \tag{18.35}$$

and the diagonal elements are

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq}) \tag{18.36}$$

The off-diagonal and diagonal elements of J_4 respectively are

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq},\ q \neq p \tag{18.37}$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}) \tag{18.38}$$

The off-diagonal and diagonal elements of J_5 are

$$\frac{\partial |V_p|^2}{\partial e_q} = 0,\ q \neq p \tag{18.39}$$

and

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p \tag{18.40}$$

The off-diagonal and diagonal elements of J_6 are

$$\frac{\partial |V_p|^2}{\partial f_q} = 0,\ q \neq p \tag{18.41}$$

and

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p \tag{18.42}$$

Next, we calculate the residual column vector consisting of ΔP, ΔQ and $|\Delta V|^2$. Let P_{sp}, Q_{sp}, and $|V_{sp}|$ be the specified quantities at the bus p. Assuming a suitable value of the solution (flat voltage profile in our case) the value of P, Q and $|V|$ at the various buses are calculated. Then

$$\Delta P_p = P_{sp} - P_p^0$$

$$\Delta Q_p = Q_{sp} - Q_p^0$$

$$|\Delta V_p|^2 = |V_{sp}|^2 - |V_p^0|^2 \tag{18.43}$$

where the superscript zero means the value calculated corresponding to initial guess *i.e.*, zeroth iteration.

Having calculated the Jacobian matrix and the residual column vector corresponding to the initial guess (initial solution) the desired increment voltage vector $\begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$ can be calculated by using any standard technique (preferably Gauss elimination with sparsity techniques). The next better solution will be

$$e_p^1 = e_p^0 + \Delta e_p^0$$

$$f_p^1 = f_p^0 + \Delta f_p^0$$

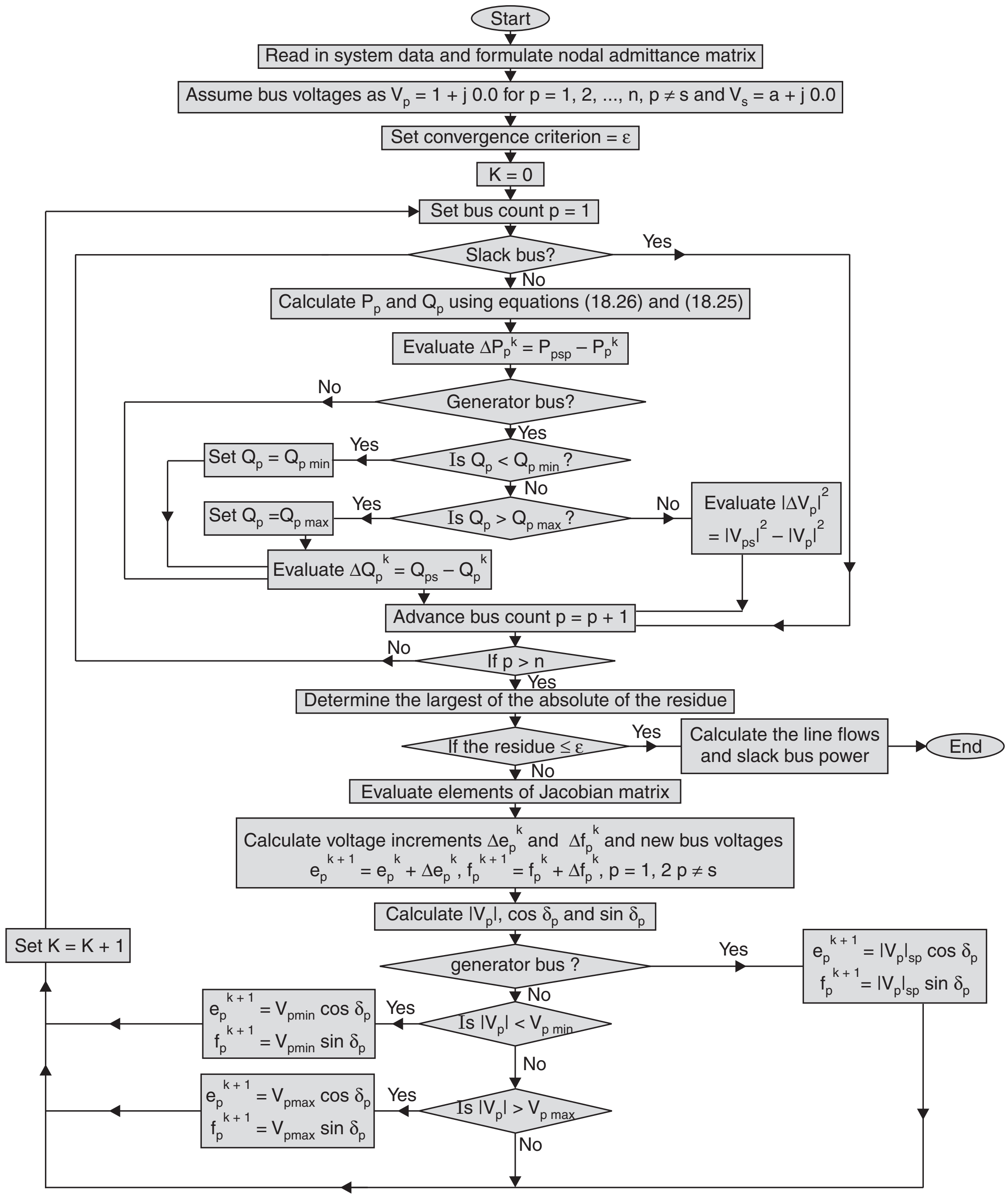

Fig. 18.9 Flow chart for load flow solution using Newton-Raphson method. Voltage controlled buses included. Voltage constraints included.

These values of voltages will be used in the next iteration. The process will be repeated and in general the new better estimates for bus voltages will be

$$e_p^{k+1} = e_p^k + \Delta e_p^k$$

$$f_p^{k+1} = f_p^k + \Delta f_p^k$$

The process is repeated till the magnitude of the largest element in the residual column vector is less than the prespecified value. The sequence of steps for the solution of load flow problem using Newton-Raphson method is explained as follows (flow chart in Fig. 18.9):

1. Assume a suitable solution for all buses except the slack bus. Let $V_p = 1 + j0.0$ for $p = 1, 2, ..., n,\ p \neq s,\ V_s = a + j0.0$.

2. Set convergence criterion = ε *i.e.*, if the largest of absolute of the residues exceeds ε the process is repeated, otherwise it is terminated.

3. Set iteration count $K = 0$

4. Set bus count $p = 1$.

5. Check if p is a slack bus. If yes, go to step 10.

6. Calculate the real and reactive powers P_p and Q_p respectively using equations (18.26) and (18.25).

7. Evaluate $\Delta P_p^k = P_{sp} - P_p^k$.

8. Check if the bus in question is a generator bus. If yes, compare the Q_p^k with the limits. If it exceeds the limit, fix the reactive power generation to the corresponding limit and treat the bus as a load bus for that iteration and go to next step. If the lower limit is violated set $Q_{psp} = Q_{p\,min}$. If the limit is not violated evaluate the voltage residue.

$$|\Delta V_p|^2 = |V_p|^2_{spec} - |V_p^k|^2$$

and go to step 10.

9. Evaluate $\Delta Q_p^k = Q_{ps} - Q_p^k$.

10. Advance the bus count by 1, *i.e.*, $p = p + 1$ and check if all the buses have been accounted. If not, go to step 5.

11. Determine the largest of the absolute value of the residue.

12. If the largest of the absolute value of the residue is less than ε, go to step 17.

13. Evaluate elements for Jacobian matrix.

14. Calculate voltage increments Δe_p^k and Δf_p^k.

15. Calculate new bus voltages $e_p^{k+1} = e_p^k + \Delta e_p^k$ and $f_p^{k+1} = f_p^k + \Delta f_p^k$.

Evaluate cos δ and sin δ of all voltages.

16. Advance iteration count $K = K + 1$ and go to step 4.

17. Evaluate bus and line powers and print the results.

Example 18.4: The load flow data for the sample power system are given below. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator at bus 2 are 0.35 and 0.0 p.u. respectively. Determine the set of load flow equations at the end of first iteration by using Newton-Raphson method.

Impedance for sample system:

Bus code	*Impedance*	*Line charging admittance*
1–2	$0.08 + j0.24$	0.0
1–3	$0.02 + j0.06$	0.0
2–3	$0.06 + j0.18$	0.0

Schedule of generation and loads:

Bus code	*Assumed voltages*	*Generation*		*Load*	
		MW	*MVAR*	*MW*	*MVAR*
1	$1.06 + j0.0$	0	0	0	0
2	$1.0 + j0.0$	0.2	0.0	0.0	0.0
3	$1.0 + j0.0$	0	0	0.6	0.25

Solution:

$$y_{12} = \frac{1}{Z_{12}} = \frac{(0.08 - j0.24)}{(0.08 + j0.24)(0.08 - j0.24)}$$
$$= 1.25 \ - j3.75$$

Similarly $y_{13} = 5 - j15$ and $y_{23} = 1.6667 - j5.0$ and the nodal admittance matrix

$$Y = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.916 - j8.75 & -1.666 + j5.0 \\ -5 + j15 & -1.666 + j5.0 & 6.666 - j20 \end{bmatrix}$$

Assuming a flat voltage profile for bus 2 and 3 and for bus 1.

$$V_1 = 1.06 + j0.0$$

From the nodal admittance matrix and the assumed voltage solution,

$G_{11} = 6.25$	$B_{11} = 18.75$	$e_1 = 1.06$	$f_1 = 0.0$
$G_{12} = -1.25$	$B_{12} = -3.75$	$e_2 = 1.0$	$f_2 = 0.0$
$G_{13} = -5.0$	$B_{13} = -15.0$	$e_3 = 1.0$	$f_3 = 0.0$
$G_{22} = 2.916$	$B_{22} = 8.75$		
$G_{23} = -1.666$	$B_{23} = -5.0$		
$G_{33} = 6.666$	$B_{33} = 20.0$		

$$\begin{aligned} P_2 &= e_2(e_1G_{21} + f_1B_{21}) + f_2(f_1G_{21} - e_1B_{21}) + e_2(e_2G_{22} + f_2B_{22}) \\ &\quad + f_2(f_2G_{22} - e_2B_{22}) + e_2(e_3G_{23} + f_3B_{23}) + f_2(f_3G_{23} - e_3B_{23}) \\ &= 1.0\{1.06(-1.25) + 0.0\ (-3.75)\} + 0.0\{0.0\ (-1.25) - 1.06\ (-375)\} \\ &\quad + 1.0\{1.0(2.916)\} + 0.0\{\ \} + 1.0\{1.0\ (-1.666)\} + 0.0\ \{\ \} \\ &= -1.325 + 2.916 - 1.666 = -0.075 \end{aligned}$$

Similarly, $P_3 = -0.3$

$$Q_2 = f_2(\ ) - e_2(f_1 G_{21} - e_1 B_{21}) + f_2(\ ) - e_2(f_2 G_{22} - e_2 B_{22})$$
$$+ f_2(\ ) - e_2(f_3 G_{23} - e_3 B_{23}) = -0.225$$

$$Q_3 = -0.9$$

$\therefore$
$$\Delta P_2 = P_{2\,sp} - \mathrm{P}_{2\,cal} = 0.2 - (-0.075) = 0.275$$
$$\Delta P_3 = -0.6 - (-0.3) = -0.3$$

Since the lower limit on Q_2 is 0.0 and the value of Q_2 as calculated above violates this limit, bus 2 is treated as a load bus where $Q_{2\,sp} = 0.0$

$\therefore$
$$\Delta Q_2 = 0.0 - (-0.225) = 0.225$$
$$\Delta Q_3 = -0.25 - (-0.9) = 0.65$$

Diagonal elements

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_2}{\partial e_2} = 2e_2 G_{22} + e_1 G_{21} + f_1 B_{21} + e_3 G_{23} + f_3 B_{23}$$
$$= 2 \times 1.0 \times 2.916 + 1.06(-1.25) + 0.0(-3.75) + 1.0(-1.666)$$
$$+ 0.0(-5.0) = 2.848$$

$$\frac{\partial P_3}{\partial e_3} = 2e_3 G_{33} + e_1 G_{31} + f_1 B_{31} + e_2 G_{32} + f_2 B_{32}$$
$$= 2(1.0)(6.666) + 1.06(-5.0) + 0.0(-15.0)$$
$$+ 1.0(-1.666) + 0.0(-5.0) = 6.3666$$

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} + e_q B_{pq})$$

$$\frac{\partial P_2}{\partial f_2} = 2f_2 G_{22} + f_1 G_{21} - e_1 B_{21} + f_3 G_{23} - e_3 B_{23}$$
$$= 2(0.0)(2.916) + (0.0)(G_{21}) - 1.06(-3.75) - 1.0(-5.0) = 8.975$$

$$\frac{\partial P_3}{\partial f_3} = 20.90$$

Off-diagonal elements:

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_2}{\partial e_3} = e_2 G_{23} - f_2 B_{23} = -1.666$$

$$\frac{\partial P_3}{\partial e_2} = -1.666$$

$$\frac{\partial P_p}{\partial f_q} = +e_p B_{pq} + f_p G_{pq}$$

$$\frac{\partial P_2}{\partial f_3} = -5.0, \frac{\partial P_3}{\partial f_2} = -5.0$$

Similarly we find out the partial derivatives of the reactive power.

Diagonal elements:

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial Q_2}{\partial e_2} = 2e_2 B_{22} - f_1 G_{21} + e_1 B_{21} - f_3 G_{23} + e_3 B_{23}$$

$$= 2(1.0)(8.75) + 1.06\,(-3.75) + 1.0(-5.0) = 8.525$$

$$\frac{\partial Q_3}{\partial e_3} = 19.1$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial Q_2}{\partial f_2} = -2.991$$

$$\frac{\partial Q_3}{\partial f_3} = -6.966$$

Similarly off-diagonal elements are calculated and the final set of linear equations at the end of iteration 1 are

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.225 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5.0 \\ -1.666 & 6.366 & -5.0 & 20.90 \\ 8.525 & -5.0 & -2.991 & 1.666 \\ -5.0 & 19.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

Example 18.5: In case the reactive power constraint at bus 2 in the previous problem is $-0.3 \leq Q_2 \leq 0.3$. Determine the equations at the end of first iteration.

Solution: Since $Q_2 = -0.225$ and the lower limit is -0.3, therefore, the bus 2 behaves like a generator bus and

$$\Delta P_2 = 0.2 - (-0.075) = 0.275$$
$$\Delta P_3 = -0.6 - (-0.3) = -0.3$$
$$\Delta Q_3 = -0.25 - (-0.9) = 0.65$$

Since bus 2 behaves like a generator bus therefore

$$|\Delta V_2|^2 = |V_{2\,\text{sp}}|^2 - |V_{2\,\text{cal}}|^2 = 1.04^2 - 1.0^2 = 0.0816$$

The Jacobian elements corresponding to rows of ΔP_2, ΔP_3 and ΔQ_3 remain same as in previous problem, those of Q_2 will change and they are calculated as follows:

$$\frac{\partial |V_2|^2}{\partial e_2} = 2e_2 = 2.0, \frac{\partial |V_2|^2}{\partial e_3} = 0.0$$

$$\frac{\partial |V_2|^2}{\partial f_2} = 2f_2 = 0.0, \frac{\partial |V_2|^2}{\partial f_3} = 0.0$$

The set of equations will be

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.65 \\ 0.0816 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5.0 \\ -1.666 & 6.366 & -5.0 & 20.90 \\ -5.0 & 19.1 & 1.666 & -6.966 \\ 2.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

18.6 COMPARISON OF SOLUTION METHODS

Since the Gauss-Seidel is undoubtedly superior to Gauss method, the comparison is restricted only between G-S method and Newton-Raphson method and that too when Y bus matrix is used for problem formulation. From the view point of computer memory requirements, polar coordinates are preferred for solution based on N-R method and rectangular coordinates for the G-S method.

The time taken to perform one iteration of the computation is relatively smaller in case of G-S method as compared to N-R method but the number of iterations required by G-S method for a particular system are greater as compared to N-R method and they increase with the increase in the size of the system. In case of N-R method the number of iterations is more or less independent of the size of the system and vary between 3 to 5 iterations. The convergence characteristics of N-R method are not affected by the selection of a slack bus whereas that of G-S method is sometimes very seriously affected and the selection of a particular bus may result in poor convergence.

The main advantage of G-S method as compared to N-R method is its ease in programming and most efficient use of core memory. Nevertheless, for large power systems N-R method is found to be more efficient and practical from the view point of computational time and convergence characteristics. Even though N-R method can solve most of the practical problems, it may fail in respect of some ill-conditioned problem where other advanced mathematical programming techniques like the non-linear programming techniques can be used. For the readers to have an approximate idea of the computation time taken by N-R method in solving the load flow problem is that, IBM 360/PS 44 system takes less than 10 seconds to obtain a load flow solution of a 14 bus system starting with a flat voltage solution of $(1 + j0.0)$. This includes the formulation of nodal admittance matrix and its storage time.

18.7 APPROXIMATION TO NEWTON-RAPHSON METHOD

It is well known that a small change in phase angle changes the flow of active power and does not affect much the flow of reactive power. Similarly a small change in nodal voltage affects the flow of reactive power whereas active power practically does not change. Keeping these facts in mind and using the polar coordinates, the set of linear load flow equations can be written in matrix form as follows:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ |\Delta V| \end{bmatrix} \tag{18.44}$$

Here J_1 corresponds to the elements $\dfrac{\partial P}{\partial \delta}$ which exist

J_2 corresponds to the elements $\dfrac{\partial P}{\partial |V|}$ which do not exist and, therefore, are zero.

J_3 corresponds to the elements $\dfrac{\partial Q}{\partial \delta}$ which do not exist and, therefore, are zero.

J_4 corresponds to the elements $\dfrac{\partial Q}{\partial |V|}$ which exist.

This certainly simplifies the calculation and results in smaller computation time.

18.8 LINE FLOW EQUATIONS

After the iterative solution of bus voltages is completed, line flows can be calculated. The current at bus p (Fig. 18.10) in the line connecting bus p to q is

$$i_{pq} = (V_p - V_q)y_{pq} + V_p \frac{y'_{pq}}{2} \tag{18.45}$$

Now $$P_{pq} - jQ_{pq} = V^*_{pq}\, i_{pq}$$

or $$P_{pq} - jQ_{pq} = V_p^* \left[(V_p - V_q)y_{pq} + V_p \frac{y'_{pq}}{2} \right]$$

$$= V_p^*(V_p - V_q)y_{pq} + V_p^* V_p \frac{y'_{pq}}{2} \tag{18.46}$$

Here P_{pq} is the real power flow from bus p to q and Q_{pq} is the reactive power flow from bus p to q.

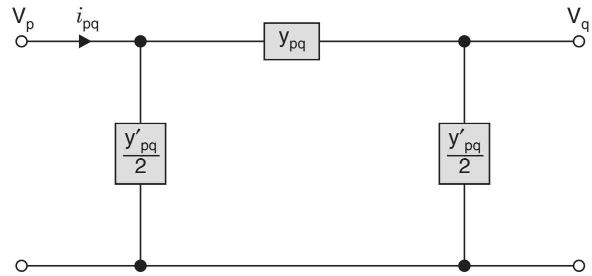

Fig. 18.10 Equivalent circuit of a transmission link for evaluating line flows.

Similarly, at bus q the power flow from bus q to p is

$$P_{qp} - jQ_{qp} = V_q^* (V_q - V_p) y_{pq} + V_q^* V_q \frac{y'_{pq}}{2} \tag{18.47}$$

The power loss in line pq is the algebraic sum of the power flows $(P_{qp} - jQ_{pq})$ and $(P_{pq} - jQ_{qp})$.

18.9 FAST-DECOUPLED LOAD FLOW

This is an extension of Newton-Raphson method formulated in polar coordinates with certain approximation which results into a fast algorithm for load flow solution. Before we discuss this method, we derive load flow equations in polar coordinates.

We know that,

$$P_p - jQ_p = V_p^* I_p \qquad \text{and} \qquad I_p = \sum_{q=1}^{n} Y_{pq} V_q$$

$$\therefore \qquad P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q \tag{18.48}$$

The voltage and admittance in polar coordinates are expressed as

$$V_p = | V_p | \exp (j\delta_{\text{p}}) \qquad \text{and} \qquad Y_{pq} = | Y_{pq} | \exp (-j\theta_{pq})$$

Substituting these values in equation (18.48), we obtain

$$P_p - jQ_p = | V_p | \exp (-j\delta_p) \sum_{q=1}^{n} | Y_{pq} | \exp (-j\theta_{pq}) | V_q | \exp (j\delta_q)$$

$$= \sum_{q=1}^{n} | V_p | | V_q | | Y_{pq} | \exp \{-j(\theta_{pq} + \delta_p - \delta_q)\}$$

$$P_p = \sum_{q=1}^{n} | V_p V_q Y_{pq} | \cos (\theta_{pq} + \delta_p - \delta_q) \tag{18.49}$$

and $$Q_p = \sum_{q=1}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \tag{18.50}$$

$$p = 1, 2, ..., n$$

Equations (18.49) and (18.50) are rewritten as

$$P_p = |V_p V_p Y_{pp}| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \tag{18.51}$$

$$Q_p = |V_p V_p Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \tag{18.52}$$

These equations after linearization can be rewritten in matrix form as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \tag{18.53}$$

Here H, N, M and L are elements of Jacobian matrix.

The first assumption under decoupled load flow method is that real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to angle. Similarly, the reactive power changes are less sensitive to change in angle but mainly sensitive to change in voltage magnitude. With these assumptions, equation (18.53) reduce to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & O \\ O & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \tag{18.54}$$

The equation (18.54) is decoupled equation which can be expanded as

$$[\Delta P] = [H][\Delta\delta] \tag{18.54 a}$$

and $$[\Delta Q] = [L][\Delta|E|/|E|] \tag{18.54 b}$$

Using equations (18.51) and (18.52) the elements of the Jacobian matrices H and L are obtained as follows:

Off-diagonal element of H is

$$\begin{aligned}
H_{pq} = \frac{\partial P_p}{\partial \delta_q} &= |V_p V_q Y_{pq}| \sin[\theta_{pq} + \delta_p - \delta_q] \\
&= |V_p V_q Y_{pq}| [\sin\theta_{pq} \cos(\delta_p - \delta_q) + \sin(\delta_p - \delta_q) \cos\theta_{pq}] \\
&= |V_p V_q| [|Y_{pq}| \sin\theta_{pq} \cos(\delta_p - \delta_q) + |Y_{pq}| \cos\theta_{pq} \sin(\delta_p - \delta_q)] \\
&= |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)]
\end{aligned} \tag{18.55}$$

Similarly off-diagonal element of L is

$$\begin{aligned}
L_{pq} = \frac{\partial Q_p |V_q|}{\partial V_q} &= |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \\
&= [V_p V_q] [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)]
\end{aligned} \tag{18.56}$$

From equations (18.55) and (18.56), it is seen that

$$H_{pq} = L_{pq} = |\, V_p\, V_q \,|\ [G_{pq} \sin (\delta_p - \delta_q) - B_{pq} \cos (\delta_p - \delta_q)]$$

The diagonal elements of H are given as

$$H_{pp} = \frac{\partial P_p}{\partial \delta p} = -\sum_{\substack{q=1\\ q \neq p}}^{n} |\, V_p\, V_q\, Y_{pq} \,| \sin (\theta_{pq} + \delta_p - \delta_q)$$

$$= -Q_p + |\, V_p\, V_p\, V_{pp} \,| \sin \theta_{pp}$$

$$= -Q_p - V_p^2\, B_{pp} \qquad (18.57)$$

Similarly diagonal elements for the matrix are given by:

$$L_{pp} = \frac{\partial Q_p\, |V_p|}{\partial V_p} = |\, 2V_p^2\, Y_{pp} \,| \sin \theta_{pp} + \sum_{\substack{q=1\\ q \neq p}}^{n} |\, V_p\, V_q\, Y_{pq} \,| \sin (\theta_{pq} + \delta_p - \delta_q)$$

$$= |\, 2V_p^2\, Y_{pp} \,| \sin \theta_{pp} + Q_p - |\, V_p^2\, Y_{pp} \,| \sin \theta_{pp}$$

$$= Q_p + |\, V_p^2\, Y_{pp} \,| \sin \theta_{pp} = Q_p - V_p^2\, B_{pp} \qquad (18.58)$$

In the case of fast decoupled load flow method following approximations are further made for evaluating Jacobian element:

$$\cos (\delta_p - \delta_q) \simeq 1$$

$$G_{pq} \sin (\delta_p - \delta_q) \leq B_{pq}$$

and $$Q_p << B_{pp}\, V_p^2$$

∴ The Jacobian elements now become

$$L_{pq} = H_{pq} = -\,|\, V_p\, V_q \,|\ \mathrm{B}_{pq} \text{ for } q \neq p$$

and $$L_{pp} = H_{pp} = -B_{pp}\, |V_p|^2$$

With these Jacobian elements equations 18.54 (*a*) and 18.54 (*b*) become

$$[\Delta P_p] = [V_p][V_q][\mathrm{B}'_{pq}][\Delta\delta_q] \qquad (18.59)$$

and $$[\Delta Q_p] = [V_p][V_q][\mathrm{B}''_{pq}]\ \frac{\Delta |E_q|}{E_q} \qquad (18.60)$$

where B'_{pq} and B''_{pq} are the elements of $[-B_{pq}]$ matrix.

Further decoupling is obtained as follows:

(*i*) Omit from B'' the angle shifting effects of phase shifters.

(*ii*) Omit from B' the representation of those network elements that affect MVAr flows *i.e.*, shunt reactors and off-nominal in phase transformer taps.

(*iii*) Divide equations (18.59) and (18.60) by V_p and assuming $V_q = 1$ p.u. and also neglecting the series resistance in calculating the elements of B'.

With these assumptions, equations (18.59) and (18.60) for the load flow solution take the form

$$\left[\frac{\Delta P_p}{E_p}\right] = [B'][\Delta\delta] \tag{18.61}$$

and

$$\left[\frac{\Delta Q_p}{E_p}\right] = [B''][\Delta E] \tag{18.62}$$

It is to be noted that $[B']$ and $[B'']$ are real and sparse and have similar structures as those of H and L respectively. Since the two matrices are constant and do not change during successive iterations for solution of the load flow problem, they need be evaluated only once and inverted once during the first iteration and then used in all successive iterations. It is because of the nature of Jacobian matrices $[B']$ and $[B'']$ and the sparsity of these matrices that the method is fast.

PROBLEMS

18.1. What is load flow solution? Explain its significance in power system analysis.

18.2. Classify various types of buses in a power system for load flow studies. Justify the classification.

18.3. Explain clearly how the nodal admittance matrix of a system is changed when an on-load tap changing transformer is introduced in a line connected between two buses.

18.4. Explain clearly with a flow chart the computational procedure for load flow solution using Gauss-Seidel method when the system contains all types of buses.

18.5. Explain clearly with a flow chart the computational procedure for load flow solution using Newton-Raphson method when the system contains all types of buses.

18.6. Develop load flow equations suitable for solution by (*i*) Gauss-Seidel method, (*ii*) Newton-Raphson method, using nodal admittance approach.

18.7. Compare the performance of Gauss-Seidal method and Newton-Raphson method for load flow solution using nodal admittance approach for the formulation of load flow equations.

18.8. The load flow data for a three-bus system is given in tables I and II. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits for bus 2 are 0.3 and 0.0 p.u. respectively. Taking bus 1 as the slack bus, determine the voltages of the various buses at the end of first iteration starting with a flat voltage profile for all buses except slack bus using (*i*) Gauss-Seidel method with acceleration factors 1.6, and (*ii*) Newton-Raphson method (only the set of equations are required).

Table I. Impedances for sample system of Problem 18.8

Bus code	*Impedance*	*Bus code*	*Line changing admittance* $\frac{y'_{pq}}{2}$
1–2	$0.06 + j0.18$	1	$j\,0.05$
1–3	$0.02 + j0.06$	2	$j0.06$
2–3	$0.04 + j0.12$	3	$j0.05$

Table II. Assumed bus voltages, generation and loads

Bus code	*Assumed voltages*	*Generation*		*Load*	
		MW p.u.	*MV Ar p.u.*	*MW p.u.*	*MV Ar p.u*
1	1.06 + *j*0.0	0.0	0.0	0.0	0.00
2	1.0 + *j*0.0	0.2	0.0	0.0	0.00
3	1.0 + *j*0.0	0.0	0.0	0.6	0.25

18.9. Repeat Problem 18.8 when the constraints on reactive power are:

$$0.2 \leq Q_2 \leq 0.5$$

18.10. The load flow data of a four-bus system is given in Tables III and IV. Taking bus 1 as slack bus determine the voltages of all buses at the end of first iteration starting with a flat voltage profile using (*i*) Gauss-Seidel method, (*ii*) Newton-Raphson method (only the set of equations are required).

Table III. Impedance and line charging admittances

Bus Code	*Impedance*	*Line charging* $\frac{y_{pq}}{2}$
1–2	0.02 + *j*0.08	0.0 + *j*0.040
1–3	0.06 + *j*0.24	0.0 + *j*0.030
2–3	0.04 + *j*0.16	0.0 + *j*0.025
2–4	0.04 + *j*0.16	0.0 + *j*0.025
3–4	0.01 + *j*0.04	0.0 + *j*0.015

Table IV. Assumed bus voltages, generations and loads

Bus code	*Assumed voltages*	*Generation*		*Load*	
		MW	*MV Ar*	*MW*	*MV Ar*
1	1.06 + *j*0.0	0.0	0.0	0.0	0.0
2	1.0 + *j*0.0	0.0	0.0	0.2	0.1
3	1.0 + *j*0.0	0.0	0.0	0.5	0.2
4	1.0 + *j*0.0	0.0	0.0	0.4	0.05

18.11. The load flow data of a four-bus system is given in Table III of Problem 18.10 and Table V. Assume the bus voltage of No. 3 as 1.04 p.u. and the maximum and minimum reactive power constraints for bus 3 are 0.3 and 0.0 p.u. respectively. Taking bus 1 as slack bus determine the voltages of all the buses at the end of first iteration starting with a flat voltage profile for all buses except the slack bus using (*i*) Gauss-Seidel method, and (*ii*) Newton-Raphson method.

Table V. Assumed voltages, generation and loads

Bus code	*Assumed voltages*	*Generation*		*Loads*	
		MW	*MV Ar*	*MW*	*MV Ar*
1	$1.06 + j0.0$	0.0	0.0	0.0	0.0
2	$1.0 + j0.0$	0.0	0.0	0.2	0.1
3	$1.0 + j0.0$	0.6	0.3	0.5	0.2
4	$1.0 + j0.0$	0.0	0.0	0.4	0.05

18.12. Repeat Problem 18.11, when the reactive power constraints of bus 3 are

$$-0.1 \le Q_3 \le 1.0.$$

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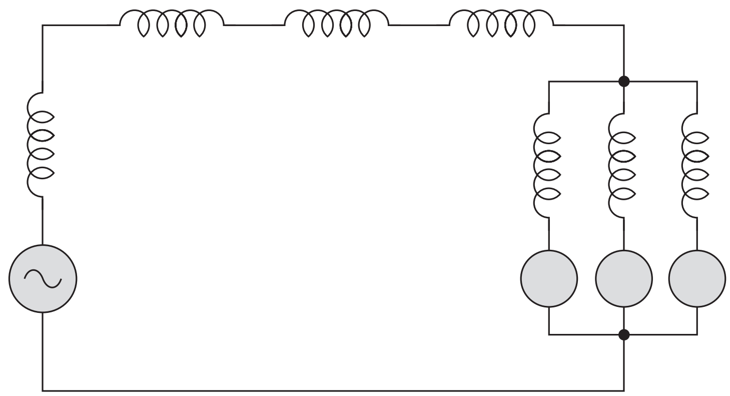
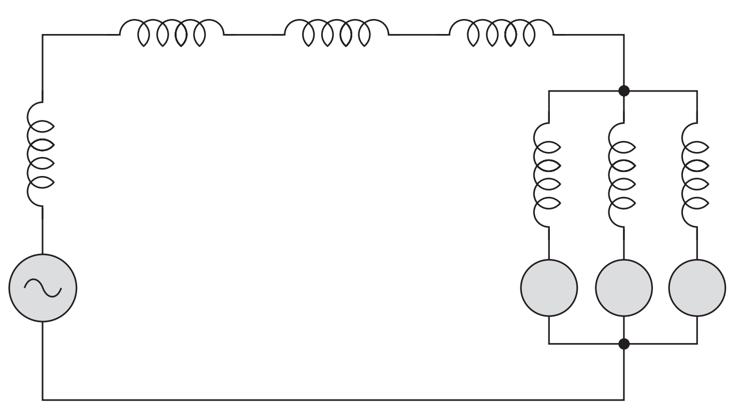


19

ECONOMIC LOAD DISPATCH



19

Economic Load Dispatch

INTRODUCTION

The economic load dispatch problem involves the solution of two different problems. The first of these is the Unit Commitment or predispatch problem wherein it is required to select optimally out of the available generating sources to operate, to meet the expected load and provide a specified margin of operating reserve over a specified period of time. The second aspect of economic dispatch is the on-line economic dispatch wherein it is required to distribute the load among the generating units actually paralleled with the system in such manner as to minimize the total cost of supplying the minute-to-minute requirements of the system.

It is this aspect of the economic dispatch problem that has been discussed in this chapter.

With large interconnection of the electric networks, the energy crisis in the world and continuous rise in prices, it is very essential to reduce the running charges of the electric energy *i.e.*, reduce the fuel consumption for meeting a particular load demand.

In the last chapter we made load flow study for the power system, in that for a particular load demand the generation at all the generator buses are fixed except at one generator bus known as slack, reference or swing bus where we allow the generation to take value within certain limits. In case of economic load dispatch the generations are not fixed but they are allowed to take values again within certain limits so as to meet a particular load demand with minimum fuel consumption. This means economic load dispatch problem is really the solution of a large number of load flow problems and choosing the one which is optimal in the sense that it needs minimum cost of generation. It is clear from this that since total cost of generation is a function of the individual generation of the sources which can take values within certain constraints, the cost of generation will depend upon the system constraint for a particular load demand. This means the cost of generation is not fixed for a particular load demand but depends upon the operating constraints of the sources. In fact the modern power system has to operate under various operational and network constraints. It is, therefore, best to understand the various constraints before actually taking up the economic load dispatching problem.

19.1 SYSTEM CONSTRAINTS

Broadly speaking there are two types of constraints: (*i*) Equality constraints, and (*ii*) Inequality constraints. Inequality constraints are of two types: (*i*) Hard type, and (*ii*) Soft type. The hard type are those which are definite and specific like the tapping range of an on-load tap changing transformer whereas soft type are those which have some flexibility associated with them like the nodal voltages and phase angles between the nodal voltages, etc. Soft inequality constraints have been very efficiently handled by penalty function methods.

Equality Constraints

The equality constraints are the basic load flow equations given by

$$P_p = \sum_{q=1}^{n} \{e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})\}$$

$$Q_p = \sum_{q=1}^{n} \{f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_p G_{pq} - e_q B_{pq})\}$$

$$p = 1, 2, \ldots, n$$

where e_p and f_p are the real and imaginary components of voltage at the pth node and G_{pq} and B_{pq} are the nodal conductance and susceptance between the pth and qth nodes.

Inequality Constraints

(*a*) *Generator Constraints:* The kVA loading on a generator is given by $\sqrt{P_p^2 + Q_p^2}$ and this should not exceed a prespecified value C_p because of the temperature rise conditions, *i.e.,*

$$P_p^2 + Q_p^2 \le C_p^2$$

The maximum active power generation of a source is limited again by thermal consideration and also minimum power generation is limited by the flame instability of a boiler. If the power output of a generator for optimum operation of the system is less than a prespecified value $P_{\min}$ the unit is not put on the bus bar because it is not possible to generate that low value of power from that unit. Hence the generator powers P_p cannot be outside the range stated by the inequality, *i.e.,*

$$P_{p\,\min} \le P_p \le P_{p\,\max}$$

Similarly the maximum and minimum reactive power generation of a source are limited. The maximum reactive power is limited because of overheating of the rotor and minimum is limited because of the stability limit of the machine. Hence the generator reactive power Q_p cannot be outside the range stated by the inequality, *i.e.,*

$$Q_{p\,\min} \le Q_p \le Q_{p\,\max}$$

(*b*) *Voltage Constraints:* It is essential that the voltage magnitudes and phase angles at various nodes should vary within certain limits. The voltage magnitudes should vary within certain limits because otherwise most of the equipments connected to the system will not operate satisfactorily or additional use of voltage regulating devices will make the system uneconomical. Thus

$$| V_{p\,\min} | \le | V_p | \le | V_{p\,\max} |$$
$$\delta_{p\,\min} \le \delta_p \le \delta_{p\,\max}$$

where $| V_p |$ and δ_p stand for the voltage magnitude and phase angle at the pth node.

The normal operating angle of transmission line lies between 30° to 45° for transient stability reasons; therefore a higher limit is imposed on angle δ. A lower limit of δ assures proper utilization of transmission facility.

(*c*) *Running Spare Capacity Constraints:* These constraints are required to meet: (*i*) the forced outages of one or more alternators on the system, and (*ii*) the unexpected load on the system.

The total generation should be such that in addition to meeting load demand and losses a minimum spare capacity should be available *i.e.,*

$$G \ge P_p + P_{so}$$

where G is the total generation and P_{so} is some prespecified power. A well planned system is one in which this space capacity P_{so} is minimum.

(*d*) *Transformer Tap Settings:* If an auto-transformer is used, the minimum tap setting could be zero and the maximum one, *i.e.,*

$$0 \le t \le 1.0$$

Similarly for a two winding transformer if tappings are provided on the secondary side,

$$0 \le t \le n$$

where n is the ratio of transformation. Phase shift limits of the phase shifting transformer

$$\theta_{p\,\min} \le \theta_p \le \theta_{p\,\max}$$

(*e*) *Transmission Line Constraints:* The flow of active and reactive power through the transmission line circuit is limited by the thermal capability of the circuit and is expressed as

$$C_p \le C_{p\,\max}$$

where $C_{p\,\max}$ is the maximum loading capacity of the pth line.

(*f*) *Network Security Constraints:* If initially a system is operating satisfactorily and there is an outage, may be scheduled or forced one, it is natural that some of the constraints of the system will be violated. The complexity of these constraints (in terms of number of constraints) is increased when a large system is under study. In this case a study is to be made with outage of one branch at a time and then more than one branches at a time. The nature of constraints are same as voltage and transmission line constraints.

This completes more or less the study of various constraints. We now switch over to the actual problem of economic load dispatch. Since we are considering a system which is already in existence, the fixed costs like salaries and capital costs are neglected from the cost criterion. Also, we consider here only the thermal plants and, therefore, we are concerned only with the cost of fuel input to different generating stations.

This chapter will deal into the problem of optimization using the conventional B-coefficient method for loss evaluation.

In the past the practice was to load the machines such that the incremental cost of production for all the machines was same and thus transmission line was not considered an integral part of the system. Later on, it was realized that a plant far from the load has to supply in addition to the load, transmission losses also and, therefore, this plant must be penalized. These transmission line losses are calculated by making use of the B-coefficients.

Before we take up the economic operation problem a few definitions are warranted. A simplified performance curve of a boiler turbine generator unit is given in Fig. 19.1. From the curve heat rate is defined as the ratio of fuel input to the corresponding power output and hence the units are million Btu per MWhr. From the heat rate curve another characteristic known as incremental fuel curve can be obtained. Incremental fuel rate is defined as

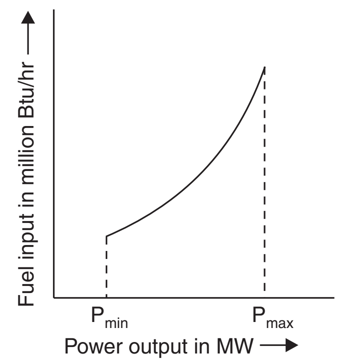


Fig. 19.1 Input-output curve.

$$\text{Incremental fuel rate} = \frac{\Delta \text{ input}}{\Delta \text{ output}}$$

which means it is a ratio equal to a small change in input to the corresponding small change in output. Now as the incremental quantities tend to zero, incremental fuel rate tends to

$$\text{Incremental fuel rate} = \frac{d(\text{input})}{d(\text{output})} = \frac{dF}{dP}$$

where F is the fuel input in million Btu per hr and P is the power output in MW. The units associated with incremental fuel rate are, therefore, million Btu per MWhr and the incremental fuel cost is expressed in terms of Rs. per MWhr which is obtained by multiplying the incremental fuel rate by the fuel cost in Rs. per Btu.

Incremental production cost consists of the incremental fuel cost plus the incremental cost of labour, supplies, maintenance and water. Since it is difficult to express exactly these costs as a function of output and also since they form generally a small fraction of the incremental cost of fuel, the incremental cost of production will hitherto be considered equal to the incremental cost of fuel. The incremental efficiency is defined as the reciprocal of incremental fuel rate or incremental production cost and is given as

$$\text{Incremental efficiency} = \frac{(\text{output})}{(\text{input})} = \frac{dP}{dF}$$

Merit Order Scheduling

When this method is applied to economic scheduling of power plants, it assumes that the incremental cost of all the generators is constant over the full range or over successive discrete portions within the range. The economical way of meeting a load will be to load the machines in order of highest incremental efficiency. This method, therefore, needs forming of a table which could be looked into, for any load condition and does not need any complicated calculations. Normally the incremental cost curves are not constant; therefore, this method is not used and will not be discussed henceforth.

19.2 ECONOMIC DISPATCH NEGLECTING LOSSES

The economic dispatch problem is defined as

$$\text{Min } F_T = \sum_{n=1}^{n} F_n \tag{19.1}$$

$$\text{Subject to } P_D = \sum_{n=1}^{n} P_n \tag{19.2}$$

where F_T is total fuel input to the system, F_n the fuel input to nth unit, P_D the total load demand and P_n the generation of nth unit.

By making use of Lagrangian multiplier the auxiliary function is obtained as

$$F = F_T + \lambda(P_D - \sum_{n=1}^{n} P_n)$$

where λ is the Lagrangian multiplier.

Differentiating F with respect to the generation P_n and equating to zero gives the condition for optimal operation of the system.

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda(0 - 1) = 0$$

$$= \frac{\partial F_T}{\partial P_n} - \lambda = 0$$

Since $\quad F_T = F_1 + F_2 + \dots + F_n,$

$\therefore \quad \dfrac{\partial F_T}{\partial P_n} = \dfrac{dF_n}{dP_n} = \lambda$

and therefore the condition for optimum operation is

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \dots = \frac{dF_n}{dP_n} = \lambda \tag{19.3}$$

Here $\dfrac{dF_n}{dP_n}$ = incremental production cost of plant n in Rs. per MWhr.

The incremental production cost of a given plant over a limited range is represented by

$$\frac{dF_n}{dP_n} = F_{nn}P_n + f_n$$

where F_{nn} = slope of incremental production cost curve, and

f_n = intercept of incremental production cost curve.

The equation (19.3) mean that the machines be so loaded that the incremental cost of production of each machine is same. It is to be noted here that the active power generation constraints are taken into account while solving the equations which are derived above. If these constraints are violated for any generator it is tied to the corresponding limit and the rest of the load is distributed to the remaining generator units according to the equal incremental cost of production. The simultaneous solution of equations (19.2) and (19.3) gives the economic operating schedule. Therefore, any good technique for solving a set of linear equations can be used but since the inequality constraints have also to be taken into account, the following iterative method is used:

1. Assume a suitable value of λ^0. This value should be more than the largest intercept of the incremental cost characteristics of the various generators.

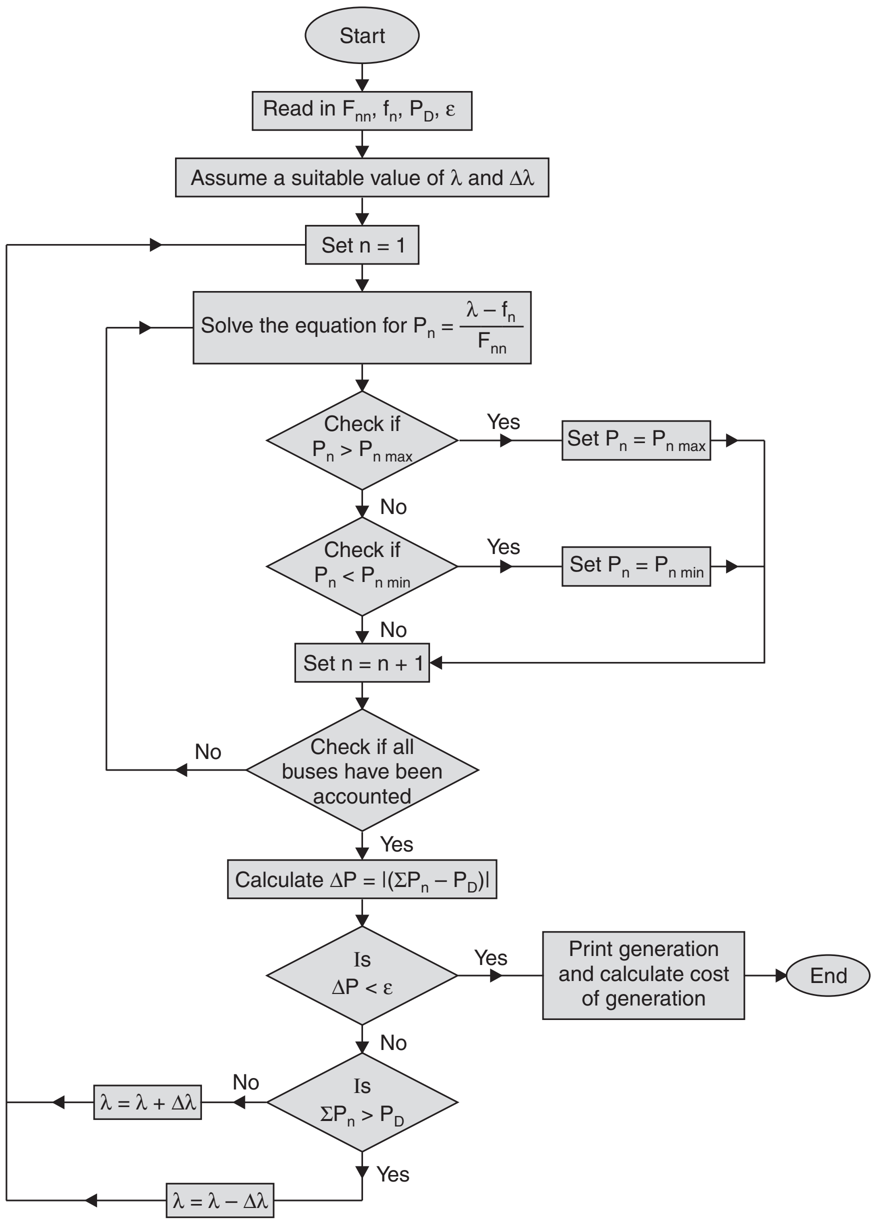


Fig. 19.2 Flow chart for economic scheduling: transmission loss neglected.

2. Compute the individual generations P_1, P_2, ... , P_n corresponding to incremental cost of production from equation (19.3). In case generation at any of the buses is violated, the generation of that generator is fixed at the limit violated during that iteration and the remaining load is distributed among the remaining generators.

3. Check if the equality

$$\sum_{n=1}^{n} P_n = P_D \text{ is satisfied.}$$

4. If not, make a second guess λ' and repeat the above steps. The selection of λ' in this step must of course be guided by the result in step (3). For example, if we find that the total generation is less than P_D then correct value of λ to be selected would be

$$\lambda_{corr} > \lambda^0$$

If equality is satisfied, the generations as calculated in step (2) give the optimum operating strategy. The flow chart is given in Fig. 19.2.

To illustrate the above procedure, the following problem is considered. There are two generators of 100 MW each with incremental characteristics:

$$\frac{dF_1}{dP_1} = 2 + 0.012\,P_1$$

$$\frac{dF_2}{dP_2} = 1.5 + 0.015\,P_2$$

Minimum load on each unit is 10 MW, total load to be supplied is 150 MW. Determine the economic operating schedule.

Solution: Assume $\lambda = 2.8$ (more than the largest of the intercepts (2, 1.5). Determine generations P_1 and P_2.

$$P_1 = \frac{2.8 - 2.0}{0.012} = 66.6 \text{ MW}$$

$$P_2 = \frac{2.8 - 1.5}{0.015} = 86.6 \text{ MW}$$

$$\therefore \qquad P_1 + P_2 = 66.6 + 86.6 = 153.2$$

Since generation is greater than the demand, therefore next approximation of λ should be less than 2.8. Also since load demand is 150 MW, the difference between P_D and P_n is not much. Therefore, value of λ should not be very much different from 2.8. Let it be 2.7.

With this, the generations are 58.4 + 80 = 138.4 MW, which is less than the demand. Therefore, next approximation of λ should be greater than 2.7; say it is 2.75. Corresponding to this the generations are 62.5 + 83.4 = 145.9 MW *i.e.,* the generations are smaller than the demand. Next approximation to λ = 2.78 which gives generations as 65 + 85.4 = 150.4 MW. It is to be seen here that the generations are converging as better approximations to λ are chosen and finally a value between 2.77 and 2.78 will give the optimal generation schedule.

It can be seen that this method does not sense the location of changes in the loads. As long as the total load is fixed, irrespective of the location of loads, the solution will always be the same and in fact for this reason the solution may not be feasible in the sense that the load voltages may not be within specified limits, the reactive power generations required also may not be within limits.

In case of an urban area where the load density is very high and the transmission distances are very small, the transmission loss could be neglected and the optimum strategy of generation could be based on the equal incremental production cost as outlined above. Whereas if the energy to be transported is over relatively larger distances with low load density, the transmission losses in some cases may amount 20 to 30% of the total load and it then becomes very essential to take these losses into account when developing an economic dispatch strategy.

19.3 OPTIMUM LOAD DISPATCH INCLUDING TRANSMISSION LOSSES

Before an optimum strategy for load scheduling is derived, the need for inclusion of losses is further stressed by the following example.

Consider Fig. 19.3 which consists of two identical generators *i.e.*, generators with identical incremental production cost. If generator 2 has a local load, according to equal incremental production criterion, the total load must be shared equally by both the generators, *i.e.*, each generator should supply half of the total load. The common sense tells us that it is more economical to let generator 2 supply most of the local load because generator 1 has to supply in addition to the load, the transmission losses also. Therefore, the criterion of sharing load by equal incremental production cost does not hold good under such situation and a strategy must be evolved which takes into account the transmission losses also.

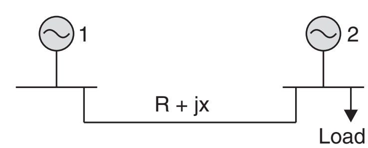


Fig. 19.3 Two identical generators connected through a transmission link.

The optimal load dispatch problem including transmission losses is defined as

$$\text{Min } F_T = \sum_{n=1}^{n} F_n \tag{19.4}$$

$$\text{Subject to } P_D + P_L - \sum_{n=1}^{n} P_n = 0 \tag{19.5}$$

where P_L is the total system loss which is assumed to be a function of generation and the other term have their usual significance.

Making use of the Lagrangian multiplier λ, the auxiliary function is given by

$$F = F_T + \lambda(P_D + P_L - \sum P_n)$$

The partial differential of this expression when equated to zero gives the condition for optimal load dispatch, *i.e.,*

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda\left(\frac{\partial P_L}{\partial P_n} - 1\right) = 0$$

or
$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \tag{19.6}$$

Here the term $\frac{\partial P_L}{\partial P_n}$ is known as the incremental transmission loss at plant n and λ is known as the incremental cost of received power in Rs. per MWhr.

The equation (19.6) is a set of n equations with $(n + 1)$ unknowns. Here n generations are unknown and λ is also unknown. These equations are known as coordination equations because they coordinate the incremental transmission losses with the incremental cost of production.

To solve these equations the loss formula equation (19.7) is expressed in terms of generations and is approximately expressed as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \tag{19.7}$$

where P_m and P_n are the source loadings, B_{mn} the transmission loss coefficients. The formula is derived under the following assumptions:

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current.

2. The generator bus voltage magnitudes and angles are constant.

3. The power factor of each source is constant.

The solution of coordination equation (19.6) requires the calculation of $\partial P_L/\partial P_n$ which is obtained from equation (19.7) as

$$\frac{\partial P_L}{\partial P_n} = 2\sum_m B_{mn} P_m \tag{19.8}$$

Also
$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n$$

∴ The coordination equations can be rewritten as

$$F_{nn} P_n + f_n + \lambda \Sigma 2B_{mn} P_m = \lambda \tag{19.9}$$

Collecting all coefficients of P_n, we obtain

$$P_n (F_{nn} + 2\lambda B_{nn}) = -\lambda \Big(\sum_{m \neq n} 2B_{mn} P_m\Big) - f_n + \lambda$$

Solving for P_n we obtain

$$P_n = \frac{1 - \dfrac{f_n}{\lambda} \displaystyle\sum_{m \neq n} 2B_{mn} P_m}{\dfrac{F_{nn}}{\lambda} + 2B_{nn}} \tag{19.10}$$

To arrive at an optimal load dispatching solution, the simultaneous solution of the coordination equations along with the equality constraint (19.5) should suffice and any standard matrix inversion subroutine could be used. But, because of the fact that plants might go beyond their loading conditions, it becomes necessary to solve a new set of equations and thus by the

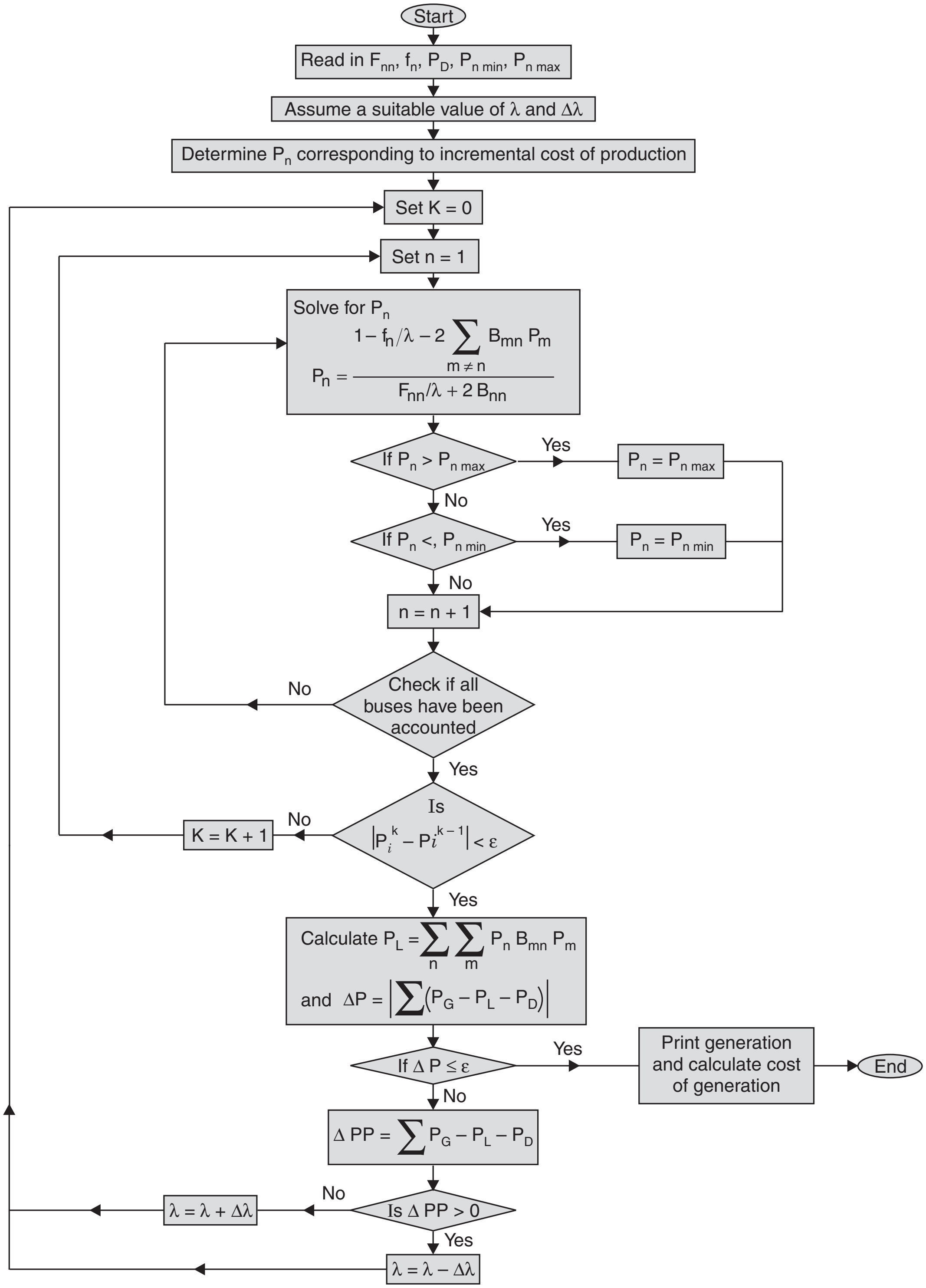


Fig. 19.4 Flow chart for the solution of coordination equations.

process of elimination this could be done. This would be very time consuming in a large interconnected system. Therefore, an iterative procedure would be used. The iterative procedure involves a method of successive approximation which rapidly converge to the correct solution. The following steps are required for the iterative procedure:

1. Assume a suitable value of λ^0. This value should be more than the largest intercept of the incremental production cost of the various generators.

2. Calculate the generations based on equal incremental production cost.

3. Calculate the generation at all the buses using the equation

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m \neq n} 2B_{mn}P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}}$$

It is to be noted that the powers to be substituted on the right hand side during zeroth iteration correspond to the values as calculated in step 2. For subsequent iterations the values of powers to be substituted correspond to the powers as calculated in the previous iteration. In case any of the generations violates the limit the generation of that generator is fixed at the limit violated.

4. Check if the difference in power at all generator buses between two consecutive iterations is less than a prespecified value. If not, go back to step 3.

5. Calculate losses using the relation

$$P_L = \sum_m \sum_n P_n B_{mn} P_m$$

and calculate

$$\Delta P = |\, \Sigma P_G - P_L - P_D \,|$$

6. If ΔP is less than ε, stop calculation and calculate cost of generation with these values of powers.

7. Update value of λ and go back to step 3. The flow chart is given in Fig. 19.4.

The iterative procedure is illustrated for a simple two-plant system. Assume that the loss formula coefficients in 1/MW units are given by

m	n	B_{mn}
1	1	0.0015
1	2	– 0.0005
2	2	0.0025

Also assume

$$\frac{dF_1}{dP_1} = 0.01P_1 + 2.0$$

and

$$\frac{dF_2}{dP_2} = 0.01P_2 + 1.5$$

The objective is to determine the operating schedule corresponding to $\lambda = 2.6$ (say).

Now
$$P_1 = \frac{1 - \frac{f_1}{\lambda} - 2B_{12}P_2}{\frac{F_{11}}{\lambda} + 2B_{11}}$$

$$P_2 = \frac{1 - \frac{f_2}{\lambda} - 2B_{12}P_1}{\frac{F_{22}}{\lambda} + 2B_{22}}$$

Substituting the values for f_1, f_2, F_{11}, F_{22} and loss coefficients,

$$P_1 = \frac{1 - \frac{2}{2.6} + 2 \times 0.0005P_2}{\frac{0.01}{2.6} + 2 \times 0.0015} = \frac{0.230769 + 0.001P_2}{0.00384615 + 0.0030}$$

$$= \frac{0.230769 + 0.001P_2}{0.00684615}$$

and
$$P_2 = \frac{1 - \frac{1.5}{2.6} + 2 \times 0.005P_1}{\frac{0.01}{2.6} + 2 \times 0.0025} = \frac{0.423077 + 0.001P_1}{0.00384615 + 0.005}$$

$$= \frac{0.423077 + 0.001P_1}{0.00884615}$$

Now generation corresponding to equal incremental cost of production is calculated as follows:

$$P_1 = 0.01P_1 + 2 \text{ or } P_1 = \frac{0.6}{0.01} = 60 \text{ MW}$$

$$P_2 = \frac{1.1}{0.01} = 110 \text{ MW}$$

Substituting these values of generations in the expressions for P_1 and P_2, we get

$$P_1 = \frac{0.230769 + 0.001 \times 110}{0.00684615} = 49.77 \text{ MW}$$

$$P_2 = \frac{0.423077 + 0.001 \times 60}{0.00884615} = 54.61 \text{ MW}$$

$$P_1 = \frac{0.230769 + 0.001 \times 54.61}{0.00684615} = 41.68 \text{ MW}$$

$$P_2 = \frac{0.423077 + 0.001 \times 49.77}{0.00884615} = 52.53 \text{ MW}$$

$P_1 = 41.38\ P_1 = 41.37$

$P_2 = 52.50\ P_2 = 52.50$

$\therefore$ $\text{Loss} = 0.0015 \times (41.37)^2 - 2 \times 0.0005 \times 41.37 \times 52.50 + 0.0025 \times (52.5)^2$

$= 2.5672 - 2.1719 + 6.8906 = 7.28 \text{ MW}$

$\therefore$ Load $P_D = 93.87 - 7.28 = 86.59$ MW

In the above problem λ is given and we are asked to find out the operating schedule. Normally in an actual problem the load demand is given and we are asked to find out the economic operating schedule. To understand the utility of coordination equations let us say that the load to be met is 160 MW. With loss coefficients and the incremental production cost given as in the problem above, we are asked to determine the economic operating schedule.

The generation corresponding to ICP will be

$$P_1 + P_2 = 160 \quad \text{and} \quad 0.01\ P_1 + 2 = 0.01\ P_2 + 1.5$$

$$\therefore \quad P_1 = 55 \text{ MW} \quad \text{and} \quad P_2 = 105 \text{ MW}$$

With $P_2 = 105$ MW,

$$\lambda = 0.01 + 105 + 1.5 = 2.55$$

Substituting this value of λ in coordination equations we have

$$P_1 = \frac{1 - \dfrac{2}{2.55} + 2 \times 0.0005\, P_2}{\dfrac{0.01}{2.55} + 2 \times 0.0015} = \frac{0.215686 + 0.001 P_2}{0.00692156}$$

and

$$P_2 = \frac{1 - \dfrac{1.5}{2.55} + 2 \times 0.0005\, P_1}{\dfrac{0.01}{2.55} + 2 \times 0.0025} = \frac{0.4117648 + 0.001 P_1}{0.00892156}$$

Starting with $P_2 = 105$, the powers from these equations are

$P_1 = 46.33$ MW	$P_1 = 38.56$ MW	$P_1 = 38.45$ MW
$P_2 = 51.34$ MW	$P_2 = 50.47$ MW	$P_2 = 50.46$ MW

At this juncture we should give a thought to our solution. Since $P_1 + P_2 < 160$ MW, this value of $\lambda = 2.55$ is not correct and since $P_1 + P_2 << 160$ therefore λ should be greater than 2.55. Let this be 5.0. With this value of λ, $P_2 = 350$ MW from the ICP equation, and the coordination equation becomes

$$P_1 = \frac{1 - \dfrac{2}{5} + 2 \times 0.0005\, P_2}{\dfrac{0.01}{5} + 2 \times 0.0015} = \frac{0.6 + 0.001 P_2}{0.005}$$

and

$$P_2 = \frac{1 - \dfrac{1.5}{5} + 2 \times 0.0005\, P_2}{\dfrac{0.01}{5} + 2 \times 0.0025} = \frac{0.7 + 0.001 P_1}{0.007}$$

Starting with $P_2 = 350$ MW, the values of P_1 and P_2 in various iterations using coordination equations are

$P_1 = 190$	$P_1 = 145.42$	$P_1 = 144.14$
$P_2 = 127.14$	$P_2 = 120.7$	$P_2 = 120.59$

Again we should give a thought to our solution. This time the sum of generations $P_1 + P_2 >> 160$ MW. Therefore, a lower value of λ is desired. Let this be $\lambda = 3.75$.

With this value of λ the coordination equations are

$$P_1 = \frac{0.46667 + 0.001P_2}{0.0056667}$$

and $$P_2 = \frac{0.6 + 0.001P_1}{0.0076667}$$

and the generations during various iterations are

$P_1 = 122.05$	$P_1 = 98.973$	$P_1 = 98.44$	$P_1 = 98.42$
$P_2 = 94.18$	$P_2 = 91.17$	$P_2 = 91.10$	$P_2 = 91.09$

Since the difference between two consecutive generations is less than 0.1 MW, we stop here and since $P_1 + P_2 = 189.51$ MW which is not far off from the load of 160 MW. We, therefore, evaluate the losses:

$$P_L = 0.0015 \times (98.42)^2 + 0.0025 \times (91.09)^2 - 2 \times 98.42 \times 91.09 \times 0.0005$$
$$= 26.31 \text{ MW}$$

$$\therefore \quad P_D = P_1 + P_2 - P_L = 189.51 - 26.31 = 163.2 \text{ MW}$$

This is 3.2 MW more than the actual demand, therefore, we cannot terminate here. We are guided by the P_D calculated for assuming a suitable value of λ. Since P_D calculated is more than the actual one, therefore, we take a lower value of λ.

The reader will find that $\lambda = 3.69$ gives the final solution of the problem. With $\lambda = 3.69$, $P_2 = 219$ MW, the coordination equations become

$$P_1 = \frac{0.4579946 + 0.001P_2}{0.005710027}$$

$$P_2 = \frac{0.593496 + 0.001P_1}{0.007710027}$$

and with $P_2 = 219$ MW as initial value, the generations during various iterations are

$P_1 = 118.56$ MW	$P_1 = 96.40$ MW	$P_1 = 95.88$ MW	$P_1 = 95.86$ MW
$P_2 = 92.35$ MW	$P_2 = 89.48$ MW	$P_2 = 89.41$ MW	$P_2 = 89.41$ MW

We terminate the process here and evaluate the losses.

$$P_L = 0.0015 \times (95.86)^2 + 0.0025 \times (89.41)^2 - 2 \times 95.86 \times 89.41 \times 0.0005$$
$$= 25.19$$

$$\therefore \quad P_D = 95.86 + 89.41 - 25.19 = 160.08$$

$$\therefore \quad |P_D \text{ calculated} - P_D| \le 0.1 \text{ MW}$$

and the solution is obtained.

$$\text{Cost of generation} = 0.005\,P_1^2 + 2\,P_1 + 0.005\,P_2^2 + 1.5\,P_2$$
$$= \text{Rs. } 411.74/\text{hr.}$$

Another possibility is that we may take the transmission losses into account but they need not be coordinated. If we don't coordinate the losses with the incremental production cost, the generation schedule so obtained will be costlier as compared to when they are coordinated. An illustration with the help of previous example will make clear the advantage of the coordination of losses.

The set of equations when losses are included but not coordinated will be

$$0.01\,P_1 + 2 = 0.01\,P_2 + 1.5$$

and
$$P_1 + P_2 - 0.0015\,P_1^2 - 0.0025\,P_2^2 + 0.001\,P_1P_2 = 160$$

or
$$P_1 + 200 = P_2 + 150$$

or
$$P_2 = P_1 + 50$$

Substituting this value of P_2 in the quadratic equation, we obtain

$$P_1^2 - 600\,P_1 + 38750 = 0$$

or
$$P_1 = 73.6 \text{ MW}$$

Neglecting the higher value as the higher value is much more than the total load on the system.

$\therefore$
$$P_2 = 73.6 + 50 = 123.6 \text{ MW}$$

$\therefore$
$$\text{Cost} = \text{Rs. } 436.06/\text{hr.}$$

Comparing the cost between the two, it is seen that cost with losses coordinated is less than when they are not coordinated and the difference in cost is

$$436.06 - 411.74 = \text{Rs. } 24.32 \text{ per hr.} \quad \textbf{Ans.}$$

Penalty factor method: From the coordination equation

$$\frac{dF_n}{dP_n} + \lambda\frac{\partial P_L}{\partial P_n} = \lambda$$

or
$$\frac{dF_n}{dP_n} = \lambda\left(1 - \frac{\partial P_L}{\partial P_n}\right)$$

or
$$\frac{dF_n}{dP_n}\cdot\frac{1}{1 - \dfrac{\partial P_L}{\partial P_n}} = \lambda$$

or
$$\frac{dF_n}{dP_n}\cdot L_n = \lambda \tag{19.11}$$

where L_n is the penalty factor of plant n and is given by

$$L_n = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_n}} \tag{19.12}$$

$$\simeq \left(1 + \frac{\partial P_L}{\partial P_n}\right) \text{ approximate penalty factor}$$

$$= L_n'$$

Since the penalty factor has been derived from the coordination equation, the solution of the problem using penalty factor gives precisely the same results.

Example 19.1: The fuel inputs per hour of plants 1 and 2 are given as

$$F_1 = 0.2\, P_1^2 + 40\, P_1 + 120 \text{ Rs. per hr}$$

$$F_2 = 0.25\, P_2^{\,2} + 30\, P_2 + 150 \text{ Rs. per hr}$$

Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading on each unit is 100 MW and 25 MW, the demand is 180 MW, and transmission losses are neglected. If the load is equally shared by both the units, determine the saving obtained by loading the units as per equal incremental production cost.

Solution: The incremental production costs of both the units are

$$\frac{dF_1}{dP_1} = 0.4\, P_1 + 40 \text{ Rs. per MWhr}$$

and

$$\frac{dF_2}{dP_2} = 0.5\, P_2 + 30 \text{ Rs. per MWhr}$$

Now for economic operation of the units

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

i.e.,

$$0.4P_1 + 40 = 0.5P_2 + 30$$

and

$$P_1 + P_2 = 180$$

Solution of these equations gives

$$P_1 = 88.89 \text{ MW} \quad \text{and} \quad P_2 = 91.11 \text{ MW}$$

Now cost of generation = $F_1 + F_2$

$$F_1 = 0.2\, P_1^2 + 40\, P_1 + 120 = \text{Rs. } 5255.88/\text{hr.}$$

$$F_2 = 0.25\, P_2^{\,2} + 30\, P_2 + 150 = \text{Rs. } 4958.55/\text{hr.}$$

Total cost = Rs. 10214.43/hr.

(*b*) If the load on each unit is 90 MW, the cost of generation will be

$$F_1 = \text{Rs. } 5340/\text{hr.}$$

$$F_2 = \text{Rs. } 4875/\text{hr.}$$

Total cost = Rs. 10215/hr.

∴ Saving will be Rs. 0.57/hr.

Example 19.2: Determine the incremental cost of received power and the penalty factor of the plant shown in Fig. E.19.2 below if the incremental cost of production is

$$\frac{dF_1}{dP_1} = 0.1\, P_1 + 3.0 \text{ Rs./MWhr.}$$

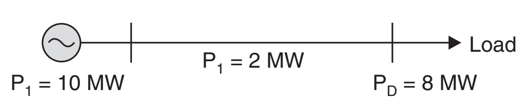


Fig. E.19.2

Solution: The penalty factor = $\dfrac{10}{8}$ **Ans.**

$\therefore$ Cost of received power = $\dfrac{dF_1}{dP_1}L_1 = (0.1 \times 10 + 3).\dfrac{10}{8}$

= Rs. 5/MWhr **Ans.**

Example 19.3: A two-bus system is shown in Fig. E.19.3. If a load of 125 MW is transmitted from plant 1 to the load, a loss of 15.625 MW is incurred. Determine the generation schedule and the load demand if the cost of received power is Rs. 24/MWhr. Solve the problem using coordination equations and the penalty factor method approach. The incremental production costs of the plants are

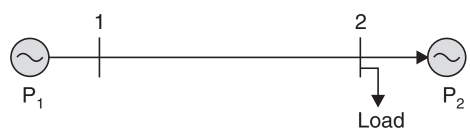


Fig. E.19.3

$$\frac{dF_1}{dP_1} = 0.025\, P_1 + 15$$

$$\frac{dF_2}{dP_2} = 0.05\, P_2 + 20$$

Solution: Since the load is at bus 2 alone, therefore, the losses in the line will not be affected by generator of plant 2.

$\therefore$ $P_L = B_{11}P_1^2$ as $B_{12} = B_{21} = 0$ and $B_{22} = 0$

$\therefore$ $15.625 = B_{11} \times 125^2$

or $B_{11} = 0.001$

Now coordination equation

$$\frac{dF_1}{dP_1} + \lambda\frac{\partial P_L}{\partial P_1} = \lambda$$

where $P_L = 0.001\, P_1^2$ or $\dfrac{dP_L}{dP_1} = 0.002\, P_1$

Substituting in the coordination equation for plant 1 we get

$$0.025\, P_1 + 15 + \lambda.0.002\, P_1 = \lambda$$

$$0.025\, P_1 + 0.048\, P_1 + 15 = 24$$

or $0.073\, P_1 = 9$

or $P_1 = 123.28$ MW

and from the coordination equation for plant 2,

$$0.05P_2 + 20 = 24 \text{ or } P_2 = 80 \text{ MW}$$

$\therefore$ The transmission loss $P_L = 0.001 \times (123.28)^2 = 15.19$ MW

$\therefore$ The load $P_D = 123.28 + 80 - 15.19 = 188.1$ MW

The solution using penalty factor is as follows: The penalty factor for plant 1 is

$$\frac{1}{1-\dfrac{\partial P_L}{\partial P_1}} = \frac{1}{(1-0.002P_1)}$$

$$\therefore \qquad \frac{dF_1}{dP_1}\frac{1}{1-0.002P_1} = 24$$

or $$\frac{0.025P_1+15}{1-0.002P_1} = 24$$

$$\therefore \qquad P_1 = 123.28 \text{ MW}$$

Similarly, since $\dfrac{dP_L}{dP_2}$ is zero, $\therefore$ L_2 = unity, *i.e.*, the incremental cost of received power equals the incremental cost of production.

$$\therefore \qquad 0.05\,P_2 + 20 = 24 \qquad \text{or} \qquad P_2 = 80 \text{ MW}$$

Example 19.4: Assume that the fuel input in Btu per hour for units 1 and 2 are given by

$$F_1 = (8P_1 + 0.024\,P_1^2 + 80)10^6$$

$$F_2 = (6P_2 + 0.04\,P_2^2 + 120)10^6$$

The maximum and minimum loads on the units are 100 MW and 10 MW respectively. Determine the minimum cost of generation when the following load (Fig. E.19.4) is supplied. The cost of fuel is Rs. 2 per million Btu.

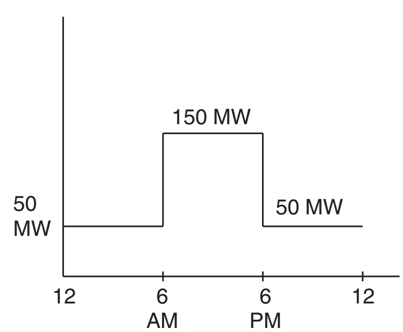


Fig. E.19.4

Solution: From the fuel input characteristics

$$\frac{dF_1}{dP_1} = 0.048\,P_1 + 8$$

and $$\frac{dF_2}{dP_2} = 0.08\,P_2 + 6$$

(*i*) When load is 50 MW, for economic loading the conditions are

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

and $$P_1 + P_2 = 50$$

i.e., $$0.048\, P_1 + 8 = 0.08\, P_2 + 6$$

and $$P_1 + P_2 = 50$$

From these equations,

$$P_1 = 15.625 \text{ MW and } P_2 = 34.375 \text{ MW}$$

$\therefore$ $$F_1 = 210.868 \text{ million Btu per hr.}$$

and $$F_2 = 373.5 \text{ million Btu per hr.}$$

(*ii*) When load is 150 MW, the equations are

$$0.048\, P_1 + 8 = 0.08 P_2 + 6$$

and $$P_1 + P_2 = 150$$

$\therefore$ $$P_1 = 78.126 \text{ MW} \quad \text{and} \quad P_2 = 71.874 \text{ MW}$$

and $$F_1 = 851.496 \text{ million Btu/hr.}$$

$$F_2 = 757.87 \text{ million Btu/hr.}$$

$\therefore$ $$\text{Total cost} = \text{Rs. } (210.868 + 373.5 + 851.496 + 757.87) \times 12 \times 2$$

$$= \text{Rs. } 52649.61$$ **Ans.**

Physical Interpretation of Coordination Equations

The physical interpretation of the coordination equations can be understood with the help of Fig. 19.5. There are n number of plants connected to a hypothetical load through a transmission network. The incremental cost of production of nth plant at the busbar is dF_n/dP_n. The plant n incurs an incremental transmission loss of $\partial P_L/\partial P_n$ in supplying the next increment of load. Let ΔP_D be the change in load and if this change is supplied by plant n alone and say in doing so the generation at plant n required is ΔP_n. This ΔP_n includes the transmission losses in addition to the increase in demand by ΔP_D. The cost of power at the plant bus will be

$$\frac{dF_n}{dP_n}\Delta P_n = \text{Rs./hr}$$

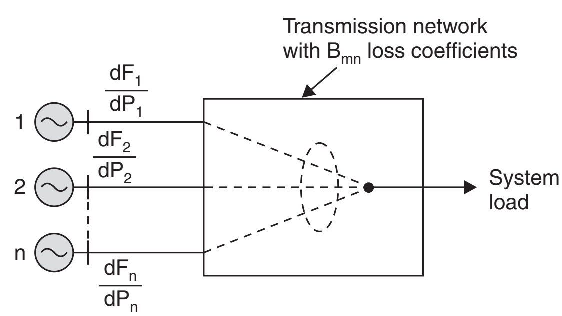


Fig. 19.5 Physical interpretation of coordination equations.

But since power received is only ΔP_D, therefore the cost of received power will be

$$\lambda = \frac{dF_n}{dP_n}.\Delta P_n.\frac{1}{\Delta P_D} = \frac{dF_n}{dP_n}.\frac{\Delta P_n}{\Delta P_D} \text{ Rs./MWhr}$$

Since $\Delta P_D = \Delta P_n - \Delta P_L$

$\therefore$
$$\lambda = \frac{dF_n}{dP_n}\frac{\Delta P_n}{\Delta P_n - \Delta P_L} = \frac{dF_n}{dP_n}\frac{1}{1-\dfrac{\Delta P_L}{\Delta P_n}}$$

and in the limit when $\Delta P_n \to 0$

$$\lambda = \frac{dF_n}{dP_n}\frac{1}{1-\dfrac{\partial P_L}{\partial P_n}} = \frac{dF_n}{dP_n}.L_n$$

which is same as the coordination equation and from this treatment penalty factor for plant n can be defined as the ratio of the small change in power at plant n to the small change in received power when generation at plant n alone is changed for meeting this change in load.

When transmission losses are included and coordinated, the following points must be kept in mind for economic load dispatch solution:

1. Whereas the incremental cost of production of a plant is always positive, the incremental transmission losses can be both positive or negative.

2. The individual generators will operate at different incremental costs of production.

3. The generation with highest positive incremental transmission loss will operate at the lowest incremental cost of production.

The B_{mn} coefficients are the loss coefficients and for an n generator system the coefficient is an $n \times n$ symmetric matrix

$$B_{mn} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix}$$

The diagonal elements are all positive and strong as compared with the off diagonal elements which mostly are negative and are relatively weaker. These coefficients are determined for a large system by an elaborate computer programme starting from the assembly of the open circuit impedance matrix of the transmission network which is quite lengthy and time consuming and is beyond the scope of this book. Besides, the formulations of B-coefficients are based on several assumptions and do not take into account the actual conditions of the system, the solution for the plant generations cannot be expected to be the best for minimum cost of generation.

B-coefficients have been developed by applying tensors to power system wherein the interconnected system is reduced to one with sources equal to the actual number of sources but loads equal to one equivalent hypothetical load.

We develop here simple equations for loss coefficients based on several assumptions for systems having any number of loads and sources. We will develop, to begin with, the equations for two generating stations and an indefinite number of loads and finally will generalize for any number of generators and loads. Consider Fig. 19.6 which indicates two generating plants connected to a transmission network which is terminated by an arbitrary number of loads.

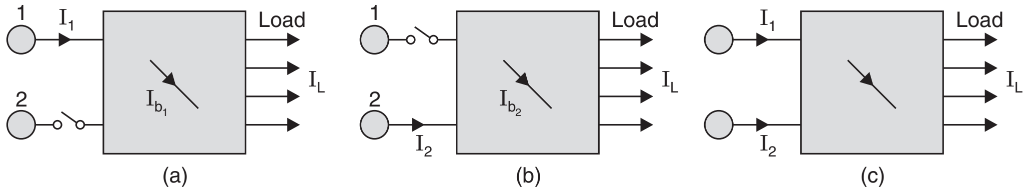


Fig. 19.6 Schematic diagram of a network with two generators and large number of loads. One branch of the network is shown.

Consider one three phase line b. The total load current I_L is supplied by source 1 and the current in line b is I_{b_1}

Let $$\alpha_{b_1} = \frac{I_{b_1}}{I_L} \tag{19.13}$$

Similarly when source 2 alone supplies load current I_L and if I_{b_2} is the current through the same line b, then

$$\alpha_{b_2} = \frac{I_{b_2}}{I_L} \tag{19.14}$$

When both the plants are connected to supply the load current, the current through the branch b, using the principle of superposition, will be

$$I_b = \alpha_{b_1} I_1 + \alpha_{b_2} I_2 \tag{19.15}$$

where I_1 and I_2 are the currents from plant 1 and 2 respectively and α_{b_1} and α_{b_2} are the current distribution factors.

Here we make two simplifying assumptions for deriving the loss coefficients.

(*i*) The ratio *X*/*R* for all the transmission lines is the same.

(*ii*) The phase angle of all the load currents is the same.

The net effect of these two assumptions is that the load currents and branch currents are in phase and hence the current distribution factors are real.

Let $$I_1 = |I_1| \cos \theta_1 + j|I_1| \sin \theta_1$$

and $$I_2 = |I_2| \cos \theta_2 + j|I_2| \sin \theta_2 \tag{19.16}$$

where θ_1 and θ_2 are the phase angles of currents I_1 and I_2 with reference to a common phasor.

$\therefore$ $$I_b = \alpha_{b_1} I_1 + \alpha_{b_2} I_2$$

$$= \alpha_{b_1} |I_1| \cos \theta_1 + \alpha_{b_2} |I_2| \cos \theta_2 + j[\alpha_{b_1} |I_1| \sin \theta_1 + \alpha_{b_2} |I_2| \sin \theta_2]$$

or $$I_b^2 = \alpha_{b_1}^2 |I_1|^2 + \alpha_{b_2}^2 |I_2|^2 + 2\alpha_{b_1}\alpha_{b_2} |I_1||I_2| \cos(\theta_1 - \theta_2) \tag{19.17}$$

Since $I_1 = \dfrac{P_1}{\sqrt{3}|V_1|\cos\phi_1}$ and $I_2 = \dfrac{P_1}{\sqrt{3}|V_2|\cos\phi_2}$ and if R_b is the resistance of branch b, the total loss will be

$$P_L = \sum_{b=1}^{b} 3 I_b^2 R_b$$

where Σ is the summation of losses in all the branches and ϕ_1 and ϕ_2 are the p.f. angles of plants 1 and 2 respectively.

$$P_L' = \sum \alpha_{b_1}^2 \frac{P_1^2 R_b}{|V_1|^2 \cos^2 \phi_1} + \sum \alpha_{b_2}^2 \frac{P_2^2 R_b}{|V_2|^2 \cos^2 \phi_2} + \sum \frac{2P_1P_2R_b\alpha_{b_1}\alpha_{b_2}\cos(\theta_1 - \theta_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2} \quad (19.18)$$

$$= B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

where $$B_{11} = \sum \frac{\alpha_{b_1}^2 R_b}{|V_1|^2 \cos^2 \phi_1} = \frac{1}{|V_1|^2 \cos^2 \phi_1} \sum_b \alpha_{b_1}^2 R_b$$

$$B_{22} = \frac{1}{|V_2|^2 \cos^2 \phi_2} \sum_b \alpha_{b_2}^2 R_b$$

and $$B_{12} = \frac{1}{|V_1||V_2|\cos\phi_1\cos\phi_2} \sum_b \alpha_{b_1}\alpha_{b_2} R_b$$

Therefore, if there are n number of sources the general loss coefficients will be

$$B_{nn} = \frac{1}{|V_n|^2 \cos^2 \phi_n} \sum_b \alpha_{b_n}^2 R_b \quad (19.19)$$

and $$B_{mn} = \frac{1}{|V_m||V_n|\cos\phi_m\cos\phi_n} \sum_b \alpha_{b_m}\alpha_{b_n} R_b \quad (19.20)$$

Whenever there are wide variations in the operating conditions, the various assumptions for evaluation of B-coefficients may cause errors in loss calculation, it is desirable to obtain one or two additional sets of loss coefficients for such conditions.

Many power companies, however, obtain reasonably accurate results assuming one set of coefficients corresponding to a typical operating condition.

19.4 EXACT TRANSMISSION LOSS FORMULA

Dopezo et al. have derived an exact formula for calculating transmission losses by making use of the bus powers and the system parameters. Let S_i be the total injected bus power at bus i and is equal to the generated power minus the load at bus i. The summation of all such powers over all the buses gives the total losses of the system, *i.e.*,

$$P_L + jQ_L = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} V_i I_i^* = V_{\text{bus}}^T I_{\text{bus}}^* \quad (19.21)$$

Here P_L and Q_L are the real and reactive power loss of the system. V_{bus} and I_{bus} are the column vectors of voltages and currents of all the buses.

Now $$V_{\text{bus}} = Z_{\text{bus}} I_{\text{bus}} \quad (19.22)$$

where Z_{bus} is the bus impedance matrix of the transmission network and is given by

$$Z_{\text{bus}} = R + jX = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix}$$

From equations (19.21) and (19.22), we have

$$P_L + jQ_L = I_{\text{bus}}^T Z_{\text{bus}}^T I_{\text{bus}}^*$$
$$= I_{\text{bus}}^T Z_{\text{bus}} I_{\text{bus}}^* \qquad (19.23)$$

The last step can be written because Z_{bus} is a symmetric matrix and, therefore,

$$Z_{\text{bus}}^T = Z_{\text{bus}}.$$

The bus current vector I_{bus} can also be written as the sum of a real and reactive component of current vectors, *i.e.,*

$$I_{\text{bus}} = I_p + jI_q = \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

The equation for the loss can be written as

$$P_L + jQ_L = (I_p + jI_q)^T (R + jX)(I_p - jI_q)$$

Separating out the real part P_L from the above matrix product, we get

$$P_L = I_p^T R I_p + I_p^T X I_q + I_q^T R I_q - I_q^T X I_p$$

Since X is a symmetric matrix,

$$I_p^T X I_q = I_q^T X I_p$$

Hence
$$P_L = I_p^T R I_p + I_q^T R I_q \qquad (19.24)$$

This expression (19.24) can be rewritten by using index notation as

$$P_L = \sum_{\substack{i=1 \\ k=1}}^{n} r_{jk}(I_{pj} I_{pk} + I_{qj} I_{qk}) \qquad (19.25)$$

The transmission loss has been expressed in terms of bus currents. In an actual power plant, the system operators usually know the bus powers and the nodal voltages; therefore, it is more practical to express P_L in terms of these quantities.

For bus powers at bus i we have

$$P_i + jQ_i = V_i I_i^* = V_i(I_{pi} - jI_{qi}) = |V_i|(\cos\delta_i + j\sin\delta_i)(I_{pi} - jI_{qi})$$

where δ_i is the phase angle of voltage V_i with respect to the reference voltage *i.e.*, the slack bus voltage. Now separating the real and imaginary parts, we have

$$P_i = |V_i| \cos\delta_i \, I_{pi} + |V_i| \, I_{qi} \sin\delta_i$$

and
$$Q_i = |V_i| \, I_{pi} \sin\delta_i - |V_i| I_{qi} \cos\delta_i$$

Solving for I_{pi} and I_{qi}, we get

$$I_{pi} = \frac{1}{|V_i|}(P_i \cos\delta_i + Q_i \sin\delta_i)$$

$$I_{qi} = \frac{1}{|V_i|}(P_i \sin\delta_i - Q_i \cos\delta_i)$$

We have expressed here the real and imaginary components of bus currents in terms of bus powers and the bus voltages. Substituting these values in the expression for power loss, we get after some algebraic manipulations,

$$P_L = \sum_{\substack{j=1\\k=1}}^{n} \alpha_{jk}(P_j P_k + Q_j Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k) \tag{19.26}$$

where $$\alpha_{jk} = \frac{r_{jk}}{|V_j|\,|V_k|}\cos(\delta_j - \delta_k)$$

and $$\beta_{jk} = \frac{r_{jk}}{|V_j|\,|V_k|}\sin(\delta_j - \delta_k)$$

By making use of this expression for transmission losses along with the coordination equations, the problem of optimal load dispatch can be solved.

It can be seen that even though the formulation for the transmission loss is exact, the method requires the calculation of bus impedance matrix which is time consuming and needs more computer memory.

19.5 MODIFIED COORDINATION EQUATIONS

Another technique for solving the optimal load dispatch problem as suggested by Kirchmayer is through the use of modified coordination equations. These equations are derived as follows:

Transmission loss is the algebraic sum of the powers at all the nodes *i.e.*, it is function of powers at all the nodes and therefore

$$P_L = \sum_{i=1}^{n} P_i \tag{19.27}$$

Here P_i is positive for generator bus and negative for load bus.

$$\therefore \qquad dP_L = \sum_{i=1}^{n} \frac{\partial P_L}{\partial P_i} dP_i \tag{19.28}$$

Say, in an interconnected system the bus powers of only two plants P_j and P_n can be changed by small amounts keeping the powers at all other buses fixed then,

$$dP_{Lj,\,n} = \frac{\partial P_L}{\partial P_j} dP_j + \frac{\partial P_L}{\partial P_n}.dP_n \tag{19.29}$$

The incremental loss $dP_{Lj,\,n}/dP_j$ is defined as the ratio of change in loss to the change in generation at plant j when power is transferred from plant j to plant n, with the generations of other plants and load keeping fixed. It is clear that

$$dP_{Lj,\,n} = dP_j + dP_n \tag{19.30}$$

Using equations (19.29) and (19.30), we get

$$dP_j + dP_n = \frac{\partial P_L}{\partial P_j}.dP_j + \frac{\partial P_L}{\partial P_n}.dP_n$$

or $$dP_j\left(1 - \frac{\partial P_L}{\partial P_j}\right) + dP_n\left(1 - \frac{\partial P_L}{\partial P_n}\right) = 0$$

or
$$\frac{dP_j}{dP_n} = -\frac{1-\partial P_L/\partial P_n}{1-\partial P_L/dP_j} \tag{19.31}$$

Also since from the coordination equations,

$$\frac{dF_j}{dP_j}\cdot\frac{1}{\left(1-\dfrac{\partial P_L}{\partial P_j}\right)} = \lambda = \frac{dF_n}{dP_n}\cdot\frac{1}{1-\dfrac{\partial P_L}{\partial P_n}}$$

$$\frac{\dfrac{dF_n}{dP_n}}{\dfrac{dF_j}{dP_j}} = \frac{1-\dfrac{\partial P_L}{\partial P_n}}{1-\dfrac{\partial P_L}{\partial P_j}} = -\frac{dP_j}{dP_n} = -\frac{dP_j}{dP_{Lj,\,n}-dP_j}$$

$$= \frac{1}{1-\dfrac{dP_{Lj,\,n}}{dP_j}}$$

or
$$\frac{dF_n}{dP_n} = \frac{1}{1-\dfrac{dP_{Lj,\,n}}{dP_j}}\cdot\frac{dF_j}{dP_j} = \mu \tag{19.32}$$

Here plant n is taken as the reference plant.

From the above expression it is clear that for economic load dispatch the condition required is that the incremental cost of power at plant bus n is equal to the incremental cost of power at plant bus j corrected for the effect of the incremental transmission loss involved in swinging generation between plants j and n. These equations are known as modified coordination equations. The modified coordination equation can be rewritten as

$$\frac{dF_n}{dP_n} = -\frac{dF_j}{dP_j}\cdot\frac{\Delta P_j}{\Delta P_n} = \mu$$

The solution of modified coordination equation requires the calculation of $\Delta P_j/\Delta P_n$ terms. The solution of these equations is explained as follows: The flow chart is given in Fig. 19.7.

1. Given a demand, assuming no transmission losses, the generations are determined according to equal incremental production cost, say P_1, P_2, ..., P_n are the generations of n plants. Thus solving the equation

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \ldots = \frac{dF_n}{dP_n}$$

and
$$\sum_{n=1}^{n} P_n = P_D$$

where P_D is the given system load demand. The generation P_1, P_2, ..., P_n are determined. Choosing nth plant as the reference plant, the incremental cost μ at the reference plant is given by

$$\mu = \frac{dF_n}{dP_n} = F_{nn}P_n + f_n$$

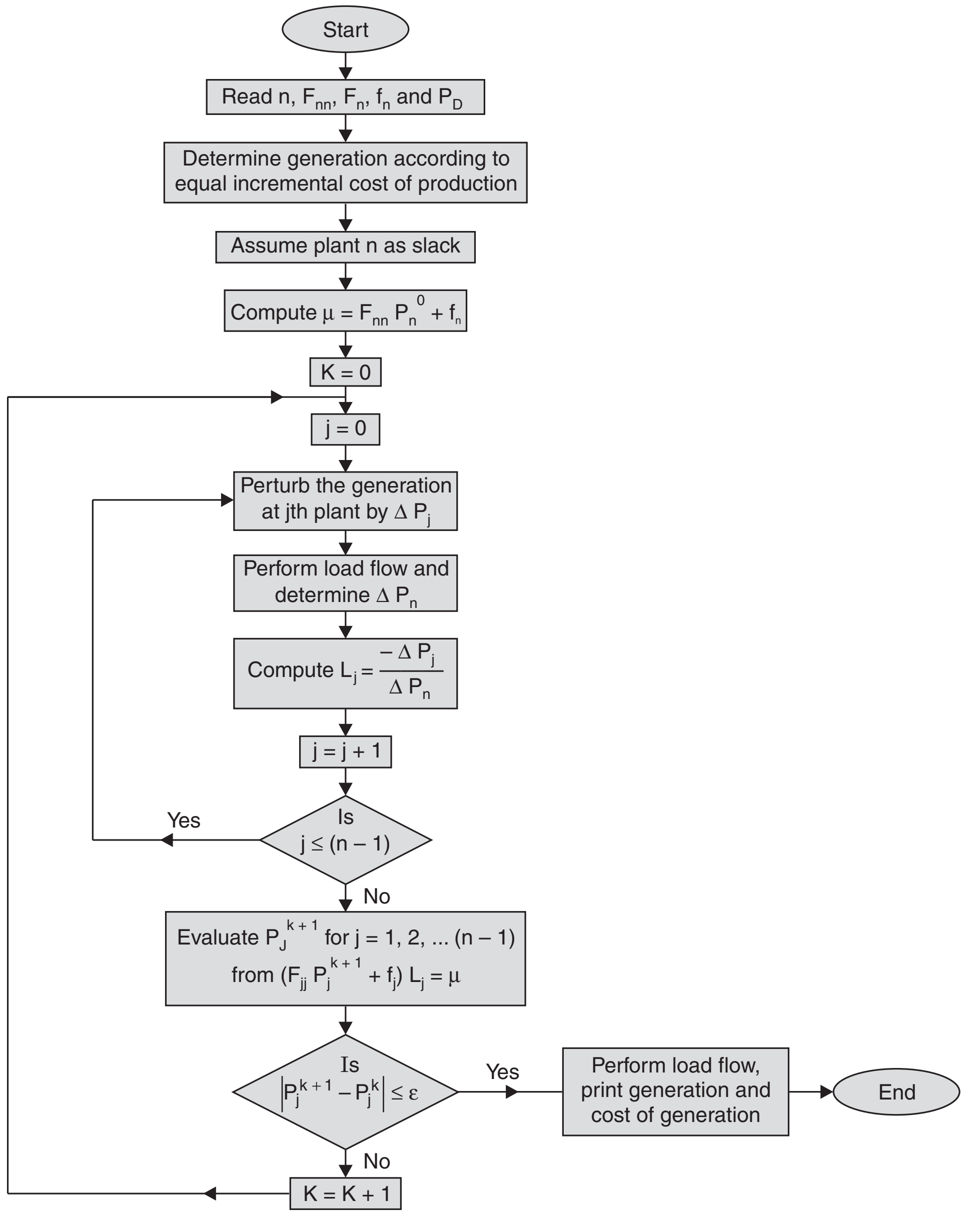


Fig. 19.7 Flow chart for the solution of modified coordination equations.

2. Taking the nth plant bus as the reference or slack bus, a load flow using Newton-Raphson technique is performed with specified generations as obtained in step 1. Let the slack bus power be P_n'.

3. Increase the generation at bus 1 by a small amount ΔP_1 and keeping all other generations constant as per step 1 and taking plant n as the slack bus perform the load flow again. Say the slack bus power be P_n''.

The procedure is repeated by swinging generation between other plants and the reference plant n, one at a time, for obtaining the quantities $-\dfrac{\Delta P_j}{\Delta P_n}$.

4. The equations $\dfrac{dF_n}{dP_n} = -\dfrac{\Delta P_j}{\Delta P_n} \cdot \dfrac{dF_j}{dP_j}$ are solved. The solution gives new generation $P_1, P_2, \ldots, P_j$. The first iteration for generation schedule is complete.

5. With the reference bus remaining unchanged and with the value of μ remaining constant as chosen in step 1, the procedure from 1 to 4 is repeated until the generation of two consecutive iterations lie within a certain specified tolerance, *i.e.,* mathematically,

$$\left| P_j^{k+1} - P_j^k \right| \le \varepsilon$$

where K is the iteration count and ε is the prespecified tolerance. When convergence is reached the next step is followed, otherwise steps 2 to 4 are repeated.

6. With P_j^{k+1} as generation, the final load flow solution is obtained and hence the cost of generation is evaluated.

19.6 AUTOMATIC LOAD DISPATCHING

It has already been explained that economic load dispatching is that aspect of power system operation wherein it is required to distribute the load among the generating units actually paralleled with the system in such a manner as to minimise the cost of supplying the minute-to-minute requirements of the system. In a large interconnected system it is humanly impossible to calculate and adjust such generations and hence the help of digital computer system along with analogue devices is sought and the whole process is carried out automatically; hence the term automatic load dispatching is used for the purpose.

An energy system is normally controlled by a central control centre which coordinates the operations of several area centres. The objective is to minimise the cost of supplying electricity to the load points while ensuring security of supply against loss of generation and transmission capacity and also maintaining the voltage and frequency of the system within specified limits. Since the interconnection is growing bigger and bigger in size with time, the control engineer has to make adjustments to various parameters in the system. Hence it has become imperative to make use of automatic control for load dispatch problem.

The chosen control system is invariably based on a digital computer working on-line. The presence of the control engineer is required only to enter certain data. The rest of the control calculations can proceed and the generation instructions calculated, transmitted and implemented at the power stations automatically.

The block diagram of an automatic control system used for load dispatching is shown in Fig. 19.8.

Computer: The computer predicts the load and suggests economic loading. It transmits information to machine controllers.

Data Input: The computer receives a lot of essential data from the telemetering system and from the paper tape. Telemetering data comes to the computer either as analogue signals representing line power flows, plant outputs or as signal bits indicating switch or isolator positions. The system is entirely automatic. Paper tape stores all the basic data required, *e.g.,* the system parameters, load prediction, security constraints, etc. and finally asking the stations to switch the machine controllers to automatic control.

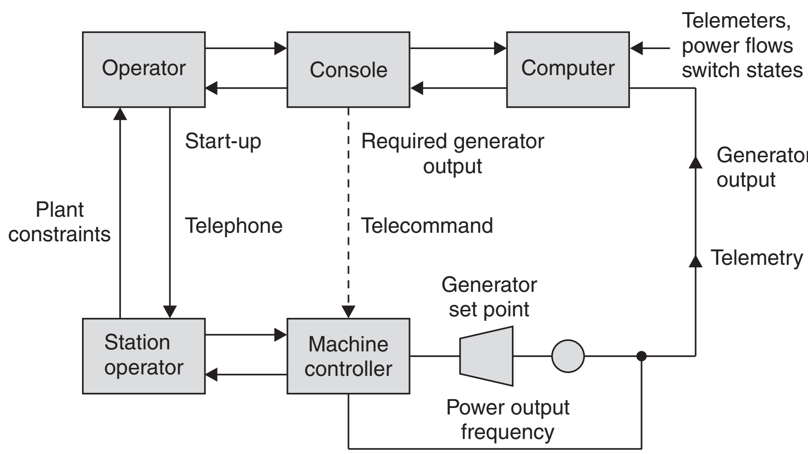


Fig. 19.8 Schematic diagram of automatic load dispatching components.

Console: The console is the component through which the operator can converse with the computer. He can obtain certain information required for some action to be taken under emergency condition should such a condition arise in the system, or he can put data into it if needed. The instructions being issued and the state of the energy system (tie-line loadings, plant outputs, nodal voltages, etc.) can be examined on cathode ray displays associated with the console.

The console has the facilities of security checking and load flows for the network calculations. Such a facility can be called manually at almost any instant of time and the results may be recorded automatically after a specified interval of time, if required. The security check (loss of generator or loss of one or more than one transmission links) reveals and displays that if one or more than one particular branches suffer outage, at least one of the remaining branches will be overloaded.

Machine Controllers: The computer sends instructions regarding the optimal generation to the machine controller at regular intervals which in turn implements them. Control on each machine is applied by a closed loop system which uses a measure of actual power generated and which operates through a conventional speeder motor. These are referred to as controller power loops.

In the power frequency loop an error signal proportional to the difference between the derived and actual frequency and power is developed. A summed error signal is formed from these two components and is converted in the motor controller to a train of pulses that are applied to a speed governor reference setting motor called the speeder motor. The duration and amplitude of these pulses are fixed but the pulse rate is made proportional to the summed error signal. The pulses are applied as 'raise' or 'lower' command to the speeder motor in accordance with the error signal and thus the output of the generators is increased or decreased accordingly.

19.7 POWER LINE CARRIER COMMUNICATION (PLCC)

The term power line carrier is used to represent the entire process of communication which uses high voltage overhead power lines as the means of transmission. The power lines offer the following advantages:

(*i*) Lines have thicker cross section of wire and hence attenuation of signals is not much.

(*ii*) Leakance is negligible even under wet weather conditions as lines are insulated with high voltage insulators.

(*iii*) Lines are disrupted during foul weather conditions as conductors are strung on very robust tower structures.

(*iv*) As the phases are separated from each other quite appreciably, cross-talks between lines is practically avoided.

(*v*) The cost of providing extra lines for the purpose is avoided.

However, there are certain difficulties associated with the transmission of communication signal over the power lines:

(*i*) Since the power is being transmitted at relatively high voltages, the operation on these lines may prove to be dangerous to human lives and also to telephone apparatus.

(*ii*) During transient operation of system *e.g.*, switching transients, surge voltages or corona phenomenon etc. the presence of higher harmonics in power currents may interfer with the communication signal.

These difficulties, of course, have been overcome by selecting suitable coupling capacitors and carrier frequencies.

The power line carrier communication finds application in telemetering, power line protection, telecontrol etc. The application of PLCC to power line protection has already been discussed in Chapter 14.

The power lines may have higher harmonics due to switching surges and corona loss on overhead lines. These frequencies generally lie between 100 Hz to 50 kHz and, therefore, if carrier frequencies are chosen in this region, noise introduced in carrier signal would be very large and hence carrier frequency in the range 30 kHz to 500 kHz are used. In order that these carrier signals do not interfere into the adjoining section of lines and also that these carrier signals are not lost by being shorted by low impedance of transformer or generator at the end of the line, line traps are used which offer very high impedance to carrier frequency and low impedance to power frequency. Similarly the coupling capacitors are so designed that they offer very high impedance to power frequency but low to carrier frequencies. The two most commonly employed coupling methods are centre-phase to ground and adjacent-phase to phase as shown in Fig. 19.9.

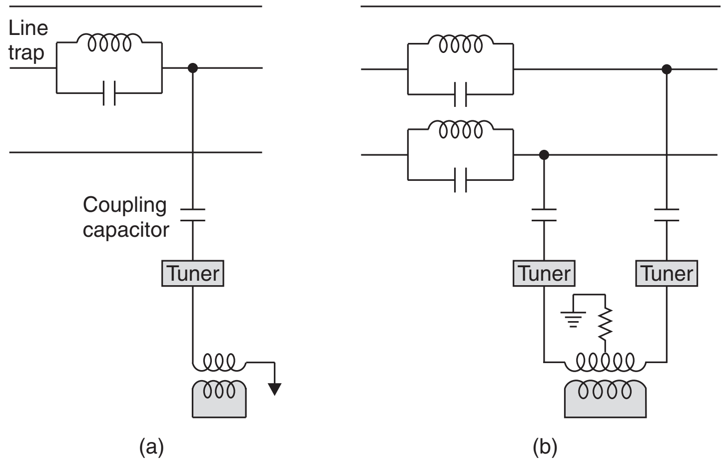


Fig. 19.9 Coupling circuits (*a*) centre-phase to ground and (*b*) adjacent phase of phase.

For centre-phase to ground only one phase line is required and ground is used as the return path whereas for adjacent phase to phase connection two phase lines are used for the go and return paths. Thus only half of the coupling components are required in the former method as compared with the latter one. Also with centre phase to ground method three separate carrier sets can be connected with three phase lines. But the adjacent phase to phase method has got less attenuation and transmission characteristics are fairly constant, radiation loss is smaller and signal to noise ratio (S/N) is high. For these reasons, even though adjacent phase to phase method is costlier but is normally used.

For efficient coupling at carrier frequencies, it is necessary to cancel the reactance of the coupling capacitor by means of inductance in the line matching unit and to match the impedance of the power line. Power line impedances are of the order of 300 to 500 ohms (phase to ground and phase to phase) respectively. The transformer included in the line matching unit are provided with several taps to facilitate optimum impedance matching between the terminal and the power line. Proper, impedance match at both parts of the tuning unit is important not only to assure maximum power transfer but also to prevent undesirable standing waves due to reflections at a mismatched junction. The line matching unit is normally located close to the coupling capacitor in the switchyard, and it must be weather proof. For phase to phase coupling, it is common to employ two line matching units.

Different Methods of Carrier Transmission Over Power Lines

There are two methods of transmitting carrier signals over power lines between different stations in the power grid:

(*i*) Fixed Frequency System; (*ii*) Wave Change-Over System.

In the first system, transmission of signals in different sections are carried out by different carrier frequencies as shown in Fig. 19.10.

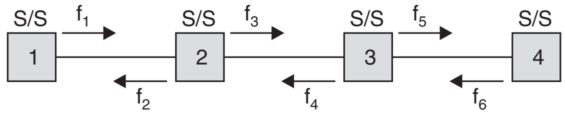


Fig. 19.10 Fixed frequency system.

Transmission between S/S 1 and 2 is carried out by carrier frequencies f_1, f_2 that between S/S 2 and 3 f_3, f_4 and between S/S 3 and 4, f_5 and f_6 and so on. In this method signals are sent between two consecutive stations by the allotted carrier frequency and the carrier is demodulated at the second station and the signal is used to modulate again the carrier frequency allotted for the second section and is retransmitted on this carrier and so on. The fixed frequency system has an advantage in that it is possible to have simultaneous transmission of signals in different sections and quality of reception is better as signals are amplified at each station. But it is a costlier system as two carrier sets are required at each intermediate substation.

In the second system (Fig. 19.11), two carrier frequencies f_1 and f_2 are used for transmission in two directions between any two stations. It is so planned that the carrier oscillator of frequency f_1 is used for sending the signal from the transmitter and the carrier oscillator of frequency f_2 is used for demodulation purpose to get back the signal in the receiver in the case of the calling station. As soon as the calling station picks up the telephone set for calling, the transmitter and the receiver side of the calling station are connected to these carrier oscillators. When the called station receives ring, its transmitter is provided with the carrier oscillator of frequency f_2 and its receiver is provided with the carrier oscillators of frequency f_1.

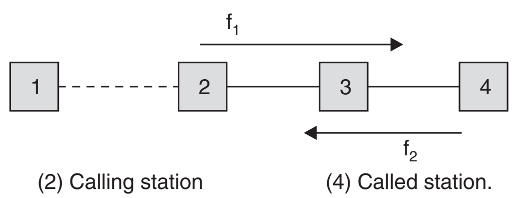


Fig. 19.11 Wave change over system.

In this way, transmission in both the directions are carried out by two carrier frequencies in all sections between any two substations.

The ground wire for protection of power lines against lightning stroke can also be used as a transmission medium by insulating the conductors from the tower with insulators having breakdown (equivalent air gap) of 15 kV to 25 kV. With this, the main function of the ground wire as a shield wire is not sacrificed. A benefit from using overhead ground wire for communication is the reduction of power frequency drainage currents induced in them due to the power lines.

The ground wire when insulated results in a slight increase in zero-sequence impedance of the power line which may cause an increase in over voltages when a short circuit involving ground occurs on the power line.

A suitable coupling device must be used at each ground wire terminal to provide a drainage path to ground for the induced power frequency influence and for lightning currents while coupling the carrier signal to the shield wires. The cost of coupling equipments in case of ground wire is much smaller and is a constant irrespective of the operating voltage. Another advantage of ground wire as a medium of transmission is that it does not require a power line outage when maintenance is required whereas this is always required for maintenance of phase wire equipment.

Whenever two ground wires are used, it is desirable to transpose the wires after every 10 to 30 km. (depending upon the experience about a particular system). With this the interference of power lines with the ground wires used as communication channels will be eliminated.

The insulated ground wires are more prone to foul weather conditions in regard to attenuation of signals as compared to the phase wires. These wires have been used for supervising control, telemetry and alarm functions, but there is limited experience in regard to its application for protective relay channels.

Bundled conductors may also be used as transmission channel for communications. Conductors of the same phase insulated from each other would be used and then ground will not be used as the return path. This scheme is being operated in USSR. The technique offers the following advantages:

(*i*) Less interference with or from radio services in the PLC band.

(*ii*) Lower fair weather lines loss.

(*iii*) Heavy reduction in cross talk.

It is hoped that bundled conductors may prove to be an important technique in future application of power line carrier particularly in ice free areas.

Modern power system has grown quite complex. The awareness of power system engineer for a good quality of reliable supply has forced him to adopt some automatic means so that he not only operates his system reliably but most economically. For this he has to know the state of the system so that he can regulate the system in that direction. The state (frequency, voltage, active power, reactive power flows, position of C.B., switches etc.) of such a complex system can be obtained by telemetering only. No manual method can handle such a complex job.

For the sake of convenience, the whole system is divided into a number of areas or groups each area having an area (or group) control station and finally these stations are centrally controlled (or coordinated) by what are known as master controlled stations. These different control stations are located at suitable points in the network and they control such items as voltage, power, frequency on getting information from distant points regarding condition of the system by telemetring.

There are some informations which are to be telemetered to group control stations only; others are to be telemetered to master control station only; yet there are other informations to be telemetered to both the stations.

Informations telemetered to group control station:

(*i*) Generated active and reactive power of each station in the group.

(*ii*) Net active and reactive power transfer from the group.

(*iii*) Active and reactive power flow through the tie lines, alongwith direction of flow and an overload alarm.

Information to be telemetered to control station:

(*i*) Voltage and frequency from selected points.

(*ii*) Total generated active and reactive power of the system.

(*iii*) Tie-line loadings (active and reactive powers) from selected points.

Informations to be telemetered to both group and central control stations:

(*i*) System frequency and rate of change of frequency.

(*ii*) Generated active and reactive power area wise.

(*iii*) Area-interconnection active and reactive power.

All the above informations are sent by telemetering to the control stations where they are displayed by suitable meters and some of these informations are fed to the computer for processing and generating commanding signals to be followed by various equipments (*e.g.*, automatic voltage regulators and other controller) for good quality of reliable and economic supply.

PROBLEMS

19.1. Incremental fuel costs in Rs. per megawatt hour for two units in a plant are given by

$$\frac{dF_1}{dP_1} = 0.1\, P_1 + 20$$

$$\frac{dF_2}{dP_2} = 0.12\, P_2 + 16$$

The minimum and maximum loads on each unit are to be 20 MW and 125 MW respectively. Determine the incremental fuel cost and the allocation of load between units for the minimum cost when loads are (*i*) 100 MW, (*ii*) 150 MW. Assume both the units are operating.

19.2. Determine the saving in fuel cost in Rs. per hour for the economic distribution of a total load of 200 MW between the two units of the plant described in Problem 19.1 compared with equal distribution of the same total load.

19.3. A system consists of two plants connected by a tie line and a load is located at plant 2. When 100 MW are transmitted from plant 1, a loss of 10 MW takes place on the tie-line. Determine the generation schedule at both the plants and the power received by the load when λ for the system is Rs. 25 per megawatt hour and the incremental fuel costs are given by the equation

$$\frac{dF_1}{dP_1} = 0.03\, P_1 + 17 \text{ Rs./MWhr}$$

$$\frac{dF_2}{dP_2} = 0.06\, P_2 + 19 \text{ Rs./MWhr}$$

19.4. If the power received by the load in the previous example is 200 MW, determine the savings in rupees per hour obtained by coordinating rather than simply including the transmission loss and not coordinating them in determining the loading of the plant.

19.5. The incremental fuel costs for two plants are given by

$$\frac{dF_1}{dP_1} = 0.1\, P_1 + 22$$

$$\frac{dF_2}{dP_2} = 0.12\, P_2 + 16$$

where F is in Rs./hr and P is in MW. If both the units operate at all times and maximum and minimum loads on each unit are 100 MW and 20 MW respectively. Determine the economic operating schedule of the plants for loads of 40 KW, 60 MW, 80 MW, 120 MW, 160 MW and 180 MW, neglecting the transmission line losses.

19.6. The fuel inputs to two plants are given by

$$F_1 = 0.015\, P_1^2 + 16\, P_1 + 50$$
$$P_2 = 0.025\, P_2^2 + 12\, P_2 + 30$$

The loss coefficients of the system are given by $B_{11} = 0.005$, $B_{12} = -\,0.0012$ and $B_{22} = 0.002$. The load to be met is 200 MW, determine the economic operating schedule and the corresponding cost of generation if (*i*) the transmission line losses are coordinated, (*ii*) the losses are included but not coordinated.

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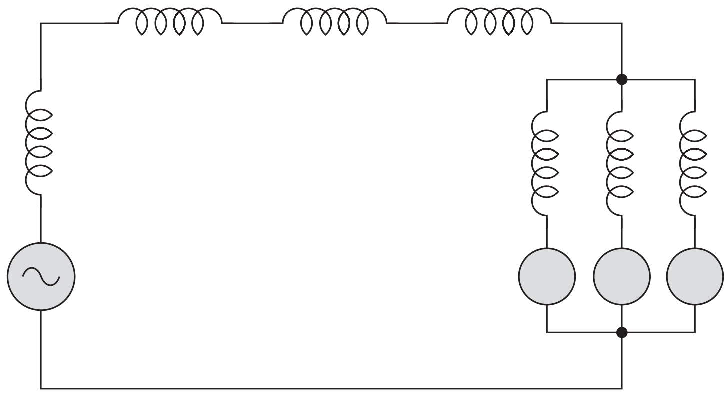
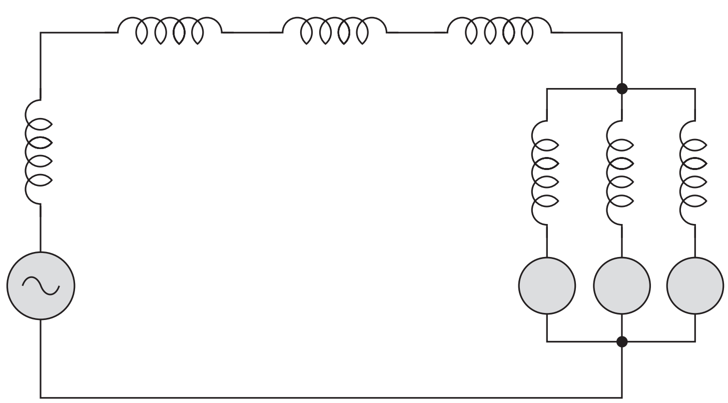


20

LOAD FREQUENCY CONTROL



20

Load Frequency Control

INTRODUCTION

The wide-spread use of electric clocks, the need for satisfactory operation of power stations running in parallel and the relation between system frequency and the speed of the motors has led to the requirement of close regulation of power system frequency. Since the control of system frequency and load depends upon the governors of the prime movers we must understand governor operation. Fig. 20.1 shows the basic characteristics of a governor. It is seen that with a given setting there is a definite relationship between turbine speed and the load being carried by the turbine. If the load carried by the turbine increases the speed decreases. For example in Fig. 20.1 if the turbine is operating at 25% load the speed is about 99% and if the load is increased to 50% the speed drops to 98%. Now if A is the initial operating point and the load is dropped to 25% the speed increases. In order to keep the speed same as at A, the governor setting by changing the spring tension in the flyball type of governor will be resorted to and the characteristic of the governor will be indicated by the dotted line as shown in Fig. 20.1. In practice the change in characteristic is obtained by remotely operating the governor control motor from the control room. A turbine can be adjusted to carry any given load at a desired speed. The governor characteristic as shown in Fig. 20.1 is an ideal one whereas the actual characteristic departs from the ideal one due to valve openings at different loadings.

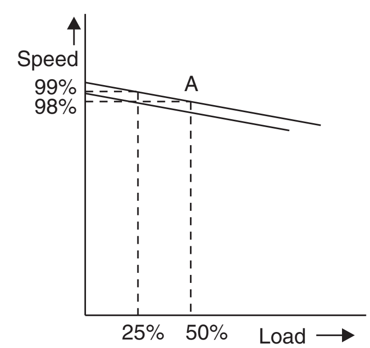


Fig. 20.1 Governor characteristic.

20.1 LOAD FREQUENCY PROBLEM

If the system consists of a single machine connected to a group of loads the speed and frequency change in accordance with the governor characteristics as the load changes. If it is not important to keep frequency constant no regulation control is required. The frequency normally would vary by about 5% between light load and full load conditions. On the other hand if constant frequency is required the operator can adjust the speed of the turbine by changing the governor characteristic as and when desired.

If a change in load is taken care of by two machines running in parallel as shown in Fig. 20.2, the complexity of the system is increased. The possibility of sharing the load by the two machines is as follows: Say, there are two stations S_1 and S_2 interconnected through a tie-line. If the change in load is either at S_1 or S_2 and if the generation of S_1 alone is regulated to adjust this change so as to have constant frequency, the method of regulation is known as Flat Frequency Regulation. Under such situation station S_2 is said to be operating on base load. The major drawback of flat frequency regulation is that S_1 must absorb all load changes for the entire system thereby the tie-line between the two stations would have to absorb all load changes at station S_2 since the generator at S_2 would maintain its output constant. The operation of generator S_2 on base load has the advantages when S_2 is much more efficient than the other station and it is desirable to obtain maximum output of S_2.

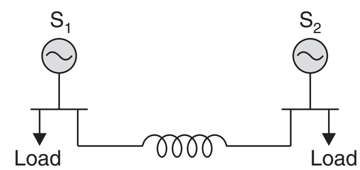


Fig. 20.2 Two plants connected through a tie-line.

The other possibility of sharing the change in load is that both S_1 and S_2 would regulate their generations to maintain the frequency constant. This is known as parallel frequency regulation. The third possibility is that the change in a particular area is taken care of by the generator in that area thereby the tie-line loading remains constant. This method of regulating the generation for keeping constant frequency is known as flat-tie line loading control. This arrangement has the advantage that load swings on station S_1 and the tie-line would be reduced as compared with the flat frequency regulation. Automatic equipment permits various types of system control. The various methods discussed above can be performed with the help of automatic control equipments. Besides these, two other types of controls are widely used in automatic arrangements. They are (*i*) Selective Frequency Control and (*ii*) Tie-line Load-bias Control.

The common method of operating a large interconnected system assigns frequency control to a central system, the other systems are then controlled on the basis of system frequency and tie-line loading. The tie-line loading as the basis of automatic control is used in three different ways. One of these is known as Selective Frequency Control. Here each system in the group takes care of the load changes on its own system and does not aid the other systems in the group for changes outside its own limits.

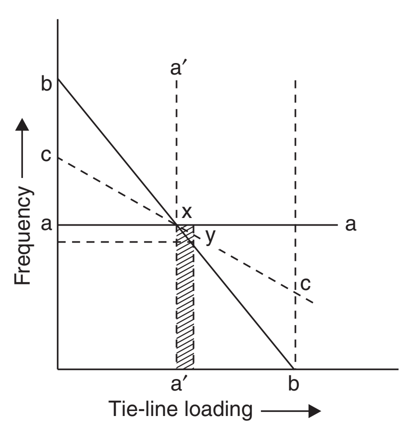


Fig. 20.3 Tie-line loading frequency characteristic.

The most commonly used method is the tie-line load-bias control in which all power systems in the interconnection aid in regulating frequency, regardless of where the frequency change originates. The equipment consists of a master load frequency controller and a tie-line recorder measuring the power input on the tie, as for selective frequency control. The tie-line instrument biases the load frequency controller by changing the control point until the desired relationship exists between tie-line loading and system frequency. This is illustrated in Fig. 20.3.

The solid line *bb* represents a characteristic similar to the governor characteristic. The solid line *aa* represents a constant frequency characteristic. If the station is trying to maintain constant frequency, the tie-line load changes would vary over very wide limits whereas if it were working along *a′a′*, the tie-line load would be held constant regardless of the frequency. The actual operating point in tie-line load-bias control lies between these two extreme limits such as *bb* or *cc*.

In order to explain this method refer to Fig. 20.3 and consider that there is increase in load on station S_1 as a result there is reduction in frequency, the control at S_2 , being immediately responsive to frequency, increases its generation to restore frequency to normal. The amount of load picked up depends on the bias for which the regulating equipment has been set. If initially *x* is the operating frequency and if it drops to *y* due to increase in load, the load picked up on the tie-line will be shown by the shaded area under *xy*. The tie-line has definitely gone off schedule, but this power is contributed to the interconnection in a direction to improve system regulation. The amount of bias is adjustable, depending on the extent to which station S_2 is scheduled to contribute to system regulation.

In a large interconnected system, as has been stressed previously that manual regulation is not feasible and, therefore, load frequency equipment is installed for each generator. Similarly for voltage control also voltage regulation equipment is installed on each generator. Figure 20.4 gives the schematic diagram of load frequency and excitation voltage regulators of a turbo-generator. The controllers are set for a particular operating condition and they take care of small changes in load demand without voltage and frequency exceeding the pre-specified limits. If the operating conditions change materially the controllers must be re-set either manually or automatically.

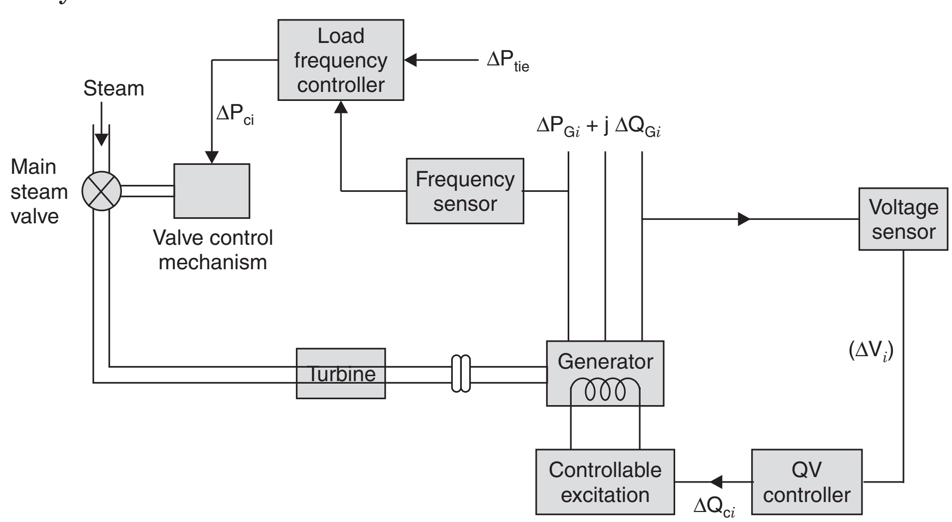


Fig. 20.4 Schematic diagram of L-F and Q-V regulators.

It is known that small changes in load depend upon the change in rotor angle δ and is independent of the bus voltage whereas the bus voltage is dependent on machine excitation (*i.e.*, on the reactive generation Q, and is independent of rotor angle δ. Therefore, the two controls, *i.e.,* load frequency and excitation voltage controls are non-interactive for small changes and can be modelled and analysed independently. Besides, the load frequency controller is slow acting because of the large time constant contributed by the turbine and generator moment of inertia, and excitation voltage control is fast acting as the time constant of the field winding is relatively smaller, thus the transients in excitation voltage control vanish much faster and do not affect the dynamics of load frequency control. We will consider here only the load frequency control aspect of regulation. It is to be noted here that the regulator designed for control should not be insensitive to fast random changes, otherwise the system will be prone to hunting, resulting in excessive wear and tear of control equipments and the rotating machines.

The main objective of the load frequency controller is to exert control of frequency and at the same time of real power exchange via the outgoing lines. The change in frequency and the tie-line real power are sensed which is a measure of the change in rotor angle δ, *i.e.*, the error $\Delta\delta_i$ to be corrected. The error signals, *i.e.,* Δf_i and ΔP_{tie} are amplified mixed and transformed into a real power command signal ΔP_{ci} which is sent to the prime mover to call for an increment in the torque. The prime mover, therefore, brings change in the generator output by an amount ΔP_{Gi} which will change the values of Δf_i and ΔP_{tie}. The process continues till the deviation Δf_i and ΔP_{tie} are well below the specified tolerances.

20.2 SPEED GOVERNING SYSTEM

Figure 20.5 shows the schematic diagram of a speed governing system which controls the real power flow in the power system. The speed governing system consists of the following parts:

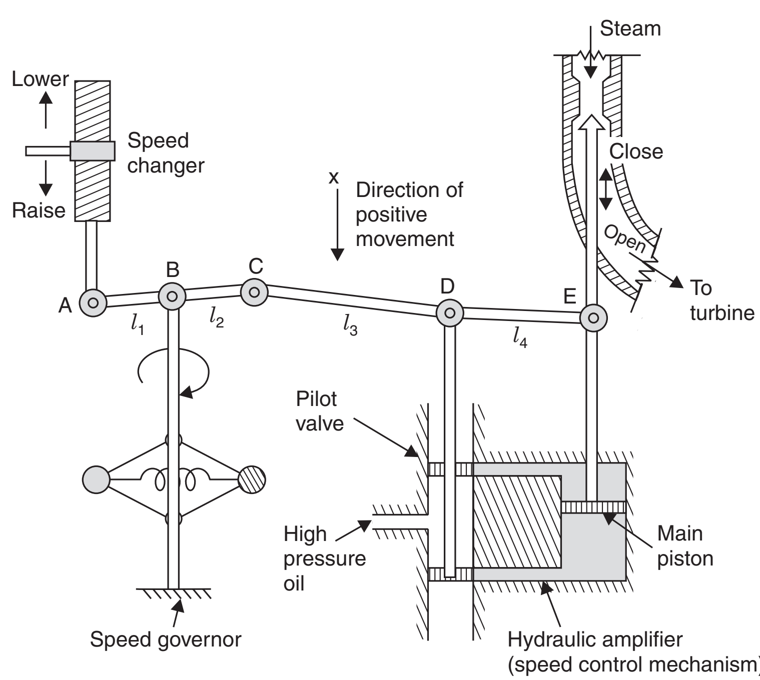


Fig. 20.5 Turbine speed governing system.

1. *Speed Governor*: This is a fly-ball type of speed governor and constitutes the heart of the system as it senses the change in speed or frequency. With the increase in speed the fly-balls move outwards and the point B on linkage mechanism moves downwards and vice-versa.

2. *Linkage Mechanism*: ABC and CDE are the rigid links pivoted at B and D respectively. The mechanism provides a movement to the control valve in the proportion to change in speed. Link 4 (l_4) provides a feedback from the steam valve movement.

3. *Hydraulic Amplifier*: This consists of the main piston and pilot value. Low power level pilot valve movement is converted into high power level piston valve movement which is necessary to open or close the steam valve against high pressure steam.

4. *Speed Changer*: The speed changer provides a steady state power output setting for the turbine. The downward movement of the speed changer opens the upper pilot valve so that more steam is admitted to the turbine under steady condition. The reverse happens when the speed changer moves upward.

Model of Speed Governing System

We consider the steady state condition by assuming that the linkage mechanism is stationary, pilot valve closed, steam valve opened by a definite magnitude, the turbine output balances the generator output and the turbine or generator is running at a particular speed when the frequency of the system is f^0, the generator output P_{G0} and let the steam valve setting corresponding to these conditions be X_E^0 (Fig. 20.5). We now proceed to obtain the linear model of the system around these operating conditions.

Let the point A of the speed changer lower down by an amount ΔX_A as a result the commanded increase in power is ΔP_c then $\Delta X_A = K_1 \Delta P_c$. The movement of linkage point A causes small position changes ΔX_c and ΔX_D of the linkage points C and D. With the movement of D upwards by ΔX_D high pressure oil flows into the hydraulic amplifier from the top of the main piston thereby the steam valve will move downwards a small distance ΔX_E which results in increased turbine torque and hence power increase ΔP_G. This further results in increase in speed and hence the frequency of generation. The increase in frequency Δf causes the link point B to move downward a small distance ΔX_B proportional to Δf. We assume that the movements are positive if the points move downwards.

Since all the movements are small, we have the linear relationship.

Two factors contribute to the movement of C:

(*i*) Increase in frequency causes B to move by ΔX_B when the frequency changes by Δf as then the fly-ball moves outward and B is lowered by ΔX_B. Therefore, this contribution is positive and is given by $K_1 \Delta f$.

(*ii*) The lowering of the speed changer by an amount ΔX_A lifts the point C upwards by an amount proportional to ΔX_A, *i.e.*, let this be $K_2' \Delta X_A$ or $K_2 \Delta P_c$.

$$\therefore \qquad \Delta X_C = K_1 \Delta f - K_2 \Delta P_c \qquad (20.1)$$

The positive constants K_1 and K_2 depend upon the length of the linkage arms AB and BC and upon the proportional constants of the speed changer and the speed governor.

The movement of D is contributed by the movement of C and E. Since C and E move downwards when D moves upwards, therefore,

$$\Delta X_D = K_3 \, \Delta X_C + K_4 \, \Delta X_E \qquad (20.2)$$

The positive constants K_3 and K_4 depend upon the length of the linkage *CD* and *DE*.

Assuming that the oil flow into the hydraulic cylinder is proportional to position ΔX_D of the pilot valve, the value of ΔX_E is given by

$$\Delta X_E = K_5 \int_0^t -(\Delta X_D)\, dt \tag{20.3}$$

The constant K_5 depends upon the fluid pressure and the geometries of the orifice and the cylinder.

Taking Laplace transform of equations (20.1–20.3), we obtain

$$\Delta X_C(s) = K_1\, \Delta F(s) - K_2\, \Delta P_c(s) \tag{20.4}$$

$$\Delta X_D(s) = K_3\, \Delta X_C(s) + K_4\, \Delta X_E(s) \tag{20.5}$$

$$\Delta X_E(s) = -\frac{K_5}{s} \Delta X_D(s) \tag{20.6}$$

Eliminating the variables ΔX_C and ΔX_D, we obtain

$$\Delta X_E(s) = \frac{K_2 K_3 \Delta P_c(s) - K_1 K_3 \Delta F(s)}{K_4 + s / K_5}$$

$$= \frac{K_G}{1 + sT_G}\left[\Delta P_c(s) - \frac{1}{R}\Delta F(s)\right] \tag{20.7}$$

where $R = \dfrac{K_2}{K_1}$ speed regulation of the governor.

$$K_G = \frac{K_2 K_3}{K_4} = \text{gain of speed governor}$$

$$T_G = \frac{1}{K_4 K_5} = \text{time constant of speed governor}$$

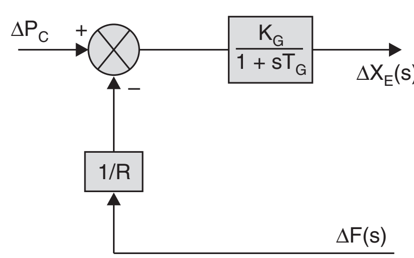


Fig. 20.6 Block diagram representation of speed governing system for steam turbine.

Turbine Model

The model requires a relation between changes in power output of the steam turbine to changes in its steam valve opening ΔX_E. We consider here a non-reheat turbine with a single gain factor K_T and a single time constant T_T and thus in the crudest model representation of the turbine the transfer function is given as

$$G_T(s) = \frac{\Delta P_t(s)}{\Delta X_E(s)} = \frac{K_T}{1 + sT_T} \tag{20.8}$$

Typically the time constant T_T lies in the range 0.2 to 2.0 sec. Fig. 20.7 shows the linearized model of a non-reheat turbine controller including the speed governor mechanism.

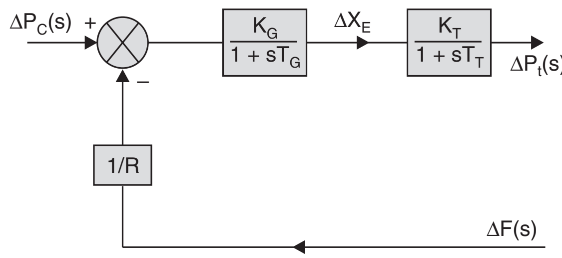


Fig. 20.7 Transfer function representation of power control mechanism of turbine.

Generator-load Model

The model gives relation between the change in frequency as a result of change in generation when the load changes by a small amount. Let ΔP_D be the change in load, as a result the generation also swings by an amount ΔP_G. The net power surplus at the bus bar is $\Delta P_G - \Delta P_D$ and this power will be absorbed by the system in two ways:

(*i*) By increasing the kinetic energy of the generator rotor at a rate $\frac{dW}{dt}$, where W is the new value of kinetic energy. Let W^0 be the K.E. before the change in load occurs when the frequency is f^0 and let W be the K.E. when the frequency is $f^0 + \Delta f$. Since the K.E. is proportional to square of the speed of the generator, therefore, the two kinetic energies can be correlated as

$$W = W^0\left(\frac{f^0 + \Delta f}{f^0}\right)^2 \tag{20.9}$$

$$W = W^0\left(1 + \frac{2\Delta f}{f^0}\right) \tag{20.10}$$

neglecting the higher terms as $\frac{\Delta f}{f^0}$ is small.

$$\therefore \qquad \frac{dW}{dt} = \frac{2W^0}{f^0} \cdot \frac{d}{dt} \cdot (\Delta f) \tag{20.11}$$

(*ii*) The load on the motors increases with increase in speed. The load on the system being mostly motor load the rate of change of load with respect to frequency can be regarded as nearly constant for small changes in frequency, *i.e.,* $D = \frac{\partial P_D}{\partial f}$, where D can be obtained empirically. Therefore, the net power surplus at the bus bar is given by

$$\Delta P_G - \Delta P_D = \frac{2W^0}{f^0}\frac{d}{dt}(\Delta f) + D \cdot \Delta f \ \ldots \tag{20.12}$$

If H is the inertia constant of the generator in MW-sec/MVA and P is the rating in MVA, then $W_0 = HP$.

Rewriting the balance equation (20.12)

$$\Delta P_G - \Delta P_D = \frac{2HP}{f^0}\frac{d}{dt}(\Delta f) + D \cdot \Delta f \quad \ldots \tag{20.13}$$

Dividing throughout by P we get

$$\Delta P_G(\text{p.u.}) - \Delta P_D\,(\text{p.u.}) = \frac{2H}{f^0}\, s\Delta F(s) + D(s)\Delta F(s)$$

$$= \Delta F(s)\left[\frac{2H}{f^0}s + D(s)\right]$$

or

$$\Delta F(s) = \frac{\Delta P_G(s) - \Delta P_D(s)}{\dfrac{2H}{f^0}s + D(s)}$$

or

$$\Delta F(s) = [\Delta P_G(s) - \Delta P_D(s)]\frac{K_P}{1 + sT_P} \tag{20.15}$$

where $T_p = \dfrac{2H}{Df^0}$ = power system time constant

and $K_p = \dfrac{1}{D}$ = power system gain

The block diagram representing equation (20.15) is given in Fig. 20.8.

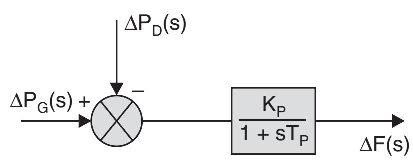


Fig. 20.8 Generator load model.

The complete block diagram representing the load frequency control of an isolated power system is given in Fig. 20.9.

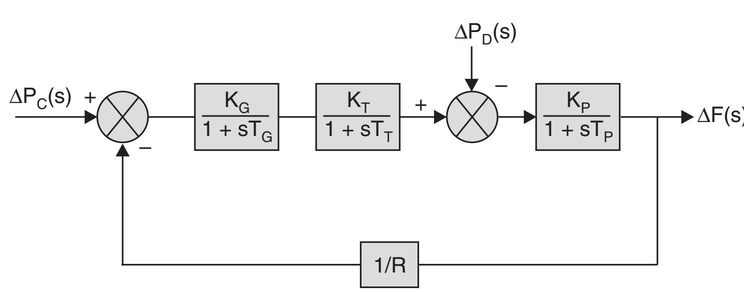


Fig. 20.9 Isolated power system load frequency control block diagram.

20.3 REASONS FOR LIMITS ON FREQUENCY

The following are the reasons for keeping strict limits on the system frequency variations:

(*i*) The speed of a.c. motors are directly related to the frequency. Even though most of the a.c. drives are not much affected for a frequency variation of even 50 ± 1.5 Hz but there are certain applications where speed consistency must be of high order.

(*ii*) The electric clocks are driven by synchronous motors and the accuracy of these clocks is not only a function of frequency error but is actually of the integral of this error.

(*iii*) If the normal frequency is 50 Hz and the turbines are run at speeds corresponding to frequency less than 47.5 Hz or more than 52.5 Hz, the blades of the turbine are likely to get damaged, hence a strict limit on frequency must be adhered to as stalling of the generator will further aggravate the problem if the system is operating at the lower limit of frequency.

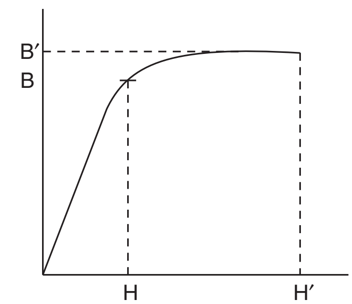


Fig. 20.9(a) B-H curve.

(*iv*) The under frequency operation of the power transformer is not desirable. For constant system voltage if the frequency is below the normal value the flux in the core increases. Since we design these transformers corresponding to the 'knee point' on the *B-H* curve (B) a small increase in B drives the transformer into saturation region (B').

As a result the magnetising current even exceeds the normal full load current. The sustained under frequency operation of the power transformer results not only in low efficiency but it may even damage the transformer winding due to overheating. The problem is further aggravated by the fact that to transmit one MW of power from generating station to the consumer end 4 MW equivalent capacity transformers are installed. Hence a strict limit on the frequency operation of power system is desirable.

(*v*) The system operation at subnormal frequency and voltage leads to loss of revenue to the suppliers due to accompanying reduction in load demand.

(*vi*) The most serious effect of subnormal frequency is on the operation of thermal power plants. With reduced frequency the blast by ID and the FD fans decreases, as a result of which the generation also decreases and thus it becomes a cumulative action and may result in complete shut-down of the plant if corrective measures like load shedding is not resorted to. Load shedding is done with the help of under frequency relay which automatically disconnects blocks of loads. The setting of the under frequency relays is so adjusted that the least important load is disconnected at a relatively higher frequency and vice versa.

(*vii*) The overall operation of power system can be better controlled if a strict limit on frequency deviation is maintained. The frequency is closely related to the real power balance in the overall network. Under normal operating conditions the generators run synchronously and the generated power equals the load demand plus the losses at any instant of time. When a generator is connected to the grid, its speed gets locked to the grid system. Now if we want to control the real power output from this generator, we must control the torque from 'its' primemover. By opening the steam valve and thus increasing the steam pressure on the turbine blade, greater torque can be applied to the generator thereby tending to accelerate the generator. However, its speed is tied to the grid and hence the motor advances its torque angle by few degrees depending upon the load increase requirements. This results in increased delivered current and power. The increased current thus develops a decelerating torque within the machine which exactly balances the increase in accelerating torque. Unfortunately the counterbalancing electromagnetic torque is not developed instantly. The duration for which the unbalance exists depends upon the change in load and the total inertia of the system. If the change in load is small as compared to the total inertia of the system, the frequency remains almost constant throughout the operation of the system. However the load fluctuations are entirely random and also are the operating conditions of the system network and hence it is impossible to obtain a perfect instant by instant match between generation and the load. There will always be a surplus or deficiency in the generation and hence this ever present mismatch will cause frequency fluctuation.

Suppose initially there is perfect matching between the power generated and the load demand and say the system frequency is 50 Hz. Let the system load decrease by some amount. The prime mover does not respond to the load change instantly as it is ignorant of the load change. The decrease in load results in decrease in current supplied by the generators which results in decrease in electromechanical torques in all the generators. Every generator would thus experience a small surplus accelerating torque which results in increase in speed of generators and thus the increase in frequency. All the motors which are fed by the network would experience increase in speed and thus their torques would increase. As the torque developed by the motors increases the power drawn by the motors from the network increases. If the prime-mover remains ineffective during this period, the frequency would level off at a new higher value.

In a single area uncontrolled system whenever a load increase takes place it is taken care of by the system in the following three ways:

(*i*) 'Borrowed' kinetic energy from the rotating machines of the system *i.e.*, initially the increase in load is supplied from the stored energy of the synchronous generators, as a result the speed of the machine goes down and the system frequency decreases.

(*ii*) 'Released' customer load *i.e.*, the reduction in the effective 'old' load. Since the frequency of the system decreases, the speed of the various motors decreases and hence the effective 'old load' decreases. Thus allowing the already available generation to partly meet the load demand.

(*iii*) Increased generation: The reduction in system frequency actuates the speed governing system of the generating units which then increases the input to the prime movers causing increased generation which subsequently arrests a further drop in frequency. The units behave coherently, maintaining thereby equal frequency deviations among them.

Initially corresponding to synchronous speed the last two components are zero and as the speed decreases, they contribute to the increased generations. However, it is to be noted that the contribution due to the last two factor is very small and the major contribution is due to the increased generation due to governor action.

In case the load is decreased the sequence of behaviour will be reversed *i.e.*, the frequency will increase and thereby the effective load will increase which then is decreased by the governor action.

The energy stored in a machine can be given as $W_0 = Kf_0^2$, where f_0 is the nominal frequency. Say the load is increased as a result the frequency decreases. Say the new energy of the rotating system is

$$W = Kf^2 \text{ such that}$$

$$f = f_0 + \Delta f$$

Now
$$\frac{dW}{df} = 2kf$$

or
$$dW = 2kf\, df$$

$$= \frac{2W_0}{f_0^{\,2}}(f_0 + \Delta f)\,\Delta f$$

$$dW \cong \frac{2\Delta f}{f_0} W_0$$

Dividing by the capacity of the machine

$$\frac{dW}{P} = \frac{2\Delta f}{f_0} \cdot \frac{W_0}{P} = \frac{2\Delta f}{f_0} H$$

Here H is the inertia constant of the machine and its units are kW-sec/kVA or it is in secs. Suppose a system has a capacity of 200 MW and assuming $H = 4$ sec. the stored energy will be 800 MJ. Also, let the load be changed suddenly by 4 MW.

Then
$$\frac{4 \times 4}{200} = \frac{2\Delta f}{50} \times 4$$

$$\frac{16}{200} = \frac{2\Delta f}{50} \times 4$$

$$\Delta f = 0.5 \text{ Hz}$$

While studying load frequency control, the system voltage in assumed to be constant. A change in the load with change in frequency is given by

$$\Delta P_D = \frac{dP_D}{\partial f} \Delta f = D\Delta f$$

where $D = \dfrac{dP_D}{df}$ and is known as the damping constant due to the frequency and has the unit MW per Hz. It is also known as frequency coefficient of load and characterises the frequency characteristic of the load. It is usually expressed in per cent of connected load per 0.1 Hz and the typical values lie between 0.3 to 0.5% per 0.1 Hz.

The percentage speed variation from no load to full load is about 3 to 6% for the generators. Since the frequency and speed are directly related, the frequency variation also will be within 3 to 6%. If the frequency variation is 4% for 50 Hz system, there will be a drop of 2 Hz in frequency on full load. If the generation is increased by ΔP_G due to frequency drop of Δf,

$$\Delta P_G = \frac{dP_G}{df} \times \Delta f = R \times \Delta f.$$

or
$$R = \frac{\Delta f}{\Delta P_G} \text{ Hz/MW}$$

Two different controls are carried out on the governor characteristics. The parameter R is adjusted during off-line condition of the unit to ensure its proper coordination with the other units. The second control shifts the straight line characteristic parallel to itself to change the load distribution among the generators connected in parallel as well as to maintain the system frequency. The second control also known as supplementary control is explained as follows: Refer to Fig. 20.10 where the governor characteristics of two units are shown. Suppose initially the total load is P and it is so shared by the two units that $P = P_1 + P_2$ and the frequency is f_n. Now if the load is increased to P', the frequency falls to f_n' as the units can increase their output by giving away some of its kinetic energy *i.e.*, by reducing the speed. In order to restore the system frequency, characteristic of both the units (as shown in Fig. 20.10) or one of the two units can be raised as shown by the dotted lines so that the total load P' is shared by both the units keeping the system frequency to f_n. The power supplied here is $P' = P_1' + P_2'$. The raising (in case of increased load) or lowering of governor characteristic is carried out with a device known as speed changer which is operated either manually or by a servometer.

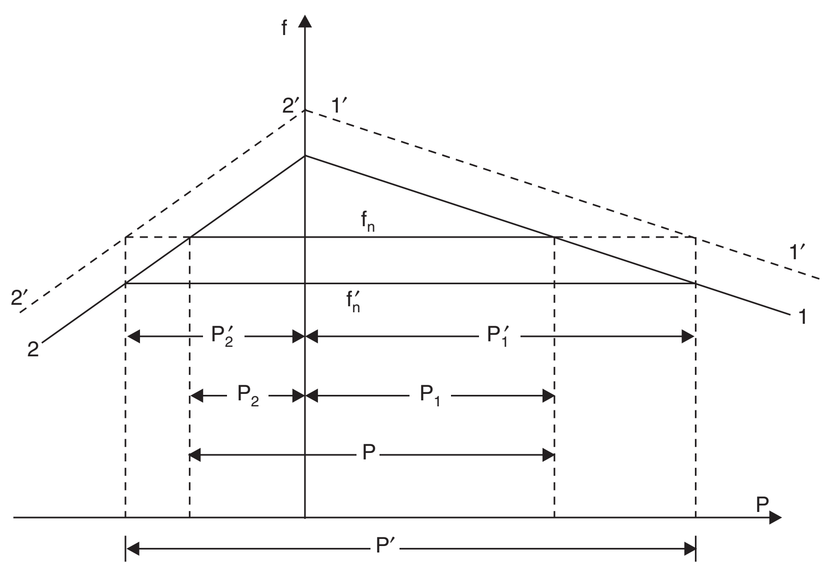


Fig. 20.10 Parallel operation of two alternators.

The system performance in terms of how the change in power affects the change in frequency is evaluated through what is known as area frequency response characteristic (AFRC). Its unit is p.u. MW/Hz. It gives the combined effects of the frequency characteristic of the load of the area and the inherent governing system of the area. This can be determined by measuring the change in frequency as a result of tripping out some load of the area. This depends upon the level of area load and the spinning capacity. AFRC is expressed in per cent of the area spinning capacity and its typical values lie in the range 1–4% per 0.1 Hz.

It is to be noted that if frequency of two areas is to be controlled, the static frequency drop is just one-half of that of the isolated operation of two systems. Also, if there is change in load in any area, half of it will be shared by the other area. It is found that if a load changes in an area, the frequency and interchange errors in that area have the same sign while these have opposite signs for the other area. Thus the relative signs of the frequency and interchange deviations help to identify the area where the load has changed.

Example 20.1: Two generating stations *A* and *B* have full load capacities of 200 MW and 75 MW respectively. The interconnector connecting the two stations has an induction motor/synchronous generator (plant *C*) of full load capacity 25 MW. Percentage changes of speeds of *A*, *B* and *C* are 5, 4 and 3 respectively. The loads on bus bars *A* and *B* are 75 MW and 30 MW respectively. Determine the load taken by the set *C* and indicate the direction in which the energy is flowing.

Solution: Refer to Fig. E.20.1 for the system.

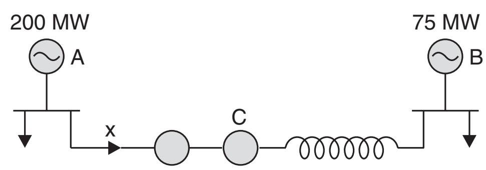


Fig. E.20.1

Assume that xMW flows from *A* to *B*.

$\therefore$ Load on station $A = (75 + x)$

$$\therefore \quad \% \text{ drop in speed} = \frac{5}{200}(75 + x)$$

Load on station $B = (30 - x)$

$$\therefore \quad \% \text{ drop in speed} = (30 - x)\frac{4}{75}$$

Since the output of *A* corresponding to the power flow through the inter-connector is the input to *C*, therefore, there is further reduction in frequency, which is given approximately by $\frac{3x}{25}$. The total reduction in frequency both from *A* and *C* and the reduction in frequency from *B* when referred to bus *B*, should be same

$$\therefore \quad \frac{5}{200}(75 + x) + \frac{3x}{25} = (30 - x)\frac{4}{75}$$

$$1.875 + 0.025x + 0.12x = 1.6 - 0.0533x$$

$$x = -\,1.38 \text{ MW}$$

which means that the power of magnitude 1.38 MW will be from *B* to *A*.

Example 20.2: Two turboalternators rated for 110 MW and 210 MW have governor drop characteristics of 5 per cent from no load to full load. They are connected in parallel to share a load of 250 MW. Determine the load shared by each machine assuming free governor action.

Solution: Since the two machines are working in parallel the per cent drop in frequency from both the machines due to different loadings must be same. Let x be the power supplied by 110 MW unit.

The per cent drop in speed = $\dfrac{5x}{110}$. Similarly per cent drop in speed of 210 MW unit will be $\dfrac{5}{210}(250 - x)$.

$$\therefore \qquad \frac{5x}{110} = \frac{5x}{210}(250 - x)$$

or
$$\frac{x}{11} = \frac{250 - x}{21}$$

or
$$x = 85.93 \text{ MW}$$

$\therefore$ Power shared by 210 MW unit will be 250 – 85.93 = 164.07 MW

Power supplied by 110 MW unit = 85.93 MW

Power supplied by 210 MW unit = 164.07 MW. **Ans.**

Example 20.3: A 100 MVA 50 Hz turboalternator operates at no load at 3000 r.p.m. A load of 25 MW is suddenly applied to the machine and the steam valves to the turbine commence to open after 0.6 secs due to the time-lag in the governor system. Assuming inertia constant H of 4.5 kW-sec per kVA of generator capacity, calculate the frequency to which the generated voltage drops before the steam flow commences to increase to meet the new load.

Solution: By definition $H = \dfrac{\text{Stored energy}}{\text{Capacity of machine}}$

$\therefore$ The energy stored at no load = 4.5 × 100 = 450 MJ

Before the steam valves open the energy lost by the rotor = 25 × 0.6 = 15 MJ

As a result of this there is reduction in speed of the rotor and, therefore, reduction in frequency.

From equation (20.9) the new frequency will be

$$f_{\text{new}} = \sqrt{\frac{450 - 15}{450}} \times 50 \text{ Hz} = 49.16 \text{ Hz.} \quad \textbf{Ans.}$$

PROBLEMS

20.1. Two generators rated at 120 MW and 250 MW are operating in parallel. The governor settings on the machines are such that they have 4 per cent and 3 per cent drops. Determine (*i*) the load taken by each machine for a total load of 200 MW (*ii*) the percentage adjustment in the no load speed to be made by the speeder motor if the machines are to share the load equally.

20.2. An interconnector with inductive reactance of 25 ohm and negligible resistance has two units of generations with voltages of 33 kV and 30 kV at its ends. A load of 6 MW is to be transferred from 33 kV to 30 kV side of the interconnector. Determine the power factor of the power transmitted and other necessary conditions between the two ends.

20.3. Two generating stations *A* and *B* have full load capacities of 500 MW and 210 MW respectively. The interconnector connecting the two stations has a motor-generator set (plant-*C*) near station *A* of full load capacity 50 MW. Percentage changes of speeds of *A, B* and *C* are 5, 4 and 2.5 respectively. The loads on bus bars *A* and *B* are 250 MW and 100 MW respectively. Determine the load taken by set *C* and indicate the direction in which the energy is flowing.

20.4. A 210 MVA, 50 Hz turboalternator operates at no load at 3000 r.p.m. A load of 75 MW is suddenly applied to the machine and the steam valves to the turbine commence to open after 1 sec due to the time lag in the governor system. Assuming inertia constant *H* of 5 kW-sec per kVA of generator capacity, calculate the frequency to which the generated voltage drops before the steam flow commences to increase to meet the new load.

20.5. Two generating stations *A* and *B* are of capacities 20 MW and 10 MW and speed regulation of 2% and 3% respectively. The two stations are connected through an interconnector and a motor-generator set. The set is connected to bus bars of *A* and has a capacity of 3 MW and full load slip of 4%. Determine the load on the interconnector when there is a load of 8 MW on *B* bus bars due to its own consumer but *A* has no external load.

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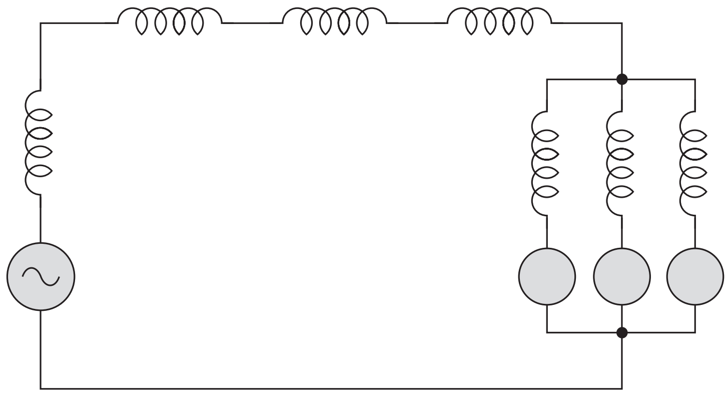
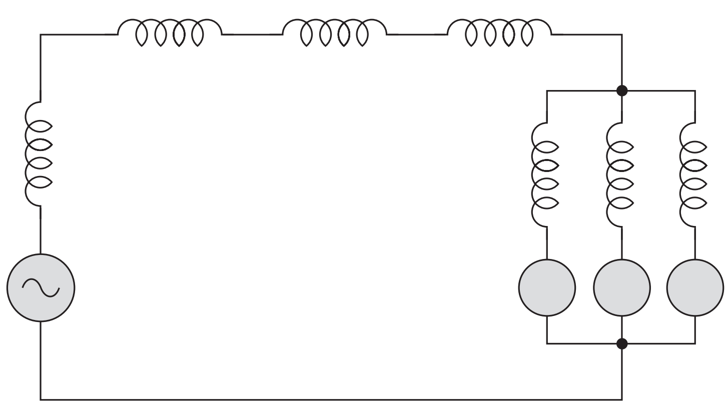


21

COMPENSATION IN POWER SYSTEM



21

Compensation in Power System

INTRODUCTION

A power system is expected to operate under widely varying conditions from no load to overloading to short-circuits and it is desired that the quality of supply should be maintained under all conditions *i.e.*, the constancy of voltage magnitude and frequency of system should be maintained. Also, it is desirable to maintain the three phase currents and voltages as balanced as possible so that undue heating of various rotating machines due to unbalancing could be avoided.

Good quality power supply also requires distortion loss voltage and current waveform of the a.c. system. These waveforms get distorted due to the presence of non-linear loads viz., adjusted speed drives, furnaces, HVDC converters and inverters, chemical processes such as electroplating power supplies, heating and welding, the use of digital computers and other digital control devices etc. The waveform can be made sinusoidal by the use of hybrid filters (passive and active filters).

We study here some of the characteristics of power systems and their loads which can deteriorate the quality of supply and then we identify those which can be corrected by compensation that is by generation or absorption of a suitable quantity of reactive power. There are mainly two types of compensations carried out (*i*) Load Compensation; (*ii*) Line Compensation.

21.1 LOAD COMPENSATION

Load compensation is the management of reactive power to improve the quality of supply especially the voltage and p.f. levels. Here the reactive power is adjusted with respect to an individual load and the compensating device is connected to the load itself. There are three main objectives in Load Compensation:

(*i*) Better voltage profile (*ii*) p.f. correction (*iii*) Load balancing.

The voltage profile must remain within ± 5% of the rated value for better and efficient operation of various electrical equipments. We know voltage variation is due to imbalance in the generation and consumption of reactive power in the system. If the generated reactive power is more than what is being consumed, voltage levels go up and vice versa. However, if the two are equal the voltage profile becomes flat. Unfortunately, the reactive power in a system keeps on varying and hence if the reactive power generation is also simultaneously controlled, a more or less near flat voltage profile could be maintained.

1. One of the obvious methods would be to have the system of large strength *i.e.*, it interconnects large size machines and a large number of lines so that the effective impedance as seen from any point into the network is negligibly small and hence the voltage profile could be improved. However, this would result in high fault levels and would require switchgears of relatively large capacity and hence is uneconomical from view point of improving voltage profile. In fact, the network should be designed based on active power transfer capability and the reactive power should be met locally by installing shunt compensating elements (capacitor and inductors). It is to be noted that these elements do not contribute to the fault level and yet provide and maintain proper balance between the generated and consumed reactive power.

2. It is desirable both economically and technically to operate the system at near unity p.f. Usually p.f. correction means to generate reactive power as close as possible to the load which requires it rather than generate it at a distance and transmit it to the load, as this results not only in large conductor size but also in increased losses. In fact, in order to operate the system at near unity p.f., some of the electric utilities impose certain penalty on account of operating loads at low p.f.

3. A very important concept of load compensation is load balancing. It is desirable to operate the three phase system under balanced condition as unbalanced operation results in flow of negative sequence current in the system and is highly dangerous especially for the rotating machines.

An *ideal load compensator* would perform the following functions: (*i*) It would provide controllable and variable reactive power almost instantaneously as required by the load; (*ii*) It should operate independently in all the three phases; and (*iii*) It would maintain constant voltage at its terminals.

Typical loads which require compensation are as follows: arc furnaces, induction furnaces, steel rolling mills, very large induction motors, arc welders, induction welders etc. These loads can be further classified as non-linear and those which cause disturbance by being switched on and off. The non-linear loads such as arc furnace generate harmonics in the system and hence arc furnace compensators usually have harmonic filters to filter out these harmonics, so that the voltage remains mostly of fundamental frequency. It is to be noted that synchronous motors as loads improve the voltage profile and p.f. of the system as this can be operated as a synchronous capacitor.

It is well known that the approximate p.u. voltage regulation for a series impedance carrying a load current I is given as

$$\Delta V = \frac{IR}{V} \cos \phi \pm \frac{IX}{V} \sin \phi \qquad (21.1)$$

where ϕ is the p.f. angle of the load and V is the load voltage. Here positive sign is for inductive load and –ve sign for the capacitive load. The load can be made capacitive effectively by connecting a capacitance across an otherwise inductive load. With this the value of capacitance can be so adjusted that $\frac{IR}{V}$ cos ϕ can be made equal to $\frac{IX}{V}$ sin ϕ and thus change in load voltage ΔV can be made equal to zero. Therefore, a purely reactive compensator can eliminate supply voltage variations caused by changes in both the real and the reactive power of the load. Also, it is clear from the above expression that with a reactive compensator it is not possible to achieve simultaneously both the unity p.f. and zero voltage regulation.

An alternative way of studying voltage regulation is through the use of short circuit capacity of the bus, X : R of the system, the active and reactive load powers. The voltage drop across the series impedance is given as

$$\Delta V = (R + jX)\left(\frac{P - jQ}{V}\right)$$

$$= \frac{RP + XQ}{V} + j\,\frac{XP - RQ}{V} = \Delta V_r + j\Delta V_x \qquad (21.2)$$

If a 3-phase short circuit is created at the load bus, the short circuit apparent power

$$S_{sc} = EI_{sc}\ \frac{E^2}{Z_{sc}}$$

where $Z_{sc} = R + jX$ and I_{sc} is the short circuit current.

Now $R = |\,Z\,| \cos \phi_{sc}$ | $X = |\,Z\,| \sin \phi_{sc}$

$$= \frac{E^2}{S_{sc}} \cos \phi_{sc} \qquad\Big|\qquad = \frac{E^2}{S_{sc}} \sin \phi_{sc}$$

Substituting for R, and X and letting $E \cong V$, we have

$$\Delta V = \frac{V}{S_{sc}}(P \cos \phi_{sc} + Q \sin \phi_{sc}) + j\,\frac{V}{S_{sc}}(P \sin \phi_{sc} - Q \cos \phi_{sc}) \qquad (21.3)$$

or

$$\frac{\Delta V}{V} = \frac{1}{S_{sc}}(P \cos \phi_{sc} + Q \sin \phi_{sc}) + j\,\frac{1}{S_{sc}}(P \sin \phi_{sc} - Q \cos \phi_{sc}) \qquad (21.4)$$

$$= \frac{\Delta V_r}{V} + j\,\frac{\Delta V_x}{V}$$

Usually ΔV_x is ignored as this does not affect the voltage magnitude much, rather this gives phase shift to the voltage.

Therefore,
$$\frac{\Delta V_r}{V} = \frac{1}{S_{sc}}\ (P \cos \phi_{sc} + Q \sin \phi_{sc}) \qquad (21.5)$$

The above equations are not only valid for any value of P and Q but these are also valid for small changes in P and Q *i.e.*,

$$\frac{\Delta V_r}{V} = \frac{1}{S_{sc}}\ (\Delta P \cos \phi_{sc} + \Delta Q \sin \phi_{sc}) \qquad (21.6)$$

Normally the real power flow does not affect much the voltage of bus and, therefore,

$$\frac{\Delta V_r}{V} \cong \frac{\Delta Q}{S_{sc}} \sin \phi_{sc} \qquad (21.7)$$

and since $\phi_{sc} = \tan^{-1} \dfrac{X}{R}$ and the ratio $\dfrac{X}{R}$ is usually more than 4, therefore $\sin \phi_{sc} \cong 1$, and, therefore,

$$\frac{\Delta V_r}{V} \cong \frac{\Delta Q}{S_{sc}} \sin \phi_{sc} \cong \frac{\Delta Q}{S_{sc}} \tag{21.8}$$

This means the per unit change in voltage equals the ratio of change in reactive power to the short circuit capacity of the bus.

Example: A load of 30 MW, 45 MVAR is connected to a line where X to R ratio is 5 and the short circuit capacity of the load bus is 250 MVA. The supply voltage is 11 kV and the load is star connected. Determine the load bus voltage.

Solution: The load per phase is $(10 + j15)$ MVA and SCC. $\dfrac{250}{3}$ MVA, voltage L – N = $\dfrac{11}{\sqrt{3}}$ kV.

The equivalent short circuit impedance $\dfrac{(11/\sqrt{3})^2}{250/3}$

$$= \frac{121}{250} = 0.484\ \Omega$$

$$\phi_{sc} = \tan^{-1} 5 = 78.69°$$

$$\therefore \qquad R = 0.0949 \text{ and } X = 0.4746\ \Omega$$

$$\left(\frac{11}{\sqrt{3}}\right)^2 = \left(V \cos \phi + \frac{P}{V} R\right)^2 + \left(V \sin \phi + \frac{Q}{V} X\right)^2$$

$$= \left(V \times 0.5547 + \frac{10 \times 10^6}{V} \times 0.0949\right)^2 + \left(V \times 0.832 + \frac{15 \times 10^6}{V \times 10^3} \times 0.4746\right)^2$$

$$\frac{121}{3} = \left(0.5547\, V + \frac{10}{V} \times 0.0949\right)^2 + \left(0.832\, V + \frac{15 \times 0.4746}{V}\right)^2$$

$$= \left(0.5547\, V + \frac{0.949}{V}\right)^2 + \left(0.832\, V + \frac{7.119}{V}\right)^2$$

$$40.33\, V^2 = (0.5547\, V^2 + 0.949)^2 + (0.832\, V^2 + 7.119)^2$$

$$40.33\, V^2 = 0.3077\, V^4 + 0.9 + 1.053\, V^2 + 0.69\, V^4 + 50.68 + 11.846\, V^2$$

or
$$0.9977\, V^4 - 27.43\, V^2 + 51.59 = 0$$

$$V^4 - 27.5\, V^2 + 51.7 = 0$$

$$V^2 = \frac{27.5 \pm \sqrt{756.25 - 206.32}}{2} = \frac{27.5 + 23.45}{2} = 25.47$$

or
$$V = 5.047 \text{ kV}$$

The drop in volts is 6.350 – 5.047 = 1.303 kV

$\therefore$ pu drop 0.2583.

Now using the expression

$$\frac{\Delta V}{V} = \frac{1}{S_{sc}}(P\cos\phi_{sc} + Q\sin\phi_{sc}) + j\frac{1}{S_{sc}}(P\sin\phi_{sc} - Q\cos\phi_{sc})$$

$$= \frac{1}{250}(30 \times 0.196 + 45 \times 0.98) + j\frac{1}{250}(30 \times 0.98 - 45 \times 0.196)$$

$$= \frac{1}{250}\{(5.88 + 44) + j(29.4 - 8.82)\}$$

$$= \frac{1}{250}(49.88 + j\,20.58)$$

$$= \frac{53.96}{250} = 0.2158$$

The difference in the two values, one obtained through the exact calculation and the other through approximate calculation, is because of the fact that $E \neq V$. In fact, the real problem lies in the amount of power being transmitted. For 11 kV system, it is not desirable to transmit such a large amount of power. Also X : R assumed as 5 is high and is not true in actual system designs. Overloading of the system results in larger voltage sags which leads to lower power transfer capability of the system and hence lower margin for stable operation of the system.

Stability and voltage levels of the system are the basic requirements of power system. By stability is meant the tendency of the power system to continue to operate in the desired mode (voltage and frequency to remain within desired limits) even when the system is subjected to a fault, sudden change of load or any other external condition which may have tendency to make the system unstable. Like any other system, it is the inherent property of the system and is a measure of developing restoring forces within the system so as to counter the disturbing forces *e.g.,* short circuits or increase in loads etc. During disturbances there is exchange of power or energy amongst various components of power system. Various synchronous machines can run and deliver or absorb energy only at synchronous speeds (under normal operation) or around synchronous speed (during a short time of disturbance). Transmission line is one of the most important components of power system through which exchange of energy takes place amongst various synchronous machines.

Like any other physical system, the stable or unstable operation of power system depends mainly on two factors (*i*) initial operating condition (*ii*) the level of perturbation *i.e.*, small perturbation or large perturbation *i.e.*, a small increase in load or a large increase in load or a short circuit.

It is possible (even though not desirable) that in a particular operating condition all the equipments are at their maximum limits and even a small perturbation might lead to unstable operation of the system. On the other hand there could be another initial operating condition for which even a large perturbation of specific amount might retain the system in stable operating condition. Therefore, larger the margin between the initial operating condition and the limiting operating condition of the system, the greater are the chances of the system to operate under stable operating condition for a specific disturbance.

As mentioned earlier, the exchange of energy amongst various synchronous machines takes place through transmission lines. One of the limitations of the line is that for a given length of the line the stability tends to become less as the initial transmitted power is increased. That is, if the initial transmission angle δ is large the smaller is the stability margin. This is why normally the transmission lines are loaded equivalent to 30° angle. A still smaller angle of transmission is desirable from stability point of view but it will be under utilisation of the power transfer capability of the line. The capital investment in transmission lines is very large these days and these should be used optimally.

As the power transfer through the line is increased a limit is reached when an attempt to increase power results actually reduction in power. Thus the two synchronous machines at the two ends of the line would lose synchronism. This limit of power transfer is known as steady state stability limit because it is the maximum steady power that can be transferred stably. The steady state stability limit of a link can be increased by increasing the excitation level of the synchronous machines at the two ends, by interconnecting another line between the two ends, by the use of series capacitors etc.

A transmission line has three critical loadings (*i*) Natural loading; (*ii*) Steady state stability limit; (*iii*) Thermal limit loading. Out of the three for a compensated line, the Natural Loading is the lowest and before thermal limit loading is reached, steady state stability limit is achieved.

21.2 LOADABILITY CHARACTERISTIC OF O/H LINES

In order to have proper understanding of the power transfer capability of the lines which is affected by voltage levels and line length, a useful concept of line loadability is defined as the degree of line loading permissible expressed in per cent of SIL for a given thermal or voltage drop or S.S. stability limits. This concept was first introduced by St. Clair who developed transmission power transfer capability curves for voltages up to 330 kV and line lengths up to 640 kms. These curves have been useful to power system engineers for quickly estimating the maximum line loading limits.

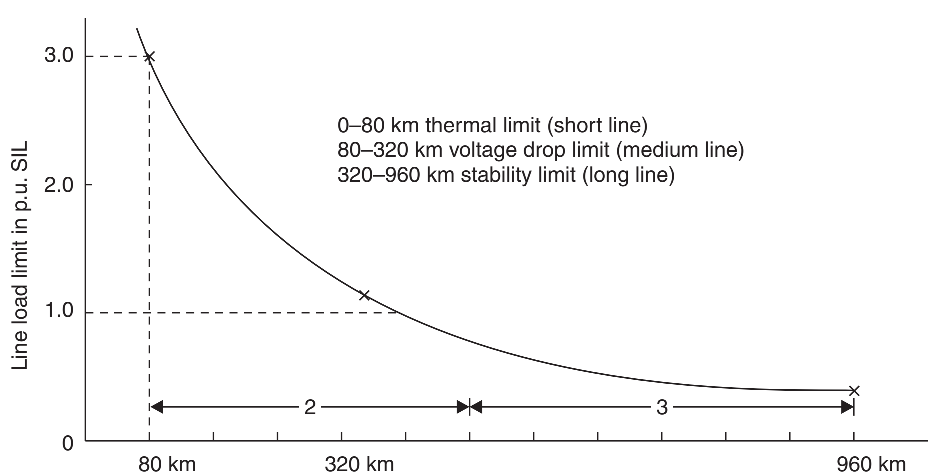


Fig. 21.1 Transmission line loadability curve.

Figure 21.1 shows the universal loadability curve for uncompensated lines applicable to all voltage levels. The curve shows the limiting values of power that can be transmitted as a function of line length. The curve has been drawn taking into account the maximum allowable voltage drop along the line to be 5% and that the minimum allowable steady state stability margin is 30% for which the load angle is about 44°.

Since the ratio X/R for extra high and ultra high voltages is very high (more than 20), the resistance of these lines is negligible as compared to its reactance and these lines could be considered as lossless lines. Therefore, the parameter β is same for all such lines and, therefore, the loadability expressed in per unit of SIL is universally applicable to lines of all voltage classes. It can be seen from the figure that lines longer than 480 km. have their loadability less than SIL and, therefore, their loadability can be improved by compensating the lines.

From the Fig. 21.1 it is clear that for short length line the steady state limit loading will be greater than the thermal loading and hence the loadability is determined by the thermal loading rather than steady state stability loading.

For medium length uncompensated lines voltage levels are the considerations for line loadings and for long length uncompensated lines steady state stability limit loading is the consideration as the loading is even less than the surge impedance loading.

As mentioned here, it is not desirable to operate transmission lines close to its steady state stability limit and a margin in the power transfer should be allowed to take care of any sudden changes in the system. In determining an appropriate margin, the concepts of dynamic and transient stability are useful. A system is said to be dynamically stable if it recovers its normal operation after it is subjected to a small perturbation. The degree of dynamic stability is expressed in terms of the rate of damping of transient component of voltages, currents and the load angles of the synchronous machines. The rate of damping is of main consideration and, therefore, small perturbation theory and eigen value analysis is normally used to study dynamic stability limits of the system.

The analysis of the system under large perturbation is known as transient state stability analysis. If a system recovers to its normal operating condition after being subjected to a large perturbation such as failure of a major component of the system (failure of a large generator, an important tie-line or a power transformer), the system is said to have achieved transient stability. The transient stability limit of a system is the maximum amount of power that can be transmitted prior to occurrance of the disturbance and still the system recovers to its normal operating condition after the disturbance. Therefore, it can be concluded that whereas the steady-state stability limit of a system is a fixed value for a particular system, the transient stability limit will have different values for different perturbations. For further understanding of the subject the reader is requested to refer to Chapter 17 of this book.

The second basic requirement in a.c. power transmission is to have proper voltage levels. An undervoltage operation results in heavy loading, reduction in steady state stability limit of the system, degradation in the performance of induction motors which forms almost 70% of the total load on the system.

A power transformer is normally operated at the knee-point of its B-H curve due to economic reasons. If some how the system voltage increases due to Ferranti effect or due to rejection of load the transformer operates along the saturation region of the B-H curve where the inductance offered by the winding decreases and keeps on changing depending upon the dynamic behaviour of increase of system voltage. Saturation of transformers subjected to over voltages can produce high currents rich in harmonics and in the presence of sufficient capacitance, there is risk of phenomenon of resonance taking place which may result in further increase in system voltages which may prove disastrous for the system insulation. This is known as Ferro-resonance as it is due to the non-linear behaviour of the iron core of the transformer. In such a situation it may be necessary to disconnect shunt capacitors very quickly to reduce the chances of occurrences of Ferro-resonance. Over voltage may also result in flash over and/or puncture of the insulation of the system. A small over voltage if allowed to exist on the system for a long time may not puncture the insulation but it would certainly shorten the useful life of insulation of the system.

Some compensating devices are used to achieve the above mentioned basic requirements in a.c. transmission. Before we do that some characteristics of uncompensated lines will be studied. In order to maintain continuity in the subject, there may be some repetition from Chapter 4 of the book.

21.3 UNCOMPENSATED TRANSMISSION LINE

The basic equation describing a transmission line is given as

$$\frac{\partial^2 V}{\partial x^2} = \gamma^2 V \tag{21.9}$$

$$\frac{\partial^2 I}{\partial x^2} = \gamma^2 I \tag{21.10}$$

and

$$\frac{\partial V}{\partial x} = Iz \tag{21.11}$$

where $\gamma^2 = (r + j\omega L)\,(g + j\omega C)$

and $z = (r + j\omega L),\ y = (g + j\omega C)$

where x is the distance from the sending end. If l is the length of the line, the solution of the above equation for a lossless line *i.e.,*

$$r = 0 \text{ and } g = 0$$

$$V(x) = V_r \cos(l - x)\,\beta + jZ_c I_r \sin \beta(l - x) \tag{21.12}$$

where β is the phase constant *i.e.,*

$\gamma = \alpha + j\beta$ and for a lossless system

$\alpha = 0$

Since $\gamma^2 = (r + j\omega L)\,(g + j\omega C) = -\,\omega^2 LC$

$\gamma = j\beta$

where $\beta = \omega\sqrt{LC} = \dfrac{2\pi f}{v} = \dfrac{2\pi}{\lambda}$

using equations (21.11) and (21.12) we get

$$I(x) = I_r \cos \beta(l - x) + j\,\frac{V_r}{Z_c} \sin \beta(l - x) \tag{21.13}$$

β is also known as wave number *i.e.,* the number of complete waves per unit of line length.

At 50 Hz the wave length $\lambda = \dfrac{3 \times 10^8}{50}$ = 6000 kms and β can be expressed as one wave length per 6000 kms *i.e.,* 360° per 6000 kms or = $\dfrac{360}{6000}$ = 0.06° per km or 1.0466×10^{-3} rad per km. The quantity βl is the electrical length in radians.

Z_c in equation (21.13) is known as the surge impedance or natural impedance. Its value depends upon the line design. Its value decreases with increase in operating voltage. Also with vertical configuration, the surge impedance is lower as compared to horizontal or delta configuration of the conductors. The usual value of Z_c lies in the range 200 to 400 ohms. Table 21.1 gives various parameters of lines operating at different voltages.

Table 21.1 Line parameters of typical EHV-UHV overhead lines

	345 kV		*500 kV*		*765 kV*		*1100 kV*		*1500 kV*
Characteristics	*Hori.*	*Delta*	*Hori.*	*Delta*	*Hori.*	*Delta*	*Hori.*	*Delta*	*Horizontal*
r Ω/km	0.04	0.04	0.0016	0.001	0.001	0.001	0.0005	0.0005	0.00045
X_l Ω/km	0.30	0.30	0.30	0.27	0.27	0.27	0.25	0.25	0.25
ωC μS/km	3.8	3.8	3.8	4.3	4.1	4.1	4.66	4.66	4.9
Charging MVA/km	0.45	0.45	0.9	1.08	2.5	2.5	5.8	5.8	10.8
β(rad/km × 10^{-3})	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
Z_c in ohm	285	283	287	247	258	257	232	231	225
SIL (MW)	417	420	870	1010	2270	2280	5220	5250	10000
Line current at SIL	700	700	1000	1170	1710	1720	2740	2750	3850

The surge impedance of a line is defined as the apparent impedance of an infinitely long line and is given by the ratio of voltage to the current at any point along it. Now an infinite line can be simulated by terminating any line by its surge impedance. In that case $V_r = I_r Z_c$ and the apparent impedance of the line at any point is

$$Z(x) = \frac{V(x)}{I(x)} = \frac{V_r \cos(l-x)\beta + jI_r Z_c \sin\beta(l-x)}{I_r \cos\beta(l-x) + j\dfrac{V_r}{Z_c}\sin\beta(l-x)}$$

$$= \frac{I_r Z_c[\cos(l-x)\beta + j\sin\beta(l-x)]}{I_r[\cos\beta(l-x) + j\sin\beta(l-x)]} = Z_c \qquad (21.14)$$

It can be seen that when the line is terminated through Z_c, the voltage and current magnitude remain same all along the length. Therefore, the line is said to have flat voltage profile. The voltages at the two ends are of equal magnitude but displaced in phase by β_0, the sending end voltage leading the receiving end voltage. For a 450 kms long line at 50 Hz the angle is 0.06 × 450 = 27°.

From equation (21.14) it is clear that the ratio of $\frac{V}{I}$ everywhere is a real number Z_c and therefore, voltage and current are in phase at all points of the line *i.e.,* power factor is unity at all points. This also means that at every point of the line, the reactive power generated equals the reactive power absorbed. Therefore, whereas the natural active power loading of the line is given by V^2/Z_c the natural reactive power loading is zero.

One of the major advantages of operating the line corresponding to natural loading is that because of flat voltage profile, the insulation all along the line is uniformly stressed.

Suppose the line is open circuited at the receiving end *i.e.,* $I_r = 0$. Therefore,

$$V(x) = V_r \cos \beta(l - x) \tag{21.15}$$

and

$$I(x) = j \frac{V_r}{Z_c} \sin \beta(l - x) \tag{21.16}$$

At $x = 0, V(x) = V_s$ and $I(x) = I_s$

$$\therefore \quad V_s = V_r \cos \beta l \tag{21.17}$$

and

$$I_s = j \frac{V_r}{Z_c} \sin \beta l$$

$$= j \frac{V_s}{Z_c} \tan \beta l \tag{21.18}$$

Suppose the length of the line is 100 km cos (0.06 × 100) = cos 6° ≅ 1.0 and therefore no Ferranti effect is experienced whereas for say 300 km cos (0.06 × 300) = cos 18° = 0.95.

i.e., $\frac{V_r}{V_s} = 1.05$ a rise of 5% of voltage at the receiving end. This could be tolerated. However, if the length is 800 km cos 48° = 0.66, and $\frac{V_r}{V_s} = 1.494$ a rise of 49.4% in the receiving end voltage as compared to the sending end voltage. Such a high rise in receiving end voltage is unacceptable and some suitable compensating devices must be used to avoid this situation.

In actual practice the receiving end voltage rises more than what is calculated above as the sending end voltage will increase with the increase in charging current due to rise in open circuit voltage at the sending end generator.

From equation 21.18 under no load condition the sending end current (charging current) is $I_s = jI_c \sin \beta l$ and for 300 km long line it will be $I_s = j\,1.05\, I_c \sin 18° = j\,0.324\, I_c$ and this will lead the sending end voltage by 90°. Here 1.05 is used, as the receiving end voltage under no load condition will be 1.05 times that under natural loading condition assuming sending end voltage as 1.0 p.u. every time.

When a line is to be energized, the generator has to supply leading vars to the line and the reactive power is given by

$$Q_s = I_m(V_s I_s^*) \tag{21.19}$$

Using value of I_s from equation (21.18), we have $Q_s = -P_c \tan \beta l$ (21.20)

For 300 km long line

$$Q_s = -P_c \tan 18° = -0.325\, P_c$$

i.e., the generator should be able to supply so much of leading vars or absorb lagging vars. Under-excited operation of the generator below a certain minimum excitation is not desirable for two reasons.

(*i*) Lower excitation results in reduced internal e.m.f. of the generator and hence reduced stability limits.

(*ii*) Lower excitation increases heating of the ends of the stator core.

Therefore, it is the under-excited operation of the alternator rather than voltage rise due to Ferranti effect that stringent limit is imposed on the length of uncompensated lines. Suppose the rating of alternators is P and their maximum reactive power absorption is Q *i.e.*, the equivalent power factor of the alternators is $\cos\left(\tan^{-1}\frac{Q}{P}\right)$ leading. Say this p.f. for a typical system is 0.95 lead, therefore $Q = 0.3286\,P$ and if the loading of the generators equals the surge impedance loading of the line, then $P = P_c$ and using equation 21.20, we have

$$0.3286\,P = -\,P\tan\beta l$$

or length of the line will be $\frac{18.19}{0.06} = 303$ km. *i.e.,* 303 km is the maximum length of transmission which can be energised by these generators under these conditions or if the p.f. of the alternators remains 0.95 and the length of the line is known to be say 400 kms, then the total generator capacity should be

$$0.3286\,P = P_c\tan(0.06 \times 400)$$

$$P = 1.35\,P_c$$

That is to charge the line 35% extra generating capacity would be required which is not desirable and hence instead this situation can be met by having the same generator capacity $p = P_c$ but using shunt reactors at the end of the line or at intermediate points of the line during charging.

21.4 SYMMETRICAL LINE

A symmetrical line is one with identical synchronous machines at both the ends and $|V_s| = |V_r|$ and under no load condition these are in phase also indicating that there is no transfer of power (no load condition) otherwise V_s shall lead V_r, phase lead depending upon the loading. With regard to such a line following observations are made:

(*i*) If the loading is less than the surge impedance loading of the line, the mid-point voltage is higher than the terminal voltage as there is a net of excess reactive power generated by the line.

(*ii*) However, if the loading is greater than the SIL of the line, due to deficit reactive power on the line the mid-point voltage is lower than the terminal voltages.

(*iii*) If the loading equals SIL of the line, the line has flat voltage profile. The corrective measures in cases (*i*) and (*ii*) above can be taken up by connecting suitable compensating network at suitable locations.

Assuming the line to be lossless, let us analyse the system when a load of $P_L + jQ_L$ is connected. Using equation (21.12) the voltages at the two ends can be correlated as

$$V_s = V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \tag{21.21}$$

The above equation holds good irrespective of whether the load is synchronous or asynchronous. Let us assume that the load is synchronous and taking V_r as reference, say V_s leads V_r by an angle δ known as load angle. Therefore,

$$V_s = |V_s| (\cos \delta + j \sin \delta) \tag{21.22}$$

Substituting for V_s in (21.21), we have

$$|V_s| [\cos \delta + j \sin \delta] = V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l$$

$$= V_r \cos \beta l + Z_c \frac{Q_L}{V_r} \sin \beta l + jZ_c \frac{P_L}{V_r} \sin \beta l \tag{21.23}$$

Equating the real and imaginary parts on the two sides, we have

$$V_s \cos \delta = V_r \cos \beta l + Z_c \frac{Q_L}{V_r} \sin \beta l \tag{21.24}$$

and $$V_s \sin \delta = Z_c \frac{P_L}{V_r} \sin \beta l$$

or $$P_L = \frac{V_s V_r}{Z_c \sin \beta l} . \sin \delta \tag{21.25}$$

If the length of the line is short $\sin \beta l \cong \beta l = \omega \sqrt{LC} \, . \, l$

and $Z_c \sin \beta l = \sqrt{\frac{L}{C}} \; \omega \sqrt{LC} \, . \, l = l\text{L}\omega = X$ = Total reactance of the line. Equation (21.25), therefore, reduces to a very well known equation

$$P_L = \frac{V_s V_r}{X} \sin \delta \tag{21.26}$$

Let $V_s = V_r$, then equation (21.25) becomes

$$P_L = \frac{V_r^2}{Z_c \sin \beta l} . \sin \delta$$

$$= \frac{P_c}{\sin \beta l} . \sin \delta$$

For certain geometry and operating voltage of the line P_L is maximum when $\delta = 90°$ and this maximum depends upon the length of the line, the longer the length the smaller is the maximum value of P_L. The ratio $\frac{P_{L\max}}{P_c}$ for a few lengths is tabulated here.

Table 21.2. $\frac{P_{L\,max}}{P_c}$ as a function of length

Length in km	*Length in degrees*	$P_{L\,max}/P_c$
300	18	3.236
600	36	1.701
900	54	1.236
1200	72	1.051
1500	90	1.00

21.5 RADIAL LINE WITH ASYNCHRONOUS LOAD

Even when the load is asynchronous, there is a maximum power that can be transmitted over a line. This can be calculated as follows for a unity p.f. load. Let us consider that sending end source and line form a voltage source with open circuit voltage V_0 and impedance $(R + jX)$ and at the receiving end a variable resistive load (unity p.f.) R_1 is connected. Our objective is to find out value of R_1 for which the power transfer is maximum.

Now short circuit current $I_{sc} = \frac{V_0}{Z} = \frac{V_0}{R + jX}$ and short circuit p.f. $\cos \phi_{sc} = \frac{R}{Z}$

The load current $I = \frac{V_0}{(R + R_1) + jX}$

The power delivered $= \frac{V_0^2}{(R + R_1)^2 + X^2} . R_1 = P$

The power delivered is maximum when $\frac{dP}{dR_1} = 0$

$$[(R + R_1)^2 + X^2] - R_1 . 2(R + R_1) = 0$$

or
$$R_1^2 + R^2 2RR_1 + X^2 - 2RR_1 - 2R_1^2 = 0$$

or
$$R_1 = Z$$

Therefore,
$$P_{max} = \frac{V_0^2 Z}{(R + Z)^2 + X^2} = \frac{V_0 I_{sc} Z^2}{R^2 + Z^2 + 2RZ + X^2}$$

$$= \frac{V_0 I_{sc} Z^2}{2Z^2 + 2RZ}$$

$$= \frac{V_0 I_{sc} Z}{2(Z + R)}$$

$$= \frac{V_0 I_s}{2(1 + \cos \phi_{sc})} \qquad (21.27)$$

Now V_0 is the open circuit voltage *i.e.*, V_r when $I_r = 0$.

Therefore, from equation (21.12)

$$V_s = V_0 \cos \beta l$$

or
$$V_0 = \frac{V_s}{\cos \beta l} \tag{21.28}$$

Similarly, short circuit current I_{sc} is the value of I_r when $V_r = 0$

Using equation 21.12 again, we have

$$V_s = jI_{sc} Z_c \sin \beta l$$

or
$$I_{sc} = \frac{V_s}{j\, Z_c \sin \beta l} \tag{21.29}$$

Assuming the line to be lossless $\cos \phi_{sc} = 0$

$$\therefore \quad P_{\max} = \frac{V_0 I_{sc}}{2} = \frac{V_s}{2\cos \beta l} \cdot \frac{V_s}{Z_c \sin \beta l} = \frac{V_s^2}{Z_c \sin 2\beta l} \tag{21.30}$$

Equation (21.30) represents loci of maximum power for different length of lines at unity p.f.

Let us consider a general load $P_L + jQ_L$ at the receiving end keeping the sending end voltage fixed. The receiving end current

$$I_r = \frac{P_L - jQ_L}{V_r}$$

Using equation (21.12) and assuming line to be lossless, we have

$$V_s = V_r \cos \beta l + jZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \tag{21.31}$$

For a fixed V_s and line length, equation (21.31) is quadratic in V_r and thus will have two roots. Figure 21.2 shows relation between $\frac{V_r}{V_s}$ as a function of normalised loading $\frac{P_L}{P_c}$

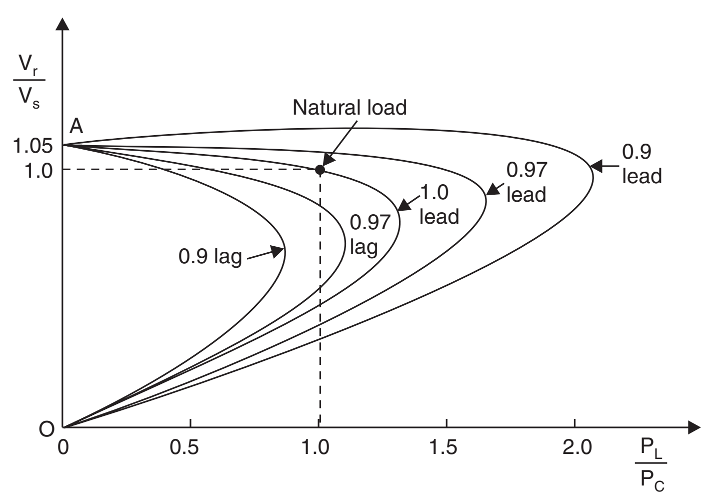


Fig. 21.2 $\frac{V_r}{V_s}$ as a function of normalised loading.

From the figure it is clear that there is a maximum power that can be transmitted for each load p.f. and for any loading, there are two different values of V_r. Normal operation of the power system is along the upper part of the curve where the receiving end voltage is nearly 1.0 p.u. The load is increased by decreasing the effective resistance of the load up to the maximum power, the product of load voltage and current increases and the system is stable. As the maximum power point is reached, a further reduction in effective load resistance, reduces the voltage much more than the increase in current and, therefore, there is an effective reduction in power transmission. The voltage finally collapses to zero and the system at the receiving end is effectively short circuited and, therefore, the power transmitted is zero (point 0). From Fig. 21.2, it is again clear that power transmitted is zero both at Point A and O. Point A corresponds to open circuit and O to short circuit and in either case the power transmitted is zero. However, at A the V_r is 1.05 p.u. and at O it is zero.

Whereas short length lines are operated without any problem, long length lines cannot be operated without the use of compensating devices because of (*i*) Ferranti effect, (*ii*) under-excited and hence unstable operation of alternators during charging of the line, and (*iii*) reduction of power transfer capability of the lines which reduces the margin between the stable and unstable operation of the system. Therefore, we not study various compensating devices for compensation of lines.

21.6 COMPENSATION OF LINES

By compensation of lines is meant the use of electrical circuits to modify the electrical characteristics of the lines such that the compensated lines will achieve the following objectives:

(*i*) Ferranti effect is minimised so that a flat voltage profile will exist on the line for all loading condition.

(*ii*) Underexcited operation of alternator will be avoided and an economical means of reactive power management will be achieved.

(*iii*) The power transfer capability of the system will be enhanced and hence stability margins increase.

In order to assess the effectiveness of a compensated system a performance index in terms of product of length of line and the power to be transmitted is evaluated. This criterion is selected as we know that for longer length lines it is not even possible to load lines to their natural loadings without compensation.

We know that flat voltage profile on the line can be achieved if the loading of the line corresponds to its surge impedance loading. Therefore, to achieve flat voltage profile, the compensating device should be so chosen that the effective or virtual surge impedance Z_c of the line should give virtual natural loading equal to the actual load. Since in actual practice the actual load keeps on changing with time, the compensating devices should also vary without delay so that every time the effective surge impedance matches with the actual loading *i.e.,*

$$P_c' = \frac{V_r^2}{Z_c'} = P_L$$

where P_L is the actual load. The compensating devices are the suitable connection of capacitors and/or inductors on to the lines. Compensation when carried out with the sole objective of modifying the surge impedance of the line, is known as surge impedance compensation or Z_c compensation.

Under-excited operation of the alternator or charging current problem of the line can be avoided by dividing the line into shorter sections and this is known as compensation by sectioning. It is achieved by connecting constant voltage compensators at intervals along the line. Since the power transmitted will be same through all sections, the maximum power will be decided by the smallest section and, therefore, there is increase in power transfer capability of the system and hence stability limit is increased.

The third aspect of the compensation *i.e.,* to increase power transfer capability of the system is achieved by inserting capacitor at suitable location in series with the line so that the net inductive reactance of the line is reduced which is equivalent to reducing the effective length of the line. This method of compensation is known as line-length compensation.

In any long length line, all the three types of compensations can be used if required.

Compensators are classified as passive and active compensators. Shunt reactors and capacitors and series capacitors are the passive compensators and synchronous capacitors and thyristorised controlled capacitors and reactors are the active compensators.

Rapid response excitation system used for synchronous machines also acts as strong compensating system and improves both voltage profile and stability.

Even though compensators are used at discrete locations along the line, it is useful to obtain certain relation assuming the effect of compensators as uniformly distributed. The relations so desired are more or less true for practical system.

We know that,

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{x_L x_C} \tag{21.32}$$

Suppose shunt inductance L_{sh} per unit is used as a compensator, the net shunt susceptance will be

$$j\omega C' = j\omega C + \frac{1}{j\omega L_{sh}} = j\omega C - \frac{j}{\omega L_{sh}} \cdot \frac{\omega C}{\omega C}$$

$$= j\,\omega C\left(1 - \frac{1}{\omega^2 C L_{sh}}\right)$$

$$= j\omega C(1 - \gamma_{sh}) \tag{21.33}$$

where $$\gamma_{sh} = \frac{1}{\omega^2 C L_{sh}} = \frac{x_c}{x_{Lsh}}$$

and γ_{sh} is known as the degree of shunt compensation.

The modified value of surge impedance will be

$$Z_c' = \sqrt{\frac{j\omega L}{j\omega C(1 - \gamma_{sh})}} = \frac{Z_c}{\sqrt{1 - \gamma_{sh}}} \tag{21.34}$$

However, if shunt capacitance is added, then γ_{sh} will be negative and hence it can be concluded that shunt inductance increases the virtual surge impedance of the line and hence reduces the virtual surge impedance loading of the line and shunt capacitance reduces the virtual surge impedance.

Let us consider the effect of series compensation on the surge impedance loading.

Suppose C_{se} is the series capacitance per unit length for series compensation. Therefore, the series reactance will be

$$
\begin{aligned}
j\omega L - \frac{j}{\omega C_{se}} &= j\omega L - \frac{j}{\omega C_{se}} \cdot \frac{j\omega L}{j\omega L} \\
&= j\omega L\left(1 - \frac{1}{\omega^2 L C_{se}}\right) \\
&= j\omega L\left(1 - \frac{X_{cse}}{X_L}\right) \\
&= j\omega L(1 - \gamma_{se}) \qquad (21.35)
\end{aligned}
$$

where γ_{se} is known as degree of series compensation. Therefore, virtual surge impedance

$$
\begin{aligned}
Z_c' &= \sqrt{\frac{j\omega L(1-\gamma_{se})}{j\omega C}} \\
&= Z_c\sqrt{(1-\gamma_{se})} \qquad (21.36)
\end{aligned}
$$

Taking into consideration both shunt and series compensation simultaneously, we have

$$Z_0' = \sqrt{\frac{j\omega L'}{j\omega C}} = Z_c\sqrt{\frac{1-\gamma_{se}}{1-\gamma_{sh}}} \qquad (21.37)$$

Therefore, the virtual surge impedance loading

$$P_c' = P_c\sqrt{\frac{1-\gamma_{sh}}{1-\gamma_{se}}} \qquad (21.38)$$

The wave number β is also modified to

$$\beta' = \beta\sqrt{(1-\gamma_{se})(1-\gamma_{sh})} \qquad (21.39)$$

The equation derived earlier for uncompensated lines are valid for compensated lines except the equivalent parameters *e.g.,* inductance, capacitance etc., are to be substituted as obtained in this section.

From equations (21.37) and (21.38) it is clear that for a fixed degree of series compensation, capacitive shunt compensation decreases the virtual surge impedance loading of the line. However, inductive shunt compensation increases the virtual surge impedance and decreases the virtual surge impedance loading of the line. If inductive shunt compensation is 100% the virtual surge impedance becomes infinite and the loading zero which implies that a flat voltage profile exists at zero load and Ferranti effect can be eliminated by the use of shunt reactors. However, under heavy loading condition, flat voltage profile can be obtained by using shunt capacitances. Suppose, we want flat voltage profile corresponding to 1.2 P_c without series compensation, the shunt capacitance compensation required will be by using equation (21.38).

$$1.2 = \sqrt{1-\gamma_{sh}}$$

or $$1.44 = 1 - \gamma_{sh} \quad \text{or} \quad \gamma_{sh} = -\,0.44 \text{ p.u.}$$

Flat voltage profile can also be obtained by series compensation for heavy loading condition. Again using equation (21.38), and assuming shunt compensation to be zero, the series compensation required for a loading 1.2 P_c is

$$1.2 = \sqrt{\frac{1}{1-\gamma_{se}}}$$

or $$1.44 = \frac{1}{(1-\gamma_{se})}$$

or $$\gamma_{se} = 0.306 \text{ p.u.}$$

However, because of the lumped nature of series capacitors voltage control using series capacitors is normally not recommended. These are normally useful for improving the stability limits of the system.

Next we study the effect of distributed compensation on the line charging reactive power. Rewriting equation (21.20)

$$Q_s = -P_c \tan \beta l$$

where Q_s is the leading reactive power supplied by the generator for a radial line and for a symmetrical line,

$$Q_s = -\,P_c \tan \beta l/2$$

However, for compensated lines, these equations become

$$Q_s' = -\,P_c \tan (\beta l)' - \text{radial line} \tag{21.40}$$

and $Q_s = -\,P_c \tan(\beta l/2)'$ – Symmetrical line, (21.41)

From equation (21.39) it is clear that by using inductive shunt compensation and or series capacitive compensation, the effective length of the line can be reduced and hence $(\beta l)'$ becomes small so that $\tan (\beta l)' \cong (\beta l)'$ and $\tan (\beta l/2)' \cong (\beta l/2)'$. Substituting values of P_c' and $(\beta l)'$ from equations (21.38) and (21.39) into equations (21.40) and (21.41), we have

$$Q_s' = P_c \sqrt{\frac{1-\gamma_{sh}}{1-\gamma_{se}}}\,.\,\beta l \sqrt{(1-\gamma_{sh})(1-\gamma_{se})} \tag{21.42}$$

$$= P_c\, \beta l (1 - \gamma_{sh})$$

for a radial line and

$$Q_s' = P_c \frac{\beta l}{2}(1-\gamma_{sh}) \tag{21.43}$$

for a symmetrical line.

From the above equations, it is clear that series compensation has no effect on the no load reactive power requirements of the generator and, therefore, the series compensated line generates roughly as much line charging reactive power at no load as a completely uncompensated line (for $\gamma_{sh} = 0$) of the same length. However, if the length of the line is large and needs series compensation from stability point of view, the generators at the two ends will have to absorb excessive reactive power and, therefore, it is important that shunt compensation (inductive) must be associated with series compensation.

Let us consider shunt compensation not only uniformaly distributed but regulated as well which is more or less similar to compensation by sectioning.

Suppose the line is operating at its natural loading. From equation (21.25)

$$\frac{P}{\sin \delta} = \frac{P_c}{\sin \beta l} \text{ and if } P = P_c,\ \delta = \beta l$$

i.e., the transmission angle δ equals the electrical length of the line. Considering series compensation to be absent and assuming that the shunt compensation could be regulated continuously such that $P_c' = P$ at all times then $\beta' l = \delta$ at all times. Therefore,

$$\frac{P}{\delta} = \frac{P_c'}{\beta' l}$$

substituting for P_c' and β' from equations (21.38) and (21.39) respectively, we have

$$\frac{P}{\delta} = \frac{P_c\sqrt{1-\gamma_{sh}}}{\beta\sqrt{1-\gamma_{sh}}\,.l} = \frac{P_c}{\beta l} = \text{constant} \qquad (21.44)$$

Equation (21.44) shows a linear relation between P and δ and the slope is given by

$$\frac{P_c}{\beta l} = \frac{V_0^{\,2}}{Z_c \beta l} = \frac{V_0^{\,2}}{\sqrt{L/C}\,.\,\omega\sqrt{LC}\,.l} = \frac{V_0^{\,2}}{X_l} \qquad (21.45)$$

Power angle relation is given as

$$P = \frac{V_0^{\,2}}{X_l} \sin \delta$$

Therefore, it's slope at δ = 0 is $\frac{V_0^{\,2}}{X_l}$ and the $P - \delta$ straight line is tangent to $P - \delta$ characteristic of the 100% shunt compensated line as shown in Fig. 21.3 (*a*) since the line with 100% shunt compensation behaves exactly as a series inductance.

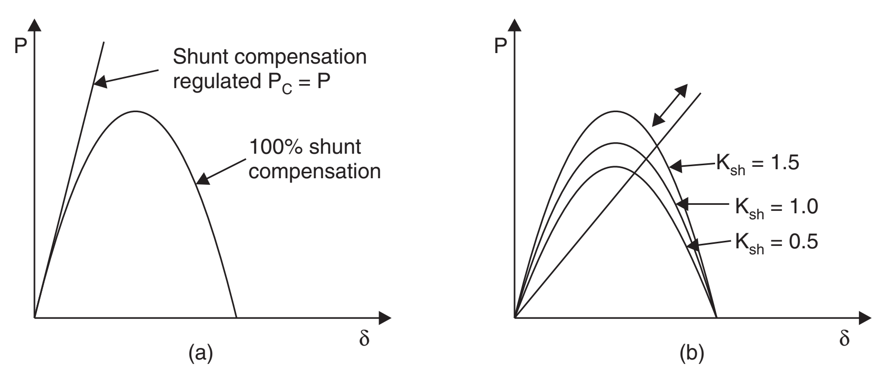


Fig. 21.3 (*a*) Effect of regulated shunt compensation in power transmission (*b*) P – δ curves for different γ_{sh}.

Figure 21.3 (*a*) suggests that if shunt compensation could be adjusted continuously, the power transmitted could be infinite.

Figure 21.3 (*b*) shows power angle characteristics for different values of shunt compensation γ_{sh} where the peak value of power is given by

$$P'_{\max} = \frac{V_0^2}{Z_c \sin \beta' l} \tag{21.46}$$

It can be seen that as power varies, to operate along the straight line characteristic the shunt compensation should also be varied. In order to operate stably for angles $\delta > \frac{\pi}{2}$ it is essential that not only the compensation should be varied continuously but rapidly so as to keep pace with the change in power P. If P is varied faster as compared to the shunt compensation the system would operate along the 'current' $P - \delta$ characteristics and the system would go unstable. A line with ideal regulated shunt compensation is said to be dynamically stabilised.

A practical ideal regulated shunt compensation is to design the compensation as constant voltage regulators as, if voltage everywhere could be kept constant to V_0, it is equivalent to loading the line corresponding to its virtual impedance loading whatever be the value of P. These constant voltage compensators are active compensators *e.g.,* synchronous condensers, thyristor-controlled compensators etc.

Whenever $P > P'_c$, the shunt compensator required is capacitive. Theoretically it appears that with shunt compensation, infinite power can be transmitted but in actual practice it requires a large amount of compensating capacitors and hence if power to be transmitted is to be increased, it is better to adopt alternative methods *e.g.,* higher transmission voltage, use of series capacitors, HVDC etc.

The other limitations of the method are:

(*i*) High speed of response of compensators.

(*ii*) If regulators fails, the whole system may become unstable.

It has been reported earlier that shunt compensation increases the virtual surge impedance of the line and hence decreases the surge impedance loading. In fact with $K_{sh} = 1.0$ *i.e.,* 100% shunt compensated line the voltage profile is flat at no load.

However, it is to be noted that shunt compensating reactors can't be distributed uniformly along the line. They are normally connected at the end of the line and/or at the mid-point of the line usually at the intermediate substation as shown in Fig. 21.4.

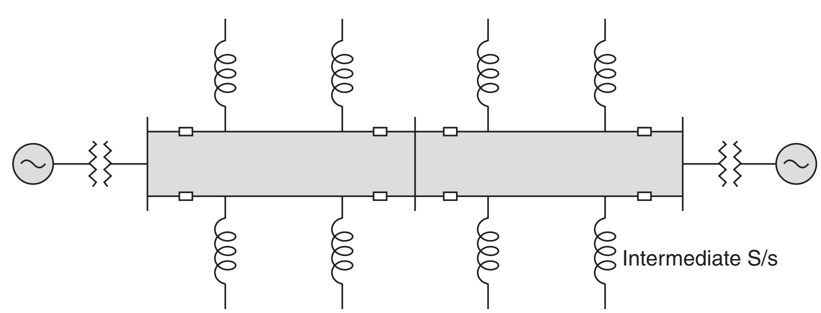


Fig. 21.4 Shunt reactors alongwith intermediate switching *S/s* on a long length a.c. line.

If switched shunt capacitors are used on such lines, these should be disconnected immediately under lightly loaded condition, otherwise this may lead to ferroresonance when the transformers are present in the system. The main idea of series compensation is to cancel part of the series inductive reactance of the line by the use of series capacitors which results into (*i*) increase in maximum transmittable power (*ii*) reduction in transmission angle for certain amount of power transfer and (*iii*) increase in virtual surge impedance loading. However, since the effective series inductive reactance is decreased, now absorbs less of the line charging reactive power and hence requires shunt compensation especially if the line is long, shunt inductive compensation is necessary in order to limit the line voltage.

From practical point of view, it is desirable not to exceed series compensation beyond 80%. If the line is 100% compensated, the line will behave as a purely resistive element and would result into series resonance even at fundamental frequency as then capacitive reactance equals the inductive reactance and it would be difficult to control voltages and currents during disturbances. Even a small disturbance in the rotor angles of the terminal synchronous machines would result into flow of large currents.

The location of series capacitors is decided partly by economical factors and partly by the severity of fault currents as fault currents would depend upon the location of the series capacitor. The voltage rating of the capacitor will depend upon the maximum fault current likely to flow through the capacitor.

Therefore, it is desirable not to use a large number of smaller unit capacitors along the line and the normal practice is to instal one or two large unit capacitances. This, however, results in an uneven voltage profile along the line as there will be sharp change in voltage at the two terminals of the capacitor *i.e.,* the line will have some voltage on one side of the capacitor and materially different voltage on the other side.

21.7 SUBSYNCHRONOUS RESONANCE

If X_l is the inductive reactance of the line and X_c the reactance of the series capacitor, the net inductive reactance of the line in $X_l' = X_l - X_c$. The inductive reactance in series with the capacitance of the series capacitor forms a series resonance circuit with the natural frequency of oscillation given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where L is the inductance of the line.

or

$$f_0 = \frac{1}{2\pi\sqrt{\dfrac{X_l}{2\pi f}\cdot\dfrac{2\pi fC}{2\pi f}}} = f\sqrt{\frac{X_c}{X_l}}$$

Here $\dfrac{X_c}{X_l}$ is the degree of series compensation and varies between 25 to 65%. Therefore $f_0 < f$ *i.e.,* the resonance takes place at subharmonic frequency. Here X_l of course includes the inductive reactance of the generator and the load of the system connected at the ends of the line.

In a series compensated network whenever a change in the form of a switching operation or a fault takes place, transient currents at sub-harmonic frequency are excited. These currents are superimposed over the power frequency current and these die out depending upon the effective damping of the system over a period of few cycles. However, under certain conditions, these subharmonic currents may assume dangerously high values and even become unstable in the absence of corrective measures. The unstable operation is exhibited in the form of negative resistance in the equivalent circuit of synchronous and induction motor. Considering subharmonic frequency operation and a round rotor synchronous machine, the equivalent circuit is given in Fig. 21.5.

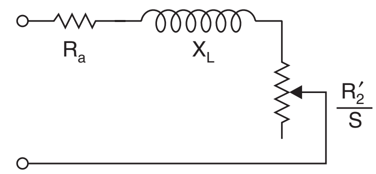


Fig. 21.5 Simplified equivalent circuit of a synchronous machine for subharmonic operation.

For simplicity the field winding has been omitted in the above figure. Suppose due to some disturbance subharmonic currents are excited in the stator winding and these currents in general would be unbalanced. Considering positive sequence component of these current which will produce a magnetic field which will rotate in the direction of rotation of the rotor but with a speed $n_0 < n_s$ *i.e.,* the rotor is rotating at a higher speed as compared to magnetic field due to subharmonic currents and the machine behaves as an induction generator as far as subharmonic currents are concerned. The rotor is slipping with respect to the field with slips given as

$$s = \frac{f_0 - f}{f_0}$$

Since $f_0 < f$, the slip is negative. $\frac{R_2'}{s}$ is the equivalent resistance of the damper winding and solid rotor resistance when referred to the stator side. $\frac{R_2'}{s}$ is negative for subharmonic currents and, therefore, this provides negative damping. If series compensation is very high, s would turn out to be very small and hence equivalent resistance very large and may become large enough to have total resistance of the system as negative which will, therefore, provide negative damping to the subharmonic currents and voltage may build up to dangerously high values. Even though this situation rarely occurs in the system but when it does, necessary care must be taken to avoid it. The measures taken are similar to those which are used to prevent subsynchronous resonance in the system.

The field due to subharmonic currents rotates in the backward direction with respect to the rotor and the main field, and, therefore, produces alternating torque on the rotor at frequency $f - f_0$. If this difference frequency coincides with one of the natural torsional resonances of the machine's shaft system, torsional Oscillations may be excited. This operation is known as subsynchronous resonance (SSR). This means whenever the natural frequency of mechanical Oscillation of the shaft of the rotor equals the difference of electrical frequency $(f - f_0)$, mechanical resonance would take place. SSR, therefore, is a combined electrical-mechanical resonance phenomenon.

Large multiple stage steam turbines which have more than one, torsional modes in the frequency range 0.50 Hz, are more susceptible to SSR. If we consider the entire system consisting of the turbine cylinder and the generator as a single mass, this will have the lowest frequency of natural Oscillation (mechanical) and this is known as swing frequency. Higher frequency currents may produce torques in some of the shafts which may have the same natural frequency as the torque frequency and hence these shafts may breakdown due to twisting action and, therefore, resonant frequencies may extend up to hundreds of Hz. The damping of these modes is usually extremely small.

If SSR phenomenon is allowed to exist for some time and if the resonance frequency coincides with swing frequency, the whole turbine generator assembly may come out from its foundation and/or if the frequency of the torque developed coincides with the natural frequency of oscillation of some shafts and if oscillations build up sufficiently, this may result in breaking of the shaft. However, even if the oscillations are damped, the disturbances like any switching operation or occurrence of a fault which create SSR condition in the system may reduce the life of the shafts and hence the whole mechanical system (turbine–generator system). This slow deterioration of the mechanical system is known as 'low-cycle fatigue' and in recent times lot of research has gone into the study of SSR phenomenon. Some of the corrective measures for SSR are:

(*i*) By-passing of series capacitors.

(*ii*) Use of very sensitive relays to detect even small levels of subharmonic currents.

(*iii*) Modulation of generator field current to provide increased positive damping at subharmonic frequency.

To provide compensation both for series and shunt compensation, large capacity capacitors are required. It has been found more economical to use individual units assembled in appropriate series and parallel connected groups to achieve desired voltage and reactive power ratings. The optimal rating these days is between 200–300 kVA_R and the dielectric used is paper/polypropylene film/askarel. The use of all polypropylene reduces dielectric loss and the probability of case rupture. Series as well as shunt capacitances have external fuses one for each unit or internal fuses. The internal fuse system has the advantage that failure of a single element or 'roll' within the unit does not cause the entire unit to fail. However, this arrangement requires higher maintenance.

21.8 ACTIVE SHUNT COMPENSATOR

Synchronous condensers are the active shunt compensators and have been used both at transmission and subtransmission levels to improve voltage profile and system stability. Whereas in static capacitors the reactive power supplied decreases with decrease in system voltage, the synchronous capacitors regulate reactive power independant of system voltage. This is the main reason for the superiority of synchronous capacitors over static capacitors for transmission lines. Yet another application of synchronous capacitor is with high voltage d.c. transmission where these supply a portion of the converter reactive power requirements and provide necessary reactive power support to an a.c. network with low short-circuit capacity.

A synchronous capacitor is a synchronous machine synchronised to the power grid and controlled to absorb or generate lagging var on the system. Most of the synchronous capacitors are out-door and unattended. These are automatically controlled *i.e.,* these are associated with automatic controls for start up, shut down and on-line monitoring. The efficiency of a synchronous capacitor including its auxiliary is 99%. The inertia constant H of synchronous capacitor is relatively unimportant and is very small as compared to those of generators as there is no turbine connected to it.

A synchronous condenser is said to have normal excitation if the terminal voltage under no load condition is its rated voltage. If the excitation is more than its normal excitation, the condenser is said to be overexcited and if less than the normal, it is underexcited. Figure 21.6 shows the flow of reactive power (lag) and the corresponding phasor diagrams under the two operating conditions.

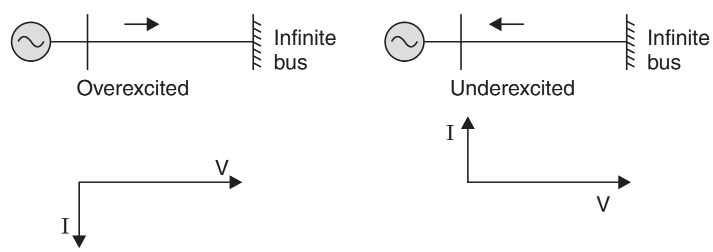


Fig. 21.6 (*a*) Flow of reactive power (*b*) Phasor diagram.

When the machine is overexcited, it acts as a shunt capacitor as it supplies lagging vars to the system and when underexcited it acts as a shunt coil as it absorbs reactive power to maintain terminal voltage.

Since the condenser needs active component of current to meet losses in the machine, the conventional generator reactive power capability curve (pull out curve discussed in Chapter 17) is not applicable and V-curves are used to explain condenser operation. The synchronous condenser provides continuous (stepless) adjustment of the reactive power in both the underexcited and overexcited modes. In overexcited case there is both a continuous 'name plate' capability and short-time overload capabilities. Steady state operation is explained with the help of V-curves shown in Fig. 21.7.

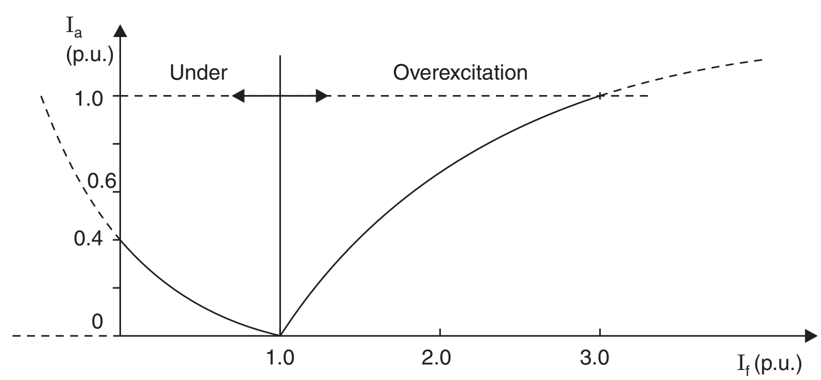


Fig. 21.7 V-curves for the condenser.

The right hand portion corresponds to overexcitation whereas the left hand portion represents the underexcitation operation. Normal continuous operation may be at any point on the V-curves below the rated stator current. The dashed portion to the right represents short-time overload operation obtained through overexcitation. The underexcited operation may even extend into the negative field current as shown in Fig. 21.7. This operation within limits is possible without pole slipping because of the presence of reluctance torque associated with salient pole machines. The limiting value of reactive current to avoid pole slipping occurs at $\dfrac{V}{X_q}$, where V is the terminal voltage.

The synchronous condensers are used for (*i*) Power system voltage control both under normal and emergency operating conditions and (*ii*) HVDC application. The emergency operating conditions result from a system fault or a sudden loss of a major transmission link or a major generating station which may result in a breakdown of the system or islanding. By islanding of the system is meant that in a large interconnected system if there is a major breakdown a part of the system may be isolated from the system and keep running before total breakdown takes place. The system which is isolated from the rest of the unhealthy system is known as islanded system and the process is known as islanding. Under islanding condition voltage extremes in either direction may occur depending upon the imbalance in the reactive power in the islanded system.

Synchronous condensers under voltage regulator control automatically change their output in a direction to minimise voltage variations. Figure 21.8 shows the short-time emergency reactive power output.

It is clear from the Fig. 21.8 that condensers have short-time capabilities well in excess of their nameplate ratings. The short-time rating may be 150% of the rated value for 1 min. In normal practice the condenser output depends upon the system voltage and the excitation ceiling. The higher the excitation ceiling, the faster is the response of the condenser for a particular reactive power output.

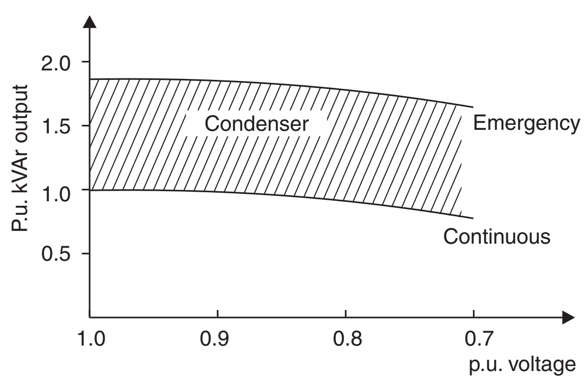


Fig. 21.8 Condenser output (*i*) Continuous (*ii*) Emergency condition.

The time-constant of field winding of a condenser is very large and, therefore, in order to have effective use of synchronous condenser in correcting, transient voltage swings, the excitation system must be equipped with supplementary control which will provide an error signal proportional to the rate of change of voltage.

Figure 21.9 shows the application of a synchronous condenser at the receiving end of an HVDC line. The condenser serves the following purposes:

(*i*) A part of the reactive power requirement of the converter.

(*ii*) A system with low short circuit capacity is subjected to large voltage dips in case of shunt faults. For such system the condenser allows d.c. converter control to maintain acceptable control of the a.c.

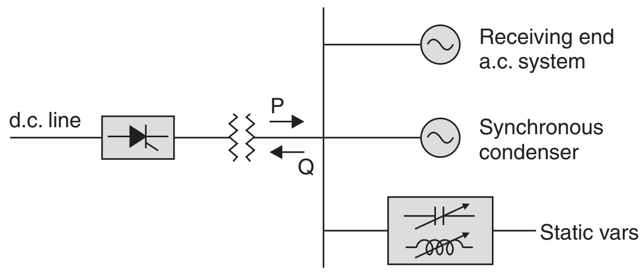


Fig. 21.9 Use of condenser for HVDC line.

When the a.c. system has low short circuit capacity, the synchronous condenser increases the short circuit capacity of the system besides providing a portion of the reactive power requirements of the inverting process. Of the available sources of reactive power (static var compensators and filters), the synchronous condenser has the unique property of providing short time voltage stability in the subtransient range.

Almost all the synchronous condensers are automated with remote start-stop and voltage control devices. The protective relaying requirements are more or less same as for a turbo-alternator except a few special requirements. Since a near zero field current is a normal operation for a synchronous condenser, a different type of loss of field relay is required. For a synchronous condenser, a low terminal voltage and low field currents is an abnormal operation and suitable relay is used to take care of this situation. Under-frequency relay is used to protect from a situation where synchronous condenser becomes isolated from the system by a remote trip out.

21.9 STATIC COMPENSATORS

Static compensators are those which have no rotating parts and are used for surge impedance compensation and for compensation by sectioning a long transmission line. These are also used for load compensation wherein these maintain constant voltage (*i*) under slowly varying load changes (*ii*) load rejection, outages of generator and line (*iii*) Under rapidly varying loads *e.g.,* arc furnaces. These improve system stability and system power factor. Since these regulate the flow of reactive power, these are used for minimisation of transmission losses and control a.c. voltage near HVDC converter terminal.

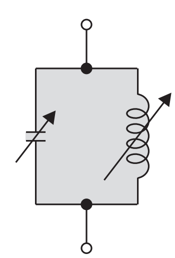


Fig. 21.10

Figure 21.10 shows an ideal static reactive power compensator capable of stepless adjustment of reactive power over an unlimited range (lagging and leading) without any time delay. Even though a fast response is generally desirable, it is not essential for all operating conditions of the systems *e.g.,* a fast compensator is not required for system stability when other factors limit the stability of the system. Similarly, in actual practice, the reactive current is limited in both lagging and leading regions because of the current carrying capacity of the compensator.

Some of the important compensators used in transmission and distribution networks are:

(*a*) Thyristor controlled reactor (TCR),

(*b*) Thyristor switched capacitors (TSC), and

(*c*) Saturated reactors (SR).

Figure 21.11 shows single line diagram of the above mentioned compensators. It is interesting to find the following common features in these circuits:

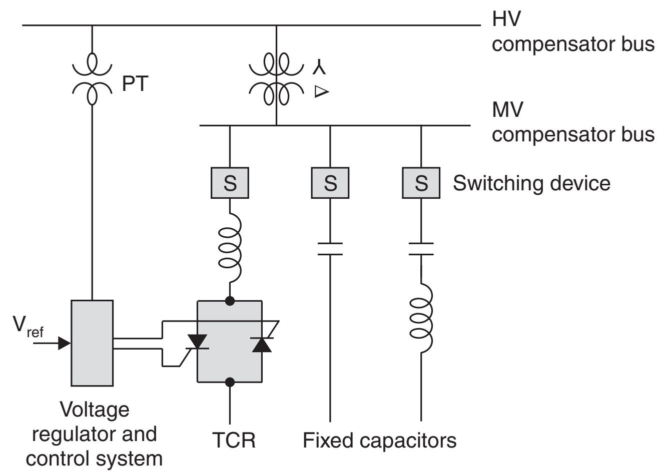


Fig. 21.11 (*a*) Thyristor controlled reactor (TCR).

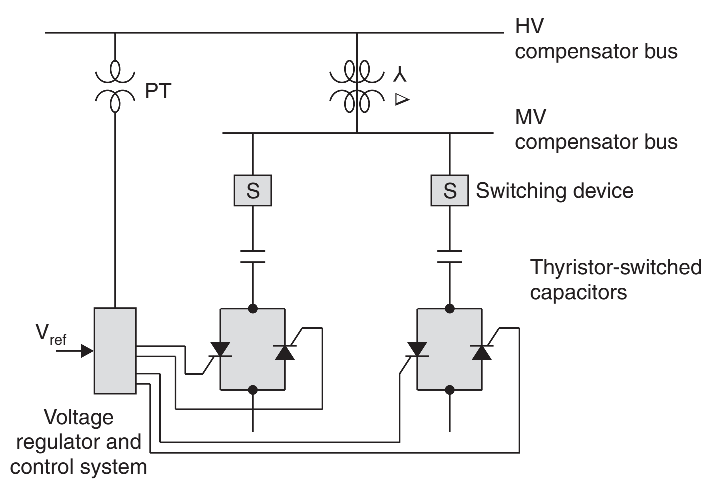


Fig. 21.11 (*b*) Thyristor switched capacitors (TSC).

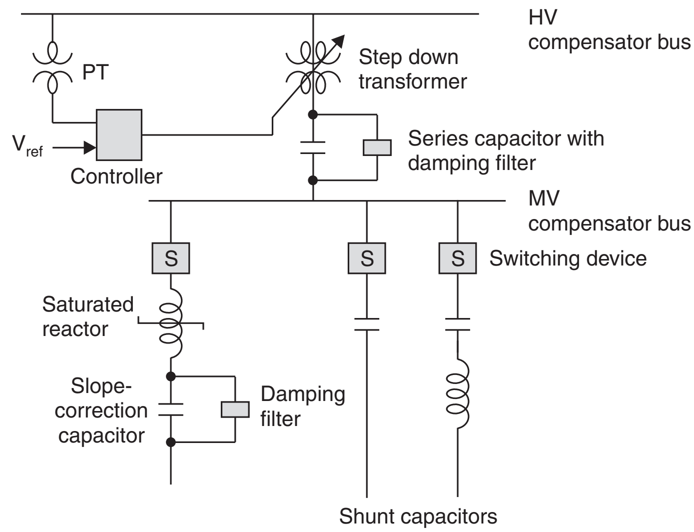


Fig. 21.11 (*c*) Saturated reactors.

(*a*) A fixed shunt capacitor in parallel with the controlled susceptance. The fixed capacitors are usually tuned with small reactors to harmonic frequencies to absorb harmonics generated by the controlled susceptance (TCR or SR) or to avoid harmful resonances.

(*b*) A step down transformer which significantly affects the performance of the transformer especially with respect to losses, harmonics and over voltages.

Figure 21.12 shows a basic thyristor controlled reactor. The controlled element is the reactor and the controlling element is the thyristor controller consisting of two oppositely poled thyristors which conduct every alternate half cycles of the supply frequency.

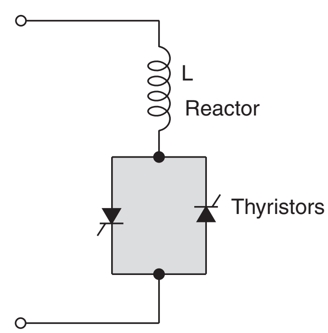


Fig. 21.12 Basic thyristor controlled reactor.

The reactive power absorbed by the reactor will depend upon the instant of switching on the voltage wave. If the voltage is passing through its peak value at the instant of switching of thyristor, maximum reactive power will be absorbed and as the delay angle increases between 90° to 180°, the reactive power absorbed decreases which really means as the conduction angle increases between 90° to 180° the effective reactance offered by the reactor increases with minimum being at 90° and maximum reactance at 180°. The fundamental component of current will be maximum when delay angle is 90° and decreases with increase in delay angle between 90° and 180°. Therefore, as far as the fundamental component of current is concerned, a thyristor controlled reactor acts as a controllable reactor. Assuming the time origin to coincide with a positive going zero crossing of the voltage, the delay angle α and the conduction angle ρ so that the conduction period lies between α and $\alpha + \rho$.

i.e.,
$$v = V_m \sin \omega t$$

Therefore, instantaneous current i is given as

$$i = \frac{1}{X_l}\int_{\alpha}^{\omega t} V_m \sin \omega t \; d(\omega t)$$

where X_l is the inductive reactance of the reactor at fundamental frequency.

$$i = \frac{V_m}{X_l}[\cos \alpha - \cos \omega t]$$

and
$$i = 0 \text{ for } \alpha + \rho < \omega t < \alpha + \pi$$

Using Fourier series, the fundamental component of current is given by

$$I_1 = \frac{\rho - \sin \rho}{\pi X_l} \cdot \frac{V_m}{\sqrt{2}} \, A \text{ r.m.s.}$$

Here ρ is related to α through the relation

$$\alpha + \frac{\rho}{2} = \pi$$

The above equation can be rewritten as

$$I_1 = B_l(\rho). \, V$$

where V is the r.m.s. voltage and $B_l(\rho)$ is an adjustable fundamental frequency susceptance controlled by the conduction angle according to the law.

$$B_l(\rho) = \frac{\rho - \sin \rho}{\pi X_l}$$

It can be seen that the susceptance is maximum when $\rho = \pi$ and its value is given as

$$B_l(\pi) = \frac{\pi - 0}{\pi X_l} = \frac{1}{X_l}$$

and its value is zero when $\rho = 0$.

The variation of B_l as a function of ρ is shown in Fig. 21.13.

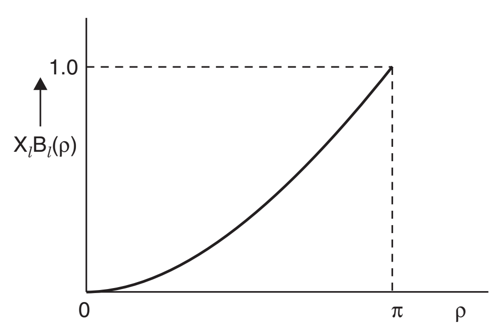


Fig. 21.13 Control law of basis TCR.

This control principle is known as phase control. Two controls are possible for the TCR.

(1) To determine ρ and fire the thyristor corresponding to this angle to achieve certain B_l or a control that responds to a signal that directly represents the desired susceptance B_l.

(2) The control processes with various measured parameters of the system to be compensated *e.g.,* voltage, and generates the gating pulses directly without using an explicit signal for B_l.

Increasing the delay angle or reducing the conduction angle decreases the power loss in both the reactors and the thyristors and generates harmonic currents. If both the thyristors are fired at the same angle, all odd order harmonics are produced. Of course, higher the order of harmonic, the smaller is the peak value.

Figure 21.14 shows a 3-phase arrangement of TCR where the three single TCRs are connected in delta. Under balanced operation, all the triplen harmonics circulate within the delta and, therefore, all these are absent from the line circuit and all the other harmonics are present in the line circuit.

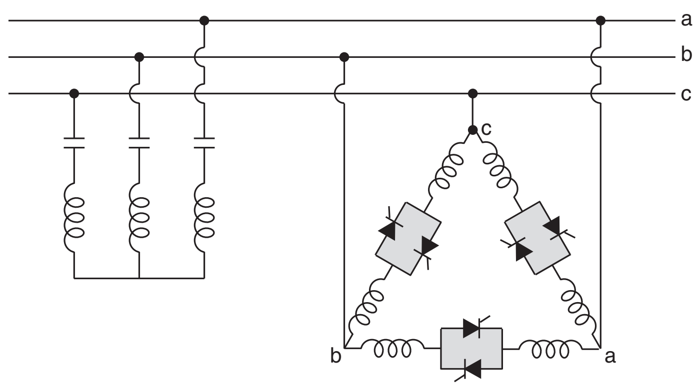


Fig. 21.14 Three phase arrangement of TCR.

If the conduction angles in the two oppositely poled thyristors are unequal, both even harmonics and d.c. component of currents are produced. This would also cause unequal thermal stresses in the two thyristors as these would carry unequal currents.

However, if the conduction angles are equal, the conduction angle is limited to 180° only. However, if the reactor in Fig. 21.12 is divided into two halves as shown in Fig. 21.15, the conduction angle can be increased to even 360°. This arrangement even though has lower harmonics yet the losses are increased as the currents circulate between the two halves.

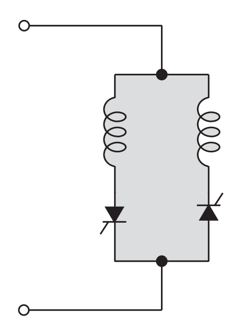


Fig. 21.15 TCR with ρ > 180°

In order to eliminate 5th and 7th harmonics the TCR is split into two parts and is fed from two secondaries on the step down transformer, one being in star and the other in delta as shown in Fig. 21.16. The star-delta connections produce a phase shift of 30° between the voltages and currents of the TCRs and practically eliminate 5th and 7th harmonics from the primary line currents. Since there are twelve thyristor gatings every period, the arrangement is known as 12-pulse arrangement. Here the lowest order characteristic harmonics are 11th and 13th and, therefore, can be used without filters for the 5th and 7th harmonics. This phase-multiplication technique is used in HVDC rectifier transformers for harmonic cancellation.

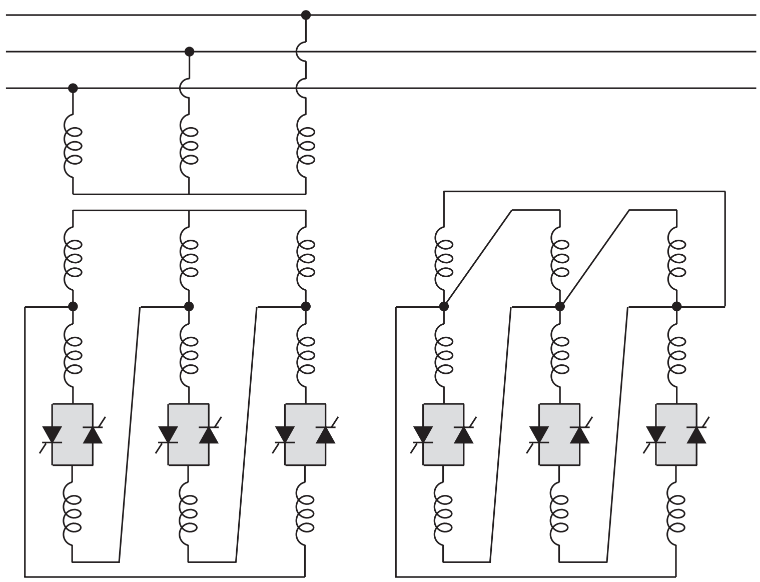


Fig. 21.16 12-pulse TCR arrangement with double secondary transformer.

It is to be noted that with TCR, current is lagging in nature and, therefore, reactive power can be absorbed only. If reactive power is to be generated a shunt capacitor of suitable value can be connected. If a fixed capacitor is connected, it is known as TCR-Fixed capacitor system.

Another variant of a TCR is a transformer designed with high leakage reactance whose secondary windings are short-circuited through the thyristor controllers as shown in Fig. 21.17. In order to obtain high leakage reactance, the transformer should have air gap in its core and, therefore, it is desirable to have three single phase transformers. This arrangement is known as Thyristor Controlled Transformer. There is no secondary bus and shunt capacitance if any, is to be connected at the primary voltage. In case of a fault on the secondary side, the high leakage reactance protects the transformer against short-circuit stresses.

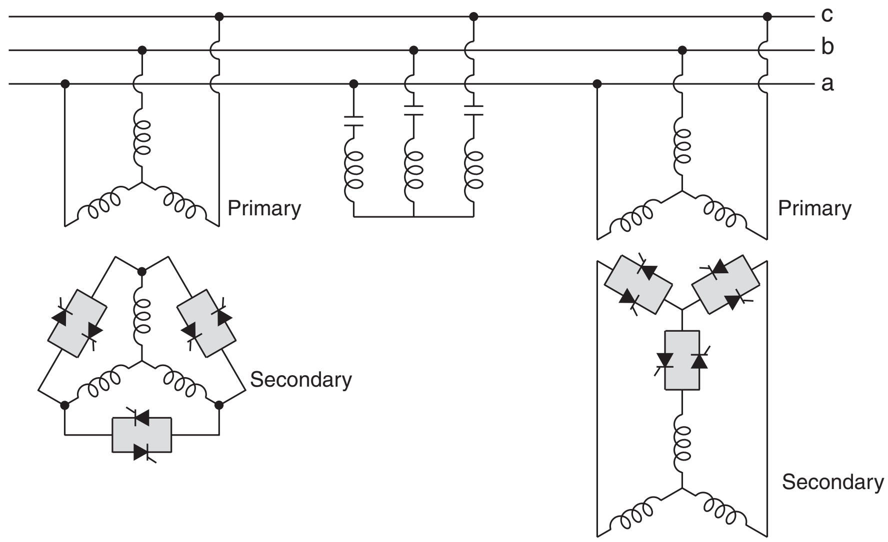


Fig. 21.17 Thyristor-controlled transformer compensator with *Y*-connection reactors and delta connected thyristor.

21.9.1 Thyristor Switched Capacitors

Figure 21.18 shows the arrangement of thyristor switched capacitor where the susceptance is adjusted by controlling the number of parallel capacitors in conduction. Each capacitor always

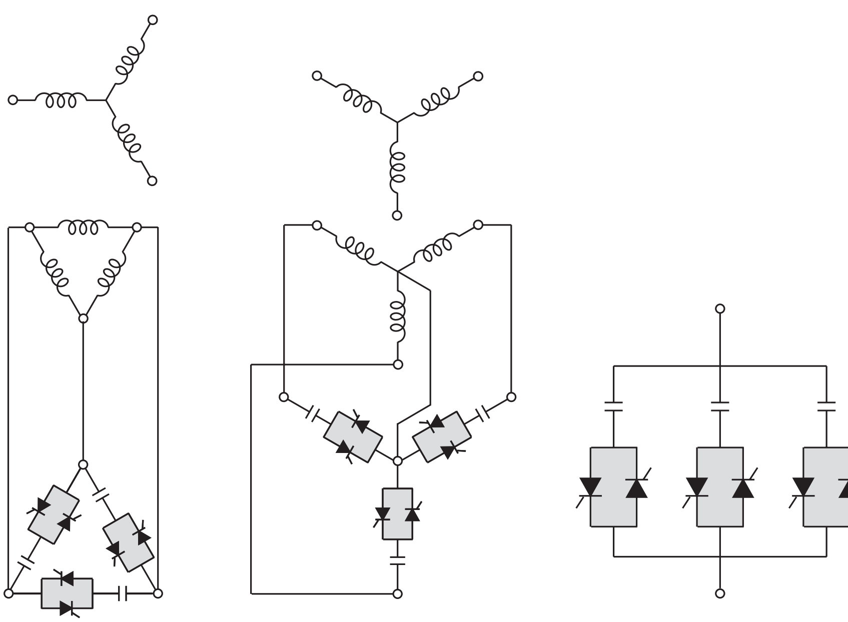


Fig. 21.18 Thyristor switched capacitor.
(*a*) Delta-connected secondary, delta connected TSC.
(*b*) Star connected secondary & TSC (4 wire system).
(*c*) Each phase of (*a*) and (*b*).

conducts for an integral number of cycles. Normally the capacitors used are of the same capacity except one of the capacitors which has its susceptance half the susceptance of the other capacitors. Therefore, if there are (n – 1) number of equal capacitances and one of half value, $2n$ number of combinations of capacitors is possible *i.e.*, $2n$ number of susceptances can be obtained. In order to have transient free switching-in or switching-out of capacitor in the circuit it is desirable to carry out the operation when the capacitor is charged to either the positive or the negative system peak voltage. Figure 21.19 shows the current and voltage wave forms under transient free switching-in and switching-out of capacitor.

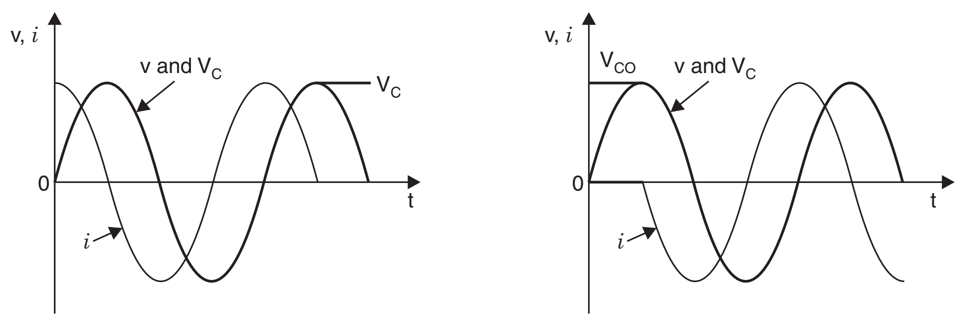


Fig. 21.19 Transient free switching operation. (*a*) Switching-in (*b*) Switching-out.

21.9.2 Saturated-reactor Compensator

The plain saturated reactor is unsuitable for use in transmission systems as the voltage or the current contain lot of harmonics. Figure 21.20 shows a three-phase saturated reactor having a short circuited delta winding which eliminates third harmonic currents from the primary winding as these currents are supplied by the delta winding. The secondary current are predominantly third harmonic and the circuit is thus an elementary frequency tripler.

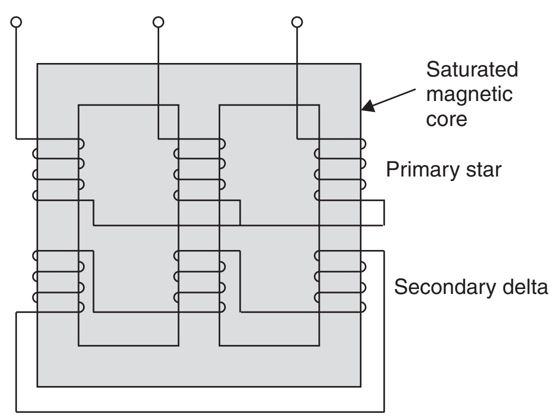


Fig. 21.20 Saturated-reactor compensator (Approx. constant voltage).

The voltage-current characteristics and harmonic performance of the above frequency tripler are not good enough to be used in power systems and much better characteristics can be obtained with higher frequency multipliers.

Figure 21.21 shows a typical volt-amp characteristic of a saturated reactor, the slope of which depends upon the after-saturation inductance of the winding. The slope of the characteristic varies between 5 and 15%. The characteristic is linear above about 10% of the rated current. A lower slope characteristic can be obtained by connecting a capacitor in series with the saturated reactor. In fact, zero or negative slopes can also be obtained by suitably sizing the capacitor to be connected in series. The slope correcting capacitor, however, can make the saturated reactor compensator susceptible to subharmonic instability especially on weak systems and, therefore, it is normally designed to connect a harmonic damping filter across the capacitor.

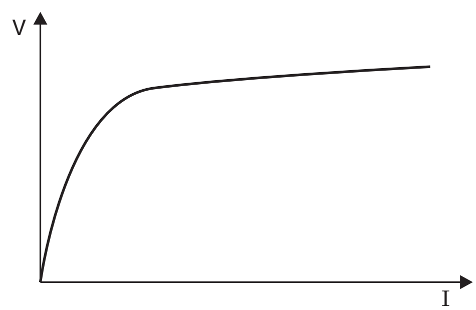


Fig. 21.21 Characteristic of a saturated reactor compensator.

The TCR compensator has the maximum control flexibility and when used along with TSC generally results in a loss characteristic which is lower in the lagging region and this also helps in having higher flexibility in controls. The TCR can be designed to have higher over-voltage limiting capability which is not available in the plain TSC.

The saturated reactor compensator even though is maintenance free, it has no control flexibility and it may require costly damping circuits to avoid any possibility of subharmonic instability. The saturated reactor compensator has the overload capability which is useful in limiting overvoltages.

21.10 FLEXIBLE A.C. TRANSMISSION SYSTEM (FACTS)

With ever increasing demand of electric power, the existing transmission networks even in the developed countries are found to be weak which results in poor quality of unreliable supply. Also, it is seen that in order to expand or enhance the power transfer capability of the existing transmission network huge sum of finances are required and sometimes even difficulties are encountered in finding right-of-way for the new lines. Lot of research has gone into developing new technologies over the past few years to gain increased efficiency from the existing power system. This programme is known as Flexible a.c. Transmission System abbreviated as FACTS. The new technologies employ high speed thyristors for switching in or out transmission line components such as capacitors, reactors or phase shifting transformer for some desirable performance of the systems.

The main objective of FACTS devices is to replace the existing slow acting mechanical controls required to react to the changing system conditions by rather fast acting electronic controls. The mechanical controls require power system operators and designers to provide generous margins to assure a stable and reliable operation of the system. As a result the existing systems cannot be made use of to their full capacity. However, with the use of fast acting controls, the power system margins could be reduced and power system capability could be more fully utilised while maintaining the present levels of quality and reliability.

The concept of FACTS is explaining as follows:

We know that, the power transfer between two systems interconnected through a tie-line is given as

$$p = \frac{V_1 V_2}{X} \sin \delta$$

It can be seen that the power flow can be controlled by three parameters, the voltages at the two systems, the reactance of the tie-line and the difference in the voltage angles at the two ends. The FACTS devices can be used to control one or more of these parameters. The various devices used are (*i*) Static var compensators (SVC) (*ii*) Controlled series compensation (*iii*) Static condensers (STATCON) (*iv*) Advanced controlled series compensation (*v*) Thyristor controlled phase shifting transformer.

Static var compensators have already been discussed in this Chapter. We take the remaining and give a brief outline of these devices.

21.10.1 Static Condenser (STATCON)

Figure 21.22 shows a basic circuit of a STATCON which is GTO (gate turn-off) based compensation system. These devices are known as STATCON or static synchronous condensers as these exhibit characteristics similar to conventional synchronous condensers without the moving parts.

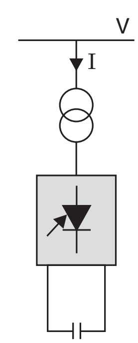


Fig. 21.22 Basic statcon.

The basic elements of a Voltage Source Inverter (VSI) based STATCON are an inverter, a d.c. capacitor and a transformer to match the line voltage as shown in Fig. 21.22. Voltage source inverter inverts a d.c. voltage (PWM inverter) with a balanced set of three quasi-square voltage waveforms by connecting the d.c. source sequentially to three output terminals. The three phase a.c. generated by inverter is synchronised to the a.c. line through a small tie reactance, which is the leakage reactance of a transformer.

When the inverter fundamental output voltage is higher than the system line voltage, the STATCON works as a capacitor and reactive vars are generated. However when the inverter voltage is lower than the system line voltage, the STATCON acts as an inductor thereby absorbing the reactive Vars from the system. To control the reactive current, thus the magnitude of d.c. voltage is raised or lower by adjusting the phase angle of the inverter output voltage. The capacitor here does not play an active role in the var generation. It is only required to maintain a smooth d.c. voltage while carrying the ripple current drawn by the inverter.

The main difference between the SVC and STATCON is whereas in case of SVC devices the current injected into the system depends upon the system voltage, in case of STATCON it is independent of the system voltage. The STATCON current I is made perpendicular to the system voltage V with the help of a STATCON coordinator which adjusts the phase of the current source I so that it is perpendicular to the STATCON terminal voltage V.

STATCON devices are used on the distribution system and have the following advantages:

(*i*) The steady state loadability of the lines is improved.

(*ii*) The voltage rises due to capacitance switching is substantially reduced both in magnitude and duration.

(*iii*) Voltage variation due to customer's loading is reduced.

STATCON is more expensive than switched capacitors or static VAR compensators on a per unit steady-state MVA basis, however, the performance of the STATCON outweights the increase in cost.

21.10.2 Advanced Thyristor Controlled Series Compensation (ATCSC)

Just as static var compensator can be improved to STATCON device using GTO converter, a controlled series compensator (CSC) can be improved to an ATCSC using a voltage-driven GTO converter is series with the line as shown in Fig. 21.23. Here the line current I is made perpenditure to the injected voltage V with the help of an ATCSC coordinator which forms a part of the whole control scheme.

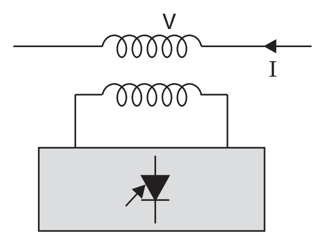


Fig. 21.23 ATCSC.

21.10.3 Thyristor Controlled Phase Shifting Transformer

Thyristor controlled phase shifting transformer or phase angle regulator consists of a shunt transformer and a boosting transformer inserted in the line as shown in Fig. 21.24.

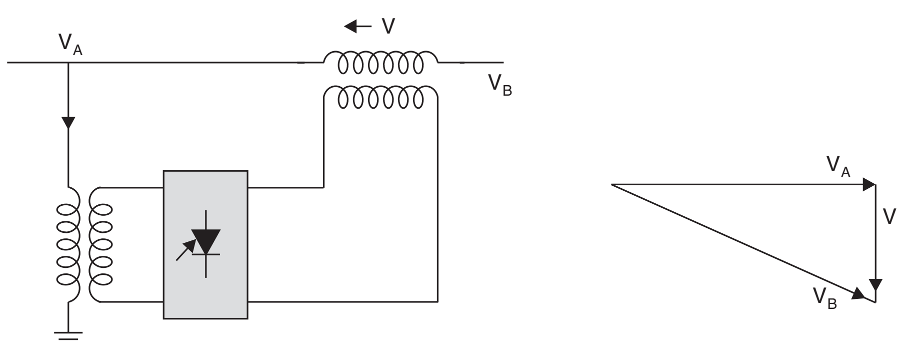


Fig. 21.24 Thyristor controller phase shifting transformer.

The voltage V is perpendicular to the terminal voltage V_A as shown in phasor diagram. Its' magnitude can be controlled by the thyristor converter. The reactive power required to induce voltage V is transmitted via the shunt transformer and the thyristor converter to the boosting transformer.

PROBLEMS

21.1. Describe briefly various compensations carried out in Power Systems.

21.2. What is load compensation ? Discuss its objectives in power systems.

21.3. Explain reasons for variation of Voltage in a power system and suggest methods to improve voltage profile.

21.4. Mention various loads which require compensation.

21.5. Comment on the statement "It is not possible to achieve simultaneously both the unity p.f. and zero voltage regulation".

21.6. Show that the approx per unit change in voltage equals the ratio of change in reactive power to the short circuit capacity of the bus.

21.7. Explain why a line is loaded normally to about 40° and not 70° – 80°.

21.8. Explain what you mean by loadability of overhead lines and discuss loadability characteristic of these lines.

21.9. Explain and differentiate between steady state, transient state and dynamic stability limits of power system.

21.10. Discuss briefly some of the disadvantages of operating the system under-voltage and overvoltage.

21.11. Define surge impedance of a line. Starting from first-principles, develop expression for surge impedance and discuss the advantages of operating a line corresponding to surge impedance loading.

21.12. Explain the terms: Infinite line, flat line.

21.13. Discuss the problem associated with charging long uncompensated line and suggest the method to overcome this problem.

21.14. What is an unsymmetrical line ? Discuss its characteristics.

21.15. A radial long uncompensated line with constant sending end voltage is terminated through an asynchronous load derive an expression for maximum power transfer when termination is through a variable resistance. Hence discuss the voltage instability problem.

21.16. Explain clearly what you mean by compensation of line and discuss briefly different methods of compensation.

21.17. Explain series and shunt compensation of lines and discuss their effect on the surge impedance loading of the lines. If shunt compensation is 100%, what happens to SIL and voltage profile.

21.18. Show mathematically that under certain operating condition, the power-angle characteristic could be a straight line characteristic.

21.19. Explain the phenomenon of subsynchronous resonance in power system operations and suggest remedies to overcome this problem.

21.20. Describe the constructional features of a synchronous capacitor. Explain its operation and discuss various applications in power system operation.

21.21. What is a static compensator ? Explain with neat diagrams working principle of various types of the static compensators.

21.22. Explain with neat diagrams the operation of a basic TCR and derive expression for the control law of the basic TCR and explain the control law.

21.23. Explain with diagrams some arrangements whereby certain harmonics can be eliminated in TCR circuits.

21.24. Explain with neat diagrams the operation of (*i*) thyrister switched capacitors (*ii*) saturated reactor compensator.

21.25. What is Flexible a.c. transmission system (FACTS) ? Describe briefly various devices used in this system.

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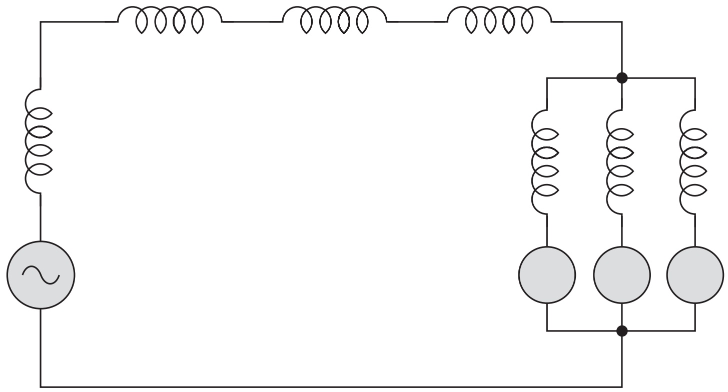
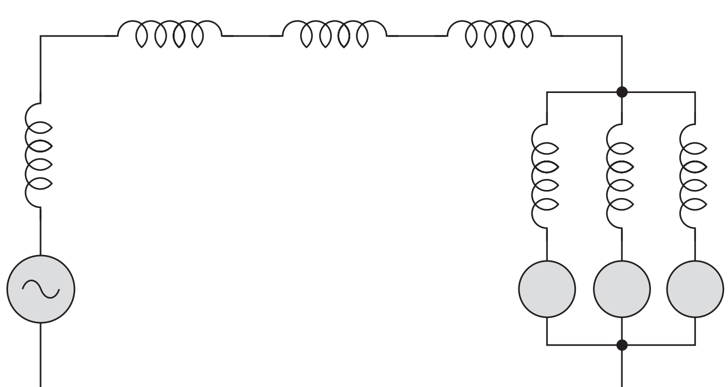


22

POWER SYSTEM VOLTAGE STABILITY



22

Power System Voltage Stability

INTRODUCTION

The maximum power transfer problem becomes much more complicated by the presence of synchronous machines in power system. The power transfer for long lines is limited by the magnitude of voltage at the two ends, the reactance between the two ends and the sine of the angle between the two voltages.

However for short transmission lines power transfer is limited by thermal capabilities. The rotor angle stability has been mainly associated with transient stability which has been practically sorted out by employing fast circuit breakers, powerful excitation systems and various other special stability controls. The rotor angle stability has been discussed in detail in Chapter 17 of the book and this stability is also referred to generator stability or synchronous stability.

The voltage stability also known as load stability, however, is now a major concern in planning and operating electric power system as the power to be transferred is increasing, the interconnection of networks is also increasing because of obvious advantages and there is need for more intense use of available transmission facilities. More and more electric utilities are facing voltage stability imposed limits. Voltage instability and collapse have resulted in several major system failures or blackouts.

Power system voltage stability involves generation, transmission and distribution. Voltage control, reactive power compensation and management, rotor angle (synchronous) stability, protective relaying and control centre operations, all influence voltage stability.

22.1 REACTIVE POWER FLOW

We know that, the angle stability problem is directly associated with the active power transfer. The voltage stability problem is associated with reactive power transfer. We develop mathematical expression for reactive power flow using a simplified model of the system as shown in Fig. 22.1.

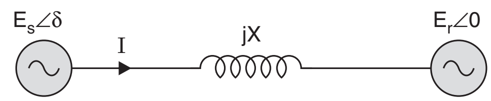


Fig. 22.1 Simplified model of system

The current $$I = \frac{E_s \angle\delta - E_r \angle 0}{jX} = \frac{E_s}{X}\angle(\delta - 90) - \frac{E_r}{X}\angle -90°$$

$$P = \text{Re}\,[E_r I^*] = \text{Re}\left[E_r \cdot \frac{E_s}{X}\angle(90-\delta) - \frac{E_r}{X}\angle 90°\right]$$

$$= \text{Re}\left[\frac{E_r}{X}\left\{E_s \cos(90-\delta) + jE_s \sin(90-\delta) - j\frac{E_r}{X}\sin 90\right\}\right]$$

$$P = \frac{E_s E_r}{X}\sin\delta \tag{22.1}$$

and $$Q_r = I_m[E_r I^*] = \frac{E_s E_r}{X}\cos\delta - \frac{E_r E_r}{X}$$

$$= \frac{E_s E_r \cos\delta - E_r^2}{X} \tag{22.2}$$

$$Q_s = I_m[E_s I^*] = I_m\left[(E_s\cos\delta + jE_s\sin\delta)\left(\frac{E_s}{X}\angle(90-\delta) - \frac{E_r}{X}\angle 90°\right)\right]$$

$$= I_m\left[(E_s\cos\delta + jE_s\sin\delta)\left(\frac{E_s}{X}\cos(90-\delta) + j\frac{E_s}{X}\sin(90-\delta) - j\frac{E_r}{X}\right)\right]$$

$$= \frac{E_s^2}{X}\cos^2\delta - \frac{E_s E_r}{X}\cos\delta + \frac{E_s^2}{X}\sin^2\delta = \frac{E_s^2 - E_s E_r\cos\delta}{X} \tag{22.3}$$

If the power angle δ is small $\cos\delta = 1$ and hence the above equations for Q_r and Q_s are rewritten as

$$Q_r = \frac{E_r(E_s - E_r)}{X} \tag{22.4}$$

and $$Q_s = \frac{E_s(E_s - E_r)}{X} \tag{22.5}$$

From equations (22.4) and (22.5) it is clear that reactive power flow depends mainly on difference between voltage magnitudes and it flows from higher voltage to lower voltage.

On the contrary active power flow depends upon the power angle and active power flows from leading voltage bus bar to lagging voltage bus bar. A typical example is that of a lightly loaded line. The power flow is from a leading voltage bus bar even though the magnitude of the voltage at this bus bar is smaller than that of the receiving end bus bar. Similarly under this loading conditions, the reactive power flows from receiving end (higher voltage) to the sending end (low voltage). Under this condition, the reactive power generated by the line is more (due to shunt capacitance) as compared to the reactive power absorbed by the line (due to series inductance) and hence reactive power is supplied to the sending end.

From equations (22.1) and (22.4) and (22.5) we also conclude that:

(*i*) P and δ are closely coupled, and

(*ii*) Q and V are closely coupled.

We make use of these relations while analyzing power system using fast decoupled power flow technique as discussed in Chapter 18.

We now demonstrate with the help of a few typical problems how these relationships breakdown during high stresses *i.e.,* high power transfers and angles. This is important as voltage stability problems normally occur during highly stressed conditions (usually following outages).

Example 22.1: Assume angular difference between the voltages at sending end and receiving end being 30°. Also assume sending end voltage 1.0 p.u. and receiving end as 0.9 p.u. Therefore we have significant voltage difference between the sending end and receiving end, a difference of 10% and hence we should expect large reactive power flow over the line.

Solution: We calculate as follows:

Using equations (22.2) and (22.3) we have

$$Q_r = \frac{E_s E_r \cos\delta - E_r^2}{X} = \frac{1.0 \times 0.9 \times 0.866 - (0.9)^2}{X}$$

$$= \frac{0.7794 - 0.81}{X} = -\frac{0.0306}{X}$$

$$Q_s = \frac{E_s^2 - E_s E_r \cos\delta}{X} = \frac{1^2 - 1 \times 0.9 \times 0.866}{X} = \frac{0.22}{X}$$

It is a peculiar situation. Even though lot of reactive power is going into the line $Q_s = \dfrac{0.22}{X}$ but nothing is coming out of the line. Rather at the receiving end the line is demanding a reactive power of $\dfrac{0.0306}{X}$ p.u. (because of –ve sign) and hence rather than transferring reactive power the transmission line has become a drain on the power system.

We expected the reactive power to flow from sending end to receiving end as we provided 10% voltage drop between the sending end and receiving end. The transmission line reactive power loss is the sum of the reactive power going into the line which turns out to be $\dfrac{0.2506}{X}$ p.u.

We calculate Q_s and Q_r for two other values of δ

Let $\delta = 0$

$$Q_r = \frac{E_r (E_s - E_r)}{X} = \frac{0.9(1 - 0.9)}{X} = \frac{0.09}{X}$$

and $$Q_s = \frac{E_r(E_s - E_r)}{X} = \frac{1(1-0.9)}{X} = \frac{0.1}{X}$$

Let $\delta = 45°$

$$Q_r = \frac{E_s E_r \cos\delta - E_r^2}{X} = \frac{1\times 0.9\times 0.707 - (0.9)^2}{X} = \frac{0.6363 - 0.81}{X}$$

$$= -\frac{0.1737}{X}$$

and $$Q_s = \frac{E_s^{\,2} - E_s E_r \cos\delta}{X} = \frac{1 - 1\times 0.9\times 0.707}{X} = \frac{0.3637}{X}$$

When $\delta = 0$ the surplus reactive power is $\frac{0.01}{X}$ p.u. Whereas for $\delta = 45°$, the transmission line acts as a drain as far as reactive power is concerned and the total reactive power loss is

$$Q_s - Q_r = \frac{0.3637}{X} + \frac{0.1737}{X} = \frac{0.5374}{X}$$

Now, δ is an indication of active power loading of the line. The higher the value of δ, the higher is the active power loading and we find that when loading corresponds to $\delta = 30°$, the reactive loss is half of what it is when $\delta = 45°$ and the concept that reactive power will flow from a higher voltage level to a lower voltage level fails.

A more general and precise method of understanding power transmission limitation is the power circle diagram which has been explained in detail in Chapter 10 of the book. We will restrict our discussion only to receiving end power circle diagram under article 10.2 of the book. Figure 22.2 is the reproduction of Fig. 10.15. The reader is advised to go through this article

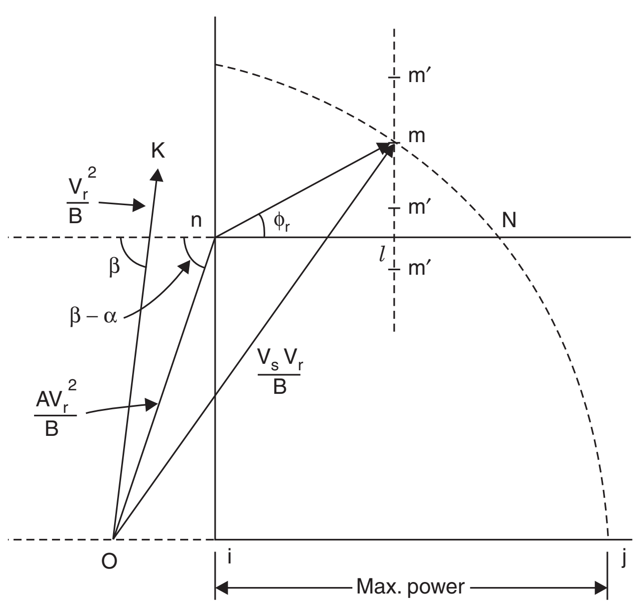


Fig. 22.2 Receiving end power circle diagram.

to understand the problem of reactive power flow. It can be seen that for active power transfer beyond the point N in Fig. 22.2 the reactive power requirement increases steeply *i.e.*, more than one megavar is required for each additional megawatt transmitted. In fact numerically also we have seen that when

$$\delta = 30°, \quad P_1 = \frac{1}{2} \text{ p.u.} \quad \text{and} \quad Q_1 = 0.25 \text{ p.u. whereas for}$$

$$\delta = 45°, \quad P_2 = 0.707 \quad \text{and} \quad Q_2 = 0.5374$$

i.e., the ratio $= \dfrac{P_2}{P_1} = 1.414$ that of $\dfrac{Q_2}{Q_1} > 2$

In fact if we go for higher loadings *i.e.,* higher load angle we find that the above statement holds good.

22.2 DIFFICULTIES WITH REACTIVE POWER TRANSMISSION

1. We have seen in the previous section that reactive power transmission is limited when the power angle is large even though sufficient voltage gradient between sending end and receiving end voltage is provided. The large power angle is due to large active power transfer and the longer length of line. However, the voltage at the receiving end in transmission system is allowed to vary between ± 5% which further restricts the transfer of reactive power. Active power transfer is limited mainly by the power angle, larger the power angle more is the active power transmitted. Also if receiving end voltage is maintained within ± 5%, better transmission of active power takes place. In contrast with real power transfer, reactive power transfer cannot be transmitted long distances.

2. Sometimes we are interested in minimising transmission losses. Active power losses can be minimised by using loss minimisation techniques and in any case active power component of current has to be transmitted over the lines as this cannot be generated at the S/S location. However, reactive power losses should be minimised to reduce investment in reactive power devices such as shunt capacitors. It is to be noted that the economic load dispatching techniques provides loading of various plants for minimum cost of generation for meeting a certain load and whatever losses occur corresponding to the loading are taken into consideration. However, if we want to minimise real power loss then surely the loadings will not correspond to minimum cost of generation.

If R and X are the series resistance and reactance of the line and I the current loading, the losses in the line are I^2R and I^2X.

Now
$$I^2 = \frac{P^2 + Q^2}{V^2} \quad \text{and}$$

$$P_L = I^2R = \frac{P^2 + Q^2}{V^2} R \quad \text{and} \tag{22.6}$$

$$Q_L = I^2X = \frac{P^2 + Q^2}{V^2} X \tag{22.7}$$

From equations (22.6) and (22.7) it is clear that in order to decrease reactive power loss, we must minimise reactive power transfer over the line and also the operating voltages should be high. Keeping operating voltages high to minimise reactive power loss helps maintain voltage stability.

3. The third reason which limits the flow of reactive power over the transmission line is the development of overvoltage during load rejection. The most onerous case is opening the receiving end circuit breakers when the transmission line is still energized from the sending end.

Consider Fig. 22.3(*a*) where a simple system consisting of a generator connected to a load through a transmission line is shown.

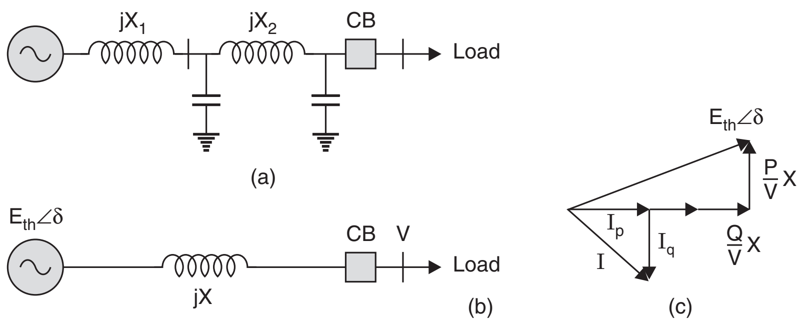


Fig. 22.3 Equivalent system for load rejection calculation
(*a*) Original system (*b*) Equivalent Thevenin's system
(*c*) Phasor diagram for (*b*).

Neglecting the shunt capacitance, the equivalent Thevenin's circuit is shown in Fig. 22.3 (*b*) and Fig. 22.3 (*c*) shows the phasor diagram for the Thevenin's equivalent system. Before the *CB* operates for rejecting (or disconnecting) the load, the Thevenin's voltage is

$$E_{th} \angle\delta = V \angle 0 + jIX, \quad \text{where } X = X_1 + X_2$$

$$= V \angle 0 + jX \frac{P_r - jQ_r}{V} = V + \frac{Q_r}{V} X + j \frac{P_r}{V} X \tag{22.8}$$

From equation (22.8) and the phasor diagram in Fig. 22.3 (*c*) indicate that the rise in voltage at the sending end is mainly due to the reactive power and hence the Thevenin's equivalent voltage source is determined mainly by reactive power flow. The active power flow component of voltage is perpendicular to *V* and hence does not affect much the sending end voltage. However, angle δ of E_{th} depends upon this quadrature component of voltage drop. So we see that when the load is suddenly disconnected there is a temporary overvoltage in the system equal to $E_{th} \angle\delta$ which is mostly affected by the reactive power flow before the load is disconnected. Following examples will further illustrate this phenomenon.

Example 22.2: A 100 km, 500 kV line has a series reactance of 0.35 Ω/km and power transmitted is 1000 MW. It is connected to a source bus which has a short circuit capacity of 5000 MW. Determine the source voltage when the load is disconnected to load p.f. (*i*) unity (*ii*) 0.8 lag.

Solution: Assuming base voltage of 500 kV and base power of 1000 MW, the base impedance is

$$\frac{500 \times 500}{1000} = 250\ \Omega$$

The per unit reactance of the line is $\frac{0.35 \times 100}{250}$ = 0.14 p.u. and since the short circuit capacity of source bus is 5000 MW, the equivalent p.u. Thevenin's source impedance is

$$\frac{1000}{5000} = 0.2 \text{ p.u.}$$

Since the source p.u. impedance is greater than that of the transmission line, the source is relatively weak. The total reactance of the system is 0.2 + 0.14 = 0.34 p.u.

Let V = 1 p.u. = 500 kV

(*i*) For p.f. unity $\phi = 0$

$$E_{th} = V + \frac{Q}{V} X + j \frac{P}{V} X = 1 + j . \frac{1}{1} 0.34 = 1.056 \angle 18.8°$$

(*ii*) For p.f. 0.8, $Q_r = P_r \tan \phi$, = 0.75 p.u.

$$E_{th} \angle\delta = 1 + \frac{0.75 \times 0.34}{1} + j\,0.34 = 1.255 + j\,0.34 = 1.30 \angle 15.2°$$

We find that if the load disconnected is operating at unity p.f. the temporary overvoltage at the sending end is 5.6% higher than the receiving end voltage. However, if the p.f. is 0.8 lag, this voltage is 30% more than the receiving end voltage. Such a rise in voltage may affect the insulations of various equipments operating in the system.

We know that in a combined system consisting of d.c. and a.c. circuit connection, the converter and inverters used in dc circuits consume large amount of reactive power and in such a situation, if d.c. circuit is shut down or blocked, the a.c. side voltage will increase to intolerable values. This is illustrated in the following example.

Example 22.3: Consider a high voltage direct current (HVDC) transmission link connected to a weak power system as shown in Fig. E22.3.

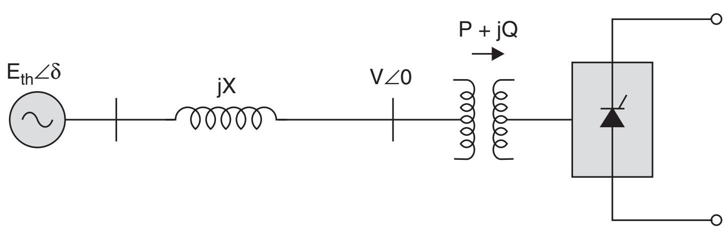


Fig. E22.3

The equivalent Thevenin's reactance of the line is 0.625 p.u. The converters consume reactive power of 60% of the dc power. Assume converter commutating bus voltage (V∠0°) to be 1 p.u. determine the ac system voltage when the d.c. system is disconnected or shutdown.

Solution: Several existing HVDC links have a high network reactance. In this case it is given to be 0.625 p.u. Hence Thevenin's voltage is given as

$$E_{th} = 1 + \frac{0.6}{1} \times 0.625 + j\,\frac{1}{1} \times 0.625 = 1.51\ \angle 24°$$

The sending end voltage is 51% higher than the receiving end voltage which is an intolerable level. Therefore, some measures must be taken to reduce the load rejection voltage. The most effective method is to produce this reactive power locally with a synchronous capacitor. It is to be noted that the synchronous capacitor increases the short circuit capacity of the bus to which it is connected and hence it reduces the effective Thevenin's reactance and, therefore, voltage rise due to load rejection is further reduced.

From the discussion above following observations are made in regard to minimising reactive power transfer. Wherever possible reactive power should be generated near the point of consumption.

1. When the active power to be transferred is large *i.e.,* the torque angle is large, it is not desirable to transfer reactive power over the transmission system as it requires substantial voltage magnitude gradients (Normally ± 5% is permissible).

2. The flow of reactive power causes high real and reactive power losses.

3. During load rejection the reactive power may lead to damaging temporary over voltages (Damaging the insulation of various equipments operating in the system).

4. It requires larger equipment sizes for transformers and cables.

22.2.1 Short Circuit Capacity and Voltage Regulation

The concept of short circuit capacity has been discussed in detail in Article 13.13 of the book.

The short circuit capacity is defined as the product of the magnitude of prefault voltage and post fault current. The strength of a bus is directly related to its short circuit capacity, the higher the short circuit capacity of the bus the more it is able to maintain its voltage in case of a fault on any other bus. Also it can be seen that higher the short circuit capacity, lower will be the Thevenin's equivalent impedance as seen between the faulted bus and the zero potential bus. Consider say three buses in a large integrated network with SCC as 1500 MVA, 1200 MVA and 1000 MVA. If a three phase fault takes place on bus 3(1000 MVA SCC) the voltage of bus 3 of course reduces to zero. The voltages of bus 1 and 2 sag. The sag in voltage of bus 2 will be more than that in bus 1 as the SCC of bus 2 is smaller than that of bus 1. A low short circuit capacity means a weak network. Switching on a load or a shunt capacitor or reactor will not change the voltage materially in a high short capacity system. However, if the system is weak *i.e.,* it has a low short circuit capacity, a large size motor switched on in such a system may stall or have difficulty in reaccelerating following fault. Motor starting will cause system voltage dip.

Sometimes it is desirable to compare the size of the equipment to the strength of the power system. A simple comparison is to divide the system strength by the device size. Comparing a 1000 MW HVDC converter to 5000 MVA short circuit capacity power system results in a short circuit ratio of 5000/1000 or 5. A high short circuit ratio means good performance in terms of voltage regulation and vice versa.

A related term, used especially with HVDC, is effective short circuit ratio (ESCR). The basic SCR accounts for only the network strength while ESCR accounts for shunt reactive equipment at the device location. A synchronous condenser clearly increases the fault current and, therefore, the effective short circuit capacity. On the other hand, shunt capacitors and harmonic filters (which are capacitive at fundamental frequencies) reduce the ESCR.

Several widely used approximate formulae for voltage regulation have been derived in article 21.1 of the book and the formula for voltage regulation involving short circuit capacity of a bus when switching in or out a reactive equipment is given by equation (21.8) or reproduced here as equation (22.9).

$$\frac{\Delta V}{V} \simeq \frac{\Delta Q}{S_{sc}} \tag{22.9}$$

Which means the per unit change in voltage at a bus equals the ratio of change in reactive power at the bus to the short circuit capacity of the bus.

Suppose a bus has a short circuit capacity of 10,000 MVA and a capacitor of 300 MVA*r* is connected to the bus, the change in voltage (increase) will be 300/10,000 = 3%.

Conversely, suppose a bus experiences a voltage change of 3% (decrease) the short circuit capacity of the bus is 5000 MVA, the size of the capacitor required to be connected to the bus would be $\Delta Q = \Delta V S_{sc} = \dfrac{3}{100} \times \dfrac{5000}{1}$ = 150 MVA*r* to compensate for 3% voltage dip.

From Fig. E 22.3 and equation (22.9) we have

$$E = V + \Delta V$$

or

$$V = E - \Delta V = E - \frac{\Delta Q}{S_{sc}} V \simeq E\left(1 - \frac{\Delta Q}{S_{sc}}\right)$$

Assume $V \simeq E$

Hence

$$V = E\left(1 - \frac{\Delta Q}{S_{sc}}\right) \tag{22.10}$$

Equation (22.10) approximates simple equation expressing the voltage drop from the source to the load point. The variation is shown in Fig. 22.4.

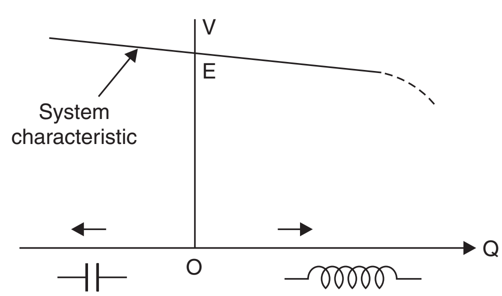


Fig. 22.4 System approximate voltage/reactive power characteristic.

Equation (22.10) represents a straight line with slope $-\dfrac{E}{S_{sc}}$ and ordinate E as shown in Fig. 22.4. The slope of the load line is related to stiffness, a nearly flat slope means a strong system. The voltage reactive power characteristics of shunt reactive devices (capacitor, reactor or static var compensators) can be superimposed on the system characteristic. The system characteristic will not be linear at high inductive loading. The dashed portion of the characteristic in Fig. 22.4 is an indication of voltage problem for large value of inductive reactive power.

22.3 VOLTAGE STABILITY: DEFINITION AND CONCEPT

Voltage stability covers a very wide range of phenomena from a slow phenomenon involving the mechanical tap changing etc. to a fast phenomenon involving the induction motors, air conditioning loads or HVDC links. Engineers and researchers have done lot of work in the field over the past number of years. A few references are in place at the end of the Chapter.

Voltage stability or voltage collapse has been seen as a steady state problem involving static power flow studies for analysis. The ability to transfer reactive power from sources to sinks during steady operating conditions is a major aspect of voltage stability. It is to be noted that the network maximum power transfer limit is not necessarily the voltage stability limit.

Voltage instability or collapse is a dynamic process. The word stability itself implies a dynamic system and power system is a dynamic system. In a large complex power system, the voltages of various busses, the flow of active and reactive power, etc. keep on changing with time. It is to be noted that, in contrast to rotor angle (synchronous) stability, the dynamics mainly involves the loads and the means for voltage control. Voltage stability, hence, has been called load stability.

Voltage stability is a very important aspect of overall power system stability. The voltage stability is analogous to stability of any other physical system.

Some of the definitions of power system voltage stability are given hereunder.

Definitions

1. A power system at a given operating state is small disturbance voltage stable if, following any small disturbance, voltages near loads are identical or close to the predisturbance values.
2. A power system at a given operating state and subject to a given disturbance is voltage stable if voltages near loads approach post disturbance equilibrium values. The disturbed state is within the region of attraction of the stable post disturbance equilibrium.
3. A power system at a given operating state and subject to a given disturbance undergoes voltage collapse if post-disturbance equilibrium voltages are below acceptable limits. Voltage collapse may be total (black out) or partial.

Voltage stability results in progressive voltage decrease or increase. Destabilizing controls reaching limits or other control actions *e.g.,* load disconnection however, may establish global stability.

Voltage stability studies normally requires a sudden increase in load or transfer of power and is almost always an aperiodic decrease in voltage. Oscillatory voltage instability may be possible but control instabilities, which may be due to too high a gain on a static var compensator or too small a deadband in a voltage relay controlling a shunt capacitor bank, are excluded. Overvoltage phenomena and instability such as self excitation of rotating machines are outside the scope of definitions. Overvoltages are normally more of an equipment problem than a power system stability problem.

Sometimes the term voltage security is used which means the ability of a system not only to operate stably, but also to remain stable following large contingencies or load increases. This, infact, requires the presence of large margin between the operating point and the voltage instability point or to the maximum power transfer point following large contingencies.

Even though voltage stability requires analysis of the power system under dynamic conditions, static power flow methods are often useful for fast but approximate analysis.

The phrases used 'Equilibrium points' and 'region of attractions' used in the definition are further explained in the succeeding section.

22.3.1 Voltage Instability-time Frames and Mechanism

The time frame for voltage instability or collapse ranges from a fraction of a second to tens of minutes. Figure 22.5 shows a classification of voltage stability into transient and longer-term time frames. There is almost always a clear separation between the two time frames. From the figure it is also clear that many power system components and controls play a role in voltage stability. However, only some will play an important role in a particular incident.

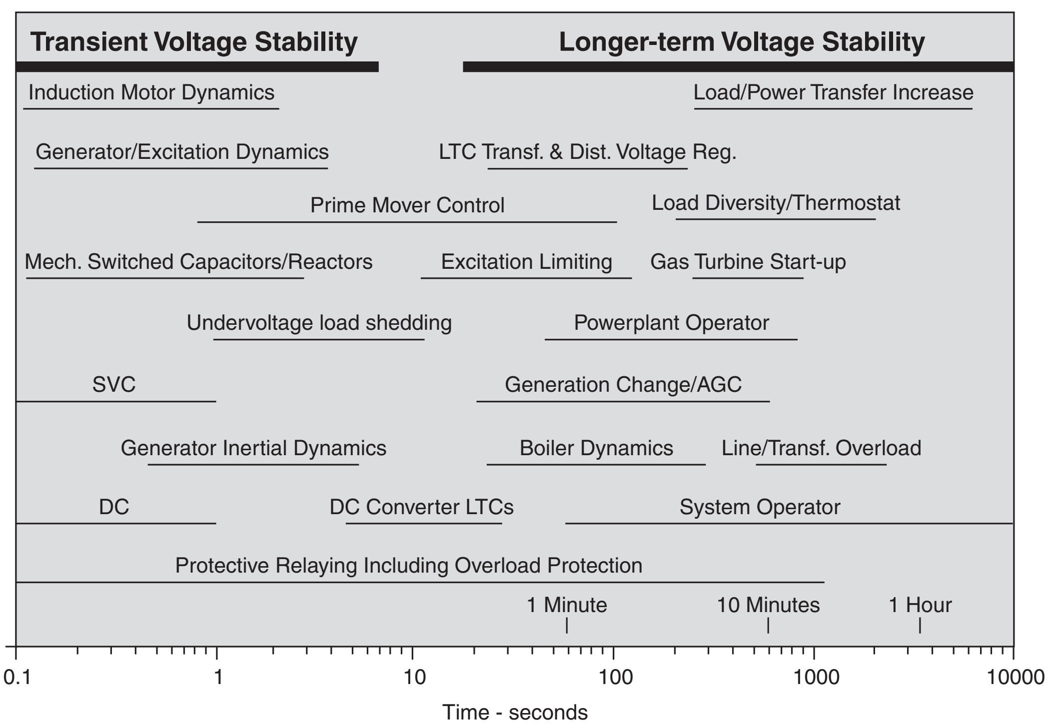


Fig. 22.5 Voltage stability phenomena and time responses.

We now describe the two classifications of voltage instability the transient and longer term voltage stability.

1. **Transient voltage stability:** The time frame for transient voltage stability and transient angle stability is almost same and varies between zero to about ten seconds. Because of this overlap of period, it is difficult to differentiate between transient voltage stability or transient angle stability and as such aspects of both phenomena may exist. Therefore, it may not be clear whether angle rotor instability led to voltage instability or vice versa. Voltage collapse is caused by unfavourable fast acting load component such as a large induction motors and d.c. converters.

It has been observed that during electric system islanding and under frequency load shedding, there is a possibility that the system voltage may collapse whenever the imbalance in the system reactive power is more than 50%. It is also found that voltage decays much faster than the frequency, the voltage decay affects voltage sensitive loads, thereby the frequency decay slows down and the load shedding due to under frequency relay is delayed. Also, the under frequency relays require certain minimum voltage to operate, and during voltage decay these relays may not operate and hence undervoltage load shedding may be necessary.

HVDC lines have been used in power system to enhance the power transfer capability of the existing network or are used to link to ac system operating at different frequencies. These HVDC circuits hence caused transient voltage stability problems as the converters and inverters require large amount of reactive power for their operation and pose voltage stability problem specially when power transfer is large (large angle δ) and for a large disturbance under such situation it is necessary to reduce dc power to support voltage levels.

2. **Longer term voltage stability:** The time frame is a few minutes typically two-three minutes. Hence operator intervention is not possible.

The longer term voltage stability involves high loads, high power imports from remote generation and a sudden large disturbance which could be in the form of loss of large generators in a load area or loss of a major transmission line (large capacity line). The system is transiently stable. However, the disturbance causes high reactive power losses and voltage sags in load areas. The distribution voltage regulators and the tap changers on bulk power delivery LTC transformers sense the low voltages and act in a way to restore distribution voltages and thus the load power levels are restored. This restoration of load further causes sags of transmission voltages and nearby generators are overexcited and overloaded. Now over excitation is allowed for one or two minutes to avoid over heating of the field winding and hence the field currents return back to normal rated value (through field current limiters or power plant operator). Therefore, the generators farther away from load are required to provide the reactive power but as mentioned in the previous article, this method of transmitting reactive power is both inefficient and ineffective. The generation and transmission system can no longer support the loads and the reactive power losses and rapid voltage decay ensues. Partial or complete voltage collapse follows. Finally the induction motors may stall and protective relays may operate and depending upon the type of loads including means for disconnection at low voltage, the collapse may be partial or total.

3. **Longer term voltage instability:** Here the time frame is in terms of tens of minutes and involves a very rapid built-up of load in terms of MW/min. and hence a large rapid power transfer increase: The operator actions in terms of timely load shedding and application of reactive power equipments to avoid voltage instability is required. The other important function

affecting the voltage stability are the time overload limit of transmission lines (tens of minutes) and loss of load diversity due to low voltage (due to constant energy, thermostatically controlled loads). Loads such as industrial process heating, space heating, water heating and air-conditioning are controlled by thermostats, behaving the loads to be constant energy. For heating loads, low voltage results in loss of load diversity since individual loads stay on longer. Over time, the aggregated load changes from resistive to constant power. Electric space heating may be a large load during cold weather. The effective time constant for loss of diversity becomes shorter during cold weather depending on factors such as temperature, wind and building thermal time constant. The heater-on cycle time will be longer and the off cycle time will be shorter. The final stages of instability involve actions of faster equipment as described in the other two forms of voltage stability.

Various equipments interact during this time frame as shown in Fig. 22.5 *e.g.,* voltage regulation using tap changers will prevent loss of diversity by thermostatic regulation of constant energy levels as explained above and over excitation limited operation prevent normal generator voltage regulation.

Example 22.4: There are four thermostatically-controlled electric space heating loads of 1 p.u. power each. It is found that under certain weather conditions and for 1 p.u. voltage, they are on for four minutes and off for four minutes. The on-off cycle of the four heaters are initially symmetrically distributed so that two heaters are on at any point of time. The voltage suddenly drops to 0.894 p.u. Calculate the new on and off times for constant energy.

Solution: Since the power is proportional to square of voltage, the new power per heater will be $1 \times (0.894)^2 = 0.8$ p.u.

The average power per heater is 0.5 p.u which must be maintained for constant energy. After the voltage reduction, the off-time will remain at four minutes. However, the on-time will increase which is calculated as follows:

$$\text{Average power } 0.5 = \frac{0.8\,(t_{\text{on}}) + 0\,(t_{\text{off}})}{t_{\text{on}} + t_{\text{off}}}$$

$$= \frac{0.8\,(t_{\text{on}})}{t_{\text{on}} + 4}$$

or $$0.5\, t_{\text{on}} + 2 = 0.8\,(t_{\text{on}})$$

or $$0.3\, t_{\text{on}} = 2 \quad \text{or} \quad t_{on} = 6.67 \text{ min.} \quad \textbf{Ans.}$$

Airconditioning and other compressor loads are constant energy loads. However, since they are nearly a constant power loads before thermostatic control, there will be little change in load diversity with voltage changes.

22.3.2 Relation of Voltage Stability to Rotor Angle Stability

Voltage stability is normally inter linked with the rotor angle synchronous stability. It is difficult to separate the two kinds of stabilities. Transient voltage stability is more or less inter linked with transient rotor angle stability and slower form of voltage stability is inter linked with the small disturbance rotor angle stability. The mechanism leading to these stabilities is more or less the same and is difficult to seperate.

However, these are extreme situations when voltage stability could be distinguished from rotor angle stability and vice versa. Consider Fig. 22.6 (*a*) where a synchronous machine is connected to an infinite bus or a large system.

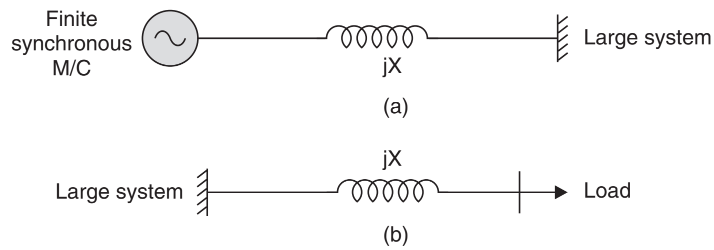


Fig. 22.6 (*a*) Pure angle stability (*b*) pure voltage stability examples.

It is a case of pure angle stability whereas in Fig. 22.6 (*b*) where an asynchronous load is connected to a large system is a case of pure voltage stability example.

Both the rotor angle stability and voltage stability are affected by reactive power control. The rotor angle steady state stability used to be a major problem before the use of fast and continuously acting automatic voltage regulators became available. By increasing the voltage levels either by automatic voltage regulator (or by overexciting generators) the steady state stability could be improved. Therefore, we see a relation between small disturbance angle stability and longer term voltage stability as the field current of the generator when overexcited prevents normal automatic voltage regulation for a longer time as the over excitation limiter brings the excitation back to normal value after two to three minutes. Therefore, generator field current limiting is detrimental to both forms of stability.

Rotor angle stability is normally concerned with integrating remote power plants to a large system over long transmission lines whereas voltage stability is concerned with load areas and load characteristics. Therefore, rotor angle stability is basically generator stability and voltage stability is basically load stability. In a large interconnected system, it is possible that there may be voltage collapse without loss of synchronism of any generators.

It can, therefore, be concluded that if voltage collapses at a point in a transmission system remote from loads, it is an angle instability situation. However, if voltage collapses in a load area it is probably mainly a voltage instability situation.

One reason for voltage instability in the power system which are in existence and in operation for a long time is intensive use of existing generation and transmission. This is because of the difficulties in building new generation in load areas and difficulties in building transmission lines from generators which are located at far off places due to nonavailability of right-of-way.

A second reason is the vast use of shunt capacitors banks for reactive power compensation. The excessive use of shunt capacitor even though enhances the power transfer limits, it results into a voltage collapse prone fragile network. The network is termed as fragile as output of shunt capacitor decreases as square of the voltage. If a transmission line is compensated to improve the transient stability limitations, then either, thermal capacity or voltage stability may dictate the transfer limits of the line.

Series capacitor compensation has conventionally been used for long transmission lines to improve transient stability of the system. However, now a days series capacitors are also used on short transmission lines to improve voltage stability. Series compensation reduces the net inductive reactance of the line. The reactive power generation I^2X_C due to series capacitance compensates for the reactive consumption I^2X_L due to series inductance of the line. Series capacitor reactive power generation increases with the current squared, thus generating reactive power when most needed. This instantaneous inherent self regulation is very valuable and because of this property, series compensation should be compared with active shunt compensation (static var compensators) rather than passive shunt capacitor banks. At light load series capacitors have little effect. Short reactors are required for long lines.

For more details on line compensation, the reader is suggested to refer to chapter 21 of the book which is wholly devoted to compensation in power system.

22.4 POWER SYSTEM LOADS

In order to study the voltage stability it is important to understand the load characteristics including the subtransmission and distribution networks that connect consumers to the transmission network and generation.

22.4.1 Transformers

The transformers with bulk power delivery may have load tap changers (LTC transformers) or there may be a voltage regulator in series with the transformer on the secondary (LT) side. For voltage stability the impedance of transformer is important. Leakage impedances of bulk power delivery transformer vary between 8 to 11% on its base.

Impedances of distribution transformers are 2 to 4%. The bulk power delivery transformers represent a major portion of the distribution system impedance. Normally, it is thought that the distribution transformer as compared to power transformers are operated at lower flux density to decrease iron losses so that all day efficiency of the distribution transformers is obtained high. However, from voltage stability point of view many distribution transformers operate with some degree of saturation, as a result the exciting current is sufficiently large even at rated voltage. Exciting current is reduced during voltage sags accounting for the high reactive power voltage sensitivity 3 to 6% reduction in reactive power for a 1% reduction in voltage.

22.4.2 Feeder Characteristics

Distribution feeder circuits leave the bulk power delivery substation. Where feeders are of different lengths, the transformer tap changing regulator is often replaced with individual feeder tap changing voltage regulator mainly the auto transformers. The *X*/*R* ratio of these feeders is much smaller as compared to those of transmission lines. Long feeders which generally serve residential or farm loads, may require additional voltage regulators. Fixed and switched shunt capacitors can also be installed along the feeder to control voltage. From voltage stability point of view we are especially interested in the characteristics of longer feeders that add significant impedance between bulk power delivery transformers and loads.

22.4.3 Voltage Reduction

Many electric utilities use intentional voltage reduction to obtain load reduction during times of generation or transmission capacity shortages and then energy is also conserved. Many utilities have also used voltage reduction technique as an emergency measure to improve voltage stability. Voltage reduction is accomplished through tap changing transformers and distribution voltage regulators.

Voltage reduction should be as close as possible to the loads and downstream from shunt capacitor banks. Often the per cent reduction in reactive power load is much more than the reduction in active load. This is because distribution transformers and some motors are operated in magnetic saturation at normal rated voltage.

22.4.4 Induction Motors

Most of residential and commercial motor use is for the compressor loads of air conditioning and refrigeration. Compressor loads require nearly constant torque at all speeds and are the most demanding from a stability point of view. Pumps, blowers and fans, and compressors account for more than half of industrial motor use.

It has been observed that motors are usually oversized with perhaps 50% of all integral horse power motors operating at less than 60% of rated load. Large industrial motors and motor supplied as part of packaged equipment, however are usually properly sized.

The steady state active power drawn by motors is fairly independent of voltage until point of stalling. Motors reactive power is more sensitive to voltage levels. As voltage drops, the reactive power will first decrease but then increase as the voltage drops further. It has been observed that for motor loads $\Delta Q/\Delta V$ varies between 0.4 and 3.5. Values of $\Delta Q/\Delta V$ greater than 2 probably reflect operation with motor core saturation. It has been observed that constant torque loads and heavy loadings are more onerous for voltage stability. The reactive power voltage sensitivity decreases with increased loading.

Voltage instability is likeliest during very high load levels caused by extreme temperatures. For summer time conditions, a major part of the load will be air conditioners. The air conditioners have low inertia constant (H) and these are prone to stall. The tests have shown that airconditioners will decelerate and stall at fault voltages below about 60%–assuming five cycles or longer fault clearing time. With slower fault clearing, air conditioners may stall at higher fault voltages. After source voltage comes to normal, stalled motors will not recover until compressor pressure bleeds off. Thermal overload protection will disconnect the air conditioners in 3 to 30 secs. Large commercial air conditioners have undervoltage relays which trip the units within five cycles after voltage drops below 70%.

Figure 22.7 shows the speed torque curves as a function of voltage and temperature for a residential central air conditioner compressor motor. The constant torque mechanical (horizontal lines) characteristic shown is the most demanding. The motor stalls whenever the electrical input (Torque speed curve) is less than the mechanical output. The stalling of the motor continues until compressor pressure bleeds off.

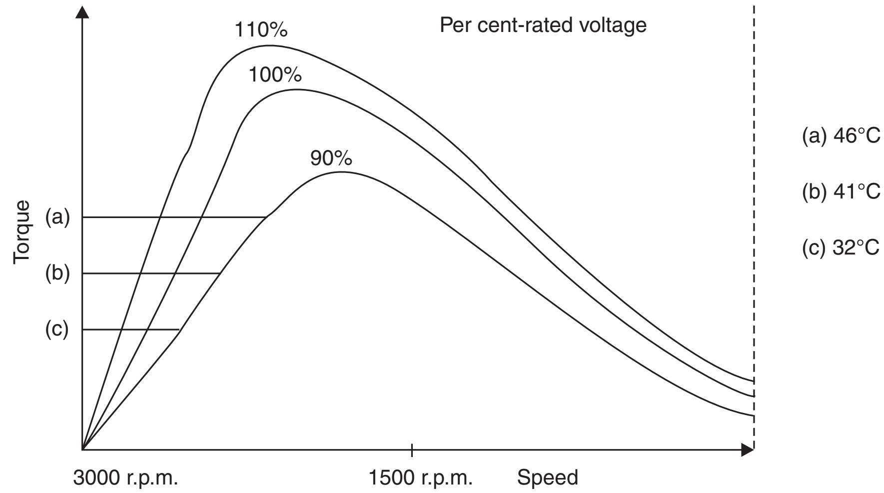


Fig. 22.7 Torque-speed curves for 5 HP single phase residential central air conditioner compressor motor.

The dynamic characteristics of induction motors are critical for voltage decay or voltage dips below about 0.9 p.u. At sustained low voltages between about 0.7 and 0.9 p.u. many motors will stall and draw large amount of reactive power. Stalling of one motor may cause nearby motors to stall as this motor draws large reactive power and hence the voltage of the bus to which this motor is connected dips further and the other nearby motors may come to a stop.

If we refer to the equivalent circuit of an induction motor, the variable is the slip. However, whenever a disturbance takes place in the system to which the induction motor is connected, the motor will first act as an impedance load as the slip s can't change instantly. It changes only after initial dynamics lasting a few tenths of a second. This "impedance jump" responses to a step change in voltage.

Example 22.5: Consider the *P-V* curve of an induction motor fed from a weak power system. Assume the motor is (*i*) at a normal operating point and (*ii*) transiently on the bottom side of the *P-V* curve. Analyse the effect of energizing a capacitor to stabilise the motor. Discuss whether the voltage will increase or decrease and state the location of the final operating point.

Solution: Figure 22.8 shows the corresponding static (constant motor power) *P-V* curves (*P-V* curves are shown in Fig. 21.2) and the motor constant slip resistance characteristics.

We know that, the power input to the rotor is $\dfrac{V^2}{R_{2e}/s} = P_e$ and for constant slip, let the resistance be R'_{2e}, hence $P_e = \dfrac{V^2}{R'_{2e}}$ or $V = \sqrt{R'_{2e}\,P_e}$ or $y = \sqrt{mx}$ and the characteristic will be as shown in Fig. 22.8.

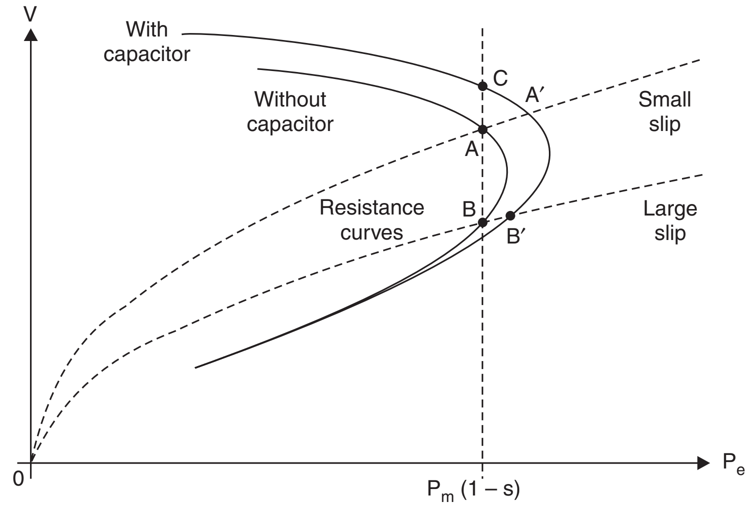


Fig. 22.8 P-V curve showing how capacitor energization will result in operation at point C for initial operation at either point A or B.

Point *A* is assumed normal initial operating point on the upper part of the *P-V* curve and point *B* is the assumed transient operating point in the bottom side before capacitor switching (the transient operating point is at a power equal to the mechanical power). Because of the initial impedance response, capacitor energisation causes increase in voltage and an immediate jump from point *A* to *A′* or from point *B* to *B′*. In either case, the electrical torque or power is grater than the mechanical torque as *A′* and *B′* lie to the right of the vertical line $P_m(1 - S)$. The motor accelerates and settles at point *C* along the *P-V* curve when capacitor is connected and thus the slip is reduced as the resistance curve passing through the origin and the point *C* having the relation $V = \sqrt{R'_{2e} P}$ will have larger slope than the previous two curves shown in the Fig. 22.8 and hence slip is reduced and also since the motor accelerates, the slip reduces.

If, however, the voltage decays to a value less than point *B* and if point *B′* (after capacitor switching) is less than the initial power *i.e., B′* lies to the left of the vertical line $P_m(1 - S)$, the motor decelerates, which finally leads to stalling of motor and voltage collapses.

Overload and under voltage protection provided in the starter of the induction motors are equally important, as the motor dynamics and possible stalling of these motors. Figure 22.9 shows a typical starter used with most of the industrial motors. The voltage coil has two operations, pick up and drop down. The ac contactors connect the motor to the supply whenever the voltage is about 90% of the rated voltage which is the pick up, whereas when the voltage is less than a prespecified voltage (drop down) the '*M*' relay releases the contactors and the supply to the motor is disconnected. The drop down voltage is not consistent. This voltage is variable and ranges between 60% to 80%. In the field, contactor drop down voltage has occasionally been experienced at even higher voltage which may be due to poor maintenance, contamination or incorrect application *e.g.*, a 600 volt starter on 400 volt motor.

There are many alternatives to the circuit shown in Fig. 22.9 which reduce unnecessary tripping of critical motors. These include mechanical latch '*M*' relays and control power from a source other than the power system. Starters are also available that will reconnect the motor following a momentary voltage dip. Very large motors have circuit breakers and relays which will trip the motor only if the damage is imminent.

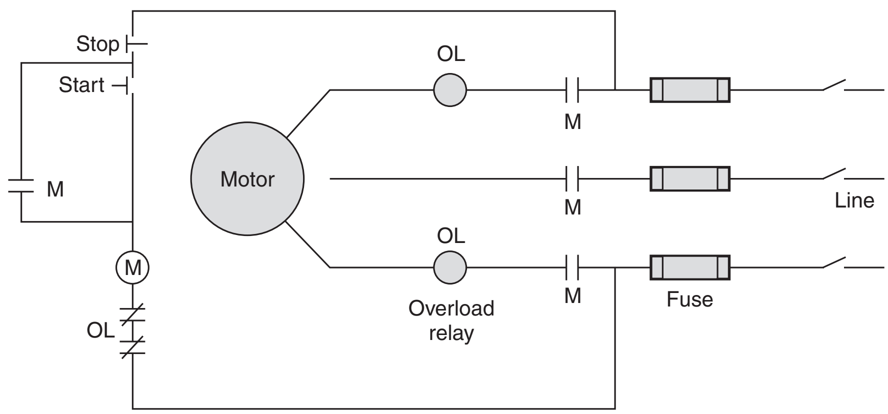


Fig. 22.9 Motor starter circuit showing contactor release. Often, control power is through a transformer with 110 volt secondary.

It is to be noted, therefore, that because of the many unknowns regarding motor control and protection, disconnection of load should not be relied on to prevent voltage instability.

22.4.5 Generation in Load Area

There are some small capacity captive power plants in the load area. The larger synchronous generators and motors with automatic voltage regulators will improve voltage stability. However, excitation control on smaller synchronous generators connected to distribution networks may instead regulate p.f. so as not to interfere with utility voltage regulation. Many non-utility generators are induction generators with shunt capacitor compensation. Performance is similar to that of induction motor.

Induction generators are simpler and cheaper. However, the synchronous generators are more efficient and are usually used for large power ratings. Network reduction software can be used to develop equivalents for sub transmission networks which include non-utility generators of significant size.

22.4.6 Reactive Compensation of Loads

The secondary or distribution side of bulk power delivery substation usually have shunt capacitor compensation for reactive power management rather than for direct voltage control. If the load on the main transformer is high, there is large reactive power demand and large reactive power loss and hence to reduce these, some capacitors are switched on. With this, the capacitors also release transformer capacity as transformers need not transfer reactive power. Again to maintain voltage levels between two bus bars to certain prespecified values reactive power transfer should be minimised. This is why reactive power compensation is provided close to the reactive power consumption.

With distribution automation, the future will probably see more centralised control of substation capacitors and feeder capacitors to optimise operation of an entire area.

The instantaneous response of the series capacitors is important when load fluctuation causes voltage flicker. However, there are certain difficulties with use of series capacitors, including ferroresonance in power transformers and sub synchronous resonance (SSR) during motor starting.

For more details on reactive power compensation of load, the reader is suggested to refer to Chapter 21.

22.4.7 LTC Transformers and Distribution Voltage Regulations

We have studied in a previous article that tap changing, a principal mechanism leads to voltage instability. Following a disturbance, the voltage sags provide temporary relief, in less than a minute, however, tap changing equipment starts to restore the load side voltage and thus the load.

The process of tap changing is explained as follows:

Bulk power delivery load tap changing transformers (LTC) and distribution voltage regulators act identically while regulating load side voltage.

A voltage relay monitors load side voltage. If the voltage goes out of certain prescribed limits, a timer relay is energized. If the relay times out (after tens of seconds), the tap changing mechanism will be energized and tapping takes place until the voltage remains within prespecified limits. Once within limits, the voltage relay and timer mechanism is reset. In the course of a voltage collapse, tap changers on individual transformers or voltage regulators may reset several times thus slowing the voltage collapse process.

Bulk power delivery LTC transformers and distribution voltage regulators usually have $\pm$ 10% tap range consisting of thirty two steps of $\frac{5}{8}\% \left(\frac{20}{32} = \frac{5}{8}\right)$ each.

Effect of tap changing on shunt compensated load: We know that, whenever there is sag in the voltage the tap changing aggrevates voltage stability by restoring the load on the system. This is true especially when the p.f. of the loads is high (Heating load). However, sometimes voltage regulation by tap changing may improve voltage stability when the p.f. of load is lagging, the loads are voltage insensitive and these are heavily shunt compensated, *e.g.,* industrial consumers with high motor load and loads with high air conditioning component. This is explained as follows: The real part of the load is nearly constant power and not affected by tap changing. The reactive part of the load may have relatively low voltage sensitivity. The shunt compensation, however, has a voltage squared reactive power sensitivity. Thus the main effect of tap changing is to support the capacitor output.

Example 22.6: An industrial load has motors which are shunt compensated. Assume that the real part of the load is insensitive to voltage variation, determine the voltage sensitivity at nominal voltage of the net reactive load assuming 75% shunt compensation. The uncompensated voltage sensitivity is $\Delta Q/\Delta V$ = 1.0. Discuss the effect of tap changing.

Solution: The net reactive power

$$\begin{aligned} Q_{net} &= Q_{load} + Q_{cap} \\ &= 1\text{ V}^1 - 0.75\text{ V}^2 \end{aligned}$$

$$\text{Voltage sensitivity} \qquad = \frac{dQ_{net}}{dV} = 1 - (0.75)(2)\text{ V} = -0.5 \text{ for } V = 1 \text{ p.u.}$$

Since the voltage sensitivity is negative voltage regulation by tap changing will reduce net reactive load and thus improve voltage stability. It is to be noted that normally reactive power voltage sensitivity of the load may be higher than unity which results in less benefit from tap changing.

Whenever possible, the capacitor should be connected on the regulated side of tap changers.

LTC transformer equivalent circuit: The main effect of tap changing on voltage stability is restoration of voltage sensitive load that is reduced during voltage sag. For an ideal transformer, and an impedance load, the load is reflected to the primary side (high voltage side say) by the square of the turns ratio (V_1/V_2).

Figure 22.10 shows an LTC transformer equivalent circuit. This circuit has already been derived in Chapter 18 on load flows in this book in Article 18.2.

If we replace $\frac{1}{(1+t)}$ by n, we have the equivalent circuit as shown in Fig. 22.10. Here $Y = \frac{1}{Z}$ where Z is the series impedance of the transformer.

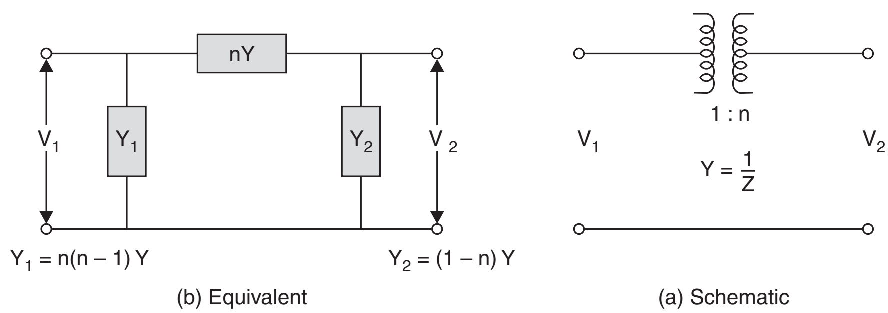


Fig. 22.10 Transformer equivalent circuit. *n* is the off nominal turns ratio.

For tapping to raise secondary voltage, the primary side shunt element is a mathematical reactor (as $n > 1$) and the secondary side shunt element is a mathematical capacitor. The secondary support depends upon additional reactive power from the primary system. The primary or source system should be strong enough to support this effect of tap changing.

Example 22.7: A 230/34.5 kV transformer has 10% leakage reactance. Assume initial $n = 1$. Determine the effect of tapping to raise the secondary voltage by 10%.

Solution: Since the secondary voltage is to be increase by 10%, the off-nominal turns ratio is 1 + 0.1 = 1.1.

Since the leakage reactance of the transformer is 10%, neglecting the resistance component, the series admittance of the transformer is $1/j0.1 = -j10$.

Hence the equivalent series admittance of the transformer is $nY = 1.1 \times (-j10) = -j11$. The primary side

$$\begin{aligned} Y_1 &= n(n-1)Y \\ &= 1.1(0.1)(-j10) \\ &= -j1.1 \end{aligned}$$

and

$$Y_2 = (1-n)Y = (1-1.1)(-j10) = j1.0$$

The shunt elements equal to a reactor of 1.1 V_1^2 size on the primary side and a capacitance of size 1.0 V_2^2 on the secondary side.

22.4.8 Line Drop Compensation

For individual feeder regulation line drop compensation is often used to regulate a point farther down the feeder. Figure 22.11 shows a circuit with line drop compensation to regulate a point $R + jX$ primary ohms down the feeder. It is to be noted that the capacitor current is subtracted in the current transformer circuit. The capacitor would be switched by a different measurement, current or time or temperature or voltage on generation side of regulator.

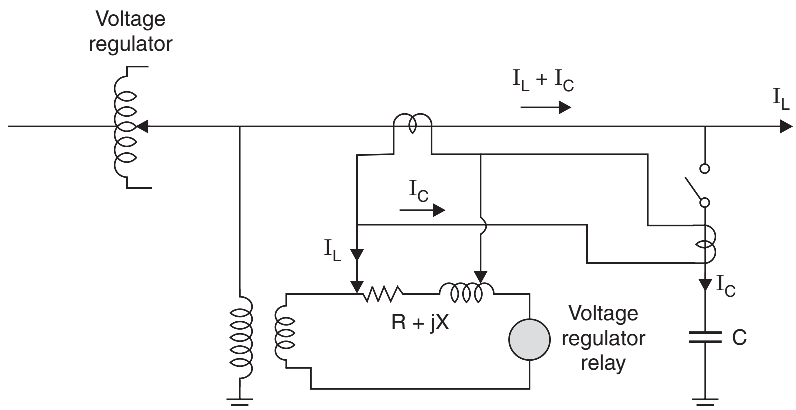


Fig. 22.11 Distribution voltage regulator with line drop compensation.

22.4.9 Load Dynamics, Equilibrium Points and Region of Attraction Mechanism

Voltage stability has been termed as load stability. The 'load' is the load seen at transmission system high voltage buses and includes the effects of sub transmission and distribution systems. Due to low voltage, there is temporary reduction in the load and the restoration of these loads during low voltage is a key aspect of voltage stability.

Active load is restored by three mechanisms:

1. Whenever, there is change in voltage the induction motors respond rapidly to match their mechanical load within a few seconds. Immediately after the sudden change in voltage the induction motor acts as an impedance load, as momentarily, the slip of the motor does not change because of motor inertia and from the equivalent circuit of the motor, if slip is constant, it acts as an impedance load. For slow voltage decay, fast responding motors track the slow dynamics of other equipment, thus acting as constant active power loads.

2. Automatic tap changing on bulk power delivery transformers and distribution voltage regulators operate over tens of seconds to several minutes to restore load side voltage and thus the voltage sensitive loads are restored. Reactive power load and reactive power output of shunt compensation are also restored.

3. Resistive loads with constant energy are restored by thermostatic or manual control. For the overall loads this results in a loss of load diversity over a period of time after a reduction in voltage takes place.

The dynamics of restoration of load by the above mentioned mechanisms, using first approximation, can be modeled as first order differential equation using a single time constant even though it is a crude approximation especially for the tap changing mechanism. Of course the time constant are different for different mechanisms. However, the load restoration mechanism can be unified for conceptual analysis. Figure 22.12 shows the three types of loads connected at one bulk power delivery load bus.

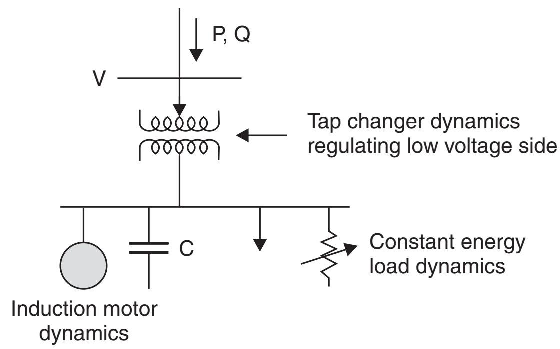


Fig. 22.12 Three mechanisms for restoration of voltage sensitive loads.

The state variable for the first order equation for the first mechanism is the motor slip *s*, for the second the turns ratio *n* of the tap changer and load conductance *G* for the third mechanism.

As state variable increases from zero, the load power increase, reaches a maximum value and then start decreasing as shown in Fig. 22.13.

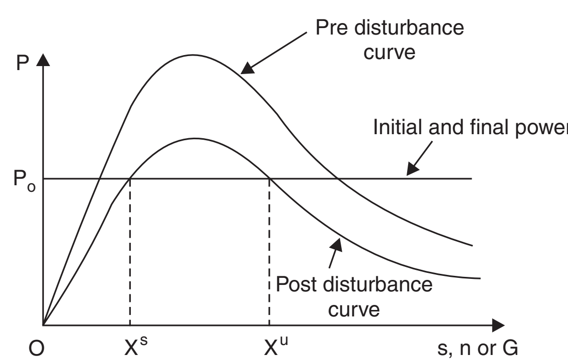


Fig. 22.13 Power vs state variables of load dynamics

As the state variable increases, the load increases and hence the voltage monotonically decreases.

Figure 22.13 is similar to torque slip characteristic of the induction motor. The applicable first order differential equation is

$$2H\omega \frac{ds}{dt} = P_o - P_e \tag{22.11}$$

where H is the inertia constant of the motor, ω the angular speed of the motor, P_o is the initial shaft output power assumed to be constant for simplicity Fig. 22.13 shows the predisturbance curve and post disturbance curve, the disturbance could be in the form of a fault and the curve represents the condition when the fault is cleared. For this curve and the shaft output, the 'region of attraction' to stable operating point lies between x^s and x^u as during this variation of slip, the electrical input to the motor is greater than the shaft output ($P_e > P_o$) and hence the motor will accelerate and hence slip s decreases and the motor will finally operate at point x^s.

The differential equation defining the operation of thermostatically controlled heating loads is given as

$$T \frac{dG}{dt} = P_o - V^2 G \tag{22.12}$$

The situation is similar to that of the induction motor. For stability following a large disturbance, the conductance at the moment of the final source system configuration (say, after circuit restoration and shunt capacitor insertion) must be within the region of attraction of the stable equilibrium point x^s. The region of attraction extends up to the point x^u as between x^u and x^s $V_L^2 G$ is greater than P_o and the thermostats will reduce conductance and hence increase voltage until point x^s is reached.

The situation is similar with the tap changers. The formulation could be changed so that the equation applies, where G is the conductance reflected to the high voltage side by tap changing.

22.5 GENERATION CHARACTERISTICS

The synchronous alternator with its controls is one of the most complex components of power system. While describing longer term voltage stability we have found two major factors which affect this stability *viz.* load restorations by tap changing and generator current limiting. The load restorations by tap changing has been discussed in the previous article. We now concentrate on the generator current limiting process. The voltage instability in power system is due to some disturbance in the form of a sudden large increase of load or a sudden failure of a large alternator which results into a generation load imbalance and causes redistribution of power flow and reactive losses. We now study how power plants respond to these imbalances.

Capability curves of alternators: A synchronous machine when over excited delivers lagging reactive power to the system whereas when under excited it draws lagging reactive power from the system *i.e.,* it operates at leading p.f. The upper excitation limit is governed by the thermal heating of the field winding whereas the lower excitation limit is governed by the stability of the synchronous machine. For certain output power P of the alternator there is going to be certain minimum excitation limit and hence the torque angle δ between the two magnetic fields *viz.,* the rotor and stator magnetic fields, below which the machines pull out of step. Therefore, it is to be noted that for a given terminal voltage V and the power P to be transferred, there is some value of Q below which the machine will go into unstable operation. For further details on this aspect, the reader is suggested to go through Article 17.12 of the book.

Similarly, the maximum active power generation of an alternator is limited by thermal consideration of the stator winding whereas minimum power generation is limited by the flame instability of a boiler (Article 19.1).

The capability curve of a synchronous machine shows the limits placed on the P and Q by the permissible temperature rise of the winding and by the mechanical system connected to the shaft (prime move, boiler) assuming operation at rated terminal voltage. Figure 22.14 shows capability curve for an alternator. In case of an alternator, the power limit is determined by the prime mover rating and is fairly fixed or definite.

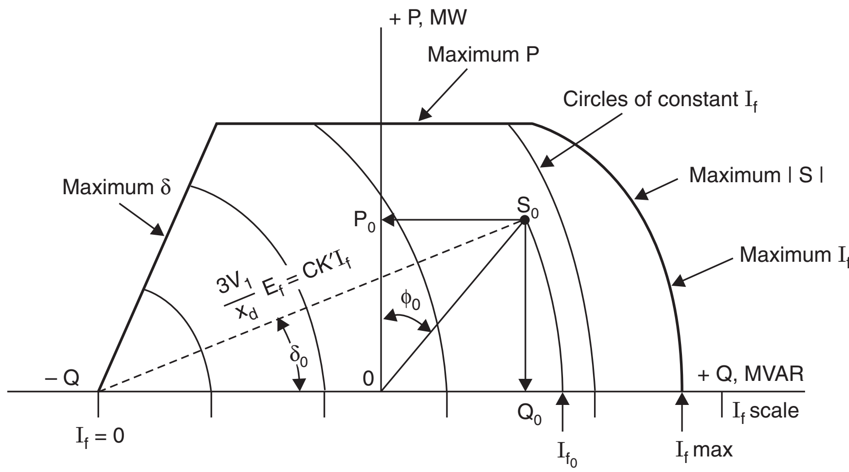


Fig. 22.14 Capability curve for an alternator.

Capability curve is plotted by taking P as the ordinate and Q the abscissa. Operation within the boundaries of the curve is safe from the view point of heating and stability. Once an operation point S_0 is located corresponding to a desired active power P_0 and Q_0, the following information is available. Here $S_0 = P_0 + jQ_0$.

1. If S_0 is inside the capability curve, the machine will not get overheated and will remain stable.
2. A line drawn from S_0 to the origin of field current I_f axis, the angle between Q-axis and the line is the torque angle δ_0.
3. The length of the line of 2, above, is a measure of the field current required to operate at the designated value of $P = P_0$ and $Q = Q_0$ with rated terminal voltage.
4. A line from S_0 to the origin of the Q-axis will be at the p.f. angle ϕ_0 from the ordinate.

The alternator capability is greatly enhanced with better cooling and hence the area of the capability curve increases and the operation zone of the alternator for safe limits increases.

It is found that for p.f. less than 0.9 lag, the limiting factor is field winding heating as the reactive power demand increases which can be met by overexciting the field winding. However, between p.f. 0.9 lag and 0.95 lead the limiting factor is the armature current and hence heating of the armature winding. For lower leading p.f. (lower than 0.95 lead) since the field winding is under excited, the limiting factor is armature core end heating or system stability.

In the portion of the capability curve where armature current is limiting the power capability obviously varies with terminal voltage and field winding curve also varies with terminal voltage.

Example 22.8: A generator is rated for 0.95 lag p.f. The turbine rating is specified to match the real power at rated p.f. Assuming the rating of turbine fixed, determine the generator rating if 0.8 lag p.f. is specified. Also, calculate the additional reactive power capability at full load.

Solution: Assuming the active power fixed and equal to P, the rating of the generator for p.f. 0.95 will be $S_1 = \dfrac{P}{0.95} = 1.0525\,P$ and that for p.f. 0.8 it will be $\dfrac{P}{0.8} = 1.25$ P MVA.

The increase in MVA rating for same MW is

$$\frac{1.25 - 1.0525}{1} = 18.75\%$$

Additional reactive power capability is calculated as follows :

Since $Q = P \tan \varphi = 0.33\,P$ for p.f. 0.95 and for p.f. 0.8, $Q = 0.75\,P$. The per cent of additional reactive power capability is $\dfrac{0.75 - 0.33}{0.33} = \dfrac{0.42}{0.33} = 1.27$ or 127%.

This additional reactive power capability could be utilised during voltage emergency. Therefore, an effective method to improve voltage stability is to use shunt capacitor banks, so nearby generators can operate near unity p.f. with substantial spinning reactive power reserve that can be rapidly activated to prevent voltage instability.

Generator *Q-V* Curves: These diagrams are constructed from a series of capability curves for different network (generator high voltage side) voltages V. The diagram is for the generator plus its step-up transformer for a particular value of active power P. From a system viewpoint, the network voltage is most important.

Figure 22.15 shows a Q-V diagram where curves for constant terminal voltage V_t are shown. If the terminal voltage V_t and network voltage V, are given the reactive power of the generator can be found from the diagram.

When the alternator is regulating voltage; the curves for constant V_t are sufficiently flat which shows that a small change in network voltage results in a large change in reactive power output. Alternators maintain the network voltage almost constant whereby voltage collapse is prevented. The characteristic is similar to an *SVC* in its active control range. If the network voltage becomes sufficiently low, either the field current limit or the armature current limit of the alternator is reached. This drastically changes the characteristic of the alternator. The slope of the rotor current limit curve is nearly vertical (nearly constant reactive power output), which means that voltage support from the alternator is lost when this limit is reached.

If armature current limit is reached, the reactive power from the alternator decreases fast if the network voltage is further reduced and under this condition voltage instability is likely. Also note that limits on acceptable terminal voltage may be violated which may adversely affect the operation of the station auxiliaries.

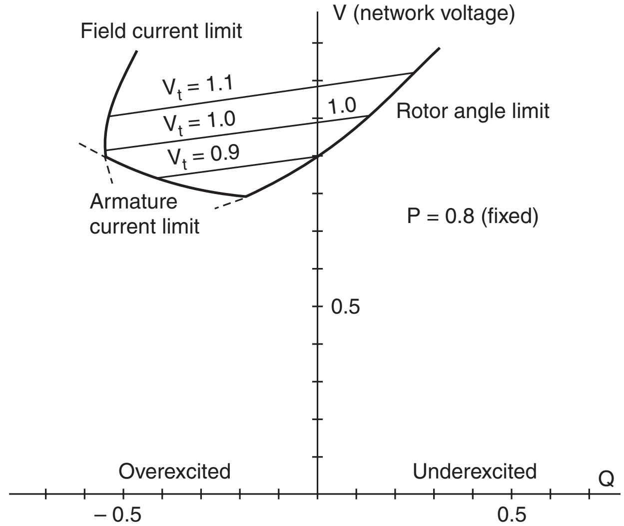


Fig. 22.15 Generator *Q-V* diagram for constant value of active power.

Operating practices: Voltage stability can sometimes be improved by reducing real power on certain alternators so that more reactive power can be made available. In some cases the gas turbine alternators are exclusively used to supply only reactive power under system sagging voltage condition under emergency condition. Gas turbines require about 5 minutes to bring to the busbar and thus voltage stability chances can be improved. If during a severe disturbance a generator is feeding a highly overloaded line, stability can be improved by reducing active power loading and rescheduling the generation at power plants serving lightly loaded lines into the load area.

Normally generators are operated at high p.f. Power plant operators should be trained in the basics of voltage stability and instructed not to reduce alternator reactive power output during rare system emergency when high levels of reactive power output are required.

22.5.1 Generator Control and Protection

Excitations control is by automatic voltage regulators (AVRs). Alternator terminal voltage is measured and compared to a desired or reference voltage. The error signal controls the exciter output which is the main alternator field voltage. Articles 17.9 and 17.10 in this book describe the role of AVRs in improving stability of system and various excitation systems respectively, the reader is suggested to refer to these articles for finer details.

Network voltage control: As mentioned earlier the voltage stability of the system can be greatly improved if alternators regulate the network voltage on the higher voltage side of the step up transformer connected to the alternator. Network voltage should be held as high as possible. There are various methods to regulate the network voltage, the most common being line drop compensation (Article 22.5.8 of the book). With line drop compensation the alternator current is measured to compute the *IZ* drop part way through the alternator step-up transformer this becomes the regulated point. For compensation usually reactive component of current is used. About 50 to 80% of the leakage reactance of the transformer is compensated in the case of the multiple alternator power plants.

When a large number of alternators are connected to the same common bus (low voltage) *e.g.,* in hydroelectric alternators, line drop compensation becomes difficult. When two or more alternators regulate the same point, reactive current compensating circuits or other methods are required to equalise reactive power sharing and control effort. It is possible to regulate a point beyond the network bus (high voltage bus).

The other method consists of a secondary outer control loop to adjust the voltage regulator set point in order to maintain the desired network voltage (*HT* side). To minimise adverse control interactions, the outer loop should be an order of magnitude slower than the primary terminal voltage regulation. The high network voltage control, therefore, should respond in around ten seconds following a disturbance. This is found to be sufficiently fast for the slower forms of voltage instability *i.e.,* it is faster than restoration of load by tap changing.

Yet another method for better network voltage control is on load tap changing of generator transformers. These transformers or LTC auxiliary equipment transformers do not have constraints on fixed tap transformers, tap settings of main and auxiliary transformer should be optimised from the view point of voltage stability performance. This optimal performance means after a sever disturbance takes place, the field current limits of all alternators should reach at the same time. These limits on field currents can generally be determined off-line. The construction and operation of LTC transformers is given in Chapter 10 of the book.

Excitation control for transient voltage stability: As discussed in Chapter 17 on power angle stability that to improve transient power angle stability fast excitation system is important and since the period for transient angle stability and transient voltage stability overlap, the fast excitation system will improve transient voltage stability especially as the problem is associated with induction motor loads or HVDC links. The speed of generator and excitation system dynamics are similar to the speed of the load dynamics. Static exciters and other high initial response exciter will improve stability. High exciter ceiling (overshooting the mark, a term used in connection with high exciter ceiling) voltages allowing momentary high field overload are important. For further details on the exciters and monitoring and controlling minimum excitation limiters (to avoid unstable operation) and maximum excitation limiters (to avoid overheating of the rotor winding) refer to Chapter 17 of the book.

There are a wide variety of over excitation control and protection devices (usually an IDMT relay is used) in service. Normally the power plant should be visited to determine the exact equipment in service and the settings. It is desirable to have an excitation system which has a continuous field current limiting control loop.

Generator armature protection and system back up protection has been discussed in detail in Section 14.11 of the book and will not be repeated here.

22.6 *HVDC* OPERATION

Mainly there are three modes of operating HVDC links (*i*) Constant voltage (*ii*) Constant current and (*iii*) Constant power. Closed loop controls regulate rectifier and inverter firing angle to indirectly control parameters such as direct current, extinction, d.c. voltage and d.c. power converter transformer tap changing, operates in a slower time frame to optimise operation.

For normal operation of long distance two-terminal HVDC links, the following strategies prevail:

1. The rectifier controls d.c. current through firing angle control.
2. The inverter sets the d.c. voltage through constant (minimum) extinction angle control and ultimately through firing angle control.
3. The rectifier tap changer operates to allow desired rectifier firing angle.
4. The inverter tap changer operates to control rectifier voltage (sending end) to maximum.
5. DC power is controlled at the rectifier as a current regulator outer loop. The required current is the scheduled power divided by the d.c. voltage. The power control may be slow compared to the basic current control.

In case of HVDC links it is desirable to have high p.f. of the system (rectifier or inverter) for the following seasons:

1. For a given current and voltage of the thyristor and transformers, the power rating of the converters is high.
2. The stresses on the thyristors and damping circuits is reduced.
3. For the same power to be transmitted the current rating of the system is reduced and also the copper losses in the a.c. lines are reduced.
4. In a.c. lines the voltage drop is reduced.

The p.f. on the a.c. side can be improved by using shunt capacitor. However, this involves cost both for the capacitors and the switching devices but this may be worth it from voltage stability point of view.

On d.c. side, the p.f. of the converter is given as

$$\cos \phi = \frac{\cos \alpha + \cos (\alpha + \gamma)}{2} \tag{22.13}$$

for a rectifier and for an inverter it is given as

$$\cos \phi = \frac{\cos \delta + \cos (\delta + \gamma)}{2} \tag{22.14}$$

where α is the firing angle or control angle, γ is the angle of overlap and δ is the extinction angle of the inverter. This equation is taken from Chapter 5 as equation (5.23).

For further details about HVDC links, refer to Chapter 5 of the book. As a typical case we study the inverter control.

Inverter control: The basic inverter control is constant extinction or commutating margin angle control at a minimum value γ_{min}. The extinction angle is so selected as to ensure sufficient time for commutation during most conditions. Equation (22.14) suggests that when extinction angle δ is increased, the p.f. $\cos \phi$ decreases and hence this results in higher reactive power equipment cost and equipment losses as current increases for the same power. This therefore, requires additional reactive power supply equipments. However, for weak system minimum extinction angle design and operation results in poor performance. The inverter characteristics are usually modified to improve performance.

Voltage collapse: We now discuss the sequence of events which take place when a large disturbance occurs near the inverter when there is heavy load on the system and the inverter is in the constant extinction angle control mode of operation:

1. Due to large disturbance a.c. interconnection will be heavily loaded and hence the power system is highly stressed.

2. As a result of disturbance the voltage sags and the voltage recovers slowly. As a result voltage swings result from electromechanical oscillation between synchronous generators.

3. Due to sag in voltage reactive power output of the capacitor banks and filters reduces.

4. In certain cases, there are controls that increase extinction angle for a.c. voltage drop. This is to maintain constant volt-second commutating margin.

5. The inverter p.f. will reduce because of the low a.c. voltage drop and the possible increase in γ. This is apparent from equation (22.14).

6. As p.f. decreases sin ϕ increases which results in high reactive current requirements. If power control is fast compared to the voltage swing, direct current will be increased as $I = \dfrac{P}{V}$ and V is changing slowly, this makes the p.f. still worse and reactive power requirements increase further. Attempting a fast dc power increase to improve transient rotor angle stability, will further increase reactive power requirement and may be counter productive from transient voltage stability point of view.

7. The combined effect of the reduced capacitor bank and filter reactive power output and the increased reactive power requirement of the inverter would further reduce the system voltage and finally might lead to voltage collapse.

8. Converter transformer tap changing may occur within few seconds. This may add more burden on a.c. system.

9. Reduction in d.c. current and power by some suitable controls may stabilise the system voltage. Reduction in d.c. power may, however, aggravate transient rotor angle stability leading to more severe voltage swings.

It is to be noted that even if the inverter were initially in constant d.c. voltage control, the minimum extinction angle mode would result because of declining a.c. voltage and increasing direct current.

Comparison between a.c. and d.c. transmission: In order to study the dynamic performance of power system, usually transient stability programs are used after a simulation model of an actual system is prepared. For this detailed models of HVDC links and motor loads are required. Normally both the rotor angle and voltage stability must be considered and appropriate trade-off made between the two types of stability as the interaction between the two forms of stability may be complex.

In planning a new long distance interconnection, one has to choose either HVDC link or an a.c. line. Dynamic performance in terms of the two forms of stability should be taken as the basic criterion. The ability to use fast acting stability enhancing controls to HVDC links is often taken as an important advantage of HVDC.

However, in case of a.c. transmission, large networks operate in synchronism. Whenever, there is a disturbance the inherent properties of a.c. transmission support the stability of interconnected synchronous alternators as, if there is outage of say one line, parallel lines will effectively instantaneously pick up the power, being transmitted by the lost line. Also, due to inertia of the synchronous machines, inertial power swings will follow to provide additional synchronising and damping powers to generators. Although reactive power losses will increase, and line charging power will decrease, the effects are relatively minor in regard to transient stability point of view especially if load characteristics are not detrimental.

D.C. transmission in the usual constant power control mode does not provide additional power without special controls. If there is a disturbance, the voltage goes down and in order to have constant power mode of operate the d.c. current increases which as mentioned earlier significantly increases the reactive power demand of the converter. We have seen that a combination of power control, constant extinction angle control at inverters and shunt capacitors may threaten voltage stability. However, we have seen in the previous article that switching from constant power to constant current control even though help the voltage stability but may cause rotor angle instability. Power modulation for synchronising or damping support will be lost in current control.

If new a.c. transmission lines are planned, they should be designed for high surge impedance loadings for enhancing voltage stability of the system. Bundled conductors are used for this purpose. The reduced reactance by increasing the no. of subconductors per phase is equivalent to uniformly distributed series compensation without bothering for the SSR (Subsynchronous resonance) or other concerns.

It is difficult to build new lines because of environmental and economic reasons. If there is an opportunity to build new lines, these should be designed for high thermal capacity, low loss and high surge impedance loadings. Double circuit lines should be encouraged.

22.7 VOLTAGE STABILITY ANALYSIS: *P-V* CURVES

The slower forms of voltage stability are analysed using load flow method. Simulations are made for an outage or for a high load built up and load flows obtained. Besides these post-disturbance power flows, two other power flow based methods are widely used, *P-V* curves and *V-Q* curves. These two methods determine steady state loadability limits which are related to voltage stability. Power flow program discussed in Chapter 18 of the book can be used for approximate analysis.

P-V curves are useful for conceptual analysis of voltage stability and for study of radial systems. The method is also used for large interconnected network, where P is the total load in an area and V is the voltage of a critical bus. P could be the power transferred over a transmission line. Voltage at several busses can be plotted. A disadvantage of the load flow solution for *P-V* curve is that it is likely to diverge near the maximum power transfer point or the nose point of the *P-V* curve. A load flow solution is for particular generations at various *P-V* buses or generator buses. However, when the load changes, the scheduling of generation at various generator buses also changes. This is yet another disadvantage of the load flow method.

The application of maximum power transfer has been made for a radial line with asynchronous load in Article 21.5 of the book. The load is a general $P_L + jQ_L$ at the receiving end keeping the sending end voltage V_s constant as shown in Fig. 22.16.

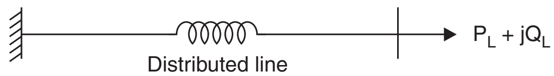


Fig. 22.16 A line terminated through lead $P_L + jQ_L$.

The relation between P and V for a long transmission line is given as

$$V_s = V_r \cos \beta l + iZ_c \frac{P_L - jQ_L}{V_r} \sin \beta l \tag{22.15}$$

This equation has been derived as equation (21.31) in this book.

For a fixed V_s and line length equation (22.15) is quadratic and thus will have two roots. Figure 21.2 shows relation between $\frac{V_r}{V_s}$ as a function of normalised loading $\frac{P_L}{P_C}$, where P_C is the surge impedance load. These curves are not reproduced here. The reader can refer to these curves in Fig. 21.2. The interpretation of the curves also can be referred from the text in Article 21.5. Further to that these *P-V* curves are for different power factors. At more leading p.f. the maximum power is higher. The leading p.f. is obtained by shunt compensation. Also with leading p.f. the critical voltage (critical voltage is the receiving end voltage for maximum power transfer or it is the nose voltage of the *P-V* curve) is higher which is a very important aspect of voltage stability.

Consider Fig. 22.17 where a purely inductive load is connected to a source through a loss less line.

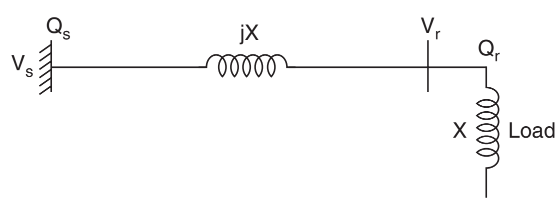


Fig. 22.17 Purely inductive load connected to the network.

Determine the voltage collapse proximity indicator $\left(VCPI = \frac{dQ_s}{dQ_r}\right)$.

Since the load is pure inductive $P = 0$ and $\delta = 0$ and using equation (22.4).

We have
$$Q_r = \frac{V_s V_r}{X} - \frac{V_r^2}{X}.$$

We first find here the maximum reactive power transferred. For this we let

$$\frac{dQ_r}{dV_r} = 0 = \frac{1}{X}(V_s - 2V_r) = 0$$

or $\quad V_{\text{cri}} = \dfrac{V_s}{2}$, where V_{cri} is the receiving end voltage for maximum reactive power transfer.

Hence
$$Q_{\max} = \frac{2V_r.V_r}{X} - \frac{V_r^2}{X} = \frac{V_r^2}{X_{\text{load}}} \tag{22.16}$$

as half of the drop will be across the line and half across the load hence $X_{\text{load}} = X$.

This, therefore, proves the maximum power transfer condition that for maximum reactive power to be transferred the load reactance should equal line reactance or source reactance.

Now the short-circuit reactive power of the line is $Q_{sc} = \dfrac{V_s^2}{X}$, hence normalised maximum reactive power is

$$q_{\max} = \frac{Q_{r\max}}{Q_{sc}} = \frac{V_r^2}{X} \cdot \frac{X}{V_s{}^2} = \frac{V_r^2}{4V_r^2} = 0.25 \tag{22.17}$$

Also
$$V_{\text{cri}} = \frac{V_{r_{cri}}}{V_s} = \frac{V_s}{2V_s} = 0.5 \tag{22.18}$$

We now calculate the voltage collapse proximity indicator (VCPI) as follows:

$$Q_s = Q_r + I^2X$$

Since $\quad P = 0$
$$I^2 = \frac{Q_s^2}{V_s^2}$$

Hence
$$Q_s = Q_r + X.\ \frac{Q_s^2}{V_s^2}$$

Multiplying the equation by $\dfrac{V_s^2}{X}$ we have

$$\frac{V_s^2}{X}Q_s = \frac{V_s^2}{X}Q_r + Q_s{}^2$$

or
$$Q_s{}^2 - \frac{V_s^2}{X}Q_s + \frac{V_s^2}{X}Q_r = 0 \tag{22.19}$$

Hence
$$Q_s = \frac{1}{2}\left[\frac{V_s^2}{X} \pm V_s\sqrt{\frac{V_s^2}{X} - \frac{4Q_r}{X}}\right] \tag{22.20}$$

$$\frac{dQ_s}{dQ_r} = \frac{V_s}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\dfrac{V_s^2}{X^2} - \dfrac{4Q}{X}}} \cdot \frac{4}{X} = \frac{1}{\sqrt{\dfrac{V_s^2}{X^2} \cdot \dfrac{X^2}{V_s^2} - 4 \cdot \dfrac{Q}{X}\,\dfrac{X^2}{V_s^2}}}$$

$$= \frac{1}{\sqrt{1 - 4.Q.\dfrac{X}{4V_r^2}}} \quad \text{as } V_s = 2V_r \text{ at max. power condition}$$

$$= \frac{1}{\sqrt{1 - \dfrac{Q_r}{Q_{\max}}}} \tag{22.21}$$

The receiving end voltage varies from V_s at no load to $\dfrac{V_s}{2}$ at maximum load ($Q_{\max}$). However VCPI is unity at no load as at no load $Q_r = 0$, hence $\left.\dfrac{dQ_s}{dQ_r}\right|_{Q_r = 0} = 1$ and it is infinity at maximum load as at this load $Q_r = Q_{r\max}$ and hence $\dfrac{dQ_s}{dQ_r} = \dfrac{1}{0} = \infty$.

This shows that near maximum load, extremely large amounts of reactive power are required at the sending end to support an incremental increase in load. The VCPI is thus a very sensitive indicator of impending voltage collapse. Besides VCPI, the reactive reserve activation and reactive losses are also sensitive indicators.

22.7.1 Voltage Stability Analysis –*V-Q* Curves

It is possible to obtain *V-Q* curves from the normalised *P-V* curves. Let us define $p = \dfrac{PX}{V_s^2}$ and $q = \dfrac{Q_r X}{V_s^2}$ and $v = \dfrac{V_r}{V_s}$. For a particular value of p, there are two pairs of values of v and q for every power factor and we plot these values. Fig. 22.18 is the result of these plots. From these curves we find that the critical voltage is very high $\left(\dfrac{V_r}{V_s} > 1\right)$ for high loadings *i.e.*, $v > 1$ for $p = 1$ p.u. The right side of the curves represents normal operation when shunt capacitors have been used to raise the voltage. The steep-sloped linear portions of the right side of the curves are equivalent to Fig. 22.4 when this figure is rotated through 90° clockwise.

For large systems, the curves are obtained by a series of load flow solutions. *V-Q* curves plot voltage at a test or critical bus versus reactive power on the same bus. Since in a *V-Q* curve the power is to be kept fixed and it is required to obtain reactive power for a set of scheduled voltage at the bus. This means we fix *P* and voltage magnitude at a representative bus and perform a load flow. In load flow terminology on a bus where *P* and *V* are fixed is known as *PV* bus or generator bus. Normally the generator bus has reactive power constraints for load flow solution. Here we connect a fictitious synchronous condenser at the bus and allow the bus to have any reactive power for a fixed *P* and *V*. We change *V* at the bus for obtaining another point on the *V-Q* curve and obtain the reactive power for different scheduled voltage at the bus. So in this process scheduled voltage at the bus is the independent variable whereas reactive power required to maintain the scheduled voltage at the bus is the dependent variable *i.e.,* voltage forms the abscissa and *Q* the ordinate for the *V-Q* curves. Capacitive reactive power is to be taken positive whereas inductive reactive power as negative. If the synchronous capacitor is not connected to the test bus, the operating point is at the zero reactive power. It is to be noted that the computation time can be reduced by choosing a suitable starting solution of the load flow problem. Once a solution for a particular scheduled voltage is obtained, this can form the starting solution for the next scheduled voltage and we find that the convergence is faster rather than starting with some conventional values.

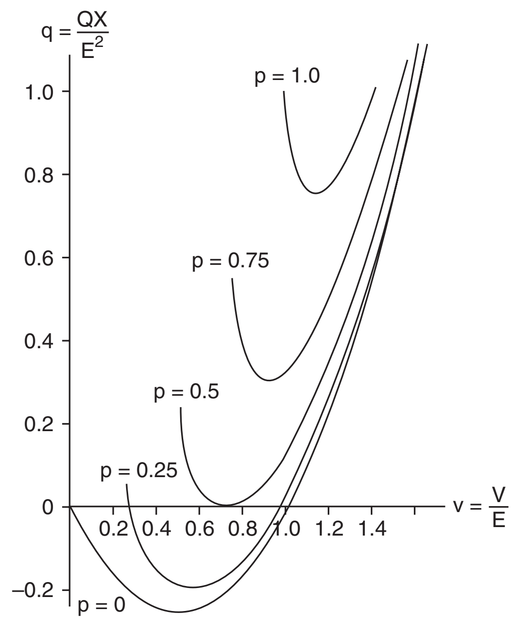


Fig. 22.18 Normalized *v-q* curves for fixed (infinite) source and reactance network Loads are constant power.

V-Q curves have the following advantages:

1. Reactive power is closely related to voltage security and it is possible to obtain reactive power margin from the *V-Q* curve of the test bus. The reactive power margin is the MVAr distance from the operating point to either the bottom of the curve, or to a point where the voltage squared ($V^2\omega C = Q$) characteristic of an applied capacitor is tangent to the *V-Q* curve as shown in Fig. 22.19. The test bus could be representative of all buses in a "Voltage control area" *i.e.,* an area where voltage magnitude changes are coherent.

2. The slope of the *V-Q* curve indicates the stiffness of the test bus *i.e.,* what is the change in *QV* for a differential change in *Q* (ΔQ) at the bus.

3. The $Q = V^2\omega C$ characteristics which could be due to a shunt capacitor or a synchronous condenser or an SVC can be drawn on the *V-Q* curve and the intersection of this curve with *V-Q* curve gives the operating point and the distance between the operating point and the

tangent point of another $V^2\omega C$ characteristics with the *V-Q* characteristic gives the reactive power margin as shown in Fig. 22.19 (*b*). This margin of reactive power is very important to voltage stability problems.

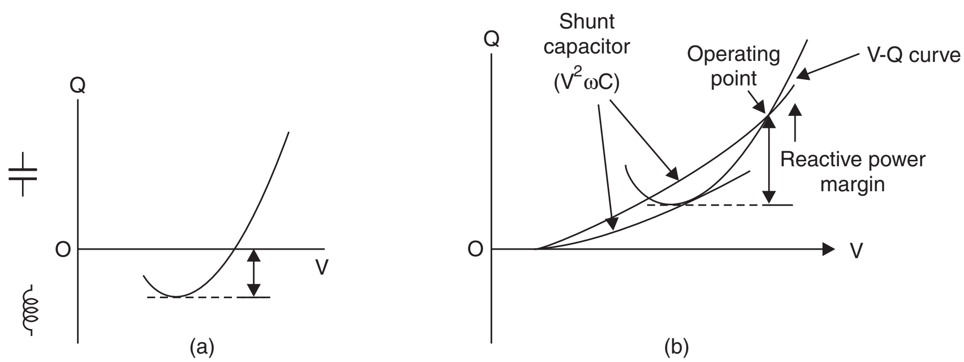


Fig. 22.19 Reactive power margins.

4. *V-Q* curves can be computed at points along a *P-V* curve to test system robustness.

5. For more insight, the reactive power of generators can be plotted on the same graph where nearby generators (close to the test bus) reach their reactive power limits, the slope of the *V-Q* curve of the test-bus becomes less steep and the bottom of the curve is approached.

From computation point of view the artificial *P-V* bus minimises power flow divergence problems. Solutions can be obtained on the left side of the curve-divergence occurs only when voltage at buses away from the *P-V* bus sag.

The effect of tap changing reaching limits or of voltage sensitive loads can be shown on *V-Q* curves. *V-Q* curves with voltage sensitive loads prior to the tap changing will have much greater reactive power margins and much lower critical voltages. When the tap changers reach limits, the curves tend to flatten out rather than turn up on the left side as shown in Fig. 22.20.

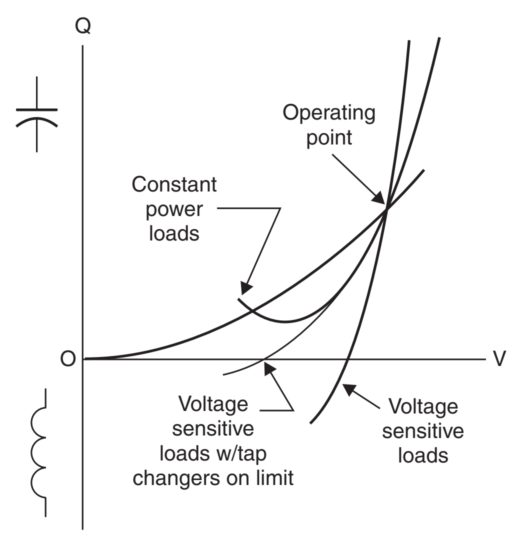


Fig. 22.20 *V-Q* curve sketches showing effect of voltage sensitive loads and tap changers on limit.

V-Q curves are presently a very useful method of analysing voltage stability problem in power system. Since the method artificially stresses a single representative bus, final decision should be taken by more realistic methods.

22.7.2 Performance Criteria and Margin to Instabality

The combination of voltage magnitude and reactive power reserves at effective locations provide good planning or operating criteria. A required margin or security measure from an operating point to stability is used. Usually the margin is to the maximum power transfer point rather than the instability point, with the tacit assumption that operation at higher load is unacceptable. The margin can be quantified in terms of MW or MVAr distance from the operating point to the critical or maximum power transfer point as shown in Fig. 22.21.

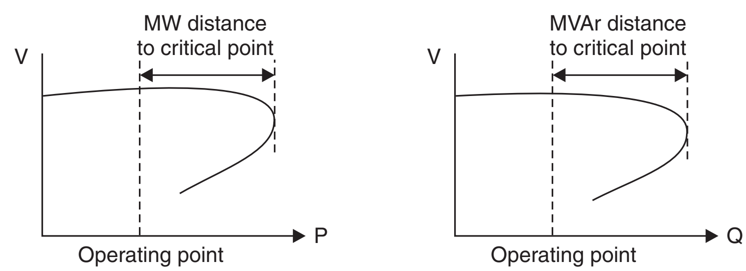


Fig. 22.21 MW and MVAr distance to voltage instability. (maximum power or critical point).

MW margin should be such that it allows for loads exceeding forecast or it should relate to forced or prior outages of generation in a load area which means that stability may be maintained for an additional contingency. If a 500 MW margin is used, stability should be maintained for loss of generators smaller than 500 MW.

MVAr margins are important as voltage and reactive power are closely related. The MVAr margin could relate to the size of shunt capacitor banks or static var compensators in the area. If a 100 MVAr shunt capacitor bank is nearby, a margin greater than 100 MVAr would ensure that stability would be maintained for the additional contingency of loss of nearby reactive power sources. MVAr margins also allow for loads exceeding forecast. During heavy loads each additional megawatt of load may require two or three MVAr of reactive power support as suggested in Fig. 22.2. The post disturbance margin must be satisfied by the system design.

22.8 METHODS OF IMPROVING VOLTAGE STABILITY

While planning and operating power system the main objective of studying voltage stability is to increase the power transfer capability of the system by eliminating the voltage stability limits.

There are many aspects of voltage stability and also has many solutions associated to the voltage stability in terms of generation, transmission and distribution. The objective is to find low cost solution whenever possible which requires special controls and effective power system operation methods.

The main objective of power system engineer is to provide good quality of reliable supply. Attempting 100% reliability is not desirable as this would involve enormous cost and hence a balance has to be struck.

22.8.1 Generation System

In order to improve power system voltage stability at the generation level we need to consider, planning control and protection and operation and maintenance.

Planning: The reliability aspect of supply can be improved by siting generating plants in the load areas. The policy makers should be advised the advantages of siting power plants near the load centres as otherwise you need to have transmission systems which reduces the reliability levels of the supply. However, environmental factors must also be kept in mind. For this gas turbines in load areas should be encouraged for fast start-up. These should normally be used for real power generation. However, under emergency conditions for voltage stability reasons, these could be used as synchronous condensers. As mentioned earlier specifying lower power factor generation, increase the fast acting reactive power reserves of the generators. If generators normally operate near unity p.f., the reduced generator losses will reduced the life cycle cost increase of the larger (Higher MVA rating) lower p.f. machines.

Load tap changing on step up generator transformer and auxiliary transformers have advantages for voltage stability as these transformers (LTC) allow the transmission side voltages to be maintained at the highest possible value without regard to terminal voltage of the alternator.

Excitation system control and protection: In order to improve transient stability, high initial response and high ceiling excitation systems help induction motors reaccelerate after the fault.

The generator high side voltage should be kept as high as possible by using line drop compensation or by an outer control loop. The high voltage operation of transmission lines minimises the increase in reactive power losses whenever a disturbance takes place. This is one of the most effective methods of improving voltage stability.

Over excitation limiter that trip the excitation system to the predisturbance excitation level especially when operating near unity p.f. may cause disasters. The excitation system should be so designed that it incorporates the features of continuously regulating field current. The over-excitation limiters should not trip the generator immediately if the field current is more than the rated current.

If fast changes in generation could counteract the fast changes in load, voltage stability can be ensured. In attempting fast changes in generation we effectively counteract the fast load restoration by tap changing and generator current limiting. The protection of alternator for voltage stability means the maloperation of alternator under minor abnormal conditions should be avoided. For this refer to Chapter 14 of the book.

Maintenance: Synchronous alternator with its controls is a very complex machine. Proper maintenance of both the main equipment and control and protection devices is very essential. Verification and maintenance of reactive power capability is especially important which includes control settings such as over excitation and under excitation limiters, alarm settings, tap changer settings protection of excitation system (especially earth fault or interturn fault) and limitations imposed by auxiliaries. If proper maintenance is carried out, it has been observed with some systems that an increase of almost 50% in reactive power capability from the generators is achieved.

Operation: During peak load period, power import over the transmission network should be reduced, instead demand should be met by using less economical sources like gas turbines within the load area.

If there is loss of generation within the load area spinning reserves should be available within the load area. The generation control should rapidly activate the spinning reserve.

It may, sometimes be advantageous to reduce real power loading of alternators in load areas to allow higher reactive power loadings and power should be rescheduled over lightly loaded lines.

Reactive power loading of alternators should be closely monitored. The control operator should know the reactive power capability of generators. Shunt capacitor banks should be used to maintain fast acting reactive power reserves at generators.

22.8.2 Transmission System

It has already been mentioned that the transmission lines should be designed for high thermal capacity, low loss and high surge impedance loadings Double Circuit EHV lines should be encouraged.

Reactive power compensation: Extra high voltage transmission lines require shunt reactors for energisation and under lightly loaded conditions. These shunt reactors should be switched off by the operator during voltage emergencies or by the under voltage relays. There are instances mentioned in literature when there have been voltage collapses because of not disconnecting the shunt reactors during voltage emergencies as these reactors further pull down the voltage resulting in voltage collapse. Mechanically switched shunt capacitors and SVC improve voltage stability.

Optimal power flow programmes using mathematical programming techniques (linear, integer, quadratic, non-linear) have been used for minimising reactive power additions subject to system constraints and equipments operating constraints. Of course, these programs do not directly address the voltage instability or collapse problem.

Controls: Automatic on-load tap changing on large EHV auto transformers can improve voltage stability. By regulating the voltage the reactive power output of shunt capacitors and the line charging increases which results in decrease in reactive power losses. Tap changing at bulk power delivery substation and at distribution voltage regulators will not occur because of the faster regulation of the high voltage system. Voltage sensitive load will, however, be restored faster and under voltage load shedding will not be as effective.

The tap changing will sag the EHV voltage which can be compensated for by capacitor bank insertion, by tripping the shunt reactors and control of EHV side voltage at generators. It is possible even to prevent starting of longer term voltage instability if remote signals are used for fast switching of shunt capacitors or shunt reactors following a disturbance.

Another control is automatic line reclosing whenever a short circuit is to be cleared. For transient stability the reclosing is very fast. However, for slower form of voltage instability this need not be as fast *e.g.,* ten seconds delay allows time for electromechanical oscillation and generator torsional oscillations to die down. The longer delay provides better opportunity for successful reclosing as more time is available for arc deionisation. Automatic reclosing should be faster than capacitor switching, tap changing or load shedding.

Protective relaying: A protective relay for overhead lines is expected to operate whenever there is a short circuit on the line. However, these relays have mal-operated even under overload conditions, thus causing black outs. The main culprit in this regard has been zone 3 impedance relay. With protective relaying provided on the system such as breaker failure relaying and bus protection local back up, there is no need to use zone 3 relays. These should be done away with. On sub transmission lines over current relays instead of impedance relays, should be used.

HVDC transmission: HVDC power control should be used to improve voltage stability. Fast reduction of d.c. power is often required to release reactive power into the power system. Controls to reduce direct current and power for low a.c. voltage can improve voltage stability.

22.8.3 Distribution and Load Systems

Voltage stability is basically load stability and effective solutions to voltage stability can be found at the problem source.

Upgrading sub transmission and distribution circuits for energy conservation will help voltage stability by reducing feeder impedance. Use of higher voltage distribution circuit will improve voltage stability.

Capacitor banks: Shunt capacitor banks should usually be located on the regulated side of LTC voltage regulators. The shunt capacitor banks thus act as constant reactive power sources.

Control of voltage using switched shunt capacitors or series capacitors rather than LTC transformers and distribution voltage regulators, will improve voltage stability.

Tap changing: A simple but effective method to improve voltage stability is to prevent LTC transfer tap changing for low unregulated side (transmission side) voltage. This is most effective at substations serving high p.f. loads or high shunt compensated loads. If the load is at some distance from the LTC transformer, tap changer blocking may not be desirable.

Blocking tap changers may be less cost-effective if a large number of distribution voltage regulators are used instead of a large LTC bulk power delivery transformer. Another possibility is that during heavy loading condition tap changing of only one or two boost steps above the value normally reached can be allowed. Another alternative is to use long intentional time delays between individual tap steps which allows extra time for corrective action. Larger tap changer setting can also be considered.

If two or more tap changer transformers or regulators are in series, the operation of the tap changers should be so coordinated that the tap changer closer to the load has a much longer time delay.

Voltage reduction: If the system is highly stressed, it is desirable to operate it at some reduced voltage as a result the load on the system reduces. From voltage stability view point, the reduction in reactive power requirement is significant. Also the distribution transformers which are normally operating in the saturation region, a 1% reduction in voltage may cause a 4 to 6% reduction in reactive power requirements.

Under voltage load shedding: It is a cost-effective decentralised voltage stability solution for infrequent disturbances. It is a valuable backup for primary solutions to voltage stability problems. If most of the load is static and highly voltage sensitive (heating and lighting), the time delays may be longer as compared to induction motor load where the time delay is from one to two seconds. The under voltage relays, however, should respond to balanced positive sequence voltage drops. They should not operate for unbalanced conditions.

Direct load control and distribution automation: It is not desirable to disconnect load abruptly by direct load tripping or under voltage load shedding. Rapidly turning off air conditioners, water heaters, electric heating or other loads for five to twenty minutes during an emergency is desirable.

For longer term voltage stability, extremely fast action is not required, tens of minutes are available. The controls provide load relief and the time needed to start gas turbines or reschedule generation. One concept is for the electric utility to communicate emergency conditions to consumer microprocessors by a large increase in the current cost of electric energy. Thermostat settings or load demand would then be changed to reduce consumption.

The reactive power reserves at generators and static var compensators can be activated to improve voltage stability. Direct load control can also improve voltage stability through switched capacitor bank control, tap changer control and voltage reduction. Direct load control can be used for inexpensive testing of load characteristics. Capacitor banks can be switched off by SCADA to lower voltage, and the resulting active and reactive power response of the loads can be monitored and analysed at control centres. Voltage reduction controls can be used for the same purpose.

22.8.4 Power System Operation

Once the power system with a large number of controls is ready, it is the responsibility of the power system operator to operate the system so that balance between reliability and economics is maintained. Energy management system (EMS) provides a variety of measured and computed data. State Estimation method is used to obtain voltage magnitude and their phase angle. This method takes care of power flow model inaccuracies especially those related to reactive power flow and proper estimate of reactive power is obtained.

Various computer programs have been developed to help the operator in reactive power management and voltage control. These programs deal in active power transmission loss minimisation subject to system operating constraints.

Artificial intelligence is another approach to centralised reactive power and voltage control. An expert system can help the operator to adjust the capacitor so that the generators operate near unity p.f. For longer term voltage stability many utilities have EMS functions to guide operator actions. Due to time constraints load flow and artificial intelligence methods will be playing major role for voltage stability in the near future. As of now, *P-V* curves are most widely used for estimating voltage security providing MW margin type indices.

It is desirable to have load dispatch centre based centralised controls. As there is insufficient time for algorithm based computations, controls must be pre-programmed based on off-line studies. Several artificial intelligence approaches have been proposed. Emergency actions are taken based mainly on measured change of generator and SVC reactive power and change of voltage magnitudes. Artificial intelligence could be used to trigger distribution automation actions.

PROBLEMS

22.1. Develop an expression for reactive power flow when a generator is connected to an infinite bus through a tie-line of reactance X. Show that the active power flow depends upon the load angle δ whereas the reactive power upon the voltage gradient

22.2. Discuss briefly various problems associated with reactive power transmission.

22.3. Explain what you mean by short circuit capacity of a bus or fault level of a bus. Explain how the short circuit capacity and voltage regulation of a bus are related.

22.4. Explain the concept of power system voltage stability and give definitions of voltage stability.

22.5. Discuss briefly various forms of voltage instability in terms of time-frame and mechanism associated with these form of instability.

22.6. Define and differentiate between rotor angle stability and voltage stability of power system.

22.7. "Rotor angle stability is associated with transmission network whereas voltage stability is associated with load." Comment on the statement.

22.8. Discuss briefly how the following components of power system affect voltage stability of the system.

(*a*) Transformer (*b*) Induction motors (*c*) Feeder

(*d*) Voltage reduction (*e*) Generation in load areas

(*f*) Reactive compensation of loads

(*g*) *LTC* transformers and distribution voltage regulators.

22.9. Explain what you mean by reactive compensation of loads and discuss how it helps in maintaining voltage stability.

22.10. Discuss briefly the effect of tap changing on shunt compensated loads from voltage stability point of view.

22.11. What is an LTC transformer ? Derive and draw the equivalent circuit of this transformer.

22.12. What is line drop compensation ? Explain with the help of a neat diagram the operation of such a scheme. Discuss its role in voltage stability.

22.13. What are capability curves of an alternator ? Discuss their role in voltage stability of the power system.

22.14. What are P-V and V-Q curves ? Discuss how these help in studying voltage stability of power system.

22.15. Explain how 'Network Voltage Control' is carried out and how it helps in voltage stability of the system.

22.16. Discuss the role of excitation control for improving transient voltage stability.

22.17. Describe the operation of an HVDC link in a large interconnected system under (*i*) normal (*ii*) abnormal operating conditions.

22.18. Discuss the sequence of events which take place when a large disturbance occurs near the inverter operating in constant extinction angle control mode.

22.19. While planning a new long distance transmission line, compare between a.c. and d.c. transmission system from stability point of view.

22.20. Explain what you mean by voltage collapse proximity indicator ? Derive an expression for VCPI for a system consisting of a large bus connected to a purely inductive load through a loss less tie-line. Show that VCPI is a very sensitive indicator of impending voltage collapse.

22.21. What are the advantages of *V-Q* curves for studying voltage stability ? Explain how you obtain a reactive power margins.

22.22. Explain the terms 'Performance criteria' and 'voltage margin to instability'.

22.23. Discuss very briefly different methods of improving voltage stability of power system.

22.24. Show that the approximate per unit change in voltage equals the ratio of change in reactive power to the short circuit capacity of the bus.

22.25. Explain clearly various mechanisms which come into action for restoration of loads when there is reduction in system voltage due to a disturbance in the system.

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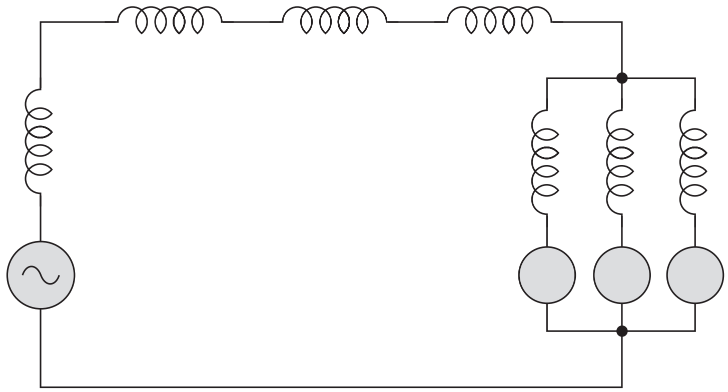
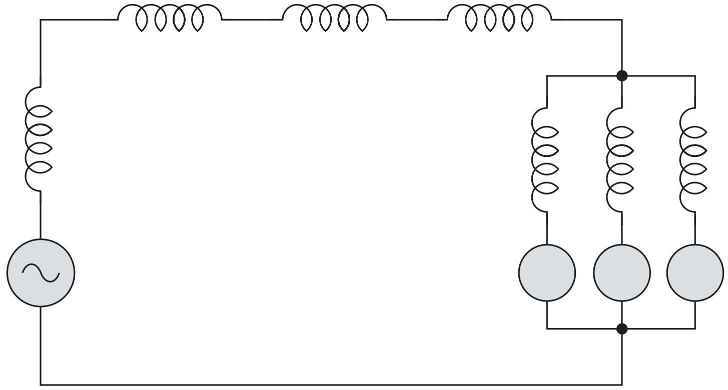


23

STATE ESTIMATION IN POWER SYSTEMS



23

State Estimation in Power Systems

23.1 INTRODUCTION

State estimation is the process of determining a set of values for a set of unknown system state variables based on certain criterion making use of the measurements made from the system under consideration. Since the measurements made may not be precise due to inherent errors associated with the transducers and also there may be some redundant measurements not required for assessment of the true value of the system state variables, statistical methods are used to estimate the true value of the state variables. State variables is the minimum number of variables required to analyse the system in all its aspects. For power system the state variables are the voltage magnitudes and relative phase angles at the system nodes. If n is the number of buses (nodes), the number of state variables should be $2n$. But we normally take one of the buses as the slack bus or reference bus where the phase angle is taken as zero *i.e.*, we measure the phase angles of all the remaining $(n - 1)$ buses with respect to this bus and hence the number of state variables is $(2n - 1)$. The measurements are required to meet the basic needs of an electric utility *i.e.,* to arrange good quality of reliable supply at reasonable cost (system security analysis and economic load dispatch or optimal power flows). Power system is the most complex and most capital intensive system and, therefore the monitoring of system voltages, and power flows etc. is very important to achieve higher index of reliability of supply and also run the system economically which are the basic requirements of the system.

While monitoring a power system, problems in measurements arise due to the following reasons:

(*i*) Transducers used for measurements of voltage, current, power etc.

(*ii*) The communication channels used for transmitting the measured values to the operations control centre.

The transducers have inherent errors associated with them. If errors are small these might go undetected and may cause wrong interpretation of those recorded values. However, if the transducers polarity is not proper the output may give negative value and thus will be cause of gross error.

The telemetry equipment often experiences periods when communication channels are completely out of service and thus this situation deprives the system operator of any information about that part of the system which are connected through these channels.

Thus this introduces gross error in the measurements. The power links running along the same route as the communication lines may introduce some noise signals in the communication lines.

The errors introduced by the measurement noise is comparable with the uncertainty of most of the operational constraints (*e.g.,* voltage constraint, the line overloading constraints etc.) against which the results of the state estimation will be checked.

However major errors (bad data) can seriously distort the results of the estimator producing non-feasible solution and hence need special type of state estimator.

A state estimator is a set of programs which obtains estimates of the static state at some required instant of time (snap shot) from telemetered values of network variables *e.g.,* voltage magnitudes, line flows, active, reactive powers etc.) and topological information. As mentioned earlier, a large amount of data may be redundant. The state estimator should incorporate all measurements to obtain the greatest possible accuracy.

The first job of the state estimator is to detect bad or grossly incorrect data and remove it from subsequent state estimates until it can be physically checked. The operator of the system is usually alarmed of this action. The establishment of acceptable error limits based on the number, types and accuracy of all measurements is an important design aspect of state estimators.

Another job of a state estimator is to detect changes in the topology of the network. If one of the three phases is tripped due to some reason, the average power flow on the intact phase is far less than a value given by the last state estimate. The operator is alerted as to this condition at the first data scan and a corrective action is taken by him.

Yet the other job of a state estimator is to complete a set of measurements so that faulty or missing data could be replaced. It is possible to determine power flows and voltages at a bus whose measurements have been lost due to communication line failure or remote terminal units (RTU). In this program the most important aspect is to determine the minimum number of measurements in order to calculate the state and to improve state estimation by additional measurements.

23.2 STATE ESTIMATION FOR LINE POWER FLOW

J.F. Dopazo *et al*. (1) were the first to introduce state estimator considering line power flow measurements. Assuming a balanced three phase system, consider a generator connected to a large no. of transmission lines where meters are connected for measurement of various quantities like power injection into the buses, bus metering and line flow measurements. Before we develop this strategy for state estimator following assumptions are made:

(*i*) The system is operating under balanced and steady state condition.

(*ii*) The full range of each meter is known *e.g.,* for a watt meter and var meter 0 to 100 MW and 0 to 100 MVAR respectively etc.

(*iii*) The analog signals are converted into digital signals for data link to the central computer and the conversion errors are of the order of 0.1% to 0.25%.

(*iv*) The accuracy of measuring instruments is known *i.e.,* the meter readings are accurate to 1%, 2% of the actual value or true value.

The three phases are monitored for balance condition and then the product of instantaneous voltages and currents are obtained after scaling down the quantities using transducers and then converted into serial digital data by the RTU. Each measurement of line flow is given as

$$S_{mi} = P_{mi} + jQ_{mi} \tag{23.1}$$

where S_{mi} is the complex power measurement i on line m in MVA. The specified weighting factor of these measurements is inversely proportional to its accuracy

i.e.,
$$W_i = \frac{50 \times 10^{-6}}{[C_1 \,|S_{mi}| + C_2\,(Fs.)]^2} \tag{23.2}$$

where C_1 = accuracy expressed as a fraction of real and reactive power flow typically 0.01 or 0.02.

C_2 = accuracy of transducer and analog to digital converter as a fraction of full scale reading of the meter typically 0.0025, 0.005 etc.

F.S. = full scale range of both the wattmeter and reactive var meter readings (MW and MVAr)

Factor 50×10^{-6} is chosen to normalise subsequent numerical quantities. It is to be noted that W_i is of be computed each time a new measurement (S_{mi}) is made.

The weighted least square method is used to obtain the best state vector $\overline{X}$ which minimises the performance index of M measurements

$$\text{Min} \quad J = \sum_{i=1}^{M} W_i \,|\, S_{mi} - S_c \,|^2 \tag{23.3}$$

where S_c is a calculated value of complex power based on state X. For power system problem state vector consists of nodal voltage magnitudes and their corresponding phase angle and is equal to $2(n-1)$ where n is the no. of buses. As in case of load flow solutions a slack bus has to be specified to satisfy the power constraints equation, here also we consider one of the buses as slack bus so that the no. of state variables is $(2n-2)$ as the voltage magnitude and phase angle (reference) at slack bus is prespecified.

Fig. 23.1 shows π-equivalent representation of a transmission line. The power flow measurements near the buses include power to the shunt elements also.

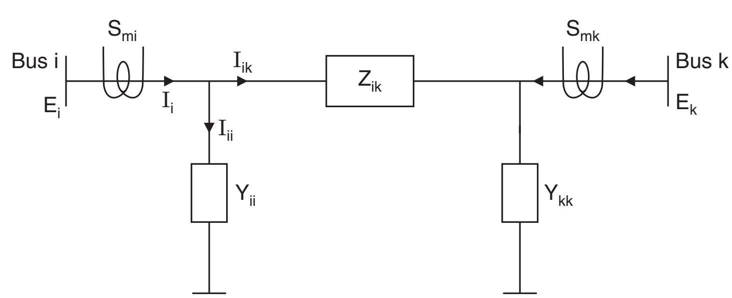


Fig. 23.1 π-equivalent of a line and power measurements at both the ends.

The power flow calculated at bus i is

$$S_{ci} = P_{ci} + jQ_{ci} = E_i I_i^* = E_i\,(I_{ii} + i_{ik})^*$$
$$= E_i\,[Y_{ii}^* E_i^* + Y_{ik}^*\,(E_i^* - E_k^*)]$$
$$= E_i \left[\frac{E_i^* - E_k^*}{Z_{ik}^*} + Y_{ii}^*\, E_i^* \right] \tag{23.4}$$

Here the parameters of the transmission line are known a priori, however the nodal voltages are unknown.

A few approximations are required to obtain an iterative algorithm to solve equation (23.3) for the nodal voltages. The approximations are identical to those for load flow solution using Jacobian matrix. It is to be noted that these approximations do not affect the accuracy of the final state estimator but it requires a few more iterations to converge *i.e.*, affect only the rate of convergence. The measured numerical values are equated to the power flow from bus voltages to be calculated. In case of load flow solution we equate the base case power quantities as prespecified, here we equate to the measured values.

Hence,
$$S_{mi} = E_i \left[\frac{E_i^* - E_k^*}{Z_{ik}^*} + Y_{ii}^*\, E_i^* \right] \tag{23.5}$$

From equation (23.5) it is clear that the power flow measurements depend mainly on difference in voltage from bus i to bus k, so the equivalent measured differences are

$$E_{im} - E_{km} = \frac{Z_{ik}^*}{E_i^*}\; S_{mi}^* - Z_{ik}^*\, Y_{ii} E_i \tag{23.6}$$
$$= H^{-1} S_{mi}^* - K$$

where
$$H = \frac{E_i}{Z_{ik}} \quad \text{and} \quad K = Z_{ik} Y_{ii} E_i$$

Equation (23.6) can be written in matrix form as

$$E_M = \begin{bmatrix} E_{1m} - E_{pm} \\ E_{2m} - E_{nm} \\ \vdots \end{bmatrix} = \mathrm{H}^{-1}\, S_M^* - K \tag{23.7}$$

Here H is an $M \times M$ diagonal matrix, K is a vector and E_M is a vector of measured differences associated with each power flow measurement. In equation (23.7) the bus voltages on the right-hand are known. However E_M can be obtained from the power measurements S_M.

The performance index J for estimation of state estimator is in matrix form as follows:

$$J = \sum_{i=1}^{M} W_i\, [S_{mi} - S_{ci}]^2$$
$$= [S_m - S_c]^{*T}\, W\, [S_M - S_c] \tag{23.8}$$

Rewriting equation (23.4)

$$S_c = E_i \left[\frac{E_i^* - E_k^*}{Z_{ik}^*} + Y_{ii}^* E_i^* \right]$$

Substituting $H = \dfrac{E_i^*}{Z_{ik}^*}$ and $K = Z_{ik}^* \, Y_{ii}^* \, E_i^*$ we have

$$S_c = (HE_c + HK)^*$$

Here E_c is difference in calculated voltages.

From equation (23.7)

$$E_M = H^{-1} S_M^* - K$$

or
$$HE_M = S_M - HK$$

or
$$S_M = (HE_M + HK) \tag{23.9}$$

Substituting the values of S_M and S_c in performance index (23.8) we have

$$\begin{aligned} J &= [S_M - S_c]^{*T} \, W \, [S_M - S_c] \\ &= [HE_M + KH - HE_c - HK)^{*T} \, W \, [HE_M + HK - HE_c - HK] \\ &= [E_M - E_c]^{*T} \, H^* \, WH \, [E_M - E_c] \end{aligned} \tag{23.10}$$

E_c, the calculated voltage difference vector can be expressed in terms of the bus voltages E_{bus} and the slack bus voltage E_r as

$$E_c = AE = BE_{bus} + bE_r \tag{23.11}$$

Here A is a double branch to bus incidence matrix *i.e.*, + 1 and – 1 in each row corresponding to a measurement along a directed branch element.

Measurement buses

$$\downarrow \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & -1 & 0 & \dots & 0 \\ -1 & 0 & 0 & 1 & 0 & \dots & \vdots \\ & & & & & & -1 \\ & \vdots & & & & & 1 \end{array} \right] = [B \quad b] \tag{23.12}$$

When measurements of power flow are made at both ends of the line, the series element (Z_{ik}) appears twice. The order of A is $M \times n$ where n is the no. of buses. A is partitioned into a bus voltages other than slack, part B, and a slack bus vector b as shown by dark vertical line. Substituting equation (23.11) into equation (23.10) we have

$$J = [E_M - BE_{bus} - bE_r]^{*T} \, H^* WH \, [E_M - B.E_{bus} - bE_r] \tag{23.13}$$

The quantity H^*WH is assumed to be constant and to minimise J we differentiate (23.13) with respect to E_{bus} and equate it to zero to obtain the condition for minimisation.

$$\frac{\partial J}{\partial E_{bus}} = - B^T H^* WH \, [E_M - BE_{bus} - bE_r] = 0$$

$$[B^T H^* WHB] \, E_{bus} = B^T H^* WH \, [E_M - bE_r] \tag{23.14}$$

Let $D = H^*WH$ be a diagonal matrix of order M and the iterative line power flow state estimator is given as

$$E_{bus}^{K+1} = [B^T DB]^{-1} B^T D\, [E_M^{\ K} - bE_r] \tag{23.15}$$

Here K is the iteration count. Here, since E_r is the slack bus or reference bus voltage it remains same through all the iterations. $E_M^{\ K}$ is evaluated using the previous value of E_{bus}. Equation (23.15) is to be solved iteratively to obtain the bus voltage vector which is the state estimator for a particular steady state operating condition of the system when only line flows are to be taken into consideration. The algorithm is given as follows:

1. Assume suitable values of voltages E_{bus}^0 for all buses. Since we are considering steady state balance operation of power system the voltages at all the buses are approximately 1 ± 0.05 pu. and hence a flat value for all buses equal to slack bus can be assumed.
2. Determine the initial voltage difference using equation (23.7), *i.e.,*
$$E_M^0 = H^{-1} S_M^* - K$$
where both H and K depend on E_{bus}^0 and system parameters.
3. Using equation (23.15) calculate $E_{bus}^{\ k+1}$
4. Repeat steps 2 and 3 till the difference in E_{bus} within a prespecified tolerance is achieved *i.e.,*
$$|\, E_{busi}^{\ k+1} - E_{busi}^{\ k} \,| \le \varepsilon$$
ε is usually selected as 10^{-4} pu.

It is to be noted that assumption of $H^*WH = D$ as constant does not affect accuracy of the final result.

Example 23.1 below illustrates the application of the algorithm for calculation of state estimator for line power flow measurements.

Example 23.1: Fig. E23.1 shows a three bus system. Bus 3 is selected as slack bus with voltage $1.05 + j0.0$, complex powers at both the ends of the lines are given as

$S_1 = 0.41 - j0.11$ $\quad S_2 = -\,0.4 + j0.10$

$S_3 = -\,0.105 + j0.11$ $\quad S_4 = -\,0.105 + j0.11$

$S_5 = 0.14 - j0.14$ $\quad S_6 = -\,0.7 + j\,0.35$

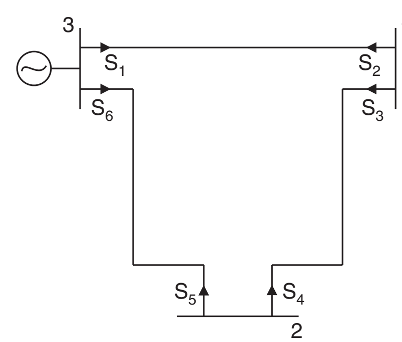


Fig. E23.1

The line pu. impedances are given as $Z_{12} = 0.08 + j\,0.24$ $Z_{23} = 0.06 + j\,0.18$ and $Z_{31} = 0.02 + j\,0.06$ on a 100 MVA base. Neglect shunt admittances. Assume meter accuracy as 2% with full range of meters as 100 MW and 100 MVAr and transducer with analogue to digital conversion error to be 0.5%. Determine the state vector at the end of first-iteration.

Solution: Using equation (23.2) for evaluation of the weighting factors $W_1 \ldots W_6$ for the six measurements (line flows).

$$W_1 = \frac{50 \times 10^{-6}}{[0.02 \times 100\,|\,0.41 - j1.1\,| + 0.05 \times 100]^2} = 27.475 \times 10^{-6}$$

$$W_2 = \frac{50 \times 10^{-6}}{[0.02 \times 100\,|-0.41 + j0.10| + 0.05 \times 100]^2} = 28.5 \times 10^{-6}$$

Similarly, the other weighting factors are

$$W_3 = 77.323 \times 10^{-6}$$

$$W_4 = 62.284 \times 10^{-6},\ W_5 = 11.135 \times 10^{-6},\ W_6 = 11.72 \times 10^{-6}$$

In order to evaluate H^*WH we assume all voltages to unity. As mentioned earlier, this approximation of H does not affect the final solution, it only slows down the process of convergence but it certainly simplifies our calculations.

The diagonal weighting factor matrix is

$$D = \text{diag}\,[d_1, d_2, d_3, d_4, d_5, d_6]$$

$$= \text{diag}\left[\frac{W_1}{|Z_{13}|^2}\ \frac{W_2}{|Z_{31}|^2}\ \frac{W_3}{|Z_{12}|^2}\ \frac{W_4}{|Z_{21}|^2}\ \frac{W_5}{|Z_{23}|^2}\ \frac{W_6}{|Z_{32}|^2}\right]$$

$$= \text{diag}\left[\frac{27.475 \times 10^{-6}}{|0.02 + j0.06|^2}\quad \frac{28.5 \times 10^{-6}}{|0.02 + j0.06|^2}\quad \frac{77.323 \times 10^{-6}}{|0.08 + j0.24|^2}\quad \frac{62.284 \times 10^{-6}}{0.08 + j0.24}\right.$$

$$\left.\frac{11.135 \times 10^{-6}}{|0.06 + j0.18|^2}\quad \frac{11.7 \times 10^{-6}}{|0.06 + j0.18|^2}\right]$$

$$= \text{diag}\,[6.869 \times 10^{-3}\quad 7.124 \times 10^{-3}\quad 1.2 \times 10^{-3}\quad 0.973 \times 10^{-3}\quad 0.309 \times 10^{-3}\quad 0.3256 \times 10^{-3}]$$

From Fig. E23.1, the double branch to bus incidence matrix is given as

$$A = \begin{array}{c} \text{Measurement} \\ \downarrow \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \overset{\text{Buses} \rightarrow}{\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}}$$

Hence,
$$E_c = AE = BE_{\text{bus}} + bE_r = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} E_{\text{bus}} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} E_r\,(E_3)$$

Here $$B = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Now $B^T D$ is,

$$\begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} d_1 & & & & & \\ & d_2 & & & & \\ & & d_3 & & & \\ & & & d_4 & & \\ & & & & d_5 & \\ & & & & & d_6 \end{bmatrix}$$

$$(2 \times 6)(6 \times 6) = 2 \times 6$$

$$= \begin{bmatrix} -d_1 & d_2 & d_3 & -d_4 & 0 & 0 \\ 0 & 0 & -d_3 & d_4 & d_5 & -d_6 \end{bmatrix}$$

Hence, $$B^T DB = \begin{bmatrix} -d_1 & d_2 & d_3 & -d_4 & 0 & 0 \\ 0 & 0 & -d_3 & d_4 & d_5 & -d_6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

$$(2 \times 6)(6 \times 2) = 2 \times 2$$

$$= \begin{bmatrix} d_1 + d_2 + d_3 + d_4 & -d_3 - d_4 \\ -d_3 - d_4 & d_3 + d_4 + d_5 + d_6 \end{bmatrix}$$

Substituting the values of diagonal elements as obtained earlier, we have

$$B^T DB = \begin{bmatrix} 0.01617 & -0.002173 \\ -0.002173 & 0.0028076 \end{bmatrix}$$

and hence $$[B^T DB]^{-1} = \begin{bmatrix} 69.02 & 53.42 \\ 53.42 & 397.5 \end{bmatrix}$$

Now $[B^T DB]^{-1} B^T D$,

$$= 10^{-3} \begin{bmatrix} 69.02 & 53.42 \\ 53.42 & 397.5 \end{bmatrix} \begin{bmatrix} -6.869 & 7.124 & 1.2 & -0.973 & 0 & 0 \\ 0 & 0 & -1.2 & 0.973 & 0.309 & -0.3256 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4741 & 0.4917 & 0.01872 & -0.01518 & 0.01654 & -0.0174 \\ -0.367 & 0.38035 & -0.4129 & 0.033478 & 0.12282 & -0.1294 \end{bmatrix}$$

According to equation (23.15), we need to calculate

$[E_M - bE_r]$ where $E_M = H^{-1}S_m{}^* - K$

Since $K = Z_{ik}Y_{ii}E_i$ and as Y_{ii} is given to be zero, hence

$E_M = H^{-1}S_{mi}{}^*$. Now $H^{-1} = \dfrac{Z_{ik}}{E_i{}^*}$ and it is an $M \times M$ dimension diagonal matrix and hence it is given as

$$H^{-1} = \begin{bmatrix} \frac{Z_{31}}{E_3} & & & & & \\ & \frac{Z_{13}}{E_1} & & & & \\ & & \frac{Z_{12}}{E_1} & & & \\ & & & \frac{Z_{21}}{E_2} & & \\ & & & & \frac{Z_{23}}{E_2} & \\ & & & & & \frac{Z_{32}}{E_2} \end{bmatrix}$$

Substituting the values where during first iteration

$E_1 = E_2 = E_3 = 1.05 + j0.0$ we have the diagonal matrix as

$$E_M{}^0 = [0.019 + j0.057,\ 0.019 + j0.057,\ 0.0762 + j0.2285,\ 0.0762 + j0.2285,\ 0.057 + j0.161,\ 0.057 + j0.161] \begin{bmatrix} 0.41 & + j0.11 \\ -0.4 & -j0.1 \\ -0.105 & -j0.11 \\ 0.14 & +j0.14 \\ 0.72 & +j0.37 \\ -0.7 & +j0.35 \end{bmatrix}$$

$$= H^{-1}S_m{}^*$$

$$= (6 \times 6)\,(6 \times 1) = 6 \times 1$$

$$= \begin{bmatrix} 0.00152 & +j0.02546 \\ -0.0019 & -j0.0247 \\ 0.0171 & -j0.03238 \\ -0.02133 & +j0.04266 \\ -0.01853 & +j0.1569 \\ 0.01645 & -j0.13265 \end{bmatrix}$$

Hence, $$[E_M^0 - bE_r] = E_M^0 - \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} (1.05 + j0.0) = \begin{bmatrix} -1.04848 & +j0.02546 \\ 1.0481 & -j0.0247 \\ 0.0171 & -j0.03238 \\ -0.02133 & +j0.04266 \\ 1.03147 & +j0.1569 \\ -1.03355 & -j0.13265 \end{bmatrix}$$

Using equation (23.15) and substituting the respective values already evaluated above, we have $\begin{bmatrix} E_1' \\ E_2' \end{bmatrix}$

$$= \begin{bmatrix} -0.4741 & 0.4917 & 0.01872 & -0.01518 & 0.01654 & -0.0174 \\ -0.367 & 0.38035 & -0.4129 & 0.033478 & 0.12282 & -0.1294 \end{bmatrix} \begin{bmatrix} -1.04848 & +j0.02546 \\ 1.0481 & -j0.0247 \\ 0.0171 & -j0.03238 \\ -0.02133 & +j0.04266 \\ 1.03147 & +j0.1569 \\ -1.03355 & -j0.13261 \end{bmatrix}$$

$$(2 \times 6)\,(6 \times 1) = 2 \times 1$$

$$= \begin{bmatrix} 1.0481224 & -j0.0205653 \\ 1.036088 & +j0.0324946 \end{bmatrix}$$

So these are the voltages of buses 1 and 2 at the end of first iteration. Usually it takes about 3 to 4 iterations to converge when the difference between the two successive iterations is less than 10^{-4} and that gives the optimum states of the system. Using these values of the state estimate, line flows (best estimate) are calculated and these are compared with the measured line flows. Most of the times there will be difference in the two values and hence some methods should be devised to detect errors in measurement for proper estimate of states of the system.

The method discussed above has certain limitations for state estimation.

1. It does not accept different ranges and accuracies for real and reactive power measurements at a single point.

2. It does not accept other available measurements *e.g.,* bus power injections and partial information *e.g.,* bus voltage and current magnitudes only and their phase angles are not known.

As mentioned earlier statistical methods will be used for static state estimation of power system. A few definitions frequently used here are in place.

Standard Deviations: The standard deviation is the most frequently used measure of deviations (*e.g.,* deviation between the measured quantity and the calculated or actual quantity). In simple terms it is defined as Root-mean-squared deviation. It is denoted by the Greek letter sigma σ and is calculated from the basic formula

$$\sigma = \sqrt{\frac{(x - \bar{x})^2}{m}} \tag{23.15a}$$

where x is any measurement and $\bar{x}$ is mean of the m measurements and σ^2 is known as the variance of the random measurements made. σ is an abstract number which means the larger the value of σ the greater the spread of values about the mean.

Standard Normal Curve: It is a smooth bell-shaped perfectly symmetrical curve based on a large number of measurements

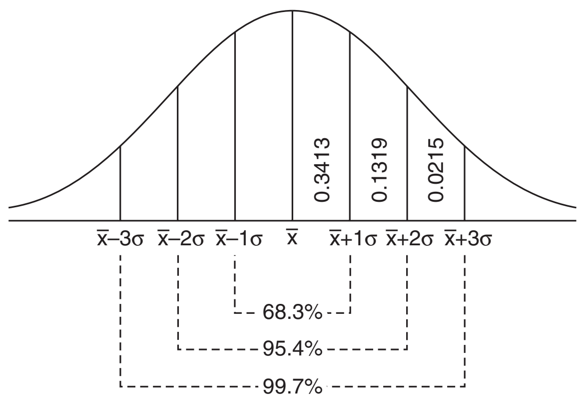


Fig. 23.2 Standard normal curve.

The total area of the curve is 1, its mean is zero and its standard deviation or variance is 1. The distance of value x from the mean of the curve in terms of standard deviation is called standard normal deviate and is usually represented by D and given by

$$D = \frac{x - \bar{x}}{\sigma} \tag{23.15b}$$

It is useful to note at this stage that in the curve in Fig. 23.2 the area between one standard deviation on either side of the mean *i.e.* ($\bar{x} \pm \sigma$) will include approximately 68% of the measured values, the area between two σ on either side of the mean includes about 95% of the values and the area between ($\bar{x} \pm 3\sigma$) will include 99.7% of the values. These limits on either side of the mean are called confidence limits.

Supposing we are considering 95% confidence limits ($\bar{x} \pm 2\sigma$). When we say this we mean that 95% of the area of the curve and hence 95% of the values in the measurements will be included between the limits $\bar{x} \pm 2\sigma$. Therefore, the probability of a measurement falling outside the 95% confidence limit is 1 in 20 *i.e.,* the probability is only 5% or 0.05 = P.

Estimation of Probability (Example). Suppose the average value of a set of measurements is 72 and σ = 2. What is the probability that a measurement chosen at random would have a value of 78 or more?

The standard normal deviate $= D = \dfrac{x - \bar{x}}{\sigma} = \dfrac{78 - 72}{2} = 3$

i.e., the confidence interval is 99.7% *i.e.,* the probability is that only 3 out of 1000 measurement would likely to have a value of 78 or more.

From the above observation it is clear that standard deviation σ provides a way to model the seriousness of the random measurements error. If σ is large the measurement is relativity inaccurate *i.e.,* poor quality measuring meters have been used, whereas a small value of σ indicates a small error spread and hence better accuracy meters have been used for measurements. The normal distribution as shown in Fig. 23.2 is commonly used for modeling measurement errors as it is the distribution that will result when many factors contribute to the overall error.

Figure 23.3 shows two sets of error distributions. In Fig. 23.3(*a*) two curves have unbiased estimates of the error distributor with two different variances.

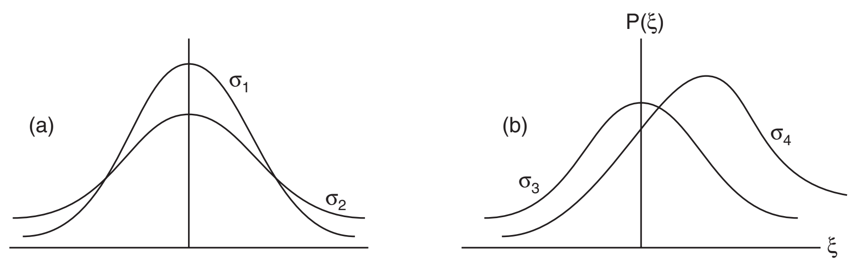


Fig. 23.3 (*a*) unbiased (*b*) biased distribution.

The variance of a random variable measures the variability of the random variable about its expected value. Hence to require an unbiased estimate to have small variance is intitutively appealing. As, if the variance is small then the value of the random variable tends to be close to its mean which in the case of an unbiased estimate means close to the true value of the parameter. Thus if $\bar{x}_1$ and $\bar{x}_2$ are two estimates for x whose probability distribution function (PDF) is as shown in Fig. 23.3(*a*) we would prefer $\bar{x}_1$ to $\bar{x}_2$. Both estimates are unbiased and $\sigma_1^2 < \sigma_2^2$.

In case of Fig. 23. 3(*b*) the decision is not clear regarding the estimates as $\bar{x}_3$ is unbiased whereas $\bar{x}_4$ is biased as $P(\xi)$ is not symmetrical as a function of ξ, the error. However $\sigma_3 > \sigma_4$. This means that while on the average $\bar{x}_3$ will be close to x, its large variance indicates that considerable deviation from x would not be surprising. $\bar{x}_4$ on the other hand would tend to be somewhat larger than x, on the average and yet might be closer to x than $\bar{x}_3$. It is to be noted that normal distribution (Fig. 23.2 and 23.3 (*a*)) are assumed to be unbiased meter error distribution. In the previous article we have studied the procedure for state estimation when line flows are considered using the weighted least square estimation formula. Next we consider the maximum likelihood formula or criterion where the objective is to maximize the probability that the estimate of the state variable $\bar{x}$ is the true value of the state variable x (*i.e.,* Max $(P(\bar{x}) = x)$. This method introduces the measurement error weighting matrix $[R]$ in a straightforward manner.

23.3 MAXIMUM LIKELIHOOD CRITERION

We will proceed to develop our estimation formula using the maximum likelihood criterion assuming normal distributions for measurement errors. The result will be a weighted least squares estimation formula even though we will develop the formulation using the maximum likelihood criteria. The measurements are assumed to be in error. Let Z_m be the value of a measurement and Z_t be the true value (calculated value) of the quantities being measured. Also, let ξ be the random measurement error such that

$$Z_m = Z_t + \xi \tag{23.16}$$

The random number ξ helps to represent the uncertainty in the measurement. If the measurement error is unbiased, the probability density function of ξ is represented as a normal distribution with zero mean. The probability density function of ξ is given as

$$PDF(\xi) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\xi^2/2\sigma^2} \tag{23.17}$$

and the plot is shown in Fig. (23.2).

Since the mean value of ξ is zero, the mean value Z_m equals Z_t. This allows us to write a probability density function for Z_m as

$$PDF(Z_m) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(Z_m - Z_t)^2}{2\sigma_1{}^2}} \tag{23.18}$$

Here we have replaced $\xi = Z_m - Z_t$.

The maximum likelihood procedure requires that we maximise the value of $P(Z_m)$ which is a function of x.

$$\text{Max PDF } (Z_m) = \text{Max}\left[\frac{1}{\sigma\sqrt{2\pi}}\, e^{-(Z_m - Z_t)^2/2\sigma^2}\right] \tag{23.19}$$

Taking natural log (ln) on both the sides we have

$$\ln\,[\text{Max PDF}(Z_m) = \max \ln \left\{\left[\frac{1}{\sigma\sqrt{2\pi}}\, e^{-(Z_m - Z_t)^2/2\sigma^2}\right]\right\}$$

or
$$\text{Max ln PDF } (Z_m) = \max\,[-\ln \sigma\sqrt{2\pi} - (Z_m - Z_t)^2/2\sigma^2] \tag{23.20}$$

and it is same as

$$\text{Min}\left[\frac{(Z_m - Z_t)^2}{2\sigma^2}\right] \tag{23.21}$$

as $\ln \sigma\sqrt{2\pi}$ is a constant quantity it is eliminated from the minimisation process and max of a negative number is min of the positive number.

From the above equations it is clear that the maximum likelihood estimate of our unknown parameters is always expressed as that value of the parameter that gives the minimum of the sum of the squares of the difference between each measured value and the true value being measured (expressed as a function of unknown parameters) with squared difference divided or "weighted" by the variance of the meter error. Therefore, if we are estimating a single parameter x using M measurements, we would write the expression

$$\text{Min } J(x) = \sum_{i=1}^{M} \frac{[Z_{im} - f_i(x)]^2}{\sigma_i^{\,2}} \tag{23.22}$$

where f_i = calculated value at x for the ith measurement

$J(x)$ = measurement residual or performance index

Z_{im} = the ith measured quantity.

However, if the state vector has n parameters and m measurements are made, we would have

$$\text{Min } J(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{M} \frac{[Z_i - f_i(x_1, x_2, \ldots, x_n)]^2}{\sigma_i^{\,2}} \tag{23.23}$$

Equations (23.22) and (23.23) indicate a weighted least square estimator which as we said earlier is equivalent to a maximum likelihood estimator if the measurement errors are modelled as random numbers having a normal distribution. Another property of ML estimate is that if $\bar{x}$ is the best estimate of x, then $f(\bar{x})$ is the best estimator. In terms of power system states if voltages and angles best estimate is given by $\bar{V}$ and $\bar{\delta}$ then $P(\bar{V}, \bar{\delta})$ and $Q(\bar{V}, \bar{\delta})$ are the best estimates of $P(V, \delta)$ and $Q(V, \delta)$ respectively.

In power system the measured quantities are MW, MVAr, MVA, voltages, currents etc. and these quantities are non-linear function of state variables except for voltage magnitude measurement.

It is to be noted that it is difficult to measure error statistics or to obtain the probability density function for error as given by equation (23.17). Power system engineers prefer to use metering equipment accuracy to define the variance σ^2 and is given as

$$\frac{1}{\sigma_i^{\,2}} = \frac{50 \times 10^{-6}}{|c_1 \,|Z_{mi}| + c_2\,(F.S.)|^2} \tag{23.24}$$

where Z_{mi} is the ith measurement and c_1 and c_2 are same as defined in equation (23.2).

Writing equation (23.23) in matrix form, the performance index to be minimised by calculating the best state estimate $\bar{X}$ for a number of measurements Z_m is

$$J(X) = [Z_M - f(X)]^T R^{-1} [Z_M - f(X)] \tag{23.25}$$

Here $f(X)$ represent the calculated value corresponding to measured quantity Z_M and R^{-1} is a diagonal weighting matrix of the variance for each of the M measurements equation (23.24).

In order to minimise $J(X)$, we equate the first derivative of $J(X)$ to zero, we have

$$\frac{\partial J(X)}{\partial X} = \frac{\partial f^T(X)}{\partial X} R^{-1} [Z_M - f(X)] = 0 \tag{23.26}$$

This results into a set of M nonlinear equations as $f(X)$ is a nonlinear function of X in state variables and $\dfrac{\partial J(X)}{\partial X}$ is known as the Jacobian matrix.

Say $J(X^{k+1})$ is the performance index during $(k + 1)$th iteration and $J(X^k)$ is the value during kth iteration. This can be written as using Taylor's series expansion

$$J(X^{k+1}) = J(X^k) + \Delta X^k J'(X^k) + \frac{(\Delta X)^2}{2} J''(X^k) + \ldots$$

or
$$J(X^{k+1}) - J(X^k) = \Delta X^k J'(X^k) + \frac{\Delta X^2}{2} J''(X^k) + \ldots \tag{23.27}$$

neglecting higher order terms. Now our convergence criterion will be when the difference between the performance indices during two consecutive iterations is less than a prespecified value or when the difference between two state variables during consecutive iterations is less than a prespecified value. We have already seen that if $\overline{X} = X$ then $f(\overline{X}) = f(X)$ for maximum likelihood condition. Hence equation (23.27) reduces to

$$0 = \Delta X^k J'(X^k) + \frac{\Delta X^{k^2}}{2} J''(X^k) \tag{23.28}$$

where $J''(X^k)$ is known as Hessian matrix or information matrix. From (23.28) we have

$$\Delta X^k = -\,2\,[J''(X^k)]^{-1} J'(X^k) \tag{23.29}$$

So to use iterative procedure we need to know expression for $J''(X^k)$.

From equation (23.26) taking second derivative, the approximate expression for

$$J''(X) \simeq \frac{\partial^2 J}{\partial X^2} = -\frac{\partial f^T(X)}{\partial X} R^{-1} \frac{\partial h}{\partial X} \tag{23.30}$$

Since the Hessian matrix has already been approximated in almost all applications, the elements of this matrix are to be taken constant and hence only one time inverse of this matrix is to be obtained and used in the following iterative scheme

$$X^{k+1} = X^k + \left\{\frac{\partial f^T(X)}{\partial X} R^{-1} \frac{\partial f(X)}{\partial X}\right\}^{-1}_{X^0} \frac{\partial f^T(X)}{\partial X} R^{-1} [Z_M - f(X)] \tag{23.31}$$

Here the inverse term is to be obtained at $X = X^0$ and then used in all successive iterations. It is to be noted that this assumption will not affect the final result of the system but only the number of iterations will be increased as compared to when exact calculation of Hessian is resorted to. Since calculation of Hessian during each iteration is a cumbersome job and time consuming, the alternative procedure suggested here is computationally more effective.

Let the measurement Z_M and the non-linear functions of state $f(X)$ be separated into real and reactive component such that

$$f(X) = \begin{bmatrix} h \\ \ldots \\ g \end{bmatrix} = \begin{bmatrix} \text{real component} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \text{reactive component} \end{bmatrix} \tag{23.32}$$

So the partial derivatives are

$$\frac{\partial f(\delta, V)}{\partial(\partial, V)} = \begin{bmatrix} \dfrac{\partial h}{\partial \delta} & \dfrac{\partial h}{\partial V} \\ \dfrac{\partial g}{\partial \delta} & \dfrac{\partial g}{\partial V} \end{bmatrix} \tag{23.33}$$

The diagonal weighting matrix R^{-1} is also separated into active and reactive component

$$R^{-1} = \begin{bmatrix} R_a^{-1} & 0 \\ 0 & R_r^{-1} \end{bmatrix} \tag{23.34}$$

Using equations (23.32), (23.33) and (23.34), the approximate Hessian matrix is written as

$$J''(\delta, V) = \left[\begin{array}{c:c} \dfrac{\partial h^T}{\partial \delta} R_a^{-1} \dfrac{\partial h}{\partial \delta} + \dfrac{\partial g^T}{\partial \delta} R_r^{-1} \dfrac{\partial g}{\partial \delta} & \dfrac{\partial h^T}{\partial \delta} R_a^{-1} \dfrac{\partial h}{\partial V} + \dfrac{\partial g^T}{\partial \delta} R_r^{-1} \dfrac{\partial g}{\partial V} \\ \hdashline \dfrac{\partial h^T}{\partial V} R_a^{-1} \dfrac{\partial h^T}{\partial \delta} + \dfrac{\partial g^T}{\partial V} R_r^{-1} \dfrac{\partial g}{\partial \delta} & \dfrac{\partial h^T}{\partial V} R_a^{-1} \dfrac{\partial h}{\partial V} + \dfrac{\partial g^T}{\partial V} R_r^{-1} \dfrac{\partial g}{\partial V} \end{array}\right] \tag{23.35}$$

Now assuming decoupling between P and V and Q and δ and also that the initial estimate for all bus voltages is $(1 + j0.0)$ *i.e.,* $V = 1.0$ and $\delta = 0.0$, the above matrix reduces to

$$J_{FD}'' = \begin{bmatrix} \dfrac{\partial h^T}{\partial \delta} R_a^{-1} \dfrac{\partial h}{\partial \delta} & 0 \\ 0 & \dfrac{\partial g^T}{\partial V} R_r^{-1} \dfrac{\partial g}{\partial V} \end{bmatrix}_{\text{Constant}} \tag{23.36}$$

as $$\frac{\partial h}{\partial V} = 0 \quad \text{and} \quad \frac{\partial g}{\partial \delta} = 0.$$

Equation (23.36) is known as a fast decoupled approximation to the Hessian or information matrix. The gradient. $\dfrac{\partial f^T(X)}{\partial X}$ in equation (23.31) should be evaluated corresponding to the current values of δ, V in that iteration for faster convergence or this could also be assumed constant as calculated during zeroth iteration.

The fast decoupled state estimator (FDSE) iterative scheme is given as follows:

$$\delta^{k+1} = \delta^k + \left[\frac{\partial h^T}{\partial \delta} R_a^{-1} \frac{\partial h}{\partial \delta}\right]^{-1} \frac{\partial h^T}{\partial \delta} R_a^{-1} \; [h_m - h(X^k)] \tag{23.37}$$

$$V^{k+1} = V^k + \left[\frac{\partial g^T}{\partial V} R_r^{-1} \frac{\partial g}{\partial V}\right]^{-1} \frac{\partial g^T}{\partial V} R_r^{-1} \; [g_m - g(X^k)] \tag{23.38}$$

where h_m corresponds to active power P and g_m the reactive power measured.

The algorithm for the fast decoupled state estimator is given below:

1. Read in system data, measured quantities, weighting factors and convergence criterion ε.
2. The Jacobian and Hessian (information) matrices are calculated corresponding to all voltages taken as $1 + j0.0$ and are kept constant throughout the process of computation.
3. Set iteration count $K = 0$.
4. Calculate the real and reactive powers $h(x)$ and $g(x)$ and hence the performance index $J(X^k)$ by assuming all voltages corresponding to the reference bus voltages if $k = 0$. Otherwise calculate these quantities corresponding to the values of voltages calculated in the previous iteration if $k > 0$.
5. Calculate new values of state vector using equations (23.37) and (23.38) and calculate $J(X^{k+1})$.
6. Compute $| J(X^{k+1}) - J(X^k) |$ and if this is less then ε stop and X^{k+1} gives the FDSE. If not go to step 4.

The following problem illustrates the implementation of the fast decoupled state estimator algorithm.

Example 23.2: Fig. E. 23.2 shows a three bus system where bus 1 is taken as reference with voltages $1.05 + j0.0$ and bus 2 and 3 are having voltages (initial) as $1 + j0.0$. The line parameters are on a 100 MVA base. The real and reactive power measurement at the three buses are

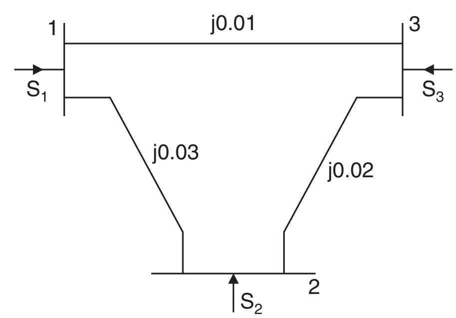


Fig. E 23.2

$$P_{m1} = 0.12 \text{ pu}, \quad P_{m2} = 0.21 \text{ pu}, P_{m3} = -0.3 \text{ pu}.$$

$$Q_{m1} = -0.24 \text{ pu}, Q_{m2} = -0.24 \text{ pu}, Q_{m3} = 0.5 \text{ pu}.$$

Assume $R_1^{-1} = 3 = W_1$, $R_2^{-1} = 5 = W_2$

and $\quad R_3^{-1} = 2 = W_3$

Determine the states of the system at the end of the first iteration.

Solution: The state vector for the system is $[\delta_2, \delta_3, V_2, V_3]^T$ and the performance index $J(\delta, V) = J(X)$ with six measurements is given as $\quad J(X) = [Z_m - f(X)]^T R^{-1} [Z_m - f(X)]$

$$= \begin{bmatrix} P_{m1} - h_1 \\ P_{m2} - h_2 \\ P_{m3} - h_3 \\ Q_{m1} - g_1 \\ Q_{m2} - g_2 \\ Q_{m3} - g_3 \end{bmatrix}^T \begin{bmatrix} R_1^{-1} & & & & & \\ & R_2^{-1} & & & 0 & \\ & & R_3^{-1} & & & \\ & 0 & & R_1^{-1} & & \\ & & & & R_2^{-1} & \\ & & & & & R_3^{-1} \end{bmatrix} \begin{bmatrix} P_{m1} - h_1 \\ P_{m2} - h_2 \\ P_{m3} - h_3 \\ Q_{m1} - g_1 \\ Q_{m2} - g_2 \\ Q_{m3} - g_3 \end{bmatrix}$$

The injection at bus 1 is

$$h_1 + jg_1 = V_1 I_1^* = V_1 [I_{12}^* + I_{13}^*] = V_1 \left[\frac{V_1 - V_2 e^{-j\delta_2}}{jX_{12}} + \frac{V_1 - V_3 e^{-j\delta_3}}{jX_{13}} \right]$$

$$h_1 = -\frac{V_1 V_2 \sin \delta_2}{X_{12}} - \frac{V_1 V_3 \sin \delta_3}{X_{13}}$$

and
$$g_1 = \frac{V_1^{\ 2}}{X_{12}} - \frac{V_1 V_2 \cos \delta_2}{X_{12}} + \frac{V_1^{\ 2}}{X_{13}} - \frac{V_1 V_3 \cos \delta_3}{X_{13}}$$

The derivatives of these quantities are

$$\frac{\partial h_1}{\partial \delta_2} = -\frac{V_1 V_2}{X_{12}} \cos \delta_2$$

$$\frac{\partial h_1}{\partial \delta_3} = -\frac{V_1 V_3}{X_{13}} \cos \delta_3$$

As mentioned earlier we assume all voltages as $1 + j0.0$ for evaluation of Jacobian elements and Hessian (information) matrices.

Hence,
$$\frac{\partial h_1}{\partial \delta_2} = -\frac{1}{X_{12}} \quad \text{and} \quad \frac{\partial h_1}{\partial \delta_3} = -\frac{1}{X_{13}}$$

Assuming decoupled relation between P and V and Q and δ

$$\frac{\partial h_1}{\partial V_2} = 0, \quad \frac{\partial h_1}{\partial V_3} = 0, \quad \frac{\partial g_1}{\partial \delta_2} = 0, \quad \frac{\partial g_1}{\partial \delta_3} = 0$$

Now
$$\frac{\partial g_1}{\partial V_2} = -\frac{V_1 \cos \delta_2}{X_{12}} = -\frac{1}{X_{12}}$$

and
$$\frac{\partial g_1}{\partial V_3} = -\frac{V_1 \cos \delta_3}{X_{13}} = -\frac{1}{X_{13}}$$

The power injections at buses 2 and 3 are

$$h_2 + jg_2 = V_2 e^{j\delta_2} \left[\frac{V_2 e^{-j\delta_2} - V_1}{-jX_{12}} + \frac{V_2 e^{-j\delta_2} - V_3 e^{-j\delta_3}}{-jX_{23}} \right]$$

$$= \frac{V_2^{\ 2} - V_1 V_2 e^{j\delta_2}}{-jX_{12}} + \frac{V_2^{\ 2} - V_2 V_3 e^{j(\delta_2 - \delta_3)}}{-jX_{23}}$$

$$= \frac{V_2^{\ 2} e^{j90}}{X_{12}} - \frac{V_1 V_2}{X_{12}} e^{j(90+\delta_2)} + \frac{V_2^{\ 2}}{X_{23}} e^{j90} - \frac{V_2 V_3}{X_{23}} e^{j(90+\delta_2-\delta_3)}$$

$$= j\frac{V_2^{\ 2}}{X_{12}} - \frac{V_1 V_2}{X_{12}} \cos (90 + \delta_2) - j\frac{V_1 V_2}{X_{12}} \sin (90 + \delta_2)$$

$$+ j\frac{V_2^{\ 2}}{X_{23}} - \frac{V_2 V_3}{X_{23}} \cos (90 + \delta_2 - \delta_3) - j\frac{V_2 V_3}{X_{23}} \sin (90 + \delta_2 - \delta_3)$$

Hence, $$h_2 = \frac{V_1V_2}{X_{12}} \sin \delta_2 + \frac{V_2V_3}{X_{23}} \sin (\delta_2 - \delta_3)$$

and $$g_2 = \frac{V_2^{\ 2}}{X_{12}} + \frac{V_2^{\ 2}}{X_{23}} - \frac{V_1V_2}{X_{12}} \cos \delta_2 - \frac{V_2V_3}{X_{23}} \cos (\delta_2 - \delta_3)$$

Similarly $$h_3 = \frac{V_1V_3}{X_{13}} \sin \delta_3 + \frac{V_2V_3}{X_{23}} \sin (\delta_3 - \delta_2)$$

and $$g_3 = \frac{V_3^{\ 2}}{X_{13}} + \frac{V_3^{\ 2}}{X_{23}} - \frac{V_1V_3}{X_{13}} \cos \delta_3 - \frac{V_2V_3}{X_{23}} \cos (\delta_3 - \delta_2)$$

Now $$\frac{\partial h_2}{\partial \delta_2} = \frac{V_1V_2}{X_{12}} \cos \delta_2 + \frac{V_2V_3}{X_{23}} \cos (\delta_2 - \delta_3) = \frac{1}{X_{12}} + \frac{1}{X_{23}}$$

$$\frac{\partial h_2}{\partial \delta_3} = -\frac{V_2V_3}{X_{23}} \cos (\delta_2 - \delta_3) = -\frac{1}{X_{23}}$$

$$\frac{\partial g_2}{\partial V_2} = \frac{2V_2}{X_{12}} + \frac{2V_2}{X_{23}} - \frac{V_1V_2}{X_{12}} - \frac{V_2V_3}{X_{23}}$$

$$= \frac{2}{X_{12}} + \frac{2}{X_{23}} - \frac{1}{X_{12}} - \frac{1}{X_{23}} = \frac{1}{X_{12}} + \frac{1}{X_{23}}$$

$$\frac{\partial g_2}{\partial V_3} = -\frac{V_2}{X_{23}} \cos (\delta_2 - \delta_3) = -\frac{1}{X_{23}}$$

$$\frac{\partial h_3}{\partial \delta_2} = -\frac{V_2V_3}{X_{23}} \cos (\delta_3 - \delta_2) = -\frac{1}{X_{23}}$$

$$\frac{\partial h_3}{\partial \delta_3} = \frac{V_1V_3}{X_{13}} \cos \delta_3 + \frac{V_2V_3}{X_{23}} \cos (\delta_3 - \delta_2) = \frac{1}{X_{13}} + \frac{1}{X_{23}}$$

$$\frac{\partial g_3}{\partial V_2} = -\frac{V_3}{X_{23}} \cos (\delta_3 - \delta_2) = -\frac{1}{X_{23}}$$

$$\frac{\partial g_3}{\partial V_3} = \frac{2V_3}{X_{13}} + \frac{2V_3}{X_{23}} - \frac{V_1}{X_{13}} \cos \delta_3 - \frac{V_2}{X_{23}} \cos(\delta_3 - \delta_2)$$

$$= \frac{2}{X_{13}} + \frac{2}{X_{23}} - \frac{1}{X_{13}} - \frac{1}{X_{23}} = \frac{1}{X_{13}} + \frac{1}{X_{23}}$$

$$\text{Hence } \frac{\partial f}{\partial X} = \begin{bmatrix} \dfrac{\partial h}{\partial \delta} & 0 \\ 0 & \dfrac{\partial g}{\partial V} \end{bmatrix} = \left[\begin{array}{cc|cc} -\dfrac{1}{X_{12}} & -\dfrac{1}{X_{13}} & & \\ \dfrac{1}{X_{12}}+\dfrac{1}{X_{23}} & -\dfrac{1}{X_{23}} & & \\ -\dfrac{1}{X_{23}} & \dfrac{1}{X_{13}}+\dfrac{1}{X_{23}} & & \\ \hline & & -\dfrac{1}{X_{12}} & -\dfrac{1}{X_{13}} \\ & & \dfrac{1}{X_{12}}+\dfrac{1}{X_{23}} & -\dfrac{1}{X_{23}} \\ & & -\dfrac{1}{X_{23}} & \dfrac{1}{X_{13}}+\dfrac{1}{X_{23}} \end{array}\right]$$

To obtain Hessian matrix $H = \dfrac{\partial f^T}{\partial X} R^{-1} \dfrac{\partial f}{\partial X}$

$$= (4 \times 6)\,(6 \times 6)\,(6 \times 4)$$

$$= (4 \times 4)$$

$$\left(\frac{\partial f}{\partial x}\right)^{\mathrm{T}} \mathrm{R}^{-1} = \begin{bmatrix} -\dfrac{1}{X_{12}} & \dfrac{1}{X_{12}}+\dfrac{1}{X_{23}} & -\dfrac{1}{X_{23}} & 0 & 0 & 0 \\ -\dfrac{1}{X_{13}} & -\dfrac{1}{X_{23}} & \dfrac{1}{X_{13}}+\dfrac{1}{X_{23}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{X_{12}} & \dfrac{1}{X_{12}}+\dfrac{1}{X_{23}} & -\dfrac{1}{X_{23}} \\ 0 & 0 & 0 & -\dfrac{1}{X_{13}} & -\dfrac{1}{X_{23}} & \dfrac{1}{X_{13}}+\dfrac{1}{X_{23}} \end{bmatrix}$$

$$\times \begin{bmatrix} W_1 & & & & & \\ & W_2 & & & 0 & \\ & & W_3 & & & \\ & & & W_1 & & \\ & 0 & & & W_2 & \\ & & & & & W_3 \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{W_1}{X_{12}} & W_2\left(\dfrac{1}{X_{12}}+\dfrac{1}{X_{23}}\right) & -\dfrac{W_3}{X_{23}} & 0 & 0 & 0 \\ -\dfrac{W_1}{X_{13}} & -\dfrac{W_2}{X_{23}} & W_3\left(\dfrac{1}{X_{13}}+\dfrac{1}{X_{23}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{W_1}{X_{12}} & W_2\left(\dfrac{1}{X_{12}}+\dfrac{1}{X_{23}}\right) & -\dfrac{W_3}{X_{23}} \\ 0 & 0 & 0 & -\dfrac{W_1}{X_{13}} & \dfrac{W_2}{X_{23}} & W_3\left(\dfrac{1}{X_{13}}+\dfrac{1}{X_{23}}\right) \end{bmatrix}$$

$$\left(\frac{\partial f}{\partial X}\right)^{T} R^{-1} . \begin{bmatrix} -\frac{1}{X_{12}} & -\frac{1}{X_{13}} & 0 & 0 \\ \frac{1}{X_{12}}+\frac{1}{X_{23}} & -\frac{1}{X_{23}} & 0 & 0 \\ -\frac{1}{X_{23}} & \frac{1}{X_{13}}+\frac{1}{X_{23}} & 0 & 0 \\ 0 & 0 & -\frac{1}{X_{12}} & -\frac{1}{X_{13}} \\ 0 & 0 & +\frac{1}{X_{12}}+\frac{1}{X_{23}} & -\frac{1}{X_{23}} \\ 0 & 0 & -\frac{1}{X_{23}} & \frac{1}{X_{13}}+\frac{1}{X_{23}} \end{bmatrix}$$

Since $\frac{\partial h}{\partial \delta}$ and $\frac{\partial g}{\partial V}$ are symmetrical. Hence

Hessian $$H = \frac{\partial f^T}{\partial X} R^{-1} \frac{\partial f}{\partial X} = \left[\begin{array}{cc:cc} & A_1 & 0 & 0 \\ & & 0 & 0 \\ \hdashline 0 & 0 & & \\ 0 & 0 & & A_2 \end{array}\right]$$

Hence,

$$A_1 = A_2 = \left[\begin{array}{c|c} \frac{W_1}{X_{12}^2}+W_2\left(\frac{1}{X_{12}}+\frac{1}{X_{23}}\right)^2+\frac{W_3}{X_{23}^2} & \frac{W_1}{X_{12}X_{13}}-\frac{W_2}{X_{23}}\left(\frac{1}{X_{12}}+\frac{1}{X_{23}}\right)-\frac{W_3}{X_{23}}\left(\frac{1}{X_{13}}+\frac{1}{X_{23}}\right) \\ \frac{W_1}{X_{12}X_{13}}-\frac{W_2}{X_{23}}\left(\frac{1}{X_{12}}+\frac{1}{X_{23}}\right)-\frac{W_3}{X_{23}}\left(\frac{1}{X_{13}}+\frac{1}{X_{23}}\right) & \frac{W_1}{X_{13}^2}+\frac{W_2}{X_{23}^2}+W_3\left(\frac{1}{X_{13}}+\frac{1}{X_{23}}\right)^2 \end{array}\right]$$

Now substituting values of X_{12}, X_{13} and X_{23} in the expression for $\frac{\partial h}{\partial \delta}$, we have

$$\frac{\partial h}{\partial \delta} = \begin{bmatrix} -\frac{1}{0.03} & -\frac{1}{0.01} \\ \frac{1}{0.03}+\frac{1}{0.02} & -\frac{1}{0.02} \\ -\frac{1}{0.02} & \frac{1}{0.01}+\frac{1}{0.02} \end{bmatrix} = \begin{bmatrix} -33.3 & -100 \\ 83.3 & -50 \\ -50 & 150 \end{bmatrix} = \frac{\partial g}{\partial V}$$

Substituting $W_1 = 3$, $W_2 = 5$ and $W_3 = 2$ in the above expression for A_1, we have

$$[A_1] = \begin{bmatrix} 3327+34700+5000 & 9990-20825-15000 \\ -25835 & 30{,}000+12500+45000 \end{bmatrix} = \begin{bmatrix} 43027 & -25835 \\ -25835 & 87500 \end{bmatrix}$$

$$[A_1]^{-1} = \begin{bmatrix} 2.825\times10^{-5} & 8.34\times10^{-6} \\ 8.34\times10^{-6} & 1.39\times10^{-5} \end{bmatrix}$$

The new value of the state variable *i.e.,* V^{k+1} and δ^{k+1} can be obtained using the relation

$$V^{k+1} = V^k + [A_1]^{-1} \frac{\partial g^T}{\partial V} R_r^{-1} [Z_m - g(X^k)]$$

and

$$\delta^{k+1} = \delta^k + [A_1]^{-1} \frac{\partial h^T}{\partial \delta} R_r^{-1} [Z_m - h(X^k)]$$

Assuming flat voltage profile for all the buses *i.e.,* taking voltages as $(1.05 + j0.0)$ for all the three buses we first find the computed values of measured quantities

$$h_1 = h_2 = h_3 = 0$$

and

$$g_1 = \frac{V_1V_2}{X_{12}} - \frac{V_1V_2}{X_{12}} \cos \delta_2 + \frac{V_1{}^2}{X_{13}} - \frac{V_1V_2 \cos \delta_3}{X_{13}} = 0$$

as

$$|\ V_1\ | = |\ V_2\ | = |\ V_3\ | = 1.05 \text{ and } \delta_1 = \delta_2 = \delta_3 = 0$$

Hence, $\quad g_1 = g_2 = g_3 = 0$

Substituting values of h^s and g^s in the iterative equations, we have

$$\begin{bmatrix} V_2{}^{k+1} \\ V_3{}^{k+1} \end{bmatrix} = \begin{bmatrix} V_2{}^k \\ V_3{}^k \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} -33.3 & 83.3 & -50 \\ -100 & -50 & 150 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -0.24 \\ -0.24 \\ 0.50 \end{bmatrix}$$

$$= \begin{bmatrix} 1.05 \\ 1.05 \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} -33.3 & 83.3 & -50 \\ -100 & -50 & 150 \end{bmatrix} \begin{bmatrix} -0.72 \\ -1.20 \\ 1.00 \end{bmatrix}$$

$$= \begin{bmatrix} 1.05 \\ 1.05 \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} -125.98 \\ 282 \end{bmatrix}$$

$$= \begin{bmatrix} 1.048793 \\ 1.0528692 \end{bmatrix}$$

Similarly,

$$\begin{bmatrix} \delta_2^1 \\ \partial_3^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} -33.3 & 83.3 & -50 \\ -100 & -50 & 150 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.12 \\ 0.21 \\ -0.30 \end{bmatrix}$$

$$= \begin{bmatrix} \delta_2{}^0 \\ \delta_3{}^0 \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} -33.3 & 83.3 & -50 \\ -100 & -50 & -150 \end{bmatrix} \begin{bmatrix} 0.36 \\ 1.05 \\ -0.60 \end{bmatrix}$$

$$= \begin{bmatrix} \delta_2{}^0 \\ \delta_3{}^0 \end{bmatrix} + 10^{-5} \begin{bmatrix} 2.825 & 0.834 \\ 0.834 & 1.39 \end{bmatrix} \begin{bmatrix} 105.477 \\ -178.5 \end{bmatrix}$$

$$= \begin{bmatrix} \delta_2{}^0 \\ \delta_3{}^0 \end{bmatrix} + \begin{bmatrix} 149.1035 \\ -161.5 \end{bmatrix} \times 10^{-5} = \begin{bmatrix} 0.001491035 \\ -0.001616 \end{bmatrix} \textbf{ Ans.}$$

Using these values of V and δ and assuming the Jacobian and Hessian matrices to remain same as in Ist iteration we calculate $h(V, \delta)$ and $g(V, \delta)$ and obtain new and better values of V and δ. We continue the process till the difference in V and δ in the consecutive iteration is less than a prespecified value we take as 10^{-4}.

It is to be noted in mathematical terms we should evaluate Jacobian and Hessian matrices corresponding to the actual value of state estimates otherwise the computational process may not converge. In power system we evaluate only once the Jacobian and Hessian matrices corresponding to state estimates as $1 + j0.0$ as our actual solution is not very far from these values. We operate our system within ± 5% variation of state *i.e.* V and δ and hence our assumption is practically justified.

After the best estimate of state is known, the best estimate for the p.u injection S_1, S_2, S_3,... etc. can be computed.

It is to be noted that there is similarity between load flow solution of power system and state estimation of power system. In both the situations we are interested in finding out the voltages both magnitude and phase at all the buses except at slack bus or reference bus. However, the state estimate provides better results as here, power even at the slack bus is specified.

This general method of state estimation can include measurement of current and voltages. However, since the phase relations are not known, inclusion of these measurements slows down the rate of convergence of final results. As a result on-line state estimation program make little use of these data and employ small weighting factors (equivalent to ignoring these data) when it is included.

23.4 DETECTION AND IDENTIFICATION OF BAD DATA

The errors associated with transducers, the analog to digital and vice versa devices, the communication links (some communication lines may be snapped and not be providing any data) bring in lot of problems for assessment of correct state estimation. The ability to detect and identity bad measurements is very important for power system operators. The measurement errors are random numbers and hence statistics theory would be employed to tackle this problem of detection and identification of bad data even though the procedure is lengthy but straight forward.

The procedure we will follow is based on this fact that for a given configuration the residual $J(X)$ calculated after the state estimator algorithm converges will be smallest if there are not bad data measurement present. When $J(X)$ is the smallest the state vector X (voltage magnitude and phase angles) has been found and corresponding to this state the various quantities like active and reactive powers generated and consumed by the load are compared with those which were measured and these are found to be close to each other. However, if there is a bad measurement the converged value of $J(X)$ will be relatively larger than expected with $x = x^{\text{est}}$.

The question then arises what value of $J(X)$ will determine whether the system has some bad measurements or not. As, if we choose a smaller value of $J(X)$, there may be wrong alarms of bad data presence when there is none and if we choose a larger value than the optimum value of $J(X)$, there may be some bad measurements which may go undetected. It is here that we make use of statistics to provide us a suitable value of $J(X)$ which if selected for comparison would help us in detecting the presence of bad measurements.

The measurement errors are random errors and hence the value of $J(X)$ is also a random number. If we assume that all the errors are described by their respective normal probability density functions, then it can be shown that $J(X)$ has a probability density function known as chi-square distribution which is written as $\chi^2(N)$ where N is known as degree of freedom of the chi-squared distribution and is given by

$$N = N_m - N_s$$

where N_m = number of measurements. It is to be noted that the measurement $P + jQ$ at a bus is taken as two measurements

N_s = number of states $(2n - 1)$ where n is the no. of buses in the system. It can be shown using statistical theory that when $X = X^{\text{est}}$ the mean value of $J(X)$ equals N and the standard deviation $\sigma_{j(\underline{X})}$ equals $\sqrt{2N}$.

Chi-squared distribution helps us to choose a threshold value t_j for $J(X)$ so that if $J(X) > t_j$ there are chances of bad data being present in the measurement. Otherwise there is again certain probability that the bad data may not be present. This can be written as follows:

$$P(J(X)) > t_i \mid J(X) \text{ is a chi-squared}) = \alpha \tag{23.39}$$

Equation (23.39) suggests that the probability that $J(X)$ is greater than t_j is equal to α given that the probability density for $J(X)$ is Chi-squared with N degrees of freedom.

This procedure usually adopted to detect whether there is bad data measurement present or not, is known as hypothesis testing and the parameter α is called the significance level of the test. Once we know the degree of freedom and we select a suitable value of α, we can find out the threshold value of $J(X)$ from the χ^2 table as shown here.

Table of χ^2

Probability (P)

DF	*0.50*	*0.10*	*0.05*	*0.02*	*0.01*	*0.005*	*0.001*
1	0.45	2.71	3.84	5.41	6.64	7.88	10.83
2	1.39	4.61	5.99	7.82	9.21	10.60	13.82
3	2.37	6.25	7.82	9.84	11.34	12.84	16.27
4	3.36	7.78	9.49	11.67	13.28	14.86	18.47
5	4.35	9.24	11.07	13.39	15.09	16.75	20.51
6	5.35	10.65	12.59	15.03	16.81	18.55	22.46
7	6.35	12.02	14.07	16.62	18.48	20.28	24.32
8	7.34	13.36	15.51	18.17	20.09	21.96	26.13
9	8.34	14.68	16.92	19.68	21.67	23.59	27.88
10	9.34	15.99	18.31	21.16	23.21	25.19	29.59

The parameter α decides what level of confidence we want in our assessment. Usually a confidence level of 95% is selected which means fake alarms will occur only in 5% of the cases. If we take 3-bus system of Example 23.2 where the no. of measurements $M = 6$ and the no. of state variables is $(2n - 1) = 5$, hence degree of freedom is $M - N_s = 6 - 5 = 1$. From the χ^2 table the threshold value t_j for dof as 1 and $\alpha = 0.05$ is 3.84 and for $\alpha = 0.01$ it is 6.64. When using a t_j derived in this manner the probability of a fake alarm is equal to α. By setting α to a small number say $\alpha = 0.05$ we would say that false alarms would occur in only 5% of the tests made. Suppose there are certain bad datas we will find that the estimator converges but at a higher value than the t_j. In fact after X^{est} the estimate is found. We substitute this value of X^{est} in the performance index and we obtain $J(X^{\text{est}})$ and we will see that the value is more than the selected value t_j.

So far we have been able to detect that our measurements have certain bad data. The next step would be to identify which datas are bad datas.

We have assumed that the error in measurement Z_i is normally distributed with zero mean value, the probability density function is centred on the true value of Z_i Fig. 23.4(*a*). Since the errors on all the measurements are assumed normal, we will assume that the estimate X^{est} is approximately normally distributed and that any quantity that is a function of X^{est} is also normally distributed quantity. Since $f(X^{\text{est}})$ is the calculated quantity of estimate X^{est}, Fig. 23.4(*b*) shows the probability density function for the calculated megawatts or megavars flows f_i. We have chosen here standard deviation for f_i smaller as compared to the measured quantities as measurements have higher redundancy or error as compared to calculated quantities.

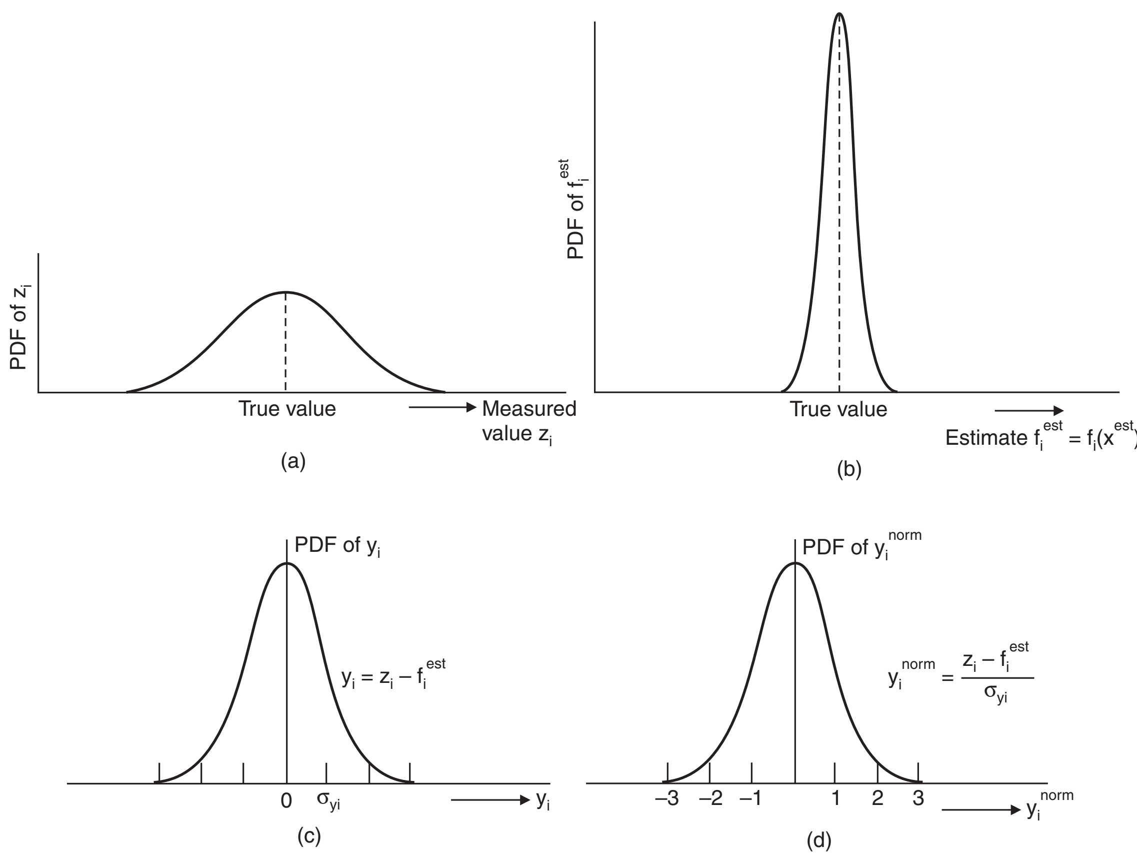


Fig. 23.4 Probability density function of the normalized measurement.

We define here now what are known as normalised (standard) quantities as explained earlier in Fig. 23.2.

The difference between the calculated quantities f_i and measured quantity is known as the measurement residual y_i. The probability density function for y_i is also normal as shown in Fig. 23.4(c) which has zero mean and a standard deviation of σ_{y_i} .

Now $$D = \frac{f_i - Z_i}{\sigma_{y_i}}$$

is known as normalised measurement residual or standard normal deviate. This may be designated as y_i^{norm} and is shown in Fig. 23.4(d). This has normal distribution with $\sigma = 1$. Corresponding to 1% probability ($P(0.01)$) the $D = y_i^{\text{norm}} = 3$. However for 5% probability $D = y_i^{\text{norm}} = 2$. If the absolute value of $y_i^{\text{norm}} > 3$, we have good reason to expect that Z_i measurement is a bad measurement value.

Once it is assumed that bad data is present, following procedure is adopted to identify the bad data. From the estimator X^{est}, all $f_i\,s$ are calculated and from the corresponding Z_i^s, y_i are calculated. From the state estimator, standard deviation σ_{yi} (equation 23.14 (a)) can be calculated and hence D or y_i^{norm} can be calculated. Measurement having the largest absolute normalised residual are labelled as prime suspects. The measurement with the largest normalised residue is eliminated and the state estimator program is repeated. This results in a different X^{est} and hence a different $J(X)$. Since one of the measurements has been eliminated, the degree of freedom has been reduced by one and hence for the same value of α, the threshold value of $J(X)$ will be different and it has to be obtained from the χ^2 table and value of $J(X)$ is to be compared against new value of t_j. If the new $J(X)$ is less than new value of t_j, we have eliminated a true bad data. If however new $J(X)$ is greater than the new t_j we must calculate $f_i(X^{\text{est}})$, σ_{yi} and y_i^{norm} for each of the remaining measurements. The measurement with the largest absolute y_i^{norm} is then again eliminated and the entire procedure repeated successively until $J(X)$ is less than t_j. It has been observed that in certain situations a group of measurements may have to be eliminated to eliminate one 'bad' measurement which means that the identification procedure often cannot pin point a single bad measurement but instead identifies a group of measurements one of which is bad.

Summarising the procedure for detection and identification of bad data, it can be said that

For detection – use chi-squared method

For identification – use normalised standard deviation or normalised residuals.

An important feature of the state estimator is its ability to calculate or estimate missing quantities (quantities not being telemetered). This is true in case of failure of some of the communication channels which cannect operation centres to data processing equipments. Many a times data from the substations are not available because no transducers or data gathering equipments were ever installed.

23.5 STATE ESTIMATOR LINEAR MODEL

For this d.c. power flow model is required which has been discussed in chapter 18. Equation (18.61) is reproduced here and is given as

$$\left[\frac{\Delta P_p}{E_p}\right] = [B'] \, [\Delta\delta] \tag{23.40}$$

Assuming E_p = 1.0 p.u., the equation is rewritten as

$$[\Delta P_p] = [B'] \, [\Delta\delta] \tag{23.41}$$

where $B_{ik} = -\dfrac{1}{x_{ik}}$ of diagonal element corresponding to ith row and jth column *i.e.,* the branch between nodes i and j and $B_{ii} = \sum\limits_{k=1}^{N} \dfrac{1}{x_{ik}}$ the diagonal element *i.e.,* the sum of susceptances of all branches connected to bus i.

Since the matrix $[B']$ is constant and does not change during successive iterations for solution of the equations, this need be evaluated only once and inverted once during their first iteration and then used in all successive iterations.

It is to be noted that the linear model representing dc power flow is only useful for calculating MW power flows on transmission lines and transformers. It does not take into account voltage magnitudes or reactive power flows on these circuits. The power flowing on each line using the dc power flow is then

$$P_{ik} = \frac{1}{x_{ik}} \, [\theta_i - \theta_k] \tag{23.42}$$

and

$$P_i = \sum_{k=1}^{N} P_{ik}$$

where k is the bus connected to bus i.

If function $f_i\,(x_1, x_2, ..., x_N)$ are linear functions, equation (23.23) has a closed form solution

We write the function $f_i\,(x_1, x_2, ..., x_N)$ as

$$f_i(x_1, x_2, ..., x_N) = f_i(X) = A_{i1}x_1 + A_{i2}x_2 + \ldots + A_{iN}x_N \tag{23.43}$$

Now if we put all f_i functions in a matrix form

$$f(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_m(X) \end{bmatrix} = [A]\, X \tag{23.44}$$

where $[A]$ is an $N_m \times N_s$ matrix consisting of the co-efficient of the linear functions $f(X)$

Similarly

$$Z_m{}^{\text{meas}} = \begin{bmatrix} Z_{1m} \\ Z_{2m} \\ \vdots \\ Z_{mm} \end{bmatrix}$$

Hence equation (23.23) can be written in compact form

$$\text{Min } J(X) = [Z^{\text{meas}} - f(X)]^T \, [R^{-1}] \, [Z^{\text{meas}} - f(X)] \tag{23.45}$$

Where $$R = \begin{bmatrix} \sigma_1^{\ 2} & & & \\ & \sigma_2^{\ 2} & & \\ & & \cdots & \\ & & & \sigma_{Nm}^{\ 2} \end{bmatrix}$$

where R is known as the covariance matrix of measurement errors. Substituting $f(X) = [A]\, X$ from equation (23.44) into (23.45) and rearranging we have

$$\text{Min } J(X) = \{Z^{\text{meas}T}[R^{-1}]\, Z^{\text{meas}} - X^T[A]^T\, [R^{-1}]\, Z^{\text{meas}} - Z^{\text{meas}T}\, [R^{-1}]\, [A]\, X + X^T\, [A]^T\, [R^{-1}]\, [A]\, X\} \quad (23.46)$$

To obtain condition for min $J(X)$ we take the first derivative of $J(X)$ and equate it to zero we have, the first term is zero, the second and third term's derivative gives

$-\,2\, [A]^T\, [R^{-1}]\, Z^{\text{meas}}$ and the last term's derivative is $2[A]^T\, [R^{-1}]\, [A]\, X$

Hence, $$0 = -\,2\, [A]^T\, [R^{-1}]\, Z^{\text{meas}} + 2\, [A]^T\, [R^{-1}]\, [A]\, X$$

or $$[A]^T\, [R^{-1}]\, [A]\, X = [A]^T\, [R^{-1}]\, Z^{\text{meas}}$$

or $$X^{\text{est}} = \{[A]^T\, [R^{-1}]\, [A]\}^{-1}\, [A]^T\, [R^{-1}]\, Z^{\text{meas}} \quad (23.47)$$

It is to be noted that the equation (23.47) holds good when the number of measurements is greater than the no. of state variables. However, if the no. of state variables equals the no. of measurements, the state variables X^{est} is given as

$$X^{\text{est}} = [A]^{-1}\, Z^{\text{meas}} \quad (23.48)$$

The third possibility is that no. of measurement is less than the no. of unknowns which is an under determined system and usually in power system state estimation such situations are not handled directly. Rather pseudo-measurements are added to the existing set of measurements to give a completely determined system or an over determined system.

Example 23.3. Consider the 3 bus system shown here: $P_{m12} = 0.12$, $P_{m13} = 0.21$, $P_{m23} = -\,0.3$, $X_{13} = 0.1$, $X_{12} = 0.3$, $X_{23} = 0.2$. Assume that the meter has ± 3 MW error over a full scale of 100 MW. Assuming that the errors are distributed according to normal probability density function as shown in Fig. 23.2 with standard deviation σ, the meter will give a reading within ± 3σ (± 3 MW) of the true value being measured for approximately 99% of the time and σ = ± 1 MW or σ = 0.01 p.u.

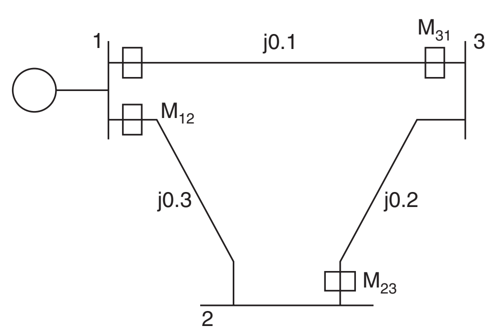


Fig. E23.3

It is to be noted that the probability of an error decreases as the error magnitude increases.

For the three bus system, we have

$$X^{\text{est}} = \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} \quad (\text{E } 23.1)$$

The elements of a matrix are obtained from the following measurements:

$$f_{12} = \frac{1}{0.3}\ (\theta_2 - \theta_2) = -\ 3.33\ \theta_2 \text{ as } \theta_1 = 0.0$$

$$f_{31} = \frac{1}{0.10}\ (\theta_3 - \theta_1) = +\ 10\ \theta_3$$

$$f_{23} = \frac{1}{0.20}\ (\theta_2 - \theta_3) = 5\theta_2 - 5\theta_3$$

$$A = \begin{bmatrix} -\ 3.33 & 0 \\ 0 & +\ 10 \\ 5 & -\ 5 \end{bmatrix} A^T = \begin{bmatrix} -\ 3.33 & 0 & 5 \\ 0 & 10 & -\ 5 \end{bmatrix}$$

The covariance matrix $[R]$ for the measurements is

$$[R] = \begin{bmatrix} 10^{-4} & & 0 \\ & 10^{-4} & \\ 0 & & 10^{-4} \end{bmatrix}. \text{ Hence, } R^{-1} = \begin{bmatrix} 10^4 & & 0 \\ & 10^4 & \\ 0 & & 10^4 \end{bmatrix}$$

and the measurements are $\begin{bmatrix} 0.12 \\ 0.21 \\ -\ 0.3 \end{bmatrix}$.

Since the no. of measurements is greater than the no. of state variables, we make use of equation (23.47) for estimation of states. Substituting the values in equation (23.47) we have

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \left\{ \begin{bmatrix} -\ 3.33 & 0 & 5 \\ 0 & +10 & -\ 5 \end{bmatrix} \begin{bmatrix} 10^4 & & \\ & 10^4 & \\ & & 10^4 \end{bmatrix} \begin{bmatrix} -\ 3.33 & 0 \\ 0 & +\ 10 \\ 5 & -\ 5 \end{bmatrix} \right\}^{-1}$$

$$\times \begin{bmatrix} -\ 3.33 & 0 & 5 \\ 0 & +\ 10 & -\ 5 \end{bmatrix} \begin{bmatrix} 10^4 & & \\ & 10^4 & \\ & & 10^4 \end{bmatrix} \begin{bmatrix} 0.12 \\ 0.21 \\ -\ 0.30 \end{bmatrix}$$

$$= \left\{ 10^4 \begin{bmatrix} 36.08 & -\ 25 \\ -\ 25 & 125 \end{bmatrix} \right\}^{-1} \begin{bmatrix} -\ 1.9 \\ +\ 3.6 \end{bmatrix} 10^4$$

$$= \begin{bmatrix} 0.032175 & 0.006435 \\ 0.006435 & 0.009287 \end{bmatrix} \begin{bmatrix} -\ 1.9 \\ 3.6 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -\ 0.0379665 \\ 0.0212067 \end{bmatrix}$$

From these estimated phase angles we can calculate power flows in various transmission lines

$$f_{12} = -\ 3.33\ (-\ 0.0379665) = 0.126428$$

$$f_{31} = 10 \times 0.0212067 = 0.212067$$

$$f_{23} = 5\,(-\,0.037965 - 0.212067) = -\,0.2958$$

Hence the performance index $J(\theta_2, \theta_3)$ is given as

$$J(\theta_2, \theta_3) = \frac{(0.12 - 0.126428)^2 + (0.21 - 0.212067)^2 + (-\,0.3 + 0.2958)^2}{10^{-4}}$$

$$= 0.41319 + 0.04272 + 0.1764$$

$$= 0.6323$$

From χ^2 table for 1 degree of freedom and probabilities $P = 0.01$, the χ^2 value is 6.64 which is much more than 0.6323 and hence the estimate is correct.

Example 23.4. For a three bus system suppose the measurements are $Z_{12} = 0.6$, $Z_{13} = 0.05$, $Z_{32} = 0.35$ and $X_{12} = 0.2$, $X_{13} = 0.4$ and $X_{23} = 0.25$. Taking bus 3 as reference and using the linear model, determine θ_1 and θ_2 and assume $\sigma_2 = 10^{-4}$.

Solution. To determine [A] we express computed powers in terms of θ_1 and θ_2 as follows:

$$f_{12} = \frac{1}{0.2}\,(\theta_1 - \theta_2) = 5\theta_1 - 5\theta_2$$

$$f_{13} = \frac{1}{0.4}\,(\theta_1 - \theta_3) = 2.5\theta_1$$

$$f_{32} = \frac{1}{0.25}\,(\theta_3 - \theta_2) = 4\theta_3 - 4\theta_2 = -\,4\theta_2$$

$$A = \begin{bmatrix} 5 & -\,5 \\ 2.5 & 0 \\ 4 & -\,4 \end{bmatrix} \text{ and } A^T = \begin{bmatrix} 5 & 2.5 & 0 \\ -\,5 & 0 & -\,4 \end{bmatrix}$$

Using equation (23.47), we have

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \left\{ \begin{bmatrix} 5 & 2.5 & 0 \\ -\,5 & 0 & -\,4 \end{bmatrix} \begin{bmatrix} 10^{-4} & & \\ & 10^{-4} & \\ & & 10^{-4} \end{bmatrix}^{-1} \begin{bmatrix} 5 & -\,5 \\ 2.5 & 0 \\ 0 & -\,4 \end{bmatrix} \right\}^{-1}$$

$$\times \begin{bmatrix} 5 & 2.5 & 0 \\ -\,5 & 0 & -\,4 \end{bmatrix} \begin{bmatrix} 10^{-4} & & \\ & 10^{-4} & \\ & & 10^{-4} \end{bmatrix}^{-1} \begin{bmatrix} 0.60 \\ 0.05 \\ 0.35 \end{bmatrix}$$

The second product term is simplified as

$$10^4 \begin{bmatrix} 5 & 2.5 & 0 \\ -\,5 & 0 & -\,4 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.05 \\ 0.35 \end{bmatrix} = \begin{bmatrix} 3.125 \\ -\,4.4 \end{bmatrix} \times 10^4$$

The first term is simplified as follows:

$$\left\{10^4 \begin{bmatrix} 5 & 2.5 & 0 \\ -5 & 0 & -4 \end{bmatrix} \begin{bmatrix} 5 & -5 \\ 2.5 & 0 \\ 0 & -4 \end{bmatrix}\right\}^{-1} = \left\{\begin{bmatrix} 31.25 & -25 \\ -25 & 41 \end{bmatrix} 10^4\right\}^{-1}$$

Hence,

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 312500 & -250000 \\ -250000 & 410000 \end{bmatrix}^{-1} \begin{bmatrix} 3.125 \times 10^4 \\ -4.4 \quad \times 10^4 \end{bmatrix}$$

$$= \begin{bmatrix} 31.25 & -25 \\ -25 & 41 \end{bmatrix}^{-1} \begin{bmatrix} 3.125 \\ -4.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0625 & 0.0381 \\ 0.0381 & 0.047637 \end{bmatrix} \begin{bmatrix} 3.125 \\ -4.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.02767 \\ -0.0905403 \end{bmatrix}$$

Now, we calculate the power flows in various lines

$$f_{12} = 5\,(\theta_1 - \theta_2) = 5\,(0.02767 + 0.0905403) = 0.59105$$

$$f_{13} = 2.5\theta_1 = 2.5 \times 0.02767 = 0.069175$$

$$f_{32} = -4\theta_2 = 4 \times 0.0905403 = 0.36216$$

Hence, the performance index $J(\theta_1, \theta_2)$ is given as

$$J(\theta_1, \theta_2) = \frac{(0.6 - 0.59106)^2 + (0.05 - 0.069175)^2 + (0.35 - 0.36216)^2}{10^{-4}}$$

$$= 0.799 + 3.67 + 1.46 = 5.929$$

From the χ^2 table for 1 degree of freedom and probability $P = 0.01$, the value of χ^2 is 6.64 which is more than 5.929 and hence the estimate is correct.

23.6 THE ROLE OF STATE ESTIMATION IN POWER SYSTEM OPERATIONS

In this section we try to learn the role of power system state estimation in the operation of power system and how state estimation, contingency evaluation and generator corrective action take place in a modern operations control center. Figure 23.5 shows a block diagram indicating the information flow between various functions to be performed in an operations control centre computer system.

The information about power system is collected from the remote terminal units (RTU) which encodes measurement transducer outputs and opened/closed status information into digital signals that are transmitted to the operations centre over communication channels. However, the control centre in turn transmits control information in terms of raise/lower commands to generators and open/close commands to circuit breakers and switches. The information coming into the control centre is divided into two parts as breaker/switch status indication and analog measurements as shown in Fig. 23.5. The analog measurements of generator output are used directly by the automatic generation control (AGC) and all other data is processed by the state estimator and the output from the state estimator which is in terms of the voltage magnitude and phase angles of buses is used in evaluating transmission line or the line loadings in terms of MW and MVAr flows, bus loads and generations calculated from the line flows. These quantities together with the electrical model developed by the network topology program (discussed below) provide the basis for contingency evaluation, economic load dispatch or optimal power flow and generation corrective action program.

Network topology means the interconnection of various components of power *viz*., generators, loads, transmission lines etc. through circuit breakers/switches. Network topology plays an important role in estimating the state of power system. Since the switches and breakers in any

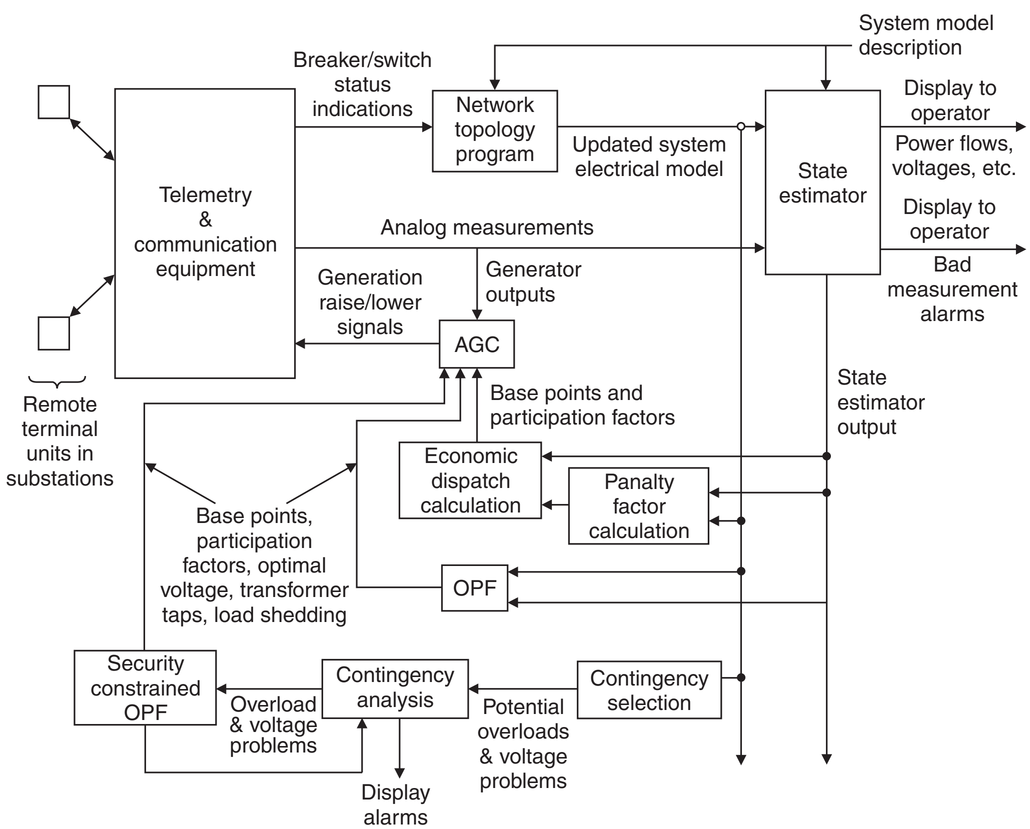


Fig. 23.5 Operations control centre computer system.

substation can cause the network topology to change, a program is provided that reads the telemetered breaker/switch status indications and restructures the electrical model of the power system. This program which reconfigures the electrical model is known as Network Topology Program. This program has complete information of each substation and how the transmission lines are connected to the substation equipments. The bus sections which are connected to other bus sections through the closed circuit breakers or bus couplers correspond to the same electrical bus. Thus the number of electrical buses and the manner in which they are interconnected can be simulated in the model to replace the actual operating conditions of the system.

QUESTIONS

23.1. Explain what you mean by state estimation of power system and how does it differ from load flow solutions.

23.2. Discuss the problems associated with measurements of various quantities on power system. How do they affect the power system state estimation process?

23.3. Discuss briefly various jobs expected of a power system state estimator.

23.4. Develop step by step the mathematical model for a state estimator using line power flows with the help of weighted least square method as suggested by Dopazo *et al*.

23.5. Explain what are weighting factors and how these affect the convergence of the computation process for state estimator.

23.6. Explain various steps of the algorithm used for estimation of states of power system using weighted least square method and discuss the limitations of this method.

23.7. Explain the terms Standard deviation, Standard normal curve, Biased and unbiased error distribution.

23.8. Show that if the probability density function of measurement errors is a normal distribution (Gaussian), the maximum likelihood criterion for state estimation is identical to weighted least square method.

23.9. What is meant by variance of meter error ? Given two normal distributions curves with different variance which one would you prefer for better estimator.

23.10. What is a fast decoupled state estimator ? List out various steps associated with algorithm of this estimator.

23.11. What do you mean by "Bad Data" in power system ? How does it creep in while obtaining a good state estimator ? Explain clearly how it is taken care of by using theory of probability.

23.12. Explain clearly how Bad Data is detected and identified.

23.13. Explain linear model of system for state estimation and derive equations for the state estimator.

23.14. Discuss the role of state estimator in power system operation.

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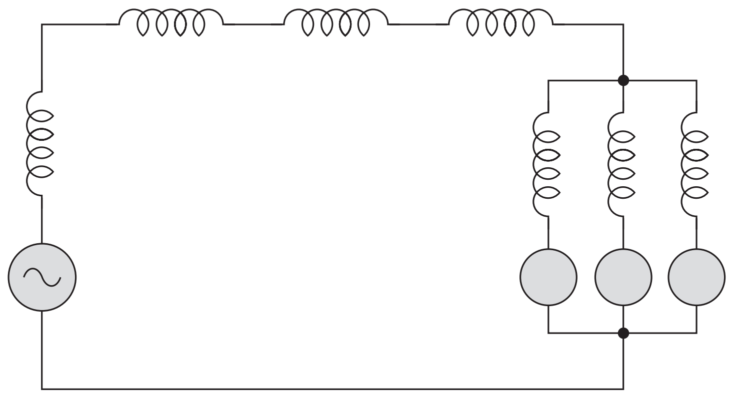
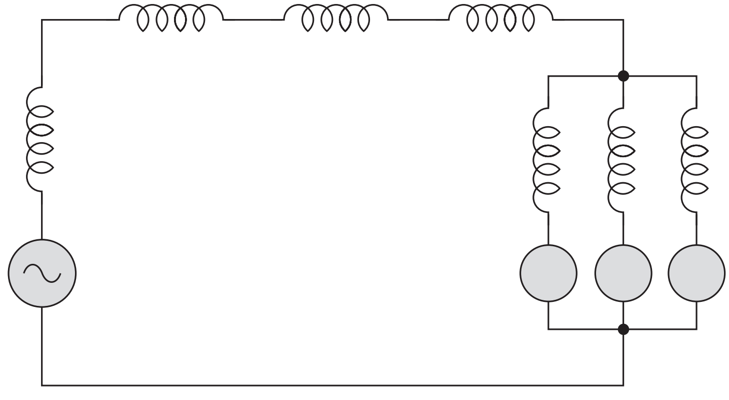


24

UNIT COMMITMENT



24

Unit Commitment

24.1 INTRODUCTION

The energy requirements on power system keeps on changing not only over the years, but it keeps on changing even over a day. Generally the energy requirement is higher during the day time and during early evenings when industrial loads are high, lights in the commercial establishments, historical monuments, national highways consume large amount of electric energy. However during late in the evenings and early mornings when most of the establishments are closed the energy requirement is low. Similarly the energy requirement during week days is higher as compared to weekend days. Also in a country like India the energy requirement during summer is much more as compared to rest of the seasons.

One option would be to connect all the available generating units on to the bus bar all the time so that any amount of energy requirement can be met. But this is highly uneconomical as during off-peak periods some of the units may not be contributing much, yet we are spending lot of fuel to keep them synchronised with the grid. A lot of saving in fuel cost can be made by switching off some of the units (generators) when these are not required. Unit commitment is, therefore, one way to suggest just sufficient number of generating units with sufficient generating capacity to meet a given load economically with sufficient reserve capacity to meet any abnormal, operating condition *e.g.,* failure of one of the generators *i.e.,* to commit generating capacity to obtain reliable and economical supply for a given load. We will for the time being consider the unit commitment problem purely from economic point of view.

Example 24.1: Suppose in a plant we have the units with specification as follows.

Unit 1 Min 160 MW Max 600 MW

$$F_1 = 600 + 7.1\, P_1 + 0.00141\, P_1^2$$

Unit 2 Min 100 MW Max 450 MW

$$F_2 = 350 + 7.80\, P_2 + 0.00195\, P_2^2$$

Unit 3 Min 50 MW Max 250 MW

$$F_3 = 80 + 8.0\, P_3 + 0.0049\, P_3^2$$

Suppose we are to feed a load of 500 MW. Our problem is what units should be committed to meet this load economically. We can meet the load with various combinations *e.g.,* we may run a combination of unit 1 and 2, unit 1 and 3 or unit 2 and 3. It is to be noted that unit with fuel cost having largest fixed component of cost will have smaller running costs *i.e.*, the coefficients with P and P^2 will be smallest. Compare the cost characteristics of the three units mentioned above. The fixed component of cost of unit 1 is 600 and is largest as compared to 350 and 80 for units 2 and 3. Similarly if we compare the co-efficients of P and P^2 for the three cost characteristics, we find that, these co-efficients are lowest for the first unit and since the maximum power generation capacity of the unit is 600 MW and our load requirement is only 500 MW, hence unit 1 can be committed for feeding the load. This unit not only supplies the load requirement, it supplies most economically out of any other combinations *e.g.,* if we take a combination of unit 1 and unit 2, this will be able to supply the load but the cost involved will be more than what unit 1 alone will cost.

One may even go for combination of unit 2 and 3, still it is possible to meet the load requirement but the cost involved will be even more than what it costs with unit combination 1 and 2. In fact for 500 MW load, unit 1 alone is the economic unit commitment. It is to be noted that unit 1 is the most cost effective, next is unit 2 and the least cost effective out of the three units is unit 3. In order to find the cost in each combination of units to meet a certain load, equal incremental cost of production strategy is used which has been discussed in detail in chapter 19.

As given in section 19.1 of the book the minimum power generation is limited by the flame instability of a boiler and maximum power generation is limited by the overheating of the armature winding. Therefore, during applying equal incremental cost of production strategy if the optimum power requirement is less then $P_{\min}$ or greater than $P_{\max}$, the output of the power from the corresponding generator is limited to its violated limit and the rest of the load is distributed through the remaining units.

Let us consider load variation from 1300 MW to 500 MW in steps of 100 MW. To obtain a "shut down rule" we use brute-force technique wherein all combination of units will be tried for each load value taken in steps of 100 MW from 1300 to 500 MW. The shut-down rule is simple.

When load is above 1050 MW, commit all the three units and between 1050 and 600 commit unit 1 and 2, and below 600 MW unit 1 is enough. The results of applying the brute-force technique are given in the following table 24.1:

Table 24.1 Shut down rule for the example

Load	*Unit 1*	*Unit 2*	*Unit 3*
1300	on	on	on
1200	on	on	on
1100	on	on	on
1000	on	on	off
900	on	on	off
800	on	on	off
700	on	on	off
600	on	off	off
500	on	off	off

If the unit commitment problem were limited only to commit sufficient units to meet a certain load, the problem is supposed to have been solved with the above commitment schedule. Unfortunately it is not true as we have considered this problem as almost unconstrained. We have considered only constraints of P_{min} and P_{max} as the generation. There are other constraints to which the system is subjected which should be considered before we can sort out the optimum unit commitment problem. Some of the constraints are listed here under.

24.2 SPINNING RESERVE

Spinning reserve is defined as the total amount of generation available from all units synchronised to the grid (or system) minus the load demand and the losses being supplied. Spinning reserve is very essential in an interconnected system as loss of a large unit in the system may result in under frequency problem. However, if sufficient spinning reserve is available in the system, the loss of one of the units can be taken care of by the remaining units in the system.

The allocation of magnitude of spinning reserves is usually decided by reliability considerations and also the economics associated with it.

Some of the electric utilities prefer to have the spinning reserve a given percentage of forecasted peak demand; some prefer to have the spinning reserve equivalent to the largest capacity unit in the system.

It is to be noted that not only sufficient spinning reserve be made available, it should be spread around the power system to avoid any problem due to transmission network failure to carry that power. Also the requisite spinning reserve should be made available even when "islanding" of system takes place. Therefore, both the allocation and location of spinning reserve is important.

The total spinning reserve should be suitably distributed among the fast responding units and slow responding units as this allows the automatic generation control (AGC) system to restore frequency and interchange quickly when one of the unit fails.

Besides spinning reserves, the unit commitment problem may involve various other types of "scheduled reserves" or "off-time reserves". These reserves include quick-start diesel or gas turbine and hydro units including pumped storage units which can be brought on to the bus bar in a few minutes. These units should be considered in the overall reserve capacity assessment. However, their time to come to full capacity should be taken into account.

Example 24.2: In order to illustrate the reserve capacity requirements consider two isolated grid systems, northern grid and southern grid, which are interconnected through tie-lines as shown in Fig. E.24.1. There are three units in the northern region and two in the southern committed to supply a load of 3320 MW. The maximum permissible tie-line loading is 550 MW in either directions.

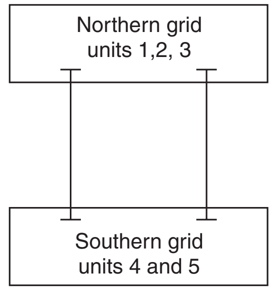


Fig. E.24.1 Two grids interconnected.

Table 24.2 shows data for the two grid interconnected system.

Table 24.2 Data for the system in the figure

Region	*Units*	*Unit capacity MW*	*Unit output MW*	*Regional generation*	*Spinning reserve*	*Regional load*	*Interchange MW*
Northern	1	1200	1000	2000	200	2200	200
	2	800	500		300		
	3	800	500		300		
Southern	4	1200	1000	1320	200	1120	200
	5	600	320		280		
Total	1—5	4600	3320	3320	1280	3320	

It can be seen that for example for unit 1 in the northern system, the loss of any unit on the system can be covered by the spinning reserve on the remaining units. Unit 1 however presents a problem. If unit 1 was lost and unit 2 and 3 were to be run at their maximum capacities which totals to 1600 MW, the local grid load is 2200, hence 600 MW should be transported over the tie line from southern grid to northern grid. Since the maximum permissible tie line loading is 550 MW only, therefore, the loss of unit 1 can't be covered even though the entire system has ample reserve. The only solution under this working condition (loading) is to commit more units to operate in the northern region.

24.3 THERMAL UNIT CONSTRAINTS

It is to be noted that thermal plants require operating personnel to take into account the temperature and pressures of steam before a unit can be switched on or off. If a unit is in operation for sometime, it cannot be switched off instantly. It takes sometime before it could be switched off. This is known as minimum up time. Similarly if a unit is decommitted *i.e.*, it is not in operation or connected to the grid, it takes certain minimum (4 to 8 hrs.) to recommit it or connect it to the grid. This is known as minimum down time. Also there is constraint on the number of operating personnel in the thermal plants especially if there are more than one unit in the plant. An optimum number of operating personnel is desirable.

A thermal unit is never connected to the grid until the steam attains certain temperature and pressure for better efficiency of the plant. The fuel input to the unit from cold to the state when it can be connected to the grid, does not produce any electric energy yet it is very much required, which is known as start-up cost. The start-up cost is not fixed as it depends upon the initial state of the unit. If the unit were initially in cold condition, the start up cost is maximum. However, if the unit was turned off recently and is still relatively close to operating temperature and pressure, the start up cost is much lower. The first type of start-up cost is known as cold start up cost and the second one is known as banking start-up cost. The two types of start-up costs are given empirically as follows:

Start-up cost cold-start = $F_c(1 - e^{-t/\alpha}) \times F + F_f$

where F_c = cold start cost (MBtu)

F = full cost

F_f = fixed cost (salary of operating personnel and maintenance expenses)

α = thermal time constant of unit

t = time in hrs. the unit was cooled

Start-up cost during banking = $F_t \times t \times F + F_f$

Where F_t is cost (MBtu/hr) of maintaining unit at operating temperature and pressure.

If we plot the two costs on a common plane as shown in Fig. 24.1, we find that up to certain number of hours the banking cost is less than cold start-up cost.

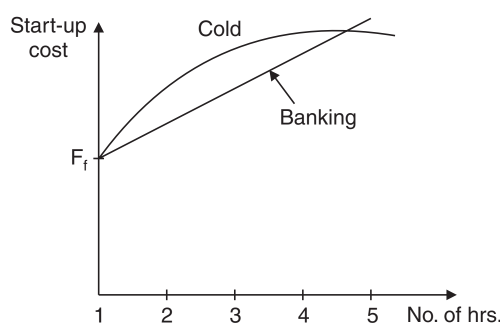


Fig. 24.1 Time dependent start-up cost.

It is to be noted that P_{min} and P_{max} of a unit also changes due to maintenance or unscheduled outages and hence this factor must be taken into consideration in unit commitment.

24.4 UNIT COMMITMENT SOLUTION METHODS

Some of the methods for solution of unit commitment problem are listed below:

1. Priority list
2. Dynamic programming
3. Lagrange multiplier

24.4.1 Priority List Method

This is the simplest of all unit commitment methods and consists of preparing a priority list of units. As discussed earlier a simple shut-down rule or priority list should be obtained after an exhaustive enumeration of all unit combinations at each load level. The priority list of the example 1 can be obtained by calculating the full load (maximum power capacity), average production cost of each unit where the full load average production cost is simply the net heat rate at full load multiplied by fuel cost.

Example 24.3: The fuel cost for the three units of example 24.1 are given here under

Fuel cost (1) = Rs 1.1/ MBtu

Fuel cost (2) = Re 1.0/MBtu

Fuel cost (3) = Rs 1.2/MBtu

The average cost for the three units is calculated as follows:

$$F_1 = 600 + 7.1\, P_1 + 0.00141\, P_1^2 \qquad P_{max} = 600 \text{ MW}$$

$$= 600 + 4260 + 507.6 = 5367.6 \text{ MBTu/hr.}$$

Hence, full load average production cost

$$= \frac{5367.6 \times 1.1}{600} = 9.84 \text{ Rs./MWhr}$$

$$F_2 = 350 + 7.80\, P_2 + 0.00195\, P_2^2 \qquad P_{2\,max} = 450 \text{ MW}$$

$$350 + 3510 + 394.87 = 4254.87$$

Hence, full load average production cost

$$= \frac{4254.87 \times 1}{450} = 9.455 \text{ Rs./MWhr}$$

$$F_3 = 80 + 8P_3 + 0.0049P_3^2 \qquad P_{3\,max} = 250 \text{ MW}$$

$$= 80 + 2000 + 306.25 = 2386.25$$

Hence, full load average production cost

$$= \frac{2386.25 \times 1.2}{250} = 11.45 \text{ Rs./MWhr}$$

Therefore, the priority list for the three units based on the average production cost is as follows:

Unit	*Rs/MWhr*	*Min MW*	*Max MW*
2	9.455	100	450
1	9.8455	160	600
3	11.45	50	250

Assuming the start-up cost, minimum up/down costs to be negligible, the unit commitment scheme is as follows:

Combination	*Min. MW from combination*	*Max. MW from combination*
2 + 1 + 3	310	1300
2 + 1	260	1050
2	100	450

Note that there is variation in the functioning of unit commitment as discussed in this example and that in example 24.1. In example 24.1, unit 2 was shut down at 600 MW leaving unit 1 on to the busbar. However, in the scheme discussed here (priority list scheme) both the units would be held on until load reached 450 MW, then unit 1 would be dropped and unit 1 will remain connected to the busbars.

Taking into consideration the cost associated with minimum up/down time of units, start-up, the priority list is built around a simple shut-down algorithm as given here.

1. When the load is dropping at each hour, find out whether dropping the next unit on the priority list will leave sufficient generation to meet the load demand and have sufficient spinning reserve capacity. If yes, go to the next step. If no, continue operating as it is without dropping any unit.
2. Determine the number of hours H, before the unit will be required again *i.e.,* assuming that the load is dropping and it will pick up again some hours later.
3. If H is less than the minimum shut down time for the unit, allow the unit to remain connected and go to last step. If not, go to next step.
4. Calculate the two costs. One is due to the hourly production costs for the next H hours with the unit up. The other is the unit down cost plus the cost associated with the start-up cost (cold unit) or start-up banking and see whichever is less expensive. If there is sufficient savings from shutting down the unit, it should be shut down, otherwise keep it committed to the system.
5. Repeat the entire procedure for the next unit on the priority list. If it is also to be dropped, go to the next unit and so on.

The priority list method discussed above provides solution which takes into consideration the full load average cost rate, as the criterion for priority fixing gives good results only if following assumptions hold good.

1. No load costs are zero.
2. The system is an unconstrained one.
3. Start-up costs are a fixed amount.

24.4.2 Dynamic Programming

The next method to consider solution of unit commitment problem is dynamic programming. In this method following assumptions are made.

1. The start-up cost of a unit is assumed to be constant and is independent of the time it has been off-line.
2. Shutting down cost of a unit is zero.

3. There is a definite priority order of units and is such that in each interval of time a specified minimum amount of capacity must be operating.

It is to be noted that unit commitment table is to be obtained for the complete load cycle. If the load increases in small but finite size steps, perhaps dynamic programming method is better as compared to other methods as this requires a fewer unit combinations to be tried and for optimal unit commitment the solution of equations due to equal incremental production cost strategy is not required.

In order to prepare optimal unit commitment table, the known quantities are: total no. of units, their minimum and maximum permissible outputs, the cost characteristics of individual units and hour to hour load cycle. Also, it is assumed that the load on each unit or combination of units changes in suitably small step sizes and uniformly. The dynamic programming is based on designing a recursive formula and it is proceeded as follows.

We start with any two arbitrarily selected units, the most economic combination is obtained for all the discrete loads of the combined output of the two units. For each load, the economic strategy would be to run either unit or both the units sharing loads economically between them. The most economic cost curve in discrete form for the two units thus obtained can be considered as the cost curve of a single equivalent unit. The third unit is now added and the procedure is repeated. For this the first two units are to be treated as a single unit as we have already prepared a composite cost characteristic for the two units and hence no combination of third unit with first or second is to be tried again. Therefore, considering the composite cost characteristic of the first two units and the cost characteristic of the third unit, for different loading, the cost characteristics of the three combined units is obtained. Thus this procedure results in considerable saving in computational effort. This procedure of adding one unit and determining the cost characteristic of the new combination is obtained till all the available units have been taken into account. The advantage of the dynamic programming algorithm is clear that if we know the optimal way of loading n units, it is easy to determine the procedure for optimal loading of $(n + 1)$ units.

Suppose we define the cost function $F_N(x)$ as follows:

$F_N(x)$ = Minimum cost in Rs/hr of generating x MW by N units.

and $f_N(y)$ = Cost of generating y MW by the Nth unit.

$F_N(x - y)$ = The minimum cost of generating $(x - y)$ MW by the remaining units $(N - 1)$.

Therefore, the application of dynamic programming algorithm results in the following recursive relation

$$F_N(x) = \min_{y} \{F_{N-1}(x - y) + f_N(y)\} \tag{24.1}$$

From the above recursive relation it is easy to find out the combination of units, which will result in minimum operating cost for loads varying in small steps from the minimum permissible load of the smallest unit to the sum of the maximum permissible capacities of all available units. Therefore, with this procedure the total minimum operating cost and the load shared by each unit of the optimal combination are automatically obtained for each load.

Example 24.4: Let us illustrate the application of dynamic programming for preparing an optimal unit commitment table for a sample system. The cost-characteristics of the first two units given in example 24.1 are reproduced here. The constant costs of the two units are eliminated as these will form a constant figure for every loading when the two units are simultaneously committed to provide the load.

$$F_1 = 7.1P_1 + 0.00141P_1^2 \qquad P_{min} = 160 \text{ MW} \qquad P_{max} = 600 \text{ M}$$

$$F_2 = 7.8P_2 + 0.00195P_2^2 \qquad P_{2\,min} = 100 \text{ MW} \quad P_{2\,max} = 450 \text{ MW}$$

Taking into consideration, the equal incremental production cost and also the minimal loading of the two units we proceed with a load of 500 MW and find out using dynamic programming the unit commitment of the two units.

Now $\qquad F_1(500) = f_1(500) = 7.1 \times 500 + 0.00141 \times 500^2 = 3902$

Since min. power output of second unit is 100 MW, therefore we find

$$F_1(400) + f_2(100) = 2840 + 225.6 + 780 + 19.5 = 3865.$$

Similarly $\qquad F_1(380) + f_2(120) = 2698 + 203.6 + 936 + 28.08 = 3865.68$

Next $\qquad F_1(360) + f_2(140) = 2556 + 182.7 + 1092 + 38.2 = 3868$

Therefore, for a load of 500 MW, the load commitment on unit 1 is 400 MW and that on 2 is 100 MW which gives minimum cost. Any other combination gives higher cost. So this is one point on the combined cost characteristic of the two units. Next we increase the load by 50 MW and make the load 550 MW.

If we load unit 1 which is more economical as compared to unit 2, the total cost is

$$F_1(550) = 3905 + 426.5 = 4331.5$$

However, if we distribute a part of load to unit 2, say we calculate

$$F_1(450) + f_2(100) = 3195 + 285.5 + 780 + 19.5 = 4280$$

$$F_1(400) + f_2(150) = 2840 + 225.6 + 1170 + 43.8 = 4279$$

$$F_1(350) + f_2(200) = 2485 + 172.7 + 1560 + 78 = 4295$$

Therefore, another point on the combined cost characteristic is $F_1(400)$, $f_2(150)$ with cost Rs. 4279/-. The cost for a few more loading (higher) can be tried and optimal unit commitment can be obtained. Using these discrete points, a suitable curve fitting, a square or cubic interpolation can be obtained and the curve can be stored for future application.

Next the procedure is repeated by including third unit. As mentioned earlier, there is no need to combine third unit with 1st or 2nd unit, as the first two units are already combined into a single unit and cost characteristic has already been obtained.

The new cost characteristic for different loading is obtained using the procedure as outlined for the first two units. This procedure is repeated for all the N units in the plant.

Once economic unit commitment table is prepared for all units in the plant, as the load on the plant changes, it would only require changes in starting and stopping of units.

24.4.3 Lagrange Relaxation Method

The dynamic programming method of obtaining unit commitment solution has some difficulties especially for large power system with large number of units as this forces the dynamic programming solutions to search over a small number of commitment states to reduce the combinations which must be tried in each time interval.

In Lagrange relaxation method even though these disadvantages are not there but it has it's own computational complications which must be looked into carefully.

Since it is a unit commitment problem, there are two possibilities for a unit whether it is connected to the bus bar or not at a particular instant of time. We define a variable.

$$U_i^t = 0 \text{ if unit } i \text{ is off-line during interval } t$$
$$= 1 \text{ if unit is on-line during interval } t.$$

Since, the Lagrangian relaxation takes into account the equality and inequality constraints on the system alongwith the main objective function. These are defined as follows:

1. Equality or loading constraint

$$P_{\text{load}}^t - \sum_{i=1}^{N} P_i^t \,.\, U_i^t = 0 \text{ for } t = 1, 2, \ldots T$$

2. Inequality constraints on active power generation of units

$$U_i^t\, P_i^{\min} \le P_i^t \le U_i^t\, P_i^{\max} \text{ for } i = 1, 2, \ldots N \text{ and } t = 1, 2, \ldots T$$

where T is the total cycle period.

It is to be noted that a few more constraints in the form of generator fuel limit constraints, spinning reserve constraints etc. could be added if required.

3. Unit minimum up and down-time constraint.

4. The objective function

$$\text{Min} \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_i^t) + \text{start-up cost}_{i,t}]\; U_i^t = \min\, [F(P_i^t\, U_i^t)] \tag{24.2}$$

The augmented objective function or Lagrange function is given as [(similar to what we did in chapter XIX) under load dispatch]

$$\text{Min}\ \angle\, [P, U, \lambda] = \text{Min } F(P_i^t\, U_i^t) + \sum_{t=1}^{T} \lambda^t \left(P_{\text{load}}^t - \sum_{i=1}^{N} P_i^t\, U_i^t\right) \tag{24.3}$$

For optimal unit commitment, the objective is to minimise the Lagrange function subject to the constraints listed at 2 and 3. It is to be noted that when the power balance equation at 1 is satisfied, the minimum of Lagrange function coincides with the minimum of the objective function listed at 4.

The cost-function in (24.2), the inequality constraints at (2) and unit commitment up and down costs for individual units are independent of rest of the units. However, the equality constraints *i.e.*, the power balance equation, the generated powers of all units are interlinked *i.e.*, if power of one unit is changed, the power generated by other units will also change.

The unit commitment problem defined above is solved using Lagrangian relaxation method wherein the equality constraints are ignored (relaxed) to begin with and dual optimisation procedure is adopted.

In this procedure, attempt is made to reach the constrained optimum by maximising the Lagrangian with respect to λ, the Lagrangian multiplier, while minimising with respect to the other variables *i.e.,*

$$q^*(\lambda) = \max_{\lambda^t} q(\lambda)$$

where $$q(\lambda) = \min_{P_i^t U_i^t} \mathcal{L} (P, U, \lambda)$$

The procedure given above is summarised here.

1. Determine λ^t in each iteration so that $q(\lambda)$ increases. It is to be noted that λ will have different values during different intervals of time.

2. Having obtained optimum value of λ^t in step 1, we keep its value fixed and we obtain the minimum value of Lagrangian by varying power generation P_i^t and U_i^t *i.e.,* whether the unit i is committed or not.

Suppose we have obtained an optimum value of λ^t, the procedure to be followed for minimising the Lagrangian is as follows:

We rewrite the Lagrangian as

$$\mathcal{L} = \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_i^t) + \text{start-up cost}_{i,t}]\ U_i^t + \sum_{t=1}^{T} \lambda^t \left(P_{\text{load}}^t - \sum_{i=1}^{N} P_i^t U_i^t \right)$$

This is rewritten as

$$\mathcal{L} = \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_i^t) + \text{Start-up cost}_{i,t})\ U_i^t + \sum \lambda^t P_{\text{load}}^t - \sum_{t=1}^{T} \sum_{i=1}^{N} P_i^t U_i^t$$

The term $\sum \lambda^t P_{\text{load}}^t$ above is constant as both λ^t and P_{load}^t are constants and hence this term can be eliminated.

Therefore, the Lagrangian is given as

$$\text{Min } \mathcal{L} = \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \{[F_i(P_i^t) + \text{start-up cost}_{i,t}]\, U_i^t - \lambda^t P_i^t U_i^t\} \right)$$

From the above it is clear that for $i = 1$ *i.e.* unit 1, we consider its commitment optimally for $t = 1$, to $t = T$, *i.e.,* we have separated the units from one another.

i.e., $$\text{Min} \sum_{t=1}^{T} \{[F_i(P_i^t) + \text{start-up cost}_{i,t}]\, U_i^t - \lambda^t P_i^t U_i^t\}$$

The minimum of the Lagrangian is the minimum cost of generation of each unit summed up over the complete time period *i.e.,*

$$\text{Min } q(\lambda) = \sum_{i=1}^{N} \text{Min} \sum_{i=1}^{T} \{[F_i(P_i^t) + \text{start-up cost}_{i,t}]\, U_i^t - \lambda^t P_i^t U_i^t\} \text{ subject to}$$

$$U_i^t\ P_i^{\min} \le P_i^t \le U_i^t\ P_i^{\max} \text{ for } t = 1, \ldots T$$

and the cost associated with up and down-time constraints.

This problem can be solved as a dynamic programming problem in one variable. For a particular unit if $U_i^T = 0$ *i.e.,* the unit is not committed, its contribution to cost is zero. However, if $U_i^t = 1$, the function to be minimized is Min $[F_i(P_i) - \lambda^t P_i^t]$. The start-up cost is not included as it is independent of the power to be generated.

The condition for minimum of this function is obtained by equating the first derivative of the function to zero.

$$\frac{d}{dP_i^t}\ [F_i(P_i) - \lambda^t P_i^t] = \frac{d}{dP_i}\ F_i\,(P_i^t) - \lambda^t = 0$$

Here λ^t is known and hence P_i^{opt} can be calculated.

Depending upon the value of P_i^{opt} we obtain, there are three possibilities considering the inequality constraints on active power generation of the units.

1. $P_i^{\text{opt}} \le P_i^{\min}$, then Min $[F_i(P_i) - \lambda^t P_i^t] = F_i(P_i^{\min}) - \lambda^t P_i^{\min}$
2. $P_i^{\min} \le P_i^{\text{opt}} \le P_i^{\max}$, then Min $[F_i(P_i) - \lambda^t P_i^t] = F_i(P_i^{\text{opt}}) - \lambda^t P_i^{\text{opt}}$
3. $P_i^{\text{opt}} \ge P_i^{\max}$, then Min $[F_i(P_i) - \lambda^t P_i^t] = F_i(P_i^{\max}) - \lambda^t P_i^{\max}$

The solution of the two-state dynamic program for each unit proceeds in the same way as was done in the dynamic programming solution of the unit commitment problem in the previous section.

It is to be noted that in the unit commitment problem the dynamic program must take into account the start-up and shut down costs of individual units and the start up and down time constraints.

It is to be noted that λ is not a single number but it is a vector of values each of which should be adjusted during every iteration. The improved value of λ is given as

$$\lambda^t = \lambda^t + \alpha \left[\frac{dq(\lambda)}{d\lambda} \right]$$

where α is a parameter which is usually less than unity and is selected such that it changes λ by a small amount. The new value of λ should increase the function $q(\lambda)$. When the largest value of $q(\lambda)$ is obtained we define the convergence criterion of the overall Lagrangian relaxation technique by what is known as relative duality gap *i.e.,* $t = (J^* - q^*)/q^*$ where q^* is the maximum value of q and J^* is defined as follows.

Once each hour has enough generation committed *i.e.*, once dynamic programming has been used to get the unit commitment schedule when enough generation has been committed, the primal value J^* represents the total generation cost summed over all hours as calculated by the economic dispatch. When the relative duality gap is less than a prespecified value the computation process is terminated and the optimal unit commitment scheduled is obtained.

The flowchart for the Lagrange relaxation unit commitment algorithm is shown in Fig. 24.2.

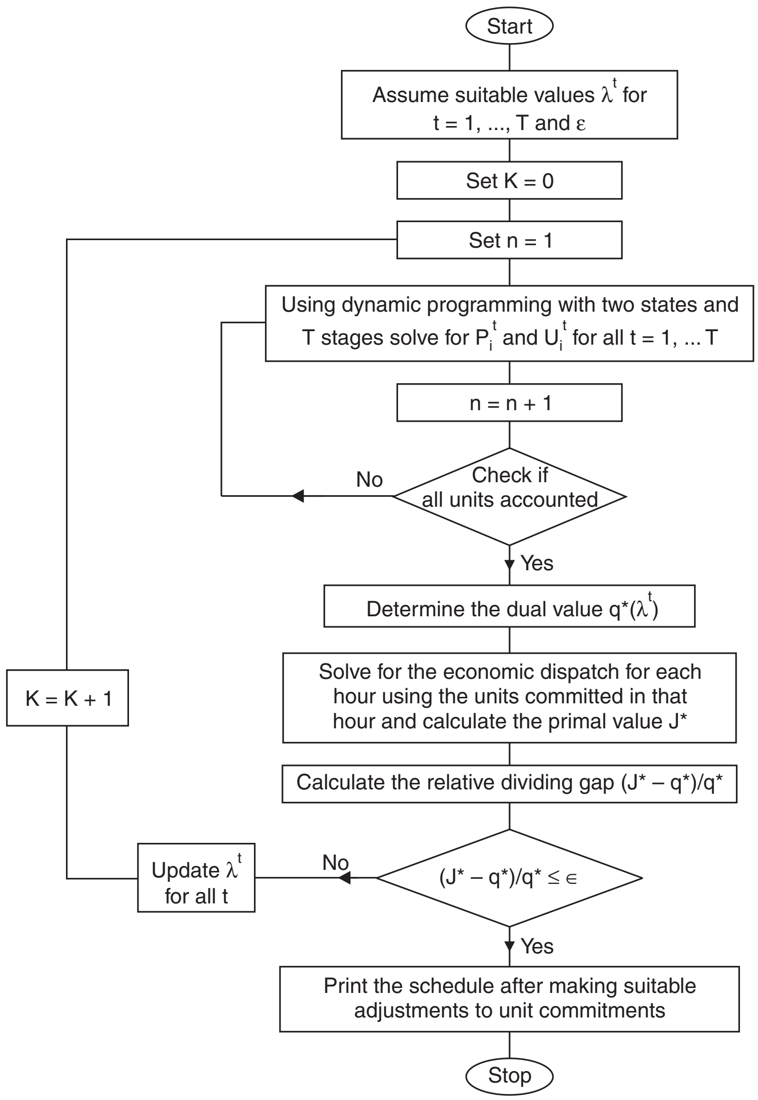


Fig. 24.2 Flowchart for Lagrange relaxation method for UC.

Following useful observations have been made about the dual optimisation technique when applied to the unit commitment problem:

1. For large power systems the relative duality gap criterion can be used for solution of optimal unit commitment problem. The larger the size of the system the smaller is the gap acceptable for convergence of the computation process.

2. When the convergence criterion is satisfied, the schedule so obtained is not necessarily a feasible schedule.

3. The convergence is unstable at the end which means that some units are being switched in and out and the process does not stabilise.

The duality gap is large at the beginning and becomes progressively smaller as the iterations increase. The solution provides unit commitment schedule with sufficient generation from the units so that an economic load dispatch schedule can be obtained.

QUESTIONS

24.1. Explain what you mean by unit commitment and how does it differ from economic load dispatch problem?

24.2. Discuss briefly various constraints imposed while solving unit commitment problem.

24.3. Discuss the importance of spinning reserve requirements in the solution of unit commitment-problem.

24.4. Discuss briefly the cold start-up cost and banking costs when referred to a thermal plant.

24.5. Explain briefly different methods of solving the unit commitment problem.

24.6. Explain with the help of a flowchart the Lagrange relaxation technique for solution of optimal unit commitment problem.

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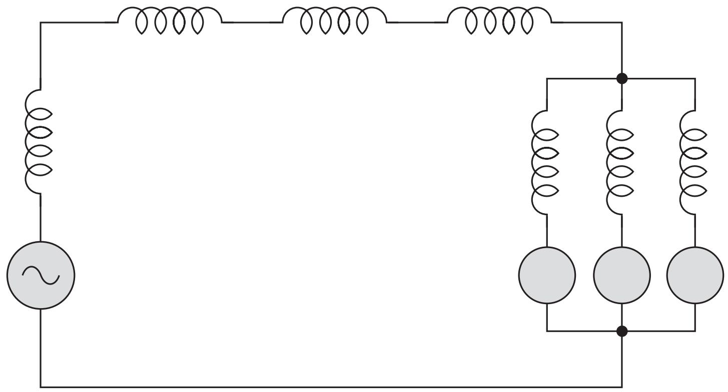
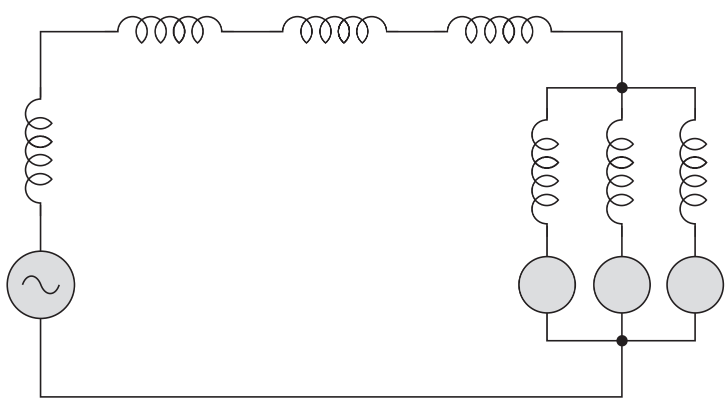


25

ECONOMIC SCHEDULING OF HYDROTHERMAL PLANTS AND OPTIMAL POWER FLOWS



25

Economic Scheduling of Hydrothermal Plants and Optimal Power Flows

25.1 INTRODUCTION

In a large interconnected power system various sources of electric energy *e.g.,* thermal, hydro, nuclear etc. are interconnected and attempt is made to optimise the operation of the system in terms of cost of generation to meet a certain load. However, here we will restrict our analysis for a hydro thermal interconnection.

The capital cost involved in a thermal power plant is much smaller as compared to a hydrostation while the running cost or operation cost for a thermal power plant is very high whereas it is practically zero for a hydroplant. In an interconnected system for a certain load duration curve, the thermal plant is used as a base load plant whereas a hydro plant is used to meet the peak portion of the curve. This approach is used as the hydro plants can be brought to bus bar very fast in a few minutes, they have higher reliability and greater speed of response and hence they can take care of fluctuating loads more efficiently and hence these are used for peak load demand whereas thermal plants which have a higher starting time in terms of hours and have slow speed of response are used as base load plants.

The main cost of operation of hydrothermal plants is the cost of fuel used in the thermal plants as the cost of water for hydroplant is negligible as compared to thermal plant. Hence the problem is to find the generation of both hydro and thermal power plants such that the total cost of generation to meet the load demand and transmission losses is minimum. We are considering here this as a short range problem and hence there will not be any appreciable change in the level of water in the reservoir during the interval and hence the head of water in the reservoir of the hydro plants will be assumed to be constant during the interval *i.e.,* the period of study. However, specified quantity of water q_i must be utilised within the interval at each hydroplant.

25.2 PROBLEM FORMULATION

The problem of economic scheduling of hydrothermal plant can be considered as one of minimising the cost of fuel of thermal plants subject to the constraint of water availability for hydrogeneration over a given period of operation *i.e.,* the problem is to determine $q(t)$ the water discharge in m^3/sec so as to minimise the cost of thermal generation

$$\text{Min } F_T = \sum F_n(P_T) \tag{25.1}$$

subject to

(*i*) The power balance equality

$$P_T + P_H - P_D - P_L = 0 \tag{25.2}$$

(*ii*) Water availability

$$W'(T) - W'(0) - \int_0^T W_i(t)\, dt + \int_0^T q(t)\, dt \tag{25.3}$$

where $W'(T)$ and $W'(0)$ are the storage at the end of the optimisation interval and at the beginning of the interval, $W_i(t)$ is the water inflow rate and $q(t)$ the water discharge rate. P_T and P_H are thermal and hydrogeneration respectively and P_D and P_L are total load demand and system loss respectively.

(*iii*) The hydrogeneration P_H is a function of the rate of discharge and water storage or head *i.e.*,

$$P_H = f(W'(t), q(t)) \tag{25.4}$$

We solve this problem by dividing the total time interval T into M subintervals each of time $\frac{T}{M} = \Delta T$. This is done to simplify the analysis as we assume that during each subinterval, all the variables remain fixed. The problem is therefore, redefined as

$$\underset{q^m (m = 1, \ldots\ldots m)}{\text{Min}} \sum_{m=1}^{M} F(P_T{}^m) \tag{25.5}$$

subject to the operating constraints

(*i*) Power balance equation

$$P_T{}^m + P_H{}^m - P_L{}^m - P_D{}^m = 0 \tag{25.6}$$

where the superscript m indicates the quantity during mth interval.

(*ii*) Water availability equation

$$W^m - W^{m-1} - W_i{}^m + q^m = 0 \tag{25.7}$$

where $W^m = W'^m/\Delta T$ and W^m and W^{m-1} are the specified storages at the beginning and end of the optimisation interval. $W_i{}^m$ and q^m are water inflow and water discharge in the mth interval.

(*iii*) Hydrogeneration in any sub interval can be expressed as

$$P_H{}^m = h_0 \{1 + 0.5\, e\, (W'^m + W'^{m-1})\}\, (q^m - \rho) \tag{25.8}$$

where $$h_0 = 9.81 \times 10^{-3}\, h_0'$$

h_0' is the height of the storage tank

$(W'^m + W'^{m-1})/2$ is the average additional height due to storage of water.

e is the water head correction factor to account for head variation with storage. ρ is the water discharge required to run hydrogenerator at no load.

The factor 9.81 is to account for the gravitation effect on the fall of water on turbine blades (mgh).

In hydrothermal economic scheduling we take water discharge in all sub-intervals except one as independent variables whereas the hydro, thermal generation, and water storages in all subintervals are treated as dependent variables. The water discharge in one of the subintervals is taken as a dependent variable as the sum total of discharges is a prespecified value and hence, out of the total discharge, the sum of discharges during $(M - 1)$ intervals will give the desired available discharge and one of the discharges is taken as dependent variable. Usually q^1 is chosen as a dependent variable and hence equation (25.7) corresponding to water availability can be rewritten as

$$q^1 = W^0 - W^M + \sum_m W_i^m - \sum_{m=2}^{M} q^m \tag{25.9}$$

The problem of economic hydrothermal scheduling is handled by making use of Lagrangian multiplier. Here the original objective function given by equation (25.5) is augmented by the addition of the equality constraints alongwith Lagrangian multiplier. The Lagrangian multiplier associated with hydrogeneration converts the water consumption in m³/hr to the cost in Rs/hr. which is termed as water value or water worth.

The augmented cost function is given as

$$F = \sum [F_n (P_T) - \lambda_1^m (P_T^m + P_H^m - P_L^m - P_D^m) + \lambda_2^m (W_m - W^{m-1} - W_i^m + q^m) + \lambda_3^m \{P_H^m - h_0(1 + 0.5\, e(W^{1m} + W^{1m-1})\,(q^m - \rho)\} \tag{25.10}$$

The Lagrangian multipliers can be obtained differentiating the augmented function w.r. t. dependent variables and equating it to zero.

$$\frac{\partial F}{\partial P_G{}^m} = \frac{\partial F_T(P_T)}{\partial P_G{}^m} - \lambda_1{}^m \left\{1 - \frac{\partial P_L{}^m}{\partial P_T}\right\} = 0$$

or

$$\frac{\partial F_T(P_T)}{\partial P_G{}^m} + \lambda_1 \frac{\partial P_L{}^m}{\partial P_T} = \lambda_1{}^m \tag{25.11}$$

This is the coordination equation as obtained in equation (19.6) of this book.

$$\frac{\partial F}{\partial P_H^m} = \lambda_3 - \lambda_1{}^m \left(1 - \frac{\partial P_L{}^m}{\partial P_H{}^m}\right) = 0$$

or

$$\lambda_1{}^m \frac{\partial P_L{}^m}{\partial P_H{}^m} = \lambda_1{}^m - \lambda_3{}^m \tag{25.12}$$

and

$$\frac{\partial F}{\partial W^m} = \lambda_2{}^m - \lambda_2{}^{m+1} - \lambda_3{}^m \{0.5\, h_0 e(q^m - \rho)\} - \lambda_3{}^{m+1} \{0.5\, h_0 e\, (q^{m-1} - \rho)\} \tag{25.13}$$

Now using equation (25.9) in equation (25.10) and differentiating w.r.t. dependent variable q'_s we have

$$\frac{\partial F}{\partial q'} = \lambda_2' - \lambda_3' \, h_0 \, \{1 + 0.5 \, e \, (2 \, W^0 + W_i^1 - 2q^1 + \rho)\} = 0 \tag{25.14}$$

Following procedure is used to obtain Lagrangian multipliers

(*i*) λ_1^m from equation (25.11)

(*ii*) λ_3^m from equation (25.12)

(*iii*) λ_2' from equation (25.14) and other values of λ_2^m $(m \neq 1)$ from equation (25.13)

For minimisation of the augmented function we differentiate the function with respect to the independent variable and obtain the gradient vector which should be zero if there are no inequality constraints

$$\left.\frac{\partial F}{\partial q^m}\right|_{m \neq 1} = \lambda_2^m - \lambda_3^m h_0 \, \{1 + 0.5e \, (2W^{m-1} + W_i^m - 2q^m + \rho)\} \tag{25.15}$$

For optimality the gradient vector should be zero.

The algorithm for the hydrothermal economic scheduling problem is given as follows:

1. Assume initial solution q^m $(m \neq 1)$ for all subintervals except the first.
2. The dependent variables W^m, P_H, P_T and q^1 are obtained using equations (25.7), (25.8),(25.6) and (25.1) respectively.
3. The Lagrangian multipliers λ_1^m, λ_3^m, λ_2^m $(m \neq 1)$ and λ_2^1 are obtained using equation (25.11), (25.12), (25.14) and (25.13).
4. The gradient vector using equation (25.15) is calculated and checked if all its elements are equal to zero within a prespecified accuracy. If yes, the optimal solution is reached, we go to step 5.
5. Obtain new values of independent variables as follows

$$q_{\text{new}}^m \approx q_{\text{old}}^m \sim \alpha \left(\frac{\partial F}{\partial q^m} \right) \qquad m \neq 1 \tag{25.16}$$

where α is a positive scalar. Go back to step (2).

It is to be noted that while optimisation process is carried during any iteration, if any of the independent variable violates its lower or upper limit it is fixed to the corresponding limit for that iteration. However, a general optimal power flow problem is being discussed in the next section where a comprehensive procedure for handling the equality and inequality constraints has been discussed.

Example 25.1: We now take up a simple hydrothermal system consisting of one thermal plant and one hydroplant and a load as shown in Fig. E.25.1(*a*) to illustrate the procedure for economic scheduling.

The load curve is shown in Fig. E.25.1(*b*). There is no water inflow into the reservoir of the hydroplant. The initial water storage in the reservoir is 120 m^3/sec and the final water storage should be 50 m^3/sec. Basic head $h_0' = 20$ m; water head correction factor $e = 0.006$; $\rho =$ 2 m^3/sec and incremental production for thermal plant is $\frac{dF_T}{dP_T} = P_T + 20$ Rs./MWhr. Neglect transmission losses.

Solution. The total water available for hydrogenerator is

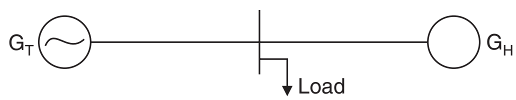


Fig. E.25.1(a)

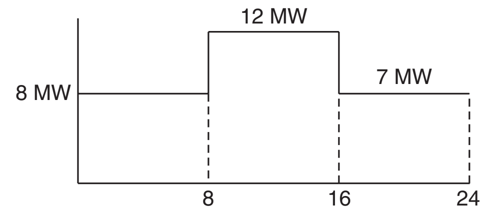


Fig. E.25.1(b)

$$120 - 50 = 70 \text{ m}^3/\text{sec.}$$

Since there are three subintervals, there are two independent variables q^2 and q^3.

Let $\quad q^2 = 25 \quad \text{and} \quad q^3 = 25 \text{ m}^3/\text{sec.}$

Hence $\quad q^1 = 70 - (25 + 25) = 20 \text{ m}^3/\text{sec.}$

It is given that $\quad W^0 = 120 \text{ m}^3/\text{sec.} \quad \text{and} \quad W^3 = 50 \text{ m}^3/\text{sec.}$

From equation (25.7) and since W_i water inflow is zero

$$W^1 = W^0 + W_i^1 - q^1 = 120 + 0 - 20 = 100 \text{ m}^3/\text{sec}$$

$$W^2 = W^1 + W_i^2 - q^2 = 100 + 0 - 25 = 75 \text{ m}^3/\text{sec.}$$

Now using equation (25.8) hydrogeneration for the subintervals can be calculated as follows

$$P_H^1 = 9.81 \times 10^{-3} \times 20 \,[1 + 0.5 \times 0.006\,(120 + 100)]\,(20 - 2)$$

$$= 9.81 \times 10^{-3} \times 20 \times 1.66 \times 18 = 5.86 \text{ MW}$$

$$P_H^2 = 9.81 \times 10^{3} \times 20 \,[1 + 0.5 \times 0.006\,(100 + 75)] \times 23$$

$$= 9.81 \times 10^{-3} \times 20 \times 1.825 \times 23 = 8.235 \text{ MW}$$

$$P_H^3 = 9.81 \times 10^{-3} \times 20 \,[1 + 0.5 \times 0.006\,(75 + 50)] \times 23$$

$$= 9.81 \times 10^{-3} \times 20 \times 1.325 \times 23 = 6.2 \text{ MW}$$

Therefore, thermal generation during three intervals is

$$P_T^1 = 8 - 5.86 = 2.14 \text{ MW}$$

$$P_T^2 = 12 - 8.23 = 3.77 \text{ MW}$$

$$P_T^3 = 7 - 6.2 = 0.8 \text{ MW}$$

Now $\quad \dfrac{dF_T}{dP_T} = P_T + 20 = \lambda_1^m$

The value of λ_1 for the three intervals

$$\lambda_1^1 = 20 + 2.14 = 22.14, \quad \lambda_1^2 = 20 + 3.77 = 23.77$$

and $\quad \lambda_1^3 = 20.8$

Since transmission losses are neglected, $\dfrac{dP_L}{dP_H}$ is zero and hence from equation (25.12)

$$\lambda_1^1 = \lambda_3^1 = 22.14, \; \lambda_1^2 = \lambda_3^2 = 23.77$$

and $\quad \lambda_1^3 = \lambda_3^3 = 20.8$

Now from equation (25.14)

$$\lambda_2{}^1 = \lambda_3{}^1\, h_0[1 + 0.5e\,(2W^0 + W_i{}^1 - 2q^1 + \rho)]$$
$$= 22.14 \times 0.1962\ [1 + 0.5 \times 0.006\ (240 + 0 - 40 + 2)]$$
$$= 22.14 \times 0.1962 \times 1.6 = 6.976$$

From equation (25.13) for $m = 1$ and 2, we have

$$\lambda_2^1 - \lambda_2^2 - \lambda_3^1\ \{0.5\ h_0 e\ (q^1 - \rho)\} - \lambda_3^2\ \{0.5\ h_0 e\ (q^2 - \rho)\} = 0$$

or $\quad \lambda_2^2 = \lambda_2^1 - \lambda_3^1\ \{0.5\ h_0 e\ (q^1 - \rho)\} - \lambda_3^2\ \{0.5\ h_0 e\ (q^2 - \rho)\}$

$$= 6.976 - 22.14\ \{0.5 \times 0.1962 \times 0.006 \times 18\} - 23.77\ \{(0.5 \times 0.1962 \times 0.006 \times (25 - 2)\}$$
$$= 6.976 - 22.14 \times 0.01 - 23.77 \times 0.0135$$
$$= 6.976 - 0.22 - 0.32 = 6.4$$

Similarly for $m = 2$

$$\lambda_2{}^3 = \lambda_2{}^2 - \lambda_3{}^2\ \{0.5\ h_0 e\ (q^2 - \rho)\} - \lambda_3{}^3\ \{0.5\ h_0 e\ (q^3 - \rho)\}$$
$$= 6.4 - 23.77\ \{0.5 \times 0.1962 \times 0.006 \times 23\} - 20.8\ \{0.5 \times 0.1962 \times 0.006 \times 23\}$$
$$= 6.4 - 0.32 - 0.28 = 5.8$$

The gradient vector is obtained using equation (25.15) for $m = 2$ and $m = 3$

$$\frac{\partial F}{\partial q^2} = \lambda_2{}^2 - \lambda_3{}^2 h_0\ \{1 + 0.5 \times 0.006\ (2 \times 100 - 2 \times 25 + 2)\}$$
$$= 6.4 - 23.77 \times 0.1962\ \{1 + 0.456\} = 6.4 - 6.79 = -\,0.39$$
$$\frac{\partial F}{\partial q^3} = \lambda_2{}^3 - \lambda_3{}^3 h_0\ \{1 + 0.5 \times 0.006\ (2W^2 + 0 - 2q^3 + \rho)\}$$
$$= 5.8 - 20.8 \times 0.1962\ \{(1 + 0.5 \times 0.006\ (2 \times 75 - 2 \times 25 + 2)\}$$
$$= 5.8 - 5.3 = 0.5$$

Since the gradient vector is not a null or near null vector, we will have to go through second iteration. However, the values of the independent variables at the start of the second iteration will be obtained using equation (25.16). Usually an optimal value of α should be selected which comes only after some experience of working on the system. Let us take $\alpha = 0.4$

$$q^2_{\text{new}} = q^2_{\text{old}} - \alpha\,\frac{\partial F}{\partial q^2} = 25 - 0.4\ (-\,0.39) = 25.156\ \text{m}^3/\text{sec}$$

$$q^3_{\text{new}} = q^3_{\text{old}} - \alpha\,\frac{\partial F}{\partial q^3} = 25 - 0.4 \times 0.5 = 24.8\ \text{m}^3/\text{sec.}$$

Hence $\qquad q^1 = 120 - 50 - (25.156 + 24.8) = 20\ \text{m}^3/\text{sec.}$

From the load curve and the values of discharges obtained at the end of the first iteration, it can be seen that we are moving in the right direction as q^2 corresponding to load 12 MW has increased whereas that for q^3 and load 7 MW, q^3 has decreased.

In fact it is desirable to have the initial discharges q^2 and q^3 keeping in mind the corresponding loads, thereby, perhaps the computation effort can be reduced.

The solution of the necessary conditions as derived from the Lagrangian gives the optimum strategies for economic load dispatch. The success of the problem depends upon a prudent grouping of the equations and methods of solving them. The formulation of the problem involves a large number of variables, the program logic used for its solution is also complex. The method of solution is quite cumbersome and time consuming.

25.3 OPTIMAL POWER FLOW

Since the equations describing the power system are inherently non-linear in nature, it is only through nonlinear programming techniques that the economic dispatch aspect of the system can be studied accurately. Carpentier was the first to formulate rigorously an exact optimisation problem using NLP techniques taking into consideration the system equality and inequality constraints. The problem is based on Lagrange Kuhn-Tucker formulation. This method has been applied for hydrothermal economic scheduling as discussed in the previous article. It is found that the method is quite cumbersome and time consuming. A brief view of the technique is given here under.

The optimisation problem using Kuhn-Tucker conditions is presented as follows.

Min $f(x)$

Subject to $h_i(x) = 0; \quad i = 1, \ldots n$ (25.17)

and $g_j(x) \le 0; \quad j = 1, \ldots N$ (25.18)

where n is the no. of equality constraints and N is number of inequality constraints.

The Lagrangian function is formulated as

$$\mathcal{L}(X, \lambda, \mu) = f(X) + \sum_{i=1}^{n} \lambda_i h_i(X) + \sum_{j=1}^{N} \mu_j g_j(X) \tag{25.19}$$

In order for Lagrangian function to be minimum, the conditions to be satisfied are

1. $\dfrac{\partial \mathcal{L}}{\partial X_i}(X, \lambda, \mu) = 0 \quad \text{for } i = 1, \ldots N.$

where N is the number of variables.

2. $h_i(X) = 0$
3. $g_j(X) \le 0$
4. $\mu_j g_j(X) = 0$

   $\mu_j \ge 0$

The first condition is that for $f(X)$ to be optimum, the partial derivative of the augmented function or Lagrangian functions should be zero. The second and third conditions are the associated equality and inequality constraints which must be satisfied at the point of optimum. The further condition known as the complimentary slackness condition gives a definite mathematical tool to handle the problem of hard and soft inequality constraints. If the product $\mu_j g_i(X)$ equals zero, either μ_j is zero or $g_j(X) = 0$ or both are equal to zero.

If μ_j is equal to zero, $g_i(X)$ is free to be nonbinding (flexible); if μ_j is positive, then $g_i(X)$ must be zero. Thus by looking at μ_j it is possible to find out whether the inequality constraint is binding or not.

To illustrate Kuhn Tucker equation consider the following problem.

$$\text{Min. } f(x_1, x_2) = 0.5\, x_1^2 + 2x_2^2$$
$$h(x_1, x_2) = 4 - x_1 - x_2 = 0$$

and
$$g(x_1, x_2) = x_1 + 0.25\, x_2 - 2 \le 0$$

First we write the Lagrangian equation

$$\mathcal{L}(x_1, x_2) = f(x_1, x_2) + \lambda\, h\,(x_1, x_2) + \mu g(x_1, x_2)$$
$$= 0.5\, x_1^2 + 2x_2^2 + \lambda\,(4 - x_1 - x_2) + \mu\,(2x_1 + 0.5x_2 - 4)$$

The first condition (partial derivatives to be zero)

$$\frac{\partial \mathcal{L}}{\partial x_1} = x_1 - \lambda + \mu = 0$$

and
$$\frac{\partial \mathcal{L}}{\partial x_2} = 4x_2 - \lambda + 0.5\,\mu = 0$$

The second condition

$$4 - x_1 - x_2 = 0$$

The third condition

$$x_1 + 0.25\, x_2 - 2 \le 0$$

The fourth condition gives

$$\mu(x_1 + 0.25\, x_2 - 2) = 0$$
$$\mu \ge 0$$

It is to be noted here that the Kuhn Tucker conditions provide only necessary conditions to obtain an optima. It does not suggest any procedure to be followed to obtain the optima. In order to solve the problem we will have to try various solutions and we accept that solution which satisfies all the four conditions.

Suppose we start with $\mu = 0$ which implies that $g(x_1, x_2)$ can be less than or equal to zero.

If $\mu = 0$, conditions 1 and 2 give

$$x_1 - \lambda = 0 \quad 4x_2 - \lambda = 0$$
$$x_1 + x_2 = 4$$
$$\lambda + \frac{\lambda}{4} = 4 \quad \text{or} \quad \frac{5\lambda}{4} = 4$$

or
$$\lambda = \frac{16}{5} = 3.2$$

Hence $\quad x_1 = 3.2 \quad \text{and} \quad x_2 = 0.8$

Substituting x_1 and x_2 in the third condition

$$3.2 + 0.25 \times 0.8 - 2 = 1.4 \nleq 0$$

Now let us take $\mu > 0$, in which case the inequality must be equal to zero and hence using condition 2 and 3 we have

$$x_1 + x_2 = 4$$

$$x_1 + 0.25\, x_2 = 2$$

$$x_1 = \frac{4}{3} \text{ and } x_2 = \frac{8}{3}$$

Substituting these values in condition 1, we have

$$\frac{4}{3} - \lambda + \mu = 0 \qquad \text{or} \qquad \lambda - \mu = 4/3$$

$$4 \times \frac{8}{3} - \lambda + 0.5\,\mu = 0 \qquad \lambda - 0.5\mu = \frac{32}{3}$$

$$0.5\,\mu = \frac{28}{3} \quad \text{or} \quad \mu = \frac{56}{3}$$

and $$\lambda = \frac{4}{3} + \frac{56}{3} = \frac{60}{3} = 20$$

The fourth condition

$$\frac{56}{3}\left(\frac{4}{3} + \frac{8}{3} \times \frac{1}{4} - 2\right) = 0 \text{ is thus satisfied.}$$

Dommel and Tinney have shown that Newton's method of load flow can be extended to optimal power flow solution feasible with respect to all relevant inequality constraints. The main features of the method are the use of the penalty functions to handle the functional inequality constraints and the application of the optimal gradient technique for actual optimisation of the functionals. Except for the two difficulties *i.e.,* step size in the gradient process and the weighting factor for the penalty function, the method is very efficient since the computational procedure is constructed around the fast Newton Raphson load flow program.

The formulation of this optimal power flow problem as suggested by Sasson is novel. The voltages are taken as independent variables and the constrained problem of optimisation is reduced to optimising a single unconstrained function using Zangwill's transformation.

25.4 PROBLEM FORMULATION

The optimal power flow or optimal load flow (economic load dispatch) problem is to minimise the cost of generation for a given load demand subject to certain system constraints (see article 19.1 for system constraints)

i.e., $$\text{Min} \sum_{i=1}^{NG} f[C_i(P_i)]$$

subject to the inequality constraints on the active and reactive power generations of the system, the load voltages and the equality constraints of the load flow equations. *NG* is the no. of generator. The inequality constraints are

$$P_{i\,\min} \le P_i \le P_{i\,\max} \qquad i = 1, \ldots NG$$
$$Q_{i\,\min} \le Q_i \le Q_{i\,\max} \qquad i = 1, \ldots. NG$$
$$V_{i\,\min} \le V_j \le V_{j\,\max} \qquad j = 1, \ldots NL$$

where NL is the number of load buses. The nodal voltages are taken as independent variables. If there are n no. of buses, $n = NL + NG$, the total no. of independent variables is $2n$, one n for the real and another n for the imaginary parts of the nodal voltages. All the real and imaginary parts of the nodal voltages can be represented by a column vector of $(2n \times 1)$ dimension.

$$[e_1, e_2, \ldots e_n, e_{n+1} \ldots e_{2n}]^T \ldots$$

where $e_1 \ldots e_n$ are real parts and $e_{n+1} \ldots e_{2n}$ are the imaginary parts of the n nodal voltages.

Let $V_p = e_P + je_{p+q}$ and $Y_{pq} = G_{pq} - jB_{pq}$

where V_p is the calculated value of voltage at pth node.

Y_{pq} is the nodal admittance between nodes p and q. The load flow equations defining the equality constraints are written in terms of the real and imaginary components of voltages as

$$K_p = \sum_{q=1}^{n} e_p\,(e_q G_{pq} + e_{q+n}\,B_{pq}) + e_{p+n}\,(e_{q+n}\,G_{pq} - e_q\,B_{pq}) \quad p = 1, 2 \ldots NL \ldots \tag{25.20}$$

$$D_p = \sum_{q=1}^{n} e_{p+n}\,(e_q G_{pq} + e_{q+n}\,B_{pq}) - e_p\,(e_{q+n}\,G_{pq} - e_q B_{pq}) \ldots p = 1, 2 \ldots NL \tag{25.21}$$

and $$e_p^{\,2} + e_{p+n}^2 - E_p^{\,2} = 0 \qquad p = 1, \ldots NG\,. \tag{25.22}$$

where K_p, D_p and E_p are the specified active power, reactive power and voltage at the pth node respectively.

The inequality constraints for upper limit are expressed as

$$P_{i\max} - P_i \ge 0,\ Q_{i\max} - Q_i \ge 0,\ V_{i\max} - V_j \ge 0 \ldots \tag{25.23}$$

Similarly, the inequality constraints for the lower limit are expressed as

$$P_i - P_{i\,\min} \ge 0,\ Q_i - Q_{i\,\min} \ge 0,\ V_j - V_{j\,\min} \ge 0 \ldots \tag{25.24}$$

The optimal load flow problem can be rewritten as

$$\text{Min } f = \sum_{p=1}^{NG} C_p \left[\sum_{q=1}^{n} e_p\,(e_q\,G_{pq} + e_{q+n}\,B_{pq}) + e_{p+n}\,(e_{q+n}\,G_{pq} - e_q B_{pq})\right] \tag{25.25}$$

subject to the constraint (25.20), (25.21), (25.22), (25.23) and (25.24)

The auxiliary function for unconstrained minimisation using Zangwills' transformation is

$$F = f + \sum_{i=1}^{m} \frac{h_i^{\,2}}{r} + \sum_{i=m+1}^{p} \frac{g_i^{\,2}}{r} \ldots \tag{25.26}$$

where the terms within the summation signs include both equality and violated inequality constraints and r is the penalty parameter. During the process of computation all the constraints must be checked. The equality constraints are always to be taken into account for every iteration.

However, during a particular iteration if it is found that some or all the inequality constraints are satisfied, the ones which are satisfied are left out from the function F. Here h_i and g_i represents equality and inequality constraints respectively.

After formulating the problem by Zangwill transformation where an original constrained optimal power flow problem is transformed into an unconstrained optimal power flow problem, the actual optimisation of F is carried out using Fletcher-Powell or Fletcher or any other suitable non-linear optimisation technique. Fletcher Powell technique requires analytical expressions for the gradients of the cost function f and the constraints $h_i(x)$ and $g_i(x)$

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x} + 2\sum_{i=1}^{m} \frac{h_i(x)}{r}\frac{dh_i}{dx} + 2\sum_{i=m+1}^{p} \frac{g_i}{r}\frac{dg_i}{dx} \ldots \tag{25.27}$$

Fig. 25.1 shows flowchart for Fletcher Powell technique. The equivalence between power system variables and ones given in FP technique are

$X \leftrightarrow e, g \rightarrow$ gradient $\frac{\partial}{\partial e}$, H is known as Hessian matrix and is the inverse of second order derivative matrix *i.e.* $G^{-1} = H$

Fletcher-Powell algorithm for optimisation is explained through the following steps. The flow chart is also given here

1. Assume a point X^0 and calculate g^0. Take H^0 as the identity matrix to startwith.
2. Determine λ such that $f(X^k - \lambda^k H^k g^k)$ is minimised in the direction of $-H^k g^k$. Let α^k be that value of λ^k where $f(X^k - \lambda^k H^k g^k)$ is minimised. Here α^k is known as the step size in the direction $-H^k g^k$ for minimisation.
3. Calculate

$$X^{k+1} = X^k - \alpha^k H^k g^k \tag{25.28}$$

$$\delta^k = X^{k+1} - X^k \tag{25.29}$$

$$y^k = g^{k+1} - g^k \tag{25.30}$$

4. Update the matrix H using the relation

$$H^{k+1} = H^k + \frac{\delta^k(\delta^k)^T}{(\delta^k)^T\, y^k} - \frac{H^k y^k\,(y^k)^T\, H^k}{(y^k)^T\, H^k y^k} \tag{25.31}$$

and go to step 2.

5. Repeat steps 2 to 4 until $|\ \delta^k\ |$ is less than a prespecified small tolerance.

It is to be noted that if the function to be minimised is quadratic positive definite, the minimisation process terminates in at most n iterations where n is the no. of variables *i.e.*, the process is quadratically convergent.

In the above process of minimisation a prudent selection of two parameters, r the penalty factor and α the step size, are very important from computational efficiency point of view.

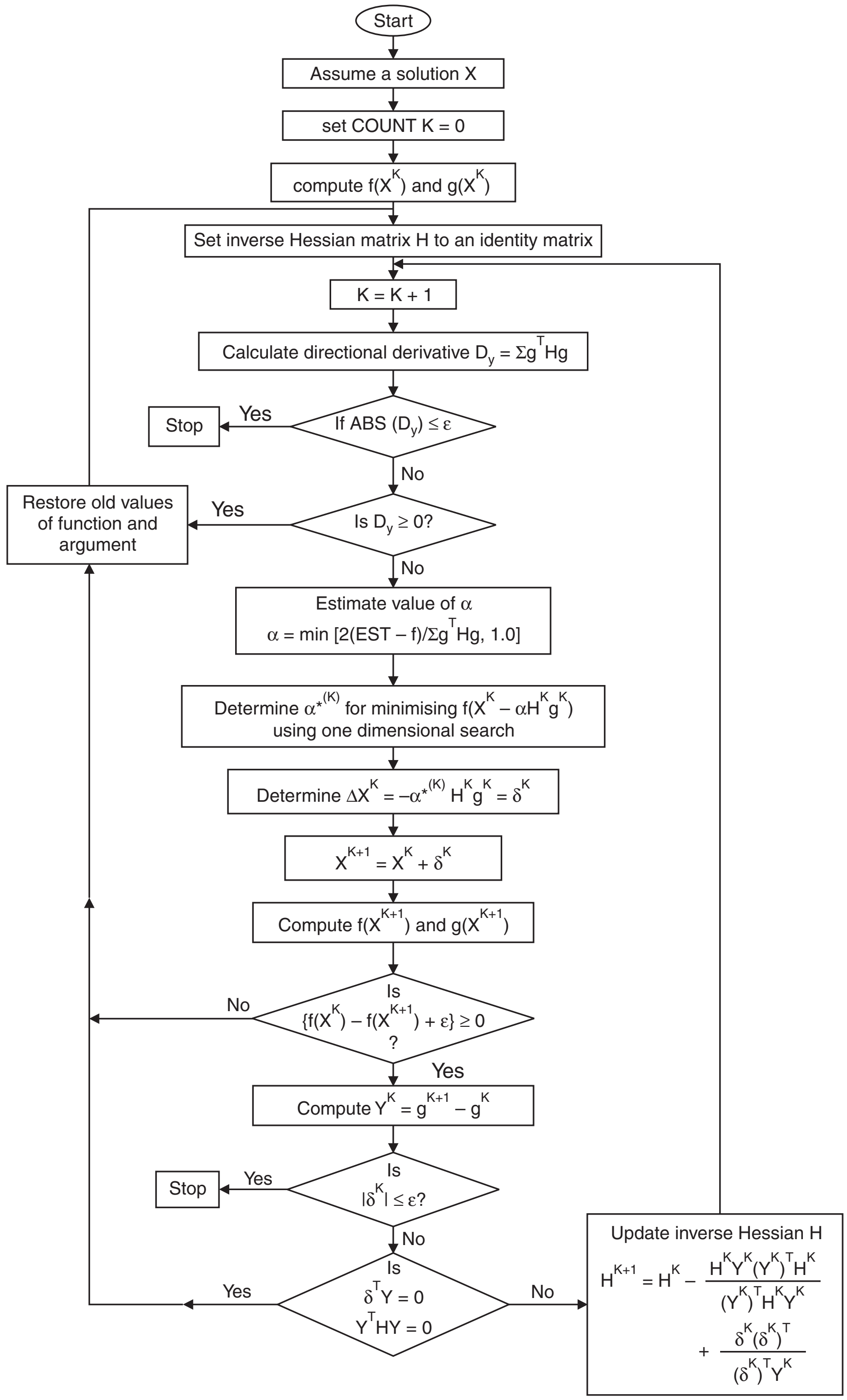


Fig. 25.1 Flowchart for standard algorithm.

During each Zangwill iteration the value of r is changed uniformly according to

$$r^{k+1} = tr^k$$

where k is Zangwill iteration and t is reduction factor. Usually for power system problems $r = \frac{1}{20}$ and $t = \frac{1}{100}$ are found to work well.

Another critical factor is the step size and it is selected according to the following scheme

$$\alpha = \text{Min}\left\{\frac{2(EST - F)}{\Sigma\, g^T\, Hg}, \frac{1}{\Sigma\,|\,Hg\,|}, 1.0\right\} \qquad (25.32)$$

where EST is the initial estimate of the objective function, F is the value of the function in that iteration.

From the above expression within the bracket it is clear that during the process of optimisation any one of the three values inside the bracket is taken up by α depending upon whichever is the minimum. It has been observed during the process of minimisation that α takes the value $\frac{2\,(EST - F)}{\Sigma\, g^T Hg}$ during the initial phase of minimisation when the total constraint violation is not large compared to the cost function; $\alpha = \frac{1}{\Sigma\,|\,Hg\,|}$ during the middle phase of optimisation when the magnitude of total constraint violation is quite comparable to the cost function and finally during the last phase of optimisation α takes a value of unity.

The other techniques used for optimal power flow problem are direct search techniques. The gradient methods in general, are quite promising and are faster than direct search techniques because, gradient being strictly a local property of the function, better approximations are possible. However, the gradient technique has the following disadvantages:

(*i*) The techniques can not be applied if the objective function is discontinuous and is non-differentiable.

(*ii*) The first or second order derivatives require a relatively large amount of problem preparation by the user before he introduces the problem into the algorithms.

Some of the direct search techniques are Nelder and Mead, flexible polyhedran etc. The discussion about these techniques and algorithms for these techniques are not within the scope of the book. However, a few references are given at the end which may prove to be quite useful to the interested readers.

Again a very brief view of multi-objective optimisation in power system is given here to bring awareness to the readers that various operating strategies of power system are possible and depending upon the requirements of a particular electric utility final decision can be made.

25.5 MULTI-OBJECTIVE OPTIMAL POWER FLOW

In general, a large scale power system possesses multiple objectives to be achieved. The ideal power system operation is achieved when various objectives like, cost of generation, system transmission loss, environment pollution etc. are simultaneously attained with minimum values.

Since these operations are conflicting in nature, it is impossible to achieve the ideal power system operation. These objectives cannot be handled by single objective optimisation. The way out lies in a multi-objective, approach to problem solving. With single objective optimisation *e.g.,* cost of generation, the values of transmission loss or level of environmental pollution may be intolerably bad. The utility would not be willing to accept such a solution. The power system analyst has nothing else to present to the utilities to facilitate the decision making process.

Multiobjective planning has the following advantages:

(*i*) Multiobjective programming and planning promotes more appropriate roles for the participants in the planning and decision-making process.

(*ii*) A wider range and alternatives are usually identified.

(*iii*) The power system analyst's perception of a problem will be more realistic if many objectives are considered.

A general rule for decision making is that more information carefully presented is better than less information. The decision to accept or reject a single optimal alternative is an uninformed decision. Informed decision making requires a knowledge of full range of possibilities provided by multiobjective analyst. Here formulation of multiobjective optimal power flow problem is given. Some of the techniques being used for its solution will be listed but not discussed as it is not within the scope of the book.

25.6 PROBLEM FORMULATION

The three aspects of the optimal power flow problem considered are

(*i*) to minimise the cost of generation

$$F_c = \sum_{i=1}^{NG} F[C_i(P_i)] \tag{25.33}$$

where P_i is the active power generation at the ith generator, C_i is the cost of generation for the ith generator and NG is the total no. of generators in the system.

(*ii*) to minimise the system transmission loss.

The objective function to minimise loss is given as

$$F_L = \sum_{p=1}^{n} P_p \tag{25.34}$$

where P_p is the active power injected at the pth node and n is the total no. of nodes in the system.

(*iii*) to minimise the environmental pollution

The objective function to minimise pollution is given as

$$F_p = \sum_{i=1}^{NG} F[S_i(P_i)] \tag{25.35}$$

where S_i is the amount of pollution in terms of oxides of nitrogen emission for the ith generator.

The overall function F is formulated as the weighted sum of F_C, F_L and F_p using Priority Goal Programming technique and is given as

$$F = W_C F_C + W_L F_L + W_P F_P \tag{25.36}$$

where W_C is the priority given to cost of generation, W_L the priority given to transmission loss and W_P is priority given to pollution.

So the objective to minimise F is subject, of course, to the equality and inequality, constraints as discussed in the previous article.

Some of the multiobjective techniques are listed here

1. Weighting Method (as given above)
2. Non-inferior Set Estimation Method
3. Minimum Distance Method
4. Surrogate Worth Trade-off Technique
5. Sequential Goal Programming

We now consider the optimal weight method in a bit detail considering cost of generation and transmission losses. The objective is

$$F = W_C F_C + W_L F_L \tag{25.37}$$

The weights W_c and W_L represent a trade off between the two objectives F_C and F_L. The trade-off depends on the range of the objective functions that each weight is implicitly representing. Weights can be used to determine the target-point which will depend on the decision maker's willingness to specify the weights. Here we try to explore the relation between the range of each objective *i.e.,* the cost of generation and transmission losses and its weights. The objective function is rewritten as

$$F = F_C + F_L \qquad \text{(ignoring the unit of } F_C \text{ and } F_L\text{)} \tag{25.38}$$

In order to scale the function we multiply F_C by the range R_L of transmission losses F_L and multiply F_L by the range R_C of the cost of generation F_C *i.e.*,

$$F = R_L F_C + R_C F_L \tag{25.39}$$

In order to normalise this function we divide throughout by R_L

$$\frac{F}{R_L} = F_C + \frac{R_C}{R_L} F_L \tag{25.40}$$

Comparing the above equation with equation (25.37) it is found that $W_C = 1$ and $W_L = R_C/R_L$ *i.e.,*

$$W_L = \frac{R_C}{R_L} = \frac{F_C - F_{C\,\min}}{F_L - F_{L\,\min}} \tag{25.41}$$

where F_C and F_L are the cost of generation and system transmission losses obtained at any randomly selected weight, $F_{C\,\min}$ is the minimum value of cost of generation obtained by setting $W_C = 1$ and $W_L = 0$ and $F_{L\,\min}$ when $W_C = 0.0$ and $W_L = 1.0$.

If the decision maker is made aware of the relation between range of objective functions and the weights associated with them, this leads to reliable methods of assessing preferences.

The optimal value of W_{Lopt} is obtained as follows:

$$W_{Lop} = \frac{F_{C \text{ at } F_{L\min}} - F_{C\min}}{F_{L \text{ at } F_{C\min}} - F_{L\min}}$$

where $F_{L \text{ at } F_{C\min}}$ is the value of F_C at $F_{L\min}$ and $F_{L \text{ at } F_{C\min}}$ is the value of transmission losses obtained at $F_{C\min}$.

And at each solution *USC* (Unit saving in cost of generation) and USL (Unit saving in loss) are evaluated which are defined as

$$USC = \frac{(F_{C \text{ at } F_{L\min}} - F_C)}{(F_{C \text{ at } F_{L\min}} - F_{C\min})}$$

where $(F_{C \text{ at} F_{L\min}} - F_{C\min})$ is *MPSC* maximum possible saving in cost of generation.

Similarly $$USL = \frac{(F_{L \text{ at} F_{C\min}} - F_L)}{(F_{L \text{ at } F_{C\min}} - F_{L\min})}$$

where $(F_{L\, at F_{C\min}} - F_{L\min}) = MPSL$ maximum possible saving in transmission losses.

At W_{Lopt} *USC* and *USL* are equal.

Table 25.1 Results of MOPF studies (5-bus system)

S. No.	W_C	W_L	F_C *\$/hr.*	$F_{C(W/L)}$ *\$/hr.*	F_L *MW*	*USC*	*USL*
1	1.00	0.00	760.95	760.95	5.18	1.00	0.00
2	0.00	1.00	764.43	764.43	5.01	0.00	1.00
3	1.00	20.81	761.82	761.82	5.05	0.75	0.75

Table shows results for the 5 bus IEEE system where it can be seen in the third row that USC and USL are equal and each equal 0.75 when $W_C = 1$ and $W_{Lop} = 20.81$.

Next we discuss here very briefly minimum distance method to multiobjective optimisation.

25.6.1 Minimum Distance Method

This method employs the concept of an Ideal Point and minimises the Euclidean distance between the ideal point and the set of feasible solutions. The ideal point is one where all the three objectives are minimum. In a 3 dimensional space where three axes represent the three objective functions having the coordinates as $F_{C\min}$, $F_{L\min}$, $F_{P\min}$ is known as ideal point which is not feasible. Therefore, one can at most achieve a point which is feasible and at a minimum distance from the ideal point. Such a point is known as the target point or the best compromise solution. This distance function is given as

$$\text{Distance} = [(F_C - F_{C\min})^2 + (F_L - F_{L\min})^2 + (F_P - F_{P\min})^2]^{1/2}$$

The procedure for evaluating $F_{C\min}$, $F_{L\min}$ and $F_{P\min}$ has already been explained.

25.6.2 Sequential Goal Programming

In this method one of the objectives which is considered most important is minimised first. For this we attach unity weight to the objective being minimised and zero weights for the remaining objectives. In the next step the objective which is minimised in the first step is kept fixed at its minimum value (equality constraint) and another objective which is considered most important out of the remaining objectives is minimised. The process is repeated till all the objectives have been considered. However, it is observed that during the process of minimisation, the values of the objectives which are already minimised increase to a small extent. In other words, in order to achieve better values of current objective being minimised, some sacrifice has to be made in the already minimised objective. At first this appears to be a limitation of this method. A little thought in this method clarifies that this is the beauty of the method.

In order to achieve minimum possible value of the objective of current interest, the technique automatically works out a reasonable trade off with the objectives already minimised. Thus, the technique includes all the previously considered objectives for making trade-off without any effort: mathematical, analytical, programming or computational.

QUESTIONS

25.1. Define Hydrothermal scheduling problem and discuss various constraints under which this problem is defined.

25.2. Using Lagrangian multiplier develop mathematical expressions for the solution of economic hydrothermal scheduling problem.

25.3. Explain the algorithm for the solution of economic hydrothermal scheduling problem.

25.4. Explain what you mean by optimal power flows. How is this different from load flow problem?

25.5. What are Kuhn Tucker conditions when referred to optimisation problem?

25.6. Develop mathematical expressions for optimal power flow for minimising cost of generation when this system is subjected to certain constraints.

25.7. Optimal power flow is a constrained problem. Explain clearly how this is converted into an unconstrained problem.

25.8. Explain with a flowchart the algorithm using. Fletcher Powell method for solution of optimal power flow problem.

25.9. Discuss clearly how the selection of the following parameters affect the convergence of solution using Fletcher Powell technique: (*i*) Initial solution (*ii*) Penalty parameter (*iii*) Step size (*iv*) directional derivative.

25.10. What is multi-objective optimal power flow problem? How does this help the electric utility to take proper decision in planning and operation of power system?

25.11. Develop mathematical expressions for multi-objective optimisation power flow when the objectives are cost of generation, transmission line loss and pollution levels. List out some of the methods for the solution of the MOPF problem.

25.12. Discuss briefly (*i*) Weighting method (*ii*) Minimum distance method and (*iii*) Sequential goal programming methods to MOPF problems.

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APPENDICES

Appendix–A

Algorithm for Formation of Bus Impedance Matrix

The bus impedance matrix Z_{BUS} relates the bus voltage V_{BUS} and the bus currents I_{BUS} through the relation.

$$V_{BUS} = Z_{BUS} I_{BUS} \tag{A.1}$$

The objective here is to develop an algorithm for building up Z_{BUS} for any given network. One way could be to formulate Y_{BUS} matrix for the system and then inverse of this matrix gives Z_{BUS}, *i.e.*,

$$Z_{BUS} = Y_{BUS}^{-1} \tag{A.2}$$

But it is found that it is numerically economical to obtain Z_{BUS} directly from the network.

Z_{BUS} matrix can be built-up starting with a single line and gradually adding one line at a time and thus modifying the existing Z_{BUS} matrix. The process is continued till all the elements are exhausted.

Assume that the Z_{BUS} matrix is known for a partial network of Fig. A.1 having m buses and a reference bus 0. The equation A.1 relates the bus quantities for the network where V_{BUS} is a column vector of bus voltages measured with respect to the reference node, I_{BUS} is a column vector $m \times 1$ of injected bus currents and Z_{BUS} is an $m \times m$ matrix.

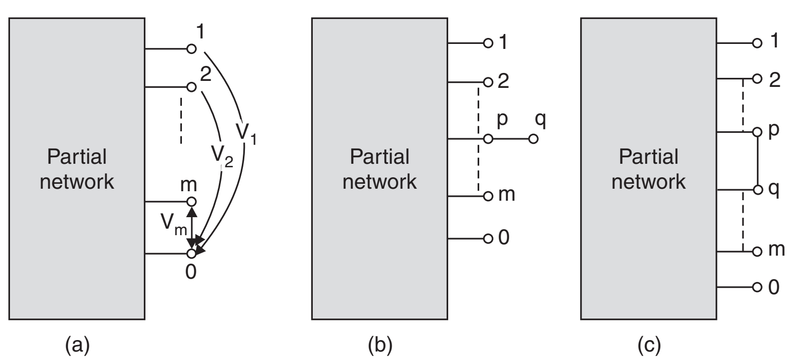


Fig. A.1 (*a*) Partial network, (*b*) Addition of a branch, (*c*) Addition of a link.

Let us add one more element to the existing partial network. There are two possibilities as far as the network changes are concerned. (*i*) The line could be a link, *i.e.*, a line between two buses already present in the partial network. (*ii*) It could be a branch, *i.e.*, a radial line from an existing bus to a new bus. In the former case the dimensions of the matrices in equation (A.1) remain unchanged but all the elements of the bus impedance matrix, must be recalculated to include the effect of the added link. In the latter case when the added element is a branch, a new bus is added to the partial network and the resultant bus impedance matrix is $(m + 1) \times (m + 1)$ dimension and both the current and voltages are of dimension $(m + 1) \times 1$. The addition of branch does not affect the original matrix and, therefore, the elements for the new row and column are required to be calculated. Let us add an element pq to the existing network as shown in Fig. A.1 (*b*) and (*c*).

Addition of a Branch

The performance equation for the partial network with an added branch pq Fig. A.1 (*b*) is

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_m \\ V_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} & \dots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \dots & Z_{2p} & \dots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \vdots \\ Z_{p1} & Z_{p2} & & Z_{pp} & \dots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \vdots \\ Z_{m1} & Z_{m2} & \dots & Z_{mp} & \dots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \dots & Z_{qp} & \dots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \tag{A.3}$$

Assuming that the network consists of bilateral passive elements $Z_{qi} = Z_{iq}$ where $i = 1, 2, \ldots\ldots, m$ and refers to the buses of the partial network.

By definition

$$Z_{qi} = \frac{V_q}{I_i} = V_q \text{ if } I_i = 1 \text{ p.u.} \tag{A.4}$$

i.e., Z_{qi} is the impedance and is numerically equal to the voltage of bus q with respect to reference bus when the current of 1 p.u. is injected at bus i and keeping all other bus currents to zero.

From equation (A.4)

$$Z_{1i} = V_1,\ Z_{2i} = V_2,\ Z_{pi} = V_p$$

$$Z_{mi} = V_m \quad \text{and} \quad Z_{qi} = V_q \tag{A.5}$$

Also from Fig. A.1 (*b*)

$$V_q = V_p - v_{pq} \tag{A.6}$$

Assuming that there is no coupling between the added branch and other elements of the partial network, then

$$i_{pq} = y_{pq,pq} v_{pq}$$

where $y_{pq,pq}$ is the self admittance of the added element. Since $i_{pq} = 0$ as q is a new node and is not connected anywhere in the network, $v_{pq} = 0$

and equation (A.6) reduces to $V_q = V_p$, *i.e.,*

$$Z_{qi} = Z_{pi} \text{ for all } i = 1, 2, \ldots\ldots, m \quad i \neq q \tag{A.7}$$

Now to evaluate

$$Z_{qq} = \frac{V_q}{I_q} = V_q \text{ if } I_q = 1 \text{ p.u.} \tag{A.8}$$

i.e., Z_{qq} is equal to V_q when a current of 1 p.u. is injected at node q and all other currents are zero. Now

$$i_{pq} = -I_q = y_{pq,pq} v_{pq} = -1$$

$$\therefore \quad v_{pq} = -\frac{1}{y_{pq,pq}}$$

Substituting this value of v_{pq} in equation (A.6) we obtain

$$V_q = V_p + \frac{1}{y_{pq,pq}} \tag{A.9}$$

Now since from equation (A.3) $V_q = Z_{qq}$ and $V_p = Z_{pq}$, equation (A.9) can be rewritten as

$$Z_{qq} = Z_{pq} + Z_{pq,pq} \tag{A.10}$$

So equation (A.7) for off diagonal and (A.10) for diagonal will generate the elements in the $(m + 1)$th row and column of the matrix when a branch is added to a matrix of $m \times m$ dimensions.

Also if node p is taken as reference.

$$Z_{pi} = 0 \text{ for } i = 1, 2, \ldots\ldots, m \quad i \neq q \tag{A.11}$$

and

$$Z_{qi} = 0 \text{ for } i = 1, 2, \ldots.., m \quad i \neq q$$

Also $Z_{pq} = 0$ and, therefore, $Z_{qq} = z_{pq,pq}$.

If there is mutual coupling between the added branch and other elements of the partial network equations (A.7) and (A.10) are respectively modified as

$$Z_{qi} = Z_{pi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}} \text{ for } i = 1, 2, \ldots\ldots, m \quad i \neq q \tag{A.12}$$

and

$$Z_{qq} = Z_{pq} + \frac{1 + y_{pq,\rho\sigma}(z_{\rho q} - Z_{\sigma q})}{y_{pq,pq}}$$

where $y_{pq,\rho\sigma}$ is the vector of mutual admittances, between the added element pq and the element $\rho\sigma$ of the partial network.

Addition of a Link

Let the added element pq be a link. The procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source e_l as shown in Fig. A.2. This creates a new node l which will be finally eliminated. The value of e_l is such that the current through the added element is zero and therefore, the element $p - l$ could be considered as a branch with node l as the new node.

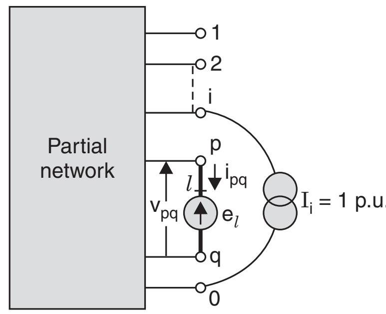


Fig. A.2 Addition of link.

The performance equation for the partial network with the added element $p - l$ and the series voltage source e_l is

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_m \\ \hline e_l \end{bmatrix} = \left[\begin{array}{cccc|c} Z_{11} & Z_{12} & \ldots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \ldots & Z_{2m} & Z_{2l} \\ \vdots & \vdots & & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \ldots & Z_{pm} & Z_{pl} \\ \vdots & \vdots & & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \ldots & Z_{mm} & Z_{ml} \\ \hline Z_{l1} & Z_{l2} & \ldots & Z_{lm} & Z_{ll} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \hline I_l \end{bmatrix} \tag{A.13}$$

Here V_1 ...... V_m are the voltages with respect to reference bus whereas e_l is with respect to node q.

The objective here is to eliminate lth node by connecting node l to node q, *i.e.*, short circuiting nodes l and q. Equation (A.13) can be rewritten in a compact form after partitioning as shown by thick line

$$\begin{bmatrix} \dfrac{V_{\text{BUS}}}{e_l} \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{\text{BUS}}}{Z_{lj}} \Big/ \dfrac{Z_{il}}{Z_{ll}} \end{bmatrix} \begin{bmatrix} \dfrac{I_{\text{BUS}}}{I_l} \end{bmatrix} \tag{A.14}$$

From equation (A.14)

$$V_{\text{BUS}} = Z_{\text{BUS}} I_{\text{BUS}} + Z_{il} I_l \tag{A.15}$$

and
$$e_l = Z_{lj} I_{\text{BUS}} + Z_{ll} I_l \tag{A.16}$$

Since e_l is to be eliminated let $e_l = 0$.

From equation (A.16)

$$e_l = Z_{lj} I_{\text{BUS}} + Z_{ll} I_l = 0$$

or
$$I_l = -I_{\text{BUS}} \frac{Z_{lj}}{Z_{ll}}$$

Substituting this value of I_l in equation (A.15), we have

$$V_{\text{BUS}} = Z_{\text{BUS}} I_{\text{BUS}} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \,.\, I_{\text{BUS}}$$

$$= \left(Z_{\text{BUS}} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \right) I_{\text{BUS}} \tag{A.17}$$

$$Z_{\text{BUS(modified)}} = \frac{V_{\text{BUS}}}{I_{\text{BUS}}} = Z_{\text{BUS}(old)} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \tag{A.18}$$

and any element of $Z_{\text{BUS(modified)}}$ can be evaluated as

$$Z_{ij\text{(modified)}} = Z_{ij\text{(old)}} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \tag{A.19}$$

Therefore, once we have the elements of the augmented part of the matrix Z_{BUS}, we will be able to find out $Z_{\text{BUS(modified)}}$. The following is the procedure for evaluating Z_{li} and Z_{ll}.

Referring to Fig. A.2, since $e_l = V_l - V_q$, the element Z_{li} can be obtained by injecting a current at the ith bus and finding out the voltage at the lth bus with respect to bus q. Since all other currents are zero, from equation (A.13)

$$V_k = Z_{ki} I_i = Z_{ki} \quad \text{if} \quad I_i = 1 \text{ p.u.}$$

and $$e_l = Z_{li} I_i = Z_{li}$$

From Fig. A.2 the series voltage source is

$$e_l = V_p - V_q - v_{pl} \tag{A.20}$$

Since element $p - l$ is being treated as a branch, $i_{pl} = 0$ and therefore $v_{pl} = 0$ and

$$\therefore \quad Z_{li} = \frac{e_l}{I_i} = \frac{V_p}{I_i} - \frac{V_q}{I_i} = Z_{pi} - Z_{qi} \tag{A.21}$$

To obtain $Z_{ll} = \dfrac{e_l}{I_l}$, take $I_l = 1$ p.u. and all other currents as zero.

Now $$i_{pl} = -I_l = -1$$

Also $i_{pl} = y_{pl,pl} v_{pl} = -1$ if mutual coupling is neglected.

Also since $y_{pl,pl} = y_{pq,pq}$

$$\therefore \quad v_{pl} = -\frac{1}{y_{pq,pq}} \tag{A.22}$$

From equations (A.20), (A.21) and (A.22)

$$Z_{ll} = \frac{e_l}{I_l} = \frac{V_p}{I_l} - \frac{V_q}{I_l} - \frac{v_{pl}}{I_l} = Z_{pl} - Z_{ql} + Z_{pq,pq} \tag{A.23}$$

Further, if bus p is taken as the reference bus

$$Z_{pi} = 0 \quad i = 1, 2, \ldots\ldots, m \quad i \neq l \tag{A.24}$$

and $$Z_{li} = -Z_{qi} \quad i = 1, 2, \ldots\ldots, m \quad i \neq l$$

Also $$Z_{pl} = 0$$

$$\therefore \quad Z_{ll} = -Z_{ql} + Z_{pq,pq} \tag{A.25}$$

Assuming that mutual coupling is there between the added element and other elements of the partial network then, equations (A.21) and (A.23) get modified as follows:

$$Z_{li} = \mathrm{Z_{pi}} - Z_{qi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}} \quad i = 1, 2, \ldots\ldots, m \tag{A.26}$$

$$i \neq l$$

and
$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + y_{pq,\rho\sigma}(Z_{\rho l} - Z_{\sigma l})}{y_{pq,pq}} \tag{A.27}$$

Modifications of Z_{BUS} for Network Changes

The Z_{BUS} matrix once built can be modified for any changes brought in the configuration of the network such as addition or removal of elements, or changes in impedance of the elements. The method described above can be used for such modifications.

The procedure to remove an element or change the impedance of an element is identical. If an element is to be removed which is not mutually coupled with other elements, it is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Similarly, if the impedance of an uncoupled element is to be changed the modified bus impedance matrix is obtained by adding a link in parallel with the element whose impedance is to be changed such that the equivalent impedance of the two elements is the desired value.

Summarising the Z_{bus} formulation under four conditions:

1. Addition of a branch: Since a new node is created the dimension of the new matrix becomes $(m + 1) \times (m + 1)$. The addition of branch does not change the original Z_{bus} matrix. Elements of the new row and column have to be calculated.

   (*i*) If the branch is connected to reference node, branch impedance is, say Z_b, then

$$Z_{\mathrm{bus(new)}} = \left[\begin{array}{ccccc:c} & & Z_{\mathrm{bus\,old}} & & & 0 \\ & & & & & 0 \\ & & & & & 0 \\ & & & & & 0 \\ & & & & & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & Z_b \end{array}\right] \tag{A.28}$$

   (*ii*) If branch is connected to node p, from equation $A.7$ and $A.10$, we have

$$\mathrm{Z_{bus(new)}} = \left[\begin{array}{ccccc:c} & & Z_{\mathrm{bus\,old}} & & & Z_{np} \\ & & & & & \vdots \\ & & & & & Z_{mp} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{p_1} & Z_{p_2} & \cdots & \cdots & Z_{pm} & Z_b + Z_{pp} \end{array}\right] \tag{A.29}$$

2. Addition of a link: Since no new node is created, the modified Z_{bus} will have the same dimension as the original Z_{bus} but all the elements of modified will be different from the original nodes.

   (*i*) If the link is connected to reference bus, using equations, *A*.(19), *A*.(24) and *A*.(25), we have

$$Z_{\text{bus new}} = [Z_{\text{bus old}}] - \frac{1}{Z_{qq} + Z_b} \begin{bmatrix} Z_{1q} \\ Z_{2q} \\ \vdots \\ \vdots \\ Z_{mg} \end{bmatrix} [Z_{q1}\ Z_{q2} \ldots Z_{qm}] \tag{A.30}$$

   (*ii*) If link is connected between nodes *p* and *q*, using equations *A*.(19), *A*.(21) and *A*.(23), we have

$$Z_{\text{bus new}} = Z_{\text{bus old}} - \frac{1}{Z_{pp} + Z_{qq} - 2Z_{pq} + Z_b} \begin{bmatrix} Z_{1p} - Z_{1q} \\ \vdots \\ \vdots \\ Z_{mp} - Z_{mq} \end{bmatrix}.$$

$$[(Z_{p1} - Z_{q1}), (Z_{p2} - Z_{q2}), \ldots (Z_{pm} - Z_{qm})] \tag{A.31}$$

Example A.1: *Consider the network shown in the following Fig. E. A.1. The bus numbers and impedances are marked. Determine the bus impedance matrix.*

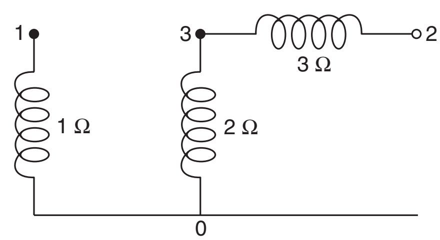


Fig. E.A.1

Solution: Taking one element at a time, say we take node 1 connected to reference bus.

Hence, $Z_{\text{bus}} = [1]$

Node 3 is connected to reference bus.

Hence, $Z_{\text{bus}} = \begin{bmatrix} 1 & 0.0 \\ 0.0 & 2 \end{bmatrix}$

$Z_b = 3\Omega$ is connected between 2 and 3

$$Z_{\text{bus}} = \begin{bmatrix} 1 & 0.0 & 0.0 \\ 0.0 & 5.0 & 2.0 \\ 0.0 & 2.0 & 2.0 \end{bmatrix}$$

It will be very convenient if we renumber the node 2 as 3 and 3 as 2 and develop the Z_{bus} matrix and later on we will transpose 2nd row with 3rd row and next 2nd column with 3rd column and we will have the same results.

With node 1 only

$$Z_{\text{bus}} = [1.0]$$

With node 3 as 2

$$Z_{\text{bus}} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

With node 2 as node 3 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2+3 \end{bmatrix}$

Transposing 2nd and 3rd column $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 5 & 2 \end{bmatrix}$

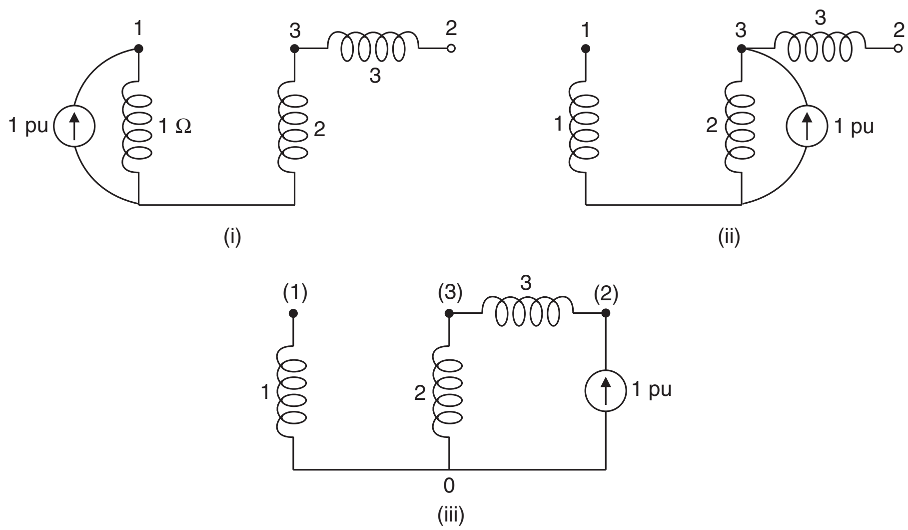
Transposing 2nd and 3 row we have $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$

Hence, the same results.

Alternatively we can use current injection method. The general entry in the bus impedance matrix is given by

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_1 = I_2 = I_j = 1\,\text{p.u.}}$$

Here we connect a source of unit current between each node and the reference node. To find out the entries in the row of the corresponding node where the source is connected we obtain values of voltage at various nodes.



Since the current is 1 A, the voltages give the impedance entries at various nodes in the row.

In Fig. (*i*) the voltages are 1 0 0 and hence thses become entries in the first-row of the Z_{bus} matrix. In Fig. (*ii*), the voltages are 0 2 2, note that voltage of node 2 is same as that of node 3. Hence these become the entries in the third row.

In Fig. (*iii*) since the source is connected between node 2 and reference, the entries will correspond to 2nd row and which is 0 5 2.

Hence, $$Z_{\text{bus}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

Example A.2: *A network consists of three elements 0-1, 1-2 and 2-0 of p.u. impedances 0.2, 0.4 and 0.4 respectively. Determine the bus impedance matrix.*

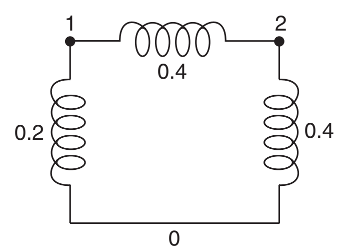


Fig. E.A.2

Solution: For element 0-1

$$Z_{\text{bus}} = [0.2]$$

Addition of branch 1-2 $\begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$

Addition of link 2-0, using equation (A.30)

$$Z_{\text{bus new}} = Z_{\text{bus}} - \frac{1}{Z_{qq} + Z_b} \begin{bmatrix} Z_{12} \\ Z_{22} \end{bmatrix} [Z_{12}\ Z_{22}]$$

$$= \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.6 \end{bmatrix} - \frac{1}{0.6 + 0.4} \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix} [0.2\ 0.6]$$

$$= \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.6 \end{bmatrix} - \begin{bmatrix} 0.04 & 0.12 \\ 0.12 & 0.36 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.08 \\ 0.08 & 0.24 \end{bmatrix}$$

Alternatively using current injection method

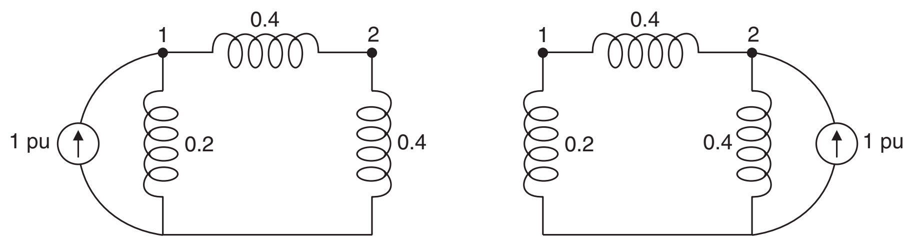


Fig. E.A-2

Current through 0-1 is $1 \times \dfrac{0.8}{1} = 0.8$ and that through 0.2 is 0.2 A. Hence, potential of 1 w.r.t. reference $0.8 \times 0.2 = 0.16$ and the potential of node 2 is $0.4 \times 0.2 = 0.08$.

In second case, current through 0-2 is $1 \times 0.6 = 0.6$A and that through 0-1 is 0.4A. Hence potential of 2 is $0.6 \times 0.4 = 0.24$ and that of 1, it is $0.4 \times 0.2 = 0.08$.

Hence, the elements are $\begin{bmatrix} 0.16 & 0.08 \\ 0.08 & 0.24 \end{bmatrix}$

Example A.3: *Develop Z_{bus} matrix for the network shown in Fig. E. A.3.*

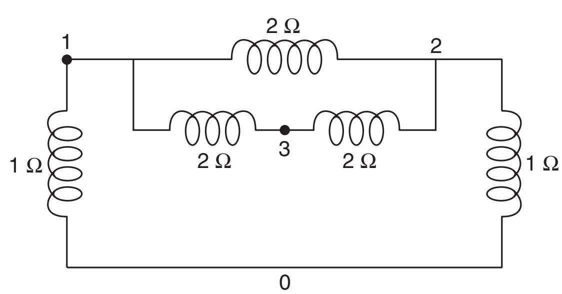


Fig. E.A-3

Solution: Z_{bus} for the element 1-0

$$Z_{\text{bus}} = [1]$$

Add branch 1-2, it is a second modification of branch addition as node 2 is created, here $p = 1$ and $q = 2$ and using equation (A.29), we have

$$\begin{bmatrix} 1 & 1 \\ 1 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

Next add element 2-0, it a link, no new node is created, hence modification 3 is to be used.

$$\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} - \frac{1}{1+3}\begin{bmatrix} 1 \\ 3 \end{bmatrix}[1 \quad 3]$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

Next add element 1-3, a new node is created, hence modification 2 is used and equation A.29, here $p = 1$, $q = 2$

$$\begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix}$$

Next add element 2-3 which is a link, $p = 2$, $q = 3$. Here modification 4 is required *i.e.,* equation $A.(31)$

$$\begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix} - \frac{1}{0.75 + 2.75 + 2 - 2 \times 0.25} \begin{bmatrix} 0.25 - 0.75 \\ 0.75 - 0.25 \\ 0.25 - 2.75 \end{bmatrix} [-0.5 \quad 0.5 \quad -2.5]$$

$$\begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 0.25 & -0.25 & 1.25 \\ -0.25 & 0.25 & -1.25 \\ 1.25 & -1.25 & 6.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.7 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 3 \end{bmatrix}$$

It is to be noted that Z_{bus} is a full matrix unlike Y_{bus} matrix which is sparse. For small systems Z_{bus} can be obtained by current injection method or the step by step formulation of Z_{bus} method. For large systems, however, the latter method is recommended.

Appendix–B

The Power Transformer

The power transformers as used in a power station or a power system could be a bank of three single phase transformers connected in either star/delta or star/star etc., or could be a single 3-phase transformer with single core. Normally for large capacity transformers, a 3-phase transformer is used for the following reasons. It is lighter and cheaper, occupies less space and is more efficient. The only disadvantage is that anything that affects the winding of one phase will affect the others also whereas in single phase transformers this is not so, as one transformer can be replaced and the operation can be continued.

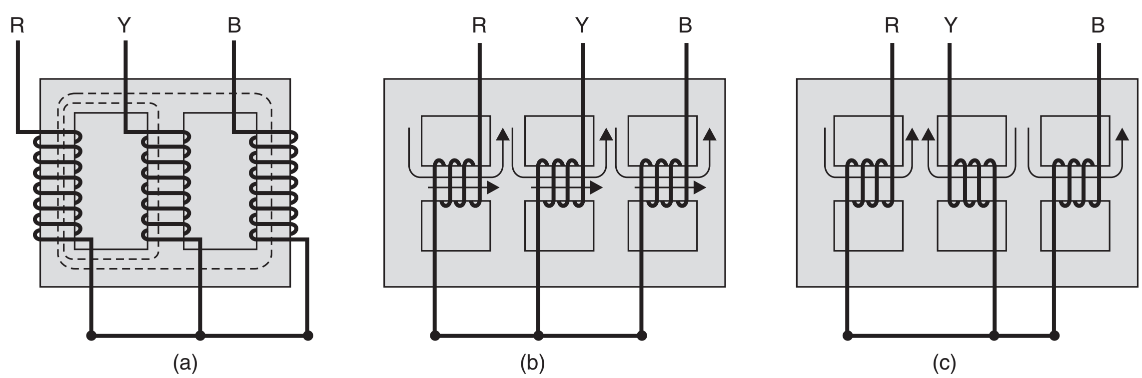


Fig. B.1 (*a*) Three phase core type transformer (*b*) Three phase shell type transformer—effect of winding direction (*c*) Three phase shell type reversed middle coil.

The magnetic circuit of a 3-phase core type transformer (Fig. B.1) is somewhat unbalanced, central limb having less reluctance than the outer two limbs, even though the unbalancing is not appreciable. For all practical purposes the flux is same in all parts of the magnetic circuit and the cross-section of the yoke and the limbs should be same in order to have uniform flux density everywhere.

The shell type construction as in Fig. B-1(*b*), with the direction of the winding the same in all three of the phases, brings in unbalancing in the flux distribution in all the limbs. In part B of the circuit the net flux is the phasor difference of fluxes due to phase R and Y which are equal in magnitude and displaced in phase by 120° *i.e.*, the net flux in part B is $\frac{\sqrt{3}}{2}\phi$ where ϕ is the flux due to each phase. Similar is the case with part C of the circuit whereas the flux in the outer limbs is $\frac{\phi}{2}$. If the connection of winding on the central limb is reversed as in Fig. B-1(*c*) the flux everywhere is $\frac{\phi}{2}$. The flux now in part B is the sum of two fluxes of magnitude $\frac{\phi}{2}$ displaced by 120° and hence the resultant is also $\frac{\phi}{2}$. It is clear from Fig. B-1(*c*) that each phase of the shell type transformer has an independent magnetic circuit. Therefore, if for certain reasons, (a fault) one of the windings is disabled and is removed from the circuit the remaining two windings can be operated in open delta. Under this condition it is preferred to short circuit both the primary and secondary of the disabled phase so that any stray flux that may find its way into its circuit from the other two circuits is reduced to a small value. In core type of construction the magnetic circuits are not independent and this kind of short circuiting of one of the phases is not done.

The type of connections for 3-phase operation of transformers normally used are delta-star, star-delta and delta-delta. Star-Star connections are normally not considered. The delta-star and star-delta connections are well suited to transformers in high voltage systems, the delta-star connection being used for stepping up the voltage and the star-delta for stepping down. Star connection is used for high voltages as then the phase voltage is equal to the line voltage divided by $\sqrt{3}$ and thus the windings can be insulated for lower voltages thereby the cost of providing insulation is reduced. The delta winding is used as primary, as the generated voltage is small and the current to be handled is large. This is because the phase voltage is the same as line to line voltage and the phase current is equal to line current divided by $\sqrt{3}$. The grounding of the neutral of the secondary of a delta-star transformer does not introduce any problem because of third harmonic, as the third harmonic component of the exciting current can flow in the primary delta winding.

When star-delta transformer is used as a step down transformer, the triple harmonic component of exciting current can't flow in the primary winding but it appears in the delta secondary winding. In other words, the main winding takes the place of the tertiary winding which is discussed later in connection with star-star windings. It is, therefore, always preferrable to have at least one delta connected winding in a three phase transformer, which will eliminate the third harmonic current in the external circuit and thus will avoid the interference of power lines with the communication networks.

It is to be noted that if the flux in a transformer magnetic circuit is sinusoidal, the exciting current must contain a third harmonic component. However, if because of transformer connections or system connections, this current can't flow, the flux will contain third harmonic component which will induce third harmonic voltages in the transformer windings. These voltages vary between 5% to 50% of fundamental frequency voltages depending upon the type of transformer used whether core type or shell type respectively.

With star-star connections, the following systems (Fig. B-2) are considered for clear understanding:

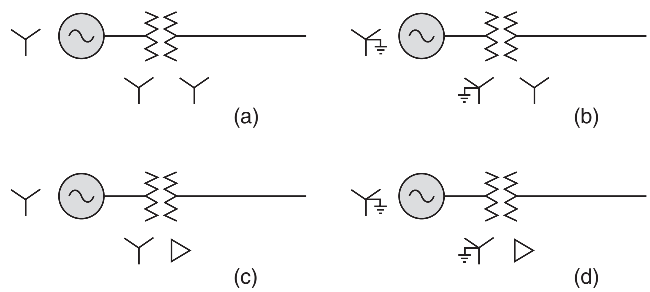


Fig. B.2

In Fig. B.2(*a*) as the third harmonic component of current does not find a path, a third harmonic component of voltage will, therefore, be present in the line to neutral voltages even though across lines the third harmonic voltage is absent.

In Fig. B.2(*b*) the third harmonic components of currents do find a path and hence third harmonic component of voltages are absent.

In Fig. B.2(*c*) and B.2(*d*) because of presence of delta winding, the delta connection provides a path for the third harmonic currents required to eliminate the third harmonic voltages. In Fig. (*c*), no third harmonic current will flow in the lines of the system whereas in (*d*) the flow will depend upon the relative magnitudes of the impedances of the system and the delta winding. However, the current is usually small and does not cause much problem with the communication networks.

In Fig. B.2(*a*) the third harmonic voltages can be suppressed by providing a third winding connected in delta. This winding is known as tertiary winding. The e.m.fs of fundamental frequency induced in these windings will be 120° apart and will, therefore, balance, but the induced e.m.fs of triple frequency will be in time phase around the closed circuit and the resultant third harmonic current will supply the magnetising component that cannot flow in the primaries.

With star-star connection, there is instability of the neutral because of the unbalance loading condition. The potential of the physical neutral is generally at some point other than the geometric centre of the voltage triangle and is greatly affected by the characteristics of the load.

Sometimes transformers with three windings are used to interconnect three circuits operating at different voltages. The tertiary winding may be used to provide voltage for auxiliary power purposes in a substation or to supply a local small distribution system. Synchronous capacitors or static capacitors may be connected to the tertiary winding for purposes of voltage control or power factor correction.

The use of tertiary alongwith star-star connection makes it possible to have single phase loading of the secondary of the transformer even though the primary neutral is isolated. Refer Fig. B.3(*a*).

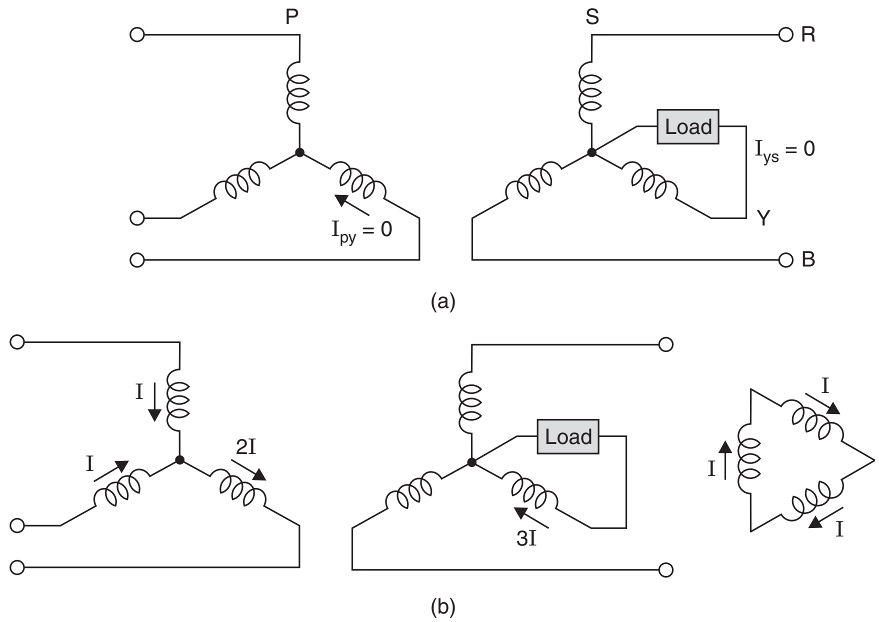


Fig. B.3

Single phase loading in Fig. B.3(*a*) is not possible as this will require current in phase *Y* of primary and hence the current in *R* and *B* of the same winding. Since there are no opposing ampere turns of the secondary winding in phases *R* and *B*, therefore, no current can flow in *Y* of primary winding and hence in *Y* of the secondary winding. Now, if tertiary winding is added as in Fig. B.3(*b*) it can be seen that the single phase loading to the same transformer is possible.

It is to be noted here that whereas the zero sequence current and third harmonic currents resemble in some aspects they differ in the sense that whereas the flow of zero sequence currents in a circuit requires ampere turns balancing, the flow of the third harmonic currents does not require the condition of balancing ampere turns.

Parallel Operation of 3-phase Transformers

It is quite evident that if two transformers are star-star connected having the same ratio of transformation will operate in parallel satisfactorily as the phasor diagram of primary and secondary voltages superpose directly.

If the Line to Line voltages of two transformers with star-star and delta-delta connection are same, they can operate in parallel satisfactorily as the phasor diagram of primary and secondary voltages likewise coincide.

If the transformers are star-star or delta-delta connected, they cannot be paralleled with star-delta or delta star as there will be a phase displacement of 30° between the secondary voltages for the same supply voltage across the primaries of the two transformers. This results in circulating currents in the windings of the transformers.

Finally, let us consider the case when one transformer is star-delta connected and the other delta-star connected, both designed for the same line voltage on the primary and secondary sides. The normal connections of star-delta and delta-star, transformers alongwith phasor diagram are shown in Fig. B.4 (*a*) and (*b*).

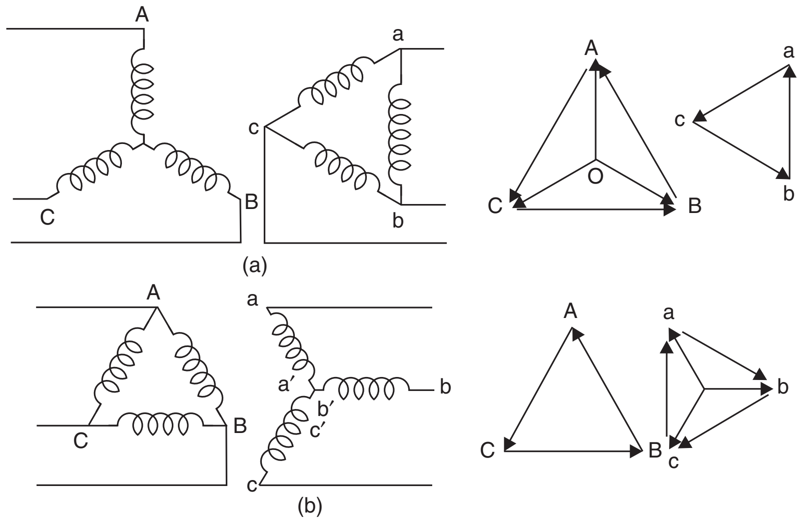


Fig. B.4 (*a*) Star delta connection (*b*) Delta star connection.

Comparing the secondary winding phasor diagrams of both the transformers, it is seen that the line voltages are not in phase, therefore, this will result in a virtual short circuit of the windings. The two voltages can be brought in phase with each other by interchanging two phases of the primary winding and then reversing the terminals (starts and finishes) of all three primary windings as shown in Fig. B.4(*c*) and now the two sets of transformers can be operated in parallel.

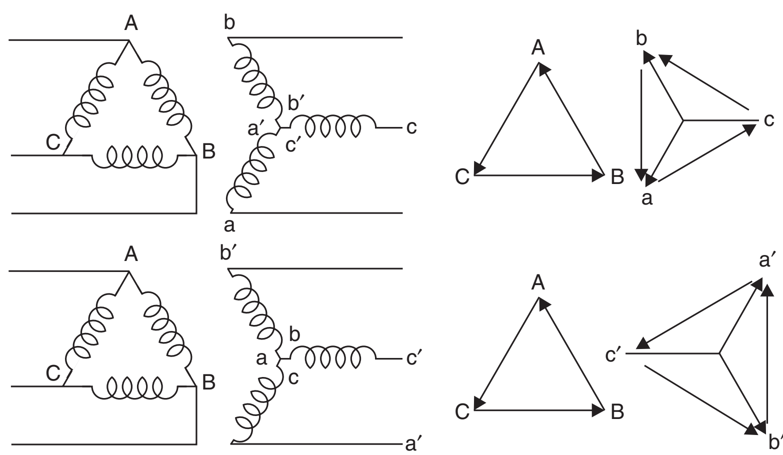


Fig. B.4 (c)

Appendix–C

Synchronous Machine

Synchronous Generator

Consider an alternator connected to an infinite bus Fig. C.1.

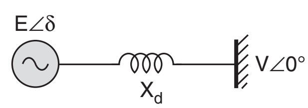


Fig. C.1 An alternator connected to an infinite bus.

An infinite bus means a large integrated power system whose frequency and voltage will not change in case some changes are stipulated in the steam input or the excitation circuit of an alternator which is connected to the system. Our objective here is to study the behaviour of the generator (incoming) when a change in its steam input or change in its excitation circuit is stipulated. It is to be noted that there is no single bus in a large interconnected system which should be considered as an infinite bus. Rather any bus could be considered as an infinite bus and the whole existing system should be considered as an infinite system with respect to the incoming alternator whose behaviour we want to study. Or in a large system we could consider one of the alternators whose behaviour we want to study as a finite machine and the rest of the system as an infinite bus.

In order to study the effect of change in steam input and change in excitation on the performance of the generator, let us derive an expression for the power delivered by the generator.

From Fig. C.1

$$I = \frac{E\angle\delta - V\angle o}{j\,X_d} = \frac{E\angle\delta - 90}{X_d} - \frac{V\angle -90}{X_d}$$

$$\text{Power } P = \text{Real } [V\,I^*]$$

$$= \text{Real}\left[V\left\{\frac{E\angle 90 - \delta}{X_d} - \frac{V\angle 90°}{X_d}\right\}\right]$$

$$= \frac{VE \cos (90 - \delta)}{X_d}$$

$$= \frac{EV}{X_d} \sin \delta \qquad \text{(C.1)}$$

and the reactive power

$$Q = I_m \, [VI^*]$$

$$= \frac{VE \sin (90 - \delta)}{X_d} - \frac{V^2}{X_d}$$

$$= \frac{EV \cos \delta}{X_d} - \frac{V^2}{X_d} \qquad \text{(C.2)}$$

If cos ϕ is the power factor of the alternator, the active and reactive powers are also given as

$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

Therefore,

$$\frac{EV}{X_d} \sin \delta = V I \cos \phi \qquad \text{(C.3)}$$

and

$$\frac{EV}{X_d} \cos \delta - \frac{V^2}{X_d} = V I \sin \phi \qquad \text{(C.4)}$$

From equation (C.2) it is clear that the alternator operates at unity p.f. if the reactive power delivered by the alternator is zero *i.e.,*

$$\frac{EV}{X_d} \cos \delta - \frac{V^2}{X_d} = 0$$

or

$$E \cos \delta = V$$

The excitation of the alternator corresponding to this operation (unity p.f.) is known as normal excitation. The atlernator delivers lagging VARS to the infinite bus if

$$\frac{EV}{X_d} \cos \delta - \frac{V^2}{X_d} > 0$$

or

$$E \cos \delta > V$$

and then the alternator is said to be over-excited as cos $\delta \le 1$ and now E should be greater than the value of E under unity p.f. operation as V is constant for an infinite bus system. Similarly, if the alternator absorbs lagging VARS from the system

$$E \cos \delta < V$$

and the alternator is said to be under-excited.

Effect of Change of Excitation

Synchronous generator—Suppose the generator is delivering power P to the infinite bus when the bus voltage, frequency, power factor and current are V, f, cos ϕ and I respectively. The reactive power delivered by the generator its power angle and induced voltages are Q, δ and E respectively. Our objective is to find out these values if excitation of the alternator is changed.

(*a*) Excitation increased:

(*i*) Initial p.f. lagging

Equation (3) is rewritten as

$$\frac{E}{X_d} \sin \delta = I \cos \phi$$

Since excitation is increased E increases and as there is no change in active power,

$$I \cos \phi = \text{Constant and, therefore,}$$

$$E \sin \delta = \text{Constant}$$

Also as E has increased sin δ should decrease, therefore, δ decreases.

As δ decreases and E increases, from equation (C.2) it becomes clear that Q increases.

Again $$P^2 + Q^2 = (VI)^2 \tag{C.5}$$

Here P and V are constant, Q has increased therefore, I increases.

Also as $I \cos \phi$ = constant (as P is constant) and I has increased $\cos \phi$ decreases. Summarising, the increase in excitation results in

P, f, V	No change
δ, $\cos \phi$	Decrease
E, Q, I	Increase

(*ii*) Unity p.f. (initial)

Results are exactly identical to when the p.f. is lagging initially.

(*iii*) p.f. leading (initial)

For the same power P, δ decreases as E increases for increased excitation since it is an under-excited case.

$$Q = \frac{EV \cos \delta}{X_d} - \frac{V^2}{X_d} < 0$$

and as E has increased and δ has decreased, the term $\frac{EV}{X_d} \cos \delta$ becomes more positive and Q less negative *i.e.*, its magnitude decreases.

From the relation (C.5), I decreases hence $\cos \phi$ increases as $I \cos \phi$ = Constant.

Summarising,

P, V, f	No change
I, δ, $\lvert Q \rvert$	Decrease
E, $\cos \phi$	Increase

(*b*) Excitation decreased

(*i*) Power factor lagging

With decrease in excitation, the induced voltage E of the generator decreases and hence for constant power P, sin δ increases and δ, therefore, increases and from equation (C.2) Q decreases as

$$Q = \frac{EV}{X_d}\cos\delta - \frac{V^2}{X_d} > 0$$

and E and cos δ decrease, therefore, Q decreases and from equation (C.5), I decreases. Since I decreases, and as $I \cos\phi$ = Constant, cos φ increases.

Summarising

P, V, f	No change
Q, I, E	Decrease
$\delta, \cos\phi$	Increase

Similarly it can be justified that if power factor is leading initially

P, V, f	No change
$E, \cos\phi$	Decrease
$I, \delta, \|Q\|$	Increase

In fact all these results can be easily explained with the help of phasor diagram. Here $E \sin\delta$ = Constant and $I \cos\phi$ = Constant *i.e.*, with increase or decrease in E its component normal to V will be constant *i.e.*, its arrow head will lie along a line parallel to V and at a distance $E \sin\delta$. Similarly the arrow head of I will lie along a line whose loci is $I \cos\phi$ = Constant.

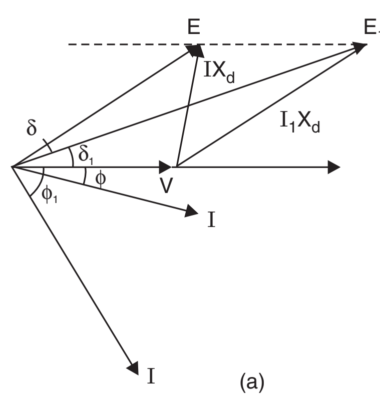


Fig. C.2 Excitation increased.

From phasor diagram

$$E_1 > E, I_1 > I, \cos\phi_1 < \cos\phi \,.\, \delta_1 < \delta$$

$\therefore$ $\qquad Q_1 > Q$, P, V and f remain unchanged.

In case the excitation is decreased, the same phasor diagrams can be used with suffix 1 as the initial operating condition.

(*c*) Steam input increased: With steam input increased, P increases, V, f and E remaining constant,

δ increases as

$$\frac{EV}{X_d}\sin\delta_1 = P_1 \text{ as } P_1 > P\,,\ \delta_1 > \delta$$

(*i*) Initial p.f. lag

$$Q = \frac{EV\cos\delta}{X_d} - \frac{V^2}{X_d} > 0$$

Since $\cos \delta$ decreases Q decreases

Now $\phi = \tan^{-1} \dfrac{Q}{P}$

Since Q has decreased and P has increased ϕ decreases and hence power factor improves. Since δ has increased and $E_1 = E$ the current increases.

Summarising

E, V, f	No change
$\cos\phi, \delta, I, P$	Increase
Q	Decreases.

(*ii*) Initially p.f. leading

E, V, f	No change

δ increases and hence

$$Q = \frac{EV}{X_d}\cos\delta - \frac{V^2}{X_d} < 0$$

becomes more negative and the $|Q|$ increases.

Since P and $|Q|$ have increased, I increases, Again, since P has increased and there is relatively larger increase in Q, the p.f. angle increases and hence p.f. decreases.

Summarising

E, V, f	No change
$P, \delta, I, \lvert Q\rvert$	Increase
$\cos\phi$	Decreases

Similarly, if the steam input is decreased its effect on the various parameters can be studied. Figure C.3 shows the effect of increase in steam input on the various parameters for an initially lagging p.f.

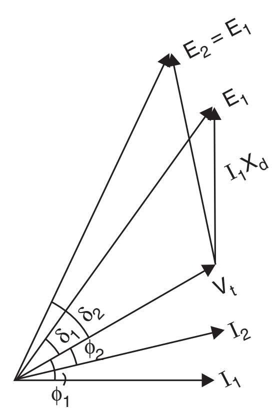


Fig. C.3 Steam input increased.

The following problem will help us understand and realise the treatment given above in a better way.

Example C.1: *An alternator delivers 1.0 p.u. current at 1.0 p.u. voltage and p.f. (i) 0.8 lag (ii) 0.8 lead to an infinite bus. The reactance of the alternator X_d is 1.0 p.u., Determine (a) P, Q, E and δ (b) If the excitation is increased by 20%, calculate P, Q, E, δ, cos φ and I.*

(*c*) If the system is as in (*a*) and the power *P* is increased by 20%, calculate *Q*, *E*, δ, cos φ, *I*, *P*

Solutions: (*i*) Given $V = 1 \angle 0°$, $\cos\phi = 0.8$ lag.

$$I = 1.0 \angle -36.9° = 0.8 - j\,0.6$$

$$E = V + j\,IX_d$$

$$= 1 + j\,0.0 + j(0.8 - j0.6) \times 1$$

$$= 1 + j0.8 + 0.6 = 1.6 + j\,0.8$$

$$= 1.79 \angle 26.56°$$

$$E = 1.79 \text{ p.u. and } \delta = 26.56°$$

$$P = \frac{1.79 \times 1}{1} \sin 26.56 = 0.800$$

$$Q = \frac{1.79 \times 1}{1} \cos 26.56 - \frac{1.0^2}{1.0}$$

$$= 0.60$$

The same results can be obtained using the relation $P = VI \cos\phi$ and $Q = V\,I \sin\phi$

(*ii*) When p.f. lead 0.8

$$E = V + j\,I\,X_d$$

$$= 1.0 + j\,(0.8 + j\,0.6) \times 1$$

$$= 0.4 + j\,0.8 = 0.8944 \angle 63.4$$

$$\therefore \quad |E| = 0.8944 \quad \delta = 63.4°$$

$$P = 0.8 \text{ p.u. and } Q = 0.4 - 1.0 = -0.6 \text{ p.u.}$$

(*b*) $P_1 = P = 0.8$ No change

$$E_1 = 1.2\,E = 1.2 \times 1.79 = 2.148 \text{ p.u.}$$

(*i*) p.f. lag 0.8

$$\therefore \quad \sin\delta_1 = \frac{0.8 \times 1.0}{2.148 \times 1.0} \text{ or } \delta_1 = 21.86$$

$$Q_1 = \frac{2.148 \times 1}{1} \times 0.928 - 1.0$$

$$= 0.994$$

$$I = \sqrt{\frac{0.8^2 + 0.994^2}{1.0^2}} = 1.276 \text{ p.u.}$$

$$\therefore \quad \cos\phi = \frac{0.8}{1 \times 1.276} = 0.627$$

(*ii*) p.f. lead 0.8

$$E_1 = 1.2 \times 0.8944 = 1.07328 \text{ p.u.}$$

$$\sin \delta_1 = \frac{0.8 \times 1}{1.07328 \times 1} = 0.74537$$

$$\delta_1 = 48.19$$

$$Q = \frac{1.07328 \times 1}{1} \cos 48.19 - 1.0$$

$$= -0.2845$$

$$I = \sqrt{\frac{0.8^2 + 0.2845^2}{1.0^2}} = 0.849$$

$$\cos \phi = \frac{0.8}{0.849} = 0.942$$

(*c*) $P_1 = 1.2 \times 0.8 = 0.96$, E, f and V constant

(*i*) p.f. lag 0.8

$$\sin \delta_1 = \frac{0.96 \times 1.0}{1.79 \times 1.0}, \ \delta_1 = 32.43$$

$$Q = \frac{1.79 \times 1.0}{1.0} \cos 32.43 - 1.0$$

$$= 0.51 \text{ p.u.}$$

$$I = \sqrt{\frac{0.96^2 + 0.51^2}{1.0^2}} = 1.087 \text{ p.u.}$$

$$\therefore \quad \cos \phi = \frac{0.96}{1.0 \times 1.087} = 0.883$$

(*ii*) p.f. lead 0.8

$$\sin \delta_1 = \frac{0.96 \times 1.0}{0.8944} > 1.0$$

Hence power 0.96 can't be transmitted during leading p.f. (under-excited case) Say the increase in steam input is 5% then new value of power = 0.8 × 1.05 = 0.84

$$\sin \delta_1 = \frac{0.84 \times 1.0}{0.8944 \times 1.0}, \ \delta_1 = 69.91°$$

$$Q = \frac{0.8944 \times 1.0}{1.0} \cos 69.91 - 1.0$$

$$= -0.6928$$

$$I = \sqrt{\frac{0.84^2 + 0.6928^2}{1.0^2}} = 1.0888 \text{ p.u.}$$

$$\cos \phi = \frac{0.84}{1 \times 1.0888} = 0.7714$$

We know that under-excitation leads to instability of the alternator, let us assume that the initial leading p.f. is 0.95 rather than 0.8 and consider the increase in steam input by 20% and then let us study the operation of the alternator. The initial conditions are:

$$E = V + j\,I\,X_d$$
$$= 1.0 + j\,(0.95 + j\,0.3122) \times 1.0$$
$$= 0.6877 + j\,0.95$$
$$= 1.1728 \angle 54°$$
$$\sin \delta_1 = \frac{0.8 \times 1.2 \times 1.0}{1.1728 \times 1.0}, \ \delta_1 = 54.94$$
$$P = \frac{1.1728 \times 1.0}{1.0} \sin 54° = 0.95 \text{ p.u.}$$
$$Q = \frac{1.1728 \times 1.0}{1.0} \cos 54° - 1.0$$
$$= 0.68935 - 1.0$$
$$= -\,0.3122$$
$$I = 1.0\,(0.95 + j\,0.3122)$$
$$\cos \phi = 0.95 \text{ lead}$$

Now if the steam input is increased by 20%, the power

$$P_1 = 1.2\,P = 1.2 \times 0.9488 = 1.14 \text{ p.u.}$$
$$\sin \delta_1 = \frac{1.14 \times 1.0}{1.1728 \times 1.0} \qquad \delta_1 = 76.4$$
$$Q = \frac{1.1728 \times 1.0}{1.0} \cos 76.4 - 1.0$$
$$= 0.2754 - 1.0$$
$$= -\,0.7245$$
$$Q = \sqrt{\frac{1.14^2 + 0.7245^2}{1.0^2}} = 1.35 \text{ p.u.}$$
$$\cos \phi = \frac{1.14}{1.35 \times 1.0} = 0.844$$

Example C.2: *An alternator is connected to an infinite bus bar and delivers 1.0 p.u. current at 1.0 p.u. voltage and p.f. (i) 0.95 lag (ii) 0.95 lead. The reactance of the alternator X_d = 1.0 p.u. Determine*

(*a*) *P, Q, E and δ*

(*b*) *If the excitation is reduced by 20%, calculate P, Q, E, δ, cos ϕ_1 and current I.*

(*c*) *If the system is as in (a) and the steam input is reduced by 20%, calculate Q, E, δ cos, I, P.*

Solution: (*i*) 0.95 lag

$$E = 1.0 + j\,1.0\,(0.95 - j\,0.3122)$$
$$= 1.3122 + j\,0.95$$
$$= 1.62\,\angle 35.9$$
$$P = \frac{1.62 \times 1.0}{1.0}\sin 35.9$$
$$= 0.95$$
$$Q = 0.3122$$

(*b*)
$$E_1 = 1.62 \times 0.8 = 1.296$$
$$\sin\delta_1 = \frac{0.95 \times 1.0}{1.296 \times 1.0}$$
$$\delta_1 = 47.14°$$
$$Q_1 = \frac{1.296 \times 1.0}{1.0}\cos 47.14 - 1.0$$
$$= -\,0.118$$
$$I = \sqrt{\frac{0.95^2 + 0.118^2}{1.0^2}} = 0.957 \text{ p.u.}$$
$$\cos\phi = \frac{0.95}{1.0 \times 0.957} = 0.9926$$

(*ii*) 0.95 lead

We know from the previous example that for this condition $E = 1.1728\,\angle 54°$

$$E_1 = 1.1728 \times 0.8 = 0.93824$$
$$\sin\delta_1 = \frac{0.95 \times 1.0}{0.93824 \times 1.0}$$

Here again a situation arises when 20% reduction in excitation below certain excitation level is not permissible.

Let us assume that the reduction is by 10% then

$$E_1 = 1.1728 \times 0.9 = 1.055 \text{ p.u.}$$

$\therefore$
$$\sin\delta_1 = \frac{0.95 \times 1.0}{1.055 \times 1.0} = 0.9$$

or
$$\delta_1 = 64.16$$
$$Q_1 = \frac{1.055 \times 1.0}{1.0}\cos 64.16 - 1.0$$
$$= -\,0.54 \text{ p.u.}$$
$$I = \sqrt{\frac{0.95^2 + 0.54^2}{1.0^2}} = 1.09 \text{ p.u.}$$
$$\cos\phi = \frac{0.95}{1.09 \times 1.0} = 0.9047$$

$\therefore$	P, V, f	No change
	$E, \cos\phi$	Decrease
	$I, \delta, \lvert Q\rvert$	Increase

(*c*) Power decreased by 20%

$$P_1 = 0.95 \times 0.8 = 0.76$$

$$E_1 = E = 1.62$$

(*i*) Power factor 0.95 lag

$$\sin\delta_1 = \frac{0.76 \times 1.0}{1.62 \times 1.0}, \ \delta_1 = 27.98 \simeq 28°$$

$$Q = \frac{1.62 \times 1.0}{1.0} \cos 28 - 1.0$$

$$= 0.43 \text{ p.u.}$$

$$I = \sqrt{\frac{0.76^2 + 0.43^2}{1.0^2}} = 0.87 \text{ p.u.}$$

$$\cos\phi = \frac{0.76}{0.87 \times 1.0} = 0.87$$

E, V, f	No change
$P, \cos\phi, I, \delta$	Decrease
Q	Increase

(*ii*) p.f. 0.95 lead

$$E_1 = E = 1.1728 \angle 54°$$

$$\sin\delta_1 = \frac{0.76 \times 1.0}{1.1728 \times 1.0}, \ \delta_1 = 40.4$$

$$Q = \frac{1.1728 \times 1.0}{1.0} \cos 40.4 - 1.0$$

$$= 0.893 - 1.0$$

$$= -0.107 \text{ p.u.}$$

$$I = \sqrt{0.76^2 + 0.107^2} = 0.768 \text{ p.u.}$$

$$\cos\phi \simeq 1.0$$

E, V, f	No change
$I, P, \delta, \lvert Q\rvert$	Decrease
$\cos\phi$	Increase.

Synchronous Motor

Let us now consider a synchronous motor connected to an infinite bus. The power is delivered to the synchronous motor from the infinite bus. Therefore V leads E, the back e.m.f. by an angle δ the torque angle depending upon the loading of the motor. The phasor diagram of the motor both for lagging and leading p.f. is given in Fig. C.4.

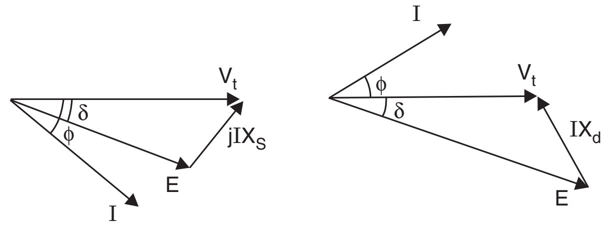


Fig. C.4 (*a*) & (*b*)

We will study the behaviour of the motor when a change in excitation or loading of the motor takes place, with the help of a problem as follows:

Example C.3: *A synchronous motor is connected to an infinite bus and draws 1.0 p.u. current at 1.0 p.u. Voltage (i) 0.9 p.f. lag and (ii) 0.9 p.f. lead. The synchronous reactance of the motor is 1.0 p.u. (a) Determine E, Q, I, δ, P (b) If the excitation is increased by 10%, determine E, Q, I, δ P, cos φ (c) If the system is as in (a) and the loading on the motor is increased by 10%, determine E, Q, I, δ, P, cos φ*

Solution: (*a*) (*i*) p.f. 0.9 lag

$$E = V - j\, I\, X_d$$

$$= 1.0 - j\,(0.9 - j\,0.436) \times 1.0$$

$$= 0.564 - j\,0.9$$

$$= 1.062 \angle -57.9°$$

$$\delta = -57.9°,\ E = 1.062$$

$$P = \frac{1.063 \times 1.0}{1.0} \sin 57.9 = 0.9 \text{ p.u.}$$

Here we are taking δ as positive, even though P is –*ve* as it is motoring action

$$Q = \frac{1.063 \times 1.0}{1.0} \cos 57.9 - 1.0$$

$$= -0.4356 \text{ p.u. (under excitation)}$$

(*b*) Excitation increased by 10%

$$E_1 = 1.1\, E = 1.1682$$

$$\sin \delta_1 = \frac{0.9 \times 1.0}{1.1682 \times 1.0},\ \delta_1 = 50.39$$

$$Q_1 = \frac{1.1682 \times 1.0}{1.0} \cos 50.39 - 1.0$$

$$= -0.255 \text{ p.u.}$$

$$I_1 = \sqrt{\frac{0.9^2 + 0.255^2}{1.0^2}} = 0.935$$

$$\cos \phi_1 = \frac{0.9}{0.935} = 0.96$$

Summarising

P_1, f, V	No change
$\|Q\|, \delta_1, I_1$	Decrease
$E, \cos\phi_1$	Increase

Similarly, if p.f. is leading 0.9, the back e.m.f. originally is

$$E = V - j\,IX_d$$
$$= 1.0 - j(0.9 + j\,0.436) \times 1.0$$
$$= 1.436 - j\,0.9$$
$$= 1.69 \angle -32°$$
$$E = 1.69,\ \delta = -32°,\ Q = 0.436 \text{ p.u.}$$

Now if $E_1 = 1.1 \times 1.69 = 1.859$

$$\sin\delta_1 = \frac{0.9 \times 1.0}{1.859 \times 1.0},\ \delta_1 = -28.9$$
$$Q = \frac{1.859 \times 1.0}{1.0} \cos 28.9 - 1.0$$
$$= 0.627 \text{ p.u.}$$
$$I = \sqrt{\frac{0.9^2 + 0.627^2}{1.0^2}} = 1.076 \text{ p.u.}$$
$$\cos\phi_1 = \frac{0.9}{1.0 \times 1.096} = 0.82$$

Summarising

P, V, f	No change
$\delta, \cos\phi$	Decrease
E, I, Q	Increase

(*c*) $P_1 = 1.1 \times 0.9 = 0.99$

(*i*) p.f. lag $E_1 = E = 1.062$

$$\sin\delta_1 = \frac{0.99 \times 1.0}{1.062 \times 1.0},\ \delta_1 = -68.78$$
$$Q = \frac{1.062 \times 1.0}{1.0} \cos 68.78 - 1.0$$
$$= -0.615 \text{ p.u.}$$
$$I = \sqrt{\frac{0.99^2 + 0.615^2}{1.0^2}} = 1.165 \text{ p.u.}$$
$$\cos\phi = \frac{0.99}{1.165} = 0.849$$

Summarising

E, V, f	No change
$\cos\phi$	Decrease
$P, \delta, \|Q\|, I$	Increase

(*ii*) p.f. 0.9 lead

$$E_1 = E = 1.69$$

$$\sin \delta_1 = \frac{0.99 \times 1.0}{1.69 \times 1.0}, \ \delta_1 = -35.86$$

$$Q = \frac{1.69 \times 1.0}{1.0} \cos 35.86 - 1.0$$

$$= 0.37 \text{ p.u.}$$

$$I = \sqrt{\frac{0.99^2 + 0.37^2}{1.0^2}} = 1.057 \text{ p.u.}$$

$$\cos \phi = \frac{0.99}{1.057} = 0.936$$

Summarising

E, V, f	No change
Q	Decrease
$P, \delta, I, \cos \phi$	Increase

Similarly, the effect of decreasing excitation and/or load on the motor performance can be studied.

OBJECTIVE QUESTIONS

Objective Questions

1. The surge impedance of 50 miles long underground cable is 50 ohms. For a 25 miles length it will be:

(*a*) 25 ohms (*b*) 50 ohms

(*c*) 100 ohms (*d*) None of the above.

2. For a load flow solution the quantities normally specified at a voltage controlled bus are:

(*a*) P and Q (*b*) P and $|V|$

(*c*) Q and $|V|$ (*d*) P and δ.

3. For reducing tower footing resistance it is better to use:

(*a*) Chemical and ground rods only

(*b*) Chemical and counterpoise only

(*c*) Ground rod and counterpoise only

(*d*) Chemical, ground rods and counterpoise.

4. The positive, negative and zero sequence impedances of a solidly grounded system under steady state condition always follow the relations:

(*a*) $Z_1 > Z_2 > Z_0$ (*b*) $Z_1 < Z_2 < Z_0$

(*c*) $Z_0 < Z_1 < Z_2$ (*d*) None of the above.

5. Mho relay is normally used for the protection of:

(*a*) Long transmission lines (*b*) Medium length lines

(*c*) Short length lines (*d*) No length criterion.

6. Reactance relay is normally preferred for protection against:

(*a*) Earth faults only (*b*) Phase faults only

(*c*) None of the above.

7. If the fault current is 2000 amps, the relay setting 50% and the C.T. ratio is 400/5, then the plug setting multiplier will be:

(*a*) 25 amps (*b*) 15 amps

(*c*) 50 amps (*d*) None of the above.

8. Carrier current protection scheme is normally used for:
(*a*) HV transmission lines only
(*b*) HV cables only
(*c*) HV transmission and cables.

9. The ratio of reset to pick up current for an induction cup relay is approx:
(*a*) 0.99 (*b*) 1.01
(*c*) 0.75 (*d*) None of the above.

10. The rotation of disc of an induction disc relay under the poles is:
(*a*) From unshaded pole to shaded pole
(*b*) From shaded pole to unshaded pole
(*c*) It depends upon the magnitude of current
(*d*) It depends upon the C.T. secondary connection.

11. If the time of operation of a relay for unity TMS is 10 secs., the time of operation for 0.5 TMS will be:
(*a*) 20 secs (*b*) 5 secs
(*c*) 10 secs (*d*) None of the above.

12. The shape of the disc of an induction disc relay is:
(*a*) Circular (*b*) Spiral
(*c*) Elliptic.

13. If the phase angle of the voltage coil of a directional relay is 50° the maximum torque angle of the relay is:
(*a*) 130° (*b*) 100°
(*c*) 25° (*d*) None of the above.

14. A reactance relay is:
(*a*) Voltage restrained directional relay
(*b*) Directional restrained over-current relay
(*c*) Voltage restrained over-current relay
(*d*) None of the above.

15. The per cent bias for a generator protection lies between:
(*a*) 5 to 10 (*b*) 10 to 15
(*c*) 15 to 20 (*d*) None of the above.

16. For protection of parallel feeders fed from one end the relays required are:
(*a*) Non-directional relays at the source end and directional relays at the load end.
(*b*) Non-directional relays at both the ends.
(*c*) Directional relays at the source end and non-directional at the load end.
(*d*) Directional relays at both the ends.

17. The number of pilot wires required for protecting 3-phase transmission lines using Translay system of protection is:

(*a*) 6 (*b*) 4

(*c*) 3 (*d*) 2.

18. A shunt fault is characterized by:

(*a*) Increase in current, frequency and p.f.

(*b*) Increase in current reduction in frequency and p.f.

(*c*) Increase in current and frequency but reduction in p.f.

(*d*) None of the above.

19. For measuring positive, negative and zero sequence voltages in a system, the reference is taken as:

(*a*) Neutral of the system only

(*b*) Ground only

(*c*) For zero sequence neutral and for positive and negative the ground

(*d*) None of the above.

20. The most economic load on an overhead line is:

(*a*) Greater than the natural load (*b*) Less than the natural load

(*c*) Equal to the natural load (*d*) None of the above is necessary.

21. The most economic load on an underground cable is:

(*a*) Greater than its surge loading (*b*) Less than the surge loading

(*c*) Equal to the surge loading (*d*) None of the above is necessary.

22. For a long transmission line, for a particular receiving end voltage, when sending end voltage is calculated, it is more than the actual value when calculated by:

(*a*) Load end capacitance method (*b*) Nominal T method

(*c*) Nominal π method (*d*) None of the above methods.

23. The p.u. impedance value of an alternator corresponding to base values 13.2 kV and 30 MVA is 0.2 p.u. The p.u. value for the base values 13.8 kV and 50 MVA will be:

(*a*) 0.306 p.u. (*b*) 0.33 p.u.

(*c*) 0.318 p.u. (*d*) 0.328 p.u.

24. For the same rupturing capacity, the actual current to be interrupted by an HRC fuse is:

(*a*) Much less than any CB (*b*) Much more than any CB

(*c*) Equal to the CB (*d*) None of the above is necessary.

25. The stability of arc in vacuum depends upon:

(*a*) The contact material only

(*b*) The contact material and its vapour pressure

(*c*) The circuit parameters only

(*d*) The combination of (*b*) and (*c*).

26. To limit current chopping in vacuum circuit breakers, the contact material used has:
(*a*) High vapour pressure and low conductivity properties
(*b*) High vapour pressure and high conductivity properties
(*c*) Low vapour pressure and high conductivity properties
(*d*) Low vapour pressure and low conductivity properties.

27. The capacitor switching is easily done with:
(*a*) Air blast circuit breaker (*b*) Oil C.B.
(*c*) Vacuum C.B. (*d*) Any one of the above.

28. Where voltages are high and current to be interrupted is low the breaker preferred is:
(*a*) Air blast C.B. (*b*) Oil C.B.
(*c*) Vacuum C.B. (*d*) Any one of the above.

29. For rural electrification in a country like India with complex network it is preferable to use:
(*a*) Air break C.B. (*b*) Oil C.B.
(*c*) Vacuum C.B. (*d*) M.O. C.B.

30. The current chopping tendency is minimised by using the SF_6 gas at relatively:
(*a*) High pressure and low velocity (*b*) High pressure and high velocity
(*c*) Low pressure and low velocity (*d*) Low pressure and high velocity.

31. SF_6 gas has excellent heat transfer properties because of its:
(*a*) Higher molecular weight (*b*) Low gaseous viscosity
(*c*) Higher dielectric strength (*d*) A combination of (*a*) and (*b*)
(*e*) A combination of (*b*) and (*c*).

32. A fault is more severe from the view point of RRRV if it is a:
(*a*) Short line fault (*b*) Medium length line fault
(*c*) Long line fault (*d*) None of the above.

33. The most suitable C.B. for short line fault without switching resistor is:
(*a*) Air blast C.B. (*b*) M.O.C.B.
(*c*) SF_6 Breaker (*d*) None of the above.

34. The voltages at the two ends of a line are 132 kV and its reactance is 40 ohms. The capacity of the line is:
(*a*) 435.6 MW (*b*) 217.5 MW
(*c*) 251.5 MW (*d*) 500 MW.

35. An overhead line with series compensation is protected using:
(*a*) Impedance relay (*b*) Reactance relay
(*c*) Mho relay (*d*) None of the above.

36. The typical value of SCR for modern alternators is:
(*a*) 1.5 (*b*) 1.2
(*c*) 1.0 (*d*) 0.5.

37. In case of a large size alternator the voltage control becomes serious if:
(*a*) SCR is high (*b*) SCR is low
(*c*) Voltage control is independent of SCR.

38. A large-size alternator is protected against overloads by providing:
(*a*) Overcurrent relays (*b*) Temperature sensitive relays
(*c*) Thermal relays (*d*) A combination of (*a*) and (*b*)
(*e*) A combination of (*b*) and (*c*).

39. The impulse ratio of a rod gap is:
(*a*) Unity (*b*) Between 1.2 and 1.5
(*c*) Between 1.6 and 1.8 (*d*) Between 2 and 2.2.

40. Shunt compensation in an EHV is resorted to:
(*a*) Improve the stability
(*b*) Reduce the fault level
(*c*) Improve the voltage profile
(*d*) As a substitute for synchronous phase modifier.

41. In order to have lower cost of electrical energy generation:
(*a*) The load factor and diversity factor should be low
(*b*) The load factor should be low but diversity factor should be high
(*c*) The load factor should be high but diversity factor low
(*d*) The load factor and diversity factors should be high.

42. The insulation of the modern EHV lines is designed based on:
(*a*) The lighting voltage (*b*) The switching voltage
(*c*) Corona (*d*) RI.

43. The size of conductor on modern EHV lines is obtained based on:
(*a*) Voltage drop (*b*) Current density
(*c*) Corona (*d*) (*a*) and (*b*).

44. A Mho relay is a:
(*a*) Voltage restrained directional relay
(*b*) Voltage controlled over current relay
(*c*) Directional restrained over current relay
(*d*) Directional restrained over voltage relay.

45. For stability and economic reasons we operate the transmission line with power angle in the range:
(*a*) 10° to 25° (*b*) 30° to 45°
(*c*) 60° to 75° (*d*) 65° to 80°.

46. High voltage d.c. testing for HV machines is resorted because:
(*a*) Certain conclusions regarding the continuous ageing of an insulation can be drawn.

(*b*) The stress distribution is a representation of the service condition

(*c*) Standardization on the magnitude of voltage to be applied is available

(*d*) The stresses do not damage the coil end insulation.

47. For a lumped inductive load, with increase in supply ferquency:

(*a*) P and Q increase (*b*) P increases, Q decreases

(*c*) P decreases, Q increases (*d*) P and Q decrease.

48. Standard impulse testing of a power transformer requires:

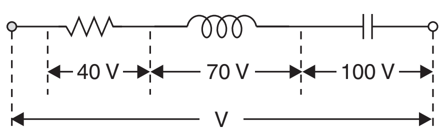
(*a*) Two applications of chopped wave followed by one application of a full wave

(*b*) One application of chopped wave followed by one application of a full wave

(*c*) One application of chopped wave followed by two applications of a full wave

(*d*) None of the above.

49. The supply voltage $|V|$ in diagram below is:



(*a*) 210 V (*b*) 70 V

(*c*) 50 V (*d*) 230 V.

50. The chances of arc interruption in subsequent current zero:

(*a*) Increases in case of *OCB*

(*b*) Decreases in case of *OCB*

(*c*) Interruption is always at first current zero (in *OCB*)

(*d*) None of the above.

51. The chances of arc interruption in subsequent current zeros:

(*a*) Increases in case of *ABCB* but decreases in *OCB*

(*b*) Decreases in case of *ABCB* but increases in *OCB*

(*c*) Decreases in both the cases

(*d*) Increases in both the cases.

52. The velocity of travelling wave through a cable of relative permittivity 9 is:

(*a*) 9×10^8 metres/sec (*b*) 3×10^8 metres/sec

(*c*) 10^8 metres/sec (*d*) 2×10^8 metres/sec.

53. An overhead line with surge impedance 400 ohms is terminated through a resistance R. A surge travelling over the line does not suffer any reflection at the junction if the value of R is:

(*a*) 20 ohms (*b*) 200 ohms

(*c*) 800 ohms (*d*) None of the above.

54. The rate of rise of restriking voltage depends upon:
(*a*) The type of circuit breaker
(*b*) The inductance of the system only
(*c*) The capacitance of the system only
(*d*) The inductance and capacitance of the system.

55. Gas turbines can be brought to the bus bar from cold in about:
(*a*) 2 minutes (*b*) 30 minutes
(*c*) 1 Hour (*d*) 2 Hours.

56. A 3-phase breaker is rated at 2000 MVA, 33 kV, its making current will be:
(*a*) 35 kA (*b*) 49 kA
(*c*) 70 kA (*d*) 89 kA.

57. If a combination of HRC fuse and circuit breaker is used, the C.B. operates for:
(*a*) Low overload currents (*b*) Short circuit current
(*c*) Under all abnormal current (*d*) The combination is never used in practice.

58. The impulse ratio of a gap of given geometry and dimension is:
(*a*) Greater with solid than with air dielectric
(*b*) Greater with air than with solid dielectric
(*c*) Same for both solid and air dielectric.

59. Tick out the correct one:
(*a*) The insulators and lightning arresters should have high impulse ratio
(*b*) The insulators and lightning arresters should have low impulse ratio
(*c*) The insulator should have high impulse ratio and lightning arrester low
(*d*) The lightning arrester should have high impulse ratio but insulator low.

60. Phase modifier is normally installed in the case of:
(*a*) Short transmission lines (*b*) Medium length lines
(*c*) Long length lines (*d*) For all length lines.

61. Series compensation on EHV lines is resorted to:
(*a*) Improve the stability
(*b*) Reduce the fault level
(*c*) Improve the voltage profile
(*d*) As a substitute for synchronous phase modifier.

62. Ferranti effect on long overhead lines is experienced when it is:
(*a*) Lightly loaded (*b*) On full load at unity p.f.
(*c*) On full load at 0.8 p.f. lag (*d*) In all these cases.

63. Stringing chart is useful for:
(*a*) Finding the sag in the conductor (*b*) In the design of tower
(*c*) In the design of insulator string (*d*) Finding the distance between the tower.

64. The presence of earth in case of overhead lines:
(*a*) Increases the capacitance
(*b*) Increases the inductance
(*c*) Decreases the capacitance
(*d*) Decreases the inductance.

65. For an existing a.c. transmission line the string efficiency is 80%. Now if d.c. voltage is supplied for the same set up, the string efficiency will be:
(*a*) 80%
(*b*) More than 80%
(*c*) Less than 80%
(*d*) 100%.

66. Effect of increase in temperature in overhead transmission lines is to:
(*a*) Increase the stress and length
(*b*) Decrease the stress and length
(*c*) Decrease the stress but increase the length
(*d*) None of the above.

67. The Buchholz relay protects a transformer from:
(*a*) All types of internal fault
(*b*) A turn to turn fault
(*c*) Winding to winding fault
(*d*) None of them.

68. The Self GMD method is used to evaluate:
(*a*) Inductance
(*b*) Capacitance
(*c*) Inductance and capacitance both
(*d*) None of the above
of the overhead transmission lines.

69. A voltage-controlled bus is treated as a load bus in subsequent iteration when its:
(*a*) Voltage limit is violated
(*b*) Active power limit is violated
(*c*) Reactive power limit is violated
(*d*) Phase angle limit is violated.

70. The voltage of a particular bus can be controlled by controlling the:
(*a*) Phase angle
(*b*) Reactive power of the bus
(*c*) Active power of the bus
(*d*) Phase angle and reactive power.

71. In case of a 3-phase short circuit in a system, the power fed into the system is:
(*a*) Mostly reactive
(*b*) Mostly active
(*c*) Active and reactive both equal
(*d*) Reactive only.

72. The load flow solution is always assured in case of:
(*a*) Newton-Raphson method
(*b*) Gauss method
(*c*) Gauss-Seidel method
(*d*) None of these methods guarantees.

73. If α is the angle of voltage wave at which an R-L circuit is switched in and θ is the impedance angle of the R-L circuit, there will be no transient when the circuit is switched in, if:
(*a*) $\alpha = \theta$
(*b*) $\alpha = 90 - \theta$
(*c*) $\alpha = 90 + \theta$
(*d*) None of the above.

74. At a particular unbalanced node, the real powers specified are:

Leaving the node 20 MW, 25 MW

Entering the node 60 MW, 30 MW

The balancing power will be:

(*a*) 30 MW leaving the node

(*b*) 45 MW leaving the node

(*c*) 45 MW entering the node

(*d*) 22.5 MW entering the node and 22.5 MW leaving the node.

75. The loss coefficients for a two-bus system are given as follows:

(*a*) $B_{11} = 0.02$, $B_{22} = 0.05$, $B_{12} = 0.01$, $B_{21} = 0.015$.

(*b*) $B_{11} = 0.02$, $B_{22} = 0.04$, $B_{12} = -0.01$, $B_{21} = 0.001$.

(*c*) $B_{11} = 0.03$, $B_{22} = 0.005$, $B_{12} = -0.001$, $B_{21} = -0.001$.

(*d*) $B_{11} = 0.03$, $B_{22} = 0.05$, $B_{12} = 0.001$, $B_{21} = -0.001$.

76. For economic operation, the generator with highest positive incremental transmission loss will operate at:

(*a*) The lowest positive incremental cost of production

(*b*) The lowest negative incremental cost of production

(*c*) The highest positive incremental cost of production

(*d*) None of the above.

77. The inertia constant H of a machine of 200 MVA is 2 p.u. its value corresponding to 400 MVA will be:

(*a*) 4.0 (*b*) 2.0

(*c*) 1.0 (*d*) 0.5.

78. The inertia constant of two groups of machines, which swing together are M_1 and M_2. The inertia constant of the system is:

(*a*) $\dfrac{M_1 M_2}{M_1 + M_2}$ (*b*) $M_1 - M_2$, $M_1 > M_2$

(*c*) $M_1 + M_2$ (*d*) $\dfrac{M_1 + M_2}{M_1 M_2}$.

79. The leakage resistance of a 50 km long cable is 1 MΩ. For a 100 km long cable it will be:

(*a*) 1 MΩ (*b*) 2 MΩ

(*c*) 0.66 MΩ (*d*) None of the above.

80. The coefficient of reflection for current for an open ended line is:

(*a*) 1.0 (*b*) 0.5

(*c*) – 1.0 (*d*) Zero.

81. The connected load of a consumer is 2 kW and his maximum demand is 1.5 kW. The load factor of the consumer is:

(*a*) 0.75 (*b*) 0.375

(*c*) 1.33 (*d*) None of the above.

82. The maximum demand of a consumer is 2 kW and his daily energy consumption is 20 units. His load factor is:

(*a*) 10% (*b*) 41.6%

(*c*) 50% (*d*) None of the above.

83. Resistance switching is normally resorted in case of:

(*a*) Bulk oil circuit breakers (*b*) Minimum oil circuit breakers

(*c*) Air blast circuit breakers (*d*) All types of breakers.

84. If the inductance and capacitance of a system are 1.0 H and 0.01 μF respectively and the instantaneous value of current interrupted is 10 amp, the voltage across the breaker contacts will be:

(*a*) 50 kV (*b*) 100 kV

(*c*) 60 kV (*d*) 57 kV.

85. The p.u. synchronous impedance of a machine is 2.0 p.u., its SCR is:

(*a*) 2 (*b*) 1.414

(*c*) 0.5 (*d*) 0.707.

86. Sheaths are used in cables to:

(*a*) Provide proper insulation (*b*) Provide mechanical strength

(*c*) Prevent ingress of moisture (*d*) None of the above.

87. Tick out the correct statement:

(*a*) The higher the initial load, the larger the critical clearing angle

(*b*) The higher the initial load, the lower the critical clearing angle

(*c*) The initial load has nothing to do with the critical clearing angle

(*d*) The higher the operating time of the circuit breaker, the larger will be the critical clearing angle.

88. The capacitance and inductance per unit length of a line operating at 110 kV are 0.01 μF and 2 mH. The surge impedance loading of the line is:

(*a*) 40 MVA (*b*) 30 MVA

(*c*) 27 MVA (*d*) None of the above.

89. An RLC series circuit remains predominantly inductive:

(*a*) At resonance frequency (*b*) Below resonance frequency

(*c*) Above resonance frequency (*d*) At the lower half power frequency.

90. An alternator is delivering a load current; its per cent regulation is found to be zero. The type of load it delivers is:

(*a*) Capacitive (*b*) Inductive

(*c*) Resistive.

91. A short length of cable between the dead end tower and the power transformer :

(*a*) Always reduces the steepness of the incident wave (*b*) Reduces the steepness of the wave under certain conditions only

(*c*) It does not change the steepness.

92. For protection of rotating machines against lightning surges a combination of:

(*a*) Lightning conductor and capacitor

(*b*) Lightning conductor and lightning arrester

(*c*) Lightning arrester and capacitor

(*d*) Lightning arrester alone is used.

93. An alternator is delivering a balanced load at unity p.f. the phase angle between line voltage and line current is:

(*a*) 90° (*b*) 60°

(*c*) 30° (*d*) 0°.

94. For the stable operation of interconnected system, the passive element that can be used a interconnecting element is:

(*a*) Reactor (*b*) Resistor

(*c*) Capacitor (*d*) Resistor and capacitor.

95. If two synchronous generators are connected, loss of synchronism will result in:

(*a*) Stalling of generators

(*b*) Wild fluctuations in current

(*c*) Wild fluctuations in current and voltage

(*d*) None of the above.

96. A generator is connected to a synchronous motor. From stability point of view it is preferable to have:

(*a*) Generator neutral reactance grounded and motor neutral resistance grounded

(*b*) Generator and motor neutrals resistance grounded

(*c*) Generator and motor neutrals reactance grounded

(*d*) Generator neutral resistance and motor neutral reactance grounded.

97. A 50 Hz, four-pole turboalternator rated at 20 MVA, 13.2 kV has an inertia constant H = 4 kW sec/kVA. The K.E. stored in the rotor at synchronous speed is:

(*a*) 80 kilojoules

(*b*) 80 megajoules

(*c*) 40 megajoules

(*d*) 20 megajoules.

98. The breakdown strength of air at STP is 21 kV/cm. Its breakdown strength at 30°C and 72 cm of Hg will be:

(*a*) 21.25 kV/cm (*b*) 20.2 kV/cm

(*c*) 23 kV (*d*) 19.5 kV/cm.

99. Corona loss is less when the shape of the conductor is:

(*a*) Circular (*b*) Flat

(*c*) Oval (*d*) Independent of shape.

100. Corona loss increases with:

(*a*) Increase in supply frequency and conductor size

(*b*) Increase in supply frequency but reduction in conductor size

(*c*) Decrease in supply frequency and conductor size

(*d*) Decrease in supply frequency but increase in conductor size.

101. The corona loss on a particular system at 50 Hz is 1 kW/phase per km. The corona loss on the same system with supply frequency 25 Hz will be:

(*a*) 1 kW/phase/km (*b*) 0.5 kW/phase/km

(*c*) 0.667 kw/phase/km (*d*) None of the above.

102. A system is said to be effectively grounded if its:

(*a*) Neutral is grounded directly (*b*) Ratio of $\frac{X_0}{X_1} > 3.0$

(*c*) Ratio of $\frac{R_0}{X_1} > 2.0$ (*d*) Ratio of $\frac{X_0}{X_1} < 3.0$.

103. For effective application of counterpoise is should be buried into the ground to a depth of:

(*a*) 1 metre (*b*) 2 metres

(*c*) Just enough to avoid theft (*d*) None of the above.

104. If r is the radius of the conductor and R the radius of the sheath of the cable, the cable operates stably from the view point of dielectric strength if:

(*a*) $\frac{r}{R} > 1.0$ (*b*) $\frac{r}{R} < 1.0$

(*c*) $\frac{r}{R} < 0.632$ (*d*) $\frac{r}{R} < 0.368$.

105. Three insulating materials with same maximum working stress and permittivities 2.5, 3.0, 4.0 are used in a single core cable. The location of the materials with respect to the core of the cable will be:

(*a*) 2.5, 3.0, 4.0 (*b*) 3.0, 2.5, 4.0

(*c*) 4.0, 3.0, 2.5 (*d*) 4.0, 2.5, 3.0.

106. Three insulating materials with breakdown strengths of 250 kV/cm, 200 kV/cm, 150 kV/cm and permittivities of 2.5, 3.0 and 3.5 are used in a single core cable. If the factor of safety for the materials is 5, the location of the materials with respect to the core of the cable will be:

(*a*) 2.5, 3.0, 3.5 (*b*) 3.0, 2.5, 3.5

(*c*) 3.5, 3.0, 2.5 (*d*) 3.5, 2.5, 3.0.

107. The coefficient of reflection of voltage for a short circuited line is:

(*a*) 1.0 (*b*) – 1.0

(*c*) 0 (*d*) 2.0.

108. The insulation resistance of a cable of length 10 km is 1 MΩ, its resistance for 50 km length will be:

(*a*) 1 MΩ (*b*) 5MΩ

(*c*) 0.2 MΩ (*d*) None of the above.

109. The capacitance of a 3-core cable between any two conductors with sheath earthed is 2 μF. The capacitance per phase will be:

(*a*) 1 μF (*b*) 4 μF

(*c*) 0.667 μF (*d*) 1.414 μF.

110. If δ is the loss angle of the cable, its power factor is:

(*a*) sin δ

(*b*) cos δ

(*c*) Power factor is independent of δ

(*d*) Power factor depends upon δ but it is not as per (*a*) and (*b*).

111. The effect of bonding the cable is:

(*a*) To increase the effective resistance and inductance

(*b*) To increase the effective resistance but reduce inductance

(*c*) To decrease the effective resistance and inductance

(*d*) To decrease the effective resistance but increase the inductance.

112. In order to eliminate sheath losses, a successful method is:

(*a*) To transpose the cable along with cross bonding

(*b*) Transpose the cables only

(*c*) Cross bonding the cables is enough

(*d*) None of the above is effective.

113. The sending end voltage of a feeder with reactance 0.2 p.u. is 1.2 p.u. If the reactive power supplied at the receiving end of the feeder is 0.3 p.u., the approximate drop of volts in the feeder is:

(*a*) 0.2 p.u. (*b*) 0.06 p.u.

(*c*) 0.05 p.u. (*d*) 0.072 p.u.

114. If a synchronous machine is overexcited, it takes lagging vars from the system when it is operated as a:

(*a*) Synchronous motor

(*b*) Synchronous generator

(*c*) Synchronous motor as well as generator

(*d*) None of the above.

115. If a synchronous machine is underexcited it takes lagging vars from the system when it is operated as a:

(*a*) Synchronous motor

(*b*) Synchronous generator

(*c*) Synchronous motor as well as generator

(*d*) None of the above.

116. A synchronous machine has higher capacity for:

(*a*) Leading p.f.

(*b*) Lagging p.f.

(*c*) It does not depend upon the p.f. of the machine

(*d*) It depends upon the p.f. of the load.

117. A machine designed to operate at full load is physically heavier and is costlier if the operating p.f. is:

(*a*) Lagging (*b*) Leading

(*c*) The size and cost do not depend on p.f.

118. For the same voltage boost, the reactive power capacity is more for a:

(*a*) Shunt capacitor (*b*) Series capacitor

(*c*) It is same for both series and shunt.

119. A synchronous phase modifier as compared to a synchronous motor used for mechanical loads has:

(*a*) Larger shaft and higher speed (*b*) Smaller shaft and higher speed

(*c*) Larger shaft and smaller speed (*d*) Smaller shaft and smaller speed.

120. If I_{a_1} is the positive sequence current of an alternator and Z_1, Z_2 and Z_0 are the sequence impedances of the alternator. The drop produced by the current I_{a_1} will be:

(*a*) $I_{a_1} Z_1$ (*b*) $I_{a_1}(Z_1 + Z_2)$

(*c*) $I_{a_1}(Z_1 + Z_2 + Z_0)$ (*d*) $I_{a_1}(Z_2 + Z_0)$.

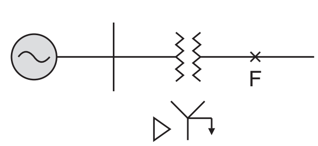
121. For the system shown in diagram below, a line-to-ground fault on the line side of the transformer is equivalent to:

(*a*) A line-to-ground fault on the generator side of the transformer

(*b*) A line-to-line fault on the generator side of the transformer

(*c*) A double line-to-ground on the generator side of the transformer

(*d*) A 3-phase fault on the generator side of the transformer.



122. The positive sequence component of voltage at the point of fault is zero when it is a:

(*a*) 3-phase fault (*b*) *L-L* fault

(*c*) *L-L-G* fault (*d*) *L-G* fault.

123. Tick out the correct statement:

(*a*) The negative and zero sequence voltages are maximum at the fault point and decrease towards the neutral

(*b*) The negative and zero sequence voltages are minimum at the fault point and increase towards the neutral

(*c*) The negative sequence is maximum and zero sequence minimum at the fault point and decrease and increase respectively towards the neutral.

(*d*) None of the above.

124. For complete protection of a 3-phase line:

(*a*) Three-phase and three-earth fault relays are required

(*b*) Three-phase and two-earth fault relays are required

(*c*) Two-phase and two-earth fault relays are required

(*d*) Two-phase and one-earth fault relays are required.

125. The phase comparators in case of static relays and electro-mechanical relays normally are:

(*a*) Sine and cosine comparators respectively

(*b*) Cosine and sine comparators respectively

(*c*) Both are cosine comparators

(*d*) Both are sine comparators.

126. The order of the lightning discharge current is:

(*a*) 10,000 amp (*b*) 100 amp

(*c*) 1 amp (*d*) 1 microampere.

127. The magnetising-inrush-current in a transformer is rich in:

(*a*) 3rd harmonics (*b*) 5th harmonics

(*c*) 7th harmonics (*d*) 2nd harmonics.

128. For effective use of a counterpoise wire:

(*a*) Its leakage resistance should be greater than the surge impedance

(*b*) Its leakage resistance should be less than the surge impedance

(*c*) Its leakage resistance should be equal to the surge impedance

(*d*) The two resistances may have any relation.

129. The inertia constants of two groups of machines which do not swing together are M_1 and M_2. The equivalent inertia constant of the system is:

(*a*) $M_1 + M_2$ (*b*) $M_1 - M_2$ if $M_1 > M_2$

(*c*) $\dfrac{M_1M_2}{M_1 + M_2}$ (*d*) $\sqrt{M_1M_2}$.

130. If the inductance and capacitance of a system are 1 H and 0.01 μF respectively and the instantaneous value of current interrupted is 10 amps, the value of shunt resistance across the breaker for critical damping is:

(*a*) 100 kΩ (*b*) 10 kΩ
(*c*) 5 kΩ (*d*) 1 kΩ.

131. For load flow solution the quantities specified at a load bus are:

(*a*) P and Q (*b*) P and $|V|$
(*c*) Q and $|V|$ (*d*) P and δ.

132. The solution of coordination equations takes into account:

(*a*) All the system constraints
(*b*) All the operational constraints
(*c*) All the system and operation constraints
(*d*) None of the above.

133. For a two-bus system if the change in load at bus 2 is 5 MW and the corresponding change in generation at bus 1 is 8 MW, the penalty factor of bus 1 is:

(*a*) 0.6 (*b*) 1.67
(*c*) 0.625 (*d*) None of the above.

134. If the penalty factor for bus 1 in a two-bus system is 1.25 and if the incremental cost of production at bus 1 is Rs. 200 per MWhr, the cost of received power at bus 2 is:

(*a*) Rs. 250/MWhr (*b*) Rs. 62.5/MWhr
(*c*) Rs. 160/MWhr (*d*) None of the above.

135. The cost of generation is theoretically minimum if:

(*a*) The system constraints are considered
(*b*) The operational constraints are considered
(*c*) (*a*) and (*b*)
(*d*) The constraints are not considered.

136. The incremental transmission loss of a plant is:

(*a*) Positive always (*b*) Negative always
(*c*) Can be positive or negative.

137. If P_m is the maximum power transferred, the loss on the system is:

(*a*) $\frac{P_m}{4}$ (*b*) $\frac{P_m}{2}$
(*c*) $\frac{3P_m}{4}$ (*d*) None of the above.

138. If X is the system reactance and R its resistance, the power transferred is maximum when:

(*a*) $X = R$ (*b*) $X = \sqrt{2R}$
(*c*) $X = \sqrt{3R}$ (*d*) $X = 2R$.

139. If the inertia constant H of a machine of 200 MVA is 2 p.u. its value corresponding to 400 MVA will be:

(*a*) 4 p.u (*b*) 2 p.u.

(*c*) 1.0 p.u. (*d*) 0.5 p.u.

140. The inertia constant of two groups of machines which do not swing together are M_1 and M_2 such that $M_1 > M_2$. It is proposed to add some inertia to one of the two groups of machines for improving the transient stability of the system. It should be added to:

(*a*) M_1 (*b*) M_2

(*c*) It does not matter whether to add to M_1 or M_2.

141. Tick out the correct one:

(*a*) Higher the SCR of a machine the heavier is its rotor

(*b*) Higher the SCR of a machine the lighter is its rotor

(*c*) The SCR and size of the rotor are not at all related.

142. A 100 V/10 V, 50 VA transformer is converted to 100 V/110 V auto transformer, the rating of the auto transformer is:

(*a*) 550 VA (*b*) 500 VA

(*c*) 110 VA (*d*) 100 VA.

143. A 100 KVA transformer has maximum efficiency of 98% when operating at half full load. Its full load losses are:

(*a*) Cu = 2.04 kW and Iron loss = 0.51 kW

(*b*) Cu = 4.08 kW and Iron loss = 1.02 kW

(*c*) Cu = 3.0 kW and Iron loss = 0.75 kW

(*d*) None of the above.

144. A voltmeter gives 120 oscillations per minute when connected to the rotor. The stator frequency is 50 Hz. The slip of the motor is:

(*a*) 2% (*b*) 4%

(*c*) 5% (*d*) 2.5%.

145. If the excitation of the synchronous generator fails, it acts as a:

(*a*) Synchronous motor (*b*) Synchronous generator

(*c*) Induction motor (*d*) Induction generator.

146. I and T are the line current and the torque respectively when DOL starter is used, these quantities when Y/Δ starter is used, are:

(*a*) $\dfrac{I}{3}, \dfrac{T}{\sqrt{3}}$ (*b*) $\dfrac{I}{\sqrt{3}}, \dfrac{T}{3}$

(*c*) $\dfrac{I}{\sqrt{2}}, \dfrac{T}{\sqrt{2}}$ (*d*) $\dfrac{I}{2}, \dfrac{T}{4}$.

147. A 100 KVA transformer has 4% impedance and 50 KVA transformer has 3% impedance. When they are operated in parallel, which transformer will reach full load first.

(*a*) 3% (*b*) 4%

(*c*) The data is insufficient to judge.

148. In EHV transmission lines, efficiency of transmission can be increased by decreasing the corona loss. This is achieved by

(*a*) Increasing the distance between the line conductors

(*b*) Using bundled conductors

(*c*) Using thick conductors

(*d*) Using thin conductors.

149. An alternator having induced emf. of 1.6 p.u. is connected to an infinite bus of 1.0 p.u. If the busbar has reactance of 0.6 p.u. and alternator has reactance of 0.2 p.u., the maximum power that can be transferred is given by

(*a*) 8 p.u. (*b*) 2 p.u.

(*c*) 2.67 p.u. (*d*) 5.0 p.u.

150. If one phase of supply goes off in the case of 3-phase induction motor, the motor

(*a*) Comes to a stop

(*b*) Draws double the initial current and continues to run

(*c*) Draws $1/\sqrt{3}$ times the initial current and continues to run

(*d*) None of the above.

151. Induction generator works between the slip

(*a*) $1 < s < 2$ (*b*) $0.1 < s < 1.0$

(*c*) $s < 0.0$ (*d*) None of the above.

152. The motor which can be used on both a.c. and d.c. is

(*a*) Reluctance motor (*b*) Induction motor

(*c*) d.c. series motor (*d*) None of the above.

153. If transformer frequency is changed from 50 Hz to 60 Hz, the ratio of eddy current loss 50 Hz to 60 Hz at constant voltage is

(*a*) 5/6 (*b*) 25/36

(*c*) 6/5 (*d*) 1.0.

154. An alternator of 300 kW is driven by a prime mover of speed regulation 4% and another alternator of 200 kW driven by a prime mover of speed regulations 3%, the total load they can take is

(*a*) 500 kW (*b*) 567 kW

(*c*) 425 kW (*d*) 257 kW.

155. The minimum oil circuit breaker has less volume of oil because:

(*a*) There is insulation between contacts

(*b*) The oil between the breaker contacts has greater strength

(*c*) Solid insulation is provided for insulating the contacts from earth

(*d*) None of the above is true.

156. Load flow study is carried out for

(*a*) Fault calculations (*b*) Stability studies

(*c*) System planning (*d*) Load frequency control.

157. Arcing on transmission lines is prevented by connecting a suitable

(*a*) Circuit breaker (*b*) Protective relay

(*c*) Inductor in the neutral (*d*) Capacitor in the neutral.

158. The in-rush current of a transformer at no load is maximum if the supply voltage is switched on

(*a*) At zero voltage value (*b*) At peak voltage value

(*c*) At $V/2$ value (*d*) At $\sqrt{3}/2$ V value.

159. If the loading of the line corresponds to the surge impedance loading the voltage at the receiving end is

(*a*) Greater than sending end (*b*) Less than sending end

(*c*) Equal to the sending end (*d*) None of the above is necessary.

160. The voltages of a generator and an infinite bus are given as $0.92\angle 10°$ and $1.0\angle 0°$ respectively. The generator acts as a

(*a*) Shunt coil (*b*) Shunt capacitor

(*c*) The data is insufficient to judge.

161. For a synchronous phase modifier the load angle is

(*a*) 0° (*b*) 25°

(*c*) 30° (*d*) None of the above.

162. The voltages of a generator and an infinite bus are given as $0.92\angle 10°$ and $1.0\angle 0°$ respectively. The active power will flow from

(*a*) Generator to infinite bus (*b*) Infinite bus to generator

(*c*) The data is insufficient to judge.

163. If the penalty factor of a plant is unity, its incremental transmission loss is

(*a*) 1.0 (*b*) – 1.0

(*c*) Zero (*d*) None of the above.

164. In a two plant system, the load is connected at plant no. 2. The loss coefficients

(*a*) B_{11}, B_{12}, B_{22} are nonzero

(*b*) B_{11} and B_{22} are nonzero but B_{12} is zero

(*c*) B_{11} and B_{12} are nonzero but B_{22} is zero

(*d*) B_{11} is nonzero but B_{12} and B_{22} are zero.

165. The diagonal elements of a nodal admittance matrix are strengthened by adding

(*a*) Shunt inductances (*b*) Shunt capacitors

(*c*) Loads (*d*) Generators.

166. The charging reactance of 50 km length of line is 1500 Ω. The charging reactance for 100 km length of line will be

(*a*) 1500 Ω
(*b*) 3000 Ω
(*c*) 750 Ω
(*d*) 600 Ω.

167. The eddy current loss in the core of a transformer is

(*a*) Inversely proportional to resistivity of the core material
(*b*) Directly proportional to resistivity of the core material
(*c*) Directly proportional to square of resistivity of the core material
(*d*) None of the above is true.

168. The normal practice to specify the making current of a circuit breaker is in terms of

(*a*) r.m.s value
(*b*) Peak value
(*c*) Average value
(*d*) Both r.m.s. and peak value.

169. A transmission line of 80 km length is operating at 400 Hz, it can be classified as

(*a*) Short length line
(*b*) Medium length line
(*c*) Long length line
(*d*) all of the above.

170. A transmission line of 210 km length has certain values of parameters A, B, C, D. If the length is made 100 km thus the parameter

(*a*) A increase B decreases
(*b*) A decreases B also decreases
(*c*) A and B both increase
(*d*) A decreases B increases

171. In the above problem, the parameters

(*a*) C increases D decreases
(*b*) C and D both decrease
(*c*) C and D both increase
(*d*) C decreases D increases

172. When two transformers of different KVA ratings are connected in parallel they share the load in proportion to their respective KVA rating only when their

(*a*) KVA ratings are identical
(*b*) Efficiencies are equal
(*c*) pu impedances are equal
(*d*) equivalent impedances are equal.

173. A salient pole synchronous motor is running at no load, its field current is switched off. The motor will

(*a*) come to a stop
(*b*) continue to run at synchronous speed
(*c*) continue to run at a speed slightly less than synchronous speed
(*d*) none of these.

174. In a network the sum of currents entering a node is $5\angle 60°$. The sum of the currents leaving the node is

(*a*) $5 \angle 60°$
(*b*) $5 \angle -60°$
(*c*) $5 \angle 240°$
(*d*) 15 A.

175. In a synchronous machine, in case the axis of field flux is in line with the armature flux, the machine is working

(*a*) as synchronous motor (*b*) as synchronous generator

(*c*) as floating machine (*d*) the m/c will not work.

176. An ideal voltage source is connected across a variable resistance. The variation of current as a function of resistance is given by

(*a*) A straight line passing through the origin

(*b*) a rectangular hyperbola

(*c*) a parabola

(*d*) it could be anyone of the above.

177. Pure inductive circuit takes power (reactive) from the a.c. line when

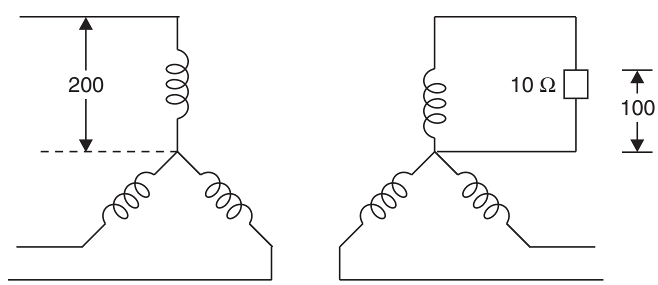
(*a*) both applied voltage and current rise

(*b*) both applied voltage and current decrease

(*c*) applied voltage decreases but current increases

(*d*) (*a*) and (*b*).

178. The current in the primary of the given transformer is



(*a*) 10 *A* (*b*) 5 *A*

(*c*) $10/\sqrt{3}\,A$ (*d*) none of the above.

179. A synchronous machine has its field winding on the stator and armature winding on the rotor. Under steady running condition, the air gap field

(*a*) rotates at synchronous speed w.r.t. stator

(*b*) rotates at synchronous speed in the direction of rotation of the rotor

(*c*) remains stationary w.r.t. stator

(*d*) remains stationary w.r.t. rotor.

180. For a given applied voltage to a transformer, if its magnetic core is replaced by a non-magnetic core

The magnitude of magnetic flux produced by the primary winding

(*a*) remains same (*b*) decreases

(*c*) increases (*d*) all the above alternatives are possible.

181. If a transformer is switched on to a voltage more than the rated voltage, for a certain loading condition (lag p.f.)

(*a*) its p.f. will improve (*b*) its p.f. will deteriorate
(*c*) no efect on p.f. (*d*) it may or may not improve.

182. Three phase transformers which can not be connected in parallel are

(*a*) YY with $\Delta\Delta$ (*b*) $Y\Delta$ with ΔY
(*c*) YY with $Y\Delta$ (*d*) (*b*) and (*c*).

183. For certain geometry and operating voltage of the uncompensated transmission line the ratio of power transfer capability to the surge impedance loading with increase in length

(*a*) increases (*b*) remains unchanged
(*c*) decreases (*d*) no fixed criterion.

184. If the normal system frequency is 50 Hz and if it is operating at 53 Hz, the equipment on the system most adversely affected is

(*a*) Power transformer (*b*) Alternator
(*c*) Turbine (*d*) all the above are equally affected.

185. The operation of the relay which is most affected due to arc resistance is

(*a*) Mho relay (*b*) reactance relay
(*c*) Impedance relay (*d*) all are equally affected.

186. The relay which is most sensitive to power swings (maloperation) is

(*a*) Mho relay (*b*) reactance relay
(*c*) Impedance relay (*d*) all are equally affected.

187. A 100/5 A bar primary current transformer supplies an O/C relay set at 25% pick up and it has a burden of 5 VA. The secondary voltage is

(*a*) 1 volt (*b*) 1.25 volt
(*c*) 2.5 volt (*d*) 4 volt.

188. A 100 : 5 current transformer has secondary winding and lead resistance of 0.2Ω and the secondary burden is 5 VA. The VA output is

(*a*) 5 VA (*b*) 10 VA
(*c*) 7.5 VA (*d*) 2.5 VA.

189. The phase angle error of a current transformer can be reduced by connecting an element across its secondary which is predominantly

(*a*) resistive (*b*) inductive
(*c*) capacitive (*d*) independent of the nature of element.

190. Auto reclosing is used in case of

(*a*) Lightning arrester (*b*) Air blast C.B.
(*c*) Bulk oil C.B. (*d*) Minimum oil C.B.

191. If the normal frequency is 50 Hz, the setting of the under-frequency relay for the most important load could be

(*a*) 49 Hz
(*b*) 48.5 Hz
(*c*) 47.5 Hz
(*d*) there is no grading for under frequency relays.

192. A distance relay is said to be inherently directional if its characteristic on R-X diagram
(*a*) is a straight line off-set from the origin
(*b*) is a circle that passes through the origin
(*c*) is a circle that encloses the origin
(*d*) always a separate directional relay is required.

193. The standard current ratings of an electromagnetic relay are
(*a*) 5 A and 15 A (*b*) 15 A and 20 A
(*c*) 1 A and 5 A (*d*) any one of the above.

194. Standard voltage ratings of the electromagnetic relay are
(*a*) 110 V, 63.5 (L-N) (*b*) 220 V 110 V
(*c*) 230 V 154 V (*d*) 400 V 231 V.

195. In order to quench the arc quickly and also optimise the dimensions generally
(*a*) oil C.B. is preferred (*b*) air blast C.B. is preferred
(*c*) SF_6 C.B. is preferred (*d*) minimum oil C.B. is preferred.

196. The frequency of the carrier in the case of carrier current-pilot scheme is in the range of
(*a*) 1 KHz to 10 KHz (*b*) 15 KHz to 25 KHz
(*c*) 25 KHz to 50 KHz (*d*) 50 KHz to 500 KHz.

197. The unit protection scheme provides
(*a*) primary protection (*b*) Back up protection
(*c*) simultaneous protection (*d*) remote protection.

198. The $X : R$ ratio for 220 KV line as compared to 400 KV line is
(*a*) greater (*b*) smaller
(*c*) equal (*d*) it could be anything.

199. If a line is 100% series compensated it may result into series resonance at
(*a*) power frequency (50 Hz) (*b*) 100 Hz
(*c*) 150 Hz (*d*) none of the above.

200. SSR phenomenon is
(*a*) purely electrical (*b*) purely mechanical
(*c*) purely hydraulic (*d*) (*a*) and (*b*).

201. The order of the subharmonic during SSR for 50 Hz normal frequency is
(*a*) 25 Hz (*b*) $16\frac{2}{3}$ Hz
(*c*) 10 Hz (*d*) none of the above.

202. A transformer rated for 500 KVA 11 KV/0.4 KV has an impedance of 10% and is connected to an infinite bus. The fault level of the transformer is

(*a*) 500 KVA (*b*) 5000 KVA

(*c*) 500 $\sqrt{3}$ KVA (*d*) none of the above.

203. With the help of a reactive compensator it is possible to have

(*a*) constant voltage operation only

(*b*) unity p.f. operation only

(*c*) both constant voltage and unity p.f.

(*d*) either constant voltage or unity p.f.

204. Efficiency of a power transformer under no load condition is approximately

(*a*) 75% (*b*) 50%

(*c*) 25% (*d*) none of the above.

205. A 3-phase induction motor is drawing 5A when a single phasing takes place. The current now drawn by the motor is

(*a*) $5\sqrt{2}$ (*b*) $5\sqrt{3}$

(*c*) $10\sqrt{3}$ (*d*) none of the above.

206. When a d.c. source is switched in a purely inductive circuit the current response is

(*a*) an exponentially rising curve

(*b*) an exponentially decaying curve

(*c*) a straight line passing through the origin

(*d*) a straight line off-set from the origin.

207. The voltage and current in a circuit are given by

$$v = 10 \sin (\omega t - \pi/6)$$
$$i = 10 \sin (\omega t + \pi/6)$$

The power consumed is given as

(*a*) 100 watts (*b*) 50 watts

(*c*) 86.6 watts (*d*) 25 watts

208. In a large interconnected power system, consider three buses having short-circuit capacities 1500 MVA (1), 1200 MVA (2) and 1000 MVA (3) respectively. The voltages of all the buses are 1.0 p.u. If a 3-phase fault takes place on bus 2, the change in bus voltage is described as

(*a*) $\Delta V_1 > \Delta V_2 > \Delta V_3$ (*b*) $\Delta V_1 < \Delta V_3 < \Delta V_2$

(*c*) $\Delta V_1 > \Delta V_3 > \Delta V_2$ (*d*) None of the above.

209. For a long uncompensated line the limit to the line loading is governed by

(*a*) Thermal limit (*b*) Voltage drop

(*c*) Stability limit (*d*) Corona loss.

210. For a lossless line terminated by its surge impedance, the natural reactive power loading is

(*a*) V^2/X (*b*) V^2/Z

(*c*) V^2/Z_c (*d*) Zero.

211. A 60 Hz 320 km lossless line has sending end voltage 1.0 p.u. The receiving end voltage on no load is

(*a*) 1.1 p.u. (*b*) 1.088 p.u.

(*c*) 1.116 p.u. (*d*) none of the above.

212. The impedance of a circuit is given by $z = 3 + j4$, its conductance is given by

(*a*) $\frac{1}{3}$ (*b*) 3/5

(*c*) 3/25 (*d*) 4/5.

213. With 100% series compensation of lines

(*a*) the circuit is series resonant at power frequency

(*b*) low transient voltage

(*c*) high transient current

(*d*) (*a*) and (*c*).

214. The effect of increasing gating angle in a thyristorised controlled reactor is

(*a*) to increase the effective inductance of the reactor

(*b*) to reduce the effective reactive power

(*c*) (*a*) and (*b*)

(*d*) none of the above.

215. The effect of increasing gating angle in a thyristorised controlled reactor is

(*a*) to decrease power loss in thyristor controller

(*b*) to decrease power loss in the reactor

(*c*) to make current wave form less sinusoidal

(*d*) all of the above.

216. The inertia constant H of a synchronous condenser is

(*a*) greater than that of hydro generator

(*b*) greater than that of turbo alternator

(*c*) equal to that of turbo alternator

(*d*) none of the above.

217. The effect of series capacitance compensation is

(*a*) to decrease the virtual surge impedance

(*b*) to decrease the effective length of the line

(*c*) to increase virtual surge impedance loading

(*d*) all of the above.

218. For any fixed degree of series compensation additional capacitive shunt compensation
(*a*) increases the effective length of line
(*b*) increases virtual surge impedance of line
(*c*) decreases virtual surge impedance loading of the line
(*d*) (*b*) and (*c*).

219. For any fixed degree of inductive shunt compensation, additional series capacitive compensation
(*a*) increases the effective length of line
(*b*) increases virtual surge impedance of line
(*c*) decreases virtual surge impedance loading of the line
(*d*) none of the above.

220. With 100% inductive shunt compensation the voltage profile is flat for
(*a*) 100% loading of line
(*b*) 50% loading of line
(*c*) Zero loading of line
(*d*) None of the above.

221. A loss less line terminated with its surge impedance has
(*a*) Flat voltage profile
(*b*) transmission line angle is greater that actual length of the line
(*c*) transmission line angle is less than the actual length
(*d*) (*a*) and (*b*).

222. The main consideration for higher and higher operating voltage of transmission is to
(*a*) increase efficiency of transmission
(*b*) reduce power losses
(*c*) increase power transfer capability
(*d*) (*a*) and (*b*).

223. A synchronous generator connected to an infinite bus delivers power at a log p.f. If its excitation is increased
(*a*) the terminal voltage increases
(*b*) voltage angle δ increases
(*c*) Current delivered increases
(*d*) (*b*) and (*c*).

224. A synchronous motor connected to an infinite bus takes power at a lag p.f. If its excitation is increased
(*a*) the terminal voltage increases
(*b*) the load angle increases
(*c*) the p.f. of motor increases
(*d*) (*c*) and (*b*).

225. In a pure LC parallel circuit under resonance condition, current drawn from the supply mains is
(*a*) very large
(*b*) $V\sqrt{LC}$
(*c*) $V/\sqrt{LC}$
(*d*) Zero.

226. The current at a given point in a certain circuit may be given a function of time as

$$i(t) = -3 + t$$

The total charge passing the point between $t = 99$ sec and $t = 102$ sec is

(*a*) 112 C (*b*) 242.5 C

(*c*) 292.5 C (*d*) 345.6 C.

227. An alternator has a phase sequence of RYB for its phase voltages. In case the field current is reversed, the phase sequence will become

(*a*) RBY (*b*) RYB

(*c*) YRB (*d*) None of the above.

228. An alternator has a phase sequence of RYB for it phase voltage. In case the direction of rotation of alternator is reversed, the phase sequence will become

(*a*) RBY (*b*) RYB

(*c*) YRB (*d*) None of the above.

229. In a circuit the voltage and current are given by $v = (10 + j5)$ and $i = (6 + j4)$. The circuit is

(*a*) inductive (*b*) capacitive

(*c*) resistive (*d*) it could be any of the above.

230. The power in the circuit of problem 229 is

(*a*) 60 watts (*b*) 20 watts

(*c*) 80 watts (*d*) 70 watts.

231. The reactive power in the circuit of problem 229 is

(*a*) 70 VArs (*b*) 60 VArs

(*c*) 10 VArs (*d*) – 10 VArs.

232. A synchronous generator is connected to an infinite bus bar and is initially operating at a lag p.f. If the steam input to the alternator is increased

(*a*) The p.f. of the alternator improves (*b*) reactive power decreases

(*c*) the frequency increase (*d*) (*a*) and (*b*).

233. A synchronous alternator is connected to an infinite bus bar and is initially operating at lead p.f. If the steam input is increased

(*a*) The p.f. increases (*b*) the current delivered increases

(*c*) the frequency increases (*d*) (*a*) and (*c*).

234. A 100 km transmission line is designed for a nominal voltage of 132 kV and consists of one conductor per phase. The line reactance is 0.726 ohm/km. The static transmission capacity of the line, in Megawatts, would be

(*a*) 132 (*b*) 240

(*c*) 416 (*d*) 720.

235. The per unit impedance of a circuit element is 0.15. If the base kV and base MVA are halved, then the new value of the per unit impedance of the circuit element will be

(*a*) 0.075 (*b*) 0.15

(*c*) 0.30 (*d*) 0.600.

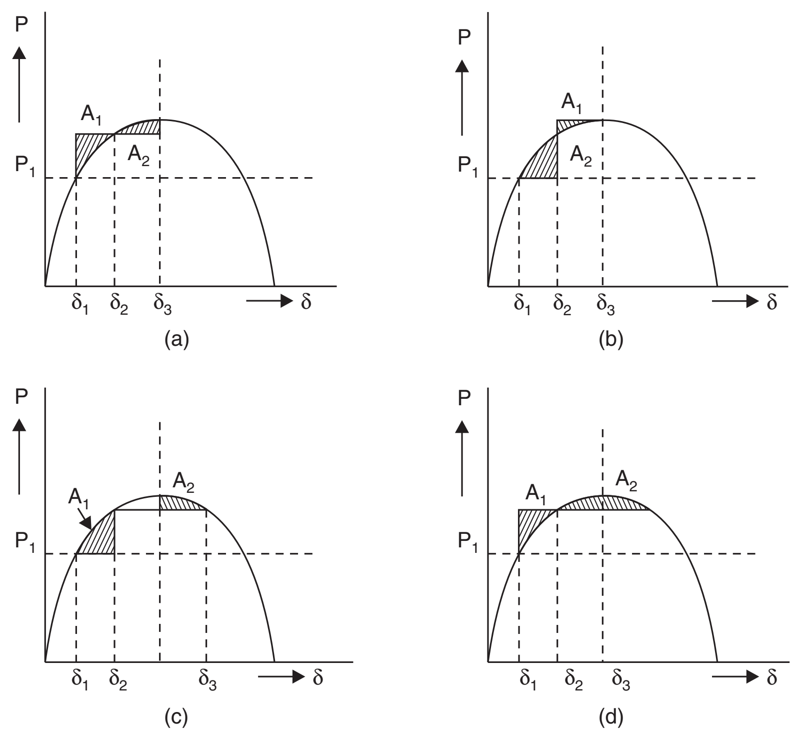
236. If the effect of earth is taken into account, then the capacitance of line to ground

(*a*) decreases
(*b*) increases
(*c*) remains unaltered
(*d*) becomes infinite.

237. A single line to ground fault occurs on an unloaded generator in phase a. If $x_d = x_2 = 0.25$ p.u., $x_0 = 0.15$ p.u., reactance connected in the neutral, $x_n = 0.05$ p.u. and the initial prefault voltage is 1.0 p.u., then the magnitude of the fault current will be

(*a*) 3.75 p.u.
(*b*) 1.54 p.u.
(*c*) 1.43 p.u.
(*d*) 1.25 p.u.

238. Principle of Equal-area Criterion is to be applied to determine, for a given initial load P_1, the maximum amount of sudden increase in load ΔP, to maintain transient stability of a cylindrical rotor synchronous motor operating from an infinite bus. Applying this criterion (in each case the area A_1 = area A_2). Which one of the following diagrams is correct ?



239. A thyrite type lightning arrestor

(*a*) blocks the surge voltage appearing in a line
(*b*) absorbs the surge voltage appearing in a line
(*c*) offers a low resistance path to the surge appearing in line
(*d*) returns the surge back to the source.

240. A 66-kV system has string insulator having five discs and the earth to disc capacitance ratio of 0.10. The string efficiency will be

(*a*) 89% (*b*) 75%

(*c*) 67% (*d*) 55%.

241. Which of the following statements regarding corona are true ?

1. It causes radio interference
2. It attenuates lightning surges
3. It amplifies switching surges
4. It causes power loss
5. It is more prevalent in the middle conductor of a transmission line employing a flat conductor configuration. Select the correct answer using the codes given below:

(*a*) 1, 3 and 5 (*b*) 2, 3 and 4

(*c*) 1, 2, 4 and 5 (*d*) 2, 3, 4 and 5.

242. ϕ_{1m}, ϕ_{2m} = the fluxes produced by the two portions of the shaded pole,

θ = the angle between ϕ_{1m} and ϕ_{2m}

R = resistance of the disc

The torque produced in an induction relay would be proportional to which of the following ?

1. ϕ_{1m} and ϕ_{2m}
2. $1/R$
3. R
4. $\sin\theta$

Select the correct answer using the codes given below:

(*a*) 1, 2 and 4 (*b*) 1, 3 and 4

(*c*) 1 and 2 (*d*) 2 and 4.

243. Match list 1 with list 2 and select the correct answer using the codes given below the lists:

List 1

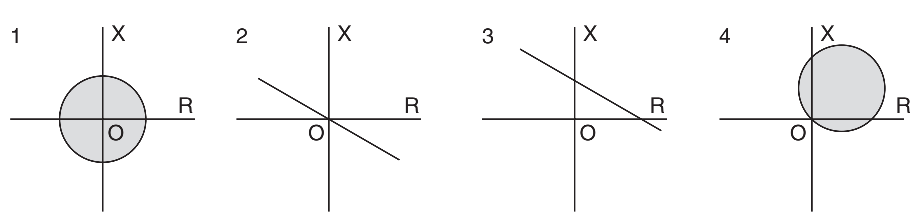
A. Mho relay

B. Plain impedance relay

C. Directional relay

D. Angle impedance relay

List 2



Codes:

(*a*)	*A*	*B*	*C*	*D*
	4	3	2	1
(*b*)	*A*	*B*	*C*	*D*
	4	1	2	3
(*c*)	*A*	*B*	*C*	*D*
	3	2	1	4
(*d*)	*A*	*B*	*C*	*D*
	3	2	4	1

244. For a *Y*-delta transformer with *Y*-side grounded, the zero sequence current

(*a*) has no path to ground

(*b*) exists in the lines on the delta side

(*c*) exists in the lines on the *Y* side

(*d*) exists in the lines on both *Y* and delta sides.

245. When there is a change in load in a power station having a number of generator units operating in parallel, the system frequency is controlled by

(*a*) adjusting the steam input to the units

(*b*) adjusting the field-excitation of the generators

(*c*) changing the load divisions between the units

(*d*) injecting reactive power at the station bus bar.

246. A 2 KVA transformer has iron loss of 150 Watts and full-load copper loss of 250 Watts. The maximum efficiency of the transformer would occur when the total loss is

(*a*) 500 W (*b*) 400 W

(*c*) 300 W (*d*) 275 W.

247. If the frequency of input voltage of a transformer is increased keeping the magnitude of voltage unchanged, then

(*a*) both hysteresis loss and eddy current loss in the core will increase

(*b*) hysteresis loss will increase but eddy current loss will decrease

(*c*) hysteresis loss will decrease but eddy current loss will increase

(*d*) hysteresis loss will decrease but eddy current loss will remain unchanged.

248. For successful parallel operation of two single-phase transformers, the most essential condition is that their

(*a*) percentage impedances are equal

(*b*) polarities are properly connected

(*c*) turns ratios are exactly equal

(*d*) KVA ratings are equal.

249. The most appropriate operating speeds in rpm of generators used in Thermal, Nuclear and Hydro-power plants would respectively be

(*a*) 3000, 300 and 1500 (*b*) 3000, 3000 and 300

(*c*) 1500, 1500 and 3000 (*d*) 1000, 900 and 750.

250. The operation of a nuclear reactor is controlled by controlling the multiplication-factor (K), defined as

$$K = \frac{\text{number of neutrons of any one Generation}}{\text{number of neutrons of immediately preceding generation}}$$

The power-level of the reactor can be increased by

(*a*) raising the value of K above 1 and, keeping it at that raised value

(*b*) raising the value of K above 1, but later bringing it back to $K = 1$

(*c*) lowering the value of K below 1 and, keeping it at that lowered value

(*d*) lowering the value of K below 1, but later bringing it back to $K = 1$.

251. The inductance of single-phase two-wire power transmission line per kilometer gets doubled when the

(*a*) distance between the wires is doubled

(*b*) distance between the wires is increased four fold.

(*c*) distance between the wires is increased as square of original distance

(*d*) radius of the wire is doubled.

252. The flow-duration curve at a given head of a hydro-electric plant is used to determine the

(*a*) total power available at the site

(*b*) total units of energy available

(*c*) load-factor at the plant

(*d*) diversity-factor for the plant.

253. Match List I with List II and select the correct answer using the codes given below the lists:

List I	*List II*
A. Thyrite arrester	1. Tower location
B. Sag template	2. Cross bonding
C. Cable sheaths	3. Restriking voltage
D. Circuit breaker	4. Non-linear resistor.

Codes:

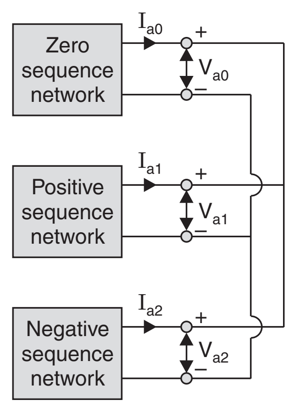
(*a*)	*A*	*B*	*C*	*D*
	4	1	3	2
(*b*)	*A*	*B*	*C*	*D*
	4	1	2	3

(*c*)	*A*	*B*	*C*	*D*
	1	4	3	2
(*d*)	*A*	*B*	*C*	*D*
	4	3	1	2

254. The connection diagram of sequence networks for a particular fault on a power system network is given in the figure. The type of the fault is

(*a*) single line to ground fault (*b*) double line to ground fault

(*c*) line to line fault (*d*) open circuit.



255. The inertia constant of a 100 MVA, 11 kV water-wheel generator is 4. The energy stored in the rotor at the synchronous speed is

(*a*) 400 MJ (*b*) 400 kJ

(*c*) 25 MJ (*d*) 25 kJ.

256. A large ac generator, supplying power to an infinite bus, has a sudden short-circuit occuring at its terminals. Assuming the prime mover input and the voltage behind the transient reactance to remain constant immediately after the fault, the acceleration of the generator rotor is

(*a*) inversely proportional to the moment of inertia of the machine

(*b*) inversely proportional to the square of the voltage

(*c*) directly proportional to the square of the short-circuit current

(*d*) directly proportional to the short-circuit power.

257. Match List I (Devices) with List II (Application) and select the correct answer using the codes given below the lists:

List I	*List II*
A. Microprocessor	1. Monitoring surge voltage
B. Breather	2. Digital protection
C. Magnetic links	3. Monitoring surge current
D. Klydonograph	4. Power transformer

Codes:

(*a*)	*A*	*B*	*C*	*D*
	4	2	1	3
(*b*)	*A*	*B*	*C*	*D*
	4	2	3	1
(*c*)	*A*	*B*	*C*	*D*
	2	4	1	3
(*d*)	*A*	*B*	*C*	*D*
	2	4	3	1

258. A lightning arrestor connected between the line and earth in a power system
(*a*) protects the terminal equipment against travelling surges
(*b*) protects the transmission line against lightning stroke
(*c*) suppresses high frequency oscillations in the line
(*d*) reflects back the travelling wave approaching it.

259. Corona loss can be reduced by the use of hollow conductor, because
(*a*) the current density is reduced
(*b*) the eddy-current in the conductor is eliminated
(*c*) for a given cross-section, the radius of the conductor is increased
(*d*) of better ventilation in the conductor.

260. The insulation of modern EHV lines is designed based on
(*a*) the lightning voltage (*b*) corona
(*c*) radio interference (*d*) switching voltage.

261. While using air-blast circuit breaker, current chopping is a phenomenon often observed when
(*a*) a long overhead line is switched off
(*b*) a bank of capacitors is switched off
(*c*) a transformer on no-load is switched off
(*d*) a heavy load is switched off.

262. The most efficient torque-producing actuating structure for induction-type relay is
(*a*) shaded-pole structure (*b*) watt-hour-meter structure
(*c*) induction-cup structure (*d*) single-induction-loop structure.

263. A power system network with a capacity of 100 MVA has a source impedance of 10% at a point. The fault level at that point is
(*a*) 10 MVA (*b*) 30 MVA
(*c*) 3000 MVA (*d*) 1000 MVA.

264. In a power station, the cost of generation of power reduces most effectively when
(*a*) diversity factor alone increases

(*b*) both diversity factor and load factor increase

(*c*) load factor alone increases

(*d*) both diversity factor and load factor decrease.

265. In the optimum generator scheduling of different power plants, the minimum fuel cost is obtained when

(*a*) only the incremental fuel cost of each plant is the same

(*b*) the penalty factor of each plant is the same

(*c*) the ratio of the incremental fuel cost to the penalty factor of each plant is the same

(*d*) the incremental fuel cost of each plant multiplied by its penalty factor is the same.

266. Shunt compensation in an EHV line is used to

(*a*) improve stability

(*b*) reduce fault level

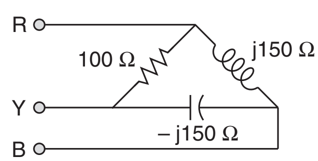
(*c*) improve the voltage profile

(*d*) substitute for synchronous phase modifier.

267. A set of 3 equal resistors, each of value R_X, connected in star across *RYB* in place of load as shown in the given figure consumes the same power as the unbalanced delta connected load. The value of R_X is

(*a*) 33.33 Ω (*b*) 100 Ω

(*c*) 173.2 Ω (*d*) 300 Ω.



268. In a 400 kV power network, 360 kV is recorded at 400 kV bus. The reactive power absorbed by a shunt reactor rated for 50 MVAR, 400 kV connected at the bus is

(*a*) 61.73 MVAR (*b*) 55.56 MVAR

(*c*) 45.0 MVAR (*d*) 40.5 MVAR.

269. Which one of the following statements is true ?

(*a*) Steady-state stability limit is greater than transient stability limit

(*b*) Steady-state stability limit is equal to transient stability limit

(*c*) Steady-state stability limit is less than the transient stability limit

(*d*) No generalisation can be made regarding the equality or otherwise of the steady-state stability limit and transient stability limit.

270. Lightning arresters are used in power systems to protect electrical equipments against

(*a*) direct strokes of lightning

(*b*) power frequency overvoltages

(*c*) overvoltages due to indirect lightning stroke

(*d*) overcurrents due to lightning stroke.

271. The voltages across the various discs of a string of suspension insulators having identical discs is different due to

(*a*) surface leakage currents (*b*) series capacitance

(*c*) shunt capacitances to ground (*d*) series and shunt capacitances.

272. Compared with a solid conductor of the same radius, corona appears on a stranded conductor at a lower voltage, because stranding

(*a*) assists ionisation

(*b*) makes the current flow spirally about the axis of the conductor

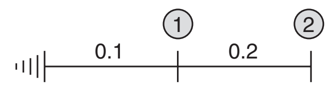
(*c*) produces oblique sections to a plane perpendicular to the axis of the conductor

(*d*) produces surfaces of smaller radius.

273. The bus admittance matrix of the network shown in the given figure, for which the marked parameters are per unit impedance, is

(*a*) $\begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.2 \end{bmatrix}^{-1}$ (*b*) $\begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}$

(*c*) $\begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.2 \end{bmatrix}$ (*d*) $\begin{bmatrix} 15 & -5 \\ -5 & 5 \end{bmatrix}$.



274. Buses for load flow studies are classified as (*i*) the load bus, (*ii*) the generator bus and (*iii*) the slack bus

The correct combination of the pair of quantities specified having their usual meaning for different buses is

	Load bus	Generator bus	Slack bus
(*a*)	$P, \|V\|$	P, Q	P, δ
(*b*)	P, Q	$P, \|V\|$	$\|V\|, \delta$
(*c*)	$\|V\|, Q$	P, δ	P, Q
(*d*)	P, δ	$Q, \|V\|$	Q, δ

275. A Bucholz relay is used for

(*a*) protection of a transformer against all internal faults

(*b*) protection of a transformer against all external faults

(*c*) protection of a transformer against both internal and external faults

(*d*) protection of induction motors.

276. The line trap unit employed in carrier current relaying

(*a*) offers high impedance to 50 Hz power frequency signals

(*b*) offers high impedance to carrier frequency signals

(*c*) offers low impedance to carrier frequency signals

(*d*) offers low impedance to carrier frequency signals.

277. Match List I with List II and select the correct answer using the codes given below the lists:

List I	*List II*
(Equipment)	*(Function)*
A. Circuit Breaker	1. Voltage control
B. Lightning Arrester	2. Power Control
C. Governor	3. Overvoltage protection
D. Exciter	4. Overcurrent protection.

Codes :

(*a*)	*A*	*B*	*C*	*D*
	1	2	3	4
(*b*)	*A*	*B*	*C*	*D*
	4	1	2	3
(*c*)	*A*	*B*	*C*	*D*
	2	3	4	1
(*d*)	*A*	*B*	*C*	*D*
	4	3	2	1

278. To prevent maloperation of differentially connected relay while energising a transformer, the relay restraining coil is biased with

(*a*) second harmonic current (*b*) third harmonic current

(*c*) fifth harmonic current (*d*) seventh harmonic current.

279. The incremental fuel cost for two generating units are given by

$IC_1 = 25 + 0.2\ PG_1$

$IC_2 = 32 + 0.2\ PG_2$, where PG_1 and PG_2 are real power generated by the units.

The economic allocation for a total load of 250 MW, neglecting transmission loss is given by

(*a*) PG_1 = 140.25 MW, PG_2 = 109.75 MW

(*b*) PG_1 = 109.75 MW, PG_2 = 140.25 MW

(*c*) $PG_1 = PG_2$ = 125 MW

(*d*) PG_1 = 100 MW, PG_2 = 150 MW.

280. The synchronisation coefficient between two area of a 2-area power system is (symbols have usual meanings)

(*a*) $\dfrac{\partial P}{\partial |V|}$ (*b*) $\dfrac{\partial P}{\partial \delta}$

(*c*) $\dfrac{\partial P}{\partial f}$ (*d*) $\dfrac{\partial P}{\partial Q}$.

281. In a high voltage dc transmission scheme, reactive power is needed both for the rectifier at the sending-end and, for the inverter at the reciving-end. During the operation of such a dc link the rectifier receives

(*a*) lagging reactive power and the inverter supplies leading reactive power

(*b*) leading reactive power and the inverter supplies lagging reactive power

(*c*) lagging reactive power and the inverter supplies lagging reactive power

(*d*) leading reactive power and the inverter supplies leading reactive power.

282. Transmission lines are transposed to

(*a*) reduce copper loss

(*b*) reduce skin effect

(*c*) prevent interference with neighbouring telephone lines

(*d*) prevent short-circuit between any two lines.

283. A single-phase transmission line of impedance j 0.8 ohm supplies a resistive load of 500 A at 300 V. The sending-end power factor is

(*a*) unity (*b*) 0.8 lagging

(*c*) 0.8 leading (*d*) 0.6 lagging.

284. The impedance value of a generator is 0.2 pu on a base value of 11 KV, 50 MVA. The impedance value for a base value of 22 KV, 150 MVA is

(*a*) 0.15 pu (*b*) 0.2 pu

(*c*) 0.3 pu (*d*) 2.4 pu.

285. For a transmission line with resistance, R, reactance X, and negligible capacitance, the transmission constant A is

(*a*) 0 (*b*) 1

(*c*) $R + jx$ (*d*) $R + X$.

286. Four identical alternators each rated for 20 MVA, 11 kV having a subtransient reactance of 16% are working in parallel. The short- circuit level at the bus-bars is

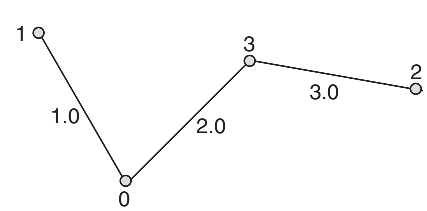
(*a*) 500 MVA (*b*) 400 MVA

(*c*) 125 MVA (*d*) 80 MVA.

287. Consider the network shown in the following figure:

The bus numbers and impedances are marked

The bus impedance matrix of this network is



(*a*) $\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 5.0 \end{bmatrix} \end{array}$

(*b*) $\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 3.0 \end{bmatrix} \end{array}$

(*c*) $\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 5.0 & 2.0 \\ 0.0 & 2.0 & 2.0 \end{bmatrix} \end{array}$

(*d*) $\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 2.0 \\ 0.0 & 2.0 & 5.0 \end{bmatrix} \end{array}$.

288. In a power system with negligible resistance, the fault current at a point is 8.00 pu. The series reactance to be included at the fault point to limit the short-circuit current to 5.00 pu is

(*a*) 3.000 pu (*b*) 0.200 pu

(*c*) 0.125 pu (*d*) 0.075 pu.

289. The 'equal area criterion' for the determination of transient stability of a synchronous machine connected to an infinite bus

(*a*) ignores line as well as synchronous machine resistances

(*b*) assumes accelerating power acting on the rotor as constant

(*c*) ignores the effect of voltage regulator and governor but considers the inherent damping present in the machine

(*d*) takes into consideration the possibility of machine losing synchronisation after it has survived during the first swing.

290. A voltage of 1000 kV is applied to an overhead line with its receiving-end open. If the surge impedance of the line is 500 ohms, then the total surge power in the line is

(*a*) 2000 MW (*b*) 500 MW

(*c*) 2 MW (*d*) 0.5 MW.

291. A short transmission line, having its line-impedance angle as θ, is delivering a given power at the receiving-end at a lagging power-factor angle of ϕ. Which one of the following is a set of conditions for which this line will have maximum- and zero-regulation ?

Maximum Regulation	Zero Regulation
(*a*) $\phi = \theta$	$\phi - \theta = \pi/2$
(*b*) $\phi - \theta = \pi/2$	$\phi = \theta$
(*c*) $\phi = \theta$	$\phi + \theta = \pi/2$
(*d*) $\phi + \theta = \pi/2$	$\phi = \theta$.

292. Impulse ratios of insulators and lightning arresters should be

(*a*) both low (*b*) high and low respectively

(*c*) low and high respectively (*d*) both high.

293. Bundled conductors in EHV transmission system provide

(*a*) reduced capacitance (*b*) increased capacitance

(*c*) increased inductance (*d*) increased voltage gradient.

294. The charging current in a transmission line increases due to corona effect because corona increases

(*a*) line current (*b*) effective line voltage

(*c*) power loss in lines (*d*) the effective conductor diameter.

295. For a 15-bus power system with 3 voltage controlled bus, the size of Jacobian matrix is

(*a*) 11 × 11 (*b*) 12 × 12

(*c*) 24 × 24 (*d*) 28 × 28.

296. If the fault current is 2 kA, the relay setting is 50% and the C.T. ratio is 400/5, then the plug setting multiplier of a relay will be

(*a*) 5 (*b*) 7

(*c*) 8 (*d*) 10.

297. The bus-bars of each of the two alternators of 15% reactance each, are interconnected through tie-bar reactors of 15% each. The equivalent impedance to fault current for a 3-phase fault in any alternator bus-bar will be

(*a*) 75% (*b*) 10%

(*c*) 11.25% (*d*) 15%.

298. A power plant has a maximum demand of 15 MW. The load factor is 50% and the plant factor is 40%. The operating reserve is

(*a*) 3.0 MW (*b*) 3.75 MW

(*c*) 6.0 MW (*d*) 7.5 MW.

299. The incremental cost characteristic of the two units in a plant are

$$IC_1 = 0.1\ P_1 + 8.0\ \text{Rs./MWh}$$
$$IC_2 = 0.15\ P_2 + 3.0\ \text{Rs./MWh}$$

When the total load is 100 MW, the optimum sharing of the load is

	P_1	P_2
(*a*)	40 MW	60 MW
(*b*)	33.3 MW	66.7 MW
(*c*)	60 MW	40 MW
(*d*)	66.7 MW	33.3 MW.

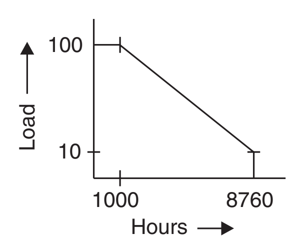
300. The load curve of a system is shown in the figure. The load factor of the system is:

(*a*) 1.66%

(*b*) 6.013%

(*c*) 16.6%

(*d*) 60.13%.



301. Control rods used in nuclear reactors are made of

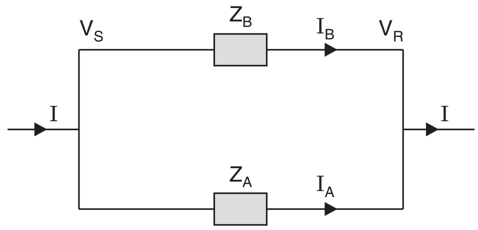
(*a*) zirconium (*b*) boron

(*c*) beryllium (*d*) lead.

302. Consider two parallel short transmission lines of impedances Z_A and Z_B respectively as shown in the figure. Currents I_A and I_B are both lagging and the sending-end voltage is V_s. If the reactance to resistance ratio of both impedances Z_A abd Z_B are equal, then the total current 'I' will

(*a*) lag both I_A and I_B (*b*) lead both I_A and I_B

(*c*) lag one of I_A and I_B but lead the other (*d*) be in phase with both I_A and I_B.



303. In a 3-phase extra-high voltage cable, a metallic screen around each core-insulation is provided to

(*a*) facilitate heat dissipation (*b*) give mechanical strength

(*c*) obtain radial electric stress (*d*) obtain longitudinal electric stress.

304. Galloping in transmission line conductors arises generally due to

(*a*) asymmetrical layers of ice formation

(*b*) vortex phenomenon in light winds

(*c*) heavy weight of the line conductors

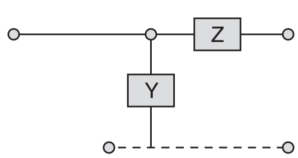
(*d*) adoption of horizontal conductor configurations.

305. In a matrix form, the equation of a 4-terminal network representing a transmission line is given by

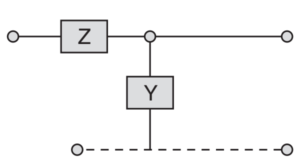
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The two networks considered are

(*A*)



(*B*)



The plausible transfer matrix for the networks (A) and (B) could be:

(*i*) $\begin{bmatrix} 1 & YZ \\ 0 & Z \end{bmatrix}$ (*ii*) $\begin{bmatrix} Y & 0 \\ YZ & 1 \end{bmatrix}$

(*iii*) $\begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}$ (*iv*) $\begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}$.

The correct combination for the two networks (*A*) and (*B*), would be:

(*a*) (*i*) and (*ii*) (*b*) (*i*) and (*iii*)

(*c*) (*ii*) and (*iv*) (*d*) (*iii*) and (*iv*).

306. The incremental generating costs of two generating units are given by

$$IC_1 = 0.1X + 20 \text{ Rs./MWhr}$$

$$IC_2 = 0.15Y + 18 \text{ Rs./MWhr.}$$

where X and Y are power (in MW) generated by the two units. For a total demand of 300 MW, the values (in MW) of X and Y will be respectively

(*a*) 172 and 128 (*b*) 128 and 172

(*c*) 175 and 125 (*d*) 200 and 100.

307. Consider the following statements:

To provide reliable protection for a distribution transformer against overvoltages using lightning arresters, it is essential that the

1. lead resistance is high
2. distance between the transformer and the arrester is small
3. transformer and the arrester have a common inter-connecting ground
4. spark overvoltage of the arrester is greater than the residual voltage.

Of these statements

(*a*) 1, 3 and 4 are correct (*b*) 2 and 3 are correct

(*c*) 2, 3 and 4 are correct (*d*) 1 and 4 are correct.

308. The reflection coefficient of a short-circuited line for voltage is

(*a*) – 1 (*b*) + 1

(*c*) 0.5 (*d*) zero.

309. The propagation constant of a transmission line is:

$$0.15 \times 10^{-3} + j1.5 \times 10^{-3}$$

The wavelength of the travelling wave is

(*a*) $\dfrac{15 \times 10^{-3}}{2\pi}$ (*b*) $\dfrac{2\pi}{15 \times 10^{-3}}$

(*c*) $\dfrac{15 \times 10^{-3}}{\pi}$ (*d*) $\dfrac{\pi}{15 \times 10^{-3}}$.

310. Hollow conductors are used in transmission lines to

(*a*) reduce weight of copper (*b*) improve stability

(*c*) reduce corona (*d*) increase power transmission capacity.

311. In the solution of load-flow equation, Newton-Raphon (NR) method is superior to the Gauss-Seidal (GS) method, because the

(*a*) time taken to perform one iteration in the NR method is less when compared to the time taken in the GS method

(*b*) number of iteration required in the NR method is more when compared to that in the GS method

(*c*) number of iterations required is not independent of the size of the system in the NR method

(*d*) convergence characteristic of the NR method are not affected by the selection of slack bus.

312. A power system network consists of three elements 0-1, 1-2 and 2-0 of per unit impedances 0.2, 0.4 and 0.4 respectively. It's bus impedance matrix is given by

(*a*) $\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 7.5 & -2.5 \\ -2.5 & 5.0 \end{bmatrix} \end{array}$ (*b*) $\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.16 & 0.08 \\ 0.08 & 0.24 \end{bmatrix} \end{array}$

(*c*) $\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.16 & -0.08 \\ -0.08 & 0.24 \end{bmatrix} \end{array}$ (*d*) $\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.8 \end{bmatrix} \end{array}$.

313. Zero sequence currents can flow from a line into a transformer bank if the windings are in

(*a*) grounded star/delta (*b*) delta/star

(*c*) star/grounded star (*d*) delta/delta.

314. When a line-to-ground fault occurs, the current in a faulted phase is 100 *A*. The zero sequence current in this case will be

(*a*) zero (*b*) 33.3 *A*

(*c*) 66.6 *A* (*d*) 100 *A*.

315. When a 50 MVA, 11 kV, 3-phase generator is subjected to a 3-phase fault, the fault current is $-j5$ pu (per unit). When it is subjected to a line-to-line fault, the positive sequence current is $-j4$ pu. The positive and negative sequence reactances are respectively

(*a*) $j0.2$ and $j0.05$ pu (*b*) $j0.2$ and $j0.25$ pu

(*c*) $j0.25$ and $j0.25$ pu (*d*) $j0.05$ and $j0.05$ pu.

316. The power generated by two plants are:

$P_1 = 50$ MW, $P_2 = 40$ MW. If the loss coefficients are $B_{11} = 0.001$, $B_{22} = 0.0025$ and $B_{12} = -0.0005$, then the power loss will be

(*a*) 5.5 MW (*b*) 6.5 MW

(*c*) 4.5 MW (*d*) 8.5 MW.

317. The following data pertain to two alternators working in parallel and supplying a total load of 80 MW:

Machine 1 : 40 MVA with 5% speed regulation

Machine 2 : 60 MVA with 5% speed regulation

The load sharing between machines 1 and 2 will be:

(*a*) $\frac{P_1}{48 \text{ MW}}, \frac{P_2}{32 \text{ MW}}$ (*b*) 40 MW, 40 MW

(*c*) 30 MW, 50 MW (*d*) 32 MW, 48 MW.

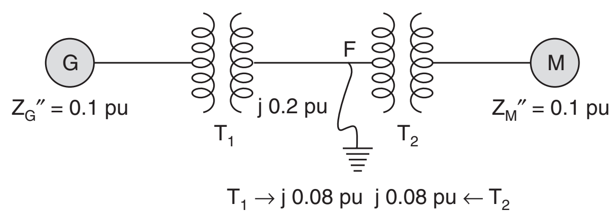
318. The per unit impedance of a synchronous machine is 0.242. If the base voltage is increased by 1.1 times, the per unit value will be

(*a*) 0.266 (*b*) 0.242

(*c*) 0.220 (*d*) 0.200

319. The following figure shows the single line diagram of a power system with all reactances marked in per unit (pu) on the same base:

The system is on no-load when a 3-phase fault occurs at 'F' on the high voltage side of the transformer T_2. The fault current will be



(*a*) $-j0.8187$ pu (*b*) $+j0.8187$ pu

(*c*) $-j8.1871$ pu (*d*) $+j8.1871$ pu.

320. Rated breaking capacity (MVA) of a circuit breaker is equal to

(*a*) the product of rated breaking current (kA) and rated voltage (kV)

(*b*) the product of rated symmetrical breaking current (kA) and rated voltage (kV)

(*c*) the product of breaking current (kA) and fault voltage (kV)

(*d*) twice the value of rated current (kA) and rated voltage (kV).

321. If, in a short transmission line, resistance and inductive reactance are found to be equal and regulation appears to be zero, then the load will

(*a*) have unity power factor (*b*) have zero power factor

(*c*) be 0.707 leading (*d*) be 0.707 lagging.

322. A hydel power plant of run-off-river type should be provided with a pondage so that the

(*a*) firm-capacity of the plant is increased

(*b*) operating head is controlled

(*c*) pressure inside the turbine casing remains constant

(*d*) kinetic energy of the running water is fully utilised.

323. If within an untransposed 3-phase circuit of a transmission line, the series impedance of each of the conductor is considered, it is found to contain resistive terms of the form

$K \log_e \left(\dfrac{d_{12}}{d_{13}}\right)$, K being a constant and d_{12} and d_{13} etc., being spacings between the conductors. These terms represent power transfer from one phase to another. The sum of these term over the three phases is

(*a*) three times the average (*b*) $\sqrt{3}$ times the average
(*c*) one-third of the average (*d*) zero.

324. Match List I (Parameter) with List II (Effect) and select the correct answer using the codes given below the Lists:

List I	*List II*
A. Percent power lost in transmission	1. Decreases with system voltage
B. For a given current density the conductor size	2. Reduces with line length
C. Power handling capacity of a line at a given voltage	3. Remains independent of line length
D. Surge impedance of a transmission line	4. Increases with line length

Code:

(*a*)	*A*	*B*	*C*	*D*
	1	2	4	3
(*b*)	*A*	*B*	*C*	*D*
	3	4	2	1
(*c*)	*A*	*B*	*C*	*D*
	3	2	4	1
(*d*)	*A*	*B*	*C*	*D*
	1	4	2	3

325. "Expanded ACSR" are conductor composed of

(*a*) larger diameter individual strands for a given cross-section of the aluminium strands
(*b*) larger diameter of the central steel strands for a given overall diameter of the conductor
(*c*) larger diameter of the aluminium strand only for a given overall diameter of the conductor
(*d*) a filler between the inner steel and the outer aluminium strands to increase the overall diameter of the conductor.

326. Which of the following are the advantages of interconnected operation of power systems ?

1. Less reserve capacity requirement
2. More reliability
3. High power factor
4. Reduction in short-circuit level.

Select the correct answer using the codes given below:

Codes:

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 3 and 4 (*d*) 1 and 4.

327. Steady-state stability of a power system is improved by

(*a*) reducing fault clearing time

(*b*) using double circuit line instead of single circuit line

(*c*) single pole switching

(*d*) decreasing generator inertia.

328. Equal area criterion gives the information regarding

(*a*) stability region (*b*) absolute stability

(*c*) relative stability (*d*) swing curves.

329. With a number of generators of MVA capacities $S_1, S_2, ..., S_n$ and inertia constants $H_1, H_2, ..., H_n$ respectively connected to the same bus bar in a station, the inertia constant of the equivalent machine on a base of S_b is given by

(*a*) $\sum_{i=1}^{n} H_i \, S_i$ (*b*) $\sum_{i=1}^{n} \frac{S_b}{S_i} . H_i$

(*c*) $\sum_{i=1}^{n} \frac{S_i}{S_b} . H_i$ (*d*) $\sum_{i=1}^{n} \frac{S_i}{S_b} . \frac{1}{H_i}$.

330. The critical clearing time of a fault in power systems is related to

(*a*) reactive power limit (*b*) short-circuit current limit

(*c*) steady-state stability limit (*d*) transient stability limit.

331. The use of fast acting relays and circuit breakers for clearing a sudden short-circuit on a transmission link between a generator and receiving-end bus improves the transient stability of the machine because

(*a*) short-circuit current becomes zero

(*b*) post-fault transfer impedance attains a value higher than that during the fault

(*c*) ordinate of the post-fault power-angle characteristic is higher than that of during-fault characteristic

(*d*) voltage behind the transient reactance increases to a higher value.

332. Consider the following statements:

The transient stability of the power system under unbalanced fault conditions can be effectively improved by

1. Excitation control
2. phase-shifting transformer
3. single-pole switching of ciruit breakers
4. increasing the turbine input.

Of these statements

(*a*) 1 and 2 are correct
(*b*) 2 and 3 are correct
(*c*) 3 and 4 are correct
(*d*) 1 and 3 are correct.

333. The non-uniform distribution of voltage across the units in a string of suspension type insulators is due to

(*a*) unequal self-capacitance of the units
(*b*) non-uniform distance of separation of the units from the tower body
(*c*) the existance of stray capacitance between the metallic junctions of the units and the tower body
(*d*) non-uniform distance between the cross-arm and the units.

334. Whenever the conductors are dead-ended or there is a change in the direction of transmission line, the insulators used are of the

(*a*) pin type
(*b*) suspension type
(*c*) strain type
(*d*) shackle type.

335. Consider the following statements:

In the case of suspension-type insulators, the string efficiency can be improved by

1. using a longer cross arm
2. using a guard ring
3. grading the insulator discs
4. reducing the cross-arm length.

Of these statements

(*a*) 1, 2 and 3 are correct
(*b*) 2, 3 and 4 are correct
(*c*) 2 and 4 are correct
(*d*) 1 and 3 are correct.

336. In a power system, each bus or node is associated with four quantities, namely

1. real power
2. reactive power
3. bus-voltage magnitude and
4. phase-angle of the bus voltage.

For load-flow solution, among these four, the number of quantities to be specified is

(*a*) any one
(*b*) any two
(*c*) any three
(*d*) all the four.

337. Match List I (Type of relay) with List II (For the protection of) and select the correct answer using the codes given below in the Lists:

List I	*List II*
A. Buchholtz relay	1. Feeder
B. Translay relay	2. Transformer
C. Negative sequence relay	3. Generator
D. Directional over current relay	4. Long Overhead line

Codes:

(*a*)	*A*	*B*	*C*	*D*
	1	2	3	4
(*b*)	*A*	*B*	*C*	*D*
	2	1	3	4
(*c*)	*A*	*B*	*C*	*D*
	2	1	4	3
(*d*)	*A*	*B*	*C*	*D*
	1	4	2	3

338. A line trap in carrier current relaying tuned to carrier frequency presents

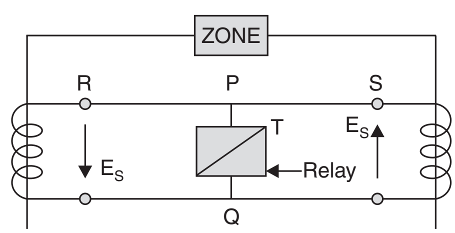
(*a*) high impedance to carrier frequency but low impedance to power frequency

(*b*) low impedance to both carrier and power frequency

(*c*) high impedance to both carrier and power frequency

(*d*) low impedance to carrier frequency but high impedance to power frequency.

339. The basic circuit of circulating current system of protection is shown in the figure. To improve the through-fault stability, a stabilizing resistor is connected between the points



(*a*) *R* and *P* in series (*b*) *P* and *S* in series

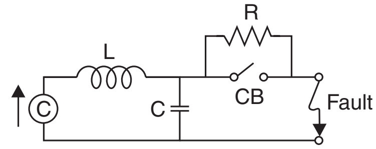
(*c*) *P* and *T* in series (*d*) *P* and *Q* in parallel.

340. One current transformer (CT) is mounted over a 3-phase 3-core cable with its sheath and armour removed from the portion covered by the CT. An ammeter placed in the CT secondary would measure

(*a*) the positive sequence current (*b*) the negative sequence current

(*c*) the zero sequence current (*d*) three times the zero sequence.

341. In connection with the arc extinction in circuit breaker, resistance switching is employed wherein a resistance is placed in parallel with the poles of the circuit breaker as shown in figure. This process introduces damping in the LC circuit. For critical damping, the value of 'R' should be equal to



(*a*) $\sqrt{\dfrac{C}{L}}$ (*b*) $0.5\sqrt{\dfrac{C}{L}}$

(*c*) $0.5\sqrt{\dfrac{L}{C}}$ (*d*) $\dfrac{1}{2\pi}\sqrt{\dfrac{L}{C}}$.

342. Load frequency control is achieved by properly matching the individual machine's

(*a*) reactive powers (*b*) generated voltages

(*c*) turbine inputs (*d*) turbine and generator ratings.

343. If a voltage-controlled bus is treated as load bus, then which one of the following limits would be violated ?

(*a*) Voltage (*b*) Active power

(*c*) Reactive power (*d*) Phase angle.

344. The self-inductance of a long cylindrical conductor due to its internal flux linkages is k H/m. If the diameter of the conductor is doubled, then the self-inductance of the conductor due to its internal flux linkages would be

(*a*) 0.5 K H/m (*b*) K H/m

(*c*) 1.414 K H/m (*d*) 4 k H/m.

345. Two arrangements of conductors are proposed for a 3-phase transmission line: one with equilateral spacing of 4 m and the other a flat with 4 m between the conductors as shown in the given figure.

(*i*) (*ii*)

The conductor diameter in each case is 2 cm. Assuming that the line is transposed in both cases. Which one of the following statements would be true ?

(C_n = capacitance in F/m line to neutral

L = inductance in H/m per phase)

(*a*) $C_{n1} = C_{n2}$ and $L_1 > L_2$ (*b*) $C_{n1} > C_{n2}$ and $L_1 < L_2$

(*c*) $C_{n1} < C_{n2}$ and $L_1 > L_2$ (*d*) $C_{n1} > C_{n2}$ and $L_1 = L_2$.

346. For a given power delivered, if the working voltage of a distributor line is increased to n times, the cross-sectional area A of the distributor line, would be reduced to

(*a*) $\dfrac{1}{n}A$ (*b*) $\dfrac{1}{n^2}A$

(*c*) $\dfrac{1}{2n^2}A$ (*d*) $\dfrac{1}{2n}A$.

347. The following sequence currents were recorded in a power system under a fault condition:

$$I_{\text{positive}} = j\ 1.653 \text{ p.u.}$$
$$I_{\text{negative}} = -j\ 0.5 \text{ p.u.}$$
$$I_{\text{zero}} = -j\ 1.153 \text{ p.u.}$$

The fault is

(*a*) line to ground (*b*) three-phase

(*c*) line to line to ground (*d*) line to line.

348. When bundle conductors are used in place of single conductors, the effective inductance and capacitance will respectively

(*a*) increase and decrease (*b*) decrease and increase

(*c*) decrease and remain unaffected (*d*) remain unaffected and increase.

349. A balanced 3-phase, 3-wire supply feeds balanced star connected resistors. If one of the resistors is disconnected, then the percentage reduction in load will be

(*a*) $33\frac{1}{3}$ (*b*) 50

(*c*) $66\frac{2}{3}$ (*d*) 75.

350. A long overhead transmission line is terminated by its characteristic impedance. Under this operating condition, the ratio of the voltage to the current at different points along the line will

(*a*) progressively increase from the sending-end to the receiving-end

(*b*) progressively increase from the receiving-end to the sending-end

(*c*) remain the same at the two ends, but is higher between the two end being maximum at the centre of the line

(*d*) remain the same at all points.

351. For a transmission line with negligible losses, the lagging reactive power (VAR) delivered at the receiving-end, for a given receiving-end voltage is directly proportional to the

(*a*) square of the line voltage drop (*b*) line voltage drop

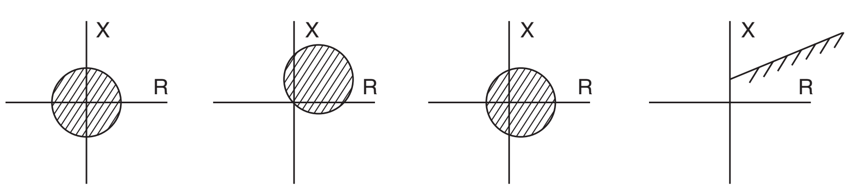
(*c*) line inductive reactance (*d*) line capacitive reactance.

352. Earth wire on EHV overhead transmission line is provided to protect the line against

(*a*) lightning surge (*b*) switching surge

(*c*) excessive fault voltages (*d*) corona effect.

353. Which one of the following is characteristic of an offset MHO relay ?



(a) (b) (c) (d)

354. Consider the following statements about distance relays

1. The effect of an arc may cause the relay to underreach

2. The effect of an intermediate current source may cause relay to overreach

3. The effect of an intermediate load may cause the relay to underreach

4. The presence of dc offset in the fault current may cause the relay to overreach.

Of these statements

(*a*) 2 and 3 are correct (*b*) 1 and 4 are correct

(*c*) 1, 2 and 3 are correct (*d*) 2, 3 and 4 are correct.

355. The inductance of a three-phase transmission line is 1.2 mH/phase/km. If the spacing of conductors and the radius of the conductor are doubled, then the inductance of the line will be

(*a*) 4.8 mH/phase/km (*b*) 1.2 mH/phase/km

(*c*) (1n 2) × 1.2 mH/phase/km (*d*) (1n 4) × 1.2 mH/phase/km.

356. A transmission line has 0.2 pu impedance on a base of 132 kV, 100 MVA. On a base of 220 kV, 50 MVA, it will have a pu impedance of

(*a*) $0.2 \times \frac{50}{100} \times \left(\frac{220}{132}\right)^2$ (*b*) $0.2 \times \frac{100}{50} \times \left(\frac{132}{220}\right)^2$

(*c*) $0.2 \times \frac{50}{100} \times \left(\frac{132}{220}\right)^2$ (*d*) $0.2 \times \frac{100}{50} \times \left(\frac{220}{132}\right)^2$.

357. A three-phase transmission line of negligible resistance and capacitance has an inductive reactance of 100 ohms per phase. When the sending-end and receiving-end voltages are maintained at 110 kV, the maximum power that can be transmitted will be

(*a*) 121 MW (*b*) $121\sqrt{3}$ MW

(*c*) 363 MW (*d*) 363 kW.

358. The per unit "regulation" of a short transmission line having a per-unit resistance voltage drop v_r and a per-unit reactance voltage drop of v_x at rated current and, with power-factor cos θ, is given by

(*a*) $(v_r + v_x) \cos \theta$ (*b*) $(v_r + v_x) \sin \theta$

(*c*) $v_r \cos \theta + v_x \sin \theta$ (*d*) $v_r \sin \theta + v_x \cos \theta$.

359. The surge impedance of a 100 km long underground cable is 50 ohms. The surge impedance of a 40 km long similar cable would be

(*a*) 20 ohms (*b*) 50 ohms

(*c*) 80 ohms (*d*) 125 ohms.

360. A string insulator has 4 units. The voltage across the bottom-most unit is 33.33% of the total voltage. Its string efficiency is

(*a*) 25% (*b*) 33.33%

(*c*) 66.7% (*d*) 75%.

361. The design of insulation for systems above 400 kV, is based upon

(*a*) lightning overvoltage (*b*) switching surges

(*c*) system voltage level (*d*) system load level.

362. Bundled conductors in EHV transmission systems provide

(*a*) increased line reactance
(*b*) reduced line capacitance
(*c*) reduced voltage gradient
(*d*) increased corona loss.

363. A power system is subjected to a fault which makes the zero sequence component of current equal to zero. The nature of fault is

(*a*) double line to ground fault
(*b*) double line fault
(*c*) line of ground fault
(*d*) three-phase to ground fault.

364. For which of the following reasons is a differential relay biased to avoid maloperation when used for transformer protection ?

1. Saturation of CTs.
2. Mismatch to CT ratios.
3. Difference in connection of both sides.
4. Current setting multiplier.

Select the correct answer using the codes given below:

(*a*) 1 and 4
(*b*) 1 and 2
(*c*) 2, 3 and 4
(*d*) 1, 2 and 3.

365. Protection scheme used for detection of loss of excitation of a very large generating unit feeding power into a grid employs

(*a*) undervoltage relay
(*b*) offset mho relay
(*c*) underfrequency relay
(*d*) percentage differential relay.

366. Which one of the following is not true regarding HVDC transmission ?

(*a*) Corona loss is much more in HVDC transmission
(*b*) The power transmission capability of bipolar line is almost the same as that of single-circuit ac line
(*c*) HVDC link can operate between two ac system whose frequencies need not be equal
(*d*) There is no distance limitation for HVDC transmission by underground cable.

367. The incremental cost characteristic of the two units in a plant are given by:

IC_1 = Rs. $(0.1\, P_1 + 0.8)$ per MWh

IC_2 = Rs. $(0.15\, P_2 + 3.0)$ per MWh

The optimum sharing of load when the total load is 100 MW is

(*a*) P_1 = 60 MW and P_2 = 40 MW
(*b*) P_1 = 33.3 MW and P_2 = 66.7 MW
(*c*) P_1 = 40 MW and P_2 = 60 MW
(*d*) P_1 = 66.7 MW and P_2 = 33.3 MW.

368. To increase the visual critical voltage of corona for an overhead line, one solid phase-conductor is replaced by a "bundle" of four smaller conductors per phase, having an aggregate cross-sectional area equal to that of the solid conductor. If the radius of the solid conductor is 40 mm, then the radius of each of the bundle conductors would be

(*a*) 10 mm
(*b*) 20 mm
(*c*) 28.2 mm
(*d*) 30 mm.

369. A single line to ground fault occurs on a three-phase isolated neutral system with a line to neutral voltage of V kV. The potentials on the healthy phases rise to a value equal to

(*a*) $\sqrt{2}$ V kV (*b*) $\sqrt{3}$ V kV

(*c*) 3 V kV (*d*) $\frac{1}{\sqrt{3}}$ V kV.

370. In a string of suspension insulators, the voltage distribution across the different units of a string could be made uniform by the use of a grading ring, because it

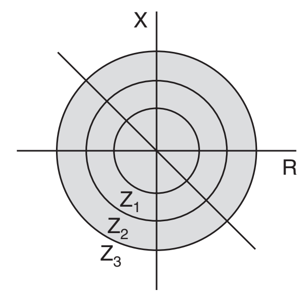
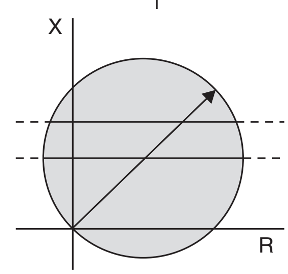
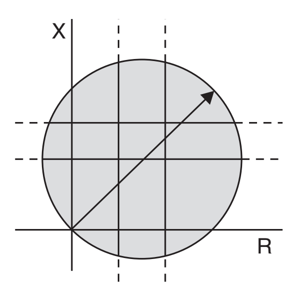
(*a*) forms capacitances with link-pins to cancel the charging current from link-pins

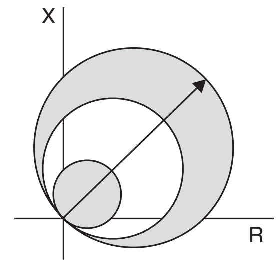
(*b*) forms capacitances which help to cancel the charging current from link-pins

(*c*) increases the capacitances of lower insulator units to cause equal voltage drop

(*d*) decreases the capacitances of upper insulator units to cause equal voltage drop.

371. The following lists relate to the relays and their characteristics for 3-zone protection. Match List I with List II and select the correct answer

List I (Relays)	*List II* (Relay characteristics)
A. Impedance relay	1. 
B. Mho relay	2. 
C. Reactance relay	3. 

4. 

	A	B	C
(*a*)	1	2	3
(*b*)	1	4	2
(*c*)	3	2	1
(*d*)	4	3	1.

372. A generator is connected to an infinite bus through a double circuit transmission. The fault occuring at the middle of one of the transmission lines is subsequently cleared by opening the circuit breakers at both the ends of the line simultaneously. The transient stability limit of the system is improved by

(*a*) decreasing the excitation of generator

(*b*) decreasing the fault-clearing time

(*c*) increasing the fault-clearing time

(*d*) increasing the transfer reactance between the generator and infinite bus.

373. The velocity of travelling wave through a cable of relative permittivity 9 is

(*a*) 9×10^8 m/s (*b*) 3×10^8 m/s

(*c*) 10^8 m/s (*d*) 2×10^8 ms.

374. The bus admittance matrix (Y_{bus}) of a power system is not

(*a*) symmetric (*b*) a square matrix

(*c*) a full matrix

(*d*) generally having dominant diagonal elements.

375. If the fault current is 3000 A, for an I.D.M.T. relay with a plug setting of 50% and C.T. ratio of 400/5, the plug setting multiplier would be

(*a*) 7.5 (*b*) 15.0

(*c*) 18.75 (*d*) 37.5.

376. A wave-trap is used at the termination of a HVAC overhead line to a station switchyard to

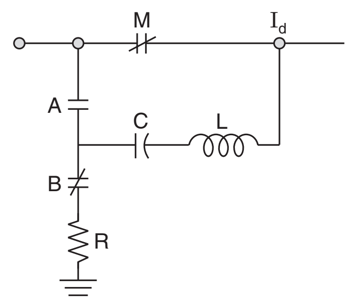
(*a*) prevent the transformer magnetising harmonics from reaching the overhead line

(*b*) damp the incoming surge waves from the overhead lines

(*c*) attenuate sub-harmonic oscillations

(*d*) provide for carrier communication.

377. The given figure shows the schematic diagram of a dc switch (oscillatory discharge type) for interrupting current (I_d) in H.V.D.C. lines, M, A and B are contactors; C, a capacitor; L, an inductor and; R, a high value resistor.



M and B are normally closed contacts, while A is normally open. For interruption of I_d, first B opens and A closes and immediately there-after contact M opens.

The current I_d is interrupted when the resultant current through M

(*a*) is positive-maximum (*b*) is negative-maximum

(*c*) passes through zero (*d*) is without the oscillatory component.

378. A 3-phase overhead transmission line has its conductors horizontally spaced with spacing between adjacent conductors equal to '*d*'. If, now, the conductors of the lines are rearranged to form an equilateral triangle of sides equal to '*d*', then the

(*a*) average capacitance as well as average inductance will increase

(*b*) average capacitance will decrease but the average inductance will increase

(*c*) average capacitance will increase but the average inductance will decrease

(*d*) surge impedance loading of the lines will increase.

379. A 50 Hz 3-phase transmission line of length 100 km has a capacitance of $0.03/\pi$, μ F per km. It is represented as a π model. The shunt admittance at each end of the transmission line will be

(*a*) $150 \times 10^{-6} \angle 90°$ mho (*b*) $100 \times 10^{-6} \angle 90°$ mho

(*c*) $50 \times 10^{-6} \angle 90°$ mho (*d*) $\dfrac{10^6}{150} \angle 90°$ mho.

380. The insulation resistance of a 20 km long underground cable is 8 mega ohm. Other things being the same, the insulation resistance of a 10 km long cable will be

(*a*) 16 mega ohm (*b*) 32 mega ohm

(*c*) 4 mega ohm (*d*) 2 mega ohm.

381. In an unbalanced 3-phase system, the currents are measured as

$$I_a = \text{zero}, I_b = 6\ \angle 60 \text{ and } I_c = 6\ \angle -120°$$

The corresponding sequence components will be

	I_{a_0}	I_{a_1}	I_{a_2}
(*a*)	Zero	$3 - j\sqrt{3}$	$-3 + j\sqrt{3}$
(*b*)	Zero	$-3 + j\sqrt{3}$	$3 - j\sqrt{3}$
(*c*)	Zero	$-9 + j3\sqrt{3}$	$9 - j3\sqrt{3}$
(*d*)	Zero	$9 - j3\sqrt{3}$	$-9 + j3\sqrt{3}$.

382. A 3-phase line having negligible resistance and 22 ohm inductive reactance per phase operates with 110 kV sending-end voltage and 100 kV receiving-end voltage. The maximum power that this line can transmit is

(*a*) $500\sqrt{3}$ MW (*b*) 500 MW

(*c*) 1500 MW (*d*) $\dfrac{500}{\sqrt{3}}$ MW.

383. A voltage control bus is characterised by the specified

(*a*) real and reactive powers (*b*) real power and voltage phase angle

(*c*) real power and voltage magnitude (*d*) reactive power and voltage magnitude.

384. A system is said to be effectively grounded only if R_0/X_1 and X_0/X_1 are respectively (symbols have the usual meanings)

(*a*) ≤ 1 and ≤ 3 (*b*) ≥ 1 and ≥ 3

(*c*) ≤ 2 and ≥ 2 (*d*) ≥ 2 and ≤ 2.

385. Match List I (Coils of a harmonic restrained percentage bias differential relay used for power transformer protection) with List II (Currents flowing through the relay coil) and select the correct answer using the codes given below the lists:

List I	*List II*
A. Operating oil	1. Through current
B. Restraining coil	2. Differential current
C. Bias coil	3. Second harmonic current

Codes:

	A	*B*	*C*
(*a*)	1	3	2
(*b*)	2	3	1
(*c*)	3	2	1
(*d*)	2	1	3

386. A composite transmission and distribution network has lines working at different voltages. Match List I (Voltage level used) with List II (Type of relays used for protection) and select the correct answer using the codes given below the lists:

List I	*List II*
A. 220 kV	1. Directional time lag overcurrent relays
B. 33 kV	2. Carrier current relays
C. 11 kV	3. Distance relays.

Codes:

(*a*)	*A*	*B*	*C*
	1	3	2
(*b*)	*A*	*B*	*C*
	3	2	1
(*c*)	*A*	*B*	*C*
	2	1	3
(*d*)	*A*	*B*	*C*
	2	3	1

387. Given that

Source voltage $e = E_m \sin(\omega t + \alpha)$, applied at $t = 0$, short-circuit current

$$i = \frac{E_m}{Z} \sin.(\omega t + \alpha - \phi) + K\varepsilon^{-Rt/L}$$

where $\phi = \tan^{-1} \dfrac{\omega L}{R}$ and K = constant.

A 3-phase short-circuit occurs at the terminals of an alternator having an effective impedance of $Z = \sqrt{R^2 + \omega^2 L^2}$. If the oscillogram of a phase current shows the presence of an exponentially decaying dc component and an ac component decaying to its steady-state value, then the dc component will be zero when

(*a*) $\alpha = \dfrac{\pi}{2}$

(*b*) $\alpha = \dfrac{\pi}{2} + \phi$

(*c*) $\alpha = \dfrac{\pi}{2} - \phi$

(*d*) $\alpha = \phi$.

388. Consider the following statements:

A differential relay is used for a 3-phase transformer protection to avoid maloperation due to

1. saturation of current transformer
2. mismatching of the current ratio for current transformers
3. difference in connections on both sides of power transformer
4. current setting multipliers

Of these statements

(*a*) 1 and 4 are correct (*b*) 1 and 2 are correct

(*c*) 1, 2 and 3 are correct (*d*) 2, 3 and 4 are correct.

389. A dc reactor is connected in series with each pole of a converter station in order to

(*a*) prevent commutation failures in the inverter

(*b*) supply reactive power to the converter

(*c*) improve system stability

(*d*) increase the power transfer capability.

390. In a HVDC transmission, the equivalent circuit representing the operation of a bridge rectifier is as shown in the given figure:

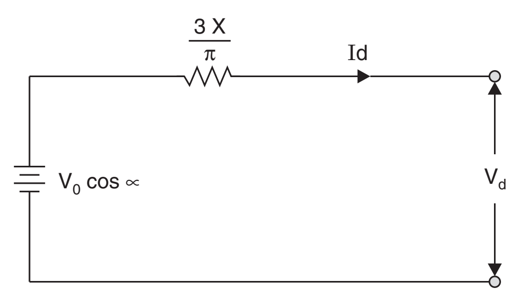
The symbols in the figure have the usual meanings. In this circuit, the voltage drop $\frac{3XI_d}{\pi}$ represents a drop due to

(*a*) ohmic resistance of the thyristors

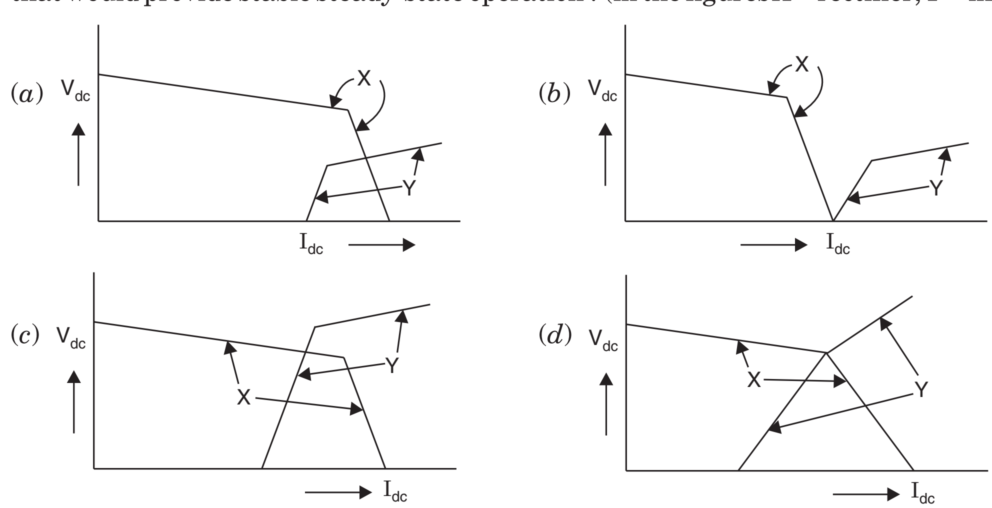
(*b*) ohmic resistance of the rectifier transformer

(*c*) commutation in the rectifier

(*d*) leakage reactance of the transformer.



391. Which one of the following V_{dc} vs I_{dc} characteristics of a HVDC link represents the one that would provide stable steady-state operation ? (in the figures X = rectifier, Y = inverter).



392. The power transmission capability of bipolar lines is approximately

(*a*) half that of 3-phase single circuit line

(*b*) the same as that of 3-phase single circuit line

(*c*) twice that of 3-phase single circuit line

(*d*) thrice that of 3-phase single circuit line.

393. In which one of the following models of transmission lines, is the full charging current assumed to flow over half the length of the line only ?

(*a*) Equivalent-π (*b*) Short line

(*c*) Nominal-π (*d*) Nominal-T.

394. If the receiving-end voltage and current are numerically equal to the corresponding sending-end values, that is, $|V_S| = |V_R|$ and $|I_S| = |I_R|$, then such a line is called

(*a*) an infinite line (*b*) a natural line

(*c*) a tuned line (*d*) a loss-less line.

395. The convergence characteristics of the Netwon-Rophson method for solving a load flow problem is

(*a*) quadratic (*b*) linear

(*c*) geometric (*d*) cubic.

396. If $|V_S| = |V_R| = 66$ KV for three-phase transmission and reactance is 11 ohms/phase, then the maximum power transmission per phase would be

(*a*) 132 MW (*b*) 396 MW

(*c*) 66 MW (*d*) None of the above.

397. For a two-machine system with losses, with the transfer-impedance being resistive, the maximum value of the sending-end power P_{1max} and the maximum receiving-end power P_{2max} will occur at power-angles (δ) in such a manner that

(*a*) both P_{1max} and P_{2max} occur at $\delta < 90°$

(*b*) both P_{1max} and P_{2max} occur at $\delta > 90°$

(*c*) P_{1max} occur at $\delta > 90°$ and P_{2max} at $\delta < 90°$

(*d*) P_{1max} occur at $\delta < 90°$ and P_{2max} at $\delta > 90°$.

398. Consider the following expression:

$$v = f_1(x - rt) + f_2(x + rt)$$

where f_1 and f_2 represent two travelling waves on a transmission line. In this case

(*a*) both waves travel in the positive direction of x

(*b*) both waves travel in the negative direction of x

(*c*) wave f_2 travels in the positive direction of x but wave f_1 travels in the negative direction of x

(*d*) wave f_1 travels in the positive direction of x but wave f_2 travels in the negative direction of x.

399. For reducing tower footing resistance, it is better to employ

(*a*) chemical and counterpoise
(*b*) chemical and ground rods
(*c*) ground rods and counterpoise
(*d*) chemical, ground rods and counterpoise.

400. A suspension type insulator has three units with self-capacitance C and ground capacitance of 0.2 C having a string efficiency of

(*a*) 78%
(*b*) 80%
(*c*) 82%
(*d*) 84%.

401. A bundled conductor line compared to a single conductor line (with same conductor cross-sectional area and same mean distance between conductors) has

Self GMD	*Mutual GMD*	*Per phase Inductance*
(*a*) Lower	Nearly same	Higher
(*b*) Higher	Lower	Nearly same
(*c*) Higher	Nearly same	Lower
(*d*) Lower	Higher	Higher.

402. A thyrite lightning arrester has

(*a*) inverse resistance characteristics
(*b*) a gap
(*c*) efficient earthing
(*d*) a combination of inverse resistance characteristics and gap.

403. Differential protection of a generator makes use of the principle that, under normal conditions, the current/currents

(*a*) at the neutral end of a phase winding is zero
(*b*) in each of the phase winding is identical
(*c*) at both ends of the phase winding are equal
(*d*) at the two ends of the phase winding are unequal.

404. The reactance relay is essentially

(*a*) an over-voltage realy with current restraint
(*b*) an over-voltage relay with directional restraint
(*c*) a directional relay with voltage restraint
(*d*) a directional relay with current restraint.

405. Severe over-voltages are produced during arcing faults in a power system with the neutral

(*a*) isolated
(*b*) solidly earthed
(*c*) earthed through a low resistance
(*d*) earthed through an inductive coil.

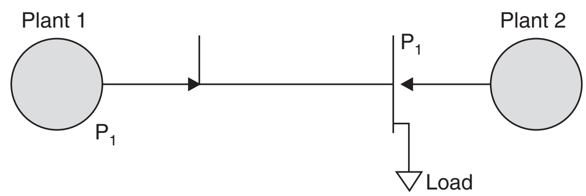
406. In HVDC transmission system, rectifier firing angle alpha is kept near

(*a*) 0°
(*b*) 15°
(*c*) 30°
(*d*) 90°.

407. A two-bus system is shown in the given figure. When 100 MW is transmitted from plant 1 to the load, the transmission loss is 10 MW. The incremental fuel costs of the two plants are

$$\frac{dC_1}{dP_1} = 0.02\,P_1 + 16 \text{ and } \frac{dC_2}{dP_2} = 0.04\,P_2 + 20$$

The optimum distribution of total load of 260 MW between the two plants when losses are included but not coordinated is



P_1	P_2
(*a*) 300 MW	50 MW
(*b*) 240 MW	20 MW
(*c*) 13 MW	130 MW
(*d*) 220 MW	60 MW.

408. The number of discs in a string of insulators for 400 KV ac overhead line lies in the range of

(*a*) 32 to 33
(*b*) 22 to 23
(*c*) 15 to 16
(*d*) 9 to 10.

409. The inertia constant of a 100 MVA, 50 Hz, 4-pole generator is, 10 MJ/MVA. If the mechanical input to the machine is suddenly raised from 50 MW to 75 MW, the rotor acceleration will be equal to

(*a*) 225 electrical degree/s^2
(*b*) 22.5 electrical degree/s^2
(*c*) 125 electrical degree/s^2
(*d*) 12.5 electrical degree/s^2.

410. Insulation coordination for UHV lines (above 500 kV) is done based on

(*a*) lightning surges
(*b*) lightning surges and switching surges
(*c*) switching surges
(*d*) none of the above.

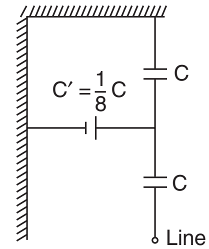
411. The number of discs in a string of insulators for 220 kV ac overhead transmission line lies in the range of

(*a*) 22 to 25
(*b*) 20 to 21
(*c*) 15 to 16
(*d*) 9 to 10.

412. The insulation resistance of a single-core cable is 200 MΩ/km. The insulation resistance for 5 km length is

(*a*) 40 MΩ
(*b*) 1000 MΩ
(*c*) 200 MΩ
(*d*) 8 MΩ.

413. The equivalent capacitor arrangement of two string insulator is shown in the above figure. The maximum voltage that each unit can withstand should not exceed 17.5 kV. The line-voltage of the complete string is



(*a*) 17.5 kV (*b*) 33 kV

(*c*) 35 kV (*d*) 37.3 kV.

414. Consider the following statements :

1. By using bundle conductors in an overhead line, the corona loss is reduced.
2. By using bundle conductors, the inductance of transmission line increases and capacitance reduces.
3. Corona loss causes interference in adjoining communication lines.

Which of these statements are correct ?

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 1 and 3 (*d*) 1, 2 and 3.

415. Which one of the following sequences of operations represents the rated operating duty cycle of a circuit breaker ?

(*O*-open ; *C*-close ; t = 3 sec., T = 3 min)

(*a*) *O–t–CO–T–CO* (*b*) *O–CO–t–CO–T–C*

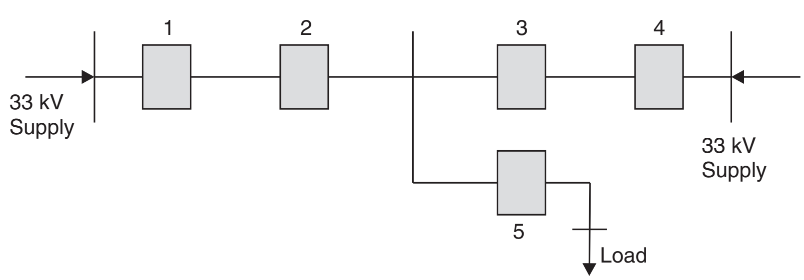
(*c*) *O–C–t–OC–T* (*d*) *O–CO–T–CO–T–C*.

416. Four alternators, each rated at 5 MVA, 11 kV with 20% reactance are working in parallel. The short-circuit level at bus bars is

(*a*) 6.25 MVA (*b*) 20 MVA

(*c*) 25 MVA (*d*) 100 MVA.

417. The distribution system shown in the figure is to be protected by over current system of protection. For proper fault discrimination, directional overcurrent relays will be required at locations



(*a*) 1 and 4 (*b*) 2 and 3

(*c*) 1, 4 and 5 (*d*) 2, 3 and 5.

418. If the fault current is 2 kA, the relay setting is 50% and the CT ratio is 400/5, then plug setting multiplier will be

(*a*) 25 (*b*) 15

(*c*) 50 (*d*) 12.5.

419. A three-phase 11/66 kV, delta/star transformer, protected by Merz-price scheme has CT ratio of 400/5 on L.T side. Ratio of C.T. on H.T. side will be equal to

(*a*) 1 : 23 (*b*) 23 : 1

(*c*) 23 : $\sqrt{3}$ (*d*) $\sqrt{3}$: 23.

420. If, for a given alternator in economic operation mode, the incremental cost is given by $(0.012P + 8)$ Rs./mwh, $\frac{dP_L}{dP} = 0.2$ and plant $\lambda = 25$, then the power generation is

(*a*) 1000 MW (*b*) 1250 MW

(*c*) 750 MW (*d*) 1500 MW.

421. Two generators rated at 200 MW and 400 MW are operating in parallel. Both the governors have a drop of 4%, when the total load is 300 MW. They share the load as (suffix '1' is used for generator 200 MW and suffix '2' is used for generator 400 MW)

(*a*) P_1 = 100 MW and P_2 = 200 MW (*b*) P_1 = 150 MW and P_2 = 150 MW

(*c*) P_1 = 200 MW and P_2 = 100 MW (*d*) P_1 = 200 MW and P_2 = 400 MW.

422. The impedance per phase of 3-phase transmission line on a base of 100 MVA, 100 kV is 2 pu. The value of this impedance on a base of 400 MVA and 400 kV would be

(*a*) 1.5 pu (*b*) 1.0 pu

(*c*) 0.5 pu (*d*) 0.25 pu.

423. The values of *A*, *B*, *C* and *D* constants for a short transmission line are respectively

(*a*) *Z*, 0, 1 and 1 (*b*) 0, 1, 1 and 1

(*c*) 1, *Z*, 0 and 1 (*d*) 1, 1, *Z* and 0.

424. If a short transmission line is delivering to a lagging *pf* load, the sending-end *pf* would be (notations have their usual meaning)

(*a*) $\frac{V_R \cos\phi + IR \sin\phi}{V_S}$ (*b*) $\frac{V_R \cos\phi + IR}{V_S}$

(*c*) $\frac{V_R \sin\phi + IR}{V_S}$ (*d*) $\frac{V_R \sin\phi + IR \cos\phi}{V_S}$.

425. If the positive, negative and zero sequence reactances of an element of a power system are 0.3, 0.3 and 0.8 respectively, then the element would be a

(*a*) synchronous generator (*b*) synchronous motor

(*c*) static load (*d*) transmission line.

426. An isolated synchronous generator with tansient reactance equal to 0.1 pu on a 100 MVA base is connected to the high voltage bus through a step up transformer of reactance 0.1 pu on 100 MVA base. The fault level at the bus is

(*a*) 1000 MVA (*b*) 500 MVA

(*c*) 50 MVA (*d*) 80 MVA.

427. If a generator of 250 MVA rating has an inertia constant of 6 MJ/MVA, its inertia constant on 100 MVA base is

(*a*) 15 MJ/MVA (*b*) 10.5 MJ/MVA
(*c*) 6 MJ/MVA (*d*) 2.4 MJ/MVA.

428. An overhead line with surge impedance of 400 Ω is terminated through a resistance 'R'. A surge travelling over the line will not suffer any reflection at the junction, if the value of R is

(*a*) 100 Ω (*b*) 200 Ω
(*c*) 400 Ω (*d*) 800 Ω.

429. An overhead line with a surge impedance of 400 Ω is connected to a transformer by a short length of cable of surge impedance 100 Ω. If a rectangular surge wave of 40 kV travels along the line towards the cable, then the voltage of the wave travelling from the junction of the overhead line through the cable towards the transformer would be

(*a*) 16 kV (*b*) 32 kV
(*c*) 30 kV (*d*) 36 kV.

430. Disruptive corona begins in smooth cylindrical conductors in air at NTP if the electric field intensity at the conductor surface goes up to

(*a*) 21.1 kV(rms)/cm (*b*) 21.1 kV(peak)/cm
(*c*) 21.1 kV(average)/cm (*d*) 21.1 kV(rms)/m.

431. In Merz Price percentage differential protection of a $\Delta - Y$ transformer, the *CT* secondaries connection in the primary and secondary windings of the transformer would be in the form of

(*a*) $\Delta - Y$ (*b*) $Y - \Delta$
(*c*) $\Delta - \Delta$ (*d*) $Y - Y$.

432. When there is interference in an overhead communication line running parallel and in close proximity to an overhead power line, the voltage induced in the communication line in the longitudinal and lateral directions by the power line are due to

(*a*) magnetic induction and electric induction respectively
(*b*) electric induction and magnetic induction respectively
(*c*) both magnetic induction and electric induction
(*d*) magnetic induction only.

433. In the case of a HVDC system, there is

(*a*) charging current but no skin effect (*b*) no charging current but skin effect
(*c*) neither charging current nor skin effect
(*d*) both charging current and skin effect.

434. Shunt compensation in an EHV line is used to improve

(*a*) stability and fault level (*b*) fault level and voltage profile
(*c*) voltage profile and stability (*d*) stability, fault level and voltage profile.

435. The cost function of a 50 MW generator is given by (P_i is the generator loading)

$$F(P_i) = 225 + 53\,P_i + 0.02\,P_i^2$$

When 100% loading is applied, the incremental fuel cost (IFC) will be

(*a*) Rs. 55 per MWh (*b*) Rs. 55 per MW

(*c*) Rs. 33 per MWh (*d*) Rs. 33 per MW.

436. In terms of power generation and B_{mn} coefficients, the transmission loss for a two-plant system is (Notations have their usual meaning)

(*a*) $P_1^2B_{11} + 2P_1P_2B_{12} + P_2^2B_{22}$ (*b*) $P_1^2B_{11} - 2P_1P_2B_{12} + P_2^2B_{22}$

(*c*) $P_2^2B_{11} + 2P_1P_2B_{12} + P_1^2B_{22}$ (*d*) $P_1^2B_{11} + P_1P_2B_{12} + P_2^2B_{22}$.

437. The *ABCD* constants of a 3-phase transmission line are

$$A = D = 0.8 \angle 1° \ ; B = 170 \angle 85° \ ; C = 0.002 \angle 90.4° \text{ mho}$$

The sending end voltage is 400 kV. The receiving end voltage under no load condition is

(*a*) 400 kV (*b*) 500 kV

(*c*) 320 kV (*d*) 417 kV.

438. In a short transmission line, voltage regulation is zero when the power factor angle of the load at the receiving end side is equal to

(*a*) $\tan^{-1}\left(\frac{X}{R}\right)$ (*b*) $\tan^{-1}\left(\frac{R}{X}\right)$

(*c*) $\tan^{-1}\left(\frac{X}{Z}\right)$ (*d*) $\tan^{-1}\left(\frac{R}{Z}\right)$.

439. Consider the following statements :

Surge impedance loading of a transmission line can be increased by

1. increasing its voltage level.
2. addition of lumped inductance in parallel.
3. addition of lumped capacitance in series.
4. reducing the length of the line.

Of these statements

(*a*) 1 and 3 are correct (*b*) 1 and 4 are correct

(*c*) 2 and 4 are correct (*d*) 3 and 4 are correct.

440. The load currents in short circuit calculation are neglected because

1. Short circuit currents are much larger than load currents.
2. Short circuit currents are greatly out of phase with load currents.

Which of these statement(s) is/are correct ?

(*a*) Neither 1 nor 2 (*b*) 2 alone

(*c*) 1 alone (*d*) 1 and 2.

441. The surge impedance of a 3-phase, 400 kV transmission line is 400 Ω. The surge impedance loading (SIL) is

(*a*) 400 MW (*b*) 100 MW

(*c*) 1600 MW (*d*) 200 MW.

442. If a travelling-wave travelling along a loss-free overhead line does not result in any reflection after it has reached the far end, then the far end of the line is

(*a*) open circuited (*b*) short circuited

(*c*) terminated into a resistance equal to surge impedance of the line

(*d*) terminated into a capacitor.

443. A travelling wave 400/1/50 means crest value of

(*a*) 400 V with rise time of 1/50 s

(*b*) 400 kV with rise time 1 s and fall time 50 s

(*c*) 400 kV with rise time 1 μs and fall time 50 μs

(*d*) 400 MV with rise time 1 μs and fall time 50 μs.

444. If a 500 MVA, 11 kV three-phase generator at 50 Hz feeds, through a transfer impedance of (0.0 + J 0.605)Ω per phase, an infinite bus also at 11 kV ; then the maximum steady state power transfer on the base of 500 MVA and 11 kV is

(*a*) 1.0 pu (*b*) 0.8 pu

(*c*) 0.5 pu (*d*) 0.4 pu.

445. Installation of capacitors at suitable locations and of optimum size in a distribution system results in

1. improved voltage regulation.
2. reduction in distribution power losses.
3. reduction of kVA rating of distribution transformers.

Select the correct answer from the codes given below :

(*a*) 1 alone (*b*) 1 and 2

(*c*) 1, 2 and 3 (*d*) 3 alone

446. In a three unit insulator string, voltage across the lowest unit is 17.5 kV and string efficiency is 84.28%. The total voltage across the string will be equal to

(*a*) 8.825 kV (*b*) 44.25 kV

(*c*) 88.25 kV (*d*) 442.5 kV.

447. Bundled conductors are used for EHV transmission lines primarily for reducing the

(*a*) corona loss (*b*) surge impedance of the line

(*c*) voltage drop across the line (*d*) I^2R losses.

448. The good effect of corona on overhead lines is to

(*a*) increase the line carrying capacity due to conducting ionised air envelop around the conductor

(*b*) increase the power factor due to corona loss

(*c*) reduce the radio interference from the conductor

(*d*) reduce the steepness of surge fronts.

449. The principal information obtained from load flow studies in a power system are

1. magnitude and phase angle of the voltage at each bus.
2. reactive and real power flows in each of lines.

3. total power loss in the network.
4. transient stability limit of the system.

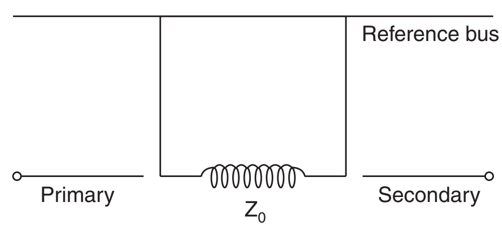
Select the correct answer from the codes given below :

(*a*) 1 and 2 (*b*) 3 and 4
(*c*) 1, 2 and 3 (*d*) 2 and 4.

450. If all the sequence voltages at the fault point in a power system are equal, then the fault is a

(*a*) three-phase fault (*b*) line to ground fault
(*c*) line to line fault (*d*) double line to ground fault.

451.



A three-phase transformer having zero-sequence impedance of Z_0 has the zero-sequence network as shown in the above figure. The connections of its windings are

(*a*) star-star (*b*) delta-delta
(*c*) star-delta (*d*) delta-star with neutral grounded.

452. Which one of the following relays has the capability of anticipating the possible major fault in a transformer ?

(*a*) Overcurrent relay (*b*) Differential relay
(*c*) Buchholz relay (*d*) Overfluxing relay.

453. In a 220 kV system, the inductance and capacitance up to the circuit breaker location are 25 mH and 0.025 μF respectively. The value of resistor required to be connected across the breaker contacts which will give no transient oscillations, is

(*a*) 25 Ω (*b*) 250 Ω
(c) 500 Ω (*d*) 1000 Ω.

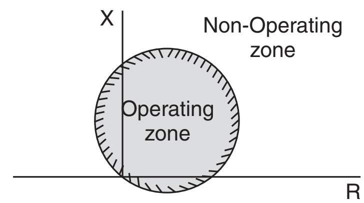
454. Match List I (Equipments) with List II (Applications) and select the correct answer using the codes given below the lists :

List I (*Equipments*)	List II (*Applications*)
A. Metal oxide arrester	1. Protects generator against short circuit faults
B. Isolator	2. Improves transient stability
C. Auto-reclosing *C.B.*	3. Allow *C.B.* maintenance.
D. Differential relay	4. Provides protection against surges

Codes :

	A	B	C	D		A	B	C	D
(*a*)	4	3	2	1	(*b*)	3	4	1	2
(*c*)	4	3	1	2	(*d*)	3	4	2	1

455. The operating characteristic of a distance relay in the *R-X* plane is shown in the given figure. It represents operating characteristic of a



(*a*) reactance relay

(*b*) directional impedance relay

(*c*) impedance relay

(*d*) mho relay.

456. The bus admittance matrix of a power system is given as

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j\,50 & +j\,10 & +j\,5 \\ +j\,10 & -j\,30 & +j\,10 \\ +j\,5 & +j\,10 & -j\,25 \end{bmatrix} \end{array}$$

The impedance of line between bus 2 and 3 will be equal to

(*a*) $+j\,0.1$ (*b*) $-j\,0.1$

(*c*) $+j\,0.2$ (*d*) $-j\,0.2$.

457. The component inductance due to the internal flux-linkage of a non-magnetic straight solid circular conductor per meter length, has a constant value, and is independent of the conductor-diameter, because

(*a*) All the internal flux due to a current remains concentrated on the peripheral region of the conductor

(*b*) The internal magnetic flux-density along the radial distance from the centre of the conductor increases proportionately to the current enclosed

(*c*) The entire current is assumed to flow along the conductor axis and the internal flux is distributed uniformly and concentrically

(*d*) The current in the conductor is assumed to be uniformly distributed throughout the conductor cross-section.

458. Match List-I with List-II and select the correct answer using the code given below the Lists :

List-I	*List-II*
A. Graetz Bridge Converter	1. EHV/UHV AC Transmission
B. Series Compensation	2. HVDC Transmission
C. Sag Templates	3. Insulators
D. Grading Ring	4. Tower Location

Codes :

	A	B	C	D		A	B	C	D
(*a*)	2	1	3	4	(*b*)	1	2	4	3
(*c*)	2	1	4	3	(*d*)	1	2	3	4

459. A rectangular voltage wave is impressed on a loss-free overhead line, with the far end of the line being short-circuited. On reaching the end of this line

(*a*) The current wave is reflected back with positive sign, but the voltage wave with negative sign

(*b*) The current wave is reflected back with negative sign, but the voltage wave with positive sign

(*c*) Both the current and the voltage waves are reflected with positive sign

(*d*) Both the current and the voltage waves are reflected with negative sign.

460. Two insulator discs of identical capacitance value C make up a string for a 22 kV, 50 Hz, single-phase overhead line insulation system. If the pin to earth capacitance is also C, then the string efficiency is

(*a*) 50% (*b*) 75%

(*c*) 90% (*d*) 86%.

461. Which one of the following statements is not correct for the use of bundled conductors in transmission lines ?

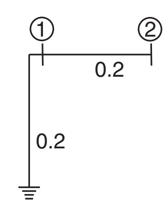
(*a*) Control of voltage gradient

(*b*) Reduction in corona loss

(*c*) Reduction in radio interference

(*d*) Increase in interference with communication lines.

462. In the network as shown here, the marked parameters are p.u. impedances. The bus-admittance matrix of the network is



(*a*) $\begin{bmatrix} 10 & -5 \\ -5 & 5 \end{bmatrix}$ (*b*) $\begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$

(*c*) $\begin{bmatrix} -10 & 5 \\ 5 & -5 \end{bmatrix}$ (*d*) $\begin{bmatrix} -5 & 5 \\ 5 & -10 \end{bmatrix}$.

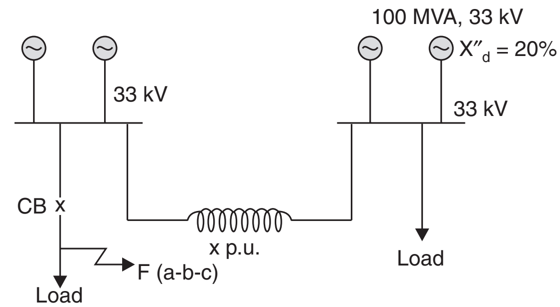
463. Consider the following statements with reference to protective relays :

1. The minimum relay coil current at which the relay operates is called pick-up value
2. The pick-up value of a relay is 7.5 *A* and fault current is 30 *A*. Therefore, its plug setting multiplier is 5
3. An earth fault current is generally lesser than the short-circuit current
4. Induction relays are used with both a.c. and d.c. quantities.

Which of these statements are correct ?

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 1 and 3 (*d*) 1, 2 and 4.



464. Four identical 100 MVA, 33 kV generators are operating in parallel, as shown above, in two bus-bar sections, interconnected through a current limiting reactor of x p.u. reactance on the generator-base. Each generator has a reactance of 0.2 p.u.

The value of the reactor x to limit a symmetrical short-circuit (a-b-c) current through the circuit breaker to 1500 MVA is

(*a*) 0.05 p.u. (*b*) 0.10 p.u.
(*c*) 0.15 p.u. (*d*) 0.20 p.u.

465. The zero sequence current of a generator for line to ground fault is j 2.4 p.u. Then the current through the neutral during the fault is

(*a*) j 2.4 p.u. (*b*) j 0.8 p.u.
(*c*) j 7.2 p.u. (*d*) j 0.24 p.u.

466. For a fixed receiving end and sending end voltages in a transmission system, what is the locus of the constant power ?

(*a*) A straight line (*b*) An ellipse
(*c*) A parabola (*d*) A circle.

467. At slack bus, which one of the following combinations of variables is specified ?

(*a*) $|V|, \delta$ (*b*) P, Q
(*c*) $P, |V|$ (*d*) $Q, |V|$.

(The symbols have their usual meaning)

468. Consider the following quantities :

1. Real power
2. Reactive power
3. Power factor
4. Input current
5. Bus voltage magnitude
6. Bus voltage phase-angle.

For the purpose of the load flow studies of a power system, each bus or node is associated with which one of the combinations of the above four quantities ?

(*a*) 1, 3, 4 and 5 (*b*) 1, 2, 3 and 4
(*c*) 2, 3, 5 and 6 (*d*) 1, 2, 5 and 6.

469. Normally Z_{BUS} matrix is a

(*a*) Null matrix (*b*) Sparse matrix
(*c*) Full matrix (*d*) Unity matrix.

470. If $\alpha = e^{j\frac{2\pi}{3}}$, and $I = AI_S$ where I is equal to phase current vector, and I_S is equal to symmetrical current vector, then which one of the following matrices is the symmetrical components transformation matrix A ?

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & \alpha \end{bmatrix}$.

471. Z_{pu}^{old} is the per unit impedance on the power base S_B^{old} and voltage base V_B^{old}. What would be the per unit impedance on the new power base S_B^{new} and voltage base V_B^{new} ?

(a) $Z_{pu}^{new} = Z_{pu}^{old} \dfrac{S_B^{old}}{S_B^{new}} \left(\dfrac{V_B^{new}}{V_B^{old}}\right)^2$ (b) $Z_{pu}^{new} = Z_{pu}^{old} \dfrac{S_B^{new}}{S_B^{old}} \left(\dfrac{V_B^{old}}{V_B^{new}}\right)^2$

(c) $Z_{pu}^{new} = Z_{pu}^{old} \dfrac{S_B^{new}}{S_B^{old}} \left(\dfrac{V_B^{new}}{V_B^{old}}\right)$ (d) $Z_{pu}^{new} = Z_{pu}^{old} \dfrac{S_B^{old}}{S_B^{new}} \left(\dfrac{V_B^{new}}{V_B^{old}}\right)$.

472. Consider the following statements regarding the fault analysis :

1. The neutral grounding impedance Z_n appears as $3Z_n$ in zero sequence equivalent circuit.
2. For faults on transmission lines, 3-phase fault is the least severe amongst other faults.
3. The positive and negative sequence networks are not affected by method of neutral grounding.

Which of the statements given above are correct ?

(a) 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 1, 2 and 3.

473. If a sudden short circuit occurs on a power system (considered as *R-L* series circuit), the current wave-form consists of

1. a decaying a.c. current.
2. a decaying d.c. current.

Let the alternator reactance be *X* and the power system resistance *R*. Which one of the following is correct ?

(a) The decay in (1) is caused by the increase in *X* but in (2) is caused by *R*
(b) The decay in (1) is caused by *R* but in (2) is caused by increase in *X*
(c) The decay in both (1) and (2) is caused by *R*
(d) The decay in both (1) and (2) is caused by the increase in *X*.

474. Match List I (Types of Relays) with List II (Types of Protection) and select the correct answer using the codes given below the lists :

List I *(Types of Relyas)*	*List II* *(Types of Protection)*
A. Directional relay	1. Relay operates for fault within certain distance of its location
B. Impedance relay	2. Relay will trip for fault in one location and block for all other locations

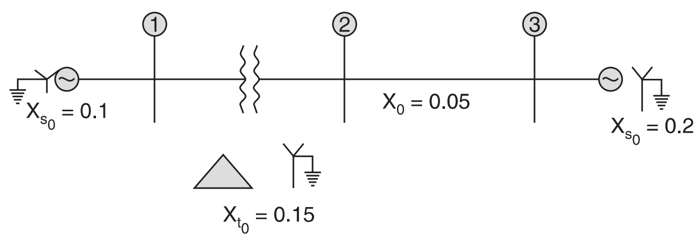
C. Differential relay — 3. High speed protection for entire transmission line

D. Pilot relay — 4. The principle of current continuity is used to devise a simple and effective relaying system over a small physical space.

Codes :

	A	B	C	D
(*a*)	1	2	4	3
(*b*)	2	1	3	4
(*c*)	2	1	4	3
(*d*)	1	2	3	4

475. The zero squence reactances (in p.u.) are indicated in the network shown in the figure. The zero sequence driving-point reactance of node 3 will be



(*a*) 0.05
(*b*) 0.1
(*c*) 0.2
(*d*) 0.3.

476. The *A*, *B*, *C* constants of a nominal *T*-circuit are given as

$$A = .95 \angle 0.5° \; ; B = 85 \angle 70° \text{ ohm} \; ; C = 0.0005 \angle 90° \text{ ohm}$$

If the two terminal voltages are held constant at 66 kV, then the steady-state stability limit of the line will be

(*a*) 0.3312 MW
(*b*) 3.312 MW
(*c*) 33.12 MW
(*d*) 331.2 MW.

477. A feeder with reactance of 0.2 p.u. has a sending-end voltage of 1.2 p.u. If the reactive power supplied at the receiving end of the feeder is 0.3 p.u., the approximate voltage drop in the feeder will be

(*a*) 0.042 p.u.
(*b*) 0.052 p.u.
(*c*) 0.060 p.u.
(*d*) 0.070 p.u.

478. Which one of the following matrices reveals the topology of the power system network ?

(*a*) Bus incidence matrix
(*b*) Primitive impedance matrix
(*c*) Primitive admittance matrix
(*d*) Bus impedance matrix.

479. The equivalent Thevenins bus admittance matrix of a two-bus system with identical generators on both buses is $\begin{bmatrix} -j30 & +j10 \\ +j10 & -j30 \end{bmatrix}$

The generator reactance and interconnecting line reactance will be respectively

(*a*) $j0.05$ and $j0.1$
(*b*) $-j0.05$ and $j0.1$
(*c*) $-j0.05$ and $-j0.1$
(*d*) $j0.1$ and $j0.05$.

480. A fault occurring on an end-supplied transmission line is more severe from the point of view of RRRV if it is a

(*a*) long line fault (*b*) short line fault

(*c*) medium line fault (*d*) generator fault.

481. The insulation strength of EHV lines is mainly governed by

(*a*) switching overvoltages (*b*) lightning overvoltages

(*c*) power frequency overvoltages (*d*) dynamic overvoltages.

482. The coefficient of reflection for current of an open-ended line is

(*a*) – 1 (*b*) 1

(*c*) 0.5 (*d*) zero.

483. A relay most likely to operate undesirably on power swings is the

(*a*) over-current relay (*b*) inverse definite minimum time relay

(*c*) mho relay (*d*) reactance relay.

484. Which of the following reasons make HRC fuses preferable in power circuits ?

1. They have inverse time characteristics.
2. They are consistent in performance.
3. They can be selected for proper discrimination.

Select the correct answer by using the codes given below :

Codes :

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 1 and 3 (*d*) 1, 2 and 3.

485. Consider the following statements about the utility of delta-connected tertiary windings in star-star connected power transformer :

1. It makes supply available for small loads.
2. It provides low reactance paths for zero sequence currents.
3. It is used to suppress harmonic voltages.
4. It is used to allow flow of earth fault current for operation of protective relays.

Which of these statements are correct ?

(*a*) 1 and 2 (*b*) 2, 3 and 4

(*c*) 1, 3 and 4 (*d*) 1, 2, 3 and 4.

486. A 3-phase transmission line conductors were arranged in horizontal spacing, with '*d*' as the distance between adjacent conductors. If these conductors are rearranged to form an equilateral triangle with sides equal '*d*', then the

(*a*) capacitance and the inductance will increase

(*b*) capacitance will increase and the inductance will decrease

(*c*) capacitance and the inductance will remain the same

(*d*) capacitance will decrease and the inductance will increase.

487. The inductance per unit length of an overhead line due to internal flux linkages

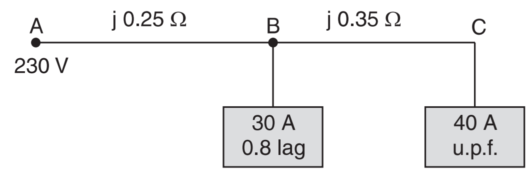
(*a*) depends on the size of the conductor

(*b*) is independent of the size of conductor and constant

(*c*) depends on the current through the conductor

(*d*) depends on distance between conductors.

488. A single-phase ac distributor supplies two single-phase loads as shown in the given figure. The voltage drop from *A* to *C* is



(*a*) $4.5 + j\,30$ V

(*b*) $30 + j\,4.5$ V

(*c*) $4.5 \;-j\,30$ V

(*d*) $j\,30$ V.

489. A synchronous generator with a synchronous reactance of 1.3 pu is connected to an infinite bus whose voltage is 1 pu, through an equivalent reactance of 0.2 pu. For maximum output of 1.2 pu, the alternator emf must be

(*a*) 1.5 pu (*b*) 1.56 pu

(*c*) 1.8 pu (*d*) 2.5 pu.

490. Consider the following statements :

Transient stability of a synchronous generator feeding power to an infinite bus through a transmission line can be increased by

1. increasing the steam supply to the turbine driving the generator during fault clearing.
2. connecting resistors at the generator terminals during fault condition.
3. employing a faster excitation system.
4. quickly throwing off the generator load.

Which of these statements are correct ?

(*a*) 2 and 3 (*b*) 3 and 4

(*c*) 1 and 2 (*d*) 2 and 4.

491. An overhead transmission line having surge impedance 'Z_1' is terminated to an underground cable of surge impedance 'Z_2'. The reflection coefficient for the travelling wave at the junction of the line and cable is

(*a*) $\dfrac{Z_1 + Z_2}{Z_1 - Z_2}$ (*b*) $\dfrac{Z_2}{Z_1 + Z_2}$

(*c*) $\dfrac{Z_2 - Z_1}{Z_1 + Z_2}$ (*d*) $\dfrac{Z_1 - Z_2}{Z_1 + Z_2}$.

492. Ring main distribution is preferred to a radial system because

(*a*) voltage drop in the feeder is less and supply is more reliable

(*b*) voltage drop in the feeder is less and power factor is high

(*c*) power factor is high and supply is more reliable

(*d*) power factor is high and system is less expensive.

493. If the fault current is 3000 A for a relay with a plug setting of 50% and CT ratio of 1000 : 1, the plug setting multiplier would be

(*a*) 1.5 (*b*) 3

(*c*) 4.5 (*d*) 6.

494. Match List I (Protective schemes) with List II (Equipment) and select the correct answer using the codes given the Lists :

List I	*List II*
A. Mho relays	1. Generators
B. Inverse time overcurrent relays	2. Transmission lines
C. Differential relays	3. Motors

Code :

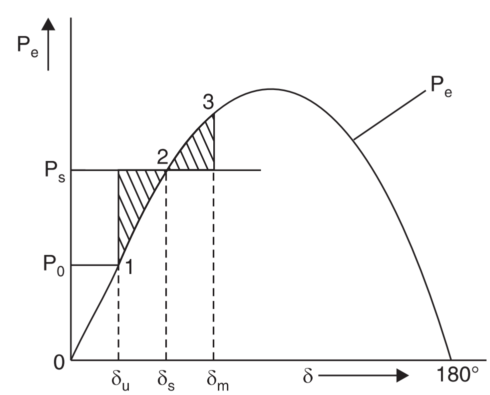
	A	*B*	*C*		*A*	*B*	*C*
(*a*)	2	1	3	(*b*)	2	3	1
(*c*)	3	2	1	(*d*)	1	3	2

495. In terms of plant powers P_n and P_m and loss coefficients B_{mn} the total transmission loss 'P_L' is

(*a*) $\sum_{m=1}^{N} \sum_{n=1}^{N} B_{mn} P_n$ (*b*) $\sum_{m=1}^{N} \sum_{n=1}^{N} P_m B_{mn}$

(*c*) $\sum_{m=1}^{N} \sum_{n=1}^{N} P_m B_{mn} P_n$ (*d*) $\sum_{m=1}^{N} \sum_{n=1}^{N} 2P_m B_{mn}$.

496. The given figure shows electric power input P_e to a lossless synchronous motor as a function of torque angle δ. The load is suddenly increased from P_0 to P_s and the motor oscillates around δ_s between δ_0 and δ_m.



Consider the following statements derived from the figure regarding the relationship between the motor speed ω and its synchronous speed ω_s at different points in the oscillating cycle :

I. At point 1, $\omega = \omega_s$.

II. At point 2, while oscillating from 1 towards 3, $\omega < \omega_s$.

III. At point 3, $\omega < \omega_s$.

IV. At point 2, while oscillating from 3 towards 1, $\omega > \omega_s$.

Which of these statement (s) is/are correct ?

(*a*) II and IV (*b*) I, II and IV

(*c*) I and III (*d*) III alone.

497. The reactance of a generator designated X″ is given as 0.25 pu based on the generator's name plate rating of 18 kV, 500 MVA. If the base for calculations is changed to 20 kV, 100 MVA, the generator reactance X'' on new base will be

(*a*) 1.025 pu (*b*) 0.05 pu

(*c*) 0.0405 pu (*d*) 0.25 pu.

498. Consider the following statements:

The calculations performed using short-line approximate model instead of nomial-π model for a medium length transmission line delivering lagging load at a given receiving-end voltage always result in higher.

1. sending-end current. 2. sending-end power.

3. regulation. 4. Efficiency.

Which of these statements are correct ?

(*a*) 1 and 2 (*b*) 3 and 4

(*c*) 1, 2 and 3 (*d*) 1, 2 and 4.

499. The severity of line to ground and three-phase faults at the terminals of an unloaded synchronous generator is to be same. If the terminal voltage is 1.0 pu, $Z_1 = Z_2 = j0.1$ pu and $Z_0 = j0.05$ pu for the alternator, then the required inductive reactance for neutral grounding is

(*a*) 0.0166 pu (*b*) 0.05 pu

(*c*) 0.1 pu (*d*) 0.15 pu.

500. A short length of cable is connected between dead-end tower and sub-station at the end of a transmission line. Which of the following will decrease, when voltage wave is entering from overhead line to cable ?

1. Velocity of propagation of voltage wave.
2. Steepness of voltage wave.
3. Magnitude of voltage wave.

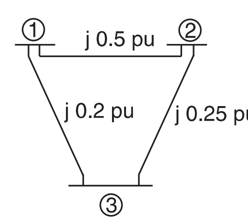
Select the correct answer using the codes given below:

Codes :

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 3 and 1 (*d*) 1, 2 and 3.

501. A single line diagram of a power system is shown in the given figure. A diagonal elements of the Y_{BUS} matrix is



(*a*) j 0.7 pu, j 0.75 pu, j 0.45 pu

(*b*) $-j$ 0.7 pu., – 0.75 pu., $-j$ 0.45 pu

(*c*) $-j$ 7.0 pu, $-j$ 6.0 pu, $-j$ 9.0 pu

(*d*) j 7.0 pu, $-j$ 6.0 pu, $-j$ 9.0 pu.

502. In load-flow analysis, the load at a bus is represented as

(*a*) a constant current drawn from the bus

(*b*) a constant impedance connected at the bus

(*c*) constant real and reactive powers drawn from the bus

(*d*) a voltage-dependent impedance at the bus.

503. Match List-I (Relay) with List-II (Equipment) and select the correct answer using the codes given below the Lists:

List-I	*List-II*
A. Mho Relay	1. Bus bar
B. Negative Sequence Relay	2. Motor
C. Thermal Relay	3. Generator
	4. Transmission line

Codes :

	A	*B*	*C*		*A*	*B*	*C*
(*a*)	1	3	2	(*b*)	2	4	1
(*c*)	4	2	3	(*d*)	4	3	2

504. Shunt resistor is connected across the contacts of a circuit-breaker in order to

(*a*) damp out the restriking transient (*b*) bypass the arc current

(*c*) limit the short-circuit current

(*d*) reduce the damage to the contacts due to arcing.

505. Maloperation of differential protection of transformers due to magnetizing inrush current is prevented by

(*a*) setting the current of the relay higher than the maximum value of inrush current

(*b*) keeping the time-setting long enough for the inrush current to fall to a value below the primary operating current of the relay

(*c*) bypassing the inrush current from the operating coil of the relay

(*d*) filtering the second harmonic content of the inrush current flowing through the operating coil and passing through the restraining coil.

506. Incremental fuel costs in Rs/MWh for a plant consisting of two units are given by

$$\frac{dF_1}{dP_1} = 0.4P_1 + 400 \text{ and } \frac{dF_2}{dP_2} = 0.48P_2 + 320.$$

The allocation of loads P_1 and P_2 between the units 1 and 2, respectively, for minimum cost of generation for a total load of 900 MW is

(*a*) 200 MW and 700 MW (*b*) 300 MW and 600 MW

(*c*) 400 MW and 500 MW (*d*) 500 MW and 400 MW.

507. For a given base voltage and base volt amperes, the per unit impedance value of an element is x. The per unit impedance value of this element when the voltage and volt amperes bases are both doubled will be

(*a*) $0.5x$ (*b*) x

(*c*) $2x$ (*d*) $4x$.

508. Consider the following statements with respect to an interconnected power system :

1. Frequency will be same at all buses in the system.
2. Voltages can be different at different buses.
3. Both frequency and voltage can be different at different buses.

Which of these statements are correct ?

(*a*) 1, 2 and 3 (*b*) 1 and 3

(*c*) 1 and 2 (*d*) 2 and 3.

509. Consider the following statements :

Overhead transmission lines are provided with earth wires

1. to protect the transmission line from direct lightning strike.
2. to protect the transmission line insulation from the indirect lightning strike.
3. to balance the line currents.
4. to provide path for neutral current.

Which of these statements is/are correct ?

(*a*) 1 and 3 (*b*) 1 and 4

(*c*) 1 only (*d*) 2 only.

510. In Gauss-Seidel method of power flow problem, the number of iterations may be reduced if the correction in voltage at each bus is multiplied by

(*a*) Gauss constant (*b*) Acceleration constant

(*c*) Deceleration constant (*d*) Blocking factor.

511. Consider the following statements :

1. It is easier to construct the Y_{BUS} matrix as compared to Z_{BUS}.
2. Z_{BUS} is a full matrix while Y_{BUS} is sparse.
3. Y_{BUS} can be easily modified whenever the network changes as compared to the Z_{BUS}.

Which of these statements are correct ?

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 1 and 3 (*d*) 1, 2 and 3.

512. Match List-I (Power system components) with List-II (Relaying schemes) and select the correct answer using the codes given below the Lists :

List-I (*Power system components*)	*List-II* (*Relaying schemes*)
A. Power Transformer	1. Differential relaying
B. Transmission Lines	2. Distance relaying
C. Alternator	

Codes :

	A	*B*	*C*		*A*	*B*	*C*
(*a*)	1	1	2	(*b*)	2	1	1
(*c*)	1	2	1	(*d*)	2	1	2

513. Steady state operating condition of a power system indicates

(*a*) a situation when the connected load is absolutely constant

(*b*) a situation when the generated power is absolutely constant (*c*) a situation when both connected load and generated power are equal to each other and remain constant

(*d*) an equilibrium state around which small fluctuations in power, both in generation and load, occur all the time.

514. Match List-I (*Insulation Type*) with List-II (*Purpose or Configuration*) and select the correct answer using the codes given below the lists :

List-I (*Insulation Type*)	*List-II* (*Purpose or Configuration*)
A. Pin type	1. Low voltage distribution lines
B. Suspension type	2. String of insulators in horizontal position
C. Strain type	3. String of insulators in vertical position
D. Shackle type	4. For voltage upto 33 kV

Codes :

	A	*B*	*C*	*D*
(*a*)	4	2	3	1
(*b*)	1	3	2	4
(*c*)	4	3	2	1
(*d*)	1	2	3	4

515. Match List-I (*Bus Types*) with List-II (*Pairs of variables*) and select the correct answer using the codes given below the lists :

List-I (*Bus Types*)	*List-II* (*Pairs of variables*)
A. Load bus	1. P and V
B. Generator bus	2. P and Q
C. Slack bus	3. V and δ

Codes :

	A	*B*	*C*
(*a*)	1	2	3
(*b*)	2	3	1
(*c*)	1	3	2
(*d*)	2	1	3

516. Match List-I (*Relays*) with List-II (*Protection*) and select the correct answer using the codes given below the lists :

List-I (*Relays*)	*List-II* (*Protection*)
A. Buchholz relay	1. Feeder
B. Translay relay	2. Transformer
C. Carrier current phase comparison relay	3. Ring main distributor
D. Directional over current relay	4. Long overhead transmission line.

Codes :

	A	*B*	*C*	*D*
(*a*)	2	3	4	1
(*b*)	4	1	2	3
(*c*)	2	1	4	3
(*d*)	4	3	2	1

517. A three-phase 50 Hz transmission line has a capacitance of line to neutral $C_n = 0.01$ μF/km. The voltage of the line is 100 kV. The charging current per kilometre of line is

(*a*) $\dfrac{314}{\sqrt{3}}$ A/km (*b*) $\dfrac{314}{\sqrt{3} \times 10^3}$ A/km

(*c*) $\dfrac{\sqrt{3} \times 10^3}{314}$ A/km (*d*) $\sqrt{3} \times 10^3$ A/km.

518. A string insulator has 5 units. The voltage across the bottom-most unit is 25% of the total voltage. The string efficiency is

(*a*) 25% (*b*) 50%

(*c*) 80% (*d*) 75%.

519. In load flow studies of a power system, a voltage control bus is specified by

(*a*) Real power and reactive power (*b*) Real power and voltage magnitude

(*c*) Voltage and voltage phase angle (*d*) Reactive power and voltage magnitude.

520. The power angle characteristic of machine-infinite bus system is

$P_c = 2 \sin \delta$ pu

It is operating at $\delta = 30°$. Which one of the following is the synchronising power coefficient at the operating point ?

(*a*) 1.0 (*b*) $\sqrt{3}$

(*c*) 2.0 (*d*) $\dfrac{1}{\sqrt{3}}$.

521. Which methods can be used for the measurement of three phase power for an unbalanced load ?

1. Three voltmeters 2. Two voltmeters and one ammeter

3. Two wattmeters 4. One wattmeter

Select the correct answer using the codes given below :

Codes :

(*a*) 1 only (*b*) 3 only

(*c*) 1 and 3 (*d*) Any one of 1, 2, 3 or 4.

522. Possible faults that may occur on a transmission line are

1. 3-phase fault 2. *L-L-G* fault

3. *L-L* fault 4. *L-G* fault.

The decreasing order of severity of the fault from the stability point of view is :

(*a*) 1-2-3-4 (*b*) 1-4-3-2

(*c*) 1-3-2-4 (*d*) 1-3-4-2.

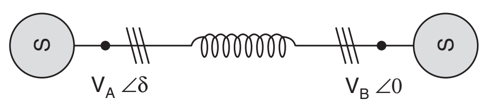
523. Match List I (*Protection Scheme*) with List II (*Type of Fault*) and select the correct answer using the codes given below the lists :

List I (*Protection Scheme*)	*List II* (*Type of Fault*)
A. Differential protection	1. Phase to phase, and phase to ground
B. Buchholz protection	2. Phase to ground
C. Earth fault protection	3. Short circuit
D. Thermal protection	4. Inter-turn
	5. Over loading

Codes :

	A	*B*	*C*	*D*
(*a*)	1	4	2	5
(*b*)	5	2	3	1
(*c*)	1	3	2	5
(*d*)	5	4	2	1

524. Two identical synchronous machines A and B, running at the same speed, are linked through inductors as shown in the given figure :



Machine A will supply active and reactive power to machine B when δ is

(*a*) positive and V_A is less than V_B

(*b*) positive and V_A is greater than V_B

(*c*) negative and V_A is less than V_B

(*d*) negative and V_A is greater than V_B.

Hint for Problem 287

The general entry in the bus impedance matrix is given by the expression

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_1 = I_2 = I_3 ... = I_n = 0, I_j = 1\text{p.u.}}$$

We connect a source of unit current between each node and the reference node. To find out the entries in the row of the corresponding node where the source is connected we obtain value of voltage of various nodes. Since the current is 1 amp. The voltages give the impedance entries at various nodes in the row. For example if we connect a current source between node 1 and the reference. The voltage of node is 1 volt and those of nodes 3 and 2 are w.r.t. reference node, zero volts. Hence entries in the first row are 1 0 0.

Now connect the source between node 2 and reference node. The voltage of node 1 w.r.t. reference is zero that of node 2 is 5 volts and 3 node it is 2 volt, therefore the entries in the second row will be 0 5 2

Similarly it can be seen that when we connect a unity current source between node 3 and reference the entires will be 0 2 2

Therefore option (*c*) is correct.

ANSWERS TO OBJECTIVE QUESTIONS

Question	*Answer*	*Question*	*Answer*	*Question*	*Answer*	*Question*	*Answer*
1.	(*b*)	40.	(*c*)	79.	(*d*)	118.	(*a*)
2.	(*b*)	41.	(*d*)	80.	(*c*)	119.	(*b*)
3.	(*c*)	42.	(*b*)	81.	(*d*)	120.	(*a*)
4.	(*a*)	43.	(*c*)	82.	(*b*)	121.	(*b*)
5.	(*a*)	44.	(*a*)	83.	(*c*)	122.	(*a*)
6.	(*a*)	45.	(*b*)	84.	(*b*)	123.	(*a*)
7.	(*d*)	46.	(*a*)	85.	(*c*)	124.	(*d*)
8.	(*a*)	47.	(*c*)	86.	(*b*)	125.	(*b*)
9.	(*a*)	48.	(*a*)	87.	(*b*)	126.	(*a*)
10.	(*a*)	49.	(*c*)	88.	(*c*)	127.	(*d*)
11.	(*b*)	50.	(*a*)	89.	(*c*)	128.	(*b*)
12.	(*b*)	51.	(*b*)	90.	(*a*)	129.	(*c*)
13.	(*d*)	52.	(*c*)	91.	(*a*)	130.	(*c*)
14.	(*b*)	53.	(*d*)	92.	(*c*)	131.	(*a*)
15.	(*a*)	54.	(*d*)	93.	(*c*)	132.	(*d*)
16.	(*a*)	55.	(*a*)	94.	(*a*)	133.	(*d*)
17.	(*d*)	56.	(*d*)	95.	(*c*)	134.	(*a*)
18.	(*b*)	57.	(*a*)	96.	(*d*)	135.	(*d*)
19.	(*d*)	58.	(*a*)	97.	(*b*)	136.	(*c*)
20.	(*a*)	59.	(*c*)	98.	(*d*)	137.	(*b*)
21.	(*b*)	60.	(*c*)	99.	(*a*)	138.	(*c*)
22.	(*c*)	61.	(*a*)	100.	(*b*)	139.	(*c*)
23.	(*a*)	62.	(*a*)	101.	(*c*)	140.	(*b*)
24.	(*a*)	63.	(*a*)	102.	(*d*)	141.	(*a*)
25.	(*d*)	64.	(*a*)	103.	(*c*)	142.	(*a*)
26.	(*a*)	65.	(*d*)	104.	(*d*)	143.	(*a*)
27.	(*c*)	66.	(*c*)	105.	(*c*)	144.	(*b*)
28.	(*c*)	67.	(*a*)	106.	(*a*)	145.	(*d*)
29.	(*c*)	68.	(*a*)	107.	(*b*)	146.	(*b*)
30.	(*c*)	69.	(*c*)	108.	(*c*)	147.	(*a*)
31.	(*d*)	70.	(*b*)	109.	(*b*)	148.	(*b*)
32.	(*a*)	71.	(*a*)	110.	(*a*)	149.	(*b*)
33.	(*c*)	72.	(*d*)	111.	(*b*)	150.	(*d*)
34.	(*a*)	73.	(*a*)	112.	(*a*)	151.	(*c*)
35.	(*d*)	74.	(*b*)	113.	(*c*)	152.	(*c*)
36.	(*d*)	75.	(*c*)	114.	(*d*)	153.	(*d*)
37.	(*b*)	76.	(*a*)	115.	(*c*)	154.	(*c*)
38.	(*b*)	77.	(*c*)	116.	(*a*)	155.	(*c*)
39.	(*b*)	78.	(*c*)	117.	(*a*)	156.	(*c*)

Question	*Answer*	*Question*	*Answer*	*Question*	*Answer*	*Question*	*Answer*
157.	(*c*)	197.	(*a*)	237.	(*a*)	277.	(*d*)
158.	(*a*)	198.	(*b*)	238.	(*d*)	278.	(*a*)
159.	(*c*)	199.	(*a*)	239.	(*c*)	279.	(*a*)
160.	(*a*)	200.	(*d*)	240.	(*c*)	280.	(*b*)
161.	(*a*)	201.	(*d*)	241.	(*c*)	281.	(*a*)
162.	(*a*)	202.	(*b*)	242.	(*a*)	282.	(*c*)
163.	(*c*)	203.	(*d*)	243.	(*b*)	283.	(*d*)
164.	(*d*)	204.	(*d*)	244.	(*c*)	284.	(*a*)
165.	(*a*)	205.	(*b*)	245.	(*a*)	285.	(*b*)
166.	(*c*)	206.	(*c*)	246.	(c)	286.	(*a*)
167.	(*a*)	207.	(*d*)	247.	(*d*)	287.	(*c*)
168.	(*b*)	208.	(*b*)	248.	(*b*)	288.	(*d*)
169.	(*c*)	209.	(*c*)	249.	(*b*)	289.	(*a*)
170.	(*a*)	210.	(*d*)	250.	(*b*)	290.	(*a*)
171.	(*d*)	211.	(*b*)	251.	(*c*)	291.	(*c*)
172.	(*c*)	212.	(*c*)	252.	(*a*)	292.	(*b*)
173.	(*b*)	213.	(*d*)	253.	(*b*)	293.	(*b*)
174.	(*c*)	214.	(*c*)	254.	(*b*)	294.	(*d*)
175.	(*c*)	215.	(*d*)	255.	(*a*)	295.	(*d*)
176.	(*b*)	216.	(*d*)	256.	(*a*)	296.	(*d*)
177.	(*c*)	217.	(*d*)	257.	(*d*)	297.	(*c*)
178.	(*d*)	218.	(*a*)	258.	(*a*)	298.	(*b*)
179.	(*c*)	219.	(*d*)	259.	(*c*)	299.	(*a*)
180.	(*a*)	220.	(*c*)	260.	(*d*)	300.	(*d*)
181.	(*b*)	221.	(*a*)	261.	(*c*)	301.	(*b*)
182.	(*c*)	222.	(*c*)	262.	(*c*)	302.	(*d*)
183.	(*c*)	223.	(*c*)	263.	(*d*)	303.	(*c*)
184.	(*c*)	224.	(*c*)	264.	(*b*)	304.	(*a*)
185.	(*c*)	225.	(*d*)	265.	(*d*)	305.	(*d*)
186.	(*b*)	226.	(*c*)	266.	(*c*)	306.	(*a*)
187.	(*d*)	227.	(*b*)	267.	(*b*)	307.	(*c*)
188.	(*b*)	228.	(*a*)	268.	(*d*)	308.	(*a*)
189.	(*b*)	229.	(*b*)	269.	(*a*)	309.	(*b*)
190.	(*b*)	230.	(*c*)	270.	(*a*)	310.	(*c*)
191.	(*c*)	231.	(*d*)	271.	(*c*)	311.	(*d*)
192.	(*b*)	232.	(*d*)	272.	(*c*)	312.	(*b*)
193.	(*c*)	233.	(*b*)	273.	(*d*)	313.	(*a*)
194.	(*a*)	234.	(*b*)	274.	(*b*)	314.	(*b*)
195.	(*b*)	235.	(*c*)	275.	(*a*)	315.	(*a*)
196.	(*d*)	236.	(*b*)	276.	(*b*)	316.	(*c*)

Question	*Answer*	*Question*	*Answer*	*Question*	*Answer*	*Question*	*Answer*
317.	(*d*)	357.	(*a*)	397.	(*a*)	437.	(*b*)
318.	(*d*)	358.	(*c*)	398.	(*d*)	438.	(*b*)
319.	(*c*)	359.	(*b*)	399.	(*c*)	439.	(*a*)
320.	(*a*)	360.	(*d*)	400.	(*a*)	440.	(*c*)
321.	(*c*)	361.	(*b*)	401.	(*c*)	441.	(*a*)
322.	(*a*)	362.	(*c*)	402.	(*d*)	442.	(*c*)
323.	(*d*)	363.	(*b*)	403.	(*c*)	443.	(*c*)
324.	(*d*)	364.	(*b*)	404.	(*b*)	444.	(*d*)
325.	(*d*)	365.	(*b*)	405.	(*a*)	445.	(*c*)
326.	(*a*)	366.	(*a*)	406.	(*b*)	446.	(*b*)
327.	(*b*)	367.	(*c*)	407.	(*a*)	447.	(*b*)
328.	(*b*)	368.	(*b*)	408.	(*b*)	448.	(*d*)
329.	(*c*)	369.	(*b*)	409.	(*a*)	449.	(*c*)
330.	(*d*)	370.	(*b*)	410.	(*c*)	450.	(*d*)
331.	(*c*)	371.	(*b*)	411.	(*c*)	451.	(*b*)
332.	(*c*)	372.	(*b*)	412.	(*a*)	452.	(*c*)
333.	(*c*)	373.	(*c*)	413.	(*b*)	453.	(*c*)
334.	(*c*)	374.	(*c*)	414.	(*c*)	454.	(*a*)
335.	(*a*)	375.	(*b*)	415.	(*a*)	455.	(*d*)
336.	(*b*)	376.	(*d*)	416.	(*d*)	456.	(*a*)
337.	(*b*)	377.	(*c*)	417.	(*b*)	457.	(*d*)
338.	(*a*)	378.	(*c*)	418.	(*d*)	458.	(*c*)
339.	(*a*)	379.	(*a*)	419.	(*b*)	459.	(*a*)
340.	(*d*)	380.	(*a*)	420.	(*a*)	460.	(*b*)
341.	(*c*)	381.	(*c*)	421.	(*a*)	461.	(*d*)
342.	(*c*)	382.	(*b*)	422.	(*c*)	462.	(*a*)
343.	(*c*)	383.	(*c*)	423.	(*c*)	463.	(*c*)
344.	(*b*)	384.	(*a*)	424.	(*b*)	464.	(*b*)
345.	(*b*)	385.	(*d*)	425.	(*d*)	465.	(*c*)
346.	(*b*)	386.	(*d*)	426.	(*b*)	466.	(*a*)
347.	(*c*)	387.	(*d*)	427.	(*a*)	467.	(*a*)
348.	(*b*)	388.	(*b*)	428.	(*c*)	468.	(*d*)
349.	(*b*)	389.	(*a*)	429.	(*a*)	469.	(*c*)
350.	(*d*)	390.	(*c*)	430.	(*a*)	470.	(*c*)
351.	(*d*)	391.	(*a*)	431.	(*b*)	471.	(*b*)
352.	(*a*)	392.	(*b*)	432.	(*a*)	472.	(*c*)
353.	(*c*)	393.	(*d*)	433.	(*c*)	473.	(*a*)
354.	(*b*)	394.	(*a*)	434.	(*b*)	474.	(*c*)
355.	(*b*)	395.	(*b*)	435.	(*a*)	475.	(*b*)
356.	(*c*)	396.	(*a*)	436.	(*a*)	476.	(*c*)

Question	*Answer*	*Question*	*Answer*	*Question*	*Answer*	*Question*	*Answer*
477.	(*b*)	489.	(*c*)	501.	(*c*)	513.	(*d*)
478.	(*a*)	490.	(*a*)	502.	(*c*)	514.	(*c*)
479.	(*a*)	491.	(*c*)	503.	(*d*)	515.	(*d*)
480.	(*b*)	492.	(*a*)	504.	(*a*)	516.	(*c*)
481.	(*a*)	493.	(*a*)	505.	(*d*)	517.	(*b*)
482.	(*a*)	494.	(*b*)	506.	(*c*)	518.	(*c*)
483.	(*d*)	495.	(*c*)	507.	(*a*)	519.	(*b*)
484.	(*d*)	496.	(*b*)	508.	(*c*)	520.	(*b*)
485.	(*d*)	497.	(*c*)	509.	(*c*)	521.	(*b*)
486.	(*d*)	498.	(*c*)	510.	(*b*)	522.	(*a*)
487.	(*b*)	499.	(*a*)	511.	(*d*)	523.	(*c*)
488.	(*a*)	500.	(*d*)	512.	(*c*)	524.	(*b*)

ANSWERS TO PROBLEMS

Chapter 1

1.1. $X_{g_1} = 0.68$, $X_{g_2} = 0.4574$, $X_{m_1} = 1.0255$, $X_{m_2} = 0.6837$

1.2. $X_{g_1} = 0.1777$, $X_{g_2} = 0.08916$, $X_{g_3} = 0.1603$

1.3. $X_{g_1} = 0.15$, $X_{g_2} = 0.1936$, $X_{g_3} = 0.224$, $X_{t_1} = 0.0706$, $X_{t_2} = 0.208$, $X_{t_3} = 0.0868$, $X_l = 0.1312$.

Chapter 2

2.2. (*a*) 1.7228*r*; (*b*) 1.703*r*

2.3. 1.339 mH/km, 0.42044 Ω/km

2.6. $L_A = 0.6915$ mH/km, $L_B = 0.9147$ mH/km, Total 1.6061 mH/km

2.7. 1.098 mH/km/phase

2.8. 0.631 mH/km/phase

2.9. 0.51027 mH/km

Chapter 3

3.4. 9.3609×10^{-9} F/km, 0.224 amp/km

3.5. 0.0117079 μF/km, 0.4669 amp/km

3.6. 0.020168 μF/km, 0.8043 amp/km

3.7. 0.0192447 μF/km, 0.7675 amp/km

Chapter 4

4.1. 86.01 kV, 163.82 A, 0.909 p.f., 15.72%, 93.65%

4.2. 58.04 A, 69.4 A, 61.08 A, 35.8 A, 300 μF, 28.46 μF, 505 μF

4.3. 226.18 volts, 210.24 V, 242.32 V

4.4. 24.198 A, 221.86 V, 221.79 V, 234.86 V

4.5. 11 kV, 57.16 kV, 57.16 kV, 16.5 kV, 1815 A, 349.28 A, 1210 A

4.6. (*i*) 93.76%, 11%. (*ii*) 93.79%, 11.14%. (*iii*) 93.96%, 10.98%

4.7. (*i*) 97.15%, 6.83%. (*ii*) 97.148%, 6.874%. (*iii*) 97.19%, 6.85%

4.8. (*i*) 7.35%. 1.0,5385 ∠68.19, 00, 1.0, – 2.93%

4.9. 0.98745 ∠0.138°, 63.138 ∠79.15°, 3.98×10^{-4} ∠90, 0.98745 ∠0.138

4.10. (*i*) 32.09 ∠79.2°, $j3.94 \times 10^{-4}$ ℧; (*ii*) 63.23 ∠79.2°, $j2.02 \times 10^{-4}$ ℧, $(1.8925 + j19.825) \times 10^{-4}$, 394.6 Ω

4.11. (*i*) 0.6906 ∠4.98°, 268.29 ∠81.15, $j\,20.00 \times 10^{-4}$ ℧, 0.6906 ∠4.98°;
(*ii*) 0.6906 ∠ 4.98°, 317.7 ∠ 79.11°, $j16.88 \times 10^{-4}$ ℧, 0.6906 ∠4.98°
(*iii*) 0.7057 ∠ 4.38°, 285.14 ∠80.3°, $j1.8 \times 10^{-3}$ ℧, 0.7057 ∠4.38°;

4.12. 285.58 ∠80.3°, $j10.55 \times 10^{-4}$, 167.59 ∠78.6°, $j1.7978 \times 10^{-3}$ ℧

4.13. 88.52%, 32.96%

Chapter 5

5.1. 144.12 kV, 112.1 kV, 50.05 kV

5.2. 94.11 kV, 0.627

5.4. 21.156 Ω

5.5. 367.42 A

Chapter 6

6.1. 139.439 kV, 157.34 kV
6.2. 0.2055 kW/km/phase, 3.064 kW/km/phase
6.3. 4.362 kW/km/phase, 118.28 kV
6.4. 158.02 kV

Chapter 7

7.8. (*i*) 2.02 m, (*ii*) 3.897 m, 3.48 m
7.9. 23.63 m
7.10. 2.4 m
7.11. 55.33 m
7.12. 46°C

Chapter 8

8.1. 5.51 kV, 6.13 kV, 7.41 kV, 85.70%
8.2. 11C, 13C, 16C, 20C, 25C, 31C, 38C
8.3. 0.099, 0.109, 0.13, 0.164, 0.213, 0.286, 58.4%

Chapter 9

9.1. 50.36, 36.6 kV, 1.374
9.2. 93.84 kV
9.5. 3.90 cm, 92 kV/cm, 20.72 A
9.7. 6.17 μF, 234.42 kVA
9.8. 71.78 A
9.9. 0.2915 μF/km, 0.388 μF/km, 0.773 A/km
9.10. 0.0155, 30.88 V
9.11. (*a*) 2.64 cm, 41.71 kV, (*b*) 0.9715 cm, 2.64 cm, 4.96 cm, 7.17 cm
9.12. 65 kV
9.13. 1.89 cm, 0.86 cm, 67.93 kV

Chapter 10

10.1. 328.92 kVA, 0.3648 lead
10.2. 42.78 MVA
10.3. 0.965
10.4. 1.14
10.5. 1.58
10.6. 23.94 MVA, 0.912
10.7. 130.909 kV, 23.96%, 2.25 MVA, 39.75 MVAr, 0.99 leading P_{max} = 111.75 MW, 37.5 MW, 0.968 lead

Chapter 11

11.4. 33.80 H, 34.84 H, 37.55 H
11.5. 884.64 Ω, 6.565 MVA
11.6. 3.38 H, 5.472 MVA

Chapter 12

12.1. 54.23 Volts negative rising, 5.42 A, 306.3 volt, 79.96°

12.2. 38.79 kV

12.9. 79.38 A

12.10. 41.3 watt sec

12.11. 38.684 kV, – 71.316 kV, 181.31 kV, 71.316 kV

12.12. 3.33 kV, 8.33 A, 83.33 A

12.13. 219.52 kV, 223.62 MW, 62.116 kJ or 14.8 kcal

12.14. 98.16 kV

12.15. 306 kV

Chapter 13

13.1. 3.2766 + j13.4967, 0.39 + j4.8367, 6.333 + j1.667

13.2. 431.8856 + j154.623, – 18.638 – j214.735, – 113.249 + j60.112

13.3. 29.99 kA, 23.326 kA, 5.744 kA

13.4. (a) $I_{fl} = -j$ 3.03 p.u., I_{fr} = 0.606 p.u.,0.0 volt, – 0.4091 – j0.8659, – 0.4091 + j0.8659; (b) – j2.856, 0.0 volt, – 0.6428 – j0.8659, – 0.6428 + j0.8659

13.5. 21.87 kA, 26.244 kA, 18.94 kA, 32.805 kA

13.6. 1202.75 A, 1443.3 A, 1041.58 A, 1804 A

13.7. 1045.67 A, 1110 A, 905 A, 1182.3 A

13.8. 21.87, 21.34 kA, 18.94 kA, 20.84 kA

13.9. 2909 A, 2959 A

13.10. 37.328 kA, 33.477 kA, 3.951 kA

13.11. 9183.32 A, 1530.55 A, 12985 A in the generator circuit

13.12. 256.97 A, 1541.85 A

13.13. 541.3 MVA

13.14. 4860 A

13.15. 1667.9 MVA, 1177.8 MVA

13.16. 446.47 A, 2232.35 A, 2232.35 A

13.17. 685.7 MVA, 900 MVA

13.18. 28.578 kA, 23.911 kA, 38.25 kA, 20.175 kA

13.19. 233.4 MVA, 59.57 MVA

13.20. 1686.64 A

13.21. I_a = 0.0, $I_b = -j$6858 A, $I_c = j$6858 A, $I_{fp} = -j$2765.6 A

13.22. 29.25 kA

13.23. 1.148 Ω

Chapter 14

14.32. 52.48 : 5, 1443 : $\frac{5}{\sqrt{3}}$

14.34. 2.56 sec

14.35. 86.9%

14.37. 95.26 Ω, 98.5%

14.41. T_{r_1} = 2 sec, T_{r_2} = 2.5 sec, TMS_2 = 0.167, 5A setting

Chapter 15

15.6. 2.5 kV/μ sec, 12.5 kHz

15.7. 17.959 kV, 15.92 kHz, 571.8 V/μ sec., 897.7 V/μ sec

15.8. 5.34 kV/μ sec

15.9. 26.24 kA, 66.92 kA, 26.24 kA, 33 kV

Chapter 17

17.1. 170 MW

17.2. 40 MJ/MVA

17.3. 0.01095 MJ sec/elect degree, 0.9859 MJ/MVA

17.4. 9.853 MJ/MVA

17.5. 36.6 MW

17.6. 250 MJ, 280.318 elect degree/sec^2, 5.606 deg., 3009.34 rpm

17.7. 51.6°, 0.111 sec, sustained fault

$\delta_1 = 37.02$, $\delta_2 = 43.46$, 53.98, 68.31, 86.18, 107.43

Fault cleared $\delta_3 = 52.34$, 61.279, 69.676, 77.104, 83.30, 88.137, 91.57, 93.6, 94.23, 93.47

17.8. $\delta_c = 48.9$, 0.111 sec, sustained fault.

$\delta_1 = 38.56$, $\delta_2 = 46.15$, $\delta_3 = 58.53$, 75.37, 96.37, 121.48.

Fault cleared $\delta_2 = 46.15$, $\delta_3 = 56.58$, 67, 76.73, 85.35, 92.67, 98.67, 103.44, 107.11, 109.81, 111.66, 112.70, 113.04, 112.68

17.17. (0, 1.996), 2.6 p.u., – 0.6143 p.u.

Chapter 18

18.8. $Y_{11} - 6.6667 - j19.95$, $Y_{12} = -1.6667 + j5.0$, $Y_{13} = -5 + j15$, $Y_{22} = 4.1667 - j12.44$, $Y_{23} = -2.5 + 7.5$, $Y_{33} = 7.5 - j22.45$

$Q_2 = 0.143104$, $1.0399606 + j0.0090506$, $1.0627124 - j0.0303737$

$$\begin{bmatrix} 0.3 \\ -0.3 \\ 0.36 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 4.0667 & -2.5 & 12.8 & -7.5 \\ -2.5 & 7.2 & -7.5 & 23.4 \\ 12.08 & -7.5 & -4.2667 & 2.5 \\ -7.5 & 21.5 & 2.5 & -7.8 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

$Q_2 = -0.36$

18.9. $1.0763625 + j0.0129963$, $1.0792612 - j0.0273225$

18.10. $Y_{11} = 3.92 - j15.61$, $Y_{12} = -2.94 + j11.76$, $Y_{13} = -0.980 + j3.92$, $Y_{14} = 0.0$,

$Y_{22} = 5.88 - j23.43$, $Y_{23} = -1.47 + j5.88$, $Y_{24} = -1.47 + j5.88$, $Y_{33} = 8.33 - j33.26$, $Y_{34} = -5.88 + j23.53$, $Y_{44} = 7.35 - j29.37$.

$1.0276904 - j0.0079658$, $1.0075717 - j0.0153434$, $1.0080743 - j0.0266215$

$$\begin{bmatrix} -0.0236 \\ -0.4412 \\ -0.4 \\ 0.6956 \\ 0.1052 \\ -0.01 \end{bmatrix} = \begin{bmatrix} 5.7036 & -1.47 & -1.47 & 24.2256 & -5.88 & -5.88 \\ -1.47 & 8.2712 & -5.88 & -5.88 & 33.56 & -23.53 \\ -1.47 & -5.88 & 7.35 & -5.88 & -23.53 & 29.41 \\ 22.63 & -5.88 & -5.88 & -6.056 & 1.47 & 1.47 \\ -5.88 & 32.95 & -23.53 & 1.47 & -8.4 & 5.88 \\ -5.88 & -23.53 & 29.33 & 1.47 & 5.88 & -7.35 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta e_4 \\ \Delta f_2 \\ \Delta f_3 \\ \Delta f_4 \end{bmatrix}$$

18.11. (*i*) $V_2^1 = 1.0276904 - j0.0079558$, $Q_3 = 0.8847129$,

$V_3^1 = 1.0231391 - j\ 0.0012026$

$V_4^1 = 1.020549 - j0.0152977$

(*ii*) Q_3 = – 0.3052, Generator bus as load bus, Q_3 = 0.0, ΔP_3 = 0.1588, ΔQ_3 = 0.3052

All other elements of residual column matrix and Jacobian matrix remain same as in question 18.10.

18.12. (*i*) Q_3 = 0.8847129, V_2^1 = 1.0276904 – j0.0079558, V_3^1 = 1.0399868 – j0.0052197, V_4^1 = 1.0344194 – j0.0170214

Chapter 19

19.1. 36.37 MW, 63.63 MW, Rs. 23.637/MWhr, 63.64 MW, 86.36 MW, Rs.26.36/MWhr

19.2. Rs. 9.1/hr

19.3. $P_1 = P_2$ = 100 MW, P_D = 190 MW, P_L = 10 MW

19.4. Rs. 289.26/hr

19.5. (*i*) 20, 20; (*ii*) 20, 40; (*iii*) 20, 60; (*iv*) 38.18, 81.82; (*v*) 60, 100; (*vi*) 80, 100 MW

19.6. (*i*) 91.97 MW, 170.9 MW, Rs. 4459.36/hr;

(*ii*) 124.39 MW, 154.63 MW Rs. 4755.6/hr

Chapter 20

20.1. 52.94 MW, 147.06 MW, 2.133% increase in no load speed of 120 MW Unit

20.2. δ = 8.7147°, 3.6 MVAr, 0.857

20.3. 7.53 MW from *B* to *A*

20.4. 48.18 Hz

20.5. 1.38 MW from *A* to *B*.

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